Computer Algebra Independent Integration Tests

Summer 2024

1-Algebraic-functions/1.2-Trinomial/1.2.5-Improper-quadratic-trinomial/138-1.2.5.2

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Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**90**]. This is test number [138].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (90)	0.00 (0)
Mathematica	100.00 (90)	0.00 (0)
Maple	97.78 (88)	2.22 (2)
Fricas	97.78 (88)	2.22 (2)
Reduce	92.22 (83)	7.78 (7)
Giac	78.89 (71)	21.11 (19)
Mupad	53.33 (48)	46.67 (42)
Sympy	36.67 (33)	63.33 (57)
Maxima	26.67 (24)	73.33 (66)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

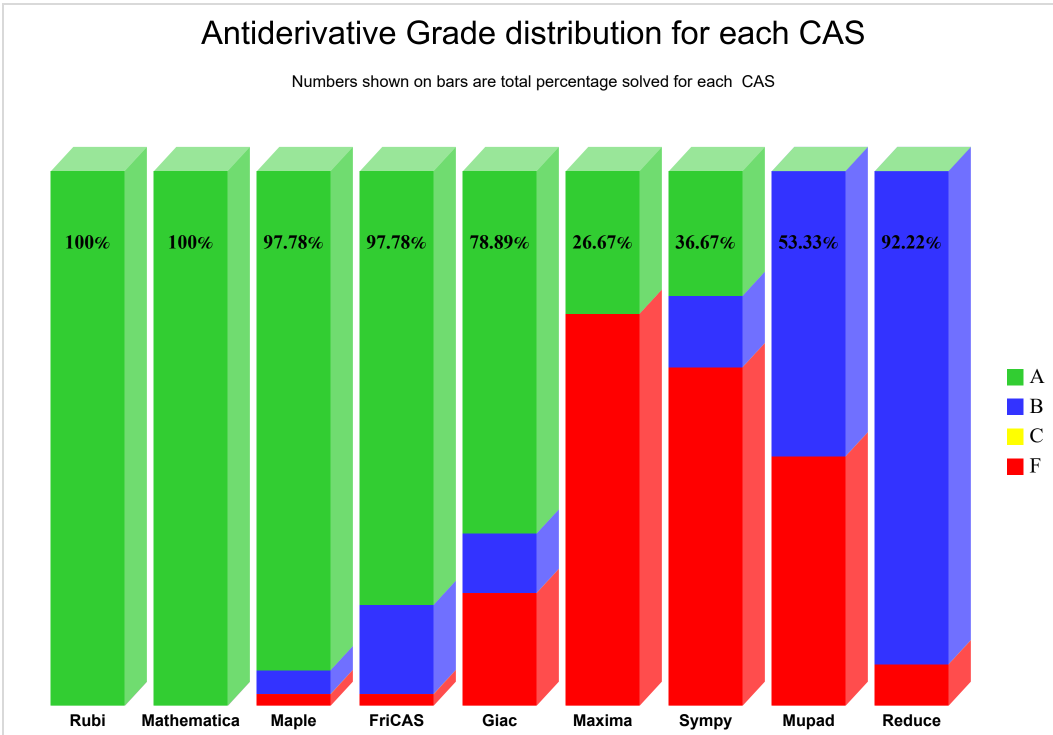
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

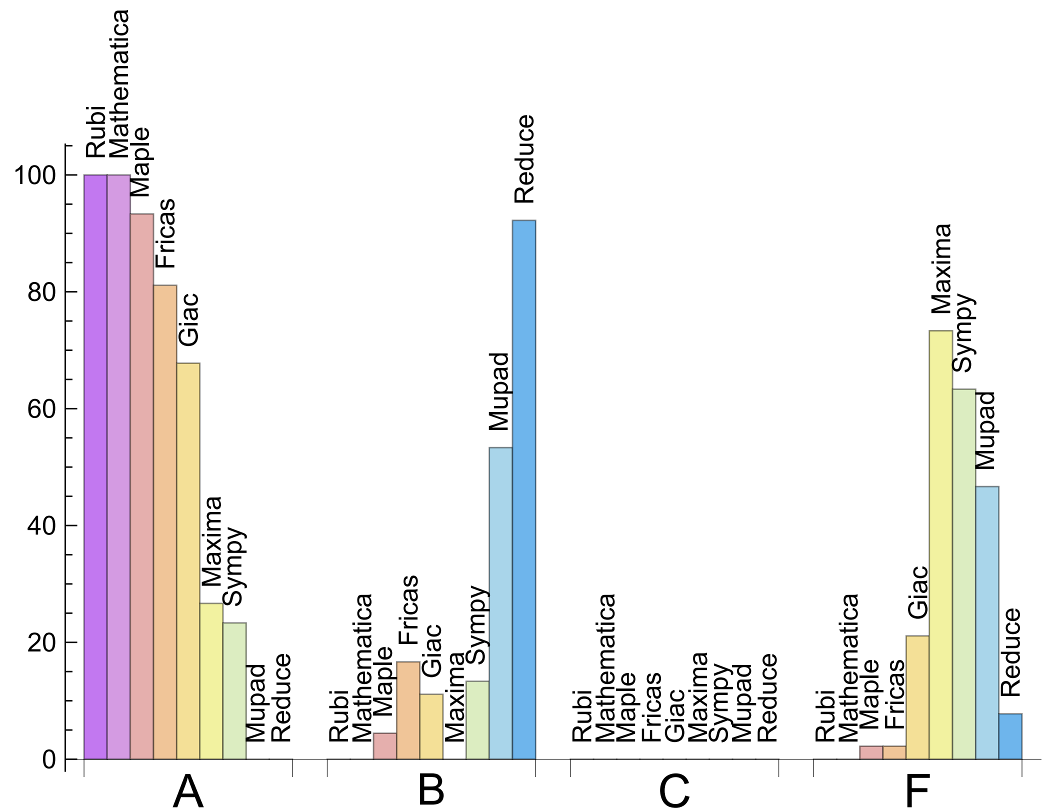
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.000	0.000	0.000	0.000
Mathematica	100.000	0.000	0.000	0.000
Maple	93.333	4.444	0.000	2.222
Fricas	81.111	16.667	0.000	2.222
Giac	67.778	11.111	0.000	21.111
Maxima	26.667	0.000	0.000	73.333
Sympy	23.333	13.333	0.000	63.333
Mupad	0.000	53.333	0.000	46.667
Reduce	0.000	92.222	0.000	7.778

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	0	0.00	0.00	0.00
Fricas	2	100.00	0.00	0.00
Maple	2	100.00	0.00	0.00
Reduce	7	100.00	0.00	0.00
Giac	19	10.53	68.42	21.05
Mupad	42	0.00	100.00	0.00
Sympy	57	78.95	21.05	0.00
Maxima	66	71.21	0.00	28.79

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.03
Fricas	0.12
Giac	0.15
Reduce	0.21
Rubi	0.33
Maple	0.41
Mathematica	0.41
Sympy	5.58
Mupad	9.98

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Maxima	47.62	0.90	25.00	0.81
Mathematica	106.08	0.96	96.00	0.95
Rubi	126.82	1.05	113.00	1.00
Maple	151.85	1.22	97.50	0.85
Sympy	203.00	3.05	48.00	0.91
Giac	276.86	2.14	79.00	1.13
Mupad	297.38	2.47	75.00	1.41
Fricas	364.82	2.74	254.50	2.49
Reduce	372.77	3.40	163.00	1.55

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

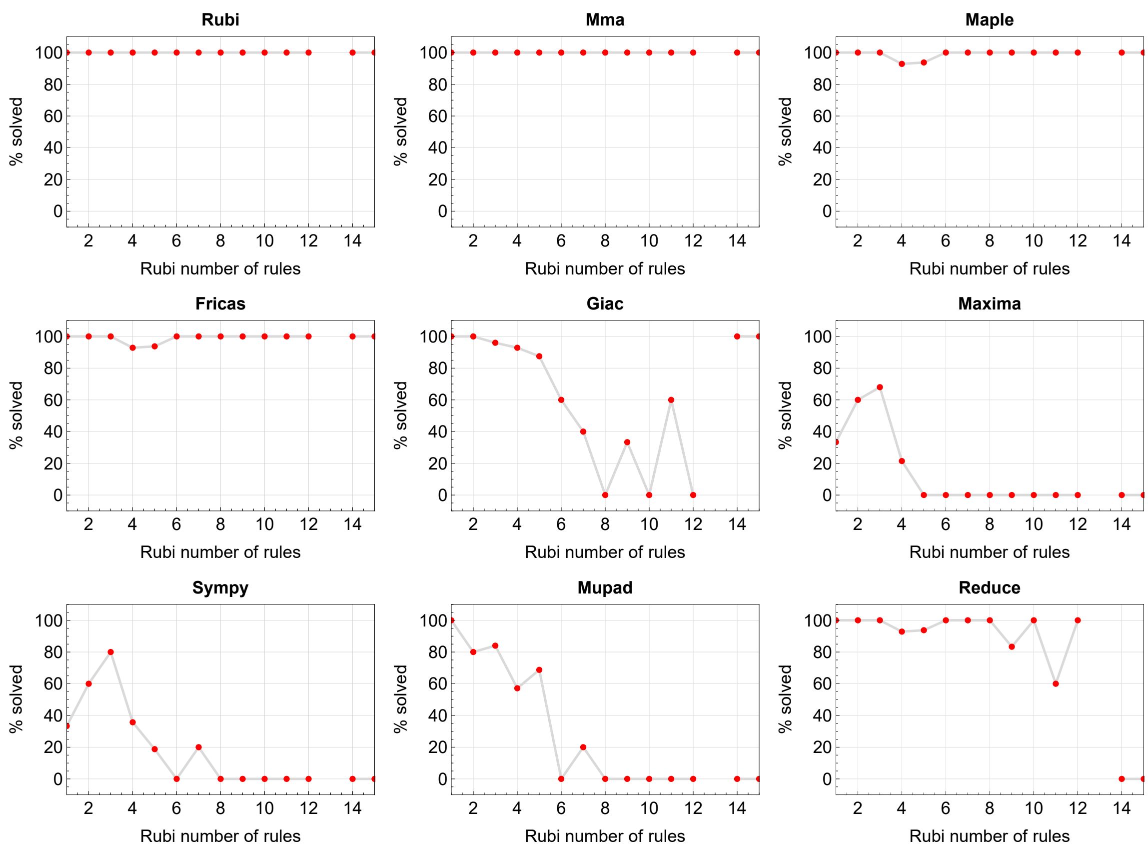


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

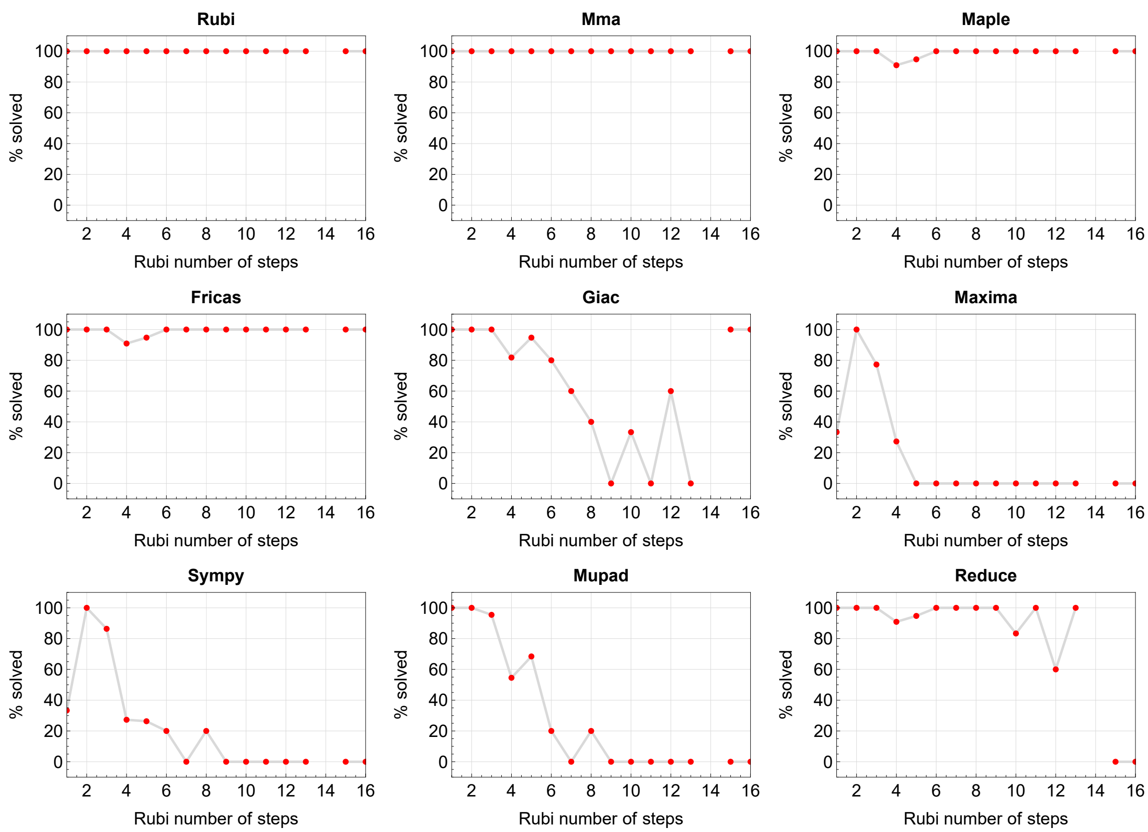


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

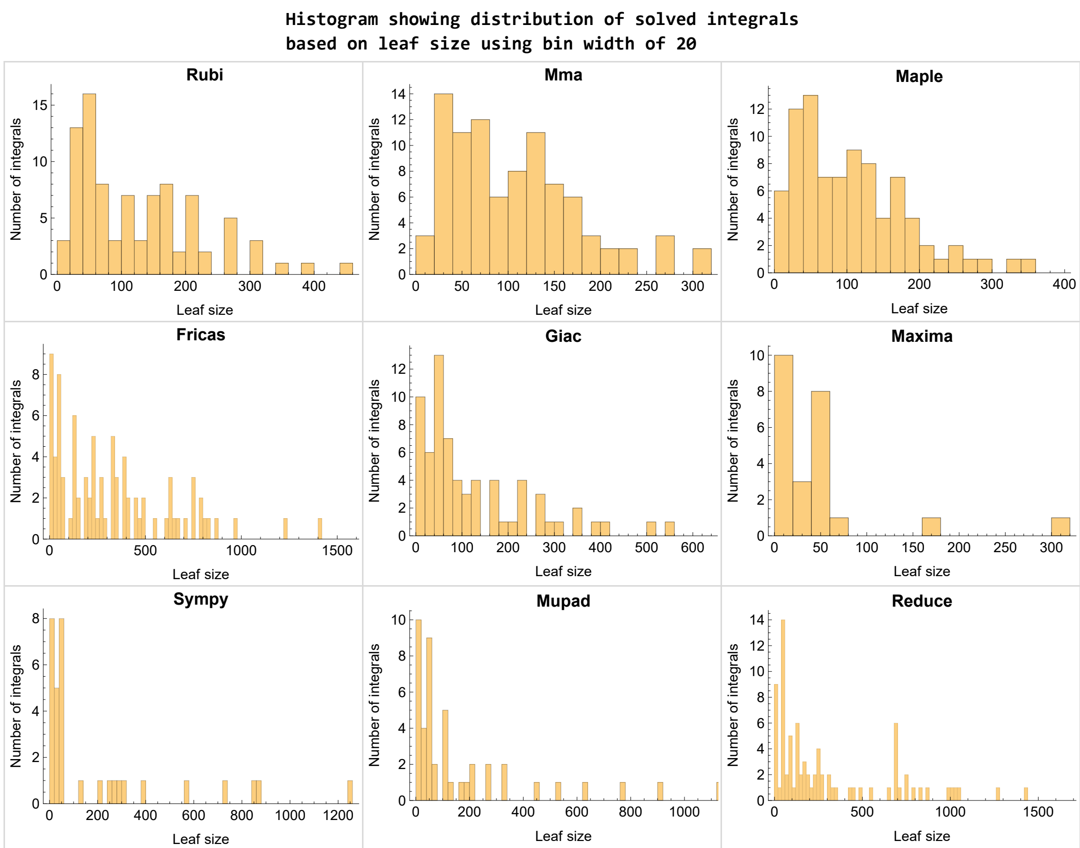


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

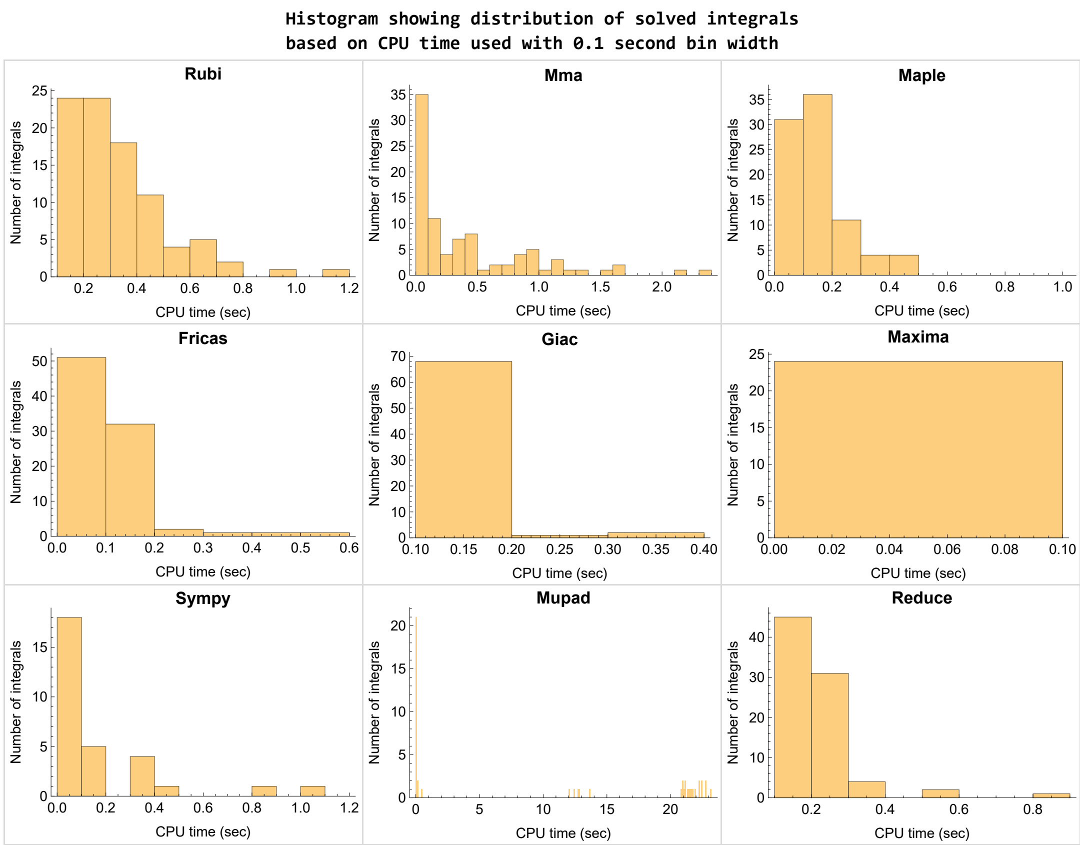


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

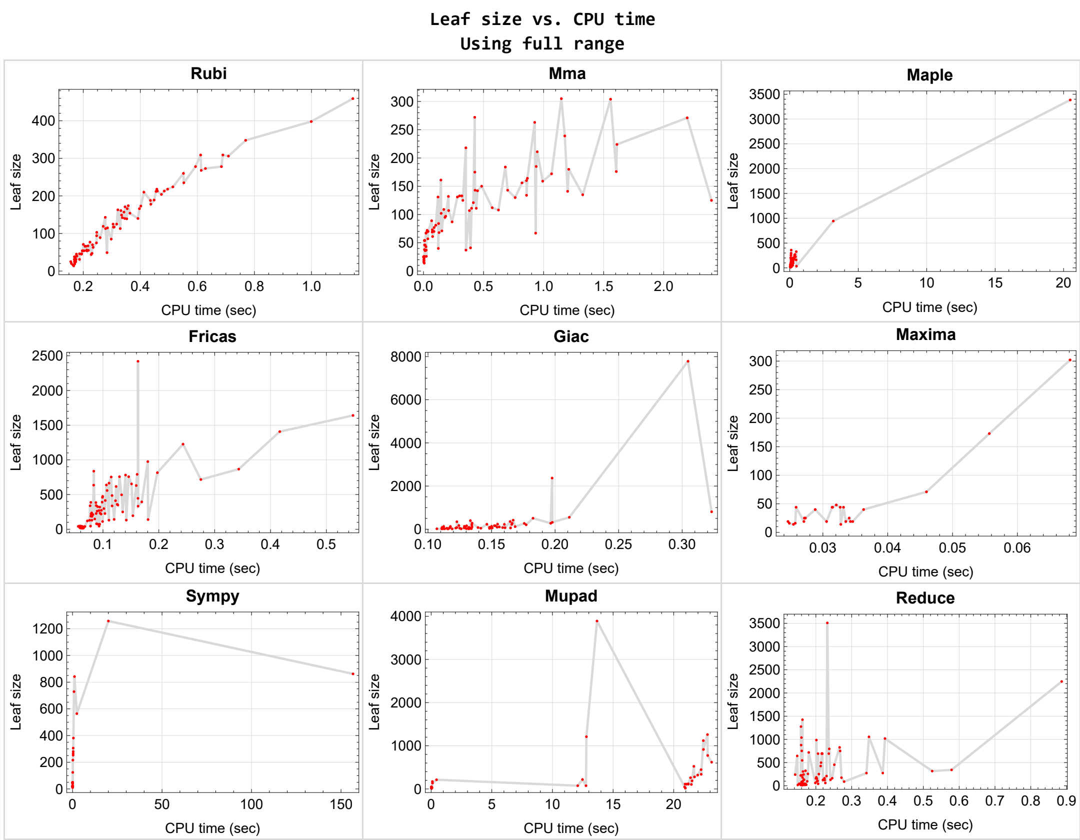


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Reduce {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {}

Mathematica {}

Maple $\{10, 11, 12, 13, 14, 15, 16, 20, 21, 22, 23, 24\}$

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
       leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

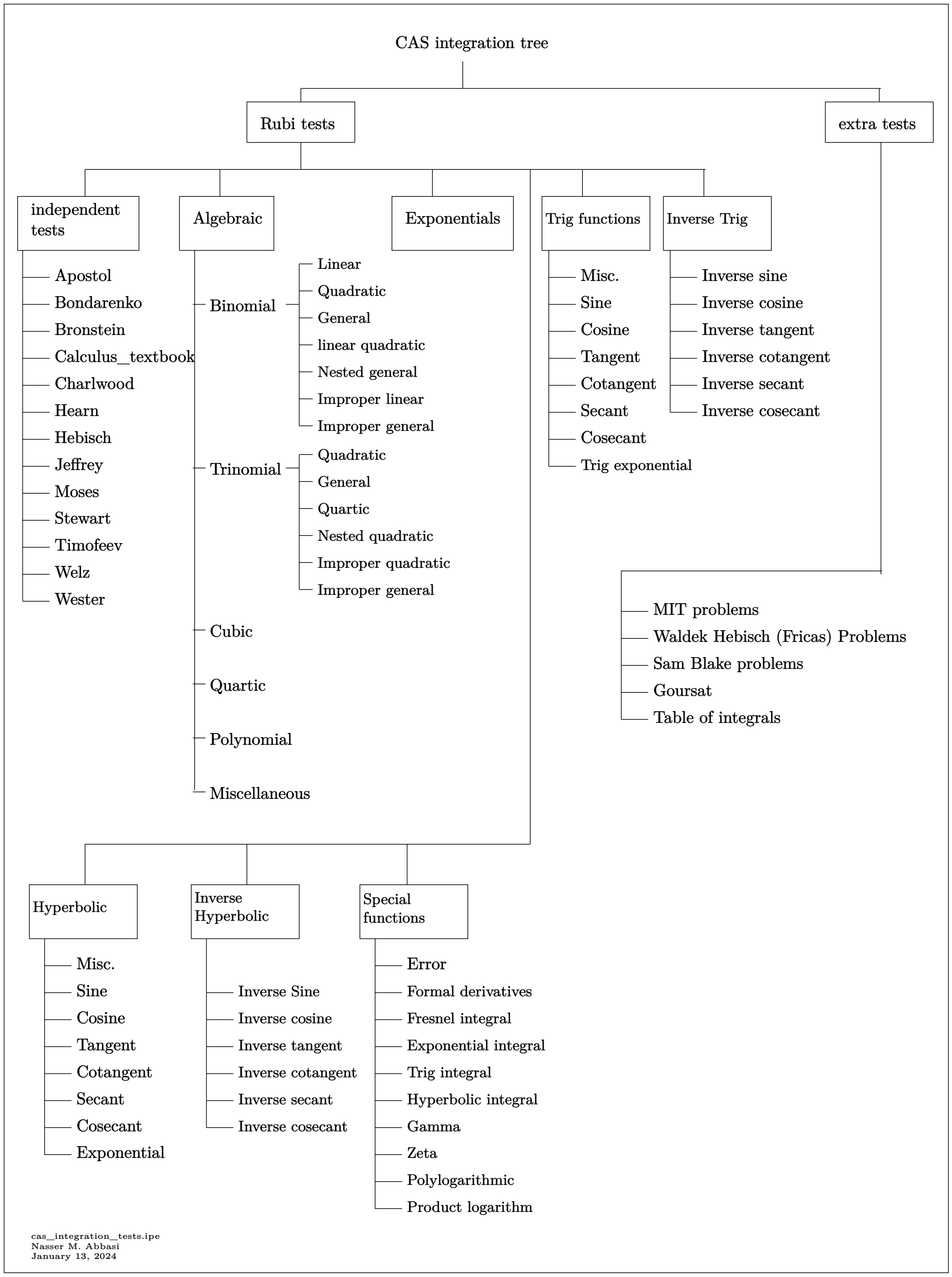
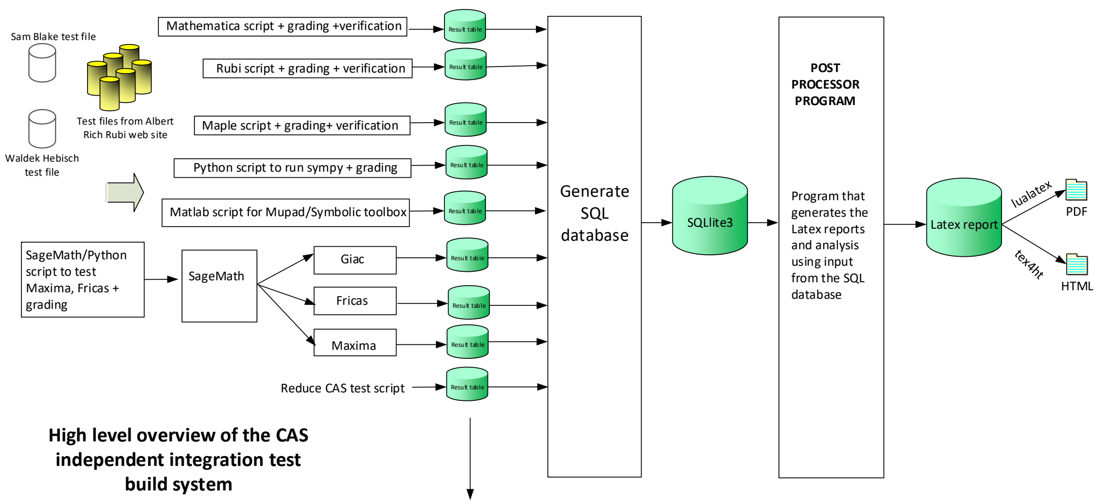


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer, the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"
13. String. Small string description of why the grade was given.
14. integer. 1 if result was verified or 0 if not verified. (For mma, rubi and maple only)

The following fields are present only in ***Rubi*** *Table file*

15. integer. Number of steps used.
16. integer. Number of rules used.
17. integer. Integrand leaf size.
18. real number. Ratio. Field 16 over field 17
19. String of form "{n,n,..}" which is list of the rules used by Rubi
20. String. The optimal antiderivative in Mathematica syntx

Nasser M. Abbasi
January 13, 2024
Design.vsdx

CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 87, 88, 89, 90 }

B grade { 4, 82, 83, 84 }

C grade { }

F normal fail { 85, 86 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 78, 79, 80, 81, 87, 88, 89, 90 }

B grade { 1, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 77, 82, 83, 84 }

C grade { }

F normal fail { 85, 86 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maxima

A grade { 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 43, 44, 45, 82, 83, 84 }

B grade { }

C grade { }

F normal fail { 1, 2, 3, 4, 5, 6, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 85, 86, 88, 89, 90 }

F(-1) timedout fail { }

F(-2) exception fail { 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 87 }

Giac

A grade { 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 55, 56, 57, 58, 59, 66, 67, 68, 69, 72, 73, 74, 75, 76, 87, 88, 89, 90 }

B grade { 1, 2, 3, 4, 5, 6, 77, 82, 83, 84 }

C grade { }

F normal fail { 85, 86 }

F(-1) timedout fail { 52, 53, 54, 61, 62, 64, 65, 70, 71, 78, 79, 80, 81 }

F(-2) exception fail { 50, 51, 60, 63 }

Mupad

A grade { }

B grade { 1, 2, 3, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 75, 76, 82, 83, 84, 87 }

C grade { }

F normal fail { }

F(-1) timedout fail { 4, 5, 6, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 77, 78, 79, 80, 81, 85, 86, 88, 89, 90 }

F(-2) exception fail { }

Sympy

A grade { 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 43, 44, 45 }

B grade { 25, 26, 27, 28, 29, 30, 33, 34, 35, 36, 37, 84 }

C grade { }

F normal fail { 1, 2, 3, 4, 5, 6, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 85, 87, 88 }

F(-1) timedout fail { 31, 32, 38, 39, 40, 41, 42, 82, 83, 86, 89, 90 }

F(-2) exception fail { }

Reduce

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 48, 49, 50, 51, 52, 53, 54, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 87, 88, 89, 90 }

C grade { }

F normal fail { 46, 47, 55, 56, 57, 85, 86 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	163	125	174	0	459	0	552	719	321
N.S.	1	1.11	0.85	1.18	0.00	3.12	0.00	3.76	4.89	2.18
time (sec)	N/A	0.321	2.397	0.346	0.000	0.099	0.000	0.211	0.180	21.976

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	113	67	97	0	219	0	269	325	188
N.S.	1	0.88	0.52	0.75	0.00	1.70	0.00	2.09	2.52	1.46
time (sec)	N/A	0.278	0.934	0.250	0.000	0.097	0.000	0.176	0.171	21.511

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	41	52	0	78	0	95	98	104
N.S.	1	1.00	0.93	1.18	0.00	1.77	0.00	2.16	2.23	2.36
time (sec)	N/A	0.228	0.392	0.214	0.000	0.088	0.000	0.154	0.163	21.456

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	75	71	109	0	141	0	105	44	0
N.S.	1	1.32	1.25	1.91	0.00	2.47	0.00	1.84	0.77	0.00
time (sec)	N/A	0.246	0.153	0.184	0.000	0.120	0.000	0.156	0.160	0.000

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	115	97	130	0	249	0	205	128	0
N.S.	1	1.07	0.91	1.21	0.00	2.33	0.00	1.92	1.20	0.00
time (sec)	N/A	0.285	0.185	0.112	0.000	0.135	0.000	0.177	0.167	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	161	135	181	0	349	0	808	257	0
N.S.	1	1.02	0.85	1.15	0.00	2.21	0.00	5.11	1.63	0.00
time (sec)	N/A	0.332	1.326	0.100	0.000	0.127	0.000	0.323	0.178	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	19	19	19	19	19	19
N.S.	1	1.00	1.00	0.80	0.76	0.76	0.76	0.76	0.76	0.76
time (sec)	N/A	0.172	0.005	0.071	0.034	0.063	0.021	0.107	0.170	0.036

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	19	19	19	19	19	19
N.S.	1	1.00	1.00	0.80	0.76	0.76	0.76	0.76	0.76	0.76
time (sec)	N/A	0.173	0.003	0.072	0.031	0.064	0.021	0.116	0.162	0.034

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	19	19	19	19	19	19
N.S.	1	1.00	1.00	0.80	0.76	0.76	0.76	0.76	0.76	0.76
time (sec)	N/A	0.156	0.000	0.069	0.027	0.060	0.022	0.116	0.150	0.032

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	19	19	19	19	19	19
N.S.	1	1.00	1.00	0.80	0.76	0.76	0.76	0.76	0.76	0.76
time (sec)	N/A	0.169	0.002	0.085	0.025	0.059	0.028	0.112	0.164	0.033

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	16	16	15	16	17	16
N.S.	1	1.00	1.00	0.85	0.80	0.80	0.75	0.80	0.85	0.80
time (sec)	N/A	0.159	0.003	0.036	0.025	0.061	0.023	0.111	0.163	0.028

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	14	14	14	15	14	14
N.S.	1	1.00	1.00	0.94	0.88	0.88	0.88	0.94	0.88	0.88
time (sec)	N/A	0.164	0.003	0.040	0.033	0.064	0.042	0.119	0.161	0.028

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	15	14	18	10	15	18	14
N.S.	1	1.00	1.00	1.07	1.00	1.29	0.71	1.07	1.29	1.00
time (sec)	N/A	0.167	0.007	0.046	0.025	0.062	0.048	0.132	0.158	0.030

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	18	16	21	17	17	21	17
N.S.	1	1.00	1.00	0.95	0.84	1.11	0.89	0.89	1.11	0.89
time (sec)	N/A	0.170	0.005	0.045	0.026	0.062	0.108	0.116	0.173	20.961

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	19	19	19	20	19	19	18
N.S.	1	1.00	1.00	0.83	0.83	0.83	0.87	0.83	0.83	0.78
time (sec)	N/A	0.171	0.005	0.043	0.034	0.065	0.103	0.113	0.158	0.035

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	19	19	19	20	19	19	19
N.S.	1	1.00	1.00	0.76	0.76	0.76	0.80	0.76	0.76	0.76
time (sec)	N/A	0.174	0.004	0.045	0.035	0.062	0.162	0.126	0.165	0.035

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	45	44	44	48	46	46	45
N.S.	1	1.00	1.00	0.83	0.81	0.81	0.89	0.85	0.85	0.83
time (sec)	N/A	0.214	0.010	0.108	0.031	0.093	0.027	0.142	0.160	0.040

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	45	44	44	49	46	46	45
N.S.	1	1.00	1.00	0.83	0.81	0.81	0.91	0.85	0.85	0.83
time (sec)	N/A	0.203	0.010	0.106	0.033	0.061	0.031	0.125	0.159	0.027

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	45	48	44	48	46	46	45
N.S.	1	1.00	1.00	0.83	0.89	0.81	0.89	0.85	0.85	0.83
time (sec)	N/A	0.212	0.009	0.079	0.032	0.060	0.024	0.115	0.167	0.025

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	45	44	44	49	46	46	45
N.S.	1	1.00	1.00	0.83	0.81	0.81	0.91	0.85	0.85	0.83
time (sec)	N/A	0.197	0.009	0.104	0.031	0.058	0.027	0.133	0.165	0.027

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	45	44	44	48	46	46	45
N.S.	1	1.00	1.00	0.83	0.81	0.81	0.89	0.85	0.85	0.83
time (sec)	N/A	0.200	0.009	0.097	0.033	0.057	0.024	0.132	0.164	0.028

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	46	45	44	44	49	46	46	45
N.S.	1	1.00	0.85	0.83	0.81	0.81	0.91	0.85	0.85	0.83
time (sec)	N/A	0.197	0.013	0.097	0.026	0.078	0.025	0.117	0.158	0.027

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	41	40	40	42	42	44	41
N.S.	1	1.00	1.00	0.89	0.87	0.87	0.91	0.91	0.96	0.89
time (sec)	N/A	0.195	0.009	0.043	0.036	0.056	0.025	0.112	0.154	0.026

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	43	42	40	40	42	42	41	40
N.S.	1	1.00	0.93	0.91	0.87	0.87	0.91	0.91	0.89	0.87
time (sec)	N/A	0.187	0.021	0.081	0.029	0.060	0.058	0.112	0.162	0.031

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	84	98	0	297	381	86	180	112
N.S.	1	1.00	0.94	1.10	0.00	3.34	4.28	0.97	2.02	1.26
time (sec)	N/A	0.259	0.126	0.104	0.000	0.102	0.482	0.132	0.163	21.049

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	73	75	0	235	306	67	136	172
N.S.	1	1.00	1.04	1.07	0.00	3.36	4.37	0.96	1.94	2.46
time (sec)	N/A	0.233	0.073	0.090	0.000	0.080	0.360	0.129	0.166	0.101

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	57	56	0	185	216	55	84	112
N.S.	1	1.00	1.02	1.00	0.00	3.30	3.86	0.98	1.50	2.00
time (sec)	N/A	0.212	0.036	0.075	0.000	0.080	0.188	0.122	0.160	20.938

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	38	35	0	120	124	34	46	46
N.S.	1	1.00	1.12	1.03	0.00	3.53	3.65	1.00	1.35	1.35
time (sec)	N/A	0.172	0.008	0.051	0.000	0.072	0.124	0.132	0.159	20.897

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	63	61	61	0	211	564	62	96	213
N.S.	1	1.02	0.98	0.98	0.00	3.40	9.10	1.00	1.55	3.44
time (sec)	N/A	0.235	0.076	0.069	0.000	0.082	2.392	0.133	0.176	0.442

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	85	77	81	0	269	862	79	158	339
N.S.	1	1.05	0.95	1.00	0.00	3.32	10.64	0.98	1.95	4.19
time (sec)	N/A	0.298	0.089	0.102	0.000	0.093	156.709	0.111	0.200	22.268

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	113	102	128	0	358	0	105	225	447
N.S.	1	1.09	0.98	1.23	0.00	3.44	0.00	1.01	2.16	4.30
time (sec)	N/A	0.330	0.147	0.128	0.000	0.125	0.000	0.135	0.160	22.253

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	140	131	157	0	445	0	136	308	524
N.S.	1	1.02	0.96	1.15	0.00	3.25	0.00	0.99	2.25	3.82
time (sec)	N/A	0.392	0.121	0.103	0.000	0.163	0.000	0.129	0.164	21.660

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	154	132	198	0	837	842	161	752	261
N.S.	1	1.11	0.95	1.42	0.00	6.02	6.06	1.16	5.41	1.88
time (sec)	N/A	0.364	0.207	0.148	0.000	0.084	1.093	0.130	0.160	21.386

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	125	109	169	0	635	729	125	547	279
N.S.	1	1.10	0.96	1.48	0.00	5.57	6.39	1.10	4.80	2.45
time (sec)	N/A	0.317	0.169	0.128	0.000	0.083	0.802	0.118	0.162	21.713

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	81	97	0	387	280	88	243	135
N.S.	1	1.00	1.21	1.45	0.00	5.78	4.18	1.31	3.63	2.01
time (sec)	N/A	0.217	0.102	0.090	0.000	0.078	0.381	0.117	0.166	0.109

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	69	70	0	338	253	76	223	110
N.S.	1	1.00	1.05	1.06	0.00	5.12	3.83	1.15	3.38	1.67
time (sec)	N/A	0.209	0.072	0.049	0.000	0.077	0.336	0.123	0.157	21.197

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	70	68	0	341	265	76	241	119
N.S.	1	1.00	1.06	1.03	0.00	5.17	4.02	1.15	3.65	1.80
time (sec)	N/A	0.206	0.087	0.049	0.000	0.088	0.341	0.139	0.142	21.199

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	139	107	177	0	781	0	126	644	620
N.S.	1	1.29	0.99	1.64	0.00	7.23	0.00	1.17	5.96	5.74
time (sec)	N/A	0.353	0.210	0.095	0.000	0.140	0.000	0.112	0.148	23.125

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	173	131	205	0	975	0	171	877	775
N.S.	1	1.17	0.89	1.39	0.00	6.59	0.00	1.16	5.93	5.24
time (sec)	N/A	0.402	0.282	0.065	0.000	0.180	0.000	0.124	0.159	22.800

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	213	175	255	0	1226	0	229	1039	914
N.S.	1	1.05	0.87	1.26	0.00	6.07	0.00	1.13	5.14	4.52
time (sec)	N/A	0.460	0.428	0.087	0.000	0.243	0.000	0.146	0.160	22.452

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	260	218	295	0	1407	0	282	1276	1120
N.S.	1	1.03	0.87	1.17	0.00	5.58	0.00	1.12	5.06	4.44
time (sec)	N/A	0.551	0.353	0.095	0.000	0.417	0.000	0.135	0.158	22.440

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	318	309	272	359	0	1640	0	347	1425	1260
N.S.	1	0.97	0.86	1.13	0.00	5.16	0.00	1.09	4.48	3.96
time (sec)	N/A	0.612	0.427	0.106	0.000	0.548	0.000	0.123	0.163	22.784

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	26	24	25	39	24	34	42	30
N.S.	1	1.00	0.84	0.77	0.81	1.26	0.77	1.10	1.35	0.97
time (sec)	N/A	0.169	0.024	0.119	0.027	0.060	0.058	0.111	0.155	0.068

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	26	24	25	39	24	27	42	23
N.S.	1	1.00	0.84	0.77	0.81	1.26	0.77	0.87	1.35	0.74
time (sec)	N/A	0.175	0.005	0.115	0.034	0.068	0.066	0.134	0.171	0.037

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	26	24	25	39	24	27	42	23
N.S.	1	1.00	0.84	0.77	0.81	1.26	0.77	0.87	1.35	0.74
time (sec)	N/A	0.187	0.005	0.028	0.027	0.064	0.064	0.130	0.158	0.041

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	278	184	133	0	390	0	275	24	0
N.S.	1	1.21	0.80	0.58	0.00	1.70	0.00	1.20	0.10	0.00
time (sec)	N/A	0.684	0.681	0.149	0.000	0.099	0.000	0.166	200.029	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	218	150	106	0	326	0	222	22	0
N.S.	1	1.23	0.85	0.60	0.00	1.84	0.00	1.25	0.12	0.00
time (sec)	N/A	0.496	0.484	0.111	0.000	0.090	0.000	0.166	200.028	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	143	121	121	0	260	0	160	163	0
N.S.	1	0.96	0.81	0.81	0.00	1.74	0.00	1.07	1.09	0.00
time (sec)	N/A	0.277	0.415	0.092	0.000	0.090	0.000	0.153	0.246	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	119	95	100	0	220	0	119	123	0
N.S.	1	1.31	1.04	1.10	0.00	2.42	0.00	1.31	1.35	0.00
time (sec)	N/A	0.270	0.179	0.089	0.000	0.079	0.000	0.169	0.225	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	140	133	91	0	638	0	0	828	0
N.S.	1	0.81	0.77	0.53	0.00	3.69	0.00	0.00	4.79	0.00
time (sec)	N/A	0.345	0.300	0.105	0.000	0.106	0.000	0.000	0.266	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	142	125	101	0	653	0	0	90	0
N.S.	1	0.82	0.72	0.58	0.00	3.77	0.00	0.00	0.52	0.00
time (sec)	N/A	0.337	0.328	0.092	0.000	0.151	0.000	0.000	0.279	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	117	111	81	0	226	0	0	130	0
N.S.	1	1.03	0.97	0.71	0.00	1.98	0.00	0.00	1.14	0.00
time (sec)	N/A	0.308	0.439	0.109	0.000	0.104	0.000	0.000	0.241	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	166	130	115	0	272	0	0	175	0
N.S.	1	1.07	0.84	0.74	0.00	1.75	0.00	0.00	1.13	0.00
time (sec)	N/A	0.397	0.763	0.117	0.000	0.097	0.000	0.000	0.271	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	224	159	149	0	336	0	0	274	0
N.S.	1	1.09	0.78	0.73	0.00	1.64	0.00	0.00	1.34	0.00
time (sec)	N/A	0.514	0.993	0.152	0.000	0.116	0.000	0.000	0.387	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	394	459	304	328	0	664	0	509	22	0
N.S.	1	1.16	0.77	0.83	0.00	1.69	0.00	1.29	0.06	0.00
time (sec)	N/A	1.145	1.558	0.474	0.000	0.111	0.000	0.183	200.023	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	336	398	239	271	0	558	0	419	20	0
N.S.	1	1.18	0.71	0.81	0.00	1.66	0.00	1.25	0.06	0.00
time (sec)	N/A	0.999	1.176	0.381	0.000	0.107	0.000	0.167	200.023	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	260	309	211	225	0	474	0	355	24	0
N.S.	1	1.19	0.81	0.87	0.00	1.82	0.00	1.37	0.09	0.00
time (sec)	N/A	0.689	0.947	0.421	0.000	0.100	0.000	0.165	200.036	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	210	161	126	0	384	0	276	317	0
N.S.	1	1.24	0.95	0.74	0.00	2.26	0.00	1.62	1.86	0.00
time (sec)	N/A	0.413	0.145	0.211	0.000	0.087	0.000	0.197	0.525	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	171	133	100	0	320	0	224	252	0
N.S.	1	1.25	0.97	0.73	0.00	2.34	0.00	1.64	1.84	0.00
time (sec)	N/A	0.347	0.319	0.179	0.000	0.094	0.000	0.160	0.208	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	204	164	192	0	791	0	0	1018	0
N.S.	1	0.90	0.72	0.85	0.00	3.48	0.00	0.00	4.48	0.00
time (sec)	N/A	0.474	0.865	0.169	0.000	0.161	0.000	0.000	0.393	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	189	156	180	0	757	0	0	162	0
N.S.	1	0.86	0.71	0.82	0.00	3.46	0.00	0.00	0.74	0.00
time (sec)	N/A	0.448	0.820	0.183	0.000	0.146	0.000	0.000	0.224	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	188	160	140	0	757	0	0	182	0
N.S.	1	0.86	0.73	0.64	0.00	3.46	0.00	0.00	0.83	0.00
time (sec)	N/A	0.436	0.857	0.208	0.000	0.130	0.000	0.000	0.201	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	232	235	172	150	0	815	0	0	210	0
N.S.	1	1.01	0.74	0.65	0.00	3.51	0.00	0.00	0.91	0.00
time (sec)	N/A	0.553	1.066	0.217	0.000	0.197	0.000	0.000	0.230	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	213	141	131	0	332	0	0	274	0
N.S.	1	1.08	0.72	0.66	0.00	1.69	0.00	0.00	1.39	0.00
time (sec)	N/A	0.484	1.200	0.219	0.000	0.163	0.000	0.000	0.341	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	249	273	176	161	0	394	0	0	343	0
N.S.	1	1.10	0.71	0.65	0.00	1.58	0.00	0.00	1.38	0.00
time (sec)	N/A	0.628	1.605	0.228	0.000	0.169	0.000	0.000	0.579	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	149	107	100	0	226	0	134	123	0
N.S.	1	1.30	0.93	0.87	0.00	1.97	0.00	1.17	1.07	0.00
time (sec)	N/A	0.334	0.382	0.166	0.000	0.088	0.000	0.150	0.220	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	103	89	50	0	188	0	90	61	0
N.S.	1	1.37	1.19	0.67	0.00	2.51	0.00	1.20	0.81	0.00
time (sec)	N/A	0.247	0.068	0.131	0.000	0.078	0.000	0.148	0.198	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	71	67	29	0	129	0	57	42	0
N.S.	1	1.65	1.56	0.67	0.00	3.00	0.00	1.33	0.98	0.00
time (sec)	N/A	0.200	0.021	0.105	0.000	0.076	0.000	0.165	0.206	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	68	42	0	130	0	59	693	0
N.S.	1	1.00	1.51	0.93	0.00	2.89	0.00	1.31	15.40	0.00
time (sec)	N/A	0.176	0.130	0.122	0.000	0.080	0.000	0.128	0.218	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	87	68	0	194	0	0	60	0
N.S.	1	1.00	1.13	0.88	0.00	2.52	0.00	0.00	0.78	0.00
time (sec)	N/A	0.224	0.238	0.142	0.000	0.154	0.000	0.000	0.226	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	125	111	117	0	232	0	0	131	0
N.S.	1	1.05	0.93	0.98	0.00	1.95	0.00	0.00	1.10	0.00
time (sec)	N/A	0.305	0.400	0.168	0.000	0.095	0.000	0.000	0.221	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	268	185	164	0	616	0	318	751	0
N.S.	1	1.15	0.79	0.70	0.00	2.63	0.00	1.36	3.21	0.00
time (sec)	N/A	0.614	0.938	0.237	0.000	0.124	0.000	0.198	0.268	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	212	143	122	0	486	0	231	499	0
N.S.	1	1.23	0.83	0.71	0.00	2.81	0.00	1.34	2.88	0.00
time (sec)	N/A	0.455	0.701	0.229	0.000	0.116	0.000	0.155	0.214	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	166	112	84	0	414	0	164	426	0
N.S.	1	1.33	0.90	0.67	0.00	3.31	0.00	1.31	3.41	0.00
time (sec)	N/A	0.354	0.572	0.178	0.000	0.104	0.000	0.152	0.213	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	37	34	0	73	0	69	105	75
N.S.	1	1.00	0.92	0.85	0.00	1.82	0.00	1.72	2.62	1.88
time (sec)	N/A	0.178	0.353	0.134	0.000	0.091	0.000	0.161	0.205	12.085

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	36	33	0	72	0	74	95	75
N.S.	1	1.00	0.92	0.85	0.00	1.85	0.00	1.90	2.44	1.92
time (sec)	N/A	0.175	0.014	0.122	0.000	0.094	0.000	0.159	0.203	12.766

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	108	107	0	411	0	199	2247	0
N.S.	1	1.00	1.15	1.14	0.00	4.37	0.00	2.12	23.90	0.00
time (sec)	N/A	0.247	0.624	0.179	0.000	0.122	0.000	0.134	0.886	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	157	134	137	0	496	0	0	454	0
N.S.	1	1.09	0.93	0.95	0.00	3.44	0.00	0.00	3.15	0.00
time (sec)	N/A	0.343	0.858	0.236	0.000	0.133	0.000	0.000	0.251	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	218	180	173	0	630	0	0	696	0
N.S.	1	1.04	0.86	0.83	0.00	3.01	0.00	0.00	3.33	0.00
time (sec)	N/A	0.458	1.209	0.267	0.000	0.160	0.000	0.000	0.217	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	271	278	224	213	0	716	0	0	796	0
N.S.	1	1.03	0.83	0.79	0.00	2.64	0.00	0.00	2.94	0.00
time (sec)	N/A	0.594	1.610	0.302	0.000	0.275	0.000	0.000	0.237	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	343	348	271	258	0	866	0	0	1055	0
N.S.	1	1.01	0.79	0.75	0.00	2.52	0.00	0.00	3.08	0.00
time (sec)	N/A	0.769	2.195	0.347	0.000	0.343	0.000	0.000	0.348	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	174	305	3384	302	2421	0	7783	3510	3890
N.S.	1	0.91	1.60	17.72	1.58	12.68	0.00	40.75	18.38	20.37
time (sec)	N/A	0.358	1.148	20.485	0.068	0.163	0.000	0.305	0.232	13.674

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	117	143	944	173	753	0	2370	986	1210
N.S.	1	0.93	1.13	7.49	1.37	5.98	0.00	18.81	7.83	9.60
time (sec)	N/A	0.306	0.430	3.179	0.056	0.115	0.000	0.198	0.201	12.801

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	40	161	71	147	1258	398	155	218
N.S.	1	1.00	0.73	2.93	1.29	2.67	22.87	7.24	2.82	3.96
time (sec)	N/A	0.218	0.124	0.464	0.046	0.085	20.017	0.133	0.203	12.477

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	175	178	142	0	0	0	0	0	32	0
N.S.	1	1.02	0.81	0.00	0.00	0.00	0.00	0.00	0.18	0.00
time (sec)	N/A	0.437	0.450	0.000	0.000	0.000	0.000	0.000	0.200	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	300	306	263	0	0	0	0	0	80	0
N.S.	1	1.02	0.88	0.00	0.00	0.00	0.00	0.00	0.27	0.00
time (sec)	N/A	0.709	0.926	0.000	0.000	0.000	0.000	0.000	0.206	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	37	35	0	111	0	35	693	34
N.S.	1	1.00	0.97	0.92	0.00	2.92	0.00	0.92	18.24	0.89
time (sec)	N/A	0.169	0.023	0.489	0.000	0.099	0.000	0.132	0.217	0.045

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	68	42	0	130	0	59	693	0
N.S.	1	1.00	1.51	0.93	0.00	2.89	0.00	1.31	15.40	0.00
time (sec)	N/A	0.194	0.024	0.105	0.000	0.111	0.000	0.150	0.206	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	70	64	0	131	0	35	693	0
N.S.	1	1.00	1.49	1.36	0.00	2.79	0.00	0.74	14.74	0.00
time (sec)	N/A	0.231	0.038	0.080	0.000	0.142	0.000	0.148	0.236	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	72	66	0	139	0	59	693	0
N.S.	1	1.00	1.47	1.35	0.00	2.84	0.00	1.20	14.14	0.00
time (sec)	N/A	0.283	0.030	0.102	0.000	0.181	0.000	0.151	0.216	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [56] had the largest ratio of [.6999999999999999956]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	5	5	1.11	26	0.192
2	A	4	4	0.88	26	0.154
3	A	3	3	1.00	26	0.115
4	A	5	4	1.32	26	0.154
5	A	6	5	1.07	26	0.192
6	A	7	6	1.02	26	0.231
7	A	3	3	1.00	20	0.150
8	A	3	3	1.00	18	0.167
9	A	1	1	1.00	16	0.062
10	A	3	3	1.00	20	0.150
11	A	2	2	1.00	20	0.100
12	A	3	3	1.00	20	0.150
13	A	3	3	1.00	20	0.150
14	A	3	3	1.00	20	0.150
15	A	3	3	1.00	20	0.150
16	A	3	3	1.00	20	0.150
17	A	3	3	1.00	22	0.136
18	A	3	3	1.00	20	0.150
19	A	3	3	1.00	18	0.167
20	A	3	3	1.00	22	0.136
21	A	3	3	1.00	22	0.136

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
22	A	3	3	1.00	22	0.136
23	A	3	3	1.00	22	0.136
24	A	3	3	1.00	22	0.136
25	A	3	3	1.00	22	0.136
26	A	3	3	1.00	22	0.136
27	A	6	5	1.00	22	0.227
28	A	4	3	1.00	22	0.136
29	A	8	7	1.02	20	0.350
30	A	5	5	1.05	18	0.278
31	A	5	5	1.09	22	0.227
32	A	5	5	1.02	22	0.227
33	A	5	5	1.11	22	0.227
34	A	4	4	1.10	22	0.182
35	A	5	4	1.00	22	0.182
36	A	5	4	1.00	22	0.182
37	A	5	4	1.00	22	0.182
38	A	5	5	1.29	22	0.227
39	A	5	5	1.17	22	0.227
40	A	5	5	1.05	20	0.250
41	A	5	5	1.03	18	0.278
42	A	5	5	0.97	22	0.227
43	A	2	2	1.00	9	0.222
44	A	3	3	1.00	14	0.214
45	A	4	4	1.00	16	0.250
46	A	12	11	1.21	24	0.458
47	A	10	9	1.23	22	0.409
48	A	6	5	0.96	20	0.250
49	A	5	4	1.31	24	0.167
50	A	8	7	0.81	24	0.292
51	A	8	7	0.82	24	0.292
52	A	6	5	1.03	24	0.208
53	A	8	7	1.07	24	0.292

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
54	A	10	9	1.09	24	0.375
55	A	16	15	1.16	22	0.682
56	A	15	14	1.18	20	0.700
57	A	12	11	1.19	24	0.458
58	A	7	6	1.24	24	0.250
59	A	6	5	1.25	24	0.208
60	A	10	9	0.90	24	0.375
61	A	10	9	0.86	24	0.375
62	A	10	9	0.86	24	0.375
63	A	12	11	1.01	24	0.458
64	A	9	8	1.08	24	0.333
65	A	12	11	1.10	24	0.458
66	A	8	7	1.30	24	0.292
67	A	5	4	1.37	24	0.167
68	A	4	3	1.65	22	0.136
69	A	3	2	1.00	20	0.100
70	A	4	3	1.00	24	0.125
71	A	7	6	1.05	24	0.250
72	A	12	11	1.15	24	0.458
73	A	10	9	1.23	24	0.375
74	A	7	6	1.33	24	0.250
75	A	1	1	1.00	24	0.042
76	A	1	1	1.00	24	0.042
77	A	4	3	1.00	24	0.125
78	A	7	6	1.09	22	0.273
79	A	9	8	1.04	20	0.400
80	A	11	10	1.03	24	0.417
81	A	13	12	1.01	24	0.500
82	A	4	4	0.91	28	0.143
83	A	4	4	0.93	28	0.143
84	A	2	2	1.00	26	0.077
85	A	4	4	1.02	28	0.143

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
86	A	5	5	1.02	28	0.179
87	A	3	2	1.00	18	0.111
88	A	4	3	1.00	18	0.167
89	A	5	4	1.00	22	0.182
90	A	5	4	1.00	24	0.167

CHAPTER 3

Listing of integrals

3.1 $\int \frac{(dx)^{7/2}}{(ax+bx^2+cx^3)^{7/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 147

$$\int \frac{(dx)^{7/2}}{(ax+bx^2+cx^3)^{7/2}}\,dx = -\frac{2d(dx)^{5/2}(b+2cx)}{5(b^2-4ac)(ax+bx^2+cx^3)^{5/2}} + \frac{32cd^2(dx)^{3/2}(b+2cx)}{15(b^2-4ac)^2(ax+bx^2+cx^3)^{3/2}} - \frac{256c^2d^3\sqrt{dx}(b+2cx)}{15(b^2-4ac)^3\sqrt{ax+bx^2+cx^3}}$$

output
```
-2/5*d*(d*x)^(5/2)*(2*c*x+b)/(-4*a*c+b^2)/(c*x^3+b*x^2+a*x)^(5/2)+32/15*c*
d^2*(d*x)^(3/2)*(2*c*x+b)/(-4*a*c+b^2)^2/(c*x^3+b*x^2+a*x)^(3/2)-256/15*c^
2*d^3*(d*x)^(1/2)*(2*c*x+b)/(-4*a*c+b^2)^3/(c*x^3+b*x^2+a*x)^(1/2)
```

Mathematica [A] (verified)

Time = 2.40 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.85

$$\int \frac{(dx)^{7/2}}{(ax+bx^2+cx^3)^{7/2}}\,dx = -\frac{2(dx)^{7/2}(b+2cx)(a+bx+cx^2)(3b^4-40ab^2c+240a^2c^2-16b^3cx+320abc^2x+112b^2c^2x^2+320ac^3x^2}{15(b^2-4ac)^3(x(a+x(b+cx)))^{7/2}}$$

input
```
Integrate[(d*x)^(7/2)/(a*x + b*x^2 + c*x^3)^(7/2),x]
```

output
```
(-2*(d*x)^(7/2)*(b + 2*c*x)*(a + b*x + c*x^2)*(3*b^4 - 40*a*b^2*c + 240*a^
2*c^2 - 16*b^3*c*x + 320*a*b*c^2*x + 112*b^2*c^2*x^2 + 320*a*c^3*x^2 + 256
*b*c^3*x^3 + 128*c^4*x^4))/(15*(b^2 - 4*a*c)^3*(x*(a + x*(b + c*x)))^(7/2)
)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.11, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {2467, 30, 1089, 1089, 1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^{7/2}}{(ax+bx^2+cx^3)^{7/2}}dx$$

$$\Big\downarrow \text{2467}$$

$$\frac{\sqrt{x}\sqrt{a+bx+cx^2}\int \frac{(dx)^{7/2}}{x^{7/2}(cx^2+bx+a)^{7/2}}dx}{\sqrt{ax+bx^2+cx^3}}$$

$$\Big\downarrow \text{30}$$

$$\frac{d^3\sqrt{dx}\sqrt{a+bx+cx^2}\int \frac{1}{(cx^2+bx+a)^{7/2}}dx}{\sqrt{ax+bx^2+cx^3}}$$

$$\Big\downarrow \text{1089}$$

$$\frac{d^3\sqrt{dx}\sqrt{a+bx+cx^2}\left(-\frac{16c\int \frac{1}{(cx^2+bx+a)^{5/2}}dx}{5(b^2-4ac)}-\frac{2(b+2cx)}{5(b^2-4ac)(a+bx+cx^2)^{5/2}}\right)}{\sqrt{ax+bx^2+cx^3}}$$

$$\Big\downarrow \text{1089}$$

$$\frac{d^3\sqrt{dx}\sqrt{a+bx+cx^2}\left(-\frac{16c\left(-\frac{8c\int\frac{1}{\left(cx^2+bx+a\right)^{3/2}}dx}{3\left(b^2-4ac\right)}-\frac{2(b+2cx)}{3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}}\right)}{5(b^2-4ac)}-\frac{2(b+2cx)}{5(b^2-4ac)(a+bx+cx^2)^{5/2}}\right)}{\sqrt{ax+bx^2+cx^3}}$$

$$\Big\downarrow \text{1088}$$

$$\frac{d^3\sqrt{dx}\sqrt{a+bx+cx^2}\left(-\frac{2(b+2cx)}{5(b^2-4ac)(a+bx+cx^2)^{5/2}}-\frac{16c\left(\frac{16c(b+2cx)}{3\left(b^2-4ac\right)^2\sqrt{a+bx+cx^2}}-\frac{2(b+2cx)}{3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}}\right)}{5(b^2-4ac)}\right)}{\sqrt{ax+bx^2+cx^3}}$$

input
```
Int[(d*x)^(7/2)/(a*x + b*x^2 + c*x^3)^(7/2),x]
```

output
```
(d^3*Sqrt[d*x]*Sqrt[a + b*x + c*x^2]*((-2*(b + 2*c*x))/(5*(b^2 - 4*a*c)*(a
 + b*x + c*x^2)^(5/2)) - (16*c*((-2*(b + 2*c*x))/(3*(b^2 - 4*a*c)*(a + b*x
 + c*x^2)^(3/2)) + (16*c*(b + 2*c*x))/(3*(b^2 - 4*a*c)^2*Sqrt[a + b*x + c*
x^2])))/(5*(b^2 - 4*a*c))))/Sqrt[a*x + b*x^2 + c*x^3]
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

rule 1089
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 2467
```
Int[(Fx_.)*(Px_)^(p_), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Simp[Px^F
racPart[p]/(x^(r*FracPart[p])*ExpandToSum[Px/x^r, x]^FracPart[p])   Int[x^(
p*r)*ExpandToSum[Px/x^r, x]^p*Fx, x], x] /; IGtQ[r, 0]] /; FreeQ[p, x] && P
olyQ[Px, x] &&  !IntegerQ[p] &&  !MonomialQ[Px, x] &&  !PolyQ[Fx, x]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.18

method	result
gosper	$\frac{2(c x^2+bx+a)(256c^5x^5+640b c^4x^4+640a c^4x^3+480b^2c^3x^3+960ab c^3x^2+80b^3c^2x^2+480a^2c^3x+240a b^2c^2x-10b^4cx+240a^2b c^2-4\ldots}{15(64a^3c^3-48a^2b^2c^2+12a b^4c-b^6)(c x^3+b x^2+xa)^{\frac{7}{2}}}$
orering	$\frac{2(c x^2+bx+a)(256c^5x^5+640b c^4x^4+640a c^4x^3+480b^2c^3x^3+960ab c^3x^2+80b^3c^2x^2+480a^2c^3x+240a b^2c^2x-10b^4cx+240a^2b c^2-4\ldots}{15(64a^3c^3-48a^2b^2c^2+12a b^4c-b^6)(c x^3+b x^2+xa)^{\frac{7}{2}}}$
default	$\frac{64d^3\sqrt{dx}\,\sqrt{x(c x^2+bx+a)}\,(256c^5x^5+640b c^4x^4+640a c^4x^3+480b^2c^3x^3+960ab c^3x^2+80b^3c^2x^2+480a^2c^3x+240a b^2c^2x-10b^4cx+2\ldots}{15x\left(2cx+\sqrt{-4ac+b^2}+b\right)^3\left(b-\sqrt{-4ac+b^2}+2cx\right)^3(4ac-b^2)^3\sqrt{d(c\ldots}}$

input
```
int((d*x)^(7/2)/(c*x^3+b*x^2+a*x)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
2/15*(c*x^2+b*x+a)*(256*c^5*x^5+640*b*c^4*x^4+640*a*c^4*x^3+480*b^2*c^3*x^
3+960*a*b*c^3*x^2+80*b^3*c^2*x^2+480*a^2*c^3*x+240*a*b^2*c^2*x-10*b^4*c*x+
240*a^2*b*c^2-40*a*b^3*c+3*b^5)*(d*x)^(7/2)/(64*a^3*c^3-48*a^2*b^2*c^2+12*
a*b^4*c-b^6)/(c*x^3+b*x^2+a*x)^(7/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 459 vs. $2(129) = 258$.

Time = 0.10 (sec) , antiderivative size = 459, normalized size of antiderivative = 3.12

$$\int \frac{(dx)^{7/2}}{(ax + bx^2 + cx^3)^{7/2}}\,dx =$$

$$-\frac{2\,(256\,c^5d^3x^5 +}{15\,((b^6c^3 - 12\,ab^4c^4 + 48\,a^2b^2c^5 - 64\,a^3c^6)x^7 + 3\,(b^7c^2 - 12\,ab^5c^3 + 48\,a^2b^3c^4 - 64\,a^3bc^5)x^6 + 3\,(b^8c - 11}$$

input
```
integrate((d*x)^(7/2)/(c*x^3+b*x^2+a*x)^(7/2),x, algorithm="fricas")
```

output
```
-2/15*(256*c^5*d^3*x^5 + 640*b*c^4*d^3*x^4 + 160*(3*b^2*c^3 + 4*a*c^4)*d^3
*x^3 + 80*(b^3*c^2 + 12*a*b*c^3)*d^3*x^2 - 10*(b^4*c - 24*a*b^2*c^2 - 48*a
^2*c^3)*d^3*x + (3*b^5 - 40*a*b^3*c + 240*a^2*b*c^2)*d^3)*sqrt(c*x^3 + b*x
^2 + a*x)*sqrt(d*x)/((b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6
)*x^7 + 3*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*x^6 + 3
*(b^8*c - 11*a*b^6*c^2 + 36*a^2*b^4*c^3 - 16*a^3*b^2*c^4 - 64*a^4*c^5)*x^5
 + (b^9 - 6*a*b^7*c - 24*a^2*b^5*c^2 + 224*a^3*b^3*c^3 - 384*a^4*b*c^4)*x^
4 + 3*(a*b^8 - 11*a^2*b^6*c + 36*a^3*b^4*c^2 - 16*a^4*b^2*c^3 - 64*a^5*c^4
)*x^3 + 3*(a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3*c^2 - 64*a^5*b*c^3)*x^2 + (
a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*x)
```

Sympy [F]

$$\int \frac{(dx)^{7/2}}{(ax + bx^2 + cx^3)^{7/2}}\,dx = \int \frac{(dx)^{\frac{7}{2}}}{(x\,(a + bx + cx^2))^{\frac{7}{2}}}\,dx$$

input
```
integrate((d*x)**(7/2)/(c*x**3+b*x**2+a*x)**(7/2),x)
```

output
```
Integral((d*x)**(7/2)/(x*(a + b*x + c*x**2))**(7/2), x)
```

Maxima [F]

$$\int \frac{(dx)^{7/2}}{(ax+bx^2+cx^3)^{7/2}}\,dx = \int \frac{(dx)^{\frac{7}{2}}}{(cx^3+bx^2+ax)^{\frac{7}{2}}}\,dx$$

input
```
integrate((d*x)^(7/2)/(c*x^3+b*x^2+a*x)^(7/2),x, algorithm="maxima")
```

output
```
integrate((d*x)^(7/2)/(c*x^3 + b*x^2 + a*x)^(7/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 552 vs. $2(129) = 258$.

Time = 0.21 (sec) , antiderivative size = 552, normalized size of antiderivative = 3.76

$$\int \frac{(dx)^{7/2}}{(ax+bx^2+cx^3)^{7/2}}\,dx =$$

$$-\frac{2}{15}\left(\frac{\left(2\left(8\left(2\left(4\left(\frac{2c^5d^2x}{b^6d^5-12ab^4cd^5+48a^2b^2c^2d^5-64a^3c^3d^5}+\frac{5bc^4d^2}{b^6d^5-12ab^4cd^5+48a^2b^2c^2d^5-64a^3c^3d^5}\right)dx+\frac{5(3b^2c^3d}{b^6d^5-12ab^4cd^5+48}\right.\right.\right.\right.}{}\right.$$

input
```
integrate((d*x)^(7/2)/(c*x^3+b*x^2+a*x)^(7/2),x, algorithm="giac")
```

output
```
-2/15*((2*(8*(2*(4*(2*c^5*d^2*x/(b^6*d^5 - 12*a*b^4*c*d^5 + 48*a^2*b^2*c^2
*d^5 - 64*a^3*c^3*d^5) + 5*b*c^4*d^2/(b^6*d^5 - 12*a*b^4*c*d^5 + 48*a^2*b^
2*c^2*d^5 - 64*a^3*c^3*d^5))*d*x + 5*(3*b^2*c^3*d^3 + 4*a*c^4*d^3)/(b^6*d^
5 - 12*a*b^4*c*d^5 + 48*a^2*b^2*c^2*d^5 - 64*a^3*c^3*d^5))*d*x + 5*(b^3*c^
2*d^4 + 12*a*b*c^3*d^4)/(b^6*d^5 - 12*a*b^4*c*d^5 + 48*a^2*b^2*c^2*d^5 - 6
4*a^3*c^3*d^5))*d*x - 5*(b^4*c*d^5 - 24*a*b^2*c^2*d^5 - 48*a^2*c^3*d^5)/(b
^6*d^5 - 12*a*b^4*c*d^5 + 48*a^2*b^2*c^2*d^5 - 64*a^3*c^3*d^5))*d*x + (3*b
^5*d^6 - 40*a*b^3*c*d^6 + 240*a^2*b*c^2*d^6)/(b^6*d^5 - 12*a*b^4*c*d^5 + 4
8*a^2*b^2*c^2*d^5 - 64*a^3*c^3*d^5))*d^8/(c*d^3*x^2 + b*d^3*x + a*d^3)^(5/
2) - (3*b^5*d^3 - 40*a*b^3*c*d^3 + 240*a^2*b*c^2*d^3)/(sqrt(a*d^3)*a^2*b^6
 - 12*sqrt(a*d^3)*a^3*b^4*c + 48*sqrt(a*d^3)*a^4*b^2*c^2 - 64*sqrt(a*d^3)*
a^5*c^3))*d^2
```

Mupad [B] (verification not implemented)

Time = 21.98 (sec) , antiderivative size = 321, normalized size of antiderivative = 2.18

$$\int \frac{(dx)^{7/2}}{(ax+bx^2+cx^3)^{7/2}}\,dx = \frac{\sqrt{c\,x^3+b\,x^2+a\,x}\,\left(\frac{64\,d^3\,x^3\,\sqrt{d\,x}\,\left(3\,b^2+4\,a\,c\right)}{3\,{\left(4\,a\,c-b^2\right)}^3}+\frac{512\,c^2\,d^3\,x^5\,\sqrt{d\,x}}{15\,{\left(4\,a\,c-b^2\right)}^3}+\frac{2\,b\,d^3\,\sqrt{d\,x}\,\left(240\,a^2\,c^2-40\,a\right.}{15\,c^3\,{\left(4\,a\,c-b^2\right)}^3}\right.}{x^7+\frac{3\,x^5\,\left(b^2+a\,c\right)}{c^2}+\frac{a^3\,x}{c^3}+\frac{3\,b\,x^6}{c}+\frac{3}{}}$$

input
```
int((d*x)^(7/2)/(a*x + b*x^2 + c*x^3)^(7/2),x)
```

output
```
((a*x + b*x^2 + c*x^3)^(1/2)*((64*d^3*x^3*(d*x)^(1/2)*(4*a*c + 3*b^2))/(3*
(4*a*c - b^2)^3) + (512*c^2*d^3*x^5*(d*x)^(1/2))/(15*(4*a*c - b^2)^3) + (2
*b*d^3*(d*x)^(1/2)*(3*b^4 + 240*a^2*c^2 - 40*a*b^2*c))/(15*c^3*(4*a*c - b^
2)^3) + (4*d^3*x*(d*x)^(1/2)*(48*a^2*c^2 - b^4 + 24*a*b^2*c))/(3*c^2*(4*a*
c - b^2)^3) + (256*b*c*d^3*x^4*(d*x)^(1/2))/(3*(4*a*c - b^2)^3) + (32*b*d^
3*x^2*(d*x)^(1/2)*(12*a*c + b^2))/(3*c*(4*a*c - b^2)^3)))/(x^7 + (3*x^5*(a
*c + b^2))/c^2 + (a^3*x)/c^3 + (3*b*x^6)/c + (3*a^2*b*x^2)/c^3 + (3*a*x^3*
(a*c + b^2))/c^3 + (b*x^4*(6*a*c + b^2))/c^3)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 719, normalized size of antiderivative = 4.89

$$\int \frac{(dx)^{7/2}}{(ax+bx^2+cx^3)^{7/2}}\,dx = \frac{2\sqrt{d}\,d^3\left(240\sqrt{c\,x^2+bx+a}\,a^2b\,c^2+480\sqrt{c\,x^2+bx+a}\,a^2c^3x-40\sqrt{c\,x^2+bx+}\right.}{960a^3c^6x^6-720a^2b^2c^5x^6+180a\,b^4c^4x^6-}$$

input
```
int((d*x)^(7/2)/(c*x^3+b*x^2+a*x)^(7/2),x)
```

output

```
(2*sqrt(d)*d**3*(240*sqrt(a + b*x + c*x**2)*a**2*b*c**2 + 480*sqrt(a + b*x
 + c*x**2)*a**2*c**3*x - 40*sqrt(a + b*x + c*x**2)*a*b**3*c + 240*sqrt(a +
 b*x + c*x**2)*a*b**2*c**2*x + 960*sqrt(a + b*x + c*x**2)*a*b*c**3*x**2 +
640*sqrt(a + b*x + c*x**2)*a*c**4*x**3 + 3*sqrt(a + b*x + c*x**2)*b**5 - 1
0*sqrt(a + b*x + c*x**2)*b**4*c*x + 80*sqrt(a + b*x + c*x**2)*b**3*c**2*x*
*2 + 480*sqrt(a + b*x + c*x**2)*b**2*c**3*x**3 + 640*sqrt(a + b*x + c*x**2
)*b*c**4*x**4 + 256*sqrt(a + b*x + c*x**2)*c**5*x**5 - 256*sqrt(c)*a**3*c*
*2 - 768*sqrt(c)*a**2*b*c**2*x - 768*sqrt(c)*a**2*c**3*x**2 - 768*sqrt(c)*
a*b**2*c**2*x**2 - 1536*sqrt(c)*a*b*c**3*x**3 - 768*sqrt(c)*a*c**4*x**4 -
256*sqrt(c)*b**3*c**2*x**3 - 768*sqrt(c)*b**2*c**3*x**4 - 768*sqrt(c)*b*c*
*4*x**5 - 256*sqrt(c)*c**5*x**6))/(15*(64*a**6*c**3 - 48*a**5*b**2*c**2 +
192*a**5*b*c**3*x + 192*a**5*c**4*x**2 + 12*a**4*b**4*c - 144*a**4*b**3*c*
*2*x + 48*a**4*b**2*c**3*x**2 + 384*a**4*b*c**4*x**3 + 192*a**4*c**5*x**4
- a**3*b**6 + 36*a**3*b**5*c*x - 108*a**3*b**4*c**2*x**2 - 224*a**3*b**3*c
**3*x**3 + 48*a**3*b**2*c**4*x**4 + 192*a**3*b*c**5*x**5 + 64*a**3*c**6*x*
*6 - 3*a**2*b**7*x + 33*a**2*b**6*c*x**2 + 24*a**2*b**5*c**2*x**3 - 108*a*
*2*b**4*c**3*x**4 - 144*a**2*b**3*c**4*x**5 - 48*a**2*b**2*c**5*x**6 - 3*a
*b**8*x**2 + 6*a*b**7*c*x**3 + 33*a*b**6*c**2*x**4 + 36*a*b**5*c**3*x**5 +
 12*a*b**4*c**4*x**6 - b**9*x**3 - 3*b**8*c*x**4 - 3*b**7*c**2*x**5 - b**6
*c**3*x**6))
```

3.2 $\int \frac{(dx)^{5/2}}{(ax+bx^2+cx^3)^{5/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 129

$$\int \frac{(dx)^{5/2}}{(ax+bx^2+cx^3)^{5/2}}\,dx = \frac{2(dx)^{5/2}\left(b^2-2ac+bcx\right)}{3a\left(b^2-4ac\right)\left(ax+bx^2+cx^3\right)^{3/2}} - \frac{2(dx)^{3/2}\left(b(b^2-12ac)-16ac^2x\right)\left(ad+bdx+cdx^2\right)}{3a\left(b^2-4ac\right)^2\left(ax+bx^2+cx^3\right)^{3/2}}$$

output
```
2/3*(d*x)^(5/2)*(b*c*x-2*a*c+b^2)/a/(-4*a*c+b^2)/(c*x^3+b*x^2+a*x)^(3/2)-2
/3*(d*x)^(3/2)*(b*(-12*a*c+b^2)-16*a*c^2*x)*(c*d*x^2+b*d*x+a*d)/a/(-4*a*c+
b^2)^2/(c*x^3+b*x^2+a*x)^(3/2)
```

Mathematica [A] (verified)

Time = 0.93 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.52

$$\int \frac{(dx)^{5/2}}{(ax+bx^2+cx^3)^{5/2}}\,dx = \frac{2d(dx)^{3/2}(b+2cx)\left(-b^2+8bcx+4c(3a+2cx^2)\right)}{3\left(b^2-4ac\right)^2\left(x(a+x(b+cx))\right)^{3/2}}$$

input
```
Integrate[(d*x)^(5/2)/(a*x + b*x^2 + c*x^3)^(5/2),x]
```

output
```
(2*d*(d*x)^(3/2)*(b + 2*c*x)*(-b^2 + 8*b*c*x + 4*c*(3*a + 2*c*x^2)))/(3*(b
^2 - 4*a*c)^2*(x*(a + x*(b + c*x)))^(3/2))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.88, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2467, 30, 1089, 1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^{5/2}}{(ax+bx^2+cx^3)^{5/2}}dx$$

$$\Big\downarrow \text{2467}$$

$$\frac{\sqrt{x}\sqrt{a+bx+cx^2}\int \frac{(dx)^{5/2}}{x^{5/2}(cx^2+bx+a)^{5/2}}dx}{\sqrt{ax+bx^2+cx^3}}$$

$$\Big\downarrow \text{30}$$

$$\frac{d^2\sqrt{dx}\sqrt{a+bx+cx^2}\int \frac{1}{(cx^2+bx+a)^{5/2}}dx}{\sqrt{ax+bx^2+cx^3}}$$

$$\Big\downarrow \text{1089}$$

$$\frac{d^2\sqrt{dx}\sqrt{a+bx+cx^2}\left(-\frac{8c\int \frac{1}{(cx^2+bx+a)^{3/2}}dx}{3(b^2-4ac)}-\frac{2(b+2cx)}{3(b^2-4ac)(a+bx+cx^2)^{3/2}}\right)}{\sqrt{ax+bx^2+cx^3}}$$

$$\Big\downarrow \text{1088}$$

$$\frac{d^2\sqrt{dx}\sqrt{a+bx+cx^2}\left(\frac{16c(b+2cx)}{3(b^2-4ac)^2\sqrt{a+bx+cx^2}}-\frac{2(b+2cx)}{3(b^2-4ac)(a+bx+cx^2)^{3/2}}\right)}{\sqrt{ax+bx^2+cx^3}}$$

input
```
Int[(d*x)^(5/2)/(a*x + b*x^2 + c*x^3)^(5/2),x]
```

output
```
(d^2*Sqrt[d*x]*Sqrt[a + b*x + c*x^2]*((-2*(b + 2*c*x))/(3*(b^2 - 4*a*c)*(a
 + b*x + c*x^2)^(3/2)) + (16*c*(b + 2*c*x))/(3*(b^2 - 4*a*c)^2*Sqrt[a + b*
x + c*x^2])))/Sqrt[a*x + b*x^2 + c*x^3]
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

rule 1089
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 2467
```
Int[(Fx_.)*(Px_)^(p_), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Simp[Px^F
racPart[p]/(x^(r*FracPart[p])*ExpandToSum[Px/x^r, x]^FracPart[p])   Int[x^(
p*r)*ExpandToSum[Px/x^r, x]^p*Fx, x], x] /; IGtQ[r, 0]] /; FreeQ[p, x] && P
olyQ[Px, x] &&  !IntegerQ[p] &&  !MonomialQ[Px, x] &&  !PolyQ[Fx, x]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.75

method	result	siz
gosper	$\frac{2(c\,x^{2}+bx+a)\left(16c^{3}x^{3}+24b\,c^{2}x^{2}+24a\,c^{2}x+6b^{2}cx+12abc-b^{3}\right)(dx)^{\frac{5}{2}}}{3(16a^{2}c^{2}-8a\,b^{2}c+b^{4})(c\,x^{3}+b\,x^{2}+xa)^{\frac{5}{2}}}$	97
orering	$\frac{2(c\,x^{2}+bx+a)\left(16c^{3}x^{3}+24b\,c^{2}x^{2}+24a\,c^{2}x+6b^{2}cx+12abc-b^{3}\right)(dx)^{\frac{5}{2}}}{3(16a^{2}c^{2}-8a\,b^{2}c+b^{4})(c\,x^{3}+b\,x^{2}+xa)^{\frac{5}{2}}}$	97
default	$\frac{16d^{2}\sqrt{dx}\,\sqrt{x(c\,x^{2}+bx+a)}\,\left(16c^{3}x^{3}+24b\,c^{2}x^{2}+24a\,c^{2}x+6b^{2}cx+12abc-b^{3}\right)c^{2}\sqrt{-\frac{d\left(2cx+\sqrt{-4ac+b^{2}}+b\right)\left(-2cx+\sqrt{-4ac+b^{2}}-b\right)}{c}}}{3x\left(2cx+\sqrt{-4ac+b^{2}}+b\right)^{2}\left(b-\sqrt{-4ac+b^{2}}+2cx\right)^{2}(4ac-b^{2})^{2}\sqrt{d(c\,x^{2}+bx+a)}}$	17

input
```
int((d*x)^(5/2)/(c*x^3+b*x^2+a*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(c*x^2+b*x+a)*(16*c^3*x^3+24*b*c^2*x^2+24*a*c^2*x+6*b^2*c*x+12*a*b*c-b
^3)*(d*x)^(5/2)/(16*a^2*c^2-8*a*b^2*c+b^4)/(c*x^3+b*x^2+a*x)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.70

$$\int \frac{(dx)^{5/2}}{(ax+bx^2+cx^3)^{5/2}}\,dx = \frac{2\,(16\,c^3d^2x^3+24\,bc^2d^2x^2+6\,(b^2c+4\,ac^2)d^2}{3\,((b^4c^2-8\,ab^2c^3+16\,a^2c^4)x^5+2\,(b^5c-8\,ab^3c^2+16\,a^2bc^3)x^4+(b^6-6\,ab^4c}$$

input
```
integrate((d*x)^(5/2)/(c*x^3+b*x^2+a*x)^(5/2),x, algorithm="fricas")
```

output
```
2/3*(16*c^3*d^2*x^3 + 24*b*c^2*d^2*x^2 + 6*(b^2*c + 4*a*c^2)*d^2*x - (b^3
- 12*a*b*c)*d^2)*sqrt(c*x^3 + b*x^2 + a*x)*sqrt(d*x)/((b^4*c^2 - 8*a*b^2*c
^3 + 16*a^2*c^4)*x^5 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^4 + (b^6 -
 6*a*b^4*c + 32*a^3*c^3)*x^3 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x^2
+ (a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2)*x)
```

Sympy [F]

$$\int \frac{(dx)^{5/2}}{(ax+bx^2+cx^3)^{5/2}}\,dx = \int \frac{(dx)^{\frac{5}{2}}}{(x\,(a+bx+cx^2))^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x)**(5/2)/(c*x**3+b*x**2+a*x)**(5/2),x)
```

output
```
Integral((d*x)**(5/2)/(x*(a + b*x + c*x**2))**(5/2), x)
```

Maxima [F]

$$\int \frac{(dx)^{5/2}}{(ax+bx^2+cx^3)^{5/2}}\,dx = \int \frac{(dx)^{\frac{5}{2}}}{(cx^3+bx^2+ax)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x)^(5/2)/(c*x^3+b*x^2+a*x)^(5/2),x, algorithm="maxima")
```

output
```
integrate((d*x)^(5/2)/(c*x^3 + b*x^2 + a*x)^(5/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 269 vs. $2(118) = 236$.

Time = 0.18 (sec) , antiderivative size = 269, normalized size of antiderivative = 2.09

$$\int \frac{(dx)^{5/2}}{(ax+bx^2+cx^3)^{5/2}}\,dx = \frac{2}{3}\left(\frac{\left(2\left(4\left(\frac{2\,c^3dx}{b^4d^3-8\,ab^2cd^3+16\,a^2c^2d^3}+\frac{3\,bc^2d}{b^4d^3-8\,ab^2cd^3+16\,a^2c^2d^3}\right)dx+\frac{3\,(b^2cd^2+4\,ac^2d^2)}{b^4d^3-8\,ab^2cd^3+16\,a^2c}\right.}{(cd^3x^2+bd^3x+ad^3)^{\frac{3}{2}}}\right.$$

input
```
integrate((d*x)^(5/2)/(c*x^3+b*x^2+a*x)^(5/2),x, algorithm="giac")
```

output
```
2/3*((2*(4*(2*c^3*d*x/(b^4*d^3 - 8*a*b^2*c*d^3 + 16*a^2*c^2*d^3) + 3*b*c^2
*d/(b^4*d^3 - 8*a*b^2*c*d^3 + 16*a^2*c^2*d^3))*d*x + 3*(b^2*c*d^2 + 4*a*c^
2*d^2)/(b^4*d^3 - 8*a*b^2*c*d^3 + 16*a^2*c^2*d^3))*d*x - (b^3*d^3 - 12*a*b
*c*d^3)/(b^4*d^3 - 8*a*b^2*c*d^3 + 16*a^2*c^2*d^3))*d^6/(c*d^3*x^2 + b*d^3
*x + a*d^3)^(3/2) + (sqrt(a*d^3)*b^3 - 12*sqrt(a*d^3)*a*b*c)/(a^2*b^4 - 8*
a^3*b^2*c + 16*a^4*c^2))*d
```

Mupad [B] (verification not implemented)

Time = 21.51 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.46

$$\int \frac{(dx)^{5/2}}{(ax+bx^2+cx^3)^{5/2}}\,dx = \frac{\sqrt{c\,x^3+b\,x^2+a\,x}\,\left(\frac{16\,b\,d^2\,x^2\,\sqrt{d\,x}}{{\left(4\,a\,c-b^2\right)}^2}+\frac{32\,c\,d^2\,x^3\,\sqrt{d\,x}}{3\,{\left(4\,a\,c-b^2\right)}^2}+\frac{4\,d^2\,x\,\sqrt{d\,x}\,\left(b^2+4\,a\,c\right)}{c\,{\left(4\,a\,c-b^2\right)}^2}+\frac{2\,b\,d^2\,\sqrt{d\,x}\,(1\cdots}{3\,c^2\,(4\,a\,c\cdots}\right.}{x^5+\frac{x^3\,\left(b^2+2\,a\,c\right)}{c^2}+\frac{a^2\,x}{c^2}+\frac{2\,b\,x^4}{c}+\frac{2\,a\,b\,x^2}{c^2}}$$

input
```
int((d*x)^(5/2)/(a*x + b*x^2 + c*x^3)^(5/2),x)
```

output
```
((a*x + b*x^2 + c*x^3)^(1/2)*((16*b*d^2*x^2*(d*x)^(1/2))/(4*a*c - b^2)^2 +
 (32*c*d^2*x^3*(d*x)^(1/2))/(3*(4*a*c - b^2)^2) + (4*d^2*x*(d*x)^(1/2)*(4*
a*c + b^2))/(c*(4*a*c - b^2)^2) + (2*b*d^2*(d*x)^(1/2)*(12*a*c - b^2))/(3*
c^2*(4*a*c - b^2)^2)))/(x^5 + (x^3*(2*a*c + b^2))/c^2 + (a^2*x)/c^2 + (2*b
*x^4)/c + (2*a*b*x^2)/c^2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 325, normalized size of antiderivative = 2.52

$$\int \frac{(dx)^{5/2}}{(ax+bx^2+cx^3)^{5/2}}\,dx = \frac{2\sqrt{d}\,d^2\left(12\sqrt{c\,x^2+bx+a}\,abc+24\sqrt{c\,x^2+bx+a}\,a\,c^2x-\sqrt{c\,x^2+bx+a}\,b^3+\cdots\right.}{48a^2c^4x^4-24a\,b^2c^3x^4+3b^4c^2x^4+96a^2b\,c\cdots}$$

input
```
int((d*x)^(5/2)/(c*x^3+b*x^2+a*x)^(5/2),x)
```

output
```
(2*sqrt(d)*d**2*(12*sqrt(a + b*x + c*x**2)*a*b*c + 24*sqrt(a + b*x + c*x**
2)*a*c**2*x - sqrt(a + b*x + c*x**2)*b**3 + 6*sqrt(a + b*x + c*x**2)*b**2*
c*x + 24*sqrt(a + b*x + c*x**2)*b*c**2*x**2 + 16*sqrt(a + b*x + c*x**2)*c*
*3*x**3 - 16*sqrt(c)*a**2*c - 32*sqrt(c)*a*b*c*x - 32*sqrt(c)*a*c**2*x**2
- 16*sqrt(c)*b**2*c*x**2 - 32*sqrt(c)*b*c**2*x**3 - 16*sqrt(c)*c**3*x**4))
/(3*(16*a**4*c**2 - 8*a**3*b**2*c + 32*a**3*b*c**2*x + 32*a**3*c**3*x**2 +
 a**2*b**4 - 16*a**2*b**3*c*x + 32*a**2*b*c**3*x**3 + 16*a**2*c**4*x**4 +
2*a*b**5*x - 6*a*b**4*c*x**2 - 16*a*b**3*c**2*x**3 - 8*a*b**2*c**3*x**4 +
b**6*x**2 + 2*b**5*c*x**3 + b**4*c**2*x**4))
```

3.3 $\int \frac{(dx)^{3/2}}{(ax+bx^2+cx^3)^{3/2}} \, dx$

Optimal result

Integrand size = 26, antiderivative size = 44

$$\int \frac{(dx)^{3/2}}{(ax+bx^2+cx^3)^{3/2}} \, dx = -\frac{2d\sqrt{dx}(b+2cx)}{(b^2-4ac)\sqrt{ax+bx^2+cx^3}}$$

output
```
-2*d*(d*x)^(1/2)*(2*c*x+b)/(-4*a*c+b^2)/(c*x^3+b*x^2+a*x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.93

$$\int \frac{(dx)^{3/2}}{(ax+bx^2+cx^3)^{3/2}} \, dx = -\frac{2d\sqrt{dx}(b+2cx)}{(b^2-4ac)\sqrt{x(a+x(b+cx))}}$$

input
```
Integrate[(d*x)^(3/2)/(a*x + b*x^2 + c*x^3)^(3/2),x]
```

output
```
(-2*d*Sqrt[d*x]*(b + 2*c*x))/((b^2 - 4*a*c)*Sqrt[x*(a + x*(b + c*x))])
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {2467, 30, 1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^{3/2}}{(ax+bx^2+cx^3)^{3/2}}dx$$

$$\Big\downarrow \text{2467}$$

$$\frac{\sqrt{x}\sqrt{a+bx+cx^2}\int \frac{(dx)^{3/2}}{x^{3/2}(cx^2+bx+a)^{3/2}}dx}{\sqrt{ax+bx^2+cx^3}}$$

$$\Big\downarrow \text{30}$$

$$\frac{d\sqrt{dx}\sqrt{a+bx+cx^2}\int \frac{1}{(cx^2+bx+a)^{3/2}}dx}{\sqrt{ax+bx^2+cx^3}}$$

$$\Big\downarrow \text{1088}$$

$$-\frac{2d\sqrt{dx}(b+2cx)}{(b^2-4ac)\sqrt{ax+bx^2+cx^3}}$$

input
```
Int[(d*x)^(3/2)/(a*x + b*x^2 + c*x^3)^(3/2),x]
```

output
```
(-2*d*Sqrt[d*x]*(b + 2*c*x))/((b^2 - 4*a*c)*Sqrt[a*x + b*x^2 + c*x^3])
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

rule 2467
```
Int[(Fx_.)*(Px_)^(p_), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Simp[Px^F
racPart[p]/(x^(r*FracPart[p])*ExpandToSum[Px/x^r, x]^FracPart[p])   Int[x^(
p*r)*ExpandToSum[Px/x^r, x]^p*Fx, x], x] /; IGtQ[r, 0]] /; FreeQ[p, x] && P
olyQ[Px, x] &&  !IntegerQ[p] &&  !MonomialQ[Px, x] &&  !PolyQ[Fx, x]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.18

method	result	size
gosper	$\frac{2(c\,x^{2}+bx+a)(2cx+b)(dx)^{\frac{3}{2}}}{(4ac-b^{2})(c\,x^{3}+b\,x^{2}+xa)^{\frac{3}{2}}}$	52
orering	$\frac{2(c\,x^{2}+bx+a)(2cx+b)(dx)^{\frac{3}{2}}}{(4ac-b^{2})(c\,x^{3}+b\,x^{2}+xa)^{\frac{3}{2}}}$	52
default	$\frac{4d\sqrt{dx}\,\sqrt{x(c\,x^{2}+bx+a)}\,c\sqrt{-\frac{d\left(2cx+\sqrt{-4ac+b^{2}}+b\right)\left(-2cx+\sqrt{-4ac+b^{2}}-b\right)}{c}}\,(2cx+b)}{x\left(2cx+\sqrt{-4ac+b^{2}}+b\right)\left(b-\sqrt{-4ac+b^{2}}+2cx\right)\sqrt{d(c\,x^{2}+bx+a)}\,(4ac-b^{2})}$	139

input
```
int((d*x)^(3/2)/(c*x^3+b*x^2+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2*(c*x^2+b*x+a)*(2*c*x+b)*(d*x)^(3/2)/(4*a*c-b^2)/(c*x^3+b*x^2+a*x)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.77

$$\int \frac{(dx)^{3/2}}{(ax+bx^2+cx^3)^{3/2}}\,dx = -\frac{2\sqrt{cx^3+bx^2+ax}(2\,cdx+bd)\sqrt{dx}}{(b^2c-4\,ac^2)x^3+(b^3-4\,abc)x^2+(ab^2-4\,a^2c)x}$$

input
```
integrate((d*x)^(3/2)/(c*x^3+b*x^2+a*x)^(3/2),x, algorithm="fricas")
```

output
```
-2*sqrt(c*x^3 + b*x^2 + a*x)*(2*c*d*x + b*d)*sqrt(d*x)/((b^2*c - 4*a*c^2)*
x^3 + (b^3 - 4*a*b*c)*x^2 + (a*b^2 - 4*a^2*c)*x)
```

Sympy [F]

$$\int \frac{(dx)^{3/2}}{(ax+bx^2+cx^3)^{3/2}}\,dx = \int \frac{(dx)^{\frac{3}{2}}}{(x\,(a+bx+cx^2))^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x)**(3/2)/(c*x**3+b*x**2+a*x)**(3/2),x)
```

output
```
Integral((d*x)**(3/2)/(x*(a + b*x + c*x**2))**(3/2), x)
```

Maxima [F]

$$\int \frac{(dx)^{3/2}}{(ax+bx^2+cx^3)^{3/2}}\,dx = \int \frac{(dx)^{\frac{3}{2}}}{(cx^3+bx^2+ax)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x)^(3/2)/(c*x^3+b*x^2+a*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*x)^(3/2)/(c*x^3 + b*x^2 + a*x)^(3/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(40) = 80$.

Time = 0.15 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.16

$$\int \frac{(dx)^{3/2}}{(ax+bx^2+cx^3)^{3/2}}\,dx = -\frac{2\left(\frac{2\,cdx}{b^2d^2-4\,acd^2}+\frac{bd}{b^2d^2-4\,acd^2}\right)d^4}{\sqrt{cd^3x^2+bd^3x+ad^3}}+\frac{2\,\sqrt{ad^3}b}{ab^2-4\,a^2c}$$

input
```
integrate((d*x)^(3/2)/(c*x^3+b*x^2+a*x)^(3/2),x, algorithm="giac")
```

output
```
-2*(2*c*d*x/(b^2*d^2 - 4*a*c*d^2) + b*d/(b^2*d^2 - 4*a*c*d^2))*d^4/sqrt(c*
d^3*x^2 + b*d^3*x + a*d^3) + 2*sqrt(a*d^3)*b/(a*b^2 - 4*a^2*c)
```

Mupad [B] (verification not implemented)

Time = 21.46 (sec) , antiderivative size = 104, normalized size of antiderivative = 2.36

$$\int \frac{(dx)^{3/2}}{(ax+bx^2+cx^3)^{3/2}}\,dx = \frac{\left(\frac{4\,d\,x\,\sqrt{d\,x}}{4\,a\,c-b^2}+\frac{2\,b\,d\,\sqrt{d\,x}}{c\,(4\,a\,c-b^2)}\right)\,\sqrt{c\,x^3+b\,x^2+a\,x}}{x^3+\frac{a\,x}{c}-\frac{x^2\,(b^3-4\,a\,b\,c)}{c\,(4\,a\,c-b^2)}}$$

input
```
int((d*x)^(3/2)/(a*x + b*x^2 + c*x^3)^(3/2),x)
```

output
```
(((4*d*x*(d*x)^(1/2))/(4*a*c - b^2) + (2*b*d*(d*x)^(1/2))/(c*(4*a*c - b^2)
))*(a*x + b*x^2 + c*x^3)^(1/2))/(x^3 + (a*x)/c - (x^2*(b^3 - 4*a*b*c))/(c*
(4*a*c - b^2)))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 98, normalized size of antiderivative = 2.23

$$\int \frac{(dx)^{3/2}}{(ax+bx^2+cx^3)^{3/2}}\,dx = \frac{2\sqrt{d}\,d\left(\sqrt{c\,x^2+bx+a}\,b+2\sqrt{c\,x^2+bx+a}\,cx+2\sqrt{c}\,a+2\sqrt{c}\,bx+2\sqrt{c}\,c\,x^2\right)}{4a\,c^2x^2-b^2c\,x^2+4abcx-b^3x+4a^2c-a\,b^2}$$

input
```
int((d*x)^(3/2)/(c*x^3+b*x^2+a*x)^(3/2),x)
```

output
```
(2*sqrt(d)*d*(sqrt(a + b*x + c*x**2)*b + 2*sqrt(a + b*x + c*x**2)*c*x + 2*
sqrt(c)*a + 2*sqrt(c)*b*x + 2*sqrt(c)*c*x**2))/(4*a**2*c - a*b**2 + 4*a*b*
c*x + 4*a*c**2*x**2 - b**3*x - b**2*c*x**2)
```

3.4 $\int \frac{\sqrt{dx}}{\sqrt{ax+bx^2+cx^3}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 57

$$\int \frac{\sqrt{dx}}{\sqrt{ax+bx^2+cx^3}}\,dx=\frac{\sqrt{d}\operatorname{arctanh}\left(\frac{\sqrt{dx}(b+2cx)}{2\sqrt{c}\sqrt{d}\sqrt{ax+bx^2+cx^3}}\right)}{\sqrt{c}}$$

output
```
d^(1/2)*arctanh(1/2*(d*x)^(1/2)*(2*c*x+b)/c^(1/2)/d^(1/2)/(c*x^3+b*x^2+a*x
)^(1/2))/c^(1/2)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.25

$$\int \frac{\sqrt{dx}}{\sqrt{ax+bx^2+cx^3}}\,dx=-\frac{\sqrt{dx}\sqrt{a+bx+cx^2}\log\left(b+2cx-2\sqrt{c}\sqrt{a+bx+cx^2}\right)}{\sqrt{c}\sqrt{x(a+x(b+cx))}}$$

input
```
Integrate[Sqrt[d*x]/Sqrt[a*x + b*x^2 + c*x^3],x]
```

output
```
-((Sqrt[d*x]*Sqrt[a + b*x + c*x^2]*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + b*x
+ c*x^2]])/(Sqrt[c]*Sqrt[x*(a + x*(b + c*x))]))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.32, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2467, 30, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{dx}}{\sqrt{ax+bx^2+cx^3}}\,dx$$

$$\Big\downarrow \text{2467}$$

$$\frac{\sqrt{x}\sqrt{a+bx+cx^2}\int \frac{\sqrt{dx}}{\sqrt{x}\sqrt{cx^2+bx+a}}dx}{\sqrt{ax+bx^2+cx^3}}$$

$$\Big\downarrow \text{30}$$

$$\frac{\sqrt{dx}\sqrt{a+bx+cx^2}\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{\sqrt{ax+bx^2+cx^3}}$$

$$\Big\downarrow \text{1092}$$

$$\frac{2\sqrt{dx}\sqrt{a+bx+cx^2}\int \frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{\sqrt{ax+bx^2+cx^3}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\sqrt{dx}\sqrt{a+bx+cx^2}\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}\sqrt{ax+bx^2+cx^3}}$$

input
```
Int[Sqrt[d*x]/Sqrt[a*x + b*x^2 + c*x^3],x]
```

output
```
(Sqrt[d*x]*Sqrt[a + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b
*x + c*x^2])])/(Sqrt[c]*Sqrt[a*x + b*x^2 + c*x^3])
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 2467
```
Int[(Fx_.)*(Px_)^(p_), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Simp[Px^F
racPart[p]/(x^(r*FracPart[p])*ExpandToSum[Px/x^r, x]^FracPart[p])   Int[x^(
p*r)*ExpandToSum[Px/x^r, x]^p*Fx, x], x] /; IGtQ[r, 0]] /; FreeQ[p, x] && P
olyQ[Px, x] &&  !IntegerQ[p] &&  !MonomialQ[Px, x] &&  !PolyQ[Fx, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 108 vs. $2(43) = 86$.

Time = 0.18 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.91

method	result	size
default	$\frac{\sqrt{dx}\,\sqrt{x(c\,x^{2}+bx+a)}\,d\ln\left(\frac{2xcd+\sqrt{-\frac{d\left(2cx+\sqrt{-4ac+b^{2}}+b\right)\left(-2cx+\sqrt{-4ac+b^{2}}-b\right)}{c}}\,\sqrt{cd}+bd}{2\sqrt{cd}}\right)}{x\sqrt{d(c\,x^{2}+bx+a)}\,\sqrt{cd}}$	109

input
```
int((d*x)^(1/2)/(c*x^3+b*x^2+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(d*x)^(1/2)*(x*(c*x^2+b*x+a))^(1/2)*d*ln(1/2*(2*x*c*d+(-1/c*d*(2*c*x+(-4*a
*c+b^2)^(1/2)+b)*(-2*c*x+(-4*a*c+b^2)^(1/2)-b))^(1/2)*(c*d)^(1/2)+b*d)/(c*
d)^(1/2))/x/(d*(c*x^2+b*x+a))^(1/2)/(c*d)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 141, normalized size of antiderivative = 2.47

$$\int \frac{\sqrt{dx}}{\sqrt{ax+bx^2+cx^3}}\,dx$$
$$= \left[\frac{1}{2}\sqrt{\frac{d}{c}}\log\left(\frac{8\,c^2dx^3+8\,bcdx^2+(b^2+4\,ac)dx+4\,\sqrt{cx^3+bx^2+ax}(2\,c^2x+bc)\sqrt{dx}\sqrt{\frac{d}{c}}}{x}\right),\right.$$
$$\left.-\sqrt{-\frac{d}{c}}\arctan\left(\frac{2\,\sqrt{cx^3+bx^2+ax}\sqrt{dx}c\sqrt{-\frac{d}{c}}}{2\,cdx^2+bdx}\right)\right]$$

input
```
integrate((d*x)^(1/2)/(c*x^3+b*x^2+a*x)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*sqrt(d/c)*log((8*c^2*d*x^3 + 8*b*c*d*x^2 + (b^2 + 4*a*c)*d*x + 4*sqrt
(c*x^3 + b*x^2 + a*x)*(2*c^2*x + b*c)*sqrt(d*x)*sqrt(d/c))/x), -sqrt(-d/c)
*arctan(2*sqrt(c*x^3 + b*x^2 + a*x)*sqrt(d*x)*c*sqrt(-d/c)/(2*c*d*x^2 + b*
d*x))]
```

Sympy [F]

$$\int \frac{\sqrt{dx}}{\sqrt{ax+bx^2+cx^3}}\,dx = \int \frac{\sqrt{dx}}{\sqrt{x\,(a+bx+cx^2)}}\,dx$$

input
```
integrate((d*x)**(1/2)/(c*x**3+b*x**2+a*x)**(1/2),x)
```

output
```
Integral(sqrt(d*x)/sqrt(x*(a + b*x + c*x**2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{dx}}{\sqrt{ax+bx^2+cx^3}}\,dx=\int \frac{\sqrt{dx}}{\sqrt{cx^3+bx^2+ax}}\,dx$$

input
```
integrate((d*x)^(1/2)/(c*x^3+b*x^2+a*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x)/sqrt(c*x^3 + b*x^2 + a*x), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 105 vs. $2(43)=86$.

Time = 0.16 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.84

$$\int \frac{\sqrt{dx}}{\sqrt{ax+bx^2+cx^3}}\,dx = -\frac{d^3\left(\frac{\sqrt{cd}\log\left(\left|-\sqrt{cd}bd-2\left(\sqrt{cd}dx-\sqrt{cd^3x^2+bd^3x+ad^3}\right)c\right|\right)}{cd}-\frac{\sqrt{cd}\log\left(\left|-\sqrt{cd}bd+2\sqrt{ad^3}c\right|\right)}{cd}\right)}{|d|^2}$$

input
```
integrate((d*x)^(1/2)/(c*x^3+b*x^2+a*x)^(1/2),x, algorithm="giac")
```

output
```
-d^3*(sqrt(c*d)*log(abs(-sqrt(c*d)*b*d - 2*(sqrt(c*d)*d*x - sqrt(c*d^3*x^2
 + b*d^3*x + a*d^3))*c))/(c*d) - sqrt(c*d)*log(abs(-sqrt(c*d)*b*d + 2*sqrt
(a*d^3)*c))/(c*d))/abs(d)^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{dx}}{\sqrt{ax+bx^2+cx^3}}\,dx = \int \frac{\sqrt{d\,x}}{\sqrt{c\,x^3+b\,x^2+a\,x}}\,dx$$

input
```
int((d*x)^(1/2)/(a*x + b*x^2 + c*x^3)^(1/2),x)
```

output
```
int((d*x)^(1/2)/(a*x + b*x^2 + c*x^3)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.77

$$\int \frac{\sqrt{dx}}{\sqrt{ax+bx^2+cx^3}}\,dx = \frac{\sqrt{d}\,\sqrt{c}\log\left(\frac{2\sqrt{c}\,\sqrt{c\,x^2+bx+a}+b+2cx}{\sqrt{4ac-b^2}}\right)}{c}$$

input
```
int((d*x)^(1/2)/(c*x^3+b*x^2+a*x)^(1/2),x)
```

output
```
(sqrt(d)*sqrt(c)*log((2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)/sqrt(4
*a*c - b**2)))/c
```

3.5 $\int \frac{\sqrt{ax+bx^2+cx^3}}{\sqrt{dx}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 107

$$\int \frac{\sqrt{ax+bx^2+cx^3}}{\sqrt{dx}}\,dx=\frac{(b+2cx)\sqrt{ax+bx^2+cx^3}}{4c\sqrt{dx}}-\frac{(b^2-4ac)\,\text{arctanh}\left(\frac{\sqrt{dx}(b+2cx)}{2\sqrt{c}\sqrt{d}\sqrt{ax+bx^2+cx^3}}\right)}{8c^{3/2}\sqrt{d}}$$

output
```
1/4*(2*c*x+b)*(c*x^3+b*x^2+a*x)^(1/2)/c/(d*x)^(1/2)-1/8*(-4*a*c+b^2)*arcta
nh(1/2*(d*x)^(1/2)*(2*c*x+b)/c^(1/2)/d^(1/2)/(c*x^3+b*x^2+a*x)^(1/2))/c^(3
/2)/d^(1/2)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{ax+bx^2+cx^3}}{\sqrt{dx}}\,dx=\frac{\sqrt{x(a+x(b+cx))}\left(\sqrt{c}(b+2cx)+\frac{(b^2-4ac)\text{arctanh}\left(\frac{\sqrt{cx}}{\sqrt{a}-\sqrt{a+x(b+cx)}}\right)}{\sqrt{a+x(b+cx)}}\right)}{4c^{3/2}\sqrt{dx}}$$

input
```
Integrate[Sqrt[a*x + b*x^2 + c*x^3]/Sqrt[d*x],x]
```

output
```
(Sqrt[x*(a + x*(b + c*x))]*(Sqrt[c]*(b + 2*c*x) + ((b^2 - 4*a*c)*ArcTanh[(
Sqrt[c]*x)/(Sqrt[a] - Sqrt[a + x*(b + c*x)])])/Sqrt[a + x*(b + c*x)]))/(4*
c^(3/2)*Sqrt[d*x])
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {2467, 30, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax+bx^2+cx^3}}{\sqrt{dx}}dx$$

$$\Big\downarrow \text{ 2467}$$

$$\frac{\sqrt{ax+bx^2+cx^3}\int \frac{\sqrt{x}\sqrt{cx^2+bx+a}}{\sqrt{dx}}dx}{\sqrt{x}\sqrt{a+bx+cx^2}}$$

$$\Big\downarrow \text{ 30}$$

$$\frac{\sqrt{ax+bx^2+cx^3}\int \sqrt{cx^2+bx+a}dx}{\sqrt{dx}\sqrt{a+bx+cx^2}}$$

$$\Big\downarrow \text{ 1087}$$

$$\frac{\sqrt{ax+bx^2+cx^3}\left(\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4c}-\frac{(b^2-4ac)\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{8c}\right)}{\sqrt{dx}\sqrt{a+bx+cx^2}}$$

$$\Big\downarrow \text{ 1092}$$

$$\frac{\sqrt{ax+bx^2+cx^3}\left(\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4c}-\frac{(b^2-4ac)\int \frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{4c}\right)}{\sqrt{dx}\sqrt{a+bx+cx^2}}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\sqrt{ax+bx^2+cx^3}\left(\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4c}-\frac{(b^2-4ac)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{3/2}}\right)}{\sqrt{dx}\sqrt{a+bx+cx^2}}$$

input
```
Int[Sqrt[a*x + b*x^2 + c*x^3]/Sqrt[d*x],x]
```

output
```
(Sqrt[a*x + b*x^2 + c*x^3]*(((b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(4*c) - ((
b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(8*c^
(3/2))))/(Sqrt[d*x]*Sqrt[a + b*x + c*x^2])
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 2467
```
Int[(Fx_.)*(Px_)^(p_), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Simp[Px^F
racPart[p]/(x^(r*FracPart[p])*ExpandToSum[Px/x^r, x]^FracPart[p])   Int[x^(
p*r)*ExpandToSum[Px/x^r, x]^p*Fx, x], x] /; IGtQ[r, 0]] /; FreeQ[p, x] && P
olyQ[Px, x] &&  !IntegerQ[p] &&  !MonomialQ[Px, x] &&  !PolyQ[Fx, x]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.21

method	result
risch	$\frac{(2cx+b)(c\,x^2+bx+a)x}{4c\sqrt{dx}\,\sqrt{x(c\,x^2+bx+a)}}+\frac{(4ac-b^2)\ln\left(\frac{\frac{1}{2}bd+xcd}{\sqrt{cd}}+\sqrt{cd\,x^2+bdx+ad}\right)\sqrt{d(c\,x^2+bx+a)}\,x}{8c\sqrt{cd}\,\sqrt{dx}\,\sqrt{x(c\,x^2+bx+a)}}$
default	$\frac{\sqrt{x(c\,x^2+bx+a)}\left(4\ln\left(\frac{2xcd+2\sqrt{d\left(c\,x^2+bx+a\right)}\,\sqrt{cd}+bd}{2\sqrt{cd}}\right)acd-\ln\left(\frac{2xcd+2\sqrt{d\left(c\,x^2+bx+a\right)}\,\sqrt{cd}+bd}{2\sqrt{cd}}\right)b^2d+4\sqrt{cd}\,\sqrt{d(c\,x^2+bx+a)}\,cx+\right.}{8\sqrt{dx}\,\sqrt{d(c\,x^2+bx+a)}\,c\sqrt{cd}}$

input
```
int((c*x^3+b*x^2+a*x)^(1/2)/(d*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*(2*c*x+b)*(c*x^2+b*x+a)/c/(d*x)^(1/2)/(x*(c*x^2+b*x+a))^(1/2)*x+1/8*(4
*a*c-b^2)/c*ln((1/2*b*d+x*c*d)/(c*d)^(1/2)+(c*d*x^2+b*d*x+a*d)^(1/2))/(c*d
)^(1/2)*(d*(c*x^2+b*x+a))^(1/2)/(d*x)^(1/2)/(x*(c*x^2+b*x+a))^(1/2)*x
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 249, normalized size of antiderivative = 2.33

$$\int \frac{\sqrt{ax+bx^2+cx^3}}{\sqrt{dx}}\,dx$$
$$=\left[-\frac{(b^2-4\,ac)\sqrt{cd}x\log\left(\frac{8\,c^2dx^3+8\,bcdx^2+(b^2+4\,ac)dx+4\,\sqrt{cx^3+bx^2+ax}\sqrt{cd}(2\,cx+b)\sqrt{dx}}{x}\right)-4\,\sqrt{cx^3+bx^2+ax}(2\,c^2x+\right.}{16\,c^2dx}$$

input
```
integrate((c*x^3+b*x^2+a*x)^(1/2)/(d*x)^(1/2),x, algorithm="fricas")
```

output
```
[-1/16*((b^2 - 4*a*c)*sqrt(c*d)*x*log((8*c^2*d*x^3 + 8*b*c*d*x^2 + (b^2 +
4*a*c)*d*x + 4*sqrt(c*x^3 + b*x^2 + a*x)*sqrt(c*d)*(2*c*x + b)*sqrt(d*x))/
x) - 4*sqrt(c*x^3 + b*x^2 + a*x)*(2*c^2*x + b*c)*sqrt(d*x))/(c^2*d*x), 1/8
*((b^2 - 4*a*c)*sqrt(-c*d)*x*arctan(1/2*sqrt(c*x^3 + b*x^2 + a*x)*sqrt(-c*
d)*(2*c*x + b)*sqrt(d*x)/(c^2*d*x^3 + b*c*d*x^2 + a*c*d*x)) + 2*sqrt(c*x^3
 + b*x^2 + a*x)*(2*c^2*x + b*c)*sqrt(d*x))/(c^2*d*x)]
```

Sympy [F]

$$\int \frac{\sqrt{ax+bx^2+cx^3}}{\sqrt{dx}}\,dx = \int \frac{\sqrt{x\,(a+bx+cx^2)}}{\sqrt{dx}}\,dx$$

input
```
integrate((c*x**3+b*x**2+a*x)**(1/2)/(d*x)**(1/2),x)
```

output
```
Integral(sqrt(x*(a + b*x + c*x**2))/sqrt(d*x), x)
```

Maxima [F]

$$\int \frac{\sqrt{ax+bx^2+cx^3}}{\sqrt{dx}}\,dx = \int \frac{\sqrt{cx^3+bx^2+ax}}{\sqrt{dx}}\,dx$$

input
```
integrate((c*x^3+b*x^2+a*x)^(1/2)/(d*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^3 + b*x^2 + a*x)/sqrt(d*x), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 205 vs. $2(85) = 170$.

Time = 0.18 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.92

$$\int \frac{\sqrt{ax+bx^2+cx^3}}{\sqrt{dx}}\,dx = \frac{\left(2\sqrt{cd^3x^2+bd^3x+ad^3}\left(2\,dx+\frac{bd}{c}\right)+\frac{(b^2d^3-4\,acd^3)\log\left(\left|-bd^2-2\left(\sqrt{cd}dx-\sqrt{cd^3x^2+bd^3x+ad^3}\right)\sqrt{cd}\right|\right)}{\sqrt{cd}c}-\frac{b^2d^3\log\left(\left|-bd^2+2\right.\right.}{}\right.}{8\,d^5}$$

input
```
integrate((c*x^3+b*x^2+a*x)^(1/2)/(d*x)^(1/2),x, algorithm="giac")
```

output
```
1/8*(2*sqrt(c*d^3*x^2 + b*d^3*x + a*d^3)*(2*d*x + b*d/c) + (b^2*d^3 - 4*a*
c*d^3)*log(abs(-b*d^2 - 2*(sqrt(c*d)*d*x - sqrt(c*d^3*x^2 + b*d^3*x + a*d^
3))*sqrt(c*d)))/(sqrt(c*d)*c) - (b^2*d^3*log(abs(-b*d^2 + 2*sqrt(a*d^3)*sq
rt(c*d))) - 4*a*c*d^3*log(abs(-b*d^2 + 2*sqrt(a*d^3)*sqrt(c*d))) + 2*sqrt(
a*d^3)*sqrt(c*d)*b*d)/(sqrt(c*d)*c))*abs(d)^2/d^5
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ax+bx^2+cx^3}}{\sqrt{dx}}\,dx = \int \frac{\sqrt{c\,x^3+b\,x^2+a\,x}}{\sqrt{d\,x}}\,dx$$

input
```
int((a*x + b*x^2 + c*x^3)^(1/2)/(d*x)^(1/2),x)
```

output
```
int((a*x + b*x^2 + c*x^3)^(1/2)/(d*x)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.20

$$\int \frac{\sqrt{ax+bx^2+cx^3}}{\sqrt{dx}}\,dx$$
$$= \frac{\sqrt{d}\left(2\sqrt{c\,x^2+bx+a}\,bc+4\sqrt{c\,x^2+bx+a}\,c^2x+4\sqrt{c}\log\left(\frac{2\sqrt{c}\,\sqrt{c\,x^2+bx+a}+b+2cx}{\sqrt{4ac-b^2}}\right)ac-\sqrt{c}\log\left(\frac{2\sqrt{c}\,\sqrt{c\,x^2+bx}}{\sqrt{4ac-}}\right.\right.}{8c^2d}$$

input
```
int((c*x^3+b*x^2+a*x)^(1/2)/(d*x)^(1/2),x)
```

output
```
(sqrt(d)*(2*sqrt(a + b*x + c*x**2)*b*c + 4*sqrt(a + b*x + c*x**2)*c**2*x +
 4*sqrt(c)*log((2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)/sqrt(4*a*c -
 b**2))*a*c - sqrt(c)*log((2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)/s
qrt(4*a*c - b**2))*b**2))/(8*c**2*d)
```

3.6 $\int \frac{\left(ax+bx^2+cx^3\right)^{3/2}}{(dx)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 158

$$\int \frac{\left(ax+bx^2+cx^3\right)^{3/2}}{(dx)^{3/2}}\,dx = -\frac{3(b^2-4ac)\,(b+2cx)\sqrt{ax+bx^2+cx^3}}{64c^2d\sqrt{dx}} + \frac{(b+2cx)\left(ax+bx^2+cx^3\right)^{3/2}}{8c(dx)^{3/2}} + \frac{3(b^2-4ac)^2\operatorname{arctanh}\left(\frac{\sqrt{dx}(b+2cx)}{2\sqrt{c}\sqrt{d}\sqrt{ax+bx^2+cx^3}}\right)}{128c^{5/2}d^{3/2}}$$

output
```
-3/64*(-4*a*c+b^2)*(2*c*x+b)*(c*x^3+b*x^2+a*x)^(1/2)/c^2/d/(d*x)^(1/2)+1/8
*(2*c*x+b)*(c*x^3+b*x^2+a*x)^(3/2)/c/(d*x)^(3/2)+3/128*(-4*a*c+b^2)^2*arct
anh(1/2*(d*x)^(1/2)*(2*c*x+b)/c^(1/2)/d^(1/2)/(c*x^3+b*x^2+a*x)^(1/2))/c^(
5/2)/d^(3/2)
```

Mathematica [A] (verified)

Time = 1.33 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.85

$$\int \frac{\left(ax+bx^2+cx^3\right)^{3/2}}{(dx)^{3/2}}\,dx = \frac{(x(a+x(b+cx)))^{3/2}\left(\frac{\sqrt{c}(b+2cx)\left(-3b^2+8bcx+4c(5a+2cx^2)\right)}{a+x(b+cx)} + \frac{3(b^2-4ac)^2\operatorname{arctanh}\left(\frac{\quad}{-\sqrt{a}\ldots}\right.}{(a+x(b+cx))^{3/\ldots}}\right.}{64c^{5/2}(dx)^{3/2}}$$

input
```
Integrate[(a*x + b*x^2 + c*x^3)^(3/2)/(d*x)^(3/2),x]
```

output
```
((x*(a + x*(b + c*x)))^(3/2)*((Sqrt[c]*(b + 2*c*x)*(-3*b^2 + 8*b*c*x + 4*c
*(5*a + 2*c*x^2)))/(a + x*(b + c*x)) + (3*(b^2 - 4*a*c)^2*ArcTanh[(Sqrt[c]
*x)/(-Sqrt[a] + Sqrt[a + x*(b + c*x)])])/(a + x*(b + c*x))^(3/2)))/(64*c^(
5/2)*(d*x)^(3/2))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2467, 30, 1087, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(ax+bx^2+cx^3\right)^{3/2}}{(dx)^{3/2}}dx$$

$$\Big\downarrow \text{2467}$$

$$\frac{\sqrt{ax+bx^2+cx^3}\int \frac{x^{3/2}\left(cx^2+bx+a\right)^{3/2}}{(dx)^{3/2}}dx}{\sqrt{x}\sqrt{a+bx+cx^2}}$$

$$\Big\downarrow \text{30}$$

$$\frac{\sqrt{ax+bx^2+cx^3}\int \left(cx^2+bx+a\right)^{3/2}dx}{d\sqrt{dx}\sqrt{a+bx+cx^2}}$$

$$\Big\downarrow \text{1087}$$

$$\frac{\sqrt{ax+bx^2+cx^3}\left(\frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\int \sqrt{cx^2+bx+a}dx}{16c}\right)}{d\sqrt{dx}\sqrt{a+bx+cx^2}}$$

$$\Big\downarrow \text{1087}$$

$$\frac{\sqrt{ax+bx^2+cx^3}\left(\frac{(b+2cx)(a+bx+cx^2)^{3/2}}{8c}-\frac{3(b^2-4ac)\left(\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4c}-\frac{(b^2-4ac)\int\frac{1}{\sqrt{cx^2+bx+a}}dx}{8c}\right)}{16c}\right)}{d\sqrt{dx}\sqrt{a+bx+cx^2}}$$

$\Big\downarrow$ 1092

$$\frac{\sqrt{ax+bx^2+cx^3}\left(\frac{(b+2cx)(a+bx+cx^2)^{3/2}}{8c}-\frac{3(b^2-4ac)\left(\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4c}-\frac{(b^2-4ac)\int\frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{4c}\right)}{16c}\right)}{d\sqrt{dx}\sqrt{a+bx+cx^2}}$$

$\Big\downarrow$ 219

$$\frac{\sqrt{ax+bx^2+cx^3}\left(\frac{(b+2cx)(a+bx+cx^2)^{3/2}}{8c}-\frac{3(b^2-4ac)\left(\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4c}-\frac{(b^2-4ac)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{3/2}}\right)}{16c}\right)}{d\sqrt{dx}\sqrt{a+bx+cx^2}}$$

input
```
Int[(a*x + b*x^2 + c*x^3)^(3/2)/(d*x)^(3/2),x]
```

output
```
(Sqrt[a*x + b*x^2 + c*x^3]*(((b + 2*c*x)*(a + b*x + c*x^2)^(3/2))/(8*c) -
(3*(b^2 - 4*a*c)*(((b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(4*c) - ((b^2 - 4*a*
c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(8*c^(3/2))))/(
16*c)))/(d*Sqrt[d*x]*Sqrt[a + b*x + c*x^2])
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 2467
```
Int[(Fx_.)*(Px_)^(p_), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Simp[Px^F
racPart[p]/(x^(r*FracPart[p])*ExpandToSum[Px/x^r, x]^FracPart[p])   Int[x^(
p*r)*ExpandToSum[Px/x^r, x]^p*Fx, x], x] /; IGtQ[r, 0]] /; FreeQ[p, x] && P
olyQ[Px, x] &&  !IntegerQ[p] &&  !MonomialQ[Px, x] &&  !PolyQ[Fx, x]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.15

method	result
risch	$\frac{\left(16c^3x^3+24b\,c^2x^2+40a\,c^2x+2b^2cx+20abc-3b^3\right)\left(c\,x^2+bx+a\right)x}{64c^2d\sqrt{dx}\,\sqrt{x(c\,x^2+bx+a)}}+\frac{3\left(16a^2c^2-8a\,b^2c+b^4\right)\ln\left(\frac{\frac{1}{2}bd+xcd}{\sqrt{cd}}+\sqrt{cd\,x^2+bdx+ad}\right)\sqrt{d(c}}{128c^2\sqrt{cd}\,d\sqrt{dx}\,\sqrt{x(c\,x^2+bx+a)}}$
default	$\frac{\sqrt{x(c\,x^2+bx+a)}\left(32c^3x^3\sqrt{cd}\,\sqrt{d(c\,x^2+bx+a)}+48b\,c^2x^2\sqrt{cd}\,\sqrt{d(c\,x^2+bx+a)}+48\ln\left(\frac{2xcd+2\sqrt{d\left(c\,x^2+bx+a\right)}\,\sqrt{cd}+bd}{2\sqrt{cd}}\right)a^2c^2d-24}{}$

input
```
int((c*x^3+b*x^2+a*x)^(3/2)/(d*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/64*(16*c^3*x^3+24*b*c^2*x^2+40*a*c^2*x+2*b^2*c*x+20*a*b*c-3*b^3)*(c*x^2+
b*x+a)/c^2/d/(d*x)^(1/2)/(x*(c*x^2+b*x+a))^(1/2)*x+3/128*(16*a^2*c^2-8*a*b
^2*c+b^4)/c^2*ln((1/2*b*d+x*c*d)/(c*d)^(1/2)+(c*d*x^2+b*d*x+a*d)^(1/2))/(c
*d)^(1/2)/d*(d*(c*x^2+b*x+a))^(1/2)/(d*x)^(1/2)/(x*(c*x^2+b*x+a))^(1/2)*x
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 349, normalized size of antiderivative = 2.21

$$\int \frac{(ax+bx^2+cx^3)^{3/2}}{(dx)^{3/2}}\,dx = \left[\frac{3\,(b^4-8\,ab^2c+16\,a^2c^2)\sqrt{cd}x\log\left(\frac{8\,c^2dx^3+8\,bcdx^2+(b^2+4\,ac)dx+4\,\sqrt{cx^3+bx^2+ax}\sqrt{cd}(2}{x}\right.}{}\right.$$

input
```
integrate((c*x^3+b*x^2+a*x)^(3/2)/(d*x)^(3/2),x, algorithm="fricas")
```

output
```
[1/256*(3*(b^4 - 8*a*b^2*c + 16*a^2*c^2)*sqrt(c*d)*x*log((8*c^2*d*x^3 + 8*
b*c*d*x^2 + (b^2 + 4*a*c)*d*x + 4*sqrt(c*x^3 + b*x^2 + a*x)*sqrt(c*d)*(2*c
*x + b)*sqrt(d*x))/x) + 4*(16*c^4*x^3 + 24*b*c^3*x^2 - 3*b^3*c + 20*a*b*c^
2 + 2*(b^2*c^2 + 20*a*c^3)*x)*sqrt(c*x^3 + b*x^2 + a*x)*sqrt(d*x))/(c^3*d^
2*x), -1/128*(3*(b^4 - 8*a*b^2*c + 16*a^2*c^2)*sqrt(-c*d)*x*arctan(1/2*sqr
t(c*x^3 + b*x^2 + a*x)*sqrt(-c*d)*(2*c*x + b)*sqrt(d*x)/(c^2*d*x^3 + b*c*d
*x^2 + a*c*d*x)) - 2*(16*c^4*x^3 + 24*b*c^3*x^2 - 3*b^3*c + 20*a*b*c^2 + 2
*(b^2*c^2 + 20*a*c^3)*x)*sqrt(c*x^3 + b*x^2 + a*x)*sqrt(d*x))/(c^3*d^2*x)]
```

Sympy [F]

$$\int \frac{(ax+bx^2+cx^3)^{3/2}}{(dx)^{3/2}}\,dx = \int \frac{(x(a+bx+cx^2))^{\frac{3}{2}}}{(dx)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*x**3+b*x**2+a*x)**(3/2)/(d*x)**(3/2),x)
```

output
```
Integral((x*(a + b*x + c*x**2))**(3/2)/(d*x)**(3/2), x)
```

Maxima [F]

$$\int \frac{(ax + bx^2 + cx^3)^{3/2}}{(dx)^{3/2}}\,dx = \int \frac{(cx^3 + bx^2 + ax)^{\frac{3}{2}}}{(dx)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*x^3+b*x^2+a*x)^(3/2)/(d*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*x^3 + b*x^2 + a*x)^(3/2)/(d*x)^(3/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 808 vs. $2(130) = 260$.

Time = 0.32 (sec) , antiderivative size = 808, normalized size of antiderivative = 5.11

$$\int \frac{(ax + bx^2 + cx^3)^{3/2}}{(dx)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((c*x^3+b*x^2+a*x)^(3/2)/(d*x)^(3/2),x, algorithm="giac")
```

output
```
1/384*(48*(2*sqrt(c*d^3*x^2 + b*d^3*x + a*d^3)*(2*d*x + b*d/c) + (b^2*d^3
- 4*a*c*d^3)*log(abs(-b*d^2 - 2*(sqrt(c*d)*d*x - sqrt(c*d^3*x^2 + b*d^3*x
+ a*d^3))*sqrt(c*d)))/(sqrt(c*d)*c) - (b^2*d^3*log(abs(-b*d^2 + 2*sqrt(a*d
^3)*sqrt(c*d))) - 4*a*c*d^3*log(abs(-b*d^2 + 2*sqrt(a*d^3)*sqrt(c*d))) + 2
*sqrt(a*d^3)*sqrt(c*d)*b*d)/(sqrt(c*d)*c))*a*abs(d)^2/d^4 + 8*(2*sqrt(c*d^
3*x^2 + b*d^3*x + a*d^3)*(2*(4*d*x + b*d/c)*d*x - (3*b^2*d^2 - 8*a*c*d^2)/
c^2) - 3*(b^3*d^4 - 4*a*b*c*d^4)*log(abs(-b*d^2 - 2*(sqrt(c*d)*d*x - sqrt(
c*d^3*x^2 + b*d^3*x + a*d^3))*sqrt(c*d)))/(sqrt(c*d)*c^2) + (3*b^3*d^4*log
(abs(-b*d^2 + 2*sqrt(a*d^3)*sqrt(c*d))) - 12*a*b*c*d^4*log(abs(-b*d^2 + 2*
sqrt(a*d^3)*sqrt(c*d))) + 6*sqrt(a*d^3)*sqrt(c*d)*b^2*d^2 - 16*sqrt(a*d^3)
*sqrt(c*d)*a*c*d^2)/(sqrt(c*d)*c^2))*b*abs(d)^2/d^5 + (2*sqrt(c*d^3*x^2 +
b*d^3*x + a*d^3)*(2*(4*(6*d*x + b*d/c)*d*x - (5*b^2*c*d^2 - 12*a*c^2*d^2)/
c^3)*d*x + (15*b^3*d^3 - 52*a*b*c*d^3)/c^3) + 3*(5*b^4*d^5 - 24*a*b^2*c*d^
5 + 16*a^2*c^2*d^5)*log(abs(-b*d^2 - 2*(sqrt(c*d)*d*x - sqrt(c*d^3*x^2 + b
*d^3*x + a*d^3))*sqrt(c*d)))/(sqrt(c*d)*c^3) - (15*b^4*d^5*log(abs(-b*d^2
+ 2*sqrt(a*d^3)*sqrt(c*d))) - 72*a*b^2*c*d^5*log(abs(-b*d^2 + 2*sqrt(a*d^3
)*sqrt(c*d))) + 48*a^2*c^2*d^5*log(abs(-b*d^2 + 2*sqrt(a*d^3)*sqrt(c*d)))
+ 30*sqrt(a*d^3)*sqrt(c*d)*b^3*d^3 - 104*sqrt(a*d^3)*sqrt(c*d)*a*b*c*d^3)/
(sqrt(c*d)*c^3))*c*abs(d)^2/d^6)/d^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ax+bx^2+cx^3)^{3/2}}{(dx)^{3/2}}\,dx=\int \frac{(c\,x^3+b\,x^2+a\,x)^{3/2}}{(d\,x)^{3/2}}\,dx$$

input
```
int((a*x + b*x^2 + c*x^3)^(3/2)/(d*x)^(3/2),x)
```

output
```
int((a*x + b*x^2 + c*x^3)^(3/2)/(d*x)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.63

$$\int \frac{(ax+bx^2+cx^3)^{3/2}}{(dx)^{3/2}}\,dx = \frac{\sqrt{d}\left(40\sqrt{c\,x^2+bx+a}\,ab\,c^2+80\sqrt{c\,x^2+bx+a}\,a\,c^3x-6\sqrt{c\,x^2+bx+a}\,b^3c+\right.}{}$$

input
```
int((c*x^3+b*x^2+a*x)^(3/2)/(d*x)^(3/2),x)
```

output
```
(sqrt(d)*(40*sqrt(a + b*x + c*x**2)*a*b*c**2 + 80*sqrt(a + b*x + c*x**2)*a
*c**3*x - 6*sqrt(a + b*x + c*x**2)*b**3*c + 4*sqrt(a + b*x + c*x**2)*b**2*
c**2*x + 48*sqrt(a + b*x + c*x**2)*b*c**3*x**2 + 32*sqrt(a + b*x + c*x**2)
*c**4*x**3 + 48*sqrt(c)*log((2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)
/sqrt(4*a*c - b**2))*a**2*c**2 - 24*sqrt(c)*log((2*sqrt(c)*sqrt(a + b*x +
c*x**2) + b + 2*c*x)/sqrt(4*a*c - b**2))*a*b**2*c + 3*sqrt(c)*log((2*sqrt(
c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)/sqrt(4*a*c - b**2))*b**4))/(128*c**
3*d**2)
```

3.7 $\int x^2(ax^2+bx^3+cx^4)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 25

$$\int x^2\left(ax^2+bx^3+cx^4\right)\,dx=\frac{ax^5}{5}+\frac{bx^6}{6}+\frac{cx^7}{7}$$

output
```
1/5*a*x^5+1/6*b*x^6+1/7*c*x^7
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int x^2\left(ax^2+bx^3+cx^4\right)\,dx=\frac{ax^5}{5}+\frac{bx^6}{6}+\frac{cx^7}{7}$$

input
```
Integrate[x^2*(a*x^2 + b*x^3 + c*x^4),x]
```

output
```
(a*x^5)/5 + (b*x^6)/6 + (c*x^7)/7
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {9, 1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a x^2+b x^3+c x^4\right) \, dx$$

$$\Big\downarrow \text{ 9}$$

$$\int x^4 \left(a+b x+c x^2\right) dx$$

$$\Big\downarrow \text{ 1140}$$

$$\int \left(a x^4+b x^5+c x^6\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{a x^5}{5}+\frac{b x^6}{6}+\frac{c x^7}{7}$$

input
```
Int[x^2*(a*x^2 + b*x^3 + c*x^4),x]
```

output
```
(a*x^5)/5 + (b*x^6)/6 + (c*x^7)/7
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
 !MonomialQ[Px, x]
```

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

method	result	size
gosper	$\frac{x^{5}\left(30 c \,x^{2}+35 b x +42 a \right)}{210}$	20
default	$\frac{1}{5} a \,x^{5}+\frac{1}{6} b \,x^{6}+\frac{1}{7} c \,x^{7}$	20
norman	$\frac{1}{5} a \,x^{5}+\frac{1}{6} b \,x^{6}+\frac{1}{7} c \,x^{7}$	20
risch	$\frac{1}{5} a \,x^{5}+\frac{1}{6} b \,x^{6}+\frac{1}{7} c \,x^{7}$	20
parallelrisch	$\frac{1}{5} a \,x^{5}+\frac{1}{6} b \,x^{6}+\frac{1}{7} c \,x^{7}$	20
orering	$\frac{x^{3}\left(30 c \,x^{2}+35 b x +42 a \right) \left(c \,x^{4}+b \,x^{3}+a \,x^{2}\right)}{210 c \,x^{2}+210 b x +210 a}$	48

input
```
int(x^2*(c*x^4+b*x^3+a*x^2),x,method=_RETURNVERBOSE)
```

output
```
1/210*x^5*(30*c*x^2+35*b*x+42*a)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int x^2 \left(ax^2 + bx^3 + cx^4\right) \, dx = \frac{1}{7} \, cx^7 + \frac{1}{6} \, bx^6 + \frac{1}{5} \, ax^5$$

input
```
integrate(x^2*(c*x^4+b*x^3+a*x^2),x, algorithm="fricas")
```

output
```
1/7*c*x^7 + 1/6*b*x^6 + 1/5*a*x^5
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int x^2(ax^2+bx^3+cx^4)\ dx=\frac{ax^5}{5}+\frac{bx^6}{6}+\frac{cx^7}{7}$$

input
```
integrate(x**2*(c*x**4+b*x**3+a*x**2),x)
```

output
```
a*x**5/5 + b*x**6/6 + c*x**7/7
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int x^2(ax^2+bx^3+cx^4)\ dx=\frac{1}{7}\,cx^7+\frac{1}{6}\,bx^6+\frac{1}{5}\,ax^5$$

input
```
integrate(x^2*(c*x^4+b*x^3+a*x^2),x, algorithm="maxima")
```

output
```
1/7*c*x^7 + 1/6*b*x^6 + 1/5*a*x^5
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int x^2(ax^2+bx^3+cx^4)\ dx=\frac{1}{7}\,cx^7+\frac{1}{6}\,bx^6+\frac{1}{5}\,ax^5$$

input
```
integrate(x^2*(c*x^4+b*x^3+a*x^2),x, algorithm="giac")
```

output
```
1/7*c*x^7 + 1/6*b*x^6 + 1/5*a*x^5
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int x^2 \left(ax^2+bx^3+cx^4\right) \, dx = \frac{x^5 \left(30\, c\, x^2+35\, b\, x+42\, a\right)}{210}$$

input
```
int(x^2*(a*x^2 + b*x^3 + c*x^4),x)
```

output
```
(x^5*(42*a + 35*b*x + 30*c*x^2))/210
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int x^2 \left(ax^2+bx^3+cx^4\right) \, dx = \frac{x^5 (30c\, x^2+35bx+42a)}{210}$$

input
```
int(x^2*(c*x^4+b*x^3+a*x^2),x)
```

output
```
(x**5*(42*a + 35*b*x + 30*c*x**2))/210
```

3.8 $\int x(ax^2+bx^3+cx^4)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 25

$$\int x\left(ax^2+bx^3+cx^4\right)\,dx=\frac{ax^4}{4}+\frac{bx^5}{5}+\frac{cx^6}{6}$$

output
```
1/4*a*x^4+1/5*b*x^5+1/6*c*x^6
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int x\left(ax^2+bx^3+cx^4\right)\,dx=\frac{ax^4}{4}+\frac{bx^5}{5}+\frac{cx^6}{6}$$

input
```
Integrate[x*(a*x^2 + b*x^3 + c*x^4),x]
```

output
```
(a*x^4)/4 + (b*x^5)/5 + (c*x^6)/6
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {9, 1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(ax^2+bx^3+cx^4\right)\,dx$$

$$\Big\downarrow \ 9$$

$$\int x^3\left(a+bx+cx^2\right)dx$$

$$\Big\downarrow \ 1140$$

$$\int \left(ax^3+bx^4+cx^5\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{ax^4}{4}+\frac{bx^5}{5}+\frac{cx^6}{6}$$

input
```
Int[x*(a*x^2 + b*x^3 + c*x^4),x]
```

output
```
(a*x^4)/4 + (b*x^5)/5 + (c*x^6)/6
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

method	result	size
gosper	$\frac{x^{4} \left(10 c \,x^{2}+12 b x +15 a \right)}{60}$	20
default	$\frac{1}{4} a \,x^{4}+\frac{1}{5} b \,x^{5}+\frac{1}{6} c \,x^{6}$	20
norman	$\frac{1}{4} a \,x^{4}+\frac{1}{5} b \,x^{5}+\frac{1}{6} c \,x^{6}$	20
risch	$\frac{1}{4} a \,x^{4}+\frac{1}{5} b \,x^{5}+\frac{1}{6} c \,x^{6}$	20
parallelrisch	$\frac{1}{4} a \,x^{4}+\frac{1}{5} b \,x^{5}+\frac{1}{6} c \,x^{6}$	20
orering	$\frac{x^{2} \left(10 c \,x^{2}+12 b x +15 a \right) \left(c \,x^{4}+b \,x^{3}+a \,x^{2}\right)}{60 c \,x^{2}+60 b x +60 a}$	48

input
```
int(x*(c*x^4+b*x^3+a*x^2),x,method=_RETURNVERBOSE)
```

output
```
1/60*x^4*(10*c*x^2+12*b*x+15*a)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int x\left(ax^2+bx^3+cx^4\right)\,dx=\frac{1}{6}\,cx^6+\frac{1}{5}\,bx^5+\frac{1}{4}\,ax^4$$

input
```
integrate(x*(c*x^4+b*x^3+a*x^2),x, algorithm="fricas")
```

output
```
1/6*c*x^6 + 1/5*b*x^5 + 1/4*a*x^4
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int x(ax^2 + bx^3 + cx^4)\ dx = \frac{ax^4}{4} + \frac{bx^5}{5} + \frac{cx^6}{6}$$

input
```
integrate(x*(c*x**4+b*x**3+a*x**2),x)
```

output
```
a*x**4/4 + b*x**5/5 + c*x**6/6
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int x(ax^2 + bx^3 + cx^4)\ dx = \frac{1}{6}\,cx^6 + \frac{1}{5}\,bx^5 + \frac{1}{4}\,ax^4$$

input
```
integrate(x*(c*x^4+b*x^3+a*x^2),x, algorithm="maxima")
```

output
```
1/6*c*x^6 + 1/5*b*x^5 + 1/4*a*x^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int x(ax^2 + bx^3 + cx^4)\ dx = \frac{1}{6}\,cx^6 + \frac{1}{5}\,bx^5 + \frac{1}{4}\,ax^4$$

input
```
integrate(x*(c*x^4+b*x^3+a*x^2),x, algorithm="giac")
```

output
```
1/6*c*x^6 + 1/5*b*x^5 + 1/4*a*x^4
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int x\left(ax^2+bx^3+cx^4\right)\,dx=\frac{x^4\,\left(10\,c\,x^2+12\,b\,x+15\,a\right)}{60}$$

input
```
int(x*(a*x^2 + b*x^3 + c*x^4),x)
```

output
```
(x^4*(15*a + 12*b*x + 10*c*x^2))/60
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int x\left(ax^2+bx^3+cx^4\right)\,dx=\frac{x^4(10c\,x^2+12bx+15a)}{60}$$

input
```
int(x*(c*x^4+b*x^3+a*x^2),x)
```

output
```
(x**4*(15*a + 12*b*x + 10*c*x**2))/60
```

3.9 $\int (ax^2 + bx^3 + cx^4) \, dx$

Optimal result

Integrand size = 16, antiderivative size = 25

$$\int (ax^2 + bx^3 + cx^4) \, dx = \frac{ax^3}{3} + \frac{bx^4}{4} + \frac{cx^5}{5}$$

output
```
1/3*a*x^3+1/4*b*x^4+1/5*c*x^5
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (ax^2 + bx^3 + cx^4) \, dx = \frac{ax^3}{3} + \frac{bx^4}{4} + \frac{cx^5}{5}$$

input
```
Integrate[a*x^2 + b*x^3 + c*x^4,x]
```

output
```
(a*x^3)/3 + (b*x^4)/4 + (c*x^5)/5
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(ax^2+bx^3+cx^4\right) \, dx$$

$$\Big\downarrow \; 2009$$

$$\frac{ax^3}{3}+\frac{bx^4}{4}+\frac{cx^5}{5}$$

input
```
Int[a*x^2 + b*x^3 + c*x^4,x]
```

output
```
(a*x^3)/3 + (b*x^4)/4 + (c*x^5)/5
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

method	result	size
gosper	$\frac{x^{3}\left(12c\,x^{2}+15bx+20a\right)}{60}$	20
default	$\frac{1}{3}a\,x^{3}+\frac{1}{4}b\,x^{4}+\frac{1}{5}c\,x^{5}$	20
norman	$\frac{1}{3}a\,x^{3}+\frac{1}{4}b\,x^{4}+\frac{1}{5}c\,x^{5}$	20
risch	$\frac{1}{3}a\,x^{3}+\frac{1}{4}b\,x^{4}+\frac{1}{5}c\,x^{5}$	20
parallelrisch	$\frac{1}{3}a\,x^{3}+\frac{1}{4}b\,x^{4}+\frac{1}{5}c\,x^{5}$	20
parts	$\frac{1}{3}a\,x^{3}+\frac{1}{4}b\,x^{4}+\frac{1}{5}c\,x^{5}$	20
orering	$\frac{x\left(12c\,x^{2}+15bx+20a\right)\left(c\,x^{4}+b\,x^{3}+a\,x^{2}\right)}{60c\,x^{2}+60bx+60a}$	46

input
```
int(c*x^4+b*x^3+a*x^2,x,method=_RETURNVERBOSE)
```

output
```
1/60*x^3*(12*c*x^2+15*b*x+20*a)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \left(ax^2+bx^3+cx^4\right)\,dx=\frac{1}{5}\,cx^5+\frac{1}{4}\,bx^4+\frac{1}{3}\,ax^3$$

input
```
integrate(c*x^4+b*x^3+a*x^2,x, algorithm="fricas")
```

output
```
1/5*c*x^5 + 1/4*b*x^4 + 1/3*a*x^3
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \left(ax^2+bx^3+cx^4\right)\,dx=\frac{ax^3}{3}+\frac{bx^4}{4}+\frac{cx^5}{5}$$

input
```
integrate(c*x**4+b*x**3+a*x**2,x)
```

output
```
a*x**3/3 + b*x**4/4 + c*x**5/5
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \left(ax^2+bx^3+cx^4\right)\,dx=\frac{1}{5}\,cx^5+\frac{1}{4}\,bx^4+\frac{1}{3}\,ax^3$$

input
```
integrate(c*x^4+b*x^3+a*x^2,x, algorithm="maxima")
```

output
```
1/5*c*x^5 + 1/4*b*x^4 + 1/3*a*x^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \left(ax^2+bx^3+cx^4\right)\,dx=\frac{1}{5}\,cx^5+\frac{1}{4}\,bx^4+\frac{1}{3}\,ax^3$$

input
```
integrate(c*x^4+b*x^3+a*x^2,x, algorithm="giac")
```

output
```
1/5*c*x^5 + 1/4*b*x^4 + 1/3*a*x^3
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \left(ax^2+bx^3+cx^4\right)\,dx=\frac{x^3\,\left(12\,c\,x^2+15\,b\,x+20\,a\right)}{60}$$

input
```
int(a*x^2 + b*x^3 + c*x^4,x)
```

output
```
(x^3*(20*a + 15*b*x + 12*c*x^2))/60
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \left(ax^2+bx^3+cx^4\right)\,dx=\frac{x^3(12c\,x^2+15bx+20a)}{60}$$

input
```
int(c*x^4+b*x^3+a*x^2,x)
```

output
```
(x**3*(20*a + 15*b*x + 12*c*x**2))/60
```

3.10 $\int \frac{ax^2+bx^3+cx^4}{x} dx$

Optimal result

Integrand size = 20, antiderivative size = 25

$$\int \frac{ax^2+bx^3+cx^4}{x} dx = \frac{ax^2}{2} + \frac{bx^3}{3} + \frac{cx^4}{4}$$

output
```
1/2*a*x^2+1/3*b*x^3+1/4*c*x^4
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{ax^2+bx^3+cx^4}{x} dx = \frac{ax^2}{2} + \frac{bx^3}{3} + \frac{cx^4}{4}$$

input
```
Integrate[(a*x^2 + b*x^3 + c*x^4)/x,x]
```

output
```
(a*x^2)/2 + (b*x^3)/3 + (c*x^4)/4
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {9, 1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ax^2+bx^3+cx^4}{x} \, dx$$

$$\Big\downarrow \text{ 9}$$

$$\int x\left(a+bx+cx^2\right) dx$$

$$\Big\downarrow \text{ 1140}$$

$$\int \left(ax+bx^2+cx^3\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{ax^2}{2}+\frac{bx^3}{3}+\frac{cx^4}{4}$$

input
```
Int[(a*x^2 + b*x^3 + c*x^4)/x,x]
```

output
```
(a*x^2)/2 + (b*x^3)/3 + (c*x^4)/4
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

method	result	size
gosper	$\frac{x^{2}\left(3 c\,x^{2}+4 b x +6 a \right)}{12}$	20
default	$\frac{1}{2} a\,x^{2}+\frac{1}{3} b\,x^{3}+\frac{1}{4} c\,x^{4}$	20
norman	$\frac{1}{2} a\,x^{2}+\frac{1}{3} b\,x^{3}+\frac{1}{4} c\,x^{4}$	20
risch	$\frac{1}{2} a\,x^{2}+\frac{1}{3} b\,x^{3}+\frac{1}{4} c\,x^{4}$	20
parallelrisch	$\frac{1}{2} a\,x^{2}+\frac{1}{3} b\,x^{3}+\frac{1}{4} c\,x^{4}$	20
orering	$\frac{\left(3 c\,x^{2}+4 b x +6 a \right) \left(c\,x^{4}+b\,x^{3}+a\,x^{2}\right)}{12 c\,x^{2}+12 b x +12 a}$	45

input
```
int((c*x^4+b*x^3+a*x^2)/x,x,method=_RETURNVERBOSE)
```

output
```
1/12*x^2*(3*c*x^2+4*b*x+6*a)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{ax^2 + bx^3 + cx^4}{x}\, dx = \frac{1}{4}\, cx^4 + \frac{1}{3}\, bx^3 + \frac{1}{2}\, ax^2$$

input
```
integrate((c*x^4+b*x^3+a*x^2)/x,x, algorithm="fricas")
```

output
```
1/4*c*x^4 + 1/3*b*x^3 + 1/2*a*x^2
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{ax^2+bx^3+cx^4}{x}\,dx=\frac{ax^2}{2}+\frac{bx^3}{3}+\frac{cx^4}{4}$$

input
```
integrate((c*x**4+b*x**3+a*x**2)/x,x)
```

output
```
a*x**2/2 + b*x**3/3 + c*x**4/4
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{ax^2+bx^3+cx^4}{x}\,dx=\frac{1}{4}\,cx^4+\frac{1}{3}\,bx^3+\frac{1}{2}\,ax^2$$

input
```
integrate((c*x^4+b*x^3+a*x^2)/x,x, algorithm="maxima")
```

output
```
1/4*c*x^4 + 1/3*b*x^3 + 1/2*a*x^2
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{ax^2+bx^3+cx^4}{x}\,dx=\frac{1}{4}\,cx^4+\frac{1}{3}\,bx^3+\frac{1}{2}\,ax^2$$

input
```
integrate((c*x^4+b*x^3+a*x^2)/x,x, algorithm="giac")
```

output
```
1/4*c*x^4 + 1/3*b*x^3 + 1/2*a*x^2
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{ax^2 + bx^3 + cx^4}{x} \, dx = \frac{x^2 \left(3\,c\,x^2 + 4\,b\,x + 6\,a\right)}{12}$$

input
```
int((a*x^2 + b*x^3 + c*x^4)/x,x)
```

output
```
(x^2*(6*a + 4*b*x + 3*c*x^2))/12
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{ax^2 + bx^3 + cx^4}{x} \, dx = \frac{x^2(3c\,x^2 + 4bx + 6a)}{12}$$

input
```
int((c*x^4+b*x^3+a*x^2)/x,x)
```

output
```
(x**2*(6*a + 4*b*x + 3*c*x**2))/12
```

3.11 $\int \frac{ax^2+bx^3+cx^4}{x^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{ax^2+bx^3+cx^4}{x^2}\,dx = ax+\frac{bx^2}{2}+\frac{cx^3}{3}$$

output
```
a*x+1/2*b*x^2+1/3*c*x^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{ax^2+bx^3+cx^4}{x^2}\,dx = ax+\frac{bx^2}{2}+\frac{cx^3}{3}$$

input
```
Integrate[(a*x^2 + b*x^3 + c*x^4)/x^2,x]
```

output
```
a*x + (b*x^2)/2 + (c*x^3)/3
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {9, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ax^2+bx^3+cx^4}{x^2} \, dx$$

$$\Big\downarrow \; 9$$

$$\int \left(a+bx+cx^2\right) dx$$

$$\Big\downarrow \; 2009$$

$$ax+\frac{bx^2}{2}+\frac{cx^3}{3}$$

input
```
Int[(a*x^2 + b*x^3 + c*x^4)/x^2,x]
```

output
```
a*x + (b*x^2)/2 + (c*x^3)/3
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

method	result	size
default	$\frac{1}{3}c\,x^{3}+\frac{1}{2}b\,x^{2}+xa$	17
risch	$\frac{1}{3}c\,x^{3}+\frac{1}{2}b\,x^{2}+xa$	17
parallelrisch	$\frac{1}{3}c\,x^{3}+\frac{1}{2}b\,x^{2}+xa$	17
parts	$\frac{1}{3}c\,x^{3}+\frac{1}{2}b\,x^{2}+xa$	17
gosper	$\frac{x\left(2c\,x^{2}+3bx+6a\right)}{6}$	18
norman	$\frac{a\,x^{2}+\frac{1}{2}b\,x^{3}+\frac{1}{3}c\,x^{4}}{x}$	23
orering	$\frac{\left(2c\,x^{2}+3bx+6a\right)\left(c\,x^{4}+b\,x^{3}+a\,x^{2}\right)}{6x\left(c\,x^{2}+bx+a\right)}$	48

input
```
int((c*x^4+b*x^3+a*x^2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*c*x^3+1/2*b*x^2+x*a
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \frac{ax^2+bx^3+cx^4}{x^2}\,dx=\frac{1}{3}\,cx^3+\frac{1}{2}\,bx^2+ax$$

input
```
integrate((c*x^4+b*x^3+a*x^2)/x^2,x, algorithm="fricas")
```

output
```
1/3*c*x^3 + 1/2*b*x^2 + a*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.75

$$\int \frac{ax^2+bx^3+cx^4}{x^2}\,dx = ax + \frac{bx^2}{2} + \frac{cx^3}{3}$$

input
```
integrate((c*x**4+b*x**3+a*x**2)/x**2,x)
```

output
```
a*x + b*x**2/2 + c*x**3/3
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \frac{ax^2+bx^3+cx^4}{x^2}\,dx = \frac{1}{3}\,cx^3 + \frac{1}{2}\,bx^2 + ax$$

input
```
integrate((c*x^4+b*x^3+a*x^2)/x^2,x, algorithm="maxima")
```

output
```
1/3*c*x^3 + 1/2*b*x^2 + a*x
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \frac{ax^2+bx^3+cx^4}{x^2}\,dx = \frac{1}{3}\,cx^3 + \frac{1}{2}\,bx^2 + ax$$

input
```
integrate((c*x^4+b*x^3+a*x^2)/x^2,x, algorithm="giac")
```

output
```
1/3*c*x^3 + 1/2*b*x^2 + a*x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \frac{ax^2 + bx^3 + cx^4}{x^2}\,dx = \frac{c\,x^3}{3} + \frac{b\,x^2}{2} + a\,x$$

input
```
int((a*x^2 + b*x^3 + c*x^4)/x^2,x)
```

output
```
a*x + (b*x^2)/2 + (c*x^3)/3
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{ax^2 + bx^3 + cx^4}{x^2}\,dx = \frac{x(2c\,x^2 + 3bx + 6a)}{6}$$

input
```
int((c*x^4+b*x^3+a*x^2)/x^2,x)
```

output
```
(x*(6*a + 3*b*x + 2*c*x**2))/6
```

3.12 $\int \frac{ax^2+bx^3+cx^4}{x^3} dx$

Optimal result

Integrand size = 20, antiderivative size = 16

$$\int \frac{ax^2+bx^3+cx^4}{x^3} dx = bx + \frac{cx^2}{2} + a\log(x)$$

output
```
b*x+1/2*c*x^2+a*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{ax^2+bx^3+cx^4}{x^3} dx = bx + \frac{cx^2}{2} + a\log(x)$$

input
```
Integrate[(a*x^2 + b*x^3 + c*x^4)/x^3,x]
```

output
```
b*x + (c*x^2)/2 + a*Log[x]
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {9, 1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ax^2+bx^3+cx^4}{x^3}\,dx$$

$$\Big\downarrow \; 9$$

$$\int \frac{a+bx+cx^2}{x}dx$$

$$\Big\downarrow \; 1140$$

$$\int \left(\frac{a}{x}+b+cx\right)dx$$

$$\Big\downarrow \; 2009$$

$$a\log(x)+bx+\frac{cx^2}{2}$$

input
```
Int[(a*x^2 + b*x^3 + c*x^4)/x^3,x]
```

output
```
b*x + (c*x^2)/2 + a*Log[x]
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result	size
default	$bx+\frac{c\,x^{2}}{2}+a\ln\left(x\right)$	15
risch	$bx+\frac{c\,x^{2}}{2}+a\ln\left(x\right)$	15
parallelrisch	$bx+\frac{c\,x^{2}}{2}+a\ln\left(x\right)$	15
norman	$\frac{b\,x^{3}+\frac{1}{2}c\,x^{4}}{x^{2}}+a\ln\left(x\right)$	22

input
```
int((c*x^4+b*x^3+a*x^2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
b*x+1/2*c*x^2+a*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{ax^2+bx^3+cx^4}{x^3}\,dx=\frac{1}{2}\,cx^2+bx+a\log\left(x\right)$$

input
```
integrate((c*x^4+b*x^3+a*x^2)/x^3,x, algorithm="fricas")
```

output
```
1/2*c*x^2 + b*x + a*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{ax^2 + bx^3 + cx^4}{x^3} \, dx = a \log(x) + bx + \frac{cx^2}{2}$$

input
```
integrate((c*x**4+b*x**3+a*x**2)/x**3,x)
```

output
```
a*log(x) + b*x + c*x**2/2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{ax^2 + bx^3 + cx^4}{x^3} \, dx = \frac{1}{2} cx^2 + bx + a \log(x)$$

input
```
integrate((c*x^4+b*x^3+a*x^2)/x^3,x, algorithm="maxima")
```

output
```
1/2*c*x^2 + b*x + a*log(x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{ax^2 + bx^3 + cx^4}{x^3} \, dx = \frac{1}{2} cx^2 + bx + a \log(|x|)$$

input
```
integrate((c*x^4+b*x^3+a*x^2)/x^3,x, algorithm="giac")
```

output
```
1/2*c*x^2 + b*x + a*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{ax^2 + bx^3 + cx^4}{x^3} \, dx = b\,x + \frac{c\,x^2}{2} + a\,\ln{(x)}$$

input
```
int((a*x^2 + b*x^3 + c*x^4)/x^3,x)
```

output
```
b*x + (c*x^2)/2 + a*log(x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{ax^2 + bx^3 + cx^4}{x^3} \, dx = \log(x)\,a + bx + \frac{c\,x^2}{2}$$

input
```
int((c*x^4+b*x^3+a*x^2)/x^3,x)
```

output
```
(2*log(x)*a + 2*b*x + c*x**2)/2
```

3.13 $\int \frac{ax^2+bx^3+cx^4}{x^4} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 14

$$\int \frac{ax^2+bx^3+cx^4}{x^4} \, dx = -\frac{a}{x}+cx+b\log(x)$$

output
```
-a/x+c*x+b*ln(x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{ax^2+bx^3+cx^4}{x^4} \, dx = -\frac{a}{x}+cx+b\log(x)$$

input
```
Integrate[(a*x^2 + b*x^3 + c*x^4)/x^4,x]
```

output
```
-(a/x) + c*x + b*Log[x]
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {9, 1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ax^2 + bx^3 + cx^4}{x^4} dx$$

$$\Big\downarrow \ 9$$

$$\int \frac{a + bx + cx^2}{x^2} dx$$

$$\Big\downarrow \ 1140$$

$$\int \left(\frac{a}{x^2} + \frac{b}{x} + c\right) dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{a}{x} + b\log(x) + cx$$

input
```
Int[(a*x^2 + b*x^3 + c*x^4)/x^4,x]
```

output
```
-(a/x) + c*x + b*Log[x]
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.05 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

method	result	size
default	$-\frac{a}{x}+cx+b\ln\left(x\right)$	15
risch	$-\frac{a}{x}+cx+b\ln\left(x\right)$	15
parallelrisch	$\frac{b\ln(x)x+c\,x^{2}-a}{x}$	19
norman	$\frac{c\,x^{4}-a\,x^{2}}{x^{3}}+b\ln\left(x\right)$	22

input
```
int((c*x^4+b*x^3+a*x^2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-a/x+c*x+b*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.29

$$\int \frac{ax^2+bx^3+cx^4}{x^4}\,dx=\frac{cx^2+bx\log\left(x\right)-a}{x}$$

input
```
integrate((c*x^4+b*x^3+a*x^2)/x^4,x, algorithm="fricas")
```

output
```
(c*x^2 + b*x*log(x) - a)/x
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{ax^2 + bx^3 + cx^4}{x^4} \, dx = -\frac{a}{x} + b\log(x) + cx$$

input
```
integrate((c*x**4+b*x**3+a*x**2)/x**4,x)
```

output
```
-a/x + b*log(x) + c*x
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{ax^2 + bx^3 + cx^4}{x^4} \, dx = cx + b\log(x) - \frac{a}{x}$$

input
```
integrate((c*x^4+b*x^3+a*x^2)/x^4,x, algorithm="maxima")
```

output
```
c*x + b*log(x) - a/x
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int \frac{ax^2 + bx^3 + cx^4}{x^4} \, dx = cx + b\log(|x|) - \frac{a}{x}$$

input
```
integrate((c*x^4+b*x^3+a*x^2)/x^4,x, algorithm="giac")
```

output
```
c*x + b*log(abs(x)) - a/x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{ax^2 + bx^3 + cx^4}{x^4}\, dx = c\, x - \frac{a}{x} + b\, \ln{(x)}$$

input
```
int((a*x^2 + b*x^3 + c*x^4)/x^4,x)
```

output
```
c*x - a/x + b*log(x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.29

$$\int \frac{ax^2 + bx^3 + cx^4}{x^4}\, dx = \frac{\log(x)\, bx - a + c\, x^2}{x}$$

input
```
int((c*x^4+b*x^3+a*x^2)/x^4,x)
```

output
```
(log(x)*b*x - a + c*x**2)/x
```

3.14 $\int \frac{ax^2+bx^3+cx^4}{x^5}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 19

$$\int \frac{ax^2+bx^3+cx^4}{x^5}\,dx=-\frac{a}{2x^2}-\frac{b}{x}+c\log(x)$$

output
```
-1/2*a/x^2-b/x+c*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{ax^2+bx^3+cx^4}{x^5}\,dx=-\frac{a}{2x^2}-\frac{b}{x}+c\log(x)$$

input
```
Integrate[(a*x^2 + b*x^3 + c*x^4)/x^5,x]
```

output
```
-1/2*a/x^2 - b/x + c*Log[x]
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {9, 1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ax^2+bx^3+cx^4}{x^5}dx$$

$$\Big\downarrow \text{9}$$

$$\int \frac{a+bx+cx^2}{x^3}dx$$

$$\Big\downarrow \text{1140}$$

$$\int \left(\frac{a}{x^3}+\frac{b}{x^2}+\frac{c}{x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a}{2x^2}-\frac{b}{x}+c\log(x)$$

input
```
Int[(a*x^2 + b*x^3 + c*x^4)/x^5,x]
```

output
```
-1/2*a/x^2 - b/x + c*Log[x]
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.04 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

method	result	size
default	$-\frac{a}{2x^2}-\frac{b}{x}+c\ln\left(x\right)$	18
risch	$\frac{-bx-\frac{a}{2}}{x^2}+c\ln\left(x\right)$	18
parallelrisch	$\frac{2c\ln(x)x^2-2bx-a}{2x^2}$	22
norman	$\frac{-\frac{1}{2}a\,x^2-b\,x^3}{x^4}+c\ln\left(x\right)$	23

input
```
int((c*x^4+b*x^3+a*x^2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a/x^2-b/x+c*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int\frac{ax^2+bx^3+cx^4}{x^5}\,dx=\frac{2\,cx^2\log\left(x\right)-2\,bx-a}{2\,x^2}$$

input
```
integrate((c*x^4+b*x^3+a*x^2)/x^5,x, algorithm="fricas")
```

output
```
1/2*(2*c*x^2*log(x) - 2*b*x - a)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{ax^2 + bx^3 + cx^4}{x^5} \, dx = c \log(x) + \frac{-a - 2bx}{2x^2}$$

input
```
integrate((c*x**4+b*x**3+a*x**2)/x**5,x)
```

output
```
c*log(x) + (-a - 2*b*x)/(2*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int \frac{ax^2 + bx^3 + cx^4}{x^5} \, dx = c \log(x) - \frac{2\,bx + a}{2\,x^2}$$

input
```
integrate((c*x^4+b*x^3+a*x^2)/x^5,x, algorithm="maxima")
```

output
```
c*log(x) - 1/2*(2*b*x + a)/x^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{ax^2 + bx^3 + cx^4}{x^5} \, dx = c \log(|x|) - \frac{2\,bx + a}{2\,x^2}$$

input
```
integrate((c*x^4+b*x^3+a*x^2)/x^5,x, algorithm="giac")
```

output
```
c*log(abs(x)) - 1/2*(2*b*x + a)/x^2
```

Mupad [B] (verification not implemented)

Time = 20.96 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{ax^2+bx^3+cx^4}{x^5}\,dx = c\,\ln(x) - \frac{\frac{a}{2}+b\,x}{x^2}$$

input
```
int((a*x^2 + b*x^3 + c*x^4)/x^5,x)
```

output
```
c*log(x) - (a/2 + b*x)/x^2
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{ax^2+bx^3+cx^4}{x^5}\,dx = \frac{2\log(x)\,c\,x^2-a-2bx}{2x^2}$$

input
```
int((c*x^4+b*x^3+a*x^2)/x^5,x)
```

output
```
(2*log(x)*c*x**2 - a - 2*b*x)/(2*x**2)
```

3.15 $\int \frac{ax^2+bx^3+cx^4}{x^6} dx$

Optimal result

Integrand size = 20, antiderivative size = 23

$$\int \frac{ax^2+bx^3+cx^4}{x^6} dx = -\frac{a}{3x^3} - \frac{b}{2x^2} - \frac{c}{x}$$

output
```
-1/3*a/x^3-1/2*b/x^2-c/x
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{ax^2+bx^3+cx^4}{x^6} dx = -\frac{a}{3x^3} - \frac{b}{2x^2} - \frac{c}{x}$$

input
```
Integrate[(a*x^2 + b*x^3 + c*x^4)/x^6,x]
```

output
```
-1/3*a/x^3 - b/(2*x^2) - c/x
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {9, 1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ax^2+bx^3+cx^4}{x^6}\,dx$$
$$\Big\downarrow \text{ 9}$$
$$\int \frac{a+bx+cx^2}{x^4}dx$$
$$\Big\downarrow \text{ 1140}$$
$$\int \left(\frac{a}{x^4}+\frac{b}{x^3}+\frac{c}{x^2}\right)dx$$
$$\Big\downarrow \text{ 2009}$$
$$-\frac{a}{3x^3}-\frac{b}{2x^2}-\frac{c}{x}$$

input
```
Int[(a*x^2 + b*x^3 + c*x^4)/x^6,x]
```

output
```
-1/3*a/x^3 - b/(2*x^2) - c/x
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

method	result	size
risch	$\frac{-c\,x^{2}-\frac{1}{2}bx-\frac{1}{3}a}{x^{3}}$	19
gosper	$-\frac{6c\,x^{2}+3bx+2a}{6x^{3}}$	20
default	$-\frac{a}{3x^{3}}-\frac{b}{2x^{2}}-\frac{c}{x}$	20
parallelrisch	$\frac{-6c\,x^{2}-3bx-2a}{6x^{3}}$	20
norman	$\frac{-\frac{1}{3}a\,x^{2}-\frac{1}{2}b\,x^{3}-c\,x^{4}}{x^{5}}$	24
orering	$-\frac{\left(6c\,x^{2}+3bx+2a\right)\left(c\,x^{4}+b\,x^{3}+a\,x^{2}\right)}{6x^{5}\left(c\,x^{2}+bx+a\right)}$	48

input
```
int((c*x^4+b*x^3+a*x^2)/x^6,x,method=_RETURNVERBOSE)
```

output
```
(-c*x^2-1/2*b*x-1/3*a)/x^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{ax^2+bx^3+cx^4}{x^6}\,dx=-\frac{6\,cx^2+3\,bx+2\,a}{6\,x^3}$$

input
```
integrate((c*x^4+b*x^3+a*x^2)/x^6,x, algorithm="fricas")
```

output
```
-1/6*(6*c*x^2 + 3*b*x + 2*a)/x^3
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{ax^2+bx^3+cx^4}{x^6}\,dx=\frac{-2a-3bx-6cx^2}{6x^3}$$

input
```
integrate((c*x**4+b*x**3+a*x**2)/x**6,x)
```

output
```
(-2*a - 3*b*x - 6*c*x**2)/(6*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{ax^2+bx^3+cx^4}{x^6}\,dx=-\frac{6\,cx^2+3\,bx+2\,a}{6\,x^3}$$

input
```
integrate((c*x^4+b*x^3+a*x^2)/x^6,x, algorithm="maxima")
```

output
```
-1/6*(6*c*x^2 + 3*b*x + 2*a)/x^3
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{ax^2+bx^3+cx^4}{x^6}\,dx=-\frac{6\,cx^2+3\,bx+2\,a}{6\,x^3}$$

input
```
integrate((c*x^4+b*x^3+a*x^2)/x^6,x, algorithm="giac")
```

output
```
-1/6*(6*c*x^2 + 3*b*x + 2*a)/x^3
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

$$\int \frac{ax^2 + bx^3 + cx^4}{x^6}\, dx = -\frac{c\,x^2 + \frac{b\,x}{2} + \frac{a}{3}}{x^3}$$

input
```
int((a*x^2 + b*x^3 + c*x^4)/x^6,x)
```

output
```
-(a/3 + (b*x)/2 + c*x^2)/x^3
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{ax^2 + bx^3 + cx^4}{x^6}\, dx = \frac{-6c\,x^2 - 3bx - 2a}{6x^3}$$

input
```
int((c*x^4+b*x^3+a*x^2)/x^6,x)
```

output
```
( - 2*a - 3*b*x - 6*c*x**2)/(6*x**3)
```

3.16 $\int \frac{ax^2+bx^3+cx^4}{x^7} dx$

Optimal result

Integrand size = 20, antiderivative size = 25

$$\int \frac{ax^2+bx^3+cx^4}{x^7} dx = -\frac{a}{4x^4} - \frac{b}{3x^3} - \frac{c}{2x^2}$$

output
```
-1/4*a/x^4-1/3*b/x^3-1/2*c/x^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{ax^2+bx^3+cx^4}{x^7} dx = -\frac{a}{4x^4} - \frac{b}{3x^3} - \frac{c}{2x^2}$$

input
```
Integrate[(a*x^2 + b*x^3 + c*x^4)/x^7,x]
```

output
```
-1/4*a/x^4 - b/(3*x^3) - c/(2*x^2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {9, 1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ax^2+bx^3+cx^4}{x^7}\,dx$$

$$\Big\downarrow \textit{9}$$

$$\int \frac{a+bx+cx^2}{x^5}dx$$

$$\Big\downarrow \textit{1140}$$

$$\int \left(\frac{a}{x^5}+\frac{b}{x^4}+\frac{c}{x^3}\right)dx$$

$$\Big\downarrow \textit{2009}$$

$$-\frac{a}{4x^4}-\frac{b}{3x^3}-\frac{c}{2x^2}$$

input
```
Int[(a*x^2 + b*x^3 + c*x^4)/x^7,x]
```

output
```
-1/4*a/x^4 - b/(3*x^3) - c/(2*x^2)
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

method	result	size
risch	$\frac{-\frac{1}{2}c\,x^2-\frac{1}{3}bx-\frac{1}{4}a}{x^4}$	19
gosper	$-\frac{6c\,x^2+4bx+3a}{12x^4}$	20
default	$-\frac{a}{4x^4}-\frac{b}{3x^3}-\frac{c}{2x^2}$	20
parallelrisch	$\frac{-6c\,x^2-4bx-3a}{12x^4}$	20
norman	$\frac{-\frac{1}{4}a\,x^2-\frac{1}{3}b\,x^3-\frac{1}{2}c\,x^4}{x^6}$	24
orering	$-\frac{(6c\,x^2+4bx+3a)\,(c\,x^4+b\,x^3+a\,x^2)}{12x^6(c\,x^2+bx+a)}$	48

input
```
int((c*x^4+b*x^3+a*x^2)/x^7,x,method=_RETURNVERBOSE)
```

output
```
(-1/2*c*x^2-1/3*b*x-1/4*a)/x^4
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{ax^2+bx^3+cx^4}{x^7}\,dx=-\frac{6\,cx^2+4\,bx+3\,a}{12\,x^4}$$

input
```
integrate((c*x^4+b*x^3+a*x^2)/x^7,x, algorithm="fricas")
```

output
```
-1/12*(6*c*x^2 + 4*b*x + 3*a)/x^4
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

$$\int \frac{ax^2 + bx^3 + cx^4}{x^7} \, dx = \frac{-3a - 4bx - 6cx^2}{12x^4}$$

input
```
integrate((c*x**4+b*x**3+a*x**2)/x**7,x)
```

output
```
(-3*a - 4*b*x - 6*c*x**2)/(12*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{ax^2 + bx^3 + cx^4}{x^7} \, dx = -\frac{6\,cx^2 + 4\,bx + 3\,a}{12\,x^4}$$

input
```
integrate((c*x^4+b*x^3+a*x^2)/x^7,x, algorithm="maxima")
```

output
```
-1/12*(6*c*x^2 + 4*b*x + 3*a)/x^4
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{ax^2 + bx^3 + cx^4}{x^7} \, dx = -\frac{6\,cx^2 + 4\,bx + 3\,a}{12\,x^4}$$

input
```
integrate((c*x^4+b*x^3+a*x^2)/x^7,x, algorithm="giac")
```

output
```
-1/12*(6*c*x^2 + 4*b*x + 3*a)/x^4
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{ax^2+bx^3+cx^4}{x^7}\,dx=-\frac{\frac{c\,x^2}{2}+\frac{b\,x}{3}+\frac{a}{4}}{x^4}$$

input
```
int((a*x^2 + b*x^3 + c*x^4)/x^7,x)
```

output
```
-(a/4 + (b*x)/3 + (c*x^2)/2)/x^4
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{ax^2+bx^3+cx^4}{x^7}\,dx=\frac{-6c\,x^2-4bx-3a}{12x^4}$$

input
```
int((c*x^4+b*x^3+a*x^2)/x^7,x)
```

output
```
( - 3*a - 4*b*x - 6*c*x**2)/(12*x**4)
```

3.17 $\int x^2(ax^2+bx^3+cx^4)^2\,dx$

Optimal result

Integrand size = 22, antiderivative size = 54

$$\int x^2\left(ax^2+bx^3+cx^4\right)^2\,dx=\frac{a^2x^7}{7}+\frac{1}{4}abx^8+\frac{1}{9}\left(b^2+2ac\right)x^9+\frac{1}{5}bcx^{10}+\frac{c^2x^{11}}{11}$$

output
```
1/7*a^2*x^7+1/4*a*b*x^8+1/9*(2*a*c+b^2)*x^9+1/5*b*c*x^10+1/11*c^2*x^11
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int x^2\left(ax^2+bx^3+cx^4\right)^2\,dx=\frac{a^2x^7}{7}+\frac{1}{4}abx^8+\frac{1}{9}\left(b^2+2ac\right)x^9+\frac{1}{5}bcx^{10}+\frac{c^2x^{11}}{11}$$

input
```
Integrate[x^2*(a*x^2 + b*x^3 + c*x^4)^2,x]
```

output
```
(a^2*x^7)/7 + (a*b*x^8)/4 + ((b^2 + 2*a*c)*x^9)/9 + (b*c*x^10)/5 + (c^2*x^
11)/11
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {9, 1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(ax^2+bx^3+cx^4\right)^2 \, dx$$

$$\Big\downarrow \text{9}$$

$$\int x^6 \left(a+bx+cx^2\right)^2dx$$

$$\Big\downarrow \text{1140}$$

$$\int \left(a^2x^6+x^8 \left(2ac+b^2\right)+2abx^7+2bcx^9+c^2x^{10}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2x^7}{7}+\frac{1}{9}x^9 \left(2ac+b^2\right)+\frac{1}{4}abx^8+\frac{1}{5}bcx^{10}+\frac{c^2x^{11}}{11}$$

input
```
Int[x^2*(a*x^2 + b*x^3 + c*x^4)^2,x]
```

output
```
(a^2*x^7)/7 + (a*b*x^8)/4 + ((b^2 + 2*a*c)*x^9)/9 + (b*c*x^10)/5 + (c^2*x^
11)/11
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{a^2x^7}{7}+\frac{ab\,x^8}{4}+\frac{(2ac+b^2)x^9}{9}+\frac{bc\,x^{10}}{5}+\frac{c^2x^{11}}{11}$	45
norman	$\frac{c^2x^{11}}{11}+\frac{bc\,x^{10}}{5}+\left(\frac{2ac}{9}+\frac{b^2}{9}\right)x^9+\frac{ab\,x^8}{4}+\frac{a^2x^7}{7}$	46
gosper	$\frac{x^7(1260c^2x^4+2772bc\,x^3+3080ac\,x^2+1540b^2x^2+3465abx+1980a^2)}{13860}$	47
risch	$\frac{1}{7}a^2x^7+\frac{1}{4}ab\,x^8+\frac{2}{9}x^9ac+\frac{1}{9}b^2x^9+\frac{1}{5}bc\,x^{10}+\frac{1}{11}c^2x^{11}$	47
parallelrisch	$\frac{1}{7}a^2x^7+\frac{1}{4}ab\,x^8+\frac{2}{9}x^9ac+\frac{1}{9}b^2x^9+\frac{1}{5}bc\,x^{10}+\frac{1}{11}c^2x^{11}$	47
orering	$\frac{x^3(1260c^2x^4+2772bc\,x^3+3080ac\,x^2+1540b^2x^2+3465abx+1980a^2)(c\,x^4+b\,x^3+a\,x^2)^2}{13860(c\,x^2+bx+a)^2}$	77

input
```
int(x^2*(c*x^4+b*x^3+a*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/7*a^2*x^7+1/4*a*b*x^8+1/9*(2*a*c+b^2)*x^9+1/5*b*c*x^10+1/11*c^2*x^11
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int x^2(ax^2+bx^3+cx^4)^2\,dx=\frac{1}{11}\,c^2x^{11}+\frac{1}{5}\,bcx^{10}+\frac{1}{4}\,abx^8+\frac{1}{9}\,(b^2+2\,ac)x^9+\frac{1}{7}\,a^2x^7$$

input
```
integrate(x^2*(c*x^4+b*x^3+a*x^2)^2,x, algorithm="fricas")
```

output
```
1/11*c^2*x^11 + 1/5*b*c*x^10 + 1/4*a*b*x^8 + 1/9*(b^2 + 2*a*c)*x^9 + 1/7*a
^2*x^7
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.89

$$\int x^2(ax^2+bx^3+cx^4)^2\,dx=\frac{a^2x^7}{7}+\frac{abx^8}{4}+\frac{bcx^{10}}{5}+\frac{c^2x^{11}}{11}+x^9\cdot\left(\frac{2ac}{9}+\frac{b^2}{9}\right)$$

input
```
integrate(x**2*(c*x**4+b*x**3+a*x**2)**2,x)
```

output
```
a**2*x**7/7 + a*b*x**8/4 + b*c*x**10/5 + c**2*x**11/11 + x**9*(2*a*c/9 + b
**2/9)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int x^2(ax^2+bx^3+cx^4)^2\,dx=\frac{1}{11}\,c^2x^{11}+\frac{1}{5}\,bcx^{10}+\frac{1}{4}\,abx^8+\frac{1}{9}\,(b^2+2\,ac)x^9+\frac{1}{7}\,a^2x^7$$

input
```
integrate(x^2*(c*x^4+b*x^3+a*x^2)^2,x, algorithm="maxima")
```

output
```
1/11*c^2*x^11 + 1/5*b*c*x^10 + 1/4*a*b*x^8 + 1/9*(b^2 + 2*a*c)*x^9 + 1/7*a
^2*x^7
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.85

$$\int x^2\left(ax^2+bx^3+cx^4\right)^2\,dx=\frac{1}{11}\,c^2x^{11}+\frac{1}{5}\,bcx^{10}+\frac{1}{9}\,b^2x^9+\frac{2}{9}\,acx^9+\frac{1}{4}\,abx^8+\frac{1}{7}\,a^2x^7$$

input
```
integrate(x^2*(c*x^4+b*x^3+a*x^2)^2,x, algorithm="giac")
```

output
```
1/11*c^2*x^11 + 1/5*b*c*x^10 + 1/9*b^2*x^9 + 2/9*a*c*x^9 + 1/4*a*b*x^8 + 1
/7*a^2*x^7
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.83

$$\int x^2\left(ax^2+bx^3+cx^4\right)^2\,dx=x^9\left(\frac{b^2}{9}+\frac{2\,a\,c}{9}\right)+\frac{a^2\,x^7}{7}+\frac{c^2\,x^{11}}{11}+\frac{a\,b\,x^8}{4}+\frac{b\,c\,x^{10}}{5}$$

input
```
int(x^2*(a*x^2 + b*x^3 + c*x^4)^2,x)
```

output
```
x^9*((2*a*c)/9 + b^2/9) + (a^2*x^7)/7 + (c^2*x^11)/11 + (a*b*x^8)/4 + (b*c
*x^10)/5
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.85

$$\int x^2\left(ax^2+bx^3+cx^4\right)^2\,dx=\frac{x^7(1260c^2x^4+2772bc\,x^3+3080ac\,x^2+1540b^2x^2+3465abx+1980a^2)}{13860}$$

input
```
int(x^2*(c*x^4+b*x^3+a*x^2)^2,x)
```

output

```
(x**7*(1980*a**2 + 3465*a*b*x + 3080*a*c*x**2 + 1540*b**2*x**2 + 2772*b*c*
x**3 + 1260*c**2*x**4))/13860
```

3.18 $\int x(ax^2+bx^3+cx^4)^2\,dx$

Optimal result

Integrand size = 20, antiderivative size = 54

$$\int x\left(ax^2+bx^3+cx^4\right)^2\,dx=\frac{a^2x^6}{6}+\frac{2}{7}abx^7+\frac{1}{8}\left(b^2+2ac\right)x^8+\frac{2}{9}bcx^9+\frac{c^2x^{10}}{10}$$

output
```
1/6*a^2*x^6+2/7*a*b*x^7+1/8*(2*a*c+b^2)*x^8+2/9*b*c*x^9+1/10*c^2*x^10
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int x\left(ax^2+bx^3+cx^4\right)^2\,dx=\frac{a^2x^6}{6}+\frac{2}{7}abx^7+\frac{1}{8}\left(b^2+2ac\right)x^8+\frac{2}{9}bcx^9+\frac{c^2x^{10}}{10}$$

input
```
Integrate[x*(a*x^2 + b*x^3 + c*x^4)^2,x]
```

output
```
(a^2*x^6)/6 + (2*a*b*x^7)/7 + ((b^2 + 2*a*c)*x^8)/8 + (2*b*c*x^9)/9 + (c^2
*x^10)/10
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {9, 1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(ax^2+bx^3+cx^4\right)^2\,dx$$

$$\Big\downarrow \; 9$$

$$\int x^5\left(a+bx+cx^2\right)^2dx$$

$$\Big\downarrow \; 1140$$

$$\int \left(a^2x^5+x^7\left(2ac+b^2\right)+2abx^6+2bcx^8+c^2x^9\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{a^2x^6}{6}+\frac{1}{8}x^8\left(2ac+b^2\right)+\frac{2}{7}abx^7+\frac{2}{9}bcx^9+\frac{c^2x^{10}}{10}$$

input
```
Int[x*(a*x^2 + b*x^3 + c*x^4)^2,x]
```

output
```
(a^2*x^6)/6 + (2*a*b*x^7)/7 + ((b^2 + 2*a*c)*x^8)/8 + (2*b*c*x^9)/9 + (c^2
*x^10)/10
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{a^2x^6}{6}+\frac{2ab\,x^7}{7}+\frac{(2ac+b^2)x^8}{8}+\frac{2bc\,x^9}{9}+\frac{c^2x^{10}}{10}$	45
norman	$\frac{c^2x^{10}}{10}+\frac{2bc\,x^9}{9}+\left(\frac{ac}{4}+\frac{b^2}{8}\right)x^8+\frac{2ab\,x^7}{7}+\frac{a^2x^6}{6}$	46
gosper	$\frac{x^6(252c^2x^4+560bc\,x^3+630ac\,x^2+315b^2x^2+720abx+420a^2)}{2520}$	47
risch	$\frac{1}{6}a^2x^6+\frac{2}{7}ab\,x^7+\frac{1}{4}x^8ac+\frac{1}{8}b^2x^8+\frac{2}{9}bc\,x^9+\frac{1}{10}c^2x^{10}$	47
parallelrisch	$\frac{1}{6}a^2x^6+\frac{2}{7}ab\,x^7+\frac{1}{4}x^8ac+\frac{1}{8}b^2x^8+\frac{2}{9}bc\,x^9+\frac{1}{10}c^2x^{10}$	47
orering	$\frac{x^2(252c^2x^4+560bc\,x^3+630ac\,x^2+315b^2x^2+720abx+420a^2)(c\,x^4+b\,x^3+a\,x^2)^2}{2520(c\,x^2+bx+a)^2}$	77

input
```
int(x*(c*x^4+b*x^3+a*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/6*a^2*x^6+2/7*a*b*x^7+1/8*(2*a*c+b^2)*x^8+2/9*b*c*x^9+1/10*c^2*x^10
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int x\left(ax^2+bx^3+cx^4\right)^2\,dx=\frac{1}{10}\,c^2x^{10}+\frac{2}{9}\,bcx^9+\frac{2}{7}\,abx^7+\frac{1}{8}\,\left(b^2+2\,ac\right)x^8+\frac{1}{6}\,a^2x^6$$

input
```
integrate(x*(c*x^4+b*x^3+a*x^2)^2,x, algorithm="fricas")
```

output
```
1/10*c^2*x^10 + 2/9*b*c*x^9 + 2/7*a*b*x^7 + 1/8*(b^2 + 2*a*c)*x^8 + 1/6*a^
2*x^6
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.91

$$\int x(ax^2+bx^3+cx^4)^2\,dx=\frac{a^2x^6}{6}+\frac{2abx^7}{7}+\frac{2bcx^9}{9}+\frac{c^2x^{10}}{10}+x^8\left(\frac{ac}{4}+\frac{b^2}{8}\right)$$

input
```
integrate(x*(c*x**4+b*x**3+a*x**2)**2,x)
```

output
```
a**2*x**6/6 + 2*a*b*x**7/7 + 2*b*c*x**9/9 + c**2*x**10/10 + x**8*(a*c/4 +
b**2/8)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int x(ax^2+bx^3+cx^4)^2\,dx=\frac{1}{10}\,c^2x^{10}+\frac{2}{9}\,bcx^9+\frac{2}{7}\,abx^7+\frac{1}{8}\,(b^2+2\,ac)x^8+\frac{1}{6}\,a^2x^6$$

input
```
integrate(x*(c*x^4+b*x^3+a*x^2)^2,x, algorithm="maxima")
```

output
```
1/10*c^2*x^10 + 2/9*b*c*x^9 + 2/7*a*b*x^7 + 1/8*(b^2 + 2*a*c)*x^8 + 1/6*a^
2*x^6
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.85

$$\int x\left(ax^2+bx^3+cx^4\right)^2\,dx=\frac{1}{10}\,c^2x^{10}+\frac{2}{9}\,bcx^9+\frac{1}{8}\,b^2x^8+\frac{1}{4}\,acx^8+\frac{2}{7}\,abx^7+\frac{1}{6}\,a^2x^6$$

input
```
integrate(x*(c*x^4+b*x^3+a*x^2)^2,x, algorithm="giac")
```

output
```
1/10*c^2*x^10 + 2/9*b*c*x^9 + 1/8*b^2*x^8 + 1/4*a*c*x^8 + 2/7*a*b*x^7 + 1/
6*a^2*x^6
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.83

$$\int x\left(ax^2+bx^3+cx^4\right)^2\,dx=x^8\left(\frac{b^2}{8}+\frac{a\,c}{4}\right)+\frac{a^2\,x^6}{6}+\frac{c^2\,x^{10}}{10}+\frac{2\,a\,b\,x^7}{7}+\frac{2\,b\,c\,x^9}{9}$$

input
```
int(x*(a*x^2 + b*x^3 + c*x^4)^2,x)
```

output
```
x^8*((a*c)/4 + b^2/8) + (a^2*x^6)/6 + (c^2*x^10)/10 + (2*a*b*x^7)/7 + (2*b
*c*x^9)/9
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.85

$$\int x\left(ax^2+bx^3+cx^4\right)^2\,dx=\frac{x^6(252c^2x^4+560bc\,x^3+630ac\,x^2+315b^2x^2+720abx+420a^2)}{2520}$$

input
```
int(x*(c*x^4+b*x^3+a*x^2)^2,x)
```

output
```
(x**6*(420*a**2 + 720*a*b*x + 630*a*c*x**2 + 315*b**2*x**2 + 560*b*c*x**3
+ 252*c**2*x**4))/2520
```

3.19 $\int (ax^2+bx^3+cx^4)^2 \, dx$

Optimal result

Integrand size = 18, antiderivative size = 54

$$\int \left(ax^2+bx^3+cx^4\right)^2 \, dx = \frac{a^2x^5}{5}+\frac{1}{3}abx^6+\frac{1}{7}\left(b^2+2ac\right)x^7+\frac{1}{4}bcx^8+\frac{c^2x^9}{9}$$

output
```
1/5*a^2*x^5+1/3*a*b*x^6+1/7*(2*a*c+b^2)*x^7+1/4*b*c*x^8+1/9*c^2*x^9
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int \left(ax^2+bx^3+cx^4\right)^2 \, dx = \frac{a^2x^5}{5}+\frac{1}{3}abx^6+\frac{1}{7}\left(b^2+2ac\right)x^7+\frac{1}{4}bcx^8+\frac{c^2x^9}{9}$$

input
```
Integrate[(a*x^2 + b*x^3 + c*x^4)^2,x]
```

output
```
(a^2*x^5)/5 + (a*b*x^6)/3 + ((b^2 + 2*a*c)*x^7)/7 + (b*c*x^8)/4 + (c^2*x^9
)/9
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1949, 1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(ax^2+bx^3+cx^4\right)^2 dx$$

$$\Big\downarrow \ 1949$$

$$\int x^4\left(a+bx+cx^2\right)^2 dx$$

$$\Big\downarrow \ 1140$$

$$\int \left(a^2x^4+x^6\left(2ac+b^2\right)+2abx^5+2bcx^7+c^2x^8\right) dx$$

$$\Big\downarrow \ 2009$$

$$\frac{a^2x^5}{5}+\frac{1}{7}x^7\left(2ac+b^2\right)+\frac{1}{3}abx^6+\frac{1}{4}bcx^8+\frac{c^2x^9}{9}$$

input
```
Int[(a*x^2 + b*x^3 + c*x^4)^2,x]
```

output
```
(a^2*x^5)/5 + (a*b*x^6)/3 + ((b^2 + 2*a*c)*x^7)/7 + (b*c*x^8)/4 + (c^2*x^9
)/9
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 1949
```
Int[((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_), x_Symbol
] :> Int[x^(p*q)*(a + b*x^(n - q) + c*x^(2*(n - q)))^p, x] /; FreeQ[{a, b,
c, n, q}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{a^2x^5}{5}+\frac{ab\,x^6}{3}+\frac{(2ac+b^2)x^7}{7}+\frac{bc\,x^8}{4}+\frac{c^2x^9}{9}$	45
norman	$\frac{c^2x^9}{9}+\frac{bc\,x^8}{4}+\left(\frac{2ac}{7}+\frac{b^2}{7}\right)x^7+\frac{ab\,x^6}{3}+\frac{a^2x^5}{5}$	46
gosper	$\frac{x^5(140c^2x^4+315bc\,x^3+360ac\,x^2+180b^2x^2+420abx+252a^2)}{1260}$	47
risch	$\frac{1}{5}a^2x^5+\frac{1}{3}ab\,x^6+\frac{2}{7}x^7ac+\frac{1}{7}b^2x^7+\frac{1}{4}bc\,x^8+\frac{1}{9}c^2x^9$	47
parallelrisch	$\frac{1}{5}a^2x^5+\frac{1}{3}ab\,x^6+\frac{2}{7}x^7ac+\frac{1}{7}b^2x^7+\frac{1}{4}bc\,x^8+\frac{1}{9}c^2x^9$	47
orering	$\frac{x(140c^2x^4+315bc\,x^3+360ac\,x^2+180b^2x^2+420abx+252a^2)(c\,x^4+b\,x^3+a\,x^2)^2}{1260(c\,x^2+bx+a)^2}$	75

input
```
int((c*x^4+b*x^3+a*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/5*a^2*x^5+1/3*a*b*x^6+1/7*(2*a*c+b^2)*x^7+1/4*b*c*x^8+1/9*c^2*x^9
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int (ax^2+bx^3+cx^4)^2\,dx=\frac{1}{9}\,c^2x^9+\frac{1}{4}\,bcx^8+\frac{1}{3}\,abx^6+\frac{1}{7}\,(b^2+2\,ac)x^7+\frac{1}{5}\,a^2x^5$$

input
```
integrate((c*x^4+b*x^3+a*x^2)^2,x, algorithm="fricas")
```

output
```
1/9*c^2*x^9 + 1/4*b*c*x^8 + 1/3*a*b*x^6 + 1/7*(b^2 + 2*a*c)*x^7 + 1/5*a^2*
x^5
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.89

$$\int (ax^2 + bx^3 + cx^4)^2 \, dx = \frac{a^2x^5}{5} + \frac{abx^6}{3} + \frac{bcx^8}{4} + \frac{c^2x^9}{9} + x^7 \cdot \left(\frac{2ac}{7} + \frac{b^2}{7}\right)$$

input
```
integrate((c*x**4+b*x**3+a*x**2)**2,x)
```

output
```
a**2*x**5/5 + a*b*x**6/3 + b*c*x**8/4 + c**2*x**9/9 + x**7*(2*a*c/7 + b**2
/7)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.89

$$\int (ax^2 + bx^3 + cx^4)^2 \, dx = \frac{1}{9}c^2x^9 + \frac{1}{4}bcx^8 + \frac{1}{7}b^2x^7 + \frac{1}{5}a^2x^5 + \frac{1}{21}(6cx^7 + 7bx^6)a$$

input
```
integrate((c*x^4+b*x^3+a*x^2)^2,x, algorithm="maxima")
```

output
```
1/9*c^2*x^9 + 1/4*b*c*x^8 + 1/7*b^2*x^7 + 1/5*a^2*x^5 + 1/21*(6*c*x^7 + 7*
b*x^6)*a
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.85

$$\int \left(ax^2+bx^3+cx^4\right)^2\,dx=\frac{1}{9}\,c^2x^9+\frac{1}{4}\,bcx^8+\frac{1}{7}\,b^2x^7+\frac{2}{7}\,acx^7+\frac{1}{3}\,abx^6+\frac{1}{5}\,a^2x^5$$

input
```
integrate((c*x^4+b*x^3+a*x^2)^2,x, algorithm="giac")
```

output
```
1/9*c^2*x^9 + 1/4*b*c*x^8 + 1/7*b^2*x^7 + 2/7*a*c*x^7 + 1/3*a*b*x^6 + 1/5*
a^2*x^5
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.83

$$\int \left(ax^2+bx^3+cx^4\right)^2\,dx=x^7\left(\frac{b^2}{7}+\frac{2\,a\,c}{7}\right)+\frac{a^2\,x^5}{5}+\frac{c^2\,x^9}{9}+\frac{a\,b\,x^6}{3}+\frac{b\,c\,x^8}{4}$$

input
```
int((a*x^2 + b*x^3 + c*x^4)^2,x)
```

output
```
x^7*((2*a*c)/7 + b^2/7) + (a^2*x^5)/5 + (c^2*x^9)/9 + (a*b*x^6)/3 + (b*c*x
^8)/4
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.85

$$\int \left(ax^2+bx^3+cx^4\right)^2\,dx$$
$$=\frac{x^5(140c^2x^4+315bc\,x^3+360ac\,x^2+180b^2x^2+420abx+252a^2)}{1260}$$

input
```
int((c*x^4+b*x^3+a*x^2)^2,x)
```

output
```
(x**5*(252*a**2 + 420*a*b*x + 360*a*c*x**2 + 180*b**2*x**2 + 315*b*c*x**3
+ 140*c**2*x**4))/1260
```

3.20 $\int \frac{(ax^2+bx^3+cx^4)^2}{x} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 54

$$\int \frac{(ax^2+bx^3+cx^4)^2}{x} \, dx = \frac{a^2x^4}{4} + \frac{2}{5}abx^5 + \frac{1}{6}\left(b^2+2ac\right)x^6 + \frac{2}{7}bcx^7 + \frac{c^2x^8}{8}$$

output
```
1/4*a^2*x^4+2/5*a*b*x^5+1/6*(2*a*c+b^2)*x^6+2/7*b*c*x^7+1/8*c^2*x^8
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int \frac{(ax^2+bx^3+cx^4)^2}{x} \, dx = \frac{a^2x^4}{4} + \frac{2}{5}abx^5 + \frac{1}{6}\left(b^2+2ac\right)x^6 + \frac{2}{7}bcx^7 + \frac{c^2x^8}{8}$$

input
```
Integrate[(a*x^2 + b*x^3 + c*x^4)^2/x,x]
```

output
```
(a^2*x^4)/4 + (2*a*b*x^5)/5 + ((b^2 + 2*a*c)*x^6)/6 + (2*b*c*x^7)/7 + (c^2
*x^8)/8
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {9, 1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax^2+bx^3+cx^4)^2}{x}dx$$

$$\Big\downarrow \ 9$$

$$\int x^3(a+bx+cx^2)^2dx$$

$$\Big\downarrow \ 1140$$

$$\int (a^2x^3+x^5(2ac+b^2)+2abx^4+2bcx^6+c^2x^7)\,dx$$

$$\Big\downarrow \ 2009$$

$$\frac{a^2x^4}{4}+\frac{1}{6}x^6(2ac+b^2)+\frac{2}{5}abx^5+\frac{2}{7}bcx^7+\frac{c^2x^8}{8}$$

input
```
Int[(a*x^2 + b*x^3 + c*x^4)^2/x,x]
```

output
```
(a^2*x^4)/4 + (2*a*b*x^5)/5 + ((b^2 + 2*a*c)*x^6)/6 + (2*b*c*x^7)/7 + (c^2
*x^8)/8
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.10 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{a^2x^4}{4}+\frac{2ab\,x^5}{5}+\frac{(2ac+b^2)\,x^6}{6}+\frac{2bc\,x^7}{7}+\frac{x^8c^2}{8}$	45
norman	$\frac{x^8c^2}{8}+\frac{2bc\,x^7}{7}+\left(\frac{ac}{3}+\frac{b^2}{6}\right)x^6+\frac{2ab\,x^5}{5}+\frac{a^2x^4}{4}$	46
gosper	$\frac{x^4(105c^2x^4+240bc\,x^3+280ac\,x^2+140b^2x^2+336abx+210a^2)}{840}$	47
risch	$\frac{1}{4}a^2x^4+\frac{2}{5}ab\,x^5+\frac{1}{3}x^6ac+\frac{1}{6}b^2x^6+\frac{2}{7}bc\,x^7+\frac{1}{8}x^8c^2$	47
parallelrisch	$\frac{1}{4}a^2x^4+\frac{2}{5}ab\,x^5+\frac{1}{3}x^6ac+\frac{1}{6}b^2x^6+\frac{2}{7}bc\,x^7+\frac{1}{8}x^8c^2$	47
orering	$\frac{(105c^2x^4+240bc\,x^3+280ac\,x^2+140b^2x^2+336abx+210a^2)\,(c\,x^4+b\,x^3+a\,x^2)^2}{840(c\,x^2+bx+a)^2}$	74

input
```
int((c*x^4+b*x^3+a*x^2)^2/x,x,method=_RETURNVERBOSE)
```

output
```
1/4*a^2*x^4+2/5*a*b*x^5+1/6*(2*a*c+b^2)*x^6+2/7*b*c*x^7+1/8*x^8*c^2
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int \frac{(ax^2+bx^3+cx^4)^2}{x}\,dx=\frac{1}{8}\,c^2x^8+\frac{2}{7}\,bcx^7+\frac{2}{5}\,abx^5+\frac{1}{6}\,(b^2+2\,ac)x^6+\frac{1}{4}\,a^2x^4$$

input
```
integrate((c*x^4+b*x^3+a*x^2)^2/x,x, algorithm="fricas")
```

output
```
1/8*c^2*x^8 + 2/7*b*c*x^7 + 2/5*a*b*x^5 + 1/6*(b^2 + 2*a*c)*x^6 + 1/4*a^2*
x^4
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.91

$$\int \frac{(ax^2 + bx^3 + cx^4)^2}{x} \, dx = \frac{a^2x^4}{4} + \frac{2abx^5}{5} + \frac{2bcx^7}{7} + \frac{c^2x^8}{8} + x^6\left(\frac{ac}{3} + \frac{b^2}{6}\right)$$

input
```
integrate((c*x**4+b*x**3+a*x**2)**2/x,x)
```

output
```
a**2*x**4/4 + 2*a*b*x**5/5 + 2*b*c*x**7/7 + c**2*x**8/8 + x**6*(a*c/3 + b*
*2/6)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int \frac{(ax^2 + bx^3 + cx^4)^2}{x} \, dx = \frac{1}{8}\,c^2x^8 + \frac{2}{7}\,bcx^7 + \frac{2}{5}\,abx^5 + \frac{1}{6}\,(b^2 + 2\,ac)x^6 + \frac{1}{4}\,a^2x^4$$

input
```
integrate((c*x^4+b*x^3+a*x^2)^2/x,x, algorithm="maxima")
```

output
```
1/8*c^2*x^8 + 2/7*b*c*x^7 + 2/5*a*b*x^5 + 1/6*(b^2 + 2*a*c)*x^6 + 1/4*a^2*
x^4
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.85

$$\int \frac{(ax^2+bx^3+cx^4)^2}{x}\,dx = \frac{1}{8}\,c^2x^8 + \frac{2}{7}\,bcx^7 + \frac{1}{6}\,b^2x^6 + \frac{1}{3}\,acx^6 + \frac{2}{5}\,abx^5 + \frac{1}{4}\,a^2x^4$$

input
```
integrate((c*x^4+b*x^3+a*x^2)^2/x,x, algorithm="giac")
```

output
```
1/8*c^2*x^8 + 2/7*b*c*x^7 + 1/6*b^2*x^6 + 1/3*a*c*x^6 + 2/5*a*b*x^5 + 1/4*
a^2*x^4
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.83

$$\int \frac{(ax^2+bx^3+cx^4)^2}{x}\,dx = x^6\left(\frac{b^2}{6}+\frac{a\,c}{3}\right)+\frac{a^2\,x^4}{4}+\frac{c^2\,x^8}{8}+\frac{2\,a\,b\,x^5}{5}+\frac{2\,b\,c\,x^7}{7}$$

input
```
int((a*x^2 + b*x^3 + c*x^4)^2/x,x)
```

output
```
x^6*((a*c)/3 + b^2/6) + (a^2*x^4)/4 + (c^2*x^8)/8 + (2*a*b*x^5)/5 + (2*b*c
*x^7)/7
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.85

$$\int \frac{(ax^2+bx^3+cx^4)^2}{x}\,dx = \frac{x^4(105c^2x^4+240bc\,x^3+280ac\,x^2+140b^2x^2+336abx+210a^2)}{840}$$

input
```
int((c*x^4+b*x^3+a*x^2)^2/x,x)
```

output
```
(x**4*(210*a**2 + 336*a*b*x + 280*a*c*x**2 + 140*b**2*x**2 + 240*b*c*x**3
+ 105*c**2*x**4))/840
```

3.21 $\int \frac{(ax^2+bx^3+cx^4)^2}{x^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 54

$$\int \frac{(ax^2+bx^3+cx^4)^2}{x^2}\,dx = \frac{a^2x^3}{3}+\frac{1}{2}abx^4+\frac{1}{5}\left(b^2+2ac\right)x^5+\frac{1}{3}bcx^6+\frac{c^2x^7}{7}$$

output
```
1/3*a^2*x^3+1/2*a*b*x^4+1/5*(2*a*c+b^2)*x^5+1/3*b*c*x^6+1/7*c^2*x^7
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int \frac{(ax^2+bx^3+cx^4)^2}{x^2}\,dx = \frac{a^2x^3}{3}+\frac{1}{2}abx^4+\frac{1}{5}\left(b^2+2ac\right)x^5+\frac{1}{3}bcx^6+\frac{c^2x^7}{7}$$

input
```
Integrate[(a*x^2 + b*x^3 + c*x^4)^2/x^2,x]
```

output
```
(a^2*x^3)/3 + (a*b*x^4)/2 + ((b^2 + 2*a*c)*x^5)/5 + (b*c*x^6)/3 + (c^2*x^7
)/7
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {9, 1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax^2+bx^3+cx^4)^2}{x^2}\,dx$$

$$\Big\downarrow \; 9$$

$$\int x^2 \left(a+bx+cx^2\right)^2 dx$$

$$\Big\downarrow \; 1140$$

$$\int \left(a^2x^2+x^4\left(2ac+b^2\right)+2abx^3+2bcx^5+c^2x^6\right) dx$$

$$\Big\downarrow \; 2009$$

$$\frac{a^2x^3}{3}+\frac{1}{5}x^5\left(2ac+b^2\right)+\frac{1}{2}abx^4+\frac{1}{3}bcx^6+\frac{c^2x^7}{7}$$

input
```
Int[(a*x^2 + b*x^3 + c*x^4)^2/x^2,x]
```

output
```
(a^2*x^3)/3 + (a*b*x^4)/2 + ((b^2 + 2*a*c)*x^5)/5 + (b*c*x^6)/3 + (c^2*x^7
)/7
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.10 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{a^2x^3}{3}+\frac{ab\,x^4}{2}+\frac{(2ac+b^2)x^5}{5}+\frac{bc\,x^6}{3}+\frac{c^2x^7}{7}$	45
gosper	$\frac{x^3(30c^2x^4+70bc\,x^3+84ac\,x^2+42b^2x^2+105abx+70a^2)}{210}$	47
risch	$\frac{1}{3}a^2x^3+\frac{1}{2}ab\,x^4+\frac{2}{5}x^5ac+\frac{1}{5}b^2x^5+\frac{1}{3}bc\,x^6+\frac{1}{7}c^2x^7$	47
parallelrisch	$\frac{1}{3}a^2x^3+\frac{1}{2}ab\,x^4+\frac{2}{5}x^5ac+\frac{1}{5}b^2x^5+\frac{1}{3}bc\,x^6+\frac{1}{7}c^2x^7$	47
norman	$\frac{\left(\frac{2ac}{5}+\frac{b^2}{5}\right)x^6+\frac{a^2x^4}{3}+\frac{x^8c^2}{7}+\frac{ab\,x^5}{2}+\frac{bc\,x^7}{3}}{x}$	50
orering	$\frac{(30c^2x^4+70bc\,x^3+84ac\,x^2+42b^2x^2+105abx+70a^2)(c\,x^4+b\,x^3+a\,x^2)^2}{210x(c\,x^2+bx+a)^2}$	77

input
```
int((c*x^4+b*x^3+a*x^2)^2/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*a^2*x^3+1/2*a*b*x^4+1/5*(2*a*c+b^2)*x^5+1/3*b*c*x^6+1/7*c^2*x^7
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int \frac{(ax^2+bx^3+cx^4)^2}{x^2}\,dx=\frac{1}{7}\,c^2x^7+\frac{1}{3}\,bcx^6+\frac{1}{2}\,abx^4+\frac{1}{5}\,(b^2+2\,ac)x^5+\frac{1}{3}\,a^2x^3$$

input
```
integrate((c*x^4+b*x^3+a*x^2)^2/x^2,x, algorithm="fricas")
```

output
```
1/7*c^2*x^7 + 1/3*b*c*x^6 + 1/2*a*b*x^4 + 1/5*(b^2 + 2*a*c)*x^5 + 1/3*a^2*
x^3
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.89

$$\int \frac{(ax^2+bx^3+cx^4)^2}{x^2}\,dx = \frac{a^2x^3}{3}+\frac{abx^4}{2}+\frac{bcx^6}{3}+\frac{c^2x^7}{7}+x^5\cdot\left(\frac{2ac}{5}+\frac{b^2}{5}\right)$$

input
```
integrate((c*x**4+b*x**3+a*x**2)**2/x**2,x)
```

output
```
a**2*x**3/3 + a*b*x**4/2 + b*c*x**6/3 + c**2*x**7/7 + x**5*(2*a*c/5 + b**2
/5)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int \frac{(ax^2+bx^3+cx^4)^2}{x^2}\,dx = \frac{1}{7}\,c^2x^7+\frac{1}{3}\,bcx^6+\frac{1}{2}\,abx^4+\frac{1}{5}\,(b^2+2\,ac)x^5+\frac{1}{3}\,a^2x^3$$

input
```
integrate((c*x^4+b*x^3+a*x^2)^2/x^2,x, algorithm="maxima")
```

output
```
1/7*c^2*x^7 + 1/3*b*c*x^6 + 1/2*a*b*x^4 + 1/5*(b^2 + 2*a*c)*x^5 + 1/3*a^2*
x^3
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.85

$$\int \frac{(ax^2 + bx^3 + cx^4)^2}{x^2}\,dx = \frac{1}{7}\,c^2x^7 + \frac{1}{3}\,bcx^6 + \frac{1}{5}\,b^2x^5 + \frac{2}{5}\,acx^5 + \frac{1}{2}\,abx^4 + \frac{1}{3}\,a^2x^3$$

input
```
integrate((c*x^4+b*x^3+a*x^2)^2/x^2,x, algorithm="giac")
```

output
```
1/7*c^2*x^7 + 1/3*b*c*x^6 + 1/5*b^2*x^5 + 2/5*a*c*x^5 + 1/2*a*b*x^4 + 1/3*
a^2*x^3
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.83

$$\int \frac{(ax^2 + bx^3 + cx^4)^2}{x^2}\,dx = x^5\left(\frac{b^2}{5} + \frac{2\,a\,c}{5}\right) + \frac{a^2\,x^3}{3} + \frac{c^2\,x^7}{7} + \frac{a\,b\,x^4}{2} + \frac{b\,c\,x^6}{3}$$

input
```
int((a*x^2 + b*x^3 + c*x^4)^2/x^2,x)
```

output
```
x^5*((2*a*c)/5 + b^2/5) + (a^2*x^3)/3 + (c^2*x^7)/7 + (a*b*x^4)/2 + (b*c*x
^6)/3
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.85

$$\int \frac{(ax^2 + bx^3 + cx^4)^2}{x^2}\,dx = \frac{x^3(30c^2x^4 + 70bc\,x^3 + 84ac\,x^2 + 42b^2x^2 + 105abx + 70a^2)}{210}$$

input
```
int((c*x^4+b*x^3+a*x^2)^2/x^2,x)
```

output
```
(x**3*(70*a**2 + 105*a*b*x + 84*a*c*x**2 + 42*b**2*x**2 + 70*b*c*x**3 + 30
*c**2*x**4))/210
```

3.22 $\int \frac{(ax^2+bx^3+cx^4)^2}{x^3} dx$

Optimal result

Integrand size = 22, antiderivative size = 54

$$\int \frac{(ax^2+bx^3+cx^4)^2}{x^3} dx = \frac{a^2x^2}{2} + \frac{2}{3}abx^3 + \frac{1}{4}(b^2+2ac)x^4 + \frac{2}{5}bcx^5 + \frac{c^2x^6}{6}$$

output
```
1/2*a^2*x^2+2/3*a*b*x^3+1/4*(2*a*c+b^2)*x^4+2/5*b*c*x^5+1/6*c^2*x^6
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.85

$$\int \frac{(ax^2+bx^3+cx^4)^2}{x^3} dx = \frac{1}{60}x^2(30a^2+40abx+15(b^2+2ac)x^2+24bcx^3+10c^2x^4)$$

input
```
Integrate[(a*x^2 + b*x^3 + c*x^4)^2/x^3,x]
```

output
```
(x^2*(30*a^2 + 40*a*b*x + 15*(b^2 + 2*a*c)*x^2 + 24*b*c*x^3 + 10*c^2*x^4))
/60
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {9, 1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax^2+bx^3+cx^4)^2}{x^3}\,dx$$

$$\Big\downarrow \text{ 9}$$

$$\int x\left(a+bx+cx^2\right)^2dx$$

$$\Big\downarrow \text{ 1140}$$

$$\int \left(a^2x+x^3\left(2ac+b^2\right)+2abx^2+2bcx^4+c^2x^5\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{a^2x^2}{2}+\frac{1}{4}x^4\left(2ac+b^2\right)+\frac{2}{3}abx^3+\frac{2}{5}bcx^5+\frac{c^2x^6}{6}$$

input
```
Int[(a*x^2 + b*x^3 + c*x^4)^2/x^3,x]
```

output
```
(a^2*x^2)/2 + (2*a*b*x^3)/3 + ((b^2 + 2*a*c)*x^4)/4 + (2*b*c*x^5)/5 + (c^2
*x^6)/6
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.10 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{a^2x^2}{2}+\frac{2a\,x^3b}{3}+\frac{(2ac+b^2)x^4}{4}+\frac{2bc\,x^5}{5}+\frac{c^2x^6}{6}$	45
gosper	$\frac{x^2(10c^2x^4+24bc\,x^3+30ac\,x^2+15b^2x^2+40abx+30a^2)}{60}$	47
risch	$\frac{1}{2}a^2x^2+\frac{2}{3}a\,x^3b+\frac{1}{2}x^4ac+\frac{1}{4}b^2x^4+\frac{2}{5}bc\,x^5+\frac{1}{6}c^2x^6$	47
parallelrisch	$\frac{1}{2}a^2x^2+\frac{2}{3}a\,x^3b+\frac{1}{2}x^4ac+\frac{1}{4}b^2x^4+\frac{2}{5}bc\,x^5+\frac{1}{6}c^2x^6$	47
norman	$\frac{\left(\frac{ac}{2}+\frac{b^2}{4}\right)x^6+\frac{a^2x^4}{2}+\frac{x^8c^2}{6}+\frac{2ab\,x^5}{3}+\frac{2bc\,x^7}{5}}{x^2}$	50
orering	$\frac{(10c^2x^4+24bc\,x^3+30ac\,x^2+15b^2x^2+40abx+30a^2)(c\,x^4+b\,x^3+a\,x^2)^2}{60x^2(c\,x^2+bx+a)^2}$	77

input
```
int((c*x^4+b*x^3+a*x^2)^2/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*a^2*x^2+2/3*a*x^3*b+1/4*(2*a*c+b^2)*x^4+2/5*b*c*x^5+1/6*c^2*x^6
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int \frac{(ax^2+bx^3+cx^4)^2}{x^3}\,dx = \frac{1}{6}\,c^2x^6+\frac{2}{5}\,bcx^5+\frac{2}{3}\,abx^3+\frac{1}{4}\,(b^2+2\,ac)x^4+\frac{1}{2}\,a^2x^2$$

input
```
integrate((c*x^4+b*x^3+a*x^2)^2/x^3,x, algorithm="fricas")
```

output
```
1/6*c^2*x^6 + 2/5*b*c*x^5 + 2/3*a*b*x^3 + 1/4*(b^2 + 2*a*c)*x^4 + 1/2*a^2*
x^2
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.91

$$\int \frac{(ax^2+bx^3+cx^4)^2}{x^3}\,dx = \frac{a^2x^2}{2} + \frac{2abx^3}{3} + \frac{2bcx^5}{5} + \frac{c^2x^6}{6} + x^4\left(\frac{ac}{2} + \frac{b^2}{4}\right)$$

input
```
integrate((c*x**4+b*x**3+a*x**2)**2/x**3,x)
```

output
```
a**2*x**2/2 + 2*a*b*x**3/3 + 2*b*c*x**5/5 + c**2*x**6/6 + x**4*(a*c/2 + b*
*2/4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int \frac{(ax^2+bx^3+cx^4)^2}{x^3}\,dx = \frac{1}{6}\,c^2x^6 + \frac{2}{5}\,bcx^5 + \frac{2}{3}\,abx^3 + \frac{1}{4}\,(b^2+2\,ac)x^4 + \frac{1}{2}\,a^2x^2$$

input
```
integrate((c*x^4+b*x^3+a*x^2)^2/x^3,x, algorithm="maxima")
```

output
```
1/6*c^2*x^6 + 2/5*b*c*x^5 + 2/3*a*b*x^3 + 1/4*(b^2 + 2*a*c)*x^4 + 1/2*a^2*
x^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.85

$$\int \frac{(ax^2+bx^3+cx^4)^2}{x^3}\,dx = \frac{1}{6}c^2x^6+\frac{2}{5}bcx^5+\frac{1}{4}b^2x^4+\frac{1}{2}acx^4+\frac{2}{3}abx^3+\frac{1}{2}a^2x^2$$

input
```
integrate((c*x^4+b*x^3+a*x^2)^2/x^3,x, algorithm="giac")
```

output
```
1/6*c^2*x^6 + 2/5*b*c*x^5 + 1/4*b^2*x^4 + 1/2*a*c*x^4 + 2/3*a*b*x^3 + 1/2*
a^2*x^2
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.83

$$\int \frac{(ax^2+bx^3+cx^4)^2}{x^3}\,dx = x^4\left(\frac{b^2}{4}+\frac{a\,c}{2}\right)+\frac{a^2\,x^2}{2}+\frac{c^2\,x^6}{6}+\frac{2\,a\,b\,x^3}{3}+\frac{2\,b\,c\,x^5}{5}$$

input
```
int((a*x^2 + b*x^3 + c*x^4)^2/x^3,x)
```

output
```
x^4*((a*c)/2 + b^2/4) + (a^2*x^2)/2 + (c^2*x^6)/6 + (2*a*b*x^3)/3 + (2*b*c
*x^5)/5
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.85

$$\int \frac{(ax^2+bx^3+cx^4)^2}{x^3}\,dx = \frac{x^2(10c^2x^4+24bc\,x^3+30ac\,x^2+15b^2x^2+40abx+30a^2)}{60}$$

input
```
int((c*x^4+b*x^3+a*x^2)^2/x^3,x)
```

output
```
(x**2*(30*a**2 + 40*a*b*x + 30*a*c*x**2 + 15*b**2*x**2 + 24*b*c*x**3 + 10*
c**2*x**4))/60
```

3.23 $\int \frac{(ax^2+bx^3+cx^4)^2}{x^4}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 46

$$\int \frac{(ax^2+bx^3+cx^4)^2}{x^4}\,dx = a^2x+abx^2+\frac{1}{3}(b^2+2ac)\,x^3+\frac{1}{2}bcx^4+\frac{c^2x^5}{5}$$

output
```
a^2*x+a*b*x^2+1/3*(2*a*c+b^2)*x^3+1/2*b*c*x^4+1/5*c^2*x^5
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \frac{(ax^2+bx^3+cx^4)^2}{x^4}\,dx = a^2x+abx^2+\frac{1}{3}(b^2+2ac)\,x^3+\frac{1}{2}bcx^4+\frac{c^2x^5}{5}$$

input
```
Integrate[(a*x^2 + b*x^3 + c*x^4)^2/x^4,x]
```

output
```
a^2*x + a*b*x^2 + ((b^2 + 2*a*c)*x^3)/3 + (b*c*x^4)/2 + (c^2*x^5)/5
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {9, 1085, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax^2+bx^3+cx^4)^2}{x^4}\,dx$$

$$\Big\downarrow \; 9$$

$$\int (a+bx+cx^2)^2\,dx$$

$$\Big\downarrow \; 1085$$

$$\int \left(a^2+b^2x^2\left(\frac{2ac}{b^2}+1\right)+2abx+2bcx^3+c^2x^4\right)dx$$

$$\Big\downarrow \; 2009$$

$$a^2x+\frac{1}{3}x^3(2ac+b^2)+abx^2+\frac{1}{2}bcx^4+\frac{c^2x^5}{5}$$

input
```
Int[(a*x^2 + b*x^3 + c*x^4)^2/x^4,x]
```

output
```
a^2*x + a*b*x^2 + ((b^2 + 2*a*c)*x^3)/3 + (b*c*x^4)/2 + (c^2*x^5)/5
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 1085
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegr
and[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && IntegerQ[p] && (G
tQ[p, 0] || EqQ[a, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.04 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.89

method	result	size
default	$x\,a^{2}+a b\,x^{2}+\frac{\left(2 a c +b^{2}\right) x^{3}}{3}+\frac{b c\,x^{4}}{2}+\frac{c^{2} x^{5}}{5}$	41
risch	$x\,a^{2}+a b\,x^{2}+\frac{2}{3} a\,x^{3} c +\frac{1}{3} b^{2} x^{3}+\frac{1}{2} b c\,x^{4}+\frac{1}{5} c^{2} x^{5}$	43
parallelrisch	$x\,a^{2}+a b\,x^{2}+\frac{2}{3} a\,x^{3} c +\frac{1}{3} b^{2} x^{3}+\frac{1}{2} b c\,x^{4}+\frac{1}{5} c^{2} x^{5}$	43
gosper	$\frac{x \left(6 c^{2} x^{4}+15 b c\,x^{3}+20 a c\,x^{2}+10 b^{2} x^{2}+30 a b x +30 a^{2}\right)}{30}$	45
norman	$\frac{a^{2} x^{4}+\left(\frac{2 a c}{3}+\frac{b^{2}}{3}\right) x^{6}+a b\,x^{5}+\frac{x^{8} c^{2}}{5}+\frac{b c\,x^{7}}{2}}{x^{3}}$	48
orering	$\frac{\left(6 c^{2} x^{4}+15 b c\,x^{3}+20 a c\,x^{2}+10 b^{2} x^{2}+30 a b x +30 a^{2}\right) \left(c\,x^{4}+b\,x^{3}+a\,x^{2}\right)^{2}}{30 x^{3} \left(c\,x^{2}+b x +a \right)^{2}}$	77

input
```
int((c*x^4+b*x^3+a*x^2)^2/x^4,x,method=_RETURNVERBOSE)
```

output
```
x*a^2+a*b*x^2+1/3*(2*a*c+b^2)*x^3+1/2*b*c*x^4+1/5*c^2*x^5
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.87

$$\int \frac{(ax^2+bx^3+cx^4)^2}{x^4}\,dx = \frac{1}{5}\,c^2x^5+\frac{1}{2}\,bcx^4+abx^2+\frac{1}{3}\,(b^2+2\,ac)x^3+a^2x$$

input
```
integrate((c*x^4+b*x^3+a*x^2)^2/x^4,x, algorithm="fricas")
```

output
```
1/5*c^2*x^5 + 1/2*b*c*x^4 + a*b*x^2 + 1/3*(b^2 + 2*a*c)*x^3 + a^2*x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.91

$$\int \frac{(ax^2+bx^3+cx^4)^2}{x^4}\,dx = a^2x + abx^2 + \frac{bcx^4}{2} + \frac{c^2x^5}{5} + x^3\cdot\left(\frac{2ac}{3}+\frac{b^2}{3}\right)$$

input
```
integrate((c*x**4+b*x**3+a*x**2)**2/x**4,x)
```

output
```
a**2*x + a*b*x**2 + b*c*x**4/2 + c**2*x**5/5 + x**3*(2*a*c/3 + b**2/3)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.87

$$\int \frac{(ax^2+bx^3+cx^4)^2}{x^4}\,dx = \frac{1}{5}\,c^2x^5 + \frac{1}{2}\,bcx^4 + abx^2 + \frac{1}{3}\,(b^2+2\,ac)x^3 + a^2x$$

input
```
integrate((c*x^4+b*x^3+a*x^2)^2/x^4,x, algorithm="maxima")
```

output
```
1/5*c^2*x^5 + 1/2*b*c*x^4 + a*b*x^2 + 1/3*(b^2 + 2*a*c)*x^3 + a^2*x
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.91

$$\int \frac{(ax^2+bx^3+cx^4)^2}{x^4}\,dx = \frac{1}{5}\,c^2x^5 + \frac{1}{2}\,bcx^4 + \frac{1}{3}\,b^2x^3 + \frac{2}{3}\,acx^3 + abx^2 + a^2x$$

input
```
integrate((c*x^4+b*x^3+a*x^2)^2/x^4,x, algorithm="giac")
```

output
```
1/5*c^2*x^5 + 1/2*b*c*x^4 + 1/3*b^2*x^3 + 2/3*a*c*x^3 + a*b*x^2 + a^2*x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.89

$$\int \frac{(ax^2+bx^3+cx^4)^2}{x^4}\,dx = a^2\,x + x^3\left(\frac{b^2}{3}+\frac{2\,a\,c}{3}\right)+\frac{c^2\,x^5}{5}+a\,b\,x^2+\frac{b\,c\,x^4}{2}$$

input
```
int((a*x^2 + b*x^3 + c*x^4)^2/x^4,x)
```

output
```
a^2*x + x^3*((2*a*c)/3 + b^2/3) + (c^2*x^5)/5 + a*b*x^2 + (b*c*x^4)/2
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.96

$$\int \frac{(ax^2+bx^3+cx^4)^2}{x^4}\,dx = \frac{x(6c^2x^4+15bc\,x^3+20ac\,x^2+10b^2x^2+30abx+30a^2)}{30}$$

input
```
int((c*x^4+b*x^3+a*x^2)^2/x^4,x)
```

output
```
(x*(30*a**2 + 30*a*b*x + 20*a*c*x**2 + 10*b**2*x**2 + 15*b*c*x**3 + 6*c**2
*x**4))/30
```

3.24 $\int \frac{(ax^2+bx^3+cx^4)^2}{x^5} dx$

Optimal result

Integrand size = 22, antiderivative size = 46

$$\int \frac{(ax^2+bx^3+cx^4)^2}{x^5} dx = 2abx + \frac{1}{2}(b^2+2ac)\,x^2 + \frac{2}{3}bcx^3 + \frac{c^2x^4}{4} + a^2\log(x)$$

output
```
2*a*b*x+1/2*(2*a*c+b^2)*x^2+2/3*b*c*x^3+1/4*c^2*x^4+a^2*ln(x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.93

$$\int \frac{(ax^2+bx^3+cx^4)^2}{x^5} dx = ax(2b+cx) + \frac{1}{12}x^2\left(6b^2+8bcx+3c^2x^2\right) + a^2\log(x)$$

input
```
Integrate[(a*x^2 + b*x^3 + c*x^4)^2/x^5,x]
```

output
```
a*x*(2*b + c*x) + (x^2*(6*b^2 + 8*b*c*x + 3*c^2*x^2))/12 + a^2*Log[x]
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {9, 1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(ax^2+bx^3+cx^4\right)^2}{x^5}dx$$

$$\Big\downarrow \text{ 9}$$

$$\int \frac{\left(a+bx+cx^2\right)^2}{x}dx$$

$$\Big\downarrow \text{ 1140}$$

$$\int \left(\frac{a^2}{x}+x\left(2ac+b^2\right)+2ab+2bcx^2+c^2x^3\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$a^2\log(x)+\frac{1}{2}x^2\left(2ac+b^2\right)+2abx+\frac{2}{3}bcx^3+\frac{c^2x^4}{4}$$

input
```
Int[(a*x^2 + b*x^3 + c*x^4)^2/x^5,x]
```

output
```
2*a*b*x + ((b^2 + 2*a*c)*x^2)/2 + (2*b*c*x^3)/3 + (c^2*x^4)/4 + a^2*Log[x]
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.08 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.91

method	result	size
default	$\frac{c^2x^4}{4}+\frac{2bc\,x^3}{3}+ac\,x^2+\frac{b^2x^2}{2}+2abx+a^2\ln\left(x\right)$	42
risch	$\frac{c^2x^4}{4}+\frac{2bc\,x^3}{3}+ac\,x^2+\frac{b^2x^2}{2}+2abx+a^2\ln\left(x\right)$	42
parallelrisch	$\frac{c^2x^4}{4}+\frac{2bc\,x^3}{3}+ac\,x^2+\frac{b^2x^2}{2}+2abx+a^2\ln\left(x\right)$	42
norman	$\frac{\left(ac+\frac{b^2}{2}\right)x^6+\frac{x^8c^2}{4}+2ab\,x^5+\frac{2bc\,x^7}{3}}{x^4}+a^2\ln\left(x\right)$	48

input
```
int((c*x^4+b*x^3+a*x^2)^2/x^5,x,method=_RETURNVERBOSE)
```

output
```
1/4*c^2*x^4+2/3*b*c*x^3+a*c*x^2+1/2*b^2*x^2+2*a*b*x+a^2*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.87

$$\int\frac{(ax^2+bx^3+cx^4)^2}{x^5}\,dx=\frac{1}{4}\,c^2x^4+\frac{2}{3}\,bcx^3+2\,abx+\frac{1}{2}\,(b^2+2\,ac)x^2+a^2\log\left(x\right)$$

input
```
integrate((c*x^4+b*x^3+a*x^2)^2/x^5,x, algorithm="fricas")
```

output
```
1/4*c^2*x^4 + 2/3*b*c*x^3 + 2*a*b*x + 1/2*(b^2 + 2*a*c)*x^2 + a^2*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.91

$$\int \frac{(ax^2 + bx^3 + cx^4)^2}{x^5}\,dx = a^2\log(x) + 2abx + \frac{2bcx^3}{3} + \frac{c^2x^4}{4} + x^2\left(ac + \frac{b^2}{2}\right)$$

input
```
integrate((c*x**4+b*x**3+a*x**2)**2/x**5,x)
```

output
```
a**2*log(x) + 2*a*b*x + 2*b*c*x**3/3 + c**2*x**4/4 + x**2*(a*c + b**2/2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.87

$$\int \frac{(ax^2 + bx^3 + cx^4)^2}{x^5}\,dx = \frac{1}{4}\,c^2x^4 + \frac{2}{3}\,bcx^3 + 2\,abx + \frac{1}{2}\,(b^2 + 2\,ac)x^2 + a^2\log(x)$$

input
```
integrate((c*x^4+b*x^3+a*x^2)^2/x^5,x, algorithm="maxima")
```

output
```
1/4*c^2*x^4 + 2/3*b*c*x^3 + 2*a*b*x + 1/2*(b^2 + 2*a*c)*x^2 + a^2*log(x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.91

$$\int \frac{(ax^2 + bx^3 + cx^4)^2}{x^5}\,dx = \frac{1}{4}\,c^2x^4 + \frac{2}{3}\,bcx^3 + \frac{1}{2}\,b^2x^2 + acx^2 + 2\,abx + a^2\log(|x|)$$

input
```
integrate((c*x^4+b*x^3+a*x^2)^2/x^5,x, algorithm="giac")
```

output
```
1/4*c^2*x^4 + 2/3*b*c*x^3 + 1/2*b^2*x^2 + a*c*x^2 + 2*a*b*x + a^2*log(abs(
x))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.87

$$\int \frac{(ax^2+bx^3+cx^4)^2}{x^5}\,dx = a^2\ln(x) + x^2\left(\frac{b^2}{2}+a\,c\right) + \frac{c^2\,x^4}{4} + 2\,a\,b\,x + \frac{2\,b\,c\,x^3}{3}$$

input
```
int((a*x^2 + b*x^3 + c*x^4)^2/x^5,x)
```

output
```
a^2*log(x) + x^2*(a*c + b^2/2) + (c^2*x^4)/4 + 2*a*b*x + (2*b*c*x^3)/3
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.89

$$\int \frac{(ax^2+bx^3+cx^4)^2}{x^5}\,dx = \log(x)\,a^2 + 2abx + ac\,x^2 + \frac{b^2x^2}{2} + \frac{2bc\,x^3}{3} + \frac{c^2x^4}{4}$$

input
```
int((c*x^4+b*x^3+a*x^2)^2/x^5,x)
```

output
```
(12*log(x)*a**2 + 24*a*b*x + 12*a*c*x**2 + 6*b**2*x**2 + 8*b*c*x**3 + 3*c*
*2*x**4)/12
```

3.25 $\int \frac{x^5}{ax^2+bx^3+cx^4}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 89

$$\int \frac{x^5}{ax^2+bx^3+cx^4}\,dx = -\frac{bx}{c^2}+\frac{x^2}{2c}+\frac{b(b^2-3ac)\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^3\sqrt{b^2-4ac}} + \frac{(b^2-ac)\log(a+bx+cx^2)}{2c^3}$$

output
```
-b*x/c^2+1/2*x^2/c+b*(-3*a*c+b^2)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/c^
3/(-4*a*c+b^2)^(1/2)+1/2*(-a*c+b^2)*ln(c*x^2+b*x+a)/c^3
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.94

$$\int \frac{x^5}{ax^2+bx^3+cx^4}\,dx = \frac{cx(-2b+cx)-\frac{2b(b^2-3ac)\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}+(b^2-ac)\log(a+x(b+cx))}{2c^3}$$

input
```
Integrate[x^5/(a*x^2 + b*x^3 + c*x^4),x]
```

output
```
(c*x*(-2*b + c*x) - (2*b*(b^2 - 3*a*c)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*
c]])/Sqrt[-b^2 + 4*a*c] + (b^2 - a*c)*Log[a + x*(b + c*x)])/(2*c^3)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {9, 1143, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{ax^2+bx^3+cx^4}\,dx$$

$$\Big\downarrow \text{9}$$

$$\int \frac{x^3}{a+bx+cx^2}dx$$

$$\Big\downarrow \text{1143}$$

$$\int \left(\frac{x(b^2-ac)+ab}{c^2\,(a+bx+cx^2)}-\frac{b}{c^2}+\frac{x}{c}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{b(b^2-3ac)\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^3\sqrt{b^2-4ac}}+\frac{(b^2-ac)\log\left(a+bx+cx^2\right)}{2c^3}-\frac{bx}{c^2}+\frac{x^2}{2c}$$

input
```
Int[x^5/(a*x^2 + b*x^3 + c*x^4),x]
```

output
```
-((b*x)/c^2) + x^2/(2*c) + (b*(b^2 - 3*a*c)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 -
 4*a*c]])/(c^3*Sqrt[b^2 - 4*a*c]) + ((b^2 - a*c)*Log[a + b*x + c*x^2])/(2*
c^3)
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 1143
```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
 :> Int[ExpandIntegrand[(d + e*x)^m/(a + b*x + c*x^2), x], x] /; FreeQ[{a,
b, c, d, e}, x] && IGtQ[m, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.10

method	result
default	$-\frac{-\frac{1}{2}c\,x^2+bx}{c^2}+\frac{\frac{\left(-ac+b^2\right)\ln\left(c\,x^2+bx+a\right)}{2c}+\frac{2\left(ab-\frac{\left(-ac+b^2\right)b}{2c}\right)\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{c^2}$
risch	$\frac{x^2}{2c}-\frac{bx}{c^2}-\frac{2\ln\left(12a^2b\,c^2-7a\,b^3c+b^5-2\sqrt{-b^2(4ac-b^2)(3ac-b^2)^2}\,cx-\sqrt{-b^2(4ac-b^2)(3ac-b^2)^2}\,b\right)a^2}{c(4ac-b^2)}+5\ln\Big(12a^2b\,c^2-7a\,b^3$

input
```
int(x^5/(c*x^4+b*x^3+a*x^2),x,method=_RETURNVERBOSE)
```

output
```
-1/c^2*(-1/2*c*x^2+b*x)+1/c^2*(1/2*(-a*c+b^2)/c*ln(c*x^2+b*x+a)+2*(a*b-1/2
*(-a*c+b^2)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 297, normalized size of antiderivative = 3.34

$$\int \frac{x^5}{ax^2+bx^3+cx^4}\,dx = \left[\frac{(b^2c^2-4\,ac^3)x^2-(b^3-3\,abc)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac-\sqrt{b^2-4\,ac}(2\,cx+b)}{cx^2+bx+a}\right)-2\,(b^3c-4\,abc^2)x+}{2\,(b^2c^3-4\,ac^4)}\right.$$

input
```
integrate(x^5/(c*x^4+b*x^3+a*x^2),x, algorithm="fricas")
```

output
```
[1/2*((b^2*c^2 - 4*a*c^3)*x^2 - (b^3 - 3*a*b*c)*sqrt(b^2 - 4*a*c)*log((2*c
^2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b
*x + a)) - 2*(b^3*c - 4*a*b*c^2)*x + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*log(c*x
^2 + b*x + a))/(b^2*c^3 - 4*a*c^4), 1/2*((b^2*c^2 - 4*a*c^3)*x^2 + 2*(b^3
- 3*a*b*c)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2
- 4*a*c)) - 2*(b^3*c - 4*a*b*c^2)*x + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*log(c*
x^2 + b*x + a))/(b^2*c^3 - 4*a*c^4)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 381 vs. $2(83) = 166$.

Time = 0.48 (sec) , antiderivative size = 381, normalized size of antiderivative = 4.28

$$\int \frac{x^5}{ax^2+bx^3+cx^4}\,dx = -\frac{bx}{c^2} + \left(-\frac{b\sqrt{-4ac+b^2}\cdot(3ac-b^2)}{2c^3\cdot(4ac-b^2)} - \frac{ac-b^2}{2c^3}\right)\log\left(x + \frac{2a^2c-ab^2+4ac^3\left(-\frac{b\sqrt{-4ac+b^2}\cdot(3ac-b^2)}{2c^3\cdot(4ac-b^2)}-\frac{ac-b^2}{2c^3}\right)-b^2c^2\left(-\frac{b\sqrt{-4ac+b^2}\cdot(3ac-b^2)}{2c^3\cdot(4ac-b^2)}-\frac{ac-b}{2c^3}\right.}{3abc-b^3}\right.$$
$$+\left(\frac{b\sqrt{-4ac+b^2}\cdot(3ac-b^2)}{2c^3\cdot(4ac-b^2)} - \frac{ac-b^2}{2c^3}\right)\log\left(x + \frac{2a^2c-ab^2+4ac^3\left(\frac{b\sqrt{-4ac+b^2}\cdot(3ac-b^2)}{2c^3\cdot(4ac-b^2)}-\frac{ac-b^2}{2c^3}\right)-b^2c^2\left(\frac{b\sqrt{-4ac+b^2}\cdot(3ac-b^2)}{2c^3\cdot(4ac-b^2)}-\frac{ac-b^2}{2c^3}\right)}{3abc-b^3}\right)$$
$$+\frac{x^2}{2c}$$

input
```
integrate(x**5/(c*x**4+b*x**3+a*x**2),x)
```

output
```
-b*x/c**2 + (-b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/(2*c**3*(4*a*c - b**2))
 - (a*c - b**2)/(2*c**3))*log(x + (2*a**2*c - a*b**2 + 4*a*c**3*(-b*sqrt(-
4*a*c + b**2)*(3*a*c - b**2)/(2*c**3*(4*a*c - b**2)) - (a*c - b**2)/(2*c**
3)) - b**2*c**2*(-b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/(2*c**3*(4*a*c - b*
*2)) - (a*c - b**2)/(2*c**3)))/(3*a*b*c - b**3)) + (b*sqrt(-4*a*c + b**2)*
(3*a*c - b**2)/(2*c**3*(4*a*c - b**2)) - (a*c - b**2)/(2*c**3))*log(x + (2
*a**2*c - a*b**2 + 4*a*c**3*(b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/(2*c**3*
(4*a*c - b**2)) - (a*c - b**2)/(2*c**3)) - b**2*c**2*(b*sqrt(-4*a*c + b**2
)*(3*a*c - b**2)/(2*c**3*(4*a*c - b**2)) - (a*c - b**2)/(2*c**3)))/(3*a*b*
c - b**3)) + x**2/(2*c)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5}{ax^2 + bx^3 + cx^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^5/(c*x^4+b*x^3+a*x^2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.97

$$\int \frac{x^5}{ax^2 + bx^3 + cx^4}\,dx = \frac{cx^2 - 2\,bx}{2\,c^2} + \frac{(b^2 - ac)\log\left(cx^2 + bx + a\right)}{2\,c^3} - \frac{(b^3 - 3\,abc)\arctan\left(\frac{2\,cx + b}{\sqrt{-b^2 + 4\,ac}}\right)}{\sqrt{-b^2 + 4\,ac}c^3}$$

input
```
integrate(x^5/(c*x^4+b*x^3+a*x^2),x, algorithm="giac")
```

output
```
1/2*(c*x^2 - 2*b*x)/c^2 + 1/2*(b^2 - a*c)*log(c*x^2 + b*x + a)/c^3 - (b^3
- 3*a*b*c)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^3)
```

Mupad [B] (verification not implemented)

Time = 21.05 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.26

$$\int \frac{x^5}{ax^2+bx^3+cx^4}\,dx = \frac{x^2}{2\,c} - \frac{\ln\left(c\,x^2+b\,x+a\right)\,\left(4\,a^2\,c^2-5\,a\,b^2\,c+b^4\right)}{2\,\left(4\,a\,c^4-b^2\,c^3\right)} - \frac{b\,x}{c^2} + \frac{b\,\mathrm{atan}\left(\frac{b+2\,c\,x}{\sqrt{4\,a\,c-b^2}}\right)\,\left(3\,a\,c-b^2\right)}{c^3\,\sqrt{4\,a\,c-b^2}}$$

input
```
int(x^5/(a*x^2 + b*x^3 + c*x^4),x)
```

output
```
x^2/(2*c) - (log(a + b*x + c*x^2)*(b^4 + 4*a^2*c^2 - 5*a*b^2*c))/(2*(4*a*c
^4 - b^2*c^3)) - (b*x)/c^2 + (b*atan((b + 2*c*x)/(4*a*c - b^2)^(1/2))*(3*a
*c - b^2))/(c^3*(4*a*c - b^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 180, normalized size of antiderivative = 2.02

$$\int \frac{x^5}{ax^2+bx^3+cx^4}\,dx = \frac{6\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)abc - 2\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)b^3 - 4\log(c\,x^2+bx+a)\,a^2c^2 + 5\log(c\,x^2+}{2c^3\left(4ac-b^2\right)}$$

input
```
int(x^5/(c*x^4+b*x^3+a*x^2),x)
```

output
```
(6*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b*c - 2*sqrt(
4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b**3 - 4*log(a + b*x +
c*x**2)*a**2*c**2 + 5*log(a + b*x + c*x**2)*a*b**2*c - log(a + b*x + c*x**
2)*b**4 - 8*a*b*c**2*x + 4*a*c**3*x**2 + 2*b**3*c*x - b**2*c**2*x**2)/(2*c
**3*(4*a*c - b**2))
```

3.26 $\int \frac{x^4}{ax^2+bx^3+cx^4}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 70

$$\int \frac{x^4}{ax^2+bx^3+cx^4}\,dx = \frac{x}{c} - \frac{(b^2-2ac)\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}} - \frac{b\log\left(a+bx+cx^2\right)}{2c^2}$$

output
```
x/c-(-2*a*c+b^2)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/c^2/(-4*a*c+b^2)^(1
/2)-1/2*b*ln(c*x^2+b*x+a)/c^2
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.04

$$\int \frac{x^4}{ax^2+bx^3+cx^4}\,dx = \frac{x}{c} + \frac{(b^2-2ac)\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{c^2\sqrt{-b^2+4ac}} - \frac{b\log\left(a+bx+cx^2\right)}{2c^2}$$

input
```
Integrate[x^4/(a*x^2 + b*x^3 + c*x^4),x]
```

output
```
x/c + ((b^2 - 2*a*c)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(c^2*Sqrt[-b^
2 + 4*a*c]) - (b*Log[a + b*x + c*x^2])/(2*c^2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {9, 1143, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{ax^2+bx^3+cx^4}\,dx$$
$$\Big\downarrow \text{ 9}$$
$$\int \frac{x^2}{a+bx+cx^2}dx$$
$$\Big\downarrow \text{ 1143}$$
$$\int \left(\frac{1}{c}-\frac{a+bx}{c\,(a+bx+cx^2)}\right)dx$$
$$\Big\downarrow \text{ 2009}$$
$$-\frac{(b^2-2ac)\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}}-\frac{b\log\left(a+bx+cx^2\right)}{2c^2}+\frac{x}{c}$$

input
```
Int[x^4/(a*x^2 + b*x^3 + c*x^4),x]
```

output
```
x/c - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^2*Sqrt[b^2
 - 4*a*c]) - (b*Log[a + b*x + c*x^2])/(2*c^2)
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 1143
```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
 :> Int[ExpandIntegrand[(d + e*x)^m/(a + b*x + c*x^2), x], x] /; FreeQ[{a,
b, c, d, e}, x] && IGtQ[m, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.07

method	result
default	$\frac{x}{c}+\frac{-\frac{b\ln\left(c\,x^{2}+bx+a\right)}{2c}+\frac{2\left(-a+\frac{b^{2}}{2c}\right)\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^{2}}}\right)}{\sqrt{4ac-b^{2}}}}{c}$
risch	$\frac{x}{c}-\frac{2\ln\left(-8a^{2}c^{2}+6a\,b^{2}c-b^{4}-2\sqrt{-(4ac-b^{2})(2ac-b^{2})^{2}}\,cx-\sqrt{-(4ac-b^{2})(2ac-b^{2})^{2}}\,b\right)ab}{c(4ac-b^{2})}+\frac{\ln\left(-8a^{2}c^{2}+6a\,b^{2}c-b^{4}-2\sqrt{-(4a\right.}{}$

input
```
int(x^4/(c*x^4+b*x^3+a*x^2),x,method=_RETURNVERBOSE)
```

output
```
x/c+1/c*(-1/2*b/c*ln(c*x^2+b*x+a)+2*(-a+1/2*b^2/c)/(4*a*c-b^2)^(1/2)*arcta
n((2*c*x+b)/(4*a*c-b^2)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 235, normalized size of antiderivative = 3.36

$$\int\frac{x^4}{ax^2+bx^3+cx^4}\,dx$$
$$=\left[-\frac{(b^2-2\,ac)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac+\sqrt{b^2-4\,ac}(2\,cx+b)}{cx^2+bx+a}\right)-2\,(b^2c-4\,ac^2)x+(b^3-4\,abc)\log\left(cx^{2}\right.}{2\,(b^2c^2-4\,ac^3)}\right.$$
$$-\frac{2\,(b^2-2\,ac)\sqrt{-b^2+4\,ac}\arctan\left(-\frac{\sqrt{-b^2+4\,ac}(2\,cx+b)}{b^2-4\,ac}\right)-2\,(b^2c-4\,ac^2)x+(b^3-4\,abc)\log\left(cx^2+bx+\right.}{2\,(b^2c^2-4\,ac^3)}$$

input
```
integrate(x^4/(c*x^4+b*x^3+a*x^2),x, algorithm="fricas")
```

output
```
[-1/2*((b^2 - 2*a*c)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*
a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) - 2*(b^2*c - 4*a*c
^2)*x + (b^3 - 4*a*b*c)*log(c*x^2 + b*x + a))/(b^2*c^2 - 4*a*c^3), -1/2*(2
*(b^2 - 2*a*c)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(
b^2 - 4*a*c)) - 2*(b^2*c - 4*a*c^2)*x + (b^3 - 4*a*b*c)*log(c*x^2 + b*x +
a))/(b^2*c^2 - 4*a*c^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 306 vs. $2(65) = 130$.

Time = 0.36 (sec) , antiderivative size = 306, normalized size of antiderivative = 4.37

$$\int \frac{x^4}{ax^2+bx^3+cx^4}\,dx = \left(-\frac{b}{2c^2} - \frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{2c^2\cdot(4ac-b^2)}\right)\log\left(x+\frac{-ab-4ac^2\left(-\frac{b}{2c^2}-\frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{2c^2\cdot(4ac-b^2)}\right)+b^2c\left(-\frac{b}{2c^2}-\frac{\sqrt{-4ac+b^2}\cdot(2ac}{2c^2\cdot(4ac-b}\right.}{2ac-b^2}\right. + \left(-\frac{b}{2c^2} + \frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{2c^2\cdot(4ac-b^2)}\right)\log\left(x+\frac{-ab-4ac^2\left(-\frac{b}{2c^2}+\frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{2c^2\cdot(4ac-b^2)}\right)+b^2c\left(-\frac{b}{2c^2}+\frac{\sqrt{-4ac+b^2}\cdot(2a}{2c^2\cdot(4ac-b}\right.}{2ac-b^2}\right. + \frac{x}{c}$$

input
```
integrate(x**4/(c*x**4+b*x**3+a*x**2),x)
```

output
```
(-b/(2*c**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(2*c**2*(4*a*c - b**2)))
*log(x + (-a*b - 4*a*c**2*(-b/(2*c**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2
)/(2*c**2*(4*a*c - b**2))) + b**2*c*(-b/(2*c**2) - sqrt(-4*a*c + b**2)*(2*
a*c - b**2)/(2*c**2*(4*a*c - b**2))))/(2*a*c - b**2)) + (-b/(2*c**2) + sqr
t(-4*a*c + b**2)*(2*a*c - b**2)/(2*c**2*(4*a*c - b**2)))*log(x + (-a*b - 4
*a*c**2*(-b/(2*c**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(2*c**2*(4*a*c -
 b**2))) + b**2*c*(-b/(2*c**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(2*c**
2*(4*a*c - b**2))))/(2*a*c - b**2)) + x/c
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^4}{ax^2 + bx^3 + cx^4} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^4/(c*x^4+b*x^3+a*x^2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.96

$$\int \frac{x^4}{ax^2 + bx^3 + cx^4} \, dx = \frac{x}{c} - \frac{b \log(cx^2 + bx + a)}{2 c^2} + \frac{(b^2 - 2 ac) \arctan\left(\frac{2 cx + b}{\sqrt{-b^2 + 4 ac}}\right)}{\sqrt{-b^2 + 4 ac} c^2}$$

input
```
integrate(x^4/(c*x^4+b*x^3+a*x^2),x, algorithm="giac")
```

output
```
x/c - 1/2*b*log(c*x^2 + b*x + a)/c^2 + (b^2 - 2*a*c)*arctan((2*c*x + b)/sq
rt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^2)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 172, normalized size of antiderivative = 2.46

$$\int \frac{x^4}{ax^2+bx^3+cx^4}\,dx = \frac{x}{c}+\frac{b^3\,\ln\left(c\,x^2+b\,x+a\right)}{2\,\left(4\,a\,c^3-b^2\,c^2\right)}-\frac{2\,a\,\mathrm{atan}\left(\frac{b}{\sqrt{4\,a\,c-b^2}}+\frac{2\,c\,x}{\sqrt{4\,a\,c-b^2}}\right)}{c\,\sqrt{4\,a\,c-b^2}} + \frac{b^2\,\mathrm{atan}\left(\frac{b}{\sqrt{4\,a\,c-b^2}}+\frac{2\,c\,x}{\sqrt{4\,a\,c-b^2}}\right)}{c^2\,\sqrt{4\,a\,c-b^2}}-\frac{2\,a\,b\,c\,\ln\left(c\,x^2+b\,x+a\right)}{4\,a\,c^3-b^2\,c^2}$$

input
```
int(x^4/(a*x^2 + b*x^3 + c*x^4),x)
```

output
```
x/c + (b^3*log(a + b*x + c*x^2))/(2*(4*a*c^3 - b^2*c^2)) - (2*a*atan(b/(4*
a*c - b^2)^(1/2) + (2*c*x)/(4*a*c - b^2)^(1/2)))/(c*(4*a*c - b^2)^(1/2)) +
 (b^2*atan(b/(4*a*c - b^2)^(1/2) + (2*c*x)/(4*a*c - b^2)^(1/2)))/(c^2*(4*a
*c - b^2)^(1/2)) - (2*a*b*c*log(a + b*x + c*x^2))/(4*a*c^3 - b^2*c^2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.94

$$\int \frac{x^4}{ax^2+bx^3+cx^4}\,dx = \frac{-4\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)ac+2\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)b^2-4\log\left(c\,x^2+bx+a\right)abc+\log\left(c\,x^2+b\right.}{2c^2\left(4ac-b^2\right)}$$

input
```
int(x^4/(c*x^4+b*x^3+a*x^2),x)
```

output
```
( - 4*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*c + 2*sqrt
(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b**2 - 4*log(a + b*x +
 c*x**2)*a*b*c + log(a + b*x + c*x**2)*b**3 + 8*a*c**2*x - 2*b**2*c*x)/(2*
c**2*(4*a*c - b**2))
```

3.27 $\int \frac{x^3}{ax^2+bx^3+cx^4} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 56

$$\int \frac{x^3}{ax^2+bx^3+cx^4} \, dx = \frac{b \operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}} + \frac{\log(a+bx+cx^2)}{2c}$$

output
```
b*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/c/(-4*a*c+b^2)^(1/2)+1/2*ln(c*x^2+
b*x+a)/c
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.02

$$\int \frac{x^3}{ax^2+bx^3+cx^4} \, dx = \frac{-\frac{2b \arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}} + \log(a+x(b+cx))}{2c}$$

input
```
Integrate[x^3/(a*x^2 + b*x^3 + c*x^4),x]
```

output
```
((-2*b*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + Log[a
+ x*(b + c*x)])/(2*c)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {9, 1142, 1083, 219, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{ax^2+bx^3+cx^4}dx$$

$$\Big\downarrow \text{ 9}$$

$$\int \frac{x}{a+bx+cx^2}dx$$

$$\Big\downarrow \text{ 1142}$$

$$\frac{\int \frac{b+2cx}{cx^2+bx+a}dx}{2c}-\frac{b\int \frac{1}{cx^2+bx+a}dx}{2c}$$

$$\Big\downarrow \text{ 1083}$$

$$\frac{b\int \frac{1}{b^2-(b+2cx)^2-4ac}d(b+2cx)}{c}+\frac{\int \frac{b+2cx}{cx^2+bx+a}dx}{2c}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\int \frac{b+2cx}{cx^2+bx+a}dx}{2c}+\frac{b\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}}$$

$$\Big\downarrow \text{ 1103}$$

$$\frac{b\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}}+\frac{\log\left(a+bx+cx^2\right)}{2c}$$

input
```
Int[x^3/(a*x^2 + b*x^3 + c*x^4),x]
```

output
```
(b*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c*Sqrt[b^2 - 4*a*c]) + Log[a +
 b*x + c*x^2]/(2*c)
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00

method	result
default	$\frac{\ln\left(c\,x^{2}+b x+a\right)}{2 c}-\frac{b \arctan\left(\frac{2 c x+b}{\sqrt{4 a c-b^{2}}}\right)}{c\sqrt{4 a c-b^{2}}}$
risch	$\frac{2\ln\left(-2\sqrt{-b^{2}\left(4 a c-b^{2}\right)}\,c x-4 a b c+b^{3}-\sqrt{-b^{2}\left(4 a c-b^{2}\right)}\,b\right)a}{4 a c-b^{2}}-\frac{\ln\left(-2\sqrt{-b^{2}\left(4 a c-b^{2}\right)}\,c x-4 a b c+b^{3}-\sqrt{-b^{2}\left(4 a c-b^{2}\right)}\,b\right)b^{2}}{2 c\left(4 a c-b^{2}\right)}+\ln\Big(-$

input
```
int(x^3/(c*x^4+b*x^3+a*x^2),x,method=_RETURNVERBOSE)
```

output
```
1/2*ln(c*x^2+b*x+a)/c-b/c/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(
1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 185, normalized size of antiderivative = 3.30

$$\int \frac{x^3}{ax^2 + bx^3 + cx^4}\,dx$$
$$= \left[\frac{\sqrt{b^2-4\,ac}b\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac+\sqrt{b^2-4\,ac}(2\,cx+b)}{cx^2+bx+a}\right)+(b^2-4\,ac)\log\left(cx^2+bx+a\right)}{2\,(b^2c-4\,ac^2)}, \frac{2\,\sqrt{-b^2+4\,ac}b\,\mathrm{arc}\right.$$

input
```
integrate(x^3/(c*x^4+b*x^3+a*x^2),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(b^2 - 4*a*c)*b*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^
2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + (b^2 - 4*a*c)*log(c*x^2 + b*x
 + a))/(b^2*c - 4*a*c^2), 1/2*(2*sqrt(-b^2 + 4*a*c)*b*arctan(-sqrt(-b^2 +
4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + (b^2 - 4*a*c)*log(c*x^2 + b*x + a))/(b
^2*c - 4*a*c^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 216 vs. $2(49) = 98$.

Time = 0.19 (sec) , antiderivative size = 216, normalized size of antiderivative = 3.86

$$\int \frac{x^3}{ax^2+bx^3+cx^4}\,dx = \left(-\frac{b\sqrt{-4ac+b^2}}{2c\,(4ac-b^2)} + \frac{1}{2c}\right)\log\left(x + \frac{-4ac\left(-\frac{b\sqrt{-4ac+b^2}}{2c(4ac-b^2)}+\frac{1}{2c}\right)+2a+b^2\left(-\frac{b\sqrt{-4ac+b^2}}{2c(4ac-b^2)}+\frac{1}{2c}\right)}{b}\right) + \left(\frac{b\sqrt{-4ac+b^2}}{2c\,(4ac-b^2)} + \frac{1}{2c}\right)\log\left(x + \frac{-4ac\left(\frac{b\sqrt{-4ac+b^2}}{2c(4ac-b^2)}+\frac{1}{2c}\right)+2a+b^2\left(\frac{b\sqrt{-4ac+b^2}}{2c(4ac-b^2)}+\frac{1}{2c}\right)}{b}\right)$$

input
```
integrate(x**3/(c*x**4+b*x**3+a*x**2),x)
```

output
```
(-b*sqrt(-4*a*c + b**2)/(2*c*(4*a*c - b**2)) + 1/(2*c))*log(x + (-4*a*c*(-
b*sqrt(-4*a*c + b**2)/(2*c*(4*a*c - b**2)) + 1/(2*c)) + 2*a + b**2*(-b*sqr
t(-4*a*c + b**2)/(2*c*(4*a*c - b**2)) + 1/(2*c)))/b) + (b*sqrt(-4*a*c + b*
*2)/(2*c*(4*a*c - b**2)) + 1/(2*c))*log(x + (-4*a*c*(b*sqrt(-4*a*c + b**2)
/(2*c*(4*a*c - b**2)) + 1/(2*c)) + 2*a + b**2*(b*sqrt(-4*a*c + b**2)/(2*c*
(4*a*c - b**2)) + 1/(2*c)))/b)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3}{ax^2+bx^3+cx^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3/(c*x^4+b*x^3+a*x^2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.98

$$\int \frac{x^3}{ax^2+bx^3+cx^4}\,dx=-\frac{b\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,acc}}+\frac{\log\left(cx^2+bx+a\right)}{2\,c}$$

input
```
integrate(x^3/(c*x^4+b*x^3+a*x^2),x, algorithm="giac")
```

output
```
-b*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c) + 1/2*log
(c*x^2 + b*x + a)/c
```

Mupad [B] (verification not implemented)

Time = 20.94 (sec) , antiderivative size = 112, normalized size of antiderivative = 2.00

$$\int \frac{x^3}{ax^2+bx^3+cx^4}\,dx=\frac{2\,a\,c\,\ln\left(c\,x^2+b\,x+a\right)}{4\,a\,c^2-b^2\,c}-\frac{b\,\mathrm{atan}\left(\frac{b}{\sqrt{4\,a\,c-b^2}}+\frac{2\,c\,x}{\sqrt{4\,a\,c-b^2}}\right)}{c\,\sqrt{4\,a\,c-b^2}}-\frac{b^2\,\ln\left(c\,x^2+b\,x+a\right)}{2\,\left(4\,a\,c^2-b^2\,c\right)}$$

input
```
int(x^3/(a*x^2 + b*x^3 + c*x^4),x)
```

output
```
(2*a*c*log(a + b*x + c*x^2))/(4*a*c^2 - b^2*c) - (b*atan(b/(4*a*c - b^2)^(
1/2) + (2*c*x)/(4*a*c - b^2)^(1/2)))/(c*(4*a*c - b^2)^(1/2)) - (b^2*log(a
+ b*x + c*x^2))/(2*(4*a*c^2 - b^2*c))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.50

$$\int \frac{x^3}{ax^2+bx^3+cx^4}\,dx$$
$$= \frac{-2\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)b+4\log(c\,x^2+bx+a)\,ac-\log(c\,x^2+bx+a)\,b^2}{2c\,(4ac-b^2)}$$

input
```
int(x^3/(c*x^4+b*x^3+a*x^2),x)
```

output
```
( - 2*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b + 4*log(a
+ b*x + c*x**2)*a*c - log(a + b*x + c*x**2)*b**2)/(2*c*(4*a*c - b**2))
```

3.28 $\int \frac{x^2}{ax^2+bx^3+cx^4}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 34

$$\int \frac{x^2}{ax^2+bx^3+cx^4}\,dx = -\frac{2\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

output
```
-2*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.12

$$\int \frac{x^2}{ax^2+bx^3+cx^4}\,dx = \frac{2\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}$$

input
```
Integrate[x^2/(a*x^2 + b*x^3 + c*x^4),x]
```

output
```
(2*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c]
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {9, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{ax^2+bx^3+cx^4}dx$$
$$\Big\downarrow \text{ 9}$$
$$\int \frac{1}{a+bx+cx^2}dx$$
$$\Big\downarrow \text{ 1083}$$
$$-2\int \frac{1}{b^2-(b+2cx)^2-4ac}d(b+2cx)$$
$$\Big\downarrow \text{ 219}$$
$$-\frac{2\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

input
```
Int[x^2/(a*x^2 + b*x^3 + c*x^4),x]
```

output
```
(-2*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/Sqrt[b^2 - 4*a*c]
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p, x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] && !MonomialQ[Px, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.03

method	result	size
default	$\frac{2\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}$	35
risch	$-\frac{\ln\left(2cx+\sqrt{-4ac+b^2}+b\right)}{\sqrt{-4ac+b^2}}+\frac{\ln\left(-2cx+\sqrt{-4ac+b^2}-b\right)}{\sqrt{-4ac+b^2}}$	61

input
```
int(x^2/(c*x^4+b*x^3+a*x^2),x,method=_RETURNVERBOSE)
```

output
```
2/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 120, normalized size of antiderivative = 3.53

$$\int \frac{x^2}{ax^2+bx^3+cx^4}\,dx = \left[\frac{\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac-\sqrt{b^2-4\,ac}(2\,cx+b)}{cx^2+bx+a}\right)}{\sqrt{b^2-4\,ac}}, -\frac{2\,\sqrt{-b^2+4\,ac}\arctan\left(-\frac{\sqrt{-b^2+4\,ac}(2\,cx+b)}{b^2-4\,ac}\right)}{b^2-4\,ac}\right]$$

input
```
integrate(x^2/(c*x^4+b*x^3+a*x^2),x, algorithm="fricas")
```

output
```
[log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(
c*x^2 + b*x + a))/sqrt(b^2 - 4*a*c), -2*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b
^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c))/(b^2 - 4*a*c)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 124 vs. $2(34) = 68$.

Time = 0.12 (sec) , antiderivative size = 124, normalized size of antiderivative = 3.65

$$\int \frac{x^2}{ax^2+bx^3+cx^4}\,dx = -\sqrt{-\frac{1}{4ac-b^2}}\log\left(x+\frac{-4ac\sqrt{-\frac{1}{4ac-b^2}}+b^2\sqrt{-\frac{1}{4ac-b^2}}+b}{2c}\right) + \sqrt{-\frac{1}{4ac-b^2}}\log\left(x+\frac{4ac\sqrt{-\frac{1}{4ac-b^2}}-b^2\sqrt{-\frac{1}{4ac-b^2}}+b}{2c}\right)$$

input
```
integrate(x**2/(c*x**4+b*x**3+a*x**2),x)
```

output
```
-sqrt(-1/(4*a*c - b**2))*log(x + (-4*a*c*sqrt(-1/(4*a*c - b**2)) + b**2*sq
rt(-1/(4*a*c - b**2)) + b)/(2*c)) + sqrt(-1/(4*a*c - b**2))*log(x + (4*a*c
*sqrt(-1/(4*a*c - b**2)) - b**2*sqrt(-1/(4*a*c - b**2)) + b)/(2*c))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2}{ax^2+bx^3+cx^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2/(c*x^4+b*x^3+a*x^2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{ax^2 + bx^3 + cx^4}\,dx = \frac{2\,\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}}$$

input
```
integrate(x^2/(c*x^4+b*x^3+a*x^2),x, algorithm="giac")
```

output
```
2*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/sqrt(-b^2 + 4*a*c)
```

Mupad [B] (verification not implemented)

Time = 20.90 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.35

$$\int \frac{x^2}{ax^2 + bx^3 + cx^4}\,dx = \frac{2\,\mathrm{atan}\left(\frac{b}{\sqrt{4\,a\,c-b^2}} + \frac{2\,c\,x}{\sqrt{4\,a\,c-b^2}}\right)}{\sqrt{4\,a\,c-b^2}}$$

input
```
int(x^2/(a*x^2 + b*x^3 + c*x^4),x)
```

output
```
(2*atan(b/(4*a*c - b^2)^(1/2) + (2*c*x)/(4*a*c - b^2)^(1/2)))/(4*a*c - b^2
)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.35

$$\int \frac{x^2}{ax^2 + bx^3 + cx^4}\,dx = \frac{2\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{4ac-b^2}$$

input
```
int(x^2/(c*x^4+b*x^3+a*x^2),x)
```

output
```
(2*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2)))/(4*a*c - b**2)
```

3.29 $\int \frac{x}{ax^2+bx^3+cx^4}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 62

$$\int \frac{x}{ax^2+bx^3+cx^4}\,dx=\frac{b\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}}+\frac{\log(x)}{a}-\frac{\log\left(a+bx+cx^2\right)}{2a}$$

output
```
b*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/a/(-4*a*c+b^2)^(1/2)+ln(x)/a-1/2*l
n(c*x^2+b*x+a)/a
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.98

$$\int \frac{x}{ax^2+bx^3+cx^4}\,dx=-\frac{\frac{2b\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}-2\log(x)+\log(a+x(b+cx))}{2a}$$

input
```
Integrate[x/(a*x^2 + b*x^3 + c*x^4),x]
```

output
```
-1/2*((2*b*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] - 2*
Log[x] + Log[a + x*(b + c*x)])/a
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {9, 1144, 25, 1142, 1083, 219, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{ax^2+bx^3+cx^4}dx$$

$$\Big\downarrow \text{9}$$

$$\int \frac{1}{x(a+bx+cx^2)}dx$$

$$\Big\downarrow \text{1144}$$

$$\frac{\int -\frac{b+cx}{cx^2+bx+a}dx}{a}+\frac{\log(x)}{a}$$

$$\Big\downarrow \text{25}$$

$$\frac{\log(x)}{a}-\frac{\int \frac{b+cx}{cx^2+bx+a}dx}{a}$$

$$\Big\downarrow \text{1142}$$

$$\frac{\log(x)}{a}-\frac{\frac{1}{2}b\int \frac{1}{cx^2+bx+a}dx+\frac{1}{2}\int \frac{b+2cx}{cx^2+bx+a}dx}{a}$$

$$\Big\downarrow \text{1083}$$

$$\frac{\log(x)}{a}-\frac{\frac{1}{2}\int \frac{b+2cx}{cx^2+bx+a}dx-b\int \frac{1}{b^2-(b+2cx)^2-4ac}d(b+2cx)}{a}$$

$$\Big\downarrow \text{219}$$

$$\frac{\log(x)}{a}-\frac{\frac{1}{2}\int \frac{b+2cx}{cx^2+bx+a}dx-\frac{b\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}}{a}$$

$$\Big\downarrow \text{1103}$$

$$\frac{\log(x)}{a}-\frac{\frac{1}{2}\log\left(a+bx+cx^2\right)-\frac{b\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}}{a}$$

input
```
Int[x/(a*x^2 + b*x^3 + c*x^4),x]
```

output
```
Log[x]/a - (-((b*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/Sqrt[b^2 - 4*a*c]
) + Log[a + b*x + c*x^2]/2)/a
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1144
```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Simp[e*(Log[RemoveContent[d + e*x, x]]/(c*d^2 - b*d*e + a*e^2)), x] + S
imp[1/(c*d^2 - b*d*e + a*e^2)   Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2),
x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.98

method	result	s
default	$\frac{-\frac{\ln\left(c\,x^{2}+bx+a\right)}{2}-\frac{b\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^{2}}}\right)}{\sqrt{4ac-b^{2}}}}{a}+\frac{\ln(x)}{a}$	6
risch	$\frac{\ln(x)}{a}+\left(\sum_{\_R=\operatorname{RootOf}\left(\left(4a^{2}c-b^{2}a\right)\_Z^{2}+\left(4ac-b^{2}\right)\_Z+c\right)}\_R\ln\left(\left(\left(6ac-2b^{2}\right)\_R+3c\right)x-\_Rab+b\right)\right)$	7

input
```
int(x/(c*x^4+b*x^3+a*x^2),x,method=_RETURNVERBOSE)
```

output
```
1/a*(-1/2*ln(c*x^2+b*x+a)-b/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)
^(1/2)))+ln(x)/a
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 211, normalized size of antiderivative = 3.40

$$\int \frac{x}{ax^2+bx^3+cx^4}\,dx$$
$$=\left[\frac{\sqrt{b^2-4\,ac}b\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac+\sqrt{b^2-4\,ac}(2\,cx+b)}{cx^2+bx+a}\right)-(b^2-4\,ac)\log\left(cx^2+bx+a\right)+2\,(b^2-4\,ac)\log(\right.}{2\,(ab^2-4\,a^2c)}\right.$$

input
```
integrate(x/(c*x^4+b*x^3+a*x^2),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(b^2 - 4*a*c)*b*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^
2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) - (b^2 - 4*a*c)*log(c*x^2 + b*x
 + a) + 2*(b^2 - 4*a*c)*log(x))/(a*b^2 - 4*a^2*c), 1/2*(2*sqrt(-b^2 + 4*a*
c)*b*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) - (b^2 - 4*a*c)
*log(c*x^2 + b*x + a) + 2*(b^2 - 4*a*c)*log(x))/(a*b^2 - 4*a^2*c)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 564 vs. $2(54) = 108$.

Time = 2.39 (sec) , antiderivative size = 564, normalized size of antiderivative = 9.10

$$\int \frac{x}{ax^2 + bx^3 + cx^4}\, dx = \left(-\frac{b\sqrt{-4ac+b^2}}{2a\,(4ac-b^2)} - \frac{1}{2a}\right)\log\left(x + \frac{24a^4c^2\left(-\frac{b\sqrt{-4ac+b^2}}{2a(4ac-b^2)} - \frac{1}{2a}\right)^2 - 14a^3b^2c\left(-\frac{b\sqrt{-4ac+b^2}}{2a(4ac-b^2)} - \frac{1}{2a}\right)^2 - 12a^3c^2\left(-\frac{b\sqrt{-4ac+b^2}}{2a(4ac-b^2)} - \frac{1}{2a}\right)}{9abc^2 -}\right.$$

$$+ \left(\frac{b\sqrt{-4ac+b^2}}{2a\,(4ac-b^2)} - \frac{1}{2a}\right)\log\left(x + \frac{24a^4c^2\left(\frac{b\sqrt{-4ac+b^2}}{2a(4ac-b^2)} - \frac{1}{2a}\right)^2 - 14a^3b^2c\left(\frac{b\sqrt{-4ac+b^2}}{2a(4ac-b^2)} - \frac{1}{2a}\right)^2 - 12a^3c^2\left(\frac{b\sqrt{-4ac+b^2}}{2a(4ac-b^2)} - \frac{1}{2a}\right) + 2a}{9abc^2 - 2b^3c}\right.$$

$$+ \frac{\log(x)}{a}$$

input
```
integrate(x/(c*x**4+b*x**3+a*x**2),x)
```

output
```
(-b*sqrt(-4*a*c + b**2)/(2*a*(4*a*c - b**2)) - 1/(2*a))*log(x + (24*a**4*c
**2*(-b*sqrt(-4*a*c + b**2)/(2*a*(4*a*c - b**2)) - 1/(2*a))**2 - 14*a**3*b
**2*c*(-b*sqrt(-4*a*c + b**2)/(2*a*(4*a*c - b**2)) - 1/(2*a))**2 - 12*a**3
*c**2*(-b*sqrt(-4*a*c + b**2)/(2*a*(4*a*c - b**2)) - 1/(2*a)) + 2*a**2*b**
4*(-b*sqrt(-4*a*c + b**2)/(2*a*(4*a*c - b**2)) - 1/(2*a))**2 + 3*a**2*b**2
*c*(-b*sqrt(-4*a*c + b**2)/(2*a*(4*a*c - b**2)) - 1/(2*a)) - 12*a**2*c**2
+ 11*a*b**2*c - 2*b**4)/(9*a*b*c**2 - 2*b**3*c)) + (b*sqrt(-4*a*c + b**2)/
(2*a*(4*a*c - b**2)) - 1/(2*a))*log(x + (24*a**4*c**2*(b*sqrt(-4*a*c + b**
2)/(2*a*(4*a*c - b**2)) - 1/(2*a))**2 - 14*a**3*b**2*c*(b*sqrt(-4*a*c + b*
*2)/(2*a*(4*a*c - b**2)) - 1/(2*a))**2 - 12*a**3*c**2*(b*sqrt(-4*a*c + b**
2)/(2*a*(4*a*c - b**2)) - 1/(2*a)) + 2*a**2*b**4*(b*sqrt(-4*a*c + b**2)/(2
*a*(4*a*c - b**2)) - 1/(2*a))**2 + 3*a**2*b**2*c*(b*sqrt(-4*a*c + b**2)/(2
*a*(4*a*c - b**2)) - 1/(2*a)) - 12*a**2*c**2 + 11*a*b**2*c - 2*b**4)/(9*a*
b*c**2 - 2*b**3*c)) + log(x)/a
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{ax^2 + bx^3 + cx^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x/(c*x^4+b*x^3+a*x^2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00

$$\int \frac{x}{ax^2 + bx^3 + cx^4}\,dx = -\frac{b \arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}a} - \frac{\log\left(cx^2+bx+a\right)}{2\,a} + \frac{\log\left(|x|\right)}{a}$$

input
```
integrate(x/(c*x^4+b*x^3+a*x^2),x, algorithm="giac")
```

output
```
-b*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a) - 1/2*log
(c*x^2 + b*x + a)/a + log(abs(x))/a
```

Mupad [B] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 213, normalized size of antiderivative = 3.44

$$\int \frac{x}{ax^2 + bx^3 + cx^4}\,dx = \frac{\ln(x)}{a} - \ln\left(b\,c - \left(x\left(6\,a\,c^2 - 2\,b^2\,c\right) - a\,b\,c\right)\left(\frac{1}{2\,a} - \frac{b\sqrt{b^2 - 4\,a\,c}}{2\,\left(a\,b^2 - 4\,a^2\,c\right)}\right) + 3\,c^2\,x\right)\left(\frac{1}{2\,a} - \frac{b\sqrt{b^2 - 4\,a\,c}}{2\,\left(a\,b^2 - 4\,a^2\,c\right)}\right) - \ln\left(\left(x\left(6\,a\,c^2 - 2\,b^2\,c\right) - a\,b\,c\right)\left(\frac{1}{2\,a} + \frac{b\sqrt{b^2 - 4\,a\,c}}{2\,\left(a\,b^2 - 4\,a^2\,c\right)}\right) - b\,c - 3\,c^2\,x\right)\left(\frac{1}{2\,a} + \frac{b\sqrt{b^2 - 4\,a\,c}}{2\,\left(a\,b^2 - 4\,a^2\,c\right)}\right)$$

input
```
int(x/(a*x^2 + b*x^3 + c*x^4),x)
```

output
```
log(x)/a - log(b*c - (x*(6*a*c^2 - 2*b^2*c) - a*b*c)*(1/(2*a) - (b*(b^2 -
4*a*c)^(1/2))/(2*(a*b^2 - 4*a^2*c))) + 3*c^2*x)*(1/(2*a) - (b*(b^2 - 4*a*c
)^(1/2))/(2*(a*b^2 - 4*a^2*c))) - log((x*(6*a*c^2 - 2*b^2*c) - a*b*c)*(1/(
2*a) + (b*(b^2 - 4*a*c)^(1/2))/(2*(a*b^2 - 4*a^2*c))) - b*c - 3*c^2*x)*(1/
(2*a) + (b*(b^2 - 4*a*c)^(1/2))/(2*(a*b^2 - 4*a^2*c)))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.55

$$\int \frac{x}{ax^2+bx^3+cx^4}\,dx = \frac{-2\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)b-4\log(c\,x^2+bx+a)\,ac+\log(c\,x^2+bx+a)\,b^2+8\log(x)\,ac-2\log(x)\,b}{2a\,(4ac-b^2)}$$

input
```
int(x/(c*x^4+b*x^3+a*x^2),x)
```

output
```
( - 2*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b - 4*log(a
+ b*x + c*x**2)*a*c + log(a + b*x + c*x**2)*b**2 + 8*log(x)*a*c - 2*log(x)
*b**2)/(2*a*(4*a*c - b**2))
```

3.30 $\int \frac{1}{ax^2+bx^3+cx^4}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 81

$$\int \frac{1}{ax^2+bx^3+cx^4}\,dx = -\frac{1}{ax}-\frac{(b^2-2ac)\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^2\sqrt{b^2-4ac}} - \frac{b\log(x)}{a^2}+\frac{b\log(a+bx+cx^2)}{2a^2}$$

output
```
-1/a/x-(-2*a*c+b^2)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^2)
^(1/2)-b*ln(x)/a^2+1/2*b*ln(c*x^2+b*x+a)/a^2
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.95

$$\int \frac{1}{ax^2+bx^3+cx^4}\,dx = \frac{-\frac{2a}{x}+\frac{2(b^2-2ac)\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}-2b\log(x)+b\log(a+x(b+cx))}{2a^2}$$

input
```
Integrate[(a*x^2 + b*x^3 + c*x^4)^(-1),x]
```

output
```
((-2*a)/x + (2*(b^2 - 2*a*c)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[
-b^2 + 4*a*c] - 2*b*Log[x] + b*Log[a + x*(b + c*x)])/(2*a^2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {1949, 1145, 25, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{ax^2+bx^3+cx^4}dx$$

$$\Big\downarrow \text{1949}$$

$$\int \frac{1}{x^2(a+bx+cx^2)}dx$$

$$\Big\downarrow \text{1145}$$

$$\frac{\int -\frac{b+cx}{x(cx^2+bx+a)}dx}{a}-\frac{1}{ax}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{b+cx}{x(cx^2+bx+a)}dx}{a}-\frac{1}{ax}$$

$$\Big\downarrow \text{1200}$$

$$-\frac{\int \left(\frac{b}{ax}+\frac{-b^2-cxb+ac}{a(cx^2+bx+a)}\right)dx}{a}-\frac{1}{ax}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{(b^2-2ac)\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}}-\frac{b\log(a+bx+cx^2)}{2a}+\frac{b\log(x)}{a}}{a}-\frac{1}{ax}$$

input
```
Int[(a*x^2 + b*x^3 + c*x^4)^(-1),x]
```

output
```
-(1/(a*x)) - (((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a*Sq
rt[b^2 - 4*a*c]) + (b*Log[x])/a - (b*Log[a + b*x + c*x^2])/(2*a))/a
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1145
```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
 :> Simp[e*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp
[1/(c*d^2 - b*d*e + a*e^2)   Int[(d + e*x)^(m + 1)*(Simp[c*d - b*e - c*e*x,
 x]/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[m, -1]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1949
```
Int[((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_), x_Symbol
] :> Int[x^(p*q)*(a + b*x^(n - q) + c*x^(2*(n - q)))^p, x] /; FreeQ[{a, b,
c, n, q}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00

method	result	size
default	$\frac{\frac{b\ln\left(c\,x^{2}+b x+a\right)}{2}+\frac{2\left(-a c+\frac{b^{2}}{2}\right)\arctan\left(\frac{2 c x+b}{\sqrt{4 a c-b^{2}}}\right)}{\sqrt{4 a c-b^{2}}}}{a^{2}}-\frac{1}{a x}-\frac{b\ln\left(x\right)}{a^{2}}$	81
risch	Expression too large to display	1295

input
```
int(1/(c*x^4+b*x^3+a*x^2),x,method=_RETURNVERBOSE)
```

output
```
1/a^2*(1/2*b*ln(c*x^2+b*x+a)+2*(-a*c+1/2*b^2)/(4*a*c-b^2)^(1/2)*arctan((2*
c*x+b)/(4*a*c-b^2)^(1/2)))-1/a/x-b*ln(x)/a^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 269, normalized size of antiderivative = 3.32

$$\int \frac{1}{ax^2+bx^3+cx^4}\,dx$$
$$= \left[-\frac{(b^2-2\,ac)\sqrt{b^2-4\,ac}x\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac+\sqrt{b^2-4\,ac}(2\,cx+b)}{cx^2+bx+a}\right)+2\,ab^2-8\,a^2c-(b^3-4\,abc)x\log\left(cx^2\right.}{2\,(a^2b^2-4\,a^3c)x}\right.$$
$$-\frac{2\,(b^2-2\,ac)\sqrt{-b^2+4\,ac}x\arctan\left(-\frac{\sqrt{-b^2+4\,ac}(2\,cx+b)}{b^2-4\,ac}\right)+2\,ab^2-8\,a^2c-(b^3-4\,abc)x\log\left(cx^2+bx+\right.}{2\,(a^2b^2-4\,a^3c)x}$$

input
```
integrate(1/(c*x^4+b*x^3+a*x^2),x, algorithm="fricas")
```

output
```
[-1/2*((b^2 - 2*a*c)*sqrt(b^2 - 4*a*c)*x*log((2*c^2*x^2 + 2*b*c*x + b^2 -
2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + 2*a*b^2 - 8*a^
2*c - (b^3 - 4*a*b*c)*x*log(c*x^2 + b*x + a) + 2*(b^3 - 4*a*b*c)*x*log(x))
/((a^2*b^2 - 4*a^3*c)*x), -1/2*(2*(b^2 - 2*a*c)*sqrt(-b^2 + 4*a*c)*x*arcta
n(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 2*a*b^2 - 8*a^2*c - (b^
3 - 4*a*b*c)*x*log(c*x^2 + b*x + a) + 2*(b^3 - 4*a*b*c)*x*log(x))/((a^2*b^
2 - 4*a^3*c)*x)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 862 vs. $2(75) = 150$.

Time = 156.71 (sec) , antiderivative size = 862, normalized size of antiderivative = 10.64

$$\int \frac{1}{ax^2 + bx^3 + cx^4}\,dx = \text{Too large to display}$$

input
```
integrate(1/(c*x**4+b*x**3+a*x**2),x)
```

output
```
(b/(2*a**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(2*a**2*(4*a*c - b**2)))*
log(x + (-28*a**6*b*c**2*(b/(2*a**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/
(2*a**2*(4*a*c - b**2)))**2 + 15*a**5*b**3*c*(b/(2*a**2) - sqrt(-4*a*c + b
**2)*(2*a*c - b**2)/(2*a**2*(4*a*c - b**2)))**2 - 4*a**5*c**3*(b/(2*a**2)
- sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(2*a**2*(4*a*c - b**2))) - 2*a**4*b**
5*(b/(2*a**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(2*a**2*(4*a*c - b**2))
)**2 - 3*a**4*b**2*c**2*(b/(2*a**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(
2*a**2*(4*a*c - b**2))) + a**3*b**4*c*(b/(2*a**2) - sqrt(-4*a*c + b**2)*(2
*a*c - b**2)/(2*a**2*(4*a*c - b**2))) - 4*a**3*b*c**3 + 25*a**2*b**3*c**2
- 14*a*b**5*c + 2*b**7)/(2*a**3*c**4 + 15*a**2*b**2*c**3 - 12*a*b**4*c**2
+ 2*b**6*c)) + (b/(2*a**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(2*a**2*(4
*a*c - b**2)))*log(x + (-28*a**6*b*c**2*(b/(2*a**2) + sqrt(-4*a*c + b**2)*
(2*a*c - b**2)/(2*a**2*(4*a*c - b**2)))**2 + 15*a**5*b**3*c*(b/(2*a**2) +
sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(2*a**2*(4*a*c - b**2)))**2 - 4*a**5*c*
*3*(b/(2*a**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(2*a**2*(4*a*c - b**2)
)) - 2*a**4*b**5*(b/(2*a**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(2*a**2*
(4*a*c - b**2)))**2 - 3*a**4*b**2*c**2*(b/(2*a**2) + sqrt(-4*a*c + b**2)*(
2*a*c - b**2)/(2*a**2*(4*a*c - b**2))) + a**3*b**4*c*(b/(2*a**2) + sqrt(-4
*a*c + b**2)*(2*a*c - b**2)/(2*a**2*(4*a*c - b**2))) - 4*a**3*b*c**3 + 25*
a**2*b**3*c**2 - 14*a*b**5*c + 2*b**7)/(2*a**3*c**4 + 15*a**2*b**2*c**3...
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{ax^2 + bx^3 + cx^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(c*x^4+b*x^3+a*x^2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.98

$$\int \frac{1}{ax^2 + bx^3 + cx^4}\,dx = \frac{b\log\left(cx^2 + bx + a\right)}{2\,a^2} - \frac{b\log\left(|x|\right)}{a^2} + \frac{\left(b^2 - 2\,ac\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2 + 4\,ac}a^2} - \frac{1}{ax}$$

input
```
integrate(1/(c*x^4+b*x^3+a*x^2),x, algorithm="giac")
```

output
```
1/2*b*log(c*x^2 + b*x + a)/a^2 - b*log(abs(x))/a^2 + (b^2 - 2*a*c)*arctan(
(2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a^2) - 1/(a*x)
```

Mupad [B] (verification not implemented)

Time = 22.27 (sec) , antiderivative size = 339, normalized size of antiderivative = 4.19

$$\int \frac{1}{ax^2+bx^3+cx^4}\,dx$$
$$= \frac{\ln\left(2\,a\,b^3+2\,b^4\,x-2\,a\,b^2\,\sqrt{b^2-4\,a\,c}+a^2\,c\,\sqrt{b^2-4\,a\,c}-2\,b^3\,x\,\sqrt{b^2-4\,a\,c}+2\,a^2\,c^2\,x-7\,a^2\,b\,c-8\,a\,b^2\,c}{4\,a^3\,c-a^2\,b^2}$$
$$-\frac{1}{a\,x}$$
$$-\frac{\ln\left(2\,a\,b^3+2\,b^4\,x+2\,a\,b^2\,\sqrt{b^2-4\,a\,c}-a^2\,c\,\sqrt{b^2-4\,a\,c}+2\,b^3\,x\,\sqrt{b^2-4\,a\,c}+2\,a^2\,c^2\,x-7\,a^2\,b\,c-8\,a\,b}{4\,a^3\,c-a^2\,b^2}$$
$$-\frac{b\,\ln(x)}{a^2}$$

input
```
int(1/(a*x^2 + b*x^3 + c*x^4),x)
```

output
```
(log(2*a*b^3 + 2*b^4*x - 2*a*b^2*(b^2 - 4*a*c)^(1/2) + a^2*c*(b^2 - 4*a*c)
^(1/2) - 2*b^3*x*(b^2 - 4*a*c)^(1/2) + 2*a^2*c^2*x - 7*a^2*b*c - 8*a*b^2*c
*x + 4*a*b*c*x*(b^2 - 4*a*c)^(1/2))*(a*(2*b*c - c*(b^2 - 4*a*c)^(1/2)) - b
^3/2 + (b^2*(b^2 - 4*a*c)^(1/2))/2))/(4*a^3*c - a^2*b^2) - 1/(a*x) - (log(
2*a*b^3 + 2*b^4*x + 2*a*b^2*(b^2 - 4*a*c)^(1/2) - a^2*c*(b^2 - 4*a*c)^(1/2
) + 2*b^3*x*(b^2 - 4*a*c)^(1/2) + 2*a^2*c^2*x - 7*a^2*b*c - 8*a*b^2*c*x -
4*a*b*c*x*(b^2 - 4*a*c)^(1/2))*(b^3/2 - a*(2*b*c + c*(b^2 - 4*a*c)^(1/2))
+ (b^2*(b^2 - 4*a*c)^(1/2))/2))/(4*a^3*c - a^2*b^2) - (b*log(x))/a^2
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.95

$$\int \frac{1}{ax^2+bx^3+cx^4}\,dx$$
$$= \frac{-4\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)acx+2\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)b^2x+4\log(c\,x^2+bx+a)\,abcx-\log(c\,x^2}{2a^2x\,(4ac-b^2)}$$

input
```
int(1/(c*x^4+b*x^3+a*x^2),x)
```

output
```
( - 4*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*c*x + 2*sq
rt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b**2*x + 4*log(a + b
*x + c*x**2)*a*b*c*x - log(a + b*x + c*x**2)*b**3*x - 8*log(x)*a*b*c*x + 2
*log(x)*b**3*x - 8*a**2*c + 2*a*b**2)/(2*a**2*x*(4*a*c - b**2))
```

3.31 $\int \frac{1}{x(ax^2+bx^3+cx^4)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 104

$$\int \frac{1}{x(ax^2+bx^3+cx^4)}\,dx = -\frac{1}{2ax^2}+\frac{b}{a^2x}+\frac{b(b^2-3ac)\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^3\sqrt{b^2-4ac}} + \frac{(b^2-ac)\log(x)}{a^3}-\frac{(b^2-ac)\log\left(a+bx+cx^2\right)}{2a^3}$$

output
```
-1/2/a/x^2+b/a^2/x+b*(-3*a*c+b^2)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/a^
3/(-4*a*c+b^2)^(1/2)+(-a*c+b^2)*ln(x)/a^3-1/2*(-a*c+b^2)*ln(c*x^2+b*x+a)/a
^3
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.98

$$\int \frac{1}{x(ax^2+bx^3+cx^4)}\,dx = \frac{-\frac{a^2}{x^2}+\frac{2ab}{x}-\frac{2b(b^2-3ac)\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}+2(b^2-ac)\log(x)+(-b^2+ac)\log(a+x(b+cx))}{2a^3}$$

input
```
Integrate[1/(x*(a*x^2 + b*x^3 + c*x^4)),x]
```

output
```
(-(a^2/x^2) + (2*a*b)/x - (2*b*(b^2 - 3*a*c)*ArcTan[(b + 2*c*x)/Sqrt[-b^2
+ 4*a*c]])/Sqrt[-b^2 + 4*a*c] + 2*(b^2 - a*c)*Log[x] + (-b^2 + a*c)*Log[a
+ x*(b + c*x)])/(2*a^3)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {9, 1145, 25, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(ax^2+bx^3+cx^4\right)}dx$$

$$\Big\downarrow \ 9$$

$$\int \frac{1}{x^3\left(a+bx+cx^2\right)}dx$$

$$\Big\downarrow \ 1145$$

$$\frac{\int -\frac{b+cx}{x^2(cx^2+bx+a)}dx}{a}-\frac{1}{2ax^2}$$

$$\Big\downarrow \ 25$$

$$-\frac{\int \frac{b+cx}{x^2(cx^2+bx+a)}dx}{a}-\frac{1}{2ax^2}$$

$$\Big\downarrow \ 1200$$

$$-\frac{\int \left(\frac{b}{ax^2}+\frac{ac-b^2}{a^2x}+\frac{b(b^2-2ac)+c(b^2-ac)x}{a^2(cx^2+bx+a)}\right)dx}{a}-\frac{1}{2ax^2}$$

$$\Big\downarrow \ 2009$$

$$-\frac{-\frac{b(b^2-3ac)\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^2\sqrt{b^2-4ac}}+\frac{(b^2-ac)\log(a+bx+cx^2)}{2a^2}-\frac{\log(x)(b^2-ac)}{a^2}-\frac{b}{ax}}{a}-\frac{1}{2ax^2}$$

input
```
Int[1/(x*(a*x^2 + b*x^3 + c*x^4)),x]
```

output
```
-1/2*1/(a*x^2) - (-(b/(a*x)) - (b*(b^2 - 3*a*c)*ArcTanh[(b + 2*c*x)/Sqrt[b
^2 - 4*a*c]])/(a^2*Sqrt[b^2 - 4*a*c]) - ((b^2 - a*c)*Log[x])/a^2 + ((b^2 -
 a*c)*Log[a + b*x + c*x^2])/(2*a^2))/a
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1145
```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
 :> Simp[e*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp
[1/(c*d^2 - b*d*e + a*e^2)   Int[(d + e*x)^(m + 1)*(Simp[c*d - b*e - c*e*x,
 x]/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[m, -1]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.23

method	result	size
default	$\frac{\frac{\left(a\,c^{2}-c\,b^{2}\right)\ln\left(c\,x^{2}+b x+a\right)}{2c}+\frac{2\left(2abc-b^{3}-\frac{\left(a\,c^{2}-c\,b^{2}\right)b}{2c}\right)\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^{2}}}\right)}{\sqrt{4ac-b^{2}}}}{a^{3}}-\frac{1}{2a\,x^{2}}+\frac{\left(-ac+b^{2}\right)\ln\left(x\right)}{a^{3}}+\frac{b}{a^{2}x}$	128
risch	Expression too large to display	2265

input
```
int(1/x/(c*x^4+b*x^3+a*x^2),x,method=_RETURNVERBOSE)
```

output
```
1/a^3*(1/2*(a*c^2-b^2*c)/c*ln(c*x^2+b*x+a)+2*(2*a*b*c-b^3-1/2*(a*c^2-b^2*c
)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2)))-1/2/a/x^2+(-
a*c+b^2)*ln(x)/a^3+b/a^2/x
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 358, normalized size of antiderivative = 3.44

$$\int \frac{1}{x\,(ax^2+bx^3+cx^4)}\,dx$$
$$= \left[-\frac{(b^3-3\,abc)\sqrt{b^2-4\,ac}x^2\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac-\sqrt{b^2-4\,ac}(2\,cx+b)}{cx^2+bx+a}\right)+a^2b^2-4\,a^3c+(b^4-5\,ab^2c+4\,a^2c}{2\,(a^3b^2-4\,a^4c)x^2}\right.$$

input
```
integrate(1/x/(c*x^4+b*x^3+a*x^2),x, algorithm="fricas")
```

output
```
[-1/2*((b^3 - 3*a*b*c)*sqrt(b^2 - 4*a*c)*x^2*log((2*c^2*x^2 + 2*b*c*x + b^
2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + a^2*b^2 -
4*a^3*c + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*x^2*log(c*x^2 + b*x + a) - 2*(b^4
- 5*a*b^2*c + 4*a^2*c^2)*x^2*log(x) - 2*(a*b^3 - 4*a^2*b*c)*x)/((a^3*b^2 -
 4*a^4*c)*x^2), 1/2*(2*(b^3 - 3*a*b*c)*sqrt(-b^2 + 4*a*c)*x^2*arctan(-sqrt
(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) - a^2*b^2 + 4*a^3*c - (b^4 - 5*a
*b^2*c + 4*a^2*c^2)*x^2*log(c*x^2 + b*x + a) + 2*(b^4 - 5*a*b^2*c + 4*a^2*
c^2)*x^2*log(x) + 2*(a*b^3 - 4*a^2*b*c)*x)/((a^3*b^2 - 4*a^4*c)*x^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x\,(ax^2+bx^3+cx^4)}\,dx = \text{Timed out}$$

input
```
integrate(1/x/(c*x**4+b*x**3+a*x**2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x\left(ax^2+bx^3+cx^4\right)}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(1/x/(c*x^4+b*x^3+a*x^2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.01

$$\int \frac{1}{x\left(ax^2+bx^3+cx^4\right)}\,dx=-\frac{\left(b^2-ac\right)\log\left(cx^2+bx+a\right)}{2\,a^3}+\frac{\left(b^2-ac\right)\log\left(|x|\right)}{a^3}-\frac{\left(b^3-3\,abc\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}a^3}+\frac{2\,abx-a^2}{2\,a^3x^2}$$

input
```
integrate(1/x/(c*x^4+b*x^3+a*x^2),x, algorithm="giac")
```

output
```
-1/2*(b^2 - a*c)*log(c*x^2 + b*x + a)/a^3 + (b^2 - a*c)*log(abs(x))/a^3 -
(b^3 - 3*a*b*c)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)
*a^3) + 1/2*(2*a*b*x - a^2)/(a^3*x^2)
```

Mupad [B] (verification not implemented)

Time = 22.25 (sec) , antiderivative size = 447, normalized size of antiderivative = 4.30

$$\int \frac{1}{x\,(ax^2+bx^3+cx^4)}\,dx$$
$$= \frac{\ln\left(2\,a\,b^4+2\,b^5\,x+6\,a^3\,c^2+2\,a\,b^3\,\sqrt{b^2-4\,a\,c}+2\,b^4\,x\,\sqrt{b^2-4\,a\,c}-9\,a^2\,b^2\,c-10\,a\,b^3\,c\,x-3\,a^2\,b\,c\,\sqrt{b^2-\right.}{}$$
$$-\frac{\ln\left(2\,a\,b^4+2\,b^5\,x+6\,a^3\,c^2-2\,a\,b^3\,\sqrt{b^2-4\,a\,c}-2\,b^4\,x\,\sqrt{b^2-4\,a\,c}-9\,a^2\,b^2\,c-10\,a\,b^3\,c\,x+3\,a^2\,b\,c\,\sqrt{b\right.}{}$$
$$-\frac{\frac{1}{2\,a}-\frac{b\,x}{a^2}}{x^2}-\frac{\ln(x)\,\left(a\,c-b^2\right)}{a^3}$$

input
```
int(1/(x*(a*x^2 + b*x^3 + c*x^4)),x)
```

output
```
(log(2*a*b^4 + 2*b^5*x + 6*a^3*c^2 + 2*a*b^3*(b^2 - 4*a*c)^(1/2) + 2*b^4*x
*(b^2 - 4*a*c)^(1/2) - 9*a^2*b^2*c - 10*a*b^3*c*x - 3*a^2*b*c*(b^2 - 4*a*c
)^(1/2) + 9*a^2*b*c^2*x + 3*a^2*c^2*x*(b^2 - 4*a*c)^(1/2) - 6*a*b^2*c*x*(b
^2 - 4*a*c)^(1/2))*(b^4/2 - a*((5*b^2*c)/2 + (3*b*c*(b^2 - 4*a*c)^(1/2))/2
) + (b^3*(b^2 - 4*a*c)^(1/2))/2 + 2*a^2*c^2))/(4*a^4*c - a^3*b^2) - (log(2
*a*b^4 + 2*b^5*x + 6*a^3*c^2 - 2*a*b^3*(b^2 - 4*a*c)^(1/2) - 2*b^4*x*(b^2
- 4*a*c)^(1/2) - 9*a^2*b^2*c - 10*a*b^3*c*x + 3*a^2*b*c*(b^2 - 4*a*c)^(1/2
) + 9*a^2*b*c^2*x - 3*a^2*c^2*x*(b^2 - 4*a*c)^(1/2) + 6*a*b^2*c*x*(b^2 - 4
*a*c)^(1/2))*(a*((5*b^2*c)/2 - (3*b*c*(b^2 - 4*a*c)^(1/2))/2) - b^4/2 + (b
^3*(b^2 - 4*a*c)^(1/2))/2 - 2*a^2*c^2))/(4*a^4*c - a^3*b^2) - (1/(2*a) - (
b*x)/a^2)/x^2 - (log(x)*(a*c - b^2))/a^3
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 225, normalized size of antiderivative = 2.16

$$\int \frac{1}{x\,(ax^2+bx^3+cx^4)}\,dx$$
$$= \frac{6\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)abc\,x^2-2\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)b^3x^2+4\log(c\,x^2+bx+a)\,a^2c^2x^2-5\log}{}$$

input
```
int(1/x/(c*x^4+b*x^3+a*x^2),x)
```

output
```
(6*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b*c*x**2 - 2*
sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b**3*x**2 + 4*log(
a + b*x + c*x**2)*a**2*c**2*x**2 - 5*log(a + b*x + c*x**2)*a*b**2*c*x**2 +
 log(a + b*x + c*x**2)*b**4*x**2 - 8*log(x)*a**2*c**2*x**2 + 10*log(x)*a*b
**2*c*x**2 - 2*log(x)*b**4*x**2 - 4*a**3*c + a**2*b**2 + 8*a**2*b*c*x - 2*
a*b**3*x)/(2*a**3*x**2*(4*a*c - b**2))
```

3.32 $\int \frac{1}{x^2(ax^2+bx^3+cx^4)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 137

$$\int \frac{1}{x^2(ax^2+bx^3+cx^4)}\,dx = -\frac{1}{3ax^3}+\frac{b}{2a^2x^2}-\frac{b^2-ac}{a^3x} - \frac{(b^4-4ab^2c+2a^2c^2)\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^4\sqrt{b^2-4ac}} - \frac{b(b^2-2ac)\log(x)}{a^4}+\frac{b(b^2-2ac)\log(a+bx+cx^2)}{2a^4}$$

output

```
-1/3/a/x^3+1/2*b/a^2/x^2-(-a*c+b^2)/a^3/x-(2*a^2*c^2-4*a*b^2*c+b^4)*arctan
h((2*c*x+b)/(-4*a*c+b^2)^(1/2))/a^4/(-4*a*c+b^2)^(1/2)-b*(-2*a*c+b^2)*ln(x
)/a^4+1/2*b*(-2*a*c+b^2)*ln(c*x^2+b*x+a)/a^4
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.96

$$\int \frac{1}{x^2(ax^2+bx^3+cx^4)}\,dx = \frac{-\frac{2a^3}{x^3}+\frac{3a^2b}{x^2}+\frac{6a(-b^2+ac)}{x}+\frac{6(b^4-4ab^2c+2a^2c^2)\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}-6(b^3-2abc)\log(x)+3(b^3-2abc)\log(a+}{6a^4}$$

input
```
Integrate[1/(x^2*(a*x^2 + b*x^3 + c*x^4)),x]
```

output
```
((-2*a^3)/x^3 + (3*a^2*b)/x^2 + (6*a*(-b^2 + a*c))/x + (6*(b^4 - 4*a*b^2*c
 + 2*a^2*c^2)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] -
 6*(b^3 - 2*a*b*c)*Log[x] + 3*(b^3 - 2*a*b*c)*Log[a + x*(b + c*x)])/(6*a^4
)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {9, 1145, 25, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \left(ax^2+bx^3+cx^4\right)} \, dx$$

$$\Big\downarrow \text{ 9}$$

$$\int \frac{1}{x^4 \left(a+bx+cx^2\right)}dx$$

$$\Big\downarrow \text{ 1145}$$

$$\frac{\int -\frac{b+cx}{x^3(cx^2+bx+a)}dx}{a}-\frac{1}{3ax^3}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\int \frac{b+cx}{x^3(cx^2+bx+a)}dx}{a}-\frac{1}{3ax^3}$$

$$\Big\downarrow \text{ 1200}$$

$$-\frac{\int \left(\frac{b}{ax^3}+\frac{b^3-2abc}{a^3x}+\frac{-b^4+3acb^2-c\left(b^2-2ac\right)xb-a^2c^2}{a^3\left(cx^2+bx+a\right)}+\frac{ac-b^2}{a^2x^2}\right)dx}{a}-\frac{1}{3ax^3}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{-\frac{b(b^2-2ac)\log(a+bx+cx^2)}{2a^3}+\frac{b\log(x)(b^2-2ac)}{a^3}+\frac{b^2-ac}{a^2x}+\frac{(2a^2c^2-4ab^2c+b^4)\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^3\sqrt{b^2-4ac}}-\frac{b}{2ax^2}}{a}-\frac{1}{3ax^3}$$

input

```
Int[1/(x^2*(a*x^2 + b*x^3 + c*x^4)),x]
```

output

```
-1/3*1/(a*x^3) - (-1/2*b/(a*x^2) + (b^2 - a*c)/(a^2*x) + ((b^4 - 4*a*b^2*c
 + 2*a^2*c^2)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a^3*Sqrt[b^2 - 4*a*
c]) + (b*(b^2 - 2*a*c)*Log[x])/a^3 - (b*(b^2 - 2*a*c)*Log[a + b*x + c*x^2]
)/(2*a^3))/a
```

Defintions of rubi rules used

rule 9

```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1145

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
 :> Simp[e*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp
[1/(c*d^2 - b*d*e + a*e^2)   Int[(d + e*x)^(m + 1)*(Simp[c*d - b*e - c*e*x,
 x]/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[m, -1]
```

rule 1200

```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.15

method	result
default	$\frac{\frac{\left(-2ab\,c^{2}+b^{3}c\right)\ln\left(c\,x^{2}+bx+a\right)}{2c}+\frac{2\left(a^{2}c^{2}-3a\,b^{2}c+b^{4}-\frac{\left(-2ab\,c^{2}+b^{3}c\right)b}{2c}\right)\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^{2}}}\right)}{\sqrt{4ac-b^{2}}}}{a^{4}}-\frac{1}{3a\,x^{3}}-\frac{-ac+b^{2}}{a^{3}x}+\frac{b\left(2ac-b^{2}\right)\ln\left(x\right)}{a^{4}}$
risch	$\frac{\frac{\left(ac-b^{2}\right)x^{2}}{a^{3}}+\frac{bx}{2a^{2}}-\frac{1}{3a}}{x^{3}}+\frac{2b\ln\left(x\right)c}{a^{3}}-\frac{b^{3}\ln\left(x\right)}{a^{4}}+\left(\sum_{\_R=\mathrm{RootOf}\left(\left(4c\,a^{5}-a^{4}b^{2}\right)\_Z^{2}+\left(8a^{2}b\,c^{2}-6a\,b^{3}c+b^{5}\right)\_Z+c^{4}\right)}\_R\ln\left(\left(\left(\right.\right.\right.\right.$

input
```
int(1/x^2/(c*x^4+b*x^3+a*x^2),x,method=_RETURNVERBOSE)
```

output
```
1/a^4*(1/2*(-2*a*b*c^2+b^3*c)/c*ln(c*x^2+b*x+a)+2*(a^2*c^2-3*a*b^2*c+b^4-1
/2*(-2*a*b*c^2+b^3*c)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^
(1/2)))-1/3/a/x^3-(-a*c+b^2)/a^3/x+b*(2*a*c-b^2)/a^4*ln(x)+1/2*b/a^2/x^2
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 445, normalized size of antiderivative = 3.25

$$\int \frac{1}{x^2\left(ax^2+bx^3+cx^4\right)}\,dx$$
$$= \left[\frac{3\left(b^4-4\,ab^2c+2\,a^2c^2\right)\sqrt{b^2-4\,ac}x^3\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac-\sqrt{b^2-4\,ac}(2\,cx+b)}{cx^2+bx+a}\right)-2\,a^3b^2+8\,a^4c+3\left(b^5-6\right.}{}\right.$$
$$-\frac{6\left(b^4-4\,ab^2c+2\,a^2c^2\right)\sqrt{-b^2+4\,ac}x^3\arctan\left(-\frac{\sqrt{-b^2+4\,ac}(2\,cx+b)}{b^2-4\,ac}\right)+2\,a^3b^2-8\,a^4c-3\left(b^5-6\,ab^3c+8\right.}{6\,(}$$

input
```
integrate(1/x^2/(c*x^4+b*x^3+a*x^2),x, algorithm="fricas")
```

output
```
[1/6*(3*(b^4 - 4*a*b^2*c + 2*a^2*c^2)*sqrt(b^2 - 4*a*c)*x^3*log((2*c^2*x^2
 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a
)) - 2*a^3*b^2 + 8*a^4*c + 3*(b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*x^3*log(c*x^2
 + b*x + a) - 6*(b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*x^3*log(x) - 6*(a*b^4 - 5*
a^2*b^2*c + 4*a^3*c^2)*x^2 + 3*(a^2*b^3 - 4*a^3*b*c)*x)/((a^4*b^2 - 4*a^5*
c)*x^3), -1/6*(6*(b^4 - 4*a*b^2*c + 2*a^2*c^2)*sqrt(-b^2 + 4*a*c)*x^3*arct
an(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 2*a^3*b^2 - 8*a^4*c -
3*(b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*x^3*log(c*x^2 + b*x + a) + 6*(b^5 - 6*a*
b^3*c + 8*a^2*b*c^2)*x^3*log(x) + 6*(a*b^4 - 5*a^2*b^2*c + 4*a^3*c^2)*x^2
- 3*(a^2*b^3 - 4*a^3*b*c)*x)/((a^4*b^2 - 4*a^5*c)*x^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^2\left(ax^2+bx^3+cx^4\right)}\,dx=\text{Timed out}$$

input
```
integrate(1/x**2/(c*x**4+b*x**3+a*x**2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^2\left(ax^2+bx^3+cx^4\right)}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(1/x^2/(c*x^4+b*x^3+a*x^2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.99

$$\int \frac{1}{x^2 (ax^2 + bx^3 + cx^4)} dx = \frac{(b^3 - 2abc) \log (cx^2 + bx + a)}{2a^4} - \frac{(b^3 - 2abc) \log (|x|)}{a^4} + \frac{(b^4 - 4ab^2c + 2a^2c^2) \arctan \left(\frac{2cx+b}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2 + 4ac}a^4} + \frac{3a^2bx - 2a^3 - 6(ab^2 - a^2c)x^2}{6a^4x^3}$$

input
```
integrate(1/x^2/(c*x^4+b*x^3+a*x^2),x, algorithm="giac")
```

output
```
1/2*(b^3 - 2*a*b*c)*log(c*x^2 + b*x + a)/a^4 - (b^3 - 2*a*b*c)*log(abs(x))
/a^4 + (b^4 - 4*a*b^2*c + 2*a^2*c^2)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c)
)/(sqrt(-b^2 + 4*a*c)*a^4) + 1/6*(3*a^2*b*x - 2*a^3 - 6*(a*b^2 - a^2*c)*x^
2)/(a^4*x^3)
```

Mupad [B] (verification not implemented)

Time = 21.66 (sec) , antiderivative size = 524, normalized size of antiderivative = 3.82

$$\int \frac{1}{x^2 (ax^2 + bx^3 + cx^4)} \, dx = \ln \Big(2\, a\, b^4 \sqrt{b^2 - 4\, a\, c} - 2\, b^6\, x - 2\, a\, b^5 + 2\, b^5\, x \sqrt{b^2 - 4\, a\, c} + 11\, a^2\, b^3\, c - 13\, a^3\, b\, c^2 + 2\, a^3\, c^3\, x + a^3\, c^2 \sqrt{b^2 - 4\, a\, c} - 17\, a^2\, b^2\, c^2\, x + 12\, a\, b^4\, c\, x - 5\, a^2\, b^2\, c \sqrt{b^2 - 4\, a\, c} - 8\, a\, b^3\, c\, x \sqrt{b^2 - 4\, a\, c} + 7\, a^2\, b\, c^2\, x \sqrt{b^2 - 4\, a\, c} \Big) \left(\frac{b^3}{2\, a^4} - \frac{b^2 \sqrt{b^2 - 4\, a\, c}}{2\, a^4} - \frac{b\, c}{a^3} + \frac{a^2\, c^2 \sqrt{b^2 - 4\, a\, c}}{4\, a^5\, c - a^4\, b^2} \right) + \ln \Big(2\, a\, b^5 + 2\, b^6\, x + 2\, a\, b^4 \sqrt{b^2 - 4\, a\, c} + 2\, b^5\, x \sqrt{b^2 - 4\, a\, c} - 11\, a^2\, b^3\, c + 13\, a^3\, b\, c^2 - 2\, a^3\, c^3\, x + a^3\, c^2 \sqrt{b^2 - 4\, a\, c} + 17\, a^2\, b^2\, c^2\, x - 12\, a\, b^4\, c\, x - 5\, a^2\, b^2\, c \sqrt{b^2 - 4\, a\, c} - 8\, a\, b^3\, c\, x \sqrt{b^2 - 4\, a\, c} + 7\, a^2\, b\, c^2\, x \sqrt{b^2 - 4\, a\, c} \Big) \left(\frac{b^3}{2\, a^4} + \frac{b^2 \sqrt{b^2 - 4\, a\, c}}{2\, a^4} - \frac{b\, c}{a^3} - \frac{a^2\, c^2 \sqrt{b^2 - 4\, a\, c}}{4\, a^5\, c - a^4\, b^2} \right) + \frac{\frac{x^2\, (a\, c - b^2)}{a^3} - \frac{1}{3\, a} + \frac{b\, x}{2\, a^2}}{x^3} + \frac{b\, \ln(x)\, (2\, a\, c - b^2)}{a^4}$$

input
```
int(1/(x^2*(a*x^2 + b*x^3 + c*x^4)),x)
```

output
```
log(2*a*b^4*(b^2 - 4*a*c)^(1/2) - 2*b^6*x - 2*a*b^5 + 2*b^5*x*(b^2 - 4*a*c
)^(1/2) + 11*a^2*b^3*c - 13*a^3*b*c^2 + 2*a^3*c^3*x + a^3*c^2*(b^2 - 4*a*c
)^(1/2) - 17*a^2*b^2*c^2*x + 12*a*b^4*c*x - 5*a^2*b^2*c*(b^2 - 4*a*c)^(1/2
) - 8*a*b^3*c*x*(b^2 - 4*a*c)^(1/2) + 7*a^2*b*c^2*x*(b^2 - 4*a*c)^(1/2))*(
b^3/(2*a^4) - (b^2*(b^2 - 4*a*c)^(1/2))/(2*a^4) - (b*c)/a^3 + (a^2*c^2*(b^
2 - 4*a*c)^(1/2))/(4*a^5*c - a^4*b^2)) + log(2*a*b^5 + 2*b^6*x + 2*a*b^4*(
b^2 - 4*a*c)^(1/2) + 2*b^5*x*(b^2 - 4*a*c)^(1/2) - 11*a^2*b^3*c + 13*a^3*b
*c^2 - 2*a^3*c^3*x + a^3*c^2*(b^2 - 4*a*c)^(1/2) + 17*a^2*b^2*c^2*x - 12*a
*b^4*c*x - 5*a^2*b^2*c*(b^2 - 4*a*c)^(1/2) - 8*a*b^3*c*x*(b^2 - 4*a*c)^(1/
2) + 7*a^2*b*c^2*x*(b^2 - 4*a*c)^(1/2))*(b^3/(2*a^4) + (b^2*(b^2 - 4*a*c)^
(1/2))/(2*a^4) - (b*c)/a^3 - (a^2*c^2*(b^2 - 4*a*c)^(1/2))/(4*a^5*c - a^4*
b^2)) + ((x^2*(a*c - b^2))/a^3 - 1/(3*a) + (b*x)/(2*a^2))/x^3 + (b*log(x)*
(2*a*c - b^2))/a^4
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 308, normalized size of antiderivative = 2.25

$$\int \frac{1}{x^2\left(ax^2+bx^3+cx^4\right)}\,dx$$
$$= \frac{12\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)a^2c^2x^3 - 24\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)a\,b^2c\,x^3 + 6\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{}$$

input
```
int(1/x^2/(c*x^4+b*x^3+a*x^2),x)
```

output
```
(12*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*c**2*x**3
 - 24*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b**2*c*x**
3 + 6*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b**4*x**3 -
24*log(a + b*x + c*x**2)*a**2*b*c**2*x**3 + 18*log(a + b*x + c*x**2)*a*b**
3*c*x**3 - 3*log(a + b*x + c*x**2)*b**5*x**3 + 48*log(x)*a**2*b*c**2*x**3
- 36*log(x)*a*b**3*c*x**3 + 6*log(x)*b**5*x**3 - 8*a**4*c + 2*a**3*b**2 +
12*a**3*b*c*x + 24*a**3*c**2*x**2 - 3*a**2*b**3*x - 30*a**2*b**2*c*x**2 +
6*a*b**4*x**2)/(6*a**4*x**3*(4*a*c - b**2))
```

3.33 $\int \frac{x^8}{(ax^2+bx^3+cx^4)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 139

$$\int \frac{x^8}{(ax^2+bx^3+cx^4)^2}\,dx = \frac{x}{c^2} - \frac{ab(b^2-3ac)+(b^4-4ab^2c+2a^2c^2)\,x}{c^3\,(b^2-4ac)\,(a+bx+cx^2)} - \frac{2(b^4-6ab^2c+6a^2c^2)\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^3\,(b^2-4ac)^{3/2}} - \frac{b\log(a+bx+cx^2)}{c^3}$$

output

```
x/c^2-(a*b*(-3*a*c+b^2)+(2*a^2*c^2-4*a*b^2*c+b^4)*x)/c^3/(-4*a*c+b^2)/(c*x
^2+b*x+a)-2*(6*a^2*c^2-6*a*b^2*c+b^4)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2)
)/c^3/(-4*a*c+b^2)^(3/2)-b*ln(c*x^2+b*x+a)/c^3
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.95

$$\int \frac{x^8}{(ax^2+bx^3+cx^4)^2}\,dx = \frac{cx+\frac{-b^4x-ab^2(b-4cx)+a^2c(3b-2cx)}{(b^2-4ac)(a+x(b+cx))}-\frac{2(b^4-6ab^2c+6a^2c^2)\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{3/2}}-b\log(a+x(b+cx))}{c^3}$$

input
```
Integrate[x^8/(a*x^2 + b*x^3 + c*x^4)^2,x]
```

output
```
(c*x + (-(b^4*x) - a*b^2*(b - 4*c*x) + a^2*c*(3*b - 2*c*x))/((b^2 - 4*a*c)
*(a + x*(b + c*x))) - (2*(b^4 - 6*a*b^2*c + 6*a^2*c^2)*ArcTan[(b + 2*c*x)/
Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2) - b*Log[a + x*(b + c*x)])/c^3
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.11, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {9, 1164, 27, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{(ax^2+bx^3+cx^4)^2}\,dx$$

$$\Big\downarrow \text{ 9}$$

$$\int \frac{x^4}{(a+bx+cx^2)^2}dx$$

$$\Big\downarrow \text{ 1164}$$

$$\frac{x^3(2a+bx)}{(b^2-4ac)(a+bx+cx^2)}-\frac{\int \frac{2x^2(3a+bx)}{cx^2+bx+a}dx}{b^2-4ac}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{x^3(2a+bx)}{(b^2-4ac)(a+bx+cx^2)}-\frac{2\int \frac{x^2(3a+bx)}{cx^2+bx+a}dx}{b^2-4ac}$$

$$\Big\downarrow \textbf{1200}$$

$$\frac{x^3(2a+bx)}{(b^2-4ac)(a+bx+cx^2)}-\frac{2\int \left(-\frac{b^2-3ac}{c^2}+\frac{bx}{c}+\frac{a(b^2-3ac)+b(b^2-4ac)x}{c^2(cx^2+bx+a)}\right)dx}{b^2-4ac}$$

$$\Big\downarrow \textbf{2009}$$

$$\frac{x^3(2a+bx)}{(b^2-4ac)(a+bx+cx^2)}-\frac{2\left(\frac{(6a^2c^2-6ab^2c+b^4)\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^3\sqrt{b^2-4ac}}+\frac{b(b^2-4ac)\log(a+bx+cx^2)}{2c^3}-\frac{x(b^2-3ac)}{c^2}+\frac{bx^2}{2c}\right)}{b^2-4ac}$$

input
```
Int[x^8/(a*x^2 + b*x^3 + c*x^4)^2,x]
```

output
```
(x^3*(2*a + b*x))/((b^2 - 4*a*c)*(a + b*x + c*x^2)) - (2*(-(((b^2 - 3*a*c)
*x)/c^2) + (b*x^2)/(2*c) + ((b^4 - 6*a*b^2*c + 6*a^2*c^2)*ArcTanh[(b + 2*c
*x)/Sqrt[b^2 - 4*a*c]])/(c^3*Sqrt[b^2 - 4*a*c]) + (b*(b^2 - 4*a*c)*Log[a +
 b*x + c*x^2])/(2*c^3)))/(b^2 - 4*a*c)
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1164
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*
c))   Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*
c*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p
+ 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[p, -1] && GtQ[m, 1] && Int
QuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.42

method	result	size
default	$\frac{x}{c^2}-\frac{\frac{-\frac{\left(2a^2c^2-4a\,b^2c+b^4\right)x}{c\left(4ac-b^2\right)}+\frac{ba\left(3ac-b^2\right)}{c\left(4ac-b^2\right)}}{c\,x^2+bx+a}+\frac{\frac{\left(4abc-b^3\right)\ln\left(c\,x^2+bx+a\right)}{c}+\frac{4\left(3a^2c-b^2a-\frac{\left(4abc-b^3\right)b}{2c}\right)\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{4ac-b^2}}{c^2}$	198
risch	Expression too large to display	1176

input
```
int(x^8/(c*x^4+b*x^3+a*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
x/c^2-1/c^2*((-(2*a^2*c^2-4*a*b^2*c+b^4)/c/(4*a*c-b^2)*x+b*a/c*(3*a*c-b^2)
/(4*a*c-b^2))/(c*x^2+b*x+a)+2/(4*a*c-b^2)*(1/2*(4*a*b*c-b^3)/c*ln(c*x^2+b*
x+a)+2*(3*a^2*c-b^2*a-1/2*(4*a*b*c-b^3)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c
*x+b)/(4*a*c-b^2)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 409 vs. $2(135) = 270$.

Time = 0.08 (sec) , antiderivative size = 837, normalized size of antiderivative = 6.02

$$\int \frac{x^8}{(ax^2 + bx^3 + cx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^8/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="fricas")
```

output
```
[-(a*b^5 - 7*a^2*b^3*c + 12*a^3*b*c^2 - (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^
4)*x^3 - (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^2 + (a*b^4 - 6*a^2*b^2*c +
 6*a^3*c^2 + (b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*x^2 + (b^5 - 6*a*b^3*c + 6*
a^2*b*c^2)*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + s
qrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + (b^6 - 9*a*b^4*c + 26*a
^2*b^2*c^2 - 24*a^3*c^3)*x + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2 + (b^5*c
- 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^2 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*x)*
log(c*x^2 + b*x + a))/(a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5 + (b^4*c^4 -
 8*a*b^2*c^5 + 16*a^2*c^6)*x^2 + (b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*x)
, -(a*b^5 - 7*a^2*b^3*c + 12*a^3*b*c^2 - (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c
^4)*x^3 - (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^2 + 2*(a*b^4 - 6*a^2*b^2*
c + 6*a^3*c^2 + (b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*x^2 + (b^5 - 6*a*b^3*c +
 6*a^2*b*c^2)*x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)
/(b^2 - 4*a*c)) + (b^6 - 9*a*b^4*c + 26*a^2*b^2*c^2 - 24*a^3*c^3)*x + (a*b
^5 - 8*a^2*b^3*c + 16*a^3*b*c^2 + (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^2
 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*x)*log(c*x^2 + b*x + a))/(a*b^4*c^3
- 8*a^2*b^2*c^4 + 16*a^3*c^5 + (b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*x^2 +
(b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*x)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 842 vs. $2(134) = 268$.

Time = 1.09 (sec) , antiderivative size = 842, normalized size of antiderivative = 6.06

$$\int \frac{x^8}{(ax^2 + bx^3 + cx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x**8/(c*x**4+b*x**3+a*x**2)**2,x)
```

output
```
(-b/c**3 - sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 - 6*a*b**2*c + b**4)/(c**
3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)))*log(x + (-10*a
**2*b*c - 16*a**2*c**4*(-b/c**3 - sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 -
6*a*b**2*c + b**4)/(c**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c -
 b**6))) + 2*a*b**3 + 8*a*b**2*c**3*(-b/c**3 - sqrt(-(4*a*c - b**2)**3)*(6
*a**2*c**2 - 6*a*b**2*c + b**4)/(c**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 +
12*a*b**4*c - b**6))) - b**4*c**2*(-b/c**3 - sqrt(-(4*a*c - b**2)**3)*(6*a
**2*c**2 - 6*a*b**2*c + b**4)/(c**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12
*a*b**4*c - b**6))))/(12*a**2*c**2 - 12*a*b**2*c + 2*b**4)) + (-b/c**3 + s
qrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 - 6*a*b**2*c + b**4)/(c**3*(64*a**3*c
**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)))*log(x + (-10*a**2*b*c - 16
*a**2*c**4*(-b/c**3 + sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 - 6*a*b**2*c +
 b**4)/(c**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))) + 2
*a*b**3 + 8*a*b**2*c**3*(-b/c**3 + sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 -
 6*a*b**2*c + b**4)/(c**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c
- b**6))) - b**4*c**2*(-b/c**3 + sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 - 6
*a*b**2*c + b**4)/(c**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c -
b**6))))/(12*a**2*c**2 - 12*a*b**2*c + 2*b**4)) + (-3*a**2*b*c + a*b**3 +
x*(2*a**2*c**2 - 4*a*b**2*c + b**4))/(4*a**2*c**4 - a*b**2*c**3 + x**2*(4*
a*c**5 - b**2*c**4) + x*(4*a*b*c**4 - b**3*c**3)) + x/c**2
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^8}{(ax^2+bx^3+cx^4)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^8/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.16

$$\int \frac{x^8}{(ax^2+bx^3+cx^4)^2}\,dx = \frac{2\,(b^4-6\,ab^2c+6\,a^2c^2)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^2c^3-4\,ac^4)\sqrt{-b^2+4\,ac}}+\frac{x}{c^2} - \frac{b\log\left(cx^2+bx+a\right)}{c^3} - \frac{\frac{(b^4-4\,ab^2c+2\,a^2c^2)x}{c}+\frac{ab^3-3\,a^2bc}{c}}{(cx^2+bx+a)(b^2-4\,ac)c^2}$$

input
```
integrate(x^8/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="giac")
```

output
```
2*(b^4 - 6*a*b^2*c + 6*a^2*c^2)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b
^2*c^3 - 4*a*c^4)*sqrt(-b^2 + 4*a*c)) + x/c^2 - b*log(c*x^2 + b*x + a)/c^3
 - ((b^4 - 4*a*b^2*c + 2*a^2*c^2)*x/c + (a*b^3 - 3*a^2*b*c)/c)/((c*x^2 + b
*x + a)*(b^2 - 4*a*c)*c^2)
```

Mupad [B] (verification not implemented)

Time = 21.39 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.88

$$\int \frac{x^8}{(ax^2+bx^3+cx^4)^2}\,dx = \frac{x}{c^2}+\frac{\frac{a\,(b^3-3\,a\,b\,c)}{c\,(4\,a\,c-b^2)}+\frac{x\,(2\,a^2\,c^2-4\,a\,b^2\,c+b^4)}{c\,(4\,a\,c-b^2)}}{c^3\,x^2+b\,c^2\,x+a\,c^2} + \frac{\ln\left(c\,x^2+b\,x+a\right)\,\left(-128\,a^3\,b\,c^3+96\,a^2\,b^3\,c^2-24\,a\,b^5\,c+2\,b^7\right)}{2\,\left(64\,a^3\,c^6-48\,a^2\,b^2\,c^5+12\,a\,b^4\,c^4-b^6\,c^3\right)} - \frac{2\,\mathrm{atan}\left(\frac{2\,c\,x}{\sqrt{4\,a\,c-b^2}}-\frac{b^3\,c^2-4\,a\,b\,c^3}{c^2\,{(4\,a\,c-b^2)}^{3/2}}\right)\,\left(6\,a^2\,c^2-6\,a\,b^2\,c+b^4\right)}{c^3\,{\left(4\,a\,c-b^2\right)}^{3/2}}$$

input
```
int(x^8/(a*x^2 + b*x^3 + c*x^4)^2,x)
```

output
```
x/c^2 + ((a*(b^3 - 3*a*b*c))/(c*(4*a*c - b^2)) + (x*(b^4 + 2*a^2*c^2 - 4*a
*b^2*c))/(c*(4*a*c - b^2)))/(a*c^2 + c^3*x^2 + b*c^2*x) + (log(a + b*x + c
*x^2)*(2*b^7 - 128*a^3*b*c^3 + 96*a^2*b^3*c^2 - 24*a*b^5*c))/(2*(64*a^3*c^
6 - b^6*c^3 + 12*a*b^4*c^4 - 48*a^2*b^2*c^5)) - (2*atan((2*c*x)/(4*a*c - b
^2)^(1/2) - (b^3*c^2 - 4*a*b*c^3)/(c^2*(4*a*c - b^2)^(3/2)))*(b^4 + 6*a^2*
c^2 - 6*a*b^2*c))/(c^3*(4*a*c - b^2)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 752, normalized size of antiderivative = 5.41

$$\int \frac{x^8}{(ax^2 + bx^3 + cx^4)^2} \, dx = \frac{-\log(c x^2 + bx + a) \, a \, b^6 - \log(c x^2 + bx + a) \, b^7 x - \log(c x^2 + bx + a) \, b^6 c \, x^2 - 24a^4c^3 + 14a^3b^2c^2 - 24a^3}{}$$

input
```
int(x^8/(c*x^4+b*x^3+a*x^2)^2,x)
```

output
```
( - 12*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**3*b*c**2
 + 12*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*b**3*c
- 12*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*b**2*c**
2*x - 12*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*b*c*
*3*x**2 - 2*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b**5
 + 12*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b**4*c*x +
 12*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b**3*c**2*x*
*2 - 2*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b**6*x - 2*
sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b**5*c*x**2 - 16*l
og(a + b*x + c*x**2)*a**3*b**2*c**2 + 8*log(a + b*x + c*x**2)*a**2*b**4*c
- 16*log(a + b*x + c*x**2)*a**2*b**3*c**2*x - 16*log(a + b*x + c*x**2)*a**
2*b**2*c**3*x**2 - log(a + b*x + c*x**2)*a*b**6 + 8*log(a + b*x + c*x**2)*
a*b**5*c*x + 8*log(a + b*x + c*x**2)*a*b**4*c**2*x**2 - log(a + b*x + c*x*
*2)*b**7*x - log(a + b*x + c*x**2)*b**6*c*x**2 - 24*a**4*c**3 + 14*a**3*b*
*2*c**2 - 24*a**3*c**4*x**2 - 2*a**2*b**4*c + 42*a**2*b**2*c**3*x**2 + 16*
a**2*b*c**4*x**3 - 17*a*b**4*c**2*x**2 - 8*a*b**3*c**3*x**3 + 2*b**6*c*x**
2 + b**5*c**2*x**3)/(b*c**3*(16*a**3*c**2 - 8*a**2*b**2*c + 16*a**2*b*c**2
*x + 16*a**2*c**3*x**2 + a*b**4 - 8*a*b**3*c*x - 8*a*b**2*c**2*x**2 + b**5
*x + b**4*c*x**2))
```

3.34 $\int \frac{x^7}{(ax^2+bx^3+cx^4)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 114

$$\int \frac{x^7}{(ax^2+bx^3+cx^4)^2}\,dx = -\frac{bx}{c(b^2-4ac)}+\frac{x^2(2a+bx)}{(b^2-4ac)(a+bx+cx^2)} + \frac{b(b^2-6ac)\,\mathrm{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2(b^2-4ac)^{3/2}}+\frac{\log(a+bx+cx^2)}{2c^2}$$

output
```
-b*x/c/(-4*a*c+b^2)+x^2*(b*x+2*a)/(-4*a*c+b^2)/(c*x^2+b*x+a)+b*(-6*a*c+b^2
)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/c^2/(-4*a*c+b^2)^(3/2)+1/2*ln(c*x^
2+b*x+a)/c^2
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.96

$$\int \frac{x^7}{(ax^2+bx^3+cx^4)^2}\,dx = \frac{\frac{2(-2a^2c+b^3x+ab(b-3cx))}{(b^2-4ac)(a+x(b+cx))}+\frac{2b(b^2-6ac)\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{3/2}}+\log(a+x(b+cx))}{2c^2}$$

input
```
Integrate[x^7/(a*x^2 + b*x^3 + c*x^4)^2,x]
```

output
```
((2*(-2*a^2*c + b^3*x + a*b*(b - 3*c*x)))/((b^2 - 4*a*c)*(a + x*(b + c*x))
) + (2*b*(b^2 - 6*a*c)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a
*c)^(3/2) + Log[a + x*(b + c*x)])/(2*c^2)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {9, 1164, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{(ax^2+bx^3+cx^4)^2}\,dx$$

$$\Big\downarrow \; 9$$

$$\int \frac{x^3}{(a+bx+cx^2)^2}dx$$

$$\Big\downarrow \; 1164$$

$$\frac{x^2(2a+bx)}{(b^2-4ac)(a+bx+cx^2)}-\frac{\int \frac{x(4a+bx)}{cx^2+bx+a}dx}{b^2-4ac}$$

$$\Big\downarrow \; 1200$$

$$\frac{x^2(2a+bx)}{(b^2-4ac)(a+bx+cx^2)}-\frac{\int \left(\frac{b}{c}-\frac{ab+(b^2-4ac)x}{c(cx^2+bx+a)}\right)dx}{b^2-4ac}$$

$$\Big\downarrow \; 2009$$

$$\frac{x^2(2a+bx)}{(b^2-4ac)(a+bx+cx^2)}-\frac{-\frac{b(b^2-6ac)\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}}-\frac{(b^2-4ac)\log(a+bx+cx^2)}{2c^2}+\frac{bx}{c}}{b^2-4ac}$$

input
```
Int[x^7/(a*x^2 + b*x^3 + c*x^4)^2,x]
```

output
```
(x^2*(2*a + b*x))/((b^2 - 4*a*c)*(a + b*x + c*x^2)) - ((b*x)/c - (b*(b^2 -
 6*a*c)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^2*Sqrt[b^2 - 4*a*c]) -
((b^2 - 4*a*c)*Log[a + b*x + c*x^2])/(2*c^2))/(b^2 - 4*a*c)
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 1164
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*
c))   Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*
c*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p
+ 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[p, -1] && GtQ[m, 1] && Int
QuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.48

method	result
default	$\frac{\frac{b(3ac-b^2)x}{c^2(4ac-b^2)}+\frac{(2ac-b^2)a}{c^2(4ac-b^2)}}{c\,x^2+bx+a}+\frac{\frac{(4ac-b^2)\ln(c\,x^2+bx+a)}{2c}+\frac{2\left(-ab-\frac{(4ac-b^2)b}{2c}\right)\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{c(4ac-b^2)}$
risch	$\frac{\frac{b(3ac-b^2)x}{c^2(4ac-b^2)}+\frac{(2ac-b^2)a}{c^2(4ac-b^2)}}{c\,x^2+bx+a}+\frac{8\ln\left(-24a^2b\,c^2+10a\,b^3c-b^5-2\sqrt{-b^2(4ac-b^2)(6ac-b^2)^2}\,cx-\sqrt{-b^2(4ac-b^2)(6ac-b^2)^2}\,b\right)a^2}{(4ac-b^2)^2}-\frac{4}{}$

input
```
int(x^7/(c*x^4+b*x^3+a*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
(b/c^2*(3*a*c-b^2)/(4*a*c-b^2)*x+1/c^2*(2*a*c-b^2)*a/(4*a*c-b^2))/(c*x^2+b
*x+a)+1/c/(4*a*c-b^2)*(1/2*(4*a*c-b^2)/c*ln(c*x^2+b*x+a)+2*(-a*b-1/2*(4*a*
c-b^2)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 308 vs. 2(108) = 216.

Time = 0.08 (sec) , antiderivative size = 635, normalized size of antiderivative = 5.57

$$\int \frac{x^7}{(ax^2+bx^3+cx^4)^2}\,dx$$
$$= \left[\frac{2\,ab^4-12\,a^2b^2c+16\,a^3c^2+(ab^3-6\,a^2bc+(b^3c-6\,abc^2)x^2+(b^4-6\,ab^2c)x)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^2x^2+2\,bc}{}\right.}{2\,(ab^4c^2-8\,a^2b^2c}\right.$$

input
```
integrate(x^7/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="fricas")
```

output
```
[1/2*(2*a*b^4 - 12*a^2*b^2*c + 16*a^3*c^2 + (a*b^3 - 6*a^2*b*c + (b^3*c -
6*a*b*c^2)*x^2 + (b^4 - 6*a*b^2*c)*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2
*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) +
 2*(b^5 - 7*a*b^3*c + 12*a^2*b*c^2)*x + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2
+ (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2
)*x)*log(c*x^2 + b*x + a))/(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4 + (b^4*
c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*x^2 + (b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^
4)*x), 1/2*(2*a*b^4 - 12*a^2*b^2*c + 16*a^3*c^2 + 2*(a*b^3 - 6*a^2*b*c + (
b^3*c - 6*a*b*c^2)*x^2 + (b^4 - 6*a*b^2*c)*x)*sqrt(-b^2 + 4*a*c)*arctan(-s
qrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 2*(b^5 - 7*a*b^3*c + 12*a^2
*b*c^2)*x + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*
a^2*c^3)*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x)*log(c*x^2 + b*x + a))/(
a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4 + (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c
^5)*x^2 + (b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*x)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 729 vs. $2(104) = 208$.

Time = 0.80 (sec) , antiderivative size = 729, normalized size of antiderivative = 6.39

$$\int \frac{x^7}{(ax^2 + bx^3 + cx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x**7/(c*x**4+b*x**3+a*x**2)**2,x)
```

output
```
(-b*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(2*c**2*(64*a**3*c**3 - 48*a**
2*b**2*c**2 + 12*a*b**4*c - b**6)) + 1/(2*c**2))*log(x + (-16*a**2*c**3*(-
b*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(2*c**2*(64*a**3*c**3 - 48*a**2*
b**2*c**2 + 12*a*b**4*c - b**6)) + 1/(2*c**2)) + 8*a**2*c + 8*a*b**2*c**2*
(-b*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(2*c**2*(64*a**3*c**3 - 48*a**
2*b**2*c**2 + 12*a*b**4*c - b**6)) + 1/(2*c**2)) - a*b**2 - b**4*c*(-b*sqr
t(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*
c**2 + 12*a*b**4*c - b**6)) + 1/(2*c**2)))/(6*a*b*c - b**3)) + (b*sqrt(-(4
*a*c - b**2)**3)*(6*a*c - b**2)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2
+ 12*a*b**4*c - b**6)) + 1/(2*c**2))*log(x + (-16*a**2*c**3*(b*sqrt(-(4*a*
c - b**2)**3)*(6*a*c - b**2)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 1
2*a*b**4*c - b**6)) + 1/(2*c**2)) + 8*a**2*c + 8*a*b**2*c**2*(b*sqrt(-(4*a
*c - b**2)**3)*(6*a*c - b**2)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 +
12*a*b**4*c - b**6)) + 1/(2*c**2)) - a*b**2 - b**4*c*(b*sqrt(-(4*a*c - b**
2)**3)*(6*a*c - b**2)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**
4*c - b**6)) + 1/(2*c**2)))/(6*a*b*c - b**3)) + (2*a**2*c - a*b**2 + x*(3*
a*b*c - b**3))/(4*a**2*c**3 - a*b**2*c**2 + x**2*(4*a*c**4 - b**2*c**3) +
x*(4*a*b*c**3 - b**3*c**2))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^7}{(ax^2 + bx^3 + cx^4)^2} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^7/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.10

$$\int \frac{x^7}{(ax^2+bx^3+cx^4)^2}\,dx = -\frac{(b^3-6\,abc)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^2c^2-4\,ac^3)\sqrt{-b^2+4\,ac}} + \frac{\log\left(cx^2+bx+a\right)}{2\,c^2} + \frac{ab^2-2\,a^2c+(b^3-3\,abc)x}{(cx^2+bx+a)(b^2-4\,ac)c^2}$$

input
```
integrate(x^7/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="giac")
```

output
```
-(b^3 - 6*a*b*c)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^2*c^2 - 4*a*c^
3)*sqrt(-b^2 + 4*a*c)) + 1/2*log(c*x^2 + b*x + a)/c^2 + (a*b^2 - 2*a^2*c +
 (b^3 - 3*a*b*c)*x)/((c*x^2 + b*x + a)*(b^2 - 4*a*c)*c^2)
```

Mupad [B] (verification not implemented)

Time = 21.71 (sec) , antiderivative size = 279, normalized size of antiderivative = 2.45

$$\int \frac{x^7}{(ax^2+bx^3+cx^4)^2}\,dx = \frac{\frac{a\,\left(2\,a\,c-b^2\right)}{c^2\,\left(4\,a\,c-b^2\right)}+\frac{b\,x\,\left(3\,a\,c-b^2\right)}{c^2\,\left(4\,a\,c-b^2\right)}}{c\,x^2+b\,x+a} - \frac{\ln\left(c\,x^2+b\,x+a\right)\,\left(-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6\right)}{2\,\left(64\,a^3\,c^5-48\,a^2\,b^2\,c^4+12\,a\,b^4\,c^3-b^6\,c^2\right)} + \frac{b\,\mathrm{atan}\left(\frac{c^2\,{\left(4\,a\,c-b^2\right)}^{5/2}\,\left(\frac{2\,b\,x\,\left(6\,a\,c-b^2\right)}{c\,{\left(4\,a\,c-b^2\right)}^3}+\frac{b^2\,\left(4\,a\,c^2-b^2\,c\right)\,\left(6\,a\,c-b^2\right)}{c^3\,{\left(4\,a\,c-b^2\right)}^4}\right)}{b^3-6\,a\,b\,c}\right)\,\left(6\,a\,c-b^2\right)}{c^2\,{\left(4\,a\,c-b^2\right)}^{3/2}}$$

input
```
int(x^7/(a*x^2 + b*x^3 + c*x^4)^2,x)
```

output
```
((a*(2*a*c - b^2))/(c^2*(4*a*c - b^2)) + (b*x*(3*a*c - b^2))/(c^2*(4*a*c -
 b^2)))/(a + b*x + c*x^2) - (log(a + b*x + c*x^2)*(b^6 - 64*a^3*c^3 + 48*a
^2*b^2*c^2 - 12*a*b^4*c))/(2*(64*a^3*c^5 - b^6*c^2 + 12*a*b^4*c^3 - 48*a^2
*b^2*c^4)) + (b*atan((c^2*(4*a*c - b^2)^(5/2)*((2*b*x*(6*a*c - b^2))/(c*(4
*a*c - b^2)^3) + (b^2*(4*a*c^2 - b^2*c)*(6*a*c - b^2))/(c^3*(4*a*c - b^2)^
4)))/(b^3 - 6*a*b*c))*(6*a*c - b^2))/(c^2*(4*a*c - b^2)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 547, normalized size of antiderivative = 4.80

$$\int \frac{x^7}{(ax^2+bx^3+cx^4)^2}\,dx = \frac{-12\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)a^2bc+2\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)a\,b^3-12\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)a\,b^2c}{}$$

input
```
int(x^7/(c*x^4+b*x^3+a*x^2)^2,x)
```

output
```
( - 12*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*b*c +
2*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b**3 - 12*sqrt
(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b**2*c*x - 12*sqrt(4
*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b*c**2*x**2 + 2*sqrt(4
*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b**4*x + 2*sqrt(4*a*c -
b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b**3*c*x**2 + 16*log(a + b*x +
c*x**2)*a**3*c**2 - 8*log(a + b*x + c*x**2)*a**2*b**2*c + 16*log(a + b*x +
 c*x**2)*a**2*b*c**2*x + 16*log(a + b*x + c*x**2)*a**2*c**3*x**2 + log(a +
 b*x + c*x**2)*a*b**4 - 8*log(a + b*x + c*x**2)*a*b**3*c*x - 8*log(a + b*x
 + c*x**2)*a*b**2*c**2*x**2 + log(a + b*x + c*x**2)*b**5*x + log(a + b*x +
 c*x**2)*b**4*c*x**2 - 8*a**3*c**2 + 2*a**2*b**2*c - 24*a**2*c**3*x**2 + 1
4*a*b**2*c**2*x**2 - 2*b**4*c*x**2)/(2*c**2*(16*a**3*c**2 - 8*a**2*b**2*c
+ 16*a**2*b*c**2*x + 16*a**2*c**3*x**2 + a*b**4 - 8*a*b**3*c*x - 8*a*b**2*
c**2*x**2 + b**5*x + b**4*c*x**2))
```

3.35 $\int \frac{x^6}{(ax^2+bx^3+cx^4)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 67

$$\int \frac{x^6}{(ax^2+bx^3+cx^4)^2}\,dx = \frac{x(2a+bx)}{(b^2-4ac)\,(a+bx+cx^2)} + \frac{4a\mathrm{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}$$

output
```
x*(b*x+2*a)/(-4*a*c+b^2)/(c*x^2+b*x+a)+4*a*arctanh((2*c*x+b)/(-4*a*c+b^2)^
(1/2))/(-4*a*c+b^2)^(3/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.21

$$\int \frac{x^6}{(ax^2+bx^3+cx^4)^2}\,dx = \frac{b^2x+a(b-2cx)}{c\,(-b^2+4ac)\,(a+x(b+cx))} + \frac{4a\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{3/2}}$$

input
```
Integrate[x^6/(a*x^2 + b*x^3 + c*x^4)^2,x]
```

output
```
(b^2*x + a*(b - 2*c*x))/(c*(-b^2 + 4*a*c)*(a + x*(b + c*x))) + (4*a*ArcTan
[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {9, 1153, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{(ax^2+bx^3+cx^4)^2}dx$$

$$\Big\downarrow \text{ 9}$$

$$\int \frac{x^2}{(a+bx+cx^2)^2}dx$$

$$\Big\downarrow \text{ 1153}$$

$$\frac{x(2a+bx)}{(b^2-4ac)(a+bx+cx^2)}-\frac{2a\int \frac{1}{cx^2+bx+a}dx}{b^2-4ac}$$

$$\Big\downarrow \text{ 1083}$$

$$\frac{4a\int \frac{1}{b^2-(b+2cx)^2-4ac}d(b+2cx)}{b^2-4ac}+\frac{x(2a+bx)}{(b^2-4ac)(a+bx+cx^2)}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{4a\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}+\frac{x(2a+bx)}{(b^2-4ac)(a+bx+cx^2)}$$

input
```
Int[x^6/(a*x^2 + b*x^3 + c*x^4)^2,x]
```

output
```
(x*(2*a + b*x))/((b^2 - 4*a*c)*(a + b*x + c*x^2)) + (4*a*ArcTanh[(b + 2*c*
x)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
 !MonomialQ[Px, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1153
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*(2*p + 3)*((c*d^2 -
b*d*e + a*e^2)/((p + 1)*(b^2 - 4*a*c)))   Int[(d + e*x)^(m - 2)*(a + b*x +
c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + 2*p + 2, 0]
&& LtQ[p, -1]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.45

method	result
default	$\frac{-\frac{\left(2ac-b^2\right)x}{c\left(4ac-b^2\right)}+\frac{ab}{c\left(4ac-b^2\right)}}{c\,x^2+bx+a}+\frac{4a\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{\frac{3}{2}}}$
risch	$\frac{-\frac{\left(2ac-b^2\right)x}{c\left(4ac-b^2\right)}+\frac{ab}{c\left(4ac-b^2\right)}}{c\,x^2+bx+a}+\frac{2a\ln\left(\left(-8a\,c^2+2c\,b^2\right)x+\left(-4ac+b^2\right)^{\frac{3}{2}}-4abc+b^3\right)}{(-4ac+b^2)^{\frac{3}{2}}}-\frac{2a\ln\left(\left(8a\,c^2-2c\,b^2\right)x+\left(-4ac+b^2\right)^{\frac{3}{2}}+4abc-\right.}{(-4ac+b^2)^{\frac{3}{2}}}$

input
```
int(x^6/(c*x^4+b*x^3+a*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
(-1/c*(2*a*c-b^2)/(4*a*c-b^2)*x+1/c*a*b/(4*a*c-b^2))/(c*x^2+b*x+a)+4*a/(4*
a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 183 vs. $2(63) = 126$.

Time = 0.08 (sec) , antiderivative size = 387, normalized size of antiderivative = 5.78

$$\int \frac{x^6}{(ax^2 + bx^3 + cx^4)^2} \, dx$$
$$= \left[-\frac{ab^3 - 4\,a^2bc + 2\,(ac^2x^2 + abcx + a^2c)\sqrt{b^2 - 4\,ac}\log\left(\frac{2\,c^2x^2 + 2\,bcx + b^2 - 2\,ac - \sqrt{b^2 - 4\,ac}(2\,cx + b)}{cx^2 + bx + a}\right) + (b^4 - 6\,ab^2}{ab^4c - 8\,a^2b^2c^2 + 16\,a^3c^3 + (b^4c^2 - 8\,ab^2c^3 + 16\,a^2c^4)x^2 + (b^5c - 8\,ab^3c^2 + 16\,a^2bc^3)x} \right.$$
$$-\frac{ab^3 - 4\,a^2bc - 4\,(ac^2x^2 + abcx + a^2c)\sqrt{-b^2 + 4\,ac}\arctan\left(-\frac{\sqrt{-b^2 + 4\,ac}(2\,cx + b)}{b^2 - 4\,ac}\right) + (b^4 - 6\,ab^2c + 8\,a^2c^2}{ab^4c - 8\,a^2b^2c^2 + 16\,a^3c^3 + (b^4c^2 - 8\,ab^2c^3 + 16\,a^2c^4)x^2 + (b^5c - 8\,ab^3c^2 + 16\,a^2bc^3)x}$$

input
```
integrate(x^6/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="fricas")
```

output
```
[-(a*b^3 - 4*a^2*b*c + 2*(a*c^2*x^2 + a*b*c*x + a^2*c)*sqrt(b^2 - 4*a*c)*l
og((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*
x^2 + b*x + a)) + (b^4 - 6*a*b^2*c + 8*a^2*c^2)*x)/(a*b^4*c - 8*a^2*b^2*c^
2 + 16*a^3*c^3 + (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^2 + (b^5*c - 8*a*b
^3*c^2 + 16*a^2*b*c^3)*x), -(a*b^3 - 4*a^2*b*c - 4*(a*c^2*x^2 + a*b*c*x +
a^2*c)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*
a*c)) + (b^4 - 6*a*b^2*c + 8*a^2*c^2)*x)/(a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3
*c^3 + (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^2 + (b^5*c - 8*a*b^3*c^2 + 1
6*a^2*b*c^3)*x)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 280 vs. $2(61) = 122$.

Time = 0.38 (sec) , antiderivative size = 280, normalized size of antiderivative = 4.18

$$\int \frac{x^6}{(ax^2+bx^3+cx^4)^2}\,dx = -2a\sqrt{-\frac{1}{(4ac-b^2)^3}}\log\left(x+\frac{-32a^3c^2\sqrt{-\frac{1}{(4ac-b^2)^3}}+16a^2b^2c\sqrt{-\frac{1}{(4ac-b^2)^3}}-2ab^4\sqrt{-\frac{1}{(4ac-b^2)^3}}+2ab}{4ac}\right) +2a\sqrt{-\frac{1}{(4ac-b^2)^3}}\log\left(x+\frac{32a^3c^2\sqrt{-\frac{1}{(4ac-b^2)^3}}-16a^2b^2c\sqrt{-\frac{1}{(4ac-b^2)^3}}+2ab^4\sqrt{-\frac{1}{(4ac-b^2)^3}}+2ab}{4ac}\right) + \frac{ab+x(-2ac+b^2)}{4a^2c^2-ab^2c+x^2\cdot(4ac^3-b^2c^2)+x(4abc^2-b^3c)}$$

input
```
integrate(x**6/(c*x**4+b*x**3+a*x**2)**2,x)
```

output
```
-2*a*sqrt(-1/(4*a*c - b**2)**3)*log(x + (-32*a**3*c**2*sqrt(-1/(4*a*c - b*
*2)**3) + 16*a**2*b**2*c*sqrt(-1/(4*a*c - b**2)**3) - 2*a*b**4*sqrt(-1/(4*
a*c - b**2)**3) + 2*a*b)/(4*a*c)) + 2*a*sqrt(-1/(4*a*c - b**2)**3)*log(x +
 (32*a**3*c**2*sqrt(-1/(4*a*c - b**2)**3) - 16*a**2*b**2*c*sqrt(-1/(4*a*c
- b**2)**3) + 2*a*b**4*sqrt(-1/(4*a*c - b**2)**3) + 2*a*b)/(4*a*c)) + (a*b
 + x*(-2*a*c + b**2))/(4*a**2*c**2 - a*b**2*c + x**2*(4*a*c**3 - b**2*c**2
) + x*(4*a*b*c**2 - b**3*c))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^6}{(ax^2+bx^3+cx^4)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^6/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.31

$$\int \frac{x^6}{(ax^2+bx^3+cx^4)^2}\,dx = -\frac{4\,a\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^2-4\,ac)\sqrt{-b^2+4\,ac}} - \frac{b^2x-2\,acx+ab}{(b^2c-4\,ac^2)(cx^2+bx+a)}$$

input
```
integrate(x^6/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="giac")
```

output
```
-4*a*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^2 - 4*a*c)*sqrt(-b^2 + 4*a
*c)) - (b^2*x - 2*a*c*x + a*b)/((b^2*c - 4*a*c^2)*(c*x^2 + b*x + a))
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 135, normalized size of antiderivative = 2.01

$$\int \frac{x^6}{(ax^2+bx^3+cx^4)^2}\,dx = -\frac{\frac{x\,(2\,a\,c-b^2)}{c\,(4\,a\,c-b^2)}-\frac{a\,b}{c\,(4\,a\,c-b^2)}}{c\,x^2+b\,x+a} - \frac{4\,a\,\mathrm{atan}\left(\frac{\left(\frac{2\,a\,(b^3-4\,a\,b\,c)}{(4\,a\,c-b^2)^{5/2}}-\frac{4\,a\,c\,x}{(4\,a\,c-b^2)^{3/2}}\right)(4\,a\,c-b^2)}{2\,a}\right)}{(4\,a\,c-b^2)^{3/2}}$$

input
```
int(x^6/(a*x^2 + b*x^3 + c*x^4)^2,x)
```

output
```
- ((x*(2*a*c - b^2))/(c*(4*a*c - b^2)) - (a*b)/(c*(4*a*c - b^2)))/(a + b*x
 + c*x^2) - (4*a*atan((((2*a*(b^3 - 4*a*b*c))/(4*a*c - b^2)^(5/2) - (4*a*c
*x)/(4*a*c - b^2)^(3/2))*(4*a*c - b^2))/(2*a)))/(4*a*c - b^2)^(3/2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 243, normalized size of antiderivative = 3.63

$$\int \frac{x^6}{(ax^2+bx^3+cx^4)^2}\,dx$$
$$= \frac{4\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)a^2b+4\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)a\,b^2x+4\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)abc\,x^2+}{b\,(16a^2c^3x^2-8a\,b^2c^2x^2+b^4c\,x^2+16a^2b\,c^2x-8a\,b^3cx+b^5x+16a^3c^2-8}$$

input
```
int(x^6/(c*x^4+b*x^3+a*x^2)^2,x)
```

output
```
(4*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*b + 4*sqrt
(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b**2*x + 4*sqrt(4*a*
c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b*c*x**2 + 8*a**3*c - 2*a
**2*b**2 + 8*a**2*c**2*x**2 - 6*a*b**2*c*x**2 + b**4*x**2)/(b*(16*a**3*c**
2 - 8*a**2*b**2*c + 16*a**2*b*c**2*x + 16*a**2*c**3*x**2 + a*b**4 - 8*a*b*
*3*c*x - 8*a*b**2*c**2*x**2 + b**5*x + b**4*c*x**2))
```

3.36 $\int \frac{x^5}{(ax^2+bx^3+cx^4)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 66

$$\int \frac{x^5}{(ax^2+bx^3+cx^4)^2}\,dx=\frac{2a+bx}{(b^2-4ac)\,(a+bx+cx^2)}-\frac{2b\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}$$

output
```
(b*x+2*a)/(-4*a*c+b^2)/(c*x^2+b*x+a)-2*b*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1
/2))/(-4*a*c+b^2)^(3/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.05

$$\int \frac{x^5}{(ax^2+bx^3+cx^4)^2}\,dx=\frac{2a+bx}{(b^2-4ac)\,(a+x(b+cx))}-\frac{2b\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{3/2}}$$

input
```
Integrate[x^5/(a*x^2 + b*x^3 + c*x^4)^2,x]
```

output
```
(2*a + b*x)/((b^2 - 4*a*c)*(a + x*(b + c*x))) - (2*b*ArcTan[(b + 2*c*x)/Sq
rt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {9, 1159, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(ax^2+bx^3+cx^4)^2}dx$$

$$\Big\downarrow \text{9}$$

$$\int \frac{x}{(a+bx+cx^2)^2}dx$$

$$\Big\downarrow \text{1159}$$

$$\frac{b\int \frac{1}{cx^2+bx+a}dx}{b^2-4ac}+\frac{2a+bx}{(b^2-4ac)(a+bx+cx^2)}$$

$$\Big\downarrow \text{1083}$$

$$\frac{2a+bx}{(b^2-4ac)(a+bx+cx^2)}-\frac{2b\int \frac{1}{b^2-(b+2cx)^2-4ac}d(b+2cx)}{b^2-4ac}$$

$$\Big\downarrow \text{219}$$

$$\frac{2a+bx}{(b^2-4ac)(a+bx+cx^2)}-\frac{2b\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}$$

input
```
Int[x^5/(a*x^2 + b*x^3 + c*x^4)^2,x]
```

output
```
(2*a + b*x)/((b^2 - 4*a*c)*(a + b*x + c*x^2)) - (2*b*ArcTanh[(b + 2*c*x)/S
qrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1159
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Simp[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &
& LtQ[p, -1] && NeQ[p, -3/2]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.06

method	result	si
default	$\frac{-bx-2a}{(4ac-b^2)(c\,x^2+bx+a)}-\frac{2b\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{\frac{3}{2}}}$	70
risch	$\frac{-\frac{bx}{4ac-b^2}-\frac{2a}{4ac-b^2}}{c\,x^2+bx+a}+\frac{b\ln\left(\left(-8a\,c^2+2c\,b^2\right)x-\left(-4ac+b^2\right)^{\frac{3}{2}}-4abc+b^3\right)}{\left(-4ac+b^2\right)^{\frac{3}{2}}}-\frac{b\ln\left(\left(8a\,c^2-2c\,b^2\right)x-\left(-4ac+b^2\right)^{\frac{3}{2}}+4abc-b^3\right)}{\left(-4ac+b^2\right)^{\frac{3}{2}}}$	14

input
```
int(x^5/(c*x^4+b*x^3+a*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
(-b*x-2*a)/(4*a*c-b^2)/(c*x^2+b*x+a)-2*b/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b
)/(4*a*c-b^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 159 vs. 2(62) = 124.

Time = 0.08 (sec) , antiderivative size = 338, normalized size of antiderivative = 5.12

$$\int \frac{x^5}{(ax^2+bx^3+cx^4)^2}\,dx$$
$$= \left[\frac{2\,ab^2-8\,a^2c-(bcx^2+b^2x+ab)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac+\sqrt{b^2-4\,ac}(2\,cx+b)}{cx^2+bx+a}\right)+(b^3-4\,abc)x}{ab^4-8\,a^2b^2c+16\,a^3c^2+(b^4c-8\,ab^2c^2+16\,a^2c^3)x^2+(b^5-8\,ab^3c+16\,a^2bc^2)x}, \frac{2\,a\right.$$

input
```
integrate(x^5/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="fricas")
```

output
```
[(2*a*b^2 - 8*a^2*c - (b*c*x^2 + b^2*x + a*b)*sqrt(b^2 - 4*a*c)*log((2*c^2
*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x
 + a)) + (b^3 - 4*a*b*c)*x)/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8
*a*b^2*c^2 + 16*a^2*c^3)*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x), (2*a*b
^2 - 8*a^2*c - 2*(b*c*x^2 + b^2*x + a*b)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-
b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + (b^3 - 4*a*b*c)*x)/(a*b^4 - 8*a^
2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^2 + (b^5 - 8*a
*b^3*c + 16*a^2*b*c^2)*x)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 253 vs. $2(60) = 120$.

Time = 0.34 (sec) , antiderivative size = 253, normalized size of antiderivative = 3.83

$$\int \frac{x^5}{(ax^2+bx^3+cx^4)^2}\,dx$$
$$= b\sqrt{-\frac{1}{(4ac-b^2)^3}}\log\left(x+\frac{-16a^2bc^2\sqrt{-\frac{1}{(4ac-b^2)^3}}+8ab^3c\sqrt{-\frac{1}{(4ac-b^2)^3}}-b^5\sqrt{-\frac{1}{(4ac-b^2)^3}}+b^2}{2bc}\right)$$
$$-b\sqrt{-\frac{1}{(4ac-b^2)^3}}\log\left(x+\frac{16a^2bc^2\sqrt{-\frac{1}{(4ac-b^2)^3}}-8ab^3c\sqrt{-\frac{1}{(4ac-b^2)^3}}+b^5\sqrt{-\frac{1}{(4ac-b^2)^3}}+b^2}{2bc}\right)$$
$$+\frac{-2a-bx}{4a^2c-ab^2+x^2\cdot(4ac^2-b^2c)+x(4abc-b^3)}$$

input
```
integrate(x**5/(c*x**4+b*x**3+a*x**2)**2,x)
```

output
```
b*sqrt(-1/(4*a*c - b**2)**3)*log(x + (-16*a**2*b*c**2*sqrt(-1/(4*a*c - b**
2)**3) + 8*a*b**3*c*sqrt(-1/(4*a*c - b**2)**3) - b**5*sqrt(-1/(4*a*c - b**
2)**3) + b**2)/(2*b*c)) - b*sqrt(-1/(4*a*c - b**2)**3)*log(x + (16*a**2*b*
c**2*sqrt(-1/(4*a*c - b**2)**3) - 8*a*b**3*c*sqrt(-1/(4*a*c - b**2)**3) +
b**5*sqrt(-1/(4*a*c - b**2)**3) + b**2)/(2*b*c)) + (-2*a - b*x)/(4*a**2*c
- a*b**2 + x**2*(4*a*c**2 - b**2*c) + x*(4*a*b*c - b**3))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5}{(ax^2+bx^3+cx^4)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^5/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.15

$$\int \frac{x^5}{(ax^2+bx^3+cx^4)^2}\,dx = \frac{2\,b\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^2-4\,ac)\sqrt{-b^2+4\,ac}} + \frac{bx+2\,a}{(cx^2+bx+a)(b^2-4\,ac)}$$

input
```
integrate(x^5/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="giac")
```

output
```
2*b*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^2 - 4*a*c)*sqrt(-b^2 + 4*a*
c)) + (b*x + 2*a)/((c*x^2 + b*x + a)*(b^2 - 4*a*c))
```

Mupad [B] (verification not implemented)

Time = 21.20 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.67

$$\int \frac{x^5}{(ax^2+bx^3+cx^4)^2}\,dx = -\frac{\frac{2\,a}{4\,a\,c-b^2}+\frac{b\,x}{4\,a\,c-b^2}}{c\,x^2+b\,x+a} - \frac{2\,b\,\mathrm{atan}\left(\frac{\left(\frac{b^2}{{\left(4\,a\,c-b^2\right)}^{3/2}}+\frac{2\,b\,c\,x}{{\left(4\,a\,c-b^2\right)}^{3/2}}\right)\,\left(4\,a\,c-b^2\right)}{b}\right)}{{\left(4\,a\,c-b^2\right)}^{3/2}}$$

input
```
int(x^5/(a*x^2 + b*x^3 + c*x^4)^2,x)
```

output
```
- ((2*a)/(4*a*c - b^2) + (b*x)/(4*a*c - b^2))/(a + b*x + c*x^2) - (2*b*ata
n(((b^2/(4*a*c - b^2)^(3/2) + (2*b*c*x)/(4*a*c - b^2)^(3/2))*(4*a*c - b^2)
)/b))/(4*a*c - b^2)^(3/2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 223, normalized size of antiderivative = 3.38

$$\int \frac{x^5}{(ax^2+bx^3+cx^4)^2}\,dx$$
$$= \frac{-2\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)ab-2\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)b^2x-2\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)bc\,x^2-4}{16a^2c^3x^2-8a\,b^2c^2x^2+b^4c\,x^2+16a^2b\,c^2x-8a\,b^3cx+b^5x+16a^3c^2-8a^2b^2c+}$$

input

```
int(x^5/(c*x^4+b*x^3+a*x^2)^2,x)
```

output

```
( - 2*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b - 2*sqrt
(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b**2*x - 2*sqrt(4*a*c
- b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b*c*x**2 - 4*a**2*c + a*b**2
+ 4*a*c**2*x**2 - b**2*c*x**2)/(16*a**3*c**2 - 8*a**2*b**2*c + 16*a**2*b*c
**2*x + 16*a**2*c**3*x**2 + a*b**4 - 8*a*b**3*c*x - 8*a*b**2*c**2*x**2 + b
**5*x + b**4*c*x**2)
```

3.37 $\int \frac{x^4}{\left(ax^2+bx^3+cx^4\right)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 66

$$\int \frac{x^4}{\left(ax^2+bx^3+cx^4\right)^2}\,dx=-\frac{b+2cx}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}+\frac{4c\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}$$

output
```
-(2*c*x+b)/(-4*a*c+b^2)/(c*x^2+b*x+a)+4*c*arctanh((2*c*x+b)/(-4*a*c+b^2)^(
1/2))/(-4*a*c+b^2)^(3/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.06

$$\int \frac{x^4}{\left(ax^2+bx^3+cx^4\right)^2}\,dx=-\frac{\frac{b+2cx}{a+x(b+cx)}+\frac{4c\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}}{b^2-4ac}$$

input
```
Integrate[x^4/(a*x^2 + b*x^3 + c*x^4)^2,x]
```

output
```
-(((b + 2*c*x)/(a + x*(b + c*x)) + (4*c*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a
*c]])/Sqrt[-b^2 + 4*a*c])/(b^2 - 4*a*c))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {9, 1086, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(ax^2+bx^3+cx^4)^2}dx$$

$$\Big\downarrow \text{9}$$

$$\int \frac{1}{(a+bx+cx^2)^2}dx$$

$$\Big\downarrow \text{1086}$$

$$-\frac{2c\int \frac{1}{cx^2+bx+a}dx}{b^2-4ac}-\frac{b+2cx}{(b^2-4ac)(a+bx+cx^2)}$$

$$\Big\downarrow \text{1083}$$

$$\frac{4c\int \frac{1}{b^2-(b+2cx)^2-4ac}d(b+2cx)}{b^2-4ac}-\frac{b+2cx}{(b^2-4ac)(a+bx+cx^2)}$$

$$\Big\downarrow \text{219}$$

$$\frac{4c\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}-\frac{b+2cx}{(b^2-4ac)(a+bx+cx^2)}$$

input
```
Int[x^4/(a*x^2 + b*x^3 + c*x^4)^2,x]
```

output
```
-((b + 2*c*x)/((b^2 - 4*a*c)*(a + b*x + c*x^2))) + (4*c*ArcTanh[(b + 2*c*x
)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1086
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && ILtQ[p, -1]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.03

method	result	s
default	$\frac{2cx+b}{(4ac-b^{2})(c\,x^{2}+bx+a)}+\frac{4c\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^{2}}}\right)}{(4ac-b^{2})^{\frac{3}{2}}}$	6
risch	$\frac{\frac{2cx}{4ac-b^{2}}+\frac{b}{4ac-b^{2}}}{c\,x^{2}+bx+a}+\frac{2c\ln\left(\left(-8a\,c^{2}+2c\,b^{2}\right)x+\left(-4ac+b^{2}\right)^{\frac{3}{2}}-4abc+b^{3}\right)}{\left(-4ac+b^{2}\right)^{\frac{3}{2}}}-\frac{2c\ln\left(\left(8a\,c^{2}-2c\,b^{2}\right)x+\left(-4ac+b^{2}\right)^{\frac{3}{2}}+4abc-b^{3}\right)}{\left(-4ac+b^{2}\right)^{\frac{3}{2}}}$	1

input
```
int(x^4/(c*x^4+b*x^3+a*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)+4*c/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)
/(4*a*c-b^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(62) = 124$.

Time = 0.09 (sec) , antiderivative size = 341, normalized size of antiderivative = 5.17

$$\int \frac{x^4}{(ax^2+bx^3+cx^4)^2}\,dx$$
$$= \left[-\frac{b^3-4\,abc+2\,(c^2x^2+bcx+ac)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac-\sqrt{b^2-4\,ac}(2\,cx+b)}{cx^2+bx+a}\right)+2\,(b^2c-4\,ac^2)x}{ab^4-8\,a^2b^2c+16\,a^3c^2+(b^4c-8\,ab^2c^2+16\,a^2c^3)x^2+(b^5-8\,ab^3c+16\,a^2bc^2)x},\right.$$
$$\left.-\frac{b^3-4\,abc-4\,(c^2x^2+bcx+ac)\sqrt{-b^2+4\,ac}\arctan\left(-\frac{\sqrt{-b^2+4\,ac}(2\,cx+b)}{b^2-4\,ac}\right)+2\,(b^2c-4\,ac^2)x}{ab^4-8\,a^2b^2c+16\,a^3c^2+(b^4c-8\,ab^2c^2+16\,a^2c^3)x^2+(b^5-8\,ab^3c+16\,a^2bc^2)x}\right]$$

input
```
integrate(x^4/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="fricas")
```

output
```
[-(b^3 - 4*a*b*c + 2*(c^2*x^2 + b*c*x + a*c)*sqrt(b^2 - 4*a*c)*log((2*c^2*
x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x
+ a)) + 2*(b^2*c - 4*a*c^2)*x)/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c
- 8*a*b^2*c^2 + 16*a^2*c^3)*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x), -(b
^3 - 4*a*b*c - 4*(c^2*x^2 + b*c*x + a*c)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-
b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 2*(b^2*c - 4*a*c^2)*x)/(a*b^4 -
8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^2 + (b^5 -
 8*a*b^3*c + 16*a^2*b*c^2)*x)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 265 vs. $2(61) = 122$.

Time = 0.34 (sec) , antiderivative size = 265, normalized size of antiderivative = 4.02

$$\int \frac{x^4}{(ax^2+bx^3+cx^4)^2}\,dx =$$
$$-2c\sqrt{-\frac{1}{(4ac-b^2)^3}}\log\left(x+\frac{-32a^2c^3\sqrt{-\frac{1}{(4ac-b^2)^3}}+16ab^2c^2\sqrt{-\frac{1}{(4ac-b^2)^3}}-2b^4c\sqrt{-\frac{1}{(4ac-b^2)^3}}+2bc}{4c^2}\right)$$
$$+2c\sqrt{-\frac{1}{(4ac-b^2)^3}}\log\left(x+\frac{32a^2c^3\sqrt{-\frac{1}{(4ac-b^2)^3}}-16ab^2c^2\sqrt{-\frac{1}{(4ac-b^2)^3}}+2b^4c\sqrt{-\frac{1}{(4ac-b^2)^3}}+2bc}{4c^2}\right)$$
$$+\frac{b+2cx}{4a^2c-ab^2+x^2\cdot(4ac^2-b^2c)+x\,(4abc-b^3)}$$

input
```
integrate(x**4/(c*x**4+b*x**3+a*x**2)**2,x)
```

output
```
-2*c*sqrt(-1/(4*a*c - b**2)**3)*log(x + (-32*a**2*c**3*sqrt(-1/(4*a*c - b*
*2)**3) + 16*a*b**2*c**2*sqrt(-1/(4*a*c - b**2)**3) - 2*b**4*c*sqrt(-1/(4*
a*c - b**2)**3) + 2*b*c)/(4*c**2)) + 2*c*sqrt(-1/(4*a*c - b**2)**3)*log(x
+ (32*a**2*c**3*sqrt(-1/(4*a*c - b**2)**3) - 16*a*b**2*c**2*sqrt(-1/(4*a*c
 - b**2)**3) + 2*b**4*c*sqrt(-1/(4*a*c - b**2)**3) + 2*b*c)/(4*c**2)) + (b
 + 2*c*x)/(4*a**2*c - a*b**2 + x**2*(4*a*c**2 - b**2*c) + x*(4*a*b*c - b**
3))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^4}{(ax^2+bx^3+cx^4)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^4/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.15

$$\int \frac{x^4}{(ax^2+bx^3+cx^4)^2}\,dx = -\frac{4\,c\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^2-4\,ac)\sqrt{-b^2+4\,ac}} - \frac{2\,cx+b}{(cx^2+bx+a)(b^2-4\,ac)}$$

input
```
integrate(x^4/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="giac")
```

output
```
-4*c*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^2 - 4*a*c)*sqrt(-b^2 + 4*a
*c)) - (2*c*x + b)/((c*x^2 + b*x + a)*(b^2 - 4*a*c))
```

Mupad [B] (verification not implemented)

Time = 21.20 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.80

$$\int \frac{x^4}{(ax^2+bx^3+cx^4)^2}\,dx = \frac{\frac{b}{4\,a\,c-b^2}+\frac{2\,c\,x}{4\,a\,c-b^2}}{c\,x^2+b\,x+a} - \frac{4\,c\,\mathrm{atan}\left(\frac{\left(\frac{2\,c\,\left(b^3-4\,a\,b\,c\right)}{{\left(4\,a\,c-b^2\right)}^{5/2}}-\frac{4\,c^2\,x}{{\left(4\,a\,c-b^2\right)}^{3/2}}\right)\,\left(4\,a\,c-b^2\right)}{2\,c}\right)}{{\left(4\,a\,c-b^2\right)}^{3/2}}$$

input
```
int(x^4/(a*x^2 + b*x^3 + c*x^4)^2,x)
```

output
```
(b/(4*a*c - b^2) + (2*c*x)/(4*a*c - b^2))/(a + b*x + c*x^2) - (4*c*atan(((
(2*c*(b^3 - 4*a*b*c))/(4*a*c - b^2)^(5/2) - (4*c^2*x)/(4*a*c - b^2)^(3/2))
*(4*a*c - b^2))/(2*c)))/(4*a*c - b^2)^(3/2)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 241, normalized size of antiderivative = 3.65

$$\int \frac{x^4}{(ax^2+bx^3+cx^4)^2}\,dx$$
$$= \frac{4\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)abc+4\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)b^2cx+4\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)bc^2x^2-}{b\,(16a^2c^3x^2-8a\,b^2c^2x^2+b^4c\,x^2+16a^2b\,c^2x-8a\,b^3cx+b^5x+16a^3c^2-8a^2}$$

input
```
int(x^4/(c*x^4+b*x^3+a*x^2)^2,x)
```

output
```
(4*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b*c + 4*sqrt(
4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b**2*c*x + 4*sqrt(4*a*c
 - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b*c**2*x**2 - 8*a**2*c**2 +
6*a*b**2*c - 8*a*c**3*x**2 - b**4 + 2*b**2*c**2*x**2)/(b*(16*a**3*c**2 - 8
*a**2*b**2*c + 16*a**2*b*c**2*x + 16*a**2*c**3*x**2 + a*b**4 - 8*a*b**3*c*
x - 8*a*b**2*c**2*x**2 + b**5*x + b**4*c*x**2))
```

3.38 $\int \frac{x^3}{(ax^2+bx^3+cx^4)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 108

$$\int \frac{x^3}{(ax^2+bx^3+cx^4)^2}\,dx = \frac{b^2-2ac+bcx}{a(b^2-4ac)(a+bx+cx^2)} + \frac{b(b^2-6ac)\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^2(b^2-4ac)^{3/2}} + \frac{\log(x)}{a^2} - \frac{\log(a+bx+cx^2)}{2a^2}$$

output
```
(b*c*x-2*a*c+b^2)/a/(-4*a*c+b^2)/(c*x^2+b*x+a)+b*(-6*a*c+b^2)*arctanh((2*c
*x+b)/(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^2)^(3/2)+ln(x)/a^2-1/2*ln(c*x^2+b*
x+a)/a^2
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.99

$$\int \frac{x^3}{(ax^2+bx^3+cx^4)^2}\,dx = \frac{\frac{2a(b^2-2ac+bcx)}{(b^2-4ac)(a+x(b+cx))} + \frac{2b(b^2-6ac)\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{3/2}} + 2\log(x) - \log(a+x(b+cx))}{2a^2}$$

input
```
Integrate[x^3/(a*x^2 + b*x^3 + c*x^4)^2,x]
```

output
```
((2*a*(b^2 - 2*a*c + b*c*x))/((b^2 - 4*a*c)*(a + x*(b + c*x))) + (2*b*(b^2
 - 6*a*c)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2) + 2
*Log[x] - Log[a + x*(b + c*x)])/(2*a^2)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.29, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {9, 1165, 25, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(ax^2+bx^3+cx^4)^2}dx$$

$$\Big\downarrow \text{ 9}$$

$$\int \frac{1}{x\,(a+bx+cx^2)^2}dx$$

$$\Big\downarrow \text{ 1165}$$

$$\frac{-2ac+b^2+bcx}{a\,(b^2-4ac)\,(a+bx+cx^2)}-\frac{\int -\frac{b^2+cxb-4ac}{x(cx^2+bx+a)}dx}{a\,(b^2-4ac)}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{b^2+cxb-4ac}{x(cx^2+bx+a)}dx}{a\,(b^2-4ac)}+\frac{-2ac+b^2+bcx}{a\,(b^2-4ac)\,(a+bx+cx^2)}$$

$$\Big\downarrow \text{ 1200}$$

$$\frac{\int \left(\frac{b^2-4ac}{ax}+\frac{-b(b^2-5ac)-c(b^2-4ac)x}{a(cx^2+bx+a)}\right)dx}{a\,(b^2-4ac)}+\frac{-2ac+b^2+bcx}{a\,(b^2-4ac)\,(a+bx+cx^2)}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{b(b^2-6ac)\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}}-\frac{(b^2-4ac)\log(a+bx+cx^2)}{2a}+\frac{\log(x)(b^2-4ac)}{a}}{a\left(b^2-4ac\right)}+\frac{-2ac+b^2+bcx}{a\left(b^2-4ac\right)\left(a+bx+cx^2\right)}$$

input
```
Int[x^3/(a*x^2 + b*x^3 + c*x^4)^2,x]
```

output
```
(b^2 - 2*a*c + b*c*x)/(a*(b^2 - 4*a*c)*(a + b*x + c*x^2)) + ((b*(b^2 - 6*a
*c)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a*Sqrt[b^2 - 4*a*c]) + ((b^2
- 4*a*c)*Log[x])/a - ((b^2 - 4*a*c)*Log[a + b*x + c*x^2])/(2*a))/(a*(b^2 -
 4*a*c))
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1165
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d
+ e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p
+ 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
 b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && LtQ[p, -1]
 && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.64

method	result	size
default	$-\frac{\frac{\frac{abcx}{4ac-b^2}-\frac{a\left(2ac-b^2\right)}{4ac-b^2}}{c\,x^2+bx+a}+\frac{\frac{\left(4a\,c^2-c\,b^2\right)\ln\left(c\,x^2+bx+a\right)}{2c}+\frac{2\left(5abc-b^3-\frac{\left(4a\,c^2-c\,b^2\right)b}{2c}\right)\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{4ac-b^2}}{a^2}+\frac{\ln\left(x\right)}{a^2}$	177
risch	Expression too large to display	2292

input
```
int(x^3/(c*x^4+b*x^3+a*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/a^2*((a*b*c/(4*a*c-b^2)*x-a*(2*a*c-b^2)/(4*a*c-b^2))/(c*x^2+b*x+a)+1/(4
*a*c-b^2)*(1/2*(4*a*c^2-b^2*c)/c*ln(c*x^2+b*x+a)+2*(5*a*b*c-b^3-1/2*(4*a*c
^2-b^2*c)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))))+ln(
x)/a^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 381 vs. 2(102) = 204.

Time = 0.14 (sec) , antiderivative size = 781, normalized size of antiderivative = 7.23

$$\int \frac{x^3}{(ax^2+bx^3+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^3/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="fricas")
```

output
```
[1/2*(2*a*b^4 - 12*a^2*b^2*c + 16*a^3*c^2 + (a*b^3 - 6*a^2*b*c + (b^3*c -
6*a*b*c^2)*x^2 + (b^4 - 6*a*b^2*c)*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2
*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) +
 2*(a*b^3*c - 4*a^2*b*c^2)*x - (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c
- 8*a*b^2*c^2 + 16*a^2*c^3)*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x)*log(
c*x^2 + b*x + a) + 2*(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*
c^2 + 16*a^2*c^3)*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x)*log(x))/(a^3*b
^4 - 8*a^4*b^2*c + 16*a^5*c^2 + (a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*x
^2 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x), 1/2*(2*a*b^4 - 12*a^2*b^2*
c + 16*a^3*c^2 + 2*(a*b^3 - 6*a^2*b*c + (b^3*c - 6*a*b*c^2)*x^2 + (b^4 - 6
*a*b^2*c)*x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^
2 - 4*a*c)) + 2*(a*b^3*c - 4*a^2*b*c^2)*x - (a*b^4 - 8*a^2*b^2*c + 16*a^3*
c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b
*c^2)*x)*log(c*x^2 + b*x + a) + 2*(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4
*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x)*l
og(x))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + (a^2*b^4*c - 8*a^3*b^2*c^2 +
16*a^4*c^3)*x^2 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^3}{(ax^2+bx^3+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**3/(c*x**4+b*x**3+a*x**2)**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3}{(ax^2+bx^3+cx^4)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.17

$$\int \frac{x^3}{(ax^2+bx^3+cx^4)^2}\,dx = -\frac{(b^3-6\,abc)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(a^2b^2-4\,a^3c)\sqrt{-b^2+4\,ac}} - \frac{\log\left(cx^2+bx+a\right)}{2\,a^2} + \frac{\log\left(|x|\right)}{a^2} + \frac{abcx+ab^2-2\,a^2c}{(cx^2+bx+a)(b^2-4\,ac)a^2}$$

input
```
integrate(x^3/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="giac")
```

output
```
-(b^3 - 6*a*b*c)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((a^2*b^2 - 4*a^3*
c)*sqrt(-b^2 + 4*a*c)) - 1/2*log(c*x^2 + b*x + a)/a^2 + log(abs(x))/a^2 +
(a*b*c*x + a*b^2 - 2*a^2*c)/((c*x^2 + b*x + a)*(b^2 - 4*a*c)*a^2)
```

Mupad [B] (verification not implemented)

Time = 23.13 (sec) , antiderivative size = 620, normalized size of antiderivative = 5.74

$$\int \frac{x^3}{(ax^2+bx^3+cx^4)^2}\,dx = \frac{\ln(x)}{a^2} + \frac{\frac{2\,a\,c-b^2}{a\,(4\,a\,c-b^2)} - \frac{b\,c\,x}{a\,(4\,a\,c-b^2)}}{c\,x^2+b\,x+a}$$
$$+ \frac{\ln\left(2\,a\,b^6+2\,b^7\,x-96\,a^4\,c^3+2\,a\,b^3\,\sqrt{-(4\,a\,c-b^2)^3}-23\,a^2\,b^4\,c+2\,b^4\,x\,\sqrt{-(4\,a\,c-b^2)^3}+84\,a^3\,b^2\,c^2\right.}{}$$
$$+ \frac{\ln\left(96\,a^4\,c^3-2\,b^7\,x-2\,a\,b^6+2\,a\,b^3\,\sqrt{-(4\,a\,c-b^2)^3}+23\,a^2\,b^4\,c+2\,b^4\,x\,\sqrt{-(4\,a\,c-b^2)^3}-84\,a^3\,b^2\,c^2\right.}{}$$

input
```
int(x^3/(a*x^2 + b*x^3 + c*x^4)^2,x)
```

output
```
log(x)/a^2 + ((2*a*c - b^2)/(a*(4*a*c - b^2)) - (b*c*x)/(a*(4*a*c - b^2)))
/(a + b*x + c*x^2) + (log(2*a*b^6 + 2*b^7*x - 96*a^4*c^3 + 2*a*b^3*(-(4*a*
c - b^2)^3)^(1/2) - 23*a^2*b^4*c + 2*b^4*x*(-(4*a*c - b^2)^3)^(1/2) + 84*a
^3*b^2*c^2 + 94*a^2*b^3*c^2*x + 12*a^2*c^2*x*(-(4*a*c - b^2)^3)^(1/2) - 24
*a*b^5*c*x - 9*a^2*b*c*(-(4*a*c - b^2)^3)^(1/2) - 120*a^3*b*c^3*x - 12*a*b
^2*c*x*(-(4*a*c - b^2)^3)^(1/2))*(b^6 - 64*a^3*c^3 + b^3*(-(4*a*c - b^2)^3
)^(1/2) + 48*a^2*b^2*c^2 - 12*a*b^4*c - 6*a*b*c*(-(4*a*c - b^2)^3)^(1/2)))
/(2*a^2*(4*a*c - b^2)^3) + (log(96*a^4*c^3 - 2*b^7*x - 2*a*b^6 + 2*a*b^3*(
-(4*a*c - b^2)^3)^(1/2) + 23*a^2*b^4*c + 2*b^4*x*(-(4*a*c - b^2)^3)^(1/2)
- 84*a^3*b^2*c^2 - 94*a^2*b^3*c^2*x + 12*a^2*c^2*x*(-(4*a*c - b^2)^3)^(1/2
) + 24*a*b^5*c*x - 9*a^2*b*c*(-(4*a*c - b^2)^3)^(1/2) + 120*a^3*b*c^3*x -
12*a*b^2*c*x*(-(4*a*c - b^2)^3)^(1/2))*(b^6 - 64*a^3*c^3 - b^3*(-(4*a*c -
b^2)^3)^(1/2) + 48*a^2*b^2*c^2 - 12*a*b^4*c + 6*a*b*c*(-(4*a*c - b^2)^3)^(
1/2)))/(2*a^2*(4*a*c - b^2)^3)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 644, normalized size of antiderivative = 5.96

$$\int \frac{x^3}{(ax^2+bx^3+cx^4)^2}\,dx$$
$$= \frac{2a\,b^4 - 12\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)a^2bc + 2\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)b^3c\,x^2 - 16\log(c\,x^2+bx+a)\,a^2b}{}$$

input
```
int(x^3/(c*x^4+b*x^3+a*x^2)^2,x)
```

output

```
( - 12*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*b*c +
2*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b**3 - 12*sqrt
(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b**2*c*x - 12*sqrt(4
*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b*c**2*x**2 + 2*sqrt(4
*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b**4*x + 2*sqrt(4*a*c -
b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b**3*c*x**2 - 16*log(a + b*x +
c*x**2)*a**3*c**2 + 8*log(a + b*x + c*x**2)*a**2*b**2*c - 16*log(a + b*x +
 c*x**2)*a**2*b*c**2*x - 16*log(a + b*x + c*x**2)*a**2*c**3*x**2 - log(a +
 b*x + c*x**2)*a*b**4 + 8*log(a + b*x + c*x**2)*a*b**3*c*x + 8*log(a + b*x
 + c*x**2)*a*b**2*c**2*x**2 - log(a + b*x + c*x**2)*b**5*x - log(a + b*x +
 c*x**2)*b**4*c*x**2 + 32*log(x)*a**3*c**2 - 16*log(x)*a**2*b**2*c + 32*lo
g(x)*a**2*b*c**2*x + 32*log(x)*a**2*c**3*x**2 + 2*log(x)*a*b**4 - 16*log(x
)*a*b**3*c*x - 16*log(x)*a*b**2*c**2*x**2 + 2*log(x)*b**5*x + 2*log(x)*b**
4*c*x**2 + 24*a**3*c**2 - 14*a**2*b**2*c + 8*a**2*c**3*x**2 + 2*a*b**4 - 2
*a*b**2*c**2*x**2)/(2*a**2*(16*a**3*c**2 - 8*a**2*b**2*c + 16*a**2*b*c**2*
x + 16*a**2*c**3*x**2 + a*b**4 - 8*a*b**3*c*x - 8*a*b**2*c**2*x**2 + b**5*
x + b**4*c*x**2))
```

3.39 $\int \frac{x^2}{(ax^2+bx^3+cx^4)^2} dx$

Optimal result

Integrand size = 22, antiderivative size = 148

$$\int \frac{x^2}{(ax^2+bx^3+cx^4)^2} dx = -\frac{2(b^2-3ac)}{a^2(b^2-4ac)x} + \frac{b^2-2ac+bcx}{a(b^2-4ac)x(a+bx+cx^2)} - \frac{2(b^4-6ab^2c+6a^2c^2)\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^3(b^2-4ac)^{3/2}} - \frac{2b\log(x)}{a^3} + \frac{b\log(a+bx+cx^2)}{a^3}$$

output
```
(6*a*c-2*b^2)/a^2/(-4*a*c+b^2)/x+(b*c*x-2*a*c+b^2)/a/(-4*a*c+b^2)/x/(c*x^2
+b*x+a)-2*(6*a^2*c^2-6*a*b^2*c+b^4)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/
a^3/(-4*a*c+b^2)^(3/2)-2*b*ln(x)/a^3+b*ln(c*x^2+b*x+a)/a^3
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.89

$$\int \frac{x^2}{(ax^2+bx^3+cx^4)^2}\,dx = -\frac{\frac{a}{x}+\frac{a(b^3-3abc+b^2cx-2ac^2x)}{(b^2-4ac)(a+x(b+cx))}+\frac{2(b^4-6ab^2c+6a^2c^2)\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{3/2}}+2b\log(x)-b\log(a+x(b+cx))}{a^3}$$

input
```
Integrate[x^2/(a*x^2 + b*x^3 + c*x^4)^2,x]
```

output
```
-((a/x + (a*(b^3 - 3*a*b*c + b^2*c*x - 2*a*c^2*x))/((b^2 - 4*a*c)*(a + x*(
b + c*x))) + (2*(b^4 - 6*a*b^2*c + 6*a^2*c^2)*ArcTan[(b + 2*c*x)/Sqrt[-b^2
 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2) + 2*b*Log[x] - b*Log[a + x*(b + c*x)])/a^
3)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.17, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {9, 1165, 27, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(ax^2+bx^3+cx^4)^2}dx$$

$$\Big\downarrow \; 9$$

$$\int \frac{1}{x^2(a+bx+cx^2)^2}dx$$

$$\Big\downarrow \; 1165$$

$$\frac{-2ac+b^2+bcx}{ax(b^2-4ac)(a+bx+cx^2)}-\frac{\int -\frac{2(b^2+cxb-3ac)}{x^2(cx^2+bx+a)}dx}{a(b^2-4ac)}$$

$$\Big\downarrow \; 27$$

$$\frac{2\int \frac{b^2+cxb-3ac}{x^2(cx^2+bx+a)}dx}{a(b^2-4ac)}+\frac{-2ac+b^2+bcx}{ax(b^2-4ac)(a+bx+cx^2)}$$

$$\Big\downarrow \text{ 1200}$$

$$\frac{2\int \left(\frac{b^2-3ac}{ax^2}+\frac{4abc-b^3}{a^2x}+\frac{b^4-5acb^2+c(b^2-4ac)xb+3a^2c^2}{a^2(cx^2+bx+a)}\right)dx}{a(b^2-4ac)}+\frac{-2ac+b^2+bcx}{ax(b^2-4ac)(a+bx+cx^2)}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2\left(-\frac{(6a^2c^2-6ab^2c+b^4)\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^2\sqrt{b^2-4ac}}+\frac{b(b^2-4ac)\log(a+bx+cx^2)}{2a^2}-\frac{b\log(x)(b^2-4ac)}{a^2}-\frac{b^2-3ac}{ax}\right)}{a(b^2-4ac)}+\frac{-2ac+b^2+bcx}{ax(b^2-4ac)(a+bx+cx^2)}$$

input
```
Int[x^2/(a*x^2 + b*x^3 + c*x^4)^2,x]
```

output
```
(b^2 - 2*a*c + b*c*x)/(a*(b^2 - 4*a*c)*x*(a + b*x + c*x^2)) + (2*(-((b^2 -
 3*a*c)/(a*x)) - ((b^4 - 6*a*b^2*c + 6*a^2*c^2)*ArcTanh[(b + 2*c*x)/Sqrt[b
^2 - 4*a*c]])/(a^2*Sqrt[b^2 - 4*a*c]) - (b*(b^2 - 4*a*c)*Log[x])/a^2 + (b*
(b^2 - 4*a*c)*Log[a + b*x + c*x^2])/(2*a^2)))/(a*(b^2 - 4*a*c))
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1165
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d
+ e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p
+ 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
 b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && LtQ[p, -1]
 && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.39

method	result
default	$-\frac{\frac{\frac{ac(2ac-b^2)x}{4ac-b^2}+\frac{ab(3ac-b^2)}{4ac-b^2}}{cx^2+bx+a}+\frac{\frac{(-4abc^2+b^3c)\ln(cx^2+bx+a)}{c}+\frac{4\left(3a^2c^2-5ab^2c+b^4-\frac{(-4abc^2+b^3c)b}{2c}\right)\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{4ac-b^2}}{a^3}-\frac{1}{a^2x}-$
risch	$\frac{-\frac{2c(3ac-b^2)x^2}{a^2(4ac-b^2)}-\frac{b(7ac-2b^2)x}{a^2(4ac-b^2)}-\frac{1}{a}}{x(cx^2+bx+a)}-\frac{2b\ln(x)}{a^3}+2\left(\sum_{\_R=\mathrm{RootOf}((64a^6c^3-48a^5b^2c^2+12a^4b^4c-a^3b^6)\_Z^2+(-64a^3bc^3+48a^2b}\right.$

input
```
int(x^2/(c*x^4+b*x^3+a*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/a^3*((a*c*(2*a*c-b^2)/(4*a*c-b^2)*x+a*b*(3*a*c-b^2)/(4*a*c-b^2))/(c*x^2
+b*x+a)+2/(4*a*c-b^2)*(1/2*(-4*a*b*c^2+b^3*c)/c*ln(c*x^2+b*x+a)+2*(3*a^2*c
^2-5*a*b^2*c+b^4-1/2*(-4*a*b*c^2+b^3*c)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c
*x+b)/(4*a*c-b^2)^(1/2))))-1/a^2/x-2*b*ln(x)/a^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 478 vs. $2(144) = 288$.

Time = 0.18 (sec) , antiderivative size = 975, normalized size of antiderivative = 6.59

$$\int \frac{x^2}{(ax^2 + bx^3 + cx^4)^2} \, dx = \text{Too large to display}$$

input
```
integrate(x^2/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="fricas")
```

output
```
[-(a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + 2*(a*b^4*c - 7*a^2*b^2*c^2 + 12*a^
3*c^3)*x^2 + ((b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*x^3 + (b^5 - 6*a*b^3*c + 6
*a^2*b*c^2)*x^2 + (a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2)*x)*sqrt(b^2 - 4*a*c)*l
og((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*
x^2 + b*x + a)) + (2*a*b^5 - 15*a^2*b^3*c + 28*a^3*b*c^2)*x - ((b^5*c - 8*
a*b^3*c^2 + 16*a^2*b*c^3)*x^3 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*x^2 + (
a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x)*log(c*x^2 + b*x + a) + 2*((b^5*c -
8*a*b^3*c^2 + 16*a^2*b*c^3)*x^3 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*x^2 +
 (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x)*log(x))/((a^3*b^4*c - 8*a^4*b^2*c
^2 + 16*a^5*c^3)*x^3 + (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*x^2 + (a^4*b
^4 - 8*a^5*b^2*c + 16*a^6*c^2)*x), -(a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 +
2*(a*b^4*c - 7*a^2*b^2*c^2 + 12*a^3*c^3)*x^2 + 2*((b^4*c - 6*a*b^2*c^2 + 6
*a^2*c^3)*x^3 + (b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*x^2 + (a*b^4 - 6*a^2*b^2*c
 + 6*a^3*c^2)*x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)
/(b^2 - 4*a*c)) + (2*a*b^5 - 15*a^2*b^3*c + 28*a^3*b*c^2)*x - ((b^5*c - 8*
a*b^3*c^2 + 16*a^2*b*c^3)*x^3 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*x^2 + (
a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x)*log(c*x^2 + b*x + a) + 2*((b^5*c -
8*a*b^3*c^2 + 16*a^2*b*c^3)*x^3 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*x^2 +
 (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x)*log(x))/((a^3*b^4*c - 8*a^4*b^2*c
^2 + 16*a^5*c^3)*x^3 + (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*x^2 + (a^...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2}{(ax^2+bx^3+cx^4)^2}\,dx=\text{Timed out}$$

input
```
integrate(x**2/(c*x**4+b*x**3+a*x**2)**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2}{(ax^2+bx^3+cx^4)^2}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(x^2/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.16

$$\begin{aligned}\int \frac{x^2}{(ax^2+bx^3+cx^4)^2}\,dx&=\frac{2\,(b^4-6\,ab^2c+6\,a^2c^2)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(a^3b^2-4\,a^4c)\sqrt{-b^2+4\,ac}}\\&\quad-\frac{2\,b^2cx^2-6\,ac^2x^2+2\,b^3x-7\,abcx+ab^2-4\,a^2c}{(a^2b^2-4\,a^3c)(cx^3+bx^2+ax)}\\&\quad+\frac{b\log\left(cx^2+bx+a\right)}{a^3}-\frac{2\,b\log\left(|x|\right)}{a^3}\end{aligned}$$

input
```
integrate(x^2/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="giac")
```

output
```
2*(b^4 - 6*a*b^2*c + 6*a^2*c^2)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((a
^3*b^2 - 4*a^4*c)*sqrt(-b^2 + 4*a*c)) - (2*b^2*c*x^2 - 6*a*c^2*x^2 + 2*b^3
*x - 7*a*b*c*x + a*b^2 - 4*a^2*c)/((a^2*b^2 - 4*a^3*c)*(c*x^3 + b*x^2 + a*
x)) + b*log(c*x^2 + b*x + a)/a^3 - 2*b*log(abs(x))/a^3
```

Mupad [B] (verification not implemented)

Time = 22.80 (sec) , antiderivative size = 775, normalized size of antiderivative = 5.24

$$\int \frac{x^2}{(ax^2+bx^3+cx^4)^2}\,dx = \ln\left(2\,a\,b^7+2\,b^8\,x+2\,a\,b^4\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}-23\,a^2\,b^5\,c-108\,a^4\,b\,c^3+24\,a^4\,c^4\,x+2\,b^5\,x\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}+87\,a^3\,b^3\,c^2+3\,a^3\,c^2\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}-9\,a^2\,b^2\,c\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}+97\,a^2\,b^4\,c^2\,x-138\,a^3\,b^2\,c^3\,x-24\,a\,b^6\,c\,x-12\,a\,b^3\,c\,x\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}+15\,a^2\,b\,c^2\,x\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}\right)\,\left(\frac{b^4\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}+6\,a^2\,c^2\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}-6\,a\,b^2\,c\,\sqrt{-{\left(4\,a\,c-b^2\right)}}}{-64\,a^6\,c^3+48\,a^5\,b^2\,c^2-12\,a^4\,b^4\,c+a^3\,b^6}+\frac{b}{a^3}\right)-\frac{\frac{1}{a}-\frac{x\,\left(2\,b^3-7\,a\,b\,c\right)}{a^2\,\left(4\,a\,c-b^2\right)}+\frac{2\,c\,x^2\,\left(3\,a\,c-b^2\right)}{a^2\,\left(4\,a\,c-b^2\right)}}{c\,x^3+b\,x^2+a\,x}-\ln\left(2\,a\,b^4\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}-2\,b^8\,x-2\,a\,b^7+23\,a^2\,b^5\,c+108\,a^4\,b\,c^3-24\,a^4\,c^4\,x+2\,b^5\,x\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}-87\,a^3\,b^3\,c^2+3\,a^3\,c^2\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}-9\,a^2\,b^2\,c\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}-97\,a^2\,b^4\,c^2\,x+138\,a^3\,b^2\,c^3\,x+24\,a\,b^6\,c\,x-12\,a\,b^3\,c\,x\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}+15\,a^2\,b\,c^2\,x\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}\right)\,\left(\frac{b^4\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}+6\,a^2\,c^2\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}-6\,a\,b^2\,c\,\sqrt{-{\left(4\,a\,c-b^2\right)}}}{-64\,a^6\,c^3+48\,a^5\,b^2\,c^2-12\,a^4\,b^4\,c+a^3\,b^6}-\frac{b}{a^3}\right)-\frac{2\,b\,\ln\left(x\right)}{a^3}$$

input
```
int(x^2/(a*x^2 + b*x^3 + c*x^4)^2,x)
```

output
```
log(2*a*b^7 + 2*b^8*x + 2*a*b^4*(-(4*a*c - b^2)^3)^(1/2) - 23*a^2*b^5*c -
108*a^4*b*c^3 + 24*a^4*c^4*x + 2*b^5*x*(-(4*a*c - b^2)^3)^(1/2) + 87*a^3*b
^3*c^2 + 3*a^3*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b^2*c*(-(4*a*c - b^2)^
3)^(1/2) + 97*a^2*b^4*c^2*x - 138*a^3*b^2*c^3*x - 24*a*b^6*c*x - 12*a*b^3*
c*x*(-(4*a*c - b^2)^3)^(1/2) + 15*a^2*b*c^2*x*(-(4*a*c - b^2)^3)^(1/2))*((
b^4*(-(4*a*c - b^2)^3)^(1/2) + 6*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^
2*c*(-(4*a*c - b^2)^3)^(1/2))/(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^
5*b^2*c^2) + b/a^3) - (1/a - (x*(2*b^3 - 7*a*b*c))/(a^2*(4*a*c - b^2)) + (
2*c*x^2*(3*a*c - b^2))/(a^2*(4*a*c - b^2)))/(a*x + b*x^2 + c*x^3) - log(2*
a*b^4*(-(4*a*c - b^2)^3)^(1/2) - 2*b^8*x - 2*a*b^7 + 23*a^2*b^5*c + 108*a^
4*b*c^3 - 24*a^4*c^4*x + 2*b^5*x*(-(4*a*c - b^2)^3)^(1/2) - 87*a^3*b^3*c^2
 + 3*a^3*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b^2*c*(-(4*a*c - b^2)^3)^(1/
2) - 97*a^2*b^4*c^2*x + 138*a^3*b^2*c^3*x + 24*a*b^6*c*x - 12*a*b^3*c*x*(-
(4*a*c - b^2)^3)^(1/2) + 15*a^2*b*c^2*x*(-(4*a*c - b^2)^3)^(1/2))*((b^4*(-
(4*a*c - b^2)^3)^(1/2) + 6*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*(-
(4*a*c - b^2)^3)^(1/2))/(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*
c^2) - b/a^3) - (2*b*log(x))/a^3
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 877, normalized size of antiderivative = 5.93

$$\int \frac{x^2}{(ax^2 + bx^3 + cx^4)^2} \, dx = \text{Too large to display}$$

input
```
int(x^2/(c*x^4+b*x^3+a*x^2)^2,x)
```

output
```
( - 12*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**3*b*c**2
*x + 12*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*b**3*
c*x - 12*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*b**2
*c**2*x**2 - 12*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*
*2*b*c**3*x**3 - 2*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))
*a*b**5*x + 12*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b
**4*c*x**2 + 12*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*
b**3*c**2*x**3 - 2*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))
*b**6*x**2 - 2*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b**
5*c*x**3 + 16*log(a + b*x + c*x**2)*a**3*b**2*c**2*x - 8*log(a + b*x + c*x
**2)*a**2*b**4*c*x + 16*log(a + b*x + c*x**2)*a**2*b**3*c**2*x**2 + 16*log
(a + b*x + c*x**2)*a**2*b**2*c**3*x**3 + log(a + b*x + c*x**2)*a*b**6*x -
8*log(a + b*x + c*x**2)*a*b**5*c*x**2 - 8*log(a + b*x + c*x**2)*a*b**4*c**
2*x**3 + log(a + b*x + c*x**2)*b**7*x**2 + log(a + b*x + c*x**2)*b**6*c*x*
*3 - 32*log(x)*a**3*b**2*c**2*x + 16*log(x)*a**2*b**4*c*x - 32*log(x)*a**2
*b**3*c**2*x**2 - 32*log(x)*a**2*b**2*c**3*x**3 - 2*log(x)*a*b**6*x + 16*l
og(x)*a*b**5*c*x**2 + 16*log(x)*a*b**4*c**2*x**3 - 2*log(x)*b**7*x**2 - 2*
log(x)*b**6*c*x**3 - 16*a**4*b*c**2 + 24*a**4*c**3*x + 8*a**3*b**3*c - 42*
a**3*b**2*c**2*x + 24*a**3*c**4*x**3 - a**2*b**5 + 17*a**2*b**4*c*x - 14*a
**2*b**2*c**3*x**3 - 2*a*b**6*x + 2*a*b**4*c**2*x**3)/(a**3*b*x*(16*a**...
```

3.40 $\int \frac{x}{(ax^2+bx^3+cx^4)^2} dx$

Optimal result

Integrand size = 20, antiderivative size = 202

$$\int \frac{x}{(ax^2+bx^3+cx^4)^2} dx = -\frac{3b^2-8ac}{2a^2(b^2-4ac)x^2} + \frac{b(3b^2-11ac)}{a^3(b^2-4ac)x} + \frac{b^2-2ac+bcx}{a(b^2-4ac)x^2(a+bx+cx^2)} + \frac{b(3b^4-20ab^2c+30a^2c^2)\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^4(b^2-4ac)^{3/2}} + \frac{(3b^2-2ac)\log(x)}{a^4} - \frac{(3b^2-2ac)\log(a+bx+cx^2)}{2a^4}$$

output

```
-1/2*(-8*a*c+3*b^2)/a^2/(-4*a*c+b^2)/x^2+b*(-11*a*c+3*b^2)/a^3/(-4*a*c+b^2
)/x+(b*c*x-2*a*c+b^2)/a/(-4*a*c+b^2)/x^2/(c*x^2+b*x+a)+b*(30*a^2*c^2-20*a*
b^2*c+3*b^4)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/a^4/(-4*a*c+b^2)^(3/2)+
(-2*a*c+3*b^2)*ln(x)/a^4-1/2*(-2*a*c+3*b^2)*ln(c*x^2+b*x+a)/a^4
```

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.87

$$\int \frac{x}{(ax^2+bx^3+cx^4)^2}\,dx$$
$$= \frac{-\frac{a^2}{x^2}+\frac{4ab}{x}+\frac{2a(b^4-4ab^2c+2a^2c^2+b^3cx-3abc^2x)}{(b^2-4ac)(a+x(b+cx))}+\frac{2b(3b^4-20ab^2c+30a^2c^2)\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{3/2}}+2(3b^2-2ac)\log(x)+(}{2a^4}$$

input
```
Integrate[x/(a*x^2 + b*x^3 + c*x^4)^2,x]
```

output
```
(-(a^2/x^2) + (4*a*b)/x + (2*a*(b^4 - 4*a*b^2*c + 2*a^2*c^2 + b^3*c*x - 3*
a*b*c^2*x))/((b^2 - 4*a*c)*(a + x*(b + c*x))) + (2*b*(3*b^4 - 20*a*b^2*c +
 30*a^2*c^2)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2)
+ 2*(3*b^2 - 2*a*c)*Log[x] + (-3*b^2 + 2*a*c)*Log[a + x*(b + c*x)])/(2*a^4
)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {9, 1165, 25, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(ax^2+bx^3+cx^4)^2}\,dx$$
$$\Big\downarrow \; 9$$
$$\int \frac{1}{x^3(a+bx+cx^2)^2}dx$$
$$\Big\downarrow \; 1165$$
$$\frac{-2ac+b^2+bcx}{ax^2(b^2-4ac)(a+bx+cx^2)}-\frac{\int -\frac{3b^2+3cxb-8ac}{x^3(cx^2+bx+a)}dx}{a(b^2-4ac)}$$

$$\Big\downarrow 25$$

$$\frac{\int \frac{3b^2+3cxb-8ac}{x^3(cx^2+bx+a)}dx}{a(b^2-4ac)}+\frac{-2ac+b^2+bcx}{ax^2(b^2-4ac)(a+bx+cx^2)}$$

$$\Big\downarrow 1200$$

$$\frac{\int \left(\frac{3b^2-8ac}{ax^3}+\frac{(b^2-4ac)(3b^2-2ac)}{a^3x}+\frac{-b(3b^4-17acb^2+19a^2c^2)-c(3b^4-14acb^2+8a^2c^2)x}{a^3(cx^2+bx+a)}+\frac{b(11ac-3b^2)}{a^2x^2}\right)dx}{a(b^2-4ac)}+\frac{-2ac+b^2+bcx}{ax^2(b^2-4ac)(a+bx+cx^2)}$$

$$\Big\downarrow 2009$$

$$\frac{-\frac{(b^2-4ac)(3b^2-2ac)\log(a+bx+cx^2)}{2a^3}+\frac{\log(x)(b^2-4ac)(3b^2-2ac)}{a^3}+\frac{b(3b^2-11ac)}{a^2x}+\frac{b(30a^2c^2-20ab^2c+3b^4)\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^3\sqrt{b^2-4ac}}-3}{a(b^2-4ac)}+\frac{-2ac+b^2+bcx}{ax^2(b^2-4ac)(a+bx+cx^2)}$$

input
```
Int[x/(a*x^2 + b*x^3 + c*x^4)^2,x]
```

output
```
(b^2 - 2*a*c + b*c*x)/(a*(b^2 - 4*a*c)*x^2*(a + b*x + c*x^2)) + (-1/2*(3*b
^2 - 8*a*c)/(a*x^2) + (b*(3*b^2 - 11*a*c))/(a^2*x) + (b*(3*b^4 - 20*a*b^2*
c + 30*a^2*c^2)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a^3*Sqrt[b^2 - 4*
a*c]) + ((b^2 - 4*a*c)*(3*b^2 - 2*a*c)*Log[x])/a^3 - ((b^2 - 4*a*c)*(3*b^2
 - 2*a*c)*Log[a + b*x + c*x^2])/(2*a^3))/(a*(b^2 - 4*a*c))
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1165
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d
+ e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p
+ 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
 b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && LtQ[p, -1]
 && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.26

method	result
default	$\frac{\frac{\frac{acb\left(3ac-b^{2}\right)x}{4ac-b^{2}}-\frac{a\left(2a^{2}c^{2}-4a\,b^{2}c+b^{4}\right)}{4ac-b^{2}}}{c\,x^{2}+bx+a}+\frac{\frac{\left(8a^{2}c^{3}-14a\,b^{2}c^{2}+3b^{4}c\right)\ln\left(c\,x^{2}+bx+a\right)}{2c}+\frac{2\left(19a^{2}b\,c^{2}-17a\,b^{3}c+3b^{5}-\frac{\left(8a^{2}c^{3}-14a\,b^{2}c^{2}+3b^{4}c\right)b}{2c}\right)\text{ar}}{\sqrt{4ac-b^{2}}}}{4ac-b^{2}}}{a^{4}}$
risch	$\frac{\frac{bc\left(11ac-3b^{2}\right)x^{3}}{a^{3}\left(4ac-b^{2}\right)}-\frac{\left(8a^{2}c^{2}-25a\,b^{2}c+6b^{4}\right)x^{2}}{2a^{3}\left(4ac-b^{2}\right)}+\frac{3bx}{2a^{2}}-\frac{1}{2a}}{\left(c\,x^{2}+bx+a\right)x^{2}}-\frac{2\ln\left(x\right)c}{a^{3}}+\frac{3\ln\left(x\right)b^{2}}{a^{4}}+\Bigg(\ {}_{\_R=\text{RootOf}\left(\left(64a^{7}c^{3}-48a^{6}b^{2}c^{2}+12a^{5}b^{4}c-a\right.\right.}$

input
```
int(x/(c*x^4+b*x^3+a*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/a^4*((a*c*b*(3*a*c-b^2)/(4*a*c-b^2)*x-a*(2*a^2*c^2-4*a*b^2*c+b^4)/(4*a*c
-b^2))/(c*x^2+b*x+a)+1/(4*a*c-b^2)*(1/2*(8*a^2*c^3-14*a*b^2*c^2+3*b^4*c)/c
*ln(c*x^2+b*x+a)+2*(19*a^2*b*c^2-17*a*b^3*c+3*b^5-1/2*(8*a^2*c^3-14*a*b^2*
c^2+3*b^4*c)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))))-
1/2/a^2/x^2+(-2*a*c+3*b^2)*ln(x)/a^4+2/a^3*b/x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 603 vs. $2(194) = 388$.

Time = 0.24 (sec) , antiderivative size = 1226, normalized size of antiderivative = 6.07

$$\int \frac{x}{(ax^2 + bx^3 + cx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="fricas")
```

output
```
[-1/2*(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 - 2*(3*a*b^5*c - 23*a^2*b^3*c^2
+ 44*a^3*b*c^3)*x^3 - (6*a*b^6 - 49*a^2*b^4*c + 108*a^3*b^2*c^2 - 32*a^4*c
^3)*x^2 + ((3*b^5*c - 20*a*b^3*c^2 + 30*a^2*b*c^3)*x^4 + (3*b^6 - 20*a*b^4
*c + 30*a^2*b^2*c^2)*x^3 + (3*a*b^5 - 20*a^2*b^3*c + 30*a^3*b*c^2)*x^2)*sq
rt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)
*(2*c*x + b))/(c*x^2 + b*x + a)) - 3*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2
)*x + ((3*b^6*c - 26*a*b^4*c^2 + 64*a^2*b^2*c^3 - 32*a^3*c^4)*x^4 + (3*b^7
 - 26*a*b^5*c + 64*a^2*b^3*c^2 - 32*a^3*b*c^3)*x^3 + (3*a*b^6 - 26*a^2*b^4
*c + 64*a^3*b^2*c^2 - 32*a^4*c^3)*x^2)*log(c*x^2 + b*x + a) - 2*((3*b^6*c
- 26*a*b^4*c^2 + 64*a^2*b^2*c^3 - 32*a^3*c^4)*x^4 + (3*b^7 - 26*a*b^5*c +
64*a^2*b^3*c^2 - 32*a^3*b*c^3)*x^3 + (3*a*b^6 - 26*a^2*b^4*c + 64*a^3*b^2*
c^2 - 32*a^4*c^3)*x^2)*log(x))/((a^4*b^4*c - 8*a^5*b^2*c^2 + 16*a^6*c^3)*x
^4 + (a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*x^3 + (a^5*b^4 - 8*a^6*b^2*c +
 16*a^7*c^2)*x^2), -1/2*(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 - 2*(3*a*b^5*c
 - 23*a^2*b^3*c^2 + 44*a^3*b*c^3)*x^3 - (6*a*b^6 - 49*a^2*b^4*c + 108*a^3*
b^2*c^2 - 32*a^4*c^3)*x^2 - 2*((3*b^5*c - 20*a*b^3*c^2 + 30*a^2*b*c^3)*x^4
 + (3*b^6 - 20*a*b^4*c + 30*a^2*b^2*c^2)*x^3 + (3*a*b^5 - 20*a^2*b^3*c + 3
0*a^3*b*c^2)*x^2)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b
)/(b^2 - 4*a*c)) - 3*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x + ((3*b^6*c
- 26*a*b^4*c^2 + 64*a^2*b^2*c^3 - 32*a^3*c^4)*x^4 + (3*b^7 - 26*a*b^5*c...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x}{(ax^2+bx^3+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate(x/(c*x**4+b*x**3+a*x**2)**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{(ax^2+bx^3+cx^4)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.13

$$\int \frac{x}{(ax^2+bx^3+cx^4)^2}\,dx = -\frac{(3\,b^5-20\,ab^3c+30\,a^2bc^2)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(a^4b^2-4\,a^5c)\sqrt{-b^2+4\,ac}} - \frac{(3\,b^2-2\,ac)\log\left(cx^2+bx+a\right)}{2\,a^4} + \frac{(3\,b^2-2\,ac)\log\left(|x|\right)}{a^4} - \frac{a^3b^2-4\,a^4c-2\,(3\,ab^3c-11\,a^2bc^2)x^3-(6\,ab^4-25\,a^2b^2c+8\,a^3c^2)x^2-3\,(a^2b^3-4\,a^3bc)x}{2\,(cx^2+bx+a)(b^2-4\,ac)a^4x^2}$$

input
```
integrate(x/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="giac")
```

output
```
-(3*b^5 - 20*a*b^3*c + 30*a^2*b*c^2)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c)
)/((a^4*b^2 - 4*a^5*c)*sqrt(-b^2 + 4*a*c)) - 1/2*(3*b^2 - 2*a*c)*log(c*x^2
 + b*x + a)/a^4 + (3*b^2 - 2*a*c)*log(abs(x))/a^4 - 1/2*(a^3*b^2 - 4*a^4*c
 - 2*(3*a*b^3*c - 11*a^2*b*c^2)*x^3 - (6*a*b^4 - 25*a^2*b^2*c + 8*a^3*c^2)
*x^2 - 3*(a^2*b^3 - 4*a^3*b*c)*x)/((c*x^2 + b*x + a)*(b^2 - 4*a*c)*a^4*x^2
)
```

Mupad [B] (verification not implemented)

Time = 22.45 (sec) , antiderivative size = 914, normalized size of antiderivative = 4.52

$$\int \frac{x}{(ax^2+bx^3+cx^4)^2}\,dx = \frac{\ln\left(6\,a\,b^8+6\,b^9\,x+192\,a^5\,c^4-6\,a\,b^5\,\sqrt{-(4\,a\,c-b^2)^3}-73\,a^2\,b^6\,c-6\,b^6\,x\,\sqrt{-(4\,a\,c-b^2)^3}+307\,a^3\,b^4\,c^2\right.}{} - \frac{\ln(x)\,(2\,a\,c-3\,b^2)}{a^4} - \frac{\frac{1}{2\,a}-\frac{3\,b\,x}{2\,a^2}+\frac{x^2\,(8\,a^2\,c^2-25\,a\,b^2\,c+6\,b^4)}{2\,a^3\,(4\,a\,c-b^2)}-\frac{b\,c\,x^3\,(11\,a\,c-3\,b^2)}{a^3\,(4\,a\,c-b^2)}}{c\,x^4+b\,x^3+a\,x^2} + \frac{\ln\left(6\,a\,b^8+6\,b^9\,x+192\,a^5\,c^4+6\,a\,b^5\,\sqrt{-(4\,a\,c-b^2)^3}-73\,a^2\,b^6\,c+6\,b^6\,x\,\sqrt{-(4\,a\,c-b^2)^3}+307\,a^3\,b^4\right.}{}$$

input
```
int(x/(a*x^2 + b*x^3 + c*x^4)^2,x)
```

output
```
(log(6*a*b^8 + 6*b^9*x + 192*a^5*c^4 - 6*a*b^5*(-(4*a*c - b^2)^3)^(1/2) -
73*a^2*b^6*c - 6*b^6*x*(-(4*a*c - b^2)^3)^(1/2) + 307*a^3*b^4*c^2 - 492*a^
4*b^2*c^3 + 31*a^2*b^3*c*(-(4*a*c - b^2)^3)^(1/2) - 27*a^3*b*c^2*(-(4*a*c
- b^2)^3)^(1/2) + 339*a^2*b^5*c^2*x - 602*a^3*b^3*c^3*x + 24*a^3*c^3*x*(-(
4*a*c - b^2)^3)^(1/2) - 76*a*b^7*c*x + 312*a^4*b*c^4*x + 40*a*b^4*c*x*(-(4
*a*c - b^2)^3)^(1/2) - 69*a^2*b^2*c^2*x*(-(4*a*c - b^2)^3)^(1/2))*(3*b^8 +
 128*a^4*c^4 - 3*b^5*(-(4*a*c - b^2)^3)^(1/2) + 168*a^2*b^4*c^2 - 288*a^3*
b^2*c^3 - 38*a*b^6*c - 30*a^2*b*c^2*(-(4*a*c - b^2)^3)^(1/2) + 20*a*b^3*c*
(-(4*a*c - b^2)^3)^(1/2)))/(2*a^4*(4*a*c - b^2)^3) - (log(x)*(2*a*c - 3*b^
2))/a^4 - (1/(2*a) - (3*b*x)/(2*a^2) + (x^2*(6*b^4 + 8*a^2*c^2 - 25*a*b^2*
c))/(2*a^3*(4*a*c - b^2)) - (b*c*x^3*(11*a*c - 3*b^2))/(a^3*(4*a*c - b^2))
)/(a*x^2 + b*x^3 + c*x^4) + (log(6*a*b^8 + 6*b^9*x + 192*a^5*c^4 + 6*a*b^5
*(-(4*a*c - b^2)^3)^(1/2) - 73*a^2*b^6*c + 6*b^6*x*(-(4*a*c - b^2)^3)^(1/2
) + 307*a^3*b^4*c^2 - 492*a^4*b^2*c^3 - 31*a^2*b^3*c*(-(4*a*c - b^2)^3)^(1
/2) + 27*a^3*b*c^2*(-(4*a*c - b^2)^3)^(1/2) + 339*a^2*b^5*c^2*x - 602*a^3*
b^3*c^3*x - 24*a^3*c^3*x*(-(4*a*c - b^2)^3)^(1/2) - 76*a*b^7*c*x + 312*a^4
*b*c^4*x - 40*a*b^4*c*x*(-(4*a*c - b^2)^3)^(1/2) + 69*a^2*b^2*c^2*x*(-(4*a
*c - b^2)^3)^(1/2))*(3*b^8 + 128*a^4*c^4 + 3*b^5*(-(4*a*c - b^2)^3)^(1/2)
+ 168*a^2*b^4*c^2 - 288*a^3*b^2*c^3 - 38*a*b^6*c + 30*a^2*b*c^2*(-(4*a*c -
 b^2)^3)^(1/2) - 20*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2)))/(2*a^4*(4*a*c - ...
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 1039, normalized size of antiderivative = 5.14

$$\int \frac{x}{(ax^2 + bx^3 + cx^4)^2} \, dx = \text{Too large to display}$$

input
```
int(x/(c*x^4+b*x^3+a*x^2)^2,x)
```

output

```
(60*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**3*b*c**2*x*
*2 - 40*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*b**3*
c*x**2 + 60*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*b
**2*c**2*x**3 + 60*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))
*a**2*b*c**3*x**4 + 6*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**
2))*a*b**5*x**2 - 40*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2
))*a*b**4*c*x**3 - 40*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**
2))*a*b**3*c**2*x**4 + 6*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c -
b**2))*b**6*x**3 + 6*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2
))*b**5*c*x**4 + 32*log(a + b*x + c*x**2)*a**4*c**3*x**2 - 64*log(a + b*x
+ c*x**2)*a**3*b**2*c**2*x**2 + 32*log(a + b*x + c*x**2)*a**3*b*c**3*x**3
+ 32*log(a + b*x + c*x**2)*a**3*c**4*x**4 + 26*log(a + b*x + c*x**2)*a**2*
b**4*c*x**2 - 64*log(a + b*x + c*x**2)*a**2*b**3*c**2*x**3 - 64*log(a + b*
x + c*x**2)*a**2*b**2*c**3*x**4 - 3*log(a + b*x + c*x**2)*a*b**6*x**2 + 26
*log(a + b*x + c*x**2)*a*b**5*c*x**3 + 26*log(a + b*x + c*x**2)*a*b**4*c**
2*x**4 - 3*log(a + b*x + c*x**2)*b**7*x**3 - 3*log(a + b*x + c*x**2)*b**6*
c*x**4 - 64*log(x)*a**4*c**3*x**2 + 128*log(x)*a**3*b**2*c**2*x**2 - 64*lo
g(x)*a**3*b*c**3*x**3 - 64*log(x)*a**3*c**4*x**4 - 52*log(x)*a**2*b**4*c*x
**2 + 128*log(x)*a**2*b**3*c**2*x**3 + 128*log(x)*a**2*b**2*c**3*x**4 + 6*
log(x)*a*b**6*x**2 - 52*log(x)*a*b**5*c*x**3 - 52*log(x)*a*b**4*c**2*x*...
```

3.41 $\int \frac{1}{(ax^2+bx^3+cx^4)^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 252

$$\begin{aligned}\int \frac{1}{(ax^2+bx^3+cx^4)^2}\,dx = &-\frac{2(2b^2-5ac)}{3a^2(b^2-4ac)x^3}+\frac{b(2b^2-7ac)}{a^3(b^2-4ac)x^2} \\ &-\frac{2(2b^4-9ab^2c+5a^2c^2)}{a^4(b^2-4ac)x}+\frac{b^2-2ac+bcx}{a(b^2-4ac)x^3(a+bx+cx^2)} \\ &-\frac{2(2b^6-15ab^4c+30a^2b^2c^2-10a^3c^3)\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^5(b^2-4ac)^{3/2}} \\ &-\frac{2b(2b^2-3ac)\log(x)}{a^5}+\frac{b(2b^2-3ac)\log(a+bx+cx^2)}{a^5}\end{aligned}$$

output
```
1/3*(10*a*c-4*b^2)/a^2/(-4*a*c+b^2)/x^3+b*(-7*a*c+2*b^2)/a^3/(-4*a*c+b^2)/
x^2-2*(5*a^2*c^2-9*a*b^2*c+2*b^4)/a^4/(-4*a*c+b^2)/x+(b*c*x-2*a*c+b^2)/a/(
-4*a*c+b^2)/x^3/(c*x^2+b*x+a)-2*(-10*a^3*c^3+30*a^2*b^2*c^2-15*a*b^4*c+2*b
^6)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/a^5/(-4*a*c+b^2)^(3/2)-2*b*(-3*a
*c+2*b^2)*ln(x)/a^5+b*(-3*a*c+2*b^2)*ln(c*x^2+b*x+a)/a^5
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.87

$$\int \frac{1}{\left(ax^2+bx^3+cx^4\right)^2}\,dx$$

$$= \frac{-\frac{a^3}{x^3}+\frac{3a^2b}{x^2}+\frac{3a(-3b^2+2ac)}{x}-\frac{3a(b^5-5ab^3c+5a^2bc^2+b^4cx-4ab^2c^2x+2a^2c^3x)}{(b^2-4ac)(a+x(b+cx))}-\frac{6(2b^6-15ab^4c+30a^2b^2c^2-10a^3c^3)\arctan\left(\frac{b+2c}{\sqrt{-b^2+}}\right.}{(-b^2+4ac)^{3/2}}}{3a^5}$$

input
```
Integrate[(a*x^2 + b*x^3 + c*x^4)^(-2),x]
```

output
```
(-(a^3/x^3) + (3*a^2*b)/x^2 + (3*a*(-3*b^2 + 2*a*c))/x - (3*a*(b^5 - 5*a*b
^3*c + 5*a^2*b*c^2 + b^4*c*x - 4*a*b^2*c^2*x + 2*a^2*c^3*x))/((b^2 - 4*a*c
)*(a + x*(b + c*x))) - (6*(2*b^6 - 15*a*b^4*c + 30*a^2*b^2*c^2 - 10*a^3*c^
3)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2) + 6*(-2*b^
3 + 3*a*b*c)*Log[x] + 3*(2*b^3 - 3*a*b*c)*Log[a + x*(b + c*x)])/(3*a^5)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {1949, 1165, 27, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(ax^2+bx^3+cx^4\right)^2}dx$$

$$\Big\downarrow \ 1949$$

$$\int \frac{1}{x^4\left(a+bx+cx^2\right)^2}dx$$

$$\Big\downarrow \ 1165$$

$$\frac{-2ac+b^2+bcx}{ax^3\left(b^2-4ac\right)\left(a+bx+cx^2\right)}-\frac{\int -\frac{2\left(2b^2+2cxb-5ac\right)}{x^4\left(cx^2+bx+a\right)}dx}{a\left(b^2-4ac\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{2\int \frac{2b^2+2cxb-5ac}{x^4(cx^2+bx+a)}dx}{a\left(b^2-4ac\right)}+\frac{-2ac+b^2+bcx}{ax^3\left(b^2-4ac\right)\left(a+bx+cx^2\right)}$$

$$\Big\downarrow \text{1200}$$

$$\frac{2\int \left(\frac{2b^2-5ac}{ax^4}+\frac{b\left(b^2-4ac\right)\left(3ac-2b^2\right)}{a^4x}+\frac{2b^6-13acb^4+21a^2c^2b^2+c\left(b^2-4ac\right)\left(2b^2-3ac\right)xb-5a^3c^3}{a^4(cx^2+bx+a)}+\frac{2b^4-9acb^2+5a^2c^2}{a^3x^2}+\frac{b\left(7ac-2b^2\right)}{a^2x^3}\right) d}{a\left(b^2-4ac\right)}$$
$$\frac{-2ac+b^2+bcx}{ax^3\left(b^2-4ac\right)\left(a+bx+cx^2\right)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2\left(\frac{b\left(b^2-4ac\right)\left(2b^2-3ac\right)\log\left(a+bx+cx^2\right)}{2a^4}-\frac{b\log(x)\left(b^2-4ac\right)\left(2b^2-3ac\right)}{a^4}+\frac{b\left(2b^2-7ac\right)}{2a^2x^2}-\frac{5a^2c^2-9ab^2c+2b^4}{a^3x}-\frac{\left(-10a^3c^3+30a^2b^2c^2-15a\right.}{a^4}\right.}{a\left(b^2-4ac\right)}$$
$$\frac{-2ac+b^2+bcx}{ax^3\left(b^2-4ac\right)\left(a+bx+cx^2\right)}$$

input
```
Int[(a*x^2 + b*x^3 + c*x^4)^(-2),x]
```

output
```
(b^2 - 2*a*c + b*c*x)/(a*(b^2 - 4*a*c)*x^3*(a + b*x + c*x^2)) + (2*(-1/3*(
2*b^2 - 5*a*c)/(a*x^3) + (b*(2*b^2 - 7*a*c))/(2*a^2*x^2) - (2*b^4 - 9*a*b^
2*c + 5*a^2*c^2)/(a^3*x) - ((2*b^6 - 15*a*b^4*c + 30*a^2*b^2*c^2 - 10*a^3*
c^3)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a^4*Sqrt[b^2 - 4*a*c]) - (b*
(b^2 - 4*a*c)*(2*b^2 - 3*a*c)*Log[x])/a^4 + (b*(b^2 - 4*a*c)*(2*b^2 - 3*a*
c)*Log[a + b*x + c*x^2])/(2*a^4)))/(a*(b^2 - 4*a*c))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1165
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d
+ e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p
+ 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
 b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && LtQ[p, -1]
 && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1949
```
Int[((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_), x_Symbol
] :> Int[x^(p*q)*(a + b*x^(n - q) + c*x^(2*(n - q)))^p, x] /; FreeQ[{a, b,
c, n, q}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.17

method	result
default	$\frac{\frac{\frac{ac\left(2a^2c^2-4a\,b^2c+b^4\right)x}{4ac-b^2}+\frac{ab\left(5a^2c^2-5a\,b^2c+b^4\right)}{4ac-b^2}}{c\,x^2+bx+a}+\frac{\frac{\left(-12a^2b\,c^3+11a\,b^3c^2-2b^5c\right)\ln\left(c\,x^2+bx+a\right)}{c}+\frac{4\left(5a^3c^3-21a^2b^2c^2+13a\,b^4c-2b^6-\frac{\left(-12a\right.}{}\right.}{\sqrt{4\cdots}}}{4ac-b^2}}{a^5}$
risch	$\frac{\frac{2c\left(5a^2c^2-9a\,b^2c+2b^4\right)x^4}{\left(4ac-b^2\right)a^4}+\frac{b\left(17a^2c^2-20a\,b^2c+4b^4\right)x^3}{a^4\left(4ac-b^2\right)}+\frac{\left(5ac-6b^2\right)x^2}{3a^3}+\frac{2bx}{3a^2}-\frac{1}{3a}}{x^3\left(c\,x^2+bx+a\right)}+\frac{6b\ln(x)c}{a^4}-\frac{4b^3\ln(x)}{a^5}+2\left(\sum_{\_R=\mathrm{RootOf}\left(\left(64\cdots\right.\right.}\right.$

input
```
int(1/(c*x^4+b*x^3+a*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/a^5*((a*c*(2*a^2*c^2-4*a*b^2*c+b^4)/(4*a*c-b^2)*x+a*b*(5*a^2*c^2-5*a*b^2
*c+b^4)/(4*a*c-b^2))/(c*x^2+b*x+a)+2/(4*a*c-b^2)*(1/2*(-12*a^2*b*c^3+11*a*
b^3*c^2-2*b^5*c)/c*ln(c*x^2+b*x+a)+2*(5*a^3*c^3-21*a^2*b^2*c^2+13*a*b^4*c-
2*b^6-1/2*(-12*a^2*b*c^3+11*a*b^3*c^2-2*b^5*c)*b/c)/(4*a*c-b^2)^(1/2)*arct
an((2*c*x+b)/(4*a*c-b^2)^(1/2))))-1/3/a^2/x^3-(-2*a*c+3*b^2)/x/a^4+1/a^3*b
/x^2+2*b*(3*a*c-2*b^2)/a^5*ln(x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 693 vs. $2(246) = 492$.

Time = 0.42 (sec) , antiderivative size = 1407, normalized size of antiderivative = 5.58

$$\int \frac{1}{(ax^2 + bx^3 + cx^4)^2} \, dx = \text{Too large to display}$$

input
```
integrate(1/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="fricas")
```

output
```
[-1/3*(a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2 + 6*(2*a*b^6*c - 17*a^2*b^4*c^2
+ 41*a^3*b^2*c^3 - 20*a^4*c^4)*x^4 + 3*(4*a*b^7 - 36*a^2*b^5*c + 97*a^3*b^
3*c^2 - 68*a^4*b*c^3)*x^3 + (6*a^2*b^6 - 53*a^3*b^4*c + 136*a^4*b^2*c^2 -
80*a^5*c^3)*x^2 - 3*((2*b^6*c - 15*a*b^4*c^2 + 30*a^2*b^2*c^3 - 10*a^3*c^4
)*x^5 + (2*b^7 - 15*a*b^5*c + 30*a^2*b^3*c^2 - 10*a^3*b*c^3)*x^4 + (2*a*b^
6 - 15*a^2*b^4*c + 30*a^3*b^2*c^2 - 10*a^4*c^3)*x^3)*sqrt(b^2 - 4*a*c)*log
((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^
2 + b*x + a)) - 2*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*x - 3*((2*b^7*c -
 19*a*b^5*c^2 + 56*a^2*b^3*c^3 - 48*a^3*b*c^4)*x^5 + (2*b^8 - 19*a*b^6*c +
 56*a^2*b^4*c^2 - 48*a^3*b^2*c^3)*x^4 + (2*a*b^7 - 19*a^2*b^5*c + 56*a^3*b
^3*c^2 - 48*a^4*b*c^3)*x^3)*log(c*x^2 + b*x + a) + 6*((2*b^7*c - 19*a*b^5*
c^2 + 56*a^2*b^3*c^3 - 48*a^3*b*c^4)*x^5 + (2*b^8 - 19*a*b^6*c + 56*a^2*b^
4*c^2 - 48*a^3*b^2*c^3)*x^4 + (2*a*b^7 - 19*a^2*b^5*c + 56*a^3*b^3*c^2 - 4
8*a^4*b*c^3)*x^3)*log(x))/((a^5*b^4*c - 8*a^6*b^2*c^2 + 16*a^7*c^3)*x^5 +
(a^5*b^5 - 8*a^6*b^3*c + 16*a^7*b*c^2)*x^4 + (a^6*b^4 - 8*a^7*b^2*c + 16*a
^8*c^2)*x^3), -1/3*(a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2 + 6*(2*a*b^6*c - 17
*a^2*b^4*c^2 + 41*a^3*b^2*c^3 - 20*a^4*c^4)*x^4 + 3*(4*a*b^7 - 36*a^2*b^5*
c + 97*a^3*b^3*c^2 - 68*a^4*b*c^3)*x^3 + (6*a^2*b^6 - 53*a^3*b^4*c + 136*a
^4*b^2*c^2 - 80*a^5*c^3)*x^2 + 6*((2*b^6*c - 15*a*b^4*c^2 + 30*a^2*b^2*c^3
 - 10*a^3*c^4)*x^5 + (2*b^7 - 15*a*b^5*c + 30*a^2*b^3*c^2 - 10*a^3*b*c^...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(ax^2+bx^3+cx^4\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(c*x**4+b*x**3+a*x**2)**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\left(ax^2+bx^3+cx^4\right)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.12

$$\int \frac{1}{\left(ax^2+bx^3+cx^4\right)^2}\,dx = \frac{2\,(2\,b^6-15\,ab^4c+30\,a^2b^2c^2-10\,a^3c^3)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(a^5b^2-4\,a^6c)\sqrt{-b^2+4\,ac}} + \frac{(2\,b^3-3\,abc)\log\left(cx^2+bx+a\right)}{a^5} - \frac{2\,(2\,b^3-3\,abc)\log\left(|x|\right)}{a^5} - \frac{a^4b^2-4\,a^5c+6\,(2\,ab^4c-9\,a^2b^2c^2+5\,a^3c^3)x^4+3\,(4\,ab^5-20\,a^2b^3c+17\,a^3bc^2)x^3+(6\,a^2b^4-29\,a^3b^2c}{3\,(cx^2+bx+a)(b^2-4\,ac)a^5x^3}$$

input
```
integrate(1/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="giac")
```

output
```
2*(2*b^6 - 15*a*b^4*c + 30*a^2*b^2*c^2 - 10*a^3*c^3)*arctan((2*c*x + b)/sq
rt(-b^2 + 4*a*c))/((a^5*b^2 - 4*a^6*c)*sqrt(-b^2 + 4*a*c)) + (2*b^3 - 3*a*
b*c)*log(c*x^2 + b*x + a)/a^5 - 2*(2*b^3 - 3*a*b*c)*log(abs(x))/a^5 - 1/3*
(a^4*b^2 - 4*a^5*c + 6*(2*a*b^4*c - 9*a^2*b^2*c^2 + 5*a^3*c^3)*x^4 + 3*(4*
a*b^5 - 20*a^2*b^3*c + 17*a^3*b*c^2)*x^3 + (6*a^2*b^4 - 29*a^3*b^2*c + 20*
a^4*c^2)*x^2 - 2*(a^3*b^3 - 4*a^4*b*c)*x)/((c*x^2 + b*x + a)*(b^2 - 4*a*c)
*a^5*x^3)
```

Mupad [B] (verification not implemented)

Time = 22.44 (sec) , antiderivative size = 1120, normalized size of antiderivative = 4.44

$$\int \frac{1}{(ax^2 + bx^3 + cx^4)^2} \, dx = \text{Too large to display}$$

input
```
int(1/(a*x^2 + b*x^3 + c*x^4)^2,x)
```

output
```
((x^2*(5*a*c - 6*b^2))/(3*a^3) - 1/(3*a) + (2*b*x)/(3*a^2) + (x^3*(4*b^5 +
 17*a^2*b*c^2 - 20*a*b^3*c))/(a^4*(4*a*c - b^2)) + (2*c*x^4*(2*b^4 + 5*a^2
*c^2 - 9*a*b^2*c))/(a^4*(4*a*c - b^2)))/(a*x^3 + b*x^4 + c*x^5) + (log(4*a
*b^9 + 4*b^10*x - 4*a*b^6*(-(4*a*c - b^2)^3)^(1/2) - 52*a^2*b^7*c + 308*a^
5*b*c^4 - 40*a^5*c^5*x - 4*b^7*x*(-(4*a*c - b^2)^3)^(1/2) + 243*a^3*b^5*c^
2 - 473*a^4*b^3*c^3 + 5*a^4*c^3*(-(4*a*c - b^2)^3)^(1/2) + 24*a^2*b^4*c*(-
(4*a*c - b^2)^3)^(1/2) + 266*a^2*b^6*c^2*x - 563*a^3*b^4*c^3*x + 438*a^4*b
^2*c^4*x - 54*a*b^8*c*x - 33*a^3*b^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 30*a*b
^5*c*x*(-(4*a*c - b^2)^3)^(1/2) + 41*a^3*b*c^3*x*(-(4*a*c - b^2)^3)^(1/2)
- 66*a^2*b^3*c^2*x*(-(4*a*c - b^2)^3)^(1/2))*(a^2*(132*b^5*c^2 - 30*b^2*c^
2*(-(4*a*c - b^2)^3)^(1/2)) - a^3*(272*b^3*c^3 - 10*c^3*(-(4*a*c - b^2)^3)
^(1/2)) + 2*b^9 - 2*b^6*(-(4*a*c - b^2)^3)^(1/2) - a*(27*b^7*c - 15*b^4*c*
(-(4*a*c - b^2)^3)^(1/2)) + 192*a^4*b*c^4))/(a^5*b^6 - 64*a^8*c^3 - 12*a^6
*b^4*c + 48*a^7*b^2*c^2) + (log(4*a*b^9 + 4*b^10*x + 4*a*b^6*(-(4*a*c - b^
2)^3)^(1/2) - 52*a^2*b^7*c + 308*a^5*b*c^4 - 40*a^5*c^5*x + 4*b^7*x*(-(4*a
*c - b^2)^3)^(1/2) + 243*a^3*b^5*c^2 - 473*a^4*b^3*c^3 - 5*a^4*c^3*(-(4*a*
c - b^2)^3)^(1/2) - 24*a^2*b^4*c*(-(4*a*c - b^2)^3)^(1/2) + 266*a^2*b^6*c^
2*x - 563*a^3*b^4*c^3*x + 438*a^4*b^2*c^4*x - 54*a*b^8*c*x + 33*a^3*b^2*c^
2*(-(4*a*c - b^2)^3)^(1/2) - 30*a*b^5*c*x*(-(4*a*c - b^2)^3)^(1/2) - 41*a^
3*b*c^3*x*(-(4*a*c - b^2)^3)^(1/2) + 66*a^2*b^3*c^2*x*(-(4*a*c - b^2)^3...
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 1276, normalized size of antiderivative = 5.06

$$\int \frac{1}{(ax^2 + bx^3 + cx^4)^2} \, dx = \text{Too large to display}$$

input
```
int(1/(c*x^4+b*x^3+a*x^2)^2,x)
```

output
```
(60*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**4*b*c**3*x*
*3 - 180*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**3*b**3
*c**2*x**3 + 60*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*
*3*b**2*c**3*x**4 + 60*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b*
*2))*a**3*b*c**4*x**5 + 90*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c
- b**2))*a**2*b**5*c*x**3 - 180*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4
*a*c - b**2))*a**2*b**4*c**2*x**4 - 180*sqrt(4*a*c - b**2)*atan((b + 2*c*x
)/sqrt(4*a*c - b**2))*a**2*b**3*c**3*x**5 - 12*sqrt(4*a*c - b**2)*atan((b
+ 2*c*x)/sqrt(4*a*c - b**2))*a*b**7*x**3 + 90*sqrt(4*a*c - b**2)*atan((b +
 2*c*x)/sqrt(4*a*c - b**2))*a*b**6*c*x**4 + 90*sqrt(4*a*c - b**2)*atan((b
+ 2*c*x)/sqrt(4*a*c - b**2))*a*b**5*c**2*x**5 - 12*sqrt(4*a*c - b**2)*atan
((b + 2*c*x)/sqrt(4*a*c - b**2))*b**8*x**4 - 12*sqrt(4*a*c - b**2)*atan((b
 + 2*c*x)/sqrt(4*a*c - b**2))*b**7*c*x**5 - 144*log(a + b*x + c*x**2)*a**4
*b**2*c**3*x**3 + 168*log(a + b*x + c*x**2)*a**3*b**4*c**2*x**3 - 144*log(
a + b*x + c*x**2)*a**3*b**3*c**3*x**4 - 144*log(a + b*x + c*x**2)*a**3*b**
2*c**4*x**5 - 57*log(a + b*x + c*x**2)*a**2*b**6*c*x**3 + 168*log(a + b*x
+ c*x**2)*a**2*b**5*c**2*x**4 + 168*log(a + b*x + c*x**2)*a**2*b**4*c**3*x
**5 + 6*log(a + b*x + c*x**2)*a*b**8*x**3 - 57*log(a + b*x + c*x**2)*a*b**
7*c*x**4 - 57*log(a + b*x + c*x**2)*a*b**6*c**2*x**5 + 6*log(a + b*x + c*x
**2)*b**9*x**4 + 6*log(a + b*x + c*x**2)*b**8*c*x**5 + 288*log(x)*a**4*...
```

3.42 $\int \frac{1}{x(ax^2+bx^3+cx^4)^2} dx$

Optimal result

Integrand size = 22, antiderivative size = 318

$$\begin{aligned}\int \frac{1}{x\left(ax^2+bx^3+cx^4\right)^2}\,dx = & -\frac{5b^2-12ac}{4a^2\left(b^2-4ac\right)x^4}+\frac{b(5b^2-17ac)}{3a^3\left(b^2-4ac\right)x^3} \\ & -\frac{5b^4-22ab^2c+12a^2c^2}{2a^4\left(b^2-4ac\right)x^2}+\frac{b(5b^4-27ab^2c+29a^2c^2)}{a^5\left(b^2-4ac\right)x} \\ & +\frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)x^4\left(a+bx+cx^2\right)} \\ & +\frac{b(5b^6-42ab^4c+105a^2b^2c^2-70a^3c^3)\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^6\left(b^2-4ac\right)^{3/2}} \\ & +\frac{(5b^4-12ab^2c+3a^2c^2)\log(x)}{a^6} \\ & -\frac{(5b^4-12ab^2c+3a^2c^2)\log\left(a+bx+cx^2\right)}{2a^6}\end{aligned}$$

output
```
-1/4*(-12*a*c+5*b^2)/a^2/(-4*a*c+b^2)/x^4+1/3*b*(-17*a*c+5*b^2)/a^3/(-4*a*
c+b^2)/x^3-1/2*(12*a^2*c^2-22*a*b^2*c+5*b^4)/a^4/(-4*a*c+b^2)/x^2+b*(29*a^
2*c^2-27*a*b^2*c+5*b^4)/a^5/(-4*a*c+b^2)/x+(b*c*x-2*a*c+b^2)/a/(-4*a*c+b^2
)/x^4/(c*x^2+b*x+a)+b*(-70*a^3*c^3+105*a^2*b^2*c^2-42*a*b^4*c+5*b^6)*arcta
nh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/a^6/(-4*a*c+b^2)^(3/2)+(3*a^2*c^2-12*a*b^
2*c+5*b^4)*ln(x)/a^6-1/2*(3*a^2*c^2-12*a*b^2*c+5*b^4)*ln(c*x^2+b*x+a)/a^6
```

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 272, normalized size of antiderivative = 0.86

$$\int \frac{1}{x\left(ax^2+bx^3+cx^4\right)^2}\,dx$$
$$= \frac{-\frac{3a^4}{x^4}+\frac{8a^3b}{x^3}+\frac{6a^2(-3b^2+2ac)}{x^2}-\frac{24ab(-2b^2+3ac)}{x}-\frac{12a(-b^6+6ab^4c-9a^2b^2c^2+2a^3c^3-b^5cx+5ab^3c^2x-5a^2bc^3x)}{(b^2-4ac)(a+x(b+cx))}+\frac{12b(5b^6-42ab}{}}{}$$

input
```
Integrate[1/(x*(a*x^2 + b*x^3 + c*x^4)^2),x]
```

output
```
((-3*a^4)/x^4 + (8*a^3*b)/x^3 + (6*a^2*(-3*b^2 + 2*a*c))/x^2 - (24*a*b*(-2
*b^2 + 3*a*c))/x - (12*a*(-b^6 + 6*a*b^4*c - 9*a^2*b^2*c^2 + 2*a^3*c^3 - b
^5*c*x + 5*a*b^3*c^2*x - 5*a^2*b*c^3*x))/((b^2 - 4*a*c)*(a + x*(b + c*x)))
 + (12*b*(5*b^6 - 42*a*b^4*c + 105*a^2*b^2*c^2 - 70*a^3*c^3)*ArcTan[(b + 2
*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2) + 12*(5*b^4 - 12*a*b^2*c +
 3*a^2*c^2)*Log[x] - 6*(5*b^4 - 12*a*b^2*c + 3*a^2*c^2)*Log[a + x*(b + c*x
)])/(12*a^6)
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 309, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {9, 1165, 25, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(ax^2+bx^3+cx^4\right)^2}\,dx$$
$$\Big\downarrow \; 9$$
$$\int \frac{1}{x^5\left(a+bx+cx^2\right)^2}dx$$
$$\Big\downarrow \; 1165$$

$$\frac{-2ac+b^2+bcx}{ax^4\left(b^2-4ac\right)\left(a+bx+cx^2\right)}-\frac{\int -\frac{5b^2+5cxb-12ac}{x^5\left(cx^2+bx+a\right)}dx}{a\left(b^2-4ac\right)}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{5b^2+5cxb-12ac}{x^5\left(cx^2+bx+a\right)}dx}{a\left(b^2-4ac\right)}+\frac{-2ac+b^2+bcx}{ax^4\left(b^2-4ac\right)\left(a+bx+cx^2\right)}$$

$$\Big\downarrow \text{ 1200}$$

$$\frac{\int \left(\frac{5b^2-12ac}{ax^5}+\frac{\left(b^2-4ac\right)\left(5b^4-12acb^2+3a^2c^2\right)}{a^5x}+\frac{-b\left(5b^6-37acb^4+78a^2c^2b^2-41a^3c^3\right)-c\left(b^2-4ac\right)\left(5b^4-12acb^2+3a^2c^2\right)x}{a^5\left(cx^2+bx+a\right)}+\frac{-5b^5+27acb^3}{a^4x}\right.}{a\left(b^2-4ac\right)}$$
$$\frac{-2ac+b^2+bcx}{ax^4\left(b^2-4ac\right)\left(a+bx+cx^2\right)}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{b\left(5b^2-17ac\right)}{3a^2x^3}-\frac{\left(b^2-4ac\right)\left(3a^2c^2-12ab^2c+5b^4\right)\log\left(a+bx+cx^2\right)}{2a^5}+\frac{\log(x)\left(b^2-4ac\right)\left(3a^2c^2-12ab^2c+5b^4\right)}{a^5}+\frac{b\left(29a^2c^2-27ab^2c+5b^4\right)}{a^4x}-\frac{12a^2}{}}{a\left(b^2-4ac\right)}$$
$$\frac{-2ac+b^2+bcx}{ax^4\left(b^2-4ac\right)\left(a+bx+cx^2\right)}$$

input
```
Int[1/(x*(a*x^2 + b*x^3 + c*x^4)^2),x]
```

output
```
(b^2 - 2*a*c + b*c*x)/(a*(b^2 - 4*a*c)*x^4*(a + b*x + c*x^2)) + (-1/4*(5*b
^2 - 12*a*c)/(a*x^4) + (b*(5*b^2 - 17*a*c))/(3*a^2*x^3) - (5*b^4 - 22*a*b^
2*c + 12*a^2*c^2)/(2*a^3*x^2) + (b*(5*b^4 - 27*a*b^2*c + 29*a^2*c^2))/(a^4
*x) + (b*(5*b^6 - 42*a*b^4*c + 105*a^2*b^2*c^2 - 70*a^3*c^3)*ArcTanh[(b +
2*c*x)/Sqrt[b^2 - 4*a*c]])/(a^5*Sqrt[b^2 - 4*a*c]) + ((b^2 - 4*a*c)*(5*b^4
 - 12*a*b^2*c + 3*a^2*c^2)*Log[x])/a^5 - ((b^2 - 4*a*c)*(5*b^4 - 12*a*b^2*
c + 3*a^2*c^2)*Log[a + b*x + c*x^2])/(2*a^5))/(a*(b^2 - 4*a*c))
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1165
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d
+ e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p
+ 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
 b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && LtQ[p, -1]
 && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.13

method	result
default	$-\frac{\frac{acb\left(5a^2c^2-5a\,b^2c+b^4\right)x}{4ac-b^2}-\frac{a\left(2a^3c^3-9a^2b^2c^2+6a\,b^4c-b^6\right)}{4ac-b^2}}{c\,x^2+bx+a}+\frac{\left(12a^3c^4-51a^2c^3b^2+32a\,b^4c^2-5b^6c\right)\ln\left(c\,x^2+bx+a\right)}{2c}+\frac{2\left(41a^3b\,c^3-78a^2b^3c^2\right.}{4ac-}$ … a^6
risch	$\frac{-\frac{bc\left(29a^2c^2-27a\,b^2c+5b^4\right)x^5}{a^5\left(4ac-b^2\right)}+\frac{\left(12a^3c^3-80a^2b^2c^2+59a\,b^4c-10b^6\right)x^4}{2a^5\left(4ac-b^2\right)}-\frac{b\left(26ac-15b^2\right)x^3}{6a^4}+\frac{\left(9ac-10b^2\right)x^2}{12a^3}+\frac{5bx}{12a^2}-\frac{1}{4a}}{x^4\left(c\,x^2+bx+a\right)}+\frac{3\ln(x)c^2}{a^4}-$ 1

input
```
int(1/x/(c*x^4+b*x^3+a*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/a^6*((a*c*b*(5*a^2*c^2-5*a*b^2*c+b^4)/(4*a*c-b^2)*x-a*(2*a^3*c^3-9*a^2*
b^2*c^2+6*a*b^4*c-b^6)/(4*a*c-b^2))/(c*x^2+b*x+a)+1/(4*a*c-b^2)*(1/2*(12*a
^3*c^4-51*a^2*b^2*c^3+32*a*b^4*c^2-5*b^6*c)/c*ln(c*x^2+b*x+a)+2*(41*a^3*b*
c^3-78*a^2*b^3*c^2+37*a*b^5*c-5*b^7-1/2*(12*a^3*c^4-51*a^2*b^2*c^3+32*a*b^
4*c^2-5*b^6*c)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2)))
)-1/4/a^2/x^4-1/2*(-2*a*c+3*b^2)/x^2/a^4+(3*a^2*c^2-12*a*b^2*c+5*b^4)*ln(x
)/a^6+2/3/a^3*b/x^3-2*b*(3*a*c-2*b^2)/a^5/x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 810 vs. 2(306) = 612.

Time = 0.55 (sec) , antiderivative size = 1640, normalized size of antiderivative = 5.16

$$\int \frac{1}{x\left(ax^2+bx^3+cx^4\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/x/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="fricas")
```

output
```
[-1/12*(3*a^5*b^4 - 24*a^6*b^2*c + 48*a^7*c^2 - 12*(5*a*b^7*c - 47*a^2*b^5
*c^2 + 137*a^3*b^3*c^3 - 116*a^4*b*c^4)*x^5 - 6*(10*a*b^8 - 99*a^2*b^6*c +
 316*a^3*b^4*c^2 - 332*a^4*b^2*c^3 + 48*a^5*c^4)*x^4 - 2*(15*a^2*b^7 - 146
*a^3*b^5*c + 448*a^4*b^3*c^2 - 416*a^5*b*c^3)*x^3 + (10*a^3*b^6 - 89*a^4*b
^4*c + 232*a^5*b^2*c^2 - 144*a^6*c^3)*x^2 - 6*((5*b^7*c - 42*a*b^5*c^2 + 1
05*a^2*b^3*c^3 - 70*a^3*b*c^4)*x^6 + (5*b^8 - 42*a*b^6*c + 105*a^2*b^4*c^2
 - 70*a^3*b^2*c^3)*x^5 + (5*a*b^7 - 42*a^2*b^5*c + 105*a^3*b^3*c^2 - 70*a^
4*b*c^3)*x^4)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + s
qrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) - 5*(a^4*b^5 - 8*a^5*b^3*
c + 16*a^6*b*c^2)*x + 6*((5*b^8*c - 52*a*b^6*c^2 + 179*a^2*b^4*c^3 - 216*a
^3*b^2*c^4 + 48*a^4*c^5)*x^6 + (5*b^9 - 52*a*b^7*c + 179*a^2*b^5*c^2 - 216
*a^3*b^3*c^3 + 48*a^4*b*c^4)*x^5 + (5*a*b^8 - 52*a^2*b^6*c + 179*a^3*b^4*c
^2 - 216*a^4*b^2*c^3 + 48*a^5*c^4)*x^4)*log(c*x^2 + b*x + a) - 12*((5*b^8*
c - 52*a*b^6*c^2 + 179*a^2*b^4*c^3 - 216*a^3*b^2*c^4 + 48*a^4*c^5)*x^6 + (
5*b^9 - 52*a*b^7*c + 179*a^2*b^5*c^2 - 216*a^3*b^3*c^3 + 48*a^4*b*c^4)*x^5
 + (5*a*b^8 - 52*a^2*b^6*c + 179*a^3*b^4*c^2 - 216*a^4*b^2*c^3 + 48*a^5*c^
4)*x^4)*log(x))/((a^6*b^4*c - 8*a^7*b^2*c^2 + 16*a^8*c^3)*x^6 + (a^6*b^5 -
 8*a^7*b^3*c + 16*a^8*b*c^2)*x^5 + (a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)*x^
4), -1/12*(3*a^5*b^4 - 24*a^6*b^2*c + 48*a^7*c^2 - 12*(5*a*b^7*c - 47*a^2*
b^5*c^2 + 137*a^3*b^3*c^3 - 116*a^4*b*c^4)*x^5 - 6*(10*a*b^8 - 99*a^2*b...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x\left(ax^2+bx^3+cx^4\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/x/(c*x**4+b*x**3+a*x**2)**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x\left(ax^2+bx^3+cx^4\right)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/x/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.09

$$\int \frac{1}{x\left(ax^2+bx^3+cx^4\right)^2}\,dx$$
$$= -\frac{\left(5\,b^7-42\,ab^5c+105\,a^2b^3c^2-70\,a^3bc^3\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(a^6b^2-4\,a^7c\right)\sqrt{-b^2+4\,ac}}$$
$$-\frac{\left(5\,b^4-12\,ab^2c+3\,a^2c^2\right)\log\left(cx^2+bx+a\right)}{2\,a^6}+\frac{\left(5\,b^4-12\,ab^2c+3\,a^2c^2\right)\log\left(|x|\right)}{a^6}$$
$$-\frac{3\,a^5b^2-12\,a^6c-12\,(5\,ab^5c-27\,a^2b^3c^2+29\,a^3bc^3)x^5-6\,(10\,ab^6-59\,a^2b^4c+80\,a^3b^2c^2-12\,a^4c^3)x^4\cdots}{12\,(cx^2+bx+a)(b^2\cdots}$$

input
```
integrate(1/x/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="giac")
```

output
```
-(5*b^7 - 42*a*b^5*c + 105*a^2*b^3*c^2 - 70*a^3*b*c^3)*arctan((2*c*x + b)/
sqrt(-b^2 + 4*a*c))/((a^6*b^2 - 4*a^7*c)*sqrt(-b^2 + 4*a*c)) - 1/2*(5*b^4
- 12*a*b^2*c + 3*a^2*c^2)*log(c*x^2 + b*x + a)/a^6 + (5*b^4 - 12*a*b^2*c +
 3*a^2*c^2)*log(abs(x))/a^6 - 1/12*(3*a^5*b^2 - 12*a^6*c - 12*(5*a*b^5*c -
 27*a^2*b^3*c^2 + 29*a^3*b*c^3)*x^5 - 6*(10*a*b^6 - 59*a^2*b^4*c + 80*a^3*
b^2*c^2 - 12*a^4*c^3)*x^4 - 2*(15*a^2*b^5 - 86*a^3*b^3*c + 104*a^4*b*c^2)*
x^3 + (10*a^3*b^4 - 49*a^4*b^2*c + 36*a^5*c^2)*x^2 - 5*(a^4*b^3 - 4*a^5*b*
c)*x)/((c*x^2 + b*x + a)*(b^2 - 4*a*c)*a^6*x^4)
```

Mupad [B] (verification not implemented)

Time = 22.78 (sec) , antiderivative size = 1260, normalized size of antiderivative = 3.96

$$\int \frac{1}{x(ax^2+bx^3+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
int(1/(x*(a*x^2 + b*x^3 + c*x^4)^2),x)
```

output
```
(log(x)*(5*b^4 + 3*a^2*c^2 - 12*a*b^2*c))/a^6 - (1/(4*a) - (x^2*(9*a*c - 1
0*b^2))/(12*a^3) - (5*b*x)/(12*a^2) + (x^4*(10*b^6 - 12*a^3*c^3 + 80*a^2*b
^2*c^2 - 59*a*b^4*c))/(2*a^5*(4*a*c - b^2)) + (b*x^3*(26*a*c - 15*b^2))/(6
*a^4) + (b*c*x^5*(5*b^4 + 29*a^2*c^2 - 27*a*b^2*c))/(a^5*(4*a*c - b^2)))/(
a*x^4 + b*x^5 + c*x^6) + (log(288*a^6*c^5 - 10*b^11*x - 10*a*b^10 + 10*a*b
^7*(-(4*a*c - b^2)^3)^(1/2) + 139*a^2*b^8*c + 10*b^8*x*(-(4*a*c - b^2)^3)^
(1/2) - 717*a^3*b^6*c^2 + 1643*a^4*b^4*c^3 - 1508*a^5*b^2*c^4 - 69*a^2*b^5
*c*(-(4*a*c - b^2)^3)^(1/2) - 53*a^4*b*c^3*(-(4*a*c - b^2)^3)^(1/2) - 779*
a^2*b^7*c^2*x + 1916*a^3*b^5*c^3*x - 1998*a^4*b^3*c^4*x + 36*a^4*c^4*x*(-(
4*a*c - b^2)^3)^(1/2) + 144*a*b^9*c*x + 129*a^3*b^3*c^2*(-(4*a*c - b^2)^3)
^(1/2) + 568*a^5*b*c^5*x - 84*a*b^6*c*x*(-(4*a*c - b^2)^3)^(1/2) + 225*a^2
*b^4*c^2*x*(-(4*a*c - b^2)^3)^(1/2) - 206*a^3*b^2*c^3*x*(-(4*a*c - b^2)^3)
^(1/2))*(a^3*(466*b^4*c^3 - 35*b*c^3*(-(4*a*c - b^2)^3)^(1/2)) - a^2*((387
*b^6*c^2)/2 - (105*b^3*c^2*(-(4*a*c - b^2)^3)^(1/2))/2) - (5*b^10)/2 + 96*
a^5*c^5 + (5*b^7*(-(4*a*c - b^2)^3)^(1/2))/2 + a*(36*b^8*c - 21*b^5*c*(-(4
*a*c - b^2)^3)^(1/2)) - 456*a^4*b^2*c^4))/(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b
^4*c + 48*a^8*b^2*c^2) - (log(10*a*b^10 + 10*b^11*x - 288*a^6*c^5 + 10*a*b
^7*(-(4*a*c - b^2)^3)^(1/2) - 139*a^2*b^8*c + 10*b^8*x*(-(4*a*c - b^2)^3)^
(1/2) + 717*a^3*b^6*c^2 - 1643*a^4*b^4*c^3 + 1508*a^5*b^2*c^4 - 69*a^2*b^5
*c*(-(4*a*c - b^2)^3)^(1/2) - 53*a^4*b*c^3*(-(4*a*c - b^2)^3)^(1/2) + 7...
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 1425, normalized size of antiderivative = 4.48

$$\int \frac{1}{x(ax^2+bx^3+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
int(1/x/(c*x^4+b*x^3+a*x^2)^2,x)
```

output
```
( - 840*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**4*b*c**
3*x**4 + 1260*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**3
*b**3*c**2*x**4 - 840*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**
2))*a**3*b**2*c**3*x**5 - 840*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a
*c - b**2))*a**3*b*c**4*x**6 - 504*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqr
t(4*a*c - b**2))*a**2*b**5*c*x**4 + 1260*sqrt(4*a*c - b**2)*atan((b + 2*c*
x)/sqrt(4*a*c - b**2))*a**2*b**4*c**2*x**5 + 1260*sqrt(4*a*c - b**2)*atan(
(b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*b**3*c**3*x**6 + 60*sqrt(4*a*c - b**2
)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b**7*x**4 - 504*sqrt(4*a*c - b**2
)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b**6*c*x**5 - 504*sqrt(4*a*c - b*
*2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b**5*c**2*x**6 + 60*sqrt(4*a*c
- b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b**8*x**5 + 60*sqrt(4*a*c - b
**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b**7*c*x**6 - 288*log(a + b*x +
c*x**2)*a**5*c**4*x**4 + 1296*log(a + b*x + c*x**2)*a**4*b**2*c**3*x**4 -
288*log(a + b*x + c*x**2)*a**4*b*c**4*x**5 - 288*log(a + b*x + c*x**2)*a**
4*c**5*x**6 - 1074*log(a + b*x + c*x**2)*a**3*b**4*c**2*x**4 + 1296*log(a
+ b*x + c*x**2)*a**3*b**3*c**3*x**5 + 1296*log(a + b*x + c*x**2)*a**3*b**2
*c**4*x**6 + 312*log(a + b*x + c*x**2)*a**2*b**6*c*x**4 - 1074*log(a + b*x
 + c*x**2)*a**2*b**5*c**2*x**5 - 1074*log(a + b*x + c*x**2)*a**2*b**4*c**3
*x**6 - 30*log(a + b*x + c*x**2)*a*b**8*x**4 + 312*log(a + b*x + c*x**2...
```

3.43 $\int \frac{1}{x^2(2+x)^2} dx$

Optimal result

Integrand size = 9, antiderivative size = 31

$$\int \frac{1}{x^2(2+x)^2} dx = -\frac{1}{4x} - \frac{1}{4(2+x)} - \frac{\log(x)}{4} + \frac{1}{4}\log(2+x)$$

output
```
-1/4/x-1/(8+4*x)-1/4*ln(x)+1/4*ln(2+x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \frac{1}{x^2(2+x)^2} dx = \frac{1}{4}\left(-\frac{2(1+x)}{x(2+x)} - \log(x) + \log(2+x)\right)$$

input
```
Integrate[1/(x^2*(2 + x)^2),x]
```

output
```
((-2*(1 + x))/(x*(2 + x)) - Log[x] + Log[2 + x])/4
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2(x+2)^2}\,dx$$

$$\Big\downarrow \ 54$$

$$\int \left(\frac{1}{4x^2}+\frac{1}{4(x+2)}+\frac{1}{4(x+2)^2}-\frac{1}{4x}\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{1}{4x}-\frac{1}{4(x+2)}-\frac{\log(x)}{4}+\frac{1}{4}\log(x+2)$$

input
```
Int[1/(x^2*(2 + x)^2),x]
```

output
```
-1/4*1/x - 1/(4*(2 + x)) - Log[x]/4 + Log[2 + x]/4
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.77

method	result	size
default	$-\frac{1}{4(2+x)}+\frac{\ln(2+x)}{4}-\frac{1}{4x}-\frac{\ln(x)}{4}$	24
norman	$\frac{-\frac{1}{2}-\frac{x}{2}}{x(2+x)}-\frac{\ln(x)}{4}+\frac{\ln(2+x)}{4}$	26
risch	$\frac{-\frac{1}{2}-\frac{x}{2}}{x(2+x)}-\frac{\ln(x)}{4}+\frac{\ln(2+x)}{4}$	26
meijerg	$-\frac{1}{4x}-\frac{1}{8}-\frac{\ln(x)}{4}+\frac{\ln(2)}{4}+\frac{3x}{16(\frac{3x}{2}+3)}+\frac{\ln(1+\frac{x}{2})}{4}$	34
parallelrisch	$-\frac{\ln(x)x^2-\ln(2+x)x^2+2+2\ln(x)x-2\ln(2+x)x+2x}{4x(2+x)}$	43

input
```
int(1/x^2/(2+x)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/4/(2+x)+1/4*ln(2+x)-1/4/x-1/4*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.26

$$\int \frac{1}{x^2(2+x)^2}\,dx=\frac{(x^2+2\,x)\log(x+2)-(x^2+2\,x)\log(x)-2\,x-2}{4\,(x^2+2\,x)}$$

input
```
integrate(1/x^2/(2+x)^2,x, algorithm="fricas")
```

output
```
1/4*((x^2 + 2*x)*log(x + 2) - (x^2 + 2*x)*log(x) - 2*x - 2)/(x^2 + 2*x)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.77

$$\int \frac{1}{x^2(2+x)^2}\,dx = \frac{-x-1}{2x^2+4x} - \frac{\log{(x)}}{4} + \frac{\log{(x+2)}}{4}$$

input
```
integrate(1/x**2/(2+x)**2,x)
```

output
```
(-x - 1)/(2*x**2 + 4*x) - log(x)/4 + log(x + 2)/4
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \frac{1}{x^2(2+x)^2}\,dx = -\frac{x+1}{2\left(x^2+2\,x\right)} + \frac{1}{4}\,\log\left(x+2\right) - \frac{1}{4}\,\log\left(x\right)$$

input
```
integrate(1/x^2/(2+x)^2,x, algorithm="maxima")
```

output
```
-1/2*(x + 1)/(x^2 + 2*x) + 1/4*log(x + 2) - 1/4*log(x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.10

$$\int \frac{1}{x^2(2+x)^2}\,dx = -\frac{1}{4\,(x+2)} + \frac{1}{8\,\left(\frac{2}{x+2}-1\right)} - \frac{1}{4}\,\log\left(\left|-\frac{2}{x+2}+1\right|\right)$$

input
```
integrate(1/x^2/(2+x)^2,x, algorithm="giac")
```

output
```
-1/4/(x + 2) + 1/8/(2/(x + 2) - 1) - 1/4*log(abs(-2/(x + 2) + 1))
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.97

$$\int \frac{1}{x^2(2+x)^2}\,dx = \frac{\ln\left(\frac{x+2}{x}\right)}{4} - \frac{1}{2\,(x+2)} - \frac{1}{2\,x\,(x+2)}$$

input
```
int(1/(x^2*(x + 2)^2),x)
```

output
```
log((x + 2)/x)/4 - 1/(2*(x + 2)) - 1/(2*x*(x + 2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.35

$$\int \frac{1}{x^2(2+x)^2}\,dx = \frac{\log(x+2)\,x^2 + 2\log(x+2)\,x - \log(x)\,x^2 - 2\log(x)\,x + x^2 - 2}{4x\,(x+2)}$$

input
```
int(1/x^2/(2+x)^2,x)
```

output
```
(log(x + 2)*x**2 + 2*log(x + 2)*x - log(x)*x**2 - 2*log(x)*x + x**2 - 2)/(
4*x*(x + 2))
```

3.44 $\int \frac{1}{x^2(4+4x+x^2)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 31

$$\int \frac{1}{x^2\left(4+4x+x^2\right)}\,dx=-\frac{1}{4x}-\frac{1}{4(2+x)}-\frac{\log(x)}{4}+\frac{1}{4}\log(2+x)$$

output
```
-1/4/x-1/(8+4*x)-1/4*ln(x)+1/4*ln(2+x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \frac{1}{x^2\left(4+4x+x^2\right)}\,dx=\frac{1}{4}\left(-\frac{2(1+x)}{x(2+x)}-\log(x)+\log(2+x)\right)$$

input
```
Integrate[1/(x^2*(4 + 4*x + x^2)),x]
```

output
```
((-2*(1 + x))/(x*(2 + x)) - Log[x] + Log[2 + x])/4
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {1098, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\left(x^2+4x+4\right)}dx$$

$$\Big\downarrow \text{1098}$$

$$\int \frac{1}{x^2(x+2)^2}dx$$

$$\Big\downarrow \text{54}$$

$$\int \left(\frac{1}{4x^2}+\frac{1}{4(x+2)}+\frac{1}{4(x+2)^2}-\frac{1}{4x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{4x}-\frac{1}{4(x+2)}-\frac{\log(x)}{4}+\frac{1}{4}\log(x+2)$$

input
```
Int[1/(x^2*(4 + 4*x + x^2)),x]
```

output
```
-1/4*1/x - 1/(4*(2 + x)) - Log[x]/4 + Log[2 + x]/4
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 1098
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[
{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.77

method	result	size
default	$-\frac{1}{4(2+x)}+\frac{\ln(2+x)}{4}-\frac{1}{4x}-\frac{\ln(x)}{4}$	24
norman	$\frac{-\frac{1}{2}-\frac{x}{2}}{x(2+x)}-\frac{\ln(x)}{4}+\frac{\ln(2+x)}{4}$	26
risch	$\frac{-\frac{1}{2}-\frac{x}{2}}{x(2+x)}-\frac{\ln(x)}{4}+\frac{\ln(2+x)}{4}$	26
meijerg	$-\frac{1}{4x}-\frac{1}{8}-\frac{\ln(x)}{4}+\frac{\ln(2)}{4}+\frac{3x}{16(\frac{3x}{2}+3)}+\frac{\ln(1+\frac{x}{2})}{4}$	34
parallelrisch	$-\frac{\ln(x)x^2-\ln(2+x)x^2+2+2\ln(x)x-2\ln(2+x)x+2x}{4x(2+x)}$	43

input
```
int(1/x^2/(x^2+4*x+4),x,method=_RETURNVERBOSE)
```

output
```
-1/4/(2+x)+1/4*ln(2+x)-1/4/x-1/4*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.26

$$\int \frac{1}{x^2(4+4x+x^2)}\,dx=\frac{(x^2+2x)\log(x+2)-(x^2+2x)\log(x)-2x-2}{4(x^2+2x)}$$

input
```
integrate(1/x^2/(x^2+4*x+4),x, algorithm="fricas")
```

output
```
1/4*((x^2 + 2*x)*log(x + 2) - (x^2 + 2*x)*log(x) - 2*x - 2)/(x^2 + 2*x)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.77

$$\int \frac{1}{x^2 (4+4x+x^2)}\,dx = \frac{-x-1}{2x^2+4x} - \frac{\log(x)}{4} + \frac{\log(x+2)}{4}$$

input
```
integrate(1/x**2/(x**2+4*x+4),x)
```

output
```
(-x - 1)/(2*x**2 + 4*x) - log(x)/4 + log(x + 2)/4
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \frac{1}{x^2 (4+4x+x^2)}\,dx = -\frac{x+1}{2(x^2+2x)} + \frac{1}{4}\log(x+2) - \frac{1}{4}\log(x)$$

input
```
integrate(1/x^2/(x^2+4*x+4),x, algorithm="maxima")
```

output
```
-1/2*(x + 1)/(x^2 + 2*x) + 1/4*log(x + 2) - 1/4*log(x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int \frac{1}{x^2 (4+4x+x^2)}\,dx = -\frac{x+1}{2(x^2+2x)} + \frac{1}{4}\log(|x+2|) - \frac{1}{4}\log(|x|)$$

input
```
integrate(1/x^2/(x^2+4*x+4),x, algorithm="giac")
```

output
```
-1/2*(x + 1)/(x^2 + 2*x) + 1/4*log(abs(x + 2)) - 1/4*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.74

$$\int \frac{1}{x^2 (4+4x+x^2)}\,dx = \frac{\operatorname{atanh}(x+1)}{2} - \frac{\frac{x}{2}+\frac{1}{2}}{x^2+2\,x}$$

input
```
int(1/(x^2*(4*x + x^2 + 4)),x)
```

output
```
atanh(x + 1)/2 - (x/2 + 1/2)/(2*x + x^2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.35

$$\int \frac{1}{x^2 (4+4x+x^2)}\,dx = \frac{\log(x+2)\,x^2+2\log(x+2)\,x-\log(x)\,x^2-2\log(x)\,x+x^2-2}{4x\,(x+2)}$$

input
```
int(1/x^2/(x^2+4*x+4),x)
```

output
```
(log(x + 2)*x**2 + 2*log(x + 2)*x - log(x)*x**2 - 2*log(x)*x + x**2 - 2)/(
4*x*(x + 2))
```

3.45 $\int \frac{1}{4x^2+4x^3+x^4} dx$

Optimal result

Integrand size = 16, antiderivative size = 31

$$\int \frac{1}{4x^2+4x^3+x^4} dx = -\frac{1}{4x} - \frac{1}{4(2+x)} - \frac{\log(x)}{4} + \frac{1}{4}\log(2+x)$$

output
```
-1/4/x-1/(8+4*x)-1/4*ln(x)+1/4*ln(2+x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \frac{1}{4x^2+4x^3+x^4} dx = \frac{1}{4}\left(-\frac{2(1+x)}{x(2+x)} - \log(x) + \log(2+x)\right)$$

input
```
Integrate[(4*x^2 + 4*x^3 + x^4)^(-1),x]
```

output
```
((-2*(1 + x))/(x*(2 + x)) - Log[x] + Log[2 + x])/4
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1949, 1098, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4+4x^3+4x^2}\,dx$$
$$\Big\downarrow \text{1949}$$
$$\int \frac{1}{x^2\left(x^2+4x+4\right)}dx$$
$$\Big\downarrow \text{1098}$$
$$\int \frac{1}{x^2(x+2)^2}dx$$
$$\Big\downarrow \text{54}$$
$$\int \left(\frac{1}{4x^2}+\frac{1}{4(x+2)}+\frac{1}{4(x+2)^2}-\frac{1}{4x}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$-\frac{1}{4x}-\frac{1}{4(x+2)}-\frac{\log(x)}{4}+\frac{1}{4}\log(x+2)$$

input
```
Int[(4*x^2 + 4*x^3 + x^4)^(-1),x]
```

output
```
-1/4*1/x - 1/(4*(2 + x)) - Log[x]/4 + Log[2 + x]/4
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 1098
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Simp[1/c^p   Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[
{a, b, c, d, e, m}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 1949
```
Int[((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_), x_Symbol
] :> Int[x^(p*q)*(a + b*x^(n - q) + c*x^(2*(n - q)))^p, x] /; FreeQ[{a, b,
c, n, q}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.77

method	result	size
default	$-\frac{1}{4(2+x)}+\frac{\ln(2+x)}{4}-\frac{1}{4x}-\frac{\ln(x)}{4}$	24
norman	$\frac{-\frac{1}{2}-\frac{x}{2}}{x(2+x)}-\frac{\ln(x)}{4}+\frac{\ln(2+x)}{4}$	26
risch	$\frac{-\frac{1}{2}-\frac{x}{2}}{x(2+x)}-\frac{\ln(x)}{4}+\frac{\ln(2+x)}{4}$	26
parallelrisch	$-\frac{\ln(x)x^2-\ln(2+x)x^2+2+2\ln(x)x-2\ln(2+x)x+2x}{4x(2+x)}$	43

input
```
int(1/(x^4+4*x^3+4*x^2),x,method=_RETURNVERBOSE)
```

output
```
-1/4/(2+x)+1/4*ln(2+x)-1/4/x-1/4*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.26

$$\int \frac{1}{4x^2+4x^3+x^4}\,dx = \frac{(x^2+2\,x)\log(x+2)-(x^2+2\,x)\log(x)-2\,x-2}{4\,(x^2+2\,x)}$$

input
```
integrate(1/(x^4+4*x^3+4*x^2),x, algorithm="fricas")
```

output
```
1/4*((x^2 + 2*x)*log(x + 2) - (x^2 + 2*x)*log(x) - 2*x - 2)/(x^2 + 2*x)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.77

$$\int \frac{1}{4x^2+4x^3+x^4}\,dx = \frac{-x-1}{2x^2+4x}-\frac{\log(x)}{4}+\frac{\log(x+2)}{4}$$

input
```
integrate(1/(x**4+4*x**3+4*x**2),x)
```

output
```
(-x - 1)/(2*x**2 + 4*x) - log(x)/4 + log(x + 2)/4
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \frac{1}{4x^2+4x^3+x^4}\,dx = -\frac{x+1}{2\,(x^2+2\,x)}+\frac{1}{4}\,\log(x+2)-\frac{1}{4}\,\log(x)$$

input
```
integrate(1/(x^4+4*x^3+4*x^2),x, algorithm="maxima")
```

output
```
-1/2*(x + 1)/(x^2 + 2*x) + 1/4*log(x + 2) - 1/4*log(x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int \frac{1}{4x^2 + 4x^3 + x^4}\, dx = -\frac{x+1}{2\,(x^2+2\,x)} + \frac{1}{4}\,\log(|x+2|) - \frac{1}{4}\,\log(|x|)$$

input
```
integrate(1/(x^4+4*x^3+4*x^2),x, algorithm="giac")
```

output
```
-1/2*(x + 1)/(x^2 + 2*x) + 1/4*log(abs(x + 2)) - 1/4*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.74

$$\int \frac{1}{4x^2 + 4x^3 + x^4}\, dx = \frac{\operatorname{atanh}(x+1)}{2} - \frac{\frac{x}{2} + \frac{1}{2}}{x^2 + 2\,x}$$

input
```
int(1/(4*x^2 + 4*x^3 + x^4),x)
```

output
```
atanh(x + 1)/2 - (x/2 + 1/2)/(2*x + x^2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.35

$$\int \frac{1}{4x^2 + 4x^3 + x^4}\, dx = \frac{\log(x+2)\,x^2 + 2\log(x+2)\,x - \log(x)\,x^2 - 2\log(x)\,x + x^2 - 2}{4x\,(x+2)}$$

input
```
int(1/(x^4+4*x^3+4*x^2),x)
```

output
```
(log(x + 2)*x**2 + 2*log(x + 2)*x - log(x)*x**2 - 2*log(x)*x + x**2 - 2)/(
4*x*(x + 2))
```

3.46 $\int x^2\sqrt{ax^2+bx^3+cx^4}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 229

$$\int x^2\sqrt{ax^2+bx^3+cx^4}\,dx=\frac{b(35b^2-116ac)\sqrt{ax^2+bx^3+cx^4}}{960c^3}-\frac{(105b^4-460ab^2c+256a^2c^2)\sqrt{ax^2+bx^3+cx^4}}{1920c^4x}-\frac{(7b^2-16ac)\,x\sqrt{ax^2+bx^3+cx^4}}{240c^2}+\frac{x^2(b+8cx)\sqrt{ax^2+bx^3+cx^4}}{40c}+\frac{b(7b^2-12ac)\,(b^2-4ac)\,\text{arctanh}\left(\frac{x(b+2cx)}{2\sqrt{c}\sqrt{ax^2+bx^3+cx^4}}\right)}{256c^{9/2}}$$

output
```
1/960*b*(-116*a*c+35*b^2)*(c*x^4+b*x^3+a*x^2)^(1/2)/c^3-1/1920*(256*a^2*c^
2-460*a*b^2*c+105*b^4)*(c*x^4+b*x^3+a*x^2)^(1/2)/c^4/x-1/240*(-16*a*c+7*b^
2)*x*(c*x^4+b*x^3+a*x^2)^(1/2)/c^2+1/40*x^2*(8*c*x+b)*(c*x^4+b*x^3+a*x^2)^
(1/2)/c+1/256*b*(-12*a*c+7*b^2)*(-4*a*c+b^2)*arctanh(1/2*x*(2*c*x+b)/c^(1/
2)/(c*x^4+b*x^3+a*x^2)^(1/2))/c^(9/2)
```

Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.80

$$\int x^2\sqrt{ax^2+bx^3+cx^4}\,dx$$
$$=\frac{2\sqrt{c}x(a+x(b+cx))\left(-105b^4+70b^3cx+4b^2c(115a-14cx^2)+8bc^2x(-29a+6cx^2)+128c^2(-2a^2+a\right.}{3840c^{9/2}\sqrt{x^2(}}$$

input
```
Integrate[x^2*Sqrt[a*x^2 + b*x^3 + c*x^4],x]
```

output
```
(2*Sqrt[c]*x*(a + x*(b + c*x))*(-105*b^4 + 70*b^3*c*x + 4*b^2*c*(115*a - 1
4*c*x^2) + 8*b*c^2*x*(-29*a + 6*c*x^2) + 128*c^2*(-2*a^2 + a*c*x^2 + 3*c^2
*x^4)) - 15*(7*b^5 - 40*a*b^3*c + 48*a^2*b*c^2)*x*Sqrt[a + x*(b + c*x)]*Lo
g[c^4*(b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/(3840*c^(9/2)*Sqrt[x
^2*(a + x*(b + c*x))])
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.21, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.458, Rules used = {1966, 27, 1996, 27, 1996, 27, 1996, 27, 1961, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{ax^2+bx^3+cx^4}\,dx$$
$$\Big\downarrow \text{1966}$$
$$\frac{\int -\frac{x^3(6ab+(7b^2-16ac)x)}{2\sqrt{cx^4+bx^3+ax^2}}dx}{40c}+\frac{x^2(b+8cx)\sqrt{ax^2+bx^3+cx^4}}{40c}$$
$$\Big\downarrow \text{27}$$
$$\frac{x^2(b+8cx)\sqrt{ax^2+bx^3+cx^4}}{40c}-\frac{\int \frac{x^3(6ab+(7b^2-16ac)x)}{\sqrt{cx^4+bx^3+ax^2}}dx}{80c}$$

$\downarrow$ 1996

$$\frac{x^2(b+8cx)\sqrt{ax^2+bx^3+cx^4}}{40c}-\frac{\frac{x(7b^2-16ac)\sqrt{ax^2+bx^3+cx^4}}{3c}-\frac{\int \frac{x^2\left(4a\left(7b^2-16ac\right)+b\left(35b^2-116ac\right)x\right)}{2\sqrt{cx^4+bx^3+ax^2}}dx}{3c}}{80c}$$

$\downarrow$ 27

$$\frac{x^2(b+8cx)\sqrt{ax^2+bx^3+cx^4}}{40c}-\frac{\frac{x(7b^2-16ac)\sqrt{ax^2+bx^3+cx^4}}{3c}-\frac{\int \frac{x^2\left(4a\left(7b^2-16ac\right)+b\left(35b^2-116ac\right)x\right)}{\sqrt{cx^4+bx^3+ax^2}}dx}{6c}}{80c}$$

$\downarrow$ 1996

$$\frac{x^2(b+8cx)\sqrt{ax^2+bx^3+cx^4}}{40c}-\frac{\frac{x(7b^2-16ac)\sqrt{ax^2+bx^3+cx^4}}{3c}-\frac{\frac{b\left(35b^2-116ac\right)\sqrt{ax^2+bx^3+cx^4}}{2c}-\frac{\int \frac{x\left(2ab\left(35b^2-116ac\right)+\left(105b^4-460acb^2+256a^2c^2\right)x\right)}{2\sqrt{cx^4+bx^3+ax^2}}dx}{2c}}{6c}}{80c}$$

$\downarrow$ 27

$$\frac{x^2(b+8cx)\sqrt{ax^2+bx^3+cx^4}}{40c}-\frac{\frac{x(7b^2-16ac)\sqrt{ax^2+bx^3+cx^4}}{3c}-\frac{\frac{b\left(35b^2-116ac\right)\sqrt{ax^2+bx^3+cx^4}}{2c}-\frac{\int \frac{x\left(2ab\left(35b^2-116ac\right)+\left(105b^4-460acb^2+256a^2c^2\right)x\right)}{\sqrt{cx^4+bx^3+ax^2}}dx}{4c}}{6c}}{80c}$$

$\downarrow$ 1996

$$\frac{x^2(b+8cx)\sqrt{ax^2+bx^3+cx^4}}{40c}-\frac{\frac{x(7b^2-16ac)\sqrt{ax^2+bx^3+cx^4}}{3c}-\frac{\frac{b\left(35b^2-116ac\right)\sqrt{ax^2+bx^3+cx^4}}{2c}-\frac{\frac{\left(256a^2c^2-460ab^2c+105b^4\right)\sqrt{ax^2+bx^3+cx^4}}{cx}-\frac{\int \frac{15b\left(7b^2-12ac\right)\left(b^2-4ac\right)x}{2\sqrt{cx^4+bx^3+ax^2}}dx}{c}}{4c}}{6c}}{80c}$$

$\downarrow$ 27

$$\frac{x^2(b+8cx)\sqrt{ax^2+bx^3+cx^4}}{40c}-\frac{\frac{x(7b^2-16ac)\sqrt{ax^2+bx^3+cx^4}}{3c}-\frac{\frac{b\left(35b^2-116ac\right)\sqrt{ax^2+bx^3+cx^4}}{2c}-\frac{\frac{\left(256a^2c^2-460ab^2c+105b^4\right)\sqrt{ax^2+bx^3+cx^4}}{cx}-\frac{15b\left(7b^2-12ac\right)\left(b^2-4ac\right)\int \frac{x}{\sqrt{cx^4+bx^3}}}{2c}}{4c}}{6c}}{80c}$$

$\downarrow$ 1961

$$\frac{x^2(b+8cx)\sqrt{ax^2+bx^3+cx^4}}{40c}-\frac{\frac{x(7b^2-16ac)\sqrt{ax^2+bx^3+cx^4}}{3c}-\frac{\frac{b(35b^2-116ac)\sqrt{ax^2+bx^3+cx^4}}{2c}-\frac{\frac{(256a^2c^2-460ab^2c+105b^4)\sqrt{ax^2+bx^3+cx^4}}{cx}-\frac{\frac{15bx(7b^2-12ac)(b^2-4ac)\sqrt{a+bx+cx^2}}{2c\sqrt{ax^2+bx^3+cx^4}}}{4c}}{6c}}{80c}$$

$\downarrow$ 1092

$$\frac{x^2(b+8cx)\sqrt{ax^2+bx^3+cx^4}}{40c}-\frac{\frac{x(7b^2-16ac)\sqrt{ax^2+bx^3+cx^4}}{3c}-\frac{\frac{b(35b^2-116ac)\sqrt{ax^2+bx^3+cx^4}}{2c}-\frac{\frac{(256a^2c^2-460ab^2c+105b^4)\sqrt{ax^2+bx^3+cx^4}}{cx}-\frac{\frac{15bx(7b^2-12ac)(b^2-4ac)\sqrt{a+bx+cx^2}}{c\sqrt{ax^2+bx^3}}}{4c}}{6c}}{80c}$$

$\downarrow$ 219

$$\frac{x^2(b+8cx)\sqrt{ax^2+bx^3+cx^4}}{40c}-\frac{\frac{x(7b^2-16ac)\sqrt{ax^2+bx^3+cx^4}}{3c}-\frac{\frac{b(35b^2-116ac)\sqrt{ax^2+bx^3+cx^4}}{2c}-\frac{\frac{(256a^2c^2-460ab^2c+105b^4)\sqrt{ax^2+bx^3+cx^4}}{cx}-\frac{\frac{15bx(7b^2-12ac)(b^2-4ac)\sqrt{a+bx+cx^2}}{2c^{3/2}\sqrt{ax^2+b}}}{4c}}{6c}}{80c}$$

input
```
Int[x^2*Sqrt[a*x^2 + b*x^3 + c*x^4],x]
```

output
```
(x^2*(b + 8*c*x)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(40*c) - (((7*b^2 - 16*a*c)*
x*Sqrt[a*x^2 + b*x^3 + c*x^4])/(3*c) - ((b*(35*b^2 - 116*a*c)*Sqrt[a*x^2 +
 b*x^3 + c*x^4])/(2*c) - (((105*b^4 - 460*a*b^2*c + 256*a^2*c^2)*Sqrt[a*x^
2 + b*x^3 + c*x^4])/(c*x) - (15*b*(7*b^2 - 12*a*c)*(b^2 - 4*a*c)*x*Sqrt[a
+ b*x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*
c^(3/2)*Sqrt[a*x^2 + b*x^3 + c*x^4]))/(4*c))/(6*c))/(80*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1961
```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Simp[x^(q/2)*(Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]/Sqrt[a
*x^q + b*x^n + c*x^(2*n - q)])   Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x
^(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] &&
 PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] ||
 EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))
```

rule 1966
```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[x^(m - n + q + 1)*(b*(n - q)*p + c*(m + p*q + (n - q)*
(2*p - 1) + 1)*x^(n - q))*((a*x^q + b*x^n + c*x^(2*n - q))^p/(c*(m + p*(2*n
 - q) + 1)*(m + p*q + (n - q)*(2*p - 1) + 1))), x] + Simp[(n - q)*(p/(c*(m
+ p*(2*n - q) + 1)*(m + p*q + (n - q)*(2*p - 1) + 1)))   Int[x^(m - (n - 2*
q))*Simp[(-a)*b*(m + p*q - n + q + 1) + (2*a*c*(m + p*q + (n - q)*(2*p - 1)
 + 1) - b^2*(m + p*q + (n - q)*(p - 1) + 1))*x^(n - q), x]*(a*x^q + b*x^n +
 c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] &
& PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[
p, 0] && RationalQ[m, q] && GtQ[m + p*q + 1, n - q] && NeQ[m + p*(2*n - q)
+ 1, 0] && NeQ[m + p*q + (n - q)*(2*p - 1) + 1, 0]
```

rule 1996
```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[B*x^(m - n + 1)*((a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1)/(c*(m + p*q + (n - q)*(2*p + 1) + 1))), x] -
Simp[1/(c*(m + p*q + (n - q)*(2*p + 1) + 1))   Int[x^(m - n + q)*Simp[a*B*(
m + p*q - n + q + 1) + (b*B*(m + p*q + (n - q)*p + 1) - A*c*(m + p*q + (n -
 q)*(2*p + 1) + 1))*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^p, x], x]
 /; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] &&  !Inte
gerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[p, -1] && LtQ[p, 0] &&
RationalQ[m, q] && GeQ[m + p*q, n - q - 1] && NeQ[m + p*q + (n - q)*(2*p +
1) + 1, 0]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.58

method	result
pseudoelliptic	$-\frac{2 ((-\frac{45}{32} a^{2} b \,c^{2}+\frac{75}{64} a \,b^{3} c -\frac{105}{512} b^{5}) \ln (2 \sqrt{c \,x^{2}+b x +a}\, \sqrt{c}+2 c x +b) +\sqrt{c \,x^{2}+b x +a}\, ((\frac{7}{32} b^{2} x^{2}+\frac{29}{32} a b x +a^{2}) c^{\frac{5}{2}}-\frac{115 (\frac{7 b x}{46}+a \cdots}{64} \cdots}{15 c^{\frac{9}{2}}}$
risch	$-\frac{(-384 c^{4} x^{4}-48 b \,c^{3} x^{3}-128 a \,c^{3} x^{2}+56 b^{2} c^{2} x^{2}+232 a b \,c^{2} x -70 b^{3} c x +256 a^{2} c^{2}-460 a \,b^{2} c +105 b^{4}) \sqrt{x^{2} (c \,x^{2}+b x +a)}}{1920 c^{4} x}+\frac{b (4 \cdots}{\cdots}$
default	$\frac{\sqrt{c \,x^{4}+b \,x^{3}+a \,x^{2}}\, (768 x^{2} (c \,x^{2}+b x +a)^{\frac{3}{2}} c^{\frac{9}{2}}-672 c^{\frac{7}{2}} (c \,x^{2}+b x +a)^{\frac{3}{2}} b x -512 c^{\frac{7}{2}} (c \,x^{2}+b x +a)^{\frac{3}{2}} a +720 c^{\frac{7}{2}} \sqrt{c \,x^{2}+b x +a}\, a b x + \cdots}{\cdots}$

input
```
int(x^2*(c*x^4+b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/15/c^(9/2)*((-45/32*a^2*b*c^2+75/64*a*b^3*c-105/512*b^5)*ln(2*(c*x^2+b*
x+a)^(1/2)*c^(1/2)+2*c*x+b)+(c*x^2+b*x+a)^(1/2)*((7/32*b^2*x^2+29/32*a*b*x
+a^2)*c^(5/2)-115/64*(7/46*b*x+a)*b^2*c^(3/2)-1/2*(3/8*b*x+a)*x^2*c^(7/2)-
3/2*c^(9/2)*x^4+105/256*c^(1/2)*b^4))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 390, normalized size of antiderivative = 1.70

$$\int x^2\sqrt{ax^2+bx^3+cx^4}\,dx$$
$$= \left[\frac{15\left(7b^5-40ab^3c+48a^2bc^2\right)\sqrt{c}x\log\left(-\frac{8c^2x^3+8bcx^2+4\sqrt{cx^4+bx^3+ax^2}(2cx+b)\sqrt{c}+(b^2+4ac)x}{x}\right)+4\left(384c^5x^4+\ldots\right.}{7\ldots}\right.$$
$$\left.-\frac{15\left(7b^5-40ab^3c+48a^2bc^2\right)\sqrt{-c}x\arctan\left(\frac{\sqrt{cx^4+bx^3+ax^2}(2cx+b)\sqrt{-c}}{2(c^2x^3+bcx^2+acx)}\right)-2\left(384c^5x^4+48bc^4x^3-105b^4\ldots\right.}{3840c^5x\ldots}\right]$$

input
```
integrate(x^2*(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/7680*(15*(7*b^5 - 40*a*b^3*c + 48*a^2*b*c^2)*sqrt(c)*x*log(-(8*c^2*x^3
+ 8*b*c*x^2 + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(c) + (b^2 + 4
*a*c)*x)/x) + 4*(384*c^5*x^4 + 48*b*c^4*x^3 - 105*b^4*c + 460*a*b^2*c^2 -
256*a^2*c^3 - 8*(7*b^2*c^3 - 16*a*c^4)*x^2 + 2*(35*b^3*c^2 - 116*a*b*c^3)*
x)*sqrt(c*x^4 + b*x^3 + a*x^2))/(c^5*x), -1/3840*(15*(7*b^5 - 40*a*b^3*c +
 48*a^2*b*c^2)*sqrt(-c)*x*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x +
b)*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x)) - 2*(384*c^5*x^4 + 48*b*c^4*x^3 -
 105*b^4*c + 460*a*b^2*c^2 - 256*a^2*c^3 - 8*(7*b^2*c^3 - 16*a*c^4)*x^2 +
2*(35*b^3*c^2 - 116*a*b*c^3)*x)*sqrt(c*x^4 + b*x^3 + a*x^2))/(c^5*x)]
```

Sympy [F]

$$\int x^2\sqrt{ax^2+bx^3+cx^4}\,dx=\int x^2\sqrt{x^2\left(a+bx+cx^2\right)}\,dx$$

input
```
integrate(x**2*(c*x**4+b*x**3+a*x**2)**(1/2),x)
```

output
```
Integral(x**2*sqrt(x**2*(a + b*x + c*x**2)), x)
```

Maxima [F]

$$\int x^2\sqrt{ax^2+bx^3+cx^4}\,dx=\int \sqrt{cx^4+bx^3+ax^2}x^2\,dx$$

input
```
integrate(x^2*(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^4 + b*x^3 + a*x^2)*x^2, x)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.20

$$\int x^2\sqrt{ax^2+bx^3+cx^4}\,dx = \frac{1}{1920}\sqrt{cx^2+bx+a}\left(2\left(4\left(6\left(8\,x\mathrm{sgn}(x)+\frac{b\mathrm{sgn}(x)}{c}\right)x-\frac{7\,b^2c^2\mathrm{sgn}(x)-16\,ac^3\mathrm{sgn}(x)}{c^4}\right)x+\frac{35\,b^3c\mathrm{sgn}(x)}{}\right.\right.$$
$$-\frac{(7\,b^5\mathrm{sgn}(x)-40\,ab^3c\mathrm{sgn}(x)+48\,a^2bc^2\mathrm{sgn}(x))\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)\sqrt{c}+b\right|\right)}{256\,c^{\frac{9}{2}}}$$
$$+\frac{\left(105\,b^5\log\left(\left|b-2\,\sqrt{a}\sqrt{c}\right|\right)-600\,ab^3c\log\left(\left|b-2\,\sqrt{a}\sqrt{c}\right|\right)+720\,a^2bc^2\log\left(\left|b-2\,\sqrt{a}\sqrt{c}\right|\right)+210\,\sqrt{a}b^4\right.}{3840\,c^{\frac{9}{2}}}$$

input
```
integrate(x^2*(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
1/1920*sqrt(c*x^2 + b*x + a)*(2*(4*(6*(8*x*sgn(x) + b*sgn(x)/c)*x - (7*b^2
*c^2*sgn(x) - 16*a*c^3*sgn(x))/c^4)*x + (35*b^3*c*sgn(x) - 116*a*b*c^2*sgn
(x))/c^4)*x - (105*b^4*sgn(x) - 460*a*b^2*c*sgn(x) + 256*a^2*c^2*sgn(x))/c
^4) - 1/256*(7*b^5*sgn(x) - 40*a*b^3*c*sgn(x) + 48*a^2*b*c^2*sgn(x))*log(a
bs(2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) + b))/c^(9/2) + 1/3840*(1
05*b^5*log(abs(b - 2*sqrt(a)*sqrt(c))) - 600*a*b^3*c*log(abs(b - 2*sqrt(a)
*sqrt(c))) + 720*a^2*b*c^2*log(abs(b - 2*sqrt(a)*sqrt(c))) + 210*sqrt(a)*b
^4*sqrt(c) - 920*a^(3/2)*b^2*c^(3/2) + 512*a^(5/2)*c^(5/2))*sgn(x)/c^(9/2)
```

Mupad [F(-1)]

Timed out.

$$\int x^2\sqrt{ax^2+bx^3+cx^4}\,dx=\int x^2\,\sqrt{c\,x^4+b\,x^3+a\,x^2}\,dx$$

input
```
int(x^2*(a*x^2 + b*x^3 + c*x^4)^(1/2),x)
```

output
```
int(x^2*(a*x^2 + b*x^3 + c*x^4)^(1/2), x)
```

Reduce [F]

$$\int x^2\sqrt{ax^2+bx^3+cx^4}\,dx=\int x^2\sqrt{c\,x^4+b\,x^3+a\,x^2}dx$$

input
```
int(x^2*(c*x^4+b*x^3+a*x^2)^(1/2),x)
```

output
```
int(x^2*(c*x^4+b*x^3+a*x^2)^(1/2),x)
```

3.47 $\int x\sqrt{ax^2+bx^3+cx^4}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 177

$$\int x\sqrt{ax^2+bx^3+cx^4}\,dx = -\frac{(5b^2-12ac)\sqrt{ax^2+bx^3+cx^4}}{96c^2} + \frac{b(15b^2-52ac)\sqrt{ax^2+bx^3+cx^4}}{192c^3x} + \frac{x(b+6cx)\sqrt{ax^2+bx^3+cx^4}}{24c} - \frac{(b^2-4ac)(5b^2-4ac)\,\text{arctanh}\left(\frac{x(b+2cx)}{2\sqrt{c}\sqrt{ax^2+bx^3+cx^4}}\right)}{128c^{7/2}}$$

output

```
-1/96*(-12*a*c+5*b^2)*(c*x^4+b*x^3+a*x^2)^(1/2)/c^2+1/192*b*(-52*a*c+15*b^
2)*(c*x^4+b*x^3+a*x^2)^(1/2)/c^3/x+1/24*x*(6*c*x+b)*(c*x^4+b*x^3+a*x^2)^(1
/2)/c-1/128*(-4*a*c+b^2)*(-4*a*c+5*b^2)*arctanh(1/2*x*(2*c*x+b)/c^(1/2)/(c
*x^4+b*x^3+a*x^2)^(1/2))/c^(7/2)
```

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.85

$$\int x\sqrt{ax^2+bx^3+cx^4}\,dx$$
$$=\frac{2\sqrt{c}x(a+x(b+cx))\left(15b^3-10b^2cx+24c^2x(a+2cx^2)+b(-52ac+8c^2x^2)\right)+3(5b^4-24ab^2c+16a^2c^2}{384c^{7/2}\sqrt{x^2(a+x(b+cx))}}$$

input
```
Integrate[x*Sqrt[a*x^2 + b*x^3 + c*x^4],x]
```

output
```
(2*Sqrt[c]*x*(a + x*(b + c*x))*(15*b^3 - 10*b^2*c*x + 24*c^2*x*(a + 2*c*x^
2) + b*(-52*a*c + 8*c^2*x^2)) + 3*(5*b^4 - 24*a*b^2*c + 16*a^2*c^2)*x*Sqrt
[a + x*(b + c*x)]*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]])/(384*c
^(7/2)*Sqrt[x^2*(a + x*(b + c*x))])
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.23, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {1966, 27, 1996, 27, 1996, 27, 1961, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{ax^2+bx^3+cx^4}\,dx$$

$$\Big\downarrow \ 1966$$

$$\frac{\int -\frac{x^2\left(4ab+(5b^2-12ac)x\right)}{2\sqrt{cx^4+bx^3+ax^2}}dx}{24c}+\frac{x(b+6cx)\sqrt{ax^2+bx^3+cx^4}}{24c}$$

$$\Big\downarrow \ 27$$

$$\frac{x(b+6cx)\sqrt{ax^2+bx^3+cx^4}}{24c}-\frac{\int \frac{x^2\left(4ab+(5b^2-12ac)x\right)}{\sqrt{cx^4+bx^3+ax^2}}dx}{48c}$$

$$\Big\downarrow \ 1996$$

$$\frac{x(b+6cx)\sqrt{ax^2+bx^3+cx^4}}{24c}-\frac{\frac{(5b^2-12ac)\sqrt{ax^2+bx^3+cx^4}}{2c}-\frac{\int\frac{x\left(2a\left(5b^2-12ac\right)+b\left(15b^2-52ac\right)x\right)}{2\sqrt{cx^4+bx^3+ax^2}}dx}{2c}}{48c}$$

$$\Big\downarrow \; 27$$

$$\frac{x(b+6cx)\sqrt{ax^2+bx^3+cx^4}}{24c}-\frac{\frac{(5b^2-12ac)\sqrt{ax^2+bx^3+cx^4}}{2c}-\frac{\int\frac{x\left(2a\left(5b^2-12ac\right)+b\left(15b^2-52ac\right)x\right)}{\sqrt{cx^4+bx^3+ax^2}}dx}{4c}}{48c}$$

$$\Big\downarrow \; 1996$$

$$\frac{x(b+6cx)\sqrt{ax^2+bx^3+cx^4}}{24c}-\frac{\frac{(5b^2-12ac)\sqrt{ax^2+bx^3+cx^4}}{2c}-\frac{\frac{b\left(15b^2-52ac\right)\sqrt{ax^2+bx^3+cx^4}}{cx}-\frac{\int\frac{3\left(b^2-4ac\right)\left(5b^2-4ac\right)x}{2\sqrt{cx^4+bx^3+ax^2}}dx}{c}}{4c}}{48c}$$

$$\Big\downarrow \; 27$$

$$\frac{x(b+6cx)\sqrt{ax^2+bx^3+cx^4}}{24c}-\frac{\frac{(5b^2-12ac)\sqrt{ax^2+bx^3+cx^4}}{2c}-\frac{\frac{b\left(15b^2-52ac\right)\sqrt{ax^2+bx^3+cx^4}}{cx}-\frac{3\left(b^2-4ac\right)\left(5b^2-4ac\right)\int\frac{x}{\sqrt{cx^4+bx^3+ax^2}}dx}{2c}}{4c}}{48c}$$

$$\Big\downarrow \; 1961$$

$$\frac{x(b+6cx)\sqrt{ax^2+bx^3+cx^4}}{24c}-\frac{\frac{(5b^2-12ac)\sqrt{ax^2+bx^3+cx^4}}{2c}-\frac{\frac{b\left(15b^2-52ac\right)\sqrt{ax^2+bx^3+cx^4}}{cx}-\frac{3x\left(b^2-4ac\right)\left(5b^2-4ac\right)\sqrt{a+bx+cx^2}\int\frac{1}{\sqrt{cx^2+bx+a}}dx}{2c\sqrt{ax^2+bx^3+cx^4}}}{4c}}{48c}$$

$$\Big\downarrow \; 1092$$

$$\frac{x(b+6cx)\sqrt{ax^2+bx^3+cx^4}}{24c}-\frac{\frac{(5b^2-12ac)\sqrt{ax^2+bx^3+cx^4}}{2c}-\frac{\frac{b\left(15b^2-52ac\right)\sqrt{ax^2+bx^3+cx^4}}{cx}-\frac{3x\left(b^2-4ac\right)\left(5b^2-4ac\right)\sqrt{a+bx+cx^2}\int\frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{c\sqrt{ax^2+bx^3+cx^4}}}{4c}}{48c}$$

$$\Big\downarrow \; 219$$

$$\frac{x(b+6cx)\sqrt{ax^2+bx^3+cx^4}}{24c}-\frac{\frac{(5b^2-12ac)\sqrt{ax^2+bx^3+cx^4}}{2c}-\frac{\frac{b\left(15b^2-52ac\right)\sqrt{ax^2+bx^3+cx^4}}{cx}-\frac{3x\left(b^2-4ac\right)\left(5b^2-4ac\right)\sqrt{a+bx+cx^2}\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2c^{3/2}\sqrt{ax^2+bx^3+cx^4}}}{4c}}{48c}$$

input
```
Int[x*Sqrt[a*x^2 + b*x^3 + c*x^4],x]
```

output
```
(x*(b + 6*c*x)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(24*c) - (((5*b^2 - 12*a*c)*Sq
rt[a*x^2 + b*x^3 + c*x^4])/(2*c) - ((b*(15*b^2 - 52*a*c)*Sqrt[a*x^2 + b*x^
3 + c*x^4])/(c*x) - (3*(b^2 - 4*a*c)*(5*b^2 - 4*a*c)*x*Sqrt[a + b*x + c*x^
2]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*c^(3/2)*Sqrt
[a*x^2 + b*x^3 + c*x^4]))/(4*c))/(48*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1961
```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Simp[x^(q/2)*(Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]/Sqrt[a
*x^q + b*x^n + c*x^(2*n - q)])   Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x
^(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] &&
 PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] ||
 EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))
```

rule 1966
```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[x^(m - n + q + 1)*(b*(n - q)*p + c*(m + p*q + (n - q)*
(2*p - 1) + 1)*x^(n - q))*((a*x^q + b*x^n + c*x^(2*n - q))^p/(c*(m + p*(2*n
 - q) + 1)*(m + p*q + (n - q)*(2*p - 1) + 1))), x] + Simp[(n - q)*(p/(c*(m
+ p*(2*n - q) + 1)*(m + p*q + (n - q)*(2*p - 1) + 1)))   Int[x^(m - (n - 2*
q))*Simp[(-a)*b*(m + p*q - n + q + 1) + (2*a*c*(m + p*q + (n - q)*(2*p - 1)
 + 1) - b^2*(m + p*q + (n - q)*(p - 1) + 1))*x^(n - q), x]*(a*x^q + b*x^n +
 c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] &
& PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[
p, 0] && RationalQ[m, q] && GtQ[m + p*q + 1, n - q] && NeQ[m + p*(2*n - q)
+ 1, 0] && NeQ[m + p*q + (n - q)*(2*p - 1) + 1, 0]
```

rule 1996
```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[B*x^(m - n + 1)*((a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1)/(c*(m + p*q + (n - q)*(2*p + 1) + 1))), x] -
Simp[1/(c*(m + p*q + (n - q)*(2*p + 1) + 1))   Int[x^(m - n + q)*Simp[a*B*(
m + p*q - n + q + 1) + (b*B*(m + p*q + (n - q)*p + 1) - A*c*(m + p*q + (n -
 q)*(2*p + 1) + 1))*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^p, x], x]
 /; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] &&  !Inte
gerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[p, -1] && LtQ[p, 0] &&
RationalQ[m, q] && GeQ[m + p*q, n - q - 1] && NeQ[m + p*q + (n - q)*(2*p +
1) + 1, 0]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.60

method	result
pseudoelliptic	$-\frac{\left(a^{2} c^{2}-\frac{3}{2} a \,b^{2} c+\frac{5}{16} b^{4}\right) \ln \left(2 \sqrt{c \,x^{2}+b x+a}\, \sqrt{c}+2 c x+b \right)+\frac{13 \left(b \left(\frac{5 b x}{26}+a \right) c^{\frac{3}{2}}-\frac{6 x \left(\frac{b x}{3}+a \right) c^{\frac{5}{2}}}{13}-\frac{15 \sqrt{c}\, b^{3}}{52}-\frac{12 c^{\frac{7}{2}} x^{3}}{13}\right) \sqrt{c \,x^{2}+b x+a}}{6}}{8 c^{\frac{7}{2}}}$
risch	$-\frac{\left(-48 c^{3} x^{3}-8 b \,c^{2} x^{2}-24 a \,c^{2} x +10 b^{2} c x +52 a b c -15 b^{3}\right) \sqrt{x^{2} \left(c \,x^{2}+b x +a \right)}}{192 c^{3} x}-\frac{\left(16 a^{2} c^{2}-24 a \,b^{2} c +5 b^{4}\right) \ln \left(\frac{\frac{b}{2}+c x}{\sqrt{c}}+\sqrt{c \,x^{2}+b}\right.}{128 c^{\frac{7}{2}} x \sqrt{c \,x^{2}+b x +a}}$
default	$\frac{\sqrt{c \,x^{4}+b \,x^{3}+a \,x^{2}}\, \left(96 x \left(c \,x^{2}+b x +a \right)^{\frac{3}{2}} c^{\frac{7}{2}}-48 c^{\frac{7}{2}} \sqrt{c \,x^{2}+b x +a}\, a x -80 c^{\frac{5}{2}} \left(c \,x^{2}+b x +a \right)^{\frac{3}{2}} b +60 c^{\frac{5}{2}} \sqrt{c \,x^{2}+b x +a}\, b^{2} x -24 c^{\frac{5}{2}} \sqrt{c}\right.}{}$

input
```
int(x*(c*x^4+b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8/c^(7/2)*((a^2*c^2-3/2*a*b^2*c+5/16*b^4)*ln(2*(c*x^2+b*x+a)^(1/2)*c^(1
/2)+2*c*x+b)+13/6*(b*(5/26*b*x+a)*c^(3/2)-6/13*x*(1/3*b*x+a)*c^(5/2)-15/52
*c^(1/2)*b^3-12/13*c^(7/2)*x^3)*(c*x^2+b*x+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.84

$$\int x\sqrt{ax^2+bx^3+cx^4}\,dx = \left[\frac{3\,(5\,b^4-24\,ab^2c+16\,a^2c^2)\sqrt{c}x\log\left(-\frac{8\,c^2x^3+8\,bcx^2-4\,\sqrt{cx^4+bx^3+ax^2}(2\,cx+b)\sqrt{c}+(b^2+4\,ac)x}{x}\right)+4\,(48\,c^4x^3+8\,bc^3 \cdots}{768\,c^4x}\right.$$

input
```
integrate(x*(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/768*(3*(5*b^4 - 24*a*b^2*c + 16*a^2*c^2)*sqrt(c)*x*log(-(8*c^2*x^3 + 8*
b*c*x^2 - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(c) + (b^2 + 4*a*c
)*x)/x) + 4*(48*c^4*x^3 + 8*b*c^3*x^2 + 15*b^3*c - 52*a*b*c^2 - 2*(5*b^2*c
^2 - 12*a*c^3)*x)*sqrt(c*x^4 + b*x^3 + a*x^2))/(c^4*x), 1/384*(3*(5*b^4 -
24*a*b^2*c + 16*a^2*c^2)*sqrt(-c)*x*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)
*(2*c*x + b)*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x)) + 2*(48*c^4*x^3 + 8*b*c
^3*x^2 + 15*b^3*c - 52*a*b*c^2 - 2*(5*b^2*c^2 - 12*a*c^3)*x)*sqrt(c*x^4 +
b*x^3 + a*x^2))/(c^4*x)]
```

Sympy [F]

$$\int x\sqrt{ax^2+bx^3+cx^4}\,dx = \int x\sqrt{x^2\,(a+bx+cx^2)}\,dx$$

input
```
integrate(x*(c*x**4+b*x**3+a*x**2)**(1/2),x)
```

output
```
Integral(x*sqrt(x**2*(a + b*x + c*x**2)), x)
```

Maxima [F]

$$\int x\sqrt{ax^2+bx^3+cx^4}\,dx=\int \sqrt{cx^4+bx^3+ax^2}x\,dx$$

input
```
integrate(x*(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^4 + b*x^3 + a*x^2)*x, x)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.25

$$\int x\sqrt{ax^2+bx^3+cx^4}\,dx$$
$$=\frac{1}{192}\sqrt{cx^2+bx+a}\left(2\left(4\left(6\,x\mathrm{sgn}(x)+\frac{b\mathrm{sgn}(x)}{c}\right)x-\frac{5\,b^2c\mathrm{sgn}(x)-12\,ac^2\mathrm{sgn}(x)}{c^3}\right)x+\frac{15\,b^3\mathrm{sgn}(x)-52}{c^3}\right.$$
$$+\frac{\left(5\,b^4\mathrm{sgn}(x)-24\,ab^2c\mathrm{sgn}(x)+16\,a^2c^2\mathrm{sgn}(x)\right)\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)\sqrt{c}+b\right|\right)}{128\,c^{\frac{7}{2}}}$$
$$-\frac{\left(15\,b^4\log\left(\left|b-2\sqrt{a}\sqrt{c}\right|\right)-72\,ab^2c\log\left(\left|b-2\sqrt{a}\sqrt{c}\right|\right)+48\,a^2c^2\log\left(\left|b-2\sqrt{a}\sqrt{c}\right|\right)+30\sqrt{a}b^3\sqrt{c}-\right.}{384\,c^{\frac{7}{2}}}$$

input
```
integrate(x*(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
1/192*sqrt(c*x^2 + b*x + a)*(2*(4*(6*x*sgn(x) + b*sgn(x)/c)*x - (5*b^2*c*s
gn(x) - 12*a*c^2*sgn(x))/c^3)*x + (15*b^3*sgn(x) - 52*a*b*c*sgn(x))/c^3) +
 1/128*(5*b^4*sgn(x) - 24*a*b^2*c*sgn(x) + 16*a^2*c^2*sgn(x))*log(abs(2*(s
qrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) + b))/c^(7/2) - 1/384*(15*b^4*lo
g(abs(b - 2*sqrt(a)*sqrt(c))) - 72*a*b^2*c*log(abs(b - 2*sqrt(a)*sqrt(c)))
 + 48*a^2*c^2*log(abs(b - 2*sqrt(a)*sqrt(c))) + 30*sqrt(a)*b^3*sqrt(c) - 1
04*a^(3/2)*b*c^(3/2))*sgn(x)/c^(7/2)
```

Mupad [F(-1)]

Timed out.

$$\int x\sqrt{ax^2+bx^3+cx^4}\,dx=\int x\,\sqrt{c\,x^4+b\,x^3+a\,x^2}\,dx$$

input
```
int(x*(a*x^2 + b*x^3 + c*x^4)^(1/2),x)
```

output
```
int(x*(a*x^2 + b*x^3 + c*x^4)^(1/2), x)
```

Reduce [F]

$$\int x\sqrt{ax^2+bx^3+cx^4}\,dx=\int x\sqrt{c\,x^4+b\,x^3+a\,x^2}dx$$

input
```
int(x*(c*x^4+b*x^3+a*x^2)^(1/2),x)
```

output
```
int(x*(c*x^4+b*x^3+a*x^2)^(1/2),x)
```

3.48 $\int \sqrt{ax^2+bx^3+cx^4}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 149

$$\int \sqrt{ax^2+bx^3+cx^4}\,dx = \frac{b\sqrt{ax^2+bx^3+cx^4}}{12c} - \frac{(3b^2-8ac)\sqrt{ax^2+bx^3+cx^4}}{24c^2x} + \frac{1}{3}x\sqrt{ax^2+bx^3+cx^4} + \frac{b(b^2-4ac)\,\text{arctanh}\left(\frac{x(b+2cx)}{2\sqrt{c}\sqrt{ax^2+bx^3+cx^4}}\right)}{16c^{5/2}}$$

output
```
1/12*b*(c*x^4+b*x^3+a*x^2)^(1/2)/c-1/24*(-8*a*c+3*b^2)*(c*x^4+b*x^3+a*x^2)
^(1/2)/c^2/x+1/3*x*(c*x^4+b*x^3+a*x^2)^(1/2)+1/16*b*(-4*a*c+b^2)*arctanh(1
/2*x*(2*c*x+b)/c^(1/2)/(c*x^4+b*x^3+a*x^2)^(1/2))/c^(5/2)
```

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.81

$$\int \sqrt{ax^2+bx^3+cx^4}\,dx = \frac{2\sqrt{c}x(a+x(b+cx))\left(-3b^2+2bcx+8c(a+cx^2)\right)-3(b^3-4abc)\,x\sqrt{a+x(b+cx)}\log\left(c^2\left(b+2cx-2\right.\right.}{48c^{5/2}\sqrt{x^2(a+x(b+cx))}}$$

input
```
Integrate[Sqrt[a*x^2 + b*x^3 + c*x^4],x]
```

output
```
(2*Sqrt[c]*x*(a + x*(b + c*x))*(-3*b^2 + 2*b*c*x + 8*c*(a + c*x^2)) - 3*(b
^3 - 4*a*b*c)*x*Sqrt[a + x*(b + c*x)]*Log[c^2*(b + 2*c*x - 2*Sqrt[c]*Sqrt[
a + x*(b + c*x)]])]/(48*c^(5/2)*Sqrt[x^2*(a + x*(b + c*x))])
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1950, 1160, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{ax^2+bx^3+cx^4}\,dx$$

$$\Big\downarrow \text{1950}$$

$$\frac{\sqrt{ax^2+bx^3+cx^4}\int x\sqrt{cx^2+bx+a}dx}{x\sqrt{a+bx+cx^2}}$$

$$\Big\downarrow \text{1160}$$

$$\frac{\sqrt{ax^2+bx^3+cx^4}\left(\frac{(a+bx+cx^2)^{3/2}}{3c}-\frac{b\int \sqrt{cx^2+bx+a}dx}{2c}\right)}{x\sqrt{a+bx+cx^2}}$$

$$\Big\downarrow \text{1087}$$

$$\frac{\sqrt{ax^2+bx^3+cx^4}\left(\frac{(a+bx+cx^2)^{3/2}}{3c}-\frac{b\left(\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4c}-\frac{(b^2-4ac)\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{8c}\right)}{2c}\right)}{x\sqrt{a+bx+cx^2}}$$

$$\Big\downarrow \text{1092}$$

$$\frac{\sqrt{ax^2+bx^3+cx^4}\left(\frac{(a+bx+cx^2)^{3/2}}{3c}-\frac{b\left(\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4c}-\frac{(b^2-4ac)\int\frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{4c}\right)}{2c}\right)}{x\sqrt{a+bx+cx^2}}$$

$$\Big\downarrow \ 219$$

$$\frac{\sqrt{ax^2+bx^3+cx^4}\left(\frac{(a+bx+cx^2)^{3/2}}{3c}-\frac{b\left(\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4c}-\frac{(b^2-4ac)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{3/2}}\right)}{2c}\right)}{x\sqrt{a+bx+cx^2}}$$

input
```
Int[Sqrt[a*x^2 + b*x^3 + c*x^4],x]
```

output
```
(Sqrt[a*x^2 + b*x^3 + c*x^4]*((a + b*x + c*x^2)^(3/2)/(3*c) - (b*(((b + 2*
c*x)*Sqrt[a + b*x + c*x^2])/(4*c) - ((b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*
Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(8*c^(3/2))))/(2*c)))/(x*Sqrt[a + b*x + c
*x^2])
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1950
```
Int[Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)], x_Symbol]
 :> Simp[Sqrt[a*x^q + b*x^n + c*x^(2*n - q)]/(x^(q/2)*Sqrt[a + b*x^(n - q)
+ c*x^(2*(n - q))])   Int[x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))],
x], x] /; FreeQ[{a, b, c, n, q}, x] && EqQ[r, 2*n - q] && PosQ[n - q]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.81

method	result
risch	$\frac{\left(8c^{2}x^{2}+2bcx+8ac-3b^{2}\right)\sqrt{x^{2}\left(c\,x^{2}+bx+a\right)}}{24c^{2}x}-\frac{b\left(4ac-b^{2}\right)\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^{2}+bx+a}\right)\sqrt{x^{2}\left(c\,x^{2}+bx+a\right)}}{16c^{\frac{5}{2}}x\sqrt{c\,x^{2}+bx+a}}$
pseudoelliptic	$\frac{16x^{2}\sqrt{c\,x^{2}+bx+a}\,c^{\frac{5}{2}}+4c^{\frac{3}{2}}\sqrt{c\,x^{2}+bx+a}\,bx+16a\,c^{\frac{3}{2}}\sqrt{c\,x^{2}+bx+a}-6\sqrt{c}\,\sqrt{c\,x^{2}+bx+a}\,b^{2}-12\ln\left(2\sqrt{c\,x^{2}+bx+a}\,\sqrt{c}+2cx+b\right)}{48c^{\frac{5}{2}}}$
default	$\frac{\sqrt{c\,x^{4}+b\,x^{3}+a\,x^{2}}\left(16\left(c\,x^{2}+bx+a\right)^{\frac{3}{2}}c^{\frac{5}{2}}-12c^{\frac{5}{2}}\sqrt{c\,x^{2}+bx+a}\,bx-6c^{\frac{3}{2}}\sqrt{c\,x^{2}+bx+a}\,b^{2}-12\ln\left(\frac{2\sqrt{c\,x^{2}+bx+a}\,\sqrt{c}+2cx+b}{2\sqrt{c}}\right)ab\,c\right.}{48x\sqrt{c\,x^{2}+bx+a}\,c^{\frac{7}{2}}}$

input
```
int((c*x^4+b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/24*(8*c^2*x^2+2*b*c*x+8*a*c-3*b^2)/c^2*(x^2*(c*x^2+b*x+a))^(1/2)/x-1/16*
b*(4*a*c-b^2)/c^(5/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*(x^2*(c*
x^2+b*x+a))^(1/2)/x/(c*x^2+b*x+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.74

$$\int \sqrt{ax^2 + bx^3 + cx^4}\, dx$$
$$= \left[-\frac{3\,(b^3 - 4\,abc)\sqrt{c}x \log\left(-\frac{8\,c^2x^3 + 8\,bcx^2 - 4\,\sqrt{cx^4 + bx^3 + ax^2}(2\,cx + b)\sqrt{c} + (b^2 + 4\,ac)x}{x}\right) - 4\,(8\,c^3x^2 + 2\,bc^2x - 3\,b^2c + 8}{96\,c^3x} \right.$$
$$-\frac{3\,(b^3 - 4\,abc)\sqrt{-c}x \arctan\left(\frac{\sqrt{cx^4 + bx^3 + ax^2}(2\,cx + b)\sqrt{-c}}{2\,(c^2x^3 + bcx^2 + acx)}\right) - 2\,(8\,c^3x^2 + 2\,bc^2x - 3\,b^2c + 8\,ac^2)\sqrt{cx^4 + bx^3 -}{48\,c^3x}$$

input
```
integrate((c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
[-1/96*(3*(b^3 - 4*a*b*c)*sqrt(c)*x*log(-(8*c^2*x^3 + 8*b*c*x^2 - 4*sqrt(c
*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(c) + (b^2 + 4*a*c)*x)/x) - 4*(8*c^3
*x^2 + 2*b*c^2*x - 3*b^2*c + 8*a*c^2)*sqrt(c*x^4 + b*x^3 + a*x^2))/(c^3*x)
, -1/48*(3*(b^3 - 4*a*b*c)*sqrt(-c)*x*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^
2)*(2*c*x + b)*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x)) - 2*(8*c^3*x^2 + 2*b*
c^2*x - 3*b^2*c + 8*a*c^2)*sqrt(c*x^4 + b*x^3 + a*x^2))/(c^3*x)]
```

Sympy [F]

$$\int \sqrt{ax^2 + bx^3 + cx^4}\, dx = \int \sqrt{ax^2 + bx^3 + cx^4}\, dx$$

input
```
integrate((c*x**4+b*x**3+a*x**2)**(1/2),x)
```

output
```
Integral(sqrt(a*x**2 + b*x**3 + c*x**4), x)
```

Maxima [F]

$$\int \sqrt{ax^2 + bx^3 + cx^4}\,dx = \int \sqrt{cx^4 + bx^3 + ax^2}\,dx$$

input
```
integrate((c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^4 + b*x^3 + a*x^2), x)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.07

$$\int \sqrt{ax^2 + bx^3 + cx^4}\,dx$$
$$= \frac{1}{24}\sqrt{cx^2 + bx + a}\left(2\left(4\,x\mathrm{sgn}(x) + \frac{b\mathrm{sgn}(x)}{c}\right)x - \frac{3\,b^2\mathrm{sgn}(x) - 8\,ac\mathrm{sgn}(x)}{c^2}\right)$$
$$- \frac{(b^3\mathrm{sgn}(x) - 4\,abc\mathrm{sgn}(x))\log\left(\left|2\left(\sqrt{c}x - \sqrt{cx^2 + bx + a}\right)\sqrt{c} + b\right|\right)}{16\,c^{\frac{5}{2}}}$$
$$+ \frac{\left(3\,b^3\log\left(\left|b - 2\sqrt{a}\sqrt{c}\right|\right) - 12\,abc\log\left(\left|b - 2\sqrt{a}\sqrt{c}\right|\right) + 6\sqrt{a}b^2\sqrt{c} - 16\,a^{\frac{3}{2}}c^{\frac{3}{2}}\right)\mathrm{sgn}(x)}{48\,c^{\frac{5}{2}}}$$

input
```
integrate((c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
1/24*sqrt(c*x^2 + b*x + a)*(2*(4*x*sgn(x) + b*sgn(x)/c)*x - (3*b^2*sgn(x)
- 8*a*c*sgn(x))/c^2) - 1/16*(b^3*sgn(x) - 4*a*b*c*sgn(x))*log(abs(2*(sqrt(
c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) + b))/c^(5/2) + 1/48*(3*b^3*log(abs(
b - 2*sqrt(a)*sqrt(c))) - 12*a*b*c*log(abs(b - 2*sqrt(a)*sqrt(c))) + 6*sqr
t(a)*b^2*sqrt(c) - 16*a^(3/2)*c^(3/2))*sgn(x)/c^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{ax^2 + bx^3 + cx^4}\,dx = \int \sqrt{c\,x^4 + b\,x^3 + a\,x^2}\,dx$$

input
```
int((a*x^2 + b*x^3 + c*x^4)^(1/2),x)
```

output
```
int((a*x^2 + b*x^3 + c*x^4)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.09

$$\int \sqrt{ax^2 + bx^3 + cx^4}\,dx$$
$$= \frac{16\sqrt{c\,x^2 + bx + a}\,a\,c^2 - 6\sqrt{c\,x^2 + bx + a}\,b^2c + 4\sqrt{c\,x^2 + bx + a}\,b\,c^2x + 16\sqrt{c\,x^2 + bx + a}\,c^3x^2 - 12\sqrt{c}\,\mathrm{lo}}{48c^3}$$

input
```
int((c*x^4+b*x^3+a*x^2)^(1/2),x)
```

output
```
(16*sqrt(a + b*x + c*x**2)*a*c**2 - 6*sqrt(a + b*x + c*x**2)*b**2*c + 4*sq
rt(a + b*x + c*x**2)*b*c**2*x + 16*sqrt(a + b*x + c*x**2)*c**3*x**2 - 12*s
qrt(c)*log((2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)/sqrt(4*a*c - b**
2))*a*b*c + 3*sqrt(c)*log((2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)/s
qrt(4*a*c - b**2))*b**3)/(48*c**3)
```

3.49 $\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 91

$$\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x}\,dx = \frac{(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{4cx} - \frac{(b^2-4ac)\,\text{arctanh}\left(\frac{x(b+2cx)}{2\sqrt{c}\sqrt{ax^2+bx^3+cx^4}}\right)}{8c^{3/2}}$$

output
```
1/4*(2*c*x+b)*(c*x^4+b*x^3+a*x^2)^(1/2)/c/x-1/8*(-4*a*c+b^2)*arctanh(1/2*x
*(2*c*x+b)/c^(1/2)/(c*x^4+b*x^3+a*x^2)^(1/2))/c^(3/2)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.04

$$\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x}\,dx = \frac{\sqrt{x^2(a+x(b+cx))}\left(\sqrt{c}(b+2cx)+\frac{(b^2-4ac)\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a}-\sqrt{a+x(b+cx)}}\right)}{\sqrt{a+x(b+cx)}}\right)}{4c^{3/2}x}$$

input
```
Integrate[Sqrt[a*x^2 + b*x^3 + c*x^4]/x,x]
```

output
```
(Sqrt[x^2*(a + x*(b + c*x))]*(Sqrt[c]*(b + 2*c*x) + ((b^2 - 4*a*c)*ArcTanh
[(Sqrt[c]*x)/(Sqrt[a] - Sqrt[a + x*(b + c*x)])])/Sqrt[a + x*(b + c*x)]))/(
4*c^(3/2)*x)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.31, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1965, 1961, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x}\,dx$$

$$\Big\downarrow \text{1965}$$

$$\frac{(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{4cx}-\frac{\left(b^2-4ac\right)\int \frac{x}{\sqrt{cx^4+bx^3+ax^2}}dx}{8c}$$

$$\Big\downarrow \text{1961}$$

$$\frac{(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{4cx}-\frac{x\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{8c\sqrt{ax^2+bx^3+cx^4}}$$

$$\Big\downarrow \text{1092}$$

$$\frac{(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{4cx}-\frac{x\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\int \frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{4c\sqrt{ax^2+bx^3+cx^4}}$$

$$\Big\downarrow \text{219}$$

$$\frac{(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{4cx}-\frac{x\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{3/2}\sqrt{ax^2+bx^3+cx^4}}$$

input
```
Int[Sqrt[a*x^2 + b*x^3 + c*x^4]/x,x]
```

output
```
((b + 2*c*x)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(4*c*x) - ((b^2 - 4*a*c)*x*Sqrt[
a + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(
8*c^(3/2)*Sqrt[a*x^2 + b*x^3 + c*x^4])
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1961
```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Simp[x^(q/2)*(Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]/Sqrt[a
*x^q + b*x^n + c*x^(2*n - q)])   Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x
^(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] &&
 PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] ||
 EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))
```

rule 1965
```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[x^(m - n + q + 1)*(b + 2*c*x^(n - q))*((a*x^q + b*x^n
+ c*x^(2*n - q))^p/(2*c*(n - q)*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*
c*(2*p + 1)))   Int[x^(m + q)*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x],
x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] &&  !IntegerQ[p
] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && E
qQ[m + p*q + 1, n - q]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.10

method	result
pseudoelliptic	$\frac{4\sqrt{c\,x^2+bx+a}\,c^{\frac{3}{2}}x+4\ln\left(2\sqrt{c\,x^2+bx+a}\,\sqrt{c}+2cx+b\right)ac-\ln\left(2\sqrt{c\,x^2+bx+a}\,\sqrt{c}+2cx+b\right)b^2+2b\sqrt{c\,x^2+bx+a}\,\sqrt{c}}{8c^{\frac{3}{2}}}$
risch	$\frac{(2cx+b)\sqrt{x^2(c\,x^2+bx+a)}}{4cx}+\frac{(4ac-b^2)\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)\sqrt{x^2(c\,x^2+bx+a)}}{8c^{\frac{3}{2}}x\sqrt{c\,x^2+bx+a}}$
default	$\frac{\sqrt{c\,x^4+b\,x^3+a\,x^2}\left(4c^{\frac{5}{2}}\sqrt{c\,x^2+bx+a}\,x+2c^{\frac{3}{2}}\sqrt{c\,x^2+bx+a}\,b+4\ln\left(\frac{2\sqrt{c\,x^2+bx+a}\,\sqrt{c}+2cx+b}{2\sqrt{c}}\right)a\,c^2-\ln\left(\frac{2\sqrt{c\,x^2+bx+a}\,\sqrt{c}+2cx}{2\sqrt{c}}\right.\right.}{8c^{\frac{5}{2}}\sqrt{c\,x^2+bx+a}\,x}$

input
```
int((c*x^4+b*x^3+a*x^2)^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
1/8/c^(3/2)*(4*(c*x^2+b*x+a)^(1/2)*c^(3/2)*x+4*ln(2*(c*x^2+b*x+a)^(1/2)*c^
(1/2)+2*c*x+b)*a*c-ln(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)*b^2+2*b*(c*x^
2+b*x+a)^(1/2)*c^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 220, normalized size of antiderivative = 2.42

$$\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x}\,dx$$
$$=\left[-\frac{(b^2-4\,ac)\sqrt{c}x\log\left(-\frac{8\,c^2x^3+8\,bcx^2+4\,\sqrt{cx^4+bx^3+ax^2}(2\,cx+b)\sqrt{c}+(b^2+4\,ac)x}{x}\right)-4\,\sqrt{cx^4+bx^3+ax^2}(2\,c^2x+bc}{16\,c^2x}\right.$$

input
```
integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x,x, algorithm="fricas")
```

output
```
[-1/16*((b^2 - 4*a*c)*sqrt(c)*x*log(-(8*c^2*x^3 + 8*b*c*x^2 + 4*sqrt(c*x^4
 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(c) + (b^2 + 4*a*c)*x)/x) - 4*sqrt(c*x^4
 + b*x^3 + a*x^2)*(2*c^2*x + b*c))/(c^2*x), 1/8*((b^2 - 4*a*c)*sqrt(-c)*x*
arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(-c)/(c^2*x^3 + b*c
*x^2 + a*c*x)) + 2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c^2*x + b*c))/(c^2*x)]
```

Sympy [F]

$$\int \frac{\sqrt{ax^2 + bx^3 + cx^4}}{x}\,dx = \int \frac{\sqrt{x^2(a + bx + cx^2)}}{x}\,dx$$

input
```
integrate((c*x**4+b*x**3+a*x**2)**(1/2)/x,x)
```

output
```
Integral(sqrt(x**2*(a + b*x + c*x**2))/x, x)
```

Maxima [F]

$$\int \frac{\sqrt{ax^2 + bx^3 + cx^4}}{x}\,dx = \int \frac{\sqrt{cx^4 + bx^3 + ax^2}}{x}\,dx$$

input
```
integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x,x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^4 + b*x^3 + a*x^2)/x, x)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.31

$$\begin{aligned}\int \frac{\sqrt{ax^2 + bx^3 + cx^4}}{x}\,dx &= \frac{1}{8}\left(2\sqrt{cx^2 + bx + a}\left(2x + \frac{b}{c}\right) + \frac{(b^2 - 4ac)\log\left(\left|2\left(\sqrt{c}x - \sqrt{cx^2 + bx + a}\right)\sqrt{c} + b\right|\right)}{c^{\frac{3}{2}}}\right)\mathrm{sgn}(x) \\ &\quad - \frac{\left(b^2\log\left(\left|b - 2\sqrt{a}\sqrt{c}\right|\right) - 4ac\log\left(\left|b - 2\sqrt{a}\sqrt{c}\right|\right) + 2\sqrt{a}b\sqrt{c}\right)\mathrm{sgn}(x)}{8c^{\frac{3}{2}}}\end{aligned}$$

input
```
integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x,x, algorithm="giac")
```

output
```
1/8*(2*sqrt(c*x^2 + b*x + a)*(2*x + b/c) + (b^2 - 4*a*c)*log(abs(2*(sqrt(c
)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) + b))/c^(3/2))*sgn(x) - 1/8*(b^2*log(
abs(b - 2*sqrt(a)*sqrt(c))) - 4*a*c*log(abs(b - 2*sqrt(a)*sqrt(c))) + 2*sq
rt(a)*b*sqrt(c))*sgn(x)/c^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x}\,dx=\int \frac{\sqrt{c\,x^4+b\,x^3+a\,x^2}}{x}\,dx$$

input
```
int((a*x^2 + b*x^3 + c*x^4)^(1/2)/x,x)
```

output
```
int((a*x^2 + b*x^3 + c*x^4)^(1/2)/x, x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.35

$$\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x}\,dx = \frac{2\sqrt{c\,x^2+bx+a}\,bc+4\sqrt{c\,x^2+bx+a}\,c^2x+4\sqrt{c}\log\left(\frac{2\sqrt{c}\,\sqrt{c\,x^2+bx+a}+b+2cx}{\sqrt{4ac-b^2}}\right)ac-\sqrt{c}\log\left(\frac{2\sqrt{c}\,\sqrt{c\,x^2+bx+a}+b-}{\sqrt{4ac-b^2}}\right.}{8c^2}$$

input
```
int((c*x^4+b*x^3+a*x^2)^(1/2)/x,x)
```

output
```
(2*sqrt(a + b*x + c*x**2)*b*c + 4*sqrt(a + b*x + c*x**2)*c**2*x + 4*sqrt(c
)*log((2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)/sqrt(4*a*c - b**2))*a
*c - sqrt(c)*log((2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)/sqrt(4*a*c
 - b**2))*b**2)/(8*c**2)
```

3.50 $\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 173

$$\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^2}\,dx = \frac{\sqrt{ax^2+bx^3+cx^4}}{x} - \frac{\sqrt{a}x\sqrt{a+bx+cx^2}\operatorname{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{ax^2+bx^3+cx^4}} + \frac{bx\sqrt{a+bx+cx^2}\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}\sqrt{ax^2+bx^3+cx^4}}$$

output
```
(c*x^4+b*x^3+a*x^2)^(1/2)/x-a^(1/2)*x*(c*x^2+b*x+a)^(1/2)*arctanh(1/2*(b*x
+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))/(c*x^4+b*x^3+a*x^2)^(1/2)+1/2*b*x*(c*x^
2+b*x+a)^(1/2)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(1/2)/
(c*x^4+b*x^3+a*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.77

$$\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^2}\,dx$$
$$=\frac{x\sqrt{a+x(b+cx)}\left(2\sqrt{c}\sqrt{a+x(b+cx)}+4\sqrt{a}\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c}x-\sqrt{a+x(b+cx)}}{\sqrt{a}}\right)-b\log\left(b+2cx-2\sqrt{c}\sqrt{a\cdots}\right.\right.}{2\sqrt{c}\sqrt{x^2(a+x(b+cx))}}$$

input
```
Integrate[Sqrt[a*x^2 + b*x^3 + c*x^4]/x^2,x]
```

output
```
(x*Sqrt[a + x*(b + c*x)]*(2*Sqrt[c]*Sqrt[a + x*(b + c*x)] + 4*Sqrt[a]*Sqrt
[c]*ArcTanh[(Sqrt[c]*x - Sqrt[a + x*(b + c*x)])/Sqrt[a]] - b*Log[b + 2*c*x
 - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]]))/(2*Sqrt[c]*Sqrt[x^2*(a + x*(b + c*x)
)])
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.81, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {1968, 1980, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^2}\,dx$$

$$\Big\downarrow \text{ 1968}$$

$$\frac{1}{2}\int \frac{2a+bx}{\sqrt{cx^4+bx^3+ax^2}}dx+\frac{\sqrt{ax^2+bx^3+cx^4}}{x}$$

$$\Big\downarrow \text{ 1980}$$

$$\frac{x\sqrt{a+bx+cx^2}\int \frac{2a+bx}{x\sqrt{cx^2+bx+a}}dx}{2\sqrt{ax^2+bx^3+cx^4}}+\frac{\sqrt{ax^2+bx^3+cx^4}}{x}$$

$$\Big\downarrow \text{ 1269}$$

$$\frac{x\sqrt{a+bx+cx^2}\left(b\int\frac{1}{\sqrt{cx^2+bx+a}}dx+2a\int\frac{1}{x\sqrt{cx^2+bx+a}}dx\right)}{2\sqrt{ax^2+bx^3+cx^4}}+\frac{\sqrt{ax^2+bx^3+cx^4}}{x}$$

$$\Big\downarrow \text{ 1092}$$

$$\frac{x\sqrt{a+bx+cx^2}\left(2a\int\frac{1}{x\sqrt{cx^2+bx+a}}dx+2b\int\frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}\right)}{2\sqrt{ax^2+bx^3+cx^4}}+\frac{\sqrt{ax^2+bx^3+cx^4}}{x}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{x\sqrt{a+bx+cx^2}\left(2a\int\frac{1}{x\sqrt{cx^2+bx+a}}dx+\frac{b\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}}\right)}{2\sqrt{ax^2+bx^3+cx^4}}+\frac{\sqrt{ax^2+bx^3+cx^4}}{x}$$

$$\Big\downarrow \text{ 1154}$$

$$\frac{x\sqrt{a+bx+cx^2}\left(\frac{b\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}}-4a\int\frac{1}{4a-\frac{(2a+bx)^2}{cx^2+bx+a}}d\frac{2a+bx}{\sqrt{cx^2+bx+a}}\right)}{2\sqrt{ax^2+bx^3+cx^4}}+\frac{\sqrt{ax^2+bx^3+cx^4}}{x}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{x\sqrt{a+bx+cx^2}\left(\frac{b\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}}-2\sqrt{a}\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)\right)}{2\sqrt{ax^2+bx^3+cx^4}}+\frac{\sqrt{ax^2+bx^3+cx^4}}{x}$$

input
```
Int[Sqrt[a*x^2 + b*x^3 + c*x^4]/x^2,x]
```

output
```
Sqrt[a*x^2 + b*x^3 + c*x^4]/x + (x*Sqrt[a + b*x + c*x^2]*(-2*Sqrt[a]*ArcTa
nh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])] + (b*ArcTanh[(b + 2*c*x)
/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/Sqrt[c]))/(2*Sqrt[a*x^2 + b*x^3 + c*x
^4])
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1968
```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[x^(m + 1)*((a*x^q + b*x^n + c*x^(2*n - q))^p/(m + p*(2
*n - q) + 1)), x] + Simp[(n - q)*(p/(m + p*(2*n - q) + 1))   Int[x^(m + q)*
(2*a + b*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; Free
Q[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] &&  !IntegerQ[p] && NeQ[b
^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && GtQ[m + p*q
 + 1, -(n - q)] && NeQ[m + p*(2*n - q) + 1, 0]
```

rule 1980
```
Int[((A_) + (B_.)*(x_)^(j_.))/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c
_.)*(x_)^(r_.)], x_Symbol] :> Simp[x^(q/2)*(Sqrt[a + b*x^(n - q) + c*x^(2*(
n - q))]/Sqrt[a*x^q + b*x^n + c*x^(2*n - q)])   Int[(A + B*x^(n - q))/(x^(q
/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]), x], x] /; FreeQ[{a, b, c, A,
B, n, q}, x] && EqQ[j, n - q] && EqQ[r, 2*n - q] && PosQ[n - q] && EqQ[n, 3
] && EqQ[q, 2]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.53

method	result	size
pseudoelliptic	$\frac{\left(\left(-\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x\sqrt{a}}\right)+\ln(2)\right)\sqrt{a}+\sqrt{c\,x^2+bx+a}\right)\sqrt{c}+\frac{\ln\left(2\sqrt{c\,x^2+bx+a}\,\sqrt{c}+2cx+b\right)b}{2}}{\sqrt{c}}$	91
default	$-\frac{\sqrt{c\,x^4+b\,x^3+a\,x^2}\left(2\sqrt{a}\,\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x}\right)\sqrt{c}-b\ln\left(\frac{2\sqrt{c\,x^2+bx+a}\,\sqrt{c}+2cx+b}{2\sqrt{c}}\right)-2\sqrt{c\,x^2+bx+a}\,\sqrt{c}\right)}{2x\sqrt{c\,x^2+bx+a}\,\sqrt{c}}$	126

input
```
int((c*x^4+b*x^3+a*x^2)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/c^(1/2)*(((-ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x/a^(1/2))+ln(2))
*a^(1/2)+(c*x^2+b*x+a)^(1/2))*c^(1/2)+1/2*ln(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)
+2*c*x+b)*b)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 638, normalized size of antiderivative = 3.69

$$\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^2}\,dx$$
$$= \left[\frac{b\sqrt{c}x\log\left(-\frac{8\,c^2x^3+8\,bcx^2+4\,\sqrt{cx^4+bx^3+ax^2}(2\,cx+b)\sqrt{c}+(b^2+4\,ac)\,x}{x}\right)+2\,\sqrt{a}cx\log\left(-\frac{8\,abx^2+(b^2+4\,ac)x^3+8\,a^2x-4\,\sqrt{cx^4+}}{x^3}\right.}{4\,cx}\right.$$
$$-\frac{b\sqrt{-c}x\arctan\left(\frac{\sqrt{cx^4+bx^3+ax^2}(2\,cx+b)\sqrt{-c}}{2\,(c^2x^3+bcx^2+acx)}\right)-\sqrt{a}cx\log\left(-\frac{8\,abx^2+(b^2+4\,ac)x^3+8\,a^2x-4\,\sqrt{cx^4+bx^3+ax^2}(bx+2\,a)\sqrt{a}}{x^3}\right)}{2\,cx}$$

input
```
integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x^2,x, algorithm="fricas")
```

output
```
[1/4*(b*sqrt(c)*x*log(-(8*c^2*x^3 + 8*b*c*x^2 + 4*sqrt(c*x^4 + b*x^3 + a*x
^2)*(2*c*x + b)*sqrt(c) + (b^2 + 4*a*c)*x)/x) + 2*sqrt(a)*c*x*log(-(8*a*b*
x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2
*a)*sqrt(a))/x^3) + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*c)/(c*x), -1/2*(b*sqrt(-
c)*x*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(-c)/(c^2*x^3
+ b*c*x^2 + a*c*x)) - sqrt(a)*c*x*log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*
a^2*x - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(a))/x^3) - 2*sqrt(c
*x^4 + b*x^3 + a*x^2)*c)/(c*x), 1/4*(4*sqrt(-a)*c*x*arctan(1/2*sqrt(c*x^4
+ b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2 + a^2*x)) + b*sqr
t(c)*x*log(-(8*c^2*x^3 + 8*b*c*x^2 + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x
+ b)*sqrt(c) + (b^2 + 4*a*c)*x)/x) + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*c)/(c*x
), 1/2*(2*sqrt(-a)*c*x*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*
sqrt(-a)/(a*c*x^3 + a*b*x^2 + a^2*x)) - b*sqrt(-c)*x*arctan(1/2*sqrt(c*x^4
 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x)) + 2*sq
rt(c*x^4 + b*x^3 + a*x^2)*c)/(c*x)]
```

Sympy [F]

$$\int \frac{\sqrt{ax^2 + bx^3 + cx^4}}{x^2}\,dx = \int \frac{\sqrt{x^2\,(a + bx + cx^2)}}{x^2}\,dx$$

input
```
integrate((c*x**4+b*x**3+a*x**2)**(1/2)/x**2,x)
```

output
```
Integral(sqrt(x**2*(a + b*x + c*x**2))/x**2, x)
```

Maxima [F]

$$\int \frac{\sqrt{ax^2 + bx^3 + cx^4}}{x^2}\,dx = \int \frac{\sqrt{cx^4 + bx^3 + ax^2}}{x^2}\,dx$$

input
```
integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^4 + b*x^3 + a*x^2)/x^2, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Degree mismatch inside factorisatio
n over extensionNot implemented, e.g. for multivariate mod/approx polynomi
alsError:
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^2}\,dx = \int \frac{\sqrt{c\,x^4+b\,x^3+a\,x^2}}{x^2}\,dx$$

input
```
int((a*x^2 + b*x^3 + c*x^4)^(1/2)/x^2,x)
```

output
```
int((a*x^2 + b*x^3 + c*x^4)^(1/2)/x^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 828, normalized size of antiderivative = 4.79

$$\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^2}\,dx = \text{Too large to display}$$

input
```
int((c*x^4+b*x^3+a*x^2)^(1/2)/x^2,x)
```

output
```
( - 2*sqrt(a)*sqrt(4*sqrt(c)*sqrt(a)*b - 4*a*c - b**2)*atan((2*sqrt(c)*sqr
t(a + b*x + c*x**2) + b + 2*c*x)/sqrt(4*sqrt(c)*sqrt(a)*b - 4*a*c - b**2))
*b*c - 4*sqrt(c)*sqrt(4*sqrt(c)*sqrt(a)*b - 4*a*c - b**2)*atan((2*sqrt(c)*
sqrt(a + b*x + c*x**2) + b + 2*c*x)/sqrt(4*sqrt(c)*sqrt(a)*b - 4*a*c - b**
2))*a*c - sqrt(a)*sqrt(4*sqrt(c)*sqrt(a)*b + 4*a*c + b**2)*log( - sqrt(4*s
qrt(c)*sqrt(a)*b + 4*a*c + b**2) + 2*sqrt(c)*sqrt(a + b*x + c*x**2) + b +
2*c*x)*b*c + sqrt(a)*sqrt(4*sqrt(c)*sqrt(a)*b + 4*a*c + b**2)*log(sqrt(4*s
qrt(c)*sqrt(a)*b + 4*a*c + b**2) + 2*sqrt(c)*sqrt(a + b*x + c*x**2) + b +
2*c*x)*b*c + 2*sqrt(c)*sqrt(4*sqrt(c)*sqrt(a)*b + 4*a*c + b**2)*log( - sqr
t(4*sqrt(c)*sqrt(a)*b + 4*a*c + b**2) + 2*sqrt(c)*sqrt(a + b*x + c*x**2) +
 b + 2*c*x)*a*c - 2*sqrt(c)*sqrt(4*sqrt(c)*sqrt(a)*b + 4*a*c + b**2)*log(s
qrt(4*sqrt(c)*sqrt(a)*b + 4*a*c + b**2) + 2*sqrt(c)*sqrt(a + b*x + c*x**2)
 + b + 2*c*x)*a*c + 8*sqrt(a + b*x + c*x**2)*a*c**2 - 2*sqrt(a + b*x + c*x
**2)*b**2*c + 4*sqrt(a)*log( - sqrt(4*sqrt(c)*sqrt(a)*b + 4*a*c + b**2) +
2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)*a*c**2 - sqrt(a)*log( - sqrt
(4*sqrt(c)*sqrt(a)*b + 4*a*c + b**2) + 2*sqrt(c)*sqrt(a + b*x + c*x**2) +
b + 2*c*x)*b**2*c + 4*sqrt(a)*log(sqrt(4*sqrt(c)*sqrt(a)*b + 4*a*c + b**2)
 + 2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)*a*c**2 - sqrt(a)*log(sqrt
(4*sqrt(c)*sqrt(a)*b + 4*a*c + b**2) + 2*sqrt(c)*sqrt(a + b*x + c*x**2) +
b + 2*c*x)*b**2*c - 4*sqrt(a)*log(4*sqrt(c)*sqrt(a + b*x + c*x**2)*b + ...
```

3.51 $\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^3} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 173

$$\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^3} \, dx = -\frac{\sqrt{ax^2+bx^3+cx^4}}{x^2} - \frac{bx\sqrt{a+bx+cx^2}\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}} + \frac{\sqrt{c}x\sqrt{a+bx+cx^2}\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{ax^2+bx^3+cx^4}}$$

output
```
-(c*x^4+b*x^3+a*x^2)^(1/2)/x^2-1/2*b*x*(c*x^2+b*x+a)^(1/2)*arctanh(1/2*(b*
x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))/a^(1/2)/(c*x^4+b*x^3+a*x^2)^(1/2)+c^(1
/2)*x*(c*x^2+b*x+a)^(1/2)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2
))/(c*x^4+b*x^3+a*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.72

$$\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^3}\,dx = \frac{\sqrt{a+x(b+cx)}\left(bx\text{arctanh}\left(\frac{\sqrt{c}x-\sqrt{a+x(b+cx)}}{\sqrt{a}}\right)-\sqrt{a}\left(\sqrt{a+x(b+cx)}+\sqrt{c}x\log\left(b+2cx-2\sqrt{c}\sqrt{a+x\right.\right.\right.}{\sqrt{a}\sqrt{x^2(a+x(b+cx))}}$$

input
```
Integrate[Sqrt[a*x^2 + b*x^3 + c*x^4]/x^3,x]
```

output
```
(Sqrt[a + x*(b + c*x)]*(b*x*ArcTanh[(Sqrt[c]*x - Sqrt[a + x*(b + c*x)])/Sq
rt[a]] - Sqrt[a]*(Sqrt[a + x*(b + c*x)] + Sqrt[c]*x*Log[b + 2*c*x - 2*Sqrt
[c]*Sqrt[a + x*(b + c*x)]])))/(Sqrt[a]*Sqrt[x^2*(a + x*(b + c*x))])
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.82, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {1967, 1980, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^3}\,dx$$

$$\Big\downarrow \text{ 1967}$$

$$\frac{1}{2}\int \frac{b+2cx}{\sqrt{cx^4+bx^3+ax^2}}dx-\frac{\sqrt{ax^2+bx^3+cx^4}}{x^2}$$

$$\Big\downarrow \text{ 1980}$$

$$\frac{x\sqrt{a+bx+cx^2}\int \frac{b+2cx}{x\sqrt{cx^2+bx+a}}dx}{2\sqrt{ax^2+bx^3+cx^4}}-\frac{\sqrt{ax^2+bx^3+cx^4}}{x^2}$$

$$\Big\downarrow \text{ 1269}$$

$$\frac{x\sqrt{a+bx+cx^2}\left(2c\int \frac{1}{\sqrt{cx^2+bx+a}}dx+b\int \frac{1}{x\sqrt{cx^2+bx+a}}dx\right)}{2\sqrt{ax^2+bx^3+cx^4}}-\frac{\sqrt{ax^2+bx^3+cx^4}}{x^2}$$

$$\Big\downarrow \text{1092}$$

$$\frac{x\sqrt{a+bx+cx^2}\left(b\int \frac{1}{x\sqrt{cx^2+bx+a}}dx+4c\int \frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}\right)}{2\sqrt{ax^2+bx^3+cx^4}}-\frac{\sqrt{ax^2+bx^3+cx^4}}{x^2}$$

$$\Big\downarrow \text{219}$$

$$\frac{x\sqrt{a+bx+cx^2}\left(b\int \frac{1}{x\sqrt{cx^2+bx+a}}dx+2\sqrt{c}\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\right)}{2\sqrt{ax^2+bx^3+cx^4}}-\frac{\sqrt{ax^2+bx^3+cx^4}}{x^2}$$

$$\Big\downarrow \text{1154}$$

$$\frac{x\sqrt{a+bx+cx^2}\left(2\sqrt{c}\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)-2b\int \frac{1}{4a-\frac{(2a+bx)^2}{cx^2+bx+a}}d\frac{2a+bx}{\sqrt{cx^2+bx+a}}\right)}{2\sqrt{ax^2+bx^3+cx^4}}-\frac{\sqrt{ax^2+bx^3+cx^4}}{x^2}$$

$$\Big\downarrow \text{219}$$

$$\frac{x\sqrt{a+bx+cx^2}\left(2\sqrt{c}\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)-\frac{b\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}}\right)}{2\sqrt{ax^2+bx^3+cx^4}}-\frac{\sqrt{ax^2+bx^3+cx^4}}{x^2}$$

input
```
Int[Sqrt[a*x^2 + b*x^3 + c*x^4]/x^3,x]
```

output
```
-(Sqrt[a*x^2 + b*x^3 + c*x^4]/x^2) + (x*Sqrt[a + b*x + c*x^2]*(-((b*ArcTan
h[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/Sqrt[a]) + 2*Sqrt[c]*Arc
Tanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])]))/(2*Sqrt[a*x^2 + b*x^
3 + c*x^4])
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1967
```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[x^(m + 1)*((a*x^q + b*x^n + c*x^(2*n - q))^p/(m + p*q
+ 1)), x] - Simp[(n - q)*(p/(m + p*q + 1))   Int[x^(m + n)*(b + 2*c*x^(n -
q))*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c}, x]
&& EqQ[r, 2*n - q] && PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] &
& IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && LeQ[m + p*q + 1, -(n - q) +
 1] && NeQ[m + p*q + 1, 0]
```

rule 1980
```
Int[((A_) + (B_.)*(x_)^(j_.))/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c
_.)*(x_)^(r_.)], x_Symbol] :> Simp[x^(q/2)*(Sqrt[a + b*x^(n - q) + c*x^(2*(
n - q))]/Sqrt[a*x^q + b*x^n + c*x^(2*n - q)])   Int[(A + B*x^(n - q))/(x^(q
/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]), x], x] /; FreeQ[{a, b, c, A,
B, n, q}, x] && EqQ[j, n - q] && EqQ[r, 2*n - q] && PosQ[n - q] && EqQ[n, 3
] && EqQ[q, 2]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.58

method	result
pseudoelliptic	$\frac{2\sqrt{c}\,\ln\left(2\sqrt{c\,x^{2}+bx+a}\,\sqrt{c}+2cx+b\right)x\sqrt{a}-bx\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^{2}+bx+a}}{x\sqrt{a}}\right)+bx\ln(2)-2\sqrt{a}\,\sqrt{c\,x^{2}+bx+a}}{2x\sqrt{a}}$
risch	$-\frac{\sqrt{x^{2}(c\,x^{2}+bx+a)}}{x^{2}}+\frac{\left(-\frac{b\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^{2}+bx+a}}{x}\right)}{2\sqrt{a}}+\sqrt{c}\,\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^{2}+bx+a}\right)\right)\sqrt{x^{2}(c\,x^{2}+bx+a)}}{x\sqrt{c\,x^{2}+bx+a}}$
default	$\frac{\sqrt{c\,x^{4}+b\,x^{3}+a\,x^{2}}\,\left(2x^{2}\sqrt{c\,x^{2}+bx+a}\,c^{\frac{5}{2}}-c^{\frac{3}{2}}\sqrt{a}\,\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^{2}+bx+a}}{x}\right)bx-2\left(c\,x^{2}+bx+a\right)^{\frac{3}{2}}c^{\frac{3}{2}}+2c^{\frac{3}{2}}\sqrt{c\,x^{2}+bx+a}\,b}{2x^{2}\sqrt{c\,x^{2}+bx+a}\,a\,c^{\frac{3}{2}}}$

input
```
int((c*x^4+b*x^3+a*x^2)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*(2*c^(1/2)*ln(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)*x*a^(1/2)-b*x*ln(
(2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x/a^(1/2))+b*x*ln(2)-2*a^(1/2)*(c*
x^2+b*x+a)^(1/2))/x/a^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 653, normalized size of antiderivative = 3.77

$$\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^3}\,dx$$
$$= \left[\frac{2\,a\sqrt{c}x^2\log\left(-\frac{8\,c^2x^3+8\,bcx^2+4\,\sqrt{cx^4+bx^3+ax^2}(2\,cx+b)\sqrt{c}+(b^2+4\,ac)x}{x}\right)+\sqrt{a}bx^2\log\left(-\frac{8\,abx^2+(b^2+4\,ac)x^3+8\,a^2x-4\,\sqrt{cx}}{x^3}\right.}{4\,ax^2}\right.$$
$$-\frac{4\,a\sqrt{-c}x^2\arctan\left(\frac{\sqrt{cx^4+bx^3+ax^2}(2\,cx+b)\sqrt{-c}}{2\,(c^2x^3+bcx^2+acx)}\right)-\sqrt{a}bx^2\log\left(-\frac{8\,abx^2+(b^2+4\,ac)x^3+8\,a^2x-4\,\sqrt{cx^4+bx^3+ax^2}(bx+2\,a)\sqrt{}}{x^3}\right.}{4\,ax^2}$$

input
```
integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x^3,x, algorithm="fricas")
```

output
```
[1/4*(2*a*sqrt(c)*x^2*log(-(8*c^2*x^3 + 8*b*c*x^2 + 4*sqrt(c*x^4 + b*x^3 +
 a*x^2)*(2*c*x + b)*sqrt(c) + (b^2 + 4*a*c)*x)/x) + sqrt(a)*b*x^2*log(-(8*
a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x
 + 2*a)*sqrt(a))/x^3) - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*a)/(a*x^2), -1/4*(4*
a*sqrt(-c)*x^2*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(-c)
/(c^2*x^3 + b*c*x^2 + a*c*x)) - sqrt(a)*b*x^2*log(-(8*a*b*x^2 + (b^2 + 4*a
*c)*x^3 + 8*a^2*x - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(a))/x^3
) + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*a)/(a*x^2), 1/2*(sqrt(-a)*b*x^2*arctan(1
/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2 + a
^2*x)) + a*sqrt(c)*x^2*log(-(8*c^2*x^3 + 8*b*c*x^2 + 4*sqrt(c*x^4 + b*x^3
+ a*x^2)*(2*c*x + b)*sqrt(c) + (b^2 + 4*a*c)*x)/x) - 2*sqrt(c*x^4 + b*x^3
+ a*x^2)*a)/(a*x^2), 1/2*(sqrt(-a)*b*x^2*arctan(1/2*sqrt(c*x^4 + b*x^3 + a
*x^2)*(b*x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2 + a^2*x)) - 2*a*sqrt(-c)*x^2
*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(-c)/(c^2*x^3 + b*
c*x^2 + a*c*x)) - 2*sqrt(c*x^4 + b*x^3 + a*x^2)*a)/(a*x^2)]
```

Sympy [F]

$$\int \frac{\sqrt{ax^2 + bx^3 + cx^4}}{x^3} \, dx = \int \frac{\sqrt{x^2 (a + bx + cx^2)}}{x^3} \, dx$$

input
```
integrate((c*x**4+b*x**3+a*x**2)**(1/2)/x**3,x)
```

output
```
Integral(sqrt(x**2*(a + b*x + c*x**2))/x**3, x)
```

Maxima [F]

$$\int \frac{\sqrt{ax^2 + bx^3 + cx^4}}{x^3} \, dx = \int \frac{\sqrt{cx^4 + bx^3 + ax^2}}{x^3} \, dx$$

input
```
integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x^3,x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^4 + b*x^3 + a*x^2)/x^3, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^3}\,dx = \int \frac{\sqrt{c\,x^4+b\,x^3+a\,x^2}}{x^3}\,dx$$

input
```
int((a*x^2 + b*x^3 + c*x^4)^(1/2)/x^3,x)
```

output
```
int((a*x^2 + b*x^3 + c*x^4)^(1/2)/x^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.52

$$\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^3}\,dx$$
$$= \frac{-2\sqrt{c\,x^2+bx+a}\,a+\sqrt{a}\log\left(2\sqrt{a}\,\sqrt{c\,x^2+bx+a}-2a-bx\right)bx-\sqrt{a}\log(x)\,bx+2\sqrt{c}\log\left(-2\sqrt{c}\,\sqrt{c\,x\right.}}{2ax}$$

input
```
int((c*x^4+b*x^3+a*x^2)^(1/2)/x^3,x)
```

output
```
( - 2*sqrt(a + b*x + c*x**2)*a + sqrt(a)*log(2*sqrt(a)*sqrt(a + b*x + c*x*
*2) - 2*a - b*x)*b*x - sqrt(a)*log(x)*b*x + 2*sqrt(c)*log( - 2*sqrt(c)*sqr
t(a + b*x + c*x**2) - b - 2*c*x)*a*x)/(2*a*x)
```

3.52 $\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^4}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 114

$$\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^4}\,dx=-\frac{\sqrt{ax^2+bx^3+cx^4}}{2x^3}-\frac{b\sqrt{ax^2+bx^3+cx^4}}{4ax^2}+\frac{(b^2-4ac)\,\text{arctanh}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{8a^{3/2}}$$

output
```
-1/2*(c*x^4+b*x^3+a*x^2)^(1/2)/x^3-1/4*b*(c*x^4+b*x^3+a*x^2)^(1/2)/a/x^2+1
/8*(-4*a*c+b^2)*arctanh(1/2*x*(b*x+2*a)/a^(1/2)/(c*x^4+b*x^3+a*x^2)^(1/2))
/a^(3/2)
```

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.97

$$\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^4}\,dx=-\frac{\sqrt{x^2(a+x(b+cx))}\left(\sqrt{a}(2a+bx)\sqrt{a+x(b+cx)}+(b^2-4ac)\,x^2\text{arctanh}\left(\frac{\sqrt{c}x-\sqrt{a+x(b+cx)}}{\sqrt{a}}\right)\right)}{4a^{3/2}x^3\sqrt{a+x(b+cx)}}$$

input
```
Integrate[Sqrt[a*x^2 + b*x^3 + c*x^4]/x^4,x]
```

output
```
-1/4*(Sqrt[x^2*(a + x*(b + c*x))]*(Sqrt[a]*(2*a + b*x)*Sqrt[a + x*(b + c*x
)] + (b^2 - 4*a*c)*x^2*ArcTanh[(Sqrt[c]*x - Sqrt[a + x*(b + c*x)])/Sqrt[a]
]))/(a^(3/2)*x^3*Sqrt[a + x*(b + c*x)])
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1967, 1998, 27, 1951, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^4}\,dx$$

$$\Big\downarrow \ 1967$$

$$\frac{1}{4}\int \frac{b+2cx}{x\sqrt{cx^4+bx^3+ax^2}}dx-\frac{\sqrt{ax^2+bx^3+cx^4}}{2x^3}$$

$$\Big\downarrow \ 1998$$

$$\frac{1}{4}\left(-\frac{\int \frac{b^2-4ac}{2\sqrt{cx^4+bx^3+ax^2}}dx}{a}-\frac{b\sqrt{ax^2+bx^3+cx^4}}{ax^2}\right)-\frac{\sqrt{ax^2+bx^3+cx^4}}{2x^3}$$

$$\Big\downarrow \ 27$$

$$\frac{1}{4}\left(-\frac{\left(b^2-4ac\right)\int \frac{1}{\sqrt{cx^4+bx^3+ax^2}}dx}{2a}-\frac{b\sqrt{ax^2+bx^3+cx^4}}{ax^2}\right)-\frac{\sqrt{ax^2+bx^3+cx^4}}{2x^3}$$

$$\Big\downarrow \ 1951$$

$$\frac{1}{4}\left(\frac{\left(b^2-4ac\right)\int \frac{1}{4a-\frac{x^2(2a+bx)^2}{cx^4+bx^3+ax^2}}d\frac{x(2a+bx)}{\sqrt{cx^4+bx^3+ax^2}}}{a}-\frac{b\sqrt{ax^2+bx^3+cx^4}}{ax^2}\right)-\frac{\sqrt{ax^2+bx^3+cx^4}}{2x^3}$$

$$\Big\downarrow \ 219$$

$$\frac{1}{4}\left(\frac{\left(b^2-4ac\right)\text{arctanh}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{2a^{3/2}}-\frac{b\sqrt{ax^2+bx^3+cx^4}}{ax^2}\right)-\frac{\sqrt{ax^2+bx^3+cx^4}}{2x^3}$$

input
```
Int[Sqrt[a*x^2 + b*x^3 + c*x^4]/x^4,x]
```

output
```
-1/2*Sqrt[a*x^2 + b*x^3 + c*x^4]/x^3 + (-((b*Sqrt[a*x^2 + b*x^3 + c*x^4])/
(a*x^2)) + ((b^2 - 4*a*c)*ArcTanh[(x*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x^2 +
b*x^3 + c*x^4])])/(2*a^(3/2)))/4
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1951
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :
> Simp[-2/(n - 2)   Subst[Int[1/(4*a - x^2), x], x, x*((2*a + b*x^(n - 2))/
Sqrt[a*x^2 + b*x^n + c*x^r])], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*
n - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]
```

rule 1967
```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[x^(m + 1)*((a*x^q + b*x^n + c*x^(2*n - q))^p/(m + p*q
+ 1)), x] - Simp[(n - q)*(p/(m + p*q + 1))   Int[x^(m + n)*(b + 2*c*x^(n -
q))*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c}, x]
&& EqQ[r, 2*n - q] && PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] &
& IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && LeQ[m + p*q + 1, -(n - q) +
 1] && NeQ[m + p*q + 1, 0]
```

rule 1998
```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[A*x^(m - q + 1)*((a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1)/(a*(m + p*q + 1))), x] + Simp[1/(a*(m + p*q +
 1))   Int[x^(m + n - q)*Simp[a*B*(m + p*q + 1) - A*b*(m + p*q + (n - q)*(p
 + 1) + 1) - A*c*(m + p*q + 2*(n - q)*(p + 1) + 1)*x^(n - q), x]*(a*x^q + b
*x^n + c*x^(2*n - q))^p, x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n -
q] && EqQ[j, 2*n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
 && RationalQ[m, p, q] && ((GeQ[p, -1] && LtQ[p, 0]) || EqQ[m + p*q + (n -
q)*(2*p + 1) + 1, 0]) && LeQ[m + p*q, -(n - q)] && NeQ[m + p*q + 1, 0]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.71

method	result
pseudoelliptic	$-\frac{\sqrt{c\,x^{2}+bx+a}\,(bx+2a)}{4x^{2}a}-\frac{(4ac-b^{2})\left(-\ln(2)+\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^{2}+bx+a}}{x\sqrt{a}}\right)\right)}{8a^{\frac{3}{2}}}$
risch	$-\frac{(bx+2a)\sqrt{x^{2}(c\,x^{2}+bx+a)}}{4x^{3}a}-\frac{(4ac-b^{2})\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^{2}+bx+a}}{x}\right)\sqrt{x^{2}(c\,x^{2}+bx+a)}}{8a^{\frac{3}{2}}\,x\sqrt{c\,x^{2}+bx+a}}$
default	$-\frac{\sqrt{c\,x^{4}+b\,x^{3}+a\,x^{2}}\left(4c\,a^{\frac{3}{2}}\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^{2}+bx+a}}{x}\right)x^{2}+2c\sqrt{c\,x^{2}+bx+a}\,b\,x^{3}-4c\sqrt{c\,x^{2}+bx+a}\,a\,x^{2}-\sqrt{a}\,\ln\left(\frac{2a+bx+2\ldots}{\ldots}\right)\ldots\right)}{8x^{3}\sqrt{c\,x^{2}+bx+a}\,a^{2}}$

input
```
int((c*x^4+b*x^3+a*x^2)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/4*(c*x^2+b*x+a)^(1/2)*(b*x+2*a)/x^2/a-1/8*(4*a*c-b^2)/a^(3/2)*(-ln(2)+l
n((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x/a^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.98

$$\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^4}\,dx$$
$$= \left[-\frac{(b^2-4\,ac)\sqrt{a}x^3 \log\left(-\frac{8\,abx^2+(b^2+4\,ac)x^3+8\,a^2x-4\,\sqrt{cx^4+bx^3+ax^2}(bx+2\,a)\sqrt{a}}{x^3}\right)+4\,\sqrt{cx^4+bx^3+ax^2}(abx+2}{16\,a^2x^3}\right.$$
$$\left. -\frac{(b^2-4\,ac)\sqrt{-a}x^3 \arctan\left(\frac{\sqrt{cx^4+bx^3+ax^2}(bx+2\,a)\sqrt{-a}}{2\,(acx^3+abx^2+a^2x)}\right)+2\,\sqrt{cx^4+bx^3+ax^2}(abx+2\,a^2)}{8\,a^2x^3}\right]$$

input
```
integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x^4,x, algorithm="fricas")
```

output
```
[-1/16*((b^2 - 4*a*c)*sqrt(a)*x^3*log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*
a^2*x - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(a))/x^3) + 4*sqrt(c
*x^4 + b*x^3 + a*x^2)*(a*b*x + 2*a^2))/(a^2*x^3), -1/8*((b^2 - 4*a*c)*sqrt
(-a)*x^3*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(-a)/(a*c*
x^3 + a*b*x^2 + a^2*x)) + 2*sqrt(c*x^4 + b*x^3 + a*x^2)*(a*b*x + 2*a^2))/(
a^2*x^3)]
```

Sympy [F]

$$\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^4}\,dx = \int \frac{\sqrt{x^2\,(a+bx+cx^2)}}{x^4}\,dx$$

input
```
integrate((c*x**4+b*x**3+a*x**2)**(1/2)/x**4,x)
```

output
```
Integral(sqrt(x**2*(a + b*x + c*x**2))/x**4, x)
```

Maxima [F]

$$\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^4}\,dx = \int \frac{\sqrt{cx^4+bx^3+ax^2}}{x^4}\,dx$$

input
```
integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x^4,x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^4 + b*x^3 + a*x^2)/x^4, x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^4}\,dx = \text{Timed out}$$

input
```
integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x^4,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^4}\,dx = \int \frac{\sqrt{c\,x^4+b\,x^3+a\,x^2}}{x^4}\,dx$$

input
```
int((a*x^2 + b*x^3 + c*x^4)^(1/2)/x^4,x)
```

output
```
int((a*x^2 + b*x^3 + c*x^4)^(1/2)/x^4, x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.14

$$\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^4}\,dx$$
$$= \frac{-4\sqrt{c\,x^2+bx+a}\,a^2-2\sqrt{c\,x^2+bx+a}\,abx+4\sqrt{a}\log\left(2\sqrt{a}\,\sqrt{c\,x^2+bx+a}-2a-bx\right)ac\,x^2-\sqrt{a}\log(}{8a^2x^2}$$

input
```
int((c*x^4+b*x^3+a*x^2)^(1/2)/x^4,x)
```

output
```
( - 4*sqrt(a + b*x + c*x**2)*a**2 - 2*sqrt(a + b*x + c*x**2)*a*b*x + 4*sqr
t(a)*log(2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a - b*x)*a*c*x**2 - sqrt(a)*
log(2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a - b*x)*b**2*x**2 - 4*sqrt(a)*lo
g(x)*a*c*x**2 + sqrt(a)*log(x)*b**2*x**2)/(8*a**2*x**2)
```

3.53 $\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^5}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 155

$$\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^5}\,dx = -\frac{\sqrt{ax^2+bx^3+cx^4}}{3x^4}-\frac{b\sqrt{ax^2+bx^3+cx^4}}{12ax^3}+\frac{(3b^2-8ac)\sqrt{ax^2+bx^3+cx^4}}{24a^2x^2}-\frac{b(b^2-4ac)\,\text{arctanh}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{16a^{5/2}}$$

output

```
-1/3*(c*x^4+b*x^3+a*x^2)^(1/2)/x^4-1/12*b*(c*x^4+b*x^3+a*x^2)^(1/2)/a/x^3+
1/24*(-8*a*c+3*b^2)*(c*x^4+b*x^3+a*x^2)^(1/2)/a^2/x^2-1/16*b*(-4*a*c+b^2)*
arctanh(1/2*x*(b*x+2*a)/a^(1/2)/(c*x^4+b*x^3+a*x^2)^(1/2))/a^(5/2)
```

Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.84

$$\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^5}\,dx = \frac{\sqrt{x^2(a+x(b+cx))}\left(\sqrt{a}\sqrt{a+x(b+cx)}(-8a^2+3b^2x^2-2ax(b+4cx))+3b(b^2-4ac)\,x^3\text{arctanh}\left(\frac{\sqrt{c}x-}{}\right.\right.}{24a^{5/2}x^4\sqrt{a+x(b+cx)}}$$

input
```
Integrate[Sqrt[a*x^2 + b*x^3 + c*x^4]/x^5,x]
```

output
```
(Sqrt[x^2*(a + x*(b + c*x))]*(Sqrt[a]*Sqrt[a + x*(b + c*x)]*(-8*a^2 + 3*b^
2*x^2 - 2*a*x*(b + 4*c*x)) + 3*b*(b^2 - 4*a*c)*x^3*ArcTanh[(Sqrt[c]*x - Sq
rt[a + x*(b + c*x)])/Sqrt[a]]))/(24*a^(5/2)*x^4*Sqrt[a + x*(b + c*x)])
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {1967, 1998, 27, 1998, 27, 1951, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^5}dx$$

$$\Big\downarrow \text{1967}$$

$$\frac{1}{6}\int \frac{b+2cx}{x^2\sqrt{cx^4+bx^3+ax^2}}dx-\frac{\sqrt{ax^2+bx^3+cx^4}}{3x^4}$$

$$\Big\downarrow \text{1998}$$

$$\frac{1}{6}\left(-\frac{\int \frac{3b^2+2cxb-8ac}{2x\sqrt{cx^4+bx^3+ax^2}}dx}{2a}-\frac{b\sqrt{ax^2+bx^3+cx^4}}{2ax^3}\right)-\frac{\sqrt{ax^2+bx^3+cx^4}}{3x^4}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{6}\left(-\frac{\int \frac{3b^2+2cxb-8ac}{x\sqrt{cx^4+bx^3+ax^2}}dx}{4a}-\frac{b\sqrt{ax^2+bx^3+cx^4}}{2ax^3}\right)-\frac{\sqrt{ax^2+bx^3+cx^4}}{3x^4}$$

$$\Big\downarrow \text{1998}$$

$$\frac{1}{6}\left(-\frac{-\frac{\int \frac{3b\left(b^2-4ac\right)}{2\sqrt{cx^4+bx^3+ax^2}}dx}{a}-\frac{\left(3b^2-8ac\right)\sqrt{ax^2+bx^3+cx^4}}{ax^2}}{4a}-\frac{b\sqrt{ax^2+bx^3+cx^4}}{2ax^3}\right)-\frac{\sqrt{ax^2+bx^3+cx^4}}{3x^4}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{6}\left(-\frac{-\frac{3b(b^2-4ac)\int \frac{1}{\sqrt{cx^4+bx^3+ax^2}}dx}{2a}-\frac{(3b^2-8ac)\sqrt{ax^2+bx^3+cx^4}}{ax^2}}{4a}-\frac{b\sqrt{ax^2+bx^3+cx^4}}{2ax^3}\right)-\frac{\sqrt{ax^2+bx^3+cx^4}}{3x^4}$$

$$\Big\downarrow \text{1951}$$

$$\frac{1}{6}\left(-\frac{\frac{3b(b^2-4ac)\int \frac{1}{4a-\frac{x^2(2a+bx)^2}{cx^4+bx^3+ax^2}}d\frac{x(2a+bx)}{\sqrt{cx^4+bx^3+ax^2}}}{a}-\frac{(3b^2-8ac)\sqrt{ax^2+bx^3+cx^4}}{ax^2}}{4a}-\frac{b\sqrt{ax^2+bx^3+cx^4}}{2ax^3}\right)-\frac{\sqrt{ax^2+bx^3+cx^4}}{3x^4}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{6}\left(-\frac{\frac{3b(b^2-4ac)\text{arctanh}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{2a^{3/2}}-\frac{(3b^2-8ac)\sqrt{ax^2+bx^3+cx^4}}{ax^2}}{4a}-\frac{b\sqrt{ax^2+bx^3+cx^4}}{2ax^3}\right)-\frac{\sqrt{ax^2+bx^3+cx^4}}{3x^4}$$

input
```
Int[Sqrt[a*x^2 + b*x^3 + c*x^4]/x^5,x]
```

output
```
-1/3*Sqrt[a*x^2 + b*x^3 + c*x^4]/x^4 + (-1/2*(b*Sqrt[a*x^2 + b*x^3 + c*x^4
])/(a*x^3) - (-(((3*b^2 - 8*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(a*x^2)) + (
3*b*(b^2 - 4*a*c)*ArcTanh[(x*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x^2 + b*x^3 +
c*x^4])])/(2*a^(3/2)))/(4*a))/6
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1951
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :
> Simp[-2/(n - 2)   Subst[Int[1/(4*a - x^2), x], x, x*((2*a + b*x^(n - 2))/
Sqrt[a*x^2 + b*x^n + c*x^r])], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*
n - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]
```

rule 1967
```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[x^(m + 1)*((a*x^q + b*x^n + c*x^(2*n - q))^p/(m + p*q
+ 1)), x] - Simp[(n - q)*(p/(m + p*q + 1))   Int[x^(m + n)*(b + 2*c*x^(n -
q))*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c}, x]
&& EqQ[r, 2*n - q] && PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] &
& IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && LeQ[m + p*q + 1, -(n - q) +
 1] && NeQ[m + p*q + 1, 0]
```

rule 1998
```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[A*x^(m - q + 1)*((a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1)/(a*(m + p*q + 1))), x] + Simp[1/(a*(m + p*q +
 1))   Int[x^(m + n - q)*Simp[a*B*(m + p*q + 1) - A*b*(m + p*q + (n - q)*(p
 + 1) + 1) - A*c*(m + p*q + 2*(n - q)*(p + 1) + 1)*x^(n - q), x]*(a*x^q + b
*x^n + c*x^(2*n - q))^p, x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n -
q] && EqQ[j, 2*n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
 && RationalQ[m, p, q] && ((GeQ[p, -1] && LtQ[p, 0]) || EqQ[m + p*q + (n -
q)*(2*p + 1) + 1, 0]) && LeQ[m + p*q, -(n - q)] && NeQ[m + p*q + 1, 0]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.74

method	result
pseudoelliptic	$\frac{b\,x^{3}\left(ac-\frac{b^{2}}{4}\right)\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^{2}+bx+a}}{x\sqrt{a}}\right)+\left(-\frac{x(4cx+b)a^{\frac{3}{2}}}{3}+\frac{\sqrt{a}\,b^{2}x^{2}}{2}-\frac{4a^{\frac{5}{2}}}{3}\right)\sqrt{c\,x^{2}+bx+a}-\ln(2)b\,x^{3}\left(ac-\frac{b^{2}}{4}\right)}{4a^{\frac{5}{2}}x^{3}}$
risch	$-\frac{(8ac\,x^{2}-3b^{2}x^{2}+2abx+8a^{2})\sqrt{x^{2}(c\,x^{2}+bx+a)}}{24x^{4}a^{2}}+\frac{(4ac-b^{2})b\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^{2}+bx+a}}{x}\right)\sqrt{x^{2}(c\,x^{2}+bx+a)}}{16a^{\frac{5}{2}}x\sqrt{c\,x^{2}+bx+a}}$
default	$\frac{\sqrt{c\,x^{4}+b\,x^{3}+a\,x^{2}}\left(12c\,a^{\frac{3}{2}}\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^{2}+bx+a}}{x}\right)b\,x^{3}+6c\sqrt{c\,x^{2}+bx+a}\,b^{2}x^{4}-12c\sqrt{c\,x^{2}+bx+a}\,ab\,x^{3}-3\sqrt{a}\,\ln\left(\frac{2a+b}{}\right.\right.}{48x^{4}\sqrt{c\,x^{2}+b}}$

input
```
int((c*x^4+b*x^3+a*x^2)^(1/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
1/4/a^(5/2)*(b*x^3*(a*c-1/4*b^2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2)
)/x/a^(1/2))+(-1/3*x*(4*c*x+b)*a^(3/2)+1/2*a^(1/2)*b^2*x^2-4/3*a^(5/2))*(c
*x^2+b*x+a)^(1/2)-ln(2)*b*x^3*(a*c-1/4*b^2))/x^3
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.75

$$\int\frac{\sqrt{ax^2+bx^3+cx^4}}{x^5}\,dx$$
$$=\left[-\frac{3\,(b^3-4\,abc)\sqrt{a}x^4\log\left(-\frac{8\,abx^2+(b^2+4\,ac)x^3+8\,a^2x+4\,\sqrt{cx^4+bx^3+ax^2}(bx+2\,a)\sqrt{a}}{x^3}\right)+4\,\sqrt{cx^4+bx^3+ax^2}(2\,a^2bx}{96\,a^3x^4}\right.$$

input
```
integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x^5,x, algorithm="fricas")
```

output
```
[-1/96*(3*(b^3 - 4*a*b*c)*sqrt(a)*x^4*log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3
+ 8*a^2*x + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(a))/x^3) + 4*sq
rt(c*x^4 + b*x^3 + a*x^2)*(2*a^2*b*x + 8*a^3 - (3*a*b^2 - 8*a^2*c)*x^2))/(
a^3*x^4), 1/48*(3*(b^3 - 4*a*b*c)*sqrt(-a)*x^4*arctan(1/2*sqrt(c*x^4 + b*x
^3 + a*x^2)*(b*x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2 + a^2*x)) - 2*sqrt(c*x
^4 + b*x^3 + a*x^2)*(2*a^2*b*x + 8*a^3 - (3*a*b^2 - 8*a^2*c)*x^2))/(a^3*x^
4)]
```

Sympy [F]

$$\int \frac{\sqrt{ax^2 + bx^3 + cx^4}}{x^5}\,dx = \int \frac{\sqrt{x^2\,(a + bx + cx^2)}}{x^5}\,dx$$

input
```
integrate((c*x**4+b*x**3+a*x**2)**(1/2)/x**5,x)
```

output
```
Integral(sqrt(x**2*(a + b*x + c*x**2))/x**5, x)
```

Maxima [F]

$$\int \frac{\sqrt{ax^2 + bx^3 + cx^4}}{x^5}\,dx = \int \frac{\sqrt{cx^4 + bx^3 + ax^2}}{x^5}\,dx$$

input
```
integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x^5,x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^4 + b*x^3 + a*x^2)/x^5, x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^5}\,dx=\text{Timed out}$$

input
```
integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x^5,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^5}\,dx=\int \frac{\sqrt{c\,x^4+b\,x^3+a\,x^2}}{x^5}\,dx$$

input
```
int((a*x^2 + b*x^3 + c*x^4)^(1/2)/x^5,x)
```

output
```
int((a*x^2 + b*x^3 + c*x^4)^(1/2)/x^5, x)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.13

$$\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^5}\,dx$$
$$=\frac{-16\sqrt{c\,x^2+bx+a}\,a^3-4\sqrt{c\,x^2+bx+a}\,a^2bx-16\sqrt{c\,x^2+bx+a}\,a^2c\,x^2+6\sqrt{c\,x^2+bx+a}\,a\,b^2x^2+12}{}$$

input
```
int((c*x^4+b*x^3+a*x^2)^(1/2)/x^5,x)
```

output
```
( - 16*sqrt(a + b*x + c*x**2)*a**3 - 4*sqrt(a + b*x + c*x**2)*a**2*b*x - 1
6*sqrt(a + b*x + c*x**2)*a**2*c*x**2 + 6*sqrt(a + b*x + c*x**2)*a*b**2*x**
2 + 12*sqrt(a)*log( - 2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a - b*x)*a*b*c*
x**3 - 3*sqrt(a)*log( - 2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a - b*x)*b**3
*x**3 - 12*sqrt(a)*log(x)*a*b*c*x**3 + 3*sqrt(a)*log(x)*b**3*x**3)/(48*a**
3*x**3)
```

3.54 $\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^6}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 205

$$\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^6}\,dx = -\frac{\sqrt{ax^2+bx^3+cx^4}}{4x^5}-\frac{b\sqrt{ax^2+bx^3+cx^4}}{24ax^4}+\frac{(5b^2-12ac)\sqrt{ax^2+bx^3+cx^4}}{96a^2x^3}-\frac{b(15b^2-52ac)\sqrt{ax^2+bx^3+cx^4}}{192a^3x^2}+\frac{(b^2-4ac)\,(5b^2-4ac)\operatorname{arctanh}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{128a^{7/2}}$$

output
```
-1/4*(c*x^4+b*x^3+a*x^2)^(1/2)/x^5-1/24*b*(c*x^4+b*x^3+a*x^2)^(1/2)/a/x^4+
1/96*(-12*a*c+5*b^2)*(c*x^4+b*x^3+a*x^2)^(1/2)/a^2/x^3-1/192*b*(-52*a*c+15
*b^2)*(c*x^4+b*x^3+a*x^2)^(1/2)/a^3/x^2+1/128*(-4*a*c+b^2)*(-4*a*c+5*b^2)*
arctanh(1/2*x*(b*x+2*a)/a^(1/2)/(c*x^4+b*x^3+a*x^2)^(1/2))/a^(7/2)
```

Mathematica [A] (verified)

Time = 0.99 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.78

$$\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^6}\,dx =$$
$$-\frac{\sqrt{x^2(a+x(b+cx))}\left(\sqrt{a}\sqrt{a+x(b+cx)}(48a^3+15b^3x^3+8a^2x(b+3cx)-2abx^2(5b+26cx))+3(5b\right.}{192a^{7/2}x^5\sqrt{a+x(b+cx)}}$$

input
```
Integrate[Sqrt[a*x^2 + b*x^3 + c*x^4]/x^6,x]
```

output
```
-1/192*(Sqrt[x^2*(a + x*(b + c*x))]*(Sqrt[a]*Sqrt[a + x*(b + c*x)]*(48*a^3
 + 15*b^3*x^3 + 8*a^2*x*(b + 3*c*x) - 2*a*b*x^2*(5*b + 26*c*x)) + 3*(5*b^4
 - 24*a*b^2*c + 16*a^2*c^2)*x^4*ArcTanh[(Sqrt[c]*x - Sqrt[a + x*(b + c*x)]
)/Sqrt[a]]))/(a^(7/2)*x^5*Sqrt[a + x*(b + c*x)])
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.09, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {1967, 1998, 27, 1998, 27, 1998, 27, 1951, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^6}dx$$

$$\Big\downarrow \text{1967}$$

$$\frac{1}{8}\int \frac{b+2cx}{x^3\sqrt{cx^4+bx^3+ax^2}}dx-\frac{\sqrt{ax^2+bx^3+cx^4}}{4x^5}$$

$$\Big\downarrow \text{1998}$$

$$\frac{1}{8}\left(-\frac{\int \frac{5b^2+4cxb-12ac}{2x^2\sqrt{cx^4+bx^3+ax^2}}dx}{3a}-\frac{b\sqrt{ax^2+bx^3+cx^4}}{3ax^4}\right)-\frac{\sqrt{ax^2+bx^3+cx^4}}{4x^5}$$

$\downarrow$ 27

$$\frac{1}{8}\left(-\frac{\int \frac{5b^2+4cxb-12ac}{x^2\sqrt{cx^4+bx^3+ax^2}}dx}{6a}-\frac{b\sqrt{ax^2+bx^3+cx^4}}{3ax^4}\right)-\frac{\sqrt{ax^2+bx^3+cx^4}}{4x^5}$$

$\downarrow$ 1998

$$\frac{1}{8}\left(-\frac{-\frac{\int \frac{b\left(15b^2-52ac\right)+2c\left(5b^2-12ac\right)x}{2x\sqrt{cx^4+bx^3+ax^2}}dx}{2a}-\frac{(5b^2-12ac)\sqrt{ax^2+bx^3+cx^4}}{2ax^3}}{6a}-\frac{b\sqrt{ax^2+bx^3+cx^4}}{3ax^4}\right)-\frac{\sqrt{ax^2+bx^3+cx^4}}{4x^5}$$

$\downarrow$ 27

$$\frac{1}{8}\left(-\frac{-\frac{\int \frac{b\left(15b^2-52ac\right)+2c\left(5b^2-12ac\right)x}{x\sqrt{cx^4+bx^3+ax^2}}dx}{4a}-\frac{(5b^2-12ac)\sqrt{ax^2+bx^3+cx^4}}{2ax^3}}{6a}-\frac{b\sqrt{ax^2+bx^3+cx^4}}{3ax^4}\right)-\frac{\sqrt{ax^2+bx^3+cx^4}}{4x^5}$$

$\downarrow$ 1998

$$\frac{1}{8}\left(-\frac{-\frac{-\frac{\int \frac{3\left(b^2-4ac\right)\left(5b^2-4ac\right)}{2\sqrt{cx^4+bx^3+ax^2}}dx}{a}-\frac{b\left(15b^2-52ac\right)\sqrt{ax^2+bx^3+cx^4}}{ax^2}}{4a}-\frac{(5b^2-12ac)\sqrt{ax^2+bx^3+cx^4}}{2ax^3}}{6a}-\frac{b\sqrt{ax^2+bx^3+cx^4}}{3ax^4}\right)-\frac{\sqrt{ax^2+bx^3+cx^4}}{4x^5}$$

$\downarrow$ 27

$$\frac{1}{8}\left(-\frac{-\frac{-\frac{3\left(b^2-4ac\right)\left(5b^2-4ac\right)\int \frac{1}{\sqrt{cx^4+bx^3+ax^2}}dx}{2a}-\frac{b\left(15b^2-52ac\right)\sqrt{ax^2+bx^3+cx^4}}{ax^2}}{4a}-\frac{(5b^2-12ac)\sqrt{ax^2+bx^3+cx^4}}{2ax^3}}{6a}-\frac{b\sqrt{ax^2+bx^3+cx^4}}{3ax^4}\right.$$

$$\frac{\sqrt{ax^2+bx^3+cx^4}}{4x^5}$$

$\downarrow$ 1951

$$\frac{1}{8}\left(-\frac{-\frac{\frac{3\left(b^2-4ac\right)\left(5b^2-4ac\right)\int \frac{1}{4a-\frac{x^2(2a+bx)^2}{cx^4+bx^3+ax^2}}d\frac{x(2a+bx)}{\sqrt{cx^4+bx^3+ax^2}}}{a}-\frac{b\left(15b^2-52ac\right)\sqrt{ax^2+bx^3+cx^4}}{ax^2}}{4a}-\frac{(5b^2-12ac)\sqrt{ax^2+bx^3+cx^4}}{2ax^3}}{6a}-\frac{b\sqrt{ax^2}}{}\right.$$

$$\frac{\sqrt{ax^2+bx^3+cx^4}}{4x^5}$$

$\downarrow$ 219

$$\frac{1}{8}\left(-\frac{-\frac{\frac{3\left(b^2-4ac\right)\left(5b^2-4ac\right)\text{arctanh}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{2a^{3/2}}-\frac{b\left(15b^2-52ac\right)\sqrt{ax^2+bx^3+cx^4}}{ax^2}}{4a}-\frac{(5b^2-12ac)\sqrt{ax^2+bx^3+cx^4}}{2ax^3}}{6a}-\frac{b\sqrt{ax^2+b}}{3ax}\right.$$

$$\frac{\sqrt{ax^2+bx^3+cx^4}}{4x^5}$$

input
```
Int[Sqrt[a*x^2 + b*x^3 + c*x^4]/x^6,x]
```

output
```
-1/4*Sqrt[a*x^2 + b*x^3 + c*x^4]/x^5 + (-1/3*(b*Sqrt[a*x^2 + b*x^3 + c*x^4
])/(a*x^4) - (-1/2*((5*b^2 - 12*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(a*x^3)
- (-((b*(15*b^2 - 52*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(a*x^2)) + (3*(b^2
- 4*a*c)*(5*b^2 - 4*a*c)*ArcTanh[(x*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x^2 + b
*x^3 + c*x^4])])/(2*a^(3/2)))/(4*a))/(6*a))/8
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1951
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :
> Simp[-2/(n - 2)   Subst[Int[1/(4*a - x^2), x], x, x*((2*a + b*x^(n - 2))/
Sqrt[a*x^2 + b*x^n + c*x^r])], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*
n - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]
```

rule 1967
```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[x^(m + 1)*((a*x^q + b*x^n + c*x^(2*n - q))^p/(m + p*q
+ 1)), x] - Simp[(n - q)*(p/(m + p*q + 1))   Int[x^(m + n)*(b + 2*c*x^(n -
q))*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c}, x]
&& EqQ[r, 2*n - q] && PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] &
& IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && LeQ[m + p*q + 1, -(n - q) +
 1] && NeQ[m + p*q + 1, 0]
```

rule 1998
```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[A*x^(m - q + 1)*((a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1)/(a*(m + p*q + 1))), x] + Simp[1/(a*(m + p*q +
 1))   Int[x^(m + n - q)*Simp[a*B*(m + p*q + 1) - A*b*(m + p*q + (n - q)*(p
 + 1) + 1) - A*c*(m + p*q + 2*(n - q)*(p + 1) + 1)*x^(n - q), x]*(a*x^q + b
*x^n + c*x^(2*n - q))^p, x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n -
q] && EqQ[j, 2*n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
 && RationalQ[m, p, q] && ((GeQ[p, -1] && LtQ[p, 0]) || EqQ[m + p*q + (n -
q)*(2*p + 1) + 1, 0]) && LeQ[m + p*q, -(n - q)] && NeQ[m + p*q + 1, 0]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.73

method	result
pseudoelliptic	$\frac{\left(ac-\frac{5b^2}{4}\right)x^4\left(ac-\frac{b^2}{4}\right)\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x\sqrt{a}}\right)+\left(\frac{5\left(\frac{26cx}{5}+b\right)b\,x^2a^{\frac{3}{2}}}{12}+\left(-c\,x^2-\frac{1}{3}bx\right)a^{\frac{5}{2}}-\frac{5\sqrt{a}\,b^3x^3}{8}-2a^{\frac{7}{2}}\right)\sqrt{c\,x^2+b \cdots}}{8a^{\frac{7}{2}}x^4}$
risch	$-\frac{\left(-52abc\,x^3+15b^3x^3+24a^2c\,x^2-10a\,b^2x^2+8b\,a^2x+48a^3\right)\sqrt{x^2\left(c\,x^2+bx+a\right)}}{192x^5a^3}+\frac{\left(16a^2c^2-24a\,b^2c+5b^4\right)\ln\left(\frac{2a+bx+2\sqrt{a} \cdots}{\cdots}\right)}{128a^{\frac{7}{2}}x\sqrt{c\,x^2 \cdots}}$
default	$\frac{\sqrt{c\,x^4+b\,x^3+a\,x^2}\left(48c^2a^{\frac{5}{2}}\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x}\right)x^4+24c^2\sqrt{c\,x^2+bx+a}\,ab\,x^5-72c\,a^{\frac{3}{2}}\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x}\right) \cdots\right.}{\cdots}$

input
```
int((c*x^4+b*x^3+a*x^2)^(1/2)/x^6,x,method=_RETURNVERBOSE)
```

output
```
1/8*((a*c-5/4*b^2)*x^4*(a*c-1/4*b^2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(
1/2))/x/a^(1/2))+(5/12*(26/5*c*x+b)*b*x^2*a^(3/2)+(-c*x^2-1/3*b*x)*a^(5/2)
-5/8*a^(1/2)*b^3*x^3-2*a^(7/2))*(c*x^2+b*x+a)^(1/2)-ln(2)*(a*c-5/4*b^2)*x^
4*(a*c-1/4*b^2))/a^(7/2)/x^4
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.64

$$\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^6}\,dx$$
$$= \left[\frac{3\left(5\,b^4-24\,ab^2c+16\,a^2c^2\right)\sqrt{a}x^5\log\left(-\frac{8\,abx^2+(b^2+4\,ac)x^3+8\,a^2x+4\,\sqrt{cx^4+bx^3+ax^2}(bx+2\,a)\sqrt{a}}{x^3}\right)-4\left(8\,a^3bx+48\,a\right.}{768\,a^4x^5}\right.$$
$$-\frac{3\left(5\,b^4-24\,ab^2c+16\,a^2c^2\right)\sqrt{-a}x^5\arctan\left(\frac{\sqrt{cx^4+bx^3+ax^2}(bx+2\,a)\sqrt{-a}}{2\,(acx^3+abx^2+a^2x)}\right)+2\left(8\,a^3bx+48\,a^4+(15\,ab^3-52\right.}{384\,a^4x^5}$$

input
```
integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x^6,x, algorithm="fricas")
```

output
```
[1/768*(3*(5*b^4 - 24*a*b^2*c + 16*a^2*c^2)*sqrt(a)*x^5*log(-(8*a*b*x^2 +
(b^2 + 4*a*c)*x^3 + 8*a^2*x + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sq
rt(a))/x^3) - 4*(8*a^3*b*x + 48*a^4 + (15*a*b^3 - 52*a^2*b*c)*x^3 - 2*(5*a
^2*b^2 - 12*a^3*c)*x^2)*sqrt(c*x^4 + b*x^3 + a*x^2))/(a^4*x^5), -1/384*(3*
(5*b^4 - 24*a*b^2*c + 16*a^2*c^2)*sqrt(-a)*x^5*arctan(1/2*sqrt(c*x^4 + b*x
^3 + a*x^2)*(b*x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2 + a^2*x)) + 2*(8*a^3*b
*x + 48*a^4 + (15*a*b^3 - 52*a^2*b*c)*x^3 - 2*(5*a^2*b^2 - 12*a^3*c)*x^2)*
sqrt(c*x^4 + b*x^3 + a*x^2))/(a^4*x^5)]
```

Sympy [F]

$$\int \frac{\sqrt{a x^2 + b x^3 + c x^4}}{x^6}\, dx = \int \frac{\sqrt{x^2 (a + b x + c x^2)}}{x^6}\, dx$$

input
```
integrate((c*x**4+b*x**3+a*x**2)**(1/2)/x**6,x)
```

output
```
Integral(sqrt(x**2*(a + b*x + c*x**2))/x**6, x)
```

Maxima [F]

$$\int \frac{\sqrt{a x^2 + b x^3 + c x^4}}{x^6}\, dx = \int \frac{\sqrt{c x^4 + b x^3 + a x^2}}{x^6}\, dx$$

input
```
integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x^6,x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^4 + b*x^3 + a*x^2)/x^6, x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{a x^2 + b x^3 + c x^4}}{x^6}\, dx = \text{Timed out}$$

input
```
integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x^6,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^6}\,dx=\int \frac{\sqrt{c\,x^4+b\,x^3+a\,x^2}}{x^6}\,dx$$

input
```
int((a*x^2 + b*x^3 + c*x^4)^(1/2)/x^6,x)
```

output
```
int((a*x^2 + b*x^3 + c*x^4)^(1/2)/x^6, x)
```

Reduce [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.34

$$\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^6}\,dx = \frac{-96\sqrt{c\,x^2+bx+a}\,a^4-16\sqrt{c\,x^2+bx+a}\,a^3bx-48\sqrt{c\,x^2+bx+a}\,a^3c\,x^2+20\sqrt{c\,x^2+bx+a}\,a^2b^2x^2+}{}$$

input
```
int((c*x^4+b*x^3+a*x^2)^(1/2)/x^6,x)
```

output
```
( - 96*sqrt(a + b*x + c*x**2)*a**4 - 16*sqrt(a + b*x + c*x**2)*a**3*b*x -
48*sqrt(a + b*x + c*x**2)*a**3*c*x**2 + 20*sqrt(a + b*x + c*x**2)*a**2*b**
2*x**2 + 104*sqrt(a + b*x + c*x**2)*a**2*b*c*x**3 - 30*sqrt(a + b*x + c*x*
*2)*a*b**3*x**3 + 48*sqrt(a)*log( - 2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a
 - b*x)*a**2*c**2*x**4 - 72*sqrt(a)*log( - 2*sqrt(a)*sqrt(a + b*x + c*x**2
) - 2*a - b*x)*a*b**2*c*x**4 + 15*sqrt(a)*log( - 2*sqrt(a)*sqrt(a + b*x +
c*x**2) - 2*a - b*x)*b**4*x**4 - 48*sqrt(a)*log(x)*a**2*c**2*x**4 + 72*sqr
t(a)*log(x)*a*b**2*c*x**4 - 15*sqrt(a)*log(x)*b**4*x**4)/(384*a**4*x**4)
```

3.55 $\int x(ax^2+bx^3+cx^4)^{3/2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 394

$$\begin{aligned}\int x(ax^2+bx^3&+cx^4)^{3/2}\,dx = \frac{(1155b^6-8988ab^4c+18896a^2b^2c^2-6720a^3c^3)\sqrt{ax^2+bx^3+cx^4}}{286720c^5}\\
&-\frac{b(3465b^6-30660ab^4c+81648a^2b^2c^2-58816a^3c^3)\sqrt{ax^2+bx^3+cx^4}}{573440c^6x}\\
&-\frac{b(231b^4-1560ab^2c+2416a^2c^2)\,x\sqrt{ax^2+bx^3+cx^4}}{71680c^4}\\
&+\frac{(99b^4-568ab^2c+560a^2c^2)\,x^2\sqrt{ax^2+bx^3+cx^4}}{35840c^3}\\
&-\frac{x^3(b(11b^2+68ac)+10c(11b^2-28ac)\,x)\sqrt{ax^2+bx^3+cx^4}}{4480c^2}\\
&+\frac{x(3b+14cx)\,(ax^2+bx^3+cx^4)^{3/2}}{112c}\\
&+\frac{3(b^2-4ac)^2\,(33b^4-72ab^2c+16a^2c^2)\,\text{arctanh}\left(\frac{x(b+2cx)}{2\sqrt{c}\sqrt{ax^2+bx^3+cx^4}}\right)}{32768c^{13/2}}\end{aligned}$$

output
```
1/286720*(-6720*a^3*c^3+18896*a^2*b^2*c^2-8988*a*b^4*c+1155*b^6)*(c*x^4+b*
x^3+a*x^2)^(1/2)/c^5-1/573440*b*(-58816*a^3*c^3+81648*a^2*b^2*c^2-30660*a*
b^4*c+3465*b^6)*(c*x^4+b*x^3+a*x^2)^(1/2)/c^6/x-1/71680*b*(2416*a^2*c^2-15
60*a*b^2*c+231*b^4)*x*(c*x^4+b*x^3+a*x^2)^(1/2)/c^4+1/35840*(560*a^2*c^2-5
68*a*b^2*c+99*b^4)*x^2*(c*x^4+b*x^3+a*x^2)^(1/2)/c^3-1/4480*x^3*(b*(68*a*c
+11*b^2)+10*c*(-28*a*c+11*b^2)*x)*(c*x^4+b*x^3+a*x^2)^(1/2)/c^2+1/112*x*(1
4*c*x+3*b)*(c*x^4+b*x^3+a*x^2)^(3/2)/c+3/32768*(-4*a*c+b^2)^2*(16*a^2*c^2-
72*a*b^2*c+33*b^4)*arctanh(1/2*x*(2*c*x+b)/c^(1/2)/(c*x^4+b*x^3+a*x^2)^(1/
2))/c^(13/2)
```

Mathematica [A] (verified)

Time = 1.56 (sec) , antiderivative size = 304, normalized size of antiderivative = 0.77

$$\int x\left(ax^2+bx^3+cx^4\right)^{3/2}\,dx = \frac{x\sqrt{a+x(b+cx)}\left(2\sqrt{c}\sqrt{a+x(b+cx)}(-3465b^7+2310b^6cx+84b^5c(365a-22cx^2)+24b^4\right.}{}$$

input
```
Integrate[x*(a*x^2 + b*x^3 + c*x^4)^(3/2),x]
```

output
```
(x*Sqrt[a + x*(b + c*x)]*(2*Sqrt[c]*Sqrt[a + x*(b + c*x)]*(-3465*b^7 + 231
0*b^6*c*x + 84*b^5*c*(365*a - 22*c*x^2) + 24*b^4*c^2*x*(-749*a + 66*c*x^2)
 + 32*b^2*c^3*x*(1181*a^2 - 284*a*c*x^2 + 40*c^2*x^4) - 16*b^3*c^2*(5103*a
^2 - 780*a*c*x^2 + 88*c^2*x^4) + 4480*c^4*x*(-3*a^3 + 2*a^2*c*x^2 + 24*a*c
^2*x^4 + 16*c^3*x^6) + 64*b*c^3*(919*a^3 - 302*a^2*c*x^2 + 104*a*c^2*x^4 +
 1360*c^3*x^6)) - 105*(b^2 - 4*a*c)^2*(33*b^4 - 72*a*b^2*c + 16*a^2*c^2)*L
og[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]]))/(1146880*c^(13/2)*Sqrt[x
^2*(a + x*(b + c*x))])
```

Rubi [A] (verified)

Time = 1.14 (sec) , antiderivative size = 459, normalized size of antiderivative = 1.16, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.682, Rules used = {1966, 27, 1992, 27, 1996, 27, 1996, 27, 1996, 27, 1996, 27, 1961, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(ax^2+bx^3+cx^4\right)^{3/2}dx$$

$$\Big\downarrow \text{1966}$$

$$\frac{3\int -\frac{1}{2}x^2\left(8ab+\left(11b^2-28ac\right)x\right)\sqrt{cx^4+bx^3+ax^2}dx}{112c}+\frac{x(3b+14cx)\left(ax^2+bx^3+cx^4\right)^{3/2}}{112c}$$

$$\Big\downarrow \text{27}$$

$$\frac{x(3b+14cx)\left(ax^2+bx^3+cx^4\right)^{3/2}}{112c}-\frac{3\int x^2\left(8ab+\left(11b^2-28ac\right)x\right)\sqrt{cx^4+bx^3+ax^2}dx}{224c}$$

$$\Big\downarrow \text{1992}$$

$$\frac{x(3b+14cx)\left(ax^2+bx^3+cx^4\right)^{3/2}}{112c}-\frac{3\left(\frac{\int -\frac{x^4\left(8ab\left(11b^2-52ac\right)+\left(99b^4-568acb^2+560a^2c^2\right)x\right)}{2\sqrt{cx^4+bx^3+ax^2}}dx}{60c}+\frac{x^3\left(10cx\left(11b^2-28ac\right)+b\left(68ac+11b^2\right)\right)\sqrt{ax^2+bx^3+cx^4}}{60c}\right)}{224c}$$

$$\Big\downarrow \text{27}$$

$$\frac{x(3b+14cx)\left(ax^2+bx^3+cx^4\right)^{3/2}}{112c}-\frac{3\left(\frac{x^3\left(10cx\left(11b^2-28ac\right)+b\left(68ac+11b^2\right)\right)\sqrt{ax^2+bx^3+cx^4}}{60c}-\frac{\int \frac{x^4\left(8ab\left(11b^2-52ac\right)+\left(99b^4-568acb^2+560a^2c^2\right)x\right)}{\sqrt{cx^4+bx^3+ax^2}}dx}{120c}\right)}{224c}$$

$$\Big\downarrow \text{1996}$$

$$\frac{x(3b+14cx)\left(ax^2+bx^3+cx^4\right)^{3/2}}{112c} - \frac{3\left(\frac{x^3\left(10cx\left(11b^2-28ac\right)+b\left(68ac+11b^2\right)\right)\sqrt{ax^2+bx^3+cx^4}}{60c} - \frac{\frac{x^2\left(560a^2c^2-568ab^2c+99b^4\right)\sqrt{ax^2+bx^3+cx^4}}{4c} - \frac{\int \frac{3x^3\left(2a\left(99b^4-568acb^2+560a^2c^2\right)+b\left(\ldots\right.\right.}{2\sqrt{cx^4+bx^3\ldots}}}{4c}}{120c}\ldots\right.}{224c}$$

$$\Big\downarrow \; 27$$

$$\frac{x(3b+14cx)\left(ax^2+bx^3+cx^4\right)^{3/2}}{112c} - \frac{3\left(\frac{x^3\left(10cx\left(11b^2-28ac\right)+b\left(68ac+11b^2\right)\right)\sqrt{ax^2+bx^3+cx^4}}{60c} - \frac{\frac{x^2\left(560a^2c^2-568ab^2c+99b^4\right)\sqrt{ax^2+bx^3+cx^4}}{4c} - \frac{3\int \frac{x^3\left(2a\left(99b^4-568acb^2+560a^2c^2\right)+b\left(\ldots\right.\right.}{\sqrt{cx^4+bx^3\ldots}}}{8c}}{120c}\ldots\right.}{224c}$$

$$\Big\downarrow \; 1996$$

$$\frac{x(3b+14cx)\left(ax^2+bx^3+cx^4\right)^{3/2}}{112c} - \frac{3\left(\frac{x^3\left(10cx\left(11b^2-28ac\right)+b\left(68ac+11b^2\right)\right)\sqrt{ax^2+bx^3+cx^4}}{60c} - \frac{\frac{x^2\left(560a^2c^2-568ab^2c+99b^4\right)\sqrt{ax^2+bx^3+cx^4}}{4c} - \frac{3\left(\frac{bx\left(2416a^2c^2-1560ab^2c+231b^4\right)\sqrt{ax\ldots}}{3c}\ldots\right.}{\ldots}}{\ldots}\ldots\right.}{224c}$$

$$\Big\downarrow \; 27$$

$$\frac{x(3b+14cx)\left(ax^2+bx^3+cx^4\right)^{3/2}}{112c} - \frac{3\left(\frac{x^3\left(10cx\left(11b^2-28ac\right)+b\left(68ac+11b^2\right)\right)\sqrt{ax^2+bx^3+cx^4}}{60c} - \frac{\frac{x^2\left(560a^2c^2-568ab^2c+99b^4\right)\sqrt{ax^2+bx^3+cx^4}}{4c} - \frac{3\left(\frac{bx\left(2416a^2c^2-1560ab^2c+231b^4\right)\sqrt{ax\ldots}}{3c}\ldots\right.}{\ldots}}{\ldots}\ldots\right.}{224c}$$

$$\Big\downarrow \; 1996$$

$$\frac{x(3b+14cx)\left(ax^2+bx^3+cx^4\right)^{3/2}}{112c}-$$
$$3\left(\frac{x^3\left(10cx\left(11b^2-28ac\right)+b\left(68ac+11b^2\right)\right)\sqrt{ax^2+bx^3+cx^4}}{60c}-\frac{\frac{x^2\left(560a^2c^2-568ab^2c+99b^4\right)\sqrt{ax^2+bx^3+cx^4}}{4c}-\frac{3\left(\frac{bx\left(2416a^2c^2-1560ab^2c+231b^4\right)\sqrt{ax}}{3c}\right.}{}}{}\right.$$

$\downarrow$ 27

$$\frac{x(3b+14cx)\left(ax^2+bx^3+cx^4\right)^{3/2}}{112c}-$$
$$3\left(\frac{x^3\left(10cx\left(11b^2-28ac\right)+b\left(68ac+11b^2\right)\right)\sqrt{ax^2+bx^3+cx^4}}{60c}-\frac{\frac{x^2\left(560a^2c^2-568ab^2c+99b^4\right)\sqrt{ax^2+bx^3+cx^4}}{4c}-\frac{3\left(\frac{bx\left(2416a^2c^2-1560ab^2c+231b^4\right)\sqrt{ax}}{3c}\right.}{}}{}\right.$$

$\downarrow$ 1996

$$\frac{x(3b+14cx)\left(ax^2+bx^3+cx^4\right)^{3/2}}{112c}-$$
$$3\left(\frac{x^3\left(10cx\left(11b^2-28ac\right)+b\left(68ac+11b^2\right)\right)\sqrt{ax^2+bx^3+cx^4}}{60c}-\frac{\frac{x^2\left(560a^2c^2-568ab^2c+99b^4\right)\sqrt{ax^2+bx^3+cx^4}}{4c}-\frac{3\left(\frac{bx\left(2416a^2c^2-1560ab^2c+231b^4\right)\sqrt{ax}}{3c}\right.}{}}{}\right.$$

$\downarrow$ 27

$$\frac{x(3b+14cx)\left(ax^2+bx^3+cx^4\right)^{3/2}}{112c}-$$

$$3\left(\frac{x^3\left(10cx\left(11b^2-28ac\right)+b\left(68ac+11b^2\right)\right)\sqrt{ax^2+bx^3+cx^4}}{60c}-\frac{\frac{x^2\left(560a^2c^2-568ab^2c+99b^4\right)\sqrt{ax^2+bx^3+cx^4}}{4c}-\frac{3\left(\frac{bx\left(2416a^2c^2-1560ab^2c+231b^4\right)\sqrt{ax}}{3c}\right.}{}}{}\right.$$

$\downarrow$ 1961

$$\frac{x(3b+14cx)\left(ax^2+bx^3+cx^4\right)^{3/2}}{112c}-$$

$$3\left(\frac{x^3\left(10cx\left(11b^2-28ac\right)+b\left(68ac+11b^2\right)\right)\sqrt{ax^2+bx^3+cx^4}}{60c}-\frac{\frac{x^2\left(560a^2c^2-568ab^2c+99b^4\right)\sqrt{ax^2+bx^3+cx^4}}{4c}-\frac{3\left(\frac{bx\left(2416a^2c^2-1560ab^2c+231b^4\right)\sqrt{ax}}{3c}\right.}{}}{}\right.$$

$\downarrow$ 1092

$$\frac{x(3b+14cx)\left(ax^2+bx^3+cx^4\right)^{3/2}}{112c}-$$

$$3\left(\frac{x^3\left(10cx\left(11b^2-28ac\right)+b\left(68ac+11b^2\right)\right)\sqrt{ax^2+bx^3+cx^4}}{60c}-\frac{\frac{x^2\left(560a^2c^2-568ab^2c+99b^4\right)\sqrt{ax^2+bx^3+cx^4}}{4c}-\frac{3\left(\frac{bx\left(2416a^2c^2-1560ab^2c+231b^4\right)\sqrt{ax}}{3c}\right.}{}}{}\right.$$

$\downarrow$ 219

$$\frac{x(3b+14cx)\left(ax^2+bx^3+cx^4\right)^{3/2}}{112c}-$$

$$3\left(\frac{x^3\left(10cx\left(11b^2-28ac\right)+b\left(68ac+11b^2\right)\right)\sqrt{ax^2+bx^3+cx^4}}{60c}-\frac{\frac{x^2\left(560a^2c^2-568ab^2c+99b^4\right)\sqrt{ax^2+bx^3+cx^4}}{4c}-\frac{3\left(\frac{bx\left(2416a^2c^2-1560ab^2c+231b^4\right)\sqrt{ax}}{3c}\right.}{}}{}\right.$$

input
```
Int[x*(a*x^2 + b*x^3 + c*x^4)^(3/2),x]
```

output
```
(x*(3*b + 14*c*x)*(a*x^2 + b*x^3 + c*x^4)^(3/2))/(112*c) - (3*((x^3*(b*(11
*b^2 + 68*a*c) + 10*c*(11*b^2 - 28*a*c)*x)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(6
0*c) - (((99*b^4 - 568*a*b^2*c + 560*a^2*c^2)*x^2*Sqrt[a*x^2 + b*x^3 + c*x
^4])/(4*c) - (3*((b*(231*b^4 - 1560*a*b^2*c + 2416*a^2*c^2)*x*Sqrt[a*x^2 +
 b*x^3 + c*x^4])/(3*c) - (((1155*b^6 - 8988*a*b^4*c + 18896*a^2*b^2*c^2 -
6720*a^3*c^3)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(2*c) - ((b*(3465*b^6 - 30660*a
*b^4*c + 81648*a^2*b^2*c^2 - 58816*a^3*c^3)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(
c*x) - (105*(b^2 - 4*a*c)^2*(33*b^4 - 72*a*b^2*c + 16*a^2*c^2)*x*Sqrt[a +
b*x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*c^
(3/2)*Sqrt[a*x^2 + b*x^3 + c*x^4]))/(4*c))/(6*c)))/(8*c))/(120*c)))/(224*c
)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1961
```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Simp[x^(q/2)*(Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]/Sqrt[a
*x^q + b*x^n + c*x^(2*n - q)])   Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x
^(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] &&
 PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] ||
 EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))
```

rule 1966
```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[x^(m - n + q + 1)*(b*(n - q)*p + c*(m + p*q + (n - q)*
(2*p - 1) + 1)*x^(n - q))*((a*x^q + b*x^n + c*x^(2*n - q))^p/(c*(m + p*(2*n
 - q) + 1)*(m + p*q + (n - q)*(2*p - 1) + 1))), x] + Simp[(n - q)*(p/(c*(m
+ p*(2*n - q) + 1)*(m + p*q + (n - q)*(2*p - 1) + 1)))   Int[x^(m - (n - 2*
q))*Simp[(-a)*b*(m + p*q - n + q + 1) + (2*a*c*(m + p*q + (n - q)*(2*p - 1)
 + 1) - b^2*(m + p*q + (n - q)*(p - 1) + 1))*x^(n - q), x]*(a*x^q + b*x^n +
 c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] &
& PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[
p, 0] && RationalQ[m, q] && GtQ[m + p*q + 1, n - q] && NeQ[m + p*(2*n - q)
+ 1, 0] && NeQ[m + p*q + (n - q)*(2*p - 1) + 1, 0]
```

rule 1992
```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[x^(m + 1)*(b*B*(n - q)*p +
A*c*(m + p*q + (n - q)*(2*p + 1) + 1) + B*c*(m + p*q + 2*(n - q)*p + 1)*x^(
n - q))*((a*x^q + b*x^n + c*x^(2*n - q))^p/(c*(m + p*(2*n - q) + 1)*(m + p*
q + (n - q)*(2*p + 1) + 1))), x] + Simp[(n - q)*(p/(c*(m + p*(2*n - q) + 1)
*(m + p*q + (n - q)*(2*p + 1) + 1)))   Int[x^(m + q)*Simp[2*a*A*c*(m + p*q
+ (n - q)*(2*p + 1) + 1) - a*b*B*(m + p*q + 1) + (2*a*B*c*(m + p*q + 2*(n -
 q)*p + 1) + A*b*c*(m + p*q + (n - q)*(2*p + 1) + 1) - b^2*B*(m + p*q + (n
- q)*p + 1))*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x]
/; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] &&  !Integ
erQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q]
 && GtQ[m + p*q, -(n - q) - 1] && NeQ[m + p*(2*n - q) + 1, 0] && NeQ[m + p*
q + (n - q)*(2*p + 1) + 1, 0]
```

rule 1996
```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[B*x^(m - n + 1)*((a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1)/(c*(m + p*q + (n - q)*(2*p + 1) + 1))), x] -
Simp[1/(c*(m + p*q + (n - q)*(2*p + 1) + 1))   Int[x^(m - n + q)*Simp[a*B*(
m + p*q - n + q + 1) + (b*B*(m + p*q + (n - q)*p + 1) - A*c*(m + p*q + (n -
 q)*(2*p + 1) + 1))*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^p, x], x]
 /; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] &&  !Inte
gerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[p, -1] && LtQ[p, 0] &&
RationalQ[m, q] && GeQ[m + p*q, n - q - 1] && NeQ[m + p*q + (n - q)*(2*p +
1) + 1, 0]
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 328, normalized size of antiderivative = 0.83

method	result
risch	$\frac{(71680c^7x^7+87040b\,c^6x^6+107520a\,c^6x^5+1280b^2c^5x^5+6656ab\,c^5x^4-1408b^3c^4x^4+8960a^2c^5x^3-9088a\,b^2c^4x^3+1584b^4c^3x^3-1932}{}$
default	$\frac{(c\,x^4+b\,x^3+a\,x^2)^{\frac{3}{2}}\left(85680c^{\frac{7}{2}}\sqrt{c\,x^2+bx+a}\,a\,b^4x-80640c^{\frac{9}{2}}(c\,x^2+bx+a)^{\frac{3}{2}}a\,b^2x-127680c^{\frac{9}{2}}\sqrt{c\,x^2+bx+a}\,a^2b^2x-134400\ln\left(\frac{2\sqrt{c\,x^2}}{}\right.\right.}{}$

input
```
int(x*(c*x^4+b*x^3+a*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/573440*(71680*c^7*x^7+87040*b*c^6*x^6+107520*a*c^6*x^5+1280*b^2*c^5*x^5+
6656*a*b*c^5*x^4-1408*b^3*c^4*x^4+8960*a^2*c^5*x^3-9088*a*b^2*c^4*x^3+1584
*b^4*c^3*x^3-19328*a^2*b*c^4*x^2+12480*a*b^3*c^3*x^2-1848*b^5*c^2*x^2-1344
0*a^3*c^4*x+37792*a^2*b^2*c^3*x-17976*a*b^4*c^2*x+2310*b^6*c*x+58816*a^3*b
*c^3-81648*a^2*b^3*c^2+30660*a*b^5*c-3465*b^7)/c^6*(x^2*(c*x^2+b*x+a))^(1/
2)/x+3/32768*(256*a^4*c^4-1280*a^3*b^2*c^3+1120*a^2*b^4*c^2-336*a*b^6*c+33
*b^8)/c^(13/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*(x^2*(c*x^2+b*x
+a))^(1/2)/x/(c*x^2+b*x+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 664, normalized size of antiderivative = 1.69

$$\int x\left(ax^2+bx^3+cx^4\right)^{3/2}\,dx = \left[\frac{105\,(33\,b^8-336\,ab^6c+1120\,a^2b^4c^2-1280\,a^3b^2c^3+256\,a^4c^4)\sqrt{c}x\log\left(-\frac{8\,c^2x^3+8\,bcx^2+4\,\sqrt{cx^4\cdots}}{}\right.}{}\right.$$

$$-\frac{105\,(33\,b^8-336\,ab^6c+1120\,a^2b^4c^2-1280\,a^3b^2c^3+256\,a^4c^4)\sqrt{-c}x\arctan\left(\frac{\sqrt{cx^4+bx^3+ax^2}(2\,cx+b)\sqrt{-c}}{2\,(c^2x^3+bcx^2+acx)}\right)-2\cdots}{}$$

input
```
integrate(x*(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/2293760*(105*(33*b^8 - 336*a*b^6*c + 1120*a^2*b^4*c^2 - 1280*a^3*b^2*c^
3 + 256*a^4*c^4)*sqrt(c)*x*log(-(8*c^2*x^3 + 8*b*c*x^2 + 4*sqrt(c*x^4 + b*
x^3 + a*x^2)*(2*c*x + b)*sqrt(c) + (b^2 + 4*a*c)*x)/x) + 4*(71680*c^8*x^7
+ 87040*b*c^7*x^6 - 3465*b^7*c + 30660*a*b^5*c^2 - 81648*a^2*b^3*c^3 + 588
16*a^3*b*c^4 + 1280*(b^2*c^6 + 84*a*c^7)*x^5 - 128*(11*b^3*c^5 - 52*a*b*c^
6)*x^4 + 16*(99*b^4*c^4 - 568*a*b^2*c^5 + 560*a^2*c^6)*x^3 - 8*(231*b^5*c^
3 - 1560*a*b^3*c^4 + 2416*a^2*b*c^5)*x^2 + 2*(1155*b^6*c^2 - 8988*a*b^4*c^
3 + 18896*a^2*b^2*c^4 - 6720*a^3*c^5)*x)*sqrt(c*x^4 + b*x^3 + a*x^2))/(c^7
*x), -1/1146880*(105*(33*b^8 - 336*a*b^6*c + 1120*a^2*b^4*c^2 - 1280*a^3*b
^2*c^3 + 256*a^4*c^4)*sqrt(-c)*x*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2
*c*x + b)*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x)) - 2*(71680*c^8*x^7 + 87040
*b*c^7*x^6 - 3465*b^7*c + 30660*a*b^5*c^2 - 81648*a^2*b^3*c^3 + 58816*a^3*
b*c^4 + 1280*(b^2*c^6 + 84*a*c^7)*x^5 - 128*(11*b^3*c^5 - 52*a*b*c^6)*x^4
+ 16*(99*b^4*c^4 - 568*a*b^2*c^5 + 560*a^2*c^6)*x^3 - 8*(231*b^5*c^3 - 156
0*a*b^3*c^4 + 2416*a^2*b*c^5)*x^2 + 2*(1155*b^6*c^2 - 8988*a*b^4*c^3 + 188
96*a^2*b^2*c^4 - 6720*a^3*c^5)*x)*sqrt(c*x^4 + b*x^3 + a*x^2))/(c^7*x)]
```

Sympy [F]

$$\int x(ax^2+bx^3+cx^4)^{3/2}\,dx=\int x\left(x^2(a+bx+cx^2)\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(x*(c*x**4+b*x**3+a*x**2)**(3/2),x)
```

output
```
Integral(x*(x**2*(a + b*x + c*x**2))**(3/2), x)
```

Maxima [F]

$$\int x(ax^2+bx^3+cx^4)^{3/2}\,dx=\int (cx^4+bx^3+ax^2)^{\frac{3}{2}}x\,dx$$

input
```
integrate(x*(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*x^4 + b*x^3 + a*x^2)^(3/2)*x, x)
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 509, normalized size of antiderivative = 1.29

$$\int x(ax^2+bx^3+cx^4)^{3/2}\,dx=\text{Too large to display}$$

input
```
integrate(x*(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

output
```
1/573440*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*(10*(4*(14*c*x*sgn(x) + 17*b*sg
n(x))*x + (b^2*c^6*sgn(x) + 84*a*c^7*sgn(x))/c^7)*x - (11*b^3*c^5*sgn(x) -
 52*a*b*c^6*sgn(x))/c^7)*x + (99*b^4*c^4*sgn(x) - 568*a*b^2*c^5*sgn(x) + 5
60*a^2*c^6*sgn(x))/c^7)*x - (231*b^5*c^3*sgn(x) - 1560*a*b^3*c^4*sgn(x) +
2416*a^2*b*c^5*sgn(x))/c^7)*x + (1155*b^6*c^2*sgn(x) - 8988*a*b^4*c^3*sgn(
x) + 18896*a^2*b^2*c^4*sgn(x) - 6720*a^3*c^5*sgn(x))/c^7)*x - (3465*b^7*c*
sgn(x) - 30660*a*b^5*c^2*sgn(x) + 81648*a^2*b^3*c^3*sgn(x) - 58816*a^3*b*c
^4*sgn(x))/c^7) - 3/32768*(33*b^8*sgn(x) - 336*a*b^6*c*sgn(x) + 1120*a^2*b
^4*c^2*sgn(x) - 1280*a^3*b^2*c^3*sgn(x) + 256*a^4*c^4*sgn(x))*log(abs(2*(s
qrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) + b))/c^(13/2) + 1/1146880*(3465
*b^8*log(abs(b - 2*sqrt(a)*sqrt(c))) - 35280*a*b^6*c*log(abs(b - 2*sqrt(a)
*sqrt(c))) + 117600*a^2*b^4*c^2*log(abs(b - 2*sqrt(a)*sqrt(c))) - 134400*a
^3*b^2*c^3*log(abs(b - 2*sqrt(a)*sqrt(c))) + 26880*a^4*c^4*log(abs(b - 2*s
qrt(a)*sqrt(c))) + 6930*sqrt(a)*b^7*sqrt(c) - 61320*a^(3/2)*b^5*c^(3/2) +
163296*a^(5/2)*b^3*c^(5/2) - 117632*a^(7/2)*b*c^(7/2))*sgn(x)/c^(13/2)
```

Mupad [F(-1)]

Timed out.

$$\int x\left(ax^2+bx^3+cx^4\right)^{3/2}\,dx=\int x\,\left(c\,x^4+b\,x^3+a\,x^2\right)^{3/2}\,dx$$

input
```
int(x*(a*x^2 + b*x^3 + c*x^4)^(3/2),x)
```

output
```
int(x*(a*x^2 + b*x^3 + c*x^4)^(3/2), x)
```

Reduce [F]

$$\int x\left(ax^2+bx^3+cx^4\right)^{3/2}\,dx=\int x\left(c\,x^4+b\,x^3+a\,x^2\right)^{\frac{3}{2}}dx$$

input
```
int(x*(c*x^4+b*x^3+a*x^2)^(3/2),x)
```

output
```
int(x*(c*x^4+b*x^3+a*x^2)^(3/2),x)
```

3.56 $\int \left(ax^2+bx^3+cx^4\right)^{3/2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 336

$$\begin{aligned}\int \left(ax^2+bx^3+cx^4\right)^{3/2}\,dx = &-\frac{b(105b^4-728ab^2c+1168a^2c^2)\sqrt{ax^2+bx^3+cx^4}}{17920c^4}\\&+\frac{(315b^6-2520ab^4c+5488a^2b^2c^2-2048a^3c^3)\sqrt{ax^2+bx^3+cx^4}}{35840c^5x}\\&+\frac{(7b^2-32ac)\,(3b^2-4ac)\,x\sqrt{ax^2+bx^3+cx^4}}{4480c^3}\\&-\frac{b(9b^2-44ac)\,x^2\sqrt{ax^2+bx^3+cx^4}}{2240c^2}\\&+\frac{x^3(b^2+24ac+10bcx)\sqrt{ax^2+bx^3+cx^4}}{280c}\\&+\frac{1}{7}x\left(ax^2+bx^3+cx^4\right)^{3/2}-\frac{3b(b^2-4ac)^2\,(3b^2-4ac)\,\text{arctanh}\left(\frac{x(b+2cx)}{2\sqrt{c}\sqrt{ax^2+bx^3+cx^4}}\right)}{2048c^{11/2}}\end{aligned}$$

output
```
-1/17920*b*(1168*a^2*c^2-728*a*b^2*c+105*b^4)*(c*x^4+b*x^3+a*x^2)^(1/2)/c^
4+1/35840*(-2048*a^3*c^3+5488*a^2*b^2*c^2-2520*a*b^4*c+315*b^6)*(c*x^4+b*x
^3+a*x^2)^(1/2)/c^5/x+1/4480*(-32*a*c+7*b^2)*(-4*a*c+3*b^2)*x*(c*x^4+b*x^3
+a*x^2)^(1/2)/c^3-1/2240*b*(-44*a*c+9*b^2)*x^2*(c*x^4+b*x^3+a*x^2)^(1/2)/c
^2+1/280*x^3*(10*b*c*x+24*a*c+b^2)*(c*x^4+b*x^3+a*x^2)^(1/2)/c+1/7*x*(c*x^
4+b*x^3+a*x^2)^(3/2)-3/2048*b*(-4*a*c+b^2)^2*(-4*a*c+3*b^2)*arctanh(1/2*x*
(2*c*x+b)/c^(1/2)/(c*x^4+b*x^3+a*x^2)^(1/2))/c^(11/2)
```

Mathematica [A] (verified)

Time = 1.18 (sec) , antiderivative size = 239, normalized size of antiderivative = 0.71

$$\int (ax^2+bx^3+cx^4)^{3/2}\,dx = \frac{x\sqrt{a+x(b+cx)}\Big(2\sqrt{c}\sqrt{a+x(b+cx)}\Big(315b^6-210b^5cx+16b^3c^2x(91a-9cx^2)+168b^4c(-$$

input
```
Integrate[(a*x^2 + b*x^3 + c*x^4)^(3/2),x]
```

output
```
(x*Sqrt[a + x*(b + c*x)]*(2*Sqrt[c]*Sqrt[a + x*(b + c*x)]*(315*b^6 - 210*b
^5*c*x + 16*b^3*c^2*x*(91*a - 9*c*x^2) + 168*b^4*c*(-15*a + c*x^2) + 1024*
c^3*(a + c*x^2)^2*(-2*a + 5*c*x^2) + 16*b^2*c^2*(343*a^2 - 62*a*c*x^2 + 8*
c^2*x^4) + 32*b*c^3*x*(-73*a^2 + 22*a*c*x^2 + 200*c^2*x^4)) + 105*b*(b^2 -
 4*a*c)^2*(3*b^2 - 4*a*c)*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]]
))/(71680*c^(11/2)*Sqrt[x^2*(a + x*(b + c*x))])
```

Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 398, normalized size of antiderivative = 1.18, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used = {1953, 1992, 27, 1996, 27, 1996, 27, 1996, 27, 1996, 27, 1961, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(ax^2+bx^3+cx^4\right)^{3/2}\,dx$$

$$\Big\downarrow \; 1953$$

$$\frac{3}{14}\int x^2(2a+bx)\sqrt{cx^4+bx^3+ax^2}dx+\frac{1}{7}x\left(ax^2+bx^3+cx^4\right)^{3/2}$$

$$\Big\downarrow \; 1992$$

$$\frac{3}{14}\left(\frac{\int -\frac{x^4\left(8a\left(b^2-6ac\right)+b\left(9b^2-44ac\right)x\right)}{2\sqrt{cx^4+bx^3+ax^2}}dx}{60c}+\frac{x^3\left(24ac+b^2+10bcx\right)\sqrt{ax^2+bx^3+cx^4}}{60c}\right)+$$
$$\frac{1}{7}x\left(ax^2+bx^3+cx^4\right)^{3/2}$$

$$\Big\downarrow \; 27$$

$$\frac{3}{14}\left(\frac{x^3\left(24ac+b^2+10bcx\right)\sqrt{ax^2+bx^3+cx^4}}{60c}-\frac{\int \frac{x^4\left(8a\left(b^2-6ac\right)+b\left(9b^2-44ac\right)x\right)}{\sqrt{cx^4+bx^3+ax^2}}dx}{120c}\right)+$$
$$\frac{1}{7}x\left(ax^2+bx^3+cx^4\right)^{3/2}$$

$$\Big\downarrow \; 1996$$

$$\frac{3}{14}\left(\frac{x^3\left(24ac+b^2+10bcx\right)\sqrt{ax^2+bx^3+cx^4}}{60c}-\frac{\frac{bx^2\left(9b^2-44ac\right)\sqrt{ax^2+bx^3+cx^4}}{4c}-\frac{\int \frac{3x^3\left(2ab\left(9b^2-44ac\right)+\left(7b^2-32ac\right)\left(3b^2-4ac\right)}{2\sqrt{cx^4+bx^3+ax^2}}}{4c}}{120c}\right.$$
$$\frac{1}{7}x\left(ax^2+bx^3+cx^4\right)^{3/2}$$

$$\Big\downarrow \; 27$$

$$\frac{3}{14}\left(\frac{x^3\left(24ac+b^2+10bcx\right)\sqrt{ax^2+bx^3+cx^4}}{60c}-\frac{\frac{bx^2\left(9b^2-44ac\right)\sqrt{ax^2+bx^3+cx^4}}{4c}-\frac{3\int \frac{x^3\left(2ab\left(9b^2-44ac\right)+\left(7b^2-32ac\right)\left(3b^2-4ac\right)}{\sqrt{cx^4+bx^3+ax^2}}}{8c}}{120c}\right.$$
$$\frac{1}{7}x\left(ax^2+bx^3+cx^4\right)^{3/2}$$

$$\Big\downarrow \; 1996$$

$$\frac{3}{14}\left(\frac{x^3\left(24ac+b^2+10bcx\right)\sqrt{ax^2+bx^3+cx^4}}{60c}-\frac{\frac{bx^2\left(9b^2-44ac\right)\sqrt{ax^2+bx^3+cx^4}}{4c}-\frac{3\left(\frac{x\left(7b^2-32ac\right)\left(3b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}}{3c}-\cdots\right.}{\cdots}}{120c}\cdots\right.$$

$$\frac{1}{7}x\left(ax^2+bx^3+cx^4\right)^{3/2}$$

$\big\downarrow$ 27

$$\frac{3}{14}\left(\frac{x^3\left(24ac+b^2+10bcx\right)\sqrt{ax^2+bx^3+cx^4}}{60c}-\frac{\frac{bx^2\left(9b^2-44ac\right)\sqrt{ax^2+bx^3+cx^4}}{4c}-\frac{3\left(\frac{x\left(7b^2-32ac\right)\left(3b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}}{3c}-\cdots\right.}{\cdots}}{120c}\cdots\right.$$

$$\frac{1}{7}x\left(ax^2+bx^3+cx^4\right)^{3/2}$$

$\big\downarrow$ 1996

$$\frac{3}{14}\left(\frac{x^3\left(24ac+b^2+10bcx\right)\sqrt{ax^2+bx^3+cx^4}}{60c}-\frac{\frac{bx^2\left(9b^2-44ac\right)\sqrt{ax^2+bx^3+cx^4}}{4c}-\frac{3\left(\frac{x\left(7b^2-32ac\right)\left(3b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}}{3c}-\cdots\right.}{\cdots}}{\cdots}\cdots\right.$$

$$\frac{1}{7}x\left(ax^2+bx^3+cx^4\right)^{3/2}$$

$\big\downarrow$ 27

$$\frac{3}{14}\left(\frac{x^3\left(24ac+b^2+10bcx\right)\sqrt{ax^2+bx^3+cx^4}}{60c}-\frac{\frac{bx^2\left(9b^2-44ac\right)\sqrt{ax^2+bx^3+cx^4}}{4c}-\frac{3\left(\frac{x\left(7b^2-32ac\right)\left(3b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}}{3c}-\cdots\right.}{\cdots}}{\cdots}\right.$$

$$\frac{1}{7}x\left(ax^2+bx^3+cx^4\right)^{3/2}$$

$\downarrow$ 1996

$$\frac{3}{14}\left(\frac{x^3\left(24ac+b^2+10bcx\right)\sqrt{ax^2+bx^3+cx^4}}{60c}-\frac{\frac{bx^2\left(9b^2-44ac\right)\sqrt{ax^2+bx^3+cx^4}}{4c}-\frac{3\left(\frac{x\left(7b^2-32ac\right)\left(3b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}}{3c}-\cdots\right.}{\cdots}}{\cdots}\right.$$

$$\frac{1}{7}x\left(ax^2+bx^3+cx^4\right)^{3/2}$$

$\downarrow$ 27

$$\frac{3}{14}\left(\frac{x^3\left(24ac+b^2+10bcx\right)\sqrt{ax^2+bx^3+cx^4}}{60c}-\frac{\frac{bx^2\left(9b^2-44ac\right)\sqrt{ax^2+bx^3+cx^4}}{4c}-\frac{3\left(\frac{x\left(7b^2-32ac\right)\left(3b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}}{3c}-\cdots\right.}{\cdots}}{\cdots}\right.$$

$$\frac{1}{7}x\left(ax^2+bx^3+cx^4\right)^{3/2}$$

$\downarrow$ 1961

$$\frac{3}{14}\left(\frac{x^3\left(24ac+b^2+10bcx\right)\sqrt{ax^2+bx^3+cx^4}}{60c}-\frac{\frac{bx^2\left(9b^2-44ac\right)\sqrt{ax^2+bx^3+cx^4}}{4c}-\frac{3\left(\frac{x\left(7b^2-32ac\right)\left(3b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}}{3c}-\frac{b}{}\right.}{}}{}\right.$$

$$\frac{1}{7}x\left(ax^2+bx^3+cx^4\right)^{3/2}$$

$\downarrow$ 1092

$$\frac{3}{14}\left(\frac{x^3\left(24ac+b^2+10bcx\right)\sqrt{ax^2+bx^3+cx^4}}{60c}-\frac{\frac{bx^2\left(9b^2-44ac\right)\sqrt{ax^2+bx^3+cx^4}}{4c}-\frac{3\left(\frac{x\left(7b^2-32ac\right)\left(3b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}}{3c}-\frac{b}{}\right.}{}}{}\right.$$

$$\frac{1}{7}x\left(ax^2+bx^3+cx^4\right)^{3/2}$$

$\downarrow$ 219

$$\frac{3}{14}\left(\frac{x^3\left(24ac+b^2+10bcx\right)\sqrt{ax^2+bx^3+cx^4}}{60c}-\frac{\frac{bx^2\left(9b^2-44ac\right)\sqrt{ax^2+bx^3+cx^4}}{4c}-\frac{3\left(\frac{x\left(7b^2-32ac\right)\left(3b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}}{3c}-\frac{b}{}\right.}{}}{}\right.$$

$$\frac{1}{7}x\left(ax^2+bx^3+cx^4\right)^{3/2}$$

input
```
Int[(a*x^2 + b*x^3 + c*x^4)^(3/2),x]
```

output
```
(x*(a*x^2 + b*x^3 + c*x^4)^(3/2))/7 + (3*((x^3*(b^2 + 24*a*c + 10*b*c*x)*S
qrt[a*x^2 + b*x^3 + c*x^4])/(60*c) - ((b*(9*b^2 - 44*a*c)*x^2*Sqrt[a*x^2 +
 b*x^3 + c*x^4])/(4*c) - (3*(((7*b^2 - 32*a*c)*(3*b^2 - 4*a*c)*x*Sqrt[a*x^
2 + b*x^3 + c*x^4])/(3*c) - ((b*(105*b^4 - 728*a*b^2*c + 1168*a^2*c^2)*Sqr
t[a*x^2 + b*x^3 + c*x^4])/(2*c) - (((315*b^6 - 2520*a*b^4*c + 5488*a^2*b^2
*c^2 - 2048*a^3*c^3)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(c*x) - (105*b*(b^2 - 4*
a*c)^2*(3*b^2 - 4*a*c)*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt
[c]*Sqrt[a + b*x + c*x^2])])/(2*c^(3/2)*Sqrt[a*x^2 + b*x^3 + c*x^4]))/(4*c
))/(6*c)))/(8*c))/(120*c)))/14
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1953
```
Int[((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_), x_Symbol
] :> Simp[x*((a*x^q + b*x^n + c*x^(2*n - q))^p/(p*(2*n - q) + 1)), x] + Sim
p[(n - q)*(p/(p*(2*n - q) + 1))   Int[x^q*(2*a + b*x^(n - q))*(a*x^q + b*x^
n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c, n, q}, x] && EqQ[r, 2
*n - q] && PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0]
 && NeQ[p*(2*n - q) + 1, 0]
```

rule 1961
```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Simp[x^(q/2)*(Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]/Sqrt[a
*x^q + b*x^n + c*x^(2*n - q)])   Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x
^(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] &&
 PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] ||
 EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))
```

rule 1992
```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[x^(m + 1)*(b*B*(n - q)*p +
A*c*(m + p*q + (n - q)*(2*p + 1) + 1) + B*c*(m + p*q + 2*(n - q)*p + 1)*x^(
n - q))*((a*x^q + b*x^n + c*x^(2*n - q))^p/(c*(m + p*(2*n - q) + 1)*(m + p*
q + (n - q)*(2*p + 1) + 1))), x] + Simp[(n - q)*(p/(c*(m + p*(2*n - q) + 1)
*(m + p*q + (n - q)*(2*p + 1) + 1)))   Int[x^(m + q)*Simp[2*a*A*c*(m + p*q
+ (n - q)*(2*p + 1) + 1) - a*b*B*(m + p*q + 1) + (2*a*B*c*(m + p*q + 2*(n -
 q)*p + 1) + A*b*c*(m + p*q + (n - q)*(2*p + 1) + 1) - b^2*B*(m + p*q + (n
- q)*p + 1))*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x]
/; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] &&  !Integ
erQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q]
 && GtQ[m + p*q, -(n - q) - 1] && NeQ[m + p*(2*n - q) + 1, 0] && NeQ[m + p*
q + (n - q)*(2*p + 1) + 1, 0]
```

rule 1996
```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[B*x^(m - n + 1)*((a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1)/(c*(m + p*q + (n - q)*(2*p + 1) + 1))), x] -
Simp[1/(c*(m + p*q + (n - q)*(2*p + 1) + 1))   Int[x^(m - n + q)*Simp[a*B*(
m + p*q - n + q + 1) + (b*B*(m + p*q + (n - q)*p + 1) - A*c*(m + p*q + (n -
 q)*(2*p + 1) + 1))*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^p, x], x]
 /; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] &&  !Inte
gerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[p, -1] && LtQ[p, 0] &&
RationalQ[m, q] && GeQ[m + p*q, n - q - 1] && NeQ[m + p*q + (n - q)*(2*p +
1) + 1, 0]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 271, normalized size of antiderivative = 0.81

method	result
risch	$-\frac{\left(-5120c^{6}x^{6}-6400b\,c^{5}x^{5}-8192a\,c^{5}x^{4}-128b^{2}c^{4}x^{4}-704ab\,c^{4}x^{3}+144b^{3}c^{3}x^{3}-1024a^{2}c^{4}x^{2}+992a\,b^{2}c^{3}x^{2}-168b^{4}c^{2}x^{2}+2336a^{2}b\,c^{3}}{35840c^{5}x}$
default	$\frac{\left(c\,x^{4}+b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}\left(10240x^{2}\left(c\,x^{2}+bx+a\right)^{\frac{5}{2}}c^{\frac{11}{2}}-7680c^{\frac{9}{2}}\left(c\,x^{2}+bx+a\right)^{\frac{5}{2}}bx-4096c^{\frac{9}{2}}\left(c\,x^{2}+bx+a\right)^{\frac{5}{2}}a+4480c^{\frac{9}{2}}\left(c\,x^{2}+bx+a\right)^{\frac{3}{2}}abx-}{}$

input
```
int((c*x^4+b*x^3+a*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/35840*(-5120*c^6*x^6-6400*b*c^5*x^5-8192*a*c^5*x^4-128*b^2*c^4*x^4-704*
a*b*c^4*x^3+144*b^3*c^3*x^3-1024*a^2*c^4*x^2+992*a*b^2*c^3*x^2-168*b^4*c^2
*x^2+2336*a^2*b*c^3*x-1456*a*b^3*c^2*x+210*b^5*c*x+2048*a^3*c^3-5488*a^2*b
^2*c^2+2520*a*b^4*c-315*b^6)/c^5*(x^2*(c*x^2+b*x+a))^(1/2)/x+3/2048*b*(64*
a^3*c^3-80*a^2*b^2*c^2+28*a*b^4*c-3*b^6)/c^(11/2)*ln((1/2*b+c*x)/c^(1/2)+(
c*x^2+b*x+a)^(1/2))*(x^2*(c*x^2+b*x+a))^(1/2)/x/(c*x^2+b*x+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 558, normalized size of antiderivative = 1.66

$$\int \left(ax^2+bx^3+cx^4\right)^{3/2}\,dx=\left[-\frac{105\left(3\,b^7-28\,ab^5c+80\,a^2b^3c^2-64\,a^3bc^3\right)\sqrt{c}x\log\left(-\frac{8\,c^2x^3+8\,bcx^2+4\,\sqrt{cx^4+bx^3+ax^2}(2\,cx+b)\sqrt{c}+}{x}\right.}{}\right.$$

input
```
integrate((c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

output
```
[-1/143360*(105*(3*b^7 - 28*a*b^5*c + 80*a^2*b^3*c^2 - 64*a^3*b*c^3)*sqrt(
c)*x*log(-(8*c^2*x^3 + 8*b*c*x^2 + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x +
b)*sqrt(c) + (b^2 + 4*a*c)*x)/x) - 4*(5120*c^7*x^6 + 6400*b*c^6*x^5 + 315*
b^6*c - 2520*a*b^4*c^2 + 5488*a^2*b^2*c^3 - 2048*a^3*c^4 + 128*(b^2*c^5 +
64*a*c^6)*x^4 - 16*(9*b^3*c^4 - 44*a*b*c^5)*x^3 + 8*(21*b^4*c^3 - 124*a*b^
2*c^4 + 128*a^2*c^5)*x^2 - 2*(105*b^5*c^2 - 728*a*b^3*c^3 + 1168*a^2*b*c^4
)*x)*sqrt(c*x^4 + b*x^3 + a*x^2))/(c^6*x), 1/71680*(105*(3*b^7 - 28*a*b^5*
c + 80*a^2*b^3*c^2 - 64*a^3*b*c^3)*sqrt(-c)*x*arctan(1/2*sqrt(c*x^4 + b*x^
3 + a*x^2)*(2*c*x + b)*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x)) + 2*(5120*c^7
*x^6 + 6400*b*c^6*x^5 + 315*b^6*c - 2520*a*b^4*c^2 + 5488*a^2*b^2*c^3 - 20
48*a^3*c^4 + 128*(b^2*c^5 + 64*a*c^6)*x^4 - 16*(9*b^3*c^4 - 44*a*b*c^5)*x^
3 + 8*(21*b^4*c^3 - 124*a*b^2*c^4 + 128*a^2*c^5)*x^2 - 2*(105*b^5*c^2 - 72
8*a*b^3*c^3 + 1168*a^2*b*c^4)*x)*sqrt(c*x^4 + b*x^3 + a*x^2))/(c^6*x)]
```

Sympy [F]

$$\int \left(ax^2+bx^3+cx^4\right)^{3/2}\,dx=\int \left(ax^2+bx^3+cx^4\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((c*x**4+b*x**3+a*x**2)**(3/2),x)
```

output
```
Integral((a*x**2 + b*x**3 + c*x**4)**(3/2), x)
```

Maxima [F]

$$\int \left(ax^2+bx^3+cx^4\right)^{3/2}\,dx=\int \left(cx^4+bx^3+ax^2\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*x^4 + b*x^3 + a*x^2)^(3/2), x)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 419, normalized size of antiderivative = 1.25

$$\int (ax^2+bx^3+cx^4)^{3/2}\,dx = \frac{1}{35840}\sqrt{cx^2+bx+a}\left(2\left(4\left(2\left(8\left(10\,(4\,cx\mathrm{sgn}(x)+5\,b\mathrm{sgn}(x))x+\frac{b^2c^5\mathrm{sgn}(x)+64\,ac^6\mathrm{sgn}}{c^6}\right.\right.\right.\right.\right.$$
$$+\frac{3\,(3\,b^7\mathrm{sgn}(x)-28\,ab^5c\mathrm{sgn}(x)+80\,a^2b^3c^2\mathrm{sgn}(x)-64\,a^3bc^3\mathrm{sgn}(x))\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)\sqrt{c}+b\right.\right.}{2048\,c^{\frac{11}{2}}}$$
$$-\frac{\left(315\,b^7\log\left(\left|b-2\,\sqrt{a}\sqrt{c}\right|\right)-2940\,ab^5c\log\left(\left|b-2\,\sqrt{a}\sqrt{c}\right|\right)+8400\,a^2b^3c^2\log\left(\left|b-2\,\sqrt{a}\sqrt{c}\right|\right)-6720\,a^3b\right.}{71680\,c^{\frac{11}{2}}}$$

input
```
integrate((c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

output
```
1/35840*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*(10*(4*c*x*sgn(x) + 5*b*sgn(x))*
x + (b^2*c^5*sgn(x) + 64*a*c^6*sgn(x))/c^6)*x - (9*b^3*c^4*sgn(x) - 44*a*b
*c^5*sgn(x))/c^6)*x + (21*b^4*c^3*sgn(x) - 124*a*b^2*c^4*sgn(x) + 128*a^2*
c^5*sgn(x))/c^6)*x - (105*b^5*c^2*sgn(x) - 728*a*b^3*c^3*sgn(x) + 1168*a^2
*b*c^4*sgn(x))/c^6)*x + (315*b^6*c*sgn(x) - 2520*a*b^4*c^2*sgn(x) + 5488*a
^2*b^2*c^3*sgn(x) - 2048*a^3*c^4*sgn(x))/c^6) + 3/2048*(3*b^7*sgn(x) - 28*
a*b^5*c*sgn(x) + 80*a^2*b^3*c^2*sgn(x) - 64*a^3*b*c^3*sgn(x))*log(abs(2*(s
qrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) + b))/c^(11/2) - 1/71680*(315*b^
7*log(abs(b - 2*sqrt(a)*sqrt(c))) - 2940*a*b^5*c*log(abs(b - 2*sqrt(a)*sqr
t(c))) + 8400*a^2*b^3*c^2*log(abs(b - 2*sqrt(a)*sqrt(c))) - 6720*a^3*b*c^3
*log(abs(b - 2*sqrt(a)*sqrt(c))) + 630*sqrt(a)*b^6*sqrt(c) - 5040*a^(3/2)*
b^4*c^(3/2) + 10976*a^(5/2)*b^2*c^(5/2) - 4096*a^(7/2)*c^(7/2))*sgn(x)/c^(
11/2)
```

Mupad [F(-1)]

Timed out.

$$\int (ax^2+bx^3+cx^4)^{3/2}\,dx = \int (c\,x^4+b\,x^3+a\,x^2)^{3/2}\,dx$$

input
```
int((a*x^2 + b*x^3 + c*x^4)^(3/2),x)
```

output
```
int((a*x^2 + b*x^3 + c*x^4)^(3/2), x)
```

Reduce [F]

$$\int \left(ax^2+bx^3+cx^4\right)^{3/2}\,dx=\int \left(c\,x^4+b\,x^3+a\,x^2\right)^{\frac{3}{2}}dx$$

input
```
int((c*x^4+b*x^3+a*x^2)^(3/2),x)
```

output
```
int((c*x^4+b*x^3+a*x^2)^(3/2),x)
```

3.57 $\int \frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{x}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 260

$$\int \frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{x}\,dx = \frac{\left(35b^4-216ab^2c+240a^2c^2\right)\sqrt{ax^2+bx^3+cx^4}}{3840c^3} - \frac{b(105b^4-760ab^2c+1296a^2c^2)\sqrt{ax^2+bx^3+cx^4}}{7680c^4x} - \frac{x(b(7b^2+12ac)+6c(7b^2-20ac)\,x)\sqrt{ax^2+bx^3+cx^4}}{960c^2} + \frac{(3b+10cx)\left(ax^2+bx^3+cx^4\right)^{3/2}}{60cx} + \frac{\left(b^2-4ac\right)^2\left(7b^2-4ac\right)\operatorname{arctanh}\left(\frac{x(b+2cx)}{2\sqrt{c}\sqrt{ax^2+bx^3+cx^4}}\right)}{1024c^{9/2}}$$

output

```
1/3840*(240*a^2*c^2-216*a*b^2*c+35*b^4)*(c*x^4+b*x^3+a*x^2)^(1/2)/c^3-1/76
80*b*(1296*a^2*c^2-760*a*b^2*c+105*b^4)*(c*x^4+b*x^3+a*x^2)^(1/2)/c^4/x-1/
960*x*(b*(12*a*c+7*b^2)+6*c*(-20*a*c+7*b^2)*x)*(c*x^4+b*x^3+a*x^2)^(1/2)/c
^2+1/60*(10*c*x+3*b)*(c*x^4+b*x^3+a*x^2)^(3/2)/c/x+1/1024*(-4*a*c+b^2)^2*(
-4*a*c+7*b^2)*arctanh(1/2*x*(2*c*x+b)/c^(1/2)/(c*x^4+b*x^3+a*x^2)^(1/2))/c
^(9/2)
```

Mathematica [A] (verified)

Time = 0.95 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.81

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x}\,dx = \frac{x\sqrt{a+x(b+cx)}\Big(2\sqrt{c}\sqrt{a+x(b+cx)}(-105b^5+70b^4cx+8b^3c(95a-7cx^2)$$

input
```
Integrate[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x,x]
```

output
```
(x*Sqrt[a + x*(b + c*x)]*(2*Sqrt[c]*Sqrt[a + x*(b + c*x)]*(-105*b^5 + 70*b
^4*c*x + 8*b^3*c*(95*a - 7*c*x^2) + 48*b^2*c^2*x*(-9*a + c*x^2) + 160*c^3*
x*(3*a^2 + 14*a*c*x^2 + 8*c^2*x^4) + 16*b*c^2*(-81*a^2 + 18*a*c*x^2 + 104*
c^2*x^4)) - 15*(b^2 - 4*a*c)^2*(7*b^2 - 4*a*c)*Log[b + 2*c*x - 2*Sqrt[c]*S
qrt[a + x*(b + c*x)]]))/(15360*c^(9/2)*Sqrt[x^2*(a + x*(b + c*x))])
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.19, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$, Rules used = {1966, 27, 1981, 27, 1996, 27, 1996, 27, 1961, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x}\,dx$$

$$\Big\downarrow \text{1966}$$

$$\frac{\int -\frac{1}{2}\left(4ab+(7b^2-20ac)x\right)\sqrt{cx^4+bx^3+ax^2}dx}{20c}+\frac{(3b+10cx)\left(ax^2+bx^3+cx^4\right)^{3/2}}{60cx}$$

$$\Big\downarrow \text{27}$$

$$\frac{(3b+10cx)\left(ax^2+bx^3+cx^4\right)^{3/2}}{60cx}-\frac{\int \left(4ab+(7b^2-20ac)x\right)\sqrt{cx^4+bx^3+ax^2}dx}{40c}$$

$$\Big\downarrow \text{1981}$$

$$\frac{(3b+10cx)\left(ax^2+bx^3+cx^4\right)^{3/2}}{60cx}-\frac{\frac{\int -\frac{x^2\left(4ab\left(7b^2-36ac\right)+\left(35b^4-216acb^2+240a^2c^2\right)x\right)}{2\sqrt{cx^4+bx^3+ax^2}}dx}{24c}+\frac{x\sqrt{ax^2+bx^3+cx^4}\left(6cx\left(7b^2-20ac\right)+b\left(12ac+7b^2\right)\right)}{24c}}{40c}$$

$\Big\downarrow$ 27

$$\frac{(3b+10cx)\left(ax^2+bx^3+cx^4\right)^{3/2}}{60cx}-\frac{\frac{x\left(6cx\left(7b^2-20ac\right)+b\left(12ac+7b^2\right)\right)\sqrt{ax^2+bx^3+cx^4}}{24c}-\frac{\int \frac{x^2\left(4ab\left(7b^2-36ac\right)+\left(35b^4-216acb^2+240a^2c^2\right)x\right)}{\sqrt{cx^4+bx^3+ax^2}}dx}{48c}}{40c}$$

$\Big\downarrow$ 1996

$$\frac{(3b+10cx)\left(ax^2+bx^3+cx^4\right)^{3/2}}{60cx}-\frac{\frac{x\left(6cx\left(7b^2-20ac\right)+b\left(12ac+7b^2\right)\right)\sqrt{ax^2+bx^3+cx^4}}{24c}-\frac{\frac{\left(240a^2c^2-216ab^2c+35b^4\right)\sqrt{ax^2+bx^3+cx^4}}{2c}-\frac{\int \frac{x\left(2a\left(35b^4-216acb^2+240a^2c^2\right)+b\left(105b^4-760acb^2\cdots\right.\right.}{2\sqrt{cx^4+bx^3+ax^2}}\cdots}{2c}}{48c}}{40c}$$

$\Big\downarrow$ 27

$$\frac{(3b+10cx)\left(ax^2+bx^3+cx^4\right)^{3/2}}{60cx}-\frac{\frac{x\left(6cx\left(7b^2-20ac\right)+b\left(12ac+7b^2\right)\right)\sqrt{ax^2+bx^3+cx^4}}{24c}-\frac{\frac{\left(240a^2c^2-216ab^2c+35b^4\right)\sqrt{ax^2+bx^3+cx^4}}{2c}-\frac{\int \frac{x\left(2a\left(35b^4-216acb^2+240a^2c^2\right)+b\left(105b^4-760acb^2\cdots\right.\right.}{\sqrt{cx^4+bx^3+ax^2}}\cdots}{4c}}{48c}}{40c}$$

$\Big\downarrow$ 1996

$$\frac{(3b+10cx)\left(ax^2+bx^3+cx^4\right)^{3/2}}{60cx}-\frac{\frac{x\left(6cx\left(7b^2-20ac\right)+b\left(12ac+7b^2\right)\right)\sqrt{ax^2+bx^3+cx^4}}{24c}-\frac{\frac{\left(240a^2c^2-216ab^2c+35b^4\right)\sqrt{ax^2+bx^3+cx^4}}{2c}-\frac{\frac{b\left(1296a^2c^2-760ab^2c+105b^4\right)\sqrt{ax^2+bx^3+cx^4}}{cx}-\int \frac{1\cdots}{\cdots}}{4c}}{48c}}{40c}$$

$\Big\downarrow$ 27

$$\frac{(3b+10cx)\left(ax^2+bx^3+cx^4\right)^{3/2}}{60cx}-\frac{\frac{x\left(6cx\left(7b^2-20ac\right)+b\left(12ac+7b^2\right)\right)\sqrt{ax^2+bx^3+cx^4}}{24c}-\frac{\frac{\left(240a^2c^2-216ab^2c+35b^4\right)\sqrt{ax^2+bx^3+cx^4}}{2c}-\frac{\frac{b\left(1296a^2c^2-760ab^2c+105b^4\right)\sqrt{ax^2+bx^3+cx^4}}{cx}-15\left(\cdots\right.}{4c}}{48c}}{40c}$$

$\Big\downarrow$ 1961

$$\frac{(3b+10cx)\left(ax^2+bx^3+cx^4\right)^{3/2}}{60cx}-$$

$$\frac{\frac{x\left(6cx\left(7b^2-20ac\right)+b\left(12ac+7b^2\right)\right)\sqrt{ax^2+bx^3+cx^4}}{24c}-\frac{\frac{\left(240a^2c^2-216ab^2c+35b^4\right)\sqrt{ax^2+bx^3+cx^4}}{2c}-\frac{\frac{b\left(1296a^2c^2-760ab^2c+105b^4\right)\sqrt{ax^2+bx^3+cx^4}}{cx}-\frac{15x}{4}}{48c}}{40c}$$

$\downarrow$ 1092

$$\frac{(3b+10cx)\left(ax^2+bx^3+cx^4\right)^{3/2}}{60cx}-$$

$$\frac{\frac{x\left(6cx\left(7b^2-20ac\right)+b\left(12ac+7b^2\right)\right)\sqrt{ax^2+bx^3+cx^4}}{24c}-\frac{\frac{\left(240a^2c^2-216ab^2c+35b^4\right)\sqrt{ax^2+bx^3+cx^4}}{2c}-\frac{\frac{b\left(1296a^2c^2-760ab^2c+105b^4\right)\sqrt{ax^2+bx^3+cx^4}}{cx}-\frac{15x}{}}{48c}}{40c}$$

$\downarrow$ 219

$$\frac{(3b+10cx)\left(ax^2+bx^3+cx^4\right)^{3/2}}{60cx}-$$

$$\frac{\frac{x\left(6cx\left(7b^2-20ac\right)+b\left(12ac+7b^2\right)\right)\sqrt{ax^2+bx^3+cx^4}}{24c}-\frac{\frac{\left(240a^2c^2-216ab^2c+35b^4\right)\sqrt{ax^2+bx^3+cx^4}}{2c}-\frac{\frac{b\left(1296a^2c^2-760ab^2c+105b^4\right)\sqrt{ax^2+bx^3+cx^4}}{cx}-\frac{15x}{}}{48c}}{40c}$$

input
```
Int[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x,x]
```

output
```
((3*b + 10*c*x)*(a*x^2 + b*x^3 + c*x^4)^(3/2))/(60*c*x) - ((x*(b*(7*b^2 +
12*a*c) + 6*c*(7*b^2 - 20*a*c)*x)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(24*c) - ((
(35*b^4 - 216*a*b^2*c + 240*a^2*c^2)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(2*c) -
((b*(105*b^4 - 760*a*b^2*c + 1296*a^2*c^2)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(c
*x) - (15*(b^2 - 4*a*c)^2*(7*b^2 - 4*a*c)*x*Sqrt[a + b*x + c*x^2]*ArcTanh[
(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*c^(3/2)*Sqrt[a*x^2 + b*
x^3 + c*x^4]))/(4*c))/(48*c))/(40*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1961
```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Simp[x^(q/2)*(Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]/Sqrt[a
*x^q + b*x^n + c*x^(2*n - q)])   Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x
^(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] &&
 PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] ||
 EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))
```

rule 1966
```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[x^(m - n + q + 1)*(b*(n - q)*p + c*(m + p*q + (n - q)*
(2*p - 1) + 1)*x^(n - q))*((a*x^q + b*x^n + c*x^(2*n - q))^p/(c*(m + p*(2*n
 - q) + 1)*(m + p*q + (n - q)*(2*p - 1) + 1))), x] + Simp[(n - q)*(p/(c*(m
+ p*(2*n - q) + 1)*(m + p*q + (n - q)*(2*p - 1) + 1)))   Int[x^(m - (n - 2*
q))*Simp[(-a)*b*(m + p*q - n + q + 1) + (2*a*c*(m + p*q + (n - q)*(2*p - 1)
 + 1) - b^2*(m + p*q + (n - q)*(p - 1) + 1))*x^(n - q), x]*(a*x^q + b*x^n +
 c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] &
& PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[
p, 0] && RationalQ[m, q] && GtQ[m + p*q + 1, n - q] && NeQ[m + p*(2*n - q)
+ 1, 0] && NeQ[m + p*q + (n - q)*(2*p - 1) + 1, 0]
```

rule 1981
```
Int[((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_)*((A_) + (
B_.)*(x_)^(r_.)), x_Symbol] :> Simp[x*(b*B*(n - q)*p + A*c*(p*q + (n - q)*(
2*p + 1) + 1) + B*c*(p*(2*n - q) + 1)*x^(n - q))*((a*x^q + b*x^n + c*x^(2*n
 - q))^p/(c*(p*(2*n - q) + 1)*(p*q + (n - q)*(2*p + 1) + 1))), x] + Simp[(n
 - q)*(p/(c*(p*(2*n - q) + 1)*(p*q + (n - q)*(2*p + 1) + 1)))   Int[x^q*(2*
a*A*c*(p*q + (n - q)*(2*p + 1) + 1) - a*b*B*(p*q + 1) + (2*a*B*c*(p*(2*n -
q) + 1) + A*b*c*(p*q + (n - q)*(2*p + 1) + 1) - b^2*B*(p*q + (n - q)*p + 1)
)*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b
, c, A, B, n, q}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] &&  !IntegerQ[p] &
& NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && NeQ[p*(2*n - q) + 1, 0] && NeQ[p*q +
(n - q)*(2*p + 1) + 1, 0]
```

rule 1996
```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[B*x^(m - n + 1)*((a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1)/(c*(m + p*q + (n - q)*(2*p + 1) + 1))), x] -
Simp[1/(c*(m + p*q + (n - q)*(2*p + 1) + 1))   Int[x^(m - n + q)*Simp[a*B*(
m + p*q - n + q + 1) + (b*B*(m + p*q + (n - q)*p + 1) - A*c*(m + p*q + (n -
 q)*(2*p + 1) + 1))*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^p, x], x]
 /; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] &&  !Inte
gerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[p, -1] && LtQ[p, 0] &&
RationalQ[m, q] && GeQ[m + p*q, n - q - 1] && NeQ[m + p*q + (n - q)*(2*p +
1) + 1, 0]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.87

method	result
risch	$-\frac{\left(-1280c^5x^5-1664b\,c^4x^4-2240a\,c^4x^3-48b^2c^3x^3-288ab\,c^3x^2+56b^3c^2x^2-480a^2c^3x+432a\,b^2c^2x-70b^4cx+1296a^2b\,c^2-760a\,b^3c}{7680c^4x}$
default	$\frac{\left(c\,x^4+b\,x^3+a\,x^2\right)^{\frac{3}{2}}\left(2560x\left(c\,x^2+bx+a\right)^{\frac{5}{2}}c^{\frac{9}{2}}-640c^{\frac{9}{2}}\left(c\,x^2+bx+a\right)^{\frac{3}{2}}ax-960c^{\frac{9}{2}}\sqrt{c\,x^2+bx+a}\,a^2x-1792c^{\frac{7}{2}}\left(c\,x^2+bx+a\right)^{\frac{5}{2}}b+1120c}{}$

input
```
int((c*x^4+b*x^3+a*x^2)^(3/2)/x,x,method=_RETURNVERBOSE)
```

output
```
-1/7680*(-1280*c^5*x^5-1664*b*c^4*x^4-2240*a*c^4*x^3-48*b^2*c^3*x^3-288*a*
b*c^3*x^2+56*b^3*c^2*x^2-480*a^2*c^3*x+432*a*b^2*c^2*x-70*b^4*c*x+1296*a^2
*b*c^2-760*a*b^3*c+105*b^5)/c^4*(x^2*(c*x^2+b*x+a))^(1/2)/x-1/1024*(64*a^3
*c^3-144*a^2*b^2*c^2+60*a*b^4*c-7*b^6)/c^(9/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x
^2+b*x+a)^(1/2))*(x^2*(c*x^2+b*x+a))^(1/2)/x/(c*x^2+b*x+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 474, normalized size of antiderivative = 1.82

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x}\,dx = \left[-\frac{15\,(7\,b^6-60\,ab^4c+144\,a^2b^2c^2-64\,a^3c^3)\sqrt{c}x\log\left(-\frac{8\,c^2x^3+8\,bcx^2-4\,\sqrt{cx^4+bx^3}}{}\right.}{}\right.$$

$$-\frac{15\,(7\,b^6-60\,ab^4c+144\,a^2b^2c^2-64\,a^3c^3)\sqrt{-c}x\arctan\left(\frac{\sqrt{cx^4+bx^3+ax^2}(2\,cx+b)\sqrt{-c}}{2\,(c^2x^3+bcx^2+acx)}\right)-2\,(1280\,c^6x^5+1664\,bc}{}$$

input
```
integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x,x, algorithm="fricas")
```

output
```
[-1/30720*(15*(7*b^6 - 60*a*b^4*c + 144*a^2*b^2*c^2 - 64*a^3*c^3)*sqrt(c)*
x*log(-(8*c^2*x^3 + 8*b*c*x^2 - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*
sqrt(c) + (b^2 + 4*a*c)*x)/x) - 4*(1280*c^6*x^5 + 1664*b*c^5*x^4 - 105*b^5
*c + 760*a*b^3*c^2 - 1296*a^2*b*c^3 + 16*(3*b^2*c^4 + 140*a*c^5)*x^3 - 8*(
7*b^3*c^3 - 36*a*b*c^4)*x^2 + 2*(35*b^4*c^2 - 216*a*b^2*c^3 + 240*a^2*c^4)
*x)*sqrt(c*x^4 + b*x^3 + a*x^2))/(c^5*x), -1/15360*(15*(7*b^6 - 60*a*b^4*c
 + 144*a^2*b^2*c^2 - 64*a^3*c^3)*sqrt(-c)*x*arctan(1/2*sqrt(c*x^4 + b*x^3
+ a*x^2)*(2*c*x + b)*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x)) - 2*(1280*c^6*x
^5 + 1664*b*c^5*x^4 - 105*b^5*c + 760*a*b^3*c^2 - 1296*a^2*b*c^3 + 16*(3*b
^2*c^4 + 140*a*c^5)*x^3 - 8*(7*b^3*c^3 - 36*a*b*c^4)*x^2 + 2*(35*b^4*c^2 -
 216*a*b^2*c^3 + 240*a^2*c^4)*x)*sqrt(c*x^4 + b*x^3 + a*x^2))/(c^5*x)]
```

Sympy [F]

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x}\,dx=\int \frac{(x^2(a+bx+cx^2))^{\frac{3}{2}}}{x}\,dx$$

input
```
integrate((c*x**4+b*x**3+a*x**2)**(3/2)/x,x)
```

output
```
Integral((x**2*(a + b*x + c*x**2))**(3/2)/x, x)
```

Maxima [F]

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x}\,dx=\int \frac{(cx^4+bx^3+ax^2)^{\frac{3}{2}}}{x}\,dx$$

input
```
integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x,x, algorithm="maxima")
```

output
```
integrate((c*x^4 + b*x^3 + a*x^2)^(3/2)/x, x)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.37

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x}\,dx=\frac{1}{7680}\sqrt{cx^2+bx+a}\left(2\left(4\left(2\left(8\left(10\,cx\mathrm{sgn}(x)+13\,b\mathrm{sgn}(x)\right)x+\frac{3\,b^2c^4\mathrm{sgn}(x)}{}\right.\right.\right.\right.$$
$$-\frac{\left(7\,b^6\mathrm{sgn}(x)-60\,ab^4c\mathrm{sgn}(x)+144\,a^2b^2c^2\mathrm{sgn}(x)-64\,a^3c^3\mathrm{sgn}(x)\right)\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)\sqrt{c}+b\right|\right.}{1024\,c^{\frac{9}{2}}}$$
$$+\frac{\left(105\,b^6\log\left(\left|b-2\,\sqrt{a}\sqrt{c}\right|\right)-900\,ab^4c\log\left(\left|b-2\,\sqrt{a}\sqrt{c}\right|\right)+2160\,a^2b^2c^2\log\left(\left|b-2\,\sqrt{a}\sqrt{c}\right|\right)-960\,a^3c^3\,\mathrm{lo}\right.}{15360\,c^{\frac{9}{2}}}$$

input
```
integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x,x, algorithm="giac")
```

output
```
1/7680*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*(10*c*x*sgn(x) + 13*b*sgn(x))*x +
 (3*b^2*c^4*sgn(x) + 140*a*c^5*sgn(x))/c^5)*x - (7*b^3*c^3*sgn(x) - 36*a*b
*c^4*sgn(x))/c^5)*x + (35*b^4*c^2*sgn(x) - 216*a*b^2*c^3*sgn(x) + 240*a^2*
c^4*sgn(x))/c^5)*x - (105*b^5*c*sgn(x) - 760*a*b^3*c^2*sgn(x) + 1296*a^2*b
*c^3*sgn(x))/c^5) - 1/1024*(7*b^6*sgn(x) - 60*a*b^4*c*sgn(x) + 144*a^2*b^2
*c^2*sgn(x) - 64*a^3*c^3*sgn(x))*log(abs(2*(sqrt(c)*x - sqrt(c*x^2 + b*x +
 a))*sqrt(c) + b))/c^(9/2) + 1/15360*(105*b^6*log(abs(b - 2*sqrt(a)*sqrt(c
))) - 900*a*b^4*c*log(abs(b - 2*sqrt(a)*sqrt(c))) + 2160*a^2*b^2*c^2*log(a
bs(b - 2*sqrt(a)*sqrt(c))) - 960*a^3*c^3*log(abs(b - 2*sqrt(a)*sqrt(c))) +
 210*sqrt(a)*b^5*sqrt(c) - 1520*a^(3/2)*b^3*c^(3/2) + 2592*a^(5/2)*b*c^(5/
2))*sgn(x)/c^(9/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x}\,dx = \int \frac{(c\,x^4+b\,x^3+a\,x^2)^{3/2}}{x}\,dx$$

input
```
int((a*x^2 + b*x^3 + c*x^4)^(3/2)/x,x)
```

output
```
int((a*x^2 + b*x^3 + c*x^4)^(3/2)/x, x)
```

Reduce [F]

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x}\,dx = \int \frac{(c\,x^4+b\,x^3+a\,x^2)^{\frac{3}{2}}}{x}dx$$

input
```
int((c*x^4+b*x^3+a*x^2)^(3/2)/x,x)
```

output
```
int((c*x^4+b*x^3+a*x^2)^(3/2)/x,x)
```

3.58 $\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^2} dx$

Optimal result

Integrand size = 24, antiderivative size = 170

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^2} dx = \frac{3b(b^2-4ac)(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{128c^3x} - \frac{b(b+2cx)(ax^2+bx^3+cx^4)^{3/2}}{16c^2x^3} + \frac{(ax^2+bx^3+cx^4)^{5/2}}{5cx^5} - \frac{3b(b^2-4ac)^2 \operatorname{arctanh}\left(\frac{x(b+2cx)}{2\sqrt{c}\sqrt{ax^2+bx^3+cx^4}}\right)}{256c^{7/2}}$$

output
```
3/128*b*(-4*a*c+b^2)*(2*c*x+b)*(c*x^4+b*x^3+a*x^2)^(1/2)/c^3/x-1/16*b*(2*c
*x+b)*(c*x^4+b*x^3+a*x^2)^(3/2)/c^2/x^3+1/5*(c*x^4+b*x^3+a*x^2)^(5/2)/c/x^
5-3/256*b*(-4*a*c+b^2)^2*arctanh(1/2*x*(2*c*x+b)/c^(1/2)/(c*x^4+b*x^3+a*x^
2)^(1/2))/c^(7/2)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.95

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^2}\,dx = \frac{x\sqrt{a+x(b+cx)}\left(2\sqrt{c}\sqrt{a+x(b+cx)}\left(15b^4-10b^3cx+128c^2(a+cx^2)^2+4\right.\right.}{128\cdots}$$

input
```
Integrate[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^2,x]
```

output
```
(x*Sqrt[a + x*(b + c*x)]*(2*Sqrt[c]*Sqrt[a + x*(b + c*x)]*(15*b^4 - 10*b^3
*c*x + 128*c^2*(a + c*x^2)^2 + 4*b^2*c*(-25*a + 2*c*x^2) + 8*b*c^2*x*(7*a
+ 22*c*x^2)) + 15*b*(b^2 - 4*a*c)^2*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(
b + c*x)]]))/(1280*c^(7/2)*Sqrt[x^2*(a + x*(b + c*x))])
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.24, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1964, 1965, 1965, 1961, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^2}dx$$

$$\Big\downarrow \text{ 1964}$$

$$\frac{(ax^2+bx^3+cx^4)^{5/2}}{5cx^5}-\frac{b\int \frac{(cx^4+bx^3+ax^2)^{3/2}}{x^3}dx}{2c}$$

$$\Big\downarrow \text{ 1965}$$

$$\frac{(ax^2+bx^3+cx^4)^{5/2}}{5cx^5}-\frac{b\left(\frac{(b+2cx)(ax^2+bx^3+cx^4)^{3/2}}{8cx^3}-\frac{3(b^2-4ac)\int \frac{\sqrt{cx^4+bx^3+ax^2}}{x}dx}{16c}\right)}{2c}$$

$$\Big\downarrow \text{ 1965}$$

$$\frac{\left(ax^2+bx^3+cx^4\right)^{5/2}}{5cx^5}-\frac{b\left(\frac{(b+2cx)\left(ax^2+bx^3+cx^4\right)^{3/2}}{8cx^3}-\frac{3\left(b^2-4ac\right)\left(\frac{(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{4cx}-\frac{\left(b^2-4ac\right)\int\frac{x}{\sqrt{cx^4+bx^3+ax^2}}dx}{8c}\right)}{16c}\right)}{2c}$$

$$\Big\downarrow \text{1961}$$

$$\frac{\left(ax^2+bx^3+cx^4\right)^{5/2}}{5cx^5}-\frac{b\left(\frac{(b+2cx)\left(ax^2+bx^3+cx^4\right)^{3/2}}{8cx^3}-\frac{3\left(b^2-4ac\right)\left(\frac{(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{4cx}-\frac{x\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\int\frac{1}{\sqrt{cx^2+bx+a}}dx}{8c\sqrt{ax^2+bx^3+cx^4}}\right)}{16c}\right)}{2c}$$

$$\Big\downarrow \text{1092}$$

$$\frac{\left(ax^2+bx^3+cx^4\right)^{5/2}}{5cx^5}-\frac{b\left(\frac{(b+2cx)\left(ax^2+bx^3+cx^4\right)^{3/2}}{8cx^3}-\frac{3\left(b^2-4ac\right)\left(\frac{(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{4cx}-\frac{x\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\int\frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{4c\sqrt{ax^2+bx^3+cx^4}}\right)}{16c}\right)}{2c}$$

$$\Big\downarrow \text{219}$$

$$\frac{\left(ax^2+bx^3+cx^4\right)^{5/2}}{5cx^5}-\frac{b\left(\frac{(b+2cx)\left(ax^2+bx^3+cx^4\right)^{3/2}}{8cx^3}-\frac{3\left(b^2-4ac\right)\left(\frac{(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{4cx}-\frac{x\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{3/2}\sqrt{ax^2+bx^3+cx^4}}\right)}{16c}\right)}{2c}$$

input
```
Int[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^2,x]
```

output
```
(a*x^2 + b*x^3 + c*x^4)^(5/2)/(5*c*x^5) - (b*(((b + 2*c*x)*(a*x^2 + b*x^3
+ c*x^4)^(3/2))/(8*c*x^3) - (3*(b^2 - 4*a*c)*(((b + 2*c*x)*Sqrt[a*x^2 + b*
x^3 + c*x^4])/(4*c*x) - ((b^2 - 4*a*c)*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(b
+ 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(8*c^(3/2)*Sqrt[a*x^2 + b*x^3
 + c*x^4])))/(16*c)))/(2*c)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1961
```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Simp[x^(q/2)*(Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]/Sqrt[a
*x^q + b*x^n + c*x^(2*n - q)])   Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x
^(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] &&
 PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] ||
 EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))
```

rule 1964
```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[x^(m - n)*((a*x^(n - 1) + b*x^n + c*x^(n + 1))^(p + 1)
/(2*c*(p + 1))), x] - Simp[b/(2*c)   Int[x^(m - 1)*(a*x^(n - 1) + b*x^n + c
*x^(n + 1))^p, x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n -
 q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && RationalQ[m, p
, q] && EqQ[m + p*(n - 1) - 1, 0]
```

rule 1965
```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[x^(m - n + q + 1)*(b + 2*c*x^(n - q))*((a*x^q + b*x^n
+ c*x^(2*n - q))^p/(2*c*(n - q)*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*
c*(2*p + 1)))   Int[x^(m + q)*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x],
x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] &&  !IntegerQ[p
] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && E
qQ[m + p*q + 1, n - q]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.74

method	result
pseudoelliptic	$\frac{-\frac{15b\left(ac-\frac{b^2}{4}\right)^2\ln\left(2\sqrt{c\,x^2+bx+a}\,\sqrt{c}+2cx+b\right)}{16}+\left(\left(\frac{1}{16}b^2x^2+\frac{7}{16}abx+a^2\right)c^{\frac{5}{2}}-\frac{25\left(\frac{bx}{10}+a\right)b^2c^{\frac{3}{2}}}{32}+\left(\frac{11}{8}b\,x^3+2a\,x^2\right)c^{\frac{7}{2}}+c^{\frac{9}{2}}x^4+\frac{15}{1}\cdots\right.}{5c^{\frac{7}{2}}}$
risch	$\frac{\left(128c^4x^4+176b\,c^3x^3+256a\,c^3x^2+8b^2c^2x^2+56ab\,c^2x-10b^3cx+128a^2c^2-100a\,b^2c+15b^4\right)\sqrt{x^2\left(c\,x^2+bx+a\right)}}{640c^3x}-\frac{3b\left(16a^2c^2\cdots\right.}{\cdots}$
default	$\frac{\left(c\,x^4+b\,x^3+a\,x^2\right)^{\frac{3}{2}}\left(256\left(c\,x^2+bx+a\right)^{\frac{5}{2}}c^{\frac{7}{2}}-160c^{\frac{7}{2}}\left(c\,x^2+bx+a\right)^{\frac{3}{2}}bx-80c^{\frac{5}{2}}\left(c\,x^2+bx+a\right)^{\frac{3}{2}}b^2-240c^{\frac{7}{2}}\sqrt{c\,x^2+bx+a}\,abx+6\cdots\right.}{\cdots}$

input
```
int((c*x^4+b*x^3+a*x^2)^(3/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/5/c^(7/2)*(-15/16*b*(a*c-1/4*b^2)^2*ln(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c
*x+b)+((1/16*b^2*x^2+7/16*a*b*x+a^2)*c^(5/2)-25/32*(1/10*b*x+a)*b^2*c^(3/2
)+(11/8*b*x^3+2*a*x^2)*c^(7/2)+c^(9/2)*x^4+15/128*c^(1/2)*b^4)*(c*x^2+b*x+
a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 384, normalized size of antiderivative = 2.26

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^2}\,dx = \left[\frac{15\,(b^5-8\,ab^3c+16\,a^2bc^2)\sqrt{c}x\log\left(-\frac{8\,c^2x^3+8\,bcx^2-4\,\sqrt{cx^4+bx^3+ax^2}(2\,cx+b)\sqrt{c}+(b^2\cdots}{x}\right.}{\cdots}\right.$$

input
```
integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^2,x, algorithm="fricas")
```

output
```
[1/2560*(15*(b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*sqrt(c)*x*log(-(8*c^2*x^3 + 8
*b*c*x^2 - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(c) + (b^2 + 4*a*
c)*x)/x) + 4*(128*c^5*x^4 + 176*b*c^4*x^3 + 15*b^4*c - 100*a*b^2*c^2 + 128
*a^2*c^3 + 8*(b^2*c^3 + 32*a*c^4)*x^2 - 2*(5*b^3*c^2 - 28*a*b*c^3)*x)*sqrt
(c*x^4 + b*x^3 + a*x^2))/(c^4*x), 1/1280*(15*(b^5 - 8*a*b^3*c + 16*a^2*b*c
^2)*sqrt(-c)*x*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(-c)
/(c^2*x^3 + b*c*x^2 + a*c*x)) + 2*(128*c^5*x^4 + 176*b*c^4*x^3 + 15*b^4*c
- 100*a*b^2*c^2 + 128*a^2*c^3 + 8*(b^2*c^3 + 32*a*c^4)*x^2 - 2*(5*b^3*c^2
- 28*a*b*c^3)*x)*sqrt(c*x^4 + b*x^3 + a*x^2))/(c^4*x)]
```

Sympy [F]

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^2}\,dx = \int \frac{(x^2(a+bx+cx^2))^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((c*x**4+b*x**3+a*x**2)**(3/2)/x**2,x)
```

output
```
Integral((x**2*(a + b*x + c*x**2))**(3/2)/x**2, x)
```

Maxima [F]

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^2}\,dx = \int \frac{(cx^4+bx^3+ax^2)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^2,x, algorithm="maxima")
```

output
```
integrate((c*x^4 + b*x^3 + a*x^2)^(3/2)/x^2, x)
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.62

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^2}\,dx = \frac{1}{640}\sqrt{cx^2+bx+a}\Bigg(2\Bigg(4\Bigg(2\left(8\,cx\mathrm{sgn}(x)+11\,b\mathrm{sgn}(x)\right)x+\frac{b^2c^3\mathrm{sgn}(x)+32\,a}{c^4}$$
$$+\frac{3\left(b^5\mathrm{sgn}(x)-8\,ab^3c\mathrm{sgn}(x)+16\,a^2bc^2\mathrm{sgn}(x)\right)\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)\sqrt{c}+b\right|\right)}{256\,c^{\frac{7}{2}}}$$
$$-\frac{\left(15\,b^5\log\left(\left|b-2\sqrt{a}\sqrt{c}\right|\right)-120\,ab^3c\log\left(\left|b-2\sqrt{a}\sqrt{c}\right|\right)+240\,a^2bc^2\log\left(\left|b-2\sqrt{a}\sqrt{c}\right|\right)+30\sqrt{a}b^4\sqrt{c}-}{1280\,c^{\frac{7}{2}}}$$

input
```
integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^2,x, algorithm="giac")
```

output
```
1/640*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*c*x*sgn(x) + 11*b*sgn(x))*x + (b^2
*c^3*sgn(x) + 32*a*c^4*sgn(x))/c^4)*x - (5*b^3*c^2*sgn(x) - 28*a*b*c^3*sgn
(x))/c^4)*x + (15*b^4*c*sgn(x) - 100*a*b^2*c^2*sgn(x) + 128*a^2*c^3*sgn(x)
)/c^4) + 3/256*(b^5*sgn(x) - 8*a*b^3*c*sgn(x) + 16*a^2*b*c^2*sgn(x))*log(a
bs(2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) + b))/c^(7/2) - 1/1280*(1
5*b^5*log(abs(b - 2*sqrt(a)*sqrt(c))) - 120*a*b^3*c*log(abs(b - 2*sqrt(a)*
sqrt(c))) + 240*a^2*b*c^2*log(abs(b - 2*sqrt(a)*sqrt(c))) + 30*sqrt(a)*b^4
*sqrt(c) - 200*a^(3/2)*b^2*c^(3/2) + 256*a^(5/2)*c^(5/2))*sgn(x)/c^(7/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^2}\,dx = \int \frac{(c\,x^4+b\,x^3+a\,x^2)^{3/2}}{x^2}\,dx$$

input
```
int((a*x^2 + b*x^3 + c*x^4)^(3/2)/x^2,x)
```

output
```
int((a*x^2 + b*x^3 + c*x^4)^(3/2)/x^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.86

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^2}\,dx = \frac{256\sqrt{c\,x^2+bx+a}\,a^2c^3-200\sqrt{c\,x^2+bx+a}\,a\,b^2c^2+112\sqrt{c\,x^2+bx+a}\,ab\,c}{}$$

input
```
int((c*x^4+b*x^3+a*x^2)^(3/2)/x^2,x)
```

output
```
(256*sqrt(a + b*x + c*x**2)*a**2*c**3 - 200*sqrt(a + b*x + c*x**2)*a*b**2*
c**2 + 112*sqrt(a + b*x + c*x**2)*a*b*c**3*x + 512*sqrt(a + b*x + c*x**2)*
a*c**4*x**2 + 30*sqrt(a + b*x + c*x**2)*b**4*c - 20*sqrt(a + b*x + c*x**2)
*b**3*c**2*x + 16*sqrt(a + b*x + c*x**2)*b**2*c**3*x**2 + 352*sqrt(a + b*x
 + c*x**2)*b*c**4*x**3 + 256*sqrt(a + b*x + c*x**2)*c**5*x**4 - 240*sqrt(c
)*log((2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)/sqrt(4*a*c - b**2))*a
**2*b*c**2 + 120*sqrt(c)*log((2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x
)/sqrt(4*a*c - b**2))*a*b**3*c - 15*sqrt(c)*log((2*sqrt(c)*sqrt(a + b*x +
c*x**2) + b + 2*c*x)/sqrt(4*a*c - b**2))*b**5)/(1280*c**4)
```

3.59 $\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 137

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^3}\,dx = -\frac{3(b^2-4ac)(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{64c^2x} + \frac{(b+2cx)(ax^2+bx^3+cx^4)^{3/2}}{8cx^3} + \frac{3(b^2-4ac)^2\operatorname{arctanh}\left(\frac{x(b+2cx)}{2\sqrt{c}\sqrt{ax^2+bx^3+cx^4}}\right)}{128c^{5/2}}$$

output
```
-3/64*(-4*a*c+b^2)*(2*c*x+b)*(c*x^4+b*x^3+a*x^2)^(1/2)/c^2/x+1/8*(2*c*x+b)
*(c*x^4+b*x^3+a*x^2)^(3/2)/c/x^3+3/128*(-4*a*c+b^2)^2*arctanh(1/2*x*(2*c*x
+b)/c^(1/2)/(c*x^4+b*x^3+a*x^2)^(1/2))/c^(5/2)
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.97

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^3}\,dx = \frac{(x^2(a+x(b+cx)))^{3/2}\left(\frac{\sqrt{c}(b+2cx)(-3b^2+8bcx+4c(5a+2cx^2))}{a+x(b+cx)} + \frac{3(b^2-4ac)^2\operatorname{arctanh}\left(\cdots\right.}{(a+x(b+cx)\cdots}\right.}{64c^{5/2}x^3}$$

input
```
Integrate[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^3,x]
```

output
```
((x^2*(a + x*(b + c*x)))^(3/2)*((Sqrt[c]*(b + 2*c*x)*(-3*b^2 + 8*b*c*x + 4
*c*(5*a + 2*c*x^2)))/(a + x*(b + c*x)) + (3*(b^2 - 4*a*c)^2*ArcTanh[(Sqrt[
c]*x)/(-Sqrt[a] + Sqrt[a + x*(b + c*x)])])/(a + x*(b + c*x))^(3/2)))/(64*c
^(5/2)*x^3)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.25, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1965, 1965, 1961, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{x^3}dx$$

$$\Big\downarrow \text{ 1965}$$

$$\frac{(b+2cx)\left(ax^2+bx^3+cx^4\right)^{3/2}}{8cx^3}-\frac{3\left(b^2-4ac\right)\int \frac{\sqrt{cx^4+bx^3+ax^2}}{x}dx}{16c}$$

$$\Big\downarrow \text{ 1965}$$

$$\frac{(b+2cx)\left(ax^2+bx^3+cx^4\right)^{3/2}}{8cx^3}-\frac{3\left(b^2-4ac\right)\left(\frac{(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{4cx}-\frac{\left(b^2-4ac\right)\int \frac{x}{\sqrt{cx^4+bx^3+ax^2}}dx}{8c}\right)}{16c}$$

$$\Big\downarrow \text{ 1961}$$

$$\frac{(b+2cx)\left(ax^2+bx^3+cx^4\right)^{3/2}}{8cx^3}-\frac{3\left(b^2-4ac\right)\left(\frac{(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{4cx}-\frac{x\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{8c\sqrt{ax^2+bx^3+cx^4}}\right)}{16c}$$

$$\Big\downarrow \text{ 1092}$$

$$\frac{(b+2cx)\left(ax^2+bx^3+cx^4\right)^{3/2}}{8cx^3}-\frac{3\left(b^2-4ac\right)\left(\frac{(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{4cx}-\frac{x\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\int\frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{4c\sqrt{ax^2+bx^3+cx^4}}\right)}{16c}$$

$\downarrow$ 219

$$\frac{(b+2cx)\left(ax^2+bx^3+cx^4\right)^{3/2}}{8cx^3}-\frac{3\left(b^2-4ac\right)\left(\frac{(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{4cx}-\frac{x\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{3/2}\sqrt{ax^2+bx^3+cx^4}}\right)}{16c}$$

input
```
Int[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^3,x]
```

output
```
((b + 2*c*x)*(a*x^2 + b*x^3 + c*x^4)^(3/2))/(8*c*x^3) - (3*(b^2 - 4*a*c)*(
((b + 2*c*x)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(4*c*x) - ((b^2 - 4*a*c)*x*Sqrt[
a + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(
8*c^(3/2)*Sqrt[a*x^2 + b*x^3 + c*x^4])))/(16*c)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1961
```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Simp[x^(q/2)*(Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]/Sqrt[a
*x^q + b*x^n + c*x^(2*n - q)])   Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x
^(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] &&
 PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] ||
 EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))
```

rule 1965
```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[x^(m - n + q + 1)*(b + 2*c*x^(n - q))*((a*x^q + b*x^n
+ c*x^(2*n - q))^p/(2*c*(n - q)*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*
c*(2*p + 1)))   Int[x^(m + q)*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x],
x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] &&  !IntegerQ[p
] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && E
qQ[m + p*q + 1, n - q]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.73

method	result
pseudoelliptic	$\frac{\frac{3\left(ac-\frac{b^2}{4}\right)^2\ln\left(2\sqrt{c\,x^2+bx+a}\,\sqrt{c}+2cx+b\right)}{8}+\frac{5\left(b\left(\frac{bx}{10}+a\right)c^{\frac{3}{2}}+\left(\frac{6}{5}b\,x^2+2xa\right)c^{\frac{5}{2}}-\frac{3\sqrt{c}\,b^3}{20}+\frac{4c^{\frac{7}{2}}x^3}{5}\right)\sqrt{c\,x^2+bx+a}}{16}}{c^{\frac{5}{2}}}$
risch	$\frac{\left(16c^3x^3+24b\,c^2x^2+40a\,c^2x+2b^2cx+20abc-3b^3\right)\sqrt{x^2\left(c\,x^2+bx+a\right)}}{64c^2x}+\frac{3\left(16a^2c^2-8a\,b^2c+b^4\right)\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)\sqrt{}}{128c^{\frac{5}{2}}x\sqrt{c\,x^2+bx+a}}$
default	$\frac{\left(c\,x^4+b\,x^3+a\,x^2\right)^{\frac{3}{2}}\left(32x\left(c\,x^2+bx+a\right)^{\frac{3}{2}}c^{\frac{7}{2}}+16c^{\frac{5}{2}}\left(c\,x^2+bx+a\right)^{\frac{3}{2}}b+48c^{\frac{7}{2}}\sqrt{c\,x^2+bx+a}\,ax-12c^{\frac{5}{2}}\sqrt{c\,x^2+bx+a}\,b^2x+24c^{\frac{5}{2}}\sqrt{}\right.}{}$

input
```
int((c*x^4+b*x^3+a*x^2)^(3/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
3/8/c^(5/2)*((a*c-1/4*b^2)^2*ln(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)+5/6
*(b*(1/10*b*x+a)*c^(3/2)+(6/5*b*x^2+2*x*a)*c^(5/2)-3/20*c^(1/2)*b^3+4/5*c^
(7/2)*x^3)*(c*x^2+b*x+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 320, normalized size of antiderivative = 2.34

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^3}\,dx = \left[\frac{3\,(b^4-8\,ab^2c+16\,a^2c^2)\sqrt{c}x\log\left(-\frac{8\,c^2x^3+8\,bcx^2+4\,\sqrt{cx^4+bx^3+ax^2}(2\,cx+b)\sqrt{c}+(b^2+4}{x}\right.}{}\right.$$

$$-\frac{3\,(b^4-8\,ab^2c+16\,a^2c^2)\sqrt{-c}x\arctan\left(\frac{\sqrt{cx^4+bx^3+ax^2}(2\,cx+b)\sqrt{-c}}{2\,(c^2x^3+bcx^2+acx)}\right)-2\,(16\,c^4x^3+24\,bc^3x^2-3\,b^3c+20\,abc^2}{128\,c^3x}$$

input
```
integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^3,x, algorithm="fricas")
```

output
```
[1/256*(3*(b^4 - 8*a*b^2*c + 16*a^2*c^2)*sqrt(c)*x*log(-(8*c^2*x^3 + 8*b*c
*x^2 + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(c) + (b^2 + 4*a*c)*x
)/x) + 4*(16*c^4*x^3 + 24*b*c^3*x^2 - 3*b^3*c + 20*a*b*c^2 + 2*(b^2*c^2 +
20*a*c^3)*x)*sqrt(c*x^4 + b*x^3 + a*x^2))/(c^3*x), -1/128*(3*(b^4 - 8*a*b^
2*c + 16*a^2*c^2)*sqrt(-c)*x*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x
 + b)*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x)) - 2*(16*c^4*x^3 + 24*b*c^3*x^2
 - 3*b^3*c + 20*a*b*c^2 + 2*(b^2*c^2 + 20*a*c^3)*x)*sqrt(c*x^4 + b*x^3 + a
*x^2))/(c^3*x)]
```

Sympy [F]

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^3}\,dx = \int \frac{(x^2(a+bx+cx^2))^{\frac{3}{2}}}{x^3}\,dx$$

input
```
integrate((c*x**4+b*x**3+a*x**2)**(3/2)/x**3,x)
```

output
```
Integral((x**2*(a + b*x + c*x**2))**(3/2)/x**3, x)
```

Maxima [F]

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^3}\,dx=\int \frac{(cx^4+bx^3+ax^2)^{\frac{3}{2}}}{x^3}\,dx$$

input
```
integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^3,x, algorithm="maxima")
```

output
```
integrate((c*x^4 + b*x^3 + a*x^2)^(3/2)/x^3, x)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.64

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^3}\,dx=\frac{1}{64}\sqrt{cx^2+bx+a}\Bigg(2\left(4\,(2\,cx\mathrm{sgn}(x)+3\,b\mathrm{sgn}(x))x+\frac{b^2c^2\mathrm{sgn}(x)+20\,ac^3\mathrm{sgn}}{c^3}\right.$$
$$-\frac{3\,(b^4\mathrm{sgn}(x)-8\,ab^2c\mathrm{sgn}(x)+16\,a^2c^2\mathrm{sgn}(x))\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)\sqrt{c}+b\right|\right)}{128\,c^{\frac{5}{2}}}$$
$$+\frac{\left(3\,b^4\log\left(\left|b-2\,\sqrt{a}\sqrt{c}\right|\right)-24\,ab^2c\log\left(\left|b-2\,\sqrt{a}\sqrt{c}\right|\right)+48\,a^2c^2\log\left(\left|b-2\,\sqrt{a}\sqrt{c}\right|\right)+6\,\sqrt{a}b^3\sqrt{c}-40\,a^{\frac{3}{2}}\right.}{128\,c^{\frac{5}{2}}}$$

input
```
integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^3,x, algorithm="giac")
```

output
```
1/64*sqrt(c*x^2 + b*x + a)*(2*(4*(2*c*x*sgn(x) + 3*b*sgn(x))*x + (b^2*c^2*
sgn(x) + 20*a*c^3*sgn(x))/c^3)*x - (3*b^3*c*sgn(x) - 20*a*b*c^2*sgn(x))/c^
3) - 3/128*(b^4*sgn(x) - 8*a*b^2*c*sgn(x) + 16*a^2*c^2*sgn(x))*log(abs(2*(
sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) + b))/c^(5/2) + 1/128*(3*b^4*lo
g(abs(b - 2*sqrt(a)*sqrt(c))) - 24*a*b^2*c*log(abs(b - 2*sqrt(a)*sqrt(c)))
 + 48*a^2*c^2*log(abs(b - 2*sqrt(a)*sqrt(c))) + 6*sqrt(a)*b^3*sqrt(c) - 40
*a^(3/2)*b*c^(3/2))*sgn(x)/c^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^3}\,dx = \int \frac{(c\,x^4+b\,x^3+a\,x^2)^{3/2}}{x^3}\,dx$$

input
```
int((a*x^2 + b*x^3 + c*x^4)^(3/2)/x^3,x)
```

output
```
int((a*x^2 + b*x^3 + c*x^4)^(3/2)/x^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.84

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^3}\,dx = \frac{40\sqrt{c\,x^2+bx+a}\,ab\,c^2+80\sqrt{c\,x^2+bx+a}\,a\,c^3x-6\sqrt{c\,x^2+bx+a}\,b^3c+4\sqrt{}$$

input
```
int((c*x^4+b*x^3+a*x^2)^(3/2)/x^3,x)
```

output
```
(40*sqrt(a + b*x + c*x**2)*a*b*c**2 + 80*sqrt(a + b*x + c*x**2)*a*c**3*x -
 6*sqrt(a + b*x + c*x**2)*b**3*c + 4*sqrt(a + b*x + c*x**2)*b**2*c**2*x +
48*sqrt(a + b*x + c*x**2)*b*c**3*x**2 + 32*sqrt(a + b*x + c*x**2)*c**4*x**
3 + 48*sqrt(c)*log((2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)/sqrt(4*a
*c - b**2))*a**2*c**2 - 24*sqrt(c)*log((2*sqrt(c)*sqrt(a + b*x + c*x**2) +
 b + 2*c*x)/sqrt(4*a*c - b**2))*a*b**2*c + 3*sqrt(c)*log((2*sqrt(c)*sqrt(a
 + b*x + c*x**2) + b + 2*c*x)/sqrt(4*a*c - b**2))*b**4)/(128*c**3)
```

3.60 $\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^4} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 227

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^4} \, dx = \frac{(b^2+8ac+2bcx)\sqrt{ax^2+bx^3+cx^4}}{8cx} + \frac{(ax^2+bx^3+cx^4)^{3/2}}{3x^3} - \frac{a^{3/2}x\sqrt{a+bx+cx^2}\,\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{ax^2+bx^3+cx^4}} - \frac{b(b^2-12ac)\,x\sqrt{a+bx+cx^2}\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{3/2}\sqrt{ax^2+bx^3+cx^4}}$$

output

```
1/8*(2*b*c*x+8*a*c+b^2)*(c*x^4+b*x^3+a*x^2)^(1/2)/c/x+1/3*(c*x^4+b*x^3+a*x
^2)^(3/2)/x^3-a^(3/2)*x*(c*x^2+b*x+a)^(1/2)*arctanh(1/2*(b*x+2*a)/a^(1/2)/
(c*x^2+b*x+a)^(1/2))/(c*x^4+b*x^3+a*x^2)^(1/2)-1/16*b*(-12*a*c+b^2)*x*(c*x
^2+b*x+a)^(1/2)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(3/2)
/(c*x^4+b*x^3+a*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.87 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.72

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^4}\,dx = \frac{x\sqrt{a+x(b+cx)}\left(-3b(b^2-12ac)\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)+2\sqrt{c}\left(\sqrt{a+x(\right.\right.}{48c^{3/2}\sqrt{x^2(a}}$$

input
```
Integrate[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^4,x]
```

output
```
(x*Sqrt[a + x*(b + c*x)]*(-3*b*(b^2 - 12*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[
c]*Sqrt[a + x*(b + c*x)])] + 2*Sqrt[c]*(Sqrt[a + x*(b + c*x)]*(3*b^2 + 14*
b*c*x + 8*c*(4*a + c*x^2)) + 48*a^(3/2)*c*ArcTanh[(Sqrt[c]*x - Sqrt[a + x*
(b + c*x)])/Sqrt[a]])))/(48*c^(3/2)*Sqrt[x^2*(a + x*(b + c*x))])
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.90, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1968, 1992, 27, 1980, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^4}\,dx$$

$$\Big\downarrow \text{1968}$$

$$\frac{1}{2}\int \frac{(2a+bx)\sqrt{cx^4+bx^3+ax^2}}{x^2}dx+\frac{(ax^2+bx^3+cx^4)^{3/2}}{3x^3}$$

$$\Big\downarrow \text{1992}$$

$$\frac{1}{2}\left(\frac{\int \frac{16a^2c-b(b^2-12ac)x}{2\sqrt{cx^4+bx^3+ax^2}}dx}{4c}+\frac{\sqrt{ax^2+bx^3+cx^4}(8ac+b^2+2bcx)}{4cx}\right)+\frac{(ax^2+bx^3+cx^4)^{3/2}}{3x^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{\int \frac{16a^2c-b(b^2-12ac)x}{\sqrt{cx^4+bx^3+ax^2}}dx}{8c}+\frac{\sqrt{ax^2+bx^3+cx^4}(8ac+b^2+2bcx)}{4cx}\right)+\frac{(ax^2+bx^3+cx^4)^{3/2}}{3x^3}$$

$\Big\downarrow$ 1980

$$\frac{1}{2}\left(\frac{x\sqrt{a+bx+cx^2}\int \frac{16a^2c-b(b^2-12ac)x}{x\sqrt{cx^2+bx+a}}dx}{8c\sqrt{ax^2+bx^3+cx^4}}+\frac{\sqrt{ax^2+bx^3+cx^4}(8ac+b^2+2bcx)}{4cx}\right)+\frac{(ax^2+bx^3+cx^4)^{3/2}}{3x^3}$$

$\Big\downarrow$ 1269

$$\frac{1}{2}\left(\frac{x\sqrt{a+bx+cx^2}\left(16a^2c\int \frac{1}{x\sqrt{cx^2+bx+a}}dx-b(b^2-12ac)\int \frac{1}{\sqrt{cx^2+bx+a}}dx\right)}{8c\sqrt{ax^2+bx^3+cx^4}}+\frac{\sqrt{ax^2+bx^3+cx^4}(8ac+b^2+2bc\right.$$

$$\frac{(ax^2+bx^3+cx^4)^{3/2}}{3x^3}$$

$\Big\downarrow$ 1092

$$\frac{1}{2}\left(\frac{x\sqrt{a+bx+cx^2}\left(16a^2c\int \frac{1}{x\sqrt{cx^2+bx+a}}dx-2b(b^2-12ac)\int \frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}\right)}{8c\sqrt{ax^2+bx^3+cx^4}}+\frac{\sqrt{ax^2+bx^3+cx^4}(8c}{4cx}\right.$$

$$\frac{(ax^2+bx^3+cx^4)^{3/2}}{3x^3}$$

$\Big\downarrow$ 219

$$\frac{1}{2}\left(\frac{x\sqrt{a+bx+cx^2}\left(16a^2c\int \frac{1}{x\sqrt{cx^2+bx+a}}dx-\frac{b(b^2-12ac)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}}\right)}{8c\sqrt{ax^2+bx^3+cx^4}}+\frac{\sqrt{ax^2+bx^3+cx^4}(8ac+b^2}{4cx}\right.$$

$$\frac{(ax^2+bx^3+cx^4)^{3/2}}{3x^3}$$

$\Big\downarrow$ 1154

$$\frac{1}{2}\left(\frac{x\sqrt{a+bx+cx^2}\left(-32a^2c\int\frac{1}{4a-\frac{(2a+bx)^2}{cx^2+bx+a}}d\frac{2a+bx}{\sqrt{cx^2+bx+a}}-\frac{b(b^2-12ac)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}}\right)}{8c\sqrt{ax^2+bx^3+cx^4}}+\frac{\sqrt{ax^2+bx^3+c\ldots}}{}\right.$$

$$\frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{3x^3}$$

$\Big\downarrow$ 219

$$\frac{1}{2}\left(\frac{x\sqrt{a+bx+cx^2}\left(-16a^{3/2}c\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)-\frac{b(b^2-12ac)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}}\right)}{8c\sqrt{ax^2+bx^3+cx^4}}+\frac{\sqrt{ax^2+bx^3+c\ldots}}{}\right.$$

$$\frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{3x^3}$$

input
```
Int[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^4,x]
```

output
```
(a*x^2 + b*x^3 + c*x^4)^(3/2)/(3*x^3) + (((b^2 + 8*a*c + 2*b*c*x)*Sqrt[a*x
^2 + b*x^3 + c*x^4])/(4*c*x) + (x*Sqrt[a + b*x + c*x^2]*(-16*a^(3/2)*c*Arc
Tanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])] - (b*(b^2 - 12*a*c)*Ar
cTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/Sqrt[c]))/(8*c*Sqrt[
a*x^2 + b*x^3 + c*x^4]))/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1968
```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[x^(m + 1)*((a*x^q + b*x^n + c*x^(2*n - q))^p/(m + p*(2
*n - q) + 1)), x] + Simp[(n - q)*(p/(m + p*(2*n - q) + 1))   Int[x^(m + q)*
(2*a + b*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; Free
Q[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] &&  !IntegerQ[p] && NeQ[b
^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && GtQ[m + p*q
 + 1, -(n - q)] && NeQ[m + p*(2*n - q) + 1, 0]
```

rule 1980
```
Int[((A_) + (B_.)*(x_)^(j_.))/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c
_.)*(x_)^(r_.)], x_Symbol] :> Simp[x^(q/2)*(Sqrt[a + b*x^(n - q) + c*x^(2*(
n - q))]/Sqrt[a*x^q + b*x^n + c*x^(2*n - q)])   Int[(A + B*x^(n - q))/(x^(q
/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]), x], x] /; FreeQ[{a, b, c, A,
B, n, q}, x] && EqQ[j, n - q] && EqQ[r, 2*n - q] && PosQ[n - q] && EqQ[n, 3
] && EqQ[q, 2]
```

rule 1992
```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[x^(m + 1)*(b*B*(n - q)*p +
A*c*(m + p*q + (n - q)*(2*p + 1) + 1) + B*c*(m + p*q + 2*(n - q)*p + 1)*x^(
n - q))*((a*x^q + b*x^n + c*x^(2*n - q))^p/(c*(m + p*(2*n - q) + 1)*(m + p*
q + (n - q)*(2*p + 1) + 1))), x] + Simp[(n - q)*(p/(c*(m + p*(2*n - q) + 1)
*(m + p*q + (n - q)*(2*p + 1) + 1)))   Int[x^(m + q)*Simp[2*a*A*c*(m + p*q
+ (n - q)*(2*p + 1) + 1) - a*b*B*(m + p*q + 1) + (2*a*B*c*(m + p*q + 2*(n -
 q)*p + 1) + A*b*c*(m + p*q + (n - q)*(2*p + 1) + 1) - b^2*B*(m + p*q + (n
- q)*p + 1))*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x]
/; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] &&  !Integ
erQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q]
 && GtQ[m + p*q, -(n - q) - 1] && NeQ[m + p*(2*n - q) + 1, 0] && NeQ[m + p*
q + (n - q)*(2*p + 1) + 1, 0]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.85

method	result
pseudoelliptic	$\frac{16x^{2}\sqrt{c\,x^{2}+bx+a}\,c^{\frac{5}{2}}-48\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^{2}+bx+a}}{x\sqrt{a}}\right)a^{\frac{3}{2}}c^{\frac{3}{2}}+48\ln\left(2\right)a^{\frac{3}{2}}c^{\frac{3}{2}}+28c^{\frac{3}{2}}\sqrt{c\,x^{2}+bx+a}\,bx+64a\,c^{\frac{3}{2}}\sqrt{c\,x^{2}+bx+a}}{48c^{\frac{3}{2}}}$
default	$-\frac{\left(c\,x^{4}+b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}\left(48c^{\frac{5}{2}}a^{\frac{3}{2}}\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^{2}+bx+a}}{x}\right)-16\left(c\,x^{2}+bx+a\right)^{\frac{3}{2}}c^{\frac{5}{2}}-12c^{\frac{5}{2}}\sqrt{c\,x^{2}+bx+a}\,bx-48c^{\frac{5}{2}}\sqrt{c\,x^{2}+}\right.}{48x^{3}\left(c\,x^{2}+bx+a\right)}$

input
```
int((c*x^4+b*x^3+a*x^2)^(3/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
1/48*(16*x^2*(c*x^2+b*x+a)^(1/2)*c^(5/2)-48*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b
*x+a)^(1/2))/x/a^(1/2))*a^(3/2)*c^(3/2)+48*ln(2)*a^(3/2)*c^(3/2)+28*c^(3/2
)*(c*x^2+b*x+a)^(1/2)*b*x+64*a*c^(3/2)*(c*x^2+b*x+a)^(1/2)+6*c^(1/2)*(c*x^
2+b*x+a)^(1/2)*b^2+36*ln(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)*a*b*c-3*ln
(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)*b^3)/c^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 791, normalized size of antiderivative = 3.48

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^4}\,dx = \text{Too large to display}$$

input
```
integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^4,x, algorithm="fricas")
```

output
```
[1/96*(48*a^(3/2)*c^2*x*log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x - 4*
sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(a))/x^3) - 3*(b^3 - 12*a*b*c)
*sqrt(c)*x*log(-(8*c^2*x^3 + 8*b*c*x^2 + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*
c*x + b)*sqrt(c) + (b^2 + 4*a*c)*x)/x) + 4*(8*c^3*x^2 + 14*b*c^2*x + 3*b^2
*c + 32*a*c^2)*sqrt(c*x^4 + b*x^3 + a*x^2))/(c^2*x), 1/48*(24*a^(3/2)*c^2*
x*log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x - 4*sqrt(c*x^4 + b*x^3 + a
*x^2)*(b*x + 2*a)*sqrt(a))/x^3) + 3*(b^3 - 12*a*b*c)*sqrt(-c)*x*arctan(1/2
*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c
*x)) + 2*(8*c^3*x^2 + 14*b*c^2*x + 3*b^2*c + 32*a*c^2)*sqrt(c*x^4 + b*x^3
+ a*x^2))/(c^2*x), 1/96*(96*sqrt(-a)*a*c^2*x*arctan(1/2*sqrt(c*x^4 + b*x^3
 + a*x^2)*(b*x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2 + a^2*x)) - 3*(b^3 - 12*
a*b*c)*sqrt(c)*x*log(-(8*c^2*x^3 + 8*b*c*x^2 + 4*sqrt(c*x^4 + b*x^3 + a*x^
2)*(2*c*x + b)*sqrt(c) + (b^2 + 4*a*c)*x)/x) + 4*(8*c^3*x^2 + 14*b*c^2*x +
 3*b^2*c + 32*a*c^2)*sqrt(c*x^4 + b*x^3 + a*x^2))/(c^2*x), 1/48*(48*sqrt(-
a)*a*c^2*x*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(-a)/(a*
c*x^3 + a*b*x^2 + a^2*x)) + 3*(b^3 - 12*a*b*c)*sqrt(-c)*x*arctan(1/2*sqrt(
c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x)) +
 2*(8*c^3*x^2 + 14*b*c^2*x + 3*b^2*c + 32*a*c^2)*sqrt(c*x^4 + b*x^3 + a*x^
2))/(c^2*x)]
```

Sympy [F]

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^4}\,dx = \int \frac{\left(x^2(a+bx+cx^2)\right)^{\frac{3}{2}}}{x^4}\,dx$$

input
```
integrate((c*x**4+b*x**3+a*x**2)**(3/2)/x**4,x)
```

output
```
Integral((x**2*(a + b*x + c*x**2))**(3/2)/x**4, x)
```

Maxima [F]

$$\int \frac{(ax^2 + bx^3 + cx^4)^{3/2}}{x^4} \, dx = \int \frac{(cx^4 + bx^3 + ax^2)^{\frac{3}{2}}}{x^4} \, dx$$

input
```
integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^4,x, algorithm="maxima")
```

output
```
integrate((c*x^4 + b*x^3 + a*x^2)^(3/2)/x^4, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(ax^2 + bx^3 + cx^4)^{3/2}}{x^4} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^4,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Degree mismatch inside factorisatio
n over extensionNot implemented, e.g. for multivariate mod/approx polynomi
alsError:
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^4}\,dx=\int \frac{(c\,x^4+b\,x^3+a\,x^2)^{3/2}}{x^4}\,dx$$

input
```
int((a*x^2 + b*x^3 + c*x^4)^(3/2)/x^4,x)
```

output
```
int((a*x^2 + b*x^3 + c*x^4)^(3/2)/x^4, x)
```

Reduce [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 1018, normalized size of antiderivative = 4.48

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^4}\,dx=\text{Too large to display}$$

input
```
int((c*x^4+b*x^3+a*x^2)^(3/2)/x^4,x)
```

output
```
( - 48*sqrt(a)*sqrt(4*sqrt(c)*sqrt(a)*b - 4*a*c - b**2)*atan((2*sqrt(c)*sq
rt(a + b*x + c*x**2) + b + 2*c*x)/sqrt(4*sqrt(c)*sqrt(a)*b - 4*a*c - b**2)
)*a*b*c**2 - 96*sqrt(c)*sqrt(4*sqrt(c)*sqrt(a)*b - 4*a*c - b**2)*atan((2*s
qrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)/sqrt(4*sqrt(c)*sqrt(a)*b - 4*a*
c - b**2))*a**2*c**2 - 24*sqrt(a)*sqrt(4*sqrt(c)*sqrt(a)*b + 4*a*c + b**2)
*log( - sqrt(4*sqrt(c)*sqrt(a)*b + 4*a*c + b**2) + 2*sqrt(c)*sqrt(a + b*x
+ c*x**2) + b + 2*c*x)*a*b*c**2 + 24*sqrt(a)*sqrt(4*sqrt(c)*sqrt(a)*b + 4*
a*c + b**2)*log(sqrt(4*sqrt(c)*sqrt(a)*b + 4*a*c + b**2) + 2*sqrt(c)*sqrt(
a + b*x + c*x**2) + b + 2*c*x)*a*b*c**2 + 48*sqrt(c)*sqrt(4*sqrt(c)*sqrt(a
)*b + 4*a*c + b**2)*log( - sqrt(4*sqrt(c)*sqrt(a)*b + 4*a*c + b**2) + 2*sq
rt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)*a**2*c**2 - 48*sqrt(c)*sqrt(4*sq
rt(c)*sqrt(a)*b + 4*a*c + b**2)*log(sqrt(4*sqrt(c)*sqrt(a)*b + 4*a*c + b**
2) + 2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)*a**2*c**2 + 256*sqrt(a
+ b*x + c*x**2)*a**2*c**3 - 40*sqrt(a + b*x + c*x**2)*a*b**2*c**2 + 112*sq
rt(a + b*x + c*x**2)*a*b*c**3*x + 64*sqrt(a + b*x + c*x**2)*a*c**4*x**2 -
6*sqrt(a + b*x + c*x**2)*b**4*c - 28*sqrt(a + b*x + c*x**2)*b**3*c**2*x -
16*sqrt(a + b*x + c*x**2)*b**2*c**3*x**2 + 96*sqrt(a)*log( - sqrt(4*sqrt(c
)*sqrt(a)*b + 4*a*c + b**2) + 2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x
)*a**2*c**3 - 24*sqrt(a)*log( - sqrt(4*sqrt(c)*sqrt(a)*b + 4*a*c + b**2) +
 2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)*a*b**2*c**2 + 96*sqrt(a)...
```

3.61 $\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^5}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 219

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^5}\,dx = \frac{3(3b+2cx)\sqrt{ax^2+bx^3+cx^4}}{4x} - \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^4} - \frac{3\sqrt{a}bx\sqrt{a+bx+cx^2}\operatorname{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{ax^2+bx^3+cx^4}} + \frac{3(b^2+4ac)\,x\sqrt{a+bx+cx^2}\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8\sqrt{c}\sqrt{ax^2+bx^3+cx^4}}$$

output
```
3/4*(2*c*x+3*b)*(c*x^4+b*x^3+a*x^2)^(1/2)/x-(c*x^4+b*x^3+a*x^2)^(3/2)/x^4-
3/2*a^(1/2)*b*x*(c*x^2+b*x+a)^(1/2)*arctanh(1/2*(b*x+2*a)/a^(1/2)/(c*x^2+b
*x+a)^(1/2))/(c*x^4+b*x^3+a*x^2)^(1/2)+3/8*(4*a*c+b^2)*x*(c*x^2+b*x+a)^(1/
2)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(1/2)/(c*x^4+b*x^3
+a*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.82 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.71

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^5}\,dx = \frac{\sqrt{a+x(b+cx)}\left(2\sqrt{c}\sqrt{a+x(b+cx)}(-4a+x(5b+2cx))+24\sqrt{a}b\sqrt{c}x\text{arct}\right.}{8\sqrt{c}\sqrt{x^2(a-}}$$

input
```
Integrate[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^5,x]
```

output
```
(Sqrt[a + x*(b + c*x)]*(2*Sqrt[c]*Sqrt[a + x*(b + c*x)]*(-4*a + x*(5*b + 2
*c*x)) + 24*Sqrt[a]*b*Sqrt[c]*x*ArcTanh[(Sqrt[c]*x - Sqrt[a + x*(b + c*x)]
)/Sqrt[a]] - 3*(b^2 + 4*a*c)*x*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c
*x)]]))/(8*Sqrt[c]*Sqrt[x^2*(a + x*(b + c*x))])
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.86, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1967, 1992, 27, 1980, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^5}dx$$

$$\Big\downarrow \text{1967}$$

$$\frac{3}{2}\int \frac{(b+2cx)\sqrt{cx^4+bx^3+ax^2}}{x^2}dx-\frac{(ax^2+bx^3+cx^4)^{3/2}}{x^4}$$

$$\Big\downarrow \text{1992}$$

$$\frac{3}{2}\left(\frac{\int \frac{c(4ab+(b^2+4ac)x)}{\sqrt{cx^4+bx^3+ax^2}}dx}{4c}+\frac{(3b+2cx)\sqrt{ax^2+bx^3+cx^4}}{2x}\right)-\frac{(ax^2+bx^3+cx^4)^{3/2}}{x^4}$$

$$\Big\downarrow \text{27}$$

$$\frac{3}{2}\left(\frac{1}{4}\int \frac{4ab+\left(b^2+4ac\right)x}{\sqrt{cx^4+bx^3+ax^2}}dx+\frac{(3b+2cx)\sqrt{ax^2+bx^3+cx^4}}{2x}\right)-\frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{x^4}$$

$$\Big\downarrow \text{ 1980}$$

$$\frac{3}{2}\left(\frac{x\sqrt{a+bx+cx^2}\int \frac{4ab+\left(b^2+4ac\right)x}{x\sqrt{cx^2+bx+a}}dx}{4\sqrt{ax^2+bx^3+cx^4}}+\frac{(3b+2cx)\sqrt{ax^2+bx^3+cx^4}}{2x}\right)-\frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{x^4}$$

$$\Big\downarrow \text{ 1269}$$

$$\frac{3}{2}\left(\frac{x\sqrt{a+bx+cx^2}\left(\left(4ac+b^2\right)\int \frac{1}{\sqrt{cx^2+bx+a}}dx+4ab\int \frac{1}{x\sqrt{cx^2+bx+a}}dx\right)}{4\sqrt{ax^2+bx^3+cx^4}}+\frac{(3b+2cx)\sqrt{ax^2+bx^3+cx^4}}{2x}\right)-\frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{x^4}$$

$$\Big\downarrow \text{ 1092}$$

$$\frac{3}{2}\left(\frac{x\sqrt{a+bx+cx^2}\left(2\left(4ac+b^2\right)\int \frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}+4ab\int \frac{1}{x\sqrt{cx^2+bx+a}}dx\right)}{4\sqrt{ax^2+bx^3+cx^4}}+\frac{(3b+2cx)\sqrt{ax^2+bx^3+}}{2x} \right.$$

$$\frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{x^4}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{3}{2}\left(\frac{x\sqrt{a+bx+cx^2}\left(4ab\int \frac{1}{x\sqrt{cx^2+bx+a}}dx+\frac{\left(4ac+b^2\right)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}}\right)}{4\sqrt{ax^2+bx^3+cx^4}}+\frac{(3b+2cx)\sqrt{ax^2+bx^3+cx^4}}{2x}\right)$$

$$\frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{x^4}$$

$$\Big\downarrow \text{ 1154}$$

$$\frac{3}{2}\left(\frac{x\sqrt{a+bx+cx^2}\left(\frac{(4ac+b^2)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}}-8ab\int\frac{1}{4a-\frac{(2a+bx)^2}{cx^2+bx+a}}d\frac{2a+bx}{\sqrt{cx^2+bx+a}}\right)}{4\sqrt{ax^2+bx^3+cx^4}}+\frac{(3b+2cx)\sqrt{ax^2+bx^3}}{2x}\right.$$

$$\frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{x^4}$$

$$\Big\downarrow \; 219$$

$$\frac{3}{2}\left(\frac{x\sqrt{a+bx+cx^2}\left(\frac{(4ac+b^2)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}}-4\sqrt{a}b\,\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)\right)}{4\sqrt{ax^2+bx^3+cx^4}}+\frac{(3b+2cx)\sqrt{ax^2+bx}}{2x}\right.$$

$$\frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{x^4}$$

input
```
Int[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^5,x]
```

output
```
-((a*x^2 + b*x^3 + c*x^4)^(3/2)/x^4) + (3*(((3*b + 2*c*x)*Sqrt[a*x^2 + b*x
^3 + c*x^4])/(2*x) + (x*Sqrt[a + b*x + c*x^2]*(-4*Sqrt[a]*b*ArcTanh[(2*a +
 b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])] + ((b^2 + 4*a*c)*ArcTanh[(b + 2*c
*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/Sqrt[c]))/(4*Sqrt[a*x^2 + b*x^3 +
c*x^4])))/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1967
```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[x^(m + 1)*((a*x^q + b*x^n + c*x^(2*n - q))^p/(m + p*q
+ 1)), x] - Simp[(n - q)*(p/(m + p*q + 1))   Int[x^(m + n)*(b + 2*c*x^(n -
q))*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c}, x]
&& EqQ[r, 2*n - q] && PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] &
& IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && LeQ[m + p*q + 1, -(n - q) +
 1] && NeQ[m + p*q + 1, 0]
```

rule 1980
```
Int[((A_) + (B_.)*(x_)^(j_.))/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c
_.)*(x_)^(r_.)], x_Symbol] :> Simp[x^(q/2)*(Sqrt[a + b*x^(n - q) + c*x^(2*(
n - q))]/Sqrt[a*x^q + b*x^n + c*x^(2*n - q)])   Int[(A + B*x^(n - q))/(x^(q
/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]), x], x] /; FreeQ[{a, b, c, A,
B, n, q}, x] && EqQ[j, n - q] && EqQ[r, 2*n - q] && PosQ[n - q] && EqQ[n, 3
] && EqQ[q, 2]
```

rule 1992
```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[x^(m + 1)*(b*B*(n - q)*p +
A*c*(m + p*q + (n - q)*(2*p + 1) + 1) + B*c*(m + p*q + 2*(n - q)*p + 1)*x^(
n - q))*((a*x^q + b*x^n + c*x^(2*n - q))^p/(c*(m + p*(2*n - q) + 1)*(m + p*
q + (n - q)*(2*p + 1) + 1))), x] + Simp[(n - q)*(p/(c*(m + p*(2*n - q) + 1)
*(m + p*q + (n - q)*(2*p + 1) + 1)))   Int[x^(m + q)*Simp[2*a*A*c*(m + p*q
+ (n - q)*(2*p + 1) + 1) - a*b*B*(m + p*q + 1) + (2*a*B*c*(m + p*q + 2*(n -
 q)*p + 1) + A*b*c*(m + p*q + (n - q)*(2*p + 1) + 1) - b^2*B*(m + p*q + (n
- q)*p + 1))*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x]
/; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] &&  !Integ
erQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q]
 && GtQ[m + p*q, -(n - q) - 1] && NeQ[m + p*(2*n - q) + 1, 0] && NeQ[m + p*
q + (n - q)*(2*p + 1) + 1, 0]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.82

method	result
pseudoelliptic	$\frac{12\ln(2)bx\sqrt{c}\,\sqrt{a}-12\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x\sqrt{a}}\right)bx\sqrt{c}\,\sqrt{a}+4c^{\frac{3}{2}}x^2\sqrt{c\,x^2+bx+a}+12\ln\left(2\sqrt{c\,x^2+bx+a}\,\sqrt{c}+2cx+b\right)acx+}{8x\sqrt{c}}$
risch	$-\frac{a\sqrt{x^2(c\,x^2+bx+a)}}{x^2}+\frac{\left(\frac{3b^2\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{8\sqrt{c}}+\frac{3a\sqrt{c}\,\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{2}+\frac{cx\sqrt{c\,x^2+bx+a}}{2}+\frac{5\sqrt{c\,x^2+bx+a}\,b}{4}\right.}{x\sqrt{c\,x^2+bx+a}}$
default	$\frac{(c\,x^4+b\,x^3+a\,x^2)^{\frac{3}{2}}\left(8c^{\frac{5}{2}}(c\,x^2+bx+a)^{\frac{3}{2}}x^2+12c^{\frac{5}{2}}\sqrt{c\,x^2+bx+a}\,a\,x^2-12c^{\frac{3}{2}}a^{\frac{3}{2}}\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x}\right)bx-8(c\,x^2+bx\right.}{8x^4}$

input
```
int((c*x^4+b*x^3+a*x^2)^(3/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
1/8/x*(12*ln(2)*b*x*c^(1/2)*a^(1/2)-12*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)
^(1/2))/x/a^(1/2))*b*x*c^(1/2)*a^(1/2)+4*c^(3/2)*x^2*(c*x^2+b*x+a)^(1/2)+1
2*ln(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)*a*c*x+3*ln(2*(c*x^2+b*x+a)^(1/
2)*c^(1/2)+2*c*x+b)*b^2*x+10*b*(c*x^2+b*x+a)^(1/2)*x*c^(1/2)-8*a*(c*x^2+b*
x+a)^(1/2)*c^(1/2))/c^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 757, normalized size of antiderivative = 3.46

$$\int \frac{(ax^2 + bx^3 + cx^4)^{3/2}}{x^5} \, dx = \text{Too large to display}$$

input
```
integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^5,x, algorithm="fricas")
```

output
```
[1/16*(12*sqrt(a)*b*c*x^2*log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x -
4*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(a))/x^3) + 3*(b^2 + 4*a*c)*
sqrt(c)*x^2*log(-(8*c^2*x^3 + 8*b*c*x^2 + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2
*c*x + b)*sqrt(c) + (b^2 + 4*a*c)*x)/x) + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2
*c^2*x^2 + 5*b*c*x - 4*a*c))/(c*x^2), 1/8*(6*sqrt(a)*b*c*x^2*log(-(8*a*b*x
^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*
a)*sqrt(a))/x^3) - 3*(b^2 + 4*a*c)*sqrt(-c)*x^2*arctan(1/2*sqrt(c*x^4 + b*
x^3 + a*x^2)*(2*c*x + b)*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x)) + 2*sqrt(c*
x^4 + b*x^3 + a*x^2)*(2*c^2*x^2 + 5*b*c*x - 4*a*c))/(c*x^2), 1/16*(24*sqrt
(-a)*b*c*x^2*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(-a)/(
a*c*x^3 + a*b*x^2 + a^2*x)) + 3*(b^2 + 4*a*c)*sqrt(c)*x^2*log(-(8*c^2*x^3
+ 8*b*c*x^2 + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(c) + (b^2 + 4
*a*c)*x)/x) + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c^2*x^2 + 5*b*c*x - 4*a*c))
/(c*x^2), 1/8*(12*sqrt(-a)*b*c*x^2*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*
(b*x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2 + a^2*x)) - 3*(b^2 + 4*a*c)*sqrt(-
c)*x^2*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(-c)/(c^2*x^
3 + b*c*x^2 + a*c*x)) + 2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c^2*x^2 + 5*b*c*x
 - 4*a*c))/(c*x^2)]
```

Sympy [F]

$$\int \frac{(ax^2 + bx^3 + cx^4)^{3/2}}{x^5} \, dx = \int \frac{(x^2(a + bx + cx^2))^{\frac{3}{2}}}{x^5} \, dx$$

input
```
integrate((c*x**4+b*x**3+a*x**2)**(3/2)/x**5,x)
```

output
```
Integral((x**2*(a + b*x + c*x**2))**(3/2)/x**5, x)
```

Maxima [F]

$$\int \frac{(ax^2 + bx^3 + cx^4)^{3/2}}{x^5} \, dx = \int \frac{(cx^4 + bx^3 + ax^2)^{\frac{3}{2}}}{x^5} \, dx$$

input
```
integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^5,x, algorithm="maxima")
```

output
```
integrate((c*x^4 + b*x^3 + a*x^2)^(3/2)/x^5, x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{(ax^2 + bx^3 + cx^4)^{3/2}}{x^5} \, dx = \text{Timed out}$$

input
```
integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^5,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ax^2 + bx^3 + cx^4)^{3/2}}{x^5} \, dx = \int \frac{(c\,x^4 + b\,x^3 + a\,x^2)^{3/2}}{x^5} \, dx$$

input
```
int((a*x^2 + b*x^3 + c*x^4)^(3/2)/x^5,x)
```

output
```
int((a*x^2 + b*x^3 + c*x^4)^(3/2)/x^5, x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.74

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^5}\,dx = \frac{-8\sqrt{c\,x^2+bx+a}\,ac+10\sqrt{c\,x^2+bx+a}\,bcx+4\sqrt{c\,x^2+bx+a}\,c^2x^2+12\sqrt{}}{}$$

input
```
int((c*x^4+b*x^3+a*x^2)^(3/2)/x^5,x)
```

output
```
( - 8*sqrt(a + b*x + c*x**2)*a*c + 10*sqrt(a + b*x + c*x**2)*b*c*x + 4*sqr
t(a + b*x + c*x**2)*c**2*x**2 + 12*sqrt(a)*log(2*sqrt(a)*sqrt(a + b*x + c*
x**2) - 2*a - b*x)*b*c*x - 12*sqrt(a)*log(x)*b*c*x + 12*sqrt(c)*log( - 2*s
qrt(c)*sqrt(a + b*x + c*x**2) - b - 2*c*x)*a*c*x + 3*sqrt(c)*log( - 2*sqrt
(c)*sqrt(a + b*x + c*x**2) - b - 2*c*x)*b**2*x)/(8*c*x)
```

3.62 $\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^6}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 219

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^6}\,dx = -\frac{3(b-2cx)\sqrt{ax^2+bx^3+cx^4}}{4x^2} - \frac{(ax^2+bx^3+cx^4)^{3/2}}{2x^5} - \frac{3(b^2+4ac)\,x\sqrt{a+bx+cx^2}\,\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{8\sqrt{a}\sqrt{ax^2+bx^3+cx^4}} + \frac{3b\sqrt{c}x\sqrt{a+bx+cx^2}\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{ax^2+bx^3+cx^4}}$$

output
```
-3/4*(-2*c*x+b)*(c*x^4+b*x^3+a*x^2)^(1/2)/x^2-1/2*(c*x^4+b*x^3+a*x^2)^(3/2
)/x^5-3/8*(4*a*c+b^2)*x*(c*x^2+b*x+a)^(1/2)*arctanh(1/2*(b*x+2*a)/a^(1/2)/
(c*x^2+b*x+a)^(1/2))/a^(1/2)/(c*x^4+b*x^3+a*x^2)^(1/2)+3/2*b*c^(1/2)*x*(c*
x^2+b*x+a)^(1/2)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/(c*x^4
+b*x^3+a*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.86 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.73

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^6}\,dx = \frac{\sqrt{x^2(a+x(b+cx))}\left(3(b^2+4ac)\,x^2\text{arctanh}\left(\frac{\sqrt{c}x-\sqrt{a+x(b+cx)}}{\sqrt{a}}\right)-\sqrt{a}\left((2a+x\right.\right.}{4\sqrt{a}x^3\sqrt{a}}$$

input
```
Integrate[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^6,x]
```

output
```
(Sqrt[x^2*(a + x*(b + c*x))]*(3*(b^2 + 4*a*c)*x^2*ArcTanh[(Sqrt[c]*x - Sqr
t[a + x*(b + c*x)])/Sqrt[a]] - Sqrt[a]*((2*a + x*(5*b - 4*c*x))*Sqrt[a + x
*(b + c*x)] + 6*b*Sqrt[c]*x^2*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*
x)]])))/(4*Sqrt[a]*x^3*Sqrt[a + x*(b + c*x)])
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.86, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1967, 1988, 25, 1980, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^6}\,dx$$

$$\Big\downarrow \text{1967}$$

$$\frac{3}{4}\int \frac{(b+2cx)\sqrt{cx^4+bx^3+ax^2}}{x^3}dx-\frac{(ax^2+bx^3+cx^4)^{3/2}}{2x^5}$$

$$\Big\downarrow \text{1988}$$

$$\frac{3}{4}\left(-\frac{1}{2}\int -\frac{b^2+4cxb+4ac}{\sqrt{cx^4+bx^3+ax^2}}dx-\frac{(b-2cx)\sqrt{ax^2+bx^3+cx^4}}{x^2}\right)-\frac{(ax^2+bx^3+cx^4)^{3/2}}{2x^5}$$

$$\Big\downarrow \text{25}$$

$$\frac{3}{4}\left(\frac{1}{2}\int \frac{b^2+4cxb+4ac}{\sqrt{cx^4+bx^3+ax^2}}dx-\frac{(b-2cx)\sqrt{ax^2+bx^3+cx^4}}{x^2}\right)-\frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{2x^5}$$

$$\Big\downarrow \text{ 1980}$$

$$\frac{3}{4}\left(\frac{x\sqrt{a+bx+cx^2}\int \frac{b^2+4cxb+4ac}{x\sqrt{cx^2+bx+a}}dx}{2\sqrt{ax^2+bx^3+cx^4}}-\frac{(b-2cx)\sqrt{ax^2+bx^3+cx^4}}{x^2}\right)-\frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{2x^5}$$

$$\Big\downarrow \text{ 1269}$$

$$\frac{3}{4}\left(\frac{x\sqrt{a+bx+cx^2}\left(\left(4ac+b^2\right)\int \frac{1}{x\sqrt{cx^2+bx+a}}dx+4bc\int \frac{1}{\sqrt{cx^2+bx+a}}dx\right)}{2\sqrt{ax^2+bx^3+cx^4}}-\frac{(b-2cx)\sqrt{ax^2+bx^3+cx^4}}{x^2}\right)-\frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{2x^5}$$

$$\Big\downarrow \text{ 1092}$$

$$\frac{3}{4}\left(\frac{x\sqrt{a+bx+cx^2}\left(\left(4ac+b^2\right)\int \frac{1}{x\sqrt{cx^2+bx+a}}dx+8bc\int \frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}\right)}{2\sqrt{ax^2+bx^3+cx^4}}-\frac{(b-2cx)\sqrt{ax^2+bx^3+cx}}{x^2}\right.$$

$$\frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{2x^5}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{3}{4}\left(\frac{x\sqrt{a+bx+cx^2}\left(\left(4ac+b^2\right)\int \frac{1}{x\sqrt{cx^2+bx+a}}dx+4b\sqrt{c}\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\right)}{2\sqrt{ax^2+bx^3+cx^4}}-\frac{(b-2cx)\sqrt{ax^2+bx^3+cx}}{x^2}\right.$$

$$\frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{2x^5}$$

$$\Big\downarrow \text{ 1154}$$

$$\frac{3}{4}\left(\frac{x\sqrt{a+bx+cx^2}\left(4b\sqrt{c}\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)-2\left(4ac+b^2\right)\int \frac{1}{4a-\frac{(2a+bx)^2}{cx^2+bx+a}}d\frac{2a+bx}{\sqrt{cx^2+bx+a}}\right)}{2\sqrt{ax^2+bx^3+cx^4}}-\frac{(b-2cx)\sqrt{ax^2}}{x^2}\right.$$

$$\frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{2x^5}$$

$$\Big\downarrow \text{219}$$

$$\frac{3}{4}\left(\frac{x\sqrt{a+bx+cx^2}\left(4b\sqrt{c}\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)-\frac{(4ac+b^2)\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}}\right)}{2\sqrt{ax^2+bx^3+cx^4}}-\frac{(b-2cx)\sqrt{ax^2+bx^3}}{x^2}\right.$$

$$\frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{2x^5}$$

input
```
Int[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^6,x]
```

output
```
-1/2*(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^5 + (3*(-(((b - 2*c*x)*Sqrt[a*x^2 + b
*x^3 + c*x^4])/x^2) + (x*Sqrt[a + b*x + c*x^2]*(-(((b^2 + 4*a*c)*ArcTanh[(
2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/Sqrt[a]) + 4*b*Sqrt[c]*ArcT
anh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])]))/(2*Sqrt[a*x^2 + b*x^3
 + c*x^4])))/4
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1967
```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[x^(m + 1)*((a*x^q + b*x^n + c*x^(2*n - q))^p/(m + p*q
+ 1)), x] - Simp[(n - q)*(p/(m + p*q + 1))   Int[x^(m + n)*(b + 2*c*x^(n -
q))*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c}, x]
&& EqQ[r, 2*n - q] && PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] &
& IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && LeQ[m + p*q + 1, -(n - q) +
 1] && NeQ[m + p*q + 1, 0]
```

rule 1980
```
Int[((A_) + (B_.)*(x_)^(j_.))/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c
_.)*(x_)^(r_.)], x_Symbol] :> Simp[x^(q/2)*(Sqrt[a + b*x^(n - q) + c*x^(2*(
n - q))]/Sqrt[a*x^q + b*x^n + c*x^(2*n - q)])   Int[(A + B*x^(n - q))/(x^(q
/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]), x], x] /; FreeQ[{a, b, c, A,
B, n, q}, x] && EqQ[j, n - q] && EqQ[r, 2*n - q] && PosQ[n - q] && EqQ[n, 3
] && EqQ[q, 2]
```

rule 1988
```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[x^(m + 1)*(A*(m + p*q + (n
- q)*(2*p + 1) + 1) + B*(m + p*q + 1)*x^(n - q))*((a*x^q + b*x^n + c*x^(2*n
 - q))^p/((m + p*q + 1)*(m + p*q + (n - q)*(2*p + 1) + 1))), x] + Simp[(n -
 q)*(p/((m + p*q + 1)*(m + p*q + (n - q)*(2*p + 1) + 1)))   Int[x^(n + m)*S
imp[2*a*B*(m + p*q + 1) - A*b*(m + p*q + (n - q)*(2*p + 1) + 1) + (b*B*(m +
 p*q + 1) - 2*A*c*(m + p*q + (n - q)*(2*p + 1) + 1))*x^(n - q), x]*(a*x^q +
 b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ
[r, n - q] && EqQ[j, 2*n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IG
tQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && LeQ[m + p*q, -(n - q)] && NeQ[m
 + p*q + 1, 0] && NeQ[m + p*q + (n - q)*(2*p + 1) + 1, 0]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.64

method	result
pseudoelliptic	$-\frac{3\left(x^{2}\left(ac+\frac{b^{2}}{4}\right)\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^{2}+bx+a}}{x\sqrt{a}}\right)-\ln\left(2\sqrt{c\,x^{2}+bx+a}\,\sqrt{c}+2cx+b\right)b\,x^{2}\sqrt{a}\,\sqrt{c}+\frac{\left(a^{\frac{3}{2}}+\left(-2c\,x^{2}+\frac{5}{2}bx\right)\sqrt{a}\right)\sqrt{c\,x \cdots}}{3}\cdots\right.}{2\sqrt{a}\,x^{2}}$
risch	$-\frac{(5bx+2a)\sqrt{x^{2}(c\,x^{2}+bx+a)}}{4x^{3}}+\frac{\left(-\frac{3\sqrt{a}\,\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^{2}+bx+a}}{x}\right)c}{2}-\frac{3\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^{2}+bx+a}}{x}\right)b^{2}}{8\sqrt{a}}+\frac{3b\sqrt{c}\,\ln\left(\frac{b}{2} \cdots\right.}{\cdots}\cdots\right.}{x\sqrt{c\,x^{2}+bx+a}\cdots}$
default	$-\frac{(c\,x^{4}+b\,x^{3}+a\,x^{2})^{\frac{3}{2}}\left(12c^{\frac{5}{2}}a^{\frac{5}{2}}\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^{2}+bx+a}}{x}\right)x^{2}-2c^{\frac{5}{2}}\left(c\,x^{2}+bx+a\right)^{\frac{3}{2}}b\,x^{3}-4c^{\frac{5}{2}}\left(c\,x^{2}+bx+a\right)^{\frac{3}{2}}a\,x^{2}-6c^{\frac{5}{2}}\sqrt{\cdots}\right.}{\cdots}$

input
```
int((c*x^4+b*x^3+a*x^2)^(3/2)/x^6,x,method=_RETURNVERBOSE)
```

output
```
-3/2/a^(1/2)*(x^2*(a*c+1/4*b^2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))
/x/a^(1/2))-ln(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)*b*x^2*a^(1/2)*c^(1/2
)+1/3*(a^(3/2)+(-2*c*x^2+5/2*b*x)*a^(1/2))*(c*x^2+b*x+a)^(1/2)-ln(2)*x^2*(
a*c+1/4*b^2))/x^2
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 757, normalized size of antiderivative = 3.46

$$\int \frac{(ax^2 + bx^3 + cx^4)^{3/2}}{x^6}\,dx = \text{Too large to display}$$

input
```
integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^6,x, algorithm="fricas")
```

output

```
[1/16*(12*a*b*sqrt(c)*x^3*log(-(8*c^2*x^3 + 8*b*c*x^2 + 4*sqrt(c*x^4 + b*x
^3 + a*x^2)*(2*c*x + b)*sqrt(c) + (b^2 + 4*a*c)*x)/x) + 3*(b^2 + 4*a*c)*sq
rt(a)*x^3*log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x - 4*sqrt(c*x^4 + b
*x^3 + a*x^2)*(b*x + 2*a)*sqrt(a))/x^3) + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(4
*a*c*x^2 - 5*a*b*x - 2*a^2))/(a*x^3), -1/16*(24*a*b*sqrt(-c)*x^3*arctan(1/
2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*
c*x)) - 3*(b^2 + 4*a*c)*sqrt(a)*x^3*log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 +
8*a^2*x - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(a))/x^3) - 4*sqrt
(c*x^4 + b*x^3 + a*x^2)*(4*a*c*x^2 - 5*a*b*x - 2*a^2))/(a*x^3), 1/8*(6*a*b
*sqrt(c)*x^3*log(-(8*c^2*x^3 + 8*b*c*x^2 + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(
2*c*x + b)*sqrt(c) + (b^2 + 4*a*c)*x)/x) + 3*(b^2 + 4*a*c)*sqrt(-a)*x^3*ar
ctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x
^2 + a^2*x)) + 2*sqrt(c*x^4 + b*x^3 + a*x^2)*(4*a*c*x^2 - 5*a*b*x - 2*a^2)
)/(a*x^3), -1/8*(12*a*b*sqrt(-c)*x^3*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2
)*(2*c*x + b)*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x)) - 3*(b^2 + 4*a*c)*sqrt
(-a)*x^3*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(-a)/(a*c*
x^3 + a*b*x^2 + a^2*x)) - 2*sqrt(c*x^4 + b*x^3 + a*x^2)*(4*a*c*x^2 - 5*a*b
*x - 2*a^2))/(a*x^3)]
```

Sympy [F]

$$\int \frac{(ax^2 + bx^3 + cx^4)^{3/2}}{x^6}\,dx = \int \frac{(x^2(a + bx + cx^2))^{\frac{3}{2}}}{x^6}\,dx$$

input

```
integrate((c*x**4+b*x**3+a*x**2)**(3/2)/x**6,x)
```

output

```
Integral((x**2*(a + b*x + c*x**2))**(3/2)/x**6, x)
```

Maxima [F]

$$\int \frac{(ax^2 + bx^3 + cx^4)^{3/2}}{x^6}\,dx = \int \frac{(cx^4 + bx^3 + ax^2)^{\frac{3}{2}}}{x^6}\,dx$$

input
```
integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^6,x, algorithm="maxima")
```

output
```
integrate((c*x^4 + b*x^3 + a*x^2)^(3/2)/x^6, x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{(ax^2 + bx^3 + cx^4)^{3/2}}{x^6}\,dx = \text{Timed out}$$

input
```
integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^6,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ax^2 + bx^3 + cx^4)^{3/2}}{x^6}\,dx = \int \frac{(c\,x^4 + b\,x^3 + a\,x^2)^{3/2}}{x^6}\,dx$$

input
```
int((a*x^2 + b*x^3 + c*x^4)^(3/2)/x^6,x)
```

output
```
int((a*x^2 + b*x^3 + c*x^4)^(3/2)/x^6, x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.83

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^6}\,dx = \frac{-4\sqrt{c\,x^2+bx+a}\,a^2-10\sqrt{c\,x^2+bx+a}\,abx+8\sqrt{c\,x^2+bx+a}\,ac\,x^2+12\sqrt{}$$

input
```
int((c*x^4+b*x^3+a*x^2)^(3/2)/x^6,x)
```

output
```
( - 4*sqrt(a + b*x + c*x**2)*a**2 - 10*sqrt(a + b*x + c*x**2)*a*b*x + 8*sq
rt(a + b*x + c*x**2)*a*c*x**2 + 12*sqrt(a)*log(2*sqrt(a)*sqrt(a + b*x + c*
x**2) - 2*a - b*x)*a*c*x**2 + 3*sqrt(a)*log(2*sqrt(a)*sqrt(a + b*x + c*x**
2) - 2*a - b*x)*b**2*x**2 - 12*sqrt(a)*log(x)*a*c*x**2 - 3*sqrt(a)*log(x)*
b**2*x**2 + 12*sqrt(c)*log( - 2*sqrt(c)*sqrt(a + b*x + c*x**2) - b - 2*c*x
)*a*b*x**2)/(8*a*x**2)
```

3.63 $\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^7} dx$

Optimal result

Integrand size = 24, antiderivative size = 232

$$\begin{aligned}\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^7} dx = & -\frac{(2ab+(b^2+8ac)x)\sqrt{ax^2+bx^3+cx^4}}{8ax^3} - \frac{(ax^2+bx^3+cx^4)^{3/2}}{3x^6} \\ & + \frac{b(b^2-12ac)(ax^2+bx^3+cx^4)^{3/2}\operatorname{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{16a^{3/2}x^3(a+bx+cx^2)^{3/2}} \\ & + \frac{c^{3/2}(ax^2+bx^3+cx^4)^{3/2}\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{x^3(a+bx+cx^2)^{3/2}}\end{aligned}$$

output
```
-1/8*(2*a*b+(8*a*c+b^2)*x)*(c*x^4+b*x^3+a*x^2)^(1/2)/a/x^3-1/3*(c*x^4+b*x^
3+a*x^2)^(3/2)/x^6+1/16*b*(-12*a*c+b^2)*(c*x^4+b*x^3+a*x^2)^(3/2)*arctanh(
1/2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))/a^(3/2)/x^3/(c*x^2+b*x+a)^(3/2)
+c^(3/2)*(c*x^4+b*x^3+a*x^2)^(3/2)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*
x+a)^(1/2))/x^3/(c*x^2+b*x+a)^(3/2)
```

Mathematica [A] (verified)

Time = 1.07 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.74

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^7}\,dx =$$
$$-\frac{\sqrt{x^2(a+x(b+cx))}\left(3b(b^2-12ac)\,x^3\text{arctanh}\left(\frac{\sqrt{c}x-\sqrt{a+x(b+cx)}}{\sqrt{a}}\right)+\sqrt{a}\left(\sqrt{a+x(b+cx)}(8a^2+3b^2x^2+2\right.\right.}{24a^{3/2}x^4\sqrt{a+x(b+cx)}}$$

input
```
Integrate[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^7,x]
```

output
```
-1/24*(Sqrt[x^2*(a + x*(b + c*x))]*(3*b*(b^2 - 12*a*c)*x^3*ArcTanh[(Sqrt[c
]*x - Sqrt[a + x*(b + c*x)])/Sqrt[a]] + Sqrt[a]*(Sqrt[a + x*(b + c*x)]*(8*
a^2 + 3*b^2*x^2 + 2*a*x*(7*b + 16*c*x)) + 24*a*c^(3/2)*x^3*Log[b + 2*c*x -
 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]])))/(a^(3/2)*x^4*Sqrt[a + x*(b + c*x)])
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.01, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.458, Rules used = {1967, 1998, 27, 1988, 25, 1980, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^7}\,dx$$

$$\Big\downarrow \text{1967}$$

$$\frac{1}{2}\int \frac{(b+2cx)\sqrt{cx^4+bx^3+ax^2}}{x^4}dx-\frac{(ax^2+bx^3+cx^4)^{3/2}}{3x^6}$$

$$\Big\downarrow \text{1998}$$

$$\frac{1}{2}\left(-\frac{\int \frac{(b^2-2cxb-8ac)\sqrt{cx^4+bx^3+ax^2}}{2x^3}dx}{2a}-\frac{b(ax^2+bx^3+cx^4)^{3/2}}{2ax^5}\right)-\frac{(ax^2+bx^3+cx^4)^{3/2}}{3x^6}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{2}\left(-\frac{\int \frac{(b^2-2cxb-8ac)\sqrt{cx^4+bx^3+ax^2}}{x^3}dx}{4a}-\frac{b\left(ax^2+bx^3+cx^4\right)^{3/2}}{2ax^5}\right)-\frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{3x^6}$$

$$\Big\downarrow \; 1988$$

$$\frac{1}{2}\left(-\frac{-\frac{1}{2}\int -\frac{b(b^2-12ac)-16ac^2x}{\sqrt{cx^4+bx^3+ax^2}}dx-\frac{\sqrt{ax^2+bx^3+cx^4}(-8ac+b^2+2bcx)}{x^2}}{4a}-\frac{b\left(ax^2+bx^3+cx^4\right)^{3/2}}{2ax^5}\right)-\frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{3x^6}$$

$$\Big\downarrow \; 25$$

$$\frac{1}{2}\left(-\frac{\frac{1}{2}\int \frac{b(b^2-12ac)-16ac^2x}{\sqrt{cx^4+bx^3+ax^2}}dx-\frac{(-8ac+b^2+2bcx)\sqrt{ax^2+bx^3+cx^4}}{x^2}}{4a}-\frac{b\left(ax^2+bx^3+cx^4\right)^{3/2}}{2ax^5}\right)-\frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{3x^6}$$

$$\Big\downarrow \; 1980$$

$$\frac{1}{2}\left(-\frac{\frac{x\sqrt{a+bx+cx^2}\int \frac{b\left(b^2-12ac\right)-16ac^2x}{x\sqrt{cx^2+bx+a}}dx}{2\sqrt{ax^2+bx^3+cx^4}}-\frac{(-8ac+b^2+2bcx)\sqrt{ax^2+bx^3+cx^4}}{x^2}}{4a}-\frac{b\left(ax^2+bx^3+cx^4\right)^{3/2}}{2ax^5}\right)-\frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{3x^6}$$

$$\Big\downarrow \; 1269$$

$$\frac{1}{2}\left(-\frac{\frac{x\sqrt{a+bx+cx^2}\left(b(b^2-12ac)\int \frac{1}{x\sqrt{cx^2+bx+a}}dx-16ac^2\int \frac{1}{\sqrt{cx^2+bx+a}}dx\right)}{2\sqrt{ax^2+bx^3+cx^4}}-\frac{(-8ac+b^2+2bcx)\sqrt{ax^2+bx^3+cx^4}}{x^2}}{4a}-\frac{b\left(ax^2+bx^3+ca\right.}{2ax^5}\right.$$

$$\frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{3x^6}$$

$$\Big\downarrow \; 1092$$

$$\frac{1}{2}\left(-\frac{\frac{x\sqrt{a+bx+cx^2}\left(b\left(b^2-12ac\right)\int\frac{1}{x\sqrt{cx^2+bx+a}}dx-32ac^2\int\frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}\right)}{2\sqrt{ax^2+bx^3+cx^4}}-\frac{\left(-8ac+b^2+2bcx\right)\sqrt{ax^2+bx^3+cx^4}}{x^2}}{4a}-\frac{b\left(ax^2\right.}{}\right.$$

$$\frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{3x^6}$$

$\downarrow$ 219

$$\frac{1}{2}\left(-\frac{\frac{x\sqrt{a+bx+cx^2}\left(b\left(b^2-12ac\right)\int\frac{1}{x\sqrt{cx^2+bx+a}}dx-16ac^{3/2}\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\right)}{2\sqrt{ax^2+bx^3+cx^4}}-\frac{\left(-8ac+b^2+2bcx\right)\sqrt{ax^2+bx^3+cx^4}}{x^2}}{4a}-\frac{b\left(ax^2\right.}{}\right.$$

$$\frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{3x^6}$$

$\downarrow$ 1154

$$\frac{1}{2}\left(-\frac{\frac{x\sqrt{a+bx+cx^2}\left(-2b\left(b^2-12ac\right)\int\frac{1}{4a-\frac{(2a+bx)^2}{cx^2+bx+a}}d\frac{2a+bx}{\sqrt{cx^2+bx+a}}-16ac^{3/2}\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\right)}{2\sqrt{ax^2+bx^3+cx^4}}-\frac{\left(-8ac+b^2+2bcx\right)\sqrt{ax^2+bx^3+}}{x^2}}{4a}\right.$$

$$\frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{3x^6}$$

$\downarrow$ 219

$$\frac{1}{2}\left(-\frac{\frac{x\sqrt{a+bx+cx^2}\left(-\frac{b\left(b^2-12ac\right)\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}}-16ac^{3/2}\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\right)}{2\sqrt{ax^2+bx^3+cx^4}}-\frac{\left(-8ac+b^2+2bcx\right)\sqrt{ax^2+bx^3+cx}}{x^2}}{4a}\right.$$

$$\frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{3x^6}$$

input
```
Int[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^7,x]
```

output
```
-1/3*(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^6 + (-1/2*(b*(a*x^2 + b*x^3 + c*x^4)^
(3/2))/(a*x^5) - (-(((b^2 - 8*a*c + 2*b*c*x)*Sqrt[a*x^2 + b*x^3 + c*x^4])/
x^2) + (x*Sqrt[a + b*x + c*x^2]*(-((b*(b^2 - 12*a*c)*ArcTanh[(2*a + b*x)/(
2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/Sqrt[a]) - 16*a*c^(3/2)*ArcTanh[(b + 2*
c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])]))/(2*Sqrt[a*x^2 + b*x^3 + c*x^4]))
/(4*a))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1967
```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[x^(m + 1)*((a*x^q + b*x^n + c*x^(2*n - q))^p/(m + p*q
+ 1)), x] - Simp[(n - q)*(p/(m + p*q + 1))   Int[x^(m + n)*(b + 2*c*x^(n -
q))*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c}, x]
&& EqQ[r, 2*n - q] && PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] &
& IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && LeQ[m + p*q + 1, -(n - q) +
 1] && NeQ[m + p*q + 1, 0]
```

rule 1980
```
Int[((A_) + (B_.)*(x_)^(j_.))/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c
_.)*(x_)^(r_.)], x_Symbol] :> Simp[x^(q/2)*(Sqrt[a + b*x^(n - q) + c*x^(2*(
n - q))]/Sqrt[a*x^q + b*x^n + c*x^(2*n - q)])   Int[(A + B*x^(n - q))/(x^(q
/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]), x], x] /; FreeQ[{a, b, c, A,
B, n, q}, x] && EqQ[j, n - q] && EqQ[r, 2*n - q] && PosQ[n - q] && EqQ[n, 3
] && EqQ[q, 2]
```

rule 1988
```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[x^(m + 1)*(A*(m + p*q + (n
- q)*(2*p + 1) + 1) + B*(m + p*q + 1)*x^(n - q))*((a*x^q + b*x^n + c*x^(2*n
 - q))^p/((m + p*q + 1)*(m + p*q + (n - q)*(2*p + 1) + 1))), x] + Simp[(n -
 q)*(p/((m + p*q + 1)*(m + p*q + (n - q)*(2*p + 1) + 1)))   Int[x^(n + m)*S
imp[2*a*B*(m + p*q + 1) - A*b*(m + p*q + (n - q)*(2*p + 1) + 1) + (b*B*(m +
 p*q + 1) - 2*A*c*(m + p*q + (n - q)*(2*p + 1) + 1))*x^(n - q), x]*(a*x^q +
 b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ
[r, n - q] && EqQ[j, 2*n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IG
tQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && LeQ[m + p*q, -(n - q)] && NeQ[m
 + p*q + 1, 0] && NeQ[m + p*q + (n - q)*(2*p + 1) + 1, 0]
```

rule 1998
```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[A*x^(m - q + 1)*((a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1)/(a*(m + p*q + 1))), x] + Simp[1/(a*(m + p*q +
 1))   Int[x^(m + n - q)*Simp[a*B*(m + p*q + 1) - A*b*(m + p*q + (n - q)*(p
 + 1) + 1) - A*c*(m + p*q + 2*(n - q)*(p + 1) + 1)*x^(n - q), x]*(a*x^q + b
*x^n + c*x^(2*n - q))^p, x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n -
q] && EqQ[j, 2*n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
 && RationalQ[m, p, q] && ((GeQ[p, -1] && LtQ[p, 0]) || EqQ[m + p*q + (n -
q)*(2*p + 1) + 1, 0]) && LeQ[m + p*q, -(n - q)] && NeQ[m + p*q + 1, 0]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.65

method	result
pseudoelliptic	$-\frac{3 \left(b x^{3} \left(a c-\frac{b^{2}}{12}\right) \ln \left(\frac{2 a+b x+2 \sqrt{a}\, \sqrt{c \,x^{2}+b x+a}}{x \sqrt{a}}\right)-\frac{4 \ln \left(2 \sqrt{c \,x^{2}+b x+a}\, \sqrt{c}+2 c x+b \right) a^{\frac{3}{2}} c^{\frac{3}{2}} x^{3}}{3}+\left(\frac{7 \left(\frac{16 c x}{7}+b \right) x \,a^{\frac{3}{2}}}{9}+\frac{\sqrt{a}\, b^{2} x^{2}}{6}+\frac{4 a}{\cdots}\right.\right.}{4 a^{\frac{3}{2}} x^{3}}$
risch	$-\frac{\left(32 a c \,x^{2}+3 b^{2} x^{2}+14 a b x+8 a^{2}\right) \sqrt{x^{2} \left(c \,x^{2}+b x+a \right)}}{24 x^{4} a}+\frac{\left(-\frac{b \left(12 a c-b^{2}\right) \ln \left(\frac{2 a+b x+2 \sqrt{a}\, \sqrt{c \,x^{2}+b x+a}}{x}\right)}{\sqrt{a}}+16 a \,c^{\frac{3}{2}} \ln \left(\frac{\frac{b}{2}+c x}{\sqrt{c}}+\cdots\right.\right.}{16 a x \sqrt{c \,x^{2}+b x+a}}$
default	$-\frac{\left(c \,x^{4}+b \,x^{3}+a \,x^{2}\right)^{\frac{3}{2}} \left(36 a^{\frac{5}{2}} \ln \left(\frac{2 a+b x+2 \sqrt{a}\, \sqrt{c \,x^{2}+b x+a}}{x}\right) c^{\frac{5}{2}} b \,x^{3}-32 c^{\frac{7}{2}} \left(c \,x^{2}+b x+a \right)^{\frac{3}{2}} a \,x^{4}-48 c^{\frac{7}{2}} \sqrt{c \,x^{2}+b x+a}\, a^{2} x^{4}+2 c \cdots\right.}{\cdots}$

input
```
int((c*x^4+b*x^3+a*x^2)^(3/2)/x^7,x,method=_RETURNVERBOSE)
```

output
```
-3/4/a^(3/2)*(b*x^3*(a*c-1/12*b^2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/
2))/x/a^(1/2))-4/3*ln(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)*a^(3/2)*c^(3/
2)*x^3+(7/9*(16/7*c*x+b)*x*a^(3/2)+1/6*a^(1/2)*b^2*x^2+4/9*a^(5/2))*(c*x^2
+b*x+a)^(1/2)-ln(2)*(a*c-1/12*b^2)*b*x^3)/x^3
```

Fricas [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 815, normalized size of antiderivative = 3.51

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^7}\,dx = \text{Too large to display}$$

input
```
integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^7,x, algorithm="fricas")
```

output
```
[1/96*(48*a^2*c^(3/2)*x^4*log(-(8*c^2*x^3 + 8*b*c*x^2 + 4*sqrt(c*x^4 + b*x
^3 + a*x^2)*(2*c*x + b)*sqrt(c) + (b^2 + 4*a*c)*x)/x) - 3*(b^3 - 12*a*b*c)
*sqrt(a)*x^4*log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x - 4*sqrt(c*x^4
+ b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(a))/x^3) - 4*sqrt(c*x^4 + b*x^3 + a*x^2)
*(14*a^2*b*x + 8*a^3 + (3*a*b^2 + 32*a^2*c)*x^2))/(a^2*x^4), -1/96*(96*a^2
*sqrt(-c)*c*x^4*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(-c
)/(c^2*x^3 + b*c*x^2 + a*c*x)) + 3*(b^3 - 12*a*b*c)*sqrt(a)*x^4*log(-(8*a*
b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x +
 2*a)*sqrt(a))/x^3) + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(14*a^2*b*x + 8*a^3 +
(3*a*b^2 + 32*a^2*c)*x^2))/(a^2*x^4), 1/48*(24*a^2*c^(3/2)*x^4*log(-(8*c^2
*x^3 + 8*b*c*x^2 + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(c) + (b^
2 + 4*a*c)*x)/x) - 3*(b^3 - 12*a*b*c)*sqrt(-a)*x^4*arctan(1/2*sqrt(c*x^4 +
 b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2 + a^2*x)) - 2*sqrt
(c*x^4 + b*x^3 + a*x^2)*(14*a^2*b*x + 8*a^3 + (3*a*b^2 + 32*a^2*c)*x^2))/(
a^2*x^4), -1/48*(48*a^2*sqrt(-c)*c*x^4*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x
^2)*(2*c*x + b)*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x)) + 3*(b^3 - 12*a*b*c)
*sqrt(-a)*x^4*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(-a)/
(a*c*x^3 + a*b*x^2 + a^2*x)) + 2*sqrt(c*x^4 + b*x^3 + a*x^2)*(14*a^2*b*x +
 8*a^3 + (3*a*b^2 + 32*a^2*c)*x^2))/(a^2*x^4)]
```

Sympy [F]

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^7}\,dx = \int \frac{(x^2(a+bx+cx^2))^{\frac{3}{2}}}{x^7}\,dx$$

input
```
integrate((c*x**4+b*x**3+a*x**2)**(3/2)/x**7,x)
```

output
```
Integral((x**2*(a + b*x + c*x**2))**(3/2)/x**7, x)
```

Maxima [F]

$$\int \frac{(ax^2 + bx^3 + cx^4)^{3/2}}{x^7}\, dx = \int \frac{(cx^4 + bx^3 + ax^2)^{\frac{3}{2}}}{x^7}\, dx$$

input
```
integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^7,x, algorithm="maxima")
```

output
```
integrate((c*x^4 + b*x^3 + a*x^2)^(3/2)/x^7, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(ax^2 + bx^3 + cx^4)^{3/2}}{x^7}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^7,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ax^2 + bx^3 + cx^4)^{3/2}}{x^7}\, dx = \int \frac{(c\,x^4 + b\,x^3 + a\,x^2)^{3/2}}{x^7}\, dx$$

input
```
int((a*x^2 + b*x^3 + c*x^4)^(3/2)/x^7,x)
```

output
```
int((a*x^2 + b*x^3 + c*x^4)^(3/2)/x^7, x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.91

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^7}\,dx = \frac{-16\sqrt{c\,x^2+bx+a}\,a^3-28\sqrt{c\,x^2+bx+a}\,a^2bx-64\sqrt{c\,x^2+bx+a}\,a^2c\,x^2-}{}$$

input
```
int((c*x^4+b*x^3+a*x^2)^(3/2)/x^7,x)
```

output
```
( - 16*sqrt(a + b*x + c*x**2)*a**3 - 28*sqrt(a + b*x + c*x**2)*a**2*b*x -
64*sqrt(a + b*x + c*x**2)*a**2*c*x**2 - 6*sqrt(a + b*x + c*x**2)*a*b**2*x*
*2 + 36*sqrt(a)*log(2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a - b*x)*a*b*c*x*
*3 - 3*sqrt(a)*log(2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a - b*x)*b**3*x**3
 - 36*sqrt(a)*log(x)*a*b*c*x**3 + 3*sqrt(a)*log(x)*b**3*x**3 + 48*sqrt(c)*
log( - 2*sqrt(c)*sqrt(a + b*x + c*x**2) - b - 2*c*x)*a**2*c*x**3)/(48*a**2
*x**3)
```

3.64 $\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^8}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 197

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^8}\,dx = -\frac{(b^2-12ac)\sqrt{ax^2+bx^3+cx^4}}{32ax^3} + \frac{b(3b^2-20ac)\sqrt{ax^2+bx^3+cx^4}}{64a^2x^2} - \frac{(b+6cx)\sqrt{ax^2+bx^3+cx^4}}{8x^4} - \frac{(ax^2+bx^3+cx^4)^{3/2}}{4x^7} - \frac{3(b^2-4ac)^2\operatorname{arctanh}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{128a^{5/2}}$$

output

```
-1/32*(-12*a*c+b^2)*(c*x^4+b*x^3+a*x^2)^(1/2)/a/x^3+1/64*b*(-20*a*c+3*b^2)
*(c*x^4+b*x^3+a*x^2)^(1/2)/a^2/x^2-1/8*(6*c*x+b)*(c*x^4+b*x^3+a*x^2)^(1/2)
/x^4-1/4*(c*x^4+b*x^3+a*x^2)^(3/2)/x^7-3/128*(-4*a*c+b^2)^2*arctanh(1/2*x*
(b*x+2*a)/a^(1/2)/(c*x^4+b*x^3+a*x^2)^(1/2))/a^(5/2)
```

Mathematica [A] (verified)

Time = 1.20 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.72

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^8}\,dx = \frac{\sqrt{x^2(a+x(b+cx))}\left(-\sqrt{a}(2a+bx)\sqrt{a+x(b+cx)}(8a^2-3b^2x^2+4ax(2b+\right.}{64a^{5/2}x^5\sqrt{a+x(b+cx)}}$$

input
```
Integrate[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^8,x]
```

output
```
(Sqrt[x^2*(a + x*(b + c*x))]*(-(Sqrt[a]*(2*a + b*x)*Sqrt[a + x*(b + c*x)]*
(8*a^2 - 3*b^2*x^2 + 4*a*x*(2*b + 5*c*x))) + 3*(b^2 - 4*a*c)^2*x^4*ArcTanh
[(Sqrt[c]*x - Sqrt[a + x*(b + c*x)])/Sqrt[a]]))/(64*a^(5/2)*x^5*Sqrt[a + x
*(b + c*x)])
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.08, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1967, 1988, 1998, 27, 1998, 27, 1951, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^8}dx$$

$$\Big\downarrow \text{1967}$$

$$\frac{3}{8}\int \frac{(b+2cx)\sqrt{cx^4+bx^3+ax^2}}{x^5}dx - \frac{(ax^2+bx^3+cx^4)^{3/2}}{4x^7}$$

$$\Big\downarrow \text{1988}$$

$$\frac{3}{8}\left(\frac{1}{6}\int \frac{b^2-4cxb-12ac}{x^2\sqrt{cx^4+bx^3+ax^2}}dx - \frac{(b+6cx)\sqrt{ax^2+bx^3+cx^4}}{3x^4}\right) - \frac{(ax^2+bx^3+cx^4)^{3/2}}{4x^7}$$

$$\Big\downarrow \text{1998}$$

$$\frac{3}{8}\left(\frac{1}{6}\left(-\frac{\int \frac{b(3b^2-20ac)+2c(b^2-12ac)x}{2x\sqrt{cx^4+bx^3+ax^2}}dx}{2a}-\frac{(b^2-12ac)\sqrt{ax^2+bx^3+cx^4}}{2ax^3}\right)-\frac{(b+6cx)\sqrt{ax^2+bx^3+cx^4}}{3x^4}\right)-\frac{(ax^2+bx^3+cx^4)^{3/2}}{4x^7}$$

$\Big\downarrow$ 27

$$\frac{3}{8}\left(\frac{1}{6}\left(-\frac{\int \frac{b(3b^2-20ac)+2c(b^2-12ac)x}{x\sqrt{cx^4+bx^3+ax^2}}dx}{4a}-\frac{(b^2-12ac)\sqrt{ax^2+bx^3+cx^4}}{2ax^3}\right)-\frac{(b+6cx)\sqrt{ax^2+bx^3+cx^4}}{3x^4}\right)-\frac{(ax^2+bx^3+cx^4)^{3/2}}{4x^7}$$

$\Big\downarrow$ 1998

$$\frac{3}{8}\left(\frac{1}{6}\left(-\frac{-\frac{\int \frac{3(b^2-4ac)^2}{2\sqrt{cx^4+bx^3+ax^2}}dx}{a}-\frac{b(3b^2-20ac)\sqrt{ax^2+bx^3+cx^4}}{ax^2}}{4a}-\frac{(b^2-12ac)\sqrt{ax^2+bx^3+cx^4}}{2ax^3}\right)-\frac{(b+6cx)\sqrt{ax^2+b}{3x^4}\right)-\frac{(ax^2+bx^3+cx^4)^{3/2}}{4x^7}$$

$\Big\downarrow$ 27

$$\frac{3}{8}\left(\frac{1}{6}\left(-\frac{-\frac{3(b^2-4ac)^2\int \frac{1}{\sqrt{cx^4+bx^3+ax^2}}dx}{2a}-\frac{b(3b^2-20ac)\sqrt{ax^2+bx^3+cx^4}}{ax^2}}{4a}-\frac{(b^2-12ac)\sqrt{ax^2+bx^3+cx^4}}{2ax^3}\right)-(b+6cx)\right)-\frac{(ax^2+bx^3+cx^4)^{3/2}}{4x^7}$$

$\Big\downarrow$ 1951

$$\frac{3}{8}\left(\frac{1}{6}\left(-\frac{\frac{3(b^2-4ac)^2\int \frac{1}{4a-\frac{x^2(2a+bx)^2}{cx^4+bx^3+ax^2}}d\frac{x(2a+bx)}{\sqrt{cx^4+bx^3+ax^2}}}{a}-\frac{b(3b^2-20ac)\sqrt{ax^2+bx^3+cx^4}}{ax^2}}{4a}-\frac{(b^2-12ac)\sqrt{ax^2+bx^3+cx^4}}{2ax^3}\right)-\right)-\frac{(ax^2+bx^3+cx^4)^{3/2}}{4x^7}$$

$\Big\downarrow$ 219

$$\frac{3}{8}\left(\frac{1}{6}\left(-\frac{\frac{3(b^2-4ac)^2\operatorname{arctanh}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{2a^{3/2}}-\frac{b(3b^2-20ac)\sqrt{ax^2+bx^3+cx^4}}{ax^2}}{4a}-\frac{(b^2-12ac)\sqrt{ax^2+bx^3+cx^4}}{2ax^3}\right)-\frac{(b\right.$$

$$\frac{(ax^2+bx^3+cx^4)^{3/2}}{4x^7}$$

input
```
Int[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^8,x]
```

output
```
-1/4*(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^7 + (3*(-1/3*((b + 6*c*x)*Sqrt[a*x^2
+ b*x^3 + c*x^4])/x^4 + (-1/2*((b^2 - 12*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])
/(a*x^3) - (-((b*(3*b^2 - 20*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(a*x^2)) +
(3*(b^2 - 4*a*c)^2*ArcTanh[(x*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x^2 + b*x^3 +
 c*x^4])])/(2*a^(3/2)))/(4*a))/6))/8
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1951
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :
> Simp[-2/(n - 2)   Subst[Int[1/(4*a - x^2), x], x, x*((2*a + b*x^(n - 2))/
Sqrt[a*x^2 + b*x^n + c*x^r])], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*
n - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]
```

rule 1967
```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[x^(m + 1)*((a*x^q + b*x^n + c*x^(2*n - q))^p/(m + p*q
+ 1)), x] - Simp[(n - q)*(p/(m + p*q + 1))   Int[x^(m + n)*(b + 2*c*x^(n -
q))*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c}, x]
&& EqQ[r, 2*n - q] && PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] &
& IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && LeQ[m + p*q + 1, -(n - q) +
 1] && NeQ[m + p*q + 1, 0]
```

rule 1988
```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[x^(m + 1)*(A*(m + p*q + (n
- q)*(2*p + 1) + 1) + B*(m + p*q + 1)*x^(n - q))*((a*x^q + b*x^n + c*x^(2*n
 - q))^p/((m + p*q + 1)*(m + p*q + (n - q)*(2*p + 1) + 1))), x] + Simp[(n -
 q)*(p/((m + p*q + 1)*(m + p*q + (n - q)*(2*p + 1) + 1)))   Int[x^(n + m)*S
imp[2*a*B*(m + p*q + 1) - A*b*(m + p*q + (n - q)*(2*p + 1) + 1) + (b*B*(m +
 p*q + 1) - 2*A*c*(m + p*q + (n - q)*(2*p + 1) + 1))*x^(n - q), x]*(a*x^q +
 b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ
[r, n - q] && EqQ[j, 2*n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IG
tQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && LeQ[m + p*q, -(n - q)] && NeQ[m
 + p*q + 1, 0] && NeQ[m + p*q + (n - q)*(2*p + 1) + 1, 0]
```

rule 1998
```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[A*x^(m - q + 1)*((a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1)/(a*(m + p*q + 1))), x] + Simp[1/(a*(m + p*q +
 1))   Int[x^(m + n - q)*Simp[a*B*(m + p*q + 1) - A*b*(m + p*q + (n - q)*(p
 + 1) + 1) - A*c*(m + p*q + 2*(n - q)*(p + 1) + 1)*x^(n - q), x]*(a*x^q + b
*x^n + c*x^(2*n - q))^p, x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n -
q] && EqQ[j, 2*n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
 && RationalQ[m, p, q] && ((GeQ[p, -1] && LtQ[p, 0]) || EqQ[m + p*q + (n -
q)*(2*p + 1) + 1, 0]) && LeQ[m + p*q, -(n - q)] && NeQ[m + p*q + 1, 0]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.66

method	result
pseudoelliptic	$-\frac{3\left(x^4\left(ac-\frac{b^2}{4}\right)^2\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x\sqrt{a}}\right)+\left(\frac{b\,x^2(10cx+b)a^{\frac{3}{2}}}{12}+x\left(\frac{5cx}{3}+b\right)a^{\frac{5}{2}}-\frac{\sqrt{a}\,b^3x^3}{8}+\frac{2a^{\frac{7}{2}}}{3}\right)\sqrt{c\,x^2+bx+a}-\ln(2)$
risch	$-\frac{(20abc\,x^3-3b^3x^3+40a^2c\,x^2+2a\,b^2x^2+24b\,a^2x+16a^3)\sqrt{x^2(c\,x^2+bx+a)}}{64x^5a^2}-\frac{3(16a^2c^2-8a\,b^2c+b^4)\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x}}{x}\right.}{128a^{\frac{5}{2}}x\sqrt{c\,x^2+bx}}$
default	$-\frac{(c\,x^4+b\,x^3+a\,x^2)^{\frac{3}{2}}\left(48c^2a^{\frac{7}{2}}\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x}\right)x^4+24c^2(c\,x^2+bx+a)^{\frac{3}{2}}ab\,x^5-24c\,a^{\frac{5}{2}}\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+}}{x}\right.\right.}{}$

input
```
int((c*x^4+b*x^3+a*x^2)^(3/2)/x^8,x,method=_RETURNVERBOSE)
```

output
```
-3/8/a^(5/2)*(x^4*(a*c-1/4*b^2)^2*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2
))/x/a^(1/2))+(1/12*b*x^2*(10*c*x+b)*a^(3/2)+x*(5/3*c*x+b)*a^(5/2)-1/8*a^(
1/2)*b^3*x^3+2/3*a^(7/2))*(c*x^2+b*x+a)^(1/2)-ln(2)*x^4*(a*c-1/4*b^2)^2)/x
^4
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.69

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^8}\,dx = \left[\frac{3\,(b^4-8\,ab^2c+16\,a^2c^2)\sqrt{a}x^5\log\left(-\frac{8\,abx^2+(b^2+4\,ac)x^3+8\,a^2x-4\,\sqrt{cx^4+bx^3+ax^2}(bx+}{x^3}\right.}{}\right.$$

input
```
integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^8,x, algorithm="fricas")
```

output
```
[1/256*(3*(b^4 - 8*a*b^2*c + 16*a^2*c^2)*sqrt(a)*x^5*log(-(8*a*b*x^2 + (b^
2 + 4*a*c)*x^3 + 8*a^2*x - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(
a))/x^3) - 4*(24*a^3*b*x + 16*a^4 - (3*a*b^3 - 20*a^2*b*c)*x^3 + 2*(a^2*b^
2 + 20*a^3*c)*x^2)*sqrt(c*x^4 + b*x^3 + a*x^2))/(a^3*x^5), 1/128*(3*(b^4 -
 8*a*b^2*c + 16*a^2*c^2)*sqrt(-a)*x^5*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^
2)*(b*x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2 + a^2*x)) - 2*(24*a^3*b*x + 16*
a^4 - (3*a*b^3 - 20*a^2*b*c)*x^3 + 2*(a^2*b^2 + 20*a^3*c)*x^2)*sqrt(c*x^4
+ b*x^3 + a*x^2))/(a^3*x^5)]
```

Sympy [F]

$$\int \frac{(ax^2 + bx^3 + cx^4)^{3/2}}{x^8}\, dx = \int \frac{(x^2(a + bx + cx^2))^{\frac{3}{2}}}{x^8}\, dx$$

input
```
integrate((c*x**4+b*x**3+a*x**2)**(3/2)/x**8,x)
```

output
```
Integral((x**2*(a + b*x + c*x**2))**(3/2)/x**8, x)
```

Maxima [F]

$$\int \frac{(ax^2 + bx^3 + cx^4)^{3/2}}{x^8}\, dx = \int \frac{(cx^4 + bx^3 + ax^2)^{\frac{3}{2}}}{x^8}\, dx$$

input
```
integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^8,x, algorithm="maxima")
```

output
```
integrate((c*x^4 + b*x^3 + a*x^2)^(3/2)/x^8, x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^8}\,dx = \text{Timed out}$$

input
```
integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^8,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^8}\,dx = \int \frac{(c\,x^4+b\,x^3+a\,x^2)^{3/2}}{x^8}\,dx$$

input
```
int((a*x^2 + b*x^3 + c*x^4)^(3/2)/x^8,x)
```

output
```
int((a*x^2 + b*x^3 + c*x^4)^(3/2)/x^8, x)
```

Reduce [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.39

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^8}\,dx = \frac{-32\sqrt{c\,x^2+bx+a}\,a^4-48\sqrt{c\,x^2+bx+a}\,a^3bx-80\sqrt{c\,x^2+bx+a}\,a^3c\,x^2-}{}$$

input
```
int((c*x^4+b*x^3+a*x^2)^(3/2)/x^8,x)
```

output

```
( - 32*sqrt(a + b*x + c*x**2)*a**4 - 48*sqrt(a + b*x + c*x**2)*a**3*b*x -
80*sqrt(a + b*x + c*x**2)*a**3*c*x**2 - 4*sqrt(a + b*x + c*x**2)*a**2*b**2
*x**2 - 40*sqrt(a + b*x + c*x**2)*a**2*b*c*x**3 + 6*sqrt(a + b*x + c*x**2)
*a*b**3*x**3 + 48*sqrt(a)*log(2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a - b*x
)*a**2*c**2*x**4 - 24*sqrt(a)*log(2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a -
 b*x)*a*b**2*c*x**4 + 3*sqrt(a)*log(2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a
 - b*x)*b**4*x**4 - 48*sqrt(a)*log(x)*a**2*c**2*x**4 + 24*sqrt(a)*log(x)*a
*b**2*c*x**4 - 3*sqrt(a)*log(x)*b**4*x**4)/(128*a**3*x**4)
```

3.65 $\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^9}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 249

$$\begin{aligned}\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^9}\,dx =& -\frac{(b^2-8ac)\sqrt{ax^2+bx^3+cx^4}}{80ax^4} \\ &+\frac{b(5b^2-28ac)\sqrt{ax^2+bx^3+cx^4}}{320a^2x^3} \\ &-\frac{(15b^4-100ab^2c+128a^2c^2)\sqrt{ax^2+bx^3+cx^4}}{640a^3x^2}-\frac{3(b+4cx)\sqrt{ax^2+bx^3+cx^4}}{40x^5} \\ &-\frac{(ax^2+bx^3+cx^4)^{3/2}}{5x^8}+\frac{3b(b^2-4ac)^2\operatorname{arctanh}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{256a^{7/2}}\end{aligned}$$

output

```
-1/80*(-8*a*c+b^2)*(c*x^4+b*x^3+a*x^2)^(1/2)/a/x^4+1/320*b*(-28*a*c+5*b^2)
*(c*x^4+b*x^3+a*x^2)^(1/2)/a^2/x^3-1/640*(128*a^2*c^2-100*a*b^2*c+15*b^4)*
(c*x^4+b*x^3+a*x^2)^(1/2)/a^3/x^2-3/40*(4*c*x+b)*(c*x^4+b*x^3+a*x^2)^(1/2)
/x^5-1/5*(c*x^4+b*x^3+a*x^2)^(3/2)/x^8+3/256*b*(-4*a*c+b^2)^2*arctanh(1/2*
x*(b*x+2*a)/a^(1/2)/(c*x^4+b*x^3+a*x^2)^(1/2))/a^(7/2)
```

Mathematica [A] (verified)

Time = 1.61 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.71

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^9}\,dx =$$
$$-\frac{\sqrt{x^2(a+x(b+cx))}\left(\sqrt{a}\sqrt{a+x(b+cx)}(128a^4+15b^4x^4-10ab^2x^3(b+10cx)+16a^3x(11b+16cx)+8\right.}{640a^{7/2}x^6\sqrt{a+x(b+cx)}}$$

input
```
Integrate[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^9,x]
```

output
```
-1/640*(Sqrt[x^2*(a + x*(b + c*x))]*(Sqrt[a]*Sqrt[a + x*(b + c*x)]*(128*a^
4 + 15*b^4*x^4 - 10*a*b^2*x^3*(b + 10*c*x) + 16*a^3*x*(11*b + 16*c*x) + 8*
a^2*x^2*(b^2 + 7*b*c*x + 16*c^2*x^2)) + 15*b*(b^2 - 4*a*c)^2*x^5*ArcTanh[(
Sqrt[c]*x - Sqrt[a + x*(b + c*x)])/Sqrt[a]]))/(a^(7/2)*x^6*Sqrt[a + x*(b +
 c*x)])
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.10, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$, Rules used = {1967, 1988, 27, 1998, 27, 1998, 27, 1998, 27, 1951, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^9}dx$$

$$\Big\downarrow \text{1967}$$

$$\frac{3}{10}\int \frac{(b+2cx)\sqrt{cx^4+bx^3+ax^2}}{x^6}dx-\frac{(ax^2+bx^3+cx^4)^{3/2}}{5x^8}$$

$$\Big\downarrow \text{1988}$$

$$\frac{3}{10}\left(\frac{1}{16}\int \frac{2(b^2-2cxb-8ac)}{x^3\sqrt{cx^4+bx^3+ax^2}}dx-\frac{(b+4cx)\sqrt{ax^2+bx^3+cx^4}}{4x^5}\right)-\frac{(ax^2+bx^3+cx^4)^{3/2}}{5x^8}$$

$$\Big\downarrow \; 27$$

$$\frac{3}{10}\left(\frac{1}{8}\int \frac{b^2-2cxb-8ac}{x^3\sqrt{cx^4+bx^3+ax^2}}dx-\frac{(b+4cx)\sqrt{ax^2+bx^3+cx^4}}{4x^5}\right)-\frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{5x^8}$$

$$\Big\downarrow \; 1998$$

$$\frac{3}{10}\left(\frac{1}{8}\left(-\frac{\int \frac{b\left(5b^2-28ac\right)+4c\left(b^2-8ac\right)x}{2x^2\sqrt{cx^4+bx^3+ax^2}}dx}{3a}-\frac{\left(b^2-8ac\right)\sqrt{ax^2+bx^3+cx^4}}{3ax^4}\right)-\frac{(b+4cx)\sqrt{ax^2+bx^3+cx^4}}{4x^5}\right)-\frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{5x^8}$$

$$\Big\downarrow \; 27$$

$$\frac{3}{10}\left(\frac{1}{8}\left(-\frac{\int \frac{b\left(5b^2-28ac\right)+4c\left(b^2-8ac\right)x}{x^2\sqrt{cx^4+bx^3+ax^2}}dx}{6a}-\frac{\left(b^2-8ac\right)\sqrt{ax^2+bx^3+cx^4}}{3ax^4}\right)-\frac{(b+4cx)\sqrt{ax^2+bx^3+cx^4}}{4x^5}\right)-\frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{5x^8}$$

$$\Big\downarrow \; 1998$$

$$\frac{3}{10}\left(\frac{1}{8}\left(-\frac{-\frac{\int \frac{15b^4-100acb^2+2c\left(5b^2-28ac\right)xb+128a^2c^2}{2x\sqrt{cx^4+bx^3+ax^2}}dx}{2a}-\frac{b\left(5b^2-28ac\right)\sqrt{ax^2+bx^3+cx^4}}{2ax^3}}{6a}-\frac{\left(b^2-8ac\right)\sqrt{ax^2+bx^3+cx^4}}{3ax^4}\right)-(\ldots\right)-\frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{5x^8}$$

$$\Big\downarrow \; 27$$

$$\frac{3}{10}\left(\frac{1}{8}\left(-\frac{-\frac{\int \frac{15b^4-100acb^2+2c\left(5b^2-28ac\right)xb+128a^2c^2}{x\sqrt{cx^4+bx^3+ax^2}}dx}{4a}-\frac{b\left(5b^2-28ac\right)\sqrt{ax^2+bx^3+cx^4}}{2ax^3}}{6a}-\frac{\left(b^2-8ac\right)\sqrt{ax^2+bx^3+cx^4}}{3ax^4}\right)-(\ldots\right)-\frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{5x^8}$$

$$\Big\downarrow \; 1998$$

$$\frac{3}{10}\left(\frac{1}{8}\left(-\frac{-\frac{-\frac{\int \frac{15b\left(b^2-4ac\right)^2}{2\sqrt{cx^4+bx^3+ax^2}}dx}{a}-\frac{\left(128a^2c^2-100ab^2c+15b^4\right)\sqrt{ax^2+bx^3+cx^4}}{ax^2}}{4a}-\frac{b\left(5b^2-28ac\right)\sqrt{ax^2+bx^3+cx^4}}{2ax^3}}{6a}-\frac{\left(b^2-8ac\right)\sqrt{ax^2+}}{3ax^4}\right.\right.$$

$$\frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{5x^8}$$

$\big\downarrow$ 27

$$\frac{3}{10}\left(\frac{1}{8}\left(-\frac{-\frac{-\frac{15b\left(b^2-4ac\right)^2\int \frac{1}{\sqrt{cx^4+bx^3+ax^2}}dx}{2a}-\frac{\left(128a^2c^2-100ab^2c+15b^4\right)\sqrt{ax^2+bx^3+cx^4}}{ax^2}}{4a}-\frac{b\left(5b^2-28ac\right)\sqrt{ax^2+bx^3+cx^4}}{2ax^3}}{6a}-\frac{\left(b^2-8ac\right)}{}\right.\right.$$

$$\frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{5x^8}$$

$\big\downarrow$ 1951

$$\frac{3}{10}\left(\frac{1}{8}\left(-\frac{-\frac{\frac{15b\left(b^2-4ac\right)^2\int \frac{1}{4a-\frac{x^2(2a+bx)^2}{cx^4+bx^3+ax^2}}d\frac{x(2a+bx)}{\sqrt{cx^4+bx^3+ax^2}}}{a}-\frac{\left(128a^2c^2-100ab^2c+15b^4\right)\sqrt{ax^2+bx^3+cx^4}}{ax^2}}{4a}-\frac{b\left(5b^2-28ac\right)\sqrt{ax^2+bx^3+cx^4}}{2ax^3}}{6a}-\right.\right.$$

$$\frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{5x^8}$$

$\big\downarrow$ 219

$$\frac{3}{10}\left(\frac{1}{8}\left(-\frac{-\frac{\frac{15b\left(b^2-4ac\right)^2\text{arctanh}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{2a^{3/2}}-\frac{\left(128a^2c^2-100ab^2c+15b^4\right)\sqrt{ax^2+bx^3+cx^4}}{ax^2}}{4a}-\frac{b\left(5b^2-28ac\right)\sqrt{ax^2+bx^3+cx^4}}{2ax^3}}{6a}-\frac{(}{}\right.\right.$$

$$\frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{5x^8}$$

input
```
Int[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^9,x]
```

output
```
-1/5*(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^8 + (3*(-1/4*((b + 4*c*x)*Sqrt[a*x^2
+ b*x^3 + c*x^4])/x^5 + (-1/3*((b^2 - 8*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/
(a*x^4) - (-1/2*(b*(5*b^2 - 28*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(a*x^3) -
 (-(((15*b^4 - 100*a*b^2*c + 128*a^2*c^2)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(a*
x^2)) + (15*b*(b^2 - 4*a*c)^2*ArcTanh[(x*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x^
2 + b*x^3 + c*x^4])])/(2*a^(3/2)))/(4*a))/(6*a))/8))/10
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1951
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :
> Simp[-2/(n - 2)   Subst[Int[1/(4*a - x^2), x], x, x*((2*a + b*x^(n - 2))/
Sqrt[a*x^2 + b*x^n + c*x^r])], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*
n - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]
```

rule 1967
```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[x^(m + 1)*((a*x^q + b*x^n + c*x^(2*n - q))^p/(m + p*q
+ 1)), x] - Simp[(n - q)*(p/(m + p*q + 1))   Int[x^(m + n)*(b + 2*c*x^(n -
q))*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c}, x]
&& EqQ[r, 2*n - q] && PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] &
& IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && LeQ[m + p*q + 1, -(n - q) +
 1] && NeQ[m + p*q + 1, 0]
```

rule 1988
```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[x^(m + 1)*(A*(m + p*q + (n
- q)*(2*p + 1) + 1) + B*(m + p*q + 1)*x^(n - q))*((a*x^q + b*x^n + c*x^(2*n
 - q))^p/((m + p*q + 1)*(m + p*q + (n - q)*(2*p + 1) + 1))), x] + Simp[(n -
 q)*(p/((m + p*q + 1)*(m + p*q + (n - q)*(2*p + 1) + 1)))   Int[x^(n + m)*S
imp[2*a*B*(m + p*q + 1) - A*b*(m + p*q + (n - q)*(2*p + 1) + 1) + (b*B*(m +
 p*q + 1) - 2*A*c*(m + p*q + (n - q)*(2*p + 1) + 1))*x^(n - q), x]*(a*x^q +
 b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ
[r, n - q] && EqQ[j, 2*n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IG
tQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && LeQ[m + p*q, -(n - q)] && NeQ[m
 + p*q + 1, 0] && NeQ[m + p*q + (n - q)*(2*p + 1) + 1, 0]
```

rule 1998
```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[A*x^(m - q + 1)*((a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1)/(a*(m + p*q + 1))), x] + Simp[1/(a*(m + p*q +
 1))   Int[x^(m + n - q)*Simp[a*B*(m + p*q + 1) - A*b*(m + p*q + (n - q)*(p
 + 1) + 1) - A*c*(m + p*q + 2*(n - q)*(p + 1) + 1)*x^(n - q), x]*(a*x^q + b
*x^n + c*x^(2*n - q))^p, x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n -
q] && EqQ[j, 2*n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
 && RationalQ[m, p, q] && ((GeQ[p, -1] && LtQ[p, 0]) || EqQ[m + p*q + (n -
q)*(2*p + 1) + 1, 0]) && LeQ[m + p*q, -(n - q)] && NeQ[m + p*q + 1, 0]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.65

method	result
pseudoelliptic	$\frac{\frac{3 b x^{5} \left(a c-\frac{b^{2}}{4}\right)^{2} \ln \left(\frac{2 a+b x+2 \sqrt{a}\, \sqrt{c\, x^{2}+b x+a}}{x \sqrt{a}}\right)}{16}+\frac{3 \left(-\frac{x^{2} \left(16 c^{2} x^{2}+7 b c x+b^{2}\right) a^{\frac{5}{2}}}{15}+\frac{b^{2} x^{3} \left(10 c x+b\right) a^{\frac{3}{2}}}{12}-\frac{22 \left(\frac{16 c x}{11}+b\right) x\, a^{\frac{7}{2}}}{15}-\frac{b^{4} x^{4} \sqrt{a}}{8}-}{16}}{a^{\frac{7}{2}} x^{5}}$
risch	$-\frac{\left(128 a^{2} c^{2} x^{4}-100 a\, b^{2} c\, x^{4}+15 b^{4} x^{4}+56 a^{2} b c\, x^{3}-10 a\, b^{3} x^{3}+256 a^{3} c\, x^{2}+8 a^{2} b^{2} x^{2}+176 b\, a^{3} x+128 a^{4}\right) \sqrt{x^{2} \left(c\, x^{2}+b x+a\right)}}{640 x^{6} a^{3}}+$
default	$\frac{\left(c\, x^{4}+b\, x^{3}+a\, x^{2}\right)^{\frac{3}{2}} \left(240 c^{2} a^{\frac{7}{2}} \ln \left(\frac{2 a+b x+2 \sqrt{a}\, \sqrt{c\, x^{2}+b x+a}}{x}\right) b\, x^{5}+120 c^{2} \left(c\, x^{2}+b x+a\right)^{\frac{3}{2}} a\, b^{2} x^{6}-120 c\, a^{\frac{5}{2}} \ln \left(\frac{2 a+b x+2 \sqrt{a}\, \sqrt{c}}{x}\right.}{}$

input
```
int((c*x^4+b*x^3+a*x^2)^(3/2)/x^9,x,method=_RETURNVERBOSE)
```

output
```
3/16*(b*x^5*(a*c-1/4*b^2)^2*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x/a
^(1/2))+(-1/15*x^2*(16*c^2*x^2+7*b*c*x+b^2)*a^(5/2)+1/12*b^2*x^3*(10*c*x+b
)*a^(3/2)-22/15*(16/11*c*x+b)*x*a^(7/2)-1/8*b^4*x^4*a^(1/2)-16/15*a^(9/2))
*(c*x^2+b*x+a)^(1/2)-ln(2)*b*x^5*(a*c-1/4*b^2)^2)/a^(7/2)/x^5
```

Fricas [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 394, normalized size of antiderivative = 1.58

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^9}\,dx = \left[\frac{15\,(b^5-8\,ab^3c+16\,a^2bc^2)\sqrt{a}x^6\log\left(-\frac{8\,abx^2+(b^2+4\,ac)x^3+8\,a^2x+4\,\sqrt{cx^4+bx^3+ax^2}(b\right.}{x^3}}{}\right.$$

$$-\frac{15\,(b^5-8\,ab^3c+16\,a^2bc^2)\sqrt{-a}x^6\arctan\left(\frac{\sqrt{cx^4+bx^3+ax^2}(bx+2\,a)\sqrt{-a}}{2\,(acx^3+abx^2+a^2x)}\right)+2\,(176\,a^4bx+128\,a^5+(15\,ab^4-10}{1280\,a^4x^6}$$

input
```
integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^9,x, algorithm="fricas")
```

output
```
[1/2560*(15*(b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*sqrt(a)*x^6*log(-(8*a*b*x^2 +
 (b^2 + 4*a*c)*x^3 + 8*a^2*x + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*s
qrt(a))/x^3) - 4*(176*a^4*b*x + 128*a^5 + (15*a*b^4 - 100*a^2*b^2*c + 128*
a^3*c^2)*x^4 - 2*(5*a^2*b^3 - 28*a^3*b*c)*x^3 + 8*(a^3*b^2 + 32*a^4*c)*x^2
)*sqrt(c*x^4 + b*x^3 + a*x^2))/(a^4*x^6), -1/1280*(15*(b^5 - 8*a*b^3*c + 1
6*a^2*b*c^2)*sqrt(-a)*x^6*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*
a)*sqrt(-a)/(a*c*x^3 + a*b*x^2 + a^2*x)) + 2*(176*a^4*b*x + 128*a^5 + (15*
a*b^4 - 100*a^2*b^2*c + 128*a^3*c^2)*x^4 - 2*(5*a^2*b^3 - 28*a^3*b*c)*x^3
+ 8*(a^3*b^2 + 32*a^4*c)*x^2)*sqrt(c*x^4 + b*x^3 + a*x^2))/(a^4*x^6)]
```

Sympy [F]

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^9}\,dx = \int \frac{\left(x^2(a+bx+cx^2)\right)^{\frac{3}{2}}}{x^9}\,dx$$

input
```
integrate((c*x**4+b*x**3+a*x**2)**(3/2)/x**9,x)
```

output
```
Integral((x**2*(a + b*x + c*x**2))**(3/2)/x**9, x)
```

Maxima [F]

$$\int \frac{(ax^2 + bx^3 + cx^4)^{3/2}}{x^9}\,dx = \int \frac{(cx^4 + bx^3 + ax^2)^{\frac{3}{2}}}{x^9}\,dx$$

input
```
integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^9,x, algorithm="maxima")
```

output
```
integrate((c*x^4 + b*x^3 + a*x^2)^(3/2)/x^9, x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{(ax^2 + bx^3 + cx^4)^{3/2}}{x^9}\,dx = \text{Timed out}$$

input
```
integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^9,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ax^2 + bx^3 + cx^4)^{3/2}}{x^9}\,dx = \int \frac{(c\,x^4 + b\,x^3 + a\,x^2)^{3/2}}{x^9}\,dx$$

input
```
int((a*x^2 + b*x^3 + c*x^4)^(3/2)/x^9,x)
```

output
```
int((a*x^2 + b*x^3 + c*x^4)^(3/2)/x^9, x)
```

Reduce [B] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.38

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^9}\,dx = \frac{-256\sqrt{c\,x^2+bx+a}\,a^5-352\sqrt{c\,x^2+bx+a}\,a^4bx-512\sqrt{c\,x^2+bx+a}\,a^4c\,x}{}$$

input
```
int((c*x^4+b*x^3+a*x^2)^(3/2)/x^9,x)
```

output
```
( - 256*sqrt(a + b*x + c*x**2)*a**5 - 352*sqrt(a + b*x + c*x**2)*a**4*b*x
- 512*sqrt(a + b*x + c*x**2)*a**4*c*x**2 - 16*sqrt(a + b*x + c*x**2)*a**3*
b**2*x**2 - 112*sqrt(a + b*x + c*x**2)*a**3*b*c*x**3 - 256*sqrt(a + b*x +
c*x**2)*a**3*c**2*x**4 + 20*sqrt(a + b*x + c*x**2)*a**2*b**3*x**3 + 200*sq
rt(a + b*x + c*x**2)*a**2*b**2*c*x**4 - 30*sqrt(a + b*x + c*x**2)*a*b**4*x
**4 + 240*sqrt(a)*log( - 2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a - b*x)*a**
2*b*c**2*x**5 - 120*sqrt(a)*log( - 2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a
- b*x)*a*b**3*c*x**5 + 15*sqrt(a)*log( - 2*sqrt(a)*sqrt(a + b*x + c*x**2)
- 2*a - b*x)*b**5*x**5 - 240*sqrt(a)*log(x)*a**2*b*c**2*x**5 + 120*sqrt(a)
*log(x)*a*b**3*c*x**5 - 15*sqrt(a)*log(x)*b**5*x**5)/(1280*a**4*x**5)
```

3.66 $\int \frac{x^3}{\sqrt{ax^2+bx^3+cx^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 115

$$\int \frac{x^3}{\sqrt{ax^2+bx^3+cx^4}}\,dx = \frac{\sqrt{ax^2+bx^3+cx^4}}{2c} - \frac{3b\sqrt{ax^2+bx^3+cx^4}}{4c^2x} + \frac{(3b^2-4ac)\,\text{arctanh}\left(\frac{x(b+2cx)}{2\sqrt{c}\sqrt{ax^2+bx^3+cx^4}}\right)}{8c^{5/2}}$$

output
```
1/2*(c*x^4+b*x^3+a*x^2)^(1/2)/c-3/4*b*(c*x^4+b*x^3+a*x^2)^(1/2)/c^2/x+1/8*
(-4*a*c+3*b^2)*arctanh(1/2*x*(2*c*x+b)/c^(1/2)/(c*x^4+b*x^3+a*x^2)^(1/2))/
c^(5/2)
```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.93

$$\int \frac{x^3}{\sqrt{ax^2+bx^3+cx^4}}\,dx = \frac{x\left(2\sqrt{c}(-3b+2cx)(a+x(b+cx))+(-3b^2+4ac)\sqrt{a+x(b+cx)}\log\left(c^2\left(b+2cx-2\sqrt{c}\sqrt{a+x(b+\right.\right.\right.}{8c^{5/2}\sqrt{x^2(a+x(b+cx))}}$$

input
```
Integrate[x^3/Sqrt[a*x^2 + b*x^3 + c*x^4],x]
```

output
```
(x*(2*Sqrt[c]*(-3*b + 2*c*x)*(a + x*(b + c*x)) + (-3*b^2 + 4*a*c)*Sqrt[a +
 x*(b + c*x)]*Log[c^2*(b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]))/(8*
c^(5/2)*Sqrt[x^2*(a + x*(b + c*x))])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.30, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {1975, 27, 1996, 27, 1961, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt{ax^2+bx^3+cx^4}}dx$$

$$\Big\downarrow \text{ 1975}$$

$$\frac{\sqrt{ax^2+bx^3+cx^4}}{2c}-\frac{\int \frac{x(2a+3bx)}{2\sqrt{cx^4+bx^3+ax^2}}dx}{2c}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{ax^2+bx^3+cx^4}}{2c}-\frac{\int \frac{x(2a+3bx)}{\sqrt{cx^4+bx^3+ax^2}}dx}{4c}$$

$$\Big\downarrow \text{ 1996}$$

$$\frac{\sqrt{ax^2+bx^3+cx^4}}{2c}-\frac{\frac{3b\sqrt{ax^2+bx^3+cx^4}}{cx}-\frac{\int \frac{(3b^2-4ac)x}{2\sqrt{cx^4+bx^3+ax^2}}dx}{c}}{4c}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{ax^2+bx^3+cx^4}}{2c}-\frac{\frac{3b\sqrt{ax^2+bx^3+cx^4}}{cx}-\frac{(3b^2-4ac)\int \frac{x}{\sqrt{cx^4+bx^3+ax^2}}dx}{2c}}{4c}$$

$$\Big\downarrow \text{ 1961}$$

$$\frac{\sqrt{ax^2+bx^3+cx^4}}{2c}-\frac{\frac{3b\sqrt{ax^2+bx^3+cx^4}}{cx}-\frac{x(3b^2-4ac)\sqrt{a+bx+cx^2}\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{2c\sqrt{ax^2+bx^3+cx^4}}}{4c}$$

$$\Big\downarrow \text{ 1092}$$

$$\frac{\sqrt{ax^2+bx^3+cx^4}}{2c}-\frac{\frac{3b\sqrt{ax^2+bx^3+cx^4}}{cx}-\frac{x(3b^2-4ac)\sqrt{a+bx+cx^2}\int\frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{c\sqrt{ax^2+bx^3+cx^4}}}{4c}$$

$\Big\downarrow$ 219

$$\frac{\sqrt{ax^2+bx^3+cx^4}}{2c}-\frac{\frac{3b\sqrt{ax^2+bx^3+cx^4}}{cx}-\frac{x(3b^2-4ac)\sqrt{a+bx+cx^2}\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2c^{3/2}\sqrt{ax^2+bx^3+cx^4}}}{4c}$$

input
```
Int[x^3/Sqrt[a*x^2 + b*x^3 + c*x^4],x]
```

output
```
Sqrt[a*x^2 + b*x^3 + c*x^4]/(2*c) - ((3*b*Sqrt[a*x^2 + b*x^3 + c*x^4])/(c*
x) - ((3*b^2 - 4*a*c)*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt[
c]*Sqrt[a + b*x + c*x^2])])/(2*c^(3/2)*Sqrt[a*x^2 + b*x^3 + c*x^4]))/(4*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1961
```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Simp[x^(q/2)*(Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]/Sqrt[a
*x^q + b*x^n + c*x^(2*n - q)])   Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x
^(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] &&
 PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] ||
 EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))
```

rule 1975
```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[x^(m - 2*n + q + 1)*((a*x^q + b*x^n + c*x^(2*n - q))^(
p + 1)/(c*(m + p*q + 2*(n - q)*p + 1))), x] - Simp[1/(c*(m + p*q + 2*(n - q
)*p + 1))   Int[x^(m - 2*(n - q))*(a*(m + p*q - 2*(n - q) + 1) + b*(m + p*q
 + (n - q)*(p - 1) + 1)*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^p, x], x
] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] &&  !IntegerQ[p]
 && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[p, -1] && LtQ[p, 0] && Rationa
lQ[m, q] && GtQ[m + p*q + 1, 2*(n - q)]
```

rule 1996
```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[B*x^(m - n + 1)*((a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1)/(c*(m + p*q + (n - q)*(2*p + 1) + 1))), x] -
Simp[1/(c*(m + p*q + (n - q)*(2*p + 1) + 1))   Int[x^(m - n + q)*Simp[a*B*(
m + p*q - n + q + 1) + (b*B*(m + p*q + (n - q)*p + 1) - A*c*(m + p*q + (n -
 q)*(2*p + 1) + 1))*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^p, x], x]
 /; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] &&  !Inte
gerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[p, -1] && LtQ[p, 0] &&
RationalQ[m, q] && GeQ[m + p*q, n - q - 1] && NeQ[m + p*q + (n - q)*(2*p +
1) + 1, 0]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.87

method	result
pseudoelliptic	$\frac{4\sqrt{c\,x^{2}+bx+a}\,c^{\frac{3}{2}}x-6b\sqrt{c\,x^{2}+bx+a}\,\sqrt{c}-4\ln\left(2\sqrt{c\,x^{2}+bx+a}\,\sqrt{c}+2cx+b\right)ac+3\ln\left(2\sqrt{c\,x^{2}+bx+a}\,\sqrt{c}+2cx+b\right)b^{2}}{8c^{\frac{5}{2}}}$
risch	$-\frac{\left(-2cx+3b\right)\left(c\,x^{2}+bx+a\right)x}{4c^{2}\sqrt{x^{2}\left(c\,x^{2}+bx+a\right)}}-\frac{\left(4ac-3b^{2}\right)\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^{2}+bx+a}\right)x\sqrt{c\,x^{2}+bx+a}}{8c^{\frac{5}{2}}\sqrt{x^{2}\left(c\,x^{2}+bx+a\right)}}$
default	$\frac{x\sqrt{c\,x^{2}+bx+a}\,\left(4c^{\frac{5}{2}}\sqrt{c\,x^{2}+bx+a}\,x-6c^{\frac{3}{2}}\sqrt{c\,x^{2}+bx+a}\,b-4a\ln\left(\frac{2\sqrt{c\,x^{2}+bx+a}\,\sqrt{c}+2cx+b}{2\sqrt{c}}\right)c^{2}+3\ln\left(\frac{2\sqrt{c\,x^{2}+bx+a}\,\sqrt{c}+2cx+}{2\sqrt{c}}\right.\right.}{8\sqrt{c\,x^{4}+b\,x^{3}+a\,x^{2}}\,c^{\frac{7}{2}}}$

input
```
int(x^3/(c*x^4+b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/8*(4*(c*x^2+b*x+a)^(1/2)*c^(3/2)*x-6*b*(c*x^2+b*x+a)^(1/2)*c^(1/2)-4*ln(
2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)*a*c+3*ln(2*(c*x^2+b*x+a)^(1/2)*c^(1
/2)+2*c*x+b)*b^2)/c^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.97

$$\int \frac{x^3}{\sqrt{ax^2+bx^3+cx^4}}\,dx$$
$$= \Bigg[-\frac{(3\,b^2-4\,ac)\sqrt{c}x\log\left(-\frac{8\,c^2x^3+8\,bcx^2-4\,\sqrt{cx^4+bx^3+ax^2}(2\,cx+b)\sqrt{c}+(b^2+4\,ac)x}{x}\right)-4\,\sqrt{cx^4+bx^3+ax^2}(2\,c^2x-3}{16\,c^3x},$$
$$-\frac{(3\,b^2-4\,ac)\sqrt{-c}x\arctan\left(\frac{\sqrt{cx^4+bx^3+ax^2}(2\,cx+b)\sqrt{-c}}{2\,(c^2x^3+bcx^2+acx)}\right)-2\,\sqrt{cx^4+bx^3+ax^2}(2\,c^2x-3\,bc)}{8\,c^3x}\Bigg]$$

input
```
integrate(x^3/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
[-1/16*((3*b^2 - 4*a*c)*sqrt(c)*x*log(-(8*c^2*x^3 + 8*b*c*x^2 - 4*sqrt(c*x
^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(c) + (b^2 + 4*a*c)*x)/x) - 4*sqrt(c*x
^4 + b*x^3 + a*x^2)*(2*c^2*x - 3*b*c))/(c^3*x), -1/8*((3*b^2 - 4*a*c)*sqrt
(-c)*x*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(-c)/(c^2*x^
3 + b*c*x^2 + a*c*x)) - 2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c^2*x - 3*b*c))/(
c^3*x)]
```

Sympy [F]

$$\int \frac{x^3}{\sqrt{ax^2+bx^3+cx^4}}\,dx = \int \frac{x^3}{\sqrt{x^2\,(a+bx+cx^2)}}\,dx$$

input
```
integrate(x**3/(c*x**4+b*x**3+a*x**2)**(1/2),x)
```

output
```
Integral(x**3/sqrt(x**2*(a + b*x + c*x**2)), x)
```

Maxima [F]

$$\int \frac{x^3}{\sqrt{ax^2+bx^3+cx^4}}\,dx = \int \frac{x^3}{\sqrt{cx^4+bx^3+ax^2}}\,dx$$

input
```
integrate(x^3/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^3/sqrt(c*x^4 + b*x^3 + a*x^2), x)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.17

$$\begin{aligned}\int \frac{x^3}{\sqrt{ax^2+bx^3+cx^4}}\,dx &= \frac{1}{4}\sqrt{cx^2+bx+a}\left(\frac{2\,x}{c\mathrm{sgn}\left(x\right)}-\frac{3\,b}{c^2\mathrm{sgn}\left(x\right)}\right) \\ &+ \frac{\left(3\,b^2\log\left(\left|b-2\,\sqrt{a}\sqrt{c}\right|\right)-4\,ac\log\left(\left|b-2\,\sqrt{a}\sqrt{c}\right|\right)+6\,\sqrt{a}b\sqrt{c}\right)\mathrm{sgn}(x)}{8\,c^{\frac{5}{2}}} \\ &- \frac{\left(3\,b^2-4\,ac\right)\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)\sqrt{c}+b\right|\right)}{8\,c^{\frac{5}{2}}\mathrm{sgn}\left(x\right)}\end{aligned}$$

input
```
integrate(x^3/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
1/4*sqrt(c*x^2 + b*x + a)*(2*x/(c*sgn(x)) - 3*b/(c^2*sgn(x))) + 1/8*(3*b^2
*log(abs(b - 2*sqrt(a)*sqrt(c))) - 4*a*c*log(abs(b - 2*sqrt(a)*sqrt(c))) +
 6*sqrt(a)*b*sqrt(c))*sgn(x)/c^(5/2) - 1/8*(3*b^2 - 4*a*c)*log(abs(2*(sqrt
(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) + b))/(c^(5/2)*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\sqrt{ax^2+bx^3+cx^4}}\,dx=\int \frac{x^3}{\sqrt{c\,x^4+b\,x^3+a\,x^2}}\,dx$$

input
```
int(x^3/(a*x^2 + b*x^3 + c*x^4)^(1/2),x)
```

output
```
int(x^3/(a*x^2 + b*x^3 + c*x^4)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.07

$$\int \frac{x^3}{\sqrt{ax^2+bx^3+cx^4}}\,dx = \frac{-6\sqrt{c\,x^2+bx+a}\,bc+4\sqrt{c\,x^2+bx+a}\,c^2x-4\sqrt{c}\log\left(\frac{2\sqrt{c}\,\sqrt{c\,x^2+bx+a}+b+2cx}{\sqrt{4ac-b^2}}\right)ac+3\sqrt{c}\log\left(\frac{2\sqrt{c}\,\sqrt{c\,x^2+bx+a}+b+2cx}{\sqrt{4ac-b^2}}\right)b^2}{8c^3}$$

input
```
int(x^3/(c*x^4+b*x^3+a*x^2)^(1/2),x)
```

output
```
( - 6*sqrt(a + b*x + c*x**2)*b*c + 4*sqrt(a + b*x + c*x**2)*c**2*x - 4*sqr
t(c)*log((2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)/sqrt(4*a*c - b**2)
)*a*c + 3*sqrt(c)*log((2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)/sqrt(
4*a*c - b**2))*b**2)/(8*c**3)
```

3.67 $\int \frac{x^2}{\sqrt{ax^2+bx^3+cx^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 75

$$\int \frac{x^2}{\sqrt{ax^2+bx^3+cx^4}}\,dx=\frac{\sqrt{ax^2+bx^3+cx^4}}{cx}-\frac{b\,\text{arctanh}\left(\frac{x(b+2cx)}{2\sqrt{c}\sqrt{ax^2+bx^3+cx^4}}\right)}{2c^{3/2}}$$

output
```
(c*x^4+b*x^3+a*x^2)^(1/2)/c/x-1/2*b*arctanh(1/2*x*(2*c*x+b)/c^(1/2)/(c*x^4
+b*x^3+a*x^2)^(1/2))/c^(3/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.19

$$\int \frac{x^2}{\sqrt{ax^2+bx^3+cx^4}}\,dx=\frac{x\left(2\sqrt{c}(a+x(b+cx))-b\sqrt{a+x(b+cx)}\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)\right)}{2c^{3/2}\sqrt{x^2(a+x(b+cx))}}$$

input
```
Integrate[x^2/Sqrt[a*x^2 + b*x^3 + c*x^4],x]
```

output
```
(x*(2*Sqrt[c]*(a + x*(b + c*x)) - b*Sqrt[a + x*(b + c*x)]*ArcTanh[(b + 2*c
*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]))/(2*c^(3/2)*Sqrt[x^2*(a + x*(b + c
*x))])
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.37, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1964, 1961, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{ax^2+bx^3+cx^4}}dx$$

$$\Big\downarrow \text{1964}$$

$$\frac{\sqrt{ax^2+bx^3+cx^4}}{cx}-\frac{b\int \frac{x}{\sqrt{cx^4+bx^3+ax^2}}dx}{2c}$$

$$\Big\downarrow \text{1961}$$

$$\frac{\sqrt{ax^2+bx^3+cx^4}}{cx}-\frac{bx\sqrt{a+bx+cx^2}\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{2c\sqrt{ax^2+bx^3+cx^4}}$$

$$\Big\downarrow \text{1092}$$

$$\frac{\sqrt{ax^2+bx^3+cx^4}}{cx}-\frac{bx\sqrt{a+bx+cx^2}\int \frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{c\sqrt{ax^2+bx^3+cx^4}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\sqrt{ax^2+bx^3+cx^4}}{cx}-\frac{bx\sqrt{a+bx+cx^2}\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2c^{3/2}\sqrt{ax^2+bx^3+cx^4}}$$

input
```
Int[x^2/Sqrt[a*x^2 + b*x^3 + c*x^4],x]
```

output
```
Sqrt[a*x^2 + b*x^3 + c*x^4]/(c*x) - (b*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(b
+ 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*c^(3/2)*Sqrt[a*x^2 + b*x^3
 + c*x^4])
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1961
```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Simp[x^(q/2)*(Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]/Sqrt[a
*x^q + b*x^n + c*x^(2*n - q)])   Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x
^(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] &&
 PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] ||
 EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))
```

rule 1964
```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[x^(m - n)*((a*x^(n - 1) + b*x^n + c*x^(n + 1))^(p + 1)
/(2*c*(p + 1))), x] - Simp[b/(2*c)   Int[x^(m - 1)*(a*x^(n - 1) + b*x^n + c
*x^(n + 1))^p, x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n -
 q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && RationalQ[m, p
, q] && EqQ[m + p*(n - 1) - 1, 0]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.67

method	result	size
pseudoelliptic	$-\frac{\ln\left(2\sqrt{c\,x^{2}+bx+a}\,\sqrt{c}+2cx+b\right)b-2\sqrt{c\,x^{2}+bx+a}\,\sqrt{c}}{2c^{\frac{3}{2}}}$	50
default	$\frac{x\sqrt{c\,x^{2}+bx+a}\,\left(2\sqrt{c\,x^{2}+bx+a}\,c^{\frac{3}{2}}-b\ln\left(\frac{2\sqrt{c\,x^{2}+bx+a}\,\sqrt{c}+2cx+b}{2\sqrt{c}}\right)c\right)}{2\sqrt{c\,x^{4}+b\,x^{3}+a\,x^{2}}\,c^{\frac{5}{2}}}$	88
risch	$\frac{(c\,x^{2}+bx+a)x}{c\sqrt{x^{2}(c\,x^{2}+bx+a)}}-\frac{b\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^{2}+bx+a}\right)x\sqrt{c\,x^{2}+bx+a}}{2c^{\frac{3}{2}}\sqrt{x^{2}(c\,x^{2}+bx+a)}}$	93

input
```
int(x^2/(c*x^4+b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(ln(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)*b-2*(c*x^2+b*x+a)^(1/2)*c^
(1/2))/c^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 188, normalized size of antiderivative = 2.51

$$\int \frac{x^2}{\sqrt{ax^2+bx^3+cx^4}}\,dx = \left[\frac{b\sqrt{c}x\log\left(-\frac{8\,c^2x^3+8\,bcx^2-4\,\sqrt{cx^4+bx^3+ax^2}(2\,cx+b)\sqrt{c}+(b^2+4\,ac)x}{x}\right)+4\,\sqrt{cx^4+bx^3+ax^2}c}{4\,c^2x}, \frac{b\sqrt{-c}x\arctan\left(\frac{\sqrt{cx^4}\cdots}{\cdots}\right.}{\cdots}\right.$$

input
```
integrate(x^2/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(b*sqrt(c)*x*log(-(8*c^2*x^3 + 8*b*c*x^2 - 4*sqrt(c*x^4 + b*x^3 + a*x
^2)*(2*c*x + b)*sqrt(c) + (b^2 + 4*a*c)*x)/x) + 4*sqrt(c*x^4 + b*x^3 + a*x
^2)*c)/(c^2*x), 1/2*(b*sqrt(-c)*x*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(
2*c*x + b)*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x)) + 2*sqrt(c*x^4 + b*x^3 +
a*x^2)*c)/(c^2*x)]
```

Sympy [F]

$$\int \frac{x^2}{\sqrt{ax^2+bx^3+cx^4}}\,dx=\int \frac{x^2}{\sqrt{x^2\left(a+bx+cx^2\right)}}\,dx$$

input
```
integrate(x**2/(c*x**4+b*x**3+a*x**2)**(1/2),x)
```

output
```
Integral(x**2/sqrt(x**2*(a + b*x + c*x**2)), x)
```

Maxima [F]

$$\int \frac{x^2}{\sqrt{ax^2+bx^3+cx^4}}\,dx=\int \frac{x^2}{\sqrt{cx^4+bx^3+ax^2}}\,dx$$

input
```
integrate(x^2/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2/sqrt(c*x^4 + b*x^3 + a*x^2), x)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.20

$$\int \frac{x^2}{\sqrt{ax^2+bx^3+cx^4}}\,dx=-\frac{\left(b\log\left(\left|b-2\sqrt{a}\sqrt{c}\right|\right)+2\sqrt{a}\sqrt{c}\right)\mathrm{sgn}(x)}{2c^{\frac{3}{2}}}+\frac{b\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)\sqrt{c}+b\right|\right)}{2c^{\frac{3}{2}}\mathrm{sgn}\left(x\right)}+\frac{\sqrt{cx^2+bx+a}}{c\mathrm{sgn}\left(x\right)}$$

input
```
integrate(x^2/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
-1/2*(b*log(abs(b - 2*sqrt(a)*sqrt(c))) + 2*sqrt(a)*sqrt(c))*sgn(x)/c^(3/2
) + 1/2*b*log(abs(2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) + b))/(c^(
3/2)*sgn(x)) + sqrt(c*x^2 + b*x + a)/(c*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{ax^2+bx^3+cx^4}}\,dx=\int \frac{x^2}{\sqrt{c\,x^4+b\,x^3+a\,x^2}}\,dx$$

input
```
int(x^2/(a*x^2 + b*x^3 + c*x^4)^(1/2),x)
```

output
```
int(x^2/(a*x^2 + b*x^3 + c*x^4)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.81

$$\int \frac{x^2}{\sqrt{ax^2+bx^3+cx^4}}\,dx=\frac{2\sqrt{c\,x^2+bx+a}\,c-\sqrt{c}\log\left(\frac{2\sqrt{c}\,\sqrt{c\,x^2+bx+a}+b+2cx}{\sqrt{4ac-b^2}}\right)b}{2c^2}$$

input
```
int(x^2/(c*x^4+b*x^3+a*x^2)^(1/2),x)
```

output
```
(2*sqrt(a + b*x + c*x**2)*c - sqrt(c)*log((2*sqrt(c)*sqrt(a + b*x + c*x**2
) + b + 2*c*x)/sqrt(4*a*c - b**2))*b)/(2*c**2)
```

3.68 $\int \frac{x}{\sqrt{ax^2+bx^3+cx^4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 43

$$\int \frac{x}{\sqrt{ax^2+bx^3+cx^4}}\,dx=\frac{\text{arctanh}\left(\frac{x(b+2cx)}{2\sqrt{c}\sqrt{ax^2+bx^3+cx^4}}\right)}{\sqrt{c}}$$

output
```
arctanh(1/2*x*(2*c*x+b)/c^(1/2)/(c*x^4+b*x^3+a*x^2)^(1/2))/c^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.56

$$\int \frac{x}{\sqrt{ax^2+bx^3+cx^4}}\,dx=-\frac{x\sqrt{a+bx+cx^2}\log\left(b+2cx-2\sqrt{c}\sqrt{a+bx+cx^2}\right)}{\sqrt{c}\sqrt{x^2(a+x(b+cx))}}$$

input
```
Integrate[x/Sqrt[a*x^2 + b*x^3 + c*x^4],x]
```

output
```
-((x*Sqrt[a + b*x + c*x^2]*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + b*x + c*x^2]
])/(Sqrt[c]*Sqrt[x^2*(a + x*(b + c*x))]))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.65, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1961, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{ax^2+bx^3+cx^4}}\,dx$$
$$\Big\downarrow \text{1961}$$
$$\frac{x\sqrt{a+bx+cx^2}\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{\sqrt{ax^2+bx^3+cx^4}}$$
$$\Big\downarrow \text{1092}$$
$$\frac{2x\sqrt{a+bx+cx^2}\int \frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{\sqrt{ax^2+bx^3+cx^4}}$$
$$\Big\downarrow \text{219}$$
$$\frac{x\sqrt{a+bx+cx^2}\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}\sqrt{ax^2+bx^3+cx^4}}$$

input
```
Int[x/Sqrt[a*x^2 + b*x^3 + c*x^4],x]
```

output
```
(x*Sqrt[a + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x
^2])])/(Sqrt[c]*Sqrt[a*x^2 + b*x^3 + c*x^4])
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1961
```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Simp[x^(q/2)*(Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]/Sqrt[a
*x^q + b*x^n + c*x^(2*n - q)])   Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x
^(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] &&
 PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] ||
 EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.67

method	result	size
pseudoelliptic	$\frac{\ln\left(2\sqrt{c\,x^{2}+bx+a}\,\sqrt{c}+2cx+b\right)}{\sqrt{c}}$	29
default	$\frac{x\sqrt{c\,x^{2}+bx+a}\,\ln\left(\frac{2\sqrt{c\,x^{2}+bx+a}\,\sqrt{c}+2cx+b}{2\sqrt{c}}\right)}{\sqrt{c\,x^{4}+b\,x^{3}+a\,x^{2}}\,\sqrt{c}}$	65

input
```
int(x/(c*x^4+b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/c^(1/2)*ln(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 129, normalized size of antiderivative = 3.00

$$\int \frac{x}{\sqrt{ax^2+bx^3+cx^4}}\,dx = \left[\frac{\log\left(-\frac{8\,c^2x^3+8\,bcx^2+4\,\sqrt{cx^4+bx^3+ax^2}(2\,cx+b)\sqrt{c}+(b^2+4\,ac)x}{x}\right)}{2\,\sqrt{c}},\ -\frac{\sqrt{-c}\arctan\left(\frac{\sqrt{cx^4+bx^3+ax^2}(2\,cx+b)\sqrt{-c}}{2\,(c^2x^3+bcx^2+acx)}\right)}{c}\right]$$

input
```
integrate(x/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*log(-(8*c^2*x^3 + 8*b*c*x^2 + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x +
b)*sqrt(c) + (b^2 + 4*a*c)*x)/x)/sqrt(c), -sqrt(-c)*arctan(1/2*sqrt(c*x^4
+ b*x^3 + a*x^2)*(2*c*x + b)*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x))/c]
```

Sympy [F]

$$\int \frac{x}{\sqrt{ax^2+bx^3+cx^4}}\,dx = \int \frac{x}{\sqrt{x^2\,(a+bx+cx^2)}}\,dx$$

input
```
integrate(x/(c*x**4+b*x**3+a*x**2)**(1/2),x)
```

output
```
Integral(x/sqrt(x**2*(a + b*x + c*x**2)), x)
```

Maxima [F]

$$\int \frac{x}{\sqrt{ax^2+bx^3+cx^4}}\,dx = \int \frac{x}{\sqrt{cx^4+bx^3+ax^2}}\,dx$$

input
```
integrate(x/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/sqrt(c*x^4 + b*x^3 + a*x^2), x)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.33

$$\int \frac{x}{\sqrt{ax^2+bx^3+cx^4}}\,dx = \frac{\log\left(\left|b-2\sqrt{a}\sqrt{c}\right|\right)\operatorname{sgn}(x)}{\sqrt{c}} - \frac{\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)\sqrt{c}+b\right|\right)}{\sqrt{c}\operatorname{sgn}(x)}$$

input
```
integrate(x/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
log(abs(b - 2*sqrt(a)*sqrt(c)))*sgn(x)/sqrt(c) - log(abs(2*(sqrt(c)*x - sq
rt(c*x^2 + b*x + a))*sqrt(c) + b))/(sqrt(c)*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{ax^2+bx^3+cx^4}}\,dx = \int \frac{x}{\sqrt{c\,x^4+b\,x^3+a\,x^2}}\,dx$$

input
```
int(x/(a*x^2 + b*x^3 + c*x^4)^(1/2),x)
```

output
```
int(x/(a*x^2 + b*x^3 + c*x^4)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.98

$$\int \frac{x}{\sqrt{ax^2+bx^3+cx^4}}\,dx = \frac{\sqrt{c}\log\left(\frac{2\sqrt{c}\,\sqrt{c\,x^2+bx+a}+b+2cx}{\sqrt{4ac-b^2}}\right)}{c}$$

input
```
int(x/(c*x^4+b*x^3+a*x^2)^(1/2),x)
```

output
```
(sqrt(c)*log((2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)/sqrt(4*a*c - b
**2)))/c
```

3.69 $\int \frac{1}{\sqrt{ax^2+bx^3+cx^4}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 45

$$\int \frac{1}{\sqrt{ax^2+bx^3+cx^4}}\,dx=-\frac{\operatorname{arctanh}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{\sqrt{a}}$$

output
```
-arctanh(1/2*x*(b*x+2*a)/a^(1/2)/(c*x^4+b*x^3+a*x^2)^(1/2))/a^(1/2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.51

$$\int \frac{1}{\sqrt{ax^2+bx^3+cx^4}}\,dx=\frac{2x\sqrt{a+x(b+cx)}\operatorname{arctanh}\left(\frac{\sqrt{c}x-\sqrt{a+x(b+cx)}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{x^2(a+x(b+cx))}}$$

input
```
Integrate[1/Sqrt[a*x^2 + b*x^3 + c*x^4],x]
```

output
```
(2*x*Sqrt[a + x*(b + c*x)]*ArcTanh[(Sqrt[c]*x - Sqrt[a + x*(b + c*x)])/Sqr
t[a]])/(Sqrt[a]*Sqrt[x^2*(a + x*(b + c*x))])
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1951, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{ax^2+bx^3+cx^4}}dx$$

$$\Big\downarrow \text{1951}$$

$$-2\int \frac{1}{4a-\frac{x^2(2a+bx)^2}{cx^4+bx^3+ax^2}}d\frac{x(2a+bx)}{\sqrt{cx^4+bx^3+ax^2}}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\text{arctanh}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{\sqrt{a}}$$

input
```
Int[1/Sqrt[a*x^2 + b*x^3 + c*x^4],x]
```

output
```
-(ArcTanh[(x*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x^2 + b*x^3 + c*x^4])]/Sqrt[a]
)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1951
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :
> Simp[-2/(n - 2)   Subst[Int[1/(4*a - x^2), x], x, x*((2*a + b*x^(n - 2))/
Sqrt[a*x^2 + b*x^n + c*x^r])], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*
n - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.93

method	result	size
pseudoelliptic	$\frac{\ln(2)-\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x\sqrt{a}}\right)}{\sqrt{a}}$	42
default	$-\frac{x\sqrt{c\,x^2+bx+a}\,\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x}\right)}{\sqrt{c\,x^4+b\,x^3+a\,x^2}\,\sqrt{a}}$	66

input
```
int(1/(c*x^4+b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(ln(2)-ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x/a^(1/2)))/a^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 130, normalized size of antiderivative = 2.89

$$\int \frac{1}{\sqrt{ax^2+bx^3+cx^4}}\,dx$$
$$=\left[\frac{\log\left(-\frac{8\,abx^2+(b^2+4\,ac)x^3+8\,a^2x-4\,\sqrt{cx^4+bx^3+ax^2}(bx+2\,a)\sqrt{a}}{x^3}\right)}{2\,\sqrt{a}},\ \frac{\sqrt{-a}\arctan\left(\frac{\sqrt{cx^4+bx^3+ax^2}(bx+2\,a)\sqrt{-a}}{2\,(acx^3+abx^2+a^2x)}\right)}{a}\right]$$

input
```
integrate(1/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x - 4*sqrt(c*x^4 + b*x^3
+ a*x^2)*(b*x + 2*a)*sqrt(a))/x^3)/sqrt(a), sqrt(-a)*arctan(1/2*sqrt(c*x^4
 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2 + a^2*x))/a]
```

Sympy [F]

$$\int \frac{1}{\sqrt{ax^2 + bx^3 + cx^4}}\,dx = \int \frac{1}{\sqrt{ax^2 + bx^3 + cx^4}}\,dx$$

input
```
integrate(1/(c*x**4+b*x**3+a*x**2)**(1/2),x)
```

output
```
Integral(1/sqrt(a*x**2 + b*x**3 + c*x**4), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{ax^2 + bx^3 + cx^4}}\,dx = \int \frac{1}{\sqrt{cx^4 + bx^3 + ax^2}}\,dx$$

input
```
integrate(1/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(c*x^4 + b*x^3 + a*x^2), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.31

$$\int \frac{1}{\sqrt{ax^2 + bx^3 + cx^4}}\,dx = -\frac{2\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)\operatorname{sgn}(x)}{\sqrt{-a}} + \frac{2\arctan\left(-\frac{\sqrt{c}x - \sqrt{cx^2+bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\operatorname{sgn}(x)}$$

input
```
integrate(1/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
-2*arctan(sqrt(a)/sqrt(-a))*sgn(x)/sqrt(-a) + 2*arctan(-(sqrt(c)*x - sqrt(
c*x^2 + b*x + a))/sqrt(-a))/(sqrt(-a)*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{ax^2+bx^3+cx^4}}\,dx=\int \frac{1}{\sqrt{c\,x^4+b\,x^3+a\,x^2}}\,dx$$

input
```
int(1/(a*x^2 + b*x^3 + c*x^4)^(1/2),x)
```

output
```
int(1/(a*x^2 + b*x^3 + c*x^4)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 693, normalized size of antiderivative = 15.40

$$\int \frac{1}{\sqrt{ax^2+bx^3+cx^4}}\,dx=\text{Too large to display}$$

input
```
int(1/(c*x^4+b*x^3+a*x^2)^(1/2),x)
```

output
```
( - 2*sqrt(a)*sqrt(4*sqrt(c)*sqrt(a)*b - 4*a*c - b**2)*atan((2*sqrt(c)*sqr
t(a + b*x + c*x**2) + b + 2*c*x)/sqrt(4*sqrt(c)*sqrt(a)*b - 4*a*c - b**2))
*b - 4*sqrt(c)*sqrt(4*sqrt(c)*sqrt(a)*b - 4*a*c - b**2)*atan((2*sqrt(c)*sq
rt(a + b*x + c*x**2) + b + 2*c*x)/sqrt(4*sqrt(c)*sqrt(a)*b - 4*a*c - b**2)
)*a - sqrt(a)*sqrt(4*sqrt(c)*sqrt(a)*b + 4*a*c + b**2)*log( - sqrt(4*sqrt(
c)*sqrt(a)*b + 4*a*c + b**2) + 2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*
x)*b + sqrt(a)*sqrt(4*sqrt(c)*sqrt(a)*b + 4*a*c + b**2)*log(sqrt(4*sqrt(c)
*sqrt(a)*b + 4*a*c + b**2) + 2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)
*b + 2*sqrt(c)*sqrt(4*sqrt(c)*sqrt(a)*b + 4*a*c + b**2)*log( - sqrt(4*sqrt
(c)*sqrt(a)*b + 4*a*c + b**2) + 2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c
*x)*a - 2*sqrt(c)*sqrt(4*sqrt(c)*sqrt(a)*b + 4*a*c + b**2)*log(sqrt(4*sqrt
(c)*sqrt(a)*b + 4*a*c + b**2) + 2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c
*x)*a + 4*sqrt(a)*log( - sqrt(4*sqrt(c)*sqrt(a)*b + 4*a*c + b**2) + 2*sqrt
(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)*a*c - sqrt(a)*log( - sqrt(4*sqrt(c
)*sqrt(a)*b + 4*a*c + b**2) + 2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x
)*b**2 + 4*sqrt(a)*log(sqrt(4*sqrt(c)*sqrt(a)*b + 4*a*c + b**2) + 2*sqrt(c
)*sqrt(a + b*x + c*x**2) + b + 2*c*x)*a*c - sqrt(a)*log(sqrt(4*sqrt(c)*sqr
t(a)*b + 4*a*c + b**2) + 2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)*b**
2 - 4*sqrt(a)*log(4*sqrt(c)*sqrt(a + b*x + c*x**2)*b + 8*sqrt(c)*sqrt(a +
b*x + c*x**2)*c*x + 4*sqrt(c)*sqrt(a)*b + 8*b*c*x + 8*c**2*x**2)*a*c + ...
```

3.70 $\int \frac{1}{x\sqrt{ax^2+bx^3+cx^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 77

$$\int \frac{1}{x\sqrt{ax^2+bx^3+cx^4}}\,dx=-\frac{\sqrt{ax^2+bx^3+cx^4}}{ax^2}+\frac{b\,\text{arctanh}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{2a^{3/2}}$$

output
```
-(c*x^4+b*x^3+a*x^2)^(1/2)/a/x^2+1/2*b*arctanh(1/2*x*(b*x+2*a)/a^(1/2)/(c*
x^4+b*x^3+a*x^2)^(1/2))/a^(3/2)
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.13

$$\int \frac{1}{x\sqrt{ax^2+bx^3+cx^4}}\,dx = \frac{-\sqrt{a}(a+x(b+cx))-bx\sqrt{a+x(b+cx)}\,\text{arctanh}\left(\frac{\sqrt{c}x-\sqrt{a+x(b+cx)}}{\sqrt{a}}\right)}{a^{3/2}\sqrt{x^2(a+x(b+cx))}}$$

input
```
Integrate[1/(x*Sqrt[a*x^2 + b*x^3 + c*x^4]),x]
```

output
```
(-(Sqrt[a]*(a + x*(b + c*x))) - b*x*Sqrt[a + x*(b + c*x)]*ArcTanh[(Sqrt[c]
*x - Sqrt[a + x*(b + c*x)])/Sqrt[a]])/(a^(3/2)*Sqrt[x^2*(a + x*(b + c*x))]
)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1974, 1951, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{ax^2+bx^3+cx^4}}dx$$

$$\Big\downarrow \text{ 1974}$$

$$-\frac{b\int \frac{1}{\sqrt{cx^4+bx^3+ax^2}}dx}{2a}-\frac{\sqrt{ax^2+bx^3+cx^4}}{ax^2}$$

$$\Big\downarrow \text{ 1951}$$

$$\frac{b\int \frac{1}{4a-\frac{x^2(2a+bx)^2}{cx^4+bx^3+ax^2}}d\frac{x(2a+bx)}{\sqrt{cx^4+bx^3+ax^2}}}{a}-\frac{\sqrt{ax^2+bx^3+cx^4}}{ax^2}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{b\text{arctanh}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{2a^{3/2}}-\frac{\sqrt{ax^2+bx^3+cx^4}}{ax^2}$$

input
```
Int[1/(x*Sqrt[a*x^2 + b*x^3 + c*x^4]),x]
```

output
```
-(Sqrt[a*x^2 + b*x^3 + c*x^4]/(a*x^2)) + (b*ArcTanh[(x*(2*a + b*x))/(2*Sqr
t[a]*Sqrt[a*x^2 + b*x^3 + c*x^4])])/(2*a^(3/2))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1951
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :
> Simp[-2/(n - 2)   Subst[Int[1/(4*a - x^2), x], x, x*((2*a + b*x^(n - 2))/
Sqrt[a*x^2 + b*x^n + c*x^r])], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*
n - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]
```

rule 1974
```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[(-x^(m - q + 1))*((a*x^q + b*x^n + c*x^(2*n - q))^(p +
 1)/(2*a*(n - q)*(p + 1))), x] - Simp[b/(2*a)   Int[x^(m + n - q)*(a*x^q +
b*x^n + c*x^(2*n - q))^p, x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q]
&& PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ
[p, -1] && LtQ[p, 0] && RationalQ[m, q] && EqQ[m + p*q + 1, -2*(n - q)*(p +
 1)]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.88

method	result	size
pseudoelliptic	$\frac{-bx\ln(2)+bx\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x\sqrt{a}}\right)-2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{2a^{\frac{3}{2}}x}$	68
default	$-\frac{\sqrt{c\,x^2+bx+a}\,\left(2a^{\frac{3}{2}}\sqrt{c\,x^2+bx+a}-b\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x}\right)xa\right)}{2\sqrt{c\,x^4+b\,x^3+a\,x^2}\,a^{\frac{5}{2}}}$	88
risch	$-\frac{c\,x^2+bx+a}{a\sqrt{x^2(c\,x^2+bx+a)}}+\frac{b\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x}\right)x\sqrt{c\,x^2+bx+a}}{2a^{\frac{3}{2}}\sqrt{x^2(c\,x^2+bx+a)}}$	97

input
```
int(1/x/(c*x^4+b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(-b*x*ln(2)+b*x*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x/a^(1/2))-
2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/a^(3/2)/x
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 194, normalized size of antiderivative = 2.52

$$\int \frac{1}{x\sqrt{ax^2+bx^3+cx^4}}\,dx = \left[\frac{\sqrt{a}bx^2\log\left(-\frac{8\,abx^2+(b^2+4\,ac)x^3+8\,a^2x+4\,\sqrt{cx^4+bx^3+ax^2}(bx+2\,a)\sqrt{a}}{x^3}\right)-4\,\sqrt{cx^4+bx^3+ax^2}a}{4\,a^2x^2}, -\frac{\sqrt{-a}bx^2\arctan\left(\frac{\sqrt{cx^4+bx^3+ax^2}(bx+2\,a)\sqrt{-a}}{2\,(acx^3+abx^2+a^2x)}\right)+2\,\sqrt{cx^4+bx^3+ax^2}a}{2\,a^2x^2}\right]$$

input
```
integrate(1/x/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(sqrt(a)*b*x^2*log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x + 4*sqrt
(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(a))/x^3) - 4*sqrt(c*x^4 + b*x^3 +
 a*x^2)*a)/(a^2*x^2), -1/2*(sqrt(-a)*b*x^2*arctan(1/2*sqrt(c*x^4 + b*x^3 +
 a*x^2)*(b*x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2 + a^2*x)) + 2*sqrt(c*x^4 +
 b*x^3 + a*x^2)*a)/(a^2*x^2)]
```

Sympy [F]

$$\int \frac{1}{x\sqrt{ax^2+bx^3+cx^4}}\,dx = \int \frac{1}{x\sqrt{x^2\,(a+bx+cx^2)}}\,dx$$

input
```
integrate(1/x/(c*x**4+b*x**3+a*x**2)**(1/2),x)
```

output
```
Integral(1/(x*sqrt(x**2*(a + b*x + c*x**2))), x)
```

Maxima [F]

$$\int \frac{1}{x\sqrt{ax^2+bx^3+cx^4}}\,dx = \int \frac{1}{\sqrt{cx^4+bx^3+ax^2}\,x}\,dx$$

input
```
integrate(1/x/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(c*x^4 + b*x^3 + a*x^2)*x), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt{ax^2+bx^3+cx^4}}\,dx = \text{Timed out}$$

input
```
integrate(1/x/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt{ax^2+bx^3+cx^4}}\,dx = \int \frac{1}{x\,\sqrt{c\,x^4+b\,x^3+a\,x^2}}\,dx$$

input
```
int(1/(x*(a*x^2 + b*x^3 + c*x^4)^(1/2)),x)
```

output
```
int(1/(x*(a*x^2 + b*x^3 + c*x^4)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.78

$$\int \frac{1}{x\sqrt{ax^2+bx^3+cx^4}}\,dx$$
$$= \frac{-2\sqrt{c\,x^2+bx+a}\,a+\sqrt{a}\,\log\left(-2\sqrt{a}\,\sqrt{c\,x^2+bx+a}-2a-bx\right)bx-\sqrt{a}\,\log(x)\,bx}{2a^2x}$$

input
```
int(1/x/(c*x^4+b*x^3+a*x^2)^(1/2),x)
```

output
```
( - 2*sqrt(a + b*x + c*x**2)*a + sqrt(a)*log( - 2*sqrt(a)*sqrt(a + b*x + c
*x**2) - 2*a - b*x)*b*x - sqrt(a)*log(x)*b*x)/(2*a**2*x)
```

3.71 $\int \frac{1}{x^2\sqrt{ax^2+bx^3+cx^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 119

$$\int \frac{1}{x^2\sqrt{ax^2+bx^3+cx^4}}\,dx = -\frac{\sqrt{ax^2+bx^3+cx^4}}{2ax^3}+\frac{3b\sqrt{ax^2+bx^3+cx^4}}{4a^2x^2} - \frac{(3b^2-4ac)\operatorname{arctanh}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{8a^{5/2}}$$

output
```
-1/2*(c*x^4+b*x^3+a*x^2)^(1/2)/a/x^3+3/4*b*(c*x^4+b*x^3+a*x^2)^(1/2)/a^2/x
^2-1/8*(-4*a*c+3*b^2)*arctanh(1/2*x*(b*x+2*a)/a^(1/2)/(c*x^4+b*x^3+a*x^2)^
(1/2))/a^(5/2)
```

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.93

$$\int \frac{1}{x^2\sqrt{ax^2+bx^3+cx^4}}\,dx = \frac{-\sqrt{a}(2a-3bx)(a+x(b+cx))+(3b^2-4ac)\,x^2\sqrt{a+x(b+cx)}\operatorname{arctanh}\left(\frac{\sqrt{c}x-\sqrt{a+x(b+cx)}}{\sqrt{a}}\right)}{4a^{5/2}x\sqrt{x^2(a+x(b+cx))}}$$

input
```
Integrate[1/(x^2*Sqrt[a*x^2 + b*x^3 + c*x^4]),x]
```

output
```
(-(Sqrt[a]*(2*a - 3*b*x)*(a + x*(b + c*x))) + (3*b^2 - 4*a*c)*x^2*Sqrt[a +
 x*(b + c*x)]*ArcTanh[(Sqrt[c]*x - Sqrt[a + x*(b + c*x)])/Sqrt[a]])/(4*a^(
5/2)*x*Sqrt[x^2*(a + x*(b + c*x))])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1976, 27, 1998, 27, 1951, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{ax^2+bx^3+cx^4}}dx$$

$$\Big\downarrow \text{1976}$$

$$\frac{\int -\frac{3b+2cx}{2x\sqrt{cx^4+bx^3+ax^2}}dx}{2a}-\frac{\sqrt{ax^2+bx^3+cx^4}}{2ax^3}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{3b+2cx}{x\sqrt{cx^4+bx^3+ax^2}}dx}{4a}-\frac{\sqrt{ax^2+bx^3+cx^4}}{2ax^3}$$

$$\Big\downarrow \text{1998}$$

$$-\frac{-\frac{\int \frac{3b^2-4ac}{2\sqrt{cx^4+bx^3+ax^2}}dx}{a}-\frac{3b\sqrt{ax^2+bx^3+cx^4}}{ax^2}}{4a}-\frac{\sqrt{ax^2+bx^3+cx^4}}{2ax^3}$$

$$\Big\downarrow \text{27}$$

$$-\frac{-\frac{(3b^2-4ac)\int \frac{1}{\sqrt{cx^4+bx^3+ax^2}}dx}{2a}-\frac{3b\sqrt{ax^2+bx^3+cx^4}}{ax^2}}{4a}-\frac{\sqrt{ax^2+bx^3+cx^4}}{2ax^3}$$

$$\Big\downarrow \text{1951}$$

$$-\frac{\frac{(3b^2-4ac)\int \frac{1}{4a-\frac{x^2(2a+bx)^2}{cx^4+bx^3+ax^2}}d\frac{x(2a+bx)}{\sqrt{cx^4+bx^3+ax^2}}}{a}-\frac{3b\sqrt{ax^2+bx^3+cx^4}}{ax^2}}{4a}-\frac{\sqrt{ax^2+bx^3+cx^4}}{2ax^3}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\frac{(3b^2-4ac)\operatorname{arctanh}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{2a^{3/2}}-\frac{3b\sqrt{ax^2+bx^3+cx^4}}{ax^2}}{4a}-\frac{\sqrt{ax^2+bx^3+cx^4}}{2ax^3}$$

input
```
Int[1/(x^2*Sqrt[a*x^2 + b*x^3 + c*x^4]),x]
```

output
```
-1/2*Sqrt[a*x^2 + b*x^3 + c*x^4]/(a*x^3) - ((-3*b*Sqrt[a*x^2 + b*x^3 + c*x
^4])/(a*x^2) + ((3*b^2 - 4*a*c)*ArcTanh[(x*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*
x^2 + b*x^3 + c*x^4])])/(2*a^(3/2)))/(4*a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1951
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :
> Simp[-2/(n - 2)   Subst[Int[1/(4*a - x^2), x], x, x*((2*a + b*x^(n - 2))/
Sqrt[a*x^2 + b*x^n + c*x^r])], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*
n - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]
```

rule 1976
```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[x^(m - q + 1)*((a*x^q + b*x^n + c*x^(2*n - q))^(p + 1)
/(a*(m + p*q + 1))), x] - Simp[1/(a*(m + p*q + 1))   Int[x^(m + n - q)*(b*(
m + p*q + (n - q)*(p + 1) + 1) + c*(m + p*q + 2*(n - q)*(p + 1) + 1)*x^(n -
 q))*(a*x^q + b*x^n + c*x^(2*n - q))^p, x], x] /; FreeQ[{a, b, c}, x] && Eq
Q[r, 2*n - q] && PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGt
Q[n, 0] && GeQ[p, -1] && LtQ[p, 0] && RationalQ[m, q] && LtQ[m + p*q + 1, 0
]
```

rule 1998
```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[A*x^(m - q + 1)*((a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1)/(a*(m + p*q + 1))), x] + Simp[1/(a*(m + p*q +
 1))   Int[x^(m + n - q)*Simp[a*B*(m + p*q + 1) - A*b*(m + p*q + (n - q)*(p
 + 1) + 1) - A*c*(m + p*q + 2*(n - q)*(p + 1) + 1)*x^(n - q), x]*(a*x^q + b
*x^n + c*x^(2*n - q))^p, x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n -
q] && EqQ[j, 2*n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
 && RationalQ[m, p, q] && ((GeQ[p, -1] && LtQ[p, 0]) || EqQ[m + p*q + (n -
q)*(2*p + 1) + 1, 0]) && LeQ[m + p*q, -(n - q)] && NeQ[m + p*q + 1, 0]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.98

method	result
risch	$-\frac{\left(c\,x^{2}+b x +a \right) \left(-3 b x +2 a \right)}{4 a^{2} x \sqrt{x^{2} \left(c\,x^{2}+b x +a \right)}}+\frac{\left(4 a c -3 b^{2}\right) \ln \left(\frac{2 a +b x +2 \sqrt{a}\, \sqrt{c\,x^{2}+b x +a}}{x}\right) x \sqrt{c\,x^{2}+b x +a}}{8 a^{\frac{5}{2}} \sqrt{x^{2} \left(c\,x^{2}+b x +a \right)}}$
pseudoelliptic	$\frac{4 \ln \left(\frac{2 a +b x +2 \sqrt{a}\, \sqrt{c\,x^{2}+b x +a}}{x \sqrt{a}}\right) a c\,x^{2}-3 \ln \left(\frac{2 a +b x +2 \sqrt{a}\, \sqrt{c\,x^{2}+b x +a}}{x \sqrt{a}}\right) b^{2} x^{2}-4 \ln \left(2\right) a c\,x^{2}+3 \ln \left(2\right) b^{2} x^{2}+6 b x \sqrt{a}\, \sqrt{c\,x^{2}+b x +c}}{8 a^{\frac{5}{2}} x^{2}}$
default	$-\frac{\sqrt{c\,x^{2}+b x +a}\, \left(-6 a^{\frac{3}{2}} \sqrt{c\,x^{2}+b x +a}\, b x -4 c \ln \left(\frac{2 a +b x +2 \sqrt{a}\, \sqrt{c\,x^{2}+b x +a}}{x}\right) a^{2} x^{2}+3 \ln \left(\frac{2 a +b x +2 \sqrt{a}\, \sqrt{c\,x^{2}+b x +a}}{x}\right) a\,b^{2} x^{2}+4 a\right.}{8 x \sqrt{c\,x^{4}+b\,x^{3}+a\,x^{2}}\, a^{\frac{7}{2}}}$

input
```
int(1/x^2/(c*x^4+b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(c*x^2+b*x+a)*(-3*b*x+2*a)/a^2/x/(x^2*(c*x^2+b*x+a))^(1/2)+1/8*(4*a*c
-3*b^2)/a^(5/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)*x/(x^2*(c*x^
2+b*x+a))^(1/2)*(c*x^2+b*x+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.95

$$\int \frac{1}{x^2\sqrt{ax^2+bx^3+cx^4}}\,dx$$
$$= \left[-\frac{(3\,b^2-4\,ac)\sqrt{a}x^3\log\left(-\frac{8\,abx^2+(b^2+4\,ac)x^3+8\,a^2x+4\,\sqrt{cx^4+bx^3+ax^2}(bx+2\,a)\sqrt{a}}{x^3}\right)-4\,\sqrt{cx^4+bx^3+ax^2}(3\,abx-}{16\,a^3x^3}\right.$$

input
```
integrate(1/x^2/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
[-1/16*((3*b^2 - 4*a*c)*sqrt(a)*x^3*log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 +
8*a^2*x + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(a))/x^3) - 4*sqrt
(c*x^4 + b*x^3 + a*x^2)*(3*a*b*x - 2*a^2))/(a^3*x^3), 1/8*((3*b^2 - 4*a*c)
*sqrt(-a)*x^3*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(-a)/
(a*c*x^3 + a*b*x^2 + a^2*x)) + 2*sqrt(c*x^4 + b*x^3 + a*x^2)*(3*a*b*x - 2*
a^2))/(a^3*x^3)]
```

Sympy [F]

$$\int \frac{1}{x^2\sqrt{ax^2+bx^3+cx^4}}\,dx = \int \frac{1}{x^2\sqrt{x^2\,(a+bx+cx^2)}}\,dx$$

input
```
integrate(1/x**2/(c*x**4+b*x**3+a*x**2)**(1/2),x)
```

output
```
Integral(1/(x**2*sqrt(x**2*(a + b*x + c*x**2))), x)
```

Maxima [F]

$$\int \frac{1}{x^2\sqrt{ax^2+bx^3+cx^4}}\,dx = \int \frac{1}{\sqrt{cx^4+bx^3+ax^2}x^2}\,dx$$

input
```
integrate(1/x^2/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(c*x^4 + b*x^3 + a*x^2)*x^2), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{x^2\sqrt{ax^2+bx^3+cx^4}}\,dx = \text{Timed out}$$

input
```
integrate(1/x^2/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\sqrt{ax^2+bx^3+cx^4}}\,dx = \int \frac{1}{x^2\,\sqrt{c\,x^4+b\,x^3+a\,x^2}}\,dx$$

input
```
int(1/(x^2*(a*x^2 + b*x^3 + c*x^4)^(1/2)),x)
```

output
```
int(1/(x^2*(a*x^2 + b*x^3 + c*x^4)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.10

$$\int \frac{1}{x^2\sqrt{ax^2+bx^3+cx^4}}\,dx$$
$$= \frac{-4\sqrt{c\,x^2+bx+a}\,a^2+6\sqrt{c\,x^2+bx+a}\,abx+4\sqrt{a}\log\left(-2\sqrt{a}\,\sqrt{c\,x^2+bx+a}-2a-bx\right)ac\,x^2-3\sqrt{a}\,\mathrm{l}}{8a^3x^2}$$

input
```
int(1/x^2/(c*x^4+b*x^3+a*x^2)^(1/2),x)
```

output
```
( - 4*sqrt(a + b*x + c*x**2)*a**2 + 6*sqrt(a + b*x + c*x**2)*a*b*x + 4*sqr
t(a)*log( - 2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a - b*x)*a*c*x**2 - 3*sqr
t(a)*log( - 2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a - b*x)*b**2*x**2 - 4*sq
rt(a)*log(x)*a*c*x**2 + 3*sqrt(a)*log(x)*b**2*x**2)/(8*a**3*x**2)
```

3.72 $\int \frac{x^7}{(ax^2+bx^3+cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 234

$$\int \frac{x^7}{(ax^2+bx^3+cx^4)^{3/2}}\,dx = \frac{2x^4(2a+bx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}} + \frac{(5b^2-12ac)\sqrt{ax^2+bx^3+cx^4}}{2c^2(b^2-4ac)} - \frac{b(15b^2-52ac)\sqrt{ax^2+bx^3+cx^4}}{4c^3(b^2-4ac)x} - \frac{2bx\sqrt{ax^2+bx^3+cx^4}}{c(b^2-4ac)} + \frac{3(5b^2-4ac)\,\mathrm{arctanh}\left(\frac{x(b+2cx)}{2\sqrt{c}\sqrt{ax^2+bx^3+cx^4}}\right)}{8c^{7/2}}$$

output
```
2*x^4*(b*x+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^3+a*x^2)^(1/2)+1/2*(-12*a*c+5*b^2)
*(c*x^4+b*x^3+a*x^2)^(1/2)/c^2/(-4*a*c+b^2)-1/4*b*(-52*a*c+15*b^2)*(c*x^4+
b*x^3+a*x^2)^(1/2)/c^3/(-4*a*c+b^2)/x-2*b*x*(c*x^4+b*x^3+a*x^2)^(1/2)/c/(-
4*a*c+b^2)+3/8*(-4*a*c+5*b^2)*arctanh(1/2*x*(2*c*x+b)/c^(1/2)/(c*x^4+b*x^3
+a*x^2)^(1/2))/c^(7/2)
```

Mathematica [A] (verified)

Time = 0.94 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.79

$$\int \frac{x^7}{(ax^2+bx^3+cx^4)^{3/2}}\,dx = \frac{x\Big(2\sqrt{c}(4a^2c(-13b+6cx)+b^2x(15b^2+5bcx-2c^2x^2)+a(15b^3-62b^2cx-2\cdots}{8c^{7/2}\cdots}$$

input
```
Integrate[x^7/(a*x^2 + b*x^3 + c*x^4)^(3/2),x]
```

output
```
(x*(2*Sqrt[c]*(4*a^2*c*(-13*b + 6*c*x) + b^2*x*(15*b^2 + 5*b*c*x - 2*c^2*x
^2) + a*(15*b^3 - 62*b^2*c*x - 20*b*c^2*x^2 + 8*c^3*x^3)) + 3*(5*b^4 - 24*
a*b^2*c + 16*a^2*c^2)*Sqrt[a + x*(b + c*x)]*Log[c^3*(b + 2*c*x - 2*Sqrt[c]
*Sqrt[a + x*(b + c*x)])]))/(8*c^(7/2)*(-b^2 + 4*a*c)*Sqrt[x^2*(a + x*(b +
c*x))])
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.15, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$, Rules used = {1970, 27, 1996, 27, 1996, 27, 1996, 27, 1961, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{(ax^2+bx^3+cx^4)^{3/2}}dx$$

$$\Big\downarrow \text{1970}$$

$$\frac{2x^4(2a+bx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}-\frac{2\int \frac{3x^3(2a+bx)}{\sqrt{cx^4+bx^3+ax^2}}dx}{b^2-4ac}$$

$$\Big\downarrow \text{27}$$

$$\frac{2x^4(2a+bx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}-\frac{6\int \frac{x^3(2a+bx)}{\sqrt{cx^4+bx^3+ax^2}}dx}{b^2-4ac}$$

$$\Big\downarrow \text{1996}$$

$$\frac{2x^4(2a+bx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}-\frac{6\left(\frac{bx\sqrt{ax^2+bx^3+cx^4}}{3c}-\frac{\int\frac{x^2\left(4ab+\left(5b^2-12ac\right)x\right)}{2\sqrt{cx^4+bx^3+ax^2}}dx}{3c}\right)}{b^2-4ac}$$

$\downarrow$ 27

$$\frac{2x^4(2a+bx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}-\frac{6\left(\frac{bx\sqrt{ax^2+bx^3+cx^4}}{3c}-\frac{\int\frac{x^2\left(4ab+\left(5b^2-12ac\right)x\right)}{\sqrt{cx^4+bx^3+ax^2}}dx}{6c}\right)}{b^2-4ac}$$

$\downarrow$ 1996

$$\frac{2x^4(2a+bx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}-\frac{6\left(\frac{bx\sqrt{ax^2+bx^3+cx^4}}{3c}-\frac{\frac{\left(5b^2-12ac\right)\sqrt{ax^2+bx^3+cx^4}}{2c}-\frac{\int\frac{x\left(2a\left(5b^2-12ac\right)+b\left(15b^2-52ac\right)x\right)}{2\sqrt{cx^4+bx^3+ax^2}}dx}{2c}}{6c}\right)}{b^2-4ac}$$

$\downarrow$ 27

$$\frac{2x^4(2a+bx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}-\frac{6\left(\frac{bx\sqrt{ax^2+bx^3+cx^4}}{3c}-\frac{\frac{\left(5b^2-12ac\right)\sqrt{ax^2+bx^3+cx^4}}{2c}-\frac{\int\frac{x\left(2a\left(5b^2-12ac\right)+b\left(15b^2-52ac\right)x\right)}{\sqrt{cx^4+bx^3+ax^2}}dx}{4c}}{6c}\right)}{b^2-4ac}$$

$\downarrow$ 1996

$$\frac{2x^4(2a+bx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}-\frac{6\left(\frac{bx\sqrt{ax^2+bx^3+cx^4}}{3c}-\frac{\frac{\left(5b^2-12ac\right)\sqrt{ax^2+bx^3+cx^4}}{2c}-\frac{\frac{b\left(15b^2-52ac\right)\sqrt{ax^2+bx^3+cx^4}}{cx}-\frac{\int\frac{3\left(b^2-4ac\right)\left(5b^2-4ac\right)x}{2\sqrt{cx^4+bx^3+ax^2}}dx}{c}}{4c}}{6c}\right)}{b^2-4ac}$$

$\downarrow$ 27

$$\frac{2x^4(2a+bx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}} - \frac{6\left(\frac{bx\sqrt{ax^2+bx^3+cx^4}}{3c} - \frac{\frac{(5b^2-12ac)\sqrt{ax^2+bx^3+cx^4}}{2c} - \frac{\frac{b(15b^2-52ac)\sqrt{ax^2+bx^3+cx^4}}{cx} - \frac{3(b^2-4ac)(5b^2-4ac)\int \frac{x}{\sqrt{cx^4+bx^3+ax^2}}dx}{2c}}{4c}}{6c}\right)}{b^2-4ac}$$

$\Big\downarrow$ 1961

$$\frac{2x^4(2a+bx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}} - \frac{6\left(\frac{bx\sqrt{ax^2+bx^3+cx^4}}{3c} - \frac{\frac{(5b^2-12ac)\sqrt{ax^2+bx^3+cx^4}}{2c} - \frac{\frac{b(15b^2-52ac)\sqrt{ax^2+bx^3+cx^4}}{cx} - \frac{3x(b^2-4ac)(5b^2-4ac)\sqrt{a+bx+cx^2}\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{2c\sqrt{ax^2+bx^3+cx^4}}}{4c}}{6c}\right)}{b^2-4ac}$$

$\Big\downarrow$ 1092

$$\frac{2x^4(2a+bx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}} - \frac{6\left(\frac{bx\sqrt{ax^2+bx^3+cx^4}}{3c} - \frac{\frac{(5b^2-12ac)\sqrt{ax^2+bx^3+cx^4}}{2c} - \frac{\frac{b(15b^2-52ac)\sqrt{ax^2+bx^3+cx^4}}{cx} - \frac{3x(b^2-4ac)(5b^2-4ac)\sqrt{a+bx+cx^2}\int \frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2c}{\sqrt{cx^2+}}}{c\sqrt{ax^2+bx^3+cx^4}}}{4c}}{6c}}{b^2-4ac}$$

$\Big\downarrow$ 219

$$\frac{2x^4(2a+bx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}} - \frac{6\left(\frac{bx\sqrt{ax^2+bx^3+cx^4}}{3c} - \frac{\frac{(5b^2-12ac)\sqrt{ax^2+bx^3+cx^4}}{2c} - \frac{\frac{b(15b^2-52ac)\sqrt{ax^2+bx^3+cx^4}}{cx} - \frac{3x(b^2-4ac)(5b^2-4ac)\sqrt{a+bx+cx^2}\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+c}}\right.}{2c^{3/2}\sqrt{ax^2+bx^3+cx^4}}}{4c}}{6c}}{b^2-4ac}$$

input
```
Int[x^7/(a*x^2 + b*x^3 + c*x^4)^(3/2),x]
```

output
```
(2*x^4*(2*a + b*x))/((b^2 - 4*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4]) - (6*((b*x
*Sqrt[a*x^2 + b*x^3 + c*x^4])/(3*c) - (((5*b^2 - 12*a*c)*Sqrt[a*x^2 + b*x^
3 + c*x^4])/(2*c) - ((b*(15*b^2 - 52*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(c*
x) - (3*(b^2 - 4*a*c)*(5*b^2 - 4*a*c)*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(b +
 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*c^(3/2)*Sqrt[a*x^2 + b*x^3
+ c*x^4]))/(4*c))/(6*c)))/(b^2 - 4*a*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1961
```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Simp[x^(q/2)*(Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]/Sqrt[a
*x^q + b*x^n + c*x^(2*n - q)])   Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x
^(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] &&
 PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] ||
 EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))
```

rule 1970
```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[(-x^(m - 2*n + q + 1))*(2*a + b*x^(n - q))*((a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1)/((n - q)*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1
/((n - q)*(p + 1)*(b^2 - 4*a*c))   Int[x^(m - 2*n + q)*(2*a*(m + p*q - 2*(n
 - q) + 1) + b*(m + p*q + (n - q)*(2*p + 1) + 1)*x^(n - q))*(a*x^q + b*x^n
+ c*x^(2*n - q))^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q]
&& PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ
[p, -1] && RationalQ[m, q] && GtQ[m + p*q + 1, 2*(n - q)]
```

rule 1996
```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[B*x^(m - n + 1)*((a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1)/(c*(m + p*q + (n - q)*(2*p + 1) + 1))), x] -
Simp[1/(c*(m + p*q + (n - q)*(2*p + 1) + 1))   Int[x^(m - n + q)*Simp[a*B*(
m + p*q - n + q + 1) + (b*B*(m + p*q + (n - q)*p + 1) - A*c*(m + p*q + (n -
 q)*(2*p + 1) + 1))*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^p, x], x]
 /; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] &&  !Inte
gerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[p, -1] && LtQ[p, 0] &&
RationalQ[m, q] && GeQ[m + p*q, n - q - 1] && NeQ[m + p*q + (n - q)*(2*p +
1) + 1, 0]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.70

method	result
pseudoelliptic	$-\frac{48\left(\left(-\frac{5}{24}b^3x^2+\frac{31}{12}xa\,b^2+\frac{13}{6}b\,a^2\right)c^{\frac{3}{2}}+\left(\frac{1}{12}b^2x^3+\frac{5}{6}ab\,x^2-x\,a^2\right)c^{\frac{5}{2}}-\frac{c^{\frac{7}{2}}a\,x^3}{3}-\frac{5b^3\sqrt{c}\,(bx+a)}{8}+\frac{\ln\left(2\sqrt{c\,x^2+bx+a}\,\sqrt{c}+2cx+b\right)}{}\right.}{c^{\frac{7}{2}}\sqrt{c\,x^2+bx+a}\,(32ac-8b^2)}$
default	$\frac{x^3(c\,x^2+bx+a)\left(16c^{\frac{9}{2}}a\,x^3-4c^{\frac{7}{2}}b^2x^3-40c^{\frac{7}{2}}ab\,x^2+48c^{\frac{7}{2}}a^2x+10c^{\frac{5}{2}}b^3x^2-124c^{\frac{5}{2}}a\,b^2x-104c^{\frac{5}{2}}a^2b+30c^{\frac{3}{2}}b^4x+30c^{\frac{3}{2}}a\,b^3-4\right.}{}$
risch	$-\frac{(-2cx+7b)(c\,x^2+bx+a)x}{4c^3\sqrt{x^2(c\,x^2+bx+a)}}-\frac{\left(3c(4ac-5b^2)\left(-\frac{x}{c\sqrt{c\,x^2+bx+a}}-\frac{b\left(-\frac{1}{c\sqrt{c\,x^2+bx+a}}-\frac{b(2cx+b)}{c\left(4ac-b^2\right)\sqrt{c\,x^2+bx+a}}\right)}{2c}+\frac{\ln\left(\frac{\frac{b}{2}+c}{\sqrt{c}}\right.}{}\right.\right.}{}$

input
```
int(x^7/(c*x^4+b*x^3+a*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-48/c^(7/2)*((-5/24*b^3*x^2+31/12*x*a*b^2+13/6*b*a^2)*c^(3/2)+(1/12*b^2*x^
3+5/6*a*b*x^2-x*a^2)*c^(5/2)-1/3*c^(7/2)*a*x^3-5/8*b^3*c^(1/2)*(b*x+a)+1/1
6*ln(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)*(c*x^2+b*x+a)^(1/2)*(16*a^2*c^
2-24*a*b^2*c+5*b^4))/(c*x^2+b*x+a)^(1/2)/(32*a*c-8*b^2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 616, normalized size of antiderivative = 2.63

$$\int \frac{x^7}{(ax^2+bx^3+cx^4)^{3/2}}\,dx = \Bigg[-\frac{3\,((5\,b^4c-24\,ab^2c^2+16\,a^2c^3)x^3+(5\,b^5-24\,ab^3c+16\,a^2bc^2)x^2+(5\,ab^4}{}$$

$$-\frac{3\,((5\,b^4c-24\,ab^2c^2+16\,a^2c^3)x^3+(5\,b^5-24\,ab^3c+16\,a^2bc^2)x^2+(5\,ab^4-24\,a^2b^2c+16\,a^3c^2)x)\sqrt{-c}\,\mathrm{arc}}{8\,((b^2c^5}$$

input
```
integrate(x^7/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

output
```
[-1/16*(3*((5*b^4*c - 24*a*b^2*c^2 + 16*a^2*c^3)*x^3 + (5*b^5 - 24*a*b^3*c
 + 16*a^2*b*c^2)*x^2 + (5*a*b^4 - 24*a^2*b^2*c + 16*a^3*c^2)*x)*sqrt(c)*lo
g(-(8*c^2*x^3 + 8*b*c*x^2 - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt
(c) + (b^2 + 4*a*c)*x)/x) + 4*(15*a*b^3*c - 52*a^2*b*c^2 - 2*(b^2*c^3 - 4*
a*c^4)*x^3 + 5*(b^3*c^2 - 4*a*b*c^3)*x^2 + (15*b^4*c - 62*a*b^2*c^2 + 24*a
^2*c^3)*x)*sqrt(c*x^4 + b*x^3 + a*x^2))/((b^2*c^5 - 4*a*c^6)*x^3 + (b^3*c^
4 - 4*a*b*c^5)*x^2 + (a*b^2*c^4 - 4*a^2*c^5)*x), -1/8*(3*((5*b^4*c - 24*a*
b^2*c^2 + 16*a^2*c^3)*x^3 + (5*b^5 - 24*a*b^3*c + 16*a^2*b*c^2)*x^2 + (5*a
*b^4 - 24*a^2*b^2*c + 16*a^3*c^2)*x)*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^
3 + a*x^2)*(2*c*x + b)*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x)) + 2*(15*a*b^3
*c - 52*a^2*b*c^2 - 2*(b^2*c^3 - 4*a*c^4)*x^3 + 5*(b^3*c^2 - 4*a*b*c^3)*x^
2 + (15*b^4*c - 62*a*b^2*c^2 + 24*a^2*c^3)*x)*sqrt(c*x^4 + b*x^3 + a*x^2))
/((b^2*c^5 - 4*a*c^6)*x^3 + (b^3*c^4 - 4*a*b*c^5)*x^2 + (a*b^2*c^4 - 4*a^2
*c^5)*x)]
```

Sympy [F]

$$\int \frac{x^7}{(ax^2+bx^3+cx^4)^{3/2}}\,dx = \int \frac{x^7}{\left(x^2\left(a+bx+cx^2\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**7/(c*x**4+b*x**3+a*x**2)**(3/2),x)
```

output
```
Integral(x**7/(x**2*(a + b*x + c*x**2))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^7}{(ax^2+bx^3+cx^4)^{3/2}}\,dx = \int \frac{x^7}{\left(cx^4+bx^3+ax^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^7/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^7/(c*x^4 + b*x^3 + a*x^2)^(3/2), x)
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 318, normalized size of antiderivative = 1.36

$$\int \frac{x^7}{(ax^2+bx^3+cx^4)^{3/2}}\,dx = \frac{\left(15\,b^4\log\left(\left|b-2\,\sqrt{a}\sqrt{c}\right|\right)-72\,ab^2c\log\left(\left|b-2\,\sqrt{a}\sqrt{c}\right|\right)+48\,a^2c^2\log\left(\left|b-2\right.\right.\right.}{8\left(b^2c^{\frac{7}{2}}-4\,ac^{\frac{9}{2}}\right)} + \frac{\left(\left(\frac{2\,(b^2c^2-4\,ac^3)x}{b^2c^3\mathrm{sgn}(x)-4\,ac^4\mathrm{sgn}(x)}-\frac{5\,(b^3c-4\,abc^2)}{b^2c^3\mathrm{sgn}(x)-4\,ac^4\mathrm{sgn}(x)}\right)x-\frac{15\,b^4-62\,ab^2c+24\,a^2c^2}{b^2c^3\mathrm{sgn}(x)-4\,ac^4\mathrm{sgn}(x)}\right)x-\frac{15\,ab^3-52\,a^2bc}{b^2c^3\mathrm{sgn}(x)-4\,ac^4\mathrm{sgn}(x)}}{4\,\sqrt{cx^2+bx+a}} - \frac{3\,(5\,b^2-4\,ac)\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)\sqrt{c}+b\right|\right)}{8\,c^{\frac{7}{2}}\mathrm{sgn}\left(x\right)}$$

input
```
integrate(x^7/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

output
```
1/8*(15*b^4*log(abs(b - 2*sqrt(a)*sqrt(c))) - 72*a*b^2*c*log(abs(b - 2*sqr
t(a)*sqrt(c))) + 48*a^2*c^2*log(abs(b - 2*sqrt(a)*sqrt(c))) + 30*sqrt(a)*b
^3*sqrt(c) - 104*a^(3/2)*b*c^(3/2))*sgn(x)/(b^2*c^(7/2) - 4*a*c^(9/2)) + 1
/4*(((2*(b^2*c^2 - 4*a*c^3)*x/(b^2*c^3*sgn(x) - 4*a*c^4*sgn(x)) - 5*(b^3*c
 - 4*a*b*c^2)/(b^2*c^3*sgn(x) - 4*a*c^4*sgn(x)))*x - (15*b^4 - 62*a*b^2*c
+ 24*a^2*c^2)/(b^2*c^3*sgn(x) - 4*a*c^4*sgn(x)))*x - (15*a*b^3 - 52*a^2*b*
c)/(b^2*c^3*sgn(x) - 4*a*c^4*sgn(x)))/sqrt(c*x^2 + b*x + a) - 3/8*(5*b^2 -
 4*a*c)*log(abs(2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) + b))/(c^(7/
2)*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^7}{(ax^2+bx^3+cx^4)^{3/2}}\,dx=\int \frac{x^7}{\left(c\,x^4+b\,x^3+a\,x^2\right)^{3/2}}\,dx$$

input
```
int(x^7/(a*x^2 + b*x^3 + c*x^4)^(3/2),x)
```

output
```
int(x^7/(a*x^2 + b*x^3 + c*x^4)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 751, normalized size of antiderivative = 3.21

$$\int \frac{x^7}{(ax^2+bx^3+cx^4)^{3/2}}\,dx=\frac{72\sqrt{c}\log\left(\frac{2\sqrt{c}\,\sqrt{c\,x^2+bx+a}+b+2cx}{\sqrt{4ac-b^2}}\right)a^2b^2c-48\sqrt{c}\log\left(\frac{2\sqrt{c}\,\sqrt{c\,x^2+bx+a}+b+2cx}{\sqrt{4ac-b^2}}\right)a^2c^3x}{}$$

input
```
int(x^7/(c*x^4+b*x^3+a*x^2)^(3/2),x)
```

output
```
( - 104*sqrt(a + b*x + c*x**2)*a**2*b*c**2 + 48*sqrt(a + b*x + c*x**2)*a**
2*c**3*x + 30*sqrt(a + b*x + c*x**2)*a*b**3*c - 124*sqrt(a + b*x + c*x**2)
*a*b**2*c**2*x - 40*sqrt(a + b*x + c*x**2)*a*b*c**3*x**2 + 16*sqrt(a + b*x
 + c*x**2)*a*c**4*x**3 + 30*sqrt(a + b*x + c*x**2)*b**4*c*x + 10*sqrt(a +
b*x + c*x**2)*b**3*c**2*x**2 - 4*sqrt(a + b*x + c*x**2)*b**2*c**3*x**3 - 4
8*sqrt(c)*log((2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)/sqrt(4*a*c -
b**2))*a**3*c**2 + 72*sqrt(c)*log((2*sqrt(c)*sqrt(a + b*x + c*x**2) + b +
2*c*x)/sqrt(4*a*c - b**2))*a**2*b**2*c - 48*sqrt(c)*log((2*sqrt(c)*sqrt(a
+ b*x + c*x**2) + b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*b*c**2*x - 48*sqrt(c
)*log((2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)/sqrt(4*a*c - b**2))*a
**2*c**3*x**2 - 15*sqrt(c)*log((2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c
*x)/sqrt(4*a*c - b**2))*a*b**4 + 72*sqrt(c)*log((2*sqrt(c)*sqrt(a + b*x +
c*x**2) + b + 2*c*x)/sqrt(4*a*c - b**2))*a*b**3*c*x + 72*sqrt(c)*log((2*sq
rt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)/sqrt(4*a*c - b**2))*a*b**2*c**2*
x**2 - 15*sqrt(c)*log((2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)/sqrt(
4*a*c - b**2))*b**5*x - 15*sqrt(c)*log((2*sqrt(c)*sqrt(a + b*x + c*x**2) +
 b + 2*c*x)/sqrt(4*a*c - b**2))*b**4*c*x**2 + 32*sqrt(c)*a**3*c**2 - 64*sq
rt(c)*a**2*b**2*c + 32*sqrt(c)*a**2*b*c**2*x + 32*sqrt(c)*a**2*c**3*x**2 +
 16*sqrt(c)*a*b**4 - 64*sqrt(c)*a*b**3*c*x - 64*sqrt(c)*a*b**2*c**2*x**2 +
 16*sqrt(c)*b**5*x + 16*sqrt(c)*b**4*c*x**2)/(8*c**4*(4*a**2*c - a*b**2...
```

3.73 $\int \frac{x^6}{\left(ax^2+bx^3+cx^4\right)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 173

$$\int \frac{x^6}{\left(ax^2+bx^3+cx^4\right)^{3/2}}\,dx = \frac{2x^3(2a+bx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}} - \frac{2b\sqrt{ax^2+bx^3+cx^4}}{c\,(b^2-4ac)} + \frac{(3b^2-8ac)\sqrt{ax^2+bx^3+cx^4}}{c^2\,(b^2-4ac)\,x} - \frac{3b\operatorname{arctanh}\left(\frac{x(b+2cx)}{2\sqrt{c}\sqrt{ax^2+bx^3+cx^4}}\right)}{2c^{5/2}}$$

output
```
2*x^3*(b*x+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^3+a*x^2)^(1/2)-2*b*(c*x^4+b*x^3+a*
x^2)^(1/2)/c/(-4*a*c+b^2)+(-8*a*c+3*b^2)*(c*x^4+b*x^3+a*x^2)^(1/2)/c^2/(-4
*a*c+b^2)/x-3/2*b*arctanh(1/2*x*(2*c*x+b)/c^(1/2)/(c*x^4+b*x^3+a*x^2)^(1/2
))/c^(5/2)
```

Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.83

$$\int \frac{x^6}{\left(ax^2+bx^3+cx^4\right)^{3/2}}\,dx = \frac{x\left(2\sqrt{c}(8a^2c-b^2x(3b+cx)+a(-3b^2+10bcx+4c^2x^2))-3b(b^2-4ac)\sqrt{a\right.}{2c^{5/2}\left(-b^2+4ac\right)\sqrt{x^2(a+x(}$$

input
```
Integrate[x^6/(a*x^2 + b*x^3 + c*x^4)^(3/2),x]
```

output
```
(x*(2*Sqrt[c]*(8*a^2*c - b^2*x*(3*b + c*x) + a*(-3*b^2 + 10*b*c*x + 4*c^2*
x^2)) - 3*b*(b^2 - 4*a*c)*Sqrt[a + x*(b + c*x)]*Log[c^2*(b + 2*c*x - 2*Sqr
t[c]*Sqrt[a + x*(b + c*x)])]))/(2*c^(5/2)*(-b^2 + 4*a*c)*Sqrt[x^2*(a + x*(
b + c*x))])
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.23, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1970, 27, 1996, 27, 1996, 27, 1961, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{(ax^2+bx^3+cx^4)^{3/2}}\,dx$$

$$\Big\downarrow \text{1970}$$

$$\frac{2x^3(2a+bx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}-\frac{2\int \frac{2x^2(2a+bx)}{\sqrt{cx^4+bx^3+ax^2}}dx}{b^2-4ac}$$

$$\Big\downarrow \text{27}$$

$$\frac{2x^3(2a+bx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}-\frac{4\int \frac{x^2(2a+bx)}{\sqrt{cx^4+bx^3+ax^2}}dx}{b^2-4ac}$$

$$\Big\downarrow \text{1996}$$

$$\frac{2x^3(2a+bx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}-\frac{4\left(\frac{b\sqrt{ax^2+bx^3+cx^4}}{2c}-\frac{\int \frac{x\left(2ab+\left(3b^2-8ac\right)x\right)}{2\sqrt{cx^4+bx^3+ax^2}}dx}{2c}\right)}{b^2-4ac}$$

$$\Big\downarrow \text{27}$$

$$\frac{2x^3(2a+bx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}-\frac{4\left(\frac{b\sqrt{ax^2+bx^3+cx^4}}{2c}-\frac{\int \frac{x\left(2ab+\left(3b^2-8ac\right)x\right)}{\sqrt{cx^4+bx^3+ax^2}}dx}{4c}\right)}{b^2-4ac}$$

$$\Big\downarrow \text{1996}$$

$$\frac{2x^3(2a+bx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}-\frac{4\left(\frac{b\sqrt{ax^2+bx^3+cx^4}}{2c}-\frac{\frac{(3b^2-8ac)\sqrt{ax^2+bx^3+cx^4}}{cx}-\frac{\int\frac{3b(b^2-4ac)x}{2\sqrt{cx^4+bx^3+ax^2}}dx}{c}}{4c}\right)}{b^2-4ac}$$

$\Big\downarrow$ 27

$$\frac{2x^3(2a+bx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}-\frac{4\left(\frac{b\sqrt{ax^2+bx^3+cx^4}}{2c}-\frac{\frac{(3b^2-8ac)\sqrt{ax^2+bx^3+cx^4}}{cx}-\frac{3b(b^2-4ac)\int\frac{x}{\sqrt{cx^4+bx^3+ax^2}}dx}{2c}}{4c}\right)}{b^2-4ac}$$

$\Big\downarrow$ 1961

$$\frac{2x^3(2a+bx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}-\frac{4\left(\frac{b\sqrt{ax^2+bx^3+cx^4}}{2c}-\frac{\frac{(3b^2-8ac)\sqrt{ax^2+bx^3+cx^4}}{cx}-\frac{3bx(b^2-4ac)\sqrt{a+bx+cx^2}\int\frac{1}{\sqrt{cx^2+bx+a}}dx}{2c\sqrt{ax^2+bx^3+cx^4}}}{4c}\right)}{b^2-4ac}$$

$\Big\downarrow$ 1092

$$\frac{2x^3(2a+bx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}-\frac{4\left(\frac{b\sqrt{ax^2+bx^3+cx^4}}{2c}-\frac{\frac{(3b^2-8ac)\sqrt{ax^2+bx^3+cx^4}}{cx}-\frac{3bx(b^2-4ac)\sqrt{a+bx+cx^2}\int\frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{c\sqrt{ax^2+bx^3+cx^4}}}{4c}\right)}{b^2-4ac}$$

$\Big\downarrow$ 219

$$\frac{2x^3(2a+bx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}-\frac{4\left(\frac{b\sqrt{ax^2+bx^3+cx^4}}{2c}-\frac{\frac{(3b^2-8ac)\sqrt{ax^2+bx^3+cx^4}}{cx}-\frac{3bx(b^2-4ac)\sqrt{a+bx+cx^2}\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2c^{3/2}\sqrt{ax^2+bx^3+cx^4}}}{4c}\right)}{b^2-4ac}$$

input
```
Int[x^6/(a*x^2 + b*x^3 + c*x^4)^(3/2),x]
```

output
```
(2*x^3*(2*a + b*x))/((b^2 - 4*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4]) - (4*((b*S
qrt[a*x^2 + b*x^3 + c*x^4])/(2*c) - (((3*b^2 - 8*a*c)*Sqrt[a*x^2 + b*x^3 +
 c*x^4])/(c*x) - (3*b*(b^2 - 4*a*c)*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(b + 2
*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*c^(3/2)*Sqrt[a*x^2 + b*x^3 +
c*x^4]))/(4*c)))/(b^2 - 4*a*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1961
```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Simp[x^(q/2)*(Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]/Sqrt[a
*x^q + b*x^n + c*x^(2*n - q)])   Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x
^(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] &&
 PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] ||
 EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))
```

rule 1970
```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[(-x^(m - 2*n + q + 1))*(2*a + b*x^(n - q))*((a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1)/((n - q)*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1
/((n - q)*(p + 1)*(b^2 - 4*a*c))   Int[x^(m - 2*n + q)*(2*a*(m + p*q - 2*(n
 - q) + 1) + b*(m + p*q + (n - q)*(2*p + 1) + 1)*x^(n - q))*(a*x^q + b*x^n
+ c*x^(2*n - q))^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q]
&& PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ
[p, -1] && RationalQ[m, q] && GtQ[m + p*q + 1, 2*(n - q)]
```

rule 1996
```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[B*x^(m - n + 1)*((a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1)/(c*(m + p*q + (n - q)*(2*p + 1) + 1))), x] -
Simp[1/(c*(m + p*q + (n - q)*(2*p + 1) + 1))   Int[x^(m - n + q)*Simp[a*B*(
m + p*q - n + q + 1) + (b*B*(m + p*q + (n - q)*p + 1) - A*c*(m + p*q + (n -
 q)*(2*p + 1) + 1))*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^p, x], x]
 /; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] &&  !Inte
gerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[p, -1] && LtQ[p, 0] &&
RationalQ[m, q] && GeQ[m + p*q, n - q - 1] && NeQ[m + p*q + (n - q)*(2*p +
1) + 1, 0]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.71

method	result
pseudoelliptic	$\frac{-\frac{3\sqrt{c\,x^2+bx+a}\,bc\left(ac-\frac{b^2}{4}\right)\ln\left(2\sqrt{c\,x^2+bx+a}\,\sqrt{c}+2cx+b\right)}{2}+c^{\frac{3}{2}}\left(a\,c^2x^2+\left(-\frac{1}{4}b^2x^2+\frac{5}{2}abx+2a^2\right)c-\frac{3b^2(bx+a)}{4}\right)}{\sqrt{c\,x^2+bx+a}\,c^{\frac{7}{2}}\left(ac-\frac{b^2}{4}\right)}$
default	$\frac{x^3\left(c\,x^2+bx+a\right)\left(8c^{\frac{7}{2}}\,a\,x^2-2c^{\frac{5}{2}}\,b^2x^2+20c^{\frac{5}{2}}\,abx-6c^{\frac{3}{2}}\,b^3x+16c^{\frac{5}{2}}\,a^2-6c^{\frac{3}{2}}\,a\,b^2-12\ln\left(\frac{2\sqrt{c\,x^2+bx+a}\,\sqrt{c}+2cx+b}{2\sqrt{c}}\right)\sqrt{c\,x^2+bx+a}}{2c^{\frac{7}{2}}\left(c\,x^4+b\,x^3+a\,x^2\right)^{\frac{3}{2}}\left(4ac-b^2\right)}$
risch	$\frac{\left(c\,x^2+bx+a\right)x}{c^2\sqrt{x^2\left(c\,x^2+bx+a\right)}}+\frac{\left(\frac{a}{c^2\sqrt{c\,x^2+bx+a}}-\frac{b^2}{4c^3\sqrt{c\,x^2+bx+a}}-\frac{b^3x}{2c^2\left(4ac-b^2\right)\sqrt{c\,x^2+bx+a}}-\frac{b^4}{4c^3\left(4ac-b^2\right)\sqrt{c\,x^2+bx+a}}+\frac{3b}{2c^2\sqrt{c\,x}}\right.}{\sqrt{x^2\left(c\,x^2+bx+a\right)}}$

input
```
int(x^6/(c*x^4+b*x^3+a*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/(c*x^2+b*x+a)^(1/2)*(-3/2*(c*x^2+b*x+a)^(1/2)*b*c*(a*c-1/4*b^2)*ln(2*(c*
x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)+c^(3/2)*(a*c^2*x^2+(-1/4*b^2*x^2+5/2*a*b
*x+2*a^2)*c-3/4*b^2*(b*x+a)))/c^(7/2)/(a*c-1/4*b^2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 486, normalized size of antiderivative = 2.81

$$\int \frac{x^6}{(ax^2+bx^3+cx^4)^{3/2}}\,dx = \left[\frac{3\left((b^3c-4\,abc^2)x^3+(b^4-4\,ab^2c)x^2+(ab^3-4\,a^2bc)x\right)\sqrt{c}\log\left(-\frac{8\,c^2x^3+8\,bcx}{\ldots}\right.}{4\left((b^2c^4-\ldots\right.}\right.$$

input
```
integrate(x^6/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/4*(3*((b^3*c - 4*a*b*c^2)*x^3 + (b^4 - 4*a*b^2*c)*x^2 + (a*b^3 - 4*a^2*
b*c)*x)*sqrt(c)*log(-(8*c^2*x^3 + 8*b*c*x^2 - 4*sqrt(c*x^4 + b*x^3 + a*x^2
)*(2*c*x + b)*sqrt(c) + (b^2 + 4*a*c)*x)/x) + 4*sqrt(c*x^4 + b*x^3 + a*x^2
)*(3*a*b^2*c - 8*a^2*c^2 + (b^2*c^2 - 4*a*c^3)*x^2 + (3*b^3*c - 10*a*b*c^2
)*x))/((b^2*c^4 - 4*a*c^5)*x^3 + (b^3*c^3 - 4*a*b*c^4)*x^2 + (a*b^2*c^3 -
4*a^2*c^4)*x), 1/2*(3*((b^3*c - 4*a*b*c^2)*x^3 + (b^4 - 4*a*b^2*c)*x^2 + (
a*b^3 - 4*a^2*b*c)*x)*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c
*x + b)*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x)) + 2*sqrt(c*x^4 + b*x^3 + a*x
^2)*(3*a*b^2*c - 8*a^2*c^2 + (b^2*c^2 - 4*a*c^3)*x^2 + (3*b^3*c - 10*a*b*c
^2)*x))/((b^2*c^4 - 4*a*c^5)*x^3 + (b^3*c^3 - 4*a*b*c^4)*x^2 + (a*b^2*c^3
- 4*a^2*c^4)*x)]
```

Sympy [F]

$$\int \frac{x^6}{(ax^2+bx^3+cx^4)^{3/2}}\,dx = \int \frac{x^6}{\left(x^2\left(a+bx+cx^2\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**6/(c*x**4+b*x**3+a*x**2)**(3/2),x)
```

output
```
Integral(x**6/(x**2*(a + b*x + c*x**2))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^6}{(ax^2+bx^3+cx^4)^{3/2}}\,dx = \int \frac{x^6}{(cx^4+bx^3+ax^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^6/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^6/(c*x^4 + b*x^3 + a*x^2)^(3/2), x)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.34

$$\int \frac{x^6}{(ax^2+bx^3+cx^4)^{3/2}}\,dx = -\frac{\left(3\,b^3\log\left(\left|b-2\,\sqrt{a}\sqrt{c}\right|\right)-12\,abc\log\left(\left|b-2\,\sqrt{a}\sqrt{c}\right|\right)+6\,\sqrt{a}b^2\sqrt{c}-16\,a^{\frac{3}{2}}c^{\frac{3}{2}}\right)\mathrm{sgn}(x)}{2\left(b^2c^{\frac{5}{2}}-4\,ac^{\frac{7}{2}}\right)} + \frac{\left(\frac{(b^2c-4\,ac^2)x}{b^2c^2\mathrm{sgn}(x)-4\,ac^3\mathrm{sgn}(x)}+\frac{3\,b^3-10\,abc}{b^2c^2\mathrm{sgn}(x)-4\,ac^3\mathrm{sgn}(x)}\right)x+\frac{3\,ab^2-8\,a^2c}{b^2c^2\mathrm{sgn}(x)-4\,ac^3\mathrm{sgn}(x)}}{\sqrt{cx^2+bx+a}} + \frac{3\,b\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)\sqrt{c}+b\right|\right)}{2\,c^{\frac{5}{2}}\mathrm{sgn}\,(x)}$$

input
```
integrate(x^6/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

output
```
-1/2*(3*b^3*log(abs(b - 2*sqrt(a)*sqrt(c))) - 12*a*b*c*log(abs(b - 2*sqrt(
a)*sqrt(c))) + 6*sqrt(a)*b^2*sqrt(c) - 16*a^(3/2)*c^(3/2))*sgn(x)/(b^2*c^(
5/2) - 4*a*c^(7/2)) + (((b^2*c - 4*a*c^2)*x/(b^2*c^2*sgn(x) - 4*a*c^3*sgn(
x)) + (3*b^3 - 10*a*b*c)/(b^2*c^2*sgn(x) - 4*a*c^3*sgn(x)))*x + (3*a*b^2 -
 8*a^2*c)/(b^2*c^2*sgn(x) - 4*a*c^3*sgn(x)))/sqrt(c*x^2 + b*x + a) + 3/2*b
*log(abs(2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) + b))/(c^(5/2)*sgn(
x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{(ax^2+bx^3+cx^4)^{3/2}}\,dx=\int \frac{x^6}{(c\,x^4+b\,x^3+a\,x^2)^{3/2}}\,dx$$

input
```
int(x^6/(a*x^2 + b*x^3 + c*x^4)^(3/2),x)
```

output
```
int(x^6/(a*x^2 + b*x^3 + c*x^4)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 499, normalized size of antiderivative = 2.88

$$\int \frac{x^6}{(ax^2+bx^3+cx^4)^{3/2}}\,dx=\frac{16\sqrt{c\,x^2+bx+a}\,a^2c^2-6\sqrt{c\,x^2+bx+a}\,a\,b^2c+20\sqrt{c\,x^2+bx+a}\,ab\,c^2x+8}{}$$

input
```
int(x^6/(c*x^4+b*x^3+a*x^2)^(3/2),x)
```

output
```
(16*sqrt(a + b*x + c*x**2)*a**2*c**2 - 6*sqrt(a + b*x + c*x**2)*a*b**2*c +
 20*sqrt(a + b*x + c*x**2)*a*b*c**2*x + 8*sqrt(a + b*x + c*x**2)*a*c**3*x*
*2 - 6*sqrt(a + b*x + c*x**2)*b**3*c*x - 2*sqrt(a + b*x + c*x**2)*b**2*c**
2*x**2 - 12*sqrt(c)*log((2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)/sqr
t(4*a*c - b**2))*a**2*b*c + 3*sqrt(c)*log((2*sqrt(c)*sqrt(a + b*x + c*x**2
) + b + 2*c*x)/sqrt(4*a*c - b**2))*a*b**3 - 12*sqrt(c)*log((2*sqrt(c)*sqrt
(a + b*x + c*x**2) + b + 2*c*x)/sqrt(4*a*c - b**2))*a*b**2*c*x - 12*sqrt(c
)*log((2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)/sqrt(4*a*c - b**2))*a
*b*c**2*x**2 + 3*sqrt(c)*log((2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x
)/sqrt(4*a*c - b**2))*b**4*x + 3*sqrt(c)*log((2*sqrt(c)*sqrt(a + b*x + c*x
**2) + b + 2*c*x)/sqrt(4*a*c - b**2))*b**3*c*x**2 + 12*sqrt(c)*a**2*b*c -
4*sqrt(c)*a*b**3 + 12*sqrt(c)*a*b**2*c*x + 12*sqrt(c)*a*b*c**2*x**2 - 4*sq
rt(c)*b**4*x - 4*sqrt(c)*b**3*c*x**2)/(2*c**3*(4*a**2*c - a*b**2 + 4*a*b*c
*x + 4*a*c**2*x**2 - b**3*x - b**2*c*x**2))
```

3.74 $\int \frac{x^5}{\left(ax^2+bx^3+cx^4\right)^{3/2}} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 125

$$\int \frac{x^5}{\left(ax^2+bx^3+cx^4\right)^{3/2}} \, dx = \frac{2x^2(2a+bx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}} - \frac{2b\sqrt{ax^2+bx^3+cx^4}}{c\,(b^2-4ac)\,x} + \frac{\operatorname{arctanh}\left(\frac{x(b+2cx)}{2\sqrt{c}\sqrt{ax^2+bx^3+cx^4}}\right)}{c^{3/2}}$$

output
```
2*x^2*(b*x+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^3+a*x^2)^(1/2)-2*b*(c*x^4+b*x^3+a*
x^2)^(1/2)/c/(-4*a*c+b^2)/x+arctanh(1/2*x*(2*c*x+b)/c^(1/2)/(c*x^4+b*x^3+a
*x^2)^(1/2))/c^(3/2)
```

Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.90

$$\int \frac{x^5}{\left(ax^2+bx^3+cx^4\right)^{3/2}} \, dx = -\frac{x\left(2\sqrt{c}(-ab-b^2x+2acx)+(b^2-4ac)\sqrt{a+x(b+cx)}\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)\right)}{c^{3/2}\,(-b^2+4ac)\sqrt{x^2(a+x(b+cx))}}$$

input
```
Integrate[x^5/(a*x^2 + b*x^3 + c*x^4)^(3/2),x]
```

output
```
-((x*(2*Sqrt[c]*(-(a*b) - b^2*x + 2*a*c*x) + (b^2 - 4*a*c)*Sqrt[a + x*(b +
 c*x)]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]))/(c^(3/2)*(
-b^2 + 4*a*c)*Sqrt[x^2*(a + x*(b + c*x))]))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.33, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1970, 1996, 27, 1961, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(ax^2+bx^3+cx^4)^{3/2}}dx$$

$$\Big\downarrow \text{1970}$$

$$\frac{2x^2(2a+bx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}-\frac{2\int \frac{x(2a+bx)}{\sqrt{cx^4+bx^3+ax^2}}dx}{b^2-4ac}$$

$$\Big\downarrow \text{1996}$$

$$\frac{2x^2(2a+bx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}-\frac{2\left(\frac{b\sqrt{ax^2+bx^3+cx^4}}{cx}-\frac{\int \frac{(b^2-4ac)x}{2\sqrt{cx^4+bx^3+ax^2}}dx}{c}\right)}{b^2-4ac}$$

$$\Big\downarrow \text{27}$$

$$\frac{2x^2(2a+bx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}-\frac{2\left(\frac{b\sqrt{ax^2+bx^3+cx^4}}{cx}-\frac{(b^2-4ac)\int \frac{x}{\sqrt{cx^4+bx^3+ax^2}}dx}{2c}\right)}{b^2-4ac}$$

$$\Big\downarrow \text{1961}$$

$$\frac{2x^2(2a+bx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}-\frac{2\left(\frac{b\sqrt{ax^2+bx^3+cx^4}}{cx}-\frac{x(b^2-4ac)\sqrt{a+bx+cx^2}\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{2c\sqrt{ax^2+bx^3+cx^4}}\right)}{b^2-4ac}$$

$$\Big\downarrow \text{1092}$$

$$\frac{2x^2(2a+bx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}-\frac{2\left(\frac{b\sqrt{ax^2+bx^3+cx^4}}{cx}-\frac{x(b^2-4ac)\sqrt{a+bx+cx^2}\int\frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{c\sqrt{ax^2+bx^3+cx^4}}\right)}{b^2-4ac}$$

$$\Big\downarrow \text{219}$$

$$\frac{2x^2(2a+bx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}-\frac{2\left(\frac{b\sqrt{ax^2+bx^3+cx^4}}{cx}-\frac{x(b^2-4ac)\sqrt{a+bx+cx^2}\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2c^{3/2}\sqrt{ax^2+bx^3+cx^4}}\right)}{b^2-4ac}$$

input
```
Int[x^5/(a*x^2 + b*x^3 + c*x^4)^(3/2),x]
```

output
```
(2*x^2*(2*a + b*x))/((b^2 - 4*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4]) - (2*((b*S
qrt[a*x^2 + b*x^3 + c*x^4])/(c*x) - ((b^2 - 4*a*c)*x*Sqrt[a + b*x + c*x^2]
*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*c^(3/2)*Sqrt[a
*x^2 + b*x^3 + c*x^4])))/(b^2 - 4*a*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1961
```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Simp[x^(q/2)*(Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]/Sqrt[a
*x^q + b*x^n + c*x^(2*n - q)])   Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x
^(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] &&
 PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] ||
 EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))
```

rule 1970
```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[(-x^(m - 2*n + q + 1))*(2*a + b*x^(n - q))*((a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1)/((n - q)*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1
/((n - q)*(p + 1)*(b^2 - 4*a*c))   Int[x^(m - 2*n + q)*(2*a*(m + p*q - 2*(n
 - q) + 1) + b*(m + p*q + (n - q)*(2*p + 1) + 1)*x^(n - q))*(a*x^q + b*x^n
+ c*x^(2*n - q))^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q]
&& PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ
[p, -1] && RationalQ[m, q] && GtQ[m + p*q + 1, 2*(n - q)]
```

rule 1996
```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[B*x^(m - n + 1)*((a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1)/(c*(m + p*q + (n - q)*(2*p + 1) + 1))), x] -
Simp[1/(c*(m + p*q + (n - q)*(2*p + 1) + 1))   Int[x^(m - n + q)*Simp[a*B*(
m + p*q - n + q + 1) + (b*B*(m + p*q + (n - q)*p + 1) - A*c*(m + p*q + (n -
 q)*(2*p + 1) + 1))*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^p, x], x]
 /; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] &&  !Inte
gerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[p, -1] && LtQ[p, 0] &&
RationalQ[m, q] && GeQ[m + p*q, n - q - 1] && NeQ[m + p*q + (n - q)*(2*p +
1) + 1, 0]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.67

method	result
pseudoelliptic	$-\frac{x}{c\sqrt{c\,x^{2}+bx+a}}+\frac{b(bx+2a)}{\sqrt{c\,x^{2}+bx+a}\,c(4ac-b^{2})}+\frac{\ln\left(2\sqrt{c\,x^{2}+bx+a}\,\sqrt{c}+2cx+b\right)}{c^{\frac{3}{2}}}$
default	$-\frac{x^{3}\left(c\,x^{2}+bx+a\right)\left(4c^{\frac{5}{2}}\,ax-2c^{\frac{3}{2}}\,b^{2}x-2c^{\frac{3}{2}}\,ab-4\ln\left(\frac{2\sqrt{c\,x^{2}+bx+a}\,\sqrt{c}+2cx+b}{2\sqrt{c}}\right)\sqrt{c\,x^{2}+bx+a}\,a\,c^{2}+\ln\left(\frac{2\sqrt{c\,x^{2}+bx+a}\,\sqrt{c}+2cx+}{2\sqrt{c}}\right)\right)}{c^{\frac{5}{2}}\left(c\,x^{4}+b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}\left(4ac-b^{2}\right)}$

input
```
int(x^5/(c*x^4+b*x^3+a*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-x/c/(c*x^2+b*x+a)^(1/2)+b*(b*x+2*a)/(c*x^2+b*x+a)^(1/2)/c/(4*a*c-b^2)+1/c
^(3/2)*ln(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 414, normalized size of antiderivative = 3.31

$$\int \frac{x^5}{(ax^2+bx^3+cx^4)^{3/2}}\,dx = \left[\frac{((b^2c-4\,ac^2)x^3+(b^3-4\,abc)x^2+(ab^2-4\,a^2c)x)\sqrt{c}\log\left(-\frac{8\,c^2x^3+8\,bcx^2+4\,\sqrt{}}{}\right.}{2\left((b^2c^3-4\,ac^4)x^3+(b^3c^2-4\right.}\right.$$
$$-\frac{((b^2c-4\,ac^2)x^3+(b^3-4\,abc)x^2+(ab^2-4\,a^2c)x)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^4+bx^3+ax^2}(2\,cx+b)\sqrt{-c}}{2\,(c^2x^3+bcx^2+acx)}\right)+2\,\sqrt{cx^4+bx^3}}{(b^2c^3-4\,ac^4)x^3+(b^3c^2-4\,abc^3)x^2+(ab^2c^2-4\,a^2c^3)x}$$

input
```
integrate(x^5/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/2*(((b^2*c - 4*a*c^2)*x^3 + (b^3 - 4*a*b*c)*x^2 + (a*b^2 - 4*a^2*c)*x)*
sqrt(c)*log(-(8*c^2*x^3 + 8*b*c*x^2 + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x
 + b)*sqrt(c) + (b^2 + 4*a*c)*x)/x) - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(a*b*c
 + (b^2*c - 2*a*c^2)*x))/((b^2*c^3 - 4*a*c^4)*x^3 + (b^3*c^2 - 4*a*b*c^3)*
x^2 + (a*b^2*c^2 - 4*a^2*c^3)*x), -(((b^2*c - 4*a*c^2)*x^3 + (b^3 - 4*a*b*
c)*x^2 + (a*b^2 - 4*a^2*c)*x)*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x
^2)*(2*c*x + b)*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x)) + 2*sqrt(c*x^4 + b*x
^3 + a*x^2)*(a*b*c + (b^2*c - 2*a*c^2)*x))/((b^2*c^3 - 4*a*c^4)*x^3 + (b^3
*c^2 - 4*a*b*c^3)*x^2 + (a*b^2*c^2 - 4*a^2*c^3)*x)]
```

Sympy [F]

$$\int \frac{x^5}{(ax^2+bx^3+cx^4)^{3/2}}\,dx = \int \frac{x^5}{(x^2(a+bx+cx^2))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**5/(c*x**4+b*x**3+a*x**2)**(3/2),x)
```

output
```
Integral(x**5/(x**2*(a + b*x + c*x**2))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^5}{(ax^2+bx^3+cx^4)^{3/2}}\,dx = \int \frac{x^5}{(cx^4+bx^3+ax^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^5/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^5/(c*x^4 + b*x^3 + a*x^2)^(3/2), x)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.31

$$\int \frac{x^5}{(ax^2+bx^3+cx^4)^{3/2}}\,dx = \frac{\left(b^2\log\left(\left|b-2\sqrt{a}\sqrt{c}\right|\right)-4\,ac\log\left(\left|b-2\sqrt{a}\sqrt{c}\right|\right)+2\sqrt{a}b\sqrt{c}\right)\mathrm{sgn}(x)}{b^2c^{\frac{3}{2}}-4\,ac^{\frac{5}{2}}} - \frac{2\left(\frac{ab}{b^2c\,\mathrm{sgn}(x)-4\,ac^2\mathrm{sgn}(x)}+\frac{(b^2-2\,ac)x}{b^2c\,\mathrm{sgn}(x)-4\,ac^2\mathrm{sgn}(x)}\right)}{\sqrt{cx^2+bx+a}} - \frac{\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)\sqrt{c}+b\right|\right)}{c^{\frac{3}{2}}\mathrm{sgn}\left(x\right)}$$

input
```
integrate(x^5/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

output
```
(b^2*log(abs(b - 2*sqrt(a)*sqrt(c))) - 4*a*c*log(abs(b - 2*sqrt(a)*sqrt(c)
)) + 2*sqrt(a)*b*sqrt(c))*sgn(x)/(b^2*c^(3/2) - 4*a*c^(5/2)) - 2*(a*b/(b^2
*c*sgn(x) - 4*a*c^2*sgn(x)) + (b^2 - 2*a*c)*x/(b^2*c*sgn(x) - 4*a*c^2*sgn(
x)))/sqrt(c*x^2 + b*x + a) - log(abs(2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))
*sqrt(c) + b))/(c^(3/2)*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{(ax^2+bx^3+cx^4)^{3/2}}\,dx=\int \frac{x^5}{{\left(c\,x^4+b\,x^3+a\,x^2\right)}^{3/2}}\,dx$$

input
```
int(x^5/(a*x^2 + b*x^3 + c*x^4)^(3/2),x)
```

output
```
int(x^5/(a*x^2 + b*x^3 + c*x^4)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 426, normalized size of antiderivative = 3.41

$$\int \frac{x^5}{(ax^2+bx^3+cx^4)^{3/2}}\,dx=\frac{2\sqrt{c\,x^2+bx+a}\,abc-4\sqrt{c\,x^2+bx+a}\,a\,c^2x+2\sqrt{c\,x^2+bx+a}\,b^2cx+4\sqrt{c}\,\mathrm{l}}{}$$

input
```
int(x^5/(c*x^4+b*x^3+a*x^2)^(3/2),x)
```

output
```
(2*sqrt(a + b*x + c*x**2)*a*b*c - 4*sqrt(a + b*x + c*x**2)*a*c**2*x + 2*sq
rt(a + b*x + c*x**2)*b**2*c*x + 4*sqrt(c)*log((2*sqrt(c)*sqrt(a + b*x + c*
x**2) + b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*c - sqrt(c)*log((2*sqrt(c)*sqr
t(a + b*x + c*x**2) + b + 2*c*x)/sqrt(4*a*c - b**2))*a*b**2 + 4*sqrt(c)*lo
g((2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)/sqrt(4*a*c - b**2))*a*b*c
*x + 4*sqrt(c)*log((2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)/sqrt(4*a
*c - b**2))*a*c**2*x**2 - sqrt(c)*log((2*sqrt(c)*sqrt(a + b*x + c*x**2) +
b + 2*c*x)/sqrt(4*a*c - b**2))*b**3*x - sqrt(c)*log((2*sqrt(c)*sqrt(a + b*
x + c*x**2) + b + 2*c*x)/sqrt(4*a*c - b**2))*b**2*c*x**2 - 4*sqrt(c)*a**2*
c + 2*sqrt(c)*a*b**2 - 4*sqrt(c)*a*b*c*x - 4*sqrt(c)*a*c**2*x**2 + 2*sqrt(
c)*b**3*x + 2*sqrt(c)*b**2*c*x**2)/(c**2*(4*a**2*c - a*b**2 + 4*a*b*c*x +
4*a*c**2*x**2 - b**3*x - b**2*c*x**2))
```

3.75 $\int \frac{x^4}{\left(ax^2+bx^3+cx^4\right)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 40

$$\int \frac{x^4}{(ax^2+bx^3+cx^4)^{3/2}}\,dx = \frac{2x(2a+bx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}$$

output
```
2*x*(b*x+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^3+a*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.92

$$\int \frac{x^4}{(ax^2+bx^3+cx^4)^{3/2}}\,dx = \frac{2x(2a+bx)}{(b^2-4ac)\sqrt{x^2(a+x(b+cx))}}$$

input
```
Integrate[x^4/(a*x^2 + b*x^3 + c*x^4)^(3/2),x]
```

output
```
(2*x*(2*a + b*x))/((b^2 - 4*a*c)*Sqrt[x^2*(a + x*(b + c*x))])
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {1963}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(ax^2+bx^3+cx^4)^{3/2}}\,dx$$

$$\Big\downarrow \; 1963$$

$$\frac{2x(2a+bx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}$$

input
```
Int[x^4/(a*x^2 + b*x^3 + c*x^4)^(3/2),x]
```

output
```
(2*x*(2*a + b*x))/((b^2 - 4*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])
```

Defintions of rubi rules used

rule 1963
```
Int[(x_)^(m_.)/((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(3/
2), x_Symbol] :> Simp[x^((n - 1)/2)*((4*a + 2*b*x)/((b^2 - 4*a*c)*Sqrt[a*x^
(n - 1) + b*x^n + c*x^(n + 1)])), x] /; FreeQ[{a, b, c, n}, x] && EqQ[m, (3
*n - 1)/2] && EqQ[q, n - 1] && EqQ[r, n + 1] && NeQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85

method	result	size
pseudoelliptic	$\frac{-2bx-4a}{\sqrt{c\,x^{2}+bx+a}\,(4ac-b^{2})}$	34
gosper	$-\frac{2\left(c\,x^{2}+bx+a\right)\left(bx+2a\right)x^{3}}{\left(4ac-b^{2}\right)\left(c\,x^{4}+b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}}$	53
default	$-\frac{2\left(c\,x^{2}+bx+a\right)\left(bx+2a\right)x^{3}}{\left(4ac-b^{2}\right)\left(c\,x^{4}+b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}}$	53
orering	$-\frac{2\left(c\,x^{2}+bx+a\right)\left(bx+2a\right)x^{3}}{\left(4ac-b^{2}\right)\left(c\,x^{4}+b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}}$	53
trager	$-\frac{2(bx+2a)\sqrt{c\,x^{4}+b\,x^{3}+a\,x^{2}}}{(c\,x^{2}+bx+a)x(4ac-b^{2})}$	55

input
```
int(x^4/(c*x^4+b*x^3+a*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(-2*b*x-4*a)/(c*x^2+b*x+a)^(1/2)/(4*a*c-b^2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.82

$$\int \frac{x^4}{\left(ax^2+bx^3+cx^4\right)^{3/2}}\,dx = \frac{2\,\sqrt{cx^4+bx^3+ax^2}(bx+2\,a)}{(b^2c-4\,ac^2)x^3+(b^3-4\,abc)x^2+(ab^2-4\,a^2c)x}$$

input
```
integrate(x^4/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

output
```
2*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)/((b^2*c - 4*a*c^2)*x^3 + (b^3 -
4*a*b*c)*x^2 + (a*b^2 - 4*a^2*c)*x)
```

Sympy [F]

$$\int \frac{x^4}{(ax^2+bx^3+cx^4)^{3/2}}\,dx = \int \frac{x^4}{(x^2(a+bx+cx^2))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**4/(c*x**4+b*x**3+a*x**2)**(3/2),x)
```

output
```
Integral(x**4/(x**2*(a + b*x + c*x**2))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^4}{(ax^2+bx^3+cx^4)^{3/2}}\,dx = \int \frac{x^4}{(cx^4+bx^3+ax^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^4/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^4/(c*x^4 + b*x^3 + a*x^2)^(3/2), x)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.72

$$\int \frac{x^4}{(ax^2+bx^3+cx^4)^{3/2}}\,dx = \frac{2\left(\frac{bx}{b^2\mathrm{sgn}(x)-4\,ac\mathrm{sgn}(x)}+\frac{2\,a}{b^2\mathrm{sgn}(x)-4\,ac\mathrm{sgn}(x)}\right)}{\sqrt{cx^2+bx+a}} - \frac{4\,\sqrt{a}\mathrm{sgn}(x)}{b^2-4\,ac}$$

input
```
integrate(x^4/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

output
```
2*(b*x/(b^2*sgn(x) - 4*a*c*sgn(x)) + 2*a/(b^2*sgn(x) - 4*a*c*sgn(x)))/sqrt
(c*x^2 + b*x + a) - 4*sqrt(a)*sgn(x)/(b^2 - 4*a*c)
```

Mupad [B] (verification not implemented)

Time = 12.08 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.88

$$\int \frac{x^4}{(ax^2+bx^3+cx^4)^{3/2}}\,dx = -\frac{\left(\frac{4\,a\,c}{4\,a\,c^2-b^2\,c}+\frac{2\,b\,c\,x}{4\,a\,c^2-b^2\,c}\right)\,\sqrt{c\,x^4+b\,x^3+a\,x^2}}{x\,\left(c\,x^2+b\,x+a\right)}$$

input
```
int(x^4/(a*x^2 + b*x^3 + c*x^4)^(3/2),x)
```

output
```
-(((4*a*c)/(4*a*c^2 - b^2*c) + (2*b*c*x)/(4*a*c^2 - b^2*c))*(a*x^2 + b*x^3
 + c*x^4)^(1/2))/(x*(a + b*x + c*x^2))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 105, normalized size of antiderivative = 2.62

$$\int \frac{x^4}{(ax^2+bx^3+cx^4)^{3/2}}\,dx = \frac{-4\sqrt{c\,x^2+bx+a}\,ac-2\sqrt{c\,x^2+bx+a}\,bcx-2\sqrt{c}\,ab-2\sqrt{c}\,b^2x-2\sqrt{c}\,bc\,x^2}{c\,(4a\,c^2x^2-b^2c\,x^2+4abcx-b^3x+4a^2c-a\,b^2)}$$

input
```
int(x^4/(c*x^4+b*x^3+a*x^2)^(3/2),x)
```

output
```
(2*( - 2*sqrt(a + b*x + c*x**2)*a*c - sqrt(a + b*x + c*x**2)*b*c*x - sqrt(
c)*a*b - sqrt(c)*b**2*x - sqrt(c)*b*c*x**2))/(c*(4*a**2*c - a*b**2 + 4*a*b
*c*x + 4*a*c**2*x**2 - b**3*x - b**2*c*x**2))
```

3.76 $\int \frac{x^3}{(ax^2+bx^3+cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 39

$$\int \frac{x^3}{(ax^2+bx^3+cx^4)^{3/2}}\,dx = -\frac{2x(b+2cx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}$$

output
```
-2*x*(2*c*x+b)/(-4*a*c+b^2)/(c*x^4+b*x^3+a*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.92

$$\int \frac{x^3}{(ax^2+bx^3+cx^4)^{3/2}}\,dx = -\frac{2x(b+2cx)}{(b^2-4ac)\sqrt{x^2(a+x(b+cx))}}$$

input
```
Integrate[x^3/(a*x^2 + b*x^3 + c*x^4)^(3/2),x]
```

output
```
(-2*x*(b + 2*c*x))/((b^2 - 4*a*c)*Sqrt[x^2*(a + x*(b + c*x))])
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = $\{1962\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(ax^2+bx^3+cx^4)^{3/2}}\,dx$$

$$\Big\downarrow \; 1962$$

$$-\frac{2x(b+2cx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}$$

input
```
Int[x^3/(a*x^2 + b*x^3 + c*x^4)^(3/2),x]
```

output
```
(-2*x*(b + 2*c*x))/((b^2 - 4*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])
```

Defintions of rubi rules used

rule 1962
```
Int[(x_)^(m_.)/((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(3/
2), x_Symbol] :> Simp[-2*x^((n - 1)/2)*((b + 2*c*x)/((b^2 - 4*a*c)*Sqrt[a*x
^(n - 1) + b*x^n + c*x^(n + 1)])), x] /; FreeQ[{a, b, c, n}, x] && EqQ[m, 3
*((n - 1)/2)] && EqQ[q, n - 1] && EqQ[r, n + 1] && NeQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.85

method	result	size
pseudoelliptic	$\frac{4cx+2b}{\sqrt{c\,x^2+bx+a}\,(4ac-b^2)}$	33
gosper	$\frac{2(c\,x^2+bx+a)(2cx+b)x^3}{(4ac-b^2)(c\,x^4+b\,x^3+a\,x^2)^{\frac{3}{2}}}$	52
default	$\frac{2(c\,x^2+bx+a)(2cx+b)x^3}{(4ac-b^2)(c\,x^4+b\,x^3+a\,x^2)^{\frac{3}{2}}}$	52
orering	$\frac{2(c\,x^2+bx+a)(2cx+b)x^3}{(4ac-b^2)(c\,x^4+b\,x^3+a\,x^2)^{\frac{3}{2}}}$	52
trager	$\frac{2(2cx+b)\sqrt{c\,x^4+b\,x^3+a\,x^2}}{(c\,x^2+bx+a)x(4ac-b^2)}$	54

input
```
int(x^3/(c*x^4+b*x^3+a*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.85

$$\int \frac{x^3}{(ax^2+bx^3+cx^4)^{3/2}}\,dx = -\frac{2\sqrt{cx^4+bx^3+ax^2}(2\,cx+b)}{(b^2c-4\,ac^2)x^3+(b^3-4\,abc)x^2+(ab^2-4\,a^2c)x}$$

input
```
integrate(x^3/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

output
```
-2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)/((b^2*c - 4*a*c^2)*x^3 + (b^3 -
 4*a*b*c)*x^2 + (a*b^2 - 4*a^2*c)*x)
```

Sympy [F]

$$\int \frac{x^3}{(ax^2+bx^3+cx^4)^{3/2}}\,dx = \int \frac{x^3}{(x^2(a+bx+cx^2))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**3/(c*x**4+b*x**3+a*x**2)**(3/2),x)
```

output
```
Integral(x**3/(x**2*(a + b*x + c*x**2))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^3}{(ax^2+bx^3+cx^4)^{3/2}}\,dx = \int \frac{x^3}{(cx^4+bx^3+ax^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^3/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^3/(c*x^4 + b*x^3 + a*x^2)^(3/2), x)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.90

$$\int \frac{x^3}{(ax^2+bx^3+cx^4)^{3/2}}\,dx = \frac{2\sqrt{a}b\mathrm{sgn}(x)}{ab^2-4\,a^2c} - \frac{2\left(\frac{2\,cx}{b^2\mathrm{sgn}(x)-4\,ac\mathrm{sgn}(x)}+\frac{b}{b^2\mathrm{sgn}(x)-4\,ac\mathrm{sgn}(x)}\right)}{\sqrt{cx^2+bx+a}}$$

input
```
integrate(x^3/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

output
```
2*sqrt(a)*b*sgn(x)/(a*b^2 - 4*a^2*c) - 2*(2*c*x/(b^2*sgn(x) - 4*a*c*sgn(x)
) + b/(b^2*sgn(x) - 4*a*c*sgn(x)))/sqrt(c*x^2 + b*x + a)
```

Mupad [B] (verification not implemented)

Time = 12.77 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.92

$$\int \frac{x^3}{(ax^2+bx^3+cx^4)^{3/2}}\,dx = \frac{\left(\frac{4\,c^2\,x}{4\,a\,c^2-b^2\,c}+\frac{2\,b\,c}{4\,a\,c^2-b^2\,c}\right)\,\sqrt{c\,x^4+b\,x^3+a\,x^2}}{x\,\left(c\,x^2+b\,x+a\right)}$$

input
```
int(x^3/(a*x^2 + b*x^3 + c*x^4)^(3/2),x)
```

output
```
(((4*c^2*x)/(4*a*c^2 - b^2*c) + (2*b*c)/(4*a*c^2 - b^2*c))*(a*x^2 + b*x^3
+ c*x^4)^(1/2))/(x*(a + b*x + c*x^2))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.44

$$\int \frac{x^3}{(ax^2+bx^3+cx^4)^{3/2}}\,dx = \frac{2\sqrt{c\,x^2+bx+a}\,b+4\sqrt{c\,x^2+bx+a}\,cx+4\sqrt{c}\,a+4\sqrt{c}\,bx+4\sqrt{c}\,c\,x^2}{4a\,c^2x^2-b^2c\,x^2+4abcx-b^3x+4a^2c-a\,b^2}$$

input
```
int(x^3/(c*x^4+b*x^3+a*x^2)^(3/2),x)
```

output
```
(2*(sqrt(a + b*x + c*x**2)*b + 2*sqrt(a + b*x + c*x**2)*c*x + 2*sqrt(c)*a
+ 2*sqrt(c)*b*x + 2*sqrt(c)*c*x**2))/(4*a**2*c - a*b**2 + 4*a*b*c*x + 4*a*
c**2*x**2 - b**3*x - b**2*c*x**2)
```

3.77 $\int \frac{x^2}{(ax^2+bx^3+cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 94

$$\int \frac{x^2}{(ax^2+bx^3+cx^4)^{3/2}}\,dx = \frac{2x(b^2-2ac+bcx)}{a(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}} - \frac{\operatorname{arctanh}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{a^{3/2}}$$

output
```
2*x*(b*c*x-2*a*c+b^2)/a/(-4*a*c+b^2)/(c*x^4+b*x^3+a*x^2)^(1/2)-arctanh(1/2
*x*(b*x+2*a)/a^(1/2)/(c*x^4+b*x^3+a*x^2)^(1/2))/a^(3/2)
```

Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.15

$$\int \frac{x^2}{(ax^2+bx^3+cx^4)^{3/2}}\,dx = -\frac{2x\left(\sqrt{a}(b^2-2ac+bcx)+(b^2-4ac)\sqrt{a+x(b+cx)}\operatorname{arctanh}\left(\frac{\sqrt{c}x-\sqrt{a+x(b+cx)}}{\sqrt{a}}\right)\right)}{a^{3/2}(-b^2+4ac)\sqrt{x^2(a+x(b+cx))}}$$

input
```
Integrate[x^2/(a*x^2 + b*x^3 + c*x^4)^(3/2),x]
```

output
```
(-2*x*(Sqrt[a]*(b^2 - 2*a*c + b*c*x) + (b^2 - 4*a*c)*Sqrt[a + x*(b + c*x)]
*ArcTanh[(Sqrt[c]*x - Sqrt[a + x*(b + c*x)])/Sqrt[a]]))/(a^(3/2)*(-b^2 + 4
*a*c)*Sqrt[x^2*(a + x*(b + c*x))])
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1969, 1951, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(ax^2+bx^3+cx^4)^{3/2}}dx$$

$$\Big\downarrow \text{1969}$$

$$\frac{\int \frac{1}{\sqrt{cx^4+bx^3+ax^2}}dx}{a}+\frac{2x(-2ac+b^2+bcx)}{a(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}$$

$$\Big\downarrow \text{1951}$$

$$\frac{2x(-2ac+b^2+bcx)}{a(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}-\frac{2\int \frac{1}{4a-\frac{x^2(2a+bx)^2}{cx^4+bx^3+ax^2}}d\frac{x(2a+bx)}{\sqrt{cx^4+bx^3+ax^2}}}{a}$$

$$\Big\downarrow \text{219}$$

$$\frac{2x(-2ac+b^2+bcx)}{a(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}-\frac{\text{arctanh}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{a^{3/2}}$$

input
```
Int[x^2/(a*x^2 + b*x^3 + c*x^4)^(3/2),x]
```

output
```
(2*x*(b^2 - 2*a*c + b*c*x))/(a*(b^2 - 4*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])
- ArcTanh[(x*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x^2 + b*x^3 + c*x^4])]/a^(3/2)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1951
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :
> Simp[-2/(n - 2)   Subst[Int[1/(4*a - x^2), x], x, x*((2*a + b*x^(n - 2))/
Sqrt[a*x^2 + b*x^n + c*x^r])], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*
n - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]
```

rule 1969
```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[(-x^(m - q + 1))*(b^2 - 2*a*c + b*c*x^(n - q))*((a*x^q
 + b*x^n + c*x^(2*n - q))^(p + 1)/(a*(n - q)*(p + 1)*(b^2 - 4*a*c))), x] +
Simp[(2*a*c - b^2*(p + 2))/(a*(p + 1)*(b^2 - 4*a*c))   Int[x^(m - q)*(a*x^q
 + b*x^n + c*x^(2*n - q))^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && EqQ[r,
2*n - q] && PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n,
0] && LtQ[p, -1] && RationalQ[m, p, q] && EqQ[m + p*q + 1, (-(n - q))*(2*p
+ 3)]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.14

method	result
pseudoelliptic	$-\frac{4\left(-a^{\frac{3}{2}} c+\frac{b\sqrt{a}\,(c x+b)}{2}+\sqrt{c\,x^{2}+b x+a}\,\left(-\ln (2)+\ln \left(\frac{2 a+b x+2\sqrt{a}\,\sqrt{c\,x^{2}+b x+a}}{x\sqrt{a}}\right)\right)\left(a c-\frac{b^{2}}{4}\right)\right)}{a^{\frac{3}{2}}\sqrt{c\,x^{2}+b x+a}\,(4 a c-b^{2})}$
default	$\frac{x^{3}(c\,x^{2}+b x+a)\,(-2 a^{\frac{3}{2}} b c x+4 a^{\frac{5}{2}} c-2 a^{\frac{3}{2}} b^{2}-4\ln \left(\frac{2 a+b x+2\sqrt{a}\,\sqrt{c\,x^{2}+b x+a}}{x}\right)\sqrt{c\,x^{2}+b x+a}\,a^{2} c+\ln (\frac{2 a+b x+2\sqrt{a}\,\sqrt{c\,x^{2}+b x+a}}{x}}{(c\,x^{4}+b\,x^{3}+a\,x^{2})^{\frac{3}{2}} a^{\frac{5}{2}}(4 a c-b^{2})}$

input
```
int(x^2/(c*x^4+b*x^3+a*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-4/a^(3/2)*(-a^(3/2)*c+1/2*b*a^(1/2)*(c*x+b)+(c*x^2+b*x+a)^(1/2)*(-ln(2)+l
n((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x/a^(1/2)))*(a*c-1/4*b^2))/(c*x^
2+b*x+a)^(1/2)/(4*a*c-b^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 201 vs. 2(84) = 168.

Time = 0.12 (sec) , antiderivative size = 411, normalized size of antiderivative = 4.37

$$\int \frac{x^2}{(ax^2+bx^3+cx^4)^{3/2}}\,dx = \left[\frac{((b^2c-4\,ac^2)x^3+(b^3-4\,abc)x^2+(ab^2-4\,a^2c)x)\sqrt{a}\log\left(-\frac{8\,abx^2+(b^2+4\,ac)x}{}\right.}{2\,((a^2b^2c-4\,a^3c^2)x^3+(a^2b^3-}\right.$$

input
```
integrate(x^2/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/2*(((b^2*c - 4*a*c^2)*x^3 + (b^3 - 4*a*b*c)*x^2 + (a*b^2 - 4*a^2*c)*x)*
sqrt(a)*log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x - 4*sqrt(c*x^4 + b*x
^3 + a*x^2)*(b*x + 2*a)*sqrt(a))/x^3) + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(a*b
*c*x + a*b^2 - 2*a^2*c))/((a^2*b^2*c - 4*a^3*c^2)*x^3 + (a^2*b^3 - 4*a^3*b
*c)*x^2 + (a^3*b^2 - 4*a^4*c)*x), (((b^2*c - 4*a*c^2)*x^3 + (b^3 - 4*a*b*c
)*x^2 + (a*b^2 - 4*a^2*c)*x)*sqrt(-a)*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^
2)*(b*x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2 + a^2*x)) + 2*sqrt(c*x^4 + b*x^
3 + a*x^2)*(a*b*c*x + a*b^2 - 2*a^2*c))/((a^2*b^2*c - 4*a^3*c^2)*x^3 + (a^
2*b^3 - 4*a^3*b*c)*x^2 + (a^3*b^2 - 4*a^4*c)*x)]
```

Sympy [F]

$$\int \frac{x^2}{(ax^2+bx^3+cx^4)^{3/2}}\,dx = \int \frac{x^2}{\left(x^2\left(a+bx+cx^2\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**2/(c*x**4+b*x**3+a*x**2)**(3/2),x)
```

output
```
Integral(x**2/(x**2*(a + b*x + c*x**2))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^2}{(ax^2+bx^3+cx^4)^{3/2}}\,dx = \int \frac{x^2}{(cx^4+bx^3+ax^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^2/(c*x^4 + b*x^3 + a*x^2)^(3/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 199 vs. $2(84) = 168$.

Time = 0.13 (sec) , antiderivative size = 199, normalized size of antiderivative = 2.12

$$\int \frac{x^2}{(ax^2+bx^3+cx^4)^{3/2}}\,dx = -\frac{2\left(ab^2\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)-4\,a^2c\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)+\sqrt{-a}\sqrt{a}b^2-2\,\sqrt{-a}a^{\frac{3}{2}}c\right)\mathrm{sgn}(x)}{\sqrt{-a}a^2b^2-4\,\sqrt{-a}a^3c} + \frac{2\left(\frac{abcx\mathrm{sgn}(x)}{a^2b^2-4\,a^3c}+\frac{ab^2\mathrm{sgn}(x)-2\,a^2c\mathrm{sgn}(x)}{a^2b^2-4\,a^3c}\right)}{\sqrt{cx^2+bx+a}} + \frac{2\,\arctan\left(-\frac{\sqrt{c}x-\sqrt{cx^2+bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a\mathrm{sgn}\,(x)}$$

input
```
integrate(x^2/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

output
```
-2*(a*b^2*arctan(sqrt(a)/sqrt(-a)) - 4*a^2*c*arctan(sqrt(a)/sqrt(-a)) + sq
rt(-a)*sqrt(a)*b^2 - 2*sqrt(-a)*a^(3/2)*c)*sgn(x)/(sqrt(-a)*a^2*b^2 - 4*sq
rt(-a)*a^3*c) + 2*(a*b*c*x*sgn(x)/(a^2*b^2 - 4*a^3*c) + (a*b^2*sgn(x) - 2*
a^2*c*sgn(x))/(a^2*b^2 - 4*a^3*c))/sqrt(c*x^2 + b*x + a) + 2*arctan(-(sqrt
(c)*x - sqrt(c*x^2 + b*x + a))/sqrt(-a))/(sqrt(-a)*a*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(ax^2+bx^3+cx^4)^{3/2}}\,dx=\int \frac{x^2}{(c\,x^4+b\,x^3+a\,x^2)^{3/2}}\,dx$$

input
```
int(x^2/(a*x^2 + b*x^3 + c*x^4)^(3/2),x)
```

output
```
int(x^2/(a*x^2 + b*x^3 + c*x^4)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.89 (sec) , antiderivative size = 2247, normalized size of antiderivative = 23.90

$$\int \frac{x^2}{(ax^2+bx^3+cx^4)^{3/2}}\,dx=\text{Too large to display}$$

input
```
int(x^2/(c*x^4+b*x^3+a*x^2)^(3/2),x)
```

output
```
( - 2*sqrt(a)*sqrt(4*sqrt(c)*sqrt(a)*b - 4*a*c - b**2)*atan((2*sqrt(c)*sqr
t(a + b*x + c*x**2) + b + 2*c*x)/sqrt(4*sqrt(c)*sqrt(a)*b - 4*a*c - b**2))
*a*b - 2*sqrt(a)*sqrt(4*sqrt(c)*sqrt(a)*b - 4*a*c - b**2)*atan((2*sqrt(c)*
sqrt(a + b*x + c*x**2) + b + 2*c*x)/sqrt(4*sqrt(c)*sqrt(a)*b - 4*a*c - b**
2))*b**2*x - 2*sqrt(a)*sqrt(4*sqrt(c)*sqrt(a)*b - 4*a*c - b**2)*atan((2*sq
rt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)/sqrt(4*sqrt(c)*sqrt(a)*b - 4*a*c
 - b**2))*b*c*x**2 - 4*sqrt(c)*sqrt(4*sqrt(c)*sqrt(a)*b - 4*a*c - b**2)*at
an((2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)/sqrt(4*sqrt(c)*sqrt(a)*b
 - 4*a*c - b**2))*a**2 - 4*sqrt(c)*sqrt(4*sqrt(c)*sqrt(a)*b - 4*a*c - b**2
)*atan((2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)/sqrt(4*sqrt(c)*sqrt(
a)*b - 4*a*c - b**2))*a*b*x - 4*sqrt(c)*sqrt(4*sqrt(c)*sqrt(a)*b - 4*a*c -
 b**2)*atan((2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)/sqrt(4*sqrt(c)*
sqrt(a)*b - 4*a*c - b**2))*a*c*x**2 - sqrt(a)*sqrt(4*sqrt(c)*sqrt(a)*b + 4
*a*c + b**2)*log( - sqrt(4*sqrt(c)*sqrt(a)*b + 4*a*c + b**2) + 2*sqrt(c)*s
qrt(a + b*x + c*x**2) + b + 2*c*x)*a*b - sqrt(a)*sqrt(4*sqrt(c)*sqrt(a)*b
+ 4*a*c + b**2)*log( - sqrt(4*sqrt(c)*sqrt(a)*b + 4*a*c + b**2) + 2*sqrt(c
)*sqrt(a + b*x + c*x**2) + b + 2*c*x)*b**2*x - sqrt(a)*sqrt(4*sqrt(c)*sqrt
(a)*b + 4*a*c + b**2)*log( - sqrt(4*sqrt(c)*sqrt(a)*b + 4*a*c + b**2) + 2*
sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)*b*c*x**2 + sqrt(a)*sqrt(4*sqrt
(c)*sqrt(a)*b + 4*a*c + b**2)*log(sqrt(4*sqrt(c)*sqrt(a)*b + 4*a*c + b*...
```

3.78 $\int \frac{x}{(ax^2+bx^3+cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 144

$$\int \frac{x}{(ax^2+bx^3+cx^4)^{3/2}}\,dx = \frac{2(b^2-2ac+bcx)}{a(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}} - \frac{(3b^2-8ac)\sqrt{ax^2+bx^3+cx^4}}{a^2(b^2-4ac)x^2} + \frac{3b\operatorname{arctanh}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{2a^{5/2}}$$

output
```
2*(b*c*x-2*a*c+b^2)/a/(-4*a*c+b^2)/(c*x^4+b*x^3+a*x^2)^(1/2)-(-8*a*c+3*b^2
)*(c*x^4+b*x^3+a*x^2)^(1/2)/a^2/(-4*a*c+b^2)/x^2+3/2*b*arctanh(1/2*x*(b*x+
2*a)/a^(1/2)/(c*x^4+b*x^3+a*x^2)^(1/2))/a^(5/2)
```

Mathematica [A] (verified)

Time = 0.86 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.93

$$\int \frac{x}{(ax^2+bx^3+cx^4)^{3/2}}\,dx = \frac{\sqrt{a}(-4a^2c+3b^2x(b+cx)+a(b^2-10bcx-8c^2x^2))+3b(b^2-4ac)\,x\sqrt{a+x}}{a^{5/2}(-b^2+4ac)\sqrt{x^2(a+x(b+cx))}}$$

input
```
Integrate[x/(a*x^2 + b*x^3 + c*x^4)^(3/2),x]
```

output
```
(Sqrt[a]*(-4*a^2*c + 3*b^2*x*(b + c*x) + a*(b^2 - 10*b*c*x - 8*c^2*x^2)) +
 3*b*(b^2 - 4*a*c)*x*Sqrt[a + x*(b + c*x)]*ArcTanh[(Sqrt[c]*x - Sqrt[a + x
*(b + c*x)])/Sqrt[a]])/(a^(5/2)*(-b^2 + 4*a*c)*Sqrt[x^2*(a + x*(b + c*x))]
)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {1971, 27, 1998, 27, 1951, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(ax^2+bx^3+cx^4)^{3/2}}dx$$

$$\Big\downarrow \text{1971}$$

$$\frac{2(-2ac+b^2+bcx)}{a(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}-\frac{2\int -\frac{3b^2+2cxb-8ac}{2x\sqrt{cx^4+bx^3+ax^2}}dx}{a(b^2-4ac)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{3b^2+2cxb-8ac}{x\sqrt{cx^4+bx^3+ax^2}}dx}{a(b^2-4ac)}+\frac{2(-2ac+b^2+bcx)}{a(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}$$

$$\Big\downarrow \text{1998}$$

$$\frac{-\frac{\int \frac{3b(b^2-4ac)}{2\sqrt{cx^4+bx^3+ax^2}}dx}{a}-\frac{(3b^2-8ac)\sqrt{ax^2+bx^3+cx^4}}{ax^2}}{a(b^2-4ac)}+\frac{2(-2ac+b^2+bcx)}{a(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{-\frac{3b(b^2-4ac)\int \frac{1}{\sqrt{cx^4+bx^3+ax^2}}dx}{2a}-\frac{(3b^2-8ac)\sqrt{ax^2+bx^3+cx^4}}{ax^2}}{a(b^2-4ac)}+\frac{2(-2ac+b^2+bcx)}{a(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}$$

$$\Big\downarrow \text{1951}$$

$$\frac{\frac{3b(b^2-4ac)\int \frac{1}{4a-\frac{x^2(2a+bx)^2}{cx^4+bx^3+ax^2}}d\frac{x(2a+bx)}{\sqrt{cx^4+bx^3+ax^2}}}{a}-\frac{(3b^2-8ac)\sqrt{ax^2+bx^3+cx^4}}{ax^2}}{a\left(b^2-4ac\right)}+\frac{2\left(-2ac+b^2+bcx\right)}{a\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{3b(b^2-4ac)\operatorname{arctanh}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{2a^{3/2}}-\frac{(3b^2-8ac)\sqrt{ax^2+bx^3+cx^4}}{ax^2}}{a\left(b^2-4ac\right)}+\frac{2\left(-2ac+b^2+bcx\right)}{a\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}}$$

input
```
Int[x/(a*x^2 + b*x^3 + c*x^4)^(3/2),x]
```

output
```
(2*(b^2 - 2*a*c + b*c*x))/(a*(b^2 - 4*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4]) +
(-(((3*b^2 - 8*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(a*x^2)) + (3*b*(b^2 - 4*
a*c)*ArcTanh[(x*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x^2 + b*x^3 + c*x^4])])/(2*
a^(3/2)))/(a*(b^2 - 4*a*c))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1951
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :
> Simp[-2/(n - 2)   Subst[Int[1/(4*a - x^2), x], x, x*((2*a + b*x^(n - 2))/
Sqrt[a*x^2 + b*x^n + c*x^r])], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*
n - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]
```

rule 1971
```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[(-x^(m - q + 1))*(b^2 - 2*a*c + b*c*x^(n - q))*((a*x^q
 + b*x^n + c*x^(2*n - q))^(p + 1)/(a*(n - q)*(p + 1)*(b^2 - 4*a*c))), x] +
Simp[1/(a*(n - q)*(p + 1)*(b^2 - 4*a*c))   Int[x^(m - q)*(b^2*(m + p*q + (n
 - q)*(p + 1) + 1) - 2*a*c*(m + p*q + 2*(n - q)*(p + 1) + 1) + b*c*(m + p*q
 + (n - q)*(2*p + 3) + 1)*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^(p + 1
), x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] &&  !Int
egerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[p, -1] && RationalQ[m,
 q] && LtQ[m + p*q + 1, n - q]
```

rule 1998
```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[A*x^(m - q + 1)*((a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1)/(a*(m + p*q + 1))), x] + Simp[1/(a*(m + p*q +
 1))   Int[x^(m + n - q)*Simp[a*B*(m + p*q + 1) - A*b*(m + p*q + (n - q)*(p
 + 1) + 1) - A*c*(m + p*q + 2*(n - q)*(p + 1) + 1)*x^(n - q), x]*(a*x^q + b
*x^n + c*x^(2*n - q))^p, x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n -
q] && EqQ[j, 2*n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
 && RationalQ[m, p, q] && ((GeQ[p, -1] && LtQ[p, 0]) || EqQ[m + p*q + (n -
q)*(2*p + 1) + 1, 0]) && LeQ[m + p*q, -(n - q)] && NeQ[m + p*q + 1, 0]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.95

method	result
pseudoelliptic	$\frac{\frac{\left(-4c^2x^2-5bcx+\frac{1}{2}b^2\right)a^{\frac{3}{2}}}{2}-a^{\frac{5}{2}}c+\frac{3\left(\frac{b\sqrt{a}\,(cx+b)}{2}+\sqrt{c\,x^2+bx+a}\,\left(-\ln(2)+\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x\sqrt{a}}\right)\right)\left(ac-\frac{b^2}{4}\right)\right)bx}{2}}{a^{\frac{5}{2}}\sqrt{c\,x^2+bx+a}\,x\left(ac-\frac{b^2}{4}\right)}$
default	$-\frac{x^2\left(c\,x^2+bx+a\right)\left(16a^{\frac{5}{2}}c^2x^2-6a^{\frac{3}{2}}b^2c\,x^2+20a^{\frac{5}{2}}bcx-6a^{\frac{3}{2}}b^3x-12\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x}\right)\sqrt{c\,x^2+bx+a}\,a^2bcx+3\ln\right.}{2(c\,x^4+b\,x^3+a\,x^2)^{\frac{3}{2}}a^{\frac{7}{2}}\left(4ac-b^2\right)}$
risch	$-\frac{c\,x^2+bx+a}{a^2\sqrt{x^2(c\,x^2+bx+a)}}+\frac{\left(\frac{2b^2cx}{a^2\left(4ac-b^2\right)\sqrt{c\,x^2+bx+a}}+\frac{b^3}{a^2\left(4ac-b^2\right)\sqrt{c\,x^2+bx+a}}-\frac{b}{a^2\sqrt{c\,x^2+bx+a}}-\frac{4c^2x}{a\left(4ac-b^2\right)\sqrt{c\,x^2+bx+a}}-\right.}{\sqrt{x^2(c\,x^2+bx+a)}}$

input
```
int(x/(c*x^4+b*x^3+a*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
3/2/a^(5/2)*(1/3*(-4*c^2*x^2-5*b*c*x+1/2*b^2)*a^(3/2)-2/3*a^(5/2)*c+(1/2*b
*a^(1/2)*(c*x+b)+(c*x^2+b*x+a)^(1/2)*(-ln(2)+ln((2*a+b*x+2*a^(1/2)*(c*x^2+
b*x+a)^(1/2))/x/a^(1/2)))*(a*c-1/4*b^2))*b*x)/(c*x^2+b*x+a)^(1/2)/x/(a*c-1
/4*b^2)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 496, normalized size of antiderivative = 3.44

$$\int \frac{x}{\left(ax^2+bx^3+cx^4\right)^{3/2}}\,dx = \left[\frac{3\left(\left(b^3c-4\,abc^2\right)x^4+\left(b^4-4\,ab^2c\right)x^3+\left(ab^3-4\,a^2bc\right)x^2\right)\sqrt{a}\log\left(-\frac{8\,abx^2+(b^2 \cdots}{\cdots}\right) \cdots}{4\left(\left(a^3b^2c - \cdots\right)\right.}\right.$$

$$-\frac{3\left(\left(b^3c-4\,abc^2\right)x^4+\left(b^4-4\,ab^2c\right)x^3+\left(ab^3-4\,a^2bc\right)x^2\right)\sqrt{-a}\arctan\left(\frac{\sqrt{cx^4+bx^3+ax^2}\,(bx+2\,a)\sqrt{-a}}{2\,(acx^3+abx^2+a^2x)}\right)+2\sqrt{cx^4 \cdots}}{2\left(\left(a^3b^2c-4\,a^4c^2\right)x^4+\left(a^3b^3-4\,a^4bc\right)x^3+\left(a^4b^2 \cdots\right)\right.}$$

input
```
integrate(x/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/4*(3*((b^3*c - 4*a*b*c^2)*x^4 + (b^4 - 4*a*b^2*c)*x^3 + (a*b^3 - 4*a^2*
b*c)*x^2)*sqrt(a)*log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x + 4*sqrt(c
*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(a))/x^3) - 4*sqrt(c*x^4 + b*x^3 + a
*x^2)*(a^2*b^2 - 4*a^3*c + (3*a*b^2*c - 8*a^2*c^2)*x^2 + (3*a*b^3 - 10*a^2
*b*c)*x))/((a^3*b^2*c - 4*a^4*c^2)*x^4 + (a^3*b^3 - 4*a^4*b*c)*x^3 + (a^4*
b^2 - 4*a^5*c)*x^2), -1/2*(3*((b^3*c - 4*a*b*c^2)*x^4 + (b^4 - 4*a*b^2*c)*
x^3 + (a*b^3 - 4*a^2*b*c)*x^2)*sqrt(-a)*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*
x^2)*(b*x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2 + a^2*x)) + 2*sqrt(c*x^4 + b*
x^3 + a*x^2)*(a^2*b^2 - 4*a^3*c + (3*a*b^2*c - 8*a^2*c^2)*x^2 + (3*a*b^3 -
 10*a^2*b*c)*x))/((a^3*b^2*c - 4*a^4*c^2)*x^4 + (a^3*b^3 - 4*a^4*b*c)*x^3
+ (a^4*b^2 - 4*a^5*c)*x^2)]
```

Sympy [F]

$$\int \frac{x}{(ax^2 + bx^3 + cx^4)^{3/2}}\,dx = \int \frac{x}{(x^2\,(a + bx + cx^2))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(c*x**4+b*x**3+a*x**2)**(3/2),x)
```

output
```
Integral(x/(x**2*(a + b*x + c*x**2))**(3/2), x)
```

Maxima [F]

$$\int \frac{x}{(ax^2 + bx^3 + cx^4)^{3/2}}\,dx = \int \frac{x}{(cx^4 + bx^3 + ax^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x/(c*x^4 + b*x^3 + a*x^2)^(3/2), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{x}{(ax^2 + bx^3 + cx^4)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(x/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(ax^2+bx^3+cx^4)^{3/2}}\,dx = \int \frac{x}{(c\,x^4+b\,x^3+a\,x^2)^{3/2}}\,dx$$

input
```
int(x/(a*x^2 + b*x^3 + c*x^4)^(3/2),x)
```

output
```
int(x/(a*x^2 + b*x^3 + c*x^4)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 454, normalized size of antiderivative = 3.15

$$\int \frac{x}{(ax^2+bx^3+cx^4)^{3/2}}\,dx = \frac{-8\sqrt{c\,x^2+bx+a}\,a^3c+2\sqrt{c\,x^2+bx+a}\,a^2b^2-20\sqrt{c\,x^2+bx+a}\,a^2bcx-1\text{(}$$

input
```
int(x/(c*x^4+b*x^3+a*x^2)^(3/2),x)
```

output
```
( - 8*sqrt(a + b*x + c*x**2)*a**3*c + 2*sqrt(a + b*x + c*x**2)*a**2*b**2 -
 20*sqrt(a + b*x + c*x**2)*a**2*b*c*x - 16*sqrt(a + b*x + c*x**2)*a**2*c**
2*x**2 + 6*sqrt(a + b*x + c*x**2)*a*b**3*x + 6*sqrt(a + b*x + c*x**2)*a*b*
*2*c*x**2 + 12*sqrt(a)*log( - 2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a - b*x
)*a**2*b*c*x - 3*sqrt(a)*log( - 2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a - b
*x)*a*b**3*x + 12*sqrt(a)*log( - 2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a -
b*x)*a*b**2*c*x**2 + 12*sqrt(a)*log( - 2*sqrt(a)*sqrt(a + b*x + c*x**2) -
2*a - b*x)*a*b*c**2*x**3 - 3*sqrt(a)*log( - 2*sqrt(a)*sqrt(a + b*x + c*x**
2) - 2*a - b*x)*b**4*x**2 - 3*sqrt(a)*log( - 2*sqrt(a)*sqrt(a + b*x + c*x*
*2) - 2*a - b*x)*b**3*c*x**3 - 12*sqrt(a)*log(x)*a**2*b*c*x + 3*sqrt(a)*lo
g(x)*a*b**3*x - 12*sqrt(a)*log(x)*a*b**2*c*x**2 - 12*sqrt(a)*log(x)*a*b*c*
*2*x**3 + 3*sqrt(a)*log(x)*b**4*x**2 + 3*sqrt(a)*log(x)*b**3*c*x**3)/(2*a*
*3*x*(4*a**2*c - a*b**2 + 4*a*b*c*x + 4*a*c**2*x**2 - b**3*x - b**2*c*x**2
))
```

3.79 $\int \frac{1}{(ax^2+bx^3+cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 209

$$\int \frac{1}{(ax^2+bx^3+cx^4)^{3/2}}\,dx = \frac{2(b^2-2ac+bcx)}{a(b^2-4ac)\,x\sqrt{ax^2+bx^3+cx^4}} - \frac{(5b^2-12ac)\sqrt{ax^2+bx^3+cx^4}}{2a^2(b^2-4ac)\,x^3} + \frac{b(15b^2-52ac)\sqrt{ax^2+bx^3+cx^4}}{4a^3(b^2-4ac)\,x^2} - \frac{3(5b^2-4ac)\operatorname{arctanh}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{8a^{7/2}}$$

output
```
2*(b*c*x-2*a*c+b^2)/a/(-4*a*c+b^2)/x/(c*x^4+b*x^3+a*x^2)^(1/2)-1/2*(-12*a*
c+5*b^2)*(c*x^4+b*x^3+a*x^2)^(1/2)/a^2/(-4*a*c+b^2)/x^3+1/4*b*(-52*a*c+15*
b^2)*(c*x^4+b*x^3+a*x^2)^(1/2)/a^3/(-4*a*c+b^2)/x^2-3/8*(-4*a*c+5*b^2)*arc
tanh(1/2*x*(b*x+2*a)/a^(1/2)/(c*x^4+b*x^3+a*x^2)^(1/2))/a^(7/2)
```

Mathematica [A] (verified)

Time = 1.21 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.86

$$\int \frac{1}{(ax^2+bx^3+cx^4)^{3/2}}\,dx = -\frac{\sqrt{a}(-8a^3c-15b^3x^2(b+cx)+2a^2(b^2+10bcx-12c^2x^2)+abx(-5b^2+62bcx+52c^2x^2))-3(5b^4-24ab}{4a^{7/2}\,(b^2-4ac)\,x\sqrt{x^2(a+x(b+cx))}}$$

input
```
Integrate[(a*x^2 + b*x^3 + c*x^4)^(-3/2),x]
```

output
```
-1/4*(Sqrt[a]*(-8*a^3*c - 15*b^3*x^2*(b + c*x) + 2*a^2*(b^2 + 10*b*c*x - 1
2*c^2*x^2) + a*b*x*(-5*b^2 + 62*b*c*x + 52*c^2*x^2)) - 3*(5*b^4 - 24*a*b^2
*c + 16*a^2*c^2)*x^2*Sqrt[a + x*(b + c*x)]*ArcTanh[(Sqrt[c]*x - Sqrt[a + x
*(b + c*x)])/Sqrt[a]])/(a^(7/2)*(b^2 - 4*a*c)*x*Sqrt[x^2*(a + x*(b + c*x))
])
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.04, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {1954, 27, 1998, 27, 1998, 27, 1951, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ax^2+bx^3+cx^4)^{3/2}}dx$$

$$\Big\downarrow \; 1954$$

$$\frac{2(-2ac+b^2+bcx)}{ax\,(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}-\frac{2\int -\frac{5b^2+4cxb-12ac}{2x^2\sqrt{cx^4+bx^3+ax^2}}dx}{a\,(b^2-4ac)}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{5b^2+4cxb-12ac}{x^2\sqrt{cx^4+bx^3+ax^2}}dx}{a\,(b^2-4ac)}+\frac{2(-2ac+b^2+bcx)}{ax\,(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}$$

$$\Big\downarrow \text{1998}$$

$$\frac{-\frac{\int \frac{b(15b^2-52ac)+2c(5b^2-12ac)x}{2x\sqrt{cx^4+bx^3+ax^2}}dx}{2a}-\frac{(5b^2-12ac)\sqrt{ax^2+bx^3+cx^4}}{2ax^3}}{a(b^2-4ac)}+\frac{2(-2ac+b^2+bcx)}{ax(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{-\frac{\int \frac{b(15b^2-52ac)+2c(5b^2-12ac)x}{x\sqrt{cx^4+bx^3+ax^2}}dx}{4a}-\frac{(5b^2-12ac)\sqrt{ax^2+bx^3+cx^4}}{2ax^3}}{a(b^2-4ac)}+\frac{2(-2ac+b^2+bcx)}{ax(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}$$

$$\Big\downarrow \text{1998}$$

$$\frac{-\frac{-\frac{\int \frac{3(b^2-4ac)(5b^2-4ac)}{2\sqrt{cx^4+bx^3+ax^2}}dx}{a}-\frac{b(15b^2-52ac)\sqrt{ax^2+bx^3+cx^4}}{ax^2}}{4a}-\frac{(5b^2-12ac)\sqrt{ax^2+bx^3+cx^4}}{2ax^3}}{a(b^2-4ac)}+\frac{2(-2ac+b^2+bcx)}{ax(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{-\frac{-\frac{3(b^2-4ac)(5b^2-4ac)\int \frac{1}{\sqrt{cx^4+bx^3+ax^2}}dx}{2a}-\frac{b(15b^2-52ac)\sqrt{ax^2+bx^3+cx^4}}{ax^2}}{4a}-\frac{(5b^2-12ac)\sqrt{ax^2+bx^3+cx^4}}{2ax^3}}{a(b^2-4ac)}+\frac{2(-2ac+b^2+bcx)}{ax(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}$$

$$\Big\downarrow \text{1951}$$

$$\frac{-\frac{\frac{3(b^2-4ac)(5b^2-4ac)\int \frac{1}{4a-\frac{x^2(2a+bx)^2}{cx^4+bx^3+ax^2}}d\frac{x(2a+bx)}{\sqrt{cx^4+bx^3+ax^2}}}{a}-\frac{b(15b^2-52ac)\sqrt{ax^2+bx^3+cx^4}}{ax^2}}{4a}-\frac{(5b^2-12ac)\sqrt{ax^2+bx^3+cx^4}}{2ax^3}}{a(b^2-4ac)}+\frac{2(-2ac+b^2+bcx)}{ax(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}$$

$$\Big\downarrow \text{219}$$

$$\frac{-\frac{\frac{3(b^2-4ac)(5b^2-4ac)\text{arctanh}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{2a^{3/2}}-\frac{b(15b^2-52ac)\sqrt{ax^2+bx^3+cx^4}}{ax^2}}{4a}-\frac{(5b^2-12ac)\sqrt{ax^2+bx^3+cx^4}}{2ax^3}}{a(b^2-4ac)}+\frac{2(-2ac+b^2+bcx)}{ax(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}$$

input
```
Int[(a*x^2 + b*x^3 + c*x^4)^(-3/2),x]
```

output
```
(2*(b^2 - 2*a*c + b*c*x))/(a*(b^2 - 4*a*c)*x*Sqrt[a*x^2 + b*x^3 + c*x^4])
+ (-1/2*((5*b^2 - 12*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(a*x^3) - (-((b*(15
*b^2 - 52*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(a*x^2)) + (3*(b^2 - 4*a*c)*(5
*b^2 - 4*a*c)*ArcTanh[(x*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x^2 + b*x^3 + c*x^
4])])/(2*a^(3/2)))/(4*a))/(a*(b^2 - 4*a*c))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1951
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :
> Simp[-2/(n - 2)   Subst[Int[1/(4*a - x^2), x], x, x*((2*a + b*x^(n - 2))/
Sqrt[a*x^2 + b*x^n + c*x^r])], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*
n - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]
```

rule 1954
```
Int[((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_), x_Symbol
] :> Simp[(-x^(-q + 1))*(b^2 - 2*a*c + b*c*x^(n - q))*((a*x^q + b*x^n + c*x
^(2*n - q))^(p + 1)/(a*(n - q)*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(a*(n -
 q)*(p + 1)*(b^2 - 4*a*c))   Int[(((p*q + 1)*(b^2 - 2*a*c) + (n - q)*(p + 1
)*(b^2 - 4*a*c) + b*c*(p*q + (n - q)*(2*p + 3) + 1)*x^(n - q))*(a*x^q + b*x
^n + c*x^(2*n - q))^(p + 1))/x^q, x], x] /; FreeQ[{a, b, c, n, q}, x] && Eq
Q[r, 2*n - q] && PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && LtQ
[p, -1]
```

rule 1998
```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[A*x^(m - q + 1)*((a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1)/(a*(m + p*q + 1))), x] + Simp[1/(a*(m + p*q +
 1))   Int[x^(m + n - q)*Simp[a*B*(m + p*q + 1) - A*b*(m + p*q + (n - q)*(p
 + 1) + 1) - A*c*(m + p*q + 2*(n - q)*(p + 1) + 1)*x^(n - q), x]*(a*x^q + b
*x^n + c*x^(2*n - q))^p, x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n -
q] && EqQ[j, 2*n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
 && RationalQ[m, p, q] && ((GeQ[p, -1] && LtQ[p, 0]) || EqQ[m + p*q + (n -
q)*(2*p + 1) + 1, 0]) && LeQ[m + p*q, -(n - q)] && NeQ[m + p*q + 1, 0]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.83

method	result
pseudoelliptic	$-\frac{-\frac{5\left(-\frac{52}{5}c^{2}x^{2}-\frac{62}{5}bcx+b^{2}\right)bx\,a^{\frac{3}{2}}}{4}+6\left(-c^{2}x^{2}+\frac{5}{6}bcx+\frac{1}{12}b^{2}\right)a^{\frac{5}{2}}-2a^{\frac{7}{2}}c+6\left(-\frac{5b^{3}(cx+b)\sqrt{a}}{8}+\left(-\ln(2)+\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^{2}+b \cdots}}{x\sqrt{a}}\right.\right.\right.}{a^{\frac{7}{2}}\sqrt{c\,x^{2}+bx+a}\,(4ac-b^{2})x^{2}}$
default	$-\frac{x\left(c\,x^{2}+bx+a\right)\left(-104a^{\frac{5}{2}}b\,c^{2}x^{3}+30a^{\frac{3}{2}}b^{3}c\,x^{3}+48a^{\frac{7}{2}}c^{2}x^{2}-124a^{\frac{5}{2}}b^{2}c\,x^{2}+30a^{\frac{3}{2}}b^{4}x^{2}-48\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^{2}+bx+a}}{x}\right)\sqrt{\cdots}\right.}{\cdots}$
risch	$-\frac{\left(c\,x^{2}+bx+a\right)\left(-7bx+2a\right)}{4a^{3}x\sqrt{x^{2}\left(c\,x^{2}+bx+a\right)}}+\frac{\left(-\frac{2c\,b^{3}x}{a^{3}\left(4ac-b^{2}\right)\sqrt{c\,x^{2}+bx+a}}-\frac{b^{4}}{a^{3}\left(4ac-b^{2}\right)\sqrt{c\,x^{2}+bx+a}}-\frac{c}{a^{2}\sqrt{c\,x^{2}+bx+a}}+\frac{6c^{2}bx}{a^{2}\left(4ac-b^{2}\right)\sqrt{c\,x^{2} \cdots}}\right.}{\cdots}$

input
```
int(1/(c*x^4+b*x^3+a*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
6/(c*x^2+b*x+a)^(1/2)/a^(7/2)*(-5/24*(-52/5*c^2*x^2-62/5*b*c*x+b^2)*b*x*a^
(3/2)+(-c^2*x^2+5/6*b*c*x+1/12*b^2)*a^(5/2)-1/3*a^(7/2)*c+(-5/8*b^3*(c*x+b
)*a^(1/2)+(-ln(2)+ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x/a^(1/2)))*(
a*c-5/4*b^2)*(c*x^2+b*x+a)^(1/2)*(a*c-1/4*b^2))*x^2)/(4*a*c-b^2)/x^2
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 630, normalized size of antiderivative = 3.01

$$\int \frac{1}{\left(ax^2+bx^3+cx^4\right)^{3/2}}\,dx = \left[-\frac{3\left(\left(5\,b^4c-24\,ab^2c^2+16\,a^2c^3\right)x^5+\left(5\,b^5-24\,ab^3c+16\,a^2bc^2\right)x^4+\left(5\,ab^4\right.\right.}{}\right.$$

input
```
integrate(1/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

output
```
[-1/16*(3*((5*b^4*c - 24*a*b^2*c^2 + 16*a^2*c^3)*x^5 + (5*b^5 - 24*a*b^3*c
 + 16*a^2*b*c^2)*x^4 + (5*a*b^4 - 24*a^2*b^2*c + 16*a^3*c^2)*x^3)*sqrt(a)*
log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x + 4*sqrt(c*x^4 + b*x^3 + a*x
^2)*(b*x + 2*a)*sqrt(a))/x^3) + 4*(2*a^3*b^2 - 8*a^4*c - (15*a*b^3*c - 52*
a^2*b*c^2)*x^3 - (15*a*b^4 - 62*a^2*b^2*c + 24*a^3*c^2)*x^2 - 5*(a^2*b^3 -
 4*a^3*b*c)*x)*sqrt(c*x^4 + b*x^3 + a*x^2))/((a^4*b^2*c - 4*a^5*c^2)*x^5 +
 (a^4*b^3 - 4*a^5*b*c)*x^4 + (a^5*b^2 - 4*a^6*c)*x^3), 1/8*(3*((5*b^4*c -
24*a*b^2*c^2 + 16*a^2*c^3)*x^5 + (5*b^5 - 24*a*b^3*c + 16*a^2*b*c^2)*x^4 +
 (5*a*b^4 - 24*a^2*b^2*c + 16*a^3*c^2)*x^3)*sqrt(-a)*arctan(1/2*sqrt(c*x^4
 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2 + a^2*x)) - 2*(2
*a^3*b^2 - 8*a^4*c - (15*a*b^3*c - 52*a^2*b*c^2)*x^3 - (15*a*b^4 - 62*a^2*
b^2*c + 24*a^3*c^2)*x^2 - 5*(a^2*b^3 - 4*a^3*b*c)*x)*sqrt(c*x^4 + b*x^3 +
a*x^2))/((a^4*b^2*c - 4*a^5*c^2)*x^5 + (a^4*b^3 - 4*a^5*b*c)*x^4 + (a^5*b^
2 - 4*a^6*c)*x^3)]
```

Sympy [F]

$$\int \frac{1}{\left(ax^2+bx^3+cx^4\right)^{3/2}}\,dx = \int \frac{1}{\left(ax^2+bx^3+cx^4\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(c*x**4+b*x**3+a*x**2)**(3/2),x)
```

output
```
Integral((a*x**2 + b*x**3 + c*x**4)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(ax^2+bx^3+cx^4)^{3/2}}\,dx = \int \frac{1}{(cx^4+bx^3+ax^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*x^4 + b*x^3 + a*x^2)^(-3/2), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{(ax^2+bx^3+cx^4)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(ax^2+bx^3+cx^4)^{3/2}}\,dx = \int \frac{1}{(c\,x^4+b\,x^3+a\,x^2)^{3/2}}\,dx$$

input
```
int(1/(a*x^2 + b*x^3 + c*x^4)^(3/2),x)
```

output
```
int(1/(a*x^2 + b*x^3 + c*x^4)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 696, normalized size of antiderivative = 3.33

$$\int \frac{1}{(ax^2 + bx^3 + cx^4)^{3/2}} \, dx = \text{Too large to display}$$

input
```
int(1/(c*x^4+b*x^3+a*x^2)^(3/2),x)
```

output
```
( - 16*sqrt(a + b*x + c*x**2)*a**4*c + 4*sqrt(a + b*x + c*x**2)*a**3*b**2
+ 40*sqrt(a + b*x + c*x**2)*a**3*b*c*x - 48*sqrt(a + b*x + c*x**2)*a**3*c*
*2*x**2 - 10*sqrt(a + b*x + c*x**2)*a**2*b**3*x + 124*sqrt(a + b*x + c*x**
2)*a**2*b**2*c*x**2 + 104*sqrt(a + b*x + c*x**2)*a**2*b*c**2*x**3 - 30*sqr
t(a + b*x + c*x**2)*a*b**4*x**2 - 30*sqrt(a + b*x + c*x**2)*a*b**3*c*x**3
+ 48*sqrt(a)*log( - 2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a - b*x)*a**3*c**
2*x**2 - 72*sqrt(a)*log( - 2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a - b*x)*a
**2*b**2*c*x**2 + 48*sqrt(a)*log( - 2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a
 - b*x)*a**2*b*c**2*x**3 + 48*sqrt(a)*log( - 2*sqrt(a)*sqrt(a + b*x + c*x*
*2) - 2*a - b*x)*a**2*c**3*x**4 + 15*sqrt(a)*log( - 2*sqrt(a)*sqrt(a + b*x
 + c*x**2) - 2*a - b*x)*a*b**4*x**2 - 72*sqrt(a)*log( - 2*sqrt(a)*sqrt(a +
 b*x + c*x**2) - 2*a - b*x)*a*b**3*c*x**3 - 72*sqrt(a)*log( - 2*sqrt(a)*sq
rt(a + b*x + c*x**2) - 2*a - b*x)*a*b**2*c**2*x**4 + 15*sqrt(a)*log( - 2*s
qrt(a)*sqrt(a + b*x + c*x**2) - 2*a - b*x)*b**5*x**3 + 15*sqrt(a)*log( - 2
*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a - b*x)*b**4*c*x**4 - 48*sqrt(a)*log(
x)*a**3*c**2*x**2 + 72*sqrt(a)*log(x)*a**2*b**2*c*x**2 - 48*sqrt(a)*log(x)
*a**2*b*c**2*x**3 - 48*sqrt(a)*log(x)*a**2*c**3*x**4 - 15*sqrt(a)*log(x)*a
*b**4*x**2 + 72*sqrt(a)*log(x)*a*b**3*c*x**3 + 72*sqrt(a)*log(x)*a*b**2*c*
*2*x**4 - 15*sqrt(a)*log(x)*b**5*x**3 - 15*sqrt(a)*log(x)*b**4*c*x**4)/(8*
a**4*x**2*(4*a**2*c - a*b**2 + 4*a*b*c*x + 4*a*c**2*x**2 - b**3*x - b**...
```

3.80 $\int \frac{1}{x\left(ax^2+bx^3+cx^4\right)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 271

$$\begin{aligned}\int \frac{1}{x\left(ax^2+bx^3+cx^4\right)^{3/2}}\,dx &= \frac{2(b^2-2ac+bcx)}{a\left(b^2-4ac\right)x^2\sqrt{ax^2+bx^3+cx^4}} \\ &- \frac{(7b^2-16ac)\sqrt{ax^2+bx^3+cx^4}}{3a^2\left(b^2-4ac\right)x^4} + \frac{b(35b^2-116ac)\sqrt{ax^2+bx^3+cx^4}}{12a^3\left(b^2-4ac\right)x^3} \\ &- \frac{\left(105b^4-460ab^2c+256a^2c^2\right)\sqrt{ax^2+bx^3+cx^4}}{24a^4\left(b^2-4ac\right)x^2} \\ &+ \frac{5b(7b^2-12ac)\,\text{arctanh}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{16a^{9/2}}\end{aligned}$$

output

```
2*(b*c*x-2*a*c+b^2)/a/(-4*a*c+b^2)/x^2/(c*x^4+b*x^3+a*x^2)^(1/2)-1/3*(-16*
a*c+7*b^2)*(c*x^4+b*x^3+a*x^2)^(1/2)/a^2/(-4*a*c+b^2)/x^4+1/12*b*(-116*a*c
+35*b^2)*(c*x^4+b*x^3+a*x^2)^(1/2)/a^3/(-4*a*c+b^2)/x^3-1/24*(256*a^2*c^2-
460*a*b^2*c+105*b^4)*(c*x^4+b*x^3+a*x^2)^(1/2)/a^4/(-4*a*c+b^2)/x^2+5/16*b
*(-12*a*c+7*b^2)*arctanh(1/2*x*(b*x+2*a)/a^(1/2)/(c*x^4+b*x^3+a*x^2)^(1/2)
)/a^(9/2)
```

Mathematica [A] (verified)

Time = 1.61 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.83

$$\int \frac{1}{x\left(ax^2+bx^3+cx^4\right)^{3/2}}\,dx = \frac{\sqrt{a}(-32a^4c+105b^4x^3(b+cx)+5ab^2x^2(7b^2-106bcx-92c^2x^2)+8a^3(b^2 -}{}$$

input
```
Integrate[1/(x*(a*x^2 + b*x^3 + c*x^4)^(3/2)),x]
```

output
```
(Sqrt[a]*(-32*a^4*c + 105*b^4*x^3*(b + c*x) + 5*a*b^2*x^2*(7*b^2 - 106*b*c
*x - 92*c^2*x^2) + 8*a^3*(b^2 + 7*b*c*x + 16*c^2*x^2) + 2*a^2*x*(-7*b^3 -
86*b^2*c*x + 244*b*c^2*x^2 + 128*c^3*x^3)) + 15*b*(7*b^4 - 40*a*b^2*c + 48
*a^2*c^2)*x^3*Sqrt[a + x*(b + c*x)]*ArcTanh[(Sqrt[c]*x - Sqrt[a + x*(b + c
*x)])/Sqrt[a]])/(24*a^(9/2)*(-b^2 + 4*a*c)*x^2*Sqrt[x^2*(a + x*(b + c*x))]
)
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.03, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {1971, 27, 1998, 27, 1998, 27, 1998, 27, 1951, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(ax^2+bx^3+cx^4\right)^{3/2}}\,dx$$

$$\Big\downarrow \; 1971$$

$$\frac{2\left(-2ac+b^2+bcx\right)}{ax^2\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}}-\frac{2\int -\frac{7b^2+6cxb-16ac}{2x^3\sqrt{cx^4+bx^3+ax^2}}dx}{a\left(b^2-4ac\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{7b^2+6cxb-16ac}{x^3\sqrt{cx^4+bx^3+ax^2}}dx}{a\left(b^2-4ac\right)}+\frac{2\left(-2ac+b^2+bcx\right)}{ax^2\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}}$$

$$\Big\downarrow \; 1998$$

$$\frac{-\frac{\int \frac{b\left(35b^2-116ac\right)+4c\left(7b^2-16ac\right)x}{2x^2\sqrt{cx^4+bx^3+ax^2}}dx}{3a}-\frac{(7b^2-16ac)\sqrt{ax^2+bx^3+cx^4}}{3ax^4}}{a\left(b^2-4ac\right)}+\frac{2\left(-2ac+b^2+bcx\right)}{ax^2\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}}$$

$\Big\downarrow$ 27

$$\frac{-\frac{\int \frac{b\left(35b^2-116ac\right)+4c\left(7b^2-16ac\right)x}{x^2\sqrt{cx^4+bx^3+ax^2}}dx}{6a}-\frac{(7b^2-16ac)\sqrt{ax^2+bx^3+cx^4}}{3ax^4}}{a\left(b^2-4ac\right)}+\frac{2\left(-2ac+b^2+bcx\right)}{ax^2\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}}$$

$\Big\downarrow$ 1998

$$\frac{-\frac{-\frac{\int \frac{105b^4-460acb^2+2c\left(35b^2-116ac\right)xb+256a^2c^2}{2x\sqrt{cx^4+bx^3+ax^2}}dx}{2a}-\frac{b\left(35b^2-116ac\right)\sqrt{ax^2+bx^3+cx^4}}{2ax^3}}{6a}-\frac{(7b^2-16ac)\sqrt{ax^2+bx^3+cx^4}}{3ax^4}}{a\left(b^2-4ac\right)}+\frac{2\left(-2ac+b^2+bcx\right)}{ax^2\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}}$$

$\Big\downarrow$ 27

$$\frac{-\frac{-\frac{\int \frac{105b^4-460acb^2+2c\left(35b^2-116ac\right)xb+256a^2c^2}{x\sqrt{cx^4+bx^3+ax^2}}dx}{4a}-\frac{b\left(35b^2-116ac\right)\sqrt{ax^2+bx^3+cx^4}}{2ax^3}}{6a}-\frac{(7b^2-16ac)\sqrt{ax^2+bx^3+cx^4}}{3ax^4}}{a\left(b^2-4ac\right)}+\frac{2\left(-2ac+b^2+bcx\right)}{ax^2\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}}$$

$\Big\downarrow$ 1998

$$\frac{-\frac{-\frac{-\frac{\int \frac{15b\left(7b^2-12ac\right)\left(b^2-4ac\right)}{2\sqrt{cx^4+bx^3+ax^2}}dx}{a}-\frac{\left(256a^2c^2-460ab^2c+105b^4\right)\sqrt{ax^2+bx^3+cx^4}}{ax^2}}{4a}-\frac{b\left(35b^2-116ac\right)\sqrt{ax^2+bx^3+cx^4}}{2ax^3}}{6a}-\frac{(7b^2-16ac)\sqrt{ax^2+bx^3+cx^4}}{3ax^4}}{a\left(b^2-4ac\right)}+\frac{2\left(-2ac+b^2+bcx\right)}{ax^2\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}}$$

$\Big\downarrow$ 27

$$\frac{-\frac{-\frac{-\frac{15b\left(7b^2-12ac\right)\left(b^2-4ac\right)\int \frac{1}{\sqrt{cx^4+bx^3+ax^2}}dx}{2a}-\frac{\left(256a^2c^2-460ab^2c+105b^4\right)\sqrt{ax^2+bx^3+cx^4}}{ax^2}}{4a}-\frac{b\left(35b^2-116ac\right)\sqrt{ax^2+bx^3+cx^4}}{2ax^3}}{6a}-\frac{(7b^2-16ac)\sqrt{ax^2+bx^3+cx^4}}{3ax^4}}{a\left(b^2-4ac\right)}+\frac{2\left(-2ac+b^2+bcx\right)}{ax^2\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}}$$

$\Big\downarrow$ 1951

$$-\frac{-\frac{-\frac{15b(7b^2-12ac)(b^2-4ac)\int \frac{1}{4a-\frac{x^2(2a+bx)^2}{cx^4+bx^3+ax^2}}d\frac{x(2a+bx)}{\sqrt{cx^4+bx^3+ax^2}}}{a}-\frac{(256a^2c^2-460ab^2c+105b^4)\sqrt{ax^2+bx^3+cx^4}}{ax^2}}{4a}-\frac{b(35b^2-116ac)\sqrt{ax^2+bx^3+cx^4}}{2ax^3}}{6a}-\ldots}{a(b^2-4ac)}$$

$$\frac{2(-2ac+b^2+bcx)}{ax^2(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}$$

$$\Big\downarrow \; 219$$

$$-\frac{-\frac{-\frac{15b(7b^2-12ac)(b^2-4ac)\text{arctanh}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{2a^{3/2}}-\frac{(256a^2c^2-460ab^2c+105b^4)\sqrt{ax^2+bx^3+cx^4}}{ax^2}}{4a}-\frac{b(35b^2-116ac)\sqrt{ax^2+bx^3+cx^4}}{2ax^3}}{6a}-\frac{(7b^2\ldots}{}}{a(b^2-4ac)}$$

$$\frac{2(-2ac+b^2+bcx)}{ax^2(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}$$

input
```
Int[1/(x*(a*x^2 + b*x^3 + c*x^4)^(3/2)),x]
```

output
```
(2*(b^2 - 2*a*c + b*c*x))/(a*(b^2 - 4*a*c)*x^2*Sqrt[a*x^2 + b*x^3 + c*x^4]
) + (-1/3*((7*b^2 - 16*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(a*x^4) - (-1/2*(
b*(35*b^2 - 116*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(a*x^3) - (-(((105*b^4 -
 460*a*b^2*c + 256*a^2*c^2)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(a*x^2)) + (15*b*
(7*b^2 - 12*a*c)*(b^2 - 4*a*c)*ArcTanh[(x*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x
^2 + b*x^3 + c*x^4])])/(2*a^(3/2)))/(4*a))/(6*a))/(a*(b^2 - 4*a*c))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1951
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :
> Simp[-2/(n - 2)   Subst[Int[1/(4*a - x^2), x], x, x*((2*a + b*x^(n - 2))/
Sqrt[a*x^2 + b*x^n + c*x^r])], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*
n - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]
```

rule 1971
```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[(-x^(m - q + 1))*(b^2 - 2*a*c + b*c*x^(n - q))*((a*x^q
 + b*x^n + c*x^(2*n - q))^(p + 1)/(a*(n - q)*(p + 1)*(b^2 - 4*a*c))), x] +
Simp[1/(a*(n - q)*(p + 1)*(b^2 - 4*a*c))   Int[x^(m - q)*(b^2*(m + p*q + (n
 - q)*(p + 1) + 1) - 2*a*c*(m + p*q + 2*(n - q)*(p + 1) + 1) + b*c*(m + p*q
 + (n - q)*(2*p + 3) + 1)*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^(p + 1
), x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] &&  !Int
egerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[p, -1] && RationalQ[m,
 q] && LtQ[m + p*q + 1, n - q]
```

rule 1998
```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[A*x^(m - q + 1)*((a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1)/(a*(m + p*q + 1))), x] + Simp[1/(a*(m + p*q +
 1))   Int[x^(m + n - q)*Simp[a*B*(m + p*q + 1) - A*b*(m + p*q + (n - q)*(p
 + 1) + 1) - A*c*(m + p*q + 2*(n - q)*(p + 1) + 1)*x^(n - q), x]*(a*x^q + b
*x^n + c*x^(2*n - q))^p, x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n -
q] && EqQ[j, 2*n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
 && RationalQ[m, p, q] && ((GeQ[p, -1] && LtQ[p, 0]) || EqQ[m + p*q + (n -
q)*(2*p + 1) + 1, 0]) && LeQ[m + p*q, -(n - q)] && NeQ[m + p*q + 1, 0]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.79

method	result
pseudoelliptic	$-\frac{15\left(-\frac{7\left(-\frac{92}{7}c^2x^2-\frac{106}{7}bcx+b^2\right)b^2x^2a^{\frac{3}{2}}}{72}+\frac{7\left(-\frac{128}{7}c^3x^3-\frac{244}{7}bc^2x^2+\frac{86}{7}b^2cx+b^3\right)xa^{\frac{5}{2}}}{180}+\frac{\left(-16c^2x^2-7bcx-b^2\right)a^{\frac{7}{2}}}{45}+\frac{4a^{\frac{9}{2}}c}{45}+b\left(\cdots\right)\cdots\right)}{\sqrt{cx^2+bx+a}\,a^{\frac{9}{2}}\left(4ac-b^2\cdots\right)}$
default	$-\frac{\left(cx^2+bx+a\right)\left(-512a^{\frac{7}{2}}c^3x^4+920a^{\frac{5}{2}}b^2c^2x^4-210a^{\frac{3}{2}}b^4cx^4-976a^{\frac{7}{2}}bc^2x^3+1060a^{\frac{5}{2}}b^3cx^3-210a^{\frac{3}{2}}b^5x^3+720\ln\left(\frac{2a+bx+2\cdots}{\cdots}\right)\cdots\right)}{\cdots}$
risch	$-\frac{\left(cx^2+bx+a\right)\left(-40acx^2+57b^2x^2-22abx+8a^2\right)}{24a^4x^2\sqrt{x^2\left(cx^2+bx+a\right)}}+\frac{\left(\frac{2b^4cx}{a^4\left(4ac-b^2\right)\sqrt{cx^2+bx+a}}+\frac{b^5}{a^4\left(4ac-b^2\right)\sqrt{cx^2+bx+a}}+\frac{4c^3x}{a^2\left(4ac-b^2\right)\sqrt{\cdots}}\cdots\right)}{\cdots}$

input
```
int(1/x/(c*x^4+b*x^3+a*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-15/(c*x^2+b*x+a)^(1/2)/a^(9/2)*(-7/72*(-92/7*c^2*x^2-106/7*b*c*x+b^2)*b^2
*x^2*a^(3/2)+7/180*(-128/7*c^3*x^3-244/7*b*c^2*x^2+86/7*b^2*c*x+b^3)*x*a^(
5/2)+1/45*(-16*c^2*x^2-7*b*c*x-b^2)*a^(7/2)+4/45*a^(9/2)*c+b*(-7/24*b^3*(c
*x+b)*a^(1/2)+(c*x^2+b*x+a)^(1/2)*(-ln(2)+ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x
+a)^(1/2))/x/a^(1/2)))*(a*c-7/12*b^2)*(a*c-1/4*b^2))*x^3)/(4*a*c-b^2)/x^3
```

Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 716, normalized size of antiderivative = 2.64

$$\int \frac{1}{x\left(ax^2+bx^3+cx^4\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/x/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

output
```
[-1/96*(15*((7*b^5*c - 40*a*b^3*c^2 + 48*a^2*b*c^3)*x^6 + (7*b^6 - 40*a*b^
4*c + 48*a^2*b^2*c^2)*x^5 + (7*a*b^5 - 40*a^2*b^3*c + 48*a^3*b*c^2)*x^4)*s
qrt(a)*log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x - 4*sqrt(c*x^4 + b*x^
3 + a*x^2)*(b*x + 2*a)*sqrt(a))/x^3) + 4*(8*a^4*b^2 - 32*a^5*c + (105*a*b^
4*c - 460*a^2*b^2*c^2 + 256*a^3*c^3)*x^4 + (105*a*b^5 - 530*a^2*b^3*c + 48
8*a^3*b*c^2)*x^3 + (35*a^2*b^4 - 172*a^3*b^2*c + 128*a^4*c^2)*x^2 - 14*(a^
3*b^3 - 4*a^4*b*c)*x)*sqrt(c*x^4 + b*x^3 + a*x^2))/((a^5*b^2*c - 4*a^6*c^2
)*x^6 + (a^5*b^3 - 4*a^6*b*c)*x^5 + (a^6*b^2 - 4*a^7*c)*x^4), -1/48*(15*((
7*b^5*c - 40*a*b^3*c^2 + 48*a^2*b*c^3)*x^6 + (7*b^6 - 40*a*b^4*c + 48*a^2*
b^2*c^2)*x^5 + (7*a*b^5 - 40*a^2*b^3*c + 48*a^3*b*c^2)*x^4)*sqrt(-a)*arcta
n(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2
+ a^2*x)) + 2*(8*a^4*b^2 - 32*a^5*c + (105*a*b^4*c - 460*a^2*b^2*c^2 + 256
*a^3*c^3)*x^4 + (105*a*b^5 - 530*a^2*b^3*c + 488*a^3*b*c^2)*x^3 + (35*a^2*
b^4 - 172*a^3*b^2*c + 128*a^4*c^2)*x^2 - 14*(a^3*b^3 - 4*a^4*b*c)*x)*sqrt(
c*x^4 + b*x^3 + a*x^2))/((a^5*b^2*c - 4*a^6*c^2)*x^6 + (a^5*b^3 - 4*a^6*b*
c)*x^5 + (a^6*b^2 - 4*a^7*c)*x^4)]
```

Sympy [F]

$$\int \frac{1}{x\,(ax^2+bx^3+cx^4)^{3/2}}\,dx = \int \frac{1}{x\,(x^2\,(a+bx+cx^2))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x/(c*x**4+b*x**3+a*x**2)**(3/2),x)
```

output
```
Integral(1/(x*(x**2*(a + b*x + c*x**2))**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{x\,(ax^2+bx^3+cx^4)^{3/2}}\,dx = \int \frac{1}{(cx^4+bx^3+ax^2)^{\frac{3}{2}}\,x}\,dx$$

input
```
integrate(1/x/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((c*x^4 + b*x^3 + a*x^2)^(3/2)*x), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{x\left(ax^2+bx^3+cx^4\right)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/x/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\left(ax^2+bx^3+cx^4\right)^{3/2}}\,dx = \int \frac{1}{x\left(c\,x^4+b\,x^3+a\,x^2\right)^{3/2}}\,dx$$

input
```
int(1/(x*(a*x^2 + b*x^3 + c*x^4)^(3/2)),x)
```

output
```
int(1/(x*(a*x^2 + b*x^3 + c*x^4)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 796, normalized size of antiderivative = 2.94

$$\int \frac{1}{x\left(ax^2+bx^3+cx^4\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int(1/x/(c*x^4+b*x^3+a*x^2)^(3/2),x)
```

output
```
( - 64*sqrt(a + b*x + c*x**2)*a**5*c + 16*sqrt(a + b*x + c*x**2)*a**4*b**2
 + 112*sqrt(a + b*x + c*x**2)*a**4*b*c*x + 256*sqrt(a + b*x + c*x**2)*a**4
*c**2*x**2 - 28*sqrt(a + b*x + c*x**2)*a**3*b**3*x - 344*sqrt(a + b*x + c*
x**2)*a**3*b**2*c*x**2 + 976*sqrt(a + b*x + c*x**2)*a**3*b*c**2*x**3 + 512
*sqrt(a + b*x + c*x**2)*a**3*c**3*x**4 + 70*sqrt(a + b*x + c*x**2)*a**2*b*
*4*x**2 - 1060*sqrt(a + b*x + c*x**2)*a**2*b**3*c*x**3 - 920*sqrt(a + b*x
+ c*x**2)*a**2*b**2*c**2*x**4 + 210*sqrt(a + b*x + c*x**2)*a*b**5*x**3 + 2
10*sqrt(a + b*x + c*x**2)*a*b**4*c*x**4 + 720*sqrt(a)*log(2*sqrt(a)*sqrt(a
 + b*x + c*x**2) - 2*a - b*x)*a**3*b*c**2*x**3 - 600*sqrt(a)*log(2*sqrt(a)
*sqrt(a + b*x + c*x**2) - 2*a - b*x)*a**2*b**3*c*x**3 + 720*sqrt(a)*log(2*
sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a - b*x)*a**2*b**2*c**2*x**4 + 720*sqrt
(a)*log(2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a - b*x)*a**2*b*c**3*x**5 + 1
05*sqrt(a)*log(2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a - b*x)*a*b**5*x**3 -
 600*sqrt(a)*log(2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a - b*x)*a*b**4*c*x*
*4 - 600*sqrt(a)*log(2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a - b*x)*a*b**3*
c**2*x**5 + 105*sqrt(a)*log(2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a - b*x)*
b**6*x**4 + 105*sqrt(a)*log(2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a - b*x)*
b**5*c*x**5 - 720*sqrt(a)*log(x)*a**3*b*c**2*x**3 + 600*sqrt(a)*log(x)*a**
2*b**3*c*x**3 - 720*sqrt(a)*log(x)*a**2*b**2*c**2*x**4 - 720*sqrt(a)*log(x
)*a**2*b*c**3*x**5 - 105*sqrt(a)*log(x)*a*b**5*x**3 + 600*sqrt(a)*log(x...
```

3.81 $\int \frac{1}{x^2(ax^2+bx^3+cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 343

$$\begin{aligned}\int \frac{1}{x^2\left(ax^2+bx^3+cx^4\right)^{3/2}}\,dx &= \frac{2(b^2-2ac+bcx)}{a\left(b^2-4ac\right)x^3\sqrt{ax^2+bx^3+cx^4}} \\ &- \frac{(9b^2-20ac)\sqrt{ax^2+bx^3+cx^4}}{4a^2\left(b^2-4ac\right)x^5} + \frac{b(21b^2-68ac)\sqrt{ax^2+bx^3+cx^4}}{8a^3\left(b^2-4ac\right)x^4} \\ &- \frac{(105b^4-448ab^2c+240a^2c^2)\sqrt{ax^2+bx^3+cx^4}}{32a^4\left(b^2-4ac\right)x^3} \\ &+ \frac{b(315b^4-1680ab^2c+1808a^2c^2)\sqrt{ax^2+bx^3+cx^4}}{64a^5\left(b^2-4ac\right)x^2} \\ &- \frac{15(21b^4-56ab^2c+16a^2c^2)\,\mathrm{arctanh}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{128a^{11/2}}\end{aligned}$$

output
```
2*(b*c*x-2*a*c+b^2)/a/(-4*a*c+b^2)/x^3/(c*x^4+b*x^3+a*x^2)^(1/2)-1/4*(-20*
a*c+9*b^2)*(c*x^4+b*x^3+a*x^2)^(1/2)/a^2/(-4*a*c+b^2)/x^5+1/8*b*(-68*a*c+2
1*b^2)*(c*x^4+b*x^3+a*x^2)^(1/2)/a^3/(-4*a*c+b^2)/x^4-1/32*(240*a^2*c^2-44
8*a*b^2*c+105*b^4)*(c*x^4+b*x^3+a*x^2)^(1/2)/a^4/(-4*a*c+b^2)/x^3+1/64*b*(
1808*a^2*c^2-1680*a*b^2*c+315*b^4)*(c*x^4+b*x^3+a*x^2)^(1/2)/a^5/(-4*a*c+b
^2)/x^2-15/128*(16*a^2*c^2-56*a*b^2*c+21*b^4)*arctanh(1/2*x*(b*x+2*a)/a^(1
/2)/(c*x^4+b*x^3+a*x^2)^(1/2))/a^(11/2)
```

Mathematica [A] (verified)

Time = 2.20 (sec) , antiderivative size = 271, normalized size of antiderivative = 0.79

$$\int \frac{1}{x^2 (ax^2+bx^3+cx^4)^{3/2}}\,dx = \frac{\sqrt{a}(-64a^5c-315b^5x^4(b+cx)-105ab^3x^3(b^2-18bcx-16c^2x^2)+16a^4(b$$

input
```
Integrate[1/(x^2*(a*x^2 + b*x^3 + c*x^4)^(3/2)),x]
```

output
```
(Sqrt[a]*(-64*a^5*c - 315*b^5*x^4*(b + c*x) - 105*a*b^3*x^3*(b^2 - 18*b*c*
x - 16*c^2*x^2) + 16*a^4*(b^2 + 6*b*c*x + 10*c^2*x^2) + 2*a^2*b*x^2*(21*b^
3 + 308*b^2*c*x - 1352*b*c^2*x^2 - 904*c^3*x^3) - 8*a^3*x*(3*b^3 + 26*b^2*
c*x + 98*b*c^2*x^2 - 60*c^3*x^3)) - 15*(21*b^6 - 140*a*b^4*c + 240*a^2*b^2
*c^2 - 64*a^3*c^3)*x^4*Sqrt[a + x*(b + c*x)]*ArcTanh[(Sqrt[c]*x - Sqrt[a +
 x*(b + c*x)])/Sqrt[a]])/(64*a^(11/2)*(-b^2 + 4*a*c)*x^3*Sqrt[x^2*(a + x*(
b + c*x))])
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 348, normalized size of antiderivative = 1.01, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1971, 27, 1998, 27, 1998, 27, 1998, 27, 1998, 27, 1951, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (ax^2+bx^3+cx^4)^{3/2}}dx$$

$$\Big\downarrow \text{1971}$$

$$\frac{2(-2ac+b^2+bcx)}{ax^3(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}-\frac{2\int -\frac{9b^2+8cxb-20ac}{2x^4\sqrt{cx^4+bx^3+ax^2}}dx}{a(b^2-4ac)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{9b^2+8cxb-20ac}{x^4\sqrt{cx^4+bx^3+ax^2}}dx}{a(b^2-4ac)}+\frac{2(-2ac+b^2+bcx)}{ax^3(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}$$

$\downarrow$ 1998

$$-\frac{-\frac{\int \frac{3\left(b\left(21b^2-68ac\right)+2c\left(9b^2-20ac\right)x\right)}{2x^3\sqrt{cx^4+bx^3+ax^2}}dx}{4a}-\frac{(9b^2-20ac)\sqrt{ax^2+bx^3+cx^4}}{4ax^5}}{a\left(b^2-4ac\right)}+\frac{2\left(-2ac+b^2+bcx\right)}{ax^3\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}}$$

$\downarrow$ 27

$$-\frac{-\frac{3\int \frac{b\left(21b^2-68ac\right)+2c\left(9b^2-20ac\right)x}{x^3\sqrt{cx^4+bx^3+ax^2}}dx}{8a}-\frac{(9b^2-20ac)\sqrt{ax^2+bx^3+cx^4}}{4ax^5}}{a\left(b^2-4ac\right)}+\frac{2\left(-2ac+b^2+bcx\right)}{ax^3\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}}$$

$\downarrow$ 1998

$$-\frac{-\frac{3\left(-\frac{\int \frac{105b^4-448acb^2+4c\left(21b^2-68ac\right)xb+240a^2c^2}{2x^2\sqrt{cx^4+bx^3+ax^2}}dx}{3a}-\frac{b\left(21b^2-68ac\right)\sqrt{ax^2+bx^3+cx^4}}{3ax^4}\right)}{8a}-\frac{(9b^2-20ac)\sqrt{ax^2+bx^3+cx^4}}{4ax^5}}{a\left(b^2-4ac\right)}+\frac{2\left(-2ac+b^2+bcx\right)}{ax^3\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}}$$

$\downarrow$ 27

$$-\frac{-\frac{3\left(-\frac{\int \frac{105b^4-448acb^2+4c\left(21b^2-68ac\right)xb+240a^2c^2}{x^2\sqrt{cx^4+bx^3+ax^2}}dx}{6a}-\frac{b\left(21b^2-68ac\right)\sqrt{ax^2+bx^3+cx^4}}{3ax^4}\right)}{8a}-\frac{(9b^2-20ac)\sqrt{ax^2+bx^3+cx^4}}{4ax^5}}{a\left(b^2-4ac\right)}+\frac{2\left(-2ac+b^2+bcx\right)}{ax^3\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}}$$

$\downarrow$ 1998

$$-\frac{-\frac{3\left(-\frac{-\frac{\int \frac{b\left(315b^4-1680acb^2+1808a^2c^2\right)+2c\left(105b^4-448acb^2+240a^2c^2\right)x}{2x\sqrt{cx^4+bx^3+ax^2}}dx}{2a}-\frac{\left(240a^2c^2-448ab^2c+105b^4\right)\sqrt{ax^2+bx^3+cx^4}}{2ax^3}}{6a}-\frac{b\left(21b^2-68ac\right)\sqrt{ax^2+bx^3+cx^4}}{3ax^4}\cdots\right)}{8a}\cdots}{a\left(b^2-4ac\right)}$$

$$\frac{2\left(-2ac+b^2+bcx\right)}{ax^3\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}}$$

$\downarrow$ 27

$$-\frac{3\left(-\frac{-\frac{\int \frac{b\left(315b^4-1680acb^2+1808a^2c^2\right)+2c\left(105b^4-448acb^2+240a^2c^2\right)x}{x\sqrt{cx^4+bx^3+ax^2}}dx}{4a}-\frac{\left(240a^2c^2-448ab^2c+105b^4\right)\sqrt{ax^2+bx^3+cx^4}}{2ax^3}}{6a}-\frac{b\left(21b^2-68ac\right)\sqrt{ax^2+bx^3+cx\cdots}}{3ax^4}\cdots\right.}{8a}\cdots}{a\left(b^2-4ac\right)}$$

$$\frac{2\left(-2ac+b^2+bcx\right)}{ax^3\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}}$$

$$\Big\downarrow \ 1998$$

$$-\frac{3\left(-\frac{-\frac{-\frac{\int \frac{15\left(b^2-4ac\right)\left(21b^4-56acb^2+16a^2c^2\right)}{2\sqrt{cx^4+bx^3+ax^2}}dx}{a}-\frac{b\left(1808a^2c^2-1680ab^2c+315b^4\right)\sqrt{ax^2+bx^3+cx^4}}{ax^2}}{4a}-\frac{\left(240a^2c^2-448ab^2c+105b^4\right)\sqrt{ax^2+bx^3+cx^4}}{2ax^3}}{6a}-\frac{b\left(21\cdots\right.}{\cdots}\cdots\right.}{8a}\cdots}{a\left(b^2-4ac\right)}$$

$$\frac{2\left(-2ac+b^2+bcx\right)}{ax^3\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}}$$

$$\Big\downarrow \ 27$$

$$-\frac{3\left(-\frac{-\frac{-\frac{15\left(b^2-4ac\right)\left(16a^2c^2-56ab^2c+21b^4\right)\int \frac{1}{\sqrt{cx^4+bx^3+ax^2}}dx}{2a}-\frac{b\left(1808a^2c^2-1680ab^2c+315b^4\right)\sqrt{ax^2+bx^3+cx^4}}{ax^2}}{4a}-\frac{\left(240a^2c^2-448ab^2c+105b^4\right)\sqrt{ax^2+\cdots}}{2ax^3}}{6a}\cdots\right.}{8a}\cdots}{a\left(b^2-4ac\right)}$$

$$\frac{2\left(-2ac+b^2+bcx\right)}{ax^3\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}}$$

$$\Big\downarrow \ 1951$$

$$-\frac{3\left(-\frac{-\frac{\frac{15\left(b^2-4ac\right)\left(16a^2c^2-56ab^2c+21b^4\right)\int \frac{1}{4a-\frac{x^2(2a+bx)^2}{cx^4+bx^3+ax^2}}d\frac{x(2a+bx)}{\sqrt{cx^4+bx^3+ax^2}}}{a}-\frac{b\left(1808a^2c^2-1680ab^2c+315b^4\right)\sqrt{ax^2+bx^3+cx^4}}{ax^2}}{4a}-\frac{\left(240a^2c^2-448ab^2\cdots\right.}{\cdots}}{6a}\cdots\right.}{8a}\cdots}{a\left(b^2-4ac\right)}$$

$$\frac{2\left(-2ac+b^2+bcx\right)}{ax^3\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}}$$

$$\Big\downarrow \ 219$$

$$-\frac{3\left(-\frac{-\frac{\left(240a^2c^2-448ab^2c+105b^4\right)\sqrt{ax^2+bx^3+cx^4}}{2ax^3}-\frac{\frac{15\left(b^2-4ac\right)\left(16a^2c^2-56ab^2c+21b^4\right)\text{arctanh}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{2a^{3/2}}-\frac{b\left(1808a^2c^2-1680ab^2c+31\cdots\right.}{ax^2}}{4a}}{6a}\cdots\right.}{8a}}{a\left(b^2-4ac\right)}$$

$$\frac{2\left(-2ac+b^2+bcx\right)}{ax^3\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}}$$

input
```
Int[1/(x^2*(a*x^2 + b*x^3 + c*x^4)^(3/2)),x]
```

output
```
(2*(b^2 - 2*a*c + b*c*x))/(a*(b^2 - 4*a*c)*x^3*Sqrt[a*x^2 + b*x^3 + c*x^4]
) + (-1/4*((9*b^2 - 20*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(a*x^5) - (3*(-1/
3*(b*(21*b^2 - 68*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(a*x^4) - (-1/2*((105*
b^4 - 448*a*b^2*c + 240*a^2*c^2)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(a*x^3) - (-
((b*(315*b^4 - 1680*a*b^2*c + 1808*a^2*c^2)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(
a*x^2)) + (15*(b^2 - 4*a*c)*(21*b^4 - 56*a*b^2*c + 16*a^2*c^2)*ArcTanh[(x*
(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x^2 + b*x^3 + c*x^4])])/(2*a^(3/2)))/(4*a))
/(6*a)))/(8*a))/(a*(b^2 - 4*a*c))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1951
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :
> Simp[-2/(n - 2)   Subst[Int[1/(4*a - x^2), x], x, x*((2*a + b*x^(n - 2))/
Sqrt[a*x^2 + b*x^n + c*x^r])], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*
n - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]
```

rule 1971
```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[(-x^(m - q + 1))*(b^2 - 2*a*c + b*c*x^(n - q))*((a*x^q
 + b*x^n + c*x^(2*n - q))^(p + 1)/(a*(n - q)*(p + 1)*(b^2 - 4*a*c))), x] +
Simp[1/(a*(n - q)*(p + 1)*(b^2 - 4*a*c))   Int[x^(m - q)*(b^2*(m + p*q + (n
 - q)*(p + 1) + 1) - 2*a*c*(m + p*q + 2*(n - q)*(p + 1) + 1) + b*c*(m + p*q
 + (n - q)*(2*p + 3) + 1)*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^(p + 1
), x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] &&  !Int
egerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[p, -1] && RationalQ[m,
 q] && LtQ[m + p*q + 1, n - q]
```

rule 1998
```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[A*x^(m - q + 1)*((a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1)/(a*(m + p*q + 1))), x] + Simp[1/(a*(m + p*q +
 1))   Int[x^(m + n - q)*Simp[a*B*(m + p*q + 1) - A*b*(m + p*q + (n - q)*(p
 + 1) + 1) - A*c*(m + p*q + 2*(n - q)*(p + 1) + 1)*x^(n - q), x]*(a*x^q + b
*x^n + c*x^(2*n - q))^p, x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n -
q] && EqQ[j, 2*n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
 && RationalQ[m, p, q] && ((GeQ[p, -1] && LtQ[p, 0]) || EqQ[m + p*q + (n -
q)*(2*p + 1) + 1, 0]) && LeQ[m + p*q, -(n - q)] && NeQ[m + p*q + 1, 0]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 258, normalized size of antiderivative = 0.75

method	result
pseudoelliptic	$-\frac{15\left(\frac{2a^{\frac{11}{2}}c}{15}+\frac{7b^3x^3\left(-16c^2x^2-18bcx+b^2\right)a^{\frac{3}{2}}}{32}-\frac{7\left(-\frac{904}{21}c^3x^3-\frac{1352}{21}bc^2x^2+\frac{44}{3}b^2cx+b^3\right)bx^2a^{\frac{5}{2}}}{80}+\frac{\left(-20c^3x^3+\frac{98}{3}bc^2x^2+\frac{26}{3}b^2cx\cdots}{20}\right.}{}$
risch	$-\frac{(cx^2+bx+a)(292abcx^3-187b^3x^3-56a^2cx^2+82ab^2x^2-40ba^2x+16a^3)}{64a^5x^3\sqrt{x^2(cx^2+bx+a)}}+\frac{\left(c(112a^2c^2-456ab^2c+187b^4)\left(-\frac{1}{c\sqrt{cx^2\cdots}}\right.\right.}{}$
default	$-\frac{(cx^2+bx+a)\left(3616a^{\frac{7}{2}}bc^3x^5-3360a^{\frac{5}{2}}b^3c^2x^5+630a^{\frac{3}{2}}b^5cx^5+960\ln\left(\frac{2a+bx+2\sqrt{a}\sqrt{cx^2+bx+a}}{x}\right)\sqrt{cx^2+bx+a}\,a^4c^3x^4-3\cdots\right.}{}$

input
```
int(1/x^2/(c*x^4+b*x^3+a*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-15/8/a^(11/2)/(c*x^2+b*x+a)^(1/2)*(2/15*a^(11/2)*c+7/32*b^3*x^3*(-16*c^2*
x^2-18*b*c*x+b^2)*a^(3/2)-7/80*(-904/21*c^3*x^3-1352/21*b*c^2*x^2+44/3*b^2
*c*x+b^3)*b*x^2*a^(5/2)+1/20*(-20*c^3*x^3+98/3*b*c^2*x^2+26/3*b^2*c*x+b^3)
*x*a^(7/2)+(-1/5*b*c*x-1/30*b^2-1/3*c^2*x^2)*a^(9/2)+x^4*(21/32*b^5*(c*x+b
)*a^(1/2)+(c*x^2+b*x+a)^(1/2)*(-ln(2)+ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^
(1/2))/x/a^(1/2)))*(a^2*c^2-7/2*a*b^2*c+21/16*b^4)*(a*c-1/4*b^2)))/x^4/(a*
c-1/4*b^2)
```

Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 866, normalized size of antiderivative = 2.52

$$\int \frac{1}{x^2 (ax^2 + bx^3 + cx^4)^{3/2}} \, dx = \text{Too large to display}$$

input
```
integrate(1/x^2/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/256*(15*((21*b^6*c - 140*a*b^4*c^2 + 240*a^2*b^2*c^3 - 64*a^3*c^4)*x^7
+ (21*b^7 - 140*a*b^5*c + 240*a^2*b^3*c^2 - 64*a^3*b*c^3)*x^6 + (21*a*b^6
- 140*a^2*b^4*c + 240*a^3*b^2*c^2 - 64*a^4*c^3)*x^5)*sqrt(a)*log(-(8*a*b*x
^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*
a)*sqrt(a))/x^3) - 4*(16*a^5*b^2 - 64*a^6*c - (315*a*b^5*c - 1680*a^2*b^3*
c^2 + 1808*a^3*b*c^3)*x^5 - (315*a*b^6 - 1890*a^2*b^4*c + 2704*a^3*b^2*c^2
 - 480*a^4*c^3)*x^4 - 7*(15*a^2*b^5 - 88*a^3*b^3*c + 112*a^4*b*c^2)*x^3 +
2*(21*a^3*b^4 - 104*a^4*b^2*c + 80*a^5*c^2)*x^2 - 24*(a^4*b^3 - 4*a^5*b*c)
*x)*sqrt(c*x^4 + b*x^3 + a*x^2))/((a^6*b^2*c - 4*a^7*c^2)*x^7 + (a^6*b^3 -
 4*a^7*b*c)*x^6 + (a^7*b^2 - 4*a^8*c)*x^5), 1/128*(15*((21*b^6*c - 140*a*b
^4*c^2 + 240*a^2*b^2*c^3 - 64*a^3*c^4)*x^7 + (21*b^7 - 140*a*b^5*c + 240*a
^2*b^3*c^2 - 64*a^3*b*c^3)*x^6 + (21*a*b^6 - 140*a^2*b^4*c + 240*a^3*b^2*c
^2 - 64*a^4*c^3)*x^5)*sqrt(-a)*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x
 + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2 + a^2*x)) - 2*(16*a^5*b^2 - 64*a^6*c -
 (315*a*b^5*c - 1680*a^2*b^3*c^2 + 1808*a^3*b*c^3)*x^5 - (315*a*b^6 - 1890
*a^2*b^4*c + 2704*a^3*b^2*c^2 - 480*a^4*c^3)*x^4 - 7*(15*a^2*b^5 - 88*a^3*
b^3*c + 112*a^4*b*c^2)*x^3 + 2*(21*a^3*b^4 - 104*a^4*b^2*c + 80*a^5*c^2)*x
^2 - 24*(a^4*b^3 - 4*a^5*b*c)*x)*sqrt(c*x^4 + b*x^3 + a*x^2))/((a^6*b^2*c
- 4*a^7*c^2)*x^7 + (a^6*b^3 - 4*a^7*b*c)*x^6 + (a^7*b^2 - 4*a^8*c)*x^5)]
```

Sympy [F]

$$\int \frac{1}{x^2 \left(ax^2+bx^3+cx^4\right)^{3/2}}\,dx = \int \frac{1}{x^2 \left(x^2 \left(a+bx+cx^2\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x**2/(c*x**4+b*x**3+a*x**2)**(3/2),x)
```

output
```
Integral(1/(x**2*(x**2*(a + b*x + c*x**2))**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{x^2 \left(ax^2+bx^3+cx^4\right)^{3/2}}\,dx = \int \frac{1}{\left(cx^4+bx^3+ax^2\right)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate(1/x^2/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((c*x^4 + b*x^3 + a*x^2)^(3/2)*x^2), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{x^2 \left(ax^2+bx^3+cx^4\right)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/x^2/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (ax^2+bx^3+cx^4)^{3/2}}\,dx = \int \frac{1}{x^2 (c\,x^4+b\,x^3+a\,x^2)^{3/2}}\,dx$$

input
```
int(1/(x^2*(a*x^2 + b*x^3 + c*x^4)^(3/2)),x)
```

output
```
int(1/(x^2*(a*x^2 + b*x^3 + c*x^4)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 1055, normalized size of antiderivative = 3.08

$$\int \frac{1}{x^2 (ax^2+bx^3+cx^4)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int(1/x^2/(c*x^4+b*x^3+a*x^2)^(3/2),x)
```

output

```
( - 128*sqrt(a + b*x + c*x**2)*a**6*c + 32*sqrt(a + b*x + c*x**2)*a**5*b**
2 + 192*sqrt(a + b*x + c*x**2)*a**5*b*c*x + 320*sqrt(a + b*x + c*x**2)*a**
5*c**2*x**2 - 48*sqrt(a + b*x + c*x**2)*a**4*b**3*x - 416*sqrt(a + b*x + c
*x**2)*a**4*b**2*c*x**2 - 1568*sqrt(a + b*x + c*x**2)*a**4*b*c**2*x**3 + 9
60*sqrt(a + b*x + c*x**2)*a**4*c**3*x**4 + 84*sqrt(a + b*x + c*x**2)*a**3*
b**4*x**2 + 1232*sqrt(a + b*x + c*x**2)*a**3*b**3*c*x**3 - 5408*sqrt(a + b
*x + c*x**2)*a**3*b**2*c**2*x**4 - 3616*sqrt(a + b*x + c*x**2)*a**3*b*c**3
*x**5 - 210*sqrt(a + b*x + c*x**2)*a**2*b**5*x**3 + 3780*sqrt(a + b*x + c*
x**2)*a**2*b**4*c*x**4 + 3360*sqrt(a + b*x + c*x**2)*a**2*b**3*c**2*x**5 -
 630*sqrt(a + b*x + c*x**2)*a*b**6*x**4 - 630*sqrt(a + b*x + c*x**2)*a*b**
5*c*x**5 + 960*sqrt(a)*log(2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a - b*x)*a
**4*c**3*x**4 - 3600*sqrt(a)*log(2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a -
b*x)*a**3*b**2*c**2*x**4 + 960*sqrt(a)*log(2*sqrt(a)*sqrt(a + b*x + c*x**2
) - 2*a - b*x)*a**3*b*c**3*x**5 + 960*sqrt(a)*log(2*sqrt(a)*sqrt(a + b*x +
 c*x**2) - 2*a - b*x)*a**3*c**4*x**6 + 2100*sqrt(a)*log(2*sqrt(a)*sqrt(a +
 b*x + c*x**2) - 2*a - b*x)*a**2*b**4*c*x**4 - 3600*sqrt(a)*log(2*sqrt(a)*
sqrt(a + b*x + c*x**2) - 2*a - b*x)*a**2*b**3*c**2*x**5 - 3600*sqrt(a)*log
(2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a - b*x)*a**2*b**2*c**3*x**6 - 315*s
qrt(a)*log(2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a - b*x)*a*b**6*x**4 + 210
0*sqrt(a)*log(2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a - b*x)*a*b**5*c*x*...
```

3.82 $\int (dx)^m \left(ax^n+bx^{1+n}+cx^{2+n}\right)^3 \, dx$

Optimal result

Integrand size = 28, antiderivative size = 191

$$\int (dx)^m \left(ax^n+bx^{1+n}+cx^{2+n}\right)^3 \, dx = \frac{3a(b^2+ac)\,x^{3(1+n)}(dx)^m}{3+m+3n}+\frac{3bc^2x^{3(2+n)}(dx)^m}{6+m+3n}+\frac{a^3x^{1+3n}(dx)^m}{1+m+3n}+\frac{3a^2bx^{2+3n}(dx)^m}{2+m+3n}+\frac{b(b^2+6ac)\,x^{4+3n}(dx)^m}{4+m+3n}+\frac{3c(b^2+ac)\,x^{5+3n}(dx)^m}{5+m+3n}+\frac{c^3x^{7+3n}(dx)^m}{7+m+3n}$$

output
```
3*a*(a*c+b^2)*x^(3+3*n)*(d*x)^m/(3+m+3*n)+3*b*c^2*x^(6+3*n)*(d*x)^m/(6+m+3
*n)+a^3*x^(1+3*n)*(d*x)^m/(1+m+3*n)+3*a^2*b*x^(2+3*n)*(d*x)^m/(2+m+3*n)+b*
(6*a*c+b^2)*x^(4+3*n)*(d*x)^m/(4+m+3*n)+3*c*(a*c+b^2)*x^(5+3*n)*(d*x)^m/(5
+m+3*n)+c^3*x^(7+3*n)*(d*x)^m/(7+m+3*n)
```

Mathematica [A] (verified)

Time = 1.15 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.60

$$\int (dx)^m \left(ax^n + bx^{1+n} + cx^{2+n}\right)^3 \, dx$$

$$= \frac{x^{1+3n}(dx)^m \left(-\frac{3x(b(11+m+3n)+2c(6+m+3n)x)(a+x(b+cx))^2}{(6+m+3n)(7+m+3n)} + (a+x(b+cx))^3 + \frac{6x\left(ab\left(-b^2(1+m+3n)-\frac{20ac(5+m+3n)}{2+m+3n}\right.\right.}{}\right.}{}$$

input
```
Integrate[(d*x)^m*(a*x^n + b*x^(1 + n) + c*x^(2 + n))^3,x]
```

output
```
(x^(1 + 3*n)*(d*x)^m*((-3*x*(b*(11 + m + 3*n) + 2*c*(6 + m + 3*n)*x)*(a +
x*(b + c*x))^2)/((6 + m + 3*n)*(7 + m + 3*n)) + (a + x*(b + c*x))^3 + (6*x
*(a*b*(-(b^2*(1 + m + 3*n)) - (20*a*c*(5 + m + 3*n))/(2 + m + 3*n) + 4*a*c
*(6 + m + 3*n)) - ((10*a*b^2*c*(5 + m + 3*n) + (b^2*(3 + m + 3*n) - 2*a*c*
(4 + m + 3*n))*(b^2*(1 + m + 3*n) - 4*a*c*(6 + m + 3*n)))*x)/(3 + m + 3*n)
 + (b^3*(1 + m + 3*n) - 10*a*b*c*(5 + m + 3*n) - 4*a*b*c*(6 + m + 3*n) + c
*(4 + m + 3*n)*(b^2*(1 + m + 3*n) - 4*a*c*(6 + m + 3*n))*x)*(a + x*(b + c*
x))))/(c*(4 + m + 3*n)*(5 + m + 3*n)*(6 + m + 3*n)*(7 + m + 3*n))))/(1 + m
 + 3*n)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.91, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2028, 30, 1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^m \left(ax^n + bx^{n+1} + cx^{n+2}\right)^3 \, dx$$

$$\Big\downarrow \; 2028$$

$$\int x^{3n}(dx)^m \left(a + bx + cx^2\right)^3 \, dx$$

$$\Big\downarrow \text{30}$$

$$x^{-m}(dx)^m \int x^{m+3n}\left(cx^2+bx+a\right)^3 dx$$

$$\Big\downarrow \text{1140}$$

$$x^{-m}(dx)^m \int \left(a^3x^{m+3n}+3a^2bx^{m+3n+1}+3a\left(b^2+ac\right)x^{m+3n+2}+b\left(b^2+6ac\right)x^{m+3n+3}+3c\left(b^2+ac\right)x^{m+3n+4}\right.$$

$$\Big\downarrow \text{2009}$$

$$x^{-m}(dx)^m \left(\frac{a^3x^{m+3n+1}}{m+3n+1}+\frac{3a^2bx^{m+3n+2}}{m+3n+2}+\frac{3a\left(ac+b^2\right)x^{m+3n+3}}{m+3n+3}+\frac{b\left(6ac+b^2\right)x^{m+3n+4}}{m+3n+4}+\frac{3c\left(ac+b^2\right)x^{m+3n}}{m+3n+5}\right.$$

input
```
Int[(d*x)^m*(a*x^n + b*x^(1 + n) + c*x^(2 + n))^3,x]
```

output
```
((d*x)^m*((a^3*x^(1 + m + 3*n))/(1 + m + 3*n) + (3*a^2*b*x^(2 + m + 3*n))/
(2 + m + 3*n) + (3*a*(b^2 + a*c)*x^(3 + m + 3*n))/(3 + m + 3*n) + (b*(b^2
+ 6*a*c)*x^(4 + m + 3*n))/(4 + m + 3*n) + (3*c*(b^2 + a*c)*x^(5 + m + 3*n)
)/(5 + m + 3*n) + (3*b*c^2*x^(6 + m + 3*n))/(6 + m + 3*n) + (c^3*x^(7 + m
+ 3*n))/(7 + m + 3*n)))/x^m
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2028
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.) + (c_.)*(x_)^(t_.))^(p_.),
x_Symbol] :> Int[x^(p*r)*(a + b*x^(s - r) + c*x^(t - r))^p*Fx, x] /; FreeQ[
{a, b, c, r, s, t}, x] && IntegerQ[p] && PosQ[s - r] && PosQ[t - r] && !(E
qQ[p, 1] && EqQ[u, 1])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3383 vs. $2(191) = 382$.

Time = 20.48 (sec) , antiderivative size = 3384, normalized size of antiderivative = 17.72

method	result	size
orering	Expression too large to display	3384
risch	Expression too large to display	3396
parallelrisch	Expression too large to display	7012

input
```
int((d*x)^m*(x^n*a+b*x^(1+n)+c*x^(2+n))^3,x,method=_RETURNVERBOSE)
```

output
```
(c^3*m^6*x^6+18*c^3*m^5*n*x^6+135*c^3*m^4*n^2*x^6+540*c^3*m^3*n^3*x^6+1215
*c^3*m^2*n^4*x^6+1458*c^3*m*n^5*x^6+729*c^3*n^6*x^6+3*b*c^2*m^6*x^5+54*b*c
^2*m^5*n*x^5+405*b*c^2*m^4*n^2*x^5+1620*b*c^2*m^3*n^3*x^5+3645*b*c^2*m^2*n
^4*x^5+4374*b*c^2*m*n^5*x^5+2187*b*c^2*n^6*x^5+21*c^3*m^5*x^6+315*c^3*m^4*
n*x^6+1890*c^3*m^3*n^2*x^6+5670*c^3*m^2*n^3*x^6+8505*c^3*m*n^4*x^6+5103*c^
3*n^5*x^6+3*a*c^2*m^6*x^4+54*a*c^2*m^5*n*x^4+405*a*c^2*m^4*n^2*x^4+1620*a*
c^2*m^3*n^3*x^4+3645*a*c^2*m^2*n^4*x^4+4374*a*c^2*m*n^5*x^4+2187*a*c^2*n^6
*x^4+3*b^2*c*m^6*x^4+54*b^2*c*m^5*n*x^4+405*b^2*c*m^4*n^2*x^4+1620*b^2*c*m
^3*n^3*x^4+3645*b^2*c*m^2*n^4*x^4+4374*b^2*c*m*n^5*x^4+2187*b^2*c*n^6*x^4+
66*b*c^2*m^5*x^5+990*b*c^2*m^4*n*x^5+5940*b*c^2*m^3*n^2*x^5+17820*b*c^2*m^
2*n^3*x^5+26730*b*c^2*m*n^4*x^5+16038*b*c^2*n^5*x^5+175*c^3*m^4*x^6+2100*c
^3*m^3*n*x^6+9450*c^3*m^2*n^2*x^6+18900*c^3*m*n^3*x^6+14175*c^3*n^4*x^6+6*
a*b*c*m^6*x^3+108*a*b*c*m^5*n*x^3+810*a*b*c*m^4*n^2*x^3+3240*a*b*c*m^3*n^3
*x^3+7290*a*b*c*m^2*n^4*x^3+8748*a*b*c*m*n^5*x^3+4374*a*b*c*n^6*x^3+69*a*c
^2*m^5*x^4+1035*a*c^2*m^4*n*x^4+6210*a*c^2*m^3*n^2*x^4+18630*a*c^2*m^2*n^3
*x^4+27945*a*c^2*m*n^4*x^4+16767*a*c^2*n^5*x^4+b^3*m^6*x^3+18*b^3*m^5*n*x^
3+135*b^3*m^4*n^2*x^3+540*b^3*m^3*n^3*x^3+1215*b^3*m^2*n^4*x^3+1458*b^3*m*
n^5*x^3+729*b^3*n^6*x^3+69*b^2*c*m^5*x^4+1035*b^2*c*m^4*n*x^4+6210*b^2*c*m
^3*n^2*x^4+18630*b^2*c*m^2*n^3*x^4+27945*b^2*c*m*n^4*x^4+16767*b^2*c*n^5*x
^4+570*b*c^2*m^4*x^5+6840*b*c^2*m^3*n*x^5+30780*b*c^2*m^2*n^2*x^5+61560...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2421 vs. $2(191) = 382$.

Time = 0.16 (sec) , antiderivative size = 2421, normalized size of antiderivative = 12.68

$$\int (dx)^m \left(ax^n + bx^{1+n} + cx^{2+n}\right)^3 \, dx = \text{Too large to display}$$

input
```
integrate((d*x)^m*(a*x^n+b*x^(1+n)+c*x^(2+n))^3,x, algorithm="fricas")
```

output
```
(a^3*m^6 + 729*a^3*n^6 + 27*a^3*m^5 + 295*a^3*m^4 + (c^3*m^6 + 729*c^3*n^6
 + 21*c^3*m^5 + 175*c^3*m^4 + 735*c^3*m^3 + 729*(2*c^3*m + 7*c^3)*n^5 + 16
24*c^3*m^2 + 405*(3*c^3*m^2 + 21*c^3*m + 35*c^3)*n^4 + 1764*c^3*m + 135*(4
*c^3*m^3 + 42*c^3*m^2 + 140*c^3*m + 147*c^3)*n^3 + 720*c^3 + 9*(15*c^3*m^4
 + 210*c^3*m^3 + 1050*c^3*m^2 + 2205*c^3*m + 1624*c^3)*n^2 + 3*(6*c^3*m^5
+ 105*c^3*m^4 + 700*c^3*m^3 + 2205*c^3*m^2 + 3248*c^3*m + 1764*c^3)*n)*x^6
 + 1665*a^3*m^3 + 729*(2*a^3*m + 9*a^3)*n^5 + 3*(b*c^2*m^6 + 729*b*c^2*n^6
 + 22*b*c^2*m^5 + 190*b*c^2*m^4 + 820*b*c^2*m^3 + 486*(3*b*c^2*m + 11*b*c^
2)*n^5 + 1849*b*c^2*m^2 + 405*(3*b*c^2*m^2 + 22*b*c^2*m + 38*b*c^2)*n^4 +
2038*b*c^2*m + 540*(b*c^2*m^3 + 11*b*c^2*m^2 + 38*b*c^2*m + 41*b*c^2)*n^3
+ 840*b*c^2 + 9*(15*b*c^2*m^4 + 220*b*c^2*m^3 + 1140*b*c^2*m^2 + 2460*b*c^
2*m + 1849*b*c^2)*n^2 + 6*(3*b*c^2*m^5 + 55*b*c^2*m^4 + 380*b*c^2*m^3 + 12
30*b*c^2*m^2 + 1849*b*c^2*m + 1019*b*c^2)*n)*x^5 + 5104*a^3*m^2 + 405*(3*a
^3*m^2 + 27*a^3*m + 59*a^3)*n^4 + 3*((b^2*c + a*c^2)*m^6 + 729*(b^2*c + a*
c^2)*n^6 + 23*(b^2*c + a*c^2)*m^5 + 243*(23*b^2*c + 23*a*c^2 + 6*(b^2*c +
a*c^2)*m)*n^5 + 207*(b^2*c + a*c^2)*m^4 + 81*(207*b^2*c + 207*a*c^2 + 15*(
b^2*c + a*c^2)*m^2 + 115*(b^2*c + a*c^2)*m)*n^4 + 925*(b^2*c + a*c^2)*m^3
+ 27*(20*(b^2*c + a*c^2)*m^3 + 925*b^2*c + 925*a*c^2 + 230*(b^2*c + a*c^2)
*m^2 + 828*(b^2*c + a*c^2)*m)*n^3 + 1008*b^2*c + 1008*a*c^2 + 2144*(b^2*c
+ a*c^2)*m^2 + 9*(15*(b^2*c + a*c^2)*m^4 + 230*(b^2*c + a*c^2)*m^3 + 21...
```

Sympy [F(-1)]

Timed out.

$$\int (dx)^m \left(ax^n + bx^{1+n} + cx^{2+n}\right)^3 \, dx = \text{Timed out}$$

input
```
integrate((d*x)**m*(a*x**n+b*x**(1+n)+c*x**(2+n))**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.58

$$\int (dx)^m \left(ax^n + bx^{1+n} + cx^{2+n}\right)^3 \, dx = \frac{c^3 d^m x^7 e^{(m\log(x)+3\,n\log(x))}}{m+3\,n+7} + \frac{3\,bc^2 d^m x^6 e^{(m\log(x)+3\,n\log(x))}}{m+3\,n+6} + \frac{3\,b^2 c d^m x^5 e^{(m\log(x)+3\,n\log(x))}}{m+3\,n+5} + \frac{3\,ac^2 d^m x^5 e^{(m\log(x)+3\,n\log(x))}}{m+3\,n+5} + \frac{b^3 d^m x^4 e^{(m\log(x)+3\,n\log(x))}}{m+3\,n+4} + \frac{6\,abcd^m x^4 e^{(m\log(x)+3\,n\log(x))}}{m+3\,n+4} + \frac{3\,ab^2 d^m x^3 e^{(m\log(x)+3\,n\log(x))}}{m+3\,n+3} + \frac{3\,a^2 cd^m x^3 e^{(m\log(x)+3\,n\log(x))}}{m+3\,n+3} + \frac{3\,a^2 bd^m x^2 e^{(m\log(x)+3\,n\log(x))}}{m+3\,n+2} + \frac{a^3 d^m x e^{(m\log(x)+3\,n\log(x))}}{m+3\,n+1}$$

input
```
integrate((d*x)^m*(a*x^n+b*x^(1+n)+c*x^(2+n))^3,x, algorithm="maxima")
```

output
```
c^3*d^m*x^7*e^(m*log(x) + 3*n*log(x))/(m + 3*n + 7) + 3*b*c^2*d^m*x^6*e^(m
*log(x) + 3*n*log(x))/(m + 3*n + 6) + 3*b^2*c*d^m*x^5*e^(m*log(x) + 3*n*lo
g(x))/(m + 3*n + 5) + 3*a*c^2*d^m*x^5*e^(m*log(x) + 3*n*log(x))/(m + 3*n +
 5) + b^3*d^m*x^4*e^(m*log(x) + 3*n*log(x))/(m + 3*n + 4) + 6*a*b*c*d^m*x^
4*e^(m*log(x) + 3*n*log(x))/(m + 3*n + 4) + 3*a*b^2*d^m*x^3*e^(m*log(x) +
3*n*log(x))/(m + 3*n + 3) + 3*a^2*c*d^m*x^3*e^(m*log(x) + 3*n*log(x))/(m +
 3*n + 3) + 3*a^2*b*d^m*x^2*e^(m*log(x) + 3*n*log(x))/(m + 3*n + 2) + a^3*
d^m*x*e^(m*log(x) + 3*n*log(x))/(m + 3*n + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7783 vs. $2(191) = 382$.

Time = 0.30 (sec) , antiderivative size = 7783, normalized size of antiderivative = 40.75

$$\int (dx)^m \left(ax^n + bx^{1+n} + cx^{2+n}\right)^3 \, dx = \text{Too large to display}$$

input
```
integrate((d*x)^m*(a*x^n+b*x^(1+n)+c*x^(2+n))^3,x, algorithm="giac")
```

output
```
(c^3*m^6*x^7*x^(3*n)*e^(m*log(d) + m*log(x)) + 18*c^3*m^5*n*x^7*x^(3*n)*e^
(m*log(d) + m*log(x)) + 135*c^3*m^4*n^2*x^7*x^(3*n)*e^(m*log(d) + m*log(x)
) + 540*c^3*m^3*n^3*x^7*x^(3*n)*e^(m*log(d) + m*log(x)) + 1215*c^3*m^2*n^4
*x^7*x^(3*n)*e^(m*log(d) + m*log(x)) + 1458*c^3*m*n^5*x^7*x^(3*n)*e^(m*log
(d) + m*log(x)) + 729*c^3*n^6*x^7*x^(3*n)*e^(m*log(d) + m*log(x)) + 3*b*c^
2*m^6*x^6*x^(3*n)*e^(m*log(d) + m*log(x)) + 54*b*c^2*m^5*n*x^6*x^(3*n)*e^(
m*log(d) + m*log(x)) + 405*b*c^2*m^4*n^2*x^6*x^(3*n)*e^(m*log(d) + m*log(x
)) + 1620*b*c^2*m^3*n^3*x^6*x^(3*n)*e^(m*log(d) + m*log(x)) + 3645*b*c^2*m
^2*n^4*x^6*x^(3*n)*e^(m*log(d) + m*log(x)) + 4374*b*c^2*m*n^5*x^6*x^(3*n)*
e^(m*log(d) + m*log(x)) + 2187*b*c^2*n^6*x^6*x^(3*n)*e^(m*log(d) + m*log(x
)) + 21*c^3*m^5*x^7*x^(3*n)*e^(m*log(d) + m*log(x)) + 315*c^3*m^4*n*x^7*x^
(3*n)*e^(m*log(d) + m*log(x)) + 1890*c^3*m^3*n^2*x^7*x^(3*n)*e^(m*log(d) +
 m*log(x)) + 5670*c^3*m^2*n^3*x^7*x^(3*n)*e^(m*log(d) + m*log(x)) + 8505*c
^3*m*n^4*x^7*x^(3*n)*e^(m*log(d) + m*log(x)) + 5103*c^3*n^5*x^7*x^(3*n)*e^
(m*log(d) + m*log(x)) + 3*b^2*c*m^6*x^5*x^(3*n)*e^(m*log(d) + m*log(x)) +
3*a*c^2*m^6*x^5*x^(3*n)*e^(m*log(d) + m*log(x)) + 54*b^2*c*m^5*n*x^5*x^(3*
n)*e^(m*log(d) + m*log(x)) + 54*a*c^2*m^5*n*x^5*x^(3*n)*e^(m*log(d) + m*lo
g(x)) + 405*b^2*c*m^4*n^2*x^5*x^(3*n)*e^(m*log(d) + m*log(x)) + 405*a*c^2*
m^4*n^2*x^5*x^(3*n)*e^(m*log(d) + m*log(x)) + 1620*b^2*c*m^3*n^3*x^5*x^(3*
n)*e^(m*log(d) + m*log(x)) + 1620*a*c^2*m^3*n^3*x^5*x^(3*n)*e^(m*log(d)...
```

Mupad [B] (verification not implemented)

Time = 13.67 (sec) , antiderivative size = 3890, normalized size of antiderivative = 20.37

$$\int (dx)^m \left(ax^n + bx^{1+n} + cx^{2+n}\right)^3 \, dx = \text{Too large to display}$$

input
```
int((d*x)^m*(a*x^n + b*x^(n + 1) + c*x^(n + 2))^3,x)
```

output
```
(a^3*x*x^(3*n)*(d*x)^m*(8028*m + 24084*n + 30624*m*n + 44955*m*n^2 + 14985
*m^2*n + 31860*m*n^3 + 3540*m^3*n + 10935*m*n^4 + 405*m^4*n + 1458*m*n^5 +
 18*m^5*n + 5104*m^2 + 1665*m^3 + 295*m^4 + 27*m^5 + m^6 + 45936*n^2 + 449
55*n^3 + 23895*n^4 + 6561*n^5 + 729*n^6 + 15930*m^2*n^2 + 7290*m^2*n^3 + 2
430*m^3*n^2 + 1215*m^2*n^4 + 540*m^3*n^3 + 135*m^4*n^2 + 5040))/(13068*m +
 39204*n + 78792*m*n + 182763*m*n^2 + 60921*m^2*n + 211680*m*n^3 + 23520*m
^3*n + 130410*m*n^4 + 4830*m^4*n + 40824*m*n^5 + 504*m^5*n + 5103*m*n^6 +
21*m^6*n + 13132*m^2 + 6769*m^3 + 1960*m^4 + 322*m^5 + 28*m^6 + m^7 + 1181
88*n^2 + 182763*n^3 + 158760*n^4 + 78246*n^5 + 20412*n^6 + 2187*n^7 + 1058
40*m^2*n^2 + 86940*m^2*n^3 + 28980*m^3*n^2 + 34020*m^2*n^4 + 15120*m^3*n^3
 + 3780*m^4*n^2 + 5103*m^2*n^5 + 2835*m^3*n^4 + 945*m^4*n^3 + 189*m^5*n^2
+ 5040) + (b^3*x*x^(3*n + 3)*(d*x)^m*(2952*m + 8856*n + 15270*m*n + 28512*
m*n^2 + 9504*m^2*n + 24408*m*n^3 + 2712*m^3*n + 9720*m*n^4 + 360*m^4*n + 1
458*m*n^5 + 18*m^5*n + 2545*m^2 + 1056*m^3 + 226*m^4 + 24*m^5 + m^6 + 2290
5*n^2 + 28512*n^3 + 18306*n^4 + 5832*n^5 + 729*n^6 + 12204*m^2*n^2 + 6480*
m^2*n^3 + 2160*m^3*n^2 + 1215*m^2*n^4 + 540*m^3*n^3 + 135*m^4*n^2 + 1260))
/(13068*m + 39204*n + 78792*m*n + 182763*m*n^2 + 60921*m^2*n + 211680*m*n^
3 + 23520*m^3*n + 130410*m*n^4 + 4830*m^4*n + 40824*m*n^5 + 504*m^5*n + 51
03*m*n^6 + 21*m^6*n + 13132*m^2 + 6769*m^3 + 1960*m^4 + 322*m^5 + 28*m^6 +
 m^7 + 118188*n^2 + 182763*n^3 + 158760*n^4 + 78246*n^5 + 20412*n^6 + 2...
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 3510, normalized size of antiderivative = 18.38

$$\int (dx)^m \left(ax^n + bx^{1+n} + cx^{2+n}\right)^3 \, dx = \text{Too large to display}$$

input
```
int((d*x)^m*(a*x^n+b*x^(1+n)+c*x^(2+n))^3,x)
```

output
```
(x**(m + 3*n)*d**m*x*(a**3*m**6 + 18*a**3*m**5*n + 27*a**3*m**5 + 135*a**3
*m**4*n**2 + 405*a**3*m**4*n + 295*a**3*m**4 + 540*a**3*m**3*n**3 + 2430*a
**3*m**3*n**2 + 3540*a**3*m**3*n + 1665*a**3*m**3 + 1215*a**3*m**2*n**4 +
7290*a**3*m**2*n**3 + 15930*a**3*m**2*n**2 + 14985*a**3*m**2*n + 5104*a**3
*m**2 + 1458*a**3*m*n**5 + 10935*a**3*m*n**4 + 31860*a**3*m*n**3 + 44955*a
**3*m*n**2 + 30624*a**3*m*n + 8028*a**3*m + 729*a**3*n**6 + 6561*a**3*n**5
 + 23895*a**3*n**4 + 44955*a**3*n**3 + 45936*a**3*n**2 + 24084*a**3*n + 50
40*a**3 + 3*a**2*b*m**6*x + 54*a**2*b*m**5*n*x + 78*a**2*b*m**5*x + 405*a*
*2*b*m**4*n**2*x + 1170*a**2*b*m**4*n*x + 810*a**2*b*m**4*x + 1620*a**2*b*
m**3*n**3*x + 7020*a**2*b*m**3*n**2*x + 9720*a**2*b*m**3*n*x + 4260*a**2*b
*m**3*x + 3645*a**2*b*m**2*n**4*x + 21060*a**2*b*m**2*n**3*x + 43740*a**2*
b*m**2*n**2*x + 38340*a**2*b*m**2*n*x + 11787*a**2*b*m**2*x + 4374*a**2*b*
m*n**5*x + 31590*a**2*b*m*n**4*x + 87480*a**2*b*m*n**3*x + 115020*a**2*b*m
*n**2*x + 70722*a**2*b*m*n*x + 15822*a**2*b*m*x + 2187*a**2*b*n**6*x + 189
54*a**2*b*n**5*x + 65610*a**2*b*n**4*x + 115020*a**2*b*n**3*x + 106083*a**
2*b*n**2*x + 47466*a**2*b*n*x + 7560*a**2*b*x + 3*a**2*c*m**6*x**2 + 54*a*
*2*c*m**5*n*x**2 + 75*a**2*c*m**5*x**2 + 405*a**2*c*m**4*n**2*x**2 + 1125*
a**2*c*m**4*n*x**2 + 741*a**2*c*m**4*x**2 + 1620*a**2*c*m**3*n**3*x**2 + 6
750*a**2*c*m**3*n**2*x**2 + 8892*a**2*c*m**3*n*x**2 + 3657*a**2*c*m**3*x**
2 + 3645*a**2*c*m**2*n**4*x**2 + 20250*a**2*c*m**2*n**3*x**2 + 40014*a*...
```

3.83 $\int (dx)^m \left(ax^n+bx^{1+n}+cx^{2+n}\right)^2 \, dx$

Optimal result

Integrand size = 28, antiderivative size = 126

$$\int (dx)^m \left(ax^n+bx^{1+n}+cx^{2+n}\right)^2 \, dx = \frac{2abx^{2(1+n)}(dx)^m}{2+m+2n}+\frac{2bcx^{2(2+n)}(dx)^m}{4+m+2n} + \frac{a^2x^{1+2n}(dx)^m}{1+m+2n}+\frac{\left(b^2+2ac\right)x^{3+2n}(dx)^m}{3+m+2n} + \frac{c^2x^{5+2n}(dx)^m}{5+m+2n}$$

output
```
2*a*b*x^(2+2*n)*(d*x)^m/(2+m+2*n)+2*b*c*x^(4+2*n)*(d*x)^m/(4+m+2*n)+a^2*x^
(1+2*n)*(d*x)^m/(1+m+2*n)+(2*a*c+b^2)*x^(3+2*n)*(d*x)^m/(3+m+2*n)+c^2*x^(5
+2*n)*(d*x)^m/(5+m+2*n)
```

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.13

$$\int (dx)^m \left(ax^n+bx^{1+n}+cx^{2+n}\right)^2 \, dx = \frac{x^{1+2n}(dx)^m\left(\frac{2x\left(-\frac{6ab}{2+m+2n}+\frac{\left(b^2(1+m+2n)-4ac(4+m+2n)\right)x}{3+m+2n}\right)}{(4+m+2n)(5+m+2n)}-\frac{2x(b(7+m+2n)+2c(4+m+2n)x)(a+x(b+cx))}{(4+m+2n)(5+m+2n)}+(a+x(b+cx\right.}{1+m+2n}$$

input
```
Integrate[(d*x)^m*(a*x^n + b*x^(1 + n) + c*x^(2 + n))^2,x]
```

output
```
(x^(1 + 2*n)*(d*x)^m*((2*x*((-6*a*b)/(2 + m + 2*n) + ((b^2*(1 + m + 2*n) -
 4*a*c*(4 + m + 2*n))*x)/(3 + m + 2*n)))/((4 + m + 2*n)*(5 + m + 2*n)) - (
2*x*(b*(7 + m + 2*n) + 2*c*(4 + m + 2*n)*x)*(a + x*(b + c*x)))/((4 + m + 2
*n)*(5 + m + 2*n)) + (a + x*(b + c*x))^2))/(1 + m + 2*n)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.93, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2028, 30, 1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^m \left(ax^n + bx^{n+1} + cx^{n+2}\right)^2 dx$$

$$\Big\downarrow \text{2028}$$

$$\int x^{2n}(dx)^m \left(a + bx + cx^2\right)^2 dx$$

$$\Big\downarrow \text{30}$$

$$x^{-m}(dx)^m \int x^{m+2n}\left(cx^2 + bx + a\right)^2 dx$$

$$\Big\downarrow \text{1140}$$

$$x^{-m}(dx)^m \int \left(a^2x^{m+2n} + 2abx^{m+2n+1} + \left(b^2 + 2ac\right)x^{m+2n+2} + 2bcx^{m+2n+3} + c^2x^{m+2n+4}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$x^{-m}(dx)^m \left(\frac{a^2x^{m+2n+1}}{m+2n+1} + \frac{\left(2ac+b^2\right)x^{m+2n+3}}{m+2n+3} + \frac{2abx^{m+2n+2}}{m+2n+2} + \frac{2bcx^{m+2n+4}}{m+2n+4} + \frac{c^2x^{m+2n+5}}{m+2n+5}\right)$$

input
```
Int[(d*x)^m*(a*x^n + b*x^(1 + n) + c*x^(2 + n))^2,x]
```

output
```
((d*x)^m*((a^2*x^(1 + m + 2*n))/(1 + m + 2*n) + (2*a*b*x^(2 + m + 2*n))/(2
 + m + 2*n) + ((b^2 + 2*a*c)*x^(3 + m + 2*n))/(3 + m + 2*n) + (2*b*c*x^(4
+ m + 2*n))/(4 + m + 2*n) + (c^2*x^(5 + m + 2*n))/(5 + m + 2*n)))/x^m
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2028
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.) + (c_.)*(x_)^(t_.))^(p_.),
x_Symbol] :> Int[x^(p*r)*(a + b*x^(s - r) + c*x^(t - r))^p*Fx, x] /; FreeQ[
{a, b, c, r, s, t}, x] && IntegerQ[p] && PosQ[s - r] && PosQ[t - r] &&  !(E
qQ[p, 1] && EqQ[u, 1])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 943 vs. 2(126) = 252.

Time = 3.18 (sec) , antiderivative size = 944, normalized size of antiderivative = 7.49

method	result
orering	$\underline{(c^2m^4x^4+8c^2m^3n\,x^4+24c^2m^2n^2x^4+32c^2m\,n^3x^4+16c^2n^4x^4+2bc\,m^4x^3+16bc\,m^3n\,x^3+48bc\,m^2n^2x^3+64bcm\,n^3x^3+32bc\,n}$
risch	$\underline{x(c^2m^4x^4+8c^2m^3n\,x^4+24c^2m^2n^2x^4+32c^2m\,n^3x^4+16c^2n^4x^4+2bc\,m^4x^3+16bc\,m^3n\,x^3+48bc\,m^2n^2x^3+64bcm\,n^3x^3+32bc}$
parallelrisch	Expression too large to display

input
```
int((d*x)^m*(x^n*a+b*x^(1+n)+c*x^(2+n))^2,x,method=_RETURNVERBOSE)
```

output
```
(c^2*m^4*x^4+8*c^2*m^3*n*x^4+24*c^2*m^2*n^2*x^4+32*c^2*m*n^3*x^4+16*c^2*n^
4*x^4+2*b*c*m^4*x^3+16*b*c*m^3*n*x^3+48*b*c*m^2*n^2*x^3+64*b*c*m*n^3*x^3+3
2*b*c*n^4*x^3+10*c^2*m^3*x^4+60*c^2*m^2*n*x^4+120*c^2*m*n^2*x^4+80*c^2*n^3
*x^4+2*a*c*m^4*x^2+16*a*c*m^3*n*x^2+48*a*c*m^2*n^2*x^2+64*a*c*m*n^3*x^2+32
*a*c*n^4*x^2+b^2*m^4*x^2+8*b^2*m^3*n*x^2+24*b^2*m^2*n^2*x^2+32*b^2*m*n^3*x
^2+16*b^2*n^4*x^2+22*b*c*m^3*x^3+132*b*c*m^2*n*x^3+264*b*c*m*n^2*x^3+176*b
*c*n^3*x^3+35*c^2*m^2*x^4+140*c^2*m*n*x^4+140*c^2*n^2*x^4+2*a*b*m^4*x+16*a
*b*m^3*n*x+48*a*b*m^2*n^2*x+64*a*b*m*n^3*x+32*a*b*n^4*x+24*a*c*m^3*x^2+144
*a*c*m^2*n*x^2+288*a*c*m*n^2*x^2+192*a*c*n^3*x^2+12*b^2*m^3*x^2+72*b^2*m^2
*n*x^2+144*b^2*m*n^2*x^2+96*b^2*n^3*x^2+82*b*c*m^2*x^3+328*b*c*m*n*x^3+328
*b*c*n^2*x^3+50*c^2*m*x^4+100*c^2*n*x^4+a^2*m^4+8*a^2*m^3*n+24*a^2*m^2*n^2
+32*a^2*m*n^3+16*a^2*n^4+26*a*b*m^3*x+156*a*b*m^2*n*x+312*a*b*m*n^2*x+208*
a*b*n^3*x+98*a*c*m^2*x^2+392*a*c*m*n*x^2+392*a*c*n^2*x^2+49*b^2*m^2*x^2+19
6*b^2*m*n*x^2+196*b^2*n^2*x^2+122*b*c*m*x^3+244*b*c*n*x^3+24*c^2*x^4+14*a^
2*m^3+84*a^2*m^2*n+168*a^2*m*n^2+112*a^2*n^3+118*a*b*m^2*x+472*a*b*m*n*x+4
72*a*b*n^2*x+156*a*c*m*x^2+312*a*c*n*x^2+78*b^2*m*x^2+156*b^2*n*x^2+60*b*c
*x^3+71*a^2*m^2+284*a^2*m*n+284*a^2*n^2+214*a*b*m*x+428*a*b*n*x+80*a*c*x^2
+40*b^2*x^2+154*a^2*m+308*a^2*n+120*a*b*x+120*a^2)/(1+m+2*n)/(2+m+2*n)/(3+
m+2*n)/(4+m+2*n)/(5+m+2*n)/(c*x^2+b*x+a)^2*x*(d*x)^m*(x^n*a+b*x^(1+n)+c*x^
(2+n))^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 753 vs. $2(126) = 252$.

Time = 0.11 (sec) , antiderivative size = 753, normalized size of antiderivative = 5.98

$$\int (dx)^m \left(ax^n + bx^{1+n} + cx^{2+n}\right)^2 \, dx = \text{Too large to display}$$

input
```
integrate((d*x)^m*(a*x^n+b*x^(1+n)+c*x^(2+n))^2,x, algorithm="fricas")
```

output
```
(a^2*m^4 + 16*a^2*n^4 + 14*a^2*m^3 + (c^2*m^4 + 16*c^2*n^4 + 10*c^2*m^3 +
35*c^2*m^2 + 16*(2*c^2*m + 5*c^2)*n^3 + 50*c^2*m + 4*(6*c^2*m^2 + 30*c^2*m
 + 35*c^2)*n^2 + 24*c^2 + 4*(2*c^2*m^3 + 15*c^2*m^2 + 35*c^2*m + 25*c^2)*n
)*x^4 + 71*a^2*m^2 + 16*(2*a^2*m + 7*a^2)*n^3 + 2*(b*c*m^4 + 16*b*c*n^4 +
11*b*c*m^3 + 41*b*c*m^2 + 8*(4*b*c*m + 11*b*c)*n^3 + 61*b*c*m + 4*(6*b*c*m
^2 + 33*b*c*m + 41*b*c)*n^2 + 30*b*c + 2*(4*b*c*m^3 + 33*b*c*m^2 + 82*b*c*
m + 61*b*c)*n)*x^3 + 154*a^2*m + 4*(6*a^2*m^2 + 42*a^2*m + 71*a^2)*n^2 + (
(b^2 + 2*a*c)*m^4 + 16*(b^2 + 2*a*c)*n^4 + 12*(b^2 + 2*a*c)*m^3 + 32*(3*b^
2 + 6*a*c + (b^2 + 2*a*c)*m)*n^3 + 49*(b^2 + 2*a*c)*m^2 + 4*(6*(b^2 + 2*a*
c)*m^2 + 49*b^2 + 98*a*c + 36*(b^2 + 2*a*c)*m)*n^2 + 40*b^2 + 80*a*c + 78*
(b^2 + 2*a*c)*m + 4*(2*(b^2 + 2*a*c)*m^3 + 18*(b^2 + 2*a*c)*m^2 + 39*b^2 +
 78*a*c + 49*(b^2 + 2*a*c)*m)*n)*x^2 + 120*a^2 + 4*(2*a^2*m^3 + 21*a^2*m^2
 + 71*a^2*m + 77*a^2)*n + 2*(a*b*m^4 + 16*a*b*n^4 + 13*a*b*m^3 + 59*a*b*m^
2 + 8*(4*a*b*m + 13*a*b)*n^3 + 107*a*b*m + 4*(6*a*b*m^2 + 39*a*b*m + 59*a*
b)*n^2 + 60*a*b + 2*(4*a*b*m^3 + 39*a*b*m^2 + 118*a*b*m + 107*a*b)*n)*x)*x
^(2*n + 4)*e^(m*log(d) + m*log(x))/((m^5 + 80*(m + 3)*n^4 + 32*n^5 + 15*m^
4 + 40*(2*m^2 + 12*m + 17)*n^3 + 85*m^3 + 20*(2*m^3 + 18*m^2 + 51*m + 45)*
n^2 + 225*m^2 + 2*(5*m^4 + 60*m^3 + 255*m^2 + 450*m + 274)*n + 274*m + 120
)*x^3)
```

Sympy [F(-1)]

Timed out.

$$\int (dx)^m \left(ax^n + bx^{1+n} + cx^{2+n}\right)^2 \, dx = \text{Timed out}$$

input
```
integrate((d*x)**m*(a*x**n+b*x**(1+n)+c*x**(2+n))**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.37

$$\int (dx)^m \left(ax^n + bx^{1+n} + cx^{2+n}\right)^2 dx = \frac{c^2 d^m x^5 e^{(m\log(x)+2n\log(x))}}{m+2n+5} + \frac{2bcd^m x^4 e^{(m\log(x)+2n\log(x))}}{m+2n+4} + \frac{b^2 d^m x^3 e^{(m\log(x)+2n\log(x))}}{m+2n+3} + \frac{2acd^m x^3 e^{(m\log(x)+2n\log(x))}}{m+2n+3} + \frac{2abd^m x^2 e^{(m\log(x)+2n\log(x))}}{m+2n+2} + \frac{a^2 d^m x e^{(m\log(x)+2n\log(x))}}{m+2n+1}$$

input
```
integrate((d*x)^m*(a*x^n+b*x^(1+n)+c*x^(2+n))^2,x, algorithm="maxima")
```

output
```
c^2*d^m*x^5*e^(m*log(x) + 2*n*log(x))/(m + 2*n + 5) + 2*b*c*d^m*x^4*e^(m*l
og(x) + 2*n*log(x))/(m + 2*n + 4) + b^2*d^m*x^3*e^(m*log(x) + 2*n*log(x))/
(m + 2*n + 3) + 2*a*c*d^m*x^3*e^(m*log(x) + 2*n*log(x))/(m + 2*n + 3) + 2*
a*b*d^m*x^2*e^(m*log(x) + 2*n*log(x))/(m + 2*n + 2) + a^2*d^m*x*e^(m*log(x
) + 2*n*log(x))/(m + 2*n + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2370 vs. 2(126) = 252.

Time = 0.20 (sec) , antiderivative size = 2370, normalized size of antiderivative = 18.81

$$\int (dx)^m \left(ax^n + bx^{1+n} + cx^{2+n}\right)^2 dx = \text{Too large to display}$$

input
```
integrate((d*x)^m*(a*x^n+b*x^(1+n)+c*x^(2+n))^2,x, algorithm="giac")
```

output
```
(c^2*m^4*x^5*x^(2*n)*e^(m*log(d) + m*log(x)) + 8*c^2*m^3*n*x^5*x^(2*n)*e^(
m*log(d) + m*log(x)) + 24*c^2*m^2*n^2*x^5*x^(2*n)*e^(m*log(d) + m*log(x))
+ 32*c^2*m*n^3*x^5*x^(2*n)*e^(m*log(d) + m*log(x)) + 16*c^2*n^4*x^5*x^(2*n
)*e^(m*log(d) + m*log(x)) + 2*b*c*m^4*x^4*x^(2*n)*e^(m*log(d) + m*log(x))
+ 16*b*c*m^3*n*x^4*x^(2*n)*e^(m*log(d) + m*log(x)) + 48*b*c*m^2*n^2*x^4*x^
(2*n)*e^(m*log(d) + m*log(x)) + 64*b*c*m*n^3*x^4*x^(2*n)*e^(m*log(d) + m*l
og(x)) + 32*b*c*n^4*x^4*x^(2*n)*e^(m*log(d) + m*log(x)) + 10*c^2*m^3*x^5*x
^(2*n)*e^(m*log(d) + m*log(x)) + 60*c^2*m^2*n*x^5*x^(2*n)*e^(m*log(d) + m*
log(x)) + 120*c^2*m*n^2*x^5*x^(2*n)*e^(m*log(d) + m*log(x)) + 80*c^2*n^3*x
^5*x^(2*n)*e^(m*log(d) + m*log(x)) + b^2*m^4*x^3*x^(2*n)*e^(m*log(d) + m*l
og(x)) + 2*a*c*m^4*x^3*x^(2*n)*e^(m*log(d) + m*log(x)) + 8*b^2*m^3*n*x^3*x
^(2*n)*e^(m*log(d) + m*log(x)) + 16*a*c*m^3*n*x^3*x^(2*n)*e^(m*log(d) + m*
log(x)) + 24*b^2*m^2*n^2*x^3*x^(2*n)*e^(m*log(d) + m*log(x)) + 48*a*c*m^2*
n^2*x^3*x^(2*n)*e^(m*log(d) + m*log(x)) + 32*b^2*m*n^3*x^3*x^(2*n)*e^(m*lo
g(d) + m*log(x)) + 64*a*c*m*n^3*x^3*x^(2*n)*e^(m*log(d) + m*log(x)) + 16*b
^2*n^4*x^3*x^(2*n)*e^(m*log(d) + m*log(x)) + 32*a*c*n^4*x^3*x^(2*n)*e^(m*l
og(d) + m*log(x)) + 22*b*c*m^3*x^4*x^(2*n)*e^(m*log(d) + m*log(x)) + 132*b
*c*m^2*n*x^4*x^(2*n)*e^(m*log(d) + m*log(x)) + 264*b*c*m*n^2*x^4*x^(2*n)*e
^(m*log(d) + m*log(x)) + 176*b*c*n^3*x^4*x^(2*n)*e^(m*log(d) + m*log(x)) +
 35*c^2*m^2*x^5*x^(2*n)*e^(m*log(d) + m*log(x)) + 140*c^2*m*n*x^5*x^(2*...
```

Mupad [B] (verification not implemented)

Time = 12.80 (sec) , antiderivative size = 1210, normalized size of antiderivative = 9.60

$$\int (dx)^m \left(ax^n + bx^{1+n} + cx^{2+n}\right)^2 \, dx = \text{Too large to display}$$

input
```
int((d*x)^m*(a*x^n + b*x^(n + 1) + c*x^(n + 2))^2,x)
```

output
```
(b^2*x*x^(2*n + 2)*(d*x)^m*(78*m + 156*n + 196*m*n + 144*m*n^2 + 72*m^2*n
+ 32*m*n^3 + 8*m^3*n + 49*m^2 + 12*m^3 + m^4 + 196*n^2 + 96*n^3 + 16*n^4 +
 24*m^2*n^2 + 40))/(274*m + 548*n + 900*m*n + 1020*m*n^2 + 510*m^2*n + 480
*m*n^3 + 120*m^3*n + 80*m*n^4 + 10*m^4*n + 225*m^2 + 85*m^3 + 15*m^4 + m^5
 + 900*n^2 + 680*n^3 + 240*n^4 + 32*n^5 + 360*m^2*n^2 + 80*m^2*n^3 + 40*m^
3*n^2 + 120) + (c^2*x*x^(2*n + 4)*(d*x)^m*(50*m + 100*n + 140*m*n + 120*m*
n^2 + 60*m^2*n + 32*m*n^3 + 8*m^3*n + 35*m^2 + 10*m^3 + m^4 + 140*n^2 + 80
*n^3 + 16*n^4 + 24*m^2*n^2 + 24))/(274*m + 548*n + 900*m*n + 1020*m*n^2 +
510*m^2*n + 480*m*n^3 + 120*m^3*n + 80*m*n^4 + 10*m^4*n + 225*m^2 + 85*m^3
 + 15*m^4 + m^5 + 900*n^2 + 680*n^3 + 240*n^4 + 32*n^5 + 360*m^2*n^2 + 80*
m^2*n^3 + 40*m^3*n^2 + 120) + (a^2*x*x^(2*n)*(d*x)^m*(154*m + 308*n + 284*
m*n + 168*m*n^2 + 84*m^2*n + 32*m*n^3 + 8*m^3*n + 71*m^2 + 14*m^3 + m^4 +
284*n^2 + 112*n^3 + 16*n^4 + 24*m^2*n^2 + 120))/(274*m + 548*n + 900*m*n +
 1020*m*n^2 + 510*m^2*n + 480*m*n^3 + 120*m^3*n + 80*m*n^4 + 10*m^4*n + 22
5*m^2 + 85*m^3 + 15*m^4 + m^5 + 900*n^2 + 680*n^3 + 240*n^4 + 32*n^5 + 360
*m^2*n^2 + 80*m^2*n^3 + 40*m^3*n^2 + 120) + (2*b*c*x*x^(n + 1)*x^(n + 2)*(
d*x)^m*(61*m + 122*n + 164*m*n + 132*m*n^2 + 66*m^2*n + 32*m*n^3 + 8*m^3*n
 + 41*m^2 + 11*m^3 + m^4 + 164*n^2 + 88*n^3 + 16*n^4 + 24*m^2*n^2 + 30))/(
274*m + 548*n + 900*m*n + 1020*m*n^2 + 510*m^2*n + 480*m*n^3 + 120*m^3*n +
 80*m*n^4 + 10*m^4*n + 225*m^2 + 85*m^3 + 15*m^4 + m^5 + 900*n^2 + 680*...
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 986, normalized size of antiderivative = 7.83

$$\int (dx)^m \left(ax^n + bx^{1+n} + cx^{2+n}\right)^2 \, dx = \text{Too large to display}$$

input
```
int((d*x)^m*(a*x^n+b*x^(1+n)+c*x^(2+n))^2,x)
```

output
```
(x**(m + 2*n)*d**m*x*(a**2*m**4 + 8*a**2*m**3*n + 14*a**2*m**3 + 24*a**2*m
**2*n**2 + 84*a**2*m**2*n + 71*a**2*m**2 + 32*a**2*m*n**3 + 168*a**2*m*n**
2 + 284*a**2*m*n + 154*a**2*m + 16*a**2*n**4 + 112*a**2*n**3 + 284*a**2*n*
*2 + 308*a**2*n + 120*a**2 + 2*a*b*m**4*x + 16*a*b*m**3*n*x + 26*a*b*m**3*
x + 48*a*b*m**2*n**2*x + 156*a*b*m**2*n*x + 118*a*b*m**2*x + 64*a*b*m*n**3
*x + 312*a*b*m*n**2*x + 472*a*b*m*n*x + 214*a*b*m*x + 32*a*b*n**4*x + 208*
a*b*n**3*x + 472*a*b*n**2*x + 428*a*b*n*x + 120*a*b*x + 2*a*c*m**4*x**2 +
16*a*c*m**3*n*x**2 + 24*a*c*m**3*x**2 + 48*a*c*m**2*n**2*x**2 + 144*a*c*m*
*2*n*x**2 + 98*a*c*m**2*x**2 + 64*a*c*m*n**3*x**2 + 288*a*c*m*n**2*x**2 +
392*a*c*m*n*x**2 + 156*a*c*m*x**2 + 32*a*c*n**4*x**2 + 192*a*c*n**3*x**2 +
 392*a*c*n**2*x**2 + 312*a*c*n*x**2 + 80*a*c*x**2 + b**2*m**4*x**2 + 8*b**
2*m**3*n*x**2 + 12*b**2*m**3*x**2 + 24*b**2*m**2*n**2*x**2 + 72*b**2*m**2*
n*x**2 + 49*b**2*m**2*x**2 + 32*b**2*m*n**3*x**2 + 144*b**2*m*n**2*x**2 +
196*b**2*m*n*x**2 + 78*b**2*m*x**2 + 16*b**2*n**4*x**2 + 96*b**2*n**3*x**2
 + 196*b**2*n**2*x**2 + 156*b**2*n*x**2 + 40*b**2*x**2 + 2*b*c*m**4*x**3 +
 16*b*c*m**3*n*x**3 + 22*b*c*m**3*x**3 + 48*b*c*m**2*n**2*x**3 + 132*b*c*m
**2*n*x**3 + 82*b*c*m**2*x**3 + 64*b*c*m*n**3*x**3 + 264*b*c*m*n**2*x**3 +
 328*b*c*m*n*x**3 + 122*b*c*m*x**3 + 32*b*c*n**4*x**3 + 176*b*c*n**3*x**3
+ 328*b*c*n**2*x**3 + 244*b*c*n*x**3 + 60*b*c*x**3 + c**2*m**4*x**4 + 8*c*
*2*m**3*n*x**4 + 10*c**2*m**3*x**4 + 24*c**2*m**2*n**2*x**4 + 60*c**2*m...
```

3.84 $\int (dx)^m \left(ax^n+bx^{1+n}+cx^{2+n}\right) dx$

Optimal result

Integrand size = 26, antiderivative size = 55

$$\int (dx)^m \left(ax^n+bx^{1+n}+cx^{2+n}\right) dx=\frac{ax^{1+n}(dx)^m}{1+m+n}+\frac{bx^{2+n}(dx)^m}{2+m+n}+\frac{cx^{3+n}(dx)^m}{3+m+n}$$

output
```
a*x^(1+n)*(d*x)^m/(1+m+n)+b*x^(2+n)*(d*x)^m/(2+m+n)+c*x^(3+n)*(d*x)^m/(3+m
+n)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.73

$$\int (dx)^m \left(ax^n+bx^{1+n}+cx^{2+n}\right) dx=x^{1+n}(dx)^m\left(\frac{a}{1+m+n}+x\left(\frac{b}{2+m+n}+\frac{cx}{3+m+n}\right)\right)$$

input
```
Integrate[(d*x)^m*(a*x^n + b*x^(1 + n) + c*x^(2 + n)),x]
```

output
```
x^(1 + n)*(d*x)^m*(a/(1 + m + n) + x*(b/(2 + m + n) + (c*x)/(3 + m + n)))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^m \left(ax^n + bx^{n+1} + cx^{n+2}\right) \, dx$$

$$\Big\downarrow \ 2010$$

$$\int \left(ax^n (dx)^m + bx^{n+1} (dx)^m + cx^{n+2} (dx)^m\right) dx$$

$$\Big\downarrow \ 2009$$

$$\frac{ax^{n+1}(dx)^m}{m+n+1} + \frac{bx^{n+2}(dx)^m}{m+n+2} + \frac{cx^{n+3}(dx)^m}{m+n+3}$$

input
```
Int[(d*x)^m*(a*x^n + b*x^(1 + n) + c*x^(2 + n)),x]
```

output
```
(a*x^(1 + n)*(d*x)^m)/(1 + m + n) + (b*x^(2 + n)*(d*x)^m)/(2 + m + n) + (c
*x^(3 + n)*(d*x)^m)/(3 + m + n)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(55) = 110$.

Time = 0.46 (sec) , antiderivative size = 161, normalized size of antiderivative = 2.93

method	result
orering	$\frac{(c m^2 x^2+2cmn x^2+c n^2 x^2+b m^2 x+2bmnx+b n^2 x+3x^2 cm+3c x^2 n+a m^2+2amn+a n^2+4bxm+4bxn+2c x^2+5am+5an+3}{(1+m+n)(2+m+n)(3+m+n)(c x^2+bx+a)}$
risch	$\frac{x(c m^2 x^2+2cmn x^2+c n^2 x^2+b m^2 x+2bmnx+b n^2 x+3x^2 cm+3c x^2 n+a m^2+2amn+a n^2+4bxm+4bxn+2c x^2+5am+5an+}{(1+m+n)(2+m+n)(3+m+n)}$
parallelrisch	$\frac{2x\, x^n (dx)^m amn+2x\, x^{1+n} (dx)^m bmn+2x\, x^{2+n} (dx)^m cmn+6x\, x^n (dx)^m a+3x\, x^{1+n} (dx)^m b+2x\, x^{2+n} (dx)^m c+x\, x^n (dx)^m a\, n}{}$

input
```
int((d*x)^m*(x^n*a+b*x^(1+n)+c*x^(2+n)),x,method=_RETURNVERBOSE)
```

output
```
(c*m^2*x^2+2*c*m*n*x^2+c*n^2*x^2+b*m^2*x+2*b*m*n*x+b*n^2*x+3*c*m*x^2+3*c*n
*x^2+a*m^2+2*a*m*n+a*n^2+4*b*m*x+4*b*n*x+2*c*x^2+5*a*m+5*a*n+3*b*x+6*a)/(1
+m+n)/(2+m+n)/(3+m+n)*x/(c*x^2+b*x+a)*(d*x)^m*(x^n*a+b*x^(1+n)+c*x^(2+n))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 147 vs. $2(55) = 110$.

Time = 0.09 (sec) , antiderivative size = 147, normalized size of antiderivative = 2.67

$$\int (dx)^m \left(ax^n + bx^{1+n} + cx^{2+n}\right) dx$$
$$= \frac{(am^2 + an^2 + (cm^2 + cn^2 + 3\,cm + (2\,cm + 3\,c)n + 2\,c)x^2 + 5\,am + (2\,am + 5\,a)n + (bm^2 + bn^2 + 4\,bn}{(m^3 + 3\,(m+2)n^2 + n^3 + 6\,m^2 + (3\,m^2 + 12\,m + 11)n + 11}$$

input
```
integrate((d*x)^m*(a*x^n+b*x^(1+n)+c*x^(2+n)),x, algorithm="fricas")
```

output
```
(a*m^2 + a*n^2 + (c*m^2 + c*n^2 + 3*c*m + (2*c*m + 3*c)*n + 2*c)*x^2 + 5*a
*m + (2*a*m + 5*a)*n + (b*m^2 + b*n^2 + 4*b*m + 2*(b*m + 2*b)*n + 3*b)*x +
 6*a)*x^(n + 2)*e^(m*log(d) + m*log(x))/((m^3 + 3*(m + 2)*n^2 + n^3 + 6*m^
2 + (3*m^2 + 12*m + 11)*n + 11*m + 6)*x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1258 vs. 2(49) = 98.

Time = 20.02 (sec) , antiderivative size = 1258, normalized size of antiderivative = 22.87

$$\int (dx)^m \left(ax^n + bx^{1+n} + cx^{2+n}\right) dx = \text{Too large to display}$$

input
```
integrate((d*x)**m*(a*x**n+b*x**(1+n)+c*x**(2+n)),x)
```

output
```
Piecewise((-a*x*x**n*(d*x)**(-n - 3)/2 - b*x*x**(n + 1)*(d*x)**(-n - 3) +
c*x*x**(n + 2)*(d*x)**(-n - 3)*log(x), Eq(m, -n - 3)), (-a*x*x**n*(d*x)**(
-n - 2) + b*x*x**(n + 1)*(d*x)**(-n - 2)*log(x) + c*x*x**(n + 2)*(d*x)**(-
n - 2), Eq(m, -n - 2)), (a*x*x**n*(d*x)**(-n - 1)*log(x) + b*x*x**(n + 1)*
(d*x)**(-n - 1) + c*x*x**(n + 2)*(d*x)**(-n - 1)/2, Eq(m, -n - 1)), (a*m**
2*x*x**n*(d*x)**m/(m**3 + 3*m**2*n + 6*m**2 + 3*m*n**2 + 12*m*n + 11*m + n
**3 + 6*n**2 + 11*n + 6) + 2*a*m*n*x*x**n*(d*x)**m/(m**3 + 3*m**2*n + 6*m*
*2 + 3*m*n**2 + 12*m*n + 11*m + n**3 + 6*n**2 + 11*n + 6) + 5*a*m*x*x**n*(
d*x)**m/(m**3 + 3*m**2*n + 6*m**2 + 3*m*n**2 + 12*m*n + 11*m + n**3 + 6*n*
*2 + 11*n + 6) + a*n**2*x*x**n*(d*x)**m/(m**3 + 3*m**2*n + 6*m**2 + 3*m*n*
*2 + 12*m*n + 11*m + n**3 + 6*n**2 + 11*n + 6) + 5*a*n*x*x**n*(d*x)**m/(m*
*3 + 3*m**2*n + 6*m**2 + 3*m*n**2 + 12*m*n + 11*m + n**3 + 6*n**2 + 11*n +
 6) + 6*a*x*x**n*(d*x)**m/(m**3 + 3*m**2*n + 6*m**2 + 3*m*n**2 + 12*m*n +
11*m + n**3 + 6*n**2 + 11*n + 6) + b*m**2*x*x**(n + 1)*(d*x)**m/(m**3 + 3*
m**2*n + 6*m**2 + 3*m*n**2 + 12*m*n + 11*m + n**3 + 6*n**2 + 11*n + 6) + 2
*b*m*n*x*x**(n + 1)*(d*x)**m/(m**3 + 3*m**2*n + 6*m**2 + 3*m*n**2 + 12*m*n
 + 11*m + n**3 + 6*n**2 + 11*n + 6) + 4*b*m*x*x**(n + 1)*(d*x)**m/(m**3 +
3*m**2*n + 6*m**2 + 3*m*n**2 + 12*m*n + 11*m + n**3 + 6*n**2 + 11*n + 6) +
 b*n**2*x*x**(n + 1)*(d*x)**m/(m**3 + 3*m**2*n + 6*m**2 + 3*m*n**2 + 12*m*
n + 11*m + n**3 + 6*n**2 + 11*n + 6) + 4*b*n*x*x**(n + 1)*(d*x)**m/(m**...
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.29

$$\int (dx)^m \left(ax^n + bx^{1+n} + cx^{2+n}\right) dx = \frac{cd^m x^3 e^{(m\log(x)+n\log(x))}}{m+n+3} + \frac{bd^m x^2 e^{(m\log(x)+n\log(x))}}{m+n+2} + \frac{ad^m x e^{(m\log(x)+n\log(x))}}{m+n+1}$$

input
```
integrate((d*x)^m*(a*x^n+b*x^(1+n)+c*x^(2+n)),x, algorithm="maxima")
```

output
```
c*d^m*x^3*e^(m*log(x) + n*log(x))/(m + n + 3) + b*d^m*x^2*e^(m*log(x) + n*
log(x))/(m + n + 2) + a*d^m*x*e^(m*log(x) + n*log(x))/(m + n + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 398 vs. 2(55) = 110.

Time = 0.13 (sec) , antiderivative size = 398, normalized size of antiderivative = 7.24

$$\int (dx)^m \left(ax^n + bx^{1+n} + cx^{2+n}\right) dx$$

$$= \frac{cm^2x^3x^n e^{(m\log(d)+m\log(x))} + 2\,cmnx^3x^n e^{(m\log(d)+m\log(x))} + cn^2x^3x^n e^{(m\log(d)+m\log(x))} + bm^2x^2x^n e^{(m\log(d)+}}{}$$

input
```
integrate((d*x)^m*(a*x^n+b*x^(1+n)+c*x^(2+n)),x, algorithm="giac")
```

output
```
(c*m^2*x^3*x^n*e^(m*log(d) + m*log(x)) + 2*c*m*n*x^3*x^n*e^(m*log(d) + m*l
og(x)) + c*n^2*x^3*x^n*e^(m*log(d) + m*log(x)) + b*m^2*x^2*x^n*e^(m*log(d)
 + m*log(x)) + 2*b*m*n*x^2*x^n*e^(m*log(d) + m*log(x)) + b*n^2*x^2*x^n*e^(
m*log(d) + m*log(x)) + 3*c*m*x^3*x^n*e^(m*log(d) + m*log(x)) + 3*c*n*x^3*x
^n*e^(m*log(d) + m*log(x)) + a*m^2*x*x^n*e^(m*log(d) + m*log(x)) + 2*a*m*n
*x*x^n*e^(m*log(d) + m*log(x)) + a*n^2*x*x^n*e^(m*log(d) + m*log(x)) + 4*b
*m*x^2*x^n*e^(m*log(d) + m*log(x)) + 4*b*n*x^2*x^n*e^(m*log(d) + m*log(x))
 + 2*c*x^3*x^n*e^(m*log(d) + m*log(x)) + 5*a*m*x*x^n*e^(m*log(d) + m*log(x
)) + 5*a*n*x*x^n*e^(m*log(d) + m*log(x)) + 3*b*x^2*x^n*e^(m*log(d) + m*log
(x)) + 6*a*x*x^n*e^(m*log(d) + m*log(x)))/(m^3 + 3*m^2*n + 3*m*n^2 + n^3 +
 6*m^2 + 12*m*n + 6*n^2 + 11*m + 11*n + 6)
```

Mupad [B] (verification not implemented)

Time = 12.48 (sec) , antiderivative size = 218, normalized size of antiderivative = 3.96

$$\int (dx)^m \left(ax^n + bx^{1+n} + cx^{2+n}\right) dx$$
$$= \frac{a\,x\,x^n\,(d\,x)^m\,(m^2+2\,m\,n+5\,m+n^2+5\,n+6)}{m^3+3\,m^2\,n+6\,m^2+3\,m\,n^2+12\,m\,n+11\,m+n^3+6\,n^2+11\,n+6}$$
$$+ \frac{b\,x\,x^{n+1}\,(d\,x)^m\,(m^2+2\,m\,n+4\,m+n^2+4\,n+3)}{m^3+3\,m^2\,n+6\,m^2+3\,m\,n^2+12\,m\,n+11\,m+n^3+6\,n^2+11\,n+6}$$
$$+ \frac{c\,x\,x^{n+2}\,(d\,x)^m\,(m^2+2\,m\,n+3\,m+n^2+3\,n+2)}{m^3+3\,m^2\,n+6\,m^2+3\,m\,n^2+12\,m\,n+11\,m+n^3+6\,n^2+11\,n+6}$$

input
```
int((d*x)^m*(a*x^n + b*x^(n + 1) + c*x^(n + 2)),x)
```

output
```
(a*x*x^n*(d*x)^m*(5*m + 5*n + 2*m*n + m^2 + n^2 + 6))/(11*m + 11*n + 12*m*
n + 3*m*n^2 + 3*m^2*n + 6*m^2 + m^3 + 6*n^2 + n^3 + 6) + (b*x*x^(n + 1)*(d
*x)^m*(4*m + 4*n + 2*m*n + m^2 + n^2 + 3))/(11*m + 11*n + 12*m*n + 3*m*n^2
 + 3*m^2*n + 6*m^2 + m^3 + 6*n^2 + n^3 + 6) + (c*x*x^(n + 2)*(d*x)^m*(3*m
+ 3*n + 2*m*n + m^2 + n^2 + 2))/(11*m + 11*n + 12*m*n + 3*m*n^2 + 3*m^2*n
+ 6*m^2 + m^3 + 6*n^2 + n^3 + 6)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 155, normalized size of antiderivative = 2.82

$$\int (dx)^m \left(ax^n + bx^{1+n} + cx^{2+n}\right) dx$$
$$= \frac{x^{m+n} d^m x(c\,m^2x^2 + 2cmn\,x^2 + c\,n^2x^2 + b\,m^2x + 2bmnx + b\,n^2x + 3cm\,x^2 + 3cn\,x^2 + a\,m^2 + 2amn + a}{m^3 + 3m^2n + 3m\,n^2 + n^3 + 6m^2 + 12mn + 6n^2 + 11m +}$$

input
```
int((d*x)^m*(a*x^n+b*x^(1+n)+c*x^(2+n)),x)
```

output
```
(x**(m + n)*d**m*x*(a*m**2 + 2*a*m*n + 5*a*m + a*n**2 + 5*a*n + 6*a + b*m*
*2*x + 2*b*m*n*x + 4*b*m*x + b*n**2*x + 4*b*n*x + 3*b*x + c*m**2*x**2 + 2*
c*m*n*x**2 + 3*c*m*x**2 + c*n**2*x**2 + 3*c*n*x**2 + 2*c*x**2))/(m**3 + 3*
m**2*n + 6*m**2 + 3*m*n**2 + 12*m*n + 11*m + n**3 + 6*n**2 + 11*n + 6)
```

3.85 $\int \frac{(dx)^m}{ax^n+bx^{1+n}+cx^{2+n}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 175

$$\int \frac{(dx)^m}{ax^n+bx^{1+n}+cx^{2+n}}\,dx = \frac{2cx^{1-n}(dx)^m \,\text{Hypergeometric2F1}\left(1,1+m-n,2+m-n,-\frac{2cx}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)(1+m-n)} - \frac{2cx^{1-n}(dx)^m \,\text{Hypergeometric2F1}\left(1,1+m-n,2+m-n,-\frac{2cx}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}\right)(1+m-n)}$$

output
```
2*c*x^(1-n)*(d*x)^m*hypergeom([1, 1+m-n],[2+m-n],-2*c*x/(b-(-4*a*c+b^2)^(1
/2)))/(-4*a*c+b^2)^(1/2)/(b-(-4*a*c+b^2)^(1/2))/(1+m-n)-2*c*x^(1-n)*(d*x)^
m*hypergeom([1, 1+m-n],[2+m-n],-2*c*x/(b+(-4*a*c+b^2)^(1/2)))/(-4*a*c+b^2)
^(1/2)/(b+(-4*a*c+b^2)^(1/2))/(1+m-n)
```

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.81

$$\int \frac{(dx)^m}{ax^n + bx^{1+n} + cx^{2+n}}\,dx$$
$$= \frac{x^{1-n}(dx)^m \left(\left(b+\sqrt{b^2-4ac}\right) \text{Hypergeometric2F1}\left(1, 1+m-n, 2+m-n, \frac{2cx}{-b+\sqrt{b^2-4ac}}\right) + \left(-b+\sqrt{b^2\right.\right.}{2a\sqrt{b^2-4ac}(1+m-n)}$$

input
```
Integrate[(d*x)^m/(a*x^n + b*x^(1 + n) + c*x^(2 + n)),x]
```

output
```
(x^(1 - n)*(d*x)^m*((b + Sqrt[b^2 - 4*a*c])*Hypergeometric2F1[1, 1 + m - n
, 2 + m - n, (2*c*x)/(-b + Sqrt[b^2 - 4*a*c])] + (-b + Sqrt[b^2 - 4*a*c])*
Hypergeometric2F1[1, 1 + m - n, 2 + m - n, (-2*c*x)/(b + Sqrt[b^2 - 4*a*c]
)]))/(2*a*Sqrt[b^2 - 4*a*c]*(1 + m - n))
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2028, 30, 1150, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^m}{ax^n + bx^{n+1} + cx^{n+2}}\,dx$$

$$\Big\downarrow \; 2028$$

$$\int \frac{x^{-n}(dx)^m}{a+bx+cx^2}dx$$

$$\Big\downarrow \; 30$$

$$x^{-m}(dx)^m \int \frac{x^{m-n}}{cx^2+bx+a}dx$$

$$\Big\downarrow \; 1150$$

$$x^{-m}(dx)^m \int \left(\frac{2cx^{m-n}}{\sqrt{b^2-4ac}\left(b+2cx-\sqrt{b^2-4ac}\right)}-\frac{2cx^{m-n}}{\sqrt{b^2-4ac}\left(b+2cx+\sqrt{b^2-4ac}\right)}\right)dx$$

$$\downarrow \text{2009}$$

$$x^{-m}(dx)^m \left(\frac{2cx^{m-n+1}\,\text{Hypergeometric2F1}\left(1,m-n+1,m-n+2,-\frac{2cx}{b-\sqrt{b^2-4ac}}\right)}{(m-n+1)\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)}-\frac{2cx^{m-n+1}\,\text{Hypergeometr}}{(m-n+}\right.$$

input
```
Int[(d*x)^m/(a*x^n + b*x^(1 + n) + c*x^(2 + n)),x]
```

output
```
((d*x)^m*((2*c*x^(1 + m - n)*Hypergeometric2F1[1, 1 + m - n, 2 + m - n, (-
2*c*x)/(b - Sqrt[b^2 - 4*a*c])])/(Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c]
)*(1 + m - n)) - (2*c*x^(1 + m - n)*Hypergeometric2F1[1, 1 + m - n, 2 + m
- n, (-2*c*x)/(b + Sqrt[b^2 - 4*a*c])])/(Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 -
 4*a*c])*(1 + m - n))))/x^m
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 1150
```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Int[ExpandIntegrand[(d + e*x)^m, 1/(a + b*x + c*x^2), x], x] /; FreeQ[
{a, b, c, d, e, m}, x] &&  !IntegerQ[2*m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2028
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.) + (c_.)*(x_)^(t_.))^(p_.),
x_Symbol] :> Int[x^(p*r)*(a + b*x^(s - r) + c*x^(t - r))^p*Fx, x] /; FreeQ[
{a, b, c, r, s, t}, x] && IntegerQ[p] && PosQ[s - r] && PosQ[t - r] &&  !(E
qQ[p, 1] && EqQ[u, 1])
```

Maple [F]

$$\int \frac{(dx)^m}{x^n a + b\,x^{1+n} + c\,x^{2+n}}dx$$

input
```
int((d*x)^m/(x^n*a+b*x^(1+n)+c*x^(2+n)),x)
```

output
```
int((d*x)^m/(x^n*a+b*x^(1+n)+c*x^(2+n)),x)
```

Fricas [F]

$$\int \frac{(dx)^m}{ax^n + bx^{1+n} + cx^{2+n}}\,dx = \int \frac{(dx)^m}{cx^{n+2} + bx^{n+1} + ax^n}\,dx$$

input
```
integrate((d*x)^m/(a*x^n+b*x^(1+n)+c*x^(2+n)),x, algorithm="fricas")
```

output
```
integral((d*x)^m/(c*x^(n + 2) + b*x^(n + 1) + a*x^n), x)
```

Sympy [F]

$$\int \frac{(dx)^m}{ax^n + bx^{1+n} + cx^{2+n}}\,dx = \int \frac{(dx)^m}{ax^n + bx^{n+1} + cx^{n+2}}\,dx$$

input
```
integrate((d*x)**m/(a*x**n+b*x**(1+n)+c*x**(2+n)),x)
```

output
```
Integral((d*x)**m/(a*x**n + b*x**(n + 1) + c*x**(n + 2)), x)
```

Maxima [F]

$$\int \frac{(dx)^m}{ax^n + bx^{1+n} + cx^{2+n}}\,dx = \int \frac{(dx)^m}{cx^{n+2} + bx^{n+1} + ax^n}\,dx$$

input
```
integrate((d*x)^m/(a*x^n+b*x^(1+n)+c*x^(2+n)),x, algorithm="maxima")
```

output
```
integrate((d*x)^m/(c*x^(n + 2) + b*x^(n + 1) + a*x^n), x)
```

Giac [F]

$$\int \frac{(dx)^m}{ax^n + bx^{1+n} + cx^{2+n}}\,dx = \int \frac{(dx)^m}{cx^{n+2} + bx^{n+1} + ax^n}\,dx$$

input
```
integrate((d*x)^m/(a*x^n+b*x^(1+n)+c*x^(2+n)),x, algorithm="giac")
```

output
```
integrate((d*x)^m/(c*x^(n + 2) + b*x^(n + 1) + a*x^n), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(dx)^m}{ax^n + bx^{1+n} + cx^{2+n}}\,dx = \int \frac{(d\,x)^m}{a\,x^n + b\,x^{n+1} + c\,x^{n+2}}\,dx$$

input
```
int((d*x)^m/(a*x^n + b*x^(n + 1) + c*x^(n + 2)),x)
```

output
```
int((d*x)^m/(a*x^n + b*x^(n + 1) + c*x^(n + 2)), x)
```

Reduce [F]

$$\int \frac{(dx)^m}{ax^n + bx^{1+n} + cx^{2+n}}\,dx = d^m\left(\int \frac{x^m}{x^n a + x^n bx + x^n c\,x^2}dx\right)$$

input
```
int((d*x)^m/(a*x^n+b*x^(1+n)+c*x^(2+n)),x)
```

output
```
d**m*int(x**m/(x**n*a + x**n*b*x + x**n*c*x**2),x)
```

3.86 $\int \frac{(dx)^m}{(ax^n+bx^{1+n}+cx^{2+n})^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 300

$$\int \frac{(dx)^m}{(ax^n+bx^{1+n}+cx^{2+n})^2}\,dx = \frac{x^{1-2n}(dx)^m(b^2-2ac+bcx)}{a(b^2-4ac)(a+bx+cx^2)} - \frac{c\left(b\left(b+\sqrt{b^2-4ac}\right)(m-2n)+4ac(1-m+2n)\right)x^{1-2n}(dx)^m \operatorname{Hypergeometric2F1}\left(1,1+m-2n,2\right.}{a(b^2-4ac)^{3/2}\left(b-\sqrt{b^2-4ac}\right)(1+m-2n)} + \frac{c\left(b\left(b-\sqrt{b^2-4ac}\right)(m-2n)+4ac(1-m+2n)\right)x^{1-2n}(dx)^m \operatorname{Hypergeometric2F1}\left(1,1+m-2n,2\right.}{a(b^2-4ac)^{3/2}\left(b+\sqrt{b^2-4ac}\right)(1+m-2n)}$$

output

```
x^(1-2*n)*(d*x)^m*(b*c*x-2*a*c+b^2)/a/(-4*a*c+b^2)/(c*x^2+b*x+a)-c*(b*(b+(
-4*a*c+b^2)^(1/2))*(m-2*n)+4*a*c*(1-m+2*n))*x^(1-2*n)*(d*x)^m*hypergeom([1
, 1+m-2*n],[2+m-2*n],-2*c*x/(b-(-4*a*c+b^2)^(1/2)))/a/(-4*a*c+b^2)^(3/2)/(
b-(-4*a*c+b^2)^(1/2))/(1+m-2*n)+c*(b*(b-(-4*a*c+b^2)^(1/2))*(m-2*n)+4*a*c*
(1-m+2*n))*x^(1-2*n)*(d*x)^m*hypergeom([1, 1+m-2*n],[2+m-2*n],-2*c*x/(b+(-
4*a*c+b^2)^(1/2)))/a/(-4*a*c+b^2)^(3/2)/(b+(-4*a*c+b^2)^(1/2))/(1+m-2*n)
```

Mathematica [A] (verified)

Time = 0.93 (sec) , antiderivative size = 263, normalized size of antiderivative = 0.88

$$\int \frac{(dx)^m}{(ax^n + bx^{1+n} + cx^{2+n})^2}\, dx$$
$$= \frac{x^{1-2n}(dx)^m \left(\frac{b^2-2ac+bcx}{a+x(b+cx)} - \frac{c\left(\frac{-4ac(-1+m-2n)+b^2(m-2n)}{\sqrt{b^2-4ac}}+b(m-2n)\right) \text{Hypergeometric2F1}\left(1,1+m-2n,2+m-2n,\frac{2cx}{-b+\sqrt{b^2-4ac}}\right)}{\left(b-\sqrt{b^2-4ac}\right)(1+m-2n)} + \right.}{a\left(b^2-4ac\right)}$$

input
```
Integrate[(d*x)^m/(a*x^n + b*x^(1 + n) + c*x^(2 + n))^2,x]
```

output
```
(x^(1 - 2*n)*(d*x)^m*((b^2 - 2*a*c + b*c*x)/(a + x*(b + c*x)) - (c*((-4*a*
c*(-1 + m - 2*n) + b^2*(m - 2*n))/Sqrt[b^2 - 4*a*c] + b*(m - 2*n))*Hyperge
ometric2F1[1, 1 + m - 2*n, 2 + m - 2*n, (2*c*x)/(-b + Sqrt[b^2 - 4*a*c])])
/((b - Sqrt[b^2 - 4*a*c])*(1 + m - 2*n)) + (c*(-4*a*c*(-1 + m - 2*n) + b*(
b - Sqrt[b^2 - 4*a*c])*(m - 2*n))*Hypergeometric2F1[1, 1 + m - 2*n, 2 + m
- 2*n, (-2*c*x)/(b + Sqrt[b^2 - 4*a*c])])/(Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2
 - 4*a*c])*(1 + m - 2*n))))/(a*(b^2 - 4*a*c))
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {2028, 30, 1165, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^m}{(ax^n + bx^{n+1} + cx^{n+2})^2}\, dx$$
$$\Big\downarrow \; 2028$$
$$\int \frac{x^{-2n}(dx)^m}{(a+bx+cx^2)^2}dx$$
$$\Big\downarrow \; 30$$

$$x^{-m}(dx)^m\int \frac{x^{m-2n}}{(cx^2+bx+a)^2}dx$$

$\big\downarrow$ 1165

$$x^{-m}(dx)^m\left(\frac{x^{m-2n+1}\left(-2ac+b^2+bcx\right)}{a\left(b^2-4ac\right)\left(a+bx+cx^2\right)}-\frac{\int \frac{x^{m-2n}\left((m-2n)b^2+c(m-2n)xb+2ac(-m+2n+1)\right)}{cx^2+bx+a}dx}{a\left(b^2-4ac\right)}\right)$$

$\big\downarrow$ 1200

$$x^{-m}(dx)^m\left(\frac{x^{m-2n+1}\left(-2ac+b^2+bcx\right)}{a\left(b^2-4ac\right)\left(a+bx+cx^2\right)}-\frac{\int \left(\frac{\left(bc(m-2n)+\frac{c\left(mb^2-2nb^2+4ac-4acm+8acn\right)}{\sqrt{b^2-4ac}}\right)x^{m-2n}}{b+2cx-\sqrt{b^2-4ac}}+\frac{\left(bc(m-2n)-\frac{c\left(mb^2-2n\right.}{}\right.}{b+2cx}\right.}{a\left(b^2-4ac\right)}\right.$$

$\big\downarrow$ 2009

$$x^{-m}(dx)^m\left(\frac{x^{m-2n+1}\left(-2ac+b^2+bcx\right)}{a\left(b^2-4ac\right)\left(a+bx+cx^2\right)}-\frac{\frac{cx^{m-2n+1}\left(\frac{4ac(-m+2n+1)+b^2(m-2n)}{\sqrt{b^2-4ac}}+b(m-2n)\right)\text{Hypergeometric2F1}\left(1,m-2n+1,m\right.}{(m-2n+1)\left(b-\sqrt{b^2-4ac}\right)}}{}\right.$$

input
```
Int[(d*x)^m/(a*x^n + b*x^(1 + n) + c*x^(2 + n))^2,x]
```

output
```
((d*x)^m*((x^(1 + m - 2*n)*(b^2 - 2*a*c + b*c*x))/(a*(b^2 - 4*a*c)*(a + b*
x + c*x^2)) - ((c*(b*(m - 2*n) + (b^2*(m - 2*n) + 4*a*c*(1 - m + 2*n))/Sqr
t[b^2 - 4*a*c])*x^(1 + m - 2*n)*Hypergeometric2F1[1, 1 + m - 2*n, 2 + m -
2*n, (-2*c*x)/(b - Sqrt[b^2 - 4*a*c])])/((b - Sqrt[b^2 - 4*a*c])*(1 + m -
2*n)) - (c*(b*(b - Sqrt[b^2 - 4*a*c])*(m - 2*n) + 4*a*c*(1 - m + 2*n))*x^(
1 + m - 2*n)*Hypergeometric2F1[1, 1 + m - 2*n, 2 + m - 2*n, (-2*c*x)/(b +
Sqrt[b^2 - 4*a*c])])/(Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])*(1 + m - 2
*n)))/(a*(b^2 - 4*a*c))))/x^m
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 1165
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d
+ e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p
+ 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
 b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && LtQ[p, -1]
 && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2028
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.) + (c_.)*(x_)^(t_.))^(p_.),
x_Symbol] :> Int[x^(p*r)*(a + b*x^(s - r) + c*x^(t - r))^p*Fx, x] /; FreeQ[
{a, b, c, r, s, t}, x] && IntegerQ[p] && PosQ[s - r] && PosQ[t - r] &&  !(E
qQ[p, 1] && EqQ[u, 1])
```

Maple [F]

$$\int \frac{(dx)^m}{(x^n a + b\,x^{1+n} + c\,x^{2+n})^2} dx$$

input
```
int((d*x)^m/(x^n*a+b*x^(1+n)+c*x^(2+n))^2,x)
```

output
```
int((d*x)^m/(x^n*a+b*x^(1+n)+c*x^(2+n))^2,x)
```

Fricas [F]

$$\int \frac{(dx)^m}{(ax^n + bx^{1+n} + cx^{2+n})^2}\, dx = \int \frac{(dx)^m}{(cx^{n+2} + bx^{n+1} + ax^n)^2}\, dx$$

input
```
integrate((d*x)^m/(a*x^n+b*x^(1+n)+c*x^(2+n))^2,x, algorithm="fricas")
```

output
```
integral((d*x)^m/(2*a*b*x^(n + 1)*x^n + a^2*x^(2*n) + c^2*x^(2*n + 4) + b^
2*x^(2*n + 2) + 2*(b*c*x^(n + 1) + a*c*x^n)*x^(n + 2)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(dx)^m}{(ax^n + bx^{1+n} + cx^{2+n})^2}\, dx = \text{Timed out}$$

input
```
integrate((d*x)**m/(a*x**n+b*x**(1+n)+c*x**(2+n))**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(dx)^m}{(ax^n + bx^{1+n} + cx^{2+n})^2} \, dx = \int \frac{(dx)^m}{(cx^{n+2} + bx^{n+1} + ax^n)^2} \, dx$$

input
```
integrate((d*x)^m/(a*x^n+b*x^(1+n)+c*x^(2+n))^2,x, algorithm="maxima")
```

output
```
integrate((d*x)^m/(c*x^(n + 2) + b*x^(n + 1) + a*x^n)^2, x)
```

Giac [F]

$$\int \frac{(dx)^m}{(ax^n + bx^{1+n} + cx^{2+n})^2} \, dx = \int \frac{(dx)^m}{(cx^{n+2} + bx^{n+1} + ax^n)^2} \, dx$$

input
```
integrate((d*x)^m/(a*x^n+b*x^(1+n)+c*x^(2+n))^2,x, algorithm="giac")
```

output
```
integrate((d*x)^m/(c*x^(n + 2) + b*x^(n + 1) + a*x^n)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(dx)^m}{(ax^n + bx^{1+n} + cx^{2+n})^2} \, dx = \int \frac{(d\,x)^m}{(a\,x^n + b\,x^{n+1} + c\,x^{n+2})^2} \, dx$$

input
```
int((d*x)^m/(a*x^n + b*x^(n + 1) + c*x^(n + 2))^2,x)
```

output
```
int((d*x)^m/(a*x^n + b*x^(n + 1) + c*x^(n + 2))^2, x)
```

Reduce [F]

$$
\begin{aligned}
&\int \frac{(dx)^m}{(ax^n + bx^{1+n} + cx^{2+n})^2} \, dx \\
&= d^m \left(\int \frac{x^m}{x^{2n}a^2 + 2x^{2n}abx + 2x^{2n}ac\,x^2 + x^{2n}b^2x^2 + 2x^{2n}bc\,x^3 + x^{2n}c^2x^4} dx \right)
\end{aligned}
$$

input
```
int((d*x)^m/(a*x^n+b*x^(1+n)+c*x^(2+n))^2,x)
```

output
```
d**m*int(x**m/(x**(2*n)*a**2 + 2*x**(2*n)*a*b*x + 2*x**(2*n)*a*c*x**2 + x*
*(2*n)*b**2*x**2 + 2*x**(2*n)*b*c*x**3 + x**(2*n)*c**2*x**4),x)
```

3.87 $\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 38

$$\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}}$$

output
```
-arctanh(1/2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))/a^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.97

$$\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx = \frac{2\operatorname{arctanh}\left(\frac{\sqrt{c}x-\sqrt{a+x(b+cx)}}{\sqrt{a}}\right)}{\sqrt{a}}$$

input
```
Integrate[1/(x*Sqrt[a + b*x + c*x^2]),x]
```

output
```
(2*ArcTanh[(Sqrt[c]*x - Sqrt[a + x*(b + c*x)])/Sqrt[a]])/Sqrt[a]
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{a+bx+cx^2}}dx$$

$$\Big\downarrow \text{ 1154}$$

$$-2\int \frac{1}{4a-\frac{(2a+bx)^2}{cx^2+bx+a}}d\frac{2a+bx}{\sqrt{cx^2+bx+a}}$$

$$\Big\downarrow \text{ 219}$$

$$-\frac{\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}}$$

input
```
Int[1/(x*Sqrt[a + b*x + c*x^2]),x]
```

output
```
-(ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])]/Sqrt[a])
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.92

method	result	size
default	$-\frac{\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^{2}+bx+a}}{x}\right)}{\sqrt{a}}$	35

input
```
int(1/x/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/a^(1/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 111, normalized size of antiderivative = 2.92

$$\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx = \left[\frac{\log\left(-\frac{8\,abx+(b^2+4\,ac)x^2-4\,\sqrt{cx^2+bx+a}(bx+2\,a)\sqrt{a}+8\,a^2}{x^2}\right)}{2\,\sqrt{a}}, \frac{\sqrt{-a}\arctan\left(\frac{\sqrt{cx^2+bx+a}(bx+2\,a)\sqrt{-a}}{2\,(acx^2+abx+a^2)}\right)}{a}\right]$$

input
```
integrate(1/x/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*log(-(8*a*b*x + (b^2 + 4*a*c)*x^2 - 4*sqrt(c*x^2 + b*x + a)*(b*x + 2*
a)*sqrt(a) + 8*a^2)/x^2)/sqrt(a), sqrt(-a)*arctan(1/2*sqrt(c*x^2 + b*x + a
)*(b*x + 2*a)*sqrt(-a)/(a*c*x^2 + a*b*x + a^2))/a]
```

Sympy [F]

$$\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx = \int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate(1/x/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral(1/(x*sqrt(a + b*x + c*x**2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/x/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.92

$$\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx = \frac{2\arctan\left(-\frac{\sqrt{c}x-\sqrt{cx^2+bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}$$

input
```
integrate(1/x/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
2*arctan(-(sqrt(c)*x - sqrt(c*x^2 + b*x + a))/sqrt(-a))/sqrt(-a)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.89

$$\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx = -\frac{\ln\left(\frac{b}{2}+\frac{a}{x}+\frac{\sqrt{a}\,\sqrt{c\,x^2+b\,x+a}}{x}\right)}{\sqrt{a}}$$

input
```
int(1/(x*(a + b*x + c*x^2)^(1/2)),x)
```

output
```
-log(b/2 + a/x + (a^(1/2)*(a + b*x + c*x^2)^(1/2))/x)/a^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 693, normalized size of antiderivative = 18.24

$$\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx = \text{Too large to display}$$

input
```
int(1/x/(c*x^2+b*x+a)^(1/2),x)
```

output
```
( - 2*sqrt(a)*sqrt(4*sqrt(c)*sqrt(a)*b - 4*a*c - b**2)*atan((2*sqrt(c)*sqr
t(a + b*x + c*x**2) + b + 2*c*x)/sqrt(4*sqrt(c)*sqrt(a)*b - 4*a*c - b**2))
*b - 4*sqrt(c)*sqrt(4*sqrt(c)*sqrt(a)*b - 4*a*c - b**2)*atan((2*sqrt(c)*sq
rt(a + b*x + c*x**2) + b + 2*c*x)/sqrt(4*sqrt(c)*sqrt(a)*b - 4*a*c - b**2)
)*a - sqrt(a)*sqrt(4*sqrt(c)*sqrt(a)*b + 4*a*c + b**2)*log( - sqrt(4*sqrt(
c)*sqrt(a)*b + 4*a*c + b**2) + 2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*
x)*b + sqrt(a)*sqrt(4*sqrt(c)*sqrt(a)*b + 4*a*c + b**2)*log(sqrt(4*sqrt(c)
*sqrt(a)*b + 4*a*c + b**2) + 2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)
*b + 2*sqrt(c)*sqrt(4*sqrt(c)*sqrt(a)*b + 4*a*c + b**2)*log( - sqrt(4*sqrt
(c)*sqrt(a)*b + 4*a*c + b**2) + 2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c
*x)*a - 2*sqrt(c)*sqrt(4*sqrt(c)*sqrt(a)*b + 4*a*c + b**2)*log(sqrt(4*sqrt
(c)*sqrt(a)*b + 4*a*c + b**2) + 2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c
*x)*a + 4*sqrt(a)*log( - sqrt(4*sqrt(c)*sqrt(a)*b + 4*a*c + b**2) + 2*sqrt
(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)*a*c - sqrt(a)*log( - sqrt(4*sqrt(c
)*sqrt(a)*b + 4*a*c + b**2) + 2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x
)*b**2 + 4*sqrt(a)*log(sqrt(4*sqrt(c)*sqrt(a)*b + 4*a*c + b**2) + 2*sqrt(c
)*sqrt(a + b*x + c*x**2) + b + 2*c*x)*a*c - sqrt(a)*log(sqrt(4*sqrt(c)*sqr
t(a)*b + 4*a*c + b**2) + 2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)*b**
2 - 4*sqrt(a)*log(4*sqrt(c)*sqrt(a + b*x + c*x**2)*b + 8*sqrt(c)*sqrt(a +
b*x + c*x**2)*c*x + 4*sqrt(c)*sqrt(a)*b + 8*b*c*x + 8*c**2*x**2)*a*c + ...
```

3.88 $\int \frac{1}{\sqrt{x^2(a+bx+cx^2)}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 45

$$\int \frac{1}{\sqrt{x^2(a+bx+cx^2)}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{\sqrt{a}}$$

output
```
-arctanh(1/2*x*(b*x+2*a)/a^(1/2)/(c*x^4+b*x^3+a*x^2)^(1/2))/a^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.51

$$\int \frac{1}{\sqrt{x^2(a+bx+cx^2)}}\,dx = \frac{2x\sqrt{a+x(b+cx)}\operatorname{arctanh}\left(\frac{\sqrt{c}x-\sqrt{a+x(b+cx)}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{x^2(a+x(b+cx))}}$$

input
```
Integrate[1/Sqrt[x^2*(a + b*x + c*x^2)],x]
```

output
```
(2*x*Sqrt[a + x*(b + c*x)]*ArcTanh[(Sqrt[c]*x - Sqrt[a + x*(b + c*x)])/Sqr
t[a]])/(Sqrt[a]*Sqrt[x^2*(a + x*(b + c*x))])
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2093, 1951, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x^2 (a+bx+cx^2)}}dx$$

$$\Big\downarrow \text{ 2093}$$

$$\int \frac{1}{\sqrt{ax^2+bx^3+cx^4}}dx$$

$$\Big\downarrow \text{ 1951}$$

$$-2\int \frac{1}{4a-\frac{x^2(2a+bx)^2}{cx^4+bx^3+ax^2}}d\frac{x(2a+bx)}{\sqrt{cx^4+bx^3+ax^2}}$$

$$\Big\downarrow \text{ 219}$$

$$-\frac{\text{arctanh}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{\sqrt{a}}$$

input
```
Int[1/Sqrt[x^2*(a + b*x + c*x^2)],x]
```

output
```
-(ArcTanh[(x*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x^2 + b*x^3 + c*x^4])]/Sqrt[a]
)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1951
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :
> Simp[-2/(n - 2)   Subst[Int[1/(4*a - x^2), x], x, x*((2*a + b*x^(n - 2))/
Sqrt[a*x^2 + b*x^n + c*x^r])], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*
n - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]
```

rule 2093
```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedTrinomialQ[u, x] &&  !GeneralizedTrinomialMatchQ[u, x]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.93

method	result	size
pseudoelliptic	$\frac{\ln(2)-\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x\sqrt{a}}\right)}{\sqrt{a}}$	42
default	$-\frac{x\sqrt{c\,x^2+bx+a}\,\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x}\right)}{\sqrt{x^2(c\,x^2+bx+a)}\,\sqrt{a}}$	64

input
```
int(1/(x^2*(c*x^2+b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(ln(2)-ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x/a^(1/2)))/a^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 130, normalized size of antiderivative = 2.89

$$\int \frac{1}{\sqrt{x^2\,(a+bx+cx^2)}}\,dx = \left[\frac{\log\left(-\frac{8\,abx^2+(b^2+4\,ac)x^3+8\,a^2x-4\,\sqrt{cx^4+bx^3+ax^2}(bx+2\,a)\sqrt{a}}{x^3}\right)}{2\,\sqrt{a}}, \frac{\sqrt{-a}\arctan\left(\frac{\sqrt{cx^4+bx^3+ax^2}(bx+2\,a)\sqrt{-a}}{2\,(acx^3+abx^2+a^2x)}\right)}{a}\right]$$

input
```
integrate(1/(x^2*(c*x^2+b*x+a))^(1/2),x, algorithm="fricas")
```

output
```
[1/2*log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x - 4*sqrt(c*x^4 + b*x^3
+ a*x^2)*(b*x + 2*a)*sqrt(a))/x^3)/sqrt(a), sqrt(-a)*arctan(1/2*sqrt(c*x^4
 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2 + a^2*x))/a]
```

Sympy [F]

$$\int \frac{1}{\sqrt{x^2\left(a+bx+cx^2\right)}}\,dx=\int \frac{1}{\sqrt{x^2\left(a+bx+cx^2\right)}}\,dx$$

input
```
integrate(1/(x**2*(c*x**2+b*x+a))**(1/2),x)
```

output
```
Integral(1/sqrt(x**2*(a + b*x + c*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{x^2\left(a+bx+cx^2\right)}}\,dx=\int \frac{1}{\sqrt{(cx^2+bx+a)x^2}}\,dx$$

input
```
integrate(1/(x^2*(c*x^2+b*x+a))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt((c*x^2 + b*x + a)*x^2), x)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.31

$$\int \frac{1}{\sqrt{x^2\left(a+bx+cx^2\right)}}\,dx=-\frac{2\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)\operatorname{sgn}(x)}{\sqrt{-a}}+\frac{2\arctan\left(-\frac{\sqrt{c}x-\sqrt{cx^2+bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\operatorname{sgn}(x)}$$

input
```
integrate(1/(x^2*(c*x^2+b*x+a))^(1/2),x, algorithm="giac")
```

output
```
-2*arctan(sqrt(a)/sqrt(-a))*sgn(x)/sqrt(-a) + 2*arctan(-(sqrt(c)*x - sqrt(
c*x^2 + b*x + a))/sqrt(-a))/(sqrt(-a)*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{x^2 (a+bx+cx^2)}}\,dx = \int \frac{1}{\sqrt{x^2\,\left(c\,x^2+b\,x+a\right)}}\,dx$$

input
```
int(1/(x^2*(a + b*x + c*x^2))^(1/2),x)
```

output
```
int(1/(x^2*(a + b*x + c*x^2))^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 693, normalized size of antiderivative = 15.40

$$\int \frac{1}{\sqrt{x^2 (a+bx+cx^2)}}\,dx = \text{Too large to display}$$

input
```
int(1/(x^2*(c*x^2+b*x+a))^(1/2),x)
```

output
```
( - 2*sqrt(a)*sqrt(4*sqrt(c)*sqrt(a)*b - 4*a*c - b**2)*atan((2*sqrt(c)*sqr
t(a + b*x + c*x**2) + b + 2*c*x)/sqrt(4*sqrt(c)*sqrt(a)*b - 4*a*c - b**2))
*b - 4*sqrt(c)*sqrt(4*sqrt(c)*sqrt(a)*b - 4*a*c - b**2)*atan((2*sqrt(c)*sq
rt(a + b*x + c*x**2) + b + 2*c*x)/sqrt(4*sqrt(c)*sqrt(a)*b - 4*a*c - b**2)
)*a - sqrt(a)*sqrt(4*sqrt(c)*sqrt(a)*b + 4*a*c + b**2)*log( - sqrt(4*sqrt(
c)*sqrt(a)*b + 4*a*c + b**2) + 2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*
x)*b + sqrt(a)*sqrt(4*sqrt(c)*sqrt(a)*b + 4*a*c + b**2)*log(sqrt(4*sqrt(c)
*sqrt(a)*b + 4*a*c + b**2) + 2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)
*b + 2*sqrt(c)*sqrt(4*sqrt(c)*sqrt(a)*b + 4*a*c + b**2)*log( - sqrt(4*sqrt
(c)*sqrt(a)*b + 4*a*c + b**2) + 2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c
*x)*a - 2*sqrt(c)*sqrt(4*sqrt(c)*sqrt(a)*b + 4*a*c + b**2)*log(sqrt(4*sqrt
(c)*sqrt(a)*b + 4*a*c + b**2) + 2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c
*x)*a + 4*sqrt(a)*log( - sqrt(4*sqrt(c)*sqrt(a)*b + 4*a*c + b**2) + 2*sqrt
(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)*a*c - sqrt(a)*log( - sqrt(4*sqrt(c
)*sqrt(a)*b + 4*a*c + b**2) + 2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x
)*b**2 + 4*sqrt(a)*log(sqrt(4*sqrt(c)*sqrt(a)*b + 4*a*c + b**2) + 2*sqrt(c
)*sqrt(a + b*x + c*x**2) + b + 2*c*x)*a*c - sqrt(a)*log(sqrt(4*sqrt(c)*sqr
t(a)*b + 4*a*c + b**2) + 2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)*b**
2 - 4*sqrt(a)*log(4*sqrt(c)*sqrt(a + b*x + c*x**2)*b + 8*sqrt(c)*sqrt(a +
b*x + c*x**2)*c*x + 4*sqrt(c)*sqrt(a)*b + 8*b*c*x + 8*c**2*x**2)*a*c + ...
```

3.89 $\int \frac{1}{\sqrt{x}\sqrt{x(a+bx+cx^2)}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 47

$$\int \frac{1}{\sqrt{x}\sqrt{x(a+bx+cx^2)}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{x}(2a+bx)}{2\sqrt{a}\sqrt{ax+bx^2+cx^3}}\right)}{\sqrt{a}}$$

output
```
-arctanh(1/2*x^(1/2)*(b*x+2*a)/a^(1/2)/(c*x^3+b*x^2+a*x)^(1/2))/a^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.49

$$\int \frac{1}{\sqrt{x}\sqrt{x(a+bx+cx^2)}}\,dx = \frac{2\sqrt{x}\sqrt{a+x(b+cx)}\operatorname{arctanh}\left(\frac{\sqrt{c}x-\sqrt{a+x(b+cx)}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{x(a+x(b+cx))}}$$

input
```
Integrate[1/(Sqrt[x]*Sqrt[x*(a + b*x + c*x^2)]),x]
```

output
```
(2*Sqrt[x]*Sqrt[a + x*(b + c*x)]*ArcTanh[(Sqrt[c]*x - Sqrt[a + x*(b + c*x)
])/Sqrt[a]])/(Sqrt[a]*Sqrt[x*(a + x*(b + c*x))])
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2035, 2093, 1951, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x}\sqrt{x(a+bx+cx^2)}}dx$$

$$\Big\downarrow \ 2035$$

$$2\int \frac{1}{\sqrt{x(cx^2+bx+a)}}d\sqrt{x}$$

$$\Big\downarrow \ 2093$$

$$2\int \frac{1}{\sqrt{cx^3+bx^2+ax}}d\sqrt{x}$$

$$\Big\downarrow \ 1951$$

$$-2\int \frac{1}{4a-x}d\frac{\sqrt{x}(2a+bx)}{\sqrt{cx^3+bx^2+ax}}$$

$$\Big\downarrow \ 219$$

$$-\frac{\text{arctanh}\left(\frac{\sqrt{x}(2a+bx)}{2\sqrt{a}\sqrt{ax+bx^2+cx^3}}\right)}{\sqrt{a}}$$

input
```
Int[1/(Sqrt[x]*Sqrt[x*(a + b*x + c*x^2)]),x]
```

output
```
-(ArcTanh[(Sqrt[x]*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x + b*x^2 + c*x^3])]/Sqr
t[a])
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1951
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :
> Simp[-2/(n - 2)   Subst[Int[1/(4*a - x^2), x], x, x*((2*a + b*x^(n - 2))/
Sqrt[a*x^2 + b*x^n + c*x^r])], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*
n - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]
```

rule 2035
```
Int[(Fx_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k   Subst
[Int[x^(k*(m + 1) - 1)*SubstPower[Fx, x, k], x], x, x^(1/k)], x]] /; Fracti
onQ[m] && AlgebraicFunctionQ[Fx, x]
```

rule 2093
```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedTrinomialQ[u, x] &&  !GeneralizedTrinomialMatchQ[u, x]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.36

method	result	size
default	$-\frac{\sqrt{x}\,\sqrt{c\,x^{2}+b x+a}\,\ln\left(\frac{2 a+b x+2 \sqrt{a}\,\sqrt{c\,x^{2}+b x+a}}{x}\right)}{\sqrt{x \left(c\,x^{2}+b x+a\right)}\,\sqrt{a}}$	64

input
```
int(1/x^(1/2)/(x*(c*x^2+b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-x^(1/2)/(x*(c*x^2+b*x+a))^(1/2)*(c*x^2+b*x+a)^(1/2)/a^(1/2)*ln((2*a+b*x+2
*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 131, normalized size of antiderivative = 2.79

$$\int \frac{1}{\sqrt{x}\sqrt{x\,(a+bx+cx^2)}}\,dx$$
$$=\left[\frac{\log\left(\frac{8\,abx^2+(b^2+4\,ac)x^3+8\,a^2x-4\,\sqrt{cx^3+bx^2+ax}(bx+2\,a)\sqrt{a}\sqrt{x}}{x^3}\right)}{2\,\sqrt{a}},\ \frac{\sqrt{-a}\arctan\left(\frac{\sqrt{cx^3+bx^2+ax}(bx+2\,a)\sqrt{-a}\sqrt{x}}{2\,(acx^3+abx^2+a^2x)}\right)}{a}\right]$$

input
```
integrate(1/x^(1/2)/(x*(c*x^2+b*x+a))^(1/2),x, algorithm="fricas")
```

output
```
[1/2*log((8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x - 4*sqrt(c*x^3 + b*x^2 +
 a*x)*(b*x + 2*a)*sqrt(a)*sqrt(x))/x^3)/sqrt(a), sqrt(-a)*arctan(1/2*sqrt(
c*x^3 + b*x^2 + a*x)*(b*x + 2*a)*sqrt(-a)*sqrt(x)/(a*c*x^3 + a*b*x^2 + a^2
*x))/a]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{x}\sqrt{x\,(a+bx+cx^2)}}\,dx = \text{Timed out}$$

input
```
integrate(1/x**(1/2)/(x*(c*x**2+b*x+a))**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{\sqrt{x}\sqrt{x\left(a+bx+cx^2\right)}}\,dx=\int \frac{1}{\sqrt{(cx^2+bx+a)x}\sqrt{x}}\,dx$$

input
```
integrate(1/x^(1/2)/(x*(c*x^2+b*x+a))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt((c*x^2 + b*x + a)*x)*sqrt(x)), x)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.74

$$\int \frac{1}{\sqrt{x}\sqrt{x\left(a+bx+cx^2\right)}}\,dx=\frac{2\arctan\left(-\frac{\sqrt{c}x-\sqrt{cx^2+bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}$$

input
```
integrate(1/x^(1/2)/(x*(c*x^2+b*x+a))^(1/2),x, algorithm="giac")
```

output
```
2*arctan(-(sqrt(c)*x - sqrt(c*x^2 + b*x + a))/sqrt(-a))/sqrt(-a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{x}\sqrt{x\left(a+bx+cx^2\right)}}\,dx=\int \frac{1}{\sqrt{x}\,\sqrt{x\,\left(c\,x^2+b\,x+a\right)}}\,dx$$

input
```
int(1/(x^(1/2)*(x*(a + b*x + c*x^2))^(1/2)),x)
```

output
```
int(1/(x^(1/2)*(x*(a + b*x + c*x^2))^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 693, normalized size of antiderivative = 14.74

$$\int \frac{1}{\sqrt{x}\sqrt{x(a+bx+cx^2)}}\,dx = \text{Too large to display}$$

input
```
int(1/x^(1/2)/(x*(c*x^2+b*x+a))^(1/2),x)
```

output
```
( - 2*sqrt(a)*sqrt(4*sqrt(c)*sqrt(a)*b - 4*a*c - b**2)*atan((2*sqrt(c)*sqr
t(a + b*x + c*x**2) + b + 2*c*x)/sqrt(4*sqrt(c)*sqrt(a)*b - 4*a*c - b**2))
*b - 4*sqrt(c)*sqrt(4*sqrt(c)*sqrt(a)*b - 4*a*c - b**2)*atan((2*sqrt(c)*sq
rt(a + b*x + c*x**2) + b + 2*c*x)/sqrt(4*sqrt(c)*sqrt(a)*b - 4*a*c - b**2)
)*a - sqrt(a)*sqrt(4*sqrt(c)*sqrt(a)*b + 4*a*c + b**2)*log( - sqrt(4*sqrt(
c)*sqrt(a)*b + 4*a*c + b**2) + 2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*
x)*b + sqrt(a)*sqrt(4*sqrt(c)*sqrt(a)*b + 4*a*c + b**2)*log(sqrt(4*sqrt(c)
*sqrt(a)*b + 4*a*c + b**2) + 2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)
*b + 2*sqrt(c)*sqrt(4*sqrt(c)*sqrt(a)*b + 4*a*c + b**2)*log( - sqrt(4*sqrt
(c)*sqrt(a)*b + 4*a*c + b**2) + 2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c
*x)*a - 2*sqrt(c)*sqrt(4*sqrt(c)*sqrt(a)*b + 4*a*c + b**2)*log(sqrt(4*sqrt
(c)*sqrt(a)*b + 4*a*c + b**2) + 2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c
*x)*a + 4*sqrt(a)*log( - sqrt(4*sqrt(c)*sqrt(a)*b + 4*a*c + b**2) + 2*sqrt
(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)*a*c - sqrt(a)*log( - sqrt(4*sqrt(c
)*sqrt(a)*b + 4*a*c + b**2) + 2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x
)*b**2 + 4*sqrt(a)*log(sqrt(4*sqrt(c)*sqrt(a)*b + 4*a*c + b**2) + 2*sqrt(c
)*sqrt(a + b*x + c*x**2) + b + 2*c*x)*a*c - sqrt(a)*log(sqrt(4*sqrt(c)*sqr
t(a)*b + 4*a*c + b**2) + 2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)*b**
2 - 4*sqrt(a)*log(4*sqrt(c)*sqrt(a + b*x + c*x**2)*b + 8*sqrt(c)*sqrt(a +
b*x + c*x**2)*c*x + 4*sqrt(c)*sqrt(a)*b + 8*b*c*x + 8*c**2*x**2)*a*c + ...
```

3.90 $\int \frac{\sqrt{x}}{\sqrt{x^3(a+bx+cx^2)}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 49

$$\int \frac{\sqrt{x}}{\sqrt{x^3(a+bx+cx^2)}}\,dx=-\frac{\operatorname{arctanh}\left(\frac{x^{3/2}(2a+bx)}{2\sqrt{a}\sqrt{ax^3+bx^4+cx^5}}\right)}{\sqrt{a}}$$

output
```
-arctanh(1/2*x^(3/2)*(b*x+2*a)/a^(1/2)/(c*x^5+b*x^4+a*x^3)^(1/2))/a^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.47

$$\int \frac{\sqrt{x}}{\sqrt{x^3(a+bx+cx^2)}}\,dx=\frac{2x^{3/2}\sqrt{a+x(b+cx)}\operatorname{arctanh}\left(\frac{\sqrt{c}x-\sqrt{a+x(b+cx)}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{x^3(a+x(b+cx))}}$$

input
```
Integrate[Sqrt[x]/Sqrt[x^3*(a + b*x + c*x^2)],x]
```

output
```
(2*x^(3/2)*Sqrt[a + x*(b + c*x)]*ArcTanh[(Sqrt[c]*x - Sqrt[a + x*(b + c*x)
])/Sqrt[a]])/(Sqrt[a]*Sqrt[x^3*(a + x*(b + c*x))])
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2035, 2094, 1960, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}}{\sqrt{x^3 (a+bx+cx^2)}} \, dx$$

$$\Big\downarrow \text{ 2035}$$

$$2\int \frac{x}{\sqrt{x^3 (cx^2+bx+a)}}d\sqrt{x}$$

$$\Big\downarrow \text{ 2094}$$

$$2\int \frac{x}{\sqrt{cx^5+bx^4+ax^3}}d\sqrt{x}$$

$$\Big\downarrow \text{ 1960}$$

$$-2\int \frac{1}{4a-x}d\frac{x^{3/2}(2a+bx)}{\sqrt{cx^5+bx^4+ax^3}}$$

$$\Big\downarrow \text{ 219}$$

$$-\frac{\text{arctanh}\left(\frac{x^{3/2}(2a+bx)}{2\sqrt{a}\sqrt{ax^3+bx^4+cx^5}}\right)}{\sqrt{a}}$$

input
```
Int[Sqrt[x]/Sqrt[x^3*(a + b*x + c*x^2)],x]
```

output
```
-(ArcTanh[(x^(3/2)*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x^3 + b*x^4 + c*x^5])]/S
qrt[a])
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1960
```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Simp[-2/(n - q)   Subst[Int[1/(4*a - x^2), x], x, x^(m + 1)*
((2*a + b*x^(n - q))/Sqrt[a*x^q + b*x^n + c*x^r])], x] /; FreeQ[{a, b, c, m
, n, q, r}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && NeQ[b^2 - 4*a*c, 0] &&
EqQ[m, q/2 - 1]
```

rule 2035
```
Int[(Fx_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k   Subst
[Int[x^(k*(m + 1) - 1)*SubstPower[Fx, x, k], x], x, x^(1/k)], x]] /; Fracti
onQ[m] && AlgebraicFunctionQ[Fx, x]
```

rule 2094
```
Int[(u_)^(p_.)*((d_.)*(x_))^(m_.), x_Symbol] :> Int[(d*x)^m*ExpandToSum[u,
x]^p, x] /; FreeQ[{d, m, p}, x] && GeneralizedTrinomialQ[u, x] &&  !General
izedTrinomialMatchQ[u, x]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.35

method	result	size
default	$-\frac{x^{\frac{3}{2}} \sqrt{c\,x^{2}+b x +a}\, \ln\left(\frac{2 a +b x +2 \sqrt{a}\, \sqrt{c\,x^{2}+b x +a}}{x}\right)}{\sqrt{x^{3} \left(c\,x^{2}+b x +a \right)}\, \sqrt{a}}$	66

input
```
int(x^(1/2)/(x^3*(c*x^2+b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/(x^3*(c*x^2+b*x+a))^(1/2)*x^(3/2)*(c*x^2+b*x+a)^(1/2)/a^(1/2)*ln((2*a+b
*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)
```

Fricas [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 139, normalized size of antiderivative = 2.84

$$\int \frac{\sqrt{x}}{\sqrt{x^3 (a + bx + cx^2)}} \, dx = \left[\frac{\log\left(\frac{8\,abx^3 + (b^2 + 4\,ac)x^4 + 8\,a^2x^2 - 4\,\sqrt{cx^5 + bx^4 + ax^3}(bx + 2\,a)\sqrt{a}\sqrt{x}}{x^4}\right)}{2\,\sqrt{a}}, \frac{\sqrt{-a}\arctan\left(\frac{\sqrt{cx^5 + bx^4 + ax^3}(bx + 2\,a)\sqrt{-a}\sqrt{x}}{2\,(acx^4 + abx^3 + a^2x^2)}\right)}{a} \right]$$

input
```
integrate(x^(1/2)/(x^3*(c*x^2+b*x+a))^(1/2),x, algorithm="fricas")
```

output
```
[1/2*log((8*a*b*x^3 + (b^2 + 4*a*c)*x^4 + 8*a^2*x^2 - 4*sqrt(c*x^5 + b*x^4
 + a*x^3)*(b*x + 2*a)*sqrt(a)*sqrt(x))/x^4)/sqrt(a), sqrt(-a)*arctan(1/2*s
qrt(c*x^5 + b*x^4 + a*x^3)*(b*x + 2*a)*sqrt(-a)*sqrt(x)/(a*c*x^4 + a*b*x^3
 + a^2*x^2))/a]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{x}}{\sqrt{x^3 (a + bx + cx^2)}} \, dx = \text{Timed out}$$

input
```
integrate(x**(1/2)/(x**3*(c*x**2+b*x+a))**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sqrt{x}}{\sqrt{x^3 (a + bx + cx^2)}}\, dx = \int \frac{\sqrt{x}}{\sqrt{(cx^2 + bx + a)x^3}}\, dx$$

input
```
integrate(x^(1/2)/(x^3*(c*x^2+b*x+a))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(x)/sqrt((c*x^2 + b*x + a)*x^3), x)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.20

$$\int \frac{\sqrt{x}}{\sqrt{x^3 (a + bx + cx^2)}}\, dx = -\frac{2 \arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right) \mathrm{sgn}(x)}{\sqrt{-a}} + \frac{2 \arctan\left(-\frac{\sqrt{c}x - \sqrt{cx^2 + bx + a}}{\sqrt{-a}}\right)}{\sqrt{-a}\mathrm{sgn}\,(x)}$$

input
```
integrate(x^(1/2)/(x^3*(c*x^2+b*x+a))^(1/2),x, algorithm="giac")
```

output
```
-2*arctan(sqrt(a)/sqrt(-a))*sgn(x)/sqrt(-a) + 2*arctan(-(sqrt(c)*x - sqrt(
c*x^2 + b*x + a))/sqrt(-a))/(sqrt(-a)*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{x}}{\sqrt{x^3 (a + bx + cx^2)}}\, dx = \int \frac{\sqrt{x}}{\sqrt{x^3\,\left(c\,x^2 + b\,x + a\right)}}\, dx$$

input
```
int(x^(1/2)/(x^3*(a + b*x + c*x^2))^(1/2),x)
```

output
```
int(x^(1/2)/(x^3*(a + b*x + c*x^2))^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 693, normalized size of antiderivative = 14.14

$$\int \frac{\sqrt{x}}{\sqrt{x^3\,(a+bx+cx^2)}}\,dx = \text{Too large to display}$$

input
```
int(x^(1/2)/(x^3*(c*x^2+b*x+a))^(1/2),x)
```

output
```
( - 2*sqrt(a)*sqrt(4*sqrt(c)*sqrt(a)*b - 4*a*c - b**2)*atan((2*sqrt(c)*sqr
t(a + b*x + c*x**2) + b + 2*c*x)/sqrt(4*sqrt(c)*sqrt(a)*b - 4*a*c - b**2))
*b - 4*sqrt(c)*sqrt(4*sqrt(c)*sqrt(a)*b - 4*a*c - b**2)*atan((2*sqrt(c)*sq
rt(a + b*x + c*x**2) + b + 2*c*x)/sqrt(4*sqrt(c)*sqrt(a)*b - 4*a*c - b**2)
)*a - sqrt(a)*sqrt(4*sqrt(c)*sqrt(a)*b + 4*a*c + b**2)*log( - sqrt(4*sqrt(
c)*sqrt(a)*b + 4*a*c + b**2) + 2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*
x)*b + sqrt(a)*sqrt(4*sqrt(c)*sqrt(a)*b + 4*a*c + b**2)*log(sqrt(4*sqrt(c)
*sqrt(a)*b + 4*a*c + b**2) + 2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)
*b + 2*sqrt(c)*sqrt(4*sqrt(c)*sqrt(a)*b + 4*a*c + b**2)*log( - sqrt(4*sqrt
(c)*sqrt(a)*b + 4*a*c + b**2) + 2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c
*x)*a - 2*sqrt(c)*sqrt(4*sqrt(c)*sqrt(a)*b + 4*a*c + b**2)*log(sqrt(4*sqrt
(c)*sqrt(a)*b + 4*a*c + b**2) + 2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c
*x)*a + 4*sqrt(a)*log( - sqrt(4*sqrt(c)*sqrt(a)*b + 4*a*c + b**2) + 2*sqrt
(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)*a*c - sqrt(a)*log( - sqrt(4*sqrt(c
)*sqrt(a)*b + 4*a*c + b**2) + 2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x
)*b**2 + 4*sqrt(a)*log(sqrt(4*sqrt(c)*sqrt(a)*b + 4*a*c + b**2) + 2*sqrt(c
)*sqrt(a + b*x + c*x**2) + b + 2*c*x)*a*c - sqrt(a)*log(sqrt(4*sqrt(c)*sqr
t(a)*b + 4*a*c + b**2) + 2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)*b**
2 - 4*sqrt(a)*log(4*sqrt(c)*sqrt(a + b*x + c*x**2)*b + 8*sqrt(c)*sqrt(a +
b*x + c*x**2)*c*x + 4*sqrt(c)*sqrt(a)*b + 8*b*c*x + 8*c**2*x**2)*a*c + ...
```

CHAPTER 4

APPENDIX

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                   convert(leaf_count_result,string)," vs. $2 (",
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
      1
    else
      max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
        1
      else
        max(2,ExpnType(op(1,expn)))
      end if
    else
        max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#            Port of original Maple grading function by
#            Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#            added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
            return True
        else:
            return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),Expr
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)  #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
else:
    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
            else: #result contains complex but optimal is not
                grade = "C"
                grade_annotation ="Result contains complex when optimal does not."
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =""
            else:
                grade = "B"
                grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
    else:
        grade = "C"
        grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType


#print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
            'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
            'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
            'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
            'elliptic_pi','exp_integral_e','log_integral',
            'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
            'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
         print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
       return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
       if expn.parent() is SR:
          return expn.operator() is None
       if expn.parent() in (ZZ, QQ, AA, QQbar):
          return expn in expn.parent() # Should always return True
       if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
          return expn in expn.parent().base_ring() or expn in expn.parent().gens()

       return False

    except AttributeError as error:
       print("Exception,AttributeError in  is_atom")
       print ("cought exception" , type(error).__name__ )
       return False


def expnType(expn):

    if debug:
       print (">>>>>Enter expnType, expn=", expn)
       print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
       return 1
    elif type(expn)==list:   #isinstance(expn,list):
       return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
       if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
          return 1
       else:
          return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
       if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
```

```
        return expnType(expn.operands()[0])    #expnType(expn.args[0])
    elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
        if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expr
elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
    m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
    m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
    return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
    return max(3,expnType(expn.operands()[0]))
elif is_special_function(expn.operator()): #is_special_function(expn.func)
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(4,m1)   #max(4,m1)
elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(5,m1)   #max(5,m1)
elif is_appell_function(expn.operator()):
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(6,m1)   #max(6,m1)
elif str(expn).find("Integral") != -1: #this will never happen, since it
        #is checked before calling the grading function that is passed.
        #but kept it here.
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):



    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file