Computer Algebra
Independent Integration Tests

Summer 2024

1-Algebraic-functions/1.3-Cubic/143-1.3.2

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Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**178**]. This is test number [143].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	94.38 (168)	5.62 (10)
Mathematica	89.89 (160)	10.11 (18)
Fricas	87.08 (155)	12.92 (23)
Maple	83.71 (149)	16.29 (29)
Mupad	69.10 (123)	30.90 (55)
Reduce	58.99 (105)	41.01 (73)
Giac	54.49 (97)	45.51 (81)
Maxima	47.75 (85)	52.25 (93)
Sympy	47.75 (85)	52.25 (93)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

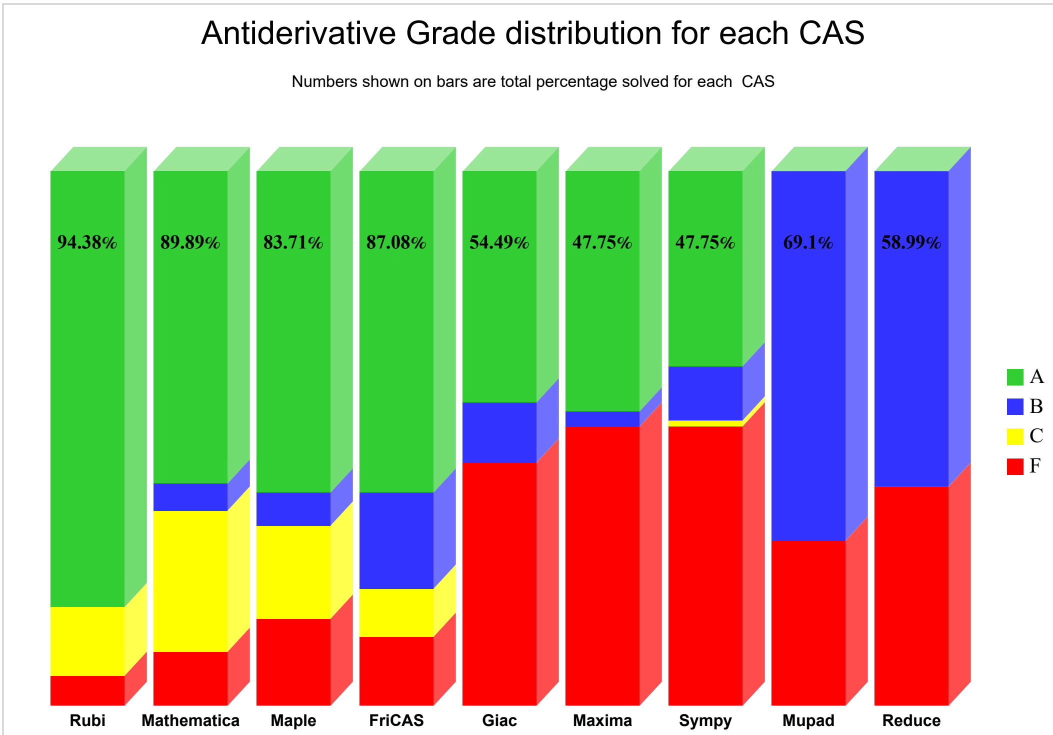
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

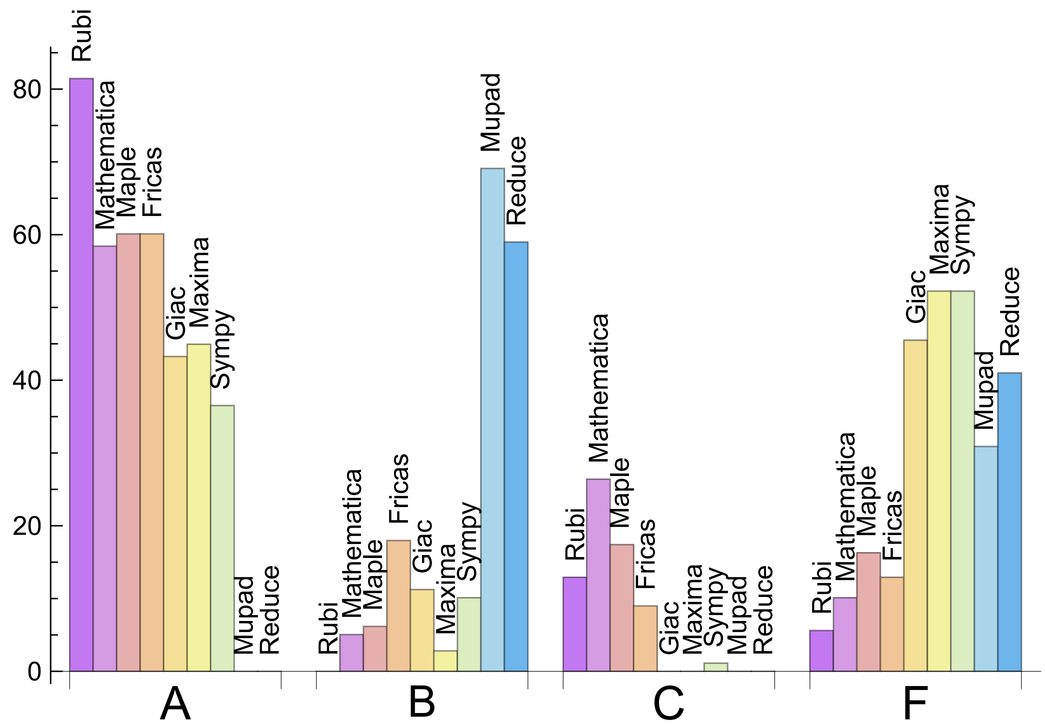
System	% A grade	% B grade	% C grade	% F grade
Rubi	81.461	0.000	12.921	5.618
Maple	60.112	6.180	17.416	16.292
Fricas	60.112	17.978	8.989	12.921
Mathematica	58.427	5.056	26.404	10.112
Maxima	44.944	2.809	0.000	52.247
Giac	43.258	11.236	0.000	45.506
Sympy	36.517	10.112	1.124	52.247
Mupad	0.000	69.101	0.000	30.899
Reduce	0.000	58.989	0.000	41.011

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	10	100.00	0.00	0.00
Mathematica	18	94.44	5.56	0.00
Fricas	23	73.91	26.09	0.00
Maple	29	100.00	0.00	0.00
Mupad	55	0.00	100.00	0.00
Reduce	73	100.00	0.00	0.00
Giac	81	56.79	0.00	43.21
Maxima	93	95.70	0.00	4.30
Sympy	93	66.67	32.26	1.08

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.06
Giac	0.13
Reduce	0.17
Maple	1.12
Fricas	2.23
Rubi	2.85
Sympy	4.05
Mathematica	4.23
Mupad	9.17

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Maple	375.07	1.03	135.00	0.91
Maxima	391.93	1.24	108.00	0.93
Rubi	491.42	1.35	265.00	1.00
Giac	538.61	1.69	152.00	1.05
Reduce	1333.39	5.09	159.00	1.19
Fricas	2204.50	4.26	205.00	1.30
Mupad	2906.28	4.91	168.00	1.03
Mathematica	3720.81	7.86	166.50	0.97
Sympy	4450.86	23.02	189.00	1.16

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

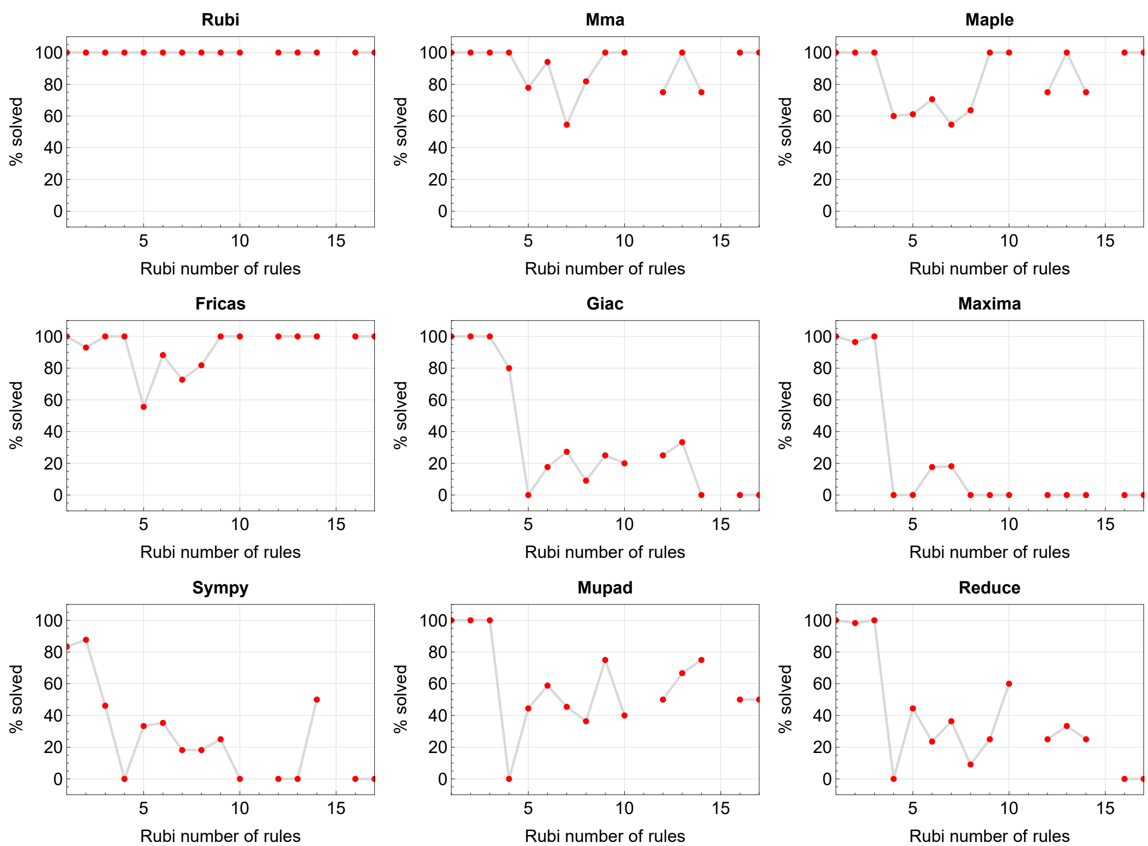


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

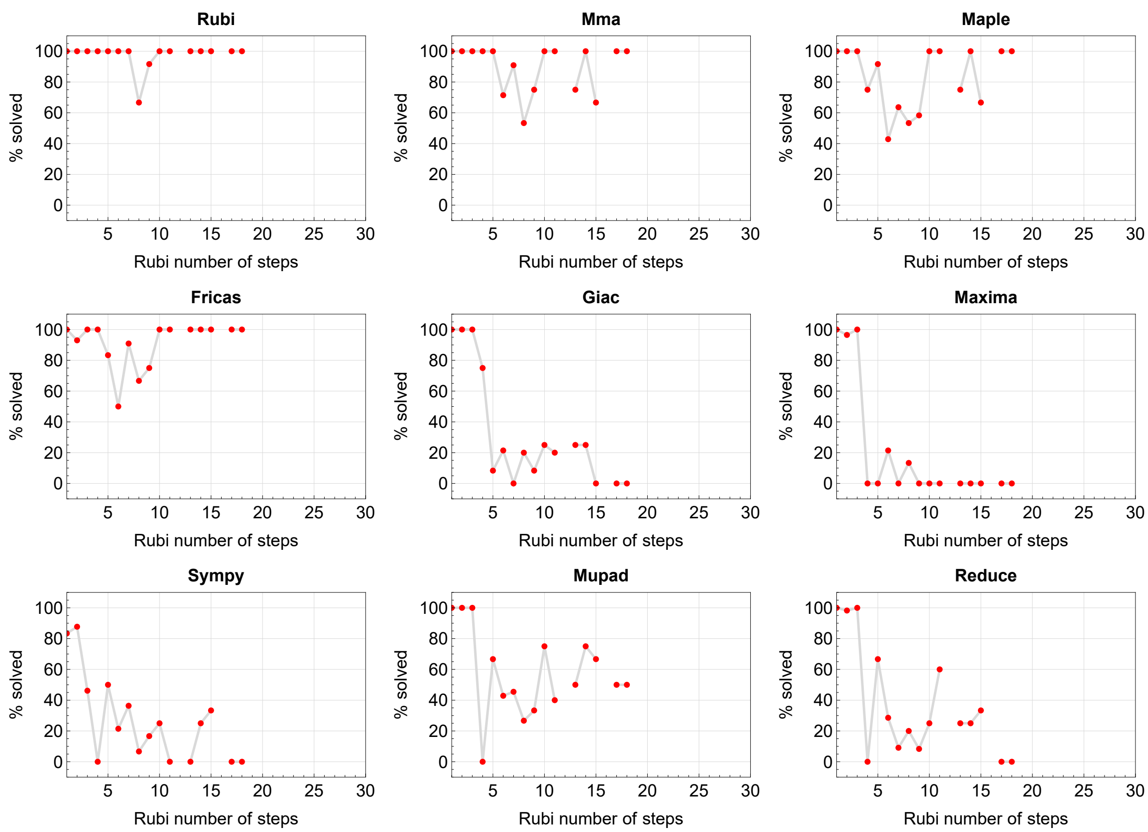


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

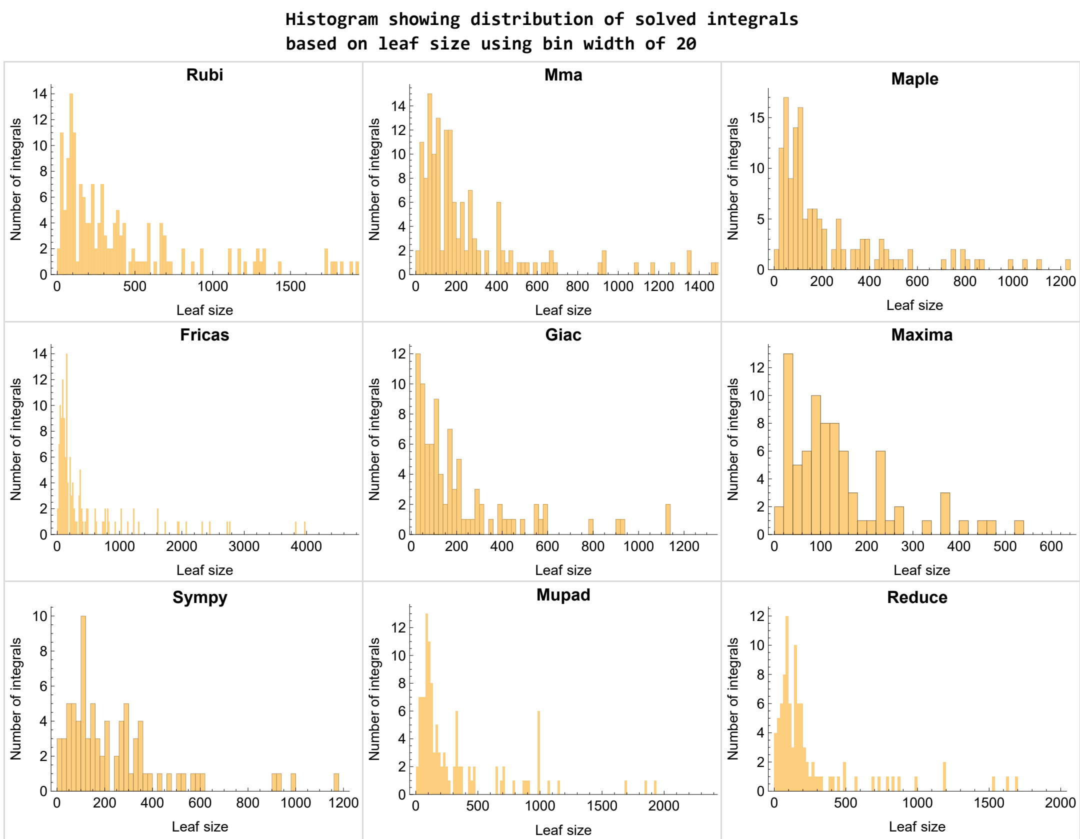


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

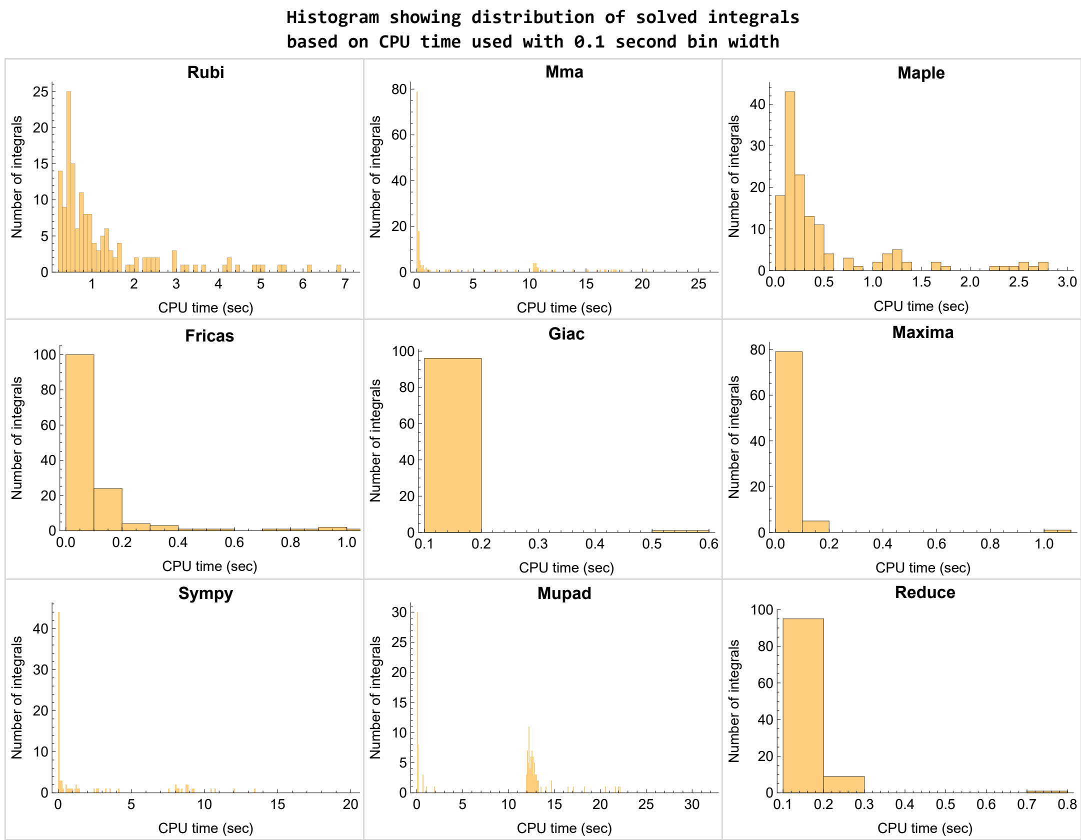


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

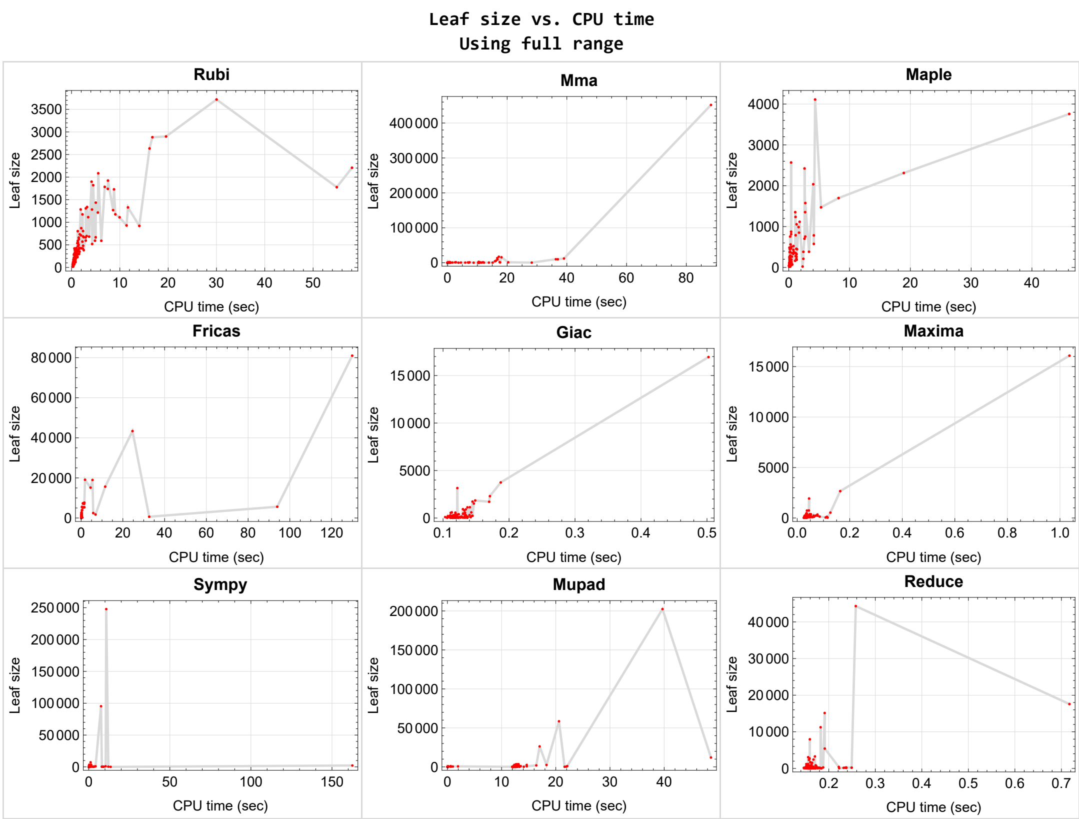


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Reduce {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {19, 20, 21, 27, 28, 46, 47, 48, 54, 74, 75, 76, 77, 78, 79, 92, 93, 94, 95, 96, 118, 119, 120, 121, 122, 132, 164, 170, 175, 176}

Mathematica {20, 27, 47, 74, 75, 76, 77, 78, 79, 81, 83, 92, 93, 94, 95, 96, 129, 131, 132, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159}

Maple $\{7, 100, 101, 102, 139, 140, 141, 142, 143\}$

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

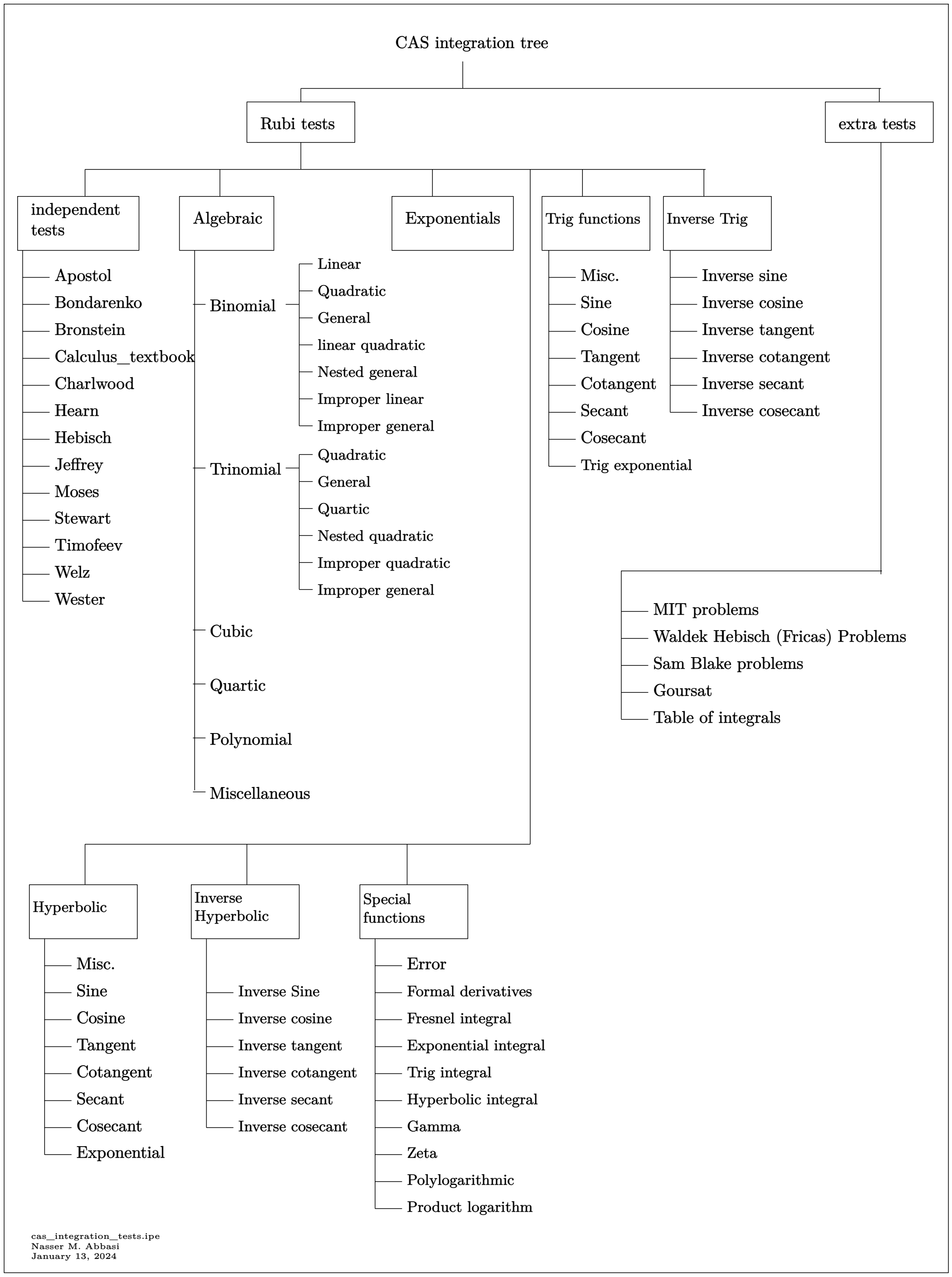
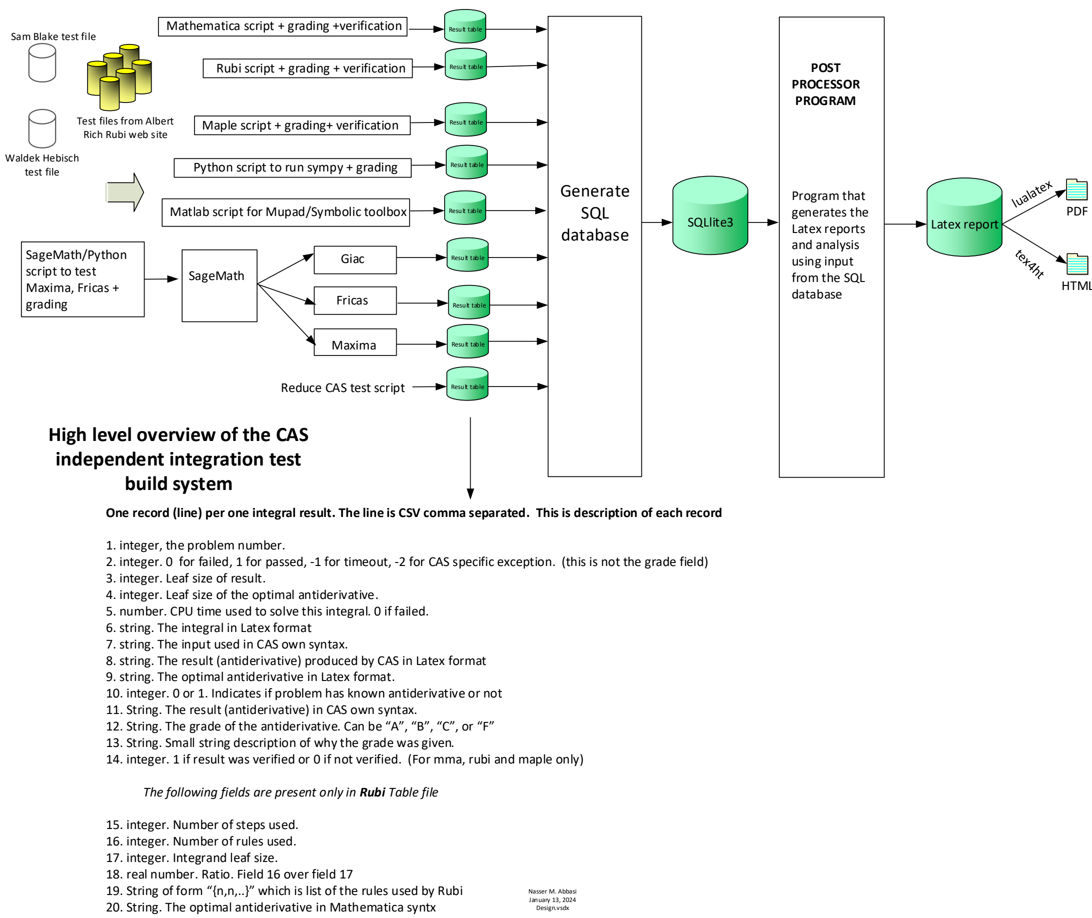


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 21, 22, 23, 24, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 48, 49, 50, 51, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 125, 126, 127, 128, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 171, 172, 173, 174, 175, 176, 177, 178 }

B grade { }

C grade { 20, 25, 26, 27, 28, 47, 52, 53, 54, 92, 93, 94, 95, 96, 118, 119, 120, 121, 122, 132, 168, 169, 170 }

F normal fail { 97, 115, 116, 117, 123, 124, 129, 130, 131, 133 }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 22, 23, 24, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 49, 50, 51, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 81, 83, 85, 86, 87, 88, 89, 91, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 118, 119, 120, 121, 122, 125, 126, 127, 128, 134, 135, 136, 137, 138, 141, 142, 146, 147, 165, 166, 167, 171, 172, 173, 177, 178 }

B grade { 55, 98, 139, 140, 143, 144, 145, 148, 149 }

C grade { 18, 19, 20, 25, 26, 27, 45, 46, 47, 52, 53, 71, 72, 73, 74, 75, 76, 77, 78, 79, 90, 92, 93, 94, 95, 96, 115, 116, 117, 129, 130, 131, 132, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 168, 169, 174, 175 }

F normal fail { 21, 28, 48, 54, 80, 82, 84, 97, 123, 124, 133, 160, 161, 162, 164, 170, 176 }

F(-1) timedout fail { 163 }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 15, 16, 17, 22, 23, 24, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 42, 43, 44, 49, 50, 51, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 68, 69, 70, 76, 77, 85, 86, 87, 88, 89, 90, 91, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 118, 119, 120, 121, 122, 125, 126, 127, 129, 130, 134, 135, 136, 137, 138, 145, 146, 147, 150, 151, 152, 165, 166, 167, 171, 172, 173, 177, 178 }

B grade { 55, 67, 74, 75, 78, 79, 117, 128, 131, 148, 149 }

C grade { 7, 18, 19, 20, 25, 26, 27, 45, 46, 47, 52, 53, 71, 72, 73, 92, 93, 94, 95, 96, 132, 139, 140, 141, 142, 143, 144, 168, 169, 174, 175 }

F normal fail { 14, 21, 28, 41, 48, 54, 80, 81, 82, 83, 84, 97, 123, 124, 133, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 170, 176 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 7, 8, 9, 10, 11, 12, 13, 15, 16, 17, 20, 22, 23, 24, 27, 29, 30, 31, 32, 33, 36, 37, 38, 39, 40, 42, 43, 44, 47, 49, 50, 51, 56, 57, 58, 60, 61, 62, 63, 64, 66, 68, 69, 70, 74, 75, 76, 77, 85, 86, 87, 88, 90, 91, 92, 93, 94, 95, 96, 100, 101, 104, 105, 106, 108, 109, 110, 111, 115, 116, 118, 119, 120, 121, 122, 125, 126, 127, 129, 130, 134, 135, 136, 137, 138, 139, 141, 142, 150, 151, 152, 153, 154, 157, 162, 165, 166, 167, 171, 172, 173, 177, 178 }

B grade { 5, 6, 34, 35, 55, 59, 65, 67, 78, 79, 89, 98, 99, 102, 103, 107, 117, 131, 140, 143, 144, 145, 146, 147, 155, 156, 158, 159, 160, 161, 163, 164 }

C grade { 18, 19, 25, 26, 45, 46, 52, 53, 71, 72, 73, 132, 168, 169, 174, 175 }

F normal fail { 14, 21, 28, 41, 48, 54, 80, 81, 82, 83, 84, 97, 123, 124, 133, 170, 176 }

F(-1) timedout fail { 112, 113, 114, 128, 148, 149 }

F(-2) exception fail { }

Maxima

A grade { 1, 2, 3, 4, 5, 6, 8, 9, 10, 15, 16, 17, 22, 23, 24, 29, 30, 31, 32, 33, 34, 35, 36, 37, 42, 43, 44, 49, 50, 51, 56, 57, 58, 60, 61, 62, 63, 64, 65, 66, 68, 69, 70, 85, 86, 87, 88, 89, 90, 91, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 125, 126, 134, 135, 136, 137, 138, 165, 166, 167, 171, 172, 173, 177, 178 }

B grade { 55, 59, 67, 113, 114 }

C grade { }

F normal fail { 7, 11, 12, 13, 14, 18, 19, 20, 21, 25, 26, 27, 28, 38, 39, 40, 41, 45, 46, 47, 48, 52, 53, 54, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 92, 93, 94, 95, 96, 97, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 129, 130, 131, 132, 133, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 162, 163, 164, 168, 169, 170, 174, 175, 176 }

F(-1) timedout fail { }

F(-2) exception fail { 127, 128, 160, 161 }

Giac

A grade { 1, 2, 3, 4, 5, 6, 11, 12, 13, 15, 16, 17, 22, 23, 24, 29, 30, 31, 32, 33, 34, 35, 38, 39, 40, 42, 43, 44, 49, 50, 51, 56, 57, 58, 59, 62, 63, 64, 68, 69, 70, 85, 86, 87, 88, 89, 90, 91, 100, 101, 102, 103, 104, 105, 106, 107, 108, 110, 111, 112, 125, 126, 127, 134, 135, 136, 137, 138, 157, 165, 166, 167, 171, 172, 173, 177, 178 }

B grade { 8, 9, 10, 36, 37, 55, 60, 61, 65, 66, 67, 98, 99, 109, 113, 114, 128, 150, 151, 152 }

C grade { }

F normal fail { 14, 20, 21, 27, 28, 41, 47, 48, 54, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 92, 93, 94, 95, 96, 97, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 129, 130, 131, 132, 133, 170, 176 }

F(-1) timedout fail { }

F(-2) exception fail { 7, 18, 19, 25, 26, 45, 46, 52, 53, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 153, 154, 155, 156, 158, 159, 160, 161, 162, 163, 164, 168, 169, 174, 175 }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 15, 16, 17, 18, 19, 20, 22, 23, 24, 25, 26, 27, 29, 30, 31, 32, 33, 34, 35, 36, 37, 42, 43, 44, 45, 46, 47, 49, 50, 51, 52, 53, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 118, 119, 120, 121, 125, 126, 127, 128, 132, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 145, 146, 147, 165, 166, 167, 168, 169, 171, 172, 173, 174, 175, 177, 178 }

C grade { }

F normal fail { }

F(-1) timedout fail { 11, 12, 13, 14, 21, 28, 38, 39, 40, 41, 48, 54, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 96, 97, 115, 116, 117, 122, 123, 124, 129, 130, 131, 133, 144, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 170, 176 }

F(-2) exception fail { }

Sympy

A grade { 1, 2, 3, 5, 6, 15, 16, 17, 18, 19, 22, 23, 24, 25, 26, 29, 30, 31, 32, 34, 35, 42, 43, 44, 45, 46, 49, 50, 51, 52, 53, 56, 57, 58, 68, 69, 70, 85, 86, 87, 100, 101, 105, 106, 110, 111, 125, 126, 134, 135, 136, 137, 138, 165, 166, 167, 168, 169, 171, 172, 173, 174, 175, 177, 178 }

B grade { 4, 7, 33, 55, 59, 67, 98, 99, 102, 103, 104, 109, 139, 140, 141, 142, 146, 147 }

C grade { 90, 91 }

F normal fail { 8, 9, 10, 11, 12, 13, 14, 20, 21, 27, 36, 37, 38, 39, 40, 41, 47, 48, 60, 61, 62, 63, 64, 65, 66, 74, 75, 76, 77, 81, 83, 92, 93, 94, 95, 96, 115, 116, 117, 118, 119, 120, 121, 122, 129, 130, 131, 132, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 162, 163, 164, 176 }

F(-1) timedout fail { 28, 54, 71, 72, 73, 78, 79, 80, 82, 84, 88, 89, 97, 108, 112, 113, 114, 123, 124, 127, 128, 133, 143, 144, 145, 148, 149, 160, 161, 170 }

F(-2) exception fail { 107 }

Reduce

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 15, 16, 17, 22, 23, 24, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 42, 43, 44, 49, 50, 51, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 85, 86, 87, 88, 89, 90, 91, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 125, 126, 127, 128, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 165, 166, 167, 171, 172, 173, 177, 178 }

C grade { }

F normal fail { 14, 18, 19, 20, 21, 25, 26, 27, 28, 41, 45, 46, 47, 48, 52, 53, 54, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 92, 93, 94, 95, 96, 97, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 129, 130, 131, 132, 133, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 168, 169, 170, 174, 175, 176 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	267	240	273	277	277	314	299	210	249
N.S.	1	1.39	1.25	1.42	1.44	1.44	1.64	1.56	1.09	1.30
time (sec)	N/A	0.728	0.069	0.254	0.046	0.071	0.054	0.116	0.249	12.270

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	171	157	177	177	177	202	191	141	162
N.S.	1	1.12	1.03	1.16	1.16	1.16	1.32	1.25	0.92	1.06
time (sec)	N/A	0.498	0.030	0.234	0.030	0.077	0.038	0.134	0.237	11.992

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	73	81	79	79	90	83	72	80
N.S.	1	1.00	0.88	0.98	0.95	0.95	1.08	1.00	0.87	0.96
time (sec)	N/A	0.299	0.010	0.223	0.041	0.078	0.025	0.122	0.222	12.076

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	98	107	107	152	243	110	151	106
N.S.	1	1.00	0.87	0.95	0.95	1.35	2.15	0.97	1.34	0.94
time (sec)	N/A	0.344	0.050	0.115	0.044	0.083	0.674	0.128	0.232	12.297

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	217	191	203	236	480	382	215	490	191
N.S.	1	1.00	0.88	0.94	1.09	2.21	1.76	0.99	2.26	0.88
time (sec)	N/A	0.501	0.115	0.180	0.045	0.105	1.280	0.134	0.174	12.034

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	335	335	290	290	368	810	522	314	838	309
N.S.	1	1.00	0.87	0.87	1.10	2.42	1.56	0.94	2.50	0.92
time (sec)	N/A	0.719	0.184	0.243	0.048	0.105	3.262	0.140	0.172	12.081

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	B	F(-2)	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	136	172	232	53	0	167	7218	0	271	140
N.S.	1	1.26	1.71	0.39	0.00	1.23	53.07	0.00	1.99	1.03
time (sec)	N/A	0.492	0.113	0.362	0.000	0.148	1.131	0.000	0.188	12.618

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	541	382	215	263	337	344	0	3745	232	442
N.S.	1	0.71	0.40	0.49	0.62	0.64	0.00	6.92	0.43	0.82
time (sec)	N/A	0.700	0.194	0.516	0.077	0.095	0.000	0.187	0.165	12.563

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	397	295	164	191	238	245	0	1846	163	333
N.S.	1	0.74	0.41	0.48	0.60	0.62	0.00	4.65	0.41	0.84
time (sec)	N/A	0.601	0.133	0.413	0.071	0.093	0.000	0.149	0.162	12.667

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	201	110	119	139	146	0	595	94	118
N.S.	1	0.80	0.44	0.47	0.55	0.58	0.00	2.37	0.37	0.47
time (sec)	N/A	0.495	0.081	0.382	0.086	0.086	0.000	0.143	0.152	12.547

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	182	135	145	0	363	0	130	157	0
N.S.	1	0.88	0.65	0.70	0.00	1.75	0.00	0.62	0.75	0.00
time (sec)	N/A	0.450	0.181	0.368	0.000	0.100	0.000	0.136	0.181	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	320	296	198	502	0	761	0	240	571	0
N.S.	1	0.92	0.62	1.57	0.00	2.38	0.00	0.75	1.78	0.00
time (sec)	N/A	0.528	0.353	0.415	0.000	0.107	0.000	0.144	0.161	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	540	421	274	874	0	1225	0	449	1195	0
N.S.	1	0.78	0.51	1.62	0.00	2.27	0.00	0.83	2.21	0.00
time (sec)	N/A	0.605	0.585	0.384	0.000	0.173	0.000	0.133	0.155	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	259	280	166	0	0	0	0	0	1607	0
N.S.	1	1.08	0.64	0.00	0.00	0.00	0.00	0.00	6.20	0.00
time (sec)	N/A	0.498	0.229	0.000	0.000	0.000	0.000	0.000	0.184	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	154	135	139	139	153	161	159	139
N.S.	1	1.00	0.99	0.87	0.90	0.90	0.99	1.04	1.03	0.90
time (sec)	N/A	0.433	0.031	0.255	0.030	0.064	0.053	0.128	0.155	0.081

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	102	92	94	94	102	106	105	93
N.S.	1	1.00	1.00	0.90	0.92	0.92	1.00	1.04	1.03	0.91
time (sec)	N/A	0.337	0.017	0.209	0.033	0.078	0.031	0.108	0.160	12.689

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	50	49	53	53	51	52	51	52
N.S.	1	1.00	0.82	0.80	0.87	0.87	0.84	0.85	0.84	0.85
time (sec)	N/A	0.251	0.013	0.178	0.037	0.066	0.025	0.130	0.176	0.018

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	C	A	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	574	571	60	41	0	7302	277	0	55	322
N.S.	1	0.99	0.10	0.07	0.00	12.72	0.48	0.00	0.10	0.56
time (sec)	N/A	2.245	0.015	0.065	0.000	0.873	8.088	0.000	0.158	12.542

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	C	A	F(-2)	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1171	1215	149	103	0	5268	337	0	485	987
N.S.	1	1.04	0.13	0.09	0.00	4.50	0.29	0.00	0.41	0.84
time (sec)	N/A	5.446	0.032	0.083	0.000	0.914	8.828	0.000	0.165	12.882

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	C	F	A	F	F	F	B
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1731	803	1260	1056	0	45	0	0	89	3117
N.S.	1	0.46	0.73	0.61	0.00	0.03	0.00	0.00	0.05	1.80
time (sec)	N/A	1.287	10.575	1.285	0.000	0.074	0.000	0.000	0.162	12.676

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	No	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	936	1175	0	0	0	0	0	0	0	0
N.S.	1	1.26	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.264	0.000	0.000	0.000	0.000	0.000	0.000	0.163	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	151	135	137	137	151	161	159	139
N.S.	1	1.00	1.00	0.89	0.91	0.91	1.00	1.07	1.05	0.92
time (sec)	N/A	0.700	0.026	0.262	0.031	0.076	0.039	0.116	0.164	12.238

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	99	92	91	91	99	106	105	93
N.S.	1	1.00	1.00	0.93	0.92	0.92	1.00	1.07	1.06	0.94
time (sec)	N/A	0.542	0.016	0.211	0.035	0.063	0.037	0.134	0.155	0.032

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	50	49	51	51	53	52	51	52
N.S.	1	1.00	0.85	0.83	0.86	0.86	0.90	0.88	0.86	0.88
time (sec)	N/A	0.395	0.013	0.187	0.033	0.065	0.026	0.117	0.154	0.018

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	C	F	C	A	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	373	930	64	43	0	7031	277	0	55	322
N.S.	1	2.49	0.17	0.12	0.00	18.85	0.74	0.00	0.15	0.86
time (sec)	N/A	11.386	0.015	0.063	0.000	1.407	9.206	0.000	0.164	12.427

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	C	F	C	A	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	885	2207	150	103	0	5181	342	0	485	995
N.S.	1	2.49	0.17	0.12	0.00	5.85	0.39	0.00	0.55	1.12
time (sec)	N/A	58.058	0.035	0.082	0.000	0.761	8.105	0.000	0.158	12.565

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	C	F	A	F	F	F	B
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	749	2085	589	788	0	45	0	0	89	3317
N.S.	1	2.78	0.79	1.05	0.00	0.06	0.00	0.00	0.12	4.43
time (sec)	N/A	5.528	10.408	1.129	0.000	0.074	0.000	0.000	0.166	12.641

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	No	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	747	1821	0	0	0	0	0	0	0	0
N.S.	1	2.44	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	4.480	0.000	0.000	0.000	0.000	0.000	0.000	0.178	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	230	353	353	369	375	375	425	407	398	335
N.S.	1	1.53	1.53	1.60	1.63	1.63	1.85	1.77	1.73	1.46
time (sec)	N/A	1.347	0.066	0.212	0.037	0.076	0.058	0.119	0.165	0.167

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	263	263	273	277	277	320	299	293	248
N.S.	1	1.37	1.37	1.42	1.44	1.44	1.67	1.56	1.53	1.29
time (sec)	N/A	1.026	0.044	0.189	0.034	0.079	0.052	0.116	0.155	12.180

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	171	171	177	177	177	204	191	188	162
N.S.	1	1.11	1.11	1.15	1.15	1.15	1.32	1.24	1.22	1.05
time (sec)	N/A	0.744	0.025	0.191	0.042	0.068	0.043	0.107	0.162	12.308

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	83	81	79	79	97	83	83	82
N.S.	1	1.00	1.00	0.98	0.95	0.95	1.17	1.00	1.00	0.99
time (sec)	N/A	0.450	0.015	0.158	0.047	0.066	0.024	0.136	0.157	0.025

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	111	105	108	152	241	111	219	104
N.S.	1	1.00	1.01	0.95	0.98	1.38	2.19	1.01	1.99	0.95
time (sec)	N/A	0.547	0.034	0.116	0.054	0.070	0.749	0.110	0.178	12.440

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	216	186	199	237	481	379	217	697	191
N.S.	1	1.00	0.86	0.92	1.10	2.23	1.75	1.00	3.23	0.88
time (sec)	N/A	0.754	0.100	0.175	0.042	0.082	1.358	0.112	0.156	12.301

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	308	308	265	286	369	811	517	316	1189	309
N.S.	1	1.00	0.86	0.93	1.20	2.63	1.68	1.03	3.86	1.00
time (sec)	N/A	1.066	0.173	0.235	0.052	0.112	3.564	0.121	0.152	12.209

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	398	389	158	191	232	247	0	1733	223	341
N.S.	1	0.98	0.40	0.48	0.58	0.62	0.00	4.35	0.56	0.86
time (sec)	N/A	1.397	0.140	0.419	0.063	0.079	0.000	0.145	0.168	12.613

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	253	283	100	119	133	148	0	554	118	118
N.S.	1	1.12	0.40	0.47	0.53	0.58	0.00	2.19	0.47	0.47
time (sec)	N/A	1.153	0.085	0.375	0.062	0.090	0.000	0.133	0.155	12.352

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	246	126	171	0	369	0	121	190	0
N.S.	1	1.19	0.61	0.83	0.00	1.79	0.00	0.59	0.92	0.00
time (sec)	N/A	1.083	0.136	0.323	0.000	0.096	0.000	0.137	0.153	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	318	376	188	466	0	768	0	224	731	0
N.S.	1	1.18	0.59	1.47	0.00	2.42	0.00	0.70	2.30	0.00
time (sec)	N/A	1.200	0.293	0.365	0.000	0.105	0.000	0.145	0.173	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	536	542	264	814	0	1232	0	429	1538	0
N.S.	1	1.01	0.49	1.52	0.00	2.30	0.00	0.80	2.87	0.00
time (sec)	N/A	1.344	0.515	0.388	0.000	0.175	0.000	0.130	0.164	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	246	378	159	0	0	0	0	0	0	0
N.S.	1	1.54	0.65	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.980	0.224	0.000	0.000	0.000	0.000	0.000	0.172	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	157	135	141	141	155	161	161	138
N.S.	1	1.00	1.00	0.86	0.90	0.90	0.99	1.03	1.03	0.88
time (sec)	N/A	0.676	0.024	0.227	0.029	0.073	0.044	0.136	0.156	0.081

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	106	92	96	96	104	106	107	92
N.S.	1	1.00	1.00	0.87	0.91	0.91	0.98	1.00	1.01	0.87
time (sec)	N/A	0.544	0.015	0.178	0.032	0.073	0.034	0.141	0.154	12.130

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	60	50	52	52	53	52	53	50
N.S.	1	1.00	1.00	0.83	0.87	0.87	0.88	0.87	0.88	0.83
time (sec)	N/A	0.401	0.009	0.135	0.036	0.083	0.033	0.114	0.157	0.017

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	C	A	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	593	591	65	45	0	7319	270	0	76	328
N.S.	1	1.00	0.11	0.08	0.00	12.34	0.46	0.00	0.13	0.55
time (sec)	N/A	4.856	0.016	0.064	0.000	0.911	8.765	0.000	0.157	12.417

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	C	A	F(-2)	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1056	1110	156	109	0	5341	338	0	606	988
N.S.	1	1.05	0.15	0.10	0.00	5.06	0.32	0.00	0.57	0.94
time (sec)	N/A	9.947	0.033	0.086	0.000	1.200	9.172	0.000	0.156	12.470

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	C	F	A	F	F	F	B
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1895	665	1597	1236	0	60	0	0	116	3476
N.S.	1	0.35	0.84	0.65	0.00	0.03	0.00	0.00	0.06	1.83
time (sec)	N/A	2.908	10.692	1.112	0.000	0.085	0.000	0.000	0.180	13.131

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	No	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	977	1308	0	0	0	0	0	0	0	0
N.S.	1	1.34	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.958	0.000	0.000	0.000	0.000	0.000	0.000	0.165	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	145	135	131	131	155	161	161	138
N.S.	1	1.00	1.00	0.93	0.90	0.90	1.07	1.11	1.11	0.95
time (sec)	N/A	0.724	0.044	0.227	0.030	0.074	0.039	0.119	0.152	12.239

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	97	92	89	89	102	106	107	92
N.S.	1	1.00	1.00	0.95	0.92	0.92	1.05	1.09	1.10	0.95
time (sec)	N/A	0.539	0.028	0.168	0.034	0.068	0.038	0.117	0.166	0.032

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	56	50	48	48	54	52	53	50
N.S.	1	1.00	1.00	0.89	0.86	0.86	0.96	0.93	0.95	0.89
time (sec)	N/A	0.373	0.016	0.132	0.033	0.064	0.029	0.137	0.149	12.148

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	C	F	C	A	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	274	667	69	47	0	7104	272	0	76	328
N.S.	1	2.43	0.25	0.17	0.00	25.93	0.99	0.00	0.28	1.20
time (sec)	N/A	4.999	0.027	0.064	0.000	1.111	8.895	0.000	0.152	12.479

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	C	F	C	A	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	724	1329	161	109	0	5266	342	0	606	996
N.S.	1	1.84	0.22	0.15	0.00	7.27	0.47	0.00	0.84	1.38
time (sec)	N/A	11.682	0.065	0.082	0.000	1.063	8.786	0.000	0.172	12.576

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	No	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	571	1729	0	0	0	0	0	0	0	0
N.S.	1	3.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	8.792	0.000	0.000	0.000	0.000	0.000	0.000	0.170	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	170	198	200	200	219	219	151	188
N.S.	1	1.00	2.46	2.87	2.90	2.90	3.17	3.17	2.19	2.72
time (sec)	N/A	0.566	0.097	0.169	0.037	0.073	0.041	0.133	0.154	0.059

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	92	101	102	102	110	110	82	100
N.S.	1	1.00	1.33	1.46	1.48	1.48	1.59	1.59	1.19	1.45
time (sec)	N/A	0.438	0.039	0.141	0.037	0.069	0.028	0.129	0.162	0.027

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	59	57	71	87	68	60	99	67
N.S.	1	1.00	0.94	0.90	1.13	1.38	1.08	0.95	1.57	1.06
time (sec)	N/A	0.426	0.028	0.070	0.032	0.079	0.291	0.116	0.156	0.042

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	52	63	101	101	107	52	90	101
N.S.	1	1.00	0.75	0.91	1.46	1.46	1.55	0.75	1.30	1.46
time (sec)	N/A	0.423	0.029	0.108	0.039	0.080	0.944	0.115	0.159	0.039

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	51	63	132	132	141	51	123	132
N.S.	1	1.00	0.74	0.91	1.91	1.91	2.04	0.74	1.78	1.91
time (sec)	N/A	0.401	0.028	0.148	0.039	0.069	2.605	0.116	0.166	12.027

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	96	64	80	193	205	0	1138	160	81
N.S.	1	0.60	0.40	0.50	1.21	1.28	0.00	7.11	1.00	0.51
time (sec)	N/A	0.450	0.111	0.447	0.052	0.076	0.000	0.141	0.153	12.264

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	96	62	80	89	106	0	399	91	79
N.S.	1	0.60	0.39	0.50	0.56	0.66	0.00	2.49	0.57	0.49
time (sec)	N/A	0.429	0.082	0.401	0.051	0.071	0.000	0.119	0.182	12.522

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	92	59	71	87	96	0	90	46	81
N.S.	1	0.59	0.38	0.46	0.56	0.62	0.00	0.58	0.29	0.52
time (sec)	N/A	0.429	0.098	0.376	0.059	0.085	0.000	0.125	0.157	12.821

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	96	63	80	128	130	0	66	76	81
N.S.	1	0.60	0.39	0.50	0.80	0.81	0.00	0.41	0.48	0.51
time (sec)	N/A	0.415	0.099	0.415	0.065	0.087	0.000	0.119	0.154	12.750

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	96	64	80	227	163	0	66	109	81
N.S.	1	0.60	0.40	0.50	1.42	1.02	0.00	0.41	0.68	0.51
time (sec)	N/A	0.448	0.102	0.373	0.078	0.088	0.000	0.120	0.169	12.873

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	113	108	146	156	230	0	399	183	219
N.S.	1	1.11	1.06	1.43	1.53	2.25	0.00	3.91	1.79	2.15
time (sec)	N/A	0.477	0.196	0.215	0.043	0.084	0.000	0.124	0.249	12.298

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	113	108	166	156	230	0	799	203	239
N.S.	1	0.68	0.65	0.99	0.93	1.38	0.00	4.78	1.22	1.43
time (sec)	N/A	0.445	0.019	0.142	0.048	0.105	0.000	0.132	0.235	12.914

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	387	684	684	756	747	747	910	909	401	718
N.S.	1	1.77	1.77	1.95	1.93	1.93	2.35	2.35	1.04	1.86
time (sec)	N/A	2.419	0.305	0.224	0.044	0.078	0.094	0.135	0.222	13.079

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	310	408	408	462	452	452	568	549	262	433
N.S.	1	1.32	1.32	1.49	1.46	1.46	1.83	1.77	0.85	1.40
time (sec)	N/A	1.491	0.159	0.190	0.036	0.075	0.069	0.112	0.238	0.137

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	227	227	240	241	241	287	279	151	225
N.S.	1	1.05	1.05	1.11	1.12	1.12	1.33	1.29	0.70	1.04
time (sec)	N/A	0.910	0.068	0.171	0.029	0.083	0.048	0.111	0.177	0.062

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	88	89	87	87	102	98	68	85
N.S.	1	1.00	1.00	1.01	0.99	0.99	1.16	1.11	0.77	0.97
time (sec)	N/A	0.479	0.020	0.161	0.034	0.093	0.025	0.142	0.160	12.588

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	C	F(-1)	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	395	382	90	67	0	19077	0	0	781	3423
N.S.	1	0.97	0.23	0.17	0.00	48.30	0.00	0.00	1.98	8.67
time (sec)	N/A	2.526	0.033	0.096	0.000	1.870	0.000	0.000	0.159	12.894

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	C	F(-1)	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	484	481	186	208	0	15642	0	0	2426	1692
N.S.	1	0.99	0.38	0.43	0.00	32.32	0.00	0.00	5.01	3.50
time (sec)	N/A	2.401	0.125	0.129	0.000	11.539	0.000	0.000	0.167	13.195

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	C	F(-1)	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	564	594	286	446	0	18983	0	0	5412	2559
N.S.	1	1.05	0.51	0.79	0.00	33.66	0.00	0.00	9.60	4.54
time (sec)	N/A	2.951	0.185	0.185	0.000	5.456	0.000	0.000	0.191	13.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	577	710	504	1576	0	365	0	0	916	0
N.S.	1	1.23	0.87	2.73	0.00	0.63	0.00	0.00	1.59	0.00
time (sec)	N/A	1.960	11.459	2.719	0.000	0.082	0.000	0.000	0.258	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	509	626	353	1353	0	203	0	0	458	0
N.S.	1	1.23	0.69	2.66	0.00	0.40	0.00	0.00	0.90	0.00
time (sec)	N/A	1.522	10.731	2.687	0.000	0.099	0.000	0.000	0.221	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	431	537	284	755	0	122	0	0	243	0
N.S.	1	1.25	0.66	1.75	0.00	0.28	0.00	0.00	0.56	0.00
time (sec)	N/A	1.206	10.662	2.708	0.000	0.083	0.000	0.000	0.220	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	486	586	928	851	0	398	0	0	0	0
N.S.	1	1.21	1.91	1.75	0.00	0.82	0.00	0.00	0.00	0.00
time (sec)	N/A	1.292	11.133	1.688	0.000	0.090	0.000	0.000	0.298	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	549	661	400	988	0	923	0	0	0	0
N.S.	1	1.20	0.73	1.80	0.00	1.68	0.00	0.00	0.00	0.00
time (sec)	N/A	1.491	12.250	1.615	0.000	0.108	0.000	0.000	0.402	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	609	733	467	1117	0	1608	0	0	0	0
N.S.	1	1.20	0.77	1.83	0.00	2.64	0.00	0.00	0.00	0.00
time (sec)	N/A	1.696	15.142	1.783	0.000	0.193	0.000	0.000	0.677	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	177	216	0	0	0	0	0	0	0	0
N.S.	1	1.22	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.769	0.000	0.000	0.000	0.000	0.000	0.000	0.193	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	295	238	549	0	0	0	0	0	0	0
N.S.	1	0.81	1.86	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.841	0.956	0.000	0.000	0.000	0.000	0.000	0.196	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	239	269	0	0	0	0	0	0	0	0
N.S.	1	1.13	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.881	0.000	0.000	0.000	0.000	0.000	0.000	0.457	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	302	280	409	0	0	0	0	0	0	0
N.S.	1	0.93	1.35	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.845	0.807	0.000	0.000	0.000	0.000	0.000	0.182	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	252	326	0	0	0	0	0	0	0	0
N.S.	1	1.29	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.945	0.000	0.000	0.000	0.000	0.000	0.000	0.477	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	419	419	419	486	461	461	585	573	456	436
N.S.	1	1.00	1.00	1.16	1.10	1.10	1.40	1.37	1.09	1.04
time (sec)	N/A	1.623	0.154	0.113	0.036	0.065	0.064	0.119	0.167	12.740

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	224	225	251	239	239	291	290	245	230
N.S.	1	1.00	1.00	1.12	1.07	1.07	1.30	1.29	1.09	1.03
time (sec)	N/A	0.897	0.082	0.103	0.037	0.062	0.058	0.135	0.156	0.061

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	85	82	83	83	97	97	92	80
N.S.	1	1.00	1.00	0.96	0.98	0.98	1.14	1.14	1.08	0.94
time (sec)	N/A	0.457	0.022	0.091	0.035	0.062	0.032	0.103	0.159	0.026

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	108	108	117	240	0	118	151	793
N.S.	1	1.00	0.90	0.90	0.98	2.00	0.00	0.98	1.26	6.61
time (sec)	N/A	0.589	0.057	0.131	0.113	0.583	0.000	0.114	0.170	14.641

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	330	375	307	376	532	2454	0	484	1699	1931
N.S.	1	1.14	0.93	1.14	1.61	7.44	0.00	1.47	5.15	5.85
time (sec)	N/A	1.320	0.253	0.151	0.126	5.797	0.000	0.131	0.161	16.402

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	101	41	39	39	925	40	64	60
N.S.	1	1.00	2.24	0.91	0.87	0.87	20.56	0.89	1.42	1.33
time (sec)	N/A	0.370	0.027	0.111	0.116	0.082	4.187	0.120	0.152	0.101

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	63	53	51	51	983	52	82	62
N.S.	1	1.00	1.11	0.93	0.89	0.89	17.25	0.91	1.44	1.09
time (sec)	N/A	0.416	0.024	0.108	0.108	0.077	10.404	0.131	0.163	13.289

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	C	F	A	F	F	F	B
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	636	2883	317	523	0	145	0	0	625	2455
N.S.	1	4.53	0.50	0.82	0.00	0.23	0.00	0.00	0.98	3.86
time (sec)	N/A	16.738	10.327	0.778	0.000	0.070	0.000	0.000	0.218	14.634

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	C	F	A	F	F	F	B
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	459	1282	271	430	0	98	0	0	427	1846
N.S.	1	2.79	0.59	0.94	0.00	0.21	0.00	0.00	0.93	4.02
time (sec)	N/A	4.238	7.144	0.750	0.000	0.075	0.000	0.000	0.213	12.452

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	C	F	A	F	F	F	B
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	336	869	231	332	0	57	0	0	180	656
N.S.	1	2.59	0.69	0.99	0.00	0.17	0.00	0.00	0.54	1.95
time (sec)	N/A	2.007	10.329	0.730	0.000	0.102	0.000	0.000	0.190	12.761

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	C	F	A	F	F	F	B
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	438	1435	275	377	0	201	0	0	1472	692
N.S.	1	3.28	0.63	0.86	0.00	0.46	0.00	0.00	3.36	1.58
time (sec)	N/A	5.019	10.526	0.832	0.000	0.073	0.000	0.000	0.209	12.903

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	C	F	A	F	F	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	555	3718	520	446	0	352	0	0	0	0
N.S.	1	6.70	0.94	0.80	0.00	0.63	0.00	0.00	0.00	0.00
time (sec)	N/A	30.023	13.889	1.363	0.000	0.075	0.000	0.000	0.212	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	405	0	0	0	0	0	0	0	0	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.249	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	49	21	20	142	163	70	159	136
N.S.	1	1.00	2.23	0.95	0.91	6.45	7.41	3.18	7.23	6.18
time (sec)	N/A	0.353	0.031	0.142	0.026	0.071	0.035	0.124	0.157	0.051

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	30	21	20	62	68	44	61	62
N.S.	1	1.00	1.36	0.95	0.91	2.82	3.09	2.00	2.77	2.82
time (sec)	N/A	0.301	0.010	0.125	0.036	0.063	0.035	0.126	0.160	12.917

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	16	19	18	18	17	27	18	18
N.S.	1	1.00	0.89	1.06	1.00	1.00	0.94	1.50	1.00	1.00
time (sec)	N/A	0.287	0.007	0.062	0.032	0.069	0.337	0.117	0.168	0.032

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	18	21	20	20	17	28	20	20
N.S.	1	1.00	0.90	1.05	1.00	1.00	0.85	1.40	1.00	1.00
time (sec)	N/A	0.277	0.012	0.081	0.026	0.067	1.281	0.112	0.163	0.048

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	20	21	20	64	71	28	75	71
N.S.	1	1.00	0.91	0.95	0.91	2.91	3.23	1.27	3.41	3.23
time (sec)	N/A	0.280	0.015	0.118	0.026	0.069	2.712	0.133	0.152	12.116

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	22	21	20	86	299	20	84	89
N.S.	1	1.00	0.92	0.88	0.83	3.58	12.46	0.83	3.50	3.71
time (sec)	N/A	0.286	0.056	0.135	0.037	0.070	0.215	0.115	0.170	12.290

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	22	21	20	39	109	20	34	20
N.S.	1	1.00	0.92	0.88	0.83	1.62	4.54	0.83	1.42	0.83
time (sec)	N/A	0.280	0.046	0.101	0.026	0.087	0.111	0.123	0.153	12.048

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	20	21	20	20	20	20	19	20
N.S.	1	1.00	0.91	0.95	0.91	0.91	0.91	0.91	0.86	0.91
time (sec)	N/A	0.280	0.040	2.271	0.032	0.069	0.100	0.106	0.157	12.034

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	20	21	20	20	22	20	36	20
N.S.	1	1.00	0.91	0.95	0.91	0.91	1.00	0.91	1.64	0.91
time (sec)	N/A	0.276	0.048	0.216	0.027	0.074	0.128	0.111	0.184	11.976

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F(-2)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	22	21	20	81	0	36	91	20
N.S.	1	1.00	0.92	0.88	0.83	3.38	0.00	1.50	3.79	0.83
time (sec)	N/A	0.278	0.055	0.229	0.033	0.076	0.000	0.118	0.181	12.213

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	24	27	26	43	0	26	39	58
N.S.	1	1.00	0.92	1.04	1.00	1.65	0.00	1.00	1.50	2.23
time (sec)	N/A	0.279	0.099	0.194	0.035	0.097	0.000	0.113	0.166	12.038

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1363	1739	1739	2569	1930	1930	3334	3151	2315	2595
N.S.	1	1.28	1.28	1.88	1.42	1.42	2.45	2.31	1.70	1.90
time (sec)	N/A	7.514	0.835	0.382	0.045	0.089	0.245	0.122	0.157	12.351

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	664	671	671	562	735	735	1166	1126	871	891
N.S.	1	1.01	1.01	0.85	1.11	1.11	1.76	1.70	1.31	1.34
time (sec)	N/A	2.507	0.466	0.293	0.037	0.070	0.102	0.137	0.152	12.183

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	160	151	156	156	219	214	187	168
N.S.	1	1.00	1.01	0.96	0.99	0.99	1.39	1.35	1.18	1.06
time (sec)	N/A	0.720	0.097	0.233	0.047	0.064	0.045	0.113	0.174	0.035

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F(-1)	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	141	141	170	0	0	170	300	166
N.S.	1	1.00	1.00	1.00	1.21	0.00	0.00	1.21	2.13	1.18
time (sec)	N/A	0.728	0.095	0.251	0.037	0.000	0.000	0.112	0.159	14.097

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	F(-1)	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	421	419	406	456	2660	0	0	2301	17569	58502
N.S.	1	1.00	0.96	1.08	6.32	0.00	0.00	5.47	41.73	138.96
time (sec)	N/A	2.325	0.558	1.240	0.164	0.000	0.000	0.171	0.717	20.592

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	F(-1)	F(-1)	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1176	1176	1162	1472	16079	0	0	16944	59	202365
N.S.	1	1.00	0.99	1.25	13.67	0.00	0.00	14.41	0.05	172.08
time (sec)	N/A	9.067	2.595	5.310	1.036	0.000	0.000	0.502	200.034	39.692

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	A	F	A	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1393	0	12161	2037	0	1943	0	0	59	0
N.S.	1	0.00	8.73	1.46	0.00	1.39	0.00	0.00	0.04	0.00
time (sec)	N/A	0.000	39.037	4.022	0.000	0.210	0.000	0.000	200.020	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	A	F	A	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	505	0	416	575	0	834	0	0	0	0
N.S.	1	0.00	0.82	1.14	0.00	1.65	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	28.242	4.123	0.000	0.148	0.000	0.000	33.768	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	B	F	B	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1151	0	9971	4111	0	5532	0	0	59	0
N.S.	1	0.00	8.66	3.57	0.00	4.81	0.00	0.00	0.05	0.00
time (sec)	N/A	0.000	36.974	4.332	0.000	0.371	0.000	0.000	200.032	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	F	A	F	F	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	498	2634	252	353	0	151	0	0	625	913
N.S.	1	5.29	0.51	0.71	0.00	0.30	0.00	0.00	1.26	1.83
time (sec)	N/A	16.146	10.600	1.125	0.000	0.074	0.000	0.000	0.211	13.532

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	F	A	F	F	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	321	1783	207	260	0	106	0	0	427	706
N.S.	1	5.55	0.64	0.81	0.00	0.33	0.00	0.00	1.33	2.20
time (sec)	N/A	6.872	8.775	1.257	0.000	0.080	0.000	0.000	0.173	12.881

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	F	A	F	F	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	1338	166	162	0	63	0	0	180	206
N.S.	1	6.53	0.81	0.79	0.00	0.31	0.00	0.00	0.88	1.00
time (sec)	N/A	3.221	10.345	1.041	0.000	0.078	0.000	0.000	0.168	0.064

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	F	A	F	F	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	319	1923	205	207	0	207	0	0	0	263
N.S.	1	6.03	0.64	0.65	0.00	0.65	0.00	0.00	0.00	0.82
time (sec)	N/A	7.546	10.398	1.248	0.000	0.085	0.000	0.000	0.191	0.042

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	F	A	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	475	2899	233	276	0	358	0	0	0	0
N.S.	1	6.10	0.49	0.58	0.00	0.75	0.00	0.00	0.00	0.00
time (sec)	N/A	19.564	10.525	1.132	0.000	0.080	0.000	0.000	0.184	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	386	0	0	0	0	0	0	0	31	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.022	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	470	0	0	0	0	0	0	0	59	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.13	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.025	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	431	431	431	382	411	411	600	597	335	462
N.S.	1	1.00	1.00	0.89	0.95	0.95	1.39	1.39	0.78	1.07
time (sec)	N/A	1.667	0.236	0.216	0.034	0.088	0.064	0.113	0.167	0.103

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	112	109	106	106	138	142	99	114
N.S.	1	1.00	1.00	0.97	0.95	0.95	1.23	1.27	0.88	1.02
time (sec)	N/A	0.521	0.055	0.181	0.037	0.063	0.026	0.114	0.155	0.029

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	192	179	0	613	0	199	10	2467
N.S.	1	1.00	0.98	0.91	0.00	3.13	0.00	1.02	0.05	12.59
time (sec)	N/A	0.848	0.185	0.183	0.000	32.620	0.000	0.117	0.172	18.285

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	F(-1)	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	676	695	661	1353	0	0	0	1517	983	26278
N.S.	1	1.03	0.98	2.00	0.00	0.00	0.00	2.24	1.45	38.87
time (sec)	N/A	3.130	1.702	1.060	0.000	0.000	0.000	0.147	0.163	17.012

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	A	F	A	F	F	F	F(-1)
verified	N/A	N/A	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	995	0	15681	1698	0	1036	0	0	45	0
N.S.	1	0.00	15.76	1.71	0.00	1.04	0.00	0.00	0.05	0.00
time (sec)	N/A	0.000	18.103	8.194	0.000	0.113	0.000	0.000	200.026	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	A	F	A	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	504	0	1086	785	0	464	0	0	45	0
N.S.	1	0.00	2.15	1.56	0.00	0.92	0.00	0.00	0.09	0.00
time (sec)	N/A	0.000	17.665	4.122	0.000	0.084	0.000	0.000	200.028	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	B	F	B	F	F	F	F(-1)
verified	N/A	N/A	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	913	0	10146	2423	0	2763	0	0	45	0
N.S.	1	0.00	11.11	2.65	0.00	3.03	0.00	0.00	0.05	0.00
time (sec)	N/A	0.000	36.315	2.592	0.000	0.197	0.000	0.000	200.025	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	C	F	C	F	F	F	B
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	252	655	325	0	57	0	0	174	879
N.S.	1	0.93	2.43	1.20	0.00	0.21	0.00	0.00	0.64	3.26
time (sec)	N/A	0.914	10.714	1.275	0.000	0.087	0.000	0.000	0.193	0.130

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	532	0	0	0	0	0	0	0	0	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.781	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	118	149	129	122	146	189	152	188	124
N.S.	1	1.07	1.35	1.17	1.11	1.33	1.72	1.38	1.71	1.13
time (sec)	N/A	0.832	0.168	0.430	0.039	0.068	0.060	0.121	0.157	0.050

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	82	96	76	72	87	107	90	96	73
N.S.	1	0.76	0.89	0.70	0.67	0.81	0.99	0.83	0.89	0.68
time (sec)	N/A	0.569	0.045	0.303	0.027	0.069	0.051	0.114	0.168	0.031

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	34	33	32	39	42	32	38	32
N.S.	1	1.00	0.97	0.94	0.91	1.11	1.20	0.91	1.09	0.91
time (sec)	N/A	0.281	0.000	0.178	0.030	0.082	0.031	0.118	0.150	0.023

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	96	94	97	91	106	107	102	144	106
N.S.	1	0.91	0.89	0.92	0.86	1.00	1.01	0.96	1.36	1.00
time (sec)	N/A	0.505	0.089	0.392	0.027	0.069	0.551	0.113	0.163	11.967

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	100	112	99	95	143	122	203	185	108
N.S.	1	0.91	1.02	0.90	0.86	1.30	1.11	1.85	1.68	0.98
time (sec)	N/A	0.507	0.071	0.431	0.035	0.074	0.889	0.114	0.155	12.235

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	F	A	B	F(-2)	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	194	230	423	70	0	316	247850	0	3061	1077
N.S.	1	1.19	2.18	0.36	0.00	1.63	1277.58	0.00	15.78	5.55
time (sec)	N/A	1.233	1.070	0.563	0.000	0.130	10.775	0.000	0.156	13.204

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	F	B	B	F(-2)	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	218	227	460	75	0	387	95302	0	3275	1147
N.S.	1	1.04	2.11	0.34	0.00	1.78	437.17	0.00	15.02	5.26
time (sec)	N/A	1.044	1.109	0.557	0.000	0.098	7.555	0.000	0.170	13.069

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	B	F(-2)	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	139	148	277	64	0	200	3281	0	1625	177
N.S.	1	1.06	1.99	0.46	0.00	1.44	23.60	0.00	11.69	1.27
time (sec)	N/A	0.704	0.683	0.481	0.000	0.096	1.464	0.000	0.160	12.263

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	B	F(-2)	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	88	97	167	56	0	102	184	0	498	132
N.S.	1	1.10	1.90	0.64	0.00	1.16	2.09	0.00	5.66	1.50
time (sec)	N/A	0.520	0.387	0.417	0.000	0.078	0.597	0.000	0.151	0.157

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	F	B	F(-1)	F(-2)	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	211	226	621	187	0	1738	0	0	11241	12009
N.S.	1	1.07	2.94	0.89	0.00	8.24	0.00	0.00	53.27	56.91
time (sec)	N/A	1.173	4.527	1.302	0.000	6.912	0.000	0.000	0.182	48.638

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	F	B	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	275	298	1347	383	0	5609	0	0	44312	0
N.S.	1	1.08	4.90	1.39	0.00	20.40	0.00	0.00	161.13	0.00
time (sec)	N/A	1.558	7.427	3.329	0.000	93.959	0.000	0.000	0.258	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F	B	F(-1)	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	404	424	914	703	0	2724	0	0	15132	645
N.S.	1	1.05	2.26	1.74	0.00	6.74	0.00	0.00	37.46	1.60
time (sec)	N/A	2.034	6.909	2.584	0.000	0.160	0.000	0.000	0.191	1.954

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	B	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	284	293	451	382	0	1615	2183	0	7954	466
N.S.	1	1.03	1.59	1.35	0.00	5.69	7.69	0.00	28.01	1.64
time (sec)	N/A	1.301	5.975	2.401	0.000	0.130	162.570	0.000	0.159	1.006

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	B	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	188	238	211	0	638	462	0	2729	346
N.S.	1	1.05	1.33	1.18	0.00	3.56	2.58	0.00	15.25	1.93
time (sec)	N/A	0.728	3.697	2.398	0.000	0.088	2.413	0.000	0.159	0.613

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	F(-1)	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	500	518	2761	2312	0	0	0	0	40	0
N.S.	1	1.04	5.52	4.62	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	4.264	10.470	18.921	0.000	0.000	0.000	0.000	200.031	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	F(-1)	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	564	590	4351	3757	0	0	0	0	40	0
N.S.	1	1.05	7.71	6.66	0.00	0.00	0.00	0.00	0.07	0.00
time (sec)	N/A	6.165	15.919	46.138	0.000	0.000	0.000	0.000	200.025	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	B	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	496	386	17362	350	0	281	0	1711	0	0
N.S.	1	0.78	35.00	0.71	0.00	0.57	0.00	3.45	0.00	0.00
time (sec)	N/A	1.247	17.221	0.572	0.000	0.124	0.000	0.170	3.125	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	B	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	312	268	9146	116	0	155	0	933	0	0
N.S.	1	0.86	29.31	0.37	0.00	0.50	0.00	2.99	0.00	0.00
time (sec)	N/A	0.861	16.739	0.408	0.000	0.113	0.000	0.130	1.303	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	B	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	189	1461	75	0	78	0	359	1228	0
N.S.	1	1.01	7.81	0.40	0.00	0.42	0.00	1.92	6.57	0.00
time (sec)	N/A	0.569	2.849	0.230	0.000	0.131	0.000	0.122	0.515	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	365	313	13435	0	0	1033	0	0	40	0
N.S.	1	0.86	36.81	0.00	0.00	2.83	0.00	0.00	0.11	0.00
time (sec)	N/A	0.948	16.693	0.000	0.000	0.229	0.000	0.000	200.024	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	435	372	5628	0	0	1309	0	0	40	0
N.S.	1	0.86	12.94	0.00	0.00	3.01	0.00	0.00	0.09	0.00
time (sec)	N/A	0.949	16.252	0.000	0.000	0.330	0.000	0.000	200.019	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	374	349	1754	0	0	2078	0	0	0	0
N.S.	1	0.93	4.69	0.00	0.00	5.56	0.00	0.00	0.00	0.00
time (sec)	N/A	1.654	17.486	0.000	0.000	0.211	0.000	0.000	29.120	0.000

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	224	226	1497	0	0	1130	0	0	0	0
N.S.	1	1.01	6.68	0.00	0.00	5.04	0.00	0.00	0.00	0.00
time (sec)	N/A	0.693	11.913	0.000	0.000	0.160	0.000	0.000	13.965	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	F	A	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	124	162	934	0	0	368	0	59	0	0
N.S.	1	1.31	7.53	0.00	0.00	2.97	0.00	0.48	0.00	0.00
time (sec)	N/A	0.484	0.424	0.000	0.000	0.107	0.000	0.124	5.110	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	329	307	1352	0	0	3961	0	0	40	0
N.S.	1	0.93	4.11	0.00	0.00	12.04	0.00	0.00	0.12	0.00
time (sec)	N/A	0.907	20.363	0.000	0.000	0.495	0.000	0.000	200.021	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	493	470	451671	0	0	15167	0	0	40	0
N.S.	1	0.95	916.17	0.00	0.00	30.76	0.00	0.00	0.08	0.00
time (sec)	N/A	1.309	88.263	0.000	0.000	4.511	0.000	0.000	200.028	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F(-2)	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	549	498	0	0	0	3837	0	0	40	0
N.S.	1	0.91	0.00	0.00	0.00	6.99	0.00	0.00	0.07	0.00
time (sec)	N/A	1.482	0.000	0.000	0.000	0.363	0.000	0.000	200.031	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F(-2)	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	461	428	0	0	0	2326	0	0	38	0
N.S.	1	0.93	0.00	0.00	0.00	5.05	0.00	0.00	0.08	0.00
time (sec)	N/A	0.975	0.000	0.000	0.000	0.284	0.000	0.000	200.034	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	399	392	0	0	0	607	0	0	32	0
N.S.	1	0.98	0.00	0.00	0.00	1.52	0.00	0.00	0.08	0.00
time (sec)	N/A	0.755	0.000	0.000	0.000	0.106	0.000	0.000	200.050	0.000

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F(-1)	F	F	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	846	806	0	0	0	43371	0	0	40	0
N.S.	1	0.95	0.00	0.00	0.00	51.27	0.00	0.00	0.05	0.00
time (sec)	N/A	2.377	0.000	0.000	0.000	24.608	0.000	0.000	200.102	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	B	F	F(-2)	F	F(-1)
verified	N/A	No	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1153	1113	0	0	0	81003	0	0	40	0
N.S.	1	0.97	0.00	0.00	0.00	70.25	0.00	0.00	0.03	0.00
time (sec)	N/A	3.447	0.000	0.000	0.000	129.875	0.000	0.000	200.022	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	171	146	151	151	172	179	177	148
N.S.	1	1.00	1.00	0.85	0.88	0.88	1.01	1.05	1.04	0.87
time (sec)	N/A	0.805	0.047	0.229	0.027	0.060	0.045	0.108	0.149	21.626

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	118	104	104	104	121	125	123	106
N.S.	1	1.00	0.99	0.87	0.87	0.87	1.02	1.05	1.03	0.89
time (sec)	N/A	0.597	0.032	0.171	0.034	0.059	0.033	0.117	0.164	0.060

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	71	58	61	61	61	69	69	57
N.S.	1	1.00	1.00	0.82	0.86	0.86	0.86	0.97	0.97	0.80
time (sec)	N/A	0.414	0.018	0.133	0.032	0.073	0.026	0.114	0.153	0.034

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	C	F	C	A	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	920	68	47	0	7641	298	0	100	369
N.S.	1	3.10	0.23	0.16	0.00	25.73	1.00	0.00	0.34	1.24
time (sec)	N/A	14.029	0.025	0.066	0.000	1.546	8.271	0.000	0.150	0.660

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	C	F	C	A	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	816	1777	161	114	0	5331	343	0	797	997
N.S.	1	2.18	0.20	0.14	0.00	6.53	0.42	0.00	0.98	1.22
time (sec)	N/A	54.904	0.053	0.087	0.000	1.500	12.045	0.000	0.160	0.601

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	No	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	520	1899	0	0	0	0	0	0	0	0
N.S.	1	3.65	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	4.148	0.000	0.000	0.000	0.000	0.000	0.000	0.161	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	171	146	151	151	172	179	177	145
N.S.	1	1.00	1.00	0.85	0.88	0.88	1.01	1.05	1.04	0.85
time (sec)	N/A	0.401	0.041	0.232	0.035	0.065	0.036	0.134	0.146	0.145

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	118	102	104	104	114	124	123	101
N.S.	1	1.00	1.00	0.86	0.88	0.88	0.97	1.05	1.04	0.86
time (sec)	N/A	0.319	0.032	0.172	0.038	0.068	0.028	0.112	0.158	0.057

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	71	56	61	61	61	68	69	55
N.S.	1	1.00	1.00	0.79	0.86	0.86	0.86	0.96	0.97	0.77
time (sec)	N/A	0.237	0.016	0.133	0.028	0.062	0.022	0.135	0.147	0.031

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	C	A	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	706	681	68	47	0	7620	296	0	100	365
N.S.	1	0.96	0.10	0.07	0.00	10.79	0.42	0.00	0.14	0.52
time (sec)	N/A	3.646	0.025	0.064	0.000	1.502	8.061	0.000	0.154	21.930

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	C	A	F(-2)	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1237	1268	160	114	0	5342	342	0	797	989
N.S.	1	1.03	0.13	0.09	0.00	4.32	0.28	0.00	0.64	0.80
time (sec)	N/A	8.601	0.054	0.088	0.000	1.540	8.461	0.000	0.168	22.116

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	No	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	960	1284	0	0	0	0	0	0	0	0
N.S.	1	1.34	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.888	0.000	0.000	0.000	0.000	0.000	0.000	0.175	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	25	18	17	17	17	20	17	17
N.S.	1	1.00	1.32	0.95	0.89	0.89	0.89	1.05	0.89	0.89
time (sec)	N/A	0.202	0.010	0.064	0.033	0.076	0.071	0.120	0.159	0.062

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	22	25	24	24	20	24	41	24
N.S.	1	1.00	0.52	0.60	0.57	0.57	0.48	0.57	0.98	0.57
time (sec)	N/A	0.330	0.066	0.113	0.035	0.077	13.482	0.137	0.152	0.123

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [26] had the largest ratio of [.6086960000000000015]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	2	1.39	34	0.059
2	A	2	2	1.12	34	0.059
3	A	2	2	1.00	32	0.062
4	A	2	2	1.00	34	0.059
5	A	2	2	1.00	34	0.059
6	A	2	2	1.00	34	0.059
7	A	6	6	1.26	35	0.171
8	A	6	6	0.71	36	0.167
9	A	6	6	0.74	36	0.167
10	A	6	6	0.80	36	0.167
11	A	8	7	0.88	36	0.194
12	A	10	9	0.92	36	0.250
13	A	13	12	0.78	36	0.333
14	A	6	6	1.08	34	0.176
15	A	2	2	1.00	23	0.087
16	A	2	2	1.00	23	0.087
17	A	2	2	1.00	21	0.095
18	A	5	5	0.99	23	0.217
19	A	6	6	1.04	23	0.261
20	C	7	6	0.46	25	0.240
21	A	6	5	1.26	23	0.217

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
22	A	2	2	1.00	23	0.087
23	A	2	2	1.00	23	0.087
24	A	2	2	1.00	21	0.095
25	C	6	6	2.49	23	0.261
26	C	14	14	2.49	23	0.609
27	C	8	7	2.78	25	0.280
28	C	7	6	2.44	23	0.261
29	A	2	2	1.53	36	0.056
30	A	2	2	1.37	36	0.056
31	A	2	2	1.11	36	0.056
32	A	2	2	1.00	34	0.059
33	A	2	2	1.00	36	0.056
34	A	2	2	1.00	36	0.056
35	A	2	2	1.00	36	0.056
36	A	8	7	0.98	38	0.184
37	A	8	7	1.12	38	0.184
38	A	9	8	1.19	38	0.211
39	A	11	10	1.18	38	0.263
40	A	14	13	1.01	38	0.342
41	A	9	8	1.54	36	0.222
42	A	2	2	1.00	25	0.080
43	A	2	2	1.00	25	0.080
44	A	2	2	1.00	23	0.087
45	A	7	6	1.00	25	0.240
46	A	9	8	1.05	25	0.320
47	C	9	8	0.35	27	0.296
48	A	8	7	1.34	25	0.280
49	A	2	2	1.00	25	0.080
50	A	2	2	1.00	25	0.080
51	A	2	2	1.00	23	0.087
52	C	8	7	2.43	25	0.280
53	C	10	9	1.84	25	0.360

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
54	C	9	8	3.03	25	0.320
55	A	3	3	1.00	40	0.075
56	A	3	3	1.00	38	0.079
57	A	3	3	1.00	40	0.075
58	A	3	3	1.00	40	0.075
59	A	3	3	1.00	40	0.075
60	A	3	3	0.60	42	0.071
61	A	3	3	0.60	42	0.071
62	A	3	3	0.59	42	0.071
63	A	3	3	0.60	42	0.071
64	A	3	3	0.60	42	0.071
65	A	3	3	1.11	20	0.150
66	A	3	3	0.68	40	0.075
67	A	2	2	1.77	38	0.053
68	A	2	2	1.32	38	0.053
69	A	2	2	1.05	38	0.053
70	A	2	2	1.00	36	0.056
71	A	11	10	0.97	38	0.263
72	A	11	10	0.99	38	0.263
73	A	15	14	1.05	38	0.368
74	A	11	10	1.23	43	0.233
75	A	9	8	1.23	43	0.186
76	A	7	6	1.25	43	0.140
77	A	7	6	1.21	43	0.140
78	A	9	8	1.20	43	0.186
79	A	11	10	1.20	43	0.233
80	A	6	5	1.22	41	0.122
81	A	6	5	0.81	34	0.147
82	A	6	5	1.13	35	0.143
83	A	6	5	0.93	34	0.147
84	A	6	5	1.29	35	0.143
85	A	2	2	1.00	34	0.059

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
86	A	2	2	1.00	34	0.059
87	A	2	2	1.00	32	0.062
88	A	2	2	1.00	34	0.059
89	A	2	2	1.14	34	0.059
90	A	2	2	1.00	22	0.091
91	A	2	2	1.00	28	0.071
92	C	18	17	4.53	30	0.567
93	C	14	13	2.79	30	0.433
94	C	10	9	2.59	30	0.300
95	C	14	13	3.28	30	0.433
96	C	18	17	6.70	30	0.567
97	F	0	0	N/A	0.000	N/A
98	A	1	1	1.00	34	0.029
99	A	1	1	1.00	32	0.031
100	A	1	1	1.00	34	0.029
101	A	1	1	1.00	34	0.029
102	A	1	1	1.00	34	0.029
103	A	1	1	1.00	36	0.028
104	A	1	1	1.00	36	0.028
105	A	1	1	1.00	36	0.028
106	A	1	1	1.00	36	0.028
107	A	1	1	1.00	36	0.028
108	A	1	1	1.00	34	0.029
109	A	2	2	1.28	57	0.035
110	A	2	2	1.01	57	0.035
111	A	2	2	1.00	55	0.036
112	A	2	2	1.00	57	0.035
113	A	2	2	1.00	57	0.035
114	A	2	2	1.00	57	0.035
115	F	0	0	N/A	0.000	N/A
116	F	0	0	N/A	0.000	N/A
117	F	0	0	N/A	0.000	N/A

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
118	C	17	16	5.29	30	0.533
119	C	13	12	5.55	30	0.400
120	C	9	8	6.53	30	0.267
121	C	13	12	6.03	30	0.400
122	C	17	16	6.10	30	0.533
123	F	0	0	N/A	0.000	N/A
124	F	0	0	N/A	0.000	N/A
125	A	2	2	1.00	43	0.047
126	A	2	2	1.00	41	0.049
127	A	2	2	1.00	43	0.047
128	A	2	2	1.03	43	0.047
129	F	0	0	N/A	0.000	N/A
130	F	0	0	N/A	0.000	N/A
131	F	0	0	N/A	0.000	N/A
132	C	10	9	0.93	30	0.300
133	F	0	0	N/A	0.000	N/A
134	A	2	2	1.07	38	0.053
135	A	2	2	0.76	36	0.056
136	A	1	1	1.00	30	0.033
137	A	2	2	0.91	38	0.053
138	A	2	2	0.91	38	0.053
139	A	7	7	1.19	43	0.163
140	A	5	5	1.04	40	0.125
141	A	5	5	1.06	38	0.132
142	A	5	5	1.10	32	0.156
143	A	5	5	1.07	40	0.125
144	A	5	5	1.08	40	0.125
145	A	5	5	1.05	40	0.125
146	A	5	5	1.03	38	0.132
147	A	5	5	1.05	32	0.156
148	A	5	5	1.04	40	0.125
149	A	5	5	1.05	40	0.125

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
150	A	4	4	0.78	42	0.095
151	A	4	4	0.86	40	0.100
152	A	4	4	1.01	34	0.118
153	A	7	6	0.86	42	0.143
154	A	7	6	0.86	42	0.143
155	A	4	4	0.93	42	0.095
156	A	6	5	1.01	40	0.125
157	A	5	4	1.31	34	0.118
158	A	7	6	0.93	42	0.143
159	A	9	8	0.95	42	0.190
160	A	9	8	0.91	42	0.190
161	A	8	7	0.93	40	0.175
162	A	8	7	0.98	34	0.206
163	A	13	12	0.95	42	0.286
164	A	15	14	0.97	42	0.333
165	A	2	2	1.00	26	0.077
166	A	2	2	1.00	26	0.077
167	A	2	2	1.00	24	0.083
168	C	7	6	3.10	26	0.231
169	C	15	14	2.18	26	0.538
170	C	8	7	3.65	26	0.269
171	A	2	2	1.00	26	0.077
172	A	2	2	1.00	26	0.077
173	A	2	2	1.00	24	0.083
174	A	7	6	0.96	26	0.231
175	A	9	8	1.03	26	0.308
176	A	8	7	1.34	26	0.269
177	A	2	2	1.00	24	0.083
178	A	3	3	1.00	50	0.060

CHAPTER 3

Listing of integrals

3.1 $\int (A+Bx+Cx^2) (27a^3+27a^2bx-4b^3x^3)^3 \, dx$

Optimal result

Integrand size = 34, antiderivative size = 192

$$\int (A+Bx+Cx^2) (27a^3+27a^2bx-4b^3x^3)^3 \, dx = \frac{729a^3(4Ab^2-6abB+9a^2C)(3a+2bx)^7}{448b^3} - \frac{243a^2(4Ab^2-12abB+27a^2C)(3a+2bx)^8}{512b^3} + \frac{3a(2Ab^2-12abB+45a^2C)(3a+2bx)^9}{32b^3} - \frac{(2Ab^2-30abB+207a^2C)(3a+2bx)^{10}}{320b^3} - \frac{(2bB-33aC)(3a+2bx)^{11}}{704b^3} - \frac{C(3a+2bx)^{12}}{768b^3}$$

output

```
729/448*a^3*(4*A*b^2-6*B*a*b+9*C*a^2)*(2*b*x+3*a)^7/b^3-243/512*a^2*(4*A*b
^2-12*B*a*b+27*C*a^2)*(2*b*x+3*a)^8/b^3+3/32*a*(2*A*b^2-12*B*a*b+45*C*a^2)
*(2*b*x+3*a)^9/b^3-1/320*(2*A*b^2-30*B*a*b+207*C*a^2)*(2*b*x+3*a)^10/b^3-1
/704*(2*B*b-33*C*a)*(2*b*x+3*a)^11/b^3-1/768*C*(2*b*x+3*a)^12/b^3
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.25

$$\int (A+Bx+Cx^2)\left(27a^3+27a^2bx-4b^3x^3\right)^3\,dx$$
$$=-\frac{16}{165}b^9x^{10}\left(66A+60Bx+55Cx^2\right)+\frac{6561}{2}a^9x(6A+x(3B+2Cx))$$
$$+\frac{19683}{4}a^8bx^2(6A+x(4B+3Cx))+\frac{19683}{20}a^7b^2x^3(20A+3x(5B+4Cx))$$
$$+\frac{729}{4}a^6b^3x^4(15A+2x(6B+5Cx))-\frac{2916}{35}a^5b^4x^5(42A+5x(7B+6Cx))$$
$$-\frac{729}{14}a^4b^5x^6(28A+3x(8B+7Cx))+\frac{18}{7}a^3b^6x^7(72A+7x(9B+8Cx))$$
$$+\frac{18}{5}a^2b^7x^8(45A+4x(10B+9Cx))$$

input
```
Integrate[(A + B*x + C*x^2)*(27*a^3 + 27*a^2*b*x - 4*b^3*x^3)^3,x]
```

output
```
(-16*b^9*x^10*(66*A + 60*B*x + 55*C*x^2))/165 + (6561*a^9*x*(6*A + x*(3*B
+ 2*C*x)))/2 + (19683*a^8*b*x^2*(6*A + x*(4*B + 3*C*x)))/4 + (19683*a^7*b^
2*x^3*(20*A + 3*x*(5*B + 4*C*x)))/20 + (729*a^6*b^3*x^4*(15*A + 2*x*(6*B +
 5*C*x)))/4 - (2916*a^5*b^4*x^5*(42*A + 5*x*(7*B + 6*C*x)))/35 - (729*a^4*
b^5*x^6*(28*A + 3*x*(8*B + 7*C*x)))/14 + (18*a^3*b^6*x^7*(72*A + 7*x*(9*B
+ 8*C*x)))/7 + (18*a^2*b^7*x^8*(45*A + 4*x*(10*B + 9*C*x)))/5
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.39, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(27a^3+27a^2bx-4b^3x^3\right)^3\left(A+Bx+Cx^2\right)\,dx$$
$$\Big\downarrow \text{2188}$$

$$\int \left(19683a^9A + 19683a^8x(aB + 3Ab) + 19683a^7x^2\left(a(aC + 3bB) + 3Ab^2\right) + 2187a^6bx^3\left(27a(aC + bB) + 5Ab^2\right)\right.$$

$$\Big\downarrow \; 2009$$

$$19683a^9Ax + \frac{19683}{2}a^8x^2(aB + 3Ab) + 6561a^7x^3\left(a(aC + 3bB) + 3Ab^2\right) + \frac{2187}{4}a^6bx^4\left(27a(aC + bB) + 5Ab^2\right) - \frac{2187}{5}a^5b^2x^5\left(8Ab^2 - a(27aC + 5bB)\right) - \frac{729}{2}a^4b^3x^6\left(a(8bB - 5aC) + 4Ab^2\right) + \frac{324}{7}a^3b^4x^7\left(4Ab^2 - 27a(2aC + bB)\right) - \frac{8}{5}b^7x^{10}\left(4Ab^2 - 81a^2C\right) + \frac{81}{2}a^2b^5x^8\left(a(4bB - 27aC) + 4Ab^2\right) + 144a^2b^6x^9(aC + bB) - \frac{64}{11}b^9Bx^{11} - \frac{16}{3}b^9Cx^{12}$$

input
```
Int[(A + B*x + C*x^2)*(27*a^3 + 27*a^2*b*x - 4*b^3*x^3)^3,x]
```

output
```
19683*a^9*A*x + (19683*a^8*(3*A*b + a*B)*x^2)/2 + 6561*a^7*(3*A*b^2 + a*(3
*b*B + a*C))*x^3 + (2187*a^6*b*(5*A*b^2 + 27*a*(b*B + a*C))*x^4)/4 - (2187
*a^5*b^2*(8*A*b^2 - a*(5*b*B + 27*a*C))*x^5)/5 - (729*a^4*b^3*(4*A*b^2 + a
*(8*b*B - 5*a*C))*x^6)/2 + (324*a^3*b^4*(4*A*b^2 - 27*a*(b*B + 2*a*C))*x^7
)/7 + (81*a^2*b^5*(4*A*b^2 + a*(4*b*B - 27*a*C))*x^8)/2 + 144*a^2*b^6*(b*B
 + a*C)*x^9 - (8*b^7*(4*A*b^2 - 81*a^2*C)*x^10)/5 - (64*b^9*B*x^11)/11 - (
16*b^9*C*x^12)/3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.42

method	result
norman	$-\frac{16C\,b^9x^{12}}{3}-\frac{64B\,b^9x^{11}}{11}+\left(-\frac{32}{5}A\,b^9+\frac{648}{5}C\,b^7a^2\right)x^{10}+\left(144B\,a^2b^7+144C\,b^6a^3\right)x^9+\left(162A\right.$
default	$-\frac{16C\,b^9x^{12}}{3}-\frac{64B\,b^9x^{11}}{11}+\frac{\left(-64A\,b^9+1296C\,b^7a^2\right)x^{10}}{10}+\frac{\left(1296B\,a^2b^7+1296C\,b^6a^3\right)x^9}{9}+\frac{\left(1296A\,a^2b^7+1296B\,a^3b\right.}{8}$
gosper	$\frac{x\left(-24640C\,b^9x^{11}-26880B\,b^9x^{10}-29568x^9A\,b^9+598752x^9C\,b^7a^2+665280B\,a^2b^7x^8+665280C\,a^3b^6x^8+748440x^7A\,a^2b^7+7\right.}{}$
risch	$\frac{648}{5}x^{10}C\,b^7a^2+162x^8A\,a^2b^7+162x^8B\,a^3b^6-\frac{2187}{2}x^8C\,b^5a^4+\frac{1296}{7}x^7A\,a^3b^6-\frac{8748}{7}x^7B\,a^4b^5$
parallelrisch	$\frac{648}{5}x^{10}C\,b^7a^2+162x^8A\,a^2b^7+162x^8B\,a^3b^6-\frac{2187}{2}x^8C\,b^5a^4+\frac{1296}{7}x^7A\,a^3b^6-\frac{8748}{7}x^7B\,a^4b^5$
orering	$\frac{x\left(-24640C\,b^9x^{11}-26880B\,b^9x^{10}-29568x^9A\,b^9+598752x^9C\,b^7a^2+665280B\,a^2b^7x^8+665280C\,a^3b^6x^8+748440x^7A\,a^2b^7+7\right.}{}$

input

```
int((C*x^2+B*x+A)*(-4*b^3*x^3+27*a^2*b*x+27*a^3)^3,x,method=_RETURNVERBOSE
)
```

output

```
-16/3*C*b^9*x^12-64/11*B*b^9*x^11+(-32/5*A*b^9+648/5*C*b^7*a^2)*x^10+(144*
B*a^2*b^7+144*C*a^3*b^6)*x^9+(162*A*a^2*b^7+162*B*a^3*b^6-2187/2*C*b^5*a^4
)*x^8+(1296/7*A*a^3*b^6-8748/7*B*a^4*b^5-17496/7*C*b^4*a^5)*x^7+(-1458*A*a
^4*b^5-2916*B*a^5*b^4+3645/2*C*b^3*a^6)*x^6+(-17496/5*A*a^5*b^4+2187*B*a^6
*b^3+59049/5*C*a^7*b^2)*x^5+(10935/4*A*a^6*b^3+59049/4*B*a^7*b^2+59049/4*C
*a^8*b)*x^4+(19683*A*a^7*b^2+19683*B*a^8*b+6561*C*a^9)*x^3+(59049/2*A*a^8*
b+19683/2*B*a^9)*x^2+19683*A*a^9*x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.44

$$\int (A+Bx+Cx^2)(27a^3+27a^2bx-4b^3x^3)^3\,dx$$
$$= -\frac{16}{3}Cb^9x^{12} - \frac{64}{11}Bb^9x^{11} + 19683\,Aa^9x + \frac{8}{5}(81\,Ca^2b^7 - 4\,Ab^9)x^{10} + 144\,(Ca^3b^6 + Ba^2b^7)x^9 - \frac{81}{2}(27\,Ca^4b^5 - 4\,Ba^3b^6 - 4\,Aa^2b^7)x^8 - \frac{324}{7}(54\,Ca^5b^4 + 27\,Ba^4b^5 - 4\,Aa^3b^6)x^7 + \frac{729}{2}(5\,Ca^6b^3 - 8\,Ba^5b^4 - 4\,Aa^4b^5)x^6 + \frac{2187}{5}(27\,Ca^7b^2 + 5\,Ba^6b^3 - 8\,Aa^5b^4)x^5 + \frac{2187}{4}(27\,Ca^8b + 27\,Ba^7b^2 + 5\,Aa^6b^3)x^4 + 6561\,(Ca^9 + 3\,Ba^8b + 3\,Aa^7b^2)x^3 + \frac{19683}{2}(Ba^9 + 3\,Aa^8b)x^2$$

input
```
integrate((C*x^2+B*x+A)*(-4*b^3*x^3+27*a^2*b*x+27*a^3)^3,x, algorithm="fri
cas")
```

output
```
-16/3*C*b^9*x^12 - 64/11*B*b^9*x^11 + 19683*A*a^9*x + 8/5*(81*C*a^2*b^7 -
4*A*b^9)*x^10 + 144*(C*a^3*b^6 + B*a^2*b^7)*x^9 - 81/2*(27*C*a^4*b^5 - 4*B
*a^3*b^6 - 4*A*a^2*b^7)*x^8 - 324/7*(54*C*a^5*b^4 + 27*B*a^4*b^5 - 4*A*a^3
*b^6)*x^7 + 729/2*(5*C*a^6*b^3 - 8*B*a^5*b^4 - 4*A*a^4*b^5)*x^6 + 2187/5*(
27*C*a^7*b^2 + 5*B*a^6*b^3 - 8*A*a^5*b^4)*x^5 + 2187/4*(27*C*a^8*b + 27*B*
a^7*b^2 + 5*A*a^6*b^3)*x^4 + 6561*(C*a^9 + 3*B*a^8*b + 3*A*a^7*b^2)*x^3 +
19683/2*(B*a^9 + 3*A*a^8*b)*x^2
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.64

$$\int (A+Bx+Cx^2)\left(27a^3+27a^2bx-4b^3x^3\right)^3 dx$$
$$= 19683Aa^9x - \frac{64Bb^9x^{11}}{11} - \frac{16Cb^9x^{12}}{3} + x^{10}\left(-\frac{32Ab^9}{5} + \frac{648Ca^2b^7}{5}\right) + x^9 \cdot \left(144Ba^2b^7 + 144Ca^3b^6\right) + x^8 \cdot \left(162Aa^2b^7 + 162Ba^3b^6 - \frac{2187Ca^4b^5}{2}\right) + x^7 \cdot \left(\frac{1296Aa^3b^6}{7} - \frac{8748Ba^4b^5}{7} - \frac{17496Ca^5b^4}{7}\right) + x^6\left(-1458Aa^4b^5 - 2916Ba^5b^4 + \frac{3645Ca^6b^3}{2}\right) + x^5\left(-\frac{17496Aa^5b^4}{5} + 2187Ba^6b^3 + \frac{59049Ca^7b^2}{5}\right) + x^4 \cdot \left(\frac{10935Aa^6b^3}{4} + \frac{59049Ba^7b^2}{4} + \frac{59049Ca^8b}{4}\right) + x^3 \cdot \left(19683Aa^7b^2 + 19683Ba^8b + 6561Ca^9\right) + x^2 \cdot \left(\frac{59049Aa^8b}{2} + \frac{19683Ba^9}{2}\right)$$

input
```
integrate((C*x**2+B*x+A)*(-4*b**3*x**3+27*a**2*b*x+27*a**3)**3,x)
```

output
```
19683*A*a**9*x - 64*B*b**9*x**11/11 - 16*C*b**9*x**12/3 + x**10*(-32*A*b**
9/5 + 648*C*a**2*b**7/5) + x**9*(144*B*a**2*b**7 + 144*C*a**3*b**6) + x**8
*(162*A*a**2*b**7 + 162*B*a**3*b**6 - 2187*C*a**4*b**5/2) + x**7*(1296*A*a
**3*b**6/7 - 8748*B*a**4*b**5/7 - 17496*C*a**5*b**4/7) + x**6*(-1458*A*a**
4*b**5 - 2916*B*a**5*b**4 + 3645*C*a**6*b**3/2) + x**5*(-17496*A*a**5*b**4
/5 + 2187*B*a**6*b**3 + 59049*C*a**7*b**2/5) + x**4*(10935*A*a**6*b**3/4 +
 59049*B*a**7*b**2/4 + 59049*C*a**8*b/4) + x**3*(19683*A*a**7*b**2 + 19683
*B*a**8*b + 6561*C*a**9) + x**2*(59049*A*a**8*b/2 + 19683*B*a**9/2)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.44

$$\int (A+Bx+Cx^2)\left(27a^3+27a^2bx-4b^3x^3\right)^3\,dx$$
$$= -\frac{16}{3}Cb^9x^{12}-\frac{64}{11}Bb^9x^{11}+19683\,Aa^9x+\frac{8}{5}\left(81\,Ca^2b^7-4\,Ab^9\right)x^{10} + 144\left(Ca^3b^6+Ba^2b^7\right)x^9-\frac{81}{2}\left(27\,Ca^4b^5-4\,Ba^3b^6-4\,Aa^2b^7\right)x^8 - \frac{324}{7}\left(54\,Ca^5b^4+27\,Ba^4b^5-4\,Aa^3b^6\right)x^7+\frac{729}{2}\left(5\,Ca^6b^3-8\,Ba^5b^4-4\,Aa^4b^5\right)x^6 + \frac{2187}{5}\left(27\,Ca^7b^2+5\,Ba^6b^3-8\,Aa^5b^4\right)x^5+\frac{2187}{4}\left(27\,Ca^8b+27\,Ba^7b^2+5\,Aa^6b^3\right)x^4 + 6561\left(Ca^9+3\,Ba^8b+3\,Aa^7b^2\right)x^3+\frac{19683}{2}\left(Ba^9+3\,Aa^8b\right)x^2$$

input
```
integrate((C*x^2+B*x+A)*(-4*b^3*x^3+27*a^2*b*x+27*a^3)^3,x, algorithm="max
ima")
```

output
```
-16/3*C*b^9*x^12 - 64/11*B*b^9*x^11 + 19683*A*a^9*x + 8/5*(81*C*a^2*b^7 -
4*A*b^9)*x^10 + 144*(C*a^3*b^6 + B*a^2*b^7)*x^9 - 81/2*(27*C*a^4*b^5 - 4*B
*a^3*b^6 - 4*A*a^2*b^7)*x^8 - 324/7*(54*C*a^5*b^4 + 27*B*a^4*b^5 - 4*A*a^3
*b^6)*x^7 + 729/2*(5*C*a^6*b^3 - 8*B*a^5*b^4 - 4*A*a^4*b^5)*x^6 + 2187/5*(
27*C*a^7*b^2 + 5*B*a^6*b^3 - 8*A*a^5*b^4)*x^5 + 2187/4*(27*C*a^8*b + 27*B*
a^7*b^2 + 5*A*a^6*b^3)*x^4 + 6561*(C*a^9 + 3*B*a^8*b + 3*A*a^7*b^2)*x^3 +
19683/2*(B*a^9 + 3*A*a^8*b)*x^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.56

$$\int (A+Bx+Cx^2)\left(27a^3+27a^2bx-4b^3x^3\right)^3\,dx$$
$$=-\frac{16}{3}Cb^9x^{12}-\frac{64}{11}Bb^9x^{11}+\frac{648}{5}Ca^2b^7x^{10}-\frac{32}{5}Ab^9x^{10}+144\,Ca^3b^6x^9+144\,Ba^2b^7x^9-\frac{2187}{2}Ca^4b^5x^8+162\,Ba^3b^6x^8+162\,Aa^2b^7x^8-\frac{17496}{7}Ca^5b^4x^7-\frac{8748}{7}Ba^4b^5x^7+\frac{1296}{7}Aa^3b^6x^7+\frac{3645}{2}Ca^6b^3x^6-2916\,Ba^5b^4x^6-1458\,Aa^4b^5x^6+\frac{59049}{5}Ca^7b^2x^5+2187\,Ba^6b^3x^5-\frac{17496}{5}Aa^5b^4x^5+\frac{59049}{4}Ca^8bx^4+\frac{59049}{4}Ba^7b^2x^4+\frac{10935}{4}Aa^6b^3x^4+6561\,Ca^9x^3+19683\,Ba^8bx^3+19683\,Aa^7b^2x^3+\frac{19683}{2}Ba^9x^2+\frac{59049}{2}Aa^8bx^2+19683\,Aa^9x$$

input
```
integrate((C*x^2+B*x+A)*(-4*b^3*x^3+27*a^2*b*x+27*a^3)^3,x, algorithm="gia
c")
```

output
```
-16/3*C*b^9*x^12 - 64/11*B*b^9*x^11 + 648/5*C*a^2*b^7*x^10 - 32/5*A*b^9*x^
10 + 144*C*a^3*b^6*x^9 + 144*B*a^2*b^7*x^9 - 2187/2*C*a^4*b^5*x^8 + 162*B*
a^3*b^6*x^8 + 162*A*a^2*b^7*x^8 - 17496/7*C*a^5*b^4*x^7 - 8748/7*B*a^4*b^5
*x^7 + 1296/7*A*a^3*b^6*x^7 + 3645/2*C*a^6*b^3*x^6 - 2916*B*a^5*b^4*x^6 -
1458*A*a^4*b^5*x^6 + 59049/5*C*a^7*b^2*x^5 + 2187*B*a^6*b^3*x^5 - 17496/5*
A*a^5*b^4*x^5 + 59049/4*C*a^8*b*x^4 + 59049/4*B*a^7*b^2*x^4 + 10935/4*A*a^
6*b^3*x^4 + 6561*C*a^9*x^3 + 19683*B*a^8*b*x^3 + 19683*A*a^7*b^2*x^3 + 196
83/2*B*a^9*x^2 + 59049/2*A*a^8*b*x^2 + 19683*A*a^9*x
```

Mupad [B] (verification not implemented)

Time = 12.27 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.30

$$\int (A+Bx+Cx^2)\left(27a^3+27a^2bx-4b^3x^3\right)^3\,dx$$
$$= \frac{19683\,a^8\,x^2\,(3\,A\,b+B\,a)}{2} - \frac{64\,B\,b^9\,x^{11}}{11} - \frac{16\,C\,b^9\,x^{12}}{3} - x^{10}\left(\frac{32\,A\,b^9}{5} - \frac{648\,C\,a^2\,b^7}{5}\right) + 6561\,a^7\,x^3\left(C\,a^2+3\,B\,a\,b+3\,A\,b^2\right) + 19683\,A\,a^9\,x + 144\,a^2\,b^6\,x^9\,(B\,b+C\,a) - \frac{729\,a^4\,b^3\,x^6\,(-5\,C\,a^2+8\,B\,a\,b+4\,A\,b^2)}{2} + \frac{81\,a^2\,b^5\,x^8\,(-27\,C\,a^2+4\,B\,a\,b+4\,A\,b^2)}{2} + \frac{2187\,a^5\,b^2\,x^5\,(27\,C\,a^2+5\,B\,a\,b-8\,A\,b^2)}{5} - \frac{324\,a^3\,b^4\,x^7\,(54\,C\,a^2+27\,B\,a\,b-4\,A\,b^2)}{7} + \frac{2187\,a^6\,b\,x^4\,(27\,C\,a^2+27\,B\,a\,b+5\,A\,b^2)}{4}$$

input
```
int((A + B*x + C*x^2)*(27*a^3 - 4*b^3*x^3 + 27*a^2*b*x)^3,x)
```

output
```
(19683*a^8*x^2*(3*A*b + B*a))/2 - (64*B*b^9*x^11)/11 - (16*C*b^9*x^12)/3 -
 x^10*((32*A*b^9)/5 - (648*C*a^2*b^7)/5) + 6561*a^7*x^3*(3*A*b^2 + C*a^2 +
 3*B*a*b) + 19683*A*a^9*x + 144*a^2*b^6*x^9*(B*b + C*a) - (729*a^4*b^3*x^6
*(4*A*b^2 - 5*C*a^2 + 8*B*a*b))/2 + (81*a^2*b^5*x^8*(4*A*b^2 - 27*C*a^2 +
4*B*a*b))/2 + (2187*a^5*b^2*x^5*(27*C*a^2 - 8*A*b^2 + 5*B*a*b))/5 - (324*a
^3*b^4*x^7*(54*C*a^2 - 4*A*b^2 + 27*B*a*b))/7 + (2187*a^6*b*x^4*(5*A*b^2 +
 27*C*a^2 + 27*B*a*b))/4
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.09

$$\int (A+Bx+Cx^2)\left(27a^3+27a^2bx-4b^3x^3\right)^3\,dx$$
$$= \frac{x(-24640b^9c\,x^{11} - 26880b^{10}x^{10} + 598752a^2b^7c\,x^9 - 29568a\,b^9x^9 + 665280a^3b^6c\,x^8 + 665280a^2b^8x^8 - 50}{}$$

input
```
int((C*x^2+B*x+A)*(-4*b^3*x^3+27*a^2*b*x+27*a^3)^3,x)
```

output
```
(x*(90935460*a**10 + 181870920*a**9*b*x + 30311820*a**9*c*x**2 + 181870920
*a**8*b**2*x**2 + 68201595*a**8*b*c*x**3 + 80831520*a**7*b**3*x**3 + 54561
276*a**7*b**2*c*x**4 - 6062364*a**6*b**4*x**4 + 8419950*a**6*b**3*c*x**5 -
 20207880*a**5*b**5*x**5 - 11547360*a**5*b**4*c*x**6 - 4918320*a**4*b**6*x
**6 - 5051970*a**4*b**5*c*x**7 + 1496880*a**3*b**7*x**7 + 665280*a**3*b**6
*c*x**8 + 665280*a**2*b**8*x**8 + 598752*a**2*b**7*c*x**9 - 29568*a*b**9*x
**9 - 26880*b**10*x**10 - 24640*b**9*c*x**11))/4620
```

3.2 $\int \left(A+Bx+Cx^2\right)\left(27a^3+27a^2bx-4b^3x^3\right)^2\,dx$

Optimal result

Integrand size = 34, antiderivative size = 153

$$\int \left(A+Bx+Cx^2\right)\left(27a^3+27a^2bx-4b^3x^3\right)^2\,dx = \frac{81a^2(4Ab^2-6abB+9a^2C)\,(3a+2bx)^5}{160b^3}-\frac{3a(4Ab^2-15abB+36a^2C)\,(3a+2bx)^6}{32b^3}+\frac{(2Ab^2-21abB+99a^2C)\,(3a+2bx)^7}{112b^3}+\frac{(bB-12aC)(3a+2bx)^8}{128b^3}+\frac{C(3a+2bx)^9}{288b^3}$$

output
```
81/160*a^2*(4*A*b^2-6*B*a*b+9*C*a^2)*(2*b*x+3*a)^5/b^3-3/32*a*(4*A*b^2-15*
B*a*b+36*C*a^2)*(2*b*x+3*a)^6/b^3+1/112*(2*A*b^2-21*B*a*b+99*C*a^2)*(2*b*x
+3*a)^7/b^3+1/128*(B*b-12*C*a)*(2*b*x+3*a)^8/b^3+1/288*C*(2*b*x+3*a)^9/b^3
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.03

$$\int \left(A+Bx+Cx^2\right)\left(27a^3+27a^2bx-4b^3x^3\right)^2\,dx = \frac{243}{2}a^6x(6A+x(3B+2Cx))+\frac{243}{2}a^5bx^2(6A+x(4B+3Cx))+\frac{243}{20}a^4b^2x^3(20A+3x(5B+4Cx))-\frac{18}{5}a^3b^3x^4(15A+2x(6B+5Cx))-\frac{36}{35}a^2b^4x^5(42A+5x(7B+6Cx))+\frac{2}{63}b^6x^7(72A+7x(9B+8Cx))$$

input
```
Integrate[(A + B*x + C*x^2)*(27*a^3 + 27*a^2*b*x - 4*b^3*x^3)^2,x]
```

output
```
(243*a^6*x*(6*A + x*(3*B + 2*C*x)))/2 + (243*a^5*b*x^2*(6*A + x*(4*B + 3*C
*x)))/2 + (243*a^4*b^2*x^3*(20*A + 3*x*(5*B + 4*C*x)))/20 - (18*a^3*b^3*x^
4*(15*A + 2*x*(6*B + 5*C*x)))/5 - (36*a^2*b^4*x^5*(42*A + 5*x*(7*B + 6*C*x
)))/35 + (2*b^6*x^7*(72*A + 7*x*(9*B + 8*C*x)))/63
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.12, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(27a^3+27a^2bx-4b^3x^3\right)^2\left(A+Bx+Cx^2\right)dx$$

$$\Big\downarrow \text{ 2188}$$

$$\int \left(729a^6A+729a^5x(aB+2Ab)+729a^4x^2\left(a(aC+2bB)+Ab^2\right)+27a^3bx^3\left(27a(2aC+bB)-8Ab^2\right)+27a^2b^2\right.$$

$$\Big\downarrow \text{ 2009}$$

$$729a^6Ax+\frac{729}{2}a^5x^2(aB+2Ab)+243a^4x^3\left(a(aC+2bB)+Ab^2\right)-\frac{27}{4}a^3bx^4\left(8Ab^2-27a(2aC+bB)\right)-\frac{27}{5}a^2b^2x^5\left(a(8bB-27aC)+8Ab^2\right)+\frac{8}{7}b^4x^7\left(2Ab^2-27a^2C\right)-36a^2b^3x^6(aC+bB)+2b^6Bx^8+\frac{16}{9}b^6Cx^9$$

input
```
Int[(A + B*x + C*x^2)*(27*a^3 + 27*a^2*b*x - 4*b^3*x^3)^2,x]
```

output
```
729*a^6*A*x + (729*a^5*(2*A*b + a*B)*x^2)/2 + 243*a^4*(A*b^2 + a*(2*b*B +
a*C))*x^3 - (27*a^3*b*(8*A*b^2 - 27*a*(b*B + 2*a*C))*x^4)/4 - (27*a^2*b^2*
(8*A*b^2 + a*(8*b*B - 27*a*C))*x^5)/5 - 36*a^2*b^3*(b*B + a*C)*x^6 + (8*b^
4*(2*A*b^2 - 27*a^2*C)*x^7)/7 + 2*b^6*B*x^8 + (16*b^6*C*x^9)/9
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.16

method	result
norman	$\frac{16C\,b^6x^9}{9}+2B\,b^6x^8+\left(\frac{16}{7}A\,b^6-\frac{216}{7}b^4C\,a^2\right)x^7+\left(-36B\,a^2b^4-36b^3C\,a^3\right)x^6+\left(-\frac{216}{5}A\,a^2b^4\right.$
default	$\frac{16C\,b^6x^9}{9}+2B\,b^6x^8+\frac{(16A\,b^6-216b^4C\,a^2)x^7}{7}+\frac{(-216B\,a^2b^4-216b^3C\,a^3)x^6}{6}+\frac{(-216A\,a^2b^4-216B\,a^3b^3+729b^2}{5}$
gosper	$\frac{x\left(2240C\,b^6x^8+2520B\,b^6x^7+2880x^6A\,b^6-38880x^6b^4C\,a^2-45360B\,a^2b^4x^5-45360C\,a^3b^3x^5-54432x^4A\,a^2b^4-54432x^4B\,a^3\right.}{}$
risch	$\frac{16}{9}C\,b^6x^9+2B\,b^6x^8+\frac{16}{7}x^7A\,b^6-\frac{216}{7}x^7b^4C\,a^2-36B\,a^2b^4x^6-36C\,a^3b^3x^6-\frac{216}{5}x^5A\,a^2b^4-$
parallelrisch	$\frac{16}{9}C\,b^6x^9+2B\,b^6x^8+\frac{16}{7}x^7A\,b^6-\frac{216}{7}x^7b^4C\,a^2-36B\,a^2b^4x^6-36C\,a^3b^3x^6-\frac{216}{5}x^5A\,a^2b^4-$
orering	$\frac{x\left(2240C\,b^6x^8+2520B\,b^6x^7+2880x^6A\,b^6-38880x^6b^4C\,a^2-45360B\,a^2b^4x^5-45360C\,a^3b^3x^5-54432x^4A\,a^2b^4-54432x^4B\,a^3\right.}{}$

input
```
int((C*x^2+B*x+A)*(-4*b^3*x^3+27*a^2*b*x+27*a^3)^2,x,method=_RETURNVERBOSE
)
```

output
```
16/9*C*b^6*x^9+2*B*b^6*x^8+(16/7*A*b^6-216/7*b^4*C*a^2)*x^7+(-36*B*a^2*b^4
-36*C*a^3*b^3)*x^6+(-216/5*A*a^2*b^4-216/5*B*a^3*b^3+729/5*b^2*C*a^4)*x^5+
(-54*A*a^3*b^3+729/4*B*a^4*b^2+729/2*C*a^5*b)*x^4+(243*A*a^4*b^2+486*B*a^5
*b+243*C*a^6)*x^3+(729*A*a^5*b+729/2*B*a^6)*x^2+729*A*a^6*x
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.16

$$\int (A + Bx + Cx^2) (27a^3 + 27a^2bx - 4b^3x^3)^2 \, dx$$
$$= \frac{16}{9} Cb^6x^9 + 2Bb^6x^8 + 729Aa^6x - \frac{8}{7} (27Ca^2b^4 - 2Ab^6)x^7 - 36 (Ca^3b^3 + Ba^2b^4)x^6 + \frac{27}{5} (27Ca^4b^2 - 8Ba^3b^3 - 8Aa^2b^4)x^5 + \frac{27}{4} (54Ca^5b + 27Ba^4b^2 - 8Aa^3b^3)x^4 + 243 (Ca^6 + 2Ba^5b + Aa^4b^2)x^3 + \frac{729}{2} (Ba^6 + 2Aa^5b)x^2$$

input
```
integrate((C*x^2+B*x+A)*(-4*b^3*x^3+27*a^2*b*x+27*a^3)^2,x, algorithm="fri
cas")
```

output
```
16/9*C*b^6*x^9 + 2*B*b^6*x^8 + 729*A*a^6*x - 8/7*(27*C*a^2*b^4 - 2*A*b^6)*
x^7 - 36*(C*a^3*b^3 + B*a^2*b^4)*x^6 + 27/5*(27*C*a^4*b^2 - 8*B*a^3*b^3 -
8*A*a^2*b^4)*x^5 + 27/4*(54*C*a^5*b + 27*B*a^4*b^2 - 8*A*a^3*b^3)*x^4 + 24
3*(C*a^6 + 2*B*a^5*b + A*a^4*b^2)*x^3 + 729/2*(B*a^6 + 2*A*a^5*b)*x^2
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.32

$$\int (A + Bx + Cx^2) (27a^3 + 27a^2bx - 4b^3x^3)^2 \, dx$$
$$= 729Aa^6x + 2Bb^6x^8 + \frac{16Cb^6x^9}{9} + x^7 \cdot \left(\frac{16Ab^6}{7} - \frac{216Ca^2b^4}{7}\right) + x^6 (-36Ba^2b^4 - 36Ca^3b^3) + x^5 \left(-\frac{216Aa^2b^4}{5} - \frac{216Ba^3b^3}{5} + \frac{729Ca^4b^2}{5}\right) + x^4 \left(-54Aa^3b^3 + \frac{729Ba^4b^2}{4} + \frac{729Ca^5b}{2}\right) + x^3 \cdot (243Aa^4b^2 + 486Ba^5b + 243Ca^6) + x^2 \cdot \left(729Aa^5b + \frac{729Ba^6}{2}\right)$$

input
```
integrate((C*x**2+B*x+A)*(-4*b**3*x**3+27*a**2*b*x+27*a**3)**2,x)
```

output
```
729*A*a**6*x + 2*B*b**6*x**8 + 16*C*b**6*x**9/9 + x**7*(16*A*b**6/7 - 216*
C*a**2*b**4/7) + x**6*(-36*B*a**2*b**4 - 36*C*a**3*b**3) + x**5*(-216*A*a*
*2*b**4/5 - 216*B*a**3*b**3/5 + 729*C*a**4*b**2/5) + x**4*(-54*A*a**3*b**3
 + 729*B*a**4*b**2/4 + 729*C*a**5*b/2) + x**3*(243*A*a**4*b**2 + 486*B*a**
5*b + 243*C*a**6) + x**2*(729*A*a**5*b + 729*B*a**6/2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.16

$$\int \left(A+Bx+Cx^2\right)\left(27a^3+27a^2bx-4b^3x^3\right)^2\,dx$$
$$=\frac{16}{9}\,Cb^6x^9+2\,Bb^6x^8+729\,Aa^6x-\frac{8}{7}\left(27\,Ca^2b^4-2\,Ab^6\right)x^7-36\left(Ca^3b^3+Ba^2b^4\right)x^6$$
$$+\frac{27}{5}\left(27\,Ca^4b^2-8\,Ba^3b^3-8\,Aa^2b^4\right)x^5+\frac{27}{4}\left(54\,Ca^5b+27\,Ba^4b^2-8\,Aa^3b^3\right)x^4$$
$$+243\left(Ca^6+2\,Ba^5b+Aa^4b^2\right)x^3+\frac{729}{2}\left(Ba^6+2\,Aa^5b\right)x^2$$

input
```
integrate((C*x^2+B*x+A)*(-4*b^3*x^3+27*a^2*b*x+27*a^3)^2,x, algorithm="max
ima")
```

output
```
16/9*C*b^6*x^9 + 2*B*b^6*x^8 + 729*A*a^6*x - 8/7*(27*C*a^2*b^4 - 2*A*b^6)*
x^7 - 36*(C*a^3*b^3 + B*a^2*b^4)*x^6 + 27/5*(27*C*a^4*b^2 - 8*B*a^3*b^3 -
8*A*a^2*b^4)*x^5 + 27/4*(54*C*a^5*b + 27*B*a^4*b^2 - 8*A*a^3*b^3)*x^4 + 24
3*(C*a^6 + 2*B*a^5*b + A*a^4*b^2)*x^3 + 729/2*(B*a^6 + 2*A*a^5*b)*x^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.25

$$\begin{aligned}&\int (A+Bx+Cx^2)\left(27a^3+27a^2bx-4b^3x^3\right)^2\,dx\\&=\frac{16}{9}Cb^6x^9+2Bb^6x^8-\frac{216}{7}Ca^2b^4x^7+\frac{16}{7}Ab^6x^7-36Ca^3b^3x^6\\&\quad-36Ba^2b^4x^6+\frac{729}{5}Ca^4b^2x^5-\frac{216}{5}Ba^3b^3x^5-\frac{216}{5}Aa^2b^4x^5\\&\quad+\frac{729}{2}Ca^5bx^4+\frac{729}{4}Ba^4b^2x^4-54Aa^3b^3x^4+243Ca^6x^3\\&\quad+486Ba^5bx^3+243Aa^4b^2x^3+\frac{729}{2}Ba^6x^2+729Aa^5bx^2+729Aa^6x\end{aligned}$$

input
```
integrate((C*x^2+B*x+A)*(-4*b^3*x^3+27*a^2*b*x+27*a^3)^2,x, algorithm="gia
c")
```

output
```
16/9*C*b^6*x^9 + 2*B*b^6*x^8 - 216/7*C*a^2*b^4*x^7 + 16/7*A*b^6*x^7 - 36*C
*a^3*b^3*x^6 - 36*B*a^2*b^4*x^6 + 729/5*C*a^4*b^2*x^5 - 216/5*B*a^3*b^3*x^
5 - 216/5*A*a^2*b^4*x^5 + 729/2*C*a^5*b*x^4 + 729/4*B*a^4*b^2*x^4 - 54*A*a
^3*b^3*x^4 + 243*C*a^6*x^3 + 486*B*a^5*b*x^3 + 243*A*a^4*b^2*x^3 + 729/2*B
*a^6*x^2 + 729*A*a^5*b*x^2 + 729*A*a^6*x
```

Mupad [B] (verification not implemented)

Time = 11.99 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.06

$$\begin{aligned}&\int (A+Bx+Cx^2)\left(27a^3+27a^2bx-4b^3x^3\right)^2\,dx\\&=x^7\left(\frac{16\,A\,b^6}{7}-\frac{216\,C\,a^2\,b^4}{7}\right)+x^2\left(\frac{729\,B\,a^6}{2}+729\,A\,b\,a^5\right)+2\,B\,b^6\,x^8+\frac{16\,C\,b^6\,x^9}{9}\\&\quad+243\,a^4\,x^3\left(C\,a^2+2\,B\,a\,b+A\,b^2\right)+729\,A\,a^6\,x-36\,a^2\,b^3\,x^6\,(B\,b+C\,a)\\&\quad-\frac{27\,a^2\,b^2\,x^5\,(-27\,C\,a^2+8\,B\,a\,b+8\,A\,b^2)}{5}+\frac{27\,a^3\,b\,x^4\,(54\,C\,a^2+27\,B\,a\,b-8\,A\,b^2)}{4}\end{aligned}$$

input
```
int((A + B*x + C*x^2)*(27*a^3 - 4*b^3*x^3 + 27*a^2*b*x)^2,x)
```

output
```
x^7*((16*A*b^6)/7 - (216*C*a^2*b^4)/7) + x^2*((729*B*a^6)/2 + 729*A*a^5*b)
 + 2*B*b^6*x^8 + (16*C*b^6*x^9)/9 + 243*a^4*x^3*(A*b^2 + C*a^2 + 2*B*a*b)
+ 729*A*a^6*x - 36*a^2*b^3*x^6*(B*b + C*a) - (27*a^2*b^2*x^5*(8*A*b^2 - 27
*C*a^2 + 8*B*a*b))/5 + (27*a^3*b*x^4*(54*C*a^2 - 8*A*b^2 + 27*B*a*b))/4
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.92

$$\int (A + Bx + Cx^2) (27a^3 + 27a^2bx - 4b^3x^3)^2 \, dx$$
$$= \frac{x(2240b^6c\,x^8 + 2520b^7x^7 - 38880a^2b^4c\,x^6 + 2880a\,b^6x^6 - 45360a^3b^3c\,x^5 - 45360a^2b^5x^5 + 183708a^4b^2c\,x}{}$$

input
```
int((C*x^2+B*x+A)*(-4*b^3*x^3+27*a^2*b*x+27*a^3)^2,x)
```

output
```
(x*(918540*a**7 + 1377810*a**6*b*x + 306180*a**6*c*x**2 + 918540*a**5*b**2
*x**2 + 459270*a**5*b*c*x**3 + 161595*a**4*b**3*x**3 + 183708*a**4*b**2*c*
x**4 - 108864*a**3*b**4*x**4 - 45360*a**3*b**3*c*x**5 - 45360*a**2*b**5*x*
*5 - 38880*a**2*b**4*c*x**6 + 2880*a*b**6*x**6 + 2520*b**7*x**7 + 2240*b**
6*c*x**8))/1260
```

3.3 $\int (A+Bx+Cx^2)(27a^3+27a^2bx-4b^3x^3)\,dx$

Optimal result

Integrand size = 32, antiderivative size = 83

$$\int (A+Bx+Cx^2)\left(27a^3+27a^2bx-4b^3x^3\right)\,dx = 27a^3Ax+\frac{27}{2}a^2(Ab+aB)x^2+9a^2(bB+aC)x^3 - \frac{1}{4}b\left(4Ab^2-27a^2C\right)x^4-\frac{4}{5}b^3Bx^5-\frac{2}{3}b^3Cx^6$$

output
```
27*a^3*A*x+27/2*a^2*(A*b+B*a)*x^2+9*a^2*(B*b+C*a)*x^3-1/4*b*(4*A*b^2-27*C*
a^2)*x^4-4/5*b^3*B*x^5-2/3*b^3*C*x^6
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.88

$$\int (A+Bx+Cx^2)\left(27a^3+27a^2bx-4b^3x^3\right)\,dx = \frac{9}{2}a^3x(6A+x(3B+2Cx))+\frac{9}{4}a^2bx^2(6A+x(4B+3Cx)) - \frac{1}{15}b^3x^4(15A+2x(6B+5Cx))$$

input
```
Integrate[(A + B*x + C*x^2)*(27*a^3 + 27*a^2*b*x - 4*b^3*x^3),x]
```

output
```
(9*a^3*x*(6*A + x*(3*B + 2*C*x)))/2 + (9*a^2*b*x^2*(6*A + x*(4*B + 3*C*x))
)/4 - (b^3*x^4*(15*A + 2*x*(6*B + 5*C*x)))/15
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(27a^3+27a^2bx-4b^3x^3\right)\left(A+Bx+Cx^2\right)\,dx$$

$$\Big\downarrow \text{2188}$$

$$\int \left(27a^3A-bx^3\left(4Ab^2-27a^2C\right)+27a^2x(aB+Ab)+27a^2x^2(aC+bB)-4b^3Bx^4-4b^3Cx^5\right)dx$$

$$\Big\downarrow \text{2009}$$

$$27a^3Ax-\frac{1}{4}bx^4\left(4Ab^2-27a^2C\right)+\frac{27}{2}a^2x^2(aB+Ab)+9a^2x^3(aC+bB)-\frac{4}{5}b^3Bx^5-\frac{2}{3}b^3Cx^6$$

input
```
Int[(A + B*x + C*x^2)*(27*a^3 + 27*a^2*b*x - 4*b^3*x^3),x]
```

output
```
27*a^3*A*x + (27*a^2*(A*b + a*B)*x^2)/2 + 9*a^2*(b*B + a*C)*x^3 - (b*(4*A*
b^2 - 27*a^2*C)*x^4)/4 - (4*b^3*B*x^5)/5 - (2*b^3*C*x^6)/3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.98

method	result
norman	$-\frac{2b^3Cx^6}{3}-\frac{4b^3Bx^5}{5}+\left(-Ab^3+\frac{27}{4}bCa^2\right)x^4+\left(9Ba^2b+9Ca^3\right)x^3+\left(\frac{27}{2}Aa^2b+\frac{27}{2}Ba^3\right)x^2$
gosper	$\frac{x\left(-40b^3Cx^5-48b^3Bx^4-60x^3Ab^3+405x^3bCa^2+540Ba^2bx^2+540Ca^3x^2+810xAa^2b+810xBa^3+1620Aa^3\right)}{60}$
default	$-\frac{2b^3Cx^6}{3}-\frac{4b^3Bx^5}{5}+\frac{\left(-4Ab^3+27bCa^2\right)x^4}{4}+\frac{\left(27Ba^2b+27Ca^3\right)x^3}{3}+\frac{\left(27Aa^2b+27Ba^3\right)x^2}{2}+27a^3Ax$
risch	$-\frac{2}{3}b^3Cx^6-\frac{4}{5}b^3Bx^5-x^4Ab^3+\frac{27}{4}x^4bCa^2+9Ba^2bx^3+9Ca^3x^3+\frac{27}{2}x^2Aa^2b+\frac{27}{2}x^2Ba$
parallelrisch	$-\frac{2}{3}b^3Cx^6-\frac{4}{5}b^3Bx^5-x^4Ab^3+\frac{27}{4}x^4bCa^2+9Ba^2bx^3+9Ca^3x^3+\frac{27}{2}x^2Aa^2b+\frac{27}{2}x^2Ba$
orering	$\frac{x\left(-40b^3Cx^5-48b^3Bx^4-60x^3Ab^3+405x^3bCa^2+540Ba^2bx^2+540Ca^3x^2+810xAa^2b+810xBa^3+1620Aa^3\right)\left(-4b^3x^3+27\right.}{60(-bx+3a)(2bx+3a)^2}$

input
```
int((C*x^2+B*x+A)*(-4*b^3*x^3+27*a^2*b*x+27*a^3),x,method=_RETURNVERBOSE)
```

output
```
-2/3*b^3*C*x^6-4/5*b^3*B*x^5+(-A*b^3+27/4*b*C*a^2)*x^4+(9*B*a^2*b+9*C*a^3)
*x^3+(27/2*A*a^2*b+27/2*B*a^3)*x^2+27*a^3*A*x
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.95

$$\int (A + Bx + Cx^2) (27a^3 + 27a^2bx - 4b^3x^3) \, dx$$
$$= -\frac{2}{3} Cb^3x^6 - \frac{4}{5} Bb^3x^5 + 27 Aa^3x + \frac{1}{4} (27 Ca^2b - 4 Ab^3)x^4 + 9 (Ca^3 + Ba^2b)x^3 + \frac{27}{2} (Ba^3 + Aa^2b)x^2$$

input
```
integrate((C*x^2+B*x+A)*(-4*b^3*x^3+27*a^2*b*x+27*a^3),x, algorithm="frica
s")
```

output
```
-2/3*C*b^3*x^6 - 4/5*B*b^3*x^5 + 27*A*a^3*x + 1/4*(27*C*a^2*b - 4*A*b^3)*x
^4 + 9*(C*a^3 + B*a^2*b)*x^3 + 27/2*(B*a^3 + A*a^2*b)*x^2
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.08

$$\int (A + Bx + Cx^2) (27a^3 + 27a^2bx - 4b^3x^3) \, dx$$
$$= 27Aa^3x - \frac{4Bb^3x^5}{5} - \frac{2Cb^3x^6}{3} + x^4 \left(-Ab^3 + \frac{27Ca^2b}{4}\right) + x^3 \cdot (9Ba^2b + 9Ca^3) + x^2 \cdot \left(\frac{27Aa^2b}{2} + \frac{27Ba^3}{2}\right)$$

input
```
integrate((C*x**2+B*x+A)*(-4*b**3*x**3+27*a**2*b*x+27*a**3),x)
```

output
```
27*A*a**3*x - 4*B*b**3*x**5/5 - 2*C*b**3*x**6/3 + x**4*(-A*b**3 + 27*C*a**
2*b/4) + x**3*(9*B*a**2*b + 9*C*a**3) + x**2*(27*A*a**2*b/2 + 27*B*a**3/2)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.95

$$\int \left(A+Bx+Cx^2\right)\left(27a^3+27a^2bx-4b^3x^3\right)\,dx$$
$$=-\frac{2}{3}\,Cb^3x^6-\frac{4}{5}\,Bb^3x^5+27\,Aa^3x+\frac{1}{4}\,\left(27\,Ca^2b-4\,Ab^3\right)x^4$$
$$+9\,\left(Ca^3+Ba^2b\right)x^3+\frac{27}{2}\,\left(Ba^3+Aa^2b\right)x^2$$

input
```
integrate((C*x^2+B*x+A)*(-4*b^3*x^3+27*a^2*b*x+27*a^3),x, algorithm="maxim
a")
```

output
```
-2/3*C*b^3*x^6 - 4/5*B*b^3*x^5 + 27*A*a^3*x + 1/4*(27*C*a^2*b - 4*A*b^3)*x
^4 + 9*(C*a^3 + B*a^2*b)*x^3 + 27/2*(B*a^3 + A*a^2*b)*x^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00

$$\int \left(A+Bx+Cx^2\right)\left(27a^3+27a^2bx-4b^3x^3\right)\,dx$$
$$=-\frac{2}{3}\,Cb^3x^6-\frac{4}{5}\,Bb^3x^5+\frac{27}{4}\,Ca^2bx^4-Ab^3x^4+9\,Ca^3x^3$$
$$+9\,Ba^2bx^3+\frac{27}{2}\,Ba^3x^2+\frac{27}{2}\,Aa^2bx^2+27\,Aa^3x$$

input
```
integrate((C*x^2+B*x+A)*(-4*b^3*x^3+27*a^2*b*x+27*a^3),x, algorithm="giac"
)
```

output
```
-2/3*C*b^3*x^6 - 4/5*B*b^3*x^5 + 27/4*C*a^2*b*x^4 - A*b^3*x^4 + 9*C*a^3*x^
3 + 9*B*a^2*b*x^3 + 27/2*B*a^3*x^2 + 27/2*A*a^2*b*x^2 + 27*A*a^3*x
```

Mupad [B] (verification not implemented)

Time = 12.08 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.96

$$\int (A+Bx+Cx^2)\left(27a^3+27a^2bx-4b^3x^3\right)\,dx$$
$$=x^2\left(\frac{27\,B\,a^3}{2}+\frac{27\,A\,b\,a^2}{2}\right)+x^3\left(9\,C\,a^3+9\,B\,b\,a^2\right)$$
$$-x^4\left(A\,b^3-\frac{27\,C\,a^2\,b}{4}\right)-\frac{4\,B\,b^3\,x^5}{5}-\frac{2\,C\,b^3\,x^6}{3}+27\,A\,a^3\,x$$

input
```
int((A + B*x + C*x^2)*(27*a^3 - 4*b^3*x^3 + 27*a^2*b*x),x)
```

output
```
x^2*((27*B*a^3)/2 + (27*A*a^2*b)/2) + x^3*(9*C*a^3 + 9*B*a^2*b) - x^4*(A*b
^3 - (27*C*a^2*b)/4) - (4*B*b^3*x^5)/5 - (2*C*b^3*x^6)/3 + 27*A*a^3*x
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.87

$$\int (A+Bx+Cx^2)\left(27a^3+27a^2bx-4b^3x^3\right)\,dx$$
$$=\frac{x(-40b^3c\,x^5-48b^4x^4+405a^2bc\,x^3-60a\,b^3x^3+540a^3c\,x^2+540a^2b^2x^2+1620a^3bx+1620a^4)}{60}$$

input
```
int((C*x^2+B*x+A)*(-4*b^3*x^3+27*a^2*b*x+27*a^3),x)
```

output
```
(x*(1620*a**4 + 1620*a**3*b*x + 540*a**3*c*x**2 + 540*a**2*b**2*x**2 + 405
*a**2*b*c*x**3 - 60*a*b**3*x**3 - 48*b**4*x**4 - 40*b**3*c*x**5))/60
```

3.4 $\int \frac{A+Bx+Cx^2}{27a^3+27a^2bx-4b^3x^3}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 113

$$\int \frac{A+Bx+Cx^2}{27a^3+27a^2bx-4b^3x^3}\,dx = \frac{-4Ab^2+6abB-9a^2C}{36ab^3(3a+2bx)} - \frac{(Ab^2+3a(bB+3aC))\log(3a-bx)}{81a^2b^3} + \frac{(4Ab^2+3a(4bB-15aC))\log(3a+2bx)}{324a^2b^3}$$

output
```
1/36*(-4*A*b^2+6*B*a*b-9*C*a^2)/a/b^3/(2*b*x+3*a)-1/81*(A*b^2+3*a*(B*b+3*C
*a))*ln(-b*x+3*a)/a^2/b^3+1/324*(4*A*b^2+3*a*(4*B*b-15*C*a))*ln(2*b*x+3*a)
/a^2/b^3
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.87

$$\int \frac{A+Bx+Cx^2}{27a^3+27a^2bx-4b^3x^3}\,dx = \frac{-\frac{9a(4Ab^2-6abB+9a^2C)}{3a+2bx} - 4(Ab^2+3a(bB+3aC))\log(3a-bx) + (4Ab^2+12abB-45a^2C)\log(3a+2bx)}{324a^2b^3}$$

input
```
Integrate[(A + B*x + C*x^2)/(27*a^3 + 27*a^2*b*x - 4*b^3*x^3),x]
```

output
```
((-9*a*(4*A*b^2 - 6*a*b*B + 9*a^2*C))/(3*a + 2*b*x) - 4*(A*b^2 + 3*a*(b*B
+ 3*a*C))*Log[3*a - b*x] + (4*A*b^2 + 12*a*b*B - 45*a^2*C)*Log[3*a + 2*b*x
])/(324*a^2*b^3)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {2462, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{27a^3+27a^2bx-4b^3x^3}\,dx$$

$$\Big\downarrow \text{2462}$$

$$\int \left(\frac{-45a^2C+12abB+4Ab^2}{162a^2b^2(3a+2bx)}+\frac{3a(3aC+bB)+Ab^2}{81a^2b^2(3a-bx)}+\frac{9a^2C-6abB+4Ab^2}{18ab^2(3a+2bx)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{9a^2C-6abB+4Ab^2}{36ab^3(3a+2bx)}-\frac{\log(3a-bx)\left(3a(3aC+bB)+Ab^2\right)}{81a^2b^3}+\frac{\log(3a+2bx)\left(3a(4bB-15aC)+4Ab^2\right)}{324a^2b^3}$$

input
```
Int[(A + B*x + C*x^2)/(27*a^3 + 27*a^2*b*x - 4*b^3*x^3),x]
```

output
```
-1/36*(4*A*b^2 - 6*a*b*B + 9*a^2*C)/(a*b^3*(3*a + 2*b*x)) - ((A*b^2 + 3*a*
(b*B + 3*a*C))*Log[3*a - b*x])/(81*a^2*b^3) + ((4*A*b^2 + 3*a*(4*b*B - 15*
a*C))*Log[3*a + 2*b*x])/(324*a^2*b^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.95

method	result
norman	$-\frac{4A\,b^{2}-6abB+9C\,a^{2}}{36b^{3}a(2bx+3a)}-\frac{(A\,b^{2}+3abB+9C\,a^{2})\ln(-bx+3a)}{81a^{2}b^{3}}+\frac{(4A\,b^{2}+12abB-45C\,a^{2})\ln(2bx+3a)}{324b^{3}a^{2}}$
default	$\frac{(-A\,b^{2}-3abB-9C\,a^{2})\ln(-bx+3a)}{81a^{2}b^{3}}-\frac{4A\,b^{2}-6abB+9C\,a^{2}}{36b^{3}a(2bx+3a)}+\frac{(4A\,b^{2}+12abB-45C\,a^{2})\ln(2bx+3a)}{324b^{3}a^{2}}$
risch	$-\frac{A}{27ba\left(\frac{2bx}{3}+a\right)}+\frac{B}{18b^{2}\left(\frac{2bx}{3}+a\right)}-\frac{aC}{12b^{3}\left(\frac{2bx}{3}+a\right)}-\frac{\ln(-bx+3a)A}{81a^{2}b}-\frac{\ln(-bx+3a)B}{27a\,b^{2}}-\frac{\ln(-bx+3a)C}{9b^{3}}+\frac{\ln(2bx+}{81b\,a}$
parallelrisch	$\frac{8A\ln\left(bx+\frac{3a}{2}\right)x\,b^{3}-8A\ln(bx-3a)x\,b^{3}+24B\ln\left(bx+\frac{3a}{2}\right)xa\,b^{2}-24B\ln(bx-3a)xa\,b^{2}-90C\ln\left(bx+\frac{3a}{2}\right)x\,a^{2}b-72C\ln(bx-3a)x}{}$

input
```
int((C*x^2+B*x+A)/(-4*b^3*x^3+27*a^2*b*x+27*a^3),x,method=_RETURNVERBOSE)
```

output
```
-1/36*(4*A*b^2-6*B*a*b+9*C*a^2)/b^3/a/(2*b*x+3*a)-1/81*(A*b^2+3*B*a*b+9*C*
a^2)/a^2/b^3*ln(-b*x+3*a)+1/324*(4*A*b^2+12*B*a*b-45*C*a^2)/b^3/a^2*ln(2*b
*x+3*a)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.35

$$\int \frac{A+Bx+Cx^2}{27a^3+27a^2bx-4b^3x^3}\,dx = -\frac{81\,Ca^3-54\,Ba^2b+36\,Aab^2+(135\,Ca^3-36\,Ba^2b-12\,Aab^2+2\,(45\,Ca^2b-12\,Bab^2-4\,Ab^3)x)\log}{324\,(2\,a^2b^4x+3\,a}$$

input
```
integrate((C*x^2+B*x+A)/(-4*b^3*x^3+27*a^2*b*x+27*a^3),x, algorithm="frica
s")
```

output
```
-1/324*(81*C*a^3 - 54*B*a^2*b + 36*A*a*b^2 + (135*C*a^3 - 36*B*a^2*b - 12*
A*a*b^2 + 2*(45*C*a^2*b - 12*B*a*b^2 - 4*A*b^3)*x)*log(2*b*x + 3*a) + 4*(2
7*C*a^3 + 9*B*a^2*b + 3*A*a*b^2 + 2*(9*C*a^2*b + 3*B*a*b^2 + A*b^3)*x)*log
(b*x - 3*a))/(2*a^2*b^4*x + 3*a^3*b^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 243 vs. 2(104) = 208.

Time = 0.67 (sec) , antiderivative size = 243, normalized size of antiderivative = 2.15

$$\int \frac{A+Bx+Cx^2}{27a^3+27a^2bx-4b^3x^3}\,dx = -\frac{4Ab^2-6Bab+9Ca^2}{108a^2b^3+72ab^4x} - \frac{(-4Ab^2-12Bab+45Ca^2)\log\left(x+\frac{6Aab^2+18Ba^2b-189Ca^3+\frac{9a\left(-4Ab^2-12Bab+45Ca^2\right)}{2}}{-8Ab^3-24Bab^2+9Ca^2b}\right)}{324a^2b^3} - \frac{(Ab^2+3Bab+9Ca^2)\log\left(x+\frac{6Aab^2+18Ba^2b-189Ca^3+18a(Ab^2+3Bab+9Ca^2)}{-8Ab^3-24Bab^2+9Ca^2b}\right)}{81a^2b^3}$$

input
```
integrate((C*x**2+B*x+A)/(-4*b**3*x**3+27*a**2*b*x+27*a**3),x)
```

output
```
-(4*A*b**2 - 6*B*a*b + 9*C*a**2)/(108*a**2*b**3 + 72*a*b**4*x) - (-4*A*b**
2 - 12*B*a*b + 45*C*a**2)*log(x + (6*A*a*b**2 + 18*B*a**2*b - 189*C*a**3 +
 9*a*(-4*A*b**2 - 12*B*a*b + 45*C*a**2)/2)/(-8*A*b**3 - 24*B*a*b**2 + 9*C*
a**2*b))/(324*a**2*b**3) - (A*b**2 + 3*B*a*b + 9*C*a**2)*log(x + (6*A*a*b*
*2 + 18*B*a**2*b - 189*C*a**3 + 18*a*(A*b**2 + 3*B*a*b + 9*C*a**2))/(-8*A*
b**3 - 24*B*a*b**2 + 9*C*a**2*b))/(81*a**2*b**3)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.95

$$\int \frac{A+Bx+Cx^2}{27a^3+27a^2bx-4b^3x^3}\,dx = -\frac{9\,Ca^2-6\,Bab+4\,Ab^2}{36\,(2\,ab^4x+3\,a^2b^3)} - \frac{(45\,Ca^2-12\,Bab-4\,Ab^2)\log(2\,bx+3\,a)}{324\,a^2b^3} - \frac{(9\,Ca^2+3\,Bab+Ab^2)\log(bx-3\,a)}{81\,a^2b^3}$$

input
```
integrate((C*x^2+B*x+A)/(-4*b^3*x^3+27*a^2*b*x+27*a^3),x, algorithm="maxim
a")
```

output
```
-1/36*(9*C*a^2 - 6*B*a*b + 4*A*b^2)/(2*a*b^4*x + 3*a^2*b^3) - 1/324*(45*C*
a^2 - 12*B*a*b - 4*A*b^2)*log(2*b*x + 3*a)/(a^2*b^3) - 1/81*(9*C*a^2 + 3*B
*a*b + A*b^2)*log(b*x - 3*a)/(a^2*b^3)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.97

$$\int \frac{A+Bx+Cx^2}{27a^3+27a^2bx-4b^3x^3}\,dx = -\frac{(45\,Ca^2-12\,Bab-4\,Ab^2)\log(|2\,bx+3\,a|)}{324\,a^2b^3} - \frac{(9\,Ca^2+3\,Bab+Ab^2)\log(|bx-3\,a|)}{81\,a^2b^3} - \frac{9\,Ca^3-6\,Ba^2b+4\,Aab^2}{36\,(2\,bx+3\,a)a^2b^3}$$

input
```
integrate((C*x^2+B*x+A)/(-4*b^3*x^3+27*a^2*b*x+27*a^3),x, algorithm="giac"
)
```

output
```
-1/324*(45*C*a^2 - 12*B*a*b - 4*A*b^2)*log(abs(2*b*x + 3*a))/(a^2*b^3) - 1
/81*(9*C*a^2 + 3*B*a*b + A*b^2)*log(abs(b*x - 3*a))/(a^2*b^3) - 1/36*(9*C*
a^3 - 6*B*a^2*b + 4*A*a*b^2)/((2*b*x + 3*a)*a^2*b^3)
```

Mupad [B] (verification not implemented)

Time = 12.30 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.94

$$\int \frac{A+Bx+Cx^2}{27a^3+27a^2bx-4b^3x^3}\,dx = \frac{\ln\left(3\,a+2\,b\,x\right)\,\left(-45\,C\,a^2+12\,B\,a\,b+4\,A\,b^2\right)}{324\,a^2\,b^3} - \frac{\ln\left(3\,a-b\,x\right)\,\left(9\,C\,a^2+3\,B\,a\,b+A\,b^2\right)}{81\,a^2\,b^3} - \frac{9\,C\,a^2-6\,B\,a\,b+4\,A\,b^2}{36\,a\,b^3\,\left(3\,a+2\,b\,x\right)}$$

input
```
int((A + B*x + C*x^2)/(27*a^3 - 4*b^3*x^3 + 27*a^2*b*x),x)
```

output
```
(log(3*a + 2*b*x)*(4*A*b^2 - 45*C*a^2 + 12*B*a*b))/(324*a^2*b^3) - (log(3*
a - b*x)*(A*b^2 + 9*C*a^2 + 3*B*a*b))/(81*a^2*b^3) - (4*A*b^2 + 9*C*a^2 -
6*B*a*b)/(36*a*b^3*(3*a + 2*b*x))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.34

$$\int \frac{A+Bx+Cx^2}{27a^3+27a^2bx-4b^3x^3}\,dx = \frac{-108\log(-bx+3a)\,a^2c-48\log(-bx+3a)\,a\,b^2-72\log(-bx+3a)\,abcx-32\log(-bx+3a)\,b^3x-135}{324a\,b^3}$$

input
```
int((C*x^2+B*x+A)/(-4*b^3*x^3+27*a^2*b*x+27*a^3),x)
```

output
```
( - 108*log(3*a - b*x)*a**2*c - 48*log(3*a - b*x)*a*b**2 - 72*log(3*a - b*
x)*a*b*c*x - 32*log(3*a - b*x)*b**3*x - 135*log(3*a + 2*b*x)*a**2*c + 48*l
og(3*a + 2*b*x)*a*b**2 - 90*log(3*a + 2*b*x)*a*b*c*x + 32*log(3*a + 2*b*x)
*b**3*x + 54*a*b*c*x - 12*b**3*x)/(324*a*b**3*(3*a + 2*b*x))
```

3.5 $\int \frac{A+Bx+Cx^2}{(27a^3+27a^2bx-4b^3x^3)^2}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 217

$$\begin{aligned}\int \frac{A+Bx+Cx^2}{(27a^3+27a^2bx-4b^3x^3)^2}\,dx &= \frac{Ab^2+3a(bB+3aC)}{6561a^4b^3(3a-bx)}-\frac{4Ab^2-6abB+9a^2C}{486a^2b^3(3a+2bx)^3}\\ &\quad -\frac{4Ab^2+3a(bB-6aC)}{1458a^3b^3(3a+2bx)^2}-\frac{2Ab+3aB}{2187a^4b^2(3a+2bx)}\\ &\quad -\frac{(8Ab^2+15abB+18a^2C)\log(3a-bx)}{59049a^5b^3}\\ &\quad +\frac{(8Ab^2+15abB+18a^2C)\log(3a+2bx)}{59049a^5b^3}\end{aligned}$$

output
```
1/6561*(A*b^2+3*a*(B*b+3*C*a))/a^4/b^3/(-b*x+3*a)-1/486*(4*A*b^2-6*B*a*b+9
*C*a^2)/a^2/b^3/(2*b*x+3*a)^3-1/1458*(4*A*b^2+3*a*(B*b-6*C*a))/a^3/b^3/(2*
b*x+3*a)^2-1/2187*(2*A*b+3*B*a)/a^4/b^2/(2*b*x+3*a)-1/59049*(8*A*b^2+15*B*
a*b+18*C*a^2)*ln(-b*x+3*a)/a^5/b^3+1/59049*(8*A*b^2+15*B*a*b+18*C*a^2)*ln(
2*b*x+3*a)/a^5/b^3
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.88

$$\int \frac{A+Bx+Cx^2}{(27a^3+27a^2bx-4b^3x^3)^2}\,dx$$
$$=\frac{\frac{18a(Ab^2+3a(bB+3aC))}{3a-bx}-\frac{243a^3(4Ab^2-6abB+9a^2C)}{(3a+2bx)^3}+\frac{81a^2(-4Ab^2+3a(-bB+6aC))}{(3a+2bx)^2}-\frac{54ab(2Ab+3aB)}{3a+2bx}-2(8Ab^2+3a(5bB}{118098a^5b^3}$$

input
```
Integrate[(A + B*x + C*x^2)/(27*a^3 + 27*a^2*b*x - 4*b^3*x^3)^2,x]
```

output
```
((18*a*(A*b^2 + 3*a*(b*B + 3*a*C)))/(3*a - b*x) - (243*a^3*(4*A*b^2 - 6*a*
b*B + 9*a^2*C))/(3*a + 2*b*x)^3 + (81*a^2*(-4*A*b^2 + 3*a*(-(b*B) + 6*a*C)
))/(3*a + 2*b*x)^2 - (54*a*b*(2*A*b + 3*a*B))/(3*a + 2*b*x) - 2*(8*A*b^2 +
 3*a*(5*b*B + 6*a*C))*Log[3*a - b*x] + 2*(8*A*b^2 + 3*a*(5*b*B + 6*a*C))*L
og[3*a + 2*b*x])/(118098*a^5*b^3)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {2462, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{(27a^3+27a^2bx-4b^3x^3)^2}\,dx$$
$$\Big\downarrow \text{2462}$$
$$\int \left(\frac{3a(3aC+bB)+Ab^2}{6561a^4b^2(3a-bx)^2}+\frac{2(3aB+2Ab)}{2187a^4b(3a+2bx)^2}+\frac{9a^2C-6abB+4Ab^2}{81a^2b^2(3a+2bx)^4}+\frac{18a^2C+15abB+8Ab^2}{59049a^5b^2(3a-bx)}+\frac{2(18a^2C}{5904}\right.$$
$$\Big\downarrow \text{2009}$$

$$-\frac{3aB+2Ab}{2187a^4b^2(3a+2bx)}+\frac{3a(3aC+bB)+Ab^2}{6561a^4b^3(3a-bx)}-\frac{3a(bB-6aC)+4Ab^2}{1458a^3b^3(3a+2bx)^2}-\frac{9a^2C-6abB+4Ab^2}{486a^2b^3(3a+2bx)^3}-\frac{\log(3a-bx)\left(18a^2C+15abB+8Ab^2\right)}{59049a^5b^3}+\frac{\log(3a+2bx)\left(18a^2C+15abB+8Ab^2\right)}{59049a^5b^3}$$

input
```
Int[(A + B*x + C*x^2)/(27*a^3 + 27*a^2*b*x - 4*b^3*x^3)^2,x]
```

output
```
(A*b^2 + 3*a*(b*B + 3*a*C))/(6561*a^4*b^3*(3*a - b*x)) - (4*A*b^2 - 6*a*b*
B + 9*a^2*C)/(486*a^2*b^3*(3*a + 2*b*x)^3) - (4*A*b^2 + 3*a*(b*B - 6*a*C))
/(1458*a^3*b^3*(3*a + 2*b*x)^2) - (2*A*b + 3*a*B)/(2187*a^4*b^2*(3*a + 2*b
*x)) - ((8*A*b^2 + 15*a*b*B + 18*a^2*C)*Log[3*a - b*x])/(59049*a^5*b^3) +
((8*A*b^2 + 15*a*b*B + 18*a^2*C)*Log[3*a + 2*b*x])/(59049*a^5*b^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.94

method	result
norman	$\frac{-\frac{2\left(2A\,b^{2}-3abB-9C\,a^{2}\right)x^{3}}{729a^{4}}+\frac{\left(14A\,b^{2}+6abB-9C\,a^{2}\right)x^{2}}{243b\,a^{3}}-\frac{4b\left(34A\,b^{2}+3abB-45C\,a^{2}\right)x^{4}}{19683a^{5}}+\frac{\left(73A\,b^{2}-15abB-18C\,a^{2}\right)x}{729a^{2}b^{2}}}{\left(2bx+3a\right)^{3}\left(-bx+3a\right)}-\frac{\left(8A\,b^{2}+15\right.}{}$
default	$\frac{\left(-8A\,b^{2}-15abB-18C\,a^{2}\right)\ln\left(-bx+3a\right)}{59049a^{5}b^{3}}+\frac{A\,b^{2}+3abB+9C\,a^{2}}{6561a^{4}b^{3}\left(-bx+3a\right)}-\frac{4Ab+6Ba}{4374a^{4}b^{2}\left(2bx+3a\right)}-\frac{4A\,b^{2}-6abB+9C\,a^{2}}{486a^{2}b^{3}\left(2bx+3a\right)^{3}}-\frac{8A\,b^{2}+}{2916a}$
risch	$\frac{\frac{4\left(8A\,b^{2}+15abB+18C\,a^{2}\right)x^{3}}{6561a^{4}}+\frac{\left(8A\,b^{2}+15abB+18C\,a^{2}\right)x^{2}}{729a^{3}b}-\frac{\left(8A\,b^{2}+15abB-63C\,a^{2}\right)x}{486a^{2}b^{2}}-\frac{34A\,b^{2}+3abB-45C\,a^{2}}{486b^{3}a}}{\left(2bx+3a\right)\left(-4b^{3}x^{3}+27b\,a^{2}x+27a^{3}\right)}-\frac{8\ln\left(-bx+3a\right)A}{59049a^{5}b}$
parallelrisch	$\frac{1458Cx\,a^{5}b-1458B\,x^{2}a^{3}b^{3}+2187C\,x^{2}a^{4}b^{2}-5913Ax\,a^{3}b^{3}+36B\,x^{4}a\,b^{5}-540C\,x^{4}a^{2}b^{4}+216C\ln\left(bx+\frac{3a}{2}\right)x^{3}a^{3}b^{3}-216C\ln\left(bx\right.}{}$

input
```
int((C*x^2+B*x+A)/(-4*b^3*x^3+27*a^2*b*x+27*a^3)^2,x,method=_RETURNVERBOSE
)
```

output
```
(-2/729*(2*A*b^2-3*B*a*b-9*C*a^2)/a^4*x^3+1/243/b*(14*A*b^2+6*B*a*b-9*C*a^
2)/a^3*x^2-4/19683*b*(34*A*b^2+3*B*a*b-45*C*a^2)/a^5*x^4+1/729*(73*A*b^2-1
5*B*a*b-18*C*a^2)/a^2/b^2*x)/(2*b*x+3*a)^3/(-b*x+3*a)-1/59049*(8*A*b^2+15*
B*a*b+18*C*a^2)*ln(-b*x+3*a)/a^5/b^3+1/59049*(8*A*b^2+15*B*a*b+18*C*a^2)*l
n(2*b*x+3*a)/a^5/b^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 480 vs. 2(205) = 410.

Time = 0.11 (sec) , antiderivative size = 480, normalized size of antiderivative = 2.21

$$\int \frac{A+Bx+Cx^2}{\left(27a^3+27a^2bx-4b^3x^3\right)^2}\,dx =$$
$$-\frac{10935\,Ca^6-729\,Ba^5b-8262\,Aa^4b^2+72\,(18\,Ca^3b^3+15\,Ba^2b^4+8\,Aab^5)x^3+162\,(18\,Ca^4b^2+15\,Ba}{}$$

input
```
integrate((C*x^2+B*x+A)/(-4*b^3*x^3+27*a^2*b*x+27*a^3)^2,x, algorithm="fri
cas")
```

output
```
-1/118098*(10935*C*a^6 - 729*B*a^5*b - 8262*A*a^4*b^2 + 72*(18*C*a^3*b^3 +
 15*B*a^2*b^4 + 8*A*a*b^5)*x^3 + 162*(18*C*a^4*b^2 + 15*B*a^3*b^3 + 8*A*a^
2*b^4)*x^2 + 243*(63*C*a^5*b - 15*B*a^4*b^2 - 8*A*a^3*b^3)*x + 2*(1458*C*a
^6 + 1215*B*a^5*b + 648*A*a^4*b^2 - 8*(18*C*a^2*b^4 + 15*B*a*b^5 + 8*A*b^6
)*x^4 - 12*(18*C*a^3*b^3 + 15*B*a^2*b^4 + 8*A*a*b^5)*x^3 + 54*(18*C*a^4*b^
2 + 15*B*a^3*b^3 + 8*A*a^2*b^4)*x^2 + 135*(18*C*a^5*b + 15*B*a^4*b^2 + 8*A
*a^3*b^3)*x)*log(2*b*x + 3*a) - 2*(1458*C*a^6 + 1215*B*a^5*b + 648*A*a^4*b
^2 - 8*(18*C*a^2*b^4 + 15*B*a*b^5 + 8*A*b^6)*x^4 - 12*(18*C*a^3*b^3 + 15*B
*a^2*b^4 + 8*A*a*b^5)*x^3 + 54*(18*C*a^4*b^2 + 15*B*a^3*b^3 + 8*A*a^2*b^4)
*x^2 + 135*(18*C*a^5*b + 15*B*a^4*b^2 + 8*A*a^3*b^3)*x)*log(b*x - 3*a))/(8
*a^5*b^7*x^4 + 12*a^6*b^6*x^3 - 54*a^7*b^5*x^2 - 135*a^8*b^4*x - 81*a^9*b^
3)
```

Sympy [A] (verification not implemented)

Time = 1.28 (sec) , antiderivative size = 382, normalized size of antiderivative = 1.76

$$\int \frac{A + Bx + Cx^2}{(27a^3 + 27a^2bx - 4b^3x^3)^2}\,dx = \frac{918Aa^3b^2 + 81Ba^4b - 1215Ca^5 + x^3(-64Ab^5 - 120Bab^4 - 144Ca^2b^3) + x^2(-144Aab^4 - 270Ba^2b^3 - 3}{-1062882a^8b^3 - 1771470a^7b^4x - 708588a^6b^5x^2 + 157464a^5b^6x^3 +} - \frac{(8Ab^2 + 15Bab + 18Ca^2)\log\left(x + \frac{-24Aab^2 - 45Ba^2b - 54Ca^3 - 9a(8Ab^2 + 15Bab + 18Ca^2)}{32Ab^3 + 60Bab^2 + 72Ca^2b}\right)}{59049a^5b^3} + \frac{(8Ab^2 + 15Bab + 18Ca^2)\log\left(x + \frac{-24Aab^2 - 45Ba^2b - 54Ca^3 + 9a(8Ab^2 + 15Bab + 18Ca^2)}{32Ab^3 + 60Bab^2 + 72Ca^2b}\right)}{59049a^5b^3}$$

input
```
integrate((C*x**2+B*x+A)/(-4*b**3*x**3+27*a**2*b*x+27*a**3)**2,x)
```

output
```
(918*A*a**3*b**2 + 81*B*a**4*b - 1215*C*a**5 + x**3*(-64*A*b**5 - 120*B*a*
b**4 - 144*C*a**2*b**3) + x**2*(-144*A*a*b**4 - 270*B*a**2*b**3 - 324*C*a*
*3*b**2) + x*(216*A*a**2*b**3 + 405*B*a**3*b**2 - 1701*C*a**4*b))/(-106288
2*a**8*b**3 - 1771470*a**7*b**4*x - 708588*a**6*b**5*x**2 + 157464*a**5*b*
*6*x**3 + 104976*a**4*b**7*x**4) - (8*A*b**2 + 15*B*a*b + 18*C*a**2)*log(x
 + (-24*A*a*b**2 - 45*B*a**2*b - 54*C*a**3 - 9*a*(8*A*b**2 + 15*B*a*b + 18
*C*a**2))/(32*A*b**3 + 60*B*a*b**2 + 72*C*a**2*b))/(59049*a**5*b**3) + (8*
A*b**2 + 15*B*a*b + 18*C*a**2)*log(x + (-24*A*a*b**2 - 45*B*a**2*b - 54*C*
a**3 + 9*a*(8*A*b**2 + 15*B*a*b + 18*C*a**2))/(32*A*b**3 + 60*B*a*b**2 + 7
2*C*a**2*b))/(59049*a**5*b**3)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.09

$$\int \frac{A+Bx+Cx^2}{(27a^3+27a^2bx-4b^3x^3)^2}\,dx = -\frac{1215\,Ca^5-81\,Ba^4b-918\,Aa^3b^2+8\,(18\,Ca^2b^3+15\,Bab^4+8\,Ab^5)x^3+18\,(18\,Ca^3b^2+15\,Ba^2b^3+8\,\ldots}{13122\,(8\,a^4b^7x^4+12\,a^5b^6x^3-54\,a^6b^5x^2-135\,a^7b^4x-81\,a\ldots} + \frac{(18\,Ca^2+15\,Bab+8\,Ab^2)\log(2\,bx+3\,a)}{59049\,a^5b^3} - \frac{(18\,Ca^2+15\,Bab+8\,Ab^2)\log(bx-3\,a)}{59049\,a^5b^3}$$

input
```
integrate((C*x^2+B*x+A)/(-4*b^3*x^3+27*a^2*b*x+27*a^3)^2,x, algorithm="max
ima")
```

output
```
-1/13122*(1215*C*a^5 - 81*B*a^4*b - 918*A*a^3*b^2 + 8*(18*C*a^2*b^3 + 15*B
*a*b^4 + 8*A*b^5)*x^3 + 18*(18*C*a^3*b^2 + 15*B*a^2*b^3 + 8*A*a*b^4)*x^2 +
 27*(63*C*a^4*b - 15*B*a^3*b^2 - 8*A*a^2*b^3)*x)/(8*a^4*b^7*x^4 + 12*a^5*b
^6*x^3 - 54*a^6*b^5*x^2 - 135*a^7*b^4*x - 81*a^8*b^3) + 1/59049*(18*C*a^2
+ 15*B*a*b + 8*A*b^2)*log(2*b*x + 3*a)/(a^5*b^3) - 1/59049*(18*C*a^2 + 15*
B*a*b + 8*A*b^2)*log(b*x - 3*a)/(a^5*b^3)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.99

$$\int \frac{A+Bx+Cx^2}{(27a^3+27a^2bx-4b^3x^3)^2}\,dx = \frac{(18\,Ca^2+15\,Bab+8\,Ab^2)\log(|2\,bx+3\,a|)}{59049\,a^5b^3} - \frac{(18\,Ca^2+15\,Bab+8\,Ab^2)\log(|bx-3\,a|)}{59049\,a^5b^3} - \frac{1215\,Ca^6-81\,Ba^5b-918\,Aa^4b^2+8\,(18\,Ca^3b^3+15\,Ba^2b^4+8\,Aab^5)x^3+18\,(18\,Ca^4b^2+15\,Ba^3b^3+}{13122\,(2\,bx+3\,a)^3(bx-3\,a)a^5b^3}$$

input
```
integrate((C*x^2+B*x+A)/(-4*b^3*x^3+27*a^2*b*x+27*a^3)^2,x, algorithm="gia
c")
```

output
```
1/59049*(18*C*a^2 + 15*B*a*b + 8*A*b^2)*log(abs(2*b*x + 3*a))/(a^5*b^3) -
1/59049*(18*C*a^2 + 15*B*a*b + 8*A*b^2)*log(abs(b*x - 3*a))/(a^5*b^3) - 1/
13122*(1215*C*a^6 - 81*B*a^5*b - 918*A*a^4*b^2 + 8*(18*C*a^3*b^3 + 15*B*a^
2*b^4 + 8*A*a*b^5)*x^3 + 18*(18*C*a^4*b^2 + 15*B*a^3*b^3 + 8*A*a^2*b^4)*x^
2 + 27*(63*C*a^5*b - 15*B*a^4*b^2 - 8*A*a^3*b^3)*x)/((2*b*x + 3*a)^3*(b*x
- 3*a)*a^5*b^3)
```

Mupad [B] (verification not implemented)

Time = 12.03 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.88

$$\int \frac{A+Bx+Cx^2}{(27a^3+27a^2bx-4b^3x^3)^2}\,dx = \frac{2\,\mathrm{atanh}\left(\frac{4\,b\,x}{9\,a}-\frac{1}{3}\right)\,(18\,C\,a^2+15\,B\,a\,b+8\,A\,b^2)}{59049\,a^5\,b^3} - \frac{\frac{-45\,C\,a^2+3\,B\,a\,b+34\,A\,b^2}{486\,a\,b^3}-\frac{4\,x^3\,(18\,C\,a^2+15\,B\,a\,b+8\,A\,b^2)}{6561\,a^4}-\frac{x^2\,(18\,C\,a^2+15\,B\,a\,b+8\,A\,b^2)}{729\,a^3\,b}+\frac{x\,(-63\,C\,a^2+15\,B\,a\,b+8\,A\,b^2)}{486\,a^2\,b^2}}{81\,a^4+135\,a^3\,b\,x+54\,a^2\,b^2\,x^2-12\,a\,b^3\,x^3-8\,b^4\,x^4}$$

input
```
int((A + B*x + C*x^2)/(27*a^3 - 4*b^3*x^3 + 27*a^2*b*x)^2,x)
```

output
```
(2*atanh((4*b*x)/(9*a) - 1/3)*(8*A*b^2 + 18*C*a^2 + 15*B*a*b))/(59049*a^5*
b^3) - ((34*A*b^2 - 45*C*a^2 + 3*B*a*b)/(486*a*b^3) - (4*x^3*(8*A*b^2 + 18
*C*a^2 + 15*B*a*b))/(6561*a^4) - (x^2*(8*A*b^2 + 18*C*a^2 + 15*B*a*b))/(72
9*a^3*b) + (x*(8*A*b^2 - 63*C*a^2 + 15*B*a*b))/(486*a^2*b^2))/(81*a^4 - 8*
b^4*x^4 - 12*a*b^3*x^3 + 54*a^2*b^2*x^2 + 135*a^3*b*x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 490, normalized size of antiderivative = 2.26

$$\int \frac{A+Bx+Cx^2}{(27a^3+27a^2bx-4b^3x^3)^2}\,dx$$
$$= \frac{-864a\,b^4c\,x^4-6210\log(-bx+3a)\,a^3b^3x-2484\log(-bx+3a)\,a^2b^4x^2+552\log(-bx+3a)\,a\,b^5x^3+62}{}$$

input
```
int((C*x^2+B*x+A)/(-4*b^3*x^3+27*a^2*b*x+27*a^3)^2,x)
```

output
```
( - 2916*log(3*a - b*x)*a**5*c - 3726*log(3*a - b*x)*a**4*b**2 - 4860*log(
3*a - b*x)*a**4*b*c*x - 6210*log(3*a - b*x)*a**3*b**3*x - 1944*log(3*a - b
*x)*a**3*b**2*c*x**2 - 2484*log(3*a - b*x)*a**2*b**4*x**2 + 432*log(3*a -
b*x)*a**2*b**3*c*x**3 + 552*log(3*a - b*x)*a*b**5*x**3 + 288*log(3*a - b*x
)*a*b**4*c*x**4 + 368*log(3*a - b*x)*b**6*x**4 + 2916*log(3*a + 2*b*x)*a**
5*c + 3726*log(3*a + 2*b*x)*a**4*b**2 + 4860*log(3*a + 2*b*x)*a**4*b*c*x +
 6210*log(3*a + 2*b*x)*a**3*b**3*x + 1944*log(3*a + 2*b*x)*a**3*b**2*c*x**
2 + 2484*log(3*a + 2*b*x)*a**2*b**4*x**2 - 432*log(3*a + 2*b*x)*a**2*b**3*
c*x**3 - 552*log(3*a + 2*b*x)*a*b**5*x**3 - 288*log(3*a + 2*b*x)*a*b**4*c*
x**4 - 368*log(3*a + 2*b*x)*b**6*x**4 + 19683*a**5*c + 2187*a**4*b**2 + 29
889*a**4*b*c*x + 13041*a**3*b**3*x + 8748*a**3*b**2*c*x**2 + 11178*a**2*b*
*4*x**2 - 864*a*b**4*c*x**4 - 1104*b**6*x**4)/(118098*a**4*b**3*(81*a**4 +
 135*a**3*b*x + 54*a**2*b**2*x**2 - 12*a*b**3*x**3 - 8*b**4*x**4))
```

3.6 $\int \frac{A+Bx+Cx^2}{\left(27a^3+27a^2bx-4b^3x^3\right)^3}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 335

$$\begin{aligned}\int \frac{A+Bx+Cx^2}{\left(27a^3+27a^2bx-4b^3x^3\right)^3}\,dx &= \frac{Ab^2+3a(bB+3aC)}{1062882a^6b^3(3a-bx)^2}+\frac{4Ab^2+9a(bB+2aC)}{1594323a^7b^3(3a-bx)}\\&\quad-\frac{4Ab^2-6abB+9a^2C}{3645a^3b^3(3a+2bx)^5}-\frac{4Ab^2-9a^2C}{8748a^4b^3(3a+2bx)^4}\\&\quad-\frac{8Ab^2+6abB-9a^2C}{59049a^5b^3(3a+2bx)^3}-\frac{40Ab^2+48abB+9a^2C}{1062882a^6b^3(3a+2bx)^2}\\&\quad-\frac{20Ab^2+30abB+27a^2C}{1594323a^7b^3(3a+2bx)}\\&\quad-\frac{(28Ab^2+48abB+63a^2C)\log(3a-bx)}{14348907a^8b^3}\\&\quad+\frac{(28Ab^2+48abB+63a^2C)\log(3a+2bx)}{14348907a^8b^3}\end{aligned}$$

output
```
1/1062882*(A*b^2+3*a*(B*b+3*C*a))/a^6/b^3/(-b*x+3*a)^2+1/1594323*(4*A*b^2+
9*a*(B*b+2*C*a))/a^7/b^3/(-b*x+3*a)-1/3645*(4*A*b^2-6*B*a*b+9*C*a^2)/a^3/b
^3/(2*b*x+3*a)^5-1/8748*(4*A*b^2-9*C*a^2)/a^4/b^3/(2*b*x+3*a)^4-1/59049*(8
*A*b^2+6*B*a*b-9*C*a^2)/a^5/b^3/(2*b*x+3*a)^3-1/1062882*(40*A*b^2+48*B*a*b
+9*C*a^2)/a^6/b^3/(2*b*x+3*a)^2-1/1594323*(20*A*b^2+30*B*a*b+27*C*a^2)/a^7
/b^3/(2*b*x+3*a)-1/14348907*(28*A*b^2+48*B*a*b+63*C*a^2)*ln(-b*x+3*a)/a^8/
b^3+1/14348907*(28*A*b^2+48*B*a*b+63*C*a^2)*ln(2*b*x+3*a)/a^8/b^3
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 290, normalized size of antiderivative = 0.87

$$\int \frac{A+Bx+Cx^2}{(27a^3+27a^2bx-4b^3x^3)^3}\,dx$$
$$= \frac{\frac{180a(4Ab^2+9a(bB+2aC))}{3a-bx}+\frac{270a^2(Ab^2+3a(bB+3aC))}{(-3a+bx)^2}-\frac{78732a^5(4Ab^2-6abB+9a^2C)}{(3a+2bx)^5}+\frac{32805a^4(-4Ab^2+9a^2C)}{(3a+2bx)^4}+\frac{4860a^3(-8Ab^2}{(3a+}}{}$$

input
```
Integrate[(A + B*x + C*x^2)/(27*a^3 + 27*a^2*b*x - 4*b^3*x^3)^3,x]
```

output
```
((180*a*(4*A*b^2 + 9*a*(b*B + 2*a*C)))/(3*a - b*x) + (270*a^2*(A*b^2 + 3*a
*(b*B + 3*a*C)))/(-3*a + b*x)^2 - (78732*a^5*(4*A*b^2 - 6*a*b*B + 9*a^2*C)
)/(3*a + 2*b*x)^5 + (32805*a^4*(-4*A*b^2 + 9*a^2*C))/(3*a + 2*b*x)^4 + (48
60*a^3*(-8*A*b^2 - 6*a*b*B + 9*a^2*C))/(3*a + 2*b*x)^3 - (270*a^2*(40*A*b^
2 + 48*a*b*B + 9*a^2*C))/(3*a + 2*b*x)^2 - (180*a*(20*A*b^2 + 3*a*(10*b*B
+ 9*a*C)))/(3*a + 2*b*x) - 20*(28*A*b^2 + 48*a*b*B + 63*a^2*C)*Log[3*a - b
*x] + 20*(28*A*b^2 + 48*a*b*B + 63*a^2*C)*Log[3*a + 2*b*x])/(286978140*a^8
*b^3)
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 335, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {2462, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{(27a^3+27a^2bx-4b^3x^3)^3}\,dx$$
$$\Big\downarrow \; 2462$$
$$\int \left(\frac{9a(2aC+bB)+4Ab^2}{1594323a^7b^2(3a-bx)^2}+\frac{3a(3aC+bB)+Ab^2}{531441a^6b^2(3a-bx)^3}+\frac{63a^2C+48abB+28Ab^2}{14348907a^8b^2(3a-bx)}+\frac{2(63a^2C+48abB+28Ab^2}{14348907a^8b^2(3a+2bx)}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{9a(2aC+bB)+4Ab^2}{1594323a^7b^3(3a-bx)}+\frac{3a(3aC+bB)+Ab^2}{1062882a^6b^3(3a-bx)^2}-\frac{\log(3a-bx)\left(63a^2C+48abB+28Ab^2\right)}{14348907a^8b^3}+\frac{\log(3a+2bx)\left(63a^2C+48abB+28Ab^2\right)}{14348907a^8b^3}-\frac{27a^2C+30abB+20Ab^2}{1594323a^7b^3(3a+2bx)}-\frac{9a^2C+48abB+40Ab^2}{1062882a^6b^3(3a+2bx)^2}-\frac{-9a^2C+6abB+8Ab^2}{59049a^5b^3(3a+2bx)^3}-\frac{4Ab^2-9a^2C}{8748a^4b^3(3a+2bx)^4}-\frac{9a^2C-6abB+4Ab^2}{3645a^3b^3(3a+2bx)^5}$$

input
```
Int[(A + B*x + C*x^2)/(27*a^3 + 27*a^2*b*x - 4*b^3*x^3)^3,x]
```

output
```
(A*b^2 + 3*a*(b*B + 3*a*C))/(1062882*a^6*b^3*(3*a - b*x)^2) + (4*A*b^2 + 9
*a*(b*B + 2*a*C))/(1594323*a^7*b^3*(3*a - b*x)) - (4*A*b^2 - 6*a*b*B + 9*a
^2*C)/(3645*a^3*b^3*(3*a + 2*b*x)^5) - (4*A*b^2 - 9*a^2*C)/(8748*a^4*b^3*(
3*a + 2*b*x)^4) - (8*A*b^2 + 6*a*b*B - 9*a^2*C)/(59049*a^5*b^3*(3*a + 2*b*
x)^3) - (40*A*b^2 + 48*a*b*B + 9*a^2*C)/(1062882*a^6*b^3*(3*a + 2*b*x)^2)
- (20*A*b^2 + 30*a*b*B + 27*a^2*C)/(1594323*a^7*b^3*(3*a + 2*b*x)) - ((28*
A*b^2 + 48*a*b*B + 63*a^2*C)*Log[3*a - b*x])/(14348907*a^8*b^3) + ((28*A*b
^2 + 48*a*b*B + 63*a^2*C)*Log[3*a + 2*b*x])/(14348907*a^8*b^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 290, normalized size of antiderivative = 0.87

method	result
norman	$\frac{\frac{(187A\,b^2+147abB+117C\,a^2)x^3}{6561a^4}+\frac{(1561A\,b^2+489abB-315C\,a^2)x^2}{13122b\,a^3}-\frac{20b(191A\,b^2+15abB-117C\,a^2)x^4}{177147a^5}-\frac{8b^2(1151A\,b^2+411abB-144C}{885735a^6}}{(2bx+3a)^5(-bx+3a)}$
default	$\frac{4A\,b^2+9abB+18C\,a^2}{1594323b^3a^7(-bx+3a)}+\frac{(-28A\,b^2-48abB-63C\,a^2)\ln(-bx+3a)}{14348907a^8b^3}-\frac{-A\,b^2-3abB-9C\,a^2}{1062882a^6b^3(-bx+3a)^2}-\frac{8A\,b^2-18C\,a^2}{17496a^4b^3(2bx+3a)^4}$
risch	$\frac{-\frac{16b^3(28A\,b^2+48abB+63C\,a^2)x^6}{1594323a^7}-\frac{4(28A\,b^2+48abB+63C\,a^2)b^2x^5}{177147a^6}+\frac{2(28A\,b^2+48abB+63C\,a^2)b\,x^4}{19683a^5}+\frac{43(28A\,b^2+48abB+63C\,a^2)x^3}{118098a^4}}{(2bx+3a)(-4b^3x^3+27b\,a^2x+27a^3)^2}$
parallelrisch	Expression too large to display

input
```
int((C*x^2+B*x+A)/(-4*b^3*x^3+27*a^2*b*x+27*a^3)^3,x,method=_RETURNVERBOSE
)
```

output
```
(1/6561*(187*A*b^2+147*B*a*b+117*C*a^2)/a^4*x^3+1/13122/b*(1561*A*b^2+489*
B*a*b-315*C*a^2)/a^3*x^2-20/177147*b*(191*A*b^2+15*B*a*b-117*C*a^2)/a^5*x^
4-8/885735*b^2*(1151*A*b^2+411*B*a*b-144*C*a^2)/a^6*x^5+4/885735*b^3*(178*
A*b^2-42*B*a*b-207*C*a^2)/a^7*x^6+8/23914845*b^4*(2162*A*b^2+582*B*a*b-603
*C*a^2)/a^8*x^7+1/6561*(701*A*b^2-48*B*a*b-63*C*a^2)/a^2/b^2*x)/(2*b*x+3*a
)^5/(-b*x+3*a)^2-1/14348907*(28*A*b^2+48*B*a*b+63*C*a^2)*ln(-b*x+3*a)/a^8/
b^3+1/14348907*(28*A*b^2+48*B*a*b+63*C*a^2)*ln(2*b*x+3*a)/a^8/b^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 810 vs. $2(315) = 630$.

Time = 0.10 (sec) , antiderivative size = 810, normalized size of antiderivative = 2.42

$$\int \frac{A+Bx+Cx^2}{(27a^3+27a^2bx-4b^3x^3)^3}\,dx = \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)/(-4*b^3*x^3+27*a^2*b*x+27*a^3)^3,x, algorithm="fri
cas")
```

output
```
1/286978140*(3956283*C*a^9 - 3818502*B*a^8*b - 14184882*A*a^7*b^2 - 2880*(
63*C*a^3*b^6 + 48*B*a^2*b^7 + 28*A*a*b^8)*x^6 - 6480*(63*C*a^4*b^5 + 48*B*
a^3*b^6 + 28*A*a^2*b^7)*x^5 + 29160*(63*C*a^5*b^4 + 48*B*a^4*b^5 + 28*A*a^
3*b^6)*x^4 + 104490*(63*C*a^6*b^3 + 48*B*a^5*b^4 + 28*A*a^4*b^5)*x^3 + 371
79*(63*C*a^7*b^2 + 48*B*a^6*b^3 + 28*A*a^5*b^4)*x^2 + 78732*(99*C*a^8*b -
156*B*a^7*b^2 - 91*A*a^6*b^3)*x + 20*(137781*C*a^9 + 104976*B*a^8*b + 6123
6*A*a^7*b^2 + 32*(63*C*a^2*b^7 + 48*B*a*b^8 + 28*A*b^9)*x^7 + 48*(63*C*a^3
*b^6 + 48*B*a^2*b^7 + 28*A*a*b^8)*x^6 - 432*(63*C*a^4*b^5 + 48*B*a^3*b^6 +
 28*A*a^2*b^7)*x^5 - 1080*(63*C*a^5*b^4 + 48*B*a^4*b^5 + 28*A*a^3*b^6)*x^4
 + 810*(63*C*a^6*b^3 + 48*B*a^5*b^4 + 28*A*a^4*b^5)*x^3 + 5103*(63*C*a^7*b
^2 + 48*B*a^6*b^3 + 28*A*a^5*b^4)*x^2 + 5832*(63*C*a^8*b + 48*B*a^7*b^2 +
28*A*a^6*b^3)*x)*log(2*b*x + 3*a) - 20*(137781*C*a^9 + 104976*B*a^8*b + 61
236*A*a^7*b^2 + 32*(63*C*a^2*b^7 + 48*B*a*b^8 + 28*A*b^9)*x^7 + 48*(63*C*a
^3*b^6 + 48*B*a^2*b^7 + 28*A*a*b^8)*x^6 - 432*(63*C*a^4*b^5 + 48*B*a^3*b^6
 + 28*A*a^2*b^7)*x^5 - 1080*(63*C*a^5*b^4 + 48*B*a^4*b^5 + 28*A*a^3*b^6)*x
^4 + 810*(63*C*a^6*b^3 + 48*B*a^5*b^4 + 28*A*a^4*b^5)*x^3 + 5103*(63*C*a^7
*b^2 + 48*B*a^6*b^3 + 28*A*a^5*b^4)*x^2 + 5832*(63*C*a^8*b + 48*B*a^7*b^2
+ 28*A*a^6*b^3)*x)*log(b*x - 3*a))/(32*a^8*b^10*x^7 + 48*a^9*b^9*x^6 - 432
*a^10*b^8*x^5 - 1080*a^11*b^7*x^4 + 810*a^12*b^6*x^3 + 5103*a^13*b^5*x^2 +
 5832*a^14*b^4*x + 2187*a^15*b^3)
```

Sympy [A] (verification not implemented)

Time = 3.26 (sec) , antiderivative size = 522, normalized size of antiderivative = 1.56

$$\int \frac{A+Bx+Cx^2}{(27a^3+27a^2bx-4b^3x^3)^3}\,dx = -\frac{1576098Aa^6b^2+424278Ba^7b-439587Ca^8+x^6\cdot(8960Ab^8+15360Bab^7+20160Ca^2b^6)+x^5\cdot(2016\ldots}{69735688020a\ldots} - \frac{(28Ab^2+48Bab+63Ca^2)\log\left(x+\frac{-84Aab^2-144Ba^2b-189Ca^3-9a(28Ab^2+48Bab+63Ca^2)}{112Ab^3+192Bab^2+252Ca^2b}\right)}{14348907a^8b^3} + \frac{(28Ab^2+48Bab+63Ca^2)\log\left(x+\frac{-84Aab^2-144Ba^2b-189Ca^3+9a(28Ab^2+48Bab+63Ca^2)}{112Ab^3+192Bab^2+252Ca^2b}\right)}{14348907a^8b^3}$$

input
```
integrate((C*x**2+B*x+A)/(-4*b**3*x**3+27*a**2*b*x+27*a**3)**3,x)
```

output
```
-(1576098*A*a**6*b**2 + 424278*B*a**7*b - 439587*C*a**8 + x**6*(8960*A*b**
8 + 15360*B*a*b**7 + 20160*C*a**2*b**6) + x**5*(20160*A*a*b**7 + 34560*B*a
**2*b**6 + 45360*C*a**3*b**5) + x**4*(-90720*A*a**2*b**6 - 155520*B*a**3*b
**5 - 204120*C*a**4*b**4) + x**3*(-325080*A*a**3*b**5 - 557280*B*a**4*b**4
 - 731430*C*a**5*b**3) + x**2*(-115668*A*a**4*b**4 - 198288*B*a**5*b**3 -
260253*C*a**6*b**2) + x*(796068*A*a**5*b**3 + 1364688*B*a**6*b**2 - 866052
*C*a**7*b))/(69735688020*a**14*b**3 + 185961834720*a**13*b**4*x + 16271660
5380*a**12*b**5*x**2 + 25828032600*a**11*b**6*x**3 - 34437376800*a**10*b**
7*x**4 - 13774950720*a**9*b**8*x**5 + 1530550080*a**8*b**9*x**6 + 10203667
20*a**7*b**10*x**7) - (28*A*b**2 + 48*B*a*b + 63*C*a**2)*log(x + (-84*A*a*
b**2 - 144*B*a**2*b - 189*C*a**3 - 9*a*(28*A*b**2 + 48*B*a*b + 63*C*a**2))
/(112*A*b**3 + 192*B*a*b**2 + 252*C*a**2*b))/(14348907*a**8*b**3) + (28*A*
b**2 + 48*B*a*b + 63*C*a**2)*log(x + (-84*A*a*b**2 - 144*B*a**2*b - 189*C*
a**3 + 9*a*(28*A*b**2 + 48*B*a*b + 63*C*a**2))/(112*A*b**3 + 192*B*a*b**2
+ 252*C*a**2*b))/(14348907*a**8*b**3)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 368, normalized size of antiderivative = 1.10

$$\int \frac{A+Bx+Cx^2}{\left(27a^3+27a^2bx-4b^3x^3\right)^3}\,dx = \frac{439587\,Ca^8 - 424278\,Ba^7b - 1576098\,Aa^6b^2 - 320\,(63\,Ca^2b^6 + 48\,Bab^7 + 28\,Ab^8)x^6 - 720\,(63\,Ca^3b^5 + \cdots}{31886460\,(\cdots} + \frac{(63\,Ca^2 + 48\,Bab + 28\,Ab^2)\log\left(2\,bx + 3\,a\right)}{14348907\,a^8b^3} - \frac{(63\,Ca^2 + 48\,Bab + 28\,Ab^2)\log\left(bx - 3\,a\right)}{14348907\,a^8b^3}$$

input
```
integrate((C*x^2+B*x+A)/(-4*b^3*x^3+27*a^2*b*x+27*a^3)^3,x, algorithm="max
ima")
```

output
```
1/31886460*(439587*C*a^8 - 424278*B*a^7*b - 1576098*A*a^6*b^2 - 320*(63*C*
a^2*b^6 + 48*B*a*b^7 + 28*A*b^8)*x^6 - 720*(63*C*a^3*b^5 + 48*B*a^2*b^6 +
28*A*a*b^7)*x^5 + 3240*(63*C*a^4*b^4 + 48*B*a^3*b^5 + 28*A*a^2*b^6)*x^4 +
11610*(63*C*a^5*b^3 + 48*B*a^4*b^4 + 28*A*a^3*b^5)*x^3 + 4131*(63*C*a^6*b^
2 + 48*B*a^5*b^3 + 28*A*a^4*b^4)*x^2 + 8748*(99*C*a^7*b - 156*B*a^6*b^2 -
91*A*a^5*b^3)*x)/(32*a^7*b^10*x^7 + 48*a^8*b^9*x^6 - 432*a^9*b^8*x^5 - 108
0*a^10*b^7*x^4 + 810*a^11*b^6*x^3 + 5103*a^12*b^5*x^2 + 5832*a^13*b^4*x +
2187*a^14*b^3) + 1/14348907*(63*C*a^2 + 48*B*a*b + 28*A*b^2)*log(2*b*x + 3
*a)/(a^8*b^3) - 1/14348907*(63*C*a^2 + 48*B*a*b + 28*A*b^2)*log(b*x - 3*a)
/(a^8*b^3)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 314, normalized size of antiderivative = 0.94

$$\int \frac{A+Bx+Cx^2}{(27a^3+27a^2bx-4b^3x^3)^3}\,dx = \frac{(63\,Ca^2+48\,Bab+28\,Ab^2)\log(|2\,bx+3\,a|)}{14348907\,a^8b^3} - \frac{(63\,Ca^2+48\,Bab+28\,Ab^2)\log(|bx-3\,a|)}{14348907\,a^8b^3} + \frac{439587\,Ca^9-424278\,Ba^8b-1576098\,Aa^7b^2-320\,(63\,Ca^3b^6+48\,Ba^2b^7+28\,Aab^8)x^6-720\,(63\,Ca^4}{}$$

input
```
integrate((C*x^2+B*x+A)/(-4*b^3*x^3+27*a^2*b*x+27*a^3)^3,x, algorithm="gia
c")
```

output
```
1/14348907*(63*C*a^2 + 48*B*a*b + 28*A*b^2)*log(abs(2*b*x + 3*a))/(a^8*b^3
) - 1/14348907*(63*C*a^2 + 48*B*a*b + 28*A*b^2)*log(abs(b*x - 3*a))/(a^8*b
^3) + 1/31886460*(439587*C*a^9 - 424278*B*a^8*b - 1576098*A*a^7*b^2 - 320*
(63*C*a^3*b^6 + 48*B*a^2*b^7 + 28*A*a*b^8)*x^6 - 720*(63*C*a^4*b^5 + 48*B*
a^3*b^6 + 28*A*a^2*b^7)*x^5 + 3240*(63*C*a^5*b^4 + 48*B*a^4*b^5 + 28*A*a^3
*b^6)*x^4 + 11610*(63*C*a^6*b^3 + 48*B*a^5*b^4 + 28*A*a^4*b^5)*x^3 + 4131*
(63*C*a^7*b^2 + 48*B*a^6*b^3 + 28*A*a^5*b^4)*x^2 + 8748*(99*C*a^8*b - 156*
B*a^7*b^2 - 91*A*a^6*b^3)*x)/((2*b*x + 3*a)^5*(b*x - 3*a)^2*a^8*b^3)
```

Mupad [B] (verification not implemented)

Time = 12.08 (sec) , antiderivative size = 309, normalized size of antiderivative = 0.92

$$\int \frac{A+Bx+Cx^2}{(27a^3+27a^2bx-4b^3x^3)^3}\,dx = \frac{2\,\mathrm{atanh}\left(\frac{4\,b\,x}{9\,a}-\frac{1}{3}\right)\,\left(63\,C\,a^2+48\,B\,a\,b+28\,A\,b^2\right)}{14348907\,a^8\,b^3}$$
$$-\frac{\frac{-603\,C\,a^2+582\,B\,a\,b+2162\,A\,b^2}{43740\,a\,b^3}-\frac{43\,x^3\,\left(63\,C\,a^2+48\,B\,a\,b+28\,A\,b^2\right)}{118098\,a^4}-\frac{17\,x^2\,\left(63\,C\,a^2+48\,B\,a\,b+28\,A\,b^2\right)}{131220\,a^3\,b}+\frac{4\,b^2\,x^5\,\left(63\,C\,a^2+48\,B\,a\,b\right.}{177147\,a^6}}{2187\,a^7+5832\,a^6\,b\,x+5103\,a^5\,b^2\,x^2+810\,a^4\,b^3\,x^3-108}$$

input
```
int((A + B*x + C*x^2)/(27*a^3 - 4*b^3*x^3 + 27*a^2*b*x)^3,x)
```

output
```
(2*atanh((4*b*x)/(9*a) - 1/3)*(28*A*b^2 + 63*C*a^2 + 48*B*a*b))/(14348907*
a^8*b^3) - ((2162*A*b^2 - 603*C*a^2 + 582*B*a*b)/(43740*a*b^3) - (43*x^3*(
28*A*b^2 + 63*C*a^2 + 48*B*a*b))/(118098*a^4) - (17*x^2*(28*A*b^2 + 63*C*a
^2 + 48*B*a*b))/(131220*a^3*b) + (4*b^2*x^5*(28*A*b^2 + 63*C*a^2 + 48*B*a*
b))/(177147*a^6) + (16*b^3*x^6*(28*A*b^2 + 63*C*a^2 + 48*B*a*b))/(1594323*
a^7) - (2*b*x^4*(28*A*b^2 + 63*C*a^2 + 48*B*a*b))/(19683*a^5) + (x*(91*A*b
^2 - 99*C*a^2 + 156*B*a*b))/(3645*a^2*b^2))/(2187*a^7 + 32*b^7*x^7 + 48*a*
b^6*x^6 + 5103*a^5*b^2*x^2 + 810*a^4*b^3*x^3 - 1080*a^3*b^4*x^4 - 432*a^2*
b^5*x^5 + 5832*a^6*b*x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 838, normalized size of antiderivative = 2.50

$$\int \frac{A+Bx+Cx^2}{(27a^3+27a^2bx-4b^3x^3)^3}\,dx = \text{Too large to display}$$

input
```
int((C*x^2+B*x+A)/(-4*b^3*x^3+27*a^2*b*x+27*a^3)^3,x)
```

output
```
( - 2755620*log(3*a - b*x)*a**8*c - 3324240*log(3*a - b*x)*a**7*b**2 - 734
8320*log(3*a - b*x)*a**7*b*c*x - 8864640*log(3*a - b*x)*a**6*b**3*x - 6429
780*log(3*a - b*x)*a**6*b**2*c*x**2 - 7756560*log(3*a - b*x)*a**5*b**4*x**
2 - 1020600*log(3*a - b*x)*a**5*b**3*c*x**3 - 1231200*log(3*a - b*x)*a**4*
b**5*x**3 + 1360800*log(3*a - b*x)*a**4*b**4*c*x**4 + 1641600*log(3*a - b*
x)*a**3*b**6*x**4 + 544320*log(3*a - b*x)*a**3*b**5*c*x**5 + 656640*log(3*
a - b*x)*a**2*b**7*x**5 - 60480*log(3*a - b*x)*a**2*b**6*c*x**6 - 72960*lo
g(3*a - b*x)*a*b**8*x**6 - 40320*log(3*a - b*x)*a*b**7*c*x**7 - 48640*log(
3*a - b*x)*b**9*x**7 + 2755620*log(3*a + 2*b*x)*a**8*c + 3324240*log(3*a +
 2*b*x)*a**7*b**2 + 7348320*log(3*a + 2*b*x)*a**7*b*c*x + 8864640*log(3*a
+ 2*b*x)*a**6*b**3*x + 6429780*log(3*a + 2*b*x)*a**6*b**2*c*x**2 + 7756560
*log(3*a + 2*b*x)*a**5*b**4*x**2 + 1020600*log(3*a + 2*b*x)*a**5*b**3*c*x*
*3 + 1231200*log(3*a + 2*b*x)*a**4*b**5*x**3 - 1360800*log(3*a + 2*b*x)*a*
*4*b**4*c*x**4 - 1641600*log(3*a + 2*b*x)*a**3*b**6*x**4 - 544320*log(3*a
+ 2*b*x)*a**3*b**5*c*x**5 - 656640*log(3*a + 2*b*x)*a**2*b**7*x**5 + 60480
*log(3*a + 2*b*x)*a**2*b**6*c*x**6 + 72960*log(3*a + 2*b*x)*a*b**8*x**6 +
40320*log(3*a + 2*b*x)*a*b**7*c*x**7 + 48640*log(3*a + 2*b*x)*b**9*x**7 +
12223143*a**8*c - 8030664*a**7*b**2 + 29839428*a**7*b*c*x + 7147116*a**6*b
**3*x + 21631617*a**6*b**2*c*x**2 + 26095284*a**5*b**4*x**2 + 9644670*a**5
*b**3*c*x**3 + 11634840*a**4*b**5*x**3 - 2245320*a**4*b**4*c*x**4 - 270...
```

3.7 $\int \frac{A+Bx+Cx^2}{2\sqrt{3}b^{3/2}-9bx+9x^3}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 136

$$\int \frac{A+Bx+Cx^2}{2\sqrt{3}b^{3/2}-9bx+9x^3}\,dx=\frac{3\sqrt{b}B+\sqrt{3}(3A+bC)}{27\sqrt{b}\left(\sqrt{3}\sqrt{b}-3x\right)}-\frac{\left(3A-2\sqrt{3}\sqrt{b}B-5bC\right)\log\left(\sqrt{b}-\sqrt{3}x\right)}{81b}+\frac{\left(3A-2\sqrt{3}\sqrt{b}B+4bC\right)\log\left(2\sqrt{b}+\sqrt{3}x\right)}{81b}$$

output
```
1/27*(3*b^(1/2)*B+3^(1/2)*(C*b+3*A))/b^(1/2)/(3^(1/2)*b^(1/2)-3*x)-1/81*(3
*A-2*3^(1/2)*b^(1/2)*B-5*C*b)*ln(b^(1/2)-x*3^(1/2))/b+1/81*(3*A-2*3^(1/2)*
b^(1/2)*B+4*C*b)*ln(2*b^(1/2)+x*3^(1/2))/b
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.71

$$\int \frac{A+Bx+Cx^2}{2\sqrt{3}b^{3/2}-9bx+9x^3}\,dx = \frac{\left(-\sqrt{3}\sqrt{b}+3x\right)\left(2\sqrt{3}\sqrt{b}+3x\right)\left(-3\left(3A\sqrt{b}+\sqrt{3}bB+b^{3/2}C\right)+\left(-5b^{3/2}C\right.\right.}{}$$

input
```
Integrate[(A + B*x + C*x^2)/(2*Sqrt[3]*b^(3/2) - 9*b*x + 9*x^3),x]
```

output
```
((-(Sqrt[3]*Sqrt[b]) + 3*x)*(2*Sqrt[3]*Sqrt[b] + 3*x)*(-3*(3*A*Sqrt[b] + S
qrt[3]*b*B + b^(3/2)*C) + (-5*b^(3/2)*C + 6*Sqrt[b]*B*x + A*(3*Sqrt[b] - 3
*Sqrt[3]*x) + Sqrt[3]*b*(-2*B + 5*C*x))*Log[-(Sqrt[3]*Sqrt[b]) + 3*x] + (-
4*b^(3/2)*C - 6*Sqrt[b]*B*x + 3*A*(-Sqrt[b] + Sqrt[3]*x) + 2*Sqrt[3]*b*(B
+ 2*C*x))*Log[2*Sqrt[3]*Sqrt[b] + 3*x]))/(81*Sqrt[3]*b*(2*Sqrt[3]*b^(3/2)
- 9*b*x + 9*x^3))
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.26, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {2525, 27, 2482, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{2\sqrt{3}b^{3/2}-9bx+9x^3}\,dx$$

$$\Big\downarrow \text{2525}$$

$$\frac{1}{27}\int \frac{9(3A+bC+3Bx)}{9x^3-9bx+2\sqrt{3}b^{3/2}}dx+\frac{1}{27}C\log\left(2\sqrt{3}b^{3/2}-9bx+9x^3\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\int \frac{3A+bC+3Bx}{9x^3-9bx+2\sqrt{3}b^{3/2}}dx+\frac{1}{27}C\log\left(2\sqrt{3}b^{3/2}-9bx+9x^3\right)$$

$$\Big\downarrow \text{2482}$$

$$108b^3\int \frac{3A+bC+3Bx}{108\sqrt{3}b^3\left(\sqrt{3}\sqrt{b}-3x\right)^2\left(\sqrt{3}x+2\sqrt{b}\right)}dx+\frac{1}{27}C\log\left(2\sqrt{3}b^{3/2}-9bx+9x^3\right)$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{3A+bC+3Bx}{\left(\sqrt{3}\sqrt{b}-3x\right)^2\left(\sqrt{3}x+2\sqrt{b}\right)}dx}{\sqrt{3}}+\frac{1}{27}C\log\left(2\sqrt{3}b^{3/2}-9bx+9x^3\right)$$

$$\Big\downarrow \ 86$$

$$\frac{\int \left(\frac{3A-2\sqrt{3}\sqrt{b}B+bC}{27b\left(\sqrt{3}x+2\sqrt{b}\right)}+\frac{3\sqrt{3}A-6\sqrt{b}B+\sqrt{3}bC}{27b\left(\sqrt{3}\sqrt{b}-3x\right)}+\frac{3A+\sqrt{3}\sqrt{b}B+bC}{3\sqrt{b}\left(\sqrt{3}\sqrt{b}-3x\right)^2}\right)dx}{\sqrt{3}}+\frac{1}{27}C\log\left(2\sqrt{3}b^{3/2}-9bx+9x^3\right)$$

$$\Big\downarrow \ 2009$$

$$\frac{\frac{3A+\sqrt{3}\sqrt{b}B+bC}{9\sqrt{b}\left(\sqrt{3}\sqrt{b}-3x\right)}+\frac{\log\left(\sqrt{b}-\sqrt{3}x\right)\left(6\sqrt{b}B-\sqrt{3}(3A+bC)\right)}{81b}-\frac{\log\left(2\sqrt{b}+\sqrt{3}x\right)\left(6\sqrt{b}B-\sqrt{3}(3A+bC)\right)}{81b}}{\sqrt{3}}+\frac{1}{27}C\log\left(2\sqrt{3}b^{3/2}-9bx+9x^3\right)$$

input
```
Int[(A + B*x + C*x^2)/(2*Sqrt[3]*b^(3/2) - 9*b*x + 9*x^3),x]
```

output
```
((3*A + Sqrt[3]*Sqrt[b]*B + b*C)/(9*Sqrt[b]*(Sqrt[3]*Sqrt[b] - 3*x)) + ((6
*Sqrt[b]*B - Sqrt[3]*(3*A + b*C))*Log[Sqrt[b] - Sqrt[3]*x])/(81*b) - ((6*S
qrt[b]*B - Sqrt[3]*(3*A + b*C))*Log[2*Sqrt[b] + Sqrt[3]*x])/(81*b))/Sqrt[3
] + (C*Log[2*Sqrt[3]*b^(3/2) - 9*b*x + 9*x^3])/27
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2482
```
Int[((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (d_.)*(x_)^3)^(p_.), x_
Symbol] :> Simp[1/(3^(3*p)*a^(2*p))   Int[(e + f*x)^m*(3*a - b*x)^p*(3*a +
2*b*x)^(2*p), x], x] /; FreeQ[{a, b, d, e, f, m}, x] && EqQ[4*b^3 + 27*a^2*
d, 0] && IntegerQ[p]
```

rule 2525
```
Int[(Pm_)/(Qn_), x_Symbol] :> With[{m = Expon[Pm, x], n = Expon[Qn, x]}, Si
mp[Coeff[Pm, x, m]*(Log[Qn]/(n*Coeff[Qn, x, n])), x] + Simp[1/(n*Coeff[Qn,
x, n])   Int[ExpandToSum[n*Coeff[Qn, x, n]*Pm - Coeff[Pm, x, m]*D[Qn, x], x
]/Qn, x], x] /; EqQ[m, n - 1]] /; PolyQ[Pm, x] && PolyQ[Qn, x]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.36 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.39

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(2\sqrt{3}\,b^{\frac{3}{2}}-9b\_Z+9\_Z^{3}\right)}\frac{\left(C\_R^{2}+B\_R+A\right)\ln\left(x-\_R\right)}{3\_R^{2}-b}\right)}{9}$	53

input
```
int((C*x^2+B*x+A)/(2*3^(1/2)*b^(3/2)-9*b*x+9*x^3),x,method=_RETURNVERBOSE)
```

output
```
1/9*sum((C*_R^2+B*_R+A)/(3*_R^2-b)*ln(x-_R),_R=RootOf(2*3^(1/2)*b^(3/2)-9*
b*_Z+9*_Z^3))
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.23

$$\int \frac{A+Bx+Cx^2}{2\sqrt{3}b^{3/2}-9bx+9x^3}\,dx =$$
$$-\frac{3\,Cb^2+9\,Bbx+3\,\sqrt{3}(Bb+(Cb+3\,A)x)\sqrt{b}+9\,Ab+\Big(4\,Cb^2-3\,(4\,Cb+3\,A)x^2+2\,\sqrt{3}(3\,Bx^2-Bb)\sqrt{}}{81}$$

input
```
integrate((C*x^2+B*x+A)/(2*3^(1/2)*b^(3/2)-9*b*x+9*x^3),x, algorithm="fric
as")
```

output
```
-1/81*(3*C*b^2 + 9*B*b*x + 3*sqrt(3)*(B*b + (C*b + 3*A)*x)*sqrt(b) + 9*A*b
 + (4*C*b^2 - 3*(4*C*b + 3*A)*x^2 + 2*sqrt(3)*(3*B*x^2 - B*b)*sqrt(b) + 3*
A*b)*log(2*sqrt(3)*sqrt(b) + 3*x) + (5*C*b^2 - 3*(5*C*b - 3*A)*x^2 - 2*sqr
t(3)*(3*B*x^2 - B*b)*sqrt(b) - 3*A*b)*log(-sqrt(3)*sqrt(b) + 3*x))/(3*b*x^
2 - b^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7218 vs. $2(122) = 244$.

Time = 1.13 (sec) , antiderivative size = 7218, normalized size of antiderivative = 53.07

$$\int \frac{A+Bx+Cx^2}{2\sqrt{3}b^{3/2}-9bx+9x^3}\,dx = \text{Too large to display}$$

input
```
integrate((C*x**2+B*x+A)/(2*3**(1/2)*b**(3/2)-9*b*x+9*x**3),x)
```

output
```
(C/18 - sqrt(36*A**2*b**8 - 48*sqrt(3)*A*B*b**(17/2) - 12*A*C*b**9 + 48*B*
*2*b**9 + 8*sqrt(3)*B*C*b**(19/2) + C**2*b**10)/(162*b**5))*log(648*sqrt(3
)*A**4*b**(25/2)/(3888*A**4*b**12 - 10368*sqrt(3)*A**3*B*b**(25/2) - 2592*
A**3*C*b**13 + 31104*A**2*B**2*b**13 + 5184*sqrt(3)*A**2*B*C*b**(27/2) + 6
48*A**2*C**2*b**14 - 13824*sqrt(3)*A*B**3*b**(27/2) - 10368*A*B**2*C*b**14
 - 864*sqrt(3)*A*B*C**2*b**(29/2) - 72*A*C**3*b**15 + 6912*B**4*b**14 + 23
04*sqrt(3)*B**3*C*b**(29/2) + 864*B**2*C**2*b**15 + 48*sqrt(3)*B*C**3*b**(
31/2) + 3*C**4*b**16) - 5184*A**3*B*b**13/(3888*A**4*b**12 - 10368*sqrt(3)
*A**3*B*b**(25/2) - 2592*A**3*C*b**13 + 31104*A**2*B**2*b**13 + 5184*sqrt(
3)*A**2*B*C*b**(27/2) + 648*A**2*C**2*b**14 - 13824*sqrt(3)*A*B**3*b**(27/
2) - 10368*A*B**2*C*b**14 - 864*sqrt(3)*A*B*C**2*b**(29/2) - 72*A*C**3*b**
15 + 6912*B**4*b**14 + 2304*sqrt(3)*B**3*C*b**(29/2) + 864*B**2*C**2*b**15
 + 48*sqrt(3)*B*C**3*b**(31/2) + 3*C**4*b**16) - 3348*sqrt(3)*A**3*C*b**(2
7/2)/(3888*A**4*b**12 - 10368*sqrt(3)*A**3*B*b**(25/2) - 2592*A**3*C*b**13
 + 31104*A**2*B**2*b**13 + 5184*sqrt(3)*A**2*B*C*b**(27/2) + 648*A**2*C**2
*b**14 - 13824*sqrt(3)*A*B**3*b**(27/2) - 10368*A*B**2*C*b**14 - 864*sqrt(
3)*A*B*C**2*b**(29/2) - 72*A*C**3*b**15 + 6912*B**4*b**14 + 2304*sqrt(3)*B
**3*C*b**(29/2) + 864*B**2*C**2*b**15 + 48*sqrt(3)*B*C**3*b**(31/2) + 3*C*
*4*b**16) + 5184*sqrt(3)*A**2*B**2*b**(27/2)/(3888*A**4*b**12 - 10368*sqrt
(3)*A**3*B*b**(25/2) - 2592*A**3*C*b**13 + 31104*A**2*B**2*b**13 + 5184...
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{2\sqrt{3}b^{3/2}-9bx+9x^3}\,dx=\int \frac{Cx^2+Bx+A}{9\,x^3+2\,\sqrt{3}b^{\frac{3}{2}}-9\,bx}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(2*3^(1/2)*b^(3/2)-9*b*x+9*x^3),x, algorithm="maxi
ma")
```

output
```
integrate((C*x^2 + B*x + A)/(9*x^3 + 2*sqrt(3)*b^(3/2) - 9*b*x), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A+Bx+Cx^2}{2\sqrt{3}b^{3/2}-9bx+9x^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((C*x^2+B*x+A)/(2*3^(1/2)*b^(3/2)-9*b*x+9*x^3),x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 12.62 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.03

$$\int \frac{A+Bx+Cx^2}{2\sqrt{3}b^{3/2}-9bx+9x^3}\,dx = \ln\left(x-\frac{\sqrt{3}\,\sqrt{b}}{3}\right)\left(\frac{C}{18}+\frac{b\left(12\,B\,\sqrt{27\,b}+\sqrt{3}\,C\,\sqrt{b}\,\sqrt{27\,b}\right)-6\,\sqrt{3}\,A\,\sqrt{b}\,\sqrt{27\,b}}{1458\,b^2}\right)+\ln\left(x+\frac{2\,\sqrt{3}\,\sqrt{b}}{3}\right)\left(\frac{C}{18}-\frac{b\left(12\,B\,\sqrt{27\,b}+\sqrt{3}\,C\,\sqrt{b}\,\sqrt{27\,b}\right)-6\,\sqrt{3}\,A\,\sqrt{b}\,\sqrt{27\,b}}{1458\,b^2}\right)-\frac{\frac{B}{27}+\frac{\sqrt{3}\left(\frac{A}{27}+\frac{C\,b}{81}\right)}{\sqrt{b}}}{x-\frac{\sqrt{3}\,\sqrt{b}}{3}}$$

input
```
int((A + B*x + C*x^2)/(2*3^(1/2)*b^(3/2) - 9*b*x + 9*x^3),x)
```

output
```
log(x - (3^(1/2)*b^(1/2))/3)*(C/18 + (b*(12*B*(27*b)^(1/2) + 3^(1/2)*C*b^(
1/2)*(27*b)^(1/2)) - 6*3^(1/2)*A*b^(1/2)*(27*b)^(1/2))/(1458*b^2)) + log(x
 + (2*3^(1/2)*b^(1/2))/3)*(C/18 - (b*(12*B*(27*b)^(1/2) + 3^(1/2)*C*b^(1/2
)*(27*b)^(1/2)) - 6*3^(1/2)*A*b^(1/2)*(27*b)^(1/2))/(1458*b^2)) - (B/27 +
(3^(1/2)*(A/27 + (C*b)/81))/b^(1/2))/(x - (3^(1/2)*b^(1/2))/3)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.99

$$\int \frac{A + Bx + Cx^2}{2\sqrt{3}b^{3/2} - 9bx + 9x^3}\, dx = \frac{-2\sqrt{b}\,\sqrt{3}\log\left(-2\sqrt{b}\,\sqrt{3} - 3x\right) b^2 + 6\sqrt{b}\,\sqrt{3}\log\left(-2\sqrt{b}\,\sqrt{3} - 3x\right) b\, x^2 + 2\sqrt{}$$

input
```
int((C*x^2+B*x+A)/(2*3^(1/2)*b^(3/2)-9*b*x+9*x^3),x)
```

output
```
( - 2*sqrt(b)*sqrt(3)*log( - 2*sqrt(b)*sqrt(3) - 3*x)*b**2 + 6*sqrt(b)*sqr
t(3)*log( - 2*sqrt(b)*sqrt(3) - 3*x)*b*x**2 + 2*sqrt(b)*sqrt(3)*log(sqrt(b
)*sqrt(3) - 3*x)*b**2 - 6*sqrt(b)*sqrt(3)*log(sqrt(b)*sqrt(3) - 3*x)*b*x**
2 + 9*sqrt(b)*sqrt(3)*a*x + 3*sqrt(b)*sqrt(3)*b*c*x + 9*sqrt(b)*sqrt(3)*b*
x**2 + 3*log( - 2*sqrt(b)*sqrt(3) - 3*x)*a*b - 9*log( - 2*sqrt(b)*sqrt(3)
- 3*x)*a*x**2 + 4*log( - 2*sqrt(b)*sqrt(3) - 3*x)*b**2*c - 12*log( - 2*sqr
t(b)*sqrt(3) - 3*x)*b*c*x**2 - 3*log(sqrt(b)*sqrt(3) - 3*x)*a*b + 9*log(sq
rt(b)*sqrt(3) - 3*x)*a*x**2 + 5*log(sqrt(b)*sqrt(3) - 3*x)*b**2*c - 15*log
(sqrt(b)*sqrt(3) - 3*x)*b*c*x**2 + 27*a*x**2 + 9*b**2*x + 9*b*c*x**2)/(81*
b*(b - 3*x**2))
```

3.8 $\int \left(A+Bx+Cx^2\right)\left(27a^3+27a^2bx-4b^3x^3\right)^{5/2}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 541

$$\begin{aligned}
&\int \left(A+Bx+Cx^2\right)\left(27a^3+27a^2bx-4b^3x^3\right)^{5/2}\,dx = \\
&-\frac{118098a^5(Ab^2+3a(bB+3aC))\,(3a-bx)\left(27a^3+27a^2bx-4b^3x^3\right)^{5/2}}{7b^3(3a+2bx)^5} \\
&+\frac{1458a^4(10Ab^2+39abB+144a^2C)\,(3a-bx)^2\left(27a^3+27a^2bx-4b^3x^3\right)^{5/2}}{b^3(3a+2bx)^5} \\
&-\frac{1458a^3(40Ab^2+210abB+981a^2C)\,(3a-bx)^3\left(27a^3+27a^2bx-4b^3x^3\right)^{5/2}}{11b^3(3a+2bx)^5} \\
&+\frac{1620a^2(8Ab^2+60abB+369a^2C)\,(3a-bx)^4\left(27a^3+27a^2bx-4b^3x^3\right)^{5/2}}{13b^3(3a+2bx)^5} \\
&-\frac{48a(2Ab^2+24abB+207a^2C)\,(3a-bx)^5\left(27a^3+27a^2bx-4b^3x^3\right)^{5/2}}{b^3(3a+2bx)^5} \\
&+\frac{32(2Ab^2+51abB+693a^2C)\,(3a-bx)^6\left(27a^3+27a^2bx-4b^3x^3\right)^{5/2}}{17b^3(3a+2bx)^5} \\
&-\frac{32(2bB+57aC)(3a-bx)^7\left(27a^3+27a^2bx-4b^3x^3\right)^{5/2}}{19b^3(3a+2bx)^5} \\
&+\frac{64C(3a-bx)^8\left(27a^3+27a^2bx-4b^3x^3\right)^{5/2}}{21b^3(3a+2bx)^5}
\end{aligned}$$

output
```
-118098/7*a^5*(A*b^2+3*a*(B*b+3*C*a))*(-b*x+3*a)*(-4*b^3*x^3+27*a^2*b*x+27
*a^3)^(5/2)/b^3/(2*b*x+3*a)^5+1458*a^4*(10*A*b^2+39*B*a*b+144*C*a^2)*(-b*x
+3*a)^2*(-4*b^3*x^3+27*a^2*b*x+27*a^3)^(5/2)/b^3/(2*b*x+3*a)^5-1458/11*a^3
*(40*A*b^2+210*B*a*b+981*C*a^2)*(-b*x+3*a)^3*(-4*b^3*x^3+27*a^2*b*x+27*a^3
)^(5/2)/b^3/(2*b*x+3*a)^5+1620/13*a^2*(8*A*b^2+60*B*a*b+369*C*a^2)*(-b*x+3
*a)^4*(-4*b^3*x^3+27*a^2*b*x+27*a^3)^(5/2)/b^3/(2*b*x+3*a)^5-48*a*(2*A*b^2
+24*B*a*b+207*C*a^2)*(-b*x+3*a)^5*(-4*b^3*x^3+27*a^2*b*x+27*a^3)^(5/2)/b^3
/(2*b*x+3*a)^5+32/17*(2*A*b^2+51*B*a*b+693*C*a^2)*(-b*x+3*a)^6*(-4*b^3*x^3
+27*a^2*b*x+27*a^3)^(5/2)/b^3/(2*b*x+3*a)^5-32/19*(2*B*b+57*C*a)*(-b*x+3*a
)^7*(-4*b^3*x^3+27*a^2*b*x+27*a^3)^(5/2)/b^3/(2*b*x+3*a)^5+64/21*C*(-b*x+3
*a)^8*(-4*b^3*x^3+27*a^2*b*x+27*a^3)^(5/2)/b^3/(2*b*x+3*a)^5
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.40

$$\int (A+Bx+Cx^2)\left(27a^3+27a^2bx-4b^3x^3\right)^{5/2}\,dx=\frac{2(-3a+bx)^3\sqrt{(3a-bx)(3a+2bx)^2}(889234200a^7C+1458a^6b(366231B+711550Cx)+\cdots}{\cdots}$$

input
```
Integrate[(A + B*x + C*x^2)*(27*a^3 + 27*a^2*b*x - 4*b^3*x^3)^(5/2),x]
```

output
```
(2*(-3*a + b*x)^3*Sqrt[(3*a - b*x)*(3*a + 2*b*x)^2]*(889234200*a^7*C + 145
8*a^6*b*(366231*B + 711550*C*x) + 48048*a*b^6*x^4*(399*A + 17*x*(21*B + 19
*C*x)) + 4576*b^7*x^5*(399*A + 17*x*(21*B + 19*C*x)) + 33264*a^2*b^5*x^3*(
2679*A + 13*x*(187*B + 171*C*x)) + 1512*a^3*b^4*x^2*(159429*A + 11*x*(1382
1*B + 13091*C*x)) + 729*a^5*b^2*(526737*A + 7*x*(122077*B + 152475*C*x)) +
 1134*a^4*b^3*x*(360411*A + x*(394689*B + 407759*C*x))))/(969969*b^3*(3*a
+ 2*b*x))
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 382, normalized size of antiderivative = 0.71, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2526, 27, 2483, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(27a^3+27a^2bx-4b^3x^3\right)^{5/2}\left(A+Bx+Cx^2\right)\,dx$$

$$\Big\downarrow \text{ 2526}$$

$$-\frac{\int -3b\left(9Ca^2+4Ab^2+4b^2Bx\right)\left(27a^3+27bxa^2-4b^3x^3\right)^{5/2}dx}{12b^3}-\frac{C\left(27a^3+27a^2bx-4b^3x^3\right)^{7/2}}{42b^3}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \left(9Ca^2+4Ab^2+4b^2Bx\right)\left(27a^3+27bxa^2-4b^3x^3\right)^{5/2}dx}{4b^2}-\frac{C\left(27a^3+27a^2bx-4b^3x^3\right)^{7/2}}{42b^3}$$

$$\Big\downarrow \text{ 2483}$$

$$\frac{\left(27a^3+27a^2bx-4b^3x^3\right)^{5/2}\int 31381059609\sqrt{3}a^{10}(3a+2bx)^5\left(3a^3-a^2bx\right)^{5/2}\left(9Ca^2+4Ab^2+4b^2Bx\right)dx}{125524238436\sqrt{3}a^{10}b^2(3a+2bx)^5\left(3a^3-a^2bx\right)^{5/2}}-\frac{C\left(27a^3+27a^2bx-4b^3x^3\right)^{7/2}}{42b^3}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\left(27a^3+27a^2bx-4b^3x^3\right)^{5/2}\int (3a+2bx)^5\left(3a^3-a^2bx\right)^{5/2}\left(9Ca^2+4Ab^2+4b^2Bx\right)dx}{4b^2(3a+2bx)^5\left(3a^3-a^2bx\right)^{5/2}}-\frac{C\left(27a^3+27a^2bx-4b^3x^3\right)^{7/2}}{42b^3}$$

$$\Big\downarrow \text{ 86}$$

$$\frac{\left(27a^3+27a^2bx-4b^3x^3\right)^{5/2}\int\left(\frac{128bB\left(3a^3-a^2bx\right)^{17/2}}{a^{12}}-\frac{32\left(9Ca^2+102bBa+4Ab^2\right)\left(3a^3-a^2bx\right)^{15/2}}{a^{10}}+\frac{720\left(9Ca^2+48bBa+4Ab^2\right)(3}{a^7}\right.}{}$$

$$\frac{C\left(27a^3+27a^2bx-4b^3x^3\right)^{7/2}}{42b^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(27a^3+27a^2bx-4b^3x^3\right)^{5/2}\left(-\frac{58320\left(3a^3-a^2bx\right)^{11/2}\left(9a^2C+21abB+4Ab^2\right)}{11a^3b}+\frac{2916\left(3a^3-a^2bx\right)^{9/2}\left(45a^2C+78abB+20Ab^2\right)}{b}-\frac{118}{}\right.}{}$$

$$\frac{C\left(27a^3+27a^2bx-4b^3x^3\right)^{7/2}}{42b^3}$$

input
```
Int[(A + B*x + C*x^2)*(27*a^3 + 27*a^2*b*x - 4*b^3*x^3)^(5/2),x]
```

output
```
-1/42*(C*(27*a^3 + 27*a^2*b*x - 4*b^3*x^3)^(7/2))/b^3 + ((27*a^3 + 27*a^2*
b*x - 4*b^3*x^3)^(5/2)*((-118098*a^3*(4*A*b^2 + 12*a*b*B + 9*a^2*C)*(3*a^3
 - a^2*b*x)^(7/2))/(7*b) + (2916*(20*A*b^2 + 78*a*b*B + 45*a^2*C)*(3*a^3 -
 a^2*b*x)^(9/2))/b - (58320*(4*A*b^2 + 21*a*b*B + 9*a^2*C)*(3*a^3 - a^2*b*
x)^(11/2))/(11*a^3*b) + (12960*(4*A*b^2 + 30*a*b*B + 9*a^2*C)*(3*a^3 - a^2
*b*x)^(13/2))/(13*a^6*b) - (96*(4*A*b^2 + 48*a*b*B + 9*a^2*C)*(3*a^3 - a^2
*b*x)^(15/2))/(a^9*b) + (64*(4*A*b^2 + 102*a*b*B + 9*a^2*C)*(3*a^3 - a^2*b
*x)^(17/2))/(17*a^12*b) - (256*B*(3*a^3 - a^2*b*x)^(19/2))/(19*a^14)))/(4*
b^2*(3*a + 2*b*x)^5*(3*a^3 - a^2*b*x)^(5/2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2483
```
Int[((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (d_.)*(x_)^3)^(p_), x_S
ymbol] :> Simp[(a + b*x + d*x^3)^p/((3*a - b*x)^p*(3*a + 2*b*x)^(2*p))   In
t[(e + f*x)^m*(3*a - b*x)^p*(3*a + 2*b*x)^(2*p), x], x] /; FreeQ[{a, b, d,
e, f, m, p}, x] && EqQ[4*b^3 + 27*a^2*d, 0] &&  !IntegerQ[p]
```

rule 2526
```
Int[(Pm_)*(Qn_)^(p_), x_Symbol] :> With[{m = Expon[Pm, x], n = Expon[Qn, x]
}, Simp[Coeff[Pm, x, m]*(Qn^(p + 1)/(n*(p + 1)*Coeff[Qn, x, n])), x] + Simp
[1/(n*Coeff[Qn, x, n])   Int[ExpandToSum[n*Coeff[Qn, x, n]*Pm - Coeff[Pm, x
, m]*D[Qn, x], x]*Qn^p, x], x] /; EqQ[m, n - 1]] /; FreeQ[p, x] && PolyQ[Pm
, x] && PolyQ[Qn, x] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 263, normalized size of antiderivative = 0.49

method	result
gosper	$-\frac{2(-bx+3a)\left(1478048C\,b^7x^7+1633632B\,b^7x^6+15519504Ca\,b^6x^6+1825824A\,b^7x^5+17153136Ba\,b^6x^5+73945872C\,a^2b^5x^5+1917\right.}{}$
default	$-\frac{2(-bx+3a)\left(1478048C\,b^7x^7+1633632B\,b^7x^6+15519504Ca\,b^6x^6+1825824A\,b^7x^5+17153136Ba\,b^6x^5+73945872C\,a^2b^5x^5+1917\right.}{}$
orering	$-\frac{2(-bx+3a)\left(1478048C\,b^7x^7+1633632B\,b^7x^6+15519504Ca\,b^6x^6+1825824A\,b^7x^5+17153136Ba\,b^6x^5+73945872C\,a^2b^5x^5+1917\right.}{}$
risch	$-\frac{2\sqrt{(-bx+3a)(2bx+3a)^2}\,\left(-1478048C\,b^{10}x^{10}-1633632B\,b^{10}x^9-2217072Ca\,b^9x^9-1825824A\,b^{10}x^8-2450448Ba\,b^9x^8+2582236\right.}{}$
trager	$-\frac{2\left(-1478048C\,b^{10}x^{10}-1633632B\,b^{10}x^9-2217072Ca\,b^9x^9-1825824A\,b^{10}x^8-2450448Ba\,b^9x^8+25822368C\,a^2b^8x^8-2738736Aa\right.}{}$

input
```
int((C*x^2+B*x+A)*(-4*b^3*x^3+27*a^2*b*x+27*a^3)^(5/2),x,method=_RETURNVER
BOSE)
```

output
```
-2/969969*(-b*x+3*a)*(1478048*C*b^7*x^7+1633632*B*b^7*x^6+15519504*C*a*b^6
*x^6+1825824*A*b^7*x^5+17153136*B*a*b^6*x^5+73945872*C*a^2*b^5*x^5+1917115
2*A*a*b^6*x^4+80864784*B*a^2*b^5*x^4+217729512*C*a^3*b^4*x^4+89114256*A*a^
2*b^5*x^3+229870872*B*a^3*b^4*x^3+462398706*C*a^4*b^3*x^3+241056648*A*a^3*
b^4*x^2+447577326*B*a^4*b^3*x^2+778079925*C*a^5*b^2*x^2+408706074*A*a^4*b^
3*x+622958931*B*a^5*b^2*x+1037439900*C*a^6*b*x+383991273*A*a^5*b^2+5339647
98*B*a^6*b+889234200*C*a^7)*(-4*b^3*x^3+27*a^2*b*x+27*a^3)^(5/2)/b^3/(2*b*
x+3*a)^5
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 344, normalized size of antiderivative = 0.64

$$\int (A+Bx+Cx^2)\left(27a^3+27a^2bx-4b^3x^3\right)^{5/2}\,dx = \frac{2\,(1478048\,Cb^{10}x^{10}-24009323400\,Ca^{10}-14417049546\,Ba^9b-10367764371\,Aa^8b^2+11$$

input
```
integrate((C*x^2+B*x+A)*(-4*b^3*x^3+27*a^2*b*x+27*a^3)^(5/2),x, algorithm=
"fricas")
```

output
```
2/969969*(1478048*C*b^10*x^10 - 24009323400*C*a^10 - 14417049546*B*a^9*b -
 10367764371*A*a^8*b^2 + 116688*(19*C*a*b^9 + 14*B*b^10)*x^9 - 6864*(3762*
C*a^2*b^8 - 357*B*a*b^9 - 266*A*b^10)*x^8 - 10296*(6669*C*a^3*b^7 + 2856*B
*a^2*b^8 - 266*A*a*b^9)*x^7 + 37422*(2147*C*a^4*b^6 - 2108*B*a^3*b^7 - 912
*A*a^2*b^8)*x^6 + 567*(879453*C*a^5*b^5 + 174522*B*a^4*b^6 - 163400*A*a^3*
b^7)*x^5 + 1701*(376713*C*a^6*b^4 + 363273*B*a^5*b^5 + 75050*A*a^4*b^6)*x^
4 + 2187*(34599*C*a^7*b^3 + 368271*B*a^6*b^4 + 369493*A*a^5*b^5)*x^3 - 218
7*(457425*C*a^8*b^2 + 32181*B*a^7*b^3 - 489535*A*a^6*b^4)*x^2 - 6561*(6099
00*C*a^9*b + 366231*B*a^8*b^2 + 101707*A*a^7*b^3)*x)*sqrt(-4*b^3*x^3 + 27*
a^2*b*x + 27*a^3)/(2*b^4*x + 3*a*b^3)
```

Sympy [F]

$$\int (A+Bx+Cx^2)\left(27a^3+27a^2bx-4b^3x^3\right)^{5/2}\,dx=\int \left(-(-3a+bx)\,(3a+2bx)^2\right)^{\frac{5}{2}}\left(A+Bx+Cx^2\right)\,dx$$

input
```
integrate((C*x**2+B*x+A)*(-4*b**3*x**3+27*a**2*b*x+27*a**3)**(5/2),x)
```

output
```
Integral((-(-3*a + b*x)*(3*a + 2*b*x)**2)**(5/2)*(A + B*x + C*x**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 337, normalized size of antiderivative = 0.62

$$\int (A+Bx+Cx^2)\left(27a^3+27a^2bx-4b^3x^3\right)^{5/2}\,dx=\frac{2\,(32032\,b^8x^8+48048\,ab^7x^7-598752\,a^2b^6x^6-1625400\,a^3b^5x^5+2239650\,a^4b^4x^4+1417}{17017\,b}+\frac{2\,(32032\,b^9x^9+48048\,ab^8x^8-576576\,a^2b^7x^7-1546776\,a^3b^6x^6+1940274\,a^4b^5x^5+12116223\,a^5b^4x^4+1}{19019\,b^2}+\frac{2\,(77792\,b^{10}x^{10}+116688\,ab^9x^9-1359072\,a^2b^8x^8-3613896\,a^3b^7x^7+4228686\,a^4b^6x^6+26244729\,a^5b^5x^5}{51}$$

input
```
integrate((C*x^2+B*x+A)*(-4*b^3*x^3+27*a^2*b*x+27*a^3)^(5/2),x, algorithm=
"maxima")
```

output
```
2/17017*(32032*b^8*x^8 + 48048*a*b^7*x^7 - 598752*a^2*b^6*x^6 - 1625400*a^
3*b^5*x^5 + 2239650*a^4*b^4*x^4 + 14176863*a^5*b^3*x^3 + 18782685*a^6*b^2*
x^2 - 11707011*a^7*b*x - 181890603*a^8)*sqrt(-b*x + 3*a)*A/b + 2/19019*(32
032*b^9*x^9 + 48048*a*b^8*x^8 - 576576*a^2*b^7*x^7 - 1546776*a^3*b^6*x^6 +
 1940274*a^4*b^5*x^5 + 12116223*a^5*b^4*x^4 + 15792327*a^6*b^3*x^3 - 13799
97*a^7*b^2*x^2 - 47114541*a^8*b*x - 282687246*a^9)*sqrt(-b*x + 3*a)*B/b^2
+ 2/51051*(77792*b^10*x^10 + 116688*a*b^9*x^9 - 1359072*a^2*b^8*x^8 - 3613
896*a^3*b^7*x^7 + 4228686*a^4*b^6*x^6 + 26244729*a^5*b^5*x^5 + 33725727*a^
6*b^4*x^4 + 3982527*a^7*b^3*x^3 - 52652025*a^8*b^2*x^2 - 210608100*a^9*b*x
 - 1263648600*a^10)*sqrt(-b*x + 3*a)*C/b^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3745 vs. $2(505) = 1010$.

Time = 0.19 (sec) , antiderivative size = 3745, normalized size of antiderivative = 6.92

$$\int (A + Bx + Cx^2) (27a^3 + 27a^2bx - 4b^3x^3)^{5/2} dx = \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)*(-4*b^3*x^3+27*a^2*b*x+27*a^3)^(5/2),x, algorithm=
"giac")
```

output
```
2/4849845*(31819833045*sqrt(-b*x + 3*a)*A*a^8*sgn(-2*b*x - 3*a) - 24748759
035*((-b*x + 3*a)^(3/2) - 9*sqrt(-b*x + 3*a)*a)*A*a^7*sgn(-2*b*x - 3*a) -
10606611015*((-b*x + 3*a)^(3/2) - 9*sqrt(-b*x + 3*a)*a)*B*a^8*sgn(-2*b*x -
 3*a)/b + 9192396213*((b*x - 3*a)^2*sqrt(-b*x + 3*a) - 10*(-b*x + 3*a)^(3/
2)*a + 45*sqrt(-b*x + 3*a)*a^2)*A*a^6*sgn(-2*b*x - 3*a) + 6363966609*((b*x
 - 3*a)^2*sqrt(-b*x + 3*a) - 10*(-b*x + 3*a)^(3/2)*a + 45*sqrt(-b*x + 3*a)
*a^2)*C*a^8*sgn(-2*b*x - 3*a)/b^2 + 14849255421*((b*x - 3*a)^2*sqrt(-b*x +
 3*a) - 10*(-b*x + 3*a)^(3/2)*a + 45*sqrt(-b*x + 3*a)*a^2)*B*a^7*sgn(-2*b*
x - 3*a)/b - 370389591*(5*(b*x - 3*a)^3*sqrt(-b*x + 3*a) + 63*(b*x - 3*a)^
2*sqrt(-b*x + 3*a)*a - 315*(-b*x + 3*a)^(3/2)*a^2 + 945*sqrt(-b*x + 3*a)*a
^3)*A*a^5*sgn(-2*b*x - 3*a) + 2121322203*(5*(b*x - 3*a)^3*sqrt(-b*x + 3*a)
 + 63*(b*x - 3*a)^2*sqrt(-b*x + 3*a)*a - 315*(-b*x + 3*a)^(3/2)*a^2 + 945*
sqrt(-b*x + 3*a)*a^3)*C*a^7*sgn(-2*b*x - 3*a)/b^2 + 1313199459*(5*(b*x - 3
*a)^3*sqrt(-b*x + 3*a) + 63*(b*x - 3*a)^2*sqrt(-b*x + 3*a)*a - 315*(-b*x +
 3*a)^(3/2)*a^2 + 945*sqrt(-b*x + 3*a)*a^3)*B*a^6*sgn(-2*b*x - 3*a)/b - 62
355150*(35*(b*x - 3*a)^4*sqrt(-b*x + 3*a) + 540*(b*x - 3*a)^3*sqrt(-b*x +
3*a)*a + 3402*(b*x - 3*a)^2*sqrt(-b*x + 3*a)*a^2 - 11340*(-b*x + 3*a)^(3/2
)*a^3 + 25515*sqrt(-b*x + 3*a)*a^4)*A*a^4*sgn(-2*b*x - 3*a) + 145911051*(3
5*(b*x - 3*a)^4*sqrt(-b*x + 3*a) + 540*(b*x - 3*a)^3*sqrt(-b*x + 3*a)*a +
3402*(b*x - 3*a)^2*sqrt(-b*x + 3*a)*a^2 - 11340*(-b*x + 3*a)^(3/2)*a^3 ...
```

Mupad [B] (verification not implemented)

Time = 12.56 (sec) , antiderivative size = 442, normalized size of antiderivative = 0.82

$$\int (A+Bx+Cx^2)\left(27a^3+27a^2bx-4b^3x^3\right)^{5/2}\,dx = \frac{2\,A\,\sqrt{27\,a^3+27\,a^2\,b\,x-4\,b^3\,x^3}\,(-9611865\,a^7+2505573\,a^6\,b\,x+4590513\,a^5\,b^2\,x^2+1665}{17017\,b}$$
$$-\frac{2\,B\,\sqrt{27\,a^3+27\,a^2\,b\,x-4\,b^3\,x^3}\,(17701578\,a^8+3903795\,a^7\,b\,x-2142531\,a^6\,b^2\,x^2-3835755\,a^5\,b^3\,x^3-1}{19019\,b^2}$$
$$-\frac{2\,C\,\sqrt{27\,a^3+27\,a^2\,b\,x-4\,b^3\,x^3}\,(76842432\,a^9+18974412\,a^8\,b\,x+4901067\,a^7\,b^2\,x^2-4594887\,a^6\,b^3\,x^3-}{51051\,b^3}$$
$$-\frac{306110016\,A\,a^8\,\sqrt{27\,a^3+27\,a^2\,b\,x-4\,b^3\,x^3}}{17017\,b\,(3\,a+2\,b\,x)}$$
$$-\frac{459165024\,B\,a^9\,\sqrt{27\,a^3+27\,a^2\,b\,x-4\,b^3\,x^3}}{19019\,b^2\,(3\,a+2\,b\,x)}$$
$$-\frac{688747536\,C\,a^{10}\,\sqrt{27\,a^3+27\,a^2\,b\,x-4\,b^3\,x^3}}{17017\,b^3\,(3\,a+2\,b\,x)}$$

input
```
int((A + B*x + C*x^2)*(27*a^3 - 4*b^3*x^3 + 27*a^2*b*x)^(5/2),x)
```

output
```
(2*A*(27*a^3 - 4*b^3*x^3 + 27*a^2*b*x)^(1/2)*(16016*b^7*x^7 - 9611865*a^7
+ 4590513*a^5*b^2*x^2 + 1665279*a^4*b^3*x^3 - 363636*a^3*b^4*x^4 - 299376*
a^2*b^5*x^5 + 2505573*a^6*b*x))/(17017*b) - (2*B*(27*a^3 - 4*b^3*x^3 + 27*
a^2*b*x)^(1/2)*(17701578*a^8 - 16016*b^8*x^8 - 2142531*a^6*b^2*x^2 - 38357
55*a^5*b^3*x^3 - 1481571*a^4*b^4*x^4 + 340956*a^3*b^5*x^5 + 288288*a^2*b^6
*x^6 + 3903795*a^7*b*x))/(19019*b^2) - (2*C*(27*a^3 - 4*b^3*x^3 + 27*a^2*b
*x)^(1/2)*(76842432*a^9 - 38896*b^9*x^9 + 4901067*a^7*b^2*x^2 - 4594887*a^
6*b^3*x^3 - 8178651*a^5*b^4*x^4 - 3295809*a^4*b^5*x^5 + 787644*a^3*b^6*x^6
 + 679536*a^2*b^7*x^7 + 18974412*a^8*b*x))/(51051*b^3) - (306110016*A*a^8*
(27*a^3 - 4*b^3*x^3 + 27*a^2*b*x)^(1/2))/(17017*b*(3*a + 2*b*x)) - (459165
024*B*a^9*(27*a^3 - 4*b^3*x^3 + 27*a^2*b*x)^(1/2))/(19019*b^2*(3*a + 2*b*x
)) - (688747536*C*a^10*(27*a^3 - 4*b^3*x^3 + 27*a^2*b*x)^(1/2))/(17017*b^3
*(3*a + 2*b*x))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.43

$$\int \left(A+Bx+Cx^2\right)\left(27a^3+27a^2bx-4b^3x^3\right)^{5/2}\,dx = \frac{2\sqrt{-bx+3a}\,(1478048b^{10}c\,x^{10}+2217072a\,b^9c\,x^9+1633632b^{11}x^9-25822368a^2b^8c\,x^8+4}{}$$

input
```
int((C*x^2+B*x+A)*(-4*b^3*x^3+27*a^2*b*x+27*a^3)^(5/2),x)
```

output
```
(2*sqrt(3*a - b*x)*( - 24009323400*a**10*c - 24784813917*a**9*b**2 - 40015
53900*a**9*b*c*x - 3070141218*a**8*b**3*x - 1000388475*a**8*b**2*c*x**2 +
1000233198*a**7*b**4*x**2 + 75668013*a**7*b**3*c*x**3 + 1613489868*a**6*b*
*5*x**3 + 640788813*a**6*b**4*c*x**4 + 745587423*a**5*b**6*x**4 + 49864985
1*a**5*b**5*c*x**5 + 6306174*a**4*b**7*x**5 + 80345034*a**4*b**6*c*x**6 -
113014440*a**3*b**8*x**6 - 68664024*a**3*b**7*c*x**7 - 26666640*a**2*b**9*
x**7 - 25822368*a**2*b**8*c*x**8 + 4276272*a*b**10*x**8 + 2217072*a*b**9*c
*x**9 + 1633632*b**11*x**9 + 1478048*b**10*c*x**10))/(969969*b**3)
```

3.9 $\int (A+Bx+Cx^2)\left(27a^3+27a^2bx-4b^3x^3\right)^{3/2}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 397

$$
\begin{aligned}
&\int (A+Bx+Cx^2)\left(27a^3+27a^2bx-4b^3x^3\right)^{3/2}\,dx = \\
&-\frac{1458a^3(Ab^2+3a(bB+3aC))\,(3a-bx)\left(27a^3+27a^2bx-4b^3x^3\right)^{3/2}}{5b^3(3a+2bx)^3} \\
&+\frac{486a^2(2Ab^2+9a(bB+4aC))\,(3a-bx)^2\left(27a^3+27a^2bx-4b^3x^3\right)^{3/2}}{7b^3(3a+2bx)^3} \\
&-\frac{6a(4Ab^2+30abB+171a^2C)\,(3a-bx)^3\left(27a^3+27a^2bx-4b^3x^3\right)^{3/2}}{b^3(3a+2bx)^3} \\
&+\frac{4(4Ab^2+66abB+603a^2C)\,(3a-bx)^4\left(27a^3+27a^2bx-4b^3x^3\right)^{3/2}}{11b^3(3a+2bx)^3} \\
&-\frac{8(2bB+39aC)(3a-bx)^5\left(27a^3+27a^2bx-4b^3x^3\right)^{3/2}}{13b^3(3a+2bx)^3} \\
&+\frac{16C(3a-bx)^6\left(27a^3+27a^2bx-4b^3x^3\right)^{3/2}}{15b^3(3a+2bx)^3}
\end{aligned}
$$

output
```
-1458/5*a^3*(A*b^2+3*a*(B*b+3*C*a))*(-b*x+3*a)*(-4*b^3*x^3+27*a^2*b*x+27*a
^3)^(3/2)/b^3/(2*b*x+3*a)^3+486/7*a^2*(2*A*b^2+9*a*(B*b+4*C*a))*(-b*x+3*a)
^2*(-4*b^3*x^3+27*a^2*b*x+27*a^3)^(3/2)/b^3/(2*b*x+3*a)^3-6*a*(4*A*b^2+30*
B*a*b+171*C*a^2)*(-b*x+3*a)^3*(-4*b^3*x^3+27*a^2*b*x+27*a^3)^(3/2)/b^3/(2*
b*x+3*a)^3+4/11*(4*A*b^2+66*B*a*b+603*C*a^2)*(-b*x+3*a)^4*(-4*b^3*x^3+27*a
^2*b*x+27*a^3)^(3/2)/b^3/(2*b*x+3*a)^3-8/13*(2*B*b+39*C*a)*(-b*x+3*a)^5*(-
4*b^3*x^3+27*a^2*b*x+27*a^3)^(3/2)/b^3/(2*b*x+3*a)^3+16/15*C*(-b*x+3*a)^6*
(-4*b^3*x^3+27*a^2*b*x+27*a^3)^(3/2)/b^3/(2*b*x+3*a)^3
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.41

$$\int (A+Bx+Cx^2)\left(27a^3+27a^2bx-4b^3x^3\right)^{3/2}\,dx =$$
$$-\frac{2(-3a+bx)^2\sqrt{(3a-bx)(3a+2bx)^2}(935064a^5C+162a^4b(3333B+4810Cx)+420ab^4x^2(195A+11x($$

input
```
Integrate[(A + B*x + C*x^2)*(27*a^3 + 27*a^2*b*x - 4*b^3*x^3)^(3/2),x]
```

output
```
(-2*(-3*a + b*x)^2*Sqrt[(3*a - b*x)*(3*a + 2*b*x)^2]*(935064*a^5*C + 162*a
^4*b*(3333*B + 4810*C*x) + 420*a*b^4*x^2*(195*A + 11*x*(15*B + 13*C*x)) +
56*b^5*x^3*(195*A + 11*x*(15*B + 13*C*x)) + 90*a^2*b^3*x*(2847*A + 7*x*(36
3*B + 325*C*x)) + 27*a^3*b^2*(14391*A + 5*x*(3333*B + 3367*C*x))))/(15015*
b^3*(3*a + 2*b*x))
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 295, normalized size of antiderivative = 0.74, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2526, 27, 2483, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(27a^3+27a^2bx-4b^3x^3\right)^{3/2}\left(A+Bx+Cx^2\right)\,dx$$

$$\Big\downarrow \text{2526}$$

$$-\frac{\int -3b\left(9Ca^2+4Ab^2+4b^2Bx\right)\left(27a^3+27bxa^2-4b^3x^3\right)^{3/2}dx}{12b^3}-\frac{C\left(27a^3+27a^2bx-4b^3x^3\right)^{5/2}}{30b^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \left(9Ca^2+4Ab^2+4b^2Bx\right)\left(27a^3+27bxa^2-4b^3x^3\right)^{3/2}dx}{4b^2}-\frac{C\left(27a^3+27a^2bx-4b^3x^3\right)^{5/2}}{30b^3}$$

$$\Big\downarrow \text{2483}$$

$$\frac{\left(27a^3+27a^2bx-4b^3x^3\right)^{3/2}\int 1594323\sqrt{3}a^6(3a+2bx)^3\left(3a^3-a^2bx\right)^{3/2}\left(9Ca^2+4Ab^2+4b^2Bx\right)dx}{6377292\sqrt{3}a^6b^2(3a+2bx)^3\left(3a^3-a^2bx\right)^{3/2}}-\frac{C\left(27a^3+27a^2bx-4b^3x^3\right)^{5/2}}{30b^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{\left(27a^3+27a^2bx-4b^3x^3\right)^{3/2}\int (3a+2bx)^3\left(3a^3-a^2bx\right)^{3/2}\left(9Ca^2+4Ab^2+4b^2Bx\right)dx}{4b^2(3a+2bx)^3\left(3a^3-a^2bx\right)^{3/2}}-\frac{C\left(27a^3+27a^2bx-4b^3x^3\right)^{5/2}}{30b^3}$$

$$\Big\downarrow \text{86}$$

$$\frac{\left(27a^3+27a^2bx-4b^3x^3\right)^{3/2}\int \left(\frac{32bB\left(3a^3-a^2bx\right)^{11/2}}{a^8}-\frac{8\left(9Ca^2+66bBa+4Ab^2\right)\left(3a^3-a^2bx\right)^{9/2}}{a^6}+\frac{108\left(9Ca^2+30bBa+4Ab^2\right)\left(3a^3-\right.}{a^3}}{4b^2(3a+2bx)^3\left(\right.}-\frac{C\left(27a^3+27a^2bx-4b^3x^3\right)^{5/2}}{30b^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(27a^3+27a^2bx-4b^3x^3\right)^{3/2}\left(\frac{972\left(3a^3-a^2bx\right)^{7/2}\left(9a(aC+2bB)+4Ab^2\right)}{7a^2b}-\frac{1458a\left(3a^3-a^2bx\right)^{5/2}\left(9a^2C+12abB+4Ab^2\right)}{5b}-\frac{64B\left(3a^3-\right.}{13a}\right.}{4b^2(3a+2bx)^3\left(3a^3-a^2bx\right)^{3/2}}-\frac{C\left(27a^3+27a^2bx-4b^3x^3\right)^{5/2}}{30b^3}$$

input
```
Int[(A + B*x + C*x^2)*(27*a^3 + 27*a^2*b*x - 4*b^3*x^3)^(3/2),x]
```

output
```
-1/30*(C*(27*a^3 + 27*a^2*b*x - 4*b^3*x^3)^(5/2))/b^3 + ((27*a^3 + 27*a^2*
b*x - 4*b^3*x^3)^(3/2)*((-1458*a*(4*A*b^2 + 12*a*b*B + 9*a^2*C)*(3*a^3 - a
^2*b*x)^(5/2))/(5*b) + (972*(4*A*b^2 + 9*a*(2*b*B + a*C))*(3*a^3 - a^2*b*x
)^(7/2))/(7*a^2*b) - (24*(4*A*b^2 + 30*a*b*B + 9*a^2*C)*(3*a^3 - a^2*b*x)^
(9/2))/(a^5*b) + (16*(4*A*b^2 + 66*a*b*B + 9*a^2*C)*(3*a^3 - a^2*b*x)^(11/
2))/(11*a^8*b) - (64*B*(3*a^3 - a^2*b*x)^(13/2))/(13*a^10)))/(4*b^2*(3*a +
 2*b*x)^3*(3*a^3 - a^2*b*x)^(3/2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2483
```
Int[((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (d_.)*(x_)^3)^(p_), x_S
ymbol] :> Simp[(a + b*x + d*x^3)^p/((3*a - b*x)^p*(3*a + 2*b*x)^(2*p))   In
t[(e + f*x)^m*(3*a - b*x)^p*(3*a + 2*b*x)^(2*p), x], x] /; FreeQ[{a, b, d,
e, f, m, p}, x] && EqQ[4*b^3 + 27*a^2*d, 0] &&  !IntegerQ[p]
```

rule 2526
```
Int[(Pm_)*(Qn_)^(p_), x_Symbol] :> With[{m = Expon[Pm, x], n = Expon[Qn, x]
}, Simp[Coeff[Pm, x, m]*(Qn^(p + 1)/(n*(p + 1)*Coeff[Qn, x, n])), x] + Simp
[1/(n*Coeff[Qn, x, n])   Int[ExpandToSum[n*Coeff[Qn, x, n]*Pm - Coeff[Pm, x
, m]*D[Qn, x], x]*Qn^p, x], x] /; EqQ[m, n - 1]] /; FreeQ[p, x] && PolyQ[Pm
, x] && PolyQ[Qn, x] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.48

method	result
gosper	$-\frac{2(-bx+3a)(8008b^5C\,x^5+9240B\,b^5x^4+60060Ca\,b^4x^4+10920A\,b^5x^3+69300Ba\,b^4x^3+204750C\,a^2b^3x^3+81900Aa\,b^4x^2+228690}{}$
default	$-\frac{2(-bx+3a)(8008b^5C\,x^5+9240B\,b^5x^4+60060Ca\,b^4x^4+10920A\,b^5x^3+69300Ba\,b^4x^3+204750C\,a^2b^3x^3+81900Aa\,b^4x^2+228690}{}$
orering	$-\frac{2(-bx+3a)(8008b^5C\,x^5+9240B\,b^5x^4+60060Ca\,b^4x^4+10920A\,b^5x^3+69300Ba\,b^4x^3+204750C\,a^2b^3x^3+81900Aa\,b^4x^2+228690}{}$
risch	$-\frac{2\sqrt{(-bx+3a)(2bx+3a)^2}\,(8008C\,b^7x^7+9240B\,b^7x^6+12012Ca\,b^6x^6+10920A\,b^7x^5+13860Ba\,b^6x^5-83538C\,a^2b^5x^5+16380Aa\,b^6}{}$
trager	$-\frac{2(8008C\,b^7x^7+9240B\,b^7x^6+12012Ca\,b^6x^6+10920A\,b^7x^5+13860Ba\,b^6x^5-83538C\,a^2b^5x^5+16380Aa\,b^6x^4-103950B\,a^2b^5x^4-2}{}$

input
```
int((C*x^2+B*x+A)*(-4*b^3*x^3+27*a^2*b*x+27*a^3)^(3/2),x,method=_RETURNVER
BOSE)
```

output
```
-2/15015*(-b*x+3*a)*(8008*C*b^5*x^5+9240*B*b^5*x^4+60060*C*a*b^4*x^4+10920
*A*b^5*x^3+69300*B*a*b^4*x^3+204750*C*a^2*b^3*x^3+81900*A*a*b^4*x^2+228690
*B*a^2*b^3*x^2+454545*C*a^3*b^2*x^2+256230*A*a^2*b^3*x+449955*B*a^3*b^2*x+
779220*C*a^4*b*x+388557*A*a^3*b^2+539946*B*a^4*b+935064*C*a^5)*(-4*b^3*x^3
+27*a^2*b*x+27*a^3)^(3/2)/b^3/(2*b*x+3*a)^3
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 245, normalized size of antiderivative = 0.62

$$\int (A+Bx+Cx^2)\left(27a^3+27a^2bx-4b^3x^3\right)^{3/2}\,dx =$$
$$-\frac{2\,(8008\,Cb^7x^7+8415576\,Ca^7+4859514\,Ba^6b+3497013\,Aa^5b^2+924\,(13\,Cab^6+10\,Bb^7)x^6-42\,(1989}{}$$

input
```
integrate((C*x^2+B*x+A)*(-4*b^3*x^3+27*a^2*b*x+27*a^3)^(3/2),x, algorithm=
"fricas")
```

output
```
-2/15015*(8008*C*b^7*x^7 + 8415576*C*a^7 + 4859514*B*a^6*b + 3497013*A*a^5
*b^2 + 924*(13*C*a*b^6 + 10*B*b^7)*x^6 - 42*(1989*C*a^2*b^5 - 330*B*a*b^6
- 260*A*b^7)*x^5 - 315*(741*C*a^3*b^4 + 330*B*a^2*b^5 - 52*A*a*b^6)*x^4 -
405*(260*C*a^4*b^3 + 737*B*a^3*b^4 + 338*A*a^2*b^5)*x^3 + 81*(4329*C*a^5*b
^2 - 1254*B*a^4*b^3 - 5083*A*a^3*b^4)*x^2 + 243*(5772*C*a^6*b + 3333*B*a^5
*b^2 - 104*A*a^4*b^3)*x)*sqrt(-4*b^3*x^3 + 27*a^2*b*x + 27*a^3)/(2*b^4*x +
 3*a*b^3)
```

Sympy [F]

$$\int (A+Bx+Cx^2)\left(27a^3+27a^2bx -4b^3x^3\right)^{3/2}\,dx=\int \left(-(-3a+bx)\,(3a+2bx)^2\right)^{\frac{3}{2}}(A+Bx+Cx^2)\,dx$$

input
```
integrate((C*x**2+B*x+A)*(-4*b**3*x**3+27*a**2*b*x+27*a**3)**(3/2),x)
```

output
```
Integral((-(-3*a + b*x)*(3*a + 2*b*x)**2)**(3/2)*(A + B*x + C*x**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.60

$$\int (A+Bx+Cx^2)\left(27a^3+27a^2bx-4b^3x^3\right)^{3/2}\,dx=$$
$$-\frac{2\,(280\,b^5x^5+420\,ab^4x^4-3510\,a^2b^3x^3-10557\,a^3b^2x^2-648\,a^4bx+89667\,a^5)\sqrt{-bx+3\,a}A}{385\,b}$$
$$-\frac{2\,(280\,b^6x^6+420\,ab^5x^5-3150\,a^2b^4x^4-9045\,a^3b^3x^3-3078\,a^4b^2x^2+24543\,a^5bx+147258\,a^6)\sqrt{-bx+3}}{455\,b^2}$$
$$-\frac{2\,(616\,b^7x^7+924\,ab^6x^6-6426\,a^2b^5x^5-17955\,a^3b^4x^4-8100\,a^4b^3x^3+26973\,a^5b^2x^2+107892\,a^6bx+64}{1155\,b^3}$$

input
```
integrate((C*x^2+B*x+A)*(-4*b^3*x^3+27*a^2*b*x+27*a^3)^(3/2),x, algorithm=
"maxima")
```

output
```
-2/385*(280*b^5*x^5 + 420*a*b^4*x^4 - 3510*a^2*b^3*x^3 - 10557*a^3*b^2*x^2
 - 648*a^4*b*x + 89667*a^5)*sqrt(-b*x + 3*a)*A/b - 2/455*(280*b^6*x^6 + 42
0*a*b^5*x^5 - 3150*a^2*b^4*x^4 - 9045*a^3*b^3*x^3 - 3078*a^4*b^2*x^2 + 245
43*a^5*b*x + 147258*a^6)*sqrt(-b*x + 3*a)*B/b^2 - 2/1155*(616*b^7*x^7 + 92
4*a*b^6*x^6 - 6426*a^2*b^5*x^5 - 17955*a^3*b^4*x^4 - 8100*a^4*b^3*x^3 + 26
973*a^5*b^2*x^2 + 107892*a^6*b*x + 647352*a^7)*sqrt(-b*x + 3*a)*C/b^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1846 vs. $2(369) = 738$.

Time = 0.15 (sec) , antiderivative size = 1846, normalized size of antiderivative = 4.65

$$\int (A+Bx+Cx^2)\left(27a^3+27a^2bx-4b^3x^3\right)^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)*(-4*b^3*x^3+27*a^2*b*x+27*a^3)^(3/2),x, algorithm=
"giac")
```

output
```
2/15015*(3648645*sqrt(-b*x + 3*a)*A*a^5*sgn(-2*b*x - 3*a) - 1621620*((-b*x
 + 3*a)^(3/2) - 9*sqrt(-b*x + 3*a)*a)*A*a^4*sgn(-2*b*x - 3*a) - 1216215*((
-b*x + 3*a)^(3/2) - 9*sqrt(-b*x + 3*a)*a)*B*a^5*sgn(-2*b*x - 3*a)/b + 8108
1*((b*x - 3*a)^2*sqrt(-b*x + 3*a) - 10*(-b*x + 3*a)^(3/2)*a + 45*sqrt(-b*x
 + 3*a)*a^2)*A*a^3*sgn(-2*b*x - 3*a) + 729729*((b*x - 3*a)^2*sqrt(-b*x + 3
*a) - 10*(-b*x + 3*a)^(3/2)*a + 45*sqrt(-b*x + 3*a)*a^2)*C*a^5*sgn(-2*b*x
- 3*a)/b^2 + 972972*((b*x - 3*a)^2*sqrt(-b*x + 3*a) - 10*(-b*x + 3*a)^(3/2
)*a + 45*sqrt(-b*x + 3*a)*a^2)*B*a^4*sgn(-2*b*x - 3*a)/b - 38610*(5*(b*x -
 3*a)^3*sqrt(-b*x + 3*a) + 63*(b*x - 3*a)^2*sqrt(-b*x + 3*a)*a - 315*(-b*x
 + 3*a)^(3/2)*a^2 + 945*sqrt(-b*x + 3*a)*a^3)*A*a^2*sgn(-2*b*x - 3*a) + 13
8996*(5*(b*x - 3*a)^3*sqrt(-b*x + 3*a) + 63*(b*x - 3*a)^2*sqrt(-b*x + 3*a)
*a - 315*(-b*x + 3*a)^(3/2)*a^2 + 945*sqrt(-b*x + 3*a)*a^3)*C*a^4*sgn(-2*b
*x - 3*a)/b^2 + 11583*(5*(b*x - 3*a)^3*sqrt(-b*x + 3*a) + 63*(b*x - 3*a)^2
*sqrt(-b*x + 3*a)*a - 315*(-b*x + 3*a)^(3/2)*a^2 + 945*sqrt(-b*x + 3*a)*a^
3)*B*a^3*sgn(-2*b*x - 3*a)/b - 572*(35*(b*x - 3*a)^4*sqrt(-b*x + 3*a) + 54
0*(b*x - 3*a)^3*sqrt(-b*x + 3*a)*a + 3402*(b*x - 3*a)^2*sqrt(-b*x + 3*a)*a
^2 - 11340*(-b*x + 3*a)^(3/2)*a^3 + 25515*sqrt(-b*x + 3*a)*a^4)*A*a*sgn(-2
*b*x - 3*a) + 1287*(35*(b*x - 3*a)^4*sqrt(-b*x + 3*a) + 540*(b*x - 3*a)^3*
sqrt(-b*x + 3*a)*a + 3402*(b*x - 3*a)^2*sqrt(-b*x + 3*a)*a^2 - 11340*(-b*x
 + 3*a)^(3/2)*a^3 + 25515*sqrt(-b*x + 3*a)*a^4)*C*a^3*sgn(-2*b*x - 3*a)...
```

Mupad [B] (verification not implemented)

Time = 12.67 (sec) , antiderivative size = 333, normalized size of antiderivative = 0.84

$$\int (A+Bx+Cx^2)\left(27a^3+27a^2bx-4b^3x^3\right)^{3/2}\,dx = -\frac{\sqrt{27\,a^3+27\,a^2\,b\,x-4\,b^3\,x^3}\,\left(\frac{89667\,A\,a^5}{385\,b^2}-\frac{10557\,A\,a^3\,x^2}{385}+\frac{8\,A\,b^3\,x^5}{11}-\frac{702\,A\,a^2\,b\,x^3}{77}-\frac{648\,A\,a^4\,x}{385\,b}+\frac{12\,A\,a\,b^2\,x^4}{11}\right)}{x+\frac{3\,a}{2\,b}} - \frac{2\,B\,\sqrt{27\,a^3+27\,a^2\,b\,x-4\,b^3\,x^3}\,\left(9720\,a^5+1701\,a^4\,b\,x-2160\,a^3\,b^2\,x^2-1575\,a^2\,b^3\,x^3+140\,b^5\,x^5\right)}{455\,b^2} - \frac{2\,C\,\sqrt{27\,a^3+27\,a^2\,b\,x-4\,b^3\,x^3}\,\left(38637\,a^6+10206\,a^5\,b\,x+2187\,a^4\,b^2\,x^2-4158\,a^3\,b^3\,x^3-3213\,a^2\,b^4\,x^4+\right.}{1155\,b^3} - \frac{236196\,B\,a^6\,\sqrt{27\,a^3+27\,a^2\,b\,x-4\,b^3\,x^3}}{455\,b^2\,\left(3\,a+2\,b\,x\right)} - \frac{354294\,C\,a^7\,\sqrt{27\,a^3+27\,a^2\,b\,x-4\,b^3\,x^3}}{385\,b^3\,\left(3\,a+2\,b\,x\right)}$$

input
```
int((A + B*x + C*x^2)*(27*a^3 - 4*b^3*x^3 + 27*a^2*b*x)^(3/2),x)
```

output
```
- ((27*a^3 - 4*b^3*x^3 + 27*a^2*b*x)^(1/2)*((89667*A*a^5)/(385*b^2) - (105
57*A*a^3*x^2)/385 + (8*A*b^3*x^5)/11 - (702*A*a^2*b*x^3)/77 - (648*A*a^4*x
)/(385*b) + (12*A*a*b^2*x^4)/11))/(x + (3*a)/(2*b)) - (2*B*(27*a^3 - 4*b^3
*x^3 + 27*a^2*b*x)^(1/2)*(9720*a^5 + 140*b^5*x^5 - 2160*a^3*b^2*x^2 - 1575
*a^2*b^3*x^3 + 1701*a^4*b*x))/(455*b^2) - (2*C*(27*a^3 - 4*b^3*x^3 + 27*a^
2*b*x)^(1/2)*(38637*a^6 + 308*b^6*x^6 + 2187*a^4*b^2*x^2 - 4158*a^3*b^3*x^
3 - 3213*a^2*b^4*x^4 + 10206*a^5*b*x))/(1155*b^3) - (236196*B*a^6*(27*a^3
- 4*b^3*x^3 + 27*a^2*b*x)^(1/2))/(455*b^2*(3*a + 2*b*x)) - (354294*C*a^7*(
27*a^3 - 4*b^3*x^3 + 27*a^2*b*x)^(1/2))/(385*b^3*(3*a + 2*b*x))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.41

$$\int (A + Bx + Cx^2) (27a^3 + 27a^2bx - 4b^3x^3)^{3/2} \, dx = \frac{2\sqrt{-bx+3a}\,(-8008b^7c\,x^7 - 12012a\,b^6c\,x^6 - 9240b^8x^6 + 83538a^2b^5c\,x^5 - 24780a\,b^7x^5 + \cdots}{\cdots}$$

input
```
int((C*x^2+B*x+A)*(-4*b^3*x^3+27*a^2*b*x+27*a^3)^(3/2),x)
```

output
```
(2*sqrt(3*a - b*x)*( - 8415576*a**7*c - 8356527*a**6*b**2 - 1402596*a**6*b
*c*x - 784647*a**5*b**3*x - 350649*a**5*b**2*c*x**2 + 513297*a**4*b**4*x**
2 + 105300*a**4*b**3*c*x**3 + 435375*a**3*b**5*x**3 + 233415*a**3*b**4*c*x
**4 + 87570*a**2*b**6*x**4 + 83538*a**2*b**5*c*x**5 - 24780*a*b**7*x**5 -
12012*a*b**6*c*x**6 - 9240*b**8*x**6 - 8008*b**7*c*x**7))/(15015*b**3)
```

3.10 $\int (A+Bx+Cx^2)\sqrt{27a^3+27a^2bx-4b^3x^3}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 251

$$\begin{aligned}\int (A+Bx+Cx^2)\sqrt{27a^3+27a^2bx-4b^3x^3}\,dx\\ &= -\frac{6a(Ab^2+3a(bB+3aC))(3a-bx)\sqrt{27a^3+27a^2bx-4b^3x^3}}{b^3(3a+2bx)}\\ &\quad+\frac{2(2Ab^2+15abB+72a^2C)(3a-bx)^2\sqrt{27a^3+27a^2bx-4b^3x^3}}{5b^3(3a+2bx)}\\ &\quad-\frac{2(2bB+21aC)(3a-bx)^3\sqrt{27a^3+27a^2bx-4b^3x^3}}{7b^3(3a+2bx)}\\ &\quad+\frac{4C(3a-bx)^4\sqrt{27a^3+27a^2bx-4b^3x^3}}{9b^3(3a+2bx)}\end{aligned}$$

output
```
-6*a*(A*b^2+3*a*(B*b+3*C*a))*(-b*x+3*a)*(-4*b^3*x^3+27*a^2*b*x+27*a^3)^(1/
2)/b^3/(2*b*x+3*a)+2/5*(2*A*b^2+15*B*a*b+72*C*a^2)*(-b*x+3*a)^2*(-4*b^3*x^
3+27*a^2*b*x+27*a^3)^(1/2)/b^3/(2*b*x+3*a)-2/7*(2*B*b+21*C*a)*(-b*x+3*a)^3
*(-4*b^3*x^3+27*a^2*b*x+27*a^3)^(1/2)/b^3/(2*b*x+3*a)+4/9*C*(-b*x+3*a)^4*(
-4*b^3*x^3+27*a^2*b*x+27*a^3)^(1/2)/b^3/(2*b*x+3*a)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.44

$$\int \left(A+Bx+Cx^2\right) \sqrt{27a^3+27a^2bx-4b^3x^3}\,dx$$
$$= \frac{2(-3a+bx)\sqrt{(3a-bx)(3a+2bx)^2}(1512a^3C+54a^2b(15B+14Cx)+9ab^2(63A+5x(9B+7Cx))+2}{315b^3(3a+2bx)}$$

input
```
Integrate[(A + B*x + C*x^2)*Sqrt[27*a^3 + 27*a^2*b*x - 4*b^3*x^3],x]
```

output
```
(2*(-3*a + b*x)*Sqrt[(3*a - b*x)*(3*a + 2*b*x)^2]*(1512*a^3*C + 54*a^2*b*(
15*B + 14*C*x) + 9*a*b^2*(63*A + 5*x*(9*B + 7*C*x)) + 2*b^3*x*(63*A + 5*x*
(9*B + 7*C*x))))/(315*b^3*(3*a + 2*b*x))
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.80, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2526, 27, 2483, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{27a^3+27a^2bx-4b^3x^3}\left(A+Bx+Cx^2\right)\,dx$$

$$\Big\downarrow \text{ 2526}$$

$$-\frac{\int -3b\left(9Ca^2+4Ab^2+4b^2Bx\right)\sqrt{27a^3+27bxa^2-4b^3x^3}dx}{12b^3}-\frac{C\left(27a^3+27a^2bx-4b^3x^3\right)^{3/2}}{18b^3}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \left(9Ca^2+4Ab^2+4b^2Bx\right)\sqrt{27a^3+27bxa^2-4b^3x^3}dx}{4b^2}-\frac{C\left(27a^3+27a^2bx-4b^3x^3\right)^{3/2}}{18b^3}$$

$$\Big\downarrow \text{ 2483}$$

$$\frac{\sqrt{27a^3+27a^2bx-4b^3x^3}\int 81\sqrt{3}a^2(3a+2bx)\sqrt{3a^3-a^2bx}\left(9Ca^2+4Ab^2+4b^2Bx\right)dx}{324\sqrt{3}a^2b^2(3a+2bx)\sqrt{3a^3-a^2bx}}-\frac{C\left(27a^3+27a^2bx-4b^3x^3\right)^{3/2}}{18b^3}$$

$\Big\downarrow$ 27

$$\frac{\sqrt{27a^3+27a^2bx-4b^3x^3}\int (3a+2bx)\sqrt{3a^3-a^2bx}\left(9Ca^2+4Ab^2+4b^2Bx\right)dx}{4b^2(3a+2bx)\sqrt{3a^3-a^2bx}}-\frac{C\left(27a^3+27a^2bx-4b^3x^3\right)^{3/2}}{18b^3}$$

$\Big\downarrow$ 86

$$\frac{\sqrt{27a^3+27a^2bx-4b^3x^3}\int \left(\frac{8bB\left(3a^3-a^2bx\right)^{5/2}}{a^4}-\frac{2\left(9Ca^2+30bBa+4Ab^2\right)\left(3a^3-a^2bx\right)^{3/2}}{a^2}+9a\left(9Ca^2+12bBa+4Ab^2\right)\sqrt{3\ldots}\right.}{4b^2(3a+2bx)\sqrt{3a^3-a^2bx}}$$
$$\frac{C\left(27a^3+27a^2bx-4b^3x^3\right)^{3/2}}{18b^3}$$

$\Big\downarrow$ 2009

$$\frac{\sqrt{27a^3+27a^2bx-4b^3x^3}\left(-6\left(3a^3-a^2bx\right)^{3/2}\left(\frac{4Ab}{a}+\frac{9aC}{b}+12B\right)-\frac{16B\left(3a^3-a^2bx\right)^{7/2}}{7a^6}+\frac{4\left(3a^3-a^2bx\right)^{5/2}\left(9a^2C+30abB\ldots\right.}{5a^4b}\right.}{4b^2(3a+2bx)\sqrt{3a^3-a^2bx}}$$
$$\frac{C\left(27a^3+27a^2bx-4b^3x^3\right)^{3/2}}{18b^3}$$

input
```
Int[(A + B*x + C*x^2)*Sqrt[27*a^3 + 27*a^2*b*x - 4*b^3*x^3],x]
```

output
```
-1/18*(C*(27*a^3 + 27*a^2*b*x - 4*b^3*x^3)^(3/2))/b^3 + (Sqrt[27*a^3 + 27*
a^2*b*x - 4*b^3*x^3]*(-6*((4*A*b)/a + 12*B + (9*a*C)/b)*(3*a^3 - a^2*b*x)^
(3/2) + (4*(4*A*b^2 + 30*a*b*B + 9*a^2*C)*(3*a^3 - a^2*b*x)^(5/2))/(5*a^4*
b) - (16*B*(3*a^3 - a^2*b*x)^(7/2))/(7*a^6)))/(4*b^2*(3*a + 2*b*x)*Sqrt[3*
a^3 - a^2*b*x])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2483
```
Int[((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (d_.)*(x_)^3)^(p_), x_S
ymbol] :> Simp[(a + b*x + d*x^3)^p/((3*a - b*x)^p*(3*a + 2*b*x)^(2*p))   In
t[(e + f*x)^m*(3*a - b*x)^p*(3*a + 2*b*x)^(2*p), x], x] /; FreeQ[{a, b, d,
e, f, m, p}, x] && EqQ[4*b^3 + 27*a^2*d, 0] &&  !IntegerQ[p]
```

rule 2526
```
Int[(Pm_)*(Qn_)^(p_), x_Symbol] :> With[{m = Expon[Pm, x], n = Expon[Qn, x]
}, Simp[Coeff[Pm, x, m]*(Qn^(p + 1)/(n*(p + 1)*Coeff[Qn, x, n])), x] + Simp
[1/(n*Coeff[Qn, x, n])   Int[ExpandToSum[n*Coeff[Qn, x, n]*Pm - Coeff[Pm, x
, m]*D[Qn, x], x]*Qn^p, x], x] /; EqQ[m, n - 1]] /; FreeQ[p, x] && PolyQ[Pm
, x] && PolyQ[Qn, x] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.47

method	result
gosper	$-\frac{2(-bx+3a)(70C\,x^3b^3+90B\,b^3x^2+315Ca\,b^2x^2+126A\,b^3x+405Ba\,b^2x+756C\,a^2bx+567Aa\,b^2+810B\,a^2b+1512C\,a^3)\sqrt{-4b^3x^3+}}{315b^3(2bx+3a)}$
default	$-\frac{2(-bx+3a)(70C\,x^3b^3+90B\,b^3x^2+315Ca\,b^2x^2+126A\,b^3x+405Ba\,b^2x+756C\,a^2bx+567Aa\,b^2+810B\,a^2b+1512C\,a^3)\sqrt{-4b^3x^3+}}{315b^3(2bx+3a)}$
orering	$-\frac{2(-bx+3a)(70C\,x^3b^3+90B\,b^3x^2+315Ca\,b^2x^2+126A\,b^3x+405Ba\,b^2x+756C\,a^2bx+567Aa\,b^2+810B\,a^2b+1512C\,a^3)\sqrt{-4b^3x^3+}}{315b^3(2bx+3a)}$
risch	$-\frac{2\sqrt{(-bx+3a)(2bx+3a)^2}\,(-70C\,b^4x^4-90B\,b^4x^3-105Ca\,b^3x^3-126A\,b^4x^2-135Ba\,b^3x^2+189C\,a^2b^2x^2-189Aa\,b^3x+405B\,a^2b^2x}{315(2bx+3a)b^3}$
trager	$-\frac{2(-70C\,b^4x^4-90B\,b^4x^3-105Ca\,b^3x^3-126A\,b^4x^2-135Ba\,b^3x^2+189C\,a^2b^2x^2-189Aa\,b^3x+405B\,a^2b^2x+756C\,a^3bx+1701A\,a^2}{315(2bx+3a)b^3}$

input
```
int((C*x^2+B*x+A)*(-4*b^3*x^3+27*a^2*b*x+27*a^3)^(1/2),x,method=_RETURNVER
BOSE)
```

output
```
-2/315*(-b*x+3*a)*(70*C*b^3*x^3+90*B*b^3*x^2+315*C*a*b^2*x^2+126*A*b^3*x+4
05*B*a*b^2*x+756*C*a^2*b*x+567*A*a*b^2+810*B*a^2*b+1512*C*a^3)*(-4*b^3*x^3
+27*a^2*b*x+27*a^3)^(1/2)/b^3/(2*b*x+3*a)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.58

$$\int \left(A+Bx+Cx^2\right)\sqrt{27a^3+27a^2bx-4b^3x^3}\,dx$$
$$=\frac{2\left(70\,Cb^4x^4-4536\,Ca^4-2430\,Ba^3b-1701\,Aa^2b^2+15\,(7\,Cab^3+6\,Bb^4)x^3-9\,(21\,Ca^2b^2-15\,Bab^3-\right.}{315\,(2\,b^4x+3\,ab^3)}$$

input
```
integrate((C*x^2+B*x+A)*(-4*b^3*x^3+27*a^2*b*x+27*a^3)^(1/2),x, algorithm=
"fricas")
```

output
```
2/315*(70*C*b^4*x^4 - 4536*C*a^4 - 2430*B*a^3*b - 1701*A*a^2*b^2 + 15*(7*C
*a*b^3 + 6*B*b^4)*x^3 - 9*(21*C*a^2*b^2 - 15*B*a*b^3 - 14*A*b^4)*x^2 - 27*
(28*C*a^3*b + 15*B*a^2*b^2 - 7*A*a*b^3)*x)*sqrt(-4*b^3*x^3 + 27*a^2*b*x +
27*a^3)/(2*b^4*x + 3*a*b^3)
```

Sympy [F]

$$\int (A+Bx+Cx^2)\sqrt{27a^3+27a^2bx-4b^3x^3}\,dx$$
$$=\int \sqrt{-(-3a+bx)(3a+2bx)^2}\left(A+Bx+Cx^2\right)\,dx$$

input
```
integrate((C*x**2+B*x+A)*(-4*b**3*x**3+27*a**2*b*x+27*a**3)**(1/2),x)
```

output
```
Integral(sqrt(-(-3*a + b*x)*(3*a + 2*b*x)**2)*(A + B*x + C*x**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.55

$$\int (A+Bx+Cx^2)\sqrt{27a^3+27a^2bx-4b^3x^3}\,dx$$
$$=\frac{2\,(2\,b^2x^2+3\,abx-27\,a^2)\sqrt{-bx+3\,a}A}{5\,b}$$
$$+\frac{2\,(2\,b^3x^3+3\,ab^2x^2-9\,a^2bx-54\,a^3)\sqrt{-bx+3\,a}B}{7\,b^2}$$
$$+\frac{2\,(10\,b^4x^4+15\,ab^3x^3-27\,a^2b^2x^2-108\,a^3bx-648\,a^4)\sqrt{-bx+3\,a}C}{45\,b^3}$$

input
```
integrate((C*x^2+B*x+A)*(-4*b^3*x^3+27*a^2*b*x+27*a^3)^(1/2),x, algorithm=
"maxima")
```

output
```
2/5*(2*b^2*x^2 + 3*a*b*x - 27*a^2)*sqrt(-b*x + 3*a)*A/b + 2/7*(2*b^3*x^3 +
 3*a*b^2*x^2 - 9*a^2*b*x - 54*a^3)*sqrt(-b*x + 3*a)*B/b^2 + 2/45*(10*b^4*x
^4 + 15*a*b^3*x^3 - 27*a^2*b^2*x^2 - 108*a^3*b*x - 648*a^4)*sqrt(-b*x + 3*
a)*C/b^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 595 vs. $2(233) = 466$.

Time = 0.14 (sec) , antiderivative size = 595, normalized size of antiderivative = 2.37

$$\int (A + Bx + Cx^2) \sqrt{27a^3 + 27a^2bx - 4b^3x^3} \, dx = \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)*(-4*b^3*x^3+27*a^2*b*x+27*a^3)^(1/2),x, algorithm=
"giac")
```

output
```
2/315*(2835*sqrt(-b*x + 3*a)*A*a^2*sgn(-2*b*x - 3*a) - 315*((-b*x + 3*a)^(
3/2) - 9*sqrt(-b*x + 3*a)*a)*A*a*sgn(-2*b*x - 3*a) - 945*((-b*x + 3*a)^(3/
2) - 9*sqrt(-b*x + 3*a)*a)*B*a^2*sgn(-2*b*x - 3*a)/b - 126*((b*x - 3*a)^2*
sqrt(-b*x + 3*a) - 10*(-b*x + 3*a)^(3/2)*a + 45*sqrt(-b*x + 3*a)*a^2)*A*sg
n(-2*b*x - 3*a) + 567*((b*x - 3*a)^2*sqrt(-b*x + 3*a) - 10*(-b*x + 3*a)^(3
/2)*a + 45*sqrt(-b*x + 3*a)*a^2)*C*a^2*sgn(-2*b*x - 3*a)/b^2 + 189*((b*x -
 3*a)^2*sqrt(-b*x + 3*a) - 10*(-b*x + 3*a)^(3/2)*a + 45*sqrt(-b*x + 3*a)*a
^2)*B*a*sgn(-2*b*x - 3*a)/b + 27*(5*(b*x - 3*a)^3*sqrt(-b*x + 3*a) + 63*(b
*x - 3*a)^2*sqrt(-b*x + 3*a)*a - 315*(-b*x + 3*a)^(3/2)*a^2 + 945*sqrt(-b*
x + 3*a)*a^3)*C*a*sgn(-2*b*x - 3*a)/b^2 - 18*(5*(b*x - 3*a)^3*sqrt(-b*x +
3*a) + 63*(b*x - 3*a)^2*sqrt(-b*x + 3*a)*a - 315*(-b*x + 3*a)^(3/2)*a^2 +
945*sqrt(-b*x + 3*a)*a^3)*B*sgn(-2*b*x - 3*a)/b - 2*(35*(b*x - 3*a)^4*sqrt
(-b*x + 3*a) + 540*(b*x - 3*a)^3*sqrt(-b*x + 3*a)*a + 3402*(b*x - 3*a)^2*s
qrt(-b*x + 3*a)*a^2 - 11340*(-b*x + 3*a)^(3/2)*a^3 + 25515*sqrt(-b*x + 3*a
)*a^4)*C*sgn(-2*b*x - 3*a)/b^2)/b
```

Mupad [B] (verification not implemented)

Time = 12.55 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.47

$$\int (A+Bx+Cx^2)\sqrt{27a^3+27a^2bx-4b^3x^3}\,dx = -\frac{2\,(3\,a-b\,x)\,\sqrt{27\,a^3+27\,a^2\,b\,x-4\,b^3\,x^3}\,(1512\,C\,a^3+756\,C\,a^2\,b\,x+810\,B\,a^2\,b+315\,C\,a\,b^2\,x^2+405\,B}{315\,b^3\,(3\,a+2\,b\,x)}$$

input
```
int((A + B*x + C*x^2)*(27*a^3 - 4*b^3*x^3 + 27*a^2*b*x)^(1/2),x)
```

output
```
-(2*(3*a - b*x)*(27*a^3 - 4*b^3*x^3 + 27*a^2*b*x)^(1/2)*(1512*C*a^3 + 90*B
*b^3*x^2 + 70*C*b^3*x^3 + 567*A*a*b^2 + 810*B*a^2*b + 126*A*b^3*x + 405*B*
a*b^2*x + 756*C*a^2*b*x + 315*C*a*b^2*x^2))/(315*b^3*(3*a + 2*b*x))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.37

$$\int (A+Bx+Cx^2)\sqrt{27a^3+27a^2bx-4b^3x^3}\,dx = \frac{2\sqrt{-bx+3a}\,(70b^4c\,x^4+105a\,b^3c\,x^3+90b^5x^3-189a^2b^2c\,x^2+261a\,b^4x^2-756a^3bcx-216a^2b^3x-4536}{315b^3}$$

input
```
int((C*x^2+B*x+A)*(-4*b^3*x^3+27*a^2*b*x+27*a^3)^(1/2),x)
```

output
```
(2*sqrt(3*a - b*x)*( - 4536*a**4*c - 4131*a**3*b**2 - 756*a**3*b*c*x - 216
*a**2*b**3*x - 189*a**2*b**2*c*x**2 + 261*a*b**4*x**2 + 105*a*b**3*c*x**3
+ 90*b**5*x**3 + 70*b**4*c*x**4))/(315*b**3)
```

3.11 $\int \frac{A+Bx+Cx^2}{\sqrt{27a^3+27a^2bx-4b^3x^3}} \, dx$

Optimal result

Integrand size = 36, antiderivative size = 208

$$\int \frac{A+Bx+Cx^2}{\sqrt{27a^3+27a^2bx-4b^3x^3}} \, dx = -\frac{(2bB+3aC)(3a-bx)(3a+2bx)}{2b^3\sqrt{27a^3+27a^2bx-4b^3x^3}} + \frac{C(3a-bx)^2(3a+2bx)}{3b^3\sqrt{27a^3+27a^2bx-4b^3x^3}} - \frac{(4Ab^2-6abB+9a^2C)(3a+2bx)\sqrt{3-\frac{bx}{a}}\operatorname{arctanh}\left(\frac{1}{3}\sqrt{2}\sqrt{3-\frac{bx}{a}}\right)}{6\sqrt{2}b^3\sqrt{27a^3+27a^2bx-4b^3x^3}}$$

output

```
-1/2*(2*B*b+3*C*a)*(-b*x+3*a)*(2*b*x+3*a)/b^3/(-4*b^3*x^3+27*a^2*b*x+27*a^
3)^(1/2)+1/3*C*(-b*x+3*a)^2*(2*b*x+3*a)/b^3/(-4*b^3*x^3+27*a^2*b*x+27*a^3)
^(1/2)-1/12*(4*A*b^2-6*B*a*b+9*C*a^2)*(2*b*x+3*a)*(3-b*x/a)^(1/2)*arctanh(
1/3*2^(1/2)*(3-b*x/a)^(1/2))*2^(1/2)/b^3/(-4*b^3*x^3+27*a^2*b*x+27*a^3)^(1
/2)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.65

$$\int \frac{A+Bx+Cx^2}{\sqrt{27a^3+27a^2bx-4b^3x^3}}\,dx = -\frac{\sqrt{3a-bx}(3a+2bx)\left(2\sqrt{a}\sqrt{3a-bx}(6bB+3aC+2bCx)+\sqrt{2}(4Ab^2-6abB+9a^2C)\operatorname{arctanh}\left(\frac{\sqrt{6a-}}{3\sqrt{}}\right.\right.}{12\sqrt{a}b^3\sqrt{(3a-bx)(3a+2bx)^2}}$$

input
```
Integrate[(A + B*x + C*x^2)/Sqrt[27*a^3 + 27*a^2*b*x - 4*b^3*x^3],x]
```

output
```
-1/12*(Sqrt[3*a - b*x]*(3*a + 2*b*x)*(2*Sqrt[a]*Sqrt[3*a - b*x]*(6*b*B + 3
*a*C + 2*b*C*x) + Sqrt[2]*(4*A*b^2 - 6*a*b*B + 9*a^2*C)*ArcTanh[Sqrt[6*a -
 2*b*x]/(3*Sqrt[a])]))/(Sqrt[a]*b^3*Sqrt[(3*a - b*x)*(3*a + 2*b*x)^2])
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.88, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {2526, 27, 2483, 27, 90, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{\sqrt{27a^3+27a^2bx-4b^3x^3}}dx$$

$$\Big\downarrow \text{2526}$$

$$-\frac{\int -\frac{3b(9Ca^2+4Ab^2+4b^2Bx)}{\sqrt{27a^3+27bxa^2-4b^3x^3}}dx}{12b^3}-\frac{C\sqrt{27a^3+27a^2bx-4b^3x^3}}{6b^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{9Ca^2+4Ab^2+4b^2Bx}{\sqrt{27a^3+27bxa^2-4b^3x^3}}dx}{4b^2}-\frac{C\sqrt{27a^3+27a^2bx-4b^3x^3}}{6b^3}$$

$$\Big\downarrow \text{2483}$$

$$\frac{81\sqrt{3}a^2(3a+2bx)\sqrt{3a^3-a^2bx}\int \frac{9Ca^2+4Ab^2+4b^2Bx}{81\sqrt{3}a^2(3a+2bx)\sqrt{3a^3-a^2bx}}dx}{4b^2\sqrt{27a^3+27a^2bx-4b^3x^3}}-\frac{C\sqrt{27a^3+27a^2bx-4b^3x^3}}{6b^3}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{(3a+2bx)\sqrt{3a^3-a^2bx}\int \frac{9Ca^2+4Ab^2+4b^2Bx}{(3a+2bx)\sqrt{3a^3-a^2bx}}dx}{4b^2\sqrt{27a^3+27a^2bx-4b^3x^3}}-\frac{C\sqrt{27a^3+27a^2bx-4b^3x^3}}{6b^3}$$

$$\Big\downarrow \textbf{90}$$

$$\frac{(3a+2bx)\sqrt{3a^3-a^2bx}\left(\left(9a^2C-6abB+4Ab^2\right)\int \frac{1}{(3a+2bx)\sqrt{3a^3-a^2bx}}dx-\frac{4B\sqrt{3a^3-a^2bx}}{a^2}\right)}{4b^2\sqrt{27a^3+27a^2bx-4b^3x^3}}-\frac{C\sqrt{27a^3+27a^2bx-4b^3x^3}}{6b^3}$$

$$\Big\downarrow \textbf{73}$$

$$\frac{(3a+2bx)\sqrt{3a^3-a^2bx}\left(-\frac{2\left(9a^2C-6abB+4Ab^2\right)\int \frac{1}{9a-\frac{2\left(3a^3-a^2bx\right)}{a^2}}d\sqrt{3a^3-a^2bx}}{a^2b}-\frac{4B\sqrt{3a^3-a^2bx}}{a^2}\right)}{4b^2\sqrt{27a^3+27a^2bx-4b^3x^3}}-\frac{C\sqrt{27a^3+27a^2bx-4b^3x^3}}{6b^3}$$

$$\Big\downarrow \textbf{221}$$

$$\frac{(3a+2bx)\sqrt{3a^3-a^2bx}\left(-\frac{4B\sqrt{3a^3-a^2bx}}{a^2}-\frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{3a^3-a^2bx}}{3a^{3/2}}\right)\left(9a^2C-6abB+4Ab^2\right)}{3a^{3/2}b}\right)}{4b^2\sqrt{27a^3+27a^2bx-4b^3x^3}}-\frac{C\sqrt{27a^3+27a^2bx-4b^3x^3}}{6b^3}$$

input
```
Int[(A + B*x + C*x^2)/Sqrt[27*a^3 + 27*a^2*b*x - 4*b^3*x^3],x]
```

output
```
-1/6*(C*Sqrt[27*a^3 + 27*a^2*b*x - 4*b^3*x^3])/b^3 + ((3*a + 2*b*x)*Sqrt[3
*a^3 - a^2*b*x]*((-4*B*Sqrt[3*a^3 - a^2*b*x])/a^2 - (Sqrt[2]*(4*A*b^2 - 6*
a*b*B + 9*a^2*C)*ArcTanh[(Sqrt[2]*Sqrt[3*a^3 - a^2*b*x])/(3*a^(3/2))])/(3*
a^(3/2)*b)))/(4*b^2*Sqrt[27*a^3 + 27*a^2*b*x - 4*b^3*x^3])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 2483
```
Int[((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (d_.)*(x_)^3)^(p_), x_S
ymbol] :> Simp[(a + b*x + d*x^3)^p/((3*a - b*x)^p*(3*a + 2*b*x)^(2*p))   In
t[(e + f*x)^m*(3*a - b*x)^p*(3*a + 2*b*x)^(2*p), x], x] /; FreeQ[{a, b, d,
e, f, m, p}, x] && EqQ[4*b^3 + 27*a^2*d, 0] &&  !IntegerQ[p]
```

rule 2526
```
Int[(Pm_)*(Qn_)^(p_), x_Symbol] :> With[{m = Expon[Pm, x], n = Expon[Qn, x]
}, Simp[Coeff[Pm, x, m]*(Qn^(p + 1)/(n*(p + 1)*Coeff[Qn, x, n])), x] + Simp
[1/(n*Coeff[Qn, x, n])   Int[ExpandToSum[n*Coeff[Qn, x, n]*Pm - Coeff[Pm, x
, m]*D[Qn, x], x]*Qn^p, x], x] /; EqQ[m, n - 1]] /; FreeQ[p, x] && PolyQ[Pm
, x] && PolyQ[Qn, x] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.70

method	result
risch	$-\frac{(2Cxb+6Bb+3Ca)(-bx+3a)(2bx+3a)}{6b^3\sqrt{(-bx+3a)(2bx+3a)^2}}-\frac{(4A\,b^2-6abB+9C\,a^2)\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-bx+3a}\,\sqrt{2}}{3\sqrt{a}}\right)\sqrt{-bx+3a}\,(2bx+3a)}{12b^3\sqrt{a}\,\sqrt{(-bx+3a)(2bx+3a)^2}}$
default	$-\frac{(2bx+3a)\sqrt{-bx+3a}\,\left(4A\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-bx+3a}\,\sqrt{2}}{3\sqrt{a}}\right)b^2-6B\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-bx+3a}\,\sqrt{2}}{3\sqrt{a}}\right)ab+9C\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-bx+3a}\,\sqrt{2}}{3\sqrt{a}}\right)a^2-4C\right.}{12\sqrt{-4b^3x^3+27b\,a^2x+27a^3}\,b^3\sqrt{a}}$

input
```
int((C*x^2+B*x+A)/(-4*b^3*x^3+27*a^2*b*x+27*a^3)^(1/2),x,method=_RETURNVER
BOSE)
```

output
```
-1/6*(2*C*b*x+6*B*b+3*C*a)/b^3*(-b*x+3*a)/((-b*x+3*a)*(2*b*x+3*a)^2)^(1/2)
*(2*b*x+3*a)-1/12*(4*A*b^2-6*B*a*b+9*C*a^2)/b^3/a^(1/2)*2^(1/2)*arctanh(1/
3*(-b*x+3*a)^(1/2)*2^(1/2)/a^(1/2))/((-b*x+3*a)*(2*b*x+3*a)^2)^(1/2)*(-b*x
+3*a)^(1/2)*(2*b*x+3*a)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 363, normalized size of antiderivative = 1.75

$$\int \frac{A+Bx+Cx^2}{\sqrt{27a^3+27a^2bx-4b^3x^3}}\,dx$$
$$=\left[\frac{\sqrt{2}(27\,Ca^3-18\,Ba^2b+12\,Aab^2+2\,(9\,Ca^2b-6\,Bab^2+4\,Ab^3)x)\sqrt{a}\log\left(\frac{4\,b^2x^2-24\,abx-45\,a^2+6\,\sqrt{2}\sqrt{-4\,b^3x^3}}{4\,b^2x^2+12\,abx+9\,a^2}\right.}{24\,(2\,ab^4x+3\,a^2b^3)}\right.$$
$$-\frac{\sqrt{2}(27\,Ca^3-18\,Ba^2b+12\,Aab^2+2\,(9\,Ca^2b-6\,Bab^2+4\,Ab^3)x)\sqrt{-a}\arctan\left(\frac{3\,\sqrt{2}\sqrt{-4\,b^3x^3+27\,a^2bx+27\,a}}{2\,(2\,b^2x^2-3\,abx-9\,a^2)}\right.}{12\,(2\,ab^4x+3\,a^2b^3)}$$

input
```
integrate((C*x^2+B*x+A)/(-4*b^3*x^3+27*a^2*b*x+27*a^3)^(1/2),x, algorithm=
"fricas")
```

output
```
[1/24*(sqrt(2)*(27*C*a^3 - 18*B*a^2*b + 12*A*a*b^2 + 2*(9*C*a^2*b - 6*B*a*
b^2 + 4*A*b^3)*x)*sqrt(a)*log((4*b^2*x^2 - 24*a*b*x - 45*a^2 + 6*sqrt(2)*s
qrt(-4*b^3*x^3 + 27*a^2*b*x + 27*a^3)*sqrt(a))/(4*b^2*x^2 + 12*a*b*x + 9*a
^2)) - 4*sqrt(-4*b^3*x^3 + 27*a^2*b*x + 27*a^3)*(2*C*a*b*x + 3*C*a^2 + 6*B
*a*b))/(2*a*b^4*x + 3*a^2*b^3), -1/12*(sqrt(2)*(27*C*a^3 - 18*B*a^2*b + 12
*A*a*b^2 + 2*(9*C*a^2*b - 6*B*a*b^2 + 4*A*b^3)*x)*sqrt(-a)*arctan(3/2*sqrt
(2)*sqrt(-4*b^3*x^3 + 27*a^2*b*x + 27*a^3)*sqrt(-a)/(2*b^2*x^2 - 3*a*b*x -
 9*a^2)) + 2*sqrt(-4*b^3*x^3 + 27*a^2*b*x + 27*a^3)*(2*C*a*b*x + 3*C*a^2 +
 6*B*a*b))/(2*a*b^4*x + 3*a^2*b^3)]
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt{27a^3+27a^2bx-4b^3x^3}}\,dx=\int \frac{A+Bx+Cx^2}{\sqrt{-\left(-3a+bx\right)\left(3a+2bx\right)^2}}\,dx$$

input
```
integrate((C*x**2+B*x+A)/(-4*b**3*x**3+27*a**2*b*x+27*a**3)**(1/2),x)
```

output
```
Integral((A + B*x + C*x**2)/sqrt(-(-3*a + b*x)*(3*a + 2*b*x)**2), x)
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt{27a^3+27a^2bx-4b^3x^3}}\,dx=\int \frac{Cx^2+Bx+A}{\sqrt{-4\,b^3x^3+27\,a^2bx+27\,a^3}}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(-4*b^3*x^3+27*a^2*b*x+27*a^3)^(1/2),x, algorithm=
"maxima")
```

output
```
integrate((C*x^2 + B*x + A)/sqrt(-4*b^3*x^3 + 27*a^2*b*x + 27*a^3), x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.62

$$\int \frac{A+Bx+Cx^2}{\sqrt{27a^3+27a^2bx-4b^3x^3}}\,dx = -\frac{\sqrt{2}(9\,Ca^2-6\,Bab+4\,Ab^2)\arctan\left(\frac{\sqrt{2}\sqrt{-bx+3\,a}}{3\,\sqrt{-a}}\right)}{12\,\sqrt{-a}b^3\mathrm{sgn}\left(-2\,bx-3\,a\right)} - \frac{2\,(-bx+3\,a)^{\frac{3}{2}}Cb^6-9\,\sqrt{-bx+3\,a}Cab^6-6\,\sqrt{-bx+3\,a}Bb^7}{6\,b^9\mathrm{sgn}\left(-2\,bx-3\,a\right)}$$

input
```
integrate((C*x^2+B*x+A)/(-4*b^3*x^3+27*a^2*b*x+27*a^3)^(1/2),x, algorithm=
"giac")
```

output
```
-1/12*sqrt(2)*(9*C*a^2 - 6*B*a*b + 4*A*b^2)*arctan(1/3*sqrt(2)*sqrt(-b*x +
 3*a)/sqrt(-a))/(sqrt(-a)*b^3*sgn(-2*b*x - 3*a)) - 1/6*(2*(-b*x + 3*a)^(3/
2)*C*b^6 - 9*sqrt(-b*x + 3*a)*C*a*b^6 - 6*sqrt(-b*x + 3*a)*B*b^7)/(b^9*sgn
(-2*b*x - 3*a))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{\sqrt{27a^3+27a^2bx-4b^3x^3}}\,dx = \int \frac{C\,x^2+B\,x+A}{\sqrt{27\,a^3+27\,a^2\,b\,x-4\,b^3\,x^3}}\,dx$$

input
```
int((A + B*x + C*x^2)/(27*a^3 - 4*b^3*x^3 + 27*a^2*b*x)^(1/2),x)
```

output
```
int((A + B*x + C*x^2)/(27*a^3 - 4*b^3*x^3 + 27*a^2*b*x)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.75

$$\int \frac{A + Bx + Cx^2}{\sqrt{27a^3 + 27a^2bx - 4b^3x^3}}\,dx$$
$$= \frac{-12\sqrt{-bx+3a}\,ac - 24\sqrt{-bx+3a}\,b^2 - 8\sqrt{-bx+3a}\,bcx + 9\sqrt{a}\,\sqrt{2}\log\left(2\sqrt{-bx+3a} - 3\sqrt{a}\,\sqrt{2}\right)ac - 2$$

input
```
int((C*x^2+B*x+A)/(-4*b^3*x^3+27*a^2*b*x+27*a^3)^(1/2),x)
```

output
```
( - 12*sqrt(3*a - b*x)*a*c - 24*sqrt(3*a - b*x)*b**2 - 8*sqrt(3*a - b*x)*b
*c*x + 9*sqrt(a)*sqrt(2)*log(2*sqrt(3*a - b*x) - 3*sqrt(a)*sqrt(2))*a*c -
2*sqrt(a)*sqrt(2)*log(2*sqrt(3*a - b*x) - 3*sqrt(a)*sqrt(2))*b**2 - 9*sqrt
(a)*sqrt(2)*log(2*sqrt(3*a - b*x) + 3*sqrt(a)*sqrt(2))*a*c + 2*sqrt(a)*sqr
t(2)*log(2*sqrt(3*a - b*x) + 3*sqrt(a)*sqrt(2))*b**2)/(24*b**3)
```

3.12 $\int \frac{A+Bx+Cx^2}{(27a^3+27a^2bx-4b^3x^3)^{3/2}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 320

$$\begin{aligned}\int \frac{A+Bx+Cx^2}{(27a^3+27a^2bx-4b^3x^3)^{3/2}}\,dx = &-\frac{(4Ab^2-6abB+9a^2C)\,(3a-bx)^2(3a+2bx)}{324a^2b^3\,(27a^3+27a^2bx-4b^3x^3)^{3/2}}\\ &-\frac{(28Ab^2+3a(10bB-51aC))\,(3a-bx)^2(3a+2bx)^2}{5832a^3b^3\,(27a^3+27a^2bx-4b^3x^3)^{3/2}}\\ &+\frac{2(Ab^2+3a(bB+3aC))\,(3a-bx)(3a+2bx)^3}{729a^3b^3\,(27a^3+27a^2bx-4b^3x^3)^{3/2}}\\ &-\frac{(20Ab^2+42abB+45a^2C)\,(3a+2bx)^3\left(3-\frac{bx}{a}\right)^{3/2}\operatorname{arctanh}\left(\frac{1}{3}\sqrt{2}\sqrt{3-\frac{bx}{a}}\right)}{5832\sqrt{2}a^2b^3\,(27a^3+27a^2bx-4b^3x^3)^{3/2}}\end{aligned}$$

output

```
-1/324*(4*A*b^2-6*B*a*b+9*C*a^2)*(-b*x+3*a)^2*(2*b*x+3*a)/a^2/b^3/(-4*b^3*
x^3+27*a^2*b*x+27*a^3)^(3/2)-1/5832*(28*A*b^2+3*a*(10*B*b-51*C*a))*(-b*x+3
*a)^2*(2*b*x+3*a)^2/a^3/b^3/(-4*b^3*x^3+27*a^2*b*x+27*a^3)^(3/2)+2/729*(A*
b^2+3*a*(B*b+3*C*a))*(-b*x+3*a)*(2*b*x+3*a)^3/a^3/b^3/(-4*b^3*x^3+27*a^2*b
*x+27*a^3)^(3/2)-1/11664*(20*A*b^2+42*B*a*b+45*C*a^2)*(2*b*x+3*a)^3*(3-b*x
/a)^(3/2)*arctanh(1/3*2^(1/2)*(3-b*x/a)^(1/2))*2^(1/2)/a^2/b^3/(-4*b^3*x^3
+27*a^2*b*x+27*a^3)^(3/2)
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.62

$$\int \frac{A+Bx+Cx^2}{(27a^3+27a^2bx-4b^3x^3)^{3/2}}\,dx = \frac{(3a+2bx)^3\left(\frac{6\sqrt{a}(3a-bx)(729a^4C+40Ab^4x^2+12ab^3x(5A+7Bx)+27a^3b(6B+29Cx)-18a}{(3a+2bx)^2}\right.}{11664a^{7/2}b^3}$$

input
```
Integrate[(A + B*x + C*x^2)/(27*a^3 + 27*a^2*b*x - 4*b^3*x^3)^(3/2),x]
```

output
```
((3*a + 2*b*x)^3*((6*Sqrt[a]*(3*a - b*x)*(729*a^4*C + 40*A*b^4*x^2 + 12*a*
b^3*x*(5*A + 7*B*x) + 27*a^3*b*(6*B + 29*C*x) - 18*a^2*b^2*(6*A - x*(7*B +
 5*C*x))))/(3*a + 2*b*x)^2 - Sqrt[2]*(20*A*b^2 + 42*a*b*B + 45*a^2*C)*(3*a
 - b*x)^(3/2)*ArcTanh[Sqrt[6*a - 2*b*x]/(3*Sqrt[a])]))/(11664*a^(7/2)*b^3*
((3*a - b*x)*(3*a + 2*b*x)^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 296, normalized size of antiderivative = 0.92, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2526, 27, 2483, 27, 87, 52, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{(27a^3+27a^2bx-4b^3x^3)^{3/2}}\,dx$$

$$\Big\downarrow \text{2526}$$

$$\frac{C}{6b^3\sqrt{27a^3+27a^2bx-4b^3x^3}} - \frac{\int -\frac{3b(9Ca^2+4Ab^2+4b^2Bx)}{(27a^3+27bxa^2-4b^3x^3)^{3/2}}dx}{12b^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{9Ca^2+4Ab^2+4b^2Bx}{(27a^3+27bxa^2-4b^3x^3)^{3/2}}dx}{4b^2} + \frac{C}{6b^3\sqrt{27a^3+27a^2bx-4b^3x^3}}$$

$$\Big\downarrow \text{2483}$$

$$\frac{1594323\sqrt{3}a^6(3a+2bx)^3\left(3a^3-a^2bx\right)^{3/2}\int \frac{9Ca^2+4Ab^2+4b^2Bx}{1594323\sqrt{3}a^6(3a+2bx)^3(3a^3-a^2bx)^{3/2}}dx}{4b^2\left(27a^3+27a^2bx-4b^3x^3\right)^{3/2}}+\frac{C}{6b^3\sqrt{27a^3+27a^2bx-4b^3x^3}}$$

$\Big\downarrow$ 27

$$\frac{\left(3a^3-a^2bx\right)^{3/2}(3a+2bx)^3\int \frac{9Ca^2+4Ab^2+4b^2Bx}{(3a+2bx)^3(3a^3-a^2bx)^{3/2}}dx}{4b^2\left(27a^3+27a^2bx-4b^3x^3\right)^{3/2}}+\frac{C}{6b^3\sqrt{27a^3+27a^2bx-4b^3x^3}}$$

$\Big\downarrow$ 87

$$\frac{\left(3a^3-a^2bx\right)^{3/2}(3a+2bx)^3\left(\frac{(45a^2C+42abB+20Ab^2)\int \frac{1}{(3a+2bx)^2\left(3a^3-a^2bx\right)^{3/2}}dx}{36a}-\frac{9a^2C-6abB+4Ab^2}{18a^3b(3a+2bx)^2\sqrt{3a^3-a^2bx}}\right)}{4b^2\left(27a^3+27a^2bx-4b^3x^3\right)^{3/2}}+\frac{C}{6b^3\sqrt{27a^3+27a^2bx-4b^3x^3}}$$

$\Big\downarrow$ 52

$$\frac{\left(3a^3-a^2bx\right)^{3/2}(3a+2bx)^3\left(\frac{(45a^2C+42abB+20Ab^2)\left(\frac{\int \frac{1}{(3a+2bx)\left(3a^3-a^2bx\right)^{3/2}}dx}{6a}-\frac{1}{9a^3b(3a+2bx)\sqrt{3a^3-a^2bx}}\right)}{36a}-\frac{9a^2C-6abB\cdots}{18a^3b(3a+2bx)^2\cdots}\cdots\right.}{4b^2\left(27a^3+27a^2bx-4b^3x^3\right)^{3/2}}\cdots$$

$$\frac{C}{6b^3\sqrt{27a^3+27a^2bx-4b^3x^3}}$$

$\Big\downarrow$ 61

$$\frac{\left(3a^3-a^2bx\right)^{3/2}(3a+2bx)^3\left(\frac{(45a^2C+42abB+20Ab^2)\left(\frac{\frac{2\int \frac{1}{(3a+2bx)\sqrt{3a^3-a^2bx}}dx}{9a^3}+\frac{2}{9a^3b\sqrt{3a^3-a^2bx}}}{6a}-\frac{1}{9a^3b(3a+2bx)\sqrt{3a^3-a^2bx}}\right)}{36a}-\cdots\right.}{4b^2\left(27a^3+27a^2bx-4b^3x^3\right)^{3/2}}\cdots$$

$$\frac{C}{6b^3\sqrt{27a^3+27a^2bx-4b^3x^3}}$$

$\Big\downarrow$ 73

$$\frac{\left(3a^3-a^2bx\right)^{3/2}(3a+2bx)^3\left(\frac{(45a^2C+42abB+20Ab^2)\left(\frac{\frac{2}{9a^3b\sqrt{3a^3-a^2bx}}-\frac{4\int\frac{1}{9a-\frac{2\left(3a^3-a^2bx\right)}{a^2}}d\sqrt{3a^3-a^2bx}}{9a^5b}}{6a}-\frac{1}{9a^3b(3a+2bx)\sqrt{3a^3-a^2bx}}\cdots\right.}{36a}\cdots\right.}{4b^2\left(27a^3+27a^2bx-4b^3x^3\right)^{3/2}}$$

$$\frac{C}{6b^3\sqrt{27a^3+27a^2bx-4b^3x^3}}$$

$$\Big\downarrow \text{221}$$

$$\frac{C}{6b^3\sqrt{27a^3+27a^2bx-4b^3x^3}}+$$

$$\frac{\left(3a^3-a^2bx\right)^{3/2}(3a+2bx)^3\left(\frac{\left(\frac{\frac{2}{9a^3b\sqrt{3a^3-a^2bx}}-\frac{2\sqrt{2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{3a^3-a^2bx}}{3a^{3/2}}\right)}{27a^{9/2}b}}{6a}-\frac{1}{9a^3b(3a+2bx)\sqrt{3a^3-a^2bx}}\right)(45a^2C+42abB+20Ab^2\cdots}{36a}\cdots\right.}{4b^2\left(27a^3+27a^2bx-4b^3x^3\right)^{3/2}}$$

input
```
Int[(A + B*x + C*x^2)/(27*a^3 + 27*a^2*b*x - 4*b^3*x^3)^(3/2),x]
```

output
```
C/(6*b^3*Sqrt[27*a^3 + 27*a^2*b*x - 4*b^3*x^3]) + ((3*a + 2*b*x)^3*(3*a^3
- a^2*b*x)^(3/2)*(-1/18*(4*A*b^2 - 6*a*b*B + 9*a^2*C)/(a^3*b*(3*a + 2*b*x)
^2*Sqrt[3*a^3 - a^2*b*x]) + ((20*A*b^2 + 42*a*b*B + 45*a^2*C)*(-1/9*1/(a^3
*b*(3*a + 2*b*x)*Sqrt[3*a^3 - a^2*b*x]) + (2/(9*a^3*b*Sqrt[3*a^3 - a^2*b*x
]) - (2*Sqrt[2]*ArcTanh[(Sqrt[2]*Sqrt[3*a^3 - a^2*b*x])/(3*a^(3/2))])/(27*
a^(9/2)*b))/(6*a)))/(36*a)))/(4*b^2*(27*a^3 + 27*a^2*b*x - 4*b^3*x^3)^(3/2
))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 2483
```
Int[((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (d_.)*(x_)^3)^(p_), x_S
ymbol] :> Simp[(a + b*x + d*x^3)^p/((3*a - b*x)^p*(3*a + 2*b*x)^(2*p))   In
t[(e + f*x)^m*(3*a - b*x)^p*(3*a + 2*b*x)^(2*p), x], x] /; FreeQ[{a, b, d,
e, f, m, p}, x] && EqQ[4*b^3 + 27*a^2*d, 0] &&  !IntegerQ[p]
```

rule 2526
```
Int[(Pm_)*(Qn_)^(p_), x_Symbol] :> With[{m = Expon[Pm, x], n = Expon[Qn, x]
}, Simp[Coeff[Pm, x, m]*(Qn^(p + 1)/(n*(p + 1)*Coeff[Qn, x, n])), x] + Simp
[1/(n*Coeff[Qn, x, n])   Int[ExpandToSum[n*Coeff[Qn, x, n]*Pm - Coeff[Pm, x
, m]*D[Qn, x], x]*Qn^p, x], x] /; EqQ[m, n - 1]] /; FreeQ[p, x] && PolyQ[Pm
, x] && PolyQ[Qn, x] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 502, normalized size of antiderivative = 1.57

method	result
default	$-\frac{\left(80A\sqrt{-bx+3a}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-bx+3a}\,\sqrt{2}}{3\sqrt{a}}\right)b^4x^2-240A\sqrt{a}\,b^4x^2+648A\,a^{\frac{5}{2}}b^2-972B\,a^{\frac{7}{2}}b+168B\sqrt{-bx+3a}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-bx+3}}{3\sqrt{a}}\right.\right.}{}$

input
```
int((C*x^2+B*x+A)/(-4*b^3*x^3+27*a^2*b*x+27*a^3)^(3/2),x,method=_RETURNVER
BOSE)
```

output
```
-1/11664*(80*A*(-b*x+3*a)^(1/2)*2^(1/2)*arctanh(1/3*(-b*x+3*a)^(1/2)*2^(1/
2)/a^(1/2))*b^4*x^2-240*A*a^(1/2)*b^4*x^2+648*A*a^(5/2)*b^2-972*B*a^(7/2)*
b+168*B*(-b*x+3*a)^(1/2)*2^(1/2)*arctanh(1/3*(-b*x+3*a)^(1/2)*2^(1/2)/a^(1
/2))*a*b^3*x^2+180*C*(-b*x+3*a)^(1/2)*2^(1/2)*arctanh(1/3*(-b*x+3*a)^(1/2)
*2^(1/2)/a^(1/2))*a^2*b^2*x^2+240*A*(-b*x+3*a)^(1/2)*2^(1/2)*arctanh(1/3*(
-b*x+3*a)^(1/2)*2^(1/2)/a^(1/2))*a*b^3*x+504*B*(-b*x+3*a)^(1/2)*2^(1/2)*ar
ctanh(1/3*(-b*x+3*a)^(1/2)*2^(1/2)/a^(1/2))*a^2*b^2*x+540*C*(-b*x+3*a)^(1/
2)*2^(1/2)*arctanh(1/3*(-b*x+3*a)^(1/2)*2^(1/2)/a^(1/2))*a^3*b*x-360*A*a^(
3/2)*b^3*x-756*B*a^(5/2)*b^2*x-4698*C*a^(7/2)*b*x+180*A*(-b*x+3*a)^(1/2)*2
^(1/2)*arctanh(1/3*(-b*x+3*a)^(1/2)*2^(1/2)/a^(1/2))*a^2*b^2+378*B*(-b*x+3
*a)^(1/2)*2^(1/2)*arctanh(1/3*(-b*x+3*a)^(1/2)*2^(1/2)/a^(1/2))*a^3*b-504*
B*a^(3/2)*b^3*x^2+405*C*(-b*x+3*a)^(1/2)*2^(1/2)*arctanh(1/3*(-b*x+3*a)^(1
/2)*2^(1/2)/a^(1/2))*a^4-540*C*a^(5/2)*b^2*x^2-4374*C*a^(9/2))*(-b*x+3*a)*
(2*b*x+3*a)/a^(7/2)/b^3/(-4*b^3*x^3+27*a^2*b*x+27*a^3)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 761, normalized size of antiderivative = 2.38

$$\int \frac{A+Bx+Cx^2}{(27a^3+27a^2bx-4b^3x^3)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)/(-4*b^3*x^3+27*a^2*b*x+27*a^3)^(3/2),x, algorithm=
"fricas")
```

output
```
[-1/23328*(sqrt(2)*(3645*C*a^6 + 3402*B*a^5*b + 1620*A*a^4*b^2 - 8*(45*C*a
^2*b^4 + 42*B*a*b^5 + 20*A*b^6)*x^4 - 12*(45*C*a^3*b^3 + 42*B*a^2*b^4 + 20
*A*a*b^5)*x^3 + 54*(45*C*a^4*b^2 + 42*B*a^3*b^3 + 20*A*a^2*b^4)*x^2 + 135*
(45*C*a^5*b + 42*B*a^4*b^2 + 20*A*a^3*b^3)*x)*sqrt(a)*log((4*b^2*x^2 - 24*
a*b*x - 45*a^2 + 6*sqrt(2)*sqrt(-4*b^3*x^3 + 27*a^2*b*x + 27*a^3)*sqrt(a))
/(4*b^2*x^2 + 12*a*b*x + 9*a^2)) + 12*(729*C*a^5 + 162*B*a^4*b - 108*A*a^3
*b^2 + 2*(45*C*a^3*b^2 + 42*B*a^2*b^3 + 20*A*a*b^4)*x^2 + 3*(261*C*a^4*b +
 42*B*a^3*b^2 + 20*A*a^2*b^3)*x)*sqrt(-4*b^3*x^3 + 27*a^2*b*x + 27*a^3))/(
8*a^4*b^7*x^4 + 12*a^5*b^6*x^3 - 54*a^6*b^5*x^2 - 135*a^7*b^4*x - 81*a^8*b
^3), 1/11664*(sqrt(2)*(3645*C*a^6 + 3402*B*a^5*b + 1620*A*a^4*b^2 - 8*(45*
C*a^2*b^4 + 42*B*a*b^5 + 20*A*b^6)*x^4 - 12*(45*C*a^3*b^3 + 42*B*a^2*b^4 +
 20*A*a*b^5)*x^3 + 54*(45*C*a^4*b^2 + 42*B*a^3*b^3 + 20*A*a^2*b^4)*x^2 + 1
35*(45*C*a^5*b + 42*B*a^4*b^2 + 20*A*a^3*b^3)*x)*sqrt(-a)*arctan(3/2*sqrt(
2)*sqrt(-4*b^3*x^3 + 27*a^2*b*x + 27*a^3)*sqrt(-a)/(2*b^2*x^2 - 3*a*b*x -
9*a^2)) - 6*(729*C*a^5 + 162*B*a^4*b - 108*A*a^3*b^2 + 2*(45*C*a^3*b^2 + 4
2*B*a^2*b^3 + 20*A*a*b^4)*x^2 + 3*(261*C*a^4*b + 42*B*a^3*b^2 + 20*A*a^2*b
^3)*x)*sqrt(-4*b^3*x^3 + 27*a^2*b*x + 27*a^3))/(8*a^4*b^7*x^4 + 12*a^5*b^6
*x^3 - 54*a^6*b^5*x^2 - 135*a^7*b^4*x - 81*a^8*b^3)]
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2}{(27a^3+27a^2bx-4b^3x^3)^{3/2}}\,dx = \int \frac{A+Bx+Cx^2}{\left(-\left(-3a+bx\right)\left(3a+2bx\right)^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((C*x**2+B*x+A)/(-4*b**3*x**3+27*a**2*b*x+27*a**3)**(3/2),x)
```

output
```
Integral((A + B*x + C*x**2)/(-(-3*a + b*x)*(3*a + 2*b*x)**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{(27a^3+27a^2bx-4b^3x^3)^{3/2}}\,dx = \int \frac{Cx^2+Bx+A}{(-4\,b^3x^3+27\,a^2bx+27\,a^3)^{\frac{3}{2}}}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(-4*b^3*x^3+27*a^2*b*x+27*a^3)^(3/2),x, algorithm=
"maxima")
```

output
```
integrate((C*x^2 + B*x + A)/(-4*b^3*x^3 + 27*a^2*b*x + 27*a^3)^(3/2), x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 240, normalized size of antiderivative = 0.75

$$\int \frac{A+Bx+Cx^2}{(27a^3+27a^2bx-4b^3x^3)^{3/2}}\,dx = -\frac{\sqrt{2}(45\,Ca^2+42\,Bab+20\,Ab^2)\arctan\left(\frac{\sqrt{2}\sqrt{-bx+3\,a}}{3\,\sqrt{-a}}\right)}{11664\,\sqrt{-a}a^3b^3\mathrm{sgn}\left(-2\,bx-3\,a\right)} - \frac{2\,(9\,Ca^2+3\,Bab+Ab^2)}{729\,\sqrt{-bx+3\,a}a^3b^3\mathrm{sgn}\left(-2\,bx-3\,a\right)} + \frac{306\,(-bx+3\,a)^{\frac{3}{2}}Ca^2-1215\,\sqrt{-bx+3\,a}Ca^3-60\,(-bx+3\,a)^{\frac{3}{2}}Bab+162\,\sqrt{-bx+3\,a}Ba^2b-56\,(-bx-}{5832\,(2\,bx+3\,a)^2a^3b^3\mathrm{sgn}\left(-2\,bx-3\,a\right)}$$

input
```
integrate((C*x^2+B*x+A)/(-4*b^3*x^3+27*a^2*b*x+27*a^3)^(3/2),x, algorithm=
"giac")
```

output
```
-1/11664*sqrt(2)*(45*C*a^2 + 42*B*a*b + 20*A*b^2)*arctan(1/3*sqrt(2)*sqrt(
-b*x + 3*a)/sqrt(-a))/(sqrt(-a)*a^3*b^3*sgn(-2*b*x - 3*a)) - 2/729*(9*C*a^
2 + 3*B*a*b + A*b^2)/(sqrt(-b*x + 3*a)*a^3*b^3*sgn(-2*b*x - 3*a)) + 1/5832
*(306*(-b*x + 3*a)^(3/2)*C*a^2 - 1215*sqrt(-b*x + 3*a)*C*a^3 - 60*(-b*x +
3*a)^(3/2)*B*a*b + 162*sqrt(-b*x + 3*a)*B*a^2*b - 56*(-b*x + 3*a)^(3/2)*A*
b^2 + 324*sqrt(-b*x + 3*a)*A*a*b^2)/((2*b*x + 3*a)^2*a^3*b^3*sgn(-2*b*x -
3*a))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(27a^3+27a^2bx-4b^3x^3)^{3/2}}\,dx=\int \frac{C\,x^2+B\,x+A}{{\left(27\,a^3+27\,a^2\,b\,x-4\,b^3\,x^3\right)}^{3/2}}\,dx$$

input
```
int((A + B*x + C*x^2)/(27*a^3 - 4*b^3*x^3 + 27*a^2*b*x)^(3/2),x)
```

output
```
int((A + B*x + C*x^2)/(27*a^3 - 4*b^3*x^3 + 27*a^2*b*x)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 571, normalized size of antiderivative = 1.78

$$\int \frac{A+Bx+Cx^2}{(27a^3+27a^2bx-4b^3x^3)^{3/2}}\,dx=\frac{405\sqrt{a}\,\sqrt{-bx+3a}\,\sqrt{2}\log\left(2\sqrt{-bx+3a}-3\sqrt{a}\,\sqrt{2}\right)a^3c+558\sqrt{a}\,\sqrt{-b}}{}$$

input
```
int((C*x^2+B*x+A)/(-4*b^3*x^3+27*a^2*b*x+27*a^3)^(3/2),x)
```

output
```
(405*sqrt(a)*sqrt(3*a - b*x)*sqrt(2)*log(2*sqrt(3*a - b*x) - 3*sqrt(a)*sqr
t(2))*a**3*c + 558*sqrt(a)*sqrt(3*a - b*x)*sqrt(2)*log(2*sqrt(3*a - b*x) -
 3*sqrt(a)*sqrt(2))*a**2*b**2 + 540*sqrt(a)*sqrt(3*a - b*x)*sqrt(2)*log(2*
sqrt(3*a - b*x) - 3*sqrt(a)*sqrt(2))*a**2*b*c*x + 744*sqrt(a)*sqrt(3*a - b
*x)*sqrt(2)*log(2*sqrt(3*a - b*x) - 3*sqrt(a)*sqrt(2))*a*b**3*x + 180*sqrt
(a)*sqrt(3*a - b*x)*sqrt(2)*log(2*sqrt(3*a - b*x) - 3*sqrt(a)*sqrt(2))*a*b
**2*c*x**2 + 248*sqrt(a)*sqrt(3*a - b*x)*sqrt(2)*log(2*sqrt(3*a - b*x) - 3
*sqrt(a)*sqrt(2))*b**4*x**2 - 405*sqrt(a)*sqrt(3*a - b*x)*sqrt(2)*log(2*sq
rt(3*a - b*x) + 3*sqrt(a)*sqrt(2))*a**3*c - 558*sqrt(a)*sqrt(3*a - b*x)*sq
rt(2)*log(2*sqrt(3*a - b*x) + 3*sqrt(a)*sqrt(2))*a**2*b**2 - 540*sqrt(a)*s
qrt(3*a - b*x)*sqrt(2)*log(2*sqrt(3*a - b*x) + 3*sqrt(a)*sqrt(2))*a**2*b*c
*x - 744*sqrt(a)*sqrt(3*a - b*x)*sqrt(2)*log(2*sqrt(3*a - b*x) + 3*sqrt(a)
*sqrt(2))*a*b**3*x - 180*sqrt(a)*sqrt(3*a - b*x)*sqrt(2)*log(2*sqrt(3*a -
b*x) + 3*sqrt(a)*sqrt(2))*a*b**2*c*x**2 - 248*sqrt(a)*sqrt(3*a - b*x)*sqrt
(2)*log(2*sqrt(3*a - b*x) + 3*sqrt(a)*sqrt(2))*b**4*x**2 + 8748*a**4*c + 6
48*a**3*b**2 + 9396*a**3*b*c*x + 2232*a**2*b**3*x + 1080*a**2*b**2*c*x**2
+ 1488*a*b**4*x**2)/(23328*sqrt(3*a - b*x)*a**3*b**3*(9*a**2 + 12*a*b*x +
4*b**2*x**2))
```

3.13 $\int \frac{A+Bx+Cx^2}{\left(27a^3+27a^2bx-4b^3x^3\right)^{5/2}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 540

$$
\begin{aligned}
\int \frac{A+Bx+Cx^2}{\left(27a^3+27a^2bx-4b^3x^3\right)^{5/2}}\,dx = &-\frac{\left(4Ab^2-6abB+9a^2C\right)(3a-bx)^3(3a+2bx)}{2916a^3b^3\left(27a^3+27a^2bx-4b^3x^3\right)^{5/2}} \\
&-\frac{\left(92Ab^2+3a(2bB-75aC)\right)(3a-bx)^3(3a+2bx)^2}{157464a^4b^3\left(27a^3+27a^2bx-4b^3x^3\right)^{5/2}} \\
&-\frac{\left(1036Ab^2+3a(298bB-375aC)\right)(3a-bx)^3(3a+2bx)^3}{5668704a^5b^3\left(27a^3+27a^2bx-4b^3x^3\right)^{5/2}} \\
&-\frac{\left(2060Ab^2+2814abB+1179a^2C\right)(3a-bx)^3(3a+2bx)^4}{34012224a^6b^3\left(27a^3+27a^2bx-4b^3x^3\right)^{5/2}} \\
&+\frac{2\left(Ab^2+3a(bB+3aC)\right)(3a-bx)(3a+2bx)^5}{177147a^5b^3\left(27a^3+27a^2bx-4b^3x^3\right)^{5/2}} \\
&+\frac{2\left(10Ab^2+21abB+36a^2C\right)(3a-bx)^2(3a+2bx)^5}{531441a^6b^3\left(27a^3+27a^2bx-4b^3x^3\right)^{5/2}} \\
&-\frac{35\left(44Ab^2+78abB+99a^2C\right)(3a+2bx)^5\left(3-\frac{bx}{a}\right)^{5/2}\operatorname{arctanh}\left(\frac{1}{3}\sqrt{2}\sqrt{3-\frac{bx}{a}}\right)}{34012224\sqrt{2}a^4b^3\left(27a^3+27a^2bx-4b^3x^3\right)^{5/2}}
\end{aligned}
$$

output
```
-1/2916*(4*A*b^2-6*B*a*b+9*C*a^2)*(-b*x+3*a)^3*(2*b*x+3*a)/a^3/b^3/(-4*b^3
*x^3+27*a^2*b*x+27*a^3)^(5/2)-1/157464*(92*A*b^2+3*a*(2*B*b-75*C*a))*(-b*x
+3*a)^3*(2*b*x+3*a)^2/a^4/b^3/(-4*b^3*x^3+27*a^2*b*x+27*a^3)^(5/2)-1/56687
04*(1036*A*b^2+3*a*(298*B*b-375*C*a))*(-b*x+3*a)^3*(2*b*x+3*a)^3/a^5/b^3/(
-4*b^3*x^3+27*a^2*b*x+27*a^3)^(5/2)-1/34012224*(2060*A*b^2+2814*B*a*b+1179
*C*a^2)*(-b*x+3*a)^3*(2*b*x+3*a)^4/a^6/b^3/(-4*b^3*x^3+27*a^2*b*x+27*a^3)^
(5/2)+2/177147*(A*b^2+3*a*(B*b+3*C*a))*(-b*x+3*a)*(2*b*x+3*a)^5/a^5/b^3/(-
4*b^3*x^3+27*a^2*b*x+27*a^3)^(5/2)+2/531441*(10*A*b^2+21*B*a*b+36*C*a^2)*(
-b*x+3*a)^2*(2*b*x+3*a)^5/a^6/b^3/(-4*b^3*x^3+27*a^2*b*x+27*a^3)^(5/2)-35/
68024448*(44*A*b^2+78*B*a*b+99*C*a^2)*(2*b*x+3*a)^5*(3-b*x/a)^(5/2)*arctan
h(1/3*2^(1/2)*(3-b*x/a)^(1/2))*2^(1/2)/a^4/b^3/(-4*b^3*x^3+27*a^2*b*x+27*a
^3)^(5/2)
```

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 274, normalized size of antiderivative = 0.51

$$\int \frac{A+Bx+Cx^2}{(27a^3+27a^2bx-4b^3x^3)^{5/2}}\,dx = \frac{(3a+2bx)^5\left(\frac{6\sqrt{a}(3a-bx)(675783a^7C-12320Ab^7x^5-1680ab^6x^4(11A+13Bx)+1458a^6b(5}{}\right.}{}$$

input
```
Integrate[(A + B*x + C*x^2)/(27*a^3 + 27*a^2*b*x - 4*b^3*x^3)^(5/2),x]
```

output
```
((3*a + 2*b*x)^5*((6*Sqrt[a]*(3*a - b*x)*(675783*a^7*C - 12320*A*b^7*x^5 -
 1680*a*b^6*x^4*(11*A + 13*B*x) + 1458*a^6*b*(51*B + 908*C*x) - 504*a^2*b^
5*x^3*(-231*A + 65*B*x + 55*C*x^2) + 108*a^3*b^4*x^2*(2717*A + 7*x*(273*B
- 55*C*x)) - 243*a^5*b^2*(2220*A - 13*x*(16*B + 209*C*x)) + 162*a^4*b^3*x*
(176*A + x*(3211*B + 1617*C*x))))/(3*a + 2*b*x)^4 - 35*Sqrt[2]*(44*A*b^2 +
 78*a*b*B + 99*a^2*C)*(3*a - b*x)^(5/2)*ArcTanh[Sqrt[6*a - 2*b*x]/(3*Sqrt[
a])]))/(68024448*a^(13/2)*b^3*((3*a - b*x)*(3*a + 2*b*x)^2)^(5/2))
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 421, normalized size of antiderivative = 0.78, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2526, 27, 2483, 27, 87, 52, 52, 52, 61, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{\left(27a^3+27a^2bx-4b^3x^3\right)^{5/2}}dx$$

$$\Big\downarrow \text{2526}$$

$$\frac{C}{18b^3\left(27a^3+27a^2bx-4b^3x^3\right)^{3/2}}-\frac{\int -\frac{3b\left(9Ca^2+4Ab^2+4b^2Bx\right)}{\left(27a^3+27bxa^2-4b^3x^3\right)^{5/2}}dx}{12b^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{9Ca^2+4Ab^2+4b^2Bx}{\left(27a^3+27bxa^2-4b^3x^3\right)^{5/2}}dx}{4b^2}+\frac{C}{18b^3\left(27a^3+27a^2bx-4b^3x^3\right)^{3/2}}$$

$$\Big\downarrow \text{2483}$$

$$\frac{31381059609\sqrt{3}a^{10}(3a+2bx)^5\left(3a^3-a^2bx\right)^{5/2}\int \frac{9Ca^2+4Ab^2+4b^2Bx}{31381059609\sqrt{3}a^{10}(3a+2bx)^5\left(3a^3-a^2bx\right)^{5/2}}dx}{4b^2\left(27a^3+27a^2bx-4b^3x^3\right)^{5/2}}+\frac{C}{18b^3\left(27a^3+27a^2bx-4b^3x^3\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\left(3a^3-a^2bx\right)^{5/2}(3a+2bx)^5\int \frac{9Ca^2+4Ab^2+4b^2Bx}{(3a+2bx)^5\left(3a^3-a^2bx\right)^{5/2}}dx}{4b^2\left(27a^3+27a^2bx-4b^3x^3\right)^{5/2}}+\frac{C}{18b^3\left(27a^3+27a^2bx-4b^3x^3\right)^{3/2}}$$

$$\Big\downarrow \text{87}$$

$$\frac{\left(3a^3-a^2bx\right)^{5/2}(3a+2bx)^5\left(\frac{\left(99a^2C+78abB+44Ab^2\right)\int \frac{1}{(3a+2bx)^4\left(3a^3-a^2bx\right)^{5/2}}dx}{72a}-\frac{9a^2C-6abB+4Ab^2}{36a^3b(3a+2bx)^4\left(3a^3-a^2bx\right)^{3/2}}\right)}{4b^2\left(27a^3+27a^2bx-4b^3x^3\right)^{5/2}}+\frac{C}{18b^3\left(27a^3+27a^2bx-4b^3x^3\right)^{3/2}}$$

$$\Big\downarrow \; 52$$

$$\frac{\left(3a^3-a^2bx\right)^{5/2}(3a+2bx)^5\left(\frac{(99a^2C+78abB+44Ab^2)\left(\frac{\int \frac{1}{(3a+2bx)^3\left(3a^3-a^2bx\right)^{5/2}}dx}{6a}-\frac{1}{27a^3b(3a+2bx)^3\left(3a^3-a^2bx\right)^{3/2}}\right)}{72a}-\frac{9a^2\cdots}{36a^3b(3a\cdots}\cdots\right.}{4b^2\left(27a^3+27a^2bx-4b^3x^3\right)^{5/2}}-\frac{C}{18b^3\left(27a^3+27a^2bx-4b^3x^3\right)^{3/2}}$$

$$\Big\downarrow \; 52$$

$$\frac{\left(3a^3-a^2bx\right)^{5/2}(3a+2bx)^5\left(\frac{(99a^2C+78abB+44Ab^2)\left(\frac{\frac{7\int \frac{1}{(3a+2bx)^2\left(3a^3-a^2bx\right)^{5/2}}dx}{36a}-\frac{1}{18a^3b(3a+2bx)^2\left(3a^3-a^2bx\right)^{3/2}}}{6a}-\frac{\cdots}{27a^3b(3a+2b\cdots}\cdots\right.}{72a}\cdots\right.}{4b^2\left(27a^3+27a^2bx-4b^3x^3\right)^{5/2}}-\frac{C}{18b^3\left(27a^3+27a^2bx-4b^3x^3\right)^{3/2}}$$

$$\Big\downarrow \; 52$$

$$\frac{\left(3a^3-a^2bx\right)^{5/2}(3a+2bx)^5\left(\frac{\left(99a^2C+78abB+44Ab^2\right)\left(\frac{7\left(\frac{5\int\frac{1}{(3a+2bx)\left(3a^3-a^2bx\right)^{5/2}}dx}{18a}-\frac{1}{9a^3b(3a+2bx)\left(3a^3-a^2bx\right)^{3/2}}\right)}{36a}-\frac{1}{18a^3b(3a+2}{6a}}{72a}\right.}{4b^2\left(27a^3+27a^2bx-4b^3x^3\right)^{5/2}}$$

$$\frac{C}{18b^3\left(27a^3+27a^2bx-4b^3x^3\right)^{3/2}}$$

$\downarrow$ 61

$$\frac{\left(3a^3-a^2bx\right)^{5/2}(3a+2bx)^5\left(\frac{\left(99a^2C+78abB+44Ab^2\right)\left(\frac{\frac{\left(\frac{7\left(\frac{5\left(\frac{2\int\frac{1}{(3a+2bx)\left(3a^3-a^2bx\right)^{3/2}}dx}{9a^3}+\frac{2}{27a^3b\left(3a^3-a^2bx\right)^{3/2}}\right)}{18a}-\frac{1}{9a^3b(3a+2bx)\left(3a\cdots\right.}\right.}{36a}\cdots}{6a}\cdots\right.}{72a}\cdots\right.}{4b^2\left(27a^3+27a^2bx-4b^3\cdots\right.}$$

$$\frac{C}{18b^3\left(27a^3+27a^2bx-4b^3x^3\right)^{3/2}}$$

$\downarrow$ 61

$$\left(3a^3-a^2bx\right)^{5/2}(3a+2bx)^5 \left(\frac{\left(99a^2C+78abB+44Ab^2\right)\left(\frac{7\left(\frac{5\left(\frac{2\left(\frac{2\int \frac{1}{(3a+2bx)\sqrt{3a^3-a^2bx}}dx}{9a^3}+\frac{2}{9a^3b\sqrt{3a^3-a^2bx}}\right)}{9a^3}+\frac{2}{27a^3b\left(3a^3-a^2bx\right)^{3/2}}\right.}{18a}\right.}{36a}\right.}{6a} \right.$$

$$\frac{\ldots}{72a} \quad \frac{\ldots}{4b^2\left(27a^3+27a^2b\ldots\right.}$$

$$\frac{C}{18b^3\left(27a^3+27a^2bx-4b^3x^3\right)^{3/2}}$$

$\downarrow$ 73

$$\left(3a^3-a^2bx\right)^{5/2}(3a+2bx)^5\left(\frac{\left(99a^2C+78abB+44Ab^2\right)\left(\frac{7\left(\frac{5\left(\frac{2\left(\frac{2}{9a^3b\sqrt{3a^3-a^2bx}}-\frac{4\int\frac{1}{9a-\frac{2\left(3a^3-a^2bx\right)}{a^2}}d\sqrt{3a^3-a^2bx}}{9a^5b}\right)}{9a^3}+\frac{2}{27a^3b\left(3a^3-a^2\cdots\right.}\cdots\right)}{18a}\cdots\right)}{36a}\cdots\right)}{6a}\cdots}{7\cdots}\right.$$

$\downarrow$ 221

$$\frac{C}{18b^3\left(27a^3+27a^2bx-4b^3x^3\right)^{3/2}}+$$

$$\frac{\left(3a^3-a^2bx\right)^{5/2}(3a+2bx)^5\left(\frac{\left(\frac{7\left(\frac{5\left(\frac{2}{27a^3b\left(3a^3-a^2bx\right)^{3/2}}+\frac{2\left(\frac{2}{9a^3b\sqrt{3a^3-a^2bx}}-\frac{2\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{3a^3-a^2bx}}{3a^{3/2}}\right)}{27a^{9/2}b}\right)}{9a^3}\right)}{18a}-\frac{1}{9a^3b(3a+2bx)\left(3a\right.}}{36a}}{6a}}{7\ldots}\right.}{4b^2\left(27a^3+27a\ldots\right.}$$

input
```
Int[(A + B*x + C*x^2)/(27*a^3 + 27*a^2*b*x - 4*b^3*x^3)^(5/2),x]
```

output
```
C/(18*b^3*(27*a^3 + 27*a^2*b*x - 4*b^3*x^3)^(3/2)) + ((3*a + 2*b*x)^5*(3*a
^3 - a^2*b*x)^(5/2)*(-1/36*(4*A*b^2 - 6*a*b*B + 9*a^2*C)/(a^3*b*(3*a + 2*b
*x)^4*(3*a^3 - a^2*b*x)^(3/2)) + ((44*A*b^2 + 78*a*b*B + 99*a^2*C)*(-1/27*
1/(a^3*b*(3*a + 2*b*x)^3*(3*a^3 - a^2*b*x)^(3/2)) + (-1/18*1/(a^3*b*(3*a +
 2*b*x)^2*(3*a^3 - a^2*b*x)^(3/2)) + (7*(-1/9*1/(a^3*b*(3*a + 2*b*x)*(3*a^
3 - a^2*b*x)^(3/2)) + (5*(2/(27*a^3*b*(3*a^3 - a^2*b*x)^(3/2)) + (2*(2/(9*
a^3*b*Sqrt[3*a^3 - a^2*b*x]) - (2*Sqrt[2]*ArcTanh[(Sqrt[2]*Sqrt[3*a^3 - a^
2*b*x])/(3*a^(3/2))])/(27*a^(9/2)*b)))/(9*a^3)))/(18*a)))/(36*a))/(6*a)))/
(72*a)))/(4*b^2*(27*a^3 + 27*a^2*b*x - 4*b^3*x^3)^(5/2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 2483
```
Int[((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (d_.)*(x_)^3)^(p_), x_S
ymbol] :> Simp[(a + b*x + d*x^3)^p/((3*a - b*x)^p*(3*a + 2*b*x)^(2*p))   In
t[(e + f*x)^m*(3*a - b*x)^p*(3*a + 2*b*x)^(2*p), x], x] /; FreeQ[{a, b, d,
e, f, m, p}, x] && EqQ[4*b^3 + 27*a^2*d, 0] &&  !IntegerQ[p]
```

rule 2526
```
Int[(Pm_)*(Qn_)^(p_), x_Symbol] :> With[{m = Expon[Pm, x], n = Expon[Qn, x]
}, Simp[Coeff[Pm, x, m]*(Qn^(p + 1)/(n*(p + 1)*Coeff[Qn, x, n])), x] + Simp
[1/(n*Coeff[Qn, x, n])   Int[ExpandToSum[n*Coeff[Qn, x, n]*Pm - Coeff[Pm, x
, m]*D[Qn, x], x]*Qn^p, x], x] /; EqQ[m, n - 1]] /; FreeQ[p, x] && PolyQ[Pm
, x] && PolyQ[Qn, x] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 874, normalized size of antiderivative = 1.62

method	result	size
default	Expression too large to display	874

input
```
int((C*x^2+B*x+A)/(-4*b^3*x^3+27*a^2*b*x+27*a^3)^(5/2),x,method=_RETURNVER
BOSE)
```

output
```
-1/68024448*(-3121092*B*a^(9/2)*b^3*x^2-3961386*C*a^(11/2)*b^2*x^2-171072*
A*a^(9/2)*b^3*x-303264*B*a^(11/2)*b^2*x-7943184*C*a^(13/2)*b*x+221130*B*(-
b*x+3*a)^(3/2)*2^(1/2)*arctanh(1/3*(-b*x+3*a)^(1/2)*2^(1/2)/a^(1/2))*a^5*b
+124740*A*(-b*x+3*a)^(3/2)*2^(1/2)*arctanh(1/3*(-b*x+3*a)^(1/2)*2^(1/2)/a^
(1/2))*a^4*b^2+110880*A*a^(3/2)*b^6*x^4+196560*B*a^(5/2)*b^5*x^4+249480*C*
a^(7/2)*b^4*x^4-698544*A*a^(5/2)*b^5*x^3-1238328*B*a^(7/2)*b^4*x^3+280665*
C*(-b*x+3*a)^(3/2)*2^(1/2)*arctanh(1/3*(-b*x+3*a)^(1/2)*2^(1/2)/a^(1/2))*a
^6-1571724*C*a^(9/2)*b^3*x^3+748440*C*(-b*x+3*a)^(3/2)*2^(1/2)*arctanh(1/3
*(-b*x+3*a)^(1/2)*2^(1/2)/a^(1/2))*a^4*b^2*x^2+332640*A*(-b*x+3*a)^(3/2)*2
^(1/2)*arctanh(1/3*(-b*x+3*a)^(1/2)*2^(1/2)/a^(1/2))*a^3*b^3*x+589680*B*(-
b*x+3*a)^(3/2)*2^(1/2)*arctanh(1/3*(-b*x+3*a)^(1/2)*2^(1/2)/a^(1/2))*a^4*b
^2*x+748440*C*(-b*x+3*a)^(3/2)*2^(1/2)*arctanh(1/3*(-b*x+3*a)^(1/2)*2^(1/2
)/a^(1/2))*a^5*b*x-1760616*A*a^(7/2)*b^4*x^2+3236760*A*a^(11/2)*b^2-446148
*B*a^(13/2)*b+332640*A*(-b*x+3*a)^(3/2)*2^(1/2)*arctanh(1/3*(-b*x+3*a)^(1/
2)*2^(1/2)/a^(1/2))*a^2*b^4*x^2+589680*B*(-b*x+3*a)^(3/2)*2^(1/2)*arctanh(
1/3*(-b*x+3*a)^(1/2)*2^(1/2)/a^(1/2))*a^3*b^3*x^2+166320*C*a^(5/2)*b^5*x^5
+131040*B*a^(3/2)*b^6*x^5+73920*A*a^(1/2)*b^7*x^5+43680*B*(-b*x+3*a)^(3/2)
*2^(1/2)*arctanh(1/3*(-b*x+3*a)^(1/2)*2^(1/2)/a^(1/2))*a*b^5*x^4+55440*C*(
-b*x+3*a)^(3/2)*2^(1/2)*arctanh(1/3*(-b*x+3*a)^(1/2)*2^(1/2)/a^(1/2))*a^2*
b^4*x^4+147840*A*(-b*x+3*a)^(3/2)*2^(1/2)*arctanh(1/3*(-b*x+3*a)^(1/2)*...
```

Fricas [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 1225, normalized size of antiderivative = 2.27

$$\int \frac{A+Bx+Cx^2}{(27a^3+27a^2bx-4b^3x^3)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)/(-4*b^3*x^3+27*a^2*b*x+27*a^3)^(5/2),x, algorithm=
"fricas")
```

output
```
[1/136048896*(35*sqrt(2)*(216513*C*a^9 + 170586*B*a^8*b + 96228*A*a^7*b^2
+ 32*(99*C*a^2*b^7 + 78*B*a*b^8 + 44*A*b^9)*x^7 + 48*(99*C*a^3*b^6 + 78*B*
a^2*b^7 + 44*A*a*b^8)*x^6 - 432*(99*C*a^4*b^5 + 78*B*a^3*b^6 + 44*A*a^2*b^
7)*x^5 - 1080*(99*C*a^5*b^4 + 78*B*a^4*b^5 + 44*A*a^3*b^6)*x^4 + 810*(99*C
*a^6*b^3 + 78*B*a^5*b^4 + 44*A*a^4*b^5)*x^3 + 5103*(99*C*a^7*b^2 + 78*B*a^
6*b^3 + 44*A*a^5*b^4)*x^2 + 5832*(99*C*a^8*b + 78*B*a^7*b^2 + 44*A*a^6*b^3
)*x)*sqrt(a)*log((4*b^2*x^2 - 24*a*b*x - 45*a^2 + 6*sqrt(2)*sqrt(-4*b^3*x^
3 + 27*a^2*b*x + 27*a^3)*sqrt(a))/(4*b^2*x^2 + 12*a*b*x + 9*a^2)) + 12*(67
5783*C*a^8 + 74358*B*a^7*b - 539460*A*a^6*b^2 - 280*(99*C*a^3*b^5 + 78*B*a
^2*b^6 + 44*A*a*b^7)*x^5 - 420*(99*C*a^4*b^4 + 78*B*a^3*b^5 + 44*A*a^2*b^6
)*x^4 + 2646*(99*C*a^5*b^3 + 78*B*a^4*b^4 + 44*A*a^3*b^5)*x^3 + 6669*(99*C
*a^6*b^2 + 78*B*a^5*b^3 + 44*A*a^4*b^4)*x^2 + 648*(2043*C*a^7*b + 78*B*a^6
*b^2 + 44*A*a^5*b^3)*x)*sqrt(-4*b^3*x^3 + 27*a^2*b*x + 27*a^3))/(32*a^7*b^
10*x^7 + 48*a^8*b^9*x^6 - 432*a^9*b^8*x^5 - 1080*a^10*b^7*x^4 + 810*a^11*b
^6*x^3 + 5103*a^12*b^5*x^2 + 5832*a^13*b^4*x + 2187*a^14*b^3), -1/68024448
*(35*sqrt(2)*(216513*C*a^9 + 170586*B*a^8*b + 96228*A*a^7*b^2 + 32*(99*C*a
^2*b^7 + 78*B*a*b^8 + 44*A*b^9)*x^7 + 48*(99*C*a^3*b^6 + 78*B*a^2*b^7 + 44
*A*a*b^8)*x^6 - 432*(99*C*a^4*b^5 + 78*B*a^3*b^6 + 44*A*a^2*b^7)*x^5 - 108
0*(99*C*a^5*b^4 + 78*B*a^4*b^5 + 44*A*a^3*b^6)*x^4 + 810*(99*C*a^6*b^3 + 7
8*B*a^5*b^4 + 44*A*a^4*b^5)*x^3 + 5103*(99*C*a^7*b^2 + 78*B*a^6*b^3 + 4...
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2}{\left(27a^3+27a^2bx-4b^3x^3\right)^{5/2}}\,dx=\int \frac{A+Bx+Cx^2}{\left(-\left(-3a+bx\right)\left(3a+2bx\right)^2\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((C*x**2+B*x+A)/(-4*b**3*x**3+27*a**2*b*x+27*a**3)**(5/2),x)
```

output
```
Integral((A + B*x + C*x**2)/(-(-3*a + b*x)*(3*a + 2*b*x)**2)**(5/2), x)
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{(27a^3+27a^2bx-4b^3x^3)^{5/2}}\,dx=\int \frac{Cx^2+Bx+A}{(-4\,b^3x^3+27\,a^2bx+27\,a^3)^{\frac{5}{2}}}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(-4*b^3*x^3+27*a^2*b*x+27*a^3)^(5/2),x, algorithm=
"maxima")
```

output
```
integrate((C*x^2 + B*x + A)/(-4*b^3*x^3 + 27*a^2*b*x + 27*a^3)^(5/2), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 449, normalized size of antiderivative = 0.83

$$\int \frac{A+Bx+Cx^2}{(27a^3+27a^2bx-4b^3x^3)^{5/2}}\,dx=$$
$$-\frac{35\sqrt{2}(99\,Ca^2+78\,Bab+44\,Ab^2)\arctan\left(\frac{\sqrt{2}\sqrt{-bx+3\,a}}{3\,\sqrt{-a}}\right)}{68024448\,\sqrt{-a}a^6b^3\mathrm{sgn}\left(-2\,bx-3\,a\right)}$$
$$-\frac{2\,(36\,(bx-3\,a)Ca^2-27\,Ca^3+21\,(bx-3\,a)Bab-9\,Ba^2b+10\,(bx-3\,a)Ab^2-3\,Aab^2)}{531441\,(bx-3\,a)\sqrt{-bx+3\,a}a^6b^3\mathrm{sgn}\left(-2\,bx-3\,a\right)}$$
$$+\frac{9432\,(bx-3\,a)^3\sqrt{-bx+3\,a}Ca^2+100332\,(bx-3\,a)^2\sqrt{-bx+3\,a}Ca^3-232794\,(-bx+3\,a)^{\frac{3}{2}}Ca^4-1968}{}$$

input
```
integrate((C*x^2+B*x+A)/(-4*b^3*x^3+27*a^2*b*x+27*a^3)^(5/2),x, algorithm=
"giac")
```

output
```
-35/68024448*sqrt(2)*(99*C*a^2 + 78*B*a*b + 44*A*b^2)*arctan(1/3*sqrt(2)*s
qrt(-b*x + 3*a)/sqrt(-a))/(sqrt(-a)*a^6*b^3*sgn(-2*b*x - 3*a)) - 2/531441*
(36*(b*x - 3*a)*C*a^2 - 27*C*a^3 + 21*(b*x - 3*a)*B*a*b - 9*B*a^2*b + 10*(
b*x - 3*a)*A*b^2 - 3*A*a*b^2)/((b*x - 3*a)*sqrt(-b*x + 3*a)*a^6*b^3*sgn(-2
*b*x - 3*a)) + 1/34012224*(9432*(b*x - 3*a)^3*sqrt(-b*x + 3*a)*C*a^2 + 100
332*(b*x - 3*a)^2*sqrt(-b*x + 3*a)*C*a^3 - 232794*(-b*x + 3*a)^(3/2)*C*a^4
 - 19683*sqrt(-b*x + 3*a)*C*a^5 + 22512*(b*x - 3*a)^3*sqrt(-b*x + 3*a)*B*a
*b + 325368*(b*x - 3*a)^2*sqrt(-b*x + 3*a)*B*a^2*b - 1563300*(-b*x + 3*a)^
(3/2)*B*a^3*b + 2427570*sqrt(-b*x + 3*a)*B*a^4*b + 16480*(b*x - 3*a)^3*sqr
t(-b*x + 3*a)*A*b^2 + 247344*(b*x - 3*a)^2*sqrt(-b*x + 3*a)*A*a*b^2 - 1264
680*(-b*x + 3*a)^(3/2)*A*a^2*b^2 + 2230740*sqrt(-b*x + 3*a)*A*a^3*b^2)/((2
*b*x + 3*a)^4*a^6*b^3*sgn(-2*b*x - 3*a))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{\left(27a^3+27a^2bx-4b^3x^3\right)^{5/2}}\,dx=\int \frac{C\,x^2+B\,x+A}{\left(27\,a^3+27\,a^2\,b\,x-4\,b^3\,x^3\right)^{5/2}}\,dx$$

input
```
int((A + B*x + C*x^2)/(27*a^3 - 4*b^3*x^3 + 27*a^2*b*x)^(5/2),x)
```

output
```
int((A + B*x + C*x^2)/(27*a^3 - 4*b^3*x^3 + 27*a^2*b*x)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 1195, normalized size of antiderivative = 2.21

$$\int \frac{A+Bx+Cx^2}{\left(27a^3+27a^2bx-4b^3x^3\right)^{5/2}}\,dx=\text{Too large to display}$$

input
```
int((C*x^2+B*x+A)/(-4*b^3*x^3+27*a^2*b*x+27*a^3)^(5/2),x)
```

output

```
(841995*sqrt(a)*sqrt(3*a - b*x)*sqrt(2)*log(2*sqrt(3*a - b*x) - 3*sqrt(a)*
sqrt(2))*a**6*c + 1037610*sqrt(a)*sqrt(3*a - b*x)*sqrt(2)*log(2*sqrt(3*a -
 b*x) - 3*sqrt(a)*sqrt(2))*a**5*b**2 + 1964655*sqrt(a)*sqrt(3*a - b*x)*sqr
t(2)*log(2*sqrt(3*a - b*x) - 3*sqrt(a)*sqrt(2))*a**5*b*c*x + 2421090*sqrt(
a)*sqrt(3*a - b*x)*sqrt(2)*log(2*sqrt(3*a - b*x) - 3*sqrt(a)*sqrt(2))*a**4
*b**3*x + 1496880*sqrt(a)*sqrt(3*a - b*x)*sqrt(2)*log(2*sqrt(3*a - b*x) -
3*sqrt(a)*sqrt(2))*a**4*b**2*c*x**2 + 1844640*sqrt(a)*sqrt(3*a - b*x)*sqrt
(2)*log(2*sqrt(3*a - b*x) - 3*sqrt(a)*sqrt(2))*a**3*b**4*x**2 + 249480*sqr
t(a)*sqrt(3*a - b*x)*sqrt(2)*log(2*sqrt(3*a - b*x) - 3*sqrt(a)*sqrt(2))*a*
*3*b**3*c*x**3 + 307440*sqrt(a)*sqrt(3*a - b*x)*sqrt(2)*log(2*sqrt(3*a - b
*x) - 3*sqrt(a)*sqrt(2))*a**2*b**5*x**3 - 166320*sqrt(a)*sqrt(3*a - b*x)*s
qrt(2)*log(2*sqrt(3*a - b*x) - 3*sqrt(a)*sqrt(2))*a**2*b**4*c*x**4 - 20496
0*sqrt(a)*sqrt(3*a - b*x)*sqrt(2)*log(2*sqrt(3*a - b*x) - 3*sqrt(a)*sqrt(2
))*a*b**6*x**4 - 55440*sqrt(a)*sqrt(3*a - b*x)*sqrt(2)*log(2*sqrt(3*a - b*
x) - 3*sqrt(a)*sqrt(2))*a*b**5*c*x**5 - 68320*sqrt(a)*sqrt(3*a - b*x)*sqrt
(2)*log(2*sqrt(3*a - b*x) - 3*sqrt(a)*sqrt(2))*b**7*x**5 - 841995*sqrt(a)*
sqrt(3*a - b*x)*sqrt(2)*log(2*sqrt(3*a - b*x) + 3*sqrt(a)*sqrt(2))*a**6*c
- 1037610*sqrt(a)*sqrt(3*a - b*x)*sqrt(2)*log(2*sqrt(3*a - b*x) + 3*sqrt(a
)*sqrt(2))*a**5*b**2 - 1964655*sqrt(a)*sqrt(3*a - b*x)*sqrt(2)*log(2*sqrt(
3*a - b*x) + 3*sqrt(a)*sqrt(2))*a**5*b*c*x - 2421090*sqrt(a)*sqrt(3*a -...
```

3.14 $\int (A+Bx+Cx^2)\left(27a^3+27a^2bx-4b^3x^3\right)^p\,dx$

Optimal result

Integrand size = 34, antiderivative size = 259

$$\int (A+Bx+Cx^2)\left(27a^3+27a^2bx-4b^3x^3\right)^p\,dx$$
$$= -\frac{(2bB(1+p)+3aC(2+3p))(3a-bx)(3a+2bx)\left(27a^3+27a^2bx-4b^3x^3\right)^p}{4b^3(1+p)(2+3p)}$$
$$+\frac{C(3a-bx)^2(3a+2bx)\left(27a^3+27a^2bx-4b^3x^3\right)^p}{6b^3(1+p)}$$
$$-\frac{3^{2p}(4Ab^2(2+3p)+3a(3aC(2+3p)+2b(B+3Bp)))(3a-bx)\left(1+\frac{2bx}{3a}\right)^{-2p}\left(27a^3+27a^2bx-4b^3x^3\right.}{4b^3(1+p)(2+3p)}$$

output
```
-1/4*(2*b*B*(p+1)+3*a*C*(2+3*p))*(-b*x+3*a)*(2*b*x+3*a)*(-4*b^3*x^3+27*a^2
*b*x+27*a^3)^p/b^3/(p+1)/(2+3*p)+1/6*C*(-b*x+3*a)^2*(2*b*x+3*a)*(-4*b^3*x^
3+27*a^2*b*x+27*a^3)^p/b^3/(p+1)-1/4*3^(2*p)*(4*A*b^2*(2+3*p)+3*a*(3*a*C*(
2+3*p)+2*b*(3*B*p+B)))*(-b*x+3*a)*(-4*b^3*x^3+27*a^2*b*x+27*a^3)^p*hyperge
om([-2*p, p+1],[2+p],2/9*(-b*x+3*a)/a)/b^3/(p+1)/(2+3*p)/((1+2/3*b*x/a)^(2
*p))
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.64

$$\int (A+Bx+Cx^2)\left(27a^3+27a^2bx-4b^3x^3\right)^p\,dx$$
$$=\frac{81^p(-3a+bx)\left((3a-bx)(3a+2bx)^2\right)^p\left(3+\frac{2bx}{a}\right)^{-2p}\left(18a(bB-3aC)\text{Hypergeometric2F1}\left(-1-2p,1+\right.\right.}{}$$

input
```
Integrate[(A + B*x + C*x^2)*(27*a^3 + 27*a^2*b*x - 4*b^3*x^3)^p,x]
```

output
```
(81^p*(-3*a + b*x)*((3*a - b*x)*(3*a + 2*b*x)^2)^p*(18*a*(b*B - 3*a*C)*Hyp
ergeometric2F1[-1 - 2*p, 1 + p, 2 + p, 2/3 - (2*b*x)/(9*a)] + (4*A*b^2 - 6
*a*b*B + 9*a^2*C)*Hypergeometric2F1[-2*p, 1 + p, 2 + p, 2/3 - (2*b*x)/(9*a
)] + 81*a^2*C*Hypergeometric2F1[-2*(1 + p), 1 + p, 2 + p, 2/3 - (2*b*x)/(9
*a)]))/(4*b^3*(1 + p)*(3 + (2*b*x)/a)^(2*p))
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {2526, 27, 2483, 90, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(27a^3+27a^2bx-4b^3x^3\right)^p\left(A+Bx+Cx^2\right)dx$$

$$\Big\downarrow \text{2526}$$

$$-\frac{\int -3b\left(9Ca^2+4Ab^2+4b^2Bx\right)\left(27a^3+27bxa^2-4b^3x^3\right)^p dx}{12b^3}-\frac{C\left(27a^3+27a^2bx-4b^3x^3\right)^{p+1}}{12b^3(p+1)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \left(9Ca^2+4Ab^2+4b^2Bx\right)\left(27a^3+27bxa^2-4b^3x^3\right)^p dx}{4b^2}-\frac{C\left(27a^3+27a^2bx-4b^3x^3\right)^{p+1}}{12b^3(p+1)}$$

$\downarrow$ 2483

$$\frac{\left(81a^3-27a^2bx\right)^{-p}\left(81a^3+54a^2bx\right)^{-2p}\left(27a^3+27a^2bx-4b^3x^3\right)^p\int\left(81a^3-27a^2bx\right)^p\left(81a^3+54bxa^2\right)^{2p}\left(9Ca^2\right.}{4b^2}$$
$$\frac{C\left(27a^3+27a^2bx-4b^3x^3\right)^{p+1}}{12b^3(p+1)}$$

$\downarrow$ 90

$$\frac{\left(81a^3-27a^2bx\right)^{-p}\left(81a^3+54a^2bx\right)^{-2p}\left(27a^3+27a^2bx-4b^3x^3\right)^p\left(\left(9a^2C+\frac{6abB(3p+1)}{3p+2}+4Ab^2\right)\int\left(81a^3-27a\right.\right.}{4b^2}$$
$$\frac{C\left(27a^3+27a^2bx-4b^3x^3\right)^{p+1}}{12b^3(p+1)}$$

$\downarrow$ 80

$$\frac{\left(81a^3-27a^2bx\right)^{-p}\left(81a^3+54a^2bx\right)^{-2p}\left(27a^3+27a^2bx-4b^3x^3\right)^p\left(81^p\left(\frac{3a+2bx}{a}\right)^{-2p}\left(81a^3+54a^2bx\right)^{2p}\left(9a^2C+\right.\right.}{4b^2}$$
$$\frac{C\left(27a^3+27a^2bx-4b^3x^3\right)^{p+1}}{12b^3(p+1)}$$

$\downarrow$ 79

$$\frac{\left(81a^3-27a^2bx\right)^{-p}\left(81a^3+54a^2bx\right)^{-2p}\left(27a^3+27a^2bx-4b^3x^3\right)^p\left(-\frac{3^{4p-3}\left(81a^3-27a^2bx\right)^{p+1}\left(81a^3+54a^2bx\right)^{2p}\left(\frac{3a+2bx}{a}\right)}{}\right.}{4b^2}$$
$$\frac{C\left(27a^3+27a^2bx-4b^3x^3\right)^{p+1}}{12b^3(p+1)}$$

input
```
Int[(A + B*x + C*x^2)*(27*a^3 + 27*a^2*b*x - 4*b^3*x^3)^p,x]
```

output
```
-1/12*(C*(27*a^3 + 27*a^2*b*x - 4*b^3*x^3)^(1 + p))/(b^3*(1 + p)) + ((27*a
^3 + 27*a^2*b*x - 4*b^3*x^3)^p*((-2*B*(81*a^3 - 27*a^2*b*x)^(1 + p)*(81*a^
3 + 54*a^2*b*x)^(1 + 2*p))/(729*a^4*(2 + 3*p)) - (3^(-3 + 4*p)*(4*A*b^2 +
9*a^2*C + (6*a*b*B*(1 + 3*p))/(2 + 3*p))*(81*a^3 - 27*a^2*b*x)^(1 + p)*(81
*a^3 + 54*a^2*b*x)^(2*p)*Hypergeometric2F1[-2*p, 1 + p, 2 + p, (2*(3*a - b
*x))/(9*a)])/(a^2*b*(1 + p)*((3*a + 2*b*x)/a)^(2*p))))/(4*b^2*(81*a^3 - 27
*a^2*b*x)^p*(81*a^3 + 54*a^2*b*x)^(2*p))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 2483
```
Int[((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (d_.)*(x_)^3)^(p_), x_S
ymbol] :> Simp[(a + b*x + d*x^3)^p/((3*a - b*x)^p*(3*a + 2*b*x)^(2*p))   In
t[(e + f*x)^m*(3*a - b*x)^p*(3*a + 2*b*x)^(2*p), x], x] /; FreeQ[{a, b, d,
e, f, m, p}, x] && EqQ[4*b^3 + 27*a^2*d, 0] &&  !IntegerQ[p]
```

rule 2526
```
Int[(Pm_)*(Qn_)^(p_), x_Symbol] :> With[{m = Expon[Pm, x], n = Expon[Qn, x]
}, Simp[Coeff[Pm, x, m]*(Qn^(p + 1)/(n*(p + 1)*Coeff[Qn, x, n])), x] + Simp
[1/(n*Coeff[Qn, x, n])   Int[ExpandToSum[n*Coeff[Qn, x, n]*Pm - Coeff[Pm, x
, m]*D[Qn, x], x]*Qn^p, x], x] /; EqQ[m, n - 1]] /; FreeQ[p, x] && PolyQ[Pm
, x] && PolyQ[Qn, x] && NeQ[p, -1]
```

Maple [F]

$$\int \left(C\,x^{2}+Bx+A\right)\left(-4b^{3}x^{3}+27b\,a^{2}x+27a^{3}\right)^{p}dx$$

input
```
int((C*x^2+B*x+A)*(-4*b^3*x^3+27*a^2*b*x+27*a^3)^p,x)
```

output
```
int((C*x^2+B*x+A)*(-4*b^3*x^3+27*a^2*b*x+27*a^3)^p,x)
```

Fricas [F]

$$\int (A+Bx+Cx^2)\left(27a^3+27a^2bx-4b^3x^3\right)^p\,dx = \int \left(Cx^{2}+Bx+A\right)\left(-4\,b^{3}x^{3}+27\,a^{2}bx+27\,a^{3}\right)^{p}dx$$

input
```
integrate((C*x^2+B*x+A)*(-4*b^3*x^3+27*a^2*b*x+27*a^3)^p,x, algorithm="fri
cas")
```

output
```
integral((C*x^2 + B*x + A)*(-4*b^3*x^3 + 27*a^2*b*x + 27*a^3)^p, x)
```

Sympy [F]

$$\int (A + Bx + Cx^2)\,(27a^3 + 27a^2bx - 4b^3x^3)^p\,dx = \int \left(-(-3a + bx)\,(3a + 2bx)^2\right)^p (A + Bx + Cx^2)\,dx$$

input
```
integrate((C*x**2+B*x+A)*(-4*b**3*x**3+27*a**2*b*x+27*a**3)**p,x)
```

output
```
Integral((-(-3*a + b*x)*(3*a + 2*b*x)**2)**p*(A + B*x + C*x**2), x)
```

Maxima [F]

$$\int (A + Bx + Cx^2)\,(27a^3 + 27a^2bx - 4b^3x^3)^p\,dx = \int (Cx^2 + Bx + A)\left(-4\,b^3x^3 + 27\,a^2bx + 27\,a^3\right)^p\,dx$$

input
```
integrate((C*x^2+B*x+A)*(-4*b^3*x^3+27*a^2*b*x+27*a^3)^p,x, algorithm="max
ima")
```

output
```
integrate((C*x^2 + B*x + A)*(-4*b^3*x^3 + 27*a^2*b*x + 27*a^3)^p, x)
```

Giac [F]

$$\int (A + Bx + Cx^2)\,(27a^3 + 27a^2bx - 4b^3x^3)^p\,dx = \int (Cx^2 + Bx + A)\left(-4\,b^3x^3 + 27\,a^2bx + 27\,a^3\right)^p\,dx$$

input
```
integrate((C*x^2+B*x+A)*(-4*b^3*x^3+27*a^2*b*x+27*a^3)^p,x, algorithm="gia
c")
```

output
```
integrate((C*x^2 + B*x + A)*(-4*b^3*x^3 + 27*a^2*b*x + 27*a^3)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(A+Bx+Cx^2\right) \left(27a^3+27a^2bx-4b^3x^3\right)^p \, dx = \int \left(C\,x^2+B\,x+A\right)\,\left(27\,a^3+27\,a^2\,b\,x-4\,b^3\,x^3\right)^p \,dx$$

input
```
int((A + B*x + C*x^2)*(27*a^3 - 4*b^3*x^3 + 27*a^2*b*x)^p,x)
```

output
```
int((A + B*x + C*x^2)*(27*a^3 - 4*b^3*x^3 + 27*a^2*b*x)^p, x)
```

Reduce [F]

$$\int \left(A+Bx+Cx^2\right) \left(27a^3+27a^2bx-4b^3x^3\right)^p \, dx = \text{Too large to display}$$

input
```
int((C*x^2+B*x+A)*(-4*b^3*x^3+27*a^2*b*x+27*a^3)^p,x)
```

output

```
(81*(27*a**3 + 27*a**2*b*x - 4*b**3*x**3)**p*a**3*c*p + 54*(27*a**3 + 27*a
**2*b*x - 4*b**3*x**3)**p*a**3*c + 54*(27*a**3 + 27*a**2*b*x - 4*b**3*x**3
)**p*a**2*b**2*p**2 + 90*(27*a**3 + 27*a**2*b*x - 4*b**3*x**3)**p*a**2*b**
2*p + 36*(27*a**3 + 27*a**2*b*x - 4*b**3*x**3)**p*a**2*b**2 - 81*(27*a**3
+ 27*a**2*b*x - 4*b**3*x**3)**p*a**2*b*c*p**2*x - 54*(27*a**3 + 27*a**2*b*
x - 4*b**3*x**3)**p*a**2*b*c*p*x + 18*(27*a**3 + 27*a**2*b*x - 4*b**3*x**3
)**p*a*b**3*p**2*x + 30*(27*a**3 + 27*a**2*b*x - 4*b**3*x**3)**p*a*b**3*p*
x + 12*(27*a**3 + 27*a**2*b*x - 4*b**3*x**3)**p*a*b**3*x + 18*(27*a**3 + 2
7*a**2*b*x - 4*b**3*x**3)**p*b**4*p**2*x**2 + 24*(27*a**3 + 27*a**2*b*x -
4*b**3*x**3)**p*b**4*p*x**2 + 6*(27*a**3 + 27*a**2*b*x - 4*b**3*x**3)**p*b
**4*x**2 + 18*(27*a**3 + 27*a**2*b*x - 4*b**3*x**3)**p*b**3*c*p**2*x**3 +
18*(27*a**3 + 27*a**2*b*x - 4*b**3*x**3)**p*b**3*c*p*x**3 + 4*(27*a**3 + 2
7*a**2*b*x - 4*b**3*x**3)**p*b**3*c*x**3 + 6561*int(((27*a**3 + 27*a**2*b*
x - 4*b**3*x**3)**p*x)/(81*a**2*p**2 + 81*a**2*p + 18*a**2 + 27*a*b*p**2*x
 + 27*a*b*p*x + 6*a*b*x - 18*b**2*p**2*x**2 - 18*b**2*p*x**2 - 4*b**2*x**2
),x)*a**3*b**2*c*p**5 + 17496*int(((27*a**3 + 27*a**2*b*x - 4*b**3*x**3)**
p*x)/(81*a**2*p**2 + 81*a**2*p + 18*a**2 + 27*a*b*p**2*x + 27*a*b*p*x + 6*
a*b*x - 18*b**2*p**2*x**2 - 18*b**2*p*x**2 - 4*b**2*x**2),x)*a**3*b**2*c*p
**4 + 16767*int(((27*a**3 + 27*a**2*b*x - 4*b**3*x**3)**p*x)/(81*a**2*p**2
 + 81*a**2*p + 18*a**2 + 27*a*b*p**2*x + 27*a*b*p*x + 6*a*b*x - 18*b**2...
```

3.15 $\int (A+Bx+Cx^2)(2-4x+3x^3)^3\,dx$

Optimal result

Integrand size = 23, antiderivative size = 155

$$\begin{aligned}\int (A+Bx+Cx^2)\left(2-4x+3x^3\right)^3\,dx = {} & 8Ax-4(6A-B)x^2+\frac{8}{3}(12A-6B+C)x^3 \\ & -(7A-24B+12C)x^4 \\ & -\frac{4}{5}(36A+7B-24C)x^5 \\ & +\frac{2}{3}(36A-36B-7C)x^6 \\ & +\frac{18}{7}(3A+8B-8C)x^7 \\ & -\frac{9}{4}(6A-3B-8C)x^8-6(2B-C)x^9 \\ & +\frac{27}{10}(A-4C)x^{10}+\frac{27Bx^{11}}{11}+\frac{9Cx^{12}}{4}\end{aligned}$$

output
```
8*A*x-4*(6*A-B)*x^2+8/3*(12*A-6*B+C)*x^3-(7*A-24*B+12*C)*x^4-4/5*(36*A+7*B
-24*C)*x^5+2/3*(36*A-36*B-7*C)*x^6+18/7*(3*A+8*B-8*C)*x^7-9/4*(6*A-3*B-8*C
)*x^8-6*(2*B-C)*x^9+27/10*(A-4*C)*x^10+27/11*B*x^11+9/4*C*x^12
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.99

$$\int (A+Bx+Cx^2)\left(2-4x+3x^3\right)^3\,dx = 8Ax-4(6A-B)x^2+\frac{8}{3}(12A-6B+C)x^3+(-7A+24B-12C)x^4-\frac{4}{5}(36A+7B-24C)x^5+\frac{2}{3}(36A-36B-7C)x^6+\frac{18}{7}(3A+8B-8C)x^7-\frac{9}{4}(6A-3B-8C)x^8-6(2B-C)x^9+\frac{27}{10}(A-4C)x^{10}+\frac{27Bx^{11}}{11}+\frac{9Cx^{12}}{4}$$

input
```
Integrate[(A + B*x + C*x^2)*(2 - 4*x + 3*x^3)^3,x]
```

output
```
8*A*x - 4*(6*A - B)*x^2 + (8*(12*A - 6*B + C)*x^3)/3 + (-7*A + 24*B - 12*C
)*x^4 - (4*(36*A + 7*B - 24*C)*x^5)/5 + (2*(36*A - 36*B - 7*C)*x^6)/3 + (1
8*(3*A + 8*B - 8*C)*x^7)/7 - (9*(6*A - 3*B - 8*C)*x^8)/4 - 6*(2*B - C)*x^9
 + (27*(A - 4*C)*x^10)/10 + (27*B*x^11)/11 + (9*C*x^12)/4
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(3x^3-4x+2\right)^3\left(A+Bx+Cx^2\right)\,dx$$

$$\Big\downarrow \text{2188}$$

$$\int \left(-18x^7(6A-3B-8C)+18x^6(3A+8B-8C)+4x^5(36A-36B-7C)-4x^4(36A+7B-24C)-4x^3(7A\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{9}{4}x^8(6A-3B-8C)+\frac{18}{7}x^7(3A+8B-8C)+\frac{2}{3}x^6(36A-36B-7C)-\frac{4}{5}x^5(36A+7B-24C)-x^4(7A-24B+12C)+\frac{8}{3}x^3(12A-6B+C)-4x^2(6A-B)+\frac{27}{10}x^{10}(A-4C)+8Ax-6x^9(2B-C)+\frac{27Bx^{11}}{11}+\frac{9Cx^{12}}{4}$$

input
```
Int[(A + B*x + C*x^2)*(2 - 4*x + 3*x^3)^3,x]
```

output
```
8*A*x - 4*(6*A - B)*x^2 + (8*(12*A - 6*B + C)*x^3)/3 - (7*A - 24*B + 12*C)
*x^4 - (4*(36*A + 7*B - 24*C)*x^5)/5 + (2*(36*A - 36*B - 7*C)*x^6)/3 + (18
*(3*A + 8*B - 8*C)*x^7)/7 - (9*(6*A - 3*B - 8*C)*x^8)/4 - 6*(2*B - C)*x^9
+ (27*(A - 4*C)*x^10)/10 + (27*B*x^11)/11 + (9*C*x^12)/4
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.87

method	result
norman	$\frac{9C\,x^{12}}{4}+\frac{27B\,x^{11}}{11}+\left(\frac{27A}{10}-\frac{54C}{5}\right)x^{10}+\left(-12B+6C\right)x^{9}+\left(-\frac{27A}{2}+\frac{27B}{4}+18C\right)x^{8}+\left(\frac{54A}{7}+\right.$
default	$\frac{9C\,x^{12}}{4}+\frac{27B\,x^{11}}{11}+\frac{(27A-108C)x^{10}}{10}+\frac{(-108B+54C)x^{9}}{9}+\frac{(-108A+54B+144C)x^{8}}{8}+\frac{(54A+144B-144C)x^{7}}{7}+$
orering	$\frac{x(10395C\,x^{11}+11340B\,x^{10}+12474A\,x^{9}-49896C\,x^{9}-55440B\,x^{8}+27720x^{8}C-62370x^{7}A+31185x^{7}B+83160x^{7}C+35640x^{6}}{}$
gosper	$-12C\,x^{4}+\frac{8}{3}C\,x^{3}+4B\,x^{2}-24A\,x^{2}-7x^{4}A-\frac{144}{5}x^{5}A+18x^{8}C+\frac{54}{7}x^{7}A-12x^{9}B-\frac{27}{2}x^{8}$
risch	$-12C\,x^{4}+\frac{8}{3}C\,x^{3}+4B\,x^{2}-24A\,x^{2}-7x^{4}A-\frac{144}{5}x^{5}A+18x^{8}C+\frac{54}{7}x^{7}A-12x^{9}B-\frac{27}{2}x^{8}$
parallelrisch	$-12C\,x^{4}+\frac{8}{3}C\,x^{3}+4B\,x^{2}-24A\,x^{2}-7x^{4}A-\frac{144}{5}x^{5}A+18x^{8}C+\frac{54}{7}x^{7}A-12x^{9}B-\frac{27}{2}x^{8}$

input
```
int((C*x^2+B*x+A)*(3*x^3-4*x+2)^3,x,method=_RETURNVERBOSE)
```

output
```
9/4*C*x^12+27/11*B*x^11+(27/10*A-54/5*C)*x^10+(-12*B+6*C)*x^9+(-27/2*A+27/
4*B+18*C)*x^8+(54/7*A+144/7*B-144/7*C)*x^7+(24*A-24*B-14/3*C)*x^6+(-144/5*
A-28/5*B+96/5*C)*x^5+(-7*A+24*B-12*C)*x^4+(32*A-16*B+8/3*C)*x^3+(-24*A+4*B
)*x^2+8*A*x
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.90

$$\begin{aligned}\int (A+Bx+Cx^2)\left(2-4x+3x^3\right)^3\,dx &= \frac{9}{4}\,Cx^{12}+\frac{27}{11}\,Bx^{11}+\frac{27}{10}\,(A-4\,C)x^{10}-6\,(2\,B-C)x^9-\frac{9}{4}\,(6\,A-3\,B-8\,C)x^8\\ &\quad+\frac{18}{7}\,(3\,A+8\,B-8\,C)x^7+\frac{2}{3}\,(36\,A-36\,B-7\,C)x^6-\frac{4}{5}\,(36\,A+7\,B-24\,C)x^5\\ &\quad-(7\,A-24\,B+12\,C)x^4+\frac{8}{3}\,(12\,A-6\,B+C)x^3-4\,(6\,A-B)x^2+8\,Ax\end{aligned}$$

input
```
integrate((C*x^2+B*x+A)*(3*x^3-4*x+2)^3,x, algorithm="fricas")
```

output
```
9/4*C*x^12 + 27/11*B*x^11 + 27/10*(A - 4*C)*x^10 - 6*(2*B - C)*x^9 - 9/4*(
6*A - 3*B - 8*C)*x^8 + 18/7*(3*A + 8*B - 8*C)*x^7 + 2/3*(36*A - 36*B - 7*C
)*x^6 - 4/5*(36*A + 7*B - 24*C)*x^5 - (7*A - 24*B + 12*C)*x^4 + 8/3*(12*A
- 6*B + C)*x^3 - 4*(6*A - B)*x^2 + 8*A*x
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.99

$$\int (A+Bx+Cx^2)\left(2-4x+3x^3\right)^3\,dx = 8Ax+\frac{27Bx^{11}}{11}+\frac{9Cx^{12}}{4}+x^{10}\cdot\left(\frac{27A}{10}-\frac{54C}{5}\right)+x^9(-12B+6C)+x^8\left(-\frac{27A}{2}+\frac{27B}{4}+18C\right)+x^7\cdot\left(\frac{54A}{7}+\frac{144B}{7}-\frac{144C}{7}\right)+x^6\cdot\left(24A-24B-\frac{14C}{3}\right)+x^5\left(-\frac{144A}{5}-\frac{28B}{5}+\frac{96C}{5}\right)+x^4(-7A+24B-12C)+x^3\cdot\left(32A-16B+\frac{8C}{3}\right)+x^2(-24A+4B)$$

input
```
integrate((C*x**2+B*x+A)*(3*x**3-4*x+2)**3,x)
```

output
```
8*A*x + 27*B*x**11/11 + 9*C*x**12/4 + x**10*(27*A/10 - 54*C/5) + x**9*(-12
*B + 6*C) + x**8*(-27*A/2 + 27*B/4 + 18*C) + x**7*(54*A/7 + 144*B/7 - 144*
C/7) + x**6*(24*A - 24*B - 14*C/3) + x**5*(-144*A/5 - 28*B/5 + 96*C/5) + x
**4*(-7*A + 24*B - 12*C) + x**3*(32*A - 16*B + 8*C/3) + x**2*(-24*A + 4*B)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.90

$$\int (A+Bx+Cx^2)\left(2-4x+3x^3\right)^3\,dx = \frac{9}{4}Cx^{12}+\frac{27}{11}Bx^{11}+\frac{27}{10}(A-4C)x^{10}-6(2B-C)x^9-\frac{9}{4}(6A-3B-8C)x^8 + \frac{18}{7}(3A+8B-8C)x^7+\frac{2}{3}(36A-36B-7C)x^6-\frac{4}{5}(36A+7B-24C)x^5 - (7A-24B+12C)x^4+\frac{8}{3}(12A-6B+C)x^3-4(6A-B)x^2+8Ax$$

input
```
integrate((C*x^2+B*x+A)*(3*x^3-4*x+2)^3,x, algorithm="maxima")
```

output
```
9/4*C*x^12 + 27/11*B*x^11 + 27/10*(A - 4*C)*x^10 - 6*(2*B - C)*x^9 - 9/4*(
6*A - 3*B - 8*C)*x^8 + 18/7*(3*A + 8*B - 8*C)*x^7 + 2/3*(36*A - 36*B - 7*C
)*x^6 - 4/5*(36*A + 7*B - 24*C)*x^5 - (7*A - 24*B + 12*C)*x^4 + 8/3*(12*A
- 6*B + C)*x^3 - 4*(6*A - B)*x^2 + 8*A*x
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.04

$$\int (A+Bx+Cx^2)\left(2-4x+3x^3\right)^3\,dx = \frac{9}{4}Cx^{12}+\frac{27}{11}Bx^{11}+\frac{27}{10}Ax^{10}-\frac{54}{5}Cx^{10} -12Bx^9+6Cx^9-\frac{27}{2}Ax^8+\frac{27}{4}Bx^8+18Cx^8 + \frac{54}{7}Ax^7+\frac{144}{7}Bx^7-\frac{144}{7}Cx^7+24Ax^6 - 24Bx^6-\frac{14}{3}Cx^6-\frac{144}{5}Ax^5-\frac{28}{5}Bx^5 + \frac{96}{5}Cx^5-7Ax^4+24Bx^4-12Cx^4+32Ax^3 - 16Bx^3+\frac{8}{3}Cx^3-24Ax^2+4Bx^2+8Ax$$

input
```
integrate((C*x^2+B*x+A)*(3*x^3-4*x+2)^3,x, algorithm="giac")
```

output
```
9/4*C*x^12 + 27/11*B*x^11 + 27/10*A*x^10 - 54/5*C*x^10 - 12*B*x^9 + 6*C*x^
9 - 27/2*A*x^8 + 27/4*B*x^8 + 18*C*x^8 + 54/7*A*x^7 + 144/7*B*x^7 - 144/7*
C*x^7 + 24*A*x^6 - 24*B*x^6 - 14/3*C*x^6 - 144/5*A*x^5 - 28/5*B*x^5 + 96/5
*C*x^5 - 7*A*x^4 + 24*B*x^4 - 12*C*x^4 + 32*A*x^3 - 16*B*x^3 + 8/3*C*x^3 -
 24*A*x^2 + 4*B*x^2 + 8*A*x
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.90

$$\int (A+Bx+Cx^2)\,(2-4x+3x^3)^3\,dx$$
$$= \frac{9\,C\,x^{12}}{4}+\frac{27\,B\,x^{11}}{11}+\left(\frac{27\,A}{10}-\frac{54\,C}{5}\right)\,x^{10}+(6\,C-12\,B)\,x^9$$
$$+\left(\frac{27\,B}{4}-\frac{27\,A}{2}+18\,C\right)\,x^8+\left(\frac{54\,A}{7}+\frac{144\,B}{7}-\frac{144\,C}{7}\right)\,x^7$$
$$+\left(24\,A-24\,B-\frac{14\,C}{3}\right)\,x^6+\left(\frac{96\,C}{5}-\frac{28\,B}{5}-\frac{144\,A}{5}\right)\,x^5$$
$$+(24\,B-7\,A-12\,C)\,x^4+\left(32\,A-16\,B+\frac{8\,C}{3}\right)\,x^3+(4\,B-24\,A)\,x^2+8\,A\,x$$

input
```
int((A + B*x + C*x^2)*(3*x^3 - 4*x + 2)^3,x)
```

output
```
8*A*x + (27*B*x^11)/11 + (9*C*x^12)/4 - x^4*(7*A - 24*B + 12*C) + x^3*(32*
A - 16*B + (8*C)/3) - x^6*(24*B - 24*A + (14*C)/3) + x^8*((27*B)/4 - (27*A
)/2 + 18*C) - x^5*((144*A)/5 + (28*B)/5 - (96*C)/5) + x^7*((54*A)/7 + (144
*B)/7 - (144*C)/7) - x^2*(24*A - 4*B) + x^10*((27*A)/10 - (54*C)/5) - x^9*
(12*B - 6*C)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.03

$$\int \left(A+Bx+Cx^2\right)\left(2-4x+3x^3\right)^3\,dx$$
$$= \frac{x(10395c\,x^{11}+11340b\,x^{10}+12474a\,x^9-49896c\,x^9-55440b\,x^8+27720c\,x^8-62370a\,x^7+31185b\,x^7+}{}$$

input
```
int((C*x^2+B*x+A)*(3*x^3-4*x+2)^3,x)
```

output
```
(x*(12474*a*x**9 - 62370*a*x**7 + 35640*a*x**6 + 110880*a*x**5 - 133056*a*
x**4 - 32340*a*x**3 + 147840*a*x**2 - 110880*a*x + 36960*a + 11340*b*x**10
 - 55440*b*x**8 + 31185*b*x**7 + 95040*b*x**6 - 110880*b*x**5 - 25872*b*x*
*4 + 110880*b*x**3 - 73920*b*x**2 + 18480*b*x + 10395*c*x**11 - 49896*c*x*
*9 + 27720*c*x**8 + 83160*c*x**7 - 95040*c*x**6 - 21560*c*x**5 + 88704*c*x
**4 - 55440*c*x**3 + 12320*c*x**2))/4620
```

3.16 $\int (A+Bx+Cx^2)(2-4x+3x^3)^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 102

$$\int (A+Bx+Cx^2)(2-4x+3x^3)^2\,dx = 4Ax-2(4A-B)x^2+\frac{4}{3}(4A-4B+C)x^3+(3A+4B-4C)x^4-\frac{4}{5}(6A-3B-4C)x^5-2(2B-C)x^6+\frac{3}{7}(3A-8C)x^7+\frac{9Bx^8}{8}+Cx^9$$

output

```
4*A*x-2*(4*A-B)*x^2+4/3*(4*A-4*B+C)*x^3+(3*A+4*B-4*C)*x^4-4/5*(6*A-3*B-4*C
)*x^5-2*(2*B-C)*x^6+3/7*(3*A-8*C)*x^7+9/8*B*x^8+C*x^9
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00

$$\int (A+Bx+Cx^2)(2-4x+3x^3)^2\,dx = 4Ax-2(4A-B)x^2+\frac{4}{3}(4A-4B+C)x^3+(3A+4B-4C)x^4-\frac{4}{5}(6A-3B-4C)x^5-2(2B-C)x^6+\frac{3}{7}(3A-8C)x^7+\frac{9Bx^8}{8}+Cx^9$$

input
```
Integrate[(A + B*x + C*x^2)*(2 - 4*x + 3*x^3)^2,x]
```

output
```
4*A*x - 2*(4*A - B)*x^2 + (4*(4*A - 4*B + C)*x^3)/3 + (3*A + 4*B - 4*C)*x^
4 - (4*(6*A - 3*B - 4*C)*x^5)/5 - 2*(2*B - C)*x^6 + (3*(3*A - 8*C)*x^7)/7
+ (9*B*x^8)/8 + C*x^9
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(3x^3-4x+2\right)^2 \left(A+Bx+Cx^2\right) \, dx$$

$$\Big\downarrow \; 2188$$

$$\int \left(-4x^4(6A-3B-4C)+4x^3(3A+4B-4C)+4x^2(4A-4B+C)-4x(4A-B)+3x^6(3A-8C)+4A-1\right.$$

$$\Big\downarrow \; 2009$$

$$-\frac{4}{5}x^5(6A-3B-4C)+x^4(3A+4B-4C)+\frac{4}{3}x^3(4A-4B+C)-2x^2(4A-B)+\frac{3}{7}x^7(3A-8C)+4Ax-2x^6(2B-C)+\frac{9Bx^8}{8}+Cx^9$$

input
```
Int[(A + B*x + C*x^2)*(2 - 4*x + 3*x^3)^2,x]
```

output
```
4*A*x - 2*(4*A - B)*x^2 + (4*(4*A - 4*B + C)*x^3)/3 + (3*A + 4*B - 4*C)*x^
4 - (4*(6*A - 3*B - 4*C)*x^5)/5 - 2*(2*B - C)*x^6 + (3*(3*A - 8*C)*x^7)/7
+ (9*B*x^8)/8 + C*x^9
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.90

method	result
norman	$C\,x^{9}+\frac{9B\,x^{8}}{8}+\left(\frac{9A}{7}-\frac{24C}{7}\right)x^{7}+\left(-4B+2C\right)x^{6}+\left(-\frac{24A}{5}+\frac{12B}{5}+\frac{16C}{5}\right)x^{5}+\left(3A+4B-\right.$
default	$C\,x^{9}+\frac{9B\,x^{8}}{8}+\frac{(9A-24C)x^{7}}{7}+\frac{(-24B+12C)x^{6}}{6}+\frac{(-24A+12B+16C)x^{5}}{5}+\frac{(12A+16B-16C)x^{4}}{4}+\frac{(16A-16B}{3}$
orering	$\frac{x(840x^{8}C+945x^{7}B+1080x^{6}A-2880C\,x^{6}-3360B\,x^{5}+1680x^{5}C-4032x^{4}A+2016x^{4}B+2688C\,x^{4}+2520x^{3}A+3360B\,x^{3}-33}{840}$
gosper	$C\,x^{9}+\frac{9}{8}B\,x^{8}+\frac{9}{7}x^{7}A-\frac{24}{7}x^{7}C-4x^{6}B+2C\,x^{6}-\frac{24}{5}x^{5}A+\frac{12}{5}B\,x^{5}+\frac{16}{5}x^{5}C+3x^{4}A+4x$
risch	$C\,x^{9}+\frac{9}{8}B\,x^{8}+\frac{9}{7}x^{7}A-\frac{24}{7}x^{7}C-4x^{6}B+2C\,x^{6}-\frac{24}{5}x^{5}A+\frac{12}{5}B\,x^{5}+\frac{16}{5}x^{5}C+3x^{4}A+4x$
parallelrisch	$C\,x^{9}+\frac{9}{8}B\,x^{8}+\frac{9}{7}x^{7}A-\frac{24}{7}x^{7}C-4x^{6}B+2C\,x^{6}-\frac{24}{5}x^{5}A+\frac{12}{5}B\,x^{5}+\frac{16}{5}x^{5}C+3x^{4}A+4x$

input
```
int((C*x^2+B*x+A)*(3*x^3-4*x+2)^2,x,method=_RETURNVERBOSE)
```

output
```
C*x^9+9/8*B*x^8+(9/7*A-24/7*C)*x^7+(-4*B+2*C)*x^6+(-24/5*A+12/5*B+16/5*C)*
x^5+(3*A+4*B-4*C)*x^4+(16/3*A-16/3*B+4/3*C)*x^3+(-8*A+2*B)*x^2+4*A*x
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.92

$$\int (A+Bx+Cx^2)\left(2-4x+3x^3\right)^2\,dx = Cx^9+\frac{9}{8}Bx^8+\frac{3}{7}(3A-8C)x^7 - 2(2B-C)x^6-\frac{4}{5}(6A-3B-4C)x^5 + (3A+4B-4C)x^4+\frac{4}{3}(4A-4B+C)x^3 - 2(4A-B)x^2+4Ax$$

input
```
integrate((C*x^2+B*x+A)*(3*x^3-4*x+2)^2,x, algorithm="fricas")
```

output
```
C*x^9 + 9/8*B*x^8 + 3/7*(3*A - 8*C)*x^7 - 2*(2*B - C)*x^6 - 4/5*(6*A - 3*B
 - 4*C)*x^5 + (3*A + 4*B - 4*C)*x^4 + 4/3*(4*A - 4*B + C)*x^3 - 2*(4*A - B
)*x^2 + 4*A*x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00

$$\int (A+Bx+Cx^2)\left(2-4x+3x^3\right)^2\,dx = 4Ax+\frac{9Bx^8}{8}+Cx^9+x^7\cdot\left(\frac{9A}{7}-\frac{24C}{7}\right) + x^6(-4B+2C)+x^5\left(-\frac{24A}{5}+\frac{12B}{5}+\frac{16C}{5}\right) + x^4\cdot(3A+4B-4C)+x^3 \cdot\left(\frac{16A}{3}-\frac{16B}{3}+\frac{4C}{3}\right)+x^2(-8A+2B)$$

input
```
integrate((C*x**2+B*x+A)*(3*x**3-4*x+2)**2,x)
```

output
```
4*A*x + 9*B*x**8/8 + C*x**9 + x**7*(9*A/7 - 24*C/7) + x**6*(-4*B + 2*C) +
x**5*(-24*A/5 + 12*B/5 + 16*C/5) + x**4*(3*A + 4*B - 4*C) + x**3*(16*A/3 -
 16*B/3 + 4*C/3) + x**2*(-8*A + 2*B)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.92

$$\int (A+Bx+Cx^2)\left(2-4x+3x^3\right)^2\,dx = Cx^9+\frac{9}{8}Bx^8+\frac{3}{7}(3A-8C)x^7 - 2(2B-C)x^6-\frac{4}{5}(6A-3B-4C)x^5 + (3A+4B-4C)x^4+\frac{4}{3}(4A-4B+C)x^3 - 2(4A-B)x^2+4Ax$$

input
```
integrate((C*x^2+B*x+A)*(3*x^3-4*x+2)^2,x, algorithm="maxima")
```

output
```
C*x^9 + 9/8*B*x^8 + 3/7*(3*A - 8*C)*x^7 - 2*(2*B - C)*x^6 - 4/5*(6*A - 3*B
 - 4*C)*x^5 + (3*A + 4*B - 4*C)*x^4 + 4/3*(4*A - 4*B + C)*x^3 - 2*(4*A - B
)*x^2 + 4*A*x
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.04

$$\int (A+Bx+Cx^2)\left(2-4x+3x^3\right)^2\,dx = Cx^9+\frac{9}{8}Bx^8+\frac{9}{7}Ax^7-\frac{24}{7}Cx^7-4Bx^6 + 2Cx^6-\frac{24}{5}Ax^5+\frac{12}{5}Bx^5+\frac{16}{5}Cx^5 + 3Ax^4+4Bx^4-4Cx^4+\frac{16}{3}Ax^3 - \frac{16}{3}Bx^3+\frac{4}{3}Cx^3-8Ax^2+2Bx^2+4Ax$$

input
```
integrate((C*x^2+B*x+A)*(3*x^3-4*x+2)^2,x, algorithm="giac")
```

output
```
C*x^9 + 9/8*B*x^8 + 9/7*A*x^7 - 24/7*C*x^7 - 4*B*x^6 + 2*C*x^6 - 24/5*A*x^
5 + 12/5*B*x^5 + 16/5*C*x^5 + 3*A*x^4 + 4*B*x^4 - 4*C*x^4 + 16/3*A*x^3 - 1
6/3*B*x^3 + 4/3*C*x^3 - 8*A*x^2 + 2*B*x^2 + 4*A*x
```

Mupad [B] (verification not implemented)

Time = 12.69 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.91

$$\int (A+Bx+Cx^2)\left(2-4x+3x^3\right)^2\,dx = C\,x^9+\frac{9\,B\,x^8}{8}+\left(\frac{9\,A}{7}-\frac{24\,C}{7}\right)\,x^7 +(2\,C-4\,B)\,x^6+\left(\frac{12\,B}{5}-\frac{24\,A}{5}+\frac{16\,C}{5}\right)\,x^5 +(3\,A+4\,B-4\,C)\,x^4 +\left(\frac{16\,A}{3}-\frac{16\,B}{3}+\frac{4\,C}{3}\right)\,x^3 +(2\,B-8\,A)\,x^2+4\,A\,x$$

input
```
int((A + B*x + C*x^2)*(3*x^3 - 4*x + 2)^2,x)
```

output
```
4*A*x + (9*B*x^8)/8 + C*x^9 + x^4*(3*A + 4*B - 4*C) + x^3*((16*A)/3 - (16*
B)/3 + (4*C)/3) + x^5*((12*B)/5 - (24*A)/5 + (16*C)/5) - x^2*(8*A - 2*B) +
 x^7*((9*A)/7 - (24*C)/7) - x^6*(4*B - 2*C)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.03

$$\int (A+Bx+Cx^2)\left(2-4x+3x^3\right)^2\,dx = \frac{x(840c\,x^8+945b\,x^7+1080a\,x^6-2880c\,x^6-3360b\,x^5+1680c\,x^5-4032a\,x^4+2016b\,x^4+2688c\,x^4+2}{840}$$

input
```
int((C*x^2+B*x+A)*(3*x^3-4*x+2)^2,x)
```

output
```
(x*(1080*a*x**6 - 4032*a*x**4 + 2520*a*x**3 + 4480*a*x**2 - 6720*a*x + 336
0*a + 945*b*x**7 - 3360*b*x**5 + 2016*b*x**4 + 3360*b*x**3 - 4480*b*x**2 +
 1680*b*x + 840*c*x**8 - 2880*c*x**6 + 1680*c*x**5 + 2688*c*x**4 - 3360*c*
x**3 + 1120*c*x**2))/840
```

3.17 $\int (A+Bx+Cx^2)(2-4x+3x^3)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 61

$$\int (A+Bx+Cx^2)(2-4x+3x^3)\,dx=2Ax-(2A-B)x^2-\frac{2}{3}(2B-C)x^3+\frac{1}{4}(3A-4C)x^4+\frac{3Bx^5}{5}+\frac{Cx^6}{2}$$

output
```
2*A*x-(2*A-B)*x^2-2/3*(2*B-C)*x^3+1/4*(3*A-4*C)*x^4+3/5*B*x^5+1/2*C*x^6
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.82

$$\int (A+Bx+Cx^2)(2-4x+3x^3)\,dx=\frac{1}{60}x\left(15A(8-8x+3x^3)+2x\left(5Cx(4-6x+3x^3)+2B(15-20x+9x^3)\right)\right)$$

input
```
Integrate[(A + B*x + C*x^2)*(2 - 4*x + 3*x^3),x]
```

output
```
(x*(15*A*(8 - 8*x + 3*x^3) + 2*x*(5*C*x*(4 - 6*x + 3*x^3) + 2*B*(15 - 20*x
 + 9*x^3))))/60
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (3x^3-4x+2)\left(A+Bx+Cx^2\right)\,dx$$

$$\Big\downarrow \text{2188}$$

$$\int \left(-2x(2A-B)+x^3(3A-4C)+2A-2x^2(2B-C)+3Bx^4+3Cx^5\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-x^2(2A-B)+\frac{1}{4}x^4(3A-4C)+2Ax-\frac{2}{3}x^3(2B-C)+\frac{3Bx^5}{5}+\frac{Cx^6}{2}$$

input
```
Int[(A + B*x + C*x^2)*(2 - 4*x + 3*x^3),x]
```

output
```
2*A*x - (2*A - B)*x^2 - (2*(2*B - C)*x^3)/3 + ((3*A - 4*C)*x^4)/4 + (3*B*x
^5)/5 + (C*x^6)/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.80

method	result	size
norman	$\frac{C\,x^{6}}{2}+\frac{3B\,x^{5}}{5}+\left(\frac{3A}{4}-C\right)x^{4}+\left(-\frac{4B}{3}+\frac{2C}{3}\right)x^{3}+\left(-2A+B\right)x^{2}+2Ax$	49
orering	$\frac{x\left(30x^{5}C+36x^{4}B+45x^{3}A-60C\,x^{3}-80B\,x^{2}+40C\,x^{2}-120Ax+60Bx+120A\right)}{60}$	52
gosper	$\frac{1}{2}C\,x^{6}+\frac{3}{5}B\,x^{5}+\frac{3}{4}x^{4}A-C\,x^{4}-\frac{4}{3}B\,x^{3}+\frac{2}{3}C\,x^{3}-2A\,x^{2}+B\,x^{2}+2Ax$	53
risch	$\frac{1}{2}C\,x^{6}+\frac{3}{5}B\,x^{5}+\frac{3}{4}x^{4}A-C\,x^{4}-\frac{4}{3}B\,x^{3}+\frac{2}{3}C\,x^{3}-2A\,x^{2}+B\,x^{2}+2Ax$	53
parallelrisch	$\frac{1}{2}C\,x^{6}+\frac{3}{5}B\,x^{5}+\frac{3}{4}x^{4}A-C\,x^{4}-\frac{4}{3}B\,x^{3}+\frac{2}{3}C\,x^{3}-2A\,x^{2}+B\,x^{2}+2Ax$	53
default	$\frac{C\,x^{6}}{2}+\frac{3B\,x^{5}}{5}+\frac{(3A-4C)x^{4}}{4}+\frac{(-4B+2C)x^{3}}{3}+\frac{(-4A+2B)x^{2}}{2}+2Ax$	54

input
```
int((C*x^2+B*x+A)*(3*x^3-4*x+2),x,method=_RETURNVERBOSE)
```

output
```
1/2*C*x^6+3/5*B*x^5+(3/4*A-C)*x^4+(-4/3*B+2/3*C)*x^3+(-2*A+B)*x^2+2*A*x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.87

$$\int \left(A+Bx+Cx^2\right)\left(2-4x+3x^3\right)\,dx=\frac{1}{2}\,Cx^6+\frac{3}{5}\,Bx^5+\frac{1}{4}\,(3\,A-4\,C)x^4-\frac{2}{3}\,(2\,B-C)x^3-(2\,A-B)x^2+2\,Ax$$

input
```
integrate((C*x^2+B*x+A)*(3*x^3-4*x+2),x, algorithm="fricas")
```

output
```
1/2*C*x^6 + 3/5*B*x^5 + 1/4*(3*A - 4*C)*x^4 - 2/3*(2*B - C)*x^3 - (2*A - B
)*x^2 + 2*A*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.84

$$\int (A+Bx+Cx^2)\,(2-4x+3x^3)\;dx = 2Ax+\frac{3Bx^5}{5}+\frac{Cx^6}{2}+x^4\cdot\left(\frac{3A}{4}-C\right)+x^3\left(-\frac{4B}{3}+\frac{2C}{3}\right)+x^2(-2A+B)$$

input
```
integrate((C*x**2+B*x+A)*(3*x**3-4*x+2),x)
```

output
```
2*A*x + 3*B*x**5/5 + C*x**6/2 + x**4*(3*A/4 - C) + x**3*(-4*B/3 + 2*C/3) +
 x**2*(-2*A + B)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.87

$$\int (A+Bx+Cx^2)\,(2-4x+3x^3)\;dx = \frac{1}{2}\,Cx^6+\frac{3}{5}\,Bx^5+\frac{1}{4}\,(3\,A-4\,C)x^4-\frac{2}{3}\,(2\,B-C)x^3-(2\,A-B)x^2+2\,Ax$$

input
```
integrate((C*x^2+B*x+A)*(3*x^3-4*x+2),x, algorithm="maxima")
```

output
```
1/2*C*x^6 + 3/5*B*x^5 + 1/4*(3*A - 4*C)*x^4 - 2/3*(2*B - C)*x^3 - (2*A - B
)*x^2 + 2*A*x
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.85

$$\int (A+Bx+Cx^2)\left(2-4x+3x^3\right)\,dx = \frac{1}{2}Cx^6+\frac{3}{5}Bx^5+\frac{3}{4}Ax^4-Cx^4-\frac{4}{3}Bx^3+\frac{2}{3}Cx^3-2Ax^2+Bx^2+2Ax$$

input
```
integrate((C*x^2+B*x+A)*(3*x^3-4*x+2),x, algorithm="giac")
```

output
```
1/2*C*x^6 + 3/5*B*x^5 + 3/4*A*x^4 - C*x^4 - 4/3*B*x^3 + 2/3*C*x^3 - 2*A*x^
2 + B*x^2 + 2*A*x
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.85

$$\int (A+Bx+Cx^2)\left(2-4x+3x^3\right)\,dx = \frac{C\,x^6}{2}+\frac{3\,B\,x^5}{5}+\left(\frac{3\,A}{4}-C\right)\,x^4+\left(\frac{2\,C}{3}-\frac{4\,B}{3}\right)\,x^3+\left(B-2\,A\right)\,x^2+2\,A\,x$$

input
```
int((A + B*x + C*x^2)*(3*x^3 - 4*x + 2),x)
```

output
```
2*A*x + (3*B*x^5)/5 + (C*x^6)/2 - x^2*(2*A - B) + x^4*((3*A)/4 - C) - x^3*
((4*B)/3 - (2*C)/3)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.84

$$\int (A + Bx + Cx^2) (2 - 4x + 3x^3) \, dx$$
$$= \frac{x(30c\,x^5 + 36b\,x^4 + 45a\,x^3 - 60c\,x^3 - 80b\,x^2 + 40c\,x^2 - 120ax + 60bx + 120a)}{60}$$

input
```
int((C*x^2+B*x+A)*(3*x^3-4*x+2),x)
```

output
```
(x*(45*a*x**3 - 120*a*x + 120*a + 36*b*x**4 - 80*b*x**2 + 60*b*x + 30*c*x*
*5 - 60*c*x**3 + 40*c*x**2))/60
```

3.18 $\int \frac{A+Bx+Cx^2}{2-4x+3x^3} \, dx$

Optimal result

Integrand size = 23, antiderivative size = 574

$$\int \frac{A+Bx+Cx^2}{2-4x+3x^3} \, dx =$$

$$-\frac{\left(9(9-\sqrt{17})\left(4+(9-\sqrt{17})^{2/3}\right)A+2\left(3\left(49-9\sqrt{17}+8(9-\sqrt{17})^{2/3}\right)B+2(9-\sqrt{17})\left(4+(9-\cdots\right.\right.\right.}{3\sqrt{6\left(49-9\sqrt{17}+8\left(9-\sqrt{17}\right)^{2/3}-4\left(9-\sqrt{17}\right)^{4/3}\right)}\left(16+4\left(9\cdots\right.\right.}$$

$$+\frac{\sqrt[3]{9-\sqrt{17}}\left(9\sqrt[3]{9-\sqrt{17}}A-3\left(4+\left(9-\sqrt{17}\right)^{2/3}\right)B+4\sqrt[3]{9-\sqrt{17}}C\right)\log\left(4+\left(9-\sqrt{17}\right)^{2/3}+3\sqrt[3]{\cdots}\right.}{9\left(16+4\left(9-\sqrt{17}\right)^{2/3}+\left(9-\sqrt{17}\right)^{4/3}\right)}$$

$$-\frac{\left(9(9-\sqrt{17})^{2/3}A-3\left(9-\sqrt{17}+4\sqrt[3]{9-\sqrt{17}}\right)B+4(9-\sqrt{17})^{2/3}C\right)\log\left(16-4(9-\sqrt{17})^{2/3}+\cdots\right.}{18\left(16+4\left(9-\sqrt{17}\right)^{2/3}+\left(9-\sqrt{17}\cdots\right.\right.}$$

$$+\frac{1}{9}C\log\left(2-4x+3x^3\right)$$

output
```
-1/3*(9*(9-17^(1/2))*(4+(9-17^(1/2))^(2/3))*A+6*(49-9*17^(1/2)+8*(9-17^(1/
2))^(2/3))*B+4*(9-17^(1/2))*(4+(9-17^(1/2))^(2/3))*C)*arctan((9-17^(1/2)+4
*(9-17^(1/2))^(1/3)-6*(9-17^(1/2))^(2/3)*x)/(294-54*17^(1/2)+48*(9-17^(1/2
))^(2/3)-24*(9-17^(1/2))^(4/3))^(1/2))/(294-54*17^(1/2)+48*(9-17^(1/2))^(2
/3)-24*(9-17^(1/2))^(4/3))^(1/2)/(16+4*(9-17^(1/2))^(2/3)+(9-17^(1/2))^(4/
3))+(9-17^(1/2))^(1/3)*(9*(9-17^(1/2))^(1/3)*A-3*(4+(9-17^(1/2))^(2/3))*B+
4*(9-17^(1/2))^(1/3)*C)*ln(4+(9-17^(1/2))^(2/3)+3*(9-17^(1/2))^(1/3)*x)/(1
44+36*(9-17^(1/2))^(2/3)+9*(9-17^(1/2))^(4/3))-(9*(9-17^(1/2))^(2/3)*A-3*(
9-17^(1/2)+4*(9-17^(1/2))^(1/3))*B+4*(9-17^(1/2))^(2/3)*C)*ln(16-4*(9-17^(
1/2))^(2/3)+(9-17^(1/2))^(4/3)-3*(9-17^(1/2)+4*(9-17^(1/2))^(1/3))*x+9*(9-
17^(1/2))^(2/3)*x^2)/(288+72*(9-17^(1/2))^(2/3)+18*(9-17^(1/2))^(4/3))+1/9
*C*ln(3*x^3-4*x+2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.10

$$\int \frac{A+Bx+Cx^2}{2-4x+3x^3}\,dx = \text{RootSum}\left[2-4\#1+3\#1^3\&,\frac{A\log(x-\#1)+B\log(x-\#1)\#1+C\log(x-\#1)\#1^2}{-4+9\#1^2}\&\right]$$

input
```
Integrate[(A + B*x + C*x^2)/(2 - 4*x + 3*x^3),x]
```

output
```
RootSum[2 - 4*#1 + 3*#1^3 & , (A*Log[x - #1] + B*Log[x - #1]*#1 + C*Log[x
- #1]*#1^2)/(-4 + 9*#1^2) & ]
```

Rubi [A] (verified)

Time = 2.25 (sec) , antiderivative size = 571, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {2525, 2485, 25, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{3x^3-4x+2}dx$$

$$\Big\downarrow \text{ 2525}$$

$$\frac{1}{9}\int \frac{9A+4C+9Bx}{3x^3-4x+2}dx+\frac{1}{9}C\log\left(3x^3-4x+2\right)$$

$$\Big\downarrow \text{ 2485}$$

$$\int -\frac{9A+4C+9Bx}{\left(3x+\frac{4+\left(9-\sqrt{17}\right)^{2/3}}{\sqrt[3]{9-\sqrt{17}}}\right)\left(-9x^2+\frac{3\left(4+\left(9-\sqrt{17}\right)^{2/3}\right)x}{\sqrt[3]{9-\sqrt{17}}}-\left(9-\sqrt{17}\right)^{2/3}-\frac{16}{\left(9-\sqrt{17}\right)^{2/3}}+4\right)}dx+\frac{1}{9}C\log\left(3x^3-4x+2\right)$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{9}C\log\left(3x^3-4x+2\right)-\int \frac{9A+4C+9Bx}{\left(3x+\frac{4+\left(9-\sqrt{17}\right)^{2/3}}{\sqrt[3]{9-\sqrt{17}}}\right)\left(-9x^2+\frac{3\left(4+\left(9-\sqrt{17}\right)^{2/3}\right)x}{\sqrt[3]{9-\sqrt{17}}}-\left(9-\sqrt{17}\right)^{2/3}-\frac{16}{\left(9-\sqrt{17}\right)^{2/3}}+4\right)}dx$$

$$\Big\downarrow \text{ 1200}$$

$$\frac{1}{9}C\log\left(3x^3-4x+2\right)-\int \left(\frac{\left(9-\sqrt{17}\right)^{2/3}\left(-9\sqrt[3]{9-\sqrt{17}}A+3\left(4+\left(9-\sqrt{17}\right)^{2/3}\right)B-4\sqrt[3]{9-\sqrt{17}}C\right)}{3\left(16+4\left(9-\sqrt{17}\right)^{2/3}+\left(9-\sqrt{17}\right)^{4/3}\right)\left(3\sqrt[3]{9-\sqrt{17}}x+\left(9-\sqrt{17}\right)^{2/3}+4\right)}+\frac{\left(9-\sqrt{17}\right)^{2/3}\left(-18\left(9-\right.\right.}{}\right.$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{\arctan\left(\frac{-6\left(9-\sqrt{17}\right)^{2/3}x+4\sqrt[3]{9-\sqrt{17}}-\sqrt{17}+9}{\sqrt{6\left(49-9\sqrt{17}+8\left(9-\sqrt{17}\right)^{2/3}-4\left(9-\sqrt{17}\right)^{4/3}\right)}}\right)\left(9(9-\sqrt{17})\left(4+(9-\sqrt{17})^{2/3}\right)A+6\left(49-9\sqrt{17}+8(9-\right.\right.}{3\sqrt{6\left(49-9\sqrt{17}+8\left(9-\sqrt{17}\right)^{2/3}-4\left(9-\sqrt{17}\right)^{4/3}\right)}\left(16+4\left(9-\sqrt{17}\right)^{2/3}\right.}$$

$$\frac{\log\left(9(9-\sqrt{17})^{2/3}x^2-3\left(9-\sqrt{17}+4\sqrt[3]{9-\sqrt{17}}\right)x+(9-\sqrt{17})^{4/3}-4(9-\sqrt{17})^{2/3}+16\right)\left(9(9-\sqrt{17})^{2/3}\right.}{18\left(16+4\left(9-\sqrt{17}\right)^{2/3}+\left(9-\sqrt{17}\right)^{4/3}\right)}$$

$$\frac{\sqrt[3]{9-\sqrt{17}}\log\left(3\sqrt[3]{9-\sqrt{17}}x+(9-\sqrt{17})^{2/3}+4\right)\left(9\sqrt[3]{9-\sqrt{17}}A-3\left(4+(9-\sqrt{17})^{2/3}\right)B+4\sqrt[3]{9-\sqrt{17}}C\right)}{9\left(16+4\left(9-\sqrt{17}\right)^{2/3}+\left(9-\sqrt{17}\right)^{4/3}\right)}$$

$$\frac{1}{9}C\log\left(3x^3-4x+2\right)$$

input

```
Int[(A + B*x + C*x^2)/(2 - 4*x + 3*x^3),x]
```

output

```
-1/3*((9*(9 - Sqrt[17])*(4 + (9 - Sqrt[17])^(2/3))*A + 6*(49 - 9*Sqrt[17]
+ 8*(9 - Sqrt[17])^(2/3))*B + 4*(9 - Sqrt[17])*(4 + (9 - Sqrt[17])^(2/3))*
C)*ArcTan[(9 - Sqrt[17] + 4*(9 - Sqrt[17])^(1/3) - 6*(9 - Sqrt[17])^(2/3)*
x)/Sqrt[6*(49 - 9*Sqrt[17] + 8*(9 - Sqrt[17])^(2/3) - 4*(9 - Sqrt[17])^(4/
3))]])/(Sqrt[6*(49 - 9*Sqrt[17] + 8*(9 - Sqrt[17])^(2/3) - 4*(9 - Sqrt[17]
)^(4/3))]*(16 + 4*(9 - Sqrt[17])^(2/3) + (9 - Sqrt[17])^(4/3))) + ((9 - Sq
rt[17])^(1/3)*(9*(9 - Sqrt[17])^(1/3)*A - 3*(4 + (9 - Sqrt[17])^(2/3))*B +
 4*(9 - Sqrt[17])^(1/3)*C)*Log[4 + (9 - Sqrt[17])^(2/3) + 3*(9 - Sqrt[17])
^(1/3)*x])/(9*(16 + 4*(9 - Sqrt[17])^(2/3) + (9 - Sqrt[17])^(4/3))) - ((9*
(9 - Sqrt[17])^(2/3)*A - 3*(9 - Sqrt[17] + 4*(9 - Sqrt[17])^(1/3))*B + 4*(
9 - Sqrt[17])^(2/3)*C)*Log[16 - 4*(9 - Sqrt[17])^(2/3) + (9 - Sqrt[17])^(4
/3) - 3*(9 - Sqrt[17] + 4*(9 - Sqrt[17])^(1/3))*x + 9*(9 - Sqrt[17])^(2/3)
*x^2])/(18*(16 + 4*(9 - Sqrt[17])^(2/3) + (9 - Sqrt[17])^(4/3))) + (C*Log[
2 - 4*x + 3*x^3])/9
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2485
```
Int[((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (d_.)*(x_)^3)^(p_), x_S
ymbol] :> With[{r = Rt[-9*a*d^2 + Sqrt[3]*d*Sqrt[4*b^3*d + 27*a^2*d^2], 3]}
, Simp[1/d^(2*p)   Int[(e + f*x)^m*Simp[18^(1/3)*b*(d/(3*r)) - r/18^(1/3) +
 d*x, x]^p*Simp[b*(d/3) + 12^(1/3)*b^2*(d^2/(3*r^2)) + r^2/(3*12^(1/3)) - d
*(2^(1/3)*b*(d/(3^(1/3)*r)) - r/18^(1/3))*x + d^2*x^2, x]^p, x], x]] /; Fre
eQ[{a, b, d, e, f, m}, x] && NeQ[4*b^3 + 27*a^2*d, 0] && ILtQ[p, 0]
```

rule 2525
```
Int[(Pm_)/(Qn_), x_Symbol] :> With[{m = Expon[Pm, x], n = Expon[Qn, x]}, Si
mp[Coeff[Pm, x, m]*(Log[Qn]/(n*Coeff[Qn, x, n])), x] + Simp[1/(n*Coeff[Qn,
x, n])   Int[ExpandToSum[n*Coeff[Qn, x, n]*Pm - Coeff[Pm, x, m]*D[Qn, x], x
]/Qn, x], x] /; EqQ[m, n - 1]] /; PolyQ[Pm, x] && PolyQ[Qn, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.07

method	result	size
default	$\sum_{\_R=\operatorname{RootOf}(3\_Z^3-4\_Z+2)}\frac{(C\_R^2+B\_R+A)\ln(x-\_R)}{9\_R^2-4}$	41
risch	$\sum_{\_R=\operatorname{RootOf}(3\_Z^3-4\_Z+2)}\frac{(C\_R^2+B\_R+A)\ln(x-\_R)}{9\_R^2-4}$	41

input
```
int((C*x^2+B*x+A)/(3*x^3-4*x+2),x,method=_RETURNVERBOSE)
```

output
```
sum((C*_R^2+B*_R+A)/(9*_R^2-4)*ln(x-_R),_R=RootOf(3*_Z^3-4*_Z+2))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.87 (sec) , antiderivative size = 7302, normalized size of antiderivative = 12.72

$$\int \frac{A + Bx + Cx^2}{2 - 4x + 3x^3} \, dx = \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)/(3*x^3-4*x+2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [A] (verification not implemented)

Time = 8.09 (sec) , antiderivative size = 277, normalized size of antiderivative = 0.48

$$\int \frac{A + Bx + Cx^2}{2 - 4x + 3x^3} \, dx$$
$$= \operatorname{RootSum}\left(612t^3 - 204t^2C + t\left(108A^2 + 162AB + 96AC + 48B^2 + 72BC + 44C^2\right) - 9A^3 - 24A^2C +$$

input
```
integrate((C*x**2+B*x+A)/(3*x**3-4*x+2),x)
```

output
```
RootSum(612*_t**3 - 204*_t**2*C + _t*(108*A**2 + 162*A*B + 96*A*C + 48*B**
2 + 72*B*C + 44*C**2) - 9*A**3 - 24*A**2*C + 12*A*B**2 - 18*A*B*C - 16*A*C
**2 + 6*B**3 - 8*B*C**2 - 4*C**3, Lambda(_t, _t*log(x + (7344*_t**2*A + 55
08*_t**2*B + 3264*_t**2*C + 918*_t*A**2 - 816*_t*A*C - 408*_t*B**2 - 1224*
_t*B*C - 544*_t*C**2 + 864*A**3 + 1944*A**2*B + 1050*A**2*C + 1356*A*B**2
+ 1728*A*B*C + 512*A*C**2 + 288*B**3 + 648*B**2*C + 452*B*C**2 + 96*C**3)/
(729*A**3 + 1296*A**2*B + 972*A**2*C + 972*A*B**2 + 1152*A*B*C + 432*A*C**
2 + 294*B**3 + 432*B**2*C + 256*B*C**2 + 64*C**3)))
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{2-4x+3x^3}\,dx=\int \frac{Cx^2+Bx+A}{3\,x^3-4\,x+2}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(3*x^3-4*x+2),x, algorithm="maxima")
```

output
```
integrate((C*x^2 + B*x + A)/(3*x^3 - 4*x + 2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A+Bx+Cx^2}{2-4x+3x^3}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((C*x^2+B*x+A)/(3*x^3-4*x+2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 12.54 (sec) , antiderivative size = 322, normalized size of antiderivative = 0.56

$$\int \frac{A+Bx+Cx^2}{2-4x+3x^3}\,dx = \sum_{k=1}^{3} \ln\left(-x\left(-3\,B^2+4\,C^2+3\,A\,C\right)+2\,C^2+\mathrm{root}\left(z^3-\frac{C\,z^2}{3}+\frac{z\left(108\,A^2+48\,B^2+44\,C^2+162\,A\,B+96\,A\,C+72\,B\,C\right)}{612}-\frac{A\,B\,C}{34}-\frac{2\,B\,C^2}{153}-\frac{4\,A\,C^2}{153}-\frac{2\,A^2\,C}{51}+\frac{A\,B^2}{51}-\frac{A^3}{68}-\frac{C^3}{153}+\frac{B^3}{102},z,k\right)\left(12\,B-36\,C+x\left(27\,A+60\,C\right)-\mathrm{root}\left(z^3-\frac{C\,z^2}{3}+\frac{z\left(108\,A^2+48\,B^2+44\,C^2+162\,A\,B+96\,A\,C+72\,B\,C\right)}{612}-\frac{A\,B\,C}{34}-\frac{2\,B\,C^2}{153}-\frac{4\,A\,C}{153}\right.\right.\right.$$
$$\left.+3\,A\,B\right)\,\mathrm{root}\left(z^3-\frac{C\,z^2}{3}+\frac{z\left(108\,A^2+48\,B^2+44\,C^2+162\,A\,B+96\,A\,C+72\,B\,C\right)}{612}-\frac{A\,B\,C}{34}-\frac{2\,B\,C^2}{153}-\frac{4\,A\,C^2}{153}-\frac{2\,A^2\,C}{51}+\frac{A\,B^2}{51}-\frac{A^3}{68}-\frac{C^3}{153}+\frac{B^3}{102},z,k\right)$$

input
```
int((A + B*x + C*x^2)/(3*x^3 - 4*x + 2),x)
```

output
```
symsum(log(2*C^2 - x*(4*C^2 - 3*B^2 + 3*A*C) + root(z^3 - (C*z^2)/3 + (z*(
108*A^2 + 48*B^2 + 44*C^2 + 162*A*B + 96*A*C + 72*B*C))/612 - (A*B*C)/34 -
 (2*B*C^2)/153 - (4*A*C^2)/153 - (2*A^2*C)/51 + (A*B^2)/51 - A^3/68 - C^3/
153 + B^3/102, z, k)*(12*B - 36*C + x*(27*A + 60*C) - root(z^3 - (C*z^2)/3
 + (z*(108*A^2 + 48*B^2 + 44*C^2 + 162*A*B + 96*A*C + 72*B*C))/612 - (A*B*
C)/34 - (2*B*C^2)/153 - (4*A*C^2)/153 - (2*A^2*C)/51 + (A*B^2)/51 - A^3/68
 - C^3/153 + B^3/102, z, k)*(216*x - 162)) + 3*A*B)*root(z^3 - (C*z^2)/3 +
 (z*(108*A^2 + 48*B^2 + 44*C^2 + 162*A*B + 96*A*C + 72*B*C))/612 - (A*B*C)
/34 - (2*B*C^2)/153 - (4*A*C^2)/153 - (2*A^2*C)/51 + (A*B^2)/51 - A^3/68 -
 C^3/153 + B^3/102, z, k), k, 1, 3)
```

Reduce [F]

$$\int \frac{A+Bx+Cx^2}{2-4x+3x^3}\,dx = \left(\int \frac{x^2}{3x^3-4x+2}dx\right)c + \left(\int \frac{x}{3x^3-4x+2}dx\right)b + \left(\int \frac{1}{3x^3-4x+2}dx\right)a$$

input
```
int((C*x^2+B*x+A)/(3*x^3-4*x+2),x)
```

output
```
int(x**2/(3*x**3 - 4*x + 2),x)*c + int(x/(3*x**3 - 4*x + 2),x)*b + int(1/(
3*x**3 - 4*x + 2),x)*a
```

3.19 $\int \frac{A+Bx+Cx^2}{(2-4x+3x^3)^2} \, dx$

Optimal result

Integrand size = 23, antiderivative size = 1171

$$\int \frac{A+Bx+Cx^2}{(2-4x+3x^3)^2} \, dx = \text{Too large to display}$$

output
```
(9-17^(1/2))^(1/3)*(36*A+27*B+16*C)/(136+34*(9-17^(1/2))^(2/3)+102*(9-17^(
1/2))^(1/3)*x)+1/34*(-17+9*17^(1/2))^(1/3)*(72*A+54*B+32*C-3*(9*(9-17^(1/2
)+4*(9-17^(1/2))^(1/3))*A+3*(16+(9-17^(1/2))^(4/3))*B+4*(9-17^(1/2)+4*(9-1
7^(1/2))^(1/3))*C)*x/(9-17^(1/2))^(2/3))*17^(1/3)/(4-(9-17^(1/2))^(2/3))/(
(4+(9-17^(1/2))^(2/3))/(9-17^(1/2))^(1/3)+3*x)/(4-16/(9-17^(1/2))^(2/3)-(9
-17^(1/2))^(2/3)+3*(4+(9-17^(1/2))^(2/3))*x/(9-17^(1/2))^(1/3)-9*x^2)-C/(2
7*x^3-36*x+18)-1/17*(9*(117000-27592*17^(1/2)+(21753-5009*17^(1/2))*(9-17^
(1/2))^(2/3))*A+3*(231137-54585*17^(1/2)+(40916-9396*17^(1/2))*(9-17^(1/2)
)^(2/3))*B+4*(117000-27592*17^(1/2)+(21753-5009*17^(1/2))*(9-17^(1/2))^(2/
3))*C)*arctan((9-17^(1/2)+4*(9-17^(1/2))^(1/3)-6*(9-17^(1/2))^(2/3)*x)/(29
4-54*17^(1/2)+48*(9-17^(1/2))^(2/3)-24*(9-17^(1/2))^(4/3))^(1/2))/(2376-52
0*17^(1/2)+(1889-441*17^(1/2))*(9-17^(1/2))^(1/3)+(594-130*17^(1/2))*(9-17
^(1/2))^(2/3))/(294-54*17^(1/2)+48*(9-17^(1/2))^(2/3)-24*(9-17^(1/2))^(4/3
))^(1/2)-4/3*(9-17^(1/2))*(9*(196-36*17^(1/2)+8*(9-17^(1/2))^(4/3)-9*(9-17
^(1/2))^(5/3))*A+3*(441-81*17^(1/2)+18*(9-17^(1/2))^(4/3)-16*(9-17^(1/2))^
(5/3))*B+4*(196-36*17^(1/2)+8*(9-17^(1/2))^(4/3)-9*(9-17^(1/2))^(5/3))*C)*
ln(4+(9-17^(1/2))^(2/3)+3*(9-17^(1/2))^(1/3)*x)/(4-(9-17^(1/2))^(2/3))^2/(
16+4*(9-17^(1/2))^(2/3)+(9-17^(1/2))^(4/3))^3+8/3*(9*(7556-1764*17^(1/2)+8
*(297-65*17^(1/2))*(9-17^(1/2))^(1/3)-9*(297-65*17^(1/2))*(9-17^(1/2))^(2/
3))*A+3*(17001-3969*17^(1/2)+18*(297-65*17^(1/2))*(9-17^(1/2))^(1/3)-16...
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.13

$$\int \frac{A+Bx+Cx^2}{(2-4x+3x^3)^2}\,dx$$
$$= \frac{1}{34}\left(\frac{2C(-9+6x+8x^2)+B(-24+16x+27x^2)+A(-32+27x+36x^2)}{2-4x+3x^3}\right.$$
$$+\text{RootSum}\Big[2-4\#1$$
$$+3\#1^3\&,\frac{54A\log(x-\#1)+32B\log(x-\#1)+24C\log(x-\#1)+36A\log(x-\#1)\#1+27B\log(x}{-4+9\#1^2}$$

input
```
Integrate[(A + B*x + C*x^2)/(2 - 4*x + 3*x^3)^2,x]
```

output
```
((2*C*(-9 + 6*x + 8*x^2) + B*(-24 + 16*x + 27*x^2) + A*(-32 + 27*x + 36*x^
2))/(2 - 4*x + 3*x^3) + RootSum[2 - 4*#1 + 3*#1^3 & , (54*A*Log[x - #1] +
32*B*Log[x - #1] + 24*C*Log[x - #1] + 36*A*Log[x - #1]*#1 + 27*B*Log[x - #
1]*#1 + 16*C*Log[x - #1]*#1)/(-4 + 9*#1^2) & ])/34
```

Rubi [A] (warning: unable to verify)

Time = 5.45 (sec) , antiderivative size = 1215, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {2526, 2485, 1235, 27, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{(3x^3-4x+2)^2}dx$$

$$\Big\downarrow \text{ 2526}$$

$$\frac{1}{9}\int \frac{9A+4C+9Bx}{(3x^3-4x+2)^2}dx-\frac{C}{9(3x^3-4x+2)}$$

$$\Big\downarrow \text{ 2485}$$

$$9\int \frac{9A+4C+9Bx}{\left(3x+\frac{4+\left(9-\sqrt{17}\right)^{2/3}}{\sqrt[3]{9-\sqrt{17}}}\right)^2\left(-9x^2+\frac{3\left(4+\left(9-\sqrt{17}\right)^{2/3}\right)x}{\sqrt[3]{9-\sqrt{17}}}-\left(9-\sqrt{17}\right)^{2/3}-\frac{16}{\left(9-\sqrt{17}\right)^{2/3}}+4\right)^2}dx-\frac{C}{9(3x^3-4x+2)}$$

$$\Big\downarrow \text{ 1235}$$

$$9\left(\frac{\sqrt[3]{9\sqrt{17}-17}\left(2(36A+27B+16C)-\frac{3x\left(9\left(9-\sqrt{17}+4\sqrt[3]{9-\sqrt{17}}\right)A+3\left(16+\left(9-\sqrt{17}\right)^{4/3}\right)B+4\left(9-\sqrt{17}+4\sqrt[3]{9-\sqrt{}}\right.\right.}{\left(9-\sqrt{17}\right)^{2/3}}\right.}{18\ 17^{2/3}\left(4-\left(9-\sqrt{17}\right)^{2/3}\right)\left(3x+\frac{4+\left(9-\sqrt{17}\right)^{2/3}}{\sqrt[3]{9-\sqrt{17}}}\right)\left(-9x^2+\frac{3\left(4+\left(9-\sqrt{17}\right)^{2/3}\right)x}{\sqrt[3]{9-\sqrt{17}}}-\left(9-\sqrt{17}\right)^{2/3}-\frac{16}{\left(9-\sqrt{17}\right)}\right.}\right.$$

$$\frac{C}{9\left(3x^3-4x+2\right)}$$

$$\Big\downarrow\ 27$$

$$9\left(\frac{\sqrt[3]{9\sqrt{17}-17}\int\frac{-\frac{\left(4+\left(9-\sqrt{17}\right)^{2/3}\right)^3B}{9-\sqrt{17}}+45B-\frac{\left(16-4\left(9-\sqrt{17}\right)^{2/3}+\left(9-\sqrt{17}\right)^{4/3}\right)(9A+4C)}{\left(9-\sqrt{17}\right)^{2/3}}-\frac{3\left(9\left(9-\sqrt{17}+4\sqrt[3]{9-\sqrt{17}}\right)A+3\left(16+\left(9-\sqrt{}\right.\right.\right.}{\left(9-\sqrt{17}\right.}}{\left(3x+\frac{4+\left(9-\sqrt{17}\right)^{2/3}}{\sqrt[3]{9-\sqrt{17}}}\right)^2\left(-9x^2+\frac{3\left(4+\left(9-\sqrt{17}\right)^{2/3}\right)x}{\sqrt[3]{9-\sqrt{17}}}-\left(9-\sqrt{17}\right)^{2/3}-\frac{16}{\left(9-\sqrt{17}\right)^{2/3}}\right.}}{3\ 17^{2/3}\left(4-\left(9-\sqrt{17}\right)^{2/3}\right)}\right.$$

$$\frac{C}{9\left(3x^3-4x+2\right)}$$

$$\Big\downarrow\ 1200$$

$$9\left(\frac{\sqrt[3]{-17+9\sqrt{17}}\left(2(36A+27B+16C)-\frac{3\left(9\left(9-\sqrt{17}+4\sqrt[3]{9-\sqrt{17}}\right)A+3\left(16+\left(9-\sqrt{17}\right)^{4/3}\right)B+4\left(9-\sqrt{17}+4\sqrt[3]{9-\sqrt{17}}\right.\right.}{\left(9-\sqrt{17}\right)^{2/3}}\right.}{18\ 17^{2/3}\left(4-\left(9-\sqrt{17}\right)^{2/3}\right)\left(3x+\frac{4+\left(9-\sqrt{17}\right)^{2/3}}{\sqrt[3]{9-\sqrt{17}}}\right)\left(-9x^2+\frac{3\left(4+\left(9-\sqrt{17}\right)^{2/3}\right)x}{\sqrt[3]{9-\sqrt{17}}}-\left(9-\sqrt{17}\right)^{2/3}-\frac{16}{\left(9-\sqrt{17}\right.}\right.}\right.$$

$$\frac{C}{9\left(3x^3-4x+2\right)}$$

$\big\downarrow$ 2009

$$9\left(\frac{\sqrt[3]{-17+9\sqrt{17}}\left(2(36A+27B+16C)-\frac{3\left(9\left(9-\sqrt{17}+4\sqrt[3]{9-\sqrt{17}}\right)A+3\left(16+\left(9-\sqrt{17}\right)^{4/3}\right)B+4\left(9-\sqrt{17}+4\sqrt[3]{9-\sqrt{17}}\right.\right.}{\left(9-\sqrt{17}\right)^{2/3}}\right.}{18\ 17^{2/3}\left(4-\left(9-\sqrt{17}\right)^{2/3}\right)\left(3x+\frac{4+\left(9-\sqrt{17}\right)^{2/3}}{\sqrt[3]{9-\sqrt{17}}}\right)\left(-9x^2+\frac{3\left(4+\left(9-\sqrt{17}\right)^{2/3}\right)x}{\sqrt[3]{9-\sqrt{17}}}-\left(9-\sqrt{17}\right)^{2/3}-\frac{16}{\left(9-\sqrt{17}\right.}\right.}\right.$$

$$\frac{C}{9\left(3x^3-4x+2\right)}$$

input
```
Int[(A + B*x + C*x^2)/(2 - 4*x + 3*x^3)^2,x]
```

output
```
-1/9*C/(2 - 4*x + 3*x^3) + 9*(((-17 + 9*Sqrt[17])^(1/3)*(2*(36*A + 27*B +
16*C) - (3*(9*(9 - Sqrt[17] + 4*(9 - Sqrt[17])^(1/3))*A + 3*(16 + (9 - Sqr
t[17])^(4/3))*B + 4*(9 - Sqrt[17] + 4*(9 - Sqrt[17])^(1/3))*C)*x)/(9 - Sqr
t[17])^(2/3)))/(18*17^(2/3)*(4 - (9 - Sqrt[17])^(2/3))*((4 + (9 - Sqrt[17]
)^(2/3))/(9 - Sqrt[17])^(1/3) + 3*x)*(4 - 16/(9 - Sqrt[17])^(2/3) - (9 - S
qrt[17])^(2/3) + (3*(4 + (9 - Sqrt[17])^(2/3))*x)/(9 - Sqrt[17])^(1/3) - 9
*x^2)) + ((-17 + 9*Sqrt[17])^(1/3)*((2*(49 - 9*Sqrt[17])*(36*A + 27*B + 16
*C))/(3*(9 - Sqrt[17])*(16 + 4*(9 - Sqrt[17])^(2/3) + (9 - Sqrt[17])^(4/3)
)*(4 + (9 - Sqrt[17])^(2/3) + 3*(9 - Sqrt[17])^(1/3)*x)) - (4*Sqrt[2/(3*(4
9 - 9*Sqrt[17] + 8*(9 - Sqrt[17])^(2/3) - 4*(9 - Sqrt[17])^(4/3)))]*(9*(18
248 - 4104*Sqrt[17] + (3457 - 729*Sqrt[17])*(9 - Sqrt[17])^(2/3))*A + 3*(3
6009 - 8129*Sqrt[17] + (6516 - 1364*Sqrt[17])*(9 - Sqrt[17])^(2/3))*B + 4*
(18248 - 4104*Sqrt[17] + (3457 - 729*Sqrt[17])*(9 - Sqrt[17])^(2/3))*C)*Ar
cTan[(9 - Sqrt[17] + 4*(9 - Sqrt[17])^(1/3) - 6*(9 - Sqrt[17])^(2/3)*x)/Sq
rt[6*(49 - 9*Sqrt[17] + 8*(9 - Sqrt[17])^(2/3) - 4*(9 - Sqrt[17])^(4/3))]]
)/(3*(9 - Sqrt[17])^(4/3)*(16 + 4*(9 - Sqrt[17])^(2/3) + (9 - Sqrt[17])^(4
/3))^2) - (2*(9*(196 - 36*Sqrt[17] + 8*(9 - Sqrt[17])^(4/3) - 9*(9 - Sqrt[
17])^(5/3))*A + 3*(441 - 81*Sqrt[17] + 18*(9 - Sqrt[17])^(4/3) - 16*(9 - S
qrt[17])^(5/3))*B + 4*(196 - 36*Sqrt[17] + 8*(9 - Sqrt[17])^(4/3) - 9*(9 -
 Sqrt[17])^(5/3))*C)*Log[4 + (9 - Sqrt[17])^(2/3) + 3*(9 - Sqrt[17])^(1...
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1235
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^m
*(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*
m + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) -
f*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
)
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2485
```
Int[((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (d_.)*(x_)^3)^(p_), x_S
ymbol] :> With[{r = Rt[-9*a*d^2 + Sqrt[3]*d*Sqrt[4*b^3*d + 27*a^2*d^2], 3]}
, Simp[1/d^(2*p)   Int[(e + f*x)^m*Simp[18^(1/3)*b*(d/(3*r)) - r/18^(1/3) +
 d*x, x]^p*Simp[b*(d/3) + 12^(1/3)*b^2*(d^2/(3*r^2)) + r^2/(3*12^(1/3)) - d
*(2^(1/3)*b*(d/(3^(1/3)*r)) - r/18^(1/3))*x + d^2*x^2, x]^p, x], x]] /; Fre
eQ[{a, b, d, e, f, m}, x] && NeQ[4*b^3 + 27*a^2*d, 0] && ILtQ[p, 0]
```

rule 2526
```
Int[(Pm_)*(Qn_)^(p_), x_Symbol] :> With[{m = Expon[Pm, x], n = Expon[Qn, x]
}, Simp[Coeff[Pm, x, m]*(Qn^(p + 1)/(n*(p + 1)*Coeff[Qn, x, n])), x] + Simp
[1/(n*Coeff[Qn, x, n])   Int[ExpandToSum[n*Coeff[Qn, x, n]*Pm - Coeff[Pm, x
, m]*D[Qn, x], x]*Qn^p, x], x] /; EqQ[m, n - 1]] /; FreeQ[p, x] && PolyQ[Pm
, x] && PolyQ[Qn, x] && NeQ[p, -1]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.09

method	result
default	$\frac{\left(\frac{8C}{51}+\frac{6A}{17}+\frac{9B}{34}\right)x^2+\left(\frac{2C}{17}+\frac{9A}{34}+\frac{8B}{51}\right)x-\frac{3C}{17}-\frac{16A}{51}-\frac{4B}{17}}{x^3-\frac{4}{3}x+\frac{2}{3}}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(3\_Z^3-4\_Z+2\right)}\frac{\left(36A\_R+27B\_R+16C\_R+54A+32B\right.}{9\_R^2-4}\right.}{34}$
risch	$\frac{\left(\frac{8C}{51}+\frac{6A}{17}+\frac{9B}{34}\right)x^2+\left(\frac{2C}{17}+\frac{9A}{34}+\frac{8B}{51}\right)x-\frac{3C}{17}-\frac{16A}{51}-\frac{4B}{17}}{x^3-\frac{4}{3}x+\frac{2}{3}}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(3\_Z^3-4\_Z+2\right)}\frac{\left((36A+27B+16C)\_R+24C+54A+32B\right)\ln}{9\_R^2-4}\right.}{34}$

input
```
int((C*x^2+B*x+A)/(3*x^3-4*x+2)^2,x,method=_RETURNVERBOSE)
```

output
```
((8/51*C+6/17*A+9/34*B)*x^2+(2/17*C+9/34*A+8/51*B)*x-3/17*C-16/51*A-4/17*B
)/(x^3-4/3*x+2/3)+1/34*sum((36*A*_R+27*B*_R+16*C*_R+54*A+32*B+24*C)/(9*_R^
2-4)*ln(x-_R),_R=RootOf(3*_Z^3-4*_Z+2))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.91 (sec) , antiderivative size = 5268, normalized size of antiderivative = 4.50

$$\int \frac{A+Bx+Cx^2}{\left(2-4x+3x^3\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)/(3*x^3-4*x+2)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [A] (verification not implemented)

Time = 8.83 (sec) , antiderivative size = 337, normalized size of antiderivative = 0.29

$$\int \frac{A+Bx+Cx^2}{(2-4x+3x^3)^2}\,dx$$
$$=\operatorname{RootSum}\left(235824t^3+t(230688A^2+296460AB+205056AC+95184B^2+131760BC+45568C^2)-\right.$$
$$+\frac{-32A-24B-18C+x^2\cdot(36A+27B+16C)+x(27A+16B+12C)}{102x^3-136x+68}$$

input
```
integrate((C*x**2+B*x+A)/(3*x**3-4*x+2)**2,x)
```

output
```
RootSum(235824*_t**3 + _t*(230688*A**2 + 296460*A*B + 205056*A*C + 95184*B
**2 + 131760*B*C + 45568*C**2) - 2916*A**3 - 1296*A**2*B - 3888*A**2*C + 1
944*A*B**2 - 1152*A*B*C - 1728*A*C**2 + 1011*B**3 + 864*B**2*C - 256*B*C**
2 - 256*C**3, Lambda(_t, _t*log(x + (114610464*_t**2*A + 73930824*_t**2*B
+ 50937984*_t**2*C + 12172680*_t*A**2 + 13483584*_t*A*B + 10820160*_t*A*C
+ 3641400*_t*B**2 + 5992704*_t*B*C + 2404480*_t*C**2 + 74742912*A**3 + 144
266832*A**2*B + 99657216*A**2*C + 92799756*A*B**2 + 128237184*A*B*C + 4429
2096*A*C**2 + 19893456*B**3 + 41244336*B**2*C + 28497152*B*C**2 + 6561792*
C**3)/(55430244*A**3 + 106760592*A**2*B + 73906992*A**2*C + 68586264*A*B**
2 + 94898304*A*B*C + 32847552*A*C**2 + 14696883*B**3 + 30482784*B**2*C + 2
1088512*B*C**2 + 4866304*C**3)))) + (-32*A - 24*B - 18*C + x**2*(36*A + 27
*B + 16*C) + x*(27*A + 16*B + 12*C))/(102*x**3 - 136*x + 68)
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{(2-4x+3x^3)^2}\,dx=\int \frac{Cx^2+Bx+A}{(3\,x^3-4\,x+2)^2}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(3*x^3-4*x+2)^2,x, algorithm="maxima")
```

output
```
1/34*((36*A + 27*B + 16*C)*x^2 + (27*A + 16*B + 12*C)*x - 32*A - 24*B - 18
*C)/(3*x^3 - 4*x + 2) + 1/34*integrate(((36*A + 27*B + 16*C)*x + 54*A + 32
*B + 24*C)/(3*x^3 - 4*x + 2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A+Bx+Cx^2}{(2-4x+3x^3)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((C*x^2+B*x+A)/(3*x^3-4*x+2)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 12.88 (sec) , antiderivative size = 987, normalized size of antiderivative = 0.84

$$\int \frac{A+Bx+Cx^2}{(2-4x+3x^3)^2}\,dx = \text{Too large to display}$$

input
```
int((A + B*x + C*x^2)/(3*x^3 - 4*x + 2)^2,x)
```

output
```
symsum(log((972*A^2*x)/289 + (2187*B^2*x)/1156 + (192*C^2*x)/289 - 216*roo
t(z^3 + z*((4806*A^2)/4913 + (1983*B^2)/4913 + (2848*C^2)/14739 + (24705*A
*B)/19652 + (4272*A*C)/4913 + (2745*B*C)/4913) - (24*A*B*C)/4913 - (16*B*C
^2)/14739 + (18*B^2*C)/4913 - (81*A^2*C)/4913 - (36*A*C^2)/4913 + (81*A*B^
2)/9826 - (27*A^2*B)/4913 - (243*A^3)/19652 - (16*C^3)/14739 + (337*B^3)/7
8608, z, k)^2*x + (1458*A^2)/289 + (648*B^2)/289 + (288*C^2)/289 + 162*roo
t(z^3 + z*((4806*A^2)/4913 + (1983*B^2)/4913 + (2848*C^2)/14739 + (24705*A
*B)/19652 + (4272*A*C)/4913 + (2745*B*C)/4913) - (24*A*B*C)/4913 - (16*B*C
^2)/14739 + (18*B^2*C)/4913 - (81*A^2*C)/4913 - (36*A*C^2)/4913 + (81*A*B^
2)/9826 - (27*A^2*B)/4913 - (243*A^3)/19652 - (16*C^3)/14739 + (337*B^3)/7
8608, z, k)^2 + (3915*A*B)/578 + (1296*A*C)/289 + (870*B*C)/289 + (216*A*r
oot(z^3 + z*((4806*A^2)/4913 + (1983*B^2)/4913 + (2848*C^2)/14739 + (24705
*A*B)/19652 + (4272*A*C)/4913 + (2745*B*C)/4913) - (24*A*B*C)/4913 - (16*B
*C^2)/14739 + (18*B^2*C)/4913 - (81*A^2*C)/4913 - (36*A*C^2)/4913 + (81*A*
B^2)/9826 - (27*A^2*B)/4913 - (243*A^3)/19652 - (16*C^3)/14739 + (337*B^3)
/78608, z, k))/17 + (162*B*root(z^3 + z*((4806*A^2)/4913 + (1983*B^2)/4913
 + (2848*C^2)/14739 + (24705*A*B)/19652 + (4272*A*C)/4913 + (2745*B*C)/491
3) - (24*A*B*C)/4913 - (16*B*C^2)/14739 + (18*B^2*C)/4913 - (81*A^2*C)/491
3 - (36*A*C^2)/4913 + (81*A*B^2)/9826 - (27*A^2*B)/4913 - (243*A^3)/19652
- (16*C^3)/14739 + (337*B^3)/78608, z, k))/17 + (96*C*root(z^3 + z*((48...
```

Reduce [F]

$$\int \frac{A+Bx+Cx^2}{(2-4x+3x^3)^2}\,dx = \frac{81\left(\int \frac{x^4}{9x^6-24x^4+12x^3+16x^2-16x+4}dx\right) b\,x^3-108\left(\int \frac{x^4}{9x^6-24x^4+12x^3+16x^2-16x+4}dx\right) bx+54\left(\int \frac{x^4}{9x^6-24x^4+12x^3+16x}\right.}{}$$

input
```
int((C*x^2+B*x+A)/(3*x^3-4*x+2)^2,x)
```

output

```
(81*int(x**4/(9*x**6 - 24*x**4 + 12*x**3 + 16*x**2 - 16*x + 4),x)*b*x**3 -
 108*int(x**4/(9*x**6 - 24*x**4 + 12*x**3 + 16*x**2 - 16*x + 4),x)*b*x + 5
4*int(x**4/(9*x**6 - 24*x**4 + 12*x**3 + 16*x**2 - 16*x + 4),x)*b + 324*in
t(x**3/(9*x**6 - 24*x**4 + 12*x**3 + 16*x**2 - 16*x + 4),x)*a*x**3 - 432*i
nt(x**3/(9*x**6 - 24*x**4 + 12*x**3 + 16*x**2 - 16*x + 4),x)*a*x + 216*int
(x**3/(9*x**6 - 24*x**4 + 12*x**3 + 16*x**2 - 16*x + 4),x)*a + 144*int(x**
3/(9*x**6 - 24*x**4 + 12*x**3 + 16*x**2 - 16*x + 4),x)*b*x**3 - 192*int(x*
*3/(9*x**6 - 24*x**4 + 12*x**3 + 16*x**2 - 16*x + 4),x)*b*x + 96*int(x**3/
(9*x**6 - 24*x**4 + 12*x**3 + 16*x**2 - 16*x + 4),x)*b + 144*int(x**3/(9*x
**6 - 24*x**4 + 12*x**3 + 16*x**2 - 16*x + 4),x)*c*x**3 - 192*int(x**3/(9*
x**6 - 24*x**4 + 12*x**3 + 16*x**2 - 16*x + 4),x)*c*x + 96*int(x**3/(9*x**
6 - 24*x**4 + 12*x**3 + 16*x**2 - 16*x + 4),x)*c + 18*a*x + 9*b*x**2 + 8*b
*x - 4*b + 6*c*x**3)/(36*(3*x**3 - 4*x + 2))
```

3.20 $\int \frac{A+Bx+Cx^2}{\sqrt{2-4x+3x^3}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 1731

$$\int \frac{A+Bx+Cx^2}{\sqrt{2-4x+3x^3}}\,dx = \text{Too large to display}$$

output
```
2/9*B*((4+(9-17^(1/2))^(2/3))/(9-17^(1/2))^(1/3)+3*x)*(-4+16/(9-17^(1/2))^
(2/3)+(9-17^(1/2))^(2/3)-3*(4+(9-17^(1/2))^(2/3))*x/(9-17^(1/2))^(1/3)+9*x
^2)^(1/2)*(16-4*(9-17^(1/2))^(2/3)+(9-17^(1/2))^(4/3)-3*(9-17^(1/2)+4*(9-1
7^(1/2))^(1/3))*x+9*(9-17^(1/2))^(2/3)*x^2)^(1/2)/(9-17^(1/2))^(1/3)/((4+(
9-17^(1/2))^(2/3)+(48+12*(9-17^(1/2))^(2/3)+3*(9-17^(1/2))^(4/3))^(1/2))/(
9-17^(1/2))^(1/3)+3*x)/(3*x^3-4*x+2)^(1/2)+2/9*C*(3*x^3-4*x+2)^(1/2)-4/3*(
585-97*17^(1/2)+4*(65-9*17^(1/2))*(9-17^(1/2))^(1/3)+12*(9-17^(1/2))^(5/3)
)^(1/2)*B*((4+(9-17^(1/2))^(2/3))/(9-17^(1/2))^(1/3)+3*x)^(1/2)*(-4+16/(9-
17^(1/2))^(2/3)+(9-17^(1/2))^(2/3)-3*(4+(9-17^(1/2))^(2/3))*x/(9-17^(1/2))
^(1/3)+9*x^2)^(1/2)*((16-4*(9-17^(1/2))^(2/3)+(9-17^(1/2))^(4/3)-3*(9-17^(
1/2)+4*(9-17^(1/2))^(1/3))*x+9*(9-17^(1/2))^(2/3)*x^2)/(1+(4+(9-17^(1/2))^
(2/3)+3*(9-17^(1/2))^(1/3)*x)/(48+12*(9-17^(1/2))^(2/3)+3*(9-17^(1/2))^(4/
3))^(1/2))^2)^(1/2)*(1+(4+(9-17^(1/2))^(2/3)+3*(9-17^(1/2))^(1/3)*x)/(48+1
2*(9-17^(1/2))^(2/3)+3*(9-17^(1/2))^(4/3))^(1/2))*EllipticE(sin(2*arctan((
9-17^(1/2))^(1/6)*((4+(9-17^(1/2))^(2/3))/(9-17^(1/2))^(1/3)+3*x)^(1/2)/(4
8+12*(9-17^(1/2))^(2/3)+3*(9-17^(1/2))^(4/3))^(1/4))),1/2*(2+(4+(9-17^(1/2
))^(2/3))*3^(1/2)/(16+4*(9-17^(1/2))^(2/3)+(9-17^(1/2))^(4/3))^(1/2))^(1/2
))/(9-17^(1/2))^(1/3)/(48+12*(9-17^(1/2))^(2/3)+3*(9-17^(1/2))^(4/3))^(3/4
)/(16-4*(9-17^(1/2))^(2/3)+(9-17^(1/2))^(4/3)-3*(9-17^(1/2)+4*(9-17^(1/2))
^(1/3))*x+9*(9-17^(1/2))^(2/3)*x^2)^(1/2)/(3*x^3-4*x+2)^(1/2)+1/81*(9*(...
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 10.58 (sec) , antiderivative size = 1260, normalized size of antiderivative = 0.73

$$\int \frac{A+Bx+Cx^2}{\sqrt{2-4x+3x^3}}\,dx = \text{Too large to display}$$

input
```
Integrate[(A + B*x + C*x^2)/Sqrt[2 - 4*x + 3*x^3],x]
```

output
```
(2*(C*(2 - 4*x + 3*x^3) + (9*A*EllipticF[ArcSin[Sqrt[(-x + Root[2 - 4*#1 +
 3*#1^3 & , 3, 0])/(-Root[2 - 4*#1 + 3*#1^3 & , 2, 0] + Root[2 - 4*#1 + 3*
#1^3 & , 3, 0])]], (Root[2 - 4*#1 + 3*#1^3 & , 2, 0] - Root[2 - 4*#1 + 3*#
1^3 & , 3, 0])/(Root[2 - 4*#1 + 3*#1^3 & , 1, 0] - Root[2 - 4*#1 + 3*#1^3
& , 3, 0])]*(x - Root[2 - 4*#1 + 3*#1^3 & , 3, 0])*Sqrt[(-x + Root[2 - 4*#
1 + 3*#1^3 & , 1, 0])/(Root[2 - 4*#1 + 3*#1^3 & , 1, 0] - Root[2 - 4*#1 +
3*#1^3 & , 3, 0])]*Sqrt[(-x + Root[2 - 4*#1 + 3*#1^3 & , 2, 0])/(Root[2 -
4*#1 + 3*#1^3 & , 2, 0] - Root[2 - 4*#1 + 3*#1^3 & , 3, 0])])/Sqrt[(x - Ro
ot[2 - 4*#1 + 3*#1^3 & , 3, 0])/(Root[2 - 4*#1 + 3*#1^3 & , 2, 0] - Root[2
 - 4*#1 + 3*#1^3 & , 3, 0])] + (4*C*EllipticF[ArcSin[Sqrt[(-x + Root[2 - 4
*#1 + 3*#1^3 & , 3, 0])/(-Root[2 - 4*#1 + 3*#1^3 & , 2, 0] + Root[2 - 4*#1
 + 3*#1^3 & , 3, 0])]], (Root[2 - 4*#1 + 3*#1^3 & , 2, 0] - Root[2 - 4*#1
+ 3*#1^3 & , 3, 0])/(Root[2 - 4*#1 + 3*#1^3 & , 1, 0] - Root[2 - 4*#1 + 3*
#1^3 & , 3, 0])]*(x - Root[2 - 4*#1 + 3*#1^3 & , 3, 0])*Sqrt[(-x + Root[2
- 4*#1 + 3*#1^3 & , 1, 0])/(Root[2 - 4*#1 + 3*#1^3 & , 1, 0] - Root[2 - 4*
#1 + 3*#1^3 & , 3, 0])]*Sqrt[(-x + Root[2 - 4*#1 + 3*#1^3 & , 2, 0])/(Root
[2 - 4*#1 + 3*#1^3 & , 2, 0] - Root[2 - 4*#1 + 3*#1^3 & , 3, 0])])/Sqrt[(x
 - Root[2 - 4*#1 + 3*#1^3 & , 3, 0])/(Root[2 - 4*#1 + 3*#1^3 & , 2, 0] - R
oot[2 - 4*#1 + 3*#1^3 & , 3, 0])] - (9*B*(x - Root[2 - 4*#1 + 3*#1^3 & , 2
, 0])*Sqrt[(-x + Root[2 - 4*#1 + 3*#1^3 & , 1, 0])/(Root[2 - 4*#1 + 3*#...
```

Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 1.29 (sec) , antiderivative size = 803, normalized size of antiderivative = 0.46, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {2526, 2486, 1269, 1172, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{\sqrt{3x^3-4x+2}}\,dx$$

$$\Big\downarrow \text{2526}$$

$$\frac{1}{9}\int \frac{9A+4C+9Bx}{\sqrt{3x^3-4x+2}}dx+\frac{2}{9}C\sqrt{3x^3-4x+2}$$

$$\Big\downarrow \text{2486}$$

$$\frac{\sqrt{3x+\frac{4+\left(9-\sqrt{17}\right)^{2/3}}{\sqrt[3]{9-\sqrt{17}}}}\sqrt{9x^2-\frac{3\left(4+\left(9-\sqrt{17}\right)^{2/3}\right)x}{\sqrt[3]{9-\sqrt{17}}}+\left(9-\sqrt{17}\right)^{2/3}+\frac{16}{\left(9-\sqrt{17}\right)^{2/3}}-4}\int\frac{9}{\sqrt{3x+\frac{4+\left(9-\sqrt{17}\right)^{2/3}}{\sqrt[3]{9-\sqrt{17}}}}\sqrt{9x^2-\frac{3\left(4+\right.}{\sqrt[3]{}}}}}{9\sqrt{3x^3-4x+2}}$$

$$\frac{2}{9}C\sqrt{3x^3-4x+2}$$

$\downarrow$ 1269

$$\frac{\sqrt{3x+\frac{4+\left(9-\sqrt{17}\right)^{2/3}}{\sqrt[3]{9-\sqrt{17}}}}\sqrt{9x^2-\frac{3\left(4+\left(9-\sqrt{17}\right)^{2/3}\right)x}{\sqrt[3]{9-\sqrt{17}}}+\left(9-\sqrt{17}\right)^{2/3}+\frac{16}{\left(9-\sqrt{17}\right)^{2/3}}-4}\left(\left(9A-\frac{3\left(4+\left(9-\sqrt{17}\right)^{2/3}\right)B}{\sqrt[3]{9-\sqrt{17}}}+\right.\right.}{}$$

$$\frac{2}{9}C\sqrt{3x^3-4x+2}$$

$\downarrow$ 1172

$$\frac{2}{9}\sqrt{3x^3-4x+2}C+$$

$$\frac{\sqrt{3x+\frac{4+\left(9-\sqrt{17}\right)^{2/3}}{\sqrt[3]{9-\sqrt{17}}}}\sqrt{9x^2-\frac{3\left(4+\left(9-\sqrt{17}\right)^{2/3}\right)x}{\sqrt[3]{9-\sqrt{17}}}+\left(9-\sqrt{17}\right)^{2/3}+\frac{16}{\left(9-\sqrt{17}\right)^{2/3}}-4}\left(-\frac{2i\sqrt{2}\sqrt[6]{9-\sqrt{17}}\left(9A-\frac{3\left(4+\left(9\right.\right.}{\sqrt[3]{9}}\right.}{}\right.}{}$$

$\downarrow$ 321

$$\frac{2}{9}\sqrt{3x^3-4x+2}C+$$

$$\sqrt{3x+\frac{4+\left(9-\sqrt{17}\right)^{2/3}}{\sqrt[3]{9-\sqrt{17}}}}\sqrt{9x^2-\frac{3\left(4+\left(9-\sqrt{17}\right)^{2/3}\right)x}{\sqrt[3]{9-\sqrt{17}}}+\left(9-\sqrt{17}\right)^{2/3}+\frac{16}{\left(9-\sqrt{17}\right)^{2/3}}-4}\left(-\frac{2i\sqrt{2}\sqrt[6]{9-\sqrt{17}}\left(9A-\frac{3\left(4+\left(9\right.\right.}{\sqrt[3]{9}}\right.}{}\right.$$

$\downarrow$ 327

$$\frac{2}{9}\sqrt{3x^3-4x+2}C+$$

$$\sqrt{3x+\frac{4+\left(9-\sqrt{17}\right)^{2/3}}{\sqrt[3]{9-\sqrt{17}}}}\sqrt{9x^2-\frac{3\left(4+\left(9-\sqrt{17}\right)^{2/3}\right)x}{\sqrt[3]{9-\sqrt{17}}}+\left(9-\sqrt{17}\right)^{2/3}+\frac{16}{\left(9-\sqrt{17}\right)^{2/3}}-4}\left(-\frac{i\sqrt{2}B\sqrt{3x+\frac{4+\left(9-\sqrt{17}\right)^{2/3}}{\sqrt[3]{9-\sqrt{17}}}}E\left(\text{ar}\right.}{}\right.$$

input
```
Int[(A + B*x + C*x^2)/Sqrt[2 - 4*x + 3*x^3],x]
```

output
```
(2*C*Sqrt[2 - 4*x + 3*x^3])/9 + (Sqrt[(4 + (9 - Sqrt[17])^(2/3))/(9 - Sqrt
[17])^(1/3) + 3*x]*Sqrt[-4 + 16/(9 - Sqrt[17])^(2/3) + (9 - Sqrt[17])^(2/3
) - (3*(4 + (9 - Sqrt[17])^(2/3))*x)/(9 - Sqrt[17])^(1/3) + 9*x^2]*(((-I)*
Sqrt[2]*B*Sqrt[(4 + (9 - Sqrt[17])^(2/3))/(9 - Sqrt[17])^(1/3) + 3*x]*Elli
pticE[ArcSin[((9 - Sqrt[17])^(1/6)*Sqrt[(-I)*((4 + I*(4*Sqrt[3] - (I + Sqr
t[3])*(9 - Sqrt[17])^(2/3)))/(9 - Sqrt[17])^(1/3) - 6*x)])/(3^(1/4)*Sqrt[2
*(4 - (9 - Sqrt[17])^(2/3))])], ((2*I)*(4 - (9 - Sqrt[17])^(2/3)))/(4*(I +
 Sqrt[3]) - (I - Sqrt[3])*(9 - Sqrt[17])^(2/3))])/((9 - Sqrt[17])^(1/6)*Sq
rt[(I*((4 + (9 - Sqrt[17])^(2/3))/(9 - Sqrt[17])^(1/3) + 3*x))/(4*(3*I - S
qrt[3]) + (3*I + Sqrt[3])*(9 - Sqrt[17])^(2/3))]) - (((2*I)/3)*Sqrt[2]*(9
- Sqrt[17])^(1/6)*(9*A - (3*(4 + (9 - Sqrt[17])^(2/3))*B)/(9 - Sqrt[17])^(
1/3) + 4*C)*Sqrt[(I*((4 + (9 - Sqrt[17])^(2/3))/(9 - Sqrt[17])^(1/3) + 3*x
))/(4*(3*I - Sqrt[3]) + (3*I + Sqrt[3])*(9 - Sqrt[17])^(2/3))]*EllipticF[A
rcSin[((9 - Sqrt[17])^(1/6)*Sqrt[(-I)*((4 + I*(4*Sqrt[3] - (I + Sqrt[3])*(
9 - Sqrt[17])^(2/3)))/(9 - Sqrt[17])^(1/3) - 6*x)])/(3^(1/4)*Sqrt[2*(4 - (
9 - Sqrt[17])^(2/3))])], ((2*I)*(4 - (9 - Sqrt[17])^(2/3)))/(4*(I + Sqrt[3
]) - (I - Sqrt[3])*(9 - Sqrt[17])^(2/3))])/Sqrt[(4 + (9 - Sqrt[17])^(2/3))
/(9 - Sqrt[17])^(1/3) + 3*x]))/(9*Sqrt[2 - 4*x + 3*x^3])
```

Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 1172
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Simp[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m))   Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^
2 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[m^2, 1/4]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2486
```
Int[((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (d_.)*(x_)^3)^(p_), x_S
ymbol] :> With[{r = Rt[-9*a*d^2 + Sqrt[3]*d*Sqrt[4*b^3*d + 27*a^2*d^2], 3]}
, Simp[(a + b*x + d*x^3)^p/(Simp[18^(1/3)*b*(d/(3*r)) - r/18^(1/3) + d*x, x
]^p*Simp[b*(d/3) + 12^(1/3)*b^2*(d^2/(3*r^2)) + r^2/(3*12^(1/3)) - d*(2^(1/
3)*b*(d/(3^(1/3)*r)) - r/18^(1/3))*x + d^2*x^2, x]^p)   Int[(e + f*x)^m*Sim
p[18^(1/3)*b*(d/(3*r)) - r/18^(1/3) + d*x, x]^p*Simp[b*(d/3) + 12^(1/3)*b^2
*(d^2/(3*r^2)) + r^2/(3*12^(1/3)) - d*(2^(1/3)*b*(d/(3^(1/3)*r)) - r/18^(1/
3))*x + d^2*x^2, x]^p, x], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && NeQ[4*
b^3 + 27*a^2*d, 0] &&  !IntegerQ[p]
```

rule 2526
```
Int[(Pm_)*(Qn_)^(p_), x_Symbol] :> With[{m = Expon[Pm, x], n = Expon[Qn, x]
}, Simp[Coeff[Pm, x, m]*(Qn^(p + 1)/(n*(p + 1)*Coeff[Qn, x, n])), x] + Simp
[1/(n*Coeff[Qn, x, n])   Int[ExpandToSum[n*Coeff[Qn, x, n]*Pm - Coeff[Pm, x
, m]*D[Qn, x], x]*Qn^p, x], x] /; EqQ[m, n - 1]] /; FreeQ[p, x] && PolyQ[Pm
, x] && PolyQ[Qn, x] && NeQ[p, -1]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.28 (sec) , antiderivative size = 1056, normalized size of antiderivative = 0.61

method	result	size
elliptic	Expression too large to display	1056
risch	Expression too large to display	1460
default	Expression too large to display	1461

input
```
int((C*x^2+B*x+A)/(3*x^3-4*x+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/9*C*(3*x^3-4*x+2)^(1/2)+2/3*I*(A+4/9*C)*3^(1/2)*(-1/3*(9+17^(1/2))^(1/3)
+4/3/(9+17^(1/2))^(1/3))*(I*(x-1/6*(9+17^(1/2))^(1/3)-2/3/(9+17^(1/2))^(1/
3)+1/2*I*3^(1/2)*(-1/3*(9+17^(1/2))^(1/3)+4/3/(9+17^(1/2))^(1/3)))*3^(1/2)
/(1/3*(9+17^(1/2))^(1/3)-4/3/(9+17^(1/2))^(1/3)))^(1/2)*((x+1/3*(9+17^(1/2
))^(1/3)+4/3/(9+17^(1/2))^(1/3))/(1/2*(9+17^(1/2))^(1/3)+2/(9+17^(1/2))^(1
/3)-1/2*I*3^(1/2)*(-1/3*(9+17^(1/2))^(1/3)+4/3/(9+17^(1/2))^(1/3))))^(1/2)
*(-I*(x-1/6*(9+17^(1/2))^(1/3)-2/3/(9+17^(1/2))^(1/3)-1/2*I*3^(1/2)*(-1/3*
(9+17^(1/2))^(1/3)+4/3/(9+17^(1/2))^(1/3)))*3^(1/2)/(1/3*(9+17^(1/2))^(1/3
)-4/3/(9+17^(1/2))^(1/3)))^(1/2)/(3*x^3-4*x+2)^(1/2)*EllipticF(1/3*3^(1/2)
*(I*(x-1/6*(9+17^(1/2))^(1/3)-2/3/(9+17^(1/2))^(1/3)+1/2*I*3^(1/2)*(-1/3*(
9+17^(1/2))^(1/3)+4/3/(9+17^(1/2))^(1/3)))*3^(1/2)/(1/3*(9+17^(1/2))^(1/3)
-4/3/(9+17^(1/2))^(1/3)))^(1/2),(I*3^(1/2)*(1/3*(9+17^(1/2))^(1/3)-4/3/(9+
17^(1/2))^(1/3))/(1/2*(9+17^(1/2))^(1/3)+2/(9+17^(1/2))^(1/3)-1/2*I*3^(1/2
)*(-1/3*(9+17^(1/2))^(1/3)+4/3/(9+17^(1/2))^(1/3))))^(1/2))+2/3*I*B*3^(1/2
)*(-1/3*(9+17^(1/2))^(1/3)+4/3/(9+17^(1/2))^(1/3))*(I*(x-1/6*(9+17^(1/2))^
(1/3)-2/3/(9+17^(1/2))^(1/3)+1/2*I*3^(1/2)*(-1/3*(9+17^(1/2))^(1/3)+4/3/(9
+17^(1/2))^(1/3)))*3^(1/2)/(1/3*(9+17^(1/2))^(1/3)-4/3/(9+17^(1/2))^(1/3))
)^(1/2)*((x+1/3*(9+17^(1/2))^(1/3)+4/3/(9+17^(1/2))^(1/3))/(1/2*(9+17^(1/2
))^(1/3)+2/(9+17^(1/2))^(1/3)-1/2*I*3^(1/2)*(-1/3*(9+17^(1/2))^(1/3)+4/3/(
9+17^(1/2))^(1/3))))^(1/2)*(-I*(x-1/6*(9+17^(1/2))^(1/3)-2/3/(9+17^(1/2...
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.03

$$\int \frac{A+Bx+Cx^2}{\sqrt{2-4x+3x^3}}\,dx = \frac{2}{27}\sqrt{3}(9A+4C)\text{weierstrassPInverse}\left(\frac{16}{3},-\frac{8}{3},x\right) - \frac{2}{3}\sqrt{3}B\text{weierstrassZeta}\left(\frac{16}{3},-\frac{8}{3},\text{weierstrassPInverse}\left(\frac{16}{3},-\frac{8}{3},x\right)\right) + \frac{2}{9}\sqrt{3x^3-4x+2}C$$

input
```
integrate((C*x^2+B*x+A)/(3*x^3-4*x+2)^(1/2),x, algorithm="fricas")
```

output
```
2/27*sqrt(3)*(9*A + 4*C)*weierstrassPInverse(16/3, -8/3, x) - 2/3*sqrt(3)*
B*weierstrassZeta(16/3, -8/3, weierstrassPInverse(16/3, -8/3, x)) + 2/9*sq
rt(3*x^3 - 4*x + 2)*C
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt{2-4x+3x^3}}\,dx = \int \frac{A+Bx+Cx^2}{\sqrt{3x^3-4x+2}}\,dx$$

input
```
integrate((C*x**2+B*x+A)/(3*x**3-4*x+2)**(1/2),x)
```

output
```
Integral((A + B*x + C*x**2)/sqrt(3*x**3 - 4*x + 2), x)
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt{2-4x+3x^3}}\,dx = \int \frac{Cx^2+Bx+A}{\sqrt{3\,x^3-4\,x+2}}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(3*x^3-4*x+2)^(1/2),x, algorithm="maxima")
```

output
```
integrate((C*x^2 + B*x + A)/sqrt(3*x^3 - 4*x + 2), x)
```

Giac [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt{2-4x+3x^3}}\,dx = \int \frac{Cx^2+Bx+A}{\sqrt{3\,x^3-4\,x+2}}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(3*x^3-4*x+2)^(1/2),x, algorithm="giac")
```

output
```
integrate((C*x^2 + B*x + A)/sqrt(3*x^3 - 4*x + 2), x)
```

Mupad [B] (verification not implemented)

Time = 12.68 (sec) , antiderivative size = 3117, normalized size of antiderivative = 1.80

$$\int \frac{A+Bx+Cx^2}{\sqrt{2-4x+3x^3}}\,dx = \text{Too large to display}$$

input
```
int((A + B*x + C*x^2)/(3*x^3 - 4*x + 2)^(1/2),x)
```

output
```
(C*((2*(x^3 - (4*x)/3 + 2/3)^(1/2))/3 - (8*(-(x - 2/(9*(1/3 - 17^(1/2)/27)
^(1/3)) - (1/3 - 17^(1/2)/27)^(1/3)/2 + (3^(1/2)*(4/(9*(1/3 - 17^(1/2)/27)
^(1/3)) - (1/3 - 17^(1/2)/27)^(1/3))*1i)/2)/(2/(3*(1/3 - 17^(1/2)/27)^(1/3
)) + (3*(1/3 - 17^(1/2)/27)^(1/3))/2 - (3^(1/2)*(4/(9*(1/3 - 17^(1/2)/27)^
(1/3)) - (1/3 - 17^(1/2)/27)^(1/3))*1i)/2))^(1/2)*ellipticF(asin((-(x - 2/
(9*(1/3 - 17^(1/2)/27)^(1/3)) - (1/3 - 17^(1/2)/27)^(1/3)/2 + (3^(1/2)*(4/
(9*(1/3 - 17^(1/2)/27)^(1/3)) - (1/3 - 17^(1/2)/27)^(1/3))*1i)/2)/(2/(3*(1
/3 - 17^(1/2)/27)^(1/3)) + (3*(1/3 - 17^(1/2)/27)^(1/3))/2 - (3^(1/2)*(4/(
9*(1/3 - 17^(1/2)/27)^(1/3)) - (1/3 - 17^(1/2)/27)^(1/3))*1i)/2))^(1/2)),
(3^(1/2)*(2/(3*(1/3 - 17^(1/2)/27)^(1/3)) + (3*(1/3 - 17^(1/2)/27)^(1/3))/
2 - (3^(1/2)*(4/(9*(1/3 - 17^(1/2)/27)^(1/3)) - (1/3 - 17^(1/2)/27)^(1/3))
*1i)/2)*1i)/(3*(4/(9*(1/3 - 17^(1/2)/27)^(1/3)) - (1/3 - 17^(1/2)/27)^(1/3
))))*((x + 4/(9*(1/3 - 17^(1/2)/27)^(1/3)) + (1/3 - 17^(1/2)/27)^(1/3))/(2
/(3*(1/3 - 17^(1/2)/27)^(1/3)) + (3*(1/3 - 17^(1/2)/27)^(1/3))/2 - (3^(1/2
)*(4/(9*(1/3 - 17^(1/2)/27)^(1/3)) - (1/3 - 17^(1/2)/27)^(1/3))*1i)/2))^(1
/2)*(2/(3*(1/3 - 17^(1/2)/27)^(1/3)) + (3*(1/3 - 17^(1/2)/27)^(1/3))/2 - (
3^(1/2)*(4/(9*(1/3 - 17^(1/2)/27)^(1/3)) - (1/3 - 17^(1/2)/27)^(1/3))*1i)/
2)*(-(3^(1/2)*(2/(9*(1/3 - 17^(1/2)/27)^(1/3)) - x + (1/3 - 17^(1/2)/27)^(
1/3)/2 + (3^(1/2)*(4/(9*(1/3 - 17^(1/2)/27)^(1/3)) - (1/3 - 17^(1/2)/27)^(
1/3))*1i)/2)*1i)/(3*(4/(9*(1/3 - 17^(1/2)/27)^(1/3)) - (1/3 - 17^(1/2)/...
```

Reduce [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt{2-4x+3x^3}}\,dx = \left(\int \frac{\sqrt{3x^3-4x+2}}{3x^3-4x+2}dx\right)a+\left(\int \frac{\sqrt{3x^3-4x+2}\,x^2}{3x^3-4x+2}dx\right)c + \left(\int \frac{\sqrt{3x^3-4x+2}\,x}{3x^3-4x+2}dx\right)b$$

input
```
int((C*x^2+B*x+A)/(3*x^3-4*x+2)^(1/2),x)
```

output
```
int(sqrt(3*x**3 - 4*x + 2)/(3*x**3 - 4*x + 2),x)*a + int((sqrt(3*x**3 - 4*
x + 2)*x**2)/(3*x**3 - 4*x + 2),x)*c + int((sqrt(3*x**3 - 4*x + 2)*x)/(3*x
**3 - 4*x + 2),x)*b
```

3.21 $\int (A+Bx+Cx^2)(2-4x+3x^3)^p\,dx$

Optimal result

Integrand size = 23, antiderivative size = 936

$$\int (A+Bx+Cx^2)(2-4x+3x^3)^p\,dx = \text{Too large to display}$$

output
```
C*(3*x^3-4*x+2)^(p+1)/(9*p+9)+1/27*(9*A-3*(4+(9-17^(1/2))^(2/3))*B/(9-17^(
1/2))^(1/3)+4*C)*((4+(9-17^(1/2))^(2/3))/(9-17^(1/2))^(1/3)+3*x)*(3*x^3-4*
x+2)^p*AppellF1(p+1,-p,-p,2+p,2*I*(9-17^(1/2))^(1/3)*((4+(9-17^(1/2))^(2/3
))/(9-17^(1/2))^(1/3)+3*x)/(12*I-4*3^(1/2)+(3*I+3^(1/2))*(9-17^(1/2))^(2/3
)),2*I*(9-17^(1/2))^(1/3)*((4+(9-17^(1/2))^(2/3))/(9-17^(1/2))^(1/3)+3*x)/
(12*I+4*3^(1/2)+(3*I-3^(1/2))*(9-17^(1/2))^(2/3)))/(p+1)/((1-2*I*(9-17^(1/
2))^(1/3)*((4+(9-17^(1/2))^(2/3))/(9-17^(1/2))^(1/3)+3*x)/(12*I+4*3^(1/2)+
(3*I-3^(1/2))*(9-17^(1/2))^(2/3)))^p)/((1-2*I*(9-17^(1/2))^(1/3)*((4+(9-17
^(1/2))^(2/3))/(9-17^(1/2))^(1/3)+3*x)/(12*I-4*3^(1/2)+(3*I+3^(1/2))*(9-17
^(1/2))^(2/3)))^p)+1/9*B*((4+(9-17^(1/2))^(2/3))/(9-17^(1/2))^(1/3)+3*x)^2
*(3*x^3-4*x+2)^p*AppellF1(2+p,-p,-p,3+p,2*I*(9-17^(1/2))^(1/3)*((4+(9-17^(
1/2))^(2/3))/(9-17^(1/2))^(1/3)+3*x)/(12*I-4*3^(1/2)+(3*I+3^(1/2))*(9-17^(
1/2))^(2/3)),2*I*(9-17^(1/2))^(1/3)*((4+(9-17^(1/2))^(2/3))/(9-17^(1/2))^(
1/3)+3*x)/(12*I+4*3^(1/2)+(3*I-3^(1/2))*(9-17^(1/2))^(2/3)))/(2+p)/((1-2*I
*(9-17^(1/2))^(1/3)*((4+(9-17^(1/2))^(2/3))/(9-17^(1/2))^(1/3)+3*x)/(12*I+
4*3^(1/2)+(3*I-3^(1/2))*(9-17^(1/2))^(2/3)))^p)/((1-2*I*(9-17^(1/2))^(1/3)
*((4+(9-17^(1/2))^(2/3))/(9-17^(1/2))^(1/3)+3*x)/(12*I-4*3^(1/2)+(3*I+3^(1
/2))*(9-17^(1/2))^(2/3)))^p)
```

Mathematica [F]

$$\int \left(A+Bx+Cx^2\right)\left(2-4x+3x^3\right)^p\,dx=\int \left(A+Bx+Cx^2\right)\left(2-4x+3x^3\right)^p\,dx$$

input
```
Integrate[(A + B*x + C*x^2)*(2 - 4*x + 3*x^3)^p,x]
```

output
```
Integrate[(A + B*x + C*x^2)*(2 - 4*x + 3*x^3)^p, x]
```

Rubi [A] (warning: unable to verify)

Time = 2.26 (sec) , antiderivative size = 1175, normalized size of antiderivative = 1.26, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2526, 2486, 1269, 1179, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(3x^3-4x+2\right)^p\left(A+Bx+Cx^2\right)dx$$

$$\Big\downarrow \text{ 2526}$$

$$\frac{1}{9}\int (9A+4C+9Bx)\left(3x^3-4x+2\right)^p dx+\frac{C\left(3x^3-4x+2\right)^{p+1}}{9(p+1)}$$

$$\Big\downarrow \text{ 2486}$$

$$\frac{1}{9}\left(3x+\frac{4+\left(9-\sqrt{17}\right)^{2/3}}{\sqrt[3]{9-\sqrt{17}}}\right)^{-p}\left(3x^3-4x+2\right)^p\left(9x^2-\frac{3\left(4+\left(9-\sqrt{17}\right)^{2/3}\right)x}{\sqrt[3]{9-\sqrt{17}}}+\left(9-\sqrt{17}\right)^{2/3}+\frac{16}{\left(9-\sqrt{17}\right)^{2}}\right.$$
$$\left.4C+9Bx\right)\left(9x^2-\frac{3\left(4+\left(9-\sqrt{17}\right)^{2/3}\right)x}{\sqrt[3]{9-\sqrt{17}}}+\left(9-\sqrt{17}\right)^{2/3}+\frac{16}{\left(9-\sqrt{17}\right)^{2/3}}-4\right)^p dx+$$
$$\frac{C\left(3x^3-4x+2\right)^{p+1}}{9(p+1)}$$

$$\Big\downarrow \text{ 1269}$$

$$\frac{1}{9}\left(3x+\frac{4+(9-\sqrt{17})^{2/3}}{\sqrt[3]{9-\sqrt{17}}}\right)^{-p}\left(3x^3-4x+2\right)^p\left(9x^2-\frac{3\left(4+(9-\sqrt{17})^{2/3}\right)x}{\sqrt[3]{9-\sqrt{17}}}+\left(9-\sqrt{17}\right)^{2/3}+\frac{16}{(9-\sqrt{17})^{2}}\right.$$

$$\frac{C\left(3x^3-4x+2\right)^{p+1}}{9(p+1)}$$

$\big\downarrow$ 1179

$$\frac{1}{9}\left(3x+\frac{4+(9-\sqrt{17})^{2/3}}{\sqrt[3]{9-\sqrt{17}}}\right)^{-p}\left(3x^3-4x+2\right)^p\left(\frac{1}{3}\left(9A-\frac{3\left(4+(9-\sqrt{17})^{2/3}\right)B}{\sqrt[3]{9-\sqrt{17}}}+4C\right)\left(9x^2-\frac{3\left(4+(9-\right.}{\sqrt[3]{9}}\right.\right.$$

$$\frac{C\left(3x^3-4x+2\right)^{p+1}}{9(p+1)}$$

$\big\downarrow$ 150

$$\frac{1}{9}\left(3x+\frac{4+(9-\sqrt{17})^{2/3}}{\sqrt[3]{9-\sqrt{17}}}\right)^{-p}\left(3x^3-4x+2\right)^p\left(\frac{\left(9A-\frac{3\left(4+\left(9-\sqrt{17}\right)^{2/3}\right)B}{\sqrt[3]{9-\sqrt{17}}}+4C\right)\left(3x+\frac{4+\left(9-\sqrt{17}\right)^{2/3}}{\sqrt[3]{9-\sqrt{17}}}\right)^{p+1}}{\quad}\right.$$

$$\frac{C\left(3x^3-4x+2\right)^{p+1}}{9(p+1)}$$

input
```
Int[(A + B*x + C*x^2)*(2 - 4*x + 3*x^3)^p,x]
```

output
```
(C*(2 - 4*x + 3*x^3)^(1 + p))/(9*(1 + p)) + ((2 - 4*x + 3*x^3)^p*(((9*A -
(3*(4 + (9 - Sqrt[17])^(2/3))*B)/(9 - Sqrt[17])^(1/3) + 4*C)*((4 + (9 - Sq
rt[17])^(2/3))/(9 - Sqrt[17])^(1/3) + 3*x)^(1 + p)*(-4 + 16/(9 - Sqrt[17])
^(2/3) + (9 - Sqrt[17])^(2/3) - (3*(4 + (9 - Sqrt[17])^(2/3))*x)/(9 - Sqrt
[17])^(1/3) + 9*x^2)^p*AppellF1[1 + p, -p, -p, 2 + p, ((2*I)*(9 - Sqrt[17]
)^(1/3)*((4 + (9 - Sqrt[17])^(2/3))/(9 - Sqrt[17])^(1/3) + 3*x))/(4*(3*I +
 Sqrt[3]) + (3*I - Sqrt[3])*(9 - Sqrt[17])^(2/3)), ((2*I)*(9 - Sqrt[17])^(
1/3)*((4 + (9 - Sqrt[17])^(2/3))/(9 - Sqrt[17])^(1/3) + 3*x))/(4*(3*I - Sq
rt[3]) + (3*I + Sqrt[3])*(9 - Sqrt[17])^(2/3))])/(3*(1 + p)*(1 - ((2*I)*(9
 - Sqrt[17])^(1/3)*((4 + (9 - Sqrt[17])^(2/3))/(9 - Sqrt[17])^(1/3) + 3*x)
)/(4*(3*I + Sqrt[3]) + (3*I - Sqrt[3])*(9 - Sqrt[17])^(2/3)))^p*(1 - ((2*I
)*(9 - Sqrt[17])^(1/3)*((4 + (9 - Sqrt[17])^(2/3))/(9 - Sqrt[17])^(1/3) +
3*x))/(4*(3*I - Sqrt[3]) + (3*I + Sqrt[3])*(9 - Sqrt[17])^(2/3)))^p) + (B*
((4 + (9 - Sqrt[17])^(2/3))/(9 - Sqrt[17])^(1/3) + 3*x)^(2 + p)*(-4 + 16/(
9 - Sqrt[17])^(2/3) + (9 - Sqrt[17])^(2/3) - (3*(4 + (9 - Sqrt[17])^(2/3))
*x)/(9 - Sqrt[17])^(1/3) + 9*x^2)^p*AppellF1[2 + p, -p, -p, 3 + p, ((2*I)*
(9 - Sqrt[17])^(1/3)*((4 + (9 - Sqrt[17])^(2/3))/(9 - Sqrt[17])^(1/3) + 3*
x))/(4*(3*I + Sqrt[3]) + (3*I - Sqrt[3])*(9 - Sqrt[17])^(2/3)), ((2*I)*(9
- Sqrt[17])^(1/3)*((4 + (9 - Sqrt[17])^(2/3))/(9 - Sqrt[17])^(1/3) + 3*x))
/(4*(3*I - Sqrt[3]) + (3*I + Sqrt[3])*(9 - Sqrt[17])^(2/3))])/((2 + p)*...
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 1179
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(a + b*x + c*x^2)^p/(e*(1 - (
d + e*x)/(d - e*((b - q)/(2*c))))^p*(1 - (d + e*x)/(d - e*((b + q)/(2*c))))
^p)   Subst[Int[x^m*Simp[1 - x/(d - e*((b - q)/(2*c))), x]^p*Simp[1 - x/(d
 - e*((b + q)/(2*c))), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, b, c, d, e, m
, p}, x]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2486
```
Int[((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (d_.)*(x_)^3)^(p_), x_S
ymbol] :> With[{r = Rt[-9*a*d^2 + Sqrt[3]*d*Sqrt[4*b^3*d + 27*a^2*d^2], 3]}
, Simp[(a + b*x + d*x^3)^p/(Simp[18^(1/3)*b*(d/(3*r)) - r/18^(1/3) + d*x, x
]^p*Simp[b*(d/3) + 12^(1/3)*b^2*(d^2/(3*r^2)) + r^2/(3*12^(1/3)) - d*(2^(1/
3)*b*(d/(3^(1/3)*r)) - r/18^(1/3))*x + d^2*x^2, x]^p)   Int[(e + f*x)^m*Sim
p[18^(1/3)*b*(d/(3*r)) - r/18^(1/3) + d*x, x]^p*Simp[b*(d/3) + 12^(1/3)*b^2
*(d^2/(3*r^2)) + r^2/(3*12^(1/3)) - d*(2^(1/3)*b*(d/(3^(1/3)*r)) - r/18^(1/
3))*x + d^2*x^2, x]^p, x], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && NeQ[4*
b^3 + 27*a^2*d, 0] &&  !IntegerQ[p]
```

rule 2526
```
Int[(Pm_)*(Qn_)^(p_), x_Symbol] :> With[{m = Expon[Pm, x], n = Expon[Qn, x]
}, Simp[Coeff[Pm, x, m]*(Qn^(p + 1)/(n*(p + 1)*Coeff[Qn, x, n])), x] + Simp
[1/(n*Coeff[Qn, x, n])   Int[ExpandToSum[n*Coeff[Qn, x, n]*Pm - Coeff[Pm, x
, m]*D[Qn, x], x]*Qn^p, x], x] /; EqQ[m, n - 1]] /; FreeQ[p, x] && PolyQ[Pm
, x] && PolyQ[Qn, x] && NeQ[p, -1]
```

Maple [F]

$$\int \left(C\,x^{2}+Bx+A\right)\left(3x^{3}-4x+2\right)^{p}dx$$

input
```
int((C*x^2+B*x+A)*(3*x^3-4*x+2)^p,x)
```

output
```
int((C*x^2+B*x+A)*(3*x^3-4*x+2)^p,x)
```

Fricas [F]

$$\int \left(A+Bx+Cx^2\right)\left(2-4x+3x^3\right)^p\,dx=\int \left(Cx^2+Bx+A\right)\left(3\,x^3-4\,x+2\right)^p\,dx$$

input
```
integrate((C*x^2+B*x+A)*(3*x^3-4*x+2)^p,x, algorithm="fricas")
```

output
```
integral((C*x^2 + B*x + A)*(3*x^3 - 4*x + 2)^p, x)
```

Sympy [F]

$$\int \left(A+Bx+Cx^2\right)\left(2-4x+3x^3\right)^p\,dx=\int \left(A+Bx+Cx^2\right)\left(3x^3-4x+2\right)^p\,dx$$

input
```
integrate((C*x**2+B*x+A)*(3*x**3-4*x+2)**p,x)
```

output
```
Integral((A + B*x + C*x**2)*(3*x**3 - 4*x + 2)**p, x)
```

Maxima [F]

$$\int \left(A+Bx+Cx^2\right)\left(2-4x+3x^3\right)^p\,dx=\int \left(Cx^2+Bx+A\right)\left(3\,x^3-4\,x+2\right)^p\,dx$$

input
```
integrate((C*x^2+B*x+A)*(3*x^3-4*x+2)^p,x, algorithm="maxima")
```

output
```
integrate((C*x^2 + B*x + A)*(3*x^3 - 4*x + 2)^p, x)
```

Giac [F]

$$\int (A+Bx+Cx^2)(2-4x+3x^3)^p\,dx=\int (Cx^2+Bx+A)(3x^3-4x+2)^p\,dx$$

input
```
integrate((C*x^2+B*x+A)*(3*x^3-4*x+2)^p,x, algorithm="giac")
```

output
```
integrate((C*x^2 + B*x + A)*(3*x^3 - 4*x + 2)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (A+Bx+Cx^2)(2-4x+3x^3)^p\,dx=\int (C\,x^2+B\,x+A)\,(3\,x^3-4\,x+2)^p\,dx$$

input
```
int((A + B*x + C*x^2)*(3*x^3 - 4*x + 2)^p,x)
```

output
```
int((A + B*x + C*x^2)*(3*x^3 - 4*x + 2)^p, x)
```

Reduce [F]

$$\int (A+Bx+Cx^2)(2-4x+3x^3)^p\,dx=\text{too large to display}$$

input
```
int((C*x^2+B*x+A)*(3*x^3-4*x+2)^p,x)
```

output

```
(27*(3*x**3 - 4*x + 2)**p*a*p**2*x + 45*(3*x**3 - 4*x + 2)**p*a*p*x + 18*(
3*x**3 - 4*x + 2)**p*a*x + 27*(3*x**3 - 4*x + 2)**p*b*p**2*x**2 - 24*(3*x*
*3 - 4*x + 2)**p*b*p**2 + 36*(3*x**3 - 4*x + 2)**p*b*p*x**2 - 32*(3*x**3 -
 4*x + 2)**p*b*p + 9*(3*x**3 - 4*x + 2)**p*b*x**2 - 8*(3*x**3 - 4*x + 2)**
p*b + 27*(3*x**3 - 4*x + 2)**p*c*p**2*x**3 - 24*(3*x**3 - 4*x + 2)**p*c*p*
*2*x + 18*(3*x**3 - 4*x + 2)**p*c*p**2 + 27*(3*x**3 - 4*x + 2)**p*c*p*x**3
 - 16*(3*x**3 - 4*x + 2)**p*c*p*x + 18*(3*x**3 - 4*x + 2)**p*c*p + 6*(3*x*
*3 - 4*x + 2)**p*c*x**3 + 4*(3*x**3 - 4*x + 2)**p*c + 1458*int((3*x**3 - 4
*x + 2)**p/(27*p**2*x**3 - 36*p**2*x + 18*p**2 + 27*p*x**3 - 36*p*x + 18*p
 + 6*x**3 - 8*x + 4),x)*a*p**5 + 3888*int((3*x**3 - 4*x + 2)**p/(27*p**2*x
**3 - 36*p**2*x + 18*p**2 + 27*p*x**3 - 36*p*x + 18*p + 6*x**3 - 8*x + 4),
x)*a*p**4 + 3726*int((3*x**3 - 4*x + 2)**p/(27*p**2*x**3 - 36*p**2*x + 18*
p**2 + 27*p*x**3 - 36*p*x + 18*p + 6*x**3 - 8*x + 4),x)*a*p**3 + 1512*int(
(3*x**3 - 4*x + 2)**p/(27*p**2*x**3 - 36*p**2*x + 18*p**2 + 27*p*x**3 - 36
*p*x + 18*p + 6*x**3 - 8*x + 4),x)*a*p**2 + 216*int((3*x**3 - 4*x + 2)**p/
(27*p**2*x**3 - 36*p**2*x + 18*p**2 + 27*p*x**3 - 36*p*x + 18*p + 6*x**3 -
 8*x + 4),x)*a*p - 864*int((3*x**3 - 4*x + 2)**p/(27*p**2*x**3 - 36*p**2*x
 + 18*p**2 + 27*p*x**3 - 36*p*x + 18*p + 6*x**3 - 8*x + 4),x)*b*p**5 - 201
6*int((3*x**3 - 4*x + 2)**p/(27*p**2*x**3 - 36*p**2*x + 18*p**2 + 27*p*x**
3 - 36*p*x + 18*p + 6*x**3 - 8*x + 4),x)*b*p**4 - 1632*int((3*x**3 - 4*...
```

3.22 $\int (A+Bx+Cx^2)(2-6x+3x^3)^3\,dx$

Optimal result

Integrand size = 23, antiderivative size = 151

$$\int (A+Bx+Cx^2)(2-6x+3x^3)^3\,dx = 8Ax-4(9A-B)x^2+\frac{8}{3}(27A-9B+C)x^3 -9(5A-6B+2C)x^4-\frac{36}{5}(6A+5B-6C)x^5 +6(9A-6B-5C)x^6+\frac{54}{7}(A+6B-4C)x^7 -\frac{27}{4}(3A-B-6C)x^8-6(3B-C)x^9 +\frac{27}{10}(A-6C)x^{10}+\frac{27Bx^{11}}{11}+\frac{9Cx^{12}}{4}$$

output
```
8*A*x-4*(9*A-B)*x^2+8/3*(27*A-9*B+C)*x^3-9*(5*A-6*B+2*C)*x^4-36/5*(6*A+5*B
-6*C)*x^5+6*(9*A-6*B-5*C)*x^6+54/7*(A+6*B-4*C)*x^7-27/4*(3*A-B-6*C)*x^8-6*
(3*B-C)*x^9+27/10*(A-6*C)*x^10+27/11*B*x^11+9/4*C*x^12
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.00

$$\int (A+Bx+Cx^2)\left(2-6x+3x^3\right)^3\,dx = 8Ax-4(9A-B)x^2+\frac{8}{3}(27A-9B+C)x^3 -9(5A-6B+2C)x^4-\frac{36}{5}(6A+5B-6C)x^5 +6(9A-6B-5C)x^6+\frac{54}{7}(A+6B-4C)x^7 -\frac{27}{4}(3A-B-6C)x^8-6(3B-C)x^9 +\frac{27}{10}(A-6C)x^{10}+\frac{27Bx^{11}}{11}+\frac{9Cx^{12}}{4}$$

input
```
Integrate[(A + B*x + C*x^2)*(2 - 6*x + 3*x^3)^3,x]
```

output
```
8*A*x - 4*(9*A - B)*x^2 + (8*(27*A - 9*B + C)*x^3)/3 - 9*(5*A - 6*B + 2*C)
*x^4 - (36*(6*A + 5*B - 6*C)*x^5)/5 + 6*(9*A - 6*B - 5*C)*x^6 + (54*(A + 6
*B - 4*C)*x^7)/7 - (27*(3*A - B - 6*C)*x^8)/4 - 6*(3*B - C)*x^9 + (27*(A -
 6*C)*x^10)/10 + (27*B*x^11)/11 + (9*C*x^12)/4
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(3x^3-6x+2\right)^3\left(A+Bx+Cx^2\right)\,dx$$

$$\Big\downarrow \text{ 2188}$$

$$\int \left(-54x^7(3A-B-6C)+54x^6(A+6B-4C)+36x^5(9A-6B-5C)-36x^4(6A+5B-6C)-36x^3(5A-\right.$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{27}{4}x^8(3A-B-6C)+\frac{54}{7}x^7(A+6B-4C)+6x^6(9A-6B-5C)-\frac{36}{5}x^5(6A+5B-6C)-9x^4(5A-6B+2C)+\frac{8}{3}x^3(27A-9B+C)-4x^2(9A-B)+\frac{27}{10}x^{10}(A-6C)+8Ax-6x^9(3B-C)+\frac{27Bx^{11}}{11}+\frac{9Cx^{12}}{4}$$

input
```
Int[(A + B*x + C*x^2)*(2 - 6*x + 3*x^3)^3,x]
```

output
```
8*A*x - 4*(9*A - B)*x^2 + (8*(27*A - 9*B + C)*x^3)/3 - 9*(5*A - 6*B + 2*C)
*x^4 - (36*(6*A + 5*B - 6*C)*x^5)/5 + 6*(9*A - 6*B - 5*C)*x^6 + (54*(A + 6
*B - 4*C)*x^7)/7 - (27*(3*A - B - 6*C)*x^8)/4 - 6*(3*B - C)*x^9 + (27*(A -
 6*C)*x^10)/10 + (27*B*x^11)/11 + (9*C*x^12)/4
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.89

method	result
norman	$\frac{9C\,x^{12}}{4}+\frac{27B\,x^{11}}{11}+\left(\frac{27A}{10}-\frac{81C}{5}\right)x^{10}+\left(-18B+6C\right)x^9+\left(-\frac{81A}{4}+\frac{27B}{4}+\frac{81C}{2}\right)x^8+\left(\frac{54A}{7}+\right.$
default	$\frac{9C\,x^{12}}{4}+\frac{27B\,x^{11}}{11}+\frac{(27A-162C)x^{10}}{10}+\frac{(-162B+54C)x^9}{9}+\frac{(-162A+54B+324C)x^8}{8}+\frac{(54A+324B-216C)x^7}{7}+$
orering	$\frac{x\left(10395C\,x^{11}+11340B\,x^{10}+12474A\,x^9-74844C\,x^9-83160B\,x^8+27720x^8C-93555x^7A+31185x^7B+187110x^7C+35640x^6\right.}{}$
gosper	$-18C\,x^4+\frac{8}{3}C\,x^3+4B\,x^2-36A\,x^2-45x^4A-\frac{216}{5}x^5A+\frac{81}{2}x^8C+\frac{54}{7}x^7A-18x^9B-\frac{81}{4}x$
risch	$-18C\,x^4+\frac{8}{3}C\,x^3+4B\,x^2-36A\,x^2-45x^4A-\frac{216}{5}x^5A+\frac{81}{2}x^8C+\frac{54}{7}x^7A-18x^9B-\frac{81}{4}x$
parallelrisch	$-18C\,x^4+\frac{8}{3}C\,x^3+4B\,x^2-36A\,x^2-45x^4A-\frac{216}{5}x^5A+\frac{81}{2}x^8C+\frac{54}{7}x^7A-18x^9B-\frac{81}{4}x$

input
```
int((C*x^2+B*x+A)*(3*x^3-6*x+2)^3,x,method=_RETURNVERBOSE)
```

output
```
9/4*C*x^12+27/11*B*x^11+(27/10*A-81/5*C)*x^10+(-18*B+6*C)*x^9+(-81/4*A+27/
4*B+81/2*C)*x^8+(54/7*A+324/7*B-216/7*C)*x^7+(54*A-36*B-30*C)*x^6+(-216/5*
A-36*B+216/5*C)*x^5+(-45*A+54*B-18*C)*x^4+(72*A-24*B+8/3*C)*x^3+(-36*A+4*B
)*x^2+8*A*x
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.91

$$\begin{aligned}\int (A+Bx+Cx^2)\left(2-6x+3x^3\right)^3\,dx &= \frac{9}{4}\,Cx^{12}+\frac{27}{11}\,Bx^{11}+\frac{27}{10}\,(A-6\,C)x^{10}-6\,(3\,B-C)x^9-\frac{27}{4}\,(3\,A-B-6\,C)x^8\\&\quad+\frac{54}{7}\,(A+6\,B-4\,C)x^7+6\,(9\,A-6\,B-5\,C)x^6-\frac{36}{5}\,(6\,A+5\,B-6\,C)x^5\\&\quad-9\,(5\,A-6\,B+2\,C)x^4+\frac{8}{3}\,(27\,A-9\,B+C)x^3-4\,(9\,A-B)x^2+8\,Ax\end{aligned}$$

input
```
integrate((C*x^2+B*x+A)*(3*x^3-6*x+2)^3,x, algorithm="fricas")
```

output
```
9/4*C*x^12 + 27/11*B*x^11 + 27/10*(A - 6*C)*x^10 - 6*(3*B - C)*x^9 - 27/4*
(3*A - B - 6*C)*x^8 + 54/7*(A + 6*B - 4*C)*x^7 + 6*(9*A - 6*B - 5*C)*x^6 -
 36/5*(6*A + 5*B - 6*C)*x^5 - 9*(5*A - 6*B + 2*C)*x^4 + 8/3*(27*A - 9*B +
C)*x^3 - 4*(9*A - B)*x^2 + 8*A*x
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.00

$$\int (A+Bx+Cx^2)\left(2-6x+3x^3\right)^3\,dx = 8Ax+\frac{27Bx^{11}}{11}+\frac{9Cx^{12}}{4}+x^{10}\cdot\left(\frac{27A}{10}-\frac{81C}{5}\right)+x^9(-18B+6C)+x^8\left(-\frac{81A}{4}+\frac{27B}{4}+\frac{81C}{2}\right)+x^7\cdot\left(\frac{54A}{7}+\frac{324B}{7}-\frac{216C}{7}\right)+x^6\cdot(54A-36B-30C)+x^5\left(-\frac{216A}{5}-36B+\frac{216C}{5}\right)+x^4(-45A+54B-18C)+x^3\cdot\left(72A-24B+\frac{8C}{3}\right)+x^2(-36A+4B)$$

input
```
integrate((C*x**2+B*x+A)*(3*x**3-6*x+2)**3,x)
```

output
```
8*A*x + 27*B*x**11/11 + 9*C*x**12/4 + x**10*(27*A/10 - 81*C/5) + x**9*(-18
*B + 6*C) + x**8*(-81*A/4 + 27*B/4 + 81*C/2) + x**7*(54*A/7 + 324*B/7 - 21
6*C/7) + x**6*(54*A - 36*B - 30*C) + x**5*(-216*A/5 - 36*B + 216*C/5) + x*
*4*(-45*A + 54*B - 18*C) + x**3*(72*A - 24*B + 8*C/3) + x**2*(-36*A + 4*B)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.91

$$\int (A+Bx+Cx^2)\left(2-6x+3x^3\right)^3\,dx = \frac{9}{4}\,Cx^{12}+\frac{27}{11}\,Bx^{11}+\frac{27}{10}\,(A-6\,C)x^{10}-6\,(3\,B-C)x^9-\frac{27}{4}\,(3\,A-B-6\,C)x^8+\frac{54}{7}\,(A+6\,B-4\,C)x^7+6\,(9\,A-6\,B-5\,C)x^6-\frac{36}{5}\,(6\,A+5\,B-6\,C)x^5-9\,(5\,A-6\,B+2\,C)x^4+\frac{8}{3}\,(27\,A-9\,B+C)x^3-4\,(9\,A-B)x^2+8\,Ax$$

input
```
integrate((C*x^2+B*x+A)*(3*x^3-6*x+2)^3,x, algorithm="maxima")
```

output
```
9/4*C*x^12 + 27/11*B*x^11 + 27/10*(A - 6*C)*x^10 - 6*(3*B - C)*x^9 - 27/4*
(3*A - B - 6*C)*x^8 + 54/7*(A + 6*B - 4*C)*x^7 + 6*(9*A - 6*B - 5*C)*x^6 -
 36/5*(6*A + 5*B - 6*C)*x^5 - 9*(5*A - 6*B + 2*C)*x^4 + 8/3*(27*A - 9*B +
C)*x^3 - 4*(9*A - B)*x^2 + 8*A*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.07

$$\int (A+Bx+Cx^2)\left(2-6x+3x^3\right)^3\,dx = \frac{9}{4}Cx^{12} + \frac{27}{11}Bx^{11} + \frac{27}{10}Ax^{10} - \frac{81}{5}Cx^{10} - 18\,Bx^9 + 6\,Cx^9 - \frac{81}{4}Ax^8 + \frac{27}{4}Bx^8 + \frac{81}{2}Cx^8 + \frac{54}{7}Ax^7 + \frac{324}{7}Bx^7 - \frac{216}{7}Cx^7 + 54\,Ax^6 - 36\,Bx^6 - 30\,Cx^6 - \frac{216}{5}Ax^5 - 36\,Bx^5 + \frac{216}{5}Cx^5 - 45\,Ax^4 + 54\,Bx^4 - 18\,Cx^4 + 72\,Ax^3 - 24\,Bx^3 + \frac{8}{3}Cx^3 - 36\,Ax^2 + 4\,Bx^2 + 8\,Ax$$

input
```
integrate((C*x^2+B*x+A)*(3*x^3-6*x+2)^3,x, algorithm="giac")
```

output
```
9/4*C*x^12 + 27/11*B*x^11 + 27/10*A*x^10 - 81/5*C*x^10 - 18*B*x^9 + 6*C*x^
9 - 81/4*A*x^8 + 27/4*B*x^8 + 81/2*C*x^8 + 54/7*A*x^7 + 324/7*B*x^7 - 216/
7*C*x^7 + 54*A*x^6 - 36*B*x^6 - 30*C*x^6 - 216/5*A*x^5 - 36*B*x^5 + 216/5*
C*x^5 - 45*A*x^4 + 54*B*x^4 - 18*C*x^4 + 72*A*x^3 - 24*B*x^3 + 8/3*C*x^3 -
 36*A*x^2 + 4*B*x^2 + 8*A*x
```

Mupad [B] (verification not implemented)

Time = 12.24 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.92

$$\int (A+Bx+Cx^2)(2-6x+3x^3)^3\,dx$$
$$=\frac{9\,C\,x^{12}}{4}+\frac{27\,B\,x^{11}}{11}+\left(\frac{27\,A}{10}-\frac{81\,C}{5}\right)x^{10}+(6\,C-18\,B)\,x^9+\left(\frac{27\,B}{4}-\frac{81\,A}{4}+\frac{81\,C}{2}\right)x^8+\left(\frac{54\,A}{7}+\frac{324\,B}{7}-\frac{216\,C}{7}\right)x^7+(54\,A-36\,B-30\,C)\,x^6+\left(\frac{216\,C}{5}-36\,B-\frac{216\,A}{5}\right)x^5+(54\,B-45\,A-18\,C)\,x^4+\left(72\,A-24\,B+\frac{8\,C}{3}\right)x^3+(4\,B-36\,A)\,x^2+8\,A\,x$$

input
```
int((A + B*x + C*x^2)*(3*x^3 - 6*x + 2)^3,x)
```

output
```
8*A*x + (27*B*x^11)/11 + (9*C*x^12)/4 + x^3*(72*A - 24*B + (8*C)/3) - x^4*
(45*A - 54*B + 18*C) - x^6*(36*B - 54*A + 30*C) + x^8*((27*B)/4 - (81*A)/4
 + (81*C)/2) - x^5*((216*A)/5 + 36*B - (216*C)/5) + x^7*((54*A)/7 + (324*B
)/7 - (216*C)/7) - x^2*(36*A - 4*B) + x^10*((27*A)/10 - (81*C)/5) - x^9*(1
8*B - 6*C)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.05

$$\int (A+Bx+Cx^2)(2-6x+3x^3)^3\,dx$$
$$=\frac{x(10395c\,x^{11}+11340b\,x^{10}+12474a\,x^9-74844c\,x^9-83160b\,x^8+27720c\,x^8-93555a\,x^7+31185b\,x^7+}{}$$

input
```
int((C*x^2+B*x+A)*(3*x^3-6*x+2)^3,x)
```

output
```
(x*(12474*a*x**9 - 93555*a*x**7 + 35640*a*x**6 + 249480*a*x**5 - 199584*a*
x**4 - 207900*a*x**3 + 332640*a*x**2 - 166320*a*x + 36960*a + 11340*b*x**1
0 - 83160*b*x**8 + 31185*b*x**7 + 213840*b*x**6 - 166320*b*x**5 - 166320*b
*x**4 + 249480*b*x**3 - 110880*b*x**2 + 18480*b*x + 10395*c*x**11 - 74844*
c*x**9 + 27720*c*x**8 + 187110*c*x**7 - 142560*c*x**6 - 138600*c*x**5 + 19
9584*c*x**4 - 83160*c*x**3 + 12320*c*x**2))/4620
```

3.23 $\int (A+Bx+Cx^2)(2-6x+3x^3)^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 99

$$\int (A+Bx+Cx^2)(2-6x+3x^3)^2\,dx = 4Ax-2(6A-B)x^2+\frac{4}{3}(9A-6B+C)x^3+3(A+3B-2C)x^4-\frac{12}{5}(3A-B-3C)x^5-2(3B-C)x^6+\frac{9}{7}(A-4C)x^7+\frac{9Bx^8}{8}+Cx^9$$

output
```
4*A*x-2*(6*A-B)*x^2+4/3*(9*A-6*B+C)*x^3+3*(A+3*B-2*C)*x^4-12/5*(3*A-B-3*C)
*x^5-2*(3*B-C)*x^6+9/7*(A-4*C)*x^7+9/8*B*x^8+C*x^9
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00

$$\int (A+Bx+Cx^2)(2-6x+3x^3)^2\,dx = 4Ax-2(6A-B)x^2+\frac{4}{3}(9A-6B+C)x^3+3(A+3B-2C)x^4-\frac{12}{5}(3A-B-3C)x^5-2(3B-C)x^6+\frac{9}{7}(A-4C)x^7+\frac{9Bx^8}{8}+Cx^9$$

input
```
Integrate[(A + B*x + C*x^2)*(2 - 6*x + 3*x^3)^2,x]
```

output
```
4*A*x - 2*(6*A - B)*x^2 + (4*(9*A - 6*B + C)*x^3)/3 + 3*(A + 3*B - 2*C)*x^
4 - (12*(3*A - B - 3*C)*x^5)/5 - 2*(3*B - C)*x^6 + (9*(A - 4*C)*x^7)/7 + (
9*B*x^8)/8 + C*x^9
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(3x^3-6x+2\right)^2 \left(A+Bx+Cx^2\right) \, dx$$

$$\Big\downarrow \text{2188}$$

$$\int \left(-12x^4(3A-B-3C)+12x^3(A+3B-2C)+4x^2(9A-6B+C)-4x(6A-B)+9x^6(A-4C)+4A-12\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{12}{5}x^5(3A-B-3C)+3x^4(A+3B-2C)+\frac{4}{3}x^3(9A-6B+C)-2x^2(6A-B)+\frac{9}{7}x^7(A-4C)+4Ax-2x^6(3B-C)+\frac{9Bx^8}{8}+Cx^9$$

input
```
Int[(A + B*x + C*x^2)*(2 - 6*x + 3*x^3)^2,x]
```

output
```
4*A*x - 2*(6*A - B)*x^2 + (4*(9*A - 6*B + C)*x^3)/3 + 3*(A + 3*B - 2*C)*x^
4 - (12*(3*A - B - 3*C)*x^5)/5 - 2*(3*B - C)*x^6 + (9*(A - 4*C)*x^7)/7 + (
9*B*x^8)/8 + C*x^9
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.93

method	result
norman	$C\,x^{9}+\frac{9B\,x^{8}}{8}+\left(\frac{9A}{7}-\frac{36C}{7}\right)x^{7}+\left(-6B+2C\right)x^{6}+\left(-\frac{36A}{5}+\frac{12B}{5}+\frac{36C}{5}\right)x^{5}+\left(3A+9B-6\right.$
default	$C\,x^{9}+\frac{9B\,x^{8}}{8}+\frac{(9A-36C)x^{7}}{7}+\frac{(-36B+12C)x^{6}}{6}+\frac{(-36A+12B+36C)x^{5}}{5}+\frac{(12A+36B-24C)x^{4}}{4}+\frac{(36A-24B}{3}$
orering	$\frac{x\left(840x^{8}C+945x^{7}B+1080x^{6}A-4320C\,x^{6}-5040B\,x^{5}+1680x^{5}C-6048x^{4}A+2016x^{4}B+6048C\,x^{4}+2520x^{3}A+7560B\,x^{3}-50\right.}{840}$
gosper	$C\,x^{9}+\frac{9}{8}B\,x^{8}+\frac{9}{7}x^{7}A-\frac{36}{7}x^{7}C-6x^{6}B+2C\,x^{6}-\frac{36}{5}x^{5}A+\frac{12}{5}B\,x^{5}+\frac{36}{5}x^{5}C+3x^{4}A+9x$
risch	$C\,x^{9}+\frac{9}{8}B\,x^{8}+\frac{9}{7}x^{7}A-\frac{36}{7}x^{7}C-6x^{6}B+2C\,x^{6}-\frac{36}{5}x^{5}A+\frac{12}{5}B\,x^{5}+\frac{36}{5}x^{5}C+3x^{4}A+9x$
parallelrisch	$C\,x^{9}+\frac{9}{8}B\,x^{8}+\frac{9}{7}x^{7}A-\frac{36}{7}x^{7}C-6x^{6}B+2C\,x^{6}-\frac{36}{5}x^{5}A+\frac{12}{5}B\,x^{5}+\frac{36}{5}x^{5}C+3x^{4}A+9x$

input
```
int((C*x^2+B*x+A)*(3*x^3-6*x+2)^2,x,method=_RETURNVERBOSE)
```

output
```
C*x^9+9/8*B*x^8+(9/7*A-36/7*C)*x^7+(-6*B+2*C)*x^6+(-36/5*A+12/5*B+36/5*C)*
x^5+(3*A+9*B-6*C)*x^4+(12*A-8*B+4/3*C)*x^3+(-12*A+2*B)*x^2+4*A*x
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.92

$$\int (A+Bx+Cx^2)\left(2-6x+3x^3\right)^2\,dx = Cx^9+\frac{9}{8}Bx^8+\frac{9}{7}(A-4C)x^7 - 2(3B-C)x^6-\frac{12}{5}(3A-B-3C)x^5 + 3(A+3B-2C)x^4+\frac{4}{3}(9A-6B+C)x^3 - 2(6A-B)x^2+4Ax$$

input
```
integrate((C*x^2+B*x+A)*(3*x^3-6*x+2)^2,x, algorithm="fricas")
```

output
```
C*x^9 + 9/8*B*x^8 + 9/7*(A - 4*C)*x^7 - 2*(3*B - C)*x^6 - 12/5*(3*A - B -
3*C)*x^5 + 3*(A + 3*B - 2*C)*x^4 + 4/3*(9*A - 6*B + C)*x^3 - 2*(6*A - B)*x
^2 + 4*A*x
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00

$$\int (A+Bx+Cx^2)\left(2-6x+3x^3\right)^2\,dx = 4Ax+\frac{9Bx^8}{8}+Cx^9+x^7\cdot\left(\frac{9A}{7}-\frac{36C}{7}\right) + x^6(-6B+2C)+x^5\left(-\frac{36A}{5}+\frac{12B}{5}+\frac{36C}{5}\right) + x^4\cdot(3A+9B-6C)+x^3 \cdot\left(12A-8B+\frac{4C}{3}\right)+x^2(-12A+2B)$$

input
```
integrate((C*x**2+B*x+A)*(3*x**3-6*x+2)**2,x)
```

output
```
4*A*x + 9*B*x**8/8 + C*x**9 + x**7*(9*A/7 - 36*C/7) + x**6*(-6*B + 2*C) +
x**5*(-36*A/5 + 12*B/5 + 36*C/5) + x**4*(3*A + 9*B - 6*C) + x**3*(12*A - 8
*B + 4*C/3) + x**2*(-12*A + 2*B)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.92

$$\int (A+Bx+Cx^2)\left(2-6x+3x^3\right)^2\,dx = Cx^9+\frac{9}{8}Bx^8+\frac{9}{7}(A-4C)x^7 - 2(3B-C)x^6-\frac{12}{5}(3A-B-3C)x^5 + 3(A+3B-2C)x^4+\frac{4}{3}(9A-6B+C)x^3 - 2(6A-B)x^2+4Ax$$

input
```
integrate((C*x^2+B*x+A)*(3*x^3-6*x+2)^2,x, algorithm="maxima")
```

output
```
C*x^9 + 9/8*B*x^8 + 9/7*(A - 4*C)*x^7 - 2*(3*B - C)*x^6 - 12/5*(3*A - B -
3*C)*x^5 + 3*(A + 3*B - 2*C)*x^4 + 4/3*(9*A - 6*B + C)*x^3 - 2*(6*A - B)*x
^2 + 4*A*x
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.07

$$\int (A+Bx+Cx^2)\left(2-6x+3x^3\right)^2\,dx = Cx^9+\frac{9}{8}Bx^8+\frac{9}{7}Ax^7-\frac{36}{7}Cx^7-6Bx^6 + 2Cx^6-\frac{36}{5}Ax^5+\frac{12}{5}Bx^5+\frac{36}{5}Cx^5 + 3Ax^4+9Bx^4-6Cx^4+12Ax^3 - 8Bx^3+\frac{4}{3}Cx^3-12Ax^2+2Bx^2+4Ax$$

input
```
integrate((C*x^2+B*x+A)*(3*x^3-6*x+2)^2,x, algorithm="giac")
```

output
```
C*x^9 + 9/8*B*x^8 + 9/7*A*x^7 - 36/7*C*x^7 - 6*B*x^6 + 2*C*x^6 - 36/5*A*x^
5 + 12/5*B*x^5 + 36/5*C*x^5 + 3*A*x^4 + 9*B*x^4 - 6*C*x^4 + 12*A*x^3 - 8*B
*x^3 + 4/3*C*x^3 - 12*A*x^2 + 2*B*x^2 + 4*A*x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.94

$$\int (A+Bx+Cx^2)\,(2-6x+3x^3)^2\,dx = C\,x^9+\frac{9\,B\,x^8}{8}+\left(\frac{9\,A}{7}-\frac{36\,C}{7}\right)\,x^7+(2\,C-6\,B)\,x^6+\left(\frac{12\,B}{5}-\frac{36\,A}{5}+\frac{36\,C}{5}\right)\,x^5+(3\,A+9\,B-6\,C)\,x^4+\left(12\,A-8\,B+\frac{4\,C}{3}\right)\,x^3+(2\,B-12\,A)\,x^2+4\,A\,x$$

input
```
int((A + B*x + C*x^2)*(3*x^3 - 6*x + 2)^2,x)
```

output
```
4*A*x + (9*B*x^8)/8 + C*x^9 + x^4*(3*A + 9*B - 6*C) + x^3*(12*A - 8*B + (4
*C)/3) + x^5*((12*B)/5 - (36*A)/5 + (36*C)/5) - x^2*(12*A - 2*B) + x^7*((9
*A)/7 - (36*C)/7) - x^6*(6*B - 2*C)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.06

$$\int (A+Bx+Cx^2)\,(2-6x+3x^3)^2\,dx = \frac{x(840c\,x^8+945b\,x^7+1080a\,x^6-4320c\,x^6-5040b\,x^5+1680c\,x^5-6048a\,x^4+2016b\,x^4+6048c\,x^4+2}{840}$$

input
```
int((C*x^2+B*x+A)*(3*x^3-6*x+2)^2,x)
```

output
```
(x*(1080*a*x**6 - 6048*a*x**4 + 2520*a*x**3 + 10080*a*x**2 - 10080*a*x + 3
360*a + 945*b*x**7 - 5040*b*x**5 + 2016*b*x**4 + 7560*b*x**3 - 6720*b*x**2
 + 1680*b*x + 840*c*x**8 - 4320*c*x**6 + 1680*c*x**5 + 6048*c*x**4 - 5040*
c*x**3 + 1120*c*x**2))/840
```

3.24 $\int (A+Bx+Cx^2)(2-6x+3x^3)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 59

$$\int (A+Bx+Cx^2)(2-6x+3x^3)\,dx = 2Ax-(3A-B)x^2-\frac{2}{3}(3B-C)x^3+\frac{3}{4}(A-2C)x^4+\frac{3Bx^5}{5}+\frac{Cx^6}{2}$$

output
```
2*A*x-(3*A-B)*x^2-2/3*(3*B-C)*x^3+3/4*(A-2*C)*x^4+3/5*B*x^5+1/2*C*x^6
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.85

$$\int (A+Bx+Cx^2)(2-6x+3x^3)\,dx = \frac{1}{60}x\left(15A(8-12x+3x^3)+2x\left(6B(5-10x+3x^3)+5Cx(4-9x+3x^3)\right)\right)$$

input
```
Integrate[(A + B*x + C*x^2)*(2 - 6*x + 3*x^3),x]
```

output
```
(x*(15*A*(8 - 12*x + 3*x^3) + 2*x*(6*B*(5 - 10*x + 3*x^3) + 5*C*x*(4 - 9*x
 + 3*x^3))))/60
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (3x^3-6x+2) (A+Bx+Cx^2) \, dx$$

$$\Big\downarrow \text{2188}$$

$$\int \left(-2x(3A-B)+3x^3(A-2C)+2A-2x^2(3B-C)+3Bx^4+3Cx^5\right) dx$$

$$\Big\downarrow \text{2009}$$

$$-x^2(3A-B)+\frac{3}{4}x^4(A-2C)+2Ax-\frac{2}{3}x^3(3B-C)+\frac{3Bx^5}{5}+\frac{Cx^6}{2}$$

input
```
Int[(A + B*x + C*x^2)*(2 - 6*x + 3*x^3),x]
```

output
```
2*A*x - (3*A - B)*x^2 - (2*(3*B - C)*x^3)/3 + (3*(A - 2*C)*x^4)/4 + (3*B*x
^5)/5 + (C*x^6)/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.83

method	result	size
norman	$\frac{C\,x^{6}}{2}+\frac{3B\,x^{5}}{5}+\left(\frac{3A}{4}-\frac{3C}{2}\right)x^{4}+\left(-2B+\frac{2C}{3}\right)x^{3}+\left(-3A+B\right)x^{2}+2Ax$	49
orering	$\frac{x\left(30x^{5}C+36x^{4}B+45x^{3}A-90C\,x^{3}-120B\,x^{2}+40C\,x^{2}-180Ax+60Bx+120A\right)}{60}$	52
gosper	$\frac{1}{2}C\,x^{6}+\frac{3}{5}B\,x^{5}+\frac{3}{4}x^{4}A-\frac{3}{2}C\,x^{4}-2B\,x^{3}+\frac{2}{3}C\,x^{3}-3A\,x^{2}+B\,x^{2}+2Ax$	53
risch	$\frac{1}{2}C\,x^{6}+\frac{3}{5}B\,x^{5}+\frac{3}{4}x^{4}A-\frac{3}{2}C\,x^{4}-2B\,x^{3}+\frac{2}{3}C\,x^{3}-3A\,x^{2}+B\,x^{2}+2Ax$	53
parallelrisch	$\frac{1}{2}C\,x^{6}+\frac{3}{5}B\,x^{5}+\frac{3}{4}x^{4}A-\frac{3}{2}C\,x^{4}-2B\,x^{3}+\frac{2}{3}C\,x^{3}-3A\,x^{2}+B\,x^{2}+2Ax$	53
default	$\frac{C\,x^{6}}{2}+\frac{3B\,x^{5}}{5}+\frac{(3A-6C)x^{4}}{4}+\frac{(-6B+2C)x^{3}}{3}+\frac{(-6A+2B)x^{2}}{2}+2Ax$	54

input
```
int((C*x^2+B*x+A)*(3*x^3-6*x+2),x,method=_RETURNVERBOSE)
```

output
```
1/2*C*x^6+3/5*B*x^5+(3/4*A-3/2*C)*x^4+(-2*B+2/3*C)*x^3+(-3*A+B)*x^2+2*A*x
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.86

$$\int \left(A+Bx+Cx^2\right)\left(2-6x+3x^3\right)\,dx=\frac{1}{2}\,Cx^{6}+\frac{3}{5}\,Bx^{5}+\frac{3}{4}\,(A-2\,C)x^{4}-\frac{2}{3}\,(3\,B-C)x^{3}-(3\,A-B)x^{2}+2\,Ax$$

input
```
integrate((C*x^2+B*x+A)*(3*x^3-6*x+2),x, algorithm="fricas")
```

output
```
1/2*C*x^6 + 3/5*B*x^5 + 3/4*(A - 2*C)*x^4 - 2/3*(3*B - C)*x^3 - (3*A - B)*
x^2 + 2*A*x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.90

$$\int (A+Bx+Cx^2)\,(2-6x+3x^3)\,dx = 2Ax + \frac{3Bx^5}{5} + \frac{Cx^6}{2} + x^4\cdot\left(\frac{3A}{4}-\frac{3C}{2}\right) + x^3\left(-2B+\frac{2C}{3}\right) + x^2(-3A+B)$$

input
```
integrate((C*x**2+B*x+A)*(3*x**3-6*x+2),x)
```

output
```
2*A*x + 3*B*x**5/5 + C*x**6/2 + x**4*(3*A/4 - 3*C/2) + x**3*(-2*B + 2*C/3)
 + x**2*(-3*A + B)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.86

$$\int (A+Bx+Cx^2)\,(2-6x+3x^3)\,dx = \frac{1}{2}\,Cx^6 + \frac{3}{5}\,Bx^5 + \frac{3}{4}\,(A-2\,C)x^4 - \frac{2}{3}\,(3\,B-C)x^3 - (3\,A-B)x^2 + 2\,Ax$$

input
```
integrate((C*x^2+B*x+A)*(3*x^3-6*x+2),x, algorithm="maxima")
```

output
```
1/2*C*x^6 + 3/5*B*x^5 + 3/4*(A - 2*C)*x^4 - 2/3*(3*B - C)*x^3 - (3*A - B)*
x^2 + 2*A*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.88

$$\int (A+Bx+Cx^2)(2-6x+3x^3)\,dx = \frac{1}{2}Cx^6+\frac{3}{5}Bx^5+\frac{3}{4}Ax^4-\frac{3}{2}Cx^4-2Bx^3+\frac{2}{3}Cx^3-3Ax^2+Bx^2+2Ax$$

input
```
integrate((C*x^2+B*x+A)*(3*x^3-6*x+2),x, algorithm="giac")
```

output
```
1/2*C*x^6 + 3/5*B*x^5 + 3/4*A*x^4 - 3/2*C*x^4 - 2*B*x^3 + 2/3*C*x^3 - 3*A*
x^2 + B*x^2 + 2*A*x
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.88

$$\int (A+Bx+Cx^2)(2-6x+3x^3)\,dx = \frac{C\,x^6}{2}+\frac{3\,B\,x^5}{5}+\left(\frac{3\,A}{4}-\frac{3\,C}{2}\right)x^4+\left(\frac{2\,C}{3}-2\,B\right)x^3+(B-3\,A)\,x^2+2\,A\,x$$

input
```
int((A + B*x + C*x^2)*(3*x^3 - 6*x + 2),x)
```

output
```
2*A*x + (3*B*x^5)/5 + (C*x^6)/2 - x^2*(3*A - B) + x^4*((3*A)/4 - (3*C)/2)
- x^3*(2*B - (2*C)/3)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.86

$$\int (A+Bx+Cx^2)\left(2-6x+3x^3\right)\,dx$$
$$=\frac{x(30c\,x^5+36b\,x^4+45a\,x^3-90c\,x^3-120b\,x^2+40c\,x^2-180ax+60bx+120a)}{60}$$

input
```
int((C*x^2+B*x+A)*(3*x^3-6*x+2),x)
```

output
```
(x*(45*a*x**3 - 180*a*x + 120*a + 36*b*x**4 - 120*b*x**2 + 60*b*x + 30*c*x
**5 - 90*c*x**3 + 40*c*x**2))/60
```

3.25 $\int \frac{A+Bx+Cx^2}{2-6x+3x^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 373

$$\int \frac{A+Bx+Cx^2}{2-6x+3x^3}\,dx = \frac{1}{9}C\log\left(2-6x+3x^3\right)$$

$$+\frac{\left(3A+2\left(C+\sqrt{6}B\cos\left(\frac{1}{6}\left(\pi+2\arcsin\left(\frac{\sqrt{\frac{3}{2}}}{2}\right)\right)\right)\right)\right)\log\left(3x-2\sqrt{6}\cos\left(\frac{1}{6}\left(\pi+2\arcsin\left(\frac{\sqrt{\frac{3}{2}}}{2}\right)\right)\right.\right.}{24\sqrt{3}\left(\cos\left(\frac{1}{6}\left(\pi+2\arcsin\left(\frac{\sqrt{\frac{3}{2}}}{2}\right)\right)\right)-\sin\left(\frac{1}{3}\arcsin\left(\frac{\sqrt{\frac{3}{2}}}{2}\right)\right)\right)}$$

$$+\frac{\log\left(3x-2\sqrt{6}\sin\left(\frac{1}{3}\arcsin\left(\frac{\sqrt{\frac{3}{2}}}{2}\right)\right)\right)\left(3A+2\left(C+\sqrt{6}B\sin\left(\frac{1}{3}\arcsin\left(\frac{\sqrt{\frac{3}{2}}}{2}\right)\right)\right)\right)}{18\left(1-2\cos\left(\frac{2}{3}\arcsin\left(\frac{\sqrt{\frac{3}{2}}}{2}\right)\right)\right)}$$

$$+\frac{\log\left(3x+2\sqrt{6}\sin\left(\frac{1}{3}\left(\pi+\arcsin\left(\frac{\sqrt{\frac{3}{2}}}{2}\right)\right)\right)\right)\sec\left(\frac{1}{3}\arcsin\left(\frac{\sqrt{\frac{3}{2}}}{2}\right)\right)\left(3A+2C-2\sqrt{6}B\sin\left(\frac{1}{3}\left(\pi+\right.\right.\right.}{24\sqrt{3}\left(\sin\left(\frac{1}{3}\arcsin\left(\frac{\sqrt{\frac{3}{2}}}{2}\right)\right)+\sin\left(\frac{1}{3}\left(\pi+\arcsin\left(\frac{\sqrt{\frac{3}{2}}}{2}\right)\right)\right)\right)}$$

output
```
1/9*C*ln(3*x^3-6*x+2)+1/72*(3*A+2*C+2*6^(1/2)*B*cos(1/6*Pi+1/3*arcsin(1/4*
6^(1/2))))*ln(3*x-2*6^(1/2)*cos(1/6*Pi+1/3*arcsin(1/4*6^(1/2))))*sec(1/3*a
rcsin(1/4*6^(1/2)))*3^(1/2)/(cos(1/6*Pi+1/3*arcsin(1/4*6^(1/2)))-sin(1/3*a
rcsin(1/4*6^(1/2))))+ln(3*x-2*6^(1/2)*sin(1/3*arcsin(1/4*6^(1/2))))*(3*A+2
*C+2*6^(1/2)*B*sin(1/3*arcsin(1/4*6^(1/2))))/(18-36*cos(2/3*arcsin(1/4*6^(
1/2))))+1/72*ln(3*x+2*6^(1/2)*sin(1/3*Pi+1/3*arcsin(1/4*6^(1/2))))*sec(1/3
*arcsin(1/4*6^(1/2)))*(3*A+2*C-2*6^(1/2)*B*sin(1/3*Pi+1/3*arcsin(1/4*6^(1/
2))))*3^(1/2)/(sin(1/3*arcsin(1/4*6^(1/2)))+sin(1/3*Pi+1/3*arcsin(1/4*6^(1
/2))))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.17

$$\int \frac{A+Bx+Cx^2}{2-6x+3x^3}\,dx = \frac{1}{3}\text{RootSum}\left[2-6\#1+3\#1^3\&,\frac{A\log(x-\#1)+B\log(x-\#1)\#1+C\log(x-\#1)\#1^2}{-2+3\#1^2}\&\right]$$

input
```
Integrate[(A + B*x + C*x^2)/(2 - 6*x + 3*x^3),x]
```

output
```
RootSum[2 - 6*#1 + 3*#1^3 & , (A*Log[x - #1] + B*Log[x - #1]*#1 + C*Log[x
- #1]*#1^2)/(-2 + 3*#1^2) & ]/3
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 11.39 (sec) , antiderivative size = 930, normalized size of antiderivative = 2.49, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {2525, 27, 2485, 25, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{3x^3-6x+2}\,dx$$

$\big\downarrow$ 2525

$$\frac{1}{9}\int \frac{3(3A+2C+3Bx)}{3x^3-6x+2}dx+\frac{1}{9}C\log\left(3x^3-6x+2\right)$$

$\big\downarrow$ 27

$$\frac{1}{3}\int \frac{3A+2C+3Bx}{3x^3-6x+2}dx+\frac{1}{9}C\log\left(3x^3-6x+2\right)$$

$\big\downarrow$ 2485

$$\frac{1}{9}C\log\left(3x^3-6x+2\right)+$$
$$3\int -\frac{3A+2C+3Bx}{\left(3x+\sqrt[3]{9-3i\sqrt{15}}+\frac{2\ 3^{2/3}}{\sqrt[3]{3-i\sqrt{15}}}\right)\left(-9x^2+3\left(\frac{2\ 3^{2/3}}{\sqrt[3]{3-i\sqrt{15}}}+\sqrt[3]{9-3i\sqrt{15}}\right)x-\left(9-3i\sqrt{15}\right)^{2/3}-\ldots\right.}$$

$\big\downarrow$ 25

$$\frac{1}{9}C\log\left(3x^3-6x+2\right)-$$
$$3\int \frac{3A+2C+3Bx}{\left(3x+\sqrt[3]{9-3i\sqrt{15}}+\frac{2\ 3^{2/3}}{\sqrt[3]{3-i\sqrt{15}}}\right)\left(-9x^2+3\left(\frac{2\ 3^{2/3}}{\sqrt[3]{3-i\sqrt{15}}}+\sqrt[3]{9-3i\sqrt{15}}\right)x-\left(9-3i\sqrt{15}\right)^{2/3}-\ldots\right.}$$

$\big\downarrow$ 1200

$$\frac{1}{9}C\log\left(3x^3-6x+2\right)-$$
$$3\int\left(\frac{\left(3-i\sqrt{15}\right)^{2/3}\left(-3\sqrt[3]{3-i\sqrt{15}}A+\sqrt[3]{3}\left(2\sqrt[3]{3}+\left(3-i\sqrt{15}\right)^{2/3}\right)B-2\sqrt[3]{3-i\sqrt{15}}C\right)}{9\left(4\sqrt[3]{3}+\left(3^{2/3}-i\sqrt[6]{3}\sqrt{5}\right)\sqrt[3]{3-i\sqrt{15}}+2\left(3-i\sqrt{15}\right)^{2/3}\right)\left(3\sqrt[3]{3-i\sqrt{15}}x+\sqrt[3]{3}\left(3-i\sqrt{15}\right)^{2/3}+2\ 3^{2/3}\ldots\right.}\right.$$

$\big\downarrow$ 2009

$$\frac{1}{9}C\log\left(3x^3-6x+2\right)-$$

$$3\left(\frac{\left(3\left(6i\sqrt{3}+6\sqrt{5}+3i\sqrt[6]{3}\left(3-i\sqrt{15}\right)^{2/3}+\sqrt{5}\left(9-3i\sqrt{15}\right)^{2/3}\right)A-6\left(i\sqrt{3}-3\sqrt{5}-2i\sqrt[6]{3}\left(3-i\sqrt{15}\right)^{2/3}\right)B+}{9\ 3^{5/6}\left(4\sqrt[3]{3}+\left(3^{2/3}-i\sqrt[6]{3}\sqrt{5}\right)\sqrt[3]{3-i\sqrt{15}}+2\left(3-i\sqrt{\right.}\right.}\right.$$

input
```
Int[(A + B*x + C*x^2)/(2 - 6*x + 3*x^3),x]
```

output
```
-3*(((3*((6*I)*Sqrt[3] + 6*Sqrt[5] + (3*I)*3^(1/6)*(3 - I*Sqrt[15])^(2/3)
+ Sqrt[5]*(9 - (3*I)*Sqrt[15])^(2/3))*A - 6*(I*Sqrt[3] - 3*Sqrt[5] - (2*I)
*3^(1/6)*(3 - I*Sqrt[15])^(2/3))*B + 2*((6*I)*Sqrt[3] + 6*Sqrt[5] + (3*I)*
3^(1/6)*(3 - I*Sqrt[15])^(2/3) + Sqrt[5]*(9 - (3*I)*Sqrt[15])^(2/3))*C)*Ar
cTan[(3^(1/3)*(3*I + Sqrt[15] + (2*I)*(9 - (3*I)*Sqrt[15])^(1/3)) - (6*I)*
(3 - I*Sqrt[15])^(2/3)*x)/(3*Sqrt[2*(3^(2/3) + (3*I)*3^(1/6)*Sqrt[5] - 2*3
^(1/3)*(3 - I*Sqrt[15])^(2/3) + 2*(3 - I*Sqrt[15])^(4/3))])])/(9*3^(5/6)*(
4*3^(1/3) + (3^(2/3) - I*3^(1/6)*Sqrt[5])*(3 - I*Sqrt[15])^(1/3) + 2*(3 -
I*Sqrt[15])^(2/3))*Sqrt[2*(3^(2/3) + (3*I)*3^(1/6)*Sqrt[5] - 2*3^(1/3)*(3
- I*Sqrt[15])^(2/3) + 2*(3 - I*Sqrt[15])^(4/3))]) - ((3 - I*Sqrt[15])^(1/3
)*(3*(3 - I*Sqrt[15])^(1/3)*A - 3^(1/3)*(2*3^(1/3) + (3 - I*Sqrt[15])^(2/3
))*B + 2*(3 - I*Sqrt[15])^(1/3)*C)*Log[2*3^(1/3) + (3 - I*Sqrt[15])^(2/3)
+ 3^(2/3)*(3 - I*Sqrt[15])^(1/3)*x])/(27*(4*3^(1/3) + (3^(2/3) - I*3^(1/6)
*Sqrt[5])*(3 - I*Sqrt[15])^(1/3) + 2*(3 - I*Sqrt[15])^(2/3))) + ((3*(3 - I
*Sqrt[15])^(2/3)*A - 3^(1/3)*(3 - I*Sqrt[15] + 2*(9 - (3*I)*Sqrt[15])^(1/3
))*B + 2*(3 - I*Sqrt[15])^(2/3)*C)*Log[I*(4*3^(1/3) + (3^(2/3) - I*3^(1/6)
*Sqrt[5])*(3 - I*Sqrt[15])^(1/3) - 2*(3 - I*Sqrt[15])^(2/3)) - 3^(2/3)*(I*
3^(2/3) + 3^(1/6)*Sqrt[5] + (2*I)*(3 - I*Sqrt[15])^(1/3))*x + (3*I)*(3 - I
*Sqrt[15])^(2/3)*x^2])/(54*(4*3^(1/3) + (3^(2/3) - I*3^(1/6)*Sqrt[5])*(3 -
 I*Sqrt[15])^(1/3) + 2*(3 - I*Sqrt[15])^(2/3)))) + (C*Log[2 - 6*x + 3*x...
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2485
```
Int[((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (d_.)*(x_)^3)^(p_), x_S
ymbol] :> With[{r = Rt[-9*a*d^2 + Sqrt[3]*d*Sqrt[4*b^3*d + 27*a^2*d^2], 3]}
, Simp[1/d^(2*p)   Int[(e + f*x)^m*Simp[18^(1/3)*b*(d/(3*r)) - r/18^(1/3) +
 d*x, x]^p*Simp[b*(d/3) + 12^(1/3)*b^2*(d^2/(3*r^2)) + r^2/(3*12^(1/3)) - d
*(2^(1/3)*b*(d/(3^(1/3)*r)) - r/18^(1/3))*x + d^2*x^2, x]^p, x], x]] /; Fre
eQ[{a, b, d, e, f, m}, x] && NeQ[4*b^3 + 27*a^2*d, 0] && ILtQ[p, 0]
```

rule 2525
```
Int[(Pm_)/(Qn_), x_Symbol] :> With[{m = Expon[Pm, x], n = Expon[Qn, x]}, Si
mp[Coeff[Pm, x, m]*(Log[Qn]/(n*Coeff[Qn, x, n])), x] + Simp[1/(n*Coeff[Qn,
x, n])   Int[ExpandToSum[n*Coeff[Qn, x, n]*Pm - Coeff[Pm, x, m]*D[Qn, x], x
]/Qn, x], x] /; EqQ[m, n - 1]] /; PolyQ[Pm, x] && PolyQ[Qn, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.12

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(3\_Z^{3}-6\_Z+2\right)}\frac{\left(C\_R^{2}+B\_R+A\right)\ln\left(x-\_R\right)}{3\_R^{2}-2}\right)}{3}$	43
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(3\_Z^{3}-6\_Z+2\right)}\frac{\left(C\_R^{2}+B\_R+A\right)\ln\left(x-\_R\right)}{3\_R^{2}-2}\right)}{3}$	43

input
```
int((C*x^2+B*x+A)/(3*x^3-6*x+2),x,method=_RETURNVERBOSE)
```

output
```
1/3*sum((C*_R^2+B*_R+A)/(3*_R^2-2)*ln(x-_R),_R=RootOf(3*_Z^3-6*_Z+2))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.41 (sec) , antiderivative size = 7031, normalized size of antiderivative = 18.85

$$\int \frac{A+Bx+Cx^2}{2-6x+3x^3}\,dx = \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)/(3*x^3-6*x+2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [A] (verification not implemented)

Time = 9.21 (sec) , antiderivative size = 277, normalized size of antiderivative = 0.74

$$\int \frac{A+Bx+Cx^2}{2-6x+3x^3}\,dx$$
$$= \mathrm{RootSum}\Big(4860t^3 - 1620t^2C + t\left(-162A^2 - 162AB - 216AC - 108B^2 - 108BC + 108C^2\right) + 9A^3 + 3$$

input
```
integrate((C*x**2+B*x+A)/(3*x**3-6*x+2),x)
```

output
```
RootSum(4860*_t**3 - 1620*_t**2*C + _t*(-162*A**2 - 162*A*B - 216*A*C - 10
8*B**2 - 108*B*C + 108*C**2) + 9*A**3 + 36*A**2*C - 18*A*B**2 + 18*A*B*C +
 36*A*C**2 - 6*B**3 + 12*B*C**2 + 4*C**3, Lambda(_t, _t*log(x + (-3240*_t*
*2*A - 1620*_t**2*B - 2160*_t**2*C - 270*_t*A**2 + 360*_t*A*C + 180*_t*B**
2 + 360*_t*B*C + 360*_t*C**2 + 72*A**3 + 108*A**2*B + 174*A**2*C + 84*A*B*
*2 + 144*A*B*C + 96*A*C**2 + 24*B**3 + 36*B**2*C + 28*B*C**2 + 8*C**3)/(27
*A**3 + 108*A**2*B + 54*A**2*C + 54*A*B**2 + 144*A*B*C + 36*A*C**2 - 6*B**
3 + 36*B**2*C + 48*B*C**2 + 8*C**3))))
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{2-6x+3x^3}\,dx=\int \frac{Cx^2+Bx+A}{3\,x^3-6\,x+2}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(3*x^3-6*x+2),x, algorithm="maxima")
```

output
```
integrate((C*x^2 + B*x + A)/(3*x^3 - 6*x + 2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A+Bx+Cx^2}{2-6x+3x^3}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((C*x^2+B*x+A)/(3*x^3-6*x+2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 12.43 (sec) , antiderivative size = 322, normalized size of antiderivative = 0.86

$$\int \frac{A+Bx+Cx^2}{2-6x+3x^3}\,dx = \sum_{k=1}^{3} \ln\left(-x\left(-3B^2+6C^2+3AC\right)+2C^2+\mathrm{root}\left(z^3-\frac{Cz^2}{3}-\frac{z\left(162A^2+108B^2-108C^2+162AB+216AC+108BC\right)}{4860}+\frac{ABC}{270}+\frac{BC^2}{405}+\frac{A^2C}{135}+\frac{AC^2}{135}-\frac{AB^2}{270}+\frac{A^3}{540}+\frac{C^3}{1215}-\frac{B^3}{810},z,k\right)\left(18B-36C+x\left(27A+90C\right)-\mathrm{root}\left(z^3-\frac{Cz^2}{3}-\frac{z\left(162A^2+108B^2-108C^2+162AB+216AC+108BC\right)}{4860}+\frac{ABC}{270}+\frac{BC^2}{405}+\frac{A}{1}\right.\right.\right.$$

$$\left.+3AB\right)\mathrm{root}\left(z^3-\frac{Cz^2}{3}-\frac{z\left(162A^2+108B^2-108C^2+162AB+216AC+108BC\right)}{4860}+\frac{ABC}{270}+\frac{BC^2}{405}+\frac{A^2C}{135}+\frac{AC^2}{135}-\frac{AB^2}{270}+\frac{A^3}{540}+\frac{C^3}{1215}-\frac{B^3}{810},z,k\right)$$

input
```
int((A + B*x + C*x^2)/(3*x^3 - 6*x + 2),x)
```

output
```
symsum(log(2*C^2 - x*(6*C^2 - 3*B^2 + 3*A*C) + root(z^3 - (C*z^2)/3 - (z*(
162*A^2 + 108*B^2 - 108*C^2 + 162*A*B + 216*A*C + 108*B*C))/4860 + (A*B*C)
/270 + (B*C^2)/405 + (A^2*C)/135 + (A*C^2)/135 - (A*B^2)/270 + A^3/540 + C
^3/1215 - B^3/810, z, k)*(18*B - 36*C + x*(27*A + 90*C) - root(z^3 - (C*z^
2)/3 - (z*(162*A^2 + 108*B^2 - 108*C^2 + 162*A*B + 216*A*C + 108*B*C))/486
0 + (A*B*C)/270 + (B*C^2)/405 + (A^2*C)/135 + (A*C^2)/135 - (A*B^2)/270 +
A^3/540 + C^3/1215 - B^3/810, z, k)*(324*x - 162)) + 3*A*B)*root(z^3 - (C*
z^2)/3 - (z*(162*A^2 + 108*B^2 - 108*C^2 + 162*A*B + 216*A*C + 108*B*C))/4
860 + (A*B*C)/270 + (B*C^2)/405 + (A^2*C)/135 + (A*C^2)/135 - (A*B^2)/270
+ A^3/540 + C^3/1215 - B^3/810, z, k), k, 1, 3)
```

Reduce [F]

$$\int \frac{A+Bx+Cx^2}{2-6x+3x^3}\,dx = \left(\int \frac{x^2}{3x^3-6x+2}dx\right)c + \left(\int \frac{x}{3x^3-6x+2}dx\right)b + \left(\int \frac{1}{3x^3-6x+2}dx\right)a$$

input
```
int((C*x^2+B*x+A)/(3*x^3-6*x+2),x)
```

output
```
int(x**2/(3*x**3 - 6*x + 2),x)*c + int(x/(3*x**3 - 6*x + 2),x)*b + int(1/(
3*x**3 - 6*x + 2),x)*a
```

3.26 $\int \frac{A+Bx+Cx^2}{\left(2-6x+3x^3\right)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 885

$$\int \frac{A+Bx+Cx^2}{\left(2-6x+3x^3\right)^2}\,dx = \text{Too large to display}$$

output
```
-1/9*C/(3*x^3-6*x+2)-1/192*(3*A+2*C+2*6^(1/2)*B*cos(1/6*Pi+1/3*arcsin(1/4*
6^(1/2))))*sec(1/3*arcsin(1/4*6^(1/2)))^2/(3*x-2*6^(1/2)*cos(1/6*Pi+1/3*ar
csin(1/4*6^(1/2))))/(cos(1/6*Pi+1/3*arcsin(1/4*6^(1/2)))-sin(1/3*arcsin(1/
4*6^(1/2))))^2-1/108*ln(3*x-2*6^(1/2)*sin(1/3*arcsin(1/4*6^(1/2))))*(B*(9-
6*cos(2/3*arcsin(1/4*6^(1/2))))+2*6^(1/2)*(3*A+2*C)*sin(1/3*arcsin(1/4*6^(
1/2))))/(1-2*cos(2/3*arcsin(1/4*6^(1/2))))^3-1/36*(3*A+2*C+2*6^(1/2)*B*sin
(1/3*arcsin(1/4*6^(1/2))))/(1-2*cos(2/3*arcsin(1/4*6^(1/2))))^2/(3*x-2*6^(
1/2)*sin(1/3*arcsin(1/4*6^(1/2))))-1/864*ln(3*x-2*6^(1/2)*cos(1/6*Pi+1/3*a
rcsin(1/4*6^(1/2))))*sec(1/3*arcsin(1/4*6^(1/2)))^3*(2^(1/2)*(3*A+2*C)*(3*
cos(1/3*arcsin(1/4*6^(1/2)))-3^(1/2)*sin(1/3*arcsin(1/4*6^(1/2))))+3*B*(3+
cos(2/3*arcsin(1/4*6^(1/2)))-3^(1/2)*sin(2/3*arcsin(1/4*6^(1/2)))))/(cos(1
/3*arcsin(1/4*6^(1/2)))-3^(1/2)*sin(1/3*arcsin(1/4*6^(1/2))))^3+1/864*ln(3
*x+2*6^(1/2)*sin(1/3*Pi+1/3*arcsin(1/4*6^(1/2))))*sec(1/3*arcsin(1/4*6^(1/
2)))^3*(2^(1/2)*(3*A+2*C)*(3*cos(1/3*arcsin(1/4*6^(1/2)))+3^(1/2)*sin(1/3*
arcsin(1/4*6^(1/2))))-3*B*(3+cos(2/3*arcsin(1/4*6^(1/2)))+3^(1/2)*sin(2/3*
arcsin(1/4*6^(1/2)))))/(cos(1/3*arcsin(1/4*6^(1/2)))+3^(1/2)*sin(1/3*arcsi
n(1/4*6^(1/2))))^3-1/192*sec(1/3*arcsin(1/4*6^(1/2)))^2*(3*A+2*C-2*6^(1/2)
*B*sin(1/3*Pi+1/3*arcsin(1/4*6^(1/2))))/(sin(1/3*arcsin(1/4*6^(1/2)))+sin(
1/3*Pi+1/3*arcsin(1/4*6^(1/2))))^2/(3*x+2*6^(1/2)*sin(1/3*Pi+1/3*arcsin(1/
4*6^(1/2))))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.17

$$\int \frac{A+Bx+Cx^2}{(2-6x+3x^3)^2}\,dx$$
$$=\frac{1}{90}\left(-\frac{3(2C(-1+x+2x^2)+B(-4+4x+3x^2)+A(-8+3x+6x^2))}{2-6x+3x^3}\right.$$
$$-\text{RootSum}\Big[2-6\#1$$
$$+3\#1^3\&,\frac{6A\log(x-\#1)+8B\log(x-\#1)+4C\log(x-\#1)+6A\log(x-\#1)\#1+3B\log(x-\#1}{-2+3\#1^2}$$

input
```
Integrate[(A + B*x + C*x^2)/(2 - 6*x + 3*x^3)^2,x]
```

output
```
((-3*(2*C*(-1 + x + 2*x^2) + B*(-4 + 4*x + 3*x^2) + A*(-8 + 3*x + 6*x^2)))
/(2 - 6*x + 3*x^3) - RootSum[2 - 6*#1 + 3*#1^3 & , (6*A*Log[x - #1] + 8*B*
Log[x - #1] + 4*C*Log[x - #1] + 6*A*Log[x - #1]*#1 + 3*B*Log[x - #1]*#1 +
4*C*Log[x - #1]*#1)/(-2 + 3*#1^2) & ])/90
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 58.06 (sec) , antiderivative size = 2207, normalized size of antiderivative = 2.49, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.609$, Rules used = {2526, 27, 2485, 1235, 27, 1200, 7239, 27, 25, 27, 1237, 27, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{(3x^3-6x+2)^2}dx$$

$$\Big\downarrow \; 2526$$

$$\frac{1}{9}\int \frac{3(3A+2C+3Bx)}{(3x^3-6x+2)^2}dx-\frac{C}{9(3x^3-6x+2)}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{3}\int \frac{3A+2C+3Bx}{(3x^3-6x+2)^2}dx-\frac{C}{9(3x^3-6x+2)}$$

$$\Big\downarrow \; 2485$$

$$-\frac{C}{9(3x^3-6x+2)}+$$

$$27\int \frac{3A+2C+3Bx}{\left(3x+\sqrt[3]{9-3i\sqrt{15}}+\frac{2\ 3^{2/3}}{\sqrt[3]{3-i\sqrt{15}}}\right)^2\left(-9x^2+3\left(\frac{2\ 3^{2/3}}{\sqrt[3]{3-i\sqrt{15}}}+\sqrt[3]{9-3i\sqrt{15}}\right)x-\left(9-3i\sqrt{15}\right)^{2/3}-\right.}$$

$$\Big\downarrow \; 1235$$

$$27\left(\frac{\left(3-i\sqrt{15}\right)^{4/3}\left(2(6A+3B+4C)-\frac{\left(3\sqrt[3]{3}\left(3-i\sqrt{15}+2\sqrt[3]{9-3i\sqrt{15}}\right)A+3\left(4\sqrt[3]{3}+\left(3^{2/3}-\right.\right.\right.}{\left(\right.}\right.}{54\sqrt[6]{3}\left(10\sqrt{3}+6i\sqrt{5}-5\sqrt[6]{3}\left(3-i\sqrt{15}\right)^{2/3}-i\sqrt{5}\left(9-3i\sqrt{15}\right)^{2/3}\right)\left(3x+\sqrt[3]{9-3i\sqrt{15}}+\frac{2\ 3^{2/3}}{\sqrt[3]{3-i\sqrt{15}}}\right)\left(\right.}\right.$$

$$\frac{C}{9\left(3x^3-6x+2\right)}$$

$\downarrow$ 27

$$27\left(\frac{\left(3-i\sqrt{15}\right)^{4/3}\left(2(6A+3B+4C)-\frac{\left(3\sqrt[3]{3}\left(3-i\sqrt{15}+2\sqrt[3]{9-3i\sqrt{15}}\right)A+3\left(4\sqrt[3]{3}+\left(3^{2/3}-\right.\right.\right.}{\left(\right.}\right.}{54\sqrt[6]{3}\left(10\sqrt{3}+6i\sqrt{5}-5\sqrt[6]{3}\left(3-i\sqrt{15}\right)^{2/3}-i\sqrt{5}\left(9-3i\sqrt{15}\right)^{2/3}\right)\left(3x+\sqrt[3]{9-3i\sqrt{15}}+\frac{2\ 3^{2/3}}{\sqrt[3]{3-i\sqrt{15}}}\right)\left(\right.}\right.$$

$$\frac{C}{9\left(3x^3-6x+2\right)}$$

$\downarrow$ 1200

$$27\left(\frac{\left(3-i\sqrt{15}\right)^{4/3}\left(2(6A+3B+4C)-\frac{\left(3\sqrt[3]{3}\left(3-i\sqrt{15}+2\sqrt[3]{9-3i\sqrt{15}}\right)A+3\left(4\sqrt[3]{3}+\left(3^{2/3}-\cdots\right.\right.\right.}{\left(\cdots\right.}\right.}{54\sqrt[6]{3}\left(10\sqrt{3}+6i\sqrt{5}-5\sqrt[6]{3}\left(3-i\sqrt{15}\right)^{2/3}-i\sqrt{5}\left(9-3i\sqrt{15}\right)^{2/3}\right)\left(3x+\sqrt[3]{9-3i\sqrt{15}}+\frac{2\ 3^{2/3}}{\sqrt[3]{3-i\sqrt{15}}}\right)\left(\cdots\right.}\right.$$

$$\frac{C}{9\left(3x^3-6x+2\right)}$$

$\downarrow$ 7239

$$27\left(\frac{\left(3-i\sqrt{15}\right)^{4/3}\left(2(6A+3B+4C)-\frac{\left(3\sqrt[3]{3}\left(3-i\sqrt{15}+2\sqrt[3]{9-3i\sqrt{15}}\right)A+3\left(4\sqrt[3]{3}+\left(3^{2/3}-\cdots\right.\right.\right.}{\left(\cdots\right.}\right.}{54\sqrt[6]{3}\left(10\sqrt{3}+6i\sqrt{5}-5\sqrt[6]{3}\left(3-i\sqrt{15}\right)^{2/3}-i\sqrt{5}\left(9-3i\sqrt{15}\right)^{2/3}\right)\left(3x+\sqrt[3]{9-3i\sqrt{15}}+\frac{2\ 3^{2/3}}{\sqrt[3]{3-i\sqrt{15}}}\right)\left(\cdots\right.}\right.$$

$$\frac{C}{9\left(3x^3-6x+2\right)}$$

$\downarrow$ 27

$$27\left(\frac{\left(3-i\sqrt{15}\right)^{4/3}\left(2(6A+3B+4C)-\frac{\left(3\sqrt[3]{3}\left(3-i\sqrt{15}+2\sqrt[3]{9-3i\sqrt{15}}\right)A+3\left(4\sqrt[3]{3}+\left(3^{2/3}-\cdots\right.\right.\right.}{\left(\cdots\right.}\right.}{54\sqrt[6]{3}\left(10\sqrt{3}+6i\sqrt{5}-5\sqrt[6]{3}\left(3-i\sqrt{15}\right)^{2/3}-i\sqrt{5}\left(9-3i\sqrt{15}\right)^{2/3}\right)\left(3x+\sqrt[3]{9-3i\sqrt{15}}+\frac{2\ 3^{2/3}}{\sqrt[3]{3-i\sqrt{15}}}\right)\left(\cdots\right.}\right.$$

$$\frac{C}{9\left(3x^3-6x+2\right)}$$

$\downarrow$ 25

$$27\left(\frac{\left(3-i\sqrt{15}\right)^{4/3}\left(2(6A+3B+4C)-\frac{\left(3\sqrt[3]{3}\left(3-i\sqrt{15}+2\sqrt[3]{9-3i\sqrt{15}}\right)A+3\left(4\sqrt[3]{3}+\left(3^{2/3}-\right.\right.\right.}{\left(\right.}\right.}{54\sqrt[6]{3}\left(10\sqrt{3}+6i\sqrt{5}-5\sqrt[6]{3}\left(3-i\sqrt{15}\right)^{2/3}-i\sqrt{5}\left(9-3i\sqrt{15}\right)^{2/3}\right)\left(3x+\sqrt[3]{9-3i\sqrt{15}}+\frac{2\ 3^{2/3}}{\sqrt[3]{3-i\sqrt{15}}}\right)\left(\right.}\right.$$

$$\frac{C}{9\left(3x^3-6x+2\right)}$$

$\downarrow$ 27

$$27\left(\frac{\left(3-i\sqrt{15}\right)^{4/3}\left(2(6A+3B+4C)-\frac{\left(3\sqrt[3]{3}\left(3-i\sqrt{15}+2\sqrt[3]{9-3i\sqrt{15}}\right)A+3\left(4\sqrt[3]{3}+\left(3^{2/3}-\right.\right.\right.}{\left(\right.}\right.}{54\sqrt[6]{3}\left(10\sqrt{3}+6i\sqrt{5}-5\sqrt[6]{3}\left(3-i\sqrt{15}\right)^{2/3}-i\sqrt{5}\left(9-3i\sqrt{15}\right)^{2/3}\right)\left(3x+\sqrt[3]{9-3i\sqrt{15}}+\frac{2\ 3^{2/3}}{\sqrt[3]{3-i\sqrt{15}}}\right)\left(\right.}\right.$$

$$\frac{C}{9\left(3x^3-6x+2\right)}$$

$\downarrow$ 1237

$$27\left(\frac{\left(3-i\sqrt{15}\right)^{4/3}\left(2(6A+3B+4C)-\frac{\left(3\sqrt[3]{3}\left(3-i\sqrt{15}+2\sqrt[3]{9-3i\sqrt{15}}\right)A+3\left(4\sqrt[3]{3}+\left(3^{2/3}-\cdots\right.\right.\right.}{\left(\cdots\right.}\right.}{54\sqrt[6]{3}\left(10\sqrt{3}+6i\sqrt{5}-5\sqrt[6]{3}\left(3-i\sqrt{15}\right)^{2/3}-i\sqrt{5}\left(9-3i\sqrt{15}\right)^{2/3}\right)\left(3x+\sqrt[3]{9-3i\sqrt{15}}+\frac{2\ 3^{2/3}}{\sqrt[3]{3-i\sqrt{15}}}\right)\left(\cdots\right.}\right.$$

$$\frac{C}{9\left(3x^3-6x+2\right)}$$

$$\Big\downarrow\ 27$$

$$27\left(\frac{\left(3-i\sqrt{15}\right)^{4/3}\left(2(6A+3B+4C)-\frac{\left(3\sqrt[3]{3}\left(3-i\sqrt{15}+2\sqrt[3]{9-3i\sqrt{15}}\right)A+3\left(4\sqrt[3]{3}+\left(3^{2/3}-\cdots\right.\right.\right.}{\left(\cdots\right.}\right.}{54\sqrt[6]{3}\left(10\sqrt{3}+6i\sqrt{5}-5\sqrt[6]{3}\left(3-i\sqrt{15}\right)^{2/3}-i\sqrt{5}\left(9-3i\sqrt{15}\right)^{2/3}\right)\left(3x+\sqrt[3]{9-3i\sqrt{15}}+\frac{2\ 3^{2/3}}{\sqrt[3]{3-i\sqrt{15}}}\right)\left(\cdots\right.}\right.$$

$$\frac{C}{9\left(3x^3-6x+2\right)}$$

$$\Big\downarrow\ 1200$$

$$27\left(\frac{\left(3-i\sqrt{15}\right)^{4/3}\left(2(6A+3B+4C)-\frac{\left(3\sqrt[3]{3}\left(3-i\sqrt{15}+2\sqrt[3]{9-3i\sqrt{15}}\right)A+3\left(4\sqrt[3]{3}+\left(3^{2/3}-\right.\right.\right.}{\left(\right.}\right.}{54\sqrt[6]{3}\left(10\sqrt{3}+6i\sqrt{5}-5\sqrt[6]{3}\left(3-i\sqrt{15}\right)^{2/3}-i\sqrt{5}\left(9-3i\sqrt{15}\right)^{2/3}\right)\left(3x+\sqrt[3]{9-3i\sqrt{15}}+\frac{2\ 3^{2/3}}{\sqrt[3]{3-i\sqrt{15}}}\right)\left(\right.}\right.$$

$$\frac{C}{9\left(3x^3-6x+2\right)}$$

$\downarrow$ 2009

$$27\left(\frac{\left(3-i\sqrt{15}\right)^{4/3}\left(2(6A+3B+4C)-\frac{\left(3\sqrt[3]{3}\left(3-i\sqrt{15}+2\sqrt[3]{9-3i\sqrt{15}}\right)A+3\left(4\sqrt[3]{3}+\left(3^{2/3}-\right.\right.\right.}{\left(\right.}\right.}{54\sqrt[6]{3}\left(10\sqrt{3}+6i\sqrt{5}-5\sqrt[6]{3}\left(3-i\sqrt{15}\right)^{2/3}-i\sqrt{5}\left(9-3i\sqrt{15}\right)^{2/3}\right)\left(3x+\sqrt[3]{9-3i\sqrt{15}}+\frac{2\ 3^{2/3}}{\sqrt[3]{3-i\sqrt{15}}}\right)\left(\right.}\right.$$

$$\frac{C}{9\left(3x^3-6x+2\right)}$$

input
```
Int[(A + B*x + C*x^2)/(2 - 6*x + 3*x^3)^2,x]
```

output
```
-1/9*C/(2 - 6*x + 3*x^3) + 27*(((3 - I*Sqrt[15])^(4/3)*(2*(6*A + 3*B + 4*C
) - ((3*3^(1/3)*(3 - I*Sqrt[15] + 2*(9 - (3*I)*Sqrt[15])^(1/3))*A + 3*(4*3
^(1/3) + (3^(2/3) - I*3^(1/6)*Sqrt[5])*(3 - I*Sqrt[15])^(1/3))*B + 2*3^(1/
3)*(3 - I*Sqrt[15] + 2*(9 - (3*I)*Sqrt[15])^(1/3))*C)*x)/(3 - I*Sqrt[15])^
(2/3)))/(54*3^(1/6)*(10*Sqrt[3] + (6*I)*Sqrt[5] - 5*3^(1/6)*(3 - I*Sqrt[15
])^(2/3) - I*Sqrt[5]*(9 - (3*I)*Sqrt[15])^(2/3))*((2*3^(2/3))/(3 - I*Sqrt[
15])^(1/3) + (9 - (3*I)*Sqrt[15])^(1/3) + 3*x)*(6 - (12*3^(1/3))/(3 - I*Sq
rt[15])^(2/3) - (9 - (3*I)*Sqrt[15])^(2/3) + 3*((2*3^(2/3))/(3 - I*Sqrt[15
])^(1/3) + (9 - (3*I)*Sqrt[15])^(1/3))*x - 9*x^2)) - (16*(3 - I*Sqrt[15])^
(1/3)*(-1/2*((6*3^(1/6)*((7*I)*Sqrt[3] + 3*Sqrt[5]) + (16*I)*3^(1/3)*(3 -
I*Sqrt[15])^(2/3) + (3 - I*Sqrt[15])^(1/3)*(39*I + Sqrt[15]))*(6*A + 3*B +
 4*C))/((3*I - 3*Sqrt[15] - 3^(5/6)*Sqrt[5]*(3 - I*Sqrt[15])^(1/3) - (3*I)
*(9 - (3*I)*Sqrt[15])^(1/3) - (2*I)*(9 - (3*I)*Sqrt[15])^(2/3))*(2*3^(1/3)
 + (3 - I*Sqrt[15])^(2/3) + 3^(2/3)*(3 - I*Sqrt[15])^(1/3)*x)) - (3*((-2*(
6*((622*I)*3^(2/3) - 90*3^(1/6)*Sqrt[5] + (669*I - 101*Sqrt[15])*(3 - I*Sq
rt[15])^(1/3) + 16*3^(1/3)*(11*I - 3*Sqrt[15])*(3 - I*Sqrt[15])^(2/3))*A +
 3*(4*3^(1/6)*((313*I)*Sqrt[3] - 35*Sqrt[5]) + 8*(163*I - 27*Sqrt[15])*(3
- I*Sqrt[15])^(1/3) + 31*3^(1/3)*(11*I - 3*Sqrt[15])*(3 - I*Sqrt[15])^(2/3
))*B + 4*((622*I)*3^(2/3) - 90*3^(1/6)*Sqrt[5] + (669*I - 101*Sqrt[15])*(3
 - I*Sqrt[15])^(1/3) + 16*3^(1/3)*(11*I - 3*Sqrt[15])*(3 - I*Sqrt[15])^...
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1235
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^m
*(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*
m + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) -
f*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
)
```

rule 1237
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[
(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && LtQ[m, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2485
```
Int[((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (d_.)*(x_)^3)^(p_), x_S
ymbol] :> With[{r = Rt[-9*a*d^2 + Sqrt[3]*d*Sqrt[4*b^3*d + 27*a^2*d^2], 3]}
, Simp[1/d^(2*p)   Int[(e + f*x)^m*Simp[18^(1/3)*b*(d/(3*r)) - r/18^(1/3) +
 d*x, x]^p*Simp[b*(d/3) + 12^(1/3)*b^2*(d^2/(3*r^2)) + r^2/(3*12^(1/3)) - d
*(2^(1/3)*b*(d/(3^(1/3)*r)) - r/18^(1/3))*x + d^2*x^2, x]^p, x], x]] /; Fre
eQ[{a, b, d, e, f, m}, x] && NeQ[4*b^3 + 27*a^2*d, 0] && ILtQ[p, 0]
```

rule 2526
```
Int[(Pm_)*(Qn_)^(p_), x_Symbol] :> With[{m = Expon[Pm, x], n = Expon[Qn, x]
}, Simp[Coeff[Pm, x, m]*(Qn^(p + 1)/(n*(p + 1)*Coeff[Qn, x, n])), x] + Simp
[1/(n*Coeff[Qn, x, n])   Int[ExpandToSum[n*Coeff[Qn, x, n]*Pm - Coeff[Pm, x
, m]*D[Qn, x], x]*Qn^p, x], x] /; EqQ[m, n - 1]] /; FreeQ[p, x] && PolyQ[Pm
, x] && PolyQ[Qn, x] && NeQ[p, -1]
```

rule 7239
```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.12

method	result
default	$\frac{\left(-\frac{2C}{45}-\frac{A}{15}-\frac{B}{30}\right)x^2+\left(-\frac{C}{45}-\frac{A}{30}-\frac{2B}{45}\right)x+\frac{C}{45}+\frac{4A}{45}+\frac{2B}{45}}{x^3-2x+\frac{2}{3}}+\frac{\left(\sum_{\_R=\text{RootOf}\left(3\_Z^3-6\_Z+2\right)}\frac{\left(-6A\_R-3B\_R-4C\_R-6A-8B-4C\right.}{3\_R^2-2}\right.}{90}$
risch	$\frac{\left(-\frac{2C}{45}-\frac{A}{15}-\frac{B}{30}\right)x^2+\left(-\frac{C}{45}-\frac{A}{30}-\frac{2B}{45}\right)x+\frac{C}{45}+\frac{4A}{45}+\frac{2B}{45}}{x^3-2x+\frac{2}{3}}+\frac{\left(\sum_{\_R=\text{RootOf}\left(3\_Z^3-6\_Z+2\right)}\frac{\left((-6A-3B-4C)\_R-4C-6A-8B\right)\ln\left(x-\right.}{3\_R^2-2}\right.}{90}$

input
```
int((C*x^2+B*x+A)/(3*x^3-6*x+2)^2,x,method=_RETURNVERBOSE)
```

output
```
((-2/45*C-1/15*A-1/30*B)*x^2+(-1/45*C-1/30*A-2/45*B)*x+1/45*C+4/45*A+2/45*
B)/(x^3-2*x+2/3)+1/90*sum((-6*A*_R-3*B*_R-4*C*_R-6*A-8*B-4*C)/(3*_R^2-2)*l
n(x-_R),_R=RootOf(3*_Z^3-6*_Z+2))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.76 (sec) , antiderivative size = 5181, normalized size of antiderivative = 5.85

$$\int \frac{A+Bx+Cx^2}{(2-6x+3x^3)^2}\,dx = \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)/(3*x^3-6*x+2)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [A] (verification not implemented)

Time = 8.10 (sec) , antiderivative size = 342, normalized size of antiderivative = 0.39

$$\int \frac{A+Bx+Cx^2}{(2-6x+3x^3)^2}\,dx$$
$$=\operatorname{RootSum}\left(4374000t^3+t\left(-15552A^2-30132AB-20736AC-15228B^2-20088BC-6912C^2\right)+108\right.$$
$$+\frac{8A+4B+2C+x^2(-6A-3B-4C)+x(-3A-4B-2C)}{90x^3-180x+60}$$

input
```
integrate((C*x**2+B*x+A)/(3*x**3-6*x+2)**2,x)
```

output
```
RootSum(4374000*_t**3 + _t*(-15552*A**2 - 30132*A*B - 20736*A*C - 15228*B*
*2 - 20088*B*C - 6912*C**2) + 108*A**3 + 108*A**2*B + 216*A**2*C - 108*A*B
**2 + 144*A*B*C + 144*A*C**2 - 105*B**3 - 72*B**2*C + 48*B*C**2 + 32*C**3,
 Lambda(_t, _t*log(x + (-4374000*_t**2*A - 4617000*_t**2*B - 2916000*_t**2
*C + 16200*_t*A**2 + 97200*_t*A*B + 21600*_t*A*C + 78300*_t*B**2 + 64800*_
t*B*C + 7200*_t*C**2 + 10368*A**3 + 31032*A**2*B + 20736*A**2*C + 31356*A*
B**2 + 41376*A*B*C + 13824*A*C**2 + 10716*B**3 + 20904*B**2*C + 13792*B*C*
*2 + 3072*C**3)/(6588*A**3 + 20412*A**2*B + 13176*A**2*C + 20196*A*B**2 +
27216*A*B*C + 8784*A*C**2 + 6381*B**3 + 13464*B**2*C + 9072*B*C**2 + 1952*
C**3)))) + (8*A + 4*B + 2*C + x**2*(-6*A - 3*B - 4*C) + x*(-3*A - 4*B - 2*
C))/(90*x**3 - 180*x + 60)
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{(2-6x+3x^3)^2}\,dx = \int \frac{Cx^2+Bx+A}{(3\,x^3-6\,x+2)^2}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(3*x^3-6*x+2)^2,x, algorithm="maxima")
```

output
```
-1/30*((6*A + 3*B + 4*C)*x^2 + (3*A + 4*B + 2*C)*x - 8*A - 4*B - 2*C)/(3*x
^3 - 6*x + 2) - 1/30*integrate(((6*A + 3*B + 4*C)*x + 6*A + 8*B + 4*C)/(3*
x^3 - 6*x + 2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A+Bx+Cx^2}{(2-6x+3x^3)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((C*x^2+B*x+A)/(3*x^3-6*x+2)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 12.56 (sec) , antiderivative size = 995, normalized size of antiderivative = 1.12

$$\int \frac{A+Bx+Cx^2}{(2-6x+3x^3)^2}\,dx = \text{Too large to display}$$

input
```
int((A + B*x + C*x^2)/(3*x^3 - 6*x + 2)^2,x)
```

output
```
((4*A)/15 + (2*B)/15 + C/15 - x*(A/10 + (2*B)/15 + C/15) - x^2*(A/5 + B/10
 + (2*C)/15))/(3*x^3 - 6*x + 2) + symsum(log((3*A^2*x)/25 + (3*B^2*x)/100
+ (4*C^2*x)/75 - 324*root(z^3 - z*((4*A^2)/1125 + (47*B^2)/13500 + (16*C^2
)/10125 + (31*A*B)/4500 + (16*A*C)/3375 + (31*B*C)/6750) + (A*B*C)/30375 +
 (B*C^2)/91125 - (B^2*C)/60750 + (A*C^2)/30375 + (A^2*C)/20250 + (A^2*B)/4
0500 - (A*B^2)/40500 + A^3/40500 + (2*C^3)/273375 - (7*B^3)/291600, z, k)^
2*x + (3*A^2)/25 + (2*B^2)/25 + (4*C^2)/75 + 162*root(z^3 - z*((4*A^2)/112
5 + (47*B^2)/13500 + (16*C^2)/10125 + (31*A*B)/4500 + (16*A*C)/3375 + (31*
B*C)/6750) + (A*B*C)/30375 + (B*C^2)/91125 - (B^2*C)/60750 + (A*C^2)/30375
 + (A^2*C)/20250 + (A^2*B)/40500 - (A*B^2)/40500 + A^3/40500 + (2*C^3)/273
375 - (7*B^3)/291600, z, k)^2 + (11*A*B)/50 + (4*A*C)/25 + (11*B*C)/75 - (
18*A*root(z^3 - z*((4*A^2)/1125 + (47*B^2)/13500 + (16*C^2)/10125 + (31*A*
B)/4500 + (16*A*C)/3375 + (31*B*C)/6750) + (A*B*C)/30375 + (B*C^2)/91125 -
 (B^2*C)/60750 + (A*C^2)/30375 + (A^2*C)/20250 + (A^2*B)/40500 - (A*B^2)/4
0500 + A^3/40500 + (2*C^3)/273375 - (7*B^3)/291600, z, k))/5 - (9*B*root(z
^3 - z*((4*A^2)/1125 + (47*B^2)/13500 + (16*C^2)/10125 + (31*A*B)/4500 + (
16*A*C)/3375 + (31*B*C)/6750) + (A*B*C)/30375 + (B*C^2)/91125 - (B^2*C)/60
750 + (A*C^2)/30375 + (A^2*C)/20250 + (A^2*B)/40500 - (A*B^2)/40500 + A^3/
40500 + (2*C^3)/273375 - (7*B^3)/291600, z, k))/5 - (12*C*root(z^3 - z*((4
*A^2)/1125 + (47*B^2)/13500 + (16*C^2)/10125 + (31*A*B)/4500 + (16*A*C)...
```

Reduce [F]

$$\int \frac{A+Bx+Cx^2}{(2-6x+3x^3)^2}\,dx$$
$$= \frac{27\left(\int \frac{x^4}{9x^6-36x^4+12x^3+36x^2-24x+4}dx\right) b\,x^3 - 54\left(\int \frac{x^4}{9x^6-36x^4+12x^3+36x^2-24x+4}dx\right) bx + 18\left(\int \frac{x^4}{9x^6-36x^4+12x^3+36x^2\cdots}\right.}{}$$

input
```
int((C*x^2+B*x+A)/(3*x^3-6*x+2)^2,x)
```

output
```
(27*int(x**4/(9*x**6 - 36*x**4 + 12*x**3 + 36*x**2 - 24*x + 4),x)*b*x**3 -
 54*int(x**4/(9*x**6 - 36*x**4 + 12*x**3 + 36*x**2 - 24*x + 4),x)*b*x + 18
*int(x**4/(9*x**6 - 36*x**4 + 12*x**3 + 36*x**2 - 24*x + 4),x)*b + 108*int
(x**3/(9*x**6 - 36*x**4 + 12*x**3 + 36*x**2 - 24*x + 4),x)*a*x**3 - 216*in
t(x**3/(9*x**6 - 36*x**4 + 12*x**3 + 36*x**2 - 24*x + 4),x)*a*x + 72*int(x
**3/(9*x**6 - 36*x**4 + 12*x**3 + 36*x**2 - 24*x + 4),x)*a + 108*int(x**3/
(9*x**6 - 36*x**4 + 12*x**3 + 36*x**2 - 24*x + 4),x)*b*x**3 - 216*int(x**3
/(9*x**6 - 36*x**4 + 12*x**3 + 36*x**2 - 24*x + 4),x)*b*x + 72*int(x**3/(9
*x**6 - 36*x**4 + 12*x**3 + 36*x**2 - 24*x + 4),x)*b + 72*int(x**3/(9*x**6
 - 36*x**4 + 12*x**3 + 36*x**2 - 24*x + 4),x)*c*x**3 - 144*int(x**3/(9*x**
6 - 36*x**4 + 12*x**3 + 36*x**2 - 24*x + 4),x)*c*x + 48*int(x**3/(9*x**6 -
 36*x**4 + 12*x**3 + 36*x**2 - 24*x + 4),x)*c + 6*a*x + 3*b*x**2 + 6*b*x -
 2*b + 2*c*x**3)/(12*(3*x**3 - 6*x + 2))
```

3.27 $\int \frac{A+Bx+Cx^2}{\sqrt{2-6x+3x^3}} dx$

Optimal result

Integrand size = 25, antiderivative size = 749

$$\int \frac{A+Bx+Cx^2}{\sqrt{2-6x+3x^3}} dx = \text{Too large to display}$$

output
```
2/9*C*(3*x^3-6*x+2)^(1/2)-1/9*2^(3/4)*(3*x-2*6^(1/2)*cos(1/6*Pi+1/3*arcsin
(1/4*6^(1/2))))^(1/2)*(3*A+2*C+2*6^(1/2)*B*cos(1/6*Pi+1/3*arcsin(1/4*6^(1/
2))))*EllipticF(2^(1/4)*(3*cos(1/3*arcsin(1/4*6^(1/2)))+3*3^(1/2)*sin(1/3*
arcsin(1/4*6^(1/2))))^(1/2)/(3*x+2*6^(1/2)*sin(1/3*Pi+1/3*arcsin(1/4*6^(1/
2))))^(1/2),(3^(1/2)*cos(1/3*arcsin(1/4*6^(1/2)))/(sin(1/3*arcsin(1/4*6^(1
/2)))+sin(1/3*Pi+1/3*arcsin(1/4*6^(1/2)))))^(1/2))*(3*x-2*6^(1/2)*sin(1/3*
arcsin(1/4*6^(1/2))))^(1/2)*(3*x+2*6^(1/2)*sin(1/3*Pi+1/3*arcsin(1/4*6^(1/
2))))^(1/2)/(3*x^3-6*x+2)^(1/2)/(3*cos(1/3*arcsin(1/4*6^(1/2)))+3*3^(1/2)*
sin(1/3*arcsin(1/4*6^(1/2))))^(1/2)+2/9*2^(1/4)*3^(3/4)*B*(3*x-2*6^(1/2)*c
os(1/6*Pi+1/3*arcsin(1/4*6^(1/2))))*EllipticE(1/2*(3*x+2*6^(1/2)*sin(1/3*P
i+1/3*arcsin(1/4*6^(1/2))))^(1/2)*2^(3/4)/(3*cos(1/3*arcsin(1/4*6^(1/2)))+
3*3^(1/2)*sin(1/3*arcsin(1/4*6^(1/2))))^(1/2),1/2*(2+2*3^(1/2)*tan(1/3*arc
sin(1/4*6^(1/2))))^(1/2))*(-3*x+2*6^(1/2)*sin(1/3*arcsin(1/4*6^(1/2))))^(1
/2)*((1+cos(2/3*arcsin(1/4*6^(1/2)))+3^(1/2)*sin(2/3*arcsin(1/4*6^(1/2))))
/(3^(1/2)*cos(1/3*arcsin(1/4*6^(1/2)))+3*sin(1/3*arcsin(1/4*6^(1/2)))))^(1
/2)*(3*x+2*6^(1/2)*sin(1/3*Pi+1/3*arcsin(1/4*6^(1/2))))^(1/2)/(3*x^3-6*x+2
)^(1/2)/(-3*x+2*6^(1/2)*cos(1/6*Pi+1/3*arcsin(1/4*6^(1/2))))^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 10.41 (sec) , antiderivative size = 589, normalized size of antiderivative = 0.79

$$\int \frac{A+Bx+Cx^2}{\sqrt{2-6x+3x^3}}\,dx = \text{Too large to display}$$

input
```
Integrate[(A + B*x + C*x^2)/Sqrt[2 - 6*x + 3*x^3],x]
```

output
```
(2*(-3*EllipticF[ArcSin[Sqrt[(x - Root[2 - 6*#1 + 3*#1^3 & , 3, 0])/(Root[
2 - 6*#1 + 3*#1^3 & , 2, 0] - Root[2 - 6*#1 + 3*#1^3 & , 3, 0])]], (Root[2
 - 6*#1 + 3*#1^3 & , 2, 0] - Root[2 - 6*#1 + 3*#1^3 & , 3, 0])/(Root[2 - 6
*#1 + 3*#1^3 & , 1, 0] - Root[2 - 6*#1 + 3*#1^3 & , 3, 0])]*Sqrt[x - Root[
2 - 6*#1 + 3*#1^3 & , 1, 0]]*(3*A + 2*C + 3*B*Root[2 - 6*#1 + 3*#1^3 & , 1
, 0])*Sqrt[-((x - Root[2 - 6*#1 + 3*#1^3 & , 2, 0])*(x - Root[2 - 6*#1 + 3
*#1^3 & , 3, 0]))] + 9*B*EllipticE[ArcSin[Sqrt[(x - Root[2 - 6*#1 + 3*#1^3
 & , 3, 0])/(Root[2 - 6*#1 + 3*#1^3 & , 2, 0] - Root[2 - 6*#1 + 3*#1^3 & ,
 3, 0])]], (Root[2 - 6*#1 + 3*#1^3 & , 2, 0] - Root[2 - 6*#1 + 3*#1^3 & ,
3, 0])/(Root[2 - 6*#1 + 3*#1^3 & , 1, 0] - Root[2 - 6*#1 + 3*#1^3 & , 3, 0
])]*Sqrt[x - Root[2 - 6*#1 + 3*#1^3 & , 1, 0]]*Sqrt[-((x - Root[2 - 6*#1 +
 3*#1^3 & , 2, 0])*(x - Root[2 - 6*#1 + 3*#1^3 & , 3, 0]))]*(Root[2 - 6*#1
 + 3*#1^3 & , 1, 0] - Root[2 - 6*#1 + 3*#1^3 & , 3, 0]) + C*(2 - 6*x + 3*x
^3)*Sqrt[-Root[2 - 6*#1 + 3*#1^3 & , 1, 0] + Root[2 - 6*#1 + 3*#1^3 & , 3,
 0]]))/(9*Sqrt[2 - 6*x + 3*x^3]*Sqrt[-Root[2 - 6*#1 + 3*#1^3 & , 1, 0] + R
oot[2 - 6*#1 + 3*#1^3 & , 3, 0]])
```

Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 5.53 (sec) , antiderivative size = 2085, normalized size of antiderivative = 2.78, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {2526, 27, 2486, 1269, 1172, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{\sqrt{3x^3-6x+2}}\,dx$$

$$\Big\downarrow\ 2526$$

$$\frac{1}{9}\int \frac{3(3A+2C+3Bx)}{\sqrt{3x^3-6x+2}}dx+\frac{2}{9}C\sqrt{3x^3-6x+2}$$

$$\Big\downarrow\ 27$$

$$\frac{1}{3}\int \frac{3A+2C+3Bx}{\sqrt{3x^3-6x+2}}dx+\frac{2}{9}C\sqrt{3x^3-6x+2}$$

$$\Big\downarrow\ 2486$$

$$\frac{2}{9}C\sqrt{3x^3-6x+2}+$$

$$\frac{\sqrt{3x+\sqrt[3]{9-3i\sqrt{15}}+\frac{2\ 3^{2/3}}{\sqrt[3]{3-i\sqrt{15}}}}\sqrt{9x^2-3\left(\frac{2\ 3^{2/3}}{\sqrt[3]{3-i\sqrt{15}}}+\sqrt[3]{9-3i\sqrt{15}}\right)x+\left(9-3i\sqrt{15}\right)^{2/3}+\frac{12\sqrt[3]{3}}{\left(3-i\sqrt{15}\right)^{2/}}}}{3\sqrt{3x^3-}}$$

$$\Big\downarrow\ 1269$$

$$\frac{2}{9}C\sqrt{3x^3-6x+2}+$$

$$\frac{\sqrt{3x+\sqrt[3]{9-3i\sqrt{15}}+\frac{2\ 3^{2/3}}{\sqrt[3]{3-i\sqrt{15}}}}\sqrt{9x^2-3\left(\frac{2\ 3^{2/3}}{\sqrt[3]{3-i\sqrt{15}}}+\sqrt[3]{9-3i\sqrt{15}}\right)x+\left(9-3i\sqrt{15}\right)^{2/3}+\frac{12\sqrt[3]{3}}{\left(3-i\sqrt{15}\right)^{2/}}}}{}$$

$$\Big\downarrow\ 1172$$

$$\frac{2}{9}\sqrt{3x^3-6x+2}C+$$

$$\sqrt{3x+\sqrt[3]{9-3i\sqrt{15}}+\frac{2\ 3^{2/3}}{\sqrt[3]{3-i\sqrt{15}}}}\sqrt{9x^2-3\left(\frac{2\ 3^{2/3}}{\sqrt[3]{3-i\sqrt{15}}}+\sqrt[3]{9-3i\sqrt{15}}\right)x+\left(9-3i\sqrt{15}\right)^{2/3}+\frac{12\sqrt[3]{3}}{\left(3-i\sqrt{15}\right)^{2/}}}$$

$\downarrow$ 321

$$\frac{2}{9}\sqrt{3x^3-6x+2}C+$$

$$\sqrt{3x+\sqrt[3]{9-3i\sqrt{15}}+\frac{2\ 3^{2/3}}{\sqrt[3]{3-i\sqrt{15}}}}\sqrt{9x^2-3\left(\frac{2\ 3^{2/3}}{\sqrt[3]{3-i\sqrt{15}}}+\sqrt[3]{9-3i\sqrt{15}}\right)x+\left(9-3i\sqrt{15}\right)^{2/3}+\frac{12\sqrt[3]{3}}{\left(3-i\sqrt{15}\right)^{2/}}}$$

↓ 327

$$\frac{2}{9}\sqrt{3x^3-6x+2}C+$$

$$\sqrt{3x+\sqrt[3]{9-3i\sqrt{15}}+\frac{2\ 3^{2/3}}{\sqrt[3]{3-i\sqrt{15}}}}\sqrt{9x^2-3\left(\frac{2\ 3^{2/3}}{\sqrt[3]{3-i\sqrt{15}}}+\sqrt[3]{9-3i\sqrt{15}}\right)x+\left(9-3i\sqrt{15}\right)^{2/3}+\frac{12\sqrt[3]{3}}{\left(3-i\sqrt{15}\right)^{2/}}}$$

---

input
```
Int[(A + B*x + C*x^2)/Sqrt[2 - 6*x + 3*x^3],x]
```

output
```
(2*C*Sqrt[2 - 6*x + 3*x^3])/9 + (Sqrt[(2*3^(2/3))/(3 - I*Sqrt[15])^(1/3) +
 (9 - (3*I)*Sqrt[15])^(1/3) + 3*x]*Sqrt[-6 + (12*3^(1/3))/(3 - I*Sqrt[15])
^(2/3) + (9 - (3*I)*Sqrt[15])^(2/3) - 3*((2*3^(2/3))/(3 - I*Sqrt[15])^(1/3
) + (9 - (3*I)*Sqrt[15])^(1/3))*x + 9*x^2]*((Sqrt[(2*(-4*3^(1/3) - (3^(2/3
) - I*3^(1/6)*Sqrt[5])*(3 - I*Sqrt[15])^(1/3) + 4*(3 - I*Sqrt[15])^(2/3)))
/3]*B*Sqrt[(2*3^(2/3))/(3 - I*Sqrt[15])^(1/3) + (9 - (3*I)*Sqrt[15])^(1/3)
 + 3*x]*Sqrt[-(((3 - I*Sqrt[15])^(2/3)*(6 - (12*3^(1/3))/(3 - I*Sqrt[15])^
(2/3) - (9 - (3*I)*Sqrt[15])^(2/3) + 3*((2*3^(2/3))/(3 - I*Sqrt[15])^(1/3)
 + (9 - (3*I)*Sqrt[15])^(1/3))*x - 9*x^2))/(4*3^(1/3) + (3^(2/3) - I*3^(1/
6)*Sqrt[5])*(3 - I*Sqrt[15])^(1/3) - 4*(3 - I*Sqrt[15])^(2/3)))]*EllipticE
[ArcSin[Sqrt[-(((3 - I*Sqrt[15])^(1/3)*((2*3^(2/3) + 3^(1/3)*(3 - I*Sqrt[1
5])^(2/3) - 3*Sqrt[-4*3^(1/3) - (3^(2/3) - I*3^(1/6)*Sqrt[5])*(3 - I*Sqrt[
15])^(1/3) + 4*(3 - I*Sqrt[15])^(2/3)])/(3 - I*Sqrt[15])^(1/3) - 6*x))/Sqr
t[-4*3^(1/3) - (3^(2/3) - I*3^(1/6)*Sqrt[5])*(3 - I*Sqrt[15])^(1/3) + 4*(3
 - I*Sqrt[15])^(2/3)])]/Sqrt[6]], (-2*Sqrt[-4*3^(1/3) - (3^(2/3) - I*3^(1/
6)*Sqrt[5])*(3 - I*Sqrt[15])^(1/3) + 4*(3 - I*Sqrt[15])^(2/3)])/(2*3^(2/3)
 + 3^(1/3)*(3 - I*Sqrt[15])^(2/3) - Sqrt[-4*3^(1/3) - (3^(2/3) - I*3^(1/6)
*Sqrt[5])*(3 - I*Sqrt[15])^(1/3) + 4*(3 - I*Sqrt[15])^(2/3)])])/((3 - I*Sq
rt[15])^(1/3)*Sqrt[((3 - I*Sqrt[15])^(1/3)*((2*3^(2/3))/(3 - I*Sqrt[15])^(
1/3) + (9 - (3*I)*Sqrt[15])^(1/3) + 3*x))/(2*3^(2/3) + 3^(1/3)*(3 - I*S...
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 1172
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Simp[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m))   Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^
2 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[m^2, 1/4]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2486
```
Int[((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (d_.)*(x_)^3)^(p_), x_S
ymbol] :> With[{r = Rt[-9*a*d^2 + Sqrt[3]*d*Sqrt[4*b^3*d + 27*a^2*d^2], 3]}
, Simp[(a + b*x + d*x^3)^p/(Simp[18^(1/3)*b*(d/(3*r)) - r/18^(1/3) + d*x, x
]^p*Simp[b*(d/3) + 12^(1/3)*b^2*(d^2/(3*r^2)) + r^2/(3*12^(1/3)) - d*(2^(1/
3)*b*(d/(3^(1/3)*r)) - r/18^(1/3))*x + d^2*x^2, x]^p)   Int[(e + f*x)^m*Sim
p[18^(1/3)*b*(d/(3*r)) - r/18^(1/3) + d*x, x]^p*Simp[b*(d/3) + 12^(1/3)*b^2
*(d^2/(3*r^2)) + r^2/(3*12^(1/3)) - d*(2^(1/3)*b*(d/(3^(1/3)*r)) - r/18^(1/
3))*x + d^2*x^2, x]^p, x], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && NeQ[4*
b^3 + 27*a^2*d, 0] &&  !IntegerQ[p]
```

rule 2526
```
Int[(Pm_)*(Qn_)^(p_), x_Symbol] :> With[{m = Expon[Pm, x], n = Expon[Qn, x]
}, Simp[Coeff[Pm, x, m]*(Qn^(p + 1)/(n*(p + 1)*Coeff[Qn, x, n])), x] + Simp
[1/(n*Coeff[Qn, x, n])   Int[ExpandToSum[n*Coeff[Qn, x, n]*Pm - Coeff[Pm, x
, m]*D[Qn, x], x]*Qn^p, x], x] /; EqQ[m, n - 1]] /; FreeQ[p, x] && PolyQ[Pm
, x] && PolyQ[Qn, x] && NeQ[p, -1]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.13 (sec) , antiderivative size = 788, normalized size of antiderivative = 1.05

method	result	size
elliptic	Expression too large to display	788
risch	Expression too large to display	1086
default	Expression too large to display	1087

input

```
int((C*x^2+B*x+A)/(3*x^3-6*x+2)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
2/9*C*(3*x^3-6*x+2)^(1/2)+1/9*(A+2/3*C)*6^(1/2)*sin(1/3*arctan(1/3*15^(1/2
)))*((x+1/3*6^(1/2)*sin(1/3*arctan(1/3*15^(1/2)))*3^(1/2)-1/3*6^(1/2)*cos(
1/3*arctan(1/3*15^(1/2))))*6^(1/2)/sin(1/3*arctan(1/3*15^(1/2)))*3^(1/2))^
(1/2)*((x+2/3*6^(1/2)*cos(1/3*arctan(1/3*15^(1/2))))/(-1/3*6^(1/2)*sin(1/3
*arctan(1/3*15^(1/2)))*3^(1/2)+6^(1/2)*cos(1/3*arctan(1/3*15^(1/2)))))^(1/
2)*(-3*(x-1/3*6^(1/2)*sin(1/3*arctan(1/3*15^(1/2)))*3^(1/2)-1/3*6^(1/2)*co
s(1/3*arctan(1/3*15^(1/2))))*6^(1/2)/sin(1/3*arctan(1/3*15^(1/2)))*3^(1/2)
)^(1/2)/(3*x^3-6*x+2)^(1/2)*EllipticF(1/6*3^(1/2)*((x+1/3*6^(1/2)*sin(1/3*
arctan(1/3*15^(1/2)))*3^(1/2)-1/3*6^(1/2)*cos(1/3*arctan(1/3*15^(1/2))))*6
^(1/2)/sin(1/3*arctan(1/3*15^(1/2)))*3^(1/2))^(1/2),1/3*I*6^(1/2)*(6^(1/2)
*sin(1/3*arctan(1/3*15^(1/2)))*3^(1/2)/(-1/3*6^(1/2)*sin(1/3*arctan(1/3*15
^(1/2)))*3^(1/2)+6^(1/2)*cos(1/3*arctan(1/3*15^(1/2)))))^(1/2))+1/9*B*6^(1
/2)*sin(1/3*arctan(1/3*15^(1/2)))*((x+1/3*6^(1/2)*sin(1/3*arctan(1/3*15^(1
/2)))*3^(1/2)-1/3*6^(1/2)*cos(1/3*arctan(1/3*15^(1/2))))*6^(1/2)/sin(1/3*a
rctan(1/3*15^(1/2)))*3^(1/2))^(1/2)*((x+2/3*6^(1/2)*cos(1/3*arctan(1/3*15^
(1/2))))/(-1/3*6^(1/2)*sin(1/3*arctan(1/3*15^(1/2)))*3^(1/2)+6^(1/2)*cos(1
/3*arctan(1/3*15^(1/2)))))^(1/2)*(-3*(x-1/3*6^(1/2)*sin(1/3*arctan(1/3*15^
(1/2)))*3^(1/2)-1/3*6^(1/2)*cos(1/3*arctan(1/3*15^(1/2))))*6^(1/2)/sin(1/3
*arctan(1/3*15^(1/2)))*3^(1/2))^(1/2)/(3*x^3-6*x+2)^(1/2)*((-1/3*6^(1/2)*s
in(1/3*arctan(1/3*15^(1/2)))*3^(1/2)+6^(1/2)*cos(1/3*arctan(1/3*15^(1/2...
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.06

$$\int \frac{A+Bx+Cx^2}{\sqrt{2-6x+3x^3}}\,dx = \frac{2}{9}\sqrt{3}(3\,A+2\,C)\text{weierstrassPInverse}\left(8,-\frac{8}{3},x\right) - \frac{2}{3}\sqrt{3}B\text{weierstrassZeta}\left(8, -\frac{8}{3},\text{weierstrassPInverse}\left(8,-\frac{8}{3},x\right)\right) + \frac{2}{9}\sqrt{3\,x^3-6\,x+2}C$$

input
```
integrate((C*x^2+B*x+A)/(3*x^3-6*x+2)^(1/2),x, algorithm="fricas")
```

output
```
2/9*sqrt(3)*(3*A + 2*C)*weierstrassPInverse(8, -8/3, x) - 2/3*sqrt(3)*B*we
ierstrassZeta(8, -8/3, weierstrassPInverse(8, -8/3, x)) + 2/9*sqrt(3*x^3 -
 6*x + 2)*C
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt{2-6x+3x^3}}\,dx = \int \frac{A+Bx+Cx^2}{\sqrt{3x^3-6x+2}}\,dx$$

input
```
integrate((C*x**2+B*x+A)/(3*x**3-6*x+2)**(1/2),x)
```

output
```
Integral((A + B*x + C*x**2)/sqrt(3*x**3 - 6*x + 2), x)
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt{2-6x+3x^3}}\,dx = \int \frac{Cx^2+Bx+A}{\sqrt{3\,x^3-6\,x+2}}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(3*x^3-6*x+2)^(1/2),x, algorithm="maxima")
```

output
```
integrate((C*x^2 + B*x + A)/sqrt(3*x^3 - 6*x + 2), x)
```

Giac [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt{2-6x+3x^3}}\,dx = \int \frac{Cx^2+Bx+A}{\sqrt{3\,x^3-6\,x+2}}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(3*x^3-6*x+2)^(1/2),x, algorithm="giac")
```

output
```
integrate((C*x^2 + B*x + A)/sqrt(3*x^3 - 6*x + 2), x)
```

Mupad [B] (verification not implemented)

Time = 12.64 (sec) , antiderivative size = 3317, normalized size of antiderivative = 4.43

$$\int \frac{A+Bx+Cx^2}{\sqrt{2-6x+3x^3}}\,dx = \text{Too large to display}$$

input
```
int((A + B*x + C*x^2)/(3*x^3 - 6*x + 2)^(1/2),x)
```

output

```
(C*((2*(x^3 - 2*x + 2/3)^(1/2))/3 + (4*ellipticF(asin(((x + 1/(3*((15^(1/2
)*1i)/9 - 1/3)^(1/3)) + ((15^(1/2)*1i)/9 - 1/3)^(1/3)/2 - 3^(1/2)*(1/(3*((
15^(1/2)*1i)/9 - 1/3)^(1/3)) - ((15^(1/2)*1i)/9 - 1/3)^(1/3)/2)*1i)/(1/((1
5^(1/2)*1i)/9 - 1/3)^(1/3) + (3*((15^(1/2)*1i)/9 - 1/3)^(1/3))/2 - 3^(1/2)
*(1/(3*((15^(1/2)*1i)/9 - 1/3)^(1/3)) - ((15^(1/2)*1i)/9 - 1/3)^(1/3)/2)*1
i))^(1/2)), (3^(1/2)*(1/(3*((15^(1/2)*1i)/9 - 1/3)^(1/3)) + ((15^(1/2)*1i)
/9 - 1/3)^(1/3)/2 - (3^(1/2)*(1/(3*((15^(1/2)*1i)/9 - 1/3)^(1/3)) - ((15^(
1/2)*1i)/9 - 1/3)^(1/3)/2)*1i)/3)*1i)/(2/(3*((15^(1/2)*1i)/9 - 1/3)^(1/3))
 - ((15^(1/2)*1i)/9 - 1/3)^(1/3)))*((x + 1/(3*((15^(1/2)*1i)/9 - 1/3)^(1/3
)) + ((15^(1/2)*1i)/9 - 1/3)^(1/3)/2 - 3^(1/2)*(1/(3*((15^(1/2)*1i)/9 - 1/
3)^(1/3)) - ((15^(1/2)*1i)/9 - 1/3)^(1/3)/2)*1i)/(1/((15^(1/2)*1i)/9 - 1/3
)^(1/3) + (3*((15^(1/2)*1i)/9 - 1/3)^(1/3))/2 - 3^(1/2)*(1/(3*((15^(1/2)*1
i)/9 - 1/3)^(1/3)) - ((15^(1/2)*1i)/9 - 1/3)^(1/3)/2)*1i))^(1/2)*((2/(3*((
15^(1/2)*1i)/9 - 1/3)^(1/3)) - x + ((15^(1/2)*1i)/9 - 1/3)^(1/3))/(1/((15^
(1/2)*1i)/9 - 1/3)^(1/3) + (3*((15^(1/2)*1i)/9 - 1/3)^(1/3))/2 - 3^(1/2)*(
1/(3*((15^(1/2)*1i)/9 - 1/3)^(1/3)) - ((15^(1/2)*1i)/9 - 1/3)^(1/3)/2)*1i)
)^(1/2)*(1/((15^(1/2)*1i)/9 - 1/3)^(1/3) + (3*((15^(1/2)*1i)/9 - 1/3)^(1/3
))/2 - 3^(1/2)*(1/(3*((15^(1/2)*1i)/9 - 1/3)^(1/3)) - ((15^(1/2)*1i)/9 - 1
/3)^(1/3)/2)*1i)*(-(3^(1/2)*(x/3 + 1/(9*((15^(1/2)*1i)/9 - 1/3)^(1/3)) + (
(15^(1/2)*1i)/9 - 1/3)^(1/3)/6 + (3^(1/2)*(1/(3*((15^(1/2)*1i)/9 - 1/3)...
```

Reduce [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt{2-6x+3x^3}}\,dx = \left(\int \frac{\sqrt{3x^3-6x+2}}{3x^3-6x+2}dx\right)a + \left(\int \frac{\sqrt{3x^3-6x+2}\,x^2}{3x^3-6x+2}dx\right)c + \left(\int \frac{\sqrt{3x^3-6x+2}\,x}{3x^3-6x+2}dx\right)b$$

input

```
int((C*x^2+B*x+A)/(3*x^3-6*x+2)^(1/2),x)
```

output

```
int(sqrt(3*x**3 - 6*x + 2)/(3*x**3 - 6*x + 2),x)*a + int((sqrt(3*x**3 - 6*
x + 2)*x**2)/(3*x**3 - 6*x + 2),x)*c + int((sqrt(3*x**3 - 6*x + 2)*x)/(3*x
**3 - 6*x + 2),x)*b
```

3.28 $\int (A+Bx+Cx^2)(2-6x+3x^3)^p\,dx$

Optimal result

Integrand size = 23, antiderivative size = 747

$$\int (A+Bx+Cx^2)(2-6x+3x^3)^p\,dx = \text{Too large to display}$$

output
```
C*(3*x^3-6*x+2)^(p+1)/(9*p+9)+1/3*2^(3/2+2*p)*B*(3*x^3-6*x+2)^p*AppellF1(p
+1,-p,-1-p,2+p,-1/6*(3*x-2*6^(1/2)*cos(1/6*Pi+1/3*arcsin(1/4*6^(1/2))))*6^
(1/2)/(3^(1/2)*cos(1/3*arcsin(1/4*6^(1/2)))-3*sin(1/3*arcsin(1/4*6^(1/2)))
),-1/12*(3*x-2*6^(1/2)*cos(1/6*Pi+1/3*arcsin(1/4*6^(1/2))))*sec(1/3*arcsin
(1/4*6^(1/2)))*2^(1/2))*cos(1/3*arcsin(1/4*6^(1/2)))^(p+1)*(3*x-2*6^(1/2)*
cos(1/6*Pi+1/3*arcsin(1/4*6^(1/2))))*(cos(1/3*arcsin(1/4*6^(1/2)))-3^(1/2)
*sin(1/3*arcsin(1/4*6^(1/2))))^p/(p+1)/((x-2/3*6^(1/2)*sin(1/3*arcsin(1/4*
6^(1/2))))^p)/((x+2/3*6^(1/2)*sin(1/3*Pi+1/3*arcsin(1/4*6^(1/2))))^p)+1/9*
(3*x^3-6*x+2)^p*AppellF1(p+1,-p,-p,2+p,-1/6*(3*x-2*6^(1/2)*cos(1/6*Pi+1/3*
arcsin(1/4*6^(1/2))))*6^(1/2)/(3^(1/2)*cos(1/3*arcsin(1/4*6^(1/2)))-3*sin(
1/3*arcsin(1/4*6^(1/2)))),-1/12*(3*x-2*6^(1/2)*cos(1/6*Pi+1/3*arcsin(1/4*6
^(1/2))))*sec(1/3*arcsin(1/4*6^(1/2)))*2^(1/2))*(3*x-2*6^(1/2)*cos(1/6*Pi+
1/3*arcsin(1/4*6^(1/2))))*(4*cos(1/3*arcsin(1/4*6^(1/2)))*(cos(1/3*arcsin(
1/4*6^(1/2)))-3^(1/2)*sin(1/3*arcsin(1/4*6^(1/2)))))^p*(3*A+2*C-2*6^(1/2)*
B*sin(1/3*Pi+1/3*arcsin(1/4*6^(1/2))))/(p+1)/((x-2/3*6^(1/2)*sin(1/3*arcsi
n(1/4*6^(1/2))))^p)/((x+2/3*6^(1/2)*sin(1/3*Pi+1/3*arcsin(1/4*6^(1/2))))^p
)
```

Mathematica [F]

$$\int \left(A+Bx+Cx^2\right) \left(2-6x+3x^3\right)^p \, dx=\int \left(A+Bx+Cx^2\right) \left(2-6x+3x^3\right)^p \, dx$$

input
```
Integrate[(A + B*x + C*x^2)*(2 - 6*x + 3*x^3)^p,x]
```

output
```
Integrate[(A + B*x + C*x^2)*(2 - 6*x + 3*x^3)^p, x]
```

Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 4.48 (sec) , antiderivative size = 1821, normalized size of antiderivative = 2.44, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2526, 27, 2486, 1269, 1179, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(3x^3-6x+2\right)^p \left(A+Bx+Cx^2\right) \, dx$$

$$\Big\downarrow \text{ 2526}$$

$$\frac{1}{9} \int 3(3A+2C+3Bx) \left(3x^3-6x+2\right)^p dx+\frac{C\left(3x^3-6x+2\right)^{p+1}}{9(p+1)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{3} \int (3A+2C+3Bx) \left(3x^3-6x+2\right)^p dx+\frac{C\left(3x^3-6x+2\right)^{p+1}}{9(p+1)}$$

$$\Big\downarrow \text{ 2486}$$

$$\frac{C\left(3x^3-6x+2\right)^{p+1}}{9(p+1)}+$$

$$\frac{1}{3}\left(3x+\sqrt[3]{9-3i\sqrt{15}}+\frac{2\ 3^{2/3}}{\sqrt[3]{3-i\sqrt{15}}}\right)^{-p}\left(3x^3-6x+2\right)^p\left(9x^2-3\left(\frac{2\ 3^{2/3}}{\sqrt[3]{3-i\sqrt{15}}}+\sqrt[3]{9-3i\sqrt{15}}\right)x+\left(9-3i\right.\right.$$

$$2C+$$

$$3Bx)\left(9x^2-3\left(\frac{2\ 3^{2/3}}{\sqrt[3]{3-i\sqrt{15}}}+\sqrt[3]{9-3i\sqrt{15}}\right)x+\left(9-3i\sqrt{15}\right)^{2/3}+\frac{12\sqrt[3]{3}}{\left(3-i\sqrt{15}\right)^{2/3}}-6\right)^p dx$$

$\downarrow$ 1269

$$\frac{C\left(3x^3-6x+2\right)^{p+1}}{9(p+1)}+$$

$$\frac{1}{3}\left(3x+\sqrt[3]{9-3i\sqrt{15}}+\frac{2\ 3^{2/3}}{\sqrt[3]{3-i\sqrt{15}}}\right)^{-p}\left(3x^3-6x+2\right)^p\left(9x^2-3\left(\frac{2\ 3^{2/3}}{\sqrt[3]{3-i\sqrt{15}}}+\sqrt[3]{9-3i\sqrt{15}}\right)x+\left(9-3i\right.\right.$$

$\downarrow$ 1179

$$\frac{1}{3}\left(3x+\sqrt[3]{9-3i\sqrt{15}}+\frac{2\ 3^{2/3}}{\sqrt[3]{3-i\sqrt{15}}}\right)^{-p}\left(3x^3-6x+2\right)^p\left(\frac{1}{3}\left(3A-\left(\frac{2\ 3^{2/3}}{\sqrt[3]{3-i\sqrt{15}}}+\sqrt[3]{9-3i\sqrt{15}}\right)B+2C\right)\right.$$

$$\frac{C\left(3x^3-6x+2\right)^{p+1}}{9(p+1)}$$

$\downarrow$ 150

$$\frac{1}{3}\left(3x+\sqrt[3]{9-3i\sqrt{15}}+\frac{2\ 3^{2/3}}{\sqrt[3]{3-i\sqrt{15}}}\right)^{-p}\left(3x^3-6x+2\right)^p\left(\frac{\left(3A-\left(\frac{2\ 3^{2/3}}{\sqrt[3]{3-i\sqrt{15}}}+\sqrt[3]{9-3i\sqrt{15}}\right)B+2C\right)}{}\right.$$

$$\frac{C\left(3x^3-6x+2\right)^{p+1}}{9(p+1)}$$

input
```
Int[(A + B*x + C*x^2)*(2 - 6*x + 3*x^3)^p,x]
```

output
```
(C*(2 - 6*x + 3*x^3)^(1 + p))/(9*(1 + p)) + ((2 - 6*x + 3*x^3)^p*(((3*A -
((2*3^(2/3))/(3 - I*Sqrt[15])^(1/3) + (9 - (3*I)*Sqrt[15])^(1/3))*B + 2*C)
*((2*3^(2/3))/(3 - I*Sqrt[15])^(1/3) + (9 - (3*I)*Sqrt[15])^(1/3) + 3*x)^(
1 + p)*(-6 + (12*3^(1/3))/(3 - I*Sqrt[15])^(2/3) + (9 - (3*I)*Sqrt[15])^(2
/3) - 3*((2*3^(2/3))/(3 - I*Sqrt[15])^(1/3) + (9 - (3*I)*Sqrt[15])^(1/3))*
x + 9*x^2)^p*AppellF1[1 + p, -p, -p, 2 + p, (2*(3 - I*Sqrt[15])^(1/3)*((2*
3^(2/3))/(3 - I*Sqrt[15])^(1/3) + (9 - (3*I)*Sqrt[15])^(1/3) + 3*x))/(3*(2
*3^(2/3) + 3^(1/3)*(3 - I*Sqrt[15])^(2/3) - Sqrt[-4*3^(1/3) - (3^(2/3) - I
*3^(1/6)*Sqrt[5])*(3 - I*Sqrt[15])^(1/3) + 4*(3 - I*Sqrt[15])^(2/3)])), (2
*(3 - I*Sqrt[15])^(1/3)*((2*3^(2/3))/(3 - I*Sqrt[15])^(1/3) + (9 - (3*I)*S
qrt[15])^(1/3) + 3*x))/(3*(2*3^(2/3) + 3^(1/3)*(3 - I*Sqrt[15])^(2/3) + Sq
rt[-4*3^(1/3) - (3^(2/3) - I*3^(1/6)*Sqrt[5])*(3 - I*Sqrt[15])^(1/3) + 4*(
3 - I*Sqrt[15])^(2/3)]))])/(3*(1 + p)*(1 - (2*(3 - I*Sqrt[15])^(1/3)*((2*3
^(2/3))/(3 - I*Sqrt[15])^(1/3) + (9 - (3*I)*Sqrt[15])^(1/3) + 3*x))/(3*(2*
3^(2/3) + 3^(1/3)*(3 - I*Sqrt[15])^(2/3) - Sqrt[-4*3^(1/3) - (3^(2/3) - I*
3^(1/6)*Sqrt[5])*(3 - I*Sqrt[15])^(1/3) + 4*(3 - I*Sqrt[15])^(2/3)])))^p*(
1 - (2*(3 - I*Sqrt[15])^(1/3)*((2*3^(2/3))/(3 - I*Sqrt[15])^(1/3) + (9 - (
3*I)*Sqrt[15])^(1/3) + 3*x))/(3*(2*3^(2/3) + 3^(1/3)*(3 - I*Sqrt[15])^(2/3
) + Sqrt[-4*3^(1/3) - (3^(2/3) - I*3^(1/6)*Sqrt[5])*(3 - I*Sqrt[15])^(1/3)
 + 4*(3 - I*Sqrt[15])^(2/3)])))^p) + (B*((2*3^(2/3))/(3 - I*Sqrt[15])^(...
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 1179
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(a + b*x + c*x^2)^p/(e*(1 - (
d + e*x)/(d - e*((b - q)/(2*c))))^p*(1 - (d + e*x)/(d - e*((b + q)/(2*c))))
^p)   Subst[Int[x^m*Simp[1 - x/(d - e*((b - q)/(2*c))), x]^p*Simp[1 - x/(d
- e*((b + q)/(2*c))), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, b, c, d, e, m
, p}, x]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2486
```
Int[((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (d_.)*(x_)^3)^(p_), x_S
ymbol] :> With[{r = Rt[-9*a*d^2 + Sqrt[3]*d*Sqrt[4*b^3*d + 27*a^2*d^2], 3]}
, Simp[(a + b*x + d*x^3)^p/(Simp[18^(1/3)*b*(d/(3*r)) - r/18^(1/3) + d*x, x
]^p*Simp[b*(d/3) + 12^(1/3)*b^2*(d^2/(3*r^2)) + r^2/(3*12^(1/3)) - d*(2^(1/
3)*b*(d/(3^(1/3)*r)) - r/18^(1/3))*x + d^2*x^2, x]^p)   Int[(e + f*x)^m*Sim
p[18^(1/3)*b*(d/(3*r)) - r/18^(1/3) + d*x, x]^p*Simp[b*(d/3) + 12^(1/3)*b^2
*(d^2/(3*r^2)) + r^2/(3*12^(1/3)) - d*(2^(1/3)*b*(d/(3^(1/3)*r)) - r/18^(1/
3))*x + d^2*x^2, x]^p, x], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && NeQ[4*
b^3 + 27*a^2*d, 0] &&  !IntegerQ[p]
```

rule 2526
```
Int[(Pm_)*(Qn_)^(p_), x_Symbol] :> With[{m = Expon[Pm, x], n = Expon[Qn, x]
}, Simp[Coeff[Pm, x, m]*(Qn^(p + 1)/(n*(p + 1)*Coeff[Qn, x, n])), x] + Simp
[1/(n*Coeff[Qn, x, n])   Int[ExpandToSum[n*Coeff[Qn, x, n]*Pm - Coeff[Pm, x
, m]*D[Qn, x], x]*Qn^p, x], x] /; EqQ[m, n - 1]] /; FreeQ[p, x] && PolyQ[Pm
, x] && PolyQ[Qn, x] && NeQ[p, -1]
```

Maple [F]

$$\int (C\,x^2+Bx+A)\,(3x^3-6x+2)^p\,dx$$

input
```
int((C*x^2+B*x+A)*(3*x^3-6*x+2)^p,x)
```

output
```
int((C*x^2+B*x+A)*(3*x^3-6*x+2)^p,x)
```

Fricas [F]

$$\int (A+Bx+Cx^2)\,(2-6x+3x^3)^p\,dx=\int (Cx^2+Bx+A)(3\,x^3-6\,x+2)^p\,dx$$

input
```
integrate((C*x^2+B*x+A)*(3*x^3-6*x+2)^p,x, algorithm="fricas")
```

output
```
integral((C*x^2 + B*x + A)*(3*x^3 - 6*x + 2)^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int (A+Bx+Cx^2)\,(2-6x+3x^3)^p\,dx=\text{Timed out}$$

input
```
integrate((C*x**2+B*x+A)*(3*x**3-6*x+2)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (A+Bx+Cx^2) (2-6x+3x^3)^p \, dx = \int (Cx^2+Bx+A)(3x^3-6x+2)^p \, dx$$

input
```
integrate((C*x^2+B*x+A)*(3*x^3-6*x+2)^p,x, algorithm="maxima")
```

output
```
integrate((C*x^2 + B*x + A)*(3*x^3 - 6*x + 2)^p, x)
```

Giac [F]

$$\int (A+Bx+Cx^2) (2-6x+3x^3)^p \, dx = \int (Cx^2+Bx+A)(3x^3-6x+2)^p \, dx$$

input
```
integrate((C*x^2+B*x+A)*(3*x^3-6*x+2)^p,x, algorithm="giac")
```

output
```
integrate((C*x^2 + B*x + A)*(3*x^3 - 6*x + 2)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (A+Bx+Cx^2) (2-6x+3x^3)^p \, dx = \int (C x^2+B x+A) (3x^3-6x+2)^p \, dx$$

input
```
int((A + B*x + C*x^2)*(3*x^3 - 6*x + 2)^p,x)
```

output
```
int((A + B*x + C*x^2)*(3*x^3 - 6*x + 2)^p, x)
```

Reduce [F]

$$\int (A+Bx+Cx^2) \left(2-6x+3x^3\right)^p \, dx = \text{too large to display}$$

input
```
int((C*x^2+B*x+A)*(3*x^3-6*x+2)^p,x)
```

output
```
(27*(3*x**3 - 6*x + 2)**p*a*p**2*x + 45*(3*x**3 - 6*x + 2)**p*a*p*x + 18*(
3*x**3 - 6*x + 2)**p*a*x + 27*(3*x**3 - 6*x + 2)**p*b*p**2*x**2 - 36*(3*x*
*3 - 6*x + 2)**p*b*p**2 + 36*(3*x**3 - 6*x + 2)**p*b*p*x**2 - 48*(3*x**3 -
 6*x + 2)**p*b*p + 9*(3*x**3 - 6*x + 2)**p*b*x**2 - 12*(3*x**3 - 6*x + 2)*
*p*b + 27*(3*x**3 - 6*x + 2)**p*c*p**2*x**3 - 36*(3*x**3 - 6*x + 2)**p*c*p
**2*x + 18*(3*x**3 - 6*x + 2)**p*c*p**2 + 27*(3*x**3 - 6*x + 2)**p*c*p*x**
3 - 24*(3*x**3 - 6*x + 2)**p*c*p*x + 18*(3*x**3 - 6*x + 2)**p*c*p + 6*(3*x
**3 - 6*x + 2)**p*c*x**3 + 4*(3*x**3 - 6*x + 2)**p*c + 1458*int((3*x**3 -
6*x + 2)**p/(27*p**2*x**3 - 54*p**2*x + 18*p**2 + 27*p*x**3 - 54*p*x + 18*
p + 6*x**3 - 12*x + 4),x)*a*p**5 + 3888*int((3*x**3 - 6*x + 2)**p/(27*p**2
*x**3 - 54*p**2*x + 18*p**2 + 27*p*x**3 - 54*p*x + 18*p + 6*x**3 - 12*x +
4),x)*a*p**4 + 3726*int((3*x**3 - 6*x + 2)**p/(27*p**2*x**3 - 54*p**2*x +
18*p**2 + 27*p*x**3 - 54*p*x + 18*p + 6*x**3 - 12*x + 4),x)*a*p**3 + 1512*
int((3*x**3 - 6*x + 2)**p/(27*p**2*x**3 - 54*p**2*x + 18*p**2 + 27*p*x**3
- 54*p*x + 18*p + 6*x**3 - 12*x + 4),x)*a*p**2 + 216*int((3*x**3 - 6*x + 2
)**p/(27*p**2*x**3 - 54*p**2*x + 18*p**2 + 27*p*x**3 - 54*p*x + 18*p + 6*x
**3 - 12*x + 4),x)*a*p - 1944*int((3*x**3 - 6*x + 2)**p/(27*p**2*x**3 - 54
*p**2*x + 18*p**2 + 27*p*x**3 - 54*p*x + 18*p + 6*x**3 - 12*x + 4),x)*b*p*
*5 - 4536*int((3*x**3 - 6*x + 2)**p/(27*p**2*x**3 - 54*p**2*x + 18*p**2 +
27*p*x**3 - 54*p*x + 18*p + 6*x**3 - 12*x + 4),x)*b*p**4 - 3672*int((3*...
```

3.29 $\int (A+Bx+Cx^2) \left(4c^3-27cd^2x^2-27d^3x^3\right)^4 \, dx$

Optimal result

Integrand size = 36, antiderivative size = 230

$$\begin{aligned}
&\int (A+Bx+Cx^2) \left(4c^3-27cd^2x^2-27d^3x^3\right)^4 \, dx \\
&= \frac{c^4(4c^2C-6Bcd+9Ad^2)(2c+3dx)^9}{3d^3} - \frac{c^3(28c^2C-33Bcd+36Ad^2)(2c+3dx)^{10}}{10d^3} \\
&+ \frac{3c^2(9c^2C-8Bcd+6Ad^2)(2c+3dx)^{11}}{11d^3} \\
&- \frac{c(62c^2C-39Bcd+18Ad^2)(2c+3dx)^{12}}{54d^3} \\
&+ \frac{(106c^2C-42Bcd+9Ad^2)(2c+3dx)^{13}}{351d^3} \\
&- \frac{(16cC-3Bd)(2c+3dx)^{14}}{378d^3} + \frac{C(2c+3dx)^{15}}{405d^3}
\end{aligned}$$

output

```
1/3*c^4*(9*A*d^2-6*B*c*d+4*C*c^2)*(3*d*x+2*c)^9/d^3-1/10*c^3*(36*A*d^2-33*
B*c*d+28*C*c^2)*(3*d*x+2*c)^10/d^3+3/11*c^2*(6*A*d^2-8*B*c*d+9*C*c^2)*(3*d
*x+2*c)^11/d^3-1/54*c*(18*A*d^2-39*B*c*d+62*C*c^2)*(3*d*x+2*c)^12/d^3+1/35
1*(9*A*d^2-42*B*c*d+106*C*c^2)*(3*d*x+2*c)^13/d^3-1/378*(-3*B*d+16*C*c)*(3
*d*x+2*c)^14/d^3+1/405*C*(3*d*x+2*c)^15/d^3
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.53

$$\int (A+Bx+Cx^2)\left(4c^3-27cd^2x^2-27d^3x^3\right)^4\,dx$$
$$=256Ac^{12}x+128Bc^{12}x^2+\frac{256}{3}c^{10}\left(c^2C-27Ad^2\right)x^3-1728c^9d^2(Bc+Ad)x^4$$
$$-\frac{864}{5}c^8d^2\left(8c^2C+8Bcd-81Ad^2\right)x^5-144c^7d^3\left(8c^2C-81Bcd-162Ad^2\right)x^6$$
$$+\frac{34992}{7}c^6d^4\left(2c^2C+4Bcd-7Ad^2\right)x^7+4374c^5d^5\left(4c^2C-7Bcd-27Ad^2\right)x^8$$
$$-243c^4d^6\left(112c^2C+432Bcd+189Ad^2\right)x^9-\frac{19683}{10}c^3d^7\left(48c^2C+21Bcd-92Ad^2\right)x^{10}$$
$$-\frac{19683}{11}c^2d^8\left(21c^2C-92Bcd-162Ad^2\right)x^{11}+\frac{6561}{2}cd^9\left(46c^2C+81Bcd+54Ad^2\right)x^{12}$$
$$+\frac{531441}{13}d^{10}\left(6c^2C+4Bcd+Ad^2\right)x^{13}+\frac{531441}{14}d^{11}(4cC+Bd)x^{14}+\frac{177147}{5}Cd^{12}x^{15}$$

input
```
Integrate[(A + B*x + C*x^2)*(4*c^3 - 27*c*d^2*x^2 - 27*d^3*x^3)^4,x]
```

output
```
256*A*c^12*x + 128*B*c^12*x^2 + (256*c^10*(c^2*C - 27*A*d^2)*x^3)/3 - 1728
*c^9*d^2*(B*c + A*d)*x^4 - (864*c^8*d^2*(8*c^2*C + 8*B*c*d - 81*A*d^2)*x^5
)/5 - 144*c^7*d^3*(8*c^2*C - 81*B*c*d - 162*A*d^2)*x^6 + (34992*c^6*d^4*(2
*c^2*C + 4*B*c*d - 7*A*d^2)*x^7)/7 + 4374*c^5*d^5*(4*c^2*C - 7*B*c*d - 27*
A*d^2)*x^8 - 243*c^4*d^6*(112*c^2*C + 432*B*c*d + 189*A*d^2)*x^9 - (19683*
c^3*d^7*(48*c^2*C + 21*B*c*d - 92*A*d^2)*x^10)/10 - (19683*c^2*d^8*(21*c^2
*C - 92*B*c*d - 162*A*d^2)*x^11)/11 + (6561*c*d^9*(46*c^2*C + 81*B*c*d + 5
4*A*d^2)*x^12)/2 + (531441*d^10*(6*c^2*C + 4*B*c*d + A*d^2)*x^13)/13 + (53
1441*d^11*(4*c*C + B*d)*x^14)/14 + (177147*C*d^12*x^15)/5
```

Rubi [A] (verified)

Time = 1.35 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.53, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(4c^3-27cd^2x^2-27d^3x^3\right)^4\left(A+Bx+Cx^2\right)\,dx$$

$$\Big\downarrow \text{2188}$$

$$\int \left(-6912c^9d^2x^3(Ad+Bc)+531441d^{10}x^{12}\left(Ad^2+4Bcd+6c^2C\right)+39366cd^9x^{11}\left(54Ad^2+81Bcd+46c^2C\right)-1\right.$$

$$\Big\downarrow \text{2009}$$

$$-1728c^9d^2x^4(Ad+Bc)+\frac{531441}{13}d^{10}x^{13}\left(Ad^2+4Bcd+6c^2C\right)+\frac{6561}{2}cd^9x^{12}\left(54Ad^2+81Bcd+46c^2C\right)-\frac{19683}{11}c^2d^8x^{11}\left(-162Ad^2-92Bcd+21c^2C\right)-\frac{864}{5}c^8d^2x^5\left(-81Ad^2+8Bcd+8c^2C\right)-144c^7d^3x^6\left(-162Ad^2-81Bcd+8c^2C\right)+\frac{34992}{7}c^6d^4x^7\left(-7Ad^2+4Bcd+2c^2C\right)+4374c^5d^5x^8\left(-27Ad^2-7Bcd+4c^2C\right)-243c^4d^6x^9\left(189Ad^2+432Bcd+112c^2C\right)-\frac{19683}{10}c^3d^7x^{10}\left(-92Ad^2+21Bcd+48c^2C\right)+256Ac^{12}x+\frac{256}{3}c^{10}x^3\left(c^2C-27Ad^2\right)+128Bc^{12}x^2+\frac{531441}{14}d^{11}x^{14}(Bd+4cC)+\frac{177147}{5}Cd^{12}x^{15}$$

input

```
Int[(A + B*x + C*x^2)*(4*c^3 - 27*c*d^2*x^2 - 27*d^3*x^3)^4,x]
```

output

```
256*A*c^12*x + 128*B*c^12*x^2 + (256*c^10*(c^2*C - 27*A*d^2)*x^3)/3 - 1728
*c^9*d^2*(B*c + A*d)*x^4 - (864*c^8*d^2*(8*c^2*C + 8*B*c*d - 81*A*d^2)*x^5
)/5 - 144*c^7*d^3*(8*c^2*C - 81*B*c*d - 162*A*d^2)*x^6 + (34992*c^6*d^4*(2
*c^2*C + 4*B*c*d - 7*A*d^2)*x^7)/7 + 4374*c^5*d^5*(4*c^2*C - 7*B*c*d - 27*
A*d^2)*x^8 - 243*c^4*d^6*(112*c^2*C + 432*B*c*d + 189*A*d^2)*x^9 - (19683*
c^3*d^7*(48*c^2*C + 21*B*c*d - 92*A*d^2)*x^10)/10 - (19683*c^2*d^8*(21*c^2
*C - 92*B*c*d - 162*A*d^2)*x^11)/11 + (6561*c*d^9*(46*c^2*C + 81*B*c*d + 5
4*A*d^2)*x^12)/2 + (531441*d^10*(6*c^2*C + 4*B*c*d + A*d^2)*x^13)/13 + (53
1441*d^11*(4*c*C + B*d)*x^14)/14 + (177147*C*d^12*x^15)/5
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 369, normalized size of antiderivative = 1.60

method	result
norman	$256c^{12}Ax+128B\,c^{12}x^2+\left(-2304c^{10}A\,d^2+\frac{256}{3}C\,c^{12}\right)x^3+\left(-1728c^9A\,d^3-1728B\,c^{10}d^2\right)x$
default	$\frac{177147C\,d^{12}x^{15}}{5}+\frac{(531441B\,d^{12}+2125764C\,d^{11}c)x^{14}}{14}+\frac{(531441A\,d^{12}+2125764Bc\,d^{11}+3188646C\,d^{10}c^2)x^{13}}{13}+\frac{(2125}{}$
gosper	$x(1063944882C\,d^{12}x^{14}+1139940945x^{13}B\,d^{12}+4559763780x^{13}C\,d^{11}c+1227628710x^{12}A\,d^{12}+4910514840x^{12}Bc\,d^{11}+73657$
risch	$\frac{1062882}{7}x^{14}C\,d^{11}c+\frac{2125764}{13}x^{13}Bc\,d^{11}+\frac{3188646}{13}x^{13}C\,d^{10}c^2+177147x^{12}Ac\,d^{11}+\frac{531441}{2}x^{12}B\,c^2$
parallelrisch	$\frac{1062882}{7}x^{14}C\,d^{11}c+\frac{2125764}{13}x^{13}Bc\,d^{11}+\frac{3188646}{13}x^{13}C\,d^{10}c^2+177147x^{12}Ac\,d^{11}+\frac{531441}{2}x^{12}B\,c^2$
orering	$x(1063944882C\,d^{12}x^{14}+1139940945x^{13}B\,d^{12}+4559763780x^{13}C\,d^{11}c+1227628710x^{12}A\,d^{12}+4910514840x^{12}Bc\,d^{11}+73657$

input
```
int((C*x^2+B*x+A)*(-27*d^3*x^3-27*c*d^2*x^2+4*c^3)^4,x,method=_RETURNVERBO
SE)
```

output
```
256*c^12*A*x+128*B*c^12*x^2+(-2304*c^10*A*d^2+256/3*C*c^12)*x^3+(-1728*A*c
^9*d^3-1728*B*c^10*d^2)*x^4+(69984/5*c^8*A*d^4-6912/5*B*c^9*d^3-6912/5*C*c
^10*d^2)*x^5+(23328*A*c^7*d^5+11664*B*c^8*d^4-1152*C*c^9*d^3)*x^6+(-34992*
c^6*A*d^6+139968/7*B*c^7*d^5+69984/7*C*d^4*c^8)*x^7+(-118098*A*c^5*d^7-306
18*B*c^6*d^6+17496*C*c^7*d^5)*x^8+(-45927*A*c^4*d^8-104976*B*c^5*d^7-27216
*C*c^6*d^6)*x^9+(905418/5*A*c^3*d^9-413343/10*B*c^4*d^8-472392/5*C*d^7*c^5
)*x^10+(3188646/11*A*c^2*d^10+1810836/11*B*c^3*d^9-413343/11*C*d^8*c^4)*x^
11+(177147*A*c*d^11+531441/2*B*c^2*d^10+150903*C*d^9*c^3)*x^12+(531441/13*
A*d^12+2125764/13*B*c*d^11+3188646/13*C*d^10*c^2)*x^13+(531441/14*B*d^12+1
062882/7*C*d^11*c)*x^14+177147/5*C*d^12*x^15
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 375, normalized size of antiderivative = 1.63

$$\begin{aligned}
&\int \left(A+Bx+Cx^2\right)\left(4c^3-27cd^2x^2-27d^3x^3\right)^4\,dx \\
&= \frac{177147}{5}Cd^{12}x^{15}+128\,Bc^{12}x^2+\frac{531441}{14}\left(4\,Ccd^{11}+Bd^{12}\right)x^{14} \\
&\quad+256\,Ac^{12}x+\frac{531441}{13}\left(6\,Cc^2d^{10}+4\,Bcd^{11}+Ad^{12}\right)x^{13} \\
&\quad+\frac{6561}{2}\left(46\,Cc^3d^9+81\,Bc^2d^{10}+54\,Acd^{11}\right)x^{12} \\
&\quad-\frac{19683}{11}\left(21\,Cc^4d^8-92\,Bc^3d^9-162\,Ac^2d^{10}\right)x^{11} \\
&\quad-\frac{19683}{10}\left(48\,Cc^5d^7+21\,Bc^4d^8-92\,Ac^3d^9\right)x^{10} \\
&\quad-243\left(112\,Cc^6d^6+432\,Bc^5d^7+189\,Ac^4d^8\right)x^9 \\
&\quad+4374\left(4\,Cc^7d^5-7\,Bc^6d^6-27\,Ac^5d^7\right)x^8+\frac{34992}{7}\left(2\,Cc^8d^4+4\,Bc^7d^5-7\,Ac^6d^6\right)x^7 \\
&\quad-144\left(8\,Cc^9d^3-81\,Bc^8d^4-162\,Ac^7d^5\right)x^6-\frac{864}{5}\left(8\,Cc^{10}d^2+8\,Bc^9d^3-81\,Ac^8d^4\right)x^5 \\
&\quad-1728\left(Bc^{10}d^2+Ac^9d^3\right)x^4+\frac{256}{3}\left(Cc^{12}-27\,Ac^{10}d^2\right)x^3
\end{aligned}$$

input
```
integrate((C*x^2+B*x+A)*(-27*d^3*x^3-27*c*d^2*x^2+4*c^3)^4,x, algorithm="f
ricas")
```

output
```
177147/5*C*d^12*x^15 + 128*B*c^12*x^2 + 531441/14*(4*C*c*d^11 + B*d^12)*x^
14 + 256*A*c^12*x + 531441/13*(6*C*c^2*d^10 + 4*B*c*d^11 + A*d^12)*x^13 +
6561/2*(46*C*c^3*d^9 + 81*B*c^2*d^10 + 54*A*c*d^11)*x^12 - 19683/11*(21*C*
c^4*d^8 - 92*B*c^3*d^9 - 162*A*c^2*d^10)*x^11 - 19683/10*(48*C*c^5*d^7 + 2
1*B*c^4*d^8 - 92*A*c^3*d^9)*x^10 - 243*(112*C*c^6*d^6 + 432*B*c^5*d^7 + 18
9*A*c^4*d^8)*x^9 + 4374*(4*C*c^7*d^5 - 7*B*c^6*d^6 - 27*A*c^5*d^7)*x^8 + 3
4992/7*(2*C*c^8*d^4 + 4*B*c^7*d^5 - 7*A*c^6*d^6)*x^7 - 144*(8*C*c^9*d^3 -
81*B*c^8*d^4 - 162*A*c^7*d^5)*x^6 - 864/5*(8*C*c^10*d^2 + 8*B*c^9*d^3 - 81
*A*c^8*d^4)*x^5 - 1728*(B*c^10*d^2 + A*c^9*d^3)*x^4 + 256/3*(C*c^12 - 27*A
*c^10*d^2)*x^3
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 425, normalized size of antiderivative = 1.85

$$\int (A+Bx+Cx^2)\,(4c^3-27cd^2x^2-27d^3x^3)^4\,dx$$
$$= 256Ac^{12}x+128Bc^{12}x^2+\frac{177147Cd^{12}x^{15}}{5}+x^{14}\cdot\left(\frac{531441Bd^{12}}{14}+\frac{1062882Ccd^{11}}{7}\right)$$
$$+x^{13}\cdot\left(\frac{531441Ad^{12}}{13}+\frac{2125764Bcd^{11}}{13}+\frac{3188646Cc^2d^{10}}{13}\right)$$
$$+x^{12}\cdot\left(177147Acd^{11}+\frac{531441Bc^2d^{10}}{2}+150903Cc^3d^9\right)$$
$$+x^{11}\cdot\left(\frac{3188646Ac^2d^{10}}{11}+\frac{1810836Bc^3d^9}{11}-\frac{413343Cc^4d^8}{11}\right)$$
$$+x^{10}\cdot\left(\frac{905418Ac^3d^9}{5}-\frac{413343Bc^4d^8}{10}-\frac{472392Cc^5d^7}{5}\right)$$
$$+x^9(-45927Ac^4d^8-104976Bc^5d^7-27216Cc^6d^6)$$
$$+x^8(-118098Ac^5d^7-30618Bc^6d^6+17496Cc^7d^5)$$
$$+x^7\left(-34992Ac^6d^6+\frac{139968Bc^7d^5}{7}+\frac{69984Cc^8d^4}{7}\right)$$
$$+x^6\cdot(23328Ac^7d^5+11664Bc^8d^4-1152Cc^9d^3)$$
$$+x^5\cdot\left(\frac{69984Ac^8d^4}{5}-\frac{6912Bc^9d^3}{5}-\frac{6912Cc^{10}d^2}{5}\right)$$
$$+x^4(-1728Ac^9d^3-1728Bc^{10}d^2)+x^3\left(-2304Ac^{10}d^2+\frac{256Cc^{12}}{3}\right)$$

input
```
integrate((C*x**2+B*x+A)*(-27*d**3*x**3-27*c*d**2*x**2+4*c**3)**4,x)
```

output
```
256*A*c**12*x + 128*B*c**12*x**2 + 177147*C*d**12*x**15/5 + x**14*(531441*
B*d**12/14 + 1062882*C*c*d**11/7) + x**13*(531441*A*d**12/13 + 2125764*B*c
*d**11/13 + 3188646*C*c**2*d**10/13) + x**12*(177147*A*c*d**11 + 531441*B*
c**2*d**10/2 + 150903*C*c**3*d**9) + x**11*(3188646*A*c**2*d**10/11 + 1810
836*B*c**3*d**9/11 - 413343*C*c**4*d**8/11) + x**10*(905418*A*c**3*d**9/5
- 413343*B*c**4*d**8/10 - 472392*C*c**5*d**7/5) + x**9*(-45927*A*c**4*d**8
 - 104976*B*c**5*d**7 - 27216*C*c**6*d**6) + x**8*(-118098*A*c**5*d**7 - 3
0618*B*c**6*d**6 + 17496*C*c**7*d**5) + x**7*(-34992*A*c**6*d**6 + 139968*
B*c**7*d**5/7 + 69984*C*c**8*d**4/7) + x**6*(23328*A*c**7*d**5 + 11664*B*c
**8*d**4 - 1152*C*c**9*d**3) + x**5*(69984*A*c**8*d**4/5 - 6912*B*c**9*d**
3/5 - 6912*C*c**10*d**2/5) + x**4*(-1728*A*c**9*d**3 - 1728*B*c**10*d**2)
+ x**3*(-2304*A*c**10*d**2 + 256*C*c**12/3)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 375, normalized size of antiderivative = 1.63

$$\int \left(A+Bx+Cx^2\right)\left(4c^3-27cd^2x^2-27d^3x^3\right)^4\,dx$$
$$\begin{aligned}
&= \frac{177147}{5}\,Cd^{12}x^{15}+128\,Bc^{12}x^2+\frac{531441}{14}\,\left(4\,Ccd^{11}+Bd^{12}\right)x^{14}\\
&\quad+256\,Ac^{12}x+\frac{531441}{13}\,\left(6\,Cc^2d^{10}+4\,Bcd^{11}+Ad^{12}\right)x^{13}\\
&\quad+\frac{6561}{2}\,\left(46\,Cc^3d^9+81\,Bc^2d^{10}+54\,Acd^{11}\right)x^{12}\\
&\quad-\frac{19683}{11}\,\left(21\,Cc^4d^8-92\,Bc^3d^9-162\,Ac^2d^{10}\right)x^{11}\\
&\quad-\frac{19683}{10}\,\left(48\,Cc^5d^7+21\,Bc^4d^8-92\,Ac^3d^9\right)x^{10}\\
&\quad-243\,\left(112\,Cc^6d^6+432\,Bc^5d^7+189\,Ac^4d^8\right)x^9\\
&\quad+4374\,\left(4\,Cc^7d^5-7\,Bc^6d^6-27\,Ac^5d^7\right)x^8+\frac{34992}{7}\,\left(2\,Cc^8d^4+4\,Bc^7d^5-7\,Ac^6d^6\right)x^7\\
&\quad-144\,\left(8\,Cc^9d^3-81\,Bc^8d^4-162\,Ac^7d^5\right)x^6-\frac{864}{5}\,\left(8\,Cc^{10}d^2+8\,Bc^9d^3-81\,Ac^8d^4\right)x^5\\
&\quad-1728\,\left(Bc^{10}d^2+Ac^9d^3\right)x^4+\frac{256}{3}\,\left(Cc^{12}-27\,Ac^{10}d^2\right)x^3
\end{aligned}$$

input
```
integrate((C*x^2+B*x+A)*(-27*d^3*x^3-27*c*d^2*x^2+4*c^3)^4,x, algorithm="m
axima")
```

output
```
177147/5*C*d^12*x^15 + 128*B*c^12*x^2 + 531441/14*(4*C*c*d^11 + B*d^12)*x^
14 + 256*A*c^12*x + 531441/13*(6*C*c^2*d^10 + 4*B*c*d^11 + A*d^12)*x^13 +
6561/2*(46*C*c^3*d^9 + 81*B*c^2*d^10 + 54*A*c*d^11)*x^12 - 19683/11*(21*C*
c^4*d^8 - 92*B*c^3*d^9 - 162*A*c^2*d^10)*x^11 - 19683/10*(48*C*c^5*d^7 + 2
1*B*c^4*d^8 - 92*A*c^3*d^9)*x^10 - 243*(112*C*c^6*d^6 + 432*B*c^5*d^7 + 18
9*A*c^4*d^8)*x^9 + 4374*(4*C*c^7*d^5 - 7*B*c^6*d^6 - 27*A*c^5*d^7)*x^8 + 3
4992/7*(2*C*c^8*d^4 + 4*B*c^7*d^5 - 7*A*c^6*d^6)*x^7 - 144*(8*C*c^9*d^3 -
81*B*c^8*d^4 - 162*A*c^7*d^5)*x^6 - 864/5*(8*C*c^10*d^2 + 8*B*c^9*d^3 - 81
*A*c^8*d^4)*x^5 - 1728*(B*c^10*d^2 + A*c^9*d^3)*x^4 + 256/3*(C*c^12 - 27*A
*c^10*d^2)*x^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 407, normalized size of antiderivative = 1.77

$$\int (A+Bx+Cx^2)\left(4c^3-27cd^2x^2-27d^3x^3\right)^4\,dx$$
$$= \frac{177147}{5}Cd^{12}x^{15} + \frac{1062882}{7}Ccd^{11}x^{14} + \frac{531441}{14}Bd^{12}x^{14} + \frac{3188646}{13}Cc^2d^{10}x^{13} + \frac{2125764}{13}Bcd^{11}x^{13} + \frac{531441}{13}Ad^{12}x^{13} + 150903\,Cc^3d^9x^{12} + \frac{531441}{2}Bc^2d^{10}x^{12} + 177147\,Acd^{11}x^{12} - \frac{413343}{11}Cc^4d^8x^{11} + \frac{1810836}{11}Bc^3d^9x^{11} + \frac{3188646}{11}Ac^2d^{10}x^{11} - \frac{472392}{5}Cc^5d^7x^{10} - \frac{413343}{10}Bc^4d^8x^{10} + \frac{905418}{5}Ac^3d^9x^{10} - 27216\,Cc^6d^6x^9 - 104976\,Bc^5d^7x^9 - 45927\,Ac^4d^8x^9 + 17496\,Cc^7d^5x^8 - 30618\,Bc^6d^6x^8 - 118098\,Ac^5d^7x^8 + \frac{69984}{7}Cc^8d^4x^7 + \frac{139968}{7}Bc^7d^5x^7 - 34992\,Ac^6d^6x^7 - 1152\,Cc^9d^3x^6 + 11664\,Bc^8d^4x^6 + 23328\,Ac^7d^5x^6 - \frac{6912}{5}Cc^{10}d^2x^5 - \frac{6912}{5}Bc^9d^3x^5 + \frac{69984}{5}Ac^8d^4x^5 - 1728\,Bc^{10}d^2x^4 - 1728\,Ac^9d^3x^4 + \frac{256}{3}Cc^{12}x^3 - 2304\,Ac^{10}d^2x^3 + 128\,Bc^{12}x^2 + 256\,Ac^{12}x$$

input
```
integrate((C*x^2+B*x+A)*(-27*d^3*x^3-27*c*d^2*x^2+4*c^3)^4,x, algorithm="g
iac")
```

output
```
177147/5*C*d^12*x^15 + 1062882/7*C*c*d^11*x^14 + 531441/14*B*d^12*x^14 + 3
188646/13*C*c^2*d^10*x^13 + 2125764/13*B*c*d^11*x^13 + 531441/13*A*d^12*x^
13 + 150903*C*c^3*d^9*x^12 + 531441/2*B*c^2*d^10*x^12 + 177147*A*c*d^11*x^
12 - 413343/11*C*c^4*d^8*x^11 + 1810836/11*B*c^3*d^9*x^11 + 3188646/11*A*c
^2*d^10*x^11 - 472392/5*C*c^5*d^7*x^10 - 413343/10*B*c^4*d^8*x^10 + 905418
/5*A*c^3*d^9*x^10 - 27216*C*c^6*d^6*x^9 - 104976*B*c^5*d^7*x^9 - 45927*A*c
^4*d^8*x^9 + 17496*C*c^7*d^5*x^8 - 30618*B*c^6*d^6*x^8 - 118098*A*c^5*d^7*
x^8 + 69984/7*C*c^8*d^4*x^7 + 139968/7*B*c^7*d^5*x^7 - 34992*A*c^6*d^6*x^7
 - 1152*C*c^9*d^3*x^6 + 11664*B*c^8*d^4*x^6 + 23328*A*c^7*d^5*x^6 - 6912/5
*C*c^10*d^2*x^5 - 6912/5*B*c^9*d^3*x^5 + 69984/5*A*c^8*d^4*x^5 - 1728*B*c^
10*d^2*x^4 - 1728*A*c^9*d^3*x^4 + 256/3*C*c^12*x^3 - 2304*A*c^10*d^2*x^3 +
 128*B*c^12*x^2 + 256*A*c^12*x
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 335, normalized size of antiderivative = 1.46

$$\int (A+Bx+Cx^2)\left(4c^3-27cd^2x^2-27d^3x^3\right)^4\,dx$$
$$= x^3\left(\frac{256\,C\,c^{12}}{3}-2304\,A\,c^{10}\,d^2\right)+128\,B\,c^{12}\,x^2+\frac{177147\,C\,d^{12}\,x^{15}}{5}+\frac{531441\,d^{11}\,x^{14}\,(B\,d+4\,C\,c)}{14}+\frac{531441\,d^{10}\,x^{13}\,(6\,C\,c^2+4\,B\,c\,d+A\,d^2)}{13}+256\,A\,c^{12}\,x-1728\,c^9\,d^2\,x^4\,(A\,d+B\,c)+\frac{34992\,c^6\,d^4\,x^7\,(2\,C\,c^2+4\,B\,c\,d-7\,A\,d^2)}{7}-4374\,c^5\,d^5\,x^8\left(-4\,C\,c^2+7\,B\,c\,d+27\,A\,d^2\right)-\frac{864\,c^8\,d^2\,x^5\,(8\,C\,c^2+8\,B\,c\,d-81\,A\,d^2)}{5}-\frac{19683\,c^3\,d^7\,x^{10}\,(48\,C\,c^2+21\,B\,c\,d-92\,A\,d^2)}{10}+144\,c^7\,d^3\,x^6\left(-8\,C\,c^2+81\,B\,c\,d+162\,A\,d^2\right)+\frac{19683\,c^2\,d^8\,x^{11}\,(-21\,C\,c^2+92\,B\,c\,d+162\,A\,d^2)}{11}-243\,c^4\,d^6\,x^9\left(112\,C\,c^2+432\,B\,c\,d+189\,A\,d^2\right)+\frac{6561\,c\,d^9\,x^{12}\,(46\,C\,c^2+81\,B\,c\,d+54\,A\,d^2)}{2}$$

input
```
int((A + B*x + C*x^2)*(27*d^3*x^3 - 4*c^3 + 27*c*d^2*x^2)^4,x)
```

output
```
x^3*((256*C*c^12)/3 - 2304*A*c^10*d^2) + 128*B*c^12*x^2 + (177147*C*d^12*x
^15)/5 + (531441*d^11*x^14*(B*d + 4*C*c))/14 + (531441*d^10*x^13*(A*d^2 +
6*C*c^2 + 4*B*c*d))/13 + 256*A*c^12*x - 1728*c^9*d^2*x^4*(A*d + B*c) + (34
992*c^6*d^4*x^7*(2*C*c^2 - 7*A*d^2 + 4*B*c*d))/7 - 4374*c^5*d^5*x^8*(27*A*
d^2 - 4*C*c^2 + 7*B*c*d) - (864*c^8*d^2*x^5*(8*C*c^2 - 81*A*d^2 + 8*B*c*d)
)/5 - (19683*c^3*d^7*x^10*(48*C*c^2 - 92*A*d^2 + 21*B*c*d))/10 + 144*c^7*d
^3*x^6*(162*A*d^2 - 8*C*c^2 + 81*B*c*d) + (19683*c^2*d^8*x^11*(162*A*d^2 -
 21*C*c^2 + 92*B*c*d))/11 - 243*c^4*d^6*x^9*(189*A*d^2 + 112*C*c^2 + 432*B
*c*d) + (6561*c*d^9*x^12*(54*A*d^2 + 46*C*c^2 + 81*B*c*d))/2
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 398, normalized size of antiderivative = 1.73

$$\int (A + Bx + Cx^2) \left(4c^3 - 27cd^2x^2 - 27d^3x^3\right)^4 \, dx$$
$$= \frac{x(1063944882c\,d^{12}x^{14} + 1139940945b\,d^{12}x^{13} + 4559763780c^2d^{11}x^{13} + 1227628710a\,d^{12}x^{12} + 491051484}{}$$

input
```
int((C*x^2+B*x+A)*(-27*d^3*x^3-27*c*d^2*x^2+4*c^3)^4,x)
```

output
```
(x*(7687680*a*c**12 - 69189120*a*c**10*d**2*x**2 - 51891840*a*c**9*d**3*x*
*3 + 420323904*a*c**8*d**4*x**4 + 700539840*a*c**7*d**5*x**5 - 1050809760*
a*c**6*d**6*x**6 - 3546482940*a*c**5*d**7*x**7 - 1379187810*a*c**4*d**8*x*
*8 + 5437940508*a*c**3*d**9*x**9 + 8705003580*a*c**2*d**10*x**10 + 5319724
410*a*c*d**11*x**11 + 1227628710*a*d**12*x**12 + 3843840*b*c**12*x - 51891
840*b*c**10*d**2*x**3 - 41513472*b*c**9*d**3*x**4 + 350269920*b*c**8*d**4*
x**5 + 600462720*b*c**7*d**5*x**6 - 919458540*b*c**6*d**6*x**7 - 315242928
0*b*c**5*d**7*x**8 - 1241269029*b*c**4*d**8*x**9 + 4943582280*b*c**3*d**9*
x**10 + 7979586615*b*c**2*d**10*x**11 + 4910514840*b*c*d**11*x**12 + 11399
40945*b*d**12*x**13 + 2562560*c**13*x**2 - 41513472*c**11*d**2*x**4 - 3459
4560*c**10*d**3*x**5 + 300231360*c**9*d**4*x**6 + 525404880*c**8*d**5*x**7
 - 817296480*c**7*d**6*x**8 - 2837186352*c**6*d**7*x**9 - 1128426390*c**5*
d**8*x**10 + 4531617090*c**4*d**9*x**11 + 7365772260*c**3*d**10*x**12 + 45
59763780*c**2*d**11*x**13 + 1063944882*c*d**12*x**14))/30030
```

3.30 $\int (A+Bx+Cx^2)(4c^3-27cd^2x^2-27d^3x^3)^3\,dx$

Optimal result

Integrand size = 36, antiderivative size = 192

$$\begin{aligned}
&\int (A+Bx+Cx^2)(4c^3-27cd^2x^2-27d^3x^3)^3\,dx \\
&= \frac{c^3(4c^2C-6Bcd+9Ad^2)(2c+3dx)^7}{7d^3} - \frac{c^2(8c^2C-9Bcd+9Ad^2)(2c+3dx)^8}{8d^3} \\
&\quad + \frac{c(19c^2C-15Bcd+9Ad^2)(2c+3dx)^9}{27d^3} - \frac{(67c^2C-33Bcd+9Ad^2)(2c+3dx)^{10}}{270d^3} \\
&\quad + \frac{(13cC-3Bd)(2c+3dx)^{11}}{297d^3} - \frac{C(2c+3dx)^{12}}{324d^3}
\end{aligned}$$

output
```
1/7*c^3*(9*A*d^2-6*B*c*d+4*C*c^2)*(3*d*x+2*c)^7/d^3-1/8*c^2*(9*A*d^2-9*B*c
*d+8*C*c^2)*(3*d*x+2*c)^8/d^3+1/27*c*(9*A*d^2-15*B*c*d+19*C*c^2)*(3*d*x+2*
c)^9/d^3-1/270*(9*A*d^2-33*B*c*d+67*C*c^2)*(3*d*x+2*c)^10/d^3+1/297*(-3*B*
d+13*C*c)*(3*d*x+2*c)^11/d^3-1/324*C*(3*d*x+2*c)^12/d^3
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.37

$$\int (A+Bx+Cx^2)\left(4c^3-27cd^2x^2-27d^3x^3\right)^3\,dx$$
$$=64Ac^9x+32Bc^9x^2+\frac{16}{3}c^7\left(4c^2C-81Ad^2\right)x^3$$
$$-324c^6d^2(Bc+Ad)x^4-\frac{324}{5}c^5d^2\left(4c^2C+4Bcd-27Ad^2\right)x^5$$
$$-54c^4d^3\left(4c^2C-27Bcd-54Ad^2\right)x^6+\frac{2187}{7}c^3d^4\left(4c^2C+8Bcd-5Ad^2\right)x^7$$
$$+\frac{2187}{8}c^2d^5\left(8c^2C-5Bcd-27Ad^2\right)x^8-243cd^6\left(5c^2C+27Bcd+27Ad^2\right)x^9$$
$$-\frac{19683}{10}d^7\left(3c^2C+3Bcd+Ad^2\right)x^{10}-\frac{19683}{11}d^8(3cC+Bd)x^{11}-\frac{6561}{4}Cd^9x^{12}$$

input
```
Integrate[(A + B*x + C*x^2)*(4*c^3 - 27*c*d^2*x^2 - 27*d^3*x^3)^3,x]
```

output
```
64*A*c^9*x + 32*B*c^9*x^2 + (16*c^7*(4*c^2*C - 81*A*d^2)*x^3)/3 - 324*c^6*
d^2*(B*c + A*d)*x^4 - (324*c^5*d^2*(4*c^2*C + 4*B*c*d - 27*A*d^2)*x^5)/5 -
 54*c^4*d^3*(4*c^2*C - 27*B*c*d - 54*A*d^2)*x^6 + (2187*c^3*d^4*(4*c^2*C +
 8*B*c*d - 5*A*d^2)*x^7)/7 + (2187*c^2*d^5*(8*c^2*C - 5*B*c*d - 27*A*d^2)*
x^8)/8 - 243*c*d^6*(5*c^2*C + 27*B*c*d + 27*A*d^2)*x^9 - (19683*d^7*(3*c^2
*C + 3*B*c*d + A*d^2)*x^10)/10 - (19683*d^8*(3*c*C + B*d)*x^11)/11 - (6561
*C*d^9*x^12)/4
```

Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.37, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}=0.056$, Rules used = {2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(4c^3-27cd^2x^2-27d^3x^3\right)^3\left(A+Bx+Cx^2\right)\,dx$$

$$\Big\downarrow \text{2188}$$

$$\int \left(-1296c^6d^2x^3(Ad+Bc)-19683d^7x^9\left(Ad^2+3Bcd+3c^2C\right)-2187cd^6x^8\left(27Ad^2+27Bcd+5c^2C\right)+2187c^2\right.$$

$$\Big\downarrow \text{2009}$$

$$-324c^6d^2x^4(Ad+Bc)-\frac{19683}{10}d^7x^{10}\left(Ad^2+3Bcd+3c^2C\right)-243cd^6x^9\left(27Ad^2+27Bcd+5c^2C\right)+\frac{2187}{8}c^2d^5x^8\left(-27Ad^2-5Bcd+8c^2C\right)-\frac{324}{5}c^5d^2x^5\left(-27Ad^2+4Bcd+4c^2C\right)-54c^4d^3x^6\left(-54Ad^2-27Bcd+4c^2C\right)+\frac{2187}{7}c^3d^4x^7\left(-5Ad^2+8Bcd+4c^2C\right)+64Ac^9x+\frac{16}{3}c^7x^3\left(4c^2C-81Ad^2\right)+32Bc^9x^2-\frac{19683}{11}d^8x^{11}(Bd+3cC)-\frac{6561}{4}Cd^9x^{12}$$

input
```
Int[(A + B*x + C*x^2)*(4*c^3 - 27*c*d^2*x^2 - 27*d^3*x^3)^3,x]
```

output
```
64*A*c^9*x + 32*B*c^9*x^2 + (16*c^7*(4*c^2*C - 81*A*d^2)*x^3)/3 - 324*c^6*
d^2*(B*c + A*d)*x^4 - (324*c^5*d^2*(4*c^2*C + 4*B*c*d - 27*A*d^2)*x^5)/5 -
 54*c^4*d^3*(4*c^2*C - 27*B*c*d - 54*A*d^2)*x^6 + (2187*c^3*d^4*(4*c^2*C +
 8*B*c*d - 5*A*d^2)*x^7)/7 + (2187*c^2*d^5*(8*c^2*C - 5*B*c*d - 27*A*d^2)*
x^8)/8 - 243*c*d^6*(5*c^2*C + 27*B*c*d + 27*A*d^2)*x^9 - (19683*d^7*(3*c^2
*C + 3*B*c*d + A*d^2)*x^10)/10 - (19683*d^8*(3*c*C + B*d)*x^11)/11 - (6561
*C*d^9*x^12)/4
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.42

method	result
norman	$-\frac{6561 C\,d^{9} x^{12}}{4}+\left(-\frac{19683}{11} B\,d^{9}-\frac{59049}{11} C\,d^{8} c\right) x^{11}+\left(-\frac{19683}{10} A\,d^{9}-\frac{59049}{10} B c\,d^{8}-\frac{59049}{10} C\,d^{7} c^{2}\right) x$
default	$-\frac{6561 C\,d^{9} x^{12}}{4}+\frac{\left(-19683 B\,d^{9}-59049 C\,d^{8} c\right) x^{11}}{11}+\frac{\left(-19683 A\,d^{9}-59049 B c\,d^{8}-59049 C\,d^{7} c^{2}\right) x^{10}}{10}+\frac{\left(-59049 A c\,d^{8}-5\right.}{}$
gosper	$\frac{x\left(-15155910 C\,d^{9} x^{11}-16533720 x^{10} B\,d^{9}-49601160 x^{10} C\,d^{8} c-18187092 x^{9} A\,d^{9}-54561276 x^{9} B c\,d^{8}-54561276 x^{9} C\,d^{7} c^{2}-60\right.}{}$
risch	$-\frac{59049}{11} x^{11} C\,d^{8} c-\frac{59049}{10} x^{10} B c\,d^{8}-\frac{59049}{10} x^{10} C\,d^{7} c^{2}-\frac{59049}{8} x^{8} A\,c^{2} d^{7}-\frac{10935}{8} x^{8} B\,c^{3} d^{6}+2187 x$
parallelrisch	$-\frac{59049}{11} x^{11} C\,d^{8} c-\frac{59049}{10} x^{10} B c\,d^{8}-\frac{59049}{10} x^{10} C\,d^{7} c^{2}-\frac{59049}{8} x^{8} A\,c^{2} d^{7}-\frac{10935}{8} x^{8} B\,c^{3} d^{6}+2187 x$
orering	$\frac{x\left(-15155910 C\,d^{9} x^{11}-16533720 x^{10} B\,d^{9}-49601160 x^{10} C\,d^{8} c-18187092 x^{9} A\,d^{9}-54561276 x^{9} B c\,d^{8}-54561276 x^{9} C\,d^{7} c^{2}-60\right.}{}$

input

```
int((C*x^2+B*x+A)*(-27*d^3*x^3-27*c*d^2*x^2+4*c^3)^3,x,method=_RETURNVERBO
SE)
```

output

```
-6561/4*C*d^9*x^12+(-19683/11*B*d^9-59049/11*C*d^8*c)*x^11+(-19683/10*A*d^
9-59049/10*B*c*d^8-59049/10*C*d^7*c^2)*x^10+(-6561*A*c*d^8-6561*B*c^2*d^7-
1215*C*c^3*d^6)*x^9+(-59049/8*A*c^2*d^7-10935/8*B*c^3*d^6+2187*C*d^5*c^4)*
x^8+(-10935/7*A*c^3*d^6+17496/7*B*c^4*d^5+8748/7*C*d^4*c^5)*x^7+(2916*A*c^
4*d^5+1458*B*c^5*d^4-216*C*c^6*d^3)*x^6+(8748/5*A*c^5*d^4-1296/5*B*c^6*d^3
-1296/5*C*c^7*d^2)*x^5+(-324*A*c^6*d^3-324*B*c^7*d^2)*x^4+(-432*A*c^7*d^2+
64/3*C*c^9)*x^3+32*B*c^9*x^2+64*A*c^9*x
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.44

$$\int (A+Bx+Cx^2) \left(4c^3-27cd^2x^2-27d^3x^3\right)^3 \, dx$$
$$= -\frac{6561}{4}Cd^9x^{12} + 32Bc^9x^2 - \frac{19683}{11}\left(3Ccd^8 + Bd^9\right)x^{11} + 64Ac^9x - \frac{19683}{10}\left(3Cc^2d^7 + 3Bcd^8 + Ad^9\right)x^{10} - 243\left(5Cc^3d^6 + 27Bc^2d^7 + 27Acd^8\right)x^9 + \frac{2187}{8}\left(8Cc^4d^5 - 5Bc^3d^6 - 27Ac^2d^7\right)x^8 + \frac{2187}{7}\left(4Cc^5d^4 + 8Bc^4d^5 - 5Ac^3d^6\right)x^7 - 54\left(4Cc^6d^3 - 27Bc^5d^4 - 54Ac^4d^5\right)x^6 - \frac{324}{5}\left(4Cc^7d^2 + 4Bc^6d^3 - 27Ac^5d^4\right)x^5 - 324\left(Bc^7d^2 + Ac^6d^3\right)x^4 + \frac{16}{3}\left(4Cc^9 - 81Ac^7d^2\right)x^3$$

input
```
integrate((C*x^2+B*x+A)*(-27*d^3*x^3-27*c*d^2*x^2+4*c^3)^3,x, algorithm="f
ricas")
```

output
```
-6561/4*C*d^9*x^12 + 32*B*c^9*x^2 - 19683/11*(3*C*c*d^8 + B*d^9)*x^11 + 64
*A*c^9*x - 19683/10*(3*C*c^2*d^7 + 3*B*c*d^8 + A*d^9)*x^10 - 243*(5*C*c^3*
d^6 + 27*B*c^2*d^7 + 27*A*c*d^8)*x^9 + 2187/8*(8*C*c^4*d^5 - 5*B*c^3*d^6 -
 27*A*c^2*d^7)*x^8 + 2187/7*(4*C*c^5*d^4 + 8*B*c^4*d^5 - 5*A*c^3*d^6)*x^7
- 54*(4*C*c^6*d^3 - 27*B*c^5*d^4 - 54*A*c^4*d^5)*x^6 - 324/5*(4*C*c^7*d^2
+ 4*B*c^6*d^3 - 27*A*c^5*d^4)*x^5 - 324*(B*c^7*d^2 + A*c^6*d^3)*x^4 + 16/3
*(4*C*c^9 - 81*A*c^7*d^2)*x^3
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.67

$$\int \left(A+Bx+Cx^2\right)\left(4c^3-27cd^2x^2-27d^3x^3\right)^3\,dx$$
$$= 64Ac^9x+32Bc^9x^2-\frac{6561Cd^9x^{12}}{4}+x^{11}\left(-\frac{19683Bd^9}{11}-\frac{59049Ccd^8}{11}\right) + x^{10}\left(-\frac{19683Ad^9}{10}-\frac{59049Bcd^8}{10}-\frac{59049Cc^2d^7}{10}\right) + x^9\left(-6561Acd^8-6561Bc^2d^7-1215Cc^3d^6\right) + x^8\left(-\frac{59049Ac^2d^7}{8}-\frac{10935Bc^3d^6}{8}+2187Cc^4d^5\right) + x^7\left(-\frac{10935Ac^3d^6}{7}+\frac{17496Bc^4d^5}{7}+\frac{8748Cc^5d^4}{7}\right) + x^6\cdot\left(2916Ac^4d^5+1458Bc^5d^4-216Cc^6d^3\right) + x^5\cdot\left(\frac{8748Ac^5d^4}{5}-\frac{1296Bc^6d^3}{5}-\frac{1296Cc^7d^2}{5}\right) + x^4\left(-324Ac^6d^3-324Bc^7d^2\right)+x^3\left(-432Ac^7d^2+\frac{64Cc^9}{3}\right)$$

input

```
integrate((C*x**2+B*x+A)*(-27*d**3*x**3-27*c*d**2*x**2+4*c**3)**3,x)
```

output

```
64*A*c**9*x + 32*B*c**9*x**2 - 6561*C*d**9*x**12/4 + x**11*(-19683*B*d**9/
11 - 59049*C*c*d**8/11) + x**10*(-19683*A*d**9/10 - 59049*B*c*d**8/10 - 59
049*C*c**2*d**7/10) + x**9*(-6561*A*c*d**8 - 6561*B*c**2*d**7 - 1215*C*c**
3*d**6) + x**8*(-59049*A*c**2*d**7/8 - 10935*B*c**3*d**6/8 + 2187*C*c**4*d
**5) + x**7*(-10935*A*c**3*d**6/7 + 17496*B*c**4*d**5/7 + 8748*C*c**5*d**4
/7) + x**6*(2916*A*c**4*d**5 + 1458*B*c**5*d**4 - 216*C*c**6*d**3) + x**5*
(8748*A*c**5*d**4/5 - 1296*B*c**6*d**3/5 - 1296*C*c**7*d**2/5) + x**4*(-32
4*A*c**6*d**3 - 324*B*c**7*d**2) + x**3*(-432*A*c**7*d**2 + 64*C*c**9/3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.44

$$\int (A+Bx+Cx^2)\left(4c^3-27cd^2x^2-27d^3x^3\right)^3\,dx$$
$$= -\frac{6561}{4}Cd^9x^{12}+32\,Bc^9x^2-\frac{19683}{11}\left(3\,Ccd^8+Bd^9\right)x^{11}+64\,Ac^9x$$
$$-\frac{19683}{10}\left(3\,Cc^2d^7+3\,Bcd^8+Ad^9\right)x^{10}-243\left(5\,Cc^3d^6+27\,Bc^2d^7+27\,Acd^8\right)x^9$$
$$+\frac{2187}{8}\left(8\,Cc^4d^5-5\,Bc^3d^6-27\,Ac^2d^7\right)x^8+\frac{2187}{7}\left(4\,Cc^5d^4+8\,Bc^4d^5-5\,Ac^3d^6\right)x^7$$
$$-54\left(4\,Cc^6d^3-27\,Bc^5d^4-54\,Ac^4d^5\right)x^6-\frac{324}{5}\left(4\,Cc^7d^2+4\,Bc^6d^3-27\,Ac^5d^4\right)x^5$$
$$-324\left(Bc^7d^2+Ac^6d^3\right)x^4+\frac{16}{3}\left(4\,Cc^9-81\,Ac^7d^2\right)x^3$$

input

```
integrate((C*x^2+B*x+A)*(-27*d^3*x^3-27*c*d^2*x^2+4*c^3)^3,x, algorithm="m
axima")
```

output

```
-6561/4*C*d^9*x^12 + 32*B*c^9*x^2 - 19683/11*(3*C*c*d^8 + B*d^9)*x^11 + 64
*A*c^9*x - 19683/10*(3*C*c^2*d^7 + 3*B*c*d^8 + A*d^9)*x^10 - 243*(5*C*c^3*
d^6 + 27*B*c^2*d^7 + 27*A*c*d^8)*x^9 + 2187/8*(8*C*c^4*d^5 - 5*B*c^3*d^6 -
 27*A*c^2*d^7)*x^8 + 2187/7*(4*C*c^5*d^4 + 8*B*c^4*d^5 - 5*A*c^3*d^6)*x^7
- 54*(4*C*c^6*d^3 - 27*B*c^5*d^4 - 54*A*c^4*d^5)*x^6 - 324/5*(4*C*c^7*d^2
+ 4*B*c^6*d^3 - 27*A*c^5*d^4)*x^5 - 324*(B*c^7*d^2 + A*c^6*d^3)*x^4 + 16/3
*(4*C*c^9 - 81*A*c^7*d^2)*x^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.56

$$\int (A+Bx+Cx^2) \left(4c^3-27cd^2x^2-27d^3x^3\right)^3 \, dx$$
$$= -\frac{6561}{4}\,Cd^9x^{12} - \frac{59049}{11}\,Ccd^8x^{11} - \frac{19683}{11}\,Bd^9x^{11} - \frac{59049}{10}\,Cc^2d^7x^{10} - \frac{59049}{10}\,Bcd^8x^{10} - \frac{19683}{10}\,Ad^9x^{10} - 1215\,Cc^3d^6x^9 - 6561\,Bc^2d^7x^9 - 6561\,Acd^8x^9 + 2187\,Cc^4d^5x^8 - \frac{10935}{8}\,Bc^3d^6x^8 - \frac{59049}{8}\,Ac^2d^7x^8 + \frac{8748}{7}\,Cc^5d^4x^7 + \frac{17496}{7}\,Bc^4d^5x^7 - \frac{10935}{7}\,Ac^3d^6x^7 - 216\,Cc^6d^3x^6 + 1458\,Bc^5d^4x^6 + 2916\,Ac^4d^5x^6 - \frac{1296}{5}\,Cc^7d^2x^5 - \frac{1296}{5}\,Bc^6d^3x^5 + \frac{8748}{5}\,Ac^5d^4x^5 - 324\,Bc^7d^2x^4 - 324\,Ac^6d^3x^4 + \frac{64}{3}\,Cc^9x^3 - 432\,Ac^7d^2x^3 + 32\,Bc^9x^2 + 64\,Ac^9x$$

input
```
integrate((C*x^2+B*x+A)*(-27*d^3*x^3-27*c*d^2*x^2+4*c^3)^3,x, algorithm="g
iac")
```

output
```
-6561/4*C*d^9*x^12 - 59049/11*C*c*d^8*x^11 - 19683/11*B*d^9*x^11 - 59049/1
0*C*c^2*d^7*x^10 - 59049/10*B*c*d^8*x^10 - 19683/10*A*d^9*x^10 - 1215*C*c^
3*d^6*x^9 - 6561*B*c^2*d^7*x^9 - 6561*A*c*d^8*x^9 + 2187*C*c^4*d^5*x^8 - 1
0935/8*B*c^3*d^6*x^8 - 59049/8*A*c^2*d^7*x^8 + 8748/7*C*c^5*d^4*x^7 + 1749
6/7*B*c^4*d^5*x^7 - 10935/7*A*c^3*d^6*x^7 - 216*C*c^6*d^3*x^6 + 1458*B*c^5
*d^4*x^6 + 2916*A*c^4*d^5*x^6 - 1296/5*C*c^7*d^2*x^5 - 1296/5*B*c^6*d^3*x^
5 + 8748/5*A*c^5*d^4*x^5 - 324*B*c^7*d^2*x^4 - 324*A*c^6*d^3*x^4 + 64/3*C*
c^9*x^3 - 432*A*c^7*d^2*x^3 + 32*B*c^9*x^2 + 64*A*c^9*x
```

Mupad [B] (verification not implemented)

Time = 12.18 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.29

$$\int (A+Bx+Cx^2)\left(4c^3-27cd^2x^2-27d^3x^3\right)^3\,dx$$
$$= x^3\left(\frac{64\,C\,c^9}{3}-432\,A\,c^7\,d^2\right)+32\,B\,c^9\,x^2-\frac{6561\,C\,d^9\,x^{12}}{4}-\frac{19683\,d^8\,x^{11}\,(B\,d+3\,C\,c)}{11}-\frac{19683\,d^7\,x^{10}\,(3\,C\,c^2+3\,B\,c\,d+A\,d^2)}{10}+64\,A\,c^9\,x-324\,c^6\,d^2\,x^4\,(A\,d+B\,c)+\frac{2187\,c^3\,d^4\,x^7\,(4\,C\,c^2+8\,B\,c\,d-5\,A\,d^2)}{7}-\frac{324\,c^5\,d^2\,x^5\,(4\,C\,c^2+4\,B\,c\,d-27\,A\,d^2)}{5}-\frac{2187\,c^2\,d^5\,x^8\,(-8\,C\,c^2+5\,B\,c\,d+27\,A\,d^2)}{8}+54\,c^4\,d^3\,x^6\left(-4\,C\,c^2+27\,B\,c\,d+54\,A\,d^2\right)-243\,c\,d^6\,x^9\left(5\,C\,c^2+27\,B\,c\,d+27\,A\,d^2\right)$$

input
```
int(-(A + B*x + C*x^2)*(27*d^3*x^3 - 4*c^3 + 27*c*d^2*x^2)^3,x)
```

output
```
x^3*((64*C*c^9)/3 - 432*A*c^7*d^2) + 32*B*c^9*x^2 - (6561*C*d^9*x^12)/4 -
(19683*d^8*x^11*(B*d + 3*C*c))/11 - (19683*d^7*x^10*(A*d^2 + 3*C*c^2 + 3*B
*c*d))/10 + 64*A*c^9*x - 324*c^6*d^2*x^4*(A*d + B*c) + (2187*c^3*d^4*x^7*(
4*C*c^2 - 5*A*d^2 + 8*B*c*d))/7 - (324*c^5*d^2*x^5*(4*C*c^2 - 27*A*d^2 + 4
*B*c*d))/5 - (2187*c^2*d^5*x^8*(27*A*d^2 - 8*C*c^2 + 5*B*c*d))/8 + 54*c^4*
d^3*x^6*(54*A*d^2 - 4*C*c^2 + 27*B*c*d) - 243*c*d^6*x^9*(27*A*d^2 + 5*C*c^
2 + 27*B*c*d)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.53

$$\int (A+Bx+Cx^2)\left(4c^3-27cd^2x^2-27d^3x^3\right)^3\,dx$$
$$= \frac{x(-15155910c\,d^9x^{11}-16533720b\,d^9x^{10}-49601160c^2d^8x^{10}-18187092a\,d^9x^9-54561276bc\,d^8x^9-545}{}$$

input
```
int((C*x^2+B*x+A)*(-27*d^3*x^3-27*c*d^2*x^2+4*c^3)^3,x)
```

output
```
(x*(591360*a*c**9 - 3991680*a*c**7*d**2*x**2 - 2993760*a*c**6*d**3*x**3 +
16166304*a*c**5*d**4*x**4 + 26943840*a*c**4*d**5*x**5 - 14434200*a*c**3*d*
*6*x**6 - 68201595*a*c**2*d**7*x**7 - 60623640*a*c*d**8*x**8 - 18187092*a*
d**9*x**9 + 295680*b*c**9*x - 2993760*b*c**7*d**2*x**3 - 2395008*b*c**6*d*
*3*x**4 + 13471920*b*c**5*d**4*x**5 + 23094720*b*c**4*d**5*x**6 - 12629925
*b*c**3*d**6*x**7 - 60623640*b*c**2*d**7*x**8 - 54561276*b*c*d**8*x**9 - 1
6533720*b*d**9*x**10 + 197120*c**10*x**2 - 2395008*c**8*d**2*x**4 - 199584
0*c**7*d**3*x**5 + 11547360*c**6*d**4*x**6 + 20207880*c**5*d**5*x**7 - 112
26600*c**4*d**6*x**8 - 54561276*c**3*d**7*x**9 - 49601160*c**2*d**8*x**10
- 15155910*c*d**9*x**11))/9240
```

3.31 $\int (A+Bx+Cx^2)(4c^3-27cd^2x^2-27d^3x^3)^2 \, dx$

Optimal result

Integrand size = 36, antiderivative size = 154

$$\int (A+Bx+Cx^2)\left(4c^3-27cd^2x^2-27d^3x^3\right)^2 \, dx = \frac{c^2(4c^2C-6Bcd+9Ad^2)(2c+3dx)^5}{15d^3} - \frac{c(20c^2C-21Bcd+18Ad^2)(2c+3dx)^6}{54d^3} + \frac{(37c^2C-24Bcd+9Ad^2)(2c+3dx)^7}{189d^3} - \frac{(10cC-3Bd)(2c+3dx)^8}{216d^3} + \frac{C(2c+3dx)^9}{243d^3}$$

output
```
1/15*c^2*(9*A*d^2-6*B*c*d+4*C*c^2)*(3*d*x+2*c)^5/d^3-1/54*c*(18*A*d^2-21*B
*c*d+20*C*c^2)*(3*d*x+2*c)^6/d^3+1/189*(9*A*d^2-24*B*c*d+37*C*c^2)*(3*d*x+
2*c)^7/d^3-1/216*(-3*B*d+10*C*c)*(3*d*x+2*c)^8/d^3+1/243*C*(3*d*x+2*c)^9/d
^3
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.11

$$\int (A+Bx+Cx^2)\left(4c^3-27cd^2x^2-27d^3x^3\right)^2\,dx$$
$$=16Ac^6x+8Bc^6x^2+\frac{8}{3}c^4\left(2c^2C-27Ad^2\right)x^3-54c^3d^2(Bc+Ad)x^4$$
$$-\frac{27}{5}c^2d^2\left(8c^2C+8Bcd-27Ad^2\right)x^5-\frac{9}{2}cd^3\left(8c^2C-27Bcd-54Ad^2\right)x^6$$
$$+\frac{729}{7}d^4\left(c^2C+2Bcd+Ad^2\right)x^7+\frac{729}{8}d^5(2cC+Bd)x^8+81Cd^6x^9$$

input
```
Integrate[(A + B*x + C*x^2)*(4*c^3 - 27*c*d^2*x^2 - 27*d^3*x^3)^2,x]
```

output
```
16*A*c^6*x + 8*B*c^6*x^2 + (8*c^4*(2*c^2*C - 27*A*d^2)*x^3)/3 - 54*c^3*d^2
*(B*c + A*d)*x^4 - (27*c^2*d^2*(8*c^2*C + 8*B*c*d - 27*A*d^2)*x^5)/5 - (9*
c*d^3*(8*c^2*C - 27*B*c*d - 54*A*d^2)*x^6)/2 + (729*d^4*(c^2*C + 2*B*c*d +
 A*d^2)*x^7)/7 + (729*d^5*(2*c*C + B*d)*x^8)/8 + 81*C*d^6*x^9
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.11, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(4c^3-27cd^2x^2-27d^3x^3\right)^2\left(A+Bx+Cx^2\right)dx$$

$$\Big\downarrow \text{2188}$$

$$\int \left(-216c^3d^2x^3(Ad+Bc)-27c^2d^2x^4\left(-27Ad^2+8Bcd+8c^2C\right)+729d^4x^6\left(Ad^2+2Bcd+c^2C\right)-27cd^3x^5\left(-5\right.\right.$$

$$\Big\downarrow \text{2009}$$

$$-54c^3d^2x^4(Ad+Bc)-\frac{27}{5}c^2d^2x^5\left(-27Ad^2+8Bcd+8c^2C\right)+\frac{729}{7}d^4x^7\left(Ad^2+2Bcd+c^2C\right)-\frac{9}{2}cd^3x^6\left(-54Ad^2-27Bcd+8c^2C\right)+16Ac^6x+\frac{8}{3}c^4x^3\left(2c^2C-27Ad^2\right)+8Bc^6x^2+\frac{729}{8}d^5x^8(Bd+2cC)+81Cd^6x^9$$

input
```
Int[(A + B*x + C*x^2)*(4*c^3 - 27*c*d^2*x^2 - 27*d^3*x^3)^2,x]
```

output
```
16*A*c^6*x + 8*B*c^6*x^2 + (8*c^4*(2*c^2*C - 27*A*d^2)*x^3)/3 - 54*c^3*d^2
*(B*c + A*d)*x^4 - (27*c^2*d^2*(8*c^2*C + 8*B*c*d - 27*A*d^2)*x^5)/5 - (9*
c*d^3*(8*c^2*C - 27*B*c*d - 54*A*d^2)*x^6)/2 + (729*d^4*(c^2*C + 2*B*c*d +
 A*d^2)*x^7)/7 + (729*d^5*(2*c*C + B*d)*x^8)/8 + 81*C*d^6*x^9
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.15

method	result
norman	$81C\,d^6x^9+\left(\frac{729}{8}B\,d^6+\frac{729}{4}Cc\,d^5\right)x^8+\left(\frac{729}{7}A\,d^6+\frac{1458}{7}Bc\,d^5+\frac{729}{7}C\,c^2d^4\right)x^7+\left(243Ac\,d^5\right.$
default	$81C\,d^6x^9+\frac{(729B\,d^6+1458Cc\,d^5)x^8}{8}+\frac{(729A\,d^6+1458Bc\,d^5+729C\,c^2d^4)x^7}{7}+\frac{(1458Ac\,d^5+729B\,c^2d^4-216C\,c^3d^3}{6}$
gosper	$\frac{x\left(68040C\,d^6x^8+76545x^7B\,d^6+153090x^7Cc\,d^5+87480x^6A\,d^6+174960x^6Bc\,d^5+87480x^6C\,c^2d^4+204120x^5Ac\,d^5+102060x^5\right.}{}$
risch	$81C\,d^6x^9+\frac{729}{8}x^8B\,d^6+\frac{729}{4}x^8Cc\,d^5+\frac{729}{7}x^7A\,d^6+\frac{1458}{7}x^7Bc\,d^5+\frac{729}{7}x^7C\,c^2d^4+243x^6Ac$
parallelrisch	$81C\,d^6x^9+\frac{729}{8}x^8B\,d^6+\frac{729}{4}x^8Cc\,d^5+\frac{729}{7}x^7A\,d^6+\frac{1458}{7}x^7Bc\,d^5+\frac{729}{7}x^7C\,c^2d^4+243x^6Ac$
orering	$\frac{x\left(68040C\,d^6x^8+76545x^7B\,d^6+153090x^7Cc\,d^5+87480x^6A\,d^6+174960x^6Bc\,d^5+87480x^6C\,c^2d^4+204120x^5Ac\,d^5+102060x^5\right.}{}$

input
```
int((C*x^2+B*x+A)*(-27*d^3*x^3-27*c*d^2*x^2+4*c^3)^2,x,method=_RETURNVERBO
SE)
```

output
```
81*C*d^6*x^9+(729/8*B*d^6+729/4*C*c*d^5)*x^8+(729/7*A*d^6+1458/7*B*c*d^5+7
29/7*C*c^2*d^4)*x^7+(243*A*c*d^5+243/2*B*c^2*d^4-36*C*c^3*d^3)*x^6+(729/5*
A*c^2*d^4-216/5*B*c^3*d^3-216/5*C*d^2*c^4)*x^5+(-54*A*c^3*d^3-54*B*c^4*d^2
)*x^4+(-72*A*c^4*d^2+16/3*C*c^6)*x^3+8*B*c^6*x^2+16*A*c^6*x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.15

$$\begin{aligned}
&\int (A+Bx+Cx^2)\left(4c^3-27cd^2x^2-27d^3x^3\right)^2\,dx \\
&= 81\,Cd^6x^9+8\,Bc^6x^2+\frac{729}{8}\left(2\,Ccd^5+Bd^6\right)x^8 \\
&\quad +16\,Ac^6x+\frac{729}{7}\left(Cc^2d^4+2\,Bcd^5+Ad^6\right)x^7 \\
&\quad -\frac{9}{2}\left(8\,Cc^3d^3-27\,Bc^2d^4-54\,Acd^5\right)x^6-\frac{27}{5}\left(8\,Cc^4d^2+8\,Bc^3d^3-27\,Ac^2d^4\right)x^5 \\
&\quad -54\left(Bc^4d^2+Ac^3d^3\right)x^4+\frac{8}{3}\left(2\,Cc^6-27\,Ac^4d^2\right)x^3
\end{aligned}$$

input
```
integrate((C*x^2+B*x+A)*(-27*d^3*x^3-27*c*d^2*x^2+4*c^3)^2,x, algorithm="f
ricas")
```

output
```
81*C*d^6*x^9 + 8*B*c^6*x^2 + 729/8*(2*C*c*d^5 + B*d^6)*x^8 + 16*A*c^6*x +
729/7*(C*c^2*d^4 + 2*B*c*d^5 + A*d^6)*x^7 - 9/2*(8*C*c^3*d^3 - 27*B*c^2*d^
4 - 54*A*c*d^5)*x^6 - 27/5*(8*C*c^4*d^2 + 8*B*c^3*d^3 - 27*A*c^2*d^4)*x^5
- 54*(B*c^4*d^2 + A*c^3*d^3)*x^4 + 8/3*(2*C*c^6 - 27*A*c^4*d^2)*x^3
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.32

$$\int (A + Bx + Cx^2) (4c^3 - 27cd^2x^2 - 27d^3x^3)^2 \, dx$$
$$= 16Ac^6x + 8Bc^6x^2 + 81Cd^6x^9 + x^8 \cdot \left(\frac{729Bd^6}{8} + \frac{729Ccd^5}{4}\right) + x^7 \cdot \left(\frac{729Ad^6}{7} + \frac{1458Bcd^5}{7} + \frac{729Cc^2d^4}{7}\right) + x^6 \cdot \left(243Acd^5 + \frac{243Bc^2d^4}{2} - 36Cc^3d^3\right) + x^5 \cdot \left(\frac{729Ac^2d^4}{5} - \frac{216Bc^3d^3}{5} - \frac{216Cc^4d^2}{5}\right) + x^4(-54Ac^3d^3 - 54Bc^4d^2) + x^3\left(-72Ac^4d^2 + \frac{16Cc^6}{3}\right)$$

input
```
integrate((C*x**2+B*x+A)*(-27*d**3*x**3-27*c*d**2*x**2+4*c**3)**2,x)
```

output
```
16*A*c**6*x + 8*B*c**6*x**2 + 81*C*d**6*x**9 + x**8*(729*B*d**6/8 + 729*C*
c*d**5/4) + x**7*(729*A*d**6/7 + 1458*B*c*d**5/7 + 729*C*c**2*d**4/7) + x*
*6*(243*A*c*d**5 + 243*B*c**2*d**4/2 - 36*C*c**3*d**3) + x**5*(729*A*c**2*
d**4/5 - 216*B*c**3*d**3/5 - 216*C*c**4*d**2/5) + x**4*(-54*A*c**3*d**3 -
54*B*c**4*d**2) + x**3*(-72*A*c**4*d**2 + 16*C*c**6/3)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.15

$$\int (A + Bx + Cx^2) (4c^3 - 27cd^2x^2 - 27d^3x^3)^2 \, dx$$
$$= 81\,Cd^6x^9 + 8\,Bc^6x^2 + \frac{729}{8}\,(2\,Ccd^5 + Bd^6)x^8 + 16\,Ac^6x + \frac{729}{7}\,(Cc^2d^4 + 2\,Bcd^5 + Ad^6)x^7 - \frac{9}{2}\,(8\,Cc^3d^3 - 27\,Bc^2d^4 - 54\,Acd^5)x^6 - \frac{27}{5}\,(8\,Cc^4d^2 + 8\,Bc^3d^3 - 27\,Ac^2d^4)x^5 - 54\,(Bc^4d^2 + Ac^3d^3)x^4 + \frac{8}{3}\,(2\,Cc^6 - 27\,Ac^4d^2)x^3$$

input
```
integrate((C*x^2+B*x+A)*(-27*d^3*x^3-27*c*d^2*x^2+4*c^3)^2,x, algorithm="m
axima")
```

output
```
81*C*d^6*x^9 + 8*B*c^6*x^2 + 729/8*(2*C*c*d^5 + B*d^6)*x^8 + 16*A*c^6*x +
729/7*(C*c^2*d^4 + 2*B*c*d^5 + A*d^6)*x^7 - 9/2*(8*C*c^3*d^3 - 27*B*c^2*d^
4 - 54*A*c*d^5)*x^6 - 27/5*(8*C*c^4*d^2 + 8*B*c^3*d^3 - 27*A*c^2*d^4)*x^5
- 54*(B*c^4*d^2 + A*c^3*d^3)*x^4 + 8/3*(2*C*c^6 - 27*A*c^4*d^2)*x^3
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.24

$$\int (A+Bx+Cx^2)\left(4c^3-27cd^2x^2-27d^3x^3\right)^2\,dx$$
$$= 81\,Cd^6x^9+\frac{729}{4}\,Ccd^5x^8+\frac{729}{8}\,Bd^6x^8+\frac{729}{7}\,Cc^2d^4x^7+\frac{1458}{7}\,Bcd^5x^7+\frac{729}{7}\,Ad^6x^7-36\,Cc^3d^3x^6+\frac{243}{2}\,Bc^2d^4x^6+243\,Acd^5x^6-\frac{216}{5}\,Cc^4d^2x^5-\frac{216}{5}\,Bc^3d^3x^5+\frac{729}{5}\,Ac^2d^4x^5-54\,Bc^4d^2x^4-54\,Ac^3d^3x^4+\frac{16}{3}\,Cc^6x^3-72\,Ac^4d^2x^3+8\,Bc^6x^2+16\,Ac^6x$$

input
```
integrate((C*x^2+B*x+A)*(-27*d^3*x^3-27*c*d^2*x^2+4*c^3)^2,x, algorithm="g
iac")
```

output
```
81*C*d^6*x^9 + 729/4*C*c*d^5*x^8 + 729/8*B*d^6*x^8 + 729/7*C*c^2*d^4*x^7 +
 1458/7*B*c*d^5*x^7 + 729/7*A*d^6*x^7 - 36*C*c^3*d^3*x^6 + 243/2*B*c^2*d^4
*x^6 + 243*A*c*d^5*x^6 - 216/5*C*c^4*d^2*x^5 - 216/5*B*c^3*d^3*x^5 + 729/5
*A*c^2*d^4*x^5 - 54*B*c^4*d^2*x^4 - 54*A*c^3*d^3*x^4 + 16/3*C*c^6*x^3 - 72
*A*c^4*d^2*x^3 + 8*B*c^6*x^2 + 16*A*c^6*x
```

Mupad [B] (verification not implemented)

Time = 12.31 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.05

$$\int (A+Bx+Cx^2)\left(4c^3-27cd^2x^2-27d^3x^3\right)^2\,dx$$
$$= x^3\left(\frac{16\,C\,c^6}{3}-72\,A\,c^4\,d^2\right)+x^8\left(\frac{729\,B\,d^6}{8}+\frac{729\,C\,c\,d^5}{4}\right)+8\,B\,c^6\,x^2+81\,C\,d^6\,x^9$$
$$+\frac{729\,d^4\,x^7\,(C\,c^2+2\,B\,c\,d+A\,d^2)}{7}+16\,A\,c^6\,x-54\,c^3\,d^2\,x^4\,(A\,d+B\,c)$$
$$-\frac{27\,c^2\,d^2\,x^5\,(8\,C\,c^2+8\,B\,c\,d-27\,A\,d^2)}{5}+\frac{9\,c\,d^3\,x^6\,(-8\,C\,c^2+27\,B\,c\,d+54\,A\,d^2)}{2}$$

input
```
int((A + B*x + C*x^2)*(27*d^3*x^3 - 4*c^3 + 27*c*d^2*x^2)^2,x)
```

output
```
x^3*((16*C*c^6)/3 - 72*A*c^4*d^2) + x^8*((729*B*d^6)/8 + (729*C*c*d^5)/4)
+ 8*B*c^6*x^2 + 81*C*d^6*x^9 + (729*d^4*x^7*(A*d^2 + C*c^2 + 2*B*c*d))/7 +
 16*A*c^6*x - 54*c^3*d^2*x^4*(A*d + B*c) - (27*c^2*d^2*x^5*(8*C*c^2 - 27*A
*d^2 + 8*B*c*d))/5 + (9*c*d^3*x^6*(54*A*d^2 - 8*C*c^2 + 27*B*c*d))/2
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.22

$$\int (A+Bx+Cx^2)\left(4c^3-27cd^2x^2-27d^3x^3\right)^2\,dx$$
$$= \frac{x(68040c\,d^6x^8+76545b\,d^6x^7+153090c^2d^5x^7+87480a\,d^6x^6+174960bc\,d^5x^6+87480c^3d^4x^6+204120a}{}$$

input
```
int((C*x^2+B*x+A)*(-27*d^3*x^3-27*c*d^2*x^2+4*c^3)^2,x)
```

output
```
(x*(13440*a*c**6 - 60480*a*c**4*d**2*x**2 - 45360*a*c**3*d**3*x**3 + 12247
2*a*c**2*d**4*x**4 + 204120*a*c*d**5*x**5 + 87480*a*d**6*x**6 + 6720*b*c**
6*x - 45360*b*c**4*d**2*x**3 - 36288*b*c**3*d**3*x**4 + 102060*b*c**2*d**4
*x**5 + 174960*b*c*d**5*x**6 + 76545*b*d**6*x**7 + 4480*c**7*x**2 - 36288*
c**5*d**2*x**4 - 30240*c**4*d**3*x**5 + 87480*c**3*d**4*x**6 + 153090*c**2
*d**5*x**7 + 68040*c*d**6*x**8))/840
```

3.32 $\int (A+Bx+Cx^2)(4c^3-27cd^2x^2-27d^3x^3)\,dx$

Optimal result

Integrand size = 34, antiderivative size = 83

$$\begin{aligned}\int (A+Bx+Cx^2)\left(4c^3-27cd^2x^2-27d^3x^3\right)\,dx &= 4Ac^3x+2Bc^3x^2+\frac{1}{3}c\left(4c^2C-27Ad^2\right)x^3\\ &\quad -\frac{27}{4}d^2(Bc+Ad)x^4-\frac{27}{5}d^2(cC+Bd)x^5-\frac{9}{2}Cd^3x^6\end{aligned}$$

output
```
4*A*c^3*x+2*B*c^3*x^2+1/3*c*(-27*A*d^2+4*C*c^2)*x^3-27/4*d^2*(A*d+B*c)*x^4
-27/5*d^2*(B*d+C*c)*x^5-9/2*C*d^3*x^6
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00

$$\begin{aligned}\int (A+Bx+Cx^2)\left(4c^3-27cd^2x^2-27d^3x^3\right)\,dx &= 4Ac^3x+2Bc^3x^2+\frac{1}{3}c\left(4c^2C-27Ad^2\right)x^3\\ &\quad -\frac{27}{4}d^2(Bc+Ad)x^4-\frac{27}{5}d^2(cC+Bd)x^5-\frac{9}{2}Cd^3x^6\end{aligned}$$

input
```
Integrate[(A + B*x + C*x^2)*(4*c^3 - 27*c*d^2*x^2 - 27*d^3*x^3),x]
```

output
```
4*A*c^3*x + 2*B*c^3*x^2 + (c*(4*c^2*C - 27*A*d^2)*x^3)/3 - (27*d^2*(B*c +
A*d)*x^4)/4 - (27*d^2*(c*C + B*d)*x^5)/5 - (9*C*d^3*x^6)/2
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(4c^3-27cd^2x^2-27d^3x^3\right)\left(A+Bx+Cx^2\right)dx$$

$$\Big\downarrow \text{2188}$$

$$\int \left(-27d^2x^3(Ad+Bc)+4Ac^3+cx^2\left(4c^2C-27Ad^2\right)+4Bc^3x-27d^2x^4(Bd+cC)-27Cd^3x^5\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{27}{4}d^2x^4(Ad+Bc)+4Ac^3x+\frac{1}{3}cx^3\left(4c^2C-27Ad^2\right)+2Bc^3x^2-\frac{27}{5}d^2x^5(Bd+cC)-\frac{9}{2}Cd^3x^6$$

input
```
Int[(A + B*x + C*x^2)*(4*c^3 - 27*c*d^2*x^2 - 27*d^3*x^3),x]
```

output
```
4*A*c^3*x + 2*B*c^3*x^2 + (c*(4*c^2*C - 27*A*d^2)*x^3)/3 - (27*d^2*(B*c +
A*d)*x^4)/4 - (27*d^2*(c*C + B*d)*x^5)/5 - (9*C*d^3*x^6)/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.98

method	result
norman	$-\frac{9C\,d^3x^6}{2}+\left(-\frac{27}{5}B\,d^3-\frac{27}{5}C c\,d^2\right)x^5+\left(-\frac{27}{4}A\,d^3-\frac{27}{4}B c\,d^2\right)x^4+\left(-9A c\,d^2+\frac{4}{3}C\,c^3\right)x^3+$
gosper	$\frac{x\left(-270C\,d^3x^5-324x^4B\,d^3-324C c\,d^2x^4-405x^3A\,d^3-405B c\,d^2x^3-540A c\,d^2x^2+80x^2C\,c^3+120B\,c^3x+240A\,c^3\right)}{60}$
default	$-\frac{9C\,d^3x^6}{2}+\frac{\left(-27B\,d^3-27C c\,d^2\right)x^5}{5}+\frac{\left(-27A\,d^3-27B c\,d^2\right)x^4}{4}+\frac{\left(-27A c\,d^2+4C\,c^3\right)x^3}{3}+2B\,c^3x^2+4A\,c^3x$
risch	$-\frac{9}{2}C\,d^3x^6-\frac{27}{5}x^5B\,d^3-\frac{27}{5}x^5C c\,d^2-\frac{27}{4}x^4A\,d^3-\frac{27}{4}B\,x^4c\,d^2-9x^3A c\,d^2+\frac{4}{3}x^3C\,c^3+2B\,c$
parallelrisch	$-\frac{9}{2}C\,d^3x^6-\frac{27}{5}x^5B\,d^3-\frac{27}{5}x^5C c\,d^2-\frac{27}{4}x^4A\,d^3-\frac{27}{4}B\,x^4c\,d^2-9x^3A c\,d^2+\frac{4}{3}x^3C\,c^3+2B\,c$
orering	$\frac{x\left(-270C\,d^3x^5-324x^4B\,d^3-324C c\,d^2x^4-405x^3A\,d^3-405B c\,d^2x^3-540A c\,d^2x^2+80x^2C\,c^3+120B\,c^3x+240A\,c^3\right)\left(-27d^3x^3\cdots\right.}{60(-3dx+c)(3dx+2c)^2}$

input
```
int((C*x^2+B*x+A)*(-27*d^3*x^3-27*c*d^2*x^2+4*c^3),x,method=_RETURNVERBOSE
)
```

output
```
-9/2*C*d^3*x^6+(-27/5*B*d^3-27/5*C*c*d^2)*x^5+(-27/4*A*d^3-27/4*B*c*d^2)*x
^4+(-9*A*c*d^2+4/3*C*c^3)*x^3+2*B*c^3*x^2+4*A*c^3*x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.95

$$\int \left(A+Bx+Cx^2\right)\left(4c^3-27cd^2x^2-27d^3x^3\right)\,dx$$
$$=-\frac{9}{2}\,Cd^3x^6+2\,Bc^3x^2-\frac{27}{5}\left(Ccd^2+Bd^3\right)x^5+4\,Ac^3x-\frac{27}{4}\left(Bcd^2+Ad^3\right)x^4+\frac{1}{3}\left(4\,Cc^3-27\,Acd^2\right)x^3$$

input
```
integrate((C*x^2+B*x+A)*(-27*d^3*x^3-27*c*d^2*x^2+4*c^3),x, algorithm="fri
cas")
```

output
```
-9/2*C*d^3*x^6 + 2*B*c^3*x^2 - 27/5*(C*c*d^2 + B*d^3)*x^5 + 4*A*c^3*x - 27
/4*(B*c*d^2 + A*d^3)*x^4 + 1/3*(4*C*c^3 - 27*A*c*d^2)*x^3
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.17

$$\int \left(A+Bx+Cx^2\right)\left(4c^3-27cd^2x^2-27d^3x^3\right)\,dx$$
$$=4Ac^3x+2Bc^3x^2-\frac{9Cd^3x^6}{2}+x^5\left(-\frac{27Bd^3}{5}-\frac{27Ccd^2}{5}\right)+x^4\left(-\frac{27Ad^3}{4}-\frac{27Bcd^2}{4}\right)+x^3\left(-9Acd^2+\frac{4Cc^3}{3}\right)$$

input
```
integrate((C*x**2+B*x+A)*(-27*d**3*x**3-27*c*d**2*x**2+4*c**3),x)
```

output
```
4*A*c**3*x + 2*B*c**3*x**2 - 9*C*d**3*x**6/2 + x**5*(-27*B*d**3/5 - 27*C*c
*d**2/5) + x**4*(-27*A*d**3/4 - 27*B*c*d**2/4) + x**3*(-9*A*c*d**2 + 4*C*c
**3/3)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.95

$$\int (A+Bx+Cx^2)\left(4c^3-27cd^2x^2-27d^3x^3\right)\,dx$$
$$=-\frac{9}{2}\,Cd^3x^6+2\,Bc^3x^2-\frac{27}{5}\,(Ccd^2+Bd^3)x^5+4\,Ac^3x-\frac{27}{4}\,(Bcd^2+Ad^3)x^4+\frac{1}{3}\,(4\,Cc^3-27\,Acd^2)x^3$$

input
```
integrate((C*x^2+B*x+A)*(-27*d^3*x^3-27*c*d^2*x^2+4*c^3),x, algorithm="max
ima")
```

output
```
-9/2*C*d^3*x^6 + 2*B*c^3*x^2 - 27/5*(C*c*d^2 + B*d^3)*x^5 + 4*A*c^3*x - 27
/4*(B*c*d^2 + A*d^3)*x^4 + 1/3*(4*C*c^3 - 27*A*c*d^2)*x^3
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00

$$\int (A+Bx+Cx^2)\left(4c^3-27cd^2x^2-27d^3x^3\right)\,dx$$
$$=-\frac{9}{2}\,Cd^3x^6-\frac{27}{5}\,Ccd^2x^5-\frac{27}{5}\,Bd^3x^5-\frac{27}{4}\,Bcd^2x^4-\frac{27}{4}\,Ad^3x^4+\frac{4}{3}\,Cc^3x^3-9\,Acd^2x^3+2\,Bc^3x^2+4\,Ac^3x$$

input
```
integrate((C*x^2+B*x+A)*(-27*d^3*x^3-27*c*d^2*x^2+4*c^3),x, algorithm="gia
c")
```

output
```
-9/2*C*d^3*x^6 - 27/5*C*c*d^2*x^5 - 27/5*B*d^3*x^5 - 27/4*B*c*d^2*x^4 - 27
/4*A*d^3*x^4 + 4/3*C*c^3*x^3 - 9*A*c*d^2*x^3 + 2*B*c^3*x^2 + 4*A*c^3*x
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.99

$$\int (A+Bx+Cx^2)\left(4c^3-27cd^2x^2-27d^3x^3\right)\,dx$$
$$= x^3\left(\frac{4\,C\,c^3}{3}-9\,A\,c\,d^2\right)-x^4\left(\frac{27\,A\,d^3}{4}+\frac{27\,B\,c\,d^2}{4}\right)-x^5\left(\frac{27\,B\,d^3}{5}+\frac{27\,C\,c\,d^2}{5}\right)+2\,B\,c^3\,x^2-\frac{9\,C\,d^3\,x^6}{2}+4\,A\,c^3\,x$$

input
```
int(-(A + B*x + C*x^2)*(27*d^3*x^3 - 4*c^3 + 27*c*d^2*x^2),x)
```

output
```
x^3*((4*C*c^3)/3 - 9*A*c*d^2) - x^4*((27*A*d^3)/4 + (27*B*c*d^2)/4) - x^5*
((27*B*d^3)/5 + (27*C*c*d^2)/5) + 2*B*c^3*x^2 - (9*C*d^3*x^6)/2 + 4*A*c^3*
x
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00

$$\int (A+Bx+Cx^2)\left(4c^3-27cd^2x^2-27d^3x^3\right)\,dx$$
$$= \frac{x(-270c\,d^3x^5-324b\,d^3x^4-324c^2d^2x^4-405a\,d^3x^3-405bc\,d^2x^3-540ac\,d^2x^2+80c^4x^2+120b\,c^3x+2}{60}$$

input
```
int((C*x^2+B*x+A)*(-27*d^3*x^3-27*c*d^2*x^2+4*c^3),x)
```

output
```
(x*(240*a*c**3 - 540*a*c*d**2*x**2 - 405*a*d**3*x**3 + 120*b*c**3*x - 405*
b*c*d**2*x**3 - 324*b*d**3*x**4 + 80*c**4*x**2 - 324*c**2*d**2*x**4 - 270*
c*d**3*x**5))/60
```

3.33 $\int \frac{A+Bx+Cx^2}{4c^3-27cd^2x^2-27d^3x^3} dx$

Optimal result

Integrand size = 36, antiderivative size = 110

$$\int \frac{A+Bx+Cx^2}{4c^3-27cd^2x^2-27d^3x^3}\,dx = \frac{-4c^2C+6Bcd-9Ad^2}{81cd^3(2c+3dx)} - \frac{(c^2C+3Bcd+9Ad^2)\log(c-3dx)}{243c^2d^3} - \frac{(8c^2C-3Bcd-9Ad^2)\log(2c+3dx)}{243c^2d^3}$$

output
```
1/81*(-9*A*d^2+6*B*c*d-4*C*c^2)/c/d^3/(3*d*x+2*c)-1/243*(9*A*d^2+3*B*c*d+C
*c^2)*ln(-3*d*x+c)/c^2/d^3-1/243*(-9*A*d^2-3*B*c*d+8*C*c^2)*ln(3*d*x+2*c)/
c^2/d^3
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.01

$$\int \frac{A+Bx+Cx^2}{4c^3-27cd^2x^2-27d^3x^3}\,dx = \frac{-4c^2C+6Bcd-9Ad^2}{81cd^3(2c+3dx)} + \frac{(-c^2C-3Bcd-9Ad^2)\log(c-3dx)}{243c^2d^3} + \frac{(-8c^2C+3Bcd+9Ad^2)\log(2c+3dx)}{243c^2d^3}$$

input
```
Integrate[(A + B*x + C*x^2)/(4*c^3 - 27*c*d^2*x^2 - 27*d^3*x^3),x]
```

output
```
(-4*c^2*C + 6*B*c*d - 9*A*d^2)/(81*c*d^3*(2*c + 3*d*x)) + ((-(c^2*C) - 3*B
*c*d - 9*A*d^2)*Log[c - 3*d*x])/(243*c^2*d^3) + ((-8*c^2*C + 3*B*c*d + 9*A
*d^2)*Log[2*c + 3*d*x])/(243*c^2*d^3)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {2462, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{4c^3-27cd^2x^2-27d^3x^3} \, dx$$

$$\Big\downarrow \text{2462}$$

$$\int \left(\frac{9Ad^2-6Bcd+4c^2C}{27cd^2(2c+3dx)^2}+\frac{9Ad^2+3Bcd+c^2C}{81c^2d^2(c-3dx)}+\frac{9Ad^2+3Bcd-8c^2C}{81c^2d^2(2c+3dx)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{9Ad^2-6Bcd+4c^2C}{81cd^3(2c+3dx)}-\frac{\log(c-3dx)\left(9Ad^2+3Bcd+c^2C\right)}{243c^2d^3}-\frac{\log(2c+3dx)\left(-9Ad^2-3Bcd+8c^2C\right)}{243c^2d^3}$$

input
```
Int[(A + B*x + C*x^2)/(4*c^3 - 27*c*d^2*x^2 - 27*d^3*x^3),x]
```

output
```
-1/81*(4*c^2*C - 6*B*c*d + 9*A*d^2)/(c*d^3*(2*c + 3*d*x)) - ((c^2*C + 3*B*
c*d + 9*A*d^2)*Log[c - 3*d*x])/(243*c^2*d^3) - ((8*c^2*C - 3*B*c*d - 9*A*d
^2)*Log[2*c + 3*d*x])/(243*c^2*d^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.95

method	result
norman	$-\frac{9A\,d^{2}-6Bcd+4C\,c^{2}}{81c\,d^{3}(3dx+2c)}+\frac{(9A\,d^{2}+3Bcd-8C\,c^{2})\ln(3dx+2c)}{243c^{2}d^{3}}-\frac{(9A\,d^{2}+3Bcd+C\,c^{2})\ln(-3dx+c)}{243c^{2}d^{3}}$
default	$\frac{(-9A\,d^{2}-3Bcd-C\,c^{2})\ln(-3dx+c)}{243c^{2}d^{3}}-\frac{9A\,d^{2}-6Bcd+4C\,c^{2}}{81c\,d^{3}(3dx+2c)}+\frac{(9A\,d^{2}+3Bcd-8C\,c^{2})\ln(3dx+2c)}{243c^{2}d^{3}}$
risch	$-\frac{A}{18cd\left(\frac{3dx}{2}+c\right)}+\frac{B}{27d^{2}\left(\frac{3dx}{2}+c\right)}-\frac{2cC}{81d^{3}\left(\frac{3dx}{2}+c\right)}+\frac{\ln(3dx+2c)A}{27c^{2}d}+\frac{\ln(3dx+2c)B}{81c\,d^{2}}-\frac{8\ln(3dx+2c)C}{243d^{3}}-\frac{\ln(-3dx}{27c^{2}}$
parallelrisch	$-\frac{27A\ln\left(dx-\frac{c}{3}\right)x\,d^{3}-27A\ln\left(dx+\frac{2c}{3}\right)x\,d^{3}+9B\ln\left(dx-\frac{c}{3}\right)xc\,d^{2}-9B\ln\left(dx+\frac{2c}{3}\right)xc\,d^{2}+3C\ln\left(dx-\frac{c}{3}\right)x\,c^{2}d+24C\ln\left(dx+\frac{2c}{3}\right)x}{}$

input
```
int((C*x^2+B*x+A)/(-27*d^3*x^3-27*c*d^2*x^2+4*c^3),x,method=_RETURNVERBOSE
)
```

output
```
-1/81*(9*A*d^2-6*B*c*d+4*C*c^2)/c/d^3/(3*d*x+2*c)+1/243*(9*A*d^2+3*B*c*d-8
*C*c^2)/c^2/d^3*ln(3*d*x+2*c)-1/243*(9*A*d^2+3*B*c*d+C*c^2)*ln(-3*d*x+c)/c
^2/d^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.38

$$\int \frac{A+Bx+Cx^2}{4c^3-27cd^2x^2-27d^3x^3}\,dx = -\frac{12\,Cc^3-18\,Bc^2d+27\,Acd^2+\left(16\,Cc^3-6\,Bc^2d-18\,Acd^2+3\,(8\,Cc^2d-3\,Bcd^2-9\,Ad^3)x\right)\log\left(3\,dx\right.}{243\,(3\,c^2d^4x+2\,c^3d^3}$$

input
```
integrate((C*x^2+B*x+A)/(-27*d^3*x^3-27*c*d^2*x^2+4*c^3),x, algorithm="fri
cas")
```

output
```
-1/243*(12*C*c^3 - 18*B*c^2*d + 27*A*c*d^2 + (16*C*c^3 - 6*B*c^2*d - 18*A*
c*d^2 + 3*(8*C*c^2*d - 3*B*c*d^2 - 9*A*d^3)*x)*log(3*d*x + 2*c) + (2*C*c^3
 + 6*B*c^2*d + 18*A*c*d^2 + 3*(C*c^2*d + 3*B*c*d^2 + 9*A*d^3)*x)*log(3*d*x
 - c))/(3*c^2*d^4*x + 2*c^3*d^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 241 vs. 2(110) = 220.

Time = 0.75 (sec) , antiderivative size = 241, normalized size of antiderivative = 2.19

$$\begin{aligned}
&\int \frac{A+Bx+Cx^2}{4c^3-27cd^2x^2-27d^3x^3}\,dx \\
&= -\frac{9Ad^2-6Bcd+4Cc^2}{162c^2d^3+243cd^4x} \\
&\quad - \frac{(-9Ad^2-3Bcd+8Cc^2)\log\left(x+\frac{-9Acd^2-3Bc^2d-10Cc^3+3c(-9Ad^2-3Bcd+8Cc^2)}{-54Ad^3-18Bcd^2+21Cc^2d}\right)}{243c^2d^3} \\
&\quad - \frac{(9Ad^2+3Bcd+Cc^2)\log\left(x+\frac{-9Acd^2-3Bc^2d-10Cc^3+3c(9Ad^2+3Bcd+Cc^2)}{-54Ad^3-18Bcd^2+21Cc^2d}\right)}{243c^2d^3}
\end{aligned}$$

input
```
integrate((C*x**2+B*x+A)/(-27*d**3*x**3-27*c*d**2*x**2+4*c**3),x)
```

output
```
-(9*A*d**2 - 6*B*c*d + 4*C*c**2)/(162*c**2*d**3 + 243*c*d**4*x) - (-9*A*d*
*2 - 3*B*c*d + 8*C*c**2)*log(x + (-9*A*c*d**2 - 3*B*c**2*d - 10*C*c**3 + 3
*c*(-9*A*d**2 - 3*B*c*d + 8*C*c**2))/(-54*A*d**3 - 18*B*c*d**2 + 21*C*c**2
*d))/(243*c**2*d**3) - (9*A*d**2 + 3*B*c*d + C*c**2)*log(x + (-9*A*c*d**2
- 3*B*c**2*d - 10*C*c**3 + 3*c*(9*A*d**2 + 3*B*c*d + C*c**2))/(-54*A*d**3
- 18*B*c*d**2 + 21*C*c**2*d))/(243*c**2*d**3)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.98

$$\int \frac{A+Bx+Cx^2}{4c^3-27cd^2x^2-27d^3x^3}\,dx = -\frac{4\,Cc^2-6\,Bcd+9\,Ad^2}{81\,(3\,cd^4x+2\,c^2d^3)} - \frac{(8\,Cc^2-3\,Bcd-9\,Ad^2)\log{(3\,dx+2\,c)}}{243\,c^2d^3} - \frac{(Cc^2+3\,Bcd+9\,Ad^2)\log{(3\,dx-c)}}{243\,c^2d^3}$$

input
```
integrate((C*x^2+B*x+A)/(-27*d^3*x^3-27*c*d^2*x^2+4*c^3),x, algorithm="max
ima")
```

output
```
-1/81*(4*C*c^2 - 6*B*c*d + 9*A*d^2)/(3*c*d^4*x + 2*c^2*d^3) - 1/243*(8*C*c
^2 - 3*B*c*d - 9*A*d^2)*log(3*d*x + 2*c)/(c^2*d^3) - 1/243*(C*c^2 + 3*B*c*
d + 9*A*d^2)*log(3*d*x - c)/(c^2*d^3)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.01

$$\int \frac{A+Bx+Cx^2}{4c^3-27cd^2x^2-27d^3x^3}\,dx = -\frac{(8\,Cc^2-3\,Bcd-9\,Ad^2)\log{(|3\,dx+2\,c|)}}{243\,c^2d^3} - \frac{(Cc^2+3\,Bcd+9\,Ad^2)\log{(|3\,dx-c|)}}{243\,c^2d^3} - \frac{4\,Cc^3-6\,Bc^2d+9\,Acd^2}{81\,(3\,dx+2\,c)c^2d^3}$$

input
```
integrate((C*x^2+B*x+A)/(-27*d^3*x^3-27*c*d^2*x^2+4*c^3),x, algorithm="gia
c")
```

output
```
-1/243*(8*C*c^2 - 3*B*c*d - 9*A*d^2)*log(abs(3*d*x + 2*c))/(c^2*d^3) - 1/2
43*(C*c^2 + 3*B*c*d + 9*A*d^2)*log(abs(3*d*x - c))/(c^2*d^3) - 1/81*(4*C*c
^3 - 6*B*c^2*d + 9*A*c*d^2)/((3*d*x + 2*c)*c^2*d^3)
```

Mupad [B] (verification not implemented)

Time = 12.44 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.95

$$\int \frac{A+Bx+Cx^2}{4c^3-27cd^2x^2-27d^3x^3}\,dx = \frac{\ln(2\,c+3\,d\,x)\,\left(-8\,C\,c^2+3\,B\,c\,d+9\,A\,d^2\right)}{243\,c^2\,d^3} - \frac{4\,C\,c^2-6\,B\,c\,d+9\,A\,d^2}{81\,c\,d^3\,\left(2\,c+3\,d\,x\right)} - \frac{\ln(c-3\,d\,x)\,\left(C\,c^2+3\,B\,c\,d+9\,A\,d^2\right)}{243\,c^2\,d^3}$$

input
```
int(-(A + B*x + C*x^2)/(27*d^3*x^3 - 4*c^3 + 27*c*d^2*x^2),x)
```

output
```
(log(2*c + 3*d*x)*(9*A*d^2 - 8*C*c^2 + 3*B*c*d))/(243*c^2*d^3) - (9*A*d^2
+ 4*C*c^2 - 6*B*c*d)/(81*c*d^3*(2*c + 3*d*x)) - (log(c - 3*d*x)*(9*A*d^2 +
 C*c^2 + 3*B*c*d))/(243*c^2*d^3)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.99

$$\int \frac{A+Bx+Cx^2}{4c^3-27cd^2x^2-27d^3x^3}\,dx = \frac{36\log(3dx+2c)\,ac\,d^2+54\log(3dx+2c)\,a\,d^3x+12\log(3dx+2c)\,b\,c^2d+18\log(3dx+2c)\,bc\,d^2x-32\,\mathrm{l}}{}$$

input
```
int((C*x^2+B*x+A)/(-27*d^3*x^3-27*c*d^2*x^2+4*c^3),x)
```

output
```
(36*log(2*c + 3*d*x)*a*c*d**2 + 54*log(2*c + 3*d*x)*a*d**3*x + 12*log(2*c
+ 3*d*x)*b*c**2*d + 18*log(2*c + 3*d*x)*b*c*d**2*x - 32*log(2*c + 3*d*x)*c
**4 - 48*log(2*c + 3*d*x)*c**3*d*x - 36*log(c - 3*d*x)*a*c*d**2 - 54*log(c
 - 3*d*x)*a*d**3*x - 12*log(c - 3*d*x)*b*c**2*d - 18*log(c - 3*d*x)*b*c*d*
*2*x - 4*log(c - 3*d*x)*c**4 - 6*log(c - 3*d*x)*c**3*d*x + 81*a*d**3*x - 5
4*b*c*d**2*x + 36*c**3*d*x)/(486*c**2*d**3*(2*c + 3*d*x))
```

3.34 $\int \frac{A+Bx+Cx^2}{\left(4c^3-27cd^2x^2-27d^3x^3\right)^2}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 216

$$\int \frac{A+Bx+Cx^2}{(4c^3-27cd^2x^2-27d^3x^3)^2}\,dx = \frac{c^2C+3Bcd+9Ad^2}{2187c^4d^3(c-3dx)} - \frac{4c^2C-6Bcd+9Ad^2}{729c^2d^3(2c+3dx)^3} + \frac{4c^2C+3Bcd-18Ad^2}{1458c^3d^3(2c+3dx)^2} + \frac{c^2C-9Ad^2}{729c^4d^3(2c+3dx)} + \frac{(2c^2C-3Bcd-36Ad^2)\log(c-3dx)}{6561c^5d^3} - \frac{(2c^2C-3Bcd-36Ad^2)\log(2c+3dx)}{6561c^5d^3}$$

output
```
1/2187*(9*A*d^2+3*B*c*d+C*c^2)/c^4/d^3/(-3*d*x+c)-1/729*(9*A*d^2-6*B*c*d+4
*C*c^2)/c^2/d^3/(3*d*x+2*c)^3+1/1458*(-18*A*d^2+3*B*c*d+4*C*c^2)/c^3/d^3/(
3*d*x+2*c)^2+1/729*(-9*A*d^2+C*c^2)/c^4/d^3/(3*d*x+2*c)+1/6561*(-36*A*d^2-
3*B*c*d+2*C*c^2)*ln(-3*d*x+c)/c^5/d^3-1/6561*(-36*A*d^2-3*B*c*d+2*C*c^2)*l
n(3*d*x+2*c)/c^5/d^3
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.86

$$\int \frac{A+Bx+Cx^2}{(4c^3-27cd^2x^2-27d^3x^3)^2}\,dx = \frac{\frac{6c(c^2C+3Bcd+9Ad^2)}{c-3dx}-\frac{18c^3(4c^2C-6Bcd+9Ad^2)}{(2c+3dx)^3}+\frac{9c^2(4c^2C+3Bcd-18Ad^2)}{(2c+3dx)^2}+\frac{18c(c^2C-9Ad^2)}{2c+3dx}+2(2c^2C-3Bcd-36Ad}{13122c^5d^3}$$

input
```
Integrate[(A + B*x + C*x^2)/(4*c^3 - 27*c*d^2*x^2 - 27*d^3*x^3)^2,x]
```

output
```
((6*c*(c^2*C + 3*B*c*d + 9*A*d^2))/(c - 3*d*x) - (18*c^3*(4*c^2*C - 6*B*c*
d + 9*A*d^2))/(2*c + 3*d*x)^3 + (9*c^2*(4*c^2*C + 3*B*c*d - 18*A*d^2))/(2*
c + 3*d*x)^2 + (18*c*(c^2*C - 9*A*d^2))/(2*c + 3*d*x) + 2*(2*c^2*C - 3*B*c
*d - 36*A*d^2)*Log[c - 3*d*x] + 2*(-2*c^2*C + 3*B*c*d + 36*A*d^2)*Log[2*c
+ 3*d*x])/(13122*c^5*d^3)
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {2462, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{(4c^3-27cd^2x^2-27d^3x^3)^2}\,dx$$

$$\Big\downarrow \text{2462}$$

$$\int \left(\frac{9Ad^2-6Bcd+4c^2C}{81c^2d^2(2c+3dx)^4}+\frac{36Ad^2+3Bcd-2c^2C}{2187c^5d^2(c-3dx)}+\frac{36Ad^2+3Bcd-2c^2C}{2187c^5d^2(2c+3dx)}+\frac{9Ad^2+3Bcd+c^2C}{729c^4d^2(c-3dx)^2}+\frac{18Ad^2}{243c^3}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{9Ad^2-6Bcd+4c^2C}{729c^2d^3(2c+3dx)^3}+\frac{\log(c-3dx)\left(-36Ad^2-3Bcd+2c^2C\right)}{6561c^5d^3}-\frac{\log(2c+3dx)\left(-36Ad^2-3Bcd+2c^2C\right)}{6561c^5d^3}+\frac{9Ad^2+3Bcd+c^2C}{2187c^4d^3(c-3dx)}+\frac{-18Ad^2+3Bcd+4c^2C}{1458c^3d^3(2c+3dx)^2}+\frac{c^2C-9Ad^2}{729c^4d^3(2c+3dx)}$$

input
```
Int[(A + B*x + C*x^2)/(4*c^3 - 27*c*d^2*x^2 - 27*d^3*x^3)^2,x]
```

output
```
(c^2*C + 3*B*c*d + 9*A*d^2)/(2187*c^4*d^3*(c - 3*d*x)) - (4*c^2*C - 6*B*c*
d + 9*A*d^2)/(729*c^2*d^3*(2*c + 3*d*x)^3) + (4*c^2*C + 3*B*c*d - 18*A*d^2
)/(1458*c^3*d^3*(2*c + 3*d*x)^2) + (c^2*C - 9*A*d^2)/(729*c^4*d^3*(2*c + 3
*d*x)) + ((2*c^2*C - 3*B*c*d - 36*A*d^2)*Log[c - 3*d*x])/(6561*c^5*d^3) -
((2*c^2*C - 3*B*c*d - 36*A*d^2)*Log[2*c + 3*d*x])/(6561*c^5*d^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.92

method	result
norman	$\frac{-\frac{\left(99A\,d^2-93Bcd-28C\,c^2\right)x^3}{216c^4}+\frac{\left(99A\,d^2+69Bcd+8C\,c^2\right)x^2}{324d\,c^3}-\frac{d\left(117A\,d^2-51Bcd-20C\,c^2\right)x^4}{216c^5}+\frac{\left(441A\,d^2-24Bcd+16C\,c^2\right)x}{1458c^2d^2}}{(3dx+2c)^3(-3dx+c)}-\frac{(36A}{}$
default	$\frac{\left(-36A\,d^2-3Bcd+2C\,c^2\right)\ln(-3dx+c)}{6561c^5d^3}+\frac{9A\,d^2+3Bcd+C\,c^2}{2187c^4d^3(-3dx+c)}-\frac{18A\,d^2-3Bcd-4C\,c^2}{1458d^3c^3(3dx+2c)^2}+\frac{\left(36A\,d^2+3Bcd-2C\,c^2\right)\ln(3dx}{6561d^3c^5}$
risch	$\frac{\frac{\left(36A\,d^2+3Bcd-2C\,c^2\right)x^3}{81c^4}+\frac{\left(36A\,d^2+3Bcd-2C\,c^2\right)x^2}{54c^3d}+\frac{\left(36A\,d^2+3Bcd+4C\,c^2\right)x}{162c^2d^2}-\frac{117A\,d^2-51Bcd-20C\,c^2}{2187c\,d^3}}{(3dx+2c)\left(-27d^3x^3-27c\,d^2x^2+4c^3\right)}-\frac{4\ln(-3dx+c)A}{729d\,c^5}-$
parallelrisch	$-\frac{-3240B\ln\left(dx+\frac{2c}{3}\right)x^3c^2d^4-2160C\ln\left(dx-\frac{c}{3}\right)x^3c^3d^3+2160C\ln\left(dx+\frac{2c}{3}\right)x^3c^3d^3+15552A\ln\left(dx-\frac{c}{3}\right)x^2c^2d^4-15552A\ln(d}{}$

input
```
int((C*x^2+B*x+A)/(-27*d^3*x^3-27*c*d^2*x^2+4*c^3)^2,x,method=_RETURNVERBO
SE)
```

output
```
(-1/216*(99*A*d^2-93*B*c*d-28*C*c^2)/c^4*x^3+1/324/d*(99*A*d^2+69*B*c*d+8*
C*c^2)/c^3*x^2-1/216*d*(117*A*d^2-51*B*c*d-20*C*c^2)/c^5*x^4+1/1458*(441*A
*d^2-24*B*c*d+16*C*c^2)/c^2/d^2*x)/(3*d*x+2*c)^3/(-3*d*x+c)-1/6561/d^3*(36
*A*d^2+3*B*c*d-2*C*c^2)/c^5*ln(-3*d*x+c)+1/6561/d^3*(36*A*d^2+3*B*c*d-2*C*
c^2)/c^5*ln(3*d*x+2*c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 481 vs. 2(206) = 412.

Time = 0.08 (sec) , antiderivative size = 481, normalized size of antiderivative = 2.23

$$\int \frac{A+Bx+Cx^2}{\left(4c^3-27cd^2x^2-27d^3x^3\right)^2}\,dx =$$
$$-\frac{120\,Cc^6+306\,Bc^5d-702\,Ac^4d^2-162\,(2\,Cc^3d^3-3\,Bc^2d^4-36\,Acd^5)x^3-243\,(2\,Cc^4d^2-3\,Bc^3d^3-}{}$$

input
```
integrate((C*x^2+B*x+A)/(-27*d^3*x^3-27*c*d^2*x^2+4*c^3)^2,x, algorithm="f
ricas")
```

output
```
-1/13122*(120*C*c^6 + 306*B*c^5*d - 702*A*c^4*d^2 - 162*(2*C*c^3*d^3 - 3*B
*c^2*d^4 - 36*A*c*d^5)*x^3 - 243*(2*C*c^4*d^2 - 3*B*c^3*d^3 - 36*A*c^2*d^4
)*x^2 + 81*(4*C*c^5*d + 3*B*c^4*d^2 + 36*A*c^3*d^3)*x - 2*(16*C*c^6 - 24*B
*c^5*d - 288*A*c^4*d^2 - 81*(2*C*c^2*d^4 - 3*B*c*d^5 - 36*A*d^6)*x^4 - 135
*(2*C*c^3*d^3 - 3*B*c^2*d^4 - 36*A*c*d^5)*x^3 - 54*(2*C*c^4*d^2 - 3*B*c^3*
d^3 - 36*A*c^2*d^4)*x^2 + 12*(2*C*c^5*d - 3*B*c^4*d^2 - 36*A*c^3*d^3)*x)*l
og(3*d*x + 2*c) + 2*(16*C*c^6 - 24*B*c^5*d - 288*A*c^4*d^2 - 81*(2*C*c^2*d
^4 - 3*B*c*d^5 - 36*A*d^6)*x^4 - 135*(2*C*c^3*d^3 - 3*B*c^2*d^4 - 36*A*c*d
^5)*x^3 - 54*(2*C*c^4*d^2 - 3*B*c^3*d^3 - 36*A*c^2*d^4)*x^2 + 12*(2*C*c^5*
d - 3*B*c^4*d^2 - 36*A*c^3*d^3)*x)*log(3*d*x - c))/(81*c^5*d^7*x^4 + 135*c
^6*d^6*x^3 + 54*c^7*d^5*x^2 - 12*c^8*d^4*x - 8*c^9*d^3)
```

Sympy [A] (verification not implemented)

Time = 1.36 (sec) , antiderivative size = 379, normalized size of antiderivative = 1.75

$$\int \frac{A+Bx+Cx^2}{(4c^3-27cd^2x^2-27d^3x^3)^2}\,dx = \frac{234Ac^3d^2-102Bc^4d-40Cc^5+x^3(-1944Ad^5-162Bcd^4+108Cc^2d^3)+x^2(-2916Acd^4-243Bc^2d^3+\cdots}{-34992c^8d^3-52488c^7d^4x+236196c^6d^5x^2+590490c^5d^6x^3+3\cdots} + \frac{(-36Ad^2-3Bcd+2Cc^2)\log\left(x+\frac{-36Acd^2-3Bc^2d+2Cc^3-3c(-36Ad^2-3Bcd+2Cc^2)}{-216Ad^3-18Bcd^2+12Cc^2d}\right)}{6561c^5d^3} - \frac{(-36Ad^2-3Bcd+2Cc^2)\log\left(x+\frac{-36Acd^2-3Bc^2d+2Cc^3+3c(-36Ad^2-3Bcd+2Cc^2)}{-216Ad^3-18Bcd^2+12Cc^2d}\right)}{6561c^5d^3}$$

input
```
integrate((C*x**2+B*x+A)/(-27*d**3*x**3-27*c*d**2*x**2+4*c**3)**2,x)
```

output
```
(234*A*c**3*d**2 - 102*B*c**4*d - 40*C*c**5 + x**3*(-1944*A*d**5 - 162*B*c
*d**4 + 108*C*c**2*d**3) + x**2*(-2916*A*c*d**4 - 243*B*c**2*d**3 + 162*C*
c**3*d**2) + x*(-972*A*c**2*d**3 - 81*B*c**3*d**2 - 108*C*c**4*d))/(-34992
*c**8*d**3 - 52488*c**7*d**4*x + 236196*c**6*d**5*x**2 + 590490*c**5*d**6*
x**3 + 354294*c**4*d**7*x**4) + (-36*A*d**2 - 3*B*c*d + 2*C*c**2)*log(x +
(-36*A*c*d**2 - 3*B*c**2*d + 2*C*c**3 - 3*c*(-36*A*d**2 - 3*B*c*d + 2*C*c*
*2))/(-216*A*d**3 - 18*B*c*d**2 + 12*C*c**2*d))/(6561*c**5*d**3) - (-36*A*
d**2 - 3*B*c*d + 2*C*c**2)*log(x + (-36*A*c*d**2 - 3*B*c**2*d + 2*C*c**3 +
 3*c*(-36*A*d**2 - 3*B*c*d + 2*C*c**2))/(-216*A*d**3 - 18*B*c*d**2 + 12*C*
c**2*d))/(6561*c**5*d**3)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.10

$$\int \frac{A+Bx+Cx^2}{(4c^3-27cd^2x^2-27d^3x^3)^2}\,dx = -\frac{40\,Cc^5+102\,Bc^4d-234\,Ac^3d^2-54\,(2\,Cc^2d^3-3\,Bcd^4-36\,Ad^5)x^3-81\,(2\,Cc^3d^2-3\,Bc^2d^3-36\,Ac}{4374\,(81\,c^4d^7x^4+135\,c^5d^6x^3+54\,c^6d^5x^2-12\,c^7d^4x-8\,c^8d} - \frac{(2\,Cc^2-3\,Bcd-36\,Ad^2)\log(3\,dx+2\,c)}{6561\,c^5d^3} + \frac{(2\,Cc^2-3\,Bcd-36\,Ad^2)\log(3\,dx-c)}{6561\,c^5d^3}$$

input
```
integrate((C*x^2+B*x+A)/(-27*d^3*x^3-27*c*d^2*x^2+4*c^3)^2,x, algorithm="m
axima")
```

output
```
-1/4374*(40*C*c^5 + 102*B*c^4*d - 234*A*c^3*d^2 - 54*(2*C*c^2*d^3 - 3*B*c*
d^4 - 36*A*d^5)*x^3 - 81*(2*C*c^3*d^2 - 3*B*c^2*d^3 - 36*A*c*d^4)*x^2 + 27
*(4*C*c^4*d + 3*B*c^3*d^2 + 36*A*c^2*d^3)*x)/(81*c^4*d^7*x^4 + 135*c^5*d^6
*x^3 + 54*c^6*d^5*x^2 - 12*c^7*d^4*x - 8*c^8*d^3) - 1/6561*(2*C*c^2 - 3*B*
c*d - 36*A*d^2)*log(3*d*x + 2*c)/(c^5*d^3) + 1/6561*(2*C*c^2 - 3*B*c*d - 3
6*A*d^2)*log(3*d*x - c)/(c^5*d^3)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.00

$$\int \frac{A+Bx+Cx^2}{(4c^3-27cd^2x^2-27d^3x^3)^2}\,dx = -\frac{(2\,Cc^2-3\,Bcd-36\,Ad^2)\log(|3\,dx+2\,c|)}{6561\,c^5d^3} + \frac{(2\,Cc^2-3\,Bcd-36\,Ad^2)\log(|3\,dx-c|)}{6561\,c^5d^3} - \frac{40\,Cc^6+102\,Bc^5d-234\,Ac^4d^2-54\,(2\,Cc^3d^3-3\,Bc^2d^4-36\,Acd^5)x^3-81\,(2\,Cc^4d^2-3\,Bc^3d^3-36\,A}{4374\,(3\,dx+2\,c)^3(3\,dx-c)c^5d^3}$$

input
```
integrate((C*x^2+B*x+A)/(-27*d^3*x^3-27*c*d^2*x^2+4*c^3)^2,x, algorithm="g
iac")
```

output
```
-1/6561*(2*C*c^2 - 3*B*c*d - 36*A*d^2)*log(abs(3*d*x + 2*c))/(c^5*d^3) + 1
/6561*(2*C*c^2 - 3*B*c*d - 36*A*d^2)*log(abs(3*d*x - c))/(c^5*d^3) - 1/437
4*(40*C*c^6 + 102*B*c^5*d - 234*A*c^4*d^2 - 54*(2*C*c^3*d^3 - 3*B*c^2*d^4
- 36*A*c*d^5)*x^3 - 81*(2*C*c^4*d^2 - 3*B*c^3*d^3 - 36*A*c^2*d^4)*x^2 + 27
*(4*C*c^5*d + 3*B*c^4*d^2 + 36*A*c^3*d^3)*x)/((3*d*x + 2*c)^3*(3*d*x - c)*
c^5*d^3)
```

Mupad [B] (verification not implemented)

Time = 12.30 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.88

$$\int \frac{A+Bx+Cx^2}{(4c^3-27cd^2x^2-27d^3x^3)^2}\,dx = \frac{2\,\mathrm{atanh}\left(\frac{2\,d\,x}{c}+\frac{1}{3}\right)\,\left(-2\,C\,c^2+3\,B\,c\,d+36\,A\,d^2\right)}{6561\,c^5\,d^3} - \frac{\frac{20\,C\,c^2+51\,B\,c\,d-117\,A\,d^2}{2187\,c\,d^3}+\frac{x^3\,\left(-2\,C\,c^2+3\,B\,c\,d+36\,A\,d^2\right)}{81\,c^4}+\frac{x^2\,\left(-2\,C\,c^2+3\,B\,c\,d+36\,A\,d^2\right)}{54\,c^3\,d}+\frac{x\,\left(4\,C\,c^2+3\,B\,c\,d+36\,A\,d^2\right)}{162\,c^2\,d^2}}{-8\,c^4-12\,c^3\,d\,x+54\,c^2\,d^2\,x^2+135\,c\,d^3\,x^3+81\,d^4\,x^4}$$

input
```
int((A + B*x + C*x^2)/(27*d^3*x^3 - 4*c^3 + 27*c*d^2*x^2)^2,x)
```

output
```
(2*atanh((2*d*x)/c + 1/3)*(36*A*d^2 - 2*C*c^2 + 3*B*c*d))/(6561*c^5*d^3) -
 ((20*C*c^2 - 117*A*d^2 + 51*B*c*d)/(2187*c*d^3) + (x^3*(36*A*d^2 - 2*C*c^
2 + 3*B*c*d))/(81*c^4) + (x^2*(36*A*d^2 - 2*C*c^2 + 3*B*c*d))/(54*c^3*d) +
 (x*(36*A*d^2 + 4*C*c^2 + 3*B*c*d))/(162*c^2*d^2))/(81*d^4*x^4 - 8*c^4 + 1
35*c*d^3*x^3 + 54*c^2*d^2*x^2 - 12*c^3*d*x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 697, normalized size of antiderivative = 3.23

$$\int \frac{A+Bx+Cx^2}{(4c^3-27cd^2x^2-27d^3x^3)^2}\,dx = \text{Too large to display}$$

input
```
int((C*x^2+B*x+A)/(-27*d^3*x^3-27*c*d^2*x^2+4*c^3)^2,x)
```

output
```
(2880*log(2*c + 3*d*x)*a*c**4*d**2 + 4320*log(2*c + 3*d*x)*a*c**3*d**3*x -
 19440*log(2*c + 3*d*x)*a*c**2*d**4*x**2 - 48600*log(2*c + 3*d*x)*a*c*d**5
*x**3 - 29160*log(2*c + 3*d*x)*a*d**6*x**4 + 240*log(2*c + 3*d*x)*b*c**5*d
 + 360*log(2*c + 3*d*x)*b*c**4*d**2*x - 1620*log(2*c + 3*d*x)*b*c**3*d**3*
x**2 - 4050*log(2*c + 3*d*x)*b*c**2*d**4*x**3 - 2430*log(2*c + 3*d*x)*b*c*
d**5*x**4 - 160*log(2*c + 3*d*x)*c**7 - 240*log(2*c + 3*d*x)*c**6*d*x + 10
80*log(2*c + 3*d*x)*c**5*d**2*x**2 + 2700*log(2*c + 3*d*x)*c**4*d**3*x**3
+ 1620*log(2*c + 3*d*x)*c**3*d**4*x**4 - 2880*log(c - 3*d*x)*a*c**4*d**2 -
 4320*log(c - 3*d*x)*a*c**3*d**3*x + 19440*log(c - 3*d*x)*a*c**2*d**4*x**2
 + 48600*log(c - 3*d*x)*a*c*d**5*x**3 + 29160*log(c - 3*d*x)*a*d**6*x**4 -
 240*log(c - 3*d*x)*b*c**5*d - 360*log(c - 3*d*x)*b*c**4*d**2*x + 1620*log
(c - 3*d*x)*b*c**3*d**3*x**2 + 4050*log(c - 3*d*x)*b*c**2*d**4*x**3 + 2430
*log(c - 3*d*x)*b*c*d**5*x**4 + 160*log(c - 3*d*x)*c**7 + 240*log(c - 3*d*
x)*c**6*d*x - 1080*log(c - 3*d*x)*c**5*d**2*x**2 - 2700*log(c - 3*d*x)*c**
4*d**3*x**3 - 1620*log(c - 3*d*x)*c**3*d**4*x**4 - 1782*a*c**4*d**2 + 1717
2*a*c**3*d**3*x + 32076*a*c**2*d**4*x**2 - 17496*a*d**6*x**4 + 1674*b*c**5
*d + 1431*b*c**4*d**2*x + 2673*b*c**3*d**3*x**2 - 1458*b*c*d**5*x**4 + 504
*c**7 + 1476*c**6*d*x - 1782*c**5*d**2*x**2 + 972*c**3*d**4*x**4)/(65610*c
**5*d**3*(8*c**4 + 12*c**3*d*x - 54*c**2*d**2*x**2 - 135*c*d**3*x**3 - 81*
d**4*x**4))
```

3.35 $\int \frac{A+Bx+Cx^2}{(4c^3-27cd^2x^2-27d^3x^3)^3}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 308

$$\begin{aligned}\int \frac{A+Bx+Cx^2}{(4c^3-27cd^2x^2-27d^3x^3)^3}\,dx = {} & \frac{c^2C+3Bcd+9Ad^2}{39366c^6d^3(c-3dx)^2}+\frac{Bc+6Ad}{6561c^7d^2(c-3dx)} \\ & -\frac{4c^2C-6Bcd+9Ad^2}{3645c^3d^3(2c+3dx)^5} \\ & +\frac{Bc-3Ad}{972c^4d^2(2c+3dx)^4}+\frac{c^2C+3Bcd-18Ad^2}{6561c^5d^3(2c+3dx)^3} \\ & +\frac{5c^2C+6Bcd-90Ad^2}{39366c^6d^3(2c+3dx)^2}+\frac{2c^2C-45Ad^2}{19683c^7d^3(2c+3dx)} \\ & +\frac{(2c^2C-3Bcd-63Ad^2)\log(c-3dx)}{59049c^8d^3} \\ & -\frac{(2c^2C-3Bcd-63Ad^2)\log(2c+3dx)}{59049c^8d^3}\end{aligned}$$

output
```
1/39366*(9*A*d^2+3*B*c*d+C*c^2)/c^6/d^3/(-3*d*x+c)^2+1/6561*(6*A*d+B*c)/c^
7/d^2/(-3*d*x+c)-1/3645*(9*A*d^2-6*B*c*d+4*C*c^2)/c^3/d^3/(3*d*x+2*c)^5+1/
972*(-3*A*d+B*c)/c^4/d^2/(3*d*x+2*c)^4+1/6561*(-18*A*d^2+3*B*c*d+C*c^2)/c^
5/d^3/(3*d*x+2*c)^3+1/39366*(-90*A*d^2+6*B*c*d+5*C*c^2)/c^6/d^3/(3*d*x+2*c
)^2+1/19683*(-45*A*d^2+2*C*c^2)/c^7/d^3/(3*d*x+2*c)+1/59049*(-63*A*d^2-3*B
*c*d+2*C*c^2)*ln(-3*d*x+c)/c^8/d^3-1/59049*(-63*A*d^2-3*B*c*d+2*C*c^2)*ln(
3*d*x+2*c)/c^8/d^3
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 265, normalized size of antiderivative = 0.86

$$\int \frac{A+Bx+Cx^2}{(4c^3-27cd^2x^2-27d^3x^3)^3}\,dx$$
$$= \frac{\frac{30c^2(c^2C+3Bcd+9Ad^2)}{(c-3dx)^2}+\frac{180cd(Bc+6Ad)}{c-3dx}-\frac{324c^5(4c^2C-6Bcd+9Ad^2)}{(2c+3dx)^5}+\frac{1215c^4d(Bc-3Ad)}{(2c+3dx)^4}+\frac{180c^3(c^2C+3Bcd-18Ad^2)}{(2c+3dx)^3}+\frac{30c^2}{}}{}$$

input
```
Integrate[(A + B*x + C*x^2)/(4*c^3 - 27*c*d^2*x^2 - 27*d^3*x^3)^3,x]
```

output
```
((30*c^2*(c^2*C + 3*B*c*d + 9*A*d^2))/(c - 3*d*x)^2 + (180*c*d*(B*c + 6*A*
d))/(c - 3*d*x) - (324*c^5*(4*c^2*C - 6*B*c*d + 9*A*d^2))/(2*c + 3*d*x)^5
+ (1215*c^4*d*(B*c - 3*A*d))/(2*c + 3*d*x)^4 + (180*c^3*(c^2*C + 3*B*c*d -
 18*A*d^2))/(2*c + 3*d*x)^3 + (30*c^2*(5*c^2*C + 6*B*c*d - 90*A*d^2))/(2*c
 + 3*d*x)^2 + (60*c*(2*c^2*C - 45*A*d^2))/(2*c + 3*d*x) + 20*(2*c^2*C - 3*
B*c*d - 63*A*d^2)*Log[c - 3*d*x] + 20*(-2*c^2*C + 3*B*c*d + 63*A*d^2)*Log[
2*c + 3*d*x])/(1180980*c^8*d^3)
```

Rubi [A] (verified)

Time = 1.07 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {2462, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{(4c^3-27cd^2x^2-27d^3x^3)^3}\,dx$$
$$\Big\downarrow \; 2462$$
$$\int \left(\frac{6Ad+Bc}{2187c^7d(c-3dx)^2}+\frac{3Ad-Bc}{81c^4d(2c+3dx)^5}+\frac{63Ad^2+3Bcd-2c^2C}{19683c^8d^2(c-3dx)}+\frac{63Ad^2+3Bcd-2c^2C}{19683c^8d^2(2c+3dx)}+\frac{9Ad^2+3Bcd}{6561c^6d^2(c-}\right.$$
$$\Big\downarrow \; 2009$$

$$\frac{6Ad+Bc}{6561c^7d^2(c-3dx)}+\frac{Bc-3Ad}{972c^4d^2(2c+3dx)^4}+\frac{\log(c-3dx)\left(-63Ad^2-3Bcd+2c^2C\right)}{59049c^8d^3}-\frac{\log(2c+3dx)\left(-63Ad^2-3Bcd+2c^2C\right)}{59049c^8d^3}+\frac{9Ad^2+3Bcd+c^2C}{39366c^6d^3(c-3dx)^2}+\frac{-90Ad^2+6Bcd+5c^2C}{39366c^6d^3(2c+3dx)^2}+\frac{-18Ad^2+3Bcd+c^2C}{6561c^5d^3(2c+3dx)^3}-\frac{9Ad^2-6Bcd+4c^2C}{3645c^3d^3(2c+3dx)^5}+\frac{2c^2C-45Ad^2}{19683c^7d^3(2c+3dx)}$$

input
```
Int[(A + B*x + C*x^2)/(4*c^3 - 27*c*d^2*x^2 - 27*d^3*x^3)^3,x]
```

output
```
(c^2*C + 3*B*c*d + 9*A*d^2)/(39366*c^6*d^3*(c - 3*d*x)^2) + (B*c + 6*A*d)/
(6561*c^7*d^2*(c - 3*d*x)) - (4*c^2*C - 6*B*c*d + 9*A*d^2)/(3645*c^3*d^3*(
2*c + 3*d*x)^5) + (B*c - 3*A*d)/(972*c^4*d^2*(2*c + 3*d*x)^4) + (c^2*C + 3
*B*c*d - 18*A*d^2)/(6561*c^5*d^3*(2*c + 3*d*x)^3) + (5*c^2*C + 6*B*c*d - 9
0*A*d^2)/(39366*c^6*d^3*(2*c + 3*d*x)^2) + (2*c^2*C - 45*A*d^2)/(19683*c^7
*d^3*(2*c + 3*d*x)) + ((2*c^2*C - 3*B*c*d - 63*A*d^2)*Log[c - 3*d*x])/(590
49*c^8*d^3) - ((2*c^2*C - 3*B*c*d - 63*A*d^2)*Log[2*c + 3*d*x])/(59049*c^8
*d^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 286, normalized size of antiderivative = 0.93

method	result
norman	$\frac{-\frac{(1179A\,d^2-291Bcd-76C\,c^2)x^3}{648c^4}-\frac{d(28827A\,d^2+4497Bcd-568C\,c^2)x^4}{7776c^5}+\frac{d^2(1827A\,d^2-21783Bcd-2488C\,c^2)x^5}{8640c^6}+\frac{d^3(549A\,d^2-321Bc}{120c^7}}{(3dx+2c)^5(-3dx+}$
default	$\frac{6Ad+Bc}{6561c^7d^2(-3dx+c)}+\frac{(-63A\,d^2-3Bcd+2C\,c^2)\ln(-3dx+c)}{59049c^8d^3}-\frac{-9A\,d^2-3Bcd-C\,c^2}{39366d^3c^6(-3dx+c)^2}-\frac{45A\,d^2-2C\,c^2}{19683d^3c^7(3dx+2c)}+\frac{(63A}{}$
risch	$\frac{-\frac{d^3(63A\,d^2+3Bcd-2C\,c^2)x^6}{27c^7}-\frac{5(63A\,d^2+3Bcd-2C\,c^2)d^2x^5}{54c^6}-\frac{2(63A\,d^2+3Bcd-2C\,c^2)d\,x^4}{27c^5}-\frac{25(63A\,d^2+3Bcd-2C\,c^2)x^3}{2916c^4}+\frac{53(63A}{}}{(3dx+2c)(-27d^3x^3-27c\,d^2x^2+4c^3)^2}$
parallelrisch	Expression too large to display

input
```
int((C*x^2+B*x+A)/(-27*d^3*x^3-27*c*d^2*x^2+4*c^3)^3,x,method=_RETURNVERBO
SE)
```

output
```
(-1/648*(1179*A*d^2-291*B*c*d-76*C*c^2)/c^4*x^3-1/7776*d*(28827*A*d^2+4497
*B*c*d-568*C*c^2)/c^5*x^4+1/8640*d^2*(1827*A*d^2-21783*B*c*d-2488*C*c^2)/c
^6*x^5+1/120*d^3*(549*A*d^2-321*B*c*d-56*C*c^2)/c^7*x^6+1/2880*d^4*(7461*A
*d^2-2769*B*c*d-584*C*c^2)/c^8*x^7+1/2916*(1179*A*d^2+681*B*c*d+32*C*c^2)/
c^3/d*x^2+1/13122*(4545*A*d^2-96*B*c*d+64*C*c^2)/c^2/d^2*x)/(3*d*x+2*c)^5/
(-3*d*x+c)^2-1/59049/d^3*(63*A*d^2+3*B*c*d-2*C*c^2)/c^8*ln(-3*d*x+c)+1/590
49/d^3*(63*A*d^2+3*B*c*d-2*C*c^2)/c^8*ln(3*d*x+2*c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 811 vs. $2(294) = 588$.

Time = 0.11 (sec) , antiderivative size = 811, normalized size of antiderivative = 2.63

$$\int \frac{A+Bx+Cx^2}{(4c^3-27cd^2x^2-27d^3x^3)^3}\,dx = \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)/(-27*d^3*x^3-27*c*d^2*x^2+4*c^3)^3,x, algorithm="f
ricas")
```

output
```
1/1180980*(3504*C*c^9 + 16614*B*c^8*d - 44766*A*c^7*d^2 + 43740*(2*C*c^3*d
^6 - 3*B*c^2*d^7 - 63*A*c*d^8)*x^6 + 109350*(2*C*c^4*d^5 - 3*B*c^3*d^6 - 6
3*A*c^2*d^7)*x^5 + 87480*(2*C*c^5*d^4 - 3*B*c^4*d^5 - 63*A*c^3*d^6)*x^4 +
10125*(2*C*c^6*d^3 - 3*B*c^5*d^4 - 63*A*c^4*d^5)*x^3 - 17172*(2*C*c^7*d^2
- 3*B*c^6*d^3 - 63*A*c^5*d^4)*x^2 + 81*(136*C*c^8*d + 201*B*c^7*d^2 + 4221
*A*c^6*d^3)*x - 20*(64*C*c^9 - 96*B*c^8*d - 2016*A*c^7*d^2 + 2187*(2*C*c^2
*d^7 - 3*B*c*d^8 - 63*A*d^9)*x^7 + 5832*(2*C*c^3*d^6 - 3*B*c^2*d^7 - 63*A*
c*d^8)*x^6 + 5103*(2*C*c^4*d^5 - 3*B*c^3*d^6 - 63*A*c^2*d^7)*x^5 + 810*(2*
C*c^5*d^4 - 3*B*c^4*d^5 - 63*A*c^3*d^6)*x^4 - 1080*(2*C*c^6*d^3 - 3*B*c^5*
d^4 - 63*A*c^4*d^5)*x^3 - 432*(2*C*c^7*d^2 - 3*B*c^6*d^3 - 63*A*c^5*d^4)*x
^2 + 48*(2*C*c^8*d - 3*B*c^7*d^2 - 63*A*c^6*d^3)*x)*log(3*d*x + 2*c) + 20*
(64*C*c^9 - 96*B*c^8*d - 2016*A*c^7*d^2 + 2187*(2*C*c^2*d^7 - 3*B*c*d^8 -
63*A*d^9)*x^7 + 5832*(2*C*c^3*d^6 - 3*B*c^2*d^7 - 63*A*c*d^8)*x^6 + 5103*(
2*C*c^4*d^5 - 3*B*c^3*d^6 - 63*A*c^2*d^7)*x^5 + 810*(2*C*c^5*d^4 - 3*B*c^4
*d^5 - 63*A*c^3*d^6)*x^4 - 1080*(2*C*c^6*d^3 - 3*B*c^5*d^4 - 63*A*c^4*d^5)
*x^3 - 432*(2*C*c^7*d^2 - 3*B*c^6*d^3 - 63*A*c^5*d^4)*x^2 + 48*(2*C*c^8*d
- 3*B*c^7*d^2 - 63*A*c^6*d^3)*x)*log(3*d*x - c))/(2187*c^8*d^10*x^7 + 5832
*c^9*d^9*x^6 + 5103*c^10*d^8*x^5 + 810*c^11*d^7*x^4 - 1080*c^12*d^6*x^3 -
432*c^13*d^5*x^2 + 48*c^14*d^4*x + 32*c^15*d^3)
```

Sympy [A] (verification not implemented)

Time = 3.56 (sec) , antiderivative size = 517, normalized size of antiderivative = 1.68

$$\int \frac{A + Bx + Cx^2}{(4c^3 - 27cd^2x^2 - 27d^3x^3)^3}\, dx =$$
$$-\frac{14922Ac^6d^2 - 5538Bc^7d - 1168Cc^8 + x^6 \cdot (918540Ad^8 + 43740Bcd^7 - 29160Cc^2d^6) + x^5 \cdot (2296350\ldots}{12597120c\ldots}$$
$$+\frac{(-63Ad^2 - 3Bcd + 2Cc^2)\log\left(x + \frac{-63Acd^2 - 3Bc^2d + 2Cc^3 - 3c(-63Ad^2 - 3Bcd + 2Cc^2)}{-378Ad^3 - 18Bcd^2 + 12Cc^2d}\right)}{59049c^8d^3}$$
$$-\frac{(-63Ad^2 - 3Bcd + 2Cc^2)\log\left(x + \frac{-63Acd^2 - 3Bc^2d + 2Cc^3 + 3c(-63Ad^2 - 3Bcd + 2Cc^2)}{-378Ad^3 - 18Bcd^2 + 12Cc^2d}\right)}{59049c^8d^3}$$

input
```
integrate((C*x**2+B*x+A)/(-27*d**3*x**3-27*c*d**2*x**2+4*c**3)**3,x)
```

output

```
-(14922*A*c**6*d**2 - 5538*B*c**7*d - 1168*C*c**8 + x**6*(918540*A*d**8 +
43740*B*c*d**7 - 29160*C*c**2*d**6) + x**5*(2296350*A*c*d**7 + 109350*B*c*
*2*d**6 - 72900*C*c**3*d**5) + x**4*(1837080*A*c**2*d**6 + 87480*B*c**3*d*
*5 - 58320*C*c**4*d**4) + x**3*(212625*A*c**3*d**5 + 10125*B*c**4*d**4 - 6
750*C*c**5*d**3) + x**2*(-360612*A*c**4*d**4 - 17172*B*c**5*d**3 + 11448*C
*c**6*d**2) + x*(-113967*A*c**5*d**3 - 5427*B*c**6*d**2 - 3672*C*c**7*d))/
(12597120*c**14*d**3 + 18895680*c**13*d**4*x - 170061120*c**12*d**5*x**2 -
 425152800*c**11*d**6*x**3 + 318864600*c**10*d**7*x**4 + 2008846980*c**9*d
**8*x**5 + 2295825120*c**8*d**9*x**6 + 860934420*c**7*d**10*x**7) + (-63*A
*d**2 - 3*B*c*d + 2*C*c**2)*log(x + (-63*A*c*d**2 - 3*B*c**2*d + 2*C*c**3
- 3*c*(-63*A*d**2 - 3*B*c*d + 2*C*c**2))/(-378*A*d**3 - 18*B*c*d**2 + 12*C
*c**2*d))/(59049*c**8*d**3) - (-63*A*d**2 - 3*B*c*d + 2*C*c**2)*log(x + (-
63*A*c*d**2 - 3*B*c**2*d + 2*C*c**3 + 3*c*(-63*A*d**2 - 3*B*c*d + 2*C*c**2
))/(-378*A*d**3 - 18*B*c*d**2 + 12*C*c**2*d))/(59049*c**8*d**3)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 369, normalized size of antiderivative = 1.20

$$\int \frac{A+Bx+Cx^2}{(4c^3-27cd^2x^2-27d^3x^3)^3}\,dx$$
$$= \frac{1168\,Cc^8+5538\,Bc^7d-14922\,Ac^6d^2+14580\,(2\,Cc^2d^6-3\,Bcd^7-63\,Ad^8)x^6+36450\,(2\,Cc^3d^5-3\,Bc^2}{393660\,(2187\,c^7d}$$
$$-\frac{(2\,Cc^2-3\,Bcd-63\,Ad^2)\log(3\,dx+2\,c)}{59049\,c^8d^3}+\frac{(2\,Cc^2-3\,Bcd-63\,Ad^2)\log(3\,dx-c)}{59049\,c^8d^3}$$

input

```
integrate((C*x^2+B*x+A)/(-27*d^3*x^3-27*c*d^2*x^2+4*c^3)^3,x, algorithm="m
axima")
```

output

```
1/393660*(1168*C*c^8 + 5538*B*c^7*d - 14922*A*c^6*d^2 + 14580*(2*C*c^2*d^6
 - 3*B*c*d^7 - 63*A*d^8)*x^6 + 36450*(2*C*c^3*d^5 - 3*B*c^2*d^6 - 63*A*c*d
^7)*x^5 + 29160*(2*C*c^4*d^4 - 3*B*c^3*d^5 - 63*A*c^2*d^6)*x^4 + 3375*(2*C
*c^5*d^3 - 3*B*c^4*d^4 - 63*A*c^3*d^5)*x^3 - 5724*(2*C*c^6*d^2 - 3*B*c^5*d
^3 - 63*A*c^4*d^4)*x^2 + 27*(136*C*c^7*d + 201*B*c^6*d^2 + 4221*A*c^5*d^3)
*x)/(2187*c^7*d^10*x^7 + 5832*c^8*d^9*x^6 + 5103*c^9*d^8*x^5 + 810*c^10*d^
7*x^4 - 1080*c^11*d^6*x^3 - 432*c^12*d^5*x^2 + 48*c^13*d^4*x + 32*c^14*d^3
) - 1/59049*(2*C*c^2 - 3*B*c*d - 63*A*d^2)*log(3*d*x + 2*c)/(c^8*d^3) + 1/
59049*(2*C*c^2 - 3*B*c*d - 63*A*d^2)*log(3*d*x - c)/(c^8*d^3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.03

$$\int \frac{A+Bx+Cx^2}{(4c^3-27cd^2x^2-27d^3x^3)^3}\,dx = -\frac{(2\,Cc^2-3\,Bcd-63\,Ad^2)\log(|3\,dx+2\,c|)}{59049\,c^8d^3} + \frac{(2\,Cc^2-3\,Bcd-63\,Ad^2)\log(|3\,dx-c|)}{59049\,c^8d^3} + \frac{1168\,Cc^9+5538\,Bc^8d-14922\,Ac^7d^2+14580\,(2\,Cc^3d^6-3\,Bc^2d^7-63\,Acd^8)x^6+36450\,(2\,Cc^4d^5-3}{}$$

input
```
integrate((C*x^2+B*x+A)/(-27*d^3*x^3-27*c*d^2*x^2+4*c^3)^3,x, algorithm="g
iac")
```

output
```
-1/59049*(2*C*c^2 - 3*B*c*d - 63*A*d^2)*log(abs(3*d*x + 2*c))/(c^8*d^3) +
1/59049*(2*C*c^2 - 3*B*c*d - 63*A*d^2)*log(abs(3*d*x - c))/(c^8*d^3) + 1/3
93660*(1168*C*c^9 + 5538*B*c^8*d - 14922*A*c^7*d^2 + 14580*(2*C*c^3*d^6 -
3*B*c^2*d^7 - 63*A*c*d^8)*x^6 + 36450*(2*C*c^4*d^5 - 3*B*c^3*d^6 - 63*A*c^
2*d^7)*x^5 + 29160*(2*C*c^5*d^4 - 3*B*c^4*d^5 - 63*A*c^3*d^6)*x^4 + 3375*(
2*C*c^6*d^3 - 3*B*c^5*d^4 - 63*A*c^4*d^5)*x^3 - 5724*(2*C*c^7*d^2 - 3*B*c^
6*d^3 - 63*A*c^5*d^4)*x^2 + 27*(136*C*c^8*d + 201*B*c^7*d^2 + 4221*A*c^6*d
^3)*x)/((3*d*x + 2*c)^5*(3*d*x - c)^2*c^8*d^3)
```

Mupad [B] (verification not implemented)

Time = 12.21 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.00

$$\int \frac{A+Bx+Cx^2}{(4c^3-27cd^2x^2-27d^3x^3)^3}\,dx = \frac{2\,\mathrm{atanh}\left(\frac{2\,d\,x}{c}+\frac{1}{3}\right)\,\left(-2\,C\,c^2+3\,B\,c\,d+63\,A\,d^2\right)}{59049\,c^8\,d^3} - \frac{\frac{25\,x^3\,\left(-2\,C\,c^2+3\,B\,c\,d+63\,A\,d^2\right)}{2916\,c^4}-\frac{584\,C\,c^2+2769\,B\,c\,d-7461\,A\,d^2}{196830\,c\,d^3}-\frac{53\,x^2\,\left(-2\,C\,c^2+3\,B\,c\,d+63\,A\,d^2\right)}{3645\,c^3\,d}+\frac{5\,d^2\,x^5\,\left(-2\,C\,c^2+3\,B\,c\,d+6\right.}{54\,c^6}}{32\,c^7+48\,c^6\,d\,x-432\,c^5\,d^2\,x^2-1080\,c^4\,d^3\,x^3+810\,c^3\,d^4}$$

input
```
int(-(A + B*x + C*x^2)/(27*d^3*x^3 - 4*c^3 + 27*c*d^2*x^2)^3,x)
```

output
```
(2*atanh((2*d*x)/c + 1/3)*(63*A*d^2 - 2*C*c^2 + 3*B*c*d))/(59049*c^8*d^3)
- ((25*x^3*(63*A*d^2 - 2*C*c^2 + 3*B*c*d))/(2916*c^4) - (584*C*c^2 - 7461*
A*d^2 + 2769*B*c*d)/(196830*c*d^3) - (53*x^2*(63*A*d^2 - 2*C*c^2 + 3*B*c*d
))/(3645*c^3*d) + (5*d^2*x^5*(63*A*d^2 - 2*C*c^2 + 3*B*c*d))/(54*c^6) + (d
^3*x^6*(63*A*d^2 - 2*C*c^2 + 3*B*c*d))/(27*c^7) + (2*d*x^4*(63*A*d^2 - 2*C
*c^2 + 3*B*c*d))/(27*c^5) - (x*(4221*A*d^2 + 136*C*c^2 + 201*B*c*d))/(1458
0*c^2*d^2))/(32*c^7 + 2187*d^7*x^7 + 5832*c*d^6*x^6 - 432*c^5*d^2*x^2 - 10
80*c^4*d^3*x^3 + 810*c^3*d^4*x^4 + 5103*c^2*d^5*x^5 + 48*c^6*d*x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 1189, normalized size of antiderivative = 3.86

$$\int \frac{A + Bx + Cx^2}{(4c^3 - 27cd^2x^2 - 27d^3x^3)^3}\, dx = \text{Too large to display}$$

input
```
int((C*x^2+B*x+A)/(-27*d^3*x^3-27*c*d^2*x^2+4*c^3)^3,x)
```

output
```
(80640*log(2*c + 3*d*x)*a*c**7*d**2 + 120960*log(2*c + 3*d*x)*a*c**6*d**3*
x - 1088640*log(2*c + 3*d*x)*a*c**5*d**4*x**2 - 2721600*log(2*c + 3*d*x)*a
*c**4*d**5*x**3 + 2041200*log(2*c + 3*d*x)*a*c**3*d**6*x**4 + 12859560*log
(2*c + 3*d*x)*a*c**2*d**7*x**5 + 14696640*log(2*c + 3*d*x)*a*c*d**8*x**6 +
 5511240*log(2*c + 3*d*x)*a*d**9*x**7 + 3840*log(2*c + 3*d*x)*b*c**8*d + 5
760*log(2*c + 3*d*x)*b*c**7*d**2*x - 51840*log(2*c + 3*d*x)*b*c**6*d**3*x*
*2 - 129600*log(2*c + 3*d*x)*b*c**5*d**4*x**3 + 97200*log(2*c + 3*d*x)*b*c
**4*d**5*x**4 + 612360*log(2*c + 3*d*x)*b*c**3*d**6*x**5 + 699840*log(2*c
+ 3*d*x)*b*c**2*d**7*x**6 + 262440*log(2*c + 3*d*x)*b*c*d**8*x**7 - 2560*l
og(2*c + 3*d*x)*c**10 - 3840*log(2*c + 3*d*x)*c**9*d*x + 34560*log(2*c + 3
*d*x)*c**8*d**2*x**2 + 86400*log(2*c + 3*d*x)*c**7*d**3*x**3 - 64800*log(2
*c + 3*d*x)*c**6*d**4*x**4 - 408240*log(2*c + 3*d*x)*c**5*d**5*x**5 - 4665
60*log(2*c + 3*d*x)*c**4*d**6*x**6 - 174960*log(2*c + 3*d*x)*c**3*d**7*x**
7 - 80640*log(c - 3*d*x)*a*c**7*d**2 - 120960*log(c - 3*d*x)*a*c**6*d**3*x
 + 1088640*log(c - 3*d*x)*a*c**5*d**4*x**2 + 2721600*log(c - 3*d*x)*a*c**4
*d**5*x**3 - 2041200*log(c - 3*d*x)*a*c**3*d**6*x**4 - 12859560*log(c - 3*
d*x)*a*c**2*d**7*x**5 - 14696640*log(c - 3*d*x)*a*c*d**8*x**6 - 5511240*lo
g(c - 3*d*x)*a*d**9*x**7 - 3840*log(c - 3*d*x)*b*c**8*d - 5760*log(c - 3*d
*x)*b*c**7*d**2*x + 51840*log(c - 3*d*x)*b*c**6*d**3*x**2 + 129600*log(c -
 3*d*x)*b*c**5*d**4*x**3 - 97200*log(c - 3*d*x)*b*c**4*d**5*x**4 - 6123...
```

3.36 $\int (A+Bx+Cx^2) \left(4c^3-27cd^2x^2-27d^3x^3\right)^{3/2} dx$

Optimal result

Integrand size = 38, antiderivative size = 398

$$\int (A+Bx+Cx^2) \left(4c^3-27cd^2x^2-27d^3x^3\right)^{3/2} dx = -\frac{2c^3(c^2C+3Bcd+9Ad^2)(c-3dx)\left(4c^3-27cd^2x^2-27d^3x^3\right)^{3/2}}{5d^3(2c+3dx)^3} + \frac{6c^2(c^2C+2Bcd+3Ad^2)(c-3dx)^2\left(4c^3-27cd^2x^2-27d^3x^3\right)^{3/2}}{7d^3(2c+3dx)^3} - \frac{2c(10c^2C+12Bcd+9Ad^2)(c-3dx)^3\left(4c^3-27cd^2x^2-27d^3x^3\right)^{3/2}}{27d^3(2c+3dx)^3} + \frac{2(46c^2C+30Bcd+9Ad^2)(c-3dx)^4\left(4c^3-27cd^2x^2-27d^3x^3\right)^{3/2}}{297d^3(2c+3dx)^3} - \frac{2(11cC+3Bd)(c-3dx)^5\left(4c^3-27cd^2x^2-27d^3x^3\right)^{3/2}}{351d^3(2c+3dx)^3} + \frac{2C(c-3dx)^6\left(4c^3-27cd^2x^2-27d^3x^3\right)^{3/2}}{405d^3(2c+3dx)^3}$$

output
```
-2/5*c^3*(9*A*d^2+3*B*c*d+C*c^2)*(-3*d*x+c)*(-27*d^3*x^3-27*c*d^2*x^2+4*c^
3)^(3/2)/d^3/(3*d*x+2*c)^3+6/7*c^2*(3*A*d^2+2*B*c*d+C*c^2)*(-3*d*x+c)^2*(-
27*d^3*x^3-27*c*d^2*x^2+4*c^3)^(3/2)/d^3/(3*d*x+2*c)^3-2/27*c*(9*A*d^2+12*
B*c*d+10*C*c^2)*(-3*d*x+c)^3*(-27*d^3*x^3-27*c*d^2*x^2+4*c^3)^(3/2)/d^3/(3
*d*x+2*c)^3+2/297*(9*A*d^2+30*B*c*d+46*C*c^2)*(-3*d*x+c)^4*(-27*d^3*x^3-27
*c*d^2*x^2+4*c^3)^(3/2)/d^3/(3*d*x+2*c)^3-2/351*(3*B*d+11*C*c)*(-3*d*x+c)^
5*(-27*d^3*x^3-27*c*d^2*x^2+4*c^3)^(3/2)/d^3/(3*d*x+2*c)^3+2/405*C*(-3*d*x
+c)^6*(-27*d^3*x^3-27*c*d^2*x^2+4*c^3)^(3/2)/d^3/(3*d*x+2*c)^3
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.40

$$\int (A+Bx+Cx^2)\left(4c^3-27cd^2x^2-27d^3x^3\right)^{3/2}\,dx =$$
$$-\frac{2(c-3dx)^3(2c+3dx)\left(6400c^5C+48c^4d(801B+1000Cx)+1701d^5x^3(195A+11x(15B+13Cx))+56\right.}{405405d}$$

input
```
Integrate[(A + B*x + C*x^2)*(4*c^3 - 27*c*d^2*x^2 - 27*d^3*x^3)^(3/2),x]
```

output
```
(-2*(c - 3*d*x)^3*(2*c + 3*d*x)*(6400*c^5*C + 48*c^4*d*(801*B + 1000*C*x)
+ 1701*d^5*x^3*(195*A + 11*x*(15*B + 13*C*x)) + 5670*c*d^4*x^2*(156*A + x*
(129*B + 110*C*x)) + 540*c^2*d^3*x*(1599*A + 7*x*(186*B + 157*C*x)) + 72*c
^3*d^2*(4602*A + 5*x*(801*B + 700*C*x))))/(405405*d^3*Sqrt[(c - 3*d*x)*(2*
c + 3*d*x)^2])
```

Rubi [A] (verified)

Time = 1.40 (sec) , antiderivative size = 389, normalized size of antiderivative = 0.98, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.184$, Rules used = {2526, 27, 2490, 2483, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(4c^3-27cd^2x^2-27d^3x^3\right)^{3/2}\left(A+Bx+Cx^2\right)dx$$

$$\Big\downarrow \text{ 2526}$$

$$-\frac{\int -27d^2(3Ad-(2cC-3Bd)x)\left(4c^3-27d^2x^2c-27d^3x^3\right)^{3/2}dx}{81d^3}-\frac{2C\left(4c^3-27cd^2x^2-27d^3x^3\right)^{5/2}}{405d^3}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int (3Ad-(2cC-3Bd)x)\left(4c^3-27d^2x^2c-27d^3x^3\right)^{3/2}dx}{3d}-\frac{2C\left(4c^3-27cd^2x^2-27d^3x^3\right)^{5/2}}{405d^3}$$

$$\Big\downarrow \text{ 2490}$$

$$\frac{\int \left((3Bd-2cC)\left(\frac{c}{3d}+x\right)-\frac{27cd^2(3Bd-2cC)-243Ad^4}{81d^3}\right)\left(2c^3+9d\left(\frac{c}{3d}+x\right)c^2-27d^3\left(\frac{c}{3d}+x\right)^3\right)^{3/2}d\left(\frac{c}{3d}+x\right)}{3d}-\frac{2C\left(4c^3-27cd^2x^2-27d^3x^3\right)^{5/2}}{405d^3}$$

$$\Big\downarrow \text{ 2483}$$

$$\frac{\left(2c^3+9c^2d\left(\frac{c}{3d}+x\right)-27d^3\left(\frac{c}{3d}+x\right)^3\right)^{3/2}\int -216\sqrt{3}c^6\left(c+3d\left(\frac{c}{3d}+x\right)\right)^3\left(2c^3-3c^2d\left(\frac{c}{3d}+x\right)\right)^{3/2}\left(-\frac{2Cc^2}{d}+3B}{1944\sqrt{3}c^6d\left(3d\left(\frac{c}{3d}+x\right)+c\right)^3\left(2c^3-3c^2d\left(\frac{c}{3d}+x\right)\right)^{3/2}}$$

$$\frac{2C\left(4c^3-27cd^2x^2-27d^3x^3\right)^{5/2}}{405d^3}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{\left(2c^3+9c^2d\left(\frac{c}{3d}+x\right)-27d^3\left(\frac{c}{3d}+x\right)^3\right)^{3/2}\int \left(c+3d\left(\frac{c}{3d}+x\right)\right)^3\left(2c^3-3c^2d\left(\frac{c}{3d}+x\right)\right)^{3/2}\left(-\frac{2Cc^2}{d}+3Bc-9Ad}{9d\left(3d\left(\frac{c}{3d}+x\right)+c\right)^3\left(2c^3-3c^2d\left(\frac{c}{3d}+x\right)\right)^{3/2}}$$

$$\frac{2C\left(4c^3-27cd^2x^2-27d^3x^3\right)^{5/2}}{405d^3}$$

$$\Big\downarrow \text{ 86}$$

$$-\frac{\left(2c^3+9c^2d\left(\frac{c}{3d}+x\right)-27d^3\left(\frac{c}{3d}+x\right)^3\right)^{3/2}\int\left(\frac{(2cC-3Bd)\left(2c^3-3c^2d\left(\frac{c}{3d}+x\right)\right)^{11/2}}{c^8d}+\frac{(-20Cc^2+30Bdc+9Ad^2)\left(2c^3-3c^2d\left(\frac{c}{3d}\right.\right.}{c^6d}}{9d\left(3d\left(\right.\right.}$$

$$\frac{2C\left(4c^3-27cd^2x^2-27d^3x^3\right)^{5/2}}{405d^3}$$

$\Big\downarrow$ 2009

$$-\frac{\left(2c^3+9c^2d\left(\frac{c}{3d}+x\right)-27d^3\left(\frac{c}{3d}+x\right)^3\right)^{3/2}\left(\frac{18\left(2c^3-3c^2d\left(\frac{c}{3d}+x\right)\right)^{7/2}(-9Ad^2-6Bcd+4c^2C)}{7c^2d^2}-\frac{18c\left(2c^3-3c^2d\left(\frac{c}{3d}+x\right)\right)^{5/2}(-9\right.}{5d^2}}{9d\left(3d\left(\frac{c}{3d}\right.\right.}$$

$$\frac{2C\left(4c^3-27cd^2x^2-27d^3x^3\right)^{5/2}}{405d^3}$$

input
```
Int[(A + B*x + C*x^2)*(4*c^3 - 27*c*d^2*x^2 - 27*d^3*x^3)^(3/2),x]
```

output
```
(-2*C*(4*c^3 - 27*c*d^2*x^2 - 27*d^3*x^3)^(5/2))/(405*d^3) - ((2*c^3 + 9*c
^2*d*(c/(3*d) + x) - 27*d^3*(c/(3*d) + x)^3)^(3/2)*((-18*c*(2*c^2*C - 3*B*
c*d - 9*A*d^2)*(2*c^3 - 3*c^2*d*(c/(3*d) + x))^(5/2))/(5*d^2) + (18*(4*c^2
*C - 6*B*c*d - 9*A*d^2)*(2*c^3 - 3*c^2*d*(c/(3*d) + x))^(7/2))/(7*c^2*d^2)
 - (2*(8*c^2*C - 12*B*c*d - 9*A*d^2)*(2*c^3 - 3*c^2*d*(c/(3*d) + x))^(9/2)
)/(3*c^5*d^2) + (2*(20*c^2*C - 30*B*c*d - 9*A*d^2)*(2*c^3 - 3*c^2*d*(c/(3*
d) + x))^(11/2))/(33*c^8*d^2) - (2*(2*c*C - 3*B*d)*(2*c^3 - 3*c^2*d*(c/(3*
d) + x))^(13/2))/(39*c^10*d^2)))/(9*d*(c + 3*d*(c/(3*d) + x))^3*(2*c^3 - 3
*c^2*d*(c/(3*d) + x))^(3/2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2483
```
Int[((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (d_.)*(x_)^3)^(p_), x_S
ymbol] :> Simp[(a + b*x + d*x^3)^p/((3*a - b*x)^p*(3*a + 2*b*x)^(2*p))   In
t[(e + f*x)^m*(3*a - b*x)^p*(3*a + 2*b*x)^(2*p), x], x] /; FreeQ[{a, b, d,
e, f, m, p}, x] && EqQ[4*b^3 + 27*a^2*d, 0] &&  !IntegerQ[p]
```

rule 2490
```
Int[(P3_)^(p_.)*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] :> With[{a = Coeff[P3
, x, 0], b = Coeff[P3, x, 1], c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Su
bst[Int[((3*d*e - c*f)/(3*d) + f*x)^m*Simp[(2*c^3 - 9*b*c*d + 27*a*d^2)/(27
*d^2) - (c^2 - 3*b*d)*(x/(3*d)) + d*x^3, x]^p, x], x, x + c/(3*d)] /; NeQ[c
, 0]] /; FreeQ[{e, f, m, p}, x] && PolyQ[P3, x, 3]
```

rule 2526
```
Int[(Pm_)*(Qn_)^(p_), x_Symbol] :> With[{m = Expon[Pm, x], n = Expon[Qn, x]
}, Simp[Coeff[Pm, x, m]*(Qn^(p + 1)/(n*(p + 1)*Coeff[Qn, x, n])), x] + Simp
[1/(n*Coeff[Qn, x, n])   Int[ExpandToSum[n*Coeff[Qn, x, n]*Pm - Coeff[Pm, x
, m]*D[Qn, x], x]*Qn^p, x], x] /; EqQ[m, n - 1]] /; FreeQ[p, x] && PolyQ[Pm
, x] && PolyQ[Qn, x] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.48

method	result
gosper	$-\frac{2(-3dx+c)(243243C\,d^5x^5+280665B\,d^5x^4+623700Cc\,d^4x^4+331695A\,d^5x^3+731430Bc\,d^4x^3+593460C\,c^2d^3x^3+884520Ac\,d^4x^2}{}$
default	$-\frac{2(-3dx+c)(243243C\,d^5x^5+280665B\,d^5x^4+623700Cc\,d^4x^4+331695A\,d^5x^3+731430Bc\,d^4x^3+593460C\,c^2d^3x^3+884520Ac\,d^4x^2}{}$
orering	$-\frac{2(-3dx+c)(243243C\,d^5x^5+280665B\,d^5x^4+623700Cc\,d^4x^4+331695A\,d^5x^3+731430Bc\,d^4x^3+593460C\,c^2d^3x^3+884520Ac\,d^4x^2}{}$
trager	$-\frac{2(2189187C\,d^7x^7+2525985B\,d^7x^6+4153842Cc\,d^6x^6+2985255A\,d^7x^5+4898880Bc\,d^6x^5+1842183C\,c^2d^5x^5+5970510Ac\,d^6x^4+}{}$

input
```
int((C*x^2+B*x+A)*(-27*d^3*x^3-27*c*d^2*x^2+4*c^3)^(3/2),x,method=_RETURNV
ERBOSE)
```

output
```
-2/405405*(-3*d*x+c)*(243243*C*d^5*x^5+280665*B*d^5*x^4+623700*C*c*d^4*x^4
+331695*A*d^5*x^3+731430*B*c*d^4*x^3+593460*C*c^2*d^3*x^3+884520*A*c*d^4*x
^2+703080*B*c^2*d^3*x^2+252000*C*c^3*d^2*x^2+863460*A*c^2*d^3*x+288360*B*c
^3*d^2*x+48000*C*c^4*d*x+331344*A*c^3*d^2+38448*B*c^4*d+6400*C*c^5)*(-27*d
^3*x^3-27*c*d^2*x^2+4*c^3)^(3/2)/d^3/(3*d*x+2*c)^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 247, normalized size of antiderivative = 0.62

$$\int (A+Bx+Cx^2)\left(4c^3-27cd^2x^2-27d^3x^3\right)^{3/2}\,dx =$$
$$-\frac{2\left(2189187\,Cd^7x^7+6400\,Cc^7+38448\,Bc^6d+331344\,Ac^5d^2+56133\,(74\,Ccd^6+45\,Bd^7)x^6+5103\,(361\right.}{}$$

input
```
integrate((C*x^2+B*x+A)*(-27*d^3*x^3-27*c*d^2*x^2+4*c^3)^(3/2),x, algorith
m="fricas")
```

output
```
-2/405405*(2189187*C*d^7*x^7 + 6400*C*c^7 + 38448*B*c^6*d + 331344*A*c^5*d
^2 + 56133*(74*C*c*d^6 + 45*B*d^7)*x^6 + 5103*(361*C*c^2*d^5 + 960*B*c*d^6
 + 585*A*d^7)*x^5 - 2835*(236*C*c^3*d^4 - 783*B*c^2*d^5 - 2106*A*c*d^6)*x^
4 - 135*(3604*C*c^4*d^3 + 6606*B*c^3*d^4 - 20709*A*c^2*d^5)*x^3 + 216*(100
*C*c^5*d^2 - 3153*B*c^4*d^3 - 6084*A*c^3*d^4)*x^2 + 12*(800*C*c^6*d + 4806
*B*c^5*d^2 - 93717*A*c^4*d^3)*x)*sqrt(-27*d^3*x^3 - 27*c*d^2*x^2 + 4*c^3)/
(3*d^4*x + 2*c*d^3)
```

Sympy [F]

$$\int (A+Bx+Cx^2)\left(4c^3-27cd^2x^2-27d^3x^3\right)^{3/2}\,dx=\int\left(-(-c+3dx)\,(2c+3dx)^2\right)^{\frac{3}{2}}(A+Bx+Cx^2)\,dx$$

input
```
integrate((C*x**2+B*x+A)*(-27*d**3*x**3-27*c*d**2*x**2+4*c**3)**(3/2),x)
```

output
```
Integral((-(-c + 3*d*x)*(2*c + 3*d*x)**2)**(3/2)*(A + B*x + C*x**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.58

$$\int (A+Bx+Cx^2)\left(4c^3-27cd^2x^2-27d^3x^3\right)^{3/2}\,dx = -\frac{2\,(8505\,d^5x^5+17010\,cd^4x^4+7965\,c^2d^3x^3-3744\,c^3d^2x^2-3204\,c^4dx+944\,c^5)\sqrt{-3\,dx+c}A}{1155\,d} - \frac{2\,(93555\,d^6x^6+181440\,cd^5x^5+82215\,c^2d^4x^4-33030\,c^3d^3x^3-25224\,c^4d^2x^2+2136\,c^5dx+1424\,c^6)\sqrt{-}}{15015\,d^2} - \frac{2\,(2189187\,d^7x^7+4153842\,cd^6x^6+1842183\,c^2d^5x^5-669060\,c^3d^4x^4-486540\,c^4d^3x^3+21600\,c^5d^2x^2+9}{405405\,d^3}$$

input
```
integrate((C*x^2+B*x+A)*(-27*d^3*x^3-27*c*d^2*x^2+4*c^3)^(3/2),x, algorith
m="maxima")
```

output
```
-2/1155*(8505*d^5*x^5 + 17010*c*d^4*x^4 + 7965*c^2*d^3*x^3 - 3744*c^3*d^2*
x^2 - 3204*c^4*d*x + 944*c^5)*sqrt(-3*d*x + c)*A/d - 2/15015*(93555*d^6*x^
6 + 181440*c*d^5*x^5 + 82215*c^2*d^4*x^4 - 33030*c^3*d^3*x^3 - 25224*c^4*d
^2*x^2 + 2136*c^5*d*x + 1424*c^6)*sqrt(-3*d*x + c)*B/d^2 - 2/405405*(21891
87*d^7*x^7 + 4153842*c*d^6*x^6 + 1842183*c^2*d^5*x^5 - 669060*c^3*d^4*x^4
- 486540*c^4*d^3*x^3 + 21600*c^5*d^2*x^2 + 9600*c^6*d*x + 6400*c^7)*sqrt(-
3*d*x + c)*C/d^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1733 vs. $2(386) = 772$.

Time = 0.14 (sec) , antiderivative size = 1733, normalized size of antiderivative = 4.35

$$\int (A + Bx + Cx^2)\left(4c^3 - 27cd^2x^2 - 27d^3x^3\right)^{3/2} dx = \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)*(-27*d^3*x^3-27*c*d^2*x^2+4*c^3)^(3/2),x, algorith
m="giac")
```

output
```
2/1216215*(3243240*sqrt(-3*d*x + c)*A*c^5*sgn(-3*d*x - 2*c) + 540540*((-3*
d*x + c)^(3/2) - 3*sqrt(-3*d*x + c)*c)*A*c^4*sgn(-3*d*x - 2*c) - 360360*((
-3*d*x + c)^(3/2) - 3*sqrt(-3*d*x + c)*c)*B*c^5*sgn(-3*d*x - 2*c)/d - 2702
70*(3*(3*d*x - c)^2*sqrt(-3*d*x + c) - 10*(-3*d*x + c)^(3/2)*c + 15*sqrt(-
3*d*x + c)*c^2)*A*c^3*sgn(-3*d*x - 2*c) + 24024*(3*(3*d*x - c)^2*sqrt(-3*d
*x + c) - 10*(-3*d*x + c)^(3/2)*c + 15*sqrt(-3*d*x + c)*c^2)*C*c^5*sgn(-3*
d*x - 2*c)/d^2 - 36036*(3*(3*d*x - c)^2*sqrt(-3*d*x + c) - 10*(-3*d*x + c)
^(3/2)*c + 15*sqrt(-3*d*x + c)*c^2)*B*c^4*sgn(-3*d*x - 2*c)/d + 11583*(5*(
3*d*x - c)^3*sqrt(-3*d*x + c) + 21*(3*d*x - c)^2*sqrt(-3*d*x + c)*c - 35*(
-3*d*x + c)^(3/2)*c^2 + 35*sqrt(-3*d*x + c)*c^3)*A*c^2*sgn(-3*d*x - 2*c) -
 5148*(5*(3*d*x - c)^3*sqrt(-3*d*x + c) + 21*(3*d*x - c)^2*sqrt(-3*d*x + c
)*c - 35*(-3*d*x + c)^(3/2)*c^2 + 35*sqrt(-3*d*x + c)*c^3)*C*c^4*sgn(-3*d*
x - 2*c)/d^2 - 38610*(5*(3*d*x - c)^3*sqrt(-3*d*x + c) + 21*(3*d*x - c)^2*
sqrt(-3*d*x + c)*c - 35*(-3*d*x + c)^(3/2)*c^2 + 35*sqrt(-3*d*x + c)*c^3)*
B*c^3*sgn(-3*d*x - 2*c)/d + 5148*(35*(3*d*x - c)^4*sqrt(-3*d*x + c) + 180*
(3*d*x - c)^3*sqrt(-3*d*x + c)*c + 378*(3*d*x - c)^2*sqrt(-3*d*x + c)*c^2
- 420*(-3*d*x + c)^(3/2)*c^3 + 315*sqrt(-3*d*x + c)*c^4)*A*c*sgn(-3*d*x -
2*c) - 1430*(35*(3*d*x - c)^4*sqrt(-3*d*x + c) + 180*(3*d*x - c)^3*sqrt(-3
*d*x + c)*c + 378*(3*d*x - c)^2*sqrt(-3*d*x + c)*c^2 - 420*(-3*d*x + c)^(3
/2)*c^3 + 315*sqrt(-3*d*x + c)*c^4)*C*c^3*sgn(-3*d*x - 2*c)/d^2 + 429*(...
```

Mupad [B] (verification not implemented)

Time = 12.61 (sec) , antiderivative size = 341, normalized size of antiderivative = 0.86

$$\int \left(A+Bx+Cx^2\right)\left(4c^3-27cd^2x^2-27d^3x^3\right)^{3/2}\,dx = -\frac{\sqrt{4\,c^3-27\,c\,d^2\,x^2-27\,d^3\,x^3}\,\left(\frac{1888\,A\,c^5}{3465\,d^2}-\frac{832\,A\,c^3\,x^2}{385}+\frac{54\,A\,d^3\,x^5}{11}+\frac{354\,A\,c^2\,d\,x^3}{77}-\frac{712\,A\,c^4\,x}{385\,d}+\frac{108\,A\,c\,d^2\,x^4}{11}\right)}{x+\frac{2\,c}{3\,d}} - \frac{\sqrt{4\,c^3-27\,c\,d^2\,x^2-27\,d^3\,x^3}\,\left(\frac{2848\,B\,c^6}{45045\,d^3}-\frac{1468\,B\,c^3\,x^3}{1001}+\frac{54\,B\,d^3\,x^6}{13}+\frac{522\,B\,c^2\,d\,x^4}{143}+\frac{1152\,B\,c\,d^2\,x^5}{143}+\frac{1424\,B\,c^5\,x}{15015\,d^2}-16\right.}{x+\frac{2\,c}{3\,d}} - \frac{2\,C\,\sqrt{4\,c^3-27\,c\,d^2\,x^2-27\,d^3\,x^3}\,\left(-4576\,c^6+11664\,c^5\,d\,x-6696\,c^4\,d^2\,x^2-233226\,c^3\,d^3\,x^3+15309\,c^2\,d^4\right.}{405405\,d^3} - \frac{384\,C\,c^7\,\sqrt{4\,c^3-27\,c\,d^2\,x^2-27\,d^3\,x^3}}{5005\,d^3\,\left(2\,c+3\,d\,x\right)}$$

input
```
int((A + B*x + C*x^2)*(4*c^3 - 27*d^3*x^3 - 27*c*d^2*x^2)^(3/2),x)
```

output
```
- ((4*c^3 - 27*d^3*x^3 - 27*c*d^2*x^2)^(1/2)*((1888*A*c^5)/(3465*d^2) - (8
32*A*c^3*x^2)/385 + (54*A*d^3*x^5)/11 + (354*A*c^2*d*x^3)/77 - (712*A*c^4*
x)/(385*d) + (108*A*c*d^2*x^4)/11))/(x + (2*c)/(3*d)) - ((4*c^3 - 27*d^3*x
^3 - 27*c*d^2*x^2)^(1/2)*((2848*B*c^6)/(45045*d^3) - (1468*B*c^3*x^3)/1001
 + (54*B*d^3*x^6)/13 + (522*B*c^2*d*x^4)/143 + (1152*B*c*d^2*x^5)/143 + (1
424*B*c^5*x)/(15015*d^2) - (16816*B*c^4*x^2)/(15015*d)))/(x + (2*c)/(3*d))
 - (2*C*(4*c^3 - 27*d^3*x^3 - 27*c*d^2*x^2)^(1/2)*(729729*d^6*x^6 - 4576*c
^6 + 898128*c*d^5*x^5 - 6696*c^4*d^2*x^2 - 233226*c^3*d^3*x^3 + 15309*c^2*
d^4*x^4 + 11664*c^5*d*x))/(405405*d^3) - (384*C*c^7*(4*c^3 - 27*d^3*x^3 -
27*c*d^2*x^2)^(1/2))/(5005*d^3*(2*c + 3*d*x))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.56

$$\int (A+Bx+Cx^2)\left(4c^3-27cd^2x^2-27d^3x^3\right)^{3/2}dx = \frac{2\sqrt{-3dx+c}\,(-2189187c\,d^7x^7-2525985b\,d^7x^6-4153842c^2d^6x^6-2985255a\,d^7x^5-48$$

input
```
int((C*x^2+B*x+A)*(-27*d^3*x^3-27*c*d^2*x^2+4*c^3)^(3/2),x)
```

output
```
(2*sqrt(c - 3*d*x)*( - 331344*a*c**5*d**2 + 1124604*a*c**4*d**3*x + 131414
4*a*c**3*d**4*x**2 - 2795715*a*c**2*d**5*x**3 - 5970510*a*c*d**6*x**4 - 29
85255*a*d**7*x**5 - 38448*b*c**6*d - 57672*b*c**5*d**2*x + 681048*b*c**4*d
**3*x**2 + 891810*b*c**3*d**4*x**3 - 2219805*b*c**2*d**5*x**4 - 4898880*b*
c*d**6*x**5 - 2525985*b*d**7*x**6 - 6400*c**8 - 9600*c**7*d*x - 21600*c**6
*d**2*x**2 + 486540*c**5*d**3*x**3 + 669060*c**4*d**4*x**4 - 1842183*c**3*
d**5*x**5 - 4153842*c**2*d**6*x**6 - 2189187*c*d**7*x**7))/(405405*d**3)
```

3.37 $\int (A+Bx+Cx^2)\sqrt{4c^3-27cd^2x^2-27d^3x^3}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 253

$$\int (A+Bx+Cx^2)\sqrt{4c^3-27cd^2x^2-27d^3x^3}\,dx$$
$$= -\frac{2c(c^2C+3Bcd+9Ad^2)(c-3dx)\sqrt{4c^3-27cd^2x^2-27d^3x^3}}{27d^3(2c+3dx)}$$
$$+\frac{2(7c^2C+12Bcd+9Ad^2)(c-3dx)^2\sqrt{4c^3-27cd^2x^2-27d^3x^3}}{135d^3(2c+3dx)}$$
$$-\frac{2(5cC+3Bd)(c-3dx)^3\sqrt{4c^3-27cd^2x^2-27d^3x^3}}{189d^3(2c+3dx)}$$
$$+\frac{2C(c-3dx)^4\sqrt{4c^3-27cd^2x^2-27d^3x^3}}{243d^3(2c+3dx)}$$

output
```
-2/27*c*(9*A*d^2+3*B*c*d+C*c^2)*(-3*d*x+c)*(-27*d^3*x^3-27*c*d^2*x^2+4*c^3
)^(1/2)/d^3/(3*d*x+2*c)+2/135*(9*A*d^2+12*B*c*d+7*C*c^2)*(-3*d*x+c)^2*(-27
*d^3*x^3-27*c*d^2*x^2+4*c^3)^(1/2)/d^3/(3*d*x+2*c)-2/189*(3*B*d+5*C*c)*(-3
*d*x+c)^3*(-27*d^3*x^3-27*c*d^2*x^2+4*c^3)^(1/2)/d^3/(3*d*x+2*c)+2/243*C*(
-3*d*x+c)^4*(-27*d^3*x^3-27*c*d^2*x^2+4*c^3)^(1/2)/d^3/(3*d*x+2*c)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.40

$$\int \left(A+Bx+Cx^2\right)\sqrt{4c^3-27cd^2x^2-27d^3x^3}\,dx = -\frac{2\left((c-3dx)(2c+3dx)^2\right)^{3/2}(64c^3C+36c^2d(9B+8Cx)+27d^3x(63A+5x(9B+7Cx))+54cd^2(42A}{8505d^3(2c+3dx)^3}$$

input
```
Integrate[(A + B*x + C*x^2)*Sqrt[4*c^3 - 27*c*d^2*x^2 - 27*d^3*x^3],x]
```

output
```
(-2*((c - 3*d*x)*(2*c + 3*d*x)^2)^(3/2)*(64*c^3*C + 36*c^2*d*(9*B + 8*C*x)
 + 27*d^3*x*(63*A + 5*x*(9*B + 7*C*x)) + 54*c*d^2*(42*A + x*(27*B + 20*C*x
))))/(8505*d^3*(2*c + 3*d*x)^3)
```

Rubi [A] (verified)

Time = 1.15 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.12, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.184$, Rules used = {2526, 27, 2490, 2483, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{4c^3-27cd^2x^2-27d^3x^3}\left(A+Bx+Cx^2\right)\,dx$$

$$\Big\downarrow \text{2526}$$

$$-\frac{\int -27d^2(3Ad-(2cC-3Bd)x)\sqrt{4c^3-27d^2x^2c-27d^3x^3}dx}{81d^3}-\frac{2C\left(4c^3-27cd^2x^2-27d^3x^3\right)^{3/2}}{243d^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int (3Ad-(2cC-3Bd)x)\sqrt{4c^3-27d^2x^2c-27d^3x^3}dx}{3d}-\frac{2C\left(4c^3-27cd^2x^2-27d^3x^3\right)^{3/2}}{243d^3}$$

$$\Big\downarrow \text{2490}$$

$$\frac{\int \left((3Bd-2cC)\left(\frac{c}{3d}+x\right)-\frac{27cd^2(3Bd-2cC)-243Ad^4}{81d^3}\right)\sqrt{2c^3+9d\left(\frac{c}{3d}+x\right)c^2-27d^3\left(\frac{c}{3d}+x\right)^3}d\left(\frac{c}{3d}+x\right)}{3d}-\frac{2C\left(4c^3-27cd^2x^2-27d^3x^3\right)^{3/2}}{243d^3}$$

$\Big\downarrow$ 2483

$$\frac{\sqrt{2c^3+9c^2d\left(\frac{c}{3d}+x\right)-27d^3\left(\frac{c}{3d}+x\right)^3}\int -2\sqrt{3}c^2\left(c+3d\left(\frac{c}{3d}+x\right)\right)\sqrt{2c^3-3c^2d\left(\frac{c}{3d}+x\right)}\left(-\frac{2Cc^2}{d}+3Bc-9Ad\right.}{18\sqrt{3}c^2d\left(3d\left(\frac{c}{3d}+x\right)+c\right)\sqrt{2c^3-3c^2d\left(\frac{c}{3d}+x\right)}}$$
$$\frac{2C\left(4c^3-27cd^2x^2-27d^3x^3\right)^{3/2}}{243d^3}$$

$\Big\downarrow$ 27

$$-\frac{\sqrt{2c^3+9c^2d\left(\frac{c}{3d}+x\right)-27d^3\left(\frac{c}{3d}+x\right)^3}\int\left(c+3d\left(\frac{c}{3d}+x\right)\right)\sqrt{2c^3-3c^2d\left(\frac{c}{3d}+x\right)}\left(-\frac{2Cc^2}{d}+3Bc-9Ad+3(2c\right.}{9d\left(3d\left(\frac{c}{3d}+x\right)+c\right)\sqrt{2c^3-3c^2d\left(\frac{c}{3d}+x\right)}}$$
$$\frac{2C\left(4c^3-27cd^2x^2-27d^3x^3\right)^{3/2}}{243d^3}$$

$\Big\downarrow$ 86

$$-\frac{\sqrt{2c^3+9c^2d\left(\frac{c}{3d}+x\right)-27d^3\left(\frac{c}{3d}+x\right)^3}\int\left(\frac{(2cC-3Bd)\left(2c^3-3c^2d\left(\frac{c}{3d}+x\right)\right)^{5/2}}{c^4d}+\frac{\left(-8Cc^2+12Bdc+9Ad^2\right)\left(2c^3-3c^2d\left(\frac{c}{3d}+x\right)\right)^{3}}{c^2d}\right.}{9d\left(3d\left(\frac{c}{3d}+x\right)+c\right)\sqrt{2c^3-3c^2d\left(\frac{c}{3d}+x\right)}}$$
$$\frac{2C\left(4c^3-27cd^2x^2-27d^3x^3\right)^{3/2}}{243d^3}$$

$\Big\downarrow$ 2009

$$-\frac{\sqrt{2c^3+9c^2d\left(\frac{c}{3d}+x\right)-27d^3\left(\frac{c}{3d}+x\right)^3}\left(-\frac{2\left(2c^3-3c^2d\left(\frac{c}{3d}+x\right)\right)^{3/2}\left(-9Ad^2-3Bcd+2c^2C\right)}{3cd^2}+\frac{2\left(2c^3-3c^2d\left(\frac{c}{3d}+x\right)\right)^{5/2}\left(-9Ad^2-\right.}{15c^4d^2}\right.}{9d\left(3d\left(\frac{c}{3d}+x\right)+c\right)\sqrt{2c^3-3c^2d\left(\frac{c}{3d}+x\right)}}$$
$$\frac{2C\left(4c^3-27cd^2x^2-27d^3x^3\right)^{3/2}}{243d^3}$$

input
```
Int[(A + B*x + C*x^2)*Sqrt[4*c^3 - 27*c*d^2*x^2 - 27*d^3*x^3],x]
```

output
```
(-2*C*(4*c^3 - 27*c*d^2*x^2 - 27*d^3*x^3)^(3/2))/(243*d^3) - (Sqrt[2*c^3 +
 9*c^2*d*(c/(3*d) + x) - 27*d^3*(c/(3*d) + x)^3]*((-2*(2*c^2*C - 3*B*c*d -
 9*A*d^2)*(2*c^3 - 3*c^2*d*(c/(3*d) + x))^(3/2))/(3*c*d^2) + (2*(8*c^2*C -
 12*B*c*d - 9*A*d^2)*(2*c^3 - 3*c^2*d*(c/(3*d) + x))^(5/2))/(15*c^4*d^2) -
 (2*(2*c*C - 3*B*d)*(2*c^3 - 3*c^2*d*(c/(3*d) + x))^(7/2))/(21*c^6*d^2)))/
(9*d*(c + 3*d*(c/(3*d) + x))*Sqrt[2*c^3 - 3*c^2*d*(c/(3*d) + x)])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2483
```
Int[((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (d_.)*(x_)^3)^(p_), x_S
ymbol] :> Simp[(a + b*x + d*x^3)^p/((3*a - b*x)^p*(3*a + 2*b*x)^(2*p))   In
t[(e + f*x)^m*(3*a - b*x)^p*(3*a + 2*b*x)^(2*p), x], x] /; FreeQ[{a, b, d,
e, f, m, p}, x] && EqQ[4*b^3 + 27*a^2*d, 0] &&  !IntegerQ[p]
```

rule 2490
```
Int[(P3_)^(p_.)*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] :> With[{a = Coeff[P3
, x, 0], b = Coeff[P3, x, 1], c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Su
bst[Int[((3*d*e - c*f)/(3*d) + f*x)^m*Simp[(2*c^3 - 9*b*c*d + 27*a*d^2)/(27
*d^2) - (c^2 - 3*b*d)*(x/(3*d)) + d*x^3, x]^p, x], x, x + c/(3*d)] /; NeQ[c
, 0]] /; FreeQ[{e, f, m, p}, x] && PolyQ[P3, x, 3]
```

rule 2526
```
Int[(Pm_)*(Qn_)^(p_), x_Symbol] :> With[{m = Expon[Pm, x], n = Expon[Qn, x]
}, Simp[Coeff[Pm, x, m]*(Qn^(p + 1)/(n*(p + 1)*Coeff[Qn, x, n])), x] + Simp
[1/(n*Coeff[Qn, x, n])   Int[ExpandToSum[n*Coeff[Qn, x, n]*Pm - Coeff[Pm, x
, m]*D[Qn, x], x]*Qn^p, x], x] /; EqQ[m, n - 1]] /; FreeQ[p, x] && PolyQ[Pm
, x] && PolyQ[Qn, x] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.47

method	result
gosper	$-\frac{2(-3dx+c)\left(945C\,x^{3}d^{3}+1215B\,d^{3}x^{2}+1080Cc\,d^{2}x^{2}+1701A\,d^{3}x+1458Bc\,d^{2}x+288C\,c^{2}dx+2268Ac\,d^{2}+324d\,c^{2}B+64C\,c^{3}\right)\sqrt{-2}}{8505d^{3}(3dx+2c)}$
default	$-\frac{2(-3dx+c)\left(945C\,x^{3}d^{3}+1215B\,d^{3}x^{2}+1080Cc\,d^{2}x^{2}+1701A\,d^{3}x+1458Bc\,d^{2}x+288C\,c^{2}dx+2268Ac\,d^{2}+324d\,c^{2}B+64C\,c^{3}\right)\sqrt{-2}}{8505d^{3}(3dx+2c)}$
orering	$-\frac{2(-3dx+c)\left(945C\,x^{3}d^{3}+1215B\,d^{3}x^{2}+1080Cc\,d^{2}x^{2}+1701A\,d^{3}x+1458Bc\,d^{2}x+288C\,c^{2}dx+2268Ac\,d^{2}+324d\,c^{2}B+64C\,c^{3}\right)\sqrt{-2}}{8505d^{3}(3dx+2c)}$
trager	$-\frac{2(-2835C\,d^{4}x^{4}-3645B\,d^{4}x^{3}-2295Cc\,d^{3}x^{3}-5103A\,d^{4}x^{2}-3159Bc\,d^{3}x^{2}+216C\,c^{2}d^{2}x^{2}-5103A\,d^{3}cx+486c^{2}d^{2}Bx+96C\,c^{3}dx+2}{8505(3dx+2c)d^{3}}$

input
```
int((C*x^2+B*x+A)*(-27*d^3*x^3-27*c*d^2*x^2+4*c^3)^(1/2),x,method=_RETURNV
ERBOSE)
```

output
```
-2/8505*(-3*d*x+c)*(945*C*d^3*x^3+1215*B*d^3*x^2+1080*C*c*d^2*x^2+1701*A*d
^3*x+1458*B*c*d^2*x+288*C*c^2*d*x+2268*A*c*d^2+324*B*c^2*d+64*C*c^3)*(-27*
d^3*x^3-27*c*d^2*x^2+4*c^3)^(1/2)/d^3/(3*d*x+2*c)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.58

$$\int \left(A+Bx+Cx^2\right)\sqrt{4c^3-27cd^2x^2-27d^3x^3}\,dx$$
$$=\frac{2\left(2835\,Cd^4x^4-64\,Cc^4-324\,Bc^3d-2268\,Ac^2d^2+135\,(17\,Ccd^3+27\,Bd^4)x^3-27\,(8\,Cc^2d^2-117\,Bcd\right.}{8505\,(3\,d^4x+2\,cd^3}$$

input
```
integrate((C*x^2+B*x+A)*(-27*d^3*x^3-27*c*d^2*x^2+4*c^3)^(1/2),x, algorith
m="fricas")
```

output
```
2/8505*(2835*C*d^4*x^4 - 64*C*c^4 - 324*B*c^3*d - 2268*A*c^2*d^2 + 135*(17
*C*c*d^3 + 27*B*d^4)*x^3 - 27*(8*C*c^2*d^2 - 117*B*c*d^3 - 189*A*d^4)*x^2
- 3*(32*C*c^3*d + 162*B*c^2*d^2 - 1701*A*c*d^3)*x)*sqrt(-27*d^3*x^3 - 27*c
*d^2*x^2 + 4*c^3)/(3*d^4*x + 2*c*d^3)
```

Sympy [F]

$$\int \left(A+Bx+Cx^2\right)\sqrt{4c^3-27cd^2x^2-27d^3x^3}\,dx$$
$$=\int \sqrt{-\left(-c+3dx\right)\left(2c+3dx\right)^2}\left(A+Bx+Cx^2\right)\,dx$$

input
```
integrate((C*x**2+B*x+A)*(-27*d**3*x**3-27*c*d**2*x**2+4*c**3)**(1/2),x)
```

output
```
Integral(sqrt(-(-c + 3*d*x)*(2*c + 3*d*x)**2)*(A + B*x + C*x**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.53

$$\int \left(A+Bx+Cx^2\right)\sqrt{4c^3-27cd^2x^2-27d^3x^3}\,dx$$
$$=\frac{2\left(9\,d^2x^2+9\,cdx-4\,c^2\right)\sqrt{-3\,dx+c}A}{15\,d}$$
$$+\frac{2\left(45\,d^3x^3+39\,cd^2x^2-6\,c^2dx-4\,c^3\right)\sqrt{-3\,dx+c}B}{105\,d^2}$$
$$+\frac{2\left(2835\,d^4x^4+2295\,cd^3x^3-216\,c^2d^2x^2-96\,c^3dx-64\,c^4\right)\sqrt{-3\,dx+c}C}{8505\,d^3}$$

input
```
integrate((C*x^2+B*x+A)*(-27*d^3*x^3-27*c*d^2*x^2+4*c^3)^(1/2),x, algorith
m="maxima")
```

output
```
2/15*(9*d^2*x^2 + 9*c*d*x - 4*c^2)*sqrt(-3*d*x + c)*A/d + 2/105*(45*d^3*x^
3 + 39*c*d^2*x^2 - 6*c^2*d*x - 4*c^3)*sqrt(-3*d*x + c)*B/d^2 + 2/8505*(283
5*d^4*x^4 + 2295*c*d^3*x^3 - 216*c^2*d^2*x^2 - 96*c^3*d*x - 64*c^4)*sqrt(-
3*d*x + c)*C/d^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 554 vs. $2(245) = 490$.

Time = 0.13 (sec) , antiderivative size = 554, normalized size of antiderivative = 2.19

$$\int (A + Bx + Cx^2) \sqrt{4c^3 - 27cd^2x^2 - 27d^3x^3}\, dx = \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)*(-27*d^3*x^3-27*c*d^2*x^2+4*c^3)^(1/2),x, algorith
m="giac")
```

output
```
2/8505*(5670*sqrt(-3*d*x + c)*A*c^2*sgn(-3*d*x - 2*c) + 945*((-3*d*x + c)^
(3/2) - 3*sqrt(-3*d*x + c)*c)*A*c*sgn(-3*d*x - 2*c) - 630*((-3*d*x + c)^(3
/2) - 3*sqrt(-3*d*x + c)*c)*B*c^2*sgn(-3*d*x - 2*c)/d - 189*(3*(3*d*x - c)
^2*sqrt(-3*d*x + c) - 10*(-3*d*x + c)^(3/2)*c + 15*sqrt(-3*d*x + c)*c^2)*A
*sgn(-3*d*x - 2*c) + 42*(3*(3*d*x - c)^2*sqrt(-3*d*x + c) - 10*(-3*d*x + c
)^(3/2)*c + 15*sqrt(-3*d*x + c)*c^2)*C*c^2*sgn(-3*d*x - 2*c)/d^2 - 63*(3*(
3*d*x - c)^2*sqrt(-3*d*x + c) - 10*(-3*d*x + c)^(3/2)*c + 15*sqrt(-3*d*x +
 c)*c^2)*B*c*sgn(-3*d*x - 2*c)/d - 9*(5*(3*d*x - c)^3*sqrt(-3*d*x + c) + 2
1*(3*d*x - c)^2*sqrt(-3*d*x + c)*c - 35*(-3*d*x + c)^(3/2)*c^2 + 35*sqrt(-
3*d*x + c)*c^3)*C*c*sgn(-3*d*x - 2*c)/d^2 - 27*(5*(3*d*x - c)^3*sqrt(-3*d*
x + c) + 21*(3*d*x - c)^2*sqrt(-3*d*x + c)*c - 35*(-3*d*x + c)^(3/2)*c^2 +
 35*sqrt(-3*d*x + c)*c^3)*B*sgn(-3*d*x - 2*c)/d - (35*(3*d*x - c)^4*sqrt(-
3*d*x + c) + 180*(3*d*x - c)^3*sqrt(-3*d*x + c)*c + 378*(3*d*x - c)^2*sqrt
(-3*d*x + c)*c^2 - 420*(-3*d*x + c)^(3/2)*c^3 + 315*sqrt(-3*d*x + c)*c^4)*
C*sgn(-3*d*x - 2*c)/d^2)/d
```

Mupad [B] (verification not implemented)

Time = 12.35 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.47

$$\int (A+Bx+Cx^2)\sqrt{4c^3-27cd^2x^2-27d^3x^3}\,dx = -\frac{2\,(c-3\,d\,x)\,\sqrt{4\,c^3-27\,c\,d^2\,x^2-27\,d^3\,x^3}\,(64\,C\,c^3+288\,C\,c^2\,d\,x+324\,B\,c^2\,d+1080\,C\,c\,d^2\,x^2+1458}{8505\,d^3\,(2\,c+3\,d\,x)}$$

input
```
int((A + B*x + C*x^2)*(4*c^3 - 27*d^3*x^3 - 27*c*d^2*x^2)^(1/2),x)
```

output
```
-(2*(c - 3*d*x)*(4*c^3 - 27*d^3*x^3 - 27*c*d^2*x^2)^(1/2)*(64*C*c^3 + 1215
*B*d^3*x^2 + 945*C*d^3*x^3 + 2268*A*c*d^2 + 324*B*c^2*d + 1701*A*d^3*x + 1
458*B*c*d^2*x + 288*C*c^2*d*x + 1080*C*c*d^2*x^2))/(8505*d^3*(2*c + 3*d*x)
)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.47

$$\int (A+Bx+Cx^2)\sqrt{4c^3-27cd^2x^2-27d^3x^3}\,dx = \frac{2\sqrt{-3dx+c}\,(2835c\,d^4x^4+3645b\,d^4x^3+2295c^2d^3x^3+5103a\,d^4x^2+3159bc\,d^3x^2-216c^3d^2x^2+5103ac}{8505d^3}$$

input
```
int((C*x^2+B*x+A)*(-27*d^3*x^3-27*c*d^2*x^2+4*c^3)^(1/2),x)
```

output
```
(2*sqrt(c - 3*d*x)*( - 2268*a*c**2*d**2 + 5103*a*c*d**3*x + 5103*a*d**4*x*
*2 - 324*b*c**3*d - 486*b*c**2*d**2*x + 3159*b*c*d**3*x**2 + 3645*b*d**4*x
**3 - 64*c**5 - 96*c**4*d*x - 216*c**3*d**2*x**2 + 2295*c**2*d**3*x**3 + 2
835*c*d**4*x**4))/(8505*d**3)
```

3.38 $\int \frac{A+Bx+Cx^2}{\sqrt{4c^3-27cd^2x^2-27d^3x^3}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 206

$$\begin{aligned}\int \frac{A+Bx+Cx^2}{\sqrt{4c^3-27cd^2x^2-27d^3x^3}}\,dx &= \frac{2(cC-3Bd)(c-3dx)(2c+3dx)}{27d^3\sqrt{4c^3-27cd^2x^2-27d^3x^3}}+\frac{2C(c-3dx)^2(2c+3dx)}{81d^3\sqrt{4c^3-27cd^2x^2-27d^3x^3}} \\ &\quad -\frac{2(4c^2C-6Bcd+9Ad^2)(2c+3dx)\sqrt{1-\frac{3dx}{c}}\,\text{arctanh}\left(\frac{\sqrt{1-\frac{3dx}{c}}}{\sqrt{3}}\right)}{27\sqrt{3}d^3\sqrt{4c^3-27cd^2x^2-27d^3x^3}}\end{aligned}$$

output
```
2/27*(-3*B*d+C*c)*(-3*d*x+c)*(3*d*x+2*c)/d^3/(-27*d^3*x^3-27*c*d^2*x^2+4*c
^3)^(1/2)+2/81*C*(-3*d*x+c)^2*(3*d*x+2*c)/d^3/(-27*d^3*x^3-27*c*d^2*x^2+4*
c^3)^(1/2)-2/81*(9*A*d^2-6*B*c*d+4*C*c^2)*(3*d*x+2*c)*(1-3*d*x/c)^(1/2)*ar
ctanh(1/3*(1-3*d*x/c)^(1/2)*3^(1/2))*3^(1/2)/d^3/(-27*d^3*x^3-27*c*d^2*x^2
+4*c^3)^(1/2)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.61

$$\int \frac{A+Bx+Cx^2}{\sqrt{4c^3-27cd^2x^2-27d^3x^3}}\,dx = \frac{2(2c+3dx)\left(\sqrt{c}(c-3dx)(4cC-3d(3B+Cx))-\sqrt{3}(4c^2C-6Bcd+9Ad^2)\sqrt{c-3dx}\,\text{arctanh}\bigg(\frac{\sqrt{c-3dx}}{\sqrt{3}\sqrt{c}}\right.}{81\sqrt{c}d^3\sqrt{(c-3dx)(2c+3dx)^2}}$$

input
```
Integrate[(A + B*x + C*x^2)/Sqrt[4*c^3 - 27*c*d^2*x^2 - 27*d^3*x^3],x]
```

output
```
(2*(2*c + 3*d*x)*(Sqrt[c]*(c - 3*d*x)*(4*c*C - 3*d*(3*B + C*x)) - Sqrt[3]*
(4*c^2*C - 6*B*c*d + 9*A*d^2)*Sqrt[c - 3*d*x]*ArcTanh[Sqrt[c - 3*d*x]/(Sqr
t[3]*Sqrt[c])]))/(81*Sqrt[c]*d^3*Sqrt[(c - 3*d*x)*(2*c + 3*d*x)^2])
```

Rubi [A] (verified)

Time = 1.08 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.19, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {2526, 27, 2490, 2483, 27, 90, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{\sqrt{4c^3-27cd^2x^2-27d^3x^3}}\,dx$$

$$\Big\downarrow \ 2526$$

$$-\frac{\int -\frac{27d^2(3Ad-(2cC-3Bd)x)}{\sqrt{4c^3-27d^2x^2c-27d^3x^3}}dx}{81d^3}-\frac{2C\sqrt{4c^3-27cd^2x^2-27d^3x^3}}{81d^3}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{3Ad-(2cC-3Bd)x}{\sqrt{4c^3-27d^2x^2c-27d^3x^3}}dx}{3d}-\frac{2C\sqrt{4c^3-27cd^2x^2-27d^3x^3}}{81d^3}$$

$$\Big\downarrow \ 2490$$

$$\frac{\int \frac{(3Bd-2cC)\left(\frac{c}{3d}+x\right)-\frac{27cd^2(3Bd-2cC)-243Ad^4}{81d^3}}{\sqrt{2c^3+9d\left(\frac{c}{3d}+x\right)c^2-27d^3\left(\frac{c}{3d}+x\right)^3}}d\left(\frac{c}{3d}+x\right)}{3d}-\frac{2C\sqrt{4c^3-27cd^2x^2-27d^3x^3}}{81d^3}$$

$\Big\downarrow$ 2483

$$\frac{2\sqrt{3}c^2\left(3d\left(\frac{c}{3d}+x\right)+c\right)\sqrt{2c^3-3c^2d\left(\frac{c}{3d}+x\right)}\int -\frac{-\frac{2Cc^2}{d}+3Bc-9Ad+3(2cC-3Bd)\left(\frac{c}{3d}+x\right)}{18\sqrt{3}c^2\left(c+3d\left(\frac{c}{3d}+x\right)\right)\sqrt{2c^3-3c^2d\left(\frac{c}{3d}+x\right)}}d\left(\frac{c}{3d}+x\right)}{d\sqrt{2c^3+9c^2d\left(\frac{c}{3d}+x\right)-27d^3\left(\frac{c}{3d}+x\right)^3}}-\frac{2C\sqrt{4c^3-27cd^2x^2-27d^3x^3}}{81d^3}$$

$\Big\downarrow$ 27

$$-\frac{\left(3d\left(\frac{c}{3d}+x\right)+c\right)\sqrt{2c^3-3c^2d\left(\frac{c}{3d}+x\right)}\int \frac{-\frac{2Cc^2}{d}+3Bc-9Ad+3(2cC-3Bd)\left(\frac{c}{3d}+x\right)}{\left(c+3d\left(\frac{c}{3d}+x\right)\right)\sqrt{2c^3-3c^2d\left(\frac{c}{3d}+x\right)}}d\left(\frac{c}{3d}+x\right)}{9d\sqrt{2c^3+9c^2d\left(\frac{c}{3d}+x\right)-27d^3\left(\frac{c}{3d}+x\right)^3}}-\frac{2C\sqrt{4c^3-27cd^2x^2-27d^3x^3}}{81d^3}$$

$\Big\downarrow$ 90

$$-\frac{\left(3d\left(\frac{c}{3d}+x\right)+c\right)\sqrt{2c^3-3c^2d\left(\frac{c}{3d}+x\right)}\left(-\frac{(9Ad^2-6Bcd+4c^2C)\int \frac{1}{\left(c+3d\left(\frac{c}{3d}+x\right)\right)\sqrt{2c^3-3c^2d\left(\frac{c}{3d}+x\right)}}d\left(\frac{c}{3d}+x\right)}{d}-\frac{2\sqrt{2c^3-3c^2d(\cdots}{\cdots}\right.}{9d\sqrt{2c^3+9c^2d\left(\frac{c}{3d}+x\right)-27d^3\left(\frac{c}{3d}+x\right)^3}}\cdots\frac{2C\sqrt{4c^3-27cd^2x^2-27d^3x^3}}{81d^3}$$

$\Big\downarrow$ 73

$$-\frac{\left(3d\left(\frac{c}{3d}+x\right)+c\right)\sqrt{2c^3-3c^2d\left(\frac{c}{3d}+x\right)}\left(\frac{2(9Ad^2-6Bcd+4c^2C)\int \frac{1}{3c-\frac{2c^3-3c^2d\left(\frac{c}{3d}+x\right)}{c^2}}d\sqrt{2c^3-3c^2d\left(\frac{c}{3d}+x\right)}}{3c^2d^2}-\frac{2\sqrt{2c^3-3c^2d(\cdots}{3c\cdots}\right.}{9d\sqrt{2c^3+9c^2d\left(\frac{c}{3d}+x\right)-27d^3\left(\frac{c}{3d}+x\right)^3}}\cdots\frac{2C\sqrt{4c^3-27cd^2x^2-27d^3x^3}}{81d^3}$$

$\Big\downarrow$ 221

$$-\frac{\left(3d\left(\frac{c}{3d}+x\right)+c\right)\sqrt{2c^3-3c^2d\left(\frac{c}{3d}+x\right)}\left(\frac{2\text{arctanh}\left(\frac{\sqrt{2c^3-3c^2d\left(\frac{c}{3d}+x\right)}}{\sqrt{3}c^{3/2}}\right)(9Ad^2-6Bcd+4c^2C)}{3\sqrt{3}c^{3/2}d^2}-\frac{2\sqrt{2c^3-3c^2d\left(\frac{c}{3d}+x\right)}(2cC-}{3c^2d^2}\right.}{9d\sqrt{2c^3+9c^2d\left(\frac{c}{3d}+x\right)-27d^3\left(\frac{c}{3d}+x\right)^3}}$$
$$\frac{2C\sqrt{4c^3-27cd^2x^2-27d^3x^3}}{81d^3}$$

input
```
Int[(A + B*x + C*x^2)/Sqrt[4*c^3 - 27*c*d^2*x^2 - 27*d^3*x^3],x]
```

output
```
(-2*C*Sqrt[4*c^3 - 27*c*d^2*x^2 - 27*d^3*x^3])/(81*d^3) - ((c + 3*d*(c/(3*
d) + x))*Sqrt[2*c^3 - 3*c^2*d*(c/(3*d) + x)]*((-2*(2*c*C - 3*B*d)*Sqrt[2*c
^3 - 3*c^2*d*(c/(3*d) + x)])/(3*c^2*d^2) + (2*(4*c^2*C - 6*B*c*d + 9*A*d^2
)*ArcTanh[Sqrt[2*c^3 - 3*c^2*d*(c/(3*d) + x)]/(Sqrt[3]*c^(3/2))])/(3*Sqrt[
3]*c^(3/2)*d^2)))/(9*d*Sqrt[2*c^3 + 9*c^2*d*(c/(3*d) + x) - 27*d^3*(c/(3*d
) + x)^3])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 2483
```
Int[((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (d_.)*(x_)^3)^(p_), x_S
ymbol] :> Simp[(a + b*x + d*x^3)^p/((3*a - b*x)^p*(3*a + 2*b*x)^(2*p))   In
t[(e + f*x)^m*(3*a - b*x)^p*(3*a + 2*b*x)^(2*p), x], x] /; FreeQ[{a, b, d,
e, f, m, p}, x] && EqQ[4*b^3 + 27*a^2*d, 0] &&  !IntegerQ[p]
```

rule 2490
```
Int[(P3_)^(p_.)*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] :> With[{a = Coeff[P3
, x, 0], b = Coeff[P3, x, 1], c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Su
bst[Int[((3*d*e - c*f)/(3*d) + f*x)^m*Simp[(2*c^3 - 9*b*c*d + 27*a*d^2)/(27
*d^2) - (c^2 - 3*b*d)*(x/(3*d)) + d*x^3, x]^p, x], x, x + c/(3*d)] /; NeQ[c
, 0]] /; FreeQ[{e, f, m, p}, x] && PolyQ[P3, x, 3]
```

rule 2526
```
Int[(Pm_)*(Qn_)^(p_), x_Symbol] :> With[{m = Expon[Pm, x], n = Expon[Qn, x]
}, Simp[Coeff[Pm, x, m]*(Qn^(p + 1)/(n*(p + 1)*Coeff[Qn, x, n])), x] + Simp
[1/(n*Coeff[Qn, x, n])   Int[ExpandToSum[n*Coeff[Qn, x, n]*Pm - Coeff[Pm, x
, m]*D[Qn, x], x]*Qn^p, x], x] /; EqQ[m, n - 1]] /; FreeQ[p, x] && PolyQ[Pm
, x] && PolyQ[Qn, x] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.83

method	result
default	$-\frac{2(3dx+2c)\sqrt{-3dx+c}\left(9A\sqrt{3}\,\operatorname{arctanh}\left(\frac{\sqrt{-3dx+c}\,\sqrt{3}}{3\sqrt{c}}\right)d^2-6B\sqrt{3}\,\operatorname{arctanh}\left(\frac{\sqrt{-3dx+c}\,\sqrt{3}}{3\sqrt{c}}\right)cd+4C\sqrt{3}\,\operatorname{arctanh}\left(\frac{\sqrt{-3dx+c}\,\sqrt{3}}{3\sqrt{c}}\right)c^2-C}{81\sqrt{-27d^3x^3-27c\,d^2x^2+4c^3}\,d^3\sqrt{c}}$

input
```
int((C*x^2+B*x+A)/(-27*d^3*x^3-27*c*d^2*x^2+4*c^3)^(1/2),x,method=_RETURNV
ERBOSE)
```

output
```
-2/81*(3*d*x+2*c)*(-3*d*x+c)^(1/2)*(9*A*3^(1/2)*arctanh(1/3*(-3*d*x+c)^(1/
2)*3^(1/2)/c^(1/2))*d^2-6*B*3^(1/2)*arctanh(1/3*(-3*d*x+c)^(1/2)*3^(1/2)/c
^(1/2))*c*d+4*C*3^(1/2)*arctanh(1/3*(-3*d*x+c)^(1/2)*3^(1/2)/c^(1/2))*c^2-
C*(-3*d*x+c)^(3/2)*c^(1/2)+9*(-3*d*x+c)^(1/2)*B*d*c^(1/2)-3*(-3*d*x+c)^(1/
2)*C*c^(3/2))/(-27*d^3*x^3-27*c*d^2*x^2+4*c^3)^(1/2)/d^3/c^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 369, normalized size of antiderivative = 1.79

$$\int \frac{A+Bx+Cx^2}{\sqrt{4c^3-27cd^2x^2-27d^3x^3}}\,dx$$
$$= \left[\frac{\sqrt{3}(8\,Cc^3-12\,Bc^2d+18\,Acd^2+3\,(4\,Cc^2d-6\,Bcd^2+9\,Ad^3)x)\sqrt{c}\log\left(\frac{9\,d^2x^2-6\,cdx-8\,c^2+2\,\sqrt{3}\sqrt{-27\,d^3x^3-2}}{9\,d^2x^2+12\,cdx+4\,c^2}\right.}{81\,(3\,cd^4x+2\,c^2d^3)} \right.$$
$$-\frac{2\left(\sqrt{3}(8\,Cc^3-12\,Bc^2d+18\,Acd^2+3\,(4\,Cc^2d-6\,Bcd^2+9\,Ad^3)x)\sqrt{-c}\arctan\left(\frac{\sqrt{3}\sqrt{-27\,d^3x^3-27\,cd^2x^2+}}{9\,d^2x^2+3\,cdx-2\,c^2}\right.\right.}{81\,(3\,cd^4x+2\,c^2d^3)}$$

input
```
integrate((C*x^2+B*x+A)/(-27*d^3*x^3-27*c*d^2*x^2+4*c^3)^(1/2),x, algorith
m="fricas")
```

output
```
[1/81*(sqrt(3)*(8*C*c^3 - 12*B*c^2*d + 18*A*c*d^2 + 3*(4*C*c^2*d - 6*B*c*d
^2 + 9*A*d^3)*x)*sqrt(c)*log((9*d^2*x^2 - 6*c*d*x - 8*c^2 + 2*sqrt(3)*sqrt
(-27*d^3*x^3 - 27*c*d^2*x^2 + 4*c^3)*sqrt(c))/(9*d^2*x^2 + 12*c*d*x + 4*c^
2)) - 2*sqrt(-27*d^3*x^3 - 27*c*d^2*x^2 + 4*c^3)*(3*C*c*d*x - 4*C*c^2 + 9*
B*c*d))/(3*c*d^4*x + 2*c^2*d^3), -2/81*(sqrt(3)*(8*C*c^3 - 12*B*c^2*d + 18
*A*c*d^2 + 3*(4*C*c^2*d - 6*B*c*d^2 + 9*A*d^3)*x)*sqrt(-c)*arctan(sqrt(3)*
sqrt(-27*d^3*x^3 - 27*c*d^2*x^2 + 4*c^3)*sqrt(-c)/(9*d^2*x^2 + 3*c*d*x - 2
*c^2)) + sqrt(-27*d^3*x^3 - 27*c*d^2*x^2 + 4*c^3)*(3*C*c*d*x - 4*C*c^2 + 9
*B*c*d))/(3*c*d^4*x + 2*c^2*d^3)]
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt{4c^3-27cd^2x^2-27d^3x^3}}\,dx=\int \frac{A+Bx+Cx^2}{\sqrt{-\left(-c+3dx\right)\left(2c+3dx\right)^2}}\,dx$$

input
```
integrate((C*x**2+B*x+A)/(-27*d**3*x**3-27*c*d**2*x**2+4*c**3)**(1/2),x)
```

output
```
Integral((A + B*x + C*x**2)/sqrt(-(-c + 3*d*x)*(2*c + 3*d*x)**2), x)
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt{4c^3-27cd^2x^2-27d^3x^3}}\,dx=\int \frac{Cx^2+Bx+A}{\sqrt{-27\,d^3x^3-27\,cd^2x^2+4\,c^3}}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(-27*d^3*x^3-27*c*d^2*x^2+4*c^3)^(1/2),x, algorith
m="maxima")
```

output
```
integrate((C*x^2 + B*x + A)/sqrt(-27*d^3*x^3 - 27*c*d^2*x^2 + 4*c^3), x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.59

$$\begin{aligned}&\int \frac{A+Bx+Cx^2}{\sqrt{4c^3-27cd^2x^2-27d^3x^3}}\,dx\\&=-\frac{2\,\sqrt{3}(4\,Cc^2-6\,Bcd+9\,Ad^2)\arctan\left(\frac{\sqrt{3}\sqrt{-3\,dx+c}}{3\,\sqrt{-c}}\right)}{81\,\sqrt{-c}d^3\mathrm{sgn}\left(-3\,dx-2\,c\right)}\\&\quad-\frac{2\left(\left(-3\,dx+c\right)^{\frac{3}{2}}Cd^6+3\,\sqrt{-3\,dx+c}Ccd^6-9\,\sqrt{-3\,dx+c}Bd^7\right)}{81\,d^9\mathrm{sgn}\left(-3\,dx-2\,c\right)}\end{aligned}$$

input
```
integrate((C*x^2+B*x+A)/(-27*d^3*x^3-27*c*d^2*x^2+4*c^3)^(1/2),x, algorith
m="giac")
```

output
```
-2/81*sqrt(3)*(4*C*c^2 - 6*B*c*d + 9*A*d^2)*arctan(1/3*sqrt(3)*sqrt(-3*d*x
 + c)/sqrt(-c))/(sqrt(-c)*d^3*sgn(-3*d*x - 2*c)) - 2/81*((-3*d*x + c)^(3/2
)*C*d^6 + 3*sqrt(-3*d*x + c)*C*c*d^6 - 9*sqrt(-3*d*x + c)*B*d^7)/(d^9*sgn(
-3*d*x - 2*c))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{\sqrt{4c^3-27cd^2x^2-27d^3x^3}}\,dx=\int \frac{C\,x^2+B\,x+A}{\sqrt{4\,c^3-27\,c\,d^2\,x^2-27\,d^3\,x^3}}\,dx$$

input
```
int((A + B*x + C*x^2)/(4*c^3 - 27*d^3*x^3 - 27*c*d^2*x^2)^(1/2),x)
```

output
```
int((A + B*x + C*x^2)/(4*c^3 - 27*d^3*x^3 - 27*c*d^2*x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.92

$$\int \frac{A+Bx+Cx^2}{\sqrt{4c^3-27cd^2x^2-27d^3x^3}}\,dx = \frac{-18\sqrt{-3dx+c}\,bcd+8\sqrt{-3dx+c}\,c^3-6\sqrt{-3dx+c}\,c^2dx+9\sqrt{c}\,\sqrt{3}\log\left(\sqrt{-3dx+c}-\sqrt{c}\,\sqrt{3}\right)a\,d^2-6}{}$$

input
```
int((C*x^2+B*x+A)/(-27*d^3*x^3-27*c*d^2*x^2+4*c^3)^(1/2),x)
```

output
```
( - 18*sqrt(c - 3*d*x)*b*c*d + 8*sqrt(c - 3*d*x)*c**3 - 6*sqrt(c - 3*d*x)*
c**2*d*x + 9*sqrt(c)*sqrt(3)*log(sqrt(c - 3*d*x) - sqrt(c)*sqrt(3))*a*d**2
 - 6*sqrt(c)*sqrt(3)*log(sqrt(c - 3*d*x) - sqrt(c)*sqrt(3))*b*c*d + 4*sqrt
(c)*sqrt(3)*log(sqrt(c - 3*d*x) - sqrt(c)*sqrt(3))*c**3 - 9*sqrt(c)*sqrt(3
)*log(sqrt(c - 3*d*x) + sqrt(c)*sqrt(3))*a*d**2 + 6*sqrt(c)*sqrt(3)*log(sq
rt(c - 3*d*x) + sqrt(c)*sqrt(3))*b*c*d - 4*sqrt(c)*sqrt(3)*log(sqrt(c - 3*
d*x) + sqrt(c)*sqrt(3))*c**3)/(81*c*d**3)
```

3.39 $\int \frac{A+Bx+Cx^2}{\left(4c^3-27cd^2x^2-27d^3x^3\right)^{3/2}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 318

$$\int \frac{A+Bx+Cx^2}{\left(4c^3-27cd^2x^2-27d^3x^3\right)^{3/2}}\,dx = -\frac{\left(4c^2C-6Bcd+9Ad^2\right)(c-3dx)^2(2c+3dx)}{486c^2d^3\left(4c^3-27cd^2x^2-27d^3x^3\right)^{3/2}} + \frac{\left(20c^2C+6Bcd-63Ad^2\right)(c-3dx)^2(2c+3dx)^2}{2916c^3d^3\left(4c^3-27cd^2x^2-27d^3x^3\right)^{3/2}} + \frac{2\left(c^2C+3Bcd+9Ad^2\right)(c-3dx)(2c+3dx)^3}{729c^3d^3\left(4c^3-27cd^2x^2-27d^3x^3\right)^{3/2}} + \frac{\left(4c^2C-6Bcd-45Ad^2\right)(2c+3dx)^3\left(1-\frac{3dx}{c}\right)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{1-\frac{3dx}{c}}}{\sqrt{3}}\right)}{972\sqrt{3}c^2d^3\left(4c^3-27cd^2x^2-27d^3x^3\right)^{3/2}}$$

output
```
-1/486*(9*A*d^2-6*B*c*d+4*C*c^2)*(-3*d*x+c)^2*(3*d*x+2*c)/c^2/d^3/(-27*d^3
*x^3-27*c*d^2*x^2+4*c^3)^(3/2)+1/2916*(-63*A*d^2+6*B*c*d+20*C*c^2)*(-3*d*x
+c)^2*(3*d*x+2*c)^2/c^3/d^3/(-27*d^3*x^3-27*c*d^2*x^2+4*c^3)^(3/2)+2/729*(
9*A*d^2+3*B*c*d+C*c^2)*(-3*d*x+c)*(3*d*x+2*c)^3/c^3/d^3/(-27*d^3*x^3-27*c*
d^2*x^2+4*c^3)^(3/2)+1/2916*(-45*A*d^2-6*B*c*d+4*C*c^2)*(3*d*x+2*c)^3*(1-3
*d*x/c)^(3/2)*arctanh(1/3*(1-3*d*x/c)^(1/2)*3^(1/2))*3^(1/2)/c^2/d^3/(-27*
d^3*x^3-27*c*d^2*x^2+4*c^3)^(3/2)
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.59

$$\int \frac{A+Bx+Cx^2}{(4c^3-27cd^2x^2-27d^3x^3)^{3/2}}\,dx = \frac{(2c+3dx)\left(3\sqrt{c}(c-3dx)\left(16c^4C+405Ad^4x^2+27cd^3x(15A+2Bx\right.\right.}{}$$

input
```
Integrate[(A + B*x + C*x^2)/(4*c^3 - 27*c*d^2*x^2 - 27*d^3*x^3)^(3/2),x]
```

output
```
((2*c + 3*d*x)*(3*Sqrt[c]*(c - 3*d*x)*(16*c^4*C + 405*A*d^4*x^2 + 27*c*d^3
*x*(15*A + 2*B*x) + 12*c^3*d*(4*B + 3*C*x) + 18*c^2*d^2*(2*A + x*(3*B - 2*
C*x))) + Sqrt[3]*(4*c^2*C - 6*B*c*d - 45*A*d^2)*(c - 3*d*x)^(3/2)*(2*c + 3
*d*x)^2*ArcTanh[Sqrt[c - 3*d*x]/(Sqrt[3]*Sqrt[c])]))/(2916*c^(7/2)*d^3*((c
 - 3*d*x)*(2*c + 3*d*x)^2)^(3/2))
```

Rubi [A] (verified)

Time = 1.20 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.18, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {2526, 27, 2490, 2483, 27, 87, 52, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{(4c^3-27cd^2x^2-27d^3x^3)^{3/2}}\,dx$$

$$\Big\downarrow \ 2526$$

$$\frac{2C}{81d^3\sqrt{4c^3-27cd^2x^2-27d^3x^3}}-\frac{\int -\frac{27d^2(3Ad-(2cC-3Bd)x)}{(4c^3-27d^2x^2c-27d^3x^3)^{3/2}}dx}{81d^3}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{3Ad-(2cC-3Bd)x}{(4c^3-27d^2x^2c-27d^3x^3)^{3/2}}dx}{3d}+\frac{2C}{81d^3\sqrt{4c^3-27cd^2x^2-27d^3x^3}}$$

$$\Big\downarrow \ 2490$$

$$\frac{\int \frac{(3Bd-2cC)\left(\frac{c}{3d}+x\right)-\frac{27cd^2(3Bd-2cC)-243Ad^4}{81d^3}}{\left(2c^3+9d\left(\frac{c}{3d}+x\right)c^2-27d^3\left(\frac{c}{3d}+x\right)^3\right)^{3/2}}d\left(\frac{c}{3d}+x\right)}{3d}+\frac{2C}{81d^3\sqrt{4c^3-27cd^2x^2-27d^3x^3}}$$

$\big\downarrow$ 2483

$$\frac{216\sqrt{3}c^6\left(3d\left(\frac{c}{3d}+x\right)+c\right)^3\left(2c^3-3c^2d\left(\frac{c}{3d}+x\right)\right)^{3/2}\int -\frac{-\frac{2Cc^2}{d}+3Bc-9Ad+3(2cC-3Bd)\left(\frac{c}{3d}+x\right)}{1944\sqrt{3}c^6\left(c+3d\left(\frac{c}{3d}+x\right)\right)^3\left(2c^3-3c^2d\left(\frac{c}{3d}+x\right)\right)^{3/2}}d\left(\frac{c}{3d}+x\right)}{d\left(2c^3+9c^2d\left(\frac{c}{3d}+x\right)-27d^3\left(\frac{c}{3d}+x\right)^3\right)^{3/2}}+\frac{2C}{81d^3\sqrt{4c^3-27cd^2x^2-27d^3x^3}}$$

$\big\downarrow$ 27

$$\frac{2C}{81d^3\sqrt{4c^3-27cd^2x^2-27d^3x^3}}-\frac{\left(3d\left(\frac{c}{3d}+x\right)+c\right)^3\left(2c^3-3c^2d\left(\frac{c}{3d}+x\right)\right)^{3/2}\int \frac{-\frac{2Cc^2}{d}+3Bc-9Ad+3(2cC-3Bd)\left(\frac{c}{3d}+x\right)}{\left(c+3d\left(\frac{c}{3d}+x\right)\right)^3\left(2c^3-3c^2d\left(\frac{c}{3d}+x\right)\right)^{3/2}}d\left(\frac{c}{3d}+x\right)}{9d\left(2c^3+9c^2d\left(\frac{c}{3d}+x\right)-27d^3\left(\frac{c}{3d}+x\right)^3\right)^{3/2}}$$

$\big\downarrow$ 87

$$\frac{2C}{81d^3\sqrt{4c^3-27cd^2x^2-27d^3x^3}}-\frac{\left(3d\left(\frac{c}{3d}+x\right)+c\right)^3\left(2c^3-3c^2d\left(\frac{c}{3d}+x\right)\right)^{3/2}\left(\frac{9Ad^2-6Bcd+4c^2C}{18c^3d^2\left(3d\left(\frac{c}{3d}+x\right)+c\right)^2\sqrt{2c^3-3c^2d\left(\frac{c}{3d}+x\right)}}-\frac{1}{12}\left(\frac{45Ad}{c}+6B-\frac{4cC}{d}\right)\int \frac{}{(c+3d}\right.}{9d\left(2c^3+9c^2d\left(\frac{c}{3d}+x\right)-27d^3\left(\frac{c}{3d}+x\right)^3\right)^{3/2}}$$

$\big\downarrow$ 52

$$\frac{2C}{81d^3\sqrt{4c^3-27cd^2x^2-27d^3x^3}}-\frac{\left(3d\left(\frac{c}{3d}+x\right)+c\right)^3\left(2c^3-3c^2d\left(\frac{c}{3d}+x\right)\right)^{3/2}\left(\frac{9Ad^2-6Bcd+4c^2C}{18c^3d^2\left(3d\left(\frac{c}{3d}+x\right)+c\right)^2\sqrt{2c^3-3c^2d\left(\frac{c}{3d}+x\right)}}-\frac{1}{12}\left(\frac{45Ad}{c}+6B-\frac{4cC}{d}\right)\left(\frac{\int \frac{}{(c+}}{}\right.\right.}{9d\left(2c^3+9c^2d\left(\frac{c}{3d}+x\right)-27d^3\left(\frac{c}{3d}+x\right)^3\right)}$$

$\big\downarrow$ 61

$$\frac{2C}{81d^3\sqrt{4c^3-27cd^2x^2-27d^3x^3}}-\frac{\left(3d\left(\frac{c}{3d}+x\right)+c\right)^3\left(2c^3-3c^2d\left(\frac{c}{3d}+x\right)\right)^{3/2}\left(\frac{9Ad^2-6Bcd+4c^2C}{18c^3d^2\left(3d\left(\frac{c}{3d}+x\right)+c\right)^2\sqrt{2c^3-3c^2d\left(\frac{c}{3d}+x\right)}}-\frac{1}{12}\left(\frac{45Ad}{c}+6B-\frac{4cC}{d}\right)\left(\frac{\int\frac{\cdots}{\left(c+\cdots\right.}\cdots}{\cdots}\cdots\right.\right.}{9d\left(2c^3+9c^2d\left(\frac{c}{3d}+x\right)-27d^3\left(\frac{c}{3d}\cdots\right.\right.}$$

$\downarrow$ 73

$$\frac{2C}{81d^3\sqrt{4c^3-27cd^2x^2-27d^3x^3}}-\frac{\left(3d\left(\frac{c}{3d}+x\right)+c\right)^3\left(2c^3-3c^2d\left(\frac{c}{3d}+x\right)\right)^{3/2}\left(\frac{9Ad^2-6Bcd+4c^2C}{18c^3d^2\left(3d\left(\frac{c}{3d}+x\right)+c\right)^2\sqrt{2c^3-3c^2d\left(\frac{c}{3d}+x\right)}}-\frac{1}{12}\left(\frac{45Ad}{c}+6B-\frac{4cC}{d}\right)\left(\frac{\cdots}{9c^3d\sqrt{\cdots}}\cdots\right.\right.}{9d\left(2c^3+9c^2d\left(\frac{c}{3d}+x\right)-27d^3\left(\frac{c}{3d}\cdots\right.\right.}$$

$\downarrow$ 221

$$\frac{2C}{81d^3\sqrt{4c^3-27cd^2x^2-27d^3x^3}}-\frac{\left(3d\left(\frac{c}{3d}+x\right)+c\right)^3\left(2c^3-3c^2d\left(\frac{c}{3d}+x\right)\right)^{3/2}\left(\frac{9Ad^2-6Bcd+4c^2C}{18c^3d^2\left(3d\left(\frac{c}{3d}+x\right)+c\right)^2\sqrt{2c^3-3c^2d\left(\frac{c}{3d}+x\right)}}-\frac{1}{12}\left(\frac{\frac{2}{9c^3d\sqrt{2c^3-3c^2d\left(\frac{c}{3d}+x\right)}}-\frac{2\arctan\cdots}{\cdots}}{2c\cdots}\cdots\right.\right.}{9d\left(2c^3+9c^2d\left(\frac{c}{3d}+x\right)-27d^3\left(\frac{c}{3d}+x\cdots\right.\right.}$$

input

```
Int[(A + B*x + C*x^2)/(4*c^3 - 27*c*d^2*x^2 - 27*d^3*x^3)^(3/2),x]
```

output
```
(2*C)/(81*d^3*Sqrt[4*c^3 - 27*c*d^2*x^2 - 27*d^3*x^3]) - ((c + 3*d*(c/(3*d
) + x))^3*(2*c^3 - 3*c^2*d*(c/(3*d) + x))^(3/2)*((4*c^2*C - 6*B*c*d + 9*A*
d^2)/(18*c^3*d^2*(c + 3*d*(c/(3*d) + x))^2*Sqrt[2*c^3 - 3*c^2*d*(c/(3*d) +
 x)]) - ((6*B - (4*c*C)/d + (45*A*d)/c)*(-1/9*1/(c^3*d*(c + 3*d*(c/(3*d) +
 x))*Sqrt[2*c^3 - 3*c^2*d*(c/(3*d) + x)]) + (2/(9*c^3*d*Sqrt[2*c^3 - 3*c^2
*d*(c/(3*d) + x)]) - (2*ArcTanh[Sqrt[2*c^3 - 3*c^2*d*(c/(3*d) + x)]/(Sqrt[
3]*c^(3/2))])/(9*Sqrt[3]*c^(9/2)*d))/(2*c)))/12))/(9*d*(2*c^3 + 9*c^2*d*(c
/(3*d) + x) - 27*d^3*(c/(3*d) + x)^3)^(3/2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 2483
```
Int[((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (d_.)*(x_)^3)^(p_), x_S
ymbol] :> Simp[(a + b*x + d*x^3)^p/((3*a - b*x)^p*(3*a + 2*b*x)^(2*p))   In
t[(e + f*x)^m*(3*a - b*x)^p*(3*a + 2*b*x)^(2*p), x], x] /; FreeQ[{a, b, d,
e, f, m, p}, x] && EqQ[4*b^3 + 27*a^2*d, 0] &&  !IntegerQ[p]
```

rule 2490
```
Int[(P3_)^(p_.)*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] :> With[{a = Coeff[P3
, x, 0], b = Coeff[P3, x, 1], c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Su
bst[Int[((3*d*e - c*f)/(3*d) + f*x)^m*Simp[(2*c^3 - 9*b*c*d + 27*a*d^2)/(27
*d^2) - (c^2 - 3*b*d)*(x/(3*d)) + d*x^3, x]^p, x], x, x + c/(3*d)] /; NeQ[c
, 0]] /; FreeQ[{e, f, m, p}, x] && PolyQ[P3, x, 3]
```

rule 2526
```
Int[(Pm_)*(Qn_)^(p_), x_Symbol] :> With[{m = Expon[Pm, x], n = Expon[Qn, x]
}, Simp[Coeff[Pm, x, m]*(Qn^(p + 1)/(n*(p + 1)*Coeff[Qn, x, n])), x] + Simp
[1/(n*Coeff[Qn, x, n])   Int[ExpandToSum[n*Coeff[Qn, x, n]*Pm - Coeff[Pm, x
, m]*D[Qn, x], x]*Qn^p, x], x] /; EqQ[m, n - 1]] /; FreeQ[p, x] && PolyQ[Pm
, x] && PolyQ[Qn, x] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 466, normalized size of antiderivative = 1.47

method	result
default	$-\frac{\left(405A\sqrt{3}\,\operatorname{arctanh}\left(\frac{\sqrt{-3dx+c}\,\sqrt{3}}{3\sqrt{c}}\right)\sqrt{-3dx+c}\,d^{4}x^{2}-162B\,c^{\frac{3}{2}}\,d^{3}x^{2}-16C\sqrt{3}\,\operatorname{arctanh}\left(\frac{\sqrt{-3dx+c}\,\sqrt{3}}{3\sqrt{c}}\right)\sqrt{-3dx+c}\,c^{4}+108C\,c^{\frac{5}{2}}\,d^{2}x^{2}-\right.}{}$

input
```
int((C*x^2+B*x+A)/(-27*d^3*x^3-27*c*d^2*x^2+4*c^3)^(3/2),x,method=_RETURNV
ERBOSE)
```

output
```
-1/2916*(405*A*3^(1/2)*arctanh(1/3*(-3*d*x+c)^(1/2)*3^(1/2)/c^(1/2))*(-3*d
*x+c)^(1/2)*d^4*x^2-162*B*c^(3/2)*d^3*x^2-16*C*3^(1/2)*arctanh(1/3*(-3*d*x
+c)^(1/2)*3^(1/2)/c^(1/2))*(-3*d*x+c)^(1/2)*c^4+108*C*c^(5/2)*d^2*x^2-1215
*A*c^(3/2)*d^3*x-162*B*c^(5/2)*d^2*x-108*C*c^(7/2)*d*x-48*C*3^(1/2)*arctan
h(1/3*(-3*d*x+c)^(1/2)*3^(1/2)/c^(1/2))*(-3*d*x+c)^(1/2)*c^3*d*x+54*B*3^(1
/2)*arctanh(1/3*(-3*d*x+c)^(1/2)*3^(1/2)/c^(1/2))*(-3*d*x+c)^(1/2)*c*d^3*x
^2-36*C*3^(1/2)*arctanh(1/3*(-3*d*x+c)^(1/2)*3^(1/2)/c^(1/2))*(-3*d*x+c)^(
1/2)*c^2*d^2*x^2+540*A*3^(1/2)*arctanh(1/3*(-3*d*x+c)^(1/2)*3^(1/2)/c^(1/2
))*(-3*d*x+c)^(1/2)*c*d^3*x+72*B*3^(1/2)*arctanh(1/3*(-3*d*x+c)^(1/2)*3^(1
/2)/c^(1/2))*(-3*d*x+c)^(1/2)*c^2*d^2*x-144*B*c^(7/2)*d-108*A*c^(5/2)*d^2-
1215*A*c^(1/2)*d^4*x^2+180*A*3^(1/2)*arctanh(1/3*(-3*d*x+c)^(1/2)*3^(1/2)/
c^(1/2))*(-3*d*x+c)^(1/2)*c^2*d^2+24*B*3^(1/2)*arctanh(1/3*(-3*d*x+c)^(1/2
)*3^(1/2)/c^(1/2))*(-3*d*x+c)^(1/2)*c^3*d-48*C*c^(9/2))*(-3*d*x+c)*(3*d*x+
2*c)/c^(7/2)/d^3/(-27*d^3*x^3-27*c*d^2*x^2+4*c^3)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 768, normalized size of antiderivative = 2.42

$$\int \frac{A+Bx+Cx^2}{(4c^3-27cd^2x^2-27d^3x^3)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)/(-27*d^3*x^3-27*c*d^2*x^2+4*c^3)^(3/2),x, algorith
m="fricas")
```

output
```
[1/5832*(sqrt(3)*(32*C*c^6 - 48*B*c^5*d - 360*A*c^4*d^2 - 81*(4*C*c^2*d^4
 - 6*B*c*d^5 - 45*A*d^6)*x^4 - 135*(4*C*c^3*d^3 - 6*B*c^2*d^4 - 45*A*c*d^5)
*x^3 - 54*(4*C*c^4*d^2 - 6*B*c^3*d^3 - 45*A*c^2*d^4)*x^2 + 12*(4*C*c^5*d -
 6*B*c^4*d^2 - 45*A*c^3*d^3)*x)*sqrt(c)*log((9*d^2*x^2 - 6*c*d*x - 8*c^2 +
 2*sqrt(3)*sqrt(-27*d^3*x^3 - 27*c*d^2*x^2 + 4*c^3)*sqrt(c))/(9*d^2*x^2 +
12*c*d*x + 4*c^2)) - 6*(16*C*c^5 + 48*B*c^4*d + 36*A*c^3*d^2 - 9*(4*C*c^3*
d^2 - 6*B*c^2*d^3 - 45*A*c*d^4)*x^2 + 9*(4*C*c^4*d + 6*B*c^3*d^2 + 45*A*c^
2*d^3)*x)*sqrt(-27*d^3*x^3 - 27*c*d^2*x^2 + 4*c^3))/(81*c^4*d^7*x^4 + 135*
c^5*d^6*x^3 + 54*c^6*d^5*x^2 - 12*c^7*d^4*x - 8*c^8*d^3), -1/2916*(sqrt(3)
*(32*C*c^6 - 48*B*c^5*d - 360*A*c^4*d^2 - 81*(4*C*c^2*d^4 - 6*B*c*d^5 - 45
*A*d^6)*x^4 - 135*(4*C*c^3*d^3 - 6*B*c^2*d^4 - 45*A*c*d^5)*x^3 - 54*(4*C*c
^4*d^2 - 6*B*c^3*d^3 - 45*A*c^2*d^4)*x^2 + 12*(4*C*c^5*d - 6*B*c^4*d^2 - 4
5*A*c^3*d^3)*x)*sqrt(-c)*arctan(sqrt(3)*sqrt(-27*d^3*x^3 - 27*c*d^2*x^2 +
4*c^3)*sqrt(-c)/(9*d^2*x^2 + 3*c*d*x - 2*c^2)) + 3*(16*C*c^5 + 48*B*c^4*d
+ 36*A*c^3*d^2 - 9*(4*C*c^3*d^2 - 6*B*c^2*d^3 - 45*A*c*d^4)*x^2 + 9*(4*C*c
^4*d + 6*B*c^3*d^2 + 45*A*c^2*d^3)*x)*sqrt(-27*d^3*x^3 - 27*c*d^2*x^2 + 4*
c^3))/(81*c^4*d^7*x^4 + 135*c^5*d^6*x^3 + 54*c^6*d^5*x^2 - 12*c^7*d^4*x -
8*c^8*d^3)]
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2}{(4c^3-27cd^2x^2-27d^3x^3)^{3/2}}\,dx=\int \frac{A+Bx+Cx^2}{\left(-\left(-c+3dx\right)\left(2c+3dx\right)^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((C*x**2+B*x+A)/(-27*d**3*x**3-27*c*d**2*x**2+4*c**3)**(3/2),x)
```

output
```
Integral((A + B*x + C*x**2)/(-(-c + 3*d*x)*(2*c + 3*d*x)**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{(4c^3-27cd^2x^2-27d^3x^3)^{3/2}}\,dx=\int \frac{Cx^2+Bx+A}{(-27\,d^3x^3-27\,cd^2x^2+4\,c^3)^{\frac{3}{2}}}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(-27*d^3*x^3-27*c*d^2*x^2+4*c^3)^(3/2),x, algorith
m="maxima")
```

output
```
integrate((C*x^2 + B*x + A)/(-27*d^3*x^3 - 27*c*d^2*x^2 + 4*c^3)^(3/2), x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.70

$$\int \frac{A+Bx+Cx^2}{(4c^3-27cd^2x^2-27d^3x^3)^{3/2}}\,dx=\frac{\sqrt{3}(4\,Cc^2-6\,Bcd-45\,Ad^2)\arctan\left(\frac{\sqrt{3}\sqrt{-3\,dx+c}}{3\,\sqrt{-c}}\right)}{2916\,\sqrt{-c}c^3d^3\mathrm{sgn}\left(-3\,dx-2\,c\right)} - \frac{2\,(Cc^2+3\,Bcd+9\,Ad^2)}{729\,\sqrt{-3\,dx+c}c^3d^3\mathrm{sgn}\left(-3\,dx-2\,c\right)} + \frac{20\,(-3\,dx+c)^{\frac{3}{2}}Cc^2-36\,\sqrt{-3\,dx+c}Cc^3+6\,(-3\,dx+c)^{\frac{3}{2}}Bcd-54\,\sqrt{-3\,dx+c}Bc^2d-63\,(-3\,dx+c)^{\frac{3}{2}}}{2916\,(3\,dx+2\,c)^2c^3d^3\mathrm{sgn}\left(-3\,dx-2\,c\right)}$$

input
```
integrate((C*x^2+B*x+A)/(-27*d^3*x^3-27*c*d^2*x^2+4*c^3)^(3/2),x, algorith
m="giac")
```

output
```
1/2916*sqrt(3)*(4*C*c^2 - 6*B*c*d - 45*A*d^2)*arctan(1/3*sqrt(3)*sqrt(-3*d
*x + c)/sqrt(-c))/(sqrt(-c)*c^3*d^3*sgn(-3*d*x - 2*c)) - 2/729*(C*c^2 + 3*
B*c*d + 9*A*d^2)/(sqrt(-3*d*x + c)*c^3*d^3*sgn(-3*d*x - 2*c)) + 1/2916*(20
*(-3*d*x + c)^(3/2)*C*c^2 - 36*sqrt(-3*d*x + c)*C*c^3 + 6*(-3*d*x + c)^(3/
2)*B*c*d - 54*sqrt(-3*d*x + c)*B*c^2*d - 63*(-3*d*x + c)^(3/2)*A*d^2 + 243
*sqrt(-3*d*x + c)*A*c*d^2)/((3*d*x + 2*c)^2*c^3*d^3*sgn(-3*d*x - 2*c))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(4c^3-27cd^2x^2-27d^3x^3)^{3/2}}\,dx=\int \frac{C\,x^2+B\,x+A}{\left(4\,c^3-27\,c\,d^2\,x^2-27\,d^3\,x^3\right)^{3/2}}\,dx$$

input
```
int((A + B*x + C*x^2)/(4*c^3 - 27*d^3*x^3 - 27*c*d^2*x^2)^(3/2),x)
```

output
```
int((A + B*x + C*x^2)/(4*c^3 - 27*d^3*x^3 - 27*c*d^2*x^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 731, normalized size of antiderivative = 2.30

$$\int \frac{A+Bx+Cx^2}{(4c^3-27cd^2x^2-27d^3x^3)^{3/2}}\,dx=\text{Too large to display}$$

input
```
int((C*x^2+B*x+A)/(-27*d^3*x^3-27*c*d^2*x^2+4*c^3)^(3/2),x)
```

output

```
(180*sqrt(c)*sqrt(c - 3*d*x)*sqrt(3)*log(sqrt(c - 3*d*x) - sqrt(c)*sqrt(3)
)*a*c**2*d**2 + 540*sqrt(c)*sqrt(c - 3*d*x)*sqrt(3)*log(sqrt(c - 3*d*x) -
sqrt(c)*sqrt(3))*a*c*d**3*x + 405*sqrt(c)*sqrt(c - 3*d*x)*sqrt(3)*log(sqrt
(c - 3*d*x) - sqrt(c)*sqrt(3))*a*d**4*x**2 + 24*sqrt(c)*sqrt(c - 3*d*x)*sq
rt(3)*log(sqrt(c - 3*d*x) - sqrt(c)*sqrt(3))*b*c**3*d + 72*sqrt(c)*sqrt(c
- 3*d*x)*sqrt(3)*log(sqrt(c - 3*d*x) - sqrt(c)*sqrt(3))*b*c**2*d**2*x + 54
*sqrt(c)*sqrt(c - 3*d*x)*sqrt(3)*log(sqrt(c - 3*d*x) - sqrt(c)*sqrt(3))*b*
c*d**3*x**2 - 16*sqrt(c)*sqrt(c - 3*d*x)*sqrt(3)*log(sqrt(c - 3*d*x) - sqr
t(c)*sqrt(3))*c**5 - 48*sqrt(c)*sqrt(c - 3*d*x)*sqrt(3)*log(sqrt(c - 3*d*x
) - sqrt(c)*sqrt(3))*c**4*d*x - 36*sqrt(c)*sqrt(c - 3*d*x)*sqrt(3)*log(sqr
t(c - 3*d*x) - sqrt(c)*sqrt(3))*c**3*d**2*x**2 - 180*sqrt(c)*sqrt(c - 3*d*
x)*sqrt(3)*log(sqrt(c - 3*d*x) + sqrt(c)*sqrt(3))*a*c**2*d**2 - 540*sqrt(c
)*sqrt(c - 3*d*x)*sqrt(3)*log(sqrt(c - 3*d*x) + sqrt(c)*sqrt(3))*a*c*d**3*
x - 405*sqrt(c)*sqrt(c - 3*d*x)*sqrt(3)*log(sqrt(c - 3*d*x) + sqrt(c)*sqrt
(3))*a*d**4*x**2 - 24*sqrt(c)*sqrt(c - 3*d*x)*sqrt(3)*log(sqrt(c - 3*d*x)
+ sqrt(c)*sqrt(3))*b*c**3*d - 72*sqrt(c)*sqrt(c - 3*d*x)*sqrt(3)*log(sqrt(
c - 3*d*x) + sqrt(c)*sqrt(3))*b*c**2*d**2*x - 54*sqrt(c)*sqrt(c - 3*d*x)*s
qrt(3)*log(sqrt(c - 3*d*x) + sqrt(c)*sqrt(3))*b*c*d**3*x**2 + 16*sqrt(c)*s
qrt(c - 3*d*x)*sqrt(3)*log(sqrt(c - 3*d*x) + sqrt(c)*sqrt(3))*c**5 + 48*sq
rt(c)*sqrt(c - 3*d*x)*sqrt(3)*log(sqrt(c - 3*d*x) + sqrt(c)*sqrt(3))*c*...
```

3.40 $\int \frac{A+Bx+Cx^2}{\left(4c^3-27cd^2x^2-27d^3x^3\right)^{5/2}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 536

$$\begin{aligned}
&\int \frac{A+Bx+Cx^2}{\left(4c^3-27cd^2x^2-27d^3x^3\right)^{5/2}}\,dx = \\
&-\frac{(4c^2C-6Bcd+9Ad^2)\,(c-3dx)^3(2c+3dx)}{2916c^3d^3\left(4c^3-27cd^2x^2-27d^3x^3\right)^{5/2}} \\
&+\frac{(4c^2C+66Bcd-207Ad^2)\,(c-3dx)^3(2c+3dx)^2}{52488c^4d^3\left(4c^3-27cd^2x^2-27d^3x^3\right)^{5/2}} \\
&+\frac{(164c^2C+330Bcd-2331Ad^2)\,(c-3dx)^3(2c+3dx)^3}{629856c^5d^3\left(4c^3-27cd^2x^2-27d^3x^3\right)^{5/2}} \\
&+\frac{(292c^2C+138Bcd-4635Ad^2)\,(c-3dx)^3(2c+3dx)^4}{1259712c^6d^3\left(4c^3-27cd^2x^2-27d^3x^3\right)^{5/2}} \\
&+\frac{2(c^2C+3Bcd+9Ad^2)\,(c-3dx)(2c+3dx)^5}{19683c^5d^3\left(4c^3-27cd^2x^2-27d^3x^3\right)^{5/2}} \\
&-\frac{2(c^2C-6Bcd-45Ad^2)\,(c-3dx)^2(2c+3dx)^5}{19683c^6d^3\left(4c^3-27cd^2x^2-27d^3x^3\right)^{5/2}} \\
&+\frac{35(4c^2C-6Bcd-99Ad^2)\,(2c+3dx)^5\left(1-\frac{3dx}{c}\right)^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{1-\frac{3dx}{c}}}{\sqrt{3}}\right)}{419904\sqrt{3}c^4d^3\left(4c^3-27cd^2x^2-27d^3x^3\right)^{5/2}}
\end{aligned}$$

output
```
-1/2916*(9*A*d^2-6*B*c*d+4*C*c^2)*(-3*d*x+c)^3*(3*d*x+2*c)/c^3/d^3/(-27*d^
3*x^3-27*c*d^2*x^2+4*c^3)^(5/2)+1/52488*(-207*A*d^2+66*B*c*d+4*C*c^2)*(-3*
d*x+c)^3*(3*d*x+2*c)^2/c^4/d^3/(-27*d^3*x^3-27*c*d^2*x^2+4*c^3)^(5/2)+1/62
9856*(-2331*A*d^2+330*B*c*d+164*C*c^2)*(-3*d*x+c)^3*(3*d*x+2*c)^3/c^5/d^3/
(-27*d^3*x^3-27*c*d^2*x^2+4*c^3)^(5/2)+1/1259712*(-4635*A*d^2+138*B*c*d+29
2*C*c^2)*(-3*d*x+c)^3*(3*d*x+2*c)^4/c^6/d^3/(-27*d^3*x^3-27*c*d^2*x^2+4*c^
3)^(5/2)+2/19683*(9*A*d^2+3*B*c*d+C*c^2)*(-3*d*x+c)*(3*d*x+2*c)^5/c^5/d^3/
(-27*d^3*x^3-27*c*d^2*x^2+4*c^3)^(5/2)-2/19683*(-45*A*d^2-6*B*c*d+C*c^2)*(
-3*d*x+c)^2*(3*d*x+2*c)^5/c^6/d^3/(-27*d^3*x^3-27*c*d^2*x^2+4*c^3)^(5/2)+3
5/1259712*(-99*A*d^2-6*B*c*d+4*C*c^2)*(3*d*x+2*c)^5*(1-3*d*x/c)^(5/2)*arct
anh(1/3*(1-3*d*x/c)^(1/2)*3^(1/2))*3^(1/2)/c^4/d^3/(-27*d^3*x^3-27*c*d^2*x
^2+4*c^3)^(5/2)
```

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 264, normalized size of antiderivative = 0.49

$$\int \frac{A+Bx+Cx^2}{\left(4c^3-27cd^2x^2-27d^3x^3\right)^{5/2}}\,dx = \frac{(2c+3dx)\left(3\sqrt{c}(c-3dx)\left(704c^7C-841995Ad^7x^5-51030cd^6x^4(33\right.\right.}{}$$

input
```
Integrate[(A + B*x + C*x^2)/(4*c^3 - 27*c*d^2*x^2 - 27*d^3*x^3)^(5/2),x]
```

output
```
((2*c + 3*d*x)*(3*Sqrt[c]*(c - 3*d*x)*(704*c^7*C - 841995*A*d^7*x^5 - 5103
0*c*d^6*x^4*(33*A + B*x) + 48*c^6*d*(194*B + 57*C*x) + 72*c^5*d^2*(190*A +
 x*(159*B - 64*C*x)) + 1701*c^2*d^5*x^3*(-539*A + 20*x*(-3*B + C*x)) + 162
*c^3*d^4*x^2*(704*A + 7*x*(-49*B + 60*C*x)) + 108*c^4*d^3*x*(1749*A + x*(6
4*B + 343*C*x))) + 35*Sqrt[3]*(4*c^2*C - 6*B*c*d - 99*A*d^2)*(c - 3*d*x)^(
5/2)*(2*c + 3*d*x)^4*ArcTanh[Sqrt[c - 3*d*x]/(Sqrt[3]*Sqrt[c])]))/(1259712
*c^(13/2)*d^3*((c - 3*d*x)*(2*c + 3*d*x)^2)^(5/2))
```

Rubi [A] (verified)

Time = 1.34 (sec) , antiderivative size = 542, normalized size of antiderivative = 1.01, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.342, Rules used = {2526, 27, 2490, 2483, 27, 87, 52, 52, 52, 61, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{\left(4c^3-27cd^2x^2-27d^3x^3\right)^{5/2}}dx$$

$$\Big\downarrow \; 2526$$

$$\frac{2C}{243d^3\left(4c^3-27cd^2x^2-27d^3x^3\right)^{3/2}}-\frac{\int -\frac{27d^2(3Ad-(2cC-3Bd)x)}{\left(4c^3-27d^2x^2c-27d^3x^3\right)^{5/2}}dx}{81d^3}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{3Ad-(2cC-3Bd)x}{\left(4c^3-27d^2x^2c-27d^3x^3\right)^{5/2}}dx}{3d}+\frac{2C}{243d^3\left(4c^3-27cd^2x^2-27d^3x^3\right)^{3/2}}$$

$$\Big\downarrow \; 2490$$

$$\frac{\int \frac{(3Bd-2cC)\left(\frac{c}{3d}+x\right)-\frac{27cd^2(3Bd-2cC)-243Ad^4}{81d^3}}{\left(2c^3+9d\left(\frac{c}{3d}+x\right)c^2-27d^3\left(\frac{c}{3d}+x\right)^3\right)^{5/2}}d\left(\frac{c}{3d}+x\right)}{3d}+\frac{2C}{243d^3\left(4c^3-27cd^2x^2-27d^3x^3\right)^{3/2}}$$

$$\Big\downarrow \; 2483$$

$$\frac{23328\sqrt{3}c^{10}\left(3d\left(\frac{c}{3d}+x\right)+c\right)^5\left(2c^3-3c^2d\left(\frac{c}{3d}+x\right)\right)^{5/2}\int -\frac{-\frac{2Cc^2}{d}+3Bc-9Ad+3(2cC-3Bd)\left(\frac{c}{3d}+x\right)}{209952\sqrt{3}c^{10}\left(c+3d\left(\frac{c}{3d}+x\right)\right)^5\left(2c^3-3c^2d\left(\frac{c}{3d}+x\right)\right)^{5/2}}d\left(\frac{c}{3d}+x\right)}{d\left(2c^3+9c^2d\left(\frac{c}{3d}+x\right)-27d^3\left(\frac{c}{3d}+x\right)^3\right)^{5/2}}-$$
$$\frac{2C}{243d^3\left(4c^3-27cd^2x^2-27d^3x^3\right)^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{2C}{243d^3\left(4c^3-27cd^2x^2-27d^3x^3\right)^{3/2}}-\frac{\left(3d\left(\frac{c}{3d}+x\right)+c\right)^5\left(2c^3-3c^2d\left(\frac{c}{3d}+x\right)\right)^{5/2}\int\frac{-\frac{2Cc^2}{d}+3Bc-9Ad+3(2cC-3Bd)\left(\frac{c}{3d}+x\right)}{\left(c+3d\left(\frac{c}{3d}+x\right)\right)^5\left(2c^3-3c^2d\left(\frac{c}{3d}+x\right)\right)^{5/2}}d\left(\frac{c}{3d}+x\right)}{9d\left(2c^3+9c^2d\left(\frac{c}{3d}+x\right)-27d^3\left(\frac{c}{3d}+x\right)^3\right)^{5/2}}$$

$\big\downarrow$ 87

$$\frac{2C}{243d^3\left(4c^3-27cd^2x^2-27d^3x^3\right)^{3/2}}-\frac{\left(3d\left(\frac{c}{3d}+x\right)+c\right)^5\left(2c^3-3c^2d\left(\frac{c}{3d}+x\right)\right)^{5/2}\left(\frac{9Ad^2-6Bcd+4c^2C}{36c^3d^2\left(3d\left(\frac{c}{3d}+x\right)+c\right)^4\left(2c^3-3c^2d\left(\frac{c}{3d}+x\right)\right)^{3/2}}-\frac{1}{24}\left(\frac{99Ad}{c}+6B-\frac{4cC}{d}\right)\int\frac{}{(c+}\right.}{9d\left(2c^3+9c^2d\left(\frac{c}{3d}+x\right)-27d^3\left(\frac{c}{3d}+x\right)^3\right)^{5/2}}$$

$\big\downarrow$ 52

$$\frac{2C}{243d^3\left(4c^3-27cd^2x^2-27d^3x^3\right)^{3/2}}-\frac{\left(3d\left(\frac{c}{3d}+x\right)+c\right)^5\left(2c^3-3c^2d\left(\frac{c}{3d}+x\right)\right)^{5/2}\left(\frac{9Ad^2-6Bcd+4c^2C}{36c^3d^2\left(3d\left(\frac{c}{3d}+x\right)+c\right)^4\left(2c^3-3c^2d\left(\frac{c}{3d}+x\right)\right)^{3/2}}-\frac{1}{24}\left(\frac{99Ad}{c}+6B-\frac{4cC}{d}\right)\left(\frac{\int}{}\right.\right.}{9d\left(2c^3+9c^2d\left(\frac{c}{3d}+x\right)-27d^3\left(\frac{c}{3d}+x\right.\right.}$$

$\big\downarrow$ 52

$$\frac{2C}{243d^3\left(4c^3-27cd^2x^2-27d^3x^3\right)^{3/2}}-\frac{\left(3d\left(\frac{c}{3d}+x\right)+c\right)^5\left(2c^3-3c^2d\left(\frac{c}{3d}+x\right)\right)^{5/2}\left(\frac{9Ad^2-6Bcd+4c^2C}{36c^3d^2\left(3d\left(\frac{c}{3d}+x\right)+c\right)^4\left(2c^3-3c^2d\left(\frac{c}{3d}+x\right)\right)^{3/2}}-\frac{1}{24}\left(\frac{99Ad}{c}+6B-\frac{4cC}{d}\right)\left(\frac{7\int}{}\right.\right.}{9d\left(2c^3+9c^2d\left(\frac{c}{3d}+\right.\right.}$$

$\big\downarrow$ 52

$$\frac{2C}{243d^3\left(4c^3-27cd^2x^2-27d^3x^3\right)^{3/2}}-$$

$$\left(3d\left(\frac{c}{3d}+x\right)+c\right)^5\left(2c^3-3c^2d\left(\frac{c}{3d}+x\right)\right)^{5/2}\left(\frac{9Ad^2-6Bcd+4c^2C}{36c^3d^2\left(3d\left(\frac{c}{3d}+x\right)+c\right)^4\left(2c^3-3c^2d\left(\frac{c}{3d}+x\right)\right)^{3/2}}-\frac{1}{24}\left(\frac{99Ad}{c}+6B-\frac{4cC}{d}\right)\left(\frac{7\left(\right.}{}\right.\right.$$

$$9d$$

$\downarrow$ 61

$$\frac{2C}{243d^3\left(4c^3-27cd^2x^2-27d^3x^3\right)^{3/2}}-$$

$$\left(3d\left(\frac{c}{3d}+x\right)+c\right)^5\left(2c^3-3c^2d\left(\frac{c}{3d}+x\right)\right)^{5/2}\left(\frac{9Ad^2-6Bcd+4c^2C}{36c^3d^2\left(3d\left(\frac{c}{3d}+x\right)+c\right)^4\left(2c^3-3c^2d\left(\frac{c}{3d}+x\right)\right)^{3/2}}-\frac{1}{24}\left(\frac{99Ad}{c}+6B-\frac{4cC}{d}\right)\left(\frac{7\left(\right.}{}\right.\right.$$

$\downarrow$ 61

$$\frac{2C}{243d^3\left(4c^3-27cd^2x^2-27d^3x^3\right)^{3/2}}-$$

$$\frac{\left(\frac{9Ad^2-6Bcd+4c^2C}{36c^3d^2\left(3d\left(\frac{c}{3d}+x\right)+c\right)^4\left(2c^3-3c^2d\left(\frac{c}{3d}+x\right)\right)^{3/2}}-\frac{1}{24}\left(\frac{99Ad}{c}+6B-\frac{4cC}{d}\right)\left(\frac{7\left(\ \right)}{\ }\right)\right)}{\left(3d\left(\frac{c}{3d}+x\right)+c\right)^5\left(2c^3-3c^2d\left(\frac{c}{3d}+x\right)\right)^{5/2}}$$

↓ 73

$$\frac{2C}{243d^3\left(4c^3-27cd^2x^2-27d^3x^3\right)^{3/2}}-$$

$$\frac{\left(3d\left(\frac{c}{3d}+x\right)+c\right)^5\left(2c^3-3c^2d\left(\frac{c}{3d}+x\right)\right)^{5/2}\left(\frac{9Ad^2-6Bcd+4c^2C}{36c^3d^2\left(3d\left(\frac{c}{3d}+x\right)+c\right)^4\left(2c^3-3c^2d\left(\frac{c}{3d}+x\right)\right)^{3/2}}-\frac{1}{24}\left(\frac{99Ad}{c}+6B-\frac{4cC}{d}\right)\left(\frac{7\left(\cdots\right.}{\cdots}\right.\right.}{\cdots}$$

↓ 221

$$\frac{2C}{243d^3\left(4c^3-27cd^2x^2-27d^3x^3\right)^{3/2}}-$$

$$\frac{\left(3d\left(\frac{c}{3d}+x\right)+c\right)^5\left(2c^3-3c^2d\left(\frac{c}{3d}+x\right)\right)^{5/2}\left(\frac{9Ad^2-6Bcd+4c^2C}{36c^3d^2\left(3d\left(\frac{c}{3d}+x\right)+c\right)^4\left(2c^3-3c^2d\left(\frac{c}{3d}+x\right)\right)^{3/2}}-\frac{1}{24}\left(\frac{7\left(\frac{5\left(\frac{2}{9c^3d\sqrt{2c^3-3c^2d\left(\frac{c}{3d}+x\right)}}\right.}{\ldots}\right.}{\ldots}\right.\right.}{\ldots}$$

input
```
Int[(A + B*x + C*x^2)/(4*c^3 - 27*c*d^2*x^2 - 27*d^3*x^3)^(5/2),x]
```

output
```
(2*C)/(243*d^3*(4*c^3 - 27*c*d^2*x^2 - 27*d^3*x^3)^(3/2)) - ((c + 3*d*(c/(
3*d) + x))^5*(2*c^3 - 3*c^2*d*(c/(3*d) + x))^(5/2)*((4*c^2*C - 6*B*c*d + 9
*A*d^2)/(36*c^3*d^2*(c + 3*d*(c/(3*d) + x))^4*(2*c^3 - 3*c^2*d*(c/(3*d) +
x))^(3/2)) - ((6*B - (4*c*C)/d + (99*A*d)/c)*(-1/27*1/(c^3*d*(c + 3*d*(c/(
3*d) + x))^3*(2*c^3 - 3*c^2*d*(c/(3*d) + x))^(3/2)) + (-1/18*1/(c^3*d*(c +
 3*d*(c/(3*d) + x))^2*(2*c^3 - 3*c^2*d*(c/(3*d) + x))^(3/2)) + (7*(-1/9*1/
(c^3*d*(c + 3*d*(c/(3*d) + x))*(2*c^3 - 3*c^2*d*(c/(3*d) + x))^(3/2)) + (5
*(2/(27*c^3*d*(2*c^3 - 3*c^2*d*(c/(3*d) + x))^(3/2)) + (2/(9*c^3*d*Sqrt[2*
c^3 - 3*c^2*d*(c/(3*d) + x)]) - (2*ArcTanh[Sqrt[2*c^3 - 3*c^2*d*(c/(3*d) +
 x)]/(Sqrt[3]*c^(3/2))])/(9*Sqrt[3]*c^(9/2)*d))/(3*c^3)))/(6*c)))/(12*c))/
(2*c)))/24))/(9*d*(2*c^3 + 9*c^2*d*(c/(3*d) + x) - 27*d^3*(c/(3*d) + x)^3)
^(5/2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 2483
```
Int[((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (d_.)*(x_)^3)^(p_), x_S
ymbol] :> Simp[(a + b*x + d*x^3)^p/((3*a - b*x)^p*(3*a + 2*b*x)^(2*p))   In
t[(e + f*x)^m*(3*a - b*x)^p*(3*a + 2*b*x)^(2*p), x], x] /; FreeQ[{a, b, d,
e, f, m, p}, x] && EqQ[4*b^3 + 27*a^2*d, 0] &&  !IntegerQ[p]
```

rule 2490
```
Int[(P3_)^(p_.)*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] :> With[{a = Coeff[P3
, x, 0], b = Coeff[P3, x, 1], c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Su
bst[Int[((3*d*e - c*f)/(3*d) + f*x)^m*Simp[(2*c^3 - 9*b*c*d + 27*a*d^2)/(27
*d^2) - (c^2 - 3*b*d)*(x/(3*d)) + d*x^3, x]^p, x], x, x + c/(3*d)] /; NeQ[c
, 0]] /; FreeQ[{e, f, m, p}, x] && PolyQ[P3, x, 3]
```

rule 2526
```
Int[(Pm_)*(Qn_)^(p_), x_Symbol] :> With[{m = Expon[Pm, x], n = Expon[Qn, x]
}, Simp[Coeff[Pm, x, m]*(Qn^(p + 1)/(n*(p + 1)*Coeff[Qn, x, n])), x] + Simp
[1/(n*Coeff[Qn, x, n])   Int[ExpandToSum[n*Coeff[Qn, x, n]*Pm - Coeff[Pm, x
, m]*D[Qn, x], x]*Qn^p, x], x] /; EqQ[m, n - 1]] /; FreeQ[p, x] && PolyQ[Pm
, x] && PolyQ[Qn, x] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 814, normalized size of antiderivative = 1.52

method	result	size
default	Expression too large to display	814

input
```
int((C*x^2+B*x+A)/(-27*d^3*x^3-27*c*d^2*x^2+4*c^3)^(5/2),x,method=_RETURNV
ERBOSE)
```

output
```
-1/1259712*(45360*B*3^(1/2)*arctanh(1/3*(-3*d*x+c)^(1/2)*3^(1/2)/c^(1/2))*
(-3*d*x+c)^(3/2)*c^2*d^4*x^3-30240*C*3^(1/2)*arctanh(1/3*(-3*d*x+c)^(1/2)*
3^(1/2)/c^(1/2))*(-3*d*x+c)^(3/2)*c^3*d^3*x^3+748440*A*3^(1/2)*arctanh(1/3
*(-3*d*x+c)^(1/2)*3^(1/2)/c^(1/2))*(-3*d*x+c)^(3/2)*c^2*d^4*x^2+45360*B*3^
(1/2)*arctanh(1/3*(-3*d*x+c)^(1/2)*3^(1/2)/c^(1/2))*(-3*d*x+c)^(3/2)*c^3*d
^3*x^2-30240*C*3^(1/2)*arctanh(1/3*(-3*d*x+c)^(1/2)*3^(1/2)/c^(1/2))*(-3*d
*x+c)^(3/2)*c^4*d^2*x^2+748440*A*3^(1/2)*arctanh(1/3*(-3*d*x+c)^(1/2)*3^(1
/2)/c^(1/2))*(-3*d*x+c)^(3/2)*c*d^5*x^3+17010*B*3^(1/2)*arctanh(1/3*(-3*d*
x+c)^(1/2)*3^(1/2)/c^(1/2))*(-3*d*x+c)^(3/2)*c*d^5*x^4-11340*C*3^(1/2)*arc
tanh(1/3*(-3*d*x+c)^(1/2)*3^(1/2)/c^(1/2))*(-3*d*x+c)^(3/2)*c^2*d^4*x^4+33
2640*A*3^(1/2)*arctanh(1/3*(-3*d*x+c)^(1/2)*3^(1/2)/c^(1/2))*(-3*d*x+c)^(3
/2)*c^3*d^3*x+20160*B*3^(1/2)*arctanh(1/3*(-3*d*x+c)^(1/2)*3^(1/2)/c^(1/2)
)*(-3*d*x+c)^(3/2)*c^4*d^2*x-13440*C*3^(1/2)*arctanh(1/3*(-3*d*x+c)^(1/2)*
3^(1/2)/c^(1/2))*(-3*d*x+c)^(3/2)*c^5*d*x-41040*A*c^(11/2)*d^2-27936*B*c^(
13/2)*d-8208*C*c^(13/2)*d*x+5051970*A*c^(3/2)*d^6*x^4+306180*B*c^(5/2)*d^5
*x^4-204120*C*c^(7/2)*d^4*x^4+2750517*A*c^(5/2)*d^5*x^3+166698*B*c^(7/2)*d
^4*x^3-2240*C*3^(1/2)*arctanh(1/3*(-3*d*x+c)^(1/2)*3^(1/2)/c^(1/2))*(-3*d*
x+c)^(3/2)*c^6-111132*C*c^(9/2)*d^3*x^3-342144*A*c^(7/2)*d^4*x^2-20736*B*c
^(9/2)*d^3*x^2+13824*C*c^(11/2)*d^2*x^2-566676*A*c^(9/2)*d^3*x-34344*B*c^(
11/2)*d^2*x-102060*C*c^(5/2)*d^5*x^5+153090*B*c^(3/2)*d^6*x^5+2525985*A...
```

Fricas [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 1232, normalized size of antiderivative = 2.30

$$\int \frac{A+Bx+Cx^2}{(4c^3-27cd^2x^2-27d^3x^3)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)/(-27*d^3*x^3-27*c*d^2*x^2+4*c^3)^(5/2),x, algorith
m="fricas")
```

output
```
[-1/2519424*(35*sqrt(3)*(128*C*c^9 - 192*B*c^8*d - 3168*A*c^7*d^2 + 2187*(
4*C*c^2*d^7 - 6*B*c*d^8 - 99*A*d^9)*x^7 + 5832*(4*C*c^3*d^6 - 6*B*c^2*d^7
- 99*A*c*d^8)*x^6 + 5103*(4*C*c^4*d^5 - 6*B*c^3*d^6 - 99*A*c^2*d^7)*x^5 +
810*(4*C*c^5*d^4 - 6*B*c^4*d^5 - 99*A*c^3*d^6)*x^4 - 1080*(4*C*c^6*d^3 - 6
*B*c^5*d^4 - 99*A*c^4*d^5)*x^3 - 432*(4*C*c^7*d^2 - 6*B*c^6*d^3 - 99*A*c^5
*d^4)*x^2 + 48*(4*C*c^8*d - 6*B*c^7*d^2 - 99*A*c^6*d^3)*x)*sqrt(c)*log((9*
d^2*x^2 - 6*c*d*x - 8*c^2 + 2*sqrt(3)*sqrt(-27*d^3*x^3 - 27*c*d^2*x^2 + 4*
c^3)*sqrt(c))/(9*d^2*x^2 + 12*c*d*x + 4*c^2)) - 6*(704*C*c^8 + 9312*B*c^7*
d + 13680*A*c^6*d^2 + 8505*(4*C*c^3*d^5 - 6*B*c^2*d^6 - 99*A*c*d^7)*x^5 +
17010*(4*C*c^4*d^4 - 6*B*c^3*d^5 - 99*A*c^2*d^6)*x^4 + 9261*(4*C*c^5*d^3 -
 6*B*c^4*d^4 - 99*A*c^3*d^5)*x^3 - 1152*(4*C*c^6*d^2 - 6*B*c^5*d^3 - 99*A*
c^4*d^4)*x^2 + 36*(76*C*c^7*d + 318*B*c^6*d^2 + 5247*A*c^5*d^3)*x)*sqrt(-2
7*d^3*x^3 - 27*c*d^2*x^2 + 4*c^3))/(2187*c^7*d^10*x^7 + 5832*c^8*d^9*x^6 +
 5103*c^9*d^8*x^5 + 810*c^10*d^7*x^4 - 1080*c^11*d^6*x^3 - 432*c^12*d^5*x^
2 + 48*c^13*d^4*x + 32*c^14*d^3), 1/1259712*(35*sqrt(3)*(128*C*c^9 - 192*B
*c^8*d - 3168*A*c^7*d^2 + 2187*(4*C*c^2*d^7 - 6*B*c*d^8 - 99*A*d^9)*x^7 +
5832*(4*C*c^3*d^6 - 6*B*c^2*d^7 - 99*A*c*d^8)*x^6 + 5103*(4*C*c^4*d^5 - 6*
B*c^3*d^6 - 99*A*c^2*d^7)*x^5 + 810*(4*C*c^5*d^4 - 6*B*c^4*d^5 - 99*A*c^3*
d^6)*x^4 - 1080*(4*C*c^6*d^3 - 6*B*c^5*d^4 - 99*A*c^4*d^5)*x^3 - 432*(4*C*
c^7*d^2 - 6*B*c^6*d^3 - 99*A*c^5*d^4)*x^2 + 48*(4*C*c^8*d - 6*B*c^7*d^2...
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2}{(4c^3-27cd^2x^2-27d^3x^3)^{5/2}}\,dx=\int \frac{A+Bx+Cx^2}{\left(-\left(-c+3dx\right)\left(2c+3dx\right)^2\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((C*x**2+B*x+A)/(-27*d**3*x**3-27*c*d**2*x**2+4*c**3)**(5/2),x)
```

output
```
Integral((A + B*x + C*x**2)/(-(-c + 3*d*x)*(2*c + 3*d*x)**2)**(5/2), x)
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{(4c^3-27cd^2x^2-27d^3x^3)^{5/2}}\,dx=\int \frac{Cx^2+Bx+A}{(-27\,d^3x^3-27\,cd^2x^2+4\,c^3)^{\frac{5}{2}}}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(-27*d^3*x^3-27*c*d^2*x^2+4*c^3)^(5/2),x, algorith
m="maxima")
```

output
```
integrate((C*x^2 + B*x + A)/(-27*d^3*x^3 - 27*c*d^2*x^2 + 4*c^3)^(5/2), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 429, normalized size of antiderivative = 0.80

$$\int \frac{A+Bx+Cx^2}{(4c^3-27cd^2x^2-27d^3x^3)^{5/2}}\,dx=\frac{35\sqrt{3}(4\,Cc^2-6\,Bcd-99\,Ad^2)\arctan\left(\frac{\sqrt{3}\sqrt{-3\,dx+c}}{3\sqrt{-c}}\right)}{1259712\sqrt{-c}c^6d^3\mathrm{sgn}\left(-3\,dx-2\,c\right)}$$
$$+\frac{2\left((3\,dx-c)Cc^2+Cc^3-6\,(3\,dx-c)Bcd+3\,Bc^2d-45\,(3\,dx-c)Ad^2+9\,Acd^2\right)}{19683\,(3\,dx-c)\sqrt{-3\,dx+c}c^6d^3\mathrm{sgn}\left(-3\,dx-2\,c\right)}$$
$$-\frac{292\,(3\,dx-c)^3\sqrt{-3\,dx+c}Cc^2+2956\,(3\,dx-c)^2\sqrt{-3\,dx+c}Cc^3-9948\,(-3\,dx+c)^{\frac{3}{2}}Cc^4+9396\sqrt{-3}}{}$$

input
```
integrate((C*x^2+B*x+A)/(-27*d^3*x^3-27*c*d^2*x^2+4*c^3)^(5/2),x, algorith
m="giac")
```

output
```
35/1259712*sqrt(3)*(4*C*c^2 - 6*B*c*d - 99*A*d^2)*arctan(1/3*sqrt(3)*sqrt(
-3*d*x + c)/sqrt(-c))/(sqrt(-c)*c^6*d^3*sgn(-3*d*x - 2*c)) + 2/19683*((3*d
*x - c)*C*c^2 + C*c^3 - 6*(3*d*x - c)*B*c*d + 3*B*c^2*d - 45*(3*d*x - c)*A
*d^2 + 9*A*c*d^2)/((3*d*x - c)*sqrt(-3*d*x + c)*c^6*d^3*sgn(-3*d*x - 2*c))
 - 1/1259712*(292*(3*d*x - c)^3*sqrt(-3*d*x + c)*C*c^2 + 2956*(3*d*x - c)^
2*sqrt(-3*d*x + c)*C*c^3 - 9948*(-3*d*x + c)^(3/2)*C*c^4 + 9396*sqrt(-3*d*
x + c)*C*c^5 + 138*(3*d*x - c)^3*sqrt(-3*d*x + c)*B*c*d + 1902*(3*d*x - c)
^2*sqrt(-3*d*x + c)*B*c^2*d - 9270*(-3*d*x + c)^(3/2)*B*c^3*d + 17010*sqrt
(-3*d*x + c)*B*c^4*d - 4635*(3*d*x - c)^3*sqrt(-3*d*x + c)*A*d^2 - 46377*(
3*d*x - c)^2*sqrt(-3*d*x + c)*A*c*d^2 + 158085*(-3*d*x + c)^(3/2)*A*c^2*d^
2 - 185895*sqrt(-3*d*x + c)*A*c^3*d^2)/((3*d*x + 2*c)^4*c^6*d^3*sgn(-3*d*x
 - 2*c))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{\left(4c^3-27cd^2x^2-27d^3x^3\right)^{5/2}}\,dx=\int \frac{C\,x^2+B\,x+A}{\left(4\,c^3-27\,c\,d^2\,x^2-27\,d^3\,x^3\right)^{5/2}}\,dx$$

input
```
int((A + B*x + C*x^2)/(4*c^3 - 27*d^3*x^3 - 27*c*d^2*x^2)^(5/2),x)
```

output
```
int((A + B*x + C*x^2)/(4*c^3 - 27*d^3*x^3 - 27*c*d^2*x^2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 1538, normalized size of antiderivative = 2.87

$$\int \frac{A+Bx+Cx^2}{\left(4c^3-27cd^2x^2-27d^3x^3\right)^{5/2}}\,dx=\text{Too large to display}$$

input
```
int((C*x^2+B*x+A)/(-27*d^3*x^3-27*c*d^2*x^2+4*c^3)^(5/2),x)
```

output

```
(55440*sqrt(c)*sqrt(c - 3*d*x)*sqrt(3)*log(sqrt(c - 3*d*x) - sqrt(c)*sqrt(
3))*a*c**5*d**2 + 166320*sqrt(c)*sqrt(c - 3*d*x)*sqrt(3)*log(sqrt(c - 3*d*
x) - sqrt(c)*sqrt(3))*a*c**4*d**3*x - 249480*sqrt(c)*sqrt(c - 3*d*x)*sqrt(
3)*log(sqrt(c - 3*d*x) - sqrt(c)*sqrt(3))*a*c**3*d**4*x**2 - 1496880*sqrt(
c)*sqrt(c - 3*d*x)*sqrt(3)*log(sqrt(c - 3*d*x) - sqrt(c)*sqrt(3))*a*c**2*d
**5*x**3 - 1964655*sqrt(c)*sqrt(c - 3*d*x)*sqrt(3)*log(sqrt(c - 3*d*x) - s
qrt(c)*sqrt(3))*a*c*d**6*x**4 - 841995*sqrt(c)*sqrt(c - 3*d*x)*sqrt(3)*log
(sqrt(c - 3*d*x) - sqrt(c)*sqrt(3))*a*d**7*x**5 + 3360*sqrt(c)*sqrt(c - 3*
d*x)*sqrt(3)*log(sqrt(c - 3*d*x) - sqrt(c)*sqrt(3))*b*c**6*d + 10080*sqrt(
c)*sqrt(c - 3*d*x)*sqrt(3)*log(sqrt(c - 3*d*x) - sqrt(c)*sqrt(3))*b*c**5*d
**2*x - 15120*sqrt(c)*sqrt(c - 3*d*x)*sqrt(3)*log(sqrt(c - 3*d*x) - sqrt(c
)*sqrt(3))*b*c**4*d**3*x**2 - 90720*sqrt(c)*sqrt(c - 3*d*x)*sqrt(3)*log(sq
rt(c - 3*d*x) - sqrt(c)*sqrt(3))*b*c**3*d**4*x**3 - 119070*sqrt(c)*sqrt(c
- 3*d*x)*sqrt(3)*log(sqrt(c - 3*d*x) - sqrt(c)*sqrt(3))*b*c**2*d**5*x**4 -
 51030*sqrt(c)*sqrt(c - 3*d*x)*sqrt(3)*log(sqrt(c - 3*d*x) - sqrt(c)*sqrt(
3))*b*c*d**6*x**5 - 2240*sqrt(c)*sqrt(c - 3*d*x)*sqrt(3)*log(sqrt(c - 3*d*
x) - sqrt(c)*sqrt(3))*c**8 - 6720*sqrt(c)*sqrt(c - 3*d*x)*sqrt(3)*log(sqrt
(c - 3*d*x) - sqrt(c)*sqrt(3))*c**7*d*x + 10080*sqrt(c)*sqrt(c - 3*d*x)*sq
rt(3)*log(sqrt(c - 3*d*x) - sqrt(c)*sqrt(3))*c**6*d**2*x**2 + 60480*sqrt(c
)*sqrt(c - 3*d*x)*sqrt(3)*log(sqrt(c - 3*d*x) - sqrt(c)*sqrt(3))*c**5*d...
```

3.41 $\int (A+Bx+Cx^2)(4c^3-27cd^2x^2-27d^3x^3)^p\,dx$

Optimal result

Integrand size = 36, antiderivative size = 246

$$\begin{aligned}
&\int (A+Bx+Cx^2)\left(4c^3-27cd^2x^2-27d^3x^3\right)^p\,dx \\
&= -\frac{(cCp+3Bd(1+p))(c-3dx)(2c+3dx)\left(4c^3-27cd^2x^2-27d^3x^3\right)^p}{27d^3(1+p)(2+3p)} \\
&\quad+\frac{C(c-3dx)^2(2c+3dx)\left(4c^3-27cd^2x^2-27d^3x^3\right)^p}{81d^3(1+p)} \\
&\quad-\frac{3^{-3+2p}4^{-p}(2c^2C-3Bcd+9Ad^2(2+3p))(c-3dx)\left(1+\frac{3dx}{2c}\right)^{-2p}\left(4c^3-27cd^2x^2-27d^3x^3\right)^p \text{Hypergeo}}{d^3(1+p)(2+3p)}
\end{aligned}$$

output

```
-1/27*(c*C*p+3*B*d*(p+1))*(-3*d*x+c)*(3*d*x+2*c)*(-27*d^3*x^3-27*c*d^2*x^2
+4*c^3)^p/d^3/(p+1)/(2+3*p)+1/81*C*(-3*d*x+c)^2*(3*d*x+2*c)*(-27*d^3*x^3-2
7*c*d^2*x^2+4*c^3)^p/d^3/(p+1)-3^(-3+2*p)*(2*C*c^2-3*B*c*d+9*A*d^2*(2+3*p)
)*(-3*d*x+c)*(-27*d^3*x^3-27*c*d^2*x^2+4*c^3)^p*hypergeom([-2*p, p+1],[2+p
],1/3*(-3*d*x+c)/c)/(4^p)/d^3/(p+1)/(2+3*p)/((1+3/2*d*x/c)^(2*p))
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.65

$$\int \left(A+Bx+Cx^2\right)\left(4c^3-27cd^2x^2-27d^3x^3\right)^p\,dx$$
$$=\frac{3^{-3+2p}(c-3dx)\left((c-3dx)(2c+3dx)^2\right)^p\left(2+\frac{3dx}{c}\right)^{-2p}\left(3c(4cC-3Bd)\,\text{Hypergeometric2F1}\left(-1-2p,1\right.\right.}{}$$

input
```
Integrate[(A + B*x + C*x^2)*(4*c^3 - 27*c*d^2*x^2 - 27*d^3*x^3)^p,x]
```

output
```
(3^(-3 + 2*p)*(c - 3*d*x)*((c - 3*d*x)*(2*c + 3*d*x)^2)^p*(3*c*(4*c*C - 3*
B*d)*Hypergeometric2F1[-1 - 2*p, 1 + p, 2 + p, 1/3 - (d*x)/c] + (-4*c^2*C
+ 6*B*c*d - 9*A*d^2)*Hypergeometric2F1[-2*p, 1 + p, 2 + p, 1/3 - (d*x)/c]
- 9*c^2*C*Hypergeometric2F1[-2*(1 + p), 1 + p, 2 + p, 1/3 - (d*x)/c]))/(d^
3*(1 + p)*(2 + (3*d*x)/c)^(2*p))
```

Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.54, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}=0.222$, Rules used = {2526, 27, 2490, 2483, 27, 90, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(A+Bx+Cx^2\right)\left(4c^3-27cd^2x^2-27d^3x^3\right)^p\,dx$$

$$\Big\downarrow \ 2526$$

$$-\frac{\int -27d^2(3Ad-(2cC-3Bd)x)\left(4c^3-27d^2x^2c-27d^3x^3\right)^p\,dx}{81d^3}-\frac{C\left(4c^3-27cd^2x^2-27d^3x^3\right)^{p+1}}{81d^3(p+1)}$$

$$\Big\downarrow \ 27$$

$$\frac{\int (3Ad-(2cC-3Bd)x)\left(4c^3-27d^2x^2c-27d^3x^3\right)^p\,dx}{3d}-\frac{C\left(4c^3-27cd^2x^2-27d^3x^3\right)^{p+1}}{81d^3(p+1)}$$

$$\Big\downarrow \text{2490}$$

$$\frac{\int \left((3Bd-2cC)\left(\frac{c}{3d}+x\right)-\frac{27cd^2(3Bd-2cC)-243Ad^4}{81d^3}\right)\left(2c^3+9d\left(\frac{c}{3d}+x\right)c^2-27d^3\left(\frac{c}{3d}+x\right)^3\right)^p d\left(\frac{c}{3d}+x\right)}{3d}-\frac{C\left(4c^3-27cd^2x^2-27d^3x^3\right)^{p+1}}{81d^3(p+1)}$$

$$\Big\downarrow \text{2483}$$

$$\frac{\left(6c^3-9c^2d\left(\frac{c}{3d}+x\right)\right)^{-p}\left(6c^3+18c^2d\left(\frac{c}{3d}+x\right)\right)^{-2p}\left(2c^3+9c^2d\left(\frac{c}{3d}+x\right)-27d^3\left(\frac{c}{3d}+x\right)^3\right)^p\int -\frac{1}{3}\left(6c^3-9c^2d\left(\frac{c}{3d}\right.\right.}{3d}$$

$$\frac{C\left(4c^3-27cd^2x^2-27d^3x^3\right)^{p+1}}{81d^3(p+1)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\left(6c^3-9c^2d\left(\frac{c}{3d}+x\right)\right)^{-p}\left(2c^3+9c^2d\left(\frac{c}{3d}+x\right)-27d^3\left(\frac{c}{3d}+x\right)^3\right)^p\left(6c^3+18c^2d\left(\frac{c}{3d}+x\right)\right)^{-2p}\int\left(6c^3-9c^2d\left(\frac{c}{3d}+\right.\right.}{9d}$$

$$\frac{C\left(4c^3-27cd^2x^2-27d^3x^3\right)^{p+1}}{81d^3(p+1)}$$

$$\Big\downarrow \text{90}$$

$$-\frac{\left(6c^3-9c^2d\left(\frac{c}{3d}+x\right)\right)^{-p}\left(2c^3+9c^2d\left(\frac{c}{3d}+x\right)-27d^3\left(\frac{c}{3d}+x\right)^3\right)^p\left(6c^3+18c^2d\left(\frac{c}{3d}+x\right)\right)^{-2p}\left(-\frac{(9Ad^2(3p+2)-3Bc\right.}{9d}$$

$$\frac{C\left(4c^3-27cd^2x^2-27d^3x^3\right)^{p+1}}{81d^3(p+1)}$$

$$\Big\downarrow \text{80}$$

$$-\frac{\left(6c^3-9c^2d\left(\frac{c}{3d}+x\right)\right)^{-p}\left(2c^3+9c^2d\left(\frac{c}{3d}+x\right)-27d^3\left(\frac{c}{3d}+x\right)^3\right)^p\left(6c^3+18c^2d\left(\frac{c}{3d}+x\right)\right)^{-2p}\left(-\frac{9^p\left(6c^3+18c^2d\left(\frac{c}{3d}\right.\right.}{}\right.}{}$$

$$\frac{C\left(4c^3-27cd^2x^2-27d^3x^3\right)^{p+1}}{81d^3(p+1)}$$

$$\Big\downarrow \text{79}$$

$$-\frac{\left(6c^3-9c^2d\left(\frac{c}{3d}+x\right)\right)^{-p}\left(2c^3+9c^2d\left(\frac{c}{3d}+x\right)-27d^3\left(\frac{c}{3d}+x\right)^3\right)^p\left(6c^3+18c^2d\left(\frac{c}{3d}+x\right)\right)^{-2p}\left(\frac{9^{p-1}\left(\frac{3d\left(\frac{c}{3d}+x\right)+c}{c}\right)}{}\right.}{}$$

$$\frac{C\left(4c^3-27cd^2x^2-27d^3x^3\right)^{p+1}}{81d^3(p+1)}$$

input
```
Int[(A + B*x + C*x^2)*(4*c^3 - 27*c*d^2*x^2 - 27*d^3*x^3)^p,x]
```

output
```
-1/81*(C*(4*c^3 - 27*c*d^2*x^2 - 27*d^3*x^3)^(1 + p))/(d^3*(1 + p)) - ((2*
c^3 + 9*c^2*d*(c/(3*d) + x) - 27*d^3*(c/(3*d) + x)^3)^p*(-1/54*((2*c*C - 3
*B*d)*(6*c^3 - 9*c^2*d*(c/(3*d) + x))^(1 + p)*(6*c^3 + 18*c^2*d*(c/(3*d) +
 x))^(1 + 2*p))/(c^4*d^2*(2 + 3*p)) + (9^(-1 + p)*(2*c^2*C - 3*B*c*d + 9*A
*d^2*(2 + 3*p))*(6*c^3 - 9*c^2*d*(c/(3*d) + x))^(1 + p)*(6*c^3 + 18*c^2*d*
(c/(3*d) + x))^(2*p)*Hypergeometric2F1[-2*p, 1 + p, 2 + p, (2*c - 3*d*(c/(
3*d) + x))/(3*c)])/(c^2*d^2*(1 + p)*(2 + 3*p)*((c + 3*d*(c/(3*d) + x))/c)^
(2*p))))/(9*d*(6*c^3 - 9*c^2*d*(c/(3*d) + x))^p*(6*c^3 + 18*c^2*d*(c/(3*d)
 + x))^(2*p))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 2483
```
Int[((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (d_.)*(x_)^3)^(p_), x_S
ymbol] :> Simp[(a + b*x + d*x^3)^p/((3*a - b*x)^p*(3*a + 2*b*x)^(2*p))   In
t[(e + f*x)^m*(3*a - b*x)^p*(3*a + 2*b*x)^(2*p), x], x] /; FreeQ[{a, b, d,
e, f, m, p}, x] && EqQ[4*b^3 + 27*a^2*d, 0] &&  !IntegerQ[p]
```

rule 2490
```
Int[(P3_)^(p_.)*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] :> With[{a = Coeff[P3
, x, 0], b = Coeff[P3, x, 1], c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Su
bst[Int[((3*d*e - c*f)/(3*d) + f*x)^m*Simp[(2*c^3 - 9*b*c*d + 27*a*d^2)/(27
*d^2) - (c^2 - 3*b*d)*(x/(3*d)) + d*x^3, x]^p, x], x, x + c/(3*d)] /; NeQ[c
, 0]] /; FreeQ[{e, f, m, p}, x] && PolyQ[P3, x, 3]
```

rule 2526
```
Int[(Pm_)*(Qn_)^(p_), x_Symbol] :> With[{m = Expon[Pm, x], n = Expon[Qn, x]
}, Simp[Coeff[Pm, x, m]*(Qn^(p + 1)/(n*(p + 1)*Coeff[Qn, x, n])), x] + Simp
[1/(n*Coeff[Qn, x, n])   Int[ExpandToSum[n*Coeff[Qn, x, n]*Pm - Coeff[Pm, x
, m]*D[Qn, x], x]*Qn^p, x], x] /; EqQ[m, n - 1]] /; FreeQ[p, x] && PolyQ[Pm
, x] && PolyQ[Qn, x] && NeQ[p, -1]
```

Maple [F]

$$\int \left(C\,x^{2}+Bx+A\right)\left(-27d^{3}x^{3}-27c\,d^{2}x^{2}+4c^{3}\right)^{p}dx$$

input
```
int((C*x^2+B*x+A)*(-27*d^3*x^3-27*c*d^2*x^2+4*c^3)^p,x)
```

output
```
int((C*x^2+B*x+A)*(-27*d^3*x^3-27*c*d^2*x^2+4*c^3)^p,x)
```

Fricas [F]

$$\int \left(A+Bx+Cx^2\right)\left(4c^3-27cd^2x^2-27d^3x^3\right)^p\,dx$$
$$=\int \left(Cx^{2}+Bx+A\right)\left(-27\,d^{3}x^{3}-27\,cd^{2}x^{2}+4\,c^{3}\right)^{p}dx$$

input
```
integrate((C*x^2+B*x+A)*(-27*d^3*x^3-27*c*d^2*x^2+4*c^3)^p,x, algorithm="f
ricas")
```

output
```
integral((C*x^2 + B*x + A)*(-27*d^3*x^3 - 27*c*d^2*x^2 + 4*c^3)^p, x)
```

Sympy [F]

$$\int \left(A+Bx+Cx^2\right)\left(4c^3-27cd^2x^2-27d^3x^3\right)^p\,dx$$
$$=\int \left(-(-c+3dx)\left(2c+3dx\right)^{2}\right)^{p}\left(A+Bx+Cx^{2}\right)\,dx$$

input
```
integrate((C*x**2+B*x+A)*(-27*d**3*x**3-27*c*d**2*x**2+4*c**3)**p,x)
```

output
```
Integral((-(-c + 3*d*x)*(2*c + 3*d*x)**2)**p*(A + B*x + C*x**2), x)
```

Maxima [F]

$$\int (A+Bx+Cx^2)\left(4c^3-27cd^2x^2-27d^3x^3\right)^p\,dx = \int (Cx^2+Bx+A)\left(-27\,d^3x^3-27\,cd^2x^2+4\,c^3\right)^p\,dx$$

input
```
integrate((C*x^2+B*x+A)*(-27*d^3*x^3-27*c*d^2*x^2+4*c^3)^p,x, algorithm="m
axima")
```

output
```
integrate((C*x^2 + B*x + A)*(-27*d^3*x^3 - 27*c*d^2*x^2 + 4*c^3)^p, x)
```

Giac [F]

$$\int (A+Bx+Cx^2)\left(4c^3-27cd^2x^2-27d^3x^3\right)^p\,dx = \int (Cx^2+Bx+A)\left(-27\,d^3x^3-27\,cd^2x^2+4\,c^3\right)^p\,dx$$

input
```
integrate((C*x^2+B*x+A)*(-27*d^3*x^3-27*c*d^2*x^2+4*c^3)^p,x, algorithm="g
iac")
```

output
```
integrate((C*x^2 + B*x + A)*(-27*d^3*x^3 - 27*c*d^2*x^2 + 4*c^3)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (A+Bx+Cx^2)\left(4c^3-27cd^2x^2-27d^3x^3\right)^p\,dx = \int \left(C\,x^2+B\,x+A\right)\,\left(4\,c^3-27\,c\,d^2\,x^2-27\,d^3\,x^3\right)^p\,dx$$

input
```
int((A + B*x + C*x^2)*(4*c^3 - 27*d^3*x^3 - 27*c*d^2*x^2)^p,x)
```

output
```
int((A + B*x + C*x^2)*(4*c^3 - 27*d^3*x^3 - 27*c*d^2*x^2)^p, x)
```

Reduce [F]

$$\int (A+Bx+Cx^2)\,(4c^3-27cd^2x^2-27d^3x^3)^p\,dx=\text{too large to display}$$

input
```
int((C*x^2+B*x+A)*(-27*d^3*x^3-27*c*d^2*x^2+4*c^3)^p,x)
```

output
```
(81*(4*c**3 - 27*c*d**2*x**2 - 27*d**3*x**3)**p*a*c*d**2*p**2 + 135*(4*c**
3 - 27*c*d**2*x**2 - 27*d**3*x**3)**p*a*c*d**2*p + 54*(4*c**3 - 27*c*d**2*
x**2 - 27*d**3*x**3)**p*a*c*d**2 + 243*(4*c**3 - 27*c*d**2*x**2 - 27*d**3*
x**3)**p*a*d**3*p**2*x + 405*(4*c**3 - 27*c*d**2*x**2 - 27*d**3*x**3)**p*a
*d**3*p*x + 162*(4*c**3 - 27*c*d**2*x**2 - 27*d**3*x**3)**p*a*d**3*x - 54*
(4*c**3 - 27*c*d**2*x**2 - 27*d**3*x**3)**p*b*c**2*d*p**2 - 81*(4*c**3 - 2
7*c*d**2*x**2 - 27*d**3*x**3)**p*b*c**2*d*p - 27*(4*c**3 - 27*c*d**2*x**2
- 27*d**3*x**3)**p*b*c**2*d + 81*(4*c**3 - 27*c*d**2*x**2 - 27*d**3*x**3)*
*p*b*c*d**2*p**2*x + 81*(4*c**3 - 27*c*d**2*x**2 - 27*d**3*x**3)**p*b*c*d*
*2*p*x + 243*(4*c**3 - 27*c*d**2*x**2 - 27*d**3*x**3)**p*b*d**3*p**2*x**2
+ 324*(4*c**3 - 27*c*d**2*x**2 - 27*d**3*x**3)**p*b*d**3*p*x**2 + 81*(4*c*
*3 - 27*c*d**2*x**2 - 27*d**3*x**3)**p*b*d**3*x**2 + 18*(4*c**3 - 27*c*d**
2*x**2 - 27*d**3*x**3)**p*c**4*p + 10*(4*c**3 - 27*c*d**2*x**2 - 27*d**3*x
**3)**p*c**4 - 54*(4*c**3 - 27*c*d**2*x**2 - 27*d**3*x**3)**p*c**3*d*p**2*
x - 54*(4*c**3 - 27*c*d**2*x**2 - 27*d**3*x**3)**p*c**3*d*p*x + 81*(4*c**3
 - 27*c*d**2*x**2 - 27*d**3*x**3)**p*c**2*d**2*p**2*x**2 + 27*(4*c**3 - 27
*c*d**2*x**2 - 27*d**3*x**3)**p*c**2*d**2*p*x**2 + 243*(4*c**3 - 27*c*d**2
*x**2 - 27*d**3*x**3)**p*c*d**3*p**2*x**3 + 243*(4*c**3 - 27*c*d**2*x**2 -
 27*d**3*x**3)**p*c*d**3*p*x**3 + 54*(4*c**3 - 27*c*d**2*x**2 - 27*d**3*x*
*3)**p*c*d**3*x**3 + 13122*int((4*c**3 - 27*c*d**2*x**2 - 27*d**3*x**3)...
```

3.42 $\int (A+Bx+Cx^2) (2-4x^2+3x^3)^3 \, dx$

Optimal result

Integrand size = 25, antiderivative size = 157

$$\begin{aligned}
&\int (A+Bx+Cx^2) (2-4x^2+3x^3)^3 \, dx \\
&= 8Ax+4Bx^2-\frac{8}{3}(6A-C)x^3+3(3A-4B)x^4+\frac{12}{5}(8A+3B-4C)x^5 \\
&\quad -2(12A-8B-3C)x^6-\frac{2}{7}(5A+72B-48C)x^7+\frac{1}{4}(72A-5B-72C)x^8 \\
&\quad -\frac{2}{9}(54A-72B+5C)x^9+\frac{9}{10}(3A-12B+16C)x^{10}+\frac{27}{11}(B-4C)x^{11}+\frac{9Cx^{12}}{4}
\end{aligned}$$

output
```
8*A*x+4*B*x^2-8/3*(6*A-C)*x^3+3*(3*A-4*B)*x^4+12/5*(8*A+3*B-4*C)*x^5-2*(12
*A-8*B-3*C)*x^6-2/7*(5*A+72*B-48*C)*x^7+1/4*(72*A-5*B-72*C)*x^8-2/9*(54*A-
72*B+5*C)*x^9+9/10*(3*A-12*B+16*C)*x^10+27/11*(B-4*C)*x^11+9/4*C*x^12
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.00

$$\int (A+Bx+Cx^2) (2-4x^2+3x^3)^3 \, dx$$
$$= 8Ax+4Bx^2-\frac{8}{3}(6A-C)x^3+3(3A-4B)x^4+\frac{12}{5}(8A+3B-4C)x^5 - 2(12A-8B-3C)x^6-\frac{2}{7}(5A+72B-48C)x^7+\frac{1}{4}(72A-5B-72C)x^8 - \frac{2}{9}(54A-72B+5C)x^9+\frac{9}{10}(3A-12B+16C)x^{10}+\frac{27}{11}(B-4C)x^{11}+\frac{9Cx^{12}}{4}$$

input
```
Integrate[(A + B*x + C*x^2)*(2 - 4*x^2 + 3*x^3)^3,x]
```

output
```
8*A*x + 4*B*x^2 - (8*(6*A - C)*x^3)/3 + 3*(3*A - 4*B)*x^4 + (12*(8*A + 3*B
 - 4*C)*x^5)/5 - 2*(12*A - 8*B - 3*C)*x^6 - (2*(5*A + 72*B - 48*C)*x^7)/7
+ ((72*A - 5*B - 72*C)*x^8)/4 - (2*(54*A - 72*B + 5*C)*x^9)/9 + (9*(3*A -
12*B + 16*C)*x^10)/10 + (27*(B - 4*C)*x^11)/11 + (9*C*x^12)/4
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (3x^3-4x^2+2)^3 (A+Bx+Cx^2) \, dx$$
$$\Big\downarrow \text{2188}$$
$$\int (9x^9(3A-12B+16C)-2x^8(54A-72B+5C)+2x^7(72A-5B-72C)-2x^6(5A+72B-48C)-12x^5(1$$
$$\Big\downarrow \text{2009}$$

$$\frac{9}{10}x^{10}(3A-12B+16C)-\frac{2}{9}x^9(54A-72B+5C)+\frac{1}{4}x^8(72A-5B-72C)-\frac{2}{7}x^7(5A+72B-48C)-2x^6(12A-8B-3C)+\frac{12}{5}x^5(8A+3B-4C)+3x^4(3A-4B)-\frac{8}{3}x^3(6A-C)+8Ax+\frac{27}{11}x^{11}(B-4C)+4Bx^2+\frac{9Cx^{12}}{4}$$

input
```
Int[(A + B*x + C*x^2)*(2 - 4*x^2 + 3*x^3)^3,x]
```

output
```
8*A*x + 4*B*x^2 - (8*(6*A - C)*x^3)/3 + 3*(3*A - 4*B)*x^4 + (12*(8*A + 3*B
 - 4*C)*x^5)/5 - 2*(12*A - 8*B - 3*C)*x^6 - (2*(5*A + 72*B - 48*C)*x^7)/7
+ ((72*A - 5*B - 72*C)*x^8)/4 - (2*(54*A - 72*B + 5*C)*x^9)/9 + (9*(3*A -
12*B + 16*C)*x^10)/10 + (27*(B - 4*C)*x^11)/11 + (9*C*x^12)/4
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.86

method	result
norman	$\frac{9C\,x^{12}}{4}+\left(\frac{27B}{11}-\frac{108C}{11}\right)x^{11}+\left(\frac{27A}{10}-\frac{54B}{5}+\frac{72C}{5}\right)x^{10}+\left(-12A+16B-\frac{10C}{9}\right)x^9+\left(18A-\right.$
default	$\frac{9C\,x^{12}}{4}+\frac{(27B-108C)x^{11}}{11}+\frac{(27A-108B+144C)x^{10}}{10}+\frac{(-108A+144B-10C)x^9}{9}+\frac{(144A-10B-144C)x^8}{8}+\frac{(-10}{}$
gosper	$\frac{8}{3}C\,x^3+4B\,x^2+9x^4A+\frac{96}{5}x^5A-12A\,x^9-\frac{54}{5}B\,x^{10}-18x^8C-\frac{10}{7}x^7A+16x^9B+18x^8A$
risch	$\frac{8}{3}C\,x^3+4B\,x^2+9x^4A+\frac{96}{5}x^5A-12A\,x^9-\frac{54}{5}B\,x^{10}-18x^8C-\frac{10}{7}x^7A+16x^9B+18x^8A$
parallelrisch	$\frac{8}{3}C\,x^3+4B\,x^2+9x^4A+\frac{96}{5}x^5A-12A\,x^9-\frac{54}{5}B\,x^{10}-18x^8C-\frac{10}{7}x^7A+16x^9B+18x^8A$
orering	$\frac{x(31185C\,x^{11}+34020B\,x^{10}-136080x^{10}C+37422A\,x^9-149688x^9B+199584C\,x^9-166320x^8A+221760B\,x^8-15400x^8C+24}{}$

input
```
int((C*x^2+B*x+A)*(3*x^3-4*x^2+2)^3,x,method=_RETURNVERBOSE)
```

output
```
9/4*C*x^12+(27/11*B-108/11*C)*x^11+(27/10*A-54/5*B+72/5*C)*x^10+(-12*A+16*
B-10/9*C)*x^9+(18*A-5/4*B-18*C)*x^8+(-10/7*A-144/7*B+96/7*C)*x^7+(-24*A+16
*B+6*C)*x^6+(96/5*A+36/5*B-48/5*C)*x^5+(9*A-12*B)*x^4+(-16*A+8/3*C)*x^3+4*
B*x^2+8*A*x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.90

$$\begin{aligned}\int (A+Bx+Cx^2)\,(2-4x^2+3x^3)^3\,dx &= \frac{9}{4}\,Cx^{12}+\frac{27}{11}\,(B-4\,C)x^{11}+\frac{9}{10}\,(3\,A-12\,B+16\,C)x^{10}-\frac{2}{9}\,(54\,A-72\,B+5\,C)x^9\\&\quad+\frac{1}{4}\,(72\,A-5\,B-72\,C)x^8-\frac{2}{7}\,(5\,A+72\,B-48\,C)x^7-2\,(12\,A-8\,B-3\,C)x^6\\&\quad+\frac{12}{5}\,(8\,A+3\,B-4\,C)x^5+3\,(3\,A-4\,B)x^4-\frac{8}{3}\,(6\,A-C)x^3+4\,Bx^2+8\,Ax\end{aligned}$$

input
```
integrate((C*x^2+B*x+A)*(3*x^3-4*x^2+2)^3,x, algorithm="fricas")
```

output
```
9/4*C*x^12 + 27/11*(B - 4*C)*x^11 + 9/10*(3*A - 12*B + 16*C)*x^10 - 2/9*(5
4*A - 72*B + 5*C)*x^9 + 1/4*(72*A - 5*B - 72*C)*x^8 - 2/7*(5*A + 72*B - 48
*C)*x^7 - 2*(12*A - 8*B - 3*C)*x^6 + 12/5*(8*A + 3*B - 4*C)*x^5 + 3*(3*A -
 4*B)*x^4 - 8/3*(6*A - C)*x^3 + 4*B*x^2 + 8*A*x
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.99

$$\int (A+Bx+Cx^2)\left(2-4x^2+3x^3\right)^3\,dx = 8Ax+4Bx^2+\frac{9Cx^{12}}{4}+x^{11}\cdot\left(\frac{27B}{11}-\frac{108C}{11}\right)+x^{10}\cdot\left(\frac{27A}{10}-\frac{54B}{5}+\frac{72C}{5}\right)+x^9\left(-12A+16B-\frac{10C}{9}\right)+x^8\cdot\left(18A-\frac{5B}{4}-18C\right)+x^7\left(-\frac{10A}{7}-\frac{144B}{7}+\frac{96C}{7}\right)+x^6(-24A+16B+6C)+x^5\cdot\left(\frac{96A}{5}+\frac{36B}{5}-\frac{48C}{5}\right)+x^4\cdot(9A-12B)+x^3\left(-16A+\frac{8C}{3}\right)$$

input
```
integrate((C*x**2+B*x+A)*(3*x**3-4*x**2+2)**3,x)
```

output
```
8*A*x + 4*B*x**2 + 9*C*x**12/4 + x**11*(27*B/11 - 108*C/11) + x**10*(27*A/
10 - 54*B/5 + 72*C/5) + x**9*(-12*A + 16*B - 10*C/9) + x**8*(18*A - 5*B/4
- 18*C) + x**7*(-10*A/7 - 144*B/7 + 96*C/7) + x**6*(-24*A + 16*B + 6*C) +
x**5*(96*A/5 + 36*B/5 - 48*C/5) + x**4*(9*A - 12*B) + x**3*(-16*A + 8*C/3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.90

$$\int (A+Bx+Cx^2)\left(2-4x^2+3x^3\right)^3\,dx = \frac{9}{4}Cx^{12}+\frac{27}{11}(B-4C)x^{11}+\frac{9}{10}(3A-12B+16C)x^{10}-\frac{2}{9}(54A-72B+5C)x^9+\frac{1}{4}(72A-5B-72C)x^8-\frac{2}{7}(5A+72B-48C)x^7-2(12A-8B-3C)x^6+\frac{12}{5}(8A+3B-4C)x^5+3(3A-4B)x^4-\frac{8}{3}(6A-C)x^3+4Bx^2+8Ax$$

input
```
integrate((C*x^2+B*x+A)*(3*x^3-4*x^2+2)^3,x, algorithm="maxima")
```

output
```
9/4*C*x^12 + 27/11*(B - 4*C)*x^11 + 9/10*(3*A - 12*B + 16*C)*x^10 - 2/9*(5
4*A - 72*B + 5*C)*x^9 + 1/4*(72*A - 5*B - 72*C)*x^8 - 2/7*(5*A + 72*B - 48
*C)*x^7 - 2*(12*A - 8*B - 3*C)*x^6 + 12/5*(8*A + 3*B - 4*C)*x^5 + 3*(3*A -
 4*B)*x^4 - 8/3*(6*A - C)*x^3 + 4*B*x^2 + 8*A*x
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.03

$$\int (A+Bx+Cx^2)\left(2-4x^2+3x^3\right)^3\,dx = \frac{9}{4}\,Cx^{12}+\frac{27}{11}\,Bx^{11}-\frac{108}{11}\,Cx^{11}+\frac{27}{10}\,Ax^{10}-\frac{54}{5}\,Bx^{10}+\frac{72}{5}\,Cx^{10}-12\,Ax^9+16\,Bx^9-\frac{10}{9}\,Cx^9+18\,Ax^8-\frac{5}{4}\,Bx^8-18\,Cx^8-\frac{10}{7}\,Ax^7-\frac{144}{7}\,Bx^7+\frac{96}{7}\,Cx^7-24\,Ax^6+16\,Bx^6+6\,Cx^6+\frac{96}{5}\,Ax^5+\frac{36}{5}\,Bx^5-\frac{48}{5}\,Cx^5+9\,Ax^4-12\,Bx^4-16\,Ax^3+\frac{8}{3}\,Cx^3+4\,Bx^2+8\,Ax$$

input
```
integrate((C*x^2+B*x+A)*(3*x^3-4*x^2+2)^3,x, algorithm="giac")
```

output
```
9/4*C*x^12 + 27/11*B*x^11 - 108/11*C*x^11 + 27/10*A*x^10 - 54/5*B*x^10 + 7
2/5*C*x^10 - 12*A*x^9 + 16*B*x^9 - 10/9*C*x^9 + 18*A*x^8 - 5/4*B*x^8 - 18*
C*x^8 - 10/7*A*x^7 - 144/7*B*x^7 + 96/7*C*x^7 - 24*A*x^6 + 16*B*x^6 + 6*C*
x^6 + 96/5*A*x^5 + 36/5*B*x^5 - 48/5*C*x^5 + 9*A*x^4 - 12*B*x^4 - 16*A*x^3
 + 8/3*C*x^3 + 4*B*x^2 + 8*A*x
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.88

$$\int (A+Bx+Cx^2) (2-4x^2+3x^3)^3 \, dx = \frac{9\,C\,x^{12}}{4}+\left(\frac{27\,B}{11}-\frac{108\,C}{11}\right)\,x^{11}+\left(\frac{27\,A}{10}-\frac{54\,B}{5}+\frac{72\,C}{5}\right)\,x^{10} +\left(16\,B-12\,A-\frac{10\,C}{9}\right)\,x^9+\left(18\,A-\frac{5\,B}{4}-18\,C\right)\,x^8 +\left(\frac{96\,C}{7}-\frac{144\,B}{7}-\frac{10\,A}{7}\right)\,x^7+\left(16\,B-24\,A+6\,C\right)\,x^6 +\left(\frac{96\,A}{5}+\frac{36\,B}{5}-\frac{48\,C}{5}\right)\,x^5+\left(9\,A-12\,B\right)\,x^4+\left(\frac{8\,C}{3}-16\,A\right)\,x^3+4\,B\,x^2+8\,A\,x$$

input
```
int((A + B*x + C*x^2)*(3*x^3 - 4*x^2 + 2)^3,x)
```

output
```
8*A*x + 4*B*x^2 + (9*C*x^12)/4 + x^6*(16*B - 24*A + 6*C) - x^8*((5*B)/4 -
18*A + 18*C) - x^9*(12*A - 16*B + (10*C)/9) + x^10*((27*A)/10 - (54*B)/5 +
 (72*C)/5) + x^5*((96*A)/5 + (36*B)/5 - (48*C)/5) - x^7*((10*A)/7 + (144*B
)/7 - (96*C)/7) + x^4*(9*A - 12*B) - x^3*(16*A - (8*C)/3) + x^11*((27*B)/1
1 - (108*C)/11)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.03

$$\int (A+Bx+Cx^2) (2-4x^2+3x^3)^3 \, dx = \frac{x(31185c\,x^{11}+34020b\,x^{10}-136080c\,x^{10}+37422a\,x^9-149688b\,x^9+199584c\,x^9-166320a\,x^8+22176}{}$$

input
```
int((C*x^2+B*x+A)*(3*x^3-4*x^2+2)^3,x)
```

output

```
(x*(37422*a*x**9 - 166320*a*x**8 + 249480*a*x**7 - 19800*a*x**6 - 332640*a
*x**5 + 266112*a*x**4 + 124740*a*x**3 - 221760*a*x**2 + 110880*a + 34020*b
*x**10 - 149688*b*x**9 + 221760*b*x**8 - 17325*b*x**7 - 285120*b*x**6 + 22
1760*b*x**5 + 99792*b*x**4 - 166320*b*x**3 + 55440*b*x + 31185*c*x**11 - 1
36080*c*x**10 + 199584*c*x**9 - 15400*c*x**8 - 249480*c*x**7 + 190080*c*x*
*6 + 83160*c*x**5 - 133056*c*x**4 + 36960*c*x**2))/13860
```

3.43 $\int (A+Bx+Cx^2)(2-4x^2+3x^3)^2\,dx$

Optimal result

Integrand size = 25, antiderivative size = 106

$$\int (A+Bx+Cx^2)(2-4x^2+3x^3)^2\,dx = 4Ax+2Bx^2-\frac{4}{3}(4A-C)x^3+(3A-4B)x^4+\frac{4}{5}(4A+3B-4C)x^5-\frac{2}{3}(6A-4B-3C)x^6+\frac{1}{7}(9A-24B+16C)x^7+\frac{3}{8}(3B-8C)x^8+Cx^9$$

output
```
4*A*x+2*B*x^2-4/3*(4*A-C)*x^3+(3*A-4*B)*x^4+4/5*(4*A+3*B-4*C)*x^5-2/3*(6*A
-4*B-3*C)*x^6+1/7*(9*A-24*B+16*C)*x^7+3/8*(3*B-8*C)*x^8+C*x^9
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00

$$\int \left(A+Bx+Cx^2\right) \left(2-4x^2+3x^3\right)^2 \, dx = 4Ax+2Bx^2-\frac{4}{3}(4A-C)x^3+(3A-4B)x^4 +\frac{4}{5}(4A+3B-4C)x^5-\frac{2}{3}(6A-4B-3C)x^6 +\frac{1}{7}(9A-24B+16C)x^7 +\frac{3}{8}(3B-8C)x^8+Cx^9$$

input
```
Integrate[(A + B*x + C*x^2)*(2 - 4*x^2 + 3*x^3)^2,x]
```

output
```
4*A*x + 2*B*x^2 - (4*(4*A - C)*x^3)/3 + (3*A - 4*B)*x^4 + (4*(4*A + 3*B -
4*C)*x^5)/5 - (2*(6*A - 4*B - 3*C)*x^6)/3 + ((9*A - 24*B + 16*C)*x^7)/7 +
(3*(3*B - 8*C)*x^8)/8 + C*x^9
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(3x^3-4x^2+2\right)^2 \left(A+Bx+Cx^2\right) \, dx$$

$$\Big\downarrow \text{ 2188}$$

$$\int \left(x^6(9A-24B+16C)-4x^5(6A-4B-3C)+4x^4(4A+3B-4C)+4x^3(3A-4B)-4x^2(4A-C)+4A+\right.$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{7}x^7(9A-24B+16C)-\frac{2}{3}x^6(6A-4B-3C)+\frac{4}{5}x^5(4A+3B-4C)+x^4(3A-4B)-\frac{4}{3}x^3(4A-C)+4Ax+\frac{3}{8}x^8(3B-8C)+2Bx^2+Cx^9$$

input
```
Int[(A + B*x + C*x^2)*(2 - 4*x^2 + 3*x^3)^2,x]
```

output
```
4*A*x + 2*B*x^2 - (4*(4*A - C)*x^3)/3 + (3*A - 4*B)*x^4 + (4*(4*A + 3*B -
4*C)*x^5)/5 - (2*(6*A - 4*B - 3*C)*x^6)/3 + ((9*A - 24*B + 16*C)*x^7)/7 +
(3*(3*B - 8*C)*x^8)/8 + C*x^9
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.87

method	result
norman	$C\,x^9+\left(\frac{9B}{8}-3C\right)x^8+\left(\frac{9A}{7}-\frac{24B}{7}+\frac{16C}{7}\right)x^7+\left(-4A+\frac{8B}{3}+2C\right)x^6+\left(\frac{16A}{5}+\frac{12B}{5}-\frac{16C}{5}\right)$
default	$C\,x^9+\frac{(9B-24C)x^8}{8}+\frac{(9A-24B+16C)x^7}{7}+\frac{(-24A+16B+12C)x^6}{6}+\frac{(16A+12B-16C)x^5}{5}+\frac{(12A-16B)x^4}{4}+$
gosper	$C\,x^9+\frac{9}{8}B\,x^8-3x^8C+\frac{9}{7}x^7A-\frac{24}{7}x^7B+\frac{16}{7}x^7C-4x^6A+\frac{8}{3}x^6B+2C\,x^6+\frac{16}{5}x^5A+\frac{12}{5}B$
risch	$C\,x^9+\frac{9}{8}B\,x^8-3x^8C+\frac{9}{7}x^7A-\frac{24}{7}x^7B+\frac{16}{7}x^7C-4x^6A+\frac{8}{3}x^6B+2C\,x^6+\frac{16}{5}x^5A+\frac{12}{5}B$
parallelrisch	$C\,x^9+\frac{9}{8}B\,x^8-3x^8C+\frac{9}{7}x^7A-\frac{24}{7}x^7B+\frac{16}{7}x^7C-4x^6A+\frac{8}{3}x^6B+2C\,x^6+\frac{16}{5}x^5A+\frac{12}{5}B$
orering	$\frac{x\left(840x^8C+945x^7B-2520x^7C+1080x^6A-2880x^6B+1920C\,x^6-3360x^5A+2240B\,x^5+1680x^5C+2688x^4A+2016x^4B-268\right.}{840}$

input
```
int((C*x^2+B*x+A)*(3*x^3-4*x^2+2)^2,x,method=_RETURNVERBOSE)
```

output
```
C*x^9+(9/8*B-3*C)*x^8+(9/7*A-24/7*B+16/7*C)*x^7+(-4*A+8/3*B+2*C)*x^6+(16/5
*A+12/5*B-16/5*C)*x^5+(3*A-4*B)*x^4+(-16/3*A+4/3*C)*x^3+2*B*x^2+4*A*x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.91

$$\int (A+Bx+Cx^2)\left(2-4x^2+3x^3\right)^2\,dx = Cx^9+\frac{3}{8}(3\,B-8\,C)x^8 + \frac{1}{7}(9\,A-24\,B+16\,C)x^7 - \frac{2}{3}(6\,A-4\,B-3\,C)x^6 + \frac{4}{5}(4\,A+3\,B-4\,C)x^5+(3\,A-4\,B)x^4 - \frac{4}{3}(4\,A-C)x^3+2\,Bx^2+4\,Ax$$

input
```
integrate((C*x^2+B*x+A)*(3*x^3-4*x^2+2)^2,x, algorithm="fricas")
```

output
```
C*x^9 + 3/8*(3*B - 8*C)*x^8 + 1/7*(9*A - 24*B + 16*C)*x^7 - 2/3*(6*A - 4*B
 - 3*C)*x^6 + 4/5*(4*A + 3*B - 4*C)*x^5 + (3*A - 4*B)*x^4 - 4/3*(4*A - C)*
x^3 + 2*B*x^2 + 4*A*x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.98

$$\int (A+Bx+Cx^2)\left(2-4x^2+3x^3\right)^2\,dx = 4Ax+2Bx^2+Cx^9+x^8\cdot\left(\frac{9B}{8}-3C\right) + x^7\cdot\left(\frac{9A}{7}-\frac{24B}{7}+\frac{16C}{7}\right) + x^6\left(-4A+\frac{8B}{3}+2C\right)+x^5 \cdot\left(\frac{16A}{5}+\frac{12B}{5}-\frac{16C}{5}\right)+x^4 \cdot(3A-4B)+x^3\left(-\frac{16A}{3}+\frac{4C}{3}\right)$$

input
```
integrate((C*x**2+B*x+A)*(3*x**3-4*x**2+2)**2,x)
```

output
```
4*A*x + 2*B*x**2 + C*x**9 + x**8*(9*B/8 - 3*C) + x**7*(9*A/7 - 24*B/7 + 16
*C/7) + x**6*(-4*A + 8*B/3 + 2*C) + x**5*(16*A/5 + 12*B/5 - 16*C/5) + x**4
*(3*A - 4*B) + x**3*(-16*A/3 + 4*C/3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.91

$$\int (A+Bx+Cx^2) \left(2-4x^2+3x^3\right)^2 dx = Cx^9 + \frac{3}{8}(3B-8C)x^8 + \frac{1}{7}(9A-24B+16C)x^7 - \frac{2}{3}(6A-4B-3C)x^6 + \frac{4}{5}(4A+3B-4C)x^5 + (3A-4B)x^4 - \frac{4}{3}(4A-C)x^3 + 2Bx^2 + 4Ax$$

input
```
integrate((C*x^2+B*x+A)*(3*x^3-4*x^2+2)^2,x, algorithm="maxima")
```

output
```
C*x^9 + 3/8*(3*B - 8*C)*x^8 + 1/7*(9*A - 24*B + 16*C)*x^7 - 2/3*(6*A - 4*B
 - 3*C)*x^6 + 4/5*(4*A + 3*B - 4*C)*x^5 + (3*A - 4*B)*x^4 - 4/3*(4*A - C)*
x^3 + 2*B*x^2 + 4*A*x
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00

$$\int (A+Bx+Cx^2) \left(2-4x^2+3x^3\right)^2 dx = Cx^9 + \frac{9}{8}Bx^8 - 3Cx^8 + \frac{9}{7}Ax^7 - \frac{24}{7}Bx^7 + \frac{16}{7}Cx^7 - 4Ax^6 + \frac{8}{3}Bx^6 + 2Cx^6 + \frac{16}{5}Ax^5 + \frac{12}{5}Bx^5 - \frac{16}{5}Cx^5 + 3Ax^4 - 4Bx^4 - \frac{16}{3}Ax^3 + \frac{4}{3}Cx^3 + 2Bx^2 + 4Ax$$

input
```
integrate((C*x^2+B*x+A)*(3*x^3-4*x^2+2)^2,x, algorithm="giac")
```

output
```
C*x^9 + 9/8*B*x^8 - 3*C*x^8 + 9/7*A*x^7 - 24/7*B*x^7 + 16/7*C*x^7 - 4*A*x^
6 + 8/3*B*x^6 + 2*C*x^6 + 16/5*A*x^5 + 12/5*B*x^5 - 16/5*C*x^5 + 3*A*x^4 -
 4*B*x^4 - 16/3*A*x^3 + 4/3*C*x^3 + 2*B*x^2 + 4*A*x
```

Mupad [B] (verification not implemented)

Time = 12.13 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.87

$$\int (A+Bx+Cx^2)(2-4x^2+3x^3)^2 \, dx = C\,x^9+\left(\frac{9\,B}{8}-3\,C\right)\,x^8+\left(\frac{9\,A}{7}-\frac{24\,B}{7}+\frac{16\,C}{7}\right)\,x^7+\left(\frac{8\,B}{3}-4\,A+2\,C\right)\,x^6+\left(\frac{16\,A}{5}+\frac{12\,B}{5}-\frac{16\,C}{5}\right)\,x^5+\left(3\,A-4\,B\right)\,x^4+\left(\frac{4\,C}{3}-\frac{16\,A}{3}\right)\,x^3+2\,B\,x^2+4\,A\,x$$

input
```
int((A + B*x + C*x^2)*(3*x^3 - 4*x^2 + 2)^2,x)
```

output
```
4*A*x + 2*B*x^2 + C*x^9 + x^6*((8*B)/3 - 4*A + 2*C) + x^5*((16*A)/5 + (12*
B)/5 - (16*C)/5) + x^7*((9*A)/7 - (24*B)/7 + (16*C)/7) + x^4*(3*A - 4*B) -
 x^3*((16*A)/3 - (4*C)/3) + x^8*((9*B)/8 - 3*C)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.01

$$\int \left(A+Bx+Cx^2\right)\left(2-4x^2+3x^3\right)^2\,dx$$
$$=\frac{x(840c\,x^8+945b\,x^7-2520c\,x^7+1080a\,x^6-2880b\,x^6+1920c\,x^6-3360a\,x^5+2240b\,x^5+1680c\,x^5+2}{840}$$

input
```
int((C*x^2+B*x+A)*(3*x^3-4*x^2+2)^2,x)
```

output
```
(x*(1080*a*x**6 - 3360*a*x**5 + 2688*a*x**4 + 2520*a*x**3 - 4480*a*x**2 +
3360*a + 945*b*x**7 - 2880*b*x**6 + 2240*b*x**5 + 2016*b*x**4 - 3360*b*x**
3 + 1680*b*x + 840*c*x**8 - 2520*c*x**7 + 1920*c*x**6 + 1680*c*x**5 - 2688
*c*x**4 + 1120*c*x**2))/840
```

3.44 $\int (A+Bx+Cx^2)(2-4x^2+3x^3)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 60

$$\int (A+Bx+Cx^2)(2-4x^2+3x^3)\,dx = 2Ax+Bx^2-\frac{2}{3}(2A-C)x^3 + \frac{1}{4}(3A-4B)x^4+\frac{1}{5}(3B-4C)x^5+\frac{Cx^6}{2}$$

output
```
2*A*x+B*x^2-2/3*(2*A-C)*x^3+1/4*(3*A-4*B)*x^4+1/5*(3*B-4*C)*x^5+1/2*C*x^6
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00

$$\int (A+Bx+Cx^2)(2-4x^2+3x^3)\,dx = 2Ax+Bx^2-\frac{2}{3}(2A-C)x^3 + \frac{1}{4}(3A-4B)x^4+\frac{1}{5}(3B-4C)x^5+\frac{Cx^6}{2}$$

input
```
Integrate[(A + B*x + C*x^2)*(2 - 4*x^2 + 3*x^3),x]
```

output
```
2*A*x + B*x^2 - (2*(2*A - C)*x^3)/3 + ((3*A - 4*B)*x^4)/4 + ((3*B - 4*C)*x
^5)/5 + (C*x^6)/2
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(3x^3-4x^2+2\right)\left(A+Bx+Cx^2\right)\,dx$$

$$\Big\downarrow \text{ 2188}$$

$$\int \left(x^3(3A-4B)-2x^2(2A-C)+2A+x^4(3B-4C)+2Bx+3Cx^5\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{4}x^4(3A-4B)-\frac{2}{3}x^3(2A-C)+2Ax+\frac{1}{5}x^5(3B-4C)+Bx^2+\frac{Cx^6}{2}$$

input
```
Int[(A + B*x + C*x^2)*(2 - 4*x^2 + 3*x^3),x]
```

output
```
2*A*x + B*x^2 - (2*(2*A - C)*x^3)/3 + ((3*A - 4*B)*x^4)/4 + ((3*B - 4*C)*x
^5)/5 + (C*x^6)/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.83

method	result	size
norman	$\frac{C x^6}{2}+\left(\frac{3B}{5}-\frac{4C}{5}\right)x^5+\left(\frac{3A}{4}-B\right)x^4+\left(-\frac{4A}{3}+\frac{2C}{3}\right)x^3+B x^2+2Ax$	50
gosper	$\frac{1}{2}C x^6+\frac{3}{5}B x^5-\frac{4}{5}x^5C+\frac{3}{4}x^4A-x^4B-\frac{4}{3}x^3A+\frac{2}{3}C x^3+B x^2+2Ax$	53
default	$\frac{C x^6}{2}+\frac{(3B-4C)x^5}{5}+\frac{(3A-4B)x^4}{4}+\frac{(-4A+2C)x^3}{3}+B x^2+2Ax$	53
risch	$\frac{1}{2}C x^6+\frac{3}{5}B x^5-\frac{4}{5}x^5C+\frac{3}{4}x^4A-x^4B-\frac{4}{3}x^3A+\frac{2}{3}C x^3+B x^2+2Ax$	53
parallelrisch	$\frac{1}{2}C x^6+\frac{3}{5}B x^5-\frac{4}{5}x^5C+\frac{3}{4}x^4A-x^4B-\frac{4}{3}x^3A+\frac{2}{3}C x^3+B x^2+2Ax$	53
orering	$\frac{x(30x^5C+36x^4B-48C x^4+45x^3A-60B x^3-80A x^2+40C x^2+60Bx+120A)}{60}$	54

input
```
int((C*x^2+B*x+A)*(3*x^3-4*x^2+2),x,method=_RETURNVERBOSE)
```

output
```
1/2*C*x^6+(3/5*B-4/5*C)*x^5+(3/4*A-B)*x^4+(-4/3*A+2/3*C)*x^3+B*x^2+2*A*x
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.87

$$\int (A+Bx+Cx^2)(2-4x^2+3x^3)\,dx=\frac{1}{2}Cx^6+\frac{1}{5}(3B-4C)x^5+\frac{1}{4}(3A-4B)x^4-\frac{2}{3}(2A-C)x^3+Bx^2+2Ax$$

input
```
integrate((C*x^2+B*x+A)*(3*x^3-4*x^2+2),x, algorithm="fricas")
```

output
```
1/2*C*x^6 + 1/5*(3*B - 4*C)*x^5 + 1/4*(3*A - 4*B)*x^4 - 2/3*(2*A - C)*x^3
+ B*x^2 + 2*A*x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.88

$$\int (A+Bx+Cx^2)(2-4x^2+3x^3)\,dx = 2Ax+Bx^2+\frac{Cx^6}{2}+x^5\cdot\left(\frac{3B}{5}-\frac{4C}{5}\right)+x^4\cdot\left(\frac{3A}{4}-B\right)+x^3\left(-\frac{4A}{3}+\frac{2C}{3}\right)$$

input
```
integrate((C*x**2+B*x+A)*(3*x**3-4*x**2+2),x)
```

output
```
2*A*x + B*x**2 + C*x**6/2 + x**5*(3*B/5 - 4*C/5) + x**4*(3*A/4 - B) + x**3
*(-4*A/3 + 2*C/3)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.87

$$\int (A+Bx+Cx^2)(2-4x^2+3x^3)\,dx = \frac{1}{2}Cx^6+\frac{1}{5}(3B-4C)x^5+\frac{1}{4}(3A-4B)x^4-\frac{2}{3}(2A-C)x^3+Bx^2+2Ax$$

input
```
integrate((C*x^2+B*x+A)*(3*x^3-4*x^2+2),x, algorithm="maxima")
```

output
```
1/2*C*x^6 + 1/5*(3*B - 4*C)*x^5 + 1/4*(3*A - 4*B)*x^4 - 2/3*(2*A - C)*x^3
+ B*x^2 + 2*A*x
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.87

$$\int \left(A+Bx+Cx^2\right)\left(2-4x^2+3x^3\right)\,dx=\frac{1}{2}Cx^6+\frac{3}{5}Bx^5-\frac{4}{5}Cx^5+\frac{3}{4}Ax^4-Bx^4-\frac{4}{3}Ax^3+\frac{2}{3}Cx^3+Bx^2+2\,Ax$$

input
```
integrate((C*x^2+B*x+A)*(3*x^3-4*x^2+2),x, algorithm="giac")
```

output
```
1/2*C*x^6 + 3/5*B*x^5 - 4/5*C*x^5 + 3/4*A*x^4 - B*x^4 - 4/3*A*x^3 + 2/3*C*
x^3 + B*x^2 + 2*A*x
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.83

$$\int \left(A+Bx+Cx^2\right)\left(2-4x^2+3x^3\right)\,dx=\frac{C\,x^6}{2}+\left(\frac{3\,B}{5}-\frac{4\,C}{5}\right)\,x^5+\left(\frac{3\,A}{4}-B\right)\,x^4+\left(\frac{2\,C}{3}-\frac{4\,A}{3}\right)\,x^3+B\,x^2+2\,A\,x$$

input
```
int((A + B*x + C*x^2)*(3*x^3 - 4*x^2 + 2),x)
```

output
```
2*A*x + B*x^2 + (C*x^6)/2 + x^4*((3*A)/4 - B) - x^3*((4*A)/3 - (2*C)/3) +
x^5*((3*B)/5 - (4*C)/5)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.88

$$\int (A+Bx+Cx^2)(2-4x^2+3x^3)\,dx = \frac{x(30c\,x^5+36b\,x^4-48c\,x^4+45a\,x^3-60b\,x^3-80a\,x^2+40c\,x^2+60bx+120a)}{60}$$

input
```
int((C*x^2+B*x+A)*(3*x^3-4*x^2+2),x)
```

output
```
(x*(45*a*x**3 - 80*a*x**2 + 120*a + 36*b*x**4 - 60*b*x**3 + 60*b*x + 30*c*
x**5 - 48*c*x**4 + 40*c*x**2))/60
```

3.45 $\int \frac{A+Bx+Cx^2}{2-4x^2+3x^3} \, dx$

Optimal result

Integrand size = 25, antiderivative size = 593

$$\int \frac{A+Bx+Cx^2}{2-4x^2+3x^3} \, dx =$$

$$-\frac{\sqrt{\frac{3}{2\left(29993-1611\sqrt{345}+128\left(179-9\sqrt{345}\right)^{2/3}-16\left(179-9\sqrt{345}\right)^{4/3}\right)}}\left(9\left(179-9\sqrt{345}\right)\left(16+\left(179-9\sqrt{345}\right)^{2/3}\right)A+\right.}{256+1}$$

$$+\frac{\sqrt[3]{179-9\sqrt{345}}\left(81\sqrt[3]{179-9\sqrt{345}}A-\left(16-4\sqrt[3]{179-9\sqrt{345}}+\left(179-9\sqrt{345}\right)^{2/3}\right)(9B+8C)\right)\mathrm{l}}{9\left(256+16\left(179-9\sqrt{345}\right)^{2/3}+\left(179-9\sqrt{345}\right)\right.}$$

$$-\frac{\left(81\left(179-9\sqrt{345}\right)^{2/3}A-\left(179-9\sqrt{345}+16\sqrt[3]{179-9\sqrt{345}}-4\left(179-9\sqrt{345}\right)^{2/3}\right)(9B+8C)\right)\mathrm{l}}{18\left(256+\right.}$$

$$+\frac{1}{9}C\log\left(2-4x^2+3x^3\right)$$

output
```
-3^(1/2)/(59986-3222*345^(1/2)+256*(179-9*345^(1/2))^(2/3)-32*(179-9*345^(
1/2))^(4/3))^(1/2)*(9*(179-9*345^(1/2))*(16+(179-9*345^(1/2))^(2/3))*A+2*(
3969-211*345^(1/2)+2*(179-9*345^(1/2))^(2/3)*(27-345^(1/2)))*(9*B+8*C))*ar
ctan((179-9*345^(1/2)+16*(179-9*345^(1/2))^(1/3)+2*(179-9*345^(1/2))^(2/3)
*(4-9*x))/(179958-9666*345^(1/2)+768*(179-9*345^(1/2))^(2/3)-96*(179-9*345
^(1/2))^(4/3))^(1/2))/(256+16*(179-9*345^(1/2))^(2/3)+(179-9*345^(1/2))^(4
/3))+(179-9*345^(1/2))^(1/3)*(81*(179-9*345^(1/2))^(1/3)*A-((179-9*345^(1/
2))^(2/3)-4*(179-9*345^(1/2))^(1/3)+16)*(9*B+8*C))*ln(16+(179-9*345^(1/2))
^(2/3)-(179-9*345^(1/2))^(1/3)*(4-9*x))/(2304+144*(179-9*345^(1/2))^(2/3)+
9*(179-9*345^(1/2))^(4/3))-(81*(179-9*345^(1/2))^(2/3)*A-(179-9*345^(1/2)+
16*(179-9*345^(1/2))^(1/3)-4*(179-9*345^(1/2))^(2/3))*(9*B+8*C))*ln((27-34
5^(1/2))*(4+(179-9*345^(1/2))^(1/3))-179*x+9*x*345^(1/2)-16*(179-9*345^(1/
2))^(1/3)*x-8*(179-9*345^(1/2))^(2/3)*x+9*(179-9*345^(1/2))^(2/3)*x^2)/(46
08+288*(179-9*345^(1/2))^(2/3)+18*(179-9*345^(1/2))^(4/3))+1/9*C*ln(3*x^3-
4*x^2+2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.11

$$\int \frac{A+Bx+Cx^2}{2-4x^2+3x^3}\,dx = \text{RootSum}\left[2-4\#1^2+3\#1^3\&,\frac{A\log(x-\#1)+B\log(x-\#1)\#1+C\log(x-\#1)\#1^2}{-8\#1+9\#1^2}\&\right]$$

input
```
Integrate[(A + B*x + C*x^2)/(2 - 4*x^2 + 3*x^3),x]
```

output
```
RootSum[2 - 4*#1^2 + 3*#1^3 & , (A*Log[x - #1] + B*Log[x - #1]*#1 + C*Log[
x - #1]*#1^2)/(-8*#1 + 9*#1^2) & ]
```

Rubi [A] (verified)

Time = 4.86 (sec) , antiderivative size = 591, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {2525, 2490, 2485, 27, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{3x^3-4x^2+2}\,dx$$

$$\Big\downarrow \text{ 2525}$$

$$\frac{1}{9}\int \frac{9A+(9B+8C)x}{3x^3-4x^2+2}dx+\frac{1}{9}C\log\left(3x^3-4x^2+2\right)$$

$$\Big\downarrow \text{ 2490}$$

$$\frac{1}{9}\int \frac{\frac{1}{9}(81A+4(9B+8C))+(9B+8C)\left(x-\frac{4}{9}\right)}{3\left(x-\frac{4}{9}\right)^3-\frac{16}{9}\left(x-\frac{4}{9}\right)+\frac{358}{243}}d\left(x-\frac{4}{9}\right)+\frac{1}{9}C\log\left(3x^3-4x^2+2\right)$$

$$\Big\downarrow \text{ 2485}$$

$$\int -\frac{3\left(81A+36B+32C+9(9B+8C)\left(x-\frac{4}{9}\right)\right)}{\left(9\left(x-\frac{4}{9}\right)+\frac{16+\left(179-9\sqrt{345}\right)^{2/3}}{\sqrt[3]{179-9\sqrt{345}}}\right)\left(-81\left(x-\frac{4}{9}\right)^2+\frac{9\left(16+\left(179-9\sqrt{345}\right)^{2/3}\right)\left(x-\frac{4}{9}\right)}{\sqrt[3]{179-9\sqrt{345}}}-\left(179-9\sqrt{345}\right)^{2/3}-\frac{\cdots}{(179\cdots}\right.}$$

$$\frac{1}{9}C\log\left(3x^3-4x^2+2\right)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{9}C\log\left(3x^3-4x^2+2\right)-$$

$$3\int \frac{81A+36B+32C+9(9B+8C)\left(x-\frac{4}{9}\right)}{\left(9\left(x-\frac{4}{9}\right)+\frac{16+\left(179-9\sqrt{345}\right)^{2/3}}{\sqrt[3]{179-9\sqrt{345}}}\right)\left(-81\left(x-\frac{4}{9}\right)^2+\frac{9\left(16+\left(179-9\sqrt{345}\right)^{2/3}\right)\left(x-\frac{4}{9}\right)}{\sqrt[3]{179-9\sqrt{345}}}-\left(179-9\sqrt{345}\right)^{2/3}-\frac{\cdots}{(179\cdots}\right.}$$

$$\Big\downarrow \text{ 1200}$$

$$\frac{1}{9}C\log\left(3x^3-4x^2+2\right)-$$
$$3\int\left(\frac{-81\left(179-9\sqrt{345}\right)A-\left(716-36\sqrt{345}-16\left(179-9\sqrt{345}\right)^{2/3}-\left(179-9\sqrt{345}\right)^{4/3}\right)(9B+8C)}{3\left(256+16\left(179-9\sqrt{345}\right)^{2/3}+\left(179-9\sqrt{345}\right)^{4/3}\right)\left(9\sqrt[3]{179-9\sqrt{345}}\left(x-\frac{4}{9}\right)+\left(179-9\sqrt{345}\right)^{2/3}+16\right.}\right.$$

$\big\downarrow$ 2009

$$\frac{1}{9}C\log\left(3x^3-4x^2+2\right)-$$
$$3\left(\frac{\arctan\left(\frac{-18\left(179-9\sqrt{345}\right)^{2/3}\left(x-\frac{4}{9}\right)+16\sqrt[3]{179-9\sqrt{345}}-9\sqrt{345}+179}{\sqrt{6\left(29993-1611\sqrt{345}+128\left(179-9\sqrt{345}\right)^{2/3}-16\left(179-9\sqrt{345}\right)^{4/3}\right)}}\right)\left(9\left(179-9\sqrt{345}\right)\left(16+\left(179-9\sqrt{345}\right)^{2/\right.\right.}{\sqrt{6\left(29993-1611\sqrt{345}+128\left(179-9\sqrt{345}\right)^{2/3}-16\left(179-9\sqrt{345}\right)^{4/3}\right)}\left(256\right.}\right.$$

input
```
Int[(A + B*x + C*x^2)/(2 - 4*x^2 + 3*x^3),x]
```

output
```
-3*(((9*(179 - 9*Sqrt[345])*(16 + (179 - 9*Sqrt[345])^(2/3))*A + 2*(3969 -
 211*Sqrt[345] + 2*(179 - 9*Sqrt[345])^(2/3)*(27 - Sqrt[345]))*(9*B + 8*C)
)*ArcTan[(179 - 9*Sqrt[345] + 16*(179 - 9*Sqrt[345])^(1/3) - 18*(179 - 9*S
qrt[345])^(2/3)*(-4/9 + x))/Sqrt[6*(29993 - 1611*Sqrt[345] + 128*(179 - 9*
Sqrt[345])^(2/3) - 16*(179 - 9*Sqrt[345])^(4/3))]])/(Sqrt[6*(29993 - 1611*
Sqrt[345] + 128*(179 - 9*Sqrt[345])^(2/3) - 16*(179 - 9*Sqrt[345])^(4/3))]
*(256 + 16*(179 - 9*Sqrt[345])^(2/3) + (179 - 9*Sqrt[345])^(4/3))) - ((81*
(179 - 9*Sqrt[345])*A + (716 - 36*Sqrt[345] - 16*(179 - 9*Sqrt[345])^(2/3)
 - (179 - 9*Sqrt[345])^(4/3))*(9*B + 8*C))*Log[16 + (179 - 9*Sqrt[345])^(2
/3) + 9*(179 - 9*Sqrt[345])^(1/3)*(-4/9 + x)])/(27*(179 - 9*Sqrt[345])^(1/
3)*(256 + 16*(179 - 9*Sqrt[345])^(2/3) + (179 - 9*Sqrt[345])^(4/3))) + ((8
1*(179 - 9*Sqrt[345])^(2/3)*A - (179 - 9*Sqrt[345] + 16*(179 - 9*Sqrt[345]
)^(1/3) - 4*(179 - 9*Sqrt[345])^(2/3))*(9*B + 8*C))*Log[256 - 16*(179 - 9*
Sqrt[345])^(2/3) + (179 - 9*Sqrt[345])^(4/3) - 9*(179 - 9*Sqrt[345] + 16*(
179 - 9*Sqrt[345])^(1/3))*(-4/9 + x) + 81*(179 - 9*Sqrt[345])^(2/3)*(-4/9
+ x)^2])/(54*(256 + 16*(179 - 9*Sqrt[345])^(2/3) + (179 - 9*Sqrt[345])^(4/
3)))) + (C*Log[2 - 4*x^2 + 3*x^3])/9
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2485
```
Int[((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (d_.)*(x_)^3)^(p_), x_S
ymbol] :> With[{r = Rt[-9*a*d^2 + Sqrt[3]*d*Sqrt[4*b^3*d + 27*a^2*d^2], 3]}
, Simp[1/d^(2*p)   Int[(e + f*x)^m*Simp[18^(1/3)*b*(d/(3*r)) - r/18^(1/3) +
 d*x, x]^p*Simp[b*(d/3) + 12^(1/3)*b^2*(d^2/(3*r^2)) + r^2/(3*12^(1/3)) - d
*(2^(1/3)*b*(d/(3^(1/3)*r)) - r/18^(1/3))*x + d^2*x^2, x]^p, x], x]] /; Fre
eQ[{a, b, d, e, f, m}, x] && NeQ[4*b^3 + 27*a^2*d, 0] && ILtQ[p, 0]
```

rule 2490
```
Int[(P3_)^(p_.)*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] :> With[{a = Coeff[P3
, x, 0], b = Coeff[P3, x, 1], c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Su
bst[Int[((3*d*e - c*f)/(3*d) + f*x)^m*Simp[(2*c^3 - 9*b*c*d + 27*a*d^2)/(27
*d^2) - (c^2 - 3*b*d)*(x/(3*d)) + d*x^3, x]^p, x], x, x + c/(3*d)] /; NeQ[c
, 0]] /; FreeQ[{e, f, m, p}, x] && PolyQ[P3, x, 3]
```

rule 2525
```
Int[(Pm_)/(Qn_), x_Symbol] :> With[{m = Expon[Pm, x], n = Expon[Qn, x]}, Si
mp[Coeff[Pm, x, m]*(Log[Qn]/(n*Coeff[Qn, x, n])), x] + Simp[1/(n*Coeff[Qn,
x, n])   Int[ExpandToSum[n*Coeff[Qn, x, n]*Pm - Coeff[Pm, x, m]*D[Qn, x], x
]/Qn, x], x] /; EqQ[m, n - 1]] /; PolyQ[Pm, x] && PolyQ[Qn, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.08

method	result	size
default	$\sum_{\_R=\mathrm{RootOf}(3\_Z^3-4\_Z^2+2)} \frac{(C\_R^2+B\_R+A)\ln(x-\_R)}{9\_R^2-8\_R}$	45
risch	$\sum_{\_R=\mathrm{RootOf}(3\_Z^3-4\_Z^2+2)} \frac{(C\_R^2+B\_R+A)\ln(x-\_R)}{9\_R^2-8\_R}$	45

input
```
int((C*x^2+B*x+A)/(3*x^3-4*x^2+2),x,method=_RETURNVERBOSE)
```

output
```
sum((C*_R^2+B*_R+A)/(9*_R^2-8*_R)*ln(x-_R),_R=RootOf(3*_Z^3-4*_Z^2+2))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.91 (sec) , antiderivative size = 7319, normalized size of antiderivative = 12.34

$$\int \frac{A+Bx+Cx^2}{2-4x^2+3x^3}\,dx = \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)/(3*x^3-4*x^2+2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [A] (verification not implemented)

Time = 8.77 (sec) , antiderivative size = 270, normalized size of antiderivative = 0.46

$$\int \frac{A+Bx+Cx^2}{2-4x^2+3x^3}\,dx$$
$$=\text{RootSum}\left(1380t^3-460t^2C+t\left(48A^2+162AB+144AC+72B^2+128BC+108C^2\right)-9A^3-12A^2B\right.$$

input
```
integrate((C*x**2+B*x+A)/(3*x**3-4*x**2+2),x)
```

output
```
RootSum(1380*_t**3 - 460*_t**2*C + _t*(48*A**2 + 162*A*B + 144*A*C + 72*B*
*2 + 128*B*C + 108*C**2) - 9*A**3 - 12*A**2*B - 16*A**2*C - 18*A*B*C - 16*
A*C**2 + 6*B**3 + 8*B**2*C - 4*C**3, Lambda(_t, _t*log(x + (22080*_t**2*A
 + 37260*_t**2*B + 33120*_t**2*C + 6210*_t*A**2 + 5520*_t*A*B - 8280*_t*B*C
 - 7360*_t*C**2 - 204*A**3 + 1296*A**2*B + 462*A**2*C + 2532*A*B**2 + 3888
*A*B*C + 1728*A*C**2 + 648*B**3 + 1728*B**2*C + 1996*B*C**2 + 864*C**3)/(1
611*A**3 + 2916*A**2*B + 2592*A**2*C + 2592*A*B**2 + 4608*A*B*C + 2048*A*C
**2 + 1458*B**3 + 3888*B**2*C + 3456*B*C**2 + 1024*C**3))))
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{2-4x^2+3x^3}\,dx = \int \frac{Cx^2+Bx+A}{3\,x^3-4\,x^2+2}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(3*x^3-4*x^2+2),x, algorithm="maxima")
```

output
```
integrate((C*x^2 + B*x + A)/(3*x^3 - 4*x^2 + 2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A+Bx+Cx^2}{2-4x^2+3x^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((C*x^2+B*x+A)/(3*x^3-4*x^2+2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 12.42 (sec) , antiderivative size = 328, normalized size of antiderivative = 0.55

$$\int \frac{A+Bx+Cx^2}{2-4x^2+3x^3}\,dx = \sum_{k=1}^{3} \ln\left(-\mathrm{root}\left(z^3 - \frac{C\,z^2}{3} + \frac{z\,(48\,A^2+72\,B^2+108\,C^2+162\,A\,B+144\,A\,C+128\,B\,C)}{1380} - \frac{3\,A\,B\,C}{230} + \frac{2\,B^2\,C}{345} - \frac{4\,A^2\,C}{345} - \frac{4\,A\,C^2}{345} - \frac{A^2\,B}{115} - \frac{3\,A^3}{460} - \frac{C^3}{345} + \frac{B^3}{230}, z, k\right)\left(12\,A+36\,C - x\,(27\,A+12\,B+32\,C) + \mathrm{root}\left(z^3 - \frac{C\,z^2}{3} + \frac{z\,(48\,A^2+72\,B^2+108\,C^2+162\,A\,B+144\,A\,C+128\,B\,C)}{1380} - \frac{3\,A\,B\,C}{230} + \frac{2\,B^2\,C}{345} - 4\right.\right.\right.$$

$$+2\,C^2+3\,A\,B+4\,A\,C+x\,\left(3\,B^2+4\,C\,B-3\,A\,C\right)\Big)\ \mathrm{root}\left(z^3 - \frac{C\,z^2}{3} + \frac{z\,(48\,A^2+72\,B^2+108\,C^2+162\,A\,B+144\,A\,C+128\,B\,C)}{1380} - \frac{3\,A\,B\,C}{230} + \frac{2\,B^2\,C}{345} - \frac{4\,A^2\,C}{345} - \frac{4\,A\,C^2}{345} - \frac{A^2\,B}{115} - \frac{3\,A^3}{460} - \frac{C^3}{345} + \frac{B^3}{230}, z, k\right)$$

input
```
int((A + B*x + C*x^2)/(3*x^3 - 4*x^2 + 2),x)
```

output
```
symsum(log(2*C^2 - root(z^3 - (C*z^2)/3 + (z*(48*A^2 + 72*B^2 + 108*C^2 +
162*A*B + 144*A*C + 128*B*C))/1380 - (3*A*B*C)/230 + (2*B^2*C)/345 - (4*A^
2*C)/345 - (4*A*C^2)/345 - (A^2*B)/115 - (3*A^3)/460 - C^3/345 + B^3/230,
z, k)*(12*A + 36*C - x*(27*A + 12*B + 32*C) + root(z^3 - (C*z^2)/3 + (z*(4
8*A^2 + 72*B^2 + 108*C^2 + 162*A*B + 144*A*C + 128*B*C))/1380 - (3*A*B*C)/
230 + (2*B^2*C)/345 - (4*A^2*C)/345 - (4*A*C^2)/345 - (A^2*B)/115 - (3*A^3
)/460 - C^3/345 + B^3/230, z, k)*(96*x - 162)) + 3*A*B + 4*A*C + x*(3*B^2
- 3*A*C + 4*B*C))*root(z^3 - (C*z^2)/3 + (z*(48*A^2 + 72*B^2 + 108*C^2 + 1
62*A*B + 144*A*C + 128*B*C))/1380 - (3*A*B*C)/230 + (2*B^2*C)/345 - (4*A^2
*C)/345 - (4*A*C^2)/345 - (A^2*B)/115 - (3*A^3)/460 - C^3/345 + B^3/230, z
, k), k, 1, 3)
```

Reduce [F]

$$\int \frac{A+Bx+Cx^2}{2-4x^2+3x^3}\,dx = \left(\int \frac{x}{3x^3-4x^2+2}dx\right)b + \frac{8\left(\int \frac{x}{3x^3-4x^2+2}dx\right)c}{9} + \left(\int \frac{1}{3x^3-4x^2+2}dx\right)a + \frac{\log(3x^3-4x^2+2)\,c}{9}$$

input
```
int((C*x^2+B*x+A)/(3*x^3-4*x^2+2),x)
```

output
```
(9*int(x/(3*x**3 - 4*x**2 + 2),x)*b + 8*int(x/(3*x**3 - 4*x**2 + 2),x)*c +
 9*int(1/(3*x**3 - 4*x**2 + 2),x)*a + log(3*x**3 - 4*x**2 + 2)*c)/9
```

3.46 $\int \frac{A+Bx+Cx^2}{(2-4x^2+3x^3)^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 1056

$$\int \frac{A+Bx+Cx^2}{(2-4x^2+3x^3)^2}\,dx = \text{Too large to display}$$

output
```
9*(179-9*345^(1/2))^(1/3)*(16*A+27*B+24*C)/(3680+230*(179-9*345^(1/2))^(2/
3)-230*(179-9*345^(1/2))^(1/3)*(4-9*x))-9/230*(-9315+537*345^(1/2))^(1/3)*
(288*A+486*B+432*C+(9*(179-9*345^(1/2)+16*(179-9*345^(1/2))^(1/3))*A+(27-3
45^(1/2))*(4+(179-9*345^(1/2))^(1/3))*(9*B+8*C))*(4-9*x)/(179-9*345^(1/2))
^(2/3))*115^(1/3)/(16-(179-9*345^(1/2))^(2/3))/(16-256/(179-9*345^(1/2))^(
2/3)-(179-9*345^(1/2))^(2/3)-(16+(179-9*345^(1/2))^(2/3))*(4-9*x)/(179-9*3
45^(1/2))^(1/3)-(4-9*x)^2)/(4-16/(179-9*345^(1/2))^(1/3)-(179-9*345^(1/2))
^(1/3)-9*x)-C/(27*x^3-36*x^2+18)-17496*6^(1/2)/(29993-1611*345^(1/2)+128*(
179-9*345^(1/2))^(2/3)-16*(179-9*345^(1/2))^(4/3))^(1/2)*((9736952672-5241
97920*345^(1/2)+(319636187-17206017*345^(1/2))*(179-9*345^(1/2))^(2/3))*A+
(1229351761-66184515*345^(1/2)+4*(5665339-304929*345^(1/2))*(179-9*345^(1/
2))^(2/3))*(9*B+8*C))*arctan((179-9*345^(1/2)+16*(179-9*345^(1/2))^(1/3)+2
*(179-9*345^(1/2))^(2/3)*(4-9*x))/(179958-9666*345^(1/2)+768*(179-9*345^(1
/2))^(2/3)-96*(179-9*345^(1/2))^(4/3))^(1/2))/(16-(179-9*345^(1/2))^(2/3))
^2/(256+16*(179-9*345^(1/2))^(2/3)+(179-9*345^(1/2))^(4/3))^3-2916*(179-9*
345^(1/2))*((479888-25776*345^(1/2)+128*(179-9*345^(1/2))^(4/3)-179*(179-9
*345^(1/2))^(5/3))*A+3*(29993-1611*345^(1/2)+8*(179-9*345^(1/2))^(4/3)-4*(
179-9*345^(1/2))^(5/3))*(9*B+8*C))*ln(16+(179-9*345^(1/2))^(2/3)-(179-9*34
5^(1/2))^(1/3)*(4-9*x))/(16-(179-9*345^(1/2))^(2/3))^2/(256+16*(179-9*345^
(1/2))^(2/3)+(179-9*345^(1/2))^(4/3))^3+2916*((82967216-4466448*345^(1/...
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.15

$$\int \frac{A+Bx+Cx^2}{(2-4x^2+3x^3)^2}\,dx = \frac{1}{230}\left(\frac{2C(-27-16x+36x^2)+A(-36+17x+48x^2)+B(-32-36x+81x^2)}{2-4x^2+3x^3} + \text{RootSum}\left[2-4\#1^2 +3\#1^3\&, \frac{98A\log(x-\#1)+36B\log(x-\#1)+32C\log(x-\#1)+48A\log(x-\#1)\#1+81B\log(x}{-8\#1+9\#1^2}\right.\right.$$

input
```
Integrate[(A + B*x + C*x^2)/(2 - 4*x^2 + 3*x^3)^2,x]
```

output
```
((2*C*(-27 - 16*x + 36*x^2) + A*(-36 + 17*x + 48*x^2) + B*(-32 - 36*x + 81
*x^2))/(2 - 4*x^2 + 3*x^3) + RootSum[2 - 4*#1^2 + 3*#1^3 & , (98*A*Log[x -
 #1] + 36*B*Log[x - #1] + 32*C*Log[x - #1] + 48*A*Log[x - #1]*#1 + 81*B*Lo
g[x - #1]*#1 + 72*C*Log[x - #1]*#1)/(-8*#1 + 9*#1^2) & ])/230
```

Rubi [A] (warning: unable to verify)

Time = 9.95 (sec) , antiderivative size = 1110, normalized size of antiderivative = 1.05, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {2526, 2490, 2485, 27, 1235, 27, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{\left(3x^3-4x^2+2\right)^2}dx$$

$$\Big\downarrow \text{2526}$$

$$\frac{1}{9}\int \frac{9A+(9B+8C)x}{\left(3x^3-4x^2+2\right)^2}dx-\frac{C}{9\left(3x^3-4x^2+2\right)}$$

$$\Big\downarrow \text{2490}$$

$$\frac{1}{9}\int \frac{\frac{1}{9}(81A+4(9B+8C))+(9B+8C)\left(x-\frac{4}{9}\right)}{\left(3\left(x-\frac{4}{9}\right)^3-\frac{16}{9}\left(x-\frac{4}{9}\right)+\frac{358}{243}\right)^2}d\left(x-\frac{4}{9}\right)-\frac{C}{9\left(3x^3-4x^2+2\right)}$$

$$\Big\downarrow \text{2485}$$

$$9\int \frac{81\left(81A+36B+32C+9(9B+8C)\left(x-\frac{4}{9}\right)\right)}{\left(9\left(x-\frac{4}{9}\right)+\frac{16+\left(179-9\sqrt{345}\right)^{2/3}}{\sqrt[3]{179-9\sqrt{345}}}\right)^2\left(-81\left(x-\frac{4}{9}\right)^2+\frac{9\left(16+\left(179-9\sqrt{345}\right)^{2/3}\right)\left(x-\frac{4}{9}\right)}{\sqrt[3]{179-9\sqrt{345}}}-\left(179-9\sqrt{345}\right)^{2/3}-\cdots\right.}$$

$$\frac{C}{9\left(3x^3-4x^2+2\right)}$$

$$\Big\downarrow \text{27}$$

$$729\int \frac{81A+36B+32C+9(9B+8C)\left(x-\frac{4}{9}\right)}{\left(9\left(x-\frac{4}{9}\right)+\frac{16+\left(179-9\sqrt{345}\right)^{2/3}}{\sqrt[3]{179-9\sqrt{345}}}\right)^2\left(-81\left(x-\frac{4}{9}\right)^2+\frac{9\left(16+\left(179-9\sqrt{345}\right)^{2/3}\right)\left(x-\frac{4}{9}\right)}{\sqrt[3]{179-9\sqrt{345}}}-\left(179-9\sqrt{345}\right)^{2/3}-\right.}$$

$$\frac{C}{9\left(3x^3-4x^2+2\right)}$$

$\big\downarrow$ 1235

$$729\left(\frac{\sqrt[3]{\frac{179}{345\sqrt{345}}-\frac{3}{115}}\left(2(16A+27B+24C)-\frac{\left(x-\frac{4}{9}\right)\left(9\left(179-9\sqrt{345}+16\sqrt[3]{179-9\sqrt{345}}\right)A+\left(27-\sqrt{3}\right.\right.}{\left(179-9\sqrt{345}\right)^{2/3}}\right.}{6\left(16-\left(179-9\sqrt{345}\right)^{2/3}\right)\left(9\left(x-\frac{4}{9}\right)+\frac{16+\left(179-9\sqrt{345}\right)^{2/3}}{\sqrt[3]{179-9\sqrt{345}}}\right)\left(-81\left(x-\frac{4}{9}\right)^2+\frac{9\left(16+\left(179-9\sqrt{345}\right)^{2/3}\right)\left(x-\frac{4}{9}\right.}{\sqrt[3]{179-9\sqrt{345}}}\right.}\right.$$

$$\frac{C}{9\left(3x^3-4x^2+2\right)}$$

$\big\downarrow$ 27

$$729\left(\frac{\sqrt[3]{\frac{179}{345\sqrt{345}}-\frac{3}{115}}\int\frac{\frac{9\left(2864-144\sqrt{345}-256\sqrt[3]{179-9\sqrt{345}}-\left(179-9\sqrt{345}\right)^{5/3}\right)A+\left(4833-243\sqrt{345}-16\sqrt[3]{179-9\sqrt{345}}\left(27-\sqrt{\right.\right.}{179-9\sqrt{345}}}{\left(9\left(x-\frac{4}{9}\right)+\frac{16+\left(179-9\sqrt{345}\right)^{2/3}}{\sqrt[3]{179-9\sqrt{345}}}\right)^2\left(-81\left(x-\right.\right.}}{3\left(16-\right.}\right.$$

$$\frac{C}{9\left(3x^3-4x^2+2\right)}$$

$\downarrow$ 1200

$$729\left(\frac{\sqrt[3]{-\frac{3}{115}+\frac{179}{345\sqrt{345}}}\left(2(16A+27B+24C)-\frac{\left(9\left(179-9\sqrt{345}+16\sqrt[3]{179-9\sqrt{345}}\right)A+\left(27-\sqrt{345}\right)\left(\right.\right.}{\left(179-9\sqrt{345}\right)^{2/3}}\right.}{6\left(16-\left(179-9\sqrt{345}\right)^{2/3}\right)\left(9\left(x-\frac{4}{9}\right)+\frac{16+\left(179-9\sqrt{345}\right)^{2/3}}{\sqrt[3]{179-9\sqrt{345}}}\right)\left(-81\left(x-\frac{4}{9}\right)^2+\frac{9\left(16+\left(179-9\sqrt{345}\right)^{2/3}\right)\left(x-\frac{4}{9}\right.}{\sqrt[3]{179-9\sqrt{345}}}\right.}\right.$$

$$\frac{C}{9\left(3x^3-4x^2+2\right)}$$

$\downarrow$ 2009

$$729\left(\frac{\sqrt[3]{-\frac{3}{115}+\frac{179}{345\sqrt{345}}}\left(2(16A+27B+24C)-\frac{\left(9\left(179-9\sqrt{345}+16\sqrt[3]{179-9\sqrt{345}}\right)A+\left(27-\sqrt{345}\right)\left(\right.\right.}{\left(179-9\sqrt{345}\right)^{2/3}}\right.}{6\left(16-\left(179-9\sqrt{345}\right)^{2/3}\right)\left(9\left(x-\frac{4}{9}\right)+\frac{16+\left(179-9\sqrt{345}\right)^{2/3}}{\sqrt[3]{179-9\sqrt{345}}}\right)\left(-81\left(x-\frac{4}{9}\right)^2+\frac{9\left(16+\left(179-9\sqrt{345}\right)^{2/3}\right)\left(x-\frac{4}{9}\right.}{\sqrt[3]{179-9\sqrt{345}}}\right.}\right.$$

$$\frac{C}{9\left(3x^3-4x^2+2\right)}$$

input
```
Int[(A + B*x + C*x^2)/(2 - 4*x^2 + 3*x^3)^2,x]
```

output
```
-1/9*C/(2 - 4*x^2 + 3*x^3) + 729*(((-3/115 + 179/(345*Sqrt[345]))^(1/3)*(2
*(16*A + 27*B + 24*C) - ((9*(179 - 9*Sqrt[345] + 16*(179 - 9*Sqrt[345])^(1
/3))*A + (27 - Sqrt[345])*(4 + (179 - 9*Sqrt[345])^(1/3))*(9*B + 8*C))*(-4
/9 + x))/(179 - 9*Sqrt[345])^(2/3)))/(6*(16 - (179 - 9*Sqrt[345])^(2/3))*(
(16 + (179 - 9*Sqrt[345])^(2/3))/(179 - 9*Sqrt[345])^(1/3) + 9*(-4/9 + x))
*(16 - 256/(179 - 9*Sqrt[345])^(2/3) - (179 - 9*Sqrt[345])^(2/3) + (9*(16
+ (179 - 9*Sqrt[345])^(2/3))*(-4/9 + x))/(179 - 9*Sqrt[345])^(1/3) - 81*(-
4/9 + x)^2)) + ((-3/115 + 179/(345*Sqrt[345]))^(1/3)*((2*(29993 - 1611*Sqr
t[345])*(16*A + 27*B + 24*C))/((179 - 9*Sqrt[345])*(256 + 16*(179 - 9*Sqrt
[345])^(2/3) + (179 - 9*Sqrt[345])^(4/3))*(16 + (179 - 9*Sqrt[345])^(2/3)
+ 9*(179 - 9*Sqrt[345])^(1/3)*(-4/9 + x))) - (4*Sqrt[6/(29993 - 1611*Sqrt[
345] + 128*(179 - 9*Sqrt[345])^(2/3) - 16*(179 - 9*Sqrt[345])^(4/3))]*((97
36952672 - 524197920*Sqrt[345] + (319636187 - 17206017*Sqrt[345])*(179 - 9
*Sqrt[345])^(2/3))*A + (1229351761 - 66184515*Sqrt[345] + 4*(5665339 - 304
929*Sqrt[345])*(179 - 9*Sqrt[345])^(2/3))*(9*B + 8*C))*ArcTan[(179 - 9*Sqr
t[345] + 16*(179 - 9*Sqrt[345])^(1/3) - 18*(179 - 9*Sqrt[345])^(2/3)*(-4/9
 + x))/Sqrt[6*(29993 - 1611*Sqrt[345] + 128*(179 - 9*Sqrt[345])^(2/3) - 16
*(179 - 9*Sqrt[345])^(4/3))]])/((179 - 9*Sqrt[345])^(4/3)*(256 + 16*(179 -
 9*Sqrt[345])^(2/3) + (179 - 9*Sqrt[345])^(4/3))^2) - (2*((479888 - 25776*
Sqrt[345] + 128*(179 - 9*Sqrt[345])^(4/3) - 179*(179 - 9*Sqrt[345])^(5/...
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1235
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^m
*(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*
m + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) -
f*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
)
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2485
```
Int[((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (d_.)*(x_)^3)^(p_), x_S
ymbol] :> With[{r = Rt[-9*a*d^2 + Sqrt[3]*d*Sqrt[4*b^3*d + 27*a^2*d^2], 3]}
, Simp[1/d^(2*p)   Int[(e + f*x)^m*Simp[18^(1/3)*b*(d/(3*r)) - r/18^(1/3) +
 d*x, x]^p*Simp[b*(d/3) + 12^(1/3)*b^2*(d^2/(3*r^2)) + r^2/(3*12^(1/3)) - d
*(2^(1/3)*b*(d/(3^(1/3)*r)) - r/18^(1/3))*x + d^2*x^2, x]^p, x], x]] /; Fre
eQ[{a, b, d, e, f, m}, x] && NeQ[4*b^3 + 27*a^2*d, 0] && ILtQ[p, 0]
```

rule 2490
```
Int[(P3_)^(p_.)*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] :> With[{a = Coeff[P3
, x, 0], b = Coeff[P3, x, 1], c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Su
bst[Int[((3*d*e - c*f)/(3*d) + f*x)^m*Simp[(2*c^3 - 9*b*c*d + 27*a*d^2)/(27
*d^2) - (c^2 - 3*b*d)*(x/(3*d)) + d*x^3, x]^p, x], x, x + c/(3*d)] /; NeQ[c
, 0]] /; FreeQ[{e, f, m, p}, x] && PolyQ[P3, x, 3]
```

rule 2526
```
Int[(Pm_)*(Qn_)^(p_), x_Symbol] :> With[{m = Expon[Pm, x], n = Expon[Qn, x]
}, Simp[Coeff[Pm, x, m]*(Qn^(p + 1)/(n*(p + 1)*Coeff[Qn, x, n])), x] + Simp
[1/(n*Coeff[Qn, x, n])   Int[ExpandToSum[n*Coeff[Qn, x, n]*Pm - Coeff[Pm, x
, m]*D[Qn, x], x]*Qn^p, x], x] /; EqQ[m, n - 1]] /; FreeQ[p, x] && PolyQ[Pm
, x] && PolyQ[Qn, x] && NeQ[p, -1]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.09 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.10

method	result
default	$\frac{\left(\frac{27B}{230}+\frac{8A}{115}+\frac{12C}{115}\right)x^2+\left(-\frac{6B}{115}+\frac{17A}{690}-\frac{16C}{345}\right)x-\frac{16B}{345}-\frac{6A}{115}-\frac{9C}{115}}{x^3-\frac{4}{3}x^2+\frac{2}{3}}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(3\_Z^3-4\_Z^2+2\right)}\frac{\left(48A\_R+81B\_R+72C\_R+\ldots\right.}{9\_R^2-\ldots}\right.}{230}$ …
risch	$\frac{\left(\frac{27B}{230}+\frac{8A}{115}+\frac{12C}{115}\right)x^2+\left(-\frac{6B}{115}+\frac{17A}{690}-\frac{16C}{345}\right)x-\frac{16B}{345}-\frac{6A}{115}-\frac{9C}{115}}{x^3-\frac{4}{3}x^2+\frac{2}{3}}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(3\_Z^3-4\_Z^2+2\right)}\frac{\left(3(16A+27B+24C)\_R+36B+98\ldots\right.}{9\_R^2-8\_\ldots}\right.}{230}$ …

input
```
int((C*x^2+B*x+A)/(3*x^3-4*x^2+2)^2,x,method=_RETURNVERBOSE)
```

output
```
((27/230*B+8/115*A+12/115*C)*x^2+(-6/115*B+17/690*A-16/345*C)*x-16/345*B-6
/115*A-9/115*C)/(x^3-4/3*x^2+2/3)+1/230*sum((48*A*_R+81*B*_R+72*C*_R+98*A+
36*B+32*C)/(9*_R^2-8*_R)*ln(x-_R),_R=RootOf(3*_Z^3-4*_Z^2+2))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.20 (sec) , antiderivative size = 5341, normalized size of antiderivative = 5.06

$$\int \frac{A+Bx+Cx^2}{\left(2-4x^2+3x^3\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)/(3*x^3-4*x^2+2)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [A] (verification not implemented)

Time = 9.17 (sec) , antiderivative size = 338, normalized size of antiderivative = 0.32

$$\int \frac{A+Bx+Cx^2}{(2-4x^2+3x^3)^2}\,dx$$
$$= \operatorname{RootSum}\Big(24334000t^3 + t(462976A^2 + 821484AB + 730208AC + 335664B^2 + 596736BC + 265216C^2$$
$$+ \frac{-36A-32B-54C+x^2\cdot(48A+81B+72C)+x(17A-36B-32C)}{690x^3-920x^2+460}$$

input
```
integrate((C*x**2+B*x+A)/(3*x**3-4*x**2+2)**2,x)
```

output
```
RootSum(24334000*_t**3 + _t*(462976*A**2 + 821484*A*B + 730208*A*C + 33566
4*B**2 + 596736*B*C + 265216*C**2) - 19332*A**3 - 28080*A**2*B - 24960*A**
2*C - 7776*A*B**2 - 13824*A*B*C - 6144*A*C**2 + 2187*B**3 + 5832*B**2*C +
5184*B*C**2 + 1536*C**3, Lambda(_t, _t*log(x + (34846288000*_t**2*A + 3361
7421000*_t**2*B + 29882152000*_t**2*C + 3281598600*_t*A**2 + 3725006400*_t
*A*B + 3311116800*_t*A*C + 925538400*_t*B**2 + 1645401600*_t*B*C + 7312896
00*_t*C**2 + 174801904*A**3 + 622975104*A**2*B + 553755648*A**2*C + 615440
268*A*B**2 + 1094116032*A*B*C + 486273792*A*C**2 + 178158852*B**3 + 475090
272*B**2*C + 422302464*B*C**2 + 125126656*C**3)/(601168164*A**3 + 13222556
64*A**2*B + 1175338368*A**2*C + 1038585888*A*B**2 + 1846374912*A*B*C + 820
611072*A*C**2 + 294722307*B**3 + 785926152*B**2*C + 698601024*B*C**2 + 206
992896*C**3)))) + (-36*A - 32*B - 54*C + x**2*(48*A + 81*B + 72*C) + x*(17
*A - 36*B - 32*C))/(690*x**3 - 920*x**2 + 460)
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{(2-4x^2+3x^3)^2}\,dx = \int \frac{Cx^2+Bx+A}{(3\,x^3-4\,x^2+2)^2}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(3*x^3-4*x^2+2)^2,x, algorithm="maxima")
```

output
```
1/230*(3*(16*A + 27*B + 24*C)*x^2 + (17*A - 36*B - 32*C)*x - 36*A - 32*B -
 54*C)/(3*x^3 - 4*x^2 + 2) + 1/230*integrate((3*(16*A + 27*B + 24*C)*x + 9
8*A + 36*B + 32*C)/(3*x^3 - 4*x^2 + 2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A+Bx+Cx^2}{(2-4x^2+3x^3)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((C*x^2+B*x+A)/(3*x^3-4*x^2+2)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 12.47 (sec) , antiderivative size = 988, normalized size of antiderivative = 0.94

$$\int \frac{A+Bx+Cx^2}{(2-4x^2+3x^3)^2}\,dx = \text{Too large to display}$$

input
```
int((A + B*x + C*x^2)/(3*x^3 - 4*x^2 + 2)^2,x)
```

output
```
symsum(log((1728*A^2*x)/13225 + (19683*B^2*x)/52900 + (3888*C^2*x)/13225 -
 96*root(z^3 + z*((28936*A^2)/1520875 + (20979*B^2)/1520875 + (16576*C^2)/
1520875 + (205371*A*B)/6083500 + (45638*A*C)/1520875 + (37296*B*C)/1520875
) - (864*A*B*C)/1520875 + (729*B^2*C)/3041750 + (324*B*C^2)/1520875 - (384
*A*C^2)/1520875 - (312*A^2*C)/304175 - (486*A*B^2)/1520875 - (351*A^2*B)/3
04175 + (2187*B^3)/24334000 + (96*C^3)/1520875 - (4833*A^3)/6083500, z, k)
^2*x + (3528*A^2)/13225 + (2187*B^2)/13225 + (1728*C^2)/13225 + 162*root(z
^3 + z*((28936*A^2)/1520875 + (20979*B^2)/1520875 + (16576*C^2)/1520875 +
(205371*A*B)/6083500 + (45638*A*C)/1520875 + (37296*B*C)/1520875) - (864*A
*B*C)/1520875 + (729*B^2*C)/3041750 + (324*B*C^2)/1520875 - (384*A*C^2)/15
20875 - (312*A^2*C)/304175 - (486*A*B^2)/1520875 - (351*A^2*B)/304175 + (2
187*B^3)/24334000 + (96*C^3)/1520875 - (4833*A^3)/6083500, z, k)^2 + (1449
9*A*B)/26450 + (6444*A*C)/13225 + (3888*B*C)/13225 - (588*A*root(z^3 + z*(
(28936*A^2)/1520875 + (20979*B^2)/1520875 + (16576*C^2)/1520875 + (205371*
A*B)/6083500 + (45638*A*C)/1520875 + (37296*B*C)/1520875) - (864*A*B*C)/15
20875 + (729*B^2*C)/3041750 + (324*B*C^2)/1520875 - (384*A*C^2)/1520875 -
(312*A^2*C)/304175 - (486*A*B^2)/1520875 - (351*A^2*B)/304175 + (2187*B^3)
/24334000 + (96*C^3)/1520875 - (4833*A^3)/6083500, z, k))/115 - (216*B*roo
t(z^3 + z*((28936*A^2)/1520875 + (20979*B^2)/1520875 + (16576*C^2)/1520875
 + (205371*A*B)/6083500 + (45638*A*C)/1520875 + (37296*B*C)/1520875) - ...
```

Reduce [F]

$$\int \frac{A+Bx+Cx^2}{(2-4x^2+3x^3)^2}\,dx$$
$$= \frac{1458\left(\int \frac{x^3}{9x^6-24x^5+16x^4+12x^3-16x^2+4}dx\right)b\,x^3-1944\left(\int \frac{x^3}{9x^6-24x^5+16x^4+12x^3-16x^2+4}dx\right)b\,x^2+972\left(\int \frac{}{9x^6-24x^5+}\right.}{}$$

input
```
int((C*x^2+B*x+A)/(3*x^3-4*x^2+2)^2,x)
```

output
```
(1458*int(x**3/(9*x**6 - 24*x**5 + 16*x**4 + 12*x**3 - 16*x**2 + 4),x)*b*x
**3 - 1944*int(x**3/(9*x**6 - 24*x**5 + 16*x**4 + 12*x**3 - 16*x**2 + 4),x
)*b*x**2 + 972*int(x**3/(9*x**6 - 24*x**5 + 16*x**4 + 12*x**3 - 16*x**2 +
4),x)*b + 1296*int(x**3/(9*x**6 - 24*x**5 + 16*x**4 + 12*x**3 - 16*x**2 +
4),x)*c*x**3 - 1728*int(x**3/(9*x**6 - 24*x**5 + 16*x**4 + 12*x**3 - 16*x*
*2 + 4),x)*c*x**2 + 864*int(x**3/(9*x**6 - 24*x**5 + 16*x**4 + 12*x**3 - 1
6*x**2 + 4),x)*c + 864*int(1/(9*x**6 - 24*x**5 + 16*x**4 + 12*x**3 - 16*x*
*2 + 4),x)*a*x**3 - 1152*int(1/(9*x**6 - 24*x**5 + 16*x**4 + 12*x**3 - 16*
x**2 + 4),x)*a*x**2 + 576*int(1/(9*x**6 - 24*x**5 + 16*x**4 + 12*x**3 - 16
*x**2 + 4),x)*a - 486*int(1/(9*x**6 - 24*x**5 + 16*x**4 + 12*x**3 - 16*x**
2 + 4),x)*b*x**3 + 648*int(1/(9*x**6 - 24*x**5 + 16*x**4 + 12*x**3 - 16*x*
*2 + 4),x)*b*x**2 - 324*int(1/(9*x**6 - 24*x**5 + 16*x**4 + 12*x**3 - 16*x
**2 + 4),x)*b - 432*int(1/(9*x**6 - 24*x**5 + 16*x**4 + 12*x**3 - 16*x**2
+ 4),x)*c*x**3 + 576*int(1/(9*x**6 - 24*x**5 + 16*x**4 + 12*x**3 - 16*x**2
 + 4),x)*c*x**2 - 288*int(1/(9*x**6 - 24*x**5 + 16*x**4 + 12*x**3 - 16*x**
2 + 4),x)*c + 81*b*x + 36*b + 48*c*x**3 - 64*c*x**2 + 72*c*x + 32*c)/(288*
(3*x**3 - 4*x**2 + 2))
```

3.47 $\int \frac{A+Bx+Cx^2}{\sqrt{2-4x^2+3x^3}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 1895

$$\int \frac{A+Bx+Cx^2}{\sqrt{2-4x^2+3x^3}}\,dx = \text{Too large to display}$$

output

```
2/9*C*(3*x^3-4*x^2+2)^(1/2)+2/729*(9*B+8*C)*(256-16*(179-9*345^(1/2))^(2/3
)+(179-9*345^(1/2))^(4/3)+(179-9*345^(1/2)+16*(179-9*345^(1/2))^(1/3))*(4-
9*x)+(179-9*345^(1/2))^(2/3)*(4-9*x)^2)^(1/2)*(4-16/(179-9*345^(1/2))^(1/3
)-(179-9*345^(1/2))^(1/3)-9*x)*(-16+256/(179-9*345^(1/2))^(2/3)+(179-9*345
^(1/2))^(2/3)+(16+(179-9*345^(1/2))^(2/3))*(4-9*x)/(179-9*345^(1/2))^(1/3)
+(-4+9*x)^2)^(1/2)/(179-9*345^(1/2))^(1/3)/(4-(179-9*345^(1/2))^(1/3)-(16+
(768+(537-27*345^(1/2))*(179-9*345^(1/2))^(1/3)+48*(179-9*345^(1/2))^(2/3)
)^(1/2))/(179-9*345^(1/2))^(1/3)-9*x)/(3*x^3-4*x^2+2)^(1/2)-4/729*(1850681
-99195*345^(1/2)+(165424-8592*345^(1/2))*(179-9*345^(1/2))^(1/3)+64*(179-9
*345^(1/2))^(5/3))^(1/2)*(9*B+8*C)*((256-16*(179-9*345^(1/2))^(2/3)+(179-9
*345^(1/2))^(4/3)+(179-9*345^(1/2)+16*(179-9*345^(1/2))^(1/3))*(4-9*x)+(17
9-9*345^(1/2))^(2/3)*(4-9*x)^2)/(1-(179-9*345^(1/2))^(1/3)*(4-16/(179-9*34
5^(1/2))^(1/3)-(179-9*345^(1/2))^(1/3)-9*x)/(768+48*(179-9*345^(1/2))^(2/3
)+3*(179-9*345^(1/2))^(4/3))^(1/2))^2)^(1/2)*(1-(179-9*345^(1/2))^(1/3)*(4
-16/(179-9*345^(1/2))^(1/3)-(179-9*345^(1/2))^(1/3)-9*x)/(768+48*(179-9*34
5^(1/2))^(2/3)+3*(179-9*345^(1/2))^(4/3))^(1/2))*(-4+16/(179-9*345^(1/2))^
(1/3)+(179-9*345^(1/2))^(1/3)+9*x)^(1/2)*(-16+256/(179-9*345^(1/2))^(2/3)+
(179-9*345^(1/2))^(2/3)+(16+(179-9*345^(1/2))^(2/3))*(4-9*x)/(179-9*345^(1
/2))^(1/3)+(-4+9*x)^2)^(1/2)*EllipticE(sin(2*arctan((179-9*345^(1/2))^(1/6
)*(-4+16/(179-9*345^(1/2))^(1/3)+(179-9*345^(1/2))^(1/3)+9*x)^(1/2)/(76...
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 10.69 (sec) , antiderivative size = 1597, normalized size of antiderivative = 0.84

$$\int \frac{A+Bx+Cx^2}{\sqrt{2-4x^2+3x^3}}\,dx = \text{Too large to display}$$

input

```
Integrate[(A + B*x + C*x^2)/Sqrt[2 - 4*x^2 + 3*x^3],x]
```

output
```
(2*(C*(2 - 4*x^2 + 3*x^3) + (9*A*EllipticF[ArcSin[Sqrt[(-x + Root[2 - 4*#1
^2 + 3*#1^3 & , 3, 0])/(-Root[2 - 4*#1^2 + 3*#1^3 & , 2, 0] + Root[2 - 4*#
1^2 + 3*#1^3 & , 3, 0])]], (Root[2 - 4*#1^2 + 3*#1^3 & , 2, 0] - Root[2 -
4*#1^2 + 3*#1^3 & , 3, 0])/(Root[2 - 4*#1^2 + 3*#1^3 & , 1, 0] - Root[2 -
4*#1^2 + 3*#1^3 & , 3, 0])]*(x - Root[2 - 4*#1^2 + 3*#1^3 & , 3, 0])*Sqrt[
(-x + Root[2 - 4*#1^2 + 3*#1^3 & , 1, 0])/(Root[2 - 4*#1^2 + 3*#1^3 & , 1,
 0] - Root[2 - 4*#1^2 + 3*#1^3 & , 3, 0])]*Sqrt[(-x + Root[2 - 4*#1^2 + 3*
#1^3 & , 2, 0])/(Root[2 - 4*#1^2 + 3*#1^3 & , 2, 0] - Root[2 - 4*#1^2 + 3*
#1^3 & , 3, 0])])/Sqrt[(x - Root[2 - 4*#1^2 + 3*#1^3 & , 3, 0])/(Root[2 -
4*#1^2 + 3*#1^3 & , 2, 0] - Root[2 - 4*#1^2 + 3*#1^3 & , 3, 0])] - (9*B*(x
 - Root[2 - 4*#1^2 + 3*#1^3 & , 2, 0])*Sqrt[(-x + Root[2 - 4*#1^2 + 3*#1^3
 & , 1, 0])/(Root[2 - 4*#1^2 + 3*#1^3 & , 1, 0] - Root[2 - 4*#1^2 + 3*#1^3
 & , 3, 0])]*Sqrt[(-x + Root[2 - 4*#1^2 + 3*#1^3 & , 3, 0])/(-Root[2 - 4*#
1^2 + 3*#1^3 & , 2, 0] + Root[2 - 4*#1^2 + 3*#1^3 & , 3, 0])]*(EllipticF[A
rcSin[Sqrt[(-x + Root[2 - 4*#1^2 + 3*#1^3 & , 3, 0])/(-Root[2 - 4*#1^2 + 3
*#1^3 & , 2, 0] + Root[2 - 4*#1^2 + 3*#1^3 & , 3, 0])]], (Root[2 - 4*#1^2
+ 3*#1^3 & , 2, 0] - Root[2 - 4*#1^2 + 3*#1^3 & , 3, 0])/(Root[2 - 4*#1^2
+ 3*#1^3 & , 1, 0] - Root[2 - 4*#1^2 + 3*#1^3 & , 3, 0])]*Root[2 - 4*#1^2
+ 3*#1^3 & , 1, 0] + EllipticE[ArcSin[Sqrt[(-x + Root[2 - 4*#1^2 + 3*#1^3
& , 3, 0])/(-Root[2 - 4*#1^2 + 3*#1^3 & , 2, 0] + Root[2 - 4*#1^2 + 3*#...
```

Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 2.91 (sec) , antiderivative size = 665, normalized size of antiderivative = 0.35, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {2526, 2490, 2486, 27, 1269, 1172, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{\sqrt{3x^3-4x^2+2}}dx$$

$$\Big\downarrow \text{2526}$$

$$\frac{1}{9}\int \frac{9A+(9B+8C)x}{\sqrt{3x^3-4x^2+2}}dx+\frac{2}{9}C\sqrt{3x^3-4x^2+2}$$

$$\Big\downarrow \text{2490}$$

$$\frac{1}{9}\int \frac{\frac{1}{9}(81A+4(9B+8C))+(9B+8C)\left(x-\frac{4}{9}\right)}{\sqrt{3\left(x-\frac{4}{9}\right)^3-\frac{16}{9}\left(x-\frac{4}{9}\right)+\frac{358}{243}}}d\left(x-\frac{4}{9}\right)+\frac{2}{9}C\sqrt{3x^3-4x^2+2}$$

$\big\downarrow$ 2486

$$\frac{\sqrt{9\left(x-\frac{4}{9}\right)+\frac{16+\left(179-9\sqrt{345}\right)^{2/3}}{\sqrt[3]{179-9\sqrt{345}}}}\sqrt{81\left(x-\frac{4}{9}\right)^2-\frac{9\left(16+\left(179-9\sqrt{345}\right)^{2/3}\right)\left(x-\frac{4}{9}\right)}{\sqrt[3]{179-9\sqrt{345}}}+\left(179-9\sqrt{345}\right)^{2/3}+\frac{256}{\left(179-9\sqrt{345}\right)}}}{3\sqrt{729\left(x-\frac{4}{9}\right)^3}}$$

$$\frac{2}{9}C\sqrt{3x^3-4x^2+2}$$

$\big\downarrow$ 27

$$\frac{\sqrt{9\left(x-\frac{4}{9}\right)+\frac{16+\left(179-9\sqrt{345}\right)^{2/3}}{\sqrt[3]{179-9\sqrt{345}}}}\sqrt{81\left(x-\frac{4}{9}\right)^2-\frac{9\left(16+\left(179-9\sqrt{345}\right)^{2/3}\right)\left(x-\frac{4}{9}\right)}{\sqrt[3]{179-9\sqrt{345}}}+\left(179-9\sqrt{345}\right)^{2/3}+\frac{256}{\left(179-9\sqrt{345}\right)}}}{3\sqrt{3}\sqrt{729\left(x-\frac{4}{9}\right)^3}}$$

$$\frac{2}{9}C\sqrt{3x^3-4x^2+2}$$

$\big\downarrow$ 1269

$$\frac{\sqrt{9\left(x-\frac{4}{9}\right)+\frac{16+\left(179-9\sqrt{345}\right)^{2/3}}{\sqrt[3]{179-9\sqrt{345}}}}\sqrt{81\left(x-\frac{4}{9}\right)^2-\frac{9\left(16+\left(179-9\sqrt{345}\right)^{2/3}\right)\left(x-\frac{4}{9}\right)}{\sqrt[3]{179-9\sqrt{345}}}+\left(179-9\sqrt{345}\right)^{2/3}+\frac{256}{\left(179-9\sqrt{345}\right)}}}{}$$

$$\frac{2}{9}C\sqrt{3x^3-4x^2+2}$$

$\big\downarrow$ 1172

$$\frac{2}{9}\sqrt{3x^3-4x^2+2}C+$$

$$\sqrt{9\left(x-\frac{4}{9}\right)+\frac{16+\left(179-9\sqrt{345}\right)^{2/3}}{\sqrt[3]{179-9\sqrt{345}}}}\sqrt{81\left(x-\frac{4}{9}\right)^2-\frac{9\left(16+\left(179-9\sqrt{345}\right)^{2/3}\right)\left(x-\frac{4}{9}\right)}{\sqrt[3]{179-9\sqrt{345}}}+\left(179-9\sqrt{345}\right)^{2/3}+\frac{256}{\left(179-9\sqrt{345}\right)}}$$

$\downarrow$ 321

$$\frac{2}{9}\sqrt{3x^3-4x^2+2}C+$$

$$\sqrt{9\left(x-\frac{4}{9}\right)+\frac{16+\left(179-9\sqrt{345}\right)^{2/3}}{\sqrt[3]{179-9\sqrt{345}}}}\sqrt{81\left(x-\frac{4}{9}\right)^2-\frac{9\left(16+\left(179-9\sqrt{345}\right)^{2/3}\right)\left(x-\frac{4}{9}\right)}{\sqrt[3]{179-9\sqrt{345}}}+\left(179-9\sqrt{345}\right)^{2/3}+\frac{256}{\left(179-9\sqrt{345}\right)}}$$

$$\Big\downarrow \; 327$$

$$\frac{2}{9}C\sqrt{3x^3-4x^2+2}+$$

$$\sqrt{9\left(x-\tfrac{4}{9}\right)+\frac{16+\left(179-9\sqrt{345}\right)^{2/3}}{\sqrt[3]{179-9\sqrt{345}}}}\sqrt{81\left(x-\tfrac{4}{9}\right)^2-\frac{9\left(16+\left(179-9\sqrt{345}\right)^{2/3}\right)\left(x-\frac{4}{9}\right)}{\sqrt[3]{179-9\sqrt{345}}}+\left(179-9\sqrt{345}\right)^{2/3}+\frac{256}{\left(179-9\sqrt{345}\right)}$$

input
```
Int[(A + B*x + C*x^2)/Sqrt[2 - 4*x^2 + 3*x^3],x]
```

output
```
(2*C*Sqrt[2 - 4*x^2 + 3*x^3])/9 + (Sqrt[(16 + (179 - 9*Sqrt[345])^(2/3))/(
179 - 9*Sqrt[345])^(1/3) + 9*(-4/9 + x)]*Sqrt[-16 + 256/(179 - 9*Sqrt[345]
)^(2/3) + (179 - 9*Sqrt[345])^(2/3) - (9*(16 + (179 - 9*Sqrt[345])^(2/3))*
(-4/9 + x))/(179 - 9*Sqrt[345])^(1/3) + 81*(-4/9 + x)^2]*(((I/9)*Sqrt[2]*(
9*B + 8*C)*Sqrt[(16 + (179 - 9*Sqrt[345])^(2/3))/(179 - 9*Sqrt[345])^(1/3)
 + 9*(-4/9 + x)]*EllipticE[ArcSin[4/9 - x], ((2*I)*(16 - (179 - 9*Sqrt[345
])^(2/3)))/(16*(I + Sqrt[3]) - (I - Sqrt[3])*(179 - 9*Sqrt[345])^(2/3))])/
((179 - 9*Sqrt[345])^(1/6)*Sqrt[(I*((16 + (179 - 9*Sqrt[345])^(2/3))/(179
- 9*Sqrt[345])^(1/3) + 9*(-4/9 + x)))/(16*(3*I - Sqrt[3]) + (3*I + Sqrt[3]
)*(179 - 9*Sqrt[345])^(2/3))]) + (((2*I)/9)*Sqrt[2]*(179 - 9*Sqrt[345])^(1
/6)*(81*A + 36*B + 32*C - ((16 + (179 - 9*Sqrt[345])^(2/3))*(9*B + 8*C))/(
179 - 9*Sqrt[345])^(1/3))*Sqrt[(I*((16 + (179 - 9*Sqrt[345])^(2/3))/(179 -
 9*Sqrt[345])^(1/3) + 9*(-4/9 + x)))/(16*(3*I - Sqrt[3]) + (3*I + Sqrt[3])
*(179 - 9*Sqrt[345])^(2/3))]*EllipticF[ArcSin[4/9 - x], ((2*I)*(16 - (179
- 9*Sqrt[345])^(2/3)))/(16*(I + Sqrt[3]) - (I - Sqrt[3])*(179 - 9*Sqrt[345
])^(2/3))])/Sqrt[(16 + (179 - 9*Sqrt[345])^(2/3))/(179 - 9*Sqrt[345])^(1/3
) + 9*(-4/9 + x)]))/(3*Sqrt[3]*Sqrt[358 - 432*(-4/9 + x) + 729*(-4/9 + x)^
3])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 1172
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Simp[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m))   Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^
2 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[m^2, 1/4]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2486
```
Int[((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (d_.)*(x_)^3)^(p_), x_S
ymbol] :> With[{r = Rt[-9*a*d^2 + Sqrt[3]*d*Sqrt[4*b^3*d + 27*a^2*d^2], 3]}
, Simp[(a + b*x + d*x^3)^p/(Simp[18^(1/3)*b*(d/(3*r)) - r/18^(1/3) + d*x, x
]^p*Simp[b*(d/3) + 12^(1/3)*b^2*(d^2/(3*r^2)) + r^2/(3*12^(1/3)) - d*(2^(1/
3)*b*(d/(3^(1/3)*r)) - r/18^(1/3))*x + d^2*x^2, x]^p)   Int[(e + f*x)^m*Sim
p[18^(1/3)*b*(d/(3*r)) - r/18^(1/3) + d*x, x]^p*Simp[b*(d/3) + 12^(1/3)*b^2
*(d^2/(3*r^2)) + r^2/(3*12^(1/3)) - d*(2^(1/3)*b*(d/(3^(1/3)*r)) - r/18^(1/
3))*x + d^2*x^2, x]^p, x], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && NeQ[4*
b^3 + 27*a^2*d, 0] &&  !IntegerQ[p]
```

rule 2490
```
Int[(P3_)^(p_.)*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] :> With[{a = Coeff[P3
, x, 0], b = Coeff[P3, x, 1], c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Su
bst[Int[((3*d*e - c*f)/(3*d) + f*x)^m*Simp[(2*c^3 - 9*b*c*d + 27*a*d^2)/(27
*d^2) - (c^2 - 3*b*d)*(x/(3*d)) + d*x^3, x]^p, x], x, x + c/(3*d)] /; NeQ[c
, 0]] /; FreeQ[{e, f, m, p}, x] && PolyQ[P3, x, 3]
```

rule 2526
```
Int[(Pm_)*(Qn_)^(p_), x_Symbol] :> With[{m = Expon[Pm, x], n = Expon[Qn, x]
}, Simp[Coeff[Pm, x, m]*(Qn^(p + 1)/(n*(p + 1)*Coeff[Qn, x, n])), x] + Simp
[1/(n*Coeff[Qn, x, n])   Int[ExpandToSum[n*Coeff[Qn, x, n]*Pm - Coeff[Pm, x
, m]*D[Qn, x], x]*Qn^p, x], x] /; EqQ[m, n - 1]] /; FreeQ[p, x] && PolyQ[Pm
, x] && PolyQ[Qn, x] && NeQ[p, -1]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.11 (sec) , antiderivative size = 1236, normalized size of antiderivative = 0.65

method	result	size
elliptic	Expression too large to display	1236
risch	Expression too large to display	1238
default	Expression too large to display	1968

input
```
int((C*x^2+B*x+A)/(3*x^3-4*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/9*C*(3*x^3-4*x^2+2)^(1/2)+2/3*I*A*3^(1/2)*(-1/9*(179+9*345^(1/2))^(1/3)+
16/9/(179+9*345^(1/2))^(1/3))*(I*(x-1/18*(179+9*345^(1/2))^(1/3)-8/9/(179+
9*345^(1/2))^(1/3)-4/9+1/2*I*3^(1/2)*(-1/9*(179+9*345^(1/2))^(1/3)+16/9/(1
79+9*345^(1/2))^(1/3)))*3^(1/2)/(1/9*(179+9*345^(1/2))^(1/3)-16/9/(179+9*3
45^(1/2))^(1/3)))^(1/2)*((x+1/9*(179+9*345^(1/2))^(1/3)+16/9/(179+9*345^(1
/2))^(1/3)-4/9)/(1/6*(179+9*345^(1/2))^(1/3)+8/3/(179+9*345^(1/2))^(1/3)-1
/2*I*3^(1/2)*(-1/9*(179+9*345^(1/2))^(1/3)+16/9/(179+9*345^(1/2))^(1/3))))
^(1/2)*(-I*(x-1/18*(179+9*345^(1/2))^(1/3)-8/9/(179+9*345^(1/2))^(1/3)-4/9
-1/2*I*3^(1/2)*(-1/9*(179+9*345^(1/2))^(1/3)+16/9/(179+9*345^(1/2))^(1/3))
)*3^(1/2)/(1/9*(179+9*345^(1/2))^(1/3)-16/9/(179+9*345^(1/2))^(1/3)))^(1/2
)/(3*x^3-4*x^2+2)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x-1/18*(179+9*345^(1/2))
^(1/3)-8/9/(179+9*345^(1/2))^(1/3)-4/9+1/2*I*3^(1/2)*(-1/9*(179+9*345^(1/2
))^(1/3)+16/9/(179+9*345^(1/2))^(1/3)))*3^(1/2)/(1/9*(179+9*345^(1/2))^(1/
3)-16/9/(179+9*345^(1/2))^(1/3)))^(1/2),(I*3^(1/2)*(1/9*(179+9*345^(1/2))^
(1/3)-16/9/(179+9*345^(1/2))^(1/3))/(1/6*(179+9*345^(1/2))^(1/3)+8/3/(179+
9*345^(1/2))^(1/3)-1/2*I*3^(1/2)*(-1/9*(179+9*345^(1/2))^(1/3)+16/9/(179+9
*345^(1/2))^(1/3))))^(1/2))+2/3*I*(B+8/9*C)*3^(1/2)*(-1/9*(179+9*345^(1/2)
)^(1/3)+16/9/(179+9*345^(1/2))^(1/3))*(I*(x-1/18*(179+9*345^(1/2))^(1/3)-8
/9/(179+9*345^(1/2))^(1/3)-4/9+1/2*I*3^(1/2)*(-1/9*(179+9*345^(1/2))^(1/3)
+16/9/(179+9*345^(1/2))^(1/3)))*3^(1/2)/(1/9*(179+9*345^(1/2))^(1/3)-16...
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.03

$$\int \frac{A+Bx+Cx^2}{\sqrt{2-4x^2+3x^3}}\,dx = \frac{2}{243}\sqrt{3}(81\,A+36\,B+32\,C)\text{weierstrassPInverse}\left(\frac{64}{27},-\frac{1432}{729},x-\frac{4}{9}\right)-\frac{2}{27}\sqrt{3}(9\,B+8\,C)\text{weierstrassZeta}\left(\frac{64}{27},-\frac{1432}{729},\text{weierstrassPInverse}\left(\frac{64}{27},-\frac{1432}{729},x-\frac{4}{9}\right)\right)+\frac{2}{9}\sqrt{3\,x^3-4\,x^2+2}C$$

input
```
integrate((C*x^2+B*x+A)/(3*x^3-4*x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
2/243*sqrt(3)*(81*A + 36*B + 32*C)*weierstrassPInverse(64/27, -1432/729, x
 - 4/9) - 2/27*sqrt(3)*(9*B + 8*C)*weierstrassZeta(64/27, -1432/729, weier
strassPInverse(64/27, -1432/729, x - 4/9)) + 2/9*sqrt(3*x^3 - 4*x^2 + 2)*C
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt{2-4x^2+3x^3}}\,dx=\int \frac{A+Bx+Cx^2}{\sqrt{3x^3-4x^2+2}}\,dx$$

input
```
integrate((C*x**2+B*x+A)/(3*x**3-4*x**2+2)**(1/2),x)
```

output
```
Integral((A + B*x + C*x**2)/sqrt(3*x**3 - 4*x**2 + 2), x)
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt{2-4x^2+3x^3}}\,dx=\int \frac{Cx^2+Bx+A}{\sqrt{3\,x^3-4\,x^2+2}}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(3*x^3-4*x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
integrate((C*x^2 + B*x + A)/sqrt(3*x^3 - 4*x^2 + 2), x)
```

Giac [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt{2-4x^2+3x^3}}\,dx=\int \frac{Cx^2+Bx+A}{\sqrt{3\,x^3-4\,x^2+2}}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(3*x^3-4*x^2+2)^(1/2),x, algorithm="giac")
```

output
```
integrate((C*x^2 + B*x + A)/sqrt(3*x^3 - 4*x^2 + 2), x)
```

Mupad [B] (verification not implemented)

Time = 13.13 (sec) , antiderivative size = 3476, normalized size of antiderivative = 1.83

$$\int \frac{A+Bx+Cx^2}{\sqrt{2-4x^2+3x^3}}\,dx = \text{Too large to display}$$

input
```
int((A + B*x + C*x^2)/(3*x^3 - 4*x^2 + 2)^(1/2),x)
```

output
```
(C*((2*(x^3 - (4*x^2)/3 + 2/3)^(1/2))/3 - (16*(ellipticF(asin(((8/(81*(179
/729 - 345^(1/2)/81)^(1/3)) - x + (179/729 - 345^(1/2)/81)^(1/3)/2 - 3^(1/
2)*(8/(81*(179/729 - 345^(1/2)/81)^(1/3)) - (179/729 - 345^(1/2)/81)^(1/3)
/2)*1i + 4/9)/(8/(27*(179/729 - 345^(1/2)/81)^(1/3)) + (3*(179/729 - 345^(
1/2)/81)^(1/3))/2 - 3^(1/2)*(8/(81*(179/729 - 345^(1/2)/81)^(1/3)) - (179/
729 - 345^(1/2)/81)^(1/3)/2)*1i))^(1/2)), (3^(1/2)*(8/(81*(179/729 - 345^(
1/2)/81)^(1/3)) + (179/729 - 345^(1/2)/81)^(1/3)/2 - (3^(1/2)*(8/(81*(179/
729 - 345^(1/2)/81)^(1/3)) - (179/729 - 345^(1/2)/81)^(1/3)/2)*1i)/3)*1i)/
(16/(81*(179/729 - 345^(1/2)/81)^(1/3)) - (179/729 - 345^(1/2)/81)^(1/3)))
*(8/(81*(179/729 - 345^(1/2)/81)^(1/3)) + (179/729 - 345^(1/2)/81)^(1/3)/2
 + 3^(1/2)*(8/(81*(179/729 - 345^(1/2)/81)^(1/3)) - (179/729 - 345^(1/2)/8
1)^(1/3)/2)*1i + 4/9) - 3^(1/2)*(16/(81*(179/729 - 345^(1/2)/81)^(1/3)) -
(179/729 - 345^(1/2)/81)^(1/3))*ellipticE(asin(((8/(81*(179/729 - 345^(1/2
)/81)^(1/3)) - x + (179/729 - 345^(1/2)/81)^(1/3)/2 - 3^(1/2)*(8/(81*(179/
729 - 345^(1/2)/81)^(1/3)) - (179/729 - 345^(1/2)/81)^(1/3)/2)*1i + 4/9)/(
8/(27*(179/729 - 345^(1/2)/81)^(1/3)) + (3*(179/729 - 345^(1/2)/81)^(1/3))
/2 - 3^(1/2)*(8/(81*(179/729 - 345^(1/2)/81)^(1/3)) - (179/729 - 345^(1/2)
/81)^(1/3)/2)*1i))^(1/2)), (3^(1/2)*(8/(81*(179/729 - 345^(1/2)/81)^(1/3))
 + (179/729 - 345^(1/2)/81)^(1/3)/2 - (3^(1/2)*(8/(81*(179/729 - 345^(1/2)
/81)^(1/3)) - (179/729 - 345^(1/2)/81)^(1/3)/2)*1i)/3)*1i)/(16/(81*(179...
```

Reduce [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt{2-4x^2+3x^3}}\,dx = \frac{2\sqrt{3x^3-4x^2+2}\,c}{9} + \left(\int \frac{\sqrt{3x^3-4x^2+2}}{3x^3-4x^2+2}dx\right)a + \left(\int \frac{\sqrt{3x^3-4x^2+2}\,x}{3x^3-4x^2+2}dx\right)b + \frac{8\left(\int \frac{\sqrt{3x^3-4x^2+2}\,x}{3x^3-4x^2+2}dx\right)c}{9}$$

input
```
int((C*x^2+B*x+A)/(3*x^3-4*x^2+2)^(1/2),x)
```

output
```
(2*sqrt(3*x**3 - 4*x**2 + 2)*c + 9*int(sqrt(3*x**3 - 4*x**2 + 2)/(3*x**3 -
 4*x**2 + 2),x)*a + 9*int((sqrt(3*x**3 - 4*x**2 + 2)*x)/(3*x**3 - 4*x**2 +
 2),x)*b + 8*int((sqrt(3*x**3 - 4*x**2 + 2)*x)/(3*x**3 - 4*x**2 + 2),x)*c)
/9
```

3.48 $\int (A+Bx+Cx^2)(2-4x^2+3x^3)^p\,dx$

Optimal result

Integrand size = 25, antiderivative size = 977

$$\int (A+Bx+Cx^2)(2-4x^2+3x^3)^p\,dx = \text{Too large to display}$$

output
```
C*(3*x^3-4*x^2+2)^(p+1)/(9*p+9)-1/729*(81*A+36*B+32*C-(16+(179-9*345^(1/2)
)^(2/3))*(9*B+8*C)/(179-9*345^(1/2))^(1/3))*(4-16/(179-9*345^(1/2))^(1/3)-
(179-9*345^(1/2))^(1/3)-9*x)*(3*x^3-4*x^2+2)^p*AppellF1(p+1,-p,-p,2+p,-2*I
*(179-9*345^(1/2))^(1/3)*(4-16/(179-9*345^(1/2))^(1/3)-(179-9*345^(1/2))^(
1/3)-9*x)/(48*I-16*3^(1/2)+(3*I+3^(1/2))*(179-9*345^(1/2))^(2/3)),-2*I*(17
9-9*345^(1/2))^(1/3)*(4-16/(179-9*345^(1/2))^(1/3)-(179-9*345^(1/2))^(1/3)
-9*x)/(48*I+16*3^(1/2)+(3*I-3^(1/2))*(179-9*345^(1/2))^(2/3)))/(p+1)/((1+2
*I*(179-9*345^(1/2))^(1/3)*(4-16/(179-9*345^(1/2))^(1/3)-(179-9*345^(1/2))
^(1/3)-9*x)/(48*I+16*3^(1/2)+(3*I-3^(1/2))*(179-9*345^(1/2))^(2/3)))^p)/((
1+2*I*(179-9*345^(1/2))^(1/3)*(4-16/(179-9*345^(1/2))^(1/3)-(179-9*345^(1/
2))^(1/3)-9*x)/(48*I-16*3^(1/2)+(3*I+3^(1/2))*(179-9*345^(1/2))^(2/3)))^p)
+1/729*(9*B+8*C)*(4-16/(179-9*345^(1/2))^(1/3)-(179-9*345^(1/2))^(1/3)-9*x
)^2*(3*x^3-4*x^2+2)^p*AppellF1(2+p,-p,-p,3+p,-2*I*(179-9*345^(1/2))^(1/3)*
(4-16/(179-9*345^(1/2))^(1/3)-(179-9*345^(1/2))^(1/3)-9*x)/(48*I-16*3^(1/2
)+(3*I+3^(1/2))*(179-9*345^(1/2))^(2/3)),-2*I*(179-9*345^(1/2))^(1/3)*(4-1
6/(179-9*345^(1/2))^(1/3)-(179-9*345^(1/2))^(1/3)-9*x)/(48*I+16*3^(1/2)+(3
*I-3^(1/2))*(179-9*345^(1/2))^(2/3)))/(2+p)/((1+2*I*(179-9*345^(1/2))^(1/3
)*(4-16/(179-9*345^(1/2))^(1/3)-(179-9*345^(1/2))^(1/3)-9*x)/(48*I+16*3^(1
/2)+(3*I-3^(1/2))*(179-9*345^(1/2))^(2/3)))^p)/((1+2*I*(179-9*345^(1/2))^(
1/3)*(4-16/(179-9*345^(1/2))^(1/3)-(179-9*345^(1/2))^(1/3)-9*x)/(48*I-1...
```

Mathematica [F]

$$\int \left(A+Bx+Cx^2\right)\left(2-4x^2+3x^3\right)^p\,dx=\int \left(A+Bx+Cx^2\right)\left(2-4x^2+3x^3\right)^p\,dx$$

input
```
Integrate[(A + B*x + C*x^2)*(2 - 4*x^2 + 3*x^3)^p,x]
```

output
```
Integrate[(A + B*x + C*x^2)*(2 - 4*x^2 + 3*x^3)^p, x]
```

Rubi [A] (warning: unable to verify)

Time = 2.96 (sec) , antiderivative size = 1308, normalized size of antiderivative = 1.34, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {2526, 2490, 2486, 27, 1269, 1179, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(3x^3-4x^2+2\right)^p \left(A+Bx+Cx^2\right) dx$$

$$\Big\downarrow \text{2526}$$

$$\frac{1}{9}\int (9A+(9B+8C)x)\left(3x^3-4x^2+2\right)^p dx+\frac{C\left(3x^3-4x^2+2\right)^{p+1}}{9(p+1)}$$

$$\Big\downarrow \text{2490}$$

$$\frac{1}{9}\int \left(\frac{1}{9}(81A+4(9B+8C))+(9B+8C)\left(x-\frac{4}{9}\right)\right)\left(3\left(x-\frac{4}{9}\right)^3-\frac{16}{9}\left(x-\frac{4}{9}\right)+\frac{358}{243}\right)^p d\left(x-\frac{4}{9}\right)+\frac{C\left(3x^3-4x^2+2\right)^{p+1}}{9(p+1)}$$

$$\Big\downarrow \text{2486}$$

$$\frac{1}{9}\left(3\left(x-\frac{4}{9}\right)+\frac{16+\left(179-9\sqrt{345}\right)^{2/3}}{3\sqrt[3]{179-9\sqrt{345}}}\right)^{-p}\left(3\left(x-\frac{4}{9}\right)^3-\frac{16}{9}\left(x-\frac{4}{9}\right)+\frac{358}{243}\right)^p\left(9\left(x-\frac{4}{9}\right)^2-\frac{\left(16+(179\right.}{\sqrt[3]{}}\right.$$
$$\frac{C\left(3x^3-4x^2+2\right)^{p+1}}{9(p+1)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{81}\left(3\left(x-\frac{4}{9}\right)+\frac{16+\left(179-9\sqrt{345}\right)^{2/3}}{3\sqrt[3]{179-9\sqrt{345}}}\right)^{-p}\left(3\left(x-\frac{4}{9}\right)^3-\frac{16}{9}\left(x-\frac{4}{9}\right)+\frac{358}{243}\right)^p\left(9\left(x-\frac{4}{9}\right)^2-\frac{\left(16+(17\right.}{}\right.$$
$$\frac{C\left(3x^3-4x^2+2\right)^{p+1}}{9(p+1)}$$

$$\Big\downarrow \text{1269}$$

$$\frac{1}{81}\left(3\left(x-\frac{4}{9}\right)+\frac{16+\left(179-9\sqrt{345}\right)^{2/3}}{3\sqrt[3]{179-9\sqrt{345}}}\right)^{-p}\left(3\left(x-\frac{4}{9}\right)^3-\frac{16}{9}\left(x-\frac{4}{9}\right)+\frac{358}{243}\right)^p\left(9\left(x-\frac{4}{9}\right)^2-\frac{\left(16+(17\right.}{}\right.$$

$$\frac{C\left(3x^3-4x^2+2\right)^{p+1}}{9(p+1)}$$

$\Big\downarrow$ 1179

$$\frac{1}{81}\left(3\left(x-\frac{4}{9}\right)+\frac{16+\left(179-9\sqrt{345}\right)^{2/3}}{3\sqrt[3]{179-9\sqrt{345}}}\right)^{-p}\left(3\left(x-\frac{4}{9}\right)^3-\frac{16}{9}\left(x-\frac{4}{9}\right)+\frac{358}{243}\right)^p\left(\frac{1}{3}\left(81A+36B+32C-\right.\right.$$

$$\frac{C\left(3x^3-4x^2+2\right)^{p+1}}{9(p+1)}$$

$\Big\downarrow$ 150

$$\frac{1}{81}\left(3\left(x-\frac{4}{9}\right)+\frac{16+\left(179-9\sqrt{345}\right)^{2/3}}{3\sqrt[3]{179-9\sqrt{345}}}\right)^{-p}\left(3\left(x-\frac{4}{9}\right)^3-\frac{16}{9}\left(x-\frac{4}{9}\right)+\frac{358}{243}\right)^p\left(\frac{\left(81A+36B+32C-\right.}{}\right.$$

$$\frac{C\left(3x^3-4x^2+2\right)^{p+1}}{9(p+1)}$$

input
```
Int[(A + B*x + C*x^2)*(2 - 4*x^2 + 3*x^3)^p,x]
```

output
```
(C*(2 - 4*x^2 + 3*x^3)^(1 + p))/(9*(1 + p)) + ((358/243 - (16*(-4/9 + x))/
9 + 3*(-4/9 + x)^3)^p*(((81*A + 36*B + 32*C - ((16 + (179 - 9*Sqrt[345])^(
2/3))*(9*B + 8*C))/(179 - 9*Sqrt[345])^(1/3))*((16 + (179 - 9*Sqrt[345])^(
2/3))/(3*(179 - 9*Sqrt[345])^(1/3)) + 3*(-4/9 + x))^(1 + p)*((-16 + 256/(1
79 - 9*Sqrt[345])^(2/3) + (179 - 9*Sqrt[345])^(2/3))/9 - ((16 + (179 - 9*S
qrt[345])^(2/3))*(-4/9 + x))/(179 - 9*Sqrt[345])^(1/3) + 9*(-4/9 + x)^2)^p
*AppellF1[1 + p, -p, -p, 2 + p, ((6*I)*(179 - 9*Sqrt[345])^(1/3)*((16 + (1
79 - 9*Sqrt[345])^(2/3))/(3*(179 - 9*Sqrt[345])^(1/3)) + 3*(-4/9 + x)))/(1
6*(3*I + Sqrt[3]) + (3*I - Sqrt[3])*(179 - 9*Sqrt[345])^(2/3)), ((6*I)*(17
9 - 9*Sqrt[345])^(1/3)*((16 + (179 - 9*Sqrt[345])^(2/3))/(3*(179 - 9*Sqrt[
345])^(1/3)) + 3*(-4/9 + x)))/(16*(3*I - Sqrt[3]) + (3*I + Sqrt[3])*(179 -
 9*Sqrt[345])^(2/3))])/(3*(1 + p)*(1 - ((2*I)*(179 - 9*Sqrt[345])^(1/3)*((
16 + (179 - 9*Sqrt[345])^(2/3))/(179 - 9*Sqrt[345])^(1/3) + 9*(-4/9 + x)))
/(16*(3*I + Sqrt[3]) + (3*I - Sqrt[3])*(179 - 9*Sqrt[345])^(2/3)))^p*(1 -
((2*I)*(179 - 9*Sqrt[345])^(1/3)*((16 + (179 - 9*Sqrt[345])^(2/3))/(179 -
9*Sqrt[345])^(1/3) + 9*(-4/9 + x)))/(16*(3*I - Sqrt[3]) + (3*I + Sqrt[3])*
(179 - 9*Sqrt[345])^(2/3)))^p) + ((9*B + 8*C)*((16 + (179 - 9*Sqrt[345])^(
2/3))/(3*(179 - 9*Sqrt[345])^(1/3)) + 3*(-4/9 + x))^(2 + p)*((-16 + 256/(1
79 - 9*Sqrt[345])^(2/3) + (179 - 9*Sqrt[345])^(2/3))/9 - ((16 + (179 - 9*S
qrt[345])^(2/3))*(-4/9 + x))/(179 - 9*Sqrt[345])^(1/3) + 9*(-4/9 + x)^2...
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 1179
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(a + b*x + c*x^2)^p/(e*(1 - (
d + e*x)/(d - e*((b - q)/(2*c))))^p*(1 - (d + e*x)/(d - e*((b + q)/(2*c))))
^p)   Subst[Int[x^m*Simp[1 - x/(d - e*((b - q)/(2*c))), x]^p*Simp[1 - x/(d
 - e*((b + q)/(2*c))), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, b, c, d, e, m
, p}, x]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2486
```
Int[((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (d_.)*(x_)^3)^(p_), x_S
ymbol] :> With[{r = Rt[-9*a*d^2 + Sqrt[3]*d*Sqrt[4*b^3*d + 27*a^2*d^2], 3]}
, Simp[(a + b*x + d*x^3)^p/(Simp[18^(1/3)*b*(d/(3*r)) - r/18^(1/3) + d*x, x
]^p*Simp[b*(d/3) + 12^(1/3)*b^2*(d^2/(3*r^2)) + r^2/(3*12^(1/3)) - d*(2^(1/
3)*b*(d/(3^(1/3)*r)) - r/18^(1/3))*x + d^2*x^2, x]^p)   Int[(e + f*x)^m*Sim
p[18^(1/3)*b*(d/(3*r)) - r/18^(1/3) + d*x, x]^p*Simp[b*(d/3) + 12^(1/3)*b^2
*(d^2/(3*r^2)) + r^2/(3*12^(1/3)) - d*(2^(1/3)*b*(d/(3^(1/3)*r)) - r/18^(1/
3))*x + d^2*x^2, x]^p, x], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && NeQ[4*
b^3 + 27*a^2*d, 0] &&  !IntegerQ[p]
```

rule 2490
```
Int[(P3_)^(p_.)*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] :> With[{a = Coeff[P3
, x, 0], b = Coeff[P3, x, 1], c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Su
bst[Int[((3*d*e - c*f)/(3*d) + f*x)^m*Simp[(2*c^3 - 9*b*c*d + 27*a*d^2)/(27
*d^2) - (c^2 - 3*b*d)*(x/(3*d)) + d*x^3, x]^p, x], x, x + c/(3*d)] /; NeQ[c
, 0]] /; FreeQ[{e, f, m, p}, x] && PolyQ[P3, x, 3]
```

rule 2526
```
Int[(Pm_)*(Qn_)^(p_), x_Symbol] :> With[{m = Expon[Pm, x], n = Expon[Qn, x]
}, Simp[Coeff[Pm, x, m]*(Qn^(p + 1)/(n*(p + 1)*Coeff[Qn, x, n])), x] + Simp
[1/(n*Coeff[Qn, x, n])   Int[ExpandToSum[n*Coeff[Qn, x, n]*Pm - Coeff[Pm, x
, m]*D[Qn, x], x]*Qn^p, x], x] /; EqQ[m, n - 1]] /; FreeQ[p, x] && PolyQ[Pm
, x] && PolyQ[Qn, x] && NeQ[p, -1]
```

Maple [F]

$$\int (C\,x^2 + Bx + A)\,(3x^3 - 4x^2 + 2)^p\,dx$$

input
```
int((C*x^2+B*x+A)*(3*x^3-4*x^2+2)^p,x)
```

output
```
int((C*x^2+B*x+A)*(3*x^3-4*x^2+2)^p,x)
```

Fricas [F]

$$\int (A+Bx+Cx^2)\left(2-4x^2+3x^3\right)^p\,dx=\int (Cx^2+Bx+A)\left(3\,x^3-4\,x^2+2\right)^p\,dx$$

input
```
integrate((C*x^2+B*x+A)*(3*x^3-4*x^2+2)^p,x, algorithm="fricas")
```

output
```
integral((C*x^2 + B*x + A)*(3*x^3 - 4*x^2 + 2)^p, x)
```

Sympy [F]

$$\int (A+Bx+Cx^2)\left(2-4x^2+3x^3\right)^p\,dx=\int (A+Bx+Cx^2)\left(3x^3-4x^2+2\right)^p\,dx$$

input
```
integrate((C*x**2+B*x+A)*(3*x**3-4*x**2+2)**p,x)
```

output
```
Integral((A + B*x + C*x**2)*(3*x**3 - 4*x**2 + 2)**p, x)
```

Maxima [F]

$$\int (A+Bx+Cx^2)\left(2-4x^2+3x^3\right)^p\,dx=\int (Cx^2+Bx+A)\left(3\,x^3-4\,x^2+2\right)^p\,dx$$

input
```
integrate((C*x^2+B*x+A)*(3*x^3-4*x^2+2)^p,x, algorithm="maxima")
```

output
```
integrate((C*x^2 + B*x + A)*(3*x^3 - 4*x^2 + 2)^p, x)
```

Giac [F]

$$\int (A+Bx+Cx^2)\left(2-4x^2+3x^3\right)^p\,dx=\int \left(Cx^2+Bx+A\right)\left(3\,x^3-4\,x^2+2\right)^p\,dx$$

input
```
integrate((C*x^2+B*x+A)*(3*x^3-4*x^2+2)^p,x, algorithm="giac")
```

output
```
integrate((C*x^2 + B*x + A)*(3*x^3 - 4*x^2 + 2)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (A+Bx+Cx^2)\left(2-4x^2+3x^3\right)^p\,dx=\int \left(C\,x^2+B\,x+A\right)\,\left(3\,x^3-4\,x^2+2\right)^p\,dx$$

input
```
int((A + B*x + C*x^2)*(3*x^3 - 4*x^2 + 2)^p,x)
```

output
```
int((A + B*x + C*x^2)*(3*x^3 - 4*x^2 + 2)^p, x)
```

Reduce [F]

$$\int (A+Bx+Cx^2)\left(2-4x^2+3x^3\right)^p\,dx=\text{too large to display}$$

input
```
int((C*x^2+B*x+A)*(3*x^3-4*x^2+2)^p,x)
```

output

```
(729*(3*x**3 - 4*x**2 + 2)**p*a*p**2*x - 324*(3*x**3 - 4*x**2 + 2)**p*a*p*
*2 + 1215*(3*x**3 - 4*x**2 + 2)**p*a*p*x - 540*(3*x**3 - 4*x**2 + 2)**p*a*
p + 486*(3*x**3 - 4*x**2 + 2)**p*a*x - 216*(3*x**3 - 4*x**2 + 2)**p*a + 72
9*(3*x**3 - 4*x**2 + 2)**p*b*p**2*x**2 - 324*(3*x**3 - 4*x**2 + 2)**p*b*p*
*2*x - 288*(3*x**3 - 4*x**2 + 2)**p*b*p**2 + 972*(3*x**3 - 4*x**2 + 2)**p*
b*p*x**2 - 324*(3*x**3 - 4*x**2 + 2)**p*b*p*x - 432*(3*x**3 - 4*x**2 + 2)*
*p*b*p + 243*(3*x**3 - 4*x**2 + 2)**p*b*x**2 - 144*(3*x**3 - 4*x**2 + 2)**
p*b + 729*(3*x**3 - 4*x**2 + 2)**p*c*p**2*x**3 - 324*(3*x**3 - 4*x**2 + 2)
**p*c*p**2*x**2 - 288*(3*x**3 - 4*x**2 + 2)**p*c*p**2*x + 230*(3*x**3 - 4*
x**2 + 2)**p*c*p**2 + 729*(3*x**3 - 4*x**2 + 2)**p*c*p*x**3 - 108*(3*x**3
- 4*x**2 + 2)**p*c*p*x**2 - 288*(3*x**3 - 4*x**2 + 2)**p*c*p*x + 102*(3*x*
*3 - 4*x**2 + 2)**p*c*p + 162*(3*x**3 - 4*x**2 + 2)**p*c*x**3 - 20*(3*x**3
 - 4*x**2 + 2)**p*c + 39366*int((3*x**3 - 4*x**2 + 2)**p/(27*p**2*x**3 - 3
6*p**2*x**2 + 18*p**2 + 27*p*x**3 - 36*p*x**2 + 18*p + 6*x**3 - 8*x**2 + 4
),x)*a*p**5 + 104976*int((3*x**3 - 4*x**2 + 2)**p/(27*p**2*x**3 - 36*p**2*
x**2 + 18*p**2 + 27*p*x**3 - 36*p*x**2 + 18*p + 6*x**3 - 8*x**2 + 4),x)*a*
p**4 + 100602*int((3*x**3 - 4*x**2 + 2)**p/(27*p**2*x**3 - 36*p**2*x**2 +
18*p**2 + 27*p*x**3 - 36*p*x**2 + 18*p + 6*x**3 - 8*x**2 + 4),x)*a*p**3 +
40824*int((3*x**3 - 4*x**2 + 2)**p/(27*p**2*x**3 - 36*p**2*x**2 + 18*p**2
+ 27*p*x**3 - 36*p*x**2 + 18*p + 6*x**3 - 8*x**2 + 4),x)*a*p**2 + 5832*...
```

3.49 $\int (A+Bx+Cx^2)(2-6x^2+3x^3)^3\,dx$

Optimal result

Integrand size = 25, antiderivative size = 145

$$\begin{aligned}\int (A+Bx+Cx^2)\left(2-6x^2+3x^3\right)^3\,dx = {} & 8Ax+4Bx^2-\frac{8}{3}(9A-C)x^3 \\ & +9(A-2B)x^4+\frac{36}{5}(6A+B-2C)x^5 \\ & -6(6A-6B-C)x^6-\frac{54}{7}(3A+4B-4C)x^7 \\ & +\frac{27}{4}(6A-3B-4C)x^8-18(A-2B+C)x^9 \\ & +\frac{27}{10}(A-6B+12C)x^{10} \\ & +\frac{27}{11}(B-6C)x^{11}+\frac{9Cx^{12}}{4}\end{aligned}$$

output

```
8*A*x+4*B*x^2-8/3*(9*A-C)*x^3+9*(A-2*B)*x^4+36/5*(6*A+B-2*C)*x^5-6*(6*A-6*
B-C)*x^6-54/7*(3*A+4*B-4*C)*x^7+27/4*(6*A-3*B-4*C)*x^8-18*(A-2*B+C)*x^9+27
/10*(A-6*B+12*C)*x^10+27/11*(B-6*C)*x^11+9/4*C*x^12
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.00

$$\int (A+Bx+Cx^2)\left(2-6x^2+3x^3\right)^3\,dx = 8Ax+4Bx^2-\frac{8}{3}(9A-C)x^3 + 9(A-2B)x^4+\frac{36}{5}(6A+B-2C)x^5 - 6(6A-6B-C)x^6-\frac{54}{7}(3A+4B-4C)x^7 + \frac{27}{4}(6A-3B-4C)x^8-18(A-2B+C)x^9 + \frac{27}{10}(A-6B+12C)x^{10} + \frac{27}{11}(B-6C)x^{11}+\frac{9Cx^{12}}{4}$$

input
```
Integrate[(A + B*x + C*x^2)*(2 - 6*x^2 + 3*x^3)^3,x]
```

output
```
8*A*x + 4*B*x^2 - (8*(9*A - C)*x^3)/3 + 9*(A - 2*B)*x^4 + (36*(6*A + B - 2
*C)*x^5)/5 - 6*(6*A - 6*B - C)*x^6 - (54*(3*A + 4*B - 4*C)*x^7)/7 + (27*(6
*A - 3*B - 4*C)*x^8)/4 - 18*(A - 2*B + C)*x^9 + (27*(A - 6*B + 12*C)*x^10)
/10 + (27*(B - 6*C)*x^11)/11 + (9*C*x^12)/4
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(3x^3-6x^2+2\right)^3 (A+Bx+Cx^2)\,dx$$

$$\Big\downarrow \text{ 2188}$$

$$\int \left(27x^9(A-6B+12C)-162x^8(A-2B+C)+54x^7(6A-3B-4C)-54x^6(3A+4B-4C)-36x^5(6A-6\right.$$

$\downarrow$ 2009

$$\frac{27}{10}x^{10}(A-6B+12C)-18x^9(A-2B+C)+\frac{27}{4}x^8(6A-3B-4C)-\frac{54}{7}x^7(3A+4B-4C)-6x^6(6A-6B-C)+\frac{36}{5}x^5(6A+B-2C)+9x^4(A-2B)-\frac{8}{3}x^3(9A-C)+8Ax+\frac{27}{11}x^{11}(B-6C)+4Bx^2+\frac{9Cx^{12}}{4}$$

input
```
Int[(A + B*x + C*x^2)*(2 - 6*x^2 + 3*x^3)^3,x]
```

output
```
8*A*x + 4*B*x^2 - (8*(9*A - C)*x^3)/3 + 9*(A - 2*B)*x^4 + (36*(6*A + B - 2
*C)*x^5)/5 - 6*(6*A - 6*B - C)*x^6 - (54*(3*A + 4*B - 4*C)*x^7)/7 + (27*(6
*A - 3*B - 4*C)*x^8)/4 - 18*(A - 2*B + C)*x^9 + (27*(A - 6*B + 12*C)*x^10)
/10 + (27*(B - 6*C)*x^11)/11 + (9*C*x^12)/4
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.93

method	result
norman	$\frac{9C\,x^{12}}{4}+\left(\frac{27B}{11}-\frac{162C}{11}\right)x^{11}+\left(\frac{27A}{10}-\frac{81B}{5}+\frac{162C}{5}\right)x^{10}+\left(-18A+36B-18C\right)x^{9}+\left(\frac{81A}{2}-\right.$
default	$\frac{9C\,x^{12}}{4}+\frac{(27B-162C)x^{11}}{11}+\frac{(27A-162B+324C)x^{10}}{10}+\frac{(-162A+324B-162C)x^{9}}{9}+\frac{(324A-162B-216C)x^{8}}{8}+$
gosper	$\frac{8}{3}C\,x^{3}+4B\,x^{2}+9x^{4}A+\frac{216}{5}x^{5}A-18A\,x^{9}-\frac{81}{5}B\,x^{10}-27x^{8}C-\frac{162}{7}x^{7}A+36x^{9}B+\frac{81}{2}x^{8}$
risch	$\frac{8}{3}C\,x^{3}+4B\,x^{2}+9x^{4}A+\frac{216}{5}x^{5}A-18A\,x^{9}-\frac{81}{5}B\,x^{10}-27x^{8}C-\frac{162}{7}x^{7}A+36x^{9}B+\frac{81}{2}x^{8}$
parallelrisch	$\frac{8}{3}C\,x^{3}+4B\,x^{2}+9x^{4}A+\frac{216}{5}x^{5}A-18A\,x^{9}-\frac{81}{5}B\,x^{10}-27x^{8}C-\frac{162}{7}x^{7}A+36x^{9}B+\frac{81}{2}x^{8}$
orering	$x\left(10395C\,x^{11}+11340B\,x^{10}-68040x^{10}C+12474A\,x^{9}-74844x^{9}B+149688C\,x^{9}-83160x^{8}A+166320B\,x^{8}-83160x^{8}C+18711\right.$

input
```
int((C*x^2+B*x+A)*(3*x^3-6*x^2+2)^3,x,method=_RETURNVERBOSE)
```

output
```
9/4*C*x^12+(27/11*B-162/11*C)*x^11+(27/10*A-81/5*B+162/5*C)*x^10+(-18*A+36
*B-18*C)*x^9+(81/2*A-81/4*B-27*C)*x^8+(-162/7*A-216/7*B+216/7*C)*x^7+(-36*
A+36*B+6*C)*x^6+(216/5*A+36/5*B-72/5*C)*x^5+(9*A-18*B)*x^4+(-24*A+8/3*C)*x
^3+4*B*x^2+8*A*x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.90

$$\begin{aligned}
&\int (A+Bx+Cx^2)\left(2-6x^2+3x^3\right)^3 dx \\
&= \frac{9}{4}\,Cx^{12}+\frac{27}{11}\,(B-6\,C)x^{11}+\frac{27}{10}\,(A-6\,B+12\,C)x^{10}-18\,(A-2\,B+C)x^9 \\
&\quad+\frac{27}{4}\,(6\,A-3\,B-4\,C)x^8-\frac{54}{7}\,(3\,A+4\,B-4\,C)x^7-6\,(6\,A-6\,B-C)x^6 \\
&\quad+\frac{36}{5}\,(6\,A+B-2\,C)x^5+9\,(A-2\,B)x^4-\frac{8}{3}\,(9\,A-C)x^3+4\,Bx^2+8\,Ax
\end{aligned}$$

input
```
integrate((C*x^2+B*x+A)*(3*x^3-6*x^2+2)^3,x, algorithm="fricas")
```

output
```
9/4*C*x^12 + 27/11*(B - 6*C)*x^11 + 27/10*(A - 6*B + 12*C)*x^10 - 18*(A -
2*B + C)*x^9 + 27/4*(6*A - 3*B - 4*C)*x^8 - 54/7*(3*A + 4*B - 4*C)*x^7 - 6
*(6*A - 6*B - C)*x^6 + 36/5*(6*A + B - 2*C)*x^5 + 9*(A - 2*B)*x^4 - 8/3*(9
*A - C)*x^3 + 4*B*x^2 + 8*A*x
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.07

$$\begin{aligned}&\int (A+Bx+Cx^2)\left(2-6x^2+3x^3\right)^3\,dx\\&=8Ax+4Bx^2+\frac{9Cx^{12}}{4}+x^{11}\cdot\left(\frac{27B}{11}-\frac{162C}{11}\right)+x^{10}\cdot\left(\frac{27A}{10}-\frac{81B}{5}+\frac{162C}{5}\right)\\&\quad+x^9(-18A+36B-18C)+x^8\cdot\left(\frac{81A}{2}-\frac{81B}{4}-27C\right)\\&\quad+x^7\left(-\frac{162A}{7}-\frac{216B}{7}+\frac{216C}{7}\right)+x^6(-36A+36B+6C)+x^5\\&\quad\cdot\left(\frac{216A}{5}+\frac{36B}{5}-\frac{72C}{5}\right)+x^4\cdot(9A-18B)+x^3\left(-24A+\frac{8C}{3}\right)\end{aligned}$$

input
```
integrate((C*x**2+B*x+A)*(3*x**3-6*x**2+2)**3,x)
```

output
```
8*A*x + 4*B*x**2 + 9*C*x**12/4 + x**11*(27*B/11 - 162*C/11) + x**10*(27*A/
10 - 81*B/5 + 162*C/5) + x**9*(-18*A + 36*B - 18*C) + x**8*(81*A/2 - 81*B/
4 - 27*C) + x**7*(-162*A/7 - 216*B/7 + 216*C/7) + x**6*(-36*A + 36*B + 6*C
) + x**5*(216*A/5 + 36*B/5 - 72*C/5) + x**4*(9*A - 18*B) + x**3*(-24*A + 8
*C/3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.90

$$\begin{aligned}&\int (A+Bx+Cx^2)\left(2-6x^2+3x^3\right)^3\,dx\\&=\frac{9}{4}\,Cx^{12}+\frac{27}{11}\,(B-6\,C)x^{11}+\frac{27}{10}\,(A-6\,B+12\,C)x^{10}-18\,(A-2\,B+C)x^9\\&\quad+\frac{27}{4}\,(6\,A-3\,B-4\,C)x^8-\frac{54}{7}\,(3\,A+4\,B-4\,C)x^7-6\,(6\,A-6\,B-C)x^6\\&\quad+\frac{36}{5}\,(6\,A+B-2\,C)x^5+9\,(A-2\,B)x^4-\frac{8}{3}\,(9\,A-C)x^3+4\,Bx^2+8\,Ax\end{aligned}$$

input
```
integrate((C*x^2+B*x+A)*(3*x^3-6*x^2+2)^3,x, algorithm="maxima")
```

output
```
9/4*C*x^12 + 27/11*(B - 6*C)*x^11 + 27/10*(A - 6*B + 12*C)*x^10 - 18*(A -
2*B + C)*x^9 + 27/4*(6*A - 3*B - 4*C)*x^8 - 54/7*(3*A + 4*B - 4*C)*x^7 - 6
*(6*A - 6*B - C)*x^6 + 36/5*(6*A + B - 2*C)*x^5 + 9*(A - 2*B)*x^4 - 8/3*(9
*A - C)*x^3 + 4*B*x^2 + 8*A*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.11

$$\int (A+Bx+Cx^2)(2-6x^2+3x^3)^3\,dx = \frac{9}{4}Cx^{12}+\frac{27}{11}Bx^{11}-\frac{162}{11}Cx^{11}+\frac{27}{10}Ax^{10}-\frac{81}{5}Bx^{10}+\frac{162}{5}Cx^{10}-18Ax^9+36Bx^9-18Cx^9+\frac{81}{2}Ax^8-\frac{81}{4}Bx^8-27Cx^8-\frac{162}{7}Ax^7-\frac{216}{7}Bx^7+\frac{216}{7}Cx^7-36Ax^6+36Bx^6+6Cx^6+\frac{216}{5}Ax^5+\frac{36}{5}Bx^5-\frac{72}{5}Cx^5+9Ax^4-18Bx^4-24Ax^3+\frac{8}{3}Cx^3+4Bx^2+8Ax$$

input
```
integrate((C*x^2+B*x+A)*(3*x^3-6*x^2+2)^3,x, algorithm="giac")
```

output
```
9/4*C*x^12 + 27/11*B*x^11 - 162/11*C*x^11 + 27/10*A*x^10 - 81/5*B*x^10 + 1
62/5*C*x^10 - 18*A*x^9 + 36*B*x^9 - 18*C*x^9 + 81/2*A*x^8 - 81/4*B*x^8 - 2
7*C*x^8 - 162/7*A*x^7 - 216/7*B*x^7 + 216/7*C*x^7 - 36*A*x^6 + 36*B*x^6 +
6*C*x^6 + 216/5*A*x^5 + 36/5*B*x^5 - 72/5*C*x^5 + 9*A*x^4 - 18*B*x^4 - 24*
A*x^3 + 8/3*C*x^3 + 4*B*x^2 + 8*A*x
```

Mupad [B] (verification not implemented)

Time = 12.24 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.95

$$\int (A+Bx+Cx^2)\left(2-6x^2+3x^3\right)^3\,dx$$
$$=\frac{9\,C\,x^{12}}{4}+\left(\frac{27\,B}{11}-\frac{162\,C}{11}\right)\,x^{11}+\left(\frac{27\,A}{10}-\frac{81\,B}{5}+\frac{162\,C}{5}\right)\,x^{10}+(36\,B-18\,A-18\,C)\,x^9+\left(\frac{81\,A}{2}-\frac{81\,B}{4}-27\,C\right)\,x^8+\left(\frac{216\,C}{7}-\frac{216\,B}{7}-\frac{162\,A}{7}\right)\,x^7+(36\,B-36\,A+6\,C)\,x^6+\left(\frac{216\,A}{5}+\frac{36\,B}{5}-\frac{72\,C}{5}\right)\,x^5+(9\,A-18\,B)\,x^4+\left(\frac{8\,C}{3}-24\,A\right)\,x^3+4\,B\,x^2+8\,A\,x$$

input
```
int((A + B*x + C*x^2)*(3*x^3 - 6*x^2 + 2)^3,x)
```

output
```
8*A*x + 4*B*x^2 + (9*C*x^12)/4 - x^9*(18*A - 36*B + 18*C) + x^6*(36*B - 36
*A + 6*C) - x^8*((81*B)/4 - (81*A)/2 + 27*C) + x^10*((27*A)/10 - (81*B)/5
+ (162*C)/5) + x^5*((216*A)/5 + (36*B)/5 - (72*C)/5) - x^7*((162*A)/7 + (2
16*B)/7 - (216*C)/7) + x^4*(9*A - 18*B) - x^3*(24*A - (8*C)/3) + x^11*((27
*B)/11 - (162*C)/11)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.11

$$\int (A+Bx+Cx^2)\left(2-6x^2+3x^3\right)^3\,dx$$
$$=\frac{x(10395c\,x^{11}+11340b\,x^{10}-68040c\,x^{10}+12474a\,x^9-74844b\,x^9+149688c\,x^9-83160a\,x^8+166320b\,x$$

input
```
int((C*x^2+B*x+A)*(3*x^3-6*x^2+2)^3,x)
```

output

```
(x*(12474*a*x**9 - 83160*a*x**8 + 187110*a*x**7 - 106920*a*x**6 - 166320*a
*x**5 + 199584*a*x**4 + 41580*a*x**3 - 110880*a*x**2 + 36960*a + 11340*b*x
**10 - 74844*b*x**9 + 166320*b*x**8 - 93555*b*x**7 - 142560*b*x**6 + 16632
0*b*x**5 + 33264*b*x**4 - 83160*b*x**3 + 18480*b*x + 10395*c*x**11 - 68040
*c*x**10 + 149688*c*x**9 - 83160*c*x**8 - 124740*c*x**7 + 142560*c*x**6 +
27720*c*x**5 - 66528*c*x**4 + 12320*c*x**2))/4620
```

3.50 $\int (A+Bx+Cx^2)(2-6x^2+3x^3)^2\,dx$

Optimal result

Integrand size = 25, antiderivative size = 97

$$\int (A+Bx+Cx^2)\left(2-6x^2+3x^3\right)^2\,dx=4Ax+2Bx^2-\frac{4}{3}(6A-C)x^3+3(A-2B)x^4+\frac{12}{5}(3A+B-2C)x^5-2(3A-3B-C)x^6+\frac{9}{7}(A-4B+4C)x^7+\frac{9}{8}(B-4C)x^8+Cx^9$$

output
```
4*A*x+2*B*x^2-4/3*(6*A-C)*x^3+3*(A-2*B)*x^4+12/5*(3*A+B-2*C)*x^5-2*(3*A-3*
B-C)*x^6+9/7*(A-4*B+4*C)*x^7+9/8*(B-4*C)*x^8+C*x^9
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00

$$\int (A+Bx+Cx^2)\left(2-6x^2+3x^3\right)^2\,dx=4Ax+2Bx^2-\frac{4}{3}(6A-C)x^3+3(A-2B)x^4+\frac{12}{5}(3A+B-2C)x^5-2(3A-3B-C)x^6+\frac{9}{7}(A-4B+4C)x^7+\frac{9}{8}(B-4C)x^8+Cx^9$$

input
```
Integrate[(A + B*x + C*x^2)*(2 - 6*x^2 + 3*x^3)^2,x]
```

output
```
4*A*x + 2*B*x^2 - (4*(6*A - C)*x^3)/3 + 3*(A - 2*B)*x^4 + (12*(3*A + B - 2
*C)*x^5)/5 - 2*(3*A - 3*B - C)*x^6 + (9*(A - 4*B + 4*C)*x^7)/7 + (9*(B - 4
*C)*x^8)/8 + C*x^9
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(3x^3-6x^2+2\right)^2 \left(A+Bx+Cx^2\right) dx$$

$$\Big\downarrow \; 2188$$

$$\int \left(9x^6(A-4B+4C)-12x^5(3A-3B-C)+12x^4(3A+B-2C)+12x^3(A-2B)-4x^2(6A-C)+4A+9$$

$$\Big\downarrow \; 2009$$

$$\frac{9}{7}x^7(A-4B+4C)-2x^6(3A-3B-C)+\frac{12}{5}x^5(3A+B-2C)+3x^4(A-2B)-\frac{4}{3}x^3(6A-C)+4Ax+\frac{9}{8}x^8(B-4C)+2Bx^2+Cx^9$$

input
```
Int[(A + B*x + C*x^2)*(2 - 6*x^2 + 3*x^3)^2,x]
```

output
```
4*A*x + 2*B*x^2 - (4*(6*A - C)*x^3)/3 + 3*(A - 2*B)*x^4 + (12*(3*A + B - 2
*C)*x^5)/5 - 2*(3*A - 3*B - C)*x^6 + (9*(A - 4*B + 4*C)*x^7)/7 + (9*(B - 4
*C)*x^8)/8 + C*x^9
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.95

method	result
norman	$C\,x^9+\left(\frac{9B}{8}-\frac{9C}{2}\right)x^8+\left(\frac{9A}{7}-\frac{36B}{7}+\frac{36C}{7}\right)x^7+\left(-6A+6B+2C\right)x^6+\left(\frac{36A}{5}+\frac{12B}{5}-\frac{24C}{5}\right)$
default	$C\,x^9+\frac{(9B-36C)x^8}{8}+\frac{(9A-36B+36C)x^7}{7}+\frac{(-36A+36B+12C)x^6}{6}+\frac{(36A+12B-24C)x^5}{5}+\frac{(12A-24B)x^4}{4}+$
gosper	$C\,x^9+\frac{9}{8}B\,x^8-\frac{9}{2}x^8C+\frac{9}{7}x^7A-\frac{36}{7}x^7B+\frac{36}{7}x^7C-6x^6A+6x^6B+2C\,x^6+\frac{36}{5}x^5A+\frac{12}{5}B$
risch	$C\,x^9+\frac{9}{8}B\,x^8-\frac{9}{2}x^8C+\frac{9}{7}x^7A-\frac{36}{7}x^7B+\frac{36}{7}x^7C-6x^6A+6x^6B+2C\,x^6+\frac{36}{5}x^5A+\frac{12}{5}B$
parallelrisch	$C\,x^9+\frac{9}{8}B\,x^8-\frac{9}{2}x^8C+\frac{9}{7}x^7A-\frac{36}{7}x^7B+\frac{36}{7}x^7C-6x^6A+6x^6B+2C\,x^6+\frac{36}{5}x^5A+\frac{12}{5}B$
orering	$\frac{x(840x^8C+945x^7B-3780x^7C+1080x^6A-4320x^6B+4320C\,x^6-5040x^5A+5040B\,x^5+1680x^5C+6048x^4A+2016x^4B-403}{840}$

input
```
int((C*x^2+B*x+A)*(3*x^3-6*x^2+2)^2,x,method=_RETURNVERBOSE)
```

output
```
C*x^9+(9/8*B-9/2*C)*x^8+(9/7*A-36/7*B+36/7*C)*x^7+(-6*A+6*B+2*C)*x^6+(36/5
*A+12/5*B-24/5*C)*x^5+(3*A-6*B)*x^4+(-8*A+4/3*C)*x^3+2*B*x^2+4*A*x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.92

$$\int (A+Bx+Cx^2)\left(2-6x^2+3x^3\right)^2\,dx = Cx^9+\frac{9}{8}\,(B-4\,C)x^8+\frac{9}{7}\,(A-4\,B+4\,C)x^7 - 2\,(3\,A-3\,B-C)x^6 + \frac{12}{5}\,(3\,A+B-2\,C)x^5+3\,(A-2\,B)x^4 - \frac{4}{3}\,(6\,A-C)x^3+2\,Bx^2+4\,Ax$$

input
```
integrate((C*x^2+B*x+A)*(3*x^3-6*x^2+2)^2,x, algorithm="fricas")
```

output
```
C*x^9 + 9/8*(B - 4*C)*x^8 + 9/7*(A - 4*B + 4*C)*x^7 - 2*(3*A - 3*B - C)*x^
6 + 12/5*(3*A + B - 2*C)*x^5 + 3*(A - 2*B)*x^4 - 4/3*(6*A - C)*x^3 + 2*B*x
^2 + 4*A*x
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.05

$$\int (A+Bx+Cx^2)\left(2-6x^2+3x^3\right)^2\,dx = 4Ax+2Bx^2+Cx^9+x^8\cdot\left(\frac{9B}{8}-\frac{9C}{2}\right)+x^7 \cdot\left(\frac{9A}{7}-\frac{36B}{7}+\frac{36C}{7}\right)+x^6(-6A+6B+2C) + x^5\cdot\left(\frac{36A}{5}+\frac{12B}{5}-\frac{24C}{5}\right)+x^4 \cdot(3A-6B)+x^3\left(-8A+\frac{4C}{3}\right)$$

input
```
integrate((C*x**2+B*x+A)*(3*x**3-6*x**2+2)**2,x)
```

output
```
4*A*x + 2*B*x**2 + C*x**9 + x**8*(9*B/8 - 9*C/2) + x**7*(9*A/7 - 36*B/7 +
36*C/7) + x**6*(-6*A + 6*B + 2*C) + x**5*(36*A/5 + 12*B/5 - 24*C/5) + x**4
*(3*A - 6*B) + x**3*(-8*A + 4*C/3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.92

$$\int \left(A+Bx+Cx^2\right)\left(2-6x^2+3x^3\right)^2\,dx = Cx^9+\frac{9}{8}\,(B-4\,C)x^8+\frac{9}{7}\,(A-4\,B+4\,C)x^7 - 2\,(3\,A-3\,B-C)x^6 + \frac{12}{5}\,(3\,A+B-2\,C)x^5+3\,(A-2\,B)x^4 - \frac{4}{3}\,(6\,A-C)x^3+2\,Bx^2+4\,Ax$$

input
```
integrate((C*x^2+B*x+A)*(3*x^3-6*x^2+2)^2,x, algorithm="maxima")
```

output
```
C*x^9 + 9/8*(B - 4*C)*x^8 + 9/7*(A - 4*B + 4*C)*x^7 - 2*(3*A - 3*B - C)*x^
6 + 12/5*(3*A + B - 2*C)*x^5 + 3*(A - 2*B)*x^4 - 4/3*(6*A - C)*x^3 + 2*B*x
^2 + 4*A*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.09

$$\int \left(A+Bx+Cx^2\right)\left(2-6x^2+3x^3\right)^2\,dx = Cx^9+\frac{9}{8}\,Bx^8-\frac{9}{2}\,Cx^8+\frac{9}{7}\,Ax^7-\frac{36}{7}\,Bx^7 + \frac{36}{7}\,Cx^7-6\,Ax^6+6\,Bx^6+2\,Cx^6 + \frac{36}{5}\,Ax^5+\frac{12}{5}\,Bx^5-\frac{24}{5}\,Cx^5+3\,Ax^4 - 6\,Bx^4-8\,Ax^3+\frac{4}{3}\,Cx^3+2\,Bx^2+4\,Ax$$

input
```
integrate((C*x^2+B*x+A)*(3*x^3-6*x^2+2)^2,x, algorithm="giac")
```

output
```
C*x^9 + 9/8*B*x^8 - 9/2*C*x^8 + 9/7*A*x^7 - 36/7*B*x^7 + 36/7*C*x^7 - 6*A*
x^6 + 6*B*x^6 + 2*C*x^6 + 36/5*A*x^5 + 12/5*B*x^5 - 24/5*C*x^5 + 3*A*x^4 -
 6*B*x^4 - 8*A*x^3 + 4/3*C*x^3 + 2*B*x^2 + 4*A*x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.95

$$\int (A+Bx+Cx^2) \left(2-6x^2+3x^3\right)^2 \, dx = C\,x^9 + \left(\frac{9\,B}{8}-\frac{9\,C}{2}\right)\,x^8 + \left(\frac{9\,A}{7}-\frac{36\,B}{7}+\frac{36\,C}{7}\right)\,x^7 + (6\,B-6\,A+2\,C)\,x^6 + \left(\frac{36\,A}{5}+\frac{12\,B}{5}-\frac{24\,C}{5}\right)\,x^5 + (3\,A-6\,B)\,x^4 + \left(\frac{4\,C}{3}-8\,A\right)\,x^3 + 2\,B\,x^2 + 4\,A\,x$$

input
```
int((A + B*x + C*x^2)*(3*x^3 - 6*x^2 + 2)^2,x)
```

output
```
4*A*x + 2*B*x^2 + C*x^9 + x^6*(6*B - 6*A + 2*C) + x^5*((36*A)/5 + (12*B)/5
 - (24*C)/5) + x^7*((9*A)/7 - (36*B)/7 + (36*C)/7) + x^4*(3*A - 6*B) - x^3
*(8*A - (4*C)/3) + x^8*((9*B)/8 - (9*C)/2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.10

$$\int (A+Bx+Cx^2) \left(2-6x^2+3x^3\right)^2 \, dx = \frac{x(840c\,x^8+945b\,x^7-3780c\,x^7+1080a\,x^6-4320b\,x^6+4320c\,x^6-5040a\,x^5+5040b\,x^5+1680c\,x^5+6}{840}$$

input
```
int((C*x^2+B*x+A)*(3*x^3-6*x^2+2)^2,x)
```

output
```
(x*(1080*a*x**6 - 5040*a*x**5 + 6048*a*x**4 + 2520*a*x**3 - 6720*a*x**2 +
3360*a + 945*b*x**7 - 4320*b*x**6 + 5040*b*x**5 + 2016*b*x**4 - 5040*b*x**
3 + 1680*b*x + 840*c*x**8 - 3780*c*x**7 + 4320*c*x**6 + 1680*c*x**5 - 4032
*c*x**4 + 1120*c*x**2))/840
```

3.51 $\int (A+Bx+Cx^2)(2-6x^2+3x^3)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 56

$$\int (A+Bx+Cx^2)(2-6x^2+3x^3)\,dx = 2Ax+Bx^2-\frac{2}{3}(3A-C)x^3+\frac{3}{4}(A-2B)x^4+\frac{3}{5}(B-2C)x^5+\frac{Cx^6}{2}$$

output
```
2*A*x+B*x^2-2/3*(3*A-C)*x^3+3/4*(A-2*B)*x^4+3/5*(B-2*C)*x^5+1/2*C*x^6
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00

$$\int (A+Bx+Cx^2)(2-6x^2+3x^3)\,dx = 2Ax+Bx^2-\frac{2}{3}(3A-C)x^3+\frac{3}{4}(A-2B)x^4+\frac{3}{5}(B-2C)x^5+\frac{Cx^6}{2}$$

input
```
Integrate[(A + B*x + C*x^2)*(2 - 6*x^2 + 3*x^3),x]
```

output
```
2*A*x + B*x^2 - (2*(3*A - C)*x^3)/3 + (3*(A - 2*B)*x^4)/4 + (3*(B - 2*C)*x
^5)/5 + (C*x^6)/2
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(3x^3-6x^2+2\right)\left(A+Bx+Cx^2\right)\,dx$$

$$\Big\downarrow \text{ 2188}$$

$$\int \left(3x^3(A-2B)-2x^2(3A-C)+2A+3x^4(B-2C)+2Bx+3Cx^5\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{3}{4}x^4(A-2B)-\frac{2}{3}x^3(3A-C)+2Ax+\frac{3}{5}x^5(B-2C)+Bx^2+\frac{Cx^6}{2}$$

input
```
Int[(A + B*x + C*x^2)*(2 - 6*x^2 + 3*x^3),x]
```

output
```
2*A*x + B*x^2 - (2*(3*A - C)*x^3)/3 + (3*(A - 2*B)*x^4)/4 + (3*(B - 2*C)*x
^5)/5 + (C*x^6)/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.89

method	result	size
norman	$\frac{C x^6}{2}+\left(\frac{3B}{5}-\frac{6C}{5}\right)x^5+\left(\frac{3A}{4}-\frac{3B}{2}\right)x^4+\left(-2A+\frac{2C}{3}\right)x^3+B x^2+2Ax$	50
gosper	$\frac{1}{2}C x^6+\frac{3}{5}B x^5-\frac{6}{5}x^5C+\frac{3}{4}x^4A-\frac{3}{2}x^4B-2x^3A+\frac{2}{3}C x^3+B x^2+2Ax$	53
default	$\frac{C x^6}{2}+\frac{(3B-6C)x^5}{5}+\frac{(3A-6B)x^4}{4}+\frac{(-6A+2C)x^3}{3}+B x^2+2Ax$	53
risch	$\frac{1}{2}C x^6+\frac{3}{5}B x^5-\frac{6}{5}x^5C+\frac{3}{4}x^4A-\frac{3}{2}x^4B-2x^3A+\frac{2}{3}C x^3+B x^2+2Ax$	53
parallelrisch	$\frac{1}{2}C x^6+\frac{3}{5}B x^5-\frac{6}{5}x^5C+\frac{3}{4}x^4A-\frac{3}{2}x^4B-2x^3A+\frac{2}{3}C x^3+B x^2+2Ax$	53
orering	$\frac{x(30x^5C+36x^4B-72C x^4+45x^3A-90B x^3-120A x^2+40C x^2+60Bx+120A)}{60}$	54

input
```
int((C*x^2+B*x+A)*(3*x^3-6*x^2+2),x,method=_RETURNVERBOSE)
```

output
```
1/2*C*x^6+(3/5*B-6/5*C)*x^5+(3/4*A-3/2*B)*x^4+(-2*A+2/3*C)*x^3+B*x^2+2*A*x
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.86

$$\int (A+Bx+Cx^2)(2-6x^2+3x^3)\,dx=\frac{1}{2}Cx^6+\frac{3}{5}(B-2C)x^5+\frac{3}{4}(A-2B)x^4-\frac{2}{3}(3A-C)x^3+Bx^2+2Ax$$

input
```
integrate((C*x^2+B*x+A)*(3*x^3-6*x^2+2),x, algorithm="fricas")
```

output
```
1/2*C*x^6 + 3/5*(B - 2*C)*x^5 + 3/4*(A - 2*B)*x^4 - 2/3*(3*A - C)*x^3 + B*
x^2 + 2*A*x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.96

$$\int (A+Bx+Cx^2)\left(2-6x^2+3x^3\right)\,dx = 2Ax+Bx^2+\frac{Cx^6}{2}+x^5\cdot\left(\frac{3B}{5}-\frac{6C}{5}\right)+x^4\cdot\left(\frac{3A}{4}-\frac{3B}{2}\right)+x^3\left(-2A+\frac{2C}{3}\right)$$

input
```
integrate((C*x**2+B*x+A)*(3*x**3-6*x**2+2),x)
```

output
```
2*A*x + B*x**2 + C*x**6/2 + x**5*(3*B/5 - 6*C/5) + x**4*(3*A/4 - 3*B/2) +
x**3*(-2*A + 2*C/3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.86

$$\int (A+Bx+Cx^2)\left(2-6x^2+3x^3\right)\,dx = \frac{1}{2}\,Cx^6+\frac{3}{5}\,(B-2\,C)x^5+\frac{3}{4}\,(A-2\,B)x^4-\frac{2}{3}\,(3\,A-C)x^3+Bx^2+2\,Ax$$

input
```
integrate((C*x^2+B*x+A)*(3*x^3-6*x^2+2),x, algorithm="maxima")
```

output
```
1/2*C*x^6 + 3/5*(B - 2*C)*x^5 + 3/4*(A - 2*B)*x^4 - 2/3*(3*A - C)*x^3 + B*
x^2 + 2*A*x
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.93

$$\int (A+Bx+Cx^2)(2-6x^2+3x^3)\,dx = \frac{1}{2}Cx^6+\frac{3}{5}Bx^5-\frac{6}{5}Cx^5+\frac{3}{4}Ax^4-\frac{3}{2}Bx^4-2Ax^3+\frac{2}{3}Cx^3+Bx^2+2Ax$$

input
```
integrate((C*x^2+B*x+A)*(3*x^3-6*x^2+2),x, algorithm="giac")
```

output
```
1/2*C*x^6 + 3/5*B*x^5 - 6/5*C*x^5 + 3/4*A*x^4 - 3/2*B*x^4 - 2*A*x^3 + 2/3*
C*x^3 + B*x^2 + 2*A*x
```

Mupad [B] (verification not implemented)

Time = 12.15 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.89

$$\int (A+Bx+Cx^2)(2-6x^2+3x^3)\,dx = \frac{C\,x^6}{2}+\left(\frac{3\,B}{5}-\frac{6\,C}{5}\right)x^5+\left(\frac{3\,A}{4}-\frac{3\,B}{2}\right)x^4+\left(\frac{2\,C}{3}-2\,A\right)x^3+B\,x^2+2\,A\,x$$

input
```
int((A + B*x + C*x^2)*(3*x^3 - 6*x^2 + 2),x)
```

output
```
2*A*x + B*x^2 + (C*x^6)/2 + x^4*((3*A)/4 - (3*B)/2) - x^3*(2*A - (2*C)/3)
+ x^5*((3*B)/5 - (6*C)/5)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.95

$$\int (A+Bx+Cx^2)(2-6x^2+3x^3)\,dx = \frac{x(30c\,x^5+36b\,x^4-72c\,x^4+45a\,x^3-90b\,x^3-120a\,x^2+40c\,x^2+60bx+120a)}{60}$$

input
```
int((C*x^2+B*x+A)*(3*x^3-6*x^2+2),x)
```

output
```
(x*(45*a*x**3 - 120*a*x**2 + 120*a + 36*b*x**4 - 90*b*x**3 + 60*b*x + 30*c
*x**5 - 72*c*x**4 + 40*c*x**2))/60
```

3.52 $\int \frac{A+Bx+Cx^2}{2-6x^2+3x^3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 274

$$\int \frac{A+Bx+Cx^2}{2-6x^2+3x^3}\,dx = \frac{1}{9}C\log\left(2-6x^2+3x^3\right) + \frac{(9A+2(3B+4C)\left(1+2\cos\left(\frac{1}{6}\left(\pi+2\arcsin\left(\frac{1}{8}\right)\right)\right)\right))\log\left(3x-2\left(1+2\cos\left(\frac{1}{6}\left(\pi+2\arcsin\left(\frac{1}{8}\right)\right)\right)\right)\right)\operatorname{se}}{48\sqrt{3}\left(\cos\left(\frac{1}{6}\left(\pi+2\arcsin\left(\frac{1}{8}\right)\right)\right)-\sin\left(\frac{1}{3}\arcsin\left(\frac{1}{8}\right)\right)\right)} + \frac{\log\left(3x-2\left(1+2\sin\left(\frac{1}{3}\arcsin\left(\frac{1}{8}\right)\right)\right)\right)\left(9A+2(3B+4C)\left(1+2\sin\left(\frac{1}{3}\arcsin\left(\frac{1}{8}\right)\right)\right)\right)}{36\left(1-2\cos\left(\frac{2}{3}\arcsin\left(\frac{1}{8}\right)\right)\right)} + \frac{\log\left(3x-2\left(1-2\sin\left(\frac{1}{3}\left(\pi+\arcsin\left(\frac{1}{8}\right)\right)\right)\right)\right)\sec\left(\frac{1}{3}\arcsin\left(\frac{1}{8}\right)\right)\left(9A+2(3B+4C)\left(1-2\sin\left(\frac{1}{3}\left(\pi+\operatorname{ar}\right.\right.\right.\right.}{48\sqrt{3}\left(\sin\left(\frac{1}{3}\arcsin\left(\frac{1}{8}\right)\right)+\sin\left(\frac{1}{3}\left(\pi+\arcsin\left(\frac{1}{8}\right)\right)\right)\right)}$$

output
```
1/9*C*ln(3*x^3-6*x^2+2)+1/144*(9*A+2*(3*B+4*C)*(1+2*cos(1/6*Pi+1/3*arcsin(
1/8))))*ln(3*x-2-4*cos(1/6*Pi+1/3*arcsin(1/8)))*sec(1/3*arcsin(1/8))*3^(1/
2)/(cos(1/6*Pi+1/3*arcsin(1/8))-sin(1/3*arcsin(1/8)))+ln(3*x-2-4*sin(1/3*a
rcsin(1/8)))*(9*A+2*(3*B+4*C)*(1+2*sin(1/3*arcsin(1/8))))/(36-72*cos(2/3*a
rcsin(1/8)))+1/144*ln(3*x-2+4*sin(1/3*Pi+1/3*arcsin(1/8)))*sec(1/3*arcsin(
1/8))*(9*A+2*(3*B+4*C)*(1-2*sin(1/3*Pi+1/3*arcsin(1/8))))*3^(1/2)/(sin(1/3
*arcsin(1/8))+sin(1/3*Pi+1/3*arcsin(1/8)))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.25

$$\int \frac{A+Bx+Cx^2}{2-6x^2+3x^3}\,dx$$
$$=\frac{1}{3}\text{RootSum}\left[2-6\#1^2+3\#1^3\&,\frac{A\log(x-\#1)+B\log(x-\#1)\#1+C\log(x-\#1)\#1^2}{-4\#1+3\#1^2}\&\right]$$

input
```
Integrate[(A + B*x + C*x^2)/(2 - 6*x^2 + 3*x^3),x]
```

output
```
RootSum[2 - 6*#1^2 + 3*#1^3 & , (A*Log[x - #1] + B*Log[x - #1]*#1 + C*Log[
x - #1]*#1^2)/(-4*#1 + 3*#1^2) & ]/3
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 5.00 (sec) , antiderivative size = 667, normalized size of antiderivative = 2.43, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {2525, 27, 2490, 2485, 27, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{3x^3-6x^2+2}\,dx$$
$$\Big\downarrow \text{2525}$$
$$\frac{1}{9}\int \frac{3(3A+(3B+4C)x)}{3x^3-6x^2+2}dx+\frac{1}{9}C\log\left(3x^3-6x^2+2\right)$$
$$\Big\downarrow \text{27}$$
$$\frac{1}{3}\int \frac{3A+(3B+4C)x}{3x^3-6x^2+2}dx+\frac{1}{9}C\log\left(3x^3-6x^2+2\right)$$

$$\Big\downarrow \text{2490}$$

$$\frac{1}{3}\int \frac{\frac{1}{9}(27A+6(3B+4C))+(3B+4C)\left(x-\frac{2}{3}\right)}{3\left(x-\frac{2}{3}\right)^3-4\left(x-\frac{2}{3}\right)+\frac{2}{9}}d\left(x-\frac{2}{3}\right)+\frac{1}{9}C\log\left(3x^3-6x^2+2\right)$$

$$\Big\downarrow \text{2485}$$

$$\frac{1}{9}C\log\left(3x^3-6x^2+2\right)+$$

$$3\int -\frac{9A+6B+8C+3(3B+4C)\left(x-\frac{2}{3}\right)}{3\left(3\left(x-\frac{2}{3}\right)+\frac{4+\left(1-3i\sqrt{7}\right)^{2/3}}{\sqrt[3]{1-3i\sqrt{7}}}\right)\left(-9\left(x-\frac{2}{3}\right)^2+\frac{3\left(4+\left(1-3i\sqrt{7}\right)^{2/3}\right)\left(x-\frac{2}{3}\right)}{\sqrt[3]{1-3i\sqrt{7}}}-\left(1-3i\sqrt{7}\right)^{2/3}-\frac{16}{\left(1-3i\sqrt{7}\right)^{2/3}}+$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{9}C\log\left(3x^3-6x^2+2\right)-$$

$$\int \frac{9A+6B+8C+3(3B+4C)\left(x-\frac{2}{3}\right)}{\left(3\left(x-\frac{2}{3}\right)+\frac{4+\left(1-3i\sqrt{7}\right)^{2/3}}{\sqrt[3]{1-3i\sqrt{7}}}\right)\left(-9\left(x-\frac{2}{3}\right)^2+\frac{3\left(4+\left(1-3i\sqrt{7}\right)^{2/3}\right)\left(x-\frac{2}{3}\right)}{\sqrt[3]{1-3i\sqrt{7}}}-\left(1-3i\sqrt{7}\right)^{2/3}-\frac{16}{\left(1-3i\sqrt{7}\right)^{2/3}}+4\right)}d$$

$$\Big\downarrow \text{1200}$$

$$\frac{1}{9}C\log\left(3x^3-6x^2+2\right)-$$

$$\int\left(\frac{-\left(\left(9-27i\sqrt{7}\right)A\right)-\left(2-6i\sqrt{7}-4\left(1-3i\sqrt{7}\right)^{2/3}-\left(1-3i\sqrt{7}\right)^{4/3}\right)(3B+4C)}{3\left(16+4\left(1-3i\sqrt{7}\right)^{2/3}+\left(1-3i\sqrt{7}\right)^{4/3}\right)\left(3\sqrt[3]{1-3i\sqrt{7}}\left(x-\frac{2}{3}\right)+\left(1-3i\sqrt{7}\right)^{2/3}+4\right)}+\frac{\left(1-3i\sqrt{7}\right)^{2/3}(}{}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\arctan\left(\frac{-6\left(1-3i\sqrt{7}\right)^{2/3}\left(x-\frac{2}{3}\right)+4\sqrt[3]{1-3i\sqrt{7}}-3i\sqrt{7}+1}{\sqrt{6\left(-31-3i\sqrt{7}+8\left(1-3i\sqrt{7}\right)^{2/3}-4\left(1-3i\sqrt{7}\right)^{4/3}\right)}}\right)\left(3i(3\sqrt{7}+i)\left(4+\left(1-3i\sqrt{7}\right)^{2/3}\right)A+2\left(9+5i\sqrt{7}+i(1-\right.\right.}{\left(16+4\left(1-3i\sqrt{7}\right)^{2/3}+\left(1-3i\sqrt{7}\right)^{4/3}\right)\sqrt{6\left(-31-3i\sqrt{7}+8\left(1-3i\sqrt{7}\right)^{2/3}-\sqrt[3]{1-3i\sqrt{7}}\right.}}$$

$$\frac{\log\left(3\sqrt[3]{1-3i\sqrt{7}}(x-\frac{2}{3})+(1-3i\sqrt{7})^{2/3}+4\right)\left(9(1-3i\sqrt{7})A+\left(2-6i\sqrt{7}-4(1-3i\sqrt{7})^{2/3}-(1-3i\sqrt{7})^{4}\right.\right.}{9\left(4-12i\sqrt{7}+16\sqrt[3]{1-3i\sqrt{7}}+\left(1-3i\sqrt{7}\right)^{5/3}\right)}$$

$$\frac{\log\left(9(1-3i\sqrt{7})^{2/3}\left(x-\frac{2}{3}\right)^2-3\left(1-3i\sqrt{7}+4\sqrt[3]{1-3i\sqrt{7}}\right)\left(x-\frac{2}{3}\right)+(1-3i\sqrt{7})^{4/3}-4(1-3i\sqrt{7})^{2/3}+16\right)}{18\left(16+4\left(1-3i\sqrt{7}\right)^{2/3}+\left(1-3i\sqrt{\right.\right.}}$$

$$\frac{1}{9}C\log\left(3x^3-6x^2+2\right)$$

input
```
Int[(A + B*x + C*x^2)/(2 - 6*x^2 + 3*x^3),x]
```

output
```
(((3*I)*(I + 3*Sqrt[7])*(4 + (1 - (3*I)*Sqrt[7])^(2/3))*A + 2*(9 + (5*I)*S
qrt[7] + I*(1 - (3*I)*Sqrt[7])^(2/3)*(3*I + Sqrt[7]))*(3*B + 4*C))*ArcTan[
(1 - (3*I)*Sqrt[7] + 4*(1 - (3*I)*Sqrt[7])^(1/3) - 6*(1 - (3*I)*Sqrt[7])^(
2/3)*(-2/3 + x))/Sqrt[6*(-31 - (3*I)*Sqrt[7] + 8*(1 - (3*I)*Sqrt[7])^(2/3)
 - 4*(1 - (3*I)*Sqrt[7])^(4/3))]])/((16 + 4*(1 - (3*I)*Sqrt[7])^(2/3) + (1
 - (3*I)*Sqrt[7])^(4/3))*Sqrt[6*(-31 - (3*I)*Sqrt[7] + 8*(1 - (3*I)*Sqrt[7
])^(2/3) - (1 - (3*I)*Sqrt[7])^(1/3)*(4 - (12*I)*Sqrt[7]))]) + ((9*(1 - (3
*I)*Sqrt[7])*A + (2 - (6*I)*Sqrt[7] - 4*(1 - (3*I)*Sqrt[7])^(2/3) - (1 - (
3*I)*Sqrt[7])^(4/3))*(3*B + 4*C))*Log[4 + (1 - (3*I)*Sqrt[7])^(2/3) + 3*(1
 - (3*I)*Sqrt[7])^(1/3)*(-2/3 + x)])/(9*(4 - (12*I)*Sqrt[7] + 16*(1 - (3*I
)*Sqrt[7])^(1/3) + (1 - (3*I)*Sqrt[7])^(5/3))) - ((9*(1 - (3*I)*Sqrt[7])^(
2/3)*A - (1 - (3*I)*Sqrt[7] + 4*(1 - (3*I)*Sqrt[7])^(1/3) - 2*(1 - (3*I)*S
qrt[7])^(2/3))*(3*B + 4*C))*Log[16 - 4*(1 - (3*I)*Sqrt[7])^(2/3) + (1 - (3
*I)*Sqrt[7])^(4/3) - 3*(1 - (3*I)*Sqrt[7] + 4*(1 - (3*I)*Sqrt[7])^(1/3))*(
-2/3 + x) + 9*(1 - (3*I)*Sqrt[7])^(2/3)*(-2/3 + x)^2])/(18*(16 + 4*(1 - (3
*I)*Sqrt[7])^(2/3) + (1 - (3*I)*Sqrt[7])^(4/3))) + (C*Log[2 - 6*x^2 + 3*x^
3])/9
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2485
```
Int[((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (d_.)*(x_)^3)^(p_), x_S
ymbol] :> With[{r = Rt[-9*a*d^2 + Sqrt[3]*d*Sqrt[4*b^3*d + 27*a^2*d^2], 3]}
, Simp[1/d^(2*p)   Int[(e + f*x)^m*Simp[18^(1/3)*b*(d/(3*r)) - r/18^(1/3) +
 d*x, x]^p*Simp[b*(d/3) + 12^(1/3)*b^2*(d^2/(3*r^2)) + r^2/(3*12^(1/3)) - d
*(2^(1/3)*b*(d/(3^(1/3)*r)) - r/18^(1/3))*x + d^2*x^2, x]^p, x], x]] /; Fre
eQ[{a, b, d, e, f, m}, x] && NeQ[4*b^3 + 27*a^2*d, 0] && ILtQ[p, 0]
```

rule 2490
```
Int[(P3_)^(p_.)*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] :> With[{a = Coeff[P3
, x, 0], b = Coeff[P3, x, 1], c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Su
bst[Int[((3*d*e - c*f)/(3*d) + f*x)^m*Simp[(2*c^3 - 9*b*c*d + 27*a*d^2)/(27
*d^2) - (c^2 - 3*b*d)*(x/(3*d)) + d*x^3, x]^p, x], x, x + c/(3*d)] /; NeQ[c
, 0]] /; FreeQ[{e, f, m, p}, x] && PolyQ[P3, x, 3]
```

rule 2525
```
Int[(Pm_)/(Qn_), x_Symbol] :> With[{m = Expon[Pm, x], n = Expon[Qn, x]}, Si
mp[Coeff[Pm, x, m]*(Log[Qn]/(n*Coeff[Qn, x, n])), x] + Simp[1/(n*Coeff[Qn,
x, n])   Int[ExpandToSum[n*Coeff[Qn, x, n]*Pm - Coeff[Pm, x, m]*D[Qn, x], x
]/Qn, x], x] /; EqQ[m, n - 1]] /; PolyQ[Pm, x] && PolyQ[Qn, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.17

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(3\_Z^{3}-6\_Z^{2}+2\right)}\frac{\left(C\_R^{2}+B\_R+A\right)\ln\left(x-\_R\right)}{3\_R^{2}-4\_R}\right)}{3}$	47
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(3\_Z^{3}-6\_Z^{2}+2\right)}\frac{\left(C\_R^{2}+B\_R+A\right)\ln\left(x-\_R\right)}{3\_R^{2}-4\_R}\right)}{3}$	47

input
```
int((C*x^2+B*x+A)/(3*x^3-6*x^2+2),x,method=_RETURNVERBOSE)
```

output
```
1/3*sum((C*_R^2+B*_R+A)/(3*_R^2-4*_R)*ln(x-_R),_R=RootOf(3*_Z^3-6*_Z^2+2))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.11 (sec) , antiderivative size = 7104, normalized size of antiderivative = 25.93

$$\int \frac{A+Bx+Cx^2}{2-6x^2+3x^3}\,dx = \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)/(3*x^3-6*x^2+2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [A] (verification not implemented)

Time = 8.89 (sec) , antiderivative size = 272, normalized size of antiderivative = 0.99

$$\int \frac{A+Bx+Cx^2}{2-6x^2+3x^3}\,dx$$
$$=\text{RootSum}\left(2268t^3-756t^2C+t\left(-108A^2-162AB-216AC-108B^2-288BC-108C^2\right)+9A^3+18\right.$$

input
```
integrate((C*x**2+B*x+A)/(3*x**3-6*x**2+2),x)
```

output
```
RootSum(2268*_t**3 - 756*_t**2*C + _t*(-108*A**2 - 162*A*B - 216*A*C - 108
*B**2 - 288*B*C - 108*C**2) + 9*A**3 + 18*A**2*B + 36*A**2*C + 18*A*B*C +
24*A*C**2 - 6*B**3 - 12*B**2*C + 4*C**3, Lambda(_t, _t*log(x + (-3024*_t**
2*A - 2268*_t**2*B - 3024*_t**2*C - 378*_t*A**2 - 504*_t*A*B + 504*_t*B*C
+ 672*_t*C**2 + 90*A**3 + 108*A**2*B + 186*A**2*C + 60*A*B**2 + 216*A*B*C
+ 144*A*C**2 + 36*B**3 + 144*B**2*C + 164*B*C**2 + 48*C**3)/(9*A**3 + 162*
A**2*B + 216*A**2*C + 216*A*B**2 + 576*A*B*C + 384*A*C**2 + 54*B**3 + 216*
B**2*C + 288*B*C**2 + 128*C**3))))
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{2-6x^2+3x^3}\,dx=\int \frac{Cx^2+Bx+A}{3\,x^3-6\,x^2+2}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(3*x^3-6*x^2+2),x, algorithm="maxima")
```

output
```
integrate((C*x^2 + B*x + A)/(3*x^3 - 6*x^2 + 2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A+Bx+Cx^2}{2-6x^2+3x^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((C*x^2+B*x+A)/(3*x^3-6*x^2+2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 12.48 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.20

$$\int \frac{A+Bx+Cx^2}{2-6x^2+3x^3}\,dx = \sum_{k=1}^{3} \ln\left(-\mathrm{root}\left(z^3-\frac{C\,z^2}{3}-\frac{z\,(108\,A^2+108\,B^2+108\,C^2+162\,A\,B+216\,A\,C+288\,B\,C)}{2268}+\frac{A\,B\,C}{126}-\frac{B^2\,C}{189}+\frac{2\,A\,C^2}{189}+\frac{A^2\,C}{63}+\frac{A^2\,B}{126}+\frac{A^3}{252}+\frac{C^3}{567}-\frac{B^3}{378},z,k\right)\left(18\,A+36\,C-x\,(27\,A+18\,B+72\,C)\right.\right.$$

$$+\mathrm{root}\left(z^3-\frac{C\,z^2}{3}-\frac{z\,(108\,A^2+108\,B^2+108\,C^2+162\,A\,B+216\,A\,C+288\,B\,C)}{2268}+\frac{A\,B\,C}{126}-\frac{B^2\,C}{189}+\frac{2}{\ldots}\right.$$

$$\left.\left.+2\,C^2+3\,A\,B+6\,A\,C+x\,\left(3\,B^2+6\,C\,B-3\,A\,C\right)\right)\mathrm{root}\left(z^3-\frac{C\,z^2}{3}-\frac{z\,(108\,A^2+108\,B^2+108\,C^2+162\,A\,B+216\,A\,C+288\,B\,C)}{2268}+\frac{A\,B\,C}{126}-\frac{B^2\,C}{189}+\frac{2\,A\,C^2}{189}+\frac{A^2\,C}{63}+\frac{A^2\,B}{126}+\frac{A^3}{252}+\frac{C^3}{567}-\frac{B^3}{378},z,k\right)$$

input
```
int((A + B*x + C*x^2)/(3*x^3 - 6*x^2 + 2),x)
```

output
```
symsum(log(2*C^2 - root(z^3 - (C*z^2)/3 - (z*(108*A^2 + 108*B^2 + 108*C^2
+ 162*A*B + 216*A*C + 288*B*C))/2268 + (A*B*C)/126 - (B^2*C)/189 + (2*A*C^
2)/189 + (A^2*C)/63 + (A^2*B)/126 + A^3/252 + C^3/567 - B^3/378, z, k)*(18
*A + 36*C - x*(27*A + 18*B + 72*C) + root(z^3 - (C*z^2)/3 - (z*(108*A^2 +
108*B^2 + 108*C^2 + 162*A*B + 216*A*C + 288*B*C))/2268 + (A*B*C)/126 - (B^
2*C)/189 + (2*A*C^2)/189 + (A^2*C)/63 + (A^2*B)/126 + A^3/252 + C^3/567 -
B^3/378, z, k)*(216*x - 162)) + 3*A*B + 6*A*C + x*(3*B^2 - 3*A*C + 6*B*C))
*root(z^3 - (C*z^2)/3 - (z*(108*A^2 + 108*B^2 + 108*C^2 + 162*A*B + 216*A*
C + 288*B*C))/2268 + (A*B*C)/126 - (B^2*C)/189 + (2*A*C^2)/189 + (A^2*C)/6
3 + (A^2*B)/126 + A^3/252 + C^3/567 - B^3/378, z, k), k, 1, 3)
```

Reduce [F]

$$\int \frac{A+Bx+Cx^2}{2-6x^2+3x^3}\,dx = \left(\int \frac{x}{3x^3-6x^2+2}dx\right) b + \frac{4\left(\int \frac{x}{3x^3-6x^2+2}dx\right) c}{3} + \left(\int \frac{1}{3x^3-6x^2+2}dx\right) a + \frac{\log(3x^3-6x^2+2)\, c}{9}$$

input
```
int((C*x^2+B*x+A)/(3*x^3-6*x^2+2),x)
```

output
```
(9*int(x/(3*x**3 - 6*x**2 + 2),x)*b + 12*int(x/(3*x**3 - 6*x**2 + 2),x)*c
+ 9*int(1/(3*x**3 - 6*x**2 + 2),x)*a + log(3*x**3 - 6*x**2 + 2)*c)/9
```

3.53 $\int \frac{A+Bx+Cx^2}{(2-6x^2+3x^3)^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 724

$$\int \frac{A+Bx+Cx^2}{(2-6x^2+3x^3)^2}\,dx = \text{Too large to display}$$

output
```
-1/9*C/(3*x^3-6*x^2+2)-1/256*(9*A+2*(3*B+4*C)*(1+2*cos(1/6*Pi+1/3*arcsin(1
/8))))*sec(1/3*arcsin(1/8))^2/(3*x-2-4*cos(1/6*Pi+1/3*arcsin(1/8)))/(cos(1
/6*Pi+1/3*arcsin(1/8))-sin(1/3*arcsin(1/8)))^2-1/384*ln(3*x-2+4*sin(1/3*Pi
+1/3*arcsin(1/8)))*sec(1/3*arcsin(1/8))^3*(6*B*(1-sin(1/3*arcsin(1/8)))*(2
-3^(1/2)*cos(1/3*arcsin(1/8))+sin(1/3*arcsin(1/8)))+8*C*(1-sin(1/3*arcsin(
1/8)))*(2-3^(1/2)*cos(1/3*arcsin(1/8))+sin(1/3*arcsin(1/8)))-9*A*(3^(1/2)*
cos(1/3*arcsin(1/8))+sin(1/3*arcsin(1/8))))/(cos(1/3*arcsin(1/8))+3^(1/2)*
sin(1/3*arcsin(1/8)))^3-1/384*ln(3*x-2-4*cos(1/6*Pi+1/3*arcsin(1/8)))*sec(
1/3*arcsin(1/8))^3*(9*A*(3^(1/2)*cos(1/3*arcsin(1/8))-sin(1/3*arcsin(1/8))
)+6*B*(1-sin(1/3*arcsin(1/8)))*(2+3^(1/2)*cos(1/3*arcsin(1/8))+sin(1/3*arc
sin(1/8)))+8*C*(1-sin(1/3*arcsin(1/8)))*(2+3^(1/2)*cos(1/3*arcsin(1/8))+si
n(1/3*arcsin(1/8))))/(cos(1/3*arcsin(1/8))-3^(1/2)*sin(1/3*arcsin(1/8)))^3
-1/48*(9*A+2*(3*B+4*C)*(1+2*sin(1/3*arcsin(1/8))))/(1-2*cos(2/3*arcsin(1/8
)))^2/(3*x-2-4*sin(1/3*arcsin(1/8)))-1/48*ln(3*x-2-4*sin(1/3*arcsin(1/8)))
*(3*B*(1+2*sin(1/3*arcsin(1/8)))^2+18*A*sin(1/3*arcsin(1/8))+4*C*(1+2*sin(
1/3*arcsin(1/8)))^2)/(1-2*cos(2/3*arcsin(1/8)))^3-1/256*sec(1/3*arcsin(1/8
))^2*(9*A+2*(3*B+4*C)*(1-2*sin(1/3*Pi+1/3*arcsin(1/8))))/(3*x-2+4*sin(1/3*
Pi+1/3*arcsin(1/8)))/(sin(1/3*arcsin(1/8))+sin(1/3*Pi+1/3*arcsin(1/8)))^2
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.06 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.22

$$\int \frac{A+Bx+Cx^2}{(2-6x^2+3x^3)^2}\,dx = \frac{6A+8B+6C+15Ax+6Bx+8Cx-12Ax^2-9Bx^2-12Cx^2}{42(2-6x^2+3x^3)} - \frac{1}{126}\text{RootSum}\Big[2-6\#1^2 +3\#1^3\&, \frac{-6A\log(x-\#1)+6B\log(x-\#1)+8C\log(x-\#1)+12A\log(x-\#1)\#1+9B\log(x-$$

$$-4\#1+3\#1^2$$

input
```
Integrate[(A + B*x + C*x^2)/(2 - 6*x^2 + 3*x^3)^2,x]
```

output
```
(6*A + 8*B + 6*C + 15*A*x + 6*B*x + 8*C*x - 12*A*x^2 - 9*B*x^2 - 12*C*x^2)
/(42*(2 - 6*x^2 + 3*x^3)) - RootSum[2 - 6*#1^2 + 3*#1^3 & , (-6*A*Log[x -
#1] + 6*B*Log[x - #1] + 8*C*Log[x - #1] + 12*A*Log[x - #1]*#1 + 9*B*Log[x
- #1]*#1 + 12*C*Log[x - #1]*#1)/(-4*#1 + 3*#1^2) & ]/126
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 11.68 (sec) , antiderivative size = 1329, normalized size of antiderivative = 1.84, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {2526, 27, 2490, 2485, 27, 1235, 27, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{(3x^3-6x^2+2)^2}\,dx$$

$$\Big\downarrow \text{2526}$$

$$\frac{1}{9}\int \frac{3(3A+(3B+4C)x)}{(3x^3-6x^2+2)^2}dx-\frac{C}{9(3x^3-6x^2+2)}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{3}\int \frac{3A+(3B+4C)x}{(3x^3-6x^2+2)^2}dx-\frac{C}{9(3x^3-6x^2+2)}$$

$$\Big\downarrow \; 2490$$

$$\frac{1}{3}\int \frac{\frac{1}{9}(27A+6(3B+4C))+(3B+4C)\left(x-\frac{2}{3}\right)}{\left(3\left(x-\frac{2}{3}\right)^3-4\left(x-\frac{2}{3}\right)+\frac{2}{9}\right)^2}d\left(x-\frac{2}{3}\right)-\frac{C}{9(3x^3-6x^2+2)}$$

$$\Big\downarrow \; 2485$$

$$-\frac{C}{9(3x^3-6x^2+2)}+$$

$$27\int \frac{9A+6B+8C+3(3B+4C)\left(x-\frac{2}{3}\right)}{3\left(3\left(x-\frac{2}{3}\right)+\frac{4+\left(1-3i\sqrt{7}\right)^{2/3}}{\sqrt[3]{1-3i\sqrt{7}}}\right)^2\left(-9\left(x-\frac{2}{3}\right)^2+\frac{3\left(4+\left(1-3i\sqrt{7}\right)^{2/3}\right)\left(x-\frac{2}{3}\right)}{\sqrt[3]{1-3i\sqrt{7}}}-\left(1-3i\sqrt{7}\right)^{2/3}-\frac{16}{\left(1-3i\sqrt{7}\right)^{2/3}}+\right.}$$

$$\Big\downarrow \; 27$$

$$-\frac{C}{9(3x^3-6x^2+2)}+$$

$$9\int \frac{9A+6B+8C+3(3B+4C)\left(x-\frac{2}{3}\right)}{\left(3\left(x-\frac{2}{3}\right)+\frac{4+\left(1-3i\sqrt{7}\right)^{2/3}}{\sqrt[3]{1-3i\sqrt{7}}}\right)^2\left(-9\left(x-\frac{2}{3}\right)^2+\frac{3\left(4+\left(1-3i\sqrt{7}\right)^{2/3}\right)\left(x-\frac{2}{3}\right)}{\sqrt[3]{1-3i\sqrt{7}}}-\left(1-3i\sqrt{7}\right)^{2/3}-\frac{16}{\left(1-3i\sqrt{7}\right)^{2/3}}+4\right.}$$

$$\Big\downarrow \; 1235$$

$$-\frac{C}{9\left(3x^3-6x^2+2\right)}+$$

$$9\left(\frac{i\sqrt[3]{1-3i\sqrt{7}}\int\frac{1458\left(\frac{3\left(16i\sqrt[3]{1-3i\sqrt{7}}-4\left(i+3\sqrt{7}\right)+\left(1-3i\sqrt{7}\right)^{2/3}\left(i+3\sqrt{7}\right)\right)A-\left(3i+9\sqrt{7}-4\sqrt[3]{1-3i\sqrt{7}}\left(3i+\sqrt{7}\right)-2\left(1-3i\sqrt{7}\right)^{2/3}\left(3i+\right.\right.}{i+3\sqrt{7}}\right.}{\left(3\left(x-\frac{2}{3}\right)+\frac{4+\left(1-3i\sqrt{7}\right)^{2/3}}{\sqrt[3]{1-3i\sqrt{7}}}\right)^2\left(-9\left(x-\frac{2}{3}\right)^2+\frac{3\left(4+\left(1-3i\sqrt{7}\right)^{2/3}\right.}{\sqrt[3]{1-3i\right.}}\right.}}{4374\sqrt{7}\left(4-\left(1-3i\sqrt{7}\right)^{2}\right.}\right.$$

$$\Big\downarrow \; 27$$

$$-\frac{C}{9\left(3x^3-6x^2+2\right)}+$$

$$9\left(\frac{i\sqrt[3]{1-3i\sqrt{7}}\int\frac{\frac{3\left(16i\sqrt[3]{1-3i\sqrt{7}}-4\left(i+3\sqrt{7}\right)+\left(1-3i\sqrt{7}\right)^{2/3}\left(i+3\sqrt{7}\right)\right)A-\left(3i+9\sqrt{7}-4\sqrt[3]{1-3i\sqrt{7}}\left(3i+\sqrt{7}\right)-2\left(1-3i\sqrt{7}\right)^{2/3}\left(3i+\sqrt{7}\right)\right)(3}{i+3\sqrt{7}}}{\left(3\left(x-\frac{2}{3}\right)+\frac{4+\left(1-3i\sqrt{7}\right)^{2/3}}{\sqrt[3]{1-3i\sqrt{7}}}\right)^2\left(-9\left(x-\frac{2}{3}\right)^2+\frac{3\left(4+\left(1-3i\sqrt{7}\right)^{2/3}\right)\left(x-\right.}{\sqrt[3]{1-3i\sqrt{7}}}\right.}}{3\sqrt{7}\left(4-\left(1-3i\sqrt{7}\right)^{2/3}\right)}\right.$$

$$\Big\downarrow \; 1200$$

$$9\left(\frac{i\sqrt[3]{1-3i\sqrt{7}}\int\left(\frac{6\sqrt[3]{1-3i\sqrt{7}}\left(31+3i\sqrt{7}\right)(4A+3B+4C)}{\left(i+3\sqrt{7}\right)\left(16i+4i\left(1-3i\sqrt{7}\right)^{2/3}+\sqrt[3]{1-3i\sqrt{7}}\left(i+3\sqrt{7}\right)\right)\left(3\sqrt[3]{1-3i\sqrt{7}}(x-\frac{2}{3})+\left(1-3i\sqrt{7}\right)^{2/3}+4\right)^2}+\frac{2\left(-\right.}{}\right.}{}\right.$$

$$\frac{C}{9\left(3x^3-6x^2+2\right)}$$

$\downarrow$ 2009

$$9\left(\frac{i\sqrt[3]{1-3i\sqrt{7}}\left(-\frac{2\left(31+3i\sqrt{7}\right)(4A+3B+4C)}{\left(i+3\sqrt{7}\right)\left(16i+4i\left(1-3i\sqrt{7}\right)^{2/3}+\sqrt[3]{1-3i\sqrt{7}}\left(i+3\sqrt{7}\right)\right)\left(3\sqrt[3]{1-3i\sqrt{7}}(x-\frac{2}{3})+\left(1-3i\sqrt{7}\right)^{2/3}+4\right)}+\frac{4\left(3\left(\right.\right.}{}\right.}{}\right.$$

$$\frac{C}{9\left(3x^3-6x^2+2\right)}$$

input
```
Int[(A + B*x + C*x^2)/(2 - 6*x^2 + 3*x^3)^2,x]
```

output
```
-1/9*C/(2 - 6*x^2 + 3*x^3) + 9*(((-1/6*I)*(1 - (3*I)*Sqrt[7])^(1/3)*(2*(4*
A + 3*B + 4*C) - ((3*(1 - (3*I)*Sqrt[7] + 4*(1 - (3*I)*Sqrt[7])^(1/3))*A +
 (3 - I*Sqrt[7])*(2 + (1 - (3*I)*Sqrt[7])^(1/3))*(3*B + 4*C))*(-2/3 + x))/
(1 - (3*I)*Sqrt[7])^(2/3)))/(Sqrt[7]*(4 - (1 - (3*I)*Sqrt[7])^(2/3))*((4 +
 (1 - (3*I)*Sqrt[7])^(2/3))/(1 - (3*I)*Sqrt[7])^(1/3) + 3*(-2/3 + x))*(4 -
 16/(1 - (3*I)*Sqrt[7])^(2/3) - (1 - (3*I)*Sqrt[7])^(2/3) + (3*(4 + (1 - (
3*I)*Sqrt[7])^(2/3))*(-2/3 + x))/(1 - (3*I)*Sqrt[7])^(1/3) - 9*(-2/3 + x)^
2)) + ((I/3)*(1 - (3*I)*Sqrt[7])^(1/3)*((-2*(31 + (3*I)*Sqrt[7])*(4*A + 3*
B + 4*C))/((I + 3*Sqrt[7])*(16*I + (4*I)*(1 - (3*I)*Sqrt[7])^(2/3) + (1 -
(3*I)*Sqrt[7])^(1/3)*(I + 3*Sqrt[7]))*(4 + (1 - (3*I)*Sqrt[7])^(2/3) + 3*(
1 - (3*I)*Sqrt[7])^(1/3)*(-2/3 + x))) + (4*(3*(536 + (184*I)*Sqrt[7] - (18
1 + I*Sqrt[7])*(1 - (3*I)*Sqrt[7])^(2/3))*A + (73 + (453*I)*Sqrt[7] - (1 -
 (3*I)*Sqrt[7])^(2/3)*(218 - (78*I)*Sqrt[7]))*(3*B + 4*C))*ArcTanh[(I + 3*
Sqrt[7] + (4*I)*(1 - (3*I)*Sqrt[7])^(1/3) - (6*I)*(1 - (3*I)*Sqrt[7])^(2/3
)*(-2/3 + x))/Sqrt[6*(-31 - (3*I)*Sqrt[7] + 8*(1 - (3*I)*Sqrt[7])^(2/3) -
(1 - (3*I)*Sqrt[7])^(1/3)*(4 - (12*I)*Sqrt[7]))]])/((1 - (3*I)*Sqrt[7])^(1
/3)*(I + 3*Sqrt[7])*(16 + 4*(1 - (3*I)*Sqrt[7])^(2/3) + (1 - (3*I)*Sqrt[7]
)^(4/3))^2*Sqrt[(3*(-31 - (3*I)*Sqrt[7] + 8*(1 - (3*I)*Sqrt[7])^(2/3) - (1
 - (3*I)*Sqrt[7])^(1/3)*(4 - (12*I)*Sqrt[7])))/2]) - (2*((124 + (12*I)*Sqr
t[7] - 8*(1 - (3*I)*Sqrt[7])^(4/3) + (1 - (3*I)*Sqrt[7])^(5/3))*A + (31...
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1235
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^m
*(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*
m + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) -
f*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
)
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2485
```
Int[((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (d_.)*(x_)^3)^(p_), x_S
ymbol] :> With[{r = Rt[-9*a*d^2 + Sqrt[3]*d*Sqrt[4*b^3*d + 27*a^2*d^2], 3]}
, Simp[1/d^(2*p)   Int[(e + f*x)^m*Simp[18^(1/3)*b*(d/(3*r)) - r/18^(1/3) +
 d*x, x]^p*Simp[b*(d/3) + 12^(1/3)*b^2*(d^2/(3*r^2)) + r^2/(3*12^(1/3)) - d
*(2^(1/3)*b*(d/(3^(1/3)*r)) - r/18^(1/3))*x + d^2*x^2, x]^p, x], x]] /; Fre
eQ[{a, b, d, e, f, m}, x] && NeQ[4*b^3 + 27*a^2*d, 0] && ILtQ[p, 0]
```

rule 2490
```
Int[(P3_)^(p_.)*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] :> With[{a = Coeff[P3
, x, 0], b = Coeff[P3, x, 1], c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Su
bst[Int[((3*d*e - c*f)/(3*d) + f*x)^m*Simp[(2*c^3 - 9*b*c*d + 27*a*d^2)/(27
*d^2) - (c^2 - 3*b*d)*(x/(3*d)) + d*x^3, x]^p, x], x, x + c/(3*d)] /; NeQ[c
, 0]] /; FreeQ[{e, f, m, p}, x] && PolyQ[P3, x, 3]
```

rule 2526
```
Int[(Pm_)*(Qn_)^(p_), x_Symbol] :> With[{m = Expon[Pm, x], n = Expon[Qn, x]
}, Simp[Coeff[Pm, x, m]*(Qn^(p + 1)/(n*(p + 1)*Coeff[Qn, x, n])), x] + Simp
[1/(n*Coeff[Qn, x, n])   Int[ExpandToSum[n*Coeff[Qn, x, n]*Pm - Coeff[Pm, x
, m]*D[Qn, x], x]*Qn^p, x], x] /; EqQ[m, n - 1]] /; FreeQ[p, x] && PolyQ[Pm
, x] && PolyQ[Qn, x] && NeQ[p, -1]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.15

method	result
default	$\frac{\left(-\frac{2C}{21}-\frac{2A}{21}-\frac{B}{14}\right)x^2+\left(\frac{4C}{63}+\frac{5A}{42}+\frac{B}{21}\right)x+\frac{C}{21}+\frac{A}{21}+\frac{4B}{63}}{x^3-2x^2+\frac{2}{3}}+\frac{\left(\sum_{\_R=\text{RootOf}\left(3\_Z^3-6\_Z^2+2\right)}\frac{\left(-12A\_R-9B\_R-12C\_R+6A-6B-\right.}{3\_R^2-4\_R}\right.}{126}$
risch	$\frac{\left(-\frac{2C}{21}-\frac{2A}{21}-\frac{B}{14}\right)x^2+\left(\frac{4C}{63}+\frac{5A}{42}+\frac{B}{21}\right)x+\frac{C}{21}+\frac{A}{21}+\frac{4B}{63}}{x^3-2x^2+\frac{2}{3}}+\frac{\left(\sum_{\_R=\text{RootOf}\left(3\_Z^3-6\_Z^2+2\right)}\frac{\left(3(-4A-3B-4C)\_R-8C+6A-6B\right)\ln\left(x-\right.}{3\_R^2-4\_R}\right.}{126}$

input
```
int((C*x^2+B*x+A)/(3*x^3-6*x^2+2)^2,x,method=_RETURNVERBOSE)
```

output
```
((-2/21*C-2/21*A-1/14*B)*x^2+(4/63*C+5/42*A+1/21*B)*x+1/21*C+1/21*A+4/63*B
)/(x^3-2*x^2+2/3)+1/126*sum((-12*A*_R-9*B*_R-12*C*_R+6*A-6*B-8*C)/(3*_R^2-
4*_R)*ln(x-_R),_R=RootOf(3*_Z^3-6*_Z^2+2))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.06 (sec) , antiderivative size = 5266, normalized size of antiderivative = 7.27

$$\int \frac{A+Bx+Cx^2}{(2-6x^2+3x^3)^2}\,dx = \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)/(3*x^3-6*x^2+2)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [A] (verification not implemented)

Time = 8.79 (sec) , antiderivative size = 342, normalized size of antiderivative = 0.47

$$\int \frac{A+Bx+Cx^2}{\left(2-6x^2+3x^3\right)^2}\,dx$$
$$=\text{RootSum}\left(1333584t^3+t\left(-2592A^2-6156AB-8208AC-7128B^2-19008BC-12672C^2\right)+36A^3\right.$$
$$+\frac{6A+8B+6C+x^2(-12A-9B-12C)+x(15A+6B+8C)}{126x^3-252x^2+84}$$

input
```
integrate((C*x**2+B*x+A)/(3*x**3-6*x**2+2)**2,x)
```

output
```
RootSum(1333584*_t**3 + _t*(-2592*A**2 - 6156*A*B - 8208*A*C - 7128*B**2 -
 19008*B*C - 12672*C**2) + 36*A**3 + 216*A**2*B + 288*A**2*C + 216*A*B**2
+ 576*A*B*C + 384*A*C**2 - 27*B**3 - 108*B**2*C - 144*B*C**2 - 64*C**3, La
mbda(_t, _t*log(x + (889056*_t**2*A + 3778488*_t**2*B + 5037984*_t**2*C +
52920*_t*A**2 + 42336*_t*A*B + 56448*_t*A*C - 95256*_t*B**2 - 254016*_t*B*
C - 169344*_t*C**2 - 2088*A**3 - 6336*A**2*B - 8448*A**2*C - 3996*A*B**2 -
 10656*A*B*C - 7104*A*C**2 - 6102*B**3 - 24408*B**2*C - 32544*B*C**2 - 144
64*C**3)/(1404*A**3 - 1944*A**2*B - 2592*A**2*C - 16200*A*B**2 - 43200*A*B
*C - 28800*A*C**2 - 11043*B**3 - 44172*B**2*C - 58896*B*C**2 - 26176*C**3)
))) + (6*A + 8*B + 6*C + x**2*(-12*A - 9*B - 12*C) + x*(15*A + 6*B + 8*C))
/(126*x**3 - 252*x**2 + 84)
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{\left(2-6x^2+3x^3\right)^2}\,dx=\int \frac{Cx^2+Bx+A}{\left(3\,x^3-6\,x^2+2\right)^2}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(3*x^3-6*x^2+2)^2,x, algorithm="maxima")
```

output
```
-1/42*(3*(4*A + 3*B + 4*C)*x^2 - (15*A + 6*B + 8*C)*x - 6*A - 8*B - 6*C)/(
3*x^3 - 6*x^2 + 2) - 1/42*integrate((3*(4*A + 3*B + 4*C)*x - 6*A + 6*B + 8
*C)/(3*x^3 - 6*x^2 + 2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A+Bx+Cx^2}{\left(2-6x^2+3x^3\right)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((C*x^2+B*x+A)/(3*x^3-6*x^2+2)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 12.58 (sec) , antiderivative size = 996, normalized size of antiderivative = 1.38

$$\int \frac{A+Bx+Cx^2}{\left(2-6x^2+3x^3\right)^2}\,dx = \text{Too large to display}$$

input
```
int((A + B*x + C*x^2)/(3*x^3 - 6*x^2 + 2)^2,x)
```

output
```
(A/7 + (4*B)/21 + C/7 + x*((5*A)/14 + B/7 + (4*C)/21) - x^2*((2*A)/7 + (3*
B)/14 + (2*C)/7))/(3*x^3 - 6*x^2 + 2) + symsum(log((12*A^2*x)/49 + (27*B^2
*x)/196 + (12*C^2*x)/49 - 216*root(z^3 - z*((2*A^2)/1029 + (11*B^2)/2058 +
 (88*C^2)/9261 + (19*A*B)/4116 + (19*A*C)/3087 + (44*B*C)/3087) + (4*A*B*C
)/9261 - (B^2*C)/12348 - (B*C^2)/9261 + (8*A*C^2)/27783 + (2*A^2*C)/9261 +
 (A^2*B)/6174 + (A*B^2)/6174 + A^3/37044 - (4*C^3)/83349 - B^3/49392, z, k
)^2*x - (6*A^2)/49 + (9*B^2)/98 + (8*C^2)/49 + 162*root(z^3 - z*((2*A^2)/1
029 + (11*B^2)/2058 + (88*C^2)/9261 + (19*A*B)/4116 + (19*A*C)/3087 + (44*
B*C)/3087) + (4*A*B*C)/9261 - (B^2*C)/12348 - (B*C^2)/9261 + (8*A*C^2)/277
83 + (2*A^2*C)/9261 + (A^2*B)/6174 + (A*B^2)/6174 + A^3/37044 - (4*C^3)/83
349 - B^3/49392, z, k)^2 + (3*A*B)/98 + (2*A*C)/49 + (12*B*C)/49 - (18*A*r
oot(z^3 - z*((2*A^2)/1029 + (11*B^2)/2058 + (88*C^2)/9261 + (19*A*B)/4116
+ (19*A*C)/3087 + (44*B*C)/3087) + (4*A*B*C)/9261 - (B^2*C)/12348 - (B*C^2
)/9261 + (8*A*C^2)/27783 + (2*A^2*C)/9261 + (A^2*B)/6174 + (A*B^2)/6174 +
A^3/37044 - (4*C^3)/83349 - B^3/49392, z, k))/7 + (18*B*root(z^3 - z*((2*A
^2)/1029 + (11*B^2)/2058 + (88*C^2)/9261 + (19*A*B)/4116 + (19*A*C)/3087 +
 (44*B*C)/3087) + (4*A*B*C)/9261 - (B^2*C)/12348 - (B*C^2)/9261 + (8*A*C^2
)/27783 + (2*A^2*C)/9261 + (A^2*B)/6174 + (A*B^2)/6174 + A^3/37044 - (4*C^
3)/83349 - B^3/49392, z, k))/7 + (24*C*root(z^3 - z*((2*A^2)/1029 + (11*B^
2)/2058 + (88*C^2)/9261 + (19*A*B)/4116 + (19*A*C)/3087 + (44*B*C)/3087...
```

Reduce [F]

$$\int \frac{A+Bx+Cx^2}{(2-6x^2+3x^3)^2}\,dx$$
$$= \frac{162\left(\int \frac{x^3}{9x^6-36x^5+36x^4+12x^3-24x^2+4}dx\right) b\,x^3 - 324\left(\int \frac{x^3}{9x^6-36x^5+36x^4+12x^3-24x^2+4}dx\right) b\,x^2 + 108\left(\int \frac{}{9x^6-36x^5+36x}\right.}{}$$

input
```
int((C*x^2+B*x+A)/(3*x^3-6*x^2+2)^2,x)
```

output

```
(162*int(x**3/(9*x**6 - 36*x**5 + 36*x**4 + 12*x**3 - 24*x**2 + 4),x)*b*x*
*3 - 324*int(x**3/(9*x**6 - 36*x**5 + 36*x**4 + 12*x**3 - 24*x**2 + 4),x)*
b*x**2 + 108*int(x**3/(9*x**6 - 36*x**5 + 36*x**4 + 12*x**3 - 24*x**2 + 4)
,x)*b + 216*int(x**3/(9*x**6 - 36*x**5 + 36*x**4 + 12*x**3 - 24*x**2 + 4),
x)*c*x**3 - 432*int(x**3/(9*x**6 - 36*x**5 + 36*x**4 + 12*x**3 - 24*x**2 +
 4),x)*c*x**2 + 144*int(x**3/(9*x**6 - 36*x**5 + 36*x**4 + 12*x**3 - 24*x*
*2 + 4),x)*c + 216*int(1/(9*x**6 - 36*x**5 + 36*x**4 + 12*x**3 - 24*x**2 +
 4),x)*a*x**3 - 432*int(1/(9*x**6 - 36*x**5 + 36*x**4 + 12*x**3 - 24*x**2
+ 4),x)*a*x**2 + 144*int(1/(9*x**6 - 36*x**5 + 36*x**4 + 12*x**3 - 24*x**2
 + 4),x)*a - 54*int(1/(9*x**6 - 36*x**5 + 36*x**4 + 12*x**3 - 24*x**2 + 4)
,x)*b*x**3 + 108*int(1/(9*x**6 - 36*x**5 + 36*x**4 + 12*x**3 - 24*x**2 + 4
),x)*b*x**2 - 36*int(1/(9*x**6 - 36*x**5 + 36*x**4 + 12*x**3 - 24*x**2 + 4
),x)*b - 72*int(1/(9*x**6 - 36*x**5 + 36*x**4 + 12*x**3 - 24*x**2 + 4),x)*
c*x**3 + 144*int(1/(9*x**6 - 36*x**5 + 36*x**4 + 12*x**3 - 24*x**2 + 4),x)
*c*x**2 - 48*int(1/(9*x**6 - 36*x**5 + 36*x**4 + 12*x**3 - 24*x**2 + 4),x)
*c + 9*b*x + 6*b + 12*c*x**3 - 24*c*x**2 + 12*c*x + 8*c)/(72*(3*x**3 - 6*x
**2 + 2))
```

3.54 $\int (A+Bx+Cx^2)(2-6x^2+3x^3)^p\,dx$

Optimal result

Integrand size = 25, antiderivative size = 571

$$\int (A+Bx+Cx^2)(2-6x^2+3x^3)^p\,dx = \text{Too large to display}$$

output
```
C*(3*x^3-6*x^2+2)^(p+1)/(9*p+9)+3^(-5/2+3/2*p)*(3*B+4*C)*(3*x^3-6*x^2+2)^p
*AppellF1(p+1,-p,-1-p,2+p,-1/2*(3*x-2-4*cos(1/6*Pi+1/3*arcsin(1/8)))/(3^(1
/2)*cos(1/3*arcsin(1/8))-3*sin(1/3*arcsin(1/8))),-1/12*(3*x-2-4*cos(1/6*Pi
+1/3*arcsin(1/8)))*sec(1/3*arcsin(1/8))*3^(1/2))*(4*cos(1/3*arcsin(1/8)))^
(p+1)*(3*x-2-4*cos(1/6*Pi+1/3*arcsin(1/8)))*(2/3*3^(1/2)*cos(1/3*arcsin(1/
8))-2*sin(1/3*arcsin(1/8)))^p/(p+1)/((3*x-2-4*sin(1/3*arcsin(1/8)))^p)/((3
*x-2+4*sin(1/3*Pi+1/3*arcsin(1/8)))^p)+3^(-3+1/2*p)*(3*x^3-6*x^2+2)^p*Appe
llF1(p+1,-p,-p,2+p,-1/2*(3*x-2-4*cos(1/6*Pi+1/3*arcsin(1/8)))/(3^(1/2)*cos
(1/3*arcsin(1/8))-3*sin(1/3*arcsin(1/8))),-1/12*(3*x-2-4*cos(1/6*Pi+1/3*ar
csin(1/8)))*sec(1/3*arcsin(1/8))*3^(1/2))*(3*x-2-4*cos(1/6*Pi+1/3*arcsin(1
/8)))*(8*cos(1/3*arcsin(1/8))*(3^(1/2)*cos(1/3*arcsin(1/8))-3*sin(1/3*arcs
in(1/8))))^p*(9*A+2*(3*B+4*C)*(1-2*sin(1/3*Pi+1/3*arcsin(1/8))))/(p+1)/((3
*x-2-4*sin(1/3*arcsin(1/8)))^p)/((3*x-2+4*sin(1/3*Pi+1/3*arcsin(1/8)))^p)
```

Mathematica [F]

$$\int \left(A+Bx+Cx^2\right)\left(2-6x^2+3x^3\right)^p\,dx=\int \left(A+Bx+Cx^2\right)\left(2-6x^2+3x^3\right)^p\,dx$$

input
```
Integrate[(A + B*x + C*x^2)*(2 - 6*x^2 + 3*x^3)^p,x]
```

output
```
Integrate[(A + B*x + C*x^2)*(2 - 6*x^2 + 3*x^3)^p, x]
```

Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 8.79 (sec) , antiderivative size = 1729, normalized size of antiderivative = 3.03, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {2526, 27, 2490, 2486, 27, 1269, 1179, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(3x^3-6x^2+2\right)^p\left(A+Bx+Cx^2\right)dx$$

$$\Big\downarrow \text{2526}$$

$$\frac{1}{9}\int 3(3A+(3B+4C)x)\left(3x^3-6x^2+2\right)^pdx+\frac{C\left(3x^3-6x^2+2\right)^{p+1}}{9(p+1)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\int (3A+(3B+4C)x)\left(3x^3-6x^2+2\right)^pdx+\frac{C\left(3x^3-6x^2+2\right)^{p+1}}{9(p+1)}$$

$$\Big\downarrow \text{2490}$$

$$\frac{1}{3}\int \left(\frac{1}{9}(27A+6(3B+4C))+(3B+4C)\left(x-\frac{2}{3}\right)\right)\left(3\left(x-\frac{2}{3}\right)^3-4\left(x-\frac{2}{3}\right)+\frac{2}{9}\right)^pd\left(x-\frac{2}{3}\right)+\frac{C\left(3x^3-6x^2+2\right)^{p+1}}{9(p+1)}$$

$\downarrow$ 2486

$$\frac{C(3x^3-6x^2+2)^{p+1}}{9(p+1)}+$$
$$\frac{1}{3}\left(3\left(x-\frac{2}{3}\right)+\frac{4+(1-3i\sqrt{7})^{2/3}}{\sqrt[3]{1-3i\sqrt{7}}}\right)^{-p}\left(3\left(x-\frac{2}{3}\right)^3-4\left(x-\frac{2}{3}\right)+\frac{2}{9}\right)^p\left(9\left(x-\frac{2}{3}\right)^2-\frac{3\left(4+(1-3i\sqrt{7})^{2/}\right.}{\sqrt[3]{1-3i\sqrt{}}}\right.$$

$\downarrow$ 27

$$\frac{C(3x^3-6x^2+2)^{p+1}}{9(p+1)}+$$
$$\frac{1}{9}\left(3\left(x-\frac{2}{3}\right)+\frac{4+(1-3i\sqrt{7})^{2/3}}{\sqrt[3]{1-3i\sqrt{7}}}\right)^{-p}\left(3\left(x-\frac{2}{3}\right)^3-4\left(x-\frac{2}{3}\right)+\frac{2}{9}\right)^p\left(9\left(x-\frac{2}{3}\right)^2-\frac{3\left(4+(1-3i\sqrt{7})^{2/}\right.}{\sqrt[3]{1-3i\sqrt{}}}\right.$$

$\downarrow$ 1269

$$\frac{C(3x^3-6x^2+2)^{p+1}}{9(p+1)}+$$
$$\frac{1}{9}\left(3\left(x-\frac{2}{3}\right)+\frac{4+(1-3i\sqrt{7})^{2/3}}{\sqrt[3]{1-3i\sqrt{7}}}\right)^{-p}\left(3\left(x-\frac{2}{3}\right)^3-4\left(x-\frac{2}{3}\right)+\frac{2}{9}\right)^p\left(9\left(x-\frac{2}{3}\right)^2-\frac{3\left(4+(1-3i\sqrt{7})^{2/}\right.}{\sqrt[3]{1-3i\sqrt{}}}\right.$$

$\downarrow$ 1179

$$\frac{1}{9}\left(3\left(x-\frac{2}{3}\right)+\frac{4+(1-3i\sqrt{7})^{2/3}}{\sqrt[3]{1-3i\sqrt{7}}}\right)^{-p}\left(3\left(x-\frac{2}{3}\right)^3-4\left(x-\frac{2}{3}\right)+\frac{2}{9}\right)^p\left(\frac{1}{3}\left(9A+6B+8C-\frac{\left(4+(1-3i\right.}{\sqrt[3]{}}\right.\right.$$
$$\frac{C(3x^3-6x^2+2)^{p+1}}{9(p+1)}$$

$\downarrow$ 150

$$\frac{1}{9}\left(3\left(x-\frac{2}{3}\right)+\frac{4+\left(1-3i\sqrt{7}\right)^{2/3}}{\sqrt[3]{1-3i\sqrt{7}}}\right)^{-p}\left(3\left(x-\frac{2}{3}\right)^{3}-4\left(x-\frac{2}{3}\right)+\frac{2}{9}\right)^{p}\left(\frac{\left(9A+6B+8C-\frac{\left(4+\left(1-3i\sqrt{7}\right)^{2/3}\right.}{\sqrt[3]{1-3i}}\right.}{}\right.$$

$$\frac{C\left(3x^{3}-6x^{2}+2\right)^{p+1}}{9(p+1)}$$

input
```
Int[(A + B*x + C*x^2)*(2 - 6*x^2 + 3*x^3)^p,x]
```

output
```
(C*(2 - 6*x^2 + 3*x^3)^(1 + p))/(9*(1 + p)) + ((2/9 - 4*(-2/3 + x) + 3*(-2
/3 + x)^3)^p*(((9*A + 6*B + 8*C - ((4 + (1 - (3*I)*Sqrt[7])^(2/3))*(3*B +
4*C))/(1 - (3*I)*Sqrt[7])^(1/3))*((4 + (1 - (3*I)*Sqrt[7])^(2/3))/(1 - (3*
I)*Sqrt[7])^(1/3) + 3*(-2/3 + x))^(1 + p)*(-4 + 16/(1 - (3*I)*Sqrt[7])^(2/
3) + (1 - (3*I)*Sqrt[7])^(2/3) - (3*(4 + (1 - (3*I)*Sqrt[7])^(2/3))*(-2/3
+ x))/(1 - (3*I)*Sqrt[7])^(1/3) + 9*(-2/3 + x)^2)^p*AppellF1[1 + p, -p, -p
, 2 + p, (-2*(I + 3*Sqrt[7])*((4 + (1 - (3*I)*Sqrt[7])^(2/3))/(1 - (3*I)*S
qrt[7])^(1/3) + 3*(-2/3 + x)))/(Sqrt[-3*(1 - (3*I)*Sqrt[7])^(2/3)]*(I + 3*
Sqrt[7] - (4*I)*(1 - (3*I)*Sqrt[7])^(1/3)) - (3*I)*(1 - (3*I)*Sqrt[7])^(2/
3)*(4 + (1 - (3*I)*Sqrt[7])^(2/3))), (2*(I + 3*Sqrt[7])*((4 + (1 - (3*I)*S
qrt[7])^(2/3))/(1 - (3*I)*Sqrt[7])^(1/3) + 3*(-2/3 + x)))/(Sqrt[-3*(1 - (3
*I)*Sqrt[7])^(2/3)]*(I + 3*Sqrt[7] - (4*I)*(1 - (3*I)*Sqrt[7])^(1/3)) + (3
*I)*(1 - (3*I)*Sqrt[7])^(2/3)*(4 + (1 - (3*I)*Sqrt[7])^(2/3)))])/(3*(1 + p
)*(1 + (2*(I + 3*Sqrt[7])*((4 + (1 - (3*I)*Sqrt[7])^(2/3))/(1 - (3*I)*Sqrt
[7])^(1/3) + 3*(-2/3 + x)))/(Sqrt[-3*(1 - (3*I)*Sqrt[7])^(2/3)]*(I + 3*Sqr
t[7] - (4*I)*(1 - (3*I)*Sqrt[7])^(1/3)) - (3*I)*(1 - (3*I)*Sqrt[7])^(2/3)*
(4 + (1 - (3*I)*Sqrt[7])^(2/3))))^p*(1 - (2*(I + 3*Sqrt[7])*((4 + (1 - (3*
I)*Sqrt[7])^(2/3))/(1 - (3*I)*Sqrt[7])^(1/3) + 3*(-2/3 + x)))/(Sqrt[-3*(1
- (3*I)*Sqrt[7])^(2/3)]*(I + 3*Sqrt[7] - (4*I)*(1 - (3*I)*Sqrt[7])^(1/3))
+ (3*I)*(1 - (3*I)*Sqrt[7])^(2/3)*(4 + (1 - (3*I)*Sqrt[7])^(2/3))))^p) ...
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 1179
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(a + b*x + c*x^2)^p/(e*(1 - (
d + e*x)/(d - e*((b - q)/(2*c))))^p*(1 - (d + e*x)/(d - e*((b + q)/(2*c))))
^p)   Subst[Int[x^m*Simp[1 - x/(d - e*((b - q)/(2*c))), x]^p*Simp[1 - x/(d
- e*((b + q)/(2*c))), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, b, c, d, e, m
, p}, x]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2486
```
Int[((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (d_.)*(x_)^3)^(p_), x_S
ymbol] :> With[{r = Rt[-9*a*d^2 + Sqrt[3]*d*Sqrt[4*b^3*d + 27*a^2*d^2], 3]}
, Simp[(a + b*x + d*x^3)^p/(Simp[18^(1/3)*b*(d/(3*r)) - r/18^(1/3) + d*x, x
]^p*Simp[b*(d/3) + 12^(1/3)*b^2*(d^2/(3*r^2)) + r^2/(3*12^(1/3)) - d*(2^(1/
3)*b*(d/(3^(1/3)*r)) - r/18^(1/3))*x + d^2*x^2, x]^p)   Int[(e + f*x)^m*Sim
p[18^(1/3)*b*(d/(3*r)) - r/18^(1/3) + d*x, x]^p*Simp[b*(d/3) + 12^(1/3)*b^2
*(d^2/(3*r^2)) + r^2/(3*12^(1/3)) - d*(2^(1/3)*b*(d/(3^(1/3)*r)) - r/18^(1/
3))*x + d^2*x^2, x]^p, x], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && NeQ[4*
b^3 + 27*a^2*d, 0] &&  !IntegerQ[p]
```

rule 2490
```
Int[(P3_)^(p_.)*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] :> With[{a = Coeff[P3
, x, 0], b = Coeff[P3, x, 1], c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Su
bst[Int[((3*d*e - c*f)/(3*d) + f*x)^m*Simp[(2*c^3 - 9*b*c*d + 27*a*d^2)/(27
*d^2) - (c^2 - 3*b*d)*(x/(3*d)) + d*x^3, x]^p, x], x, x + c/(3*d)] /; NeQ[c
, 0]] /; FreeQ[{e, f, m, p}, x] && PolyQ[P3, x, 3]
```

rule 2526
```
Int[(Pm_)*(Qn_)^(p_), x_Symbol] :> With[{m = Expon[Pm, x], n = Expon[Qn, x]
}, Simp[Coeff[Pm, x, m]*(Qn^(p + 1)/(n*(p + 1)*Coeff[Qn, x, n])), x] + Simp
[1/(n*Coeff[Qn, x, n])   Int[ExpandToSum[n*Coeff[Qn, x, n]*Pm - Coeff[Pm, x
, m]*D[Qn, x], x]*Qn^p, x], x] /; EqQ[m, n - 1]] /; FreeQ[p, x] && PolyQ[Pm
, x] && PolyQ[Qn, x] && NeQ[p, -1]
```

Maple [F]

$$\int \left(C\,x^{2}+Bx+A\right)\left(3x^{3}-6x^{2}+2\right)^{p}dx$$

input
```
int((C*x^2+B*x+A)*(3*x^3-6*x^2+2)^p,x)
```

output
```
int((C*x^2+B*x+A)*(3*x^3-6*x^2+2)^p,x)
```

Fricas [F]

$$\int \left(A+Bx+Cx^2\right)\left(2-6x^2+3x^3\right)^p\,dx=\int \left(Cx^{2}+Bx+A\right)\left(3\,x^{3}-6\,x^{2}+2\right)^{p}dx$$

input
```
integrate((C*x^2+B*x+A)*(3*x^3-6*x^2+2)^p,x, algorithm="fricas")
```

output
```
integral((C*x^2 + B*x + A)*(3*x^3 - 6*x^2 + 2)^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int (A+Bx+Cx^2)\left(2-6x^2+3x^3\right)^p\,dx = \text{Timed out}$$

input
```
integrate((C*x**2+B*x+A)*(3*x**3-6*x**2+2)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (A+Bx+Cx^2)\left(2-6x^2+3x^3\right)^p\,dx = \int (Cx^2+Bx+A)\left(3\,x^3-6\,x^2+2\right)^p\,dx$$

input
```
integrate((C*x^2+B*x+A)*(3*x^3-6*x^2+2)^p,x, algorithm="maxima")
```

output
```
integrate((C*x^2 + B*x + A)*(3*x^3 - 6*x^2 + 2)^p, x)
```

Giac [F]

$$\int (A+Bx+Cx^2)\left(2-6x^2+3x^3\right)^p\,dx = \int (Cx^2+Bx+A)\left(3\,x^3-6\,x^2+2\right)^p\,dx$$

input
```
integrate((C*x^2+B*x+A)*(3*x^3-6*x^2+2)^p,x, algorithm="giac")
```

output
```
integrate((C*x^2 + B*x + A)*(3*x^3 - 6*x^2 + 2)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(A+Bx+Cx^2\right)\left(2-6x^2+3x^3\right)^p\,dx=\int \left(C\,x^2+B\,x+A\right)\,\left(3\,x^3-6\,x^2+2\right)^p\,dx$$

input
```
int((A + B*x + C*x^2)*(3*x^3 - 6*x^2 + 2)^p,x)
```

output
```
int((A + B*x + C*x^2)*(3*x^3 - 6*x^2 + 2)^p, x)
```

Reduce [F]

$$\int \left(A+Bx+Cx^2\right)\left(2-6x^2+3x^3\right)^p\,dx=\text{too large to display}$$

input
```
int((C*x^2+B*x+A)*(3*x^3-6*x^2+2)^p,x)
```

output

```
(27*(3*x**3 - 6*x**2 + 2)**p*a*p**2*x - 18*(3*x**3 - 6*x**2 + 2)**p*a*p**2
 + 45*(3*x**3 - 6*x**2 + 2)**p*a*p*x - 30*(3*x**3 - 6*x**2 + 2)**p*a*p + 1
8*(3*x**3 - 6*x**2 + 2)**p*a*x - 12*(3*x**3 - 6*x**2 + 2)**p*a + 27*(3*x**
3 - 6*x**2 + 2)**p*b*p**2*x**2 - 18*(3*x**3 - 6*x**2 + 2)**p*b*p**2*x - 24
*(3*x**3 - 6*x**2 + 2)**p*b*p**2 + 36*(3*x**3 - 6*x**2 + 2)**p*b*p*x**2 -
18*(3*x**3 - 6*x**2 + 2)**p*b*p*x - 36*(3*x**3 - 6*x**2 + 2)**p*b*p + 9*(3
*x**3 - 6*x**2 + 2)**p*b*x**2 - 12*(3*x**3 - 6*x**2 + 2)**p*b + 27*(3*x**3
 - 6*x**2 + 2)**p*c*p**2*x**3 - 18*(3*x**3 - 6*x**2 + 2)**p*c*p**2*x**2 -
24*(3*x**3 - 6*x**2 + 2)**p*c*p**2*x - 14*(3*x**3 - 6*x**2 + 2)**p*c*p**2
+ 27*(3*x**3 - 6*x**2 + 2)**p*c*p*x**3 - 6*(3*x**3 - 6*x**2 + 2)**p*c*p*x*
*2 - 24*(3*x**3 - 6*x**2 + 2)**p*c*p*x - 30*(3*x**3 - 6*x**2 + 2)**p*c*p +
 6*(3*x**3 - 6*x**2 + 2)**p*c*x**3 - 12*(3*x**3 - 6*x**2 + 2)**p*c + 1458*
int((3*x**3 - 6*x**2 + 2)**p/(27*p**2*x**3 - 54*p**2*x**2 + 18*p**2 + 27*p
*x**3 - 54*p*x**2 + 18*p + 6*x**3 - 12*x**2 + 4),x)*a*p**5 + 3888*int((3*x
**3 - 6*x**2 + 2)**p/(27*p**2*x**3 - 54*p**2*x**2 + 18*p**2 + 27*p*x**3 -
54*p*x**2 + 18*p + 6*x**3 - 12*x**2 + 4),x)*a*p**4 + 3726*int((3*x**3 - 6*
x**2 + 2)**p/(27*p**2*x**3 - 54*p**2*x**2 + 18*p**2 + 27*p*x**3 - 54*p*x**
2 + 18*p + 6*x**3 - 12*x**2 + 4),x)*a*p**3 + 1512*int((3*x**3 - 6*x**2 + 2
)**p/(27*p**2*x**3 - 54*p**2*x**2 + 18*p**2 + 27*p*x**3 - 54*p*x**2 + 18*p
 + 6*x**3 - 12*x**2 + 4),x)*a*p**2 + 216*int((3*x**3 - 6*x**2 + 2)**p/(...
```

3.55 $\int (A+Bx+Cx^2) (a^3+3a^2bx+3ab^2x^2+b^3x^3)^2 \, dx$

Optimal result

Integrand size = 40, antiderivative size = 69

$$\int (A+Bx+Cx^2) (a^3+3a^2bx+3ab^2x^2+b^3x^3)^2 \, dx = \frac{(Ab^2-a(bB-aC))(a+bx)^7}{7b^3}+\frac{(bB-2aC)(a+bx)^8}{8b^3}+\frac{C(a+bx)^9}{9b^3}$$

output
```
1/7*(A*b^2-a*(B*b-C*a))*(b*x+a)^7/b^3+1/8*(B*b-2*C*a)*(b*x+a)^8/b^3+1/9*C*
(b*x+a)^9/b^3
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 170 vs. 2(69) = 138.

Time = 0.10 (sec) , antiderivative size = 170, normalized size of antiderivative = 2.46

$$\int (A+Bx+Cx^2) (a^3+3a^2bx+3ab^2x^2+b^3x^3)^2 \, dx = \frac{1}{504}x\big(84a^6(6A+x(3B+2Cx))+252a^5bx(6A+x(4B+3Cx)) +126a^4b^2x^2(20A+3x(5B+4Cx))+168a^3b^3x^3(15A+2x(6B+5Cx)) +36a^2b^4x^4(42A+5x(7B+6Cx))+18ab^5x^5(28A+3x(8B+7Cx)) +b^6x^6(72A+7x(9B+8Cx))\big)$$

input
```
Integrate[(A + B*x + C*x^2)*(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^2,x]
```

output
```
(x*(84*a^6*(6*A + x*(3*B + 2*C*x)) + 252*a^5*b*x*(6*A + x*(4*B + 3*C*x)) +
 126*a^4*b^2*x^2*(20*A + 3*x*(5*B + 4*C*x)) + 168*a^3*b^3*x^3*(15*A + 2*x*
(6*B + 5*C*x)) + 36*a^2*b^4*x^4*(42*A + 5*x*(7*B + 6*C*x)) + 18*a*b^5*x^5*
(28*A + 3*x*(8*B + 7*C*x)) + b^6*x^6*(72*A + 7*x*(9*B + 8*C*x))))/504
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.075$, Rules used = {2006, 1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)^2 \left(A+Bx+Cx^2\right) \, dx$$

$$\Big\downarrow \text{2006}$$

$$\int (a+bx)^6 \left(A+Bx+Cx^2\right) dx$$

$$\Big\downarrow \text{1140}$$

$$\int \left(\frac{(a+bx)^6 \left(Ab^2-a(bB-aC)\right)}{b^2}+\frac{(a+bx)^7(bB-2aC)}{b^2}+\frac{C(a+bx)^8}{b^2}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{(a+bx)^7 \left(Ab^2-a(bB-aC)\right)}{7b^3}+\frac{(a+bx)^8(bB-2aC)}{8b^3}+\frac{C(a+bx)^9}{9b^3}$$

input
```
Int[(A + B*x + C*x^2)*(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^2,x]
```

output
```
((A*b^2 - a*(b*B - a*C))*(a + b*x)^7)/(7*b^3) + ((b*B - 2*a*C)*(a + b*x)^8
)/(8*b^3) + (C*(a + b*x)^9)/(9*b^3)
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2006
```
Int[(u_.)*(Px_), x_Symbol] :> With[{a = Rt[Coeff[Px, x, 0], Expon[Px, x]],
b = Rt[Coeff[Px, x, Expon[Px, x]], Expon[Px, x]]}, Int[u*(a + b*x)^Expon[Px
, x], x] /; EqQ[Px, (a + b*x)^Expon[Px, x]]] /; PolyQ[Px, x] && GtQ[Expon[P
x, x], 1] && NeQ[Coeff[Px, x, 0], 0] &&  !MatchQ[Px, (a_.)*(v_)^Expon[Px, x
] /; FreeQ[a, x] && LinearQ[v, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 197 vs. $2(63) = 126$.

Time = 0.17 (sec) , antiderivative size = 198, normalized size of antiderivative = 2.87

method	result
norman	$\frac{C\,b^{6}x^{9}}{9}+\left(\frac{1}{8}B\,b^{6}+\frac{3}{4}C\,b^{5}a\right)x^{8}+\left(\frac{1}{7}A\,b^{6}+\frac{6}{7}B\,b^{5}a+\frac{15}{7}b^{4}C\,a^{2}\right)x^{7}+\left(A\,b^{5}a+\frac{5}{2}B\,b^{4}a^{2}+\frac{10}{3}C\right.$
default	$\frac{C\,b^{6}x^{9}}{9}+\frac{\left(B\,b^{6}+6C\,b^{5}a\right)x^{8}}{8}+\frac{\left(A\,b^{6}+6B\,b^{5}a+15b^{4}C\,a^{2}\right)x^{7}}{7}+\frac{\left(6A\,b^{5}a+15B\,b^{4}a^{2}+20C\,b^{3}a^{3}\right)x^{6}}{6}+\frac{(15A\,a^{2}b^{4}+20B\,a}{5}$
gosper	$\frac{x\left(56C\,b^{6}x^{8}+63x^{7}B\,b^{6}+378x^{7}C\,b^{5}a+72x^{6}A\,b^{6}+432x^{6}B\,b^{5}a+1080x^{6}b^{4}C\,a^{2}+504x^{5}A\,b^{5}a+1260B\,a^{2}b^{4}x^{5}+1680x^{5}C\,b^{3}a^{3}+\right.}{}$
risch	$\frac{1}{9}C\,b^{6}x^{9}+\frac{1}{8}B\,b^{6}x^{8}+\frac{3}{4}x^{8}C\,b^{5}a+\frac{1}{7}x^{7}A\,b^{6}+\frac{6}{7}x^{7}B\,b^{5}a+\frac{15}{7}x^{7}b^{4}C\,a^{2}+x^{6}A\,b^{5}a+\frac{5}{2}B\,a^{2}b^{4}x$
parallelrisch	$\frac{1}{9}C\,b^{6}x^{9}+\frac{1}{8}B\,b^{6}x^{8}+\frac{3}{4}x^{8}C\,b^{5}a+\frac{1}{7}x^{7}A\,b^{6}+\frac{6}{7}x^{7}B\,b^{5}a+\frac{15}{7}x^{7}b^{4}C\,a^{2}+x^{6}A\,b^{5}a+\frac{5}{2}B\,a^{2}b^{4}x$
orering	$\frac{x\left(56C\,b^{6}x^{8}+63x^{7}B\,b^{6}+378x^{7}C\,b^{5}a+72x^{6}A\,b^{6}+432x^{6}B\,b^{5}a+1080x^{6}b^{4}C\,a^{2}+504x^{5}A\,b^{5}a+1260B\,a^{2}b^{4}x^{5}+1680x^{5}C\,b^{3}a^{3}+\right.}{}$

input
```
int((C*x^2+B*x+A)*(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^2,x,method=_RETURNVE
RBOSE)
```

output
```
1/9*C*b^6*x^9+(1/8*B*b^6+3/4*C*b^5*a)*x^8+(1/7*A*b^6+6/7*B*b^5*a+15/7*b^4*
C*a^2)*x^7+(A*b^5*a+5/2*B*b^4*a^2+10/3*C*b^3*a^3)*x^6+(3*A*a^2*b^4+4*B*a^3
*b^3+3*C*a^4*b^2)*x^5+(5*A*a^3*b^3+15/4*B*a^4*b^2+3/2*C*a^5*b)*x^4+(5*A*a^
4*b^2+2*B*a^5*b+1/3*C*a^6)*x^3+(3*A*a^5*b+1/2*B*a^6)*x^2+A*a^6*x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 200 vs. $2(63) = 126$.

Time = 0.07 (sec) , antiderivative size = 200, normalized size of antiderivative = 2.90

$$\begin{aligned}\int (A+Bx+Cx^2)\left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)^2\,dx &= \frac{1}{9}Cb^6x^9+\frac{1}{8}\left(6\,Cab^5+Bb^6\right)x^8+Aa^6x \\ &+\frac{1}{7}\left(15\,Ca^2b^4+6\,Bab^5+Ab^6\right)x^7+\frac{1}{6}\left(20\,Ca^3b^3+15\,Ba^2b^4+6\,Aab^5\right)x^6 \\ &+\left(3\,Ca^4b^2+4\,Ba^3b^3+3\,Aa^2b^4\right)x^5+\frac{1}{4}\left(6\,Ca^5b+15\,Ba^4b^2+20\,Aa^3b^3\right)x^4 \\ &+\frac{1}{3}\left(Ca^6+6\,Ba^5b+15\,Aa^4b^2\right)x^3+\frac{1}{2}\left(Ba^6+6\,Aa^5b\right)x^2\end{aligned}$$

input
```
integrate((C*x^2+B*x+A)*(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^2,x, algorithm
="fricas")
```

output
```
1/9*C*b^6*x^9 + 1/8*(6*C*a*b^5 + B*b^6)*x^8 + A*a^6*x + 1/7*(15*C*a^2*b^4
+ 6*B*a*b^5 + A*b^6)*x^7 + 1/6*(20*C*a^3*b^3 + 15*B*a^2*b^4 + 6*A*a*b^5)*x
^6 + (3*C*a^4*b^2 + 4*B*a^3*b^3 + 3*A*a^2*b^4)*x^5 + 1/4*(6*C*a^5*b + 15*B
*a^4*b^2 + 20*A*a^3*b^3)*x^4 + 1/3*(C*a^6 + 6*B*a^5*b + 15*A*a^4*b^2)*x^3
+ 1/2*(B*a^6 + 6*A*a^5*b)*x^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 219 vs. $2(60) = 120$.

Time = 0.04 (sec) , antiderivative size = 219, normalized size of antiderivative = 3.17

$$\int (A+Bx+Cx^2)\left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)^2\,dx = Aa^6x+\frac{Cb^6x^9}{9}+x^8\left(\frac{Bb^6}{8}+\frac{3Cab^5}{4}\right)+x^7\left(\frac{Ab^6}{7}+\frac{6Bab^5}{7}+\frac{15Ca^2b^4}{7}\right) + x^6\left(Aab^5+\frac{5Ba^2b^4}{2}+\frac{10Ca^3b^3}{3}\right)+x^5\cdot\left(3Aa^2b^4+4Ba^3b^3+3Ca^4b^2\right)+x^4 \cdot\left(5Aa^3b^3+\frac{15Ba^4b^2}{4}+\frac{3Ca^5b}{2}\right)+x^3\cdot\left(5Aa^4b^2+2Ba^5b+\frac{Ca^6}{3}\right)+x^2\cdot\left(3Aa^5b+\frac{Ba^6}{2}\right)$$

input
```
integrate((C*x**2+B*x+A)*(b**3*x**3+3*a*b**2*x**2+3*a**2*b*x+a**3)**2,x)
```

output
```
A*a**6*x + C*b**6*x**9/9 + x**8*(B*b**6/8 + 3*C*a*b**5/4) + x**7*(A*b**6/7
 + 6*B*a*b**5/7 + 15*C*a**2*b**4/7) + x**6*(A*a*b**5 + 5*B*a**2*b**4/2 + 1
0*C*a**3*b**3/3) + x**5*(3*A*a**2*b**4 + 4*B*a**3*b**3 + 3*C*a**4*b**2) +
x**4*(5*A*a**3*b**3 + 15*B*a**4*b**2/4 + 3*C*a**5*b/2) + x**3*(5*A*a**4*b*
*2 + 2*B*a**5*b + C*a**6/3) + x**2*(3*A*a**5*b + B*a**6/2)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 200 vs. $2(63) = 126$.

Time = 0.04 (sec) , antiderivative size = 200, normalized size of antiderivative = 2.90

$$\int (A+Bx+Cx^2)\left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)^2\,dx = \frac{1}{9}Cb^6x^9+\frac{1}{8}\left(6\,Cab^5+Bb^6\right)x^8+Aa^6x + \frac{1}{7}\left(15\,Ca^2b^4+6\,Bab^5+Ab^6\right)x^7+\frac{1}{6}\left(20\,Ca^3b^3+15\,Ba^2b^4+6\,Aab^5\right)x^6 + \left(3\,Ca^4b^2+4\,Ba^3b^3+3\,Aa^2b^4\right)x^5+\frac{1}{4}\left(6\,Ca^5b+15\,Ba^4b^2+20\,Aa^3b^3\right)x^4 + \frac{1}{3}\left(Ca^6+6\,Ba^5b+15\,Aa^4b^2\right)x^3+\frac{1}{2}\left(Ba^6+6\,Aa^5b\right)x^2$$

input
```
integrate((C*x^2+B*x+A)*(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^2,x, algorithm
="maxima")
```

output
```
1/9*C*b^6*x^9 + 1/8*(6*C*a*b^5 + B*b^6)*x^8 + A*a^6*x + 1/7*(15*C*a^2*b^4
+ 6*B*a*b^5 + A*b^6)*x^7 + 1/6*(20*C*a^3*b^3 + 15*B*a^2*b^4 + 6*A*a*b^5)*x
^6 + (3*C*a^4*b^2 + 4*B*a^3*b^3 + 3*A*a^2*b^4)*x^5 + 1/4*(6*C*a^5*b + 15*B
*a^4*b^2 + 20*A*a^3*b^3)*x^4 + 1/3*(C*a^6 + 6*B*a^5*b + 15*A*a^4*b^2)*x^3
+ 1/2*(B*a^6 + 6*A*a^5*b)*x^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 219 vs. $2(63) = 126$.

Time = 0.13 (sec) , antiderivative size = 219, normalized size of antiderivative = 3.17

$$\int (A + Bx + Cx^2) (a^3 + 3a^2bx + 3ab^2x^2 + b^3x^3)^2 \, dx$$
$$= \frac{1}{9}Cb^6x^9 + \frac{3}{4}Cab^5x^8 + \frac{1}{8}Bb^6x^8 + \frac{15}{7}Ca^2b^4x^7 + \frac{6}{7}Bab^5x^7 + \frac{1}{7}Ab^6x^7 + \frac{10}{3}Ca^3b^3x^6 + \frac{5}{2}Ba^2b^4x^6 + Aab^5x^6 + 3Ca^4b^2x^5 + 4Ba^3b^3x^5 + 3Aa^2b^4x^5 + \frac{3}{2}Ca^5bx^4 + \frac{15}{4}Ba^4b^2x^4 + 5Aa^3b^3x^4 + \frac{1}{3}Ca^6x^3 + 2Ba^5bx^3 + 5Aa^4b^2x^3 + \frac{1}{2}Ba^6x^2 + 3Aa^5bx^2 + Aa^6x$$

input
```
integrate((C*x^2+B*x+A)*(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^2,x, algorithm
="giac")
```

output
```
1/9*C*b^6*x^9 + 3/4*C*a*b^5*x^8 + 1/8*B*b^6*x^8 + 15/7*C*a^2*b^4*x^7 + 6/7
*B*a*b^5*x^7 + 1/7*A*b^6*x^7 + 10/3*C*a^3*b^3*x^6 + 5/2*B*a^2*b^4*x^6 + A*
a*b^5*x^6 + 3*C*a^4*b^2*x^5 + 4*B*a^3*b^3*x^5 + 3*A*a^2*b^4*x^5 + 3/2*C*a^
5*b*x^4 + 15/4*B*a^4*b^2*x^4 + 5*A*a^3*b^3*x^4 + 1/3*C*a^6*x^3 + 2*B*a^5*b
*x^3 + 5*A*a^4*b^2*x^3 + 1/2*B*a^6*x^2 + 3*A*a^5*b*x^2 + A*a^6*x
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 188, normalized size of antiderivative = 2.72

$$\int (A+Bx+Cx^2)\left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)^2\,dx = x^2\left(\frac{B\,a^6}{2}+3\,A\,b\,a^5\right)+x^8\left(\frac{B\,b^6}{8}+\frac{3\,C\,a\,b^5}{4}\right) + x^3\left(\frac{C\,a^6}{3}+2\,B\,a^5\,b+5\,A\,a^4\,b^2\right)+x^7\left(\frac{15\,C\,a^2\,b^4}{7}+\frac{6\,B\,a\,b^5}{7}+\frac{A\,b^6}{7}\right) + \frac{C\,b^6\,x^9}{9}+A\,a^6\,x+a^2\,b^2\,x^5\left(3\,C\,a^2+4\,B\,a\,b+3\,A\,b^2\right) + \frac{a^3\,b\,x^4\,(6\,C\,a^2+15\,B\,a\,b+20\,A\,b^2)}{4}+\frac{a\,b^3\,x^6\,(20\,C\,a^2+15\,B\,a\,b+6\,A\,b^2)}{6}$$

input
```
int((A + B*x + C*x^2)*(a^3 + b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x)^2,x)
```

output
```
x^2*((B*a^6)/2 + 3*A*a^5*b) + x^8*((B*b^6)/8 + (3*C*a*b^5)/4) + x^3*((C*a^
6)/3 + 5*A*a^4*b^2 + 2*B*a^5*b) + x^7*((A*b^6)/7 + (15*C*a^2*b^4)/7 + (6*B
*a*b^5)/7) + (C*b^6*x^9)/9 + A*a^6*x + a^2*b^2*x^5*(3*A*b^2 + 3*C*a^2 + 4*
B*a*b) + (a^3*b*x^4*(20*A*b^2 + 6*C*a^2 + 15*B*a*b))/4 + (a*b^3*x^6*(6*A*b
^2 + 20*C*a^2 + 15*B*a*b))/6
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 151, normalized size of antiderivative = 2.19

$$\int (A+Bx+Cx^2)\left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)^2\,dx = \frac{x(56b^6c\,x^8+378a\,b^5c\,x^7+63b^7x^7+1080a^2b^4c\,x^6+504a\,b^6x^6+1680a^3b^3c\,x^5+1764a^2b^5x^5+1512a^4b^2c}{504}$$

input
```
int((C*x^2+B*x+A)*(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^2,x)
```

output
```
(x*(504*a**7 + 1764*a**6*b*x + 168*a**6*c*x**2 + 3528*a**5*b**2*x**2 + 756
*a**5*b*c*x**3 + 4410*a**4*b**3*x**3 + 1512*a**4*b**2*c*x**4 + 3528*a**3*b
**4*x**4 + 1680*a**3*b**3*c*x**5 + 1764*a**2*b**5*x**5 + 1080*a**2*b**4*c*
x**6 + 504*a*b**6*x**6 + 378*a*b**5*c*x**7 + 63*b**7*x**7 + 56*b**6*c*x**8
))/504
```

3.56 $\int \left(A+Bx+Cx^2\right)\left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)\,dx$

Optimal result

Integrand size = 38, antiderivative size = 69

$$\int \left(A+Bx+Cx^2\right)\left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)\,dx = \frac{\left(Ab^2-a(bB-aC)\right)(a+bx)^4}{4b^3}+\frac{(bB-2aC)(a+bx)^5}{5b^3}+\frac{C(a+bx)^6}{6b^3}$$

output
```
1/4*(A*b^2-a*(B*b-C*a))*(b*x+a)^4/b^3+1/5*(B*b-2*C*a)*(b*x+a)^5/b^3+1/6*C*
(b*x+a)^6/b^3
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.33

$$\int \left(A+Bx+Cx^2\right)\left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)\,dx = \frac{1}{60}x\left(10a^3(6A+x(3B+2Cx))+15a^2bx(6A+x(4B+3Cx))+3ab^2x^2(20A+3x(5B+4Cx))+b^3x^3(15A+2x(6B+5Cx))\right)$$

input
```
Integrate[(A + B*x + C*x^2)*(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3),x]
```

output
```
(x*(10*a^3*(6*A + x*(3*B + 2*C*x)) + 15*a^2*b*x*(6*A + x*(4*B + 3*C*x)) +
3*a*b^2*x^2*(20*A + 3*x*(5*B + 4*C*x)) + b^3*x^3*(15*A + 2*x*(6*B + 5*C*x)
)))/60
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.079$, Rules used = {2006, 1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)\left(A+Bx+Cx^2\right)dx$$

$$\Big\downarrow \text{2006}$$

$$\int (a+bx)^3\left(A+Bx+Cx^2\right)dx$$

$$\Big\downarrow \text{1140}$$

$$\int \left(\frac{(a+bx)^3\left(Ab^2-a(bB-aC)\right)}{b^2}+\frac{(a+bx)^4(bB-2aC)}{b^2}+\frac{C(a+bx)^5}{b^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{(a+bx)^4\left(Ab^2-a(bB-aC)\right)}{4b^3}+\frac{(a+bx)^5(bB-2aC)}{5b^3}+\frac{C(a+bx)^6}{6b^3}$$

input
```
Int[(A + B*x + C*x^2)*(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3),x]
```

output
```
((A*b^2 - a*(b*B - a*C))*(a + b*x)^4)/(4*b^3) + ((b*B - 2*a*C)*(a + b*x)^5
)/(5*b^3) + (C*(a + b*x)^6)/(6*b^3)
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2006
```
Int[(u_.)*(Px_), x_Symbol] :> With[{a = Rt[Coeff[Px, x, 0], Expon[Px, x]],
b = Rt[Coeff[Px, x, Expon[Px, x]], Expon[Px, x]]}, Int[u*(a + b*x)^Expon[Px
, x], x] /; EqQ[Px, (a + b*x)^Expon[Px, x]]] /; PolyQ[Px, x] && GtQ[Expon[P
x, x], 1] && NeQ[Coeff[Px, x, 0], 0] &&  !MatchQ[Px, (a_.)*(v_)^Expon[Px, x
] /; FreeQ[a, x] && LinearQ[v, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.46

method	result
norman	$\frac{b^3 C x^6}{6}+\left(\frac{1}{5} B b^3+\frac{3}{5} b^2 C a\right) x^5+\left(\frac{1}{4} A b^3+\frac{3}{4} B b^2 a+\frac{3}{4} b C a^2\right) x^4+\left(A a b^2+B a^2 b+\frac{1}{3} C a^3\right) x$
default	$\frac{b^3 C x^6}{6}+\frac{\left(B b^3+3 b^2 C a\right) x^5}{5}+\frac{\left(A b^3+3 B b^2 a+3 b C a^2\right) x^4}{4}+\frac{\left(3 A a b^2+3 B a^2 b+C a^3\right) x^3}{3}+\frac{\left(3 A a^2 b+B a^3\right) x^2}{2}+a^3 A$
risch	$\frac{1}{6} b^3 C x^6+\frac{1}{5} b^3 B x^5+\frac{3}{5} x^5 b^2 C a+\frac{1}{4} x^4 A b^3+\frac{3}{4} x^4 B b^2 a+\frac{3}{4} x^4 b C a^2+x^3 A a b^2+B a^2 b x^3+$
parallelrisch	$\frac{1}{6} b^3 C x^6+\frac{1}{5} b^3 B x^5+\frac{3}{5} x^5 b^2 C a+\frac{1}{4} x^4 A b^3+\frac{3}{4} x^4 B b^2 a+\frac{3}{4} x^4 b C a^2+x^3 A a b^2+B a^2 b x^3+$
gosper	$\frac{x\left(10 b^3 C x^5+12 x^4 B b^3+36 x^4 b^2 C a+15 x^3 A b^3+45 x^3 B b^2 a+45 x^3 b C a^2+60 x^2 A a b^2+60 x^2 B a^2 b+20 x^2 C a^3+90 x A a^2 b+30 x\right.}{60}$
orering	$\frac{x\left(10 b^3 C x^5+12 x^4 B b^3+36 x^4 b^2 C a+15 x^3 A b^3+45 x^3 B b^2 a+45 x^3 b C a^2+60 x^2 A a b^2+60 x^2 B a^2 b+20 x^2 C a^3+90 x A a^2 b+30 x\right.}{60(b x+a)^3}$

input
```
int((C*x^2+B*x+A)*(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3),x,method=_RETURNVERB
OSE)
```

output
```
1/6*b^3*C*x^6+(1/5*B*b^3+3/5*b^2*C*a)*x^5+(1/4*A*b^3+3/4*B*b^2*a+3/4*b*C*a
^2)*x^4+(A*a*b^2+B*a^2*b+1/3*C*a^3)*x^3+(3/2*A*a^2*b+1/2*B*a^3)*x^2+a^3*A*
x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.48

$$\int \left(A+Bx+Cx^2\right)\left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)\,dx$$
$$=\frac{1}{6}\,Cb^3x^6+\frac{1}{5}\,\left(3\,Cab^2+Bb^3\right)x^5+Aa^3x+\frac{1}{4}\,\left(3\,Ca^2b+3\,Bab^2+Ab^3\right)x^4+\frac{1}{3}\,\left(Ca^3+3\,Ba^2b+3\,Aab^2\right)x^3+\frac{1}{2}\,\left(Ba^3+3\,Aa^2b\right)x^2$$

input
```
integrate((C*x^2+B*x+A)*(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3),x, algorithm="
fricas")
```

output
```
1/6*C*b^3*x^6 + 1/5*(3*C*a*b^2 + B*b^3)*x^5 + A*a^3*x + 1/4*(3*C*a^2*b + 3
*B*a*b^2 + A*b^3)*x^4 + 1/3*(C*a^3 + 3*B*a^2*b + 3*A*a*b^2)*x^3 + 1/2*(B*a
^3 + 3*A*a^2*b)*x^2
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.59

$$\int \left(A+Bx+Cx^2\right)\left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)\,dx$$
$$=Aa^3x+\frac{Cb^3x^6}{6}+x^5\left(\frac{Bb^3}{5}+\frac{3Cab^2}{5}\right)+x^4\left(\frac{Ab^3}{4}+\frac{3Bab^2}{4}+\frac{3Ca^2b}{4}\right)+x^3\left(Aab^2+Ba^2b+\frac{Ca^3}{3}\right)+x^2\cdot\left(\frac{3Aa^2b}{2}+\frac{Ba^3}{2}\right)$$

input
```
integrate((C*x**2+B*x+A)*(b**3*x**3+3*a*b**2*x**2+3*a**2*b*x+a**3),x)
```

output
```
A*a**3*x + C*b**3*x**6/6 + x**5*(B*b**3/5 + 3*C*a*b**2/5) + x**4*(A*b**3/4
 + 3*B*a*b**2/4 + 3*C*a**2*b/4) + x**3*(A*a*b**2 + B*a**2*b + C*a**3/3) +
x**2*(3*A*a**2*b/2 + B*a**3/2)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.48

$$\int (A+Bx+Cx^2)\left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)\,dx$$
$$=\frac{1}{6}Cb^3x^6+\frac{1}{5}\left(3Cab^2+Bb^3\right)x^5+Aa^3x+\frac{1}{4}\left(3Ca^2b+3Bab^2+Ab^3\right)x^4+\frac{1}{3}\left(Ca^3+3Ba^2b+3Aab^2\right)x^3+\frac{1}{2}\left(Ba^3+3Aa^2b\right)x^2$$

input
```
integrate((C*x^2+B*x+A)*(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3),x, algorithm="
maxima")
```

output
```
1/6*C*b^3*x^6 + 1/5*(3*C*a*b^2 + B*b^3)*x^5 + A*a^3*x + 1/4*(3*C*a^2*b + 3
*B*a*b^2 + A*b^3)*x^4 + 1/3*(C*a^3 + 3*B*a^2*b + 3*A*a*b^2)*x^3 + 1/2*(B*a
^3 + 3*A*a^2*b)*x^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.59

$$\int (A+Bx+Cx^2)\left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)\,dx$$
$$=\frac{1}{6}Cb^3x^6+\frac{3}{5}Cab^2x^5+\frac{1}{5}Bb^3x^5+\frac{3}{4}Ca^2bx^4+\frac{3}{4}Bab^2x^4+\frac{1}{4}Ab^3x^4+\frac{1}{3}Ca^3x^3+Ba^2bx^3+Aab^2x^3+\frac{1}{2}Ba^3x^2+\frac{3}{2}Aa^2bx^2+Aa^3x$$

input
```
integrate((C*x^2+B*x+A)*(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3),x, algorithm="
giac")
```

output
```
1/6*C*b^3*x^6 + 3/5*C*a*b^2*x^5 + 1/5*B*b^3*x^5 + 3/4*C*a^2*b*x^4 + 3/4*B*
a*b^2*x^4 + 1/4*A*b^3*x^4 + 1/3*C*a^3*x^3 + B*a^2*b*x^3 + A*a*b^2*x^3 + 1/
2*B*a^3*x^2 + 3/2*A*a^2*b*x^2 + A*a^3*x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.45

$$\int \left(A+Bx+Cx^2\right)\left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)\,dx$$
$$=x^3\left(\frac{C\,a^3}{3}+B\,a^2\,b+A\,a\,b^2\right)+x^4\left(\frac{3\,C\,a^2\,b}{4}+\frac{3\,B\,a\,b^2}{4}+\frac{A\,b^3}{4}\right)$$
$$+x^2\left(\frac{B\,a^3}{2}+\frac{3\,A\,b\,a^2}{2}\right)+x^5\left(\frac{B\,b^3}{5}+\frac{3\,C\,a\,b^2}{5}\right)+\frac{C\,b^3\,x^6}{6}+A\,a^3\,x$$

input
```
int((A + B*x + C*x^2)*(a^3 + b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x),x)
```

output
```
x^3*((C*a^3)/3 + A*a*b^2 + B*a^2*b) + x^4*((A*b^3)/4 + (3*B*a*b^2)/4 + (3*
C*a^2*b)/4) + x^2*((B*a^3)/2 + (3*A*a^2*b)/2) + x^5*((B*b^3)/5 + (3*C*a*b^
2)/5) + (C*b^3*x^6)/6 + A*a^3*x
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.19

$$\int \left(A+Bx+Cx^2\right)\left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)\,dx$$
$$=\frac{x(10b^3c\,x^5+36a\,b^2c\,x^4+12b^4x^4+45a^2bc\,x^3+60a\,b^3x^3+20a^3c\,x^2+120a^2b^2x^2+120a^3bx+60a^4)}{60}$$

input
```
int((C*x^2+B*x+A)*(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3),x)
```

output
```
(x*(60*a**4 + 120*a**3*b*x + 20*a**3*c*x**2 + 120*a**2*b**2*x**2 + 45*a**2
*b*c*x**3 + 60*a*b**3*x**3 + 36*a*b**2*c*x**4 + 12*b**4*x**4 + 10*b**3*c*x
**5))/60
```

3.57 $\int \frac{A+Bx+Cx^2}{a^3+3a^2bx+3ab^2x^2+b^3x^3}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 63

$$\int \frac{A+Bx+Cx^2}{a^3+3a^2bx+3ab^2x^2+b^3x^3}\,dx = \frac{-Ab^2+a(bB-aC)}{2b^3(a+bx)^2} - \frac{bB-2aC}{b^3(a+bx)} + \frac{C\log(a+bx)}{b^3}$$

output
```
1/2*(-A*b^2+a*(B*b-C*a))/b^3/(b*x+a)^2-(B*b-2*C*a)/b^3/(b*x+a)+C*ln(b*x+a)
/b^3
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.94

$$\int \frac{A+Bx+Cx^2}{a^3+3a^2bx+3ab^2x^2+b^3x^3}\,dx = -\frac{Ab^2+abB-3a^2C+2b^2Bx-4abCx-2C(a+bx)^2\log(a+bx)}{2b^3(a+bx)^2}$$

input
```
Integrate[(A + B*x + C*x^2)/(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3),x]
```

output
```
-1/2*(A*b^2 + a*b*B - 3*a^2*C + 2*b^2*B*x - 4*a*b*C*x - 2*C*(a + b*x)^2*Lo
g[a + b*x])/(b^3*(a + b*x)^2)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.075$, Rules used = {2007, 1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{a^3+3a^2bx+3ab^2x^2+b^3x^3}dx$$

$$\Big\downarrow \text{2007}$$

$$\int \frac{A+Bx+Cx^2}{(a+bx)^3}dx$$

$$\Big\downarrow \text{1140}$$

$$\int \left(\frac{Ab^2-a(bB-aC)}{b^2(a+bx)^3}+\frac{bB-2aC}{b^2(a+bx)^2}+\frac{C}{b^2(a+bx)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{Ab^2-a(bB-aC)}{2b^3(a+bx)^2}-\frac{bB-2aC}{b^3(a+bx)}+\frac{C\log(a+bx)}{b^3}$$

input
```
Int[(A + B*x + C*x^2)/(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3),x]
```

output
```
-1/2*(A*b^2 - a*(b*B - a*C))/(b^3*(a + b*x)^2) - (b*B - 2*a*C)/(b^3*(a + b
*x)) + (C*Log[a + b*x])/b^3
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2007
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{a = Rt[Coeff[Px, x, 0], Expon[Px,
x]], b = Rt[Coeff[Px, x, Expon[Px, x]], Expon[Px, x]]}, Int[u*(a + b*x)^(Ex
pon[Px, x]*p), x] /; EqQ[Px, (a + b*x)^Expon[Px, x]]] /; IntegerQ[p] && Pol
yQ[Px, x] && GtQ[Expon[Px, x], 1] && NeQ[Coeff[Px, x, 0], 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.90

method	result	size
norman	$\frac{-\frac{A\,b^{2}+abB-3C\,a^{2}}{2b^{3}}-\frac{(Bb-2Ca)x}{b^{2}}}{(bx+a)^{2}}+\frac{C\ln(bx+a)}{b^{3}}$	57
default	$-\frac{A\,b^{2}-abB+C\,a^{2}}{2b^{3}(bx+a)^{2}}+\frac{C\ln(bx+a)}{b^{3}}-\frac{Bb-2Ca}{b^{3}(bx+a)}$	61
risch	$\frac{-\frac{A\,b^{2}+abB-3C\,a^{2}}{2b^{3}}-\frac{(Bb-2Ca)x}{b^{2}}}{b^{2}x^{2}+2abx+a^{2}}+\frac{C\ln(bx+a)}{b^{3}}$	68
parallelrisch	$-\frac{-2C\ln(bx+a)x^{2}b^{2}-4C\ln(bx+a)xab+2B\,b^{2}x-2C\ln(bx+a)a^{2}-4Cabx+A\,b^{2}+abB-3C\,a^{2}}{2b^{3}(b^{2}x^{2}+2abx+a^{2})}$	92

input
```
int((C*x^2+B*x+A)/(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3),x,method=_RETURNVERB
OSE)
```

output
```
(-1/2*(A*b^2+B*a*b-3*C*a^2)/b^3-(B*b-2*C*a)/b^2*x)/(b*x+a)^2+C*ln(b*x+a)/b
^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.38

$$\int \frac{A+Bx+Cx^2}{a^3+3a^2bx+3ab^2x^2+b^3x^3}\,dx = \frac{3\,Ca^2-Bab-Ab^2+2\,(2\,Cab-Bb^2)x+2\,(Cb^2x^2+2\,Cabx+Ca^2)\log\left(bx+a\right)}{2\,(b^5x^2+2\,ab^4x+a^2b^3)}$$

input
```
integrate((C*x^2+B*x+A)/(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3),x, algorithm="
fricas")
```

output
```
1/2*(3*C*a^2 - B*a*b - A*b^2 + 2*(2*C*a*b - B*b^2)*x + 2*(C*b^2*x^2 + 2*C*
a*b*x + C*a^2)*log(b*x + a))/(b^5*x^2 + 2*a*b^4*x + a^2*b^3)
```

Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.08

$$\int \frac{A+Bx+Cx^2}{a^3+3a^2bx+3ab^2x^2+b^3x^3}\,dx = \frac{C\log{(a+bx)}}{b^3} + \frac{-Ab^2-Bab+3Ca^2+x(-2Bb^2+4Cab)}{2a^2b^3+4ab^4x+2b^5x^2}$$

input
```
integrate((C*x**2+B*x+A)/(b**3*x**3+3*a*b**2*x**2+3*a**2*b*x+a**3),x)
```

output
```
C*log(a + b*x)/b**3 + (-A*b**2 - B*a*b + 3*C*a**2 + x*(-2*B*b**2 + 4*C*a*b
))/(2*a**2*b**3 + 4*a*b**4*x + 2*b**5*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.13

$$\int \frac{A+Bx+Cx^2}{a^3+3a^2bx+3ab^2x^2+b^3x^3}\,dx = \frac{3\,Ca^2-Bab-Ab^2+2\,(2\,Cab-Bb^2)x}{2\,(b^5x^2+2\,ab^4x+a^2b^3)} + \frac{C\log{(bx+a)}}{b^3}$$

input
```
integrate((C*x^2+B*x+A)/(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3),x, algorithm="
maxima")
```

output
```
1/2*(3*C*a^2 - B*a*b - A*b^2 + 2*(2*C*a*b - B*b^2)*x)/(b^5*x^2 + 2*a*b^4*x
 + a^2*b^3) + C*log(b*x + a)/b^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.95

$$\int \frac{A+Bx+Cx^2}{a^3+3a^2bx+3ab^2x^2+b^3x^3}\,dx=\frac{C\log\left(|bx+a|\right)}{b^3}+\frac{2\left(2\,Ca-Bb\right)x+\frac{3\,Ca^2-Bab-Ab^2}{b}}{2\left(bx+a\right)^2b^2}$$

input
```
integrate((C*x^2+B*x+A)/(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3),x, algorithm="
giac")
```

output
```
C*log(abs(b*x + a))/b^3 + 1/2*(2*(2*C*a - B*b)*x + (3*C*a^2 - B*a*b - A*b^
2)/b)/((b*x + a)^2*b^2)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.06

$$\int \frac{A+Bx+Cx^2}{a^3+3a^2bx+3ab^2x^2+b^3x^3}\,dx=\frac{C\,\ln\left(a+b\,x\right)}{b^3}-\frac{\frac{-3\,C\,a^2+B\,a\,b+A\,b^2}{2\,b^3}+\frac{x\,\left(B\,b-2\,C\,a\right)}{b^2}}{a^2+2\,a\,b\,x+b^2\,x^2}$$

input
```
int((A + B*x + C*x^2)/(a^3 + b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x),x)
```

output
```
(C*log(a + b*x))/b^3 - ((A*b^2 - 3*C*a^2 + B*a*b)/(2*b^3) + (x*(B*b - 2*C*
a))/b^2)/(a^2 + b^2*x^2 + 2*a*b*x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.57

$$\int \frac{A+Bx+Cx^2}{a^3+3a^2bx+3ab^2x^2+b^3x^3}\,dx$$
$$=\frac{2\log(bx+a)\,a^3c+4\log(bx+a)\,a^2bcx+2\log(bx+a)\,a\,b^2c\,x^2+a^3c-a^2b^2-2a\,b^2c\,x^2+b^4x^2}{2a\,b^3\left(b^2x^2+2abx+a^2\right)}$$

input
```
int((C*x^2+B*x+A)/(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3),x)
```

output

```
(2*log(a + b*x)*a**3*c + 4*log(a + b*x)*a**2*b*c*x + 2*log(a + b*x)*a*b**2
*c*x**2 + a**3*c - a**2*b**2 - 2*a*b**2*c*x**2 + b**4*x**2)/(2*a*b**3*(a**
2 + 2*a*b*x + b**2*x**2))
```

3.58 $\int \frac{A+Bx+Cx^2}{\left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)^2}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 69

$$\int \frac{A+Bx+Cx^2}{\left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)^2}\,dx = \frac{-Ab^2+a(bB-aC)}{5b^3(a+bx)^5}-\frac{bB-2aC}{4b^3(a+bx)^4}-\frac{C}{3b^3(a+bx)^3}$$

output
```
1/5*(-A*b^2+a*(B*b-C*a))/b^3/(b*x+a)^5-1/4*(B*b-2*C*a)/b^3/(b*x+a)^4-1/3*C
/b^3/(b*x+a)^3
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.75

$$\int \frac{A+Bx+Cx^2}{\left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)^2}\,dx = -\frac{12Ab^2+2a^2C+5b^2x(3B+4Cx)+ab(3B+10Cx)}{60b^3(a+bx)^5}$$

input
```
Integrate[(A + B*x + C*x^2)/(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^2,x]
```

output
```
-1/60*(12*A*b^2 + 2*a^2*C + 5*b^2*x*(3*B + 4*C*x) + a*b*(3*B + 10*C*x))/(b
^3*(a + b*x)^5)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.075$, Rules used = {2007, 1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{(a^3+3a^2bx+3ab^2x^2+b^3x^3)^2}\,dx$$

$$\Big\downarrow \text{2007}$$

$$\int \frac{A+Bx+Cx^2}{(a+bx)^6}dx$$

$$\Big\downarrow \text{1140}$$

$$\int \left(\frac{Ab^2-a(bB-aC)}{b^2(a+bx)^6}+\frac{bB-2aC}{b^2(a+bx)^5}+\frac{C}{b^2(a+bx)^4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{Ab^2-a(bB-aC)}{5b^3(a+bx)^5}-\frac{bB-2aC}{4b^3(a+bx)^4}-\frac{C}{3b^3(a+bx)^3}$$

input
```
Int[(A + B*x + C*x^2)/(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^2,x]
```

output
```
-1/5*(A*b^2 - a*(b*B - a*C))/(b^3*(a + b*x)^5) - (b*B - 2*a*C)/(4*b^3*(a +
 b*x)^4) - C/(3*b^3*(a + b*x)^3)
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2007
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{a = Rt[Coeff[Px, x, 0], Expon[Px,
x]], b = Rt[Coeff[Px, x, Expon[Px, x]], Expon[Px, x]]}, Int[u*(a + b*x)^(Ex
pon[Px, x]*p), x] /; EqQ[Px, (a + b*x)^Expon[Px, x]]] /; IntegerQ[p] && Pol
yQ[Px, x] && GtQ[Expon[Px, x], 1] && NeQ[Coeff[Px, x, 0], 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.91

method	result	size
default	$-\frac{Bb-2Ca}{4b^{3}\left(bx+a\right)^{4}}-\frac{A\,b^{2}-abB+C\,a^{2}}{5b^{3}\left(bx+a\right)^{5}}-\frac{C}{3b^{3}\left(bx+a\right)^{3}}$	63
norman	$\frac{-\frac{C\,x^{2}}{3b}-\frac{\left(3B\,b^{3}+2b^{2}Ca\right)x}{12b^{4}}-\frac{12A\,b^{4}+3aB\,b^{3}+2C\,a^{2}b^{2}}{60b^{5}}}{\left(bx+a\right)^{5}}$	67
orering	$-\frac{\left(20C\,b^{2}x^{2}+15B\,b^{2}x+10Cabx+12A\,b^{2}+3abB+2C\,a^{2}\right)\left(bx+a\right)}{60b^{3}\left(b^{3}x^{3}+3a\,b^{2}x^{2}+3b\,a^{2}x+a^{3}\right)^{2}}$	80
gosper	$-\frac{20C\,b^{2}x^{2}+15B\,b^{2}x+10Cabx+12A\,b^{2}+3abB+2C\,a^{2}}{60b^{3}\left(b^{2}x^{2}+2abx+a^{2}\right)\left(b^{3}x^{3}+3a\,b^{2}x^{2}+3b\,a^{2}x+a^{3}\right)}$	93
risch	$\frac{-\frac{C\,x^{2}}{3b}-\frac{\left(3Bb+2Ca\right)x}{12b^{2}}-\frac{12A\,b^{2}+3abB+2C\,a^{2}}{60b^{3}}}{\left(b^{2}x^{2}+2abx+a^{2}\right)\left(b^{3}x^{3}+3a\,b^{2}x^{2}+3b\,a^{2}x+a^{3}\right)}$	97
parallelrisch	$-\frac{20C\,x^{2}b^{4}+15B\,b^{4}x+10Ca\,b^{3}x+12A\,b^{4}+3aB\,b^{3}+2C\,a^{2}b^{2}}{60b^{5}\left(b^{3}x^{3}+3a\,b^{2}x^{2}+3b\,a^{2}x+a^{3}\right)\left(b^{2}x^{2}+2abx+a^{2}\right)}$	100

input
```
int((C*x^2+B*x+A)/(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^2,x,method=_RETURNVE
RBOSE)
```

output
```
-1/4*(B*b-2*C*a)/b^3/(b*x+a)^4-1/5*(A*b^2-B*a*b+C*a^2)/b^3/(b*x+a)^5-1/3*C
/b^3/(b*x+a)^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.46

$$\int \frac{A+Bx+Cx^2}{(a^3+3a^2bx+3ab^2x^2+b^3x^3)^2}\,dx = -\frac{20\,Cb^2x^2+2\,Ca^2+3\,Bab+12\,Ab^2+5\,(2\,Cab+3\,Bb^2)x}{60\,(b^8x^5+5\,ab^7x^4+10\,a^2b^6x^3+10\,a^3b^5x^2+5\,a^4b^4x+a^5b^3)}$$

input
```
integrate((C*x^2+B*x+A)/(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^2,x, algorithm
="fricas")
```

output
```
-1/60*(20*C*b^2*x^2 + 2*C*a^2 + 3*B*a*b + 12*A*b^2 + 5*(2*C*a*b + 3*B*b^2)
*x)/(b^8*x^5 + 5*a*b^7*x^4 + 10*a^2*b^6*x^3 + 10*a^3*b^5*x^2 + 5*a^4*b^4*x
 + a^5*b^3)
```

Sympy [A] (verification not implemented)

Time = 0.94 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.55

$$\int \frac{A+Bx+Cx^2}{(a^3+3a^2bx+3ab^2x^2+b^3x^3)^2}\,dx = \frac{-12Ab^2-3Bab-2Ca^2-20Cb^2x^2+x(-15Bb^2-10Cab)}{60a^5b^3+300a^4b^4x+600a^3b^5x^2+600a^2b^6x^3+300ab^7x^4+60b^8x^5}$$

input
```
integrate((C*x**2+B*x+A)/(b**3*x**3+3*a*b**2*x**2+3*a**2*b*x+a**3)**2,x)
```

output
```
(-12*A*b**2 - 3*B*a*b - 2*C*a**2 - 20*C*b**2*x**2 + x*(-15*B*b**2 - 10*C*a
*b))/(60*a**5*b**3 + 300*a**4*b**4*x + 600*a**3*b**5*x**2 + 600*a**2*b**6*
x**3 + 300*a*b**7*x**4 + 60*b**8*x**5)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.46

$$\int \frac{A+Bx+Cx^2}{(a^3+3a^2bx+3ab^2x^2+b^3x^3)^2}\,dx = -\frac{20\,Cb^2x^2+2\,Ca^2+3\,Bab+12\,Ab^2+5\,(2\,Cab+3\,Bb^2)x}{60\,(b^8x^5+5\,ab^7x^4+10\,a^2b^6x^3+10\,a^3b^5x^2+5\,a^4b^4x+a^5b^3)}$$

input
```
integrate((C*x^2+B*x+A)/(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^2,x, algorithm
="maxima")
```

output
```
-1/60*(20*C*b^2*x^2 + 2*C*a^2 + 3*B*a*b + 12*A*b^2 + 5*(2*C*a*b + 3*B*b^2)
*x)/(b^8*x^5 + 5*a*b^7*x^4 + 10*a^2*b^6*x^3 + 10*a^3*b^5*x^2 + 5*a^4*b^4*x
 + a^5*b^3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.75

$$\int \frac{A+Bx+Cx^2}{(a^3+3a^2bx+3ab^2x^2+b^3x^3)^2}\,dx = -\frac{20\,Cb^2x^2+10\,Cabx+15\,Bb^2x+2\,Ca^2+3\,Bab+12\,Ab^2}{60\,(bx+a)^5b^3}$$

input
```
integrate((C*x^2+B*x+A)/(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^2,x, algorithm
="giac")
```

output
```
-1/60*(20*C*b^2*x^2 + 10*C*a*b*x + 15*B*b^2*x + 2*C*a^2 + 3*B*a*b + 12*A*b
^2)/((b*x + a)^5*b^3)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.46

$$\int \frac{A+Bx+Cx^2}{(a^3+3a^2bx+3ab^2x^2+b^3x^3)^2}\,dx = -\frac{\frac{2\,C\,a^2+3\,B\,a\,b+12\,A\,b^2}{60\,b^3}+\frac{C\,x^2}{3\,b}+\frac{x\,(3\,B\,b+2\,C\,a)}{12\,b^2}}{a^5+5\,a^4\,b\,x+10\,a^3\,b^2\,x^2+10\,a^2\,b^3\,x^3+5\,a\,b^4\,x^4+b^5\,x^5}$$

input
```
int((A + B*x + C*x^2)/(a^3 + b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x)^2,x)
```

output
```
-((12*A*b^2 + 2*C*a^2 + 3*B*a*b)/(60*b^3) + (C*x^2)/(3*b) + (x*(3*B*b + 2*
C*a))/(12*b^2))/(a^5 + b^5*x^5 + 5*a*b^4*x^4 + 10*a^3*b^2*x^2 + 10*a^2*b^3
*x^3 + 5*a^4*b*x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.30

$$\int \frac{A+Bx+Cx^2}{(a^3+3a^2bx+3ab^2x^2+b^3x^3)^2}\,dx = \frac{-20b^2c\,x^2-10abcx-15b^3x-2a^2c-15a\,b^2}{60b^3\,(b^5x^5+5a\,b^4x^4+10a^2b^3x^3+10a^3b^2x^2+5a^4bx+a^5)}$$

input
```
int((C*x^2+B*x+A)/(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^2,x)
```

output
```
( - 2*a**2*c - 15*a*b**2 - 10*a*b*c*x - 15*b**3*x - 20*b**2*c*x**2)/(60*b*
*3*(a**5 + 5*a**4*b*x + 10*a**3*b**2*x**2 + 10*a**2*b**3*x**3 + 5*a*b**4*x
**4 + b**5*x**5))
```

3.59 $\int \frac{A+Bx+Cx^2}{\left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)^3} \, dx$

Optimal result

Integrand size = 40, antiderivative size = 69

$$\int \frac{A+Bx+Cx^2}{\left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)^3} \, dx = \frac{-Ab^2+a(bB-aC)}{8b^3(a+bx)^8}-\frac{bB-2aC}{7b^3(a+bx)^7}-\frac{C}{6b^3(a+bx)^6}$$

output
```
1/8*(-A*b^2+a*(B*b-C*a))/b^3/(b*x+a)^8-1/7*(B*b-2*C*a)/b^3/(b*x+a)^7-1/6*C
/b^3/(b*x+a)^6
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.74

$$\int \frac{A+Bx+Cx^2}{\left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)^3} \, dx = -\frac{21Ab^2+a^2C+4b^2x(6B+7Cx)+ab(3B+8Cx)}{168b^3(a+bx)^8}$$

input
```
Integrate[(A + B*x + C*x^2)/(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^3,x]
```

output
```
-1/168*(21*A*b^2 + a^2*C + 4*b^2*x*(6*B + 7*C*x) + a*b*(3*B + 8*C*x))/(b^3
*(a + b*x)^8)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.075$, Rules used = {2007, 1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{(a^3+3a^2bx+3ab^2x^2+b^3x^3)^3}\,dx$$
$$\Big\downarrow \text{2007}$$
$$\int \frac{A+Bx+Cx^2}{(a+bx)^9}dx$$
$$\Big\downarrow \text{1140}$$
$$\int \left(\frac{Ab^2-a(bB-aC)}{b^2(a+bx)^9}+\frac{bB-2aC}{b^2(a+bx)^8}+\frac{C}{b^2(a+bx)^7}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$-\frac{Ab^2-a(bB-aC)}{8b^3(a+bx)^8}-\frac{bB-2aC}{7b^3(a+bx)^7}-\frac{C}{6b^3(a+bx)^6}$$

input
```
Int[(A + B*x + C*x^2)/(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^3,x]
```

output
```
-1/8*(A*b^2 - a*(b*B - a*C))/(b^3*(a + b*x)^8) - (b*B - 2*a*C)/(7*b^3*(a +
 b*x)^7) - C/(6*b^3*(a + b*x)^6)
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2007
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{a = Rt[Coeff[Px, x, 0], Expon[Px,
x]], b = Rt[Coeff[Px, x, Expon[Px, x]], Expon[Px, x]]}, Int[u*(a + b*x)^(Ex
pon[Px, x]*p), x] /; EqQ[Px, (a + b*x)^Expon[Px, x]]] /; IntegerQ[p] && Pol
yQ[Px, x] && GtQ[Expon[Px, x], 1] && NeQ[Coeff[Px, x, 0], 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.91

method	result	size
default	$-\frac{Bb-2Ca}{7b^{3}\left(bx+a\right)^{7}}-\frac{C}{6b^{3}\left(bx+a\right)^{6}}-\frac{A\,b^{2}-abB+C\,a^{2}}{8b^{3}\left(bx+a\right)^{8}}$	63
norman	$\frac{-\frac{C\,x^{2}}{6b}-\frac{\left(3B\,b^{6}+C\,b^{5}a\right)x}{21b^{7}}-\frac{21A\,b^{7}+3Ba\,b^{6}+C\,a^{2}b^{5}}{168b^{8}}}{\left(bx+a\right)^{8}}$	65
orering	$-\frac{\left(28C\,b^{2}x^{2}+24B\,b^{2}x+8Cabx+21A\,b^{2}+3abB+C\,a^{2}\right)\left(bx+a\right)}{168b^{3}\left(b^{3}x^{3}+3a\,b^{2}x^{2}+3b\,a^{2}x+a^{3}\right)^{3}}$	79
gosper	$-\frac{28C\,b^{2}x^{2}+24B\,b^{2}x+8Cabx+21A\,b^{2}+3abB+C\,a^{2}}{168b^{3}\left(b^{2}x^{2}+2abx+a^{2}\right)\left(b^{3}x^{3}+3a\,b^{2}x^{2}+3b\,a^{2}x+a^{3}\right)^{2}}$	92
risch	$\frac{-\frac{C\,x^{2}}{6b}-\frac{\left(3Bb+Ca\right)x}{21b^{2}}-\frac{21A\,b^{2}+3abB+C\,a^{2}}{168b^{3}}}{\left(b^{2}x^{2}+2abx+a^{2}\right)\left(b^{3}x^{3}+3a\,b^{2}x^{2}+3b\,a^{2}x+a^{3}\right)^{2}}$	95
parallelrisch	$-\frac{28C\,x^{2}b^{7}+24B\,b^{7}x+8Ca\,b^{6}x+21A\,b^{7}+3Ba\,b^{6}+C\,a^{2}b^{5}}{168b^{8}\left(b^{3}x^{3}+3a\,b^{2}x^{2}+3b\,a^{2}x+a^{3}\right)^{2}\left(b^{2}x^{2}+2abx+a^{2}\right)}$	99

input
```
int((C*x^2+B*x+A)/(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^3,x,method=_RETURNVE
RBOSE)
```

output
```
-1/7*(B*b-2*C*a)/b^3/(b*x+a)^7-1/6*C/b^3/(b*x+a)^6-1/8*(A*b^2-B*a*b+C*a^2)
/b^3/(b*x+a)^8
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 132 vs. $2(63) = 126$.

Time = 0.07 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.91

$$\int \frac{A + Bx + Cx^2}{(a^3 + 3a^2bx + 3ab^2x^2 + b^3x^3)^3}\,dx = -\frac{28\,Cb^2x^2 + Ca^2 + 3\,Bab + 21\,Ab^2 + 8\,(Cab + 3\,Bb^2)x}{168\,(b^{11}x^8 + 8\,ab^{10}x^7 + 28\,a^2b^9x^6 + 56\,a^3b^8x^5 + 70\,a^4b^7x^4 + 56\,a^5b^6x^3 + 28\,a^6b^5x^2 + 8\,a^7b^4x + a^8b^3)}$$

input
```
integrate((C*x^2+B*x+A)/(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^3,x, algorithm
="fricas")
```

output
```
-1/168*(28*C*b^2*x^2 + C*a^2 + 3*B*a*b + 21*A*b^2 + 8*(C*a*b + 3*B*b^2)*x)
/(b^11*x^8 + 8*a*b^10*x^7 + 28*a^2*b^9*x^6 + 56*a^3*b^8*x^5 + 70*a^4*b^7*x
^4 + 56*a^5*b^6*x^3 + 28*a^6*b^5*x^2 + 8*a^7*b^4*x + a^8*b^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 141 vs. $2(61) = 122$.

Time = 2.61 (sec) , antiderivative size = 141, normalized size of antiderivative = 2.04

$$\int \frac{A + Bx + Cx^2}{(a^3 + 3a^2bx + 3ab^2x^2 + b^3x^3)^3}\,dx = \frac{-21Ab^2 - 3Bab - Ca^2 - 28Cb^2x^2 + x(-24Bb^2 - 8Cab)}{168a^8b^3 + 1344a^7b^4x + 4704a^6b^5x^2 + 9408a^5b^6x^3 + 11760a^4b^7x^4 + 9408a^3b^8x^5 + 4704a^2b^9x^6 + 1344ab^{10}}$$

input
```
integrate((C*x**2+B*x+A)/(b**3*x**3+3*a*b**2*x**2+3*a**2*b*x+a**3)**3,x)
```

output
```
(-21*A*b**2 - 3*B*a*b - C*a**2 - 28*C*b**2*x**2 + x*(-24*B*b**2 - 8*C*a*b)
)/(168*a**8*b**3 + 1344*a**7*b**4*x + 4704*a**6*b**5*x**2 + 9408*a**5*b**6
*x**3 + 11760*a**4*b**7*x**4 + 9408*a**3*b**8*x**5 + 4704*a**2*b**9*x**6 +
 1344*a*b**10*x**7 + 168*b**11*x**8)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 132 vs. $2(63) = 126$.

Time = 0.04 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.91

$$\int \frac{A + Bx + Cx^2}{(a^3 + 3a^2bx + 3ab^2x^2 + b^3x^3)^3} dx = -\frac{28\,Cb^2x^2 + Ca^2 + 3\,Bab + 21\,Ab^2 + 8\,(Cab + 3\,Bb^2)x}{168\,(b^{11}x^8 + 8\,ab^{10}x^7 + 28\,a^2b^9x^6 + 56\,a^3b^8x^5 + 70\,a^4b^7x^4 + 56\,a^5b^6x^3 + 28\,a^6b^5x^2 + 8\,a^7b^4x + a^8b^3)}$$

input
```
integrate((C*x^2+B*x+A)/(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^3,x, algorithm
="maxima")
```

output
```
-1/168*(28*C*b^2*x^2 + C*a^2 + 3*B*a*b + 21*A*b^2 + 8*(C*a*b + 3*B*b^2)*x)
/(b^11*x^8 + 8*a*b^10*x^7 + 28*a^2*b^9*x^6 + 56*a^3*b^8*x^5 + 70*a^4*b^7*x
^4 + 56*a^5*b^6*x^3 + 28*a^6*b^5*x^2 + 8*a^7*b^4*x + a^8*b^3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.74

$$\int \frac{A + Bx + Cx^2}{(a^3 + 3a^2bx + 3ab^2x^2 + b^3x^3)^3} dx = -\frac{28\,Cb^2x^2 + 8\,Cabx + 24\,Bb^2x + Ca^2 + 3\,Bab + 21\,Ab^2}{168\,(bx + a)^8b^3}$$

input
```
integrate((C*x^2+B*x+A)/(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^3,x, algorithm
="giac")
```

output
```
-1/168*(28*C*b^2*x^2 + 8*C*a*b*x + 24*B*b^2*x + C*a^2 + 3*B*a*b + 21*A*b^2
)/((b*x + a)^8*b^3)
```

Mupad [B] (verification not implemented)

Time = 12.03 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.91

$$\int \frac{A+Bx+Cx^2}{(a^3+3a^2bx+3ab^2x^2+b^3x^3)^3}\,dx = -\frac{\frac{C\,a^2+3\,B\,a\,b+21\,A\,b^2}{168\,b^3}+\frac{C\,x^2}{6\,b}+\frac{x\,(3\,B\,b+C\,a)}{21\,b^2}}{a^8+8\,a^7\,b\,x+28\,a^6\,b^2\,x^2+56\,a^5\,b^3\,x^3+70\,a^4\,b^4\,x^4+56\,a^3\,b^5\,x^5+28\,a^2\,b^6\,x^6+8\,a\,b^7\,x^7+b^8\,x^8}$$

input
```
int((A + B*x + C*x^2)/(a^3 + b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x)^3,x)
```

output
```
-((21*A*b^2 + C*a^2 + 3*B*a*b)/(168*b^3) + (C*x^2)/(6*b) + (x*(3*B*b + C*a
))/(21*b^2))/(a^8 + b^8*x^8 + 8*a*b^7*x^7 + 28*a^6*b^2*x^2 + 56*a^5*b^3*x^
3 + 70*a^4*b^4*x^4 + 56*a^3*b^5*x^5 + 28*a^2*b^6*x^6 + 8*a^7*b*x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.78

$$\int \frac{A+Bx+Cx^2}{(a^3+3a^2bx+3ab^2x^2+b^3x^3)^3}\,dx = \frac{-28b^2c\,x^2-8abcx-24b^3x-a^2c-24a\,b^2}{168b^3\,(b^8x^8+8a\,b^7x^7+28a^2b^6x^6+56a^3b^5x^5+70a^4b^4x^4+56a^5b^3x^3+28a^6b^2x^2+8a^7bx+a^8)}$$

input
```
int((C*x^2+B*x+A)/(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^3,x)
```

output
```
( - a**2*c - 24*a*b**2 - 8*a*b*c*x - 24*b**3*x - 28*b**2*c*x**2)/(168*b**3
*(a**8 + 8*a**7*b*x + 28*a**6*b**2*x**2 + 56*a**5*b**3*x**3 + 70*a**4*b**4
*x**4 + 56*a**3*b**5*x**5 + 28*a**2*b**6*x**6 + 8*a*b**7*x**7 + b**8*x**8)
)
```

3.60 $\int (A+Bx+Cx^2)(a^3+3a^2bx+3ab^2x^2+b^3x^3)^{3/2}$ d

Optimal result

Integrand size = 42, antiderivative size = 160

$$\int (A+Bx+Cx^2)\left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)^{3/2}\,dx=\frac{2(Ab^2-a(bB-aC))(a+bx)\left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)^{3/2}}{11b^3}+\frac{2(bB-2aC)(a+bx)^2\left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)^{3/2}}{13b^3}+\frac{2C(a+bx)^3\left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)^{3/2}}{15b^3}$$

output
```
2/11*(A*b^2-a*(B*b-C*a))*(b*x+a)*(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^(3/2)
/b^3+2/13*(B*b-2*C*a)*(b*x+a)^2*(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^(3/2)/
b^3+2/15*C*(b*x+a)^3*(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^(3/2)/b^3
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.40

$$\int \left(A+Bx+Cx^2\right)\left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)^{3/2}\,dx=\frac{2(a+bx)^4\sqrt{(a+bx)^3}(195Ab^2+8a^2C+11b^2x(15B+13Cx)-2ab(15B+22Cx))}{2145b^3}$$

input
```
Integrate[(A + B*x + C*x^2)*(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^(3/2
),x]
```

output
```
(2*(a + b*x)^4*Sqrt[(a + b*x)^3]*(195*A*b^2 + 8*a^2*C + 11*b^2*x*(15*B + 1
3*C*x) - 2*a*b*(15*B + 22*C*x)))/(2145*b^3)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.60, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2008, 1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)^{3/2}\left(A+Bx+Cx^2\right)dx$$

$$\Big\downarrow \text{2008}$$

$$\frac{\left((a+bx)^3\right)^{3/2}\int (a+bx)^{9/2}\left(Cx^2+Bx+A\right)dx}{(a+bx)^{9/2}}$$

$$\Big\downarrow \text{1140}$$

$$\frac{\left((a+bx)^3\right)^{3/2}\int \left(\frac{C(a+bx)^{13/2}}{b^2}+\frac{(bB-2aC)(a+bx)^{11/2}}{b^2}+\frac{(Ab^2-a(bB-aC))(a+bx)^{9/2}}{b^2}\right)dx}{(a+bx)^{9/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left((a+bx)^3\right)^{3/2}\left(\frac{2(a+bx)^{11/2}(Ab^2-a(bB-aC))}{11b^3}+\frac{2(a+bx)^{13/2}(bB-2aC)}{13b^3}+\frac{2C(a+bx)^{15/2}}{15b^3}\right)}{(a+bx)^{9/2}}$$

input
```
Int[(A + B*x + C*x^2)*(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^(3/2),x]
```

output
```
(((a + b*x)^3)^(3/2)*((2*(A*b^2 - a*(b*B - a*C))*(a + b*x)^(11/2))/(11*b^3
) + (2*(b*B - 2*a*C)*(a + b*x)^(13/2))/(13*b^3) + (2*C*(a + b*x)^(15/2))/(
15*b^3)))/(a + b*x)^(9/2)
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2008
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{a = Rt[Coeff[Px, x, 0], Expon[Px,
x]], b = Rt[Coeff[Px, x, Expon[Px, x]], Expon[Px, x]]}, Simp[((a + b*x)^Exp
on[Px, x])^p/(a + b*x)^(Expon[Px, x]*p)   Int[u*(a + b*x)^(Expon[Px, x]*p),
 x], x] /; EqQ[Px, (a + b*x)^Expon[Px, x]]] /;  !IntegerQ[p] && PolyQ[Px, x
] && GtQ[Expon[Px, x], 1] && NeQ[Coeff[Px, x, 0], 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.50

method	result
gosper	$\frac{2(bx+a)\left(143C\,b^{2}x^{2}+165B\,b^{2}x-44Cabx+195A\,b^{2}-30abB+8C\,a^{2}\right)\left(b^{3}x^{3}+3a\,b^{2}x^{2}+3b\,a^{2}x+a^{3}\right)^{\frac{3}{2}}}{2145b^{3}}$
default	$\frac{2(bx+a)\left(143C\,b^{2}x^{2}+165B\,b^{2}x-44Cabx+195A\,b^{2}-30abB+8C\,a^{2}\right)\left(b^{3}x^{3}+3a\,b^{2}x^{2}+3b\,a^{2}x+a^{3}\right)^{\frac{3}{2}}}{2145b^{3}}$
orering	$\frac{2(bx+a)\left(143C\,b^{2}x^{2}+165B\,b^{2}x-44Cabx+195A\,b^{2}-30abB+8C\,a^{2}\right)\left(b^{3}x^{3}+3a\,b^{2}x^{2}+3b\,a^{2}x+a^{3}\right)^{\frac{3}{2}}}{2145b^{3}}$
trager	$\frac{2\left(143C\,b^{6}x^{6}+165B\,b^{6}x^{5}+528C\,b^{5}a\,x^{5}+195A\,x^{4}b^{6}+630B\,x^{4}a\,b^{5}+690C\,x^{4}a^{2}b^{4}+780A\,x^{3}a\,b^{5}+870B\,x^{3}a^{2}b^{4}+340C\,x^{3}a^{3}b^{3}+1170A\right.}{21}$
risch	$\frac{2\sqrt{(bx+a)^{3}}\,\left(143C\,b^{7}x^{7}+165B\,b^{7}x^{6}+671Ca\,b^{6}x^{6}+195A\,b^{7}x^{5}+795Ba\,b^{6}x^{5}+1218C\,a^{2}b^{5}x^{5}+975Aa\,b^{6}x^{4}+1500B\,a^{2}b^{5}x^{4}+1030C\,a\right.}{}$

input
```
int((C*x^2+B*x+A)*(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^(3/2),x,method=_RETU
RNVERBOSE)
```

output
```
2/2145*(b*x+a)*(143*C*b^2*x^2+165*B*b^2*x-44*C*a*b*x+195*A*b^2-30*B*a*b+8*
C*a^2)*(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^(3/2)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.28

$$\int \left(A+Bx+Cx^{2}\right)\left(a^{3}+3a^{2}bx+3ab^{2}x^{2}+b^{3}x^{3}\right)^{3/2}\,dx=\frac{2\left(143\,Cb^{6}x^{6}+8\,Ca^{6}-30\,Ba^{5}b+195\,Aa^{4}b^{2}+33\,(16\,Cab^{5}+5\,Bb^{6})x^{5}+15\,(46\,Ca^{2}b^{4}+4\right.}{}$$

input
```
integrate((C*x^2+B*x+A)*(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^(3/2),x, algor
ithm="fricas")
```

output
```
2/2145*(143*C*b^6*x^6 + 8*C*a^6 - 30*B*a^5*b + 195*A*a^4*b^2 + 33*(16*C*a*
b^5 + 5*B*b^6)*x^5 + 15*(46*C*a^2*b^4 + 42*B*a*b^5 + 13*A*b^6)*x^4 + 10*(3
4*C*a^3*b^3 + 87*B*a^2*b^4 + 78*A*a*b^5)*x^3 + 15*(C*a^4*b^2 + 32*B*a^3*b^
3 + 78*A*a^2*b^4)*x^2 - 3*(4*C*a^5*b - 15*B*a^4*b^2 - 260*A*a^3*b^3)*x)*sq
rt(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)/b^3
```

Sympy [F]

$$\int (A+Bx+Cx^2)\left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)^{3/2}\,dx=\int (A+Bx+Cx^2)\left((a+bx)^3\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((C*x**2+B*x+A)*(b**3*x**3+3*a*b**2*x**2+3*a**2*b*x+a**3)**(3/2),
x)
```

output
```
Integral((A + B*x + C*x**2)*((a + b*x)**3)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.21

$$\int (A+Bx+Cx^2)\left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)^{3/2}\,dx=\frac{2\,(b^4x^4+4\,ab^3x^3+6\,a^2b^2x^2+4\,a^3bx+a^4)(bx+a)^{\frac{3}{2}}A}{11\,b}+\frac{2\,(11\,b^5x^5+42\,ab^4x^4+58\,a^2b^3x^3+32\,a^3b^2x^2+3\,a^4bx-2\,a^5)(bx+a)^{\frac{3}{2}}B}{143\,b^2}+\frac{2\,(143\,b^6x^6+528\,ab^5x^5+690\,a^2b^4x^4+340\,a^3b^3x^3+15\,a^4b^2x^2-12\,a^5bx+8\,a^6)(bx+a)^{\frac{3}{2}}C}{2145\,b^3}$$

input
```
integrate((C*x^2+B*x+A)*(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^(3/2),x, algor
ithm="maxima")
```

output
```
2/11*(b^4*x^4 + 4*a*b^3*x^3 + 6*a^2*b^2*x^2 + 4*a^3*b*x + a^4)*(b*x + a)^(
3/2)*A/b + 2/143*(11*b^5*x^5 + 42*a*b^4*x^4 + 58*a^2*b^3*x^3 + 32*a^3*b^2*
x^2 + 3*a^4*b*x - 2*a^5)*(b*x + a)^(3/2)*B/b^2 + 2/2145*(143*b^6*x^6 + 528
*a*b^5*x^5 + 690*a^2*b^4*x^4 + 340*a^3*b^3*x^3 + 15*a^4*b^2*x^2 - 12*a^5*b
*x + 8*a^6)*(b*x + a)^(3/2)*C/b^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1138 vs. $2(148) = 296$.

Time = 0.14 (sec) , antiderivative size = 1138, normalized size of antiderivative = 7.11

$$\int (A + Bx + Cx^2) \left(a^3 + 3a^2bx + 3ab^2x^2 + b^3x^3\right)^{3/2} dx = \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)*(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^(3/2),x, algor
ithm="giac")
```

output
```
2/45045*(45045*sqrt(b*x + a)*A*a^5*sgn(b*x + a) + 75075*((b*x + a)^(3/2) -
 3*sqrt(b*x + a)*a)*A*a^4*sgn(b*x + a) + 15015*((b*x + a)^(3/2) - 3*sqrt(b
*x + a)*a)*B*a^5*sgn(b*x + a)/b + 30030*(3*(b*x + a)^(5/2) - 10*(b*x + a)^
(3/2)*a + 15*sqrt(b*x + a)*a^2)*A*a^3*sgn(b*x + a) + 3003*(3*(b*x + a)^(5/
2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)*C*a^5*sgn(b*x + a)/b^2 +
 15015*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)*B
*a^4*sgn(b*x + a)/b + 12870*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35
*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*A*a^2*sgn(b*x + a) + 6435*(5*
(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2 - 35*sqrt(
b*x + a)*a^3)*C*a^4*sgn(b*x + a)/b^2 + 12870*(5*(b*x + a)^(7/2) - 21*(b*x
+ a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*B*a^3*sgn(b*
x + a)/b + 715*(35*(b*x + a)^(9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*x + a)
^(5/2)*a^2 - 420*(b*x + a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)*A*a*sgn(b*x
+ a) + 1430*(35*(b*x + a)^(9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*x + a)^(5
/2)*a^2 - 420*(b*x + a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)*C*a^3*sgn(b*x +
 a)/b^2 + 1430*(35*(b*x + a)^(9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*x + a)
^(5/2)*a^2 - 420*(b*x + a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)*B*a^2*sgn(b*
x + a)/b + 65*(63*(b*x + a)^(11/2) - 385*(b*x + a)^(9/2)*a + 990*(b*x + a)
^(7/2)*a^2 - 1386*(b*x + a)^(5/2)*a^3 + 1155*(b*x + a)^(3/2)*a^4 - 693*sqr
t(b*x + a)*a^5)*A*sgn(b*x + a) + 650*(63*(b*x + a)^(11/2) - 385*(b*x + ...
```

Mupad [B] (verification not implemented)

Time = 12.26 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.51

$$\int (A+Bx+Cx^2)\left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)^{3/2}\,dx=\frac{2\,(a+b\,x)^4\,\sqrt{a^3+3\,a^2\,b\,x+3\,a\,b^2\,x^2+b^3\,x^3}\,\left(8\,C\,a^2-44\,C\,a\,b\,x-30\,B\,a\,b+143\,C\,b^2\,x^2+\right.}{2145\,b^3}$$

input
```
int((A + B*x + C*x^2)*(a^3 + b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x)^(3/2),x)
```

output
```
(2*(a + b*x)^4*(a^3 + b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x)^(1/2)*(195*A*b^2
+ 8*C*a^2 + 143*C*b^2*x^2 - 30*B*a*b + 165*B*b^2*x - 44*C*a*b*x))/(2145*b^
3)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.00

$$\int (A+Bx+Cx^2)\left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)^{3/2}\,dx=\frac{2\sqrt{bx+a}\,(143b^7c\,x^7+671a\,b^6c\,x^6+165b^8x^6+1218a^2b^5c\,x^5+990a\,b^7x^5+1030a^3b^4c\,x^4+}{}$$

input
```
int((C*x^2+B*x+A)*(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^(3/2),x)
```

output
```
(2*sqrt(a + b*x)*(8*a**7*c + 165*a**6*b**2 - 4*a**6*b*c*x + 990*a**5*b**3*
x + 3*a**5*b**2*c*x**2 + 2475*a**4*b**4*x**2 + 355*a**4*b**3*c*x**3 + 3300
*a**3*b**5*x**3 + 1030*a**3*b**4*c*x**4 + 2475*a**2*b**6*x**4 + 1218*a**2*
b**5*c*x**5 + 990*a*b**7*x**5 + 671*a*b**6*c*x**6 + 165*b**8*x**6 + 143*b*
*7*c*x**7))/(2145*b**3)
```

3.61 $\int (A+Bx+Cx^2)\sqrt{a^3+3a^2bx+3ab^2x^2+b^3x^3}\,dx$

Optimal result

Integrand size = 42, antiderivative size = 160

$$\begin{aligned}\int (A+Bx+Cx^2)\sqrt{a^3+3a^2bx+3ab^2x^2+b^3x^3}\,dx &= \frac{2(Ab^2-a(bB-aC))\,(a+bx)\sqrt{a^3+3a^2bx+3ab^2x^2+b^3x^3}}{5b^3}\\ &+\frac{2(bB-2aC)(a+bx)^2\sqrt{a^3+3a^2bx+3ab^2x^2+b^3x^3}}{7b^3}\\ &+\frac{2C(a+bx)^3\sqrt{a^3+3a^2bx+3ab^2x^2+b^3x^3}}{9b^3}\end{aligned}$$

output
```
2/5*(A*b^2-a*(B*b-C*a))*(b*x+a)*(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^(1/2)/
b^3+2/7*(B*b-2*C*a)*(b*x+a)^2*(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^(1/2)/b^
3+2/9*C*(b*x+a)^3*(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^(1/2)/b^3
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.39

$$\int \left(A+Bx+Cx^2\right)\sqrt{a^3+3a^2bx+3ab^2x^2+b^3x^3}\,dx = \frac{2(a+bx)\sqrt{(a+bx)^3}(63Ab^2+8a^2C+5b^2x(9B+7Cx)-2ab(9B+10Cx))}{315b^3}$$

input
```
Integrate[(A + B*x + C*x^2)*Sqrt[a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3],
x]
```

output
```
(2*(a + b*x)*Sqrt[(a + b*x)^3]*(63*A*b^2 + 8*a^2*C + 5*b^2*x*(9*B + 7*C*x)
 - 2*a*b*(9*B + 10*C*x)))/(315*b^3)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.60, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2008, 1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a^3+3a^2bx+3ab^2x^2+b^3x^3}\left(A+Bx+Cx^2\right)\,dx$$

$$\Big\downarrow \text{2008}$$

$$\frac{\sqrt{(a+bx)^3}\int (a+bx)^{3/2}\left(Cx^2+Bx+A\right)dx}{(a+bx)^{3/2}}$$

$$\Big\downarrow \text{1140}$$

$$\frac{\sqrt{(a+bx)^3}\int \left(\frac{C(a+bx)^{7/2}}{b^2}+\frac{(bB-2aC)(a+bx)^{5/2}}{b^2}+\frac{(Ab^2-a(bB-aC))(a+bx)^{3/2}}{b^2}\right)dx}{(a+bx)^{3/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{(a+bx)^3}\left(\frac{2(a+bx)^{5/2}(Ab^2-a(bB-aC))}{5b^3}+\frac{2(a+bx)^{7/2}(bB-2aC)}{7b^3}+\frac{2C(a+bx)^{9/2}}{9b^3}\right)}{(a+bx)^{3/2}}$$

input
```
Int[(A + B*x + C*x^2)*Sqrt[a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3],x]
```

output
```
(Sqrt[(a + b*x)^3]*((2*(A*b^2 - a*(b*B - a*C))*(a + b*x)^(5/2))/(5*b^3) +
(2*(b*B - 2*a*C)*(a + b*x)^(7/2))/(7*b^3) + (2*C*(a + b*x)^(9/2))/(9*b^3))
)/(a + b*x)^(3/2)
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2008
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{a = Rt[Coeff[Px, x, 0], Expon[Px,
x]], b = Rt[Coeff[Px, x, Expon[Px, x]], Expon[Px, x]]}, Simp[((a + b*x)^Exp
on[Px, x])^p/(a + b*x)^(Expon[Px, x]*p)   Int[u*(a + b*x)^(Expon[Px, x]*p),
 x], x] /; EqQ[Px, (a + b*x)^Expon[Px, x]]] /;  !IntegerQ[p] && PolyQ[Px, x
] && GtQ[Expon[Px, x], 1] && NeQ[Coeff[Px, x, 0], 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.50

method	result
gosper	$\frac{2(bx+a)\left(35C\,b^{2}x^{2}+45B\,b^{2}x-20Cabx+63A\,b^{2}-18abB+8C\,a^{2}\right)\sqrt{b^{3}x^{3}+3a\,b^{2}x^{2}+3b\,a^{2}x+a^{3}}}{315b^{3}}$
default	$\frac{2(bx+a)\left(35C\,b^{2}x^{2}+45B\,b^{2}x-20Cabx+63A\,b^{2}-18abB+8C\,a^{2}\right)\sqrt{b^{3}x^{3}+3a\,b^{2}x^{2}+3b\,a^{2}x+a^{3}}}{315b^{3}}$
orering	$\frac{2(bx+a)\left(35C\,b^{2}x^{2}+45B\,b^{2}x-20Cabx+63A\,b^{2}-18abB+8C\,a^{2}\right)\sqrt{b^{3}x^{3}+3a\,b^{2}x^{2}+3b\,a^{2}x+a^{3}}}{315b^{3}}$
trager	$\frac{2\left(35C\,x^{3}b^{3}+45B\,b^{3}x^{2}+15Ca\,b^{2}x^{2}+63A\,b^{3}x+27Ba\,b^{2}x-12C\,a^{2}bx+63Aa\,b^{2}-18B\,a^{2}b+8C\,a^{3}\right)\sqrt{b^{3}x^{3}+3a\,b^{2}x^{2}+3b\,a^{2}x+a^{3}}}{315b^{3}}$
risch	$\frac{2\sqrt{(bx+a)^{3}}\,(35C\,x^{4}b^{4}+45B\,b^{4}x^{3}+50Ca\,b^{3}x^{3}+63A\,b^{4}x^{2}+72Ba\,b^{3}x^{2}+3C\,a^{2}b^{2}x^{2}+126Aa\,b^{3}x+9B\,a^{2}b^{2}x-4C\,a^{3}bx+63A\,a^{2}b^{2}-18}{315(bx+a)b^{3}}$

input
```
int((C*x^2+B*x+A)*(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^(1/2),x,method=_RETU
RNVERBOSE)
```

output
```
2/315*(b*x+a)*(35*C*b^2*x^2+45*B*b^2*x-20*C*a*b*x+63*A*b^2-18*B*a*b+8*C*a^
2)*(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^(1/2)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.66

$$\int (A+Bx+Cx^2)\sqrt{a^3+3a^2bx+3ab^2x^2+b^3x^3}\,dx$$
$$=\frac{2\,(35\,Cb^3x^3+8\,Ca^3-18\,Ba^2b+63\,Aab^2+15\,(Cab^2+3\,Bb^3)x^2-3\,(4\,Ca^2b-9\,Bab^2-21\,Ab^3)x)\sqrt{b^3x}}{315\,b^3}$$

input
```
integrate((C*x^2+B*x+A)*(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^(1/2),x, algor
ithm="fricas")
```

output
```
2/315*(35*C*b^3*x^3 + 8*C*a^3 - 18*B*a^2*b + 63*A*a*b^2 + 15*(C*a*b^2 + 3*
B*b^3)*x^2 - 3*(4*C*a^2*b - 9*B*a*b^2 - 21*A*b^3)*x)*sqrt(b^3*x^3 + 3*a*b^
2*x^2 + 3*a^2*b*x + a^3)/b^3
```

Sympy [F]

$$\int \left(A+Bx+Cx^2\right)\sqrt{a^3+3a^2bx+3ab^2x^2+b^3x^3}\,dx=\int \left(A+Bx+Cx^2\right)\sqrt{\left(a+bx\right)^3}\,dx$$

input
```
integrate((C*x**2+B*x+A)*(b**3*x**3+3*a*b**2*x**2+3*a**2*b*x+a**3)**(1/2),
x)
```

output
```
Integral((A + B*x + C*x**2)*sqrt((a + b*x)**3), x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.56

$$\int \left(A+Bx+Cx^2\right)\sqrt{a^3+3a^2bx+3ab^2x^2+b^3x^3}\,dx = \frac{2\,(bx+a)^{\frac{5}{2}}A}{5\,b}+\frac{2\,(5\,b^2x^2+3\,abx-2\,a^2)(bx+a)^{\frac{3}{2}}B}{35\,b^2} + \frac{2\,(35\,b^3x^3+15\,ab^2x^2-12\,a^2bx+8\,a^3)(bx+a)^{\frac{3}{2}}C}{315\,b^3}$$

input
```
integrate((C*x^2+B*x+A)*(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^(1/2),x, algor
ithm="maxima")
```

output
```
2/5*(b*x + a)^(5/2)*A/b + 2/35*(5*b^2*x^2 + 3*a*b*x - 2*a^2)*(b*x + a)^(3/
2)*B/b^2 + 2/315*(35*b^3*x^3 + 15*a*b^2*x^2 - 12*a^2*b*x + 8*a^3)*(b*x + a
)^(3/2)*C/b^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 399 vs. $2(148) = 296$.

Time = 0.12 (sec) , antiderivative size = 399, normalized size of antiderivative = 2.49

$$\int (A + Bx + Cx^2)\sqrt{a^3 + 3a^2bx + 3ab^2x^2 + b^3x^3}\,dx$$
$$= \frac{2\left(315\sqrt{bx+a}Aa^2\mathrm{sgn}(bx+a) + 210\left((bx+a)^{\frac{3}{2}} - 3\sqrt{bx+a}a\right)Aa\mathrm{sgn}(bx+a) + \frac{105\left((bx+a)^{\frac{3}{2}}-3\sqrt{bx+a}a\right)\dots}{b}\right.}{}$$

input
```
integrate((C*x^2+B*x+A)*(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^(1/2),x, algor
ithm="giac")
```

output
```
2/315*(315*sqrt(b*x + a)*A*a^2*sgn(b*x + a) + 210*((b*x + a)^(3/2) - 3*sqr
t(b*x + a)*a)*A*a*sgn(b*x + a) + 105*((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a)
*B*a^2*sgn(b*x + a)/b + 21*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*
sqrt(b*x + a)*a^2)*A*sgn(b*x + a) + 21*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(
3/2)*a + 15*sqrt(b*x + a)*a^2)*C*a^2*sgn(b*x + a)/b^2 + 42*(3*(b*x + a)^(5
/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)*B*a*sgn(b*x + a)/b + 18
*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2 - 35*s
qrt(b*x + a)*a^3)*C*a*sgn(b*x + a)/b^2 + 9*(5*(b*x + a)^(7/2) - 21*(b*x +
a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*B*sgn(b*x + a)
/b + (35*(b*x + a)^(9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2
 - 420*(b*x + a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)*C*sgn(b*x + a)/b^2)/b
```

Mupad [B] (verification not implemented)

Time = 12.52 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.49

$$\int (A + Bx + Cx^2)\sqrt{a^3 + 3a^2bx + 3ab^2x^2 + b^3x^3}\,dx$$
$$= \frac{2\,(a+b\,x)\,\sqrt{a^3+3\,a^2\,b\,x+3\,a\,b^2\,x^2+b^3\,x^3}\,(8\,C\,a^2-20\,C\,a\,b\,x-18\,B\,a\,b+35\,C\,b^2\,x^2+45\,B\,b^2\,x+63\dots}{315\,b^3}$$

input
```
int((A + B*x + C*x^2)*(a^3 + b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x)^(1/2),x)
```

output
```
(2*(a + b*x)*(a^3 + b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x)^(1/2)*(63*A*b^2 + 8
*C*a^2 + 35*C*b^2*x^2 - 18*B*a*b + 45*B*b^2*x - 20*C*a*b*x))/(315*b^3)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.57

$$\int (A + Bx + Cx^2) \sqrt{a^3 + 3a^2bx + 3ab^2x^2 + b^3x^3}\,dx$$
$$= \frac{2\sqrt{bx+a}\,(35b^4c\,x^4 + 50a\,b^3c\,x^3 + 45b^5x^3 + 3a^2b^2c\,x^2 + 135a\,b^4x^2 - 4a^3bcx + 135a^2b^3x + 8a^4c + 45a^3b^2}{315b^3}$$

input
```
int((C*x^2+B*x+A)*(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^(1/2),x)
```

output
```
(2*sqrt(a + b*x)*(8*a**4*c + 45*a**3*b**2 - 4*a**3*b*c*x + 135*a**2*b**3*x
 + 3*a**2*b**2*c*x**2 + 135*a*b**4*x**2 + 50*a*b**3*c*x**3 + 45*b**5*x**3
+ 35*b**4*c*x**4))/(315*b**3)
```

3.62 $\int \frac{A+Bx+Cx^2}{\sqrt{a^3+3a^2bx+3ab^2x^2+b^3x^3}}\,dx$

Optimal result

Integrand size = 42, antiderivative size = 156

$$\int \frac{A+Bx+Cx^2}{\sqrt{a^3+3a^2bx+3ab^2x^2+b^3x^3}}\,dx = -\frac{2(Ab^2-a(bB-aC))\,(a+bx)}{b^3\sqrt{a^3+3a^2bx+3ab^2x^2+b^3x^3}} + \frac{2(bB-2aC)(a+bx)^2}{b^3\sqrt{a^3+3a^2bx+3ab^2x^2+b^3x^3}} + \frac{2C(a+bx)^3}{3b^3\sqrt{a^3+3a^2bx+3ab^2x^2+b^3x^3}}$$

output
```
-2*(A*b^2-a*(B*b-C*a))*(b*x+a)/b^3/(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^(1/
2)+2*(B*b-2*C*a)*(b*x+a)^2/b^3/(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^(1/2)+2
/3*C*(b*x+a)^3/b^3/(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^(1/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.38

$$\int \frac{A+Bx+Cx^2}{\sqrt{a^3+3a^2bx+3ab^2x^2+b^3x^3}}\,dx = \frac{2(a+bx)\,(-3Ab^2-8a^2C+ab(6B-4Cx)+b^2x(3B+Cx))}{3b^3\sqrt{(a+bx)^3}}$$

input
```
Integrate[(A + B*x + C*x^2)/Sqrt[a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3],
x]
```

output
```
(2*(a + b*x)*(-3*A*b^2 - 8*a^2*C + a*b*(6*B - 4*C*x) + b^2*x*(3*B + C*x)))
/(3*b^3*Sqrt[(a + b*x)^3])
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.59, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2008, 1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{\sqrt{a^3+3a^2bx+3ab^2x^2+b^3x^3}}dx$$

$$\Big\downarrow \text{ 2008}$$

$$\frac{(a+bx)^{3/2}\int \frac{Cx^2+Bx+A}{(a+bx)^{3/2}}dx}{\sqrt{(a+bx)^3}}$$

$$\Big\downarrow \text{ 1140}$$

$$\frac{(a+bx)^{3/2}\int \left(\frac{\sqrt{a+bx}C}{b^2}+\frac{bB-2aC}{b^2\sqrt{a+bx}}+\frac{Ab^2-a(bB-aC)}{b^2(a+bx)^{3/2}}\right)dx}{\sqrt{(a+bx)^3}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{(a+bx)^{3/2}\left(-\frac{2(Ab^2-a(bB-aC))}{b^3\sqrt{a+bx}}+\frac{2\sqrt{a+bx}(bB-2aC)}{b^3}+\frac{2C(a+bx)^{3/2}}{3b^3}\right)}{\sqrt{(a+bx)^3}}$$

input
```
Int[(A + B*x + C*x^2)/Sqrt[a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3],x]
```

output
```
((a + b*x)^(3/2)*((-2*(A*b^2 - a*(b*B - a*C)))/(b^3*Sqrt[a + b*x]) + (2*(b
*B - 2*a*C)*Sqrt[a + b*x])/b^3 + (2*C*(a + b*x)^(3/2))/(3*b^3)))/Sqrt[(a +
 b*x)^3]
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2008
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{a = Rt[Coeff[Px, x, 0], Expon[Px,
x]], b = Rt[Coeff[Px, x, Expon[Px, x]], Expon[Px, x]]}, Simp[((a + b*x)^Exp
on[Px, x])^p/(a + b*x)^(Expon[Px, x]*p)   Int[u*(a + b*x)^(Expon[Px, x]*p),
 x], x] /; EqQ[Px, (a + b*x)^Expon[Px, x]]] /;  !IntegerQ[p] && PolyQ[Px, x
] && GtQ[Expon[Px, x], 1] && NeQ[Coeff[Px, x, 0], 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.46

method	result	size
risch	$\frac{2(Cxb+3Bb-5Ca)(bx+a)^2}{3b^3\sqrt{(bx+a)^3}}-\frac{2(A\,b^2-abB+C\,a^2)(bx+a)}{b^3\sqrt{(bx+a)^3}}$	71
gosper	$-\frac{2(bx+a)\left(-C\,b^2x^2-3B\,b^2x+4Cabx+3A\,b^2-6abB+8C\,a^2\right)}{3b^3\sqrt{b^3x^3+3a\,b^2x^2+3b\,a^2x+a^3}}$	80
default	$-\frac{2(bx+a)\left(-C\,b^2x^2-3B\,b^2x+4Cabx+3A\,b^2-6abB+8C\,a^2\right)}{3b^3\sqrt{b^3x^3+3a\,b^2x^2+3b\,a^2x+a^3}}$	80
orering	$-\frac{2(bx+a)\left(-C\,b^2x^2-3B\,b^2x+4Cabx+3A\,b^2-6abB+8C\,a^2\right)}{3b^3\sqrt{b^3x^3+3a\,b^2x^2+3b\,a^2x+a^3}}$	80
trager	$-\frac{2\left(-C\,b^2x^2-3B\,b^2x+4Cabx+3A\,b^2-6abB+8C\,a^2\right)\sqrt{b^3x^3+3a\,b^2x^2+3b\,a^2x+a^3}}{3(bx+a)^2b^3}$	82

input
```
int((C*x^2+B*x+A)/(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^(1/2),x,method=_RETU
RNVERBOSE)
```

output
```
2/3*(C*b*x+3*B*b-5*C*a)*(b*x+a)^2/b^3/((b*x+a)^3)^(1/2)-2/b^3*(A*b^2-B*a*b
+C*a^2)*(b*x+a)/((b*x+a)^3)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.62

$$\int \frac{A + Bx + Cx^2}{\sqrt{a^3 + 3a^2bx + 3ab^2x^2 + b^3x^3}} \, dx = \frac{2\sqrt{b^3x^3 + 3\,ab^2x^2 + 3\,a^2bx + a^3}(Cb^2x^2 - 8\,Ca^2 + 6\,Bab - 3\,Ab^2 - (4\,Cab - 3\,Bb^2)x)}{3\,(b^5x^2 + 2\,ab^4x + a^2b^3)}$$

input
```
integrate((C*x^2+B*x+A)/(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^(1/2),x, algor
ithm="fricas")
```

output
```
2/3*sqrt(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)*(C*b^2*x^2 - 8*C*a^2 + 6
*B*a*b - 3*A*b^2 - (4*C*a*b - 3*B*b^2)*x)/(b^5*x^2 + 2*a*b^4*x + a^2*b^3)
```

Sympy [F]

$$\int \frac{A + Bx + Cx^2}{\sqrt{a^3 + 3a^2bx + 3ab^2x^2 + b^3x^3}} \, dx = \int \frac{A + Bx + Cx^2}{\sqrt{\left(a + bx\right)^3}} \, dx$$

input
```
integrate((C*x**2+B*x+A)/(b**3*x**3+3*a*b**2*x**2+3*a**2*b*x+a**3)**(1/2),
x)
```

output
```
Integral((A + B*x + C*x**2)/sqrt((a + b*x)**3), x)
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.56

$$\int \frac{A+Bx+Cx^2}{\sqrt{a^3+3a^2bx+3ab^2x^2+b^3x^3}}\,dx = -\frac{2\,A}{\sqrt{bx+a}b} + \frac{2\,(b^2x^2+3\,abx+2\,a^2)B}{(bx+a)^{\frac{3}{2}}b^2} + \frac{2\,(b^3x^3-3\,ab^2x^2-12\,a^2bx-8\,a^3)C}{3\,(bx+a)^{\frac{3}{2}}b^3}$$

input
```
integrate((C*x^2+B*x+A)/(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^(1/2),x, algor
ithm="maxima")
```

output
```
-2*A/(sqrt(b*x + a)*b) + 2*(b^2*x^2 + 3*a*b*x + 2*a^2)*B/((b*x + a)^(3/2)*
b^2) + 2/3*(b^3*x^3 - 3*a*b^2*x^2 - 12*a^2*b*x - 8*a^3)*C/((b*x + a)^(3/2)
*b^3)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.58

$$\int \frac{A+Bx+Cx^2}{\sqrt{a^3+3a^2bx+3ab^2x^2+b^3x^3}}\,dx = -\frac{2\,(Ca^2-Bab+Ab^2)}{\sqrt{bx+a}b^3\mathrm{sgn}\,(bx+a)} + \frac{2\,\left((bx+a)^{\frac{3}{2}}Cb^6-6\,\sqrt{bx+a}Cab^6+3\,\sqrt{bx+a}Bb^7\right)}{3\,b^9\mathrm{sgn}\,(bx+a)}$$

input
```
integrate((C*x^2+B*x+A)/(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^(1/2),x, algor
ithm="giac")
```

output
```
-2*(C*a^2 - B*a*b + A*b^2)/(sqrt(b*x + a)*b^3*sgn(b*x + a)) + 2/3*((b*x +
a)^(3/2)*C*b^6 - 6*sqrt(b*x + a)*C*a*b^6 + 3*sqrt(b*x + a)*B*b^7)/(b^9*sgn
(b*x + a))
```

Mupad [B] (verification not implemented)

Time = 12.82 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.52

$$\int \frac{A+Bx+Cx^2}{\sqrt{a^3+3a^2bx+3ab^2x^2+b^3x^3}}\,dx = -\frac{2\sqrt{a^3+3\,a^2\,b\,x+3\,a\,b^2\,x^2+b^3\,x^3}\,\left(8\,C\,a^2+4\,C\,a\,b\,x-6\,B\,a\,b-C\,b^2\,x^2-3\,B\,b^2\,x+3\,A\,b^2\right)}{3\,b^3\,{\left(a+b\,x\right)}^2}$$

input
```
int((A + B*x + C*x^2)/(a^3 + b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x)^(1/2),x)
```

output
```
-(2*(a^3 + b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x)^(1/2)*(3*A*b^2 + 8*C*a^2 - C
*b^2*x^2 - 6*B*a*b - 3*B*b^2*x + 4*C*a*b*x))/(3*b^3*(a + b*x)^2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.29

$$\int \frac{A+Bx+Cx^2}{\sqrt{a^3+3a^2bx+3ab^2x^2+b^3x^3}}\,dx = \frac{\frac{2}{3}b^2c\,x^2-\frac{8}{3}abcx+2b^3x-\frac{16}{3}a^2c+2a\,b^2}{\sqrt{bx+a}\,b^3}$$

input
```
int((C*x^2+B*x+A)/(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^(1/2),x)
```

output
```
(2*( - 8*a**2*c + 3*a*b**2 - 4*a*b*c*x + 3*b**3*x + b**2*c*x**2))/(3*sqrt(
a + b*x)*b**3)
```

3.63 $\int \frac{A+Bx+Cx^2}{\left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)^{3/2}}\,dx$

Optimal result

Integrand size = 42, antiderivative size = 160

$$\int \frac{A+Bx+Cx^2}{\left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)^{3/2}}\,dx=-\frac{2(Ab^2-a(bB-aC))\,(a+bx)}{7b^3\left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)^{3/2}}-\frac{2(bB-2aC)(a+bx)^2}{5b^3\left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)^{3/2}}-\frac{2C(a+bx)^3}{3b^3\left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)^{3/2}}$$

output
```
-2/7*(A*b^2-a*(B*b-C*a))*(b*x+a)/b^3/(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^(
3/2)-2/5*(B*b-2*C*a)*(b*x+a)^2/b^3/(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^(3/
2)-2/3*C*(b*x+a)^3/b^3/(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^(3/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.39

$$\int \frac{A+Bx+Cx^2}{\left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)^{3/2}}\,dx=-\frac{2(15Ab^2+8a^2C+7b^2x(3B+5Cx)+ab(6B+28Cx))}{105b^3(a+bx)^2\sqrt{(a+bx)^3}}$$

input
```
Integrate[(A + B*x + C*x^2)/(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^(3/2
),x]
```

output
```
(-2*(15*A*b^2 + 8*a^2*C + 7*b^2*x*(3*B + 5*C*x) + a*b*(6*B + 28*C*x)))/(10
5*b^3*(a + b*x)^2*Sqrt[(a + b*x)^3])
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.60, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2008, 1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{(a^3+3a^2bx+3ab^2x^2+b^3x^3)^{3/2}}dx$$

$$\Big\downarrow \text{ 2008}$$

$$\frac{(a+bx)^{9/2}\int \frac{Cx^2+Bx+A}{(a+bx)^{9/2}}dx}{((a+bx)^3)^{3/2}}$$

$$\Big\downarrow \text{ 1140}$$

$$\frac{(a+bx)^{9/2}\int \left(\frac{C}{b^2(a+bx)^{5/2}}+\frac{bB-2aC}{b^2(a+bx)^{7/2}}+\frac{Ab^2-a(bB-aC)}{b^2(a+bx)^{9/2}}\right)dx}{((a+bx)^3)^{3/2}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{(a+bx)^{9/2}\left(-\frac{2(Ab^2-a(bB-aC))}{7b^3(a+bx)^{7/2}}-\frac{2(bB-2aC)}{5b^3(a+bx)^{5/2}}-\frac{2C}{3b^3(a+bx)^{3/2}}\right)}{((a+bx)^3)^{3/2}}$$

input
```
Int[(A + B*x + C*x^2)/(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^(3/2),x]
```

output
```
((a + b*x)^(9/2)*((-2*(A*b^2 - a*(b*B - a*C)))/(7*b^3*(a + b*x)^(7/2)) - (
2*(b*B - 2*a*C))/(5*b^3*(a + b*x)^(5/2)) - (2*C)/(3*b^3*(a + b*x)^(3/2))))
/((a + b*x)^3)^(3/2)
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2008
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{a = Rt[Coeff[Px, x, 0], Expon[Px,
x]], b = Rt[Coeff[Px, x, Expon[Px, x]], Expon[Px, x]]}, Simp[((a + b*x)^Exp
on[Px, x])^p/(a + b*x)^(Expon[Px, x]*p)   Int[u*(a + b*x)^(Expon[Px, x]*p),
 x], x] /; EqQ[Px, (a + b*x)^Expon[Px, x]]] /;  !IntegerQ[p] && PolyQ[Px, x
] && GtQ[Expon[Px, x], 1] && NeQ[Coeff[Px, x, 0], 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.50

method	result	size
gosper	$-\frac{2(bx+a)\left(35C\,b^{2}x^{2}+21B\,b^{2}x+28Cabx+15A\,b^{2}+6abB+8C\,a^{2}\right)}{105b^{3}\left(b^{3}x^{3}+3a\,b^{2}x^{2}+3b\,a^{2}x+a^{3}\right)^{\frac{3}{2}}}$	80
default	$-\frac{2(bx+a)\left(35C\,b^{2}x^{2}+21B\,b^{2}x+28Cabx+15A\,b^{2}+6abB+8C\,a^{2}\right)}{105b^{3}\left(b^{3}x^{3}+3a\,b^{2}x^{2}+3b\,a^{2}x+a^{3}\right)^{\frac{3}{2}}}$	80
orering	$-\frac{2(bx+a)\left(35C\,b^{2}x^{2}+21B\,b^{2}x+28Cabx+15A\,b^{2}+6abB+8C\,a^{2}\right)}{105b^{3}\left(b^{3}x^{3}+3a\,b^{2}x^{2}+3b\,a^{2}x+a^{3}\right)^{\frac{3}{2}}}$	80
trager	$-\frac{2\left(35C\,b^{2}x^{2}+21B\,b^{2}x+28Cabx+15A\,b^{2}+6abB+8C\,a^{2}\right)\sqrt{b^{3}x^{3}+3a\,b^{2}x^{2}+3b\,a^{2}x+a^{3}}}{105\left(bx+a\right)^{5}b^{3}}$	82

input
```
int((C*x^2+B*x+A)/(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^(3/2),x,method=_RETU
RNVERBOSE)
```

output
```
-2/105*(b*x+a)*(35*C*b^2*x^2+21*B*b^2*x+28*C*a*b*x+15*A*b^2+6*B*a*b+8*C*a^
2)/b^3/(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.81

$$\int \frac{A+Bx+Cx^2}{(a^3+3a^2bx+3ab^2x^2+b^3x^3)^{3/2}}\,dx = -\frac{2\sqrt{b^3x^3+3\,ab^2x^2+3\,a^2bx+a^3}(35\,Cb^2x^2+8\,Ca^2+6\,Bab+15\,Ab^2+7\,(4\,Cab+3\,Bb^2)x)}{105\,(b^8x^5+5\,ab^7x^4+10\,a^2b^6x^3+10\,a^3b^5x^2+5\,a^4b^4x+a^5b^3)}$$

input
```
integrate((C*x^2+B*x+A)/(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^(3/2),x, algor
ithm="fricas")
```

output
```
-2/105*sqrt(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)*(35*C*b^2*x^2 + 8*C*a
^2 + 6*B*a*b + 15*A*b^2 + 7*(4*C*a*b + 3*B*b^2)*x)/(b^8*x^5 + 5*a*b^7*x^4
+ 10*a^2*b^6*x^3 + 10*a^3*b^5*x^2 + 5*a^4*b^4*x + a^5*b^3)
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2}{(a^3+3a^2bx+3ab^2x^2+b^3x^3)^{3/2}}\,dx = \int \frac{A+Bx+Cx^2}{\left((a+bx)^3\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((C*x**2+B*x+A)/(b**3*x**3+3*a*b**2*x**2+3*a**2*b*x+a**3)**(3/2),
x)
```

output
```
Integral((A + B*x + C*x**2)/((a + b*x)**3)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.80

$$\int \frac{A+Bx+Cx^2}{(a^3+3a^2bx+3ab^2x^2+b^3x^3)^{3/2}}\,dx = -\frac{2\,(7\,bx+2\,a)B}{35\,(b^4x^2+2\,ab^3x+a^2b^2)(bx+a)^{\frac{3}{2}}} - \frac{2\,(35\,b^2x^2+28\,abx+8\,a^2)C}{105\,(b^5x^2+2\,ab^4x+a^2b^3)(bx+a)^{\frac{3}{2}}} - \frac{2\,A}{7\,(b^3x^2+2\,ab^2x+a^2b)(bx+a)^{\frac{3}{2}}}$$

input

```
integrate((C*x^2+B*x+A)/(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^(3/2),x, algor
ithm="maxima")
```

output

```
-2/35*(7*b*x + 2*a)*B/((b^4*x^2 + 2*a*b^3*x + a^2*b^2)*(b*x + a)^(3/2)) -
2/105*(35*b^2*x^2 + 28*a*b*x + 8*a^2)*C/((b^5*x^2 + 2*a*b^4*x + a^2*b^3)*(
b*x + a)^(3/2)) - 2/7*A/((b^3*x^2 + 2*a*b^2*x + a^2*b)*(b*x + a)^(3/2))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.41

$$\int \frac{A+Bx+Cx^2}{(a^3+3a^2bx+3ab^2x^2+b^3x^3)^{3/2}}\,dx = -\frac{2\left(35\,(bx+a)^2C-42\,(bx+a)Ca+15\,Ca^2+21\,(bx+a)Bb-15\,Bab+15\,Ab^2\right)}{105\,(bx+a)^{\frac{7}{2}}b^3\mathrm{sgn}\,(bx+a)}$$

input

```
integrate((C*x^2+B*x+A)/(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^(3/2),x, algor
ithm="giac")
```

output

```
-2/105*(35*(b*x + a)^2*C - 42*(b*x + a)*C*a + 15*C*a^2 + 21*(b*x + a)*B*b
- 15*B*a*b + 15*A*b^2)/((b*x + a)^(7/2)*b^3*sgn(b*x + a))
```

Mupad [B] (verification not implemented)

Time = 12.75 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.51

$$\int \frac{A+Bx+Cx^2}{(a^3+3a^2bx+3ab^2x^2+b^3x^3)^{3/2}}\,dx = -\frac{2\sqrt{a^3+3\,a^2\,b\,x+3\,a\,b^2\,x^2+b^3\,x^3}\,\left(8\,C\,a^2+28\,C\,a\,b\,x+6\,B\,a\,b+35\,C\,b^2\,x^2+21\,B\,b^2\,x+15\,A\,b^2\right)}{105\,b^3\,{\left(a+b\,x\right)}^5}$$

input
```
int((A + B*x + C*x^2)/(a^3 + b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x)^(3/2),x)
```

output
```
-(2*(a^3 + b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x)^(1/2)*(15*A*b^2 + 8*C*a^2 +
35*C*b^2*x^2 + 6*B*a*b + 21*B*b^2*x + 28*C*a*b*x))/(105*b^3*(a + b*x)^5)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.48

$$\int \frac{A+Bx+Cx^2}{(a^3+3a^2bx+3ab^2x^2+b^3x^3)^{3/2}}\,dx = \frac{-\frac{2}{3}b^2c\,x^2-\frac{8}{15}abcx-\frac{2}{5}b^3x-\frac{16}{105}a^2c-\frac{2}{5}a\,b^2}{\sqrt{bx+a}\,b^3\left(b^3x^3+3a\,b^2x^2+3a^2bx+a^3\right)}$$

input
```
int((C*x^2+B*x+A)/(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^(3/2),x)
```

output
```
(2*( - 8*a**2*c - 21*a*b**2 - 28*a*b*c*x - 21*b**3*x - 35*b**2*c*x**2))/(1
05*sqrt(a + b*x)*b**3*(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3))
```

3.64 $\int \frac{A+Bx+Cx^2}{(a^3+3a^2bx+3ab^2x^2+b^3x^3)^{5/2}}\,dx$

Optimal result

Integrand size = 42, antiderivative size = 160

$$\int \frac{A+Bx+Cx^2}{(a^3+3a^2bx+3ab^2x^2+b^3x^3)^{5/2}}\,dx = -\frac{2(Ab^2-a(bB-aC))\,(a+bx)}{13b^3\,(a^3+3a^2bx+3ab^2x^2+b^3x^3)^{5/2}} - \frac{2(bB-2aC)(a+bx)^2}{11b^3\,(a^3+3a^2bx+3ab^2x^2+b^3x^3)^{5/2}} - \frac{2C(a+bx)^3}{9b^3\,(a^3+3a^2bx+3ab^2x^2+b^3x^3)^{5/2}}$$

output
```
-2/13*(A*b^2-a*(B*b-C*a))*(b*x+a)/b^3/(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^
(5/2)-2/11*(B*b-2*C*a)*(b*x+a)^2/b^3/(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^(
5/2)-2/9*C*(b*x+a)^3/b^3/(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^(5/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.40

$$\int \frac{A+Bx+Cx^2}{(a^3+3a^2bx+3ab^2x^2+b^3x^3)^{5/2}}\,dx = -\frac{2(99Ab^2+8a^2C+13b^2x(9B+11Cx)+2ab(9B+26Cx))}{1287b^3(a+bx)^5\sqrt{(a+bx)^3}}$$

input
```
Integrate[(A + B*x + C*x^2)/(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^(5/2
),x]
```

output
```
(-2*(99*A*b^2 + 8*a^2*C + 13*b^2*x*(9*B + 11*C*x) + 2*a*b*(9*B + 26*C*x)))
/(1287*b^3*(a + b*x)^5*Sqrt[(a + b*x)^3])
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.60, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2008, 1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{(a^3+3a^2bx+3ab^2x^2+b^3x^3)^{5/2}}dx$$

$$\Big\downarrow \text{2008}$$

$$\frac{(a+bx)^{15/2}\int \frac{Cx^2+Bx+A}{(a+bx)^{15/2}}dx}{((a+bx)^3)^{5/2}}$$

$$\Big\downarrow \text{1140}$$

$$\frac{(a+bx)^{15/2}\int \left(\frac{C}{b^2(a+bx)^{11/2}}+\frac{bB-2aC}{b^2(a+bx)^{13/2}}+\frac{Ab^2-a(bB-aC)}{b^2(a+bx)^{15/2}}\right)dx}{((a+bx)^3)^{5/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(a+bx)^{15/2}\left(-\frac{2(Ab^2-a(bB-aC))}{13b^3(a+bx)^{13/2}}-\frac{2(bB-2aC)}{11b^3(a+bx)^{11/2}}-\frac{2C}{9b^3(a+bx)^{9/2}}\right)}{((a+bx)^3)^{5/2}}$$

input
```
Int[(A + B*x + C*x^2)/(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^(5/2),x]
```

output
```
((a + b*x)^(15/2)*((-2*(A*b^2 - a*(b*B - a*C)))/(13*b^3*(a + b*x)^(13/2))
- (2*(b*B - 2*a*C))/(11*b^3*(a + b*x)^(11/2)) - (2*C)/(9*b^3*(a + b*x)^(9/
2))))/((a + b*x)^3)^(5/2)
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2008
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{a = Rt[Coeff[Px, x, 0], Expon[Px,
x]], b = Rt[Coeff[Px, x, Expon[Px, x]], Expon[Px, x]]}, Simp[((a + b*x)^Exp
on[Px, x])^p/(a + b*x)^(Expon[Px, x]*p)   Int[u*(a + b*x)^(Expon[Px, x]*p),
 x], x] /; EqQ[Px, (a + b*x)^Expon[Px, x]]] /;  !IntegerQ[p] && PolyQ[Px, x
] && GtQ[Expon[Px, x], 1] && NeQ[Coeff[Px, x, 0], 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.50

method	result	size
gosper	$-\frac{2(bx+a)\left(143C\,b^2x^2+117B\,b^2x+52Cabx+99A\,b^2+18abB+8C\,a^2\right)}{1287b^3\left(b^3x^3+3a\,b^2x^2+3b\,a^2x+a^3\right)^{\frac{5}{2}}}$	80
default	$-\frac{2(bx+a)\left(143C\,b^2x^2+117B\,b^2x+52Cabx+99A\,b^2+18abB+8C\,a^2\right)}{1287b^3\left(b^3x^3+3a\,b^2x^2+3b\,a^2x+a^3\right)^{\frac{5}{2}}}$	80
orering	$-\frac{2(bx+a)\left(143C\,b^2x^2+117B\,b^2x+52Cabx+99A\,b^2+18abB+8C\,a^2\right)}{1287b^3\left(b^3x^3+3a\,b^2x^2+3b\,a^2x+a^3\right)^{\frac{5}{2}}}$	80
trager	$-\frac{2\left(143C\,b^2x^2+117B\,b^2x+52Cabx+99A\,b^2+18abB+8C\,a^2\right)\sqrt{b^3x^3+3a\,b^2x^2+3b\,a^2x+a^3}}{1287(bx+a)^8b^3}$	82

input
```
int((C*x^2+B*x+A)/(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^(5/2),x,method=_RETU
RNVERBOSE)
```

output
```
-2/1287*(b*x+a)*(143*C*b^2*x^2+117*B*b^2*x+52*C*a*b*x+99*A*b^2+18*B*a*b+8*
C*a^2)/b^3/(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.02

$$\int \frac{A+Bx+Cx^2}{(a^3+3a^2bx+3ab^2x^2+b^3x^3)^{5/2}}\,dx = -\frac{2\sqrt{b^3x^3+3\,ab^2x^2+3\,a^2bx+a^3}(143\,Cb^2x^2+8\,Ca^2+18\,Bab+99\,Ab^2+13\,(4\,Cab+9\,Bb^2)x)}{1287\,(b^{11}x^8+8\,ab^{10}x^7+28\,a^2b^9x^6+56\,a^3b^8x^5+70\,a^4b^7x^4+56\,a^5b^6x^3+28\,a^6b^5x^2+8\,a^7b^4x+a^8b^3)}$$

input
```
integrate((C*x^2+B*x+A)/(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^(5/2),x, algor
ithm="fricas")
```

output
```
-2/1287*sqrt(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)*(143*C*b^2*x^2 + 8*C
*a^2 + 18*B*a*b + 99*A*b^2 + 13*(4*C*a*b + 9*B*b^2)*x)/(b^11*x^8 + 8*a*b^1
0*x^7 + 28*a^2*b^9*x^6 + 56*a^3*b^8*x^5 + 70*a^4*b^7*x^4 + 56*a^5*b^6*x^3
+ 28*a^6*b^5*x^2 + 8*a^7*b^4*x + a^8*b^3)
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2}{(a^3+3a^2bx+3ab^2x^2+b^3x^3)^{5/2}}\,dx = \int \frac{A+Bx+Cx^2}{\left((a+bx)^3\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((C*x**2+B*x+A)/(b**3*x**3+3*a*b**2*x**2+3*a**2*b*x+a**3)**(5/2),
x)
```

output
```
Integral((A + B*x + C*x**2)/((a + b*x)**3)**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.42

$$\int \frac{A+Bx+Cx^2}{\left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)^{5/2}}\,dx = -\frac{2\,(13\,bx+2\,a)B}{143\,(b^7x^5+5\,ab^6x^4+10\,a^2b^5x^3+10\,a^3b^4x^2+5\,a^4b^3x+a^5b^2)(bx+a)^{\frac{3}{2}}} - \frac{2\,(143\,b^2x^2+52\,abx+8\,a^2)C}{1287\,(b^8x^5+5\,ab^7x^4+10\,a^2b^6x^3+10\,a^3b^5x^2+5\,a^4b^4x+a^5b^3)(bx+a)^{\frac{3}{2}}} - \frac{2\,A}{13\,(b^6x^5+5\,ab^5x^4+10\,a^2b^4x^3+10\,a^3b^3x^2+5\,a^4b^2x+a^5b)(bx+a)^{\frac{3}{2}}}$$

input
```
integrate((C*x^2+B*x+A)/(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^(5/2),x, algor
ithm="maxima")
```

output
```
-2/143*(13*b*x + 2*a)*B/((b^7*x^5 + 5*a*b^6*x^4 + 10*a^2*b^5*x^3 + 10*a^3*
b^4*x^2 + 5*a^4*b^3*x + a^5*b^2)*(b*x + a)^(3/2)) - 2/1287*(143*b^2*x^2 +
52*a*b*x + 8*a^2)*C/((b^8*x^5 + 5*a*b^7*x^4 + 10*a^2*b^6*x^3 + 10*a^3*b^5*
x^2 + 5*a^4*b^4*x + a^5*b^3)*(b*x + a)^(3/2)) - 2/13*A/((b^6*x^5 + 5*a*b^5
*x^4 + 10*a^2*b^4*x^3 + 10*a^3*b^3*x^2 + 5*a^4*b^2*x + a^5*b)*(b*x + a)^(3
/2))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.41

$$\int \frac{A+Bx+Cx^2}{\left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)^{5/2}}\,dx = -\frac{2\left(143\,(bx+a)^2C-234\,(bx+a)Ca+99\,Ca^2+117\,(bx+a)Bb-99\,Bab+99\,Ab^2\right)}{1287\,(bx+a)^{\frac{13}{2}}\,b^3\mathrm{sgn}\,(bx+a)}$$

input
```
integrate((C*x^2+B*x+A)/(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^(5/2),x, algor
ithm="giac")
```

output
```
-2/1287*(143*(b*x + a)^2*C - 234*(b*x + a)*C*a + 99*C*a^2 + 117*(b*x + a)*
B*b - 99*B*a*b + 99*A*b^2)/((b*x + a)^(13/2)*b^3*sgn(b*x + a))
```

Mupad [B] (verification not implemented)

Time = 12.87 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.51

$$\int \frac{A+Bx+Cx^2}{(a^3+3a^2bx+3ab^2x^2+b^3x^3)^{5/2}}\,dx = -\frac{2\sqrt{a^3+3\,a^2\,b\,x+3\,a\,b^2\,x^2+b^3\,x^3}\,(8\,C\,a^2+52\,C\,a\,b\,x+18\,B\,a\,b+143\,C\,b^2\,x^2+117\,B\,b^2\,x+99\,A\,b^2)}{1287\,b^3\,(a+b\,x)^8}$$

input
```
int((A + B*x + C*x^2)/(a^3 + b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x)^(5/2),x)
```

output
```
-(2*(a^3 + b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x)^(1/2)*(99*A*b^2 + 8*C*a^2 +
143*C*b^2*x^2 + 18*B*a*b + 117*B*b^2*x + 52*C*a*b*x))/(1287*b^3*(a + b*x)^
8)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.68

$$\int \frac{A+Bx+Cx^2}{(a^3+3a^2bx+3ab^2x^2+b^3x^3)^{5/2}}\,dx = \frac{-\frac{2}{9}b^2c\,x^2-\frac{8}{99}abcx-\frac{2}{11}b^3x-\frac{16}{1287}a^2c-\frac{2}{11}a\,b^2}{\sqrt{bx+a}\,b^3\,(b^6x^6+6a\,b^5x^5+15a^2b^4x^4+20a^3b^3x^3+15a^4b^2x^2+6}$$

input
```
int((C*x^2+B*x+A)/(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^(5/2),x)
```

output
```
(2*( - 8*a**2*c - 117*a*b**2 - 52*a*b*c*x - 117*b**3*x - 143*b**2*c*x**2))
/(1287*sqrt(a + b*x)*b**3*(a**6 + 6*a**5*b*x + 15*a**4*b**2*x**2 + 20*a**3
*b**3*x**3 + 15*a**2*b**4*x**4 + 6*a*b**5*x**5 + b**6*x**6))
```

3.65 $\int \left((a+bx)^3\right)^p \left(A+Bx+Cx^2\right) dx$

Optimal result

Integrand size = 20, antiderivative size = 102

$$\int \left((a+bx)^3\right)^p \left(A+Bx+Cx^2\right) dx = \frac{(Ab^2-a(bB-aC))\,(a+bx)\left((a+bx)^3\right)^p}{b^3(1+3p)} + \frac{(bB-2aC)(a+bx)^2\left((a+bx)^3\right)^p}{b^3(2+3p)} + \frac{C\left((a+bx)^3\right)^{1+p}}{3b^3(1+p)}$$

output
```
(A*b^2-a*(B*b-C*a))*(b*x+a)*((b*x+a)^3)^p/b^3/(1+3*p)+(B*b-2*C*a)*(b*x+a)^
2*((b*x+a)^3)^p/b^3/(2+3*p)+1/3*C*((b*x+a)^3)^(p+1)/b^3/(p+1)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.06

$$\int \left((a+bx)^3\right)^p \left(A+Bx+Cx^2\right) dx = \frac{(a+bx)\left((a+bx)^3\right)^p \left(2a^2C+3Ab^2(2+5p+3p^2)-ab(3B(1+p)+2C(1+3p)x)+b^2(1+3p)x(3B(1\right.}{3b^3(1+p)(1+3p)(2+3p)}$$

input
```
Integrate[((a + b*x)^3)^p*(A + B*x + C*x^2),x]
```

output
```
((a + b*x)*((a + b*x)^3)^p*(2*a^2*C + 3*A*b^2*(2 + 5*p + 3*p^2) - a*b*(3*B
*(1 + p) + 2*C*(1 + 3*p)*x) + b^2*(1 + 3*p)*x*(3*B*(1 + p) + C*(2 + 3*p)*x
)))/(3*b^3*(1 + p)*(1 + 3*p)*(2 + 3*p))
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.11, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {2008, 1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left((a+bx)^3\right)^p \left(A+Bx+Cx^2\right) dx$$

$$\Big\downarrow \text{2008}$$

$$(a+bx)^{-3p} \left((a+bx)^3\right)^p \int (a+bx)^{3p} \left(Cx^2+Bx+A\right) dx$$

$$\Big\downarrow \text{1140}$$

$$(a+bx)^{-3p} \left((a+bx)^3\right)^p \int \left(\frac{\left(Ab^2-a(bB-aC)\right) (a+bx)^{3p}}{b^2}+\frac{(bB-2aC)(a+bx)^{3p+1}}{b^2}+\frac{C(a+bx)^{3p+2}}{b^2}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$(a+bx)^{-3p} \left((a+bx)^3\right)^p \left(\frac{(a+bx)^{3p+1} \left(Ab^2-a(bB-aC)\right)}{b^3(3p+1)}+\frac{(bB-2aC)(a+bx)^{3p+2}}{b^3(3p+2)}+\frac{C(a+bx)^{3(p+1)}}{3b^3(p+1)}\right)$$

input
```
Int[((a + b*x)^3)^p*(A + B*x + C*x^2),x]
```

output
```
(((a + b*x)^3)^p*((C*(a + b*x)^(3*(1 + p)))/(3*b^3*(1 + p)) + ((A*b^2 - a*
(b*B - a*C))*(a + b*x)^(1 + 3*p))/(b^3*(1 + 3*p)) + ((b*B - 2*a*C)*(a + b*
x)^(2 + 3*p))/(b^3*(2 + 3*p))))/(a + b*x)^(3*p)
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2008
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{a = Rt[Coeff[Px, x, 0], Expon[Px,
x]], b = Rt[Coeff[Px, x, Expon[Px, x]], Expon[Px, x]]}, Simp[((a + b*x)^Exp
on[Px, x])^p/(a + b*x)^(Expon[Px, x]*p)   Int[u*(a + b*x)^(Expon[Px, x]*p),
 x], x] /; EqQ[Px, (a + b*x)^Expon[Px, x]]] /;  !IntegerQ[p] && PolyQ[Px, x
] && GtQ[Expon[Px, x], 1] && NeQ[Coeff[Px, x, 0], 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.43

method	result
gosper	$\frac{(bx+a)\left(9C\,b^2p^2x^2+9B\,b^2p^2x+9C\,b^2p\,x^2+9A\,b^2p^2+12B\,b^2px-6Cabpx+2C\,b^2x^2+15A\,b^2p-3Babp+3B\,b^2x-2Cabx+6A\,b^2}{3b^3(9p^3+18p^2+11p+2)}$
orering	$\frac{(bx+a)\left(9C\,b^2p^2x^2+9B\,b^2p^2x+9C\,b^2p\,x^2+9A\,b^2p^2+12B\,b^2px-6Cabpx+2C\,b^2x^2+15A\,b^2p-3Babp+3B\,b^2x-2Cabx+6A\,b^2}{3b^3(9p^3+18p^2+11p+2)}$
norman	$\frac{(Bbp+Cap+Bb)x^2\mathrm{e}^{p\ln\left((bx+a)^3\right)}}{b(3p^2+5p+2)}+\frac{(3A\,b^2p^2+3Bab\,p^2+5A\,b^2p+3Babp-2C\,a^2p+2A\,b^2)x\,\mathrm{e}^{p\ln\left((bx+a)^3\right)}}{b^2(9p^3+18p^2+11p+2)}+\frac{C\,x^3\mathrm{e}^{p\ln((b}{3p+3}$
risch	$\frac{(9C\,b^3p^2x^3+9B\,b^3p^2x^2+9Ca\,b^2p^2x^2+9C\,b^3p\,x^3+9A\,b^3p^2x+9Ba\,b^2p^2x+12B\,b^3p\,x^2+3Ca\,b^2p\,x^2+2C\,x^3b^3+9Aa\,b^2p^2+15A}{3(2+3p)(p+1)(1+3p}$
parallelrisch	$9C\,x^3\left((bx+a)^3\right)^pb^3p^2+9B\,x^2\left((bx+a)^3\right)^pb^3p^2+9C\,x^3\left((bx+a)^3\right)^pb^3p+9C\,x^2\left((bx+a)^3\right)^pa\,b^2p^2+9Ax\left((bx+a)^3\right)^pb^3p^2+1$

input
```
int(((b*x+a)^3)^p*(C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

output
```
1/3*(b*x+a)*(9*C*b^2*p^2*x^2+9*B*b^2*p^2*x+9*C*b^2*p*x^2+9*A*b^2*p^2+12*B*
b^2*p*x-6*C*a*b*p*x+2*C*b^2*x^2+15*A*b^2*p-3*B*a*b*p+3*B*b^2*x-2*C*a*b*x+6
*A*b^2-3*B*a*b+2*C*a^2)*((b*x+a)^3)^p/b^3/(9*p^3+18*p^2+11*p+2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 230 vs. $2(101) = 202$.

Time = 0.08 (sec) , antiderivative size = 230, normalized size of antiderivative = 2.25

$$\int \left((a+bx)^3\right)^p \left(A+Bx+Cx^2\right)\,dx$$
$$= \frac{(9\,Aab^2p^2 + 2\,Ca^3 - 3\,Ba^2b + 6\,Aab^2 + (9\,Cb^3p^2 + 9\,Cb^3p + 2\,Cb^3)x^3 + 3\,(Bb^3 + 3\,(Cab^2 + Bb^3)p^2 + (}{3}$$

input
```
integrate(((b*x+a)^3)^p*(C*x^2+B*x+A),x, algorithm="fricas")
```

output
```
1/3*(9*A*a*b^2*p^2 + 2*C*a^3 - 3*B*a^2*b + 6*A*a*b^2 + (9*C*b^3*p^2 + 9*C*
b^3*p + 2*C*b^3)*x^3 + 3*(B*b^3 + 3*(C*a*b^2 + B*b^3)*p^2 + (C*a*b^2 + 4*B
*b^3)*p)*x^2 - 3*(B*a^2*b - 5*A*a*b^2)*p + 3*(2*A*b^3 + 3*(B*a*b^2 + A*b^3
)*p^2 - (2*C*a^2*b - 3*B*a*b^2 - 5*A*b^3)*p)*x)*(b^3*x^3 + 3*a*b^2*x^2 + 3
*a^2*b*x + a^3)^p/(9*b^3*p^3 + 18*b^3*p^2 + 11*b^3*p + 2*b^3)
```

Sympy [F]

$$\int \left((a+bx)^3\right)^p \left(A+Bx+Cx^2\right)\,dx = \text{Too large to display}$$

input
```
integrate(((b*x+a)**3)**p*(C*x**2+B*x+A),x)
```

output
```
Piecewise(((A*x + B*x**2/2 + C*x**3/3)*(a**3)**p, Eq(b, 0)), (-A*b**2/(2*a
**2*b**3 + 4*a*b**4*x + 2*b**5*x**2) - B*a*b/(2*a**2*b**3 + 4*a*b**4*x + 2
*b**5*x**2) - 2*B*b**2*x/(2*a**2*b**3 + 4*a*b**4*x + 2*b**5*x**2) + 2*C*a*
*2*log(a/b + x)/(2*a**2*b**3 + 4*a*b**4*x + 2*b**5*x**2) + 3*C*a**2/(2*a**
2*b**3 + 4*a*b**4*x + 2*b**5*x**2) + 4*C*a*b*x*log(a/b + x)/(2*a**2*b**3 +
 4*a*b**4*x + 2*b**5*x**2) + 4*C*a*b*x/(2*a**2*b**3 + 4*a*b**4*x + 2*b**5*
x**2) + 2*C*b**2*x**2*log(a/b + x)/(2*a**2*b**3 + 4*a*b**4*x + 2*b**5*x**2
), Eq(p, -1)), (Integral((A + B*x + C*x**2)/((a + b*x)**3)**(2/3), x), Eq(
p, -2/3)), (Integral((A + B*x + C*x**2)/((a + b*x)**3)**(1/3), x), Eq(p, -
1/3)), (9*A*a*b**2*p**2*(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3)**p
/(27*b**3*p**3 + 54*b**3*p**2 + 33*b**3*p + 6*b**3) + 15*A*a*b**2*p*(a**3
+ 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3)**p/(27*b**3*p**3 + 54*b**3*p**2
+ 33*b**3*p + 6*b**3) + 6*A*a*b**2*(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b*
*3*x**3)**p/(27*b**3*p**3 + 54*b**3*p**2 + 33*b**3*p + 6*b**3) + 9*A*b**3*
p**2*x*(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3)**p/(27*b**3*p**3 +
54*b**3*p**2 + 33*b**3*p + 6*b**3) + 15*A*b**3*p*x*(a**3 + 3*a**2*b*x + 3*
a*b**2*x**2 + b**3*x**3)**p/(27*b**3*p**3 + 54*b**3*p**2 + 33*b**3*p + 6*b
**3) + 6*A*b**3*x*(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3)**p/(27*b
**3*p**3 + 54*b**3*p**2 + 33*b**3*p + 6*b**3) - 3*B*a**2*b*p*(a**3 + 3*a**
2*b*x + 3*a*b**2*x**2 + b**3*x**3)**p/(27*b**3*p**3 + 54*b**3*p**2 + 33...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.53

$$\int \left((a+bx)^3\right)^p \left(A+Bx+Cx^2\right)\,dx = \frac{(bx+a)(bx+a)^{3\,p}A}{b(3\,p+1)} + \frac{(b^2(3\,p+1)x^2+3\,abpx-a^2)(bx+a)^{3\,p}B}{(9\,p^2+9\,p+2)b^2} + \frac{((9\,p^2+9\,p+2)b^3x^3+3\,(3\,p^2+p)ab^2x^2-6\,a^2bpx+2\,a^3)(bx+a)^{3\,p}C}{3\,(9\,p^3+18\,p^2+11\,p+2)b^3}$$

input
```
integrate(((b*x+a)^3)^p*(C*x^2+B*x+A),x, algorithm="maxima")
```

output
```
(b*x + a)*(b*x + a)^(3*p)*A/(b*(3*p + 1)) + (b^2*(3*p + 1)*x^2 + 3*a*b*p*x
 - a^2)*(b*x + a)^(3*p)*B/((9*p^2 + 9*p + 2)*b^2) + 1/3*((9*p^2 + 9*p + 2)
*b^3*x^3 + 3*(3*p^2 + p)*a*b^2*x^2 - 6*a^2*b*p*x + 2*a^3)*(b*x + a)^(3*p)*
C/((9*p^3 + 18*p^2 + 11*p + 2)*b^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 399 vs. $2(101) = 202$.

Time = 0.12 (sec) , antiderivative size = 399, normalized size of antiderivative = 3.91

$$\int \left((a+bx)^3\right)^p \left(A+Bx+Cx^2\right)\,dx = \frac{9\,(bx+a)^{3\,p}Cb^3p^2x^3+9\,(bx+a)^{3\,p}Cab^2p^2x^2+9\,(bx+a)^{3\,p}Bb^3p^2x^2+9\,(bx+a)^{3\,p}Cb^3px^3+9\,(bx+a)^{3}}{}$$

input
```
integrate(((b*x+a)^3)^p*(C*x^2+B*x+A),x, algorithm="giac")
```

output
```
1/3*(9*(b*x + a)^(3*p)*C*b^3*p^2*x^3 + 9*(b*x + a)^(3*p)*C*a*b^2*p^2*x^2 +
 9*(b*x + a)^(3*p)*B*b^3*p^2*x^2 + 9*(b*x + a)^(3*p)*C*b^3*p*x^3 + 9*(b*x
+ a)^(3*p)*B*a*b^2*p^2*x + 9*(b*x + a)^(3*p)*A*b^3*p^2*x + 3*(b*x + a)^(3*
p)*C*a*b^2*p*x^2 + 12*(b*x + a)^(3*p)*B*b^3*p*x^2 + 2*(b*x + a)^(3*p)*C*b^
3*x^3 + 9*(b*x + a)^(3*p)*A*a*b^2*p^2 - 6*(b*x + a)^(3*p)*C*a^2*b*p*x + 9*
(b*x + a)^(3*p)*B*a*b^2*p*x + 15*(b*x + a)^(3*p)*A*b^3*p*x + 3*(b*x + a)^(
3*p)*B*b^3*x^2 - 3*(b*x + a)^(3*p)*B*a^2*b*p + 15*(b*x + a)^(3*p)*A*a*b^2*
p + 6*(b*x + a)^(3*p)*A*b^3*x + 2*(b*x + a)^(3*p)*C*a^3 - 3*(b*x + a)^(3*p
)*B*a^2*b + 6*(b*x + a)^(3*p)*A*a*b^2)/(9*b^3*p^3 + 18*b^3*p^2 + 11*b^3*p
+ 2*b^3)
```

Mupad [B] (verification not implemented)

Time = 12.30 (sec) , antiderivative size = 219, normalized size of antiderivative = 2.15

$$\int \left((a+bx)^3\right)^p \left(A+Bx+Cx^2\right)\,dx = \left((a+b\,x)^3\right)^p\,\left(\frac{x\,\left(-6\,C\,a^2\,b\,p+9\,B\,a\,b^2\,p^2+9\,B\,a\,b^2\,p+9\,A\,b^3\,p^2+15\,A\,b^3\,p+6\,A\,b^3\right)}{3\,b^3\,\left(9\,p^3+18\,p^2+11\,p+2\right)}+\frac{C\,x^3\,\left(9\,p^2+9\,p+2\right)}{3\,\left(9\,p^3+18\,p^2+11\,p+2\right)}+\frac{a\,\left(2\,C\,a^2-3\,B\,a\,b\,p-3\,B\,a\,b+9\,A\,b^2\,p^2+15\,A\,b^2\,p+6\,A\,b^2\right)}{3\,b^3\,\left(9\,p^3+18\,p^2+11\,p+2\right)}+\frac{x^2\,\left(3\,p+1\right)\,\left(B\,b+B\,b\,p+C\,a\,p\right)}{b\,\left(9\,p^3+18\,p^2+11\,p+2\right)}\right)$$

input
```
int(((a + b*x)^3)^p*(A + B*x + C*x^2),x)
```

output
```
((a + b*x)^3)^p*((x*(6*A*b^3 + 9*A*b^3*p^2 + 15*A*b^3*p + 9*B*a*b^2*p - 6*
C*a^2*b*p + 9*B*a*b^2*p^2))/(3*b^3*(11*p + 18*p^2 + 9*p^3 + 2)) + (C*x^3*(
9*p + 9*p^2 + 2))/(3*(11*p + 18*p^2 + 9*p^3 + 2)) + (a*(6*A*b^2 + 2*C*a^2
+ 9*A*b^2*p^2 - 3*B*a*b + 15*A*b^2*p - 3*B*a*b*p))/(3*b^3*(11*p + 18*p^2 +
 9*p^3 + 2)) + (x^2*(3*p + 1)*(B*b + B*b*p + C*a*p))/(b*(11*p + 18*p^2 + 9
*p^3 + 2)))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.79

$$\int \left((a+bx)^3\right)^p \left(A+Bx+Cx^2\right)\,dx$$
$$= \frac{(bx+a)^{3p}\left(9b^3c\,p^2x^3+9a\,b^2c\,p^2x^2+9b^4p^2x^2+9b^3cp\,x^3+18a\,b^3p^2x+3a\,b^2cp\,x^2+12b^4p\,x^2+2b^3c\,x^3+\right.}{3b^3\left(9p^3+18p^2+11p+2\right)}$$

input
```
int(((b*x+a)^3)^p*(C*x^2+B*x+A),x)
```

output
```
((a + b*x)**(3*p)*(2*a**3*c + 9*a**2*b**2*p**2 + 12*a**2*b**2*p + 3*a**2*b
**2 - 6*a**2*b*c*p*x + 18*a*b**3*p**2*x + 24*a*b**3*p*x + 6*a*b**3*x + 9*a
*b**2*c*p**2*x**2 + 3*a*b**2*c*p*x**2 + 9*b**4*p**2*x**2 + 12*b**4*p*x**2
+ 3*b**4*x**2 + 9*b**3*c*p**2*x**3 + 9*b**3*c*p*x**3 + 2*b**3*c*x**3))/(3*
b**3*(9*p**3 + 18*p**2 + 11*p + 2))
```

3.66 $\int (A+Bx+Cx^2) (a^3+3a^2bx+3ab^2x^2+b^3x^3)^p \, dx$

Optimal result

Integrand size = 40, antiderivative size = 167

$$\int (A+Bx+Cx^2) (a^3+3a^2bx+3ab^2x^2+b^3x^3)^p \, dx = \frac{(Ab^2-a(bB-aC)) (a+bx) (a^3+3a^2bx+3ab^2x^2+b^3x^3)^p}{b^3(1+3p)} + \frac{(bB-2aC)(a+bx)^2 (a^3+3a^2bx+3ab^2x^2+b^3x^3)^p}{b^3(2+3p)} + \frac{C(a+bx)^3 (a^3+3a^2bx+3ab^2x^2+b^3x^3)^p}{3b^3(1+p)}$$

output
```
(A*b^2-a*(B*b-C*a))*(b*x+a)*(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^p/b^3/(1+3
*p)+(B*b-2*C*a)*(b*x+a)^2*(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^p/b^3/(2+3*p
)+1/3*C*(b*x+a)^3*(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^p/b^3/(p+1)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.65

$$\int \left(A+Bx+Cx^2\right)\left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)^p\,dx$$
$$=\frac{(a+bx)\left((a+bx)^3\right)^p\left(2a^2C+3Ab^2(2+5p+3p^2)-ab(3B(1+p)+2C(1+3p)x)+b^2(1+3p)x(3B(1\right.}{3b^3(1+p)(1+3p)(2+3p)}$$

input
```
Integrate[(A + B*x + C*x^2)*(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^p,x]
```

output
```
((a + b*x)*((a + b*x)^3)^p*(2*a^2*C + 3*A*b^2*(2 + 5*p + 3*p^2) - a*b*(3*B
*(1 + p) + 2*C*(1 + 3*p)*x) + b^2*(1 + 3*p)*x*(3*B*(1 + p) + C*(2 + 3*p)*x
)))/(3*b^3*(1 + p)*(1 + 3*p)*(2 + 3*p))
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.68, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.075, Rules used = {2008, 1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)^p\left(A+Bx+Cx^2\right)dx$$

$$\Big\downarrow \text{ 2008}$$

$$(a+bx)^{-3p}\left((a+bx)^3\right)^p\int (a+bx)^{3p}\left(Cx^2+Bx+A\right)dx$$

$$\Big\downarrow \text{ 1140}$$

$$(a+bx)^{-3p}\left((a+bx)^3\right)^p\int \left(\frac{\left(Ab^2-a(bB-aC)\right)(a+bx)^{3p}}{b^2}+\frac{(bB-2aC)(a+bx)^{3p+1}}{b^2}+\frac{C(a+bx)^{3p+2}}{b^2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$(a+bx)^{-3p}\left((a+bx)^3\right)^p\left(\frac{(a+bx)^{3p+1}\left(Ab^2-a(bB-aC)\right)}{b^3(3p+1)}+\frac{(bB-2aC)(a+bx)^{3p+2}}{b^3(3p+2)}+\frac{C(a+bx)^{3(p+1)}}{3b^3(p+1)}\right)$$

input
```
Int[(A + B*x + C*x^2)*(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)^p,x]
```

output
```
(((a + b*x)^3)^p*((C*(a + b*x)^(3*(1 + p)))/(3*b^3*(1 + p)) + ((A*b^2 - a*
(b*B - a*C))*(a + b*x)^(1 + 3*p))/(b^3*(1 + 3*p)) + ((b*B - 2*a*C)*(a + b*
x)^(2 + 3*p))/(b^3*(2 + 3*p))))/(a + b*x)^(3*p)
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2008
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{a = Rt[Coeff[Px, x, 0], Expon[Px,
x]], b = Rt[Coeff[Px, x, Expon[Px, x]], Expon[Px, x]]}, Simp[((a + b*x)^Exp
on[Px, x])^p/(a + b*x)^(Expon[Px, x]*p)   Int[u*(a + b*x)^(Expon[Px, x]*p),
 x], x] /; EqQ[Px, (a + b*x)^Expon[Px, x]]] /;  !IntegerQ[p] && PolyQ[Px, x
] && GtQ[Expon[Px, x], 1] && NeQ[Coeff[Px, x, 0], 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.99

method	result
gosper	$\frac{(bx+a)\left(9C\,b^{2}p^{2}x^{2}+9B\,b^{2}p^{2}x+9C\,b^{2}p\,x^{2}+9A\,b^{2}p^{2}+12B\,b^{2}px-6Cabpx+2C\,b^{2}x^{2}+15A\,b^{2}p-3Babp+3B\,b^{2}x-2Cabx+6A\,b^{2}}{3b^{3}\left(9p^{3}+18p^{2}+11p+2\right)}$
orering	$\frac{(bx+a)\left(9C\,b^{2}p^{2}x^{2}+9B\,b^{2}p^{2}x+9C\,b^{2}p\,x^{2}+9A\,b^{2}p^{2}+12B\,b^{2}px-6Cabpx+2C\,b^{2}x^{2}+15A\,b^{2}p-3Babp+3B\,b^{2}x-2Cabx+6A\,b^{2}}{3b^{3}\left(9p^{3}+18p^{2}+11p+2\right)}$
risch	$\frac{\left(9C\,b^{3}p^{2}x^{3}+9B\,b^{3}p^{2}x^{2}+9Ca\,b^{2}p^{2}x^{2}+9C\,b^{3}p\,x^{3}+9A\,b^{3}p^{2}x+9Ba\,b^{2}p^{2}x+12B\,b^{3}p\,x^{2}+3Ca\,b^{2}p\,x^{2}+2C\,x^{3}b^{3}+9Aa\,b^{2}p^{2}+15A}{3(2+3p)(p+1)(1+3p}$
norman	$\frac{(Bbp+Cap+Bb)x^{2}e^{p\ln\left(b^{3}x^{3}+3a\,b^{2}x^{2}+3b\,a^{2}x+a^{3}\right)}}{b(3p^{2}+5p+2)}+\frac{(3A\,b^{2}p^{2}+3Bab\,p^{2}+5A\,b^{2}p+3Babp-2C\,a^{2}p+2A\,b^{2})x\,e^{p\ln\left(b^{3}x^{3}+3a\,b\right.}}{b^{2}(9p^{3}+18p^{2}+11p+2)}$
parallelrisch	$9C\,x^{3}\left(b^{3}x^{3}+3a\,b^{2}x^{2}+3b\,a^{2}x+a^{3}\right)^{p}b^{3}p^{2}+9B\,x^{2}\left(b^{3}x^{3}+3a\,b^{2}x^{2}+3b\,a^{2}x+a^{3}\right)^{p}b^{3}p^{2}+9C\,x^{3}\left(b^{3}x^{3}+3a\,b^{2}x^{2}+3b\,a^{2}x+a^{3}\right)^{p}b^{3}p-$

input
```
int((C*x^2+B*x+A)*(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^p,x,method=_RETURNVE
RBOSE)
```

output
```
1/3*(b*x+a)*(9*C*b^2*p^2*x^2+9*B*b^2*p^2*x+9*C*b^2*p*x^2+9*A*b^2*p^2+12*B*
b^2*p*x-6*C*a*b*p*x+2*C*b^2*x^2+15*A*b^2*p-3*B*a*b*p+3*B*b^2*x-2*C*a*b*x+6
*A*b^2-3*B*a*b+2*C*a^2)*(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^p/b^3/(9*p^3+1
8*p^2+11*p+2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.38

$$\int\left(A+Bx+Cx^{2}\right)\left(a^{3}+3a^{2}bx+3ab^{2}x^{2}+b^{3}x^{3}\right)^{p}\,dx$$
$$=\frac{(9\,Aab^{2}p^{2}+2\,Ca^{3}-3\,Ba^{2}b+6\,Aab^{2}+(9\,Cb^{3}p^{2}+9\,Cb^{3}p+2\,Cb^{3})x^{3}+3\,(Bb^{3}+3\,(Cab^{2}+Bb^{3})p^{2}+(C}{3}$$

input
```
integrate((C*x^2+B*x+A)*(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^p,x, algorithm
="fricas")
```

output
```
1/3*(9*A*a*b^2*p^2 + 2*C*a^3 - 3*B*a^2*b + 6*A*a*b^2 + (9*C*b^3*p^2 + 9*C*
b^3*p + 2*C*b^3)*x^3 + 3*(B*b^3 + 3*(C*a*b^2 + B*b^3)*p^2 + (C*a*b^2 + 4*B
*b^3)*p)*x^2 - 3*(B*a^2*b - 5*A*a*b^2)*p + 3*(2*A*b^3 + 3*(B*a*b^2 + A*b^3
)*p^2 - (2*C*a^2*b - 3*B*a*b^2 - 5*A*b^3)*p)*x)*(b^3*x^3 + 3*a*b^2*x^2 + 3
*a^2*b*x + a^3)^p/(9*b^3*p^3 + 18*b^3*p^2 + 11*b^3*p + 2*b^3)
```

Sympy [F]

$$\int (A + Bx + Cx^2) (a^3 + 3a^2bx + 3ab^2x^2 + b^3x^3)^p \, dx = \text{Too large to display}$$

input
```
integrate((C*x**2+B*x+A)*(b**3*x**3+3*a*b**2*x**2+3*a**2*b*x+a**3)**p,x)
```

output
```
Piecewise(((A*x + B*x**2/2 + C*x**3/3)*(a**3)**p, Eq(b, 0)), (-A*b**2/(2*a
**2*b**3 + 4*a*b**4*x + 2*b**5*x**2) - B*a*b/(2*a**2*b**3 + 4*a*b**4*x + 2
*b**5*x**2) - 2*B*b**2*x/(2*a**2*b**3 + 4*a*b**4*x + 2*b**5*x**2) + 2*C*a*
*2*log(a/b + x)/(2*a**2*b**3 + 4*a*b**4*x + 2*b**5*x**2) + 3*C*a**2/(2*a**
2*b**3 + 4*a*b**4*x + 2*b**5*x**2) + 4*C*a*b*x*log(a/b + x)/(2*a**2*b**3 +
 4*a*b**4*x + 2*b**5*x**2) + 4*C*a*b*x/(2*a**2*b**3 + 4*a*b**4*x + 2*b**5*
x**2) + 2*C*b**2*x**2*log(a/b + x)/(2*a**2*b**3 + 4*a*b**4*x + 2*b**5*x**2
), Eq(p, -1)), (Integral((A + B*x + C*x**2)/((a + b*x)**3)**(2/3), x), Eq(
p, -2/3)), (Integral((A + B*x + C*x**2)/((a + b*x)**3)**(1/3), x), Eq(p, -
1/3)), (9*A*a*b**2*p**2*(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3)**p
/(27*b**3*p**3 + 54*b**3*p**2 + 33*b**3*p + 6*b**3) + 15*A*a*b**2*p*(a**3
+ 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3)**p/(27*b**3*p**3 + 54*b**3*p**2
+ 33*b**3*p + 6*b**3) + 6*A*a*b**2*(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b*
*3*x**3)**p/(27*b**3*p**3 + 54*b**3*p**2 + 33*b**3*p + 6*b**3) + 9*A*b**3*
p**2*x*(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3)**p/(27*b**3*p**3 +
54*b**3*p**2 + 33*b**3*p + 6*b**3) + 15*A*b**3*p*x*(a**3 + 3*a**2*b*x + 3*
a*b**2*x**2 + b**3*x**3)**p/(27*b**3*p**3 + 54*b**3*p**2 + 33*b**3*p + 6*b
**3) + 6*A*b**3*x*(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3)**p/(27*b
**3*p**3 + 54*b**3*p**2 + 33*b**3*p + 6*b**3) - 3*B*a**2*b*p*(a**3 + 3*a**
2*b*x + 3*a*b**2*x**2 + b**3*x**3)**p/(27*b**3*p**3 + 54*b**3*p**2 + 33...
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.93

$$\int \left(A+Bx+Cx^2\right)\left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)^p\,dx$$
$$=\frac{(bx+a)(bx+a)^{3p}A}{b(3p+1)}+\frac{(b^2(3p+1)x^2+3abpx-a^2)(bx+a)^{3p}B}{(9p^2+9p+2)b^2}$$
$$+\frac{((9p^2+9p+2)b^3x^3+3(3p^2+p)ab^2x^2-6a^2bpx+2a^3)(bx+a)^{3p}C}{3(9p^3+18p^2+11p+2)b^3}$$

input
```
integrate((C*x^2+B*x+A)*(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^p,x, algorithm
="maxima")
```

output
```
(b*x + a)*(b*x + a)^(3*p)*A/(b*(3*p + 1)) + (b^2*(3*p + 1)*x^2 + 3*a*b*p*x
 - a^2)*(b*x + a)^(3*p)*B/((9*p^2 + 9*p + 2)*b^2) + 1/3*((9*p^2 + 9*p + 2)
*b^3*x^3 + 3*(3*p^2 + p)*a*b^2*x^2 - 6*a^2*b*p*x + 2*a^3)*(b*x + a)^(3*p)*
C/((9*p^3 + 18*p^2 + 11*p + 2)*b^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 799 vs. $2(166) = 332$.

Time = 0.13 (sec) , antiderivative size = 799, normalized size of antiderivative = 4.78

$$\int \left(A+Bx+Cx^2\right)\left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)^p\,dx = \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)*(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^p,x, algorithm
="giac")
```

output
```
1/3*(9*(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)^p*C*b^3*p^2*x^3 + 9*(b^3*
x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)^p*C*a*b^2*p^2*x^2 + 9*(b^3*x^3 + 3*a*
b^2*x^2 + 3*a^2*b*x + a^3)^p*B*b^3*p^2*x^2 + 9*(b^3*x^3 + 3*a*b^2*x^2 + 3*
a^2*b*x + a^3)^p*C*b^3*p*x^3 + 9*(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)
^p*B*a*b^2*p^2*x + 9*(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)^p*A*b^3*p^2
*x + 3*(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)^p*C*a*b^2*p*x^2 + 12*(b^3
*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)^p*B*b^3*p*x^2 + 2*(b^3*x^3 + 3*a*b^2
*x^2 + 3*a^2*b*x + a^3)^p*C*b^3*x^3 + 9*(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x
 + a^3)^p*A*a*b^2*p^2 - 6*(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)^p*C*a^
2*b*p*x + 9*(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)^p*B*a*b^2*p*x + 15*(
b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)^p*A*b^3*p*x + 3*(b^3*x^3 + 3*a*b^
2*x^2 + 3*a^2*b*x + a^3)^p*B*b^3*x^2 - 3*(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*
x + a^3)^p*B*a^2*b*p + 15*(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)^p*A*a*
b^2*p + 6*(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)^p*A*b^3*x + 2*(b^3*x^3
 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)^p*C*a^3 - 3*(b^3*x^3 + 3*a*b^2*x^2 + 3*a
^2*b*x + a^3)^p*B*a^2*b + 6*(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)^p*A*
a*b^2)/(9*b^3*p^3 + 18*b^3*p^2 + 11*b^3*p + 2*b^3)
```

Mupad [B] (verification not implemented)

Time = 12.91 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.43

$$\int (A+Bx+Cx^2)\left(a^3+3a^2bx+3ab^2x^2+b^3x^3\right)^p\,dx$$
$$= \left(a^3+3\,a^2\,b\,x+3\,a\,b^2\,x^2+b^3\,x^3\right)^p\left(\frac{x\,(-6\,C\,a^2\,b\,p+9\,B\,a\,b^2\,p^2+9\,B\,a\,b^2\,p+9\,A\,b^3\,p^2+15\,A\,b^3\,p+6\,A}{3\,b^3\,\left(9\,p^3+18\,p^2+11\,p+2\right)} + \frac{C\,x^3\,\left(9\,p^2+9\,p+2\right)}{3\,\left(9\,p^3+18\,p^2+11\,p+2\right)} + \frac{a\,(2\,C\,a^2-3\,B\,a\,b\,p-3\,B\,a\,b+9\,A\,b^2\,p^2+15\,A\,b^2\,p+6\,A\,b^2)}{3\,b^3\,\left(9\,p^3+18\,p^2+11\,p+2\right)} + \frac{x^2\,(3\,p+1)\,\left(B\,b+B\,b\,p+C\,a\,p\right)}{b\,\left(9\,p^3+18\,p^2+11\,p+2\right)}\right)$$

input
```
int((A + B*x + C*x^2)*(a^3 + b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x)^p,x)
```

output
```
(a^3 + b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x)^p*((x*(6*A*b^3 + 9*A*b^3*p^2 + 1
5*A*b^3*p + 9*B*a*b^2*p - 6*C*a^2*b*p + 9*B*a*b^2*p^2))/(3*b^3*(11*p + 18*
p^2 + 9*p^3 + 2)) + (C*x^3*(9*p + 9*p^2 + 2))/(3*(11*p + 18*p^2 + 9*p^3 +
2)) + (a*(6*A*b^2 + 2*C*a^2 + 9*A*b^2*p^2 - 3*B*a*b + 15*A*b^2*p - 3*B*a*b
*p))/(3*b^3*(11*p + 18*p^2 + 9*p^3 + 2)) + (x^2*(3*p + 1)*(B*b + B*b*p + C
*a*p))/(b*(11*p + 18*p^2 + 9*p^3 + 2)))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.22

$$\int (A + Bx + Cx^2) (a^3 + 3a^2bx + 3ab^2x^2 + b^3x^3)^p \, dx$$
$$= \frac{(b^3x^3 + 3a\,b^2x^2 + 3a^2bx + a^3)^p\,(9b^3c\,p^2x^3 + 9a\,b^2c\,p^2x^2 + 9b^4p^2x^2 + 9b^3cp\,x^3 + 18a\,b^3p^2x + 3a\,b^2cp\,x^2 + \cdots}{3b^3\,(9p^3 + 18p^2 + \cdots}$$

input
```
int((C*x^2+B*x+A)*(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)^p,x)
```

output
```
((a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3)**p*(2*a**3*c + 9*a**2*b**
2*p**2 + 12*a**2*b**2*p + 3*a**2*b**2 - 6*a**2*b*c*p*x + 18*a*b**3*p**2*x
+ 24*a*b**3*p*x + 6*a*b**3*x + 9*a*b**2*c*p**2*x**2 + 3*a*b**2*c*p*x**2 +
9*b**4*p**2*x**2 + 12*b**4*p*x**2 + 3*b**4*x**2 + 9*b**3*c*p**2*x**3 + 9*b
**3*c*p*x**3 + 2*b**3*c*x**3))/(3*b**3*(9*p**3 + 18*p**2 + 11*p + 2))
```

3.67 $\int (A+Bx+Cx^2) \left(3ab+3b^2x+3bcx^2+c^2x^3\right)^4 \, dx$

Optimal result

Integrand size = 38, antiderivative size = 387

$$\begin{aligned}
&\int (A+Bx+Cx^2) \left(3ab+3b^2x+3bcx^2+c^2x^3\right)^4 \, dx \\
&= -\frac{b^4(b^2-3ac)^4 \left(bBc-Ac^2-b^2C\right) x}{c^6}+\frac{b^4(b^2-3ac)^4 (Bc-2bC)(b+cx)^2}{2c^7} \\
&\quad +\frac{b^3(b^2-3ac)^3 \left(bBc-Ac^2-b^2C\right) (b+cx)^4}{c^7}-\frac{4b^3(b^2-3ac)^3 (Bc-2bC)(b+cx)^5}{5c^7} \\
&\quad -\frac{6b^2(b^2-3ac)^2 \left(bBc-Ac^2-b^2C\right) (b+cx)^7}{7c^7} \\
&\quad +\frac{3b^2(b^2-3ac)^2 (Bc-2bC)(b+cx)^8}{4c^7}+\frac{2b(b^2-3ac) \left(bBc-Ac^2-b^2C\right) (b+cx)^{10}}{5c^7} \\
&\quad -\frac{4b(b^2-3ac) (Bc-2bC)(b+cx)^{11}}{11c^7}-\frac{\left(bBc-Ac^2-b^2C\right) (b+cx)^{13}}{13c^7} \\
&\quad +\frac{(Bc-2bC)(b+cx)^{14}}{14c^7}+\frac{C\left(b\left(3a-\frac{b^2}{c}\right)+\frac{(b+cx)^3}{c}\right)^5}{15c^2}
\end{aligned}$$

output
```
-b^4*(-3*a*c+b^2)^4*(-A*c^2+B*b*c-C*b^2)*x/c^6+1/2*b^4*(-3*a*c+b^2)^4*(B*c
-2*C*b)*(c*x+b)^2/c^7+b^3*(-3*a*c+b^2)^3*(-A*c^2+B*b*c-C*b^2)*(c*x+b)^4/c^
7-4/5*b^3*(-3*a*c+b^2)^3*(B*c-2*C*b)*(c*x+b)^5/c^7-6/7*b^2*(-3*a*c+b^2)^2*
(-A*c^2+B*b*c-C*b^2)*(c*x+b)^7/c^7+3/4*b^2*(-3*a*c+b^2)^2*(B*c-2*C*b)*(c*x
+b)^8/c^7+2/5*b*(-3*a*c+b^2)*(-A*c^2+B*b*c-C*b^2)*(c*x+b)^10/c^7-4/11*b*(-
3*a*c+b^2)*(B*c-2*C*b)*(c*x+b)^11/c^7-1/13*(-A*c^2+B*b*c-C*b^2)*(c*x+b)^13
/c^7+1/14*(B*c-2*C*b)*(c*x+b)^14/c^7+1/15*C*(b*(3*a-b^2/c)+(c*x+b)^3/c)^5/
c^2
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 684, normalized size of antiderivative = 1.77

$$\int (A+Bx+Cx^2)\left(3ab+3b^2x+3bcx^2+c^2x^3\right)^4\,dx$$

$$= 81a^4Ab^4x + \frac{81}{2}a^3b^4(4Ab+aB)x^2 + 27a^2b^4\left(6Ab^2+4abB+4aAc+a^2C\right)x^3 + \frac{27}{2}ab^3\left(2A\left(3b^4+9ab^2c+a^2c^2\right)+3ab\left(3b^2B+2aBc+2abC\right)\right)x^4 + \frac{27}{5}b^3\left(3A\left(b^5+12ab^3c+10a^2bc^2\right) + 2a\left(6b^4B+18ab^2Bc+2a^2Bc^2+9ab^3C+6a^2bcC\right)\right)x^5 + \frac{9}{2}b^3\left(3b^5B+36ab^3Bc + 30a^2bBc^2+12b^4(Ac+aC)+4a^2c^2(3Ac+aC)+12ab^2c(4Ac+3aC)\right)x^6 + \frac{27}{7}b^2\left(12b^5Bc+48ab^3Bc^2+12a^2bBc^3+2a^2Ac^4+3b^6C+6ab^2c^2(6Ac+5aC) + 2b^4c(11Ac+18aC)\right)x^7 + \frac{27}{4}b^2c\left(11b^4Bc+18ab^2Bc^2+a^2Bc^3+6b^5C + 12b^3c(Ac+2aC)+2abc^2(4Ac+3aC)\right)x^8 + 3b^2c^2\left(24b^3Bc+16abBc^2+22b^4C+2ac^2(2Ac+aC)+b^2c(17Ac+36aC)\right)x^9 + \frac{3}{10}bc^3\left(153b^3Bc+36abBc^2+4aAc^3+216b^4C+72b^2c(Ac+2aC)\right)x^{10} + \frac{3}{11}bc^4\left(72b^2Bc+4aBc^2+153b^3C+2bc(11Ac+18aC)\right)x^{11} + \frac{1}{2}bc^5\left(11bBc+36b^2C+2c(Ac+aC)\right)x^{12} + \frac{1}{13}c^6\left(12bBc+Ac^2+66b^2C\right)x^{13} + \frac{1}{14}c^7(Bc+12bC)x^{14} + \frac{1}{15}c^8Cx^{15}$$

input
```
Integrate[(A + B*x + C*x^2)*(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3)^4,x]
```

output
```
81*a^4*A*b^4*x + (81*a^3*b^4*(4*A*b + a*B)*x^2)/2 + 27*a^2*b^4*(6*A*b^2 +
4*a*b*B + 4*a*A*c + a^2*C)*x^3 + (27*a*b^3*(2*A*(3*b^4 + 9*a*b^2*c + a^2*c
^2) + 3*a*b*(3*b^2*B + 2*a*B*c + 2*a*b*C))*x^4)/2 + (27*b^3*(3*A*(b^5 + 12
*a*b^3*c + 10*a^2*b*c^2) + 2*a*(6*b^4*B + 18*a*b^2*B*c + 2*a^2*B*c^2 + 9*a
*b^3*C + 6*a^2*b*c*C))*x^5)/5 + (9*b^3*(3*b^5*B + 36*a*b^3*B*c + 30*a^2*b*
B*c^2 + 12*b^4*(A*c + a*C) + 4*a^2*c^2*(3*A*c + a*C) + 12*a*b^2*c*(4*A*c +
 3*a*C))*x^6)/2 + (27*b^2*(12*b^5*B*c + 48*a*b^3*B*c^2 + 12*a^2*b*B*c^3 +
2*a^2*A*c^4 + 3*b^6*C + 6*a*b^2*c^2*(6*A*c + 5*a*C) + 2*b^4*c*(11*A*c + 18
*a*C))*x^7)/7 + (27*b^2*c*(11*b^4*B*c + 18*a*b^2*B*c^2 + a^2*B*c^3 + 6*b^5
*C + 12*b^3*c*(A*c + 2*a*C) + 2*a*b*c^2*(4*A*c + 3*a*C))*x^8)/4 + 3*b^2*c^
2*(24*b^3*B*c + 16*a*b*B*c^2 + 22*b^4*C + 2*a*c^2*(2*A*c + a*C) + b^2*c*(1
7*A*c + 36*a*C))*x^9 + (3*b*c^3*(153*b^3*B*c + 36*a*b*B*c^2 + 4*a*A*c^3 +
216*b^4*C + 72*b^2*c*(A*c + 2*a*C))*x^10)/10 + (3*b*c^4*(72*b^2*B*c + 4*a*
B*c^2 + 153*b^3*C + 2*b*c*(11*A*c + 18*a*C))*x^11)/11 + (b*c^5*(11*b*B*c +
 36*b^2*C + 2*c*(A*c + a*C))*x^12)/2 + (c^6*(12*b*B*c + A*c^2 + 66*b^2*C)*
x^13)/13 + (c^7*(B*c + 12*b*C)*x^14)/14 + (c^8*C*x^15)/15
```

Rubi [A] (verified)

Time = 2.42 (sec) , antiderivative size = 684, normalized size of antiderivative = 1.77, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(3ab+3b^2x+3bcx^2+c^2x^3\right)^4 \left(A+Bx+Cx^2\right) \, dx$$

$$\Big\downarrow \text{2188}$$

$$\int \left(81a^4Ab^4+81a^3b^4x(aB+4Ab)+81a^2b^4x^2\left(a^2C+4aAc+4abB+6Ab^2\right)+54ab^3x^3\left(2A\left(a^2c^2+9ab^2c+3b^4\right.\right.\right.$$

$$\Big\downarrow \text{2009}$$

$$81a^4Ab^4x+\frac{81}{2}a^3b^4x^2(aB+4Ab)+27a^2b^4x^3\left(a^2C+4aAc+4abB+6Ab^2\right)+\frac{27}{2}ab^3x^4\left(2A\left(a^2c^2+9ab^2c+3b^4\right)+3ab\left(2abC+2aBc+3b^2B\right)\right)+\frac{9}{2}b^3x^6\left(4a^2c^2(aC+3Ac)+30a^2bBc^2+12b^4(aC+Ac)+12ab^2c(3aC+4Ac)+36ab^3Bc+3b^5B\right)+\frac{27}{5}b^3x^5\left(3A\left(10a^2bc^2+12ab^3c+b^5\right)+2a\left(6a^2bcC+2a^2Bc^2+9ab^3C+18ab^2Bc+6b^4B\right)\right)+\frac{27}{4}b^2cx^8\left(a^2Bc^3+12b^3c(2aC+Ac)+2abc^2(3aC+4Ac)+18ab^2Bc^2+6b^5C+11b^4Bc\right)+\frac{27}{7}b^2x^7\left(2a^2Ac^4+12a^2bBc^3+2b^4c(18aC+11Ac)+6ab^2c^2(5aC+6Ac)+48ab^3Bc^2+3b^6C+12b^5Bc\right)+\frac{1}{2}bc^5x^{12}\left(2c(aC+Ac)+36b^2C+11bBc\right)+\frac{3}{11}bc^4x^{11}\left(2bc(18aC+11Ac)+4aBc^2+153b^3C+72b^2Bc\right)+3b^2c^2x^9\left(b^2c(36aC+17Ac)+2ac^2(aC+2Ac)+16abBc^2+22b^4C+24b^3Bc\right)+\frac{3}{10}bc^3x^{10}\left(72b^2c(2aC+Ac)+4aAc^3+36abBc^2+216b^4C+153b^3Bc\right)+\frac{1}{13}c^6x^{13}\left(Ac^2+66b^2C+12bBc\right)+\frac{1}{14}c^7x^{14}(12bC+Bc)+\frac{1}{15}c^8Cx^{15}$$

input
```
Int[(A + B*x + C*x^2)*(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3)^4,x]
```

output
```
81*a^4*A*b^4*x + (81*a^3*b^4*(4*A*b + a*B)*x^2)/2 + 27*a^2*b^4*(6*A*b^2 +
4*a*b*B + 4*a*A*c + a^2*C)*x^3 + (27*a*b^3*(2*A*(3*b^4 + 9*a*b^2*c + a^2*c
^2) + 3*a*b*(3*b^2*B + 2*a*B*c + 2*a*b*C))*x^4)/2 + (27*b^3*(3*A*(b^5 + 12
*a*b^3*c + 10*a^2*b*c^2) + 2*a*(6*b^4*B + 18*a*b^2*B*c + 2*a^2*B*c^2 + 9*a
*b^3*C + 6*a^2*b*c*C))*x^5)/5 + (9*b^3*(3*b^5*B + 36*a*b^3*B*c + 30*a^2*b*
B*c^2 + 12*b^4*(A*c + a*C) + 4*a^2*c^2*(3*A*c + a*C) + 12*a*b^2*c*(4*A*c +
 3*a*C))*x^6)/2 + (27*b^2*(12*b^5*B*c + 48*a*b^3*B*c^2 + 12*a^2*b*B*c^3 +
2*a^2*A*c^4 + 3*b^6*C + 6*a*b^2*c^2*(6*A*c + 5*a*C) + 2*b^4*c*(11*A*c + 18
*a*C))*x^7)/7 + (27*b^2*c*(11*b^4*B*c + 18*a*b^2*B*c^2 + a^2*B*c^3 + 6*b^5
*C + 12*b^3*c*(A*c + 2*a*C) + 2*a*b*c^2*(4*A*c + 3*a*C))*x^8)/4 + 3*b^2*c^
2*(24*b^3*B*c + 16*a*b*B*c^2 + 22*b^4*C + 2*a*c^2*(2*A*c + a*C) + b^2*c*(1
7*A*c + 36*a*C))*x^9 + (3*b*c^3*(153*b^3*B*c + 36*a*b*B*c^2 + 4*a*A*c^3 +
216*b^4*C + 72*b^2*c*(A*c + 2*a*C))*x^10)/10 + (3*b*c^4*(72*b^2*B*c + 4*a*
B*c^2 + 153*b^3*C + 2*b*c*(11*A*c + 18*a*C))*x^11)/11 + (b*c^5*(11*b*B*c +
 36*b^2*C + 2*c*(A*c + a*C))*x^12)/2 + (c^6*(12*b*B*c + A*c^2 + 66*b^2*C)*
x^13)/13 + (c^7*(B*c + 12*b*C)*x^14)/14 + (c^8*C*x^15)/15
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 755 vs. $2(369) = 738$.

Time = 0.22 (sec) , antiderivative size = 756, normalized size of antiderivative = 1.95

method	result
norman	$81A\,a^4b^4x+\left(162A\,a^3b^5+\frac{81}{2}B\,a^4b^4\right)x^2+\left(108A\,a^3b^4c+162A\,a^2b^6+108B\,a^3b^5+27C\,a^4b\right.$
gosper	$216x^6Aa\,b^5c^2+135x^6B\,a^2b^4c^2+162x^6Ba\,b^6c+18x^6C\,a^3b^3c^2+\frac{54}{7}x^7A\,a^2b^2c^4+\frac{972}{7}x^7Aa\,b$
risch	$216x^6Aa\,b^5c^2+135x^6B\,a^2b^4c^2+162x^6Ba\,b^6c+18x^6C\,a^3b^3c^2+\frac{54}{7}x^7A\,a^2b^2c^4+\frac{972}{7}x^7Aa\,b$
parallelrisch	$216x^6Aa\,b^5c^2+135x^6B\,a^2b^4c^2+162x^6Ba\,b^6c+18x^6C\,a^3b^3c^2+\frac{54}{7}x^7A\,a^2b^2c^4+\frac{972}{7}x^7Aa\,b$
orering	$\frac{x(4004C\,c^8x^{14}+4290B\,c^8x^{13}+51480Cb\,c^7x^{13}+4620A\,c^8x^{12}+55440Bb\,c^7x^{12}+304920C\,b^2c^6x^{12}+60060Ab\,c^7x^{11}+330330B}{}$
default	Expression too large to display

input
```
int((C*x^2+B*x+A)*(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^4,x,method=_RETURNVERB
OSE)
```

output
```
81*A*a^4*b^4*x+(162*A*a^3*b^5+81/2*B*a^4*b^4)*x^2+(108*A*a^3*b^4*c+162*A*a
^2*b^6+108*B*a^3*b^5+27*C*a^4*b^4)*x^3+(27*A*a^3*b^3*c^2+243*A*a^2*b^5*c+8
1*A*a*b^7+81*B*a^3*b^4*c+243/2*B*a^2*b^6+81*C*a^3*b^5)*x^4+(162*A*a^2*b^4*
c^2+972/5*A*a*b^6*c+81/5*A*b^8+108/5*B*a^3*b^3*c^2+972/5*B*a^2*b^5*c+324/5
*B*a*b^7+324/5*C*a^3*b^4*c+486/5*C*a^2*b^6)*x^5+(54*A*a^2*b^3*c^3+216*A*a*
b^5*c^2+54*A*b^7*c+135*B*a^2*b^4*c^2+162*B*a*b^6*c+27/2*B*b^8+18*C*a^3*b^3
*c^2+162*C*a^2*b^5*c+54*C*a*b^7)*x^6+(54/7*A*a^2*b^2*c^4+972/7*A*a*b^4*c^3
+594/7*A*b^6*c^2+324/7*B*a^2*b^3*c^3+1296/7*B*a*b^5*c^2+324/7*B*b^7*c+810/
7*C*a^2*b^4*c^2+972/7*C*a*b^6*c+81/7*C*b^8)*x^7+(54*A*a*b^3*c^4+81*A*b^5*c
^3+27/4*B*a^2*b^2*c^4+243/2*B*a*b^4*c^3+297/4*B*b^6*c^2+81/2*C*a^2*b^3*c^3
+162*C*a*b^5*c^2+81/2*C*b^7*c)*x^8+(12*A*a*b^2*c^5+51*A*b^4*c^4+48*B*a*b^3
*c^4+72*B*b^5*c^3+6*C*a^2*b^2*c^4+108*C*a*b^4*c^3+66*C*b^6*c^2)*x^9+(6/5*A
*a*b*c^6+108/5*A*b^3*c^5+54/5*B*a*b^2*c^5+459/10*B*b^4*c^4+216/5*C*a*b^3*c
^4+324/5*C*b^5*c^3)*x^10+(6*A*b^2*c^6+12/11*B*a*b*c^6+216/11*B*b^3*c^5+108
/11*C*a*b^2*c^5+459/11*C*b^4*c^4)*x^11+(A*b*c^7+11/2*B*b^2*c^6+C*a*b*c^6+1
8*C*b^3*c^5)*x^12+(1/13*A*c^8+12/13*B*b*c^7+66/13*C*b^2*c^6)*x^13+(1/14*B*
c^8+6/7*C*b*c^7)*x^14+1/15*C*c^8*x^15
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 747 vs. $2(369) = 738$.

Time = 0.08 (sec) , antiderivative size = 747, normalized size of antiderivative = 1.93

$$\int (A + Bx + Cx^2)\left(3ab + 3b^2x + 3bcx^2 + c^2x^3\right)^4 \, dx = \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)*(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^4,x, algorithm="
fricas")
```

output
```
1/15*C*c^8*x^15 + 1/14*(12*C*b*c^7 + B*c^8)*x^14 + 1/13*(66*C*b^2*c^6 + 12
*B*b*c^7 + A*c^8)*x^13 + 1/2*(36*C*b^3*c^5 + 2*A*b*c^7 + (2*C*a*b + 11*B*b
^2)*c^6)*x^12 + 3/11*(153*C*b^4*c^4 + 2*(2*B*a*b + 11*A*b^2)*c^6 + 36*(C*a
*b^2 + 2*B*b^3)*c^5)*x^11 + 3/10*(216*C*b^5*c^3 + 4*A*a*b*c^6 + 36*(B*a*b^
2 + 2*A*b^3)*c^5 + 9*(16*C*a*b^3 + 17*B*b^4)*c^4)*x^10 + 81*A*a^4*b^4*x +
3*(22*C*b^6*c^2 + 4*A*a*b^2*c^5 + (2*C*a^2*b^2 + 16*B*a*b^3 + 17*A*b^4)*c^
4 + 12*(3*C*a*b^4 + 2*B*b^5)*c^3)*x^9 + 27/4*(6*C*b^7*c + (B*a^2*b^2 + 8*A
*a*b^3)*c^4 + 6*(C*a^2*b^3 + 3*B*a*b^4 + 2*A*b^5)*c^3 + (24*C*a*b^5 + 11*B
*b^6)*c^2)*x^8 + 27/7*(3*C*b^8 + 2*A*a^2*b^2*c^4 + 12*(B*a^2*b^3 + 3*A*a*b
^4)*c^3 + 2*(15*C*a^2*b^4 + 24*B*a*b^5 + 11*A*b^6)*c^2 + 12*(3*C*a*b^6 + B
*b^7)*c)*x^7 + 9/2*(12*C*a*b^7 + 3*B*b^8 + 12*A*a^2*b^3*c^3 + 2*(2*C*a^3*b
^3 + 15*B*a^2*b^4 + 24*A*a*b^5)*c^2 + 12*(3*C*a^2*b^5 + 3*B*a*b^6 + A*b^7)
*c)*x^6 + 27/5*(18*C*a^2*b^6 + 12*B*a*b^7 + 3*A*b^8 + 2*(2*B*a^3*b^3 + 15*
A*a^2*b^4)*c^2 + 12*(C*a^3*b^4 + 3*B*a^2*b^5 + 3*A*a*b^6)*c)*x^5 + 27/2*(6
*C*a^3*b^5 + 9*B*a^2*b^6 + 6*A*a*b^7 + 2*A*a^3*b^3*c^2 + 6*(B*a^3*b^4 + 3*
A*a^2*b^5)*c)*x^4 + 27*(C*a^4*b^4 + 4*B*a^3*b^5 + 6*A*a^2*b^6 + 4*A*a^3*b^
4*c)*x^3 + 81/2*(B*a^4*b^4 + 4*A*a^3*b^5)*x^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 910 vs. $2(369) = 738$.

Time = 0.09 (sec) , antiderivative size = 910, normalized size of antiderivative = 2.35

$$\int (A + Bx + Cx^2) \left(3ab + 3b^2x + 3bcx^2 + c^2x^3\right)^4 \, dx = \text{Too large to display}$$

input
```
integrate((C*x**2+B*x+A)*(c**2*x**3+3*b*c*x**2+3*b**2*x+3*a*b)**4,x)
```

output
```
81*A*a**4*b**4*x + C*c**8*x**15/15 + x**14*(B*c**8/14 + 6*C*b*c**7/7) + x*
*13*(A*c**8/13 + 12*B*b*c**7/13 + 66*C*b**2*c**6/13) + x**12*(A*b*c**7 + 1
1*B*b**2*c**6/2 + C*a*b*c**6 + 18*C*b**3*c**5) + x**11*(6*A*b**2*c**6 + 12
*B*a*b*c**6/11 + 216*B*b**3*c**5/11 + 108*C*a*b**2*c**5/11 + 459*C*b**4*c*
*4/11) + x**10*(6*A*a*b*c**6/5 + 108*A*b**3*c**5/5 + 54*B*a*b**2*c**5/5 +
459*B*b**4*c**4/10 + 216*C*a*b**3*c**4/5 + 324*C*b**5*c**3/5) + x**9*(12*A
*a*b**2*c**5 + 51*A*b**4*c**4 + 48*B*a*b**3*c**4 + 72*B*b**5*c**3 + 6*C*a*
*2*b**2*c**4 + 108*C*a*b**4*c**3 + 66*C*b**6*c**2) + x**8*(54*A*a*b**3*c**
4 + 81*A*b**5*c**3 + 27*B*a**2*b**2*c**4/4 + 243*B*a*b**4*c**3/2 + 297*B*b
**6*c**2/4 + 81*C*a**2*b**3*c**3/2 + 162*C*a*b**5*c**2 + 81*C*b**7*c/2) +
x**7*(54*A*a**2*b**2*c**4/7 + 972*A*a*b**4*c**3/7 + 594*A*b**6*c**2/7 + 32
4*B*a**2*b**3*c**3/7 + 1296*B*a*b**5*c**2/7 + 324*B*b**7*c/7 + 810*C*a**2*
b**4*c**2/7 + 972*C*a*b**6*c/7 + 81*C*b**8/7) + x**6*(54*A*a**2*b**3*c**3
+ 216*A*a*b**5*c**2 + 54*A*b**7*c + 135*B*a**2*b**4*c**2 + 162*B*a*b**6*c
+ 27*B*b**8/2 + 18*C*a**3*b**3*c**2 + 162*C*a**2*b**5*c + 54*C*a*b**7) + x
**5*(162*A*a**2*b**4*c**2 + 972*A*a*b**6*c/5 + 81*A*b**8/5 + 108*B*a**3*b*
*3*c**2/5 + 972*B*a**2*b**5*c/5 + 324*B*a*b**7/5 + 324*C*a**3*b**4*c/5 + 4
86*C*a**2*b**6/5) + x**4*(27*A*a**3*b**3*c**2 + 243*A*a**2*b**5*c + 81*A*a
*b**7 + 81*B*a**3*b**4*c + 243*B*a**2*b**6/2 + 81*C*a**3*b**5) + x**3*(108
*A*a**3*b**4*c + 162*A*a**2*b**6 + 108*B*a**3*b**5 + 27*C*a**4*b**4) + ...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 747 vs. $2(369) = 738$.

Time = 0.04 (sec) , antiderivative size = 747, normalized size of antiderivative = 1.93

$$\int \left(A+Bx+Cx^2\right)\left(3ab+3b^2x+3bcx^2+c^2x^3\right)^4\,dx=\text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)*(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^4,x, algorithm="
maxima")
```

output
```
1/15*C*c^8*x^15 + 1/14*(12*C*b*c^7 + B*c^8)*x^14 + 1/13*(66*C*b^2*c^6 + 12
*B*b*c^7 + A*c^8)*x^13 + 1/2*(36*C*b^3*c^5 + 2*A*b*c^7 + (2*C*a*b + 11*B*b
^2)*c^6)*x^12 + 3/11*(153*C*b^4*c^4 + 2*(2*B*a*b + 11*A*b^2)*c^6 + 36*(C*a
*b^2 + 2*B*b^3)*c^5)*x^11 + 3/10*(216*C*b^5*c^3 + 4*A*a*b*c^6 + 36*(B*a*b^
2 + 2*A*b^3)*c^5 + 9*(16*C*a*b^3 + 17*B*b^4)*c^4)*x^10 + 81*A*a^4*b^4*x +
3*(22*C*b^6*c^2 + 4*A*a*b^2*c^5 + (2*C*a^2*b^2 + 16*B*a*b^3 + 17*A*b^4)*c^
4 + 12*(3*C*a*b^4 + 2*B*b^5)*c^3)*x^9 + 27/4*(6*C*b^7*c + (B*a^2*b^2 + 8*A
*a*b^3)*c^4 + 6*(C*a^2*b^3 + 3*B*a*b^4 + 2*A*b^5)*c^3 + (24*C*a*b^5 + 11*B
*b^6)*c^2)*x^8 + 27/7*(3*C*b^8 + 2*A*a^2*b^2*c^4 + 12*(B*a^2*b^3 + 3*A*a*b
^4)*c^3 + 2*(15*C*a^2*b^4 + 24*B*a*b^5 + 11*A*b^6)*c^2 + 12*(3*C*a*b^6 + B
*b^7)*c)*x^7 + 9/2*(12*C*a*b^7 + 3*B*b^8 + 12*A*a^2*b^3*c^3 + 2*(2*C*a^3*b
^3 + 15*B*a^2*b^4 + 24*A*a*b^5)*c^2 + 12*(3*C*a^2*b^5 + 3*B*a*b^6 + A*b^7)
*c)*x^6 + 27/5*(18*C*a^2*b^6 + 12*B*a*b^7 + 3*A*b^8 + 2*(2*B*a^3*b^3 + 15*
A*a^2*b^4)*c^2 + 12*(C*a^3*b^4 + 3*B*a^2*b^5 + 3*A*a*b^6)*c)*x^5 + 27/2*(6
*C*a^3*b^5 + 9*B*a^2*b^6 + 6*A*a*b^7 + 2*A*a^3*b^3*c^2 + 6*(B*a^3*b^4 + 3*
A*a^2*b^5)*c)*x^4 + 27*(C*a^4*b^4 + 4*B*a^3*b^5 + 6*A*a^2*b^6 + 4*A*a^3*b^
4*c)*x^3 + 81/2*(B*a^4*b^4 + 4*A*a^3*b^5)*x^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 909 vs. $2(369) = 738$.

Time = 0.13 (sec) , antiderivative size = 909, normalized size of antiderivative = 2.35

$$\int (A + Bx + Cx^2) (3ab + 3b^2x + 3bcx^2 + c^2x^3)^4 \, dx = \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)*(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^4,x, algorithm="
giac")
```

output
```
1/15*C*c^8*x^15 + 6/7*C*b*c^7*x^14 + 1/14*B*c^8*x^14 + 66/13*C*b^2*c^6*x^1
3 + 12/13*B*b*c^7*x^13 + 1/13*A*c^8*x^13 + 18*C*b^3*c^5*x^12 + C*a*b*c^6*x
^12 + 11/2*B*b^2*c^6*x^12 + A*b*c^7*x^12 + 459/11*C*b^4*c^4*x^11 + 108/11*
C*a*b^2*c^5*x^11 + 216/11*B*b^3*c^5*x^11 + 12/11*B*a*b*c^6*x^11 + 6*A*b^2*
c^6*x^11 + 324/5*C*b^5*c^3*x^10 + 216/5*C*a*b^3*c^4*x^10 + 459/10*B*b^4*c^
4*x^10 + 54/5*B*a*b^2*c^5*x^10 + 108/5*A*b^3*c^5*x^10 + 6/5*A*a*b*c^6*x^10
 + 66*C*b^6*c^2*x^9 + 108*C*a*b^4*c^3*x^9 + 72*B*b^5*c^3*x^9 + 6*C*a^2*b^2
*c^4*x^9 + 48*B*a*b^3*c^4*x^9 + 51*A*b^4*c^4*x^9 + 12*A*a*b^2*c^5*x^9 + 81
/2*C*b^7*c*x^8 + 162*C*a*b^5*c^2*x^8 + 297/4*B*b^6*c^2*x^8 + 81/2*C*a^2*b^
3*c^3*x^8 + 243/2*B*a*b^4*c^3*x^8 + 81*A*b^5*c^3*x^8 + 27/4*B*a^2*b^2*c^4*
x^8 + 54*A*a*b^3*c^4*x^8 + 81/7*C*b^8*x^7 + 972/7*C*a*b^6*c*x^7 + 324/7*B*
b^7*c*x^7 + 810/7*C*a^2*b^4*c^2*x^7 + 1296/7*B*a*b^5*c^2*x^7 + 594/7*A*b^6
*c^2*x^7 + 324/7*B*a^2*b^3*c^3*x^7 + 972/7*A*a*b^4*c^3*x^7 + 54/7*A*a^2*b^
2*c^4*x^7 + 54*C*a*b^7*x^6 + 27/2*B*b^8*x^6 + 162*C*a^2*b^5*c*x^6 + 162*B*
a*b^6*c*x^6 + 54*A*b^7*c*x^6 + 18*C*a^3*b^3*c^2*x^6 + 135*B*a^2*b^4*c^2*x^
6 + 216*A*a*b^5*c^2*x^6 + 54*A*a^2*b^3*c^3*x^6 + 486/5*C*a^2*b^6*x^5 + 324
/5*B*a*b^7*x^5 + 81/5*A*b^8*x^5 + 324/5*C*a^3*b^4*c*x^5 + 972/5*B*a^2*b^5*
c*x^5 + 972/5*A*a*b^6*c*x^5 + 108/5*B*a^3*b^3*c^2*x^5 + 162*A*a^2*b^4*c^2*
x^5 + 81*C*a^3*b^5*x^4 + 243/2*B*a^2*b^6*x^4 + 81*A*a*b^7*x^4 + 81*B*a^3*b
^4*c*x^4 + 243*A*a^2*b^5*c*x^4 + 27*A*a^3*b^3*c^2*x^4 + 27*C*a^4*b^4*x^...
```

Mupad [B] (verification not implemented)

Time = 13.08 (sec) , antiderivative size = 718, normalized size of antiderivative = 1.86

$$\int (A+Bx+Cx^2)\left(3ab+3b^2x+3bcx^2+c^2x^3\right)^4\,dx$$
$$= x^8\left(\frac{81\,C\,a^2\,b^3\,c^3}{2}+\frac{27\,B\,a^2\,b^2\,c^4}{4}+162\,C\,a\,b^5\,c^2+\frac{243\,B\,a\,b^4\,c^3}{2}+54\,A\,a\,b^3\,c^4+\frac{81\,C\,b^7\,c}{2}+\frac{297\,B\,b^6\,c^2}{4}+81\,A\,b^5\,c^3\right)+x^{10}\left(\frac{324\,C\,b^5\,c^3}{5}+\frac{459\,B\,b^4\,c^4}{10}+\frac{108\,A\,b^3\,c^5}{5}+\frac{216\,C\,a\,b^3\,c^4}{5}+\frac{54\,B\,a\,b^2\,c^5}{5}+\frac{6\,A\,a\,b\,c^6}{5}\right)$$
$$+x^6\left(18\,C\,a^3\,b^3\,c^2+162\,C\,a^2\,b^5\,c+135\,B\,a^2\,b^4\,c^2+54\,A\,a^2\,b^3\,c^3+54\,C\,a\,b^7+162\,B\,a\,b^6\,c+216\,A\,a\,b^5\,c^2+\frac{27\,B\,b^8}{2}+54\,A\,b^7\,c\right)+x^{14}\left(\frac{B\,c^8}{14}+\frac{6\,C\,b\,c^7}{7}\right)$$
$$+x^7\left(\frac{810\,C\,a^2\,b^4\,c^2}{7}+\frac{324\,B\,a^2\,b^3\,c^3}{7}+\frac{54\,A\,a^2\,b^2\,c^4}{7}+\frac{972\,C\,a\,b^6\,c}{7}+\frac{1296\,B\,a\,b^5\,c^2}{7}+\frac{972\,A\,a\,b^4\,c^3}{7}+\frac{81\,C\,b^8}{7}+\frac{324\,B\,b^7\,c}{7}+\frac{594\,A\,b^6\,c^2}{7}\right)$$
$$+x^5\left(\frac{324\,C\,a^3\,b^4\,c}{5}+\frac{108\,B\,a^3\,b^3\,c^2}{5}+\frac{486\,C\,a^2\,b^6}{5}+\frac{972\,B\,a^2\,b^5\,c}{5}+162\,A\,a^2\,b^4\,c^2+\frac{324\,B\,a\,b^7}{5}+\frac{972\,A\,a\,b^6\,c}{5}+\frac{81\,A\,b^8}{5}\right)$$
$$+x^{13}\left(\frac{66\,C\,b^2\,c^6}{13}+\frac{12\,B\,b\,c^7}{13}+\frac{A\,c^8}{13}\right)+\frac{C\,c^8\,x^{15}}{15}+\frac{81\,a^3\,b^4\,x^2\,(4\,A\,b+B\,a)}{2}$$
$$+\frac{3\,b\,c^4\,x^{11}\,(153\,C\,b^3+72\,B\,b^2\,c+22\,A\,b\,c^2+36\,C\,a\,b\,c+4\,B\,a\,c^2)}{11}$$
$$+\frac{b\,c^5\,x^{12}\,(36\,C\,b^2+11\,B\,b\,c+2\,A\,c^2+2\,C\,a\,c)}{2}$$
$$+\frac{27\,a\,b^3\,x^4\,(6\,C\,a^2\,b^2+6\,B\,a^2\,b\,c+2\,A\,a^2\,c^2+9\,B\,a\,b^3+18\,A\,a\,b^2\,c+6\,A\,b^4)}{2}$$
$$+81\,A\,a^4\,b^4\,x+3\,b^2\,c^2\,x^9\left(2\,C\,a^2\,c^2+36\,C\,a\,b^2\,c+16\,B\,a\,b\,c^2+4\,A\,a\,c^3+22\,C\,b^4+24\,B\,b^3\,c+17\,A\,b^2\,c^2\right)+27\,a^2\,b^4\,x^3\left(C\,a^2+4\,B\,a\,b+4\,A\,c\,a+6\,A\,b^2\right)$$

input
```
int((A + B*x + C*x^2)*(3*a*b + 3*b^2*x + c^2*x^3 + 3*b*c*x^2)^4,x)
```

output
```
x^8*(81*A*b^5*c^3 + (297*B*b^6*c^2)/4 + (81*C*b^7*c)/2 + 54*A*a*b^3*c^4 +
(243*B*a*b^4*c^3)/2 + 162*C*a*b^5*c^2 + (27*B*a^2*b^2*c^4)/4 + (81*C*a^2*b
^3*c^3)/2) + x^10*((108*A*b^3*c^5)/5 + (459*B*b^4*c^4)/10 + (324*C*b^5*c^3
)/5 + (6*A*a*b*c^6)/5 + (54*B*a*b^2*c^5)/5 + (216*C*a*b^3*c^4)/5) + x^6*((
27*B*b^8)/2 + 54*A*b^7*c + 54*C*a*b^7 + 162*B*a*b^6*c + 216*A*a*b^5*c^2 +
162*C*a^2*b^5*c + 54*A*a^2*b^3*c^3 + 135*B*a^2*b^4*c^2 + 18*C*a^3*b^3*c^2)
 + x^14*((B*c^8)/14 + (6*C*b*c^7)/7) + x^7*((81*C*b^8)/7 + (594*A*b^6*c^2)
/7 + (324*B*b^7*c)/7 + (972*C*a*b^6*c)/7 + (972*A*a*b^4*c^3)/7 + (1296*B*a
*b^5*c^2)/7 + (54*A*a^2*b^2*c^4)/7 + (324*B*a^2*b^3*c^3)/7 + (810*C*a^2*b^
4*c^2)/7) + x^5*((81*A*b^8)/5 + (486*C*a^2*b^6)/5 + (324*B*a*b^7)/5 + (972
*A*a*b^6*c)/5 + (972*B*a^2*b^5*c)/5 + (324*C*a^3*b^4*c)/5 + 162*A*a^2*b^4*
c^2 + (108*B*a^3*b^3*c^2)/5) + x^13*((A*c^8)/13 + (66*C*b^2*c^6)/13 + (12*
B*b*c^7)/13) + (C*c^8*x^15)/15 + (81*a^3*b^4*x^2*(4*A*b + B*a))/2 + (3*b*c
^4*x^11*(153*C*b^3 + 22*A*b*c^2 + 4*B*a*c^2 + 72*B*b^2*c + 36*C*a*b*c))/11
 + (b*c^5*x^12*(2*A*c^2 + 36*C*b^2 + 11*B*b*c + 2*C*a*c))/2 + (27*a*b^3*x^
4*(6*A*b^4 + 2*A*a^2*c^2 + 6*C*a^2*b^2 + 9*B*a*b^3 + 18*A*a*b^2*c + 6*B*a^
2*b*c))/2 + 81*A*a^4*b^4*x + 3*b^2*c^2*x^9*(22*C*b^4 + 17*A*b^2*c^2 + 2*C*
a^2*c^2 + 4*A*a*c^3 + 24*B*b^3*c + 16*B*a*b*c^2 + 36*C*a*b^2*c) + 27*a^2*b
^4*x^3*(6*A*b^2 + C*a^2 + 4*A*a*c + 4*B*a*b)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 401, normalized size of antiderivative = 1.04

$$\int (A + Bx + Cx^2) (3ab + 3b^2x + 3bcx^2 + c^2x^3)^4 \, dx$$
$$= \frac{x(4004c^9x^{14} + 55770b\,c^8x^{13} + 4620a\,c^8x^{12} + 360360b^2c^7x^{12} + 120120ab\,c^7x^{11} + 1411410b^3c^6x^{11} + 10155}{}$$

input
```
int((C*x^2+B*x+A)*(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^4,x)
```

output
```
(x*(4864860*a**5*b**4 + 12162150*a**4*b**5*x + 8108100*a**4*b**4*c*x**2 +
1621620*a**4*b**3*c**2*x**3 + 16216200*a**3*b**6*x**2 + 24324300*a**3*b**5
*c*x**3 + 14918904*a**3*b**4*c**2*x**4 + 4324320*a**3*b**3*c**3*x**5 + 463
320*a**3*b**2*c**4*x**6 + 12162150*a**2*b**7*x**3 + 29189160*a**2*b**6*c*x
**4 + 30810780*a**2*b**5*c**2*x**5 + 18069480*a**2*b**4*c**3*x**6 + 608107
5*a**2*b**3*c**4*x**7 + 1081080*a**2*b**2*c**5*x**8 + 72072*a**2*b*c**6*x*
*9 + 4864860*a*b**8*x**4 + 16216200*a*b**7*c*x**5 + 24555960*a*b**6*c**2*x
**6 + 21891870*a*b**5*c**3*x**7 + 12432420*a*b**4*c**4*x**8 + 4540536*a*b*
*3*c**5*x**9 + 1015560*a*b**2*c**6*x**10 + 120120*a*b*c**7*x**11 + 4620*a*
c**8*x**12 + 810810*b**9*x**5 + 3474900*b**8*c*x**6 + 6891885*b**7*c**2*x*
*7 + 8288280*b**6*c**3*x**8 + 6648642*b**5*c**4*x**9 + 3685500*b**4*c**5*x
**10 + 1411410*b**3*c**6*x**11 + 360360*b**2*c**7*x**12 + 55770*b*c**8*x**
13 + 4004*c**9*x**14))/60060
```

3.68 $\int (A+Bx+Cx^2)\left(3ab+3b^2x+3bcx^2+c^2x^3\right)^3\,dx$

Optimal result

Integrand size = 38, antiderivative size = 310

$$\begin{aligned}
&\int (A+Bx+Cx^2)\left(3ab+3b^2x+3bcx^2+c^2x^3\right)^3\,dx \\
&= \frac{b^3(b^2-3ac)^3\,(bBc-Ac^2-b^2C)\,x}{c^5}-\frac{b^3(b^2-3ac)^3\,(Bc-2bC)(b+cx)^2}{2c^6} \\
&\quad -\frac{3b^2(b^2-3ac)^2\,(bBc-Ac^2-b^2C)\,(b+cx)^4}{4c^6} \\
&\quad +\frac{3b^2(b^2-3ac)^2\,(Bc-2bC)(b+cx)^5}{5c^6}+\frac{3b(b^2-3ac)\,(bBc-Ac^2-b^2C)\,(b+cx)^7}{7c^6} \\
&\quad -\frac{3b(b^2-3ac)\,(Bc-2bC)(b+cx)^8}{8c^6}-\frac{(bBc-Ac^2-b^2C)\,(b+cx)^{10}}{10c^6} \\
&\quad +\frac{(Bc-2bC)(b+cx)^{11}}{11c^6}+\frac{C\left(b\left(3a-\frac{b^2}{c}\right)+\frac{(b+cx)^3}{c}\right)^4}{12c^2}
\end{aligned}$$

output

```
b^3*(-3*a*c+b^2)^3*(-A*c^2+B*b*c-C*b^2)*x/c^5-1/2*b^3*(-3*a*c+b^2)^3*(B*c-
2*C*b)*(c*x+b)^2/c^6-3/4*b^2*(-3*a*c+b^2)^2*(-A*c^2+B*b*c-C*b^2)*(c*x+b)^4
/c^6+3/5*b^2*(-3*a*c+b^2)^2*(B*c-2*C*b)*(c*x+b)^5/c^6+3/7*b*(-3*a*c+b^2)*(
-A*c^2+B*b*c-C*b^2)*(c*x+b)^7/c^6-3/8*b*(-3*a*c+b^2)*(B*c-2*C*b)*(c*x+b)^8
/c^6-1/10*(-A*c^2+B*b*c-C*b^2)*(c*x+b)^10/c^6+1/11*(B*c-2*C*b)*(c*x+b)^11/
c^6+1/12*C*(b*(3*a-b^2/c)+(c*x+b)^3/c)^4/c^2
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 408, normalized size of antiderivative = 1.32

$$\int (A+Bx+Cx^2)\left(3ab+3b^2x+3bcx^2+c^2x^3\right)^3\,dx$$
$$= 27a^3Ab^3x+\frac{27}{2}a^2b^3(3Ab+aB)x^2+9ab^3\left(3A\left(b^2+ac\right)+a(3bB+aC)\right)x^3$$
$$+\frac{27}{4}b^2\left(A\left(b^4+6ab^2c+a^2c^2\right)+3ab\left(b^2B+aBc+abC\right)\right)x^4$$
$$+\frac{27}{5}b^2\left(b^4B+6ab^2Bc+a^2Bc^2+3b^3(Ac+aC)+abc(5Ac+3aC)\right)x^5$$
$$+\frac{9}{2}b^2\left(3b^3Bc+5abBc^2+b^4C+ac^2(2Ac+aC)+2b^2c(2Ac+3aC)\right)x^6$$
$$+\frac{9}{7}bc\left(12b^3Bc+6abBc^2+aAc^3+9b^4C+3b^2c(3Ac+5aC)\right)x^7$$
$$+\frac{9}{8}bc^2\left(9b^2Bc+aBc^2+12b^3C+2bc(2Ac+3aC)\right)x^8$$
$$+bc^3\left(4bBc+9b^2C+c(Ac+aC)\right)x^9$$
$$+\frac{1}{10}c^4\left(9bBc+Ac^2+36b^2C\right)x^{10}+\frac{1}{11}c^5(Bc+9bC)x^{11}+\frac{1}{12}c^6Cx^{12}$$

input
```
Integrate[(A + B*x + C*x^2)*(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3)^3,x]
```

output
```
27*a^3*A*b^3*x + (27*a^2*b^3*(3*A*b + a*B)*x^2)/2 + 9*a*b^3*(3*A*(b^2 + a*
c) + a*(3*b*B + a*C))*x^3 + (27*b^2*(A*(b^4 + 6*a*b^2*c + a^2*c^2) + 3*a*b
*(b^2*B + a*B*c + a*b*C))*x^4)/4 + (27*b^2*(b^4*B + 6*a*b^2*B*c + a^2*B*c^
2 + 3*b^3*(A*c + a*C) + a*b*c*(5*A*c + 3*a*C))*x^5)/5 + (9*b^2*(3*b^3*B*c
+ 5*a*b*B*c^2 + b^4*C + a*c^2*(2*A*c + a*C) + 2*b^2*c*(2*A*c + 3*a*C))*x^6
)/2 + (9*b*c*(12*b^3*B*c + 6*a*b*B*c^2 + a*A*c^3 + 9*b^4*C + 3*b^2*c*(3*A*
c + 5*a*C))*x^7)/7 + (9*b*c^2*(9*b^2*B*c + a*B*c^2 + 12*b^3*C + 2*b*c*(2*A
*c + 3*a*C))*x^8)/8 + b*c^3*(4*b*B*c + 9*b^2*C + c*(A*c + a*C))*x^9 + (c^4
*(9*b*B*c + A*c^2 + 36*b^2*C)*x^10)/10 + (c^5*(B*c + 9*b*C)*x^11)/11 + (c^
6*C*x^12)/12
```

Rubi [A] (verified)

Time = 1.49 (sec) , antiderivative size = 408, normalized size of antiderivative = 1.32, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(3ab+3b^2x+3bcx^2+c^2x^3\right)^3 \left(A+Bx+Cx^2\right) \, dx$$

$$\Big\downarrow \text{2188}$$

$$\int \left(27a^3Ab^3+27a^2b^3x(aB+3Ab)+27b^2x^3\left(A\left(a^2c^2+6ab^2c+b^4\right)+3ab\left(abC+aBc+b^2B\right)\right)+27b^2x^4\left(a^2Bc^2\right.\right.$$

$$\Big\downarrow \text{2009}$$

$$27a^3Ab^3x+\frac{27}{2}a^2b^3x^2(aB+3Ab)+\frac{27}{4}b^2x^4\left(A\left(a^2c^2+6ab^2c+b^4\right)+3ab\left(abC+aBc+b^2B\right)\right)+\frac{27}{5}b^2x^5\left(a^2Bc^2+3b^3(aC+Ac)+abc(3aC+5Ac)+6ab^2Bc+b^4B\right)+bc^3x^9\left(c(aC+Ac)+9b^2C+4bBc\right)+\frac{9}{8}bc^2x^8\left(2bc(3aC+2Ac)+aBc^2+12b^3C+9b^2Bc\right)+9ab^3x^3\left(3A\left(ac+b^2\right)+a(aC+3bB)\right)+\frac{9}{2}b^2x^6\left(2b^2c(3aC+2Ac)+ac^2(aC+2Ac)+5abBc^2+b^4C+3b^3Bc\right)+\frac{9}{7}bcx^7\left(3b^2c(5aC+3Ac)+aAc^3+6abBc^2+9b^4C+12b^3Bc\right)+\frac{1}{10}c^4x^{10}\left(Ac^2+36b^2C+9bBc\right)+\frac{1}{11}c^5x^{11}(9bC+Bc)+\frac{1}{12}c^6Cx^{12}$$

input
```
Int[(A + B*x + C*x^2)*(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3)^3,x]
```

output
```
27*a^3*A*b^3*x + (27*a^2*b^3*(3*A*b + a*B)*x^2)/2 + 9*a*b^3*(3*A*(b^2 + a*
c) + a*(3*b*B + a*C))*x^3 + (27*b^2*(A*(b^4 + 6*a*b^2*c + a^2*c^2) + 3*a*b
*(b^2*B + a*B*c + a*b*C))*x^4)/4 + (27*b^2*(b^4*B + 6*a*b^2*B*c + a^2*B*c^
2 + 3*b^3*(A*c + a*C) + a*b*c*(5*A*c + 3*a*C))*x^5)/5 + (9*b^2*(3*b^3*B*c
+ 5*a*b*B*c^2 + b^4*C + a*c^2*(2*A*c + a*C) + 2*b^2*c*(2*A*c + 3*a*C))*x^6
)/2 + (9*b*c*(12*b^3*B*c + 6*a*b*B*c^2 + a*A*c^3 + 9*b^4*C + 3*b^2*c*(3*A*
c + 5*a*C))*x^7)/7 + (9*b*c^2*(9*b^2*B*c + a*B*c^2 + 12*b^3*C + 2*b*c*(2*A
*c + 3*a*C))*x^8)/8 + b*c^3*(4*b*B*c + 9*b^2*C + c*(A*c + a*C))*x^9 + (c^4
*(9*b*B*c + A*c^2 + 36*b^2*C)*x^10)/10 + (c^5*(B*c + 9*b*C)*x^11)/11 + (c^
6*C*x^12)/12
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 462, normalized size of antiderivative = 1.49

method	result
norman	$\frac{C\,c^6x^{12}}{12}+\left(\frac{1}{11}B\,c^6+\frac{9}{11}Cb\,c^5\right)x^{11}+\left(\frac{1}{10}A\,c^6+\frac{9}{10}Bb\,c^5+\frac{18}{5}C\,b^2c^4\right)x^{10}+\left(Ab\,c^5+4B\,b^2c^4+\right.$
gosper	$\frac{27}{2}B\,x^2a^3b^3+27Ax\,a^3b^3+\frac{81}{4}B\,x^4a\,b^5+\frac{81}{4}C\,x^4a^2b^4+27A\,a^2b^3c\,x^3+\frac{27}{4}x^4A\,a^2b^2c^2+\frac{81}{2}x^4$
risch	$\frac{27}{2}B\,x^2a^3b^3+27Ax\,a^3b^3+\frac{81}{4}B\,x^4a\,b^5+\frac{81}{4}C\,x^4a^2b^4+27A\,a^2b^3c\,x^3+\frac{27}{4}x^4A\,a^2b^2c^2+\frac{81}{2}x^4$
parallelrisch	$\frac{27}{2}B\,x^2a^3b^3+27Ax\,a^3b^3+\frac{81}{4}B\,x^4a\,b^5+\frac{81}{4}C\,x^4a^2b^4+27A\,a^2b^3c\,x^3+\frac{27}{4}x^4A\,a^2b^2c^2+\frac{81}{2}x^4$
orering	$\frac{x(770C\,c^6x^{11}+840B\,c^6x^{10}+7560Cb\,c^5x^{10}+924A\,c^6x^9+8316Bb\,c^5x^9+33264C\,b^2c^4x^9+9240Ab\,c^5x^8+36960B\,b^2c^4x^8+9240}{}$
default	$\frac{C\,c^6x^{12}}{12}+\frac{(B\,c^6+9Cb\,c^5)x^{11}}{11}+\frac{(A\,c^6+9Bb\,c^5+36C\,b^2c^4)x^{10}}{10}+\frac{(9Ab\,c^5+36B\,b^2c^4+C(3ab\,c^4+63b^3c^3+c^2(6ab\,c^2+18b}{9}$

input
```
int((C*x^2+B*x+A)*(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^3,x,method=_RETURNVERB
OSE)
```

output
```
1/12*C*c^6*x^12+(1/11*B*c^6+9/11*C*b*c^5)*x^11+(1/10*A*c^6+9/10*B*b*c^5+18
/5*C*b^2*c^4)*x^10+(A*b*c^5+4*B*b^2*c^4+C*a*b*c^4+9*C*b^3*c^3)*x^9+(9/2*A*
b^2*c^4+9/8*B*a*b*c^4+81/8*B*b^3*c^3+27/4*C*a*b^2*c^3+27/2*C*b^4*c^2)*x^8+
(9/7*A*a*b*c^4+81/7*A*b^3*c^3+54/7*B*a*b^2*c^3+108/7*B*b^4*c^2+135/7*C*a*b
^3*c^2+81/7*C*b^5*c)*x^7+(9*A*a*b^2*c^3+18*A*b^4*c^2+45/2*B*a*b^3*c^2+27/2
*B*b^5*c+9/2*C*a^2*b^2*c^2+27*C*a*b^4*c+9/2*C*b^6)*x^6+(27*A*a*b^3*c^2+81/
5*A*b^5*c+27/5*B*a^2*b^2*c^2+162/5*B*a*b^4*c+27/5*B*b^6+81/5*C*a^2*b^3*c+8
1/5*C*b^5*a)*x^5+(27/4*A*a^2*b^2*c^2+81/2*A*a*b^4*c+27/4*A*b^6+81/4*B*a^2*
b^3*c+81/4*B*b^5*a+81/4*b^4*C*a^2)*x^4+(27*A*a^2*b^3*c+27*A*a*b^5+27*B*a^2
*b^4+9*C*a^3*b^3)*x^3+(81/2*A*a^2*b^4+27/2*B*a^3*b^3)*x^2+27*A*x*a^3*b^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 452, normalized size of antiderivative = 1.46

$$\begin{aligned}
&\int \left(A+Bx+Cx^2\right)\left(3ab+3b^2x+3bcx^2+c^2x^3\right)^3\,dx \\
&= \frac{1}{12}\,Cc^6x^{12}+\frac{1}{11}\left(9\,Cbc^5+Bc^6\right)x^{11}+\frac{1}{10}\left(36\,Cb^2c^4+9\,Bbc^5+Ac^6\right)x^{10} \\
&\quad+\left(9\,Cb^3c^3+Abc^5+\left(Cab+4\,Bb^2\right)c^4\right)x^9 \\
&\quad+\frac{9}{8}\left(12\,Cb^4c^2+\left(Bab+4\,Ab^2\right)c^4+3\left(2\,Cab^2+3\,Bb^3\right)c^3\right)x^8+27\,Aa^3b^3x \\
&\quad+\frac{9}{7}\left(9\,Cb^5c+Aabc^4+3\left(2\,Bab^2+3\,Ab^3\right)c^3+3\left(5\,Cab^3+4\,Bb^4\right)c^2\right)x^7 \\
&\quad+\frac{9}{2}\left(Cb^6+2\,Aab^2c^3+\left(Ca^2b^2+5\,Bab^3+4\,Ab^4\right)c^2+3\left(2\,Cab^4+Bb^5\right)c\right)x^6 \\
&\quad+\frac{27}{5}\left(3\,Cab^5+Bb^6+\left(Ba^2b^2+5\,Aab^3\right)c^2+3\left(Ca^2b^3+2\,Bab^4+Ab^5\right)c\right)x^5 \\
&\quad+\frac{27}{4}\left(3\,Ca^2b^4+3\,Bab^5+Ab^6+Aa^2b^2c^2+3\left(Ba^2b^3+2\,Aab^4\right)c\right)x^4 \\
&\quad+9\left(Ca^3b^3+3\,Ba^2b^4+3\,Aab^5+3\,Aa^2b^3c\right)x^3+\frac{27}{2}\left(Ba^3b^3+3\,Aa^2b^4\right)x^2
\end{aligned}$$

input
```
integrate((C*x^2+B*x+A)*(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^3,x, algorithm="
fricas")
```

output
```
1/12*C*c^6*x^12 + 1/11*(9*C*b*c^5 + B*c^6)*x^11 + 1/10*(36*C*b^2*c^4 + 9*B
*b*c^5 + A*c^6)*x^10 + (9*C*b^3*c^3 + A*b*c^5 + (C*a*b + 4*B*b^2)*c^4)*x^9
 + 9/8*(12*C*b^4*c^2 + (B*a*b + 4*A*b^2)*c^4 + 3*(2*C*a*b^2 + 3*B*b^3)*c^3
)*x^8 + 27*A*a^3*b^3*x + 9/7*(9*C*b^5*c + A*a*b*c^4 + 3*(2*B*a*b^2 + 3*A*b
^3)*c^3 + 3*(5*C*a*b^3 + 4*B*b^4)*c^2)*x^7 + 9/2*(C*b^6 + 2*A*a*b^2*c^3 +
(C*a^2*b^2 + 5*B*a*b^3 + 4*A*b^4)*c^2 + 3*(2*C*a*b^4 + B*b^5)*c)*x^6 + 27/
5*(3*C*a*b^5 + B*b^6 + (B*a^2*b^2 + 5*A*a*b^3)*c^2 + 3*(C*a^2*b^3 + 2*B*a*
b^4 + A*b^5)*c)*x^5 + 27/4*(3*C*a^2*b^4 + 3*B*a*b^5 + A*b^6 + A*a^2*b^2*c^
2 + 3*(B*a^2*b^3 + 2*A*a*b^4)*c)*x^4 + 9*(C*a^3*b^3 + 3*B*a^2*b^4 + 3*A*a*
b^5 + 3*A*a^2*b^3*c)*x^3 + 27/2*(B*a^3*b^3 + 3*A*a^2*b^4)*x^2
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 568, normalized size of antiderivative = 1.83

$$\int (A+Bx+Cx^2)\left(3ab+3b^2x+3bcx^2+c^2x^3\right)^3\,dx$$
$$= 27Aa^3b^3x + \frac{Cc^6x^{12}}{12} + x^{11}\left(\frac{Bc^6}{11}+\frac{9Cbc^5}{11}\right) + x^{10}\left(\frac{Ac^6}{10}+\frac{9Bbc^5}{10}+\frac{18Cb^2c^4}{5}\right)$$
$$+ x^9\left(Abc^5+4Bb^2c^4+Cabc^4+9Cb^3c^3\right) + x^8$$
$$\cdot\left(\frac{9Ab^2c^4}{2}+\frac{9Babc^4}{8}+\frac{81Bb^3c^3}{8}+\frac{27Cab^2c^3}{4}+\frac{27Cb^4c^2}{2}\right) + x^7$$
$$\cdot\left(\frac{9Aabc^4}{7}+\frac{81Ab^3c^3}{7}+\frac{54Bab^2c^3}{7}+\frac{108Bb^4c^2}{7}+\frac{135Cab^3c^2}{7}+\frac{81Cb^5c}{7}\right) + x^6$$
$$\cdot\left(9Aab^2c^3+18Ab^4c^2+\frac{45Bab^3c^2}{2}+\frac{27Bb^5c}{2}+\frac{9Ca^2b^2c^2}{2}+27Cab^4c+\frac{9Cb^6}{2}\right) + x^5$$
$$\cdot\left(27Aab^3c^2+\frac{81Ab^5c}{5}+\frac{27Ba^2b^2c^2}{5}+\frac{162Bab^4c}{5}+\frac{27Bb^6}{5}+\frac{81Ca^2b^3c}{5}+\frac{81Cab^5}{5}\right)$$
$$+ x^4\cdot\left(\frac{27Aa^2b^2c^2}{4}+\frac{81Aab^4c}{2}+\frac{27Ab^6}{4}+\frac{81Ba^2b^3c}{4}+\frac{81Bab^5}{4}+\frac{81Ca^2b^4}{4}\right) + x^3$$
$$\cdot\left(27Aa^2b^3c+27Aab^5+27Ba^2b^4+9Ca^3b^3\right) + x^2\cdot\left(\frac{81Aa^2b^4}{2}+\frac{27Ba^3b^3}{2}\right)$$

input
```
integrate((C*x**2+B*x+A)*(c**2*x**3+3*b*c*x**2+3*b**2*x+3*a*b)**3,x)
```

output
```
27*A*a**3*b**3*x + C*c**6*x**12/12 + x**11*(B*c**6/11 + 9*C*b*c**5/11) + x
**10*(A*c**6/10 + 9*B*b*c**5/10 + 18*C*b**2*c**4/5) + x**9*(A*b*c**5 + 4*B
*b**2*c**4 + C*a*b*c**4 + 9*C*b**3*c**3) + x**8*(9*A*b**2*c**4/2 + 9*B*a*b
*c**4/8 + 81*B*b**3*c**3/8 + 27*C*a*b**2*c**3/4 + 27*C*b**4*c**2/2) + x**7
*(9*A*a*b*c**4/7 + 81*A*b**3*c**3/7 + 54*B*a*b**2*c**3/7 + 108*B*b**4*c**2
/7 + 135*C*a*b**3*c**2/7 + 81*C*b**5*c/7) + x**6*(9*A*a*b**2*c**3 + 18*A*b
**4*c**2 + 45*B*a*b**3*c**2/2 + 27*B*b**5*c/2 + 9*C*a**2*b**2*c**2/2 + 27*
C*a*b**4*c + 9*C*b**6/2) + x**5*(27*A*a*b**3*c**2 + 81*A*b**5*c/5 + 27*B*a
**2*b**2*c**2/5 + 162*B*a*b**4*c/5 + 27*B*b**6/5 + 81*C*a**2*b**3*c/5 + 81
*C*a*b**5/5) + x**4*(27*A*a**2*b**2*c**2/4 + 81*A*a*b**4*c/2 + 27*A*b**6/4
 + 81*B*a**2*b**3*c/4 + 81*B*a*b**5/4 + 81*C*a**2*b**4/4) + x**3*(27*A*a**
2*b**3*c + 27*A*a*b**5 + 27*B*a**2*b**4 + 9*C*a**3*b**3) + x**2*(81*A*a**2
*b**4/2 + 27*B*a**3*b**3/2)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 452, normalized size of antiderivative = 1.46

$$
\begin{aligned}
&\int (A+Bx+Cx^2)\left(3ab+3b^2x+3bcx^2+c^2x^3\right)^3\,dx \\
&= \frac{1}{12}\,Cc^6x^{12}+\frac{1}{11}\left(9\,Cbc^5+Bc^6\right)x^{11}+\frac{1}{10}\left(36\,Cb^2c^4+9\,Bbc^5+Ac^6\right)x^{10} \\
&\quad+\left(9\,Cb^3c^3+Abc^5+\left(Cab+4\,Bb^2\right)c^4\right)x^9 \\
&\quad+\frac{9}{8}\left(12\,Cb^4c^2+\left(Bab+4\,Ab^2\right)c^4+3\left(2\,Cab^2+3\,Bb^3\right)c^3\right)x^8+27\,Aa^3b^3x \\
&\quad+\frac{9}{7}\left(9\,Cb^5c+Aabc^4+3\left(2\,Bab^2+3\,Ab^3\right)c^3+3\left(5\,Cab^3+4\,Bb^4\right)c^2\right)x^7 \\
&\quad+\frac{9}{2}\left(Cb^6+2\,Aab^2c^3+\left(Ca^2b^2+5\,Bab^3+4\,Ab^4\right)c^2+3\left(2\,Cab^4+Bb^5\right)c\right)x^6 \\
&\quad+\frac{27}{5}\left(3\,Cab^5+Bb^6+\left(Ba^2b^2+5\,Aab^3\right)c^2+3\left(Ca^2b^3+2\,Bab^4+Ab^5\right)c\right)x^5 \\
&\quad+\frac{27}{4}\left(3\,Ca^2b^4+3\,Bab^5+Ab^6+Aa^2b^2c^2+3\left(Ba^2b^3+2\,Aab^4\right)c\right)x^4 \\
&\quad+9\left(Ca^3b^3+3\,Ba^2b^4+3\,Aab^5+3\,Aa^2b^3c\right)x^3+\frac{27}{2}\left(Ba^3b^3+3\,Aa^2b^4\right)x^2
\end{aligned}
$$

input
```
integrate((C*x^2+B*x+A)*(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^3,x, algorithm="
maxima")
```

output
```
1/12*C*c^6*x^12 + 1/11*(9*C*b*c^5 + B*c^6)*x^11 + 1/10*(36*C*b^2*c^4 + 9*B
*b*c^5 + A*c^6)*x^10 + (9*C*b^3*c^3 + A*b*c^5 + (C*a*b + 4*B*b^2)*c^4)*x^9
 + 9/8*(12*C*b^4*c^2 + (B*a*b + 4*A*b^2)*c^4 + 3*(2*C*a*b^2 + 3*B*b^3)*c^3
)*x^8 + 27*A*a^3*b^3*x + 9/7*(9*C*b^5*c + A*a*b*c^4 + 3*(2*B*a*b^2 + 3*A*b
^3)*c^3 + 3*(5*C*a*b^3 + 4*B*b^4)*c^2)*x^7 + 9/2*(C*b^6 + 2*A*a*b^2*c^3 +
(C*a^2*b^2 + 5*B*a*b^3 + 4*A*b^4)*c^2 + 3*(2*C*a*b^4 + B*b^5)*c)*x^6 + 27/
5*(3*C*a*b^5 + B*b^6 + (B*a^2*b^2 + 5*A*a*b^3)*c^2 + 3*(C*a^2*b^3 + 2*B*a*
b^4 + A*b^5)*c)*x^5 + 27/4*(3*C*a^2*b^4 + 3*B*a*b^5 + A*b^6 + A*a^2*b^2*c^
2 + 3*(B*a^2*b^3 + 2*A*a*b^4)*c)*x^4 + 9*(C*a^3*b^3 + 3*B*a^2*b^4 + 3*A*a*
b^5 + 3*A*a^2*b^3*c)*x^3 + 27/2*(B*a^3*b^3 + 3*A*a^2*b^4)*x^2
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 549, normalized size of antiderivative = 1.77

$$\int (A+Bx+Cx^2)\left(3ab+3b^2x+3bcx^2+c^2x^3\right)^3\,dx$$
$$= \frac{1}{12}Cc^6x^{12}+\frac{9}{11}Cbc^5x^{11}+\frac{1}{11}Bc^6x^{11}+\frac{18}{5}Cb^2c^4x^{10}+\frac{9}{10}Bbc^5x^{10}+\frac{1}{10}Ac^6x^{10}+9Cb^3c^3x^9+Cabc^4x^9+4Bb^2c^4x^9+Abc^5x^9+\frac{27}{2}Cb^4c^2x^8+\frac{27}{4}Cab^2c^3x^8+\frac{81}{8}Bb^3c^3x^8+\frac{9}{8}Babc^4x^8+\frac{9}{2}Ab^2c^4x^8+\frac{81}{7}Cb^5cx^7+\frac{135}{7}Cab^3c^2x^7+\frac{108}{7}Bb^4c^2x^7+\frac{54}{7}Bab^2c^3x^7+\frac{81}{7}Ab^3c^3x^7+\frac{9}{7}Aabc^4x^7+\frac{9}{2}Cb^6x^6+27Cab^4cx^6+\frac{27}{2}Bb^5cx^6+\frac{9}{2}Ca^2b^2c^2x^6+\frac{45}{2}Bab^3c^2x^6+18Ab^4c^2x^6+9Aab^2c^3x^6+\frac{81}{5}Cab^5x^5+\frac{27}{5}Bb^6x^5+\frac{81}{5}Ca^2b^3cx^5+\frac{162}{5}Bab^4cx^5+\frac{81}{5}Ab^5cx^5+\frac{27}{5}Ba^2b^2c^2x^5+27Aab^3c^2x^5+\frac{81}{4}Ca^2b^4x^4+\frac{81}{4}Bab^5x^4+\frac{27}{4}Ab^6x^4+\frac{81}{4}Ba^2b^3cx^4+\frac{81}{2}Aab^4cx^4+\frac{27}{4}Aa^2b^2c^2x^4+9Ca^3b^3x^3+27Ba^2b^4x^3+27Aab^5x^3+27Aa^2b^3cx^3+\frac{27}{2}Ba^3b^3x^2+\frac{81}{2}Aa^2b^4x^2+27Aa^3b^3x$$

input
```
integrate((C*x^2+B*x+A)*(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^3,x, algorithm="
giac")
```

output
```
1/12*C*c^6*x^12 + 9/11*C*b*c^5*x^11 + 1/11*B*c^6*x^11 + 18/5*C*b^2*c^4*x^1
0 + 9/10*B*b*c^5*x^10 + 1/10*A*c^6*x^10 + 9*C*b^3*c^3*x^9 + C*a*b*c^4*x^9
+ 4*B*b^2*c^4*x^9 + A*b*c^5*x^9 + 27/2*C*b^4*c^2*x^8 + 27/4*C*a*b^2*c^3*x^
8 + 81/8*B*b^3*c^3*x^8 + 9/8*B*a*b*c^4*x^8 + 9/2*A*b^2*c^4*x^8 + 81/7*C*b^
5*c*x^7 + 135/7*C*a*b^3*c^2*x^7 + 108/7*B*b^4*c^2*x^7 + 54/7*B*a*b^2*c^3*x
^7 + 81/7*A*b^3*c^3*x^7 + 9/7*A*a*b*c^4*x^7 + 9/2*C*b^6*x^6 + 27*C*a*b^4*c
*x^6 + 27/2*B*b^5*c*x^6 + 9/2*C*a^2*b^2*c^2*x^6 + 45/2*B*a*b^3*c^2*x^6 + 1
8*A*b^4*c^2*x^6 + 9*A*a*b^2*c^3*x^6 + 81/5*C*a*b^5*x^5 + 27/5*B*b^6*x^5 +
81/5*C*a^2*b^3*c*x^5 + 162/5*B*a*b^4*c*x^5 + 81/5*A*b^5*c*x^5 + 27/5*B*a^2
*b^2*c^2*x^5 + 27*A*a*b^3*c^2*x^5 + 81/4*C*a^2*b^4*x^4 + 81/4*B*a*b^5*x^4
+ 27/4*A*b^6*x^4 + 81/4*B*a^2*b^3*c*x^4 + 81/2*A*a*b^4*c*x^4 + 27/4*A*a^2*
b^2*c^2*x^4 + 9*C*a^3*b^3*x^3 + 27*B*a^2*b^4*x^3 + 27*A*a*b^5*x^3 + 27*A*a
^2*b^3*c*x^3 + 27/2*B*a^3*b^3*x^2 + 81/2*A*a^2*b^4*x^2 + 27*A*a^3*b^3*x
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 433, normalized size of antiderivative = 1.40

$$\int (A+Bx+Cx^2)\left(3ab+3b^2x+3bcx^2+c^2x^3\right)^3\,dx$$
$$= x^4\left(\frac{81\,C\,a^2\,b^4}{4}+\frac{81\,B\,a^2\,b^3\,c}{4}+\frac{27\,A\,a^2\,b^2\,c^2}{4}+\frac{81\,B\,a\,b^5}{4}+\frac{81\,A\,a\,b^4\,c}{2}+\frac{27\,A\,b^6}{4}\right) + x^5\left(\frac{81\,C\,a^2\,b^3\,c}{5}+\frac{27\,B\,a^2\,b^2\,c^2}{5}+\frac{81\,C\,a\,b^5}{5}+\frac{162\,B\,a\,b^4\,c}{5}+27\,A\,a\,b^3\,c^2+\frac{27\,B\,b^6}{5}+\frac{81\,A\,b^5\,c}{5}\right)+x^{11}\left(\frac{B\,c^6}{11}+\frac{9\,C\,b\,c^5}{11}\right) + x^7\left(\frac{81\,C\,b^5\,c}{7}+\frac{108\,B\,b^4\,c^2}{7}+\frac{81\,A\,b^3\,c^3}{7}+\frac{135\,C\,a\,b^3\,c^2}{7}+\frac{54\,B\,a\,b^2\,c^3}{7}+\frac{9\,A\,a\,b\,c^4}{7}\right) + x^{10}\left(\frac{18\,C\,b^2\,c^4}{5}+\frac{9\,B\,b\,c^5}{10}+\frac{A\,c^6}{10}\right)+x^6\left(\frac{9\,C\,a^2\,b^2\,c^2}{2}+27\,C\,a\,b^4\,c+\frac{45\,B\,a\,b^3\,c^2}{2}+9\,A\,a\,b^2\,c^3+\frac{9\,C\,b^6}{2}+\frac{27\,B\,b^5\,c}{2}+18\,A\,b^4\,c^2\right)+\frac{C\,c^6\,x^{12}}{12}+\frac{27\,a^2\,b^3\,x^2\,(3\,A\,b+B\,a)}{2} + \frac{9\,b\,c^2\,x^8\,(12\,C\,b^3+9\,B\,b^2\,c+4\,A\,b\,c^2+6\,C\,a\,b\,c+B\,a\,c^2)}{8} + 9\,a\,b^3\,x^3\,(C\,a^2+3\,B\,a\,b+3\,A\,c\,a+3\,A\,b^2) + b\,c^3\,x^9\,(9\,C\,b^2+4\,B\,b\,c+A\,c^2+C\,a\,c)+27\,A\,a^3\,b^3\,x$$

input
```
int((A + B*x + C*x^2)*(3*a*b + 3*b^2*x + c^2*x^3 + 3*b*c*x^2)^3,x)
```

output
```
x^4*((27*A*b^6)/4 + (81*C*a^2*b^4)/4 + (81*B*a*b^5)/4 + (81*A*a*b^4*c)/2 +
 (81*B*a^2*b^3*c)/4 + (27*A*a^2*b^2*c^2)/4) + x^5*((27*B*b^6)/5 + (81*A*b^
5*c)/5 + (81*C*a*b^5)/5 + (162*B*a*b^4*c)/5 + 27*A*a*b^3*c^2 + (81*C*a^2*b
^3*c)/5 + (27*B*a^2*b^2*c^2)/5) + x^11*((B*c^6)/11 + (9*C*b*c^5)/11) + x^7
*((81*A*b^3*c^3)/7 + (108*B*b^4*c^2)/7 + (81*C*b^5*c)/7 + (9*A*a*b*c^4)/7
+ (54*B*a*b^2*c^3)/7 + (135*C*a*b^3*c^2)/7) + x^10*((A*c^6)/10 + (18*C*b^2
*c^4)/5 + (9*B*b*c^5)/10) + x^6*((9*C*b^6)/2 + 18*A*b^4*c^2 + (27*B*b^5*c)
/2 + 27*C*a*b^4*c + 9*A*a*b^2*c^3 + (45*B*a*b^3*c^2)/2 + (9*C*a^2*b^2*c^2)
/2) + (C*c^6*x^12)/12 + (27*a^2*b^3*x^2*(3*A*b + B*a))/2 + (9*b*c^2*x^8*(1
2*C*b^3 + 4*A*b*c^2 + B*a*c^2 + 9*B*b^2*c + 6*C*a*b*c))/8 + 9*a*b^3*x^3*(3
*A*b^2 + C*a^2 + 3*A*a*c + 3*B*a*b) + b*c^3*x^9*(A*c^2 + 9*C*b^2 + 4*B*b*c
 + C*a*c) + 27*A*a^3*b^3*x
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 262, normalized size of antiderivative = 0.85

$$\int \left(A+Bx+Cx^2\right)\left(3ab+3b^2x+3bcx^2+c^2x^3\right)^3\,dx$$
$$=\frac{x(770c^7x^{11}+8400b\,c^6x^{10}+924a\,c^6x^9+41580b^2c^5x^9+18480ab\,c^5x^8+120120b^3c^4x^8+114345a\,b^2c^4x^7}{}$$

input
```
int((C*x^2+B*x+A)*(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^3,x)
```

output
```
(x*(249480*a**4*b**3 + 498960*a**3*b**4*x + 332640*a**3*b**3*c*x**2 + 6237
0*a**3*b**2*c**2*x**3 + 498960*a**2*b**5*x**2 + 748440*a**2*b**4*c*x**3 +
449064*a**2*b**3*c**2*x**4 + 124740*a**2*b**2*c**3*x**5 + 11880*a**2*b*c**
4*x**6 + 249480*a*b**6*x**3 + 598752*a*b**5*c*x**4 + 623700*a*b**4*c**2*x*
*5 + 356400*a*b**3*c**3*x**6 + 114345*a*b**2*c**4*x**7 + 18480*a*b*c**5*x*
*8 + 924*a*c**6*x**9 + 49896*b**7*x**4 + 166320*b**6*c*x**5 + 249480*b**5*
c**2*x**6 + 218295*b**4*c**3*x**7 + 120120*b**3*c**4*x**8 + 41580*b**2*c**
5*x**9 + 8400*b*c**6*x**10 + 770*c**7*x**11))/9240
```

3.69 $\int (A+Bx+Cx^2) (3ab+3b^2x+3bcx^2+c^2x^3)^2 \, dx$

Optimal result

Integrand size = 38, antiderivative size = 216

$$\begin{aligned}
&\int (A+Bx+Cx^2) (3ab+3b^2x+3bcx^2+c^2x^3)^2 \, dx \\
&= \frac{b(b^2-3ac)(bBc-Ac^2-b^2C)(b+cx)^4}{2c^5} - \frac{2b(b^2-3ac)(Bc-2bC)(b+cx)^5}{5c^5} \\
&\quad - \frac{(bBc-Ac^2-b^2C)(b+cx)^7}{7c^5} + \frac{(Bc-2bC)(b+cx)^8}{8c^5} \\
&\quad + \frac{b^2(b^2-3ac)^2 (Ac^2-b^2C+c(Bc-2bC)x)^2}{2c^5(Bc-2bC)} + \frac{C\left(b\left(3a-\frac{b^2}{c}\right)+\frac{(b+cx)^3}{c}\right)^3}{9c^2}
\end{aligned}$$

output

```
1/2*b*(-3*a*c+b^2)*(-A*c^2+B*b*c-C*b^2)*(c*x+b)^4/c^5-2/5*b*(-3*a*c+b^2)*(
B*c-2*C*b)*(c*x+b)^5/c^5-1/7*(-A*c^2+B*b*c-C*b^2)*(c*x+b)^7/c^5+1/8*(B*c-2
*C*b)*(c*x+b)^8/c^5+1/2*b^2*(-3*a*c+b^2)^2*(A*c^2-C*b^2+c*(B*c-2*C*b)*x)^2
/c^5/(B*c-2*C*b)+1/9*C*(b*(3*a-b^2/c)+(c*x+b)^3/c)^3/c^2
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.05

$$\int (A+Bx+Cx^2)\left(3ab+3b^2x+3bcx^2+c^2x^3\right)^2\,dx$$
$$=9a^2Ab^2x+\frac{9}{2}ab^2(2Ab+aB)x^2+3b^2\left(A\left(b^2+2ac\right)+a(2bB+aC)\right)x^3$$
$$+\frac{3}{4}b\left(3b^3B+6abBc+2aAc^2+6b^2(Ac+aC)\right)x^4$$
$$+\frac{3}{5}b\left(6b^2Bc+2aBc^2+3b^3C+bc(5Ac+6aC)\right)x^5$$
$$+\frac{1}{2}bc\left(5bBc+6b^2C+2c(Ac+aC)\right)x^6$$
$$+\frac{1}{7}c^2\left(6bBc+Ac^2+15b^2C\right)x^7+\frac{1}{8}c^3(Bc+6bC)x^8+\frac{1}{9}c^4Cx^9$$

input
```
Integrate[(A + B*x + C*x^2)*(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3)^2,x]
```

output
```
9*a^2*A*b^2*x + (9*a*b^2*(2*A*b + a*B)*x^2)/2 + 3*b^2*(A*(b^2 + 2*a*c) + a
*(2*b*B + a*C))*x^3 + (3*b*(3*b^3*B + 6*a*b*B*c + 2*a*A*c^2 + 6*b^2*(A*c +
 a*C))*x^4)/4 + (3*b*(6*b^2*B*c + 2*a*B*c^2 + 3*b^3*C + b*c*(5*A*c + 6*a*C
))*x^5)/5 + (b*c*(5*b*B*c + 6*b^2*C + 2*c*(A*c + a*C))*x^6)/2 + (c^2*(6*b*
B*c + A*c^2 + 15*b^2*C)*x^7)/7 + (c^3*(B*c + 6*b*C)*x^8)/8 + (c^4*C*x^9)/9
```

Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.05, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(3ab+3b^2x+3bcx^2+c^2x^3\right)^2(A+Bx+Cx^2)\,dx$$

$$\Big\downarrow \text{ 2188}$$

$$\int \left(9a^2Ab^2 + 3bcx^5\left(2c(aC + Ac) + 6b^2C + 5bBc\right) + 9b^2x^2\left(A\left(2ac + b^2\right) + a(aC + 2bB)\right) + 9ab^2x(aB + 2Ab) + \right.$$

$$\Big\downarrow \text{2009}$$

$$9a^2Ab^2x + \frac{1}{2}bcx^6\left(2c(aC + Ac) + 6b^2C + 5bBc\right) + 3b^2x^3\left(A\left(2ac + b^2\right) + a(aC + 2bB)\right) + \frac{9}{2}ab^2x^2(aB + 2Ab) + \frac{3}{5}bx^5\left(bc(6aC + 5Ac) + 2aBc^2 + 3b^3C + 6b^2Bc\right) + \frac{3}{4}bx^4\left(6b^2(aC + Ac) + 2aAc^2 + 6abBc + 3b^3B\right) + \frac{1}{7}c^2x^7\left(Ac^2 + 15b^2C + 6bBc\right) + \frac{1}{8}c^3x^8(6bC + Bc) + \frac{1}{9}c^4Cx^9$$

input
```
Int[(A + B*x + C*x^2)*(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3)^2,x]
```

output
```
9*a^2*A*b^2*x + (9*a*b^2*(2*A*b + a*B)*x^2)/2 + 3*b^2*(A*(b^2 + 2*a*c) + a
*(2*b*B + a*C))*x^3 + (3*b*(3*b^3*B + 6*a*b*B*c + 2*a*A*c^2 + 6*b^2*(A*c +
 a*C))*x^4)/4 + (3*b*(6*b^2*B*c + 2*a*B*c^2 + 3*b^3*C + b*c*(5*A*c + 6*a*C
))*x^5)/5 + (b*c*(5*b*B*c + 6*b^2*C + 2*c*(A*c + a*C))*x^6)/2 + (c^2*(6*b*
B*c + A*c^2 + 15*b^2*C)*x^7)/7 + (c^3*(B*c + 6*b*C)*x^8)/8 + (c^4*C*x^9)/9
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.11

method	result
norman	$\frac{C\,c^4x^9}{9}+\left(\frac{1}{8}B\,c^4+\frac{3}{4}Cb\,c^3\right)x^8+\left(\frac{1}{7}c^4A+\frac{6}{7}Bb\,c^3+\frac{15}{7}C\,b^2c^2\right)x^7+\left(Ab\,c^3+\frac{5}{2}B\,b^2c^2+Cab\,c\right.$
default	$\frac{C\,c^4x^9}{9}+\frac{(B\,c^4+6Cb\,c^3)x^8}{8}+\frac{(c^4A+6Bb\,c^3+15C\,b^2c^2)x^7}{7}+\frac{(6Ab\,c^3+15B\,b^2c^2+C(6ab\,c^2+18b^3c))x^6}{6}+\frac{(15A\,b^2c^2+}{}$
gosper	$3C\,a^2b^2x^3+9x^2A\,b^3a+\frac{9}{2}x^2B\,a^2b^2+6Ba\,b^3x^3+3x^5A\,b^2c^2+\frac{18}{5}x^5B\,b^3c+\frac{9}{2}x^4A\,b^3c+\frac{9}{2}x^4$
risch	$3C\,a^2b^2x^3+9x^2A\,b^3a+\frac{9}{2}x^2B\,a^2b^2+6Ba\,b^3x^3+3x^5A\,b^2c^2+\frac{18}{5}x^5B\,b^3c+\frac{9}{2}x^4A\,b^3c+\frac{9}{2}x^4$
parallelrisch	$3C\,a^2b^2x^3+9x^2A\,b^3a+\frac{9}{2}x^2B\,a^2b^2+6Ba\,b^3x^3+3x^5A\,b^2c^2+\frac{18}{5}x^5B\,b^3c+\frac{9}{2}x^4A\,b^3c+\frac{9}{2}x^4$
orering	$\frac{x(280C\,c^4x^8+315B\,c^4x^7+1890Cb\,c^3x^7+360A\,c^4x^6+2160Bb\,c^3x^6+5400C\,b^2c^2x^6+2520Ab\,c^3x^5+6300B\,b^2c^2x^5+2520Cab\,c}{}$

input
```
int((C*x^2+B*x+A)*(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^2,x,method=_RETURNVERB
OSE)
```

output
```
1/9*C*c^4*x^9+(1/8*B*c^4+3/4*C*b*c^3)*x^8+(1/7*c^4*A+6/7*B*b*c^3+15/7*C*b^
2*c^2)*x^7+(A*b*c^3+5/2*B*b^2*c^2+C*a*b*c^2+3*C*b^3*c)*x^6+(3*A*b^2*c^2+6/
5*B*a*b*c^2+18/5*B*b^3*c+18/5*C*a*b^2*c+9/5*C*b^4)*x^5+(3/2*A*a*b*c^2+9/2*
A*b^3*c+9/2*B*a*b^2*c+9/4*B*b^4+9/2*C*a*b^3)*x^4+(6*A*a*b^2*c+3*A*b^4+6*B*
a*b^3+3*C*a^2*b^2)*x^3+(9*A*b^3*a+9/2*B*a^2*b^2)*x^2+9*A*a^2*b^2*x
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.12

$$\begin{aligned}
&\int \left(A+Bx+Cx^2\right)\left(3ab+3b^2x+3bcx^2+c^2x^3\right)^2\,dx \\
&= \frac{1}{9}\,Cc^4x^9+\frac{1}{8}\left(6\,Cbc^3+Bc^4\right)x^8+\frac{1}{7}\left(15\,Cb^2c^2+6\,Bbc^3+Ac^4\right)x^7 \\
&\quad+\frac{1}{2}\left(6\,Cb^3c+2\,Abc^3+\left(2\,Cab+5\,Bb^2\right)c^2\right)x^6+9\,Aa^2b^2x \\
&\quad+\frac{3}{5}\left(3\,Cb^4+\left(2\,Bab+5\,Ab^2\right)c^2+6\left(Cab^2+Bb^3\right)c\right)x^5 \\
&\quad+\frac{3}{4}\left(6\,Cab^3+3\,Bb^4+2\,Aabc^2+6\left(Bab^2+Ab^3\right)c\right)x^4 \\
&\quad+3\left(Ca^2b^2+2\,Bab^3+Ab^4+2\,Aab^2c\right)x^3+\frac{9}{2}\left(Ba^2b^2+2\,Aab^3\right)x^2
\end{aligned}$$

input
```
integrate((C*x^2+B*x+A)*(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^2,x, algorithm="
fricas")
```

output
```
1/9*C*c^4*x^9 + 1/8*(6*C*b*c^3 + B*c^4)*x^8 + 1/7*(15*C*b^2*c^2 + 6*B*b*c^
3 + A*c^4)*x^7 + 1/2*(6*C*b^3*c + 2*A*b*c^3 + (2*C*a*b + 5*B*b^2)*c^2)*x^6
 + 9*A*a^2*b^2*x + 3/5*(3*C*b^4 + (2*B*a*b + 5*A*b^2)*c^2 + 6*(C*a*b^2 + B
*b^3)*c)*x^5 + 3/4*(6*C*a*b^3 + 3*B*b^4 + 2*A*a*b*c^2 + 6*(B*a*b^2 + A*b^3
)*c)*x^4 + 3*(C*a^2*b^2 + 2*B*a*b^3 + A*b^4 + 2*A*a*b^2*c)*x^3 + 9/2*(B*a^
2*b^2 + 2*A*a*b^3)*x^2
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.33

$$\int (A+Bx+Cx^2)\left(3ab+3b^2x+3bcx^2+c^2x^3\right)^2\,dx = 9Aa^2b^2x+\frac{Cc^4x^9}{9}+x^8\left(\frac{Bc^4}{8}+\frac{3Cbc^3}{4}\right)+x^7\left(\frac{Ac^4}{7}+\frac{6Bbc^3}{7}+\frac{15Cb^2c^2}{7}\right)+x^6\left(Abc^3+\frac{5Bb^2c^2}{2}+Cabc^2+3Cb^3c\right)+x^5\cdot\left(3Ab^2c^2+\frac{6Babc^2}{5}+\frac{18Bb^3c}{5}+\frac{18Cab^2c}{5}+\frac{9Cb^4}{5}\right)+x^4\cdot\left(\frac{3Aabc^2}{2}+\frac{9Ab^3c}{2}+\frac{9Bab^2c}{2}+\frac{9Bb^4}{4}+\frac{9Cab^3}{2}\right)+x^3\cdot\left(6Aab^2c+3Ab^4+6Bab^3+3Ca^2b^2\right)+x^2\cdot\left(9Aab^3+\frac{9Ba^2b^2}{2}\right)$$

input
```
integrate((C*x**2+B*x+A)*(c**2*x**3+3*b*c*x**2+3*b**2*x+3*a*b)**2,x)
```

output
```
9*A*a**2*b**2*x + C*c**4*x**9/9 + x**8*(B*c**4/8 + 3*C*b*c**3/4) + x**7*(A
*c**4/7 + 6*B*b*c**3/7 + 15*C*b**2*c**2/7) + x**6*(A*b*c**3 + 5*B*b**2*c**
2/2 + C*a*b*c**2 + 3*C*b**3*c) + x**5*(3*A*b**2*c**2 + 6*B*a*b*c**2/5 + 18
*B*b**3*c/5 + 18*C*a*b**2*c/5 + 9*C*b**4/5) + x**4*(3*A*a*b*c**2/2 + 9*A*b
**3*c/2 + 9*B*a*b**2*c/2 + 9*B*b**4/4 + 9*C*a*b**3/2) + x**3*(6*A*a*b**2*c
 + 3*A*b**4 + 6*B*a*b**3 + 3*C*a**2*b**2) + x**2*(9*A*a*b**3 + 9*B*a**2*b*
*2/2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.12

$$\begin{aligned}
&\int \left(A+Bx+Cx^2\right)\left(3ab+3b^2x+3bcx^2+c^2x^3\right)^2\,dx\\
&=\frac{1}{9}\,Cc^4x^9+\frac{1}{8}\left(6\,Cbc^3+Bc^4\right)x^8+\frac{1}{7}\left(15\,Cb^2c^2+6\,Bbc^3+Ac^4\right)x^7\\
&\quad+\frac{1}{2}\left(6\,Cb^3c+2\,Abc^3+\left(2\,Cab+5\,Bb^2\right)c^2\right)x^6+9\,Aa^2b^2x\\
&\quad+\frac{3}{5}\left(3\,Cb^4+\left(2\,Bab+5\,Ab^2\right)c^2+6\left(Cab^2+Bb^3\right)c\right)x^5\\
&\quad+\frac{3}{4}\left(6\,Cab^3+3\,Bb^4+2\,Aabc^2+6\left(Bab^2+Ab^3\right)c\right)x^4\\
&\quad+3\left(Ca^2b^2+2\,Bab^3+Ab^4+2\,Aab^2c\right)x^3+\frac{9}{2}\left(Ba^2b^2+2\,Aab^3\right)x^2
\end{aligned}$$

input
```
integrate((C*x^2+B*x+A)*(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^2,x, algorithm="
maxima")
```

output
```
1/9*C*c^4*x^9 + 1/8*(6*C*b*c^3 + B*c^4)*x^8 + 1/7*(15*C*b^2*c^2 + 6*B*b*c^
3 + A*c^4)*x^7 + 1/2*(6*C*b^3*c + 2*A*b*c^3 + (2*C*a*b + 5*B*b^2)*c^2)*x^6
 + 9*A*a^2*b^2*x + 3/5*(3*C*b^4 + (2*B*a*b + 5*A*b^2)*c^2 + 6*(C*a*b^2 + B
*b^3)*c)*x^5 + 3/4*(6*C*a*b^3 + 3*B*b^4 + 2*A*a*b*c^2 + 6*(B*a*b^2 + A*b^3
)*c)*x^4 + 3*(C*a^2*b^2 + 2*B*a*b^3 + A*b^4 + 2*A*a*b^2*c)*x^3 + 9/2*(B*a^
2*b^2 + 2*A*a*b^3)*x^2
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.29

$$\begin{aligned}
&\int \left(A+Bx+Cx^2\right)\left(3ab+3b^2x+3bcx^2+c^2x^3\right)^2\,dx\\
&=\frac{1}{9}\,Cc^4x^9+\frac{3}{4}\,Cbc^3x^8+\frac{1}{8}\,Bc^4x^8+\frac{15}{7}\,Cb^2c^2x^7+\frac{6}{7}\,Bbc^3x^7+\frac{1}{7}\,Ac^4x^7+3\,Cb^3cx^6\\
&\quad+Cabc^2x^6+\frac{5}{2}\,Bb^2c^2x^6+Abc^3x^6+\frac{9}{5}\,Cb^4x^5+\frac{18}{5}\,Cab^2cx^5+\frac{18}{5}\,Bb^3cx^5+\frac{6}{5}\,Babc^2x^5\\
&\quad+3\,Ab^2c^2x^5+\frac{9}{2}\,Cab^3x^4+\frac{9}{4}\,Bb^4x^4+\frac{9}{2}\,Bab^2cx^4+\frac{9}{2}\,Ab^3cx^4+\frac{3}{2}\,Aabc^2x^4\\
&\quad+3\,Ca^2b^2x^3+6\,Bab^3x^3+3\,Ab^4x^3+6\,Aab^2cx^3+\frac{9}{2}\,Ba^2b^2x^2+9\,Aab^3x^2+9\,Aa^2b^2x
\end{aligned}$$

input
```
integrate((C*x^2+B*x+A)*(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^2,x, algorithm="
giac")
```

output
```
1/9*C*c^4*x^9 + 3/4*C*b*c^3*x^8 + 1/8*B*c^4*x^8 + 15/7*C*b^2*c^2*x^7 + 6/7
*B*b*c^3*x^7 + 1/7*A*c^4*x^7 + 3*C*b^3*c*x^6 + C*a*b*c^2*x^6 + 5/2*B*b^2*c
^2*x^6 + A*b*c^3*x^6 + 9/5*C*b^4*x^5 + 18/5*C*a*b^2*c*x^5 + 18/5*B*b^3*c*x
^5 + 6/5*B*a*b*c^2*x^5 + 3*A*b^2*c^2*x^5 + 9/2*C*a*b^3*x^4 + 9/4*B*b^4*x^4
 + 9/2*B*a*b^2*c*x^4 + 9/2*A*b^3*c*x^4 + 3/2*A*a*b*c^2*x^4 + 3*C*a^2*b^2*x
^3 + 6*B*a*b^3*x^3 + 3*A*b^4*x^3 + 6*A*a*b^2*c*x^3 + 9/2*B*a^2*b^2*x^2 + 9
*A*a*b^3*x^2 + 9*A*a^2*b^2*x
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.04

$$\int \left(A+Bx+Cx^2\right)\left(3ab+3b^2x+3bcx^2+c^2x^3\right)^2\,dx$$
$$= x^5\left(\frac{9\,C\,b^4}{5}+\frac{18\,B\,b^3\,c}{5}+3\,A\,b^2\,c^2+\frac{18\,C\,a\,b^2\,c}{5}+\frac{6\,B\,a\,b\,c^2}{5}\right) + x^8\left(\frac{B\,c^4}{8}+\frac{3\,C\,b\,c^3}{4}\right)+x^4\left(\frac{9\,B\,b^4}{4}+\frac{9\,A\,b^3\,c}{2}+\frac{9\,C\,a\,b^3}{2}+\frac{9\,B\,a\,b^2\,c}{2}+\frac{3\,A\,a\,b\,c^2}{2}\right) + x^7\left(\frac{15\,C\,b^2\,c^2}{7}+\frac{6\,B\,b\,c^3}{7}+\frac{A\,c^4}{7}\right)+\frac{C\,c^4\,x^9}{9}+3\,b^2\,x^3\left(C\,a^2+2\,B\,a\,b+2\,A\,c\,a+A\,b^2\right) + 9\,A\,a^2\,b^2\,x+\frac{9\,a\,b^2\,x^2\,(2\,A\,b+B\,a)}{2}+\frac{b\,c\,x^6\,(6\,C\,b^2+5\,B\,b\,c+2\,A\,c^2+2\,C\,a\,c)}{2}$$

input
```
int((A + B*x + C*x^2)*(3*a*b + 3*b^2*x + c^2*x^3 + 3*b*c*x^2)^2,x)
```

output
```
x^5*((9*C*b^4)/5 + 3*A*b^2*c^2 + (18*B*b^3*c)/5 + (6*B*a*b*c^2)/5 + (18*C*
a*b^2*c)/5) + x^8*((B*c^4)/8 + (3*C*b*c^3)/4) + x^4*((9*B*b^4)/4 + (9*A*b^
3*c)/2 + (9*C*a*b^3)/2 + (3*A*a*b*c^2)/2 + (9*B*a*b^2*c)/2) + x^7*((A*c^4)
/7 + (15*C*b^2*c^2)/7 + (6*B*b*c^3)/7) + (C*c^4*x^9)/9 + 3*b^2*x^3*(A*b^2
+ C*a^2 + 2*A*a*c + 2*B*a*b) + 9*A*a^2*b^2*x + (9*a*b^2*x^2*(2*A*b + B*a))
/2 + (b*c*x^6*(2*A*c^2 + 6*C*b^2 + 5*B*b*c + 2*C*a*c))/2
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.70

$$\int \left(A+Bx+Cx^2\right)\left(3ab+3b^2x+3bcx^2+c^2x^3\right)^2\,dx$$
$$=\frac{x(280c^5x^8+2205b\,c^4x^7+360a\,c^4x^6+7560b^2c^3x^6+5040ab\,c^3x^5+13860b^3c^2x^5+19656a\,b^2c^2x^4+1360}{}$$

input
```
int((C*x^2+B*x+A)*(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^2,x)
```

output
```
(x*(22680*a**3*b**2 + 34020*a**2*b**3*x + 22680*a**2*b**2*c*x**2 + 3780*a*
*2*b*c**2*x**3 + 22680*a*b**4*x**2 + 34020*a*b**3*c*x**3 + 19656*a*b**2*c*
*2*x**4 + 5040*a*b*c**3*x**5 + 360*a*c**4*x**6 + 5670*b**5*x**3 + 13608*b*
*4*c*x**4 + 13860*b**3*c**2*x**5 + 7560*b**2*c**3*x**6 + 2205*b*c**4*x**7
+ 280*c**5*x**8))/2520
```

3.70 $\int (A+Bx+Cx^2) (3ab+3b^2x+3bcx^2+c^2x^3) \, dx$

Optimal result

Integrand size = 36, antiderivative size = 88

$$\begin{aligned}\int (A+Bx+Cx^2) \left(3ab+3b^2x+3bcx^2+c^2x^3\right) \, dx &= 3aAbx+\frac{3}{2}b(Ab+aB)x^2+b(bB+Ac+aC)x^3 \\ &\quad +\frac{1}{4}\left(3bBc+Ac^2+3b^2C\right)x^4+\frac{1}{5}c(Bc+3bC)x^5+\frac{1}{6}c^2Cx^6\end{aligned}$$

output
```
3*a*A*b*x+3/2*b*(A*b+B*a)*x^2+b*(A*c+B*b+C*a)*x^3+1/4*(A*c^2+3*B*b*c+3*C*b
^2)*x^4+1/5*c*(B*c+3*C*b)*x^5+1/6*c^2*C*x^6
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00

$$\begin{aligned}\int (A+Bx+Cx^2) \left(3ab+3b^2x+3bcx^2+c^2x^3\right) \, dx &= 3aAbx+\frac{3}{2}b(Ab+aB)x^2+b(bB+Ac+aC)x^3 \\ &\quad +\frac{1}{4}\left(3bBc+Ac^2+3b^2C\right)x^4+\frac{1}{5}c(Bc+3bC)x^5+\frac{1}{6}c^2Cx^6\end{aligned}$$

input
```
Integrate[(A + B*x + C*x^2)*(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3),x]
```

output
```
3*a*A*b*x + (3*b*(A*b + a*B)*x^2)/2 + b*(b*B + A*c + a*C)*x^3 + ((3*b*B*c
+ A*c^2 + 3*b^2*C)*x^4)/4 + (c*(B*c + 3*b*C)*x^5)/5 + (c^2*C*x^6)/6
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(3ab+3b^2x+3bcx^2+c^2x^3\right)\left(A+Bx+Cx^2\right)dx$$

$$\Big\downarrow \ 2188$$

$$\int \left(3bx^2(aC+Ac+bB)+3bx(aB+Ab)+3aAb+x^3\left(Ac^2+3b^2C+3bBc\right)+cx^4(3bC+Bc)+c^2Cx^5\right)dx$$

$$\Big\downarrow \ 2009$$

$$bx^3(aC+Ac+bB)+\frac{3}{2}bx^2(aB+Ab)+3aAbx+\frac{1}{4}x^4\left(Ac^2+3b^2C+3bBc\right)+\frac{1}{5}cx^5(3bC+Bc)+\frac{1}{6}c^2Cx^6$$

input
```
Int[(A + B*x + C*x^2)*(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3),x]
```

output
```
3*a*A*b*x + (3*b*(A*b + a*B)*x^2)/2 + b*(b*B + A*c + a*C)*x^3 + ((3*b*B*c
+ A*c^2 + 3*b^2*C)*x^4)/4 + (c*(B*c + 3*b*C)*x^5)/5 + (c^2*C*x^6)/6
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.01

method	result
norman	$\frac{c^2 C\,x^6}{6}+\left(\frac{1}{5}B\,c^2+\frac{3}{5}Cbc\right)x^5+\left(\frac{1}{4}A\,c^2+\frac{3}{4}bBc+\frac{3}{4}b^2C\right)x^4+\left(Abc+B\,b^2+Cab\right)x^3+\left(\frac{3}{2}A\,b\right.$
default	$\frac{c^2 C\,x^6}{6}+\frac{\left(B\,c^2+3Cbc\right)x^5}{5}+\frac{\left(A\,c^2+3bBc+3b^2C\right)x^4}{4}+\frac{\left(3Abc+3B\,b^2+3Cab\right)x^3}{3}+\frac{\left(3A\,b^2+3abB\right)x^2}{2}+3aAbx$
gosper	$\frac{1}{6}c^2C\,x^6+\frac{1}{5}B\,c^2x^5+\frac{3}{5}x^5Cbc+\frac{1}{4}A\,c^2x^4+\frac{3}{4}x^4bBc+\frac{3}{4}C\,b^2x^4+Abc\,x^3+B\,b^2x^3+Cab\,x^3$
risch	$\frac{1}{6}c^2C\,x^6+\frac{1}{5}B\,c^2x^5+\frac{3}{5}x^5Cbc+\frac{1}{4}A\,c^2x^4+\frac{3}{4}x^4bBc+\frac{3}{4}C\,b^2x^4+Abc\,x^3+B\,b^2x^3+Cab\,x^3$
parallelrisch	$\frac{1}{6}c^2C\,x^6+\frac{1}{5}B\,c^2x^5+\frac{3}{5}x^5Cbc+\frac{1}{4}A\,c^2x^4+\frac{3}{4}x^4bBc+\frac{3}{4}C\,b^2x^4+Abc\,x^3+B\,b^2x^3+Cab\,x^3$
orering	$\frac{x\left(10C\,c^2x^5+12B\,c^2x^4+36Cbc\,x^4+15A\,c^2x^3+45Bbc\,x^3+45C\,b^2x^3+60Abc\,x^2+60B\,b^2x^2+60Cab\,x^2+90A\,b^2x+90Babx+18\right.}{60}$

input
```
int((C*x^2+B*x+A)*(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b),x,method=_RETURNVERBOS
E)
```

output
```
1/6*c^2*C*x^6+(1/5*B*c^2+3/5*C*b*c)*x^5+(1/4*A*c^2+3/4*b*B*c+3/4*b^2*C)*x^
4+(A*b*c+B*b^2+C*a*b)*x^3+(3/2*A*b^2+3/2*a*b*B)*x^2+3*a*A*b*x
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.99

$$\int \left(A+Bx+Cx^2\right)\left(3ab+3b^2x+3bcx^2+c^2x^3\right)\,dx$$
$$=\frac{1}{6}\,Cc^2x^6+\frac{1}{5}\left(3\,Cbc+Bc^2\right)x^5+\frac{1}{4}\left(3\,Cb^2+3\,Bbc+Ac^2\right)x^4+3\,Aabx+\left(Cab+Bb^2+Abc\right)x^3+\frac{3}{2}\left(Bab+Ab^2\right)x^2$$

input
```
integrate((C*x^2+B*x+A)*(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b),x, algorithm="fr
icas")
```

output
```
1/6*C*c^2*x^6 + 1/5*(3*C*b*c + B*c^2)*x^5 + 1/4*(3*C*b^2 + 3*B*b*c + A*c^2
)*x^4 + 3*A*a*b*x + (C*a*b + B*b^2 + A*b*c)*x^3 + 3/2*(B*a*b + A*b^2)*x^2
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.16

$$\int \left(A+Bx+Cx^2\right)\left(3ab+3b^2x+3bcx^2+c^2x^3\right)\,dx$$
$$=3Aabx+\frac{Cc^2x^6}{6}+x^5\left(\frac{Bc^2}{5}+\frac{3Cbc}{5}\right)+x^4\left(\frac{Ac^2}{4}+\frac{3Bbc}{4}+\frac{3Cb^2}{4}\right)+x^3\left(Abc+Bb^2+Cab\right)+x^2\cdot\left(\frac{3Ab^2}{2}+\frac{3Bab}{2}\right)$$

input
```
integrate((C*x**2+B*x+A)*(c**2*x**3+3*b*c*x**2+3*b**2*x+3*a*b),x)
```

output
```
3*A*a*b*x + C*c**2*x**6/6 + x**5*(B*c**2/5 + 3*C*b*c/5) + x**4*(A*c**2/4 +
 3*B*b*c/4 + 3*C*b**2/4) + x**3*(A*b*c + B*b**2 + C*a*b) + x**2*(3*A*b**2/
2 + 3*B*a*b/2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.99

$$\int (A+Bx+Cx^2)\left(3ab+3b^2x+3bcx^2+c^2x^3\right)\,dx$$
$$=\frac{1}{6}\,Cc^2x^6+\frac{1}{5}\,(3\,Cbc+Bc^2)x^5+\frac{1}{4}\,(3\,Cb^2+3\,Bbc+Ac^2)x^4$$
$$+3\,Aabx+(Cab+Bb^2+Abc)x^3+\frac{3}{2}\,(Bab+Ab^2)x^2$$

input
```
integrate((C*x^2+B*x+A)*(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b),x, algorithm="ma
xima")
```

output
```
1/6*C*c^2*x^6 + 1/5*(3*C*b*c + B*c^2)*x^5 + 1/4*(3*C*b^2 + 3*B*b*c + A*c^2
)*x^4 + 3*A*a*b*x + (C*a*b + B*b^2 + A*b*c)*x^3 + 3/2*(B*a*b + A*b^2)*x^2
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.11

$$\int (A+Bx+Cx^2)\left(3ab+3b^2x+3bcx^2+c^2x^3\right)\,dx$$
$$=\frac{1}{6}\,Cc^2x^6+\frac{3}{5}\,Cbcx^5+\frac{1}{5}\,Bc^2x^5+\frac{3}{4}\,Cb^2x^4+\frac{3}{4}\,Bbcx^4+\frac{1}{4}\,Ac^2x^4$$
$$+Cabx^3+Bb^2x^3+Abcx^3+\frac{3}{2}\,Babx^2+\frac{3}{2}\,Ab^2x^2+3\,Aabx$$

input
```
integrate((C*x^2+B*x+A)*(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b),x, algorithm="gi
ac")
```

output
```
1/6*C*c^2*x^6 + 3/5*C*b*c*x^5 + 1/5*B*c^2*x^5 + 3/4*C*b^2*x^4 + 3/4*B*b*c*
x^4 + 1/4*A*c^2*x^4 + C*a*b*x^3 + B*b^2*x^3 + A*b*c*x^3 + 3/2*B*a*b*x^2 +
3/2*A*b^2*x^2 + 3*A*a*b*x
```

Mupad [B] (verification not implemented)

Time = 12.59 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.97

$$\int (A+Bx+Cx^2)\left(3ab+3b^2x+3bcx^2+c^2x^3\right)\,dx$$
$$= x^2\left(\frac{3\,A\,b^2}{2}+\frac{3\,B\,a\,b}{2}\right)+x^5\left(\frac{B\,c^2}{5}+\frac{3\,C\,b\,c}{5}\right)$$
$$+x^4\left(\frac{3\,C\,b^2}{4}+\frac{3\,B\,b\,c}{4}+\frac{A\,c^2}{4}\right)+b\,x^3\,(A\,c+B\,b+C\,a)+\frac{C\,c^2\,x^6}{6}+3\,A\,a\,b\,x$$

input
```
int((A + B*x + C*x^2)*(3*a*b + 3*b^2*x + c^2*x^3 + 3*b*c*x^2),x)
```

output
```
x^2*((3*A*b^2)/2 + (3*B*a*b)/2) + x^5*((B*c^2)/5 + (3*C*b*c)/5) + x^4*((A*
c^2)/4 + (3*C*b^2)/4 + (3*B*b*c)/4) + b*x^3*(A*c + B*b + C*a) + (C*c^2*x^6
)/6 + 3*A*a*b*x
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.77

$$\int (A+Bx+Cx^2)\left(3ab+3b^2x+3bcx^2+c^2x^3\right)\,dx$$
$$=\frac{x(10c^3x^5+48b\,c^2x^4+15a\,c^2x^3+90b^2c\,x^3+120abc\,x^2+60b^3x^2+180a\,b^2x+180a^2b)}{60}$$

input
```
int((C*x^2+B*x+A)*(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b),x)
```

output
```
(x*(180*a**2*b + 180*a*b**2*x + 120*a*b*c*x**2 + 15*a*c**2*x**3 + 60*b**3*
x**2 + 90*b**2*c*x**3 + 48*b*c**2*x**4 + 10*c**3*x**5))/60
```

3.71 $\int \frac{A+Bx+Cx^2}{3ab+3b^2x+3bcx^2+c^2x^3}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 395

$$\int \frac{A+Bx+Cx^2}{3ab+3b^2x+3bcx^2+c^2x^3}\,dx$$
$$= \frac{\left(bBc-Ac^2+\sqrt[3]{b}Bc\sqrt[3]{b^2-3ac}-b^2C-2b^{4/3}\sqrt[3]{b^2-3ac}C\right)\arctan\left(\frac{\sqrt[3]{b}+\frac{2(b+cx)}{\sqrt[3]{b^2-3ac}}}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt{3}b^{2/3}c^2\left(b^2-3ac\right)^{2/3}}$$
$$-\frac{\left(bBc-Ac^2-\sqrt[3]{b}Bc\sqrt[3]{b^2-3ac}-b^2C+2b^{4/3}\sqrt[3]{b^2-3ac}C\right)\log\left(\sqrt[3]{b}\left(b^{2/3}-\sqrt[3]{b^2-3ac}\right)+cx\right)}{3b^{2/3}c^2\left(b^2-3ac\right)^{2/3}}$$
$$+\frac{\left(bBc-Ac^2-\sqrt[3]{b}Bc\sqrt[3]{b^2-3ac}-b^2C+2b^{4/3}\sqrt[3]{b^2-3ac}C\right)\log\left(b^{2/3}(b^2-3ac)^{2/3}+\sqrt[3]{b}\sqrt[3]{b^2-3ac}(b+\right.}{6b^{2/3}c^2\left(b^2-3ac\right)^{2/3}}$$
$$+\frac{C\log\left(b(b^2-3ac)-(b+cx)^3\right)}{3c^2}$$

output
```
1/3*(B*b*c-A*c^2+b^(1/3)*B*c*(-3*a*c+b^2)^(1/3)-C*b^2-2*b^(4/3)*(-3*a*c+b^
2)^(1/3)*C)*arctan(1/3*(b^(1/3)+2*(c*x+b)/(-3*a*c+b^2)^(1/3))*3^(1/2)/b^(1
/3))*3^(1/2)/b^(2/3)/c^2/(-3*a*c+b^2)^(2/3)-1/3*(B*b*c-A*c^2-b^(1/3)*B*c*(
-3*a*c+b^2)^(1/3)-C*b^2+2*b^(4/3)*(-3*a*c+b^2)^(1/3)*C)*ln(b^(1/3)*(b^(2/3
)-(-3*a*c+b^2)^(1/3))+c*x)/b^(2/3)/c^2/(-3*a*c+b^2)^(2/3)+1/6*(B*b*c-A*c^2
-b^(1/3)*B*c*(-3*a*c+b^2)^(1/3)-C*b^2+2*b^(4/3)*(-3*a*c+b^2)^(1/3)*C)*ln(b
^(2/3)*(-3*a*c+b^2)^(2/3)+b^(1/3)*(-3*a*c+b^2)^(1/3)*(c*x+b)+(c*x+b)^2)/b^
(2/3)/c^2/(-3*a*c+b^2)^(2/3)+1/3*C*ln(b*(-3*a*c+b^2)-(c*x+b)^3)/c^2
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.23

$$\int \frac{A+Bx+Cx^2}{3ab+3b^2x+3bcx^2+c^2x^3}\,dx = \frac{1}{3}\text{RootSum}\left[3ab+3b^2\#1+3bc\#1^2 + c^2\#1^3\&,\frac{A\log(x-\#1)+B\log(x-\#1)\#1+C\log(x-\#1)\#1^2}{b^2+2bc\#1+c^2\#1^2}\&\right]$$

input
```
Integrate[(A + B*x + C*x^2)/(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3),x]
```

output
```
RootSum[3*a*b + 3*b^2*#1 + 3*b*c*#1^2 + c^2*#1^3 & , (A*Log[x - #1] + B*Lo
g[x - #1]*#1 + C*Log[x - #1]*#1^2)/(b^2 + 2*b*c*#1 + c^2*#1^2) & ]/3
```

Rubi [A] (verified)

Time = 2.53 (sec) , antiderivative size = 382, normalized size of antiderivative = 0.97, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {2459, 2410, 792, 2400, 16, 1142, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{3ab+3b^2x+3bcx^2+c^2x^3}\,dx$$

$\Big\downarrow$ 2459

$$\int \frac{A-\frac{b(Bc-bC)}{c^2}+\left(\frac{b}{c}+x\right)\left(B-\frac{2bC}{c}\right)+C\left(\frac{b}{c}+x\right)^2}{b\left(3a-\frac{b^2}{c}\right)+c^2\left(\frac{b}{c}+x\right)^3}d\left(\frac{b}{c}+x\right)$$

$\Big\downarrow$ 2410

$$\int \frac{A-\frac{b(Bc-bC)}{c^2}+\left(B-\frac{2bC}{c}\right)\left(\frac{b}{c}+x\right)}{c^2\left(\frac{b}{c}+x\right)^3+b\left(3a-\frac{b^2}{c}\right)}d\left(\frac{b}{c}+x\right)+C\int \frac{\left(\frac{b}{c}+x\right)^2}{c^2\left(\frac{b}{c}+x\right)^3+b\left(3a-\frac{b^2}{c}\right)}d\left(\frac{b}{c}+x\right)$$

$\Big\downarrow$ 792

$$\int \frac{A-\frac{b(Bc-bC)}{c^2}+\left(B-\frac{2bC}{c}\right)\left(\frac{b}{c}+x\right)}{c^2\left(\frac{b}{c}+x\right)^3+b\left(3a-\frac{b^2}{c}\right)}d\left(\frac{b}{c}+x\right)+\frac{C\log\left(b(b^2-3ac)-c^3\left(\frac{b}{c}+x\right)^3\right)}{3c^2}$$

$\Big\downarrow$ 2400

$$\frac{\left(-\sqrt[3]{b}Bc\sqrt[3]{b^2-3ac}+2b^{4/3}C\sqrt[3]{b^2-3ac}-Ac^2+b^2(-C)+bBc\right)\int \frac{1}{\sqrt[3]{b}\sqrt[3]{b^2-3ac}-c\left(\frac{b}{c}+x\right)}d\left(\frac{b}{c}+x\right)}{3b^{2/3}c\left(b^2-3ac\right)^{2/3}}+$$
$$\frac{\int \frac{\sqrt[3]{b}\sqrt[3]{b^2-3ac}\left(\sqrt[3]{b}\sqrt[3]{b^2-3ac}\left(B-\frac{2bC}{c}\right)-2c\left(A-\frac{b(Bc-bC)}{c^2}\right)\right)-c\left(Ac+\sqrt[3]{b}\sqrt[3]{b^2-3ac}\left(B-\frac{2bC}{c}\right)-\frac{b(Bc-bC)}{c}\right)\left(\frac{b}{c}+x\right)}{c^2\left(\frac{b}{c}+x\right)^2+\sqrt[3]{b}c\sqrt[3]{b^2-3ac}\left(\frac{b}{c}+x\right)+b^{2/3}(b^2-3ac)^{2/3}}d\left(\frac{b}{c}+x\right)}{3b^{2/3}\left(b^2-3ac\right)^{2/3}}$$
$$\frac{C\log\left(b(b^2-3ac)-c^3\left(\frac{b}{c}+x\right)^3\right)}{3c^2}$$

$\Big\downarrow$ 16

$$\frac{\int \frac{\sqrt[3]{b}\sqrt[3]{b^2-3ac}\left(\sqrt[3]{b}\sqrt[3]{b^2-3ac}\left(B-\frac{2bC}{c}\right)-2c\left(A-\frac{b(Bc-bC)}{c^2}\right)\right)-c\left(Ac+\sqrt[3]{b}\sqrt[3]{b^2-3ac}\left(B-\frac{2bC}{c}\right)-\frac{b(Bc-bC)}{c}\right)\left(\frac{b}{c}+x\right)}{c^2\left(\frac{b}{c}+x\right)^2+\sqrt[3]{b}c\sqrt[3]{b^2-3ac}\left(\frac{b}{c}+x\right)+b^{2/3}(b^2-3ac)^{2/3}}d\left(\frac{b}{c}+x\right)}{3b^{2/3}\left(b^2-3ac\right)^{2/3}}$$
$$\frac{\log\left(\sqrt[3]{b}\sqrt[3]{b^2-3ac}-c\left(\frac{b}{c}+x\right)\right)\left(-\sqrt[3]{b}Bc\sqrt[3]{b^2-3ac}+2b^{4/3}C\sqrt[3]{b^2-3ac}-Ac^2+b^2(-C)+bBc\right)}{3b^{2/3}c^2\left(b^2-3ac\right)^{2/3}}+$$
$$\frac{C\log\left(b(b^2-3ac)-c^3\left(\frac{b}{c}+x\right)^3\right)}{3c^2}$$

$\Big\downarrow$ 1142

$$\frac{\frac{3\sqrt[3]{b}\sqrt[3]{b^2-3ac}\left(\sqrt[3]{b}Bc\sqrt[3]{b^2-3ac}-2b^{4/3}C\sqrt[3]{b^2-3ac}-Ac^2+b^2(-C)+bBc\right)\int\frac{1}{c^2\left(\frac{b}{c}+x\right)^2+\sqrt[3]{b}c\sqrt[3]{b^2-3ac}\left(\frac{b}{c}+x\right)+b^{2/3}\left(b^2-3ac\right)^{2/3}}}{2c}}{3b^{2/3}\left(b^2-3a\right.}$$

$$\frac{\log\left(\sqrt[3]{b}\sqrt[3]{b^2-3ac}-c\left(\frac{b}{c}+x\right)\right)\left(-\sqrt[3]{b}Bc\sqrt[3]{b^2-3ac}+2b^{4/3}C\sqrt[3]{b^2-3ac}-Ac^2+b^2(-C)+bBc\right)}{3b^{2/3}c^2\left(b^2-3ac\right)^{2/3}}+\frac{C\log\left(b\left(b^2-3ac\right)-c^3\left(\frac{b}{c}+x\right)^3\right)}{3c^2}$$

$\big\downarrow$ 27

$$\frac{\frac{3\sqrt[3]{b}\sqrt[3]{b^2-3ac}\left(\sqrt[3]{b}Bc\sqrt[3]{b^2-3ac}-2b^{4/3}C\sqrt[3]{b^2-3ac}-Ac^2+b^2(-C)+bBc\right)\int\frac{1}{c^2\left(\frac{b}{c}+x\right)^2+\sqrt[3]{b}c\sqrt[3]{b^2-3ac}\left(\frac{b}{c}+x\right)+b^{2/3}\left(b^2-3ac\right)^{2}}}{2c}}{3b^{2/3}\left(b\right.}$$

$$\frac{\log\left(\sqrt[3]{b}\sqrt[3]{b^2-3ac}-c\left(\frac{b}{c}+x\right)\right)\left(-\sqrt[3]{b}Bc\sqrt[3]{b^2-3ac}+2b^{4/3}C\sqrt[3]{b^2-3ac}-Ac^2+b^2(-C)+bBc\right)}{3b^{2/3}c^2\left(b^2-3ac\right)^{2/3}}+\frac{C\log\left(b\left(b^2-3ac\right)-c^3\left(\frac{b}{c}+x\right)^3\right)}{3c^2}$$

$\big\downarrow$ 1082

$$\frac{-\frac{1}{2}\left(\sqrt[3]{b}\sqrt[3]{b^2-3ac}\left(B-\frac{2bC}{c}\right)+Ac-\frac{b(Bc-bC)}{c}\right)\int\frac{2c\left(\frac{b}{c}+x\right)+\sqrt[3]{b}\sqrt[3]{b^2-3ac}}{c^2\left(\frac{b}{c}+x\right)^2+\sqrt[3]{b}c\sqrt[3]{b^2-3ac}\left(\frac{b}{c}+x\right)+b^{2/3}\left(b^2-3ac\right)^{2/3}}d\left(\frac{b}{c}+x\right)-\frac{3\left(\sqrt[3]{\right.}}{}}{3b^{2/3}\left(b^2-3ac\right)^{2}}$$

$$\frac{\log\left(\sqrt[3]{b}\sqrt[3]{b^2-3ac}-c\left(\frac{b}{c}+x\right)\right)\left(-\sqrt[3]{b}Bc\sqrt[3]{b^2-3ac}+2b^{4/3}C\sqrt[3]{b^2-3ac}-Ac^2+b^2(-C)+bBc\right)}{3b^{2/3}c^2\left(b^2-3ac\right)^{2/3}}+\frac{C\log\left(b\left(b^2-3ac\right)-c^3\left(\frac{b}{c}+x\right)^3\right)}{3c^2}$$

$\big\downarrow$ 217

$$\frac{\frac{\sqrt{3}\arctan\left(\frac{\frac{2c\left(\frac{b}{c}+x\right)}{\sqrt[3]{b}\sqrt[3]{b^2-3ac}}+1}{\sqrt{3}}\right)\left(\sqrt[3]{b}Bc\sqrt[3]{b^2-3ac}-2b^{4/3}C\sqrt[3]{b^2-3ac}-Ac^2+b^2(-C)+bBc\right)}{c^2}-\frac{1}{2}\left(\sqrt[3]{b}\sqrt[3]{b^2-3ac}\left(B-\frac{2bC}{c}\right.\right.}{3b^{2/3}\left(b^2-3ac\right)^{2/3}}$$

$$\frac{\log\left(\sqrt[3]{b}\sqrt[3]{b^2-3ac}-c\left(\frac{b}{c}+x\right)\right)\left(-\sqrt[3]{b}Bc\sqrt[3]{b^2-3ac}+2b^{4/3}C\sqrt[3]{b^2-3ac}-Ac^2+b^2(-C)+bBc\right)}{3b^{2/3}c^2\left(b^2-3ac\right)^{2/3}}+$$

$$\frac{C\log\left(b\left(b^2-3ac\right)-c^3\left(\frac{b}{c}+x\right)^3\right)}{3c^2}$$

$$\Big\downarrow \text{1103}$$

$$\frac{\frac{\sqrt{3}\arctan\left(\frac{\frac{2c\left(\frac{b}{c}+x\right)}{\sqrt[3]{b}\sqrt[3]{b^2-3ac}}+1}{\sqrt{3}}\right)\left(\sqrt[3]{b}Bc\sqrt[3]{b^2-3ac}-2b^{4/3}C\sqrt[3]{b^2-3ac}-Ac^2+b^2(-C)+bBc\right)}{c^2}-\frac{\log\left(\sqrt[3]{b}c\sqrt[3]{b^2-3ac}\left(\frac{b}{c}+x\right)+\right.}{}}{3b^{2/3}\left(b^2-3ac\right)^{2/3}}$$

$$\frac{\log\left(\sqrt[3]{b}\sqrt[3]{b^2-3ac}-c\left(\frac{b}{c}+x\right)\right)\left(-\sqrt[3]{b}Bc\sqrt[3]{b^2-3ac}+2b^{4/3}C\sqrt[3]{b^2-3ac}-Ac^2+b^2(-C)+bBc\right)}{3b^{2/3}c^2\left(b^2-3ac\right)^{2/3}}+$$

$$\frac{C\log\left(b\left(b^2-3ac\right)-c^3\left(\frac{b}{c}+x\right)^3\right)}{3c^2}$$

input
```
Int[(A + B*x + C*x^2)/(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3),x]
```

output
```
-1/3*((b*B*c - A*c^2 - b^(1/3)*B*c*(b^2 - 3*a*c)^(1/3) - b^2*C + 2*b^(4/3)
*(b^2 - 3*a*c)^(1/3)*C)*Log[b^(1/3)*(b^2 - 3*a*c)^(1/3) - c*(b/c + x)])/(b
^(2/3)*c^2*(b^2 - 3*a*c)^(2/3)) + ((Sqrt[3]*(b*B*c - A*c^2 + b^(1/3)*B*c*(
b^2 - 3*a*c)^(1/3) - b^2*C - 2*b^(4/3)*(b^2 - 3*a*c)^(1/3)*C)*ArcTan[(1 +
(2*c*(b/c + x))/(b^(1/3)*(b^2 - 3*a*c)^(1/3)))/Sqrt[3]])/c^2 - ((A*c - (b*
(B*c - b*C))/c + b^(1/3)*(b^2 - 3*a*c)^(1/3)*(B - (2*b*C)/c))*Log[b^(2/3)*
(b^2 - 3*a*c)^(2/3) + b^(1/3)*c*(b^2 - 3*a*c)^(1/3)*(b/c + x) + c^2*(b/c +
 x)^2])/(2*c))/(3*b^(2/3)*(b^2 - 3*a*c)^(2/3)) + (C*Log[b*(b^2 - 3*a*c) -
c^3*(b/c + x)^3])/(3*c^2)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 792
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2400
```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[-a/b, 3]], s = Denominator[Rt[-a/b, 3]]}, Simp[r*((B*r + A*s)/(3*a*
s))   Int[1/(r - s*x), x], x] - Simp[r/(3*a*s)   Int[(r*(B*r - 2*A*s) - s*(
B*r + A*s)*x)/(r^2 + r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
 NeQ[a*B^3 - b*A^3, 0] && NegQ[a/b]
```

rule 2410
```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Si
mp[C   Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[
a/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

rule 2459
```
Int[(Pn_)^(p_.)*(Qx_), x_Symbol] :> With[{S = Coeff[Pn, x, Expon[Pn, x] - 1
]/(Expon[Pn, x]*Coeff[Pn, x, Expon[Pn, x]])}, Subst[Int[ExpandToSum[Pn /. x
 -> x - S, x]^p*ExpandToSum[Qx /. x -> x - S, x], x], x, x + S] /; Binomial
Q[Pn /. x -> x - S, x] || (IntegerQ[Expon[Pn, x]/2] && TrinomialQ[Pn /. x -
> x - S, x])] /; FreeQ[p, x] && PolyQ[Pn, x] && GtQ[Expon[Pn, x], 2] && NeQ
[Coeff[Pn, x, Expon[Pn, x] - 1], 0] && PolyQ[Qx, x] &&  !(MonomialQ[Qx, x]
&& IGtQ[p, 0])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.10 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.17

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{3}c^{2}+3\_Z^{2}bc+3b^{2}\_Z+3ab\right)}\frac{\left(C\_R^{2}+B\_R+A\right)\ln\left(x-\_R\right)}{\_R^{2}c^{2}+2\_Rbc+b^{2}}\right)}{3}$	67
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{3}c^{2}+3\_Z^{2}bc+3b^{2}\_Z+3ab\right)}\frac{\left(C\_R^{2}+B\_R+A\right)\ln\left(x-\_R\right)}{\_R^{2}c^{2}+2\_Rbc+b^{2}}\right)}{3}$	67

input
```
int((C*x^2+B*x+A)/(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b),x,method=_RETURNVERBOS
E)
```

output
```
1/3*sum((C*_R^2+B*_R+A)/(_R^2*c^2+2*_R*b*c+b^2)*ln(x-_R),_R=RootOf(_Z^3*c^
2+3*_Z^2*b*c+3*_Z*b^2+3*a*b))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.87 (sec) , antiderivative size = 19077, normalized size of antiderivative = 48.30

$$\int \frac{A + Bx + Cx^2}{3ab + 3b^2x + 3bcx^2 + c^2x^3} \, dx = \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)/(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b),x, algorithm="fr
icas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + Bx + Cx^2}{3ab + 3b^2x + 3bcx^2 + c^2x^3} \, dx = \text{Timed out}$$

input
```
integrate((C*x**2+B*x+A)/(c**2*x**3+3*b*c*x**2+3*b**2*x+3*a*b),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A + Bx + Cx^2}{3ab + 3b^2x + 3bcx^2 + c^2x^3} \, dx = \int \frac{Cx^2 + Bx + A}{c^2x^3 + 3\,bcx^2 + 3\,b^2x + 3\,ab} \, dx$$

input
```
integrate((C*x^2+B*x+A)/(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b),x, algorithm="ma
xima")
```

output
```
integrate((C*x^2 + B*x + A)/(c^2*x^3 + 3*b*c*x^2 + 3*b^2*x + 3*a*b), x)
```

Giac [F]

$$\int \frac{A+Bx+Cx^2}{3ab+3b^2x+3bcx^2+c^2x^3}\,dx = \int \frac{Cx^2+Bx+A}{c^2x^3+3\,bcx^2+3\,b^2x+3\,ab}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b),x, algorithm="gi
ac")
```

output
```
integrate((C*x^2 + B*x + A)/(c^2*x^3 + 3*b*c*x^2 + 3*b^2*x + 3*a*b), x)
```

Mupad [B] (verification not implemented)

Time = 12.89 (sec) , antiderivative size = 3423, normalized size of antiderivative = 8.67

$$\int \frac{A+Bx+Cx^2}{3ab+3b^2x+3bcx^2+c^2x^3}\,dx = \text{Too large to display}$$

input
```
int((A + B*x + C*x^2)/(3*a*b + 3*b^2*x + c^2*x^3 + 3*b*c*x^2),x)
```

output
```
symsum(log(B^2*c^2*x + 3*C^2*b^2*x - 9*root(243*a^2*b^2*c^6*z^3 - 162*a*b^
4*c^5*z^3 + 27*b^6*c^4*z^3 - 243*C*a^2*b^2*c^4*z^2 + 162*C*a*b^4*c^3*z^2 -
 27*C*b^6*c^2*z^2 + 27*A*B*a*b*c^4*z + 81*B*C*a*b^3*c^2*z - 54*A*C*a*b^2*c
^3*z + 18*A*C*b^4*c^2*z - 9*A*B*b^3*c^3*z + 81*C^2*a^2*b^2*c^2*z - 27*B*C*
b^5*c*z - 27*B^2*a*b^2*c^3*z - 108*C^2*a*b^4*c*z + 9*B^2*b^4*c^2*z + 27*C^
2*b^6*z - 9*A*B*C*a*b*c^2 - 9*B^2*C*a*b^2*c + 18*A*C^2*a*b^2*c + 9*A*B*C*b
^3*c - 3*A^2*C*b^2*c^2 - 3*A*B^2*b^2*c^2 + 3*B^3*a*b*c^2 + 9*B*C^2*a*b^3 +
 3*A^2*B*b*c^3 - 9*C^3*a^2*b^2 - 9*A*C^2*b^4 - A^3*c^4, z, k)^2*b^3*c^3 +
A*B*c^2 + 3*C^2*a*b + 3*A*root(243*a^2*b^2*c^6*z^3 - 162*a*b^4*c^5*z^3 + 2
7*b^6*c^4*z^3 - 243*C*a^2*b^2*c^4*z^2 + 162*C*a*b^4*c^3*z^2 - 27*C*b^6*c^2
*z^2 + 27*A*B*a*b*c^4*z + 81*B*C*a*b^3*c^2*z - 54*A*C*a*b^2*c^3*z + 18*A*C
*b^4*c^2*z - 9*A*B*b^3*c^3*z + 81*C^2*a^2*b^2*c^2*z - 27*B*C*b^5*c*z - 27*
B^2*a*b^2*c^3*z - 108*C^2*a*b^4*c*z + 9*B^2*b^4*c^2*z + 27*C^2*b^6*z - 9*A
*B*C*a*b*c^2 - 9*B^2*C*a*b^2*c + 18*A*C^2*a*b^2*c + 9*A*B*C*b^3*c - 3*A^2*
C*b^2*c^2 - 3*A*B^2*b^2*c^2 + 3*B^3*a*b*c^2 + 9*B*C^2*a*b^3 + 3*A^2*B*b*c^
3 - 9*C^3*a^2*b^2 - 9*A*C^2*b^4 - A^3*c^4, z, k)*b*c^3 + 9*C*root(243*a^2*
b^2*c^6*z^3 - 162*a*b^4*c^5*z^3 + 27*b^6*c^4*z^3 - 243*C*a^2*b^2*c^4*z^2 +
 162*C*a*b^4*c^3*z^2 - 27*C*b^6*c^2*z^2 + 27*A*B*a*b*c^4*z + 81*B*C*a*b^3*
c^2*z - 54*A*C*a*b^2*c^3*z + 18*A*C*b^4*c^2*z - 9*A*B*b^3*c^3*z + 81*C^2*a
^2*b^2*c^2*z - 27*B*C*b^5*c*z - 27*B^2*a*b^2*c^3*z - 108*C^2*a*b^4*c*z ...
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 781, normalized size of antiderivative = 1.98

$$\int \frac{A+Bx+Cx^2}{3ab+3b^2x+3bcx^2+c^2x^3}\,dx = \text{Too large to display}$$

input
```
int((C*x^2+B*x+A)/(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b),x)
```

output
```
( - 2*b**(2/3)*(3*a*c - b**2)**(2/3)*sqrt(3)*atan((b**(1/3)*(3*a*c - b**2)
**(1/3) - 2*b - 2*c*x)/(b**(1/3)*(3*a*c - b**2)**(1/3)*sqrt(3)))*a*c + 6*s
qrt(3)*atan((b**(1/3)*(3*a*c - b**2)**(1/3) - 2*b - 2*c*x)/(b**(1/3)*(3*a*
c - b**2)**(1/3)*sqrt(3)))*a*b**2*c - 2*sqrt(3)*atan((b**(1/3)*(3*a*c - b*
*2)**(1/3) - 2*b - 2*c*x)/(b**(1/3)*(3*a*c - b**2)**(1/3)*sqrt(3)))*b**4 -
 b**(2/3)*(3*a*c - b**2)**(2/3)*log(b**(2/3)*(3*a*c - b**2)**(2/3) - b**(1
/3)*(3*a*c - b**2)**(1/3)*b - b**(1/3)*(3*a*c - b**2)**(1/3)*c*x + b**2 +
2*b*c*x + c**2*x**2)*a*c + 2*b**(2/3)*(3*a*c - b**2)**(2/3)*log(b**(1/3)*(
3*a*c - b**2)**(1/3) + b + c*x)*a*c + 6*b**(1/3)*(3*a*c - b**2)**(1/3)*log
(b**(2/3)*(3*a*c - b**2)**(2/3) - b**(1/3)*(3*a*c - b**2)**(1/3)*b - b**(1
/3)*(3*a*c - b**2)**(1/3)*c*x + b**2 + 2*b*c*x + c**2*x**2)*a*b*c - 2*b**(
1/3)*(3*a*c - b**2)**(1/3)*log(b**(2/3)*(3*a*c - b**2)**(2/3) - b**(1/3)*(
3*a*c - b**2)**(1/3)*b - b**(1/3)*(3*a*c - b**2)**(1/3)*c*x + b**2 + 2*b*c
*x + c**2*x**2)*b**3 + 6*b**(1/3)*(3*a*c - b**2)**(1/3)*log(b**(1/3)*(3*a*
c - b**2)**(1/3) + b + c*x)*a*b*c - 2*b**(1/3)*(3*a*c - b**2)**(1/3)*log(b
**(1/3)*(3*a*c - b**2)**(1/3) + b + c*x)*b**3 - 3*log(b**(2/3)*(3*a*c - b*
*2)**(2/3) - b**(1/3)*(3*a*c - b**2)**(1/3)*b - b**(1/3)*(3*a*c - b**2)**(
1/3)*c*x + b**2 + 2*b*c*x + c**2*x**2)*a*b**2*c + log(b**(2/3)*(3*a*c - b*
*2)**(2/3) - b**(1/3)*(3*a*c - b**2)**(1/3)*b - b**(1/3)*(3*a*c - b**2)**(
1/3)*c*x + b**2 + 2*b*c*x + c**2*x**2)*b**4 + 6*log(b**(1/3)*(3*a*c - b...
```

3.72 $\int \frac{A+Bx+Cx^2}{(3ab+3b^2x+3bcx^2+c^2x^3)^2} dx$

Optimal result

Integrand size = 38, antiderivative size = 484

$$\int \frac{A+Bx+Cx^2}{(3ab+3b^2x+3bcx^2+c^2x^3)^2} dx$$
$$= -\frac{C}{3c^2\left(b\left(3a-\frac{b^2}{c}\right)+\frac{(b+cx)^3}{c}\right)} - \frac{(b+cx)\,(Ac^2-b^2C+c(Bc-2bC)x)}{3bc^2\,(b^2-3ac)\left(b\left(3a-\frac{b^2}{c}\right)+\frac{(b+cx)^3}{c}\right)}$$
$$-\frac{\left(2bBc-2Ac^2+\sqrt[3]{b}Bc\sqrt[3]{b^2-3ac}-2b^2C-2b^{4/3}\sqrt[3]{b^2-3ac}C\right)\arctan\left(\frac{\sqrt[3]{b}+\frac{2(b+cx)}{\sqrt[3]{b^2-3ac}}}{\sqrt{3}\sqrt[3]{b}}\right)}{3\sqrt{3}b^{5/3}c\,(b^2-3ac)^{5/3}}$$
$$+\frac{\left(2bBc-2Ac^2-\sqrt[3]{b}Bc\sqrt[3]{b^2-3ac}-2b^2C+2b^{4/3}\sqrt[3]{b^2-3ac}C\right)\log\left(\sqrt[3]{b}\left(b^{2/3}-\sqrt[3]{b^2-3ac}\right)+cx\right)}{9b^{5/3}c\,(b^2-3ac)^{5/3}}$$
$$-\frac{\left(2bBc-2Ac^2-\sqrt[3]{b}Bc\sqrt[3]{b^2-3ac}-2b^2C+2b^{4/3}\sqrt[3]{b^2-3ac}C\right)\log\left(b^{2/3}(b^2-3ac)^{2/3}+\sqrt[3]{b}\sqrt[3]{b^2-3ac}\right.}{18b^{5/3}c\,(b^2-3ac)^{5/3}}$$

output
```
-1/3*C/c^2/(b*(3*a-b^2/c)+(c*x+b)^3/c)-1/3*(c*x+b)*(A*c^2-C*b^2+c*(B*c-2*C
*b)*x)/b/c^2/(-3*a*c+b^2)/(b*(3*a-b^2/c)+(c*x+b)^3/c)-1/9*(2*B*b*c-2*A*c^2
+b^(1/3)*B*c*(-3*a*c+b^2)^(1/3)-2*C*b^2-2*b^(4/3)*(-3*a*c+b^2)^(1/3)*C)*ar
ctan(1/3*(b^(1/3)+2*(c*x+b)/(-3*a*c+b^2)^(1/3))*3^(1/2)/b^(1/3))*3^(1/2)/b
^(5/3)/c/(-3*a*c+b^2)^(5/3)+1/9*(2*B*b*c-2*A*c^2-b^(1/3)*B*c*(-3*a*c+b^2)^
(1/3)-2*C*b^2+2*b^(4/3)*(-3*a*c+b^2)^(1/3)*C)*ln(b^(1/3)*(b^(2/3)-(-3*a*c+
b^2)^(1/3))+c*x)/b^(5/3)/c/(-3*a*c+b^2)^(5/3)-1/18*(2*B*b*c-2*A*c^2-b^(1/3
)*B*c*(-3*a*c+b^2)^(1/3)-2*C*b^2+2*b^(4/3)*(-3*a*c+b^2)^(1/3)*C)*ln(b^(2/3
)*(-3*a*c+b^2)^(2/3)+b^(1/3)*(-3*a*c+b^2)^(1/3)*(c*x+b)+(c*x+b)^2)/b^(5/3)
/c/(-3*a*c+b^2)^(5/3)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.12 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.38

$$\int \frac{A+Bx+Cx^2}{(3ab+3b^2x+3bcx^2+c^2x^3)^2}\,dx$$
$$= \frac{-\frac{3(-3abC+Ac(b+cx)+x(-3b^2C+Bc^2x+bc(B-2Cx)))}{c(3ab+x(3b^2+3bcx+c^2x^2))}+\text{RootSum}\left[3ab+3b^2\#1+3bc\#1^2+c^2\#1^3\&,\frac{bB\log(x-\#1)-\cdots}{\cdots}\right]}{9(b^3-3abc)}$$

input
```
Integrate[(A + B*x + C*x^2)/(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3)^2,x]
```

output
```
((-3*(-3*a*b*C + A*c*(b + c*x) + x*(-3*b^2*C + B*c^2*x + b*c*(B - 2*C*x)))
)/(c*(3*a*b + x*(3*b^2 + 3*b*c*x + c^2*x^2))) + RootSum[3*a*b + 3*b^2*#1 +
 3*b*c*#1^2 + c^2*#1^3 & , (b*B*Log[x - #1] - 2*A*c*Log[x - #1] - B*c*Log[
x - #1]*#1 + 2*b*C*Log[x - #1]*#1)/(b^2 + 2*b*c*#1 + c^2*#1^2) & ])/(9*(b^
3 - 3*a*b*c))
```

Rubi [A] (verified)

Time = 2.40 (sec) , antiderivative size = 481, normalized size of antiderivative = 0.99, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {2459, 2393, 25, 2400, 16, 1142, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{(3ab+3b^2x+3bcx^2+c^2x^3)^2}dx$$

$$\Big\downarrow \text{2459}$$

$$\int \frac{A-\frac{b(Bc-bC)}{c^2}+\left(\frac{b}{c}+x\right)\left(B-\frac{2bC}{c}\right)+C\left(\frac{b}{c}+x\right)^2}{\left(b\left(3a-\frac{b^2}{c}\right)+c^2\left(\frac{b}{c}+x\right)^3\right)^2}d\left(\frac{b}{c}+x\right)$$

$$\Big\downarrow \text{2393}$$

$$\frac{c\int -\frac{2\left(A-\frac{b(Bc-bC)}{c^2}\right)+\left(B-\frac{2bC}{c}\right)\left(\frac{b}{c}+x\right)}{c^2\left(\frac{b}{c}+x\right)^3+b\left(3a-\frac{b^2}{c}\right)}d\left(\frac{b}{c}+x\right)}{3b\left(b^2-3ac\right)}+\frac{bC\left(3a-\frac{b^2}{c}\right)-c^2\left(\frac{b}{c}+x\right)\left(A-\frac{b(Bc-bC)}{c^2}+\left(\frac{b}{c}+x\right)\left(B-\frac{2bC}{c}\right)\right)}{3bc\left(b^2-3ac\right)\left(b\left(3a-\frac{b^2}{c}\right)+c^2\left(\frac{b}{c}+x\right)^3\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{bC\left(3a-\frac{b^2}{c}\right)-c^2\left(\frac{b}{c}+x\right)\left(A-\frac{b(Bc-bC)}{c^2}+\left(\frac{b}{c}+x\right)\left(B-\frac{2bC}{c}\right)\right)}{3bc\left(b^2-3ac\right)\left(b\left(3a-\frac{b^2}{c}\right)+c^2\left(\frac{b}{c}+x\right)^3\right)}-\frac{c\int \frac{2\left(A-\frac{b(Bc-bC)}{c^2}\right)+\left(B-\frac{2bC}{c}\right)\left(\frac{b}{c}+x\right)}{c^2\left(\frac{b}{c}+x\right)^3+b\left(3a-\frac{b^2}{c}\right)}d\left(\frac{b}{c}+x\right)}{3b\left(b^2-3ac\right)}$$

$$\Big\downarrow \text{2400}$$

$$\frac{bC\left(3a-\frac{b^2}{c}\right)-c^2\left(\frac{b}{c}+x\right)\left(A-\frac{b(Bc-bC)}{c^2}+\left(\frac{b}{c}+x\right)\left(B-\frac{2bC}{c}\right)\right)}{3bc\left(b^2-3ac\right)\left(b\left(3a-\frac{b^2}{c}\right)+c^2\left(\frac{b}{c}+x\right)^3\right)}-$$

$$\frac{c\left(\frac{\left(-\sqrt[3]{b}Bc\sqrt[3]{b^2-3ac}+2b^{4/3}C\sqrt[3]{b^2-3ac}-2Ac^2-2b^2C+2bBc\right)\int\frac{1}{\sqrt[3]{b}\sqrt[3]{b^2-3ac}-c\left(\frac{b}{c}+x\right)}d\left(\frac{b}{c}+x\right)}{3b^{2/3}c\left(b^2-3ac\right)^{2/3}}+\frac{\int\frac{\sqrt[3]{b}\sqrt[3]{b^2-3ac}\left(\sqrt[3]{\cdots}\right.}{\cdots}}{\cdots}\right.}{3b\left(b^2-3ac\right)}$$

$\downarrow$ 16

$$\frac{bC\left(3a-\frac{b^2}{c}\right)-c^2\left(\frac{b}{c}+x\right)\left(A-\frac{b(Bc-bC)}{c^2}+\left(\frac{b}{c}+x\right)\left(B-\frac{2bC}{c}\right)\right)}{3bc\left(b^2-3ac\right)\left(b\left(3a-\frac{b^2}{c}\right)+c^2\left(\frac{b}{c}+x\right)^3\right)}-$$

$$\frac{c\left(\frac{\int\frac{\sqrt[3]{b}\sqrt[3]{b^2-3ac}\left(\sqrt[3]{b}\sqrt[3]{b^2-3ac}\left(B-\frac{2bC}{c}\right)-4c\left(A-\frac{b(Bc-bC)}{c^2}\right)\right)-c\left(2Ac+\sqrt[3]{b}\sqrt[3]{b^2-3ac}\left(B-\frac{2bC}{c}\right)-\frac{2b(Bc-bC)}{c}\right)\left(\frac{b}{c}+x\right)}{c^2\left(\frac{b}{c}+x\right)^2+\sqrt[3]{b}c\sqrt[3]{b^2-3ac}\left(\frac{b}{c}+x\right)+b^{2/3}\left(b^2-3ac\right)^{2/3}}d\left(\frac{b}{c}+x\right)}{3b^{2/3}\left(b^2-3ac\right)^{2/3}}\cdots\right.}{3b\left(b^2-3ac\right)}$$

$\downarrow$ 1142

$$\frac{bC\left(3a-\frac{b^2}{c}\right)-c^2\left(\frac{b}{c}+x\right)\left(A-\frac{b(Bc-bC)}{c^2}+\left(\frac{b}{c}+x\right)\left(B-\frac{2bC}{c}\right)\right)}{3bc\left(b^2-3ac\right)\left(b\left(3a-\frac{b^2}{c}\right)+c^2\left(\frac{b}{c}+x\right)^3\right)}-$$

$$c\left(\frac{\frac{3\sqrt[3]{b}\sqrt[3]{b^2-3ac}\left(\sqrt[3]{b}Bc\sqrt[3]{b^2-3ac}-2b^{4/3}C\sqrt[3]{b^2-3ac}-2Ac^2-2b^2C+2bBc\right)\int\frac{1}{c^2\left(\frac{b}{c}+x\right)^2+\sqrt[3]{b}c\sqrt[3]{b^2-3ac}\left(\frac{b}{c}+x\right)+b^{2/3}\left(b^2-3ac\right)\cdots}\cdots}{2c}}{3b^{2/3}\left(b^2-3ac\right)\cdots}\cdots\right.$$

$\downarrow$ 27

$$\frac{bC\left(3a-\frac{b^2}{c}\right)-c^2\left(\frac{b}{c}+x\right)\left(A-\frac{b(Bc-bC)}{c^2}+\left(\frac{b}{c}+x\right)\left(B-\frac{2bC}{c}\right)\right)}{3bc\left(b^2-3ac\right)\left(b\left(3a-\frac{b^2}{c}\right)+c^2\left(\frac{b}{c}+x\right)^3\right)}-$$

$$c\left(\frac{\frac{3\sqrt[3]{b}\sqrt[3]{b^2-3ac}\left(\sqrt[3]{b}Bc\sqrt[3]{b^2-3ac}-2b^{4/3}C\sqrt[3]{b^2-3ac}-2Ac^2-2b^2C+2bBc\right)\int\frac{1}{c^2\left(\frac{b}{c}+x\right)^2+\sqrt[3]{b}c\sqrt[3]{b^2-3ac}\left(\frac{b}{c}+x\right)+b^{2/3}\left(b^2-3ac\right)}}{2c}}{3b^{2/3}\left(b^2-3a\right.}\right.$$

$$\Big\downarrow \ 1082$$

$$\frac{bC\left(3a-\frac{b^2}{c}\right)-c^2\left(\frac{b}{c}+x\right)\left(A-\frac{b(Bc-bC)}{c^2}+\left(\frac{b}{c}+x\right)\left(B-\frac{2bC}{c}\right)\right)}{3bc\left(b^2-3ac\right)\left(b\left(3a-\frac{b^2}{c}\right)+c^2\left(\frac{b}{c}+x\right)^3\right)}-$$

$$c\left(\frac{-\frac{1}{2}\left(\sqrt[3]{b}\sqrt[3]{b^2-3ac}\left(B-\frac{2bC}{c}\right)+2Ac-\frac{2b(Bc-bC)}{c}\right)\int\frac{2c\left(\frac{b}{c}+x\right)+\sqrt[3]{b}\sqrt[3]{b^2-3ac}}{c^2\left(\frac{b}{c}+x\right)^2+\sqrt[3]{b}c\sqrt[3]{b^2-3ac}\left(\frac{b}{c}+x\right)+b^{2/3}\left(b^2-3ac\right)^{2/3}}d\left(\frac{b}{c}+x\right)-\frac{3\left(\sqrt[3]{b}Bc\sqrt[3]{b^2\right.}{}}{3b^{2/3}\left(b^2-3ac\right)^{2/3}}\right.$$

$$\Big\downarrow \ 217$$

$$\frac{bC\left(3a-\frac{b^2}{c}\right)-c^2\left(\frac{b}{c}+x\right)\left(A-\frac{b(Bc-bC)}{c^2}+\left(\frac{b}{c}+x\right)\left(B-\frac{2bC}{c}\right)\right)}{3bc\left(b^2-3ac\right)\left(b\left(3a-\frac{b^2}{c}\right)+c^2\left(\frac{b}{c}+x\right)^3\right)}-$$

$$c\left(\frac{\frac{\sqrt{3}\arctan\left(\frac{\frac{2c\left(\frac{b}{c}+x\right)}{\sqrt[3]{b}\sqrt[3]{b^2-3ac}}+1}{\sqrt{3}}\right)\left(\sqrt[3]{b}Bc\sqrt[3]{b^2-3ac}-2b^{4/3}C\sqrt[3]{b^2-3ac}-2Ac^2-2b^2C+2bBc\right)}{c^2}-\frac{1}{2}\left(\sqrt[3]{b}\sqrt[3]{b^2-3ac}\left(B-\frac{2bC}{c}\right)+2A\right.}{3b^{2/3}\left(b^2-3ac\right)^{2/3}}\right.$$

$$\Big\downarrow \ 1103$$

$$\frac{bC\left(3a-\frac{b^2}{c}\right)-c^2\left(\frac{b}{c}+x\right)\left(A-\frac{b(Bc-bC)}{c^2}+\left(\frac{b}{c}+x\right)\left(B-\frac{2bC}{c}\right)\right)}{3bc\left(b^2-3ac\right)\left(b\left(3a-\frac{b^2}{c}\right)+c^2\left(\frac{b}{c}+x\right)^3\right)}-$$

$$c\left(\frac{\frac{\sqrt{3}\arctan\left(\frac{\frac{2c\left(\frac{b}{c}+x\right)}{\sqrt[3]{b}\sqrt[3]{b^2-3ac}}+1}{\sqrt{3}}\right)\left(\sqrt[3]{b}Bc\sqrt[3]{b^2-3ac}-2b^{4/3}C\sqrt[3]{b^2-3ac}-2Ac^2-2b^2C+2bBc\right)}{c^2}-\frac{\log\left(\sqrt[3]{b}c\sqrt[3]{b^2-3ac}\left(\frac{b}{c}+x\right)+b^{2/3}\left(\right.\right.}{}}{3b^{2/3}\left(b^2-3ac\right)^{2/3}}\right.$$

input
```
Int[(A + B*x + C*x^2)/(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3)^2,x]
```

output
```
(b*(3*a - b^2/c)*C - c^2*(b/c + x)*(A - (b*(B*c - b*C))/c^2 + (B - (2*b*C)
/c)*(b/c + x)))/(3*b*c*(b^2 - 3*a*c)*(b*(3*a - b^2/c) + c^2*(b/c + x)^3))
- (c*(-1/3*((2*b*B*c - 2*A*c^2 - b^(1/3)*B*c*(b^2 - 3*a*c)^(1/3) - 2*b^2*C
 + 2*b^(4/3)*(b^2 - 3*a*c)^(1/3)*C)*Log[b^(1/3)*(b^2 - 3*a*c)^(1/3) - c*(b
/c + x)])/(b^(2/3)*c^2*(b^2 - 3*a*c)^(2/3)) + ((Sqrt[3]*(2*b*B*c - 2*A*c^2
 + b^(1/3)*B*c*(b^2 - 3*a*c)^(1/3) - 2*b^2*C - 2*b^(4/3)*(b^2 - 3*a*c)^(1/
3)*C)*ArcTan[(1 + (2*c*(b/c + x))/(b^(1/3)*(b^2 - 3*a*c)^(1/3)))/Sqrt[3]])
/c^2 - ((2*A*c - (2*b*(B*c - b*C))/c + b^(1/3)*(b^2 - 3*a*c)^(1/3)*(B - (2
*b*C)/c))*Log[b^(2/3)*(b^2 - 3*a*c)^(2/3) + b^(1/3)*c*(b^2 - 3*a*c)^(1/3)*
(b/c + x) + c^2*(b/c + x)^2])/(2*c))/(3*b^(2/3)*(b^2 - 3*a*c)^(2/3))))/(3*
b*(b^2 - 3*a*c))
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2393
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   In
t[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(
p + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n
, 0] && LtQ[p, -1]
```

rule 2400
```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[-a/b, 3]], s = Denominator[Rt[-a/b, 3]]}, Simp[r*((B*r + A*s)/(3*a*
s))   Int[1/(r - s*x), x], x] - Simp[r/(3*a*s)   Int[(r*(B*r - 2*A*s) - s*(
B*r + A*s)*x)/(r^2 + r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
 NeQ[a*B^3 - b*A^3, 0] && NegQ[a/b]
```

rule 2459
```
Int[(Pn_)^(p_.)*(Qx_), x_Symbol] :> With[{S = Coeff[Pn, x, Expon[Pn, x] - 1
]/(Expon[Pn, x]*Coeff[Pn, x, Expon[Pn, x]])}, Subst[Int[ExpandToSum[Pn /. x
 -> x - S, x]^p*ExpandToSum[Qx /. x -> x - S, x], x], x, x + S] /; Binomial
Q[Pn /. x -> x - S, x] || (IntegerQ[Expon[Pn, x]/2] && TrinomialQ[Pn /. x -
> x - S, x])] /; FreeQ[p, x] && PolyQ[Pn, x] && GtQ[Expon[Pn, x], 2] && NeQ
[Coeff[Pn, x, Expon[Pn, x] - 1], 0] && PolyQ[Qx, x] &&  !(MonomialQ[Qx, x]
&& IGtQ[p, 0])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.13 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.43

method	result
default	$\frac{\frac{(Bc-2bC)x^2}{9b(3ac-b^2)}+\frac{(A c^2+bBc-3b^2C)x}{9bc(3ac-b^2)}+\frac{Ac-3Ca}{9c(3ac-b^2)}}{\frac{1}{3}c^2x^3+bc\,x^2+b^2x+ab}+\frac{\sum_{\_R=\mathrm{RootOf}(\_Z^3c^2+3\_Z^2bc+3b^2\_Z+3ab)}\frac{(B\_Rc-2C\_Rb+2Ac-Bb)\ln(x-\_}{\_R^2c^2+2\_Rbc+b^2}}{9b(3ac-b^2)}$
risch	$\frac{\frac{(Bc-2bC)x^2}{9b(3ac-b^2)}+\frac{(A c^2+bBc-3b^2C)x}{9bc(3ac-b^2)}+\frac{Ac-3Ca}{9c(3ac-b^2)}}{\frac{1}{3}c^2x^3+bc\,x^2+b^2x+ab}+\frac{\sum_{\_R=\mathrm{RootOf}(\_Z^3c^2+3\_Z^2bc+3b^2\_Z+3ab)}\frac{\left(\frac{(Bc-2bC)\_R}{3ac-b^2}+\frac{2Ac-Bb}{3ac-b^2}\right)\ln(x-\_}{\_R^2c^2+2\_Rbc+b^2}}{9b}$

input
```
int((C*x^2+B*x+A)/(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^2,x,method=_RETURNVERB
OSE)
```

output
```
(1/9*(B*c-2*C*b)/b/(3*a*c-b^2)*x^2+1/9*(A*c^2+B*b*c-3*C*b^2)/b/c/(3*a*c-b^
2)*x+1/9/c*(A*c-3*C*a)/(3*a*c-b^2))/(1/3*c^2*x^3+b*c*x^2+b^2*x+a*b)+1/9/b/
(3*a*c-b^2)*sum((B*_R*c-2*C*_R*b+2*A*c-B*b)/(_R^2*c^2+2*_R*b*c+b^2)*ln(x-_
R),_R=RootOf(_Z^3*c^2+3*_Z^2*b*c+3*_Z*b^2+3*a*b))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 11.54 (sec) , antiderivative size = 15642, normalized size of antiderivative = 32.32

$$\int \frac{A+Bx+Cx^2}{(3ab+3b^2x+3bcx^2+c^2x^3)^2}\,dx = \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)/(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^2,x, algorithm="
fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(3ab+3b^2x+3bcx^2+c^2x^3)^2}\,dx = \text{Timed out}$$

input
```
integrate((C*x**2+B*x+A)/(c**2*x**3+3*b*c*x**2+3*b**2*x+3*a*b)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{(3ab+3b^2x+3bcx^2+c^2x^3)^2}\,dx = \int \frac{Cx^2+Bx+A}{(c^2x^3+3\,bcx^2+3\,b^2x+3\,ab)^2}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^2,x, algorithm="
maxima")
```

output
```
1/3*(3*C*a*b - A*b*c + (2*C*b*c - B*c^2)*x^2 + (3*C*b^2 - B*b*c - A*c^2)*x
)/(3*a*b^4*c - 9*a^2*b^2*c^2 + (b^3*c^3 - 3*a*b*c^4)*x^3 + 3*(b^4*c^2 - 3*
a*b^2*c^3)*x^2 + 3*(b^5*c - 3*a*b^3*c^2)*x) - 1/3*integrate(-(B*b - 2*A*c
+ (2*C*b - B*c)*x)/(c^2*x^3 + 3*b*c*x^2 + 3*b^2*x + 3*a*b), x)/(b^3 - 3*a*
b*c)
```

Giac [F]

$$\int \frac{A+Bx+Cx^2}{\left(3ab+3b^2x+3bcx^2+c^2x^3\right)^2}\,dx=\int \frac{Cx^2+Bx+A}{\left(c^2x^3+3\,bcx^2+3\,b^2x+3\,ab\right)^2}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^2,x, algorithm="
giac")
```

output
```
integrate((C*x^2 + B*x + A)/(c^2*x^3 + 3*b*c*x^2 + 3*b^2*x + 3*a*b)^2, x)
```

Mupad [B] (verification not implemented)

Time = 13.19 (sec) , antiderivative size = 1692, normalized size of antiderivative = 3.50

$$\int \frac{A+Bx+Cx^2}{\left(3ab+3b^2x+3bcx^2+c^2x^3\right)^2}\,dx=\text{Too large to display}$$

input
```
int((A + B*x + C*x^2)/(3*a*b + 3*b^2*x + c^2*x^3 + 3*b*c*x^2)^2,x)
```

output
```
((A*c - 3*C*a)/(3*c*(3*a*c - b^2)) + (x^2*(B*c - 2*C*b))/(3*b*(3*a*c - b^2
)) + (x*(A*c^2 - 3*C*b^2 + B*b*c))/(3*b*c*(3*a*c - b^2)))/(3*a*b + 3*b^2*x
 + c^2*x^3 + 3*b*c*x^2) + symsum(log((x*(B^2*c^4 + 4*C^2*b^2*c^2 - 4*B*C*b
*c^3))/(9*(b^6 + 9*a^2*b^2*c^2 - 6*a*b^4*c)) - root(295245*a^4*b^7*c^6*z^3
 - 196830*a^3*b^9*c^5*z^3 - 177147*a^5*b^5*c^7*z^3 + 65610*a^2*b^11*c^4*z^
3 - 10935*a*b^13*c^3*z^3 + 729*b^15*c^2*z^3 - 1458*B*C*a^2*b^4*c^3*z + 972
*A*C*a^2*b^3*c^4*z - 486*A*B*a^2*b^2*c^5*z + 972*B*C*a*b^6*c^2*z - 648*A*C
*a*b^5*c^3*z + 324*A*B*a*b^4*c^4*z + 108*A*C*b^7*c^2*z - 54*A*B*b^6*c^3*z
+ 972*C^2*a^2*b^5*c^2*z + 486*B^2*a^2*b^3*c^4*z - 162*B*C*b^8*c*z - 324*B^
2*a*b^5*c^3*z - 648*C^2*a*b^7*c*z + 54*B^2*b^7*c^2*z + 108*C^2*b^9*z - 36*
B*C^2*a*b^3*c - 48*A*B*C*b^3*c^2 + 18*B^2*C*a*b^2*c^2 + 24*A^2*C*b^2*c^3 +
 24*A*B^2*b^2*c^3 - 3*B^3*a*b*c^3 + 18*B^2*C*b^4*c + 24*A*C^2*b^4*c - 24*A
^2*B*b*c^4 - 7*B^3*b^3*c^2 + 24*C^3*a*b^4 - 12*B*C^2*b^5 + 8*A^3*c^5, z, k
)*((18*A*b^4*c^4 - 18*B*b^5*c^3 + 18*C*b^6*c^2 - 54*A*a*b^2*c^5 + 54*B*a*b
^3*c^4 - 54*C*a*b^4*c^3)/(9*(b^6 + 9*a^2*b^2*c^2 - 6*a*b^4*c)) + (root(295
245*a^4*b^7*c^6*z^3 - 196830*a^3*b^9*c^5*z^3 - 177147*a^5*b^5*c^7*z^3 + 65
610*a^2*b^11*c^4*z^3 - 10935*a*b^13*c^3*z^3 + 729*b^15*c^2*z^3 - 1458*B*C*
a^2*b^4*c^3*z + 972*A*C*a^2*b^3*c^4*z - 486*A*B*a^2*b^2*c^5*z + 972*B*C*a*
b^6*c^2*z - 648*A*C*a*b^5*c^3*z + 324*A*B*a*b^4*c^4*z + 108*A*C*b^7*c^2*z
- 54*A*B*b^6*c^3*z + 972*C^2*a^2*b^5*c^2*z + 486*B^2*a^2*b^3*c^4*z - 16...
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 2426, normalized size of antiderivative = 5.01

$$\int \frac{A+Bx+Cx^2}{(3ab+3b^2x+3bcx^2+c^2x^3)^2}\,dx = \text{Too large to display}$$

input
```
int((C*x^2+B*x+A)/(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^2,x)
```

output

```
( - 12*b**(2/3)*(3*a*c - b**2)**(2/3)*sqrt(3)*atan((b**(1/3)*(3*a*c - b**2
)**(1/3) - 2*b - 2*c*x)/(b**(1/3)*(3*a*c - b**2)**(1/3)*sqrt(3)))*a**2*b*c
 - 12*b**(2/3)*(3*a*c - b**2)**(2/3)*sqrt(3)*atan((b**(1/3)*(3*a*c - b**2)
**(1/3) - 2*b - 2*c*x)/(b**(1/3)*(3*a*c - b**2)**(1/3)*sqrt(3)))*a*b**2*c*
x - 12*b**(2/3)*(3*a*c - b**2)**(2/3)*sqrt(3)*atan((b**(1/3)*(3*a*c - b**2
)**(1/3) - 2*b - 2*c*x)/(b**(1/3)*(3*a*c - b**2)**(1/3)*sqrt(3)))*a*b*c**2
*x**2 - 4*b**(2/3)*(3*a*c - b**2)**(2/3)*sqrt(3)*atan((b**(1/3)*(3*a*c - b
**2)**(1/3) - 2*b - 2*c*x)/(b**(1/3)*(3*a*c - b**2)**(1/3)*sqrt(3)))*a*c**
3*x**3 + 18*sqrt(3)*atan((b**(1/3)*(3*a*c - b**2)**(1/3) - 2*b - 2*c*x)/(b
**(1/3)*(3*a*c - b**2)**(1/3)*sqrt(3)))*a**2*b**3*c - 6*sqrt(3)*atan((b**(
1/3)*(3*a*c - b**2)**(1/3) - 2*b - 2*c*x)/(b**(1/3)*(3*a*c - b**2)**(1/3)*
sqrt(3)))*a*b**5 + 18*sqrt(3)*atan((b**(1/3)*(3*a*c - b**2)**(1/3) - 2*b -
 2*c*x)/(b**(1/3)*(3*a*c - b**2)**(1/3)*sqrt(3)))*a*b**4*c*x + 18*sqrt(3)*
atan((b**(1/3)*(3*a*c - b**2)**(1/3) - 2*b - 2*c*x)/(b**(1/3)*(3*a*c - b**
2)**(1/3)*sqrt(3)))*a*b**3*c**2*x**2 + 6*sqrt(3)*atan((b**(1/3)*(3*a*c - b
**2)**(1/3) - 2*b - 2*c*x)/(b**(1/3)*(3*a*c - b**2)**(1/3)*sqrt(3)))*a*b**
2*c**3*x**3 - 6*sqrt(3)*atan((b**(1/3)*(3*a*c - b**2)**(1/3) - 2*b - 2*c*x
)/(b**(1/3)*(3*a*c - b**2)**(1/3)*sqrt(3)))*b**6*x - 6*sqrt(3)*atan((b**(1
/3)*(3*a*c - b**2)**(1/3) - 2*b - 2*c*x)/(b**(1/3)*(3*a*c - b**2)**(1/3)*s
qrt(3)))*b**5*c*x**2 - 2*sqrt(3)*atan((b**(1/3)*(3*a*c - b**2)**(1/3) -...
```

3.73 $\int \frac{A+Bx+Cx^2}{(3ab+3b^2x+3bcx^2+c^2x^3)^3}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 564

$$\int \frac{A+Bx+Cx^2}{(3ab+3b^2x+3bcx^2+c^2x^3)^3}\,dx$$

$$= -\frac{C}{6c^2\left(b\left(3a-\frac{b^2}{c}\right)+\frac{(b+cx)^3}{c}\right)^2} - \frac{(b+cx)\left(Ac^2-b^2C+c(Bc-2bC)x\right)}{6bc^2\left(b^2-3ac\right)\left(b\left(3a-\frac{b^2}{c}\right)+\frac{(b+cx)^3}{c}\right)^2}$$

$$- \frac{(b+cx)\left(5(bBc-Ac^2-b^2C)-4(Bc-2bC)(b+cx)\right)}{18b^2c\left(b^2-3ac\right)^2\left(b\left(3a-\frac{b^2}{c}\right)+\frac{(b+cx)^3}{c}\right)}$$

$$+ \frac{\left(5bBc-5Ac^2+2\sqrt[3]{b}Bc\sqrt[3]{b^2-3ac}-5b^2C-4b^{4/3}\sqrt[3]{b^2-3ac}C\right)\arctan\left(\frac{\sqrt[3]{b}+\frac{2(b+cx)}{\sqrt[3]{b^2-3ac}}}{\sqrt{3}\sqrt[3]{b}}\right)}{9\sqrt{3}b^{8/3}\left(b^2-3ac\right)^{8/3}}$$

$$- \frac{\left(5bBc-5Ac^2-2\sqrt[3]{b}Bc\sqrt[3]{b^2-3ac}-5b^2C+4b^{4/3}\sqrt[3]{b^2-3ac}C\right)\log\left(\sqrt[3]{b}\left(b^{2/3}-\sqrt[3]{b^2-3ac}\right)+cx\right)}{27b^{8/3}\left(b^2-3ac\right)^{8/3}}$$

$$+ \frac{\left(5bBc-5Ac^2-2\sqrt[3]{b}Bc\sqrt[3]{b^2-3ac}-5b^2C+4b^{4/3}\sqrt[3]{b^2-3ac}C\right)\log\left(b^{2/3}(b^2-3ac)^{2/3}+\sqrt[3]{b}\sqrt[3]{b^2-3a}\right.}{54b^{8/3}\left(b^2-3ac\right)^{8/3}}$$

output
```
-1/6*C/c^2/(b*(3*a-b^2/c)+(c*x+b)^3/c)^2-1/6*(c*x+b)*(A*c^2-C*b^2+c*(B*c-2
*C*b)*x)/b/c^2/(-3*a*c+b^2)/(b*(3*a-b^2/c)+(c*x+b)^3/c)^2-1/18*(c*x+b)*(-5
*A*c^2+5*B*b*c-5*C*b^2-4*(B*c-2*C*b)*(c*x+b))/b^2/c/(-3*a*c+b^2)^2/(b*(3*a
-b^2/c)+(c*x+b)^3/c)+1/27*(5*B*b*c-5*A*c^2+2*b^(1/3)*B*c*(-3*a*c+b^2)^(1/3
)-5*C*b^2-4*b^(4/3)*(-3*a*c+b^2)^(1/3)*C)*arctan(1/3*(b^(1/3)+2*(c*x+b)/(-
3*a*c+b^2)^(1/3))*3^(1/2)/b^(1/3))*3^(1/2)/b^(8/3)/(-3*a*c+b^2)^(8/3)-1/27
*(5*B*b*c-5*A*c^2-2*b^(1/3)*B*c*(-3*a*c+b^2)^(1/3)-5*C*b^2+4*b^(4/3)*(-3*a
*c+b^2)^(1/3)*C)*ln(b^(1/3)*(b^(2/3)-(-3*a*c+b^2)^(1/3))+c*x)/b^(8/3)/(-3*
a*c+b^2)^(8/3)+1/54*(5*B*b*c-5*A*c^2-2*b^(1/3)*B*c*(-3*a*c+b^2)^(1/3)-5*C*
b^2+4*b^(4/3)*(-3*a*c+b^2)^(1/3)*C)*ln(b^(2/3)*(-3*a*c+b^2)^(2/3)+b^(1/3)*
(-3*a*c+b^2)^(1/3)*(c*x+b)+(c*x+b)^2)/b^(8/3)/(-3*a*c+b^2)^(8/3)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.18 (sec) , antiderivative size = 286, normalized size of antiderivative = 0.51

$$\int \frac{A+Bx+Cx^2}{\left(3ab+3b^2x+3bcx^2+c^2x^3\right)^3}\,dx$$

$$= \frac{\frac{9b(b^2-3ac)\left(3abC-Ac(b+cx)+x\left(-bBc+3b^2C-Bc^2x+2bcCx\right)\right)}{c(3ab+x(3b^2+3bcx+c^2x^2))^2} - \frac{3(b+cx)\left(3b^2C-c^2(5A+4Bx)+bc(B+8Cx)\right)}{c(3ab+x(3b^2+3bcx+c^2x^2))} + 2\text{RootSum}\Big[3ab+}{54\,(b}$$

input
```
Integrate[(A + B*x + C*x^2)/(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3)^3,x]
```

output
```
((9*b*(b^2 - 3*a*c)*(3*a*b*C - A*c*(b + c*x) + x*(-(b*B*c) + 3*b^2*C - B*c
^2*x + 2*b*c*C*x)))/(c*(3*a*b + x*(3*b^2 + 3*b*c*x + c^2*x^2))^2) - (3*(b
+ c*x)*(3*b^2*C - c^2*(5*A + 4*B*x) + b*c*(B + 8*C*x)))/(c*(3*a*b + x*(3*b
^2 + 3*b*c*x + c^2*x^2))) + 2*RootSum[3*a*b + 3*b^2*#1 + 3*b*c*#1^2 + c^2*
#1^3 & , (-3*b*B*c*Log[x - #1] + 5*A*c^2*Log[x - #1] + b^2*C*Log[x - #1] +
 2*B*c^2*Log[x - #1]*#1 - 4*b*c*C*Log[x - #1]*#1)/(b^2 + 2*b*c*#1 + c^2*#1
^2) & ])/(54*(b^3 - 3*a*b*c)^2)
```

Rubi [A] (verified)

Time = 2.95 (sec) , antiderivative size = 594, normalized size of antiderivative = 1.05, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {2459, 2393, 25, 2394, 27, 2400, 16, 25, 27, 1142, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{\left(3ab+3b^2x+3bcx^2+c^2x^3\right)^3}dx$$

$$\Big\downarrow \text{2459}$$

$$\int \frac{A-\frac{b(Bc-bC)}{c^2}+\left(\frac{b}{c}+x\right)\left(B-\frac{2bC}{c}\right)+C\left(\frac{b}{c}+x\right)^2}{\left(b\left(3a-\frac{b^2}{c}\right)+c^2\left(\frac{b}{c}+x\right)^3\right)^3}d\left(\frac{b}{c}+x\right)$$

$$\Big\downarrow \text{2393}$$

$$\frac{c\int -\frac{5\left(A-\frac{b(Bc-bC)}{c^2}\right)+4\left(B-\frac{2bC}{c}\right)\left(\frac{b}{c}+x\right)}{\left(c^2\left(\frac{b}{c}+x\right)^3+b\left(3a-\frac{b^2}{c}\right)\right)^2}d\left(\frac{b}{c}+x\right)}{6b\left(b^2-3ac\right)}+\frac{bC\left(3a-\frac{b^2}{c}\right)-c^2\left(\frac{b}{c}+x\right)\left(A-\frac{b(Bc-bC)}{c^2}+\left(\frac{b}{c}+x\right)\left(B-\frac{2bC}{c}\right)\right)}{6bc\left(b^2-3ac\right)\left(b\left(3a-\frac{b^2}{c}\right)+c^2\left(\frac{b}{c}+x\right)^3\right)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{bC\left(3a-\frac{b^2}{c}\right)-c^2\left(\frac{b}{c}+x\right)\left(A-\frac{b(Bc-bC)}{c^2}+\left(\frac{b}{c}+x\right)\left(B-\frac{2bC}{c}\right)\right)}{6bc\left(b^2-3ac\right)\left(b\left(3a-\frac{b^2}{c}\right)+c^2\left(\frac{b}{c}+x\right)^3\right)^2}-\frac{c\int \frac{5\left(A-\frac{b(Bc-bC)}{c^2}\right)+4\left(B-\frac{2bC}{c}\right)\left(\frac{b}{c}+x\right)}{\left(c^2\left(\frac{b}{c}+x\right)^3+b\left(3a-\frac{b^2}{c}\right)\right)^2}d\left(\frac{b}{c}+x\right)}{6b\left(b^2-3ac\right)}$$

$$\Big\downarrow \text{2394}$$

$$\frac{bC\left(3a-\frac{b^2}{c}\right)-c^2\left(\frac{b}{c}+x\right)\left(A-\frac{b(Bc-bC)}{c^2}+\left(\frac{b}{c}+x\right)\left(B-\frac{2bC}{c}\right)\right)}{6bc\left(b^2-3ac\right)\left(b\left(3a-\frac{b^2}{c}\right)+c^2\left(\frac{b}{c}+x\right)^3\right)^2}-\frac{c\left(\frac{c\int \frac{2\left(5\left(-Cb^2+Bcb-Ac^2\right)-2c^2\left(B-\frac{2bC}{c}\right)\left(\frac{b}{c}+x\right)\right)}{c^2\left(c^2\left(\frac{b}{c}+x\right)^3+b\left(3a-\frac{b^2}{c}\right)\right)}d\left(\frac{b}{c}+x\right)}{3b(b^2-3ac)}-\frac{c\left(\frac{b}{c}+x\right)\left(5\left(A-\frac{b(Bc-bC)}{c^2}\right)+4\left(\frac{b}{c}+x\right)\left(B-\frac{2bC}{c}\right)\right)}{3b(b^2-3ac)\left(b\left(3a-\frac{b^2}{c}\right)+c^2\left(\frac{b}{c}+x\right)^3\right)}\right)}{6b\left(b^2-3ac\right)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{bC\left(3a-\frac{b^2}{c}\right)-c^2\left(\frac{b}{c}+x\right)\left(A-\frac{b(Bc-bC)}{c^2}+\left(\frac{b}{c}+x\right)\left(B-\frac{2bC}{c}\right)\right)}{6bc\left(b^2-3ac\right)\left(b\left(3a-\frac{b^2}{c}\right)+c^2\left(\frac{b}{c}+x\right)^3\right)^2}-\frac{c\left(\frac{2\int \frac{5\left(-Cb^2+Bcb-Ac^2\right)-2c(Bc-2bC)\left(\frac{b}{c}+x\right)}{c^2\left(\frac{b}{c}+x\right)^3+b\left(3a-\frac{b^2}{c}\right)}d\left(\frac{b}{c}+x\right)}{3bc(b^2-3ac)}-\frac{c\left(\frac{b}{c}+x\right)\left(5\left(A-\frac{b(Bc-bC)}{c^2}\right)+4\left(\frac{b}{c}+x\right)\left(B-\frac{2bC}{c}\right)\right)}{3b(b^2-3ac)\left(b\left(3a-\frac{b^2}{c}\right)+c^2\left(\frac{b}{c}+x\right)^3\right)}\right)}{6b\left(b^2-3ac\right)}$$

$$\Big\downarrow \text{ 2400}$$

$$\frac{bC\left(3a-\frac{b^2}{c}\right)-c^2\left(\frac{b}{c}+x\right)\left(A-\frac{b(Bc-bC)}{c^2}+\left(\frac{b}{c}+x\right)\left(B-\frac{2bC}{c}\right)\right)}{6bc\left(b^2-3ac\right)\left(b\left(3a-\frac{b^2}{c}\right)+c^2\left(\frac{b}{c}+x\right)^3\right)^2}-$$

$$c\left(\frac{2\left(\frac{\int -\frac{c\left(2\sqrt[3]{b}\sqrt[3]{b^2-3ac}\left(\sqrt[3]{b}\sqrt[3]{b^2-3ac}(Bc-2bC)+5\left(-Cb^2+Bcb-Ac^2\right)\right)-c\left(2\sqrt[3]{b}\sqrt[3]{b^2-3ac}(Bc-2bC)-5\left(-Cb^2+Bcb-Ac^2\right)\right)\left(\frac{b}{c}+\right.}{c^2\left(\frac{b}{c}+x\right)^2+\sqrt[3]{b}c\sqrt[3]{b^2-3ac}\left(\frac{b}{c}+x\right)+b^{2/3}\left(b^2-3ac\right)^{2/3}}}{3b^{2/3}\left(b^2-3ac\right)^{2/3}}\right.}{3bc(b^2-3ac)}\right.$$

$$\Big\downarrow \text{ 16}$$

$$\frac{bC\left(3a-\frac{b^2}{c}\right)-c^2\left(\frac{b}{c}+x\right)\left(A-\frac{b(Bc-bC)}{c^2}+\left(\frac{b}{c}+x\right)\left(B-\frac{2bC}{c}\right)\right)}{6bc\left(b^2-3ac\right)\left(b\left(3a-\frac{b^2}{c}\right)+c^2\left(\frac{b}{c}+x\right)^3\right)^2}-$$

$$c\left(\frac{2\left(\frac{\int -\frac{c\left(2\sqrt[3]{b}\sqrt[3]{b^2-3ac}\left(\sqrt[3]{b}\sqrt[3]{b^2-3ac}(Bc-2bC)+5\left(-Cb^2+Bcb-Ac^2\right)\right)-c\left(2\sqrt[3]{b}\sqrt[3]{b^2-3ac}(Bc-2bC)-5\left(-Cb^2+Bcb-Ac^2\right)\right)\left(\frac{b}{c}+\right.}{c^2\left(\frac{b}{c}+x\right)^2+\sqrt[3]{b}c\sqrt[3]{b^2-3ac}\left(\frac{b}{c}+x\right)+b^{2/3}\left(b^2-3ac\right)^{2/3}}}{3b^{2/3}\left(b^2-3ac\right)^{2/3}}\right.}{3bc(b^2-3ac)}\right.$$

$$\downarrow 25$$

$$\frac{bC\left(3a-\frac{b^2}{c}\right)-c^2\left(\frac{b}{c}+x\right)\left(A-\frac{b(Bc-bC)}{c^2}+\left(\frac{b}{c}+x\right)\left(B-\frac{2bC}{c}\right)\right)}{6bc\left(b^2-3ac\right)\left(b\left(3a-\frac{b^2}{c}\right)+c^2\left(\frac{b}{c}+x\right)^3\right)^2}-$$

$$c\left(\frac{2\left(\frac{\log\left(\sqrt[3]{b}\sqrt[3]{b^2-3ac}-c\left(\frac{b}{c}+x\right)\right)\left(-2\sqrt[3]{b}Bc\sqrt[3]{b^2-3ac}+4b^{4/3}C\sqrt[3]{b^2-3ac}-5Ac^2-5b^2C+5bBc\right)}{3b^{2/3}\left(b^2-3ac\right)^{2/3}}-\frac{\int \frac{c\left(2\sqrt[3]{b}\sqrt[3]{b^2-3ac}\left(\sqrt[3]{b}\sqrt[3]{\right.\right.}{}}{}\right.}{3bc(b^2-3ac)}\right.$$

$$\downarrow 27$$

$$\frac{bC\left(3a-\frac{b^2}{c}\right)-c^2\left(\frac{b}{c}+x\right)\left(A-\frac{b(Bc-bC)}{c^2}+\left(\frac{b}{c}+x\right)\left(B-\frac{2bC}{c}\right)\right)}{6bc\left(b^2-3ac\right)\left(b\left(3a-\frac{b^2}{c}\right)+c^2\left(\frac{b}{c}+x\right)^3\right)^2}-$$

$$c\left(\frac{2\left(\frac{\log\left(\sqrt[3]{b}\sqrt[3]{b^2-3ac}-c\left(\frac{b}{c}+x\right)\right)\left(-2\sqrt[3]{b}Bc\sqrt[3]{b^2-3ac}+4b^{4/3}C\sqrt[3]{b^2-3ac}-5Ac^2-5b^2C+5bBc\right)}{3b^{2/3}\left(b^2-3ac\right)^{2/3}}-\frac{c\int\frac{2\sqrt[3]{b}\sqrt[3]{b^2-3ac}\left(\sqrt[3]{b}\sqrt[3]{\cdots}\right.}{\cdots}}{\cdots}\right.}{3bc\left(b^2-3ac\right)\cdots}\right.$$

$\downarrow$ 1142

$$\frac{bC\left(3a-\frac{b^2}{c}\right)-c^2\left(\frac{b}{c}+x\right)\left(A-\frac{b(Bc-bC)}{c^2}+\left(\frac{b}{c}+x\right)\left(B-\frac{2bC}{c}\right)\right)}{6bc\left(b^2-3ac\right)\left(b\left(3a-\frac{b^2}{c}\right)+c^2\left(\frac{b}{c}+x\right)^3\right)^2}-$$

$$c\left(\frac{2\left(\frac{\log\left(\sqrt[3]{b}\sqrt[3]{b^2-3ac}-c\left(\frac{b}{c}+x\right)\right)\left(-2\sqrt[3]{b}Bc\sqrt[3]{b^2-3ac}+4b^{4/3}C\sqrt[3]{b^2-3ac}-5Ac^2-5b^2C+5bBc\right)}{3b^{2/3}\left(b^2-3ac\right)^{2/3}}-\frac{c\left(\frac{3}{2}\sqrt[3]{b}\sqrt[3]{b^2-3ac}\left(2\sqrt[3]{b}Bc\cdots\right.\right.}{\cdots}\right.}{\cdots}\right.$$

$$\Big\downarrow 27$$

$$\frac{bC\left(3a-\frac{b^2}{c}\right)-c^2\left(\frac{b}{c}+x\right)\left(A-\frac{b(Bc-bC)}{c^2}+\left(\frac{b}{c}+x\right)\left(B-\frac{2bC}{c}\right)\right)}{6bc\left(b^2-3ac\right)\left(b\left(3a-\frac{b^2}{c}\right)+c^2\left(\frac{b}{c}+x\right)^3\right)^2}-$$

$$c\left(\frac{2\left(\frac{\log\left(\sqrt[3]{b}\sqrt[3]{b^2-3ac}-c\left(\frac{b}{c}+x\right)\right)\left(-2\sqrt[3]{b}Bc\sqrt[3]{b^2-3ac}+4b^{4/3}C\sqrt[3]{b^2-3ac}-5Ac^2-5b^2C+5bBc\right)}{3b^{2/3}\left(b^2-3ac\right)^{2/3}}-\frac{c\left(\frac{3}{2}\sqrt[3]{b}\sqrt[3]{b^2-3ac}\left(2\sqrt[3]{b}Bc\right.\right.}{}\right.}{}\right.$$

$$\Big\downarrow 1082$$

$$\frac{bC\left(3a-\frac{b^2}{c}\right)-c^2\left(\frac{b}{c}+x\right)\left(A-\frac{b(Bc-bC)}{c^2}+\left(\frac{b}{c}+x\right)\left(B-\frac{2bC}{c}\right)\right)}{6bc\left(b^2-3ac\right)\left(b\left(3a-\frac{b^2}{c}\right)+c^2\left(\frac{b}{c}+x\right)^3\right)^2}-$$

$$c\left(\frac{2\left(\frac{\log\left(\sqrt[3]{b}\sqrt[3]{b^2-3ac}-c\left(\frac{b}{c}+x\right)\right)\left(-2\sqrt[3]{b}Bc\sqrt[3]{b^2-3ac}+4b^{4/3}C\sqrt[3]{b^2-3ac}-5Ac^2-5b^2C+5bBc\right)}{3b^{2/3}\left(b^2-3ac\right)^{2/3}}-\frac{c\left(-\frac{1}{2}\left(2\sqrt[3]{b}\sqrt[3]{b^2-3ac}(Bc-\right.\right.}{}\right.}{}\right.$$

↓ 217

$$\frac{bC\left(3a-\frac{b^2}{c}\right)-c^2\left(\frac{b}{c}+x\right)\left(A-\frac{b(Bc-bC)}{c^2}+\left(\frac{b}{c}+x\right)\left(B-\frac{2bC}{c}\right)\right)}{6bc\left(b^2-3ac\right)\left(b\left(3a-\frac{b^2}{c}\right)+c^2\left(\frac{b}{c}+x\right)^3\right)^2}-$$

$$c\left(\frac{2\left(\frac{\log\left(\sqrt[3]{b}\sqrt[3]{b^2-3ac}-c\left(\frac{b}{c}+x\right)\right)\left(-2\sqrt[3]{b}Bc\sqrt[3]{b^2-3ac}+4b^{4/3}C\sqrt[3]{b^2-3ac}-5Ac^2-5b^2C+5bBc\right)}{3b^{2/3}\left(b^2-3ac\right)^{2/3}}-\frac{c\left(\sqrt{3}\arctan\left(\frac{\frac{2c\left(\frac{b}{c}+x\right)}{\sqrt[3]{b}\sqrt[3]{b^2-3ac}}}{\sqrt{3}}\right.\right.}{}\right.}{}\right.$$

↓ 1103

$$\frac{bC\left(3a-\frac{b^2}{c}\right)-c^2\left(\frac{b}{c}+x\right)\left(A-\frac{b(Bc-bC)}{c^2}+\left(\frac{b}{c}+x\right)\left(B-\frac{2bC}{c}\right)\right)}{6bc\left(b^2-3ac\right)\left(b\left(3a-\frac{b^2}{c}\right)+c^2\left(\frac{b}{c}+x\right)^3\right)^2}-$$

$$c\left(2\left(\frac{\log\left(\sqrt[3]{b}\sqrt[3]{b^2-3ac}-c\left(\frac{b}{c}+x\right)\right)\left(-2\sqrt[3]{b}Bc\sqrt[3]{b^2-3ac}+4b^{4/3}C\sqrt[3]{b^2-3ac}-5Ac^2-5b^2C+5bBc\right)}{3b^{2/3}\left(b^2-3ac\right)^{2/3}}-\frac{c\left(\frac{\sqrt{3}\arctan\left(\frac{\frac{2c\left(\frac{b}{c}+x\right)}{\sqrt[3]{b}\sqrt[3]{b^2-3ac}}}{\sqrt{3}}\right.}{}\right.}{}\right.\right.$$

input
```
Int[(A + B*x + C*x^2)/(3*a*b + 3*b^2*x + 3*b*c*x^2 + c^2*x^3)^3,x]
```

output

```
(b*(3*a - b^2/c)*C - c^2*(b/c + x)*(A - (b*(B*c - b*C))/c^2 + (B - (2*b*C)
/c)*(b/c + x)))/(6*b*c*(b^2 - 3*a*c)*(b*(3*a - b^2/c) + c^2*(b/c + x)^3)^2
) - (c*(-1/3*(c*(b/c + x)*(5*(A - (b*(B*c - b*C))/c^2) + 4*(B - (2*b*C)/c)
*(b/c + x)))/(b*(b^2 - 3*a*c)*(b*(3*a - b^2/c) + c^2*(b/c + x)^3)) + (2*((
(5*b*B*c - 5*A*c^2 - 2*b^(1/3)*B*c*(b^2 - 3*a*c)^(1/3) - 5*b^2*C + 4*b^(4/
3)*(b^2 - 3*a*c)^(1/3)*C)*Log[b^(1/3)*(b^2 - 3*a*c)^(1/3) - c*(b/c + x)])/
(3*b^(2/3)*(b^2 - 3*a*c)^(2/3)) - (c*((Sqrt[3]*(5*b*B*c - 5*A*c^2 + 2*b^(1
/3)*B*c*(b^2 - 3*a*c)^(1/3) - 5*b^2*C - 4*b^(4/3)*(b^2 - 3*a*c)^(1/3)*C)*A
rcTan[(1 + (2*c*(b/c + x))/(b^(1/3)*(b^2 - 3*a*c)^(1/3)))/Sqrt[3]])/c - ((
2*b^(1/3)*(b^2 - 3*a*c)^(1/3)*(B*c - 2*b*C) - 5*(b*B*c - A*c^2 - b^2*C))*L
og[b^(2/3)*(b^2 - 3*a*c)^(2/3) + b^(1/3)*c*(b^2 - 3*a*c)^(1/3)*(b/c + x) +
 c^2*(b/c + x)^2])/(2*c)))/(3*b^(2/3)*(b^2 - 3*a*c)^(2/3))))/(3*b*c*(b^2 -
 3*a*c))))/(6*b*(b^2 - 3*a*c))
```

Defintions of rubi rules used

rule 16

```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2393
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   In
t[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(
p + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n
, 0] && LtQ[p, -1]
```

rule 2394
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   Int[ExpandToSum[n
*(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x
] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

rule 2400
```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[-a/b, 3]], s = Denominator[Rt[-a/b, 3]]}, Simp[r*((B*r + A*s)/(3*a*
s))   Int[1/(r - s*x), x], x] - Simp[r/(3*a*s)   Int[(r*(B*r - 2*A*s) - s*(
B*r + A*s)*x)/(r^2 + r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &&
 NeQ[a*B^3 - b*A^3, 0] && NegQ[a/b]
```

rule 2459
```
Int[(Pn_)^(p_.)*(Qx_), x_Symbol] :> With[{S = Coeff[Pn, x, Expon[Pn, x] - 1
]/(Expon[Pn, x]*Coeff[Pn, x, Expon[Pn, x]])}, Subst[Int[ExpandToSum[Pn /. x
 -> x - S, x]^p*ExpandToSum[Qx /. x -> x - S, x], x], x, x + S] /; Binomial
Q[Pn /. x -> x - S, x] || (IntegerQ[Expon[Pn, x]/2] && TrinomialQ[Pn /. x -
> x - S, x])] /; FreeQ[p, x] && PolyQ[Pn, x] && GtQ[Expon[Pn, x], 2] && NeQ
[Coeff[Pn, x, Expon[Pn, x] - 1], 0] && PolyQ[Qx, x] &&  !(MonomialQ[Qx, x]
&& IGtQ[p, 0])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.18 (sec) , antiderivative size = 446, normalized size of antiderivative = 0.79

method	result
default	$\frac{\frac{2c^3(Bc-2bC)x^5}{9b^2(9a^2c^2-6a\,b^2c+b^4)}+\frac{5c^2(A\,c^2+3bBc-7b^2C)x^4}{18b^2(9a^2c^2-6a\,b^2c+b^4)}+\frac{10c(A\,c^2+bBc-3b^2C)x^3}{9b(9a^2c^2-6a\,b^2c+b^4)}+\frac{(10c^2bA+7B\,c^2a+B\,b^2c-14Cabc-12C\,b^3)x^2}{6b(9a^2c^2-6a\,b^2c+b^4)}+\frac{(4Aa\,c^2+2}{3}}{(c^2x^3+3bc\,x^2+3b^2x+3ab)^2}$
risch	$\frac{\frac{2c^3(Bc-2bC)x^5}{9b^2(9a^2c^2-6a\,b^2c+b^4)}+\frac{5c^2(A\,c^2+3bBc-7b^2C)x^4}{18b^2(9a^2c^2-6a\,b^2c+b^4)}+\frac{10c(A\,c^2+bBc-3b^2C)x^3}{9b(9a^2c^2-6a\,b^2c+b^4)}+\frac{(10c^2bA+7B\,c^2a+B\,b^2c-14Cabc-12C\,b^3)x^2}{6b(9a^2c^2-6a\,b^2c+b^4)}+\frac{(4Aa\,c^2+2}{3}}{(c^2x^3+3bc\,x^2+3b^2x+3ab)^2}$

input
```
int((C*x^2+B*x+A)/(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^3,x,method=_RETURNVERB
OSE)
```

output
```
9*(2/81*c^3*(B*c-2*C*b)/b^2/(9*a^2*c^2-6*a*b^2*c+b^4)*x^5+5/162*c^2*(A*c^2
+3*B*b*c-7*C*b^2)/b^2/(9*a^2*c^2-6*a*b^2*c+b^4)*x^4+10/81*c*(A*c^2+B*b*c-3
*C*b^2)/b/(9*a^2*c^2-6*a*b^2*c+b^4)*x^3+1/54*(10*A*b*c^2+7*B*a*c^2+B*b^2*c
-14*C*a*b*c-12*C*b^3)/b/(9*a^2*c^2-6*a*b^2*c+b^4)*x^2+1/27/b*(4*A*a*c^2+2*
A*b^2*c+3*B*a*b*c-B*b^3-10*C*a*b^2)/(9*a^2*c^2-6*a*b^2*c+b^4)*x+1/54*(8*A*
a*c-A*b^2-B*a*b-9*C*a^2)/(9*a^2*c^2-6*a*b^2*c+b^4))/(c^2*x^3+3*b*c*x^2+3*b
^2*x+3*a*b)^2+1/27/b^2/(9*a^2*c^2-6*a*b^2*c+b^4)*sum((2*B*_R*c^2-4*C*_R*b*
c+5*A*c^2-3*B*b*c+C*b^2)/(_R^2*c^2+2*_R*b*c+b^2)*ln(x-_R),_R=RootOf(_Z^3*c
^2+3*_Z^2*b*c+3*_Z*b^2+3*a*b))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 5.46 (sec) , antiderivative size = 18983, normalized size of antiderivative = 33.66

$$\int \frac{A+Bx+Cx^2}{(3ab+3b^2x+3bcx^2+c^2x^3)^3}\,dx = \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)/(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^3,x, algorithm="
fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(3ab+3b^2x+3bcx^2+c^2x^3)^3}\,dx = \text{Timed out}$$

input
```
integrate((C*x**2+B*x+A)/(c**2*x**3+3*b*c*x**2+3*b**2*x+3*a*b)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{(3ab+3b^2x+3bcx^2+c^2x^3)^3}\,dx = \int \frac{Cx^2+Bx+A}{(c^2x^3+3\,bcx^2+3\,b^2x+3\,ab)^3}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^3,x, algorithm="
maxima")
```

output
```
-1/18*(4*(2*C*b*c^3 - B*c^4)*x^5 + 27*C*a^2*b^2 + 3*B*a*b^3 + 3*A*b^4 - 24
*A*a*b^2*c + 5*(7*C*b^2*c^2 - 3*B*b*c^3 - A*c^4)*x^4 + 20*(3*C*b^3*c - B*b
^2*c^2 - A*b*c^3)*x^3 + 3*(12*C*b^4 - (7*B*a*b + 10*A*b^2)*c^2 + (14*C*a*b
^2 - B*b^3)*c)*x^2 + 6*(10*C*a*b^3 + B*b^4 - 4*A*a*b*c^2 - (3*B*a*b^2 + 2*
A*b^3)*c)*x)/(9*a^2*b^8 - 54*a^3*b^6*c + 81*a^4*b^4*c^2 + (b^6*c^4 - 6*a*b
^4*c^5 + 9*a^2*b^2*c^6)*x^6 + 6*(b^7*c^3 - 6*a*b^5*c^4 + 9*a^2*b^3*c^5)*x^
5 + 15*(b^8*c^2 - 6*a*b^6*c^3 + 9*a^2*b^4*c^4)*x^4 + 6*(3*b^9*c - 17*a*b^7
*c^2 + 21*a^2*b^5*c^3 + 9*a^3*b^3*c^4)*x^3 + 9*(b^10 - 4*a*b^8*c - 3*a^2*b
^6*c^2 + 18*a^3*b^4*c^3)*x^2 + 18*(a*b^9 - 6*a^2*b^7*c + 9*a^3*b^5*c^2)*x)
 + 1/9*integrate((C*b^2 - 3*B*b*c + 5*A*c^2 - 2*(2*C*b*c - B*c^2)*x)/(c^2*
x^3 + 3*b*c*x^2 + 3*b^2*x + 3*a*b), x)/(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2)
```

Giac [F]

$$\int \frac{A+Bx+Cx^2}{(3ab+3b^2x+3bcx^2+c^2x^3)^3}\,dx = \int \frac{Cx^2+Bx+A}{(c^2x^3+3\,bcx^2+3\,b^2x+3\,ab)^3}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^3,x, algorithm="
giac")
```

output
```
integrate((C*x^2 + B*x + A)/(c^2*x^3 + 3*b*c*x^2 + 3*b^2*x + 3*a*b)^3, x)
```

Mupad [B] (verification not implemented)

Time = 13.00 (sec) , antiderivative size = 2559, normalized size of antiderivative = 4.54

$$\int \frac{A+Bx+Cx^2}{(3ab+3b^2x+3bcx^2+c^2x^3)^3}\,dx = \text{Too large to display}$$

input
```
int((A + B*x + C*x^2)/(3*a*b + 3*b^2*x + c^2*x^3 + 3*b*c*x^2)^3,x)
```

output
```
symsum(log(root(472392*a*b^22*c*z^3 - 401769396*a^6*b^12*c^6*z^3 + 3443737
68*a^7*b^10*c^7*z^3 + 267846264*a^5*b^14*c^5*z^3 - 129140163*a^8*b^8*c^8*z
^3 - 111602610*a^4*b^16*c^4*z^3 + 29760696*a^3*b^18*c^3*z^3 - 4960116*a^2*
b^20*c^2*z^3 - 19683*b^24*z^3 - 65610*B*C*a^3*b^5*c^4*z + 65610*B*C*a^2*b^
7*c^3*z + 43740*A*C*a^3*b^4*c^5*z - 43740*A*C*a^2*b^6*c^4*z - 21870*A*B*a^
3*b^3*c^6*z + 21870*A*B*a^2*b^5*c^5*z - 21870*B*C*a*b^9*c^2*z + 14580*A*C*
a*b^8*c^3*z - 7290*A*B*a*b^7*c^4*z - 1620*A*C*b^10*c^2*z + 810*A*B*b^9*c^3
*z + 43740*C^2*a^3*b^6*c^3*z - 43740*C^2*a^2*b^8*c^2*z + 21870*B^2*a^3*b^4
*c^5*z - 21870*B^2*a^2*b^6*c^4*z + 2430*B*C*b^11*c*z + 7290*B^2*a*b^8*c^3*
z + 14580*C^2*a*b^10*c*z - 810*B^2*b^10*c^2*z - 1620*C^2*b^12*z - 750*A*B*
C*b^3*c^3 - 288*B*C^2*a*b^3*c^2 + 144*B^2*C*a*b^2*c^3 + 327*B^2*C*b^4*c^2
+ 375*A^2*C*b^2*c^4 + 375*A*C^2*b^4*c^2 + 375*A*B^2*b^2*c^4 + 192*C^3*a*b^
4*c - 24*B^3*a*b*c^4 - 279*B*C^2*b^5*c - 375*A^2*B*b*c^5 - 117*B^3*b^3*c^3
 + 61*C^3*b^6 + 125*A^3*c^6, z, k)*((135*A*b^7*c^5 - 135*B*b^8*c^4 + 135*C
*b^9*c^3 - 810*A*a*b^5*c^6 + 810*B*a*b^6*c^5 - 810*C*a*b^7*c^4 + 1215*A*a^
2*b^3*c^7 - 1215*B*a^2*b^4*c^6 + 1215*C*a^2*b^5*c^5)/(81*(b^12 + 54*a^2*b^
8*c^2 - 108*a^3*b^6*c^3 + 81*a^4*b^4*c^4 - 12*a*b^10*c)) - (root(472392*a*
b^22*c*z^3 - 401769396*a^6*b^12*c^6*z^3 + 344373768*a^7*b^10*c^7*z^3 + 267
846264*a^5*b^14*c^5*z^3 - 129140163*a^8*b^8*c^8*z^3 - 111602610*a^4*b^16*c
^4*z^3 + 29760696*a^3*b^18*c^3*z^3 - 4960116*a^2*b^20*c^2*z^3 - 19683*b...
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 5412, normalized size of antiderivative = 9.60

$$\int \frac{A + Bx + Cx^2}{(3ab + 3b^2x + 3bcx^2 + c^2x^3)^3} \, dx = \text{Too large to display}$$

input
```
int((C*x^2+B*x+A)/(c^2*x^3+3*b*c*x^2+3*b^2*x+3*a*b)^3,x)
```

output
```
( - 90*b**(2/3)*(3*a*c - b**2)**(2/3)*sqrt(3)*atan((b**(1/3)*(3*a*c - b**2
)**(1/3) - 2*b - 2*c*x)/(b**(1/3)*(3*a*c - b**2)**(1/3)*sqrt(3)))*a**3*b**
2*c**2 - 180*b**(2/3)*(3*a*c - b**2)**(2/3)*sqrt(3)*atan((b**(1/3)*(3*a*c
- b**2)**(1/3) - 2*b - 2*c*x)/(b**(1/3)*(3*a*c - b**2)**(1/3)*sqrt(3)))*a*
*2*b**3*c**2*x - 180*b**(2/3)*(3*a*c - b**2)**(2/3)*sqrt(3)*atan((b**(1/3)
*(3*a*c - b**2)**(1/3) - 2*b - 2*c*x)/(b**(1/3)*(3*a*c - b**2)**(1/3)*sqrt
(3)))*a**2*b**2*c**3*x**2 - 60*b**(2/3)*(3*a*c - b**2)**(2/3)*sqrt(3)*atan
((b**(1/3)*(3*a*c - b**2)**(1/3) - 2*b - 2*c*x)/(b**(1/3)*(3*a*c - b**2)**
(1/3)*sqrt(3)))*a**2*b*c**4*x**3 - 90*b**(2/3)*(3*a*c - b**2)**(2/3)*sqrt(
3)*atan((b**(1/3)*(3*a*c - b**2)**(1/3) - 2*b - 2*c*x)/(b**(1/3)*(3*a*c -
b**2)**(1/3)*sqrt(3)))*a*b**4*c**2*x**2 - 180*b**(2/3)*(3*a*c - b**2)**(2/
3)*sqrt(3)*atan((b**(1/3)*(3*a*c - b**2)**(1/3) - 2*b - 2*c*x)/(b**(1/3)*(
3*a*c - b**2)**(1/3)*sqrt(3)))*a*b**3*c**3*x**3 - 150*b**(2/3)*(3*a*c - b*
*2)**(2/3)*sqrt(3)*atan((b**(1/3)*(3*a*c - b**2)**(1/3) - 2*b - 2*c*x)/(b*
*(1/3)*(3*a*c - b**2)**(1/3)*sqrt(3)))*a*b**2*c**4*x**4 - 60*b**(2/3)*(3*a
*c - b**2)**(2/3)*sqrt(3)*atan((b**(1/3)*(3*a*c - b**2)**(1/3) - 2*b - 2*c
*x)/(b**(1/3)*(3*a*c - b**2)**(1/3)*sqrt(3)))*a*b*c**5*x**5 - 10*b**(2/3)*
(3*a*c - b**2)**(2/3)*sqrt(3)*atan((b**(1/3)*(3*a*c - b**2)**(1/3) - 2*b -
 2*c*x)/(b**(1/3)*(3*a*c - b**2)**(1/3)*sqrt(3)))*a*c**6*x**6 + 108*sqrt(3
)*atan((b**(1/3)*(3*a*c - b**2)**(1/3) - 2*b - 2*c*x)/(b**(1/3)*(3*a*c ...
```

3.74 $\int (A+Bx+Cx^2)\left(-64+b^3+3b^2cx+3bc^2x^2+c^3x$

Optimal result

Integrand size = 43, antiderivative size = 577

$$\int (A+Bx+Cx^2)\left(-64+b^3+3b^2cx+3bc^2x^2+c^3x^3\right)^{3/2}\,dx =$$

$$-\frac{221184(Bc-2bC)\sqrt{-64+(b+cx)^3}}{91c^3\left(4-4\sqrt{3}-b-cx\right)}$$

$$+\frac{1152\left(\frac{91(bBc-Ac^2-b^2C)(b+cx)}{c}-\frac{55(Bc-2bC)(b+cx)^2}{c}\right)\sqrt{-64+(b+cx)^3}}{5005c^2}$$

$$-\frac{2\left(\frac{13(bBc-Ac^2-b^2C)(b+cx)}{c}-\frac{11(Bc-2bC)(b+cx)^2}{c}\right)\left(-64+(b+cx)^3\right)^{3/2}}{143c^2}$$

$$+\frac{2C\left(-64+(b+cx)^3\right)^{5/2}}{15c^3}$$

$$+\frac{221184\sqrt[4]{3}\sqrt{2+\sqrt{3}}(Bc-2bC)(4-b-cx)\sqrt{\frac{16+4(b+cx)+(b+cx)^2}{\left(4-4\sqrt{3}-b-cx\right)^2}}E\left(\arcsin\left(\frac{4+4\sqrt{3}-b-cx}{4-4\sqrt{3}-b-cx}\right)|-7+4\sqrt{3}\right)}{91c^3\sqrt{-\frac{4-b-cx}{\left(4-4\sqrt{3}-b-cx\right)^2}}\sqrt{-64+(b+cx)^3}}$$

$$-\frac{36864\,3^{3/4}\sqrt{2-\sqrt{3}}\left(220\left(1+\sqrt{3}\right)(Bc-2bC)-91(bBc-Ac^2-b^2C)\right)(4-b-cx)\sqrt{\frac{16+4(b+cx)+(b+cx)^2}{\left(4-4\sqrt{3}-b-cx\right)^2}}}{5005c^3\sqrt{-\frac{4-b-cx}{\left(4-4\sqrt{3}-b-cx\right)^2}}\sqrt{-64+(b+cx)^3}}$$

output
```
-221184/91*(B*c-2*C*b)*(-64+(c*x+b)^3)^(1/2)/c^3/(4-4*3^(1/2)-b-c*x)+1152/
5005*(91*(-A*c^2+B*b*c-C*b^2)*(c*x+b)/c-55*(B*c-2*C*b)*(c*x+b)^2/c)*(-64+(
c*x+b)^3)^(1/2)/c^2-2/143*(13*(-A*c^2+B*b*c-C*b^2)*(c*x+b)/c-11*(B*c-2*C*b
)*(c*x+b)^2/c)*(-64+(c*x+b)^3)^(3/2)/c^2+2/15*C*(-64+(c*x+b)^3)^(5/2)/c^3+
221184/91*3^(1/4)*(1/2*6^(1/2)+1/2*2^(1/2))*(B*c-2*C*b)*(-c*x-b+4)*((16+4*
c*x+4*b+(c*x+b)^2)/(4-4*3^(1/2)-b-c*x)^2)^(1/2)*EllipticE((4+4*3^(1/2)-b-c
*x)/(4-4*3^(1/2)-b-c*x),2*I-I*3^(1/2))/c^3/(-(-c*x-b+4)/(4-4*3^(1/2)-b-c*x
)^2)^(1/2)/(-64+(c*x+b)^3)^(1/2)-36864/5005*3^(3/4)*(1/2*6^(1/2)-1/2*2^(1/
2))*(220*(1+3^(1/2))*(B*c-2*C*b)+91*A*c^2-91*B*b*c+91*C*b^2)*(-c*x-b+4)*((
16+4*c*x+4*b+(c*x+b)^2)/(4-4*3^(1/2)-b-c*x)^2)^(1/2)*EllipticF((4+4*3^(1/2
)-b-c*x)/(4-4*3^(1/2)-b-c*x),2*I-I*3^(1/2))/c^3/(-(-c*x-b+4)/(4-4*3^(1/2)-
b-c*x)^2)^(1/2)/(-64+(c*x+b)^3)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 11.46 (sec) , antiderivative size = 504, normalized size of antiderivative = 0.87

$$\int (A+Bx+Cx^2)\left(-64+b^3+3b^2cx+3bc^2x^2+c^3x^3\right)^{3/2}dx = \frac{2\left(-\left((64-b^3-3b^2cx-3bc^2x^2-c^3x^3)\left(56b^6C-42b^5c(5B+2Cx)+1001C(-64+c^3x^3\right.\right.\right.}{}$$

input
```
Integrate[(A + B*x + C*x^2)*(-64 + b^3 + 3*b^2*c*x + 3*b*c^2*x^2 + c^3*x^3
)^(3/2),x]
```

output
```
(2*(-((64 - b^3 - 3*b^2*c*x - 3*b*c^2*x^2 - c^3*x^3)*(56*b^6*C - 42*b^5*c*
(5*B + 2*C*x) + 1001*C*(-64 + c^3*x^3)^2 + 105*b^4*c^2*(13*A + x*(3*B + C*
x)) + 3*c^3*x*(91*A*(-896 + 5*c^3*x^3) + 55*B*x*(-1024 + 7*c^3*x^3)) + 4*b
^3*(105*c^3*x*(13*A + 8*B*x) + 17*C*(-512 + 35*c^3*x^3)) + 6*b^2*c*(B*(126
08 + 1015*c^3*x^3) + x*(7808*C + 1365*A*c^3*x + 805*c^3*C*x^3)) + 6*b*c^2*
(182*A*(-224 + 5*c^3*x^3) + x*(88*C*x*(-88 + 7*c^3*x^3) + 3*B*(-5184 + 245
*c^3*x^3))))) - 165888*Sqrt[2]*Sqrt[((-I)*(-4 + b + c*x))/(3*I + Sqrt[3])]
*Sqrt[16 + b^2 + 4*c*x + c^2*x^2 + 2*b*(2 + c*x)]*(-110*(3*I + Sqrt[3])*(B
*c - 2*b*C)*EllipticE[ArcSin[Sqrt[2*I + 2*Sqrt[3] + I*b + I*c*x]/(2*3^(1/4
))], (2*Sqrt[3])/(3*I + Sqrt[3])] + I*((220 - 91*b)*B*c + 91*A*c^2 + b*(-4
40 + 91*b)*C)*EllipticF[ArcSin[Sqrt[2*I + 2*Sqrt[3] + I*b + I*c*x]/(2*3^(1
/4))], (2*Sqrt[3])/(3*I + Sqrt[3])])))/(15015*c^3*Sqrt[-64 + b^3 + 3*b^2*c
*x + 3*b*c^2*x^2 + c^3*x^3])
```

Rubi [A] (warning: unable to verify)

Time = 1.96 (sec) , antiderivative size = 710, normalized size of antiderivative = 1.23, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {2459, 2392, 27, 2392, 27, 2425, 793, 2419, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(b^3+3b^2cx+3bc^2x^2+c^3x^3-64\right)^{3/2}\left(A+Bx+Cx^2\right)dx$$

$$\Big\downarrow \text{2459}$$

$$\int \left(c^3\left(\frac{b}{c}+x\right)^3-64\right)^{3/2}\left(A-\frac{b(Bc-bC)}{c^2}+\left(\frac{b}{c}+x\right)\left(B-\frac{2bC}{c}\right)+C\left(\frac{b}{c}+x\right)^2\right)d\left(\frac{b}{c}+x\right)$$

$$\Big\downarrow \text{2392}$$

$$\frac{2\left(c^3\left(\frac{b}{c}+x\right)^3-64\right)^{3/2}\left(195\left(\frac{b}{c}+x\right)\left(A-\frac{b(Bc-bC)}{c^2}\right)+165\left(\frac{b}{c}+x\right)^2\left(B-\frac{2bC}{c}\right)+143C\left(\frac{b}{c}+x\right)^3\right)}{2145}-288\int \frac{2\left(143C\left(\frac{b}{c}+x\right)^2+165\left(B-\frac{2bC}{c}\right)\left(\frac{b}{c}+x\right)+195\left(A-\frac{b(Bc-bC)}{c^2}\right)\right)\sqrt{c^3\left(\frac{b}{c}+x\right)^3-64}}{2145}d\left(\frac{b}{c}+x\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{2\left(c^3\left(\frac{b}{c}+x\right)^3-64\right)^{3/2}\left(195\left(\frac{b}{c}+x\right)\left(A-\frac{b(Bc-bC)}{c^2}\right)+165\left(\frac{b}{c}+x\right)^2\left(B-\frac{2bC}{c}\right)+143C\left(\frac{b}{c}+x\right)^3\right)}{2145}-$$
$$\frac{192}{715}\int\left(143C\left(\frac{b}{c}+x\right)^2+165\left(B-\frac{2bC}{c}\right)\left(\frac{b}{c}+x\right)+195\left(A-\frac{b(Bc-bC)}{c^2}\right)\right)\sqrt{c^3\left(\frac{b}{c}+x\right)^3-64}d\left(\frac{b}{c}+x\right.$$

$$\Big\downarrow \text{2392}$$

$$\frac{2\left(c^3\left(\frac{b}{c}+x\right)^3-64\right)^{3/2}\left(195\left(\frac{b}{c}+x\right)\left(A-\frac{b(Bc-bC)}{c^2}\right)+165\left(\frac{b}{c}+x\right)^2\left(B-\frac{2bC}{c}\right)+143C\left(\frac{b}{c}+x\right)^3\right)}{2145}-$$
$$\frac{192}{715}\left(\frac{2}{63}\sqrt{c^3\left(\frac{b}{c}+x\right)^3-64}\left(2457\left(\frac{b}{c}+x\right)\left(A-\frac{b(Bc-bC)}{c^2}\right)+1485\left(\frac{b}{c}+x\right)^2\left(B-\frac{2bC}{c}\right)+1001C\left(\frac{b}{c}+\right.\right.\right.$$

$$\Big\downarrow \text{27}$$

$$\frac{2\left(c^3\left(\frac{b}{c}+x\right)^3-64\right)^{3/2}\left(195\left(\frac{b}{c}+x\right)\left(A-\frac{b(Bc-bC)}{c^2}\right)+165\left(\frac{b}{c}+x\right)^2\left(B-\frac{2bC}{c}\right)+143C\left(\frac{b}{c}+x\right)^3\right)}{2145}-$$
$$\frac{192}{715}\left(\frac{2}{63}\sqrt{c^3\left(\frac{b}{c}+x\right)^3-64}\left(2457\left(\frac{b}{c}+x\right)\left(A-\frac{b(Bc-bC)}{c^2}\right)+1485\left(\frac{b}{c}+x\right)^2\left(B-\frac{2bC}{c}\right)+1001C\left(\frac{b}{c}+\right.\right.\right.$$

$$\Big\downarrow \text{2425}$$

$$\frac{2\left(c^3\left(\frac{b}{c}+x\right)^3-64\right)^{3/2}\left(195\left(\frac{b}{c}+x\right)\left(A-\frac{b(Bc-bC)}{c^2}\right)+165\left(\frac{b}{c}+x\right)^2\left(B-\frac{2bC}{c}\right)+143C\left(\frac{b}{c}+x\right)^3\right)}{2145}-$$
$$\frac{192}{715}\left(\frac{2}{63}\sqrt{c^3\left(\frac{b}{c}+x\right)^3-64}\left(2457\left(\frac{b}{c}+x\right)\left(A-\frac{b(Bc-bC)}{c^2}\right)+1485\left(\frac{b}{c}+x\right)^2\left(B-\frac{2bC}{c}\right)+1001C\left(\frac{b}{c}+\right.\right.\right.$$

$$\Big\downarrow \text{793}$$

$$\frac{2\left(c^3\left(\frac{b}{c}+x\right)^3-64\right)^{3/2}\left(195\left(\frac{b}{c}+x\right)\left(A-\frac{b(Bc-bC)}{c^2}\right)+165\left(\frac{b}{c}+x\right)^2\left(B-\frac{2bC}{c}\right)+143C\left(\frac{b}{c}+x\right)^3\right)}{2145}-$$
$$\frac{192}{715}\left(\frac{2}{63}\sqrt{c^3\left(\frac{b}{c}+x\right)^3-64}\left(2457\left(\frac{b}{c}+x\right)\left(A-\frac{b(Bc-bC)}{c^2}\right)+1485\left(\frac{b}{c}+x\right)^2\left(B-\frac{2bC}{c}\right)+1001C\left(\frac{b}{c}+\right.\right.\right.$$

$$\Big\downarrow \text{2419}$$

$$\frac{2\left(c^3\left(\frac{b}{c}+x\right)^3-64\right)^{3/2}\left(195\left(\frac{b}{c}+x\right)\left(A-\frac{b(Bc-bC)}{c^2}\right)+165\left(\frac{b}{c}+x\right)^2\left(B-\frac{2bC}{c}\right)+143C\left(\frac{b}{c}+x\right)^3\right)}{2145}-$$

$$\frac{192}{715}\left(\frac{2}{63}\sqrt{c^3\left(\frac{b}{c}+x\right)^3-64}\left(2457\left(\frac{b}{c}+x\right)\left(A-\frac{b(Bc-bC)}{c^2}\right)+1485\left(\frac{b}{c}+x\right)^2\left(B-\frac{2bC}{c}\right)+1001C\left(\frac{b}{c}+\right.\right.\right.$$

↓ 760

$$\frac{2\left(c^3\left(\frac{b}{c}+x\right)^3-64\right)^{3/2}\left(195\left(\frac{b}{c}+x\right)\left(A-\frac{b(Bc-bC)}{c^2}\right)+165\left(\frac{b}{c}+x\right)^2\left(B-\frac{2bC}{c}\right)+143C\left(\frac{b}{c}+x\right)^3\right)}{2145}-$$

$$\frac{192}{715}\left(\frac{2}{63}\sqrt{c^3\left(\frac{b}{c}+x\right)^3-64}\left(2457\left(\frac{b}{c}+x\right)\left(A-\frac{b(Bc-bC)}{c^2}\right)+1485\left(\frac{b}{c}+x\right)^2\left(B-\frac{2bC}{c}\right)+1001C\left(\frac{b}{c}+\right.\right.\right.$$

↓ 2418

$$\frac{2\left(c^3\left(\frac{b}{c}+x\right)^3-64\right)^{3/2}\left(195\left(\frac{b}{c}+x\right)\left(A-\frac{b(Bc-bC)}{c^2}\right)+165\left(\frac{b}{c}+x\right)^2\left(B-\frac{2bC}{c}\right)+143C\left(\frac{b}{c}+x\right)^3\right)}{2145}-$$

$$\frac{192}{715}\left(\frac{2}{63}\sqrt{c^3\left(\frac{b}{c}+x\right)^3-64}\left(2457\left(\frac{b}{c}+x\right)\left(A-\frac{b(Bc-bC)}{c^2}\right)+1485\left(\frac{b}{c}+x\right)^2\left(B-\frac{2bC}{c}\right)+1001C\left(\frac{b}{c}+\right.\right.\right.$$

input
```
Int[(A + B*x + C*x^2)*(-64 + b^3 + 3*b^2*c*x + 3*b*c^2*x^2 + c^3*x^3)^(3/2
),x]
```

output
```
(2*(-64 + c^3*(b/c + x)^3)^(3/2)*(195*(A - (b*(B*c - b*C))/c^2)*(b/c + x)
+ 165*(B - (2*b*C)/c)*(b/c + x)^2 + 143*C*(b/c + x)^3))/2145 - (192*((2*Sq
rt[-64 + c^3*(b/c + x)^3]*(2457*(A - (b*(B*c - b*C))/c^2)*(b/c + x) + 1485
*(B - (2*b*C)/c)*(b/c + x)^2 + 1001*C*(b/c + x)^3))/63 - (64*((2002*C*Sqrt
[-64 + c^3*(b/c + x)^3])/(3*c^3) - (1485*(B*c - 2*b*C)*((2*Sqrt[-64 + c^3*
(b/c + x)^3])/(c*(4*(1 - Sqrt[3]) - c*(b/c + x))) - (2*3^(1/4)*Sqrt[2 + Sq
rt[3]]*(4 - c*(b/c + x))*Sqrt[(16 + 4*c*(b/c + x) + c^2*(b/c + x)^2)/(4*(1
 - Sqrt[3]) - c*(b/c + x))^2]*EllipticE[ArcSin[(4*(1 + Sqrt[3]) - c*(b/c +
 x))/(4*(1 - Sqrt[3]) - c*(b/c + x))], -7 + 4*Sqrt[3]])/(c*Sqrt[-((4 - c*(
b/c + x))/(4*(1 - Sqrt[3]) - c*(b/c + x))^2)]*Sqrt[-64 + c^3*(b/c + x)^3])
))/c^2 - (9*3^(3/4)*Sqrt[2 - Sqrt[3]]*(220*(1 + Sqrt[3])*(B - (2*b*C)/c) +
 91*c*(A - (b*(B*c - b*C))/c^2))*(4 - c*(b/c + x))*Sqrt[(16 + 4*c*(b/c + x
) + c^2*(b/c + x)^2)/(4*(1 - Sqrt[3]) - c*(b/c + x))^2]*EllipticF[ArcSin[(
4*(1 + Sqrt[3]) - c*(b/c + x))/(4*(1 - Sqrt[3]) - c*(b/c + x))], -7 + 4*Sq
rt[3]])/(c^2*Sqrt[-((4 - c*(b/c + x))/(4*(1 - Sqrt[3]) - c*(b/c + x))^2)]*
Sqrt[-64 + c^3*(b/c + x)^3])))/21))/715
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

rule 2392
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{q = Expon[Pq
, x], i}, Simp[(a + b*x^n)^p*Sum[Coeff[Pq, x, i]*(x^(i + 1)/(n*p + i + 1)),
 {i, 0, q}], x] + Simp[a*n*p   Int[(a + b*x^n)^(p - 1)*Sum[Coeff[Pq, x, i]*
(x^i/(n*p + i + 1)), {i, 0, q}], x], x]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x
] && IGtQ[(n - 1)/2, 0] && GtQ[p, 0]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

rule 2419
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(c*r - (1 + Sqrt[3])*d*s)/r
  Int[1/Sqrt[a + b*x^3], x], x] + Simp[d/r   Int[((1 + Sqrt[3])*s + r*x)/Sq
rt[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] && NeQ[b*c^3 -
2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

rule 2425
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Pq, x, n -
 1]   Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq,
x, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[P
q, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

rule 2459
```
Int[(Pn_)^(p_.)*(Qx_), x_Symbol] :> With[{S = Coeff[Pn, x, Expon[Pn, x] - 1
]/(Expon[Pn, x]*Coeff[Pn, x, Expon[Pn, x]])}, Subst[Int[ExpandToSum[Pn /. x
 -> x - S, x]^p*ExpandToSum[Qx /. x -> x - S, x], x], x, x + S] /; Binomial
Q[Pn /. x -> x - S, x] || (IntegerQ[Expon[Pn, x]/2] && TrinomialQ[Pn /. x -
> x - S, x])] /; FreeQ[p, x] && PolyQ[Pn, x] && GtQ[Expon[Pn, x], 2] && NeQ
[Coeff[Pn, x, Expon[Pn, x] - 1], 0] && PolyQ[Qx, x] &&  !(MonomialQ[Qx, x]
&& IGtQ[p, 0])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1575 vs. $2(515) = 1030$.

Time = 2.72 (sec) , antiderivative size = 1576, normalized size of antiderivative = 2.73

method	result	size
risch	Expression too large to display	1576
default	Expression too large to display	5082
elliptic	Expression too large to display	5698

input

```
int((C*x^2+B*x+A)*(c^3*x^3+3*b*c^2*x^2+3*b^2*c*x+b^3-64)^(3/2),x,method=_R
ETURNVERBOSE)
```

output

```
2/15015/c^3*(1001*C*c^6*x^6+1155*B*c^6*x^5+3696*C*b*c^5*x^5+1365*A*c^6*x^4
+4410*B*b*c^5*x^4+4830*C*b^2*c^4*x^4+5460*A*b*c^5*x^3+6090*B*b^2*c^4*x^3+2
380*C*b^3*c^3*x^3+8190*A*b^2*c^4*x^2+3360*B*b^3*c^3*x^2+105*C*b^4*c^2*x^2+
5460*A*b^3*c^3*x+315*B*b^4*c^2*x-84*C*b^5*c*x+1365*A*b^4*c^2-210*B*b^5*c+5
6*C*b^6-128128*C*c^3*x^3-168960*B*c^3*x^2-46464*C*b*c^2*x^2-244608*A*c^3*x
-93312*B*b*c^2*x+46848*C*b^2*c*x-244608*A*b*c^2+75648*B*b^2*c-34816*C*b^3+
4100096*C)*(c^3*x^3+3*b*c^2*x^2+3*b^2*c*x+b^3-64)^(1/2)+110592/5005/c^2*(1
10*c*(B*c-2*C*b)*((-b-2-2*I*3^(1/2))/c+(b-4)/c)*((x+(b-4)/c)/((-b-2-2*I*3^
(1/2))/c+(b-4)/c))^(1/2)*((x-(-b-2+2*I*3^(1/2))/c)/(-(b-4)/c-(-b-2+2*I*3^(
1/2))/c))^(1/2)*((x-(-b-2-2*I*3^(1/2))/c)/(-(b-4)/c-(-b-2-2*I*3^(1/2))/c))
^(1/2)/(c^3*x^3+3*b*c^2*x^2+3*b^2*c*x+b^3-64)^(1/2)*((-(b-4)/c-(-b-2+2*I*3
^(1/2))/c)*EllipticE(((x+(b-4)/c)/((-b-2-2*I*3^(1/2))/c+(b-4)/c))^(1/2),((
-(b-4)/c-(-b-2-2*I*3^(1/2))/c)/(-(b-4)/c-(-b-2+2*I*3^(1/2))/c))^(1/2))+(-b
-2+2*I*3^(1/2))/c*EllipticF(((x+(b-4)/c)/((-b-2-2*I*3^(1/2))/c+(b-4)/c))^(
1/2),((-(b-4)/c-(-b-2-2*I*3^(1/2))/c)/(-(b-4)/c-(-b-2+2*I*3^(1/2))/c))^(1/
2)))+182*A*c^2*((-b-2-2*I*3^(1/2))/c+(b-4)/c)*((x+(b-4)/c)/((-b-2-2*I*3^(1
/2))/c+(b-4)/c))^(1/2)*((x-(-b-2+2*I*3^(1/2))/c)/(-(b-4)/c-(-b-2+2*I*3^(1/
2))/c))^(1/2)*((x-(-b-2-2*I*3^(1/2))/c)/(-(b-4)/c-(-b-2-2*I*3^(1/2))/c))^(
1/2)/(c^3*x^3+3*b*c^2*x^2+3*b^2*c*x+b^3-64)^(1/2)*EllipticF(((x+(b-4)/c)/(
(-b-2-2*I*3^(1/2))/c+(b-4)/c))^(1/2),((-(b-4)/c-(-b-2-2*I*3^(1/2))/c)/(...
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 365, normalized size of antiderivative = 0.63

$$\int (A+Bx+Cx^2)\left(-64+b^3+3b^2cx+3bc^2x^2+c^3x^3\right)^{3/2}\,dx = \frac{2\left(30191616\,(Cb^2-Bbc+Ac^2)\sqrt{c^3}\text{weierstrassPInverse}\left(0,\frac{256}{c^3},\frac{cx+b}{c}\right)+18247680\,(2\,Cbc-\right.}{}$$

input
```
integrate((C*x^2+B*x+A)*(c^3*x^3+3*b*c^2*x^2+3*b^2*c*x+b^3-64)^(3/2),x, al
gorithm="fricas")
```

output
```
2/15015*(30191616*(C*b^2 - B*b*c + A*c^2)*sqrt(c^3)*weierstrassPInverse(0,
 256/c^3, (c*x + b)/c) + 18247680*(2*C*b*c - B*c^2)*sqrt(c^3)*weierstrassZ
eta(0, 256/c^3, weierstrassPInverse(0, 256/c^3, (c*x + b)/c)) + (1001*C*c^
8*x^6 + 231*(16*C*b*c^7 + 5*B*c^8)*x^5 + 273*(5*A*b^4 - 896*A*b)*c^4 + 105
*(46*C*b^2*c^6 + 42*B*b*c^7 + 13*A*c^8)*x^4 - 6*(35*B*b^5 - 12608*B*b^2)*c
^3 + 14*(435*B*b^2*c^6 + 390*A*b*c^7 + 2*(85*C*b^3 - 4576*C)*c^5)*x^3 + 8*
(7*C*b^6 - 4352*C*b^3 + 512512*C)*c^2 + 3*(2730*A*b^2*c^6 + 160*(7*B*b^3 -
 352*B)*c^5 + (35*C*b^4 - 15488*C*b)*c^4)*x^2 + 3*(364*(5*A*b^3 - 224*A)*c
^5 + 3*(35*B*b^4 - 10368*B*b)*c^4 - 4*(7*C*b^5 - 3904*C*b^2)*c^3)*x)*sqrt(
c^3*x^3 + 3*b*c^2*x^2 + 3*b^2*c*x + b^3 - 64))/c^5
```

Sympy [F]

$$\int (A+Bx+Cx^2)\left(-64+b^3+3b^2cx+3bc^2x^2+c^3x^3\right)^{3/2}\,dx = \int \left((b+cx-4)\left(b^2+2bcx+4b+c^2x^2+4cx+16\right)\right)^{\frac{3}{2}}\left(A+Bx+Cx^2\right)\,dx$$

input
```
integrate((C*x**2+B*x+A)*(c**3*x**3+3*b*c**2*x**2+3*b**2*c*x+b**3-64)**(3/
2),x)
```

output
```
Integral(((b + c*x - 4)*(b**2 + 2*b*c*x + 4*b + c**2*x**2 + 4*c*x + 16))**
(3/2)*(A + B*x + C*x**2), x)
```

Maxima [F]

$$\int (A+Bx+Cx^2)\left(-64+b^3+3b^2cx+3bc^2x^2+c^3x^3\right)^{3/2}\,dx=\int \left(c^3x^3+3\,bc^2x^2+3\,b^2cx+b^3-64\right)^{\frac{3}{2}}\left(Cx^2+Bx+A\right)dx$$

input
```
integrate((C*x^2+B*x+A)*(c^3*x^3+3*b*c^2*x^2+3*b^2*c*x+b^3-64)^(3/2),x, al
gorithm="maxima")
```

output
```
integrate((c^3*x^3 + 3*b*c^2*x^2 + 3*b^2*c*x + b^3 - 64)^(3/2)*(C*x^2 + B*
x + A), x)
```

Giac [F]

$$\int (A+Bx+Cx^2)\left(-64+b^3+3b^2cx+3bc^2x^2+c^3x^3\right)^{3/2}\,dx=\int \left(c^3x^3+3\,bc^2x^2+3\,b^2cx+b^3-64\right)^{\frac{3}{2}}\left(Cx^2+Bx+A\right)dx$$

input
```
integrate((C*x^2+B*x+A)*(c^3*x^3+3*b*c^2*x^2+3*b^2*c*x+b^3-64)^(3/2),x, al
gorithm="giac")
```

output
```
integrate((c^3*x^3 + 3*b*c^2*x^2 + 3*b^2*c*x + b^3 - 64)^(3/2)*(C*x^2 + B*
x + A), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(A+Bx+Cx^2\right)\left(-64+b^3+3b^2cx+3bc^2x^2+c^3x^3\right)^{3/2}\,dx=\int \left(C\,x^2+B\,x+A\right)\,\left(b^3+3\,b^2\,c\,x+3\,b\,c^2\,x^2+c^3\,x^3-64\right)^{3/2}\,dx$$

input
```
int((A + B*x + C*x^2)*(b^3 + c^3*x^3 + 3*b*c^2*x^2 + 3*b^2*c*x - 64)^(3/2)
,x)
```

output
```
int((A + B*x + C*x^2)*(b^3 + c^3*x^3 + 3*b*c^2*x^2 + 3*b^2*c*x - 64)^(3/2)
, x)
```

Reduce [F]

$$\int \left(A+Bx+Cx^2\right)\left(-64+b^3+3b^2cx+3bc^2x^2+c^3x^3\right)^{3/2}\,dx=\text{Too large to display}$$

input
```
int((C*x^2+B*x+A)*(c^3*x^3+3*b*c^2*x^2+3*b^2*c*x+b^3-64)^(3/2),x)
```

output

```
(2*(1365*sqrt(b**3 + 3*b**2*c*x + 3*b*c**2*x**2 + c**3*x**3 - 64)*a*b**4*c
 + 5460*sqrt(b**3 + 3*b**2*c*x + 3*b*c**2*x**2 + c**3*x**3 - 64)*a*b**3*c*
*2*x + 8190*sqrt(b**3 + 3*b**2*c*x + 3*b*c**2*x**2 + c**3*x**3 - 64)*a*b**
2*c**3*x**2 + 5460*sqrt(b**3 + 3*b**2*c*x + 3*b*c**2*x**2 + c**3*x**3 - 64
)*a*b*c**4*x**3 - 244608*sqrt(b**3 + 3*b**2*c*x + 3*b*c**2*x**2 + c**3*x**
3 - 64)*a*b*c + 1365*sqrt(b**3 + 3*b**2*c*x + 3*b*c**2*x**2 + c**3*x**3 -
64)*a*c**5*x**4 - 244608*sqrt(b**3 + 3*b**2*c*x + 3*b*c**2*x**2 + c**3*x**
3 - 64)*a*c**2*x - 154*sqrt(b**3 + 3*b**2*c*x + 3*b*c**2*x**2 + c**3*x**3
- 64)*b**6 + 231*sqrt(b**3 + 3*b**2*c*x + 3*b*c**2*x**2 + c**3*x**3 - 64)*
b**5*c*x + 3465*sqrt(b**3 + 3*b**2*c*x + 3*b*c**2*x**2 + c**3*x**3 - 64)*b
**4*c**2*x**2 + 8470*sqrt(b**3 + 3*b**2*c*x + 3*b*c**2*x**2 + c**3*x**3 -
64)*b**3*c**3*x**3 + 40832*sqrt(b**3 + 3*b**2*c*x + 3*b*c**2*x**2 + c**3*x
**3 - 64)*b**3 + 9240*sqrt(b**3 + 3*b**2*c*x + 3*b*c**2*x**2 + c**3*x**3 -
 64)*b**2*c**4*x**4 - 46464*sqrt(b**3 + 3*b**2*c*x + 3*b*c**2*x**2 + c**3*
x**3 - 64)*b**2*c*x + 4851*sqrt(b**3 + 3*b**2*c*x + 3*b*c**2*x**2 + c**3*x
**3 - 64)*b*c**5*x**5 - 215424*sqrt(b**3 + 3*b**2*c*x + 3*b*c**2*x**2 + c*
*3*x**3 - 64)*b*c**2*x**2 + 1001*sqrt(b**3 + 3*b**2*c*x + 3*b*c**2*x**2 +
c**3*x**3 - 64)*c**6*x**6 - 128128*sqrt(b**3 + 3*b**2*c*x + 3*b*c**2*x**2
+ c**3*x**3 - 64)*c**3*x**3 + 1058816*sqrt(b**3 + 3*b**2*c*x + 3*b*c**2*x*
*2 + c**3*x**3 - 64) + 15095808*int(sqrt(b**3 + 3*b**2*c*x + 3*b*c**2*x...
```

3.75 $\int (A+Bx+Cx^2)\sqrt{-64+b^3+3b^2cx+3bc^2x^2+c^3$

Optimal result

Integrand size = 43, antiderivative size = 509

$$\int (A+Bx+Cx^2)\sqrt{-64+b^3+3b^2cx+3bc^2x^2+c^3x^3}\,dx$$
$$= \frac{384(Bc-2bC)\sqrt{-64+(b+cx)^3}}{7c^3\left(4-4\sqrt{3}-b-cx\right)} - \frac{2\left(\frac{7(bBc-Ac^2-b^2C)(b+cx)}{c}-\frac{5(Bc-2bC)(b+cx)^2}{c}\right)\sqrt{-64+(b+cx)^3}}{35c^2}$$
$$+ \frac{2C\left(-64+(b+cx)^3\right)^{3/2}}{9c^3}$$
$$- \frac{384\sqrt[4]{3}\sqrt{2+\sqrt{3}}(Bc-2bC)(4-b-cx)\sqrt{\frac{16+4(b+cx)+(b+cx)^2}{\left(4-4\sqrt{3}-b-cx\right)^2}}E\left(\arcsin\left(\frac{4+4\sqrt{3}-b-cx}{4-4\sqrt{3}-b-cx}\right)|-7+4\sqrt{3}\right)}{7c^3\sqrt{-\frac{4-b-cx}{\left(4-4\sqrt{3}-b-cx\right)^2}}\sqrt{-64+(b+cx)^3}}$$
$$+ \frac{64\ 3^{3/4}\sqrt{2-\sqrt{3}}\left(20\left(1+\sqrt{3}\right)(Bc-2bC)-7(bBc-Ac^2-b^2C)\right)(4-b-cx)\sqrt{\frac{16+4(b+cx)+(b+cx)^2}{\left(4-4\sqrt{3}-b-cx\right)^2}}\,\mathrm{Ell}}{35c^3\sqrt{-\frac{4-b-cx}{\left(4-4\sqrt{3}-b-cx\right)^2}}\sqrt{-64+(b+cx)^3}}$$

output

```
384/7*(B*c-2*C*b)*(-64+(c*x+b)^3)^(1/2)/c^3/(4-4*3^(1/2)-b-c*x)-2/35*(7*(-
A*c^2+B*b*c-C*b^2)*(c*x+b)/c-5*(B*c-2*C*b)*(c*x+b)^2/c)*(-64+(c*x+b)^3)^(1
/2)/c^2+2/9*C*(-64+(c*x+b)^3)^(3/2)/c^3-384/7*3^(1/4)*(1/2*6^(1/2)+1/2*2^(
1/2))*(B*c-2*C*b)*(-c*x-b+4)*((16+4*c*x+4*b+(c*x+b)^2)/(4-4*3^(1/2)-b-c*x)
^2)^(1/2)*EllipticE((4+4*3^(1/2)-b-c*x)/(4-4*3^(1/2)-b-c*x),2*I-I*3^(1/2))
/c^3/(-(-c*x-b+4)/(4-4*3^(1/2)-b-c*x)^2)^(1/2)/(-64+(c*x+b)^3)^(1/2)+64/35
*3^(3/4)*(1/2*6^(1/2)-1/2*2^(1/2))*(20*(1+3^(1/2))*(B*c-2*C*b)+7*A*c^2-7*B
*b*c+7*C*b^2)*(-c*x-b+4)*((16+4*c*x+4*b+(c*x+b)^2)/(4-4*3^(1/2)-b-c*x)^2)^
(1/2)*EllipticF((4+4*3^(1/2)-b-c*x)/(4-4*3^(1/2)-b-c*x),2*I-I*3^(1/2))/c^3
/(-(-c*x-b+4)/(4-4*3^(1/2)-b-c*x)^2)^(1/2)/(-64+(c*x+b)^3)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.73 (sec) , antiderivative size = 353, normalized size of antiderivative = 0.69

$$\int (A+Bx+Cx^2)\sqrt{-64+b^3+3b^2cx+3bc^2x^2+c^3x^3}\,dx$$

$$= \frac{2\Big((-64+b^3+3b^2cx+3bc^2x^2+c^3x^3)\left(8b^3C+9c^3x(7A+5Bx)-6b^2c(3B+2Cx)+35C(-64+c^3x^3\right.}{}$$

input

```
Integrate[(A + B*x + C*x^2)*Sqrt[-64 + b^3 + 3*b^2*c*x + 3*b*c^2*x^2 + c^3
*x^3],x]
```

output

```
(2*((-64 + b^3 + 3*b^2*c*x + 3*b*c^2*x^2 + c^3*x^3)*(8*b^3*C + 9*c^3*x*(7*
A + 5*B*x) - 6*b^2*c*(3*B + 2*C*x) + 35*C*(-64 + c^3*x^3) + 3*b*c^2*(21*A
+ x*(9*B + 5*C*x))) + 864*Sqrt[2]*Sqrt[((-I)*(-4 + b + c*x))/(3*I + Sqrt[3
])]*Sqrt[16 + b^2 + 4*c*x + c^2*x^2 + 2*b*(2 + c*x)]*(-10*(3*I + Sqrt[3])*
(B*c - 2*b*C)*EllipticE[ArcSin[Sqrt[2*I + 2*Sqrt[3] + I*b + I*c*x]/(2*3^(1
/4))], (2*Sqrt[3])/(3*I + Sqrt[3])] + I*((20 - 7*b)*B*c + 7*A*c^2 + b*(-40
 + 7*b)*C)*EllipticF[ArcSin[Sqrt[2*I + 2*Sqrt[3] + I*b + I*c*x]/(2*3^(1/4)
)], (2*Sqrt[3])/(3*I + Sqrt[3])])))/(315*c^3*Sqrt[-64 + b^3 + 3*b^2*c*x +
3*b*c^2*x^2 + c^3*x^3])
```

Rubi [A] (warning: unable to verify)

Time = 1.52 (sec) , antiderivative size = 626, normalized size of antiderivative = 1.23, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.186, Rules used = {2459, 2392, 27, 2425, 793, 2419, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{b^3+3b^2cx+3bc^2x^2+c^3x^3-64}\left(A+Bx+Cx^2\right)\,dx$$

$$\Big\downarrow \text{2459}$$

$$\int \sqrt{c^3\left(\frac{b}{c}+x\right)^3-64}\left(A-\frac{b(Bc-bC)}{c^2}+\left(\frac{b}{c}+x\right)\left(B-\frac{2bC}{c}\right)+C\left(\frac{b}{c}+x\right)^2\right)d\left(\frac{b}{c}+x\right)$$

$$\Big\downarrow \text{2392}$$

$$\frac{2}{315}\sqrt{c^3\left(\frac{b}{c}+x\right)^3-64}\left(63\left(\frac{b}{c}+x\right)\left(A-\frac{b(Bc-bC)}{c^2}\right)+45\left(\frac{b}{c}+x\right)^2\left(B-\frac{2bC}{c}\right)+35C\left(\frac{b}{c}+x\right)^3\right)-96\int \frac{2\left(35C\left(\frac{b}{c}+x\right)^2+45\left(B-\frac{2bC}{c}\right)\left(\frac{b}{c}+x\right)+63\left(A-\frac{b(Bc-bC)}{c^2}\right)\right)}{315\sqrt{c^3\left(\frac{b}{c}+x\right)^3-64}}d\left(\frac{b}{c}+x\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{315}\sqrt{c^3\left(\frac{b}{c}+x\right)^3-64}\left(63\left(\frac{b}{c}+x\right)\left(A-\frac{b(Bc-bC)}{c^2}\right)+45\left(\frac{b}{c}+x\right)^2\left(B-\frac{2bC}{c}\right)+35C\left(\frac{b}{c}+x\right)^3\right)-\frac{64}{105}\int \frac{35C\left(\frac{b}{c}+x\right)^2+45\left(B-\frac{2bC}{c}\right)\left(\frac{b}{c}+x\right)+63\left(A-\frac{b(Bc-bC)}{c^2}\right)}{\sqrt{c^3\left(\frac{b}{c}+x\right)^3-64}}d\left(\frac{b}{c}+x\right)$$

$$\Big\downarrow \text{2425}$$

$$\frac{2}{315}\sqrt{c^3\left(\frac{b}{c}+x\right)^3-64}\left(63\left(\frac{b}{c}+x\right)\left(A-\frac{b(Bc-bC)}{c^2}\right)+45\left(\frac{b}{c}+x\right)^2\left(B-\frac{2bC}{c}\right)+35C\left(\frac{b}{c}+x\right)^3\right)-\frac{64}{105}\left(\int \frac{63\left(A-\frac{b(Bc-bC)}{c^2}\right)+45\left(B-\frac{2bC}{c}\right)\left(\frac{b}{c}+x\right)}{\sqrt{c^3\left(\frac{b}{c}+x\right)^3-64}}d\left(\frac{b}{c}+x\right)+35C\int \frac{\left(\frac{b}{c}+x\right)^2}{\sqrt{c^3\left(\frac{b}{c}+x\right)^3-64}}d\left(\frac{b}{c}+x\right)\right)$$

$$\Big\downarrow \text{793}$$

$$\frac{2}{315}\sqrt{c^3\left(\frac{b}{c}+x\right)^3-64}\left(63\left(\frac{b}{c}+x\right)\left(A-\frac{b(Bc-bC)}{c^2}\right)+45\left(\frac{b}{c}+x\right)^2\left(B-\frac{2bC}{c}\right)+35C\left(\frac{b}{c}+x\right)^3\right)-\frac{64}{105}\left(\int\frac{63\left(A-\frac{b(Bc-bC)}{c^2}\right)+45\left(B-\frac{2bC}{c}\right)\left(\frac{b}{c}+x\right)}{\sqrt{c^3\left(\frac{b}{c}+x\right)^3-64}}d\left(\frac{b}{c}+x\right)+\frac{70C\sqrt{c^3\left(\frac{b}{c}+x\right)^3-64}}{3c^3}\right)$$

$$\Big\downarrow \text{2419}$$

$$\frac{2}{315}\sqrt{c^3\left(\frac{b}{c}+x\right)^3-64}\left(63\left(\frac{b}{c}+x\right)\left(A-\frac{b(Bc-bC)}{c^2}\right)+45\left(\frac{b}{c}+x\right)^2\left(B-\frac{2bC}{c}\right)+35C\left(\frac{b}{c}+x\right)^3\right)-\frac{64}{105}\left(\frac{9\left(7Ac^2+\left(20\left(1+\sqrt{3}\right)-7b\right)Bc-\left(40\left(1+\sqrt{3}\right)-7b\right)bC\right)\int\frac{1}{\sqrt{c^3\left(\frac{b}{c}+x\right)^3-64}}d\left(\frac{b}{c}+x\right)}{c^2}-\frac{45(Bc-2bC)\int\frac{4\left(\right.}{\sqrt{}}}{}\right.$$

$$\Big\downarrow \text{760}$$

$$\frac{2}{315}\sqrt{c^3\left(\frac{b}{c}+x\right)^3-64}\left(63\left(\frac{b}{c}+x\right)\left(A-\frac{b(Bc-bC)}{c^2}\right)+45\left(\frac{b}{c}+x\right)^2\left(B-\frac{2bC}{c}\right)+35C\left(\frac{b}{c}+x\right)^3\right)-\frac{64}{105}\left(-\frac{45(Bc-2bC)\int\frac{4\left(1+\sqrt{3}\right)-c\left(\frac{b}{c}+x\right)}{\sqrt{c^3\left(\frac{b}{c}+x\right)^3-64}}d\left(\frac{b}{c}+x\right)}{c^2}-\frac{3\,3^{3/4}\sqrt{2-\sqrt{3}}\left(4-c\left(\frac{b}{c}+x\right)\right)\sqrt{\frac{c^2\left(\frac{b}{c}+x\right)^2+4c\left(\frac{b}{c}+x\right)+16}{\left(4\left(1-\sqrt{3}\right)-c\left(\frac{b}{c}+x\right)\right)^2}}\left(7Ac\right.}{}\right.$$

$$\Big\downarrow \text{2418}$$

$$\frac{2}{315}\sqrt{c^3\left(\frac{b}{c}+x\right)^3-64}\left(63\left(\frac{b}{c}+x\right)\left(A-\frac{b(Bc-bC)}{c^2}\right)+45\left(\frac{b}{c}+x\right)^2\left(B-\frac{2bC}{c}\right)+35C\left(\frac{b}{c}+x\right)^3\right)-$$

$$\frac{64}{105}\left(-\frac{3\,3^{3/4}\sqrt{2-\sqrt{3}}\left(4-c\left(\frac{b}{c}+x\right)\right)\sqrt{\frac{c^2\left(\frac{b}{c}+x\right)^2+4c\left(\frac{b}{c}+x\right)+16}{\left(4\left(1-\sqrt{3}\right)-c\left(\frac{b}{c}+x\right)\right)^2}}\left(7Ac^2+\left(20\left(1+\sqrt{3}\right)-7b\right)Bc-\left(40\left(1+\sqrt{3}\right)-\right.\right.}{c^3\sqrt{-\frac{4-c\left(\frac{b}{c}+x\right)}{\left(4\left(1-\sqrt{3}\right)-c\left(\frac{b}{c}+x\right)\right)^2}}\sqrt{c^3\left(\frac{b}{c}+x\right)^3-64}}\right.$$

input
```
Int[(A + B*x + C*x^2)*Sqrt[-64 + b^3 + 3*b^2*c*x + 3*b*c^2*x^2 + c^3*x^3],
x]
```

output
```
(2*Sqrt[-64 + c^3*(b/c + x)^3]*(63*(A - (b*(B*c - b*C))/c^2)*(b/c + x) + 4
5*(B - (2*b*C)/c)*(b/c + x)^2 + 35*C*(b/c + x)^3))/315 - (64*((70*C*Sqrt[-
64 + c^3*(b/c + x)^3])/(3*c^3) - (45*(B*c - 2*b*C)*((2*Sqrt[-64 + c^3*(b/c
 + x)^3])/(c*(4*(1 - Sqrt[3]) - c*(b/c + x))) - (2*3^(1/4)*Sqrt[2 + Sqrt[3
]]*(4 - c*(b/c + x))*Sqrt[(16 + 4*c*(b/c + x) + c^2*(b/c + x)^2)/(4*(1 - S
qrt[3]) - c*(b/c + x))^2]*EllipticE[ArcSin[(4*(1 + Sqrt[3]) - c*(b/c + x))
/(4*(1 - Sqrt[3]) - c*(b/c + x))], -7 + 4*Sqrt[3]])/(c*Sqrt[-((4 - c*(b/c
+ x))/(4*(1 - Sqrt[3]) - c*(b/c + x))^2)]*Sqrt[-64 + c^3*(b/c + x)^3])))/c
^2 - (3*3^(3/4)*Sqrt[2 - Sqrt[3]]*((20*(1 + Sqrt[3]) - 7*b)*B*c + 7*A*c^2
- (40*(1 + Sqrt[3]) - 7*b)*b*C)*(4 - c*(b/c + x))*Sqrt[(16 + 4*c*(b/c + x)
 + c^2*(b/c + x)^2)/(4*(1 - Sqrt[3]) - c*(b/c + x))^2]*EllipticF[ArcSin[(4
*(1 + Sqrt[3]) - c*(b/c + x))/(4*(1 - Sqrt[3]) - c*(b/c + x))], -7 + 4*Sqr
t[3]])/(c^3*Sqrt[-((4 - c*(b/c + x))/(4*(1 - Sqrt[3]) - c*(b/c + x))^2)]*S
qrt[-64 + c^3*(b/c + x)^3])))/105
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

rule 2392
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{q = Expon[Pq
, x], i}, Simp[(a + b*x^n)^p*Sum[Coeff[Pq, x, i]*(x^(i + 1)/(n*p + i + 1)),
 {i, 0, q}], x] + Simp[a*n*p   Int[(a + b*x^n)^(p - 1)*Sum[Coeff[Pq, x, i]*
(x^i/(n*p + i + 1)), {i, 0, q}], x], x]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x
] && IGtQ[(n - 1)/2, 0] && GtQ[p, 0]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

rule 2419
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(c*r - (1 + Sqrt[3])*d*s)/r
  Int[1/Sqrt[a + b*x^3], x], x] + Simp[d/r   Int[((1 + Sqrt[3])*s + r*x)/Sq
rt[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] && NeQ[b*c^3 -
2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

rule 2425
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Pq, x, n -
 1]   Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq,
x, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[P
q, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

rule 2459
```
Int[(Pn_)^(p_.)*(Qx_), x_Symbol] :> With[{S = Coeff[Pn, x, Expon[Pn, x] - 1
]/(Expon[Pn, x]*Coeff[Pn, x, Expon[Pn, x]])}, Subst[Int[ExpandToSum[Pn /. x
 -> x - S, x]^p*ExpandToSum[Qx /. x -> x - S, x], x], x, x + S] /; Binomial
Q[Pn /. x -> x - S, x] || (IntegerQ[Expon[Pn, x]/2] && TrinomialQ[Pn /. x -
> x - S, x])] /; FreeQ[p, x] && PolyQ[Pn, x] && GtQ[Expon[Pn, x], 2] && NeQ
[Coeff[Pn, x, Expon[Pn, x] - 1], 0] && PolyQ[Qx, x] &&  !(MonomialQ[Qx, x]
&& IGtQ[p, 0])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1352 vs. $2(451) = 902$.

Time = 2.69 (sec) , antiderivative size = 1353, normalized size of antiderivative = 2.66

method	result	size
elliptic	Expression too large to display	1353
risch	Expression too large to display	1397
default	Expression too large to display	2104

input
```
int((C*x^2+B*x+A)*(c^3*x^3+3*b*c^2*x^2+3*b^2*c*x+b^3-64)^(1/2),x,method=_R
ETURNVERBOSE)
```

output
```
2/9*C*x^3*(c^3*x^3+3*b*c^2*x^2+3*b^2*c*x+b^3-64)^(1/2)+2/7*(B*c^3+1/3*C*b*
c^2)/c^3*x^2*(c^3*x^3+3*b*c^2*x^2+3*b^2*c*x+b^3-64)^(1/2)+2/5*(A*c^3+3*B*b
*c^2+2/3*C*b^2*c-18/7*b/c*(B*c^3+1/3*C*b*c^2))/c^3*x*(c^3*x^3+3*b*c^2*x^2+
3*b^2*c*x+b^3-64)^(1/2)+2/3*(3*c^2*b*A+3*B*b^2*c+C*b^3-64*C-2/9*C*(3*b^3-1
92)-15/7*b^2/c^2*(B*c^3+1/3*C*b*c^2)-12/5*b/c*(A*c^3+3*B*b*c^2+2/3*C*b^2*c
-18/7*b/c*(B*c^3+1/3*C*b*c^2)))/c^3*(c^3*x^3+3*b*c^2*x^2+3*b^2*c*x+b^3-64)
^(1/2)+2*(A*b^3-64*A-2/5*(b^3-64)/c^3*(A*c^3+3*B*b*c^2+2/3*C*b^2*c-18/7*b/
c*(B*c^3+1/3*C*b*c^2))-(3*c^2*b*A+3*B*b^2*c+C*b^3-64*C-2/9*C*(3*b^3-192)-1
5/7*b^2/c^2*(B*c^3+1/3*C*b*c^2)-12/5*b/c*(A*c^3+3*B*b*c^2+2/3*C*b^2*c-18/7
*b/c*(B*c^3+1/3*C*b*c^2)))/c^2*b^2)*((-b-2-2*I*3^(1/2))/c+(b-4)/c)*((x+(b-
4)/c)/((-b-2-2*I*3^(1/2))/c+(b-4)/c))^(1/2)*((x-(-b-2+2*I*3^(1/2))/c)/(-(b
-4)/c-(-b-2+2*I*3^(1/2))/c))^(1/2)*((x-(-b-2-2*I*3^(1/2))/c)/(-(b-4)/c-(-b
-2-2*I*3^(1/2))/c))^(1/2)/(c^3*x^3+3*b*c^2*x^2+3*b^2*c*x+b^3-64)^(1/2)*Ell
ipticF(((x+(b-4)/c)/((-b-2-2*I*3^(1/2))/c+(b-4)/c))^(1/2),((-(b-4)/c-(-b-2
-2*I*3^(1/2))/c)/(-(b-4)/c-(-b-2+2*I*3^(1/2))/c))^(1/2))+2*(3*A*b^2*c+B*b^
3-64*B-2/7*(B*c^3+1/3*C*b*c^2)/c^3*(2*b^3-128)-9/5*b^2/c^2*(A*c^3+3*B*b*c^
2+2/3*C*b^2*c-18/7*b/c*(B*c^3+1/3*C*b*c^2))-2*(3*c^2*b*A+3*B*b^2*c+C*b^3-6
4*C-2/9*C*(3*b^3-192)-15/7*b^2/c^2*(B*c^3+1/3*C*b*c^2)-12/5*b/c*(A*c^3+3*B
*b*c^2+2/3*C*b^2*c-18/7*b/c*(B*c^3+1/3*C*b*c^2)))/c*b)*((-b-2-2*I*3^(1/2))
/c+(b-4)/c)*((x+(b-4)/c)/((-b-2-2*I*3^(1/2))/c+(b-4)/c))^(1/2)*((x-(-b-...
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.40

$$\int (A + Bx + Cx^2)\sqrt{-64 + b^3 + 3b^2cx + 3bc^2x^2 + c^3x^3}\,dx =$$

$$-\frac{2\left(12096\,(Cb^2 - Bbc + Ac^2)\sqrt{c^3}\text{weierstrassPInverse}\left(0, \frac{256}{c^3}, \frac{cx+b}{c}\right) + 8640\,(2\,Cbc - Bc^2)\sqrt{c^3}\text{weierstras}\right.}{}$$

input
```
integrate((C*x^2+B*x+A)*(c^3*x^3+3*b*c^2*x^2+3*b^2*c*x+b^3-64)^(1/2),x, al
gorithm="fricas")
```

output
```
-2/315*(12096*(C*b^2 - B*b*c + A*c^2)*sqrt(c^3)*weierstrassPInverse(0, 256
/c^3, (c*x + b)/c) + 8640*(2*C*b*c - B*c^2)*sqrt(c^3)*weierstrassZeta(0, 2
56/c^3, weierstrassPInverse(0, 256/c^3, (c*x + b)/c)) - (35*C*c^5*x^3 - 18
*B*b^2*c^3 + 63*A*b*c^4 + 8*(C*b^3 - 280*C)*c^2 + 15*(C*b*c^4 + 3*B*c^5)*x
^2 - 3*(4*C*b^2*c^3 - 9*B*b*c^4 - 21*A*c^5)*x)*sqrt(c^3*x^3 + 3*b*c^2*x^2
+ 3*b^2*c*x + b^3 - 64))/c^5
```

Sympy [F]

$$\int (A+Bx+Cx^2)\sqrt{-64+b^3+3b^2cx+3bc^2x^2+c^3x^3}\,dx$$
$$=\int \sqrt{(b+cx-4)\,(b^2+2bcx+4b+c^2x^2+4cx+16)}\,(A+Bx+Cx^2)\;dx$$

input
```
integrate((C*x**2+B*x+A)*(c**3*x**3+3*b*c**2*x**2+3*b**2*c*x+b**3-64)**(1/
2),x)
```

output
```
Integral(sqrt((b + c*x - 4)*(b**2 + 2*b*c*x + 4*b + c**2*x**2 + 4*c*x + 16
))*(A + B*x + C*x**2), x)
```

Maxima [F]

$$\int (A+Bx+Cx^2)\sqrt{-64+b^3+3b^2cx+3bc^2x^2+c^3x^3}\,dx$$
$$=\int \sqrt{c^3x^3+3\,bc^2x^2+3\,b^2cx+b^3-64}\,(Cx^2+Bx+A)\,dx$$

input
```
integrate((C*x^2+B*x+A)*(c^3*x^3+3*b*c^2*x^2+3*b^2*c*x+b^3-64)^(1/2),x, al
gorithm="maxima")
```

output
```
integrate(sqrt(c^3*x^3 + 3*b*c^2*x^2 + 3*b^2*c*x + b^3 - 64)*(C*x^2 + B*x
+ A), x)
```

Giac [F]

$$\int (A+Bx+Cx^2)\sqrt{-64+b^3+3b^2cx+3bc^2x^2+c^3x^3}\,dx$$
$$=\int \sqrt{c^3x^3+3\,bc^2x^2+3\,b^2cx+b^3-64}(Cx^2+Bx+A)\,dx$$

input
```
integrate((C*x^2+B*x+A)*(c^3*x^3+3*b*c^2*x^2+3*b^2*c*x+b^3-64)^(1/2),x, al
gorithm="giac")
```

output
```
integrate(sqrt(c^3*x^3 + 3*b*c^2*x^2 + 3*b^2*c*x + b^3 - 64)*(C*x^2 + B*x
+ A), x)
```

Mupad [F(-1)]

Timed out.

$$\int (A+Bx+Cx^2)\sqrt{-64+b^3+3b^2cx+3bc^2x^2+c^3x^3}\,dx$$
$$=\int \left(C\,x^2+B\,x+A\right)\sqrt{b^3+3\,b^2\,c\,x+3\,b\,c^2\,x^2+c^3\,x^3-64}\,dx$$

input
```
int((A + B*x + C*x^2)*(b^3 + c^3*x^3 + 3*b*c^2*x^2 + 3*b^2*c*x - 64)^(1/2)
,x)
```

output
```
int((A + B*x + C*x^2)*(b^3 + c^3*x^3 + 3*b*c^2*x^2 + 3*b^2*c*x - 64)^(1/2)
, x)
```

Reduce [F]

$$\int \left(A+Bx+Cx^2\right)\sqrt{-64+b^3+3b^2cx+3bc^2x^2+c^3x^3}\,dx$$

$$=\frac{\frac{2\sqrt{c^3x^3+3b\,c^2x^2+3b^2cx+b^3-64}\,abc}{5}+\frac{2\sqrt{c^3x^3+3b\,c^2x^2+3b^2cx+b^3-64}\,a\,c^2x}{5}-\frac{4\sqrt{c^3x^3+3b\,c^2x^2+3b^2cx+b^3-64}\,b^3}{63}+\frac{2\sqrt{c^3x^3+3b\,c^2x^2+3}}{2}$$

input
```
int((C*x^2+B*x+A)*(c^3*x^3+3*b*c^2*x^2+3*b^2*c*x+b^3-64)^(1/2),x)
```

output
```
(2*(63*sqrt(b**3 + 3*b**2*c*x + 3*b*c**2*x**2 + c**3*x**3 - 64)*a*b*c + 63
*sqrt(b**3 + 3*b**2*c*x + 3*b*c**2*x**2 + c**3*x**3 - 64)*a*c**2*x - 10*sq
rt(b**3 + 3*b**2*c*x + 3*b*c**2*x**2 + c**3*x**3 - 64)*b**3 + 15*sqrt(b**3
 + 3*b**2*c*x + 3*b*c**2*x**2 + c**3*x**3 - 64)*b**2*c*x + 60*sqrt(b**3 +
3*b**2*c*x + 3*b*c**2*x**2 + c**3*x**3 - 64)*b*c**2*x**2 + 35*sqrt(b**3 +
3*b**2*c*x + 3*b*c**2*x**2 + c**3*x**3 - 64)*c**3*x**3 - 800*sqrt(b**3 + 3
*b**2*c*x + 3*b*c**2*x**2 + c**3*x**3 - 64) - 6048*int(sqrt(b**3 + 3*b**2*
c*x + 3*b*c**2*x**2 + c**3*x**3 - 64)/(b**3 + 3*b**2*c*x + 3*b*c**2*x**2 +
 c**3*x**3 - 64),x)*a*c**2 + 2160*int(sqrt(b**3 + 3*b**2*c*x + 3*b*c**2*x*
*2 + c**3*x**3 - 64)/(b**3 + 3*b**2*c*x + 3*b*c**2*x**2 + c**3*x**3 - 64),
x)*b**2*c - 2160*int((sqrt(b**3 + 3*b**2*c*x + 3*b*c**2*x**2 + c**3*x**3 -
 64)*x**2)/(b**3 + 3*b**2*c*x + 3*b*c**2*x**2 + c**3*x**3 - 64),x)*c**3))/
(315*c**2)
```

3.76 $\int \frac{A+Bx+Cx^2}{\sqrt{-64+b^3+3b^2cx+3bc^2x^2+c^3x^3}}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 431

$$\int \frac{A+Bx+Cx^2}{\sqrt{-64+b^3+3b^2cx+3bc^2x^2+c^3x^3}}\,dx = \frac{2C\sqrt{-64+(b+cx)^3}}{3c^3}-\frac{2(Bc-2bC)\sqrt{-64+(b+cx)^3}}{c^3\left(4-4\sqrt{3}-b-cx\right)} + \frac{2\sqrt[4]{3}\sqrt{2+\sqrt{3}}(Bc-2bC)(4-b-cx)\sqrt{\frac{16+4(b+cx)+(b+cx)^2}{\left(4-4\sqrt{3}-b-cx\right)^2}}E\left(\arcsin\left(\frac{4+4\sqrt{3}-b-cx}{4-4\sqrt{3}-b-cx}\right)|-7+4\sqrt{3}\right)}{c^3\sqrt{-\frac{4-b-cx}{\left(4-4\sqrt{3}-b-cx\right)^2}}\sqrt{-64+(b+cx)^3}} + \frac{\sqrt{2-\sqrt{3}}\left(bBc-Ac^2-b^2C-4\left(1+\sqrt{3}\right)(Bc-2bC)\right)(4-b-cx)\sqrt{\frac{16+4(b+cx)+(b+cx)^2}{\left(4-4\sqrt{3}-b-cx\right)^2}}\,\mathrm{EllipticF}\left(\mathrm{arc}\right.}{\sqrt[4]{3}c^3\sqrt{-\frac{4-b-cx}{\left(4-4\sqrt{3}-b-cx\right)^2}}\sqrt{-64+(b+cx)^3}}$$

output
```
2/3*C*(-64+(c*x+b)^3)^(1/2)/c^3-2*(B*c-2*C*b)*(-64+(c*x+b)^3)^(1/2)/c^3/(4
-4*3^(1/2)-b-c*x)+2*3^(1/4)*(1/2*6^(1/2)+1/2*2^(1/2))*(B*c-2*C*b)*(-c*x-b+
4)*((16+4*c*x+4*b+(c*x+b)^2)/(4-4*3^(1/2)-b-c*x)^2)^(1/2)*EllipticE((4+4*3
^(1/2)-b-c*x)/(4-4*3^(1/2)-b-c*x),2*I-I*3^(1/2))/c^3/(-(-c*x-b+4)/(4-4*3^(
1/2)-b-c*x)^2)^(1/2)/(-64+(c*x+b)^3)^(1/2)+1/3*(1/2*6^(1/2)-1/2*2^(1/2))*(
B*b*c-A*c^2-C*b^2-4*(1+3^(1/2))*(B*c-2*C*b))*(-c*x-b+4)*((16+4*c*x+4*b+(c*
x+b)^2)/(4-4*3^(1/2)-b-c*x)^2)^(1/2)*EllipticF((4+4*3^(1/2)-b-c*x)/(4-4*3^
(1/2)-b-c*x),2*I-I*3^(1/2))*3^(3/4)/c^3/(-(-c*x-b+4)/(4-4*3^(1/2)-b-c*x)^2
)^(1/2)/(-64+(c*x+b)^3)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.66 (sec) , antiderivative size = 284, normalized size of antiderivative = 0.66

$$\int \frac{A+Bx+Cx^2}{\sqrt{-64+b^3+3b^2cx+3bc^2x^2+c^3x^3}}\,dx$$

$$= \frac{2C(-64+b^3+3b^2cx+3bc^2x^2+c^3x^3)-3\sqrt{2}\sqrt{-\frac{i(-4+b+cx)}{3i+\sqrt{3}}}\sqrt{16+b^2+4cx+c^2x^2+2b(2+cx)}\Big(-2(3\cdots}{\cdots}$$

input
```
Integrate[(A + B*x + C*x^2)/Sqrt[-64 + b^3 + 3*b^2*c*x + 3*b*c^2*x^2 + c^3
*x^3],x]
```

output
```
(2*C*(-64 + b^3 + 3*b^2*c*x + 3*b*c^2*x^2 + c^3*x^3) - 3*Sqrt[2]*Sqrt[((-I
)*(-4 + b + c*x))/(3*I + Sqrt[3])]*Sqrt[16 + b^2 + 4*c*x + c^2*x^2 + 2*b*(
2 + c*x)]*(-2*(3*I + Sqrt[3])*(B*c - 2*b*C)*EllipticE[ArcSin[Sqrt[2*I + 2*
Sqrt[3] + I*b + I*c*x]/(2*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])] + I*(-((
-4 + b)*B*c) + A*c^2 + (-8 + b)*b*C)*EllipticF[ArcSin[Sqrt[2*I + 2*Sqrt[3]
 + I*b + I*c*x]/(2*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])]))/(3*c^3*Sqrt[-
64 + b^3 + 3*b^2*c*x + 3*b*c^2*x^2 + c^3*x^3])
```

Rubi [A] (warning: unable to verify)

Time = 1.21 (sec) , antiderivative size = 537, normalized size of antiderivative = 1.25, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.140, Rules used = {2459, 2425, 793, 2419, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{\sqrt{b^3+3b^2cx+3bc^2x^2+c^3x^3-64}}dx$$

$$\Big\downarrow \text{ 2459}$$

$$\int \frac{A-\frac{b(Bc-bC)}{c^2}+\left(\frac{b}{c}+x\right)\left(B-\frac{2bC}{c}\right)+C\left(\frac{b}{c}+x\right)^2}{\sqrt{c^3\left(\frac{b}{c}+x\right)^3-64}}d\left(\frac{b}{c}+x\right)$$

$$\Big\downarrow \text{ 2425}$$

$$\int \frac{A-\frac{b(Bc-bC)}{c^2}+\left(B-\frac{2bC}{c}\right)\left(\frac{b}{c}+x\right)}{\sqrt{c^3\left(\frac{b}{c}+x\right)^3-64}}d\left(\frac{b}{c}+x\right)+C\int \frac{\left(\frac{b}{c}+x\right)^2}{\sqrt{c^3\left(\frac{b}{c}+x\right)^3-64}}d\left(\frac{b}{c}+x\right)$$

$$\Big\downarrow \text{ 793}$$

$$\int \frac{A-\frac{b(Bc-bC)}{c^2}+\left(B-\frac{2bC}{c}\right)\left(\frac{b}{c}+x\right)}{\sqrt{c^3\left(\frac{b}{c}+x\right)^3-64}}d\left(\frac{b}{c}+x\right)+\frac{2C\sqrt{c^3\left(\frac{b}{c}+x\right)^3-64}}{3c^3}$$

$$\Big\downarrow \text{ 2419}$$

$$\frac{\left(Ac^2+\left(-b+4\sqrt{3}+4\right)Bc-\left(-b+8\sqrt{3}+8\right)bC\right)\int \frac{1}{\sqrt{c^3\left(\frac{b}{c}+x\right)^3-64}}d\left(\frac{b}{c}+x\right)}{c^2}-\frac{(Bc-2bC)\int \frac{4\left(1+\sqrt{3}\right)-c\left(\frac{b}{c}+x\right)}{\sqrt{c^3\left(\frac{b}{c}+x\right)^3-64}}d\left(\frac{b}{c}+x\right)}{c^2}+\frac{2C\sqrt{c^3\left(\frac{b}{c}+x\right)^3-64}}{3c^3}$$

$$\Big\downarrow \text{ 760}$$

$$-\frac{(Bc-2bC)\int \frac{4\left(1+\sqrt{3}\right)-c\left(\frac{b}{c}+x\right)}{\sqrt{c^3\left(\frac{b}{c}+x\right)^3-64}}d\left(\frac{b}{c}+x\right)}{c^2}-$$

$$\frac{\sqrt{2-\sqrt{3}}\left(4-c\left(\frac{b}{c}+x\right)\right)\sqrt{\frac{c^2\left(\frac{b}{c}+x\right)^2+4c\left(\frac{b}{c}+x\right)+16}{\left(4\left(1-\sqrt{3}\right)-c\left(\frac{b}{c}+x\right)\right)^2}}\left(Ac^2+\left(-b+4\sqrt{3}+4\right)Bc-\left(-b+8\sqrt{3}+8\right)bC\right)\text{EllipticF}\left(\text{a}\right.}{\sqrt[4]{3}c^3\sqrt{-\frac{4-c\left(\frac{b}{c}+x\right)}{\left(4\left(1-\sqrt{3}\right)-c\left(\frac{b}{c}+x\right)\right)^2}}\sqrt{c^3\left(\frac{b}{c}+x\right)^3-64}}$$

$$\frac{2C\sqrt{c^3\left(\frac{b}{c}+x\right)^3-64}}{3c^3}$$

$\downarrow$ 2418

$$-\frac{\sqrt{2-\sqrt{3}}\left(4-c\left(\frac{b}{c}+x\right)\right)\sqrt{\frac{c^2\left(\frac{b}{c}+x\right)^2+4c\left(\frac{b}{c}+x\right)+16}{\left(4\left(1-\sqrt{3}\right)-c\left(\frac{b}{c}+x\right)\right)^2}}\left(Ac^2+\left(-b+4\sqrt{3}+4\right)Bc-\left(-b+8\sqrt{3}+8\right)bC\right)\text{EllipticF}\left(\right.}{\sqrt[4]{3}c^3\sqrt{-\frac{4-c\left(\frac{b}{c}+x\right)}{\left(4\left(1-\sqrt{3}\right)-c\left(\frac{b}{c}+x\right)\right)^2}}\sqrt{c^3\left(\frac{b}{c}+x\right)^3-64}}$$

$$\frac{(Bc-2bC)\left(\frac{2\sqrt{c^3\left(\frac{b}{c}+x\right)^3-64}}{c\left(4\left(1-\sqrt{3}\right)-c\left(\frac{b}{c}+x\right)\right)}-\frac{2\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(4-c\left(\frac{b}{c}+x\right)\right)\sqrt{\frac{c^2\left(\frac{b}{c}+x\right)^2+4c\left(\frac{b}{c}+x\right)+16}{\left(4\left(1-\sqrt{3}\right)-c\left(\frac{b}{c}+x\right)\right)^2}}E\left(\arcsin\left(\frac{4\left(1+\sqrt{3}\right)-c\left(\frac{b}{c}+x\right)}{4\left(1-\sqrt{3}\right)-c\left(\frac{b}{c}+x\right)}\right)|-7+4\sqrt{3}\right.}{c\sqrt{-\frac{4-c\left(\frac{b}{c}+x\right)}{\left(4\left(1-\sqrt{3}\right)-c\left(\frac{b}{c}+x\right)\right)^2}}\sqrt{c^3\left(\frac{b}{c}+x\right)^3-64}}\right.}{c^2}$$

$$\frac{2C\sqrt{c^3\left(\frac{b}{c}+x\right)^3-64}}{3c^3}$$

input

```
Int[(A + B*x + C*x^2)/Sqrt[-64 + b^3 + 3*b^2*c*x + 3*b*c^2*x^2 + c^3*x^3],
x]
```

output

```
(2*C*Sqrt[-64 + c^3*(b/c + x)^3])/(3*c^3) - ((B*c - 2*b*C)*((2*Sqrt[-64 +
c^3*(b/c + x)^3])/(c*(4*(1 - Sqrt[3]) - c*(b/c + x))) - (2*3^(1/4)*Sqrt[2
+ Sqrt[3]]*(4 - c*(b/c + x))*Sqrt[(16 + 4*c*(b/c + x) + c^2*(b/c + x)^2)/(
4*(1 - Sqrt[3]) - c*(b/c + x))^2]*EllipticE[ArcSin[(4*(1 + Sqrt[3]) - c*(b
/c + x))/(4*(1 - Sqrt[3]) - c*(b/c + x))], -7 + 4*Sqrt[3]])/(c*Sqrt[-((4 -
 c*(b/c + x))/(4*(1 - Sqrt[3]) - c*(b/c + x))^2)]*Sqrt[-64 + c^3*(b/c + x)
^3])))/c^2 - (Sqrt[2 - Sqrt[3]]*((4 + 4*Sqrt[3] - b)*B*c + A*c^2 - (8 + 8*
Sqrt[3] - b)*b*C)*(4 - c*(b/c + x))*Sqrt[(16 + 4*c*(b/c + x) + c^2*(b/c +
x)^2)/(4*(1 - Sqrt[3]) - c*(b/c + x))^2]*EllipticF[ArcSin[(4*(1 + Sqrt[3])
 - c*(b/c + x))/(4*(1 - Sqrt[3]) - c*(b/c + x))], -7 + 4*Sqrt[3]])/(3^(1/4
)*c^3*Sqrt[-((4 - c*(b/c + x))/(4*(1 - Sqrt[3]) - c*(b/c + x))^2)]*Sqrt[-6
4 + c^3*(b/c + x)^3])
```

Defintions of rubi rules used

rule 760

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 793

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

rule 2418

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

rule 2419
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(c*r - (1 + Sqrt[3])*d*s)/r
  Int[1/Sqrt[a + b*x^3], x], x] + Simp[d/r   Int[((1 + Sqrt[3])*s + r*x)/Sq
rt[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] && NeQ[b*c^3 -
2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

rule 2425
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Pq, x, n -
 1]   Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq,
x, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[P
q, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

rule 2459
```
Int[(Pn_)^(p_.)*(Qx_), x_Symbol] :> With[{S = Coeff[Pn, x, Expon[Pn, x] - 1
]/(Expon[Pn, x]*Coeff[Pn, x, Expon[Pn, x]])}, Subst[Int[ExpandToSum[Pn /. x
 -> x - S, x]^p*ExpandToSum[Qx /. x -> x - S, x], x], x, x + S] /; Binomial
Q[Pn /. x -> x - S, x] || (IntegerQ[Expon[Pn, x]/2] && TrinomialQ[Pn /. x -
> x - S, x])] /; FreeQ[p, x] && PolyQ[Pn, x] && GtQ[Expon[Pn, x], 2] && NeQ
[Coeff[Pn, x, Expon[Pn, x] - 1], 0] && PolyQ[Qx, x] &&  !(MonomialQ[Qx, x]
&& IGtQ[p, 0])
```

Maple [A] (verified)

Time = 2.71 (sec) , antiderivative size = 755, normalized size of antiderivative = 1.75

method	result
elliptic	$\frac{2C\sqrt{c^3x^3+3b\,c^2x^2+3b^2cx+b^3-64}}{3c^3}+\frac{2\left(A-\frac{b^2C}{c^2}\right)\left(\frac{-b-2-2i\sqrt{3}}{c}+\frac{b-4}{c}\right)\sqrt{\frac{x+\frac{b-4}{c}}{\frac{-b-2-2i\sqrt{3}}{c}+\frac{b-4}{c}}}\sqrt{\frac{x-\frac{-b-2+2i\sqrt{3}}{c}}{-\frac{b-4}{c}-\frac{-b-2+2i\sqrt{3}}{c}}}\sqrt{\frac{x-\frac{-b-2-}{c}}{-\frac{b-4}{c}-\frac{-b-}{}}}}{\sqrt{c^3x^3+3b\,c^2x^2+3b^2cx+b^3-}}$
risch	Expression too large to display
default	Expression too large to display

input
```
int((C*x^2+B*x+A)/(c^3*x^3+3*b*c^2*x^2+3*b^2*c*x+b^3-64)^(1/2),x,method=_R
ETURNVERBOSE)
```

output
```
2/3*C/c^3*(c^3*x^3+3*b*c^2*x^2+3*b^2*c*x+b^3-64)^(1/2)+2*(A-b^2/c^2*C)*((-
b-2-2*I*3^(1/2))/c+(b-4)/c)*((x+(b-4)/c)/((-b-2-2*I*3^(1/2))/c+(b-4)/c))^(
1/2)*((x-(-b-2+2*I*3^(1/2))/c)/(-(b-4)/c-(-b-2+2*I*3^(1/2))/c))^(1/2)*((x-
(-b-2-2*I*3^(1/2))/c)/(-(b-4)/c-(-b-2-2*I*3^(1/2))/c))^(1/2)/(c^3*x^3+3*b*
c^2*x^2+3*b^2*c*x+b^3-64)^(1/2)*EllipticF(((x+(b-4)/c)/((-b-2-2*I*3^(1/2))
/c+(b-4)/c))^(1/2),((-(b-4)/c-(-b-2-2*I*3^(1/2))/c)/(-(b-4)/c-(-b-2+2*I*3^
(1/2))/c))^(1/2))+2*(B-2*b/c*C)*((-b-2-2*I*3^(1/2))/c+(b-4)/c)*((x+(b-4)/c
)/((-b-2-2*I*3^(1/2))/c+(b-4)/c))^(1/2)*((x-(-b-2+2*I*3^(1/2))/c)/(-(b-4)/
c-(-b-2+2*I*3^(1/2))/c))^(1/2)*((x-(-b-2-2*I*3^(1/2))/c)/(-(b-4)/c-(-b-2-2
*I*3^(1/2))/c))^(1/2)/(c^3*x^3+3*b*c^2*x^2+3*b^2*c*x+b^3-64)^(1/2)*((-(b-4
)/c-(-b-2+2*I*3^(1/2))/c)*EllipticE(((x+(b-4)/c)/((-b-2-2*I*3^(1/2))/c+(b-
4)/c))^(1/2),((-(b-4)/c-(-b-2-2*I*3^(1/2))/c)/(-(b-4)/c-(-b-2+2*I*3^(1/2))
/c))^(1/2))+(-b-2+2*I*3^(1/2))/c*EllipticF(((x+(b-4)/c)/((-b-2-2*I*3^(1/2)
)/c+(b-4)/c))^(1/2),((-(b-4)/c-(-b-2-2*I*3^(1/2))/c)/(-(b-4)/c-(-b-2+2*I*3
^(1/2))/c))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.28

$$\int \frac{A+Bx+Cx^2}{\sqrt{-64+b^3+3b^2cx+3bc^2x^2+c^3x^3}}\,dx$$
$$= \frac{2\left(\sqrt{c^3x^3+3\,bc^2x^2+3\,b^2cx+b^3-64}Cc^2+3\left(Cb^2-Bbc+Ac^2\right)\sqrt{c^3}\text{weierstrassPInverse}\left(0,\frac{256}{c^3},\frac{cx+b}{c}\right)\right.}{3\,c^5}$$

input
```
integrate((C*x^2+B*x+A)/(c^3*x^3+3*b*c^2*x^2+3*b^2*c*x+b^3-64)^(1/2),x, al
gorithm="fricas")
```

output
```
2/3*(sqrt(c^3*x^3 + 3*b*c^2*x^2 + 3*b^2*c*x + b^3 - 64)*C*c^2 + 3*(C*b^2 -
 B*b*c + A*c^2)*sqrt(c^3)*weierstrassPInverse(0, 256/c^3, (c*x + b)/c) + 3
*(2*C*b*c - B*c^2)*sqrt(c^3)*weierstrassZeta(0, 256/c^3, weierstrassPInver
se(0, 256/c^3, (c*x + b)/c)))/c^5
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt{-64+b^3+3b^2cx+3bc^2x^2+c^3x^3}}\,dx = \int \frac{A+Bx+Cx^2}{\sqrt{(b+cx-4)\,(b^2+2bcx+4b+c^2x^2+4cx+16)}}\,dx$$

input
```
integrate((C*x**2+B*x+A)/(c**3*x**3+3*b*c**2*x**2+3*b**2*c*x+b**3-64)**(1/
2),x)
```

output
```
Integral((A + B*x + C*x**2)/sqrt((b + c*x - 4)*(b**2 + 2*b*c*x + 4*b + c**
2*x**2 + 4*c*x + 16)), x)
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt{-64+b^3+3b^2cx+3bc^2x^2+c^3x^3}}\,dx = \int \frac{Cx^2+Bx+A}{\sqrt{c^3x^3+3\,bc^2x^2+3\,b^2cx+b^3-64}}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(c^3*x^3+3*b*c^2*x^2+3*b^2*c*x+b^3-64)^(1/2),x, al
gorithm="maxima")
```

output
```
integrate((C*x^2 + B*x + A)/sqrt(c^3*x^3 + 3*b*c^2*x^2 + 3*b^2*c*x + b^3 -
 64), x)
```

Giac [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt{-64+b^3+3b^2cx+3bc^2x^2+c^3x^3}}\,dx = \int \frac{Cx^2+Bx+A}{\sqrt{c^3x^3+3\,bc^2x^2+3\,b^2cx+b^3-64}}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(c^3*x^3+3*b*c^2*x^2+3*b^2*c*x+b^3-64)^(1/2),x, al
gorithm="giac")
```

output
```
integrate((C*x^2 + B*x + A)/sqrt(c^3*x^3 + 3*b*c^2*x^2 + 3*b^2*c*x + b^3 -
 64), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{\sqrt{-64+b^3+3b^2cx+3bc^2x^2+c^3x^3}}\,dx=\int \frac{C\,x^2+B\,x+A}{\sqrt{b^3+3\,b^2\,c\,x+3\,b\,c^2\,x^2+c^3\,x^3-64}}\,dx$$

input
```
int((A + B*x + C*x^2)/(b^3 + c^3*x^3 + 3*b*c^2*x^2 + 3*b^2*c*x - 64)^(1/2)
,x)
```

output
```
int((A + B*x + C*x^2)/(b^3 + c^3*x^3 + 3*b*c^2*x^2 + 3*b^2*c*x - 64)^(1/2)
, x)
```

Reduce [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt{-64+b^3+3b^2cx+3bc^2x^2+c^3x^3}}\,dx = \frac{2\sqrt{c^3x^3+3b\,c^2x^2+3b^2cx+b^3-64}+6\left(\int \frac{\sqrt{c^3x^3+3b\,c^2x^2+3b^2cx+b^3-64}}{c^3x^3+3b\,c^2x^2+3b^2cx+b^3-64}dx\right)a\,c^2-3\left(\int \frac{\sqrt{c^3x^3+3b\,c^2x^2+3b^2cx+b^3-}}{c^3x^3+3b\,c^2x^2+3b^2cx+b^3-6}\right.}{6c^2}$$

input
```
int((C*x^2+B*x+A)/(c^3*x^3+3*b*c^2*x^2+3*b^2*c*x+b^3-64)^(1/2),x)
```

output
```
(2*sqrt(b**3 + 3*b**2*c*x + 3*b*c**2*x**2 + c**3*x**3 - 64) + 6*int(sqrt(b
**3 + 3*b**2*c*x + 3*b*c**2*x**2 + c**3*x**3 - 64)/(b**3 + 3*b**2*c*x + 3*
b*c**2*x**2 + c**3*x**3 - 64),x)*a*c**2 - 3*int(sqrt(b**3 + 3*b**2*c*x + 3
*b*c**2*x**2 + c**3*x**3 - 64)/(b**3 + 3*b**2*c*x + 3*b*c**2*x**2 + c**3*x
**3 - 64),x)*b**2*c + 3*int((sqrt(b**3 + 3*b**2*c*x + 3*b*c**2*x**2 + c**3
*x**3 - 64)*x**2)/(b**3 + 3*b**2*c*x + 3*b*c**2*x**2 + c**3*x**3 - 64),x)*
c**3)/(6*c**2)
```

3.77 $\int \frac{A+Bx+Cx^2}{\left(-64+b^3+3b^2cx+3bc^2x^2+c^3x^3\right)^{3/2}}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 486

$$\begin{aligned}
\int \frac{A+Bx+Cx^2}{\left(-64+b^3+3b^2cx+3bc^2x^2+c^3x^3\right)^{3/2}}\,dx &= -\frac{2C}{3c^3\sqrt{-64+(b+cx)^3}} \\
&- \frac{(b+cx)\left(Ac^2-b^2C+c(Bc-2bC)x\right)}{96c^3\sqrt{-64+(b+cx)^3}} - \frac{(Bc-2bC)\sqrt{-64+(b+cx)^3}}{96c^3\left(4-4\sqrt{3}-b-cx\right)} \\
&+ \frac{\sqrt{2+\sqrt{3}}(Bc-2bC)(4-b-cx)\sqrt{\frac{16+4(b+cx)+(b+cx)^2}{\left(4-4\sqrt{3}-b-cx\right)^2}}E\left(\arcsin\left(\frac{4+4\sqrt{3}-b-cx}{4-4\sqrt{3}-b-cx}\right)|-7+4\sqrt{3}\right)}{32\ 3^{3/4}c^3\sqrt{-\frac{4-b-cx}{\left(4-4\sqrt{3}-b-cx\right)^2}}\sqrt{-64+(b+cx)^3}} \\
&- \frac{\sqrt{2-\sqrt{3}}\left(\left(4+4\sqrt{3}+b\right)Bc-Ac^2-b\left(8+8\sqrt{3}+b\right)C\right)(4-b-cx)\sqrt{\frac{16+4(b+cx)+(b+cx)^2}{\left(4-4\sqrt{3}-b-cx\right)^2}}\ \text{EllipticF}\left(\text{arcs}\right.}{192\sqrt[4]{3}c^3\sqrt{-\frac{4-b-cx}{\left(4-4\sqrt{3}-b-cx\right)^2}}\sqrt{-64+(b+cx)^3}}
\end{aligned}$$

output
```
-2/3*C/c^3/(-64+(c*x+b)^3)^(1/2)-1/96*(c*x+b)*(A*c^2-C*b^2+c*(B*c-2*C*b)*x
)/c^3/(-64+(c*x+b)^3)^(1/2)-1/96*(B*c-2*C*b)*(-64+(c*x+b)^3)^(1/2)/c^3/(4-
4*3^(1/2)-b-c*x)+1/96*3^(1/4)*(1/2*6^(1/2)+1/2*2^(1/2))*(B*c-2*C*b)*(-c*x-
b+4)*((16+4*c*x+4*b+(c*x+b)^2)/(4-4*3^(1/2)-b-c*x)^2)^(1/2)*EllipticE((4+4
*3^(1/2)-b-c*x)/(4-4*3^(1/2)-b-c*x),2*I-I*3^(1/2))/c^3/(-(-c*x-b+4)/(4-4*3
^(1/2)-b-c*x)^2)^(1/2)/(-64+(c*x+b)^3)^(1/2)-1/576*(1/2*6^(1/2)-1/2*2^(1/2
))*((4+4*3^(1/2)+b)*B*c-A*c^2-b*(8+8*3^(1/2)+b)*C)*(-c*x-b+4)*((16+4*c*x+4
*b+(c*x+b)^2)/(4-4*3^(1/2)-b-c*x)^2)^(1/2)*EllipticF((4+4*3^(1/2)-b-c*x)/(
4-4*3^(1/2)-b-c*x),2*I-I*3^(1/2))*3^(3/4)/c^3/(-(-c*x-b+4)/(4-4*3^(1/2)-b-
c*x)^2)^(1/2)/(-64+(c*x+b)^3)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 11.13 (sec) , antiderivative size = 928, normalized size of antiderivative = 1.91

$$\int \frac{A+Bx+Cx^2}{(-64+b^3+3b^2cx+3bc^2x^2+c^3x^3)^{3/2}}\,dx = \frac{-2\sqrt{16+b^2+4cx+c^2x^2+2b(2+cx)}(Ac^2(b+cx)+Bc}{}$$

input
```
Integrate[(A + B*x + C*x^2)/(-64 + b^3 + 3*b^2*c*x + 3*b*c^2*x^2 + c^3*x^3
)^(3/2),x]
```

output
```
(-2*Sqrt[16 + b^2 + 4*c*x + c^2*x^2 + 2*b*(2 + c*x)]*(A*c^2*(b + c*x) + B*
c^2*x*(b + c*x) - C*(-64 + b^3 + 3*b^2*c*x + 2*b*c^2*x^2)) + 2*Sqrt[2]*b*B
*c*(-2*I + 2*Sqrt[3] - I*b - I*c*x)*Sqrt[((-I)*(-4 + b + c*x))/(3*I + Sqrt
[3])]*(2 - (2*I)*Sqrt[3] + b + c*x)*EllipticF[ArcSin[Sqrt[2*I + 2*Sqrt[3]
+ I*b + I*c*x]/(2*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])] + Sqrt[2]*A*c^2*
(2*I + 2*Sqrt[3] + I*b + I*c*x)*Sqrt[((-I)*(-4 + b + c*x))/(3*I + Sqrt[3])
]*(2 + (2*I)*Sqrt[3] + b + c*x)*EllipticF[ArcSin[Sqrt[2*I + 2*Sqrt[3] + I*
b + I*c*x]/(2*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])] + 3*Sqrt[2]*b^2*C*(2
*I + 2*Sqrt[3] + I*b + I*c*x)*Sqrt[((-I)*(-4 + b + c*x))/(3*I + Sqrt[3])]*
(2 + (2*I)*Sqrt[3] + b + c*x)*EllipticF[ArcSin[Sqrt[2*I + 2*Sqrt[3] + I*b
+ I*c*x]/(2*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])] - Sqrt[2]*B*c*(-2 - (2
*I)*Sqrt[3] - b - c*x)*(2*I + 2*Sqrt[3] + I*b + I*c*x)*Sqrt[((-I)*(-4 + b
+ c*x))/(3*I + Sqrt[3])]*((6 - (2*I)*Sqrt[3])*EllipticE[ArcSin[Sqrt[2*I +
2*Sqrt[3] + I*b + I*c*x]/(2*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])] + (-4
+ b)*EllipticF[ArcSin[Sqrt[2*I + 2*Sqrt[3] + I*b + I*c*x]/(2*3^(1/4))], (2
*Sqrt[3])/(3*I + Sqrt[3])]) + 2*Sqrt[2]*b*C*(-2 - (2*I)*Sqrt[3] - b - c*x)
*(2*I + 2*Sqrt[3] + I*b + I*c*x)*Sqrt[((-I)*(-4 + b + c*x))/(3*I + Sqrt[3]
)]*((6 - (2*I)*Sqrt[3])*EllipticE[ArcSin[Sqrt[2*I + 2*Sqrt[3] + I*b + I*c*
x]/(2*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])] + (-4 + b)*EllipticF[ArcSin[
Sqrt[2*I + 2*Sqrt[3] + I*b + I*c*x]/(2*3^(1/4))], (2*Sqrt[3])/(3*I + Sq...
```

Rubi [A] (warning: unable to verify)

Time = 1.29 (sec) , antiderivative size = 586, normalized size of antiderivative = 1.21, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.140$, Rules used = {2459, 2393, 27, 2419, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{(b^3+3b^2cx+3bc^2x^2+c^3x^3-64)^{3/2}}dx$$

$$\Big\downarrow \text{2459}$$

$$\int \frac{A-\frac{b(Bc-bC)}{c^2}+\left(\frac{b}{c}+x\right)\left(B-\frac{2bC}{c}\right)+C\left(\frac{b}{c}+x\right)^2}{\left(c^3\left(\frac{b}{c}+x\right)^3-64\right)^{3/2}}d\left(\frac{b}{c}+x\right)$$

$$\Big\downarrow \text{2393}$$

$$\frac{1}{96}\int -\frac{A-\frac{b(Bc-bC)}{c^2}-\left(B-\frac{2bC}{c}\right)\left(\frac{b}{c}+x\right)}{2\sqrt{c^3\left(\frac{b}{c}+x\right)^3-64}}d\left(\frac{b}{c}+x\right)-\frac{c^3\left(\frac{b}{c}+x\right)\left(A-\frac{b(Bc-bC)}{c^2}+\left(\frac{b}{c}+x\right)\left(B-\frac{2bC}{c}\right)\right)+64C}{96c^3\sqrt{c^3\left(\frac{b}{c}+x\right)^3-64}}$$

$$\Big\downarrow \; 27$$

$$-\frac{1}{192}\int \frac{A-\frac{b(Bc-bC)}{c^2}-\left(B-\frac{2bC}{c}\right)\left(\frac{b}{c}+x\right)}{\sqrt{c^3\left(\frac{b}{c}+x\right)^3-64}}d\left(\frac{b}{c}+x\right)-\frac{c^3\left(\frac{b}{c}+x\right)\left(A-\frac{b(Bc-bC)}{c^2}+\left(\frac{b}{c}+x\right)\left(B-\frac{2bC}{c}\right)\right)+64C}{96c^3\sqrt{c^3\left(\frac{b}{c}+x\right)^3-64}}$$

$$\Big\downarrow \; 2419$$

$$\frac{1}{192}\left(\frac{\left(-Ac^2+\left(b+4\sqrt{3}+4\right)Bc-b\left(b+8\sqrt{3}+8\right)C\right)\int \frac{1}{\sqrt{c^3\left(\frac{b}{c}+x\right)^3-64}}d\left(\frac{b}{c}+x\right)}{c^2}-\frac{(Bc-2bC)\int \frac{4\left(1+\sqrt{3}\right)-c\left(\frac{b}{c}+ \right.}{\sqrt{c^3\left(\frac{b}{c}+x\right)^3-6}}}{c^2}\right.$$

$$\frac{c^3\left(\frac{b}{c}+x\right)\left(A-\frac{b(Bc-bC)}{c^2}+\left(\frac{b}{c}+x\right)\left(B-\frac{2bC}{c}\right)\right)+64C}{96c^3\sqrt{c^3\left(\frac{b}{c}+x\right)^3-64}}$$

$$\Big\downarrow \; 760$$

$$\frac{1}{192}\left(-\frac{(Bc-2bC)\int \frac{4\left(1+\sqrt{3}\right)-c\left(\frac{b}{c}+x\right)}{\sqrt{c^3\left(\frac{b}{c}+x\right)^3-64}}d\left(\frac{b}{c}+x\right)}{c^2}-\frac{\sqrt{2-\sqrt{3}}\left(4-c\left(\frac{b}{c}+x\right)\right)\sqrt{\frac{c^2\left(\frac{b}{c}+x\right)^2+4c\left(\frac{b}{c}+x\right)+16}{\left(4\left(1-\sqrt{3}\right)-c\left(\frac{b}{c}+x\right)\right)^2}}\left(-Ac^2+(b\right.}{\sqrt[4]{3}c^3\sqrt{-\ldots}}\right.$$

$$\frac{c^3\left(\frac{b}{c}+x\right)\left(A-\frac{b(Bc-bC)}{c^2}+\left(\frac{b}{c}+x\right)\left(B-\frac{2bC}{c}\right)\right)+64C}{96c^3\sqrt{c^3\left(\frac{b}{c}+x\right)^3-64}}$$

$$\Big\downarrow \; 2418$$

$$\frac{1}{192}\left(-\frac{\sqrt{2-\sqrt{3}}\left(4-c\left(\frac{b}{c}+x\right)\right)\sqrt{\frac{c^2\left(\frac{b}{c}+x\right)^2+4c\left(\frac{b}{c}+x\right)+16}{\left(4\left(1-\sqrt{3}\right)-c\left(\frac{b}{c}+x\right)\right)^2}}\left(-Ac^2+\left(b+4\sqrt{3}+4\right)Bc-b\left(b+8\sqrt{3}+8\right)C\right)\text{Ellipti}}{\sqrt[4]{3}c^3\sqrt{-\frac{4-c\left(\frac{b}{c}+x\right)}{\left(4\left(1-\sqrt{3}\right)-c\left(\frac{b}{c}+x\right)\right)^2}}\sqrt{c^3\left(\frac{b}{c}+x\right)^3-64}}\right.$$

$$\frac{c^3\left(\frac{b}{c}+x\right)\left(A-\frac{b(Bc-bC)}{c^2}+\left(\frac{b}{c}+x\right)\left(B-\frac{2bC}{c}\right)\right)+64C}{96c^3\sqrt{c^3\left(\frac{b}{c}+x\right)^3-64}}$$

input
```
Int[(A + B*x + C*x^2)/(-64 + b^3 + 3*b^2*c*x + 3*b*c^2*x^2 + c^3*x^3)^(3/2
),x]
```

output
```
-1/96*(64*C + c^3*(b/c + x)*(A - (b*(B*c - b*C))/c^2 + (B - (2*b*C)/c)*(b/
c + x)))/(c^3*Sqrt[-64 + c^3*(b/c + x)^3]) + (-(((B*c - 2*b*C)*((2*Sqrt[-6
4 + c^3*(b/c + x)^3])/(c*(4*(1 - Sqrt[3]) - c*(b/c + x))) - (2*3^(1/4)*Sqr
t[2 + Sqrt[3]]*(4 - c*(b/c + x))*Sqrt[(16 + 4*c*(b/c + x) + c^2*(b/c + x)^
2)/(4*(1 - Sqrt[3]) - c*(b/c + x))^2]*EllipticE[ArcSin[(4*(1 + Sqrt[3]) -
c*(b/c + x))/(4*(1 - Sqrt[3]) - c*(b/c + x))], -7 + 4*Sqrt[3]])/(c*Sqrt[-(
(4 - c*(b/c + x))/(4*(1 - Sqrt[3]) - c*(b/c + x))^2)]*Sqrt[-64 + c^3*(b/c
+ x)^3])))/c^2) - (Sqrt[2 - Sqrt[3]]*((4 + 4*Sqrt[3] + b)*B*c - A*c^2 - b*
(8 + 8*Sqrt[3] + b)*C)*(4 - c*(b/c + x))*Sqrt[(16 + 4*c*(b/c + x) + c^2*(b
/c + x)^2)/(4*(1 - Sqrt[3]) - c*(b/c + x))^2]*EllipticF[ArcSin[(4*(1 + Sqr
t[3]) - c*(b/c + x))/(4*(1 - Sqrt[3]) - c*(b/c + x))], -7 + 4*Sqrt[3]])/(3
^(1/4)*c^3*Sqrt[-((4 - c*(b/c + x))/(4*(1 - Sqrt[3]) - c*(b/c + x))^2)]*Sq
rt[-64 + c^3*(b/c + x)^3]))/192
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 2393
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   In
t[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(
p + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n
, 0] && LtQ[p, -1]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

rule 2419
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(c*r - (1 + Sqrt[3])*d*s)/r
  Int[1/Sqrt[a + b*x^3], x], x] + Simp[d/r   Int[((1 + Sqrt[3])*s + r*x)/Sq
rt[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] && NeQ[b*c^3 -
2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

rule 2459
```
Int[(Pn_)^(p_.)*(Qx_), x_Symbol] :> With[{S = Coeff[Pn, x, Expon[Pn, x] - 1
]/(Expon[Pn, x]*Coeff[Pn, x, Expon[Pn, x]])}, Subst[Int[ExpandToSum[Pn /. x
 -> x - S, x]^p*ExpandToSum[Qx /. x -> x - S, x], x], x, x + S] /; Binomial
Q[Pn /. x -> x - S, x] || (IntegerQ[Expon[Pn, x]/2] && TrinomialQ[Pn /. x -
> x - S, x])] /; FreeQ[p, x] && PolyQ[Pn, x] && GtQ[Expon[Pn, x], 2] && NeQ
[Coeff[Pn, x, Expon[Pn, x] - 1], 0] && PolyQ[Qx, x] && !(MonomialQ[Qx, x]
&& IGtQ[p, 0])
```

Maple [A] (verified)

Time = 1.69 (sec) , antiderivative size = 851, normalized size of antiderivative = 1.75

method	result
elliptic	$-\frac{2c^3\left(\frac{(Bc-2bC)x^2}{192c^4}+\frac{\left(A\,c^2+bBc-3b^2C\right)x}{192c^5}+\frac{c^2bA-C\,b^3+64C}{192c^6}\right)}{\sqrt{\left(x^3+\frac{3b\,x^2}{c}+\frac{3b^2x}{c^2}+\frac{b^3-64}{c^3}\right)c^3}}+\frac{2\left(-\frac{A\,c^2-b^2C}{64c^2}+\frac{A\,c^2+bBc-3b^2C}{96c^2}\right)\left(\frac{-b-2-2i\sqrt{3}}{c}+\frac{b-4}{c}\right)\sqrt{\frac{x+}{\frac{-b-2-}{c}}}}{}$
default	Expression too large to display

input
```
int((C*x^2+B*x+A)/(c^3*x^3+3*b*c^2*x^2+3*b^2*c*x+b^3-64)^(3/2),x,method=_R
ETURNVERBOSE)
```

output
```
-2*c^3*(1/192*(B*c-2*C*b)/c^4*x^2+1/192*(A*c^2+B*b*c-3*C*b^2)/c^5*x+1/192*
(A*b*c^2-C*b^3+64*C)/c^6)/((x^3+3*b/c*x^2+3*b^2/c^2*x+(b^3-64)/c^3)*c^3)^(
1/2)+2*(-1/64*(A*c^2-C*b^2)/c^2+1/96/c^2*(A*c^2+B*b*c-3*C*b^2))*((-b-2-2*I
*3^(1/2))/c+(b-4)/c)*((x+(b-4)/c)/((-b-2-2*I*3^(1/2))/c+(b-4)/c))^(1/2)*((
x-(-b-2+2*I*3^(1/2))/c)/(-(b-4)/c-(-b-2+2*I*3^(1/2))/c))^(1/2)*((x-(-b-2-2
*I*3^(1/2))/c)/(-(b-4)/c-(-b-2-2*I*3^(1/2))/c))^(1/2)/(c^3*x^3+3*b*c^2*x^2
+3*b^2*c*x+b^3-64)^(1/2)*EllipticF(((x+(b-4)/c)/((-b-2-2*I*3^(1/2))/c+(b-4
)/c))^(1/2),((-(b-4)/c-(-b-2-2*I*3^(1/2))/c)/(-(b-4)/c-(-b-2+2*I*3^(1/2))/
c))^(1/2))+1/96/c*(B*c-2*C*b)*((-b-2-2*I*3^(1/2))/c+(b-4)/c)*((x+(b-4)/c)/
((-b-2-2*I*3^(1/2))/c+(b-4)/c))^(1/2)*((x-(-b-2+2*I*3^(1/2))/c)/(-(b-4)/c-
(-b-2+2*I*3^(1/2))/c))^(1/2)*((x-(-b-2-2*I*3^(1/2))/c)/(-(b-4)/c-(-b-2-2*I
*3^(1/2))/c))^(1/2)/(c^3*x^3+3*b*c^2*x^2+3*b^2*c*x+b^3-64)^(1/2)*((-(b-4)/
c-(-b-2+2*I*3^(1/2))/c)*EllipticE(((x+(b-4)/c)/((-b-2-2*I*3^(1/2))/c+(b-4)
/c))^(1/2),((-(b-4)/c-(-b-2-2*I*3^(1/2))/c)/(-(b-4)/c-(-b-2+2*I*3^(1/2))/c
))^(1/2))+(-b-2+2*I*3^(1/2))/c*EllipticF(((x+(b-4)/c)/((-b-2-2*I*3^(1/2))/
c+(b-4)/c))^(1/2),((-(b-4)/c-(-b-2-2*I*3^(1/2))/c)/(-(b-4)/c-(-b-2+2*I*3^(
1/2))/c))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 398, normalized size of antiderivative = 0.82

$$\int \frac{A+Bx+Cx^2}{\left(-64+b^3+3b^2cx+3bc^2x^2+c^3x^3\right)^{3/2}}\,dx =$$
$$-\frac{(Cb^5+(Cb^2c^3-Bbc^4+Ac^5)x^3-64\,Cb^2+(Ab^3-64\,A)c^2+3\,(Cb^3c^2-Bb^2c^3+Abc^4)x^2-(Bb^4-64$$

input
```
integrate((C*x^2+B*x+A)/(c^3*x^3+3*b*c^2*x^2+3*b^2*c*x+b^3-64)^(3/2),x, al
gorithm="fricas")
```

output
```
-1/96*((C*b^5 + (C*b^2*c^3 - B*b*c^4 + A*c^5)*x^3 - 64*C*b^2 + (A*b^3 - 64
*A)*c^2 + 3*(C*b^3*c^2 - B*b^2*c^3 + A*b*c^4)*x^2 - (B*b^4 - 64*B*b)*c + 3
*(C*b^4*c - B*b^3*c^2 + A*b^2*c^3)*x)*sqrt(c^3)*weierstrassPInverse(0, 256
/c^3, (c*x + b)/c) - ((2*C*b*c^4 - B*c^5)*x^3 - (B*b^3 - 64*B)*c^2 + 3*(2*
C*b^2*c^3 - B*b*c^4)*x^2 + 2*(C*b^4 - 64*C*b)*c + 3*(2*C*b^3*c^2 - B*b^2*c
^3)*x)*sqrt(c^3)*weierstrassZeta(0, 256/c^3, weierstrassPInverse(0, 256/c^
3, (c*x + b)/c)) + (A*b*c^4 - (C*b^3 - 64*C)*c^2 - (2*C*b*c^4 - B*c^5)*x^2
 - (3*C*b^2*c^3 - B*b*c^4 - A*c^5)*x)*sqrt(c^3*x^3 + 3*b*c^2*x^2 + 3*b^2*c
*x + b^3 - 64))/(c^8*x^3 + 3*b*c^7*x^2 + 3*b^2*c^6*x + (b^3 - 64)*c^5)
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2}{\left(-64+b^3+3b^2cx+3bc^2x^2+c^3x^3\right)^{3/2}}\,dx = \int \frac{A+Bx+Cx^2}{\left((b+cx-4)\left(b^2+2bcx+4b+c^2x^2+4cx+16\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((C*x**2+B*x+A)/(c**3*x**3+3*b*c**2*x**2+3*b**2*c*x+b**3-64)**(3/
2),x)
```

output
```
Integral((A + B*x + C*x**2)/((b + c*x - 4)*(b**2 + 2*b*c*x + 4*b + c**2*x*
*2 + 4*c*x + 16))**(3/2), x)
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{(-64+b^3+3b^2cx+3bc^2x^2+c^3x^3)^{3/2}}\,dx = \int \frac{Cx^2+Bx+A}{(c^3x^3+3\,bc^2x^2+3\,b^2cx+b^3-64)^{\frac{3}{2}}}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(c^3*x^3+3*b*c^2*x^2+3*b^2*c*x+b^3-64)^(3/2),x, al
gorithm="maxima")
```

output
```
integrate((C*x^2 + B*x + A)/(c^3*x^3 + 3*b*c^2*x^2 + 3*b^2*c*x + b^3 - 64)
^(3/2), x)
```

Giac [F]

$$\int \frac{A+Bx+Cx^2}{(-64+b^3+3b^2cx+3bc^2x^2+c^3x^3)^{3/2}}\,dx = \int \frac{Cx^2+Bx+A}{(c^3x^3+3\,bc^2x^2+3\,b^2cx+b^3-64)^{\frac{3}{2}}}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(c^3*x^3+3*b*c^2*x^2+3*b^2*c*x+b^3-64)^(3/2),x, al
gorithm="giac")
```

output
```
integrate((C*x^2 + B*x + A)/(c^3*x^3 + 3*b*c^2*x^2 + 3*b^2*c*x + b^3 - 64)
^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(-64+b^3+3b^2cx+3bc^2x^2+c^3x^3)^{3/2}}\,dx = \int \frac{C\,x^2+B\,x+A}{(b^3+3\,b^2\,c\,x+3\,b\,c^2\,x^2+c^3\,x^3-64)^{3/2}}\,dx$$

input
```
int((A + B*x + C*x^2)/(b^3 + c^3*x^3 + 3*b*c^2*x^2 + 3*b^2*c*x - 64)^(3/2)
,x)
```

output
```
int((A + B*x + C*x^2)/(b^3 + c^3*x^3 + 3*b*c^2*x^2 + 3*b^2*c*x - 64)^(3/2)
, x)
```

Reduce [F]

$$\int \frac{A+Bx+Cx^2}{(-64+b^3+3b^2cx+3bc^2x^2+c^3x^3)^{3/2}}\,dx = \text{too large to display}$$

input
```
int((C*x^2+B*x+A)/(c^3*x^3+3*b*c^2*x^2+3*b^2*c*x+b^3-64)^(3/2),x)
```

output
```
(12*sqrt(b**3 + 3*b**2*c*x + 3*b*c**2*x**2 + c**3*x**3 - 64)*a*c**2*x - 4*
sqrt(b**3 + 3*b**2*c*x + 3*b*c**2*x**2 + c**3*x**3 - 64)*b**3 - 6*sqrt(b**
3 + 3*b**2*c*x + 3*b*c**2*x**2 + c**3*x**3 - 64)*b**2*c*x + 256*sqrt(b**3
+ 3*b**2*c*x + 3*b*c**2*x**2 + c**3*x**3 - 64) + 6*int((sqrt(b**3 + 3*b**2
*c*x + 3*b*c**2*x**2 + c**3*x**3 - 64)*x**3)/(b**9 + 6*b**8*c*x + 15*b**7*
c**2*x**2 + 20*b**6*c**3*x**3 - 192*b**6 + 15*b**5*c**4*x**4 - 768*b**5*c*
x + 6*b**4*c**5*x**5 - 1344*b**4*c**2*x**2 + b**3*c**6*x**6 - 1408*b**3*c*
*3*x**3 + 12288*b**3 - 960*b**2*c**4*x**4 + 24576*b**2*c*x - 384*b*c**5*x*
*5 + 24576*b*c**2*x**2 - 64*c**6*x**6 + 8192*c**3*x**3 - 262144),x)*a*b**6
*c**5 + 18*int((sqrt(b**3 + 3*b**2*c*x + 3*b*c**2*x**2 + c**3*x**3 - 64)*x
**3)/(b**9 + 6*b**8*c*x + 15*b**7*c**2*x**2 + 20*b**6*c**3*x**3 - 192*b**6
 + 15*b**5*c**4*x**4 - 768*b**5*c*x + 6*b**4*c**5*x**5 - 1344*b**4*c**2*x*
*2 + b**3*c**6*x**6 - 1408*b**3*c**3*x**3 + 12288*b**3 - 960*b**2*c**4*x**
4 + 24576*b**2*c*x - 384*b*c**5*x**5 + 24576*b*c**2*x**2 - 64*c**6*x**6 +
8192*c**3*x**3 - 262144),x)*a*b**5*c**6*x + 18*int((sqrt(b**3 + 3*b**2*c*x
 + 3*b*c**2*x**2 + c**3*x**3 - 64)*x**3)/(b**9 + 6*b**8*c*x + 15*b**7*c**2
*x**2 + 20*b**6*c**3*x**3 - 192*b**6 + 15*b**5*c**4*x**4 - 768*b**5*c*x +
6*b**4*c**5*x**5 - 1344*b**4*c**2*x**2 + b**3*c**6*x**6 - 1408*b**3*c**3*x
**3 + 12288*b**3 - 960*b**2*c**4*x**4 + 24576*b**2*c*x - 384*b*c**5*x**5 +
 24576*b*c**2*x**2 - 64*c**6*x**6 + 8192*c**3*x**3 - 262144),x)*a*b**4*...
```

3.78 $\int \frac{A+Bx+Cx^2}{\left(-64+b^3+3b^2cx+3bc^2x^2+c^3x^3\right)^{5/2}}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 549

$$\int \frac{A+Bx+Cx^2}{\left(-64+b^3+3b^2cx+3bc^2x^2+c^3x^3\right)^{5/2}}\,dx =$$
$$-\frac{2C}{9c^3\left(-64+(b+cx)^3\right)^{3/2}}-\frac{(b+cx)\left(Ac^2-b^2C+c(Bc-2bC)x\right)}{288c^3\left(-64+(b+cx)^3\right)^{3/2}}$$
$$-\frac{(b+cx)\left(7(bBc-Ac^2-b^2C)-5(Bc-2bC)(b+cx)\right)}{55296c^3\sqrt{-64+(b+cx)^3}}$$
$$+\frac{5(Bc-2bC)\sqrt{-64+(b+cx)^3}}{55296c^3\left(4-4\sqrt{3}-b-cx\right)}$$
$$-\frac{5\sqrt{2+\sqrt{3}}(Bc-2bC)(4-b-cx)\sqrt{\frac{16+4(b+cx)+(b+cx)^2}{\left(4-4\sqrt{3}-b-cx\right)^2}}E\left(\arcsin\left(\frac{4+4\sqrt{3}-b-cx}{4-4\sqrt{3}-b-cx}\right)|-7+4\sqrt{3}\right)}{18432\,3^{3/4}c^3\sqrt{-\frac{4-b-cx}{\left(4-4\sqrt{3}-b-cx\right)^2}}\sqrt{-64+(b+cx)^3}}$$
$$+\frac{\sqrt{2-\sqrt{3}}\left(20\left(1+\sqrt{3}\right)(Bc-2bC)+7(bBc-Ac^2-b^2C)\right)(4-b-cx)\sqrt{\frac{16+4(b+cx)+(b+cx)^2}{\left(4-4\sqrt{3}-b-cx\right)^2}}\,\text{EllipticF}\left(\text{ar}\right.}{110592\sqrt[4]{3}c^3\sqrt{-\frac{4-b-cx}{\left(4-4\sqrt{3}-b-cx\right)^2}}\sqrt{-64+(b+cx)^3}}$$

output
```
-2/9*C/c^3/(-64+(c*x+b)^3)^(3/2)-1/288*(c*x+b)*(A*c^2-C*b^2+c*(B*c-2*C*b)*
x)/c^3/(-64+(c*x+b)^3)^(3/2)-1/55296*(c*x+b)*(-7*A*c^2+7*B*b*c-7*C*b^2-5*(
B*c-2*C*b)*(c*x+b))/c^3/(-64+(c*x+b)^3)^(1/2)+5/55296*(B*c-2*C*b)*(-64+(c*
x+b)^3)^(1/2)/c^3/(4-4*3^(1/2)-b-c*x)-5/55296*3^(1/4)*(1/2*6^(1/2)+1/2*2^(
1/2))*(B*c-2*C*b)*(-c*x-b+4)*((16+4*c*x+4*b+(c*x+b)^2)/(4-4*3^(1/2)-b-c*x)
^2)^(1/2)*EllipticE((4+4*3^(1/2)-b-c*x)/(4-4*3^(1/2)-b-c*x),2*I-I*3^(1/2))
/c^3/(-(-c*x-b+4)/(4-4*3^(1/2)-b-c*x)^2)^(1/2)/(-64+(c*x+b)^3)^(1/2)+1/331
776*(1/2*6^(1/2)-1/2*2^(1/2))*(20*(1+3^(1/2))*(B*c-2*C*b)-7*A*c^2+7*B*b*c-
7*C*b^2)*(-c*x-b+4)*((16+4*c*x+4*b+(c*x+b)^2)/(4-4*3^(1/2)-b-c*x)^2)^(1/2)
*EllipticF((4+4*3^(1/2)-b-c*x)/(4-4*3^(1/2)-b-c*x),2*I-I*3^(1/2))*3^(3/4)/
c^3/(-(-c*x-b+4)/(4-4*3^(1/2)-b-c*x)^2)^(1/2)/(-64+(c*x+b)^3)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 12.25 (sec) , antiderivative size = 400, normalized size of antiderivative = 0.73

$$\int \frac{A+Bx+Cx^2}{(-64+b^3+3b^2cx+3bc^2x^2+c^3x^3)^{5/2}}\,dx = \frac{2\left(-2b^2Bc+7Abc^2-3b^3C+3bBc^2x+7Ac^3x-13b^2cCx\right.}{}$$

input
```
Integrate[(A + B*x + C*x^2)/(-64 + b^3 + 3*b^2*c*x + 3*b*c^2*x^2 + c^3*x^3
)^(5/2),x]
```

output
```
(2*(-2*b^2*B*c + 7*A*b*c^2 - 3*b^3*C + 3*b*B*c^2*x + 7*A*c^3*x - 13*b^2*c*
C*x + 5*B*c^3*x^2 - 10*b*c^2*C*x^2 - (192*(A*c^2*(b + c*x) + B*c^2*x*(b +
c*x) - C*(-64 + b^3 + 3*b^2*c*x + 2*b*c^2*x^2)))/(-64 + b^3 + 3*b^2*c*x +
3*b*c^2*x^2 + c^3*x^3)) + Sqrt[2]*Sqrt[((-I)*(-4 + b + c*x))/(3*I + Sqrt[3
])]*Sqrt[16 + b^2 + 4*c*x + c^2*x^2 + 2*b*(2 + c*x)]*(-10*(3*I + Sqrt[3])*
(B*c - 2*b*C)*EllipticE[ArcSin[Sqrt[2*I + 2*Sqrt[3] + I*b + I*c*x]/(2*3^(1
/4))], (2*Sqrt[3])/(3*I + Sqrt[3])] + I*((20 + 7*b)*B*c - 7*A*c^2 - b*(40
+ 7*b)*C)*EllipticF[ArcSin[Sqrt[2*I + 2*Sqrt[3] + I*b + I*c*x]/(2*3^(1/4))
], (2*Sqrt[3])/(3*I + Sqrt[3])]))/(110592*c^3*Sqrt[-64 + b^3 + 3*b^2*c*x +
 3*b*c^2*x^2 + c^3*x^3])
```

Rubi [A] (warning: unable to verify)

Time = 1.49 (sec) , antiderivative size = 661, normalized size of antiderivative = 1.20, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.186, Rules used = {2459, 2393, 27, 2394, 27, 2419, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{(b^3+3b^2cx+3bc^2x^2+c^3x^3-64)^{5/2}}\,dx$$

$$\Big\downarrow \; 2459$$

$$\int \frac{A-\frac{b(Bc-bC)}{c^2}+\left(\frac{b}{c}+x\right)\left(B-\frac{2bC}{c}\right)+C\left(\frac{b}{c}+x\right)^2}{\left(c^3\left(\frac{b}{c}+x\right)^3-64\right)^{5/2}}d\left(\frac{b}{c}+x\right)$$

$$\Big\downarrow \; 2393$$

$$\frac{1}{288}\int -\frac{7\left(A-\frac{b(Bc-bC)}{c^2}\right)+5\left(B-\frac{2bC}{c}\right)\left(\frac{b}{c}+x\right)}{2\left(c^3\left(\frac{b}{c}+x\right)^3-64\right)^{3/2}}d\left(\frac{b}{c}+x\right)-\frac{c^3\left(\frac{b}{c}+x\right)\left(A-\frac{b(Bc-bC)}{c^2}+\left(\frac{b}{c}+x\right)\left(B-\frac{2bC}{c}\right)\right)+64C}{288c^3\left(c^3\left(\frac{b}{c}+x\right)^3-64\right)^{3/2}}$$

$$\Big\downarrow \; 27$$

$$-\frac{1}{576}\int \frac{7\left(A-\frac{b(Bc-bC)}{c^2}\right)+5\left(B-\frac{2bC}{c}\right)\left(\frac{b}{c}+x\right)}{\left(c^3\left(\frac{b}{c}+x\right)^3-64\right)^{3/2}}d\left(\frac{b}{c}+x\right)-\frac{c^3\left(\frac{b}{c}+x\right)\left(A-\frac{b(Bc-bC)}{c^2}+\left(\frac{b}{c}+x\right)\left(B-\frac{2bC}{c}\right)\right)+64C}{288c^3\left(c^3\left(\frac{b}{c}+x\right)^3-64\right)^{3/2}}$$

$$\Big\downarrow \; 2394$$

$$\frac{1}{576}\left(\frac{\left(\frac{b}{c}+x\right)\left(7\left(A-\frac{b(Bc-bC)}{c^2}\right)+5\left(\frac{b}{c}+x\right)\left(B-\frac{2bC}{c}\right)\right)}{96\sqrt{c^3\left(\frac{b}{c}+x\right)^3-64}}-\frac{1}{96}\int\frac{5\left(B-\frac{2bC}{c}\right)\left(\frac{b}{c}+x\right)c^2+7\left(-Cb^2+Bcb-Ac\right.}{2c^2\sqrt{c^3\left(\frac{b}{c}+x\right)^3-64}}\right.$$

$$\frac{c^3\left(\frac{b}{c}+x\right)\left(A-\frac{b(Bc-bC)}{c^2}+\left(\frac{b}{c}+x\right)\left(B-\frac{2bC}{c}\right)\right)+64C}{288c^3\left(c^3\left(\frac{b}{c}+x\right)^3-64\right)^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{576}\left(\frac{\left(\frac{b}{c}+x\right)\left(7\left(A-\frac{b(Bc-bC)}{c^2}\right)+5\left(\frac{b}{c}+x\right)\left(B-\frac{2bC}{c}\right)\right)}{96\sqrt{c^3\left(\frac{b}{c}+x\right)^3-64}}-\frac{\int\frac{7(-Cb^2+Bcb-Ac^2)+5c(Bc-2bC)\left(\frac{b}{c}+x\right)}{\sqrt{c^3\left(\frac{b}{c}+x\right)^3-64}}d\left(\frac{b}{c}+x\right)}{192c^2}\right)-$$

$$\frac{c^3\left(\frac{b}{c}+x\right)\left(A-\frac{b(Bc-bC)}{c^2}+\left(\frac{b}{c}+x\right)\left(B-\frac{2bC}{c}\right)\right)+64C}{288c^3\left(c^3\left(\frac{b}{c}+x\right)^3-64\right)^{3/2}}$$

$$\Big\downarrow \; 2419$$

$$\frac{1}{576}\left(\frac{\left(\frac{b}{c}+x\right)\left(7\left(A-\frac{b(Bc-bC)}{c^2}\right)+5\left(\frac{b}{c}+x\right)\left(B-\frac{2bC}{c}\right)\right)}{96\sqrt{c^3\left(\frac{b}{c}+x\right)^3-64}}-\frac{\left(7\left(-Ac^2+b^2(-C)+bBc\right)+20\left(1+\sqrt{3}\right)\left(Bc-2b\right.\right.}{}\right.$$

$$\frac{c^3\left(\frac{b}{c}+x\right)\left(A-\frac{b(Bc-bC)}{c^2}+\left(\frac{b}{c}+x\right)\left(B-\frac{2bC}{c}\right)\right)+64C}{288c^3\left(c^3\left(\frac{b}{c}+x\right)^3-64\right)^{3/2}}$$

$$\Big\downarrow \; 760$$

$$\frac{1}{576}\left(\frac{\left(\frac{b}{c}+x\right)\left(7\left(A-\frac{b(Bc-bC)}{c^2}\right)+5\left(\frac{b}{c}+x\right)\left(B-\frac{2bC}{c}\right)\right)}{96\sqrt{c^3\left(\frac{b}{c}+x\right)^3-64}}-\frac{-5(Bc-2bC)\int\frac{4\left(1+\sqrt{3}\right)-c\left(\frac{b}{c}+x\right)}{\sqrt{c^3\left(\frac{b}{c}+x\right)^3-64}}d\left(\frac{b}{c}+x\right)-\frac{\sqrt{2-\sqrt{3}}\cdots}{\cdots}}{\cdots}\right.$$

$$\frac{c^3\left(\frac{b}{c}+x\right)\left(A-\frac{b(Bc-bC)}{c^2}+\left(\frac{b}{c}+x\right)\left(B-\frac{2bC}{c}\right)\right)+64C}{288c^3\left(c^3\left(\frac{b}{c}+x\right)^3-64\right)^{3/2}}$$

$\Big\downarrow$ 2418

$$\frac{1}{576}\left(\frac{\left(\frac{b}{c}+x\right)\left(7\left(A-\frac{b(Bc-bC)}{c^2}\right)+5\left(\frac{b}{c}+x\right)\left(B-\frac{2bC}{c}\right)\right)}{96\sqrt{c^3\left(\frac{b}{c}+x\right)^3-64}}-\frac{-\frac{\sqrt{2-\sqrt{3}}\left(4-c\left(\frac{b}{c}+x\right)\right)\sqrt{\frac{c^2\left(\frac{b}{c}+x\right)^2+4c\left(\frac{b}{c}+x\right)+16}{\left(4\left(1-\sqrt{3}\right)-c\left(\frac{b}{c}+x\right)\right)^2}}\left(7\left(-Ac^2+b^2\left(\cdots\right.\right.\right.}{\sqrt[4]{3}c\sqrt{-\frac{\cdots}{\left(4\cdots\right.}}}}{\cdots}\right.$$

$$\frac{c^3\left(\frac{b}{c}+x\right)\left(A-\frac{b(Bc-bC)}{c^2}+\left(\frac{b}{c}+x\right)\left(B-\frac{2bC}{c}\right)\right)+64C}{288c^3\left(c^3\left(\frac{b}{c}+x\right)^3-64\right)^{3/2}}$$

input
```
Int[(A + B*x + C*x^2)/(-64 + b^3 + 3*b^2*c*x + 3*b*c^2*x^2 + c^3*x^3)^(5/2
),x]
```

output
```
-1/288*(64*C + c^3*(b/c + x)*(A - (b*(B*c - b*C))/c^2 + (B - (2*b*C)/c)*(b
/c + x)))/(c^3*(-64 + c^3*(b/c + x)^3)^(3/2)) + (((b/c + x)*(7*(A - (b*(B*
c - b*C))/c^2) + 5*(B - (2*b*C)/c)*(b/c + x)))/(96*Sqrt[-64 + c^3*(b/c + x
)^3]) - (-5*(B*c - 2*b*C)*((2*Sqrt[-64 + c^3*(b/c + x)^3])/(c*(4*(1 - Sqrt
[3]) - c*(b/c + x))) - (2*3^(1/4)*Sqrt[2 + Sqrt[3]]*(4 - c*(b/c + x))*Sqrt
[(16 + 4*c*(b/c + x) + c^2*(b/c + x)^2)/(4*(1 - Sqrt[3]) - c*(b/c + x))^2]
*EllipticE[ArcSin[(4*(1 + Sqrt[3]) - c*(b/c + x))/(4*(1 - Sqrt[3]) - c*(b/
c + x))], -7 + 4*Sqrt[3]])/(c*Sqrt[-((4 - c*(b/c + x))/(4*(1 - Sqrt[3]) -
c*(b/c + x))^2)]*Sqrt[-64 + c^3*(b/c + x)^3])) - (Sqrt[2 - Sqrt[3]]*(20*(1
 + Sqrt[3])*(B*c - 2*b*C) + 7*(b*B*c - A*c^2 - b^2*C))*(4 - c*(b/c + x))*S
qrt[(16 + 4*c*(b/c + x) + c^2*(b/c + x)^2)/(4*(1 - Sqrt[3]) - c*(b/c + x))
^2]*EllipticF[ArcSin[(4*(1 + Sqrt[3]) - c*(b/c + x))/(4*(1 - Sqrt[3]) - c*
(b/c + x))], -7 + 4*Sqrt[3]])/(3^(1/4)*c*Sqrt[-((4 - c*(b/c + x))/(4*(1 -
Sqrt[3]) - c*(b/c + x))^2)]*Sqrt[-64 + c^3*(b/c + x)^3]))/(192*c^2))/576
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 2393
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   In
t[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(
p + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n
, 0] && LtQ[p, -1]
```

rule 2394
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   Int[ExpandToSum[n
*(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x
] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

rule 2419
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(c*r - (1 + Sqrt[3])*d*s)/r
  Int[1/Sqrt[a + b*x^3], x], x] + Simp[d/r   Int[((1 + Sqrt[3])*s + r*x)/Sq
rt[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] && NeQ[b*c^3 -
2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

rule 2459
```
Int[(Pn_)^(p_.)*(Qx_), x_Symbol] :> With[{S = Coeff[Pn, x, Expon[Pn, x] - 1
]/(Expon[Pn, x]*Coeff[Pn, x, Expon[Pn, x]])}, Subst[Int[ExpandToSum[Pn /. x
 -> x - S, x]^p*ExpandToSum[Qx /. x -> x - S, x], x], x, x + S] /; Binomial
Q[Pn /. x -> x - S, x] || (IntegerQ[Expon[Pn, x]/2] && TrinomialQ[Pn /. x -
> x - S, x])] /; FreeQ[p, x] && PolyQ[Pn, x] && GtQ[Expon[Pn, x], 2] && NeQ
[Coeff[Pn, x, Expon[Pn, x] - 1], 0] && PolyQ[Qx, x] &&  !(MonomialQ[Qx, x]
&& IGtQ[p, 0])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 987 vs. $2(485) = 970$.

Time = 1.62 (sec) , antiderivative size = 988, normalized size of antiderivative = 1.80

method	result
elliptic	$\frac{\left(-\frac{(Bc-2bC)x^2}{288c^7}-\frac{\left(A c^2+bBc-3b^2C\right)x}{288c^8}-\frac{c^2bA-C b^3+64C}{288c^9}\right)\sqrt{c^3x^3+3b c^2x^2+3b^2cx+b^3-64}}{\left(x^3+\frac{3b x^2}{c}+\frac{3b^2x}{c^2}+\frac{b^3-64}{c^3}\right)^2}-\frac{2c^3\left(-\frac{5(Bc-2bC)x^2}{110592c^4}-\frac{\left(7A c^2+3bBc-1\cdots\right.}{110592c^5\cdots}\cdots\right.}{\sqrt{\left(x^3+\frac{3b x^2}{c}+\frac{3b^2\cdots}{c^2\cdots}\cdots\right.}}$
default	Expression too large to display

input
```
int((C*x^2+B*x+A)/(c^3*x^3+3*b*c^2*x^2+3*b^2*c*x+b^3-64)^(5/2),x,method=_R
ETURNVERBOSE)
```

output
```
(-1/288*(B*c-2*C*b)/c^7*x^2-1/288*(A*c^2+B*b*c-3*C*b^2)/c^8*x-1/288*(A*b*c
^2-C*b^3+64*C)/c^9)*(c^3*x^3+3*b*c^2*x^2+3*b^2*c*x+b^3-64)^(1/2)/(x^3+3*b/
c*x^2+3*b^2/c^2*x+(b^3-64)/c^3)^2-2*c^3*(-5/110592*(B*c-2*C*b)/c^4*x^2-1/1
10592/c^5*(7*A*c^2+3*B*b*c-13*C*b^2)*x-1/110592*b/c^6*(7*A*c^2-2*B*b*c-3*C
*b^2))/((x^3+3*b/c*x^2+3*b^2/c^2*x+(b^3-64)/c^3)*c^3)^(1/2)+2*(1/36864/c^2
*(7*A*c^2-2*B*b*c-3*C*b^2)-1/55296/c^2*(7*A*c^2+3*B*b*c-13*C*b^2))*((-b-2-
2*I*3^(1/2))/c+(b-4)/c)*((x+(b-4)/c)/((-b-2-2*I*3^(1/2))/c+(b-4)/c))^(1/2)
*((x-(-b-2+2*I*3^(1/2))/c)/(-(b-4)/c-(-b-2+2*I*3^(1/2))/c))^(1/2)*((x-(-b-
2-2*I*3^(1/2))/c)/(-(b-4)/c-(-b-2-2*I*3^(1/2))/c))^(1/2)/(c^3*x^3+3*b*c^2*
x^2+3*b^2*c*x+b^3-64)^(1/2)*EllipticF(((x+(b-4)/c)/((-b-2-2*I*3^(1/2))/c+(
b-4)/c))^(1/2),((-(b-4)/c-(-b-2-2*I*3^(1/2))/c)/(-(b-4)/c-(-b-2+2*I*3^(1/2
))/c))^(1/2))-5/55296/c*(B*c-2*C*b)*((-b-2-2*I*3^(1/2))/c+(b-4)/c)*((x+(b-
4)/c)/((-b-2-2*I*3^(1/2))/c+(b-4)/c))^(1/2)*((x-(-b-2+2*I*3^(1/2))/c)/(-(b
-4)/c-(-b-2+2*I*3^(1/2))/c))^(1/2)*((x-(-b-2-2*I*3^(1/2))/c)/(-(b-4)/c-(-b
-2-2*I*3^(1/2))/c))^(1/2)/(c^3*x^3+3*b*c^2*x^2+3*b^2*c*x+b^3-64)^(1/2)*((-
(b-4)/c-(-b-2+2*I*3^(1/2))/c)*EllipticE(((x+(b-4)/c)/((-b-2-2*I*3^(1/2))/c
+(b-4)/c))^(1/2),((-(b-4)/c-(-b-2-2*I*3^(1/2))/c)/(-(b-4)/c-(-b-2+2*I*3^(1
/2))/c))^(1/2))+(-b-2+2*I*3^(1/2))/c*EllipticF(((x+(b-4)/c)/((-b-2-2*I*3^(
1/2))/c+(b-4)/c))^(1/2),((-(b-4)/c-(-b-2-2*I*3^(1/2))/c)/(-(b-4)/c-(-b-2+2
*I*3^(1/2))/c))^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 923 vs. $2(439) = 878$.

Time = 0.11 (sec) , antiderivative size = 923, normalized size of antiderivative = 1.68

$$\int \frac{A + Bx + Cx^2}{(-64 + b^3 + 3b^2cx + 3bc^2x^2 + c^3x^3)^{5/2}} \, dx = \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)/(c^3*x^3+3*b*c^2*x^2+3*b^2*c*x+b^3-64)^(5/2),x, al
gorithm="fricas")
```

output
```
1/55296*(7*(C*b^8 + (C*b^2*c^6 - B*b*c^7 + A*c^8)*x^6 - 128*C*b^5 + 6*(C*b
^3*c^5 - B*b^2*c^6 + A*b*c^7)*x^5 + 15*(C*b^4*c^4 - B*b^3*c^5 + A*b^2*c^6)
*x^4 + 4*((5*A*b^3 - 32*A)*c^5 - (5*B*b^4 - 32*B*b)*c^4 + (5*C*b^5 - 32*C*
b^2)*c^3)*x^3 + 4096*C*b^2 + (A*b^6 - 128*A*b^3 + 4096*A)*c^2 + 3*((5*A*b^
4 - 128*A*b)*c^4 - (5*B*b^5 - 128*B*b^2)*c^3 + (5*C*b^6 - 128*C*b^3)*c^2)*
x^2 - (B*b^7 - 128*B*b^4 + 4096*B*b)*c + 6*((A*b^5 - 64*A*b^2)*c^3 - (B*b^
6 - 64*B*b^3)*c^2 + (C*b^7 - 64*C*b^4)*c)*x)*sqrt(c^3)*weierstrassPInverse
(0, 256/c^3, (c*x + b)/c) - 5*((2*C*b*c^7 - B*c^8)*x^6 + 6*(2*C*b^2*c^6 -
B*b*c^7)*x^5 + 15*(2*C*b^3*c^5 - B*b^2*c^6)*x^4 - 4*((5*B*b^3 - 32*B)*c^5
- 2*(5*C*b^4 - 32*C*b)*c^4)*x^3 - (B*b^6 - 128*B*b^3 + 4096*B)*c^2 - 3*((5
*B*b^4 - 128*B*b)*c^4 - 2*(5*C*b^5 - 128*C*b^2)*c^3)*x^2 + 2*(C*b^7 - 128*
C*b^4 + 4096*C*b)*c - 6*((B*b^5 - 64*B*b^2)*c^3 - 2*(C*b^6 - 64*C*b^3)*c^2
)*x)*sqrt(c^3)*weierstrassZeta(0, 256/c^3, weierstrassPInverse(0, 256/c^3,
 (c*x + b)/c)) - sqrt(c^3*x^3 + 3*b*c^2*x^2 + 3*b^2*c*x + b^3 - 64)*(5*(2*
C*b*c^7 - B*c^8)*x^5 - (7*A*b^4 - 640*A*b)*c^4 + (43*C*b^2*c^6 - 18*B*b*c^
7 - 7*A*c^8)*x^4 + 2*(B*b^5 - 64*B*b^2)*c^3 + 2*(36*C*b^3*c^5 - 11*B*b^2*c
^6 - 14*A*b*c^7)*x^3 + 3*(C*b^6 - 128*C*b^3 + 4096*C)*c^2 - 2*(21*A*b^2*c^
6 + 4*(B*b^3 - 64*B)*c^5 - (29*C*b^4 - 512*C*b)*c^4)*x^2 - (4*(7*A*b^3 - 1
60*A)*c^5 - 3*(B*b^4 + 128*B*b)*c^4 - 22*(C*b^5 - 64*C*b^2)*c^3)*x))/(c^11
*x^6 + 6*b*c^10*x^5 + 15*b^2*c^9*x^4 + 4*(5*b^3 - 32)*c^8*x^3 + 3*(5*b^...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(-64+b^3+3b^2cx+3bc^2x^2+c^3x^3)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((C*x**2+B*x+A)/(c**3*x**3+3*b*c**2*x**2+3*b**2*c*x+b**3-64)**(5/
2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{(-64+b^3+3b^2cx+3bc^2x^2+c^3x^3)^{5/2}}\,dx = \int \frac{Cx^2+Bx+A}{(c^3x^3+3\,bc^2x^2+3\,b^2cx+b^3-64)^{\frac{5}{2}}}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(c^3*x^3+3*b*c^2*x^2+3*b^2*c*x+b^3-64)^(5/2),x, al
gorithm="maxima")
```

output
```
integrate((C*x^2 + B*x + A)/(c^3*x^3 + 3*b*c^2*x^2 + 3*b^2*c*x + b^3 - 64)
^(5/2), x)
```

Giac [F]

$$\int \frac{A+Bx+Cx^2}{(-64+b^3+3b^2cx+3bc^2x^2+c^3x^3)^{5/2}}\,dx = \int \frac{Cx^2+Bx+A}{(c^3x^3+3\,bc^2x^2+3\,b^2cx+b^3-64)^{\frac{5}{2}}}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(c^3*x^3+3*b*c^2*x^2+3*b^2*c*x+b^3-64)^(5/2),x, al
gorithm="giac")
```

output
```
integrate((C*x^2 + B*x + A)/(c^3*x^3 + 3*b*c^2*x^2 + 3*b^2*c*x + b^3 - 64)
^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(-64+b^3+3b^2cx+3bc^2x^2+c^3x^3)^{5/2}}\,dx = \int \frac{C\,x^2+B\,x+A}{\left(b^3+3\,b^2\,c\,x+3\,b\,c^2\,x^2+c^3\,x^3-64\right)^{5/2}}\,dx$$

input
```
int((A + B*x + C*x^2)/(b^3 + c^3*x^3 + 3*b*c^2*x^2 + 3*b^2*c*x - 64)^(5/2)
,x)
```

output
```
int((A + B*x + C*x^2)/(b^3 + c^3*x^3 + 3*b*c^2*x^2 + 3*b^2*c*x - 64)^(5/2)
, x)
```

Reduce [F]

$$\int \frac{A+Bx+Cx^2}{(-64+b^3+3b^2cx+3bc^2x^2+c^3x^3)^{5/2}}\,dx = \text{too large to display}$$

input
```
int((C*x^2+B*x+A)/(c^3*x^3+3*b*c^2*x^2+3*b^2*c*x+b^3-64)^(5/2),x)
```

output
```
( - 2*sqrt(b**3 + 3*b**2*c*x + 3*b*c**2*x**2 + c**3*x**3 - 64) + 18*int(sq
rt(b**3 + 3*b**2*c*x + 3*b*c**2*x**2 + c**3*x**3 - 64)/(b**9 + 9*b**8*c*x
+ 36*b**7*c**2*x**2 + 84*b**6*c**3*x**3 - 192*b**6 + 126*b**5*c**4*x**4 -
1152*b**5*c*x + 126*b**4*c**5*x**5 - 2880*b**4*c**2*x**2 + 84*b**3*c**6*x*
*6 - 3840*b**3*c**3*x**3 + 12288*b**3 + 36*b**2*c**7*x**7 - 2880*b**2*c**4
*x**4 + 36864*b**2*c*x + 9*b*c**8*x**8 - 1152*b*c**5*x**5 + 36864*b*c**2*x
**2 + c**9*x**9 - 192*c**6*x**6 + 12288*c**3*x**3 - 262144),x)*a*b**6*c**2
 + 108*int(sqrt(b**3 + 3*b**2*c*x + 3*b*c**2*x**2 + c**3*x**3 - 64)/(b**9
+ 9*b**8*c*x + 36*b**7*c**2*x**2 + 84*b**6*c**3*x**3 - 192*b**6 + 126*b**5
*c**4*x**4 - 1152*b**5*c*x + 126*b**4*c**5*x**5 - 2880*b**4*c**2*x**2 + 84
*b**3*c**6*x**6 - 3840*b**3*c**3*x**3 + 12288*b**3 + 36*b**2*c**7*x**7 - 2
880*b**2*c**4*x**4 + 36864*b**2*c*x + 9*b*c**8*x**8 - 1152*b*c**5*x**5 + 3
6864*b*c**2*x**2 + c**9*x**9 - 192*c**6*x**6 + 12288*c**3*x**3 - 262144),x
)*a*b**5*c**3*x + 270*int(sqrt(b**3 + 3*b**2*c*x + 3*b*c**2*x**2 + c**3*x*
*3 - 64)/(b**9 + 9*b**8*c*x + 36*b**7*c**2*x**2 + 84*b**6*c**3*x**3 - 192*
b**6 + 126*b**5*c**4*x**4 - 1152*b**5*c*x + 126*b**4*c**5*x**5 - 2880*b**4
*c**2*x**2 + 84*b**3*c**6*x**6 - 3840*b**3*c**3*x**3 + 12288*b**3 + 36*b**
2*c**7*x**7 - 2880*b**2*c**4*x**4 + 36864*b**2*c*x + 9*b*c**8*x**8 - 1152*
b*c**5*x**5 + 36864*b*c**2*x**2 + c**9*x**9 - 192*c**6*x**6 + 12288*c**3*x
**3 - 262144),x)*a*b**4*c**4*x**2 + 360*int(sqrt(b**3 + 3*b**2*c*x + 3*...
```

3.79 $\int \frac{A+Bx+Cx^2}{(-64+b^3+3b^2cx+3bc^2x^2+c^3x^3)^{7/2}}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 609

$$\int \frac{A+Bx+Cx^2}{(-64+b^3+3b^2cx+3bc^2x^2+c^3x^3)^{7/2}}\,dx =$$

$$-\frac{2C}{15c^3(-64+(b+cx)^3)^{5/2}}-\frac{(b+cx)(Ac^2-b^2C+c(Bc-2bC)x)}{480c^3(-64+(b+cx)^3)^{5/2}}$$

$$-\frac{(b+cx)(13(bBc-Ac^2-b^2C)-11(Bc-2bC)(b+cx))}{276480c^3(-64+(b+cx)^3)^{3/2}}$$

$$+\frac{(b+cx)(91(bBc-Ac^2-b^2C)-55(Bc-2bC)(b+cx))}{53084160c^3\sqrt{-64+(b+cx)^3}}$$

$$-\frac{11(Bc-2bC)\sqrt{-64+(b+cx)^3}}{10616832c^3\left(4-4\sqrt{3}-b-cx\right)}$$

$$+\frac{11\sqrt{2+\sqrt{3}}(Bc-2bC)(4-b-cx)\sqrt{\frac{16+4(b+cx)+(b+cx)^2}{\left(4-4\sqrt{3}-b-cx\right)^2}}E\left(\arcsin\left(\frac{4+4\sqrt{3}-b-cx}{4-4\sqrt{3}-b-cx}\right)|-7+4\sqrt{3}\right)}{3538944\ 3^{3/4}c^3\sqrt{-\frac{4-b-cx}{\left(4-4\sqrt{3}-b-cx\right)^2}}\sqrt{-64+(b+cx)^3}}$$

$$-\frac{\sqrt{2-\sqrt{3}}\left(220\left(1+\sqrt{3}\right)(Bc-2bC)+91(bBc-Ac^2-b^2C)\right)(4-b-cx)\sqrt{\frac{16+4(b+cx)+(b+cx)^2}{\left(4-4\sqrt{3}-b-cx\right)^2}}\,\text{EllipticF}(}{106168320\sqrt[4]{3}c^3\sqrt{-\frac{4-b-cx}{\left(4-4\sqrt{3}-b-cx\right)^2}}\sqrt{-64+(b+cx)^3}}$$

output
```
-2/15*C/c^3/(-64+(c*x+b)^3)^(5/2)-1/480*(c*x+b)*(A*c^2-C*b^2+c*(B*c-2*C*b)
*x)/c^3/(-64+(c*x+b)^3)^(5/2)-1/276480*(c*x+b)*(-13*A*c^2+13*B*b*c-13*C*b^
2-11*(B*c-2*C*b)*(c*x+b))/c^3/(-64+(c*x+b)^3)^(3/2)+1/53084160*(c*x+b)*(-9
1*A*c^2+91*B*b*c-91*C*b^2-55*(B*c-2*C*b)*(c*x+b))/c^3/(-64+(c*x+b)^3)^(1/2
)-11/10616832*(B*c-2*C*b)*(-64+(c*x+b)^3)^(1/2)/c^3/(4-4*3^(1/2)-b-c*x)+11
/10616832*3^(1/4)*(1/2*6^(1/2)+1/2*2^(1/2))*(B*c-2*C*b)*(-c*x-b+4)*((16+4*
c*x+4*b+(c*x+b)^2)/(4-4*3^(1/2)-b-c*x)^2)^(1/2)*EllipticE((4+4*3^(1/2)-b-c
*x)/(4-4*3^(1/2)-b-c*x),2*I-I*3^(1/2))/c^3/(-(-c*x-b+4)/(4-4*3^(1/2)-b-c*x
)^2)^(1/2)/(-64+(c*x+b)^3)^(1/2)-1/318504960*(1/2*6^(1/2)-1/2*2^(1/2))*(22
0*(1+3^(1/2))*(B*c-2*C*b)-91*A*c^2+91*B*b*c-91*C*b^2)*(-c*x-b+4)*((16+4*c*
x+4*b+(c*x+b)^2)/(4-4*3^(1/2)-b-c*x)^2)^(1/2)*EllipticF((4+4*3^(1/2)-b-c*x
)/(4-4*3^(1/2)-b-c*x),2*I-I*3^(1/2))*3^(3/4)/c^3/(-(-c*x-b+4)/(4-4*3^(1/2)
-b-c*x)^2)^(1/2)/(-64+(c*x+b)^3)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 15.14 (sec) , antiderivative size = 467, normalized size of antiderivative = 0.77

$$\int \frac{A+Bx+Cx^2}{(-64+b^3+3b^2cx+3bc^2x^2+c^3x^3)^{7/2}}\,dx = \frac{2\Big(36b^2Bc-91Abc^2+19b^3C-19bBc^2x-91Ac^3x+129b}{}$$

input
```
Integrate[(A + B*x + C*x^2)/(-64 + b^3 + 3*b^2*c*x + 3*b*c^2*x^2 + c^3*x^3
)^(7/2),x]
```

output
```
(2*(36*b^2*B*c - 91*A*b*c^2 + 19*b^3*C - 19*b*B*c^2*x - 91*A*c^3*x + 129*b
^2*c*C*x - 55*B*c^3*x^2 + 110*b*c^2*C*x^2 - (192*(b + c*x)*(9*b^2*C - c^2*
(13*A + 11*B*x) + 2*b*c*(B + 11*C*x)))/(-64 + b^3 + 3*b^2*c*x + 3*b*c^2*x^
2 + c^3*x^3) - (110592*(A*c^2*(b + c*x) + B*c^2*x*(b + c*x) - C*(-64 + b^3
 + 3*b^2*c*x + 2*b*c^2*x^2)))/(-64 + b^3 + 3*b^2*c*x + 3*b*c^2*x^2 + c^3*x
^3)^2) + Sqrt[2]*Sqrt[((-I)*(-4 + b + c*x))/(3*I + Sqrt[3])]*Sqrt[16 + b^2
 + 4*c*x + c^2*x^2 + 2*b*(2 + c*x)]*(110*(3*I + Sqrt[3])*(B*c - 2*b*C)*Ell
ipticE[ArcSin[Sqrt[2*I + 2*Sqrt[3] + I*b + I*c*x]/(2*3^(1/4))], (2*Sqrt[3]
)/(3*I + Sqrt[3])] - I*((220 + 91*b)*B*c - 91*A*c^2 - b*(440 + 91*b)*C)*El
lipticF[ArcSin[Sqrt[2*I + 2*Sqrt[3] + I*b + I*c*x]/(2*3^(1/4))], (2*Sqrt[3
])/(3*I + Sqrt[3])]))/(106168320*c^3*Sqrt[-64 + b^3 + 3*b^2*c*x + 3*b*c^2*
x^2 + c^3*x^3])
```

Rubi [A] (warning: unable to verify)

Time = 1.70 (sec) , antiderivative size = 733, normalized size of antiderivative = 1.20, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {2459, 2393, 27, 2394, 27, 2394, 27, 2419, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{(b^3+3b^2cx+3bc^2x^2+c^3x^3-64)^{7/2}}dx$$

$$\Big\downarrow \text{2459}$$

$$\int \frac{A-\frac{b(Bc-bC)}{c^2}+\left(\frac{b}{c}+x\right)\left(B-\frac{2bC}{c}\right)+C\left(\frac{b}{c}+x\right)^2}{\left(c^3\left(\frac{b}{c}+x\right)^3-64\right)^{7/2}}d\left(\frac{b}{c}+x\right)$$

$$\Big\downarrow \text{2393}$$

$$\frac{1}{480}\int -\frac{13\left(A-\frac{b(Bc-bC)}{c^2}\right)+11\left(B-\frac{2bC}{c}\right)\left(\frac{b}{c}+x\right)}{2\left(c^3\left(\frac{b}{c}+x\right)^3-64\right)^{5/2}}d\left(\frac{b}{c}+x\right)-\frac{c^3\left(\frac{b}{c}+x\right)\left(A-\frac{b(Bc-bC)}{c^2}+\left(\frac{b}{c}+x\right)\left(B-\frac{2bC}{c}\right)\right)+64C}{480c^3\left(c^3\left(\frac{b}{c}+x\right)^3-64\right)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{960}\int \frac{13\left(A-\frac{b(Bc-bC)}{c^2}\right)+11\left(B-\frac{2bC}{c}\right)\left(\frac{b}{c}+x\right)}{\left(c^3\left(\frac{b}{c}+x\right)^3-64\right)^{5/2}}d\left(\frac{b}{c}+x\right)-\frac{c^3\left(\frac{b}{c}+x\right)\left(A-\frac{b(Bc-bC)}{c^2}+\left(\frac{b}{c}+x\right)\left(B-\frac{2bC}{c}\right)\right)+64C}{480c^3\left(c^3\left(\frac{b}{c}+x\right)^3-64\right)^{5/2}}$$

$\downarrow$ 2394

$$\frac{1}{960}\left(\frac{\left(\frac{b}{c}+x\right)\left(13\left(A-\frac{b(Bc-bC)}{c^2}\right)+11\left(\frac{b}{c}+x\right)\left(B-\frac{2bC}{c}\right)\right)}{288\left(c^3\left(\frac{b}{c}+x\right)^3-64\right)^{3/2}}-\frac{1}{288}\int\frac{91\left(-Cb^2+Bcb-Ac^2\right)-55c^2\left(B-\frac{2bC}{c}\right)}{2c^2\left(c^3\left(\frac{b}{c}+x\right)^3-64\right)^{3/2}}\right.$$
$$\frac{c^3\left(\frac{b}{c}+x\right)\left(A-\frac{b(Bc-bC)}{c^2}+\left(\frac{b}{c}+x\right)\left(B-\frac{2bC}{c}\right)\right)+64C}{480c^3\left(c^3\left(\frac{b}{c}+x\right)^3-64\right)^{5/2}}$$

$\downarrow$ 27

$$\frac{1}{960}\left(\frac{\left(\frac{b}{c}+x\right)\left(13\left(A-\frac{b(Bc-bC)}{c^2}\right)+11\left(\frac{b}{c}+x\right)\left(B-\frac{2bC}{c}\right)\right)}{288\left(c^3\left(\frac{b}{c}+x\right)^3-64\right)^{3/2}}-\frac{\int\frac{91\left(-Cb^2+Bcb-Ac^2\right)-55c(Bc-2bC)\left(\frac{b}{c}+x\right)}{\left(c^3\left(\frac{b}{c}+x\right)^3-64\right)^{3/2}}d\left(\frac{b}{c}+x\right)}{576c^2}\right)$$
$$\frac{c^3\left(\frac{b}{c}+x\right)\left(A-\frac{b(Bc-bC)}{c^2}+\left(\frac{b}{c}+x\right)\left(B-\frac{2bC}{c}\right)\right)+64C}{480c^3\left(c^3\left(\frac{b}{c}+x\right)^3-64\right)^{5/2}}$$

$\downarrow$ 2394

$$\frac{1}{960}\left(\frac{\left(\frac{b}{c}+x\right)\left(13\left(A-\frac{b(Bc-bC)}{c^2}\right)+11\left(\frac{b}{c}+x\right)\left(B-\frac{2bC}{c}\right)\right)}{288\left(c^3\left(\frac{b}{c}+x\right)^3-64\right)^{3/2}}-\frac{\frac{1}{96}\int-\frac{91\left(-Cb^2+Bcb-Ac^2\right)+55c(Bc-2bC)\left(\frac{b}{c}+x\right)}{2\sqrt{c^3\left(\frac{b}{c}+x\right)^3-64}}d\left(\frac{b}{c}+\right.}{57}\right.$$
$$\frac{c^3\left(\frac{b}{c}+x\right)\left(A-\frac{b(Bc-bC)}{c^2}+\left(\frac{b}{c}+x\right)\left(B-\frac{2bC}{c}\right)\right)+64C}{480c^3\left(c^3\left(\frac{b}{c}+x\right)^3-64\right)^{5/2}}$$

$\downarrow$ 27

$$\frac{1}{960}\left(\frac{\left(\frac{b}{c}+x\right)\left(13\left(A-\frac{b(Bc-bC)}{c^2}\right)+11\left(\frac{b}{c}+x\right)\left(B-\frac{2bC}{c}\right)\right)}{288\left(c^3\left(\frac{b}{c}+x\right)^3-64\right)^{3/2}}-\frac{-\frac{1}{192}\int\frac{91(-Cb^2+Bcb-Ac^2)+55c(Bc-2bC)\left(\frac{b}{c}+x\right)}{\sqrt{c^3\left(\frac{b}{c}+x\right)^3-64}}d\left(\frac{b}{c}+\right.}{5}\right.$$

$$\frac{c^3\left(\frac{b}{c}+x\right)\left(A-\frac{b(Bc-bC)}{c^2}+\left(\frac{b}{c}+x\right)\left(B-\frac{2bC}{c}\right)\right)+64C}{480c^3\left(c^3\left(\frac{b}{c}+x\right)^3-64\right)^{5/2}}$$

$$\Big\downarrow \text{2419}$$

$$\frac{1}{960}\left(\frac{\left(\frac{b}{c}+x\right)\left(13\left(A-\frac{b(Bc-bC)}{c^2}\right)+11\left(\frac{b}{c}+x\right)\left(B-\frac{2bC}{c}\right)\right)}{288\left(c^3\left(\frac{b}{c}+x\right)^3-64\right)^{3/2}}-\frac{\frac{1}{192}\left(55(Bc-2bC)\int\frac{4\left(1+\sqrt{3}\right)-c\left(\frac{b}{c}+x\right)}{\sqrt{c^3\left(\frac{b}{c}+x\right)^3-64}}d\left(\frac{b}{c}+x\right)-\right.}{}\right.$$

$$\frac{c^3\left(\frac{b}{c}+x\right)\left(A-\frac{b(Bc-bC)}{c^2}+\left(\frac{b}{c}+x\right)\left(B-\frac{2bC}{c}\right)\right)+64C}{480c^3\left(c^3\left(\frac{b}{c}+x\right)^3-64\right)^{5/2}}$$

$$\Big\downarrow \text{760}$$

$$\frac{1}{960}\left(\frac{\left(\frac{b}{c}+x\right)\left(13\left(A-\frac{b(Bc-bC)}{c^2}\right)+11\left(\frac{b}{c}+x\right)\left(B-\frac{2bC}{c}\right)\right)}{288\left(c^3\left(\frac{b}{c}+x\right)^3-64\right)^{3/2}}-\frac{\frac{1}{192}\left(55(Bc-2bC)\int\frac{4\left(1+\sqrt{3}\right)-c\left(\frac{b}{c}+x\right)}{\sqrt{c^3\left(\frac{b}{c}+x\right)^3-64}}d\left(\frac{b}{c}+x\right)+\right.}{}\right.$$

$$\frac{c^3\left(\frac{b}{c}+x\right)\left(A-\frac{b(Bc-bC)}{c^2}+\left(\frac{b}{c}+x\right)\left(B-\frac{2bC}{c}\right)\right)+64C}{480c^3\left(c^3\left(\frac{b}{c}+x\right)^3-64\right)^{5/2}}$$

$$\Big\downarrow \text{2418}$$

$$\frac{1}{960}\left(\frac{\left(\frac{b}{c}+x\right)\left(13\left(A-\frac{b(Bc-bC)}{c^2}\right)+11\left(\frac{b}{c}+x\right)\left(B-\frac{2bC}{c}\right)\right)}{288\left(c^3\left(\frac{b}{c}+x\right)^3-64\right)^{3/2}}-\frac{\frac{1}{192}\left(\frac{\sqrt{2-\sqrt{3}}\left(4-c\left(\frac{b}{c}+x\right)\right)\sqrt{\frac{c^2\left(\frac{b}{c}+x\right)^2+4c\left(\frac{b}{c}+x\right)+16}{\left(4\left(1-\sqrt{3}\right)-c\left(\frac{b}{c}+x\right)\right)^2}}\left(91(-\cdots\right.}{\sqrt[4]{3}\cdots}\right.}{\cdots}\right.$$

$$\frac{c^3\left(\frac{b}{c}+x\right)\left(A-\frac{b(Bc-bC)}{c^2}+\left(\frac{b}{c}+x\right)\left(B-\frac{2bC}{c}\right)\right)+64C}{480c^3\left(c^3\left(\frac{b}{c}+x\right)^3-64\right)^{5/2}}$$

input
```
Int[(A + B*x + C*x^2)/(-64 + b^3 + 3*b^2*c*x + 3*b*c^2*x^2 + c^3*x^3)^(7/2
),x]
```

output
```
-1/480*(64*C + c^3*(b/c + x)*(A - (b*(B*c - b*C))/c^2 + (B - (2*b*C)/c)*(b
/c + x)))/(c^3*(-64 + c^3*(b/c + x)^3)^(5/2)) + (((b/c + x)*(13*(A - (b*(B
*c - b*C))/c^2) + 11*(B - (2*b*C)/c)*(b/c + x)))/(288*(-64 + c^3*(b/c + x)
^3)^(3/2)) - (-1/96*((b/c + x)*(91*(b*B*c - A*c^2 - b^2*C) - 55*c*(B*c - 2
*b*C)*(b/c + x)))/Sqrt[-64 + c^3*(b/c + x)^3] + (55*(B*c - 2*b*C)*((2*Sqrt
[-64 + c^3*(b/c + x)^3])/(c*(4*(1 - Sqrt[3]) - c*(b/c + x))) - (2*3^(1/4)*
Sqrt[2 + Sqrt[3]]*(4 - c*(b/c + x))*Sqrt[(16 + 4*c*(b/c + x) + c^2*(b/c +
x)^2)/(4*(1 - Sqrt[3]) - c*(b/c + x))^2]*EllipticE[ArcSin[(4*(1 + Sqrt[3])
 - c*(b/c + x))/(4*(1 - Sqrt[3]) - c*(b/c + x))], -7 + 4*Sqrt[3]])/(c*Sqrt
[-((4 - c*(b/c + x))/(4*(1 - Sqrt[3]) - c*(b/c + x))^2)]*Sqrt[-64 + c^3*(b
/c + x)^3])) + (Sqrt[2 - Sqrt[3]]*(220*(1 + Sqrt[3])*(B*c - 2*b*C) + 91*(b
*B*c - A*c^2 - b^2*C))*(4 - c*(b/c + x))*Sqrt[(16 + 4*c*(b/c + x) + c^2*(b
/c + x)^2)/(4*(1 - Sqrt[3]) - c*(b/c + x))^2]*EllipticF[ArcSin[(4*(1 + Sqr
t[3]) - c*(b/c + x))/(4*(1 - Sqrt[3]) - c*(b/c + x))], -7 + 4*Sqrt[3]])/(3
^(1/4)*c*Sqrt[-((4 - c*(b/c + x))/(4*(1 - Sqrt[3]) - c*(b/c + x))^2)]*Sqrt
[-64 + c^3*(b/c + x)^3]))/192)/(576*c^2))/960
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 2393
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   In
t[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(
p + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n
, 0] && LtQ[p, -1]
```

rule 2394
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   Int[ExpandToSum[n
*(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x
] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

rule 2419
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(c*r - (1 + Sqrt[3])*d*s)/r
  Int[1/Sqrt[a + b*x^3], x], x] + Simp[d/r   Int[((1 + Sqrt[3])*s + r*x)/Sq
rt[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] && NeQ[b*c^3 -
2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

rule 2459
```
Int[(Pn_)^(p_.)*(Qx_), x_Symbol] :> With[{S = Coeff[Pn, x, Expon[Pn, x] - 1
]/(Expon[Pn, x]*Coeff[Pn, x, Expon[Pn, x]])}, Subst[Int[ExpandToSum[Pn /. x
 -> x - S, x]^p*ExpandToSum[Qx /. x -> x - S, x], x], x, x + S] /; Binomial
Q[Pn /. x -> x - S, x] || (IntegerQ[Expon[Pn, x]/2] && TrinomialQ[Pn /. x -
> x - S, x])] /; FreeQ[p, x] && PolyQ[Pn, x] && GtQ[Expon[Pn, x], 2] && NeQ
[Coeff[Pn, x, Expon[Pn, x] - 1], 0] && PolyQ[Qx, x] &&  !(MonomialQ[Qx, x]
&& IGtQ[p, 0])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1116 vs. 2(539) = 1078.

Time = 1.78 (sec) , antiderivative size = 1117, normalized size of antiderivative = 1.83

method	result	size
elliptic	Expression too large to display	1117
default	Expression too large to display	2396

input
```
int((C*x^2+B*x+A)/(c^3*x^3+3*b*c^2*x^2+3*b^2*c*x+b^3-64)^(7/2),x,method=_R
ETURNVERBOSE)
```

output
```
(-1/480*(B*c-2*C*b)/c^10*x^2-1/480*(A*c^2+B*b*c-3*C*b^2)/c^11*x-1/480*(A*b
*c^2-C*b^3+64*C)/c^12)*(c^3*x^3+3*b*c^2*x^2+3*b^2*c*x+b^3-64)^(1/2)/(x^3+3
*b/c*x^2+3*b^2/c^2*x+(b^3-64)/c^3)^3+(11/276480*(B*c-2*C*b)/c^7*x^2+1/2764
80/c^8*(13*A*c^2+9*B*b*c-31*C*b^2)*x+1/276480*b/c^9*(13*A*c^2-2*B*b*c-9*C*
b^2))*(c^3*x^3+3*b*c^2*x^2+3*b^2*c*x+b^3-64)^(1/2)/(x^3+3*b/c*x^2+3*b^2/c^
2*x+(b^3-64)/c^3)^2-2*c^3*(11/21233664*(B*c-2*C*b)/c^4*x^2+1/106168320/c^5
*(91*A*c^2+19*B*b*c-129*C*b^2)*x+1/106168320*b/c^6*(91*A*c^2-36*B*b*c-19*C
*b^2))/((x^3+3*b/c*x^2+3*b^2/c^2*x+(b^3-64)/c^3)*c^3)^(1/2)+2*(-1/35389440
/c^2*(91*A*c^2-36*B*b*c-19*C*b^2)+1/53084160/c^2*(91*A*c^2+19*B*b*c-129*C*
b^2))*((-b-2-2*I*3^(1/2))/c+(b-4)/c)*((x+(b-4)/c)/((-b-2-2*I*3^(1/2))/c+(b
-4)/c))^(1/2)*((x-(-b-2+2*I*3^(1/2))/c)/(-(b-4)/c-(-b-2+2*I*3^(1/2))/c))^(
1/2)*((x-(-b-2-2*I*3^(1/2))/c)/(-(b-4)/c-(-b-2-2*I*3^(1/2))/c))^(1/2)/(c^3
*x^3+3*b*c^2*x^2+3*b^2*c*x+b^3-64)^(1/2)*EllipticF(((x+(b-4)/c)/((-b-2-2*I
*3^(1/2))/c+(b-4)/c))^(1/2),((-(b-4)/c-(-b-2-2*I*3^(1/2))/c)/(-(b-4)/c-(-b
-2+2*I*3^(1/2))/c))^(1/2))+11/10616832/c*(B*c-2*C*b)*((-b-2-2*I*3^(1/2))/c
+(b-4)/c)*((x+(b-4)/c)/((-b-2-2*I*3^(1/2))/c+(b-4)/c))^(1/2)*((x-(-b-2+2*I
*3^(1/2))/c)/(-(b-4)/c-(-b-2+2*I*3^(1/2))/c))^(1/2)*((x-(-b-2-2*I*3^(1/2))
/c)/(-(b-4)/c-(-b-2-2*I*3^(1/2))/c))^(1/2)/(c^3*x^3+3*b*c^2*x^2+3*b^2*c*x+
b^3-64)^(1/2)*((-(b-4)/c-(-b-2+2*I*3^(1/2))/c)*EllipticE(((x+(b-4)/c)/((-b
-2-2*I*3^(1/2))/c+(b-4)/c))^(1/2),((-(b-4)/c-(-b-2-2*I*3^(1/2))/c)/(-(b...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1608 vs. $2(494) = 988$.

Time = 0.19 (sec) , antiderivative size = 1608, normalized size of antiderivative = 2.64

$$\int \frac{A + Bx + Cx^2}{(-64 + b^3 + 3b^2cx + 3bc^2x^2 + c^3x^3)^{7/2}}\, dx = \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)/(c^3*x^3+3*b*c^2*x^2+3*b^2*c*x+b^3-64)^(7/2),x, al
gorithm="fricas")
```

output
```
-1/53084160*(91*(C*b^11 + (C*b^2*c^9 - B*b*c^10 + A*c^11)*x^9 - 192*C*b^8
+ 9*(C*b^3*c^8 - B*b^2*c^9 + A*b*c^10)*x^8 + 36*(C*b^4*c^7 - B*b^3*c^8 + A
*b^2*c^9)*x^7 + 12*((7*A*b^3 - 16*A)*c^8 - (7*B*b^4 - 16*B*b)*c^7 + (7*C*b
^5 - 16*C*b^2)*c^6)*x^6 + 12288*C*b^5 + 18*((7*A*b^4 - 64*A*b)*c^7 - (7*B*
b^5 - 64*B*b^2)*c^6 + (7*C*b^6 - 64*C*b^3)*c^5)*x^5 + 18*((7*A*b^5 - 160*A
*b^2)*c^6 - (7*B*b^6 - 160*B*b^3)*c^5 + (7*C*b^7 - 160*C*b^4)*c^4)*x^4 + 1
2*((7*A*b^6 - 320*A*b^3 + 1024*A)*c^5 - (7*B*b^7 - 320*B*b^4 + 1024*B*b)*c
^4 + (7*C*b^8 - 320*C*b^5 + 1024*C*b^2)*c^3)*x^3 - 262144*C*b^2 + (A*b^9 -
 192*A*b^6 + 12288*A*b^3 - 262144*A)*c^2 + 36*((A*b^7 - 80*A*b^4 + 1024*A*
b)*c^4 - (B*b^8 - 80*B*b^5 + 1024*B*b^2)*c^3 + (C*b^9 - 80*C*b^6 + 1024*C*
b^3)*c^2)*x^2 - (B*b^10 - 192*B*b^7 + 12288*B*b^4 - 262144*B*b)*c + 9*((A*
b^8 - 128*A*b^5 + 4096*A*b^2)*c^3 - (B*b^9 - 128*B*b^6 + 4096*B*b^3)*c^2 +
 (C*b^10 - 128*C*b^7 + 4096*C*b^4)*c)*x)*sqrt(c^3)*weierstrassPInverse(0,
256/c^3, (c*x + b)/c) - 55*((2*C*b*c^10 - B*c^11)*x^9 + 9*(2*C*b^2*c^9 - B
*b*c^10)*x^8 + 36*(2*C*b^3*c^8 - B*b^2*c^9)*x^7 - 12*((7*B*b^3 - 16*B)*c^8
 - 2*(7*C*b^4 - 16*C*b)*c^7)*x^6 - 18*((7*B*b^4 - 64*B*b)*c^7 - 2*(7*C*b^5
 - 64*C*b^2)*c^6)*x^5 - 18*((7*B*b^5 - 160*B*b^2)*c^6 - 2*(7*C*b^6 - 160*C
*b^3)*c^5)*x^4 - 12*((7*B*b^6 - 320*B*b^3 + 1024*B)*c^5 - 2*(7*C*b^7 - 320
*C*b^4 + 1024*C*b)*c^4)*x^3 - (B*b^9 - 192*B*b^6 + 12288*B*b^3 - 262144*B)
*c^2 - 36*((B*b^7 - 80*B*b^4 + 1024*B*b)*c^4 - 2*(C*b^8 - 80*C*b^5 + 10...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{\left(-64+b^3+3b^2cx+3bc^2x^2+c^3x^3\right)^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate((C*x**2+B*x+A)/(c**3*x**3+3*b*c**2*x**2+3*b**2*c*x+b**3-64)**(7/
2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{\left(-64+b^3+3b^2cx+3bc^2x^2+c^3x^3\right)^{7/2}}\,dx=\int \frac{Cx^2+Bx+A}{\left(c^3x^3+3\,bc^2x^2+3\,b^2cx+b^3-64\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(c^3*x^3+3*b*c^2*x^2+3*b^2*c*x+b^3-64)^(7/2),x, al
gorithm="maxima")
```

output
```
integrate((C*x^2 + B*x + A)/(c^3*x^3 + 3*b*c^2*x^2 + 3*b^2*c*x + b^3 - 64)
^(7/2), x)
```

Giac [F]

$$\int \frac{A+Bx+Cx^2}{\left(-64+b^3+3b^2cx+3bc^2x^2+c^3x^3\right)^{7/2}}\,dx=\int \frac{Cx^2+Bx+A}{\left(c^3x^3+3\,bc^2x^2+3\,b^2cx+b^3-64\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(c^3*x^3+3*b*c^2*x^2+3*b^2*c*x+b^3-64)^(7/2),x, al
gorithm="giac")
```

output
```
integrate((C*x^2 + B*x + A)/(c^3*x^3 + 3*b*c^2*x^2 + 3*b^2*c*x + b^3 - 64)
^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{\left(-64+b^3+3b^2cx+3bc^2x^2+c^3x^3\right)^{7/2}}\,dx=\int \frac{C\,x^2+B\,x+A}{{\left(b^3+3\,b^2\,c\,x+3\,b\,c^2\,x^2+c^3\,x^3-64\right)}^{7/2}}\,dx$$

input
```
int((A + B*x + C*x^2)/(b^3 + c^3*x^3 + 3*b*c^2*x^2 + 3*b^2*c*x - 64)^(7/2)
,x)
```

output
```
int((A + B*x + C*x^2)/(b^3 + c^3*x^3 + 3*b*c^2*x^2 + 3*b^2*c*x - 64)^(7/2)
, x)
```

Reduce [F]

$$\int \frac{A+Bx+Cx^2}{(-64+b^3+3b^2cx+3bc^2x^2+c^3x^3)^{7/2}}\,dx = \text{too large to display}$$

input
```
int((C*x^2+B*x+A)/(c^3*x^3+3*b*c^2*x^2+3*b^2*c*x+b^3-64)^(7/2),x)
```

output
```
( - 2*sqrt(b**3 + 3*b**2*c*x + 3*b*c**2*x**2 + c**3*x**3 - 64) + 30*int(sq
rt(b**3 + 3*b**2*c*x + 3*b*c**2*x**2 + c**3*x**3 - 64)/(b**12 + 12*b**11*c
*x + 66*b**10*c**2*x**2 + 220*b**9*c**3*x**3 - 256*b**9 + 495*b**8*c**4*x*
*4 - 2304*b**8*c*x + 792*b**7*c**5*x**5 - 9216*b**7*c**2*x**2 + 924*b**6*c
**6*x**6 - 21504*b**6*c**3*x**3 + 24576*b**6 + 792*b**5*c**7*x**7 - 32256*
b**5*c**4*x**4 + 147456*b**5*c*x + 495*b**4*c**8*x**8 - 32256*b**4*c**5*x*
*5 + 368640*b**4*c**2*x**2 + 220*b**3*c**9*x**9 - 21504*b**3*c**6*x**6 + 4
91520*b**3*c**3*x**3 - 1048576*b**3 + 66*b**2*c**10*x**10 - 9216*b**2*c**7
*x**7 + 368640*b**2*c**4*x**4 - 3145728*b**2*c*x + 12*b*c**11*x**11 - 2304
*b*c**8*x**8 + 147456*b*c**5*x**5 - 3145728*b*c**2*x**2 + c**12*x**12 - 25
6*c**9*x**9 + 24576*c**6*x**6 - 1048576*c**3*x**3 + 16777216),x)*a*b**9*c*
*2 + 270*int(sqrt(b**3 + 3*b**2*c*x + 3*b*c**2*x**2 + c**3*x**3 - 64)/(b**
12 + 12*b**11*c*x + 66*b**10*c**2*x**2 + 220*b**9*c**3*x**3 - 256*b**9 + 4
95*b**8*c**4*x**4 - 2304*b**8*c*x + 792*b**7*c**5*x**5 - 9216*b**7*c**2*x*
*2 + 924*b**6*c**6*x**6 - 21504*b**6*c**3*x**3 + 24576*b**6 + 792*b**5*c**
7*x**7 - 32256*b**5*c**4*x**4 + 147456*b**5*c*x + 495*b**4*c**8*x**8 - 322
56*b**4*c**5*x**5 + 368640*b**4*c**2*x**2 + 220*b**3*c**9*x**9 - 21504*b**
3*c**6*x**6 + 491520*b**3*c**3*x**3 - 1048576*b**3 + 66*b**2*c**10*x**10 -
 9216*b**2*c**7*x**7 + 368640*b**2*c**4*x**4 - 3145728*b**2*c*x + 12*b*c**
11*x**11 - 2304*b*c**8*x**8 + 147456*b*c**5*x**5 - 3145728*b*c**2*x**2 ...
```

3.80 $\int (A+Bx+Cx^2)(-64+b^3+3b^2cx+3bc^2x^2+c^3x$

Optimal result

Integrand size = 41, antiderivative size = 177

$$\int (A+Bx+Cx^2)\left(-64+b^3+3b^2cx+3bc^2x^2+c^3x^3\right)^p\,dx=\frac{C\left(-64+(b+cx)^3\right)^{1+p}}{3c^3(1+p)}$$
$$-\frac{64^p(bBc-Ac^2-b^2C)(b+cx)\left(64-(b+cx)^3\right)^{-p}\left(-64+(b+cx)^3\right)^p\text{Hypergeometric2F1}\left(\frac{1}{3},-p,\frac{4}{3},\frac{1}{6}\right.}{c^3}$$
$$+\frac{2^{-1+6p}(Bc-2bC)(b+cx)^2\left(64-(b+cx)^3\right)^{-p}\left(-64+(b+cx)^3\right)^p\text{Hypergeometric2F1}\left(\frac{2}{3},-p,\frac{5}{3},\frac{1}{64}(b\right.}{c^3}$$

output
```
1/3*C*(-64+(c*x+b)^3)^(p+1)/c^3/(p+1)-64^p*(-A*c^2+B*b*c-C*b^2)*(c*x+b)*(-
64+(c*x+b)^3)^p*hypergeom([1/3, -p],[4/3],1/64*(c*x+b)^3)/c^3/((64-(c*x+b)
^3)^p)+2^(-1+6*p)*(B*c-2*C*b)*(c*x+b)^2*(-64+(c*x+b)^3)^p*hypergeom([2/3,
-p],[5/3],1/64*(c*x+b)^3)/c^3/((64-(c*x+b)^3)^p)
```

Mathematica [F]

$$\int (A+Bx+Cx^2)\left(-64+b^3+3b^2cx+3bc^2x^2+c^3x^3\right)^p\,dx$$
$$=\int (A+Bx+Cx^2)\left(-64+b^3+3b^2cx+3bc^2x^2+c^3x^3\right)^p\,dx$$

input
```
Integrate[(A + B*x + C*x^2)*(-64 + b^3 + 3*b^2*c*x + 3*b*c^2*x^2 + c^3*x^3
)^p,x]
```

output
```
Integrate[(A + B*x + C*x^2)*(-64 + b^3 + 3*b^2*c*x + 3*b*c^2*x^2 + c^3*x^3
)^p, x]
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.22, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.122$, Rules used = {2459, 2425, 793, 2432, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (A+Bx+Cx^2)\left(b^3+3b^2cx+3bc^2x^2+c^3x^3-64\right)^p\,dx$$

$$\Big\downarrow \text{2459}$$

$$\int \left(c^3\left(\frac{b}{c}+x\right)^3-64\right)^p\left(A-\frac{b(Bc-bC)}{c^2}+\left(\frac{b}{c}+x\right)\left(B-\frac{2bC}{c}\right)+C\left(\frac{b}{c}+x\right)^2\right)d\left(\frac{b}{c}+x\right)$$

$$\Big\downarrow \text{2425}$$

$$\int \left(A-\frac{b(Bc-bC)}{c^2}+\left(B-\frac{2bC}{c}\right)\left(\frac{b}{c}+x\right)\right)\left(c^3\left(\frac{b}{c}+x\right)^3-64\right)^p d\left(\frac{b}{c}+x\right)+ C\int \left(\frac{b}{c}+x\right)^2\left(c^3\left(\frac{b}{c}+x\right)^3-64\right)^p d\left(\frac{b}{c}+x\right)$$

$$\Big\downarrow \text{793}$$

$$\int \left(A-\frac{b(Bc-bC)}{c^2}+\left(B-\frac{2bC}{c}\right)\left(\frac{b}{c}+x\right)\right)\left(c^3\left(\frac{b}{c}+x\right)^3-64\right)^p d\left(\frac{b}{c}+x\right)+\frac{C\left(c^3\left(\frac{b}{c}+x\right)^3-64\right)^{p+1}}{3c^3(p+1)}$$

$$\Big\downarrow \text{ 2432}$$

$$\int \left(A\left(\frac{b(bC-Bc)}{Ac^2}+1\right)\left(c^3\left(\frac{b}{c}+x\right)^3-64\right)^p+\frac{(Bc-2bC)\left(\frac{b}{c}+x\right)\left(c^3\left(\frac{b}{c}+x\right)^3-64\right)^p}{c}\right)d\left(\frac{b}{c}+x\right)+\frac{C\left(c^3\left(\frac{b}{c}+x\right)^3-64\right)^{p+1}}{3c^3(p+1)}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{\left(\frac{b}{c}+x\right)\left(c^3\left(\frac{b}{c}+x\right)^3-64\right)^p\left(1-\frac{1}{64}c^3\left(\frac{b}{c}+x\right)^3\right)^{-p}\left(-Ac^2+b^2(-C)+bBc\right)\text{Hypergeometric2F1}\left(\frac{1}{3},-p,\frac{4}{3},\frac{1}{64}\right.}{c^2}$$
$$\frac{\left(\frac{b}{c}+x\right)^2(Bc-2bC)\left(c^3\left(\frac{b}{c}+x\right)^3-64\right)^p\left(1-\frac{1}{64}c^3\left(\frac{b}{c}+x\right)^3\right)^{-p}\text{Hypergeometric2F1}\left(\frac{2}{3},-p,\frac{5}{3},\frac{1}{64}c^3\left(\frac{b}{c}+x\right)^3\right)}{2c}+\frac{C\left(c^3\left(\frac{b}{c}+x\right)^3-64\right)^{p+1}}{3c^3(p+1)}$$

input
```
Int[(A + B*x + C*x^2)*(-64 + b^3 + 3*b^2*c*x + 3*b*c^2*x^2 + c^3*x^3)^p,x]
```

output
```
(C*(-64 + c^3*(b/c + x)^3)^(1 + p))/(3*c^3*(1 + p)) - ((b*B*c - A*c^2 - b^
2*C)*(b/c + x)*(-64 + c^3*(b/c + x)^3)^p*Hypergeometric2F1[1/3, -p, 4/3, (
c^3*(b/c + x)^3)/64])/(c^2*(1 - (c^3*(b/c + x)^3)/64)^p) + ((B*c - 2*b*C)*
(b/c + x)^2*(-64 + c^3*(b/c + x)^3)^p*Hypergeometric2F1[2/3, -p, 5/3, (c^3
*(b/c + x)^3)/64])/(2*c*(1 - (c^3*(b/c + x)^3)/64)^p)
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2425
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Pq, x, n -
 1]   Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq,
x, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[P
q, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

rule 2432
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

rule 2459
```
Int[(Pn_)^(p_.)*(Qx_), x_Symbol] :> With[{S = Coeff[Pn, x, Expon[Pn, x] - 1
]/(Expon[Pn, x]*Coeff[Pn, x, Expon[Pn, x]])}, Subst[Int[ExpandToSum[Pn /. x
 -> x - S, x]^p*ExpandToSum[Qx /. x -> x - S, x], x], x, x + S] /; Binomial
Q[Pn /. x -> x - S, x] || (IntegerQ[Expon[Pn, x]/2] && TrinomialQ[Pn /. x -
> x - S, x])] /; FreeQ[p, x] && PolyQ[Pn, x] && GtQ[Expon[Pn, x], 2] && NeQ
[Coeff[Pn, x, Expon[Pn, x] - 1], 0] && PolyQ[Qx, x] &&  !(MonomialQ[Qx, x]
&& IGtQ[p, 0])
```

Maple [F]

$$\int (C\,x^{2}+Bx+A)\,(c^{3}x^{3}+3b\,c^{2}x^{2}+3b^{2}cx+b^{3}-64)^{p}\,dx$$

input
```
int((C*x^2+B*x+A)*(c^3*x^3+3*b*c^2*x^2+3*b^2*c*x+b^3-64)^p,x)
```

output
```
int((C*x^2+B*x+A)*(c^3*x^3+3*b*c^2*x^2+3*b^2*c*x+b^3-64)^p,x)
```

Fricas [F]

$$\int (A+Bx+Cx^2)\left(-64+b^3+3b^2cx+3bc^2x^2+c^3x^3\right)^p\,dx$$
$$=\int (Cx^2+Bx+A)\left(c^3x^3+3\,bc^2x^2+3\,b^2cx+b^3-64\right)^p\,dx$$

input
```
integrate((C*x^2+B*x+A)*(c^3*x^3+3*b*c^2*x^2+3*b^2*c*x+b^3-64)^p,x, algori
thm="fricas")
```

output
```
integral((C*x^2 + B*x + A)*(c^3*x^3 + 3*b*c^2*x^2 + 3*b^2*c*x + b^3 - 64)^
p, x)
```

Sympy [F(-1)]

Timed out.

$$\int (A+Bx+Cx^2)\left(-64+b^3+3b^2cx+3bc^2x^2+c^3x^3\right)^p\,dx=\text{Timed out}$$

input
```
integrate((C*x**2+B*x+A)*(c**3*x**3+3*b*c**2*x**2+3*b**2*c*x+b**3-64)**p,x
)
```

output
```
Timed out
```

Maxima [F]

$$\int (A+Bx+Cx^2)\left(-64+b^3+3b^2cx+3bc^2x^2+c^3x^3\right)^p\,dx$$
$$=\int (Cx^2+Bx+A)\left(c^3x^3+3\,bc^2x^2+3\,b^2cx+b^3-64\right)^p\,dx$$

input
```
integrate((C*x^2+B*x+A)*(c^3*x^3+3*b*c^2*x^2+3*b^2*c*x+b^3-64)^p,x, algori
thm="maxima")
```

output
```
integrate((C*x^2 + B*x + A)*(c^3*x^3 + 3*b*c^2*x^2 + 3*b^2*c*x + b^3 - 64)
^p, x)
```

Giac [F]

$$\int (A+Bx+Cx^2) (-64+b^3+3b^2cx+3bc^2x^2+c^3x^3)^p dx$$
$$= \int (Cx^2+Bx+A)(c^3x^3+3bc^2x^2+3b^2cx+b^3-64)^p dx$$

input
```
integrate((C*x^2+B*x+A)*(c^3*x^3+3*b*c^2*x^2+3*b^2*c*x+b^3-64)^p,x, algori
thm="giac")
```

output
```
integrate((C*x^2 + B*x + A)*(c^3*x^3 + 3*b*c^2*x^2 + 3*b^2*c*x + b^3 - 64)
^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (A+Bx+Cx^2) (-64+b^3+3b^2cx+3bc^2x^2+c^3x^3)^p dx$$
$$= \int (Cx^2+Bx+A) (b^3+3b^2cx+3bc^2x^2+c^3x^3-64)^p dx$$

input
```
int((A + B*x + C*x^2)*(b^3 + c^3*x^3 + 3*b*c^2*x^2 + 3*b^2*c*x - 64)^p,x)
```

output
```
int((A + B*x + C*x^2)*(b^3 + c^3*x^3 + 3*b*c^2*x^2 + 3*b^2*c*x - 64)^p, x)
```

Reduce [F]

$$\int (A+Bx+Cx^2)\left(-64+b^3+3b^2cx+3bc^2x^2+c^3x^3\right)^p\,dx=\text{too large to display}$$

input
```
int((C*x^2+B*x+A)*(c^3*x^3+3*b*c^2*x^2+3*b^2*c*x+b^3-64)^p,x)
```

output
```
(9*(b**3 + 3*b**2*c*x + 3*b*c**2*x**2 + c**3*x**3 - 64)**p*a*b*c*p**2 + 15
*(b**3 + 3*b**2*c*x + 3*b*c**2*x**2 + c**3*x**3 - 64)**p*a*b*c*p + 6*(b**3
 + 3*b**2*c*x + 3*b*c**2*x**2 + c**3*x**3 - 64)**p*a*b*c + 9*(b**3 + 3*b**
2*c*x + 3*b*c**2*x**2 + c**3*x**3 - 64)**p*a*c**2*p**2*x + 15*(b**3 + 3*b*
*2*c*x + 3*b*c**2*x**2 + c**3*x**3 - 64)**p*a*c**2*p*x + 6*(b**3 + 3*b**2*
c*x + 3*b*c**2*x**2 + c**3*x**3 - 64)**p*a*c**2*x - 3*(b**3 + 3*b**2*c*x +
 3*b*c**2*x**2 + c**3*x**3 - 64)**p*b**3*p - (b**3 + 3*b**2*c*x + 3*b*c**2
*x**2 + c**3*x**3 - 64)**p*b**3 + 9*(b**3 + 3*b**2*c*x + 3*b*c**2*x**2 + c
**3*x**3 - 64)**p*b**2*c*p**2*x + 3*(b**3 + 3*b**2*c*x + 3*b*c**2*x**2 + c
**3*x**3 - 64)**p*b**2*c*p*x + 18*(b**3 + 3*b**2*c*x + 3*b*c**2*x**2 + c**
3*x**3 - 64)**p*b*c**2*p**2*x**2 + 15*(b**3 + 3*b**2*c*x + 3*b*c**2*x**2 +
 c**3*x**3 - 64)**p*b*c**2*p*x**2 + 3*(b**3 + 3*b**2*c*x + 3*b*c**2*x**2 +
 c**3*x**3 - 64)**p*b*c**2*x**2 + 9*(b**3 + 3*b**2*c*x + 3*b*c**2*x**2 + c
**3*x**3 - 64)**p*c**3*p**2*x**3 + 9*(b**3 + 3*b**2*c*x + 3*b*c**2*x**2 +
c**3*x**3 - 64)**p*c**3*p*x**3 + 2*(b**3 + 3*b**2*c*x + 3*b*c**2*x**2 + c*
*3*x**3 - 64)**p*c**3*x**3 - 288*(b**3 + 3*b**2*c*x + 3*b*c**2*x**2 + c**3
*x**3 - 64)**p*p**2 - 192*(b**3 + 3*b**2*c*x + 3*b*c**2*x**2 + c**3*x**3 -
 64)**p*p - 32*(b**3 + 3*b**2*c*x + 3*b*c**2*x**2 + c**3*x**3 - 64)**p - 1
5552*int((b**3 + 3*b**2*c*x + 3*b*c**2*x**2 + c**3*x**3 - 64)**p/(9*b**3*p
**2 + 9*b**3*p + 2*b**3 + 27*b**2*c*p**2*x + 27*b**2*c*p*x + 6*b**2*c*x...
```

3.81 $\int (A+Bx+Cx^2)(3c^2x+3cdx^2+d^2x^3)^p\,dx$

Optimal result

Integrand size = 34, antiderivative size = 295

$$\int (A+Bx+Cx^2)(3c^2x+3cdx^2+d^2x^3)^p\,dx=\frac{C(3c^2x+3cdx^2+d^2x^3)^{1+p}}{3d^2(1+p)}$$
$$-\frac{(2cC-Bd)x^2\left(1+\frac{2dx}{3c-\sqrt{3}\sqrt{-c^2}}\right)^{-p}\left(1+\frac{2dx}{3c+\sqrt{3}\sqrt{-c^2}}\right)^{-p}(3c^2x+3cdx^2+d^2x^3)^p\operatorname{AppellF1}\left(2+p,-p,-\right.}{d(2+p)}$$
$$-\frac{(c^2C-Ad^2)(c+dx)(3c^2x+3cdx^2+d^2x^3)^p\left(1-\frac{(c+dx)^3}{c^3}\right)^{-p}\operatorname{Hypergeometric2F1}\left(\frac{1}{3},-p,\frac{4}{3},\frac{(c+dx)^3}{c^3}\right)}{d^3}$$

output
```
1/3*C*(d^2*x^3+3*c*d*x^2+3*c^2*x)^(p+1)/d^2/(p+1)-(-B*d+2*C*c)*x^2*(d^2*x^
3+3*c*d*x^2+3*c^2*x)^p*AppellF1(2+p,-p,-p,3+p,-2*d*x/(3*c-3^(1/2)*(-c^2)^(
1/2)),-2*d*x/(3*c+3^(1/2)*(-c^2)^(1/2)))/d/(2+p)/((1+2*d*x/(3*c-3^(1/2)*(-
c^2)^(1/2)))^p)/((1+2*d*x/(3*c+3^(1/2)*(-c^2)^(1/2)))^p)-(-A*d^2+C*c^2)*(d
*x+c)*(d^2*x^3+3*c*d*x^2+3*c^2*x)^p*hypergeom([1/3, -p],[4/3],(d*x+c)^3/c^
3)/d^3/((1-(d*x+c)^3/c^3)^p)
```

Mathematica [A] (warning: unable to verify)

Time = 0.96 (sec) , antiderivative size = 549, normalized size of antiderivative = 1.86

$$\int (A+Bx+Cx^2)\left(3c^2x+3cdx^2+d^2x^3\right)^p\,dx$$
$$= \frac{x\left(\frac{3cd-\sqrt{3}\sqrt{-c^2d^2}}{2d^2}+x\right)^{-p}\left(\frac{3c}{d}+\frac{\sqrt{3}c^2}{\sqrt{-c^2d^2}}+2x\right)^p\left(\frac{3c}{d}+\frac{\sqrt{3}\sqrt{-c^2d^2}}{d^2}+2x\right)^p\left(\frac{6c}{d}+\frac{2\sqrt{3}\sqrt{-c^2d^2}}{d^2}+4x\right)^{-p}\left(\frac{-3cd+\sqrt{3}\cdots}{-3cd+\cdots}\right.}{}$$

input
```
Integrate[(A + B*x + C*x^2)*(3*c^2*x + 3*c*d*x^2 + d^2*x^3)^p,x]
```

output
```
(x*((3*c)/d + (Sqrt[3]*c^2)/Sqrt[-(c^2*d^2)] + 2*x)^p*((3*c)/d + (Sqrt[3]*
Sqrt[-(c^2*d^2)])/d^2 + 2*x)^p*(x*(3*c^2 + 3*c*d*x + d^2*x^2))^p*(A*(6 + 5
*p + p^2)*AppellF1[1 + p, -p, -p, 2 + p, (-2*d^2*x)/(3*c*d + Sqrt[3]*Sqrt[
-(c^2*d^2)]), (2*d^2*x)/(-3*c*d + Sqrt[3]*Sqrt[-(c^2*d^2)])] + (1 + p)*x*(
B*(3 + p)*AppellF1[2 + p, -p, -p, 3 + p, (-2*d^2*x)/(3*c*d + Sqrt[3]*Sqrt[
-(c^2*d^2)]), (2*d^2*x)/(-3*c*d + Sqrt[3]*Sqrt[-(c^2*d^2)])] + C*(2 + p)*x
*AppellF1[3 + p, -p, -p, 4 + p, (-2*d^2*x)/(3*c*d + Sqrt[3]*Sqrt[-(c^2*d^2
)]), (2*d^2*x)/(-3*c*d + Sqrt[3]*Sqrt[-(c^2*d^2)])])))/((1 + p)*(2 + p)*(3
 + p)*((3*c*d - Sqrt[3]*Sqrt[-(c^2*d^2)])/(2*d^2) + x)^p*((6*c)/d + (2*Sqr
t[3]*Sqrt[-(c^2*d^2)])/d^2 + 4*x)^p*((-3*c*d + Sqrt[3]*Sqrt[-(c^2*d^2)] -
2*d^2*x)/(-3*c*d + Sqrt[3]*Sqrt[-(c^2*d^2)]))^p*((3*c*d + Sqrt[3]*Sqrt[-(c
^2*d^2)] + 2*d^2*x)/(3*c*d + Sqrt[3]*Sqrt[-(c^2*d^2)]))^p)
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.81, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$, Rules used = {2459, 2425, 793, 2432, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (A+Bx+Cx^2)\left(3c^2x+3cdx^2+d^2x^3\right)^p\,dx$$
$$\Big\downarrow \; 2459$$

$$\int \left(d^2\left(\frac{c}{d}+x\right)^3-\frac{c^3}{d}\right)^p \left(A+\frac{c(cC-Bd)}{d^2}+\left(\frac{c}{d}+x\right)\left(B-\frac{2cC}{d}\right)+C\left(\frac{c}{d}+x\right)^2\right)d\left(\frac{c}{d}+x\right)$$

$$\Big\downarrow \text{2425}$$

$$\int \left(A+\frac{c(cC-Bd)}{d^2}+\left(B-\frac{2cC}{d}\right)\left(\frac{c}{d}+x\right)\right)\left(d^2\left(\frac{c}{d}+x\right)^3-\frac{c^3}{d}\right)^p d\left(\frac{c}{d}+x\right)+ C\int \left(\frac{c}{d}+x\right)^2\left(d^2\left(\frac{c}{d}+x\right)^3-\frac{c^3}{d}\right)^p d\left(\frac{c}{d}+x\right)$$

$$\Big\downarrow \text{793}$$

$$\int \left(A+\frac{c(cC-Bd)}{d^2}+\left(B-\frac{2cC}{d}\right)\left(\frac{c}{d}+x\right)\right)\left(d^2\left(\frac{c}{d}+x\right)^3-\frac{c^3}{d}\right)^p d\left(\frac{c}{d}+x\right)+ \frac{C\left(d^2\left(\frac{c}{d}+x\right)^3-\frac{c^3}{d}\right)^{p+1}}{3d^2(p+1)}$$

$$\Big\downarrow \text{2432}$$

$$\int \left(A\left(\frac{c(cC-Bd)}{Ad^2}+1\right)\left(d^2\left(\frac{c}{d}+x\right)^3-\frac{c^3}{d}\right)^p+\frac{(Bd-2cC)\left(\frac{c}{d}+x\right)\left(d^2\left(\frac{c}{d}+x\right)^3-\frac{c^3}{d}\right)^p}{d}\right)d\left(\frac{c}{d}+x\right)+ \frac{C\left(d^2\left(\frac{c}{d}+x\right)^3-\frac{c^3}{d}\right)^{p+1}}{3d^2(p+1)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(\frac{c}{d}+x\right)\left(d^2\left(\frac{c}{d}+x\right)^3-\frac{c^3}{d}\right)^p\left(1-\frac{d^3\left(\frac{c}{d}+x\right)^3}{c^3}\right)^{-p}(Ad^2-Bcd+c^2C)\,\text{Hypergeometric2F1}\left(\frac{1}{3},-p,\frac{4}{3},\frac{d^3\left(\frac{c}{d}+x\right)^3}{c^3}\right)}{d^2}- \frac{\left(\frac{c}{d}+x\right)^2(2cC-Bd)\left(d^2\left(\frac{c}{d}+x\right)^3-\frac{c^3}{d}\right)^p\left(1-\frac{d^3\left(\frac{c}{d}+x\right)^3}{c^3}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{2}{3},-p,\frac{5}{3},\frac{d^3\left(\frac{c}{d}+x\right)^3}{c^3}\right)}{2d}+ \frac{C\left(d^2\left(\frac{c}{d}+x\right)^3-\frac{c^3}{d}\right)^{p+1}}{3d^2(p+1)}$$

input
```
Int[(A + B*x + C*x^2)*(3*c^2*x + 3*c*d*x^2 + d^2*x^3)^p,x]
```

output
```
(C*(-(c^3/d) + d^2*(c/d + x)^3)^(1 + p))/(3*d^2*(1 + p)) + ((c^2*C - B*c*d
 + A*d^2)*(c/d + x)*(-(c^3/d) + d^2*(c/d + x)^3)^p*Hypergeometric2F1[1/3,
-p, 4/3, (d^3*(c/d + x)^3)/c^3])/(d^2*(1 - (d^3*(c/d + x)^3)/c^3)^p) - ((2
*c*C - B*d)*(c/d + x)^2*(-(c^3/d) + d^2*(c/d + x)^3)^p*Hypergeometric2F1[2
/3, -p, 5/3, (d^3*(c/d + x)^3)/c^3])/(2*d*(1 - (d^3*(c/d + x)^3)/c^3)^p)
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2425
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Pq, x, n -
 1]   Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq,
x, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[P
q, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

rule 2432
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

rule 2459
```
Int[(Pn_)^(p_.)*(Qx_), x_Symbol] :> With[{S = Coeff[Pn, x, Expon[Pn, x] - 1
]/(Expon[Pn, x]*Coeff[Pn, x, Expon[Pn, x]])}, Subst[Int[ExpandToSum[Pn /. x
 -> x - S, x]^p*ExpandToSum[Qx /. x -> x - S, x], x], x, x + S] /; Binomial
Q[Pn /. x -> x - S, x] || (IntegerQ[Expon[Pn, x]/2] && TrinomialQ[Pn /. x -
> x - S, x])] /; FreeQ[p, x] && PolyQ[Pn, x] && GtQ[Expon[Pn, x], 2] && NeQ
[Coeff[Pn, x, Expon[Pn, x] - 1], 0] && PolyQ[Qx, x] &&  !(MonomialQ[Qx, x]
&& IGtQ[p, 0])
```

Maple [F]

$$\int \left(C\,x^2+Bx+A\right)\left(d^2x^3+3cd\,x^2+3c^2x\right)^p dx$$

input
```
int((C*x^2+B*x+A)*(d^2*x^3+3*c*d*x^2+3*c^2*x)^p,x)
```

output
```
int((C*x^2+B*x+A)*(d^2*x^3+3*c*d*x^2+3*c^2*x)^p,x)
```

Fricas [F]

$$\int \left(A+Bx+Cx^2\right)\left(3c^2x+3cdx^2+d^2x^3\right)^p\,dx = \int \left(Cx^2+Bx+A\right)\left(d^2x^3+3\,cdx^2+3\,c^2x\right)^p dx$$

input
```
integrate((C*x^2+B*x+A)*(d^2*x^3+3*c*d*x^2+3*c^2*x)^p,x, algorithm="fricas
")
```

output
```
integral((C*x^2 + B*x + A)*(d^2*x^3 + 3*c*d*x^2 + 3*c^2*x)^p, x)
```

Sympy [F]

$$\int \left(A+Bx+Cx^2\right)\left(3c^2x+3cdx^2+d^2x^3\right)^p\,dx = \int \left(x\left(3c^2+3cdx+d^2x^2\right)\right)^p\left(A+Bx+Cx^2\right)\,dx$$

input
```
integrate((C*x**2+B*x+A)*(d**2*x**3+3*c*d*x**2+3*c**2*x)**p,x)
```

output
```
Integral((x*(3*c**2 + 3*c*d*x + d**2*x**2))**p*(A + B*x + C*x**2), x)
```

Maxima [F]

$$\int (A+Bx+Cx^2)\left(3c^2x+3cdx^2+d^2x^3\right)^p\,dx = \int (Cx^2+Bx+A)\left(d^2x^3+3\,cdx^2+3\,c^2x\right)^p\,dx$$

input
```
integrate((C*x^2+B*x+A)*(d^2*x^3+3*c*d*x^2+3*c^2*x)^p,x, algorithm="maxima
")
```

output
```
integrate((C*x^2 + B*x + A)*(d^2*x^3 + 3*c*d*x^2 + 3*c^2*x)^p, x)
```

Giac [F]

$$\int (A+Bx+Cx^2)\left(3c^2x+3cdx^2+d^2x^3\right)^p\,dx = \int (Cx^2+Bx+A)\left(d^2x^3+3\,cdx^2+3\,c^2x\right)^p\,dx$$

input
```
integrate((C*x^2+B*x+A)*(d^2*x^3+3*c*d*x^2+3*c^2*x)^p,x, algorithm="giac")
```

output
```
integrate((C*x^2 + B*x + A)*(d^2*x^3 + 3*c*d*x^2 + 3*c^2*x)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (A+Bx+Cx^2)\left(3c^2x+3cdx^2+d^2x^3\right)^p\,dx = \int \left(C\,x^2+B\,x+A\right)\,\left(3\,c^2\,x+3\,c\,d\,x^2+d^2\,x^3\right)^p\,dx$$

input
```
int((A + B*x + C*x^2)*(3*c^2*x + d^2*x^3 + 3*c*d*x^2)^p,x)
```

output
```
int((A + B*x + C*x^2)*(3*c^2*x + d^2*x^3 + 3*c*d*x^2)^p, x)
```

Reduce [F]

$$\int (A+Bx+Cx^2)\left(3c^2x+3cdx^2+d^2x^3\right)^p\,dx=\text{too large to display}$$

input
```
int((C*x^2+B*x+A)*(d^2*x^3+3*c*d*x^2+3*c^2*x)^p,x)
```

output
```
(18*(3*c**2*x + 3*c*d*x**2 + d**2*x**3)**p*a*c*d**2*p**2 + 30*(3*c**2*x +
3*c*d*x**2 + d**2*x**3)**p*a*c*d**2*p + 12*(3*c**2*x + 3*c*d*x**2 + d**2*x
**3)**p*a*c*d**2 + 18*(3*c**2*x + 3*c*d*x**2 + d**2*x**3)**p*a*d**3*p**2*x
 + 30*(3*c**2*x + 3*c*d*x**2 + d**2*x**3)**p*a*d**3*p*x + 12*(3*c**2*x + 3
*c*d*x**2 + d**2*x**3)**p*a*d**3*x - 9*(3*c**2*x + 3*c*d*x**2 + d**2*x**3)
**p*b*c**2*d*p**2 - 18*(3*c**2*x + 3*c*d*x**2 + d**2*x**3)**p*b*c**2*d*p -
 9*(3*c**2*x + 3*c*d*x**2 + d**2*x**3)**p*b*c**2*d + 18*(3*c**2*x + 3*c*d*
x**2 + d**2*x**3)**p*b*c*d**2*p**2*x + 18*(3*c**2*x + 3*c*d*x**2 + d**2*x*
*3)**p*b*c*d**2*p*x + 18*(3*c**2*x + 3*c*d*x**2 + d**2*x**3)**p*b*d**3*p**
2*x**2 + 24*(3*c**2*x + 3*c*d*x**2 + d**2*x**3)**p*b*d**3*p*x**2 + 6*(3*c*
*2*x + 3*c*d*x**2 + d**2*x**3)**p*b*d**3*x**2 + 6*(3*c**2*x + 3*c*d*x**2 +
 d**2*x**3)**p*c**4*p + 6*(3*c**2*x + 3*c*d*x**2 + d**2*x**3)**p*c**4 - 12
*(3*c**2*x + 3*c*d*x**2 + d**2*x**3)**p*c**3*d*p*x + 18*(3*c**2*x + 3*c*d*
x**2 + d**2*x**3)**p*c**2*d**2*p**2*x**2 + 6*(3*c**2*x + 3*c*d*x**2 + d**2
*x**3)**p*c**2*d**2*p*x**2 + 18*(3*c**2*x + 3*c*d*x**2 + d**2*x**3)**p*c*d
**3*p**2*x**3 + 18*(3*c**2*x + 3*c*d*x**2 + d**2*x**3)**p*c*d**3*p*x**3 +
4*(3*c**2*x + 3*c*d*x**2 + d**2*x**3)**p*c*d**3*x**3 - 486*int((3*c**2*x +
 3*c*d*x**2 + d**2*x**3)**p/(27*c**2*p**2*x + 27*c**2*p*x + 6*c**2*x + 27*
c*d*p**2*x**2 + 27*c*d*p*x**2 + 6*c*d*x**2 + 9*d**2*p**2*x**3 + 9*d**2*p*x
**3 + 2*d**2*x**3),x)*a*c**3*d**2*p**5 - 1296*int((3*c**2*x + 3*c*d*x**...
```

3.82 $\int (A+Bx+Cx^2)(a+3c^2x+3cdx^2+d^2x^3)^p \, dx$

Optimal result

Integrand size = 35, antiderivative size = 239

$$\int (A+Bx+Cx^2)\left(a+3c^2x+3cdx^2+d^2x^3\right)^p \, dx = \frac{C\left(a-\frac{c^3}{d}+\frac{(c+dx)^3}{d}\right)^{1+p}}{3d^2(1+p)} + \frac{(c^2C-Bcd+Ad^2)(c+dx)\left(a-\frac{c^3}{d}+\frac{(c+dx)^3}{d}\right)^p\left(1-\frac{(c+dx)^3}{c^3-ad}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{3},-p,\frac{4}{3},\frac{(c+dx)^3}{c^3-ad}\right)}{d^3} - \frac{(2cC-Bd)(c+dx)^2\left(a-\frac{c^3}{d}+\frac{(c+dx)^3}{d}\right)^p\left(1-\frac{(c+dx)^3}{c^3-ad}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{2}{3},-p,\frac{5}{3},\frac{(c+dx)^3}{c^3-ad}\right)}{2d^3}$$

output
```
1/3*C*(a-c^3/d+(d*x+c)^3/d)^(p+1)/d^2/(p+1)+(A*d^2-B*c*d+C*c^2)*(d*x+c)*(a
-c^3/d+(d*x+c)^3/d)^p*hypergeom([1/3, -p],[4/3],(d*x+c)^3/(c^3-a*d))/d^3/(
(1-(d*x+c)^3/(c^3-a*d))^p)-1/2*(-B*d+2*C*c)*(d*x+c)^2*(a-c^3/d+(d*x+c)^3/d
)^p*hypergeom([2/3, -p],[5/3],(d*x+c)^3/(c^3-a*d))/d^3/((1-(d*x+c)^3/(c^3-
a*d))^p)
```

Mathematica [F]

$$\int (A+Bx+Cx^2)\left(a+3c^2x+3cdx^2+d^2x^3\right)^p\,dx$$
$$=\int (A+Bx+Cx^2)\left(a+3c^2x+3cdx^2+d^2x^3\right)^p\,dx$$

input
```
Integrate[(A + B*x + C*x^2)*(a + 3*c^2*x + 3*c*d*x^2 + d^2*x^3)^p,x]
```

output
```
Integrate[(A + B*x + C*x^2)*(a + 3*c^2*x + 3*c*d*x^2 + d^2*x^3)^p, x]
```

Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.13, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2459, 2425, 793, 2432, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (A+Bx+Cx^2)\left(a+3c^2x+3cdx^2+d^2x^3\right)^p\,dx$$

$$\Big\downarrow \text{2459}$$

$$\int \left(a-\frac{c^3}{d}+d^2\left(\frac{c}{d}+x\right)^3\right)^p\left(A+\frac{c(cC-Bd)}{d^2}+\left(\frac{c}{d}+x\right)\left(B-\frac{2cC}{d}\right)+C\left(\frac{c}{d}+x\right)^2\right)d\left(\frac{c}{d}+x\right)$$

$$\Big\downarrow \text{2425}$$

$$\int \left(A+\frac{c(cC-Bd)}{d^2}+\left(B-\frac{2cC}{d}\right)\left(\frac{c}{d}+x\right)\right)\left(-\frac{c^3}{d}+d^2\left(\frac{c}{d}+x\right)^3+a\right)^p d\left(\frac{c}{d}+x\right)+C\int \left(\frac{c}{d}+x\right)^2\left(-\frac{c^3}{d}+d^2\left(\frac{c}{d}+x\right)^3+a\right)^p d\left(\frac{c}{d}+x\right)$$

$$\Big\downarrow \text{793}$$

$$\int \left(A+\frac{c(cC-Bd)}{d^2}+\left(B-\frac{2cC}{d}\right)\left(\frac{c}{d}+x\right)\right)\left(-\frac{c^3}{d}+d^2\left(\frac{c}{d}+x\right)^3+a\right)^p d\left(\frac{c}{d}+x\right)+\frac{C\left(a-\frac{c^3}{d}+d^2\left(\frac{c}{d}+x\right)^3\right)^{p+1}}{3d^2(p+1)}$$

$\downarrow$ 2432

$$\int \left(A\left(\frac{c(cC-Bd)}{Ad^2}+1\right)\left(-\frac{c^3}{d}+d^2\left(\frac{c}{d}+x\right)^3+a\right)^p+\frac{(Bd-2cC)\left(\frac{c}{d}+x\right)\left(-\frac{c^3}{d}+d^2\left(\frac{c}{d}+x\right)^3+a\right)^p}{d}\right)d\left(\frac{c}{d}\right.$$
$$\frac{C\left(a-\frac{c^3}{d}+d^2\left(\frac{c}{d}+x\right)^3\right)^{p+1}}{3d^2(p+1)}$$

$\downarrow$ 2009

$$\frac{\left(\frac{c}{d}+x\right)\left(a-\frac{c^3}{d}+d^2\left(\frac{c}{d}+x\right)^3\right)^p\left(1-\frac{d^3\left(\frac{c}{d}+x\right)^3}{c^3-ad}\right)^{-p}\left(Ad^2-Bcd+c^2C\right)\text{Hypergeometric2F1}\left(\frac{1}{3},-p,\frac{4}{3},\frac{d^3\left(\frac{c}{d}+x\right)^3}{c^3-ad}\right.}{d^2}$$
$$\frac{\left(\frac{c}{d}+x\right)^2(2cC-Bd)\left(a-\frac{c^3}{d}+d^2\left(\frac{c}{d}+x\right)^3\right)^p\left(1-\frac{d^3\left(\frac{c}{d}+x\right)^3}{c^3-ad}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{2}{3},-p,\frac{5}{3},\frac{d^3\left(\frac{c}{d}+x\right)^3}{c^3-ad}\right)}{2d}+$$
$$\frac{C\left(a-\frac{c^3}{d}+d^2\left(\frac{c}{d}+x\right)^3\right)^{p+1}}{3d^2(p+1)}$$

input
```
Int[(A + B*x + C*x^2)*(a + 3*c^2*x + 3*c*d*x^2 + d^2*x^3)^p,x]
```

output
```
(C*(a - c^3/d + d^2*(c/d + x)^3)^(1 + p))/(3*d^2*(1 + p)) + ((c^2*C - B*c*
d + A*d^2)*(c/d + x)*(a - c^3/d + d^2*(c/d + x)^3)^p*Hypergeometric2F1[1/3
, -p, 4/3, (d^3*(c/d + x)^3)/(c^3 - a*d)])/(d^2*(1 - (d^3*(c/d + x)^3)/(c^
3 - a*d))^p) - ((2*c*C - B*d)*(c/d + x)^2*(a - c^3/d + d^2*(c/d + x)^3)^p*
Hypergeometric2F1[2/3, -p, 5/3, (d^3*(c/d + x)^3)/(c^3 - a*d)])/(2*d*(1 -
(d^3*(c/d + x)^3)/(c^3 - a*d))^p)
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2425
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Pq, x, n -
 1]   Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq,
x, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[P
q, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

rule 2432
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

rule 2459
```
Int[(Pn_)^(p_.)*(Qx_), x_Symbol] :> With[{S = Coeff[Pn, x, Expon[Pn, x] - 1
]/(Expon[Pn, x]*Coeff[Pn, x, Expon[Pn, x]])}, Subst[Int[ExpandToSum[Pn /. x
 -> x - S, x]^p*ExpandToSum[Qx /. x -> x - S, x], x], x, x + S] /; Binomial
Q[Pn /. x -> x - S, x] || (IntegerQ[Expon[Pn, x]/2] && TrinomialQ[Pn /. x -
> x - S, x])] /; FreeQ[p, x] && PolyQ[Pn, x] && GtQ[Expon[Pn, x], 2] && NeQ
[Coeff[Pn, x, Expon[Pn, x] - 1], 0] && PolyQ[Qx, x] &&  !(MonomialQ[Qx, x]
&& IGtQ[p, 0])
```

Maple [F]

$$\int \left(C\,x^{2}+Bx+A\right)\left(d^{2}x^{3}+3cd\,x^{2}+3c^{2}x+a\right)^{p}dx$$

input
```
int((C*x^2+B*x+A)*(d^2*x^3+3*c*d*x^2+3*c^2*x+a)^p,x)
```

output
```
int((C*x^2+B*x+A)*(d^2*x^3+3*c*d*x^2+3*c^2*x+a)^p,x)
```

Fricas [F]

$$\int (A+Bx+Cx^2)\left(a+3c^2x+3cdx^2+d^2x^3\right)^p dx$$
$$=\int (Cx^2+Bx+A)\left(d^2x^3+3\,cdx^2+3\,c^2x+a\right)^p dx$$

input
```
integrate((C*x^2+B*x+A)*(d^2*x^3+3*c*d*x^2+3*c^2*x+a)^p,x, algorithm="fric
as")
```

output
```
integral((C*x^2 + B*x + A)*(d^2*x^3 + 3*c*d*x^2 + 3*c^2*x + a)^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int (A+Bx+Cx^2)\left(a+3c^2x+3cdx^2+d^2x^3\right)^p dx = \text{Timed out}$$

input
```
integrate((C*x**2+B*x+A)*(d**2*x**3+3*c*d*x**2+3*c**2*x+a)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (A+Bx+Cx^2)\left(a+3c^2x+3cdx^2+d^2x^3\right)^p dx$$
$$=\int (Cx^2+Bx+A)\left(d^2x^3+3\,cdx^2+3\,c^2x+a\right)^p dx$$

input
```
integrate((C*x^2+B*x+A)*(d^2*x^3+3*c*d*x^2+3*c^2*x+a)^p,x, algorithm="maxi
ma")
```

output
```
integrate((C*x^2 + B*x + A)*(d^2*x^3 + 3*c*d*x^2 + 3*c^2*x + a)^p, x)
```

Giac [F]

$$\int \left(A+Bx+Cx^2\right)\left(a+3c^2x+3cdx^2+d^2x^3\right)^p\,dx$$
$$=\int \left(Cx^2+Bx+A\right)\left(d^2x^3+3\,cdx^2+3\,c^2x+a\right)^p\,dx$$

input
```
integrate((C*x^2+B*x+A)*(d^2*x^3+3*c*d*x^2+3*c^2*x+a)^p,x, algorithm="giac
")
```

output
```
integrate((C*x^2 + B*x + A)*(d^2*x^3 + 3*c*d*x^2 + 3*c^2*x + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(A+Bx+Cx^2\right)\left(a+3c^2x+3cdx^2+d^2x^3\right)^p\,dx$$
$$=\int \left(C\,x^2+B\,x+A\right)\,\left(3\,c^2\,x+3\,c\,d\,x^2+d^2\,x^3+a\right)^p\,dx$$

input
```
int((A + B*x + C*x^2)*(a + 3*c^2*x + d^2*x^3 + 3*c*d*x^2)^p,x)
```

output
```
int((A + B*x + C*x^2)*(a + 3*c^2*x + d^2*x^3 + 3*c*d*x^2)^p, x)
```

Reduce [F]

$$\int \left(A+Bx+Cx^2\right)\left(a+3c^2x+3cdx^2+d^2x^3\right)^p\,dx=\text{too large to display}$$

input
```
int((C*x^2+B*x+A)*(d^2*x^3+3*c*d*x^2+3*c^2*x+a)^p,x)
```

output
```
(9*(a + 3*c**2*x + 3*c*d*x**2 + d**2*x**3)**p*a*b*d**2*p**2 + 12*(a + 3*c*
*2*x + 3*c*d*x**2 + d**2*x**3)**p*a*b*d**2*p + 3*(a + 3*c**2*x + 3*c*d*x**
2 + d**2*x**3)**p*a*b*d**2 + 18*(a + 3*c**2*x + 3*c*d*x**2 + d**2*x**3)**p
*a*c**2*d**2*p**2 + 30*(a + 3*c**2*x + 3*c*d*x**2 + d**2*x**3)**p*a*c**2*d
**2*p + 12*(a + 3*c**2*x + 3*c*d*x**2 + d**2*x**3)**p*a*c**2*d**2 - 6*(a +
 3*c**2*x + 3*c*d*x**2 + d**2*x**3)**p*a*c**2*d*p - 2*(a + 3*c**2*x + 3*c*
d*x**2 + d**2*x**3)**p*a*c**2*d + 18*(a + 3*c**2*x + 3*c*d*x**2 + d**2*x**
3)**p*a*c*d**3*p**2*x + 30*(a + 3*c**2*x + 3*c*d*x**2 + d**2*x**3)**p*a*c*
d**3*p*x + 12*(a + 3*c**2*x + 3*c*d*x**2 + d**2*x**3)**p*a*c*d**3*x - 9*(a
 + 3*c**2*x + 3*c*d*x**2 + d**2*x**3)**p*b*c**3*d*p**2 - 18*(a + 3*c**2*x
+ 3*c*d*x**2 + d**2*x**3)**p*b*c**3*d*p - 9*(a + 3*c**2*x + 3*c*d*x**2 + d
**2*x**3)**p*b*c**3*d + 18*(a + 3*c**2*x + 3*c*d*x**2 + d**2*x**3)**p*b*c*
*2*d**2*p**2*x + 18*(a + 3*c**2*x + 3*c*d*x**2 + d**2*x**3)**p*b*c**2*d**2
*p*x + 18*(a + 3*c**2*x + 3*c*d*x**2 + d**2*x**3)**p*b*c*d**3*p**2*x**2 +
24*(a + 3*c**2*x + 3*c*d*x**2 + d**2*x**3)**p*b*c*d**3*p*x**2 + 6*(a + 3*c
**2*x + 3*c*d*x**2 + d**2*x**3)**p*b*c*d**3*x**2 + 6*(a + 3*c**2*x + 3*c*d
*x**2 + d**2*x**3)**p*c**5*p + 6*(a + 3*c**2*x + 3*c*d*x**2 + d**2*x**3)**
p*c**5 - 12*(a + 3*c**2*x + 3*c*d*x**2 + d**2*x**3)**p*c**4*d*p*x + 18*(a
+ 3*c**2*x + 3*c*d*x**2 + d**2*x**3)**p*c**3*d**2*p**2*x**2 + 6*(a + 3*c**
2*x + 3*c*d*x**2 + d**2*x**3)**p*c**3*d**2*p*x**2 + 18*(a + 3*c**2*x + ...
```

3.83 $\int (A+Bx+Cx^2)(c^2x+3cdx^2+3d^2x^3)^p \, dx$

Optimal result

Integrand size = 34, antiderivative size = 302

$$\int (A+Bx+Cx^2)(c^2x+3cdx^2+3d^2x^3)^p \, dx = \frac{C(c^2x+3cdx^2+3d^2x^3)^{1+p}}{9d^2(1+p)}$$
$$-\frac{(2cC-3Bd)x^2\left(1+\frac{6dx}{3c-\sqrt{3}\sqrt{-c^2}}\right)^{-p}\left(1+\frac{6dx}{3c+\sqrt{3}\sqrt{-c^2}}\right)^{-p}(c^2x+3cdx^2+3d^2x^3)^p \operatorname{AppellF1}\left(2+p,-p,-\right.}{3d(2+p)}$$
$$-\frac{(c^2C-9Ad^2)(c+3dx)(c^2x+3cdx^2+3d^2x^3)^p\left(1-\frac{(c+3dx)^3}{c^3}\right)^{-p}\operatorname{Hypergeometric2F1}\left(\frac{1}{3},-p,\frac{4}{3},\frac{(c+3dx}{c^3}\right.}{27d^3}$$

output
```
1/9*C*(3*d^2*x^3+3*c*d*x^2+c^2*x)^(p+1)/d^2/(p+1)-1/3*(-3*B*d+2*C*c)*x^2*(
3*d^2*x^3+3*c*d*x^2+c^2*x)^p*AppellF1(2+p,-p,-p,3+p,-6*d*x/(3*c-3^(1/2)*(-
c^2)^(1/2)),-6*d*x/(3*c+3^(1/2)*(-c^2)^(1/2)))/d/(2+p)/((1+6*d*x/(3*c-3^(1
/2)*(-c^2)^(1/2)))^p)/((1+6*d*x/(3*c+3^(1/2)*(-c^2)^(1/2)))^p)-1/27*(-9*A*
d^2+C*c^2)*(3*d*x+c)*(3*d^2*x^3+3*c*d*x^2+c^2*x)^p*hypergeom([1/3, -p],[4/
3],(3*d*x+c)^3/c^3)/d^3/((1-(3*d*x+c)^3/c^3)^p)
```

Mathematica [A] (warning: unable to verify)

Time = 0.81 (sec) , antiderivative size = 409, normalized size of antiderivative = 1.35

$$\int \left(A+Bx+Cx^2\right)\left(c^2x+3cdx^2+3d^2x^3\right)^p\,dx$$
$$= \frac{x\left(\frac{-3cd+\sqrt{3}\sqrt{-c^2d^2}-6d^2x}{-3cd+\sqrt{3}\sqrt{-c^2d^2}}\right)^{-p}\left(\frac{3cd+\sqrt{3}\sqrt{-c^2d^2}+6d^2x}{3cd+\sqrt{3}\sqrt{-c^2d^2}}\right)^{-p}\left(x(c^2+3cdx+3d^2x^2)\right)^p\left(A(6+5p+p^2)\,\text{AppellF1}\left(1\right.\right.}{}$$

input
```
Integrate[(A + B*x + C*x^2)*(c^2*x + 3*c*d*x^2 + 3*d^2*x^3)^p,x]
```

output
```
(x*(x*(c^2 + 3*c*d*x + 3*d^2*x^2))^p*(A*(6 + 5*p + p^2)*AppellF1[1 + p, -p
, -p, 2 + p, (-6*d^2*x)/(3*c*d + Sqrt[3]*Sqrt[-(c^2*d^2)]), (6*d^2*x)/(-3*
c*d + Sqrt[3]*Sqrt[-(c^2*d^2)])] + (1 + p)*x*(B*(3 + p)*AppellF1[2 + p, -p
, -p, 3 + p, (-6*d^2*x)/(3*c*d + Sqrt[3]*Sqrt[-(c^2*d^2)]), (6*d^2*x)/(-3*
c*d + Sqrt[3]*Sqrt[-(c^2*d^2)])] + C*(2 + p)*x*AppellF1[3 + p, -p, -p, 4 +
 p, (-6*d^2*x)/(3*c*d + Sqrt[3]*Sqrt[-(c^2*d^2)]), (6*d^2*x)/(-3*c*d + Sqr
t[3]*Sqrt[-(c^2*d^2)])])))/((1 + p)*(2 + p)*(3 + p)*((-3*c*d + Sqrt[3]*Sqr
t[-(c^2*d^2)] - 6*d^2*x)/(-3*c*d + Sqrt[3]*Sqrt[-(c^2*d^2)]))^p*((3*c*d +
Sqrt[3]*Sqrt[-(c^2*d^2)] + 6*d^2*x)/(3*c*d + Sqrt[3]*Sqrt[-(c^2*d^2)]))^p)
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 280, normalized size of antiderivative = 0.93, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$, Rules used = {2459, 2425, 793, 2432, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(A+Bx+Cx^2\right)\left(c^2x+3cdx^2+3d^2x^3\right)^p\,dx$$
$$\Big\downarrow \; 2459$$
$$\int \left(3d^2\left(\frac{c}{3d}+x\right)^3-\frac{c^3}{9d}\right)^p\left(\frac{1}{9}\left(9A+\frac{c(cC-3Bd)}{d^2}\right)+\frac{1}{3}\left(\frac{c}{3d}+x\right)\left(3B-\frac{2cC}{d}\right)+C\left(\frac{c}{3d}+x\right)^2\right)d\left(\frac{c}{3d}+x\right)$$

$$\Big\downarrow \text{2425}$$

$$\int \left(\frac{1}{9}\left(9A+\frac{c(cC-3Bd)}{d^2}\right)+\frac{1}{3}\left(3B-\frac{2cC}{d}\right)\left(\frac{c}{3d}+x\right)\right)\left(3d^2\left(\frac{c}{3d}+x\right)^3-\frac{c^3}{9d}\right)^p d\left(\frac{c}{3d}+x\right)+ C\int \left(\frac{c}{3d}+x\right)^2\left(3d^2\left(\frac{c}{3d}+x\right)^3-\frac{c^3}{9d}\right)^p d\left(\frac{c}{3d}+x\right)$$

$$\Big\downarrow \text{793}$$

$$\int \left(\frac{1}{9}\left(9A+\frac{c(cC-3Bd)}{d^2}\right)+\frac{1}{3}\left(3B-\frac{2cC}{d}\right)\left(\frac{c}{3d}+x\right)\right)\left(3d^2\left(\frac{c}{3d}+x\right)^3-\frac{c^3}{9d}\right)^p d\left(\frac{c}{3d}+x\right)+ \frac{C\left(3d^2\left(\frac{c}{3d}+x\right)^3-\frac{c^3}{9d}\right)^{p+1}}{9d^2(p+1)}$$

$$\Big\downarrow \text{2432}$$

$$\int \left(\frac{(Cc^2-3Bdc+9Ad^2)\left(3d^2\left(\frac{c}{3d}+x\right)^3-\frac{c^3}{9d}\right)^p}{9d^2}+\frac{(3Bd-2cC)\left(\frac{c}{3d}+x\right)\left(3d^2\left(\frac{c}{3d}+x\right)^3-\frac{c^3}{9d}\right)^p}{3d}\right) d\left(\frac{c}{3d}+x\right) \frac{C\left(3d^2\left(\frac{c}{3d}+x\right)^3-\frac{c^3}{9d}\right)^{p+1}}{9d^2(p+1)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(\frac{c}{3d}+x\right)\left(3d^2\left(\frac{c}{3d}+x\right)^3-\frac{c^3}{9d}\right)^p\left(1-\frac{27d^3\left(\frac{c}{3d}+x\right)^3}{c^3}\right)^{-p}(9Ad^2-3Bcd+c^2C)\,\text{Hypergeometric2F1}\left(\frac{1}{3},-p,\frac{4}{3},\frac{27d^3(}{}\right.}{9d^2} \frac{\left(\frac{c}{3d}+x\right)^2(2cC-3Bd)\left(3d^2\left(\frac{c}{3d}+x\right)^3-\frac{c^3}{9d}\right)^p\left(1-\frac{27d^3\left(\frac{c}{3d}+x\right)^3}{c^3}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{2}{3},-p,\frac{5}{3},\frac{27d^3\left(\frac{c}{3d}+x\right)^3}{c^3}\right)}{6d} \frac{C\left(3d^2\left(\frac{c}{3d}+x\right)^3-\frac{c^3}{9d}\right)^{p+1}}{9d^2(p+1)}$$

input
```
Int[(A + B*x + C*x^2)*(c^2*x + 3*c*d*x^2 + 3*d^2*x^3)^p,x]
```

output
```
(C*(-1/9*c^3/d + 3*d^2*(c/(3*d) + x)^3)^(1 + p))/(9*d^2*(1 + p)) + ((c^2*C
 - 3*B*c*d + 9*A*d^2)*(c/(3*d) + x)*(-1/9*c^3/d + 3*d^2*(c/(3*d) + x)^3)^p
*Hypergeometric2F1[1/3, -p, 4/3, (27*d^3*(c/(3*d) + x)^3)/c^3])/(9*d^2*(1
- (27*d^3*(c/(3*d) + x)^3)/c^3)^p) - ((2*c*C - 3*B*d)*(c/(3*d) + x)^2*(-1/
9*c^3/d + 3*d^2*(c/(3*d) + x)^3)^p*Hypergeometric2F1[2/3, -p, 5/3, (27*d^3
*(c/(3*d) + x)^3)/c^3])/(6*d*(1 - (27*d^3*(c/(3*d) + x)^3)/c^3)^p)
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2425
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Pq, x, n -
 1]   Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq,
x, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[P
q, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

rule 2432
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

rule 2459
```
Int[(Pn_)^(p_.)*(Qx_), x_Symbol] :> With[{S = Coeff[Pn, x, Expon[Pn, x] - 1
]/(Expon[Pn, x]*Coeff[Pn, x, Expon[Pn, x]])}, Subst[Int[ExpandToSum[Pn /. x
 -> x - S, x]^p*ExpandToSum[Qx /. x -> x - S, x], x], x, x + S] /; Binomial
Q[Pn /. x -> x - S, x] || (IntegerQ[Expon[Pn, x]/2] && TrinomialQ[Pn /. x -
> x - S, x])] /; FreeQ[p, x] && PolyQ[Pn, x] && GtQ[Expon[Pn, x], 2] && NeQ
[Coeff[Pn, x, Expon[Pn, x] - 1], 0] && PolyQ[Qx, x] &&  !(MonomialQ[Qx, x]
&& IGtQ[p, 0])
```

Maple [F]

$$\int \left(C\,x^{2}+Bx+A\right)\left(3d^{2}x^{3}+3cd\,x^{2}+c^{2}x\right)^{p}dx$$

input
```
int((C*x^2+B*x+A)*(3*d^2*x^3+3*c*d*x^2+c^2*x)^p,x)
```

output
```
int((C*x^2+B*x+A)*(3*d^2*x^3+3*c*d*x^2+c^2*x)^p,x)
```

Fricas [F]

$$\int (A+Bx+Cx^2)\left(c^2x+3cdx^2+3d^2x^3\right)^p\,dx = \int (Cx^2+Bx+A)\left(3\,d^2x^3+3\,cdx^2+c^2x\right)^p\,dx$$

input
```
integrate((C*x^2+B*x+A)*(3*d^2*x^3+3*c*d*x^2+c^2*x)^p,x, algorithm="fricas
")
```

output
```
integral((C*x^2 + B*x + A)*(3*d^2*x^3 + 3*c*d*x^2 + c^2*x)^p, x)
```

Sympy [F]

$$\int (A+Bx+Cx^2)\left(c^2x+3cdx^2+3d^2x^3\right)^p\,dx = \int \left(x\left(c^2+3cdx+3d^2x^2\right)\right)^p\left(A+Bx+Cx^2\right)\,dx$$

input
```
integrate((C*x**2+B*x+A)*(3*d**2*x**3+3*c*d*x**2+c**2*x)**p,x)
```

output
```
Integral((x*(c**2 + 3*c*d*x + 3*d**2*x**2))**p*(A + B*x + C*x**2), x)
```

Maxima [F]

$$\int (A+Bx+Cx^2)\left(c^2x+3cdx^2+3d^2x^3\right)^p\,dx = \int (Cx^2+Bx+A)\left(3\,d^2x^3+3\,cdx^2+c^2x\right)^p\,dx$$

input
```
integrate((C*x^2+B*x+A)*(3*d^2*x^3+3*c*d*x^2+c^2*x)^p,x, algorithm="maxima
")
```

output
```
integrate((C*x^2 + B*x + A)*(3*d^2*x^3 + 3*c*d*x^2 + c^2*x)^p, x)
```

Giac [F]

$$\int (A+Bx+Cx^2)\left(c^2x+3cdx^2+3d^2x^3\right)^p\,dx = \int (Cx^2+Bx+A)\left(3\,d^2x^3+3\,cdx^2+c^2x\right)^p\,dx$$

input
```
integrate((C*x^2+B*x+A)*(3*d^2*x^3+3*c*d*x^2+c^2*x)^p,x, algorithm="giac")
```

output
```
integrate((C*x^2 + B*x + A)*(3*d^2*x^3 + 3*c*d*x^2 + c^2*x)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (A+Bx+Cx^2)\left(c^2x+3cdx^2+3d^2x^3\right)^p\,dx = \int (C\,x^2+B\,x+A)\,\left(c^2\,x+3\,c\,d\,x^2+3\,d^2\,x^3\right)^p\,dx$$

input
```
int((A + B*x + C*x^2)*(c^2*x + 3*d^2*x^3 + 3*c*d*x^2)^p,x)
```

output
```
int((A + B*x + C*x^2)*(c^2*x + 3*d^2*x^3 + 3*c*d*x^2)^p, x)
```

Reduce [F]

$$\int (A + Bx + Cx^2) (c^2x + 3cdx^2 + 3d^2x^3)^p \, dx = \text{too large to display}$$

input
```
int((C*x^2+B*x+A)*(3*d^2*x^3+3*c*d*x^2+c^2*x)^p,x)
```

output
```
(54*(c**2*x + 3*c*d*x**2 + 3*d**2*x**3)**p*a*c*d**2*p**2 + 90*(c**2*x + 3*
c*d*x**2 + 3*d**2*x**3)**p*a*c*d**2*p + 36*(c**2*x + 3*c*d*x**2 + 3*d**2*x
**3)**p*a*c*d**2 + 162*(c**2*x + 3*c*d*x**2 + 3*d**2*x**3)**p*a*d**3*p**2*
x + 270*(c**2*x + 3*c*d*x**2 + 3*d**2*x**3)**p*a*d**3*p*x + 108*(c**2*x +
3*c*d*x**2 + 3*d**2*x**3)**p*a*d**3*x - 9*(c**2*x + 3*c*d*x**2 + 3*d**2*x*
*3)**p*b*c**2*d*p**2 - 18*(c**2*x + 3*c*d*x**2 + 3*d**2*x**3)**p*b*c**2*d*
p - 9*(c**2*x + 3*c*d*x**2 + 3*d**2*x**3)**p*b*c**2*d + 54*(c**2*x + 3*c*d
*x**2 + 3*d**2*x**3)**p*b*c*d**2*p**2*x + 54*(c**2*x + 3*c*d*x**2 + 3*d**2
*x**3)**p*b*c*d**2*p*x + 162*(c**2*x + 3*c*d*x**2 + 3*d**2*x**3)**p*b*d**3
*p**2*x**2 + 216*(c**2*x + 3*c*d*x**2 + 3*d**2*x**3)**p*b*d**3*p*x**2 + 54
*(c**2*x + 3*c*d*x**2 + 3*d**2*x**3)**p*b*d**3*x**2 + 2*(c**2*x + 3*c*d*x*
*2 + 3*d**2*x**3)**p*c**4*p + 2*(c**2*x + 3*c*d*x**2 + 3*d**2*x**3)**p*c**
4 - 12*(c**2*x + 3*c*d*x**2 + 3*d**2*x**3)**p*c**3*d*p*x + 54*(c**2*x + 3*
c*d*x**2 + 3*d**2*x**3)**p*c**2*d**2*p**2*x**2 + 18*(c**2*x + 3*c*d*x**2 +
 3*d**2*x**3)**p*c**2*d**2*p*x**2 + 162*(c**2*x + 3*c*d*x**2 + 3*d**2*x**3
)**p*c*d**3*p**2*x**3 + 162*(c**2*x + 3*c*d*x**2 + 3*d**2*x**3)**p*c*d**3*
p*x**3 + 36*(c**2*x + 3*c*d*x**2 + 3*d**2*x**3)**p*c*d**3*x**3 - 486*int((
c**2*x + 3*c*d*x**2 + 3*d**2*x**3)**p/(9*c**2*p**2*x + 9*c**2*p*x + 2*c**2
*x + 27*c*d*p**2*x**2 + 27*c*d*p*x**2 + 6*c*d*x**2 + 27*d**2*p**2*x**3 + 2
7*d**2*p*x**3 + 6*d**2*x**3),x)*a*c**3*d**2*p**5 - 1296*int((c**2*x + 3...
```

3.84 $\int (A+Bx+Cx^2)(a+c^2x+3cdx^2+3d^2x^3)^p\,dx$

Optimal result

Integrand size = 35, antiderivative size = 252

$$\int (A+Bx+Cx^2)(a+c^2x+3cdx^2+3d^2x^3)^p\,dx = \frac{C(a+c^2x+3cdx^2+3d^2x^3)^{1+p}}{9d^2(1+p)} + \frac{(c^2C-3Bcd+9Ad^2)(c+3dx)(a+c^2x+3cdx^2+3d^2x^3)^p\left(1-\frac{(c+3dx)^3}{c^3-9ad}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{3},\right.}{27d^3} - \frac{(2cC-3Bd)(c+3dx)^2(a+c^2x+3cdx^2+3d^2x^3)^p\left(1-\frac{(c+3dx)^3}{c^3-9ad}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{2}{3},-p,\frac{5}{3},\right.}{54d^3}$$

output
```
1/9*C*(3*d^2*x^3+3*c*d*x^2+c^2*x+a)^(p+1)/d^2/(p+1)+1/27*(9*A*d^2-3*B*c*d+
C*c^2)*(3*d*x+c)*(3*d^2*x^3+3*c*d*x^2+c^2*x+a)^p*hypergeom([1/3, -p],[4/3]
,(3*d*x+c)^3/(c^3-9*a*d))/d^3/((1-(3*d*x+c)^3/(c^3-9*a*d))^p)-1/54*(-3*B*d
+2*C*c)*(3*d*x+c)^2*(3*d^2*x^3+3*c*d*x^2+c^2*x+a)^p*hypergeom([2/3, -p],[5
/3],(3*d*x+c)^3/(c^3-9*a*d))/d^3/((1-(3*d*x+c)^3/(c^3-9*a*d))^p)
```

Mathematica [F]

$$\int \left(A+Bx+Cx^2\right)\left(a+c^2x+3cdx^2+3d^2x^3\right)^p\,dx = \int \left(A+Bx+Cx^2\right)\left(a+c^2x+3cdx^2+3d^2x^3\right)^p\,dx$$

input
```
Integrate[(A + B*x + C*x^2)*(a + c^2*x + 3*c*d*x^2 + 3*d^2*x^3)^p,x]
```

output
```
Integrate[(A + B*x + C*x^2)*(a + c^2*x + 3*c*d*x^2 + 3*d^2*x^3)^p, x]
```

Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.29, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2459, 2425, 793, 2432, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(A+Bx+Cx^2\right)\left(a+c^2x+3cdx^2+3d^2x^3\right)^p\,dx$$

$$\Big\downarrow \text{2459}$$

$$\int \left(\frac{1}{9}\left(9a-\frac{c^3}{d}\right)+3d^2\left(\frac{c}{3d}+x\right)^3\right)^p\left(\frac{1}{9}\left(9A+\frac{c(cC-3Bd)}{d^2}\right)+\frac{1}{3}\left(\frac{c}{3d}+x\right)\left(3B-\frac{2cC}{d}\right)+C\left(\frac{c}{3d}+x\right)^2\right)d$$

$$\Big\downarrow \text{2425}$$

$$\int \left(\frac{1}{9}\left(9A+\frac{c(cC-3Bd)}{d^2}\right)+\frac{1}{3}\left(3B-\frac{2cC}{d}\right)\left(\frac{c}{3d}+x\right)\right)\left(3d^2\left(\frac{c}{3d}+x\right)^3+\frac{1}{9}\left(9a-\frac{c^3}{d}\right)\right)^p d\left(\frac{c}{3d}+x\right)+ C\int \left(\frac{c}{3d}+x\right)^2\left(3d^2\left(\frac{c}{3d}+x\right)^3+\frac{1}{9}\left(9a-\frac{c^3}{d}\right)\right)^p d\left(\frac{c}{3d}+x\right)$$

$$\Big\downarrow \text{793}$$

$$\int \left(\frac{1}{9}\left(9A+\frac{c(cC-3Bd)}{d^2}\right)+\frac{1}{3}\left(3B-\frac{2cC}{d}\right)\left(\frac{c}{3d}+x\right)\right)\left(3d^2\left(\frac{c}{3d}+x\right)^3+\frac{1}{9}\left(9a-\frac{c^3}{d}\right)\right)^p d\left(\frac{c}{3d}+x\right)+\frac{C\left(\frac{1}{9}\left(9a-\frac{c^3}{d}\right)+3d^2\left(\frac{c}{3d}+x\right)^3\right)^{p+1}}{9d^2(p+1)}$$

$\Big\downarrow$ 2432

$$\int \left(\frac{(Cc^2-3Bdc+9Ad^2)\left(3d^2\left(\frac{c}{3d}+x\right)^3+\frac{1}{9}\left(9a-\frac{c^3}{d}\right)\right)^p}{9d^2}+\frac{(3Bd-2cC)\left(\frac{c}{3d}+x\right)\left(3d^2\left(\frac{c}{3d}+x\right)^3+\frac{1}{9}\left(9a-\frac{c}{d}\right.\right.}{3d}\right.$$
$$\frac{C\left(\frac{1}{9}\left(9a-\frac{c^3}{d}\right)+3d^2\left(\frac{c}{3d}+x\right)^3\right)^{p+1}}{9d^2(p+1)}$$

$\Big\downarrow$ 2009

$$\frac{\left(\frac{c}{3d}+x\right)\left(\frac{1}{9}\left(9a-\frac{c^3}{d}\right)+3d^2\left(\frac{c}{3d}+x\right)^3\right)^p\left(1-\frac{27d^3\left(\frac{c}{3d}+x\right)^3}{c^3-9ad}\right)^{-p}(9Ad^2-3Bcd+c^2C)\,\text{Hypergeometric2F1}\left(\frac{1}{3},-\right.}{9d^2}$$
$$\frac{\left(\frac{c}{3d}+x\right)^2(2cC-3Bd)\left(\frac{1}{9}\left(9a-\frac{c^3}{d}\right)+3d^2\left(\frac{c}{3d}+x\right)^3\right)^p\left(1-\frac{27d^3\left(\frac{c}{3d}+x\right)^3}{c^3-9ad}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{2}{3},-p,\frac{5}{3},\frac{27d}{c}\right.}{6d}$$
$$\frac{C\left(\frac{1}{9}\left(9a-\frac{c^3}{d}\right)+3d^2\left(\frac{c}{3d}+x\right)^3\right)^{p+1}}{9d^2(p+1)}$$

input
```
Int[(A + B*x + C*x^2)*(a + c^2*x + 3*c*d*x^2 + 3*d^2*x^3)^p,x]
```

output
```
(C*((9*a - c^3/d)/9 + 3*d^2*(c/(3*d) + x)^3)^(1 + p))/(9*d^2*(1 + p)) + ((
c^2*C - 3*B*c*d + 9*A*d^2)*(c/(3*d) + x)*((9*a - c^3/d)/9 + 3*d^2*(c/(3*d)
 + x)^3)^p*Hypergeometric2F1[1/3, -p, 4/3, (27*d^3*(c/(3*d) + x)^3)/(c^3 -
 9*a*d)])/(9*d^2*(1 - (27*d^3*(c/(3*d) + x)^3)/(c^3 - 9*a*d))^p) - ((2*c*C
 - 3*B*d)*(c/(3*d) + x)^2*((9*a - c^3/d)/9 + 3*d^2*(c/(3*d) + x)^3)^p*Hype
rgeometric2F1[2/3, -p, 5/3, (27*d^3*(c/(3*d) + x)^3)/(c^3 - 9*a*d)])/(6*d*
(1 - (27*d^3*(c/(3*d) + x)^3)/(c^3 - 9*a*d))^p)
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2425
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Pq, x, n -
 1]   Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq,
x, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[P
q, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

rule 2432
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

rule 2459
```
Int[(Pn_)^(p_.)*(Qx_), x_Symbol] :> With[{S = Coeff[Pn, x, Expon[Pn, x] - 1
]/(Expon[Pn, x]*Coeff[Pn, x, Expon[Pn, x]])}, Subst[Int[ExpandToSum[Pn /. x
 -> x - S, x]^p*ExpandToSum[Qx /. x -> x - S, x], x], x, x + S] /; Binomial
Q[Pn /. x -> x - S, x] || (IntegerQ[Expon[Pn, x]/2] && TrinomialQ[Pn /. x -
> x - S, x])] /; FreeQ[p, x] && PolyQ[Pn, x] && GtQ[Expon[Pn, x], 2] && NeQ
[Coeff[Pn, x, Expon[Pn, x] - 1], 0] && PolyQ[Qx, x] &&  !(MonomialQ[Qx, x]
&& IGtQ[p, 0])
```

Maple [F]

$$\int \left(C\,x^{2}+Bx+A\right)\left(3d^{2}x^{3}+3cd\,x^{2}+c^{2}x+a\right)^{p}dx$$

input
```
int((C*x^2+B*x+A)*(3*d^2*x^3+3*c*d*x^2+c^2*x+a)^p,x)
```

output
```
int((C*x^2+B*x+A)*(3*d^2*x^3+3*c*d*x^2+c^2*x+a)^p,x)
```

Fricas [F]

$$\int \left(A+Bx+Cx^2\right)\left(a+c^2x+3cdx^2+3d^2x^3\right)^p\,dx = \int \left(Cx^2+Bx+A\right)\left(3\,d^2x^3+3\,cdx^2+c^2x+a\right)^p\,dx$$

input
```
integrate((C*x^2+B*x+A)*(3*d^2*x^3+3*c*d*x^2+c^2*x+a)^p,x, algorithm="fric
as")
```

output
```
integral((C*x^2 + B*x + A)*(3*d^2*x^3 + 3*c*d*x^2 + c^2*x + a)^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int \left(A+Bx+Cx^2\right)\left(a+c^2x+3cdx^2+3d^2x^3\right)^p\,dx = \text{Timed out}$$

input
```
integrate((C*x**2+B*x+A)*(3*d**2*x**3+3*c*d*x**2+c**2*x+a)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \left(A+Bx+Cx^2\right)\left(a+c^2x+3cdx^2+3d^2x^3\right)^p\,dx = \int \left(Cx^2+Bx+A\right)\left(3\,d^2x^3+3\,cdx^2+c^2x+a\right)^p\,dx$$

input
```
integrate((C*x^2+B*x+A)*(3*d^2*x^3+3*c*d*x^2+c^2*x+a)^p,x, algorithm="maxi
ma")
```

output
```
integrate((C*x^2 + B*x + A)*(3*d^2*x^3 + 3*c*d*x^2 + c^2*x + a)^p, x)
```

Giac [F]

$$\int (A+Bx+Cx^2) (a+c^2x+3cdx^2+3d^2x^3)^p dx$$
$$= \int (Cx^2+Bx+A)(3d^2x^3+3cdx^2+c^2x+a)^p dx$$

input
```
integrate((C*x^2+B*x+A)*(3*d^2*x^3+3*c*d*x^2+c^2*x+a)^p,x, algorithm="giac
")
```

output
```
integrate((C*x^2 + B*x + A)*(3*d^2*x^3 + 3*c*d*x^2 + c^2*x + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (A+Bx+Cx^2) (a+c^2x+3cdx^2+3d^2x^3)^p dx$$
$$= \int (C x^2+B x+A) (c^2 x+3 c d x^2+3 d^2 x^3+a)^p dx$$

input
```
int((A + B*x + C*x^2)*(a + c^2*x + 3*d^2*x^3 + 3*c*d*x^2)^p,x)
```

output
```
int((A + B*x + C*x^2)*(a + c^2*x + 3*d^2*x^3 + 3*c*d*x^2)^p, x)
```

Reduce [F]

$$\int (A+Bx+Cx^2) (a+c^2x+3cdx^2+3d^2x^3)^p dx = \text{too large to display}$$

input
```
int((C*x^2+B*x+A)*(3*d^2*x^3+3*c*d*x^2+c^2*x+a)^p,x)
```

output
```
(81*(a + c**2*x + 3*c*d*x**2 + 3*d**2*x**3)**p*a*b*d**2*p**2 + 108*(a + c*
*2*x + 3*c*d*x**2 + 3*d**2*x**3)**p*a*b*d**2*p + 27*(a + c**2*x + 3*c*d*x*
*2 + 3*d**2*x**3)**p*a*b*d**2 + 54*(a + c**2*x + 3*c*d*x**2 + 3*d**2*x**3)
**p*a*c**2*d**2*p**2 + 90*(a + c**2*x + 3*c*d*x**2 + 3*d**2*x**3)**p*a*c**
2*d**2*p + 36*(a + c**2*x + 3*c*d*x**2 + 3*d**2*x**3)**p*a*c**2*d**2 - 18*
(a + c**2*x + 3*c*d*x**2 + 3*d**2*x**3)**p*a*c**2*d*p - 6*(a + c**2*x + 3*
c*d*x**2 + 3*d**2*x**3)**p*a*c**2*d + 162*(a + c**2*x + 3*c*d*x**2 + 3*d**
2*x**3)**p*a*c*d**3*p**2*x + 270*(a + c**2*x + 3*c*d*x**2 + 3*d**2*x**3)**
p*a*c*d**3*p*x + 108*(a + c**2*x + 3*c*d*x**2 + 3*d**2*x**3)**p*a*c*d**3*x
 - 9*(a + c**2*x + 3*c*d*x**2 + 3*d**2*x**3)**p*b*c**3*d*p**2 - 18*(a + c*
*2*x + 3*c*d*x**2 + 3*d**2*x**3)**p*b*c**3*d*p - 9*(a + c**2*x + 3*c*d*x**
2 + 3*d**2*x**3)**p*b*c**3*d + 54*(a + c**2*x + 3*c*d*x**2 + 3*d**2*x**3)*
*p*b*c**2*d**2*p**2*x + 54*(a + c**2*x + 3*c*d*x**2 + 3*d**2*x**3)**p*b*c*
*2*d**2*p*x + 162*(a + c**2*x + 3*c*d*x**2 + 3*d**2*x**3)**p*b*c*d**3*p**2
*x**2 + 216*(a + c**2*x + 3*c*d*x**2 + 3*d**2*x**3)**p*b*c*d**3*p*x**2 + 5
4*(a + c**2*x + 3*c*d*x**2 + 3*d**2*x**3)**p*b*c*d**3*x**2 + 2*(a + c**2*x
 + 3*c*d*x**2 + 3*d**2*x**3)**p*c**5*p + 2*(a + c**2*x + 3*c*d*x**2 + 3*d*
*2*x**3)**p*c**5 - 12*(a + c**2*x + 3*c*d*x**2 + 3*d**2*x**3)**p*c**4*d*p*
x + 54*(a + c**2*x + 3*c*d*x**2 + 3*d**2*x**3)**p*c**3*d**2*p**2*x**2 + 18
*(a + c**2*x + 3*c*d*x**2 + 3*d**2*x**3)**p*c**3*d**2*p*x**2 + 162*(a +...
```

3.85 $\int (A+Bx+Cx^2)\left(bc+bdx+cdx^2+d^2x^3\right)^3\,dx$

Optimal result

Integrand size = 34, antiderivative size = 419

$$\begin{aligned}
&\int (A+Bx+Cx^2)\left(bc+bdx+cdx^2+d^2x^3\right)^3\,dx\\
&= Ab^3c^3x+\frac{1}{2}b^3c^2(Bc+3Ad)x^2+\frac{1}{3}b^2c\left(3Ac^2d+b\left(c^2C+3Bcd+3Ad^2\right)\right)x^3\\
&\quad+\frac{1}{4}b^2d\left(3bc^2C+9Ac^2d+Abd^2+3B\left(c^3+bcd\right)\right)x^4\\
&\quad+\frac{1}{5}bd\left(3Ac^3d+b^2d(3cC+Bd)+3bc\left(c^2C+3Bcd+3Ad^2\right)\right)x^5\\
&\quad+\frac{1}{6}bd^2\left(9bc^2C+9Ac^2d+b^2Cd+3Abd^2+3B\left(c^3+3bcd\right)\right)x^6\\
&\quad+\frac{1}{7}d^2\left(Ac^3d+3b^2d(3cC+Bd)+3bc\left(c^2C+3Bcd+3Ad^2\right)\right)x^7\\
&\quad+\frac{1}{8}d^3\left(B\left(c^3+9bcd\right)+3\left(3bc^2C+Ac^2d+b^2Cd+Abd^2\right)\right)x^8\\
&\quad+\frac{1}{9}d^3\left(c^3C+3Bc^2d+3bBd^2+3cd(3bC+Ad)\right)x^9\\
&\quad+\frac{1}{10}d^4\left(3c^2C+3Bcd+d(3bC+Ad)\right)x^{10}+\frac{1}{11}d^5(3cC+Bd)x^{11}+\frac{1}{12}Cd^6x^{12}
\end{aligned}$$

output
```
A*b^3*c^3*x+1/2*b^3*c^2*(3*A*d+B*c)*x^2+1/3*b^2*c*(3*A*c^2*d+b*(3*A*d^2+3*
B*c*d+C*c^2))*x^3+1/4*b^2*d*(3*b*c^2*C+9*A*c^2*d+A*b*d^2+3*B*(b*c*d+c^3))*
x^4+1/5*b*d*(3*A*c^3*d+b^2*d*(B*d+3*C*c)+3*b*c*(3*A*d^2+3*B*c*d+C*c^2))*x^
5+1/6*b*d^2*(9*b*c^2*C+9*A*c^2*d+b^2*C*d+3*A*b*d^2+3*B*(3*b*c*d+c^3))*x^6+
1/7*d^2*(A*c^3*d+3*b^2*d*(B*d+3*C*c)+3*b*c*(3*A*d^2+3*B*c*d+C*c^2))*x^7+1/
8*d^3*(B*(9*b*c*d+c^3)+3*A*b*d^2+3*A*c^2*d+3*b^2*C*d+9*b*c^2*C)*x^8+1/9*d^
3*(c^3*C+3*B*c^2*d+3*b*B*d^2+3*c*d*(A*d+3*C*b))*x^9+1/10*d^4*(3*C*c^2+3*B*
c*d+d*(A*d+3*C*b))*x^10+1/11*d^5*(B*d+3*C*c)*x^11+1/12*C*d^6*x^12
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 419, normalized size of antiderivative = 1.00

$$\begin{aligned}
&\int (A+Bx+Cx^2)\left(bc+bdx+cdx^2+d^2x^3\right)^3\,dx \\
&= Ab^3c^3x+\frac{1}{2}b^3c^2(Bc+3Ad)x^2+\frac{1}{3}b^2c\left(3Ac^2d+b\left(c^2C+3Bcd+3Ad^2\right)\right)x^3 \\
&\quad+\frac{1}{4}b^2d\left(3bc^2C+9Ac^2d+Abd^2+3B\left(c^3+bcd\right)\right)x^4 \\
&\quad+\frac{1}{5}bd\left(3Ac^3d+b^2d(3cC+Bd)+3bc\left(c^2C+3Bcd+3Ad^2\right)\right)x^5 \\
&\quad+\frac{1}{6}bd^2\left(9bc^2C+9Ac^2d+b^2Cd+3Abd^2+3B\left(c^3+3bcd\right)\right)x^6 \\
&\quad+\frac{1}{7}d^2\left(Ac^3d+3b^2d(3cC+Bd)+3bc\left(c^2C+3Bcd+3Ad^2\right)\right)x^7 \\
&\quad+\frac{1}{8}d^3\left(B\left(c^3+9bcd\right)+3\left(3bc^2C+Ac^2d+b^2Cd+Abd^2\right)\right)x^8 \\
&\quad+\frac{1}{9}d^3\left(c^3C+3Bc^2d+3bBd^2+3cd(3bC+Ad)\right)x^9 \\
&\quad+\frac{1}{10}d^4\left(3c^2C+3Bcd+d(3bC+Ad)\right)x^{10}+\frac{1}{11}d^5(3cC+Bd)x^{11}+\frac{1}{12}Cd^6x^{12}
\end{aligned}$$

input
```
Integrate[(A + B*x + C*x^2)*(b*c + b*d*x + c*d*x^2 + d^2*x^3)^3,x]
```

output
```
A*b^3*c^3*x + (b^3*c^2*(B*c + 3*A*d)*x^2)/2 + (b^2*c*(3*A*c^2*d + b*(c^2*C
 + 3*B*c*d + 3*A*d^2))*x^3)/3 + (b^2*d*(3*b*c^2*C + 9*A*c^2*d + A*b*d^2 +
3*B*(c^3 + b*c*d))*x^4)/4 + (b*d*(3*A*c^3*d + b^2*d*(3*c*C + B*d) + 3*b*c*
(c^2*C + 3*B*c*d + 3*A*d^2))*x^5)/5 + (b*d^2*(9*b*c^2*C + 9*A*c^2*d + b^2*
C*d + 3*A*b*d^2 + 3*B*(c^3 + 3*b*c*d))*x^6)/6 + (d^2*(A*c^3*d + 3*b^2*d*(3
*c*C + B*d) + 3*b*c*(c^2*C + 3*B*c*d + 3*A*d^2))*x^7)/7 + (d^3*(B*(c^3 + 9
*b*c*d) + 3*(3*b*c^2*C + A*c^2*d + b^2*C*d + A*b*d^2))*x^8)/8 + (d^3*(c^3*
C + 3*B*c^2*d + 3*b*B*d^2 + 3*c*d*(3*b*C + A*d))*x^9)/9 + (d^4*(3*c^2*C +
3*B*c*d + d*(3*b*C + A*d))*x^10)/10 + (d^5*(3*c*C + B*d)*x^11)/11 + (C*d^6
*x^12)/12
```

Rubi [A] (verified)

Time = 1.62 (sec) , antiderivative size = 419, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(A+Bx+Cx^2\right)\left(bc+bdx+cdx^2+d^2x^3\right)^3\,dx$$

$$\Big\downarrow \text{ 2188}$$

$$\int \left(b^3c^2x(3Ad+Bc)+Ab^3c^3+b^2cx^2\left(b\left(3Ad^2+3Bcd+c^2C\right)+3Ac^2d\right)+d^2x^6\left(3bc\left(3Ad^2+3Bcd+c^2C\right)+Ac\right.\right.$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}b^3c^2x^2(3Ad+Bc)+Ab^3c^3x+\frac{1}{3}b^2cx^3\left(b\left(3Ad^2+3Bcd+c^2C\right)+3Ac^2d\right)+\frac{1}{7}d^2x^7\left(3bc\left(3Ad^2+3Bcd+c^2C\right)+Ac^3d+3b^2d(Bd+3cC)\right)+\frac{1}{6}bd^2x^6\left(3Abd^2+9Ac^2d+b^2Cd+3B\left(3bcd+c^3\right)+9bc^2C\right)+\frac{1}{5}bdx^5\left(3bc\left(3Ad^2+3Bcd+c^2C\right)+3Ac^3d+b^2d(Bd+3cC)\right)+\frac{1}{4}b^2dx^4\left(Abd^2+9Ac^2d+3B\left(bcd+c^3\right)+3bc^2C\right)+\frac{1}{8}d^3x^8\left(3\left(Abd^2+Ac^2d+b^2Cd+3bc^2C\right)+B\left(9bcd+c^3\right)\right)+\frac{1}{10}d^4x^{10}\left(d(Ad+3bC)+3Bcd+3c^2C\right)+\frac{1}{9}d^3x^9\left(3cd(Ad+3bC)+3bBd^2+3Bc^2d+c^3C\right)+\frac{1}{11}d^5x^{11}(Bd+3cC)+\frac{1}{12}Cd^6x^{12}$$

input
```
Int[(A + B*x + C*x^2)*(b*c + b*d*x + c*d*x^2 + d^2*x^3)^3,x]
```

output
```
A*b^3*c^3*x + (b^3*c^2*(B*c + 3*A*d)*x^2)/2 + (b^2*c*(3*A*c^2*d + b*(c^2*C
 + 3*B*c*d + 3*A*d^2))*x^3)/3 + (b^2*d*(3*b*c^2*C + 9*A*c^2*d + A*b*d^2 +
3*B*(c^3 + b*c*d))*x^4)/4 + (b*d*(3*A*c^3*d + b^2*d*(3*c*C + B*d) + 3*b*c*
(c^2*C + 3*B*c*d + 3*A*d^2))*x^5)/5 + (b*d^2*(9*b*c^2*C + 9*A*c^2*d + b^2*
C*d + 3*A*b*d^2 + 3*B*(c^3 + 3*b*c*d))*x^6)/6 + (d^2*(A*c^3*d + 3*b^2*d*(3
*c*C + B*d) + 3*b*c*(c^2*C + 3*B*c*d + 3*A*d^2))*x^7)/7 + (d^3*(B*(c^3 + 9
*b*c*d) + 3*(3*b*c^2*C + A*c^2*d + b^2*C*d + A*b*d^2))*x^8)/8 + (d^3*(c^3*
C + 3*B*c^2*d + 3*b*B*d^2 + 3*c*d*(3*b*C + A*d))*x^9)/9 + (d^4*(3*c^2*C +
3*B*c*d + d*(3*b*C + A*d))*x^10)/10 + (d^5*(3*c*C + B*d)*x^11)/11 + (C*d^6
*x^12)/12
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 486, normalized size of antiderivative = 1.16

method	result
norman	$\frac{C\,d^6x^{12}}{12}+\left(\frac{1}{11}B\,d^6+\frac{3}{11}Cc\,d^5\right)x^{11}+\left(\frac{1}{10}A\,d^6+\frac{3}{10}Bc\,d^5+\frac{3}{10}Cb\,d^5+\frac{3}{10}C\,c^2d^4\right)x^{10}+\left(\frac{1}{3}Ac\,d\right.$
risch	$A\,b^3c^3x+\frac{3}{8}x^8C\,b^2d^4+\frac{1}{7}x^7A\,c^3d^3+\frac{3}{7}x^7b^2B\,d^4+\frac{1}{3}x^9B\,c^2d^4+\frac{1}{9}x^9C\,c^3d^3+\frac{3}{8}x^8Ab\,d^5+\frac{3}{8}x^{\cdots}$
parallelrisch	$A\,b^3c^3x+\frac{3}{8}x^8C\,b^2d^4+\frac{1}{7}x^7A\,c^3d^3+\frac{3}{7}x^7b^2B\,d^4+\frac{1}{3}x^9B\,c^2d^4+\frac{1}{9}x^9C\,c^3d^3+\frac{3}{8}x^8Ab\,d^5+\frac{3}{8}x^{\cdots}$
gosper	$\frac{x(2310C\,d^6x^{11}+2520x^{10}B\,d^6+7560x^{10}Cc\,d^5+2772x^9A\,d^6+8316x^9Bc\,d^5+8316x^9Cb\,d^5+8316x^9C\,c^2d^4+9240x^8Ac\,d^5+924\cdots}{}$
orering	$\frac{x(2310C\,d^6x^{11}+2520x^{10}B\,d^6+7560x^{10}Cc\,d^5+2772x^9A\,d^6+8316x^9Bc\,d^5+8316x^9Cb\,d^5+8316x^9C\,c^2d^4+9240x^8Ac\,d^5+924\cdots}{}$
default	$\frac{C\,d^6x^{12}}{12}+\frac{(B\,d^6+3Cc\,d^5)x^{11}}{11}+\frac{(A\,d^6+3Bc\,d^5+C(b\,d^5+2c^2d^4+d^2(2b\,d^3+c^2d^2)))x^{10}}{10}+\frac{(3Ac\,d^5+B(b\,d^5+2c^2d^4+d^2(\cdots}{}$

input
```
int((C*x^2+B*x+A)*(d^2*x^3+c*d*x^2+b*d*x+b*c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/12*C*d^6*x^12+(1/11*B*d^6+3/11*C*c*d^5)*x^11+(1/10*A*d^6+3/10*B*c*d^5+3/
10*C*b*d^5+3/10*C*c^2*d^4)*x^10+(1/3*A*c*d^5+1/3*b*B*d^5+1/3*B*c^2*d^4+C*b
*c*d^4+1/9*C*c^3*d^3)*x^9+(3/8*A*b*d^5+3/8*A*c^2*d^4+9/8*b*B*d^4*c+1/8*B*c
^3*d^3+3/8*C*b^2*d^4+9/8*C*b*c^2*d^3)*x^8+(9/7*A*b*c*d^4+1/7*A*c^3*d^3+3/7
*b^2*B*d^4+9/7*b*B*d^3*c^2+9/7*C*b^2*c*d^3+3/7*C*b*c^3*d^2)*x^7+(1/2*A*b^2
*d^4+3/2*A*b*c^2*d^3+3/2*b^2*B*d^3*c+1/2*b*B*d^2*c^3+1/6*C*b^3*d^3+3/2*C*b
^2*c^2*d^2)*x^6+(9/5*A*b^2*c*d^3+3/5*A*b*c^3*d^2+1/5*B*b^3*d^3+9/5*b^2*B*d
^2*c^2+3/5*C*b^3*c*d^2+3/5*C*b^2*c^3*d)*x^5+(1/4*A*b^3*d^3+9/4*A*b^2*c^2*d
^2+3/4*B*b^3*c*d^2+3/4*b^2*B*d*c^3+3/4*C*b^3*c^2*d)*x^4+(A*b^3*c*d^2+A*b^2
*c^3*d+B*b^3*c^2*d+1/3*C*b^3*c^3)*x^3+(3/2*A*b^3*c^2*d+1/2*B*b^3*c^3)*x^2+
A*b^3*c^3*x
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 461, normalized size of antiderivative = 1.10

$$\int (A+Bx+Cx^2)\left(bc+bdx+cdx^2+d^2x^3\right)^3\,dx$$
$$=\frac{1}{12}Cd^6x^{12}+\frac{1}{11}\left(3Ccd^5+Bd^6\right)x^{11}+\frac{1}{10}\left(3Cc^2d^4+Ad^6+3(Cb+Bc)d^5\right)x^{10}$$
$$+\frac{1}{9}\left(Cc^3d^3+3(Bb+Ac)d^5+3\left(3Cbc+Bc^2\right)d^4\right)x^9$$
$$+\frac{1}{8}\left(3Abd^5+3\left(Cb^2+3Bbc+Ac^2\right)d^4+\left(9Cbc^2+Bc^3\right)d^3\right)x^8+Ab^3c^3x$$
$$+\frac{1}{7}\left(3Cbc^3d^2+3\left(Bb^2+3Abc\right)d^4+\left(9Cb^2c+9Bbc^2+Ac^3\right)d^3\right)x^7$$
$$+\frac{1}{6}\left(3Ab^2d^4+\left(Cb^3+9Bb^2c+9Abc^2\right)d^3+3\left(3Cb^2c^2+Bbc^3\right)d^2\right)x^6$$
$$+\frac{1}{5}\left(3Cb^2c^3d+\left(Bb^3+9Ab^2c\right)d^3+3\left(Cb^3c+3Bb^2c^2+Abc^3\right)d^2\right)x^5$$
$$+\frac{1}{4}\left(Ab^3d^3+3\left(Bb^3c+3Ab^2c^2\right)d^2+3\left(Cb^3c^2+Bb^2c^3\right)d\right)x^4$$
$$+\frac{1}{3}\left(Cb^3c^3+3Ab^3cd^2+3\left(Bb^3c^2+Ab^2c^3\right)d\right)x^3+\frac{1}{2}\left(Bb^3c^3+3Ab^3c^2d\right)x^2$$

input
```
integrate((C*x^2+B*x+A)*(d^2*x^3+c*d*x^2+b*d*x+b*c)^3,x, algorithm="fricas
")
```

output
```
1/12*C*d^6*x^12 + 1/11*(3*C*c*d^5 + B*d^6)*x^11 + 1/10*(3*C*c^2*d^4 + A*d^
6 + 3*(C*b + B*c)*d^5)*x^10 + 1/9*(C*c^3*d^3 + 3*(B*b + A*c)*d^5 + 3*(3*C*
b*c + B*c^2)*d^4)*x^9 + 1/8*(3*A*b*d^5 + 3*(C*b^2 + 3*B*b*c + A*c^2)*d^4 +
 (9*C*b*c^2 + B*c^3)*d^3)*x^8 + A*b^3*c^3*x + 1/7*(3*C*b*c^3*d^2 + 3*(B*b^
2 + 3*A*b*c)*d^4 + (9*C*b^2*c + 9*B*b*c^2 + A*c^3)*d^3)*x^7 + 1/6*(3*A*b^2
*d^4 + (C*b^3 + 9*B*b^2*c + 9*A*b*c^2)*d^3 + 3*(3*C*b^2*c^2 + B*b*c^3)*d^2
)*x^6 + 1/5*(3*C*b^2*c^3*d + (B*b^3 + 9*A*b^2*c)*d^3 + 3*(C*b^3*c + 3*B*b^
2*c^2 + A*b*c^3)*d^2)*x^5 + 1/4*(A*b^3*d^3 + 3*(B*b^3*c + 3*A*b^2*c^2)*d^2
 + 3*(C*b^3*c^2 + B*b^2*c^3)*d)*x^4 + 1/3*(C*b^3*c^3 + 3*A*b^3*c*d^2 + 3*(
B*b^3*c^2 + A*b^2*c^3)*d)*x^3 + 1/2*(B*b^3*c^3 + 3*A*b^3*c^2*d)*x^2
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 585, normalized size of antiderivative = 1.40

$$
\begin{aligned}
&\int (A+Bx+Cx^2)\left(bc+bdx+cdx^2+d^2x^3\right)^3\,dx \\
&= Ab^3c^3x+\frac{Cd^6x^{12}}{12}+x^{11}\left(\frac{Bd^6}{11}+\frac{3Ccd^5}{11}\right)+x^{10}\left(\frac{Ad^6}{10}+\frac{3Bcd^5}{10}+\frac{3Cbd^5}{10}+\frac{3Cc^2d^4}{10}\right) \\
&\quad+x^9\left(\frac{Acd^5}{3}+\frac{Bbd^5}{3}+\frac{Bc^2d^4}{3}+Cbcd^4+\frac{Cc^3d^3}{9}\right)+x^8 \\
&\quad\cdot\left(\frac{3Abd^5}{8}+\frac{3Ac^2d^4}{8}+\frac{9Bbcd^4}{8}+\frac{Bc^3d^3}{8}+\frac{3Cb^2d^4}{8}+\frac{9Cbc^2d^3}{8}\right)+x^7 \\
&\quad\cdot\left(\frac{9Abcd^4}{7}+\frac{Ac^3d^3}{7}+\frac{3Bb^2d^4}{7}+\frac{9Bbc^2d^3}{7}+\frac{9Cb^2cd^3}{7}+\frac{3Cbc^3d^2}{7}\right) \\
&\quad+x^6\left(\frac{Ab^2d^4}{2}+\frac{3Abc^2d^3}{2}+\frac{3Bb^2cd^3}{2}+\frac{Bbc^3d^2}{2}+\frac{Cb^3d^3}{6}+\frac{3Cb^2c^2d^2}{2}\right)+x^5 \\
&\quad\cdot\left(\frac{9Ab^2cd^3}{5}+\frac{3Abc^3d^2}{5}+\frac{Bb^3d^3}{5}+\frac{9Bb^2c^2d^2}{5}+\frac{3Cb^3cd^2}{5}+\frac{3Cb^2c^3d}{5}\right) \\
&\quad+x^4\left(\frac{Ab^3d^3}{4}+\frac{9Ab^2c^2d^2}{4}+\frac{3Bb^3cd^2}{4}+\frac{3Bb^2c^3d}{4}+\frac{3Cb^3c^2d}{4}\right) \\
&\quad+x^3\left(Ab^3cd^2+Ab^2c^3d+Bb^3c^2d+\frac{Cb^3c^3}{3}\right)+x^2\cdot\left(\frac{3Ab^3c^2d}{2}+\frac{Bb^3c^3}{2}\right)
\end{aligned}
$$

input

```
integrate((C*x**2+B*x+A)*(d**2*x**3+c*d*x**2+b*d*x+b*c)**3,x)
```

output

```
A*b**3*c**3*x + C*d**6*x**12/12 + x**11*(B*d**6/11 + 3*C*c*d**5/11) + x**1
0*(A*d**6/10 + 3*B*c*d**5/10 + 3*C*b*d**5/10 + 3*C*c**2*d**4/10) + x**9*(A
*c*d**5/3 + B*b*d**5/3 + B*c**2*d**4/3 + C*b*c*d**4 + C*c**3*d**3/9) + x**
8*(3*A*b*d**5/8 + 3*A*c**2*d**4/8 + 9*B*b*c*d**4/8 + B*c**3*d**3/8 + 3*C*b
**2*d**4/8 + 9*C*b*c**2*d**3/8) + x**7*(9*A*b*c*d**4/7 + A*c**3*d**3/7 + 3
*B*b**2*d**4/7 + 9*B*b*c**2*d**3/7 + 9*C*b**2*c*d**3/7 + 3*C*b*c**3*d**2/7
) + x**6*(A*b**2*d**4/2 + 3*A*b*c**2*d**3/2 + 3*B*b**2*c*d**3/2 + B*b*c**3
*d**2/2 + C*b**3*d**3/6 + 3*C*b**2*c**2*d**2/2) + x**5*(9*A*b**2*c*d**3/5
+ 3*A*b*c**3*d**2/5 + B*b**3*d**3/5 + 9*B*b**2*c**2*d**2/5 + 3*C*b**3*c*d*
*2/5 + 3*C*b**2*c**3*d/5) + x**4*(A*b**3*d**3/4 + 9*A*b**2*c**2*d**2/4 + 3
*B*b**3*c*d**2/4 + 3*B*b**2*c**3*d/4 + 3*C*b**3*c**2*d/4) + x**3*(A*b**3*c
*d**2 + A*b**2*c**3*d + B*b**3*c**2*d + C*b**3*c**3/3) + x**2*(3*A*b**3*c*
*2*d/2 + B*b**3*c**3/2)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 461, normalized size of antiderivative = 1.10

$$\int \left(A+Bx+Cx^2\right)\left(bc+bdx+cdx^2+d^2x^3\right)^3\,dx$$
$$=\frac{1}{12}Cd^6x^{12}+\frac{1}{11}\left(3\,Ccd^5+Bd^6\right)x^{11}+\frac{1}{10}\left(3\,Cc^2d^4+Ad^6+3\,(Cb+Bc)d^5\right)x^{10}$$
$$+\frac{1}{9}\left(Cc^3d^3+3\,(Bb+Ac)d^5+3\left(3\,Cbc+Bc^2\right)d^4\right)x^9$$
$$+\frac{1}{8}\left(3\,Abd^5+3\left(Cb^2+3\,Bbc+Ac^2\right)d^4+\left(9\,Cbc^2+Bc^3\right)d^3\right)x^8+Ab^3c^3x$$
$$+\frac{1}{7}\left(3\,Cbc^3d^2+3\left(Bb^2+3\,Abc\right)d^4+\left(9\,Cb^2c+9\,Bbc^2+Ac^3\right)d^3\right)x^7$$
$$+\frac{1}{6}\left(3\,Ab^2d^4+\left(Cb^3+9\,Bb^2c+9\,Abc^2\right)d^3+3\left(3\,Cb^2c^2+Bbc^3\right)d^2\right)x^6$$
$$+\frac{1}{5}\left(3\,Cb^2c^3d+\left(Bb^3+9\,Ab^2c\right)d^3+3\left(Cb^3c+3\,Bb^2c^2+Abc^3\right)d^2\right)x^5$$
$$+\frac{1}{4}\left(Ab^3d^3+3\left(Bb^3c+3\,Ab^2c^2\right)d^2+3\left(Cb^3c^2+Bb^2c^3\right)d\right)x^4$$
$$+\frac{1}{3}\left(Cb^3c^3+3\,Ab^3cd^2+3\left(Bb^3c^2+Ab^2c^3\right)d\right)x^3+\frac{1}{2}\left(Bb^3c^3+3\,Ab^3c^2d\right)x^2$$

input
```
integrate((C*x^2+B*x+A)*(d^2*x^3+c*d*x^2+b*d*x+b*c)^3,x, algorithm="maxima
")
```

output
```
1/12*C*d^6*x^12 + 1/11*(3*C*c*d^5 + B*d^6)*x^11 + 1/10*(3*C*c^2*d^4 + A*d^
6 + 3*(C*b + B*c)*d^5)*x^10 + 1/9*(C*c^3*d^3 + 3*(B*b + A*c)*d^5 + 3*(3*C*
b*c + B*c^2)*d^4)*x^9 + 1/8*(3*A*b*d^5 + 3*(C*b^2 + 3*B*b*c + A*c^2)*d^4 +
 (9*C*b*c^2 + B*c^3)*d^3)*x^8 + A*b^3*c^3*x + 1/7*(3*C*b*c^3*d^2 + 3*(B*b^
2 + 3*A*b*c)*d^4 + (9*C*b^2*c + 9*B*b*c^2 + A*c^3)*d^3)*x^7 + 1/6*(3*A*b^2
*d^4 + (C*b^3 + 9*B*b^2*c + 9*A*b*c^2)*d^3 + 3*(3*C*b^2*c^2 + B*b*c^3)*d^2
)*x^6 + 1/5*(3*C*b^2*c^3*d + (B*b^3 + 9*A*b^2*c)*d^3 + 3*(C*b^3*c + 3*B*b^
2*c^2 + A*b*c^3)*d^2)*x^5 + 1/4*(A*b^3*d^3 + 3*(B*b^3*c + 3*A*b^2*c^2)*d^2
 + 3*(C*b^3*c^2 + B*b^2*c^3)*d)*x^4 + 1/3*(C*b^3*c^3 + 3*A*b^3*c*d^2 + 3*(
B*b^3*c^2 + A*b^2*c^3)*d)*x^3 + 1/2*(B*b^3*c^3 + 3*A*b^3*c^2*d)*x^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 573, normalized size of antiderivative = 1.37

$$\int (A+Bx+Cx^2)(bc+bdx+cdx^2+d^2x^3)^3\,dx = \frac{1}{12}Cd^6x^{12}+\frac{3}{11}Ccd^5x^{11}+\frac{1}{11}Bd^6x^{11}+\frac{3}{10}Cc^2d^4x^{10}+\frac{3}{10}Cbd^5x^{10}+\frac{3}{10}Bcd^5x^{10} + \frac{1}{10}Ad^6x^{10}+\frac{1}{9}Cc^3d^3x^9+Cbcd^4x^9+\frac{1}{3}Bc^2d^4x^9+\frac{1}{3}Bbd^5x^9+\frac{1}{3}Acd^5x^9 + \frac{9}{8}Cbc^2d^3x^8+\frac{1}{8}Bc^3d^3x^8+\frac{3}{8}Cb^2d^4x^8+\frac{9}{8}Bbcd^4x^8+\frac{3}{8}Ac^2d^4x^8+\frac{3}{8}Abd^5x^8 + \frac{3}{7}Cbc^3d^2x^7+\frac{9}{7}Cb^2cd^3x^7+\frac{9}{7}Bbc^2d^3x^7+\frac{1}{7}Ac^3d^3x^7+\frac{3}{7}Bb^2d^4x^7+\frac{9}{7}Abcd^4x^7 + \frac{3}{2}Cb^2c^2d^2x^6+\frac{1}{2}Bbc^3d^2x^6+\frac{1}{6}Cb^3d^3x^6+\frac{3}{2}Bb^2cd^3x^6+\frac{3}{2}Abc^2d^3x^6+\frac{1}{2}Ab^2d^4x^6 + \frac{3}{5}Cb^2c^3dx^5+\frac{3}{5}Cb^3cd^2x^5+\frac{9}{5}Bb^2c^2d^2x^5+\frac{3}{5}Abc^3d^2x^5+\frac{1}{5}Bb^3d^3x^5+\frac{9}{5}Ab^2cd^3x^5 + \frac{3}{4}Cb^3c^2dx^4+\frac{3}{4}Bb^2c^3dx^4+\frac{3}{4}Bb^3cd^2x^4+\frac{9}{4}Ab^2c^2d^2x^4+\frac{1}{4}Ab^3d^3x^4+\frac{1}{3}Cb^3c^3x^3 + Bb^3c^2dx^3+Ab^2c^3dx^3+Ab^3cd^2x^3+\frac{1}{2}Bb^3c^3x^2+\frac{3}{2}Ab^3c^2dx^2+Ab^3c^3x$$

input
```
integrate((C*x^2+B*x+A)*(d^2*x^3+c*d*x^2+b*d*x+b*c)^3,x, algorithm="giac")
```

output
```
1/12*C*d^6*x^12 + 3/11*C*c*d^5*x^11 + 1/11*B*d^6*x^11 + 3/10*C*c^2*d^4*x^1
0 + 3/10*C*b*d^5*x^10 + 3/10*B*c*d^5*x^10 + 1/10*A*d^6*x^10 + 1/9*C*c^3*d^
3*x^9 + C*b*c*d^4*x^9 + 1/3*B*c^2*d^4*x^9 + 1/3*B*b*d^5*x^9 + 1/3*A*c*d^5*
x^9 + 9/8*C*b*c^2*d^3*x^8 + 1/8*B*c^3*d^3*x^8 + 3/8*C*b^2*d^4*x^8 + 9/8*B*
b*c*d^4*x^8 + 3/8*A*c^2*d^4*x^8 + 3/8*A*b*d^5*x^8 + 3/7*C*b*c^3*d^2*x^7 +
9/7*C*b^2*c*d^3*x^7 + 9/7*B*b*c^2*d^3*x^7 + 1/7*A*c^3*d^3*x^7 + 3/7*B*b^2*
d^4*x^7 + 9/7*A*b*c*d^4*x^7 + 3/2*C*b^2*c^2*d^2*x^6 + 1/2*B*b*c^3*d^2*x^6
+ 1/6*C*b^3*d^3*x^6 + 3/2*B*b^2*c*d^3*x^6 + 3/2*A*b*c^2*d^3*x^6 + 1/2*A*b^
2*d^4*x^6 + 3/5*C*b^2*c^3*d*x^5 + 3/5*C*b^3*c*d^2*x^5 + 9/5*B*b^2*c^2*d^2*
x^5 + 3/5*A*b*c^3*d^2*x^5 + 1/5*B*b^3*d^3*x^5 + 9/5*A*b^2*c*d^3*x^5 + 3/4*
C*b^3*c^2*d*x^4 + 3/4*B*b^2*c^3*d*x^4 + 3/4*B*b^3*c*d^2*x^4 + 9/4*A*b^2*c^
2*d^2*x^4 + 1/4*A*b^3*d^3*x^4 + 1/3*C*b^3*c^3*x^3 + B*b^3*c^2*d*x^3 + A*b^
2*c^3*d*x^3 + A*b^3*c*d^2*x^3 + 1/2*B*b^3*c^3*x^2 + 3/2*A*b^3*c^2*d*x^2 +
A*b^3*c^3*x
```

Mupad [B] (verification not implemented)

Time = 12.74 (sec) , antiderivative size = 436, normalized size of antiderivative = 1.04

$$\int (A+Bx+Cx^2)(bc+bdx+cdx^2+d^2x^3)^3\,dx$$
$$= x^9\left(\frac{C\,c^3\,d^3}{9}+\frac{B\,c^2\,d^4}{3}+\frac{A\,c\,d^5}{3}+C\,b\,c\,d^4+\frac{B\,b\,d^5}{3}\right)$$
$$+x^7\left(\frac{9\,C\,b^2\,c\,d^3}{7}+\frac{3\,B\,b^2\,d^4}{7}+\frac{3\,C\,b\,c^3\,d^2}{7}+\frac{9\,B\,b\,c^2\,d^3}{7}+\frac{9\,A\,b\,c\,d^4}{7}+\frac{A\,c^3\,d^3}{7}\right)$$
$$+x^{10}\left(\frac{3\,C\,c^2\,d^4}{10}+\frac{3\,B\,c\,d^5}{10}+\frac{A\,d^6}{10}+\frac{3\,C\,b\,d^5}{10}\right)+x^{11}\left(\frac{B\,d^6}{11}+\frac{3\,C\,c\,d^5}{11}\right)$$
$$+x^5\left(\frac{3\,C\,b^3\,c\,d^2}{5}+\frac{B\,b^3\,d^3}{5}+\frac{3\,C\,b^2\,c^3\,d}{5}+\frac{9\,B\,b^2\,c^2\,d^2}{5}+\frac{9\,A\,b^2\,c\,d^3}{5}+\frac{3\,A\,b\,c^3\,d^2}{5}\right)$$
$$+\frac{C\,d^6\,x^{12}}{12}+\frac{d^3\,x^8\,(3\,C\,b^2\,d+9\,C\,b\,c^2+9\,B\,b\,c\,d+3\,A\,b\,d^2+B\,c^3+3\,A\,c^2\,d)}{8}$$
$$+\frac{b\,d^2\,x^6\,(C\,b^2\,d+9\,C\,b\,c^2+9\,B\,b\,c\,d+3\,A\,b\,d^2+3\,B\,c^3+9\,A\,c^2\,d)}{6}$$
$$+\frac{b^3\,c^2\,x^2\,(3\,A\,d+B\,c)}{2}+\frac{b^2\,d\,x^4\,(3\,B\,c^3+9\,A\,c^2\,d+3\,C\,b\,c^2+3\,B\,b\,c\,d+A\,b\,d^2)}{4}$$
$$+A\,b^3\,c^3\,x+\frac{b^2\,c\,x^3\,(3\,A\,c^2\,d+C\,b\,c^2+3\,B\,b\,c\,d+3\,A\,b\,d^2)}{3}$$

input
```
int((A + B*x + C*x^2)*(b*c + d^2*x^3 + b*d*x + c*d*x^2)^3,x)
```

output
```
x^9*((B*c^2*d^4)/3 + (C*c^3*d^3)/9 + (A*c*d^5)/3 + (B*b*d^5)/3 + C*b*c*d^4
) + x^7*((A*c^3*d^3)/7 + (3*B*b^2*d^4)/7 + (9*A*b*c*d^4)/7 + (9*B*b*c^2*d^
3)/7 + (3*C*b*c^3*d^2)/7 + (9*C*b^2*c*d^3)/7) + x^10*((A*d^6)/10 + (3*C*c^
2*d^4)/10 + (3*B*c*d^5)/10 + (3*C*b*d^5)/10) + x^11*((B*d^6)/11 + (3*C*c*d
^5)/11) + x^5*((B*b^3*d^3)/5 + (3*A*b*c^3*d^2)/5 + (9*A*b^2*c*d^3)/5 + (3*
C*b^2*c^3*d)/5 + (3*C*b^3*c*d^2)/5 + (9*B*b^2*c^2*d^2)/5) + (C*d^6*x^12)/1
2 + (d^3*x^8*(B*c^3 + 3*A*b*d^2 + 3*A*c^2*d + 9*C*b*c^2 + 3*C*b^2*d + 9*B*
b*c*d))/8 + (b*d^2*x^6*(3*B*c^3 + 3*A*b*d^2 + 9*A*c^2*d + 9*C*b*c^2 + C*b^
2*d + 9*B*b*c*d))/6 + (b^3*c^2*x^2*(3*A*d + B*c))/2 + (b^2*d*x^4*(3*B*c^3
+ A*b*d^2 + 9*A*c^2*d + 3*C*b*c^2 + 3*B*b*c*d))/4 + A*b^3*c^3*x + (b^2*c*x
^3*(3*A*b*d^2 + 3*A*c^2*d + C*b*c^2 + 3*B*b*c*d))/3
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 456, normalized size of antiderivative = 1.09

$$\int \left(A+Bx+Cx^2\right)\left(bc+bdx+cdx^2+d^2x^3\right)^3\,dx$$
$$=\frac{x(2310c\,d^6x^{11}+2520b\,d^6x^{10}+7560c^2d^5x^{10}+2772a\,d^6x^9+16632bc\,d^5x^9+8316c^3d^4x^9+9240ac\,d^5x^8+\cdots}{}$$

input
```
int((C*x^2+B*x+A)*(d^2*x^3+c*d*x^2+b*d*x+b*c)^3,x)
```

output
```
(x*(27720*a*b**3*c**3 + 41580*a*b**3*c**2*d*x + 27720*a*b**3*c*d**2*x**2 +
 6930*a*b**3*d**3*x**3 + 27720*a*b**2*c**3*d*x**2 + 62370*a*b**2*c**2*d**2
*x**3 + 49896*a*b**2*c*d**3*x**4 + 13860*a*b**2*d**4*x**5 + 16632*a*b*c**3
*d**2*x**4 + 41580*a*b*c**2*d**3*x**5 + 35640*a*b*c*d**4*x**6 + 10395*a*b*
d**5*x**7 + 3960*a*c**3*d**3*x**6 + 10395*a*c**2*d**4*x**7 + 9240*a*c*d**5
*x**8 + 2772*a*d**6*x**9 + 13860*b**4*c**3*x + 27720*b**4*c**2*d*x**2 + 20
790*b**4*c*d**2*x**3 + 5544*b**4*d**3*x**4 + 9240*b**3*c**4*x**2 + 41580*b
**3*c**3*d*x**3 + 66528*b**3*c**2*d**2*x**4 + 46200*b**3*c*d**3*x**5 + 118
80*b**3*d**4*x**6 + 16632*b**2*c**4*d*x**4 + 55440*b**2*c**3*d**2*x**5 + 7
1280*b**2*c**2*d**3*x**6 + 41580*b**2*c*d**4*x**7 + 9240*b**2*d**5*x**8 +
11880*b*c**4*d**2*x**6 + 34650*b*c**3*d**3*x**7 + 36960*b*c**2*d**4*x**8 +
 16632*b*c*d**5*x**9 + 2520*b*d**6*x**10 + 3080*c**4*d**3*x**8 + 8316*c**3
*d**4*x**9 + 7560*c**2*d**5*x**10 + 2310*c*d**6*x**11))/27720
```

3.86 $\int (A+Bx+Cx^2) (bc+bdx+cdx^2+d^2x^3)^2 \, dx$

Optimal result

Integrand size = 34, antiderivative size = 224

$$\begin{aligned}
&\int (A+Bx+Cx^2) \left(bc+bdx+cdx^2+d^2x^3\right)^2 \, dx \\
&= Ab^2c^2x+\frac{1}{2}b^2c(Bc+2Ad)x^2+\frac{1}{3}b\left(2Ac^2d+b\left(c^2C+2Bcd+Ad^2\right)\right)x^3 \\
&\quad+\frac{1}{4}bd\left(2Bc^2+2bcC+bBd+4Acd\right)x^4 \\
&\quad+\frac{1}{5}d\left(Ac^2d+b^2Cd+2b\left(c^2C+2Bcd+Ad^2\right)\right)x^5+\frac{1}{6}d^2\left(Bc^2+4bcC+2bBd+2Acd\right)x^6 \\
&\quad+\frac{1}{7}d^2\left(c^2C+2Bcd+d(2bC+Ad)\right)x^7+\frac{1}{8}d^3(2cC+Bd)x^8+\frac{1}{9}Cd^4x^9
\end{aligned}$$

output
```
A*b^2*c^2*x+1/2*b^2*c*(2*A*d+B*c)*x^2+1/3*b*(2*A*c^2*d+b*(A*d^2+2*B*c*d+C*
c^2))*x^3+1/4*b*d*(4*A*c*d+B*b*d+2*B*c^2+2*C*b*c)*x^4+1/5*d*(A*c^2*d+b^2*C
*d+2*b*(A*d^2+2*B*c*d+C*c^2))*x^5+1/6*d^2*(2*A*c*d+2*B*b*d+B*c^2+4*C*b*c)*
x^6+1/7*d^2*(C*c^2+2*B*c*d+d*(A*d+2*C*b))*x^7+1/8*d^3*(B*d+2*C*c)*x^8+1/9*
C*d^4*x^9
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.00

$$\int \left(A+Bx+Cx^2\right)\left(bc+bdx+cdx^2+d^2x^3\right)^2\,dx$$
$$=Ab^2c^2x+\frac{1}{2}b^2c(Bc+2Ad)x^2+\frac{1}{3}b\left(bc^2C+2bBcd+2Ac^2d+Abd^2\right)x^3$$
$$+\frac{1}{4}bd\left(2Bc^2+2bcC+bBd+4Acd\right)x^4$$
$$+\frac{1}{5}d\left(2bc^2C+4bBcd+Ac^2d+b^2Cd+2Abd^2\right)x^5+\frac{1}{6}d^2\left(Bc^2+4bcC+2bBd+2Acd\right)x^6$$
$$+\frac{1}{7}d^2\left(c^2C+2Bcd+2bCd+Ad^2\right)x^7+\frac{1}{8}d^3(2cC+Bd)x^8+\frac{1}{9}Cd^4x^9$$

input
```
Integrate[(A + B*x + C*x^2)*(b*c + b*d*x + c*d*x^2 + d^2*x^3)^2,x]
```

output
```
A*b^2*c^2*x + (b^2*c*(B*c + 2*A*d)*x^2)/2 + (b*(b*c^2*C + 2*b*B*c*d + 2*A*
c^2*d + A*b*d^2)*x^3)/3 + (b*d*(2*B*c^2 + 2*b*c*C + b*B*d + 4*A*c*d)*x^4)/
4 + (d*(2*b*c^2*C + 4*b*B*c*d + A*c^2*d + b^2*C*d + 2*A*b*d^2)*x^5)/5 + (d
^2*(B*c^2 + 4*b*c*C + 2*b*B*d + 2*A*c*d)*x^6)/6 + (d^2*(c^2*C + 2*B*c*d +
2*b*C*d + A*d^2)*x^7)/7 + (d^3*(2*c*C + B*d)*x^8)/8 + (C*d^4*x^9)/9
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(A+Bx+Cx^2\right)\left(bc+bdx+cdx^2+d^2x^3\right)^2\,dx$$
$$\Big\downarrow \text{2188}$$
$$\int \left(dx^4\left(2b\left(Ad^2+2Bcd+c^2C\right)+Ac^2d+b^2Cd\right)+b^2cx(2Ad+Bc)+Ab^2c^2+d^2x^6\left(d(Ad+2bC)+2Bcd+c^2C\right.\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{5}dx^5\left(2b\left(Ad^2+2Bcd+c^2C\right)+Ac^2d+b^2Cd\right)+\frac{1}{2}b^2cx^2(2Ad+Bc)+Ab^2c^2x+\frac{1}{7}d^2x^7\left(d(Ad+2bC)+2Bcd+c^2C\right)+\frac{1}{6}d^2x^6\left(2Acd+2bBd+4bcC+Bc^2\right)+\frac{1}{3}bx^3\left(b\left(Ad^2+2Bcd+c^2C\right)+2Ac^2d\right)+\frac{1}{4}bdx^4\left(4Acd+bBd+2bcC+2Bc^2\right)+\frac{1}{8}d^3x^8(Bd+2cC)+\frac{1}{9}Cd^4x^9$$

input
```
Int[(A + B*x + C*x^2)*(b*c + b*d*x + c*d*x^2 + d^2*x^3)^2,x]
```

output
```
A*b^2*c^2*x + (b^2*c*(B*c + 2*A*d)*x^2)/2 + (b*(2*A*c^2*d + b*(c^2*C + 2*B
*c*d + A*d^2))*x^3)/3 + (b*d*(2*B*c^2 + 2*b*c*C + b*B*d + 4*A*c*d)*x^4)/4
+ (d*(A*c^2*d + b^2*C*d + 2*b*(c^2*C + 2*B*c*d + A*d^2))*x^5)/5 + (d^2*(B*
c^2 + 4*b*c*C + 2*b*B*d + 2*A*c*d)*x^6)/6 + (d^2*(c^2*C + 2*B*c*d + d*(2*b
*C + A*d))*x^7)/7 + (d^3*(2*c*C + B*d)*x^8)/8 + (C*d^4*x^9)/9
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.12

method	result
norman	$\frac{C d^4 x^9}{9} + \left(\frac{1}{8}B d^4 + \frac{1}{4}C c d^3\right) x^8 + \left(\frac{1}{7}A d^4 + \frac{2}{7}B c d^3 + \frac{2}{7}C d^3 b + \frac{1}{7}C c^2 d^2\right) x^7 + \big(\frac{1}{3}A d^3 c + \frac{1}{3}B\cdots$
default	$\frac{C d^4 x^9}{9} + \frac{(B d^4+2C c d^3)x^8}{8} + \frac{(A d^4+2B c d^3+C(2b d^3+c^2 d^2))x^7}{7} + \frac{(2A d^3 c+B(2b d^3+c^2 d^2)+4C b c d^2)x^6}{6} + \frac{(A(2b\cdots}{}$
risch	$\frac{2}{3}x^6 C b c d^2 + \frac{4}{5}x^5 B b c d^2 + \frac{2}{5}x^5 C b c^2 d + \frac{1}{4}x^8 C c d^3 + \frac{2}{7}x^7 B c d^3 + \frac{2}{7}x^7 C d^3 b + \frac{1}{7}x^7 C c^2 d^2 + \frac{1}{3}\cdots$
parallelrisch	$\frac{2}{3}x^6 C b c d^2 + \frac{4}{5}x^5 B b c d^2 + \frac{2}{5}x^5 C b c^2 d + \frac{1}{4}x^8 C c d^3 + \frac{2}{7}x^7 B c d^3 + \frac{2}{7}x^7 C d^3 b + \frac{1}{7}x^7 C c^2 d^2 + \frac{1}{3}\cdots$
gosper	$\frac{x(280C d^4 x^8+315x^7 B d^4+630x^7 C c d^3+360x^6 A d^4+720x^6 B c d^3+720x^6 C d^3 b+360x^6 C c^2 d^2+840x^5 A d^3 c+840x^5 B b d^3+42\cdots}{}$
orering	$\frac{x(280C d^4 x^8+315x^7 B d^4+630x^7 C c d^3+360x^6 A d^4+720x^6 B c d^3+720x^6 C d^3 b+360x^6 C c^2 d^2+840x^5 A d^3 c+840x^5 B b d^3+42\cdots}{}$

input
```
int((C*x^2+B*x+A)*(d^2*x^3+c*d*x^2+b*d*x+b*c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/9*C*d^4*x^9+(1/8*B*d^4+1/4*C*c*d^3)*x^8+(1/7*A*d^4+2/7*B*c*d^3+2/7*C*d^3
*b+1/7*C*c^2*d^2)*x^7+(1/3*A*d^3*c+1/3*B*b*d^3+1/6*B*c^2*d^2+2/3*C*b*c*d^2
)*x^6+(2/5*A*b*d^3+1/5*A*c^2*d^2+4/5*B*b*c*d^2+1/5*C*d^2*b^2+2/5*C*b*c^2*d
)*x^5+(A*b*c*d^2+1/4*B*b^2*d^2+1/2*B*b*c^2*d+1/2*C*b^2*c*d)*x^4+(1/3*A*b^2
*d^2+2/3*A*b*c^2*d+2/3*B*b^2*c*d+1/3*C*b^2*c^2)*x^3+(A*b^2*c*d+1/2*B*b^2*c
^2)*x^2+A*b^2*c^2*x
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.07

$$\begin{aligned}\int \left(A+Bx+Cx^2\right)\left(bc+bdx+cdx^2+d^2x^3\right)^2\,dx &= \frac{1}{9}Cd^4x^9 + \frac{1}{8}\left(2Ccd^3+Bd^4\right)x^8 + \frac{1}{7}\left(Cc^2d^2+Ad^4+2\left(Cb+Bc\right)d^3\right)x^7 \\ &+ \frac{1}{6}\left(2\left(Bb+Ac\right)d^3+\left(4Cbc+Bc^2\right)d^2\right)x^6 + Ab^2c^2x \\ &+ \frac{1}{5}\left(2Cbc^2d+2Abd^3+\left(Cb^2+4Bbc+Ac^2\right)d^2\right)x^5 \\ &+ \frac{1}{4}\left(\left(Bb^2+4Abc\right)d^2+2\left(Cb^2c+Bbc^2\right)d\right)x^4 \\ &+ \frac{1}{3}\left(Cb^2c^2+Ab^2d^2+2\left(Bb^2c+Abc^2\right)d\right)x^3 + \frac{1}{2}\left(Bb^2c^2+2Ab^2cd\right)x^2\end{aligned}$$

input
```
integrate((C*x^2+B*x+A)*(d^2*x^3+c*d*x^2+b*d*x+b*c)^2,x, algorithm="fricas
")
```

output
```
1/9*C*d^4*x^9 + 1/8*(2*C*c*d^3 + B*d^4)*x^8 + 1/7*(C*c^2*d^2 + A*d^4 + 2*(
C*b + B*c)*d^3)*x^7 + 1/6*(2*(B*b + A*c)*d^3 + (4*C*b*c + B*c^2)*d^2)*x^6
+ A*b^2*c^2*x + 1/5*(2*C*b*c^2*d + 2*A*b*d^3 + (C*b^2 + 4*B*b*c + A*c^2)*d
^2)*x^5 + 1/4*((B*b^2 + 4*A*b*c)*d^2 + 2*(C*b^2*c + B*b*c^2)*d)*x^4 + 1/3*
(C*b^2*c^2 + A*b^2*d^2 + 2*(B*b^2*c + A*b*c^2)*d)*x^3 + 1/2*(B*b^2*c^2 + 2
*A*b^2*c*d)*x^2
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.30

$$\int (A+Bx+Cx^2)\left(bc+bdx+cdx^2+d^2x^3\right)^2\,dx$$
$$= Ab^2c^2x+\frac{Cd^4x^9}{9}+x^8\left(\frac{Bd^4}{8}+\frac{Ccd^3}{4}\right)+x^7\left(\frac{Ad^4}{7}+\frac{2Bcd^3}{7}+\frac{2Cbd^3}{7}+\frac{Cc^2d^2}{7}\right)$$
$$+x^6\left(\frac{Acd^3}{3}+\frac{Bbd^3}{3}+\frac{Bc^2d^2}{6}+\frac{2Cbcd^2}{3}\right)+x^5$$
$$\cdot\left(\frac{2Abd^3}{5}+\frac{Ac^2d^2}{5}+\frac{4Bbcd^2}{5}+\frac{Cb^2d^2}{5}+\frac{2Cbc^2d}{5}\right)$$
$$+x^4\left(Abcd^2+\frac{Bb^2d^2}{4}+\frac{Bbc^2d}{2}+\frac{Cb^2cd}{2}\right)$$
$$+x^3\left(\frac{Ab^2d^2}{3}+\frac{2Abc^2d}{3}+\frac{2Bb^2cd}{3}+\frac{Cb^2c^2}{3}\right)+x^2\left(Ab^2cd+\frac{Bb^2c^2}{2}\right)$$

input
```
integrate((C*x**2+B*x+A)*(d**2*x**3+c*d*x**2+b*d*x+b*c)**2,x)
```

output
```
A*b**2*c**2*x + C*d**4*x**9/9 + x**8*(B*d**4/8 + C*c*d**3/4) + x**7*(A*d**
4/7 + 2*B*c*d**3/7 + 2*C*b*d**3/7 + C*c**2*d**2/7) + x**6*(A*c*d**3/3 + B*
b*d**3/3 + B*c**2*d**2/6 + 2*C*b*c*d**2/3) + x**5*(2*A*b*d**3/5 + A*c**2*d
**2/5 + 4*B*b*c*d**2/5 + C*b**2*d**2/5 + 2*C*b*c**2*d/5) + x**4*(A*b*c*d**
2 + B*b**2*d**2/4 + B*b*c**2*d/2 + C*b**2*c*d/2) + x**3*(A*b**2*d**2/3 + 2
*A*b*c**2*d/3 + 2*B*b**2*c*d/3 + C*b**2*c**2/3) + x**2*(A*b**2*c*d + B*b**
2*c**2/2)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.07

$$\int (A+Bx+Cx^2)\left(bc+bdx+cdx^2+d^2x^3\right)^2\,dx$$
$$=\frac{1}{9}Cd^4x^9+\frac{1}{8}\left(2Ccd^3+Bd^4\right)x^8+\frac{1}{7}\left(Cc^2d^2+Ad^4+2(Cb+Bc)d^3\right)x^7+\frac{1}{6}\left(2(Bb+Ac)d^3+\left(4Cbc+Bc^2\right)d^2\right)x^6+Ab^2c^2x+\frac{1}{5}\left(2Cbc^2d+2Abd^3+\left(Cb^2+4Bbc+Ac^2\right)d^2\right)x^5+\frac{1}{4}\left(\left(Bb^2+4Abc\right)d^2+2\left(Cb^2c+Bbc^2\right)d\right)x^4+\frac{1}{3}\left(Cb^2c^2+Ab^2d^2+2\left(Bb^2c+Abc^2\right)d\right)x^3+\frac{1}{2}\left(Bb^2c^2+2Ab^2cd\right)x^2$$

input
```
integrate((C*x^2+B*x+A)*(d^2*x^3+c*d*x^2+b*d*x+b*c)^2,x, algorithm="maxima
")
```

output
```
1/9*C*d^4*x^9 + 1/8*(2*C*c*d^3 + B*d^4)*x^8 + 1/7*(C*c^2*d^2 + A*d^4 + 2*(
C*b + B*c)*d^3)*x^7 + 1/6*(2*(B*b + A*c)*d^3 + (4*C*b*c + B*c^2)*d^2)*x^6
+ A*b^2*c^2*x + 1/5*(2*C*b*c^2*d + 2*A*b*d^3 + (C*b^2 + 4*B*b*c + A*c^2)*d
^2)*x^5 + 1/4*((B*b^2 + 4*A*b*c)*d^2 + 2*(C*b^2*c + B*b*c^2)*d)*x^4 + 1/3*
(C*b^2*c^2 + A*b^2*d^2 + 2*(B*b^2*c + A*b*c^2)*d)*x^3 + 1/2*(B*b^2*c^2 + 2
*A*b^2*c*d)*x^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.29

$$\int (A+Bx+Cx^2)\left(bc+bdx+cdx^2+d^2x^3\right)^2\,dx$$
$$=\frac{1}{9}Cd^4x^9+\frac{1}{4}Ccd^3x^8+\frac{1}{8}Bd^4x^8+\frac{1}{7}Cc^2d^2x^7+\frac{2}{7}Cbd^3x^7+\frac{2}{7}Bcd^3x^7+\frac{1}{7}Ad^4x^7+\frac{2}{3}Cbcd^2x^6+\frac{1}{6}Bc^2d^2x^6+\frac{1}{3}Bbd^3x^6+\frac{1}{3}Acd^3x^6+\frac{2}{5}Cbc^2dx^5+\frac{1}{5}Cb^2d^2x^5+\frac{4}{5}Bbcd^2x^5+\frac{1}{5}Ac^2d^2x^5+\frac{2}{5}Abd^3x^5+\frac{1}{2}Cb^2cdx^4+\frac{1}{2}Bbc^2dx^4+\frac{1}{4}Bb^2d^2x^4+Abcd^2x^4+\frac{1}{3}Cb^2c^2x^3+\frac{2}{3}Bb^2cdx^3+\frac{2}{3}Abc^2dx^3+\frac{1}{3}Ab^2d^2x^3+\frac{1}{2}Bb^2c^2x^2+Ab^2cdx^2+Ab^2c^2x$$

input
```
integrate((C*x^2+B*x+A)*(d^2*x^3+c*d*x^2+b*d*x+b*c)^2,x, algorithm="giac")
```

output
```
1/9*C*d^4*x^9 + 1/4*C*c*d^3*x^8 + 1/8*B*d^4*x^8 + 1/7*C*c^2*d^2*x^7 + 2/7*
C*b*d^3*x^7 + 2/7*B*c*d^3*x^7 + 1/7*A*d^4*x^7 + 2/3*C*b*c*d^2*x^6 + 1/6*B*
c^2*d^2*x^6 + 1/3*B*b*d^3*x^6 + 1/3*A*c*d^3*x^6 + 2/5*C*b*c^2*d*x^5 + 1/5*
C*b^2*d^2*x^5 + 4/5*B*b*c*d^2*x^5 + 1/5*A*c^2*d^2*x^5 + 2/5*A*b*d^3*x^5 +
1/2*C*b^2*c*d*x^4 + 1/2*B*b*c^2*d*x^4 + 1/4*B*b^2*d^2*x^4 + A*b*c*d^2*x^4
+ 1/3*C*b^2*c^2*x^3 + 2/3*B*b^2*c*d*x^3 + 2/3*A*b*c^2*d*x^3 + 1/3*A*b^2*d^
2*x^3 + 1/2*B*b^2*c^2*x^2 + A*b^2*c*d*x^2 + A*b^2*c^2*x
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.03

$$\begin{aligned}
&\int (A + Bx + Cx^2) \left(bc + bdx + cdx^2 + d^2x^3\right)^2 \, dx \\
&= x^3 \left(\frac{C\,b^2\,c^2}{3} + \frac{2\,B\,b^2\,c\,d}{3} + \frac{A\,b^2\,d^2}{3} + \frac{2\,A\,b\,c^2\,d}{3}\right) \\
&\quad + x^7 \left(\frac{C\,c^2\,d^2}{7} + \frac{2\,B\,c\,d^3}{7} + \frac{A\,d^4}{7} + \frac{2\,C\,b\,d^3}{7}\right) + x^8 \left(\frac{B\,d^4}{8} + \frac{C\,c\,d^3}{4}\right) \\
&\quad + x^5 \left(\frac{C\,b^2\,d^2}{5} + \frac{2\,C\,b\,c^2\,d}{5} + \frac{4\,B\,b\,c\,d^2}{5} + \frac{2\,A\,b\,d^3}{5} + \frac{A\,c^2\,d^2}{5}\right) \\
&\quad + \frac{C\,d^4\,x^9}{9} + \frac{d^2\,x^6\,(B\,c^2 + 2\,A\,c\,d + 2\,B\,b\,d + 4\,C\,b\,c)}{6} + A\,b^2\,c^2\,x \\
&\quad + \frac{b^2\,c\,x^2\,(2\,A\,d + B\,c)}{2} + \frac{b\,d\,x^4\,(2\,B\,c^2 + 4\,A\,c\,d + B\,b\,d + 2\,C\,b\,c)}{4}
\end{aligned}$$

input
```
int((A + B*x + C*x^2)*(b*c + d^2*x^3 + b*d*x + c*d*x^2)^2,x)
```

output
```
x^3*((A*b^2*d^2)/3 + (C*b^2*c^2)/3 + (2*A*b*c^2*d)/3 + (2*B*b^2*c*d)/3) +
x^7*((A*d^4)/7 + (C*c^2*d^2)/7 + (2*B*c*d^3)/7 + (2*C*b*d^3)/7) + x^8*((B*
d^4)/8 + (C*c*d^3)/4) + x^5*((A*c^2*d^2)/5 + (C*b^2*d^2)/5 + (2*A*b*d^3)/5
 + (4*B*b*c*d^2)/5 + (2*C*b*c^2*d)/5) + (C*d^4*x^9)/9 + (d^2*x^6*(B*c^2 +
2*A*c*d + 2*B*b*d + 4*C*b*c))/6 + A*b^2*c^2*x + (b^2*c*x^2*(2*A*d + B*c))/
2 + (b*d*x^4*(2*B*c^2 + 4*A*c*d + B*b*d + 2*C*b*c))/4
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.09

$$\int (A + Bx + Cx^2) (bc + bdx + cdx^2 + d^2x^3)^2 \, dx$$

$$= \frac{x(280c\,d^4x^8 + 315b\,d^4x^7 + 630c^2d^3x^7 + 360a\,d^4x^6 + 1440bc\,d^3x^6 + 360c^3d^2x^6 + 840ac\,d^3x^5 + 840b^2d^3x^5}{}$$

input
```
int((C*x^2+B*x+A)*(d^2*x^3+c*d*x^2+b*d*x+b*c)^2,x)
```

output
```
(x*(2520*a*b**2*c**2 + 2520*a*b**2*c*d*x + 840*a*b**2*d**2*x**2 + 1680*a*b
*c**2*d*x**2 + 2520*a*b*c*d**2*x**3 + 1008*a*b*d**3*x**4 + 504*a*c**2*d**2
*x**4 + 840*a*c*d**3*x**5 + 360*a*d**4*x**6 + 1260*b**3*c**2*x + 1680*b**3
*c*d*x**2 + 630*b**3*d**2*x**3 + 840*b**2*c**3*x**2 + 2520*b**2*c**2*d*x**
3 + 2520*b**2*c*d**2*x**4 + 840*b**2*d**3*x**5 + 1008*b*c**3*d*x**4 + 2100
*b*c**2*d**2*x**5 + 1440*b*c*d**3*x**6 + 315*b*d**4*x**7 + 360*c**3*d**2*x
**6 + 630*c**2*d**3*x**7 + 280*c*d**4*x**8))/2520
```

3.87 $\int (A+Bx+Cx^2)(bc+bdx+cdx^2+d^2x^3)\,dx$

Optimal result

Integrand size = 32, antiderivative size = 85

$$\int (A+Bx+Cx^2)(bc+bdx+cdx^2+d^2x^3)\,dx = Abcx+\frac{1}{2}b(Bc+Ad)x^2+\frac{1}{3}(bcC+bBd+Acd)x^3 + \frac{1}{4}d(Bc+bC+Ad)x^4+\frac{1}{5}d(cC+Bd)x^5+\frac{1}{6}Cd^2x^6$$

output
```
A*b*c*x+1/2*b*(A*d+B*c)*x^2+1/3*(A*c*d+B*b*d+C*b*c)*x^3+1/4*d*(A*d+B*c+C*b
)*x^4+1/5*d*(B*d+C*c)*x^5+1/6*C*d^2*x^6
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00

$$\int (A+Bx+Cx^2)(bc+bdx+cdx^2+d^2x^3)\,dx = Abcx+\frac{1}{2}b(Bc+Ad)x^2+\frac{1}{3}(bcC+bBd+Acd)x^3 + \frac{1}{4}d(Bc+bC+Ad)x^4+\frac{1}{5}d(cC+Bd)x^5+\frac{1}{6}Cd^2x^6$$

input
```
Integrate[(A + B*x + C*x^2)*(b*c + b*d*x + c*d*x^2 + d^2*x^3),x]
```

output
```
A*b*c*x + (b*(B*c + A*d)*x^2)/2 + ((b*c*C + b*B*d + A*c*d)*x^3)/3 + (d*(B*
c + b*C + A*d)*x^4)/4 + (d*(c*C + B*d)*x^5)/5 + (C*d^2*x^6)/6
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (A+Bx+Cx^2)\left(bc+bdx+cdx^2+d^2x^3\right)\,dx$$

$$\Big\downarrow \; 2188$$

$$\int \left(dx^3(Ad+bC+Bc)+x^2(Acd+bBd+bcC)+bx(Ad+Bc)+Abc+dx^4(Bd+cC)+Cd^2x^5\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{4}dx^4(Ad+bC+Bc)+\frac{1}{3}x^3(Acd+bBd+bcC)+\frac{1}{2}bx^2(Ad+Bc)+Abcx+\frac{1}{5}dx^5(Bd+cC)+\frac{1}{6}Cd^2x^6$$

input
```
Int[(A + B*x + C*x^2)*(b*c + b*d*x + c*d*x^2 + d^2*x^3),x]
```

output
```
A*b*c*x + (b*(B*c + A*d)*x^2)/2 + ((b*c*C + b*B*d + A*c*d)*x^3)/3 + (d*(B*
c + b*C + A*d)*x^4)/4 + (d*(c*C + B*d)*x^5)/5 + (C*d^2*x^6)/6
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.96

method	result
default	$\frac{C\,d^2x^6}{6}+\frac{(B\,d^2+Ccd)x^5}{5}+\frac{(A\,d^2+Bcd+Cbd)x^4}{4}+\frac{(Acd+Bbd+Cbc)x^3}{3}+\frac{(Abd+bBc)x^2}{2}+Abcx$
norman	$\frac{C\,d^2x^6}{6}+\left(\frac{1}{5}B\,d^2+\frac{1}{5}Ccd\right)x^5+\left(\frac{1}{4}A\,d^2+\frac{1}{4}Bcd+\frac{1}{4}Cbd\right)x^4+\left(\frac{1}{3}Acd+\frac{1}{3}Bbd+\frac{1}{3}Cbc\right)x^3+$
gosper	$\frac{x(10C\,d^2x^5+12B\,d^2x^4+12Ccd\,x^4+15A\,d^2x^3+15Bcd\,x^3+15Cbd\,x^3+20Acd\,x^2+20Bbd\,x^2+20Cbc\,x^2+30Abdx+30cBxb+60}{60}$
risch	$\frac{1}{6}C\,d^2x^6+\frac{1}{5}x^5B\,d^2+\frac{1}{5}x^5Ccd+\frac{1}{4}x^4A\,d^2+\frac{1}{4}Bcd\,x^4+\frac{1}{4}x^4Cbd+\frac{1}{3}x^3Acd+\frac{1}{3}x^3Bbd+\frac{1}{3}C$
parallelrisch	$\frac{1}{6}C\,d^2x^6+\frac{1}{5}x^5B\,d^2+\frac{1}{5}x^5Ccd+\frac{1}{4}x^4A\,d^2+\frac{1}{4}Bcd\,x^4+\frac{1}{4}x^4Cbd+\frac{1}{3}x^3Acd+\frac{1}{3}x^3Bbd+\frac{1}{3}C$
orering	$\frac{x(10C\,d^2x^5+12B\,d^2x^4+12Ccd\,x^4+15A\,d^2x^3+15Bcd\,x^3+15Cbd\,x^3+20Acd\,x^2+20Bbd\,x^2+20Cbc\,x^2+30Abdx+30cBxb+60}{60(dx+c)(d\,x^2+b)}$

input
```
int((C*x^2+B*x+A)*(d^2*x^3+c*d*x^2+b*d*x+b*c),x,method=_RETURNVERBOSE)
```

output
```
1/6*C*d^2*x^6+1/5*(B*d^2+C*c*d)*x^5+1/4*(A*d^2+B*c*d+C*b*d)*x^4+1/3*(A*c*d
+B*b*d+C*b*c)*x^3+1/2*(A*b*d+B*b*c)*x^2+A*b*c*x
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.98

$$\int (A+Bx+Cx^2)\left(bc+bdx+cdx^2+d^2x^3\right)\,dx$$
$$=\frac{1}{6}Cd^2x^6+\frac{1}{5}\left(Ccd+Bd^2\right)x^5+\frac{1}{4}\left(Ad^2+(Cb+Bc)d\right)x^4+Abcx+\frac{1}{3}(Cbc+(Bb+Ac)d)x^3+\frac{1}{2}(Bbc+Abd)x^2$$

input
```
integrate((C*x^2+B*x+A)*(d^2*x^3+c*d*x^2+b*d*x+b*c),x, algorithm="fricas")
```

output
```
1/6*C*d^2*x^6 + 1/5*(C*c*d + B*d^2)*x^5 + 1/4*(A*d^2 + (C*b + B*c)*d)*x^4
+ A*b*c*x + 1/3*(C*b*c + (B*b + A*c)*d)*x^3 + 1/2*(B*b*c + A*b*d)*x^2
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.14

$$\int (A+Bx+Cx^2)\left(bc+bdx+cdx^2+d^2x^3\right)\,dx$$
$$=Abcx+\frac{Cd^2x^6}{6}+x^5\left(\frac{Bd^2}{5}+\frac{Ccd}{5}\right)+x^4\left(\frac{Ad^2}{4}+\frac{Bcd}{4}+\frac{Cbd}{4}\right)+x^3\left(\frac{Acd}{3}+\frac{Bbd}{3}+\frac{Cbc}{3}\right)+x^2\left(\frac{Abd}{2}+\frac{Bbc}{2}\right)$$

input
```
integrate((C*x**2+B*x+A)*(d**2*x**3+c*d*x**2+b*d*x+b*c),x)
```

output
```
A*b*c*x + C*d**2*x**6/6 + x**5*(B*d**2/5 + C*c*d/5) + x**4*(A*d**2/4 + B*c
*d/4 + C*b*d/4) + x**3*(A*c*d/3 + B*b*d/3 + C*b*c/3) + x**2*(A*b*d/2 + B*b
*c/2)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.98

$$\int (A+Bx+Cx^2)\left(bc+bdx+cdx^2+d^2x^3\right)\,dx$$
$$=\frac{1}{6}Cd^2x^6+\frac{1}{5}\left(Ccd+Bd^2\right)x^5+\frac{1}{4}\left(Ad^2+(Cb+Bc)d\right)x^4+Abcx+\frac{1}{3}(Cbc+(Bb+Ac)d)x^3+\frac{1}{2}(Bbc+Abd)x^2$$

input
```
integrate((C*x^2+B*x+A)*(d^2*x^3+c*d*x^2+b*d*x+b*c),x, algorithm="maxima")
```

output
```
1/6*C*d^2*x^6 + 1/5*(C*c*d + B*d^2)*x^5 + 1/4*(A*d^2 + (C*b + B*c)*d)*x^4
+ A*b*c*x + 1/3*(C*b*c + (B*b + A*c)*d)*x^3 + 1/2*(B*b*c + A*b*d)*x^2
```

Giac [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.14

$$\int (A+Bx+Cx^2)\left(bc+bdx+cdx^2+d^2x^3\right)\,dx$$
$$=\frac{1}{6}Cd^2x^6+\frac{1}{5}Ccdx^5+\frac{1}{5}Bd^2x^5+\frac{1}{4}Cbdx^4+\frac{1}{4}Bcdx^4+\frac{1}{4}Ad^2x^4+\frac{1}{3}Cbcx^3+\frac{1}{3}Bbdx^3+\frac{1}{3}Acdx^3+\frac{1}{2}Bbcx^2+\frac{1}{2}Abdx^2+Abcx$$

input
```
integrate((C*x^2+B*x+A)*(d^2*x^3+c*d*x^2+b*d*x+b*c),x, algorithm="giac")
```

output
```
1/6*C*d^2*x^6 + 1/5*C*c*d*x^5 + 1/5*B*d^2*x^5 + 1/4*C*b*d*x^4 + 1/4*B*c*d*
x^4 + 1/4*A*d^2*x^4 + 1/3*C*b*c*x^3 + 1/3*B*b*d*x^3 + 1/3*A*c*d*x^3 + 1/2*
B*b*c*x^2 + 1/2*A*b*d*x^2 + A*b*c*x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.94

$$\int \left(A+Bx+Cx^2\right)\left(bc+bdx+cdx^2+d^2x^3\right)\,dx$$
$$= x^5\left(\frac{B\,d^2}{5}+\frac{C\,c\,d}{5}\right)+x^3\left(\frac{A\,c\,d}{3}+\frac{B\,b\,d}{3}+\frac{C\,b\,c}{3}\right) + \frac{b\,x^2\,(A\,d+B\,c)}{2}+\frac{d\,x^4\,(A\,d+B\,c+C\,b)}{4}+\frac{C\,d^2\,x^6}{6}+A\,b\,c\,x$$

input
```
int((A + B*x + C*x^2)*(b*c + d^2*x^3 + b*d*x + c*d*x^2),x)
```

output
```
x^5*((B*d^2)/5 + (C*c*d)/5) + x^3*((A*c*d)/3 + (B*b*d)/3 + (C*b*c)/3) + (b
*x^2*(A*d + B*c))/2 + (d*x^4*(A*d + B*c + C*b))/4 + (C*d^2*x^6)/6 + A*b*c*
x
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.08

$$\int \left(A+Bx+Cx^2\right)\left(bc+bdx+cdx^2+d^2x^3\right)\,dx$$
$$= \frac{x(10c\,d^2x^5+12b\,d^2x^4+12c^2d\,x^4+15a\,d^2x^3+30bcd\,x^3+20acd\,x^2+20b^2d\,x^2+20b\,c^2x^2+30abdx+30}{60}$$

input
```
int((C*x^2+B*x+A)*(d^2*x^3+c*d*x^2+b*d*x+b*c),x)
```

output
```
(x*(60*a*b*c + 30*a*b*d*x + 20*a*c*d*x**2 + 15*a*d**2*x**3 + 30*b**2*c*x +
 20*b**2*d*x**2 + 20*b*c**2*x**2 + 30*b*c*d*x**3 + 12*b*d**2*x**4 + 12*c**
2*d*x**4 + 10*c*d**2*x**5))/60
```

3.88 $\int \frac{A+Bx+Cx^2}{bc+bdx+cdx^2+d^2x^3}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 120

$$\int \frac{A+Bx+Cx^2}{bc+bdx+cdx^2+d^2x^3}\,dx = -\frac{(bcC-bBd-Acd)\arctan\left(\frac{\sqrt{d}x}{\sqrt{b}}\right)}{\sqrt{b}d^{3/2}\left(c^2+bd\right)} + \frac{\left(c^2C-Bcd+Ad^2\right)\log(c+dx)}{d^2\left(c^2+bd\right)} + \frac{(Bc+bC-Ad)\log\left(b+dx^2\right)}{2d\left(c^2+bd\right)}$$

output
```
-(-A*c*d-B*b*d+C*b*c)*arctan(d^(1/2)*x/b^(1/2))/b^(1/2)/d^(3/2)/(b*d+c^2)+
(A*d^2-B*c*d+C*c^2)*ln(d*x+c)/d^2/(b*d+c^2)+1/2*(-A*d+B*c+C*b)*ln(d*x^2+b)
/d/(b*d+c^2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.90

$$\int \frac{A+Bx+Cx^2}{bc+bdx+cdx^2+d^2x^3}\,dx = \frac{2\sqrt{d}(-bcC+bBd+Acd)\arctan\left(\frac{\sqrt{d}x}{\sqrt{b}}\right)+\sqrt{b}(2(c^2C-Bcd+Ad^2)\log(c+dx)+d(Bc+bC-Ad)\log}{2\sqrt{b}d^2\left(c^2+bd\right)}$$

input
```
Integrate[(A + B*x + C*x^2)/(b*c + b*d*x + c*d*x^2 + d^2*x^3),x]
```

output
```
(2*Sqrt[d]*(-(b*c*C) + b*B*d + A*c*d)*ArcTan[(Sqrt[d]*x)/Sqrt[b]] + Sqrt[b
]*(2*(c^2*C - B*c*d + A*d^2)*Log[c + d*x] + d*(B*c + b*C - A*d)*Log[b + d*
x^2]))/(2*Sqrt[b]*d^2*(c^2 + b*d))
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {2462, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{bc+bdx+cdx^2+d^2x^3}\,dx$$
$$\Big\downarrow \text{2462}$$
$$\int \left(\frac{Ad^2-Bcd+c^2C}{d\left(bd+c^2\right)(c+dx)}+\frac{dx(-Ad+bC+Bc)+Acd+bBd-bcC}{d\left(bd+c^2\right)\left(b+dx^2\right)}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$-\frac{\arctan\left(\frac{\sqrt{d}x}{\sqrt{b}}\right)(-Acd-bBd+bcC)}{\sqrt{b}d^{3/2}\left(bd+c^2\right)}+\frac{\log(c+dx)\left(Ad^2-Bcd+c^2C\right)}{d^2\left(bd+c^2\right)}+\frac{\log\left(b+dx^2\right)(-Ad+bC+Bc)}{2d\left(bd+c^2\right)}$$

input
```
Int[(A + B*x + C*x^2)/(b*c + b*d*x + c*d*x^2 + d^2*x^3),x]
```

output
```
-(((b*c*C - b*B*d - A*c*d)*ArcTan[(Sqrt[d]*x)/Sqrt[b]])/(Sqrt[b]*d^(3/2)*(
c^2 + b*d))) + ((c^2*C - B*c*d + A*d^2)*Log[c + d*x])/(d^2*(c^2 + b*d)) +
((B*c + b*C - A*d)*Log[b + d*x^2])/(2*d*(c^2 + b*d))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.90

method	result	size
default	$\frac{\left(A\,d^{2}-Bcd+C\,c^{2}\right)\ln\left(dx+c\right)}{d^{2}\left(bd+c^{2}\right)}+\frac{\frac{\left(-A\,d^{2}+Bcd+Cbd\right)\ln\left(d\,x^{2}+b\right)}{2d}+\frac{\left(Acd+Bbd-Cbc\right)\arctan\left(\frac{dx}{\sqrt{bd}}\right)}{\sqrt{bd}}}{d\left(bd+c^{2}\right)}$	108
risch	Expression too large to display	5978

input
```
int((C*x^2+B*x+A)/(d^2*x^3+c*d*x^2+b*d*x+b*c),x,method=_RETURNVERBOSE)
```

output
```
(A*d^2-B*c*d+C*c^2)*ln(d*x+c)/d^2/(b*d+c^2)+1/d/(b*d+c^2)*(1/2*(-A*d^2+B*c
*d+C*b*d)/d*ln(d*x^2+b)+(A*c*d+B*b*d-C*b*c)/(b*d)^(1/2)*arctan(d*x/(b*d)^(
1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 240, normalized size of antiderivative = 2.00

$$\int \frac{A+Bx+Cx^2}{bc+bdx+cdx^2+d^2x^3}\,dx$$

$$= \left[\frac{(Cbc-(Bb+Ac)d)\sqrt{-bd}\log\left(\frac{dx^2-2\sqrt{-bd}x-b}{dx^2+b}\right)-(Abd^2-(Cb^2+Bbc)d)\log(dx^2+b)+2\,(Cbc^2-Bb}{2\,(bc^2d^2+b^2d^3)}\right.$$

$$-\frac{2\,(Cbc-(Bb+Ac)d)\sqrt{bd}\arctan\left(\frac{\sqrt{bd}x}{b}\right)+(Abd^2-(Cb^2+Bbc)d)\log(dx^2+b)-2\,(Cbc^2-Bbcd+}{2\,(bc^2d^2+b^2d^3)}$$

input
```
integrate((C*x^2+B*x+A)/(d^2*x^3+c*d*x^2+b*d*x+b*c),x, algorithm="fricas")
```

output
```
[1/2*((C*b*c - (B*b + A*c)*d)*sqrt(-b*d)*log((d*x^2 - 2*sqrt(-b*d)*x - b)/
(d*x^2 + b)) - (A*b*d^2 - (C*b^2 + B*b*c)*d)*log(d*x^2 + b) + 2*(C*b*c^2 -
 B*b*c*d + A*b*d^2)*log(d*x + c))/(b*c^2*d^2 + b^2*d^3), -1/2*(2*(C*b*c -
(B*b + A*c)*d)*sqrt(b*d)*arctan(sqrt(b*d)*x/b) + (A*b*d^2 - (C*b^2 + B*b*c
)*d)*log(d*x^2 + b) - 2*(C*b*c^2 - B*b*c*d + A*b*d^2)*log(d*x + c))/(b*c^2
*d^2 + b^2*d^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + Bx + Cx^2}{bc + bdx + cdx^2 + d^2x^3}\,dx = \text{Timed out}$$

input
```
integrate((C*x**2+B*x+A)/(d**2*x**3+c*d*x**2+b*d*x+b*c),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.98

$$\int \frac{A + Bx + Cx^2}{bc + bdx + cdx^2 + d^2x^3}\,dx = \frac{(Cb + Bc - Ad)\log(dx^2 + b)}{2(c^2d + bd^2)} + \frac{(Cc^2 - Bcd + Ad^2)\log(dx + c)}{c^2d^2 + bd^3} - \frac{(Cbc - (Bb + Ac)d)\arctan\left(\frac{dx}{\sqrt{bd}}\right)}{(c^2d + bd^2)\sqrt{bd}}$$

input
```
integrate((C*x^2+B*x+A)/(d^2*x^3+c*d*x^2+b*d*x+b*c),x, algorithm="maxima")
```

output
```
1/2*(C*b + B*c - A*d)*log(d*x^2 + b)/(c^2*d + b*d^2) + (C*c^2 - B*c*d + A*
d^2)*log(d*x + c)/(c^2*d^2 + b*d^3) - (C*b*c - (B*b + A*c)*d)*arctan(d*x/s
qrt(b*d))/((c^2*d + b*d^2)*sqrt(b*d))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.98

$$\int \frac{A+Bx+Cx^2}{bc+bdx+cdx^2+d^2x^3}\,dx = \frac{(Cb+Bc-Ad)\log\left(dx^2+b\right)}{2\left(c^2d+bd^2\right)} + \frac{\left(Cc^2-Bcd+Ad^2\right)\log\left(|dx+c|\right)}{c^2d^2+bd^3} - \frac{(Cbc-Bbd-Acd)\arctan\left(\frac{dx}{\sqrt{bd}}\right)}{\left(c^2d+bd^2\right)\sqrt{bd}}$$

input
```
integrate((C*x^2+B*x+A)/(d^2*x^3+c*d*x^2+b*d*x+b*c),x, algorithm="giac")
```

output
```
1/2*(C*b + B*c - A*d)*log(d*x^2 + b)/(c^2*d + b*d^2) + (C*c^2 - B*c*d + A*
d^2)*log(abs(d*x + c))/(c^2*d^2 + b*d^3) - (C*b*c - B*b*d - A*c*d)*arctan(
d*x/sqrt(b*d))/((c^2*d + b*d^2)*sqrt(b*d))
```

Mupad [B] (verification not implemented)

Time = 14.64 (sec) , antiderivative size = 793, normalized size of antiderivative = 6.61

$$\int \frac{A+Bx+Cx^2}{bc+bdx+cdx^2+d^2x^3}\,dx = \text{Too large to display}$$

input
```
int((A + B*x + C*x^2)/(b*c + d^2*x^3 + b*d*x + c*d*x^2),x)
```

output
```
(log(x*(B^2*d^2 - A*C*d^2 + C^2*b*d - B*C*c*d) + ((d^2*((C*b^2)/2 + (B*b*c
)/2) + d*((A*c*(-b*d^3)^(1/2))/2 + (B*b*(-b*d^3)^(1/2))/2) - (A*b*d^3)/2 -
 (C*b*c*(-b*d^3)^(1/2))/2)*(x*(3*A*d^4 + 2*C*c^2*d^2 - B*c*d^3 - 5*C*b*d^3
) + A*c*d^3 - B*b*d^3 + ((x*(6*b*d^5 - 2*c^2*d^4) + 8*b*c*d^4)*(d^2*((C*b^
2)/2 + (B*b*c)/2) + d*((A*c*(-b*d^3)^(1/2))/2 + (B*b*(-b*d^3)^(1/2))/2) -
(A*b*d^3)/2 - (C*b*c*(-b*d^3)^(1/2))/2))/(b^2*d^4 + b*c^2*d^3) - 5*C*b*c*d
^2))/(b^2*d^4 + b*c^2*d^3) + A*B*d^2 + C^2*b*c - A*C*c*d)*(d^2*((C*b^2)/2
+ (B*b*c)/2) + d*((A*c*(-b*d^3)^(1/2))/2 + (B*b*(-b*d^3)^(1/2))/2) - (A*b*
d^3)/2 - (C*b*c*(-b*d^3)^(1/2))/2))/(b^2*d^4 + b*c^2*d^3) + (log(x*(B^2*d^
2 - A*C*d^2 + C^2*b*d - B*C*c*d) + ((d^2*((C*b^2)/2 + (B*b*c)/2) - d*((A*c
*(-b*d^3)^(1/2))/2 + (B*b*(-b*d^3)^(1/2))/2) - (A*b*d^3)/2 + (C*b*c*(-b*d^
3)^(1/2))/2)*(x*(3*A*d^4 + 2*C*c^2*d^2 - B*c*d^3 - 5*C*b*d^3) + A*c*d^3 -
B*b*d^3 + ((x*(6*b*d^5 - 2*c^2*d^4) + 8*b*c*d^4)*(d^2*((C*b^2)/2 + (B*b*c)
/2) - d*((A*c*(-b*d^3)^(1/2))/2 + (B*b*(-b*d^3)^(1/2))/2) - (A*b*d^3)/2 +
(C*b*c*(-b*d^3)^(1/2))/2))/(b^2*d^4 + b*c^2*d^3) - 5*C*b*c*d^2))/(b^2*d^4
+ b*c^2*d^3) + A*B*d^2 + C^2*b*c - A*C*c*d)*(d^2*((C*b^2)/2 + (B*b*c)/2) -
 d*((A*c*(-b*d^3)^(1/2))/2 + (B*b*(-b*d^3)^(1/2))/2) - (A*b*d^3)/2 + (C*b*
c*(-b*d^3)^(1/2))/2))/(b^2*d^4 + b*c^2*d^3) + (log(c + d*x)*(A*d^2 + C*c^2
 - B*c*d))/(d^2*(b*d + c^2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.26

$$\int \frac{A+Bx+Cx^2}{bc+bdx+cdx^2+d^2x^3}\,dx = \frac{2\sqrt{d}\,\sqrt{b}\,atan\left(\frac{dx}{\sqrt{d}\,\sqrt{b}}\right)acd + 2\sqrt{d}\,\sqrt{b}\,atan\left(\frac{dx}{\sqrt{d}\,\sqrt{b}}\right)b^2d - 2\sqrt{d}\,\sqrt{b}\,atan\left(\frac{dx}{\sqrt{d}\,\sqrt{b}}\right)b\,c^2 - \log(d\,x^2+b)\,ab\,d^2 +}{2b\,d^2\,(bd+c^2)}$$

input
```
int((C*x^2+B*x+A)/(d^2*x^3+c*d*x^2+b*d*x+b*c),x)
```

output
```
(2*sqrt(d)*sqrt(b)*atan((d*x)/(sqrt(d)*sqrt(b)))*a*c*d + 2*sqrt(d)*sqrt(b)
*atan((d*x)/(sqrt(d)*sqrt(b)))*b**2*d - 2*sqrt(d)*sqrt(b)*atan((d*x)/(sqrt
(d)*sqrt(b)))*b*c**2 - log(b + d*x**2)*a*b*d**2 + 2*log(b + d*x**2)*b**2*c
*d + 2*log(c + d*x)*a*b*d**2 - 2*log(c + d*x)*b**2*c*d + 2*log(c + d*x)*b*
c**3)/(2*b*d**2*(b*d + c**2))
```

3.89 $\int \frac{A+Bx+Cx^2}{\left(bc+bdx+cdx^2+d^2x^3\right)^2} dx$

Optimal result

Integrand size = 34, antiderivative size = 330

$$\begin{aligned}
&\int \frac{A+Bx+Cx^2}{\left(bc+bdx+cdx^2+d^2x^3\right)^2} dx \\
&= \frac{Ac^2d+b^2Cd-b(3c^2C-4Bcd+3Ad^2)}{2bd\left(c^2+bd\right)^2(c+dx)} - \frac{b(Bc+bC-Ad)+(bcC-bBd-Acd)x}{2bd\left(c^2+bd\right)(c+dx)\left(b+dx^2\right)} \\
&+ \frac{\left(Ac^4d+b^3Cd^2-3b^2d(2c^2C-2Bcd+Ad^2)+bc^2(c^2C-2Bcd+6Ad^2)\right)\arctan\left(\frac{\sqrt{d}x}{\sqrt{b}}\right)}{2b^{3/2}d^{3/2}\left(c^2+bd\right)^3} \\
&+ \frac{\left(2c^3C-3Bc^2d+bBd^2-2cd(bC-2Ad)\right)\log(c+dx)}{d\left(c^2+bd\right)^3} \\
&- \frac{\left(2c^3C-3Bc^2d+bBd^2-2cd(bC-2Ad)\right)\log\left(b+dx^2\right)}{2d\left(c^2+bd\right)^3}
\end{aligned}$$

output
```
1/2*(A*c^2*d+b^2*C*d-b*(3*A*d^2-4*B*c*d+3*C*c^2))/b/d/(b*d+c^2)^2/(d*x+c)-
1/2*(b*(-A*d+B*c+C*b)+(-A*c*d-B*b*d+C*b*c)*x)/b/d/(b*d+c^2)/(d*x+c)/(d*x^2
+b)+1/2*(A*c^4*d+b^3*C*d^2-3*b^2*d*(A*d^2-2*B*c*d+2*C*c^2)+b*c^2*(6*A*d^2-
2*B*c*d+C*c^2))*arctan(d^(1/2)*x/b^(1/2))/b^(3/2)/d^(3/2)/(b*d+c^2)^3+(2*c
^3*C-3*B*c^2*d+b*B*d^2-2*c*d*(-2*A*d+C*b))*ln(d*x+c)/d/(b*d+c^2)^3-1/2*(2*
c^3*C-3*B*c^2*d+b*B*d^2-2*c*d*(-2*A*d+C*b))*ln(d*x^2+b)/d/(b*d+c^2)^3
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 307, normalized size of antiderivative = 0.93

$$\int \frac{A+Bx+Cx^2}{(bc+bdx+cdx^2+d^2x^3)^2}\,dx$$
$$= \frac{-\frac{2\sqrt{d}(c^2+bd)(c^2C-Bcd+Ad^2)}{c+dx}+\frac{\sqrt{d}(c^2+bd)(Ac^2dx+b^2(-2cC+d(B+Cx))-b(c^2Cx+Bc(c-2dx)+Ad(-2c+dx)))}{b(b+dx^2)}+\frac{(Ac^4d+b^3Cd^2-3}{}}{}$$

input
```
Integrate[(A + B*x + C*x^2)/(b*c + b*d*x + c*d*x^2 + d^2*x^3)^2,x]
```

output
```
((-2*Sqrt[d]*(c^2 + b*d)*(c^2*C - B*c*d + A*d^2))/(c + d*x) + (Sqrt[d]*(c^
2 + b*d)*(A*c^2*d*x + b^2*(-2*c*C + d*(B + C*x)) - b*(c^2*C*x + B*c*(c - 2
*d*x) + A*d*(-2*c + d*x))))/(b*(b + d*x^2)) + ((A*c^4*d + b^3*C*d^2 - 3*b^
2*d*(2*c^2*C - 2*B*c*d + A*d^2) + b*c^2*(c^2*C - 2*B*c*d + 6*A*d^2))*ArcTa
n[(Sqrt[d]*x)/Sqrt[b]])/b^(3/2) + 2*Sqrt[d]*(2*c^3*C - 3*B*c^2*d + b*B*d^2
 + 2*c*d*(-(b*C) + 2*A*d))*Log[c + d*x] - Sqrt[d]*(2*c^3*C - 3*B*c^2*d + b
*B*d^2 + 2*c*d*(-(b*C) + 2*A*d))*Log[b + d*x^2])/(2*d^(3/2)*(c^2 + b*d)^3)
```

Rubi [A] (verified)

Time = 1.32 (sec) , antiderivative size = 375, normalized size of antiderivative = 1.14, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {2462, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{(bc+bdx+cdx^2+d^2x^3)^2}\,dx$$

$$\Big\downarrow \text{2462}$$

$$\int \left(\frac{-b(Ad^2-2Bcd+c^2C)+dx(-2Acd-bBd+2bcC+Bc^2)+Ac^2d+b^2Cd}{d(bd+c^2)^2(b+dx^2)^2}+\frac{Ad^2-Bcd+c^2C}{(bd+c^2)^2(c+dx)^2}+\frac{-2c}{}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{\arctan\left(\frac{\sqrt{d}x}{\sqrt{b}}\right)\left(-b\left(Ad^2-2Bcd+c^2C\right)+Ac^2d+b^2Cd\right)}{2b^{3/2}d^{3/2}\left(bd+c^2\right)^2}+\frac{\arctan\left(\frac{\sqrt{d}x}{\sqrt{b}}\right)\left(-3c^2d(bC-Ad)-Abd^3+2bBcd^2-2Bc^3d+c^4C\right)}{\sqrt{b}d^{3/2}\left(bd+c^2\right)^3}-\frac{b\left(-2Acd-bBd+2bcC+Bc^2\right)-x\left(-b\left(Ad^2-2Bcd+c^2C\right)+Ac^2d+b^2Cd\right)}{2bd\left(bd+c^2\right)^2\left(b+dx^2\right)}-\frac{Ad^2-Bcd+c^2C}{d\left(bd+c^2\right)^2(c+dx)}-\frac{\log\left(b+dx^2\right)\left(-2cd(bC-2Ad)+bBd^2-3Bc^2d+2c^3C\right)}{2d\left(bd+c^2\right)^3}+\frac{\log(c+dx)\left(-2cd(bC-2Ad)+bBd^2-3Bc^2d+2c^3C\right)}{d\left(bd+c^2\right)^3}$$

input
```
Int[(A + B*x + C*x^2)/(b*c + b*d*x + c*d*x^2 + d^2*x^3)^2,x]
```

output
```
-((c^2*C - B*c*d + A*d^2)/(d*(c^2 + b*d)^2*(c + d*x))) - (b*(B*c^2 + 2*b*c
*C - b*B*d - 2*A*c*d) - (A*c^2*d + b^2*C*d - b*(c^2*C - 2*B*c*d + A*d^2))*
x)/(2*b*d*(c^2 + b*d)^2*(b + d*x^2)) + ((c^4*C - 2*B*c^3*d + 2*b*B*c*d^2 -
 A*b*d^3 - 3*c^2*d*(b*C - A*d))*ArcTan[(Sqrt[d]*x)/Sqrt[b]])/(Sqrt[b]*d^(3
/2)*(c^2 + b*d)^3) + ((A*c^2*d + b^2*C*d - b*(c^2*C - 2*B*c*d + A*d^2))*Ar
cTan[(Sqrt[d]*x)/Sqrt[b]])/(2*b^(3/2)*d^(3/2)*(c^2 + b*d)^2) + ((2*c^3*C -
 3*B*c^2*d + b*B*d^2 - 2*c*d*(b*C - 2*A*d))*Log[c + d*x])/(d*(c^2 + b*d)^3
) - ((2*c^3*C - 3*B*c^2*d + b*B*d^2 - 2*c*d*(b*C - 2*A*d))*Log[b + d*x^2])
/(2*d*(c^2 + b*d)^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.14

method	result
default	$-\frac{A\,d^{2}-Bcd+C\,c^{2}}{\left(bd+c^{2}\right)^{2} d\left(dx+c\right)}+\frac{\left(4Ac\,d^{2}+bB\,d^{2}-3d\,c^{2}B-2bcCd+2C\,c^{3}\right)\ln\left(dx+c\right)}{\left(bd+c^{2}\right)^{3} d}-\frac{\frac{\left(A\,b^{2}d^{3}-A\,c^{4}d-2B\,b^{2}c\,d^{2}-2Bb\,c^{3}d-b^{3}C\,d^{2}+Cb\,c^{4}\right)x}{2bd}}{d\,x^{2}+}$
risch	Expression too large to display

input
```
int((C*x^2+B*x+A)/(d^2*x^3+c*d*x^2+b*d*x+b*c)^2,x,method=_RETURNVERBOSE)
```

output
```
-(A*d^2-B*c*d+C*c^2)/(b*d+c^2)^2/d/(d*x+c)+(4*A*c*d^2+B*b*d^2-3*B*c^2*d-2*
C*b*c*d+2*C*c^3)/(b*d+c^2)^3/d*ln(d*x+c)-1/(b*d+c^2)^3*((1/2*(A*b^2*d^3-A*
c^4*d-2*B*b^2*c*d^2-2*B*b*c^3*d-C*b^3*d^2+C*b*c^4)/b/d*x-1/2*(2*A*b*c*d^2+
2*A*c^3*d+B*b^2*d^2-B*c^4-2*C*b^2*c*d-2*C*b*c^3)/d)/(d*x^2+b)+1/2/b/d*(1/2
*(8*A*b*c*d^3+2*B*b^2*d^3-6*B*b*c^2*d^2-4*C*b^2*c*d^2+4*C*b*c^3*d)/d*ln(d*
x^2+b)+(3*A*b^2*d^3-6*A*b*c^2*d^2-A*c^4*d-6*B*b^2*c*d^2+2*B*b*c^3*d-C*b^3*
d^2+6*C*b^2*c^2*d-C*b*c^4)/(b*d)^(1/2)*arctan(d*x/(b*d)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1217 vs. 2(314) = 628.

Time = 5.80 (sec) , antiderivative size = 2454, normalized size of antiderivative = 7.44

$$\int \frac{A+Bx+Cx^2}{\left(bc+bdx+cdx^2+d^2x^3\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)/(d^2*x^3+c*d*x^2+b*d*x+b*c)^2,x, algorithm="fricas
")
```

output
```
[1/4*(6*B*b^4*c*d^3 - 4*A*b^4*d^4 - 4*(2*C*b^4*c^2 - B*b^3*c^3 - A*b^2*c^4
)*d^2 - 2*(3*C*b^2*c^4*d^2 + 3*A*b^3*d^5 - (C*b^4 + 4*B*b^3*c - 2*A*b^2*c^
2)*d^4 + (2*C*b^3*c^2 - 4*B*b^2*c^3 - A*b*c^4)*d^3)*x^2 + (C*b^2*c^5 - 3*A
*b^3*c*d^3 + (C*b*c^4*d^2 - 3*A*b^2*d^5 + (C*b^3 + 6*B*b^2*c + 6*A*b*c^2)*
d^4 - (6*C*b^2*c^2 + 2*B*b*c^3 - A*c^4)*d^3)*x^3 + (C*b^4*c + 6*B*b^3*c^2
+ 6*A*b^2*c^3)*d^2 + (C*b*c^5*d - 3*A*b^2*c*d^4 + (C*b^3*c + 6*B*b^2*c^2 +
 6*A*b*c^3)*d^3 - (6*C*b^2*c^3 + 2*B*b*c^4 - A*c^5)*d^2)*x^2 - (6*C*b^3*c^
3 + 2*B*b^2*c^4 - A*b*c^5)*d + (C*b^2*c^4*d - 3*A*b^3*d^4 + (C*b^4 + 6*B*b
^3*c + 6*A*b^2*c^2)*d^3 - (6*C*b^3*c^2 + 2*B*b^2*c^3 - A*b*c^4)*d^2)*x)*sq
rt(-b*d)*log((d*x^2 + 2*sqrt(-b*d)*x - b)/(d*x^2 + b)) - 2*(4*C*b^3*c^4 +
B*b^2*c^5)*d - 2*(C*b^2*c^5*d - (B*b^4 + A*b^3*c)*d^4 + (C*b^4*c - 2*B*b^3
*c^2 - 2*A*b^2*c^3)*d^3 + (2*C*b^3*c^3 - B*b^2*c^4 - A*b*c^5)*d^2)*x - 2*(
2*C*b^3*c^4*d + (B*b^4*c + 4*A*b^3*c^2)*d^3 + (2*C*b^2*c^3*d^3 + (B*b^3 +
4*A*b^2*c)*d^5 - (2*C*b^3*c + 3*B*b^2*c^2)*d^4)*x^3 - (2*C*b^4*c^2 + 3*B*b
^3*c^3)*d^2 + (2*C*b^2*c^4*d^2 + (B*b^3*c + 4*A*b^2*c^2)*d^4 - (2*C*b^3*c^
2 + 3*B*b^2*c^3)*d^3)*x^2 + (2*C*b^3*c^3*d^2 + (B*b^4 + 4*A*b^3*c)*d^4 - (
2*C*b^4*c + 3*B*b^3*c^2)*d^3)*x)*log(d*x^2 + b) + 4*(2*C*b^3*c^4*d + (B*b^
4*c + 4*A*b^3*c^2)*d^3 + (2*C*b^2*c^3*d^3 + (B*b^3 + 4*A*b^2*c)*d^5 - (2*C
*b^3*c + 3*B*b^2*c^2)*d^4)*x^3 - (2*C*b^4*c^2 + 3*B*b^3*c^3)*d^2 + (2*C*b^
2*c^4*d^2 + (B*b^3*c + 4*A*b^2*c^2)*d^4 - (2*C*b^3*c^2 + 3*B*b^2*c^3)*d...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(bc+bdx+cdx^2+d^2x^3)^2}\,dx = \text{Timed out}$$

input
```
integrate((C*x**2+B*x+A)/(d**2*x**3+c*d*x**2+b*d*x+b*c)**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 532, normalized size of antiderivative = 1.61

$$\int \frac{A+Bx+Cx^2}{(bc+bdx+cdx^2+d^2x^3)^2}\,dx$$
$$= -\frac{(2\,Cc^3+(Bb+4\,Ac)d^2-(2\,Cbc+3\,Bc^2)d)\log(dx^2+b)}{2\,(c^6d+3\,bc^4d^2+3\,b^2c^2d^3+b^3d^4)}$$
$$+\frac{(2\,Cc^3+(Bb+4\,Ac)d^2-(2\,Cbc+3\,Bc^2)d)\log(dx+c)}{c^6d+3\,bc^4d^2+3\,b^2c^2d^3+b^3d^4}$$
$$+\frac{(Cbc^4-3\,Ab^2d^3+(Cb^3+6\,Bb^2c+6\,Abc^2)d^2-(6\,Cb^2c^2+2\,Bbc^3-Ac^4)d)\arctan\left(\frac{dx}{\sqrt{bd}}\right)}{2\,(bc^6d+3\,b^2c^4d^2+3\,b^3c^2d^3+b^4d^4)\sqrt{bd}}$$
$$-\frac{4\,Cb^2c^2+Bbc^3+2\,Ab^2d^2+(3\,Cbc^2d+3\,Abd^3-(Cb^2+4\,Bbc+Ac^2)d^2)x^2-(3\,Bb^2c+2\,Abc^2)d+(}{2\,(b^2c^5d+2\,b^3c^3d^2+b^4cd^3+(bc^4d^3+2\,b^2c^2d^4+b^3d^5)x^3+(bc^5d^2+2\,b^2c^3d^3+b^3cd^4)}$$

input
```
integrate((C*x^2+B*x+A)/(d^2*x^3+c*d*x^2+b*d*x+b*c)^2,x, algorithm="maxima
")
```

output
```
-1/2*(2*C*c^3 + (B*b + 4*A*c)*d^2 - (2*C*b*c + 3*B*c^2)*d)*log(d*x^2 + b)/
(c^6*d + 3*b*c^4*d^2 + 3*b^2*c^2*d^3 + b^3*d^4) + (2*C*c^3 + (B*b + 4*A*c)
*d^2 - (2*C*b*c + 3*B*c^2)*d)*log(d*x + c)/(c^6*d + 3*b*c^4*d^2 + 3*b^2*c^
2*d^3 + b^3*d^4) + 1/2*(C*b*c^4 - 3*A*b^2*d^3 + (C*b^3 + 6*B*b^2*c + 6*A*b
*c^2)*d^2 - (6*C*b^2*c^2 + 2*B*b*c^3 - A*c^4)*d)*arctan(d*x/sqrt(b*d))/((b
*c^6*d + 3*b^2*c^4*d^2 + 3*b^3*c^2*d^3 + b^4*d^4)*sqrt(b*d)) - 1/2*(4*C*b^
2*c^2 + B*b*c^3 + 2*A*b^2*d^2 + (3*C*b*c^2*d + 3*A*b*d^3 - (C*b^2 + 4*B*b*
c + A*c^2)*d^2)*x^2 - (3*B*b^2*c + 2*A*b*c^2)*d + (C*b*c^3 - (B*b^2 + A*b*
c)*d^2 + (C*b^2*c - B*b*c^2 - A*c^3)*d)*x)/(b^2*c^5*d + 2*b^3*c^3*d^2 + b^
4*c*d^3 + (b*c^4*d^3 + 2*b^2*c^2*d^4 + b^3*d^5)*x^3 + (b*c^5*d^2 + 2*b^2*c
^3*d^3 + b^3*c*d^4)*x^2 + (b^2*c^4*d^2 + 2*b^3*c^2*d^3 + b^4*d^4)*x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 484, normalized size of antiderivative = 1.47

$$\int \frac{A+Bx+Cx^2}{(bc+bdx+cdx^2+d^2x^3)^2}\,dx$$
$$= -\frac{(2\,Cc^3-2\,Cbcd-3\,Bc^2d+Bbd^2+4\,Acd^2)\log(dx^2+b)}{2\,(c^6d+3\,bc^4d^2+3\,b^2c^2d^3+b^3d^4)}$$
$$+\frac{(2\,Cc^3d-2\,Cbcd^2-3\,Bc^2d^2+Bbd^3+4\,Acd^3)\log(|dx+c|)}{c^6d^2+3\,bc^4d^3+3\,b^2c^2d^4+b^3d^5}$$
$$+\frac{(Cbc^4-6\,Cb^2c^2d-2\,Bbc^3d+Ac^4d+Cb^3d^2+6\,Bb^2cd^2+6\,Abc^2d^2-3\,Ab^2d^3)\arctan\left(\frac{dx}{\sqrt{bd}}\right)}{2\,(bc^6d+3\,b^2c^4d^2+3\,b^3c^2d^3+b^4d^4)\sqrt{bd}}$$
$$-\frac{3\,Cbc^2dx^2-Cb^2d^2x^2-4\,Bbcd^2x^2-Ac^2d^2x^2+3\,Abd^3x^2+Cbc^3x+Cb^2cdx-Bbc^2dx-Ac^3dx-B}{2\,(bc^4d+2\,b^2c^2d^2+b^3d^3)(d^2x^3+cdx^2+bd}$$

input
```
integrate((C*x^2+B*x+A)/(d^2*x^3+c*d*x^2+b*d*x+b*c)^2,x, algorithm="giac")
```

output
```
-1/2*(2*C*c^3 - 2*C*b*c*d - 3*B*c^2*d + B*b*d^2 + 4*A*c*d^2)*log(d*x^2 + b
)/(c^6*d + 3*b*c^4*d^2 + 3*b^2*c^2*d^3 + b^3*d^4) + (2*C*c^3*d - 2*C*b*c*d
^2 - 3*B*c^2*d^2 + B*b*d^3 + 4*A*c*d^3)*log(abs(d*x + c))/(c^6*d^2 + 3*b*c
^4*d^3 + 3*b^2*c^2*d^4 + b^3*d^5) + 1/2*(C*b*c^4 - 6*C*b^2*c^2*d - 2*B*b*c
^3*d + A*c^4*d + C*b^3*d^2 + 6*B*b^2*c*d^2 + 6*A*b*c^2*d^2 - 3*A*b^2*d^3)*
arctan(d*x/sqrt(b*d))/((b*c^6*d + 3*b^2*c^4*d^2 + 3*b^3*c^2*d^3 + b^4*d^4)
*sqrt(b*d)) - 1/2*(3*C*b*c^2*d*x^2 - C*b^2*d^2*x^2 - 4*B*b*c*d^2*x^2 - A*c
^2*d^2*x^2 + 3*A*b*d^3*x^2 + C*b*c^3*x + C*b^2*c*d*x - B*b*c^2*d*x - A*c^3
*d*x - B*b^2*d^2*x - A*b*c*d^2*x + 4*C*b^2*c^2 + B*b*c^3 - 3*B*b^2*c*d - 2
*A*b*c^2*d + 2*A*b^2*d^2)/((b*c^4*d + 2*b^2*c^2*d^2 + b^3*d^3)*(d^2*x^3 +
c*d*x^2 + b*d*x + b*c))
```

Mupad [B] (verification not implemented)

Time = 16.40 (sec) , antiderivative size = 1931, normalized size of antiderivative = 5.85

$$\int \frac{A+Bx+Cx^2}{(bc+bdx+cdx^2+d^2x^3)^2}\,dx = \text{Too large to display}$$

input
```
int((A + B*x + C*x^2)/(b*c + d^2*x^3 + b*d*x + c*d*x^2)^2,x)
```

output
```
((x^2*(A*c^2*d - 3*A*b*d^2 - 3*C*b*c^2 + C*b^2*d + 4*B*b*c*d))/(2*b*(c^4 +
 b^2*d^2 + 2*b*c^2*d)) - (B*c^3 + 2*A*b*d^2 - 2*A*c^2*d + 4*C*b*c^2 - 3*B*
b*c*d)/(2*d*(b*d + c^2)^2) + (x*(A*c*d + B*b*d - C*b*c))/(2*b*d*(b*d + c^2
)))/(b*c + d^2*x^3 + b*d*x + c*d*x^2) + (log(c + d*x)*(2*C*c^3 + d^2*(4*A*
c + B*b) - d*(3*B*c^2 + 2*C*b*c)))/(c^6*d + b^3*d^4 + 3*b*c^4*d^2 + 3*b^2*
c^2*d^3) + (log(6*B*b^5*d^5 - 3*A*d*(-b^3*d^3)^(3/2) + 18*B*c*(-b^3*d^3)^(
3/2) + C*b*(-b^3*d^3)^(3/2) + A*c^6*d*(-b^3*d^3)^(1/2) + C*b*c^6*(-b^3*d^3
)^(1/2) + 30*A*b^4*c*d^5 - 14*C*b^5*c*d^4 + 3*A*b^4*d^6*x - C*b^5*d^5*x +
6*B*d*x*(-b^3*d^3)^(3/2) - 14*C*c*x*(-b^3*d^3)^(3/2) - 2*A*b^2*c^5*d^3 - 3
6*A*b^3*c^3*d^4 + 22*B*b^3*c^4*d^3 - 36*B*b^4*c^2*d^4 - 14*C*b^3*c^5*d^2 +
 36*C*b^4*c^3*d^3 + A*b*c^6*d^3*x - 18*B*b^4*c*d^5*x - 57*A*b^2*c^2*d^3*(-
b^3*d^3)^(1/2) + 44*B*b^2*c^3*d^2*(-b^3*d^3)^(1/2) + 31*C*b^3*c^2*d^2*(-b^
3*d^3)^(1/2) + 5*A*b^2*c^4*d^4*x - 57*A*b^3*c^2*d^5*x - 2*B*b^2*c^5*d^3*x
+ 44*B*b^3*c^3*d^4*x + C*b^2*c^6*d^2*x - 31*C*b^3*c^4*d^3*x + 31*C*b^4*c^2
*d^4*x - 2*B*b*c^5*d*(-b^3*d^3)^(1/2) + 5*A*b*c^4*d^2*(-b^3*d^3)^(1/2) - 3
1*C*b^2*c^4*d*(-b^3*d^3)^(1/2) + 2*A*c^5*d^2*x*(-b^3*d^3)^(1/2) + 36*A*b*c
^3*d^3*x*(-b^3*d^3)^(1/2) - 30*A*b^2*c*d^4*x*(-b^3*d^3)^(1/2) - 22*B*b*c^4
*d^2*x*(-b^3*d^3)^(1/2) + 36*B*b^2*c^2*d^3*x*(-b^3*d^3)^(1/2) - 36*C*b^2*c
^3*d^2*x*(-b^3*d^3)^(1/2) + 14*C*b*c^5*d*x*(-b^3*d^3)^(1/2))*(d^3*((3*B*b^
3*c^2)/2 - (3*A*b^2*(-b^3*d^3)^(1/2))/4 + C*b^4*c) - d*((B*b*c^3*(-b^3*...
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 1699, normalized size of antiderivative = 5.15

$$\int \frac{A + Bx + Cx^2}{(bc + bdx + cdx^2 + d^2x^3)^2} \, dx = \text{Too large to display}$$

input
```
int((C*x^2+B*x+A)/(d^2*x^3+c*d*x^2+b*d*x+b*c)^2,x)
```

output

```
( - 3*sqrt(d)*sqrt(b)*atan((d*x)/(sqrt(d)*sqrt(b)))*a*b**3*c**2*d**3 - 3*s
qrt(d)*sqrt(b)*atan((d*x)/(sqrt(d)*sqrt(b)))*a*b**3*c*d**4*x + 6*sqrt(d)*s
qrt(b)*atan((d*x)/(sqrt(d)*sqrt(b)))*a*b**2*c**4*d**2 + 6*sqrt(d)*sqrt(b)*
atan((d*x)/(sqrt(d)*sqrt(b)))*a*b**2*c**3*d**3*x - 3*sqrt(d)*sqrt(b)*atan(
(d*x)/(sqrt(d)*sqrt(b)))*a*b**2*c**2*d**4*x**2 - 3*sqrt(d)*sqrt(b)*atan((d
*x)/(sqrt(d)*sqrt(b)))*a*b**2*c*d**5*x**3 + sqrt(d)*sqrt(b)*atan((d*x)/(sq
rt(d)*sqrt(b)))*a*b*c**6*d + sqrt(d)*sqrt(b)*atan((d*x)/(sqrt(d)*sqrt(b)))
*a*b*c**5*d**2*x + 6*sqrt(d)*sqrt(b)*atan((d*x)/(sqrt(d)*sqrt(b)))*a*b*c**
4*d**3*x**2 + 6*sqrt(d)*sqrt(b)*atan((d*x)/(sqrt(d)*sqrt(b)))*a*b*c**3*d**
4*x**3 + sqrt(d)*sqrt(b)*atan((d*x)/(sqrt(d)*sqrt(b)))*a*c**6*d**2*x**2 +
sqrt(d)*sqrt(b)*atan((d*x)/(sqrt(d)*sqrt(b)))*a*c**5*d**3*x**3 + 7*sqrt(d)
*sqrt(b)*atan((d*x)/(sqrt(d)*sqrt(b)))*b**4*c**3*d**2 + 7*sqrt(d)*sqrt(b)*
atan((d*x)/(sqrt(d)*sqrt(b)))*b**4*c**2*d**3*x - 8*sqrt(d)*sqrt(b)*atan((d
*x)/(sqrt(d)*sqrt(b)))*b**3*c**5*d - 8*sqrt(d)*sqrt(b)*atan((d*x)/(sqrt(d)
*sqrt(b)))*b**3*c**4*d**2*x + 7*sqrt(d)*sqrt(b)*atan((d*x)/(sqrt(d)*sqrt(b
)))*b**3*c**3*d**3*x**2 + 7*sqrt(d)*sqrt(b)*atan((d*x)/(sqrt(d)*sqrt(b)))*
b**3*c**2*d**4*x**3 + sqrt(d)*sqrt(b)*atan((d*x)/(sqrt(d)*sqrt(b)))*b**2*c
**7 + sqrt(d)*sqrt(b)*atan((d*x)/(sqrt(d)*sqrt(b)))*b**2*c**6*d*x - 8*sqrt
(d)*sqrt(b)*atan((d*x)/(sqrt(d)*sqrt(b)))*b**2*c**5*d**2*x**2 - 8*sqrt(d)*
sqrt(b)*atan((d*x)/(sqrt(d)*sqrt(b)))*b**2*c**4*d**3*x**3 + sqrt(d)*sqr...
```

3.90 $\int \frac{A+Bx+Cx^2}{1+x+x^2+x^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 45

$$\int \frac{A+Bx+Cx^2}{1+x+x^2+x^3}\,dx = \frac{1}{2}(A+B-C)\arctan(x)+\frac{1}{2}(A-B+C)\log(1+x) - \frac{1}{4}(A-B-C)\log\left(1+x^2\right)$$

output
```
1/2*(A+B-C)*arctan(x)+1/2*(A-B+C)*ln(1+x)-1/4*(A-B-C)*ln(x^2+1)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.03 (sec) , antiderivative size = 101, normalized size of antiderivative = 2.24

$$\int \frac{A+Bx+Cx^2}{1+x+x^2+x^3}\,dx = \frac{1}{4}\left((1+i)(-iA+B+iC)\arctan\left(\frac{-1+x}{1+x}\right) - (1+i)(A-iB-C)\arctan\left(\frac{1+x}{-1+x}\right)+2A\log(1+x) - 2B\log(1+x)+2C\log(1+x)-A\log\left(1+x^2\right) + B\log\left(1+x^2\right)+C\log\left(1+x^2\right)\right)$$

input
```
Integrate[(A + B*x + C*x^2)/(1 + x + x^2 + x^3),x]
```

output
```
((1 + I)*((-I)*A + B + I*C)*ArcTan[(-1 + x)/(1 + x)] - (1 + I)*(A - I*B -
C)*ArcTan[(1 + x)/(-1 + x)] + 2*A*Log[1 + x] - 2*B*Log[1 + x] + 2*C*Log[1
+ x] - A*Log[1 + x^2] + B*Log[1 + x^2] + C*Log[1 + x^2])/4
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {2462, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{x^3+x^2+x+1}\,dx$$

$$\Big\downarrow \text{ 2462}$$

$$\int \left(\frac{-x(A-B-C)+A+B-C}{2\left(x^2+1\right)}+\frac{A-B+C}{2(x+1)}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\arctan(x)(A+B-C)-\frac{1}{4}\log\left(x^2+1\right)(A-B-C)+\frac{1}{2}\log(x+1)(A-B+C)$$

input
```
Int[(A + B*x + C*x^2)/(1 + x + x^2 + x^3),x]
```

output
```
((A + B - C)*ArcTan[x])/2 + ((A - B + C)*Log[1 + x])/2 - ((A - B - C)*Log[
1 + x^2])/4
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.91

method	result
default	$\frac{(-A+B+C)\ln(x^2+1)}{4}+\frac{(A+B-C)\arctan(x)}{2}+\left(\frac{A}{2}-\frac{B}{2}+\frac{C}{2}\right)\ln\left(x+1\right)$
parallelrisch	$\frac{\ln(x+1)A}{2}-\frac{\ln(x+1)B}{2}+\frac{\ln(x+1)C}{2}-\frac{\ln(x-i)A}{4}+\frac{\ln(x-i)B}{4}+\frac{\ln(x-i)C}{4}+\frac{i\ln(x+i)B}{4}-\frac{i\ln(x-i)B}{4}+$
risch	$-\frac{B\arctan\left(\frac{3A}{A+B-C}-\frac{3B}{A+B-C}+\frac{C}{A+B-C}\right)}{2}-$

input
```
int((C*x^2+B*x+A)/(x^3+x^2+x+1),x,method=_RETURNVERBOSE)
```

output
```
1/4*(-A+B+C)*ln(x^2+1)+1/2*(A+B-C)*arctan(x)+(1/2*A-1/2*B+1/2*C)*ln(x+1)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.87

$$\int \frac{A+Bx+Cx^2}{1+x+x^2+x^3}\,dx = \frac{1}{2}\left(A+B-C\right)\arctan\left(x\right)-\frac{1}{4}\left(A-B-C\right)\log\left(x^2+1\right) + \frac{1}{2}\left(A-B+C\right)\log\left(x+1\right)$$

input
```
integrate((C*x^2+B*x+A)/(x^3+x^2+x+1),x, algorithm="fricas")
```

output
```
1/2*(A + B - C)*arctan(x) - 1/4*(A - B - C)*log(x^2 + 1) + 1/2*(A - B + C)
*log(x + 1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.19 (sec) , antiderivative size = 925, normalized size of antiderivative = 20.56

$$\int \frac{A+Bx+Cx^2}{1+x+x^2+x^3}\,dx = \text{Too large to display}$$

input
```
integrate((C*x**2+B*x+A)/(x**3+x**2+x+1),x)
```

output
```
(A - B + C)*log(x + (3*A**3 - 9*A**2*B - A**2*C + 6*A**2*(A - B + C) + 11*
A*B**2 - 6*A*B*C - 4*A*B*(A - B + C) + A*C**2 + 4*A*C*(A - B + C) - 4*A*(A
 - B + C)**2 - B**3 - 3*B**2*C + 2*B**2*(A - B + C) + 7*B*C**2 - 12*B*C*(A
 - B + C) + 8*B*(A - B + C)**2 - 3*C**3 + 6*C**2*(A - B + C) - 4*C*(A - B
+ C)**2)/(5*A**3 - 3*A**2*B - 3*A**2*C - 3*A*B**2 + 6*A*B*C - A*C**2 + 5*B
**3 - 9*B**2*C + 5*B*C**2 - C**3))/2 + (-A/4 + B/4 + C/4 - I*(A + B - C)/4
)*log(x + (3*A**3 - 9*A**2*B - A**2*C + 12*A**2*(-A/4 + B/4 + C/4 - I*(A +
 B - C)/4) + 11*A*B**2 - 6*A*B*C - 8*A*B*(-A/4 + B/4 + C/4 - I*(A + B - C)
/4) + A*C**2 + 8*A*C*(-A/4 + B/4 + C/4 - I*(A + B - C)/4) - 16*A*(-A/4 + B
/4 + C/4 - I*(A + B - C)/4)**2 - B**3 - 3*B**2*C + 4*B**2*(-A/4 + B/4 + C/
4 - I*(A + B - C)/4) + 7*B*C**2 - 24*B*C*(-A/4 + B/4 + C/4 - I*(A + B - C)
/4) + 32*B*(-A/4 + B/4 + C/4 - I*(A + B - C)/4)**2 - 3*C**3 + 12*C**2*(-A/
4 + B/4 + C/4 - I*(A + B - C)/4) - 16*C*(-A/4 + B/4 + C/4 - I*(A + B - C)/
4)**2)/(5*A**3 - 3*A**2*B - 3*A**2*C - 3*A*B**2 + 6*A*B*C - A*C**2 + 5*B**
3 - 9*B**2*C + 5*B*C**2 - C**3)) + (-A/4 + B/4 + C/4 + I*(A + B - C)/4)*lo
g(x + (3*A**3 - 9*A**2*B - A**2*C + 12*A**2*(-A/4 + B/4 + C/4 + I*(A + B -
 C)/4) + 11*A*B**2 - 6*A*B*C - 8*A*B*(-A/4 + B/4 + C/4 + I*(A + B - C)/4)
+ A*C**2 + 8*A*C*(-A/4 + B/4 + C/4 + I*(A + B - C)/4) - 16*A*(-A/4 + B/4 +
 C/4 + I*(A + B - C)/4)**2 - B**3 - 3*B**2*C + 4*B**2*(-A/4 + B/4 + C/4 +
I*(A + B - C)/4) + 7*B*C**2 - 24*B*C*(-A/4 + B/4 + C/4 + I*(A + B - C)/...
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.87

$$\int \frac{A+Bx+Cx^2}{1+x+x^2+x^3}\,dx = \frac{1}{2}\,(A+B-C)\arctan{(x)} - \frac{1}{4}\,(A-B-C)\log{(x^2+1)} + \frac{1}{2}\,(A-B+C)\log{(x+1)}$$

input
```
integrate((C*x^2+B*x+A)/(x^3+x^2+x+1),x, algorithm="maxima")
```

output
```
1/2*(A + B - C)*arctan(x) - 1/4*(A - B - C)*log(x^2 + 1) + 1/2*(A - B + C)
*log(x + 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.89

$$\int \frac{A+Bx+Cx^2}{1+x+x^2+x^3}\,dx = \frac{1}{2}\,(A+B-C)\arctan{(x)} - \frac{1}{4}\,(A-B-C)\log{(x^2+1)} + \frac{1}{2}\,(A-B+C)\log{(|x+1|)}$$

input
```
integrate((C*x^2+B*x+A)/(x^3+x^2+x+1),x, algorithm="giac")
```

output
```
1/2*(A + B - C)*arctan(x) - 1/4*(A - B - C)*log(x^2 + 1) + 1/2*(A - B + C)
*log(abs(x + 1))
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.33

$$\int \frac{A+Bx+Cx^2}{1+x+x^2+x^3}\,dx = \ln(x+1)\left(\frac{A}{2}-\frac{B}{2}+\frac{C}{2}\right) + \ln(x-\mathrm{i})\left(A\left(-\frac{1}{4}-\frac{1}{4}\mathrm{i}\right)+B\left(\frac{1}{4}-\frac{1}{4}\mathrm{i}\right)+C\left(\frac{1}{4}+\frac{1}{4}\mathrm{i}\right)\right) + \ln(x+1\mathrm{i})\left(A\left(-\frac{1}{4}+\frac{1}{4}\mathrm{i}\right)+B\left(\frac{1}{4}+\frac{1}{4}\mathrm{i}\right)+C\left(\frac{1}{4}-\frac{1}{4}\mathrm{i}\right)\right)$$

input
```
int((A + B*x + C*x^2)/(x + x^2 + x^3 + 1),x)
```

output
```
log(x + 1)*(A/2 - B/2 + C/2) + log(x - 1i)*(B*(1/4 - 1i/4) - A*(1/4 + 1i/4
) + C*(1/4 + 1i/4)) + log(x + 1i)*(B*(1/4 + 1i/4) - A*(1/4 - 1i/4) + C*(1/
4 - 1i/4))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.42

$$\int \frac{A+Bx+Cx^2}{1+x+x^2+x^3}\,dx = \frac{atan(x)\,a}{2}+\frac{atan(x)\,b}{2}-\frac{atan(x)\,c}{2}-\frac{\log(x^2+1)\,a}{4}+\frac{\log(x^2+1)\,b}{4}+\frac{\log(x^2+1)\,c}{4}+\frac{\log(x+1)\,a}{2}-\frac{\log(x+1)\,b}{2}+\frac{\log(x+1)\,c}{2}$$

input
```
int((C*x^2+B*x+A)/(x^3+x^2+x+1),x)
```

output
```
(2*atan(x)*a + 2*atan(x)*b - 2*atan(x)*c - log(x**2 + 1)*a + log(x**2 + 1)
*b + log(x**2 + 1)*c + 2*log(x + 1)*a - 2*log(x + 1)*b + 2*log(x + 1)*c)/4
```

3.91 $\int \frac{A+Bx+Cx^2}{-1+4x-4x^2+16x^3} \, dx$

Optimal result

Integrand size = 28, antiderivative size = 57

$$\int \frac{A+Bx+Cx^2}{-1+4x-4x^2+16x^3} \, dx = -\frac{1}{40}(4A-4B-C)\arctan(2x) + \frac{1}{80}(16A+4B+C)\log(1-4x) - \frac{1}{40}(4A+B-C)\log\left(1+4x^2\right)$$

output
```
-1/40*(4*A-4*B-C)*arctan(2*x)+1/80*(16*A+4*B+C)*ln(1-4*x)-1/40*(4*A+B-C)*l
n(4*x^2+1)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.11

$$\int \frac{A+Bx+Cx^2}{-1+4x-4x^2+16x^3} \, dx = \frac{1}{80}\left((8A-2(4B+C))\arctan\left(\frac{2(1+x)}{-1+4x}\right) + (16A+4B+C)\log(1-4x) - 2(4A+B-C)\log\left(1+4x^2\right)\right)$$

input
```
Integrate[(A + B*x + C*x^2)/(-1 + 4*x - 4*x^2 + 16*x^3),x]
```

output
```
((8*A - 2*(4*B + C))*ArcTan[(2*(1 + x))/(-1 + 4*x)] + (16*A + 4*B + C)*Log
[1 - 4*x] - 2*(4*A + B - C)*Log[1 + 4*x^2])/80
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2462, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{16x^3-4x^2+4x-1}dx$$

$$\Big\downarrow \text{2462}$$

$$\int \left(\frac{-4x(4A+B-C)-4A+4B+C}{20(4x^2+1)}+\frac{16A+4B+C}{20(4x-1)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{40}\arctan(2x)(4A-4B-C)-\frac{1}{40}\log\left(4x^2+1\right)(4A+B-C)+\frac{1}{80}\log(1-4x)(16A+4B+C)$$

input
```
Int[(A + B*x + C*x^2)/(-1 + 4*x - 4*x^2 + 16*x^3),x]
```

output
```
-1/40*((4*A - 4*B - C)*ArcTan[2*x]) + ((16*A + 4*B + C)*Log[1 - 4*x])/80 -
 ((4*A + B - C)*Log[1 + 4*x^2])/40
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.93

method	result
default	$\frac{(-16A-4B+4C)\ln(4x^2+1)}{160}+\frac{(-4A+4B+C)\arctan(2x)}{40}+\left(\frac{C}{80}+\frac{B}{20}+\frac{A}{5}\right)\ln(4x-1)$
parallelrisch	$\frac{\ln(x-\frac{1}{4})C}{80}+\frac{\ln(x-\frac{1}{4})B}{20}+\frac{\ln(x-\frac{1}{4})A}{5}-\frac{\ln(x-\frac{i}{2})A}{10}-\frac{\ln(x-\frac{i}{2})B}{40}+\frac{\ln(x-\frac{i}{2})C}{40}+\frac{i\ln(x+\frac{i}{2})B}{20}+\frac{i\ln(x-\frac{i}{2})A}{20}$
risch	Expression too large to display

input
```
int((C*x^2+B*x+A)/(16*x^3-4*x^2+4*x-1),x,method=_RETURNVERBOSE)
```

output
```
1/160*(-16*A-4*B+4*C)*ln(4*x^2+1)+1/40*(-4*A+4*B+C)*arctan(2*x)+(1/80*C+1/
20*B+1/5*A)*ln(4*x-1)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.89

$$\int \frac{A+Bx+Cx^2}{-1+4x-4x^2+16x^3}\,dx = -\frac{1}{40}\,(4\,A-4\,B-C)\arctan\left(2\,x\right) - \frac{1}{40}\,(4\,A+B-C)\log\left(4\,x^2+1\right) + \frac{1}{80}\,(16\,A+4\,B+C)\log\left(4\,x-1\right)$$

input
```
integrate((C*x^2+B*x+A)/(16*x^3-4*x^2+4*x-1),x, algorithm="fricas")
```

output
```
-1/40*(4*A - 4*B - C)*arctan(2*x) - 1/40*(4*A + B - C)*log(4*x^2 + 1) + 1/
80*(16*A + 4*B + C)*log(4*x - 1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 10.40 (sec) , antiderivative size = 983, normalized size of antiderivative = 17.25

$$\int \frac{A+Bx+Cx^2}{-1+4x-4x^2+16x^3}\,dx = \text{Too large to display}$$

input
```
integrate((C*x**2+B*x+A)/(16*x**3-4*x**2+4*x-1),x)
```

output
```
(16*A + 4*B + C)*log(x + (-5952*A**3 - 5184*A**2*B + 1424*A**2*C - 480*A**
2*(16*A + 4*B + C) - 976*A*B**2 + 672*A*B*C - 80*A*B*(16*A + 4*B + C) + 4*
A*C**2 - 80*A*C*(16*A + 4*B + C) + 44*A*(16*A + 4*B + C)**2 + 112*B**3 + 2
04*B**2*C - 40*B**2*(16*A + 4*B + C) + 56*B*C**2 - 60*B*C*(16*A + 4*B + C)
 + 16*B*(16*A + 4*B + C)**2 + 3*C**3 - C*(16*A + 4*B + C)**2)/(9472*A**3 -
 5376*A**2*B - 3264*A**2*C - 3264*A*B**2 - 192*A*B*C + 256*A*C**2 - 832*B*
*3 - 144*B**2*C - 16*B*C**2 - 8*C**3))/80 + (-A/10 - B/40 + C/40 - I*(4*A
- 4*B - C)/80)*log(x + (-5952*A**3 - 5184*A**2*B + 1424*A**2*C - 38400*A**
2*(-A/10 - B/40 + C/40 - I*(4*A - 4*B - C)/80) - 976*A*B**2 + 672*A*B*C -
6400*A*B*(-A/10 - B/40 + C/40 - I*(4*A - 4*B - C)/80) + 4*A*C**2 - 6400*A*
C*(-A/10 - B/40 + C/40 - I*(4*A - 4*B - C)/80) + 281600*A*(-A/10 - B/40 +
C/40 - I*(4*A - 4*B - C)/80)**2 + 112*B**3 + 204*B**2*C - 3200*B**2*(-A/10
 - B/40 + C/40 - I*(4*A - 4*B - C)/80) + 56*B*C**2 - 4800*B*C*(-A/10 - B/4
0 + C/40 - I*(4*A - 4*B - C)/80) + 102400*B*(-A/10 - B/40 + C/40 - I*(4*A
- 4*B - C)/80)**2 + 3*C**3 - 6400*C*(-A/10 - B/40 + C/40 - I*(4*A - 4*B -
C)/80)**2)/(9472*A**3 - 5376*A**2*B - 3264*A**2*C - 3264*A*B**2 - 192*A*B*
C + 256*A*C**2 - 832*B**3 - 144*B**2*C - 16*B*C**2 - 8*C**3)) + (-A/10 - B
/40 + C/40 + I*(4*A - 4*B - C)/80)*log(x + (-5952*A**3 - 5184*A**2*B + 142
4*A**2*C - 38400*A**2*(-A/10 - B/40 + C/40 + I*(4*A - 4*B - C)/80) - 976*A
*B**2 + 672*A*B*C - 6400*A*B*(-A/10 - B/40 + C/40 + I*(4*A - 4*B - C)/8...
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.89

$$\int \frac{A+Bx+Cx^2}{-1+4x-4x^2+16x^3}\,dx = -\frac{1}{40}\,(4\,A-4\,B-C)\arctan\left(2\,x\right) - \frac{1}{40}\,(4\,A+B-C)\log\left(4\,x^2+1\right) + \frac{1}{80}\,(16\,A+4\,B+C)\log\left(4\,x-1\right)$$

input
```
integrate((C*x^2+B*x+A)/(16*x^3-4*x^2+4*x-1),x, algorithm="maxima")
```

output
```
-1/40*(4*A - 4*B - C)*arctan(2*x) - 1/40*(4*A + B - C)*log(4*x^2 + 1) + 1/
80*(16*A + 4*B + C)*log(4*x - 1)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.91

$$\int \frac{A+Bx+Cx^2}{-1+4x-4x^2+16x^3}\,dx = -\frac{1}{40}\,(4\,A-4\,B-C)\arctan\left(2\,x\right) - \frac{1}{40}\,(4\,A+B-C)\log\left(4\,x^2+1\right) + \frac{1}{80}\,(16\,A+4\,B+C)\log\left(|4\,x-1|\right)$$

input
```
integrate((C*x^2+B*x+A)/(16*x^3-4*x^2+4*x-1),x, algorithm="giac")
```

output
```
-1/40*(4*A - 4*B - C)*arctan(2*x) - 1/40*(4*A + B - C)*log(4*x^2 + 1) + 1/
80*(16*A + 4*B + C)*log(abs(4*x - 1))
```

Mupad [B] (verification not implemented)

Time = 13.29 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.09

$$\int \frac{A+Bx+Cx^2}{-1+4x-4x^2+16x^3}\,dx = \ln\left(x-\frac{1}{4}\right)\left(\frac{A}{5}+\frac{B}{20}+\frac{C}{80}\right) - \ln\left(x-\frac{1}{2}\mathrm{i}\right)\left(A\left(\frac{1}{10}-\frac{1}{20}\mathrm{i}\right)+B\left(\frac{1}{40}+\frac{1}{20}\mathrm{i}\right) + C\left(-\frac{1}{40}+\frac{1}{80}\mathrm{i}\right)\right) - \ln\left(x+\frac{1}{2}\mathrm{i}\right)\left(A\left(\frac{1}{10}+\frac{1}{20}\mathrm{i}\right) + B\left(\frac{1}{40}-\frac{1}{20}\mathrm{i}\right)+C\left(-\frac{1}{40}-\frac{1}{80}\mathrm{i}\right)\right)$$

input
```
int((A + B*x + C*x^2)/(4*x - 4*x^2 + 16*x^3 - 1),x)
```

output
```
log(x - 1/4)*(A/5 + B/20 + C/80) - log(x - 1i/2)*(A*(1/10 - 1i/20) + B*(1/
40 + 1i/20) - C*(1/40 - 1i/80)) - log(x + 1i/2)*(A*(1/10 + 1i/20) + B*(1/4
0 - 1i/20) - C*(1/40 + 1i/80))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.44

$$\int \frac{A+Bx+Cx^2}{-1+4x-4x^2+16x^3}\,dx = -\frac{atan(2x)\,a}{10}+\frac{atan(2x)\,b}{10}+\frac{atan(2x)\,c}{40} - \frac{\log(4x^2+1)\,a}{10}-\frac{\log(4x^2+1)\,b}{40}+\frac{\log(4x^2+1)\,c}{40} + \frac{\log(4x-1)\,a}{5}+\frac{\log(4x-1)\,b}{20}+\frac{\log(4x-1)\,c}{80}$$

input
```
int((C*x^2+B*x+A)/(16*x^3-4*x^2+4*x-1),x)
```

output
```
( - 8*atan(2*x)*a + 8*atan(2*x)*b + 2*atan(2*x)*c - 8*log(4*x**2 + 1)*a -
2*log(4*x**2 + 1)*b + 2*log(4*x**2 + 1)*c + 16*log(4*x - 1)*a + 4*log(4*x
- 1)*b + log(4*x - 1)*c)/80
```

3.92 $\int (A+Bx+Cx^2)(2+6x+3x^2+9x^3)^{3/2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 636

$$\int (A+Bx+Cx^2)\left(2+6x+3x^2+9x^3\right)^{3/2}\,dx = -\frac{2(5967A+1152B-1582C)\left(2+6x+3x^2+9x^3\right)^{3/2}}{243243(1+3x)} + \frac{2(108A-45B-14C)x(2+6x+3x^2+9x^3)^{3/2}}{891(1+3x)} - \frac{8(71955A+31869B-23072C)\left(2+6x+3x^2+9x^3\right)^{3/2}}{3648645(1+3x)\left(2+3x^2\right)} + \frac{4(64935A-23418B-9226C)x(2+6x+3x^2+9x^3)^{3/2}}{405405(1+3x)\left(2+3x^2\right)} + \frac{16(4590A-1251B-742C)\left(2+6x+3x^2+9x^3\right)^{3/2}}{25515(1+3x)\left(1+\sqrt{7}+3x\right)\left(2+3x^2\right)}$$

$$+\frac{2}{351}(9B-2C)\left(2+3x^2\right)\left(2+6x+3x^2+9x^3\right)^{3/2}+\frac{2(117A+9B-28C)\left(2+3x^2\right)\left(2+6x+3x^2+9x^3\right)^{3/2}}{1287(1+3x)}$$

output
```
-2*(5967*A+1152*B-1582*C)*(9*x^3+3*x^2+6*x+2)^(3/2)/(243243+729729*x)+2*(1
08*A-45*B-14*C)*x*(9*x^3+3*x^2+6*x+2)^(3/2)/(891+2673*x)-8/3648645*(71955*
A+31869*B-23072*C)*(9*x^3+3*x^2+6*x+2)^(3/2)/(1+3*x)/(3*x^2+2)+4/405405*(6
4935*A-23418*B-9226*C)*x*(9*x^3+3*x^2+6*x+2)^(3/2)/(1+3*x)/(3*x^2+2)+16/25
515*(4590*A-1251*B-742*C)*(9*x^3+3*x^2+6*x+2)^(3/2)/(1+3*x)/(1+7^(1/2)+3*x
)/(3*x^2+2)+2/351*(9*B-2*C)*(3*x^2+2)*(9*x^3+3*x^2+6*x+2)^(3/2)+2*(117*A+9
*B-28*C)*(3*x^2+2)*(9*x^3+3*x^2+6*x+2)^(3/2)/(1287+3861*x)+2/135*C*(1+3*x)
*(3*x^2+2)*(9*x^3+3*x^2+6*x+2)^(3/2)-16/76545*(4590*A-1251*B-742*C)*(1+7^(
1/2)+3*x)*((3*x^2+2)/(1+7^(1/2)+3*x)^2)^(1/2)*(9*x^3+3*x^2+6*x+2)^(3/2)*El
lipticE(sin(2*arctan(1/7*(1+3*x)^(1/2)*7^(3/4))),1/14*(98+14*7^(1/2))^(1/2
))*3^(1/2)*7^(1/4)/(1+3*x)^(3/2)/(3*x^2+2)^2+8/10945935*(656370*A-71955*7^
(1/2)*A-178893*B-31869*7^(1/2)*B-106106*C+23072*7^(1/2)*C)*(1+7^(1/2)+3*x)
*((3*x^2+2)/(1+7^(1/2)+3*x)^2)^(1/2)*(9*x^3+3*x^2+6*x+2)^(3/2)*InverseJaco
biAM(2*arctan(1/7*(1+3*x)^(1/2)*7^(3/4)),1/14*(98+14*7^(1/2))^(1/2))*3^(1/
2)*7^(1/4)/(1+3*x)^(3/2)/(3*x^2+2)^2
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.33 (sec) , antiderivative size = 317, normalized size of antiderivative = 0.50

$$\int (A+Bx+Cx^2)\left(2+6x+3x^2+9x^3\right)^{3/2}\,dx = \frac{2(2+3x^2)\left((1+3x)\sqrt{6+9x^2}(1755A(490+1170x+2115x^2+756x^3+1701x^4)+9B(349\cdots\right.}{}$$

input
```
Integrate[(A + B*x + C*x^2)*(2 + 6*x + 3*x^2 + 9*x^3)^(3/2),x]
```

output
```
(2*(2 + 3*x^2)*((1 + 3*x)*Sqrt[6 + 9*x^2]*(1755*A*(490 + 1170*x + 2115*x^2
 + 756*x^3 + 1701*x^4) + 9*B*(34916 + 36954*x + 153000*x^2 + 312795*x^3 +
119070*x^4 + 280665*x^5) + 7*C*(-21832 + 16920*x + 23778*x^2 + 146502*x^3
+ 322947*x^4 + 128304*x^5 + 312741*x^6)) + ((1144*I)*(4590*A - 1251*B - 74
2*C)*(1 + 3*x)*EllipticE[ArcSin[Sqrt[Sqrt[6] - (3*I)*x]/(2^(3/4)*3^(1/4))]
, (2*Sqrt[6])/(I + Sqrt[6])])/Sqrt[(I*(1 + 3*x))/(I + Sqrt[6])] - (56*I)*(
71955*A + 31869*B - 23072*C)*Sqrt[(I*(1 + 3*x))/(I + Sqrt[6])]*EllipticF[A
rcSin[Sqrt[Sqrt[6] - (3*I)*x]/(2^(3/4)*3^(1/4))], (2*Sqrt[6])/(I + Sqrt[6]
)]))/(3648645*Sqrt[6 + 9*x^2]*Sqrt[2 + 6*x + 3*x^2 + 9*x^3])
```

Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 16.74 (sec) , antiderivative size = 2883, normalized size of antiderivative = 4.53, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.567$, Rules used = {2526, 27, 2490, 2486, 27, 1236, 27, 1236, 27, 1231, 27, 1231, 27, 1269, 1172, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(9x^3+3x^2+6x+2\right)^{3/2} \left(A+Bx+Cx^2\right) dx$$

$$\Big\downarrow \text{2526}$$

$$\frac{1}{27}\int 3(9A-2C+(9B-2C)x)\left(9x^3+3x^2+6x+2\right)^{3/2}dx+\frac{2}{135}C\left(9x^3+3x^2+6x+2\right)^{5/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{9}\int (9A-2C+(9B-2C)x)\left(9x^3+3x^2+6x+2\right)^{3/2}dx+\frac{2}{135}C\left(9x^3+3x^2+6x+2\right)^{5/2}$$

$$\Big\downarrow \text{2490}$$

$$\frac{1}{9}\int \left(\frac{1}{27}(27(9A-2C)-3(9B-2C))+(9B-2C)\left(x+\frac{1}{9}\right)\right)\left(9\left(x+\frac{1}{9}\right)^3+\frac{17}{3}\left(x+\frac{1}{9}\right)+\frac{110}{81}\right)^{3/2}d\left(x+\right.$$
$$\frac{2}{135}C\left(9x^3+3x^2+6x+2\right)^{5/2}$$

$$\Big\downarrow \text{2486}$$

$$\frac{\left(729\left(x+\frac{1}{9}\right)^3+459\left(x+\frac{1}{9}\right)+110\right)^{3/2}\int \frac{1}{9}\left(9\left(x+\frac{1}{9}\right)+\frac{17-\left(-55+63\sqrt{2}\right)^{2/3}}{\sqrt[3]{-55+63\sqrt{2}}}\right)^{3/2}\left(81A-9B-16C+9(9B-2C)\right.}{6561\left(9\left(x+\frac{1}{9}\right)+\frac{17-\left(63\sqrt{2}-55\right)^{2/3}}{\sqrt[3]{63\sqrt{2}-55}}\right)^{3/2}\left(81\left(x+\frac{1}{9}\right)^2-\frac{9\left(17-\left(6\right.\right.}{\sqrt[3]{}}\right.}$$

$$\frac{2}{135}C\left(9x^3+3x^2+6x+2\right)^{5/2}$$

$\downarrow$ 27

$$\frac{\left(729\left(x+\frac{1}{9}\right)^3+459\left(x+\frac{1}{9}\right)+110\right)^{3/2}\int \left(9\left(x+\frac{1}{9}\right)+\frac{17-\left(-55+63\sqrt{2}\right)^{2/3}}{\sqrt[3]{-55+63\sqrt{2}}}\right)^{3/2}\left(81A-9B-16C+9(9B-2C)\right)(}{59049\left(9\left(x+\frac{1}{9}\right)+\frac{17-\left(63\sqrt{2}-55\right)^{2/3}}{\sqrt[3]{63\sqrt{2}-55}}\right)^{3/2}\left(81\left(x+\frac{1}{9}\right)^2-\frac{9\left(17-\left(6\right.\right.}{\sqrt[3]{}}\right.}$$

$$\frac{2}{135}C\left(9x^3+3x^2+6x+2\right)^{5/2}$$

$\downarrow$ 1236

$$\frac{\left(729\left(x+\frac{1}{9}\right)^3+459\left(x+\frac{1}{9}\right)+110\right)^{3/2}\left(\frac{2\int \frac{81}{2}\sqrt{9\left(x+\frac{1}{9}\right)+\frac{17-\left(-55+63\sqrt{2}\right)^{2/3}}{\sqrt[3]{-55+63\sqrt{2}}}}\left(\frac{1053\left(55-63\sqrt{2}+17\sqrt[3]{-55+63\sqrt{2}}\right)A-9\left(137-\right.}{}\right.}{}\right.}{}$$

$$\frac{2}{135}C\left(9x^3+3x^2+6x+2\right)^{5/2}$$

$\downarrow$ 27

$$\left(729\left(x+\frac{1}{9}\right)^3+459\left(x+\frac{1}{9}\right)+110\right)^{3/2}\left(\frac{1}{13}\int\sqrt{9\left(x+\frac{1}{9}\right)+\frac{17-\left(-55+63\sqrt{2}\right)^{2/3}}{\sqrt[3]{-55+63\sqrt{2}}}}\left(\frac{1053\left(55-63\sqrt{2}+17\sqrt[3]{-55+63\sqrt{2}}\right.}{}\right.\right.$$

$$\frac{2}{135}C\left(9x^3+3x^2+6x+2\right)^{5/2}$$

$\downarrow$ 1236

$$\frac{2}{135}C\left(9x^3+3x^2+6x+2\right)^{5/2}+$$

$$\left(729\left(x+\frac{1}{9}\right)^3+459\left(x+\frac{1}{9}\right)+110\right)^{3/2}\left(\frac{2}{117}(9B-2C)\left(9\left(x+\frac{1}{9}\right)+\frac{17-\left(-55+63\sqrt{2}\right)^{2/3}}{\sqrt[3]{-55+63\sqrt{2}}}\right)^{3/2}\left(81\left(x+\frac{1}{9}\right)^2-\frac{9\left(1\right.}{}\right.\right.$$

$\downarrow$ 27

$$\frac{2}{135}C\left(9x^3+3x^2+6x+2\right)^{5/2}+$$

$$\left(729\left(x+\frac{1}{9}\right)^3+459\left(x+\frac{1}{9}\right)+110\right)^{3/2}\left(\frac{2}{117}(9B-2C)\left(9\left(x+\frac{1}{9}\right)+\frac{17-\left(-55+63\sqrt{2}\right)^{2/3}}{\sqrt[3]{-55+63\sqrt{2}}}\right)^{3/2}\left(81\left(x+\frac{1}{9}\right)^2-\frac{9\left(1\right.}{}\right.\right.$$

$\downarrow$ 1231

$$\frac{2}{135}C\left(9x^3+3x^2+6x+2\right)^{5/2}+$$

$$\left(729\left(x+\frac{1}{9}\right)^3+459\left(x+\frac{1}{9}\right)+110\right)^{3/2}\left(\frac{2}{117}(9B-2C)\left(9\left(x+\frac{1}{9}\right)+\frac{17-\left(-55+63\sqrt{2}\right)^{2/3}}{\sqrt[3]{-55+63\sqrt{2}}}\right)^{3/2}\left(81\left(x+\frac{1}{9}\right)^2-\frac{9\left(1\right.}{}\right.\right.$$

$\downarrow$ 27

$$\frac{2}{135}C\left(9x^3+3x^2+6x+2\right)^{5/2}+$$

$$\left(729\left(x+\frac{1}{9}\right)^3+459\left(x+\frac{1}{9}\right)+110\right)^{3/2}\left(\frac{2}{117}(9B-2C)\left(9\left(x+\frac{1}{9}\right)+\frac{17-\left(-55+63\sqrt{2}\right)^{2/3}}{\sqrt[3]{-55+63\sqrt{2}}}\right)^{3/2}\left(81\left(x+\frac{1}{9}\right)^2-\frac{9\left(1\right.}{}\right.\right.$$

$\downarrow$ 1231

$$\frac{2}{135}C\left(9x^3+3x^2+6x+2\right)^{5/2}+$$

$$\left(729\left(x+\tfrac{1}{9}\right)^3+459\left(x+\tfrac{1}{9}\right)+110\right)^{3/2}\left(\tfrac{2}{117}(9B-2C)\left(9\left(x+\tfrac{1}{9}\right)+\frac{17-\left(-55+63\sqrt{2}\right)^{2/3}}{\sqrt[3]{-55+63\sqrt{2}}}\right)^{3/2}\left(81\left(x+\tfrac{1}{9}\right)^2-\frac{9\left(1\right.}{}\right.\right.$$

$\downarrow$ 27

$$\frac{2}{135}C\left(9x^3+3x^2+6x+2\right)^{5/2}+$$

$$\left(729\left(x+\tfrac{1}{9}\right)^3+459\left(x+\tfrac{1}{9}\right)+110\right)^{3/2}\left(\tfrac{2}{117}(9B-2C)\left(9\left(x+\tfrac{1}{9}\right)+\frac{17-\left(-55+63\sqrt{2}\right)^{2/3}}{\sqrt[3]{-55+63\sqrt{2}}}\right)^{3/2}\left(81\left(x+\tfrac{1}{9}\right)^2-\frac{9\left(1\right.}{}\right.\right.$$

$\downarrow$ 1269

$$\frac{2}{135}C\left(9x^3+3x^2+6x+2\right)^{5/2}+$$

$$\left(729\left(x+\tfrac{1}{9}\right)^3+459\left(x+\tfrac{1}{9}\right)+110\right)^{3/2}\left(\tfrac{2}{117}(9B-2C)\left(9\left(x+\tfrac{1}{9}\right)+\frac{17-\left(-55+63\sqrt{2}\right)^{2/3}}{\sqrt[3]{-55+63\sqrt{2}}}\right)^{3/2}\left(81\left(x+\tfrac{1}{9}\right)^2-\frac{9\left(1\right.}{}\right.\right.$$

$\downarrow$ 1172

$$\frac{2}{135}C\left(9x^3+3x^2+6x+2\right)^{5/2}+$$

$$\left(729\left(x+\tfrac{1}{9}\right)^3+459\left(x+\tfrac{1}{9}\right)+110\right)^{3/2}\left(\tfrac{2}{117}(9B-2C)\left(9\left(x+\tfrac{1}{9}\right)+\frac{17-\left(-55+63\sqrt{2}\right)^{2/3}}{\sqrt[3]{-55+63\sqrt{2}}}\right)^{3/2}\left(81\left(x+\tfrac{1}{9}\right)^2-\frac{9\left(1\right.}{}\right.\right.$$

$\downarrow$ 321

$$\frac{2}{135}C\left(9x^3+3x^2+6x+2\right)^{5/2}+$$

$$\left(729\left(x+\tfrac{1}{9}\right)^3+459\left(x+\tfrac{1}{9}\right)+110\right)^{3/2}\left(\tfrac{2}{117}(9B-2C)\left(9\left(x+\tfrac{1}{9}\right)+\frac{17-\left(-55+63\sqrt{2}\right)^{2/3}}{\sqrt[3]{-55+63\sqrt{2}}}\right)^{3/2}\left(81\left(x+\tfrac{1}{9}\right)^2-\frac{9\left(1\ldots\right.}{\ldots}\right.\right.$$

$\downarrow$ 327

$$\frac{2}{135}C\left(9x^3+3x^2+6x+2\right)^{5/2}+$$

$$\left(729\left(x+\tfrac{1}{9}\right)^3+459\left(x+\tfrac{1}{9}\right)+110\right)^{3/2}\left(\tfrac{2}{117}(9B-2C)\left(9\left(x+\tfrac{1}{9}\right)+\frac{17-\left(-55+63\sqrt{2}\right)^{2/3}}{\sqrt[3]{-55+63\sqrt{2}}}\right)^{3/2}\left(81\left(x+\tfrac{1}{9}\right)^2-\frac{9\left(1\ldots\right.}{\ldots}\right.\right.$$

input
```
Int[(A + B*x + C*x^2)*(2 + 6*x + 3*x^2 + 9*x^3)^(3/2),x]
```

output
```
(2*C*(2 + 6*x + 3*x^2 + 9*x^3)^(5/2))/135 + ((110 + 459*(1/9 + x) + 729*(1
/9 + x)^3)^(3/2)*((2*(9*B - 2*C)*((17 - (-55 + 63*Sqrt[2])^(2/3))/(-55 + 6
3*Sqrt[2])^(1/3) + 9*(1/9 + x))^(3/2)*(17 + 289/(-55 + 63*Sqrt[2])^(2/3) +
 (-55 + 63*Sqrt[2])^(2/3) - (9*(17 - (-55 + 63*Sqrt[2])^(2/3))*(1/9 + x))/
(-55 + 63*Sqrt[2])^(1/3) + 81*(1/9 + x)^2)^(5/2))/117 + ((2*(1053*A - (117
 - 1683/(-55 + 63*Sqrt[2])^(1/3) + 99*(-55 + 63*Sqrt[2])^(1/3))*B - 2*(104
 + 187/(-55 + 63*Sqrt[2])^(1/3) - 11*(-55 + 63*Sqrt[2])^(1/3))*C)*Sqrt[(17
 - (-55 + 63*Sqrt[2])^(2/3))/(-55 + 63*Sqrt[2])^(1/3) + 9*(1/9 + x)]*(17 +
 289/(-55 + 63*Sqrt[2])^(2/3) + (-55 + 63*Sqrt[2])^(2/3) - (9*(17 - (-55 +
 63*Sqrt[2])^(2/3))*(1/9 + x))/(-55 + 63*Sqrt[2])^(1/3) + 81*(1/9 + x)^2)^
(5/2))/99 - (27*((-2*Sqrt[(17 - (-55 + 63*Sqrt[2])^(2/3))/(-55 + 63*Sqrt[2
])^(1/3) + 9*(1/9 + x)]*((351*(187*(10963 - 6930*Sqrt[2]) - 7*(10963 - 693
0*Sqrt[2])*(-55 + 63*Sqrt[2])^(2/3) - 2023*(-55 + 63*Sqrt[2])^(4/3))*A - 9
*(2827*(10963 - 6930*Sqrt[2]) + 357*(28149 - 18487*Sqrt[2])*(-55 + 63*Sqrt
[2])^(1/3) + 21*(23019 - 34496*Sqrt[2])*(-55 + 63*Sqrt[2])^(2/3))*B - 28*(
319*(10963 - 6930*Sqrt[2]) - 51*(7997 - 2282*Sqrt[2])*(-55 + 63*Sqrt[2])^(
1/3) - 3*(82769 - 62293*Sqrt[2])*(-55 + 63*Sqrt[2])^(2/3))*C)/(55 - 63*Sqr
t[2])^2 + (63*(702*(55 - 63*Sqrt[2] + 17*(-55 + 63*Sqrt[2])^(1/3))*A + 9*(
583 + 546*Sqrt[2] - (349 - 231*Sqrt[2])*(-55 + 63*Sqrt[2])^(1/3) - 187*(-5
5 + 63*Sqrt[2])^(2/3))*B - 2*(4873 - 4368*Sqrt[2] - 187*(-55 + 63*Sqrt[...
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 1172
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Simp[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m))   Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^
2 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[m^2, 1/4]
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1236
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1
)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m
*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2486
```
Int[((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (d_.)*(x_)^3)^(p_), x_S
ymbol] :> With[{r = Rt[-9*a*d^2 + Sqrt[3]*d*Sqrt[4*b^3*d + 27*a^2*d^2], 3]}
, Simp[(a + b*x + d*x^3)^p/(Simp[18^(1/3)*b*(d/(3*r)) - r/18^(1/3) + d*x, x
]^p*Simp[b*(d/3) + 12^(1/3)*b^2*(d^2/(3*r^2)) + r^2/(3*12^(1/3)) - d*(2^(1/
3)*b*(d/(3^(1/3)*r)) - r/18^(1/3))*x + d^2*x^2, x]^p)   Int[(e + f*x)^m*Sim
p[18^(1/3)*b*(d/(3*r)) - r/18^(1/3) + d*x, x]^p*Simp[b*(d/3) + 12^(1/3)*b^2
*(d^2/(3*r^2)) + r^2/(3*12^(1/3)) - d*(2^(1/3)*b*(d/(3^(1/3)*r)) - r/18^(1/
3))*x + d^2*x^2, x]^p, x], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && NeQ[4*
b^3 + 27*a^2*d, 0] &&  !IntegerQ[p]
```

rule 2490
```
Int[(P3_)^(p_.)*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] :> With[{a = Coeff[P3
, x, 0], b = Coeff[P3, x, 1], c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Su
bst[Int[((3*d*e - c*f)/(3*d) + f*x)^m*Simp[(2*c^3 - 9*b*c*d + 27*a*d^2)/(27
*d^2) - (c^2 - 3*b*d)*(x/(3*d)) + d*x^3, x]^p, x], x, x + c/(3*d)] /; NeQ[c
, 0]] /; FreeQ[{e, f, m, p}, x] && PolyQ[P3, x, 3]
```

rule 2526
```
Int[(Pm_)*(Qn_)^(p_), x_Symbol] :> With[{m = Expon[Pm, x], n = Expon[Qn, x]
}, Simp[Coeff[Pm, x, m]*(Qn^(p + 1)/(n*(p + 1)*Coeff[Qn, x, n])), x] + Simp
[1/(n*Coeff[Qn, x, n])   Int[ExpandToSum[n*Coeff[Qn, x, n]*Pm - Coeff[Pm, x
, m]*D[Qn, x], x]*Qn^p, x], x] /; EqQ[m, n - 1]] /; FreeQ[p, x] && PolyQ[Pm
, x] && PolyQ[Qn, x] && NeQ[p, -1]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.78 (sec) , antiderivative size = 523, normalized size of antiderivative = 0.82

method	result
elliptic	$\frac{6C\,x^{6}\sqrt{9x^{3}+3x^{2}+6x+2}}{5}+\left(\frac{18B}{13}+\frac{32C}{65}\right)x^{5}\sqrt{9x^{3}+3x^{2}+6x+2}+\left(\frac{84B}{143}+\frac{18A}{11}+\frac{886C}{715}\right)x^{4}\sqrt{9x^{3}+3x^{2}}$
risch	$\frac{2(2189187C\,x^{6}+2525985B\,x^{5}+898128x^{5}C+2985255x^{4}A+1071630x^{4}B+2260629C\,x^{4}+1326780x^{3}A+2815155B\,x^{3}+1025514C\,x^{3}}{36}$
default	Expression too large to display

input
```
int((C*x^2+B*x+A)*(9*x^3+3*x^2+6*x+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
6/5*C*x^6*(9*x^3+3*x^2+6*x+2)^(1/2)+(18/13*B+32/65*C)*x^5*(9*x^3+3*x^2+6*x
+2)^(1/2)+(84/143*B+18/11*A+886/715*C)*x^4*(9*x^3+3*x^2+6*x+2)^(1/2)+(662/
429*B+8/11*A+10852/19305*C)*x^3*(9*x^3+3*x^2+6*x+2)^(1/2)+(6800/9009*B+470
/231*A+5284/57915*C)*x^2*(9*x^3+3*x^2+6*x+2)^(1/2)+(8212/45045*B+260/231*A
+752/11583*C)*x*(9*x^3+3*x^2+6*x+2)^(1/2)+(9976/57915*B+140/297*A-43664/52
1235*C)*(9*x^3+3*x^2+6*x+2)^(1/2)+2*(232/693*A-119104/135135*B+21104/17374
5*C)*(-1/3*I*6^(1/2)+1/3)*((x+1/3)/(-1/3*I*6^(1/2)+1/3))^(1/2)*((x-1/3*I*6
^(1/2))/(-1/3-1/3*I*6^(1/2)))^(1/2)*((x+1/3*I*6^(1/2))/(-1/3+1/3*I*6^(1/2)
))^(1/2)/(9*x^3+3*x^2+6*x+2)^(1/2)*EllipticF(((x+1/3)/(-1/3*I*6^(1/2)+1/3)
)^(1/2),((-1/3+1/3*I*6^(1/2))/(-1/3-1/3*I*6^(1/2)))^(1/2))+2*(272/63*A-111
2/945*B-848/1215*C)*(-1/3*I*6^(1/2)+1/3)*((x+1/3)/(-1/3*I*6^(1/2)+1/3))^(1
/2)*((x-1/3*I*6^(1/2))/(-1/3-1/3*I*6^(1/2)))^(1/2)*((x+1/3*I*6^(1/2))/(-1/
3+1/3*I*6^(1/2)))^(1/2)/(9*x^3+3*x^2+6*x+2)^(1/2)*((-1/3-1/3*I*6^(1/2))*El
lipticE(((x+1/3)/(-1/3*I*6^(1/2)+1/3))^(1/2),((-1/3+1/3*I*6^(1/2))/(-1/3-1
/3*I*6^(1/2)))^(1/2))+1/3*I*6^(1/2)*EllipticF(((x+1/3)/(-1/3*I*6^(1/2)+1/3
))^(1/2),((-1/3+1/3*I*6^(1/2))/(-1/3-1/3*I*6^(1/2)))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.23

$$\int \left(A+Bx+Cx^2\right)\left(2+6x+3x^2+9x^3\right)^{3/2}\,dx =$$
$$-\frac{16}{6567561}\left(39663\,A+205407\,B-54460\,C\right)\text{weierstrassPInverse}\left(-\frac{68}{27},-\frac{440}{729},x+\frac{1}{9}\right)$$
$$-\frac{16}{25515}\left(4590\,A-1251\,B-742\,C\right)\text{weierstrassZeta}\left(-\frac{68}{27},\right.$$
$$\left.-\frac{440}{729},\text{weierstrassPInverse}\left(-\frac{68}{27},-\frac{440}{729},x+\frac{1}{9}\right)\right)$$
$$+\frac{2}{3648645}\left(2189187\,Cx^6+56133\,(45\,B+16\,C)x^5+5103\,(585\,A+210\,B+443\,C)x^4+189\,(7020\,A+148\right.$$

input
```
integrate((C*x^2+B*x+A)*(9*x^3+3*x^2+6*x+2)^(3/2),x, algorithm="fricas")
```

output
```
-16/6567561*(39663*A + 205407*B - 54460*C)*weierstrassPInverse(-68/27, -44
0/729, x + 1/9) - 16/25515*(4590*A - 1251*B - 742*C)*weierstrassZeta(-68/2
7, -440/729, weierstrassPInverse(-68/27, -440/729, x + 1/9)) + 2/3648645*(
2189187*C*x^6 + 56133*(45*B + 16*C)*x^5 + 5103*(585*A + 210*B + 443*C)*x^4
 + 189*(7020*A + 14895*B + 5426*C)*x^3 + 9*(412425*A + 153000*B + 18494*C)
*x^2 + 18*(114075*A + 18477*B + 6580*C)*x + 859950*A + 314244*B - 152824*C
)*sqrt(9*x^3 + 3*x^2 + 6*x + 2)
```

Sympy [F]

$$\int \left(A+Bx+Cx^2\right)\left(2+6x+3x^2+9x^3\right)^{3/2}\,dx = \int \left((3x+1)\left(3x^2+2\right)\right)^{\frac{3}{2}}\left(A+Bx+Cx^2\right)\,dx$$

input
```
integrate((C*x**2+B*x+A)*(9*x**3+3*x**2+6*x+2)**(3/2),x)
```

output
```
Integral(((3*x + 1)*(3*x**2 + 2))**(3/2)*(A + B*x + C*x**2), x)
```

Maxima [F]

$$\int \left(A+Bx+Cx^2\right) \left(2+6x+3x^2+9x^3\right)^{3/2} dx = \int \left(Cx^2+Bx+A\right) \left(9\,x^3+3\,x^2+6\,x+2\right)^{\frac{3}{2}} dx$$

input
```
integrate((C*x^2+B*x+A)*(9*x^3+3*x^2+6*x+2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((C*x^2 + B*x + A)*(9*x^3 + 3*x^2 + 6*x + 2)^(3/2), x)
```

Giac [F]

$$\int \left(A+Bx+Cx^2\right) \left(2+6x+3x^2+9x^3\right)^{3/2} dx = \int \left(Cx^2+Bx+A\right) \left(9\,x^3+3\,x^2+6\,x+2\right)^{\frac{3}{2}} dx$$

input
```
integrate((C*x^2+B*x+A)*(9*x^3+3*x^2+6*x+2)^(3/2),x, algorithm="giac")
```

output
```
integrate((C*x^2 + B*x + A)*(9*x^3 + 3*x^2 + 6*x + 2)^(3/2), x)
```

Mupad [B] (verification not implemented)

Time = 14.63 (sec) , antiderivative size = 2455, normalized size of antiderivative = 3.86

$$\int \left(A+Bx+Cx^2\right) \left(2+6x+3x^2+9x^3\right)^{3/2} dx = \text{Too large to display}$$

input
```
int((A + B*x + C*x^2)*(6*x + 3*x^2 + 9*x^3 + 2)^(3/2),x)
```

output
```
(280*A)/(297*(6*x + 3*x^2 + 9*x^3 + 2)^(1/2)) + (19952*B)/(57915*(6*x + 3*
x^2 + 9*x^3 + 2)^(1/2)) - (87328*C)/(521235*(6*x + 3*x^2 + 9*x^3 + 2)^(1/2
)) + (320*A*x)/(63*(6*x + 3*x^2 + 9*x^3 + 2)^(1/2)) + (17176*B*x)/(12285*(
6*x + 3*x^2 + 9*x^3 + 2)^(1/2)) - (5888*C*x)/(15795*(6*x + 3*x^2 + 9*x^3 +
 2)^(1/2)) + (8480*A*x^2)/(693*(6*x + 3*x^2 + 9*x^3 + 2)^(1/2)) + (4916*A*
x^3)/(231*(6*x + 3*x^2 + 9*x^3 + 2)^(1/2)) + (1838*A*x^4)/(77*(6*x + 3*x^2
 + 9*x^3 + 2)^(1/2)) + (2334*A*x^5)/(77*(6*x + 3*x^2 + 9*x^3 + 2)^(1/2)) +
 (126*A*x^6)/(11*(6*x + 3*x^2 + 9*x^3 + 2)^(1/2)) + (162*A*x^7)/(11*(6*x +
 3*x^2 + 9*x^3 + 2)^(1/2)) + (421648*B*x^2)/(135135*(6*x + 3*x^2 + 9*x^3 +
 2)^(1/2)) + (437488*B*x^3)/(45045*(6*x + 3*x^2 + 9*x^3 + 2)^(1/2)) + (215
296*B*x^4)/(15015*(6*x + 3*x^2 + 9*x^3 + 2)^(1/2)) + (17734*B*x^5)/(1001*(
6*x + 3*x^2 + 9*x^3 + 2)^(1/2)) + (3426*B*x^6)/(143*(6*x + 3*x^2 + 9*x^3 +
 2)^(1/2)) + (1350*B*x^7)/(143*(6*x + 3*x^2 + 9*x^3 + 2)^(1/2)) + (162*B*x
^8)/(13*(6*x + 3*x^2 + 9*x^3 + 2)^(1/2)) + (11144*C*x^2)/(34749*(6*x + 3*x
^2 + 9*x^3 + 2)^(1/2)) + (64432*C*x^3)/(57915*(6*x + 3*x^2 + 9*x^3 + 2)^(1
/2)) + (25904*C*x^4)/(3861*(6*x + 3*x^2 + 9*x^3 + 2)^(1/2)) + (70316*C*x^5
)/(6435*(6*x + 3*x^2 + 9*x^3 + 2)^(1/2)) + (6062*C*x^6)/(429*(6*x + 3*x^2
+ 9*x^3 + 2)^(1/2)) + (14178*C*x^7)/(715*(6*x + 3*x^2 + 9*x^3 + 2)^(1/2))
+ (522*C*x^8)/(65*(6*x + 3*x^2 + 9*x^3 + 2)^(1/2)) + (54*C*x^9)/(5*(6*x +
3*x^2 + 9*x^3 + 2)^(1/2)) - (544*A*ellipticE(asin((x/((2^(1/2)*3^(1/2)*...
```

Reduce [F]

$$\int (A+Bx+Cx^2)\left(2+6x+3x^2+9x^3\right)^{3/2}dx = \frac{18\sqrt{9x^3+3x^2+6x+2}\,a\,x^4}{11} + \frac{8\sqrt{9x^3+3x^2+6x+2}\,a\,x^3}{11} + \frac{470\sqrt{9x^3+3x^2+6x+2}\,a\,x^2}{231} + \frac{260\sqrt{9x^3+3x^2+6x+2}\,ax}{231} + \frac{1324\sqrt{9x^3+3x^2+6x+2}\,a}{693} + \frac{18\sqrt{9x^3+3x^2+6x+2}\,b\,x^5}{13} + \frac{84\sqrt{9x^3+3x^2+6x+2}\,b\,x^4}{143} + \frac{662\sqrt{9x^3+3x^2+6x+2}\,b\,x^3}{429} + \frac{6800\sqrt{9x^3+3x^2+6x+2}\,b\,x^2}{9009} + \frac{8212\sqrt{9x^3+3x^2+6x+2}\,bx}{45045} - \frac{29728\sqrt{9x^3+3x^2+6x+2}\,b}{135135} + \frac{6\sqrt{9x^3+3x^2+6x+2}\,c\,x^6}{5} + \frac{32\sqrt{9x^3+3x^2+6x+2}\,c\,x^5}{65} + \frac{886\sqrt{9x^3+3x^2+6x+2}\,c\,x^4}{715} + \frac{10852\sqrt{9x^3+3x^2+6x+2}\,c\,x^3}{19305} + \frac{5284\sqrt{9x^3+3x^2+6x+2}\,c\,x^2}{57915} + \frac{752\sqrt{9x^3+3x^2+6x+2}\,cx}{11583} - \frac{54976\sqrt{9x^3+3x^2+6x+2}\,c}{173745} - \frac{920\left(\int \frac{\sqrt{9x^3+3x^2+6x+2}}{9x^3+3x^2+6x+2}dx\right)a}{231} + \frac{13304\left(\int \frac{\sqrt{9x^3+3x^2+6x+2}}{9x^3+3x^2+6x+2}dx\right)b}{45045} + \frac{47456\left(\int \frac{\sqrt{9x^3+3x^2+6x+2}}{9x^3+3x^2+6x+2}dx\right)c}{57915} - \frac{136\left(\int \frac{\sqrt{9x^3+3x^2+6x+2}\,x^2}{9x^3+3x^2+6x+2}dx\right)a}{7} + \frac{556\left(\int \frac{\sqrt{9x^3+3x^2+6x+2}\,x^2}{9x^3+3x^2+6x+2}dx\right)b}{105} + \frac{424\left(\int \frac{\sqrt{9x^3+3x^2+6x+2}\,x^2}{9x^3+3x^2+6x+2}dx\right)c}{135}$$

input
```
int((C*x^2+B*x+A)*(9*x^3+3*x^2+6*x+2)^(3/2),x)
```

output
```
(2*(995085*sqrt(9*x**3 + 3*x**2 + 6*x + 2)*a*x**4 + 442260*sqrt(9*x**3 + 3
*x**2 + 6*x + 2)*a*x**3 + 1237275*sqrt(9*x**3 + 3*x**2 + 6*x + 2)*a*x**2 +
 684450*sqrt(9*x**3 + 3*x**2 + 6*x + 2)*a*x + 1161810*sqrt(9*x**3 + 3*x**2
 + 6*x + 2)*a + 841995*sqrt(9*x**3 + 3*x**2 + 6*x + 2)*b*x**5 + 357210*sqr
t(9*x**3 + 3*x**2 + 6*x + 2)*b*x**4 + 938385*sqrt(9*x**3 + 3*x**2 + 6*x +
2)*b*x**3 + 459000*sqrt(9*x**3 + 3*x**2 + 6*x + 2)*b*x**2 + 110862*sqrt(9*
x**3 + 3*x**2 + 6*x + 2)*b*x - 133776*sqrt(9*x**3 + 3*x**2 + 6*x + 2)*b +
729729*sqrt(9*x**3 + 3*x**2 + 6*x + 2)*c*x**6 + 299376*sqrt(9*x**3 + 3*x**
2 + 6*x + 2)*c*x**5 + 753543*sqrt(9*x**3 + 3*x**2 + 6*x + 2)*c*x**4 + 3418
38*sqrt(9*x**3 + 3*x**2 + 6*x + 2)*c*x**3 + 55482*sqrt(9*x**3 + 3*x**2 + 6
*x + 2)*c*x**2 + 39480*sqrt(9*x**3 + 3*x**2 + 6*x + 2)*c*x - 192416*sqrt(9
*x**3 + 3*x**2 + 6*x + 2)*c - 2421900*int(sqrt(9*x**3 + 3*x**2 + 6*x + 2)/
(9*x**3 + 3*x**2 + 6*x + 2),x)*a + 179604*int(sqrt(9*x**3 + 3*x**2 + 6*x +
 2)/(9*x**3 + 3*x**2 + 6*x + 2),x)*b + 498288*int(sqrt(9*x**3 + 3*x**2 + 6
*x + 2)/(9*x**3 + 3*x**2 + 6*x + 2),x)*c - 11814660*int((sqrt(9*x**3 + 3*x
**2 + 6*x + 2)*x**2)/(9*x**3 + 3*x**2 + 6*x + 2),x)*a + 3220074*int((sqrt(
9*x**3 + 3*x**2 + 6*x + 2)*x**2)/(9*x**3 + 3*x**2 + 6*x + 2),x)*b + 190990
8*int((sqrt(9*x**3 + 3*x**2 + 6*x + 2)*x**2)/(9*x**3 + 3*x**2 + 6*x + 2),x
)*c))/1216215
```

3.93 $\int (A+Bx+Cx^2)\sqrt{2+6x+3x^2+9x^3}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 459

$$\int (A+Bx+Cx^2)\sqrt{2+6x+3x^2+9x^3}\,dx = \frac{2(189A-306B+26C)\sqrt{2+6x+3x^2+9x^3}}{8505} + \frac{2}{945}(189A+9B-44C)x\sqrt{2+6x+3x^2+9x^3} + \frac{4(3213A+468B-818C)\sqrt{2+6x+3x^2+9x^3}}{8505\left(1+\sqrt{7}+3x\right)} + \frac{2}{189}(9B-2C)\left(2+3x^2\right)\sqrt{2+6x+3x^2+9x^3} + \frac{2}{81}C(1+3x)\left(2+3x^2\right)\sqrt{2+6x+3x^2+9x^3} - \frac{4(3213A+468B-818C)\left(1+\sqrt{7}+3x\right)\sqrt{\frac{2+3x^2}{\left(1+\sqrt{7}+3x\right)^2}}\sqrt{2+6x+3x^2+9x^3}E\left(2\arctan\left(\frac{\sqrt{1+3x}}{\sqrt[4]{7}}\right)|\frac{1}{14}\right.}{1215\sqrt{3}7^{3/4}\sqrt{1+3x}\left(2+3x^2\right)} + \frac{2\left(189\left(17+\sqrt{7}\right)A+468B-306\sqrt{7}B-818C+26\sqrt{7}C\right)\left(1+\sqrt{7}+3x\right)\sqrt{\frac{2+3x^2}{\left(1+\sqrt{7}+3x\right)^2}}\sqrt{2+6x+3x}}{1215\sqrt{3}7^{3/4}\sqrt{1+3x}\left(2+3x^2\right)}$$

output
```
2/8505*(189*A-306*B+26*C)*(9*x^3+3*x^2+6*x+2)^(1/2)+2/945*(189*A+9*B-44*C)
*x*(9*x^3+3*x^2+6*x+2)^(1/2)+4*(3213*A+468*B-818*C)*(9*x^3+3*x^2+6*x+2)^(1
/2)/(8505+8505*7^(1/2)+25515*x)+2/189*(9*B-2*C)*(3*x^2+2)*(9*x^3+3*x^2+6*x
+2)^(1/2)+2/81*C*(1+3*x)*(3*x^2+2)*(9*x^3+3*x^2+6*x+2)^(1/2)-4/25515*(3213
*A+468*B-818*C)*(1+7^(1/2)+3*x)*((3*x^2+2)/(1+7^(1/2)+3*x)^2)^(1/2)*(9*x^3
+3*x^2+6*x+2)^(1/2)*EllipticE(sin(2*arctan(1/7*(1+3*x)^(1/2)*7^(3/4))),1/1
4*(98+14*7^(1/2))^(1/2))*3^(1/2)*7^(1/4)/(1+3*x)^(1/2)/(3*x^2+2)+2/25515*(
189*(17+7^(1/2))*A+468*B-306*7^(1/2)*B-818*C+26*7^(1/2)*C)*(1+7^(1/2)+3*x)
*((3*x^2+2)/(1+7^(1/2)+3*x)^2)^(1/2)*(9*x^3+3*x^2+6*x+2)^(1/2)*InverseJaco
biAM(2*arctan(1/7*(1+3*x)^(1/2)*7^(3/4)),1/14*(98+14*7^(1/2))^(1/2))*3^(1/
2)*7^(1/4)/(1+3*x)^(1/2)/(3*x^2+2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 7.14 (sec) , antiderivative size = 271, normalized size of antiderivative = 0.59

$$\int (A+Bx+Cx^2)\sqrt{2+6x+3x^2+9x^3}\,dx$$

$$= \frac{2(2+3x^2)\left((1+3x)\sqrt{6+9x^2}(189A(1+9x)+9B(56+9x+135x^2)+C(56+234x+45x^2+945x^3))\right.}{8505\sqrt{}}$$

input
```
Integrate[(A + B*x + C*x^2)*Sqrt[2 + 6*x + 3*x^2 + 9*x^3],x]
```

output
```
(2*(2 + 3*x^2)*((1 + 3*x)*Sqrt[6 + 9*x^2]*(189*A*(1 + 9*x) + 9*B*(56 + 9*x
 + 135*x^2) + C*(56 + 234*x + 45*x^2 + 945*x^3)) + ((2*I)*(3213*A + 468*B
- 818*C)*(1 + 3*x)*EllipticE[ArcSin[Sqrt[Sqrt[6] - (3*I)*x]/(2^(3/4)*3^(1/
4))], (2*Sqrt[6])/(I + Sqrt[6])])/Sqrt[(I*(1 + 3*x))/(I + Sqrt[6])] + (14*
I)*(189*A - 306*B + 26*C)*Sqrt[(I*(1 + 3*x))/(I + Sqrt[6])]*EllipticF[ArcS
in[Sqrt[Sqrt[6] - (3*I)*x]/(2^(3/4)*3^(1/4))], (2*Sqrt[6])/(I + Sqrt[6])])
)/(8505*Sqrt[6 + 9*x^2]*Sqrt[2 + 6*x + 3*x^2 + 9*x^3])
```

Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 4.24 (sec) , antiderivative size = 1282, normalized size of antiderivative = 2.79, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.433, Rules used = {2526, 27, 2490, 2486, 27, 1236, 27, 1231, 27, 1269, 1172, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{9x^3+3x^2+6x+2}\left(A+Bx+Cx^2\right)\,dx$$

$$\Big\downarrow \text{2526}$$

$$\frac{1}{27}\int 3(9A-2C+(9B-2C)x)\sqrt{9x^3+3x^2+6x+2}dx+\frac{2}{81}C\left(9x^3+3x^2+6x+2\right)^{3/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{9}\int (9A-2C+(9B-2C)x)\sqrt{9x^3+3x^2+6x+2}dx+\frac{2}{81}C\left(9x^3+3x^2+6x+2\right)^{3/2}$$

$$\Big\downarrow \text{2490}$$

$$\frac{1}{9}\int \left(\frac{1}{27}(27(9A-2C)-3(9B-2C))+(9B-2C)\left(x+\frac{1}{9}\right)\right)\sqrt{9\left(x+\frac{1}{9}\right)^3+\frac{17}{3}\left(x+\frac{1}{9}\right)+\frac{110}{81}}d\left(x+\frac{1}{9}\right)$$
$$\frac{2}{81}C\left(9x^3+3x^2+6x+2\right)^{3/2}$$

$$\Big\downarrow \text{2486}$$

$$\frac{\sqrt{729\left(x+\frac{1}{9}\right)^3+459\left(x+\frac{1}{9}\right)+110}\int \frac{1}{9}\sqrt{9\left(x+\frac{1}{9}\right)+\frac{17-\left(-55+63\sqrt{2}\right)^{2/3}}{\sqrt[3]{-55+63\sqrt{2}}}}\left(81A-9B-16C+9(9B-2C)\left(x+\frac{1}{9}\right.\right.}{81\sqrt{9\left(x+\frac{1}{9}\right)+\frac{17-\left(63\sqrt{2}-55\right)^{2/3}}{\sqrt[3]{63\sqrt{2}-55}}}\sqrt{81\left(x+\frac{1}{9}\right)^2-\frac{9\left(17-\left(63\sqrt{2}-55\right.\right.}{\sqrt[3]{63\sqrt{2}}}}}$$
$$\frac{2}{81}C\left(9x^3+3x^2+6x+2\right)^{3/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{729\left(x+\frac{1}{9}\right)^3+459\left(x+\frac{1}{9}\right)+110}\int\sqrt{9\left(x+\frac{1}{9}\right)+\frac{17-\left(-55+63\sqrt{2}\right)^{2/3}}{\sqrt[3]{-55+63\sqrt{2}}}}\left(81A-9B-16C+9(9B-2C)\left(x+\frac{1}{9}\right)\right.}{729\sqrt{9\left(x+\frac{1}{9}\right)+\frac{17-\left(63\sqrt{2}-55\right)^{2/3}}{\sqrt[3]{63\sqrt{2}-55}}}\sqrt{81\left(x+\frac{1}{9}\right)^2-\frac{9\left(17-\left(63\sqrt{2}-55\right.\right.}{\sqrt[3]{63\sqrt{2}}}}}$$

$$\frac{2}{81}C\left(9x^3+3x^2+6x+2\right)^{3/2}$$

$\downarrow$ 1236

$$\frac{\sqrt{729\left(x+\frac{1}{9}\right)^3+459\left(x+\frac{1}{9}\right)+110}\left(\frac{2}{567}\int\frac{81\left(\frac{567\left(55-63\sqrt{2}+17\sqrt[3]{-55+63\sqrt{2}}\right)A+9\left(193+441\sqrt{2}-\left(229-126\sqrt{2}\right)\sqrt[3]{-55+6}\right.}{}\right.}{}\right.}{}$$

$$\frac{2}{81}C\left(9x^3+3x^2+6x+2\right)^{3/2}$$

$\downarrow$ 27

$$\frac{\sqrt{729\left(x+\frac{1}{9}\right)^3+459\left(x+\frac{1}{9}\right)+110}\left(\frac{1}{7}\int\frac{\left(\frac{567\left(55-63\sqrt{2}+17\sqrt[3]{-55+63\sqrt{2}}\right)A+9\left(193+441\sqrt{2}-\left(229-126\sqrt{2}\right)\sqrt[3]{-55+63\sqrt{2}}\right.}{(-}\right.}{}\right.}{}$$

$$\frac{2}{81}C\left(9x^3+3x^2+6x+2\right)^{3/2}$$

$$\Big\downarrow \text{1231}$$

$$\sqrt{729\left(x+\frac{1}{9}\right)^3+459\left(x+\frac{1}{9}\right)+110}\left(\frac{1}{7}\left(-\frac{2\int -\frac{177147\left(5(2079A-1098B-218C)+9(3213A+468B-818C)\left(x+\right.\right.}{\sqrt{9\left(x+\frac{1}{9}\right)+\frac{17-\left(-55+63\sqrt{2}\right)^{2/3}}{\sqrt[3]{-55+63\sqrt{2}}}}\sqrt{81\left(x+\frac{1}{9}\right)^2-\frac{9\left(17-\left(-55+63\sqrt{2}\right)^{2/3}\right)\left(x+\frac{1}{9}\right)}{\sqrt[3]{-55+63\sqrt{2}}}+\left(-55\right.}}}{98415}\right.\right.$$

$$\frac{2}{81}C\left(9x^3+3x^2+6x+2\right)^{3/2}$$

$$\Big\downarrow \text{27}$$

$$\sqrt{729\left(x+\frac{1}{9}\right)^3+459\left(x+\frac{1}{9}\right)+110}\left(\frac{1}{7}\left(\frac{18}{5}\int \frac{5(2079A-1098B-218C)+9(3213A+468B-818C)(x}{\sqrt{9\left(x+\frac{1}{9}\right)+\frac{17-\left(-55+63\sqrt{2}\right)^{2/3}}{\sqrt[3]{-55+63\sqrt{2}}}}\sqrt{81\left(x+\frac{1}{9}\right)^2-\frac{9\left(17-\left(-55+63\sqrt{2}\right)^{2/3}\right)\left(x+\frac{1}{9}\right)}{\sqrt[3]{-55+63\sqrt{2}}}+\left(-5\right.}}\right.\right.$$

$$\frac{2}{81}C\left(9x^3+3x^2+6x+2\right)^{3/2}$$

$$\Big\downarrow \text{1269}$$

$$\frac{2}{81}C\left(9x^3+3x^2+6x+2\right)^{3/2}+$$

$$\sqrt{729\left(x+\frac{1}{9}\right)^3+459\left(x+\frac{1}{9}\right)+110}\left(\frac{2}{63}(9B-2C)\sqrt{9\left(x+\frac{1}{9}\right)+\frac{17-\left(-55+63\sqrt{2}\right)^{2/3}}{\sqrt[3]{-55+63\sqrt{2}}}}\left(81\left(x+\frac{1}{9}\right)^2-\frac{9\left(17-\left(-55+6\right.\right.}{\sqrt[3]{-55}}\right.\right.$$

$$\Big\downarrow \text{1172}$$

$$\frac{2}{81}C\left(9x^3+3x^2+6x+2\right)^{3/2}+$$

$$\sqrt{729\left(x+\tfrac{1}{9}\right)^3+459\left(x+\tfrac{1}{9}\right)+110}\left(\tfrac{2}{63}(9B-2C)\sqrt{9\left(x+\tfrac{1}{9}\right)+\frac{17-\left(-55+63\sqrt{2}\right)^{2/3}}{\sqrt[3]{-55+63\sqrt{2}}}}\left(81\left(x+\tfrac{1}{9}\right)^2-\frac{9\left(17-\left(-55+6\right.\right.}{\sqrt[3]{-55}}\right.\right.$$

↓ 321

$$\frac{2}{81}C\left(9x^3+3x^2+6x+2\right)^{3/2}+$$

$$\sqrt{729\left(x+\tfrac{1}{9}\right)^3+459\left(x+\tfrac{1}{9}\right)+110}\left(\tfrac{2}{63}(9B-2C)\sqrt{9\left(x+\tfrac{1}{9}\right)+\frac{17-\left(-55+63\sqrt{2}\right)^{2/3}}{\sqrt[3]{-55+63\sqrt{2}}}}\left(81\left(x+\tfrac{1}{9}\right)^2-\frac{9\left(17-\left(-55+6\cdots\right.\right.}{\sqrt[3]{-55\cdots}}\right.\right.$$

$\downarrow$ 327

$$\frac{2}{81}C\left(9x^3+3x^2+6x+2\right)^{3/2}+$$

$$\sqrt{729\left(x+\tfrac{1}{9}\right)^3+459\left(x+\tfrac{1}{9}\right)+110}\left(\tfrac{2}{63}(9B-2C)\sqrt{9\left(x+\tfrac{1}{9}\right)+\frac{17-\left(-55+63\sqrt{2}\right)^{2/3}}{\sqrt[3]{-55+63\sqrt{2}}}}\left(81\left(x+\tfrac{1}{9}\right)^2-\frac{9\left(17-\left(-55+6\cdots\right.\right.}{\sqrt[3]{-55\cdots}}\right.\right.$$

input
```
Int[(A + B*x + C*x^2)*Sqrt[2 + 6*x + 3*x^2 + 9*x^3],x]
```

output
```
(2*C*(2 + 6*x + 3*x^2 + 9*x^3)^(3/2))/81 + (Sqrt[110 + 459*(1/9 + x) + 729
*(1/9 + x)^3]*((2*(9*B - 2*C)*Sqrt[(17 - (-55 + 63*Sqrt[2])^(2/3))/(-55 +
63*Sqrt[2])^(1/3) + 9*(1/9 + x)]*(17 + 289/(-55 + 63*Sqrt[2])^(2/3) + (-55
 + 63*Sqrt[2])^(2/3) - (9*(17 - (-55 + 63*Sqrt[2])^(2/3))*(1/9 + x))/(-55
+ 63*Sqrt[2])^(1/3) + 81*(1/9 + x)^2)^(3/2))/63 + ((-2*Sqrt[(17 - (-55 + 6
3*Sqrt[2])^(2/3))/(-55 + 63*Sqrt[2])^(1/3) + 9*(1/9 + x)]*((5*(289 - 17*(-
55 + 63*Sqrt[2])^(2/3) + (-55 + 63*Sqrt[2])^(4/3))*(9*B - 2*C))/(-55 + 63*
Sqrt[2])^(2/3) - 9*(567*A - 9*(7 - 85/(-55 + 63*Sqrt[2])^(1/3) + 5*(-55 +
63*Sqrt[2])^(1/3))*B - 2*(56 + 85/(-55 + 63*Sqrt[2])^(1/3) - 5*(-55 + 63*S
qrt[2])^(1/3))*C)*(1/9 + x))*Sqrt[17 + 289/(-55 + 63*Sqrt[2])^(2/3) + (-55
 + 63*Sqrt[2])^(2/3) - (9*(17 - (-55 + 63*Sqrt[2])^(2/3))*(1/9 + x))/(-55
+ 63*Sqrt[2])^(1/3) + 81*(1/9 + x)^2])/45 + (18*(((I/9)*Sqrt[2]*(3213*A +
468*B - 818*C)*Sqrt[(17 - (-55 + 63*Sqrt[2])^(2/3))/(-55 + 63*Sqrt[2])^(1/
3) + 9*(1/9 + x)]*EllipticE[ArcSin[((-55 + 63*Sqrt[2])^(1/6)*Sqrt[(-I)*(((
-55 + 63*Sqrt[2])^(2/3)*(1 + I*Sqrt[3]) + (17*I)*(I + Sqrt[3]))/(-55 + 63*
Sqrt[2])^(1/3) + 18*(1/9 + x))])/(3^(1/4)*Sqrt[2*(17 + (-55 + 63*Sqrt[2])^
(2/3))])], (2*(17 + (-55 + 63*Sqrt[2])^(2/3)))/(17 + (17*I)*Sqrt[3] + (-55
 + 63*Sqrt[2])^(2/3)*(1 - I*Sqrt[3]))])/((-55 + 63*Sqrt[2])^(1/6)*Sqrt[((-
I)*((17 - (-55 + 63*Sqrt[2])^(2/3))/(-55 + 63*Sqrt[2])^(1/3) + 9*(1/9 + x)
))/((-55 + 63*Sqrt[2])^(2/3)*(3*I - Sqrt[3]) - 17*(3*I + Sqrt[3]))]) - ...
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 1172
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Simp[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m))   Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^
2 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[m^2, 1/4]
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1236
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1
)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m
*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2486
```
Int[((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (d_.)*(x_)^3)^(p_), x_S
ymbol] :> With[{r = Rt[-9*a*d^2 + Sqrt[3]*d*Sqrt[4*b^3*d + 27*a^2*d^2], 3]}
, Simp[(a + b*x + d*x^3)^p/(Simp[18^(1/3)*b*(d/(3*r)) - r/18^(1/3) + d*x, x
]^p*Simp[b*(d/3) + 12^(1/3)*b^2*(d^2/(3*r^2)) + r^2/(3*12^(1/3)) - d*(2^(1/
3)*b*(d/(3^(1/3)*r)) - r/18^(1/3))*x + d^2*x^2, x]^p)   Int[(e + f*x)^m*Sim
p[18^(1/3)*b*(d/(3*r)) - r/18^(1/3) + d*x, x]^p*Simp[b*(d/3) + 12^(1/3)*b^2
*(d^2/(3*r^2)) + r^2/(3*12^(1/3)) - d*(2^(1/3)*b*(d/(3^(1/3)*r)) - r/18^(1/
3))*x + d^2*x^2, x]^p, x], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && NeQ[4*
b^3 + 27*a^2*d, 0] &&  !IntegerQ[p]
```

rule 2490
```
Int[(P3_)^(p_.)*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] :> With[{a = Coeff[P3
, x, 0], b = Coeff[P3, x, 1], c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Su
bst[Int[((3*d*e - c*f)/(3*d) + f*x)^m*Simp[(2*c^3 - 9*b*c*d + 27*a*d^2)/(27
*d^2) - (c^2 - 3*b*d)*(x/(3*d)) + d*x^3, x]^p, x], x, x + c/(3*d)] /; NeQ[c
, 0]] /; FreeQ[{e, f, m, p}, x] && PolyQ[P3, x, 3]
```

rule 2526
```
Int[(Pm_)*(Qn_)^(p_), x_Symbol] :> With[{m = Expon[Pm, x], n = Expon[Qn, x]
}, Simp[Coeff[Pm, x, m]*(Qn^(p + 1)/(n*(p + 1)*Coeff[Qn, x, n])), x] + Simp
[1/(n*Coeff[Qn, x, n])   Int[ExpandToSum[n*Coeff[Qn, x, n]*Pm - Coeff[Pm, x
, m]*D[Qn, x], x]*Qn^p, x], x] /; EqQ[m, n - 1]] /; FreeQ[p, x] && PolyQ[Pm
, x] && PolyQ[Qn, x] && NeQ[p, -1]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.75 (sec) , antiderivative size = 430, normalized size of antiderivative = 0.94

method	result
elliptic	$\frac{2C\,x^3\sqrt{9x^3+3x^2+6x+2}}{9}+\left(\frac{2B}{7}+\frac{2C}{189}\right)x^2\sqrt{9x^3+3x^2+6x+2}+\left(\frac{2B}{105}+\frac{2A}{5}+\frac{52C}{945}\right)x\sqrt{9x^3+3x^2+6x}$
risch	$\frac{2\left(945C\,x^3+1215B\,x^2+45C\,x^2+1701Ax+81Bx+234Cx+189A+504B+56C\right)\sqrt{9x^3+3x^2+6x+2}}{8505}+\frac{4(3213A+468B-818C)\left(-\frac{i}{}\right.}{}$
default	Expression too large to display

input
```
int((C*x^2+B*x+A)*(9*x^3+3*x^2+6*x+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/9*C*x^3*(9*x^3+3*x^2+6*x+2)^(1/2)+(2/7*B+2/189*C)*x^2*(9*x^3+3*x^2+6*x+2
)^(1/2)+(2/105*B+2/5*A+52/945*C)*x*(9*x^3+3*x^2+6*x+2)^(1/2)+(16/135*B+2/4
5*A+16/1215*C)*(9*x^3+3*x^2+6*x+2)^(1/2)+2*(16/15*A-424/2835*C-124/315*B)*
(-1/3*I*6^(1/2)+1/3)*((x+1/3)/(-1/3*I*6^(1/2)+1/3))^(1/2)*((x-1/3*I*6^(1/2
))/(-1/3-1/3*I*6^(1/2)))^(1/2)*((x+1/3*I*6^(1/2))/(-1/3+1/3*I*6^(1/2)))^(1
/2)/(9*x^3+3*x^2+6*x+2)^(1/2)*EllipticF(((x+1/3)/(-1/3*I*6^(1/2)+1/3))^(1/
2),((-1/3+1/3*I*6^(1/2))/(-1/3-1/3*I*6^(1/2)))^(1/2))+2*(104/315*B+34/15*A
-1636/2835*C)*(-1/3*I*6^(1/2)+1/3)*((x+1/3)/(-1/3*I*6^(1/2)+1/3))^(1/2)*((
x-1/3*I*6^(1/2))/(-1/3-1/3*I*6^(1/2)))^(1/2)*((x+1/3*I*6^(1/2))/(-1/3+1/3*
I*6^(1/2)))^(1/2)/(9*x^3+3*x^2+6*x+2)^(1/2)*((-1/3-1/3*I*6^(1/2))*Elliptic
E(((x+1/3)/(-1/3*I*6^(1/2)+1/3))^(1/2),((-1/3+1/3*I*6^(1/2))/(-1/3-1/3*I*6
^(1/2)))^(1/2))+1/3*I*6^(1/2)*EllipticF(((x+1/3)/(-1/3*I*6^(1/2)+1/3))^(1/
2),((-1/3+1/3*I*6^(1/2))/(-1/3-1/3*I*6^(1/2)))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.21

$$\int (A+Bx+Cx^2)\sqrt{2+6x+3x^2+9x^3}\,dx$$
$$= \frac{4}{15309}(2079\,A-1098\,B-218\,C)\text{weierstrassPInverse}\left(-\frac{68}{27},-\frac{440}{729},x+\frac{1}{9}\right)$$
$$-\frac{4}{8505}(3213\,A+468\,B-818\,C)\text{weierstrassZeta}\left(-\frac{68}{27},-\frac{440}{729},\text{weierstrassPInverse}\left(-\frac{68}{27},-\frac{440}{729},x+\frac{1}{9}\right)\right)$$
$$+\frac{2}{8505}\left(945\,Cx^3+45\,(27\,B+C)x^2+9\,(189\,A+9\,B+26\,C)x+189\,A+504\,B+56\,C\right)\sqrt{9\,x^3+3\,x^2+}$$

input
```
integrate((C*x^2+B*x+A)*(9*x^3+3*x^2+6*x+2)^(1/2),x, algorithm="fricas")
```

output
```
4/15309*(2079*A - 1098*B - 218*C)*weierstrassPInverse(-68/27, -440/729, x
+ 1/9) - 4/8505*(3213*A + 468*B - 818*C)*weierstrassZeta(-68/27, -440/729,
 weierstrassPInverse(-68/27, -440/729, x + 1/9)) + 2/8505*(945*C*x^3 + 45*
(27*B + C)*x^2 + 9*(189*A + 9*B + 26*C)*x + 189*A + 504*B + 56*C)*sqrt(9*x
^3 + 3*x^2 + 6*x + 2)
```

Sympy [F]

$$\int (A+Bx+Cx^2)\sqrt{2+6x+3x^2+9x^3}\,dx = \int \sqrt{(3x+1)\,(3x^2+2)}\,(A+Bx+Cx^2)\ dx$$

input
```
integrate((C*x**2+B*x+A)*(9*x**3+3*x**2+6*x+2)**(1/2),x)
```

output
```
Integral(sqrt((3*x + 1)*(3*x**2 + 2))*(A + B*x + C*x**2), x)
```

Maxima [F]

$$\int (A+Bx+Cx^2)\sqrt{2+6x+3x^2+9x^3}\,dx = \int (Cx^2+Bx+A)\sqrt{9\,x^3+3\,x^2+6\,x+2}\,dx$$

input
```
integrate((C*x^2+B*x+A)*(9*x^3+3*x^2+6*x+2)^(1/2),x, algorithm="maxima")
```

output
```
integrate((C*x^2 + B*x + A)*sqrt(9*x^3 + 3*x^2 + 6*x + 2), x)
```

Giac [F]

$$\int (A+Bx+Cx^2)\sqrt{2+6x+3x^2+9x^3}\,dx = \int (Cx^2+Bx+A)\sqrt{9\,x^3+3\,x^2+6\,x+2}\,dx$$

input
```
integrate((C*x^2+B*x+A)*(9*x^3+3*x^2+6*x+2)^(1/2),x, algorithm="giac")
```

output
```
integrate((C*x^2 + B*x + A)*sqrt(9*x^3 + 3*x^2 + 6*x + 2), x)
```

Mupad [B] (verification not implemented)

Time = 12.45 (sec) , antiderivative size = 1846, normalized size of antiderivative = 4.02

$$\int (A+Bx+Cx^2)\sqrt{2+6x+3x^2+9x^3}\,dx = \text{Too large to display}$$

input
```
int((A + B*x + C*x^2)*(6*x + 3*x^2 + 9*x^3 + 2)^(1/2),x)
```

output
```
(((9*B)/7 + (3*C)/7)*((2*x)/3 + x^2/3 + x^3 + 2/9)^(1/2)*(2*x^2*((2*x)/3 +
 x^2/3 + x^3 + 2/9)^(1/2) - (20*((2*x)/3 + x^2/3 + x^3 + 2/9)^(1/2))/9 - (
(4*x)/5 - 16/45)*((2*x)/3 + x^2/3 + x^3 + 2/9)^(1/2) + (16*((6^(1/2)*1i)/3
 + 1/3)*(((6^(1/2)*1i)/3 - 1/3)*ellipticE(asin(((x + 1/3)/((6^(1/2)*1i)/3
+ 1/3))^(1/2)), -((6^(1/2)*1i)/3 + 1/3)/((6^(1/2)*1i)/3 - 1/3)) - (6^(1/2)
*ellipticF(asin(((x + 1/3)/((6^(1/2)*1i)/3 + 1/3))^(1/2)), -((6^(1/2)*1i)/
3 + 1/3)/((6^(1/2)*1i)/3 - 1/3))*1i)/3)*((x + (6^(1/2)*1i)/3)/((6^(1/2)*1i
)/3 - 1/3))^(1/2)*(-(x - (6^(1/2)*1i)/3)/((6^(1/2)*1i)/3 + 1/3))^(1/2)*((x
 + 1/3)/((6^(1/2)*1i)/3 + 1/3))^(1/2))/(15*((2*x)/3 + x^2/3 + x^3 + 2/9)^(
1/2)) + (8*((6^(1/2)*1i)/3 + 1/3)*((x + (6^(1/2)*1i)/3)/((6^(1/2)*1i)/3 -
1/3))^(1/2)*(-(x - (6^(1/2)*1i)/3)/((6^(1/2)*1i)/3 + 1/3))^(1/2)*((x + 1/3
)/((6^(1/2)*1i)/3 + 1/3))^(1/2)*ellipticF(asin(((x + 1/3)/((6^(1/2)*1i)/3
+ 1/3))^(1/2)), -((6^(1/2)*1i)/3 + 1/3)/((6^(1/2)*1i)/3 - 1/3)))/(5*((2*x)
/3 + x^2/3 + x^3 + 2/9)^(1/2))))/(6*x + 3*x^2 + 9*x^3 + 2)^(1/2) - (((4*((
6^(1/2)*1i)/3 + 1/3)*(((6^(1/2)*1i)/3 - 1/3)*ellipticE(asin(((x + 1/3)/((6
^(1/2)*1i)/3 + 1/3))^(1/2)), -((6^(1/2)*1i)/3 + 1/3)/((6^(1/2)*1i)/3 - 1/3
)) - (6^(1/2)*ellipticF(asin(((x + 1/3)/((6^(1/2)*1i)/3 + 1/3))^(1/2)), -(
(6^(1/2)*1i)/3 + 1/3)/((6^(1/2)*1i)/3 - 1/3))*1i)/3)*((x + (6^(1/2)*1i)/3)
/((6^(1/2)*1i)/3 - 1/3))^(1/2)*(-(x - (6^(1/2)*1i)/3)/((6^(1/2)*1i)/3 + 1/
3))^(1/2)*((x + 1/3)/((6^(1/2)*1i)/3 + 1/3))^(1/2))/(9*((2*x)/3 + x^2/3...
```

Reduce [F]

$$\begin{aligned}
&\int \left(A+Bx+Cx^2\right)\sqrt{2+6x+3x^2+9x^3}\,dx \\
&= \frac{2\sqrt{9x^3+3x^2+6x+2}\,ax}{5}+\frac{4\sqrt{9x^3+3x^2+6x+2}\,a}{5}+\frac{2\sqrt{9x^3+3x^2+6x+2}\,b\,x^2}{7} \\
&\quad+\frac{2\sqrt{9x^3+3x^2+6x+2}\,bx}{105}+\frac{8\sqrt{9x^3+3x^2+6x+2}\,b}{35}+\frac{2\sqrt{9x^3+3x^2+6x+2}\,c\,x^3}{9} \\
&\quad+\frac{2\sqrt{9x^3+3x^2+6x+2}\,c\,x^2}{189}+\frac{52\sqrt{9x^3+3x^2+6x+2}\,cx}{945} \\
&\quad-\frac{508\sqrt{9x^3+3x^2+6x+2}\,c}{2835}-\frac{6\left(\int \frac{\sqrt{9x^3+3x^2+6x+2}}{9x^3+3x^2+6x+2}dx\right)a}{5}-\frac{76\left(\int \frac{\sqrt{9x^3+3x^2+6x+2}}{9x^3+3x^2+6x+2}dx\right)b}{105} \\
&\quad+\frac{404\left(\int \frac{\sqrt{9x^3+3x^2+6x+2}}{9x^3+3x^2+6x+2}dx\right)c}{945}-\frac{51\left(\int \frac{\sqrt{9x^3+3x^2+6x+2}\,x^2}{9x^3+3x^2+6x+2}dx\right)a}{5} \\
&\quad-\frac{52\left(\int \frac{\sqrt{9x^3+3x^2+6x+2}\,x^2}{9x^3+3x^2+6x+2}dx\right)b}{35}+\frac{818\left(\int \frac{\sqrt{9x^3+3x^2+6x+2}\,x^2}{9x^3+3x^2+6x+2}dx\right)c}{315}
\end{aligned}$$

input
```
int((C*x^2+B*x+A)*(9*x^3+3*x^2+6*x+2)^(1/2),x)
```

output
```
(1134*sqrt(9*x**3 + 3*x**2 + 6*x + 2)*a*x + 2268*sqrt(9*x**3 + 3*x**2 + 6*
x + 2)*a + 810*sqrt(9*x**3 + 3*x**2 + 6*x + 2)*b*x**2 + 54*sqrt(9*x**3 + 3
*x**2 + 6*x + 2)*b*x + 648*sqrt(9*x**3 + 3*x**2 + 6*x + 2)*b + 630*sqrt(9*
x**3 + 3*x**2 + 6*x + 2)*c*x**3 + 30*sqrt(9*x**3 + 3*x**2 + 6*x + 2)*c*x**
2 + 156*sqrt(9*x**3 + 3*x**2 + 6*x + 2)*c*x - 508*sqrt(9*x**3 + 3*x**2 + 6
*x + 2)*c - 3402*int(sqrt(9*x**3 + 3*x**2 + 6*x + 2)/(9*x**3 + 3*x**2 + 6*
x + 2),x)*a - 2052*int(sqrt(9*x**3 + 3*x**2 + 6*x + 2)/(9*x**3 + 3*x**2 +
6*x + 2),x)*b + 1212*int(sqrt(9*x**3 + 3*x**2 + 6*x + 2)/(9*x**3 + 3*x**2
+ 6*x + 2),x)*c - 28917*int((sqrt(9*x**3 + 3*x**2 + 6*x + 2)*x**2)/(9*x**3
 + 3*x**2 + 6*x + 2),x)*a - 4212*int((sqrt(9*x**3 + 3*x**2 + 6*x + 2)*x**2
)/(9*x**3 + 3*x**2 + 6*x + 2),x)*b + 7362*int((sqrt(9*x**3 + 3*x**2 + 6*x
+ 2)*x**2)/(9*x**3 + 3*x**2 + 6*x + 2),x)*c)/2835
```

3.94 $\int \frac{A+Bx+Cx^2}{\sqrt{2+6x+3x^2+9x^3}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 336

$$\begin{aligned}&\int \frac{A+Bx+Cx^2}{\sqrt{2+6x+3x^2+9x^3}}\,dx\\ &=\frac{2C(1+3x)\left(2+3x^2\right)}{27\sqrt{2+6x+3x^2+9x^3}}+\frac{2(9B-2C)(1+3x)\left(2+3x^2\right)}{27\left(1+\sqrt{7}+3x\right)\sqrt{2+6x+3x^2+9x^3}}\\ &-\frac{2\sqrt[4]{7}(9B-2C)\sqrt{1+3x}\left(1+\sqrt{7}+3x\right)\sqrt{\frac{2+3x^2}{\left(1+\sqrt{7}+3x\right)^2}}E\left(2\arctan\left(\frac{\sqrt{1+3x}}{\sqrt[4]{7}}\right)|\frac{1}{14}\left(7+\sqrt{7}\right)\right)}{27\sqrt{3}\sqrt{2+6x+3x^2+9x^3}}\\ &+\frac{\left(27A-9\left(1-\sqrt{7}\right)B-2\left(2+\sqrt{7}\right)C\right)\sqrt{1+3x}\left(1+\sqrt{7}+3x\right)\sqrt{\frac{2+3x^2}{\left(1+\sqrt{7}+3x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt{1+}}{\sqrt[4]{}}\right.\right.}{27\sqrt{3}\sqrt[4]{7}\sqrt{2+6x+3x^2+9x^3}}\end{aligned}$$

output

```
2/27*C*(1+3*x)*(3*x^2+2)/(9*x^3+3*x^2+6*x+2)^(1/2)+2/27*(9*B-2*C)*(1+3*x)*
(3*x^2+2)/(1+7^(1/2)+3*x)/(9*x^3+3*x^2+6*x+2)^(1/2)-2/81*7^(1/4)*(9*B-2*C)
*(1+3*x)^(1/2)*(1+7^(1/2)+3*x)*((3*x^2+2)/(1+7^(1/2)+3*x)^2)^(1/2)*Ellipti
cE(sin(2*arctan(1/7*(1+3*x)^(1/2)*7^(3/4))),1/14*(98+14*7^(1/2))^(1/2))*3^
(1/2)/(9*x^3+3*x^2+6*x+2)^(1/2)+1/567*(27*A-9*(1-7^(1/2))*B-2*(2+7^(1/2))*
C)*(1+3*x)^(1/2)*(1+7^(1/2)+3*x)*((3*x^2+2)/(1+7^(1/2)+3*x)^2)^(1/2)*Inver
seJacobiAM(2*arctan(1/7*(1+3*x)^(1/2)*7^(3/4)),1/14*(98+14*7^(1/2))^(1/2))
*3^(1/2)*7^(3/4)/(9*x^3+3*x^2+6*x+2)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.33 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.69

$$\int \frac{A+Bx+Cx^2}{\sqrt{2+6x+3x^2+9x^3}}\,dx = \frac{2(2+3x^2)\left(C(1+3x)\sqrt{6+9x^2}+(i+\sqrt{6})(9B-2C)\sqrt{\frac{i(1+3x)}{i+\sqrt{6}}}E\left(\arcsin\left(\frac{\sqrt{\sqrt{6}-3ix}}{2^{3/4}\sqrt[4]{3}}\right)|\frac{2\sqrt{6}}{i+\sqrt{6}}\right)+i(27A}{27\sqrt{6+9x^2}\sqrt{2+6x+3x^2+9x^3}}$$

input
```
Integrate[(A + B*x + C*x^2)/Sqrt[2 + 6*x + 3*x^2 + 9*x^3],x]
```

output
```
(2*(2 + 3*x^2)*(C*(1 + 3*x)*Sqrt[6 + 9*x^2] + (I + Sqrt[6])*(9*B - 2*C)*Sq
rt[(I*(1 + 3*x))/(I + Sqrt[6])]*EllipticE[ArcSin[Sqrt[Sqrt[6] - (3*I)*x]/(
2^(3/4)*3^(1/4))], (2*Sqrt[6])/(I + Sqrt[6])] + I*(27*A - 9*B - 4*C)*Sqrt[
(I*(1 + 3*x))/(I + Sqrt[6])]*EllipticF[ArcSin[Sqrt[Sqrt[6] - (3*I)*x]/(2^(
3/4)*3^(1/4))], (2*Sqrt[6])/(I + Sqrt[6])]))/(27*Sqrt[6 + 9*x^2]*Sqrt[2 +
6*x + 3*x^2 + 9*x^3])
```

Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 2.01 (sec) , antiderivative size = 869, normalized size of antiderivative = 2.59, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {2526, 27, 2490, 2486, 27, 1269, 1172, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{\sqrt{9x^3+3x^2+6x+2}}\,dx$$

$$\Big\downarrow \text{2526}$$

$$\frac{1}{27}\int \frac{3(9A-2C+(9B-2C)x)}{\sqrt{9x^3+3x^2+6x+2}}dx+\frac{2}{27}C\sqrt{9x^3+3x^2+6x+2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{9}\int \frac{9A-2C+(9B-2C)x}{\sqrt{9x^3+3x^2+6x+2}}dx+\frac{2}{27}C\sqrt{9x^3+3x^2+6x+2}$$

$$\Big\downarrow \text{2490}$$

$$\frac{1}{9}\int \frac{\frac{1}{27}(27(9A-2C)-3(9B-2C))+(9B-2C)\left(x+\frac{1}{9}\right)}{\sqrt{9\left(x+\frac{1}{9}\right)^3+\frac{17}{3}\left(x+\frac{1}{9}\right)+\frac{110}{81}}}d\left(x+\frac{1}{9}\right)+\frac{2}{27}C\sqrt{9x^3+3x^2+6x+2}$$

$$\Big\downarrow \text{2486}$$

$$\frac{\sqrt{9\left(x+\frac{1}{9}\right)+\frac{17-\left(63\sqrt{2}-55\right)^{2/3}}{\sqrt[3]{63\sqrt{2}-55}}}\sqrt{81\left(x+\frac{1}{9}\right)^2-\frac{9\left(17-\left(63\sqrt{2}-55\right)^{2/3}\right)\left(x+\frac{1}{9}\right)}{\sqrt[3]{63\sqrt{2}-55}}+\left(63\sqrt{2}-55\right)^{2/3}+\frac{289}{\left(63\sqrt{2}-55\right)^{2/3}}+17}}{\sqrt{729\left(x+\frac{1}{9}\right)^3+459}}$$

$$\frac{2}{27}C\sqrt{9x^3+3x^2+6x+2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{9\left(x+\frac{1}{9}\right)+\frac{17-\left(63\sqrt{2}-55\right)^{2/3}}{\sqrt[3]{63\sqrt{2}-55}}}\sqrt{81\left(x+\frac{1}{9}\right)^2-\frac{9\left(17-\left(63\sqrt{2}-55\right)^{2/3}\right)\left(x+\frac{1}{9}\right)}{\sqrt[3]{63\sqrt{2}-55}}+\left(63\sqrt{2}-55\right)^{2/3}+\frac{289}{\left(63\sqrt{2}-55\right)^{2/3}}+17}}{9\sqrt{729\left(x+\frac{1}{9}\right)^3+459}}$$

$$\frac{2}{27}C\sqrt{9x^3+3x^2+6x+2}$$

$$\Big\downarrow \text{1269}$$

$$\frac{\sqrt{9\left(x+\frac{1}{9}\right)+\frac{17-\left(63\sqrt{2}-55\right)^{2/3}}{\sqrt[3]{63\sqrt{2}-55}}}\sqrt{81\left(x+\frac{1}{9}\right)^2-\frac{9\left(17-\left(63\sqrt{2}-55\right)^{2/3}\right)\left(x+\frac{1}{9}\right)}{\sqrt[3]{63\sqrt{2}-55}}+\left(63\sqrt{2}-55\right)^{2/3}+\frac{289}{\left(63\sqrt{2}-55\right)^{2/3}}+17}}{}$$

$$\frac{2}{27}C\sqrt{9x^3+3x^2+6x+2}$$

$$\Big\downarrow 1172$$

$$\frac{2}{27}\sqrt{9x^3+3x^2+6x+2}C+$$

$$\sqrt{9\left(x+\frac{1}{9}\right)+\frac{17-\left(-55+63\sqrt{2}\right)^{2/3}}{\sqrt[3]{-55+63\sqrt{2}}}}\sqrt{81\left(x+\frac{1}{9}\right)^2-\frac{9\left(17-\left(-55+63\sqrt{2}\right)^{2/3}\right)\left(x+\frac{1}{9}\right)}{\sqrt[3]{-55+63\sqrt{2}}}+\left(-55+63\sqrt{2}\right)^{2/3}+\frac{289}{\left(-55+63\sqrt{2}\right)^{2/}}}$$

$$\Big\downarrow 321$$

$$\frac{2}{27}\sqrt{9x^3+3x^2+6x+2}C+$$

$$\sqrt{9\left(x+\frac{1}{9}\right)+\frac{17-\left(-55+63\sqrt{2}\right)^{2/3}}{\sqrt[3]{-55+63\sqrt{2}}}}\sqrt{81\left(x+\frac{1}{9}\right)^2-\frac{9\left(17-\left(-55+63\sqrt{2}\right)^{2/3}\right)\left(x+\frac{1}{9}\right)}{\sqrt[3]{-55+63\sqrt{2}}}+\left(-55+63\sqrt{2}\right)^{2/3}+\frac{289}{\left(-55+63\sqrt{2}\right)^{2/}}}$$

$$\Big\downarrow \text{327}$$

$$\frac{2}{27}\sqrt{9x^3+3x^2+6x+2}C+$$

$$\sqrt{9\left(x+\frac{1}{9}\right)+\frac{17-\left(-55+63\sqrt{2}\right)^{2/3}}{\sqrt[3]{-55+63\sqrt{2}}}}\sqrt{81\left(x+\frac{1}{9}\right)^2-\frac{9\left(17-\left(-55+63\sqrt{2}\right)^{2/3}\right)\left(x+\frac{1}{9}\right)}{\sqrt[3]{-55+63\sqrt{2}}}+\left(-55+63\sqrt{2}\right)^{2/3}+\frac{289}{\left(-55+63\sqrt{2}\right)^{2/}}}$$

input
```
Int[(A + B*x + C*x^2)/Sqrt[2 + 6*x + 3*x^2 + 9*x^3],x]
```

output
```
(2*C*Sqrt[2 + 6*x + 3*x^2 + 9*x^3])/27 + (Sqrt[(17 - (-55 + 63*Sqrt[2])^(2
/3))/(-55 + 63*Sqrt[2])^(1/3) + 9*(1/9 + x)]*Sqrt[17 + 289/(-55 + 63*Sqrt[
2])^(2/3) + (-55 + 63*Sqrt[2])^(2/3) - (9*(17 - (-55 + 63*Sqrt[2])^(2/3))*
(1/9 + x))/(-55 + 63*Sqrt[2])^(1/3) + 81*(1/9 + x)^2]*(((I/9)*Sqrt[2]*(9*B
 - 2*C)*Sqrt[(17 - (-55 + 63*Sqrt[2])^(2/3))/(-55 + 63*Sqrt[2])^(1/3) + 9*
(1/9 + x)]*EllipticE[ArcSin[((-55 + 63*Sqrt[2])^(1/6)*Sqrt[(-I)*(((-55 + 6
3*Sqrt[2])^(2/3)*(1 + I*Sqrt[3]) + (17*I)*(I + Sqrt[3]))/(-55 + 63*Sqrt[2]
)^(1/3) + 18*(1/9 + x))])/(3^(1/4)*Sqrt[2*(17 + (-55 + 63*Sqrt[2])^(2/3))]
)], (2*(17 + (-55 + 63*Sqrt[2])^(2/3)))/(17 + (17*I)*Sqrt[3] + (-55 + 63*S
qrt[2])^(2/3)*(1 - I*Sqrt[3]))])/((-55 + 63*Sqrt[2])^(1/6)*Sqrt[((-I)*((17
 - (-55 + 63*Sqrt[2])^(2/3))/(-55 + 63*Sqrt[2])^(1/3) + 9*(1/9 + x)))/((-5
5 + 63*Sqrt[2])^(2/3)*(3*I - Sqrt[3]) - 17*(3*I + Sqrt[3]))]) + (((2*I)/9)
*Sqrt[2]*(-55 + 63*Sqrt[2])^(1/6)*(81*A - 9*B - ((17 - (-55 + 63*Sqrt[2])^
(2/3))*(9*B - 2*C))/(-55 + 63*Sqrt[2])^(1/3) - 16*C)*Sqrt[((-I)*((17 - (-5
5 + 63*Sqrt[2])^(2/3))/(-55 + 63*Sqrt[2])^(1/3) + 9*(1/9 + x)))/((-55 + 63
*Sqrt[2])^(2/3)*(3*I - Sqrt[3]) - 17*(3*I + Sqrt[3]))]*EllipticF[ArcSin[((
-55 + 63*Sqrt[2])^(1/6)*Sqrt[(-I)*(((-55 + 63*Sqrt[2])^(2/3)*(1 + I*Sqrt[3
]) + (17*I)*(I + Sqrt[3]))/(-55 + 63*Sqrt[2])^(1/3) + 18*(1/9 + x))])/(3^(
1/4)*Sqrt[2*(17 + (-55 + 63*Sqrt[2])^(2/3))])], (2*(17 + (-55 + 63*Sqrt[2]
)^(2/3)))/(17 + (17*I)*Sqrt[3] + (-55 + 63*Sqrt[2])^(2/3)*(1 - I*Sqrt[3...
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 1172
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Simp[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m))   Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^
2 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[m^2, 1/4]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2486
```
Int[((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (d_.)*(x_)^3)^(p_), x_S
ymbol] :> With[{r = Rt[-9*a*d^2 + Sqrt[3]*d*Sqrt[4*b^3*d + 27*a^2*d^2], 3]}
, Simp[(a + b*x + d*x^3)^p/(Simp[18^(1/3)*b*(d/(3*r)) - r/18^(1/3) + d*x, x
]^p*Simp[b*(d/3) + 12^(1/3)*b^2*(d^2/(3*r^2)) + r^2/(3*12^(1/3)) - d*(2^(1/
3)*b*(d/(3^(1/3)*r)) - r/18^(1/3))*x + d^2*x^2, x]^p)   Int[(e + f*x)^m*Sim
p[18^(1/3)*b*(d/(3*r)) - r/18^(1/3) + d*x, x]^p*Simp[b*(d/3) + 12^(1/3)*b^2
*(d^2/(3*r^2)) + r^2/(3*12^(1/3)) - d*(2^(1/3)*b*(d/(3^(1/3)*r)) - r/18^(1/
3))*x + d^2*x^2, x]^p, x], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && NeQ[4*
b^3 + 27*a^2*d, 0] &&  !IntegerQ[p]
```

rule 2490
```
Int[(P3_)^(p_.)*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] :> With[{a = Coeff[P3
, x, 0], b = Coeff[P3, x, 1], c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Su
bst[Int[((3*d*e - c*f)/(3*d) + f*x)^m*Simp[(2*c^3 - 9*b*c*d + 27*a*d^2)/(27
*d^2) - (c^2 - 3*b*d)*(x/(3*d)) + d*x^3, x]^p, x], x, x + c/(3*d)] /; NeQ[c
, 0]] /; FreeQ[{e, f, m, p}, x] && PolyQ[P3, x, 3]
```

rule 2526
```
Int[(Pm_)*(Qn_)^(p_), x_Symbol] :> With[{m = Expon[Pm, x], n = Expon[Qn, x]
}, Simp[Coeff[Pm, x, m]*(Qn^(p + 1)/(n*(p + 1)*Coeff[Qn, x, n])), x] + Simp
[1/(n*Coeff[Qn, x, n])   Int[ExpandToSum[n*Coeff[Qn, x, n]*Pm - Coeff[Pm, x
, m]*D[Qn, x], x]*Qn^p, x], x] /; EqQ[m, n - 1]] /; FreeQ[p, x] && PolyQ[Pm
, x] && PolyQ[Qn, x] && NeQ[p, -1]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.73 (sec) , antiderivative size = 332, normalized size of antiderivative = 0.99

method	result
elliptic	$\frac{2C\sqrt{9x^3+3x^2+6x+2}}{27}+\frac{2\left(A-\frac{2C}{9}\right)\left(-\frac{i\sqrt{6}}{3}+\frac{1}{3}\right)\sqrt{\frac{x+\frac{1}{3}}{-\frac{i\sqrt{6}}{3}+\frac{1}{3}}}\sqrt{\frac{x-\frac{i\sqrt{6}}{3}}{-\frac{1}{3}-\frac{i\sqrt{6}}{3}}}\sqrt{\frac{x+\frac{i\sqrt{6}}{3}}{-\frac{1}{3}+\frac{i\sqrt{6}}{3}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x+\frac{1}{3}}{-\frac{i\sqrt{6}}{3}+\frac{1}{3}}},\sqrt{\frac{-\frac{1}{3}+\frac{i\sqrt{6}}{3}}{-\frac{1}{3}-\frac{i\sqrt{6}}{3}}}\right)}{\sqrt{9x^3+3x^2+6x+2}}+$
risch	$\frac{2C\sqrt{9x^3+3x^2+6x+2}}{27}+\frac{2(9B-2C)\left(-\frac{i\sqrt{6}}{3}+\frac{1}{3}\right)\sqrt{\frac{x+\frac{1}{3}}{-\frac{i\sqrt{6}}{3}+\frac{1}{3}}}\sqrt{\frac{x-\frac{i\sqrt{6}}{3}}{-\frac{1}{3}-\frac{i\sqrt{6}}{3}}}\sqrt{\frac{x+\frac{i\sqrt{6}}{3}}{-\frac{1}{3}+\frac{i\sqrt{6}}{3}}}\left(\left(-\frac{1}{3}-\frac{i\sqrt{6}}{3}\right)\mathrm{EllipticE}\left(\sqrt{\frac{x+\frac{1}{3}}{-\frac{i\sqrt{6}}{3}+\frac{1}{3}}},\right.\right.}{9\sqrt{9x^3+3x^2+6x+2}}$
default	$\frac{2A\left(-\frac{i\sqrt{6}}{3}+\frac{1}{3}\right)\sqrt{\frac{x+\frac{1}{3}}{-\frac{i\sqrt{6}}{3}+\frac{1}{3}}}\sqrt{\frac{x-\frac{i\sqrt{6}}{3}}{-\frac{1}{3}-\frac{i\sqrt{6}}{3}}}\sqrt{\frac{x+\frac{i\sqrt{6}}{3}}{-\frac{1}{3}+\frac{i\sqrt{6}}{3}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x+\frac{1}{3}}{-\frac{i\sqrt{6}}{3}+\frac{1}{3}}},\sqrt{\frac{-\frac{1}{3}+\frac{i\sqrt{6}}{3}}{-\frac{1}{3}-\frac{i\sqrt{6}}{3}}}\right)}{\sqrt{9x^3+3x^2+6x+2}}+\frac{2B\left(-\frac{i\sqrt{6}}{3}+\frac{1}{3}\right)\sqrt{\frac{x+\frac{1}{3}}{-\frac{i\sqrt{6}}{3}+\frac{1}{3}}}\cdots}{}$

input
```
int((C*x^2+B*x+A)/(9*x^3+3*x^2+6*x+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/27*C*(9*x^3+3*x^2+6*x+2)^(1/2)+2*(A-2/9*C)*(-1/3*I*6^(1/2)+1/3)*((x+1/3)
/(-1/3*I*6^(1/2)+1/3))^(1/2)*((x-1/3*I*6^(1/2))/(-1/3-1/3*I*6^(1/2)))^(1/2
)*((x+1/3*I*6^(1/2))/(-1/3+1/3*I*6^(1/2)))^(1/2)/(9*x^3+3*x^2+6*x+2)^(1/2)
*EllipticF(((x+1/3)/(-1/3*I*6^(1/2)+1/3))^(1/2),((-1/3+1/3*I*6^(1/2))/(-1/
3-1/3*I*6^(1/2)))^(1/2))+2*(B-2/9*C)*(-1/3*I*6^(1/2)+1/3)*((x+1/3)/(-1/3*I
*6^(1/2)+1/3))^(1/2)*((x-1/3*I*6^(1/2))/(-1/3-1/3*I*6^(1/2)))^(1/2)*((x+1/
3*I*6^(1/2))/(-1/3+1/3*I*6^(1/2)))^(1/2)/(9*x^3+3*x^2+6*x+2)^(1/2)*((-1/3-
1/3*I*6^(1/2))*EllipticE(((x+1/3)/(-1/3*I*6^(1/2)+1/3))^(1/2),((-1/3+1/3*I
*6^(1/2))/(-1/3-1/3*I*6^(1/2)))^(1/2))+1/3*I*6^(1/2)*EllipticF(((x+1/3)/(-
1/3*I*6^(1/2)+1/3))^(1/2),((-1/3+1/3*I*6^(1/2))/(-1/3-1/3*I*6^(1/2)))^(1/2
)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.17

$$\int \frac{A+Bx+Cx^2}{\sqrt{2+6x+3x^2+9x^3}}\,dx = \frac{2}{243}\,(81\,A-9\,B-16\,C)\text{weierstrassPInverse}\left(-\frac{68}{27},-\frac{440}{729},x+\frac{1}{9}\right)-\frac{2}{27}\,(9\,B-2\,C)\text{weierstrassZeta}\left(-\frac{68}{27},-\frac{440}{729},\text{weierstrassPInverse}\left(-\frac{68}{27},-\frac{440}{729},x+\frac{1}{9}\right)\right)+\frac{2}{27}\,\sqrt{9\,x^3+3\,x^2+6\,x+2}C$$

input
```
integrate((C*x^2+B*x+A)/(9*x^3+3*x^2+6*x+2)^(1/2),x, algorithm="fricas")
```

output
```
2/243*(81*A - 9*B - 16*C)*weierstrassPInverse(-68/27, -440/729, x + 1/9) -
 2/27*(9*B - 2*C)*weierstrassZeta(-68/27, -440/729, weierstrassPInverse(-6
8/27, -440/729, x + 1/9)) + 2/27*sqrt(9*x^3 + 3*x^2 + 6*x + 2)*C
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt{2+6x+3x^2+9x^3}}\,dx = \int \frac{A+Bx+Cx^2}{\sqrt{(3x+1)\,(3x^2+2)}}\,dx$$

input
```
integrate((C*x**2+B*x+A)/(9*x**3+3*x**2+6*x+2)**(1/2),x)
```

output
```
Integral((A + B*x + C*x**2)/sqrt((3*x + 1)*(3*x**2 + 2)), x)
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt{2+6x+3x^2+9x^3}}\,dx=\int \frac{Cx^2+Bx+A}{\sqrt{9\,x^3+3\,x^2+6\,x+2}}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(9*x^3+3*x^2+6*x+2)^(1/2),x, algorithm="maxima")
```

output
```
integrate((C*x^2 + B*x + A)/sqrt(9*x^3 + 3*x^2 + 6*x + 2), x)
```

Giac [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt{2+6x+3x^2+9x^3}}\,dx=\int \frac{Cx^2+Bx+A}{\sqrt{9\,x^3+3\,x^2+6\,x+2}}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(9*x^3+3*x^2+6*x+2)^(1/2),x, algorithm="giac")
```

output
```
integrate((C*x^2 + B*x + A)/sqrt(9*x^3 + 3*x^2 + 6*x + 2), x)
```

Mupad [B] (verification not implemented)

Time = 12.76 (sec) , antiderivative size = 656, normalized size of antiderivative = 1.95

$$\int \frac{A+Bx+Cx^2}{\sqrt{2+6x+3x^2+9x^3}}\,dx=\text{Too large to display}$$

input
```
int((A + B*x + C*x^2)/(6*x + 3*x^2 + 9*x^3 + 2)^(1/2),x)
```

output
```
(2*A*((6^(1/2)*1i)/3 + 1/3)*((x + (6^(1/2)*1i)/3)/((6^(1/2)*1i)/3 - 1/3))^
(1/2)*(-(x - (6^(1/2)*1i)/3)/((6^(1/2)*1i)/3 + 1/3))^(1/2)*((x + 1/3)/((6^
(1/2)*1i)/3 + 1/3))^(1/2)*ellipticF(asin(((x + 1/3)/((6^(1/2)*1i)/3 + 1/3)
)^(1/2)), -((6^(1/2)*1i)/3 + 1/3)/((6^(1/2)*1i)/3 - 1/3)))/(6*x + 3*x^2 +
9*x^3 + 2)^(1/2) - (C*((4*((6^(1/2)*1i)/3 + 1/3)*(((6^(1/2)*1i)/3 - 1/3)*e
llipticE(asin(((x + 1/3)/((6^(1/2)*1i)/3 + 1/3))^(1/2)), -((6^(1/2)*1i)/3
+ 1/3)/((6^(1/2)*1i)/3 - 1/3)) - (6^(1/2)*ellipticF(asin(((x + 1/3)/((6^(1
/2)*1i)/3 + 1/3))^(1/2)), -((6^(1/2)*1i)/3 + 1/3)/((6^(1/2)*1i)/3 - 1/3))*
1i)/3)*((x + (6^(1/2)*1i)/3)/((6^(1/2)*1i)/3 - 1/3))^(1/2)*(-(x - (6^(1/2)
*1i)/3)/((6^(1/2)*1i)/3 + 1/3))^(1/2)*((x + 1/3)/((6^(1/2)*1i)/3 + 1/3))^(
1/2))/(9*((2*x)/3 + x^2/3 + x^3 + 2/9)^(1/2)) - (2*((2*x)/3 + x^2/3 + x^3
+ 2/9)^(1/2))/3 + (4*((6^(1/2)*1i)/3 + 1/3)*((x + (6^(1/2)*1i)/3)/((6^(1/2
)*1i)/3 - 1/3))^(1/2)*(-(x - (6^(1/2)*1i)/3)/((6^(1/2)*1i)/3 + 1/3))^(1/2)
*((x + 1/3)/((6^(1/2)*1i)/3 + 1/3))^(1/2)*ellipticF(asin(((x + 1/3)/((6^(1
/2)*1i)/3 + 1/3))^(1/2)), -((6^(1/2)*1i)/3 + 1/3)/((6^(1/2)*1i)/3 - 1/3)))
/(9*((2*x)/3 + x^2/3 + x^3 + 2/9)^(1/2)))*((2*x)/3 + x^2/3 + x^3 + 2/9)^(1
/2))/(6*x + 3*x^2 + 9*x^3 + 2)^(1/2) + (2*B*((6^(1/2)*1i)/3 + 1/3)*(((6^(1
/2)*1i)/3 - 1/3)*ellipticE(asin(((x + 1/3)/((6^(1/2)*1i)/3 + 1/3))^(1/2)),
 -((6^(1/2)*1i)/3 + 1/3)/((6^(1/2)*1i)/3 - 1/3)) - (6^(1/2)*ellipticF(asin
(((x + 1/3)/((6^(1/2)*1i)/3 + 1/3))^(1/2)), -((6^(1/2)*1i)/3 + 1/3)/((6...
```

Reduce [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt{2+6x+3x^2+9x^3}}\,dx = \frac{\sqrt{9x^3+3x^2+6x+2}\,b}{3} + \left(\int \frac{\sqrt{9x^3+3x^2+6x+2}}{9x^3+3x^2+6x+2}dx\right)a - \left(\int \frac{\sqrt{9x^3+3x^2+6x+2}}{9x^3+3x^2+6x+2}dx\right)b - \frac{9\left(\int \frac{\sqrt{9x^3+3x^2+6x+2}\,x^2}{9x^3+3x^2+6x+2}dx\right)b}{2} + \left(\int \frac{\sqrt{9x^3+3x^2+6x+2}\,x^2}{9x^3+3x^2+6x+2}dx\right)c$$

input
```
int((C*x^2+B*x+A)/(9*x^3+3*x^2+6*x+2)^(1/2),x)
```

output
```
(2*sqrt(9*x**3 + 3*x**2 + 6*x + 2)*b + 6*int(sqrt(9*x**3 + 3*x**2 + 6*x +
2)/(9*x**3 + 3*x**2 + 6*x + 2),x)*a - 6*int(sqrt(9*x**3 + 3*x**2 + 6*x + 2
)/(9*x**3 + 3*x**2 + 6*x + 2),x)*b - 27*int((sqrt(9*x**3 + 3*x**2 + 6*x +
2)*x**2)/(9*x**3 + 3*x**2 + 6*x + 2),x)*b + 6*int((sqrt(9*x**3 + 3*x**2 +
6*x + 2)*x**2)/(9*x**3 + 3*x**2 + 6*x + 2),x)*c)/6
```

3.95 $\int \frac{A+Bx+Cx^2}{(2+6x+3x^2+9x^3)^{3/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 438

$$\int \frac{A+Bx+Cx^2}{(2+6x+3x^2+9x^3)^{3/2}}\,dx=\frac{(1+3x)(2(3A-B-2C)+(3A+6B-2C)x)\,(2+3x^2)}{42\,(2+6x+3x^2+9x^3)^{3/2}}$$

$$-\frac{(17A-8B-2C)(1+3x)\,(2+3x^2)^2}{98\,(2+6x+3x^2+9x^3)^{3/2}}+\frac{(17A-8B-2C)(1+3x)^2\,(2+3x^2)^2}{98\,(1+\sqrt{7}+3x)\,(2+6x+3x^2+9x^3)^{3/2}}$$

$$-\frac{(17A-8B-2C)(1+3x)^{3/2}\,(1+\sqrt{7}+3x)\,(2+3x^2)\sqrt{\frac{2+3x^2}{(1+\sqrt{7}+3x)^2}}E\left(2\arctan\left(\frac{\sqrt{1+3x}}{\sqrt[4]{7}}\right)|\frac{1}{14}(7+\sqrt{7})\right)}{14\sqrt{3}7^{3/4}\,(2+6x+3x^2+9x^3)^{3/2}}$$

$$+\frac{\left(3(17+\sqrt{7})\,A-6(4-\sqrt{7})\,B-2(3+\sqrt{7})\,C\right)(1+3x)^{3/2}\,(1+\sqrt{7}+3x)\,(2+3x^2)\sqrt{\frac{2+3x^2}{(1+\sqrt{7}+3x)^2}}\text{Ellipt}}{84\sqrt{3}7^{3/4}\,(2+6x+3x^2+9x^3)^{3/2}}$$

output
```
1/42*(1+3*x)*(6*A-2*B-4*C+(3*A+6*B-2*C)*x)*(3*x^2+2)/(9*x^3+3*x^2+6*x+2)^(
3/2)-1/98*(17*A-8*B-2*C)*(1+3*x)*(3*x^2+2)^2/(9*x^3+3*x^2+6*x+2)^(3/2)+1/9
8*(17*A-8*B-2*C)*(1+3*x)^2*(3*x^2+2)^2/(1+7^(1/2)+3*x)/(9*x^3+3*x^2+6*x+2)
^(3/2)-1/294*(17*A-8*B-2*C)*(1+3*x)^(3/2)*(1+7^(1/2)+3*x)*(3*x^2+2)*((3*x^
2+2)/(1+7^(1/2)+3*x)^2)^(1/2)*EllipticE(sin(2*arctan(1/7*(1+3*x)^(1/2)*7^(
3/4))),1/14*(98+14*7^(1/2))^(1/2))*3^(1/2)*7^(1/4)/(9*x^3+3*x^2+6*x+2)^(3/
2)+1/1764*(3*(17+7^(1/2))*A-6*(4-7^(1/2))*B-2*(3+7^(1/2))*C)*(1+3*x)^(3/2)
*(1+7^(1/2)+3*x)*(3*x^2+2)*((3*x^2+2)/(1+7^(1/2)+3*x)^2)^(1/2)*InverseJaco
biAM(2*arctan(1/7*(1+3*x)^(1/2)*7^(3/4)),1/14*(98+14*7^(1/2))^(1/2))*3^(1/
2)*7^(1/4)/(9*x^3+3*x^2+6*x+2)^(3/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.53 (sec) , antiderivative size = 275, normalized size of antiderivative = 0.63

$$\int \frac{A+Bx+Cx^2}{(2+6x+3x^2+9x^3)^{3/2}}\,dx = \frac{\sqrt{6+9x^2}(2C(-8-7x+9x^2)+2B(17+21x+36x^2)-3A(20-7x+5$$

input
```
Integrate[(A + B*x + C*x^2)/(2 + 6*x + 3*x^2 + 9*x^3)^(3/2),x]
```

output
```
(Sqrt[6 + 9*x^2]*(2*C*(-8 - 7*x + 9*x^2) + 2*B*(17 + 21*x + 36*x^2) - 3*A*
(20 - 7*x + 51*x^2)) + ((3*I)*(17*A - 2*(4*B + C))*(1 + 3*x)*(2 + 3*x^2)*E
llipticE[ArcSin[Sqrt[Sqrt[6] - (3*I)*x]/(2^(3/4)*3^(1/4))], (2*Sqrt[6])/(I
 + Sqrt[6])])/Sqrt[(I*(1 + 3*x))/(I + Sqrt[6])] + (7*I)*(3*A + 6*B - 2*C)*
Sqrt[(I*(1 + 3*x))/(I + Sqrt[6])]*(2 + 3*x^2)*EllipticF[ArcSin[Sqrt[Sqrt[6
] - (3*I)*x]/(2^(3/4)*3^(1/4))], (2*Sqrt[6])/(I + Sqrt[6])])/(294*Sqrt[6 +
 9*x^2]*Sqrt[2 + 6*x + 3*x^2 + 9*x^3])
```

Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 5.02 (sec) , antiderivative size = 1435, normalized size of antiderivative = 3.28, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.433, Rules used = {2526, 27, 2490, 2486, 27, 1235, 27, 1237, 27, 1269, 1172, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{(9x^3+3x^2+6x+2)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 2526}$$

$$\frac{1}{27}\int \frac{3(9A-2C+(9B-2C)x)}{(9x^3+3x^2+6x+2)^{3/2}}dx-\frac{2C}{27\sqrt{9x^3+3x^2+6x+2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{9}\int \frac{9A-2C+(9B-2C)x}{(9x^3+3x^2+6x+2)^{3/2}}dx-\frac{2C}{27\sqrt{9x^3+3x^2+6x+2}}$$

$$\Big\downarrow \text{ 2490}$$

$$\frac{1}{9}\int \frac{\frac{1}{27}(27(9A-2C)-3(9B-2C))+(9B-2C)\left(x+\frac{1}{9}\right)}{\left(9\left(x+\frac{1}{9}\right)^3+\frac{17}{3}\left(x+\frac{1}{9}\right)+\frac{110}{81}\right)^{3/2}}d\left(x+\frac{1}{9}\right)-\frac{2C}{27\sqrt{9x^3+3x^2+6x+2}}$$

$$\Big\downarrow \text{ 2486}$$

$$\frac{81\left(9\left(x+\frac{1}{9}\right)+\frac{17-\left(63\sqrt{2}-55\right)^{2/3}}{\sqrt[3]{63\sqrt{2}-55}}\right)^{3/2}\left(81\left(x+\frac{1}{9}\right)^2-\frac{9\left(17-\left(63\sqrt{2}-55\right)^{2/3}\right)\left(x+\frac{1}{9}\right)}{\sqrt[3]{63\sqrt{2}-55}}+\left(63\sqrt{2}-55\right)^{2/3}+\frac{289}{\left(63\sqrt{2}-55\right)^{2/\cdots}}\cdots\right.}{\left(729\left(x+\frac{1}{9}\right)^{3}\cdots\right.}$$

$$\frac{2C}{27\sqrt{9x^3+3x^2+6x+2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{2C}{9\left(9\left(x+\frac{1}{9}\right)+\frac{17-\left(63\sqrt{2}-55\right)^{2/3}}{\sqrt[3]{63\sqrt{2}-55}}\right)^{3/2}\left(81\left(x+\frac{1}{9}\right)^2-\frac{9\left(17-\left(63\sqrt{2}-55\right)^{2/3}\right)\left(x+\frac{1}{9}\right)}{\sqrt[3]{63\sqrt{2}-55}}+\left(63\sqrt{2}-55\right)^{2/3}+\frac{289}{\left(63\sqrt{2}-55\right)^{2/3}}}$$

$$\left(729\left(x+\frac{1}{9}\right)^3\right.$$

$$\frac{2C}{27\sqrt{9x^3+3x^2+6x+2}}$$

$$\Big\downarrow \; 1235$$

$$\frac{2C}{9\left(9\left(x+\frac{1}{9}\right)+\frac{17-\left(-55+63\sqrt{2}\right)^{2/3}}{\sqrt[3]{-55+63\sqrt{2}}}\right)^{3/2}\left(81\left(x+\frac{1}{9}\right)^2-\frac{9\left(17-\left(-55+63\sqrt{2}\right)^{2/3}\right)\left(x+\frac{1}{9}\right)}{\sqrt[3]{-55+63\sqrt{2}}}+\left(-55+63\sqrt{2}\right)^{2/3}+\frac{289}{\left(-55+63\right.}}$$

$$\frac{2C}{27\sqrt{9x^3+3x^2+6x+2}}$$

$$\Big\downarrow \; 27$$

$$9\left(9\left(x+\frac{1}{9}\right)+\frac{17-\left(-55+63\sqrt{2}\right)^{2/3}}{\sqrt[3]{-55+63\sqrt{2}}}\right)^{3/2}\left(81\left(x+\frac{1}{9}\right)^2-\frac{9\left(17-\left(-55+63\sqrt{2}\right)^{2/3}\right)\left(x+\frac{1}{9}\right)}{\sqrt[3]{-55+63\sqrt{2}}}+\left(-55+63\sqrt{2}\right)^{2/3}+\frac{289}{\left(-55+63\right.}$$

$$\frac{2C}{27\sqrt{9x^3+3x^2+6x+2}}$$

$\downarrow$ 1237

$$9\left(9\left(x+\frac{1}{9}\right)+\frac{17-\left(-55+63\sqrt{2}\right)^{2/3}}{\sqrt[3]{-55+63\sqrt{2}}}\right)^{3/2}\left(81\left(x+\frac{1}{9}\right)^2-\frac{9\left(17-\left(-55+63\sqrt{2}\right)^{2/3}\right)\left(x+\frac{1}{9}\right)}{\sqrt[3]{-55+63\sqrt{2}}}+\left(-55+63\sqrt{2}\right)^{2/3}+\frac{289}{\left(-55+63\right.}$$

$$\frac{2C}{27\sqrt{9x^3+3x^2+6x+2}}$$

$\downarrow$ 27

$$9\left(9\left(x+\frac{1}{9}\right)+\frac{17-\left(-55+63\sqrt{2}\right)^{2/3}}{\sqrt[3]{-55+63\sqrt{2}}}\right)^{3/2}\left(81\left(x+\frac{1}{9}\right)^2-\frac{9\left(17-\left(-55+63\sqrt{2}\right)^{2/3}\right)\left(x+\frac{1}{9}\right)}{\sqrt[3]{-55+63\sqrt{2}}}+\left(-55+63\sqrt{2}\right)^{2/3}+\frac{289}{\left(-55+63\right.}\right.$$

$$\frac{2C}{27\sqrt{9x^3+3x^2+6x+2}}$$

$$\Big\downarrow\ 1269$$

$$9\left(9\left(x+\tfrac{1}{9}\right)+\frac{17-\left(-55+63\sqrt{2}\right)^{2/3}}{\sqrt[3]{-55+63\sqrt{2}}}\right)^{3/2}\left(81\left(x+\tfrac{1}{9}\right)^2-\frac{9\left(17-\left(-55+63\sqrt{2}\right)^{2/3}\right)\left(x+\frac{1}{9}\right)}{\sqrt[3]{-55+63\sqrt{2}}}+\left(-55+63\sqrt{2}\right)^{2/3}+\frac{289}{\left(-55+63\cdots\right.}\right.$$

$$\frac{2C}{27\sqrt{9x^3+3x^2+6x+2}}$$

$\downarrow$ 1172

$$9\left(9\left(x+\frac{1}{9}\right)+\frac{17-\left(-55+63\sqrt{2}\right)^{2/3}}{\sqrt[3]{-55+63\sqrt{2}}}\right)^{3/2}\left(81\left(x+\frac{1}{9}\right)^2-\frac{9\left(17-\left(-55+63\sqrt{2}\right)^{2/3}\right)\left(x+\frac{1}{9}\right)}{\sqrt[3]{-55+63\sqrt{2}}}+\left(-55+63\sqrt{2}\right)^{2/3}+\frac{289}{\left(-55+63\cdots\right.}$$

↓ 321

$$9\left(9\left(x+\frac{1}{9}\right)+\frac{17-\left(-55+63\sqrt{2}\right)^{2/3}}{\sqrt[3]{-55+63\sqrt{2}}}\right)^{3/2}\left(81\left(x+\frac{1}{9}\right)^2-\frac{9\left(17-\left(-55+63\sqrt{2}\right)^{2/3}\right)\left(x+\frac{1}{9}\right)}{\sqrt[3]{-55+63\sqrt{2}}}+\left(-55+63\sqrt{2}\right)^{2/3}+\frac{289}{\left(-55+63\right.}\right.$$

↓ 327

$$9\left(9\left(x+\frac{1}{9}\right)+\frac{17-\left(-55+63\sqrt{2}\right)^{2/3}}{\sqrt[3]{-55+63\sqrt{2}}}\right)^{3/2}\left(81\left(x+\frac{1}{9}\right)^2-\frac{9\left(17-\left(-55+63\sqrt{2}\right)^{2/3}\right)\left(x+\frac{1}{9}\right)}{\sqrt[3]{-55+63\sqrt{2}}}+\left(-55+63\sqrt{2}\right)^{2/3}+\frac{289}{\left(-55+63\right.}\right.$$

input
```
Int[(A + B*x + C*x^2)/(2 + 6*x + 3*x^2 + 9*x^3)^(3/2),x]
```

output
```
(-2*C)/(27*Sqrt[2 + 6*x + 3*x^2 + 9*x^3]) + (9*((17 - (-55 + 63*Sqrt[2])^(
2/3))/(-55 + 63*Sqrt[2])^(1/3) + 9*(1/9 + x))^(3/2)*(17 + 289/(-55 + 63*Sq
rt[2])^(2/3) + (-55 + 63*Sqrt[2])^(2/3) - (9*(17 - (-55 + 63*Sqrt[2])^(2/3
))*(1/9 + x))/(-55 + 63*Sqrt[2])^(1/3) + 81*(1/9 + x)^2)^(3/2)*(((126 - 55
*Sqrt[2])^(1/3)*(2*(17*A - 8*B - 2*C) + ((9*(55 - 63*Sqrt[2] + 17*(-55 + 6
3*Sqrt[2])^(1/3))*A + 9*(26 + 7*Sqrt[2] - (8 - 7*Sqrt[2])*(-55 + 63*Sqrt[2
])^(1/3))*B - 2*(81 - 56*Sqrt[2] + (9 + 7*Sqrt[2])*(-55 + 63*Sqrt[2])^(1/3
))*C)*(1/9 + x))/(-55 + 63*Sqrt[2])^(2/3)))/(21*2^(2/3)*(17 + (-55 + 63*Sq
rt[2])^(2/3))*Sqrt[(17 - (-55 + 63*Sqrt[2])^(2/3))/(-55 + 63*Sqrt[2])^(1/3
) + 9*(1/9 + x)]*Sqrt[17 + 289/(-55 + 63*Sqrt[2])^(2/3) + (-55 + 63*Sqrt[2
])^(2/3) - (9*(17 - (-55 + 63*Sqrt[2])^(2/3))*(1/9 + x))/(-55 + 63*Sqrt[2]
)^(1/3) + 81*(1/9 + x)^2]) + ((126 - 55*Sqrt[2])^(1/3)*((-4*(-55 + 63*Sqrt
[2])^(2/3)*(17*A - 8*B - 2*C)*Sqrt[17 + 289/(-55 + 63*Sqrt[2])^(2/3) + (-5
5 + 63*Sqrt[2])^(2/3) - (9*(17 - (-55 + 63*Sqrt[2])^(2/3))*(1/9 + x))/(-55
 + 63*Sqrt[2])^(1/3) + 81*(1/9 + x)^2])/((289 - 17*(-55 + 63*Sqrt[2])^(2/3
) + (-55 + 63*Sqrt[2])^(4/3))*Sqrt[(17 - (-55 + 63*Sqrt[2])^(2/3))/(-55 +
63*Sqrt[2])^(1/3) + 9*(1/9 + x)]) + (18*(-55 + 63*Sqrt[2])^(2/3)*(((I/9)*S
qrt[2]*(17*A - 8*B - 2*C)*Sqrt[(17 - (-55 + 63*Sqrt[2])^(2/3))/(-55 + 63*S
qrt[2])^(1/3) + 9*(1/9 + x)]*EllipticE[ArcSin[((-55 + 63*Sqrt[2])^(1/6)*Sq
rt[(-I)*(((-55 + 63*Sqrt[2])^(2/3)*(1 + I*Sqrt[3]) + (17*I)*(I + Sqrt[3...
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 1172
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Simp[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m))   Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^
2 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[m^2, 1/4]
```

rule 1235
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^m
*(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*
m + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) -
f*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
)
```

rule 1237
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[
(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && LtQ[m, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2486
```
Int[((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (d_.)*(x_)^3)^(p_), x_S
ymbol] :> With[{r = Rt[-9*a*d^2 + Sqrt[3]*d*Sqrt[4*b^3*d + 27*a^2*d^2], 3]}
, Simp[(a + b*x + d*x^3)^p/(Simp[18^(1/3)*b*(d/(3*r)) - r/18^(1/3) + d*x, x
]^p*Simp[b*(d/3) + 12^(1/3)*b^2*(d^2/(3*r^2)) + r^2/(3*12^(1/3)) - d*(2^(1/
3)*b*(d/(3^(1/3)*r)) - r/18^(1/3))*x + d^2*x^2, x]^p)   Int[(e + f*x)^m*Sim
p[18^(1/3)*b*(d/(3*r)) - r/18^(1/3) + d*x, x]^p*Simp[b*(d/3) + 12^(1/3)*b^2
*(d^2/(3*r^2)) + r^2/(3*12^(1/3)) - d*(2^(1/3)*b*(d/(3^(1/3)*r)) - r/18^(1/
3))*x + d^2*x^2, x]^p, x], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && NeQ[4*
b^3 + 27*a^2*d, 0] &&  !IntegerQ[p]
```

rule 2490
```
Int[(P3_)^(p_.)*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] :> With[{a = Coeff[P3
, x, 0], b = Coeff[P3, x, 1], c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Su
bst[Int[((3*d*e - c*f)/(3*d) + f*x)^m*Simp[(2*c^3 - 9*b*c*d + 27*a*d^2)/(27
*d^2) - (c^2 - 3*b*d)*(x/(3*d)) + d*x^3, x]^p, x], x, x + c/(3*d)] /; NeQ[c
, 0]] /; FreeQ[{e, f, m, p}, x] && PolyQ[P3, x, 3]
```

rule 2526
```
Int[(Pm_)*(Qn_)^(p_), x_Symbol] :> With[{m = Expon[Pm, x], n = Expon[Qn, x]
}, Simp[Coeff[Pm, x, m]*(Qn^(p + 1)/(n*(p + 1)*Coeff[Qn, x, n])), x] + Simp
[1/(n*Coeff[Qn, x, n])   Int[ExpandToSum[n*Coeff[Qn, x, n]*Pm - Coeff[Pm, x
, m]*D[Qn, x], x]*Qn^p, x], x] /; EqQ[m, n - 1]] /; FreeQ[p, x] && PolyQ[Pm
, x] && PolyQ[Qn, x] && NeQ[p, -1]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.83 (sec) , antiderivative size = 377, normalized size of antiderivative = 0.86

method	result
elliptic	$-\frac{18\left(\left(-\frac{2B}{147}-\frac{C}{294}+\frac{17A}{588}\right)x^2+\left(-\frac{B}{126}+\frac{C}{378}-\frac{A}{252}\right)x-\frac{17B}{2646}+\frac{4C}{1323}+\frac{5A}{441}\right)}{\sqrt{9x^3+3x^2+6x+2}}+\frac{2\left(\frac{3B}{98}-\frac{5C}{147}+\frac{6A}{49}\right)\left(-\frac{i\sqrt{6}}{3}+\frac{1}{3}\right)\sqrt{\frac{x+\frac{1}{3}}{-\frac{i\sqrt{6}}{3}+\frac{1}{3}}}\sqrt{\frac{x-\frac{i\sqrt{6}}{3}}{-\frac{1}{3}-\frac{i\sqrt{6}}{3}}}}{\sqrt{9x^3+3x^2}}$
risch	$-\frac{153A\,x^2-72B\,x^2-18C\,x^2-21Ax-42Bx+14Cx+60A-34B+16C}{294\sqrt{9x^3+3x^2+6x+2}}+\frac{(153A-72B-18C)\left(-\frac{i\sqrt{6}}{3}+\frac{1}{3}\right)\sqrt{\frac{x+\frac{1}{3}}{-\frac{i\sqrt{6}}{3}+\frac{1}{3}}}\sqrt{\frac{x-\frac{i\sqrt{6}}{3}}{-\frac{1}{3}-\frac{i\sqrt{6}}{3}}}}{}$
default	Expression too large to display

input
```
int((C*x^2+B*x+A)/(9*x^3+3*x^2+6*x+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-18*((-2/147*B-1/294*C+17/588*A)*x^2+(-1/126*B+1/378*C-1/252*A)*x-17/2646*
B+4/1323*C+5/441*A)/(9*x^3+3*x^2+6*x+2)^(1/2)+2*(3/98*B-5/147*C+6/49*A)*(-
1/3*I*6^(1/2)+1/3)*((x+1/3)/(-1/3*I*6^(1/2)+1/3))^(1/2)*((x-1/3*I*6^(1/2))
/(-1/3-1/3*I*6^(1/2)))^(1/2)*((x+1/3*I*6^(1/2))/(-1/3+1/3*I*6^(1/2)))^(1/2
)/(9*x^3+3*x^2+6*x+2)^(1/2)*EllipticF(((x+1/3)/(-1/3*I*6^(1/2)+1/3))^(1/2)
,((-1/3+1/3*I*6^(1/2))/(-1/3-1/3*I*6^(1/2)))^(1/2))+2*(-6/49*B-3/98*C+51/1
96*A)*(-1/3*I*6^(1/2)+1/3)*((x+1/3)/(-1/3*I*6^(1/2)+1/3))^(1/2)*((x-1/3*I*
6^(1/2))/(-1/3-1/3*I*6^(1/2)))^(1/2)*((x+1/3*I*6^(1/2))/(-1/3+1/3*I*6^(1/2
)))^(1/2)/(9*x^3+3*x^2+6*x+2)^(1/2)*((-1/3-1/3*I*6^(1/2))*EllipticE(((x+1/
3)/(-1/3*I*6^(1/2)+1/3))^(1/2),((-1/3+1/3*I*6^(1/2))/(-1/3-1/3*I*6^(1/2)))
^(1/2))+1/3*I*6^(1/2)*EllipticF(((x+1/3)/(-1/3*I*6^(1/2)+1/3))^(1/2),((-1/
3+1/3*I*6^(1/2))/(-1/3-1/3*I*6^(1/2)))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.46

$$\int \frac{A+Bx+Cx^2}{(2+6x+3x^2+9x^3)^{3/2}}\,dx=\frac{(9\,(55\,A+26\,B-18\,C)x^3+3\,(55\,A+26\,B-18\,C)x^2+6\,(55\,A+26\,B}{}$$

input
```
integrate((C*x^2+B*x+A)/(9*x^3+3*x^2+6*x+2)^(3/2),x, algorithm="fricas")
```

output
```
1/882*((9*(55*A + 26*B - 18*C)*x^3 + 3*(55*A + 26*B - 18*C)*x^2 + 6*(55*A
+ 26*B - 18*C)*x + 110*A + 52*B - 36*C)*weierstrassPInverse(-68/27, -440/7
29, x + 1/9) - 9*(9*(17*A - 8*B - 2*C)*x^3 + 3*(17*A - 8*B - 2*C)*x^2 + 6*
(17*A - 8*B - 2*C)*x + 34*A - 16*B - 4*C)*weierstrassZeta(-68/27, -440/729
, weierstrassPInverse(-68/27, -440/729, x + 1/9)) - 3*(9*(17*A - 8*B - 2*C
)*x^2 - 7*(3*A + 6*B - 2*C)*x + 60*A - 34*B + 16*C)*sqrt(9*x^3 + 3*x^2 + 6
*x + 2))/(9*x^3 + 3*x^2 + 6*x + 2)
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2}{(2+6x+3x^2+9x^3)^{3/2}}\,dx = \int \frac{A+Bx+Cx^2}{\left((3x+1)\left(3x^2+2\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((C*x**2+B*x+A)/(9*x**3+3*x**2+6*x+2)**(3/2),x)
```

output
```
Integral((A + B*x + C*x**2)/((3*x + 1)*(3*x**2 + 2))**(3/2), x)
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{(2+6x+3x^2+9x^3)^{3/2}}\,dx = \int \frac{Cx^2+Bx+A}{(9\,x^3+3\,x^2+6\,x+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(9*x^3+3*x^2+6*x+2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((C*x^2 + B*x + A)/(9*x^3 + 3*x^2 + 6*x + 2)^(3/2), x)
```

Giac [F]

$$\int \frac{A+Bx+Cx^2}{(2+6x+3x^2+9x^3)^{3/2}}\,dx = \int \frac{Cx^2+Bx+A}{(9\,x^3+3\,x^2+6\,x+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(9*x^3+3*x^2+6*x+2)^(3/2),x, algorithm="giac")
```

output
```
integrate((C*x^2 + B*x + A)/(9*x^3 + 3*x^2 + 6*x + 2)^(3/2), x)
```

Mupad [B] (verification not implemented)

Time = 12.90 (sec) , antiderivative size = 692, normalized size of antiderivative = 1.58

$$\int \frac{A+Bx+Cx^2}{(2+6x+3x^2+9x^3)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int((A + B*x + C*x^2)/(6*x + 3*x^2 + 9*x^3 + 2)^(3/2),x)
```

output
```
(2*((x + 1/3)/((2^(1/2)*3^(1/2)*1i)/3 + 1/3))^(1/2)*((x + (2^(1/2)*3^(1/2)
*1i)/3)/((2^(1/2)*3^(1/2)*1i)/3 - 1/3))^(1/2)*(-(x - (2^(1/2)*3^(1/2)*1i)/
3)/((2^(1/2)*3^(1/2)*1i)/3 + 1/3))^(1/2)*((2^(1/2)*3^(1/2)*1i)/3 + 1/3)*el
lipticPi(((2^(1/2)*3^(1/2)*1i)/3 + 1/3)/((6^(1/2)*1i)/3 + 1/3), asin(((x +
 1/3)/((2^(1/2)*3^(1/2)*1i)/3 + 1/3))^(1/2)), -((2^(1/2)*3^(1/2)*1i)/3 + 1
/3)/((2^(1/2)*3^(1/2)*1i)/3 - 1/3))*(A/14 - B/42 - C/21 + (6^(1/2)*B*1i)/4
2 - (6^(1/2)*C*1i)/126 + (6^(1/2)*A*1i)/84))/(((6^(1/2)*1i)/3 + 1/3)*(6*x
+ 3*x^2 + 9*x^3 + 2)^(1/2)) + (2*((x + 1/3)/((2^(1/2)*3^(1/2)*1i)/3 + 1/3)
)^(1/2)*((x + (2^(1/2)*3^(1/2)*1i)/3)/((2^(1/2)*3^(1/2)*1i)/3 - 1/3))^(1/2
)*(-(x - (2^(1/2)*3^(1/2)*1i)/3)/((2^(1/2)*3^(1/2)*1i)/3 + 1/3))^(1/2)*((2
^(1/2)*3^(1/2)*1i)/3 + 1/3)*ellipticPi(-((2^(1/2)*3^(1/2)*1i)/3 + 1/3)/((6
^(1/2)*1i)/3 - 1/3), asin(((x + 1/3)/((2^(1/2)*3^(1/2)*1i)/3 + 1/3))^(1/2)
), -((2^(1/2)*3^(1/2)*1i)/3 + 1/3)/((2^(1/2)*3^(1/2)*1i)/3 - 1/3))*(B/42 -
 A/14 + C/21 + (6^(1/2)*B*1i)/42 - (6^(1/2)*C*1i)/126 + (6^(1/2)*A*1i)/84)
)/(((6^(1/2)*1i)/3 - 1/3)*(6*x + 3*x^2 + 9*x^3 + 2)^(1/2)) + (2^(1/2)*3^(1
/2)*((x + 1/3)/((2^(1/2)*3^(1/2)*1i)/3 + 1/3))^(1/2)*(ellipticE(asin(((2^(
1/2)*3^(1/2)*(x - (2^(1/2)*3^(1/2)*1i)/3)*1i)/4)^(1/2)), (2^(1/2)*3^(1/2)*
2i)/(3*((2^(1/2)*3^(1/2)*1i)/3 + 1/3))) - (2^(1/2)*3^(1/2)*sin(2*asin(((2^
(1/2)*3^(1/2)*(x - (2^(1/2)*3^(1/2)*1i)/3)*1i)/4)^(1/2)))*1i)/(3*((2^(1/2)
*3^(1/2)*1i)/3 + 1/3)*((x - (2^(1/2)*3^(1/2)*1i)/3)/((2^(1/2)*3^(1/2)*1...
```

Reduce [F]

$$\int \frac{A + Bx + Cx^2}{(2 + 6x + 3x^2 + 9x^3)^{3/2}}\, dx = \text{Too large to display}$$

input
```
int((C*x^2+B*x+A)/(9*x^3+3*x^2+6*x+2)^(3/2),x)
```

output

```
(6*sqrt(9*x**3 + 3*x**2 + 6*x + 2)*a*x - 6*sqrt(9*x**3 + 3*x**2 + 6*x + 2)
*b*x - 4*sqrt(9*x**3 + 3*x**2 + 6*x + 2)*b + 243*int((sqrt(9*x**3 + 3*x**2
 + 6*x + 2)*x**3)/(81*x**6 + 54*x**5 + 117*x**4 + 72*x**3 + 48*x**2 + 24*x
 + 4),x)*a*x**3 + 81*int((sqrt(9*x**3 + 3*x**2 + 6*x + 2)*x**3)/(81*x**6 +
 54*x**5 + 117*x**4 + 72*x**3 + 48*x**2 + 24*x + 4),x)*a*x**2 + 162*int((s
qrt(9*x**3 + 3*x**2 + 6*x + 2)*x**3)/(81*x**6 + 54*x**5 + 117*x**4 + 72*x*
*3 + 48*x**2 + 24*x + 4),x)*a*x + 54*int((sqrt(9*x**3 + 3*x**2 + 6*x + 2)*
x**3)/(81*x**6 + 54*x**5 + 117*x**4 + 72*x**3 + 48*x**2 + 24*x + 4),x)*a -
 243*int((sqrt(9*x**3 + 3*x**2 + 6*x + 2)*x**3)/(81*x**6 + 54*x**5 + 117*x
**4 + 72*x**3 + 48*x**2 + 24*x + 4),x)*b*x**3 - 81*int((sqrt(9*x**3 + 3*x*
*2 + 6*x + 2)*x**3)/(81*x**6 + 54*x**5 + 117*x**4 + 72*x**3 + 48*x**2 + 24
*x + 4),x)*b*x**2 - 162*int((sqrt(9*x**3 + 3*x**2 + 6*x + 2)*x**3)/(81*x**
6 + 54*x**5 + 117*x**4 + 72*x**3 + 48*x**2 + 24*x + 4),x)*b*x - 54*int((sq
rt(9*x**3 + 3*x**2 + 6*x + 2)*x**3)/(81*x**6 + 54*x**5 + 117*x**4 + 72*x**
3 + 48*x**2 + 24*x + 4),x)*b - 486*int((sqrt(9*x**3 + 3*x**2 + 6*x + 2)*x*
*2)/(81*x**6 + 54*x**5 + 117*x**4 + 72*x**3 + 48*x**2 + 24*x + 4),x)*b*x**
3 - 162*int((sqrt(9*x**3 + 3*x**2 + 6*x + 2)*x**2)/(81*x**6 + 54*x**5 + 11
7*x**4 + 72*x**3 + 48*x**2 + 24*x + 4),x)*b*x**2 - 324*int((sqrt(9*x**3 +
3*x**2 + 6*x + 2)*x**2)/(81*x**6 + 54*x**5 + 117*x**4 + 72*x**3 + 48*x**2
+ 24*x + 4),x)*b*x - 108*int((sqrt(9*x**3 + 3*x**2 + 6*x + 2)*x**2)/(81...
```

3.96 $\int \frac{A+Bx+Cx^2}{(2+6x+3x^2+9x^3)^{5/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 555

$$\int \frac{A+Bx+Cx^2}{(2+6x+3x^2+9x^3)^{5/2}}\,dx = \frac{(1+3x)(2(3A-B-2C)+(3A+6B-2C)x)\,(2+3x^2)}{126\,(2+6x+3x^2+9x^3)^{5/2}}$$
$$+\frac{(1+3x)(9(17A-8B-2C)+(150A+111B-58C)x)\,(2+3x^2)^2}{1764\,(2+6x+3x^2+9x^3)^{5/2}}$$
$$-\frac{(615A-471B-32C)(1+3x)\,(2+3x^2)^3}{12348\,(2+6x+3x^2+9x^3)^{5/2}}$$
$$-\frac{(5610A+447B-1346C)(1+3x)^2\,(2+3x^2)^3}{86436\,(2+6x+3x^2+9x^3)^{5/2}}$$
$$+\frac{(5610A+447B-1346C)(1+3x)^3\,(2+3x^2)^3}{86436\,\left(1+\sqrt{7}+3x\right)(2+6x+3x^2+9x^3)^{5/2}}$$
$$-\frac{(5610A+447B-1346C)(1+3x)^{5/2}\left(1+\sqrt{7}+3x\right)(2+3x^2)^2\sqrt{\frac{2+3x^2}{\left(1+\sqrt{7}+3x\right)^2}}\,E\left(2\arctan\left(\frac{\sqrt{1+3x}}{\sqrt[4]{7}}\right)|\frac{1}{14}(7\right.}{12348\sqrt{3}\,7^{3/4}\,(2+6x+3x^2+9x^3)^{5/2}}$$
$$+\frac{\left(\left(5610-615\sqrt{7}\right)A+\left(447+471\sqrt{7}\right)B-2\left(673-16\sqrt{7}\right)C\right)(1+3x)^{5/2}\left(1+\sqrt{7}+3x\right)(2+3x^2)^2\sqrt{\frac{\ }{(1}}}{24696\sqrt{3}\,7^{3/4}\,(2+6x+3x^2+9x^3)^{5/2}}$$

output
```
1/126*(1+3*x)*(6*A-2*B-4*C+(3*A+6*B-2*C)*x)*(3*x^2+2)/(9*x^3+3*x^2+6*x+2)^
(5/2)+1/1764*(1+3*x)*(153*A-72*B-18*C+(150*A+111*B-58*C)*x)*(3*x^2+2)^2/(9
*x^3+3*x^2+6*x+2)^(5/2)-1/12348*(615*A-471*B-32*C)*(1+3*x)*(3*x^2+2)^3/(9*
x^3+3*x^2+6*x+2)^(5/2)-1/86436*(5610*A+447*B-1346*C)*(1+3*x)^2*(3*x^2+2)^3
/(9*x^3+3*x^2+6*x+2)^(5/2)+1/86436*(5610*A+447*B-1346*C)*(1+3*x)^3*(3*x^2+
2)^3/(1+7^(1/2)+3*x)/(9*x^3+3*x^2+6*x+2)^(5/2)-1/259308*(5610*A+447*B-1346
*C)*(1+3*x)^(5/2)*(1+7^(1/2)+3*x)*(3*x^2+2)^2*((3*x^2+2)/(1+7^(1/2)+3*x)^2
)^(1/2)*EllipticE(sin(2*arctan(1/7*(1+3*x)^(1/2)*7^(3/4))),1/14*(98+14*7^(
1/2))^(1/2))*3^(1/2)*7^(1/4)/(9*x^3+3*x^2+6*x+2)^(5/2)+1/518616*((5610-615
*7^(1/2))*A+(447+471*7^(1/2))*B-2*(673-16*7^(1/2))*C)*(1+3*x)^(5/2)*(1+7^(
1/2)+3*x)*(3*x^2+2)^2*((3*x^2+2)/(1+7^(1/2)+3*x)^2)^(1/2)*InverseJacobiAM(
2*arctan(1/7*(1+3*x)^(1/2)*7^(3/4)),1/14*(98+14*7^(1/2))^(1/2))*3^(1/2)*7^
(1/4)/(9*x^3+3*x^2+6*x+2)^(5/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 13.89 (sec) , antiderivative size = 520, normalized size of antiderivative = 0.94

$$\int \frac{A+Bx+Cx^2}{(2+6x+3x^2+9x^3)^{5/2}}\,dx = \frac{\frac{B(8916+28890x+70848x^2+675x^3+76950x^4-36207x^5)+6C(886+4548x+8097x^2+19965x^3+7065x}{2+6x+3x^2+9x^3}}{}$$

input
```
Integrate[(A + B*x + C*x^2)/(2 + 6*x + 3*x^2 + 9*x^3)^(5/2),x]
```

output
```
((B*(8916 + 28890*x + 70848*x^2 + 675*x^3 + 76950*x^4 - 36207*x^5) + 6*C*(
886 + 4548*x + 8097*x^2 + 19965*x^3 + 7065*x^4 + 18171*x^5) - 9*A*(6850 +
16854*x + 32163*x^2 + 59970*x^3 + 29745*x^4 + 50490*x^5))/(2 + 6*x + 3*x^2
 + 9*x^3) + (3*Sqrt[2] + I*Sqrt[3])*(5610*A + 447*B - 1346*C)*Sqrt[(I*(1 +
 3*x))/(I + Sqrt[6])]*Sqrt[2 + 3*x^2]*EllipticE[ArcSin[Sqrt[Sqrt[6] - (3*I
)*x]/(2^(3/4)*3^(1/4))], (2*Sqrt[6])/(I + Sqrt[6])] + (Sqrt[(I*(1 + 3*x))/
(I + Sqrt[6])]*((224*I)*Sqrt[3]*C*Sqrt[Sqrt[6] - (3*I)*x]*Sqrt[2 + 3*x^2]
+ 3*B*((1248*I)*Sqrt[6]*Sqrt[Sqrt[6] + (3*I)*x] + 3744*Sqrt[Sqrt[6] + (3*I
)*x]*x - (149*I)*Sqrt[3]*Sqrt[Sqrt[6] - (3*I)*x]*Sqrt[2 + 3*x^2]) + 15*A*(
(87*I)*Sqrt[6]*Sqrt[Sqrt[6] + (3*I)*x] + 261*Sqrt[Sqrt[6] + (3*I)*x]*x - (
374*I)*Sqrt[3]*Sqrt[Sqrt[6] - (3*I)*x]*Sqrt[2 + 3*x^2]))*EllipticF[ArcSin[
Sqrt[Sqrt[6] - (3*I)*x]/(2^(3/4)*3^(1/4))], (2*Sqrt[6])/(I + Sqrt[6])])/Sq
rt[Sqrt[6] - (3*I)*x])/(259308*Sqrt[2 + 6*x + 3*x^2 + 9*x^3])
```

Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 30.02 (sec) , antiderivative size = 3718, normalized size of antiderivative = 6.70, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.567$, Rules used = {2526, 27, 2490, 2486, 27, 1235, 27, 1235, 27, 1237, 27, 1237, 27, 1269, 1172, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{(9x^3+3x^2+6x+2)^{5/2}}dx$$

$$\Big\downarrow \text{ 2526}$$

$$\frac{1}{27}\int \frac{3(9A-2C+(9B-2C)x)}{(9x^3+3x^2+6x+2)^{5/2}}dx-\frac{2C}{81\left(9x^3+3x^2+6x+2\right)^{3/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{9}\int \frac{9A-2C+(9B-2C)x}{(9x^3+3x^2+6x+2)^{5/2}}dx-\frac{2C}{81\left(9x^3+3x^2+6x+2\right)^{3/2}}$$

$$\Big\downarrow \text{ 2490}$$

$$\frac{1}{9}\int \frac{\frac{1}{27}(27(9A-2C)-3(9B-2C))+(9B-2C)\left(x+\frac{1}{9}\right)}{\left(9\left(x+\frac{1}{9}\right)^3+\frac{17}{3}\left(x+\frac{1}{9}\right)+\frac{110}{81}\right)^{5/2}}d\left(x+\frac{1}{9}\right)-\frac{2C}{81\left(9x^3+3x^2+6x+2\right)^{3/2}}$$

$\downarrow$ 2486

$$\frac{6561\left(9\left(x+\frac{1}{9}\right)+\frac{17-\left(63\sqrt{2}-55\right)^{2/3}}{\sqrt[3]{63\sqrt{2}-55}}\right)^{5/2}\left(81\left(x+\frac{1}{9}\right)^2-\frac{9\left(17-\left(63\sqrt{2}-55\right)^{2/3}\right)\left(x+\frac{1}{9}\right)}{\sqrt[3]{63\sqrt{2}-55}}+\left(63\sqrt{2}-55\right)^{2/3}+\frac{289}{\left(63\sqrt{2}-55\right)\cdots}\right.}{\left(729\left(x+\frac{1}{9}\right)\cdots\right.}$$

$$\frac{2C}{81\left(9x^3+3x^2+6x+2\right)^{3/2}}$$

$\downarrow$ 27

$$\frac{729\left(9\left(x+\frac{1}{9}\right)+\frac{17-\left(63\sqrt{2}-55\right)^{2/3}}{\sqrt[3]{63\sqrt{2}-55}}\right)^{5/2}\left(81\left(x+\frac{1}{9}\right)^2-\frac{9\left(17-\left(63\sqrt{2}-55\right)^{2/3}\right)\left(x+\frac{1}{9}\right)}{\sqrt[3]{63\sqrt{2}-55}}+\left(63\sqrt{2}-55\right)^{2/3}+\frac{289}{\left(63\sqrt{2}-55\right)^{2}\cdots}\right.}{\left(729\left(x+\frac{1}{9}\right)^{3}\cdots\right.}$$

$$\frac{2C}{81\left(9x^3+3x^2+6x+2\right)^{3/2}}$$

$\downarrow$ 1235

$$729\left(9\left(x+\frac{1}{9}\right)+\frac{17-\left(-55+63\sqrt{2}\right)^{2/3}}{\sqrt[3]{-55+63\sqrt{2}}}\right)^{5/2}\left(81\left(x+\frac{1}{9}\right)^2-\frac{9\left(17-\left(-55+63\sqrt{2}\right)^{2/3}\right)\left(x+\frac{1}{9}\right)}{\sqrt[3]{-55+63\sqrt{2}}}+\left(-55+63\sqrt{2}\right)^{2/3}+\frac{}{\left(-55+\right.}\right.$$

$$\frac{2C}{81\left(9x^3+3x^2+6x+2\right)^{3/2}}$$

$$\Big\downarrow 27$$

$$729\left(9\left(x+\frac{1}{9}\right)+\frac{17-\left(-55+63\sqrt{2}\right)^{2/3}}{\sqrt[3]{-55+63\sqrt{2}}}\right)^{5/2}\left(81\left(x+\frac{1}{9}\right)^2-\frac{9\left(17-\left(-55+63\sqrt{2}\right)^{2/3}\right)\left(x+\frac{1}{9}\right)}{\sqrt[3]{-55+63\sqrt{2}}}+\left(-55+63\sqrt{2}\right)^{2/3}+\frac{}{\left(-55+\right.}\right.$$

$$\frac{2C}{81\left(9x^3+3x^2+6x+2\right)^{3/2}}$$

$$\Big\downarrow 1235$$

$$729\left(9\left(x+\frac{1}{9}\right)+\frac{17-\left(-55+63\sqrt{2}\right)^{2/3}}{\sqrt[3]{-55+63\sqrt{2}}}\right)^{5/2}\left(81\left(x+\frac{1}{9}\right)^2-\frac{9\left(17-\left(-55+63\sqrt{2}\right)^{2/3}\right)\left(x+\frac{1}{9}\right)}{\sqrt[3]{-55+63\sqrt{2}}}+\left(-55+63\sqrt{2}\right)^{2/3}+\frac{}{\left(-55+\right.}\right.$$

$$\frac{2C}{81\left(9x^3+3x^2+6x+2\right)^{3/2}}$$

$$\Big\downarrow \; 27$$

$$729\left(9\left(x+\frac{1}{9}\right)+\frac{17-\left(-55+63\sqrt{2}\right)^{2/3}}{\sqrt[3]{-55+63\sqrt{2}}}\right)^{5/2}\left(81\left(x+\frac{1}{9}\right)^2-\frac{9\left(17-\left(-55+63\sqrt{2}\right)^{2/3}\right)\left(x+\frac{1}{9}\right)}{\sqrt[3]{-55+63\sqrt{2}}}+\left(-55+63\sqrt{2}\right)^{2/3}+\frac{}{\left(-55+\right.}\right.$$

$$\frac{2C}{81\left(9x^3+3x^2+6x+2\right)^{3/2}}$$

$\downarrow$ 1237

$$729\left(9\left(x+\frac{1}{9}\right)+\frac{17-\left(-55+63\sqrt{2}\right)^{2/3}}{\sqrt[3]{-55+63\sqrt{2}}}\right)^{5/2}\left(81\left(x+\frac{1}{9}\right)^2-\frac{9\left(17-\left(-55+63\sqrt{2}\right)^{2/3}\right)\left(x+\frac{1}{9}\right)}{\sqrt[3]{-55+63\sqrt{2}}}+\left(-55+63\sqrt{2}\right)^{2/3}+\frac{}{\left(-55+\right.}\right.$$

↓ 27

$$729\left(9\left(x+\frac{1}{9}\right)+\frac{17-\left(-55+63\sqrt{2}\right)^{2/3}}{\sqrt[3]{-55+63\sqrt{2}}}\right)^{5/2}\left(81\left(x+\frac{1}{9}\right)^2-\frac{9\left(17-\left(-55+63\sqrt{2}\right)^{2/3}\right)\left(x+\frac{1}{9}\right)}{\sqrt[3]{-55+63\sqrt{2}}}+\left(-55+63\sqrt{2}\right)^{2/3}+\frac{}{\left(-55+\right.}\right.$$

↓ 1237

$\downarrow$ 27

$$729\left(9\left(x+\tfrac{1}{9}\right)+\frac{17-\left(-55+63\sqrt{2}\right)^{2/3}}{\sqrt[3]{-55+63\sqrt{2}}}\right)^{5/2}\left(81\left(x+\tfrac{1}{9}\right)^2-\frac{9\left(17-\left(-55+63\sqrt{2}\right)^{2/3}\right)\left(x+\frac{1}{9}\right)}{\sqrt[3]{-55+63\sqrt{2}}}+\left(-55+63\sqrt{2}\right)^{2/3}+\frac{}{\left(-55+\right.}\right.$$

↓ 1269

↓ 1172

↓ 321

↓ 327

input
```
Int[(A + B*x + C*x^2)/(2 + 6*x + 3*x^2 + 9*x^3)^(5/2),x]
```

output
```
(-2*C)/(81*(2 + 6*x + 3*x^2 + 9*x^3)^(3/2)) + (729*((17 - (-55 + 63*Sqrt[2
])^(2/3))/(-55 + 63*Sqrt[2])^(1/3) + 9*(1/9 + x))^(5/2)*(17 + 289/(-55 + 6
3*Sqrt[2])^(2/3) + (-55 + 63*Sqrt[2])^(2/3) - (9*(17 - (-55 + 63*Sqrt[2])^
(2/3))*(1/9 + x))/(-55 + 63*Sqrt[2])^(1/3) + 81*(1/9 + x)^2)^(5/2)*(((126
- 55*Sqrt[2])^(1/3)*(2*(17*A - 8*B - 2*C) + ((9*(55 - 63*Sqrt[2] + 17*(-55
 + 63*Sqrt[2])^(1/3))*A + 9*(26 + 7*Sqrt[2] - (8 - 7*Sqrt[2])*(-55 + 63*Sq
rt[2])^(1/3))*B - 2*(81 - 56*Sqrt[2] + (9 + 7*Sqrt[2])*(-55 + 63*Sqrt[2])^
(1/3))*C)*(1/9 + x))/(-55 + 63*Sqrt[2])^(2/3)))/(63*2^(2/3)*(17 + (-55 + 6
3*Sqrt[2])^(2/3))*((17 - (-55 + 63*Sqrt[2])^(2/3))/(-55 + 63*Sqrt[2])^(1/3
) + 9*(1/9 + x))^(3/2)*(17 + 289/(-55 + 63*Sqrt[2])^(2/3) + (-55 + 63*Sqrt
[2])^(2/3) - (9*(17 - (-55 + 63*Sqrt[2])^(2/3))*(1/9 + x))/(-55 + 63*Sqrt[
2])^(1/3) + 81*(1/9 + x)^2)^(3/2)) + ((126 - 55*Sqrt[2])^(1/3)*(((2*(126 -
 55*Sqrt[2]))^(1/3)*((17*((2329250 - 501417*Sqrt[2] + 119*(13988 - 10395*S
qrt[2])*(-55 + 63*Sqrt[2])^(1/3) + 1156*(-55 + 63*Sqrt[2])^(5/3))*A + (288
5386 - 2280852*Sqrt[2] - 14*(31697 - 13797*Sqrt[2])*(-55 + 63*Sqrt[2])^(1/
3) - 544*(-55 + 63*Sqrt[2])^(5/3))*B - 2*(7*(82772 - 44163*Sqrt[2]) + 7*(1
9378 - 16569*Sqrt[2])*(-55 + 63*Sqrt[2])^(1/3) + 68*(-55 + 63*Sqrt[2])^(5/
3))*C))/(-55 + 63*Sqrt[2])^(5/3) - 9*(55 + 63*Sqrt[2])^(1/3)*(5*(289 - 154
*(-55 + 63*Sqrt[2])^(1/3) - 17*(-55 + 63*Sqrt[2])^(2/3))*A - (680 + 364*(-
55 + 63*Sqrt[2])^(1/3) - 40*(-55 + 63*Sqrt[2])^(2/3))*B - 2*(85 - 126*(...
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 1172
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Simp[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m))   Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^
2 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[m^2, 1/4]
```

rule 1235
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^m
*(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*
m + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) -
f*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
)
```

rule 1237
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[
(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && LtQ[m, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2486
```
Int[((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (d_.)*(x_)^3)^(p_), x_S
ymbol] :> With[{r = Rt[-9*a*d^2 + Sqrt[3]*d*Sqrt[4*b^3*d + 27*a^2*d^2], 3]}
, Simp[(a + b*x + d*x^3)^p/(Simp[18^(1/3)*b*(d/(3*r)) - r/18^(1/3) + d*x, x
]^p*Simp[b*(d/3) + 12^(1/3)*b^2*(d^2/(3*r^2)) + r^2/(3*12^(1/3)) - d*(2^(1/
3)*b*(d/(3^(1/3)*r)) - r/18^(1/3))*x + d^2*x^2, x]^p)   Int[(e + f*x)^m*Sim
p[18^(1/3)*b*(d/(3*r)) - r/18^(1/3) + d*x, x]^p*Simp[b*(d/3) + 12^(1/3)*b^2
*(d^2/(3*r^2)) + r^2/(3*12^(1/3)) - d*(2^(1/3)*b*(d/(3^(1/3)*r)) - r/18^(1/
3))*x + d^2*x^2, x]^p, x], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && NeQ[4*
b^3 + 27*a^2*d, 0] &&  !IntegerQ[p]
```

rule 2490
```
Int[(P3_)^(p_.)*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] :> With[{a = Coeff[P3
, x, 0], b = Coeff[P3, x, 1], c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Su
bst[Int[((3*d*e - c*f)/(3*d) + f*x)^m*Simp[(2*c^3 - 9*b*c*d + 27*a*d^2)/(27
*d^2) - (c^2 - 3*b*d)*(x/(3*d)) + d*x^3, x]^p, x], x, x + c/(3*d)] /; NeQ[c
, 0]] /; FreeQ[{e, f, m, p}, x] && PolyQ[P3, x, 3]
```

rule 2526
```
Int[(Pm_)*(Qn_)^(p_), x_Symbol] :> With[{m = Expon[Pm, x], n = Expon[Qn, x]
}, Simp[Coeff[Pm, x, m]*(Qn^(p + 1)/(n*(p + 1)*Coeff[Qn, x, n])), x] + Simp
[1/(n*Coeff[Qn, x, n])   Int[ExpandToSum[n*Coeff[Qn, x, n]*Pm - Coeff[Pm, x
, m]*D[Qn, x], x]*Qn^p, x], x] /; EqQ[m, n - 1]] /; FreeQ[p, x] && PolyQ[Pm
, x] && PolyQ[Qn, x] && NeQ[p, -1]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.36 (sec) , antiderivative size = 446, normalized size of antiderivative = 0.80

method	result
elliptic	$\frac{\left(\left(\frac{4B}{3969}+\frac{C}{3969}-\frac{17A}{7938}\right)x^2+\left(\frac{B}{1701}-\frac{C}{5103}+\frac{A}{3402}\right)x+\frac{17B}{35721}-\frac{8C}{35721}-\frac{10A}{11907}\right)\sqrt{9x^3+3x^2+6x+2}}{\left(x^3+\frac{1}{3}x^2+\frac{2}{3}x+\frac{2}{9}\right)^2}-\frac{18\left(\left(\frac{935A}{86436}+\frac{149B}{172872}-\frac{673C}{259308}\right)x^2+\cdots\right.}{\sqrt{\cdots}}$
risch	$-\frac{151470x^5A+12069Bx^5-36342x^5C+89235x^4A-25650x^4B-14130Cx^4+179910x^3A-225Bx^3-39930Cx^3+96489Ax^2-23616\cdots}{86436(9x^3+3x^2+6x+2)^{\frac{3}{2}}}$
default	Expression too large to display

input

```
int((C*x^2+B*x+A)/(9*x^3+3*x^2+6*x+2)^(5/2),x,method=_RETURNVERBOSE)
```

output

```
((4/3969*B+1/3969*C-17/7938*A)*x^2+(1/1701*B-1/5103*C+1/3402*A)*x+17/35721
*B-8/35721*C-10/11907*A)*(9*x^3+3*x^2+6*x+2)^(1/2)/(x^3+1/3*x^2+2/3*x+2/9)
^2-18*((935/86436*A+149/172872*B-673/259308*C)*x^2+(205/74088*A-157/74088*
B-4/27783*C)*x+815/172872*A+5/43218*B-835/777924*C)/(9*x^3+3*x^2+6*x+2)^(1
/2)+2*(145/19208*A+52/2401*B-187/28812*C)*(-1/3*I*6^(1/2)+1/3)*((x+1/3)/(-
1/3*I*6^(1/2)+1/3))^(1/2)*((x-1/3*I*6^(1/2))/(-1/3-1/3*I*6^(1/2)))^(1/2)*(
(x+1/3*I*6^(1/2))/(-1/3+1/3*I*6^(1/2)))^(1/2)/(9*x^3+3*x^2+6*x+2)^(1/2)*El
lipticF(((x+1/3)/(-1/3*I*6^(1/2)+1/3))^(1/2),((-1/3+1/3*I*6^(1/2))/(-1/3-1
/3*I*6^(1/2)))^(1/2))+2*(935/9604*A+149/19208*B-673/28812*C)*(-1/3*I*6^(1/
2)+1/3)*((x+1/3)/(-1/3*I*6^(1/2)+1/3))^(1/2)*((x-1/3*I*6^(1/2))/(-1/3-1/3*
I*6^(1/2)))^(1/2)*((x+1/3*I*6^(1/2))/(-1/3+1/3*I*6^(1/2)))^(1/2)/(9*x^3+3*
x^2+6*x+2)^(1/2)*((-1/3-1/3*I*6^(1/2))*EllipticE(((x+1/3)/(-1/3*I*6^(1/2)+
1/3))^(1/2),((-1/3+1/3*I*6^(1/2))/(-1/3-1/3*I*6^(1/2)))^(1/2))+1/3*I*6^(1/
2)*EllipticF(((x+1/3)/(-1/3*I*6^(1/2)+1/3))^(1/2),((-1/3+1/3*I*6^(1/2))/(-
1/3-1/3*I*6^(1/2)))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 352, normalized size of antiderivative = 0.63

$$\int \frac{A + Bx + Cx^2}{(2 + 6x + 3x^2 + 9x^3)^{5/2}}\, dx =$$
$$-\frac{5\,(81\,(339\,A - 2157\,B + 404\,C)x^6 + 54\,(339\,A - 2157\,B + 404\,C)x^5 + 117\,(339\,A - 2157\,B + 404\,C)x^4}{}$$

input
```
integrate((C*x^2+B*x+A)/(9*x^3+3*x^2+6*x+2)^(5/2),x, algorithm="fricas")
```

output
```
-1/777924*(5*(81*(339*A - 2157*B + 404*C)*x^6 + 54*(339*A - 2157*B + 404*C
)*x^5 + 117*(339*A - 2157*B + 404*C)*x^4 + 72*(339*A - 2157*B + 404*C)*x^3
 + 48*(339*A - 2157*B + 404*C)*x^2 + 24*(339*A - 2157*B + 404*C)*x + 1356*
A - 8628*B + 1616*C)*weierstrassPInverse(-68/27, -440/729, x + 1/9) + 9*(8
1*(5610*A + 447*B - 1346*C)*x^6 + 54*(5610*A + 447*B - 1346*C)*x^5 + 117*(
5610*A + 447*B - 1346*C)*x^4 + 72*(5610*A + 447*B - 1346*C)*x^3 + 48*(5610
*A + 447*B - 1346*C)*x^2 + 24*(5610*A + 447*B - 1346*C)*x + 22440*A + 1788
*B - 5384*C)*weierstrassZeta(-68/27, -440/729, weierstrassPInverse(-68/27,
 -440/729, x + 1/9)) + 9*(27*(5610*A + 447*B - 1346*C)*x^5 + 45*(1983*A -
570*B - 314*C)*x^4 + 15*(11994*A - 15*B - 2662*C)*x^3 + 3*(32163*A - 7872*
B - 5398*C)*x^2 + 6*(8427*A - 1605*B - 1516*C)*x + 20550*A - 2972*B - 1772
*C)*sqrt(9*x^3 + 3*x^2 + 6*x + 2))/(81*x^6 + 54*x^5 + 117*x^4 + 72*x^3 + 4
8*x^2 + 24*x + 4)
```

Sympy [F]

$$\int \frac{A + Bx + Cx^2}{(2 + 6x + 3x^2 + 9x^3)^{5/2}}\, dx = \int \frac{A + Bx + Cx^2}{((3x + 1)(3x^2 + 2))^{\frac{5}{2}}}\, dx$$

input
```
integrate((C*x**2+B*x+A)/(9*x**3+3*x**2+6*x+2)**(5/2),x)
```

output
```
Integral((A + B*x + C*x**2)/((3*x + 1)*(3*x**2 + 2))**(5/2), x)
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{(2+6x+3x^2+9x^3)^{5/2}}\,dx = \int \frac{Cx^2+Bx+A}{(9\,x^3+3\,x^2+6\,x+2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(9*x^3+3*x^2+6*x+2)^(5/2),x, algorithm="maxima")
```

output
```
integrate((C*x^2 + B*x + A)/(9*x^3 + 3*x^2 + 6*x + 2)^(5/2), x)
```

Giac [F]

$$\int \frac{A+Bx+Cx^2}{(2+6x+3x^2+9x^3)^{5/2}}\,dx = \int \frac{Cx^2+Bx+A}{(9\,x^3+3\,x^2+6\,x+2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(9*x^3+3*x^2+6*x+2)^(5/2),x, algorithm="giac")
```

output
```
integrate((C*x^2 + B*x + A)/(9*x^3 + 3*x^2 + 6*x + 2)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(2+6x+3x^2+9x^3)^{5/2}}\,dx = \int \frac{C\,x^2+B\,x+A}{(9\,x^3+3\,x^2+6\,x+2)^{5/2}}\,dx$$

input
```
int((A + B*x + C*x^2)/(6*x + 3*x^2 + 9*x^3 + 2)^(5/2),x)
```

output
```
int((A + B*x + C*x^2)/(6*x + 3*x^2 + 9*x^3 + 2)^(5/2), x)
```

Reduce [F]

$$\int \frac{A + Bx + Cx^2}{(2 + 6x + 3x^2 + 9x^3)^{5/2}} \, dx = \text{too large to display}$$

input
```
int((C*x^2+B*x+A)/(9*x^3+3*x^2+6*x+2)^(5/2),x)
```

output
```
( - 2*sqrt(9*x**3 + 3*x**2 + 6*x + 2)*b + 1458*int(sqrt(9*x**3 + 3*x**2 +
6*x + 2)/(729*x**9 + 729*x**8 + 1701*x**7 + 1485*x**6 + 1458*x**5 + 1026*x
**4 + 540*x**3 + 252*x**2 + 72*x + 8),x)*a*x**6 + 972*int(sqrt(9*x**3 + 3*
x**2 + 6*x + 2)/(729*x**9 + 729*x**8 + 1701*x**7 + 1485*x**6 + 1458*x**5 +
 1026*x**4 + 540*x**3 + 252*x**2 + 72*x + 8),x)*a*x**5 + 2106*int(sqrt(9*x
**3 + 3*x**2 + 6*x + 2)/(729*x**9 + 729*x**8 + 1701*x**7 + 1485*x**6 + 145
8*x**5 + 1026*x**4 + 540*x**3 + 252*x**2 + 72*x + 8),x)*a*x**4 + 1296*int(
sqrt(9*x**3 + 3*x**2 + 6*x + 2)/(729*x**9 + 729*x**8 + 1701*x**7 + 1485*x*
*6 + 1458*x**5 + 1026*x**4 + 540*x**3 + 252*x**2 + 72*x + 8),x)*a*x**3 + 8
64*int(sqrt(9*x**3 + 3*x**2 + 6*x + 2)/(729*x**9 + 729*x**8 + 1701*x**7 +
1485*x**6 + 1458*x**5 + 1026*x**4 + 540*x**3 + 252*x**2 + 72*x + 8),x)*a*x
**2 + 432*int(sqrt(9*x**3 + 3*x**2 + 6*x + 2)/(729*x**9 + 729*x**8 + 1701*
x**7 + 1485*x**6 + 1458*x**5 + 1026*x**4 + 540*x**3 + 252*x**2 + 72*x + 8)
,x)*a*x + 72*int(sqrt(9*x**3 + 3*x**2 + 6*x + 2)/(729*x**9 + 729*x**8 + 17
01*x**7 + 1485*x**6 + 1458*x**5 + 1026*x**4 + 540*x**3 + 252*x**2 + 72*x +
 8),x)*a - 1458*int(sqrt(9*x**3 + 3*x**2 + 6*x + 2)/(729*x**9 + 729*x**8 +
 1701*x**7 + 1485*x**6 + 1458*x**5 + 1026*x**4 + 540*x**3 + 252*x**2 + 72*
x + 8),x)*b*x**6 - 972*int(sqrt(9*x**3 + 3*x**2 + 6*x + 2)/(729*x**9 + 729
*x**8 + 1701*x**7 + 1485*x**6 + 1458*x**5 + 1026*x**4 + 540*x**3 + 252*x**
2 + 72*x + 8),x)*b*x**5 - 2106*int(sqrt(9*x**3 + 3*x**2 + 6*x + 2)/(729...
```

3.97 $\int (A+Bx+Cx^2) (bc+bdx+cdx^2+d^2x^3)^p \, dx$

Optimal result

Integrand size = 34, antiderivative size = 405

$$\int (A+Bx+Cx^2) (bc+bdx+cdx^2+d^2x^3)^p \, dx$$
$$= \frac{C(c+dx)(b+dx^2)(bc+bdx+cdx^2+d^2x^3)^p}{3d^2(1+p)}$$
$$+ \frac{(2c^2C-3Bcd-d(bC-3Ad))(c+dx)(bc+bdx+cdx^2+d^2x^3)^p \left(1-\frac{c+dx}{c-\sqrt{-b}\sqrt{d}}\right)^{-p} \left(1-\frac{c+dx}{c+\sqrt{-b}\sqrt{d}}\right)^{-}}{3d^3(1+p)}$$
$$- \frac{(2cC-3Bd)(c+dx)^2(bc+bdx+cdx^2+d^2x^3)^p \left(1-\frac{c+dx}{c-\sqrt{-b}\sqrt{d}}\right)^{-p} \left(1-\frac{c+dx}{c+\sqrt{-b}\sqrt{d}}\right)^{-p} \operatorname{AppellF1}\left(2+\right.}{3d^3(2+p)}$$

output

```
1/3*C*(d*x+c)*(d*x^2+b)*(d^2*x^3+c*d*x^2+b*d*x+b*c)^p/d^2/(p+1)+1/3*(2*C*c
^2-3*B*c*d-d*(-3*A*d+C*b))*(d*x+c)*(d^2*x^3+c*d*x^2+b*d*x+b*c)^p*AppellF1(
p+1,-p,-p,2+p,(d*x+c)/(c-(-b)^(1/2)*d^(1/2)),(d*x+c)/(c+(-b)^(1/2)*d^(1/2)
))/d^3/(p+1)/((1-(d*x+c)/(c-(-b)^(1/2)*d^(1/2)))^p)/((1-(d*x+c)/(c+(-b)^(1
/2)*d^(1/2)))^p)-1/3*(-3*B*d+2*C*c)*(d*x+c)^2*(d^2*x^3+c*d*x^2+b*d*x+b*c)^
p*AppellF1(2+p,-p,-p,3+p,(d*x+c)/(c-(-b)^(1/2)*d^(1/2)),(d*x+c)/(c+(-b)^(1
/2)*d^(1/2)))/d^3/(2+p)/((1-(d*x+c)/(c-(-b)^(1/2)*d^(1/2)))^p)/((1-(d*x+c)
/(c+(-b)^(1/2)*d^(1/2)))^p)
```

Mathematica [F]

$$\int (A+Bx+Cx^2)\left(bc+bdx+cdx^2+d^2x^3\right)^p\,dx$$
$$=\int (A+Bx+Cx^2)\left(bc+bdx+cdx^2+d^2x^3\right)^p\,dx$$

input
```
Integrate[(A + B*x + C*x^2)*(b*c + b*d*x + c*d*x^2 + d^2*x^3)^p,x]
```

output
```
Integrate[(A + B*x + C*x^2)*(b*c + b*d*x + c*d*x^2 + d^2*x^3)^p, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (A+Bx+Cx^2)\left(bc+bdx+cdx^2+d^2x^3\right)^p\,dx$$

$$\Big\downarrow \text{ 2526}$$

$$\frac{\int -d(bC-3Ad+(2cC-3Bd)x)\left(d^2x^3+cdx^2+bdx+bc\right)^p\,dx}{3d^2}+\frac{C\left(bc+bdx+cdx^2+d^2x^3\right)^{p+1}}{3d^2(p+1)}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{C\left(bc+bdx+cdx^2+d^2x^3\right)^{p+1}}{3d^2(p+1)}-\frac{\int d(bC-3Ad+(2cC-3Bd)x)\left(d^2x^3+cdx^2+bdx+bc\right)^p\,dx}{3d^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{C\left(bc+bdx+cdx^2+d^2x^3\right)^{p+1}}{3d^2(p+1)}-\frac{\int (bC-3Ad+(2cC-3Bd)x)\left(d^2x^3+cdx^2+bdx+bc\right)^p\,dx}{3d}$$

$$\Big\downarrow \text{ 2490}$$

$$\frac{C\left(bc+bdx+cdx^2+d^2x^3\right)^{p+1}}{3d^2(p+1)}-$$
$$\frac{\int \left(\frac{3d^2(bC-3Ad)-cd(2cC-3Bd)}{3d^2}+(2cC-3Bd)\left(\frac{c}{3d}+x\right)\right)\left(d^2\left(\frac{c}{3d}+x\right)^3-\frac{1}{3}\left(c^2-3bd\right)\left(\frac{c}{3d}+x\right)+\frac{2c\left(c^2+9bd\right)}{27d}\right)^p d\left(\frac{c}{3d}\right.}{3d}$$

$\downarrow$ 2486

$$\frac{C\left(d^2x^3+cdx^2+bdx+bc\right)^{p+1}}{3d^2(p+1)}-$$
$$\left(\left(\frac{c}{3d}+x\right)d^2+\frac{\left(d^{2/3}c^2-3bd^{5/3}+\left(dc^3+9bd^2c-3\sqrt{3}\sqrt{bd^3\left(c^2+bd\right)^2}\right)^{2/3}\right)d^{2/3}}{3\sqrt[3]{dc^3+9bd^2c-3\sqrt{3}\sqrt{bd^3\left(c^2+bd\right)^2}}}\right)^{-p}\left(\left(\frac{c}{3d}+x\right)^2d^4-\frac{\left(d^{2/3}c^2-3bd^{5/3}+\left(dc^3+9bd^2\right.\right.}{3\sqrt[3]{dc^3+9bd^2c}}\right.$$

$\downarrow$ 27

$$\frac{C\left(d^2x^3+cdx^2+bdx+bc\right)^{p+1}}{3d^2(p+1)}-$$
$$\left(\left(\frac{c}{3d}+x\right)d^2+\frac{\left(d^{2/3}c^2-3bd^{5/3}+\left(dc^3+9bd^2c-3\sqrt{3}\sqrt{bd^3\left(c^2+bd\right)^2}\right)^{2/3}\right)d^{2/3}}{3\sqrt[3]{dc^3+9bd^2c-3\sqrt{3}\sqrt{bd^3\left(c^2+bd\right)^2}}}\right)^{-p}\left(\left(\frac{c}{3d}+x\right)^2d^4-\frac{\left(d^{2/3}c^2-3bd^{5/3}+\left(dc^3+9bd^2\right.\right.}{3\sqrt[3]{dc^3+9bd^2c}}\right.$$

$\downarrow$ 1269

$$\frac{C\left(d^2x^3+cdx^2+bdx+bc\right)^{p+1}}{3d^2(p+1)}-$$
$$\left(\left(\frac{c}{3d}+x\right)d^2+\frac{\left(d^{2/3}c^2-3bd^{5/3}+\left(dc^3+9bd^2c-3\sqrt{3}\sqrt{bd^3\left(c^2+bd\right)^2}\right)^{2/3}\right)d^{2/3}}{3\sqrt[3]{dc^3+9bd^2c-3\sqrt{3}\sqrt{bd^3\left(c^2+bd\right)^2}}}\right)^{-p}\left(\left(\frac{c}{3d}+x\right)^2d^4-\frac{\left(d^{2/3}c^2-3bd^{5/3}+\left(dc^3+9bd^2\right.\right.}{3\sqrt[3]{dc^3+9bd^2c}}\right.$$

$\downarrow$ 7299

$$\frac{C(d^2x^3+cdx^2+bdx+bc)^{p+1}}{3d^2(p+1)}-$$

$$\left(\left(\frac{c}{3d}+x\right)d^2+\frac{\left(d^{2/3}c^2-3bd^{5/3}+\left(dc^3+9bd^2c-3\sqrt{3}\sqrt{bd^3(c^2+bd)^2}\right)^{2/3}\right)d^{2/3}}{3\sqrt[3]{dc^3+9bd^2c-3\sqrt{3}\sqrt{bd^3(c^2+bd)^2}}}\right)^{-p}\left(\left(\frac{c}{3d}+x\right)^2d^4-\frac{\left(d^{2/3}c^2-3bd^{5/3}+\left(dc^3+9bd^2\right.\right.}{3\sqrt[3]{dc^3+9bd^2c}}\right.$$

input
```
Int[(A + B*x + C*x^2)*(b*c + b*d*x + c*d*x^2 + d^2*x^3)^p,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2486
```
Int[((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (d_.)*(x_)^3)^(p_), x_S
ymbol] :> With[{r = Rt[-9*a*d^2 + Sqrt[3]*d*Sqrt[4*b^3*d + 27*a^2*d^2], 3]}
, Simp[(a + b*x + d*x^3)^p/(Simp[18^(1/3)*b*(d/(3*r)) - r/18^(1/3) + d*x, x
]^p*Simp[b*(d/3) + 12^(1/3)*b^2*(d^2/(3*r^2)) + r^2/(3*12^(1/3)) - d*(2^(1/
3)*b*(d/(3^(1/3)*r)) - r/18^(1/3))*x + d^2*x^2, x]^p)   Int[(e + f*x)^m*Sim
p[18^(1/3)*b*(d/(3*r)) - r/18^(1/3) + d*x, x]^p*Simp[b*(d/3) + 12^(1/3)*b^2
*(d^2/(3*r^2)) + r^2/(3*12^(1/3)) - d*(2^(1/3)*b*(d/(3^(1/3)*r)) - r/18^(1/
3))*x + d^2*x^2, x]^p, x], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && NeQ[4*
b^3 + 27*a^2*d, 0] &&  !IntegerQ[p]
```

rule 2490
```
Int[(P3_)^(p_.)*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] :> With[{a = Coeff[P3
, x, 0], b = Coeff[P3, x, 1], c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Su
bst[Int[((3*d*e - c*f)/(3*d) + f*x)^m*Simp[(2*c^3 - 9*b*c*d + 27*a*d^2)/(27
*d^2) - (c^2 - 3*b*d)*(x/(3*d)) + d*x^3, x]^p, x], x, x + c/(3*d)] /; NeQ[c
, 0]] /; FreeQ[{e, f, m, p}, x] && PolyQ[P3, x, 3]
```

rule 2526
```
Int[(Pm_)*(Qn_)^(p_), x_Symbol] :> With[{m = Expon[Pm, x], n = Expon[Qn, x]
}, Simp[Coeff[Pm, x, m]*(Qn^(p + 1)/(n*(p + 1)*Coeff[Qn, x, n])), x] + Simp
[1/(n*Coeff[Qn, x, n])   Int[ExpandToSum[n*Coeff[Qn, x, n]*Pm - Coeff[Pm, x
, m]*D[Qn, x], x]*Qn^p, x], x] /; EqQ[m, n - 1]] /; FreeQ[p, x] && PolyQ[Pm
, x] && PolyQ[Qn, x] && NeQ[p, -1]
```

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int (C x^2 + Bx + A) (d^2x^3 + cd x^2 + bdx + bc)^p dx$$

input
```
int((C*x^2+B*x+A)*(d^2*x^3+c*d*x^2+b*d*x+b*c)^p,x)
```

output
```
int((C*x^2+B*x+A)*(d^2*x^3+c*d*x^2+b*d*x+b*c)^p,x)
```

Fricas [F]

$$\int (A+Bx+Cx^2)\left(bc+bdx+cdx^2+d^2x^3\right)^p\,dx$$
$$=\int \left(Cx^2+Bx+A\right)\left(d^2x^3+cdx^2+bdx+bc\right)^p\,dx$$

input
```
integrate((C*x^2+B*x+A)*(d^2*x^3+c*d*x^2+b*d*x+b*c)^p,x, algorithm="fricas
")
```

output
```
integral((C*x^2 + B*x + A)*(d^2*x^3 + c*d*x^2 + b*d*x + b*c)^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int (A+Bx+Cx^2)\left(bc+bdx+cdx^2+d^2x^3\right)^p\,dx=\text{Timed out}$$

input
```
integrate((C*x**2+B*x+A)*(d**2*x**3+c*d*x**2+b*d*x+b*c)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (A+Bx+Cx^2)\left(bc+bdx+cdx^2+d^2x^3\right)^p\,dx$$
$$=\int \left(Cx^2+Bx+A\right)\left(d^2x^3+cdx^2+bdx+bc\right)^p\,dx$$

input
```
integrate((C*x^2+B*x+A)*(d^2*x^3+c*d*x^2+b*d*x+b*c)^p,x, algorithm="maxima
")
```

output
```
integrate((C*x^2 + B*x + A)*(d^2*x^3 + c*d*x^2 + b*d*x + b*c)^p, x)
```

Giac [F]

$$\int (A+Bx+Cx^2) (bc+bdx+cdx^2+d^2x^3)^p \, dx = \int (Cx^2+Bx+A)(d^2x^3+cdx^2+bdx+bc)^p \, dx$$

input
```
integrate((C*x^2+B*x+A)*(d^2*x^3+c*d*x^2+b*d*x+b*c)^p,x, algorithm="giac")
```

output
```
integrate((C*x^2 + B*x + A)*(d^2*x^3 + c*d*x^2 + b*d*x + b*c)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (A+Bx+Cx^2) (bc+bdx+cdx^2+d^2x^3)^p \, dx = \int (C\,x^2+B\,x+A)\,(d^2\,x^3+c\,d\,x^2+b\,d\,x+b\,c)^p \, dx$$

input
```
int((A + B*x + C*x^2)*(b*c + d^2*x^3 + b*d*x + c*d*x^2)^p,x)
```

output
```
int((A + B*x + C*x^2)*(b*c + d^2*x^3 + b*d*x + c*d*x^2)^p, x)
```

Reduce [F]

$$\int (A+Bx+Cx^2) (bc+bdx+cdx^2+d^2x^3)^p \, dx = \text{too large to display}$$

input
```
int((C*x^2+B*x+A)*(d^2*x^3+c*d*x^2+b*d*x+b*c)^p,x)
```

output
```
(9*(b*c + b*d*x + c*d*x**2 + d**2*x**3)**p*a*b*d**2*p**2 + 15*(b*c + b*d*x
 + c*d*x**2 + d**2*x**3)**p*a*b*d**2*p + 6*(b*c + b*d*x + c*d*x**2 + d**2*
x**3)**p*a*b*d**2 + 9*(b*c + b*d*x + c*d*x**2 + d**2*x**3)**p*a*c*d**2*p**
2*x + 15*(b*c + b*d*x + c*d*x**2 + d**2*x**3)**p*a*c*d**2*p*x + 6*(b*c + b
*d*x + c*d*x**2 + d**2*x**3)**p*a*c*d**2*x + 9*(b*c + b*d*x + c*d*x**2 + d
**2*x**3)**p*b**2*c*d*p**2 + 10*(b*c + b*d*x + c*d*x**2 + d**2*x**3)**p*b*
*2*c*d*p + (b*c + b*d*x + c*d*x**2 + d**2*x**3)**p*b**2*c*d + (b*c + b*d*x
 + c*d*x**2 + d**2*x**3)**p*b*c**3*p**2 - (b*c + b*d*x + c*d*x**2 + d**2*x
**3)**p*b*c**3*p + 9*(b*c + b*d*x + c*d*x**2 + d**2*x**3)**p*b*c**2*d*p**2
*x + 7*(b*c + b*d*x + c*d*x**2 + d**2*x**3)**p*b*c**2*d*p*x + 9*(b*c + b*d
*x + c*d*x**2 + d**2*x**3)**p*b*c*d**2*p**2*x**2 + 12*(b*c + b*d*x + c*d*x
**2 + d**2*x**3)**p*b*c*d**2*p*x**2 + 3*(b*c + b*d*x + c*d*x**2 + d**2*x**
3)**p*b*c*d**2*x**2 - 2*(b*c + b*d*x + c*d*x**2 + d**2*x**3)**p*c**4*p**2*
x - 2*(b*c + b*d*x + c*d*x**2 + d**2*x**3)**p*c**4*p*x + 3*(b*c + b*d*x +
c*d*x**2 + d**2*x**3)**p*c**3*d*p**2*x**2 + (b*c + b*d*x + c*d*x**2 + d**2
*x**3)**p*c**3*d*p*x**2 + 9*(b*c + b*d*x + c*d*x**2 + d**2*x**3)**p*c**2*d
**2*p**2*x**3 + 9*(b*c + b*d*x + c*d*x**2 + d**2*x**3)**p*c**2*d**2*p*x**3
 + 2*(b*c + b*d*x + c*d*x**2 + d**2*x**3)**p*c**2*d**2*x**3 - 81*int((b*c
+ b*d*x + c*d*x**2 + d**2*x**3)**p/(9*b*c*p**2 + 9*b*c*p + 2*b*c + 9*b*d*p
**2*x + 9*b*d*p*x + 2*b*d*x + 9*c*d*p**2*x**2 + 9*c*d*p*x**2 + 2*c*d*x*...
```

3.98 $\int (be+2cex+3dex^2)(a+bx+cx^2+dx^3)^2\,dx$

Optimal result

Integrand size = 34, antiderivative size = 22

$$\int (be+2cex+3dex^2)(a+bx+cx^2+dx^3)^2\,dx=\frac{1}{3}e(a+bx+cx^2+dx^3)^3$$

output
```
1/3*e*(d*x^3+c*x^2+b*x+a)^3
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 49 vs. 2(22) = 44.

Time = 0.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.23

$$\int (be+2cex+3dex^2)(a+bx+cx^2+dx^3)^2\,dx$$
$$=\frac{1}{3}ex(b+x(c+dx))\left(3a^2+3ax(b+x(c+dx))+x^2(b+x(c+dx))^2\right)$$

input
```
Integrate[(b*e + 2*c*e*x + 3*d*e*x^2)*(a + b*x + c*x^2 + d*x^3)^2,x]
```

output
```
(e*x*(b + x*(c + d*x))*(3*a^2 + 3*a*x*(b + x*(c + d*x)) + x^2*(b + x*(c +
d*x))^2))/3
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.029$, Rules used = {2021}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx+cx^2+dx^3)^2 (be+2cex+3dex^2)\,dx$$

$$\Big\downarrow \; 2021$$

$$\frac{1}{3}e(a+bx+cx^2+dx^3)^3$$

input
```
Int[(b*e + 2*c*e*x + 3*d*e*x^2)*(a + b*x + c*x^2 + d*x^3)^2,x]
```

output
```
(e*(a + b*x + c*x^2 + d*x^3)^3)/3
```

Defintions of rubi rules used

rule 2021
```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

method	result
default	$\frac{e\left(d\,x^{3}+c\,x^{2}+bx+a\right)^{3}}{3}$
norman	$\left(a^{2}de+2abce+\frac{1}{3}b^{3}e\right)x^{3}+\left(d^{2}ea+2bcde+\frac{1}{3}e\,c^{3}\right)x^{6}+\left(a^{2}ce+a\,b^{2}e\right)x^{2}+\left(b\,d^{2}e+c^{2}de\right)$
gosper	$\frac{ex\left(d^{3}x^{8}+3c\,d^{2}x^{7}+3b\,d^{2}x^{6}+3c^{2}d\,x^{6}+3x^{5}a\,d^{2}+6x^{5}cbd+c^{3}x^{5}+6acd\,x^{4}+3b^{2}d\,x^{4}+3b\,c^{2}x^{4}+6abd\,x^{3}+3a\,c^{2}x^{3}+3b^{2}c\,x^{3}+3x^{2}d\right.}{3}$
parallelrisch	$\frac{1}{3}d^{3}e\,x^{9}+d^{2}ec\,x^{8}+b\,d^{2}e\,x^{7}+c^{2}de\,x^{7}+a\,d^{2}e\,x^{6}+2bcde\,x^{6}+\frac{1}{3}c^{3}e\,x^{6}+2acde\,x^{5}+b^{2}de\,x^{5}$
risch	$\frac{1}{3}d^{3}e\,x^{9}+d^{2}ec\,x^{8}+b\,d^{2}e\,x^{7}+c^{2}de\,x^{7}+a\,d^{2}e\,x^{6}+2bcde\,x^{6}+\frac{1}{3}c^{3}e\,x^{6}+2acde\,x^{5}+b^{2}de\,x^{5}$
orering	$\frac{x\left(d^{3}x^{8}+3c\,d^{2}x^{7}+3b\,d^{2}x^{6}+3c^{2}d\,x^{6}+3x^{5}a\,d^{2}+6x^{5}cbd+c^{3}x^{5}+6acd\,x^{4}+3b^{2}d\,x^{4}+3b\,c^{2}x^{4}+6abd\,x^{3}+3a\,c^{2}x^{3}+3b^{2}c\,x^{3}+3x^{2}d\,a\right.}{9d\,x^{2}+6cx+3b}$

input
```
int((3*d*e*x^2+2*c*e*x+b*e)*(d*x^3+c*x^2+b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*e*(d*x^3+c*x^2+b*x+a)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 142 vs. $2(20) = 40$.

Time = 0.07 (sec) , antiderivative size = 142, normalized size of antiderivative = 6.45

$$\int \left(be+2cex+3dex^2\right)\left(a+bx+cx^2+dx^3\right)^2\,dx$$
$$=\frac{1}{3}\,d^3ex^9+cd^2ex^8+\left(c^2d+bd^2\right)ex^7+\frac{1}{3}\left(c^3+6\,bcd+3\,ad^2\right)ex^6$$
$$+\left(bc^2+\left(b^2+2\,ac\right)d\right)ex^5+\left(b^2c+ac^2+2\,abd\right)ex^4$$
$$+a^2bex+\frac{1}{3}\left(b^3+6\,abc+3\,a^2d\right)ex^3+\left(ab^2+a^2c\right)ex^2$$

input
```
integrate((3*d*e*x^2+2*c*e*x+b*e)*(d*x^3+c*x^2+b*x+a)^2,x, algorithm="fric
as")
```

output
```
1/3*d^3*e*x^9 + c*d^2*e*x^8 + (c^2*d + b*d^2)*e*x^7 + 1/3*(c^3 + 6*b*c*d +
 3*a*d^2)*e*x^6 + (b*c^2 + (b^2 + 2*a*c)*d)*e*x^5 + (b^2*c + a*c^2 + 2*a*b
*d)*e*x^4 + a^2*b*e*x + 1/3*(b^3 + 6*a*b*c + 3*a^2*d)*e*x^3 + (a*b^2 + a^2
*c)*e*x^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 163 vs. $2(19) = 38$.

Time = 0.04 (sec) , antiderivative size = 163, normalized size of antiderivative = 7.41

$$\int (be + 2cex + 3dex^2)(a + bx + cx^2 + dx^3)^2 \, dx$$
$$= a^2bex + cd^2ex^8 + \frac{d^3ex^9}{3} + x^7(bd^2e + c^2de) + x^6\left(ad^2e + 2bcde + \frac{c^3e}{3}\right)$$
$$+ x^5 \cdot (2acde + b^2de + bc^2e) + x^4 \cdot (2abde + ac^2e + b^2ce)$$
$$+ x^3\left(a^2de + 2abce + \frac{b^3e}{3}\right) + x^2(a^2ce + ab^2e)$$

input
```
integrate((3*d*e*x**2+2*c*e*x+b*e)*(d*x**3+c*x**2+b*x+a)**2,x)
```

output
```
a**2*b*e*x + c*d**2*e*x**8 + d**3*e*x**9/3 + x**7*(b*d**2*e + c**2*d*e) +
x**6*(a*d**2*e + 2*b*c*d*e + c**3*e/3) + x**5*(2*a*c*d*e + b**2*d*e + b*c*
*2*e) + x**4*(2*a*b*d*e + a*c**2*e + b**2*c*e) + x**3*(a**2*d*e + 2*a*b*c*
e + b**3*e/3) + x**2*(a**2*c*e + a*b**2*e)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int (be + 2cex + 3dex^2)(a + bx + cx^2 + dx^3)^2 \, dx = \frac{1}{3}(dx^3 + cx^2 + bx + a)^3 e$$

input
```
integrate((3*d*e*x^2+2*c*e*x+b*e)*(d*x^3+c*x^2+b*x+a)^2,x, algorithm="maxi
ma")
```

output
```
1/3*(d*x^3 + c*x^2 + b*x + a)^3*e
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(20) = 40$.

Time = 0.12 (sec) , antiderivative size = 70, normalized size of antiderivative = 3.18

$$\int (be+2cex+3dex^2)\left(a+bx+cx^2+dx^3\right)^2\,dx$$
$$=(dex^3+cex^2+bex)a^2+\frac{3\left(dex^3+cex^2+bex\right)^2ae+\left(dex^3+cex^2+bex\right)^3}{3\,e^2}$$

input
```
integrate((3*d*e*x^2+2*c*e*x+b*e)*(d*x^3+c*x^2+b*x+a)^2,x, algorithm="giac
")
```

output
```
(d*e*x^3 + c*e*x^2 + b*e*x)*a^2 + 1/3*(3*(d*e*x^3 + c*e*x^2 + b*e*x)^2*a*e
 + (d*e*x^3 + c*e*x^2 + b*e*x)^3)/e^2
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 136, normalized size of antiderivative = 6.18

$$\int (be+2cex+3dex^2)\left(a+bx+cx^2+dx^3\right)^2\,dx$$
$$=\frac{d^3\,e\,x^9}{3}+e\,x^4\left(b^2\,c+2\,a\,d\,b+a\,c^2\right)+e\,x^5\left(d\,b^2+b\,c^2+2\,a\,d\,c\right)+\frac{e\,x^3\left(3\,d\,a^2+6\,c\,a\,b+b^3\right)}{3}+\frac{e\,x^6\left(c^3+6\,b\,c\,d+3\,a\,d^2\right)}{3}+a\,e\,x^2\left(b^2+a\,c\right)+d\,e\,x^7\left(c^2+b\,d\right)+a^2\,b\,e\,x+c\,d^2\,e\,x^8$$

input
```
int((b*e + 2*c*e*x + 3*d*e*x^2)*(a + b*x + c*x^2 + d*x^3)^2,x)
```

output
```
(d^3*e*x^9)/3 + e*x^4*(a*c^2 + b^2*c + 2*a*b*d) + e*x^5*(b*c^2 + b^2*d + 2
*a*c*d) + (e*x^3*(3*a^2*d + b^3 + 6*a*b*c))/3 + (e*x^6*(3*a*d^2 + c^3 + 6*
b*c*d))/3 + a*e*x^2*(a*c + b^2) + d*e*x^7*(b*d + c^2) + a^2*b*e*x + c*d^2*
e*x^8
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 159, normalized size of antiderivative = 7.23

$$\int (be + 2cex + 3dex^2) (a + bx + cx^2 + dx^3)^2 \, dx$$
$$= \frac{ex(d^3x^8 + 3c\,d^2x^7 + 3b\,d^2x^6 + 3c^2d\,x^6 + 3a\,d^2x^5 + 6bcd\,x^5 + c^3x^5 + 6acd\,x^4 + 3b^2d\,x^4 + 3b\,c^2x^4 + 6abd}{3}$$

input
```
int((3*d*e*x^2+2*c*e*x+b*e)*(d*x^3+c*x^2+b*x+a)^2,x)
```

output
```
(e*x*(3*a**2*b + 3*a**2*c*x + 3*a**2*d*x**2 + 3*a*b**2*x + 6*a*b*c*x**2 +
6*a*b*d*x**3 + 3*a*c**2*x**3 + 6*a*c*d*x**4 + 3*a*d**2*x**5 + b**3*x**2 +
3*b**2*c*x**3 + 3*b**2*d*x**4 + 3*b*c**2*x**4 + 6*b*c*d*x**5 + 3*b*d**2*x*
*6 + c**3*x**5 + 3*c**2*d*x**6 + 3*c*d**2*x**7 + d**3*x**8))/3
```

3.99 $\int (be+2cex+3dex^2)(a+bx+cx^2+dx^3)\,dx$

Optimal result

Integrand size = 32, antiderivative size = 22

$$\int (be+2cex+3dex^2)(a+bx+cx^2+dx^3)\,dx = \frac{1}{2}e(a+bx+cx^2+dx^3)^2$$

output
```
1/2*e*(d*x^3+c*x^2+b*x+a)^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.36

$$\int (be+2cex+3dex^2)(a+bx+cx^2+dx^3)\,dx = \frac{1}{2}ex(b+x(c+dx))(2a+x(b+x(c+dx)))$$

input
```
Integrate[(b*e + 2*c*e*x + 3*d*e*x^2)*(a + b*x + c*x^2 + d*x^3),x]
```

output
```
(e*x*(b + x*(c + d*x))*(2*a + x*(b + x*(c + d*x))))/2
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.031$, Rules used = {2021}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a + bx + cx^2 + dx^3) (be + 2cex + 3dex^2) \, dx$$

$$\Big\downarrow \text{2021}$$

$$\frac{1}{2}e(a + bx + cx^2 + dx^3)^2$$

input
```
Int[(b*e + 2*c*e*x + 3*d*e*x^2)*(a + b*x + c*x^2 + d*x^3),x]
```

output
```
(e*(a + b*x + c*x^2 + d*x^3)^2)/2
```

Defintions of rubi rules used

rule 2021
```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

method	result
default	$\frac{e\left(d\,x^{3}+c\,x^{2}+b x+a\right)^{2}}{2}$
gosper	$\frac{e x \left(d^{2} x^{5}+2 c d\,x^{4}+2 b d\,x^{3}+c^{2} x^{3}+2 a d\,x^{2}+2 b c\,x^{2}+2 x a c+b^{2} x+2 a b \right)}{2}$
norman	$\left(a c e+\frac{1}{2} b^{2} e \right) x^{2}+\left(b d e+\frac{1}{2} c^{2} e \right) x^{4}+\left(a d e+b c e \right) x^{3}+a b e x+c d e\,x^{5}+\frac{d^{2} e\,x^{6}}{2}$
parallelrisch	$\frac{1}{2} d^{2} e\,x^{6}+c d e\,x^{5}+b d e\,x^{4}+\frac{1}{2} c^{2} e\,x^{4}+a d e\,x^{3}+b c e\,x^{3}+a c e\,x^{2}+\frac{1}{2} b^{2} e\,x^{2}+a b e x$
risch	$\frac{1}{2} d^{2} e\,x^{6}+c d e\,x^{5}+b d e\,x^{4}+\frac{1}{2} c^{2} e\,x^{4}+a d e\,x^{3}+b c e\,x^{3}+a c e\,x^{2}+\frac{1}{2} b^{2} e\,x^{2}+a b e x+\frac{1}{2} a^{2} e$
orering	$\frac{x \left(d^{2} x^{5}+2 c d\,x^{4}+2 b d\,x^{3}+c^{2} x^{3}+2 a d\,x^{2}+2 b c\,x^{2}+2 x a c+b^{2} x+2 a b \right) \left(3 d e\,x^{2}+2 c e x+e b \right)}{6 d\,x^{2}+4 c x+2 b}$

input
```
int((3*d*e*x^2+2*c*e*x+b*e)*(d*x^3+c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*e*(d*x^3+c*x^2+b*x+a)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 62 vs. $2(20) = 40$.

Time = 0.06 (sec) , antiderivative size = 62, normalized size of antiderivative = 2.82

$$\int \left(be+2cex+3dex^2\right) \left(a+bx+cx^2+dx^3\right) \, dx = \frac{1}{2} \, d^{2} e x^{6} + c d e x^{5} + \frac{1}{2} \, {\left(c^{2} + 2 \, b d\right)} e x^{4} + {\left(b c + a d\right)} e x^{3} + a b e x + \frac{1}{2} \, {\left(b^{2} + 2 \, a c\right)} e x^{2}$$

input
```
integrate((3*d*e*x^2+2*c*e*x+b*e)*(d*x^3+c*x^2+b*x+a),x, algorithm="fricas
")
```

output
```
1/2*d^2*e*x^6 + c*d*e*x^5 + 1/2*(c^2 + 2*b*d)*e*x^4 + (b*c + a*d)*e*x^3 +
a*b*e*x + 1/2*(b^2 + 2*a*c)*e*x^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(19) = 38$.

Time = 0.03 (sec) , antiderivative size = 68, normalized size of antiderivative = 3.09

$$\int (be + 2cex + 3dex^2)(a + bx + cx^2 + dx^3)\,dx$$
$$= abex + cdex^5 + \frac{d^2ex^6}{2} + x^4\left(bde + \frac{c^2e}{2}\right) + x^3(ade + bce) + x^2\left(ace + \frac{b^2e}{2}\right)$$

input
```
integrate((3*d*e*x**2+2*c*e*x+b*e)*(d*x**3+c*x**2+b*x+a),x)
```

output
```
a*b*e*x + c*d*e*x**5 + d**2*e*x**6/2 + x**4*(b*d*e + c**2*e/2) + x**3*(a*d
*e + b*c*e) + x**2*(a*c*e + b**2*e/2)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int (be + 2cex + 3dex^2)(a + bx + cx^2 + dx^3)\,dx = \frac{1}{2}(dx^3 + cx^2 + bx + a)^2 e$$

input
```
integrate((3*d*e*x^2+2*c*e*x+b*e)*(d*x^3+c*x^2+b*x+a),x, algorithm="maxima
")
```

output
```
1/2*(d*x^3 + c*x^2 + b*x + a)^2*e
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 44 vs. $2(20) = 40$.

Time = 0.13 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.00

$$\int \left(be+2cex+3dex^2\right)\left(a+bx+cx^2+dx^3\right)\,dx = \left(dex^3+cex^2+bex\right)a+\frac{\left(dex^3+cex^2+bex\right)^2}{2\,e}$$

input
```
integrate((3*d*e*x^2+2*c*e*x+b*e)*(d*x^3+c*x^2+b*x+a),x, algorithm="giac")
```

output
```
(d*e*x^3 + c*e*x^2 + b*e*x)*a + 1/2*(d*e*x^3 + c*e*x^2 + b*e*x)^2/e
```

Mupad [B] (verification not implemented)

Time = 12.92 (sec) , antiderivative size = 62, normalized size of antiderivative = 2.82

$$\int \left(be+2cex+3dex^2\right)\left(a+bx+cx^2+dx^3\right)\,dx = e\,x^3\,(a\,d+b\,c)+\frac{d^2\,e\,x^6}{2}+\frac{e\,x^2\,(b^2+2\,a\,c)}{2}+\frac{e\,x^4\,(c^2+2\,b\,d)}{2}+c\,d\,e\,x^5+a\,b\,e\,x$$

input
```
int((b*e + 2*c*e*x + 3*d*e*x^2)*(a + b*x + c*x^2 + d*x^3),x)
```

output
```
e*x^3*(a*d + b*c) + (d^2*e*x^6)/2 + (e*x^2*(2*a*c + b^2))/2 + (e*x^4*(2*b*
d + c^2))/2 + c*d*e*x^5 + a*b*e*x
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.77

$$\int \left(be+2cex+3dex^2\right)\left(a+bx+cx^2+dx^3\right)\,dx = \frac{ex(d^2x^5+2cd\,x^4+2bd\,x^3+c^2x^3+2ad\,x^2+2bc\,x^2+2acx+b^2x+2ab)}{2}$$

input

```
int((3*d*e*x^2+2*c*e*x+b*e)*(d*x^3+c*x^2+b*x+a),x)
```

output

```
(e*x*(2*a*b + 2*a*c*x + 2*a*d*x**2 + b**2*x + 2*b*c*x**2 + 2*b*d*x**3 + c*
*2*x**3 + 2*c*d*x**4 + d**2*x**5))/2
```

3.100 $\int \frac{be+2cex+3dex^2}{a+bx+cx^2+dx^3}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 18

$$\int \frac{be+2cex+3dex^2}{a+bx+cx^2+dx^3}\,dx = e\log\left(a+bx+cx^2+dx^3\right)$$

output
```
e*ln(d*x^3+c*x^2+b*x+a)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{be+2cex+3dex^2}{a+bx+cx^2+dx^3}\,dx = e\log(a+x(b+x(c+dx)))$$

input
```
Integrate[(b*e + 2*c*e*x + 3*d*e*x^2)/(a + b*x + c*x^2 + d*x^3),x]
```

output
```
e*Log[a + x*(b + x*(c + d*x))]
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.029, Rules used = {2020}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{be+2cex+3dex^2}{a+bx+cx^2+dx^3}\,dx$$

$$\Big\downarrow \text{2020}$$

$$e\log\left(a+bx+cx^2+dx^3\right)$$

input
```
Int[(b*e + 2*c*e*x + 3*d*e*x^2)/(a + b*x + c*x^2 + d*x^3),x]
```

output
```
e*Log[a + b*x + c*x^2 + d*x^3]
```

Defintions of rubi rules used

rule 2020
```
Int[(Pp_)/(Qq_), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Si
mp[Coeff[Pp, x, p]*(Log[RemoveContent[Qq, x]]/(q*Coeff[Qq, x, q])), x] /; E
qQ[p, q - 1] && EqQ[Pp, Simplify[(Coeff[Pp, x, p]/(q*Coeff[Qq, x, q]))*D[Qq
, x]]]] /; PolyQ[Pp, x] && PolyQ[Qq, x]
```

Maple [A] (warning: unable to verify)

Time = 0.06 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

method	result	size
default	$e\ln\left(d\,x^3+c\,x^2+bx+a\right)$	19
norman	$e\ln\left(d\,x^3+c\,x^2+bx+a\right)$	19
risch	$e\ln\left(d\,x^3+c\,x^2+bx+a\right)$	19
parallelrisch	$e\ln\left(d\,x^3+c\,x^2+bx+a\right)$	19

input
```
int((3*d*e*x^2+2*c*e*x+b*e)/(d*x^3+c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

output
```
e*ln(d*x^3+c*x^2+b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{be+2cex+3dex^2}{a+bx+cx^2+dx^3}\,dx=e\log\left(dx^3+cx^2+bx+a\right)$$

input
```
integrate((3*d*e*x^2+2*c*e*x+b*e)/(d*x^3+c*x^2+b*x+a),x, algorithm="fricas
")
```

output
```
e*log(d*x^3 + c*x^2 + b*x + a)
```

Sympy [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{be+2cex+3dex^2}{a+bx+cx^2+dx^3}\,dx=e\log\left(a+bx+cx^2+dx^3\right)$$

input
```
integrate((3*d*e*x**2+2*c*e*x+b*e)/(d*x**3+c*x**2+b*x+a),x)
```

output
```
e*log(a + b*x + c*x**2 + d*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{be + 2cex + 3dex^2}{a + bx + cx^2 + dx^3} \, dx = e \log \left(dx^3 + cx^2 + bx + a\right)$$

input
```
integrate((3*d*e*x^2+2*c*e*x+b*e)/(d*x^3+c*x^2+b*x+a),x, algorithm="maxima
")
```

output
```
e*log(d*x^3 + c*x^2 + b*x + a)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.50

$$\int \frac{be + 2cex + 3dex^2}{a + bx + cx^2 + dx^3} \, dx = e \log \left(\left|a + \frac{dex^3 + cex^2 + bex}{e}\right|\right)$$

input
```
integrate((3*d*e*x^2+2*c*e*x+b*e)/(d*x^3+c*x^2+b*x+a),x, algorithm="giac")
```

output
```
e*log(abs(a + (d*e*x^3 + c*e*x^2 + b*e*x)/e))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{be + 2cex + 3dex^2}{a + bx + cx^2 + dx^3} \, dx = e \ln \left(d\,x^3 + c\,x^2 + b\,x + a\right)$$

input
```
int((b*e + 2*c*e*x + 3*d*e*x^2)/(a + b*x + c*x^2 + d*x^3),x)
```

output
```
e*log(a + b*x + c*x^2 + d*x^3)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{be + 2cex + 3dex^2}{a + bx + cx^2 + dx^3} \, dx = \log\left(d\,x^3 + c\,x^2 + bx + a\right) e$$

input
```
int((3*d*e*x^2+2*c*e*x+b*e)/(d*x^3+c*x^2+b*x+a),x)
```

output
```
log(a + b*x + c*x**2 + d*x**3)*e
```

3.101 $\int \frac{be+2cex+3dex^2}{(a+bx+cx^2+dx^3)^2}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 20

$$\int \frac{be+2cex+3dex^2}{(a+bx+cx^2+dx^3)^2}\,dx = -\frac{e}{a+bx+cx^2+dx^3}$$

output
```
-e/(d*x^3+c*x^2+b*x+a)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{be+2cex+3dex^2}{(a+bx+cx^2+dx^3)^2}\,dx = -\frac{e}{a+x(b+x(c+dx))}$$

input
```
Integrate[(b*e + 2*c*e*x + 3*d*e*x^2)/(a + b*x + c*x^2 + d*x^3)^2,x]
```

output
```
-(e/(a + x*(b + x*(c + d*x))))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.029$, Rules used = $\{2021\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{be + 2cex + 3dex^2}{(a + bx + cx^2 + dx^3)^2} dx$$

$$\Big\downarrow \text{2021}$$

$$-\frac{e}{a + bx + cx^2 + dx^3}$$

input
```
Int[(b*e + 2*c*e*x + 3*d*e*x^2)/(a + b*x + c*x^2 + d*x^3)^2,x]
```

output
```
-(e/(a + b*x + c*x^2 + d*x^3))
```

Defintions of rubi rules used

rule 2021
```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Maple [A] (warning: unable to verify)

Time = 0.08 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.05

method	result	size
gosper	$-\frac{e}{d\,x^{3}+c\,x^{2}+bx+a}$	21
default	$-\frac{e}{d\,x^{3}+c\,x^{2}+bx+a}$	21
risch	$-\frac{e}{d\,x^{3}+c\,x^{2}+bx+a}$	21
parallelrisch	$-\frac{e}{d\,x^{3}+c\,x^{2}+bx+a}$	21
norman	$\frac{\frac{bex}{a}+\frac{ce\,x^{2}}{a}+\frac{de\,x^{3}}{a}}{d\,x^{3}+c\,x^{2}+bx+a}$	45
orering	$-\frac{3de\,x^{2}+2cex+eb}{(d\,x^{3}+c\,x^{2}+bx+a)(3d\,x^{2}+2cx+b)}$	50

input
```
int((3*d*e*x^2+2*c*e*x+b*e)/(d*x^3+c*x^2+b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-e/(d*x^3+c*x^2+b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{be+2cex+3dex^2}{(a+bx+cx^2+dx^3)^2}\,dx=-\frac{e}{dx^3+cx^2+bx+a}$$

input
```
integrate((3*d*e*x^2+2*c*e*x+b*e)/(d*x^3+c*x^2+b*x+a)^2,x, algorithm="fric
as")
```

output
```
-e/(d*x^3 + c*x^2 + b*x + a)
```

Sympy [A] (verification not implemented)

Time = 1.28 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{be+2cex+3dex^2}{(a+bx+cx^2+dx^3)^2}\,dx=-\frac{e}{a+bx+cx^2+dx^3}$$

input
```
integrate((3*d*e*x**2+2*c*e*x+b*e)/(d*x**3+c*x**2+b*x+a)**2,x)
```

output
```
-e/(a + b*x + c*x**2 + d*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{be + 2cex + 3dex^2}{(a + bx + cx^2 + dx^3)^2} dx = -\frac{e}{dx^3 + cx^2 + bx + a}$$

input
```
integrate((3*d*e*x^2+2*c*e*x+b*e)/(d*x^3+c*x^2+b*x+a)^2,x, algorithm="maxi
ma")
```

output
```
-e/(d*x^3 + c*x^2 + b*x + a)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.40

$$\int \frac{be + 2cex + 3dex^2}{(a + bx + cx^2 + dx^3)^2} dx = -\frac{e}{a + \frac{dex^3 + cex^2 + bex}{e}}$$

input
```
integrate((3*d*e*x^2+2*c*e*x+b*e)/(d*x^3+c*x^2+b*x+a)^2,x, algorithm="giac
")
```

output
```
-e/(a + (d*e*x^3 + c*e*x^2 + b*e*x)/e)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{be+2cex+3dex^2}{(a+bx+cx^2+dx^3)^2}\,dx = -\frac{e}{d\,x^3+c\,x^2+b\,x+a}$$

input
```
int((b*e + 2*c*e*x + 3*d*e*x^2)/(a + b*x + c*x^2 + d*x^3)^2,x)
```

output
```
-e/(a + b*x + c*x^2 + d*x^3)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{be+2cex+3dex^2}{(a+bx+cx^2+dx^3)^2}\,dx = -\frac{e}{d\,x^3+c\,x^2+bx+a}$$

input
```
int((3*d*e*x^2+2*c*e*x+b*e)/(d*x^3+c*x^2+b*x+a)^2,x)
```

output
```
( - e)/(a + b*x + c*x**2 + d*x**3)
```

3.102 $\int \frac{be+2cex+3dex^2}{(a+bx+cx^2+dx^3)^3}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 22

$$\int \frac{be+2cex+3dex^2}{(a+bx+cx^2+dx^3)^3}\,dx = -\frac{e}{2\left(a+bx+cx^2+dx^3\right)^2}$$

output
```
-1/2*e/(d*x^3+c*x^2+b*x+a)^2
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{be+2cex+3dex^2}{(a+bx+cx^2+dx^3)^3}\,dx = -\frac{e}{2(a+x(b+x(c+dx)))^2}$$

input
```
Integrate[(b*e + 2*c*e*x + 3*d*e*x^2)/(a + b*x + c*x^2 + d*x^3)^3,x]
```

output
```
-1/2*e/(a + x*(b + x*(c + d*x)))^2
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.029$, Rules used = {2021}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{be + 2cex + 3dex^2}{(a + bx + cx^2 + dx^3)^3} \, dx$$

$$\Big\downarrow \text{2021}$$

$$-\frac{e}{2(a + bx + cx^2 + dx^3)^2}$$

input
```
Int[(b*e + 2*c*e*x + 3*d*e*x^2)/(a + b*x + c*x^2 + d*x^3)^3,x]
```

output
```
-1/2*e/(a + b*x + c*x^2 + d*x^3)^2
```

Defintions of rubi rules used

rule 2021
```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Maple [A] (warning: unable to verify)

Time = 0.12 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

method	result	size
gosper	$-\frac{e}{2(d\,x^3+c\,x^2+bx+a)^2}$	21
default	$-\frac{e}{2(d\,x^3+c\,x^2+bx+a)^2}$	21
norman	$-\frac{e}{2(d\,x^3+c\,x^2+bx+a)^2}$	21
risch	$-\frac{e}{2(d\,x^3+c\,x^2+bx+a)^2}$	21
parallelrisch	$-\frac{e}{2(d\,x^3+c\,x^2+bx+a)^2}$	21
orering	$-\frac{3de\,x^2+2cex+eb}{2(d\,x^3+c\,x^2+bx+a)^2(3d\,x^2+2cx+b)}$	50

input
```
int((3*d*e*x^2+2*c*e*x+b*e)/(d*x^3+c*x^2+b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*e/(d*x^3+c*x^2+b*x+a)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 64 vs. $2(20) = 40$.

Time = 0.07 (sec) , antiderivative size = 64, normalized size of antiderivative = 2.91

$$\int \frac{be+2cex+3dex^2}{(a+bx+cx^2+dx^3)^3}\,dx$$
$$= -\frac{e}{2\left(d^2x^6+2\,cdx^5+(c^2+2\,bd)x^4+2\,(bc+ad)x^3+2\,abx+(b^2+2\,ac)x^2+a^2\right)}$$

input
```
integrate((3*d*e*x^2+2*c*e*x+b*e)/(d*x^3+c*x^2+b*x+a)^3,x, algorithm="fric
as")
```

output
```
-1/2*e/(d^2*x^6 + 2*c*d*x^5 + (c^2 + 2*b*d)*x^4 + 2*(b*c + a*d)*x^3 + 2*a*
b*x + (b^2 + 2*a*c)*x^2 + a^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(20) = 40$.

Time = 2.71 (sec) , antiderivative size = 71, normalized size of antiderivative = 3.23

$$\int \frac{be + 2cex + 3dex^2}{(a + bx + cx^2 + dx^3)^3}\,dx = -\frac{e}{2a^2 + 4abx + 4cdx^5 + 2d^2x^6 + x^4\cdot(4bd + 2c^2) + x^3\cdot(4ad + 4bc) + x^2\cdot(4ac + 2b^2)}$$

input
```
integrate((3*d*e*x**2+2*c*e*x+b*e)/(d*x**3+c*x**2+b*x+a)**3,x)
```

output
```
-e/(2*a**2 + 4*a*b*x + 4*c*d*x**5 + 2*d**2*x**6 + x**4*(4*b*d + 2*c**2) +
x**3*(4*a*d + 4*b*c) + x**2*(4*a*c + 2*b**2))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{be + 2cex + 3dex^2}{(a + bx + cx^2 + dx^3)^3}\,dx = -\frac{e}{2\,(dx^3 + cx^2 + bx + a)^2}$$

input
```
integrate((3*d*e*x^2+2*c*e*x+b*e)/(d*x^3+c*x^2+b*x+a)^3,x, algorithm="maxi
ma")
```

output
```
-1/2*e/(d*x^3 + c*x^2 + b*x + a)^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.27

$$\int \frac{be + 2cex + 3dex^2}{(a + bx + cx^2 + dx^3)^3}\, dx = -\frac{e}{2\left(a + \frac{dex^3 + cex^2 + bex}{e}\right)^2}$$

input
```
integrate((3*d*e*x^2+2*c*e*x+b*e)/(d*x^3+c*x^2+b*x+a)^3,x, algorithm="giac
")
```

output
```
-1/2*e/(a + (d*e*x^3 + c*e*x^2 + b*e*x)/e)^2
```

Mupad [B] (verification not implemented)

Time = 12.12 (sec) , antiderivative size = 71, normalized size of antiderivative = 3.23

$$\int \frac{be + 2cex + 3dex^2}{(a + bx + cx^2 + dx^3)^3}\, dx = -\frac{e}{2\,\left(x^2\,\left(b^2 + 2\,a\,c\right) + x^4\,\left(c^2 + 2\,b\,d\right) + a^2 + x^3\,\left(2\,a\,d + 2\,b\,c\right) + d^2\,x^6 + 2\,a\,b\,x + 2\,c\,d\,x^5\right)}$$

input
```
int((b*e + 2*c*e*x + 3*d*e*x^2)/(a + b*x + c*x^2 + d*x^3)^3,x)
```

output
```
-e/(2*(x^2*(2*a*c + b^2) + x^4*(2*b*d + c^2) + a^2 + x^3*(2*a*d + 2*b*c) +
 d^2*x^6 + 2*a*b*x + 2*c*d*x^5))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 75, normalized size of antiderivative = 3.41

$$\int \frac{be + 2cex + 3dex^2}{(a + bx + cx^2 + dx^3)^3}\, dx = -\frac{e}{2d^2x^6 + 4cd\,x^5 + 4bd\,x^4 + 2c^2x^4 + 4ad\,x^3 + 4bc\,x^3 + 4ac\,x^2 + 2b^2x^2 + 4abx + 2a^2}$$

input
```
int((3*d*e*x^2+2*c*e*x+b*e)/(d*x^3+c*x^2+b*x+a)^3,x)
```

output
```
( - e)/(2*(a**2 + 2*a*b*x + 2*a*c*x**2 + 2*a*d*x**3 + b**2*x**2 + 2*b*c*x*
*3 + 2*b*d*x**4 + c**2*x**4 + 2*c*d*x**5 + d**2*x**6))
```

3.103 $\int (be+2cex+3dex^2)(a+bx+cx^2+dx^3)^{3/2}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 24

$$\int (be+2cex+3dex^2)(a+bx+cx^2+dx^3)^{3/2}\,dx=\frac{2}{5}e(a+bx+cx^2+dx^3)^{5/2}$$

output
```
2/5*e*(d*x^3+c*x^2+b*x+a)^(5/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int (be+2cex+3dex^2)(a+bx+cx^2+dx^3)^{3/2}\,dx=\frac{2}{5}e(a+x(b+x(c+dx)))^{5/2}$$

input
```
Integrate[(b*e + 2*c*e*x + 3*d*e*x^2)*(a + b*x + c*x^2 + d*x^3)^(3/2),x]
```

output
```
(2*e*(a + x*(b + x*(c + d*x)))^(5/2))/5
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.028$, Rules used = {2021}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a + bx + cx^2 + dx^3)^{3/2} (be + 2cex + 3dex^2) \, dx$$

$$\Big\downarrow \text{2021}$$

$$\frac{2}{5} e (a + bx + cx^2 + dx^3)^{5/2}$$

input
```
Int[(b*e + 2*c*e*x + 3*d*e*x^2)*(a + b*x + c*x^2 + d*x^3)^(3/2),x]
```

output
```
(2*e*(a + b*x + c*x^2 + d*x^3)^(5/2))/5
```

Defintions of rubi rules used

rule 2021
```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.88

method	result
gosper	$\frac{2e(d\,x^3+c\,x^2+bx+a)^{\frac{5}{2}}}{5}$
default	$\frac{2e(d\,x^3+c\,x^2+bx+a)^{\frac{5}{2}}}{5}$
pseudoelliptic	$\frac{2e(d\,x^3+c\,x^2+bx+a)^{\frac{5}{2}}}{5}$
orering	$\frac{2(d\,x^3+c\,x^2+bx+a)^{\frac{5}{2}}(3de\,x^2+2cex+eb)}{5(3d\,x^2+2cx+b)}$
risch	$\frac{2e(d^2x^6+2cd\,x^5+2bd\,x^4+c^2x^4+2ad\,x^3+2bc\,x^3+2ac\,x^2+b^2x^2+2abx+a^2)\sqrt{d\,x^3+c\,x^2+bx+a}}{5}$
trager	$e\left(\frac{2}{5}d^2x^6+\frac{4}{5}cd\,x^5+\frac{4}{5}bd\,x^4+\frac{2}{5}c^2x^4+\frac{4}{5}ad\,x^3+\frac{4}{5}bc\,x^3+\frac{4}{5}ac\,x^2+\frac{2}{5}b^2x^2+\frac{4}{5}abx+\frac{2}{5}a^2\right)\sqrt{}$

input
```
int((3*d*e*x^2+2*c*e*x+b*e)*(d*x^3+c*x^2+b*x+a)^(3/2),x,method=_RETURNVERB
OSE)
```

output
```
2/5*e*(d*x^3+c*x^2+b*x+a)^(5/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 86 vs. $2(20) = 40$.

Time = 0.07 (sec) , antiderivative size = 86, normalized size of antiderivative = 3.58

$$\int (be+2cex+3dex^2)\left(a+bx+cx^2+dx^3\right)^{3/2}\,dx=\frac{2}{5}\left(d^2ex^6+2\,cdex^5+\left(c^2+2\,bd\right)ex^4+2\,(bc+ad)ex^3+2\,abex+\left(b^2+2\,ac\right)ex^2+a^2e\right)\sqrt{dx}$$

input
```
integrate((3*d*e*x^2+2*c*e*x+b*e)*(d*x^3+c*x^2+b*x+a)^(3/2),x, algorithm="
fricas")
```

output
```
2/5*(d^2*e*x^6 + 2*c*d*e*x^5 + (c^2 + 2*b*d)*e*x^4 + 2*(b*c + a*d)*e*x^3 +
 2*a*b*e*x + (b^2 + 2*a*c)*e*x^2 + a^2*e)*sqrt(d*x^3 + c*x^2 + b*x + a)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 299 vs. $2(22) = 44$.

Time = 0.21 (sec) , antiderivative size = 299, normalized size of antiderivative = 12.46

$$\int \left(be + 2cex + 3dex^2\right) \left(a + bx + cx^2 + dx^3\right)^{3/2} \, dx = \frac{2a^2e\sqrt{a + bx + cx^2 + dx^3}}{5} + \frac{4abex\sqrt{a + bx + cx^2 + dx^3}}{5} + \frac{4acex^2\sqrt{a + bx + cx^2 + dx^3}}{5} + \frac{4adex^3\sqrt{a + bx + cx^2 + dx^3}}{5} + \frac{2b^2ex^2\sqrt{a + bx + cx^2 + dx^3}}{5} + \frac{4bcex^3\sqrt{a + bx + cx^2 + dx^3}}{5} + \frac{4bdex^4\sqrt{a + bx + cx^2 + dx^3}}{5} + \frac{2c^2ex^4\sqrt{a + bx + cx^2 + dx^3}}{5} + \frac{4cdex^5\sqrt{a + bx + cx^2 + dx^3}}{5} + \frac{2d^2ex^6\sqrt{a + bx + cx^2 + dx^3}}{5}$$

input
```
integrate((3*d*e*x**2+2*c*e*x+b*e)*(d*x**3+c*x**2+b*x+a)**(3/2),x)
```

output
```
2*a**2*e*sqrt(a + b*x + c*x**2 + d*x**3)/5 + 4*a*b*e*x*sqrt(a + b*x + c*x*
*2 + d*x**3)/5 + 4*a*c*e*x**2*sqrt(a + b*x + c*x**2 + d*x**3)/5 + 4*a*d*e*
x**3*sqrt(a + b*x + c*x**2 + d*x**3)/5 + 2*b**2*e*x**2*sqrt(a + b*x + c*x*
*2 + d*x**3)/5 + 4*b*c*e*x**3*sqrt(a + b*x + c*x**2 + d*x**3)/5 + 4*b*d*e*
x**4*sqrt(a + b*x + c*x**2 + d*x**3)/5 + 2*c**2*e*x**4*sqrt(a + b*x + c*x*
*2 + d*x**3)/5 + 4*c*d*e*x**5*sqrt(a + b*x + c*x**2 + d*x**3)/5 + 2*d**2*e
*x**6*sqrt(a + b*x + c*x**2 + d*x**3)/5
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \left(be + 2cex + 3dex^2\right) \left(a + bx + cx^2 + dx^3\right)^{3/2} \, dx = \frac{2}{5} \left(dx^3 + cx^2 + bx + a\right)^{\frac{5}{2}} e$$

input
```
integrate((3*d*e*x^2+2*c*e*x+b*e)*(d*x^3+c*x^2+b*x+a)^(3/2),x, algorithm="
maxima")
```

output
```
2/5*(d*x^3 + c*x^2 + b*x + a)^(5/2)*e
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \left(be+2cex+3dex^2\right)\left(a+bx+cx^2+dx^3\right)^{3/2}\,dx=\frac{2}{5}\left(dx^3+cx^2+bx+a\right)^{\frac{5}{2}}e$$

input
```
integrate((3*d*e*x^2+2*c*e*x+b*e)*(d*x^3+c*x^2+b*x+a)^(3/2),x, algorithm="
giac")
```

output
```
2/5*(d*x^3 + c*x^2 + b*x + a)^(5/2)*e
```

Mupad [B] (verification not implemented)

Time = 12.29 (sec) , antiderivative size = 89, normalized size of antiderivative = 3.71

$$\int \left(be+2cex+3dex^2\right)\left(a+bx+cx^2+dx^3\right)^{3/2}\,dx=\sqrt{d\,x^3+c\,x^2+b\,x+a}\left(\frac{2\,a^2\,e}{5}+\frac{4\,e\,x^3\,(a\,d+b\,c)}{5}+\frac{2\,d^2\,e\,x^6}{5}+\frac{2\,e\,x^2\,(b^2+2\,a\,c)}{5}+\frac{2\,e\,x^4\,(c^2+2\,b\,d)}{5}+\frac{4\,c\,d\,e\,x^5}{5}+\frac{4\,a\,b\,e\,x}{5}\right)$$

input
```
int((b*e + 2*c*e*x + 3*d*e*x^2)*(a + b*x + c*x^2 + d*x^3)^(3/2),x)
```

output
```
(a + b*x + c*x^2 + d*x^3)^(1/2)*((2*a^2*e)/5 + (4*e*x^3*(a*d + b*c))/5 + (
2*d^2*e*x^6)/5 + (2*e*x^2*(2*a*c + b^2))/5 + (2*e*x^4*(2*b*d + c^2))/5 + (
4*c*d*e*x^5)/5 + (4*a*b*e*x)/5)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 84, normalized size of antiderivative = 3.50

$$\int \left(be+2cex+3dex^2\right)\left(a+bx+cx^2+dx^3\right)^{3/2}\,dx=\frac{2\sqrt{d\,x^3+c\,x^2+bx+a}\,e(d^2x^6+2cd\,x^5+2bd\,x^4+c^2x^4+2ad\,x^3+2bc\,x^3+2ac\,x^2+b^2x^2+}{5}$$

input
```
int((3*d*e*x^2+2*c*e*x+b*e)*(d*x^3+c*x^2+b*x+a)^(3/2),x)
```

output
```
(2*sqrt(a + b*x + c*x**2 + d*x**3)*e*(a**2 + 2*a*b*x + 2*a*c*x**2 + 2*a*d*
x**3 + b**2*x**2 + 2*b*c*x**3 + 2*b*d*x**4 + c**2*x**4 + 2*c*d*x**5 + d**2
*x**6))/5
```

3.104 $\int (be+2cex+3dex^2) \sqrt{a+bx+cx^2+dx^3} \, dx$

Optimal result

Integrand size = 36, antiderivative size = 24

$$\int (be+2cex+3dex^2) \sqrt{a+bx+cx^2+dx^3} \, dx = \frac{2}{3}e\left(a+bx+cx^2+dx^3\right)^{3/2}$$

output
```
2/3*e*(d*x^3+c*x^2+b*x+a)^(3/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int (be+2cex+3dex^2) \sqrt{a+bx+cx^2+dx^3} \, dx = \frac{2}{3}e(a+x(b+x(c+dx)))^{3/2}$$

input
```
Integrate[(b*e + 2*c*e*x + 3*d*e*x^2)*Sqrt[a + b*x + c*x^2 + d*x^3],x]
```

output
```
(2*e*(a + x*(b + x*(c + d*x)))^(3/2))/3
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.028$, Rules used = {2021}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+bx+cx^2+dx^3}\left(be+2cex+3dex^2\right) dx$$

$$\Big\downarrow \text{2021}$$

$$\frac{2}{3}e\left(a+bx+cx^2+dx^3\right)^{3/2}$$

input
```
Int[(b*e + 2*c*e*x + 3*d*e*x^2)*Sqrt[a + b*x + c*x^2 + d*x^3],x]
```

output
```
(2*e*(a + b*x + c*x^2 + d*x^3)^(3/2))/3
```

Defintions of rubi rules used

rule 2021
```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.88

method	result	size
gosper	$\frac{2e\left(d\,x^{3}+c\,x^{2}+bx+a\right)^{\frac{3}{2}}}{3}$	21
default	$\frac{2e\left(d\,x^{3}+c\,x^{2}+bx+a\right)^{\frac{3}{2}}}{3}$	21
risch	$\frac{2e\left(d\,x^{3}+c\,x^{2}+bx+a\right)^{\frac{3}{2}}}{3}$	21
pseudoelliptic	$\frac{2e\left(d\,x^{3}+c\,x^{2}+bx+a\right)^{\frac{3}{2}}}{3}$	21
trager	$e\left(\frac{2}{3}d\,x^{3}+\frac{2}{3}c\,x^{2}+\frac{2}{3}bx+\frac{2}{3}a\right)\sqrt{d\,x^{3}+c\,x^{2}+bx+a}$	40
orering	$\frac{2\left(3de\,x^{2}+2cex+eb\right)\left(d\,x^{3}+c\,x^{2}+bx+a\right)^{\frac{3}{2}}}{3\left(3d\,x^{2}+2cx+b\right)}$	50

input
```
int((3*d*e*x^2+2*c*e*x+b*e)*(d*x^3+c*x^2+b*x+a)^(1/2),x,method=_RETURNVERB
OSE)
```

output
```
2/3*e*(d*x^3+c*x^2+b*x+a)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.62

$$\int \left(be+2cex+3dex^{2}\right)\sqrt{a+bx+cx^{2}+dx^{3}}\,dx = \frac{2}{3}\left(dex^{3}+cex^{2}+bex+ae\right)\sqrt{dx^{3}+cx^{2}+bx+a}$$

input
```
integrate((3*d*e*x^2+2*c*e*x+b*e)*(d*x^3+c*x^2+b*x+a)^(1/2),x, algorithm="
fricas")
```

output
```
2/3*(d*e*x^3 + c*e*x^2 + b*e*x + a*e)*sqrt(d*x^3 + c*x^2 + b*x + a)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 109 vs. $2(22) = 44$.

Time = 0.11 (sec) , antiderivative size = 109, normalized size of antiderivative = 4.54

$$\int (be + 2cex + 3dex^2)\sqrt{a + bx + cx^2 + dx^3}\,dx = \frac{2ae\sqrt{a + bx + cx^2 + dx^3}}{3} + \frac{2bex\sqrt{a + bx + cx^2 + dx^3}}{3} + \frac{2cex^2\sqrt{a + bx + cx^2 + dx^3}}{3} + \frac{2dex^3\sqrt{a + bx + cx^2 + dx^3}}{3}$$

input
```
integrate((3*d*e*x**2+2*c*e*x+b*e)*(d*x**3+c*x**2+b*x+a)**(1/2),x)
```

output
```
2*a*e*sqrt(a + b*x + c*x**2 + d*x**3)/3 + 2*b*e*x*sqrt(a + b*x + c*x**2 +
d*x**3)/3 + 2*c*e*x**2*sqrt(a + b*x + c*x**2 + d*x**3)/3 + 2*d*e*x**3*sqrt
(a + b*x + c*x**2 + d*x**3)/3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int (be + 2cex + 3dex^2)\sqrt{a + bx + cx^2 + dx^3}\,dx = \frac{2}{3}\left(dx^3 + cx^2 + bx + a\right)^{\frac{3}{2}}e$$

input
```
integrate((3*d*e*x^2+2*c*e*x+b*e)*(d*x^3+c*x^2+b*x+a)^(1/2),x, algorithm="
maxima")
```

output
```
2/3*(d*x^3 + c*x^2 + b*x + a)^(3/2)*e
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \left(be+2cex+3dex^2\right)\sqrt{a+bx+cx^2+dx^3}\,dx=\frac{2}{3}\left(dx^3+cx^2+bx+a\right)^{\frac{3}{2}}e$$

input
```
integrate((3*d*e*x^2+2*c*e*x+b*e)*(d*x^3+c*x^2+b*x+a)^(1/2),x, algorithm="
giac")
```

output
```
2/3*(d*x^3 + c*x^2 + b*x + a)^(3/2)*e
```

Mupad [B] (verification not implemented)

Time = 12.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \left(be+2cex+3dex^2\right)\sqrt{a+bx+cx^2+dx^3}\,dx=\frac{2\,e\,{\left(d\,x^3+c\,x^2+b\,x+a\right)}^{3/2}}{3}$$

input
```
int((b*e + 2*c*e*x + 3*d*e*x^2)*(a + b*x + c*x^2 + d*x^3)^(1/2),x)
```

output
```
(2*e*(a + b*x + c*x^2 + d*x^3)^(3/2))/3
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.42

$$\int \left(be+2cex+3dex^2\right)\sqrt{a+bx+cx^2+dx^3}\,dx$$
$$=\frac{2\sqrt{d\,x^3+c\,x^2+bx+a}\,e(d\,x^3+c\,x^2+bx+a)}{3}$$

input
```
int((3*d*e*x^2+2*c*e*x+b*e)*(d*x^3+c*x^2+b*x+a)^(1/2),x)
```

output
```
(2*sqrt(a + b*x + c*x**2 + d*x**3)*e*(a + b*x + c*x**2 + d*x**3))/3
```

3.105 $\int \frac{be+2cex+3dex^2}{\sqrt{a+bx+cx^2+dx^3}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 22

$$\int \frac{be+2cex+3dex^2}{\sqrt{a+bx+cx^2+dx^3}}\,dx = 2e\sqrt{a+bx+cx^2+dx^3}$$

output
```
2*e*(d*x^3+c*x^2+b*x+a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{be+2cex+3dex^2}{\sqrt{a+bx+cx^2+dx^3}}\,dx = 2e\sqrt{a+x(b+x(c+dx))}$$

input
```
Integrate[(b*e + 2*c*e*x + 3*d*e*x^2)/Sqrt[a + b*x + c*x^2 + d*x^3],x]
```

output
```
2*e*Sqrt[a + x*(b + x*(c + d*x))]
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.028, Rules used = {2021}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{be+2cex+3dex^2}{\sqrt{a+bx+cx^2+dx^3}}\,dx$$
$$\Big\downarrow \text{ 2021}$$
$$2e\sqrt{a+bx+cx^2+dx^3}$$

input
```
Int[(b*e + 2*c*e*x + 3*d*e*x^2)/Sqrt[a + b*x + c*x^2 + d*x^3],x]
```

output
```
2*e*Sqrt[a + b*x + c*x^2 + d*x^3]
```

Defintions of rubi rules used

rule 2021
```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 2.27 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

method	result	size
gosper	$2e\sqrt{d\,x^3+c\,x^2+bx+a}$	21
default	$2e\sqrt{d\,x^3+c\,x^2+bx+a}$	21
trager	$2e\sqrt{d\,x^3+c\,x^2+bx+a}$	21
risch	$2e\sqrt{d\,x^3+c\,x^2+bx+a}$	21
elliptic	$2e\sqrt{d\,x^3+c\,x^2+bx+a}$	21
pseudoelliptic	$2e\sqrt{d\,x^3+c\,x^2+bx+a}$	21
orering	$\frac{2(3de\,x^2+2cex+eb)\sqrt{d\,x^3+c\,x^2+bx+a}}{3d\,x^2+2cx+b}$	50

input
```
int((3*d*e*x^2+2*c*e*x+b*e)/(d*x^3+c*x^2+b*x+a)^(1/2),x,method=_RETURNVERB
OSE)
```

output
```
2*e*(d*x^3+c*x^2+b*x+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{be+2cex+3dex^2}{\sqrt{a+bx+cx^2+dx^3}}\,dx = 2\sqrt{dx^3+cx^2+bx+a}e$$

input
```
integrate((3*d*e*x^2+2*c*e*x+b*e)/(d*x^3+c*x^2+b*x+a)^(1/2),x, algorithm="
fricas")
```

output
```
2*sqrt(d*x^3 + c*x^2 + b*x + a)*e
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{be + 2cex + 3dex^2}{\sqrt{a + bx + cx^2 + dx^3}}\, dx = 2e\sqrt{a + bx + cx^2 + dx^3}$$

input
```
integrate((3*d*e*x**2+2*c*e*x+b*e)/(d*x**3+c*x**2+b*x+a)**(1/2),x)
```

output
```
2*e*sqrt(a + b*x + c*x**2 + d*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{be + 2cex + 3dex^2}{\sqrt{a + bx + cx^2 + dx^3}}\, dx = 2\sqrt{dx^3 + cx^2 + bx + a}e$$

input
```
integrate((3*d*e*x^2+2*c*e*x+b*e)/(d*x^3+c*x^2+b*x+a)^(1/2),x, algorithm="
maxima")
```

output
```
2*sqrt(d*x^3 + c*x^2 + b*x + a)*e
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{be + 2cex + 3dex^2}{\sqrt{a + bx + cx^2 + dx^3}}\, dx = 2\sqrt{dx^3 + cx^2 + bx + a}e$$

input
```
integrate((3*d*e*x^2+2*c*e*x+b*e)/(d*x^3+c*x^2+b*x+a)^(1/2),x, algorithm="
giac")
```

output
```
2*sqrt(d*x^3 + c*x^2 + b*x + a)*e
```

Mupad [B] (verification not implemented)

Time = 12.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{be+2cex+3dex^2}{\sqrt{a+bx+cx^2+dx^3}}\,dx=2\,e\,\sqrt{d\,x^3+c\,x^2+b\,x+a}$$

input
```
int((b*e + 2*c*e*x + 3*d*e*x^2)/(a + b*x + c*x^2 + d*x^3)^(1/2),x)
```

output
```
2*e*(a + b*x + c*x^2 + d*x^3)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

$$\int \frac{be+2cex+3dex^2}{\sqrt{a+bx+cx^2+dx^3}}\,dx=2\sqrt{d\,x^3+c\,x^2+bx+a}\,e$$

input
```
int((3*d*e*x^2+2*c*e*x+b*e)/(d*x^3+c*x^2+b*x+a)^(1/2),x)
```

output
```
2*sqrt(a + b*x + c*x**2 + d*x**3)*e
```

3.106 $\int \frac{be+2cex+3dex^2}{(a+bx+cx^2+dx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 22

$$\int \frac{be+2cex+3dex^2}{(a+bx+cx^2+dx^3)^{3/2}}\,dx=-\frac{2e}{\sqrt{a+bx+cx^2+dx^3}}$$

output
```
-2*e/(d*x^3+c*x^2+b*x+a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{be+2cex+3dex^2}{(a+bx+cx^2+dx^3)^{3/2}}\,dx=-\frac{2e}{\sqrt{a+x(b+x(c+dx))}}$$

input
```
Integrate[(b*e + 2*c*e*x + 3*d*e*x^2)/(a + b*x + c*x^2 + d*x^3)^(3/2),x]
```

output
```
(-2*e)/Sqrt[a + x*(b + x*(c + d*x))]
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.028$, Rules used = {2021}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{be+2cex+3dex^2}{(a+bx+cx^2+dx^3)^{3/2}}\,dx$$
$$\Big\downarrow \ 2021$$
$$-\frac{2e}{\sqrt{a+bx+cx^2+dx^3}}$$

input
```
Int[(b*e + 2*c*e*x + 3*d*e*x^2)/(a + b*x + c*x^2 + d*x^3)^(3/2),x]
```

output
```
(-2*e)/Sqrt[a + b*x + c*x^2 + d*x^3]
```

Defintions of rubi rules used

rule 2021
```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

method	result	size
gosper	$-\frac{2e}{\sqrt{d\,x^{3}+c\,x^{2}+bx+a}}$	21
default	$-\frac{2e}{\sqrt{d\,x^{3}+c\,x^{2}+bx+a}}$	21
trager	$-\frac{2e}{\sqrt{d\,x^{3}+c\,x^{2}+bx+a}}$	21
pseudoelliptic	$-\frac{2e}{\sqrt{d\,x^{3}+c\,x^{2}+bx+a}}$	21
elliptic	$-\frac{2e}{\sqrt{\left(x^{3}+\frac{c\,x^{2}}{d}+\frac{bx}{d}+\frac{a}{d}\right)d}}$	31
orering	$-\frac{2\left(3de\,x^{2}+2cex+eb\right)}{\sqrt{d\,x^{3}+c\,x^{2}+bx+a}\,\left(3d\,x^{2}+2cx+b\right)}$	50

input
```
int((3*d*e*x^2+2*c*e*x+b*e)/(d*x^3+c*x^2+b*x+a)^(3/2),x,method=_RETURNVERB
OSE)
```

output
```
-2*e/(d*x^3+c*x^2+b*x+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{be+2cex+3dex^2}{(a+bx+cx^2+dx^3)^{3/2}}\,dx = -\frac{2\,e}{\sqrt{dx^3+cx^2+bx+a}}$$

input
```
integrate((3*d*e*x^2+2*c*e*x+b*e)/(d*x^3+c*x^2+b*x+a)^(3/2),x, algorithm="
fricas")
```

output
```
-2*e/sqrt(d*x^3 + c*x^2 + b*x + a)
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{be + 2cex + 3dex^2}{(a + bx + cx^2 + dx^3)^{3/2}}\,dx = -\frac{2e}{\sqrt{a + bx + cx^2 + dx^3}}$$

input
```
integrate((3*d*e*x**2+2*c*e*x+b*e)/(d*x**3+c*x**2+b*x+a)**(3/2),x)
```

output
```
-2*e/sqrt(a + b*x + c*x**2 + d*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{be + 2cex + 3dex^2}{(a + bx + cx^2 + dx^3)^{3/2}}\,dx = -\frac{2\,e}{\sqrt{dx^3 + cx^2 + bx + a}}$$

input
```
integrate((3*d*e*x^2+2*c*e*x+b*e)/(d*x^3+c*x^2+b*x+a)^(3/2),x, algorithm="
maxima")
```

output
```
-2*e/sqrt(d*x^3 + c*x^2 + b*x + a)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{be + 2cex + 3dex^2}{(a + bx + cx^2 + dx^3)^{3/2}}\,dx = -\frac{2\,e}{\sqrt{dx^3 + cx^2 + bx + a}}$$

input
```
integrate((3*d*e*x^2+2*c*e*x+b*e)/(d*x^3+c*x^2+b*x+a)^(3/2),x, algorithm="
giac")
```

output
```
-2*e/sqrt(d*x^3 + c*x^2 + b*x + a)
```

Mupad [B] (verification not implemented)

Time = 11.98 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{be+2cex+3dex^2}{(a+bx+cx^2+dx^3)^{3/2}}\,dx=-\frac{2\,e}{\sqrt{d\,x^3+c\,x^2+b\,x+a}}$$

input
```
int((b*e + 2*c*e*x + 3*d*e*x^2)/(a + b*x + c*x^2 + d*x^3)^(3/2),x)
```

output
```
-(2*e)/(a + b*x + c*x^2 + d*x^3)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.64

$$\int \frac{be+2cex+3dex^2}{(a+bx+cx^2+dx^3)^{3/2}}\,dx=-\frac{2\sqrt{d\,x^3+c\,x^2+bx+a}\,e}{d\,x^3+c\,x^2+bx+a}$$

input
```
int((3*d*e*x^2+2*c*e*x+b*e)/(d*x^3+c*x^2+b*x+a)^(3/2),x)
```

output
```
( - 2*sqrt(a + b*x + c*x**2 + d*x**3)*e)/(a + b*x + c*x**2 + d*x**3)
```

3.107 $\int \frac{be+2cex+3dex^2}{(a+bx+cx^2+dx^3)^{5/2}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 24

$$\int \frac{be+2cex+3dex^2}{(a+bx+cx^2+dx^3)^{5/2}}\,dx=-\frac{2e}{3\,(a+bx+cx^2+dx^3)^{3/2}}$$

output
```
-2/3*e/(d*x^3+c*x^2+b*x+a)^(3/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{be+2cex+3dex^2}{(a+bx+cx^2+dx^3)^{5/2}}\,dx=-\frac{2e}{3(a+x(b+x(c+dx)))^{3/2}}$$

input
```
Integrate[(b*e + 2*c*e*x + 3*d*e*x^2)/(a + b*x + c*x^2 + d*x^3)^(5/2),x]
```

output
```
(-2*e)/(3*(a + x*(b + x*(c + d*x)))^(3/2))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.028, Rules used = {2021}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{be+2cex+3dex^2}{(a+bx+cx^2+dx^3)^{5/2}} \, dx$$

$$\Big\downarrow \; 2021$$

$$-\frac{2e}{3(a+bx+cx^2+dx^3)^{3/2}}$$

input
```
Int[(b*e + 2*c*e*x + 3*d*e*x^2)/(a + b*x + c*x^2 + d*x^3)^(5/2),x]
```

output
```
(-2*e)/(3*(a + b*x + c*x^2 + d*x^3)^(3/2))
```

Defintions of rubi rules used

rule 2021
```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.88

method	result	size
gosper	$-\frac{2e}{3(d\,x^3+c\,x^2+bx+a)^{\frac{3}{2}}}$	21
default	$-\frac{2e}{3(d\,x^3+c\,x^2+bx+a)^{\frac{3}{2}}}$	21
trager	$-\frac{2e}{3(d\,x^3+c\,x^2+bx+a)^{\frac{3}{2}}}$	21
pseudoelliptic	$-\frac{2e}{3(d\,x^3+c\,x^2+bx+a)^{\frac{3}{2}}}$	21
elliptic	$-\frac{2e\sqrt{d\,x^3+c\,x^2+bx+a}}{3d^2\left(x^3+\frac{c\,x^2}{d}+\frac{bx}{d}+\frac{a}{d}\right)^2}$	49
orering	$-\frac{2\left(3de\,x^2+2cex+eb\right)}{3(d\,x^3+c\,x^2+bx+a)^{\frac{3}{2}}\left(3d\,x^2+2cx+b\right)}$	50

input
```
int((3*d*e*x^2+2*c*e*x+b*e)/(d*x^3+c*x^2+b*x+a)^(5/2),x,method=_RETURNVERB
OSE)
```

output
```
-2/3*e/(d*x^3+c*x^2+b*x+a)^(3/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 81 vs. 2(20) = 40.

Time = 0.08 (sec) , antiderivative size = 81, normalized size of antiderivative = 3.38

$$\int \frac{be+2cex+3dex^2}{(a+bx+cx^2+dx^3)^{5/2}}\,dx = -\frac{2\sqrt{dx^3+cx^2+bx+a}e}{3\left(d^2x^6+2\,cdx^5+(c^2+2\,bd)x^4+2\,(bc+ad)x^3+2\,abx+(b^2+2\,ac)x^2+a^2\right)}$$

input
```
integrate((3*d*e*x^2+2*c*e*x+b*e)/(d*x^3+c*x^2+b*x+a)^(5/2),x, algorithm="
fricas")
```

output
```
-2/3*sqrt(d*x^3 + c*x^2 + b*x + a)*e/(d^2*x^6 + 2*c*d*x^5 + (c^2 + 2*b*d)*
x^4 + 2*(b*c + a*d)*x^3 + 2*a*b*x + (b^2 + 2*a*c)*x^2 + a^2)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{be + 2cex + 3dex^2}{(a + bx + cx^2 + dx^3)^{5/2}}\,dx = \text{Exception raised: RecursionError}$$

input
```
integrate((3*d*e*x**2+2*c*e*x+b*e)/(d*x**3+c*x**2+b*x+a)**(5/2),x)
```

output
```
Exception raised: RecursionError >> maximum recursion depth exceeded in co
mparison
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{be + 2cex + 3dex^2}{(a + bx + cx^2 + dx^3)^{5/2}}\,dx = -\frac{2\,e}{3\,(dx^3 + cx^2 + bx + a)^{\frac{3}{2}}}$$

input
```
integrate((3*d*e*x^2+2*c*e*x+b*e)/(d*x^3+c*x^2+b*x+a)^(5/2),x, algorithm="
maxima")
```

output
```
-2/3*e/(d*x^3 + c*x^2 + b*x + a)^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.50

$$\int \frac{be+2cex+3dex^2}{(a+bx+cx^2+dx^3)^{5/2}}\,dx = -\frac{2\,e^5}{3\,(ae^2+(dex^3+cex^2+bex)e)^{\frac{3}{2}}|e|}$$

input
```
integrate((3*d*e*x^2+2*c*e*x+b*e)/(d*x^3+c*x^2+b*x+a)^(5/2),x, algorithm="
giac")
```

output
```
-2/3*e^5/((a*e^2 + (d*e*x^3 + c*e*x^2 + b*e*x)*e)^(3/2)*abs(e))
```

Mupad [B] (verification not implemented)

Time = 12.21 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{be+2cex+3dex^2}{(a+bx+cx^2+dx^3)^{5/2}}\,dx = -\frac{2\,e}{3\,(d\,x^3+c\,x^2+b\,x+a)^{3/2}}$$

input
```
int((b*e + 2*c*e*x + 3*d*e*x^2)/(a + b*x + c*x^2 + d*x^3)^(5/2),x)
```

output
```
-(2*e)/(3*(a + b*x + c*x^2 + d*x^3)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 91, normalized size of antiderivative = 3.79

$$\int \frac{be+2cex+3dex^2}{(a+bx+cx^2+dx^3)^{5/2}}\,dx = -\frac{2\sqrt{d\,x^3+c\,x^2+bx+a}\,e}{3d^2x^6+6cd\,x^5+6bd\,x^4+3c^2x^4+6ad\,x^3+6bc\,x^3+6ac\,x^2+3b^2x^2+6abx+3a^2}$$

input
```
int((3*d*e*x^2+2*c*e*x+b*e)/(d*x^3+c*x^2+b*x+a)^(5/2),x)
```

output
```
( - 2*sqrt(a + b*x + c*x**2 + d*x**3)*e)/(3*(a**2 + 2*a*b*x + 2*a*c*x**2 +
 2*a*d*x**3 + b**2*x**2 + 2*b*c*x**3 + 2*b*d*x**4 + c**2*x**4 + 2*c*d*x**5
 + d**2*x**6))
```

3.108 $\int (be + 2cex + 3dex^2) (a + bx + cx^2 + dx^3)^p \, dx$

Optimal result

Integrand size = 34, antiderivative size = 26

$$\int (be + 2cex + 3dex^2) (a + bx + cx^2 + dx^3)^p \, dx = \frac{e(a + bx + cx^2 + dx^3)^{1+p}}{1 + p}$$

output
```
e*(d*x^3+c*x^2+b*x+a)^(p+1)/(p+1)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int (be + 2cex + 3dex^2) (a + bx + cx^2 + dx^3)^p \, dx = \frac{e(a + x(b + x(c + dx)))^{1+p}}{1 + p}$$

input
```
Integrate[(b*e + 2*c*e*x + 3*d*e*x^2)*(a + b*x + c*x^2 + d*x^3)^p,x]
```

output
```
(e*(a + x*(b + x*(c + d*x)))^(1 + p))/(1 + p)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.029$, Rules used = {2021}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (be + 2cex + 3dex^2) (a + bx + cx^2 + dx^3)^p \, dx$$

$$\Big\downarrow \text{2021}$$

$$\frac{e(a + bx + cx^2 + dx^3)^{p+1}}{p+1}$$

input
```
Int[(b*e + 2*c*e*x + 3*d*e*x^2)*(a + b*x + c*x^2 + d*x^3)^p,x]
```

output
```
(e*(a + b*x + c*x^2 + d*x^3)^(1 + p))/(1 + p)
```

Defintions of rubi rules used

rule 2021
```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.04

method	result	si
gosper	$\frac{e\left(d\,x^{3}+c\,x^{2}+bx+a\right)^{p+1}}{p+1}$	2
risch	$\frac{\left(d\,x^{3}+c\,x^{2}+bx+a\right)^{p} e\left(d\,x^{3}+c\,x^{2}+bx+a\right)}{p+1}$	4
orering	$\frac{\left(d\,x^{3}+c\,x^{2}+bx+a\right)\left(3de\,x^{2}+2cex+eb\right)\left(d\,x^{3}+c\,x^{2}+bx+a\right)^{p}}{(p+1)\left(3d\,x^{2}+2cx+b\right)}$	6
parallelrisch	$\frac{x^{3}\left(d\,x^{3}+c\,x^{2}+bx+a\right)^{p} d^{2}e+x^{2}\left(d\,x^{3}+c\,x^{2}+bx+a\right)^{p} cde+x\left(d\,x^{3}+c\,x^{2}+bx+a\right)^{p} bde+\left(d\,x^{3}+c\,x^{2}+bx+a\right)^{p} ade}{d(p+1)}$	1
norman	$\frac{ae\,{\mathrm e}^{p\ln\left(d\,x^{3}+c\,x^{2}+bx+a\right)}}{p+1}+\frac{ce\,x^{2}{\mathrm e}^{p\ln\left(d\,x^{3}+c\,x^{2}+bx+a\right)}}{p+1}+\frac{de\,x^{3}{\mathrm e}^{p\ln\left(d\,x^{3}+c\,x^{2}+bx+a\right)}}{p+1}+\frac{ebx\,{\mathrm e}^{p\ln\left(d\,x^{3}+c\,x^{2}+bx+a\right)}}{p+1}$	1

input
```
int((3*d*e*x^2+2*c*e*x+b*e)*(d*x^3+c*x^2+b*x+a)^p,x,method=_RETURNVERBOSE)
```

output
```
e*(d*x^3+c*x^2+b*x+a)^(p+1)/(p+1)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.65

$$\int \left(be+2cex+3dex^2\right)\left(a+bx+cx^2+dx^3\right)^p\,dx = \frac{(dex^3+cex^2+bex+ae)(dx^3+cx^2+bx+a)^p}{p+1}$$

input
```
integrate((3*d*e*x^2+2*c*e*x+b*e)*(d*x^3+c*x^2+b*x+a)^p,x, algorithm="fric
as")
```

output
```
(d*e*x^3 + c*e*x^2 + b*e*x + a*e)*(d*x^3 + c*x^2 + b*x + a)^p/(p + 1)
```

Sympy [F(-1)]

Timed out.

$$\int \left(be+2cex+3dex^2\right)\left(a+bx+cx^2+dx^3\right)^p\,dx=\text{Timed out}$$

input
```
integrate((3*d*e*x**2+2*c*e*x+b*e)*(d*x**3+c*x**2+b*x+a)**p,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \left(be+2cex+3dex^2\right)\left(a+bx+cx^2+dx^3\right)^p\,dx=\frac{\left(dx^3+cx^2+bx+a\right)^{p+1}e}{p+1}$$

input
```
integrate((3*d*e*x^2+2*c*e*x+b*e)*(d*x^3+c*x^2+b*x+a)^p,x, algorithm="maxi
ma")
```

output
```
(d*x^3 + c*x^2 + b*x + a)^(p + 1)*e/(p + 1)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \left(be+2cex+3dex^2\right)\left(a+bx+cx^2+dx^3\right)^p\,dx=\frac{\left(dx^3+cx^2+bx+a\right)^{p+1}e}{p+1}$$

input
```
integrate((3*d*e*x^2+2*c*e*x+b*e)*(d*x^3+c*x^2+b*x+a)^p,x, algorithm="giac
")
```

output
```
(d*x^3 + c*x^2 + b*x + a)^(p + 1)*e/(p + 1)
```

Mupad [B] (verification not implemented)

Time = 12.04 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.23

$$\int (be + 2cex + 3dex^2) (a + bx + cx^2 + dx^3)^p \, dx$$
$$= (d\,x^3 + c\,x^2 + b\,x + a)^p \left(\frac{a\,e}{p+1} + \frac{c\,e\,x^2}{p+1} + \frac{d\,e\,x^3}{p+1} + \frac{b\,e\,x}{p+1}\right)$$

input
```
int((b*e + 2*c*e*x + 3*d*e*x^2)*(a + b*x + c*x^2 + d*x^3)^p,x)
```

output
```
(a + b*x + c*x^2 + d*x^3)^p*((a*e)/(p + 1) + (c*e*x^2)/(p + 1) + (d*e*x^3)
/(p + 1) + (b*e*x)/(p + 1))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.50

$$\int (be + 2cex + 3dex^2) (a + bx + cx^2 + dx^3)^p \, dx$$
$$= \frac{(d\,x^3 + c\,x^2 + bx + a)^p\, e(d\,x^3 + c\,x^2 + bx + a)}{p+1}$$

input
```
int((3*d*e*x^2+2*c*e*x+b*e)*(d*x^3+c*x^2+b*x+a)^p,x)
```

output
```
((a + b*x + c*x**2 + d*x**3)**p*e*(a + b*x + c*x**2 + d*x**3))/(p + 1)
```

3.109 $\int (A+Bx+Cx^2)\,(ace+(bce+ade+acf)x+(b$

Optimal result

Integrand size = 57, antiderivative size = 1363

$$\int (A+Bx+Cx^2)\left(ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3\right)^3\,dx$$
$$=\text{Too large to display}$$

output
```
1/4*(A*b^2-a*(B*b-C*a))*(-a*d+b*c)^3*(-a*f+b*e)^3*(b*x+a)^4/b^9-1/5*(-a*d+
b*c)^2*(-a*f+b*e)^2*(8*a^3*C*d*f+2*a*b^2*(3*A*d*f+2*B*c*f+2*B*d*e+C*c*e)-b
^3*(B*c*e+3*A*(c*f+d*e))-a^2*b*(7*B*d*f+5*C*(c*f+d*e)))*(b*x+a)^5/b^9+1/6*
(-a*d+b*c)*(-a*f+b*e)*(28*a^4*C*d^2*f^2-7*a^3*b*d*f*(3*B*d*f+5*C*(c*f+d*e)
)+a^2*b^2*(3*d*f*(5*A*d*f+8*B*c*f+8*B*d*e)+C*(10*c^2*f^2+37*c*d*e*f+10*d^2
*e^2))+b^4*(3*A*d^2*e^2+3*c*d*e*(3*A*f+B*e)+c^2*(C*e^2+3*f*(A*f+B*e)))-a*b
^3*(3*d^2*e*(5*A*f+2*B*e)+2*c^2*f*(3*B*f+4*C*e)+c*d*(8*C*e^2+3*f*(5*A*f+7*
B*e))))*(b*x+a)^6/b^9-1/7*(56*a^5*C*d^3*f^3-35*a^4*b*d^2*f^2*(B*d*f+3*C*(c
*f+d*e))+20*a^3*b^2*d*f*(d*f*(A*d*f+3*B*c*f+3*B*d*e)+3*C*(c^2*f^2+3*c*d*e*
f+d^2*e^2))-b^5*(A*d^3*e^3+3*c*d^2*e^2*(3*A*f+B*e)+3*c^2*d*e*(C*e^2+3*f*(A
*f+B*e))+c^3*f*(3*C*e^2+f*(A*f+3*B*e)))+4*a*b^4*(d^3*e^2*(3*A*f+B*e)+c^3*f
^2*(B*f+3*C*e)+3*c*d^2*e*(C*e^2+3*f*(A*f+B*e))+3*c^2*d*f*(3*C*e^2+f*(A*f+3
*B*e)))-10*a^2*b^3*(C*(c^3*f^3+9*c^2*d*e*f^2+9*c*d^2*e^2*f+d^3*e^3)+3*d*f*
(A*d*f*(c*f+d*e)+B*(c^2*f^2+3*c*d*e*f+d^2*e^2))))*(b*x+a)^7/b^9+1/8*(70*a^
4*C*d^3*f^3-35*a^3*b*d^2*f^2*(B*d*f+3*C*(c*f+d*e))+15*a^2*b^2*d*f*(d*f*(A*
d*f+3*B*c*f+3*B*d*e)+3*C*(c^2*f^2+3*c*d*e*f+d^2*e^2))+b^4*(d^3*e^2*(3*A*f+
B*e)+c^3*f^2*(B*f+3*C*e)+3*c*d^2*e*(C*e^2+3*f*(A*f+B*e))+3*c^2*d*f*(3*C*e^
2+f*(A*f+3*B*e)))-5*a*b^3*(C*(c^3*f^3+9*c^2*d*e*f^2+9*c*d^2*e^2*f+d^3*e^3)
+3*d*f*(A*d*f*(c*f+d*e)+B*(c^2*f^2+3*c*d*e*f+d^2*e^2))))*(b*x+a)^8/b^9-1/9
*(56*a^3*C*d^3*f^3-21*a^2*b*d^2*f^2*(B*d*f+3*C*(c*f+d*e))+6*a*b^2*d*f*(...
```

Mathematica [A] (verified)

Time = 0.83 (sec) , antiderivative size = 1739, normalized size of antiderivative = 1.28

$$\int \left(A+Bx+Cx^2\right)\left(ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3\right)^3\,dx$$
$$= \text{Too large to display}$$

input
```
Integrate[(A + B*x + C*x^2)*(a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e +
b*c*f + a*d*f)*x^2 + b*d*f*x^3)^3,x]
```

output
```
a^3*A*c^3*e^3*x + (a^2*c^2*e^2*(a*B*c*e + 3*A*(b*c*e + a*d*e + a*c*f))*x^2
)/2 + (a*c*e*(a*c*e*(3*b*B*c*e + a*(c*C*e + 3*B*d*e + 3*B*c*f)) + 3*A*(b^2
*c^2*e^2 + 3*a*b*c*e*(d*e + c*f) + a^2*(d^2*e^2 + 3*c*d*e*f + c^2*f^2)))*x
^3)/3 + ((A*(b^3*c^3*e^3 + 9*a*b^2*c^2*e^2*(d*e + c*f) + 9*a^2*b*c*e*(d^2*
e^2 + 3*c*d*e*f + c^2*f^2) + a^3*(d^3*e^3 + 9*c*d^2*e^2*f + 9*c^2*d*e*f^2
+ c^3*f^3)) + 3*a*c*e*(b^2*B*c^2*e^2 + a*b*c*e*(c*C*e + 3*B*d*e + 3*B*c*f)
 + a^2*(B*d^2*e^2 + c^2*f*(C*e + B*f) + c*d*e*(C*e + 3*B*f))))*x^4)/4 + ((
b^3*c^2*e^2*(B*c*e + 3*A*(d*e + c*f)) + 3*a*b^2*c*e*(3*A*d^2*e^2 + 3*c*d*e
*(B*e + 3*A*f) + c^2*(C*e^2 + 3*f*(B*e + A*f))) + 3*a^2*b*(A*d^3*e^3 + 3*c
*d^2*e^2*(B*e + 3*A*f) + 3*c^2*d*e*(C*e^2 + 3*f*(B*e + A*f)) + c^3*f*(3*C*
e^2 + f*(3*B*e + A*f))) + a^3*(d^3*e^2*(B*e + 3*A*f) + c^3*f^2*(3*C*e + B*
f) + 3*c*d^2*e*(C*e^2 + 3*f*(B*e + A*f)) + 3*c^2*d*f*(3*C*e^2 + f*(3*B*e +
 A*f))))*x^5)/5 + ((b^3*c*e*(3*A*d^2*e^2 + 3*c*d*e*(B*e + 3*A*f) + c^2*(C*
e^2 + 3*f*(B*e + A*f))) + 3*a*b^2*(A*d^3*e^3 + 3*c*d^2*e^2*(B*e + 3*A*f) +
 3*c^2*d*e*(C*e^2 + 3*f*(B*e + A*f)) + c^3*f*(3*C*e^2 + f*(3*B*e + A*f)))
+ 3*a^2*b*(d^3*e^2*(B*e + 3*A*f) + c^3*f^2*(3*C*e + B*f) + 3*c*d^2*e*(C*e^
2 + 3*f*(B*e + A*f)) + 3*c^2*d*f*(3*C*e^2 + f*(3*B*e + A*f))) + a^3*(C*(d^
3*e^3 + 9*c*d^2*e^2*f + 9*c^2*d*e*f^2 + c^3*f^3) + 3*d*f*(A*d*f*(d*e + c*f
) + B*(d^2*e^2 + 3*c*d*e*f + c^2*f^2))))*x^6)/6 + ((a^3*d*f*(d*f*(3*B*d*e
+ 3*B*c*f + A*d*f) + 3*C*(d^2*e^2 + 3*c*d*e*f + c^2*f^2)) + b^3*(A*d^3*...
```

Rubi [A] (verified)

Time = 7.51 (sec) , antiderivative size = 1739, normalized size of antiderivative = 1.28, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.035$, Rules used = {2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (A + Bx + Cx^2) \left(x^2(adf + bcf + bde) + x(acf + ade + bce) + ace + bdfx^3\right)^3 dx$$

$$\Big\downarrow \text{ 2188}$$

$$\int \left(b^3Cd^3f^3x^{11} + b^2d^2f^2(bBdf + 3aCdf + 3bC(de + cf))x^{10} + bdf\left((df(3Bde + 3Bcf + Adf) + 3C(d^2e^2 + 3cd\right.\right.$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{12}b^3Cd^3f^3x^{12} + \frac{1}{11}b^2d^2f^2(bBdf + 3aCdf + 3bC(de + cf))x^{11} +$$
$$\frac{1}{10}bdf\left(\left(df(3Bde + 3Bcf + Adf) + 3C\left(d^2e^2 + 3cdfe + c^2f^2\right)\right)b^2 + 3adf(Bdf + 3C(de + cf))b + 3a^2Cd^2f^2\right)x^{10}$$
$$\frac{1}{9}\left(\left(C\left(d^3e^3 + 9cd^2fe^2 + 9c^2df^2e + c^3f^3\right) + 3df\left(Adf(de + cf) + B\left(d^2e^2 + 3cdfe + c^2f^2\right)\right)\right)b^3 + 3adf\left(df(3Bde +\right.\right.$$
$$\frac{1}{8}\left(d^2f^2(Bdf + 3C(de + cf))a^3 + 3bdf\left(df(3Bde + 3Bcf + Adf) + 3C\left(d^2e^2 + 3cdfe + c^2f^2\right)\right)a^2 + 3b^2\left(C\left(d^3e^3 +\right.\right.\right.$$
$$\frac{1}{7}\left(df\left(df(3Bde + 3Bcf + Adf) + 3C\left(d^2e^2 + 3cdfe + c^2f^2\right)\right)a^3 + 3b\left(C\left(d^3e^3 + 9cd^2fe^2 + 9c^2df^2e + c^3f^3\right) + 3df\right.\right.$$
$$\frac{1}{6}\left(\left(C\left(d^3e^3 + 9cd^2fe^2 + 9c^2df^2e + c^3f^3\right) + 3df\left(Adf(de + cf) + B\left(d^2e^2 + 3cdfe + c^2f^2\right)\right)\right)a^3 + 3b\left(f^2(3Ce + B\right.\right.$$
$$\frac{1}{5}\left(\left(f^2(3Ce + Bf)c^3 + 3df\left(3Ce^2 + f(3Be + Af)\right)c^2 + 3d^2e\left(Ce^2 + 3f(Be + Af)\right)c + d^3e^2(Be + 3Af)\right)a^3 + 3b\right.$$
$$\frac{1}{4}\left(A\left(\left(d^3e^3 + 9cd^2fe^2 + 9c^2df^2e + c^3f^3\right)a^3 + 9bce\left(d^2e^2 + 3cdfe + c^2f^2\right)a^2 + 9b^2c^2e^2(de + cf)a + b^3c^3e^3\right) + 3a\right.$$
$$\frac{1}{3}ace\left(ace(3bBce + a(cCe + 3Bde + 3Bcf)) + 3A\left(\left(d^2e^2 + 3cdfe + c^2f^2\right)a^2 + 3bce(de + cf)a + b^2c^2e^2\right)\right)x^3 +$$
$$\frac{1}{2}a^2c^2e^2(aBce + 3A(bce + ade + acf))x^2 + a^3Ac^3e^3x$$

input
```
Int[(A + B*x + C*x^2)*(a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*c*f
+ a*d*f)*x^2 + b*d*f*x^3)^3,x]
```

output
```
a^3*A*c^3*e^3*x + (a^2*c^2*e^2*(a*B*c*e + 3*A*(b*c*e + a*d*e + a*c*f))*x^2
)/2 + (a*c*e*(a*c*e*(3*b*B*c*e + a*(c*C*e + 3*B*d*e + 3*B*c*f)) + 3*A*(b^2
*c^2*e^2 + 3*a*b*c*e*(d*e + c*f) + a^2*(d^2*e^2 + 3*c*d*e*f + c^2*f^2)))*x
^3)/3 + ((A*(b^3*c^3*e^3 + 9*a*b^2*c^2*e^2*(d*e + c*f) + 9*a^2*b*c*e*(d^2*
e^2 + 3*c*d*e*f + c^2*f^2) + a^3*(d^3*e^3 + 9*c*d^2*e^2*f + 9*c^2*d*e*f^2
+ c^3*f^3)) + 3*a*c*e*(b^2*B*c^2*e^2 + a*b*c*e*(c*C*e + 3*B*d*e + 3*B*c*f)
 + a^2*(B*d^2*e^2 + c^2*f*(C*e + B*f) + c*d*e*(C*e + 3*B*f))))*x^4)/4 + ((
b^3*c^2*e^2*(B*c*e + 3*A*(d*e + c*f)) + 3*a*b^2*c*e*(3*A*d^2*e^2 + 3*c*d*e
*(B*e + 3*A*f) + c^2*(C*e^2 + 3*f*(B*e + A*f))) + 3*a^2*b*(A*d^3*e^3 + 3*c
*d^2*e^2*(B*e + 3*A*f) + 3*c^2*d*e*(C*e^2 + 3*f*(B*e + A*f)) + c^3*f*(3*C*
e^2 + f*(3*B*e + A*f))) + a^3*(d^3*e^2*(B*e + 3*A*f) + c^3*f^2*(3*C*e + B*
f) + 3*c*d^2*e*(C*e^2 + 3*f*(B*e + A*f)) + 3*c^2*d*f*(3*C*e^2 + f*(3*B*e +
 A*f))))*x^5)/5 + ((b^3*c*e*(3*A*d^2*e^2 + 3*c*d*e*(B*e + 3*A*f) + c^2*(C*
e^2 + 3*f*(B*e + A*f))) + 3*a*b^2*(A*d^3*e^3 + 3*c*d^2*e^2*(B*e + 3*A*f) +
 3*c^2*d*e*(C*e^2 + 3*f*(B*e + A*f)) + c^3*f*(3*C*e^2 + f*(3*B*e + A*f)))
+ 3*a^2*b*(d^3*e^2*(B*e + 3*A*f) + c^3*f^2*(3*C*e + B*f) + 3*c*d^2*e*(C*e^
2 + 3*f*(B*e + A*f)) + 3*c^2*d*f*(3*C*e^2 + f*(3*B*e + A*f))) + a^3*(C*(d^
3*e^3 + 9*c*d^2*e^2*f + 9*c^2*d*e*f^2 + c^3*f^3) + 3*d*f*(A*d*f*(d*e + c*f
) + B*(d^2*e^2 + 3*c*d*e*f + c^2*f^2))))*x^6)/6 + ((a^3*d*f*(d*f*(3*B*d*e
+ 3*B*c*f + A*d*f) + 3*C*(d^2*e^2 + 3*c*d*e*f + c^2*f^2)) + b^3*(A*d^3*...
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 2569, normalized size of antiderivative = 1.88

method	result	size
default	Expression too large to display	2569
norman	Expression too large to display	2632
risch	Expression too large to display	3152
parallelrisch	Expression too large to display	3152
gosper	Expression too large to display	3162
orering	Expression too large to display	3229

input
```
int((C*x^2+B*x+A)*(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d
*f*x^3)^3,x,method=_RETURNVERBOSE)
```

output
```
1/12*C*b^3*d^3*f^3*x^12+1/11*(B*b^3*d^3*f^3+3*C*(a*d*f+b*c*f+b*d*e)*b^2*d^
2*f^2)*x^11+1/10*(A*b^3*d^3*f^3+3*B*(a*d*f+b*c*f+b*d*e)*b^2*d^2*f^2+C*((a*
c*f+a*d*e+b*c*e)*b^2*d^2*f^2+2*(a*d*f+b*c*f+b*d*e)^2*b*d*f+b*d*f*(2*(a*c*f
+a*d*e+b*c*e)*b*d*f+(a*d*f+b*c*f+b*d*e)^2)))*x^10+1/9*(3*A*(a*d*f+b*c*f+b*
d*e)*b^2*d^2*f^2+B*((a*c*f+a*d*e+b*c*e)*b^2*d^2*f^2+2*(a*d*f+b*c*f+b*d*e)^
2*b*d*f+b*d*f*(2*(a*c*f+a*d*e+b*c*e)*b*d*f+(a*d*f+b*c*f+b*d*e)^2))+C*(a*b^
2*c*d^2*e*f^2+2*(a*c*f+a*d*e+b*c*e)*(a*d*f+b*c*f+b*d*e)*b*d*f+(a*d*f+b*c*f
+b*d*e)*(2*(a*c*f+a*d*e+b*c*e)*b*d*f+(a*d*f+b*c*f+b*d*e)^2)+b*d*f*(2*a*c*e
*b*d*f+2*(a*c*f+a*d*e+b*c*e)*(a*d*f+b*c*f+b*d*e))))*x^9+1/8*(A*((a*c*f+a*d
*e+b*c*e)*b^2*d^2*f^2+2*(a*d*f+b*c*f+b*d*e)^2*b*d*f+b*d*f*(2*(a*c*f+a*d*e+
b*c*e)*b*d*f+(a*d*f+b*c*f+b*d*e)^2))+B*(a*b^2*c*d^2*e*f^2+2*(a*c*f+a*d*e+b
*c*e)*(a*d*f+b*c*f+b*d*e)*b*d*f+(a*d*f+b*c*f+b*d*e)*(2*(a*c*f+a*d*e+b*c*e)
*b*d*f+(a*d*f+b*c*f+b*d*e)^2)+b*d*f*(2*a*c*e*b*d*f+2*(a*c*f+a*d*e+b*c*e)*(
a*d*f+b*c*f+b*d*e)))+C*(2*a*c*e*(a*d*f+b*c*f+b*d*e)*b*d*f+(a*c*f+a*d*e+b*c
*e)*(2*(a*c*f+a*d*e+b*c*e)*b*d*f+(a*d*f+b*c*f+b*d*e)^2)+(a*d*f+b*c*f+b*d*e
)*(2*a*c*e*b*d*f+2*(a*c*f+a*d*e+b*c*e)*(a*d*f+b*c*f+b*d*e))+b*d*f*(2*a*c*e
*(a*d*f+b*c*f+b*d*e)+(a*c*f+a*d*e+b*c*e)^2)))*x^8+1/7*(A*(a*b^2*c*d^2*e*f^
2+2*(a*c*f+a*d*e+b*c*e)*(a*d*f+b*c*f+b*d*e)*b*d*f+(a*d*f+b*c*f+b*d*e)*(2*(
a*c*f+a*d*e+b*c*e)*b*d*f+(a*d*f+b*c*f+b*d*e)^2)+b*d*f*(2*a*c*e*b*d*f+2*(a*
c*f+a*d*e+b*c*e)*(a*d*f+b*c*f+b*d*e)))+B*(2*a*c*e*(a*d*f+b*c*f+b*d*e)*b...
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 1930, normalized size of antiderivative = 1.42

$$\int (A + Bx + Cx^2) \left(ace + (bce + ade + acf)x + (bde + bcf + adf)x^2 + bdfx^3\right)^3 dx$$
$$= \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)*(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x
^2+b*d*f*x^3)^3,x, algorithm="fricas")
```

output
```
1/12*C*b^3*d^3*f^3*x^12 + 1/11*(3*C*b^3*d^3*e*f^2 + (3*C*b^3*c*d^2 + (3*C*
a*b^2 + B*b^3)*d^3)*f^3)*x^11 + A*a^3*c^3*e^3*x + 1/10*(3*C*b^3*d^3*e^2*f
+ 3*(3*C*b^3*c*d^2 + (3*C*a*b^2 + B*b^3)*d^3)*e*f^2 + (3*C*b^3*c^2*d + 3*(
3*C*a*b^2 + B*b^3)*c*d^2 + (3*C*a^2*b + 3*B*a*b^2 + A*b^3)*d^3)*f^3)*x^10
+ 1/9*(C*b^3*d^3*e^3 + 3*(3*C*b^3*c*d^2 + (3*C*a*b^2 + B*b^3)*d^3)*e^2*f +
 3*(3*C*b^3*c^2*d + 3*(3*C*a*b^2 + B*b^3)*c*d^2 + (3*C*a^2*b + 3*B*a*b^2 +
 A*b^3)*d^3)*e*f^2 + (C*b^3*c^3 + 3*(3*C*a*b^2 + B*b^3)*c^2*d + 3*(3*C*a^2
*b + 3*B*a*b^2 + A*b^3)*c*d^2 + (C*a^3 + 3*B*a^2*b + 3*A*a*b^2)*d^3)*f^3)*
x^9 + 1/8*((3*C*b^3*c*d^2 + (3*C*a*b^2 + B*b^3)*d^3)*e^3 + 3*(3*C*b^3*c^2*
d + 3*(3*C*a*b^2 + B*b^3)*c*d^2 + (3*C*a^2*b + 3*B*a*b^2 + A*b^3)*d^3)*e^2
*f + 3*(C*b^3*c^3 + 3*(3*C*a*b^2 + B*b^3)*c^2*d + 3*(3*C*a^2*b + 3*B*a*b^2
 + A*b^3)*c*d^2 + (C*a^3 + 3*B*a^2*b + 3*A*a*b^2)*d^3)*e*f^2 + ((3*C*a*b^2
 + B*b^3)*c^3 + 3*(3*C*a^2*b + 3*B*a*b^2 + A*b^3)*c^2*d + 3*(C*a^3 + 3*B*a
^2*b + 3*A*a*b^2)*c*d^2 + (B*a^3 + 3*A*a^2*b)*d^3)*f^3)*x^8 + 1/7*((3*C*b^
3*c^2*d + 3*(3*C*a*b^2 + B*b^3)*c*d^2 + (3*C*a^2*b + 3*B*a*b^2 + A*b^3)*d^
3)*e^3 + 3*(C*b^3*c^3 + 3*(3*C*a*b^2 + B*b^3)*c^2*d + 3*(3*C*a^2*b + 3*B*a
*b^2 + A*b^3)*c*d^2 + (C*a^3 + 3*B*a^2*b + 3*A*a*b^2)*d^3)*e^2*f + 3*((3*C
*a*b^2 + B*b^3)*c^3 + 3*(3*C*a^2*b + 3*B*a*b^2 + A*b^3)*c^2*d + 3*(C*a^3 +
 3*B*a^2*b + 3*A*a*b^2)*c*d^2 + (B*a^3 + 3*A*a^2*b)*d^3)*e*f^2 + (A*a^3*d^
3 + (3*C*a^2*b + 3*B*a*b^2 + A*b^3)*c^3 + 3*(C*a^3 + 3*B*a^2*b + 3*A*a*...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3334 vs. 2(1423) = 2846.

Time = 0.24 (sec) , antiderivative size = 3334, normalized size of antiderivative = 2.45

$$\int (A+Bx+Cx^2)\left(ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3\right)^3\,dx = \text{Too large to display}$$

input
```
integrate((C*x**2+B*x+A)*(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*
x**2+b*d*f*x**3)**3,x)
```

output
```
A*a**3*c**3*e**3*x + C*b**3*d**3*f**3*x**12/12 + x**11*(B*b**3*d**3*f**3/1
1 + 3*C*a*b**2*d**3*f**3/11 + 3*C*b**3*c*d**2*f**3/11 + 3*C*b**3*d**3*e*f*
*2/11) + x**10*(A*b**3*d**3*f**3/10 + 3*B*a*b**2*d**3*f**3/10 + 3*B*b**3*c
*d**2*f**3/10 + 3*B*b**3*d**3*e*f**2/10 + 3*C*a**2*b*d**3*f**3/10 + 9*C*a*
b**2*c*d**2*f**3/10 + 9*C*a*b**2*d**3*e*f**2/10 + 3*C*b**3*c**2*d*f**3/10
+ 9*C*b**3*c*d**2*e*f**2/10 + 3*C*b**3*d**3*e**2*f/10) + x**9*(A*a*b**2*d*
*3*f**3/3 + A*b**3*c*d**2*f**3/3 + A*b**3*d**3*e*f**2/3 + B*a**2*b*d**3*f*
*3/3 + B*a*b**2*c*d**2*f**3 + B*a*b**2*d**3*e*f**2 + B*b**3*c**2*d*f**3/3
+ B*b**3*c*d**2*e*f**2 + B*b**3*d**3*e**2*f/3 + C*a**3*d**3*f**3/9 + C*a**
2*b*c*d**2*f**3 + C*a**2*b*d**3*e*f**2 + C*a*b**2*c**2*d*f**3 + 3*C*a*b**2
*c*d**2*e*f**2 + C*a*b**2*d**3*e**2*f + C*b**3*c**3*f**3/9 + C*b**3*c**2*d
*e*f**2 + C*b**3*c*d**2*e**2*f + C*b**3*d**3*e**3/9) + x**8*(3*A*a**2*b*d*
*3*f**3/8 + 9*A*a*b**2*c*d**2*f**3/8 + 9*A*a*b**2*d**3*e*f**2/8 + 3*A*b**3
*c**2*d*f**3/8 + 9*A*b**3*c*d**2*e*f**2/8 + 3*A*b**3*d**3*e**2*f/8 + B*a**
3*d**3*f**3/8 + 9*B*a**2*b*c*d**2*f**3/8 + 9*B*a**2*b*d**3*e*f**2/8 + 9*B*
a*b**2*c**2*d*f**3/8 + 27*B*a*b**2*c*d**2*e*f**2/8 + 9*B*a*b**2*d**3*e**2*
f/8 + B*b**3*c**3*f**3/8 + 9*B*b**3*c**2*d*e*f**2/8 + 9*B*b**3*c*d**2*e**2
*f/8 + B*b**3*d**3*e**3/8 + 3*C*a**3*c*d**2*f**3/8 + 3*C*a**3*d**3*e*f**2/
8 + 9*C*a**2*b*c**2*d*f**3/8 + 27*C*a**2*b*c*d**2*e*f**2/8 + 9*C*a**2*b*d*
*3*e**2*f/8 + 3*C*a*b**2*c**3*f**3/8 + 27*C*a*b**2*c**2*d*e*f**2/8 + 27...
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 1930, normalized size of antiderivative = 1.42

$$\int (A+Bx+Cx^2)\left(ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3\right)^3\,dx$$
$$= \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)*(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x
^2+b*d*f*x^3)^3,x, algorithm="maxima")
```

output
```
1/12*C*b^3*d^3*f^3*x^12 + 1/11*(3*C*b^3*d^3*e*f^2 + (3*C*b^3*c*d^2 + (3*C*
a*b^2 + B*b^3)*d^3)*f^3)*x^11 + A*a^3*c^3*e^3*x + 1/10*(3*C*b^3*d^3*e^2*f
+ 3*(3*C*b^3*c*d^2 + (3*C*a*b^2 + B*b^3)*d^3)*e*f^2 + (3*C*b^3*c^2*d + 3*(
3*C*a*b^2 + B*b^3)*c*d^2 + (3*C*a^2*b + 3*B*a*b^2 + A*b^3)*d^3)*f^3)*x^10
+ 1/9*(C*b^3*d^3*e^3 + 3*(3*C*b^3*c*d^2 + (3*C*a*b^2 + B*b^3)*d^3)*e^2*f +
 3*(3*C*b^3*c^2*d + 3*(3*C*a*b^2 + B*b^3)*c*d^2 + (3*C*a^2*b + 3*B*a*b^2 +
 A*b^3)*d^3)*e*f^2 + (C*b^3*c^3 + 3*(3*C*a*b^2 + B*b^3)*c^2*d + 3*(3*C*a^2
*b + 3*B*a*b^2 + A*b^3)*c*d^2 + (C*a^3 + 3*B*a^2*b + 3*A*a*b^2)*d^3)*f^3)*
x^9 + 1/8*((3*C*b^3*c*d^2 + (3*C*a*b^2 + B*b^3)*d^3)*e^3 + 3*(3*C*b^3*c^2*
d + 3*(3*C*a*b^2 + B*b^3)*c*d^2 + (3*C*a^2*b + 3*B*a*b^2 + A*b^3)*d^3)*e^2
*f + 3*(C*b^3*c^3 + 3*(3*C*a*b^2 + B*b^3)*c^2*d + 3*(3*C*a^2*b + 3*B*a*b^2
 + A*b^3)*c*d^2 + (C*a^3 + 3*B*a^2*b + 3*A*a*b^2)*d^3)*e*f^2 + ((3*C*a*b^2
 + B*b^3)*c^3 + 3*(3*C*a^2*b + 3*B*a*b^2 + A*b^3)*c^2*d + 3*(C*a^3 + 3*B*a
^2*b + 3*A*a*b^2)*c*d^2 + (B*a^3 + 3*A*a^2*b)*d^3)*f^3)*x^8 + 1/7*((3*C*b^
3*c^2*d + 3*(3*C*a*b^2 + B*b^3)*c*d^2 + (3*C*a^2*b + 3*B*a*b^2 + A*b^3)*d^
3)*e^3 + 3*(C*b^3*c^3 + 3*(3*C*a*b^2 + B*b^3)*c^2*d + 3*(3*C*a^2*b + 3*B*a
*b^2 + A*b^3)*c*d^2 + (C*a^3 + 3*B*a^2*b + 3*A*a*b^2)*d^3)*e^2*f + 3*((3*C
*a*b^2 + B*b^3)*c^3 + 3*(3*C*a^2*b + 3*B*a*b^2 + A*b^3)*c^2*d + 3*(C*a^3 +
 3*B*a^2*b + 3*A*a*b^2)*c*d^2 + (B*a^3 + 3*A*a^2*b)*d^3)*e*f^2 + (A*a^3*d^
3 + (3*C*a^2*b + 3*B*a*b^2 + A*b^3)*c^3 + 3*(C*a^3 + 3*B*a^2*b + 3*A*a*...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3151 vs. 2(1344) = 2688.

Time = 0.12 (sec) , antiderivative size = 3151, normalized size of antiderivative = 2.31

$$\int (A + Bx + Cx^2) \left(ace + (bce + ade + acf)x + (bde + bcf + adf)x^2 + bdfx^3\right)^3 dx = \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)*(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x
^2+b*d*f*x^3)^3,x, algorithm="giac")
```

output
```
1/12*C*b^3*d^3*f^3*x^12 + 3/11*C*b^3*d^3*e*f^2*x^11 + 3/11*C*b^3*c*d^2*f^3
*x^11 + 3/11*C*a*b^2*d^3*f^3*x^11 + 1/11*B*b^3*d^3*f^3*x^11 + 3/10*C*b^3*d
^3*e^2*f*x^10 + 9/10*C*b^3*c*d^2*e*f^2*x^10 + 9/10*C*a*b^2*d^3*e*f^2*x^10
+ 3/10*B*b^3*d^3*e*f^2*x^10 + 3/10*C*b^3*c^2*d*f^3*x^10 + 9/10*C*a*b^2*c*d
^2*f^3*x^10 + 3/10*B*b^3*c*d^2*f^3*x^10 + 3/10*C*a^2*b*d^3*f^3*x^10 + 3/10
*B*a*b^2*d^3*f^3*x^10 + 1/10*A*b^3*d^3*f^3*x^10 + 1/9*C*b^3*d^3*e^3*x^9 +
C*b^3*c*d^2*e^2*f*x^9 + C*a*b^2*d^3*e^2*f*x^9 + 1/3*B*b^3*d^3*e^2*f*x^9 +
C*b^3*c^2*d*e*f^2*x^9 + 3*C*a*b^2*c*d^2*e*f^2*x^9 + B*b^3*c*d^2*e*f^2*x^9
+ C*a^2*b*d^3*e*f^2*x^9 + B*a*b^2*d^3*e*f^2*x^9 + 1/3*A*b^3*d^3*e*f^2*x^9
+ 1/9*C*b^3*c^3*f^3*x^9 + C*a*b^2*c^2*d*f^3*x^9 + 1/3*B*b^3*c^2*d*f^3*x^9
+ C*a^2*b*c*d^2*f^3*x^9 + B*a*b^2*c*d^2*f^3*x^9 + 1/3*A*b^3*c*d^2*f^3*x^9
+ 1/9*C*a^3*d^3*f^3*x^9 + 1/3*B*a^2*b*d^3*f^3*x^9 + 1/3*A*a*b^2*d^3*f^3*x^
9 + 3/8*C*b^3*c*d^2*e^3*x^8 + 3/8*C*a*b^2*d^3*e^3*x^8 + 1/8*B*b^3*d^3*e^3*
x^8 + 9/8*C*b^3*c^2*d*e^2*f*x^8 + 27/8*C*a*b^2*c*d^2*e^2*f*x^8 + 9/8*B*b^3
*c*d^2*e^2*f*x^8 + 9/8*C*a^2*b*d^3*e^2*f*x^8 + 9/8*B*a*b^2*d^3*e^2*f*x^8 +
 3/8*A*b^3*d^3*e^2*f*x^8 + 3/8*C*b^3*c^3*e*f^2*x^8 + 27/8*C*a*b^2*c^2*d*e*
f^2*x^8 + 9/8*B*b^3*c^2*d*e*f^2*x^8 + 27/8*C*a^2*b*c*d^2*e*f^2*x^8 + 27/8*
B*a*b^2*c*d^2*e*f^2*x^8 + 9/8*A*b^3*c*d^2*e*f^2*x^8 + 3/8*C*a^3*d^3*e*f^2*
x^8 + 9/8*B*a^2*b*d^3*e*f^2*x^8 + 9/8*A*a*b^2*d^3*e*f^2*x^8 + 3/8*C*a*b^2*
c^3*f^3*x^8 + 1/8*B*b^3*c^3*f^3*x^8 + 9/8*C*a^2*b*c^2*d*f^3*x^8 + 9/8*B...
```

Mupad [B] (verification not implemented)

Time = 12.35 (sec) , antiderivative size = 2595, normalized size of antiderivative = 1.90

$$\int (A+Bx+Cx^2) \left(ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3\right)^3 dx$$
$$= \text{Too large to display}$$

input
```
int((A + B*x + C*x^2)*(x^2*(a*d*f + b*c*f + b*d*e) + x*(a*c*f + a*d*e + b*
c*e) + a*c*e + b*d*f*x^3)^3,x)
```

output
```
x^3*((C*a^3*c^3*e^3)/3 + A*a*b^2*c^3*e^3 + B*a^2*b*c^3*e^3 + A*a^3*c*d^2*e
^3 + B*a^3*c^2*d*e^3 + A*a^3*c^3*e*f^2 + B*a^3*c^3*e^2*f + 3*A*a^2*b*c^2*d
*e^3 + 3*A*a^2*b*c^3*e^2*f + 3*A*a^3*c^2*d*e^2*f) + x^4*((A*a^3*c^3*f^3)/4
 + (A*a^3*d^3*e^3)/4 + (A*b^3*c^3*e^3)/4 + (3*B*a*b^2*c^3*e^3)/4 + (3*C*a^
2*b*c^3*e^3)/4 + (3*B*a^3*c*d^2*e^3)/4 + (3*C*a^3*c^2*d*e^3)/4 + (3*B*a^3*
c^3*e*f^2)/4 + (3*C*a^3*c^3*e^2*f)/4 + (9*A*a*b^2*c^2*d*e^3)/4 + (9*A*a^2*
b*c*d^2*e^3)/4 + (9*B*a^2*b*c^2*d*e^3)/4 + (9*A*a*b^2*c^3*e^2*f)/4 + (9*A*
a^2*b*c^3*e*f^2)/4 + (9*B*a^2*b*c^3*e^2*f)/4 + (9*A*a^3*c*d^2*e^2*f)/4 + (
9*A*a^3*c^2*d*e*f^2)/4 + (9*B*a^3*c^2*d*e^2*f)/4 + (27*A*a^2*b*c^2*d*e^2*f
)/4) + x^10*((A*b^3*d^3*f^3)/10 + (3*B*a*b^2*d^3*f^3)/10 + (3*C*a^2*b*d^3*
f^3)/10 + (3*B*b^3*c*d^2*f^3)/10 + (3*C*b^3*c^2*d*f^3)/10 + (3*B*b^3*d^3*e
*f^2)/10 + (3*C*b^3*d^3*e^2*f)/10 + (9*C*a*b^2*c*d^2*f^3)/10 + (9*C*a*b^2*
d^3*e*f^2)/10 + (9*C*b^3*c*d^2*e*f^2)/10) + x^9*((C*a^3*d^3*f^3)/9 + (C*b^
3*c^3*f^3)/9 + (C*b^3*d^3*e^3)/9 + (A*a*b^2*d^3*f^3)/3 + (B*a^2*b*d^3*f^3)
/3 + (A*b^3*c*d^2*f^3)/3 + (B*b^3*c^2*d*f^3)/3 + (A*b^3*d^3*e*f^2)/3 + (B*
b^3*d^3*e^2*f)/3 + B*a*b^2*c*d^2*f^3 + C*a*b^2*c^2*d*f^3 + C*a^2*b*c*d^2*f
^3 + B*a*b^2*d^3*e*f^2 + C*a*b^2*d^3*e^2*f + C*a^2*b*d^3*e*f^2 + B*b^3*c*d
^2*e*f^2 + C*b^3*c*d^2*e^2*f + C*b^3*c^2*d*e*f^2 + 3*C*a*b^2*c*d^2*e*f^2)
+ x^5*((B*a^3*c^3*f^3)/5 + (B*a^3*d^3*e^3)/5 + (B*b^3*c^3*e^3)/5 + (3*A*a^
2*b*c^3*f^3)/5 + (3*A*a^2*b*d^3*e^3)/5 + (3*C*a*b^2*c^3*e^3)/5 + (3*A*a...
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 2315, normalized size of antiderivative = 1.70

$$\int \left(A+Bx+Cx^2\right)\left(ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3\right)^3\,dx$$
$$= \text{Too large to display}$$

input
```
int((C*x^2+B*x+A)*(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d
*f*x^3)^3,x)
```

output
```
(x*(27720*a**4*c**3*e**3 + 41580*a**4*c**3*e**2*f*x + 27720*a**4*c**3*e*f*
*2*x**2 + 6930*a**4*c**3*f**3*x**3 + 41580*a**4*c**2*d*e**3*x + 83160*a**4
*c**2*d*e**2*f*x**2 + 62370*a**4*c**2*d*e*f**2*x**3 + 16632*a**4*c**2*d*f*
*3*x**4 + 27720*a**4*c*d**2*e**3*x**2 + 62370*a**4*c*d**2*e**2*f*x**3 + 49
896*a**4*c*d**2*e*f**2*x**4 + 13860*a**4*c*d**2*f**3*x**5 + 6930*a**4*d**3
*e**3*x**3 + 16632*a**4*d**3*e**2*f*x**4 + 13860*a**4*d**3*e*f**2*x**5 + 3
960*a**4*d**3*f**3*x**6 + 55440*a**3*b*c**3*e**3*x + 110880*a**3*b*c**3*e*
*2*f*x**2 + 83160*a**3*b*c**3*e*f**2*x**3 + 22176*a**3*b*c**3*f**3*x**4 +
110880*a**3*b*c**2*d*e**3*x**2 + 249480*a**3*b*c**2*d*e**2*f*x**3 + 199584
*a**3*b*c**2*d*e*f**2*x**4 + 55440*a**3*b*c**2*d*f**3*x**5 + 83160*a**3*b*
c*d**2*e**3*x**3 + 199584*a**3*b*c*d**2*e**2*f*x**4 + 166320*a**3*b*c*d**2
*e*f**2*x**5 + 47520*a**3*b*c*d**2*f**3*x**6 + 22176*a**3*b*d**3*e**3*x**4
 + 55440*a**3*b*d**3*e**2*f*x**5 + 47520*a**3*b*d**3*e*f**2*x**6 + 13860*a
**3*b*d**3*f**3*x**7 + 9240*a**3*c**4*e**3*x**2 + 20790*a**3*c**4*e**2*f*x
**3 + 16632*a**3*c**4*e*f**2*x**4 + 4620*a**3*c**4*f**3*x**5 + 20790*a**3*
c**3*d*e**3*x**3 + 49896*a**3*c**3*d*e**2*f*x**4 + 41580*a**3*c**3*d*e*f**
2*x**5 + 11880*a**3*c**3*d*f**3*x**6 + 16632*a**3*c**2*d**2*e**3*x**4 + 41
580*a**3*c**2*d**2*e**2*f*x**5 + 35640*a**3*c**2*d**2*e*f**2*x**6 + 10395*
a**3*c**2*d**2*f**3*x**7 + 4620*a**3*c*d**3*e**3*x**5 + 11880*a**3*c*d**3*
e**2*f*x**6 + 10395*a**3*c*d**3*e*f**2*x**7 + 3080*a**3*c*d**3*f**3*x**...
```

3.110 $\int (A+Bx+Cx^2)\left(ace+(bce+ade+acf)x+(b$

Optimal result

Integrand size = 57, antiderivative size = 664

$$\int (A+Bx+Cx^2)\left(ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3\right)^2\,dx$$
$$= \frac{(Ab^2-a(bB-aC))\,(bc-ad)^2(be-af)^2(a+bx)^3}{3b^7}$$
$$-\frac{(bc-ad)(be-af)\,(6a^3Cdf+ab^2(2cCe+3Bde+3Bcf+4Adf)-b^3(Bce+2A(de+cf))-a^2b(5B$$
$$4b^7$$
$$+\frac{(15a^4Cd^2f^2-10a^3bdf(Bdf+2C(de+cf))+b^4(Ad^2e^2+2cde(Be+2Af)+c^2(Ce^2+2Bef+Af^2)$$
$$-\frac{(20a^3Cd^2f^2-10a^2bdf(Bdf+2C(de+cf))-b^3(d^2e(Be+2Af)+c^2f(2Ce+Bf)+2cd(Ce^2+2B}{6b^7}$$
$$+\frac{(15a^2Cd^2f^2-5abdf(Bdf+2C(de+cf))+b^2(df(2Bde+2Bcf+Adf)+C(d^2e^2+4cdef+c^2f^2)))}{7b^7}$$
$$-\frac{df(6aCdf-b(Bdf+2C(de+cf)))(a+bx)^8}{8b^7}+\frac{Cd^2f^2(a+bx)^9}{9b^7}$$

output
```
1/3*(A*b^2-a*(B*b-C*a))*(-a*d+b*c)^2*(-a*f+b*e)^2*(b*x+a)^3/b^7-1/4*(-a*d+
b*c)*(-a*f+b*e)*(6*a^3*C*d*f+a*b^2*(4*A*d*f+3*B*c*f+3*B*d*e+2*C*c*e)-b^3*(
B*c*e+2*A*(c*f+d*e))-a^2*b*(5*B*d*f+4*C*(c*f+d*e)))*(b*x+a)^4/b^7+1/5*(15*
a^4*C*d^2*f^2-10*a^3*b*d*f*(B*d*f+2*C*(c*f+d*e))+b^4*(A*d^2*e^2+2*c*d*e*(2
*A*f+B*e)+c^2*(A*f^2+2*B*e*f+C*e^2))-3*a*b^3*(d^2*e*(2*A*f+B*e)+c^2*f*(B*f
+2*C*e)+2*c*d*(A*f^2+2*B*e*f+C*e^2))+6*a^2*b^2*(d*f*(A*d*f+2*B*c*f+2*B*d*e
)+C*(c^2*f^2+4*c*d*e*f+d^2*e^2)))*(b*x+a)^5/b^7-1/6*(20*a^3*C*d^2*f^2-10*a
^2*b*d*f*(B*d*f+2*C*(c*f+d*e))-b^3*(d^2*e*(2*A*f+B*e)+c^2*f*(B*f+2*C*e)+2*
c*d*(A*f^2+2*B*e*f+C*e^2))+4*a*b^2*(d*f*(A*d*f+2*B*c*f+2*B*d*e)+C*(c^2*f^2
+4*c*d*e*f+d^2*e^2)))*(b*x+a)^6/b^7+1/7*(15*a^2*C*d^2*f^2-5*a*b*d*f*(B*d*f
+2*C*(c*f+d*e))+b^2*(d*f*(A*d*f+2*B*c*f+2*B*d*e)+C*(c^2*f^2+4*c*d*e*f+d^2*
e^2)))*(b*x+a)^7/b^7-1/8*d*f*(6*a*C*d*f-b*(B*d*f+2*C*(c*f+d*e)))*(b*x+a)^8
/b^7+1/9*C*d^2*f^2*(b*x+a)^9/b^7
```

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 671, normalized size of antiderivative = 1.01

$$\int (A+Bx+Cx^2)\left(ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3\right)^2\,dx$$
$$\begin{aligned}
&= a^2Ac^2e^2x+\frac{1}{2}ace(aBce+2A(bce+ade+acf))x^2 \\
&\quad+\frac{1}{3}\left(ace(2bBce+a(cCe+2Bde+2Bcf))\right. \\
&\qquad\left.+A\left(b^2c^2e^2+4abce(de+cf)+a^2\left(d^2e^2+4cdef+c^2f^2\right)\right)\right)x^3 \\
&\quad+\frac{1}{4}\left(b^2ce(Bce+2A(de+cf))+2ab\left(Ad^2e^2+2cde(Be+2Af)+c^2\left(Ce^2+2Bef+Af^2\right)\right)\right. \\
&\qquad\left.+a^2\left(d^2e(Be+2Af)+c^2f(2Ce+Bf)+2cd\left(Ce^2+2Bef+Af^2\right)\right)\right)x^4 \\
&\quad+\frac{1}{5}\left(b^2\left(Ad^2e^2+2cde(Be+2Af)+c^2\left(Ce^2+2Bef+Af^2\right)\right)\right. \\
&\qquad+2ab\left(d^2e(Be+2Af)+c^2f(2Ce+Bf)+2cd\left(Ce^2+2Bef+Af^2\right)\right) \\
&\qquad\left.+a^2\left(df(2Bde+2Bcf+Adf)+C\left(d^2e^2+4cdef+c^2f^2\right)\right)\right)x^5 \\
&\quad+\frac{1}{6}\left(a^2df(Bdf+2C(de+cf))\right. \\
&\qquad+b^2\left(d^2e(Be+2Af)+c^2f(2Ce+Bf)+2cd\left(Ce^2+2Bef+Af^2\right)\right) \\
&\qquad\left.+2ab\left(df(2Bde+2Bcf+Adf)+C\left(d^2e^2+4cdef+c^2f^2\right)\right)\right)x^6 \\
&\quad+\frac{1}{7}\left(a^2Cd^2f^2+2abdf(Bdf+2C(de+cf))\right. \\
&\qquad\left.+b^2\left(df(2Bde+2Bcf+Adf)+C\left(d^2e^2+4cdef+c^2f^2\right)\right)\right)x^7 \\
&\quad+\frac{1}{8}bdf(bBdf+2aCdf+2bC(de+cf))x^8+\frac{1}{9}b^2Cd^2f^2x^9
\end{aligned}$$

input
```
Integrate[(A + B*x + C*x^2)*(a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e +
b*c*f + a*d*f)*x^2 + b*d*f*x^3)^2,x]
```

output
```
a^2*A*c^2*e^2*x + (a*c*e*(a*B*c*e + 2*A*(b*c*e + a*d*e + a*c*f))*x^2)/2 +
((a*c*e*(2*b*B*c*e + a*(c*C*e + 2*B*d*e + 2*B*c*f)) + A*(b^2*c^2*e^2 + 4*a
*b*c*e*(d*e + c*f) + a^2*(d^2*e^2 + 4*c*d*e*f + c^2*f^2)))*x^3)/3 + ((b^2*
c*e*(B*c*e + 2*A*(d*e + c*f)) + 2*a*b*(A*d^2*e^2 + 2*c*d*e*(B*e + 2*A*f) +
 c^2*(C*e^2 + 2*B*e*f + A*f^2)) + a^2*(d^2*e*(B*e + 2*A*f) + c^2*f*(2*C*e
+ B*f) + 2*c*d*(C*e^2 + 2*B*e*f + A*f^2)))*x^4)/4 + ((b^2*(A*d^2*e^2 + 2*c
*d*e*(B*e + 2*A*f) + c^2*(C*e^2 + 2*B*e*f + A*f^2)) + 2*a*b*(d^2*e*(B*e +
2*A*f) + c^2*f*(2*C*e + B*f) + 2*c*d*(C*e^2 + 2*B*e*f + A*f^2)) + a^2*(d*f
*(2*B*d*e + 2*B*c*f + A*d*f) + C*(d^2*e^2 + 4*c*d*e*f + c^2*f^2)))*x^5)/5
+ ((a^2*d*f*(B*d*f + 2*C*(d*e + c*f)) + b^2*(d^2*e*(B*e + 2*A*f) + c^2*f*(
2*C*e + B*f) + 2*c*d*(C*e^2 + 2*B*e*f + A*f^2)) + 2*a*b*(d*f*(2*B*d*e + 2*
B*c*f + A*d*f) + C*(d^2*e^2 + 4*c*d*e*f + c^2*f^2)))*x^6)/6 + ((a^2*C*d^2*
f^2 + 2*a*b*d*f*(B*d*f + 2*C*(d*e + c*f)) + b^2*(d*f*(2*B*d*e + 2*B*c*f +
A*d*f) + C*(d^2*e^2 + 4*c*d*e*f + c^2*f^2)))*x^7)/7 + (b*d*f*(b*B*d*f + 2*
a*C*d*f + 2*b*C*(d*e + c*f))*x^8)/8 + (b^2*C*d^2*f^2*x^9)/9
```

Rubi [A] (verified)

Time = 2.51 (sec) , antiderivative size = 671, normalized size of antiderivative = 1.01, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.035$, Rules used = {2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(A+Bx+Cx^2\right)\left(x^2(adf+bcf+bde)+x(acf+ade+bce)+ace+bdfx^3\right)^2\,dx$$

$$\Big\downarrow \text{2188}$$

$$\int \left(x^6\left(a^2Cd^2f^2+2abdf(Bdf+2C(cf+de))+b^2\left(df(Adf+2Bcf+2Bde)+C\left(c^2f^2+4cdef+d^2e^2\right)\right)\right)+x^5\left(\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{7}x^7\left(a^2Cd^2f^2+2abdf(Bdf+2C(cf+de))+b^2\left(df(Adf+2Bcf+2Bde)+C\left(c^2f^2+4cdef+d^2e^2\right)\right)\right)+$$
$$\frac{1}{6}x^6\left(a^2df(Bdf+2C(cf+de))+2ab\left(df(Adf+2Bcf+2Bde)+C\left(c^2f^2+4cdef+d^2e^2\right)\right)+b^2\left(2cd\left(Af^2+2B\right.\right.\right.$$
$$\frac{1}{5}x^5\left(a^2\left(df(Adf+2Bcf+2Bde)+C\left(c^2f^2+4cdef+d^2e^2\right)\right)+2ab\left(2cd\left(Af^2+2Bef+Ce^2\right)+d^2e(2Af+Be)\right.\right.$$
$$\frac{1}{4}x^4\left(a^2\left(2cd\left(Af^2+2Bef+Ce^2\right)+d^2e(2Af+Be)+c^2f(Bf+2Ce)\right)+2ab\left(c^2\left(Af^2+2Bef+Ce^2\right)+2cde(2\right.\right.$$
$$\frac{1}{3}x^3\left(A\left(a^2\left(c^2f^2+4cdef+d^2e^2\right)+4abce(cf+de)+b^2c^2e^2\right)+ace(a(2Bcf+2Bde+cCe)+2bBce)\right)+$$
$$a^2Ac^2e^2x+\frac{1}{2}acex^2(2A(acf+ade+bce)+aBce)+\frac{1}{8}bdfx^8(2aCdf+bBdf+2bC(cf+de))+$$
$$\frac{1}{9}b^2Cd^2f^2x^9$$

input

```
Int[(A + B*x + C*x^2)*(a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*c*f
+ a*d*f)*x^2 + b*d*f*x^3)^2,x]
```

output

```
a^2*A*c^2*e^2*x + (a*c*e*(a*B*c*e + 2*A*(b*c*e + a*d*e + a*c*f))*x^2)/2 +
((a*c*e*(2*b*B*c*e + a*(c*C*e + 2*B*d*e + 2*B*c*f)) + A*(b^2*c^2*e^2 + 4*a
*b*c*e*(d*e + c*f) + a^2*(d^2*e^2 + 4*c*d*e*f + c^2*f^2)))*x^3)/3 + ((b^2*
c*e*(B*c*e + 2*A*(d*e + c*f)) + 2*a*b*(A*d^2*e^2 + 2*c*d*e*(B*e + 2*A*f) +
 c^2*(C*e^2 + 2*B*e*f + A*f^2)) + a^2*(d^2*e*(B*e + 2*A*f) + c^2*f*(2*C*e
+ B*f) + 2*c*d*(C*e^2 + 2*B*e*f + A*f^2)))*x^4)/4 + ((b^2*(A*d^2*e^2 + 2*c
*d*e*(B*e + 2*A*f) + c^2*(C*e^2 + 2*B*e*f + A*f^2)) + 2*a*b*(d^2*e*(B*e +
2*A*f) + c^2*f*(2*C*e + B*f) + 2*c*d*(C*e^2 + 2*B*e*f + A*f^2)) + a^2*(d*f
*(2*B*d*e + 2*B*c*f + A*d*f) + C*(d^2*e^2 + 4*c*d*e*f + c^2*f^2)))*x^5)/5
+ ((a^2*d*f*(B*d*f + 2*C*(d*e + c*f)) + b^2*(d^2*e*(B*e + 2*A*f) + c^2*f*(
2*C*e + B*f) + 2*c*d*(C*e^2 + 2*B*e*f + A*f^2)) + 2*a*b*(d*f*(2*B*d*e + 2*
B*c*f + A*d*f) + C*(d^2*e^2 + 4*c*d*e*f + c^2*f^2)))*x^6)/6 + ((a^2*C*d^2*
f^2 + 2*a*b*d*f*(B*d*f + 2*C*(d*e + c*f)) + b^2*(d*f*(2*B*d*e + 2*B*c*f +
A*d*f) + C*(d^2*e^2 + 4*c*d*e*f + c^2*f^2)))*x^7)/7 + (b*d*f*(b*B*d*f + 2*
a*C*d*f + 2*b*C*(d*e + c*f))*x^8)/8 + (b^2*C*d^2*f^2*x^9)/9
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 562, normalized size of antiderivative = 0.85

method	result
default	$\frac{C f^2 d^2 b^2 x^9}{9} + \frac{(B b^2 d^2 f^2 + 2C(adf+bcf+bde)bdf)x^8}{8} + \frac{\left(A b^2 d^2 f^2 + 2B(adf+bcf+bde)bdf + C\left(2(acf+ade+bce)bdf + (ad\right.\right.}{7}$
norman	$\frac{C f^2 d^2 b^2 x^9}{9} + \left(\frac{1}{8} B b^2 d^2 f^2 + \frac{1}{4} C f^2 d^2 ab + \frac{1}{4} C b^2 cd f^2 + \frac{1}{4} C b^2 d^2 ef\right) x^8 + \left(\frac{1}{7} A b^2 d^2 f^2 + \frac{2}{7} Bab\right.$
risch	Expression too large to display
parallelrisch	Expression too large to display
gosper	Expression too large to display
orering	Expression too large to display

input
```
int((C*x^2+B*x+A)*(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d
*f*x^3)^2,x,method=_RETURNVERBOSE)
```

output
```
1/9*C*f^2*d^2*b^2*x^9+1/8*(B*b^2*d^2*f^2+2*C*(a*d*f+b*c*f+b*d*e)*b*d*f)*x^
8+1/7*(A*b^2*d^2*f^2+2*B*(a*d*f+b*c*f+b*d*e)*b*d*f+C*(2*(a*c*f+a*d*e+b*c*e
)*b*d*f+(a*d*f+b*c*f+b*d*e)^2))*x^7+1/6*(2*A*(a*d*f+b*c*f+b*d*e)*b*d*f+B*(
2*(a*c*f+a*d*e+b*c*e)*b*d*f+(a*d*f+b*c*f+b*d*e)^2)+C*(2*a*c*e*b*d*f+2*(a*c
*f+a*d*e+b*c*e)*(a*d*f+b*c*f+b*d*e)))*x^6+1/5*(A*(2*(a*c*f+a*d*e+b*c*e)*b*
d*f+(a*d*f+b*c*f+b*d*e)^2)+B*(2*a*c*e*b*d*f+2*(a*c*f+a*d*e+b*c*e)*(a*d*f+b
*c*f+b*d*e))+C*(2*a*c*e*(a*d*f+b*c*f+b*d*e)+(a*c*f+a*d*e+b*c*e)^2))*x^5+1/
4*(A*(2*a*c*e*b*d*f+2*(a*c*f+a*d*e+b*c*e)*(a*d*f+b*c*f+b*d*e))+B*(2*a*c*e*
(a*d*f+b*c*f+b*d*e)+(a*c*f+a*d*e+b*c*e)^2)+2*C*a*c*e*(a*c*f+a*d*e+b*c*e))*
x^4+1/3*(A*(2*a*c*e*(a*d*f+b*c*f+b*d*e)+(a*c*f+a*d*e+b*c*e)^2)+2*B*a*c*e*(
a*c*f+a*d*e+b*c*e)+C*a^2*c^2*e^2)*x^3+1/2*(2*A*a*c*e*(a*c*f+a*d*e+b*c*e)+B
*a^2*c^2*e^2)*x^2+A*c^2*e^2*a^2*x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 735, normalized size of antiderivative = 1.11

$$\int (A + Bx + Cx^2) \left(ace + (bce + ade + acf)x + (bde + bcf + adf)x^2 + bdfx^3\right)^2 dx$$
$$= \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)*(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x
^2+b*d*f*x^3)^2,x, algorithm="fricas")
```

output
```
1/9*C*b^2*d^2*f^2*x^9 + 1/8*(2*C*b^2*d^2*e*f + (2*C*b^2*c*d + (2*C*a*b + B
*b^2)*d^2)*f^2)*x^8 + A*a^2*c^2*e^2*x + 1/7*(C*b^2*d^2*e^2 + 2*(2*C*b^2*c*
d + (2*C*a*b + B*b^2)*d^2)*e*f + (C*b^2*c^2 + 2*(2*C*a*b + B*b^2)*c*d + (C
*a^2 + 2*B*a*b + A*b^2)*d^2)*f^2)*x^7 + 1/6*((2*C*b^2*c*d + (2*C*a*b + B*b
^2)*d^2)*e^2 + 2*(C*b^2*c^2 + 2*(2*C*a*b + B*b^2)*c*d + (C*a^2 + 2*B*a*b +
 A*b^2)*d^2)*e*f + ((2*C*a*b + B*b^2)*c^2 + 2*(C*a^2 + 2*B*a*b + A*b^2)*c*
d + (B*a^2 + 2*A*a*b)*d^2)*f^2)*x^6 + 1/5*((C*b^2*c^2 + 2*(2*C*a*b + B*b^2
)*c*d + (C*a^2 + 2*B*a*b + A*b^2)*d^2)*e^2 + 2*((2*C*a*b + B*b^2)*c^2 + 2*
(C*a^2 + 2*B*a*b + A*b^2)*c*d + (B*a^2 + 2*A*a*b)*d^2)*e*f + (A*a^2*d^2 +
(C*a^2 + 2*B*a*b + A*b^2)*c^2 + 2*(B*a^2 + 2*A*a*b)*c*d)*f^2)*x^5 + 1/4*((
(2*C*a*b + B*b^2)*c^2 + 2*(C*a^2 + 2*B*a*b + A*b^2)*c*d + (B*a^2 + 2*A*a*b
)*d^2)*e^2 + 2*(A*a^2*d^2 + (C*a^2 + 2*B*a*b + A*b^2)*c^2 + 2*(B*a^2 + 2*A
*a*b)*c*d)*e*f + (2*A*a^2*c*d + (B*a^2 + 2*A*a*b)*c^2)*f^2)*x^4 + 1/3*(A*a
^2*c^2*f^2 + (A*a^2*d^2 + (C*a^2 + 2*B*a*b + A*b^2)*c^2 + 2*(B*a^2 + 2*A*a
*b)*c*d)*e^2 + 2*(2*A*a^2*c*d + (B*a^2 + 2*A*a*b)*c^2)*e*f)*x^3 + 1/2*(2*A
*a^2*c^2*e*f + (2*A*a^2*c*d + (B*a^2 + 2*A*a*b)*c^2)*e^2)*x^2
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 1166, normalized size of antiderivative = 1.76

$$\int (A + Bx + Cx^2) \left(ace + (bce + ade + acf)x + (bde + bcf + adf)x^2 + bdfx^3\right)^2 dx$$
$$= \text{Too large to display}$$

input
```
integrate((C*x**2+B*x+A)*(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*
x**2+b*d*f*x**3)**2,x)
```

output
```
A*a**2*c**2*e**2*x + C*b**2*d**2*f**2*x**9/9 + x**8*(B*b**2*d**2*f**2/8 +
C*a*b*d**2*f**2/4 + C*b**2*c*d*f**2/4 + C*b**2*d**2*e*f/4) + x**7*(A*b**2*
d**2*f**2/7 + 2*B*a*b*d**2*f**2/7 + 2*B*b**2*c*d*f**2/7 + 2*B*b**2*d**2*e*
f/7 + C*a**2*d**2*f**2/7 + 4*C*a*b*c*d*f**2/7 + 4*C*a*b*d**2*e*f/7 + C*b**
2*c**2*f**2/7 + 4*C*b**2*c*d*e*f/7 + C*b**2*d**2*e**2/7) + x**6*(A*a*b*d**
2*f**2/3 + A*b**2*c*d*f**2/3 + A*b**2*d**2*e*f/3 + B*a**2*d**2*f**2/6 + 2*
B*a*b*c*d*f**2/3 + 2*B*a*b*d**2*e*f/3 + B*b**2*c**2*f**2/6 + 2*B*b**2*c*d*
e*f/3 + B*b**2*d**2*e**2/6 + C*a**2*c*d*f**2/3 + C*a**2*d**2*e*f/3 + C*a*b
*c**2*f**2/3 + 4*C*a*b*c*d*e*f/3 + C*a*b*d**2*e**2/3 + C*b**2*c**2*e*f/3 +
 C*b**2*c*d*e**2/3) + x**5*(A*a**2*d**2*f**2/5 + 4*A*a*b*c*d*f**2/5 + 4*A*
a*b*d**2*e*f/5 + A*b**2*c**2*f**2/5 + 4*A*b**2*c*d*e*f/5 + A*b**2*d**2*e**
2/5 + 2*B*a**2*c*d*f**2/5 + 2*B*a**2*d**2*e*f/5 + 2*B*a*b*c**2*f**2/5 + 8*
B*a*b*c*d*e*f/5 + 2*B*a*b*d**2*e**2/5 + 2*B*b**2*c**2*e*f/5 + 2*B*b**2*c*d
*e**2/5 + C*a**2*c**2*f**2/5 + 4*C*a**2*c*d*e*f/5 + C*a**2*d**2*e**2/5 + 4
*C*a*b*c**2*e*f/5 + 4*C*a*b*c*d*e**2/5 + C*b**2*c**2*e**2/5) + x**4*(A*a**
2*c*d*f**2/2 + A*a**2*d**2*e*f/2 + A*a*b*c**2*f**2/2 + 2*A*a*b*c*d*e*f + A
*a*b*d**2*e**2/2 + A*b**2*c**2*e*f/2 + A*b**2*c*d*e**2/2 + B*a**2*c**2*f**
2/4 + B*a**2*c*d*e*f + B*a**2*d**2*e**2/4 + B*a*b*c**2*e*f + B*a*b*c*d*e**
2 + B*b**2*c**2*e**2/4 + C*a**2*c**2*e*f/2 + C*a**2*c*d*e**2/2 + C*a*b*c**
2*e**2/2) + x**3*(A*a**2*c**2*f**2/3 + 4*A*a**2*c*d*e*f/3 + A*a**2*d**2...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 735, normalized size of antiderivative = 1.11

$$\int (A + Bx + Cx^2) \left(ace + (bce + ade + acf)x + (bde + bcf + adf)x^2 + bdfx^3\right)^2 dx$$
$$= \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)*(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x
^2+b*d*f*x^3)^2,x, algorithm="maxima")
```

output
```
1/9*C*b^2*d^2*f^2*x^9 + 1/8*(2*C*b^2*d^2*e*f + (2*C*b^2*c*d + (2*C*a*b + B
*b^2)*d^2)*f^2)*x^8 + A*a^2*c^2*e^2*x + 1/7*(C*b^2*d^2*e^2 + 2*(2*C*b^2*c*
d + (2*C*a*b + B*b^2)*d^2)*e*f + (C*b^2*c^2 + 2*(2*C*a*b + B*b^2)*c*d + (C
*a^2 + 2*B*a*b + A*b^2)*d^2)*f^2)*x^7 + 1/6*((2*C*b^2*c*d + (2*C*a*b + B*b
^2)*d^2)*e^2 + 2*(C*b^2*c^2 + 2*(2*C*a*b + B*b^2)*c*d + (C*a^2 + 2*B*a*b +
 A*b^2)*d^2)*e*f + ((2*C*a*b + B*b^2)*c^2 + 2*(C*a^2 + 2*B*a*b + A*b^2)*c*
d + (B*a^2 + 2*A*a*b)*d^2)*f^2)*x^6 + 1/5*((C*b^2*c^2 + 2*(2*C*a*b + B*b^2
)*c*d + (C*a^2 + 2*B*a*b + A*b^2)*d^2)*e^2 + 2*((2*C*a*b + B*b^2)*c^2 + 2*
(C*a^2 + 2*B*a*b + A*b^2)*c*d + (B*a^2 + 2*A*a*b)*d^2)*e*f + (A*a^2*d^2 +
(C*a^2 + 2*B*a*b + A*b^2)*c^2 + 2*(B*a^2 + 2*A*a*b)*c*d)*f^2)*x^5 + 1/4*((
(2*C*a*b + B*b^2)*c^2 + 2*(C*a^2 + 2*B*a*b + A*b^2)*c*d + (B*a^2 + 2*A*a*b
)*d^2)*e^2 + 2*(A*a^2*d^2 + (C*a^2 + 2*B*a*b + A*b^2)*c^2 + 2*(B*a^2 + 2*A
*a*b)*c*d)*e*f + (2*A*a^2*c*d + (B*a^2 + 2*A*a*b)*c^2)*f^2)*x^4 + 1/3*(A*a
^2*c^2*f^2 + (A*a^2*d^2 + (C*a^2 + 2*B*a*b + A*b^2)*c^2 + 2*(B*a^2 + 2*A*a
*b)*c*d)*e^2 + 2*(2*A*a^2*c*d + (B*a^2 + 2*A*a*b)*c^2)*e*f)*x^3 + 1/2*(2*A
*a^2*c^2*e*f + (2*A*a^2*c*d + (B*a^2 + 2*A*a*b)*c^2)*e^2)*x^2
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 1126, normalized size of antiderivative = 1.70

$$\int (A+Bx+Cx^2)\left(ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3\right)^2\,dx$$
$$= \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)*(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x
^2+b*d*f*x^3)^2,x, algorithm="giac")
```

output
```
1/9*C*b^2*d^2*f^2*x^9 + 1/4*C*b^2*d^2*e*f*x^8 + 1/4*C*b^2*c*d*f^2*x^8 + 1/
4*C*a*b*d^2*f^2*x^8 + 1/8*B*b^2*d^2*f^2*x^8 + 1/7*C*b^2*d^2*e^2*x^7 + 4/7*
C*b^2*c*d*e*f*x^7 + 4/7*C*a*b*d^2*e*f*x^7 + 2/7*B*b^2*d^2*e*f*x^7 + 1/7*C*
b^2*c^2*f^2*x^7 + 4/7*C*a*b*c*d*f^2*x^7 + 2/7*B*b^2*c*d*f^2*x^7 + 1/7*C*a^
2*d^2*f^2*x^7 + 2/7*B*a*b*d^2*f^2*x^7 + 1/7*A*b^2*d^2*f^2*x^7 + 1/3*C*b^2*
c*d*e^2*x^6 + 1/3*C*a*b*d^2*e^2*x^6 + 1/6*B*b^2*d^2*e^2*x^6 + 1/3*C*b^2*c^
2*e*f*x^6 + 4/3*C*a*b*c*d*e*f*x^6 + 2/3*B*b^2*c*d*e*f*x^6 + 1/3*C*a^2*d^2*
e*f*x^6 + 2/3*B*a*b*d^2*e*f*x^6 + 1/3*A*b^2*d^2*e*f*x^6 + 1/3*C*a*b*c^2*f^
2*x^6 + 1/6*B*b^2*c^2*f^2*x^6 + 1/3*C*a^2*c*d*f^2*x^6 + 2/3*B*a*b*c*d*f^2*
x^6 + 1/3*A*b^2*c*d*f^2*x^6 + 1/6*B*a^2*d^2*f^2*x^6 + 1/3*A*a*b*d^2*f^2*x^
6 + 1/5*C*b^2*c^2*e^2*x^5 + 4/5*C*a*b*c*d*e^2*x^5 + 2/5*B*b^2*c*d*e^2*x^5
+ 1/5*C*a^2*d^2*e^2*x^5 + 2/5*B*a*b*d^2*e^2*x^5 + 1/5*A*b^2*d^2*e^2*x^5 +
4/5*C*a*b*c^2*e*f*x^5 + 2/5*B*b^2*c^2*e*f*x^5 + 4/5*C*a^2*c*d*e*f*x^5 + 8/
5*B*a*b*c*d*e*f*x^5 + 4/5*A*b^2*c*d*e*f*x^5 + 2/5*B*a^2*d^2*e*f*x^5 + 4/5*
A*a*b*d^2*e*f*x^5 + 1/5*C*a^2*c^2*f^2*x^5 + 2/5*B*a*b*c^2*f^2*x^5 + 1/5*A*
b^2*c^2*f^2*x^5 + 2/5*B*a^2*c*d*f^2*x^5 + 4/5*A*a*b*c*d*f^2*x^5 + 1/5*A*a^
2*d^2*f^2*x^5 + 1/2*C*a*b*c^2*e^2*x^4 + 1/4*B*b^2*c^2*e^2*x^4 + 1/2*C*a^2*
c*d*e^2*x^4 + B*a*b*c*d*e^2*x^4 + 1/2*A*b^2*c*d*e^2*x^4 + 1/4*B*a^2*d^2*e^
2*x^4 + 1/2*A*a*b*d^2*e^2*x^4 + 1/2*C*a^2*c^2*e*f*x^4 + B*a*b*c^2*e*f*x^4
+ 1/2*A*b^2*c^2*e*f*x^4 + B*a^2*c*d*e*f*x^4 + 2*A*a*b*c*d*e*f*x^4 + 1/2...
```

Mupad [B] (verification not implemented)

Time = 12.18 (sec) , antiderivative size = 891, normalized size of antiderivative = 1.34

$$\int \left(A+Bx+Cx^2\right)\left(ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3\right)^2\,dx$$
$$= \text{Too large to display}$$

input
```
int((A + B*x + C*x^2)*(x^2*(a*d*f + b*c*f + b*d*e) + x*(a*c*f + a*d*e + b*
c*e) + a*c*e + b*d*f*x^3)^2,x)
```

output
```
x^5*((A*a^2*d^2*f^2)/5 + (A*b^2*c^2*f^2)/5 + (A*b^2*d^2*e^2)/5 + (C*a^2*c^
2*f^2)/5 + (C*a^2*d^2*e^2)/5 + (C*b^2*c^2*e^2)/5 + (2*B*a*b*c^2*f^2)/5 + (
2*B*a*b*d^2*e^2)/5 + (2*B*a^2*c*d*f^2)/5 + (2*B*b^2*c*d*e^2)/5 + (2*B*a^2*
d^2*e*f)/5 + (2*B*b^2*c^2*e*f)/5 + (4*A*a*b*c*d*f^2)/5 + (4*C*a*b*c*d*e^2)
/5 + (4*A*a*b*d^2*e*f)/5 + (4*C*a*b*c^2*e*f)/5 + (4*A*b^2*c*d*e*f)/5 + (4*
C*a^2*c*d*e*f)/5 + (8*B*a*b*c*d*e*f)/5) + x^4*((B*a^2*c^2*f^2)/4 + (B*a^2*
d^2*e^2)/4 + (B*b^2*c^2*e^2)/4 + (A*a*b*c^2*f^2)/2 + (A*a*b*d^2*e^2)/2 + (
C*a*b*c^2*e^2)/2 + (A*a^2*c*d*f^2)/2 + (A*b^2*c*d*e^2)/2 + (C*a^2*c*d*e^2)
/2 + (A*a^2*d^2*e*f)/2 + (A*b^2*c^2*e*f)/2 + (C*a^2*c^2*e*f)/2 + B*a*b*c*d
*e^2 + B*a*b*c^2*e*f + B*a^2*c*d*e*f + 2*A*a*b*c*d*e*f) + x^6*((B*a^2*d^2*
f^2)/6 + (B*b^2*c^2*f^2)/6 + (B*b^2*d^2*e^2)/6 + (A*a*b*d^2*f^2)/3 + (C*a*
b*c^2*f^2)/3 + (C*a*b*d^2*e^2)/3 + (A*b^2*c*d*f^2)/3 + (C*a^2*c*d*f^2)/3 +
 (C*b^2*c*d*e^2)/3 + (A*b^2*d^2*e*f)/3 + (C*a^2*d^2*e*f)/3 + (C*b^2*c^2*e*
f)/3 + (2*B*a*b*c*d*f^2)/3 + (2*B*a*b*d^2*e*f)/3 + (2*B*b^2*c*d*e*f)/3 + (
4*C*a*b*c*d*e*f)/3) + x^3*((A*a^2*c^2*f^2)/3 + (A*a^2*d^2*e^2)/3 + (A*b^2*
c^2*e^2)/3 + (C*a^2*c^2*e^2)/3 + (2*B*a*b*c^2*e^2)/3 + (2*B*a^2*c*d*e^2)/3
 + (2*B*a^2*c^2*e*f)/3 + (4*A*a*b*c*d*e^2)/3 + (4*A*a*b*c^2*e*f)/3 + (4*A*
a^2*c*d*e*f)/3) + x^7*((A*b^2*d^2*f^2)/7 + (C*a^2*d^2*f^2)/7 + (C*b^2*c^2*
f^2)/7 + (C*b^2*d^2*e^2)/7 + (2*B*a*b*d^2*f^2)/7 + (2*B*b^2*c*d*f^2)/7 + (
2*B*b^2*d^2*e*f)/7 + (4*C*a*b*c*d*f^2)/7 + (4*C*a*b*d^2*e*f)/7 + (4*C*b...
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 871, normalized size of antiderivative = 1.31

$$\int (A+Bx+Cx^2) \left(ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3\right)^2 dx$$
$$= \frac{x(280b^2cd^2f^2x^8+630abcd^2f^2x^7+315b^3d^2f^2x^7+630b^2c^2df^2x^7+630b^2cd^2efx^7+360a^2cd^2f^2x^6+10\cdots}{}$$

input
```
int((C*x^2+B*x+A)*(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d
*f*x^3)^2,x)
```

output
```
(x*(2520*a**3*c**2*e**2 + 2520*a**3*c**2*e*f*x + 840*a**3*c**2*f**2*x**2 +
 2520*a**3*c*d*e**2*x + 3360*a**3*c*d*e*f*x**2 + 1260*a**3*c*d*f**2*x**3 +
 840*a**3*d**2*e**2*x**2 + 1260*a**3*d**2*e*f*x**3 + 504*a**3*d**2*f**2*x*
*4 + 3780*a**2*b*c**2*e**2*x + 5040*a**2*b*c**2*e*f*x**2 + 1890*a**2*b*c**
2*f**2*x**3 + 5040*a**2*b*c*d*e**2*x**2 + 7560*a**2*b*c*d*e*f*x**3 + 3024*
a**2*b*c*d*f**2*x**4 + 1890*a**2*b*d**2*e**2*x**3 + 3024*a**2*b*d**2*e*f*x
**4 + 1260*a**2*b*d**2*f**2*x**5 + 840*a**2*c**3*e**2*x**2 + 1260*a**2*c**
3*e*f*x**3 + 504*a**2*c**3*f**2*x**4 + 1260*a**2*c**2*d*e**2*x**3 + 2016*a
**2*c**2*d*e*f*x**4 + 840*a**2*c**2*d*f**2*x**5 + 504*a**2*c*d**2*e**2*x**
4 + 840*a**2*c*d**2*e*f*x**5 + 360*a**2*c*d**2*f**2*x**6 + 2520*a*b**2*c**
2*e**2*x**2 + 3780*a*b**2*c**2*e*f*x**3 + 1512*a*b**2*c**2*f**2*x**4 + 378
0*a*b**2*c*d*e**2*x**3 + 6048*a*b**2*c*d*e*f*x**4 + 2520*a*b**2*c*d*f**2*x
**5 + 1512*a*b**2*d**2*e**2*x**4 + 2520*a*b**2*d**2*e*f*x**5 + 1080*a*b**2
*d**2*f**2*x**6 + 1260*a*b*c**3*e**2*x**3 + 2016*a*b*c**3*e*f*x**4 + 840*a
*b*c**3*f**2*x**5 + 2016*a*b*c**2*d*e**2*x**4 + 3360*a*b*c**2*d*e*f*x**5 +
 1440*a*b*c**2*d*f**2*x**6 + 840*a*b*c*d**2*e**2*x**5 + 1440*a*b*c*d**2*e*
f*x**6 + 630*a*b*c*d**2*f**2*x**7 + 630*b**3*c**2*e**2*x**3 + 1008*b**3*c*
*2*e*f*x**4 + 420*b**3*c**2*f**2*x**5 + 1008*b**3*c*d*e**2*x**4 + 1680*b**
3*c*d*e*f*x**5 + 720*b**3*c*d*f**2*x**6 + 420*b**3*d**2*e**2*x**5 + 720*b*
*3*d**2*e*f*x**6 + 315*b**3*d**2*f**2*x**7 + 504*b**2*c**3*e**2*x**4 + ...
```

3.111 $\int (A+Bx+Cx^2)\left(ace+(bce+ade+acf)x+(b$

Optimal result

Integrand size = 55, antiderivative size = 158

$$\begin{aligned}\int (A+Bx+Cx^2)\left(ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3\right)\,dx\\ =aAcex+\frac{1}{2}(aBce+A(bce+ade+acf))x^2\\ +\frac{1}{3}(b(Bce+Ade+Acf)+a(cCe+Bde+Bcf+Adf))x^3\\ +\frac{1}{4}(b(cCe+Bde+Bcf+Adf)+a(Cde+cCf+Bdf))x^4\\ +\frac{1}{5}(aCdf+b(Cde+cCf+Bdf))x^5+\frac{1}{6}bCdfx^6\end{aligned}$$

output
```
a*A*c*e*x+1/2*(a*B*c*e+A*(a*c*f+a*d*e+b*c*e))*x^2+1/3*(b*(A*c*f+A*d*e+B*c*
e)+a*(A*d*f+B*c*f+B*d*e+C*c*e))*x^3+1/4*(b*(A*d*f+B*c*f+B*d*e+C*c*e)+a*(B*
d*f+C*c*f+C*d*e))*x^4+1/5*(a*C*d*f+b*(B*d*f+C*c*f+C*d*e))*x^5+1/6*b*C*d*f*
x^6
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.01

$$\int (A+Bx+Cx^2)\left(ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3\right)\,dx$$
$$=aAcex+\frac{1}{2}(Abce+aBce+aAde+aAcf)x^2+\frac{1}{3}(bBce+acCe+Abde+aBde+Abcf+aBcf+aAdf)x^3+\frac{1}{4}(bcCe+bBde+aCde+bBcf+acCf+Abdf+aBdf)x^4+\frac{1}{5}(bCde+bcCf+bBdf+aCdf)x^5+\frac{1}{6}bCdfx^6$$

input
```
Integrate[(A + B*x + C*x^2)*(a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e +
b*c*f + a*d*f)*x^2 + b*d*f*x^3),x]
```

output
```
a*A*c*e*x + ((A*b*c*e + a*B*c*e + a*A*d*e + a*A*c*f)*x^2)/2 + ((b*B*c*e +
a*c*C*e + A*b*d*e + a*B*d*e + A*b*c*f + a*B*c*f + a*A*d*f)*x^3)/3 + ((b*c*
C*e + b*B*d*e + a*C*d*e + b*B*c*f + a*c*C*f + A*b*d*f + a*B*d*f)*x^4)/4 +
((b*C*d*e + b*c*C*f + b*B*d*f + a*C*d*f)*x^5)/5 + (b*C*d*f*x^6)/6
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (A+Bx+Cx^2)\left(x^2(adf+bcf+bde)+x(acf+ade+bce)+ace+bdfx^3\right)\,dx$$

$$\Big\downarrow \text{2188}$$

$$\int \left(x^3(a(Bdf+cCf+Cde)+b(Adf+Bcf+Bde+cCe))+x^2(a(Adf+Bcf+Bde+cCe)+b(Acf+Ade+\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}x^4(a(Bdf+cCf+Cde)+b(Adf+Bcf+Bde+cCe))+\frac{1}{3}x^3(a(Adf+Bcf+Bde+cCe)+b(Acf+Ade+Bce))+\frac{1}{2}x^2(A(acf+ade+bce)+aBce)+aAcex+\frac{1}{5}x^5(aCdf+b(Bdf+cCf+Cde))+\frac{1}{6}bCdfx^6$$

input
```
Int[(A + B*x + C*x^2)*(a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*c*f
+ a*d*f)*x^2 + b*d*f*x^3),x]
```

output
```
a*A*c*e*x + ((a*B*c*e + A*(b*c*e + a*d*e + a*c*f))*x^2)/2 + ((b*(B*c*e + A
*d*e + A*c*f) + a*(c*C*e + B*d*e + B*c*f + A*d*f))*x^3)/3 + ((b*(c*C*e + B
*d*e + B*c*f + A*d*f) + a*(C*d*e + c*C*f + B*d*f))*x^4)/4 + ((a*C*d*f + b*
(C*d*e + c*C*f + B*d*f))*x^5)/5 + (b*C*d*f*x^6)/6
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.96

method	result
default	$\frac{bCdf\,x^6}{6}+\frac{(bdfB+C(adf+bcf+bde))x^5}{5}+\frac{(bdfA+B(adf+bcf+bde)+C(acf+ade+bce))x^4}{4}+\frac{(A(adf+bcf+bde)+B(a}{3}$
norman	$\frac{bCdf\,x^6}{6}+\left(\frac{1}{5}bdfB+\frac{1}{5}aCdf+\frac{1}{5}Cbcf+\frac{1}{5}Cbde\right)x^5+\left(\frac{1}{4}bdfA+\frac{1}{4}Badf+\frac{1}{4}Bbcf+\frac{1}{4}Bbde+\right.$
gosper	$\frac{x(10Cbdf\,x^5+12Bbdf\,x^4+12Cadf\,x^4+12Cbcf\,x^4+12Cbde\,x^4+15Abdf\,x^3+15Badf\,x^3+15Bbcf\,x^3+15Bbde\,x^3+15Cacf\,x^3}{}$
risch	$\frac{1}{6}bCdf\,x^6+\frac{1}{5}x^5bdfB+\frac{1}{5}x^5aCdf+\frac{1}{5}x^5Cbcf+\frac{1}{5}x^5Cbde+\frac{1}{4}x^4bdfA+\frac{1}{4}x^4Badf+\frac{1}{4}x^4Bbc$
parallelrisch	$\frac{1}{6}bCdf\,x^6+\frac{1}{5}x^5bdfB+\frac{1}{5}x^5aCdf+\frac{1}{5}x^5Cbcf+\frac{1}{5}x^5Cbde+\frac{1}{4}x^4bdfA+\frac{1}{4}x^4Badf+\frac{1}{4}x^4Bbc$
orering	$\frac{x(10Cbdf\,x^5+12Bbdf\,x^4+12Cadf\,x^4+12Cbcf\,x^4+12Cbde\,x^4+15Abdf\,x^3+15Badf\,x^3+15Bbcf\,x^3+15Bbde\,x^3+15Cacf\,x^3}{}$

input
```
int((C*x^2+B*x+A)*(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d
*f*x^3),x,method=_RETURNVERBOSE)
```

output
```
1/6*b*C*d*f*x^6+1/5*(b*d*f*B+C*(a*d*f+b*c*f+b*d*e))*x^5+1/4*(b*d*f*A+B*(a*
d*f+b*c*f+b*d*e)+C*(a*c*f+a*d*e+b*c*e))*x^4+1/3*(A*(a*d*f+b*c*f+b*d*e)+B*(
a*c*f+a*d*e+b*c*e)+C*a*e*c)*x^3+1/2*(B*a*c*e+A*(a*c*f+a*d*e+b*c*e))*x^2+A*
a*c*e*x
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.99

$$\begin{aligned}
&\int \left(A+Bx+Cx^2\right)\left(ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3\right)\,dx \\
&= \frac{1}{6}\,Cbdfx^6+\frac{1}{5}\,(Cbde+(Cbc+(Ca+Bb)d)f)x^5+Aacex \\
&\quad+\frac{1}{4}\,((Cbc+(Ca+Bb)d)e+((Ca+Bb)c+(Ba+Ab)d)f)x^4 \\
&\quad+\frac{1}{3}\,(((Ca+Bb)c+(Ba+Ab)d)e+(Aad+(Ba+Ab)c)f)x^3 \\
&\quad+\frac{1}{2}\,(Aacf+(Aad+(Ba+Ab)c)e)x^2
\end{aligned}$$

input
```
integrate((C*x^2+B*x+A)*(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x
^2+b*d*f*x^3),x, algorithm="fricas")
```

output
```
1/6*C*b*d*f*x^6 + 1/5*(C*b*d*e + (C*b*c + (C*a + B*b)*d)*f)*x^5 + A*a*c*e*
x + 1/4*((C*b*c + (C*a + B*b)*d)*e + ((C*a + B*b)*c + (B*a + A*b)*d)*f)*x^
4 + 1/3*(((C*a + B*b)*c + (B*a + A*b)*d)*e + (A*a*d + (B*a + A*b)*c)*f)*x^
3 + 1/2*(A*a*c*f + (A*a*d + (B*a + A*b)*c)*e)*x^2
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.39

$$\int (A+Bx+Cx^2)\left(ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3\right)\,dx$$
$$= Aacex+\frac{Cbdfx^6}{6}+x^5\left(\frac{Bbdf}{5}+\frac{Cadf}{5}+\frac{Cbcf}{5}+\frac{Cbde}{5}\right)$$
$$+x^4\left(\frac{Abdf}{4}+\frac{Badf}{4}+\frac{Bbcf}{4}+\frac{Bbde}{4}+\frac{Cacf}{4}+\frac{Cade}{4}+\frac{Cbce}{4}\right)$$
$$+x^3\left(\frac{Aadf}{3}+\frac{Abcf}{3}+\frac{Abde}{3}+\frac{Bacf}{3}+\frac{Bade}{3}+\frac{Bbce}{3}+\frac{Cace}{3}\right)$$
$$+x^2\left(\frac{Aacf}{2}+\frac{Aade}{2}+\frac{Abce}{2}+\frac{Bace}{2}\right)$$

input
```
integrate((C*x**2+B*x+A)*(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*
x**2+b*d*f*x**3),x)
```

output
```
A*a*c*e*x + C*b*d*f*x**6/6 + x**5*(B*b*d*f/5 + C*a*d*f/5 + C*b*c*f/5 + C*b
*d*e/5) + x**4*(A*b*d*f/4 + B*a*d*f/4 + B*b*c*f/4 + B*b*d*e/4 + C*a*c*f/4
+ C*a*d*e/4 + C*b*c*e/4) + x**3*(A*a*d*f/3 + A*b*c*f/3 + A*b*d*e/3 + B*a*c
*f/3 + B*a*d*e/3 + B*b*c*e/3 + C*a*c*e/3) + x**2*(A*a*c*f/2 + A*a*d*e/2 +
A*b*c*e/2 + B*a*c*e/2)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.99

$$\int (A+Bx+Cx^2) \left(ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3\right) dx$$
$$= \frac{1}{6}Cbdfx^6+\frac{1}{5}(Cbde+(Cbc+(Ca+Bb)d)f)x^5+Aacex$$
$$+\frac{1}{4}((Cbc+(Ca+Bb)d)e+((Ca+Bb)c+(Ba+Ab)d)f)x^4$$
$$+\frac{1}{3}(((Ca+Bb)c+(Ba+Ab)d)e+(Aad+(Ba+Ab)c)f)x^3$$
$$+\frac{1}{2}(Aacf+(Aad+(Ba+Ab)c)e)x^2$$

input
```
integrate((C*x^2+B*x+A)*(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x
^2+b*d*f*x^3),x, algorithm="maxima")
```

output
```
1/6*C*b*d*f*x^6 + 1/5*(C*b*d*e + (C*b*c + (C*a + B*b)*d)*f)*x^5 + A*a*c*e*
x + 1/4*((C*b*c + (C*a + B*b)*d)*e + ((C*a + B*b)*c + (B*a + A*b)*d)*f)*x^
4 + 1/3*(((C*a + B*b)*c + (B*a + A*b)*d)*e + (A*a*d + (B*a + A*b)*c)*f)*x^
3 + 1/2*(A*a*c*f + (A*a*d + (B*a + A*b)*c)*e)*x^2
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.35

$$\int (A+Bx+Cx^2) \left(ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3\right) dx$$
$$= \frac{1}{6}Cbdfx^6+\frac{1}{5}Cbdex^5+\frac{1}{5}Cbcfx^5+\frac{1}{5}Cadfx^5+\frac{1}{5}Bbdfx^5+\frac{1}{4}Cbcex^4$$
$$+\frac{1}{4}Cadex^4+\frac{1}{4}Bbdex^4+\frac{1}{4}Cacfx^4+\frac{1}{4}Bbcfx^4+\frac{1}{4}Badfx^4+\frac{1}{4}Abdfx^4$$
$$+\frac{1}{3}Cacex^3+\frac{1}{3}Bbcex^3+\frac{1}{3}Badex^3+\frac{1}{3}Abdex^3+\frac{1}{3}Bacfx^3+\frac{1}{3}Abcfx^3$$
$$+\frac{1}{3}Aadfx^3+\frac{1}{2}Bacex^2+\frac{1}{2}Abcex^2+\frac{1}{2}Aadex^2+\frac{1}{2}Aacfx^2+Aacex$$

input
```
integrate((C*x^2+B*x+A)*(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x
^2+b*d*f*x^3),x, algorithm="giac")
```

output
```
1/6*C*b*d*f*x^6 + 1/5*C*b*d*e*x^5 + 1/5*C*b*c*f*x^5 + 1/5*C*a*d*f*x^5 + 1/
5*B*b*d*f*x^5 + 1/4*C*b*c*e*x^4 + 1/4*C*a*d*e*x^4 + 1/4*B*b*d*e*x^4 + 1/4*
C*a*c*f*x^4 + 1/4*B*b*c*f*x^4 + 1/4*B*a*d*f*x^4 + 1/4*A*b*d*f*x^4 + 1/3*C*
a*c*e*x^3 + 1/3*B*b*c*e*x^3 + 1/3*B*a*d*e*x^3 + 1/3*A*b*d*e*x^3 + 1/3*B*a*
c*f*x^3 + 1/3*A*b*c*f*x^3 + 1/3*A*a*d*f*x^3 + 1/2*B*a*c*e*x^2 + 1/2*A*b*c*
e*x^2 + 1/2*A*a*d*e*x^2 + 1/2*A*a*c*f*x^2 + A*a*c*e*x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.06

$$\int (A+Bx+Cx^2)\left(ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3\right)\,dx$$
$$=\frac{C\,b\,d\,f\,x^6}{6}+\left(\frac{B\,b\,d\,f}{5}+\frac{C\,a\,d\,f}{5}+\frac{C\,b\,c\,f}{5}+\frac{C\,b\,d\,e}{5}\right)x^5$$
$$+\left(\frac{A\,b\,d\,f}{4}+\frac{B\,a\,d\,f}{4}+\frac{B\,b\,c\,f}{4}+\frac{B\,b\,d\,e}{4}+\frac{C\,a\,c\,f}{4}+\frac{C\,a\,d\,e}{4}+\frac{C\,b\,c\,e}{4}\right)x^4$$
$$+\left(\frac{A\,a\,d\,f}{3}+\frac{A\,b\,c\,f}{3}+\frac{A\,b\,d\,e}{3}+\frac{B\,a\,c\,f}{3}+\frac{B\,a\,d\,e}{3}+\frac{B\,b\,c\,e}{3}+\frac{C\,a\,c\,e}{3}\right)x^3$$
$$+\left(\frac{A\,a\,c\,f}{2}+\frac{A\,a\,d\,e}{2}+\frac{A\,b\,c\,e}{2}+\frac{B\,a\,c\,e}{2}\right)x^2+A\,a\,c\,e\,x$$

input
```
int((A + B*x + C*x^2)*(x^2*(a*d*f + b*c*f + b*d*e) + x*(a*c*f + a*d*e + b*
c*e) + a*c*e + b*d*f*x^3),x)
```

output
```
x^3*((A*a*d*f)/3 + (A*b*c*f)/3 + (A*b*d*e)/3 + (B*a*c*f)/3 + (B*a*d*e)/3 +
 (B*b*c*e)/3 + (C*a*c*e)/3) + x^4*((A*b*d*f)/4 + (B*a*d*f)/4 + (B*b*c*f)/4
 + (B*b*d*e)/4 + (C*a*c*f)/4 + (C*a*d*e)/4 + (C*b*c*e)/4) + x^2*((A*a*c*f)
/2 + (A*a*d*e)/2 + (A*b*c*e)/2 + (B*a*c*e)/2) + x^5*((B*b*d*f)/5 + (C*a*d*
f)/5 + (C*b*c*f)/5 + (C*b*d*e)/5) + A*a*c*e*x + (C*b*d*f*x^6)/6
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.18

$$\int (A + Bx + Cx^2) (ace + (bce + ade + acf)x + (bde + bcf + adf)x^2 + bdfx^3) \, dx$$
$$= \frac{x(10bcdf\,x^5 + 12acdf\,x^4 + 12b^2df\,x^4 + 12b\,c^2f\,x^4 + 12bcde\,x^4 + 30abdf\,x^3 + 15a\,c^2f\,x^3 + 15acde\,x^3 + 1}{}$$

input
```
int((C*x^2+B*x+A)*(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d
*f*x^3),x)
```

output
```
(x*(60*a**2*c*e + 30*a**2*c*f*x + 30*a**2*d*e*x + 20*a**2*d*f*x**2 + 60*a*
b*c*e*x + 40*a*b*c*f*x**2 + 40*a*b*d*e*x**2 + 30*a*b*d*f*x**3 + 20*a*c**2*
e*x**2 + 15*a*c**2*f*x**3 + 15*a*c*d*e*x**3 + 12*a*c*d*f*x**4 + 20*b**2*c*
e*x**2 + 15*b**2*c*f*x**3 + 15*b**2*d*e*x**3 + 12*b**2*d*f*x**4 + 15*b*c**
2*e*x**3 + 12*b*c**2*f*x**4 + 12*b*c*d*e*x**4 + 10*b*c*d*f*x**5))/60
```

3.112 $\int \frac{A+Bx+Cx^2}{ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3}\,dx$

Optimal result

Integrand size = 57, antiderivative size = 141

$$\int \frac{A+Bx+Cx^2}{ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3}\,dx = \frac{(Ab^2-a(bB-aC))\log(a+bx)}{b(bc-ad)(be-af)} - \frac{(c^2C-Bcd+Ad^2)\log(c+dx)}{d(bc-ad)(de-cf)} + \frac{(Ce^2-Bef+Af^2)\log(e+fx)}{f(be-af)(de-cf)}$$

output
```
(A*b^2-a*(B*b-C*a))*ln(b*x+a)/b/(-a*d+b*c)/(-a*f+b*e)-(A*d^2-B*c*d+C*c^2)*
ln(d*x+c)/d/(-a*d+b*c)/(-c*f+d*e)+(A*f^2-B*e*f+C*e^2)*ln(f*x+e)/f/(-a*f+b*
e)/(-c*f+d*e)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00

$$\int \frac{A+Bx+Cx^2}{ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3}\,dx = \frac{(Ab^2-abB+a^2C)\log(a+bx)}{b(bc-ad)(be-af)} - \frac{(-c^2C+Bcd-Ad^2)\log(c+dx)}{d(bc-ad)(-de+cf)} + \frac{(Ce^2-Bef+Af^2)\log(e+fx)}{f(be-af)(de-cf)}$$

input
```
Integrate[(A + B*x + C*x^2)/(a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e +
b*c*f + a*d*f)*x^2 + b*d*f*x^3),x]
```

output
```
((A*b^2 - a*b*B + a^2*C)*Log[a + b*x])/(b*(b*c - a*d)*(b*e - a*f)) - ((-(c
^2*C) + B*c*d - A*d^2)*Log[c + d*x])/(d*(b*c - a*d)*(-(d*e) + c*f)) + ((C*
e^2 - B*e*f + A*f^2)*Log[e + f*x])/(f*(b*e - a*f)*(d*e - c*f))
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.035$, Rules used = {2462, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{x^2(adf+bcf+bde)+x(acf+ade+bce)+ace+bdfx^3}dx$$

$$\Big\downarrow \text{2462}$$

$$\int \left(\frac{Ab^2-a(bB-aC)}{(a+bx)(bc-ad)(be-af)}+\frac{Ad^2-Bcd+c^2C}{(c+dx)(bc-ad)(cf-de)}+\frac{Af^2-Bef+Ce^2}{(e+fx)(be-af)(de-cf)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\log(a+bx)\left(Ab^2-a(bB-aC)\right)}{b(bc-ad)(be-af)}-\frac{\log(c+dx)\left(Ad^2-Bcd+c^2C\right)}{d(bc-ad)(de-cf)}+\frac{\log(e+fx)\left(Af^2-Bef+Ce^2\right)}{f(be-af)(de-cf)}$$

input
```
Int[(A + B*x + C*x^2)/(a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*c*f
+ a*d*f)*x^2 + b*d*f*x^3),x]
```

output
```
((A*b^2 - a*(b*B - a*C))*Log[a + b*x])/(b*(b*c - a*d)*(b*e - a*f)) - ((c^2
*C - B*c*d + A*d^2)*Log[c + d*x])/(d*(b*c - a*d)*(d*e - c*f)) + ((C*e^2 -
B*e*f + A*f^2)*Log[e + f*x])/(f*(b*e - a*f)*(d*e - c*f))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00

method	result
default	$\frac{(-A d^2+Bcd-C c^2)\ln(dx+c)}{(ad-bc)(cf-de)d}+\frac{(A b^2-abB+C a^2)\ln(bx+a)}{(af-eb)(ad-bc)b}+\frac{(A f^2-Bef+C e^2)\ln(fx+e)}{(cf-de)(af-eb)f}$
norman	$\frac{(A f^2-Bef+C e^2)\ln(fx+e)}{f(ac f^2-adef-bcef+bd e^2)}+\frac{(A b^2-abB+C a^2)\ln(bx+a)}{(af-eb)(ad-bc)b}-\frac{(A d^2-Bcd+C c^2)\ln(dx+c)}{(ad-bc)d(cf-de)}$
parallelrisch	$A\ln(bx+a)b^2cd f^2-A\ln(bx+a)b^2d^2ef-A\ln(dx+c)ab d^2f^2+A\ln(dx+c)b^2d^2ef+A\ln(fx+e)ab d^2f^2-A\ln(fx+e)b^2cd f^2-$
risch	$-\frac{d\ln(-dx-c)A}{acdf-d^2ea-b c^2f+bcde}+\frac{\ln(-dx-c)Bc}{acdf-d^2ea-b c^2f+bcde}-\frac{\ln(-dx-c)C c^2}{d(acdf-d^2ea-b c^2f+bcde)}+\frac{b\ln(bx+a)A}{a^2df-abcf-abde+b^2ce}-$

input
```
int((C*x^2+B*x+A)/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d
*f*x^3),x,method=_RETURNVERBOSE)
```

output
```
(-A*d^2+B*c*d-C*c^2)/(a*d-b*c)/(c*f-d*e)/d*ln(d*x+c)+(A*b^2-B*a*b+C*a^2)/(
a*f-b*e)/(a*d-b*c)/b*ln(b*x+a)+(A*f^2-B*e*f+C*e^2)/(c*f-d*e)/(a*f-b*e)/f*l
n(f*x+e)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3}\,dx=\text{Timed out}$$

input
```
integrate((C*x^2+B*x+A)/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x
^2+b*d*f*x^3),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3}\,dx=\text{Timed out}$$

input
```
integrate((C*x**2+B*x+A)/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*
x**2+b*d*f*x**3),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.21

$$\begin{aligned}&\int \frac{A+Bx+Cx^2}{ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3}\,dx\\&=\frac{(Ca^2-Bab+Ab^2)\log(bx+a)}{(b^3c-ab^2d)e-(ab^2c-a^2bd)f}-\frac{(Cc^2-Bcd+Ad^2)\log(dx+c)}{(bcd^2-ad^3)e-(bc^2d-acd^2)f}\\&\quad+\frac{(Ce^2-Bef+Af^2)\log(fx+e)}{bde^2f+acf^3-(bc+ad)ef^2}\end{aligned}$$

input
```
integrate((C*x^2+B*x+A)/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x
^2+b*d*f*x^3),x, algorithm="maxima")
```

output
```
(C*a^2 - B*a*b + A*b^2)*log(b*x + a)/((b^3*c - a*b^2*d)*e - (a*b^2*c - a^2
*b*d)*f) - (C*c^2 - B*c*d + A*d^2)*log(d*x + c)/((b*c*d^2 - a*d^3)*e - (b*
c^2*d - a*c*d^2)*f) + (C*e^2 - B*e*f + A*f^2)*log(f*x + e)/(b*d*e^2*f + a*
c*f^3 - (b*c + a*d)*e*f^2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.21

$$\int \frac{A+Bx+Cx^2}{ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3}\,dx = \frac{(Ca^2-Bab+Ab^2)\log(|bx+a|)}{b^3ce-ab^2de-ab^2cf+a^2bdf} - \frac{(Cc^2-Bcd+Ad^2)\log(|dx+c|)}{bcd^2e-ad^3e-bc^2df+acd^2f} + \frac{(Ce^2-Bef+Af^2)\log(|fx+e|)}{bde^2f-bcef^2-adef^2+acf^3}$$

input
```
integrate((C*x^2+B*x+A)/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x
^2+b*d*f*x^3),x, algorithm="giac")
```

output
```
(C*a^2 - B*a*b + A*b^2)*log(abs(b*x + a))/(b^3*c*e - a*b^2*d*e - a*b^2*c*f
 + a^2*b*d*f) - (C*c^2 - B*c*d + A*d^2)*log(abs(d*x + c))/(b*c*d^2*e - a*d
^3*e - b*c^2*d*f + a*c*d^2*f) + (C*e^2 - B*e*f + A*f^2)*log(abs(f*x + e))/
(b*d*e^2*f - b*c*e*f^2 - a*d*e*f^2 + a*c*f^3)
```

Mupad [B] (verification not implemented)

Time = 14.10 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.18

$$\int \frac{A+Bx+Cx^2}{ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3}\,dx = \frac{\ln(a+b\,x)\,(C\,a^2-B\,a\,b+A\,b^2)}{b^3\,c\,e-a\,b^2\,c\,f-a\,b^2\,d\,e+a^2\,b\,d\,f} + \frac{\ln(c+d\,x)\,(C\,c^2-B\,c\,d+A\,d^2)}{a\,d^3\,e-a\,c\,d^2\,f-b\,c\,d^2\,e+b\,c^2\,d\,f} + \frac{\ln(e+f\,x)\,(C\,e^2-B\,e\,f+A\,f^2)}{a\,c\,f^3-a\,d\,e\,f^2-b\,c\,e\,f^2+b\,d\,e^2\,f}$$

input
```
int((A + B*x + C*x^2)/(x^2*(a*d*f + b*c*f + b*d*e) + x*(a*c*f + a*d*e + b*
c*e) + a*c*e + b*d*f*x^3),x)
```

output
```
(log(a + b*x)*(A*b^2 + C*a^2 - B*a*b))/(b^3*c*e - a*b^2*c*f - a*b^2*d*e +
a^2*b*d*f) + (log(c + d*x)*(A*d^2 + C*c^2 - B*c*d))/(a*d^3*e - a*c*d^2*f -
 b*c*d^2*e + b*c^2*d*f) + (log(e + f*x)*(A*f^2 + C*e^2 - B*e*f))/(a*c*f^3
- a*d*e*f^2 - b*c*e*f^2 + b*d*e^2*f)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 300, normalized size of antiderivative = 2.13

$$\int \frac{A+Bx+Cx^2}{ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3}\,dx$$
$$= \frac{\log(bx+a)\,a^2c^2d\,f^2-\log(bx+a)\,a^2c\,d^2ef-\log(dx+c)\,a^2b\,d^2f^2+\log(dx+c)\,a\,b^2cd\,f^2+\log(dx+c)}{}$$

input
```
int((C*x^2+B*x+A)/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d
*f*x^3),x)
```

output
```
(log(a + b*x)*a**2*c**2*d*f**2 - log(a + b*x)*a**2*c*d**2*e*f - log(c + d*
x)*a**2*b*d**2*f**2 + log(c + d*x)*a*b**2*c*d*f**2 + log(c + d*x)*a*b**2*d
**2*e*f - log(c + d*x)*a*b*c**3*f**2 - log(c + d*x)*b**3*c*d*e*f + log(c +
 d*x)*b**2*c**3*e*f + log(e + f*x)*a**2*b*d**2*f**2 - log(e + f*x)*a*b**2*
c*d*f**2 - log(e + f*x)*a*b**2*d**2*e*f + log(e + f*x)*a*b*c*d**2*e**2 + l
og(e + f*x)*b**3*c*d*e*f - log(e + f*x)*b**2*c**2*d*e**2)/(b*d*f*(a**2*c*d
*f**2 - a**2*d**2*e*f - a*b*c**2*f**2 + a*b*d**2*e**2 + b**2*c**2*e*f - b*
*2*c*d*e**2))
```

3.113 $\int \frac{A+Bx+Cx^2}{(ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3)^2}\,dx$

Optimal result

Integrand size = 57, antiderivative size = 421

$$\begin{aligned}
&\int \frac{A+Bx+Cx^2}{(ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3)^2}\,dx \\
&= -\frac{b(Ab^2-a(bB-aC))}{(bc-ad)^2(be-af)^2(a+bx)} \\
&\quad -\frac{d(c^2C-Bcd+Ad^2)}{(bc-ad)^2(de-cf)^2(c+dx)}-\frac{f(Ce^2-Bef+Af^2)}{(be-af)^2(de-cf)^2(e+fx)} \\
&\quad -\frac{b(3a^2bBdf-2a^3Cdf+ab^2(2cCe-Bde-Bcf-4Adf)-b^3(Bce-2A(de+cf)))\log(a+bx)}{(bc-ad)^3(be-af)^3} \\
&\quad +\frac{d(ad^2(2cCe-Bde-Bcf+2Adf)-b(2c^3Cf+Bcd(de-3cf)-2Ad^2(de-2cf)))\log(c+dx)}{(bc-ad)^3(de-cf)^3} \\
&\quad -\frac{f(af^2(2cCe-Bde-Bcf+2Adf)-b(2Cde^3+f(2Af(2de-cf)-Be(3de-cf))))\log(e+fx)}{(be-af)^3(de-cf)^3}
\end{aligned}$$

output
```
-b*(A*b^2-a*(B*b-C*a))/(-a*d+b*c)^2/(-a*f+b*e)^2/(b*x+a)-d*(A*d^2-B*c*d+C*
c^2)/(-a*d+b*c)^2/(-c*f+d*e)^2/(d*x+c)-f*(A*f^2-B*e*f+C*e^2)/(-a*f+b*e)^2/
(-c*f+d*e)^2/(f*x+e)-b*(3*a^2*b*B*d*f-2*a^3*C*d*f+a*b^2*(-4*A*d*f-B*c*f-B*
d*e+2*C*c*e)-b^3*(B*c*e-2*A*(c*f+d*e)))*ln(b*x+a)/(-a*d+b*c)^3/(-a*f+b*e)^
3+d*(a*d^2*(2*A*d*f-B*c*f-B*d*e+2*C*c*e)-b*(2*c^3*C*f+B*c*d*(-3*c*f+d*e)-2
*A*d^2*(-2*c*f+d*e)))*ln(d*x+c)/(-a*d+b*c)^3/(-c*f+d*e)^3-f*(a*f^2*(2*A*d*
f-B*c*f-B*d*e+2*C*c*e)-b*(2*C*d*e^3+f*(2*A*f*(-c*f+2*d*e)-B*e*(-c*f+3*d*e)
)))*ln(f*x+e)/(-a*f+b*e)^3/(-c*f+d*e)^3
```

Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 406, normalized size of antiderivative = 0.96

$$\int \frac{A+Bx+Cx^2}{(ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3)^2}\,dx$$
$$= -\frac{b(Ab^2+a(-bB+aC))}{(bc-ad)^2(be-af)^2(a+bx)} - \frac{d(c^2C-Bcd+Ad^2)}{(bc-ad)^2(de-cf)^2(c+dx)} - \frac{f(Ce^2+f(-Be+Af))}{(be-af)^2(de-cf)^2(e+fx)} + \frac{b(-3a^2bBdf+2a^3Cdf+ab^2(-2cCe+Bde+Bcf+4Adf)+b^3(Bce-2A(de+cf)))\log(a+bx)}{(bc-ad)^3(be-af)^3} + \frac{(ad^3(-2cCe+Bde+Bcf-2Adf)+bd(-2Ad^3e+2c^3Cf+4Acd^2f+Bcd(de-3cf)))\log(c+dx)}{(bc-ad)^3(-de+cf)^3} + \frac{f(2bCde^3+af^2(-2cCe+Bde+Bcf-2Adf)+bf(Be(-3de+cf)-2Af(-2de+cf)))\log(e+f}{(be-af)^3(de-cf)^3}$$

input
```
Integrate[(A + B*x + C*x^2)/(a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e +
b*c*f + a*d*f)*x^2 + b*d*f*x^3)^2,x]
```

output
```
-((b*(A*b^2 + a*(-(b*B) + a*C)))/((b*c - a*d)^2*(b*e - a*f)^2*(a + b*x)))
- (d*(c^2*C - B*c*d + A*d^2))/((b*c - a*d)^2*(d*e - c*f)^2*(c + d*x)) - (f
*(C*e^2 + f*(-(B*e) + A*f)))/((b*e - a*f)^2*(d*e - c*f)^2*(e + f*x)) + (b*
(-3*a^2*b*B*d*f + 2*a^3*C*d*f + a*b^2*(-2*c*C*e + B*d*e + B*c*f + 4*A*d*f)
 + b^3*(B*c*e - 2*A*(d*e + c*f)))*Log[a + b*x])/((b*c - a*d)^3*(b*e - a*f)
^3) + ((a*d^3*(-2*c*C*e + B*d*e + B*c*f - 2*A*d*f) + b*d*(-2*A*d^3*e + 2*c
^3*C*f + 4*A*c*d^2*f + B*c*d*(d*e - 3*c*f)))*Log[c + d*x])/((b*c - a*d)^3*
(-(d*e) + c*f)^3) + (f*(2*b*C*d*e^3 + a*f^2*(-2*c*C*e + B*d*e + B*c*f - 2*
A*d*f) + b*f*(B*e*(-3*d*e + c*f) - 2*A*f*(-2*d*e + c*f)))*Log[e + f*x])/((
b*e - a*f)^3*(d*e - c*f)^3)
```

Rubi [A] (verified)

Time = 2.32 (sec) , antiderivative size = 419, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.035$, Rules used = {2462, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{\left(x^2(adf+bcf+bde)+x(acf+ade+bce)+ace+bdfx^3\right)^2}\,dx$$

$$\Big\downarrow \text{2462}$$

$$\int \left(\frac{b^2\left(2a^3Cdf-3a^2bBdf-ab^2(-4Adf-Bcf-Bde+2cCe)+b^3(Bce-2A(cf+de))\right)}{(a+bx)(bc-ad)^3(be-af)^3}+\frac{b^2\left(Ab^2-a(bI\right.}{(a+bx)^2(bc-ad\right.}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{b\log(a+bx)\left(-2a^3Cdf+3a^2bBdf+ab^2(-4Adf-Bcf-Bde+2cCe)-b^3(Bce-2A(cf+de))\right)}{(bc-ad)^3(be-af)^3}-\frac{b\left(Ab^2-a(bB-aC)\right)}{(a+bx)(bc-ad)^2(be-af)^2}+\frac{d\log(c+dx)\left(ad^2(2Adf-Bcf-Bde+2cCe)-b\left(-2Ad^2(de-2cf)+Bcd(de-3cf)+2c^3Cf\right)\right)}{(bc-ad)^3(de-cf)^3}-\frac{d\left(Ad^2-Bcd+c^2C\right)}{(c+dx)(bc-ad)^2(de-cf)^2}+\frac{f\log(e+fx)\left(-af^2(2Adf-Bcf-Bde+2cCe)+bf(2Af(2de-cf)-Be(3de-cf))+2bCde^3\right)}{(be-af)^3(de-cf)^3}-\frac{f\left(Af^2-Bef+Ce^2\right)}{(e+fx)(be-af)^2(de-cf)^2}$$

input
```
Int[(A + B*x + C*x^2)/(a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*c*f
+ a*d*f)*x^2 + b*d*f*x^3)^2,x]
```

output
```
-((b*(A*b^2 - a*(b*B - a*C)))/((b*c - a*d)^2*(b*e - a*f)^2*(a + b*x))) - (
d*(c^2*C - B*c*d + A*d^2))/((b*c - a*d)^2*(d*e - c*f)^2*(c + d*x)) - (f*(C
*e^2 - B*e*f + A*f^2))/((b*e - a*f)^2*(d*e - c*f)^2*(e + f*x)) - (b*(3*a^2
*b*B*d*f - 2*a^3*C*d*f + a*b^2*(2*c*C*e - B*d*e - B*c*f - 4*A*d*f) - b^3*(
B*c*e - 2*A*(d*e + c*f)))*Log[a + b*x])/((b*c - a*d)^3*(b*e - a*f)^3) + (d
*(a*d^2*(2*c*C*e - B*d*e - B*c*f + 2*A*d*f) - b*(2*c^3*C*f + B*c*d*(d*e -
3*c*f) - 2*A*d^2*(d*e - 2*c*f)))*Log[c + d*x])/((b*c - a*d)^3*(d*e - c*f)^
3) + (f*(2*b*C*d*e^3 - a*f^2*(2*c*C*e - B*d*e - B*c*f + 2*A*d*f) + b*f*(2*
A*f*(2*d*e - c*f) - B*e*(3*d*e - c*f)))*Log[e + f*x])/((b*e - a*f)^3*(d*e
- c*f)^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

Maple [A] (verified)

Time = 1.24 (sec) , antiderivative size = 456, normalized size of antiderivative = 1.08

method	result
default	$\frac{d\left(2Aa\,d^{3}f-4Abc\,d^{2}f+2Ab\,d^{3}e-Bac\,d^{2}f-Ba\,d^{3}e+3Bb\,c^{2}df-Bbc\,d^{2}e+2Cac\,d^{2}e-2Cb\,c^{3}f\right)\ln\left(dx+c\right)}{\left(cf-de\right)^{3}\left(ad-bc\right)^{3}}-\frac{\left(A\,d^{2}-Bcd+C\right.}{\left(cf-de\right)^{2}\left(ad-bc\right.}$
norman	Expression too large to display
risch	Expression too large to display
parallelrisch	Expression too large to display

input
```
int((C*x^2+B*x+A)/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d
*f*x^3)^2,x,method=_RETURNVERBOSE)
```

output
```
d*(2*A*a*d^3*f-4*A*b*c*d^2*f+2*A*b*d^3*e-B*a*c*d^2*f-B*a*d^3*e+3*B*b*c^2*d
*f-B*b*c*d^2*e+2*C*a*c*d^2*e-2*C*b*c^3*f)/(c*f-d*e)^3/(a*d-b*c)^3*ln(d*x+c
)-(A*d^2-B*c*d+C*c^2)*d/(c*f-d*e)^2/(a*d-b*c)^2/(d*x+c)+b*(4*A*a*b^2*d*f-2
*A*b^3*c*f-2*A*b^3*d*e-3*B*a^2*b*d*f+B*a*b^2*c*f+B*a*b^2*d*e+B*b^3*c*e+2*C
*a^3*d*f-2*C*a*b^2*c*e)/(a*f-b*e)^3/(a*d-b*c)^3*ln(b*x+a)-(A*b^2-B*a*b+C*a
^2)*b/(a*f-b*e)^2/(a*d-b*c)^2/(b*x+a)-(A*f^2-B*e*f+C*e^2)*f/(c*f-d*e)^2/(a
*f-b*e)^2/(f*x+e)-f*(2*A*a*d*f^3+2*A*b*c*f^3-4*A*b*d*e*f^2-B*a*c*f^3-B*a*d
*e*f^2-B*b*c*e*f^2+3*B*b*d*e^2*f+2*C*a*c*e*f^2-2*C*b*d*e^3)/(c*f-d*e)^3/(a
*f-b*e)^3*ln(f*x+e)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{\left(ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3\right)^2}\,dx=\text{Timed out}$$

input
```
integrate((C*x^2+B*x+A)/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x
^2+b*d*f*x^3)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + Bx + Cx^2}{(ace + (bce + ade + acf)x + (bde + bcf + adf)x^2 + bdfx^3)^2} dx = \text{Timed out}$$

input
```
integrate((C*x**2+B*x+A)/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*
x**2+b*d*f*x**3)**2,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2660 vs. $2(419) = 838$.

Time = 0.16 (sec) , antiderivative size = 2660, normalized size of antiderivative = 6.32

$$\int \frac{A + Bx + Cx^2}{(ace + (bce + ade + acf)x + (bde + bcf + adf)x^2 + bdfx^3)^2} dx = \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x
^2+b*d*f*x^3)^2,x, algorithm="maxima")
```

output
```
-(((2*C*a*b^3 - B*b^4)*c - (B*a*b^3 - 2*A*b^4)*d)*e - ((B*a*b^3 - 2*A*b^4)
*c + (2*C*a^3*b - 3*B*a^2*b^2 + 4*A*a*b^3)*d)*f)*log(b*x + a)/((b^6*c^3 -
3*a*b^5*c^2*d + 3*a^2*b^4*c*d^2 - a^3*b^3*d^3)*e^3 - 3*(a*b^5*c^3 - 3*a^2*
b^4*c^2*d + 3*a^3*b^3*c*d^2 - a^4*b^2*d^3)*e^2*f + 3*(a^2*b^4*c^3 - 3*a^3*
b^3*c^2*d + 3*a^4*b^2*c*d^2 - a^5*b*d^3)*e*f^2 - (a^3*b^3*c^3 - 3*a^4*b^2*
c^2*d + 3*a^5*b*c*d^2 - a^6*d^3)*f^3) + (((2*C*a - B*b)*c*d^3 - (B*a - 2*A
*b)*d^4)*e - (2*C*b*c^3*d - 3*B*b*c^2*d^2 - 2*A*a*d^4 + (B*a + 4*A*b)*c*d^
3)*f)*log(d*x + c)/((b^3*c^3*d^3 - 3*a*b^2*c^2*d^4 + 3*a^2*b*c*d^5 - a^3*d
^6)*e^3 - 3*(b^3*c^4*d^2 - 3*a*b^2*c^3*d^3 + 3*a^2*b*c^2*d^4 - a^3*c*d^5)*
e^2*f + 3*(b^3*c^5*d - 3*a*b^2*c^4*d^2 + 3*a^2*b*c^3*d^3 - a^3*c^2*d^4)*e*
f^2 - (b^3*c^6 - 3*a*b^2*c^5*d + 3*a^2*b*c^4*d^2 - a^3*c^3*d^3)*f^3) + (2*
C*b*d*e^3*f - 3*B*b*d*e^2*f^2 - ((2*C*a - B*b)*c - (B*a + 4*A*b)*d)*e*f^3
- (2*A*a*d - (B*a - 2*A*b)*c)*f^4)*log(f*x + e)/(b^3*d^3*e^6 + a^3*c^3*f^6
 - 3*(b^3*c*d^2 + a*b^2*d^3)*e^5*f + 3*(b^3*c^2*d + 3*a*b^2*c*d^2 + a^2*b*
d^3)*e^4*f^2 - (b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*e^3*f^3
 + 3*(a*b^2*c^3 + 3*a^2*b*c^2*d + a^3*c*d^2)*e^2*f^4 - 3*(a^2*b*c^3 + a^3*
c^2*d)*e*f^5) - ((C*a*b^2*c^2*d + A*a*b^2*d^3 + (C*a^2*b - 2*B*a*b^2 + A*b
^3)*c*d^2)*e^3 + (C*a*b^2*c^3 - 2*A*a^2*b*d^3 - 2*(3*C*a^2*b - B*a*b^2 + A
*b^3)*c^2*d + (C*a^3 + 2*B*a^2*b)*c*d^2)*e^2*f - (2*B*a^3*c*d^2 - A*a^3*d^
3 - (C*a^2*b - 2*B*a*b^2 + A*b^3)*c^3 - (C*a^3 + 2*B*a^2*b)*c^2*d)*e*f^...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2301 vs. $2(419) = 838$.

Time = 0.17 (sec) , antiderivative size = 2301, normalized size of antiderivative = 5.47

$$\int \frac{A + Bx + Cx^2}{(ace + (bce + ade + acf)x + (bde + bcf + adf)x^2 + bdfx^3)^2} \, dx = \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x
^2+b*d*f*x^3)^2,x, algorithm="giac")
```

output
```
-(2*C*a*b^4*c*e - B*b^5*c*e - B*a*b^4*d*e + 2*A*b^5*d*e - B*a*b^4*c*f + 2*
A*b^5*c*f - 2*C*a^3*b^2*d*f + 3*B*a^2*b^3*d*f - 4*A*a*b^4*d*f)*log(abs(b*x
 + a))/(b^7*c^3*e^3 - 3*a*b^6*c^2*d*e^3 + 3*a^2*b^5*c*d^2*e^3 - a^3*b^4*d^
3*e^3 - 3*a*b^6*c^3*e^2*f + 9*a^2*b^5*c^2*d*e^2*f - 9*a^3*b^4*c*d^2*e^2*f
+ 3*a^4*b^3*d^3*e^2*f + 3*a^2*b^5*c^3*e*f^2 - 9*a^3*b^4*c^2*d*e*f^2 + 9*a^
4*b^3*c*d^2*e*f^2 - 3*a^5*b^2*d^3*e*f^2 - a^3*b^4*c^3*f^3 + 3*a^4*b^3*c^2*
d*f^3 - 3*a^5*b^2*c*d^2*f^3 + a^6*b*d^3*f^3) + (2*C*a*c*d^4*e - B*b*c*d^4*
e - B*a*d^5*e + 2*A*b*d^5*e - 2*C*b*c^3*d^2*f + 3*B*b*c^2*d^3*f - B*a*c*d^
4*f - 4*A*b*c*d^4*f + 2*A*a*d^5*f)*log(abs(d*x + c))/(b^3*c^3*d^4*e^3 - 3*
a*b^2*c^2*d^5*e^3 + 3*a^2*b*c*d^6*e^3 - a^3*d^7*e^3 - 3*b^3*c^4*d^3*e^2*f
+ 9*a*b^2*c^3*d^4*e^2*f - 9*a^2*b*c^2*d^5*e^2*f + 3*a^3*c*d^6*e^2*f + 3*b^
3*c^5*d^2*e*f^2 - 9*a*b^2*c^4*d^3*e*f^2 + 9*a^2*b*c^3*d^4*e*f^2 - 3*a^3*c^
2*d^5*e*f^2 - b^3*c^6*d*f^3 + 3*a*b^2*c^5*d^2*f^3 - 3*a^2*b*c^4*d^3*f^3 +
a^3*c^3*d^4*f^3) + (2*C*b*d*e^3*f^2 - 3*B*b*d*e^2*f^3 - 2*C*a*c*e*f^4 + B*
b*c*e*f^4 + B*a*d*e*f^4 + 4*A*b*d*e*f^4 + B*a*c*f^5 - 2*A*b*c*f^5 - 2*A*a*
d*f^5)*log(abs(f*x + e))/(b^3*d^3*e^6*f - 3*b^3*c*d^2*e^5*f^2 - 3*a*b^2*d^
3*e^5*f^2 + 3*b^3*c^2*d*e^4*f^3 + 9*a*b^2*c*d^2*e^4*f^3 + 3*a^2*b*d^3*e^4*
f^3 - b^3*c^3*e^3*f^4 - 9*a*b^2*c^2*d*e^3*f^4 - 9*a^2*b*c*d^2*e^3*f^4 - a^
3*d^3*e^3*f^4 + 3*a*b^2*c^3*e^2*f^5 + 9*a^2*b*c^2*d*e^2*f^5 + 3*a^3*c*d^2*
e^2*f^5 - 3*a^2*b*c^3*e*f^6 - 3*a^3*c^2*d*e*f^6 + a^3*c^3*f^7) - (2*C*b...
```

Mupad [B] (verification not implemented)

Time = 20.59 (sec) , antiderivative size = 58502, normalized size of antiderivative = 138.96

$$\int \frac{A+Bx+Cx^2}{(ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3)^2}\,dx = \text{Too large to display}$$

input
```
int((A + B*x + C*x^2)/(x^2*(a*d*f + b*c*f + b*d*e) + x*(a*c*f + a*d*e + b*
c*e) + a*c*e + b*d*f*x^3)^2,x)
```

output
```
((x^2*(2*A*a*b^2*c*d^2*f^3 - 2*A*b^3*c^2*d*f^3 - 2*A*b^3*d^3*e^2*f - 2*A*a
^2*b*d^3*f^3 + B*a*b^2*c^2*d*f^3 + B*a^2*b*c*d^2*f^3 + 2*A*a*b^2*d^3*e*f^2
 - 2*C*a^2*b*c^2*d*f^3 + B*a*b^2*d^3*e^2*f + B*a^2*b*d^3*e*f^2 + 2*A*b^3*c
*d^2*e*f^2 - 2*C*a^2*b*d^3*e^2*f + B*b^3*c*d^2*e^2*f + B*b^3*c^2*d*e*f^2 -
 2*C*b^3*c^2*d*e^2*f - 6*B*a*b^2*c*d^2*e*f^2 + 2*C*a*b^2*c*d^2*e^2*f + 2*C
*a*b^2*c^2*d*e*f^2 + 2*C*a^2*b*c*d^2*e*f^2))/(a^2*b^2*c^4*f^4 + a^2*b^2*d^
4*e^4 + a^4*c^2*d^2*f^4 + b^4*c^2*d^2*e^4 + a^4*d^4*e^2*f^2 + b^4*c^4*e^2*
f^2 - 2*a*b^3*c*d^3*e^4 - 2*a^3*b*c^3*d*f^4 - 2*a*b^3*c^4*e*f^3 - 2*a^3*b*
d^4*e^3*f - 2*a^4*c*d^3*e*f^3 - 2*b^4*c^3*d*e^3*f + 2*a*b^3*c^2*d^2*e^3*f
+ 2*a*b^3*c^3*d*e^2*f^2 + 2*a^2*b^2*c*d^3*e^3*f + 2*a^2*b^2*c^3*d*e*f^3 +
2*a^3*b*c*d^3*e^2*f^2 + 2*a^3*b*c^2*d^2*e*f^3 - 6*a^2*b^2*c^2*d^2*e^2*f^2)
 - (A*a*b^2*c^3*f^3 + A*a*b^2*d^3*e^3 + A*a^3*c*d^2*f^3 + A*b^3*c*d^2*e^3
+ A*a^3*d^3*e*f^2 + A*b^3*c^3*e*f^2 - 2*A*a^2*b*c^2*d*f^3 - 2*B*a*b^2*c*d^
2*e^3 + C*a*b^2*c^2*d*e^3 + C*a^2*b*c*d^2*e^3 - 2*A*a^2*b*d^3*e^2*f - 2*B*
a*b^2*c^3*e*f^2 + C*a*b^2*c^3*e^2*f + C*a^2*b*c^3*e*f^2 - 2*A*b^3*c^2*d*e^
2*f - 2*B*a^3*c*d^2*e*f^2 + C*a^3*c*d^2*e^2*f + C*a^3*c^2*d*e*f^2 + 2*B*a*
b^2*c^2*d*e^2*f + 2*B*a^2*b*c*d^2*e^2*f + 2*B*a^2*b*c^2*d*e*f^2 - 6*C*a^2*
b*c^2*d*e^2*f)/(a^2*b^2*c^4*f^4 + a^2*b^2*d^4*e^4 + a^4*c^2*d^2*f^4 + b^4*
c^2*d^2*e^4 + a^4*d^4*e^2*f^2 + b^4*c^4*e^2*f^2 - 2*a*b^3*c*d^3*e^4 - 2*a^
3*b*c^3*d*f^4 - 2*a*b^3*c^4*e*f^3 - 2*a^3*b*d^4*e^3*f - 2*a^4*c*d^3*e*f...
```

Reduce [B] (verification not implemented)

Time = 0.72 (sec) , antiderivative size = 17569, normalized size of antiderivative = 41.73

$$\int \frac{A+Bx+Cx^2}{(ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3)^2}\,dx = \text{Too large to display}$$

input
```
int((C*x^2+B*x+A)/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d
*f*x^3)^2,x)
```

output
```
(2*log(a + b*x)*a**5*b*c**5*d**2*e*f**5 + 2*log(a + b*x)*a**5*b*c**5*d**2*
f**6*x - 6*log(a + b*x)*a**5*b*c**4*d**3*e**2*f**4 - 4*log(a + b*x)*a**5*b
*c**4*d**3*e*f**5*x + 2*log(a + b*x)*a**5*b*c**4*d**3*f**6*x**2 + 6*log(a
+ b*x)*a**5*b*c**3*d**4*e**3*f**3 - 6*log(a + b*x)*a**5*b*c**3*d**4*e*f**5
*x**2 - 2*log(a + b*x)*a**5*b*c**2*d**5*e**4*f**2 + 4*log(a + b*x)*a**5*b*
c**2*d**5*e**3*f**3*x + 6*log(a + b*x)*a**5*b*c**2*d**5*e**2*f**4*x**2 - 2
*log(a + b*x)*a**5*b*c*d**6*e**4*f**2*x - 2*log(a + b*x)*a**5*b*c*d**6*e**
3*f**3*x**2 + log(a + b*x)*a**4*b**3*c**4*d**2*e*f**5 + log(a + b*x)*a**4*
b**3*c**4*d**2*f**6*x - 3*log(a + b*x)*a**4*b**3*c**3*d**3*e**2*f**4 - 2*l
og(a + b*x)*a**4*b**3*c**3*d**3*e*f**5*x + log(a + b*x)*a**4*b**3*c**3*d**
3*f**6*x**2 + 3*log(a + b*x)*a**4*b**3*c**2*d**4*e**3*f**3 - 3*log(a + b*x
)*a**4*b**3*c**2*d**4*e*f**5*x**2 - log(a + b*x)*a**4*b**3*c*d**5*e**4*f**
2 + 2*log(a + b*x)*a**4*b**3*c*d**5*e**3*f**3*x + 3*log(a + b*x)*a**4*b**3
*c*d**5*e**2*f**4*x**2 - log(a + b*x)*a**4*b**3*d**6*e**4*f**2*x - log(a +
 b*x)*a**4*b**3*d**6*e**3*f**3*x**2 + 2*log(a + b*x)*a**4*b**2*c**6*d*e*f*
*5 + 2*log(a + b*x)*a**4*b**2*c**6*d*f**6*x - 4*log(a + b*x)*a**4*b**2*c**
5*d**2*e**2*f**4 + 4*log(a + b*x)*a**4*b**2*c**5*d**2*f**6*x**2 - 10*log(a
 + b*x)*a**4*b**2*c**4*d**3*e**2*f**4*x - 8*log(a + b*x)*a**4*b**2*c**4*d*
*3*e*f**5*x**2 + 2*log(a + b*x)*a**4*b**2*c**4*d**3*f**6*x**3 + 4*log(a +
b*x)*a**4*b**2*c**3*d**4*e**4*f**2 + 10*log(a + b*x)*a**4*b**2*c**3*d**...
```

3.114 $\int \frac{A+Bx+Cx^2}{(ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3)^3}\,dx$

Optimal result

Integrand size = 57, antiderivative size = 1176

$$\int \frac{A+Bx+Cx^2}{(ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3)^3}\,dx = \text{Too large to display}$$

output
```
-1/2*b^3*(A*b^2-a*(B*b-C*a))/(-a*d+b*c)^3/(-a*f+b*e)^3/(b*x+a)^2-b^3*(4*a^
3*C*d*f-2*a*b^2*(-3*A*d*f-B*c*f-B*d*e+C*c*e)-a^2*b*(5*B*d*f+C*c*f+C*d*e)+b
^3*(B*c*e-3*A*(c*f+d*e)))/(-a*d+b*c)^4/(-a*f+b*e)^4/(b*x+a)+1/2*d^3*(A*d^2
-B*c*d+C*c^2)/(-a*d+b*c)^3/(-c*f+d*e)^3/(d*x+c)^2+d^3*(a*d*(c^2*C*f-d^2*(-
3*A*f+B*e)+2*c*d*(-B*f+C*e))+b*(3*A*d^3*e-4*c^3*C*f-2*c*d^2*(3*A*f+B*e)+c^
2*d*(5*B*f+C*e)))/(-a*d+b*c)^4/(-c*f+d*e)^4/(d*x+c)-1/2*f^3*(A*f^2-B*e*f+C
*e^2)/(-a*f+b*e)^3/(-c*f+d*e)^3/(f*x+e)^2+f^3*(a*f*(C*e*(2*c*f+d*e)-f*(-3*
A*d*f+B*c*f+2*B*d*e))-b*(C*e^2*(-c*f+4*d*e)-f*(B*e*(-2*c*f+5*d*e)-3*A*f*(-
c*f+2*d*e))))/(-a*f+b*e)^4/(-c*f+d*e)^4/(f*x+e)+b^3*(10*a^4*C*d^2*f^2-5*a^
3*b*d*f*(3*B*d*f+C*c*f+C*d*e)+b^4*(6*A*d^2*e^2-3*c*d*e*(-3*A*f+B*e)+c^2*(6
*A*f^2-3*B*e*f+C*e^2))+a^2*b^2*(3*d*f*(7*A*d*f+4*B*c*f+4*B*d*e)+C*(c^2*f^2
-11*c*d*e*f+d^2*e^2))-a*b^3*(3*d^2*e*(7*A*f+B*e)-c^2*f*(-3*B*f+4*C*e)-c*d*
(4*C*e^2+3*f*(-7*A*f+B*e))))*ln(b*x+a)/(-a*d+b*c)^5/(-a*f+b*e)^5+d^3*(a*b*
d*(5*c^3*C*f^2+3*d^3*e*(-3*A*f+B*e)+c^2*d*f*(-12*B*f+11*C*e)-c*d^2*(4*C*e^
2+3*f*(-7*A*f+B*e)))-b^2*(6*A*d^4*e^2+10*c^4*C*f^2-3*c*d^3*e*(7*A*f+B*e)-5
*c^3*d*f*(3*B*f+C*e)+c^2*d^2*(C*e^2+3*f*(7*A*f+4*B*e)))-a^2*d^2*(C*(c^2*f^
2+4*c*d*e*f+d^2*e^2)+3*d*f*(2*A*d*f-B*(c*f+d*e))))*ln(d*x+c)/(-a*d+b*c)^5/
(-c*f+d*e)^5+f^3*(a^2*f^2*(C*(c^2*f^2+4*c*d*e*f+d^2*e^2)+3*d*f*(2*A*d*f-B*
(c*f+d*e)))-a*b*f*(C*e*(-4*c^2*f^2+11*c*d*e*f+5*d^2*e^2)+3*f*(A*d*f*(-3*c*
f+7*d*e)-B*(-c^2*f^2+c*d*e*f+4*d^2*e^2)))+b^2*(C*e^2*(c^2*f^2-5*c*d*e*f...
```

Mathematica [A] (verified)

Time = 2.60 (sec) , antiderivative size = 1162, normalized size of antiderivative = 0.99

$$\int \frac{A+Bx+Cx^2}{(ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3)^3}\,dx = \text{Too large to display}$$

input
```
Integrate[(A + B*x + C*x^2)/(a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e +
b*c*f + a*d*f)*x^2 + b*d*f*x^3)^3,x]
```

output

```
-1/2*(b^3*(A*b^2 + a*(-(b*B) + a*C)))/((b*c - a*d)^3*(b*e - a*f)^3*(a + b*
x)^2) - (b^3*(4*a^3*C*d*f + 2*a*b^2*(-(c*C*e) + B*d*e + B*c*f + 3*A*d*f) -
 a^2*b*(C*d*e + c*C*f + 5*B*d*f) + b^3*(B*c*e - 3*A*(d*e + c*f))))/((b*c -
 a*d)^4*(b*e - a*f)^4*(a + b*x)) - (d^3*(c^2*C - B*c*d + A*d^2))/(2*(b*c -
 a*d)^3*(-(d*e) + c*f)^3*(c + d*x)^2) + (d^3*(a*d*(c^2*C*f + d^2*(-(B*e) +
 3*A*f) + 2*c*d*(C*e - B*f)) + b*(3*A*d^3*e - 4*c^3*C*f - 2*c*d^2*(B*e + 3
*A*f) + c^2*d*(C*e + 5*B*f))))/((b*c - a*d)^4*(d*e - c*f)^4*(c + d*x)) - (
f^3*(C*e^2 + f*(-(B*e) + A*f)))/(2*(b*e - a*f)^3*(d*e - c*f)^3*(e + f*x)^2
) + (f^3*(a*f*(C*e*(d*e + 2*c*f) - f*(2*B*d*e + B*c*f - 3*A*d*f)) + b*(C*e
^2*(-4*d*e + c*f) + f*(B*e*(5*d*e - 2*c*f) + 3*A*f*(-2*d*e + c*f)))))/((b*
e - a*f)^4*(d*e - c*f)^4*(e + f*x)) + (b^3*(10*a^4*C*d^2*f^2 - 5*a^3*b*d*f
*(C*d*e + c*C*f + 3*B*d*f) + b^4*(6*A*d^2*e^2 - 3*c*d*e*(B*e - 3*A*f) + c^
2*(C*e^2 - 3*B*e*f + 6*A*f^2)) + a^2*b^2*(3*d*f*(4*B*d*e + 4*B*c*f + 7*A*d
*f) + C*(d^2*e^2 - 11*c*d*e*f + c^2*f^2)) + a*b^3*(-3*d^2*e*(B*e + 7*A*f)
+ c^2*f*(4*C*e - 3*B*f) + c*d*(4*C*e^2 + 3*f*(B*e - 7*A*f))))*Log[a + b*x]
)/((b*c - a*d)^5*(b*e - a*f)^5) + (d^3*(a*b*d*(-5*c^3*C*f^2 - 3*d^3*e*(B*e
 - 3*A*f) + c^2*d*f*(-11*C*e + 12*B*f) + c*d^2*(4*C*e^2 + 3*f*(B*e - 7*A*f
))) + b^2*(6*A*d^4*e^2 + 10*c^4*C*f^2 - 3*c*d^3*e*(B*e + 7*A*f) - 5*c^3*d*
f*(C*e + 3*B*f) + c^2*d^2*(C*e^2 + 3*f*(4*B*e + 7*A*f))) + a^2*d^2*(C*(d^2
*e^2 + 4*c*d*e*f + c^2*f^2) + 3*d*f*(2*A*d*f - B*(d*e + c*f))))*Log[c +...
```

Rubi [A] (verified)

Time = 9.07 (sec) , antiderivative size = 1176, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.035$, Rules used = {2462, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{\left(x^2(adf+bcf+bde)+x(acf+ade+bce)+ace+bdfx^3\right)^3}dx$$

$$\Big\downarrow \text{2462}$$

$$\int \left(\frac{\left(10Cd^2f^2a^4-5bdf(Cde+cCf+3Bdf)a^3+b^2\left(3df(4Bde+4Bcf+7Adf)+C\left(d^2e^2-11cdfe+c^2f^2\right)\right)a\right.}{}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(10Cd^2f^2a^4-5bdf(Cde+cCf+3Bdf)a^3+b^2\left(3df(4Bde+4Bcf+7Adf)+C\left(d^2e^2-11cdfe+c^2f^2\right)\right)a^2-b}{}$$

$$\frac{\left(4Cdfa^3-b(Cde+cCf+5Bdf)a^2-2b^2(cCe-Bde-Bcf-3Adf)a+b^3(Bce-3A(de+cf))\right)b^3}{(bc-ad)^4(be-af)^4(a+bx)}-\frac{\left(Ab^2-a(bB-aC)\right)b^3}{2(bc-ad)^3(be-af)^3(a+bx)^2}+\frac{d^3\left(-\left(\left(10Cf^2c^4-5df(Ce+3Bf)c^3+d^2\left(Ce^2+3f(4Be+7Af)\right)c^2-3d^3e(Be+7Af)c+6Ad^4e^2\right)b^2\right)+ad(5}{}$$

$$\frac{f^3\left(\left(Ce^2\left(10d^2e^2-5cdfe+c^2f^2\right)-3f\left(Be\left(5d^2e^2-4cdfe+c^2f^2\right)-Af\left(7d^2e^2-7cdfe+2c^2f^2\right)\right)\right)b^2-af(Ce(}{}$$

$$\frac{d^3\left(ad(Cfc^2+2d(Ce-Bf)c-d^2(Be-3Af))+b\left(-4Cfc^3+d(Ce+5Bf)c^2-2d^2(Be+3Af)c+3Ad^3e\right)\right)}{(bc-ad)^4(de-cf)^4(c+dx)}+\frac{f^3\left(af(Ce(de+2cf)-f(2Bde+Bcf-3Adf))-b\left(Ce^2(4de-cf)-f(Be(5de-2cf)-3Af(2de-cf))\right)\right)}{(be-af)^4(de-cf)^4(e+fx)}+\frac{d^3\left(Cc^2-Bdc+Ad^2\right)}{2(bc-ad)^3(de-cf)^3(c+dx)^2}-\frac{f^3\left(Ce^2-Bfe+Af^2\right)}{2(be-af)^3(de-cf)^3(e+fx)^2}$$

input

```
Int[(A + B*x + C*x^2)/(a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*c*f
+ a*d*f)*x^2 + b*d*f*x^3)^3,x]
```

output
```
-1/2*(b^3*(A*b^2 - a*(b*B - a*C)))/((b*c - a*d)^3*(b*e - a*f)^3*(a + b*x)^
2) - (b^3*(4*a^3*C*d*f - 2*a*b^2*(c*C*e - B*d*e - B*c*f - 3*A*d*f) - a^2*b
*(C*d*e + c*C*f + 5*B*d*f) + b^3*(B*c*e - 3*A*(d*e + c*f))))/((b*c - a*d)^
4*(b*e - a*f)^4*(a + b*x)) + (d^3*(c^2*C - B*c*d + A*d^2))/(2*(b*c - a*d)^
3*(d*e - c*f)^3*(c + d*x)^2) + (d^3*(a*d*(c^2*C*f - d^2*(B*e - 3*A*f) + 2*
c*d*(C*e - B*f)) + b*(3*A*d^3*e - 4*c^3*C*f - 2*c*d^2*(B*e + 3*A*f) + c^2*
d*(C*e + 5*B*f))))/((b*c - a*d)^4*(d*e - c*f)^4*(c + d*x)) - (f^3*(C*e^2 -
 B*e*f + A*f^2))/(2*(b*e - a*f)^3*(d*e - c*f)^3*(e + f*x)^2) + (f^3*(a*f*(
C*e*(d*e + 2*c*f) - f*(2*B*d*e + B*c*f - 3*A*d*f)) - b*(C*e^2*(4*d*e - c*f
) - f*(B*e*(5*d*e - 2*c*f) - 3*A*f*(2*d*e - c*f)))))/((b*e - a*f)^4*(d*e -
 c*f)^4*(e + f*x)) + (b^3*(10*a^4*C*d^2*f^2 - 5*a^3*b*d*f*(C*d*e + c*C*f +
 3*B*d*f) + b^4*(6*A*d^2*e^2 - 3*c*d*e*(B*e - 3*A*f) + c^2*(C*e^2 - 3*B*e*
f + 6*A*f^2)) + a^2*b^2*(3*d*f*(4*B*d*e + 4*B*c*f + 7*A*d*f) + C*(d^2*e^2
- 11*c*d*e*f + c^2*f^2)) - a*b^3*(3*d^2*e*(B*e + 7*A*f) - c^2*f*(4*C*e - 3
*B*f) - c*d*(4*C*e^2 + 3*f*(B*e - 7*A*f))))*Log[a + b*x])/((b*c - a*d)^5*(
b*e - a*f)^5) + (d^3*(a*b*d*(5*c^3*C*f^2 + 3*d^3*e*(B*e - 3*A*f) + c^2*d*f
*(11*C*e - 12*B*f) - c*d^2*(4*C*e^2 + 3*f*(B*e - 7*A*f))) - b^2*(6*A*d^4*e
^2 + 10*c^4*C*f^2 - 3*c*d^3*e*(B*e + 7*A*f) - 5*c^3*d*f*(C*e + 3*B*f) + c^
2*d^2*(C*e^2 + 3*f*(4*B*e + 7*A*f))) - a^2*d^2*(C*(d^2*e^2 + 4*c*d*e*f + c
^2*f^2) + 3*d*f*(2*A*d*f - B*(d*e + c*f))))*Log[c + d*x])/((b*c - a*d)^...
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

Maple [A] (verified)

Time = 5.31 (sec) , antiderivative size = 1472, normalized size of antiderivative = 1.25

method	result	size
default	Expression too large to display	1472
norman	Expression too large to display	20951
risch	Expression too large to display	59917
parallelrisch	Expression too large to display	92024

input
```
int((C*x^2+B*x+A)/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d
*f*x^3)^3,x,method=_RETURNVERBOSE)
```

output
```
d^3*(3*A*a*d^3*f-6*A*b*c*d^2*f+3*A*b*d^3*e-2*B*a*c*d^2*f-B*a*d^3*e+5*B*b*c
^2*d*f-2*B*b*c*d^2*e+C*a*c^2*d*f+2*C*a*c*d^2*e-4*C*b*c^3*f+C*b*c^2*d*e)/(a
*d-b*c)^4/(c*f-d*e)^4/(d*x+c)-d^3*(6*A*a^2*d^4*f^2-21*A*a*b*c*d^3*f^2+9*A*
a*b*d^4*e*f+21*A*b^2*c^2*d^2*f^2-21*A*b^2*c*d^3*e*f+6*A*b^2*d^4*e^2-3*B*a^
2*c*d^3*f^2-3*B*a^2*d^4*e*f+12*B*a*b*c^2*d^2*f^2+3*B*a*b*c*d^3*e*f-3*B*a*b
*d^4*e^2-15*B*b^2*c^3*d*f^2+12*B*b^2*c^2*d^2*e*f-3*B*b^2*c*d^3*e^2+C*a^2*c
^2*d^2*f^2+4*C*a^2*c*d^3*e*f+C*a^2*d^4*e^2-5*C*a*b*c^3*d*f^2-11*C*a*b*c^2*
d^2*e*f+4*C*a*b*c*d^3*e^2+10*C*b^2*c^4*f^2-5*C*b^2*c^3*d*e*f+C*b^2*c^2*d^2
*e^2)/(a*d-b*c)^5/(c*f-d*e)^5*ln(d*x+c)+1/2*(A*d^2-B*c*d+C*c^2)*d^3/(a*d-b
*c)^3/(c*f-d*e)^3/(d*x+c)^2-b^3*(6*A*a*b^2*d*f-3*A*b^3*c*f-3*A*b^3*d*e-5*B
*a^2*b*d*f+2*B*a*b^2*c*f+2*B*a*b^2*d*e+B*b^3*c*e+4*C*a^3*d*f-C*a^2*b*c*f-C
*a^2*b*d*e-2*C*a*b^2*c*e)/(a*f-b*e)^4/(a*d-b*c)^4/(b*x+a)+b^3*(21*A*a^2*b^
2*d^2*f^2-21*A*a*b^3*c*d*f^2-21*A*a*b^3*d^2*e*f+6*A*b^4*c^2*f^2+9*A*b^4*c*
d*e*f+6*A*b^4*d^2*e^2-15*B*a^3*b*d^2*f^2+12*B*a^2*b^2*c*d*f^2+12*B*a^2*b^2
*d^2*e*f-3*B*a*b^3*c^2*f^2+3*B*a*b^3*c*d*e*f-3*B*a*b^3*d^2*e^2-3*B*b^4*c^2
*e*f-3*B*b^4*c*d*e^2+10*C*a^4*d^2*f^2-5*C*a^3*b*c*d*f^2-5*C*a^3*b*d^2*e*f+
C*a^2*b^2*c^2*f^2-11*C*a^2*b^2*c*d*e*f+C*a^2*b^2*d^2*e^2+4*C*a*b^3*c^2*e*f
+4*C*a*b^3*c*d*e^2+C*b^4*c^2*e^2)/(a*f-b*e)^5/(a*d-b*c)^5*ln(b*x+a)-1/2*(A
*b^2-B*a*b+C*a^2)*b^3/(a*f-b*e)^3/(a*d-b*c)^3/(b*x+a)^2+f^3*(6*A*a^2*d^2*f
^4+9*A*a*b*c*d*f^4-21*A*a*b*d^2*e*f^3+6*A*b^2*c^2*f^4-21*A*b^2*c*d*e*f^...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3)^3}\,dx = \text{Timed out}$$

input
```
integrate((C*x^2+B*x+A)/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x
^2+b*d*f*x^3)^3,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3)^3}\,dx = \text{Timed out}$$

input
```
integrate((C*x**2+B*x+A)/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*
x**2+b*d*f*x**3)**3,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 16079 vs. 2(1169) = 2338.

Time = 1.04 (sec) , antiderivative size = 16079, normalized size of antiderivative = 13.67

$$\int \frac{A+Bx+Cx^2}{(ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3)^3}\,dx = \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x
^2+b*d*f*x^3)^3,x, algorithm="maxima")
```

output
```
((C*b^7*c^2 + (4*C*a*b^6 - 3*B*b^7)*c*d + (C*a^2*b^5 - 3*B*a*b^6 + 6*A*b^7
)*d^2)*e^2 + ((4*C*a*b^6 - 3*B*b^7)*c^2 - (11*C*a^2*b^5 - 3*B*a*b^6 - 9*A*
b^7)*c*d - (5*C*a^3*b^4 - 12*B*a^2*b^5 + 21*A*a*b^6)*d^2)*e*f + ((C*a^2*b^
5 - 3*B*a*b^6 + 6*A*b^7)*c^2 - (5*C*a^3*b^4 - 12*B*a^2*b^5 + 21*A*a*b^6)*c
*d + (10*C*a^4*b^3 - 15*B*a^3*b^4 + 21*A*a^2*b^5)*d^2)*f^2)*log(b*x + a)/(
(b^10*c^5 - 5*a*b^9*c^4*d + 10*a^2*b^8*c^3*d^2 - 10*a^3*b^7*c^2*d^3 + 5*a^
4*b^6*c*d^4 - a^5*b^5*d^5)*e^5 - 5*(a*b^9*c^5 - 5*a^2*b^8*c^4*d + 10*a^3*b
^7*c^3*d^2 - 10*a^4*b^6*c^2*d^3 + 5*a^5*b^5*c*d^4 - a^6*b^4*d^5)*e^4*f + 1
0*(a^2*b^8*c^5 - 5*a^3*b^7*c^4*d + 10*a^4*b^6*c^3*d^2 - 10*a^5*b^5*c^2*d^3
 + 5*a^6*b^4*c*d^4 - a^7*b^3*d^5)*e^3*f^2 - 10*(a^3*b^7*c^5 - 5*a^4*b^6*c^
4*d + 10*a^5*b^5*c^3*d^2 - 10*a^6*b^4*c^2*d^3 + 5*a^7*b^3*c*d^4 - a^8*b^2*
d^5)*e^2*f^3 + 5*(a^4*b^6*c^5 - 5*a^5*b^5*c^4*d + 10*a^6*b^4*c^3*d^2 - 10*
a^7*b^3*c^2*d^3 + 5*a^8*b^2*c*d^4 - a^9*b*d^5)*e*f^4 - (a^5*b^5*c^5 - 5*a^
6*b^4*c^4*d + 10*a^7*b^3*c^3*d^2 - 10*a^8*b^2*c^2*d^3 + 5*a^9*b*c*d^4 - a^
10*d^5)*f^5) - ((C*b^2*c^2*d^5 + (4*C*a*b - 3*B*b^2)*c*d^6 + (C*a^2 - 3*B*
a*b + 6*A*b^2)*d^7)*e^2 - (5*C*b^2*c^3*d^4 + (11*C*a*b - 12*B*b^2)*c^2*d^5
 - (4*C*a^2 + 3*B*a*b - 21*A*b^2)*c*d^6 + 3*(B*a^2 - 3*A*a*b)*d^7)*e*f + (
10*C*b^2*c^4*d^3 + 6*A*a^2*d^7 - 5*(C*a*b + 3*B*b^2)*c^3*d^4 + (C*a^2 + 12
*B*a*b + 21*A*b^2)*c^2*d^5 - 3*(B*a^2 + 7*A*a*b)*c*d^6)*f^2)*log(d*x + c)/
((b^5*c^5*d^5 - 5*a*b^4*c^4*d^6 + 10*a^2*b^3*c^3*d^7 - 10*a^3*b^2*c^2*d...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 16944 vs. 2(1169) = 2338.

Time = 0.50 (sec) , antiderivative size = 16944, normalized size of antiderivative = 14.41

$$\int \frac{A+Bx+Cx^2}{\left(ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x
^2+b*d*f*x^3)^3,x, algorithm="giac")
```

output
```
(C*b^8*c^2*e^2 + 4*C*a*b^7*c*d*e^2 - 3*B*b^8*c*d*e^2 + C*a^2*b^6*d^2*e^2 -
 3*B*a*b^7*d^2*e^2 + 6*A*b^8*d^2*e^2 + 4*C*a*b^7*c^2*e*f - 3*B*b^8*c^2*e*f
 - 11*C*a^2*b^6*c*d*e*f + 3*B*a*b^7*c*d*e*f + 9*A*b^8*c*d*e*f - 5*C*a^3*b^
5*d^2*e*f + 12*B*a^2*b^6*d^2*e*f - 21*A*a*b^7*d^2*e*f + C*a^2*b^6*c^2*f^2
- 3*B*a*b^7*c^2*f^2 + 6*A*b^8*c^2*f^2 - 5*C*a^3*b^5*c*d*f^2 + 12*B*a^2*b^6
*c*d*f^2 - 21*A*a*b^7*c*d*f^2 + 10*C*a^4*b^4*d^2*f^2 - 15*B*a^3*b^5*d^2*f^
2 + 21*A*a^2*b^6*d^2*f^2)*log(abs(b*x + a))/(b^11*c^5*e^5 - 5*a*b^10*c^4*d
*e^5 + 10*a^2*b^9*c^3*d^2*e^5 - 10*a^3*b^8*c^2*d^3*e^5 + 5*a^4*b^7*c*d^4*e
^5 - a^5*b^6*d^5*e^5 - 5*a*b^10*c^5*e^4*f + 25*a^2*b^9*c^4*d*e^4*f - 50*a^
3*b^8*c^3*d^2*e^4*f + 50*a^4*b^7*c^2*d^3*e^4*f - 25*a^5*b^6*c*d^4*e^4*f +
5*a^6*b^5*d^5*e^4*f + 10*a^2*b^9*c^5*e^3*f^2 - 50*a^3*b^8*c^4*d*e^3*f^2 +
100*a^4*b^7*c^3*d^2*e^3*f^2 - 100*a^5*b^6*c^2*d^3*e^3*f^2 + 50*a^6*b^5*c*d
^4*e^3*f^2 - 10*a^7*b^4*d^5*e^3*f^2 - 10*a^3*b^8*c^5*e^2*f^3 + 50*a^4*b^7*
c^4*d*e^2*f^3 - 100*a^5*b^6*c^3*d^2*e^2*f^3 + 100*a^6*b^5*c^2*d^3*e^2*f^3
- 50*a^7*b^4*c*d^4*e^2*f^3 + 10*a^8*b^3*d^5*e^2*f^3 + 5*a^4*b^7*c^5*e*f^4
- 25*a^5*b^6*c^4*d*e*f^4 + 50*a^6*b^5*c^3*d^2*e*f^4 - 50*a^7*b^4*c^2*d^3*e
*f^4 + 25*a^8*b^3*c*d^4*e*f^4 - 5*a^9*b^2*d^5*e*f^4 - a^5*b^6*c^5*f^5 + 5*
a^6*b^5*c^4*d*f^5 - 10*a^7*b^4*c^3*d^2*f^5 + 10*a^8*b^3*c^2*d^3*f^5 - 5*a^
9*b^2*c*d^4*f^5 + a^10*b*d^5*f^5) - (C*b^2*c^2*d^6*e^2 + 4*C*a*b*c*d^7*e^2
 - 3*B*b^2*c*d^7*e^2 + C*a^2*d^8*e^2 - 3*B*a*b*d^8*e^2 + 6*A*b^2*d^8*e^...
```

Mupad [B] (verification not implemented)

Time = 39.69 (sec) , antiderivative size = 202365, normalized size of antiderivative = 172.08

$$\int \frac{A+Bx+Cx^2}{(ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3)^3}\,dx = \text{Too large to display}$$

input
```
int((A + B*x + C*x^2)/(x^2*(a*d*f + b*c*f + b*d*e) + x*(a*c*f + a*d*e + b*
c*e) + a*c*e + b*d*f*x^3)^3,x)
```

output
```
symsum(log(root(756756*a^10*b^10*c^10*d^10*e^10*f^10*z^3 + 573300*a^12*b^8
*c^9*d^11*e^9*f^11*z^3 + 573300*a^11*b^9*c^11*d^9*e^8*f^12*z^3 + 573300*a^
11*b^9*c^8*d^12*e^11*f^9*z^3 + 573300*a^9*b^11*c^12*d^8*e^9*f^11*z^3 + 573
300*a^9*b^11*c^9*d^11*e^12*f^8*z^3 + 573300*a^8*b^12*c^11*d^9*e^11*f^9*z^3
 - 343980*a^11*b^9*c^10*d^10*e^9*f^11*z^3 - 343980*a^11*b^9*c^9*d^11*e^10*
f^10*z^3 - 343980*a^10*b^10*c^11*d^9*e^9*f^11*z^3 - 343980*a^10*b^10*c^9*d
^11*e^11*f^9*z^3 - 343980*a^9*b^11*c^11*d^9*e^10*f^10*z^3 - 343980*a^9*b^1
1*c^10*d^10*e^11*f^9*z^3 + 326340*a^13*b^7*c^10*d^10*e^7*f^13*z^3 + 326340
*a^13*b^7*c^7*d^13*e^10*f^10*z^3 + 326340*a^10*b^10*c^13*d^7*e^7*f^13*z^3
+ 326340*a^10*b^10*c^7*d^13*e^13*f^7*z^3 + 326340*a^7*b^13*c^13*d^7*e^10*f
^10*z^3 + 326340*a^7*b^13*c^10*d^10*e^13*f^7*z^3 - 267540*a^12*b^8*c^10*d^
10*e^8*f^12*z^3 - 267540*a^12*b^8*c^8*d^12*e^10*f^10*z^3 - 267540*a^10*b^1
0*c^12*d^8*e^8*f^12*z^3 - 267540*a^10*b^10*c^8*d^12*e^12*f^8*z^3 - 267540*
a^8*b^12*c^12*d^8*e^10*f^10*z^3 - 267540*a^8*b^12*c^10*d^10*e^12*f^8*z^3 +
 245700*a^14*b^6*c^8*d^12*e^8*f^12*z^3 + 245700*a^12*b^8*c^12*d^8*e^6*f^14
*z^3 + 245700*a^12*b^8*c^6*d^14*e^12*f^8*z^3 + 245700*a^8*b^12*c^14*d^6*e^
8*f^12*z^3 + 245700*a^8*b^12*c^8*d^12*e^14*f^6*z^3 + 245700*a^6*b^14*c^12*
d^8*e^12*f^8*z^3 - 191100*a^13*b^7*c^9*d^11*e^8*f^12*z^3 - 191100*a^13*b^7
*c^8*d^12*e^9*f^11*z^3 - 191100*a^12*b^8*c^11*d^9*e^7*f^13*z^3 - 191100*a^
12*b^8*c^7*d^13*e^11*f^9*z^3 - 191100*a^11*b^9*c^12*d^8*e^7*f^13*z^3 - ...
```

Reduce [F]

$$\int \frac{A+Bx+Cx^2}{(ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3)^3}\,dx$$
$$=\int \frac{C\,x^2+Bx+A}{(ace+(acf+ade+bce)\,x+(adf+bcf+bde)\,x^2+bdf\,x^3)^3}dx$$

input
```
int((C*x^2+B*x+A)/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d
*f*x^3)^3,x)
```

output
```
int((C*x^2+B*x+A)/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d
*f*x^3)^3,x)
```

3.115 $\int (A+Bx+Cx^2)\sqrt{ace+(bce+ade+acf)x+($

Optimal result

Integrand size = 59, antiderivative size = 1393

$$\int (A+Bx+Cx^2)\sqrt{ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3}\,dx$$
$$= \text{Too large to display}$$

output
```
2/315*(8*a^3*C*d^3*f^3+3*a^2*b*d^2*f^2*(-4*B*d*f-C*c*f+C*d*e)-3*a*b^2*d*f^
2*((-7*A*d^2+C*c^2)*f+B*d*(-2*c*f+d*e))-b^3*(C*(-8*c^3*f^3-3*c^2*d*e*f^2+1
6*d^3*e^3)+3*d*f*(7*A*d*f*(-c*f+2*d*e)-B*(-4*c^2*f^2-c*d*e*f+8*d^2*e^2))))
*(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)^(1/2)/b^3
/d^3/f^3-2/105*(7*d*f*(-3*A*b*d*f+C*a*c*f+C*a*d*e+C*b*c*e)+(a*d*f-4*b*(c*f
+d*e))*(2*a*C*d*f-b*(3*B*d*f-2*C*(c*f+d*e)))/b)*(f*x+e)*(a*c*e+(a*c*f+a*d*
e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)^(1/2)/b/d^2/f^3+2/21*(3*B*d*
f-2*C*(a*d*f+b*c*f+b*d*e)/b)*(d*x+c)*(f*x+e)*(a*c*e+(a*c*f+a*d*e+b*c*e)*x+
(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)^(1/2)/d^2/f^2+2/9*C*(b*x+a)*(d*x+c)*(f*
x+e)*(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)^(1/2)
/b/d/f-2/315*(a*d-b*c)^(1/2)*(16*a^4*C*d^4*f^4-8*a^3*b*d^3*f^3*(3*B*d*f+C*
c*f+C*d*e)+3*a^2*b^2*d^2*f^2*(d*f*(14*A*d*f+5*B*c*f+5*B*d*e)-2*C*(c^2*f^2-
c*d*e*f+d^2*e^2))-a*b^3*d*f*(C*(8*c^3*f^3-6*c^2*d*e*f^2-6*c*d^2*e^2*f+8*d^
3*e^3)+3*d*f*(14*A*d*f*(c*f+d*e)-B*(5*c^2*f^2-6*c*d*e*f+5*d^2*e^2)))+b^4*(
2*C*(8*c^4*f^4-4*c^3*d*e*f^3-3*c^2*d^2*e^2*f^2-4*c*d^3*e^3*f+8*d^4*e^4)+3*
d*f*(14*A*d*f*(c^2*f^2-c*d*e*f+d^2*e^2)-B*(8*c^3*f^3-5*c^2*d*e*f^2-5*c*d^2
*e^2*f+8*d^3*e^3))))*(b*(d*x+c)/(-a*d+b*c))^(1/2)*(a*c*e+(a*c*f+a*d*e+b*c*
e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)^(1/2)*EllipticE(d^(1/2)*(b*x+a)^(1
/2)/(a*d-b*c)^(1/2),((-a*d+b*c)*f/d/(-a*f+b*e))^(1/2))/b^4/d^(7/2)/f^4/(b*
x+a)^(1/2)/(d*x+c)/(b*(f*x+e)/(-a*f+b*e))^(1/2)-2/315*(a*d-b*c)^(1/2)*(...
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 39.04 (sec) , antiderivative size = 12161, normalized size of antiderivative = 8.73

$$\int \left(A+Bx+Cx^2\right)\sqrt{ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3}\,dx$$
$$= \text{Result too large to show}$$

input
```
Integrate[(A + B*x + C*x^2)*Sqrt[a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*
e + b*c*f + a*d*f)*x^2 + b*d*f*x^3],x]
```

output
```
Result too large to show
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(A+Bx+Cx^2\right) \sqrt{x^2(adf+bcf+bde)+x(acf+ade+bce)+ace+bdfx^3} \, dx$$

$$\Big\downarrow \text{2526}$$

$$\frac{\int -\left((aC(de+cf)+b(cCe-3Adf)-(3bBdf-2aCdf-2bC(de+cf))x)\sqrt{bdfx^3+(bde+bcf+adf)x^2+(bc}\right.}{3bdf}$$
$$\frac{2C\left(x^2(adf+bcf+bde)+x(acf+ade+bce)+ace+bdfx^3\right)^{3/2}}{9bdf}$$

$$\Big\downarrow \text{25}$$

$$\frac{2C\left(x^2(adf+bcf+bde)+x(acf+ade+bce)+ace+bdfx^3\right)^{3/2}}{9bdf} - \frac{\int (bcCe+aCde+acCf-3Abdf-(3bBdf-2aCdf-2bC(de+cf))x)\sqrt{bdfx^3+(bde+bcf+adf)x^2+(bce+}}{3bdf}$$

$$\Big\downarrow \text{2490}$$

$$\frac{2C\left(x^2(adf+bcf+bde)+x(acf+ade+bce)+ace+bdfx^3\right)^{3/2}}{9bdf} - \frac{\int \left(\frac{3bdf(bcCe+aCde+acCf-3Abdf)-(bde+bcf+adf)(-3bBdf+2aCdf+2bC(de+cf))}{3bdf}+(-3bBdf+2aCdf+2bC(de+cf))\left(\frac{bde}{}\right.\right.}{}$$

$$\Big\downarrow \text{7292}$$

$$\frac{2C\left(x^2(adf+bcf+bde)+x(acf+ade+bce)+ace+bdfx^3\right)^{3/2}}{9bdf} - \frac{\int \left(\frac{3bdf(bcCe+aCde+acCf-3Abdf)-(bde+bcf+adf)(-3bBdf+2aCdf+2bC(de+cf))}{3bdf}+(-3bBdf+2aCdf+2bC(de+cf))\left(\frac{bde}{}\right.\right.}{}$$

$$\Big\downarrow \text{7293}$$

$$\frac{2C\left(x^2(adf+bcf+bde)+x(acf+ade+bce)+ace+bdfx^3\right)^{3/2}}{9bdf}-$$

$$\int\left(\frac{\sqrt{bdf\left(\frac{bde+bcf+adf}{3bdf}+x\right)^3+\frac{1}{3}\left(3(bce+ade+acf)-\frac{(bde+bcf+adf)^2}{bdf}\right)\left(\frac{bde+bcf+adf}{3bdf}+x\right)+\frac{(bde+bcf-2adf)(2bde-bcf-adf)(bde-2bcf+adf)}{27b^2d^2f^2}}\left(-((C\right.}{3bdf}\right.$$

$$\Big\downarrow \; 7293$$

$$\frac{2C\left(x^2(adf+bcf+bde)+x(acf+ade+bce)+ace+bdfx^3\right)^{3/2}}{9bdf}-$$

$$\int\left(\frac{\sqrt{bdf\left(\frac{bde+bcf+adf}{3bdf}+x\right)^3+\frac{1}{3}\left(-\frac{(bde+bcf+adf)^2}{bdf}+3bce+3ade+3acf\right)\left(\frac{bde+bcf+adf}{3bdf}+x\right)+\frac{(bde+bcf-2adf)(2bde-bcf-adf)(bde-2bcf+adf)}{27b^2d^2f^2}}\left(-(\right.}{3bdf}\right.$$

$$\Big\downarrow \; 7299$$

$$\frac{2C\left(x^2(adf+bcf+bde)+x(acf+ade+bce)+ace+bdfx^3\right)^{3/2}}{9bdf}-$$

$$\int\left(\frac{\sqrt{bdf\left(\frac{bde+bcf+adf}{3bdf}+x\right)^3+\frac{1}{3}\left(-\frac{(bde+bcf+adf)^2}{bdf}+3bce+3ade+3acf\right)\left(\frac{bde+bcf+adf}{3bdf}+x\right)+\frac{(bde+bcf-2adf)(2bde-bcf-adf)(bde-2bcf+adf)}{27b^2d^2f^2}}\left(-(\right.}{3bdf}\right.$$

input
```
Int[(A + B*x + C*x^2)*Sqrt[a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*
c*f + a*d*f)*x^2 + b*d*f*x^3],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2490
```
Int[(P3_)^(p_.)*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] :> With[{a = Coeff[P3
, x, 0], b = Coeff[P3, x, 1], c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Su
bst[Int[((3*d*e - c*f)/(3*d) + f*x)^m*Simp[(2*c^3 - 9*b*c*d + 27*a*d^2)/(27
*d^2) - (c^2 - 3*b*d)*(x/(3*d)) + d*x^3, x]^p, x], x, x + c/(3*d)] /; NeQ[c
, 0]] /; FreeQ[{e, f, m, p}, x] && PolyQ[P3, x, 3]
```

rule 2526
```
Int[(Pm_)*(Qn_)^(p_), x_Symbol] :> With[{m = Expon[Pm, x], n = Expon[Qn, x]
}, Simp[Coeff[Pm, x, m]*(Qn^(p + 1)/(n*(p + 1)*Coeff[Qn, x, n])), x] + Simp
[1/(n*Coeff[Qn, x, n])   Int[ExpandToSum[n*Coeff[Qn, x, n]*Pm - Coeff[Pm, x
, m]*D[Qn, x], x]*Qn^p, x], x] /; EqQ[m, n - 1]] /; FreeQ[p, x] && PolyQ[Pm
, x] && PolyQ[Qn, x] && NeQ[p, -1]
```

rule 7292
```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [A] (verified)

Time = 4.02 (sec) , antiderivative size = 2037, normalized size of antiderivative = 1.46

method	result	size
elliptic	Expression too large to display	2037
default	Expression too large to display	3074

input
```
int((C*x^2+B*x+A)*(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d
*f*x^3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/9*C*x^3*(b*d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d*e*x^2+a*c*f*x+a*d*e*x+b*c*e*x
+a*c*e)^(1/2)+2/7*(b*d*f*B+a*C*d*f+C*b*c*f+C*b*d*e-2/9*(4*a*d*f+4*b*c*f+4*
b*d*e)*C)/b/d/f*x^2*(b*d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d*e*x^2+a*c*f*x+a*d*e
*x+b*c*e*x+a*c*e)^(1/2)+2/5*(b*d*f*A+B*a*d*f+B*b*c*f+B*b*d*e+C*a*c*f+C*a*d
*e+C*b*c*e-2/9*C*(7/2*a*c*f+7/2*a*d*e+7/2*b*c*e)-2/7*(b*d*f*B+a*C*d*f+C*b*
c*f+C*b*d*e-2/9*(4*a*d*f+4*b*c*f+4*b*d*e)*C)/b/d/f*(3*a*d*f+3*b*c*f+3*b*d*
e))/b/d/f*x*(b*d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d*e*x^2+a*c*f*x+a*d*e*x+b*c*e
*x+a*c*e)^(1/2)+2/3*(A*a*d*f+A*b*c*f+A*b*d*e+B*a*c*f+a*B*d*e+B*b*c*e+1/3*C
*a*e*c-2/7*(b*d*f*B+a*C*d*f+C*b*c*f+C*b*d*e-2/9*(4*a*d*f+4*b*c*f+4*b*d*e)*
C)/b/d/f*(5/2*a*c*f+5/2*a*d*e+5/2*b*c*e)-2/5*(b*d*f*A+B*a*d*f+B*b*c*f+B*b*
d*e+C*a*c*f+C*a*d*e+C*b*c*e-2/9*C*(7/2*a*c*f+7/2*a*d*e+7/2*b*c*e)-2/7*(b*d
*f*B+a*C*d*f+C*b*c*f+C*b*d*e-2/9*(4*a*d*f+4*b*c*f+4*b*d*e)*C)/b/d/f*(3*a*d
*f+3*b*c*f+3*b*d*e))/b/d/f*(2*a*d*f+2*b*c*f+2*b*d*e))/b/d/f*(b*d*f*x^3+a*d
*f*x^2+b*c*f*x^2+b*d*e*x^2+a*c*f*x+a*d*e*x+b*c*e*x+a*c*e)^(1/2)+2*(a*A*c*e
-2/5*(b*d*f*A+B*a*d*f+B*b*c*f+B*b*d*e+C*a*c*f+C*a*d*e+C*b*c*e-2/9*C*(7/2*a
*c*f+7/2*a*d*e+7/2*b*c*e)-2/7*(b*d*f*B+a*C*d*f+C*b*c*f+C*b*d*e-2/9*(4*a*d*
f+4*b*c*f+4*b*d*e)*C)/b/d/f*(3*a*d*f+3*b*c*f+3*b*d*e))/b/d/f*a*c*e-2/3*(A*
a*d*f+A*b*c*f+A*b*d*e+B*a*c*f+a*B*d*e+B*b*c*e+1/3*C*a*e*c-2/7*(b*d*f*B+a*C
*d*f+C*b*c*f+C*b*d*e-2/9*(4*a*d*f+4*b*c*f+4*b*d*e)*C)/b/d/f*(5/2*a*c*f+5/2
*a*d*e+5/2*b*c*e)-2/5*(b*d*f*A+B*a*d*f+B*b*c*f+B*b*d*e+C*a*c*f+C*a*d*e+...
```

Fricas [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 1943, normalized size of antiderivative = 1.39

$$\int (A + Bx + Cx^2)\sqrt{ace + (bce + ade + acf)x + (bde + bcf + adf)x^2 + bdfx^3}\,dx$$
$$= \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)*(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x
^2+b*d*f*x^3)^(1/2),x, algorithm="fricas")
```

output
```
2/945*((16*C*b^5*d^5*e^5 - 8*(2*C*b^5*c*d^4 + (2*C*a*b^4 + 3*B*b^5)*d^5)*e
^4*f - (5*C*b^5*c^2*d^3 - (20*C*a*b^4 + 27*B*b^5)*c*d^4 + (5*C*a^2*b^3 - 2
7*B*a*b^4 - 42*A*b^5)*d^5)*e^3*f^2 - (5*C*b^5*c^3*d^2 - 6*(C*a*b^4 + 2*B*b
^5)*c^2*d^3 - 3*(2*C*a^2*b^3 - 14*B*a*b^4 - 21*A*b^5)*c*d^4 + (5*C*a^3*b^2
 - 12*B*a^2*b^3 + 63*A*a*b^4)*d^5)*e^2*f^3 - (16*C*b^5*c^4*d - (20*C*a*b^4
 + 27*B*b^5)*c^3*d^2 - 3*(2*C*a^2*b^3 - 14*B*a*b^4 - 21*A*b^5)*c^2*d^3 - 2
*(10*C*a^3*b^2 - 21*B*a^2*b^3 + 126*A*a*b^4)*c*d^4 + (16*C*a^4*b - 27*B*a^
3*b^2 + 63*A*a^2*b^3)*d^5)*e*f^4 + (16*C*b^5*c^5 - 8*(2*C*a*b^4 + 3*B*b^5)
*c^4*d - (5*C*a^2*b^3 - 27*B*a*b^4 - 42*A*b^5)*c^3*d^2 - (5*C*a^3*b^2 - 12
*B*a^2*b^3 + 63*A*a*b^4)*c^2*d^3 - (16*C*a^4*b - 27*B*a^3*b^2 + 63*A*a^2*b
^3)*c*d^4 + 2*(8*C*a^5 - 12*B*a^4*b + 21*A*a^3*b^2)*d^5)*f^5)*sqrt(b*d*f)*
weierstrassPInverse(4/3*(b^2*d^2*e^2 - (b^2*c*d + a*b*d^2)*e*f + (b^2*c^2
- a*b*c*d + a^2*d^2)*f^2)/(b^2*d^2*f^2), -4/27*(2*b^3*d^3*e^3 - 3*(b^3*c*d
^2 + a*b^2*d^3)*e^2*f - 3*(b^3*c^2*d - 4*a*b^2*c*d^2 + a^2*b*d^3)*e*f^2 +
(2*b^3*c^3 - 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + 2*a^3*d^3)*f^3)/(b^3*d^3*f^3)
, 1/3*(3*b*d*f*x + b*d*e + (b*c + a*d)*f)/(b*d*f)) + 3*(16*C*b^5*d^5*e^4*f
 - 8*(C*b^5*c*d^4 + (C*a*b^4 + 3*B*b^5)*d^5)*e^3*f^2 - 3*(2*C*b^5*c^2*d^3
- (2*C*a*b^4 + 5*B*b^5)*c*d^4 + (2*C*a^2*b^3 - 5*B*a*b^4 - 14*A*b^5)*d^5)*
e^2*f^3 - (8*C*b^5*c^3*d^2 - 3*(2*C*a*b^4 + 5*B*b^5)*c^2*d^3 - 6*(C*a^2*b^
3 - 3*B*a*b^4 - 7*A*b^5)*c*d^4 + (8*C*a^3*b^2 - 15*B*a^2*b^3 + 42*A*a*b...
```

Sympy [F]

$$\int (A+Bx+Cx^2)\sqrt{ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3}\,dx$$
$$=\int \sqrt{(a+bx)\,(c+dx)\,(e+fx)}\left(A+Bx+Cx^2\right)\,dx$$

input
```
integrate((C*x**2+B*x+A)*(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*
x**2+b*d*f*x**3)**(1/2),x)
```

output
```
Integral(sqrt((a + b*x)*(c + d*x)*(e + f*x))*(A + B*x + C*x**2), x)
```

Maxima [F]

$$\int (A+Bx+Cx^2)\sqrt{ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3}\,dx$$
$$=\int \sqrt{bdfx^3+ace+(bde+bcf+adf)x^2+(bce+ade+acf)x}\,(Cx^2+Bx+A)\,dx$$

input
```
integrate((C*x^2+B*x+A)*(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x
^2+b*d*f*x^3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*d*f*x^3 + a*c*e + (b*d*e + b*c*f + a*d*f)*x^2 + (b*c*e +
a*d*e + a*c*f)*x)*(C*x^2 + B*x + A), x)
```

Giac [F]

$$\int (A+Bx+Cx^2)\sqrt{ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3}\,dx$$
$$=\int \sqrt{bdfx^3+ace+(bde+bcf+adf)x^2+(bce+ade+acf)x}\,(Cx^2+Bx+A)\,dx$$

input
```
integrate((C*x^2+B*x+A)*(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x
^2+b*d*f*x^3)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*d*f*x^3 + a*c*e + (b*d*e + b*c*f + a*d*f)*x^2 + (b*c*e +
a*d*e + a*c*f)*x)*(C*x^2 + B*x + A), x)
```

Mupad [F(-1)]

Timed out.

$$\int (A+Bx+Cx^2)\sqrt{ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3}\,dx$$
$$=\int (C\,x^2+B\,x+A)\,\sqrt{b\,d\,f\,x^3+(a\,d\,f+b\,c\,f+b\,d\,e)\,x^2+(a\,c\,f+a\,d\,e+b\,c\,e)\,x+a\,c\,e}\,dx$$

input
```
int((A + B*x + C*x^2)*(x^2*(a*d*f + b*c*f + b*d*e) + x*(a*c*f + a*d*e + b*
c*e) + a*c*e + b*d*f*x^3)^(1/2),x)
```

output
```
int((A + B*x + C*x^2)*(x^2*(a*d*f + b*c*f + b*d*e) + x*(a*c*f + a*d*e + b*
c*e) + a*c*e + b*d*f*x^3)^(1/2), x)
```

Reduce [F]

$$\int (A+Bx+Cx^2)\sqrt{ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3}\,dx$$
$$=\int (C\,x^2+Bx+A)\,\sqrt{ace+(acf+ade+bce)\,x+(adf+bcf+bde)\,x^2+bdf\,x^3}\,dx$$

input
```
int((C*x^2+B*x+A)*(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d
*f*x^3)^(1/2),x)
```

output
```
int((C*x^2+B*x+A)*(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d
*f*x^3)^(1/2),x)
```

3.116 $\int \frac{A+Bx+Cx^2}{\sqrt{ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3}}\,dx$

Optimal result

Integrand size = 59, antiderivative size = 505

$$\begin{aligned}
&\int \frac{A+Bx+Cx^2}{\sqrt{ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3}}\,dx \\
&= \frac{2C(a+bx)(c+dx)(e+fx)}{3bdf\sqrt{ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3}} \\
&+ \frac{2\sqrt{-bc+ad}\left(3Bdf-\frac{2C(bde+bcf+adf)}{b}\right)\sqrt{a+bx}\sqrt{\frac{b(c+dx)}{bc-ad}}(e+fx)E\left(\arcsin\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{-bc+ad}}\right)|\frac{(bc-ad)f}{d(be-af)}\right)}{3bd^{3/2}f^2\sqrt{\frac{b(e+fx)}{be-af}}\sqrt{ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3}} \\
&+ \frac{2\sqrt{-bc+ad}(aCf(de-cf)-b(3df(Be-Af)-Ce(2de+cf)))\sqrt{a+bx}\sqrt{\frac{b(c+dx)}{bc-ad}}\sqrt{\frac{b(e+fx)}{be-af}}\,\text{EllipticF}}{3b^2d^{3/2}f^2\sqrt{ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3}}
\end{aligned}$$

output
```
2/3*C*(b*x+a)*(d*x+c)*(f*x+e)/b/d/f/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*
c*f+b*d*e)*x^2+b*d*f*x^3)^(1/2)+2/3*(a*d-b*c)^(1/2)*(3*B*d*f-2*C*(a*d*f+b*
c*f+b*d*e)/b)*(b*x+a)^(1/2)*(b*(d*x+c)/(-a*d+b*c))^(1/2)*(f*x+e)*EllipticE
(d^(1/2)*(b*x+a)^(1/2)/(a*d-b*c)^(1/2),((-a*d+b*c)*f/d/(-a*f+b*e))^(1/2))/
b/d^(3/2)/f^2/(b*(f*x+e)/(-a*f+b*e))^(1/2)/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a
*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)^(1/2)+2/3*(a*d-b*c)^(1/2)*(a*C*f*(-c*f+d*
e)-b*(3*d*f*(-A*f+B*e)-C*e*(c*f+2*d*e)))*(b*x+a)^(1/2)*(b*(d*x+c)/(-a*d+b*
c))^(1/2)*(b*(f*x+e)/(-a*f+b*e))^(1/2)*EllipticF(d^(1/2)*(b*x+a)^(1/2)/(a*
d-b*c)^(1/2),((-a*d+b*c)*f/d/(-a*f+b*e))^(1/2))/b^2/d^(3/2)/f^2/(a*c*e+(a*
c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 28.24 (sec) , antiderivative size = 416, normalized size of antiderivative = 0.82

$$\int \frac{A+Bx+Cx^2}{\sqrt{ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3}}\,dx$$

$$= \frac{(a+bx)\left(2b^2Cdf(c+dx)(e+fx)-\frac{2b^2(-3bBdf+2aCdf+2bC(de+cf))(c+dx)(e+fx)}{a+bx}+2i\sqrt{-a+\frac{bc}{d}}df(3bBdf-2\right.}{}$$

input
```
Integrate[(A + B*x + C*x^2)/Sqrt[a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*
e + b*c*f + a*d*f)*x^2 + b*d*f*x^3],x]
```

output
```
((a + b*x)*(2*b^2*C*d*f*(c + d*x)*(e + f*x) - (2*b^2*(-3*b*B*d*f + 2*a*C*d
*f + 2*b*C*(d*e + c*f))*(c + d*x)*(e + f*x))/(a + b*x) + (2*I)*Sqrt[-a + (
b*c)/d]*d*f*(3*b*B*d*f - 2*a*C*d*f - 2*b*C*(d*e + c*f))*Sqrt[a + b*x]*Sqrt
[(b*(c + d*x))/(d*(a + b*x))]*Sqrt[(b*(e + f*x))/(f*(a + b*x))]*EllipticE[
I*ArcSinh[Sqrt[-a + (b*c)/d]/Sqrt[a + b*x]], (b*d*e - a*d*f)/(b*c*f - a*d*
f)] + ((2*I)*b*f*(a*C*d*(-(d*e) + c*f) + b*(2*c^2*C*f + 3*A*d^2*f + c*d*(C
*e - 3*B*f)))*Sqrt[a + b*x]*Sqrt[(b*(c + d*x))/(d*(a + b*x))]*Sqrt[(b*(e +
 f*x))/(f*(a + b*x))]*EllipticF[I*ArcSinh[Sqrt[-a + (b*c)/d]/Sqrt[a + b*x]
], (b*d*e - a*d*f)/(b*c*f - a*d*f)])/Sqrt[-a + (b*c)/d]))/(3*b^3*d^2*f^2*S
qrt[(a + b*x)*(c + d*x)*(e + f*x)])
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{\sqrt{x^2(adf+bcf+bde)+x(acf+ade+bce)+ace+bdfx^3}}dx$$

$$\Big\downarrow \text{ 2526}$$

$$\frac{\int -\frac{aC(de+cf)+b(cCe-3Adf)-(3bBdf-2aCdf-2bC(de+cf))x}{\sqrt{bdfx^3+(bde+bcf+adf)x^2+(bce+ade+acf)x+ace}}dx}{3bdf}+\frac{2C\sqrt{x^2(adf+bcf+bde)+x(acf+ade+bce)+ace+bdfx^3}}{3bdf}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{2C\sqrt{x^2(adf+bcf+bde)+x(acf+ade+bce)+ace+bdfx^3}}{3bdf}-\frac{\int \frac{bcCe+aCde+acCf-3Abdf-(3bBdf-2aCdf-2bC(de+cf))x}{\sqrt{bdfx^3+(bde+bcf+adf)x^2+(bce+ade+acf)x+ace}}dx}{3bdf}$$

$$\Big\downarrow \text{ 2490}$$

$$\frac{2C\sqrt{x^2(adf+bcf+bde)+x(acf+ade+bce)+ace+bdfx^3}}{3bdf}-$$

$$\frac{\int \frac{\frac{3bdf(bcCe+aCde+acCf-3Abdf)-(bde+bcf+adf)(-3bBdf+2aCdf+2bC(de+cf))}{3bdf}+(-3bBdf+2aCdf+2bC(de+cf))\left(\frac{bde+bcf+adf}{3bdf}+x\right)}{\sqrt{bdf\left(\frac{bde+bcf+adf}{3bdf}+x\right)^3+\frac{\left(3bdf(bce+ade+acf)-(bde+bcf+adf)^2\right)\left(\frac{bde+bcf+adf}{3bdf}+x\right)}{3bdf}+\frac{(bde+bcf-2adf)(2bde-bcf-adf)(bde-2bcf+adf)}{27b^2d^2f^2}}}d\left(\frac{bde+bc}{3bd}\cdots\right.}{3bdf}$$

$$\Big\downarrow \text{ 7292}$$

$$\frac{2C\sqrt{x^2(adf+bcf+bde)+x(acf+ade+bce)+ace+bdfx^3}}{3bdf}-$$

$$\frac{\int \frac{\frac{3bdf(bcCe+aCde+acCf-3Abdf)-(bde+bcf+adf)(-3bBdf+2aCdf+2bC(de+cf))}{3bdf}+(-3bBdf+2aCdf+2bC(de+cf))\left(\frac{bde+bcf+adf}{3bdf}+x\right)}{\sqrt{bdf\left(\frac{bde+bcf+adf}{3bdf}+x\right)^3+\frac{1}{3}\left(3(bce+ade+acf)-\frac{(bde+bcf+adf)^2}{bdf}\right)\left(\frac{bde+bcf+adf}{3bdf}+x\right)+\frac{(bde+bcf-2adf)(2bde-bcf-adf)(bde-2bcf+adf)}{27b^2d^2f^2}}}d\left(\frac{bde+}{}\cdots\right.}{3bdf}$$

$$\Big\downarrow \text{ 7293}$$

$$\frac{2C\sqrt{x^2(adf+bcf+bde)+x(acf+ade+bce)+ace+bdfx^3}}{3bdf}-$$

$$\int \left(\frac{-\left(\left(C\left(2d^2e^2+cdfe+2c^2f^2\right)+3df(3Adf-B(de+cf))\right)b^2\right)-adf(Cde+cCf-3Bdf)b-2a^2Cd^2f^2}{3bdf\sqrt{bdf\left(\frac{bde+bcf+adf}{3bdf}+x\right)^3+\frac{1}{3}\left(3(bce+ade+acf)-\frac{(bde+bcf+adf)^2}{bdf}\right)\left(\frac{bde+bcf+adf}{3bdf}+x\right)+\frac{(bde+bcf-2adf)(2bde-bcf-adf)(bde-2bcf+adf)}{27b^2d^2f^2}}}+\right.$$

$$\Big\downarrow \ 7293$$

$$\frac{2C\sqrt{x^2(adf+bcf+bde)+x(acf+ade+bce)+ace+bdfx^3}}{3bdf}-$$

$$\int \left(\frac{\sqrt{3}\left(-\left(\left(C\left(2d^2e^2+cdfe+2c^2f^2\right)+3df(3Adf-B(de+cf))\right)b^2\right)-adf(Cde+cCf-3Bdf)b-2a^2Cd^2f^2\right)}{bdf\sqrt{\frac{27b^3d^3f^3\left(\frac{bde+bcf+adf}{3bdf}+x\right)^3-9bdf\left(\left(d^2e^2-cdfe+c^2f^2\right)b^2-adf(de+cf)b+a^2d^2f^2\right)\left(\frac{bde+bcf+adf}{3bdf}+x\right)+(bde+bcf-2adf)(2bde-bcf-adf)(bde-2b}{b^2d^2f^2}}}\right.$$

$$\Big\downarrow \ 7299$$

$$\frac{2C\sqrt{x^2(adf+bcf+bde)+x(acf+ade+bce)+ace+bdfx^3}}{3bdf}-$$

$$\int \left(\frac{\sqrt{3}\left(-\left(\left(C\left(2d^2e^2+cdfe+2c^2f^2\right)+3df(3Adf-B(de+cf))\right)b^2\right)-adf(Cde+cCf-3Bdf)b-2a^2Cd^2f^2\right)}{bdf\sqrt{\frac{27b^3d^3f^3\left(\frac{bde+bcf+adf}{3bdf}+x\right)^3-9bdf\left(\left(d^2e^2-cdfe+c^2f^2\right)b^2-adf(de+cf)b+a^2d^2f^2\right)\left(\frac{bde+bcf+adf}{3bdf}+x\right)+(bde+bcf-2adf)(2bde-bcf-adf)(bde-2b}{b^2d^2f^2}}}\right.$$

input
```
Int[(A + B*x + C*x^2)/Sqrt[a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*
c*f + a*d*f)*x^2 + b*d*f*x^3],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2490
```
Int[(P3_)^(p_.)*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] :> With[{a = Coeff[P3
, x, 0], b = Coeff[P3, x, 1], c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Su
bst[Int[((3*d*e - c*f)/(3*d) + f*x)^m*Simp[(2*c^3 - 9*b*c*d + 27*a*d^2)/(27
*d^2) - (c^2 - 3*b*d)*(x/(3*d)) + d*x^3, x]^p, x], x, x + c/(3*d)] /; NeQ[c
, 0]] /; FreeQ[{e, f, m, p}, x] && PolyQ[P3, x, 3]
```

rule 2526
```
Int[(Pm_)*(Qn_)^(p_), x_Symbol] :> With[{m = Expon[Pm, x], n = Expon[Qn, x]
}, Simp[Coeff[Pm, x, m]*(Qn^(p + 1)/(n*(p + 1)*Coeff[Qn, x, n])), x] + Simp
[1/(n*Coeff[Qn, x, n])   Int[ExpandToSum[n*Coeff[Qn, x, n]*Pm - Coeff[Pm, x
, m]*D[Qn, x], x]*Qn^p, x], x] /; EqQ[m, n - 1]] /; FreeQ[p, x] && PolyQ[Pm
, x] && PolyQ[Qn, x] && NeQ[p, -1]
```

rule 7292
```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [A] (verified)

Time = 4.12 (sec) , antiderivative size = 575, normalized size of antiderivative = 1.14

method	result
elliptic	$\frac{2C\sqrt{bdf\,x^3+adf\,x^2+bcf\,x^2+bde\,x^2+acfx+adex+bcex+ace}}{3bdf}+\frac{2\left(A-\frac{2C\left(\frac{1}{2}acf+\frac{1}{2}ade+\frac{1}{2}bce\right)}{3bdf}\right)\left(\frac{e}{f}-\frac{c}{d}\right)\sqrt{\frac{x+\frac{e}{f}}{\frac{e}{f}-\frac{c}{d}}}\sqrt{\frac{x+\frac{a}{b}}{-\frac{e}{f}+\frac{a}{b}}}\sqrt{\frac{x+}{-\frac{e}{f}}}}{\sqrt{bdf\,x^3+adf\,x^2+bcf\,x^2+bde\,x^2+acfx+a}}$
default	Expression too large to display

input
```
int((C*x^2+B*x+A)/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d
*f*x^3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*C/b/d/f*(b*d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d*e*x^2+a*c*f*x+a*d*e*x+b*c*e
*x+a*c*e)^(1/2)+2*(A-2/3*C/b/d/f*(1/2*a*c*f+1/2*a*d*e+1/2*b*c*e))*(e/f-c/d
)*((x+e/f)/(e/f-c/d))^(1/2)*((x+a/b)/(-e/f+a/b))^(1/2)*((x+c/d)/(-e/f+c/d)
)^(1/2)/(b*d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d*e*x^2+a*c*f*x+a*d*e*x+b*c*e*x+a
*c*e)^(1/2)*EllipticF(((x+e/f)/(e/f-c/d))^(1/2),((-e/f+c/d)/(-e/f+a/b))^(1
/2))+2*(B-2/3*C/b/d/f*(a*d*f+b*c*f+b*d*e))*(e/f-c/d)*((x+e/f)/(e/f-c/d))^(
1/2)*((x+a/b)/(-e/f+a/b))^(1/2)*((x+c/d)/(-e/f+c/d))^(1/2)/(b*d*f*x^3+a*d*
f*x^2+b*c*f*x^2+b*d*e*x^2+a*c*f*x+a*d*e*x+b*c*e*x+a*c*e)^(1/2)*((-e/f+a/b)
*EllipticE(((x+e/f)/(e/f-c/d))^(1/2),((-e/f+c/d)/(-e/f+a/b))^(1/2))-a/b*El
lipticF(((x+e/f)/(e/f-c/d))^(1/2),((-e/f+c/d)/(-e/f+a/b))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 834, normalized size of antiderivative = 1.65

$$\int \frac{A+Bx+Cx^2}{\sqrt{ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3}}\,dx = \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x
^2+b*d*f*x^3)^(1/2),x, algorithm="fricas")
```

output
```
2/9*(3*sqrt(b*d*f*x^3 + a*c*e + (b*d*e + (b*c + a*d)*f)*x^2 + (a*c*f + (b*
c + a*d)*e)*x)*C*b^2*d^2*f^2 + (2*C*b^2*d^2*e^2 + (C*b^2*c*d + (C*a*b - 3*
B*b^2)*d^2)*e*f + (2*C*b^2*c^2 + (C*a*b - 3*B*b^2)*c*d + (2*C*a^2 - 3*B*a*
b + 9*A*b^2)*d^2)*f^2)*sqrt(b*d*f)*weierstrassPInverse(4/3*(b^2*d^2*e^2 -
(b^2*c*d + a*b*d^2)*e*f + (b^2*c^2 - a*b*c*d + a^2*d^2)*f^2)/(b^2*d^2*f^2)
, -4/27*(2*b^3*d^3*e^3 - 3*(b^3*c*d^2 + a*b^2*d^3)*e^2*f - 3*(b^3*c^2*d -
4*a*b^2*c*d^2 + a^2*b*d^3)*e*f^2 + (2*b^3*c^3 - 3*a*b^2*c^2*d - 3*a^2*b*c*
d^2 + 2*a^3*d^3)*f^3)/(b^3*d^3*f^3), 1/3*(3*b*d*f*x + b*d*e + (b*c + a*d)*
f)/(b*d*f)) + 3*(2*C*b^2*d^2*e*f + (2*C*b^2*c*d + (2*C*a*b - 3*B*b^2)*d^2)
*f^2)*sqrt(b*d*f)*weierstrassZeta(4/3*(b^2*d^2*e^2 - (b^2*c*d + a*b*d^2)*e
*f + (b^2*c^2 - a*b*c*d + a^2*d^2)*f^2)/(b^2*d^2*f^2), -4/27*(2*b^3*d^3*e^
3 - 3*(b^3*c*d^2 + a*b^2*d^3)*e^2*f - 3*(b^3*c^2*d - 4*a*b^2*c*d^2 + a^2*b
*d^3)*e*f^2 + (2*b^3*c^3 - 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + 2*a^3*d^3)*f^3)
/(b^3*d^3*f^3), weierstrassPInverse(4/3*(b^2*d^2*e^2 - (b^2*c*d + a*b*d^2)
*e*f + (b^2*c^2 - a*b*c*d + a^2*d^2)*f^2)/(b^2*d^2*f^2), -4/27*(2*b^3*d^3*
e^3 - 3*(b^3*c*d^2 + a*b^2*d^3)*e^2*f - 3*(b^3*c^2*d - 4*a*b^2*c*d^2 + a^2
*b*d^3)*e*f^2 + (2*b^3*c^3 - 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + 2*a^3*d^3)*f^
3)/(b^3*d^3*f^3), 1/3*(3*b*d*f*x + b*d*e + (b*c + a*d)*f)/(b*d*f))))/(b^3*
d^3*f^3)
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt{ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3}}\,dx = \int \frac{A+Bx+Cx^2}{\sqrt{(a+bx)\,(c+dx)\,(e+fx)}}\,dx$$

input
```
integrate((C*x**2+B*x+A)/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*
x**2+b*d*f*x**3)**(1/2),x)
```

output
```
Integral((A + B*x + C*x**2)/sqrt((a + b*x)*(c + d*x)*(e + f*x)), x)
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt{ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3}}\,dx = \int \frac{Cx^2+Bx+A}{\sqrt{bdfx^3+ace+(bde+bcf+adf)x^2+(bce+ade+acf)x}}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x
^2+b*d*f*x^3)^(1/2),x, algorithm="maxima")
```

output
```
integrate((C*x^2 + B*x + A)/sqrt(b*d*f*x^3 + a*c*e + (b*d*e + b*c*f + a*d*
f)*x^2 + (b*c*e + a*d*e + a*c*f)*x), x)
```

Giac [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt{ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3}}\,dx = \int \frac{Cx^2+Bx+A}{\sqrt{bdfx^3+ace+(bde+bcf+adf)x^2+(bce+ade+acf)x}}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x
^2+b*d*f*x^3)^(1/2),x, algorithm="giac")
```

output
```
integrate((C*x^2 + B*x + A)/sqrt(b*d*f*x^3 + a*c*e + (b*d*e + b*c*f + a*d*
f)*x^2 + (b*c*e + a*d*e + a*c*f)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{\sqrt{ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3}}\,dx$$
$$=\int \frac{C\,x^2+B\,x+A}{\sqrt{b\,d\,f\,x^3+(a\,d\,f+b\,c\,f+b\,d\,e)\,x^2+(a\,c\,f+a\,d\,e+b\,c\,e)\,x+a\,c\,e}}\,dx$$

input
```
int((A + B*x + C*x^2)/(x^2*(a*d*f + b*c*f + b*d*e) + x*(a*c*f + a*d*e + b*
c*e) + a*c*e + b*d*f*x^3)^(1/2),x)
```

output
```
int((A + B*x + C*x^2)/(x^2*(a*d*f + b*c*f + b*d*e) + x*(a*c*f + a*d*e + b*
c*e) + a*c*e + b*d*f*x^3)^(1/2), x)
```

Reduce [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt{ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3}}\,dx=\text{too large to display}$$

input
```
int((C*x^2+B*x+A)/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d
*f*x^3)^(1/2),x)
```

output

```
(2*sqrt(e + f*x)*sqrt(c + d*x)*sqrt(a + b*x)*b + 2*int((sqrt(e + f*x)*sqrt
(c + d*x)*sqrt(a + b*x)*x**2)/(a**2*c*d*e*f + a**2*c*d*f**2*x + a**2*d**2*
e*f*x + a**2*d**2*f**2*x**2 + a*b*c**2*e*f + a*b*c**2*f**2*x + a*b*c*d*e**
2 + 3*a*b*c*d*e*f*x + 2*a*b*c*d*f**2*x**2 + a*b*d**2*e**2*x + 2*a*b*d**2*e
*f*x**2 + a*b*d**2*f**2*x**3 + b**2*c**2*e*f*x + b**2*c**2*f**2*x**2 + b**
2*c*d*e**2*x + 2*b**2*c*d*e*f*x**2 + b**2*c*d*f**2*x**3 + b**2*d**2*e**2*x
**2 + b**2*d**2*e*f*x**3),x)*a**2*c*d**2*f**2 - 3*int((sqrt(e + f*x)*sqrt(
c + d*x)*sqrt(a + b*x)*x**2)/(a**2*c*d*e*f + a**2*c*d*f**2*x + a**2*d**2*e
*f*x + a**2*d**2*f**2*x**2 + a*b*c**2*e*f + a*b*c**2*f**2*x + a*b*c*d*e**2
 + 3*a*b*c*d*e*f*x + 2*a*b*c*d*f**2*x**2 + a*b*d**2*e**2*x + 2*a*b*d**2*e*
f*x**2 + a*b*d**2*f**2*x**3 + b**2*c**2*e*f*x + b**2*c**2*f**2*x**2 + b**2
*c*d*e**2*x + 2*b**2*c*d*e*f*x**2 + b**2*c*d*f**2*x**3 + b**2*d**2*e**2*x*
*2 + b**2*d**2*e*f*x**3),x)*a*b**2*d**2*f**2 + 4*int((sqrt(e + f*x)*sqrt(c
 + d*x)*sqrt(a + b*x)*x**2)/(a**2*c*d*e*f + a**2*c*d*f**2*x + a**2*d**2*e*
f*x + a**2*d**2*f**2*x**2 + a*b*c**2*e*f + a*b*c**2*f**2*x + a*b*c*d*e**2
+ 3*a*b*c*d*e*f*x + 2*a*b*c*d*f**2*x**2 + a*b*d**2*e**2*x + 2*a*b*d**2*e*f
*x**2 + a*b*d**2*f**2*x**3 + b**2*c**2*e*f*x + b**2*c**2*f**2*x**2 + b**2*
c*d*e**2*x + 2*b**2*c*d*e*f*x**2 + b**2*c*d*f**2*x**3 + b**2*d**2*e**2*x**
2 + b**2*d**2*e*f*x**3),x)*a*b*c**2*d*f**2 + 4*int((sqrt(e + f*x)*sqrt(c +
 d*x)*sqrt(a + b*x)*x**2)/(a**2*c*d*e*f + a**2*c*d*f**2*x + a**2*d**2*e...
```

3.117 $\int \frac{A+Bx+Cx^2}{(ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 59, antiderivative size = 1151

$$\int \frac{A+Bx+Cx^2}{(ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3)^{3/2}}\,dx = \text{Too large to display}$$

output
```
-2*(A*b^2-a*(B*b-C*a))*(b*x+a)*(d*x+c)*(f*x+e)/b/(-a*d+b*c)/(-a*f+b*e)/(a*
c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)^(3/2)-2*(a^2*
C*d*(-c*f+d*e)+b^2*(c^2*C*e+2*A*d^2*e-c*d*(A*f+B*e))-a*b*((A*d^2+C*c^2)*f+
B*d*(-2*c*f+d*e)))*(b*x+a)^2*(d*x+c)*(f*x+e)/b/(-a*d+b*c)^2/(-a*f+b*e)/(-c
*f+d*e)/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)^(3
/2)+2*f*(a*b*(d^2*e*(2*A*f+B*e)+c^2*f*(B*f+2*C*e)+2*c*d*(A*f^2-3*B*e*f+C*e
^2))-b^2*(2*A*d^2*e^2-c*d*e*(2*A*f+B*e)+c^2*(2*A*f^2-B*e*f+2*C*e^2))+a^2*(
d*f*(-2*A*d*f+B*c*f+B*d*e)-2*C*(c^2*f^2-c*d*e*f+d^2*e^2)))*(b*x+a)^2*(d*x+
c)^2*(f*x+e)/(-a*d+b*c)^2/(-a*f+b*e)^2/(-c*f+d*e)^2/(a*c*e+(a*c*f+a*d*e+b*
c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)^(3/2)-2*d^(1/2)*(a*b*(d^2*e*(2*A
*f+B*e)+c^2*f*(B*f+2*C*e)+2*c*d*(A*f^2-3*B*e*f+C*e^2))-b^2*(2*A*d^2*e^2-c*
d*e*(2*A*f+B*e)+c^2*(2*A*f^2-B*e*f+2*C*e^2))+a^2*(d*f*(-2*A*d*f+B*c*f+B*d*
e)-2*C*(c^2*f^2-c*d*e*f+d^2*e^2)))*(b*x+a)^(3/2)*(d*x+c)*(b*(d*x+c)/(-a*d+
b*c))^(1/2)*(f*x+e)^2*EllipticE(d^(1/2)*(b*x+a)^(1/2)/(a*d-b*c)^(1/2),((-a
*d+b*c)*f/d/(-a*f+b*e))^(1/2))/(a*d-b*c)^(3/2)/(-a*f+b*e)^2/(-c*f+d*e)^2/(
b*(f*x+e)/(-a*f+b*e))^(1/2)/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*
e)*x^2+b*d*f*x^3)^(3/2)-2*(a^2*C*d*(-c*f+d*e)+b^2*(c^2*C*e+2*A*d^2*e-c*d*(
A*f+B*e))-a*b*((A*d^2+C*c^2)*f+B*d*(-2*c*f+d*e)))*(b*x+a)^(3/2)*(d*x+c)*(b
*(d*x+c)/(-a*d+b*c))^(1/2)*(f*x+e)*(b*(f*x+e)/(-a*f+b*e))^(1/2)*EllipticF(
d^(1/2)*(b*x+a)^(1/2)/(a*d-b*c)^(1/2),((-a*d+b*c)*f/d/(-a*f+b*e))^(1/2)...
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 36.97 (sec) , antiderivative size = 9971, normalized size of antiderivative = 8.66

$$\int \frac{A+Bx+Cx^2}{(ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3)^{3/2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(A + B*x + C*x^2)/(a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e +
b*c*f + a*d*f)*x^2 + b*d*f*x^3)^(3/2),x]
```

output
```
Result too large to show
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{\left(x^2(adf+bcf+bde)+x(acf+ade+bce)+ace+bdfx^3\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 2526}$$

$$\frac{\int -\frac{aC(de+cf)+b(cCe-3Adf)-(3bBdf-2aCdf-2bC(de+cf))x}{\left(bdfx^3+(bde+bcf+adf)x^2+(bce+ade+acf)x+ace\right)^{3/2}}dx}{3bdf}-\frac{2C}{3bdf\sqrt{x^2(adf+bcf+bde)+x(acf+ade+bce)+ace+bdfx^3}}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\int \frac{bcCe+aCde+acCf-3Abdf-(3bBdf-2aCdf-2bC(de+cf))x}{\left(bdfx^3+(bde+bcf+adf)x^2+(bce+ade+acf)x+ace\right)^{3/2}}dx}{3bdf}-\frac{2C}{3bdf\sqrt{x^2(adf+bcf+bde)+x(acf+ade+bce)+ace+bdfx^3}}$$

$$\Big\downarrow \text{ 2490}$$

$$-\frac{\int \frac{\frac{3bdf(bcCe+aCde+acCf-3Abdf)-(bde+bcf+adf)(-3bBdf+2aCdf+2bC(de+cf))}{3bdf}+(-3bBdf+2aCdf+2bC(de+cf))\left(\frac{bde+bcf+adf}{3bdf}+x\right)}{\left(bdf\left(\frac{bde+bcf+adf}{3bdf}+x\right)^3+\frac{\left(3bdf(bce+ade+acf)-(bde+bcf+adf)^2\right)\left(\frac{bde+bcf+adf}{3bdf}+x\right)}{3bdf}+\frac{(bde+bcf-2adf)(2bde-bcf-adf)(bde-2bcf+adf)}{27b^2d^2f^2}\right)^{3/2}}d\Big(b}{3bdf}$$

$$\frac{2C}{3bdf\sqrt{x^2(adf+bcf+bde)+x(acf+ade+bce)+ace+bdfx^3}}$$

$$\Big\downarrow \text{ 7292}$$

$$-\frac{\int \frac{\frac{3bdf(bcCe+aCde+acCf-3Abdf)-(bde+bcf+adf)(-3bBdf+2aCdf+2bC(de+cf))}{3bdf}+(-3bBdf+2aCdf+2bC(de+cf))\left(\frac{bde+bcf+adf}{3bdf}+x\right)}{\left(bdf\left(\frac{bde+bcf+adf}{3bdf}+x\right)^3+\frac{1}{3}\left(3(bce+ade+acf)-\frac{(bde+bcf+adf)^2}{bdf}\right)\left(\frac{bde+bcf+adf}{3bdf}+x\right)+\frac{(bde+bcf-2adf)(2bde-bcf-adf)(bde-2bcf+adf)}{27b^2d^2f^2}\right)^{3/2}}d\Big(}{3bdf}$$

$$\frac{2C}{3bdf\sqrt{x^2(adf+bcf+bde)+x(acf+ade+bce)+ace+bdfx^3}}$$

$$\Big\downarrow \text{ 7293}$$

$$-\int \left(\frac{-((C(2d^2e^2+cdfe+2c^2f^2)+3df(3Adf-B(de+cf)))b^2)-adf(Cde+cCf-3Bdf)b-2a^2Cd^2f^2}{3bdf\left(bdf\left(\frac{bde+bcf+adf}{3bdf}+x\right)^3+\frac{1}{3}\left(3(bce+ade+acf)-\frac{(bde+bcf+adf)^2}{bdf}\right)\left(\frac{bde+bcf+adf}{3bdf}+x\right)+\frac{(bde+bcf-2adf)(2bde-bcf-adf)(bde-2bcf+adf)}{27b^2d^2f^2}\right.}\right.$$

$$\frac{2C}{3bdf\sqrt{x^2(adf+bcf+bde)+x(acf+ade+bce)+ace+bdfx^3}}$$

$$\Big\downarrow \text{ 7293}$$

$$-\int \left(\frac{27\sqrt{3}(-((C(2d^2e^2+cdfe+2c^2f^2)+3df(3Adf-B(de+cf)))b^2)-adf(Cde+cCf-3Bdf)b-2a^2Cd^2f^2)}{bdf\left(\frac{27b^3d^3f^3\left(\frac{bde+bcf+adf}{3bdf}+x\right)^3-9bdf\left(\left(d^2e^2-cdfe+c^2f^2\right)b^2-adf(de+cf)b+a^2d^2f^2\right)\left(\frac{bde+bcf+adf}{3bdf}+x\right)+(bde+bcf-2adf)(2bde-bcf-adf)(bde-}{b^2d^2f^2}\right.}\right.$$

$$\frac{2C}{3bdf\sqrt{x^2(adf+bcf+bde)+x(acf+ade+bce)+ace+bdfx^3}}$$

$$\Big\downarrow \text{ 7299}$$

$$-\int \left(\frac{27\sqrt{3}(-((C(2d^2e^2+cdfe+2c^2f^2)+3df(3Adf-B(de+cf)))b^2)-adf(Cde+cCf-3Bdf)b-2a^2Cd^2f^2)}{bdf\left(\frac{27b^3d^3f^3\left(\frac{bde+bcf+adf}{3bdf}+x\right)^3-9bdf\left(\left(d^2e^2-cdfe+c^2f^2\right)b^2-adf(de+cf)b+a^2d^2f^2\right)\left(\frac{bde+bcf+adf}{3bdf}+x\right)+(bde+bcf-2adf)(2bde-bcf-adf)(bde-}{b^2d^2f^2}\right.}\right.$$

$$\frac{2C}{3bdf\sqrt{x^2(adf+bcf+bde)+x(acf+ade+bce)+ace+bdfx^3}}$$

input
```
Int[(A + B*x + C*x^2)/(a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*c*f
+ a*d*f)*x^2 + b*d*f*x^3)^(3/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2490
```
Int[(P3_)^(p_.)*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] :> With[{a = Coeff[P3
, x, 0], b = Coeff[P3, x, 1], c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Su
bst[Int[((3*d*e - c*f)/(3*d) + f*x)^m*Simp[(2*c^3 - 9*b*c*d + 27*a*d^2)/(27
*d^2) - (c^2 - 3*b*d)*(x/(3*d)) + d*x^3, x]^p, x], x, x + c/(3*d)] /; NeQ[c
, 0]] /; FreeQ[{e, f, m, p}, x] && PolyQ[P3, x, 3]
```

rule 2526
```
Int[(Pm_)*(Qn_)^(p_), x_Symbol] :> With[{m = Expon[Pm, x], n = Expon[Qn, x]
}, Simp[Coeff[Pm, x, m]*(Qn^(p + 1)/(n*(p + 1)*Coeff[Qn, x, n])), x] + Simp
[1/(n*Coeff[Qn, x, n])   Int[ExpandToSum[n*Coeff[Qn, x, n]*Pm - Coeff[Pm, x
, m]*D[Qn, x], x]*Qn^p, x], x] /; EqQ[m, n - 1]] /; FreeQ[p, x] && PolyQ[Pm
, x] && PolyQ[Qn, x] && NeQ[p, -1]
```

rule 7292
```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 4110 vs. $2(1111) = 2222$.

Time = 4.33 (sec) , antiderivative size = 4111, normalized size of antiderivative = 3.57

method	result	size
elliptic	Expression too large to display	4111
default	Expression too large to display	8188

input
```
int((C*x^2+B*x+A)/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d
*f*x^3)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*b*d*f*((2*A*a^2*d^2*f^2-2*A*a*b*c*d*f^2-2*A*a*b*d^2*e*f+2*A*b^2*c^2*f^2
-2*A*b^2*c*d*e*f+2*A*b^2*d^2*e^2-B*a^2*c*d*f^2-B*a^2*d^2*e*f-B*a*b*c^2*f^2
+6*B*a*b*c*d*e*f-B*a*b*d^2*e^2-B*b^2*c^2*e*f-B*b^2*c*d*e^2+2*C*a^2*c^2*f^2
-2*C*a^2*c*d*e*f+2*C*a^2*d^2*e^2-2*C*a*b*c^2*e*f-2*C*a*b*c*d*e^2+2*C*b^2*c
^2*e^2)/(a^4*c^2*d^2*f^4-2*a^4*c*d^3*e*f^3+a^4*d^4*e^2*f^2-2*a^3*b*c^3*d*f
^4+2*a^3*b*c^2*d^2*e*f^3+2*a^3*b*c*d^3*e^2*f^2-2*a^3*b*d^4*e^3*f+a^2*b^2*c
^4*f^4+2*a^2*b^2*c^3*d*e*f^3-6*a^2*b^2*c^2*d^2*e^2*f^2+2*a^2*b^2*c*d^3*e^3
*f+a^2*b^2*d^4*e^4-2*a*b^3*c^4*e*f^3+2*a*b^3*c^3*d*e^2*f^2+2*a*b^3*c^2*d^2
*e^3*f-2*a*b^3*c*d^3*e^4+b^4*c^4*e^2*f^2-2*b^4*c^3*d*e^3*f+b^4*c^2*d^2*e^4
)*x^2+(2*A*a^3*d^3*f^3-A*a^2*b*c*d^2*f^3-A*a^2*b*d^3*e*f^2-A*a*b^2*c^2*d*f
^3-A*a*b^2*d^3*e^2*f+2*A*b^3*c^3*f^3-A*b^3*c^2*d*e*f^2-A*b^3*c*d^2*e^2*f+2
*A*b^3*d^3*e^3-B*a^3*c*d^2*f^3-B*a^3*d^3*e*f^2+2*B*a^2*b*c*d^2*e*f^2-B*a*b
^2*c^3*f^3+2*B*a*b^2*c^2*d*e*f^2+2*B*a*b^2*c*d^2*e^2*f-B*a*b^2*d^3*e^3-B*b
^3*c^3*e*f^2-B*b^3*c*d^2*e^3+C*a^3*c^2*d*f^3+C*a^3*d^3*e^2*f+C*a^2*b*c^3*f
^3-2*C*a^2*b*c^2*d*e*f^2-2*C*a^2*b*c*d^2*e^2*f+C*a^2*b*d^3*e^3-2*C*a*b^2*c
^2*d*e^2*f+C*b^3*c^3*e^2*f+C*b^3*c^2*d*e^3)/b/d/f/(a^4*c^2*d^2*f^4-2*a^4*c
*d^3*e*f^3+a^4*d^4*e^2*f^2-2*a^3*b*c^3*d*f^4+2*a^3*b*c^2*d^2*e*f^3+2*a^3*b
*c*d^3*e^2*f^2-2*a^3*b*d^4*e^3*f+a^2*b^2*c^4*f^4+2*a^2*b^2*c^3*d*e*f^3-6*a
^2*b^2*c^2*d^2*e^2*f^2+2*a^2*b^2*c*d^3*e^3*f+a^2*b^2*d^4*e^4-2*a*b^3*c^4*e
*f^3+2*a*b^3*c^3*d*e^2*f^2+2*a*b^3*c^2*d^2*e^3*f-2*a*b^3*c*d^3*e^4+b^4*...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5532 vs. 2(1110) = 2220.

Time = 0.37 (sec) , antiderivative size = 5532, normalized size of antiderivative = 4.81

$$\int \frac{A + Bx + Cx^2}{(ace + (bce + ade + acf)x + (bde + bcf + adf)x^2 + bdfx^3)^{3/2}} \, dx = \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x
^2+b*d*f*x^3)^(3/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2}{\left(ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3\right)^{3/2}}\,dx=\int \frac{A+Bx+Cx^2}{\left((a+bx)\,(c+dx)\,(e+fx)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((C*x**2+B*x+A)/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*
x**2+b*d*f*x**3)**(3/2),x)
```

output
```
Integral((A + B*x + C*x**2)/((a + b*x)*(c + d*x)*(e + f*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{\left(ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3\right)^{3/2}}\,dx=\int \frac{Cx^2+Bx}{\left(bdfx^3+ace+(bde+bcf+adf\right.}$$

input
```
integrate((C*x^2+B*x+A)/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x
^2+b*d*f*x^3)^(3/2),x, algorithm="maxima")
```

output
```
integrate((C*x^2 + B*x + A)/(b*d*f*x^3 + a*c*e + (b*d*e + b*c*f + a*d*f)*x
^2 + (b*c*e + a*d*e + a*c*f)*x)^(3/2), x)
```

Giac [F]

$$\int \frac{A+Bx+Cx^2}{(ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3)^{3/2}}\,dx = \int \frac{Cx^2+Bx}{(bdfx^3+ace+(bde+bcf+adf}$$

input
```
integrate((C*x^2+B*x+A)/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x
^2+b*d*f*x^3)^(3/2),x, algorithm="giac")
```

output
```
integrate((C*x^2 + B*x + A)/(b*d*f*x^3 + a*c*e + (b*d*e + b*c*f + a*d*f)*x
^2 + (b*c*e + a*d*e + a*c*f)*x)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3)^{3/2}}\,dx = \int \frac{C\,x^2}{(b\,d\,f\,x^3+(a\,d\,f+b\,c\,f+b\,d\,e)}$$

input
```
int((A + B*x + C*x^2)/(x^2*(a*d*f + b*c*f + b*d*e) + x*(a*c*f + a*d*e + b*
c*e) + a*c*e + b*d*f*x^3)^(3/2),x)
```

output
```
int((A + B*x + C*x^2)/(x^2*(a*d*f + b*c*f + b*d*e) + x*(a*c*f + a*d*e + b*
c*e) + a*c*e + b*d*f*x^3)^(3/2), x)
```

Reduce [F]

$$\int \frac{A+Bx+Cx^2}{(ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3)^{3/2}}\,dx = \int \frac{C\,x^2+B}{(ace+(acf+ade+bce)\,x+(a}$$

input
```
int((C*x^2+B*x+A)/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d
*f*x^3)^(3/2),x)
```

output

```
int((C*x^2+B*x+A)/(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d
*f*x^3)^(3/2),x)
```

3.118 $\int (A+Bx+Cx^2)(70+67x-53x^2+6x^3)^{3/2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 498

$$\int (A+Bx+Cx^2)\left(70+67x-53x^2+6x^3\right)^{3/2}\,dx =$$

$$-\frac{(5429970A-13817781B-101582932C)\left(70+67x-53x^2+6x^3\right)^{3/2}}{1990170(5-2x)(7-x)}$$

$$+\frac{(1687014A+186462072B+1091774983C)\left(70+67x-53x^2+6x^3\right)^{3/2}}{673596(5-2x)(7-x)(2+3x)}$$

$$-\frac{1}{351}(9B+53C)(5-2x)(2+3x)\left(70+67x-53x^2+6x^3\right)^{3/2}+\frac{(51246A+36513B+24272C)(2+3x)(70+}{104247(7-x)}$$

output
```
-1/1990170*(5429970*A-13817781*B-101582932*C)*(6*x^3-53*x^2+67*x+70)^(3/2)
/(5-2*x)/(7-x)+1/673596*(1687014*A+186462072*B+1091774983*C)*(6*x^3-53*x^2
+67*x+70)^(3/2)/(5-2*x)/(7-x)/(2+3*x)-1/351*(9*B+53*C)*(5-2*x)*(2+3*x)*(6*
x^3-53*x^2+67*x+70)^(3/2)+(51246*A+36513*B+24272*C)*(2+3*x)*(6*x^3-53*x^2+
67*x+70)^(3/2)/(729729-104247*x)+1/2189187*(3970512*A+4397985*B+11120117*C
)*(2+3*x)*(6*x^3-53*x^2+67*x+70)^(3/2)/(5-2*x)/(7-x)+(702*A-342*B-4627*C)*
(5-2*x)*(2+3*x)*(6*x^3-53*x^2+67*x+70)^(3/2)/(162162-23166*x)+1/45*C*(5-2*
x)*(7-x)*(2+3*x)*(6*x^3-53*x^2+67*x+70)^(3/2)-1/131351220*19^(1/2)*(854332
31520*A+588536034399*B+3147821840803*C)*(6*x^3-53*x^2+67*x+70)^(3/2)*Ellip
ticE(1/23*(2+3*x)^(1/2)*23^(1/2),1/19*874^(1/2))/(5-2*x)^(3/2)/(7-x)^(3/2)
/(2+3*x)^(3/2)-1/4864860*19^(1/2)*(3070783170*A+11473156917*B+56134008934*
C)*(6*x^3-53*x^2+67*x+70)^(3/2)*EllipticF(1/2*38^(1/2)/(2+3*x)^(1/2),1/19*
874^(1/2))/(-7+x)^(3/2)/(-5+2*x)^(3/2)/(2+3*x)^(3/2)
```

Mathematica [A] (verified)

Time = 10.60 (sec) , antiderivative size = 252, normalized size of antiderivative = 0.51

$$\int (A+Bx+Cx^2)\left(70+67x-53x^2+6x^3\right)^{3/2}\,dx =$$

$$-\frac{\sqrt{5-2x}\Big(6\sqrt{5-2x}(-14-19x+3x^2)\left(24570A(30587+81630x+47538x^2-22896x^3+1944x^4)+18B\right.}{}$$

input
```
Integrate[(A + B*x + C*x^2)*(70 + 67*x - 53*x^2 + 6*x^3)^(3/2),x]
```

output
```
-1/262702440*(Sqrt[5 - 2*x]*(6*Sqrt[5 - 2*x]*(-14 - 19*x + 3*x^2)*(24570*A
*(30587 + 81630*x + 47538*x^2 - 22896*x^3 + 1944*x^4) + 18*B*(696448058 +
55894437*x + 75120120*x^2 + 47096910*x^3 - 25242840*x^4 + 2245320*x^5) + C
*(75793722863 + 7585816032*x + 763363998*x^2 + 993475476*x^3 + 661818276*x
^4 - 380806272*x^5 + 35026992*x^6)) + 2*Sqrt[46]*(85433231520*A + 58853603
4399*B + 3147821840803*C)*Sqrt[7 - x]*Sqrt[2 + 3*x]*EllipticE[ArcSin[Sqrt[
2/19]*Sqrt[2 + 3*x]], 19/46] - 27*Sqrt[46]*(6251222250*A + 35066348514*B +
 183233391763*C)*Sqrt[7 - x]*Sqrt[2 + 3*x]*EllipticF[ArcSin[Sqrt[2/19]*Sqr
t[2 + 3*x]], 19/46]))/Sqrt[70 + 67*x - 53*x^2 + 6*x^3]
```

Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 16.15 (sec) , antiderivative size = 2634, normalized size of antiderivative = 5.29, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {2526, 2490, 2486, 27, 1236, 27, 1236, 27, 1231, 27, 1231, 27, 1269, 1172, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(6x^3-53x^2+67x+70\right)^{3/2} \left(A+Bx+Cx^2\right) dx$$

$$\Big\downarrow \; 2526$$

$$\frac{1}{18}\int (18A-67C+2(9B+53C)x) \left(6x^3-53x^2+67x+70\right)^{3/2} dx+\frac{1}{45}C\left(6x^3-53x^2+67x+70\right)^{5/2}$$

$$\Big\downarrow \; 2490$$

$$\frac{1}{18}\int \left(\frac{1}{18}(18(18A-67C)+106(9B+53C))+2(9B+53C)\left(x-\frac{53}{18}\right)\right)\left(6\left(x-\frac{53}{18}\right)^3-\frac{1603}{18}\left(x-\frac{53}{18}\right)-\right.$$
$$\frac{1}{45}C\left(6x^3-53x^2+67x+70\right)^{5/2}$$

$$\Big\downarrow \; 2486$$

$$\frac{1}{45}C\left(6x^3-53x^2+67x+70\right)^{5/2}+$$
$$\frac{\left(2916\left(x-\frac{53}{18}\right)^3-43281\left(x-\frac{53}{18}\right)-18980\right)^{3/2}\int \frac{\left(18\left(x-\frac{53}{18}\right)-\frac{1603+\left(18980+35397i\sqrt{3}\right)^{2/3}}{\sqrt[3]{18980+35397i\sqrt{3}}}\right)^{3/2}(162A+477B+2206C+18(9B+5}{}}{972\sqrt{2}\left(18\left(x-\frac{53}{18}\right)-\frac{1603+\left(18980+35397i\sqrt{3}\right)^{2/3}}{\sqrt[3]{18980+35397i\sqrt{3}}}\right)^{3/2}\left(324\left(x-\frac{53}{18}\right)^2+\frac{18\left(1603+\left(189\right.\right.}{\sqrt[3]{189}}\right.}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{45}C\left(6x^3-53x^2+67x+70\right)^{5/2}+$$

$$\frac{\left(2916\left(x-\frac{53}{18}\right)^3-43281\left(x-\frac{53}{18}\right)-18980\right)^{3/2}\int\left(18\left(x-\frac{53}{18}\right)-\frac{1603+\left(18980+35397i\sqrt{3}\right)^{2/3}}{\sqrt[3]{18980+35397i\sqrt{3}}}\right)^{3/2}\left(162A+477B+\right.}{708588\sqrt{6}\left(18\left(x-\frac{53}{18}\right)-\frac{1603+\left(18980+35397i\sqrt{3}\right)^{2/3}}{\sqrt[3]{18980+35397i\sqrt{3}}}\right)^{3/2}\left(324\right.}$$

$\downarrow$ 1236

$$\frac{1}{45}C\left(6x^3-53x^2+67x+70\right)^{5/2}+$$

$$\left(2916\left(x-\frac{53}{18}\right)^3-43281\left(x-\frac{53}{18}\right)-18980\right)^{3/2}\left(\frac{\int 162\sqrt{18\left(x-\frac{53}{18}\right)-\frac{1603+\left(18980+35397i\sqrt{3}\right)^{2/3}}{\sqrt[3]{18980+35397i\sqrt{3}}}}\left(\frac{5\left(1603+\left(18980+35397i\sqrt{3}\right)^{2/3}\right)}{}\right.}{}\right.$$

$\downarrow$ 27

$$\frac{1}{45}C\left(6x^3-53x^2+67x+70\right)^{5/2}+$$

$$\left(2916\left(x-\frac{53}{18}\right)^3-43281\left(x-\frac{53}{18}\right)-18980\right)^{3/2}\left(\frac{1}{13}\int\sqrt{18\left(x-\frac{53}{18}\right)-\frac{1603+\left(18980+35397i\sqrt{3}\right)^{2/3}}{\sqrt[3]{18980+35397i\sqrt{3}}}}\left(\frac{5\left(1603+\left(18980+3\right.\right.}{}\right.\right.$$

$\downarrow$ 1236

$$\frac{1}{45}C\left(6x^3-53x^2+67x+70\right)^{5/2}+$$

$$\left(2916\left(x-\tfrac{53}{18}\right)^3-43281\left(x-\tfrac{53}{18}\right)-18980\right)^{3/2}\left(\tfrac{1}{117}(9B+53C)\left(18\left(x-\tfrac{53}{18}\right)-\frac{1603+\left(18980+35397i\sqrt{3}\right)^{2/3}}{\sqrt[3]{18980+35397i\sqrt{3}}}\right)^{3/2}\left(\right.\right.$$

$\downarrow$ 27

$$\frac{1}{45}C\left(6x^3-53x^2+67x+70\right)^{5/2}+$$

$$\left(2916\left(x-\tfrac{53}{18}\right)^3-43281\left(x-\tfrac{53}{18}\right)-18980\right)^{3/2}\left(\tfrac{1}{117}(9B+53C)\left(18\left(x-\tfrac{53}{18}\right)-\frac{1603+\left(18980+35397i\sqrt{3}\right)^{2/3}}{\sqrt[3]{18980+35397i\sqrt{3}}}\right)^{3/2}\left(\right.\right.$$

$\downarrow$ 1231

$$\frac{1}{45}C\left(6x^3-53x^2+67x+70\right)^{5/2}+$$

$$\frac{\left(2916\left(x-\frac{53}{18}\right)^3-43281\left(x-\frac{53}{18}\right)-18980\right)^{3/2}\left(\frac{1}{117}(9B+53C)\left(18\left(x-\frac{53}{18}\right)-\frac{1603+\left(18980+35397i\sqrt{3}\right)^{2/3}}{\sqrt[3]{18980+35397i\sqrt{3}}}\right)^{3/2}\left(\right.\right.}{}$$

$\downarrow$ 27

$$\frac{1}{45}C\left(6x^3-53x^2+67x+70\right)^{5/2}+$$

$$\frac{\left(2916\left(x-\frac{53}{18}\right)^3-43281\left(x-\frac{53}{18}\right)-18980\right)^{3/2}\left(\frac{1}{117}(9B+53C)\left(18\left(x-\frac{53}{18}\right)-\frac{1603+\left(18980+35397i\sqrt{3}\right)^{2/3}}{\sqrt[3]{18980+35397i\sqrt{3}}}\right)^{3/2}\left(\right.\right.}{}$$

$\downarrow$ 1231

$$\frac{1}{45}C\left(6x^3-53x^2+67x+70\right)^{5/2}+$$
$$\left(2916\left(x-\tfrac{53}{18}\right)^3-43281\left(x-\tfrac{53}{18}\right)-18980\right)^{3/2}\left(\tfrac{1}{117}(9B+53C)\left(18\left(x-\tfrac{53}{18}\right)-\frac{1603+\left(18980+35397i\sqrt{3}\right)^{2/3}}{\sqrt[3]{18980+35397i\sqrt{3}}}\right)^{3/2}\left(\right.\right.$$

$\downarrow$ 27

$$\frac{1}{45}C\left(6x^3-53x^2+67x+70\right)^{5/2}+$$
$$\left(2916\left(x-\tfrac{53}{18}\right)^3-43281\left(x-\tfrac{53}{18}\right)-18980\right)^{3/2}\left(\tfrac{1}{117}(9B+53C)\left(18\left(x-\tfrac{53}{18}\right)-\frac{1603+\left(18980+35397i\sqrt{3}\right)^{2/3}}{\sqrt[3]{18980+35397i\sqrt{3}}}\right)^{3/2}\left(\right.\right.$$

$\downarrow$ 1269

$$\frac{1}{45}C\left(6x^3-53x^2+67x+70\right)^{5/2}+$$
$$\left(2916\left(x-\tfrac{53}{18}\right)^3-43281\left(x-\tfrac{53}{18}\right)-18980\right)^{3/2}\left(\tfrac{1}{117}(9B+53C)\left(18\left(x-\tfrac{53}{18}\right)-\frac{1603+\left(18980+35397i\sqrt{3}\right)^{2/3}}{\sqrt[3]{18980+35397i\sqrt{3}}}\right)^{3/2}\left(\right.\right.$$

$\downarrow$ 1172

$$\frac{1}{45}C\left(6x^3-53x^2+67x+70\right)^{5/2}+$$

$$\frac{\left(\frac{1}{117}(9B+53C)\left(18\left(x-\frac{53}{18}\right)-\frac{1603+\left(18980+35397i\sqrt{3}\right)^{2/3}}{\sqrt[3]{18980+35397i\sqrt{3}}}\right)^{3/2}\left(\right.\right.}{\left(2916\left(x-\frac{53}{18}\right)^3-43281\left(x-\frac{53}{18}\right)-18980\right)^{3/2}}$$

↓ 321

$$\frac{1}{45}C\left(6x^3-53x^2+67x+70\right)^{5/2}+$$

$$\left(2916\left(x-\tfrac{53}{18}\right)^3-43281\left(x-\tfrac{53}{18}\right)-18980\right)^{3/2}\left(\tfrac{1}{117}(9B+53C)\left(18\left(x-\tfrac{53}{18}\right)-\frac{1603+\left(18980+35397i\sqrt{3}\right)^{2/3}}{\sqrt[3]{18980+35397i\sqrt{3}}}\right)^{3/2}\left(\right.\right.$$

$\downarrow$ 327

$$\frac{1}{45}C\left(6x^3-53x^2+67x+70\right)^{5/2}+$$

$$\left(2916\left(x-\tfrac{53}{18}\right)^3-43281\left(x-\tfrac{53}{18}\right)-18980\right)^{3/2}\left(\tfrac{1}{117}(9B+53C)\left(18\left(x-\tfrac{53}{18}\right)-\frac{1603+\left(18980+35397i\sqrt{3}\right)^{2/3}}{\sqrt[3]{18980+35397i\sqrt{3}}}\right)^{3/2}\left(\right.\right.$$

input
```
Int[(A + B*x + C*x^2)*(70 + 67*x - 53*x^2 + 6*x^3)^(3/2),x]
```

output
```
(C*(70 + 67*x - 53*x^2 + 6*x^3)^(5/2))/45 + ((-18980 - 43281*(-53/18 + x)
+ 2916*(-53/18 + x)^3)^(3/2)*(((9*B + 53*C)*(-((1603 + (18980 + (35397*I)*
Sqrt[3])^(2/3))/(18980 + (35397*I)*Sqrt[3])^(1/3)) + 18*(-53/18 + x))^(3/2
)*(-1603 + 2569609/(18980 + (35397*I)*Sqrt[3])^(2/3) + (18980 + (35397*I)*
Sqrt[3])^(2/3) + (18*(1603 + (18980 + (35397*I)*Sqrt[3])^(2/3))*(-53/18 +
x))/(18980 + (35397*I)*Sqrt[3])^(1/3) + 324*(-53/18 + x)^2)^(5/2))/117 + (
((2106*A + (6201 - 158697/(18980 + (35397*I)*Sqrt[3])^(1/3) - 99*(18980 +
(35397*I)*Sqrt[3])^(1/3))*B + (28678 - 934549/(18980 + (35397*I)*Sqrt[3])^
(1/3) - 583*(18980 + (35397*I)*Sqrt[3])^(1/3))*C)*Sqrt[-((1603 + (18980 +
(35397*I)*Sqrt[3])^(2/3))/(18980 + (35397*I)*Sqrt[3])^(1/3)) + 18*(-53/18
+ x)]*(-1603 + 2569609/(18980 + (35397*I)*Sqrt[3])^(2/3) + (18980 + (35397
*I)*Sqrt[3])^(2/3) + (18*(1603 + (18980 + (35397*I)*Sqrt[3])^(2/3))*(-53/1
8 + x))/(18980 + (35397*I)*Sqrt[3])^(1/3) + 324*(-53/18 + x)^2)^(5/2))/99
+ (27*(-1/567*(Sqrt[-((1603 + (18980 + (35397*I)*Sqrt[3])^(2/3))/(18980 +
(35397*I)*Sqrt[3])^(1/3)) + 18*(-53/18 + x)]*((4914*(2519*(18980*I - 35397
*Sqrt[3]) + (2569609*I)*(18980 + (35397*I)*Sqrt[3])^(1/3) + (18980*I - 353
97*Sqrt[3])*(18980 + (35397*I)*Sqrt[3])^(2/3))*A + 9*(3353779*(18980*I - 3
5397*Sqrt[3]) + 1918791*(18980 + (35397*I)*Sqrt[3])^(1/3)*(1339*I + 1518*S
qrt[3]) - 1197*(18980 + (35397*I)*Sqrt[3])^(2/3)*(84706*I + 47541*Sqrt[3])
)*B + 2*(65837629*(18980*I - 35397*Sqrt[3]) + 33663*(18980 + (35397*I)*...
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 1172
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Simp[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m))   Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^
2 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[m^2, 1/4]
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1236
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1
)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m
*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2486
```
Int[((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (d_.)*(x_)^3)^(p_), x_S
ymbol] :> With[{r = Rt[-9*a*d^2 + Sqrt[3]*d*Sqrt[4*b^3*d + 27*a^2*d^2], 3]}
, Simp[(a + b*x + d*x^3)^p/(Simp[18^(1/3)*b*(d/(3*r)) - r/18^(1/3) + d*x, x
]^p*Simp[b*(d/3) + 12^(1/3)*b^2*(d^2/(3*r^2)) + r^2/(3*12^(1/3)) - d*(2^(1/
3)*b*(d/(3^(1/3)*r)) - r/18^(1/3))*x + d^2*x^2, x]^p)   Int[(e + f*x)^m*Sim
p[18^(1/3)*b*(d/(3*r)) - r/18^(1/3) + d*x, x]^p*Simp[b*(d/3) + 12^(1/3)*b^2
*(d^2/(3*r^2)) + r^2/(3*12^(1/3)) - d*(2^(1/3)*b*(d/(3^(1/3)*r)) - r/18^(1/
3))*x + d^2*x^2, x]^p, x], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && NeQ[4*
b^3 + 27*a^2*d, 0] &&  !IntegerQ[p]
```

rule 2490
```
Int[(P3_)^(p_.)*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] :> With[{a = Coeff[P3
, x, 0], b = Coeff[P3, x, 1], c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Su
bst[Int[((3*d*e - c*f)/(3*d) + f*x)^m*Simp[(2*c^3 - 9*b*c*d + 27*a*d^2)/(27
*d^2) - (c^2 - 3*b*d)*(x/(3*d)) + d*x^3, x]^p, x], x, x + c/(3*d)] /; NeQ[c
, 0]] /; FreeQ[{e, f, m, p}, x] && PolyQ[P3, x, 3]
```

rule 2526
```
Int[(Pm_)*(Qn_)^(p_), x_Symbol] :> With[{m = Expon[Pm, x], n = Expon[Qn, x]
}, Simp[Coeff[Pm, x, m]*(Qn^(p + 1)/(n*(p + 1)*Coeff[Qn, x, n])), x] + Simp
[1/(n*Coeff[Qn, x, n])   Int[ExpandToSum[n*Coeff[Qn, x, n]*Pm - Coeff[Pm, x
, m]*D[Qn, x], x]*Qn^p, x], x] /; EqQ[m, n - 1]] /; FreeQ[p, x] && PolyQ[Pm
, x] && PolyQ[Qn, x] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 1.12 (sec) , antiderivative size = 353, normalized size of antiderivative = 0.71

method	result
elliptic	$\frac{4C\,x^6\sqrt{6x^3-53x^2+67x+70}}{5}+\left(\frac{12B}{13}-\frac{1696C}{195}\right)x^5\sqrt{6x^3-53x^2+67x+70}+\left(-\frac{1484B}{143}+\frac{12A}{11}+\frac{32423C}{2145}\right)x$
risch	$\frac{(35026992C\,x^6+40415760B\,x^5-380806272x^5C+47764080x^4A-454371120x^4B+661818276C\,x^4-562554720x^3A+847744380B\,x}{}$
default	$A\left(\frac{12x^4\sqrt{6x^3-53x^2+67x+70}}{11}-\frac{424x^3\sqrt{6x^3-53x^2+67x+70}}{33}+\frac{2641x^2\sqrt{6x^3-53x^2+67x+70}}{99}+\frac{4535x\sqrt{6x^3-53x^2+67x+7}}{99}\right.$

input
```
int((C*x^2+B*x+A)*(6*x^3-53*x^2+67*x+70)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
4/5*C*x^6*(6*x^3-53*x^2+67*x+70)^(1/2)+(12/13*B-1696/195*C)*x^5*(6*x^3-53*
x^2+67*x+70)^(1/2)+(-1484/143*B+12/11*A+32423/2145*C)*x^4*(6*x^3-53*x^2+67
*x+70)^(1/2)+(24919/1287*B-424/33*A+1314121/57915*C)*x^3*(6*x^3-53*x^2+67*
x+70)^(1/2)+(834668/27027*B+2641/99*A+42409111/2432430*C)*x^2*(6*x^3-53*x^
2+67*x+70)^(1/2)+(6210493/270270*B+4535/99*A+210717112/1216215*C)*x*(6*x^3
-53*x^2+67*x+70)^(1/2)+(348224029/1216215*B+30587/1782*A+75793722863/43783
740*C)*(6*x^3-53*x^2+67*x+70)^(1/2)+1/1311*(3986071/3564*A-27243620533/243
2430*B-6140193676301/87567480*C)*(76+114*x)^(1/2)*(483-69*x)^(1/2)*(285-11
4*x)^(1/2)/(6*x^3-53*x^2+67*x+70)^(1/2)*EllipticF(1/19*(76+114*x)^(1/2),1/
46*874^(1/2))+1/1311*(1738568/891*A+65392892711/4864860*B+3147821840803/43
783740*C)*(76+114*x)^(1/2)*(483-69*x)^(1/2)*(285-114*x)^(1/2)/(6*x^3-53*x^
2+67*x+70)^(1/2)*(-23/3*EllipticE(1/19*(76+114*x)^(1/2),1/46*874^(1/2))+7*
EllipticF(1/19*(76+114*x)^(1/2),1/46*874^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.30

$$\int \left(A+Bx+Cx^2\right) \left(70+67x-53x^2+6x^3\right)^{3/2} dx = \frac{1}{5196312}\sqrt{6}(11888793738\,A+49154894001\,B+245214976870\,C)\text{weierstrassPInverse}\left(\frac{16}{2} \right. - \frac{53}{18}\Big) - \frac{1}{131351220}\sqrt{6}(85433231520\,A+588536034399\,B+3147821840803\,C)\text{weierstrassZeta}\left(\frac{1603}{27},\frac{18980}{729},\text{w}\right. + \frac{1}{43783740}\left(35026992\,Cx^6+898128\,(45\,B-424\,C)x^5+20412\,(2340\,A-22260\,B+32423\,C)x^4-756\,(7\right.$$

input
```
integrate((C*x^2+B*x+A)*(6*x^3-53*x^2+67*x+70)^(3/2),x, algorithm="fricas"
)
```

output
```
1/5196312*sqrt(6)*(11888793738*A + 49154894001*B + 245214976870*C)*weierst
rassPInverse(1603/27, 18980/729, x - 53/18) - 1/131351220*sqrt(6)*(8543323
1520*A + 588536034399*B + 3147821840803*C)*weierstrassZeta(1603/27, 18980/
729, weierstrassPInverse(1603/27, 18980/729, x - 53/18)) + 1/43783740*(350
26992*C*x^6 + 898128*(45*B - 424*C)*x^5 + 20412*(2340*A - 22260*B + 32423*
C)*x^4 - 756*(744120*A - 1121355*B - 1314121*C)*x^3 + 18*(64889370*A + 751
20120*B + 42409111*C)*x^2 + 18*(111424950*A + 55894437*B + 421434224*C)*x
+ 751522590*A + 12536065044*B + 75793722863*C)*sqrt(6*x^3 - 53*x^2 + 67*x
+ 70)
```

Sympy [F]

$$\int (A+Bx+Cx^2)\left(70+67x-53x^2+6x^3\right)^{3/2}\,dx=\int ((x-7)(2x-5)(3x+2))^{\frac{3}{2}}\left(A+Bx+Cx^2\right)\,dx$$

input
```
integrate((C*x**2+B*x+A)*(6*x**3-53*x**2+67*x+70)**(3/2),x)
```

output
```
Integral(((x - 7)*(2*x - 5)*(3*x + 2))**(3/2)*(A + B*x + C*x**2), x)
```

Maxima [F]

$$\int (A+Bx+Cx^2)\left(70+67x-53x^2+6x^3\right)^{3/2}\,dx=\int \left(Cx^2+Bx+A\right)\left(6\,x^3-53\,x^2+67\,x+70\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((C*x^2+B*x+A)*(6*x^3-53*x^2+67*x+70)^(3/2),x, algorithm="maxima"
)
```

output
```
integrate((C*x^2 + B*x + A)*(6*x^3 - 53*x^2 + 67*x + 70)^(3/2), x)
```

Giac [F]

$$\int (A+Bx+Cx^2) \left(70+67x-53x^2+6x^3\right)^{3/2} dx = \int (Cx^2+Bx+A)\left(6\,x^3-53\,x^2+67\,x+70\right)^{\frac{3}{2}} dx$$

input
```
integrate((C*x^2+B*x+A)*(6*x^3-53*x^2+67*x+70)^(3/2),x, algorithm="giac")
```

output
```
integrate((C*x^2 + B*x + A)*(6*x^3 - 53*x^2 + 67*x + 70)^(3/2), x)
```

Mupad [B] (verification not implemented)

Time = 13.53 (sec) , antiderivative size = 913, normalized size of antiderivative = 1.83

$$\int (A+Bx+Cx^2) \left(70+67x-53x^2+6x^3\right)^{3/2} dx = \text{Too large to display}$$

input
```
int((A + B*x + C*x^2)*(67*x - 53*x^2 + 6*x^3 + 70)^(3/2),x)
```

output

```
(1070545*A)/(891*(67*x - 53*x^2 + 6*x^3 + 70)^(1/2)) + (696448058*B)/(3474
9*(67*x - 53*x^2 + 6*x^3 + 70)^(1/2)) + (75793722863*C)/(625482*(67*x - 53
*x^2 + 6*x^3 + 70)^(1/2)) + (7763429*A*x)/(1782*(67*x - 53*x^2 + 6*x^3 + 7
0)^(1/2)) + (25287315238*B*x)/(1216215*(67*x - 53*x^2 + 6*x^3 + 70)^(1/2))
 + (5609186554061*C*x)/(43783740*(67*x - 53*x^2 + 6*x^3 + 70)^(1/2)) + (71
75759*A*x^2)/(1782*(67*x - 53*x^2 + 6*x^3 + 70)^(1/2)) - (426757*A*x^3)/(2
97*(67*x - 53*x^2 + 6*x^3 + 70)^(1/2)) - (190427*A*x^4)/(99*(67*x - 53*x^2
 + 6*x^3 + 70)^(1/2)) + (30166*A*x^5)/(33*(67*x - 53*x^2 + 6*x^3 + 70)^(1/
2)) - (1484*A*x^6)/(11*(67*x - 53*x^2 + 6*x^3 + 70)^(1/2)) + (72*A*x^7)/(1
1*(67*x - 53*x^2 + 6*x^3 + 70)^(1/2)) - (5581682279*B*x^2)/(486486*(67*x -
 53*x^2 + 6*x^3 + 70)^(1/2)) + (3182038309*B*x^3)/(810810*(67*x - 53*x^2 +
 6*x^3 + 70)^(1/2)) - (125416976*B*x^4)/(135135*(67*x - 53*x^2 + 6*x^3 + 7
0)^(1/2)) - (13257457*B*x^5)/(9009*(67*x - 53*x^2 + 6*x^3 + 70)^(1/2)) + (
312326*B*x^6)/(429*(67*x - 53*x^2 + 6*x^3 + 70)^(1/2)) - (15900*B*x^7)/(14
3*(67*x - 53*x^2 + 6*x^3 + 70)^(1/2)) + (72*B*x^8)/(13*(67*x - 53*x^2 + 6*
x^3 + 70)^(1/2)) - (691076431547*C*x^2)/(8756748*(67*x - 53*x^2 + 6*x^3 +
70)^(1/2)) + (14450229889*C*x^3)/(3648645*(67*x - 53*x^2 + 6*x^3 + 70)^(1/
2)) + (1310519339*C*x^4)/(486486*(67*x - 53*x^2 + 6*x^3 + 70)^(1/2)) - (28
1376211*C*x^5)/(405405*(67*x - 53*x^2 + 6*x^3 + 70)^(1/2)) - (23006017*C*x
^6)/(19305*(67*x - 53*x^2 + 6*x^3 + 70)^(1/2)) + (1298278*C*x^7)/(2145*...
```

Reduce [F]

$$\int \left(A+Bx+Cx^2\right)\left(70+67x-53x^2+6x^3\right)^{3/2}\,dx = \frac{12\sqrt{6x^3-53x^2+67x+70}\,a\,x^4}{11} - \frac{424\sqrt{6x^3-53x^2+67x+70}\,a\,x^3}{33} + \frac{2641\sqrt{6x^3-53x^2+67x+70}\,a\,x^2}{99} + \frac{4535\sqrt{6x^3-53x^2+67x+70}\,ax}{99} - \frac{206225\sqrt{6x^3-53x^2+67x+70}\,a}{10494} + \frac{12\sqrt{6x^3-53x^2+67x+70}\,b\,x^5}{13} - \frac{1484\sqrt{6x^3-53x^2+67x+70}\,b\,x^4}{143} + \frac{24919\sqrt{6x^3-53x^2+67x+70}\,b\,x^3}{1287} + \frac{834668\sqrt{6x^3-53x^2+67x+70}\,b\,x^2}{27027} + \frac{6210493\sqrt{6x^3-53x^2+67x+70}\,bx}{270270} + \frac{936733493\sqrt{6x^3-53x^2+67x+70}\,b}{28648620} + \frac{4\sqrt{6x^3-53x^2+67x+70}\,c\,x^6}{5} - \frac{1696\sqrt{6x^3-53x^2+67x+70}\,c\,x^5}{195} + \frac{32423\sqrt{6x^3-53x^2+67x+70}\,c\,x^4}{2145} + \frac{1314121\sqrt{6x^3-53x^2+67x+70}\,c\,x^3}{57915} + \frac{42409111\sqrt{6x^3-53x^2+67x+70}\,c\,x^2}{2432430} + \frac{210717112\sqrt{6x^3-53x^2+67x+70}\,cx}{1216215} + \frac{24145707526\sqrt{6x^3-53x^2+67x+70}\,c}{64459395} + \frac{49358875\left(\int \frac{\sqrt{6x^3-53x^2+67x+70}}{6x^3-53x^2+67x+70}dx\right)a}{20988} - \frac{154924860151\left(\int \frac{\sqrt{6x^3-53x^2+67x+70}}{6x^3-53x^2+67x+70}dx\right)b}{57297240} - \frac{1590641687641\left(\int \frac{\sqrt{6x^3-53x^2+67x+70}}{6x^3-53x^2+67x+70}dx\right)c}{64459395} + \frac{1738568\left(\int \frac{\sqrt{6x^3-53x^2+67x+70}\,x^2}{6x^3-53x^2+67x+70}dx\right)a}{5247} + \frac{65392892711\left(\int \frac{\sqrt{6x^3-53x^2+67x+70}\,x^2}{6x^3-53x^2+67x+70}dx\right)b}{28648620} + \frac{3147821840803\left(\int \frac{\sqrt{6x^3-53x^2+67x+70}\,x^2}{6x^3-53x^2+67x+70}dx\right)c}{257837580}$$

input
```
int((C*x^2+B*x+A)*(6*x^3-53*x^2+67*x+70)^(3/2),x)
```

output
```
(562554720*sqrt(6*x**3 - 53*x**2 + 67*x + 70)*a*x**4 - 6625644480*sqrt(6*x
**3 - 53*x**2 + 67*x + 70)*a*x**3 + 13756546440*sqrt(6*x**3 - 53*x**2 + 67
*x + 70)*a*x**2 + 23622089400*sqrt(6*x**3 - 53*x**2 + 67*x + 70)*a*x - 101
33896500*sqrt(6*x**3 - 53*x**2 + 67*x + 70)*a + 476007840*sqrt(6*x**3 - 53
*x**2 + 67*x + 70)*b*x**5 - 5351482080*sqrt(6*x**3 - 53*x**2 + 67*x + 70)*
b*x**4 + 9984544920*sqrt(6*x**3 - 53*x**2 + 67*x + 70)*b*x**3 + 1592546544
0*sqrt(6*x**3 - 53*x**2 + 67*x + 70)*b*x**2 + 11849620644*sqrt(6*x**3 - 53
*x**2 + 67*x + 70)*b*x + 16861202874*sqrt(6*x**3 - 53*x**2 + 67*x + 70)*b
+ 412540128*sqrt(6*x**3 - 53*x**2 + 67*x + 70)*c*x**6 - 4485051648*sqrt(6*
x**3 - 53*x**2 + 67*x + 70)*c*x**5 + 7794748584*sqrt(6*x**3 - 53*x**2 + 67
*x + 70)*c*x**4 + 11700933384*sqrt(6*x**3 - 53*x**2 + 67*x + 70)*c*x**3 +
8990731532*sqrt(6*x**3 - 53*x**2 + 67*x + 70)*c*x**2 + 89344055488*sqrt(6*
x**3 - 53*x**2 + 67*x + 70)*c*x + 193165660208*sqrt(6*x**3 - 53*x**2 + 67*
x + 70)*c + 1212747558750*int(sqrt(6*x**3 - 53*x**2 + 67*x + 70)/(6*x**3 -
 53*x**2 + 67*x + 70),x)*a - 1394323741359*int(sqrt(6*x**3 - 53*x**2 + 67*
x + 70)/(6*x**3 - 53*x**2 + 67*x + 70),x)*b - 12725133501128*int(sqrt(6*x*
*3 - 53*x**2 + 67*x + 70)/(6*x**3 - 53*x**2 + 67*x + 70),x)*c + 1708664630
40*int((sqrt(6*x**3 - 53*x**2 + 67*x + 70)*x**2)/(6*x**3 - 53*x**2 + 67*x
+ 70),x)*a + 1177072068798*int((sqrt(6*x**3 - 53*x**2 + 67*x + 70)*x**2)/(
6*x**3 - 53*x**2 + 67*x + 70),x)*b + 6295643681606*int((sqrt(6*x**3 - 5...
```

3.119 $\int (A+Bx+Cx^2)\sqrt{70+67x-53x^2+6x^3}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 321

$$\begin{aligned}
&\int (A+Bx+Cx^2)\sqrt{70+67x-53x^2+6x^3}\,dx\\
&=-\frac{(4914A+11412B+48913C)\sqrt{70+67x-53x^2+6x^3}}{3402}\\
&\quad+\frac{(378A+18B-1301C)(2+3x)\sqrt{70+67x-53x^2+6x^3}}{2835}\\
&\quad-\frac{1}{189}(9B+53C)(5-2x)(2+3x)\sqrt{70+67x-53x^2+6x^3}\\
&\quad+\frac{1}{27}C(5-2x)(7-x)(2+3x)\sqrt{70+67x-53x^2+6x^3}\\
&\quad+\frac{\sqrt{19}(605934A+2068839B+9927742C)\sqrt{70+67x-53x^2+6x^3}\,E\left(\arcsin\left(\frac{\sqrt{2+3x}}{\sqrt{23}}\right)|\frac{46}{19}\right)}{51030\sqrt{5-2x}\sqrt{7-x}\sqrt{2+3x}}\\
&\quad+\frac{\sqrt{19}(1512A+28017B+159361C)\sqrt{70+67x-53x^2+6x^3}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{19}{2}}}{\sqrt{2+3x}}\right),\frac{46}{19}\right)}{1890\sqrt{-7+x}\sqrt{-5+2x}\sqrt{2+3x}}
\end{aligned}$$

output
```
-1/3402*(4914*A+11412*B+48913*C)*(6*x^3-53*x^2+67*x+70)^(1/2)+1/2835*(378*
A+18*B-1301*C)*(2+3*x)*(6*x^3-53*x^2+67*x+70)^(1/2)-1/189*(9*B+53*C)*(5-2*
x)*(2+3*x)*(6*x^3-53*x^2+67*x+70)^(1/2)+1/27*C*(5-2*x)*(7-x)*(2+3*x)*(6*x^
3-53*x^2+67*x+70)^(1/2)+1/51030*19^(1/2)*(605934*A+2068839*B+9927742*C)*(6
*x^3-53*x^2+67*x+70)^(1/2)*EllipticE(1/23*(2+3*x)^(1/2)*23^(1/2),1/19*874^
(1/2))/(5-2*x)^(1/2)/(7-x)^(1/2)/(2+3*x)^(1/2)+1/1890*19^(1/2)*(1512*A+280
17*B+159361*C)*(6*x^3-53*x^2+67*x+70)^(1/2)*EllipticF(1/2*38^(1/2)/(2+3*x)
^(1/2),1/19*874^(1/2))/(-7+x)^(1/2)/(-5+2*x)^(1/2)/(2+3*x)^(1/2)
```

Mathematica [A] (verified)

Time = 8.77 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.64

$$\int (A+Bx+Cx^2)\sqrt{70+67x-53x^2+6x^3}\,dx =$$

$$-\frac{\sqrt{5-2x}\Big(6\sqrt{5-2x}(-14-19x+3x^2)\left(378A(-53+18x)+18B(-3608-477x+270x^2)+C(-263\right.}{}$$

input
```
Integrate[(A + B*x + C*x^2)*Sqrt[70 + 67*x - 53*x^2 + 6*x^3],x]
```

output
```
-1/102060*(Sqrt[5 - 2*x]*(6*Sqrt[5 - 2*x]*(-14 - 19*x + 3*x^2)*(378*A*(-53
 + 18*x) + 18*B*(-3608 - 477*x + 270*x^2) + C*(-263777 - 33678*x - 4770*x^
2 + 3780*x^3)) - 2*Sqrt[46]*(605934*A + 2068839*B + 9927742*C)*Sqrt[7 - x]
*Sqrt[2 + 3*x]*EllipticE[ArcSin[Sqrt[2/19]*Sqrt[2 + 3*x]], 19/46] + 27*Sqr
t[46]*(27594*A + 113094*B + 563287*C)*Sqrt[7 - x]*Sqrt[2 + 3*x]*EllipticF[
ArcSin[Sqrt[2/19]*Sqrt[2 + 3*x]], 19/46]))/Sqrt[70 + 67*x - 53*x^2 + 6*x^3
]
```

Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 6.87 (sec) , antiderivative size = 1783, normalized size of antiderivative = 5.55, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {2526, 2490, 2486, 27, 1236, 27, 1231, 27, 1269, 1172, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{6x^3-53x^2+67x+70}\left(A+Bx+Cx^2\right)\,dx$$

$$\Big\downarrow \text{ 2526}$$

$$\frac{1}{18}\int (18A-67C+2(9B+53C)x)\sqrt{6x^3-53x^2+67x+70}dx+\frac{1}{27}C\left(6x^3-53x^2+67x+70\right)^{3/2}$$

$$\Big\downarrow \text{ 2490}$$

$$\frac{1}{18}\int \left(\frac{1}{18}(18(18A-67C)+106(9B+53C))+2(9B+53C)\left(x-\frac{53}{18}\right)\right)\sqrt{6\left(x-\frac{53}{18}\right)^3-\frac{1603}{18}\left(x-\frac{53}{18}\right)-}$$
$$\frac{1}{27}C\left(6x^3-53x^2+67x+70\right)^{3/2}$$

$$\Big\downarrow \text{ 2486}$$

$$\frac{1}{27}C\left(6x^3-53x^2+67x+70\right)^{3/2}+$$
$$\frac{\sqrt{2916\left(x-\frac{53}{18}\right)^3-43281\left(x-\frac{53}{18}\right)-18980}\int \frac{\sqrt{18\left(x-\frac{53}{18}\right)-\frac{1603+\left(18980+35397i\sqrt{3}\right)^{2/3}}{\sqrt[3]{18980+35397i\sqrt{3}}}}(162A+477B+2206C+18(9B+53C)(x-}{}}{54\sqrt{2}\sqrt{18\left(x-\frac{53}{18}\right)-\frac{1603+\left(18980+35397i\sqrt{3}\right)^{2/3}}{\sqrt[3]{18980+35397i\sqrt{3}}}}\sqrt{324\left(x-\frac{53}{18}\right)^2+\frac{18\left(1603+\left(18980+353\right.\right.}{\sqrt[3]{18980+}}}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{27}C\left(6x^3-53x^2+67x+70\right)^{3/2}+$$
$$\frac{\sqrt{2916\left(x-\frac{53}{18}\right)^3-43281\left(x-\frac{53}{18}\right)-18980}\int \sqrt{18\left(x-\frac{53}{18}\right)-\frac{1603+\left(18980+35397i\sqrt{3}\right)^{2/3}}{\sqrt[3]{18980+35397i\sqrt{3}}}}(162A+477B+2206C}{1458\sqrt{6}\sqrt{18\left(x-\frac{53}{18}\right)-\frac{1603+\left(18980+35397i\sqrt{3}\right)^{2/3}}{\sqrt[3]{18980+35397i\sqrt{3}}}}\sqrt{324\left(x-\frac{5}{1}\right.}}$$

$$\Big\downarrow \text{ 1236}$$

$$\frac{1}{27}C\left(6x^3-53x^2+67x+70\right)^{3/2}+$$

$$\sqrt{2916\left(x-\frac{53}{18}\right)^3-43281\left(x-\frac{53}{18}\right)-18980}\left(\frac{\int -\frac{162\left(\frac{\left(2569609-1603\left(18980+35397i\sqrt{3}\right)^{2/3}+\left(18980+35397i\sqrt{3}\right)^{4/3}\right)(9B+53C)+\sqrt[3]{18}}{}\right.}{}}{}\right.$$

$\downarrow$ 27

$$\frac{1}{27}C\left(6x^3-53x^2+67x+70\right)^{3/2}+$$

$$\sqrt{2916\left(x-\frac{53}{18}\right)^3-43281\left(x-\frac{53}{18}\right)-18980}\left(\frac{1}{63}\sqrt{18\left(x-\frac{53}{18}\right)-\frac{1603+\left(18980+35397i\sqrt{3}\right)^{2/3}}{\sqrt[3]{18980+35397i\sqrt{3}}}}\left(324\left(x-\frac{53}{18}\right)^2+\frac{18\left(16\right.}{}\right.\right.$$

$\downarrow$ 1231

$$\frac{1}{27}C\left(6x^3-53x^2+67x+70\right)^{3/2}+$$

$$\sqrt{2916\left(x-\frac{53}{18}\right)^3-43281\left(x-\frac{53}{18}\right)-18980}\left(\frac{1}{63}(9B+53C)\sqrt{18\left(x-\frac{53}{18}\right)-\frac{1603+\left(18980+35397i\sqrt{3}\right)^{2/3}}{\sqrt[3]{18980+35397i\sqrt{3}}}}\left(324(x-\right.\right.$$

$$\downarrow 27$$

$$\frac{1}{27}C\left(6x^3-53x^2+67x+70\right)^{3/2}+$$

$$\sqrt{2916\left(x-\frac{53}{18}\right)^3-43281\left(x-\frac{53}{18}\right)-18980}\left(\frac{1}{7}\left(-\frac{18}{5}\int\frac{35(204984A+1704825B+9276529C)}{\sqrt{18(x-\frac{53}{18})-\frac{1603+\left(18980+35397i\sqrt{3}\right)^{2/3}}{\sqrt[3]{18980+35397i\sqrt{3}}}}\sqrt{324(x-\frac{53}{18})^2+\frac{18\left(1603+\left(18\right.\right.}{\sqrt[3]{18}}}}\right.\right.$$

$$\downarrow 1269$$

$$\frac{1}{27}C\left(6x^3-53x^2+67x+70\right)^{3/2}+$$

$$\sqrt{2916\left(x-\frac{53}{18}\right)^3-43281\left(x-\frac{53}{18}\right)-18980}\left(\frac{1}{63}(9B+53C)\sqrt{18\left(x-\frac{53}{18}\right)-\frac{1603+\left(18980+35397i\sqrt{3}\right)^{2/3}}{\sqrt[3]{18980+35397i\sqrt{3}}}}\left(324(x-\right.\right.$$

$$\downarrow 1172$$

$$\frac{1}{27}C\left(6x^3-53x^2+67x+70\right)^{3/2}+$$

$$\sqrt{2916\left(x-\frac{53}{18}\right)^3-43281\left(x-\frac{53}{18}\right)-18980}\left(\frac{1}{63}(9B+53C)\sqrt{18\left(x-\frac{53}{18}\right)-\frac{1603+\left(18980+35397i\sqrt{3}\right)^{2/3}}{\sqrt[3]{18980+35397i\sqrt{3}}}}\left(324\left(x-\right.\right.\right.$$

↓ 321

$$\frac{1}{27}C\left(6x^3-53x^2+67x+70\right)^{3/2}+$$

$$\sqrt{2916\left(x-\frac{53}{18}\right)^3-43281\left(x-\frac{53}{18}\right)-18980}\left(\frac{1}{63}(9B+53C)\sqrt{18\left(x-\frac{53}{18}\right)-\frac{1603+\left(18980+35397i\sqrt{3}\right)^{2/3}}{\sqrt[3]{18980+35397i\sqrt{3}}}}\left(324(x-\right.\right.$$

$$\Big\downarrow \; 327$$

$$\frac{1}{27}C\left(6x^3-53x^2+67x+70\right)^{3/2}+$$

$$\sqrt{2916\left(x-\frac{53}{18}\right)^3-43281\left(x-\frac{53}{18}\right)-18980}\left(\frac{1}{63}(9B+53C)\sqrt{18\left(x-\frac{53}{18}\right)-\frac{1603+\left(18980+35397i\sqrt{3}\right)^{2/3}}{\sqrt[3]{18980+35397i\sqrt{3}}}}\left(324(x-\right.\right.$$

input
```
Int[(A + B*x + C*x^2)*Sqrt[70 + 67*x - 53*x^2 + 6*x^3],x]
```

output
```
(C*(70 + 67*x - 53*x^2 + 6*x^3)^(3/2))/27 + (Sqrt[-18980 - 43281*(-53/18 +
 x) + 2916*(-53/18 + x)^3]*(((9*B + 53*C)*Sqrt[-((1603 + (18980 + (35397*I
)*Sqrt[3])^(2/3))/(18980 + (35397*I)*Sqrt[3])^(1/3)) + 18*(-53/18 + x)]*(-
1603 + 2569609/(18980 + (35397*I)*Sqrt[3])^(2/3) + (18980 + (35397*I)*Sqrt
[3])^(2/3) + (18*(1603 + (18980 + (35397*I)*Sqrt[3])^(2/3))*(-53/18 + x))/
(18980 + (35397*I)*Sqrt[3])^(1/3) + 324*(-53/18 + x)^2)^(3/2))/63 + (-1/45
*(Sqrt[-((1603 + (18980 + (35397*I)*Sqrt[3])^(2/3))/(18980 + (35397*I)*Sqr
t[3])^(1/3)) + 18*(-53/18 + x)]*((5*(2569609 + 1603*(18980 + (35397*I)*Sqr
t[3])^(2/3) + (18980 + (35397*I)*Sqrt[3])^(4/3))*(9*B + 53*C))/(18980 + (3
5397*I)*Sqrt[3])^(2/3) - 18*(1134*A + 9*(371 - 8015/(18980 + (35397*I)*Sqr
t[3])^(1/3) - 5*(18980 + (35397*I)*Sqrt[3])^(1/3))*B + (15442 - 424795/(18
980 + (35397*I)*Sqrt[3])^(1/3) - 265*(18980 + (35397*I)*Sqrt[3])^(1/3))*C)
*(-53/18 + x))*Sqrt[-1603 + 2569609/(18980 + (35397*I)*Sqrt[3])^(2/3) + (1
8980 + (35397*I)*Sqrt[3])^(2/3) + (18*(1603 + (18980 + (35397*I)*Sqrt[3])^
(2/3))*(-53/18 + x))/(18980 + (35397*I)*Sqrt[3])^(1/3) + 324*(-53/18 + x)^
2]) - (18*(((1603 - (18980 + (35397*I)*Sqrt[3])^(2/3))*(605934*A + 2068839
*B + 9927742*C)*Sqrt[-((1603 + (18980 + (35397*I)*Sqrt[3])^(2/3))/(18980 +
 (35397*I)*Sqrt[3])^(1/3)) + 18*(-53/18 + x)]*Sqrt[-(((18980 + (35397*I)*S
qrt[3])^(2/3)*(1603 - 2569609/(18980 + (35397*I)*Sqrt[3])^(2/3) - (18980 +
 (35397*I)*Sqrt[3])^(2/3) - (18*(1603 + (18980 + (35397*I)*Sqrt[3])^(2/...
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 1172

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Simp[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m))   Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^
2 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[m^2, 1/4]
```

rule 1231

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1236

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1
)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m
*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

rule 1269

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2486
```
Int[((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (d_.)*(x_)^3)^(p_), x_S
ymbol] :> With[{r = Rt[-9*a*d^2 + Sqrt[3]*d*Sqrt[4*b^3*d + 27*a^2*d^2], 3]}
, Simp[(a + b*x + d*x^3)^p/(Simp[18^(1/3)*b*(d/(3*r)) - r/18^(1/3) + d*x, x
]^p*Simp[b*(d/3) + 12^(1/3)*b^2*(d^2/(3*r^2)) + r^2/(3*12^(1/3)) - d*(2^(1/
3)*b*(d/(3^(1/3)*r)) - r/18^(1/3))*x + d^2*x^2, x]^p)   Int[(e + f*x)^m*Sim
p[18^(1/3)*b*(d/(3*r)) - r/18^(1/3) + d*x, x]^p*Simp[b*(d/3) + 12^(1/3)*b^2
*(d^2/(3*r^2)) + r^2/(3*12^(1/3)) - d*(2^(1/3)*b*(d/(3^(1/3)*r)) - r/18^(1/
3))*x + d^2*x^2, x]^p, x], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && NeQ[4*
b^3 + 27*a^2*d, 0] &&  !IntegerQ[p]
```

rule 2490
```
Int[(P3_)^(p_.)*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] :> With[{a = Coeff[P3
, x, 0], b = Coeff[P3, x, 1], c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Su
bst[Int[((3*d*e - c*f)/(3*d) + f*x)^m*Simp[(2*c^3 - 9*b*c*d + 27*a*d^2)/(27
*d^2) - (c^2 - 3*b*d)*(x/(3*d)) + d*x^3, x]^p, x], x, x + c/(3*d)] /; NeQ[c
, 0]] /; FreeQ[{e, f, m, p}, x] && PolyQ[P3, x, 3]
```

rule 2526
```
Int[(Pm_)*(Qn_)^(p_), x_Symbol] :> With[{m = Expon[Pm, x], n = Expon[Qn, x]
}, Simp[Coeff[Pm, x, m]*(Qn^(p + 1)/(n*(p + 1)*Coeff[Qn, x, n])), x] + Simp
[1/(n*Coeff[Qn, x, n])   Int[ExpandToSum[n*Coeff[Qn, x, n]*Pm - Coeff[Pm, x
, m]*D[Qn, x], x]*Qn^p, x], x] /; EqQ[m, n - 1]] /; FreeQ[p, x] && PolyQ[Pm
, x] && PolyQ[Qn, x] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 1.26 (sec) , antiderivative size = 260, normalized size of antiderivative = 0.81

method	result
elliptic	$\frac{2x^3\sqrt{6x^3-53x^2+67x+70}\,C}{9}+\left(\frac{2B}{7}-\frac{53C}{189}\right)x^2\sqrt{6x^3-53x^2+67x+70}+\left(-\frac{53B}{105}+\frac{2A}{5}-\frac{1871C}{945}\right)x\sqrt{6x^3}$
risch	$\frac{\left(3780C\,x^3+4860B\,x^2-4770C\,x^2+6804Ax-8586Bx-33678Cx-20034A-64944B-263777C\right)\sqrt{6x^3-53x^2+67x+70}}{17010}-\frac{(1211868}{}$
default	$A\left(\frac{2x\sqrt{6x^3-53x^2+67x+70}}{5}-\frac{53\sqrt{6x^3-53x^2+67x+70}}{45}+\frac{7331\sqrt{76+114x}\,\sqrt{483-69x}\,\sqrt{285-114x}\,\operatorname{EllipticF}\left(\frac{\sqrt{76+114x}}{19},\frac{\sqrt{874}}{46}\right)}{117990\sqrt{6x^3-53x^2+67x+70}}\right.$

input
```
int((C*x^2+B*x+A)*(6*x^3-53*x^2+67*x+70)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/9*x^3*(6*x^3-53*x^2+67*x+70)^(1/2)*C+(2/7*B-53/189*C)*x^2*(6*x^3-53*x^2+
67*x+70)^(1/2)+(-53/105*B+2/5*A-1871/945*C)*x*(6*x^3-53*x^2+67*x+70)^(1/2)
+(-3608/945*B-53/45*A-263777/17010*C)*(6*x^3-53*x^2+67*x+70)^(1/2)+1/1311*
(7331/90*A+154258/945*B+22387979/34020*C)*(76+114*x)^(1/2)*(483-69*x)^(1/2
)*(285-114*x)^(1/2)/(6*x^3-53*x^2+67*x+70)^(1/2)*EllipticF(1/19*(76+114*x)
^(1/2),1/46*874^(1/2))+1/1311*(-1603/45*A-229871/1890*B-4963871/8505*C)*(7
6+114*x)^(1/2)*(483-69*x)^(1/2)*(285-114*x)^(1/2)/(6*x^3-53*x^2+67*x+70)^(
1/2)*(-23/3*EllipticE(1/19*(76+114*x)^(1/2),1/46*874^(1/2))+7*EllipticF(1/
19*(76+114*x)^(1/2),1/46*874^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.33

$$\int \left(A+Bx+Cx^2\right)\sqrt{70+67x-53x^2+6x^3}\,dx =$$

$$-\frac{1}{26244}\sqrt{6}(204984\,A+1704825\,B+9276529\,C)\text{weierstrassPInverse}\left(\frac{1603}{27},\frac{18980}{729},x-\frac{53}{18}\right)$$

$$+\frac{1}{51030}\sqrt{6}(605934\,A+2068839\,B+9927742\,C)\text{weierstrassZeta}\left(\frac{1603}{27},\frac{18980}{729},\text{weierstrassPInverse}\left(\ldots-\frac{53}{18}\right)\right)$$

$$+\frac{1}{17010}\left(3780\,Cx^3+90\,(54\,B-53\,C)x^2+18\,(378\,A-477\,B-1871\,C)x-20034\,A-64944\,B-263\ldots\right.$$

input
```
integrate((C*x^2+B*x+A)*(6*x^3-53*x^2+67*x+70)^(1/2),x, algorithm="fricas"
)
```

output
```
-1/26244*sqrt(6)*(204984*A + 1704825*B + 9276529*C)*weierstrassPInverse(16
03/27, 18980/729, x - 53/18) + 1/51030*sqrt(6)*(605934*A + 2068839*B + 992
7742*C)*weierstrassZeta(1603/27, 18980/729, weierstrassPInverse(1603/27, 1
8980/729, x - 53/18)) + 1/17010*(3780*C*x^3 + 90*(54*B - 53*C)*x^2 + 18*(3
78*A - 477*B - 1871*C)*x - 20034*A - 64944*B - 263777*C)*sqrt(6*x^3 - 53*x
^2 + 67*x + 70)
```

Sympy [F]

$$\int (A+Bx+Cx^2)\sqrt{70+67x-53x^2+6x^3}\,dx = \int \sqrt{(x-7)(2x-5)(3x+2)}\left(A+Bx+Cx^2\right)\,dx$$

input
```
integrate((C*x**2+B*x+A)*(6*x**3-53*x**2+67*x+70)**(1/2),x)
```

output
```
Integral(sqrt((x - 7)*(2*x - 5)*(3*x + 2))*(A + B*x + C*x**2), x)
```

Maxima [F]

$$\int (A+Bx+Cx^2)\sqrt{70+67x-53x^2+6x^3}\,dx = \int (Cx^2+Bx+A)\sqrt{6\,x^3-53\,x^2+67\,x+70}\,dx$$

input
```
integrate((C*x^2+B*x+A)*(6*x^3-53*x^2+67*x+70)^(1/2),x, algorithm="maxima"
)
```

output
```
integrate((C*x^2 + B*x + A)*sqrt(6*x^3 - 53*x^2 + 67*x + 70), x)
```

Giac [F]

$$\int (A+Bx+Cx^2)\sqrt{70+67x-53x^2+6x^3}\,dx = \int (Cx^2+Bx+A)\sqrt{6\,x^3-53\,x^2+67\,x+70}\,dx$$

input
```
integrate((C*x^2+B*x+A)*(6*x^3-53*x^2+67*x+70)^(1/2),x, algorithm="giac")
```

output
```
integrate((C*x^2 + B*x + A)*sqrt(6*x^3 - 53*x^2 + 67*x + 70), x)
```

Mupad [B] (verification not implemented)

Time = 12.88 (sec) , antiderivative size = 706, normalized size of antiderivative = 2.20

$$\int (A+Bx+Cx^2)\sqrt{70+67x-53x^2+6x^3}\,dx = \text{Too large to display}$$

input
```
int((A + B*x + C*x^2)*(67*x - 53*x^2 + 6*x^3 + 70)^(1/2),x)
```

output

```
(803*A*x^2)/(9*(67*x - 53*x^2 + 6*x^3 + 70)^(1/2)) - (7216*B)/(27*(67*x -
53*x^2 + 6*x^3 + 70)^(1/2)) - (263777*C)/(243*(67*x - 53*x^2 + 6*x^3 + 70)
^(1/2)) - (2291*A*x)/(45*(67*x - 53*x^2 + 6*x^3 + 70)^(1/2)) - (275126*B*x
)/(945*(67*x - 53*x^2 + 6*x^3 + 70)^(1/2)) - (20030519*C*x)/(17010*(67*x -
 53*x^2 + 6*x^3 + 70)^(1/2)) - (742*A)/(9*(67*x - 53*x^2 + 6*x^3 + 70)^(1/
2)) - (424*A*x^3)/(15*(67*x - 53*x^2 + 6*x^3 + 70)^(1/2)) + (12*A*x^4)/(5*
(67*x - 53*x^2 + 6*x^3 + 70)^(1/2)) + (35633*B*x^2)/(189*(67*x - 53*x^2 +
6*x^3 + 70)^(1/2)) + (7241*B*x^3)/(315*(67*x - 53*x^2 + 6*x^3 + 70)^(1/2))
 - (636*B*x^4)/(35*(67*x - 53*x^2 + 6*x^3 + 70)^(1/2)) + (12*B*x^5)/(7*(67
*x - 53*x^2 + 6*x^3 + 70)^(1/2)) + (2277971*C*x^2)/(3402*(67*x - 53*x^2 +
6*x^3 + 70)^(1/2)) + (24547*C*x^3)/(2835*(67*x - 53*x^2 + 6*x^3 + 70)^(1/2
)) + (16889*C*x^4)/(945*(67*x - 53*x^2 + 6*x^3 + 70)^(1/2)) - (848*C*x^5)/
(63*(67*x - 53*x^2 + 6*x^3 + 70)^(1/2)) + (4*C*x^6)/(3*(67*x - 53*x^2 + 6*
x^3 + 70)^(1/2)) + (36869*A*((2*x)/9 - 5/9)^(1/2)*((3*x)/23 + 2/23)^(1/2)*
(21/23 - (3*x)/23)^(1/2)*ellipticE(asin((21/23 - (3*x)/23)^(1/2)), 46/27))
/(15*(67*x - 53*x^2 + 6*x^3 + 70)^(1/2)) + (1748*A*((2*x)/9 - 5/9)^(1/2)*(
(3*x)/23 + 2/23)^(1/2)*(21/23 - (3*x)/23)^(1/2)*ellipticF(asin((21/23 - (3
*x)/23)^(1/2)), 46/27))/(15*(67*x - 53*x^2 + 6*x^3 + 70)^(1/2)) + (5287033
*B*((2*x)/9 - 5/9)^(1/2)*((3*x)/23 + 2/23)^(1/2)*(21/23 - (3*x)/23)^(1/2)*
ellipticE(asin((21/23 - (3*x)/23)^(1/2)), 46/27))/(630*(67*x - 53*x^2 +...
```

Reduce [F]

$$\int (A+Bx+Cx^2)\sqrt{70+67x-53x^2+6x^3}\,dx = \frac{2\sqrt{6x^3-53x^2+67x+70}\,ax}{5}-\frac{134\sqrt{6x^3-53x^2+67x+70}\,a}{265}+\frac{2\sqrt{6x^3-53x^2+67x+70}\,b\,x^2}{7}-\frac{53\sqrt{6x^3-53x^2+67x+70}\,bx}{105}-\frac{5651\sqrt{6x^3-53x^2+67x+70}\,b}{3710}+\frac{2\sqrt{6x^3-53x^2+67x+70}\,c\,x^3}{9}-\frac{53\sqrt{6x^3-53x^2+67x+70}\,c\,x^2}{189}-\frac{1871\sqrt{6x^3-53x^2+67x+70}\,cx}{945}-\frac{450271\sqrt{6x^3-53x^2+67x+70}\,c}{100170}+\frac{15619\left(\int \frac{\sqrt{6x^3-53x^2+67x+70}}{6x^3-53x^2+67x+70}dx\right)a}{265}+\frac{1922371\left(\int \frac{\sqrt{6x^3-53x^2+67x+70}}{6x^3-53x^2+67x+70}dx\right)b}{22260}+\frac{57933797\left(\int \frac{\sqrt{6x^3-53x^2+67x+70}}{6x^3-53x^2+67x+70}dx\right)c}{200340}-\frac{1603\left(\int \frac{\sqrt{6x^3-53x^2+67x+70}\,x^2}{6x^3-53x^2+67x+70}dx\right)a}{265}-\frac{229871\left(\int \frac{\sqrt{6x^3-53x^2+67x+70}\,x^2}{6x^3-53x^2+67x+70}dx\right)b}{11130}-\frac{4963871\left(\int \frac{\sqrt{6x^3-53x^2+67x+70}\,x^2}{6x^3-53x^2+67x+70}dx\right)c}{50085}$$

input
```
int((C*x^2+B*x+A)*(6*x^3-53*x^2+67*x+70)^(1/2),x)
```

output
```
(80136*sqrt(6*x**3 - 53*x**2 + 67*x + 70)*a*x - 101304*sqrt(6*x**3 - 53*x*
*2 + 67*x + 70)*a + 57240*sqrt(6*x**3 - 53*x**2 + 67*x + 70)*b*x**2 - 1011
24*sqrt(6*x**3 - 53*x**2 + 67*x + 70)*b*x - 305154*sqrt(6*x**3 - 53*x**2 +
 67*x + 70)*b + 44520*sqrt(6*x**3 - 53*x**2 + 67*x + 70)*c*x**3 - 56180*sq
rt(6*x**3 - 53*x**2 + 67*x + 70)*c*x**2 - 396652*sqrt(6*x**3 - 53*x**2 + 6
7*x + 70)*c*x - 900542*sqrt(6*x**3 - 53*x**2 + 67*x + 70)*c + 11807964*int
(sqrt(6*x**3 - 53*x**2 + 67*x + 70)/(6*x**3 - 53*x**2 + 67*x + 70),x)*a +
17301339*int(sqrt(6*x**3 - 53*x**2 + 67*x + 70)/(6*x**3 - 53*x**2 + 67*x +
 70),x)*b + 57933797*int(sqrt(6*x**3 - 53*x**2 + 67*x + 70)/(6*x**3 - 53*x
**2 + 67*x + 70),x)*c - 1211868*int((sqrt(6*x**3 - 53*x**2 + 67*x + 70)*x*
*2)/(6*x**3 - 53*x**2 + 67*x + 70),x)*a - 4137678*int((sqrt(6*x**3 - 53*x*
*2 + 67*x + 70)*x**2)/(6*x**3 - 53*x**2 + 67*x + 70),x)*b - 19855484*int((
sqrt(6*x**3 - 53*x**2 + 67*x + 70)*x**2)/(6*x**3 - 53*x**2 + 67*x + 70),x)
*c)/200340
```

3.120 $\int \frac{A+Bx+Cx^2}{\sqrt{70+67x-53x^2+6x^3}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 205

$$\begin{aligned}\int \frac{A+Bx+Cx^2}{\sqrt{70+67x-53x^2+6x^3}}\,dx &= \frac{C(5-2x)(7-x)(2+3x)}{9\sqrt{70+67x-53x^2+6x^3}} \\ &\quad - \frac{\sqrt{19}(9B+53C)\sqrt{5-2x}\sqrt{7-x}\sqrt{2+3x}E\left(\arcsin\left(\frac{\sqrt{2+3x}}{\sqrt{23}}\right)|\frac{46}{19}\right)}{27\sqrt{70+67x-53x^2+6x^3}} \\ &\quad - \frac{(2A+5B+22C)\sqrt{-7+x}\sqrt{-5+2x}\sqrt{2+3x}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{19}{2}}}{\sqrt{2+3x}}\right),\frac{46}{19}\right)}{\sqrt{19}\sqrt{70+67x-53x^2+6x^3}}\end{aligned}$$

output
```
1/9*C*(5-2*x)*(7-x)*(2+3*x)/(6*x^3-53*x^2+67*x+70)^(1/2)-1/27*19^(1/2)*(9*
B+53*C)*(5-2*x)^(1/2)*(7-x)^(1/2)*(2+3*x)^(1/2)*EllipticE(1/23*(2+3*x)^(1/
2)*23^(1/2),1/19*874^(1/2))/(6*x^3-53*x^2+67*x+70)^(1/2)-1/19*(2*A+5*B+22*
C)*(-7+x)^(1/2)*(-5+2*x)^(1/2)*(2+3*x)^(1/2)*EllipticF(1/2*38^(1/2)/(2+3*x
)^(1/2),1/19*874^(1/2))*19^(1/2)/(6*x^3-53*x^2+67*x+70)^(1/2)
```

Mathematica [A] (verified)

Time = 10.35 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.81

$$\int \frac{A+Bx+Cx^2}{\sqrt{70+67x-53x^2+6x^3}}\,dx =$$
$$-\frac{\sqrt{5-2x}\left(138C\sqrt{5-2x}(-14-19x+3x^2)+46\sqrt{46}(9B+53C)\sqrt{7-x}\sqrt{2+3x}E\left(\arcsin\left(\sqrt{\frac{2}{19}}\sqrt{2-\right.\right.\right.}{1242\sqrt{70+67x-}}$$

input
```
Integrate[(A + B*x + C*x^2)/Sqrt[70 + 67*x - 53*x^2 + 6*x^3],x]
```

output
```
-1/1242*(Sqrt[5 - 2*x]*(138*C*Sqrt[5 - 2*x]*(-14 - 19*x + 3*x^2) + 46*Sqrt
[46]*(9*B + 53*C)*Sqrt[7 - x]*Sqrt[2 + 3*x]*EllipticE[ArcSin[Sqrt[2/19]*Sq
rt[2 + 3*x]], 19/46] - 27*Sqrt[46]*(2*A + 14*B + 75*C)*Sqrt[7 - x]*Sqrt[2
+ 3*x]*EllipticF[ArcSin[Sqrt[2/19]*Sqrt[2 + 3*x]], 19/46]))/Sqrt[70 + 67*x
 - 53*x^2 + 6*x^3]
```

Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 3.22 (sec) , antiderivative size = 1338, normalized size of antiderivative = 6.53, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {2526, 2490, 2486, 27, 1269, 1172, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{\sqrt{6x^3-53x^2+67x+70}}dx$$

$$\Big\downarrow \text{ 2526}$$

$$\frac{1}{18}\int \frac{18A-67C+2(9B+53C)x}{\sqrt{6x^3-53x^2+67x+70}}dx+\frac{1}{9}C\sqrt{6x^3-53x^2+67x+70}$$

$$\Big\downarrow \text{ 2490}$$

$$\frac{1}{18}\int \frac{\frac{1}{18}(18(18A-67C)+106(9B+53C))+2(9B+53C)\left(x-\frac{53}{18}\right)}{\sqrt{6\left(x-\frac{53}{18}\right)^3-\frac{1603}{18}\left(x-\frac{53}{18}\right)-\frac{9490}{243}}}d\left(x-\frac{53}{18}\right)+\frac{1}{9}C\sqrt{6x^3-53x^2+67x+70}$$

$$\Big\downarrow \text{2486}$$

$$\frac{1}{9}C\sqrt{6x^3-53x^2+67x+70}+$$

$$\sqrt{18\left(x-\frac{53}{18}\right)-\frac{1603+\left(18980+35397i\sqrt{3}\right)^{2/3}}{\sqrt[3]{18980+35397i\sqrt{3}}}}\sqrt{324\left(x-\frac{53}{18}\right)^2+\frac{18\left(1603+\left(18980+35397i\sqrt{3}\right)^{2/3}\right)\left(x-\frac{53}{18}\right)}{\sqrt[3]{18980+35397i\sqrt{3}}}+\left(18980+3539\right.}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{9}C\sqrt{6x^3-53x^2+67x+70}+$$

$$\sqrt{18\left(x-\frac{53}{18}\right)-\frac{1603+\left(18980+35397i\sqrt{3}\right)^{2/3}}{\sqrt[3]{18980+35397i\sqrt{3}}}}\sqrt{324\left(x-\frac{53}{18}\right)^2+\frac{18\left(1603+\left(18980+35397i\sqrt{3}\right)^{2/3}\right)\left(x-\frac{53}{18}\right)}{\sqrt[3]{18980+35397i\sqrt{3}}}+\left(18980+3539\right.}$$

$$\Big\downarrow \text{1269}$$

$$\frac{1}{9}C\sqrt{6x^3-53x^2+67x+70}+$$

$$\sqrt{18\left(x-\frac{53}{18}\right)-\frac{1603+\left(18980+35397i\sqrt{3}\right)^{2/3}}{\sqrt[3]{18980+35397i\sqrt{3}}}}\sqrt{324\left(x-\frac{53}{18}\right)^2+\frac{18\left(1603+\left(18980+35397i\sqrt{3}\right)^{2/3}\right)\left(x-\frac{53}{18}\right)}{\sqrt[3]{18980+35397i\sqrt{3}}}+\left(18980+3539\right.}$$

$$\Big\downarrow \text{1172}$$

$$\frac{1}{9}\sqrt{6x^3-53x^2+67x+70}C+$$

$$\sqrt{18\left(x-\frac{53}{18}\right)-\frac{1603+\left(18980+35397i\sqrt{3}\right)^{2/3}}{\sqrt[3]{18980+35397i\sqrt{3}}}}\sqrt{324\left(x-\frac{53}{18}\right)^2+\frac{18\left(1603+\left(18980+35397i\sqrt{3}\right)^{2/3}\right)\left(x-\frac{53}{18}\right)}{\sqrt[3]{18980+35397i\sqrt{3}}}+\left(18980+3539\right.}$$

$\downarrow$ 321

$$\frac{1}{9}\sqrt{6x^3-53x^2+67x+70}C+$$

$$\sqrt{18\left(x-\frac{53}{18}\right)-\frac{1603+\left(18980+35397i\sqrt{3}\right)^{2/3}}{\sqrt[3]{18980+35397i\sqrt{3}}}}\sqrt{324\left(x-\frac{53}{18}\right)^2+\frac{18\left(1603+\left(18980+35397i\sqrt{3}\right)^{2/3}\right)\left(x-\frac{53}{18}\right)}{\sqrt[3]{18980+35397i\sqrt{3}}}+(18980+3539$$

$$\Big\downarrow \text{327}$$

$$\frac{1}{9}\sqrt{6x^3-53x^2+67x+70}C+$$

$$\sqrt{18\left(x-\frac{53}{18}\right)-\frac{1603+\left(18980+35397i\sqrt{3}\right)^{2/3}}{\sqrt[3]{18980+35397i\sqrt{3}}}}\sqrt{324\left(x-\frac{53}{18}\right)^2+\frac{18\left(1603+\left(18980+35397i\sqrt{3}\right)^{2/3}\right)\left(x-\frac{53}{18}\right)}{\sqrt[3]{18980+35397i\sqrt{3}}}+(18980+3539$$

---

input

```
Int[(A + B*x + C*x^2)/Sqrt[70 + 67*x - 53*x^2 + 6*x^3],x]
```

output
```
(C*Sqrt[70 + 67*x - 53*x^2 + 6*x^3])/9 + (Sqrt[-((1603 + (18980 + (35397*I
)*Sqrt[3])^(2/3))/(18980 + (35397*I)*Sqrt[3])^(1/3)) + 18*(-53/18 + x)]*Sq
rt[-1603 + 2569609/(18980 + (35397*I)*Sqrt[3])^(2/3) + (18980 + (35397*I)*
Sqrt[3])^(2/3) + (18*(1603 + (18980 + (35397*I)*Sqrt[3])^(2/3))*(-53/18 +
x))/(18980 + (35397*I)*Sqrt[3])^(1/3) + 324*(-53/18 + x)^2]*(((1603 - (189
80 + (35397*I)*Sqrt[3])^(2/3))*(9*B + 53*C)*Sqrt[-((1603 + (18980 + (35397
*I)*Sqrt[3])^(2/3))/(18980 + (35397*I)*Sqrt[3])^(1/3)) + 18*(-53/18 + x)]*
Sqrt[-(((18980 + (35397*I)*Sqrt[3])^(2/3)*(1603 - 2569609/(18980 + (35397*
I)*Sqrt[3])^(2/3) - (18980 + (35397*I)*Sqrt[3])^(2/3) - (18*(1603 + (18980
 + (35397*I)*Sqrt[3])^(2/3))*(-53/18 + x))/(18980 + (35397*I)*Sqrt[3])^(1/
3) - 324*(-53/18 + x)^2))/(1603 - (18980 + (35397*I)*Sqrt[3])^(2/3))^2)]*E
llipticE[ArcSin[53/18 - x], (2*(18980 + (35397*I)*Sqrt[3] - 1603*(18980 +
(35397*I)*Sqrt[3])^(1/3)))/(18980 + (35397*I)*Sqrt[3] - 1603*(18980 + (353
97*I)*Sqrt[3])^(1/3) + Sqrt[-3*(18980 + (35397*I)*Sqrt[3])^(2/3)]*(1603 +
(18980 + (35397*I)*Sqrt[3])^(2/3)))])/(3*Sqrt[6]*Sqrt[-(18980 + (35397*I)*
Sqrt[3])^(2/3)]*Sqrt[-(((1603 + (18980 + (35397*I)*Sqrt[3])^(2/3))/(18980
+ (35397*I)*Sqrt[3])^(1/3) - 18*(-53/18 + x))/((Sqrt[-1/3*(18980 + (35397*
I)*Sqrt[3])^(2/3)]*(324 - 519372/(18980 + (35397*I)*Sqrt[3])^(2/3)))/324 -
 (1603 + (18980 + (35397*I)*Sqrt[3])^(2/3))/(18980 + (35397*I)*Sqrt[3])^(1
/3)))]*Sqrt[-1603 + 2569609/(18980 + (35397*I)*Sqrt[3])^(2/3) + (18980 ...
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 1172
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Simp[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m))   Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^
2 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[m^2, 1/4]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2486
```
Int[((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (d_.)*(x_)^3)^(p_), x_S
ymbol] :> With[{r = Rt[-9*a*d^2 + Sqrt[3]*d*Sqrt[4*b^3*d + 27*a^2*d^2], 3]}
, Simp[(a + b*x + d*x^3)^p/(Simp[18^(1/3)*b*(d/(3*r)) - r/18^(1/3) + d*x, x
]^p*Simp[b*(d/3) + 12^(1/3)*b^2*(d^2/(3*r^2)) + r^2/(3*12^(1/3)) - d*(2^(1/
3)*b*(d/(3^(1/3)*r)) - r/18^(1/3))*x + d^2*x^2, x]^p)   Int[(e + f*x)^m*Sim
p[18^(1/3)*b*(d/(3*r)) - r/18^(1/3) + d*x, x]^p*Simp[b*(d/3) + 12^(1/3)*b^2
*(d^2/(3*r^2)) + r^2/(3*12^(1/3)) - d*(2^(1/3)*b*(d/(3^(1/3)*r)) - r/18^(1/
3))*x + d^2*x^2, x]^p, x], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && NeQ[4*
b^3 + 27*a^2*d, 0] &&  !IntegerQ[p]
```

rule 2490
```
Int[(P3_)^(p_.)*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] :> With[{a = Coeff[P3
, x, 0], b = Coeff[P3, x, 1], c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Su
bst[Int[((3*d*e - c*f)/(3*d) + f*x)^m*Simp[(2*c^3 - 9*b*c*d + 27*a*d^2)/(27
*d^2) - (c^2 - 3*b*d)*(x/(3*d)) + d*x^3, x]^p, x], x, x + c/(3*d)] /; NeQ[c
, 0]] /; FreeQ[{e, f, m, p}, x] && PolyQ[P3, x, 3]
```

rule 2526
```
Int[(Pm_)*(Qn_)^(p_), x_Symbol] :> With[{m = Expon[Pm, x], n = Expon[Qn, x]
}, Simp[Coeff[Pm, x, m]*(Qn^(p + 1)/(n*(p + 1)*Coeff[Qn, x, n])), x] + Simp
[1/(n*Coeff[Qn, x, n])   Int[ExpandToSum[n*Coeff[Qn, x, n]*Pm - Coeff[Pm, x
, m]*D[Qn, x], x]*Qn^p, x], x] /; EqQ[m, n - 1]] /; FreeQ[p, x] && PolyQ[Pm
, x] && PolyQ[Qn, x] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.79

method	result
elliptic	$\frac{\sqrt{6x^3-53x^2+67x+70}\,C}{9}+\frac{\left(A-\frac{67C}{18}\right)\sqrt{76+114x}\,\sqrt{483-69x}\,\sqrt{285-114x}\ \text{EllipticF}\left(\frac{\sqrt{76+114x}}{19},\frac{\sqrt{874}}{46}\right)}{1311\sqrt{6x^3-53x^2+67x+70}}+\frac{\left(B+\frac{53C}{9}\right)\sqrt{76+114x}}{}$
risch	$\frac{\sqrt{6x^3-53x^2+67x+70}\,C}{9}+\frac{(18B+106C)\sqrt{76+114x}\,\sqrt{483-69x}\,\sqrt{285-114x}\left(-\frac{23\,\text{EllipticE}\left(\frac{\sqrt{76+114x}}{19},\frac{\sqrt{874}}{46}\right)}{3}+7\,\text{EllipticF}\left(\frac{\sqrt{76-}}{1}\right.\right.}{23598\sqrt{6x^3-53x^2+67x+70}}$
default	$\frac{A\sqrt{76+114x}\,\sqrt{483-69x}\,\sqrt{285-114x}\ \text{EllipticF}\left(\frac{\sqrt{76+114x}}{19},\frac{\sqrt{874}}{46}\right)}{1311\sqrt{6x^3-53x^2+67x+70}}+\frac{B\sqrt{76+114x}\,\sqrt{483-69x}\,\sqrt{285-114x}\left(-\frac{23\,\text{EllipticE}\left(\frac{\sqrt{76+1}}{19}\right.}{3}\right.}{1311\sqrt{6x^3-53x^2+67}}$

input
```
int((C*x^2+B*x+A)/(6*x^3-53*x^2+67*x+70)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/9*(6*x^3-53*x^2+67*x+70)^(1/2)*C+1/1311*(A-67/18*C)*(76+114*x)^(1/2)*(48
3-69*x)^(1/2)*(285-114*x)^(1/2)/(6*x^3-53*x^2+67*x+70)^(1/2)*EllipticF(1/1
9*(76+114*x)^(1/2),1/46*874^(1/2))+1/1311*(B+53/9*C)*(76+114*x)^(1/2)*(483
-69*x)^(1/2)*(285-114*x)^(1/2)/(6*x^3-53*x^2+67*x+70)^(1/2)*(-23/3*Ellipti
cE(1/19*(76+114*x)^(1/2),1/46*874^(1/2))+7*EllipticF(1/19*(76+114*x)^(1/2)
,1/46*874^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.31

$$\int \frac{A+Bx+Cx^2}{\sqrt{70+67x-53x^2+6x^3}}\,dx = \frac{1}{486}\sqrt{6}(162\,A+477\,B+2206\,C)\text{weierstrassPInverse}\left(\frac{1603}{27},\frac{18980}{729},x-\frac{53}{18}\right) - \frac{1}{27}\sqrt{6}(9\,B+53\,C)\text{weierstrassZeta}\left(\frac{1603}{27},\frac{18980}{729},\text{weierstrassPInverse}\left(\frac{1603}{27},\frac{18980}{729},x-\frac{53}{18}\right)\right)+\frac{1}{9}\sqrt{6\,x^3-53\,x^2+67\,x+70}C$$

input
```
integrate((C*x^2+B*x+A)/(6*x^3-53*x^2+67*x+70)^(1/2),x, algorithm="fricas"
)
```

output
```
1/486*sqrt(6)*(162*A + 477*B + 2206*C)*weierstrassPInverse(1603/27, 18980/
729, x - 53/18) - 1/27*sqrt(6)*(9*B + 53*C)*weierstrassZeta(1603/27, 18980
/729, weierstrassPInverse(1603/27, 18980/729, x - 53/18)) + 1/9*sqrt(6*x^3
 - 53*x^2 + 67*x + 70)*C
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt{70+67x-53x^2+6x^3}}\,dx = \int \frac{A+Bx+Cx^2}{\sqrt{(x-7)\,(2x-5)\,(3x+2)}}\,dx$$

input
```
integrate((C*x**2+B*x+A)/(6*x**3-53*x**2+67*x+70)**(1/2),x)
```

output
```
Integral((A + B*x + C*x**2)/sqrt((x - 7)*(2*x - 5)*(3*x + 2)), x)
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt{70+67x-53x^2+6x^3}}\,dx = \int \frac{Cx^2+Bx+A}{\sqrt{6\,x^3-53\,x^2+67\,x+70}}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(6*x^3-53*x^2+67*x+70)^(1/2),x, algorithm="maxima"
)
```

output
```
integrate((C*x^2 + B*x + A)/sqrt(6*x^3 - 53*x^2 + 67*x + 70), x)
```

Giac [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt{70+67x-53x^2+6x^3}}\,dx=\int \frac{Cx^2+Bx+A}{\sqrt{6\,x^3-53\,x^2+67\,x+70}}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(6*x^3-53*x^2+67*x+70)^(1/2),x, algorithm="giac")
```

output
```
integrate((C*x^2 + B*x + A)/sqrt(6*x^3 - 53*x^2 + 67*x + 70), x)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.00

$$\int \frac{A+Bx+Cx^2}{\sqrt{70+67x-53x^2+6x^3}}\,dx = -\frac{46\,A\,\sqrt{\frac{2\,x}{9}-\frac{5}{9}}\,\sqrt{\frac{3\,x}{23}+\frac{2}{23}}\,\sqrt{\frac{21}{23}-\frac{3\,x}{23}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{21}{23}-\frac{3\,x}{23}}\right)\middle|\frac{46}{27}\right)-2\,C\left(x^3-\frac{53\,x^2}{6}+\frac{67\,x}{6}+\frac{35}{3}\right)+207\,B}{}$$

input
```
int((A + B*x + C*x^2)/(67*x - 53*x^2 + 6*x^3 + 70)^(1/2),x)
```

output
```
-(46*A*((2*x)/9 - 5/9)^(1/2)*((3*x)/23 + 2/23)^(1/2)*(21/23 - (3*x)/23)^(1
/2)*ellipticF(asin((21/23 - (3*x)/23)^(1/2)), 46/27) - 2*C*((67*x)/6 - (53
*x^2)/6 + x^3 + 35/3) + 207*B*((2*x)/9 - 5/9)^(1/2)*((3*x)/23 + 2/23)^(1/2
)*(21/23 - (3*x)/23)^(1/2)*ellipticE(asin((21/23 - (3*x)/23)^(1/2)), 46/27
) + 115*B*((2*x)/9 - 5/9)^(1/2)*((3*x)/23 + 2/23)^(1/2)*(21/23 - (3*x)/23)
^(1/2)*ellipticF(asin((21/23 - (3*x)/23)^(1/2)), 46/27) + 1219*C*((2*x)/9
- 5/9)^(1/2)*((3*x)/23 + 2/23)^(1/2)*(21/23 - (3*x)/23)^(1/2)*ellipticE(as
in((21/23 - (3*x)/23)^(1/2)), 46/27) + 506*C*((2*x)/9 - 5/9)^(1/2)*((3*x)/
23 + 2/23)^(1/2)*(21/23 - (3*x)/23)^(1/2)*ellipticF(asin((21/23 - (3*x)/23
)^(1/2)), 46/27))/(3*(67*x - 53*x^2 + 6*x^3 + 70)^(1/2))
```

Reduce [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt{70+67x-53x^2+6x^3}}\,dx = -\frac{\sqrt{6x^3-53x^2+67x+70}\,b}{53} + \left(\int \frac{\sqrt{6x^3-53x^2+67x+70}}{6x^3-53x^2+67x+70}dx\right)a + \frac{67\left(\int \frac{\sqrt{6x^3-53x^2+67x+70}}{6x^3-53x^2+67x+70}dx\right)b}{106} + \frac{9\left(\int \frac{\sqrt{6x^3-53x^2+67x+70}\,x^2}{6x^3-53x^2+67x+70}dx\right)b}{53} + \left(\int \frac{\sqrt{6x^3-53x^2+67x+70}\,x^2}{6x^3-53x^2+67x+70}dx\right)c$$

input
```
int((C*x^2+B*x+A)/(6*x^3-53*x^2+67*x+70)^(1/2),x)
```

output
```
( - 2*sqrt(6*x**3 - 53*x**2 + 67*x + 70)*b + 106*int(sqrt(6*x**3 - 53*x**2
 + 67*x + 70)/(6*x**3 - 53*x**2 + 67*x + 70),x)*a + 67*int(sqrt(6*x**3 - 5
3*x**2 + 67*x + 70)/(6*x**3 - 53*x**2 + 67*x + 70),x)*b + 18*int((sqrt(6*x
**3 - 53*x**2 + 67*x + 70)*x**2)/(6*x**3 - 53*x**2 + 67*x + 70),x)*b + 106
*int((sqrt(6*x**3 - 53*x**2 + 67*x + 70)*x**2)/(6*x**3 - 53*x**2 + 67*x +
70),x)*c)/106
```

3.121 $\int \frac{A+Bx+Cx^2}{(70+67x-53x^2+6x^3)^{3/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 319

$$\begin{aligned}\int \frac{A+Bx+Cx^2}{(70+67x-53x^2+6x^3)^{3/2}}\,dx &= -\frac{2(A+7B+49C)(5-2x)(7-x)(2+3x)}{207\,(70+67x-53x^2+6x^3)^{3/2}}\\
&+\frac{2(130A+496B+2437C)(5-2x)(7-x)^2(2+3x)}{35397\,(70+67x-53x^2+6x^3)^{3/2}}\\
&-\frac{2(3206A+7331B+31238C)(5-2x)^2(7-x)^2(2+3x)}{5156163\,(70+67x-53x^2+6x^3)^{3/2}}\\
&-\frac{2(3206A+7331B+31238C)(5-2x)^{3/2}(7-x)^{3/2}(2+3x)^{3/2}E\left(\arcsin\left(\frac{\sqrt{2+3x}}{\sqrt{23}}\right)|\frac{46}{19}\right)}{814131\sqrt{19}\,(70+67x-53x^2+6x^3)^{3/2}}\\
&-\frac{2(8A-151B-919C)(-7+x)^{3/2}(-5+2x)^{3/2}(2+3x)^{3/2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{19}{2}}}{\sqrt{2+3x}}\right),\frac{46}{19}\right)}{30153\sqrt{19}\,(70+67x-53x^2+6x^3)^{3/2}}\end{aligned}$$

output
```
-2/207*(A+7*B+49*C)*(5-2*x)*(7-x)*(2+3*x)/(6*x^3-53*x^2+67*x+70)^(3/2)+2/3
5397*(130*A+496*B+2437*C)*(5-2*x)*(7-x)^2*(2+3*x)/(6*x^3-53*x^2+67*x+70)^(
3/2)-2/5156163*(3206*A+7331*B+31238*C)*(5-2*x)^2*(7-x)^2*(2+3*x)/(6*x^3-53
*x^2+67*x+70)^(3/2)-2/15468489*(3206*A+7331*B+31238*C)*(5-2*x)^(3/2)*(7-x)
^(3/2)*(2+3*x)^(3/2)*EllipticE(1/23*(2+3*x)^(1/2)*23^(1/2),1/19*874^(1/2))
*19^(1/2)/(6*x^3-53*x^2+67*x+70)^(3/2)-2/572907*(8*A-151*B-919*C)*(-7+x)^(
3/2)*(-5+2*x)^(3/2)*(2+3*x)^(3/2)*EllipticF(1/2*38^(1/2)/(2+3*x)^(1/2),1/1
9*874^(1/2))*19^(1/2)/(6*x^3-53*x^2+67*x+70)^(3/2)
```

Mathematica [A] (verified)

Time = 10.40 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.64

$$\int \frac{A+Bx+Cx^2}{(70+67x-53x^2+6x^3)^{3/2}}\,dx = \frac{2\sqrt{5-2x}(C(513170+715597x-187428x^2)+B(224420+201115x}{}$$

input
```
Integrate[(A + B*x + C*x^2)/(70 + 67*x - 53*x^2 + 6*x^3)^(3/2),x]
```

output
```
(2*Sqrt[5 - 2*x]*(C*(513170 + 715597*x - 187428*x^2) + B*(224420 + 201115*
x - 43986*x^2) + A*(-13687 + 125932*x - 19236*x^2)) + 2*Sqrt[46]*(3206*A +
 7331*B + 31238*C)*Sqrt[7 - x]*(-5 + 2*x)*Sqrt[2 + 3*x]*EllipticE[ArcSin[S
qrt[2/19]*Sqrt[2 + 3*x]], 19/46] - 27*Sqrt[46]*(146*A + 194*B + 599*C)*Sqr
t[7 - x]*(-5 + 2*x)*Sqrt[2 + 3*x]*EllipticF[ArcSin[Sqrt[2/19]*Sqrt[2 + 3*x
]], 19/46])/(15468489*Sqrt[5 - 2*x]*Sqrt[70 + 67*x - 53*x^2 + 6*x^3])
```

Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 7.55 (sec) , antiderivative size = 1923, normalized size of antiderivative = 6.03, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {2526, 2490, 2486, 27, 1235, 27, 1237, 27, 1269, 1172, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{(6x^3-53x^2+67x+70)^{3/2}}\,dx$$

$$\Big\downarrow \text{2526}$$

$$\frac{1}{18}\int \frac{18A-67C+2(9B+53C)x}{(6x^3-53x^2+67x+70)^{3/2}}dx-\frac{C}{9\sqrt{6x^3-53x^2+67x+70}}$$

$$\Big\downarrow \text{2490}$$

$$\frac{1}{18}\int \frac{\frac{1}{18}(18(18A-67C)+106(9B+53C))+2(9B+53C)\left(x-\frac{53}{18}\right)}{\left(6\left(x-\frac{53}{18}\right)^3-\frac{1603}{18}\left(x-\frac{53}{18}\right)-\frac{9490}{243}\right)^{3/2}}d\left(x-\frac{53}{18}\right)-\frac{C}{9\sqrt{6x^3-53x^2+67x+70}}$$

$$\Big\downarrow \text{2486}$$

$$-\frac{C}{9\sqrt{6x^3-53x^2+67x+70}}+$$

$$3\sqrt{2}\left(18\left(x-\frac{53}{18}\right)-\frac{1603+\left(18980+35397i\sqrt{3}\right)^{2/3}}{\sqrt[3]{18980+35397i\sqrt{3}}}\right)^{3/2}\left(324\left(x-\frac{53}{18}\right)^2+\frac{18\left(1603+\left(18980+35397i\sqrt{3}\right)^{2/3}\right)\left(x-\frac{53}{18}\right)}{\sqrt[3]{18980+35397i\sqrt{3}}}+(18980\right.$$

$$\Big\downarrow \text{27}$$

$$-\frac{C}{9\sqrt{6x^3-53x^2+67x+70}}+$$

$$27\sqrt{6}\left(18\left(x-\frac{53}{18}\right)-\frac{1603+\left(18980+35397i\sqrt{3}\right)^{2/3}}{\sqrt[3]{18980+35397i\sqrt{3}}}\right)^{3/2}\left(324\left(x-\frac{53}{18}\right)^2+\frac{18\left(1603+\left(18980+35397i\sqrt{3}\right)^{2/3}\right)\left(x-\frac{53}{18}\right)}{\sqrt[3]{18980+35397i\sqrt{3}}}+(1898\right.$$

$$\Big\downarrow \text{1235}$$

$$27\sqrt{6}\left(18\left(x-\frac{53}{18}\right)-\frac{1603+\left(18980+35397i\sqrt{3}\right)^{2/3}}{\sqrt[3]{18980+35397i\sqrt{3}}}\right)^{3/2}\left(324\left(x-\frac{53}{18}\right)^2+\frac{18\left(1603+\left(18980+35397i\sqrt{3}\right)^{2/3}\right)\left(x-\frac{53}{18}\right)}{\sqrt[3]{18980+35397i\sqrt{3}}}+(1898\right.$$

$$\frac{C}{9\sqrt{6x^3-53x^2+67x+70}}$$

$$\Big\downarrow \; 27$$

$$27\sqrt{6}\left(18\left(x-\frac{53}{18}\right)-\frac{1603+\left(18980+35397i\sqrt{3}\right)^{2/3}}{\sqrt[3]{18980+35397i\sqrt{3}}}\right)^{3/2}\left(324\left(x-\frac{53}{18}\right)^2+\frac{18\left(1603+\left(18980+35397i\sqrt{3}\right)^{2/3}\right)\left(x-\frac{53}{18}\right)}{\sqrt[3]{18980+35397i\sqrt{3}}}+(1898\right.$$

$$\frac{C}{9\sqrt{6x^3-53x^2+67x+70}}$$

$$\Big\downarrow \; 1237$$

$$27\sqrt{6}\left(18\left(x-\frac{53}{18}\right)-\frac{1603+\left(18980+35397i\sqrt{3}\right)^{2/3}}{\sqrt[3]{18980+35397i\sqrt{3}}}\right)^{3/2}\left(324\left(x-\frac{53}{18}\right)^2+\frac{18\left(1603+\left(18980+35397i\sqrt{3}\right)^{2/3}\right)\left(x-\frac{53}{18}\right)}{\sqrt[3]{18980+35397i\sqrt{3}}}+(1898\right.$$

$$\frac{C}{9\sqrt{6x^3-53x^2+67x+70}}$$

$\downarrow$ 27

$$27\sqrt{6}\left(18\left(x-\frac{53}{18}\right)-\frac{1603+\left(18980+35397i\sqrt{3}\right)^{2/3}}{\sqrt[3]{18980+35397i\sqrt{3}}}\right)^{3/2}\left(324\left(x-\frac{53}{18}\right)^2+\frac{18\left(1603+\left(18980+35397i\sqrt{3}\right)^{2/3}\right)\left(x-\frac{53}{18}\right)}{\sqrt[3]{18980+35397i\sqrt{3}}}+(1898\right.$$

$$\frac{C}{9\sqrt{6x^3-53x^2+67x+70}}$$

$\downarrow$ 1269

$$27\sqrt{6}\left(18\left(x-\frac{53}{18}\right)-\frac{1603+\left(18980+35397i\sqrt{3}\right)^{2/3}}{\sqrt[3]{18980+35397i\sqrt{3}}}\right)^{3/2}\left(324\left(x-\frac{53}{18}\right)^2+\frac{18\left(1603+\left(18980+35397i\sqrt{3}\right)^{2/3}\right)\left(x-\frac{53}{18}\right)}{\sqrt[3]{18980+35397i\sqrt{3}}}+(1898$$

$$\frac{C}{9\sqrt{6x^3-53x^2+67x+70}}$$

$\downarrow$ 1172

$$27\sqrt{6}\left(18\left(x-\frac{53}{18}\right)-\frac{1603+\left(18980+35397i\sqrt{3}\right)^{2/3}}{\sqrt[3]{18980+35397i\sqrt{3}}}\right)^{3/2}\left(324\left(x-\frac{53}{18}\right)^{2}+\frac{18\left(1603+\left(18980+35397i\sqrt{3}\right)^{2/3}\right)\left(x-\frac{53}{18}\right)}{\sqrt[3]{18980+35397i\sqrt{3}}}+(1898\right.$$

↓ 321

$$27\sqrt{6}\left(18\left(x-\frac{53}{18}\right)-\frac{1603+\left(18980+35397i\sqrt{3}\right)^{2/3}}{\sqrt[3]{18980+35397i\sqrt{3}}}\right)^{3/2}\left(324\left(x-\frac{53}{18}\right)^2+\frac{18\left(1603+\left(18980+35397i\sqrt{3}\right)^{2/3}\right)\left(x-\frac{53}{18}\right)}{\sqrt[3]{18980+35397i\sqrt{3}}}+(1898\right.$$

↓ 327

$$27\sqrt{6}\left(18\left(x-\frac{53}{18}\right)-\frac{1603+\left(18980+35397i\sqrt{3}\right)^{2/3}}{\sqrt[3]{18980+35397i\sqrt{3}}}\right)^{3/2}\left(324\left(x-\frac{53}{18}\right)^2+\frac{18\left(1603+\left(18980+35397i\sqrt{3}\right)^{2/3}\right)\left(x-\frac{53}{18}\right)}{\sqrt[3]{18980+35397i\sqrt{3}}}+\left(1898\right.\right.$$

$$\frac{C}{9\sqrt{6x^3-53x^2+67x+70}}$$

input
```
Int[(A + B*x + C*x^2)/(70 + 67*x - 53*x^2 + 6*x^3)^(3/2),x]
```

output
```
-1/9*C/Sqrt[70 + 67*x - 53*x^2 + 6*x^3] + (27*Sqrt[6]*(-((1603 + (18980 +
(35397*I)*Sqrt[3])^(2/3))/(18980 + (35397*I)*Sqrt[3])^(1/3)) + 18*(-53/18
+ x))^(3/2)*(-1603 + 2569609/(18980 + (35397*I)*Sqrt[3])^(2/3) + (18980 +
(35397*I)*Sqrt[3])^(2/3) + (18*(1603 + (18980 + (35397*I)*Sqrt[3])^(2/3))*
(-53/18 + x))/(18980 + (35397*I)*Sqrt[3])^(1/3) + 324*(-53/18 + x)^2)^(3/2
)*(((-1/106191*I)*(18980 + (35397*I)*Sqrt[3])^(1/3)*(9*(3206*A + 7331*B +
31238*C) - (((2569609 + (18980 + (35397*I)*Sqrt[3])^(4/3))*(9*B + 53*C) -
(18980 + (35397*I)*Sqrt[3])^(1/3)*(1603 + (18980 + (35397*I)*Sqrt[3])^(2/3
))*(162*A + 477*B + 2206*C))*(-53/18 + x))/(18980 + (35397*I)*Sqrt[3])^(2/
3)))/(Sqrt[3]*(1603 - (18980 + (35397*I)*Sqrt[3])^(2/3))*Sqrt[-((1603 + (1
8980 + (35397*I)*Sqrt[3])^(2/3))/(18980 + (35397*I)*Sqrt[3])^(1/3)) + 18*(
-53/18 + x)]*Sqrt[-1603 + 2569609/(18980 + (35397*I)*Sqrt[3])^(2/3) + (189
80 + (35397*I)*Sqrt[3])^(2/3) + (18*(1603 + (18980 + (35397*I)*Sqrt[3])^(2
/3))*(-53/18 + x))/(18980 + (35397*I)*Sqrt[3])^(1/3) + 324*(-53/18 + x)^2]
) + ((I/212382)*(18980 + (35397*I)*Sqrt[3])^(1/3)*((18*(18980 + (35397*I)*
Sqrt[3])^(2/3)*(3206*A + 7331*B + 31238*C)*Sqrt[-1603 + 2569609/(18980 + (
35397*I)*Sqrt[3])^(2/3) + (18980 + (35397*I)*Sqrt[3])^(2/3) + (18*(1603 +
(18980 + (35397*I)*Sqrt[3])^(2/3))*(-53/18 + x))/(18980 + (35397*I)*Sqrt[3
])^(1/3) + 324*(-53/18 + x)^2])/((2569609 + 1603*(18980 + (35397*I)*Sqrt[3
])^(2/3) + (18980 + (35397*I)*Sqrt[3])^(4/3))*Sqrt[-((1603 + (18980 + (...
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 1172

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Simp[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m))   Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^
2 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[m^2, 1/4]
```

rule 1235

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^m
*(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*
m + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) -
f*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
)
```

rule 1237

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[
(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && LtQ[m, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2486
```
Int[((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (d_.)*(x_)^3)^(p_), x_S
ymbol] :> With[{r = Rt[-9*a*d^2 + Sqrt[3]*d*Sqrt[4*b^3*d + 27*a^2*d^2], 3]}
, Simp[(a + b*x + d*x^3)^p/(Simp[18^(1/3)*b*(d/(3*r)) - r/18^(1/3) + d*x, x
]^p*Simp[b*(d/3) + 12^(1/3)*b^2*(d^2/(3*r^2)) + r^2/(3*12^(1/3)) - d*(2^(1/
3)*b*(d/(3^(1/3)*r)) - r/18^(1/3))*x + d^2*x^2, x]^p)   Int[(e + f*x)^m*Sim
p[18^(1/3)*b*(d/(3*r)) - r/18^(1/3) + d*x, x]^p*Simp[b*(d/3) + 12^(1/3)*b^2
*(d^2/(3*r^2)) + r^2/(3*12^(1/3)) - d*(2^(1/3)*b*(d/(3^(1/3)*r)) - r/18^(1/
3))*x + d^2*x^2, x]^p, x], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && NeQ[4*
b^3 + 27*a^2*d, 0] &&  !IntegerQ[p]
```

rule 2490
```
Int[(P3_)^(p_.)*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] :> With[{a = Coeff[P3
, x, 0], b = Coeff[P3, x, 1], c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Su
bst[Int[((3*d*e - c*f)/(3*d) + f*x)^m*Simp[(2*c^3 - 9*b*c*d + 27*a*d^2)/(27
*d^2) - (c^2 - 3*b*d)*(x/(3*d)) + d*x^3, x]^p, x], x, x + c/(3*d)] /; NeQ[c
, 0]] /; FreeQ[{e, f, m, p}, x] && PolyQ[P3, x, 3]
```

rule 2526
```
Int[(Pm_)*(Qn_)^(p_), x_Symbol] :> With[{m = Expon[Pm, x], n = Expon[Qn, x]
}, Simp[Coeff[Pm, x, m]*(Qn^(p + 1)/(n*(p + 1)*Coeff[Qn, x, n])), x] + Simp
[1/(n*Coeff[Qn, x, n])   Int[ExpandToSum[n*Coeff[Qn, x, n]*Pm - Coeff[Pm, x
, m]*D[Qn, x], x]*Qn^p, x], x] /; EqQ[m, n - 1]] /; FreeQ[p, x] && PolyQ[Pm
, x] && PolyQ[Qn, x] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 1.25 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.65

method	result
elliptic	$-\frac{12\left(\left(\frac{7331B}{15468489}+\frac{31238C}{15468489}+\frac{3206A}{15468489}\right)x^2+\left(-\frac{10585B}{4884786}-\frac{37663C}{4884786}-\frac{3314A}{2442393}\right)x-\frac{112210B}{46405467}-\frac{256585C}{46405467}+\frac{13687A}{92810934}\right)}{\sqrt{6x^3-53x^2+67x+70}}+\frac{\left(-\frac{62476B}{5156163}-\frac{31}{51}\right.}{}$
risch	$-\frac{2\left(19236A\,x^2+43986B\,x^2+187428C\,x^2-125932Ax-201115Bx-715597Cx+13687A-224420B-513170C\right)}{15468489\sqrt{6x^3-53x^2+67x+70}}+\frac{(6412A+14662}{}$
default	$A\left(-\frac{12\left(\frac{3206}{15468489}x^2-\frac{3314}{2442393}x+\frac{13687}{92810934}\right)}{\sqrt{6x^3-53x^2+67x+70}}-\frac{14662\sqrt{76+114x}\,\sqrt{483-69x}\,\sqrt{285-114x}\ \operatorname{EllipticF}\left(\frac{\sqrt{76+114x}}{19},\frac{\sqrt{874}}{46}\right)}{6759729693\sqrt{6x^3-53x^2+67x+70}}+\frac{6412\sqrt{}}{}\right.$

input
```
int((C*x^2+B*x+A)/(6*x^3-53*x^2+67*x+70)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-12*((7331/15468489*B+31238/15468489*C+3206/15468489*A)*x^2+(-10585/488478
6*B-37663/4884786*C-3314/2442393*A)*x-112210/46405467*B-256585/46405467*C+
13687/92810934*A)/(6*x^3-53*x^2+67*x+70)^(1/2)+1/1311*(-62476/5156163*B-31
3339/5156163*C-14662/5156163*A)*(76+114*x)^(1/2)*(483-69*x)^(1/2)*(285-114
*x)^(1/2)/(6*x^3-53*x^2+67*x+70)^(1/2)*EllipticF(1/19*(76+114*x)^(1/2),1/4
6*874^(1/2))+1/1311*(14662/5156163*B+62476/5156163*C+6412/5156163*A)*(76+1
14*x)^(1/2)*(483-69*x)^(1/2)*(285-114*x)^(1/2)/(6*x^3-53*x^2+67*x+70)^(1/2
)*(-23/3*EllipticE(1/19*(76+114*x)^(1/2),1/46*874^(1/2))+7*EllipticF(1/19*
(76+114*x)^(1/2),1/46*874^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.65

$$\int \frac{A+Bx+Cx^2}{(70+67x-53x^2+6x^3)^{3/2}}\,dx = \frac{\sqrt{6}(6\,(37960\,A-173741\,B-1164437\,C)x^3-53\,(37960\,A-173741\,\ldots}{}$$

input
```
integrate((C*x^2+B*x+A)/(6*x^3-53*x^2+67*x+70)^(3/2),x, algorithm="fricas"
)
```

output
```
1/139216401*(sqrt(6)*(6*(37960*A - 173741*B - 1164437*C)*x^3 - 53*(37960*A
 - 173741*B - 1164437*C)*x^2 + 67*(37960*A - 173741*B - 1164437*C)*x + 265
7200*A - 12161870*B - 81510590*C)*weierstrassPInverse(1603/27, 18980/729,
x - 53/18) - 18*sqrt(6)*(6*(3206*A + 7331*B + 31238*C)*x^3 - 53*(3206*A +
7331*B + 31238*C)*x^2 + 67*(3206*A + 7331*B + 31238*C)*x + 224420*A + 5131
70*B + 2186660*C)*weierstrassZeta(1603/27, 18980/729, weierstrassPInverse(
1603/27, 18980/729, x - 53/18)) - 18*(6*(3206*A + 7331*B + 31238*C)*x^2 -
19*(6628*A + 10585*B + 37663*C)*x + 13687*A - 224420*B - 513170*C)*sqrt(6*
x^3 - 53*x^2 + 67*x + 70))/(6*x^3 - 53*x^2 + 67*x + 70)
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2}{(70+67x-53x^2+6x^3)^{3/2}}\,dx = \int \frac{A+Bx+Cx^2}{((x-7)(2x-5)(3x+2))^{\frac{3}{2}}}\,dx$$

input
```
integrate((C*x**2+B*x+A)/(6*x**3-53*x**2+67*x+70)**(3/2),x)
```

output
```
Integral((A + B*x + C*x**2)/((x - 7)*(2*x - 5)*(3*x + 2))**(3/2), x)
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{(70+67x-53x^2+6x^3)^{3/2}}\,dx = \int \frac{Cx^2+Bx+A}{(6x^3-53x^2+67x+70)^{\frac{3}{2}}}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(6*x^3-53*x^2+67*x+70)^(3/2),x, algorithm="maxima"
)
```

output
```
integrate((C*x^2 + B*x + A)/(6*x^3 - 53*x^2 + 67*x + 70)^(3/2), x)
```

Giac [F]

$$\int \frac{A+Bx+Cx^2}{(70+67x-53x^2+6x^3)^{3/2}}\,dx = \int \frac{Cx^2+Bx+A}{(6x^3-53x^2+67x+70)^{\frac{3}{2}}}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(6*x^3-53*x^2+67*x+70)^(3/2),x, algorithm="giac")
```

output
```
integrate((C*x^2 + B*x + A)/(6*x^3 - 53*x^2 + 67*x + 70)^(3/2), x)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 263, normalized size of antiderivative = 0.82

$$\int \frac{A+Bx+Cx^2}{(70+67x-53x^2+6x^3)^{3/2}}\,dx = -\frac{2\sqrt{\frac{2x}{9}-\frac{5}{9}}\sqrt{\frac{3x}{23}+\frac{2}{23}}\sqrt{\frac{21}{23}-\frac{3x}{23}}\left(\frac{3A}{437}-\frac{2B}{437}+\frac{4C}{1311}\right)\left(\frac{27\,\mathrm{E}\left(\mathrm{asin}\left(\sqrt{\frac{21}{23}-\frac{3x}{23}}\right)\middle|\frac{46}{27}\right)}{19}+\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{21}{23}-\frac{3x}{23}}\right)\middle|\frac{46}{27}\right)-\right.}{\sqrt{6x^3-53x^2+67x+70}} - \frac{92\sqrt{\frac{2x}{9}-\frac{5}{9}}\sqrt{\frac{3x}{23}+\frac{2}{23}}\sqrt{\frac{21}{23}-\frac{3x}{23}}\left(\frac{27\,\mathrm{E}\left(\mathrm{asin}\left(\sqrt{\frac{21}{23}-\frac{3x}{23}}\right)\middle|\frac{46}{27}\right)}{19}-\frac{23\sin\left(2\,\mathrm{asin}\left(\sqrt{\frac{21}{23}-\frac{3x}{23}}\right)\right)}{19\sqrt{\frac{2x}{9}-\frac{5}{9}}}\right)\left(\frac{2A}{171}+\frac{5B}{171}+\frac{25C}{342}\right)}{27\sqrt{6x^3-53x^2+67x+70}} - \frac{19\sqrt{\frac{6x}{19}+\frac{4}{19}}\sqrt{\frac{15}{19}-\frac{6x}{19}}\sqrt{\frac{21}{23}-\frac{3x}{23}}\left(\frac{46\,\mathrm{E}\left(\mathrm{asin}\left(\sqrt{\frac{6x}{19}+\frac{4}{19}}\right)\middle|\frac{19}{46}\right)}{27}-\frac{19\sin\left(2\,\mathrm{asin}\left(\sqrt{\frac{6x}{19}+\frac{4}{19}}\right)\right)}{54\sqrt{\frac{21}{23}-\frac{3x}{23}}}\right)\left(\frac{A}{207}+\frac{7B}{207}+\frac{49C}{207}\right)}{23\sqrt{6x^3-53x^2+67x+70}}$$

input
```
int((A + B*x + C*x^2)/(67*x - 53*x^2 + 6*x^3 + 70)^(3/2),x)
```

output
```
- (2*((2*x)/9 - 5/9)^(1/2)*((3*x)/23 + 2/23)^(1/2)*(21/23 - (3*x)/23)^(1/2
)*((3*A)/437 - (2*B)/437 + (4*C)/1311)*((27*ellipticE(asin((21/23 - (3*x)/
23)^(1/2)), 46/27))/19 + ellipticF(asin((21/23 - (3*x)/23)^(1/2)), 46/27)
- (27*((2*x)/9 - 5/9)^(1/2)*(21/23 - (3*x)/23)^(1/2))/(19*((3*x)/23 + 2/23
)^(1/2))))/(67*x - 53*x^2 + 6*x^3 + 70)^(1/2) - (92*((2*x)/9 - 5/9)^(1/2)*
((3*x)/23 + 2/23)^(1/2)*(21/23 - (3*x)/23)^(1/2)*((27*ellipticE(asin((21/2
3 - (3*x)/23)^(1/2)), 46/27))/19 - (23*sin(2*asin((21/23 - (3*x)/23)^(1/2)
)))/(19*((2*x)/9 - 5/9)^(1/2)))*((2*A)/171 + (5*B)/171 + (25*C)/342))/(27*
(67*x - 53*x^2 + 6*x^3 + 70)^(1/2)) - (19*((6*x)/19 + 4/19)^(1/2)*(15/19 -
 (6*x)/19)^(1/2)*(21/23 - (3*x)/23)^(1/2)*((46*ellipticE(asin(((6*x)/19 +
4/19)^(1/2)), 19/46))/27 - (19*sin(2*asin(((6*x)/19 + 4/19)^(1/2))))/(54*(
21/23 - (3*x)/23)^(1/2)))*(A/207 + (7*B)/207 + (49*C)/207))/(23*(67*x - 53
*x^2 + 6*x^3 + 70)^(1/2))
```

Reduce [F]

$$\int \frac{A + Bx + Cx^2}{(70 + 67x - 53x^2 + 6x^3)^{3/2}}\,dx = \text{too large to display}$$

input
```
int((C*x^2+B*x+A)/(6*x^3-53*x^2+67*x+70)^(3/2),x)
```

output
```
(212*sqrt(6*x**3 - 53*x**2 + 67*x + 70)*a*x + 134*sqrt(6*x**3 - 53*x**2 +
67*x + 70)*b*x + 280*sqrt(6*x**3 - 53*x**2 + 67*x + 70)*b + 3816*int((sqrt
(6*x**3 - 53*x**2 + 67*x + 70)*x**3)/(36*x**6 - 636*x**5 + 3613*x**4 - 626
2*x**3 - 2931*x**2 + 9380*x + 4900),x)*a*x**3 - 33708*int((sqrt(6*x**3 - 5
3*x**2 + 67*x + 70)*x**3)/(36*x**6 - 636*x**5 + 3613*x**4 - 6262*x**3 - 29
31*x**2 + 9380*x + 4900),x)*a*x**2 + 42612*int((sqrt(6*x**3 - 53*x**2 + 67
*x + 70)*x**3)/(36*x**6 - 636*x**5 + 3613*x**4 - 6262*x**3 - 2931*x**2 + 9
380*x + 4900),x)*a*x + 44520*int((sqrt(6*x**3 - 53*x**2 + 67*x + 70)*x**3)
/(36*x**6 - 636*x**5 + 3613*x**4 - 6262*x**3 - 2931*x**2 + 9380*x + 4900),
x)*a + 2412*int((sqrt(6*x**3 - 53*x**2 + 67*x + 70)*x**3)/(36*x**6 - 636*x
**5 + 3613*x**4 - 6262*x**3 - 2931*x**2 + 9380*x + 4900),x)*b*x**3 - 21306
*int((sqrt(6*x**3 - 53*x**2 + 67*x + 70)*x**3)/(36*x**6 - 636*x**5 + 3613*
x**4 - 6262*x**3 - 2931*x**2 + 9380*x + 4900),x)*b*x**2 + 26934*int((sqrt(
6*x**3 - 53*x**2 + 67*x + 70)*x**3)/(36*x**6 - 636*x**5 + 3613*x**4 - 6262
*x**3 - 2931*x**2 + 9380*x + 4900),x)*b*x + 28140*int((sqrt(6*x**3 - 53*x*
*2 + 67*x + 70)*x**3)/(36*x**6 - 636*x**5 + 3613*x**4 - 6262*x**3 - 2931*x
**2 + 9380*x + 4900),x)*b + 15120*int((sqrt(6*x**3 - 53*x**2 + 67*x + 70)*
x**2)/(36*x**6 - 636*x**5 + 3613*x**4 - 6262*x**3 - 2931*x**2 + 9380*x + 4
900),x)*b*x**3 - 133560*int((sqrt(6*x**3 - 53*x**2 + 67*x + 70)*x**2)/(36*
x**6 - 636*x**5 + 3613*x**4 - 6262*x**3 - 2931*x**2 + 9380*x + 4900),x)...
```

3.122 $\int \frac{A+Bx+Cx^2}{(70+67x-53x^2+6x^3)^{5/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 475

$$\int \frac{A+Bx+Cx^2}{(70+67x-53x^2+6x^3)^{5/2}}\,dx = -\frac{2(A+7B+49C)(5-2x)(7-x)(2+3x)}{621\,(70+67x-53x^2+6x^3)^{5/2}}$$
$$-\frac{2(146A+815B+4256C)(5-2x)(7-x)^2(2+3x)}{42849\,(70+67x-53x^2+6x^3)^{5/2}}$$
$$+\frac{4(11312A+57863B+298643C)(5-2x)(7-x)^3(2+3x)}{21981537\,(70+67x-53x^2+6x^3)^{5/2}}$$
$$-\frac{4(170170A+1604983B+8818156C)(5-2x)^2(7-x)^3(2+3x)}{3758842827\,(70+67x-53x^2+6x^3)^{5/2}}$$
$$-\frac{10(62482A-7760240B-45931763C)(5-2x)^3(7-x)^3(2+3x)}{547538105133\,(70+67x-53x^2+6x^3)^{5/2}}$$
$$-\frac{4(243399520A-417945287B-3367220459C)(5-2x)^3(7-x)^3(2+3x)^2}{239274151943121\,(70+67x-53x^2+6x^3)^{5/2}}$$
$$-\frac{4(243399520A-417945287B-3367220459C)(5-2x)^{5/2}(7-x)^{5/2}(2+3x)^{5/2}E\left(\arcsin\left(\frac{\sqrt{2+3x}}{\sqrt{23}}\right)\middle|\frac{46}{19}\right)}{37780129254177\sqrt{19}\,(70+67x-53x^2+6x^3)^{5/2}}$$
$$-\frac{4(8748670A+17573419B+70923418C)(-7+x)^{5/2}(-5+2x)^{5/2}(2+3x)^{5/2}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{19}{2}}}{\sqrt{2+3x}}\right.\right.}{1399264046451\sqrt{19}\,(70+67x-53x^2+6x^3)^{5/2}}$$

output
```
-2/621*(A+7*B+49*C)*(5-2*x)*(7-x)*(2+3*x)/(6*x^3-53*x^2+67*x+70)^(5/2)-2/4
2849*(146*A+815*B+4256*C)*(5-2*x)*(7-x)^2*(2+3*x)/(6*x^3-53*x^2+67*x+70)^(
5/2)+4/21981537*(11312*A+57863*B+298643*C)*(5-2*x)*(7-x)^3*(2+3*x)/(6*x^3-
53*x^2+67*x+70)^(5/2)-4/3758842827*(170170*A+1604983*B+8818156*C)*(5-2*x)^
2*(7-x)^3*(2+3*x)/(6*x^3-53*x^2+67*x+70)^(5/2)-10/547538105133*(62482*A-77
60240*B-45931763*C)*(5-2*x)^3*(7-x)^3*(2+3*x)/(6*x^3-53*x^2+67*x+70)^(5/2)
-4/239274151943121*(243399520*A-417945287*B-3367220459*C)*(5-2*x)^3*(7-x)^
3*(2+3*x)^2/(6*x^3-53*x^2+67*x+70)^(5/2)-4/717822455829363*(243399520*A-41
7945287*B-3367220459*C)*(5-2*x)^(5/2)*(7-x)^(5/2)*(2+3*x)^(5/2)*EllipticE(
1/23*(2+3*x)^(1/2)*23^(1/2),1/19*874^(1/2))*19^(1/2)/(6*x^3-53*x^2+67*x+70
)^(5/2)-4/26586016882569*(8748670*A+17573419*B+70923418*C)*(-7+x)^(5/2)*(-
5+2*x)^(5/2)*(2+3*x)^(5/2)*EllipticF(1/2*38^(1/2)/(2+3*x)^(1/2),1/19*874^(
1/2))*19^(1/2)/(6*x^3-53*x^2+67*x+70)^(5/2)
```

Mathematica [A] (verified)

Time = 10.52 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.49

$$\int \frac{A+Bx+Cx^2}{(70+67x-53x^2+6x^3)^{5/2}}\,dx =$$

$$-\frac{2\Big(15468489(A(13687-125932x+19236x^2)+B(-224420-201115x+43986x^2)+C(-513170-715}{}$$

input
```
Integrate[(A + B*x + C*x^2)/(70 + 67*x - 53*x^2 + 6*x^3)^(5/2),x]
```

output
```
(-2*(15468489*(A*(13687 - 125932*x + 19236*x^2) + B*(-224420 - 201115*x +
43986*x^2) + C*(-513170 - 715597*x + 187428*x^2)) + (70 + 67*x - 53*x^2 +
6*x^3)*(C*(210892425319 + 183141328016*x - 40406645508*x^2) + B*(526713535
12 + 13733513918*x - 5015343444*x^2) + 70*A*(381044737 - 392492842*x + 417
25632*x^2)) + Sqrt[46]*Sqrt[5 - 2*x]*Sqrt[7 - x]*Sqrt[2 + 3*x]*(70 + 67*x
- 53*x^2 + 6*x^3)*((486799040*A - 835890574*B - 6734440918*C)*EllipticE[Ar
cSin[Sqrt[2/19]*Sqrt[2 + 3*x]], 19/46] + 27*(-17809750*A + 3654362*B + 878
11979*C)*EllipticF[ArcSin[Sqrt[2/19]*Sqrt[2 + 3*x]], 19/46])))/(7178224558
29363*(70 + 67*x - 53*x^2 + 6*x^3)^(3/2))
```

Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 19.56 (sec) , antiderivative size = 2899, normalized size of antiderivative = 6.10, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {2526, 2490, 2486, 27, 1235, 27, 1235, 27, 1237, 27, 1237, 27, 1269, 1172, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{(6x^3-53x^2+67x+70)^{5/2}}dx$$

$$\Big\downarrow \text{2526}$$

$$\frac{1}{18}\int \frac{18A-67C+2(9B+53C)x}{(6x^3-53x^2+67x+70)^{5/2}}dx-\frac{C}{27(6x^3-53x^2+67x+70)^{3/2}}$$

$$\Big\downarrow \text{2490}$$

$$\frac{1}{18}\int \frac{\frac{1}{18}(18(18A-67C)+106(9B+53C))+2(9B+53C)\left(x-\frac{53}{18}\right)}{\left(6\left(x-\frac{53}{18}\right)^3-\frac{1603}{18}\left(x-\frac{53}{18}\right)-\frac{9490}{243}\right)^{5/2}}d\left(x-\frac{53}{18}\right)-\frac{C}{27(6x^3-53x^2+67x+70)^{3/2}}$$

$$\Big\downarrow \text{2486}$$

$$-\frac{C}{27(6x^3-53x^2+67x+70)^{3/2}}+$$

$$54\sqrt{2}\left(18\left(x-\frac{53}{18}\right)-\frac{1603+\left(18980+35397i\sqrt{3}\right)^{2/3}}{\sqrt[3]{18980+35397i\sqrt{3}}}\right)^{5/2}\left(324\left(x-\frac{53}{18}\right)^2+\frac{18\left(1603+\left(18980+35397i\sqrt{3}\right)^{2/3}\right)\left(x-\frac{53}{18}\right)}{\sqrt[3]{18980+35397i\sqrt{3}}}+(1898\right.$$

$$\Big\downarrow \text{27}$$

$$-\frac{C}{27\left(6x^3-53x^2+67x+70\right)^{3/2}}+$$

$$13122\sqrt{6}\left(18\left(x-\frac{53}{18}\right)-\frac{1603+\left(18980+35397i\sqrt{3}\right)^{2/3}}{\sqrt[3]{18980+35397i\sqrt{3}}}\right)^{5/2}\left(324\left(x-\frac{53}{18}\right)^2+\frac{18\left(1603+\left(18980+35397i\sqrt{3}\right)^{2/3}\right)\left(x-\frac{53}{18}\right)}{\sqrt[3]{18980+35397i\sqrt{3}}}+(1$$

$$\Big\downarrow \text{1235}$$

$$13122\sqrt{6}\left(18\left(x-\frac{53}{18}\right)-\frac{1603+\left(18980+35397i\sqrt{3}\right)^{2/3}}{\sqrt[3]{18980+35397i\sqrt{3}}}\right)^{5/2}\left(324\left(x-\frac{53}{18}\right)^2+\frac{18\left(1603+\left(18980+35397i\sqrt{3}\right)^{2/3}\right)\left(x-\frac{53}{18}\right)}{\sqrt[3]{18980+35397i\sqrt{3}}}+(1$$

$$\frac{C}{27\left(6x^3-53x^2+67x+70\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$13122\sqrt{6}\left(18\left(x-\frac{53}{18}\right)-\frac{1603+\left(18980+35397i\sqrt{3}\right)^{2/3}}{\sqrt[3]{18980+35397i\sqrt{3}}}\right)^{5/2}\left(324\left(x-\frac{53}{18}\right)^2+\frac{18\left(1603+\left(18980+35397i\sqrt{3}\right)^{2/3}\right)\left(x-\frac{53}{18}\right)}{\sqrt[3]{18980+35397i\sqrt{3}}}+(1\right.$$

$$\frac{C}{27\left(6x^3-53x^2+67x+70\right)^{3/2}}$$

$$\Big\downarrow 1235$$

$$13122\sqrt{6}\left(18\left(x-\frac{53}{18}\right)-\frac{1603+\left(18980+35397i\sqrt{3}\right)^{2/3}}{\sqrt[3]{18980+35397i\sqrt{3}}}\right)^{5/2}\left(324\left(x-\frac{53}{18}\right)^2+\frac{18\left(1603+\left(18980+35397i\sqrt{3}\right)^{2/3}\right)\left(x-\frac{53}{18}\right)}{\sqrt[3]{18980+35397i\sqrt{3}}}+(1\right.$$

$$\frac{C}{27\left(6x^3-53x^2+67x+70\right)^{3/2}}$$

$$\Big\downarrow \; 27$$

$$13122\sqrt{6}\left(18\left(x-\frac{53}{18}\right)-\frac{1603+\left(18980+35397i\sqrt{3}\right)^{2/3}}{\sqrt[3]{18980+35397i\sqrt{3}}}\right)^{5/2}\left(324\left(x-\frac{53}{18}\right)^2+\frac{18\left(1603+\left(18980+35397i\sqrt{3}\right)^{2/3}\right)\left(x-\frac{53}{18}\right)}{\sqrt[3]{18980+35397i\sqrt{3}}}+(1$$

$$\frac{C}{27\left(6x^3-53x^2+67x+70\right)^{3/2}}$$

$\downarrow$ 1237

$$13122\sqrt{6}\left(18\left(x-\tfrac{53}{18}\right)-\frac{1603+\left(18980+35397i\sqrt{3}\right)^{2/3}}{\sqrt[3]{18980+35397i\sqrt{3}}}\right)^{5/2}\left(324\left(x-\tfrac{53}{18}\right)^2+\frac{18\left(1603+\left(18980+35397i\sqrt{3}\right)^{2/3}\right)\left(x-\frac{53}{18}\right)}{\sqrt[3]{18980+35397i\sqrt{3}}}+(1$$

$$\downarrow 27$$

$$13122\sqrt{6}\left(18\left(x-\frac{53}{18}\right)-\frac{1603+\left(18980+35397i\sqrt{3}\right)^{2/3}}{\sqrt[3]{18980+35397i\sqrt{3}}}\right)^{5/2}\left(324\left(x-\frac{53}{18}\right)^2+\frac{18\left(1603+\left(18980+35397i\sqrt{3}\right)^{2/3}\right)\left(x-\frac{53}{18}\right)}{\sqrt[3]{18980+35397i\sqrt{3}}}+(1\right.$$

↓ 1237

$$13122\sqrt{6}\left(18\left(x-\frac{53}{18}\right)-\frac{1603+\left(18980+35397i\sqrt{3}\right)^{2/3}}{\sqrt[3]{18980+35397i\sqrt{3}}}\right)^{5/2}\left(324\left(x-\frac{53}{18}\right)^{2}+\frac{18\left(1603+\left(18980+35397i\sqrt{3}\right)^{2/3}\right)\left(x-\frac{53}{18}\right)}{\sqrt[3]{18980+35397i\sqrt{3}}}+(1$$

$$\Big\downarrow \text{27}$$

$$13122\sqrt{6}\left(18\left(x-\frac{53}{18}\right)-\frac{1603+\left(18980+35397i\sqrt{3}\right)^{2/3}}{\sqrt[3]{18980+35397i\sqrt{3}}}\right)^{5/2}\left(324\left(x-\frac{53}{18}\right)^2+\frac{18\left(1603+\left(18980+35397i\sqrt{3}\right)^{2/3}\right)\left(x-\frac{53}{18}\right)}{\sqrt[3]{18980+35397i\sqrt{3}}}+(1\right.$$

↓ 1269

$$13122\sqrt{6}\left(18\left(x-\tfrac{53}{18}\right)-\frac{1603+\left(18980+35397i\sqrt{3}\right)^{2/3}}{\sqrt[3]{18980+35397i\sqrt{3}}}\right)^{5/2}\left(324\left(x-\tfrac{53}{18}\right)^{2}+\frac{18\left(1603+\left(18980+35397i\sqrt{3}\right)^{2/3}\right)\left(x-\frac{53}{18}\right)}{\sqrt[3]{18980+35397i\sqrt{3}}}+(1\right.$$

↓ 1172

↓ 321

↓ 327

input
```
Int[(A + B*x + C*x^2)/(70 + 67*x - 53*x^2 + 6*x^3)^(5/2),x]
```

output
```
-1/27*C/(70 + 67*x - 53*x^2 + 6*x^3)^(3/2) + (13122*Sqrt[6]*(-((1603 + (18
980 + (35397*I)*Sqrt[3])^(2/3))/(18980 + (35397*I)*Sqrt[3])^(1/3)) + 18*(-
53/18 + x))^(5/2)*(-1603 + 2569609/(18980 + (35397*I)*Sqrt[3])^(2/3) + (18
980 + (35397*I)*Sqrt[3])^(2/3) + (18*(1603 + (18980 + (35397*I)*Sqrt[3])^(
2/3))*(-53/18 + x))/(18980 + (35397*I)*Sqrt[3])^(1/3) + 324*(-53/18 + x)^2
)^(5/2)*(((-1/318573*I)*(18980 + (35397*I)*Sqrt[3])^(1/3)*(9*(3206*A + 733
1*B + 31238*C) - (((2569609 + (18980 + (35397*I)*Sqrt[3])^(4/3))*(9*B + 53
*C) - (18980 + (35397*I)*Sqrt[3])^(1/3)*(1603 + (18980 + (35397*I)*Sqrt[3]
)^(2/3))*(162*A + 477*B + 2206*C))*(-53/18 + x))/(18980 + (35397*I)*Sqrt[3
])^(2/3)))/(Sqrt[3]*(1603 - (18980 + (35397*I)*Sqrt[3])^(2/3))*(-((1603 +
(18980 + (35397*I)*Sqrt[3])^(2/3))/(18980 + (35397*I)*Sqrt[3])^(1/3)) + 18
*(-53/18 + x))^(3/2)*(-1603 + 2569609/(18980 + (35397*I)*Sqrt[3])^(2/3) +
(18980 + (35397*I)*Sqrt[3])^(2/3) + (18*(1603 + (18980 + (35397*I)*Sqrt[3]
)^(2/3))*(-53/18 + x))/(18980 + (35397*I)*Sqrt[3])^(1/3) + 324*(-53/18 + x
)^2)^(3/2)) + ((I/637146)*(18980 + (35397*I)*Sqrt[3])^(1/3)*(((I/11799)*((
(7*I)*(2*(13674725584000*I - 171306047863719*Sqrt[3] - 1468348*(18980*I -
35397*Sqrt[3])*(18980 + (35397*I)*Sqrt[3])^(2/3) - 1603*(18980 + (35397*I)
*Sqrt[3])^(1/3)*(3038362027*I + 2015505180*Sqrt[3]))*A + (948018096198827*
I - 433351142966943*Sqrt[3] - (30201213892877*I - 7860364011*Sqrt[3])*(189
80 + (35397*I)*Sqrt[3])^(1/3) - 6715196*(18980*I - 35397*Sqrt[3])*(1898...
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 1172

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Simp[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m))   Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^
2 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[m^2, 1/4]
```

rule 1235

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^m
*(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*
m + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) -
f*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
)
```

rule 1237

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[
(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && LtQ[m, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2486
```
Int[((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (d_.)*(x_)^3)^(p_), x_S
ymbol] :> With[{r = Rt[-9*a*d^2 + Sqrt[3]*d*Sqrt[4*b^3*d + 27*a^2*d^2], 3]}
, Simp[(a + b*x + d*x^3)^p/(Simp[18^(1/3)*b*(d/(3*r)) - r/18^(1/3) + d*x, x
]^p*Simp[b*(d/3) + 12^(1/3)*b^2*(d^2/(3*r^2)) + r^2/(3*12^(1/3)) - d*(2^(1/
3)*b*(d/(3^(1/3)*r)) - r/18^(1/3))*x + d^2*x^2, x]^p)   Int[(e + f*x)^m*Sim
p[18^(1/3)*b*(d/(3*r)) - r/18^(1/3) + d*x, x]^p*Simp[b*(d/3) + 12^(1/3)*b^2
*(d^2/(3*r^2)) + r^2/(3*12^(1/3)) - d*(2^(1/3)*b*(d/(3^(1/3)*r)) - r/18^(1/
3))*x + d^2*x^2, x]^p, x], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && NeQ[4*
b^3 + 27*a^2*d, 0] &&  !IntegerQ[p]
```

rule 2490
```
Int[(P3_)^(p_.)*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] :> With[{a = Coeff[P3
, x, 0], b = Coeff[P3, x, 1], c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Su
bst[Int[((3*d*e - c*f)/(3*d) + f*x)^m*Simp[(2*c^3 - 9*b*c*d + 27*a*d^2)/(27
*d^2) - (c^2 - 3*b*d)*(x/(3*d)) + d*x^3, x]^p, x], x, x + c/(3*d)] /; NeQ[c
, 0]] /; FreeQ[{e, f, m, p}, x] && PolyQ[P3, x, 3]
```

rule 2526
```
Int[(Pm_)*(Qn_)^(p_), x_Symbol] :> With[{m = Expon[Pm, x], n = Expon[Qn, x]
}, Simp[Coeff[Pm, x, m]*(Qn^(p + 1)/(n*(p + 1)*Coeff[Qn, x, n])), x] + Simp
[1/(n*Coeff[Qn, x, n])   Int[ExpandToSum[n*Coeff[Qn, x, n]*Pm - Coeff[Pm, x
, m]*D[Qn, x], x]*Qn^p, x], x] /; EqQ[m, n - 1]] /; FreeQ[p, x] && PolyQ[Pm
, x] && PolyQ[Qn, x] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 1.13 (sec) , antiderivative size = 276, normalized size of antiderivative = 0.58

method	result
elliptic	$\frac{\left(\left(-\frac{7331B}{139216401}-\frac{31238C}{139216401}-\frac{3206A}{139216401}\right)x^2+\left(\frac{10585B}{43963074}+\frac{37663C}{43963074}+\frac{3314A}{21981537}\right)x+\frac{112210B}{417649203}+\frac{256585C}{417649203}-\frac{13687A}{835298406}\right)\sqrt{6x^3-53x^2+67}}{\left(x^3-\frac{53}{6}x^2+\frac{67}{6}x+\frac{35}{3}\right)^2}$
risch	$-\frac{2(17524765440x^5A-30092060664Bx^5-242439873048x^5C-319649088360x^4A+348214286040x^4B+3240400180020Cx^4+181}{}$
default	$A\left(\frac{\left(-\frac{3206}{139216401}x^2+\frac{3314}{21981537}x-\frac{13687}{835298406}\right)\sqrt{6x^3-53x^2+67x+70}}{\left(x^3-\frac{53}{6}x^2+\frac{67}{6}x+\frac{35}{3}\right)^2}-\frac{12\left(\frac{13336565795}{2153467367488089}+\frac{486799040}{717822455829363}x^2-\frac{723013130}{113340387762531}\right.}{\sqrt{6x^3-53x^2+67x+70}}\right.$

input
```
int((C*x^2+B*x+A)/(6*x^3-53*x^2+67*x+70)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
((-7331/139216401*B-31238/139216401*C-3206/139216401*A)*x^2+(10585/4396307
4*B+37663/43963074*C+3314/21981537*A)*x+112210/417649203*B+256585/41764920
3*C-13687/835298406*A)*(6*x^3-53*x^2+67*x+70)^(1/2)/(x^3-53/6*x^2+67/6*x+3
5/3)^2-12*((486799040/717822455829363*A-835890574/717822455829363*B-673444
0918/717822455829363*C)*x^2+(-723013130/113340387762531*A+361408261/113340
387762531*B+4819508632/113340387762531*C)*x+13336565795/2153467367488089*A
+26335676756/2153467367488089*B+210892425319/4306934734976178*C)/(6*x^3-53
*x^2+67*x+70)^(1/2)+1/1311*(558049940/239274151943121*A+10189562168/239274
151943121*B+57928013546/239274151943121*C)*(76+114*x)^(1/2)*(483-69*x)^(1/
2)*(285-114*x)^(1/2)/(6*x^3-53*x^2+67*x+70)^(1/2)*EllipticF(1/19*(76+114*x
)^(1/2),1/46*874^(1/2))+1/1311*(973598080/239274151943121*A-1671781148/239
274151943121*B-13468881836/239274151943121*C)*(76+114*x)^(1/2)*(483-69*x)^
(1/2)*(285-114*x)^(1/2)/(6*x^3-53*x^2+67*x+70)^(1/2)*(-23/3*EllipticE(1/19
*(76+114*x)^(1/2),1/46*874^(1/2))+7*EllipticF(1/19*(76+114*x)^(1/2),1/46*8
74^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 358, normalized size of antiderivative = 0.75

$$\int \frac{A+Bx+Cx^2}{(70+67x-53x^2+6x^3)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)/(6*x^3-53*x^2+67*x+70)^(5/2),x, algorithm="fricas"
)
```

output
```
2/6460402102464267*(35*sqrt(6)*(36*(440325694*A + 677197987*B + 2348953618
*C)*x^6 - 636*(440325694*A + 677197987*B + 2348953618*C)*x^5 + 3613*(44032
5694*A + 677197987*B + 2348953618*C)*x^4 - 6262*(440325694*A + 677197987*B
 + 2348953618*C)*x^3 - 2931*(440325694*A + 677197987*B + 2348953618*C)*x^2
 + 9380*(440325694*A + 677197987*B + 2348953618*C)*x + 2157595900600*A + 3
318270136300*B + 11509872728200*C)*weierstrassPInverse(1603/27, 18980/729,
 x - 53/18) - 18*sqrt(6)*(36*(243399520*A - 417945287*B - 3367220459*C)*x^
6 - 636*(243399520*A - 417945287*B - 3367220459*C)*x^5 + 3613*(243399520*A
 - 417945287*B - 3367220459*C)*x^4 - 6262*(243399520*A - 417945287*B - 336
7220459*C)*x^3 - 2931*(243399520*A - 417945287*B - 3367220459*C)*x^2 + 938
0*(243399520*A - 417945287*B - 3367220459*C)*x + 1192657648000*A - 2047931
906300*B - 16499380249100*C)*weierstrassZeta(1603/27, 18980/729, weierstra
ssPInverse(1603/27, 18980/729, x - 53/18)) - 9*(72*(243399520*A - 41794528
7*B - 3367220459*C)*x^5 - 60*(5327484806*A - 5803571434*B - 54006669667*C)
*x^4 + 10*(181188044744*A - 74787612733*B - 1114838108197*C)*x^3 - 21*(131
069521526*A + 73433970836*B - 55425390757*C)*x^2 - 9*(231565874002*A - 153
264610481*B - 1764497903840*C)*x + 2078836420243*A + 215556444460*B + 6824
505272200*C)*sqrt(6*x^3 - 53*x^2 + 67*x + 70))/(36*x^6 - 636*x^5 + 3613*x^
4 - 6262*x^3 - 2931*x^2 + 9380*x + 4900)
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2}{(70+67x-53x^2+6x^3)^{5/2}}\,dx = \int \frac{A+Bx+Cx^2}{((x-7)(2x-5)(3x+2))^{\frac{5}{2}}}\,dx$$

input
```
integrate((C*x**2+B*x+A)/(6*x**3-53*x**2+67*x+70)**(5/2),x)
```

output
```
Integral((A + B*x + C*x**2)/((x - 7)*(2*x - 5)*(3*x + 2))**(5/2), x)
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{(70+67x-53x^2+6x^3)^{5/2}}\,dx=\int \frac{Cx^2+Bx+A}{(6\,x^3-53\,x^2+67\,x+70)^{\frac{5}{2}}}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(6*x^3-53*x^2+67*x+70)^(5/2),x, algorithm="maxima"
)
```

output
```
integrate((C*x^2 + B*x + A)/(6*x^3 - 53*x^2 + 67*x + 70)^(5/2), x)
```

Giac [F]

$$\int \frac{A+Bx+Cx^2}{(70+67x-53x^2+6x^3)^{5/2}}\,dx=\int \frac{Cx^2+Bx+A}{(6\,x^3-53\,x^2+67\,x+70)^{\frac{5}{2}}}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(6*x^3-53*x^2+67*x+70)^(5/2),x, algorithm="giac")
```

output
```
integrate((C*x^2 + B*x + A)/(6*x^3 - 53*x^2 + 67*x + 70)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(70+67x-53x^2+6x^3)^{5/2}}\,dx=\int \frac{C\,x^2+B\,x+A}{(6\,x^3-53\,x^2+67\,x+70)^{5/2}}\,dx$$

input
```
int((A + B*x + C*x^2)/(67*x - 53*x^2 + 6*x^3 + 70)^(5/2),x)
```

output
```
int((A + B*x + C*x^2)/(67*x - 53*x^2 + 6*x^3 + 70)^(5/2), x)
```

Reduce [F]

$$\int \frac{A+Bx+Cx^2}{(70+67x-53x^2+6x^3)^{5/2}}\,dx = \text{too large to display}$$

input
```
int((C*x^2+B*x+A)/(6*x^3-53*x^2+67*x+70)^(5/2),x)
```

output
```
(2*sqrt(6*x**3 - 53*x**2 + 67*x + 70)*b + 11448*int(sqrt(6*x**3 - 53*x**2
+ 67*x + 70)/(216*x**9 - 5724*x**8 + 57798*x**7 - 269153*x**6 + 511851*x**
5 + 44979*x**4 - 1102457*x**3 + 163590*x**2 + 984900*x + 343000),x)*a*x**6
 - 202248*int(sqrt(6*x**3 - 53*x**2 + 67*x + 70)/(216*x**9 - 5724*x**8 + 5
7798*x**7 - 269153*x**6 + 511851*x**5 + 44979*x**4 - 1102457*x**3 + 163590
*x**2 + 984900*x + 343000),x)*a*x**5 + 1148934*int(sqrt(6*x**3 - 53*x**2 +
 67*x + 70)/(216*x**9 - 5724*x**8 + 57798*x**7 - 269153*x**6 + 511851*x**5
 + 44979*x**4 - 1102457*x**3 + 163590*x**2 + 984900*x + 343000),x)*a*x**4
- 1991316*int(sqrt(6*x**3 - 53*x**2 + 67*x + 70)/(216*x**9 - 5724*x**8 + 5
7798*x**7 - 269153*x**6 + 511851*x**5 + 44979*x**4 - 1102457*x**3 + 163590
*x**2 + 984900*x + 343000),x)*a*x**3 - 932058*int(sqrt(6*x**3 - 53*x**2 +
67*x + 70)/(216*x**9 - 5724*x**8 + 57798*x**7 - 269153*x**6 + 511851*x**5
+ 44979*x**4 - 1102457*x**3 + 163590*x**2 + 984900*x + 343000),x)*a*x**2 +
 2982840*int(sqrt(6*x**3 - 53*x**2 + 67*x + 70)/(216*x**9 - 5724*x**8 + 57
798*x**7 - 269153*x**6 + 511851*x**5 + 44979*x**4 - 1102457*x**3 + 163590*
x**2 + 984900*x + 343000),x)*a*x + 1558200*int(sqrt(6*x**3 - 53*x**2 + 67*
x + 70)/(216*x**9 - 5724*x**8 + 57798*x**7 - 269153*x**6 + 511851*x**5 + 4
4979*x**4 - 1102457*x**3 + 163590*x**2 + 984900*x + 343000),x)*a + 7236*in
t(sqrt(6*x**3 - 53*x**2 + 67*x + 70)/(216*x**9 - 5724*x**8 + 57798*x**7 -
269153*x**6 + 511851*x**5 + 44979*x**4 - 1102457*x**3 + 163590*x**2 + 9...
```

3.123 $\int ((a+bx)(c+dx)(e+fx))^p (A+Bx+Cx^2)\,dx$

Optimal result

Integrand size = 29, antiderivative size = 386

$$\int ((a+bx)(c+dx)(e+fx))^p \left(A+Bx+Cx^2\right)\,dx = \frac{C(a+bx)(c+dx)(e+fx)((a+bx)(c+dx)(e+fx))^p}{3bdf(1+p)} + \frac{(2a^2Cdf-b^2(cCe-3Adf)+ab(Cde+cCf-3Bdf))(a+bx)\left(\frac{b(c+dx)}{bc-ad}\right)^{-p}\left(\frac{b(e+fx)}{be-af}\right)^{-p}((a+bx)(c+ \cdots}{3b^3df(1+p)} - \frac{(2aCdf-b(3Bdf-2C(de+cf)))(a+bx)^2\left(\frac{b(c+dx)}{bc-ad}\right)^{-p}\left(\frac{b(e+fx)}{be-af}\right)^{-p}((a+bx)(c+dx)(e+fx))^p \operatorname{App}\cdots}{3b^3df(2+p)}$$

output
```
1/3*C*(b*x+a)*(d*x+c)*(f*x+e)*((b*x+a)*(d*x+c)*(f*x+e))^p/b/d/f/(p+1)+1/3*
(2*a^2*C*d*f-b^2*(-3*A*d*f+C*c*e)+a*b*(-3*B*d*f+C*c*f+C*d*e))*(b*x+a)*((b*
x+a)*(d*x+c)*(f*x+e))^p*AppellF1(p+1,-p,-p,2+p,-d*(b*x+a)/(-a*d+b*c),-f*(b
*x+a)/(-a*f+b*e))/b^3/d/f/(p+1)/((b*(d*x+c)/(-a*d+b*c))^p)/((b*(f*x+e)/(-a
*f+b*e))^p)-1/3*(2*a*C*d*f-b*(3*B*d*f-2*C*(c*f+d*e)))*(b*x+a)^2*((b*x+a)*(
d*x+c)*(f*x+e))^p*AppellF1(2+p,-p,-p,3+p,-d*(b*x+a)/(-a*d+b*c),-f*(b*x+a)/
(-a*f+b*e))/b^3/d/f/(2+p)/((b*(d*x+c)/(-a*d+b*c))^p)/((b*(f*x+e)/(-a*f+b*e
))^p)
```

Mathematica [F]

$$\int ((a+bx)(c+dx)(e+fx))^p \left(A+Bx+Cx^2\right) dx = \int ((a+bx)(c+dx)(e+fx))^p \left(A+Bx+Cx^2\right) dx$$

input
```
Integrate[((a + b*x)*(c + d*x)*(e + f*x))^p*(A + B*x + C*x^2),x]
```

output
```
Integrate[((a + b*x)*(c + d*x)*(e + f*x))^p*(A + B*x + C*x^2), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(A+Bx+Cx^2\right)((a+bx)(c+dx)(e+fx))^p dx$$

$$\Big\downarrow \text{2526}$$

$$\frac{\int -((a+bx)(c+dx)(e+fx))^p(aC(de+cf)+b(cCe-3Adf)-(3bBdf-2aCdf-2bC(de+cf))x)dx}{3bdf}+\frac{C((a+bx)(c+dx)(e+fx))^{p+1}}{3bdf(p+1)}$$

$$\Big\downarrow \text{25}$$

$$\frac{C((a+bx)(c+dx)(e+fx))^{p+1}}{3bdf(p+1)}-\frac{\int ((a+bx)(c+dx)(e+fx))^p(bcCe+aCde+acCf-3Abdf-(3bBdf-2aCdf-2bC(de+cf))x)dx}{3bdf}$$

$$\Big\downarrow \text{2490}$$

$$\frac{C((a+bx)(c+dx)(e+fx))^{p+1}}{3bdf(p+1)}-$$

$$\int \left(\frac{3bdf(bcCe+aCde+acCf-3Abdf)-(bde+bcf+adf)(-3bBdf+2aCdf+2bC(de+cf))}{3bdf}+(-3bBdf+2aCdf+2bC(de+cf))\left(bde\right.\right.$$

$$\Big\downarrow \text{7292}$$

$$\frac{C((a+bx)(c+dx)(e+fx))^{p+1}}{3bdf(p+1)}-$$

$$\int\left(\frac{3bdf(bcCe+aCde+acCf-3Abdf)-(bde+bcf+adf)(-3bBdf+2aCdf+2bC(de+cf))}{3bdf}+(-3bBdf+2aCdf+2bC(de+cf))\left(\frac{bde}{\cdots}\right.\right.$$

$$\Big\downarrow \text{7293}$$

$$\frac{C((a+bx)(c+dx)(e+fx))^{p+1}}{3bdf(p+1)}-$$

$$\int\left(\frac{(-((C(2d^2e^2+cdfe+2c^2f^2)+3df(3Adf-B(de+cf)))b^2)-adf(Cde+cCf-3Bdf)b-2a^2Cd^2f^2)\left(bdf\left(\frac{bde+bcf+adf}{3bdf}+x\right)^3+\frac{1}{3}\left(3(bce+ade+\cdots\right.\right.}{3bdf}\right.$$

$$\Big\downarrow \text{7293}$$

$$\frac{C((a+bx)(c+dx)(e+fx))^{p+1}}{3bdf(p+1)}-$$

$$\int\left(\frac{(-((C(2d^2e^2+cdfe+2c^2f^2)+3df(3Adf-B(de+cf)))b^2)-adf(Cde+cCf-3Bdf)b-2a^2Cd^2f^2)\left(bdf\left(\frac{bde+bcf+adf}{3bdf}+x\right)^3+\frac{1}{3}\left(-\frac{(bde+bcf+\cdots}{bdf}\right.\right.}{3bdf}\right.$$

$$\Big\downarrow \text{7299}$$

$$\frac{C((a+bx)(c+dx)(e+fx))^{p+1}}{3bdf(p+1)}-$$

$$\int\left(\frac{(-((C(2d^2e^2+cdfe+2c^2f^2)+3df(3Adf-B(de+cf)))b^2)-adf(Cde+cCf-3Bdf)b-2a^2Cd^2f^2)\left(bdf\left(\frac{bde+bcf+adf}{3bdf}+x\right)^3+\frac{1}{3}\left(-\frac{(bde+bcf+\cdots}{bdf}\right.\right.}{3bdf}\right.$$

input
```
Int[((a + b*x)*(c + d*x)*(e + f*x))^p*(A + B*x + C*x^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2490
```
Int[(P3_)^(p_.)*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] :> With[{a = Coeff[P3
, x, 0], b = Coeff[P3, x, 1], c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Su
bst[Int[((3*d*e - c*f)/(3*d) + f*x)^m*Simp[(2*c^3 - 9*b*c*d + 27*a*d^2)/(27
*d^2) - (c^2 - 3*b*d)*(x/(3*d)) + d*x^3, x]^p, x], x, x + c/(3*d)] /; NeQ[c
, 0]] /; FreeQ[{e, f, m, p}, x] && PolyQ[P3, x, 3]
```

rule 2526
```
Int[(Pm_)*(Qn_)^(p_), x_Symbol] :> With[{m = Expon[Pm, x], n = Expon[Qn, x]
}, Simp[Coeff[Pm, x, m]*(Qn^(p + 1)/(n*(p + 1)*Coeff[Qn, x, n])), x] + Simp
[1/(n*Coeff[Qn, x, n])   Int[ExpandToSum[n*Coeff[Qn, x, n]*Pm - Coeff[Pm, x
, m]*D[Qn, x], x]*Qn^p, x], x] /; EqQ[m, n - 1]] /; FreeQ[p, x] && PolyQ[Pm
, x] && PolyQ[Qn, x] && NeQ[p, -1]
```

rule 7292
```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int ((bx+a)\,(dx+c)\,(fx+e))^{p}\left(C\,x^{2}+Bx+A\right)dx$$

input
```
int(((b*x+a)*(d*x+c)*(f*x+e))^p*(C*x^2+B*x+A),x)
```

output
```
int(((b*x+a)*(d*x+c)*(f*x+e))^p*(C*x^2+B*x+A),x)
```

Fricas [F]

$$\int ((a+bx)(c+dx)(e+fx))^p \left(A+Bx+Cx^2\right) dx = \int (Cx^2+Bx+A)((bx+a)(dx+c)(fx+e))^p dx$$

input
```
integrate((((b*x+a)*(d*x+c)*(f*x+e))^p*(C*x^2+B*x+A),x, algorithm="fricas")
```

output
```
integral((C*x^2 + B*x + A)*(b*d*f*x^3 + a*c*e + (b*d*e + (b*c + a*d)*f)*x^
2 + (a*c*f + (b*c + a*d)*e)*x)^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int ((a+bx)(c+dx)(e+fx))^p \left(A+Bx+Cx^2\right) dx = \text{Timed out}$$

input
```
integrate(((b*x+a)*(d*x+c)*(f*x+e))**p*(C*x**2+B*x+A),x)
```

output
```
Timed out
```

Maxima [F]

$$\int ((a+bx)(c+dx)(e+fx))^p \left(A+Bx+Cx^2\right) dx = \int (Cx^2+Bx+A)((bx+a)(dx+c)(fx+e))^p dx$$

input
```
integrate((((b*x+a)*(d*x+c)*(f*x+e))^p*(C*x^2+B*x+A),x, algorithm="maxima")
```

output
```
integrate((C*x^2 + B*x + A)*((b*x + a)*(d*x + c)*(f*x + e))^p, x)
```

Giac [F]

$$\int ((a+bx)(c+dx)(e+fx))^p \left(A+Bx+Cx^2\right) dx = \int \left(Cx^2+Bx+A\right)((bx+a)(dx+c)(fx+e))^p \, dx$$

input
```
integrate(((b*x+a)*(d*x+c)*(f*x+e))^p*(C*x^2+B*x+A),x, algorithm="giac")
```

output
```
integrate((C*x^2 + B*x + A)*((b*x + a)*(d*x + c)*(f*x + e))^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int ((a+bx)(c+dx)(e+fx))^p \left(A+Bx+Cx^2\right) dx = \int \left(C\,x^2+B\,x+A\right)\,\left(\left(e+f\,x\right)\,\left(a+b\,x\right)\,\left(c+d\,x\right)\right)^p dx$$

input
```
int((A + B*x + C*x^2)*((e + f*x)*(a + b*x)*(c + d*x))^p,x)
```

output
```
int((A + B*x + C*x^2)*((e + f*x)*(a + b*x)*(c + d*x))^p, x)
```

Reduce [F]

$$\int ((a+bx)(c+dx)(e+fx))^p \left(A+Bx+Cx^2\right) dx = \int \left((bx+a)\,(dx+c)\,(fx+e)\right)^p \left(C\,x^2+Bx+A\right) dx$$

input
```
int(((b*x+a)*(d*x+c)*(f*x+e))^p*(C*x^2+B*x+A),x)
```

output
```
int(((b*x+a)*(d*x+c)*(f*x+e))^p*(C*x^2+B*x+A),x)
```

3.124 $\int (A+Bx+Cx^2)(ace+(bce+ade+acf)x+(b$

Optimal result

Integrand size = 57, antiderivative size = 470

$$\int (A+Bx+Cx^2)\left(ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3\right)^p\,dx$$
$$=\frac{C(a+bx)(c+dx)(e+fx)\left(ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3\right)^p}{3bdf(1+p)}$$
$$+\frac{(2a^2Cdf-b^2(cCe-3Adf)+ab(Cde+cCf-3Bdf))(a+bx)\left(\frac{b(c+dx)}{bc-ad}\right)^{-p}\left(\frac{b(e+fx)}{be-af}\right)^{-p}(ace+(bce+}{3b^3df(1}$$
$$-\frac{(2aCdf-b(3Bdf-2C(de+cf)))(a+bx)^2\left(\frac{b(c+dx)}{bc-ad}\right)^{-p}\left(\frac{b(e+fx)}{be-af}\right)^{-p}(ace+(bce+ade+acf)x+(bd}{3b^3df(2+p)}$$

output
```
1/3*C*(b*x+a)*(d*x+c)*(f*x+e)*(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*
d*e)*x^2+b*d*f*x^3)^p/b/d/f/(p+1)+1/3*(2*a^2*C*d*f-b^2*(-3*A*d*f+C*c*e)+a*
b*(-3*B*d*f+C*c*f+C*d*e))*(b*x+a)*(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*
f+b*d*e)*x^2+b*d*f*x^3)^p*AppellF1(p+1,-p,-p,2+p,-d*(b*x+a)/(-a*d+b*c),-f*
(b*x+a)/(-a*f+b*e))/b^3/d/f/(p+1)/((b*(d*x+c)/(-a*d+b*c))^p)/((b*(f*x+e)/(
-a*f+b*e))^p)-1/3*(2*a*C*d*f-b*(3*B*d*f-2*C*(c*f+d*e)))*(b*x+a)^2*(a*c*e+(
a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d*f*x^3)^p*AppellF1(2+p,-p,
-p,3+p,-d*(b*x+a)/(-a*d+b*c),-f*(b*x+a)/(-a*f+b*e))/b^3/d/f/(2+p)/((b*(d*x
+c)/(-a*d+b*c))^p)/((b*(f*x+e)/(-a*f+b*e))^p)
```

Mathematica [F]

$$\int (A+Bx+Cx^2)\left(ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3\right)^p\,dx$$
$$=\int (A+Bx+Cx^2)\left(ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3\right)^p\,dx$$

input
```
Integrate[(A + B*x + C*x^2)*(a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e +
b*c*f + a*d*f)*x^2 + b*d*f*x^3)^p,x]
```

output
```
Integrate[(A + B*x + C*x^2)*(a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e +
b*c*f + a*d*f)*x^2 + b*d*f*x^3)^p, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (A+Bx+Cx^2)\left(x^2(adf+bcf+bde)+x(acf+ade+bce)+ace+bdfx^3\right)^p\,dx$$

$$\Big\downarrow \text{2526}$$

$$\frac{\int -((aC(de+cf)+b(cCe-3Adf)-(3bBdf-2aCdf-2bC(de+cf))x)\left(bdfx^3+(bde+bcf+adf)x^2+(bce\right.}{3bdf}$$
$$\frac{C\left(x^2(adf+bcf+bde)+x(acf+ade+bce)+ace+bdfx^3\right)^{p+1}}{3bdf(p+1)}$$

$$\Big\downarrow \text{25}$$

$$\frac{C\left(x^2(adf+bcf+bde)+x(acf+ade+bce)+ace+bdfx^3\right)^{p+1}}{3bdf(p+1)}-$$
$$\frac{\int (bcCe+aCde+acCf-3Abdf-(3bBdf-2aCdf-2bC(de+cf))x)\left(bdfx^3+(bde+bcf+adf)x^2+(bce+\right.}{3bdf}$$

$$\Big\downarrow \text{2490}$$

$$\frac{C\left(x^2(adf+bcf+bde)+x(acf+ade+bce)+ace+bdfx^3\right)^{p+1}}{3bdf(p+1)}-$$

$$\int\left(\frac{3bdf(bcCe+aCde+acCf-3Abdf)-(bde+bcf+adf)(-3bBdf+2aCdf+2bC(de+cf))}{3bdf}+(-3bBdf+2aCdf+2bC(de+cf))\left(\frac{bde}{}\right.\right.$$

↓ 7292

$$\frac{C\left(x^2(adf+bcf+bde)+x(acf+ade+bce)+ace+bdfx^3\right)^{p+1}}{3bdf(p+1)}-$$

$$\int\left(\frac{3bdf(bcCe+aCde+acCf-3Abdf)-(bde+bcf+adf)(-3bBdf+2aCdf+2bC(de+cf))}{3bdf}+(-3bBdf+2aCdf+2bC(de+cf))\left(\frac{bde}{}\right.\right.$$

↓ 7293

$$\frac{C\left(x^2(adf+bcf+bde)+x(acf+ade+bce)+ace+bdfx^3\right)^{p+1}}{3bdf(p+1)}-$$

$$\int\left(\frac{(-((C(2d^2e^2+cdfe+2c^2f^2)+3df(3Adf-B(de+cf)))b^2)-adf(Cde+cCf-3Bdf)b-2a^2Cd^2f^2)\left(bdf\left(\frac{bde+bcf+adf}{3bdf}+x\right)^3+\frac{1}{3}\left(3(bce+ade+\right.\right.}{3bdf}\right.$$

↓ 7293

$$\frac{C\left(x^2(adf+bcf+bde)+x(acf+ade+bce)+ace+bdfx^3\right)^{p+1}}{3bdf(p+1)}-$$

$$\int\left(\frac{(-((C(2d^2e^2+cdfe+2c^2f^2)+3df(3Adf-B(de+cf)))b^2)-adf(Cde+cCf-3Bdf)b-2a^2Cd^2f^2)\left(bdf\left(\frac{bde+bcf+adf}{3bdf}+x\right)^3+\frac{1}{3}\left(-\frac{(bde+bcf+}{bdf}\right.\right.}{3bdf}\right.$$

↓ 7299

$$\frac{C\left(x^2(adf+bcf+bde)+x(acf+ade+bce)+ace+bdfx^3\right)^{p+1}}{3bdf(p+1)}-$$

$$\int\left(\frac{(-((C(2d^2e^2+cdfe+2c^2f^2)+3df(3Adf-B(de+cf)))b^2)-adf(Cde+cCf-3Bdf)b-2a^2Cd^2f^2)\left(bdf\left(\frac{bde+bcf+adf}{3bdf}+x\right)^3+\frac{1}{3}\left(-\frac{(bde+bcf+}{bdf}\right.\right.}{3bdf}\right.$$

input
```
Int[(A + B*x + C*x^2)*(a*c*e + (b*c*e + a*d*e + a*c*f)*x + (b*d*e + b*c*f
+ a*d*f)*x^2 + b*d*f*x^3)^p,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2490
```
Int[(P3_)^(p_.)*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] :> With[{a = Coeff[P3
, x, 0], b = Coeff[P3, x, 1], c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Su
bst[Int[((3*d*e - c*f)/(3*d) + f*x)^m*Simp[(2*c^3 - 9*b*c*d + 27*a*d^2)/(27
*d^2) - (c^2 - 3*b*d)*(x/(3*d)) + d*x^3, x]^p, x], x, x + c/(3*d)] /; NeQ[c
, 0]] /; FreeQ[{e, f, m, p}, x] && PolyQ[P3, x, 3]
```

rule 2526
```
Int[(Pm_)*(Qn_)^(p_), x_Symbol] :> With[{m = Expon[Pm, x], n = Expon[Qn, x]
}, Simp[Coeff[Pm, x, m]*(Qn^(p + 1)/(n*(p + 1)*Coeff[Qn, x, n])), x] + Simp
[1/(n*Coeff[Qn, x, n])   Int[ExpandToSum[n*Coeff[Qn, x, n]*Pm - Coeff[Pm, x
, m]*D[Qn, x], x]*Qn^p, x], x] /; EqQ[m, n - 1]] /; FreeQ[p, x] && PolyQ[Pm
, x] && PolyQ[Qn, x] && NeQ[p, -1]
```

rule 7292
```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int (C\,x^2+Bx+A)\left(ace+(acf+ade+bce)\,x+(adf+bcf+bde)\,x^2+bdf\,x^3\right)^p dx$$

input
```
int((C*x^2+B*x+A)*(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d
*f*x^3)^p,x)
```

output
```
int((C*x^2+B*x+A)*(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d
*f*x^3)^p,x)
```

Fricas [F]

$$\int (A+Bx+Cx^2)\left(ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3\right)^p\,dx$$
$$=\int (Cx^2+Bx+A)\left(bdfx^3+ace+(bde+bcf+adf)x^2+(bce+ade+acf)x\right)^p\,dx$$

input
```
integrate((C*x^2+B*x+A)*(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x
^2+b*d*f*x^3)^p,x, algorithm="fricas")
```

output
```
integral((C*x^2 + B*x + A)*(b*d*f*x^3 + a*c*e + (b*d*e + (b*c + a*d)*f)*x^
2 + (a*c*f + (b*c + a*d)*e)*x)^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int (A+Bx+Cx^2)\left(ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3\right)^p\,dx$$
$$=\text{Timed out}$$

input
```
integrate((C*x**2+B*x+A)*(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*
x**2+b*d*f*x**3)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \left(A+Bx+Cx^2\right)\left(ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3\right)^p\,dx$$
$$=\int \left(Cx^2+Bx+A\right)\left(bdfx^3+ace+(bde+bcf+adf)x^2+(bce+ade+acf)x\right)^p\,dx$$

input
```
integrate((C*x^2+B*x+A)*(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x
^2+b*d*f*x^3)^p,x, algorithm="maxima")
```

output
```
integrate((C*x^2 + B*x + A)*(b*d*f*x^3 + a*c*e + (b*d*e + b*c*f + a*d*f)*x
^2 + (b*c*e + a*d*e + a*c*f)*x)^p, x)
```

Giac [F]

$$\int \left(A+Bx+Cx^2\right)\left(ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3\right)^p\,dx$$
$$=\int \left(Cx^2+Bx+A\right)\left(bdfx^3+ace+(bde+bcf+adf)x^2+(bce+ade+acf)x\right)^p\,dx$$

input
```
integrate((C*x^2+B*x+A)*(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x
^2+b*d*f*x^3)^p,x, algorithm="giac")
```

output
```
integrate((C*x^2 + B*x + A)*(b*d*f*x^3 + a*c*e + (b*d*e + b*c*f + a*d*f)*x
^2 + (b*c*e + a*d*e + a*c*f)*x)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(A+Bx+Cx^2\right) \left(ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3\right)^p\,dx$$
$$=\int \left(C\,x^2+B\,x+A\right)\,\left(b\,d\,f\,x^3+\left(a\,d\,f+b\,c\,f+b\,d\,e\right)\,x^2+\left(a\,c\,f+a\,d\,e+b\,c\,e\right)\,x+a\,c\,e\right)^p\,dx$$

input
```
int((A + B*x + C*x^2)*(x^2*(a*d*f + b*c*f + b*d*e) + x*(a*c*f + a*d*e + b*
c*e) + a*c*e + b*d*f*x^3)^p,x)
```

output
```
int((A + B*x + C*x^2)*(x^2*(a*d*f + b*c*f + b*d*e) + x*(a*c*f + a*d*e + b*
c*e) + a*c*e + b*d*f*x^3)^p, x)
```

Reduce [F]

$$\int \left(A+Bx+Cx^2\right) \left(ace+(bce+ade+acf)x+(bde+bcf+adf)x^2+bdfx^3\right)^p\,dx$$
$$=\int \left(C\,x^2+Bx+A\right) \left(ace+\left(acf+ade+bce\right)x+\left(adf+bcf+bde\right)x^2+bdf\,x^3\right)^p dx$$

input
```
int((C*x^2+B*x+A)*(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d
*f*x^3)^p,x)
```

output
```
int((C*x^2+B*x+A)*(a*c*e+(a*c*f+a*d*e+b*c*e)*x+(a*d*f+b*c*f+b*d*e)*x^2+b*d
*f*x^3)^p,x)
```

3.125 $\int (A+Bx+Cx^2)\left(ad+(bd+ae)x+(cd+be)x^2\right.$

Optimal result

Integrand size = 43, antiderivative size = 431

$$\begin{aligned}
&\int (A+Bx+Cx^2)\left(ad+(bd+ae)x+(cd+be)x^2+cex^3\right)^2\,dx\\
&=a^2Ad^2x+\frac{1}{2}ad(aBd+2A(bd+ae))x^2\\
&\quad+\frac{1}{3}\left(ad(2bBd+aCd+2aBe)+A\left(b^2d^2+4abde+a\left(2cd^2+ae^2\right)\right)\right)x^3+\frac{1}{4}\left(b^2d(Bd+2Ae)\right.\\
&\qquad\left.+a\left(2Bcd^2+4Acde+2aCde+aBe^2\right)+2b\left(ad(Cd+2Be)+A\left(cd^2+ae^2\right)\right)\right)x^4\\
&\quad+\frac{1}{5}\left(Ac\left(cd^2+2ae^2\right)+b^2\left(Cd^2+e(2Bd+Ae)\right)+a\left(aCe^2+2cd(Cd+2Be)\right)\right.\\
&\qquad\left.+2b\left(2(Ac+aC)de+B\left(cd^2+ae^2\right)\right)\right)x^5+\frac{1}{6}\left(B\left(c^2d^2+b^2e^2+2ce(2bd+ae)\right)\right.\\
&\qquad\left.+2\left(b^2Cde+c(Ac+2aC)de+b\left(cCd^2+Ace^2+aCe^2\right)\right)\right)x^6\\
&\quad+\frac{1}{7}\left(b^2Ce^2+2ce(2bCd+bBe+aCe)+c^2\left(Cd^2+e(2Bd+Ae)\right)\right)x^7\\
&\quad+\frac{1}{8}ce(2cCd+Bce+2bCe)x^8+\frac{1}{9}c^2Ce^2x^9
\end{aligned}$$

output
```
a^2*A*d^2*x+1/2*a*d*(a*B*d+2*A*(a*e+b*d))*x^2+1/3*(a*d*(2*B*a*e+2*B*b*d+C*
a*d)+A*(b^2*d^2+4*a*b*d*e+a*(a*e^2+2*c*d^2)))*x^3+1/4*(b^2*d*(2*A*e+B*d)+a
*(4*A*c*d*e+B*a*e^2+2*B*c*d^2+2*C*a*d*e)+2*b*(a*d*(2*B*e+C*d)+A*(a*e^2+c*d
^2)))*x^4+1/5*(A*c*(2*a*e^2+c*d^2)+b^2*(C*d^2+e*(A*e+2*B*d))+a*(a*C*e^2+2*
c*d*(2*B*e+C*d))+2*b*(2*(A*c+C*a)*d*e+B*(a*e^2+c*d^2)))*x^5+1/6*(B*(c^2*d^
2+b^2*e^2+2*c*e*(a*e+2*b*d))+2*b^2*C*d*e+2*c*(A*c+2*C*a)*d*e+2*b*(A*c*e^2+
C*a*e^2+C*c*d^2))*x^6+1/7*(b^2*C*e^2+2*c*e*(B*b*e+C*a*e+2*C*b*d)+c^2*(C*d^
2+e*(A*e+2*B*d)))*x^7+1/8*c*e*(B*c*e+2*C*b*e+2*C*c*d)*x^8+1/9*c^2*C*e^2*x^
9
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 431, normalized size of antiderivative = 1.00

$$\int \left(A+Bx+Cx^2\right)\left(ad+(bd+ae)x+(cd+be)x^2+cex^3\right)^2\,dx$$
$$= a^2Ad^2x+\frac{1}{2}ad(aBd+2A(bd+ae))x^2$$
$$+\frac{1}{3}\left(ad(2bBd+aCd+2aBe)+A\left(b^2d^2+4abde+a\left(2cd^2+ae^2\right)\right)\right)x^3+\frac{1}{4}\left(b^2d(Bd+2Ae)\right.$$
$$\left.+a\left(2Bcd^2+4Acde+2aCde+aBe^2\right)+2b\left(ad(Cd+2Be)+A\left(cd^2+ae^2\right)\right)\right)x^4$$
$$+\frac{1}{5}\left(Ac\left(cd^2+2ae^2\right)+b^2\left(Cd^2+e(2Bd+Ae)\right)+a\left(aCe^2+2cd(Cd+2Be)\right)\right.$$
$$\left.+2b\left(2(Ac+aC)de+B\left(cd^2+ae^2\right)\right)\right)x^5+\frac{1}{6}\left(B\left(c^2d^2+b^2e^2+2ce(2bd+ae)\right)\right.$$
$$\left.+2\left(b^2Cde+c(Ac+2aC)de+b\left(cCd^2+Ace^2+aCe^2\right)\right)\right)x^6$$
$$+\frac{1}{7}\left(b^2Ce^2+2ce(2bCd+bBe+aCe)+c^2\left(Cd^2+e(2Bd+Ae)\right)\right)x^7$$
$$+\frac{1}{8}ce(2cCd+Bce+2bCe)x^8+\frac{1}{9}c^2Ce^2x^9$$

input
```
Integrate[(A + B*x + C*x^2)*(a*d + (b*d + a*e)*x + (c*d + b*e)*x^2 + c*e*x
^3)^2,x]
```

output
```
a^2*A*d^2*x + (a*d*(a*B*d + 2*A*(b*d + a*e))*x^2)/2 + ((a*d*(2*b*B*d + a*C
*d + 2*a*B*e) + A*(b^2*d^2 + 4*a*b*d*e + a*(2*c*d^2 + a*e^2)))*x^3)/3 + ((
b^2*d*(B*d + 2*A*e) + a*(2*B*c*d^2 + 4*A*c*d*e + 2*a*C*d*e + a*B*e^2) + 2*
b*(a*d*(C*d + 2*B*e) + A*(c*d^2 + a*e^2)))*x^4)/4 + ((A*c*(c*d^2 + 2*a*e^2
) + b^2*(C*d^2 + e*(2*B*d + A*e)) + a*(a*C*e^2 + 2*c*d*(C*d + 2*B*e)) + 2*
b*(2*(A*c + a*C)*d*e + B*(c*d^2 + a*e^2)))*x^5)/5 + ((B*(c^2*d^2 + b^2*e^2
 + 2*c*e*(2*b*d + a*e)) + 2*(b^2*C*d*e + c*(A*c + 2*a*C)*d*e + b*(c*C*d^2
+ A*c*e^2 + a*C*e^2)))*x^6)/6 + ((b^2*C*e^2 + 2*c*e*(2*b*C*d + b*B*e + a*C
*e) + c^2*(C*d^2 + e*(2*B*d + A*e)))*x^7)/7 + (c*e*(2*c*C*d + B*c*e + 2*b*
C*e)*x^8)/8 + (c^2*C*e^2*x^9)/9
```

Rubi [A] (verified)

Time = 1.67 (sec) , antiderivative size = 431, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.047$, Rules used = {2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(A+Bx+Cx^2\right)\left(x(ae+bd)+ad+x^2(be+cd)+cex^3\right)^2dx$$

$$\Big\downarrow \text{2188}$$

$$\int \left(a^2Ad^2+x^6\left(2ce(aCe+bBe+2bCd)+c^2\left(e(Ae+2Bd)+Cd^2\right)+b^2Ce^2\right)+x^5\left(2\left(b\left(aCe^2+Ace^2+cCd^2\right)+\right.\right.\right.$$

$$\Big\downarrow \text{2009}$$

$$a^2Ad^2x+\frac{1}{7}x^7\left(2ce(aCe+bBe+2bCd)+c^2\left(e(Ae+2Bd)+Cd^2\right)+b^2Ce^2\right)+$$
$$\frac{1}{6}x^6\left(2\left(b\left(aCe^2+Ace^2+cCd^2\right)+cde(2aC+Ac)+b^2Cde\right)+B\left(2ce(ae+2bd)+b^2e^2+c^2d^2\right)\right)+$$
$$\frac{1}{5}x^5\left(2b\left(2de(aC+Ac)+B\left(ae^2+cd^2\right)\right)+Ac\left(2ae^2+cd^2\right)+a\left(aCe^2+2cd(2Be+Cd)\right)+b^2\left(e(Ae+2Bd)+C\right.\right.$$
$$\frac{1}{4}x^4\left(2b\left(A\left(ae^2+cd^2\right)+ad(2Be+Cd)\right)+a\left(aBe^2+2aCde+4Acde+2Bcd^2\right)+b^2d(2Ae+Bd)\right)+$$
$$\frac{1}{3}x^3\left(A\left(4abde+a\left(ae^2+2cd^2\right)+b^2d^2\right)+ad(2aBe+aCd+2bBd)\right)+\frac{1}{2}adx^2(2A(ae+bd)+$$
$$aBd)+\frac{1}{8}cex^8(2bCe+Bce+2cCd)+\frac{1}{9}c^2Ce^2x^9$$

input
```
Int[(A + B*x + C*x^2)*(a*d + (b*d + a*e)*x + (c*d + b*e)*x^2 + c*e*x^3)^2,
x]
```

output
```
a^2*A*d^2*x + (a*d*(a*B*d + 2*A*(b*d + a*e))*x^2)/2 + ((a*d*(2*b*B*d + a*C
*d + 2*a*B*e) + A*(b^2*d^2 + 4*a*b*d*e + a*(2*c*d^2 + a*e^2)))*x^3)/3 + ((
b^2*d*(B*d + 2*A*e) + a*(2*B*c*d^2 + 4*A*c*d*e + 2*a*C*d*e + a*B*e^2) + 2*
b*(a*d*(C*d + 2*B*e) + A*(c*d^2 + a*e^2)))*x^4)/4 + ((A*c*(c*d^2 + 2*a*e^2
) + b^2*(C*d^2 + e*(2*B*d + A*e)) + a*(a*C*e^2 + 2*c*d*(C*d + 2*B*e)) + 2*
b*(2*(A*c + a*C)*d*e + B*(c*d^2 + a*e^2)))*x^5)/5 + ((B*(c^2*d^2 + b^2*e^2
 + 2*c*e*(2*b*d + a*e)) + 2*(b^2*C*d*e + c*(A*c + 2*a*C)*d*e + b*(c*C*d^2
+ A*c*e^2 + a*C*e^2)))*x^6)/6 + ((b^2*C*e^2 + 2*c*e*(2*b*C*d + b*B*e + a*C
*e) + c^2*(C*d^2 + e*(2*B*d + A*e)))*x^7)/7 + (c*e*(2*c*C*d + B*c*e + 2*b*
C*e)*x^8)/8 + (c^2*C*e^2*x^9)/9
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 382, normalized size of antiderivative = 0.89

method	result
default	$\frac{c^2 C e^2 x^9}{9}+\frac{\left(B c^2 e^2+2 C (e b+c d) c e\right) x^8}{8}+\frac{\left(A c^2 e^2+2 B (e b+c d) c e+C \left(2 c e (a e+b d)+(e b+c d)^2\right)\right) x^7}{7}+\frac{\left(2 A (e b+c d) c e+B\right.}{}$
norman	$\frac{c^2 C e^2 x^9}{9}+\left(\frac{1}{8} B c^2 e^2+\frac{1}{4} C b c e^2+\frac{1}{4} C c^2 d e\right) x^8+\left(\frac{1}{7} A c^2 e^2+\frac{2}{7} B b c e^2+\frac{2}{7} B c^2 d e+\frac{2}{7} C a c e^2+\right.$
risch	$\frac{1}{2} x^2 B a^2 d^2+\frac{4}{7} x^7 C b c d e+\frac{2}{3} x^6 B b c d e+\frac{2}{3} x^6 C a d e c+\frac{1}{8} x^8 B c^2 e^2+\frac{1}{7} x^7 A c^2 e^2+\frac{1}{7} x^7 b^2 C e^2+$
parallelrisch	$\frac{1}{2} x^2 B a^2 d^2+\frac{4}{7} x^7 C b c d e+\frac{2}{3} x^6 B b c d e+\frac{2}{3} x^6 C a d e c+\frac{1}{8} x^8 B c^2 e^2+\frac{1}{7} x^7 A c^2 e^2+\frac{1}{7} x^7 b^2 C e^2+$
gosper	$\frac{x \left(280 C c^2 e^2 x^8+315 x^7 B c^2 e^2+630 x^7 C b c e^2+630 x^7 C c^2 d e+360 x^6 A c^2 e^2+720 x^6 B b c e^2+720 x^6 B c^2 d e+720 x^6 C a c e^2+360 x^{\cdot}\right.}{}$
orering	$\frac{x \left(280 C c^2 e^2 x^8+315 x^7 B c^2 e^2+630 x^7 C b c e^2+630 x^7 C c^2 d e+360 x^6 A c^2 e^2+720 x^6 B b c e^2+720 x^6 B c^2 d e+720 x^6 C a c e^2+360 x^{\cdot}\right.}{}$

input
```
int((C*x^2+B*x+A)*(a*d+(a*e+b*d)*x+(b*e+c*d)*x^2+c*e*x^3)^2,x,method=_RETU
RNVERBOSE)
```

output
```
1/9*c^2*C*e^2*x^9+1/8*(B*c^2*e^2+2*C*(b*e+c*d)*c*e)*x^8+1/7*(A*c^2*e^2+2*B
*(b*e+c*d)*c*e+C*(2*c*e*(a*e+b*d)+(b*e+c*d)^2))*x^7+1/6*(2*A*(b*e+c*d)*c*e
+B*(2*c*e*(a*e+b*d)+(b*e+c*d)^2)+C*(2*a*c*d*e+2*(a*e+b*d)*(b*e+c*d)))*x^6+
1/5*(A*(2*c*e*(a*e+b*d)+(b*e+c*d)^2)+B*(2*a*c*d*e+2*(a*e+b*d)*(b*e+c*d))+C
*(2*a*d*(b*e+c*d)+(a*e+b*d)^2))*x^5+1/4*(A*(2*a*c*d*e+2*(a*e+b*d)*(b*e+c*d
))+B*(2*a*d*(b*e+c*d)+(a*e+b*d)^2)+2*C*a*d*(a*e+b*d))*x^4+1/3*(A*(2*a*d*(b
*e+c*d)+(a*e+b*d)^2)+2*B*a*d*(a*e+b*d)+C*a^2*d^2)*x^3+1/2*(2*A*a*d*(a*e+b*
d)+B*a^2*d^2)*x^2+a^2*A*d^2*x
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 411, normalized size of antiderivative = 0.95

$$\int (A+Bx+Cx^2)\left(ad+(bd+ae)x+(cd+be)x^2+cex^3\right)^2\,dx$$
$$= \frac{1}{9}\,Cc^2e^2x^9 + \frac{1}{8}\left(2\,Cc^2de + \left(2\,Cbc + Bc^2\right)e^2\right)x^8$$
$$+ \frac{1}{7}\left(Cc^2d^2 + 2\left(2\,Cbc + Bc^2\right)de + \left(Cb^2 + Ac^2 + 2\,(Ca + Bb)c\right)e^2\right)x^7$$
$$+ \frac{1}{6}\left(\left(2\,Cbc + Bc^2\right)d^2 + 2\left(Cb^2 + Ac^2 + 2\,(Ca + Bb)c\right)de + \left(2\,Cab + Bb^2 + 2\,(Ba + Ab)c\right)e^2\right)x^6$$
$$+ Aa^2d^2x$$
$$+ \frac{1}{5}\left(\left(Cb^2 + Ac^2 + 2\,(Ca + Bb)c\right)d^2 + 2\left(2\,Cab + Bb^2 + 2\,(Ba + Ab)c\right)de + \left(Ca^2 + 2\,Bab + Ab^2 + 2\,A\right.\right.$$
$$+ \frac{1}{4}\left(\left(2\,Cab + Bb^2 + 2\,(Ba + Ab)c\right)d^2 + 2\left(Ca^2 + 2\,Bab + Ab^2 + 2\,Aac\right)de + \left(Ba^2 + 2\,Aab\right)e^2\right)x^4$$
$$+ \frac{1}{3}\left(Aa^2e^2 + \left(Ca^2 + 2\,Bab + Ab^2 + 2\,Aac\right)d^2 + 2\left(Ba^2 + 2\,Aab\right)de\right)x^3$$
$$+ \frac{1}{2}\left(2\,Aa^2de + \left(Ba^2 + 2\,Aab\right)d^2\right)x^2$$

input
```
integrate((C*x^2+B*x+A)*(a*d+(a*e+b*d)*x+(b*e+c*d)*x^2+c*e*x^3)^2,x, algor
ithm="fricas")
```

output
```
1/9*C*c^2*e^2*x^9 + 1/8*(2*C*c^2*d*e + (2*C*b*c + B*c^2)*e^2)*x^8 + 1/7*(C
*c^2*d^2 + 2*(2*C*b*c + B*c^2)*d*e + (C*b^2 + A*c^2 + 2*(C*a + B*b)*c)*e^2
)*x^7 + 1/6*((2*C*b*c + B*c^2)*d^2 + 2*(C*b^2 + A*c^2 + 2*(C*a + B*b)*c)*d
*e + (2*C*a*b + B*b^2 + 2*(B*a + A*b)*c)*e^2)*x^6 + A*a^2*d^2*x + 1/5*((C*
b^2 + A*c^2 + 2*(C*a + B*b)*c)*d^2 + 2*(2*C*a*b + B*b^2 + 2*(B*a + A*b)*c)
*d*e + (C*a^2 + 2*B*a*b + A*b^2 + 2*A*a*c)*e^2)*x^5 + 1/4*((2*C*a*b + B*b^
2 + 2*(B*a + A*b)*c)*d^2 + 2*(C*a^2 + 2*B*a*b + A*b^2 + 2*A*a*c)*d*e + (B*
a^2 + 2*A*a*b)*e^2)*x^4 + 1/3*(A*a^2*e^2 + (C*a^2 + 2*B*a*b + A*b^2 + 2*A*
a*c)*d^2 + 2*(B*a^2 + 2*A*a*b)*d*e)*x^3 + 1/2*(2*A*a^2*d*e + (B*a^2 + 2*A*
a*b)*d^2)*x^2
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 600, normalized size of antiderivative = 1.39

$$\int (A+Bx+Cx^2)\left(ad+(bd+ae)x+(cd+be)x^2+cex^3\right)^2\,dx$$
$$= Aa^2d^2x+\frac{Cc^2e^2x^9}{9}+x^8\left(\frac{Bc^2e^2}{8}+\frac{Cbce^2}{4}+\frac{Cc^2de}{4}\right)$$
$$+x^7\left(\frac{Ac^2e^2}{7}+\frac{2Bbce^2}{7}+\frac{2Bc^2de}{7}+\frac{2Cace^2}{7}+\frac{Cb^2e^2}{7}+\frac{4Cbcde}{7}+\frac{Cc^2d^2}{7}\right)$$
$$+x^6\left(\frac{Abce^2}{3}+\frac{Ac^2de}{3}+\frac{Bace^2}{3}+\frac{Bb^2e^2}{6}+\frac{2Bbcde}{3}+\frac{Bc^2d^2}{6}+\frac{Cabe^2}{3}+\frac{2Cacde}{3}\right.$$
$$\left.+\frac{Cb^2de}{3}+\frac{Cbcd^2}{3}\right)+x^5\cdot\left(\frac{2Aace^2}{5}+\frac{Ab^2e^2}{5}+\frac{4Abcde}{5}+\frac{Ac^2d^2}{5}+\frac{2Babe^2}{5}+\frac{4Bacde}{5}\right.$$
$$\left.+\frac{2Bb^2de}{5}+\frac{2Bbcd^2}{5}+\frac{Ca^2e^2}{5}+\frac{4Cabde}{5}+\frac{2Cacd^2}{5}+\frac{Cb^2d^2}{5}\right)+x^4\left(\frac{Aabe^2}{2}+Aacde\right.$$
$$\left.+\frac{Ab^2de}{2}+\frac{Abcd^2}{2}+\frac{Ba^2e^2}{4}+Babde+\frac{Bacd^2}{2}+\frac{Bb^2d^2}{4}+\frac{Ca^2de}{2}+\frac{Cabd^2}{2}\right)$$
$$+x^3\left(\frac{Aa^2e^2}{3}+\frac{4Aabde}{3}+\frac{2Aacd^2}{3}+\frac{Ab^2d^2}{3}+\frac{2Ba^2de}{3}+\frac{2Babd^2}{3}+\frac{Ca^2d^2}{3}\right)$$
$$+x^2\left(Aa^2de+Aabd^2+\frac{Ba^2d^2}{2}\right)$$

input
```
integrate((C*x**2+B*x+A)*(a*d+(a*e+b*d)*x+(b*e+c*d)*x**2+c*e*x**3)**2,x)
```

output
```
A*a**2*d**2*x + C*c**2*e**2*x**9/9 + x**8*(B*c**2*e**2/8 + C*b*c*e**2/4 +
C*c**2*d*e/4) + x**7*(A*c**2*e**2/7 + 2*B*b*c*e**2/7 + 2*B*c**2*d*e/7 + 2*
C*a*c*e**2/7 + C*b**2*e**2/7 + 4*C*b*c*d*e/7 + C*c**2*d**2/7) + x**6*(A*b*
c*e**2/3 + A*c**2*d*e/3 + B*a*c*e**2/3 + B*b**2*e**2/6 + 2*B*b*c*d*e/3 + B
*c**2*d**2/6 + C*a*b*e**2/3 + 2*C*a*c*d*e/3 + C*b**2*d*e/3 + C*b*c*d**2/3)
 + x**5*(2*A*a*c*e**2/5 + A*b**2*e**2/5 + 4*A*b*c*d*e/5 + A*c**2*d**2/5 +
2*B*a*b*e**2/5 + 4*B*a*c*d*e/5 + 2*B*b**2*d*e/5 + 2*B*b*c*d**2/5 + C*a**2*
e**2/5 + 4*C*a*b*d*e/5 + 2*C*a*c*d**2/5 + C*b**2*d**2/5) + x**4*(A*a*b*e**
2/2 + A*a*c*d*e + A*b**2*d*e/2 + A*b*c*d**2/2 + B*a**2*e**2/4 + B*a*b*d*e
+ B*a*c*d**2/2 + B*b**2*d**2/4 + C*a**2*d*e/2 + C*a*b*d**2/2) + x**3*(A*a*
*2*e**2/3 + 4*A*a*b*d*e/3 + 2*A*a*c*d**2/3 + A*b**2*d**2/3 + 2*B*a**2*d*e/
3 + 2*B*a*b*d**2/3 + C*a**2*d**2/3) + x**2*(A*a**2*d*e + A*a*b*d**2 + B*a*
*2*d**2/2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 411, normalized size of antiderivative = 0.95

$$\int \left(A+Bx+Cx^2\right)\left(ad+(bd+ae)x+(cd+be)x^2+cex^3\right)^2\,dx$$
$$= \frac{1}{9}Cc^2e^2x^9 + \frac{1}{8}\left(2Cc^2de + \left(2Cbc + Bc^2\right)e^2\right)x^8$$
$$+ \frac{1}{7}\left(Cc^2d^2 + 2\left(2Cbc + Bc^2\right)de + \left(Cb^2 + Ac^2 + 2(Ca + Bb)c\right)e^2\right)x^7$$
$$+ \frac{1}{6}\left(\left(2Cbc + Bc^2\right)d^2 + 2\left(Cb^2 + Ac^2 + 2(Ca + Bb)c\right)de + \left(2Cab + Bb^2 + 2(Ba + Ab)c\right)e^2\right)x^6$$
$$+ Aa^2d^2x$$
$$+ \frac{1}{5}\left(\left(Cb^2 + Ac^2 + 2(Ca + Bb)c\right)d^2 + 2\left(2Cab + Bb^2 + 2(Ba + Ab)c\right)de + \left(Ca^2 + 2Bab + Ab^2 + 2A\right.\right.$$
$$+ \frac{1}{4}\left(\left(2Cab + Bb^2 + 2(Ba + Ab)c\right)d^2 + 2\left(Ca^2 + 2Bab + Ab^2 + 2Aac\right)de + \left(Ba^2 + 2Aab\right)e^2\right)x^4$$
$$+ \frac{1}{3}\left(Aa^2e^2 + \left(Ca^2 + 2Bab + Ab^2 + 2Aac\right)d^2 + 2\left(Ba^2 + 2Aab\right)de\right)x^3$$
$$+ \frac{1}{2}\left(2Aa^2de + \left(Ba^2 + 2Aab\right)d^2\right)x^2$$

input
```
integrate((C*x^2+B*x+A)*(a*d+(a*e+b*d)*x+(b*e+c*d)*x^2+c*e*x^3)^2,x, algor
ithm="maxima")
```

output
```
1/9*C*c^2*e^2*x^9 + 1/8*(2*C*c^2*d*e + (2*C*b*c + B*c^2)*e^2)*x^8 + 1/7*(C
*c^2*d^2 + 2*(2*C*b*c + B*c^2)*d*e + (C*b^2 + A*c^2 + 2*(C*a + B*b)*c)*e^2
)*x^7 + 1/6*((2*C*b*c + B*c^2)*d^2 + 2*(C*b^2 + A*c^2 + 2*(C*a + B*b)*c)*d
*e + (2*C*a*b + B*b^2 + 2*(B*a + A*b)*c)*e^2)*x^6 + A*a^2*d^2*x + 1/5*((C*
b^2 + A*c^2 + 2*(C*a + B*b)*c)*d^2 + 2*(2*C*a*b + B*b^2 + 2*(B*a + A*b)*c)
*d*e + (C*a^2 + 2*B*a*b + A*b^2 + 2*A*a*c)*e^2)*x^5 + 1/4*((2*C*a*b + B*b^
2 + 2*(B*a + A*b)*c)*d^2 + 2*(C*a^2 + 2*B*a*b + A*b^2 + 2*A*a*c)*d*e + (B*
a^2 + 2*A*a*b)*e^2)*x^4 + 1/3*(A*a^2*e^2 + (C*a^2 + 2*B*a*b + A*b^2 + 2*A*
a*c)*d^2 + 2*(B*a^2 + 2*A*a*b)*d*e)*x^3 + 1/2*(2*A*a^2*d*e + (B*a^2 + 2*A*
a*b)*d^2)*x^2
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 597, normalized size of antiderivative = 1.39

$$\begin{aligned}
&\int (A+Bx+Cx^2)\left(ad+(bd+ae)x+(cd+be)x^2+cex^3\right)^2\,dx \\
&= \frac{1}{9}Cc^2e^2x^9+\frac{1}{4}Cc^2dex^8+\frac{1}{4}Cbce^2x^8+\frac{1}{8}Bc^2e^2x^8+\frac{1}{7}Cc^2d^2x^7+\frac{4}{7}Cbcdex^7 \\
&+\frac{2}{7}Bc^2dex^7+\frac{1}{7}Cb^2e^2x^7+\frac{2}{7}Cace^2x^7+\frac{2}{7}Bbce^2x^7+\frac{1}{7}Ac^2e^2x^7+\frac{1}{3}Cbcd^2x^6 \\
&+\frac{1}{6}Bc^2d^2x^6+\frac{1}{3}Cb^2dex^6+\frac{2}{3}Cacdex^6+\frac{2}{3}Bbcdex^6+\frac{1}{3}Ac^2dex^6+\frac{1}{3}Cabe^2x^6 \\
&+\frac{1}{6}Bb^2e^2x^6+\frac{1}{3}Bace^2x^6+\frac{1}{3}Abce^2x^6+\frac{1}{5}Cb^2d^2x^5+\frac{2}{5}Cacd^2x^5+\frac{2}{5}Bbcd^2x^5 \\
&+\frac{1}{5}Ac^2d^2x^5+\frac{4}{5}Cabdex^5+\frac{2}{5}Bb^2dex^5+\frac{4}{5}Bacdex^5+\frac{4}{5}Abcdex^5+\frac{1}{5}Ca^2e^2x^5 \\
&+\frac{2}{5}Babe^2x^5+\frac{1}{5}Ab^2e^2x^5+\frac{2}{5}Aace^2x^5+\frac{1}{2}Cabd^2x^4+\frac{1}{4}Bb^2d^2x^4+\frac{1}{2}Bacd^2x^4 \\
&+\frac{1}{2}Abcd^2x^4+\frac{1}{2}Ca^2dex^4+Babdex^4+\frac{1}{2}Ab^2dex^4+Aacdex^4+\frac{1}{4}Ba^2e^2x^4 \\
&+\frac{1}{2}Aabe^2x^4+\frac{1}{3}Ca^2d^2x^3+\frac{2}{3}Babd^2x^3+\frac{1}{3}Ab^2d^2x^3+\frac{2}{3}Aacd^2x^3+\frac{2}{3}Ba^2dex^3 \\
&+\frac{4}{3}Aabdex^3+\frac{1}{3}Aa^2e^2x^3+\frac{1}{2}Ba^2d^2x^2+Aabd^2x^2+Aa^2dex^2+Aa^2d^2x
\end{aligned}$$

input
```
integrate((C*x^2+B*x+A)*(a*d+(a*e+b*d)*x+(b*e+c*d)*x^2+c*e*x^3)^2,x, algor
ithm="giac")
```

output
```
1/9*C*c^2*e^2*x^9 + 1/4*C*c^2*d*e*x^8 + 1/4*C*b*c*e^2*x^8 + 1/8*B*c^2*e^2*
x^8 + 1/7*C*c^2*d^2*x^7 + 4/7*C*b*c*d*e*x^7 + 2/7*B*c^2*d*e*x^7 + 1/7*C*b^
2*e^2*x^7 + 2/7*C*a*c*e^2*x^7 + 2/7*B*b*c*e^2*x^7 + 1/7*A*c^2*e^2*x^7 + 1/
3*C*b*c*d^2*x^6 + 1/6*B*c^2*d^2*x^6 + 1/3*C*b^2*d*e*x^6 + 2/3*C*a*c*d*e*x^
6 + 2/3*B*b*c*d*e*x^6 + 1/3*A*c^2*d*e*x^6 + 1/3*C*a*b*e^2*x^6 + 1/6*B*b^2*
e^2*x^6 + 1/3*B*a*c*e^2*x^6 + 1/3*A*b*c*e^2*x^6 + 1/5*C*b^2*d^2*x^5 + 2/5*
C*a*c*d^2*x^5 + 2/5*B*b*c*d^2*x^5 + 1/5*A*c^2*d^2*x^5 + 4/5*C*a*b*d*e*x^5
+ 2/5*B*b^2*d*e*x^5 + 4/5*B*a*c*d*e*x^5 + 4/5*A*b*c*d*e*x^5 + 1/5*C*a^2*e^
2*x^5 + 2/5*B*a*b*e^2*x^5 + 1/5*A*b^2*e^2*x^5 + 2/5*A*a*c*e^2*x^5 + 1/2*C*
a*b*d^2*x^4 + 1/4*B*b^2*d^2*x^4 + 1/2*B*a*c*d^2*x^4 + 1/2*A*b*c*d^2*x^4 +
1/2*C*a^2*d*e*x^4 + B*a*b*d*e*x^4 + 1/2*A*b^2*d*e*x^4 + A*a*c*d*e*x^4 + 1/
4*B*a^2*e^2*x^4 + 1/2*A*a*b*e^2*x^4 + 1/3*C*a^2*d^2*x^3 + 2/3*B*a*b*d^2*x^
3 + 1/3*A*b^2*d^2*x^3 + 2/3*A*a*c*d^2*x^3 + 2/3*B*a^2*d*e*x^3 + 4/3*A*a*b*
d*e*x^3 + 1/3*A*a^2*e^2*x^3 + 1/2*B*a^2*d^2*x^2 + A*a*b*d^2*x^2 + A*a^2*d*
e*x^2 + A*a^2*d^2*x
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 462, normalized size of antiderivative = 1.07

$$\int (A+Bx+Cx^2)\left(ad+(bd+ae)x+(cd+be)x^2+cex^3\right)^2\,dx$$
$$= x^4\left(\frac{C\,a^2\,d\,e}{2}+\frac{B\,a^2\,e^2}{4}+\frac{C\,a\,b\,d^2}{2}+B\,a\,b\,d\,e+\frac{A\,a\,b\,e^2}{2}+\frac{B\,c\,a\,d^2}{2}+A\,c\,a\,d\,e+\frac{B\,b^2\,d^2}{4}+\frac{A\,b^2\,d\,e}{2}+\frac{A\,c\,b\,d^2}{2}\right)+x^6\left(\frac{C\,b^2\,d\,e}{3}+\frac{B\,b^2\,e^2}{6}+\frac{C\,b\,c\,d^2}{3}+\frac{2\,B\,b\,c\,d\,e}{3}+\frac{A\,b\,c\,e^2}{3}+\frac{C\,a\,b\,e^2}{3}+\frac{B\,c^2\,d^2}{6}+\frac{A\,c^2\,d\,e}{3}+\frac{2\,C\,a\,c\,d\,e}{3}+\frac{B\,a\,c\,e^2}{3}\right)$$
$$+x^5\left(\frac{C\,a^2\,e^2}{5}+\frac{4\,C\,a\,b\,d\,e}{5}+\frac{2\,B\,a\,b\,e^2}{5}+\frac{2\,C\,a\,c\,d^2}{5}+\frac{4\,B\,a\,c\,d\,e}{5}+\frac{2\,A\,a\,c\,e^2}{5}+\frac{C\,b^2\,d^2}{5}+\frac{2\,B\,b^2\,d\,e}{5}+\frac{A\,b^2\,e^2}{5}+\frac{2\,B\,b\,c\,d^2}{5}+\frac{4\,A\,b\,c\,d\,e}{5}+\frac{A\,c^2\,d^2}{5}\right)$$
$$+x^3\left(\frac{C\,a^2\,d^2}{3}+\frac{2\,B\,a^2\,d\,e}{3}+\frac{A\,a^2\,e^2}{3}+\frac{2\,B\,a\,b\,d^2}{3}+\frac{4\,A\,a\,b\,d\,e}{3}+\frac{2\,A\,c\,a\,d^2}{3}+\frac{A\,b^2\,d^2}{3}\right)$$
$$+x^7\left(\frac{C\,b^2\,e^2}{7}+\frac{4\,C\,b\,c\,d\,e}{7}+\frac{2\,B\,b\,c\,e^2}{7}+\frac{C\,c^2\,d^2}{7}+\frac{2\,B\,c^2\,d\,e}{7}+\frac{A\,c^2\,e^2}{7}+\frac{2\,C\,a\,c\,e^2}{7}\right)$$
$$+\frac{C\,c^2\,e^2\,x^9}{9}+\frac{a\,d\,x^2\,(2\,A\,a\,e+2\,A\,b\,d+B\,a\,d)}{2}$$
$$+\frac{c\,e\,x^8\,(B\,c\,e+2\,C\,b\,e+2\,C\,c\,d)}{8}+A\,a^2\,d^2\,x$$

input
```
int((A + B*x + C*x^2)*(a*d + x*(a*e + b*d) + x^2*(b*e + c*d) + c*e*x^3)^2,
x)
```

output
```
x^4*((B*a^2*e^2)/4 + (B*b^2*d^2)/4 + (A*a*b*e^2)/2 + (A*b*c*d^2)/2 + (B*a*
c*d^2)/2 + (C*a*b*d^2)/2 + (A*b^2*d*e)/2 + (C*a^2*d*e)/2 + A*a*c*d*e + B*a
*b*d*e) + x^6*((B*b^2*e^2)/6 + (B*c^2*d^2)/6 + (A*b*c*e^2)/3 + (B*a*c*e^2)
/3 + (C*a*b*e^2)/3 + (C*b*c*d^2)/3 + (A*c^2*d*e)/3 + (C*b^2*d*e)/3 + (2*B*
b*c*d*e)/3 + (2*C*a*c*d*e)/3) + x^5*((A*b^2*e^2)/5 + (A*c^2*d^2)/5 + (C*a^
2*e^2)/5 + (C*b^2*d^2)/5 + (2*A*a*c*e^2)/5 + (2*B*a*b*e^2)/5 + (2*B*b*c*d^
2)/5 + (2*C*a*c*d^2)/5 + (2*B*b^2*d*e)/5 + (4*A*b*c*d*e)/5 + (4*B*a*c*d*e)
/5 + (4*C*a*b*d*e)/5) + x^3*((A*a^2*e^2)/3 + (A*b^2*d^2)/3 + (C*a^2*d^2)/3
 + (2*A*a*c*d^2)/3 + (2*B*a*b*d^2)/3 + (2*B*a^2*d*e)/3 + (4*A*a*b*d*e)/3)
+ x^7*((A*c^2*e^2)/7 + (C*b^2*e^2)/7 + (C*c^2*d^2)/7 + (2*B*b*c*e^2)/7 + (
2*C*a*c*e^2)/7 + (2*B*c^2*d*e)/7 + (4*C*b*c*d*e)/7) + (C*c^2*e^2*x^9)/9 +
(a*d*x^2*(2*A*a*e + 2*A*b*d + B*a*d))/2 + (c*e*x^8*(B*c*e + 2*C*b*e + 2*C*
c*d))/8 + A*a^2*d^2*x
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 335, normalized size of antiderivative = 0.78

$$\int \left(A+Bx+Cx^2\right)\left(ad+(bd+ae)x+(cd+be)x^2+cex^3\right)^2\,dx$$
$$= \frac{x(280c^3e^2x^8+945b\,c^2e^2x^7+630c^3de\,x^7+1080a\,c^2e^2x^6+1080b^2c\,e^2x^6+2160b\,c^2de\,x^6+360c^3d^2x^6+2}{}$$

input
```
int((C*x^2+B*x+A)*(a*d+(a*e+b*d)*x+(b*e+c*d)*x^2+c*e*x^3)^2,x)
```

output
```
(x*(2520*a**3*d**2 + 2520*a**3*d*e*x + 840*a**3*e**2*x**2 + 3780*a**2*b*d*
*2*x + 5040*a**2*b*d*e*x**2 + 1890*a**2*b*e**2*x**3 + 2520*a**2*c*d**2*x**
2 + 3780*a**2*c*d*e*x**3 + 1512*a**2*c*e**2*x**4 + 2520*a*b**2*d**2*x**2 +
 3780*a*b**2*d*e*x**3 + 1512*a*b**2*e**2*x**4 + 3780*a*b*c*d**2*x**3 + 604
8*a*b*c*d*e*x**4 + 2520*a*b*c*e**2*x**5 + 1512*a*c**2*d**2*x**4 + 2520*a*c
**2*d*e*x**5 + 1080*a*c**2*e**2*x**6 + 630*b**3*d**2*x**3 + 1008*b**3*d*e*
x**4 + 420*b**3*e**2*x**5 + 1512*b**2*c*d**2*x**4 + 2520*b**2*c*d*e*x**5 +
 1080*b**2*c*e**2*x**6 + 1260*b*c**2*d**2*x**5 + 2160*b*c**2*d*e*x**6 + 94
5*b*c**2*e**2*x**7 + 360*c**3*d**2*x**6 + 630*c**3*d*e*x**7 + 280*c**3*e**
2*x**8))/2520
```

3.126 $\int (A+Bx+Cx^2)(ad+(bd+ae)x+(cd+be)x^2$

Optimal result

Integrand size = 41, antiderivative size = 112

$$\begin{aligned}\int (A+Bx+Cx^2)\left(ad+(bd+ae)x+(cd+be)x^2+cex^3\right)\,dx\\ &= aAdx+\frac{1}{2}(Abd+aBd+aAe)x^2+\frac{1}{3}(bBd+Acd+aCd+Abe+aBe)x^3\\ &\quad+\frac{1}{4}(Bcd+bCd+bBe+Ace+aCe)x^4+\frac{1}{5}(cCd+Bce+bCe)x^5+\frac{1}{6}cCex^6\end{aligned}$$

output
```
a*A*d*x+1/2*(A*a*e+A*b*d+B*a*d)*x^2+1/3*(A*b*e+A*c*d+B*a*e+B*b*d+C*a*d)*x^
3+1/4*(A*c*e+B*b*e+B*c*d+C*a*e+C*b*d)*x^4+1/5*(B*c*e+C*b*e+C*c*d)*x^5+1/6*
c*C*e*x^6
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00

$$\begin{aligned}\int (A+Bx+Cx^2)\left(ad+(bd+ae)x+(cd+be)x^2+cex^3\right)\,dx\\ &= aAdx+\frac{1}{2}(Abd+aBd+aAe)x^2+\frac{1}{3}(bBd+Acd+aCd+Abe+aBe)x^3\\ &\quad+\frac{1}{4}(Bcd+bCd+bBe+Ace+aCe)x^4+\frac{1}{5}(cCd+Bce+bCe)x^5+\frac{1}{6}cCex^6\end{aligned}$$

input
```
Integrate[(A + B*x + C*x^2)*(a*d + (b*d + a*e)*x + (c*d + b*e)*x^2 + c*e*x
^3),x]
```

output
```
a*A*d*x + ((A*b*d + a*B*d + a*A*e)*x^2)/2 + ((b*B*d + A*c*d + a*C*d + A*b*
e + a*B*e)*x^3)/3 + ((B*c*d + b*C*d + b*B*e + A*c*e + a*C*e)*x^4)/4 + ((c*
C*d + B*c*e + b*C*e)*x^5)/5 + (c*C*e*x^6)/6
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.049$, Rules used = {2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(A+Bx+Cx^2\right)\left(x(ae+bd)+ad+x^2(be+cd)+cex^3\right)\,dx$$

$$\Big\downarrow \text{2188}$$

$$\int \left(x^3(aCe+Ace+bBe+bCd+Bcd)+x^2(aBe+aCd+Abe+Acd+bBd)+x(aAe+aBd+Abd)+aAd+\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}x^4(aCe+Ace+bBe+bCd+Bcd)+\frac{1}{3}x^3(aBe+aCd+Abe+Acd+bBd)+\frac{1}{2}x^2(aAe+aBd+Abd)+aAdx+\frac{1}{5}x^5(bCe+Bce+cCd)+\frac{1}{6}cCex^6$$

input
```
Int[(A + B*x + C*x^2)*(a*d + (b*d + a*e)*x + (c*d + b*e)*x^2 + c*e*x^3),x]
```

output
```
a*A*d*x + ((A*b*d + a*B*d + a*A*e)*x^2)/2 + ((b*B*d + A*c*d + a*C*d + A*b*
e + a*B*e)*x^3)/3 + ((B*c*d + b*C*d + b*B*e + A*c*e + a*C*e)*x^4)/4 + ((c*
C*d + B*c*e + b*C*e)*x^5)/5 + (c*C*e*x^6)/6
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.97

method	result
default	$\frac{cCe\,x^6}{6}+\frac{((eb+cd)C+Bce)x^5}{5}+\frac{((ae+bd)C+(eb+cd)B+Ace)x^4}{4}+\frac{(Cad+(ae+bd)B+(eb+cd)A)x^3}{3}+\frac{(Bad+A(ae+\cdots}{2}$
norman	$\frac{cCe\,x^6}{6}+\left(\frac{1}{5}Bce+\frac{1}{5}Cbe+\frac{1}{5}Ccd\right)x^5+\left(\frac{1}{4}Ace+\frac{1}{4}Bbe+\frac{1}{4}Bcd+\frac{1}{4}Cae+\frac{1}{4}Cbd\right)x^4+\left(\frac{1}{3}Ab\cdots\right.$
gosper	$\frac{x(10Cce\,x^5+12Bce\,x^4+12Cbe\,x^4+12Ccd\,x^4+15Ace\,x^3+15Bbe\,x^3+15Bcd\,x^3+15Cae\,x^3+15Cbd\,x^3+20Abe\,x^2+20Acd\,x^2+\cdots}{60}$
risch	$\frac{1}{6}cCe\,x^6+\frac{1}{5}x^5Bce+\frac{1}{5}x^5Cbe+\frac{1}{5}x^5Ccd+\frac{1}{4}Ace\,x^4+\frac{1}{4}Bbe\,x^4+\frac{1}{4}Bcd\,x^4+\frac{1}{4}x^4Cae+\frac{1}{4}x^4\cdots$
parallelrisch	$\frac{1}{6}cCe\,x^6+\frac{1}{5}x^5Bce+\frac{1}{5}x^5Cbe+\frac{1}{5}x^5Ccd+\frac{1}{4}Ace\,x^4+\frac{1}{4}Bbe\,x^4+\frac{1}{4}Bcd\,x^4+\frac{1}{4}x^4Cae+\frac{1}{4}x^4\cdots$
orering	$\frac{x(10Cce\,x^5+12Bce\,x^4+12Cbe\,x^4+12Ccd\,x^4+15Ace\,x^3+15Bbe\,x^3+15Bcd\,x^3+15Cae\,x^3+15Cbd\,x^3+20Abe\,x^2+20Acd\,x^2+\cdots}{60(ex+d)(c\,x^2+bx+\cdots}$

input
```
int((C*x^2+B*x+A)*(a*d+(a*e+b*d)*x+(b*e+c*d)*x^2+c*e*x^3),x,method=_RETURN
VERBOSE)
```

output
```
1/6*c*C*e*x^6+1/5*((b*e+c*d)*C+B*c*e)*x^5+1/4*((a*e+b*d)*C+(b*e+c*d)*B+A*c
*e)*x^4+1/3*(C*a*d+(a*e+b*d)*B+(b*e+c*d)*A)*x^3+1/2*(B*a*d+A*(a*e+b*d))*x^
2+a*A*d*x
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.95

$$\int \left(A+Bx+Cx^2\right) \left(ad+(bd+ae)x+(cd+be)x^2+cex^3\right) \, dx = \frac{1}{6}\,Ccex^6+\frac{1}{5}\,(Ccd+(Cb+Bc)e)x^5+\frac{1}{4}\,((Cb+Bc)d+(Ca+Bb+Ac)e)x^4 + Aadx+\frac{1}{3}\,((Ca+Bb+Ac)d+(Ba+Ab)e)x^3+\frac{1}{2}\,(Aae+(Ba+Ab)d)x^2$$

input
```
integrate((C*x^2+B*x+A)*(a*d+(a*e+b*d)*x+(b*e+c*d)*x^2+c*e*x^3),x, algorit
hm="fricas")
```

output
```
1/6*C*c*e*x^6 + 1/5*(C*c*d + (C*b + B*c)*e)*x^5 + 1/4*((C*b + B*c)*d + (C*
a + B*b + A*c)*e)*x^4 + A*a*d*x + 1/3*((C*a + B*b + A*c)*d + (B*a + A*b)*e
)*x^3 + 1/2*(A*a*e + (B*a + A*b)*d)*x^2
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.23

$$\int \left(A+Bx+Cx^2\right) \left(ad+(bd+ae)x+(cd+be)x^2+cex^3\right) \, dx = Aadx+\frac{Ccex^6}{6}+x^5\left(\frac{Bce}{5}+\frac{Cbe}{5}+\frac{Ccd}{5}\right)+x^4\left(\frac{Ace}{4}+\frac{Bbe}{4}+\frac{Bcd}{4}+\frac{Cae}{4}+\frac{Cbd}{4}\right) + x^3\left(\frac{Abe}{3}+\frac{Acd}{3}+\frac{Bae}{3}+\frac{Bbd}{3}+\frac{Cad}{3}\right)+x^2\left(\frac{Aae}{2}+\frac{Abd}{2}+\frac{Bad}{2}\right)$$

input
```
integrate((C*x**2+B*x+A)*(a*d+(a*e+b*d)*x+(b*e+c*d)*x**2+c*e*x**3),x)
```

output
```
A*a*d*x + C*c*e*x**6/6 + x**5*(B*c*e/5 + C*b*e/5 + C*c*d/5) + x**4*(A*c*e/
4 + B*b*e/4 + B*c*d/4 + C*a*e/4 + C*b*d/4) + x**3*(A*b*e/3 + A*c*d/3 + B*a
*e/3 + B*b*d/3 + C*a*d/3) + x**2*(A*a*e/2 + A*b*d/2 + B*a*d/2)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.95

$$\int \left(A+Bx+Cx^2\right)\left(ad+(bd+ae)x+(cd+be)x^2+cex^3\right)\,dx$$
$$=\frac{1}{6}Ccex^6+\frac{1}{5}(Ccd+(Cb+Bc)e)x^5+\frac{1}{4}((Cb+Bc)d+(Ca+Bb+Ac)e)x^4+Aadx+\frac{1}{3}((Ca+Bb+Ac)d+(Ba+Ab)e)x^3+\frac{1}{2}(Aae+(Ba+Ab)d)x^2$$

input
```
integrate((C*x^2+B*x+A)*(a*d+(a*e+b*d)*x+(b*e+c*d)*x^2+c*e*x^3),x, algorit
hm="maxima")
```

output
```
1/6*C*c*e*x^6 + 1/5*(C*c*d + (C*b + B*c)*e)*x^5 + 1/4*((C*b + B*c)*d + (C*
a + B*b + A*c)*e)*x^4 + A*a*d*x + 1/3*((C*a + B*b + A*c)*d + (B*a + A*b)*e
)*x^3 + 1/2*(A*a*e + (B*a + A*b)*d)*x^2
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.27

$$\int \left(A+Bx+Cx^2\right)\left(ad+(bd+ae)x+(cd+be)x^2+cex^3\right)\,dx$$
$$=\frac{1}{6}Ccex^6+\frac{1}{5}Ccdx^5+\frac{1}{5}Cbex^5+\frac{1}{5}Bcex^5+\frac{1}{4}Cbdx^4+\frac{1}{4}Bcdx^4+\frac{1}{4}Caex^4+\frac{1}{4}Bbex^4+\frac{1}{4}Acex^4+\frac{1}{3}Cadx^3+\frac{1}{3}Bbdx^3+\frac{1}{3}Acdx^3+\frac{1}{3}Baex^3+\frac{1}{3}Abex^3+\frac{1}{2}Badx^2+\frac{1}{2}Abdx^2+\frac{1}{2}Aaex^2+Aadx$$

input
```
integrate((C*x^2+B*x+A)*(a*d+(a*e+b*d)*x+(b*e+c*d)*x^2+c*e*x^3),x, algorit
hm="giac")
```

output
```
1/6*C*c*e*x^6 + 1/5*C*c*d*x^5 + 1/5*C*b*e*x^5 + 1/5*B*c*e*x^5 + 1/4*C*b*d*
x^4 + 1/4*B*c*d*x^4 + 1/4*C*a*e*x^4 + 1/4*B*b*e*x^4 + 1/4*A*c*e*x^4 + 1/3*
C*a*d*x^3 + 1/3*B*b*d*x^3 + 1/3*A*c*d*x^3 + 1/3*B*a*e*x^3 + 1/3*A*b*e*x^3
+ 1/2*B*a*d*x^2 + 1/2*A*b*d*x^2 + 1/2*A*a*e*x^2 + A*a*d*x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.02

$$\int (A+Bx+Cx^2)\left(ad+(bd+ae)x+(cd+be)x^2+cex^3\right)\,dx$$
$$=\frac{C\,c\,e\,x^6}{6}+\left(\frac{B\,c\,e}{5}+\frac{C\,b\,e}{5}+\frac{C\,c\,d}{5}\right)x^5+\left(\frac{A\,c\,e}{4}+\frac{B\,b\,e}{4}+\frac{B\,c\,d}{4}+\frac{C\,a\,e}{4}+\frac{C\,b\,d}{4}\right)x^4+\left(\frac{A\,b\,e}{3}+\frac{A\,c\,d}{3}+\frac{B\,a\,e}{3}+\frac{B\,b\,d}{3}+\frac{C\,a\,d}{3}\right)x^3+\left(\frac{A\,a\,e}{2}+\frac{A\,b\,d}{2}+\frac{B\,a\,d}{2}\right)x^2+A\,a\,d\,x$$

input
```
int((A + B*x + C*x^2)*(a*d + x*(a*e + b*d) + x^2*(b*e + c*d) + c*e*x^3),x)
```

output
```
x^3*((A*b*e)/3 + (A*c*d)/3 + (B*a*e)/3 + (B*b*d)/3 + (C*a*d)/3) + x^4*((A*
c*e)/4 + (B*b*e)/4 + (B*c*d)/4 + (C*a*e)/4 + (C*b*d)/4) + x^2*((A*a*e)/2 +
 (A*b*d)/2 + (B*a*d)/2) + x^5*((B*c*e)/5 + (C*b*e)/5 + (C*c*d)/5) + (C*c*e
*x^6)/6 + A*a*d*x
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.88

$$\int (A+Bx+Cx^2)\left(ad+(bd+ae)x+(cd+be)x^2+cex^3\right)\,dx$$
$$=\frac{x(10c^2e\,x^5+24bce\,x^4+12c^2d\,x^4+30ace\,x^3+15b^2e\,x^3+30bcd\,x^3+40abe\,x^2+40acd\,x^2+20b^2d\,x^2+3}{60}$$

input
```
int((C*x^2+B*x+A)*(a*d+(a*e+b*d)*x+(b*e+c*d)*x^2+c*e*x^3),x)
```

output
```
(x*(60*a**2*d + 30*a**2*e*x + 60*a*b*d*x + 40*a*b*e*x**2 + 40*a*c*d*x**2 +
 30*a*c*e*x**3 + 20*b**2*d*x**2 + 15*b**2*e*x**3 + 30*b*c*d*x**3 + 24*b*c*
e*x**4 + 12*c**2*d*x**4 + 10*c**2*e*x**5))/60
```

3.127 $\int \frac{A+Bx+Cx^2}{ad+(bd+ae)x+(cd+be)x^2+cex^3}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 196

$$\int \frac{A+Bx+Cx^2}{ad+(bd+ae)x+(cd+be)x^2+cex^3}\,dx = -\frac{(b^2Cd+2c(Acd-aCd+aBe)-b(Bcd+Ace+aCe))\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}\,(cd^2-bde+ae^2)} + \frac{(Cd^2-e(Bd-Ae))\log(d+ex)}{e\,(cd^2-bde+ae^2)} + \frac{(Bcd-bCd-Ace+aCe)\log\left(a+bx+cx^2\right)}{2c\,(cd^2-bde+ae^2)}$$

output

```
-(b^2*C*d+2*c*(A*c*d+B*a*e-C*a*d)-b*(A*c*e+B*c*d+C*a*e))*arctanh((2*c*x+b)
/(-4*a*c+b^2)^(1/2))/c/(-4*a*c+b^2)^(1/2)/(a*e^2-b*d*e+c*d^2)+(C*d^2-e*(-A
*e+B*d))*ln(e*x+d)/e/(a*e^2-b*d*e+c*d^2)+1/2*(-A*c*e+B*c*d+C*a*e-C*b*d)*ln
(c*x^2+b*x+a)/c/(a*e^2-b*d*e+c*d^2)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.98

$$\int \frac{A+Bx+Cx^2}{ad+(bd+ae)x+(cd+be)x^2+cex^3}\,dx$$
$$= \frac{-2e(-b^2Cd-2c(Acd-aCd+aBe)+b(Bcd+Ace+aCe))\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)+2c\sqrt{-b^2+4ac}(Cd^2}{2c\sqrt{-b^2+4ac}\,e\,(cd^2+e}$$

input
```
Integrate[(A + B*x + C*x^2)/(a*d + (b*d + a*e)*x + (c*d + b*e)*x^2 + c*e*x
^3),x]
```

output
```
(-2*e*(-(b^2*C*d) - 2*c*(A*c*d - a*C*d + a*B*e) + b*(B*c*d + A*c*e + a*C*e
))*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]] + 2*c*Sqrt[-b^2 + 4*a*c]*(C*d^2
+ e*(-(B*d) + A*e))*Log[d + e*x] + Sqrt[-b^2 + 4*a*c]*e*(B*c*d - b*C*d - A
*c*e + a*C*e)*Log[a + x*(b + c*x)])/(2*c*Sqrt[-b^2 + 4*a*c]*e*(c*d^2 + e*(
-(b*d) + a*e)))
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.047$, Rules used = {2462, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{x(ae+bd)+ad+x^2(be+cd)+cex^3}\,dx$$
$$\Big\downarrow \ 2462$$
$$\int \left(\frac{x(aCe-Ace-bCd+Bcd)+aBe-aCd-Abe+Acd}{(a+bx+cx^2)(ae^2-bde+cd^2)}+\frac{Ae^2-Bde+Cd^2}{(d+ex)(ae^2-bde+cd^2)}\right)dx$$
$$\Big\downarrow \ 2009$$

$$-\frac{\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)(-b(aCe+Ace+Bcd)+2c(aBe-aCd+Acd)+b^2Cd)}{c\sqrt{b^2-4ac}\,(ae^2-bde+cd^2)}+\frac{\log(a+bx+cx^2)\,(aCe-Ace-bCd+Bcd)}{2c\,(ae^2-bde+cd^2)}+\frac{\log(d+ex)\,(Cd^2-e(Bd-Ae))}{e\,(ae^2-bde+cd^2)}$$

input
```
Int[(A + B*x + C*x^2)/(a*d + (b*d + a*e)*x + (c*d + b*e)*x^2 + c*e*x^3),x]
```

output
```
-(((b^2*C*d + 2*c*(A*c*d - a*C*d + a*B*e) - b*(B*c*d + A*c*e + a*C*e))*Arc
Tanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c*Sqrt[b^2 - 4*a*c]*(c*d^2 - b*d*e +
 a*e^2))) + ((C*d^2 - e*(B*d - A*e))*Log[d + e*x])/(e*(c*d^2 - b*d*e + a*e
^2)) + ((B*c*d - b*C*d - A*c*e + a*C*e)*Log[a + b*x + c*x^2])/(2*c*(c*d^2
- b*d*e + a*e^2))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.91

method	result
default	$\frac{\frac{(-Ace+Bcd+Cae-Cbd)\ln(cx^2+bx+a)}{2c}+\frac{2\left(-Abe+Acd+Bae-Cad-\frac{(-Ace+Bcd+Cae-Cbd)b}{2c}\right)\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{ae^2-bde+cd^2}+\frac{(Ae^2-Bde+Cd^2}{(ae^2-bde+}$
risch	$\frac{e\ln(ex+d)A}{ae^2-bde+cd^2}-\frac{\ln(ex+d)Bd}{ae^2-bde+cd^2}+\frac{\ln(ex+d)Cd^2}{(ae^2-bde+cd^2)e}+\left(\sum_{\_R=\operatorname{RootOf}((4a^2c^2e^2-ab^2ce^2-4abc^2de+4ac^3d^2+b^3cde-b^2c^2d}\right.$

input
```
int((C*x^2+B*x+A)/(a*d+(a*e+b*d)*x+(b*e+c*d)*x^2+c*e*x^3),x,method=_RETURN
VERBOSE)
```

output
```
1/(a*e^2-b*d*e+c*d^2)*(1/2*(-A*c*e+B*c*d+C*a*e-C*b*d)/c*ln(c*x^2+b*x+a)+2*
(-A*b*e+A*c*d+B*a*e-C*a*d-1/2*(-A*c*e+B*c*d+C*a*e-C*b*d)*b/c)/(4*a*c-b^2)^
(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2)))+(A*e^2-B*d*e+C*d^2)/(a*e^2-b*d*
e+c*d^2)/e*ln(e*x+d)
```

Fricas [A] (verification not implemented)

Time = 32.62 (sec) , antiderivative size = 613, normalized size of antiderivative = 3.13

$$\int \frac{A+Bx+Cx^2}{ad+(bd+ae)x+(cd+be)x^2+cex^3}\,dx$$
$$= \left[-\frac{((Cb^2+2\,Ac^2-(2\,Ca+Bb)c)de-(Cab-(2\,Ba-Ab)c)e^2)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac+\sqrt{b^2-}}{cx^2+bx+a}\right.}{} \right.$$
$$-\frac{2\,((Cb^2+2\,Ac^2-(2\,Ca+Bb)c)de-(Cab-(2\,Ba-Ab)c)e^2)\sqrt{-b^2+4\,ac}\arctan\left(-\frac{\sqrt{-b^2+4\,ac}(2\,cx+b}{b^2-4\,ac}\right.}{}$$

input
```
integrate((C*x^2+B*x+A)/(a*d+(a*e+b*d)*x+(b*e+c*d)*x^2+c*e*x^3),x, algorit
hm="fricas")
```

output
```
[-1/2*(((C*b^2 + 2*A*c^2 - (2*C*a + B*b)*c)*d*e - (C*a*b - (2*B*a - A*b)*c
)*e^2)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2
 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + ((C*b^3 + 4*B*a*c^2 - (4*C*a*b
 + B*b^2)*c)*d*e - (C*a*b^2 + 4*A*a*c^2 - (4*C*a^2 + A*b^2)*c)*e^2)*log(c*
x^2 + b*x + a) - 2*((C*b^2*c - 4*C*a*c^2)*d^2 - (B*b^2*c - 4*B*a*c^2)*d*e
+ (A*b^2*c - 4*A*a*c^2)*e^2)*log(e*x + d))/((b^2*c^2 - 4*a*c^3)*d^2*e - (b
^3*c - 4*a*b*c^2)*d*e^2 + (a*b^2*c - 4*a^2*c^2)*e^3), -1/2*(2*((C*b^2 + 2*
A*c^2 - (2*C*a + B*b)*c)*d*e - (C*a*b - (2*B*a - A*b)*c)*e^2)*sqrt(-b^2 +
4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + ((C*b^3 + 4
*B*a*c^2 - (4*C*a*b + B*b^2)*c)*d*e - (C*a*b^2 + 4*A*a*c^2 - (4*C*a^2 + A*
b^2)*c)*e^2)*log(c*x^2 + b*x + a) - 2*((C*b^2*c - 4*C*a*c^2)*d^2 - (B*b^2*
c - 4*B*a*c^2)*d*e + (A*b^2*c - 4*A*a*c^2)*e^2)*log(e*x + d))/((b^2*c^2 -
4*a*c^3)*d^2*e - (b^3*c - 4*a*b*c^2)*d*e^2 + (a*b^2*c - 4*a^2*c^2)*e^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + Bx + Cx^2}{ad + (bd + ae)x + (cd + be)x^2 + cex^3}\, dx = \text{Timed out}$$

input
```
integrate((C*x**2+B*x+A)/(a*d+(a*e+b*d)*x+(b*e+c*d)*x**2+c*e*x**3),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A + Bx + Cx^2}{ad + (bd + ae)x + (cd + be)x^2 + cex^3}\, dx = \text{Exception raised: ValueError}$$

input
```
integrate((C*x^2+B*x+A)/(a*d+(a*e+b*d)*x+(b*e+c*d)*x^2+c*e*x^3),x, algorit
hm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.02

$$\int \frac{A+Bx+Cx^2}{ad+(bd+ae)x+(cd+be)x^2+cex^3}\,dx = -\frac{(Cbd-Bcd-Cae+Ace)\log(cx^2+bx+a)}{2\,(c^2d^2-bcde+ace^2)}+\frac{(Cd^2-Bde+Ae^2)\log(|ex+d|)}{cd^2e-bde^2+ae^3} + \frac{(Cb^2d-2\,Cacd-Bbcd+2\,Ac^2d-Cabe+2\,Bace-Abce)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(c^2d^2-bcde+ace^2)\sqrt{-b^2+4\,ac}}$$

input
```
integrate((C*x^2+B*x+A)/(a*d+(a*e+b*d)*x+(b*e+c*d)*x^2+c*e*x^3),x, algorit
hm="giac")
```

output
```
-1/2*(C*b*d - B*c*d - C*a*e + A*c*e)*log(c*x^2 + b*x + a)/(c^2*d^2 - b*c*d
*e + a*c*e^2) + (C*d^2 - B*d*e + A*e^2)*log(abs(e*x + d))/(c*d^2*e - b*d*e
^2 + a*e^3) + (C*b^2*d - 2*C*a*c*d - B*b*c*d + 2*A*c^2*d - C*a*b*e + 2*B*a
*c*e - A*b*c*e)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((c^2*d^2 - b*c*d*e
 + a*c*e^2)*sqrt(-b^2 + 4*a*c))
```

Mupad [B] (verification not implemented)

Time = 18.29 (sec) , antiderivative size = 2467, normalized size of antiderivative = 12.59

$$\int \frac{A+Bx+Cx^2}{ad+(bd+ae)x+(cd+be)x^2+cex^3}\,dx = \text{Too large to display}$$

input
```
int((A + B*x + C*x^2)/(a*d + x*(a*e + b*d) + x^2*(b*e + c*d) + c*e*x^3),x)
```

output
```
(log(d + e*x)*(A*e^2 + C*d^2 - B*d*e))/(a*e^3 - b*d*e^2 + c*d^2*e) - (log(
6*A*a^2*c*e^4 - 2*A*a*b^2*e^4 - 2*C*a^3*e^4 + B*a^2*b*e^4 - 4*C*a*c^2*d^4
+ C*b^2*c*d^4 + C*b^3*d^3*e - 2*A*b^3*e^4*x - B*a^2*e^4*(b^2 - 4*a*c)^(1/2
) + B*a*b^2*e^4*x - 2*B*a^2*c*e^4*x - C*a^2*b*e^4*x + 2*A*c^3*d^3*e*x + B*
b^3*d*e^3*x + A*c^2*d^3*e*(b^2 - 4*a*c)^(1/2) + 3*C*a^2*d*e^3*(b^2 - 4*a*c
)^(1/2) + C*b^2*d^3*e*(b^2 - 4*a*c)^(1/2) + 2*A*b^2*e^4*x*(b^2 - 4*a*c)^(1
/2) + C*a^2*e^4*x*(b^2 - 4*a*c)^(1/2) + 2*C*c^2*d^4*x*(b^2 - 4*a*c)^(1/2)
- 10*A*a*c^2*d^2*e^2 + A*b^2*c*d^2*e^2 - 4*C*a*b^2*d^2*e^2 + 10*C*a^2*c*d^
2*e^2 - C*b^3*d^2*e^2*x + 2*A*a*b*e^4*(b^2 - 4*a*c)^(1/2) + C*b*c*d^4*(b^2
 - 4*a*c)^(1/2) + B*a*b^2*d*e^3 + A*b*c^2*d^3*e + 6*B*a*c^2*d^3*e - 10*B*a
^2*c*d*e^3 + 3*C*a^2*b*d*e^3 - 2*B*b^2*c*d^3*e + 7*A*a*b*c*e^4*x + 5*A*c^2
*d^2*e^2*x*(b^2 - 4*a*c)^(1/2) + C*b^2*d^2*e^2*x*(b^2 - 4*a*c)^(1/2) - 7*A
*a*c*d*e^3*(b^2 - 4*a*c)^(1/2) - B*a*b*d*e^3*(b^2 - 4*a*c)^(1/2) - 2*B*b*c
*d^3*e*(b^2 - 4*a*c)^(1/2) - 5*C*a*c*d^3*e*(b^2 - 4*a*c)^(1/2) - 3*A*a*c*e
^4*x*(b^2 - 4*a*c)^(1/2) - B*a*b*e^4*x*(b^2 - 4*a*c)^(1/2) + 3*B*a*b*c*d^2
*e^2 - 14*A*a*c^2*d*e^3*x + 5*A*b^2*c*d*e^3*x - B*b*c^2*d^3*e*x - 10*C*a*c
^2*d^3*e*x + 6*C*a^2*c*d*e^3*x + 3*C*b^2*c*d^3*e*x + A*b*c*d^2*e^2*(b^2 -
4*a*c)^(1/2) + 7*B*a*c*d^2*e^2*(b^2 - 4*a*c)^(1/2) - 2*C*a*b*d^2*e^2*(b^2
- 4*a*c)^(1/2) - B*b^2*d*e^3*x*(b^2 - 4*a*c)^(1/2) - 3*B*c^2*d^3*e*x*(b^2
- 4*a*c)^(1/2) - 3*A*b*c^2*d^2*e^2*x + 14*B*a*c^2*d^2*e^2*x - 2*B*b^2*c...
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.05

$$\int \frac{A+Bx+Cx^2}{ad+(bd+ae)x+(cd+be)x^2+cex^3}\,dx=\frac{\log(ex+d)}{e}$$

input
```
int((C*x^2+B*x+A)/(a*d+(a*e+b*d)*x+(b*e+c*d)*x^2+c*e*x^3),x)
```

output
```
log(d + e*x)/e
```

3.128 $\int \frac{A+Bx+Cx^2}{(ad+(bd+ae)x+(cd+be)x^2+cex^3)^2}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 676

$$\int \frac{A+Bx+Cx^2}{(ad+(bd+ae)x+(cd+be)x^2+cex^3)^2}\,dx =$$

$$-\frac{e(b^2(2Cd^2-e(Bd-2Ae))-b(2(Ac+aC)de+B(cd^2+ae^2))+2(Ac(cd^2-3ae^2)+a(aCe^2-cd(3C}{(b^2-4ac)\left(cd^2-bde+ae^2\right)^2(d+ex)}$$

$$-\frac{A(bcd-b^2e+2ace)-a(2Bcd-bCd-bBe+2aCe)-((Bc-bC)(bd-2ae)-(Ac-aC)(2cd-be}{(b^2-4ac)\left(cd^2-bde+ae^2\right)(d+ex)\left(a+bx+cx^2\right)}$$

$$+\frac{(b^4e^3(Bd-2Ae)+6b^2ce(Bcd^3+2aCd^2e-aBde^2+2aAe^3)-b^3e(ae^2(2Cd-Be)+cd(2Cd^2+3Bc}{}$$

$$+\frac{e(e^2(bBd-2aCd-2Abe+aBe)+cd(2Cd^2-e(3Bd-4Ae)))\log(d+ex)}{\left(cd^2-bde+ae^2\right)^3}$$

$$-\frac{e(e^2(bBd-2aCd-2Abe+aBe)+cd(2Cd^2-e(3Bd-4Ae)))\log\left(a+bx+cx^2\right)}{2\left(cd^2-bde+ae^2\right)^3}$$

output
```
-e*(b^2*(2*C*d^2-e*(-2*A*e+B*d))-b*(2*(A*c+C*a)*d*e+B*(a*e^2+c*d^2))+2*A*c
*(-3*a*e^2+c*d^2)+2*a*(a*C*e^2-c*d*(-4*B*e+3*C*d)))/(-4*a*c+b^2)/(a*e^2-b*
d*e+c*d^2)^2/(e*x+d)-(A*(2*a*c*e-b^2*e+b*c*d)-a*(-B*b*e+2*B*c*d+2*C*a*e-C*
b*d)-((B*c-C*b)*(-2*a*e+b*d)-(A*c-C*a)*(-b*e+2*c*d))*x)/(-4*a*c+b^2)/(a*e^
2-b*d*e+c*d^2)/(e*x+d)/(c*x^2+b*x+a)+(b^4*e^3*(-2*A*e+B*d)+6*b^2*c*e*(2*A*
a*e^3-B*a*d*e^2+B*c*d^3+2*C*a*d^2*e)-b^3*e*(a*e^2*(-B*e+2*C*d)+c*d*(-4*A*e
^2+3*B*d*e+2*C*d^2))-2*b*c*(4*A*c*d*e*(3*a*e^2+c*d^2)+B*(3*a^2*e^4+c^2*d^4
))+4*c*(A*c*(-3*a^2*e^4+6*a*c*d^2*e^2+c^2*d^4)+a*(a^2*C*e^4+c^2*d^3*(-2*B*
e+C*d)-6*a*c*d*e^2*(-B*e+C*d))))*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/(-4
*a*c+b^2)^(3/2)/(a*e^2-b*d*e+c*d^2)^3+e*(e^2*(-2*A*b*e+B*a*e+B*b*d-2*C*a*d
)+c*d*(2*C*d^2-e*(-4*A*e+3*B*d)))*ln(e*x+d)/(a*e^2-b*d*e+c*d^2)^3-1/2*e*(e
^2*(-2*A*b*e+B*a*e+B*b*d-2*C*a*d)+c*d*(2*C*d^2-e*(-4*A*e+3*B*d)))*ln(c*x^2
+b*x+a)/(a*e^2-b*d*e+c*d^2)^3
```

Mathematica [A] (verified)

Time = 1.70 (sec) , antiderivative size = 661, normalized size of antiderivative = 0.98

$$\int \frac{A+Bx+Cx^2}{(ad+(bd+ae)x+(cd+be)x^2+cex^3)^2}\,dx = -\frac{e(Cd^2+e(-Bd+Ae))}{(cd^2+e(-bd+ae))^2\,(d+ex)}$$
$$+\frac{bc(Bc-bC)d^2x-a^2e(bCe+2c(-2Cd+Be+Cex))-A(b^3e^2+b^2ce(-2d+ex)+bc(-3ae^2+cd(\ldots}{(b^2-4ac)\,(cd^2+\ldots}$$
$$-\frac{(b^4e^3(Bd-2Ae)+6b^2ce(Bcd^3+2aCd^2e-aBde^2+2aAe^3)+b^3(ae^3(-2Cd+Be)+cde(-2Cd^2-\ldots}{\ldots}$$
$$+\frac{e(2cCd^3+cde(-3Bd+4Ae)+e^2(bBd-2aCd-2Abe+aBe))\log(d+ex)}{(cd^2+e(-bd+ae))^3}$$
$$-\frac{e(2cCd^3+cde(-3Bd+4Ae)+e^2(bBd-2aCd-2Abe+aBe))\log(a+x(b+cx))}{2\,(cd^2+e(-bd+ae))^3}$$

input
```
Integrate[(A + B*x + C*x^2)/(a*d + (b*d + a*e)*x + (c*d + b*e)*x^2 + c*e*x
^3)^2,x]
```

output
```
-((e*(C*d^2 + e*(-(B*d) + A*e)))/((c*d^2 + e*(-(b*d) + a*e))^2*(d + e*x)))
 + (b*c*(B*c - b*C)*d^2*x - a^2*e*(b*C*e + 2*c*(-2*C*d + B*e + C*e*x)) - A
*(b^3*e^2 + b^2*c*e*(-2*d + e*x) + b*c*(-3*a*e^2 + c*d*(d - 2*e*x)) + 2*c^
2*(c*d^2*x + a*e*(2*d - e*x))) + a*(c*C*d*(-(b*d) + 2*c*d*x + 2*b*e*x) + B
*(b^2*e^2 + 2*c^2*d*(d - 2*e*x) + b*c*e*(-2*d + e*x))))/((b^2 - 4*a*c)*(c*
d^2 + e*(-(b*d) + a*e))^2*(a + x*(b + c*x))) - ((b^4*e^3*(B*d - 2*A*e) + 6
*b^2*c*e*(B*c*d^3 + 2*a*C*d^2*e - a*B*d*e^2 + 2*a*A*e^3) + b^3*(a*e^3*(-2*
C*d + B*e) + c*d*e*(-2*C*d^2 - 3*B*d*e + 4*A*e^2)) - 2*b*c*(4*A*c*d*e*(c*d
^2 + 3*a*e^2) + B*(c^2*d^4 + 3*a^2*e^4)) + 4*c*(A*c*(c^2*d^4 + 6*a*c*d^2*e
^2 - 3*a^2*e^4) + a*(a^2*C*e^4 + c^2*d^3*(C*d - 2*B*e) + 6*a*c*d*e^2*(-(C*
d) + B*e))))*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/((-b^2 + 4*a*c)^(3/2)
*(-(c*d^2) + e*(b*d - a*e))^3) + (e*(2*c*C*d^3 + c*d*e*(-3*B*d + 4*A*e) +
e^2*(b*B*d - 2*a*C*d - 2*A*b*e + a*B*e))*Log[d + e*x])/(c*d^2 + e*(-(b*d)
+ a*e))^3 - (e*(2*c*C*d^3 + c*d*e*(-3*B*d + 4*A*e) + e^2*(b*B*d - 2*a*C*d
- 2*A*b*e + a*B*e))*Log[a + x*(b + c*x)])/(2*(c*d^2 + e*(-(b*d) + a*e))^3)
```

Rubi [A] (verified)

Time = 3.13 (sec) , antiderivative size = 695, normalized size of antiderivative = 1.03, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.047$, Rules used = {2462, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A + Bx + Cx^2}{(x(ae + bd) + ad + x^2(be + cd) + cex^3)^2} \, dx$$

$$\Big\downarrow \text{2462}$$

$$\int \left(\frac{cx(-aBe^2 + 2aCde + Abe^2 - 2Acde - bCd^2 + Bcd^2) - Ace(ae + 2bd) - ae^2(bB - aC) - acd(Cd - 2Be)}{(a + bx + cx^2)^2 (ae^2 - bde + cd^2)^2} \right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2c\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(-2a^2Ce^2+2bde(aC+Ac)-2Ac\left(cd^2-ae^2\right)+bB\left(ae^2+cd^2\right)+2acd(Cd-2Be)-\left(b^2\right.\right.}{\left(b^2-4ac\right)^{3/2}\left(ae^2-bde+cd^2\right)^2}$$

$$\frac{cx\left(-2a^2Ce^2+2bde(aC+Ac)-2Ac\left(cd^2-ae^2\right)+bB\left(ae^2+cd^2\right)+2acd(Cd-2Be)-\left(b^2\left(Ae^2+Cd^2\right)\right)\right)+A}{\left(b^2-4ac\right)\left(a+bx+\right.}$$

$$\frac{\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(-ce^2\left(3bd(Bd-2Ae)-2a\left(Ae^2-2Bde+3Cd^2\right)\right)+be^3(aBe-2aCd-2Abe+bBd)-2c^2\right.}{\sqrt{b^2-4ac}\left(ae^2-bde+cd^2\right)^3}$$

$$\frac{e\left(Cd^2-e(Bd-Ae)\right)}{(d+ex)\left(ae^2-bde+cd^2\right)^2}-$$

$$\frac{e\log\left(a+bx+cx^2\right)\left(e^2(aBe-2aCd-2Abe+bBd)-cde(3Bd-4Ae)+2cCd^3\right)}{2\left(ae^2-bde+cd^2\right)^3}+$$

$$\frac{e\log(d+ex)\left(e^2(aBe-2aCd-2Abe+bBd)-cde(3Bd-4Ae)+2cCd^3\right)}{\left(ae^2-bde+cd^2\right)^3}$$

input
```
Int[(A + B*x + C*x^2)/(a*d + (b*d + a*e)*x + (c*d + b*e)*x^2 + c*e*x^3)^2,
x]
```

output
```
-((e*(C*d^2 - e*(B*d - A*e)))/((c*d^2 - b*d*e + a*e^2)^2*(d + e*x))) + (A*
(2*b^2*c*d*e - 4*a*c^2*d*e - b^3*e^2 - b*c*(c*d^2 - 3*a*e^2)) - a*(C*(b*c*
d^2 - 4*a*c*d*e + a*b*e^2) - B*(2*c^2*d^2 + b^2*e^2 - 2*c*e*(b*d + a*e)))
+ c*(2*b*(A*c + a*C)*d*e - 2*a^2*C*e^2 + 2*a*c*d*(C*d - 2*B*e) - 2*A*c*(c*
d^2 - a*e^2) + b*B*(c*d^2 + a*e^2) - b^2*(C*d^2 + A*e^2))*x)/((b^2 - 4*a*c
)*(c*d^2 - b*d*e + a*e^2)^2*(a + b*x + c*x^2)) - (2*c*(2*b*(A*c + a*C)*d*e
 - 2*a^2*C*e^2 + 2*a*c*d*(C*d - 2*B*e) - 2*A*c*(c*d^2 - a*e^2) + b*B*(c*d^
2 + a*e^2) - b^2*(C*d^2 + A*e^2))*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/
((b^2 - 4*a*c)^(3/2)*(c*d^2 - b*d*e + a*e^2)^2) + ((b*e^3*(b*B*d - 2*a*C*d
 - 2*A*b*e + a*B*e) - 2*c^2*(C*d^4 - d^2*e*(2*B*d - 3*A*e)) - c*e^2*(3*b*d
*(B*d - 2*A*e) - 2*a*(3*C*d^2 - 2*B*d*e + A*e^2)))*ArcTanh[(b + 2*c*x)/Sqr
t[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*c]*(c*d^2 - b*d*e + a*e^2)^3) + (e*(2*c*C
*d^3 - c*d*e*(3*B*d - 4*A*e) + e^2*(b*B*d - 2*a*C*d - 2*A*b*e + a*B*e))*Lo
g[d + e*x])/(c*d^2 - b*d*e + a*e^2)^3 - (e*(2*c*C*d^3 - c*d*e*(3*B*d - 4*A
*e) + e^2*(b*B*d - 2*a*C*d - 2*A*b*e + a*B*e))*Log[a + b*x + c*x^2])/(2*(c
*d^2 - b*d*e + a*e^2)^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1352 vs. $2(669) = 1338$.

Time = 1.06 (sec) , antiderivative size = 1353, normalized size of antiderivative = 2.00

method	result	size
default	Expression too large to display	1353
risch	Expression too large to display	1359690

input
```
int((C*x^2+B*x+A)/(a*d+(a*e+b*d)*x+(b*e+c*d)*x^2+c*e*x^3)^2,x,method=_RETU
RNVERBOSE)
```

output
```
-1/(a*e^2-b*d*e+c*d^2)^3*((c*(2*A*a^2*c*e^4-A*a*b^2*e^4+A*b^3*d*e^3-3*A*b^
2*c*d^2*e^2+4*A*b*c^2*d^3*e-2*A*c^3*d^4+B*a^2*b*e^4-4*B*a^2*c*d*e^3-B*a*b^
2*d*e^3+6*B*a*b*c*d^2*e^2-4*B*a*c^2*d^3*e-B*b^2*c*d^3*e+B*b*c^2*d^4-2*C*a^
3*e^4+4*C*a^2*b*d*e^3-3*C*a*b^2*d^2*e^2+2*C*a*c^2*d^4+C*b^3*d^3*e-C*b^2*c*
d^4)/(4*a*c-b^2)*x+(3*A*a^2*b*c*e^4-4*A*a^2*c^2*d*e^3-A*a*b^3*e^4-A*a*b^2*
c*d*e^3+6*A*a*b*c^2*d^2*e^2-4*A*a*c^3*d^3*e+A*b^4*d*e^3-3*A*b^3*c*d^2*e^2+
3*A*b^2*c^2*d^3*e-A*b*c^3*d^4-2*B*a^3*c*e^4+B*a^2*b^2*e^4-B*a*b^3*d*e^3+3*
B*a*b^2*c*d^2*e^2-4*B*a*b*c^2*d^3*e+2*B*a*c^3*d^4-C*a^3*b*e^4+4*C*a^3*c*d*
e^3+C*a^2*b^2*d*e^3-6*C*a^2*b*c*d^2*e^2+4*C*a^2*c^2*d^3*e+C*a*b^2*c*d^3*e-
C*a*b*c^2*d^4)/(4*a*c-b^2))/(c*x^2+b*x+a)+1/(4*a*c-b^2)*(1/2*(-8*A*a*b*c^2
*e^4+16*A*a*c^3*d*e^3+2*A*b^3*c*e^4-4*A*b^2*c^2*d*e^3+4*B*a^2*c^2*e^4-B*a*
b^2*c*e^4+4*B*a*b*c^2*d*e^3-12*B*a*c^3*d^2*e^2-B*b^3*c*d*e^3+3*B*b^2*c^2*d
^2*e^2-8*C*a^2*c^2*d*e^3+2*C*a*b^2*c*d*e^3+8*C*a*c^3*d^3*e-2*C*b^2*c^2*d^3
*e)/c*ln(c*x^2+b*x+a)+2*(6*A*a^2*c^2*e^4-2*C*a^3*c*e^4-2*C*a*c^3*d^4+20*A*
a*b*c^2*d*e^3+5*B*a*b^2*c*d*e^3-6*B*a*b*c^2*d^2*e^2-4*C*a^2*b*c*d*e^3+4*C*
a*b*c^2*d^3*e-B*a*b^3*e^4-B*b^4*d*e^3-12*A*a*c^3*d^2*e^2+3*B*b^3*c*d^2*e^2
-3*B*b^2*c^2*d^3*e+12*C*a^2*c^2*d^2*e^2+2*C*a*b^3*d*e^3-10*A*a*b^2*c*e^4-4
*A*b^3*c*d*e^3+4*A*b*c^3*d^3*e+5*B*a^2*b*c*e^4-12*B*a^2*c^2*d*e^3+4*B*a*c^
3*d^3*e+B*b*c^3*d^4-6*C*a*b^2*c*d^2*e^2-2*A*c^4*d^4-1/2*(-8*A*a*b*c^2*e^4+
16*A*a*c^3*d*e^3+2*A*b^3*c*e^4-4*A*b^2*c^2*d*e^3+4*B*a^2*c^2*e^4-B*a*b^...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(ad+(bd+ae)x+(cd+be)x^2+cex^3)^2}\,dx = \text{Timed out}$$

input
```
integrate((C*x^2+B*x+A)/(a*d+(a*e+b*d)*x+(b*e+c*d)*x^2+c*e*x^3)^2,x, algor
ithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(ad+(bd+ae)x+(cd+be)x^2+cex^3)^2}\,dx=\text{Timed out}$$

input
```
integrate((C*x**2+B*x+A)/(a*d+(a*e+b*d)*x+(b*e+c*d)*x**2+c*e*x**3)**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+Bx+Cx^2}{(ad+(bd+ae)x+(cd+be)x^2+cex^3)^2}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((C*x^2+B*x+A)/(a*d+(a*e+b*d)*x+(b*e+c*d)*x^2+c*e*x^3)^2,x, algor
ithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1517 vs. $2(671) = 1342$.

Time $= 0.15$ (sec) , antiderivative size $= 1517$, normalized size of antiderivative $= 2.24$

$$\int \frac{A+Bx+Cx^2}{(ad+(bd+ae)x+(cd+be)x^2+cex^3)^2}\,dx=\text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)/(a*d+(a*e+b*d)*x+(b*e+c*d)*x^2+c*e*x^3)^2,x, algor
ithm="giac")
```

output
```
-1/2*(2*C*c*d^3*e - 3*B*c*d^2*e^2 - 2*C*a*d*e^3 + B*b*d*e^3 + 4*A*c*d*e^3
+ B*a*e^4 - 2*A*b*e^4)*log(c*x^2 + b*x + a)/(c^3*d^6 - 3*b*c^2*d^5*e + 3*b
^2*c*d^4*e^2 + 3*a*c^2*d^4*e^2 - b^3*d^3*e^3 - 6*a*b*c*d^3*e^3 + 3*a*b^2*d
^2*e^4 + 3*a^2*c*d^2*e^4 - 3*a^2*b*d*e^5 + a^3*e^6) + (2*C*c*d^3*e^2 - 3*B
*c*d^2*e^3 - 2*C*a*d*e^4 + B*b*d*e^4 + 4*A*c*d*e^4 + B*a*e^5 - 2*A*b*e^5)*
log(abs(e*x + d))/(c^3*d^6*e - 3*b*c^2*d^5*e^2 + 3*b^2*c*d^4*e^3 + 3*a*c^2
*d^4*e^3 - b^3*d^3*e^4 - 6*a*b*c*d^3*e^4 + 3*a*b^2*d^2*e^5 + 3*a^2*c*d^2*e
^5 - 3*a^2*b*d*e^6 + a^3*e^7) - (4*C*a*c^3*d^4 - 2*B*b*c^3*d^4 + 4*A*c^4*d
^4 - 2*C*b^3*c*d^3*e + 6*B*b^2*c^2*d^3*e - 8*B*a*c^3*d^3*e - 8*A*b*c^3*d^3
*e + 12*C*a*b^2*c*d^2*e^2 - 3*B*b^3*c*d^2*e^2 - 24*C*a^2*c^2*d^2*e^2 + 24*
A*a*c^3*d^2*e^2 - 2*C*a*b^3*d*e^3 + B*b^4*d*e^3 - 6*B*a*b^2*c*d*e^3 + 4*A*
b^3*c*d*e^3 + 24*B*a^2*c^2*d*e^3 - 24*A*a*b*c^2*d*e^3 + B*a*b^3*e^4 - 2*A*
b^4*e^4 + 4*C*a^3*c*e^4 - 6*B*a^2*b*c*e^4 + 12*A*a*b^2*c*e^4 - 12*A*a^2*c^
2*e^4)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^2*c^3*d^6 - 4*a*c^4*d^6
- 3*b^3*c^2*d^5*e + 12*a*b*c^3*d^5*e + 3*b^4*c*d^4*e^2 - 9*a*b^2*c^2*d^4*e
^2 - 12*a^2*c^3*d^4*e^2 - b^5*d^3*e^3 - 2*a*b^3*c*d^3*e^3 + 24*a^2*b*c^2*d
^3*e^3 + 3*a*b^4*d^2*e^4 - 9*a^2*b^2*c*d^2*e^4 - 12*a^3*c^2*d^2*e^4 - 3*a^
2*b^3*d*e^5 + 12*a^3*b*c*d*e^5 + a^3*b^2*e^6 - 4*a^4*c*e^6)*sqrt(-b^2 + 4*
a*c)) - (2*C*b^2*c*d^2*e*x^2 - 6*C*a*c^2*d^2*e*x^2 - B*b*c^2*d^2*e*x^2 + 2
*A*c^3*d^2*e*x^2 - 2*C*a*b*c*d*e^2*x^2 - B*b^2*c*d*e^2*x^2 + 8*B*a*c^2*...
```

Mupad [B] (verification not implemented)

Time = 17.01 (sec) , antiderivative size = 26278, normalized size of antiderivative = 38.87

$$\int \frac{A+Bx+Cx^2}{(ad+(bd+ae)x+(cd+be)x^2+cex^3)^2}\,dx = \text{Too large to display}$$

input
```
int((A + B*x + C*x^2)/(a*d + x*(a*e + b*d) + x^2*(b*e + c*d) + c*e*x^3)^2,
x)
```

output
```
((A*a*b^2*e^3 + A*b*c^2*d^3 - 4*A*a^2*c*e^3 - 2*B*a*c^2*d^3 + A*b^3*d*e^2
+ C*a*b*c*d^3 + 4*A*a*c^2*d^2*e - 2*B*a*b^2*d*e^2 - 2*A*b^2*c*d^2*e + 6*B*
a^2*c*d*e^2 + C*a*b^2*d^2*e + C*a^2*b*d*e^2 - 8*C*a^2*c*d^2*e - 3*A*a*b*c*
d*e^2 + 2*B*a*b*c*d^2*e)/(4*a*c^3*d^4 + 4*a^3*c*e^4 - a^2*b^2*e^4 - b^2*c^
2*d^4 - b^4*d^2*e^2 + 8*a^2*c^2*d^2*e^2 + 2*a*b^3*d*e^3 + 2*b^3*c*d^3*e -
8*a*b*c^2*d^3*e - 8*a^2*b*c*d*e^3 + 2*a*b^2*c*d^2*e^2) - (x^2*(6*A*a*c^2*e
^3 - 2*A*b^2*c*e^3 - 2*C*a^2*c*e^3 - 2*A*c^3*d^2*e + B*a*b*c*e^3 + 2*A*b*c
^2*d*e^2 - 8*B*a*c^2*d*e^2 + B*b*c^2*d^2*e + B*b^2*c*d*e^2 + 6*C*a*c^2*d^2
*e - 2*C*b^2*c*d^2*e + 2*C*a*b*c*d*e^2))/(4*a*c^3*d^4 + 4*a^3*c*e^4 - a^2*
b^2*e^4 - b^2*c^2*d^4 - b^4*d^2*e^2 + 8*a^2*c^2*d^2*e^2 + 2*a*b^3*d*e^3 +
2*b^3*c*d^3*e - 8*a*b*c^2*d^3*e - 8*a^2*b*c*d*e^3 + 2*a*b^2*c*d^2*e^2) + (
x*(2*A*b^3*e^3 + 2*A*c^3*d^3 - B*a*b^2*e^3 - B*b*c^2*d^3 + 2*B*a^2*c*e^3 -
 2*C*a*c^2*d^3 + C*a^2*b*e^3 + C*b^2*c*d^3 - B*b^3*d*e^2 + C*b^3*d^2*e - 7
*A*a*b*c*e^3 + 2*A*a*c^2*d*e^2 - A*b*c^2*d^2*e - A*b^2*c*d*e^2 + 2*B*a*c^2
*d^2*e - 2*C*a^2*c*d*e^2 + 5*B*a*b*c*d*e^2 - 5*C*a*b*c*d^2*e))/(4*a*c^3*d^
4 + 4*a^3*c*e^4 - a^2*b^2*e^4 - b^2*c^2*d^4 - b^4*d^2*e^2 + 8*a^2*c^2*d^2*
e^2 + 2*a*b^3*d*e^3 + 2*b^3*c*d^3*e - 8*a*b*c^2*d^3*e - 8*a^2*b*c*d*e^3 +
2*a*b^2*c*d^2*e^2))/(a*d + x*(a*e + b*d) + x^2*(b*e + c*d) + c*e*x^3) + sy
msum(log((x*(36*A^2*a^2*c^5*e^7 + 4*A^2*b^4*c^3*e^7 + 4*C^2*a^4*c^3*e^7 +
4*A^2*c^7*d^4*e^3 + B^2*a^2*b^2*c^3*e^7 + 12*A^2*b^2*c^5*d^2*e^5 + 64*B...
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 983, normalized size of antiderivative = 1.45

$$\int \frac{A+Bx+Cx^2}{(ad+(bd+ae)x+(cd+be)x^2+cex^3)^2}\,dx = \text{Too large to display}$$

input
```
int((C*x^2+B*x+A)/(a*d+(a*e+b*d)*x+(b*e+c*d)*x^2+c*e*x^3)^2,x)
```

output

```
( - 4*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*c*d**2*e**
2 - 4*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*c*d*e**3*x
 + 2*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b**2*d**2*e**
2 + 2*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b**2*d*e**3*
x - 4*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b*c*d**3*e -
 4*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b*c*d**2*e**2*x
 + 4*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*c**2*d**4 + 4
*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*c**2*d**3*e*x + 4
*log(a + b*x + c*x**2)*a*b*c*d**2*e**2 + 4*log(a + b*x + c*x**2)*a*b*c*d*e
**3*x - 8*log(a + b*x + c*x**2)*a*c**2*d**3*e - 8*log(a + b*x + c*x**2)*a*
c**2*d**2*e**2*x - log(a + b*x + c*x**2)*b**3*d**2*e**2 - log(a + b*x + c*
x**2)*b**3*d*e**3*x + 2*log(a + b*x + c*x**2)*b**2*c*d**3*e + 2*log(a + b*
x + c*x**2)*b**2*c*d**2*e**2*x - 8*log(d + e*x)*a*b*c*d**2*e**2 - 8*log(d
+ e*x)*a*b*c*d*e**3*x + 16*log(d + e*x)*a*c**2*d**3*e + 16*log(d + e*x)*a*
c**2*d**2*e**2*x + 2*log(d + e*x)*b**3*d**2*e**2 + 2*log(d + e*x)*b**3*d*e
**3*x - 4*log(d + e*x)*b**2*c*d**3*e - 4*log(d + e*x)*b**2*c*d**2*e**2*x +
 8*a**2*c*e**4*x - 2*a*b**2*e**4*x - 8*a*b*c*d*e**3*x + 8*a*c**2*d**2*e**2
*x + 2*b**3*d*e**3*x - 2*b**2*c*d**2*e**2*x)/(2*d*(4*a**3*c*d*e**4 + 4*a**
3*c*e**5*x - a**2*b**2*d*e**4 - a**2*b**2*e**5*x - 8*a**2*b*c*d**2*e**3 -
8*a**2*b*c*d*e**4*x + 8*a**2*c**2*d**3*e**2 + 8*a**2*c**2*d**2*e**3*x +...
```

3.129 $\int (A+Bx+Cx^2)\sqrt{ad+(bd+ae)x+(cd+be)x$

Optimal result

Integrand size = 45, antiderivative size = 995

$$\int (A+Bx+Cx^2)\sqrt{ad+(bd+ae)x+(cd+be)x^2+cex^3}\,dx$$

$$= \frac{2(8b^3Ce^3-3bce^2(bCd+4bBe-aCe)+c^3d(8Cd^2-3e(4Bd-7Ae))+3c^2e(ae(Cd-5Be)-b(Cd^2-}{}$$

$$+\frac{2\left(3Bc-\frac{2C(cd+be)}{e}\right)(a+bx+cx^2)\sqrt{ad+(bd+ae)x+(cd+be)x^2+cex^3}}{21c^2}$$

$$+\frac{2C(d+ex)(a+bx+cx^2)\sqrt{ad+(bd+ae)x+(cd+be)x^2+cex^3}}{9ce}$$

$$+\frac{\sqrt{2}\sqrt{b^2-4ac}\left(\frac{(8c^2d^2-2b^2e^2-3ce(bd-2ae))(8b^2Ce^2-ce(bCd+12bBe+7aCe)-c^2(2Cd^2-3e(Bd+7Ae)))}{e}-5c(2cd-be)(6b^2}{}$$

$$-\frac{2\sqrt{2}\sqrt{b^2-4ac}(cd^2-bde+ae^2)(8b^3Ce^3-3c^2e^2(bBd+2aCd-7Abe-10aBe)+3bce^2(bCd-4bB}{}$$

output
```
2/315*(8*b^3*C*e^3-3*b*c*e^2*(4*B*b*e-C*a*e+C*b*d)+c^3*d*(8*C*d^2-3*e*(-7*
A*e+4*B*d))+3*c^2*e*(a*e*(-5*B*e+C*d)-b*(-7*A*e^2-2*B*d*e+C*d^2))+3*c*e*(8
*b^2*C*e^2-c*e*(12*B*b*e+7*C*a*e+C*b*d)-c^2*(2*C*d^2-3*e*(7*A*e+B*d)))*x)*
(a*d+(a*e+b*d)*x+(b*e+c*d)*x^2+c*e*x^3)^(1/2)/c^3/e^3+2/21*(3*B*c-2*C*(b*e
+c*d)/e)*(c*x^2+b*x+a)*(a*d+(a*e+b*d)*x+(b*e+c*d)*x^2+c*e*x^3)^(1/2)/c^2+2
/9*C*(e*x+d)*(c*x^2+b*x+a)*(a*d+(a*e+b*d)*x+(b*e+c*d)*x^2+c*e*x^3)^(1/2)/c
/e+1/315*2^(1/2)*(-4*a*c+b^2)^(1/2)*((8*c^2*d^2-2*b^2*e^2-3*c*e*(-2*a*e+b*
d))*(8*b^2*C*e^2-c*e*(12*B*b*e+7*C*a*e+C*b*d)-c^2*(2*C*d^2-3*e*(7*A*e+B*d)
))/e-5*c*(-b*e+2*c*d)*(6*b^2*C*d*e+c*e*(21*A*c*d-3*B*a*e-5*C*a*d)+b*(2*a*C
*e^2-c*d*(9*B*e+C*d))))*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*(a*d+(a*e+b*
d)*x+(b*e+c*d)*x^2+c*e*x^3)^(1/2)*EllipticE(1/2*(1+(2*c*x+b)/(-4*a*c+b^2)^
(1/2))^(1/2)*2^(1/2),(-2*(-4*a*c+b^2)^(1/2)*e/(2*c*d-(b+(-4*a*c+b^2)^(1/2)
)*e))^(1/2))/c^4/e^3/(c*(e*x+d)/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e))^(1/2)/(c
*x^2+b*x+a)-2/315*2^(1/2)*(-4*a*c+b^2)^(1/2)*(a*e^2-b*d*e+c*d^2)*(8*b^3*C*
e^3-3*c^2*e^2*(-7*A*b*e-10*B*a*e+B*b*d+2*C*a*d)+3*b*c*e^2*(-4*B*b*e-9*C*a*
e+C*b*d)-2*c^3*d*(8*C*d^2-3*e*(-7*A*e+4*B*d)))*(c*(e*x+d)/(2*c*d-(b+(-4*a*
c+b^2)^(1/2))*e))^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*(a*d+(a*e+b*
d)*x+(b*e+c*d)*x^2+c*e*x^3)^(1/2)*EllipticF(1/2*(1+(2*c*x+b)/(-4*a*c+b^2)^
(1/2))^(1/2)*2^(1/2),(-2*(-4*a*c+b^2)^(1/2)*e/(2*c*d-(b+(-4*a*c+b^2)^(1/2)
)*e))^(1/2))/c^4/e^4/(e*x+d)/(c*x^2+b*x+a)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 18.10 (sec) , antiderivative size = 15681, normalized size of antiderivative = 15.76

$$\int (A+Bx+Cx^2)\sqrt{ad+(bd+ae)x+(cd+be)x^2+cex^3}\,dx$$
$$= \text{Result too large to show}$$

input
```
Integrate[(A + B*x + C*x^2)*Sqrt[a*d + (b*d + a*e)*x + (c*d + b*e)*x^2 + c
*e*x^3],x]
```

output
```
Result too large to show
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(A+Bx+Cx^2\right)\sqrt{x(ae+bd)+ad+x^2(be+cd)+cex^3}\,dx$$

$$\Big\downarrow \text{2526}$$

$$\frac{\int -\left((bCd-3Ace+aCe+(2cCd-3Bce+2bCe)x)\sqrt{cex^3+(cd+be)x^2+(bd+ae)x+ad}\right)dx}{3ce}+\frac{2C\left(x(ae+bd)+ad+x^2(be+cd)+cex^3\right)^{3/2}}{9ce}$$

$$\Big\downarrow \text{25}$$

$$\frac{2C\left(x(ae+bd)+ad+x^2(be+cd)+cex^3\right)^{3/2}}{9ce}-\frac{\int (bCd-3Ace+aCe+(2cCd-3Bce+2bCe)x)\sqrt{cex^3+(cd+be)x^2+(bd+ae)x+ad}dx}{3ce}$$

$$\Big\downarrow \text{2490}$$

$$\frac{2C\left(x(ae+bd)+ad+x^2(be+cd)+cex^3\right)^{3/2}}{9ce}-$$

$$\frac{\int \left(\frac{3ce(bCd-3Ace+aCe)-(cd+be)(2cCd-3Bce+2bCe)}{3ce}+(2cCd-3Bce+2bCe)\left(\frac{cd+be}{3ce}+x\right)\right)\sqrt{ce\left(\frac{cd+be}{3ce}+x\right)^3+\frac{(3ce(bd-}{}}}{3ce}$$

$$\Big\downarrow \text{7293}$$

$$\frac{2C\left(x(ae+bd)+ad+x^2(be+cd)+cex^3\right)^{3/2}}{9ce}-$$

$$\int \left(\frac{\sqrt{ce\left(\frac{cd+be}{3ce}+x\right)^3+\frac{\left(3ce(bd+ae)-(cd+be)^2\right)\left(\frac{cd+be}{3ce}+x\right)}{3ce}+\frac{(2cd-be)\left(c^2d^2-2b^2e^2-ce(bd-9ae)\right)}{27c^2e^2}}\left(-\left(\left(2Cd^2-3e(Bd-3Ae)\right)c^2\right)-e(bCd-3bBe-3a\right.}{3ce}\right.$$

$$\Big\downarrow \text{7293}$$

$$\frac{2C\left(x(ae+bd)+ad+x^2(be+cd)+cex^3\right)^{3/2}}{9ce}-$$
$$\int\left(\frac{\sqrt{\frac{\left(-2cd+be-3ce\left(\frac{cd+be}{3ce}+x\right)\right)\left(-c^2d^2+2b^2e^2-9c^2e^2\left(\frac{cd+be}{3ce}+x\right)^2+ce(bd-9ae)+3ce(2cd-be)\left(\frac{cd+be}{3ce}+x\right)\right)}{c^2e^2}}\left(-\left(\left(2Cd^2-3Bed+9Ae^2\right)c^2\right)-e(bCd-3\right.}{9\sqrt{3}ce}\right.$$

$\downarrow$ 7292

$$\frac{2C\left(x(ae+bd)+ad+x^2(be+cd)+cex^3\right)^{3/2}}{9ce}-$$
$$\int\left(\frac{\sqrt{\frac{\left(-2cd+be-3ce\left(\frac{cd+be}{3ce}+x\right)\right)\left(-c^2d^2+2b^2e^2-9c^2e^2\left(\frac{cd+be}{3ce}+x\right)^2+ce(bd-9ae)+3ce(2cd-be)\left(\frac{cd+be}{3ce}+x\right)\right)}{c^2e^2}}\left(-\left(\left(2Cd^2-3e(Bd-3Ae)\right)c^2\right)-e(bCd-\right.}{9\sqrt{3}ce}\right.$$

$\downarrow$ 7293

$$\frac{2C\left(x(ae+bd)+ad+x^2(be+cd)+cex^3\right)^{3/2}}{9ce}-$$
$$\int\left(\frac{\sqrt{\frac{\left(-2cd+be-3ce\left(\frac{cd+be}{3ce}+x\right)\right)\left(-c^2d^2+2b^2e^2-9c^2e^2\left(\frac{cd+be}{3ce}+x\right)^2+ce(bd-9ae)+3ce(2cd-be)\left(\frac{cd+be}{3ce}+x\right)\right)}{c^2e^2}}\left(-\left(\left(2Cd^2-3Bed+9Ae^2\right)c^2\right)-e(bCd-3\right.}{9\sqrt{3}ce}\right.$$

$\downarrow$ 7292

$$\frac{2C\left(x(ae+bd)+ad+x^2(be+cd)+cex^3\right)^{3/2}}{9ce}-$$
$$\int\left(\frac{\sqrt{\frac{\left(-2cd+be-3ce\left(\frac{cd+be}{3ce}+x\right)\right)\left(-c^2d^2+2b^2e^2-9c^2e^2\left(\frac{cd+be}{3ce}+x\right)^2+ce(bd-9ae)+3ce(2cd-be)\left(\frac{cd+be}{3ce}+x\right)\right)}{c^2e^2}}\left(-\left(\left(2Cd^2-3e(Bd-3Ae)\right)c^2\right)-e(bCd-\right.}{9\sqrt{3}ce}\right.$$

$\downarrow$ 7293

$$\frac{2C\left(x(ae+bd)+ad+x^2(be+cd)+cex^3\right)^{3/2}}{9ce}-$$
$$\int\left(\frac{\sqrt{\frac{\left(-2cd+be-3ce\left(\frac{cd+be}{3ce}+x\right)\right)\left(-c^2d^2+2b^2e^2-9c^2e^2\left(\frac{cd+be}{3ce}+x\right)^2+ce(bd-9ae)+3ce(2cd-be)\left(\frac{cd+be}{3ce}+x\right)\right)}{c^2e^2}}\left(-\left(\left(2Cd^2-3Bed+9Ae^2\right)c^2\right)-e(bCd-3\right.}{9\sqrt{3}ce}\right.$$

$\downarrow$ 7292

$$\frac{2C\left(x(ae+bd)+ad+x^2(be+cd)+cex^3\right)^{3/2}}{9ce}-$$

$$\int \left(\frac{\sqrt{\frac{\left(-2cd+be-3ce\left(\frac{cd+be}{3ce}+x\right)\right)\left(-c^2d^2+2b^2e^2-9c^2e^2\left(\frac{cd+be}{3ce}+x\right)^2+ce(bd-9ae)+3ce(2cd-be)\left(\frac{cd+be}{3ce}+x\right)\right)}{c^2e^2}}\left(-\left(\left(2Cd^2-3e(Bd-3Ae)\right)c^2\right)-e(bCd-}{9\sqrt{3}ce}$$

$\downarrow$ 7293

$$\frac{2C\left(x(ae+bd)+ad+x^2(be+cd)+cex^3\right)^{3/2}}{9ce}-$$

$$\int \left(\frac{\sqrt{\frac{\left(-2cd+be-3ce\left(\frac{cd+be}{3ce}+x\right)\right)\left(-c^2d^2+2b^2e^2-9c^2e^2\left(\frac{cd+be}{3ce}+x\right)^2+ce(bd-9ae)+3ce(2cd-be)\left(\frac{cd+be}{3ce}+x\right)\right)}{c^2e^2}}\left(-\left(\left(2Cd^2-3Bed+9Ae^2\right)c^2\right)-e(bCd-3}{9\sqrt{3}ce}$$

$\downarrow$ 7292

$$\frac{2C\left(x(ae+bd)+ad+x^2(be+cd)+cex^3\right)^{3/2}}{9ce}-$$

$$\int \left(\frac{\sqrt{\frac{\left(-2cd+be-3ce\left(\frac{cd+be}{3ce}+x\right)\right)\left(-c^2d^2+2b^2e^2-9c^2e^2\left(\frac{cd+be}{3ce}+x\right)^2+ce(bd-9ae)+3ce(2cd-be)\left(\frac{cd+be}{3ce}+x\right)\right)}{c^2e^2}}\left(-\left(\left(2Cd^2-3e(Bd-3Ae)\right)c^2\right)-e(bCd-}{9\sqrt{3}ce}$$

$\downarrow$ 7293

$$\frac{2C\left(x(ae+bd)+ad+x^2(be+cd)+cex^3\right)^{3/2}}{9ce}-$$

$$\int \left(\frac{\sqrt{\frac{\left(-2cd+be-3ce\left(\frac{cd+be}{3ce}+x\right)\right)\left(-c^2d^2+2b^2e^2-9c^2e^2\left(\frac{cd+be}{3ce}+x\right)^2+ce(bd-9ae)+3ce(2cd-be)\left(\frac{cd+be}{3ce}+x\right)\right)}{c^2e^2}}\left(-\left(\left(2Cd^2-3Bed+9Ae^2\right)c^2\right)-e(bCd-3}{9\sqrt{3}ce}$$

$\downarrow$ 7292

$$\frac{2C\left(x(ae+bd)+ad+x^2(be+cd)+cex^3\right)^{3/2}}{9ce}-$$

$$\int \left(\frac{\sqrt{\frac{\left(-2cd+be-3ce\left(\frac{cd+be}{3ce}+x\right)\right)\left(-c^2d^2+2b^2e^2-9c^2e^2\left(\frac{cd+be}{3ce}+x\right)^2+ce(bd-9ae)+3ce(2cd-be)\left(\frac{cd+be}{3ce}+x\right)\right)}{c^2e^2}}\left(-\left(\left(2Cd^2-3e(Bd-3Ae)\right)c^2\right)-e(bCd-}{9\sqrt{3}ce}$$

$\downarrow$ 7293

$$\frac{2C\left(x(ae+bd)+ad+x^2(be+cd)+cex^3\right)^{3/2}}{9ce}-$$

$$\int\left(\frac{\sqrt{\frac{\left(-2cd+be-3ce\left(\frac{cd+be}{3ce}+x\right)\right)\left(-c^2d^2+2b^2e^2-9c^2e^2\left(\frac{cd+be}{3ce}+x\right)^2+ce(bd-9ae)+3ce(2cd-be)\left(\frac{cd+be}{3ce}+x\right)\right)}{c^2e^2}}\left(-\left(\left(2Cd^2-3Bed+9Ae^2\right)c^2\right)-e(bCd-3\right.}{9\sqrt{3}ce}\right.$$

$$\downarrow \text{7292}$$

$$\frac{2C\left(x(ae+bd)+ad+x^2(be+cd)+cex^3\right)^{3/2}}{9ce}-$$

$$\int\left(\frac{\sqrt{\frac{\left(-2cd+be-3ce\left(\frac{cd+be}{3ce}+x\right)\right)\left(-c^2d^2+2b^2e^2-9c^2e^2\left(\frac{cd+be}{3ce}+x\right)^2+ce(bd-9ae)+3ce(2cd-be)\left(\frac{cd+be}{3ce}+x\right)\right)}{c^2e^2}}\left(-\left(\left(2Cd^2-3e(Bd-3Ae)\right)c^2\right)-e(bCd-\right.}{9\sqrt{3}ce}\right.$$

$$\downarrow \text{7293}$$

$$\frac{2C\left(x(ae+bd)+ad+x^2(be+cd)+cex^3\right)^{3/2}}{9ce}-$$

$$\int\left(\frac{\sqrt{\frac{\left(-2cd+be-3ce\left(\frac{cd+be}{3ce}+x\right)\right)\left(-c^2d^2+2b^2e^2-9c^2e^2\left(\frac{cd+be}{3ce}+x\right)^2+ce(bd-9ae)+3ce(2cd-be)\left(\frac{cd+be}{3ce}+x\right)\right)}{c^2e^2}}\left(-\left(\left(2Cd^2-3Bed+9Ae^2\right)c^2\right)-e(bCd-3\right.}{9\sqrt{3}ce}\right.$$

$$\downarrow \text{7292}$$

$$\frac{2C\left(x(ae+bd)+ad+x^2(be+cd)+cex^3\right)^{3/2}}{9ce}-$$

$$\int\left(\frac{\sqrt{\frac{\left(-2cd+be-3ce\left(\frac{cd+be}{3ce}+x\right)\right)\left(-c^2d^2+2b^2e^2-9c^2e^2\left(\frac{cd+be}{3ce}+x\right)^2+ce(bd-9ae)+3ce(2cd-be)\left(\frac{cd+be}{3ce}+x\right)\right)}{c^2e^2}}\left(-\left(\left(2Cd^2-3e(Bd-3Ae)\right)c^2\right)-e(bCd-\right.}{9\sqrt{3}ce}\right.$$

$$\downarrow \text{7293}$$

$$\frac{2C\left(x(ae+bd)+ad+x^2(be+cd)+cex^3\right)^{3/2}}{9ce}-$$

$$\int\left(\frac{\sqrt{\frac{\left(-2cd+be-3ce\left(\frac{cd+be}{3ce}+x\right)\right)\left(-c^2d^2+2b^2e^2-9c^2e^2\left(\frac{cd+be}{3ce}+x\right)^2+ce(bd-9ae)+3ce(2cd-be)\left(\frac{cd+be}{3ce}+x\right)\right)}{c^2e^2}}\left(-\left(\left(2Cd^2-3Bed+9Ae^2\right)c^2\right)-e(bCd-3\right.}{9\sqrt{3}ce}\right.$$

$$\downarrow \text{7292}$$

$$\int \left(\frac{2C\left(x(ae+bd)+ad+x^2(be+cd)+cex^3\right)^{3/2}}{9ce} - \frac{\sqrt{\frac{\left(-2cd+be-3ce\left(\frac{cd+be}{3ce}+x\right)\right)\left(-c^2d^2+2b^2e^2-9c^2e^2\left(\frac{cd+be}{3ce}+x\right)^2+ce(bd-9ae)+3ce(2cd-be)\left(\frac{cd+be}{3ce}+x\right)\right)}{c^2e^2}}\left(-\left(\left(2Cd^2-3e(Bd-3Ae)\right)c^2\right)-e(bCd-\right.}{9\sqrt{3}ce}\right.$$

$$\Big\downarrow \; 7293$$

$$\int \left(\frac{2C\left(x(ae+bd)+ad+x^2(be+cd)+cex^3\right)^{3/2}}{9ce} - \frac{\sqrt{\frac{\left(-2cd+be-3ce\left(\frac{cd+be}{3ce}+x\right)\right)\left(-c^2d^2+2b^2e^2-9c^2e^2\left(\frac{cd+be}{3ce}+x\right)^2+ce(bd-9ae)+3ce(2cd-be)\left(\frac{cd+be}{3ce}+x\right)\right)}{c^2e^2}}\left(-\left(\left(2Cd^2-3Bed+9Ae^2\right)c^2\right)-e(bCd-3\right.}{9\sqrt{3}ce}\right.$$

$$\Big\downarrow \; 7292$$

$$\int \left(\frac{2C\left(x(ae+bd)+ad+x^2(be+cd)+cex^3\right)^{3/2}}{9ce} - \frac{\sqrt{\frac{\left(-2cd+be-3ce\left(\frac{cd+be}{3ce}+x\right)\right)\left(-c^2d^2+2b^2e^2-9c^2e^2\left(\frac{cd+be}{3ce}+x\right)^2+ce(bd-9ae)+3ce(2cd-be)\left(\frac{cd+be}{3ce}+x\right)\right)}{c^2e^2}}\left(-\left(\left(2Cd^2-3e(Bd-3Ae)\right)c^2\right)-e(bCd-\right.}{9\sqrt{3}ce}\right.$$

$$\Big\downarrow \; 7293$$

$$\int \left(\frac{2C\left(x(ae+bd)+ad+x^2(be+cd)+cex^3\right)^{3/2}}{9ce} - \frac{\sqrt{\frac{\left(-2cd+be-3ce\left(\frac{cd+be}{3ce}+x\right)\right)\left(-c^2d^2+2b^2e^2-9c^2e^2\left(\frac{cd+be}{3ce}+x\right)^2+ce(bd-9ae)+3ce(2cd-be)\left(\frac{cd+be}{3ce}+x\right)\right)}{c^2e^2}}\left(-\left(\left(2Cd^2-3Bed+9Ae^2\right)c^2\right)-e(bCd-3\right.}{9\sqrt{3}ce}\right.$$

$$\Big\downarrow \; 7292$$

$$\int \left(\frac{2C\left(x(ae+bd)+ad+x^2(be+cd)+cex^3\right)^{3/2}}{9ce} - \frac{\sqrt{\frac{\left(-2cd+be-3ce\left(\frac{cd+be}{3ce}+x\right)\right)\left(-c^2d^2+2b^2e^2-9c^2e^2\left(\frac{cd+be}{3ce}+x\right)^2+ce(bd-9ae)+3ce(2cd-be)\left(\frac{cd+be}{3ce}+x\right)\right)}{c^2e^2}}\left(-\left(\left(2Cd^2-3e(Bd-3Ae)\right)c^2\right)-e(bCd-\right.}{9\sqrt{3}ce}\right.$$

$$\Big\downarrow \; 7293$$

$$\frac{2C\left(x(ae+bd)+ad+x^2(be+cd)+cex^3\right)^{3/2}}{9ce}-$$
$$\int\left(\frac{\sqrt{\frac{\left(-2cd+be-3ce\left(\frac{cd+be}{3ce}+x\right)\right)\left(-c^2d^2+2b^2e^2-9c^2e^2\left(\frac{cd+be}{3ce}+x\right)^2+ce(bd-9ae)+3ce(2cd-be)\left(\frac{cd+be}{3ce}+x\right)\right)}{c^2e^2}}\left(-\left(\left(2Cd^2-3Bed+9Ae^2\right)c^2\right)-e(bCd-3\right.}{9\sqrt{3}ce}\right.$$

$\downarrow$ 7292

$$\frac{2C\left(x(ae+bd)+ad+x^2(be+cd)+cex^3\right)^{3/2}}{9ce}-$$
$$\int\left(\frac{\sqrt{\frac{\left(-2cd+be-3ce\left(\frac{cd+be}{3ce}+x\right)\right)\left(-c^2d^2+2b^2e^2-9c^2e^2\left(\frac{cd+be}{3ce}+x\right)^2+ce(bd-9ae)+3ce(2cd-be)\left(\frac{cd+be}{3ce}+x\right)\right)}{c^2e^2}}\left(-\left(\left(2Cd^2-3e(Bd-3Ae)\right)c^2\right)-e(bCd-\right.}{9\sqrt{3}ce}\right.$$

$\downarrow$ 7293

$$\frac{2C\left(x(ae+bd)+ad+x^2(be+cd)+cex^3\right)^{3/2}}{9ce}-$$
$$\int\left(\frac{\sqrt{\frac{\left(-2cd+be-3ce\left(\frac{cd+be}{3ce}+x\right)\right)\left(-c^2d^2+2b^2e^2-9c^2e^2\left(\frac{cd+be}{3ce}+x\right)^2+ce(bd-9ae)+3ce(2cd-be)\left(\frac{cd+be}{3ce}+x\right)\right)}{c^2e^2}}\left(-\left(\left(2Cd^2-3Bed+9Ae^2\right)c^2\right)-e(bCd-3\right.}{9\sqrt{3}ce}\right.$$

$\downarrow$ 7292

$$\frac{2C\left(x(ae+bd)+ad+x^2(be+cd)+cex^3\right)^{3/2}}{9ce}-$$
$$\int\left(\frac{\sqrt{\frac{\left(-2cd+be-3ce\left(\frac{cd+be}{3ce}+x\right)\right)\left(-c^2d^2+2b^2e^2-9c^2e^2\left(\frac{cd+be}{3ce}+x\right)^2+ce(bd-9ae)+3ce(2cd-be)\left(\frac{cd+be}{3ce}+x\right)\right)}{c^2e^2}}\left(-\left(\left(2Cd^2-3e(Bd-3Ae)\right)c^2\right)-e(bCd-\right.}{9\sqrt{3}ce}\right.$$

$\downarrow$ 7293

$$\frac{2C\left(x(ae+bd)+ad+x^2(be+cd)+cex^3\right)^{3/2}}{9ce}-$$
$$\int\left(\frac{\sqrt{\frac{\left(-2cd+be-3ce\left(\frac{cd+be}{3ce}+x\right)\right)\left(-c^2d^2+2b^2e^2-9c^2e^2\left(\frac{cd+be}{3ce}+x\right)^2+ce(bd-9ae)+3ce(2cd-be)\left(\frac{cd+be}{3ce}+x\right)\right)}{c^2e^2}}\left(-\left(\left(2Cd^2-3Bed+9Ae^2\right)c^2\right)-e(bCd-3\right.}{9\sqrt{3}ce}\right.$$

$\downarrow$ 7292

$$\frac{2C\left(x(ae+bd)+ad+x^2(be+cd)+cex^3\right)^{3/2}}{9ce}-$$
$$\int \left(\frac{\sqrt{\frac{\left(-2cd+be-3ce\left(\frac{cd+be}{3ce}+x\right)\right)\left(-c^2d^2+2b^2e^2-9c^2e^2\left(\frac{cd+be}{3ce}+x\right)^2+ce(bd-9ae)+3ce(2cd-be)\left(\frac{cd+be}{3ce}+x\right)\right)}{c^2e^2}}\left(-\left(\left(2Cd^2-3e(Bd-3Ae)\right)c^2\right)-e(bCd-\right.}{9\sqrt{3}ce} \right.$$

input
```
Int[(A + B*x + C*x^2)*Sqrt[a*d + (b*d + a*e)*x + (c*d + b*e)*x^2 + c*e*x^3
],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2490
```
Int[(P3_)^(p_.)*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] :> With[{a = Coeff[P3
, x, 0], b = Coeff[P3, x, 1], c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Su
bst[Int[((3*d*e - c*f)/(3*d) + f*x)^m*Simp[(2*c^3 - 9*b*c*d + 27*a*d^2)/(27
*d^2) - (c^2 - 3*b*d)*(x/(3*d)) + d*x^3, x]^p, x], x, x + c/(3*d)] /; NeQ[c
, 0]] /; FreeQ[{e, f, m, p}, x] && PolyQ[P3, x, 3]
```

rule 2526
```
Int[(Pm_)*(Qn_)^(p_), x_Symbol] :> With[{m = Expon[Pm, x], n = Expon[Qn, x]
}, Simp[Coeff[Pm, x, m]*(Qn^(p + 1)/(n*(p + 1)*Coeff[Qn, x, n])), x] + Simp
[1/(n*Coeff[Qn, x, n])   Int[ExpandToSum[n*Coeff[Qn, x, n]*Pm - Coeff[Pm, x
, m]*D[Qn, x], x]*Qn^p, x], x] /; EqQ[m, n - 1]] /; FreeQ[p, x] && PolyQ[Pm
, x] && PolyQ[Qn, x] && NeQ[p, -1]
```

rule 7292
```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 8.19 (sec) , antiderivative size = 1698, normalized size of antiderivative = 1.71

method	result	size
elliptic	Expression too large to display	1698
default	Expression too large to display	3190
risch	Expression too large to display	7090

input
```
int((C*x^2+B*x+A)*(a*d+(a*e+b*d)*x+(b*e+c*d)*x^2+c*e*x^3)^(1/2),x,method=_
RETURNVERBOSE)
```

output
```
2/9*C*x^3*(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)+2/7*(B*c*e+C*b*e
+C*c*d-2/9*(4*b*e+4*c*d)*C)/c/e*x^2*(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a
*d)^(1/2)+2/5*(A*c*e+B*b*e+B*c*d+C*a*e+C*b*d-2/9*C*(7/2*a*e+7/2*b*d)-2/7*(
B*c*e+C*b*e+C*c*d-2/9*(4*b*e+4*c*d)*C)/c/e*(3*b*e+3*c*d))/c/e*x*(c*e*x^3+b
*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)+2/3*(A*b*e+A*c*d+B*a*e+B*b*d+1/3*C*a
*d-2/7*(B*c*e+C*b*e+C*c*d-2/9*(4*b*e+4*c*d)*C)/c/e*(5/2*a*e+5/2*b*d)-2/5*(
A*c*e+B*b*e+B*c*d+C*a*e+C*b*d-2/9*C*(7/2*a*e+7/2*b*d)-2/7*(B*c*e+C*b*e+C*c
*d-2/9*(4*b*e+4*c*d)*C)/c/e*(3*b*e+3*c*d))/c/e*(2*b*e+2*c*d))/c/e*(c*e*x^3
+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)+2*(a*d*A-2/5*(A*c*e+B*b*e+B*c*d+C*
a*e+C*b*d-2/9*C*(7/2*a*e+7/2*b*d)-2/7*(B*c*e+C*b*e+C*c*d-2/9*(4*b*e+4*c*d)
*C)/c/e*(3*b*e+3*c*d))/c/e*a*d-2/3*(A*b*e+A*c*d+B*a*e+B*b*d+1/3*C*a*d-2/7*
(B*c*e+C*b*e+C*c*d-2/9*(4*b*e+4*c*d)*C)/c/e*(5/2*a*e+5/2*b*d)-2/5*(A*c*e+B
*b*e+B*c*d+C*a*e+C*b*d-2/9*C*(7/2*a*e+7/2*b*d)-2/7*(B*c*e+C*b*e+C*c*d-2/9*
(4*b*e+4*c*d)*C)/c/e*(3*b*e+3*c*d))/c/e*(2*b*e+2*c*d))/c/e*(1/2*a*e+1/2*b*
d))*(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(
1/2))/c))^(1/2)*((x-1/2*(-b+(-4*a*c+b^2)^(1/2))/c)/(-d/e-1/2*(-b+(-4*a*c+b
^2)^(1/2))/c))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e+1/2*(b+(-4*a*
c+b^2)^(1/2))/c))^(1/2)/(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)*El
lipticF(((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-d/e+1/2*(b+(
-4*a*c+b^2)^(1/2))/c)/(-d/e-1/2*(-b+(-4*a*c+b^2)^(1/2))/c))^(1/2))+2*(A...
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 1036, normalized size of antiderivative = 1.04

$$\int (A+Bx+Cx^2)\sqrt{ad+(bd+ae)x+(cd+be)x^2+cex^3}\,dx = \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)*(a*d+(a*e+b*d)*x+(b*e+c*d)*x^2+c*e*x^3)^(1/2),x, a
lgorithm="fricas")
```

output
```
2/945*((16*C*c^5*d^5 - 8*(2*C*b*c^4 + 3*B*c^5)*d^4*e - (5*C*b^2*c^3 - 42*A
*c^5 - 3*(10*C*a + 9*B*b)*c^4)*d^3*e^2 - (5*C*b^3*c^2 + 3*(22*B*a + 21*A*b
)*c^4 - 3*(7*C*a*b + 4*B*b^2)*c^3)*d^2*e^3 - (16*C*b^4*c - 378*A*a*c^4 + 3
*(22*C*a^2 + 41*B*a*b + 21*A*b^2)*c^3 - 3*(28*C*a*b^2 + 9*B*b^3)*c^2)*d*e^
4 + (16*C*b^5 - 9*(10*B*a^2 + 21*A*a*b)*c^3 + 3*(41*C*a^2*b + 41*B*a*b^2 +
 14*A*b^3)*c^2 - 24*(4*C*a*b^3 + B*b^4)*c)*e^5)*sqrt(c*e)*weierstrassPInve
rse(4/3*(c^2*d^2 - b*c*d*e + (b^2 - 3*a*c)*e^2)/(c^2*e^2), -4/27*(2*c^3*d^
3 - 3*b*c^2*d^2*e - 3*(b^2*c - 6*a*c^2)*d*e^2 + (2*b^3 - 9*a*b*c)*e^3)/(c^
3*e^3), 1/3*(3*c*e*x + c*d + b*e)/(c*e)) + 3*(16*C*c^5*d^4*e - 8*(C*b*c^4
+ 3*B*c^5)*d^3*e^2 - 3*(2*C*b^2*c^3 - 14*A*c^5 - (6*C*a + 5*B*b)*c^4)*d^2*
e^3 - (8*C*b^3*c^2 + 6*(8*B*a + 7*A*b)*c^4 - 15*(2*C*a*b + B*b^2)*c^3)*d*e
^4 + (16*C*b^4*c - 126*A*a*c^4 + 3*(14*C*a^2 + 29*B*a*b + 14*A*b^2)*c^3 -
24*(3*C*a*b^2 + B*b^3)*c^2)*e^5)*sqrt(c*e)*weierstrassZeta(4/3*(c^2*d^2 -
b*c*d*e + (b^2 - 3*a*c)*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e -
 3*(b^2*c - 6*a*c^2)*d*e^2 + (2*b^3 - 9*a*b*c)*e^3)/(c^3*e^3), weierstrass
PInverse(4/3*(c^2*d^2 - b*c*d*e + (b^2 - 3*a*c)*e^2)/(c^2*e^2), -4/27*(2*c
^3*d^3 - 3*b*c^2*d^2*e - 3*(b^2*c - 6*a*c^2)*d*e^2 + (2*b^3 - 9*a*b*c)*e^3
)/(c^3*e^3), 1/3*(3*c*e*x + c*d + b*e)/(c*e))) + 3*(35*C*c^5*e^5*x^3 + 8*C
*c^5*d^3*e^2 - 3*(C*b*c^4 + 4*B*c^5)*d^2*e^3 - (3*C*b^2*c^3 - 21*A*c^5 - 2
*(4*C*a + 3*B*b)*c^4)*d*e^4 + (8*C*b^3*c^2 + 3*(10*B*a + 7*A*b)*c^4 - 3...
```

Sympy [F]

$$\int (A+Bx+Cx^2)\sqrt{ad+(bd+ae)x+(cd+be)x^2+cex^3}\,dx$$
$$=\int \sqrt{(d+ex)(a+bx+cx^2)}(A+Bx+Cx^2)\,dx$$

input
```
integrate((C*x**2+B*x+A)*(a*d+(a*e+b*d)*x+(b*e+c*d)*x**2+c*e*x**3)**(1/2),
x)
```

output
```
Integral(sqrt((d + e*x)*(a + b*x + c*x**2))*(A + B*x + C*x**2), x)
```

Maxima [F]

$$\int (A+Bx+Cx^2)\sqrt{ad+(bd+ae)x+(cd+be)x^2+cex^3}\,dx$$
$$=\int \sqrt{cex^3+(cd+be)x^2+ad+(bd+ae)x}\,(Cx^2+Bx+A)\,dx$$

input
```
integrate((C*x^2+B*x+A)*(a*d+(a*e+b*d)*x+(b*e+c*d)*x^2+c*e*x^3)^(1/2),x, a
lgorithm="maxima")
```

output
```
integrate(sqrt(c*e*x^3 + (c*d + b*e)*x^2 + a*d + (b*d + a*e)*x)*(C*x^2 + B
*x + A), x)
```

Giac [F]

$$\int (A+Bx+Cx^2)\sqrt{ad+(bd+ae)x+(cd+be)x^2+cex^3}\,dx$$
$$=\int \sqrt{cex^3+(cd+be)x^2+ad+(bd+ae)x}\,(Cx^2+Bx+A)\,dx$$

input
```
integrate((C*x^2+B*x+A)*(a*d+(a*e+b*d)*x+(b*e+c*d)*x^2+c*e*x^3)^(1/2),x, a
lgorithm="giac")
```

output
```
integrate(sqrt(c*e*x^3 + (c*d + b*e)*x^2 + a*d + (b*d + a*e)*x)*(C*x^2 + B
*x + A), x)
```

Mupad [F(-1)]

Timed out.

$$\int (A+Bx+Cx^2)\sqrt{ad+(bd+ae)x+(cd+be)x^2+cex^3}\,dx$$
$$=\int (C\,x^2+B\,x+A)\,\sqrt{c\,e\,x^3+(b\,e+c\,d)\,x^2+(a\,e+b\,d)\,x+a\,d}\,dx$$

input
```
int((A + B*x + C*x^2)*(a*d + x*(a*e + b*d) + x^2*(b*e + c*d) + c*e*x^3)^(1
/2),x)
```

output
```
int((A + B*x + C*x^2)*(a*d + x*(a*e + b*d) + x^2*(b*e + c*d) + c*e*x^3)^(1
/2), x)
```

Reduce [F]

$$\int (A+Bx+Cx^2)\sqrt{ad+(bd+ae)x+(cd+be)x^2+cex^3}\,dx$$
$$=\int (C\,x^2+Bx+A)\,\sqrt{ad+(ae+bd)\,x+(be+cd)\,x^2+ce\,x^3}\,dx$$

input
```
int((C*x^2+B*x+A)*(a*d+(a*e+b*d)*x+(b*e+c*d)*x^2+c*e*x^3)^(1/2),x)
```

output
```
int((C*x^2+B*x+A)*(a*d+(a*e+b*d)*x+(b*e+c*d)*x^2+c*e*x^3)^(1/2),x)
```

3.130 $\int \frac{A+Bx+Cx^2}{\sqrt{ad+(bd+ae)x+(cd+be)x^2+cex^3}}\,dx$

Optimal result

Integrand size = 45, antiderivative size = 504

$$\int \frac{A+Bx+Cx^2}{\sqrt{ad+(bd+ae)x+(cd+be)x^2+cex^3}}\,dx = \frac{2C(d+ex)\left(a+bx+cx^2\right)}{3ce\sqrt{ad+(bd+ae)x+(cd+be)x^2+cex^3}} - \frac{\sqrt{2}\sqrt{b^2-4ac}(2cCd-3Bce+2bCe)(d+ex)\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\arcsin\left(\frac{\sqrt{1+\frac{b+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ace}}{2cd-\left(b+\sqrt{b^2-4ac}\right.}\right.}{3c^2e^2\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{ad+(bd+ae)x+(cd+be)x^2+cex^3}} + \frac{2\sqrt{2}\sqrt{b^2-4ac}(Ce(bd-ae)+c(2Cd^2-3e(Bd-Ae)))\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\operatorname{EllipticF}\left(\right.}{3c^2e^2\sqrt{ad+(bd+ae)x+(cd+be)x^2+cex^3}}$$

output
```
2/3*C*(e*x+d)*(c*x^2+b*x+a)/c/e/(a*d+(a*e+b*d)*x+(b*e+c*d)*x^2+c*e*x^3)^(1
/2)-1/3*2^(1/2)*(-4*a*c+b^2)^(1/2)*(-3*B*c*e+2*C*b*e+2*C*c*d)*(e*x+d)*(-c*
(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*EllipticE(1/2*(1+(2*c*x+b)/(-4*a*c+b^2)^
(1/2))^(1/2)*2^(1/2),(-2*(-4*a*c+b^2)^(1/2)*e/(2*c*d-(b+(-4*a*c+b^2)^(1/2)
)*e))^(1/2))/c^2/e^2/(c*(e*x+d)/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e))^(1/2)/(a
*d+(a*e+b*d)*x+(b*e+c*d)*x^2+c*e*x^3)^(1/2)+2/3*2^(1/2)*(-4*a*c+b^2)^(1/2)
*(C*e*(-a*e+b*d)+c*(2*C*d^2-3*e*(-A*e+B*d)))*(c*(e*x+d)/(2*c*d-(b+(-4*a*c+
b^2)^(1/2))*e))^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*EllipticF(1/2*
(1+(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*(-4*a*c+b^2)^(1/2)*e/(2
*c*d-(b+(-4*a*c+b^2)^(1/2))*e))^(1/2))/c^2/e^2/(a*d+(a*e+b*d)*x+(b*e+c*d)*
x^2+c*e*x^3)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 17.66 (sec) , antiderivative size = 1086, normalized size of antiderivative = 2.15

$$\int \frac{A+Bx+Cx^2}{\sqrt{ad+(bd+ae)x+(cd+be)x^2+cex^3}}\,dx = \text{Too large to display}$$

input
```
Integrate[(A + B*x + C*x^2)/Sqrt[a*d + (b*d + a*e)*x + (c*d + b*e)*x^2 + c
*e*x^3],x]
```

output
```
(2*C*(d + e*x)*(a + b*x + c*x^2))/(3*c*e*Sqrt[(d + e*x)*(a + x*(b + c*x))]
) + ((d + e*x)^2*Sqrt[a + b*x + c*x^2]*(-4*(2*c*C*d - 3*B*c*e + 2*b*C*e)*S
qrt[(c*d^2 + e*(-(b*d) + a*e))/(-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])]*(
c*(-1 + d/(d + e*x))^2 + (e*(b - (b*d)/(d + e*x) + (a*e)/(d + e*x)))/(d +
e*x)) + (I*Sqrt[2]*(2*c*C*d - 3*B*c*e + 2*b*C*e)*(2*c*d - b*e + Sqrt[(b^2
- 4*a*c)*e^2])*Sqrt[(Sqrt[(b^2 - 4*a*c)*e^2] - (2*a*e^2)/(d + e*x) - 2*c*d
*(-1 + d/(d + e*x)) + b*e*(-1 + (2*d)/(d + e*x)))/(2*c*d - b*e + Sqrt[(b^2
 - 4*a*c)*e^2])]*Sqrt[(Sqrt[(b^2 - 4*a*c)*e^2] + (2*a*e^2)/(d + e*x) + 2*c
*d*(-1 + d/(d + e*x)) + b*(e - (2*d*e)/(d + e*x)))/(-2*c*d + b*e + Sqrt[(b
^2 - 4*a*c)*e^2])]*EllipticE[I*ArcSinh[(Sqrt[2]*Sqrt[(c*d^2 - b*d*e + a*e^
2)/(-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])])/Sqrt[d + e*x]], -((-2*c*d +
b*e + Sqrt[(b^2 - 4*a*c)*e^2])/(2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2]))])/
Sqrt[d + e*x] - (I*Sqrt[2]*(-2*b^2*C*e^2 + b*e*(3*B*c*e + 2*C*Sqrt[(b^2 -
4*a*c)*e^2]) + c*(-6*A*c*e^2 + 2*a*C*e^2 + Sqrt[(b^2 - 4*a*c)*e^2]*(2*C*d
- 3*B*e)))*Sqrt[(Sqrt[(b^2 - 4*a*c)*e^2] - (2*a*e^2)/(d + e*x) - 2*c*d*(-1
 + d/(d + e*x)) + b*e*(-1 + (2*d)/(d + e*x)))/(2*c*d - b*e + Sqrt[(b^2 - 4
*a*c)*e^2])]*Sqrt[(Sqrt[(b^2 - 4*a*c)*e^2] + (2*a*e^2)/(d + e*x) + 2*c*d*(
-1 + d/(d + e*x)) + b*(e - (2*d*e)/(d + e*x)))/(-2*c*d + b*e + Sqrt[(b^2 -
 4*a*c)*e^2])]*EllipticF[I*ArcSinh[(Sqrt[2]*Sqrt[(c*d^2 - b*d*e + a*e^2)/(
-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])])/Sqrt[d + e*x]], -((-2*c*d + b...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{\sqrt{x(ae+bd)+ad+x^2(be+cd)+cex^3}}dx$$

$$\Big\downarrow \text{2526}$$

$$\frac{\int -\frac{bCd-3Ace+aCe+(2cCd-3Bce+2bCe)x}{\sqrt{cex^3+(cd+be)x^2+(bd+ae)x+ad}}dx}{3ce}+\frac{2C\sqrt{x(ae+bd)+ad+x^2(be+cd)+cex^3}}{3ce}$$

$$\Big\downarrow \text{25}$$

$$\frac{2C\sqrt{x(ae+bd)+ad+x^2(be+cd)+cex^3}}{3ce}-\frac{\int \frac{bCd-3Ace+aCe+(2cCd-3Bce+2bCe)x}{\sqrt{cex^3+(cd+be)x^2+(bd+ae)x+ad}}dx}{3ce}$$

$$\Big\downarrow \text{2490}$$

$$\frac{2C\sqrt{x(ae+bd)+ad+x^2(be+cd)+cex^3}}{3ce}-\frac{\int \frac{\frac{3ce(bCd-3Ace+aCe)-(cd+be)(2cCd-3Bce+2bCe)}{3ce}+(2cCd-3Bce+2bCe)\left(\frac{cd+be}{3ce}+x\right)}{\sqrt{ce\left(\frac{cd+be}{3ce}+x\right)^3+\frac{\left(3ce(bd+ae)-(cd+be)^2\right)\left(\frac{cd+be}{3ce}+x\right)}{3ce}+\frac{(2cd-be)\left(c^2d^2-2b^2e^2-ce(bd-9ae)\right)}{27c^2e^2}}}d\left(\frac{cd+be}{3ce}+x\right)}{3ce}$$

$\downarrow$ 7293

$$\frac{2C\sqrt{x(ae+bd)+ad+x^2(be+cd)+cex^3}}{3ce}-\frac{\int \left(\frac{-\left(\left(2Cd^2-3e(Bd-3Ae)\right)c^2\right)-e(bCd-3bBe-3aCe)c-2b^2Ce^2}{3ce\sqrt{ce\left(\frac{cd+be}{3ce}+x\right)^3+\frac{\left(3ce(bd+ae)-(cd+be)^2\right)\left(\frac{cd+be}{3ce}+x\right)}{3ce}+\frac{(2cd-be)\left(c^2d^2-2b^2e^2-ce(bd-9ae)\right)}{27c^2e^2}}}+\frac{(2cCd-3Bce\cdots}{\sqrt{ce\left(\frac{cd+be}{3ce}+x\right)^3+\frac{\left(3ce(bd+ae)-(cd+be)^2\right)\cdots}{3ce}}}\right.}{3ce}$$

$\downarrow$ 7293

$$\frac{2C\sqrt{x(ae+bd)+ad+x^2(be+cd)+cex^3}}{3ce}-\frac{\int \left(\frac{\sqrt{3}\left(-\left(\left(2Cd^2-3Bed+9Ae^2\right)c^2\right)-e(bCd-3bBe-3aCe)c-2b^2Ce^2\right)}{ce\sqrt{\frac{\left(-2cd+be-3ce\left(\frac{cd+be}{3ce}+x\right)\right)\left(-c^2d^2+2b^2e^2-9c^2e^2\left(\frac{cd+be}{3ce}+x\right)^2+ce(bd-9ae)+3ce(2cd-be)\left(\frac{cd+be}{3ce}+x\right)\right)}{c^2e^2}}}+\frac{3\sqrt{3}\cdots}{\sqrt{\left(2cd-be+3ce\left(\frac{cd+be}{3ce}+x\right)\right)\left(c^2d^2\cdots\right.}}\right.}{3ce}$$

$\downarrow$ 7292

$$\frac{2C\sqrt{x(ae+bd)+ad+x^2(be+cd)+cex^3}}{3ce}-\frac{\int \left(\frac{\sqrt{3}\left(-\left(\left(2Cd^2-3e(Bd-3Ae)\right)c^2\right)-e(bCd-3bBe-3aCe)c-2b^2Ce^2\right)}{ce\sqrt{\frac{\left(-2cd+be-3ce\left(\frac{cd+be}{3ce}+x\right)\right)\left(-c^2d^2+2b^2e^2-9c^2e^2\left(\frac{cd+be}{3ce}+x\right)^2+ce(bd-9ae)+3ce(2cd-be)\left(\frac{cd+be}{3ce}+x\right)\right)}{c^2e^2}}}+\frac{3\sqrt{3}\cdots}{\sqrt{\left(2cd-be+3ce\left(\frac{cd+be}{3ce}+x\right)\right)\left(c^2d^2\cdots\right.}}\right.}{3ce}$$

$\downarrow$ 7293

$$\frac{2C\sqrt{x(ae+bd)+ad+x^2(be+cd)+cex^3}}{3ce}-\frac{\int \left(\frac{\sqrt{3}\left(-\left(\left(2Cd^2-3Bed+9Ae^2\right)c^2\right)-e(bCd-3bBe-3aCe)c-2b^2Ce^2\right)}{ce\sqrt{\frac{\left(-2cd+be-3ce\left(\frac{cd+be}{3ce}+x\right)\right)\left(-c^2d^2+2b^2e^2-9c^2e^2\left(\frac{cd+be}{3ce}+x\right)^2+ce(bd-9ae)+3ce(2cd-be)\left(\frac{cd+be}{3ce}+x\right)\right)}{c^2e^2}}}+\frac{3\sqrt{3}\cdots}{\sqrt{\left(2cd-be+3ce\left(\frac{cd+be}{3ce}+x\right)\right)\left(c^2d^2\cdots\right.}}\right.}{3ce}$$

$\downarrow$ 7292

$$\frac{2C\sqrt{x(ae+bd)+ad+x^2(be+cd)+cex^3}}{3ce}-\frac{\int\left(\frac{\sqrt{3}(-((2Cd^2-3e(Bd-3Ae))c^2)-e(bCd-3bBe-3aCe)c-2b^2Ce^2)}{ce\sqrt{\frac{\left(-2cd+be-3ce\left(\frac{cd+be}{3ce}+x\right)\right)\left(-c^2d^2+2b^2e^2-9c^2e^2\left(\frac{cd+be}{3ce}+x\right)^2+ce(bd-9ae)+3ce(2cd-be)\left(\frac{cd+be}{3ce}+x\right)\right)}{c^2e^2}}}+\frac{3\sqrt{3}\cdots}{\sqrt{\frac{\left(2cd-be+3ce\left(\frac{cd+be}{3ce}+x\right)\right)\left(c^2d^2\cdots\right)}{\cdots}}}\cdots\right.}{3ce}$$

$$\Big\downarrow \ 7293$$

$$\frac{2C\sqrt{x(ae+bd)+ad+x^2(be+cd)+cex^3}}{3ce}-\frac{\int\left(\frac{\sqrt{3}(-((2Cd^2-3Bed+9Ae^2)c^2)-e(bCd-3bBe-3aCe)c-2b^2Ce^2)}{ce\sqrt{\frac{\left(-2cd+be-3ce\left(\frac{cd+be}{3ce}+x\right)\right)\left(-c^2d^2+2b^2e^2-9c^2e^2\left(\frac{cd+be}{3ce}+x\right)^2+ce(bd-9ae)+3ce(2cd-be)\left(\frac{cd+be}{3ce}+x\right)\right)}{c^2e^2}}}+\frac{3\sqrt{3}\cdots}{\sqrt{\frac{\left(2cd-be+3ce\left(\frac{cd+be}{3ce}+x\right)\right)\left(c^2d^2\cdots\right)}{\cdots}}}\cdots\right.}{3ce}$$

$$\Big\downarrow \ 7292$$

$$\frac{2C\sqrt{x(ae+bd)+ad+x^2(be+cd)+cex^3}}{3ce}-\frac{\int\left(\frac{\sqrt{3}(-((2Cd^2-3e(Bd-3Ae))c^2)-e(bCd-3bBe-3aCe)c-2b^2Ce^2)}{ce\sqrt{\frac{\left(-2cd+be-3ce\left(\frac{cd+be}{3ce}+x\right)\right)\left(-c^2d^2+2b^2e^2-9c^2e^2\left(\frac{cd+be}{3ce}+x\right)^2+ce(bd-9ae)+3ce(2cd-be)\left(\frac{cd+be}{3ce}+x\right)\right)}{c^2e^2}}}+\frac{3\sqrt{3}\cdots}{\sqrt{\frac{\left(2cd-be+3ce\left(\frac{cd+be}{3ce}+x\right)\right)\left(c^2d^2\cdots\right)}{\cdots}}}\cdots\right.}{3ce}$$

$$\Big\downarrow \ 7293$$

$$\frac{2C\sqrt{x(ae+bd)+ad+x^2(be+cd)+cex^3}}{3ce}-\frac{\int\left(\frac{\sqrt{3}(-((2Cd^2-3Bed+9Ae^2)c^2)-e(bCd-3bBe-3aCe)c-2b^2Ce^2)}{ce\sqrt{\frac{\left(-2cd+be-3ce\left(\frac{cd+be}{3ce}+x\right)\right)\left(-c^2d^2+2b^2e^2-9c^2e^2\left(\frac{cd+be}{3ce}+x\right)^2+ce(bd-9ae)+3ce(2cd-be)\left(\frac{cd+be}{3ce}+x\right)\right)}{c^2e^2}}}+\frac{3\sqrt{3}\cdots}{\sqrt{\frac{\left(2cd-be+3ce\left(\frac{cd+be}{3ce}+x\right)\right)\left(c^2d^2\cdots\right)}{\cdots}}}\cdots\right.}{3ce}$$

$$\Big\downarrow \ 7292$$

$$\frac{2C\sqrt{x(ae+bd)+ad+x^2(be+cd)+cex^3}}{3ce}-\frac{\int\left(\frac{\sqrt{3}(-((2Cd^2-3e(Bd-3Ae))c^2)-e(bCd-3bBe-3aCe)c-2b^2Ce^2)}{ce\sqrt{\frac{\left(-2cd+be-3ce\left(\frac{cd+be}{3ce}+x\right)\right)\left(-c^2d^2+2b^2e^2-9c^2e^2\left(\frac{cd+be}{3ce}+x\right)^2+ce(bd-9ae)+3ce(2cd-be)\left(\frac{cd+be}{3ce}+x\right)\right)}{c^2e^2}}}+\frac{3\sqrt{3}\cdots}{\sqrt{\frac{\left(2cd-be+3ce\left(\frac{cd+be}{3ce}+x\right)\right)\left(c^2d^2\cdots\right)}{\cdots}}}\cdots\right.}{3ce}$$

$$\Big\downarrow \ 7293$$

$$\frac{2C\sqrt{x(ae+bd)+ad+x^2(be+cd)+cex^3}}{3ce}-$$
$$\frac{\int \left(\frac{\sqrt{3}(-((2Cd^2-3Bed+9Ae^2)c^2)-e(bCd-3bBe-3aCe)c-2b^2Ce^2)}{ce\sqrt{\frac{\left(-2cd+be-3ce\left(\frac{cd+be}{3ce}+x\right)\right)\left(-c^2d^2+2b^2e^2-9c^2e^2\left(\frac{cd+be}{3ce}+x\right)^2+ce(bd-9ae)+3ce(2cd-be)\left(\frac{cd+be}{3ce}+x\right)\right)}{c^2e^2}}}+\frac{3\sqrt{3}\cdots}{\sqrt{\left(2cd-be+3ce\left(\frac{cd+be}{3ce}+x\right)\right)\left(c^2d^2\cdots\right.}}\right.\cdots}{3ce}$$

$\downarrow$ 7292

$$\frac{2C\sqrt{x(ae+bd)+ad+x^2(be+cd)+cex^3}}{3ce}-$$
$$\frac{\int \left(\frac{\sqrt{3}(-((2Cd^2-3e(Bd-3Ae))c^2)-e(bCd-3bBe-3aCe)c-2b^2Ce^2)}{ce\sqrt{\frac{\left(-2cd+be-3ce\left(\frac{cd+be}{3ce}+x\right)\right)\left(-c^2d^2+2b^2e^2-9c^2e^2\left(\frac{cd+be}{3ce}+x\right)^2+ce(bd-9ae)+3ce(2cd-be)\left(\frac{cd+be}{3ce}+x\right)\right)}{c^2e^2}}}+\frac{3\sqrt{3}\cdots}{\sqrt{\left(2cd-be+3ce\left(\frac{cd+be}{3ce}+x\right)\right)\left(c^2d^2\cdots\right.}}\right.\cdots}{3ce}$$

$\downarrow$ 7293

$$\frac{2C\sqrt{x(ae+bd)+ad+x^2(be+cd)+cex^3}}{3ce}-$$
$$\frac{\int \left(\frac{\sqrt{3}(-((2Cd^2-3Bed+9Ae^2)c^2)-e(bCd-3bBe-3aCe)c-2b^2Ce^2)}{ce\sqrt{\frac{\left(-2cd+be-3ce\left(\frac{cd+be}{3ce}+x\right)\right)\left(-c^2d^2+2b^2e^2-9c^2e^2\left(\frac{cd+be}{3ce}+x\right)^2+ce(bd-9ae)+3ce(2cd-be)\left(\frac{cd+be}{3ce}+x\right)\right)}{c^2e^2}}}+\frac{3\sqrt{3}\cdots}{\sqrt{\left(2cd-be+3ce\left(\frac{cd+be}{3ce}+x\right)\right)\left(c^2d^2\cdots\right.}}\right.\cdots}{3ce}$$

$\downarrow$ 7292

$$\frac{2C\sqrt{x(ae+bd)+ad+x^2(be+cd)+cex^3}}{3ce}-$$
$$\frac{\int \left(\frac{\sqrt{3}(-((2Cd^2-3e(Bd-3Ae))c^2)-e(bCd-3bBe-3aCe)c-2b^2Ce^2)}{ce\sqrt{\frac{\left(-2cd+be-3ce\left(\frac{cd+be}{3ce}+x\right)\right)\left(-c^2d^2+2b^2e^2-9c^2e^2\left(\frac{cd+be}{3ce}+x\right)^2+ce(bd-9ae)+3ce(2cd-be)\left(\frac{cd+be}{3ce}+x\right)\right)}{c^2e^2}}}+\frac{3\sqrt{3}\cdots}{\sqrt{\left(2cd-be+3ce\left(\frac{cd+be}{3ce}+x\right)\right)\left(c^2d^2\cdots\right.}}\right.\cdots}{3ce}$$

$\downarrow$ 7293

$$\frac{2C\sqrt{x(ae+bd)+ad+x^2(be+cd)+cex^3}}{3ce}-$$
$$\frac{\int \left(\frac{\sqrt{3}(-((2Cd^2-3Bed+9Ae^2)c^2)-e(bCd-3bBe-3aCe)c-2b^2Ce^2)}{ce\sqrt{\frac{\left(-2cd+be-3ce\left(\frac{cd+be}{3ce}+x\right)\right)\left(-c^2d^2+2b^2e^2-9c^2e^2\left(\frac{cd+be}{3ce}+x\right)^2+ce(bd-9ae)+3ce(2cd-be)\left(\frac{cd+be}{3ce}+x\right)\right)}{c^2e^2}}}+\frac{3\sqrt{3}\cdots}{\sqrt{\left(2cd-be+3ce\left(\frac{cd+be}{3ce}+x\right)\right)\left(c^2d^2\cdots\right.}}\right.\cdots}{3ce}$$

$\downarrow$ 7292

$$\frac{2C\sqrt{x(ae+bd)+ad+x^2(be+cd)+cex^3}}{3ce}-\frac{\int\left(\frac{\sqrt{3}\left(-\left(\left(2Cd^2-3e(Bd-3Ae)\right)c^2\right)-e(bCd-3bBe-3aCe)c-2b^2Ce^2\right)}{ce\sqrt{\frac{\left(-2cd+be-3ce\left(\frac{cd+be}{3ce}+x\right)\right)\left(-c^2d^2+2b^2e^2-9c^2e^2\left(\frac{cd+be}{3ce}+x\right)^2+ce(bd-9ae)+3ce(2cd-be)\left(\frac{cd+be}{3ce}+x\right)\right)}{c^2e^2}}}+\frac{3\sqrt{3}\cdots}{\sqrt{\left(2cd-be+3ce\left(\frac{cd+be}{3ce}+x\right)\right)\left(c^2d^2\cdots\right.}}\cdots\right.}{3ce}$$

$$\Big\downarrow \; 7293$$

$$\frac{2C\sqrt{x(ae+bd)+ad+x^2(be+cd)+cex^3}}{3ce}-\frac{\int\left(\frac{\sqrt{3}\left(-\left(\left(2Cd^2-3Bed+9Ae^2\right)c^2\right)-e(bCd-3bBe-3aCe)c-2b^2Ce^2\right)}{ce\sqrt{\frac{\left(-2cd+be-3ce\left(\frac{cd+be}{3ce}+x\right)\right)\left(-c^2d^2+2b^2e^2-9c^2e^2\left(\frac{cd+be}{3ce}+x\right)^2+ce(bd-9ae)+3ce(2cd-be)\left(\frac{cd+be}{3ce}+x\right)\right)}{c^2e^2}}}+\frac{3\sqrt{3}\cdots}{\sqrt{\left(2cd-be+3ce\left(\frac{cd+be}{3ce}+x\right)\right)\left(c^2d^2\cdots\right.}}\cdots\right.}{3ce}$$

$$\Big\downarrow \; 7292$$

$$\frac{2C\sqrt{x(ae+bd)+ad+x^2(be+cd)+cex^3}}{3ce}-\frac{\int\left(\frac{\sqrt{3}\left(-\left(\left(2Cd^2-3e(Bd-3Ae)\right)c^2\right)-e(bCd-3bBe-3aCe)c-2b^2Ce^2\right)}{ce\sqrt{\frac{\left(-2cd+be-3ce\left(\frac{cd+be}{3ce}+x\right)\right)\left(-c^2d^2+2b^2e^2-9c^2e^2\left(\frac{cd+be}{3ce}+x\right)^2+ce(bd-9ae)+3ce(2cd-be)\left(\frac{cd+be}{3ce}+x\right)\right)}{c^2e^2}}}+\frac{3\sqrt{3}\cdots}{\sqrt{\left(2cd-be+3ce\left(\frac{cd+be}{3ce}+x\right)\right)\left(c^2d^2\cdots\right.}}\cdots\right.}{3ce}$$

$$\Big\downarrow \; 7293$$

$$\frac{2C\sqrt{x(ae+bd)+ad+x^2(be+cd)+cex^3}}{3ce}-\frac{\int\left(\frac{\sqrt{3}\left(-\left(\left(2Cd^2-3Bed+9Ae^2\right)c^2\right)-e(bCd-3bBe-3aCe)c-2b^2Ce^2\right)}{ce\sqrt{\frac{\left(-2cd+be-3ce\left(\frac{cd+be}{3ce}+x\right)\right)\left(-c^2d^2+2b^2e^2-9c^2e^2\left(\frac{cd+be}{3ce}+x\right)^2+ce(bd-9ae)+3ce(2cd-be)\left(\frac{cd+be}{3ce}+x\right)\right)}{c^2e^2}}}+\frac{3\sqrt{3}\cdots}{\sqrt{\left(2cd-be+3ce\left(\frac{cd+be}{3ce}+x\right)\right)\left(c^2d^2\cdots\right.}}\cdots\right.}{3ce}$$

$$\Big\downarrow \; 7292$$

$$\frac{2C\sqrt{x(ae+bd)+ad+x^2(be+cd)+cex^3}}{3ce}-\frac{\int\left(\frac{\sqrt{3}\left(-\left(\left(2Cd^2-3e(Bd-3Ae)\right)c^2\right)-e(bCd-3bBe-3aCe)c-2b^2Ce^2\right)}{ce\sqrt{\frac{\left(-2cd+be-3ce\left(\frac{cd+be}{3ce}+x\right)\right)\left(-c^2d^2+2b^2e^2-9c^2e^2\left(\frac{cd+be}{3ce}+x\right)^2+ce(bd-9ae)+3ce(2cd-be)\left(\frac{cd+be}{3ce}+x\right)\right)}{c^2e^2}}}+\frac{3\sqrt{3}\cdots}{\sqrt{\left(2cd-be+3ce\left(\frac{cd+be}{3ce}+x\right)\right)\left(c^2d^2\cdots\right.}}\cdots\right.}{3ce}$$

$$\Big\downarrow \; 7293$$

$$\frac{2C\sqrt{x(ae+bd)+ad+x^2(be+cd)+cex^3}}{3ce} - \frac{\int \left(\frac{\sqrt{3}(-((2Cd^2-3Bed+9Ae^2)c^2)-e(bCd-3bBe-3aCe)c-2b^2Ce^2)}{ce\sqrt{\frac{\left(-2cd+be-3ce\left(\frac{cd+be}{3ce}+x\right)\right)\left(-c^2d^2+2b^2e^2-9c^2e^2\left(\frac{cd+be}{3ce}+x\right)^2+ce(bd-9ae)+3ce(2cd-be)\left(\frac{cd+be}{3ce}+x\right)\right)}{c^2e^2}}} + \frac{3\sqrt{3}}{\sqrt{\left(2cd-be+3ce\left(\frac{cd+be}{3ce}+x\right)\right)\left(c^2d^2\right.}}}{3ce}$$

$$\Big\downarrow \text{ 7292}$$

$$\frac{2C\sqrt{x(ae+bd)+ad+x^2(be+cd)+cex^3}}{3ce} - \frac{\int \left(\frac{\sqrt{3}(-((2Cd^2-3e(Bd-3Ae))c^2)-e(bCd-3bBe-3aCe)c-2b^2Ce^2)}{ce\sqrt{\frac{\left(-2cd+be-3ce\left(\frac{cd+be}{3ce}+x\right)\right)\left(-c^2d^2+2b^2e^2-9c^2e^2\left(\frac{cd+be}{3ce}+x\right)^2+ce(bd-9ae)+3ce(2cd-be)\left(\frac{cd+be}{3ce}+x\right)\right)}{c^2e^2}}} + \frac{3\sqrt{3}}{\sqrt{\left(2cd-be+3ce\left(\frac{cd+be}{3ce}+x\right)\right)\left(c^2d^2\right.}}}{3ce}$$

$$\Big\downarrow \text{ 7293}$$

$$\frac{2C\sqrt{x(ae+bd)+ad+x^2(be+cd)+cex^3}}{3ce} - \frac{\int \left(\frac{\sqrt{3}(-((2Cd^2-3Bed+9Ae^2)c^2)-e(bCd-3bBe-3aCe)c-2b^2Ce^2)}{ce\sqrt{\frac{\left(-2cd+be-3ce\left(\frac{cd+be}{3ce}+x\right)\right)\left(-c^2d^2+2b^2e^2-9c^2e^2\left(\frac{cd+be}{3ce}+x\right)^2+ce(bd-9ae)+3ce(2cd-be)\left(\frac{cd+be}{3ce}+x\right)\right)}{c^2e^2}}} + \frac{3\sqrt{3}}{\sqrt{\left(2cd-be+3ce\left(\frac{cd+be}{3ce}+x\right)\right)\left(c^2d^2\right.}}}{3ce}$$

$$\Big\downarrow \text{ 7292}$$

$$\frac{2C\sqrt{x(ae+bd)+ad+x^2(be+cd)+cex^3}}{3ce} - \frac{\int \left(\frac{\sqrt{3}(-((2Cd^2-3e(Bd-3Ae))c^2)-e(bCd-3bBe-3aCe)c-2b^2Ce^2)}{ce\sqrt{\frac{\left(-2cd+be-3ce\left(\frac{cd+be}{3ce}+x\right)\right)\left(-c^2d^2+2b^2e^2-9c^2e^2\left(\frac{cd+be}{3ce}+x\right)^2+ce(bd-9ae)+3ce(2cd-be)\left(\frac{cd+be}{3ce}+x\right)\right)}{c^2e^2}}} + \frac{3\sqrt{3}}{\sqrt{\left(2cd-be+3ce\left(\frac{cd+be}{3ce}+x\right)\right)\left(c^2d^2\right.}}}{3ce}$$

$$\Big\downarrow \text{ 7293}$$

$$\frac{2C\sqrt{x(ae+bd)+ad+x^2(be+cd)+cex^3}}{3ce} - \frac{\int \left(\frac{\sqrt{3}(-((2Cd^2-3Bed+9Ae^2)c^2)-e(bCd-3bBe-3aCe)c-2b^2Ce^2)}{ce\sqrt{\frac{\left(-2cd+be-3ce\left(\frac{cd+be}{3ce}+x\right)\right)\left(-c^2d^2+2b^2e^2-9c^2e^2\left(\frac{cd+be}{3ce}+x\right)^2+ce(bd-9ae)+3ce(2cd-be)\left(\frac{cd+be}{3ce}+x\right)\right)}{c^2e^2}}} + \frac{3\sqrt{3}}{\sqrt{\left(2cd-be+3ce\left(\frac{cd+be}{3ce}+x\right)\right)\left(c^2d^2\right.}}}{3ce}$$

$$\Big\downarrow \text{ 7292}$$

$$\frac{2C\sqrt{x(ae+bd)+ad+x^2(be+cd)+cex^3}}{3ce}-$$
$$\frac{\int\left(\frac{\sqrt{3}\left(-\left(\left(2Cd^2-3e(Bd-3Ae)\right)c^2\right)-e(bCd-3bBe-3aCe)c-2b^2Ce^2\right)}{ce\sqrt{\frac{\left(-2cd+be-3ce\left(\frac{cd+be}{3ce}+x\right)\right)\left(-c^2d^2+2b^2e^2-9c^2e^2\left(\frac{cd+be}{3ce}+x\right)^2+ce(bd-9ae)+3ce(2cd-be)\left(\frac{cd+be}{3ce}+x\right)\right)}{c^2e^2}}}+\frac{3\sqrt{3}\cdots}{\sqrt{\left(2cd-be+3ce\left(\frac{cd+be}{3ce}+x\right)\right)\left(c^2d^2\cdots\right.}}\cdots\right.}{3ce}$$

$\downarrow$ 7293

$$\frac{2C\sqrt{x(ae+bd)+ad+x^2(be+cd)+cex^3}}{3ce}-$$
$$\frac{\int\left(\frac{\sqrt{3}\left(-\left(\left(2Cd^2-3Bed+9Ae^2\right)c^2\right)-e(bCd-3bBe-3aCe)c-2b^2Ce^2\right)}{ce\sqrt{\frac{\left(-2cd+be-3ce\left(\frac{cd+be}{3ce}+x\right)\right)\left(-c^2d^2+2b^2e^2-9c^2e^2\left(\frac{cd+be}{3ce}+x\right)^2+ce(bd-9ae)+3ce(2cd-be)\left(\frac{cd+be}{3ce}+x\right)\right)}{c^2e^2}}}+\frac{3\sqrt{3}\cdots}{\sqrt{\left(2cd-be+3ce\left(\frac{cd+be}{3ce}+x\right)\right)\left(c^2d^2\cdots\right.}}\cdots\right.}{3ce}$$

$\downarrow$ 7292

$$\frac{2C\sqrt{x(ae+bd)+ad+x^2(be+cd)+cex^3}}{3ce}-$$
$$\frac{\int\left(\frac{\sqrt{3}\left(-\left(\left(2Cd^2-3e(Bd-3Ae)\right)c^2\right)-e(bCd-3bBe-3aCe)c-2b^2Ce^2\right)}{ce\sqrt{\frac{\left(-2cd+be-3ce\left(\frac{cd+be}{3ce}+x\right)\right)\left(-c^2d^2+2b^2e^2-9c^2e^2\left(\frac{cd+be}{3ce}+x\right)^2+ce(bd-9ae)+3ce(2cd-be)\left(\frac{cd+be}{3ce}+x\right)\right)}{c^2e^2}}}+\frac{3\sqrt{3}\cdots}{\sqrt{\left(2cd-be+3ce\left(\frac{cd+be}{3ce}+x\right)\right)\left(c^2d^2\cdots\right.}}\cdots\right.}{3ce}$$

input
```
Int[(A + B*x + C*x^2)/Sqrt[a*d + (b*d + a*e)*x + (c*d + b*e)*x^2 + c*e*x^3
],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2490
```
Int[(P3_)^(p_.)*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] :> With[{a = Coeff[P3
, x, 0], b = Coeff[P3, x, 1], c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Su
bst[Int[((3*d*e - c*f)/(3*d) + f*x)^m*Simp[(2*c^3 - 9*b*c*d + 27*a*d^2)/(27
*d^2) - (c^2 - 3*b*d)*(x/(3*d)) + d*x^3, x]^p, x], x, x + c/(3*d)] /; NeQ[c
, 0]] /; FreeQ[{e, f, m, p}, x] && PolyQ[P3, x, 3]
```

rule 2526
```
Int[(Pm_)*(Qn_)^(p_), x_Symbol] :> With[{m = Expon[Pm, x], n = Expon[Qn, x]
}, Simp[Coeff[Pm, x, m]*(Qn^(p + 1)/(n*(p + 1)*Coeff[Qn, x, n])), x] + Simp
[1/(n*Coeff[Qn, x, n])   Int[ExpandToSum[n*Coeff[Qn, x, n]*Pm - Coeff[Pm, x
, m]*D[Qn, x], x]*Qn^p, x], x] /; EqQ[m, n - 1]] /; FreeQ[p, x] && PolyQ[Pm
, x] && PolyQ[Qn, x] && NeQ[p, -1]
```

rule 7292
```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 4.12 (sec) , antiderivative size = 785, normalized size of antiderivative = 1.56

method	result
elliptic	$\frac{2C\sqrt{ce\,x^3+x^2eb+cd\,x^2+aex+bdx+ad}}{3ce}+\frac{2\left(A-\frac{2C\left(\frac{ae}{2}+\frac{bd}{2}\right)}{3ce}\right)\left(\frac{d}{e}-\frac{b+\sqrt{-4ac+b^2}}{2c}\right)\sqrt{\frac{x+\frac{d}{e}}{\frac{d}{e}-\frac{b+\sqrt{-4ac+b^2}}{2c}}}\sqrt{\frac{x-\frac{-b+\sqrt{-4ac+b^2}}{2c}}{-\frac{d}{e}-\frac{-b+\sqrt{-4ac+b^2}}{2c}}}\sqrt{}}{\sqrt{ce\,x^3+x^2eb+cd\,x^2+a}}$
risch	Expression too large to display
default	Expression too large to display

input
```
int((C*x^2+B*x+A)/(a*d+(a*e+b*d)*x+(b*e+c*d)*x^2+c*e*x^3)^(1/2),x,method=_
RETURNVERBOSE)
```

output
```
2/3*C/c/e*(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)+2*(A-2/3*C/c/e*(
1/2*a*e+1/2*b*d))*(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+d/e)/(d/e-1/2*(b+
(-4*a*c+b^2)^(1/2))/c))^(1/2)*((x-1/2*(-b+(-4*a*c+b^2)^(1/2))/c)/(-d/e-1/2
*(-b+(-4*a*c+b^2)^(1/2))/c))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e
+1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)/(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x
+a*d)^(1/2)*EllipticF(((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),(
(-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e-1/2*(-b+(-4*a*c+b^2)^(1/2))/c))^
(1/2))+2*(B-2/3*C/c/e*(b*e+c*d))*(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+d/
e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)*((x-1/2*(-b+(-4*a*c+b^2)^(1/2
))/c)/(-d/e-1/2*(-b+(-4*a*c+b^2)^(1/2))/c))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^
(1/2))/c)/(-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)/(c*e*x^3+b*e*x^2+c*d*
x^2+a*e*x+b*d*x+a*d)^(1/2)*((-d/e-1/2*(-b+(-4*a*c+b^2)^(1/2))/c)*EllipticE
(((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-d/e+1/2*(b+(-4*a*c+
b^2)^(1/2))/c)/(-d/e-1/2*(-b+(-4*a*c+b^2)^(1/2))/c))^(1/2))+1/2*(-b+(-4*a*
c+b^2)^(1/2))/c*EllipticF(((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/
2),((-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e-1/2*(-b+(-4*a*c+b^2)^(1/2))/
c))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 464, normalized size of antiderivative = 0.92

$$\int \frac{A+Bx+Cx^2}{\sqrt{ad+(bd+ae)x+(cd+be)x^2+cex^3}}\,dx$$
$$= \frac{2\left(3\sqrt{cex^3+(cd+be)x^2+ad+(bd+ae)x}Cc^2e^2+(2\,Cc^2d^2+(Cbc-3\,Bc^2)de+(2\,Cb^2+9\,Ac^2-3\,\right.}{}$$

input
```
integrate((C*x^2+B*x+A)/(a*d+(a*e+b*d)*x+(b*e+c*d)*x^2+c*e*x^3)^(1/2),x, a
lgorithm="fricas")
```

output
```
2/9*(3*sqrt(c*e*x^3 + (c*d + b*e)*x^2 + a*d + (b*d + a*e)*x)*C*c^2*e^2 + (
2*C*c^2*d^2 + (C*b*c - 3*B*c^2)*d*e + (2*C*b^2 + 9*A*c^2 - 3*(C*a + B*b)*c
)*e^2)*sqrt(c*e)*weierstrassPInverse(4/3*(c^2*d^2 - b*c*d*e + (b^2 - 3*a*c
)*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*(b^2*c - 6*a*c^2)*d
*e^2 + (2*b^3 - 9*a*b*c)*e^3)/(c^3*e^3), 1/3*(3*c*e*x + c*d + b*e)/(c*e))
+ 3*(2*C*c^2*d*e + (2*C*b*c - 3*B*c^2)*e^2)*sqrt(c*e)*weierstrassZeta(4/3*
(c^2*d^2 - b*c*d*e + (b^2 - 3*a*c)*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*
c^2*d^2*e - 3*(b^2*c - 6*a*c^2)*d*e^2 + (2*b^3 - 9*a*b*c)*e^3)/(c^3*e^3),
weierstrassPInverse(4/3*(c^2*d^2 - b*c*d*e + (b^2 - 3*a*c)*e^2)/(c^2*e^2),
 -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*(b^2*c - 6*a*c^2)*d*e^2 + (2*b^3 - 9
*a*b*c)*e^3)/(c^3*e^3), 1/3*(3*c*e*x + c*d + b*e)/(c*e))))/(c^3*e^3)
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt{ad+(bd+ae)x+(cd+be)x^2+cex^3}}\,dx = \int \frac{A+Bx+Cx^2}{\sqrt{(d+ex)\,(a+bx+cx^2)}}\,dx$$

input
```
integrate((C*x**2+B*x+A)/(a*d+(a*e+b*d)*x+(b*e+c*d)*x**2+c*e*x**3)**(1/2),
x)
```

output
```
Integral((A + B*x + C*x**2)/sqrt((d + e*x)*(a + b*x + c*x**2)), x)
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt{ad+(bd+ae)x+(cd+be)x^2+cex^3}}\,dx$$
$$= \int \frac{Cx^2+Bx+A}{\sqrt{cex^3+(cd+be)x^2+ad+(bd+ae)x}}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(a*d+(a*e+b*d)*x+(b*e+c*d)*x^2+c*e*x^3)^(1/2),x, a
lgorithm="maxima")
```

output
```
integrate((C*x^2 + B*x + A)/sqrt(c*e*x^3 + (c*d + b*e)*x^2 + a*d + (b*d +
a*e)*x), x)
```

Giac [F]

$$\int \frac{A + Bx + Cx^2}{\sqrt{ad + (bd + ae)x + (cd + be)x^2 + cex^3}}\,dx = \int \frac{Cx^2 + Bx + A}{\sqrt{cex^3 + (cd + be)x^2 + ad + (bd + ae)x}}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(a*d+(a*e+b*d)*x+(b*e+c*d)*x^2+c*e*x^3)^(1/2),x, a
lgorithm="giac")
```

output
```
integrate((C*x^2 + B*x + A)/sqrt(c*e*x^3 + (c*d + b*e)*x^2 + a*d + (b*d +
a*e)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A + Bx + Cx^2}{\sqrt{ad + (bd + ae)x + (cd + be)x^2 + cex^3}}\,dx = \int \frac{C\,x^2 + B\,x + A}{\sqrt{c\,e\,x^3 + (b\,e + c\,d)\,x^2 + (a\,e + b\,d)\,x + a\,d}}\,dx$$

input
```
int((A + B*x + C*x^2)/(a*d + x*(a*e + b*d) + x^2*(b*e + c*d) + c*e*x^3)^(1
/2),x)
```

output
```
int((A + B*x + C*x^2)/(a*d + x*(a*e + b*d) + x^2*(b*e + c*d) + c*e*x^3)^(1
/2), x)
```

Reduce [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt{ad+(bd+ae)x+(cd+be)x^2+cex^3}}\,dx = \int \frac{C\,x^2+Bx+A}{\sqrt{ad+(ae+bd)\,x+(be+cd)\,x^2+ce\,x^3}}dx$$

input
```
int((C*x^2+B*x+A)/(a*d+(a*e+b*d)*x+(b*e+c*d)*x^2+c*e*x^3)^(1/2),x)
```

output
```
int((C*x^2+B*x+A)/(a*d+(a*e+b*d)*x+(b*e+c*d)*x^2+c*e*x^3)^(1/2),x)
```

3.131 $\int \frac{A+Bx+Cx^2}{\left(ad+(bd+ae)x+(cd+be)x^2+cex^3\right)^{3/2}}\,dx$

Optimal result

Integrand size = 45, antiderivative size = 913

$$\int \frac{A+Bx+Cx^2}{\left(ad+(bd+ae)x+(cd+be)x^2+cex^3\right)^{3/2}}\,dx =$$

$$-\frac{2(d+ex)\left(A(bcd-b^2e+2ace)-a(2Bcd-bCd-bBe+2aCe)+(b^2Cd+2c(Acd-aCd+aBe)-b(}{(b^2-4ac)\left(cd^2-bde+ae^2\right)\left(ad+(bd+ae)x+(cd+be)x^2+cex^3\right)^3}$$

$$-\frac{2e(b^2(2Cd^2-e(Bd-2Ae))-b(2(Ac+aC)de+B(cd^2+ae^2))+2(Ac(cd^2-3ae^2)+a(aCe^2-cd(3C}{(b^2-4ac)\left(cd^2-bde+ae^2\right)^2\left(ad+(bd+ae)x+(cd+be)x^2+cex^3\right)^3}$$

$$+\frac{\sqrt{2}(b^2(2Cd^2-e(Bd-2Ae))-b(2(Ac+aC)de+B(cd^2+ae^2))+2(Ac(cd^2-3ae^2)+a(aCe^2-cd(3C}{\sqrt{b^2-4ac}\left(cd^2-bde+ae^2\right)^2\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}}$$

$$-\frac{2\sqrt{2}(b^2Cd+2c(Acd-aCd+aBe)-b(Bcd+Ace+aCe))\,(d+ex)\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}(a+bx+cx^2)\,\sqrt{}}{c\sqrt{b^2-4ac}\left(cd^2-bde+ae^2\right)\left(ad+(bd+ae)x+(cd+\right.}$$

output
```
-2*(e*x+d)*(A*(2*a*c*e-b^2*e+b*c*d)-a*(-B*b*e+2*B*c*d+2*C*a*e-C*b*d)+(b^2*
C*d+2*c*(A*c*d+B*a*e-C*a*d)-b*(A*c*e+B*c*d+C*a*e))*x)*(c*x^2+b*x+a)/(-4*a*
c+b^2)/(a*e^2-b*d*e+c*d^2)/(a*d+(a*e+b*d)*x+(b*e+c*d)*x^2+c*e*x^3)^(3/2)-2
*e*(b^2*(2*C*d^2-e*(-2*A*e+B*d))-b*(2*(A*c+C*a)*d*e+B*(a*e^2+c*d^2))+2*A*c
*(-3*a*e^2+c*d^2)+2*a*(a*C*e^2-c*d*(-4*B*e+3*C*d)))*(e*x+d)*(c*x^2+b*x+a)^
2/(-4*a*c+b^2)/(a*e^2-b*d*e+c*d^2)^2/(a*d+(a*e+b*d)*x+(b*e+c*d)*x^2+c*e*x^
3)^(3/2)+2^(1/2)*(b^2*(2*C*d^2-e*(-2*A*e+B*d))-b*(2*(A*c+C*a)*d*e+B*(a*e^2
+c*d^2))+2*A*c*(-3*a*e^2+c*d^2)+2*a*(a*C*e^2-c*d*(-4*B*e+3*C*d)))*(e*x+d)^
2*(c*x^2+b*x+a)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*EllipticE(1/2*(1+(2*
c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*(-4*a*c+b^2)^(1/2)*e/(2*c*d-(
b+(-4*a*c+b^2)^(1/2))*e))^(1/2))/(-4*a*c+b^2)^(1/2)/(a*e^2-b*d*e+c*d^2)^2/
(c*(e*x+d)/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e))^(1/2)/(a*d+(a*e+b*d)*x+(b*e+c
*d)*x^2+c*e*x^3)^(3/2)-2*2^(1/2)*(b^2*C*d+2*c*(A*c*d+B*a*e-C*a*d)-b*(A*c*e
+B*c*d+C*a*e))*(e*x+d)*(c*(e*x+d)/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e))^(1/2)*
(c*x^2+b*x+a)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*EllipticF(1/2*(1+(2*c*
x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*(-4*a*c+b^2)^(1/2)*e/(2*c*d-(b+
(-4*a*c+b^2)^(1/2))*e))^(1/2))/c/(-4*a*c+b^2)^(1/2)/(a*e^2-b*d*e+c*d^2)/(a
*d+(a*e+b*d)*x+(b*e+c*d)*x^2+c*e*x^3)^(3/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 36.31 (sec) , antiderivative size = 10146, normalized size of antiderivative = 11.11

$$\int \frac{A+Bx+Cx^2}{(ad+(bd+ae)x+(cd+be)x^2+cex^3)^{3/2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(A + B*x + C*x^2)/(a*d + (b*d + a*e)*x + (c*d + b*e)*x^2 + c*e*x
^3)^(3/2),x]
```

output
```
Result too large to show
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{(x(ae+bd)+ad+x^2(be+cd)+cex^3)^{3/2}}\,dx$$

$$\Big\downarrow \text{2526}$$

$$\frac{\int -\frac{bCd-3Ace+aCe+(2cCd-3Bce+2bCe)x}{(cex^3+(cd+be)x^2+(bd+ae)x+ad)^{3/2}}dx}{3ce}-\frac{2C}{3ce\sqrt{x(ae+bd)+ad+x^2(be+cd)+cex^3}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{bCd-3Ace+aCe+(2cCd-3Bce+2bCe)x}{(cex^3+(cd+be)x^2+(bd+ae)x+ad)^{3/2}}dx}{3ce}-\frac{2C}{3ce\sqrt{x(ae+bd)+ad+x^2(be+cd)+cex^3}}$$

$$\Big\downarrow \text{2490}$$

$$-\frac{\int \frac{\frac{3ce(bCd-3Ace+aCe)-(cd+be)(2cCd-3Bce+2bCe)}{3ce}+(2cCd-3Bce+2bCe)\left(\frac{cd+be}{3ce}+x\right)}{\left(ce\left(\frac{cd+be}{3ce}+x\right)^3+\frac{\left(3ce(bd+ae)-(cd+be)^2\right)\left(\frac{cd+be}{3ce}+x\right)}{3ce}+\frac{(2cd-be)\left(c^2d^2-2b^2e^2-ce(bd-9ae)\right)}{27c^2e^2}\right)^{3/2}}d\left(\frac{cd+be}{3ce}+x\right)}{3ce}-\frac{2C}{3ce\sqrt{x(ae+bd)+ad+x^2(be+cd)+cex^3}}$$

$$\Big\downarrow \text{7293}$$

$$-\frac{\int \left(\frac{-\left(\left(2Cd^2-3e(Bd-3Ae)\right)c^2\right)-e(bCd-3bBe-3aCe)c-2b^2Ce^2}{3ce\left(ce\left(\frac{cd+be}{3ce}+x\right)^3+\frac{\left(3ce(bd+ae)-(cd+be)^2\right)\left(\frac{cd+be}{3ce}+x\right)}{3ce}+\frac{(2cd-be)\left(c^2d^2-2b^2e^2-ce(bd-9ae)\right)}{27c^2e^2}\right)^{3/2}}+\frac{(2cC}{\left(ce\left(\frac{cd+be}{3ce}+x\right)^3+\frac{\left(3ce(bd+ae)-(cd\right.}{}}\right.}{3ce}$$

$$-\frac{2C}{3ce\sqrt{x(ae+bd)+ad+x^2(be+cd)+cex^3}}$$

$$\Big\downarrow \text{7293}$$

$$-\frac{\int \left(\frac{27\sqrt{3}\left(-\left(\left(2Cd^2-3Bed+9Ae^2\right)c^2\right)-e(bCd-3bBe-3aCe)c-2b^2Ce^2\right)}{ce\left(\frac{\left(-2cd+be-3ce\left(\frac{cd+be}{3ce}+x\right)\right)\left(-c^2d^2+2b^2e^2-9c^2e^2\left(\frac{cd+be}{3ce}+x\right)^2+ce(bd-9ae)+3ce(2cd-be)\left(\frac{cd+be}{3ce}+x\right)\right)}{c^2e^2}\right)^{3/2}}+\frac{\left(\left(2cd-be+3ce\left(\frac{cd+be}{3ce}+x\right)\cdots\right.\right.}{\cdots}\right.\cdots}{3ce}}{\frac{2C}{3ce\sqrt{x(ae+bd)+ad+x^2(be+cd)+cex^3}}}$$

$\Big\downarrow$ 7292

$$-\frac{\int \left(\frac{27\sqrt{3}\left(-\left(\left(2Cd^2-3e(Bd-3Ae)\right)c^2\right)-e(bCd-3bBe-3aCe)c-2b^2Ce^2\right)}{ce\left(\frac{\left(-2cd+be-3ce\left(\frac{cd+be}{3ce}+x\right)\right)\left(-c^2d^2+2b^2e^2-9c^2e^2\left(\frac{cd+be}{3ce}+x\right)^2+ce(bd-9ae)+3ce(2cd-be)\left(\frac{cd+be}{3ce}+x\right)\right)}{c^2e^2}\right)^{3/2}}+\frac{\left(\left(2cd-be+3ce\left(\frac{cd+be}{3ce}+x\right)\cdots\right.\right.}{\cdots}\right.\cdots}{3ce}}{\frac{2C}{3ce\sqrt{x(ae+bd)+ad+x^2(be+cd)+cex^3}}}$$

$\Big\downarrow$ 7293

$$-\frac{\int \left(\frac{27\sqrt{3}\left(-\left(\left(2Cd^2-3Bed+9Ae^2\right)c^2\right)-e(bCd-3bBe-3aCe)c-2b^2Ce^2\right)}{ce\left(\frac{\left(-2cd+be-3ce\left(\frac{cd+be}{3ce}+x\right)\right)\left(-c^2d^2+2b^2e^2-9c^2e^2\left(\frac{cd+be}{3ce}+x\right)^2+ce(bd-9ae)+3ce(2cd-be)\left(\frac{cd+be}{3ce}+x\right)\right)}{c^2e^2}\right)^{3/2}}+\frac{\left(\left(2cd-be+3ce\left(\frac{cd+be}{3ce}+x\right)\cdots\right.\right.}{\cdots}\right.\cdots}{3ce}}{\frac{2C}{3ce\sqrt{x(ae+bd)+ad+x^2(be+cd)+cex^3}}}$$

$\Big\downarrow$ 7292

$$-\frac{\int \left(\frac{27\sqrt{3}\left(-\left(\left(2Cd^2-3e(Bd-3Ae)\right)c^2\right)-e(bCd-3bBe-3aCe)c-2b^2Ce^2\right)}{ce\left(\frac{\left(-2cd+be-3ce\left(\frac{cd+be}{3ce}+x\right)\right)\left(-c^2d^2+2b^2e^2-9c^2e^2\left(\frac{cd+be}{3ce}+x\right)^2+ce(bd-9ae)+3ce(2cd-be)\left(\frac{cd+be}{3ce}+x\right)\right)}{c^2e^2}\right)^{3/2}}+\frac{\left(\left(2cd-be+3ce\left(\frac{cd+be}{3ce}+x\right)\cdots\right.\right.}{\cdots}\right.\cdots}{3ce}}{\frac{2C}{3ce\sqrt{x(ae+bd)+ad+x^2(be+cd)+cex^3}}}$$

$\Big\downarrow$ 7293

$$-\frac{\int \left(\frac{27\sqrt{3}(-((2Cd^2-3Bed+9Ae^2)c^2)-e(bCd-3bBe-3aCe)c-2b^2Ce^2)}{ce\left(\frac{\left(-2cd+be-3ce\left(\frac{cd+be}{3ce}+x\right)\right)\left(-c^2d^2+2b^2e^2-9c^2e^2\left(\frac{cd+be}{3ce}+x\right)^2+ce(bd-9ae)+3ce(2cd-be)\left(\frac{cd+be}{3ce}+x\right)\right)}{c^2e^2}\right)^{3/2}}+\frac{}{\left(\frac{\left(2cd-be+3ce\left(\frac{cd+be}{3ce}+x\right)\right.}{}\right.}\right.}{3ce}$$

$$\frac{2C}{3ce\sqrt{x(ae+bd)+ad+x^2(be+cd)+cex^3}}$$

$\downarrow$ 7292

$$-\frac{\int \left(\frac{27\sqrt{3}(-((2Cd^2-3e(Bd-3Ae))c^2)-e(bCd-3bBe-3aCe)c-2b^2Ce^2)}{ce\left(\frac{\left(-2cd+be-3ce\left(\frac{cd+be}{3ce}+x\right)\right)\left(-c^2d^2+2b^2e^2-9c^2e^2\left(\frac{cd+be}{3ce}+x\right)^2+ce(bd-9ae)+3ce(2cd-be)\left(\frac{cd+be}{3ce}+x\right)\right)}{c^2e^2}\right)^{3/2}}+\frac{}{\left(\frac{\left(2cd-be+3ce\left(\frac{cd+be}{3ce}+x\right)\right.}{}\right.}\right.}{3ce}$$

$$\frac{2C}{3ce\sqrt{x(ae+bd)+ad+x^2(be+cd)+cex^3}}$$

$\downarrow$ 7293

$$-\frac{\int \left(\frac{27\sqrt{3}(-((2Cd^2-3Bed+9Ae^2)c^2)-e(bCd-3bBe-3aCe)c-2b^2Ce^2)}{ce\left(\frac{\left(-2cd+be-3ce\left(\frac{cd+be}{3ce}+x\right)\right)\left(-c^2d^2+2b^2e^2-9c^2e^2\left(\frac{cd+be}{3ce}+x\right)^2+ce(bd-9ae)+3ce(2cd-be)\left(\frac{cd+be}{3ce}+x\right)\right)}{c^2e^2}\right)^{3/2}}+\frac{}{\left(\frac{\left(2cd-be+3ce\left(\frac{cd+be}{3ce}+x\right)\right.}{}\right.}\right.}{3ce}$$

$$\frac{2C}{3ce\sqrt{x(ae+bd)+ad+x^2(be+cd)+cex^3}}$$

$\downarrow$ 7292

$$-\frac{\int \left(\frac{27\sqrt{3}(-((2Cd^2-3e(Bd-3Ae))c^2)-e(bCd-3bBe-3aCe)c-2b^2Ce^2)}{ce\left(\frac{\left(-2cd+be-3ce\left(\frac{cd+be}{3ce}+x\right)\right)\left(-c^2d^2+2b^2e^2-9c^2e^2\left(\frac{cd+be}{3ce}+x\right)^2+ce(bd-9ae)+3ce(2cd-be)\left(\frac{cd+be}{3ce}+x\right)\right)}{c^2e^2}\right)^{3/2}}+\frac{}{\left(\frac{\left(2cd-be+3ce\left(\frac{cd+be}{3ce}+x\right)\right.}{}\right.}\right.}{3ce}$$

$$\frac{2C}{3ce\sqrt{x(ae+bd)+ad+x^2(be+cd)+cex^3}}$$

$\downarrow$ 7293

$$-\frac{\int \left(\frac{27\sqrt{3}\left(-\left(\left(2Cd^2-3Bed+9Ae^2\right)c^2\right)-e(bCd-3bBe-3aCe)c-2b^2Ce^2\right)}{ce\left(\frac{\left(-2cd+be-3ce\left(\frac{cd+be}{3ce}+x\right)\right)\left(-c^2d^2+2b^2e^2-9c^2e^2\left(\frac{cd+be}{3ce}+x\right)^2+ce(bd-9ae)+3ce(2cd-be)\left(\frac{cd+be}{3ce}+x\right)\right)}{c^2e^2}\right)^{3/2}} + \frac{\left(\frac{\left(2cd-be+3ce\left(\frac{cd+be}{3ce}+x\right)\cdots\right.}{\cdots}\right.}{\cdots} \right.}{3ce \cdots}$$

$$\frac{2C}{3ce\sqrt{x(ae+bd)+ad+x^2(be+cd)+cex^3}}$$

$$\Big\downarrow \; 7292$$

$$-\frac{\int \left(\frac{27\sqrt{3}\left(-\left(\left(2Cd^2-3e(Bd-3Ae)\right)c^2\right)-e(bCd-3bBe-3aCe)c-2b^2Ce^2\right)}{ce\left(\frac{\left(-2cd+be-3ce\left(\frac{cd+be}{3ce}+x\right)\right)\left(-c^2d^2+2b^2e^2-9c^2e^2\left(\frac{cd+be}{3ce}+x\right)^2+ce(bd-9ae)+3ce(2cd-be)\left(\frac{cd+be}{3ce}+x\right)\right)}{c^2e^2}\right)^{3/2}} + \frac{\left(\frac{\left(2cd-be+3ce\left(\frac{cd+be}{3ce}+x\right)\cdots\right.}{\cdots}\right.}{\cdots} \right.}{3ce \cdots}$$

$$\frac{2C}{3ce\sqrt{x(ae+bd)+ad+x^2(be+cd)+cex^3}}$$

$$\Big\downarrow \; 7293$$

$$-\frac{\int \left(\frac{27\sqrt{3}\left(-\left(\left(2Cd^2-3Bed+9Ae^2\right)c^2\right)-e(bCd-3bBe-3aCe)c-2b^2Ce^2\right)}{ce\left(\frac{\left(-2cd+be-3ce\left(\frac{cd+be}{3ce}+x\right)\right)\left(-c^2d^2+2b^2e^2-9c^2e^2\left(\frac{cd+be}{3ce}+x\right)^2+ce(bd-9ae)+3ce(2cd-be)\left(\frac{cd+be}{3ce}+x\right)\right)}{c^2e^2}\right)^{3/2}} + \frac{\left(\frac{\left(2cd-be+3ce\left(\frac{cd+be}{3ce}+x\right)\cdots\right.}{\cdots}\right.}{\cdots} \right.}{3ce \cdots}$$

$$\frac{2C}{3ce\sqrt{x(ae+bd)+ad+x^2(be+cd)+cex^3}}$$

$$\Big\downarrow \; 7292$$

$$-\frac{\int \left(\frac{27\sqrt{3}\left(-\left(\left(2Cd^2-3e(Bd-3Ae)\right)c^2\right)-e(bCd-3bBe-3aCe)c-2b^2Ce^2\right)}{ce\left(\frac{\left(-2cd+be-3ce\left(\frac{cd+be}{3ce}+x\right)\right)\left(-c^2d^2+2b^2e^2-9c^2e^2\left(\frac{cd+be}{3ce}+x\right)^2+ce(bd-9ae)+3ce(2cd-be)\left(\frac{cd+be}{3ce}+x\right)\right)}{c^2e^2}\right)^{3/2}} + \frac{\left(\frac{\left(2cd-be+3ce\left(\frac{cd+be}{3ce}+x\right)\cdots\right.}{\cdots}\right.}{\cdots} \right.}{3ce \cdots}$$

$$\frac{2C}{3ce\sqrt{x(ae+bd)+ad+x^2(be+cd)+cex^3}}$$

$$\Big\downarrow \; 7293$$

$$-\frac{\int \left(\frac{27\sqrt{3}\left(-\left(\left(2Cd^2-3Bed+9Ae^2\right)c^2\right)-e(bCd-3bBe-3aCe)c-2b^2Ce^2\right)}{ce\left(\frac{\left(-2cd+be-3ce\left(\frac{cd+be}{3ce}+x\right)\right)\left(-c^2d^2+2b^2e^2-9c^2e^2\left(\frac{cd+be}{3ce}+x\right)^2+ce(bd-9ae)+3ce(2cd-be)\left(\frac{cd+be}{3ce}+x\right)\right)}{c^2e^2}\right)^{3/2}}+\frac{\cdots}{\left(\frac{\left(2cd-be+3ce\left(\frac{cd+be}{3ce}+x\right)\cdots\right.}{3ce}\cdots\right.}\cdots\right.}{\frac{2C}{3ce\sqrt{x(ae+bd)+ad+x^2(be+cd)+cex^3}}\cdots}$$

$\downarrow$ 7292

$$-\frac{\int \left(\frac{27\sqrt{3}\left(-\left(\left(2Cd^2-3e(Bd-3Ae)\right)c^2\right)-e(bCd-3bBe-3aCe)c-2b^2Ce^2\right)}{ce\left(\frac{\left(-2cd+be-3ce\left(\frac{cd+be}{3ce}+x\right)\right)\left(-c^2d^2+2b^2e^2-9c^2e^2\left(\frac{cd+be}{3ce}+x\right)^2+ce(bd-9ae)+3ce(2cd-be)\left(\frac{cd+be}{3ce}+x\right)\right)}{c^2e^2}\right)^{3/2}}+\frac{\cdots}{\left(\frac{\left(2cd-be+3ce\left(\frac{cd+be}{3ce}+x\right)\cdots\right.}{3ce}\cdots\right.}\cdots\right.}{\frac{2C}{3ce\sqrt{x(ae+bd)+ad+x^2(be+cd)+cex^3}}\cdots}$$

$\downarrow$ 7293

$$-\frac{\int \left(\frac{27\sqrt{3}\left(-\left(\left(2Cd^2-3Bed+9Ae^2\right)c^2\right)-e(bCd-3bBe-3aCe)c-2b^2Ce^2\right)}{ce\left(\frac{\left(-2cd+be-3ce\left(\frac{cd+be}{3ce}+x\right)\right)\left(-c^2d^2+2b^2e^2-9c^2e^2\left(\frac{cd+be}{3ce}+x\right)^2+ce(bd-9ae)+3ce(2cd-be)\left(\frac{cd+be}{3ce}+x\right)\right)}{c^2e^2}\right)^{3/2}}+\frac{\cdots}{\left(\frac{\left(2cd-be+3ce\left(\frac{cd+be}{3ce}+x\right)\cdots\right.}{3ce}\cdots\right.}\cdots\right.}{\frac{2C}{3ce\sqrt{x(ae+bd)+ad+x^2(be+cd)+cex^3}}\cdots}$$

$\downarrow$ 7292

$$-\frac{\int \left(\frac{27\sqrt{3}\left(-\left(\left(2Cd^2-3e(Bd-3Ae)\right)c^2\right)-e(bCd-3bBe-3aCe)c-2b^2Ce^2\right)}{ce\left(\frac{\left(-2cd+be-3ce\left(\frac{cd+be}{3ce}+x\right)\right)\left(-c^2d^2+2b^2e^2-9c^2e^2\left(\frac{cd+be}{3ce}+x\right)^2+ce(bd-9ae)+3ce(2cd-be)\left(\frac{cd+be}{3ce}+x\right)\right)}{c^2e^2}\right)^{3/2}}+\frac{\cdots}{\left(\frac{\left(2cd-be+3ce\left(\frac{cd+be}{3ce}+x\right)\cdots\right.}{3ce}\cdots\right.}\cdots\right.}{\frac{2C}{3ce\sqrt{x(ae+bd)+ad+x^2(be+cd)+cex^3}}\cdots}$$

$\downarrow$ 7293

$$-\frac{\int \left(\frac{27\sqrt{3}(-((2Cd^2-3Bed+9Ae^2)c^2)-e(bCd-3bBe-3aCe)c-2b^2Ce^2)}{ce\left(\frac{\left(-2cd+be-3ce\left(\frac{cd+be}{3ce}+x\right)\right)\left(-c^2d^2+2b^2e^2-9c^2e^2\left(\frac{cd+be}{3ce}+x\right)^2+ce(bd-9ae)+3ce(2cd-be)\left(\frac{cd+be}{3ce}+x\right)\right)}{c^2e^2}\right)^{3/2}}+\frac{\cdots}{\left(\frac{\left(2cd-be+3ce\left(\frac{cd+be}{3ce}+x\right)\right)\cdots}{\cdots}\right.}\cdots\right.}{3ce\cdots}$$

$$\frac{2C}{3ce\sqrt{x(ae+bd)+ad+x^2(be+cd)+cex^3}}$$

$\big\downarrow$ 7292

$$-\frac{\int \left(\frac{27\sqrt{3}(-((2Cd^2-3e(Bd-3Ae))c^2)-e(bCd-3bBe-3aCe)c-2b^2Ce^2)}{ce\left(\frac{\left(-2cd+be-3ce\left(\frac{cd+be}{3ce}+x\right)\right)\left(-c^2d^2+2b^2e^2-9c^2e^2\left(\frac{cd+be}{3ce}+x\right)^2+ce(bd-9ae)+3ce(2cd-be)\left(\frac{cd+be}{3ce}+x\right)\right)}{c^2e^2}\right)^{3/2}}+\frac{\cdots}{\left(\frac{\left(2cd-be+3ce\left(\frac{cd+be}{3ce}+x\right)\right)\cdots}{\cdots}\right.}\cdots\right.}{3ce\cdots}$$

$$\frac{2C}{3ce\sqrt{x(ae+bd)+ad+x^2(be+cd)+cex^3}}$$

$\big\downarrow$ 7293

$$-\frac{\int \left(\frac{27\sqrt{3}(-((2Cd^2-3Bed+9Ae^2)c^2)-e(bCd-3bBe-3aCe)c-2b^2Ce^2)}{ce\left(\frac{\left(-2cd+be-3ce\left(\frac{cd+be}{3ce}+x\right)\right)\left(-c^2d^2+2b^2e^2-9c^2e^2\left(\frac{cd+be}{3ce}+x\right)^2+ce(bd-9ae)+3ce(2cd-be)\left(\frac{cd+be}{3ce}+x\right)\right)}{c^2e^2}\right)^{3/2}}+\frac{\cdots}{\left(\frac{\left(2cd-be+3ce\left(\frac{cd+be}{3ce}+x\right)\right)\cdots}{\cdots}\right.}\cdots\right.}{3ce\cdots}$$

$$\frac{2C}{3ce\sqrt{x(ae+bd)+ad+x^2(be+cd)+cex^3}}$$

$\big\downarrow$ 7292

$$-\frac{\int \left(\frac{27\sqrt{3}(-((2Cd^2-3e(Bd-3Ae))c^2)-e(bCd-3bBe-3aCe)c-2b^2Ce^2)}{ce\left(\frac{\left(-2cd+be-3ce\left(\frac{cd+be}{3ce}+x\right)\right)\left(-c^2d^2+2b^2e^2-9c^2e^2\left(\frac{cd+be}{3ce}+x\right)^2+ce(bd-9ae)+3ce(2cd-be)\left(\frac{cd+be}{3ce}+x\right)\right)}{c^2e^2}\right)^{3/2}}+\frac{\cdots}{\left(\frac{\left(2cd-be+3ce\left(\frac{cd+be}{3ce}+x\right)\right)\cdots}{\cdots}\right.}\cdots\right.}{3ce\cdots}$$

$$\frac{2C}{3ce\sqrt{x(ae+bd)+ad+x^2(be+cd)+cex^3}}$$

$\big\downarrow$ 7293

$$-\frac{\int \left(\frac{27\sqrt{3}(-((2Cd^2-3Bed+9Ae^2)c^2)-e(bCd-3bBe-3aCe)c-2b^2Ce^2)}{ce\left(\frac{\left(-2cd+be-3ce\left(\frac{cd+be}{3ce}+x\right)\right)\left(-c^2d^2+2b^2e^2-9c^2e^2\left(\frac{cd+be}{3ce}+x\right)^2+ce(bd-9ae)+3ce(2cd-be)\left(\frac{cd+be}{3ce}+x\right)\right)}{c^2e^2}\right)^{3/2}}+\frac{\cdots}{\left(\frac{\left(2cd-be+3ce\left(\frac{cd+be}{3ce}+x\right)\right)\cdots}{\cdots}\right.}\right.\cdots}{3ce\cdots}$$

$$\frac{2C}{3ce\sqrt{x(ae+bd)+ad+x^2(be+cd)+cex^3}}$$

$\big\downarrow$ 7292

$$-\frac{\int \left(\frac{27\sqrt{3}(-((2Cd^2-3e(Bd-3Ae))c^2)-e(bCd-3bBe-3aCe)c-2b^2Ce^2)}{ce\left(\frac{\left(-2cd+be-3ce\left(\frac{cd+be}{3ce}+x\right)\right)\left(-c^2d^2+2b^2e^2-9c^2e^2\left(\frac{cd+be}{3ce}+x\right)^2+ce(bd-9ae)+3ce(2cd-be)\left(\frac{cd+be}{3ce}+x\right)\right)}{c^2e^2}\right)^{3/2}}+\frac{\cdots}{\left(\frac{\left(2cd-be+3ce\left(\frac{cd+be}{3ce}+x\right)\right)\cdots}{\cdots}\right.}\right.\cdots}{3ce\cdots}$$

$$\frac{2C}{3ce\sqrt{x(ae+bd)+ad+x^2(be+cd)+cex^3}}$$

$\big\downarrow$ 7293

$$-\frac{\int \left(\frac{27\sqrt{3}(-((2Cd^2-3Bed+9Ae^2)c^2)-e(bCd-3bBe-3aCe)c-2b^2Ce^2)}{ce\left(\frac{\left(-2cd+be-3ce\left(\frac{cd+be}{3ce}+x\right)\right)\left(-c^2d^2+2b^2e^2-9c^2e^2\left(\frac{cd+be}{3ce}+x\right)^2+ce(bd-9ae)+3ce(2cd-be)\left(\frac{cd+be}{3ce}+x\right)\right)}{c^2e^2}\right)^{3/2}}+\frac{\cdots}{\left(\frac{\left(2cd-be+3ce\left(\frac{cd+be}{3ce}+x\right)\right)\cdots}{\cdots}\right.}\right.\cdots}{3ce\cdots}$$

$$\frac{2C}{3ce\sqrt{x(ae+bd)+ad+x^2(be+cd)+cex^3}}$$

$\big\downarrow$ 7292

$$-\frac{\int \left(\frac{27\sqrt{3}(-((2Cd^2-3e(Bd-3Ae))c^2)-e(bCd-3bBe-3aCe)c-2b^2Ce^2)}{ce\left(\frac{\left(-2cd+be-3ce\left(\frac{cd+be}{3ce}+x\right)\right)\left(-c^2d^2+2b^2e^2-9c^2e^2\left(\frac{cd+be}{3ce}+x\right)^2+ce(bd-9ae)+3ce(2cd-be)\left(\frac{cd+be}{3ce}+x\right)\right)}{c^2e^2}\right)^{3/2}}+\frac{\cdots}{\left(\frac{\left(2cd-be+3ce\left(\frac{cd+be}{3ce}+x\right)\right)\cdots}{\cdots}\right.}\right.\cdots}{3ce\cdots}$$

$$\frac{2C}{3ce\sqrt{x(ae+bd)+ad+x^2(be+cd)+cex^3}}$$

$\big\downarrow$ 7293

$$-\frac{\int \left(\frac{27\sqrt{3}(-((2Cd^2-3Bed+9Ae^2)c^2)-e(bCd-3bBe-3aCe)c-2b^2Ce^2)}{ce\left(\frac{\left(-2cd+be-3ce\left(\frac{cd+be}{3ce}+x\right)\right)\left(-c^2d^2+2b^2e^2-9c^2e^2\left(\frac{cd+be}{3ce}+x\right)^2+ce(bd-9ae)+3ce(2cd-be)\left(\frac{cd+be}{3ce}+x\right)\right)}{c^2e^2}\right)^{3/2}}+\frac{}{\left(\frac{\left(2cd-be+3ce\left(\frac{cd+be}{3ce}+x\right)\right.}{}\right.}\right.}{3ce}$$

$$\frac{2C}{3ce\sqrt{x(ae+bd)+ad+x^2(be+cd)+cex^3}}$$

$\downarrow$ 7292

$$-\frac{\int \left(\frac{27\sqrt{3}(-((2Cd^2-3e(Bd-3Ae))c^2)-e(bCd-3bBe-3aCe)c-2b^2Ce^2)}{ce\left(\frac{\left(-2cd+be-3ce\left(\frac{cd+be}{3ce}+x\right)\right)\left(-c^2d^2+2b^2e^2-9c^2e^2\left(\frac{cd+be}{3ce}+x\right)^2+ce(bd-9ae)+3ce(2cd-be)\left(\frac{cd+be}{3ce}+x\right)\right)}{c^2e^2}\right)^{3/2}}+\frac{}{\left(\frac{\left(2cd-be+3ce\left(\frac{cd+be}{3ce}+x\right)\right.}{}\right.}\right.}{3ce}$$

$$\frac{2C}{3ce\sqrt{x(ae+bd)+ad+x^2(be+cd)+cex^3}}$$

input
```
Int[(A + B*x + C*x^2)/(a*d + (b*d + a*e)*x + (c*d + b*e)*x^2 + c*e*x^3)^(3
/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2490
```
Int[(P3_)^(p_.)*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] :> With[{a = Coeff[P3
, x, 0], b = Coeff[P3, x, 1], c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Su
bst[Int[((3*d*e - c*f)/(3*d) + f*x)^m*Simp[(2*c^3 - 9*b*c*d + 27*a*d^2)/(27
*d^2) - (c^2 - 3*b*d)*(x/(3*d)) + d*x^3, x]^p, x], x, x + c/(3*d)] /; NeQ[c
, 0]] /; FreeQ[{e, f, m, p}, x] && PolyQ[P3, x, 3]
```

rule 2526
```
Int[(Pm_)*(Qn_)^(p_), x_Symbol] :> With[{m = Expon[Pm, x], n = Expon[Qn, x]
}, Simp[Coeff[Pm, x, m]*(Qn^(p + 1)/(n*(p + 1)*Coeff[Qn, x, n])), x] + Simp
[1/(n*Coeff[Qn, x, n])   Int[ExpandToSum[n*Coeff[Qn, x, n]*Pm - Coeff[Pm, x
, m]*D[Qn, x], x]*Qn^p, x], x] /; EqQ[m, n - 1]] /; FreeQ[p, x] && PolyQ[Pm
, x] && PolyQ[Qn, x] && NeQ[p, -1]
```

rule 7292
```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2422 vs. $2(867) = 1734$.

Time = 2.59 (sec) , antiderivative size = 2423, normalized size of antiderivative = 2.65

method	result	size
elliptic	Expression too large to display	2423
default	Expression too large to display	5320

input
```
int((C*x^2+B*x+A)/(a*d+(a*e+b*d)*x+(b*e+c*d)*x^2+c*e*x^3)^(3/2),x,method=_
RETURNVERBOSE)
```

output
```
-2*c*e*((6*A*a*c*e^2-2*A*b^2*e^2+2*A*b*c*d*e-2*A*c^2*d^2+B*a*b*e^2-8*B*a*c
*d*e+B*b^2*d*e+B*b*c*d^2-2*C*a^2*e^2+2*C*a*b*d*e+6*C*a*c*d^2-2*C*b^2*d^2)/
(4*a^3*c*e^4-a^2*b^2*e^4-8*a^2*b*c*d*e^3+8*a^2*c^2*d^2*e^2+2*a*b^3*d*e^3+2
*a*b^2*c*d^2*e^2-8*a*b*c^2*d^3*e+4*a*c^3*d^4-b^4*d^2*e^2+2*b^3*c*d^3*e-b^2
*c^2*d^4)*x^2+(7*A*a*b*c*e^3-2*A*a*c^2*d*e^2-2*A*b^3*e^3+A*b^2*c*d*e^2+A*b
*c^2*d^2*e-2*A*c^3*d^3-2*B*a^2*c*e^3+B*a*b^2*e^3-5*B*a*b*c*d*e^2-2*B*a*c^2
*d^2*e+B*b^3*d*e^2+B*b*c^2*d^3-C*a^2*b*e^3+2*C*a^2*c*d*e^2+5*C*a*b*c*d^2*e
+2*C*a*c^2*d^3-C*b^3*d^2*e-C*b^2*c*d^3)/c/e/(4*a^3*c*e^4-a^2*b^2*e^4-8*a^2
*b*c*d*e^3+8*a^2*c^2*d^2*e^2+2*a*b^3*d*e^3+2*a*b^2*c*d^2*e^2-8*a*b*c^2*d^3
*e+4*a*c^3*d^4-b^4*d^2*e^2+2*b^3*c*d^3*e-b^2*c^2*d^4)*x+(4*A*a^2*c*e^3-A*a
*b^2*e^3+3*A*a*b*c*d*e^2-4*A*a*c^2*d^2*e-A*b^3*d*e^2+2*A*b^2*c*d^2*e-A*b*c
^2*d^3-6*B*a^2*c*d*e^2+2*B*a*b^2*d*e^2-2*B*a*b*c*d^2*e+2*B*a*c^2*d^3-C*a^2
*b*d*e^2+8*C*a^2*c*d^2*e-C*a*b^2*d^2*e-C*a*b*c*d^3)/c/e/(4*a^3*c*e^4-a^2*b
^2*e^4-8*a^2*b*c*d*e^3+8*a^2*c^2*d^2*e^2+2*a*b^3*d*e^3+2*a*b^2*c*d^2*e^2-8
*a*b*c^2*d^3*e+4*a*c^3*d^4-b^4*d^2*e^2+2*b^3*c*d^3*e-b^2*c^2*d^4))/((x^3+(
b*e+c*d)/c/e*x^2+(a*e+b*d)/c/e*x+a*d/c/e)*c*e)^(1/2)+2*(-(15*A*a*b*c*e^3-1
2*A*a*c^2*d*e^2-4*A*b^3*e^3+3*A*b^2*c*d*e^2+3*A*b*c^2*d^2*e-4*A*c^3*d^3-6*
B*a^2*c*e^3+2*B*a*b^2*e^3-8*B*a*b*c*d*e^2+2*B*a*c^2*d^2*e+2*B*b^3*d*e^2-2*
B*b^2*c*d^2*e+2*B*b*c^2*d^3-C*a^2*b*e^3+8*C*a^2*c*d*e^2-2*C*a*b^2*d*e^2+7*
C*a*b*c*d^2*e-C*b^3*d^2*e-C*b^2*c*d^3)/(4*a^3*c*e^4-a^2*b^2*e^4-8*a^2*b...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2763 vs. $2(874) = 1748$.

Time = 0.20 (sec) , antiderivative size = 2763, normalized size of antiderivative = 3.03

$$\int \frac{A + Bx + Cx^2}{(ad + (bd + ae)x + (cd + be)x^2 + cex^3)^{3/2}}\, dx = \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)/(a*d+(a*e+b*d)*x+(b*e+c*d)*x^2+c*e*x^3)^(3/2),x, a
lgorithm="fricas")
```

output
```
2/3*(((C*a*b^2*c - 2*A*a*c^3 - (6*C*a^2 - B*a*b)*c^2)*d^4 + (C*a*b^3 + (10
*B*a^2 + 3*A*a*b)*c^2 - (C*a^2*b + 4*B*a*b^2)*c)*d^3*e - (4*C*a^2*b^2 - B*
a*b^3 + 18*A*a^2*c^2 - (10*C*a^3 - B*a^2*b + 3*A*a*b^2)*c)*d^2*e^2 + (C*a^
3*b + B*a^2*b^2 - 2*A*a*b^3 - 3*(2*B*a^3 - 3*A*a^2*b)*c)*d*e^3 + ((C*b^2*c
^2 - 2*A*c^4 - (6*C*a - B*b)*c^3)*d^3*e + (C*b^3*c + (10*B*a + 3*A*b)*c^3
- (C*a*b + 4*B*b^2)*c^2)*d^2*e^2 - (18*A*a*c^3 - (10*C*a^2 - B*a*b + 3*A*b
^2)*c^2 + (4*C*a*b^2 - B*b^3)*c)*d*e^3 - (3*(2*B*a^2 - 3*A*a*b)*c^2 - (C*a
^2*b + B*a*b^2 - 2*A*b^3)*c)*e^4)*x^3 + ((C*b^2*c^2 - 2*A*c^4 - (6*C*a - B
*b)*c^3)*d^4 + (2*C*b^3*c + (10*B*a + A*b)*c^3 - (7*C*a*b + 3*B*b^2)*c^2)*
d^3*e + (C*b^4 - 18*A*a*c^3 + (10*C*a^2 + 9*B*a*b + 6*A*b^2)*c^2 - (5*C*a*
b^2 + 3*B*b^3)*c)*d^2*e^2 - (4*C*a*b^3 - B*b^4 + 3*(2*B*a^2 + 3*A*a*b)*c^2
 - (11*C*a^2*b + A*b^3)*c)*d*e^3 + (C*a^2*b^2 + B*a*b^3 - 2*A*b^4 - 3*(2*B
*a^2*b - 3*A*a*b^2)*c)*e^4)*x^2 + ((C*b^3*c - 2*A*b*c^3 - (6*C*a*b - B*b^2
)*c^2)*d^4 + (C*b^4 - 4*B*b^3*c - 2*A*a*c^3 - (6*C*a^2 - 11*B*a*b - 3*A*b^
2)*c^2)*d^3*e - (3*C*a*b^3 - B*b^4 - 5*(2*B*a^2 - 3*A*a*b)*c^2 - (9*C*a^2*
b - 5*B*a*b^2 + 3*A*b^3)*c)*d^2*e^2 - (3*C*a^2*b^2 - 2*B*a*b^3 + 2*A*b^4 +
 18*A*a^2*c^2 - (10*C*a^3 - 7*B*a^2*b + 12*A*a*b^2)*c)*d*e^3 + (C*a^3*b +
B*a^2*b^2 - 2*A*a*b^3 - 3*(2*B*a^3 - 3*A*a^2*b)*c)*e^4)*x)*sqrt(c*e)*weier
strassPInverse(4/3*(c^2*d^2 - b*c*d*e + (b^2 - 3*a*c)*e^2)/(c^2*e^2), -4/2
7*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*(b^2*c - 6*a*c^2)*d*e^2 + (2*b^3 - 9*a...
```

Sympy [F]

$$\int \frac{A + Bx + Cx^2}{\left(ad + (bd + ae)x + (cd + be)x^2 + cex^3\right)^{3/2}}\, dx = \int \frac{A + Bx + Cx^2}{\left((d + ex)\left(a + bx + cx^2\right)\right)^{\frac{3}{2}}}\, dx$$

input
```
integrate((C*x**2+B*x+A)/(a*d+(a*e+b*d)*x+(b*e+c*d)*x**2+c*e*x**3)**(3/2),
x)
```

output
```
Integral((A + B*x + C*x**2)/((d + e*x)*(a + b*x + c*x**2))**(3/2), x)
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{(ad+(bd+ae)x+(cd+be)x^2+cex^3)^{3/2}}\,dx=\int \frac{Cx^2+Bx+A}{(cex^3+(cd+be)x^2+ad+(bd+ae)x)^{\frac{3}{2}}}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(a*d+(a*e+b*d)*x+(b*e+c*d)*x^2+c*e*x^3)^(3/2),x, a
lgorithm="maxima")
```

output
```
integrate((C*x^2 + B*x + A)/(c*e*x^3 + (c*d + b*e)*x^2 + a*d + (b*d + a*e)
*x)^(3/2), x)
```

Giac [F]

$$\int \frac{A+Bx+Cx^2}{(ad+(bd+ae)x+(cd+be)x^2+cex^3)^{3/2}}\,dx=\int \frac{Cx^2+Bx+A}{(cex^3+(cd+be)x^2+ad+(bd+ae)x)^{\frac{3}{2}}}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(a*d+(a*e+b*d)*x+(b*e+c*d)*x^2+c*e*x^3)^(3/2),x, a
lgorithm="giac")
```

output
```
integrate((C*x^2 + B*x + A)/(c*e*x^3 + (c*d + b*e)*x^2 + a*d + (b*d + a*e)
*x)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(ad+(bd+ae)x+(cd+be)x^2+cex^3)^{3/2}}\,dx=\int \frac{C\,x^2+B\,x+A}{{\left(c\,e\,x^3+\left(b\,e+c\,d\right)\,x^2+\left(a\,e+b\,d\right)\,x+a\,d\right)}^{3/2}}\,dx$$

input
```
int((A + B*x + C*x^2)/(a*d + x*(a*e + b*d) + x^2*(b*e + c*d) + c*e*x^3)^(3
/2),x)
```

output
```
int((A + B*x + C*x^2)/(a*d + x*(a*e + b*d) + x^2*(b*e + c*d) + c*e*x^3)^(3
/2), x)
```

Reduce [F]

$$\int \frac{A+Bx+Cx^2}{(ad+(bd+ae)x+(cd+be)x^2+cex^3)^{3/2}}\,dx = \int \frac{C\,x^2+Bx+A}{(ad+(ae+bd)\,x+(be+cd)\,x^2+ce\,x^3)^{\frac{3}{2}}}dx$$

input
```
int((C*x^2+B*x+A)/(a*d+(a*e+b*d)*x+(b*e+c*d)*x^2+c*e*x^3)^(3/2),x)
```

output
```
int((C*x^2+B*x+A)/(a*d+(a*e+b*d)*x+(b*e+c*d)*x^2+c*e*x^3)^(3/2),x)
```

3.132 $\int \frac{A+Bx+Cx^2}{\sqrt{8-8x+4x^2-x^3}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 270

$$\begin{aligned}\int \frac{A+Bx+Cx^2}{\sqrt{8-8x+4x^2-x^3}}\,dx &= -\frac{2C(2-x)\left(4-2x+x^2\right)}{3\sqrt{8-8x+4x^2-x^3}}+\frac{2(3B+8C)(2-x)\left(4-2x+x^2\right)}{3(4-x)\sqrt{8-8x+4x^2-x^3}} \\ &\quad -\frac{2\sqrt{2}(3B+8C)\sqrt{2-x}(4-x)\sqrt{\frac{4-2x+x^2}{(4-x)^2}}E\left(2\arctan\left(\frac{\sqrt{2-x}}{\sqrt{2}}\right)|\frac{3}{4}\right)}{3\sqrt{8-8x+4x^2-x^3}} \\ &\quad -\frac{(3A-8C)\sqrt{2-x}(4-x)\sqrt{\frac{4-2x+x^2}{(4-x)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt{2-x}}{\sqrt{2}}\right),\frac{3}{4}\right)}{3\sqrt{2}\sqrt{8-8x+4x^2-x^3}}\end{aligned}$$

output
```
-2/3*C*(2-x)*(x^2-2*x+4)/(-x^3+4*x^2-8*x+8)^(1/2)+2/3*(3*B+8*C)*(2-x)*(x^2
-2*x+4)/(4-x)/(-x^3+4*x^2-8*x+8)^(1/2)-2/3*2^(1/2)*(3*B+8*C)*(2-x)^(1/2)*(
4-x)*((x^2-2*x+4)/(4-x)^2)^(1/2)*EllipticE(sin(2*arctan(1/2*2^(1/2)*(2-x)^
(1/2))),1/2*3^(1/2))/(-x^3+4*x^2-8*x+8)^(1/2)-1/6*(3*A-8*C)*(2-x)^(1/2)*(4
-x)*((x^2-2*x+4)/(4-x)^2)^(1/2)*InverseJacobiAM(2*arctan(1/2*2^(1/2)*(2-x)
^(1/2)),1/2*3^(1/2))*2^(1/2)/(-x^3+4*x^2-8*x+8)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.71 (sec) , antiderivative size = 655, normalized size of antiderivative = 2.43

$$\int \frac{A+Bx+Cx^2}{\sqrt{8-8x+4x^2-x^3}}\,dx$$

$$= \frac{2\left(C(-8+8x-4x^2+x^3)+\frac{3B\sqrt{i+\sqrt{3}-ix}\sqrt{\frac{i(-2+x)}{-i+\sqrt{3}}}\left(-1-i\sqrt{3}+x\right)\left(\left(1+i\sqrt{3}\right)E\left(\arcsin\left(\frac{\sqrt{i+\sqrt{3}-ix}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{-i+\sqrt{3}}\right)-2\,\text{EllipticF}\left(\right.\right.}{\sqrt{-i+\sqrt{3}+ix}}\right.}{}$$

input
```
Integrate[(A + B*x + C*x^2)/Sqrt[8 - 8*x + 4*x^2 - x^3],x]
```

output
```
(2*(C*(-8 + 8*x - 4*x^2 + x^3) + (3*B*Sqrt[I + Sqrt[3] - I*x]*Sqrt[(I*(-2
+ x))/(-I + Sqrt[3])]*(-1 - I*Sqrt[3] + x)*((1 + I*Sqrt[3])*EllipticE[ArcS
in[Sqrt[I + Sqrt[3] - I*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(-I + Sqrt[3])]
 - 2*EllipticF[ArcSin[Sqrt[I + Sqrt[3] - I*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[
3])/(-I + Sqrt[3])]))/Sqrt[-I + Sqrt[3] + I*x] + (8*C*Sqrt[I + Sqrt[3] - I
*x]*Sqrt[(I*(-2 + x))/(-I + Sqrt[3])]*(-1 - I*Sqrt[3] + x)*((1 + I*Sqrt[3]
)*EllipticE[ArcSin[Sqrt[I + Sqrt[3] - I*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])
/(-I + Sqrt[3])] - 2*EllipticF[ArcSin[Sqrt[I + Sqrt[3] - I*x]/(Sqrt[2]*3^(
1/4))], (2*Sqrt[3])/(-I + Sqrt[3])]))/Sqrt[-I + Sqrt[3] + I*x] + (3*A*Sqrt
[-I + Sqrt[3] + I*x]*Sqrt[(I*(-2 + x))/(-I + Sqrt[3])]*(-1 + I*Sqrt[3] + x
)*EllipticF[ArcSin[Sqrt[I + Sqrt[3] - I*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])
/(-I + Sqrt[3])])/Sqrt[I + Sqrt[3] - I*x] - (8*C*Sqrt[-I + Sqrt[3] + I*x]*
Sqrt[(I*(-2 + x))/(-I + Sqrt[3])]*(-1 + I*Sqrt[3] + x)*EllipticF[ArcSin[Sq
rt[I + Sqrt[3] - I*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(-I + Sqrt[3])])/Sqr
t[I + Sqrt[3] - I*x]))/(3*Sqrt[8 - 8*x + 4*x^2 - x^3])
```

Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.91 (sec) , antiderivative size = 252, normalized size of antiderivative = 0.93, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {2526, 25, 2490, 2486, 27, 1269, 1172, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{\sqrt{-x^3+4x^2-8x+8}}dx$$

$$\Big\downarrow \text{ 2526}$$

$$-\frac{1}{3}\int -\frac{3A-8C+(3B+8C)x}{\sqrt{-x^3+4x^2-8x+8}}dx-\frac{2}{3}C\sqrt{-x^3+4x^2-8x+8}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{3}\int \frac{3A-8C+(3B+8C)x}{\sqrt{-x^3+4x^2-8x+8}}dx-\frac{2}{3}C\sqrt{-x^3+4x^2-8x+8}$$

$$\Big\downarrow \text{ 2490}$$

$$\frac{1}{3}\int \frac{\frac{1}{3}(3(3A-8C)+4(3B+8C))+(3B+8C)\left(x-\frac{4}{3}\right)}{\sqrt{-\left(x-\frac{4}{3}\right)^3-\frac{8}{3}\left(x-\frac{4}{3}\right)+\frac{56}{27}}}d\left(x-\frac{4}{3}\right)-\frac{2}{3}C\sqrt{-x^3+4x^2-8x+8}$$

$$\Big\downarrow \text{ 2486}$$

$$\frac{\sqrt{2-3\left(x-\frac{4}{3}\right)}\sqrt{9\left(x-\frac{4}{3}\right)^2+6\left(x-\frac{4}{3}\right)+28}\int \frac{\sqrt{3}\left(9A+12B+8C+3(3B+8C)\left(x-\frac{4}{3}\right)\right)}{\sqrt{2-3\left(x-\frac{4}{3}\right)}\sqrt{9\left(x-\frac{4}{3}\right)^2+6\left(x-\frac{4}{3}\right)+28}}d\left(x-\frac{4}{3}\right)}{3\sqrt{-27\left(x-\frac{4}{3}\right)^3-72\left(x-\frac{4}{3}\right)+56}}-\frac{2}{3}C\sqrt{-x^3+4x^2-8x+8}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{2-3\left(x-\frac{4}{3}\right)}\sqrt{9\left(x-\frac{4}{3}\right)^2+6\left(x-\frac{4}{3}\right)+28}\int \frac{9A+12B+8C+3(3B+8C)\left(x-\frac{4}{3}\right)}{\sqrt{2-3\left(x-\frac{4}{3}\right)}\sqrt{9\left(x-\frac{4}{3}\right)^2+6\left(x-\frac{4}{3}\right)+28}}d\left(x-\frac{4}{3}\right)}{\sqrt{3}\sqrt{-27\left(x-\frac{4}{3}\right)^3-72\left(x-\frac{4}{3}\right)+56}}-\frac{2}{3}C\sqrt{-x^3+4x^2-8x+8}$$

$$\Big\downarrow \; 1269$$

$$\frac{\sqrt{2-3\left(x-\frac{4}{3}\right)}\sqrt{9\left(x-\frac{4}{3}\right)^2+6\left(x-\frac{4}{3}\right)+28}\left(3(3A+6B+8C)\int \frac{1}{\sqrt{2-3\left(x-\frac{4}{3}\right)}\sqrt{9\left(x-\frac{4}{3}\right)^2+6\left(x-\frac{4}{3}\right)+28}}d\left(x-\frac{4}{3}\right)-(\right.}{\sqrt{3}\sqrt{-27\left(x-\frac{4}{3}\right)^3-72\left(x-\frac{4}{3}\right)+56}}$$

$$\frac{2}{3}C\sqrt{-x^3+4x^2-8x+8}$$

$$\Big\downarrow \; 1172$$

$$-\frac{2}{3}C\sqrt{-x^3+4x^2-8x+8}+$$

$$\frac{\sqrt{2-3\left(x-\frac{4}{3}\right)}\sqrt{9\left(x-\frac{4}{3}\right)^2+6\left(x-\frac{4}{3}\right)+28}\left(\frac{2i\sqrt{\frac{2-3\left(x-\frac{4}{3}\right)}{1+i\sqrt{3}}}(3A+6B+8C)\int \frac{1}{\sqrt{\frac{i\left(3\left(x-\frac{4}{3}\right)+3i\sqrt{3}+1\right)}{6\sqrt{3}}+1}\sqrt{\frac{i\left(3\left(x-\frac{4}{3}\right)+3i\sqrt{3}+1\right)}{\sqrt{3}\left(3-i\sqrt{3}\right)}+1}}}{\sqrt{3}\sqrt{2-3\left(x-\frac{4}{3}\right)}}\right.}{\sqrt{3}\sqrt{-27\left(x-\frac{4}{3}\right)^3-72\left(x-\frac{4}{3}\right)}}$$

$$\Big\downarrow \; 321$$

$$-\frac{2}{3}C\sqrt{-x^3+4x^2-8x+8}+$$

$$\frac{\sqrt{2-3\left(x-\frac{4}{3}\right)}\sqrt{9\left(x-\frac{4}{3}\right)^2+6\left(x-\frac{4}{3}\right)+28}\left(-\frac{2i\sqrt{2-3\left(x-\frac{4}{3}\right)}(3B+8C)\int \frac{\sqrt{\frac{i\left(3\left(x-\frac{4}{3}\right)+3i\sqrt{3}+1\right)}{\sqrt{3}\left(3-i\sqrt{3}\right)}+1}}{\sqrt{\frac{i\left(3\left(x-\frac{4}{3}\right)+3i\sqrt{3}+1\right)}{6\sqrt{3}}+1}}d\frac{\sqrt{-i\left(3\left(x-\frac{4}{3}\right)+3i\sqrt{3}+1\right)}}{\sqrt{2}3^{3/4}}}{\sqrt{3}\sqrt{\frac{2-3\left(x-\frac{4}{3}\right)}{1+i\sqrt{3}}}}-\right.}{\sqrt{3}\sqrt{-27\left(x-\frac{4}{3}\right)^3-72\left(x-\frac{4}{3}\right)+56}}$$

$$\Big\downarrow \; 327$$

$$-\frac{2}{3}C\sqrt{-x^3+4x^2-8x+8}+$$

$$\frac{\sqrt{2-3\left(x-\frac{4}{3}\right)}\sqrt{9\left(x-\frac{4}{3}\right)^2+6\left(x-\frac{4}{3}\right)+28}\left(\frac{2i\sqrt{2-3\left(x-\frac{4}{3}\right)}(3B+8C)E\left(\arcsin\left(\frac{4}{3}-x\right)|\frac{6}{3-i\sqrt{3}}\right)}{\sqrt{3}\sqrt{\frac{2-3\left(x-\frac{4}{3}\right)}{1+i\sqrt{3}}}}-\frac{2i\sqrt{\frac{2-3\left(x-\frac{4}{3}\right)}{1+i\sqrt{3}}}(3A+6B+8}{\sqrt{3}\sqrt{}}\right.}{\sqrt{3}\sqrt{-27\left(x-\frac{4}{3}\right)^3-72\left(x-\frac{4}{3}\right)+56}}$$

input
```
Int[(A + B*x + C*x^2)/Sqrt[8 - 8*x + 4*x^2 - x^3],x]
```

output
```
(-2*C*Sqrt[8 - 8*x + 4*x^2 - x^3])/3 + (Sqrt[2 - 3*(-4/3 + x)]*Sqrt[28 + 6
*(-4/3 + x) + 9*(-4/3 + x)^2]*(((2*I)*(3*B + 8*C)*Sqrt[2 - 3*(-4/3 + x)]*E
llipticE[ArcSin[4/3 - x], 6/(3 - I*Sqrt[3])])/(Sqrt[3]*Sqrt[(2 - 3*(-4/3 +
 x))/(1 + I*Sqrt[3])]) - ((2*I)*(3*A + 6*B + 8*C)*Sqrt[(2 - 3*(-4/3 + x))/
(1 + I*Sqrt[3])]*EllipticF[ArcSin[4/3 - x], 6/(3 - I*Sqrt[3])])/(Sqrt[3]*S
qrt[2 - 3*(-4/3 + x)])))/(Sqrt[3]*Sqrt[56 - 72*(-4/3 + x) - 27*(-4/3 + x)^
3])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 1172
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Simp[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m))   Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^
2 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[m^2, 1/4]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2486
```
Int[((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (d_.)*(x_)^3)^(p_), x_S
ymbol] :> With[{r = Rt[-9*a*d^2 + Sqrt[3]*d*Sqrt[4*b^3*d + 27*a^2*d^2], 3]}
, Simp[(a + b*x + d*x^3)^p/(Simp[18^(1/3)*b*(d/(3*r)) - r/18^(1/3) + d*x, x
]^p*Simp[b*(d/3) + 12^(1/3)*b^2*(d^2/(3*r^2)) + r^2/(3*12^(1/3)) - d*(2^(1/
3)*b*(d/(3^(1/3)*r)) - r/18^(1/3))*x + d^2*x^2, x]^p)   Int[(e + f*x)^m*Sim
p[18^(1/3)*b*(d/(3*r)) - r/18^(1/3) + d*x, x]^p*Simp[b*(d/3) + 12^(1/3)*b^2
*(d^2/(3*r^2)) + r^2/(3*12^(1/3)) - d*(2^(1/3)*b*(d/(3^(1/3)*r)) - r/18^(1/
3))*x + d^2*x^2, x]^p, x], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && NeQ[4*
b^3 + 27*a^2*d, 0] &&  !IntegerQ[p]
```

rule 2490
```
Int[(P3_)^(p_.)*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] :> With[{a = Coeff[P3
, x, 0], b = Coeff[P3, x, 1], c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Su
bst[Int[((3*d*e - c*f)/(3*d) + f*x)^m*Simp[(2*c^3 - 9*b*c*d + 27*a*d^2)/(27
*d^2) - (c^2 - 3*b*d)*(x/(3*d)) + d*x^3, x]^p, x], x, x + c/(3*d)] /; NeQ[c
, 0]] /; FreeQ[{e, f, m, p}, x] && PolyQ[P3, x, 3]
```

rule 2526
```
Int[(Pm_)*(Qn_)^(p_), x_Symbol] :> With[{m = Expon[Pm, x], n = Expon[Qn, x]
}, Simp[Coeff[Pm, x, m]*(Qn^(p + 1)/(n*(p + 1)*Coeff[Qn, x, n])), x] + Simp
[1/(n*Coeff[Qn, x, n])   Int[ExpandToSum[n*Coeff[Qn, x, n]*Pm - Coeff[Pm, x
, m]*D[Qn, x], x]*Qn^p, x], x] /; EqQ[m, n - 1]] /; FreeQ[p, x] && PolyQ[Pm
, x] && PolyQ[Qn, x] && NeQ[p, -1]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.28 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.20

method	result
elliptic	$-\frac{2C\sqrt{-x^3+4x^2-8x+8}}{3}-\frac{2i\left(A-\frac{8C}{3}\right)\sqrt{3}\,\sqrt{i\left(x-1-i\sqrt{3}\right)\sqrt{3}}\,\sqrt{\frac{x-2}{-1+i\sqrt{3}}}\,\sqrt{-i\left(x-1+i\sqrt{3}\right)\sqrt{3}}\,\text{EllipticF}\left(\frac{\sqrt{6}\,\sqrt{i\left(x-1-i\sqrt{3}\right)\sqrt{3}}}{6},\sqrt{\right.}{3\sqrt{-x^3+4x^2-8x+8}}$
risch	$\frac{2C\left(x^3-4x^2+8x-8\right)}{3\sqrt{-x^3+4x^2-8x+8}}-\frac{2i(3B+8C)\sqrt{3}\,\sqrt{i\left(x-1-i\sqrt{3}\right)\sqrt{3}}\,\sqrt{\frac{x-2}{-1+i\sqrt{3}}}\,\sqrt{-i\left(x-1+i\sqrt{3}\right)\sqrt{3}}\left(\left(-1+i\sqrt{3}\right)\text{EllipticE}\left(\frac{\sqrt{6}\,\sqrt{i\left(x-1-\right.}}{6}\right.\right.}{9\sqrt{-x^3+4x^2-8x+8}}$
default	$-\frac{2iA\sqrt{3}\,\sqrt{i\left(x-1-i\sqrt{3}\right)\sqrt{3}}\,\sqrt{\frac{x-2}{-1+i\sqrt{3}}}\,\sqrt{-i\left(x-1+i\sqrt{3}\right)\sqrt{3}}\,\text{EllipticF}\left(\frac{\sqrt{6}\,\sqrt{i\left(x-1-i\sqrt{3}\right)\sqrt{3}}}{6},\sqrt{2}\,\sqrt{\frac{i\sqrt{3}}{-1+i\sqrt{3}}}\right)}{3\sqrt{-x^3+4x^2-8x+8}}-\frac{2iB\sqrt{3}\,\sqrt{i\left(x-\right.}}{}$

input

```
int((C*x^2+B*x+A)/(-x^3+4*x^2-8*x+8)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
-2/3*C*(-x^3+4*x^2-8*x+8)^(1/2)-2/3*I*(A-8/3*C)*3^(1/2)*(I*(x-1-I*3^(1/2))
*3^(1/2))^(1/2)*((x-2)/(-1+I*3^(1/2)))^(1/2)*(-I*(x-1+I*3^(1/2))*3^(1/2))^
(1/2)/(-x^3+4*x^2-8*x+8)^(1/2)*EllipticF(1/6*6^(1/2)*(I*(x-1-I*3^(1/2))*3^
(1/2))^(1/2),2^(1/2)*(I*3^(1/2)/(-1+I*3^(1/2)))^(1/2))-2/3*I*(B+8/3*C)*3^(
1/2)*(I*(x-1-I*3^(1/2))*3^(1/2))^(1/2)*((x-2)/(-1+I*3^(1/2)))^(1/2)*(-I*(x
-1+I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+4*x^2-8*x+8)^(1/2)*((-1+I*3^(1/2))*Elli
pticE(1/6*6^(1/2)*(I*(x-1-I*3^(1/2))*3^(1/2))^(1/2),2^(1/2)*(I*3^(1/2)/(-1
+I*3^(1/2)))^(1/2))+2*EllipticF(1/6*6^(1/2)*(I*(x-1-I*3^(1/2))*3^(1/2))^(1
/2),2^(1/2)*(I*3^(1/2)/(-1+I*3^(1/2)))^(1/2)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.21

$$\int \frac{A+Bx+Cx^2}{\sqrt{8-8x+4x^2-x^3}}\,dx = -\frac{2}{9}\,(9i\,A+12i\,B+8i\,C)\text{weierstrassPInverse}\left(-\frac{32}{3},\frac{224}{27},x-\frac{4}{3}\right) - \frac{2}{3}\,(-3i\,B-8i\,C)\text{weierstrassZeta}\left(-\frac{32}{3},\frac{224}{27},\text{weierstrassPInverse}\left(-\frac{32}{3},\frac{224}{27},x-\frac{4}{3}\right)\right) - \frac{2}{3}\,\sqrt{-x^3+4\,x^2-8\,x+8}C$$

input
```
integrate((C*x^2+B*x+A)/(-x^3+4*x^2-8*x+8)^(1/2),x, algorithm="fricas")
```

output
```
-2/9*(9*I*A + 12*I*B + 8*I*C)*weierstrassPInverse(-32/3, 224/27, x - 4/3)
- 2/3*(-3*I*B - 8*I*C)*weierstrassZeta(-32/3, 224/27, weierstrassPInverse(
-32/3, 224/27, x - 4/3)) - 2/3*sqrt(-x^3 + 4*x^2 - 8*x + 8)*C
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt{8-8x+4x^2-x^3}}\,dx = \int \frac{A+Bx+Cx^2}{\sqrt{-(x-2)\,(x^2-2x+4)}}\,dx$$

input
```
integrate((C*x**2+B*x+A)/(-x**3+4*x**2-8*x+8)**(1/2),x)
```

output
```
Integral((A + B*x + C*x**2)/sqrt(-(x - 2)*(x**2 - 2*x + 4)), x)
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt{8-8x+4x^2-x^3}}\,dx=\int \frac{Cx^2+Bx+A}{\sqrt{-x^3+4\,x^2-8\,x+8}}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(-x^3+4*x^2-8*x+8)^(1/2),x, algorithm="maxima")
```

output
```
integrate((C*x^2 + B*x + A)/sqrt(-x^3 + 4*x^2 - 8*x + 8), x)
```

Giac [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt{8-8x+4x^2-x^3}}\,dx=\int \frac{Cx^2+Bx+A}{\sqrt{-x^3+4\,x^2-8\,x+8}}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(-x^3+4*x^2-8*x+8)^(1/2),x, algorithm="giac")
```

output
```
integrate((C*x^2 + B*x + A)/sqrt(-x^3 + 4*x^2 - 8*x + 8), x)
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 879, normalized size of antiderivative = 3.26

$$\int \frac{A+Bx+Cx^2}{\sqrt{8-8x+4x^2-x^3}}\,dx=\text{Too large to display}$$

input
```
int((A + B*x + C*x^2)/(4*x^2 - 8*x - x^3 + 8)^(1/2),x)
```

output
```
(C*((2*(8*x - 4*x^2 + x^3 - 8)^(1/2))/3 - (16*(3^(1/2)*1i - 1)*((x + 3^(1/
2)*1i - 1)/(3^(1/2)*1i + 1))^(1/2)*((x - 2)/(3^(1/2)*1i - 1))^(1/2)*((3^(1
/2)*1i - x + 1)/(3^(1/2)*1i - 1))^(1/2)*ellipticF(asin(((x - 2)/(3^(1/2)*1
i - 1))^(1/2)), -(3^(1/2)*1i - 1)/(3^(1/2)*1i + 1)))/(3*(2*(3^(1/2)*1i - 1
)*(3^(1/2)*1i + 1) - x*((3^(1/2)*1i - 1)*(3^(1/2)*1i + 1) - 4) - 4*x^2 + x
^3)^(1/2)) + (16*((3^(1/2)*1i + 1)*ellipticE(asin(((x - 2)/(3^(1/2)*1i - 1
))^(1/2)), -(3^(1/2)*1i - 1)/(3^(1/2)*1i + 1)) - (3^(1/2)*1i - 1)*elliptic
F(asin(((x - 2)/(3^(1/2)*1i - 1))^(1/2)), -(3^(1/2)*1i - 1)/(3^(1/2)*1i +
1)))*(3^(1/2)*1i - 1)*((x + 3^(1/2)*1i - 1)/(3^(1/2)*1i + 1))^(1/2)*((x -
2)/(3^(1/2)*1i - 1))^(1/2)*((3^(1/2)*1i - x + 1)/(3^(1/2)*1i - 1))^(1/2))/
(3*(2*(3^(1/2)*1i - 1)*(3^(1/2)*1i + 1) - x*((3^(1/2)*1i - 1)*(3^(1/2)*1i
+ 1) - 4) - 4*x^2 + x^3)^(1/2)))*(8*x - 4*x^2 + x^3 - 8)^(1/2))/(4*x^2 - 8
*x - x^3 + 8)^(1/2) + (2*A*(3^(1/2)*1i - 1)*((x + 3^(1/2)*1i - 1)/(3^(1/2)
*1i + 1))^(1/2)*((x - 2)/(3^(1/2)*1i - 1))^(1/2)*((3^(1/2)*1i - x + 1)/(3^
(1/2)*1i - 1))^(1/2)*ellipticF(asin(((x - 2)/(3^(1/2)*1i - 1))^(1/2)), -(3
^(1/2)*1i - 1)/(3^(1/2)*1i + 1))*(8*x - 4*x^2 + x^3 - 8)^(1/2))/((4*x^2 -
8*x - x^3 + 8)^(1/2)*(2*(3^(1/2)*1i - 1)*(3^(1/2)*1i + 1) - x*((3^(1/2)*1i
 - 1)*(3^(1/2)*1i + 1) - 4) - 4*x^2 + x^3)^(1/2)) + (2*B*((3^(1/2)*1i + 1)
*ellipticE(asin(((x - 2)/(3^(1/2)*1i - 1))^(1/2)), -(3^(1/2)*1i - 1)/(3^(1
/2)*1i + 1)) - (3^(1/2)*1i - 1)*ellipticF(asin(((x - 2)/(3^(1/2)*1i - 1...
```

Reduce [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt{8-8x+4x^2-x^3}}\,dx = \frac{\sqrt{-x^3+4x^2-8x+8}\,b}{4} - \left(\int \frac{\sqrt{-x^3+4x^2-8x+8}}{x^3-4x^2+8x-8}dx\right)a - \left(\int \frac{\sqrt{-x^3+4x^2-8x+8}}{x^3-4x^2+8x-8}dx\right)b - \frac{3\left(\int \frac{\sqrt{-x^3+4x^2-8x+8}\,x^2}{x^3-4x^2+8x-8}dx\right)b}{8} - \left(\int \frac{\sqrt{-x^3+4x^2-8x+8}\,x^2}{x^3-4x^2+8x-8}dx\right)c$$

input
```
int((C*x^2+B*x+A)/(-x^3+4*x^2-8*x+8)^(1/2),x)
```

output

```
(2*sqrt( - x**3 + 4*x**2 - 8*x + 8)*b - 8*int(sqrt( - x**3 + 4*x**2 - 8*x
+ 8)/(x**3 - 4*x**2 + 8*x - 8),x)*a - 8*int(sqrt( - x**3 + 4*x**2 - 8*x +
8)/(x**3 - 4*x**2 + 8*x - 8),x)*b - 3*int((sqrt( - x**3 + 4*x**2 - 8*x + 8
)*x**2)/(x**3 - 4*x**2 + 8*x - 8),x)*b - 8*int((sqrt( - x**3 + 4*x**2 - 8*
x + 8)*x**2)/(x**3 - 4*x**2 + 8*x - 8),x)*c)/8
```

3.133 $\int (A+Bx+Cx^2)(ad+(bd+ae)x+(cd+be)x^2$

Optimal result

Integrand size = 43, antiderivative size = 532

$$\int (A+Bx+Cx^2)\left(ad+(bd+ae)x+(cd+be)x^2+cex^3\right)^p\,dx$$

$$=\frac{C(d+ex)\left(a+bx+cx^2\right)\left(ad+(bd+ae)x+(cd+be)x^2+cex^3\right)^p}{3ce(1+p)}$$

$$+\frac{(Ce(bd-ae)+c(2Cd^2-3e(Bd-Ae)))\,(d+ex)\left(ad+(bd+ae)x+(cd+be)x^2+cex^3\right)^p\left(1-\frac{\quad}{2cd-}\right.}{3}$$

$$-\frac{(2cCd-3Bce+2bCe)(d+ex)^2\left(ad+(bd+ae)x+(cd+be)x^2+cex^3\right)^p\left(1-\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)^{-p}\left(\right.}{3ce^3(2+p}$$

output
```
1/3*C*(e*x+d)*(c*x^2+b*x+a)*(a*d+(a*e+b*d)*x+(b*e+c*d)*x^2+c*e*x^3)^p/c/e/
(p+1)+1/3*(C*e*(-a*e+b*d)+c*(2*C*d^2-3*e*(-A*e+B*d)))*(e*x+d)*(a*d+(a*e+b*
d)*x+(b*e+c*d)*x^2+c*e*x^3)^p*AppellF1(p+1,-p,-p,2+p,2*c*(e*x+d)/(2*c*d-(b
-(-4*a*c+b^2)^(1/2))*e),2*c*(e*x+d)/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e))/c/e^
3/(p+1)/((1-2*c*(e*x+d)/(2*c*d-(b-(-4*a*c+b^2)^(1/2))*e))^p)/((1-2*c*(e*x+
d)/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e))^p)-1/3*(-3*B*c*e+2*C*b*e+2*C*c*d)*(e*
x+d)^2*(a*d+(a*e+b*d)*x+(b*e+c*d)*x^2+c*e*x^3)^p*AppellF1(2+p,-p,-p,3+p,2*
c*(e*x+d)/(2*c*d-(b-(-4*a*c+b^2)^(1/2))*e),2*c*(e*x+d)/(2*c*d-(b+(-4*a*c+b
^2)^(1/2))*e))/c/e^3/(2+p)/((1-2*c*(e*x+d)/(2*c*d-(b-(-4*a*c+b^2)^(1/2))*e
))^p)/((1-2*c*(e*x+d)/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e))^p)
```

Mathematica [F]

$$\int (A+Bx+Cx^2)\left(ad+(bd+ae)x+(cd+be)x^2+cex^3\right)^p\,dx$$
$$=\int (A+Bx+Cx^2)\left(ad+(bd+ae)x+(cd+be)x^2+cex^3\right)^p\,dx$$

input
```
Integrate[(A + B*x + C*x^2)*(a*d + (b*d + a*e)*x + (c*d + b*e)*x^2 + c*e*x
^3)^p,x]
```

output
```
Integrate[(A + B*x + C*x^2)*(a*d + (b*d + a*e)*x + (c*d + b*e)*x^2 + c*e*x
^3)^p, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (A+Bx+Cx^2)\left(x(ae+bd)+ad+x^2(be+cd)+cex^3\right)^p\,dx$$

$$\Big\downarrow \text{2526}$$

$$\frac{\int -\left((bCd-3Ace+aCe+(2cCd-3Bce+2bCe)x)\left(cex^3+(cd+be)x^2+(bd+ae)x+ad\right)^p\right)dx}{3ce}+\frac{C\left(x(ae+bd)+ad+x^2(be+cd)+cex^3\right)^{p+1}}{3ce(p+1)}$$

$$\Big\downarrow \text{25}$$

$$\frac{C\left(x(ae+bd)+ad+x^2(be+cd)+cex^3\right)^{p+1}}{3ce(p+1)}-\frac{\int (bCd-3Ace+aCe+(2cCd-3Bce+2bCe)x)\left(cex^3+(cd+be)x^2+(bd+ae)x+ad\right)^p\,dx}{3ce}$$

$$\Big\downarrow \text{2490}$$

$$\frac{C\left(x(ae+bd)+ad+x^2(be+cd)+cex^3\right)^{p+1}}{3ce(p+1)}-$$

$$\frac{\int \left(\frac{3ce(bCd-3Ace+aCe)-(cd+be)(2cCd-3Bce+2bCe)}{3ce}+(2cCd-3Bce+2bCe)\left(\frac{cd+be}{3ce}+x\right)\right)\left(ce\left(\frac{cd+be}{3ce}+x\right)^3+\frac{(3ce(bd+}{}\right.}{3ce}$$

$\downarrow$ 7293

$$\frac{C\left(x(ae+bd)+ad+x^2(be+cd)+cex^3\right)^{p+1}}{3ce(p+1)}-$$

$$\int \left(\frac{\left(-\left(\left(2Cd^2-3e(Bd-3Ae)\right)c^2\right)-e(bCd-3bBe-3aCe)c-2b^2Ce^2\right)\left(ce\left(\frac{cd+be}{3ce}+x\right)^3+\frac{\left(3ce(bd+ae)-(cd+be)^2\right)\left(\frac{cd+be}{3ce}+x\right)}{3ce}+\frac{(2cd-be)\left(c^2d^2-2b^2e\right.}{27c^2e^2}\right.}{3ce}\right.$$

$\downarrow$ 7293

$$\frac{C\left(x(ae+bd)+ad+x^2(be+cd)+cex^3\right)^{p+1}}{3ce(p+1)}-$$

$$\int \left(\frac{3^{-3p-1}\left(-\left(\left(2Cd^2-3Bed+9Ae^2\right)c^2\right)-e(bCd-3bBe-3aCe)c-2b^2Ce^2\right)\left(\frac{\left(2cd-be+3ce\left(\frac{cd+be}{3ce}+x\right)\right)\left(c^2d^2-2b^2e^2+9c^2e^2\left(\frac{cd+be}{3ce}+x\right)^2-ce(bd-9\right.}{c^2e^2}\right.}{ce}\right.$$

$\downarrow$ 7292

$$\frac{C\left(x(ae+bd)+ad+x^2(be+cd)+cex^3\right)^{p+1}}{3ce(p+1)}-$$

$$\int \left(\frac{3^{-3p-1}\left(-\left(\left(2Cd^2-3e(Bd-3Ae)\right)c^2\right)-e(bCd-3bBe-3aCe)c-2b^2Ce^2\right)\left(\frac{\left(2cd-be+3ce\left(\frac{cd+be}{3ce}+x\right)\right)\left(c^2d^2-2b^2e^2+9c^2e^2\left(\frac{cd+be}{3ce}+x\right)^2-ce(bd-9\right.}{c^2e^2}\right.}{ce}\right.$$

$\downarrow$ 7293

$$\frac{C\left(x(ae+bd)+ad+x^2(be+cd)+cex^3\right)^{p+1}}{3ce(p+1)}-$$

$$\int \left(\frac{3^{-3p-1}\left(-\left(\left(2Cd^2-3Bed+9Ae^2\right)c^2\right)-e(bCd-3bBe-3aCe)c-2b^2Ce^2\right)\left(\frac{\left(2cd-be+3ce\left(\frac{cd+be}{3ce}+x\right)\right)\left(c^2d^2-2b^2e^2+9c^2e^2\left(\frac{cd+be}{3ce}+x\right)^2-ce(bd-9\right.}{c^2e^2}\right.}{ce}\right.$$

$$\Big\downarrow \text{7292}$$

$$\frac{C\left(x(ae+bd)+ad+x^2(be+cd)+cex^3\right)^{p+1}}{3ce(p+1)}-$$

$$\int \left(\frac{3^{-3p-1}\left(-\left(\left(2Cd^2-3e(Bd-3Ae)\right)c^2\right)-e(bCd-3bBe-3aCe)c-2b^2Ce^2\right)\left(\frac{\left(2cd-be+3ce\left(\frac{cd+be}{3ce}+x\right)\right)\left(c^2d^2-2b^2e^2+9c^2e^2\left(\frac{cd+be}{3ce}+x\right)^2-ce(bd-9\right.}{c^2e^2}\right.}{ce}\right.$$

$$\Big\downarrow \text{7293}$$

$$\frac{C\left(x(ae+bd)+ad+x^2(be+cd)+cex^3\right)^{p+1}}{3ce(p+1)}-$$

$$\int \left(\frac{3^{-3p-1}\left(-\left(\left(2Cd^2-3Bed+9Ae^2\right)c^2\right)-e(bCd-3bBe-3aCe)c-2b^2Ce^2\right)\left(\frac{\left(2cd-be+3ce\left(\frac{cd+be}{3ce}+x\right)\right)\left(c^2d^2-2b^2e^2+9c^2e^2\left(\frac{cd+be}{3ce}+x\right)^2-ce(bd-9c\right.}{c^2e^2}\right.}{ce}\right.$$

$$\Big\downarrow \text{7292}$$

$$\frac{C\left(x(ae+bd)+ad+x^2(be+cd)+cex^3\right)^{p+1}}{3ce(p+1)}-$$

$$\int \left(\frac{3^{-3p-1}\left(-\left(\left(2Cd^2-3e(Bd-3Ae)\right)c^2\right)-e(bCd-3bBe-3aCe)c-2b^2Ce^2\right)\left(\frac{\left(2cd-be+3ce\left(\frac{cd+be}{3ce}+x\right)\right)\left(c^2d^2-2b^2e^2+9c^2e^2\left(\frac{cd+be}{3ce}+x\right)^2-ce(bd-9\right.}{c^2e^2}\right.}{ce}\right.$$

$$\Big\downarrow \text{7293}$$

$$\frac{C\left(x(ae+bd)+ad+x^2(be+cd)+cex^3\right)^{p+1}}{3ce(p+1)}-$$

$$\int \left(\frac{3^{-3p-1}\left(-\left(\left(2Cd^2-3Bed+9Ae^2\right)c^2\right)-e(bCd-3bBe-3aCe)c-2b^2Ce^2\right)\left(\frac{\left(2cd-be+3ce\left(\frac{cd+be}{3ce}+x\right)\right)\left(c^2d^2-2b^2e^2+9c^2e^2\left(\frac{cd+be}{3ce}+x\right)^2-ce(bd-9c\right.}{c^2e^2}\right.}{ce}\right.$$

$$\Big\downarrow \text{7292}$$

$$\frac{C\left(x(ae+bd)+ad+x^2(be+cd)+cex^3\right)^{p+1}}{3ce(p+1)}-$$

$$\int \left(\frac{3^{-3p-1}\left(-\left(\left(2Cd^2-3e(Bd-3Ae)\right)c^2\right)-e(bCd-3bBe-3aCe)c-2b^2Ce^2\right)\left(\frac{\left(2cd-be+3ce\left(\frac{cd+be}{3ce}+x\right)\right)\left(c^2d^2-2b^2e^2+9c^2e^2\left(\frac{cd+be}{3ce}+x\right)^2-ce(bd-9\right.}{c^2e^2}\right.}{ce}\right.$$

$\downarrow$ 7293

$$\frac{C\left(x(ae+bd)+ad+x^2(be+cd)+cex^3\right)^{p+1}}{3ce(p+1)}-$$

$$\int \left(\frac{3^{-3p-1}\left(-\left(\left(2Cd^2-3Bed+9Ae^2\right)c^2\right)-e(bCd-3bBe-3aCe)c-2b^2Ce^2\right)\left(\frac{\left(2cd-be+3ce\left(\frac{cd+be}{3ce}+x\right)\right)\left(c^2d^2-2b^2e^2+9c^2e^2\left(\frac{cd+be}{3ce}+x\right)^2-ce(bd-9e\right.}{c^2e^2}\right.}{ce}\right.$$

$\downarrow$ 7292

$$\frac{C\left(x(ae+bd)+ad+x^2(be+cd)+cex^3\right)^{p+1}}{3ce(p+1)}-$$

$$\int \left(\frac{3^{-3p-1}\left(-\left(\left(2Cd^2-3e(Bd-3Ae)\right)c^2\right)-e(bCd-3bBe-3aCe)c-2b^2Ce^2\right)\left(\frac{\left(2cd-be+3ce\left(\frac{cd+be}{3ce}+x\right)\right)\left(c^2d^2-2b^2e^2+9c^2e^2\left(\frac{cd+be}{3ce}+x\right)^2-ce(bd-9\right.}{c^2e^2}\right.}{ce}\right.$$

$\downarrow$ 7293

$$\frac{C\left(x(ae+bd)+ad+x^2(be+cd)+cex^3\right)^{p+1}}{3ce(p+1)}-$$

$$\int \left(\frac{3^{-3p-1}\left(-\left(\left(2Cd^2-3Bed+9Ae^2\right)c^2\right)-e(bCd-3bBe-3aCe)c-2b^2Ce^2\right)\left(\frac{\left(2cd-be+3ce\left(\frac{cd+be}{3ce}+x\right)\right)\left(c^2d^2-2b^2e^2+9c^2e^2\left(\frac{cd+be}{3ce}+x\right)^2-ce(bd-9e\right.}{c^2e^2}\right.}{ce}\right.$$

$\downarrow$ 7292

$$\frac{C\left(x(ae+bd)+ad+x^2(be+cd)+cex^3\right)^{p+1}}{3ce(p+1)}-$$

$$\int \left(\frac{3^{-3p-1}\left(-\left(\left(2Cd^2-3e(Bd-3Ae)\right)c^2\right)-e(bCd-3bBe-3aCe)c-2b^2Ce^2\right)\left(\frac{\left(2cd-be+3ce\left(\frac{cd+be}{3ce}+x\right)\right)\left(c^2d^2-2b^2e^2+9c^2e^2\left(\frac{cd+be}{3ce}+x\right)^2-ce(bd-9\right.}{c^2e^2}\right.}{ce}\right.$$

$\downarrow$ 7293

$$\frac{C\left(x(ae+bd)+ad+x^2(be+cd)+cex^3\right)^{p+1}}{3ce(p+1)}-$$

$$\int \left(\frac{3^{-3p-1}\left(-\left(\left(2Cd^2-3Bed+9Ae^2\right)c^2\right)-e(bCd-3bBe-3aCe)c-2b^2Ce^2\right)\left(\frac{\left(2cd-be+3ce\left(\frac{cd+be}{3ce}+x\right)\right)\left(c^2d^2-2b^2e^2+9c^2e^2\left(\frac{cd+be}{3ce}+x\right)^2-ce(bd-9\right.}{c^2e^2}\right.}{ce}\right.$$

$\downarrow$ 7292

$$\frac{C\left(x(ae+bd)+ad+x^2(be+cd)+cex^3\right)^{p+1}}{3ce(p+1)}-$$

$$\int \left(\frac{3^{-3p-1}\left(-\left(\left(2Cd^2-3e(Bd-3Ae)\right)c^2\right)-e(bCd-3bBe-3aCe)c-2b^2Ce^2\right)\left(\frac{\left(2cd-be+3ce\left(\frac{cd+be}{3ce}+x\right)\right)\left(c^2d^2-2b^2e^2+9c^2e^2\left(\frac{cd+be}{3ce}+x\right)^2-ce(bd-9\right.}{c^2e^2}\right.}{ce}\right.$$

$\downarrow$ 7293

$$\frac{C\left(x(ae+bd)+ad+x^2(be+cd)+cex^3\right)^{p+1}}{3ce(p+1)}-$$

$$\int \left(\frac{3^{-3p-1}\left(-\left(\left(2Cd^2-3Bed+9Ae^2\right)c^2\right)-e(bCd-3bBe-3aCe)c-2b^2Ce^2\right)\left(\frac{\left(2cd-be+3ce\left(\frac{cd+be}{3ce}+x\right)\right)\left(c^2d^2-2b^2e^2+9c^2e^2\left(\frac{cd+be}{3ce}+x\right)^2-ce(bd-9\right.}{c^2e^2}\right.}{ce}\right.$$

$\downarrow$ 7292

$$\frac{C\left(x(ae+bd)+ad+x^2(be+cd)+cex^3\right)^{p+1}}{3ce(p+1)}-$$

$$\int\left(\frac{3^{-3p-1}\left(-\left(\left(2Cd^2-3e(Bd-3Ae)\right)c^2\right)-e(bCd-3bBe-3aCe)c-2b^2Ce^2\right)\left(\frac{\left(2cd-be+3ce\left(\frac{cd+be}{3ce}+x\right)\right)\left(c^2d^2-2b^2e^2+9c^2e^2\left(\frac{cd+be}{3ce}+x\right)^2-ce(bd-9\right.}{c^2e^2}\right.}{ce}\right.$$

$$\Big\downarrow \; 7293$$

$$\frac{C\left(x(ae+bd)+ad+x^2(be+cd)+cex^3\right)^{p+1}}{3ce(p+1)}-$$

$$\int\left(\frac{3^{-3p-1}\left(-\left(\left(2Cd^2-3Bed+9Ae^2\right)c^2\right)-e(bCd-3bBe-3aCe)c-2b^2Ce^2\right)\left(\frac{\left(2cd-be+3ce\left(\frac{cd+be}{3ce}+x\right)\right)\left(c^2d^2-2b^2e^2+9c^2e^2\left(\frac{cd+be}{3ce}+x\right)^2-ce(bd-9\right.}{c^2e^2}\right.}{ce}\right.$$

$$\Big\downarrow \; 7292$$

$$\frac{C\left(x(ae+bd)+ad+x^2(be+cd)+cex^3\right)^{p+1}}{3ce(p+1)}-$$

$$\int\left(\frac{3^{-3p-1}\left(-\left(\left(2Cd^2-3e(Bd-3Ae)\right)c^2\right)-e(bCd-3bBe-3aCe)c-2b^2Ce^2\right)\left(\frac{\left(2cd-be+3ce\left(\frac{cd+be}{3ce}+x\right)\right)\left(c^2d^2-2b^2e^2+9c^2e^2\left(\frac{cd+be}{3ce}+x\right)^2-ce(bd-9\right.}{c^2e^2}\right.}{ce}\right.$$

$$\Big\downarrow \; 7293$$

$$\frac{C\left(x(ae+bd)+ad+x^2(be+cd)+cex^3\right)^{p+1}}{3ce(p+1)}-$$

$$\int\left(\frac{3^{-3p-1}\left(-\left(\left(2Cd^2-3Bed+9Ae^2\right)c^2\right)-e(bCd-3bBe-3aCe)c-2b^2Ce^2\right)\left(\frac{\left(2cd-be+3ce\left(\frac{cd+be}{3ce}+x\right)\right)\left(c^2d^2-2b^2e^2+9c^2e^2\left(\frac{cd+be}{3ce}+x\right)^2-ce(bd-9\right.}{c^2e^2}\right.}{ce}\right.$$

$$\Big\downarrow \; 7292$$

$$\frac{C\left(x(ae+bd)+ad+x^2(be+cd)+cex^3\right)^{p+1}}{3ce(p+1)}-$$
$$\int\left(\frac{3^{-3p-1}\left(-\left(\left(2Cd^2-3e(Bd-3Ae)\right)c^2\right)-e(bCd-3bBe-3aCe)c-2b^2Ce^2\right)\left(\frac{\left(2cd-be+3ce\left(\frac{cd+be}{3ce}+x\right)\right)\left(c^2d^2-2b^2e^2+9c^2e^2\left(\frac{cd+be}{3ce}+x\right)^2-ce(bd-9\right.}{c^2e^2}\right.}{ce}\right.$$

$\downarrow$ 7293

$$\frac{C\left(x(ae+bd)+ad+x^2(be+cd)+cex^3\right)^{p+1}}{3ce(p+1)}-$$
$$\int\left(\frac{3^{-3p-1}\left(-\left(\left(2Cd^2-3Bed+9Ae^2\right)c^2\right)-e(bCd-3bBe-3aCe)c-2b^2Ce^2\right)\left(\frac{\left(2cd-be+3ce\left(\frac{cd+be}{3ce}+x\right)\right)\left(c^2d^2-2b^2e^2+9c^2e^2\left(\frac{cd+be}{3ce}+x\right)^2-ce(bd-9c\right.}{c^2e^2}\right.}{ce}\right.$$

$\downarrow$ 7292

$$\frac{C\left(x(ae+bd)+ad+x^2(be+cd)+cex^3\right)^{p+1}}{3ce(p+1)}-$$
$$\int\left(\frac{3^{-3p-1}\left(-\left(\left(2Cd^2-3e(Bd-3Ae)\right)c^2\right)-e(bCd-3bBe-3aCe)c-2b^2Ce^2\right)\left(\frac{\left(2cd-be+3ce\left(\frac{cd+be}{3ce}+x\right)\right)\left(c^2d^2-2b^2e^2+9c^2e^2\left(\frac{cd+be}{3ce}+x\right)^2-ce(bd-9\right.}{c^2e^2}\right.}{ce}\right.$$

$\downarrow$ 7293

$$\frac{C\left(x(ae+bd)+ad+x^2(be+cd)+cex^3\right)^{p+1}}{3ce(p+1)}-$$
$$\int\left(\frac{3^{-3p-1}\left(-\left(\left(2Cd^2-3Bed+9Ae^2\right)c^2\right)-e(bCd-3bBe-3aCe)c-2b^2Ce^2\right)\left(\frac{\left(2cd-be+3ce\left(\frac{cd+be}{3ce}+x\right)\right)\left(c^2d^2-2b^2e^2+9c^2e^2\left(\frac{cd+be}{3ce}+x\right)^2-ce(bd-9c\right.}{c^2e^2}\right.}{ce}\right.$$

$\downarrow$ 7292

$$\frac{C\left(x(ae+bd)+ad+x^2(be+cd)+cex^3\right)^{p+1}}{3ce(p+1)} -$$

$$\int \left(\frac{3^{-3p-1}\left(-\left(\left(2Cd^2-3e(Bd-3Ae)\right)c^2\right)-e(bCd-3bBe-3aCe)c-2b^2Ce^2\right)\left(\frac{\left(2cd-be+3ce\left(\frac{cd+be}{3ce}+x\right)\right)\left(c^2d^2-2b^2e^2+9c^2e^2\left(\frac{cd+be}{3ce}+x\right)^2-ce(bd-9 \cdots}{c^2e^2}\right.}{ce} \right.$$

$$\Big\downarrow \; 7293$$

$$\frac{C\left(x(ae+bd)+ad+x^2(be+cd)+cex^3\right)^{p+1}}{3ce(p+1)} -$$

$$\int \left(\frac{3^{-3p-1}\left(-\left(\left(2Cd^2-3Bed+9Ae^2\right)c^2\right)-e(bCd-3bBe-3aCe)c-2b^2Ce^2\right)\left(\frac{\left(2cd-be+3ce\left(\frac{cd+be}{3ce}+x\right)\right)\left(c^2d^2-2b^2e^2+9c^2e^2\left(\frac{cd+be}{3ce}+x\right)^2-ce(bd-9c \cdots}{c^2e^2}\right.}{ce} \right.$$

$$\Big\downarrow \; 7292$$

$$\frac{C\left(x(ae+bd)+ad+x^2(be+cd)+cex^3\right)^{p+1}}{3ce(p+1)} -$$

$$\int \left(\frac{3^{-3p-1}\left(-\left(\left(2Cd^2-3e(Bd-3Ae)\right)c^2\right)-e(bCd-3bBe-3aCe)c-2b^2Ce^2\right)\left(\frac{\left(2cd-be+3ce\left(\frac{cd+be}{3ce}+x\right)\right)\left(c^2d^2-2b^2e^2+9c^2e^2\left(\frac{cd+be}{3ce}+x\right)^2-ce(bd-9 \cdots}{c^2e^2}\right.}{ce} \right.$$

input
```
Int[(A + B*x + C*x^2)*(a*d + (b*d + a*e)*x + (c*d + b*e)*x^2 + c*e*x^3)^p,
x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2490
```
Int[(P3_)^(p_.)*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] :> With[{a = Coeff[P3
, x, 0], b = Coeff[P3, x, 1], c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Su
bst[Int[((3*d*e - c*f)/(3*d) + f*x)^m*Simp[(2*c^3 - 9*b*c*d + 27*a*d^2)/(27
*d^2) - (c^2 - 3*b*d)*(x/(3*d)) + d*x^3, x]^p, x], x, x + c/(3*d)] /; NeQ[c
, 0]] /; FreeQ[{e, f, m, p}, x] && PolyQ[P3, x, 3]
```

rule 2526
```
Int[(Pm_)*(Qn_)^(p_), x_Symbol] :> With[{m = Expon[Pm, x], n = Expon[Qn, x]
}, Simp[Coeff[Pm, x, m]*(Qn^(p + 1)/(n*(p + 1)*Coeff[Qn, x, n])), x] + Simp
[1/(n*Coeff[Qn, x, n])   Int[ExpandToSum[n*Coeff[Qn, x, n]*Pm - Coeff[Pm, x
, m]*D[Qn, x], x]*Qn^p, x], x] /; EqQ[m, n - 1]] /; FreeQ[p, x] && PolyQ[Pm
, x] && PolyQ[Qn, x] && NeQ[p, -1]
```

rule 7292
```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [F]

$$\int \left(C\,x^{2}+Bx+A\right)\left(ad+\left(ae+bd\right)x+\left(eb+cd\right)x^{2}+ce\,x^{3}\right)^{p}dx$$

input
```
int((C*x^2+B*x+A)*(a*d+(a*e+b*d)*x+(b*e+c*d)*x^2+c*e*x^3)^p,x)
```

output
```
int((C*x^2+B*x+A)*(a*d+(a*e+b*d)*x+(b*e+c*d)*x^2+c*e*x^3)^p,x)
```

Fricas [F]

$$\int (A+Bx+Cx^2)\left(ad+(bd+ae)x+(cd+be)x^2+cex^3\right)^p\,dx$$
$$=\int (Cx^2+Bx+A)\left(cex^3+(cd+be)x^2+ad+(bd+ae)x\right)^p\,dx$$

input
```
integrate((C*x^2+B*x+A)*(a*d+(a*e+b*d)*x+(b*e+c*d)*x^2+c*e*x^3)^p,x, algor
ithm="fricas")
```

output
```
integral((C*x^2 + B*x + A)*(c*e*x^3 + (c*d + b*e)*x^2 + a*d + (b*d + a*e)*
x)^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int (A+Bx+Cx^2)\left(ad+(bd+ae)x+(cd+be)x^2+cex^3\right)^p\,dx=\text{Timed out}$$

input
```
integrate((C*x**2+B*x+A)*(a*d+(a*e+b*d)*x+(b*e+c*d)*x**2+c*e*x**3)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (A+Bx+Cx^2)\left(ad+(bd+ae)x+(cd+be)x^2+cex^3\right)^p\,dx$$
$$=\int (Cx^2+Bx+A)\left(cex^3+(cd+be)x^2+ad+(bd+ae)x\right)^p\,dx$$

input
```
integrate((C*x^2+B*x+A)*(a*d+(a*e+b*d)*x+(b*e+c*d)*x^2+c*e*x^3)^p,x, algor
ithm="maxima")
```

output
```
integrate((C*x^2 + B*x + A)*(c*e*x^3 + (c*d + b*e)*x^2 + a*d + (b*d + a*e)
*x)^p, x)
```

Giac [F]

$$\int (A+Bx+Cx^2)\left(ad+(bd+ae)x+(cd+be)x^2+cex^3\right)^p\,dx$$
$$=\int (Cx^2+Bx+A)\left(cex^3+(cd+be)x^2+ad+(bd+ae)x\right)^p dx$$

input
```
integrate((C*x^2+B*x+A)*(a*d+(a*e+b*d)*x+(b*e+c*d)*x^2+c*e*x^3)^p,x, algor
ithm="giac")
```

output
```
integrate((C*x^2 + B*x + A)*(c*e*x^3 + (c*d + b*e)*x^2 + a*d + (b*d + a*e)
*x)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (A+Bx+Cx^2)\left(ad+(bd+ae)x+(cd+be)x^2+cex^3\right)^p\,dx$$
$$=\int \left(C\,x^2+B\,x+A\right)\,\left(c\,e\,x^3+\left(b\,e+c\,d\right)\,x^2+\left(a\,e+b\,d\right)\,x+a\,d\right)^p\,dx$$

input
```
int((A + B*x + C*x^2)*(a*d + x*(a*e + b*d) + x^2*(b*e + c*d) + c*e*x^3)^p,
x)
```

output
```
int((A + B*x + C*x^2)*(a*d + x*(a*e + b*d) + x^2*(b*e + c*d) + c*e*x^3)^p,
 x)
```

Reduce [F]

$$\int (A + Bx + Cx^2) \left(ad + (bd + ae)x + (cd + be)x^2 + cex^3\right)^p \, dx = \text{too large to display}$$

input
```
int((C*x^2+B*x+A)*(a*d+(a*e+b*d)*x+(b*e+c*d)*x^2+c*e*x^3)^p,x)
```

output
```
(12*(a*d + a*e*x + b*d*x + b*e*x**2 + c*d*x**2 + c*e*x**3)**p*a**2*c*e**3*
p**2 + 20*(a*d + a*e*x + b*d*x + b*e*x**2 + c*d*x**2 + c*e*x**3)**p*a**2*c
*e**3*p + 8*(a*d + a*e*x + b*d*x + b*e*x**2 + c*d*x**2 + c*e*x**3)**p*a**2
*c*e**3 - (a*d + a*e*x + b*d*x + b*e*x**2 + c*d*x**2 + c*e*x**3)**p*a*b**2
*e**3*p**2 - 2*(a*d + a*e*x + b*d*x + b*e*x**2 + c*d*x**2 + c*e*x**3)**p*a
*b**2*e**3*p - (a*d + a*e*x + b*d*x + b*e*x**2 + c*d*x**2 + c*e*x**3)**p*a
*b**2*e**3 + 34*(a*d + a*e*x + b*d*x + b*e*x**2 + c*d*x**2 + c*e*x**3)**p*
a*b*c*d*e**2*p**2 + 42*(a*d + a*e*x + b*d*x + b*e*x**2 + c*d*x**2 + c*e*x*
*3)**p*a*b*c*d*e**2*p + 12*(a*d + a*e*x + b*d*x + b*e*x**2 + c*d*x**2 + c*
e*x**3)**p*a*b*c*d*e**2 + 30*(a*d + a*e*x + b*d*x + b*e*x**2 + c*d*x**2 +
c*e*x**3)**p*a*b*c*e**3*p**2*x + 38*(a*d + a*e*x + b*d*x + b*e*x**2 + c*d*
x**2 + c*e*x**3)**p*a*b*c*e**3*p*x + 12*(a*d + a*e*x + b*d*x + b*e*x**2 +
c*d*x**2 + c*e*x**3)**p*a*b*c*e**3*x + 2*(a*d + a*e*x + b*d*x + b*e*x**2 +
 c*d*x**2 + c*e*x**3)**p*a*c**2*d**2*e*p**2 - 2*(a*d + a*e*x + b*d*x + b*e
*x**2 + c*d*x**2 + c*e*x**3)**p*a*c**2*d**2*e*p + 30*(a*d + a*e*x + b*d*x
+ b*e*x**2 + c*d*x**2 + c*e*x**3)**p*a*c**2*d*e**2*p**2*x + 38*(a*d + a*e*
x + b*d*x + b*e*x**2 + c*d*x**2 + c*e*x**3)**p*a*c**2*d*e**2*p*x + 12*(a*d
 + a*e*x + b*d*x + b*e*x**2 + c*d*x**2 + c*e*x**3)**p*a*c**2*d*e**2*x - (a
*d + a*e*x + b*d*x + b*e*x**2 + c*d*x**2 + c*e*x**3)**p*b**3*d*e**2*p**2 -
 2*(a*d + a*e*x + b*d*x + b*e*x**2 + c*d*x**2 + c*e*x**3)**p*b**3*d*e**...
```

3.134 $\int (e+fx)^2 \left(1-(1-6b)^{3/2}-9b+54bx-54x^2+10$

Optimal result

Integrand size = 38, antiderivative size = 110

$$\int (e+fx)^2 \left(1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3\right) dx = 27be^2x^2 - 18e(e-2bf)x^3 + \frac{27}{2}\left(2e^2-2ef+bf^2\right)x^4 + \frac{54}{5}(4e-f)fx^5 + 18f^2x^6 + \frac{\left(1-\sqrt{1-6b}-\left(9-6\sqrt{1-6b}\right)b\right)(e+fx)^3}{3f}$$

output
```
27*b*e^2*x^2-18*e*(-2*b*f+e)*x^3+27/2*(b*f^2+2*e^2-2*e*f)*x^4+54/5*(4*e-f)
*f*x^5+18*f^2*x^6+1/3*(1-(1-6*b)^(1/2)-(9-6*(1-6*b)^(1/2))*b)*(f*x+e)^3/f
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.35

$$\int (e+fx)^2 \left(1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3\right) dx = \frac{1}{30}x\left(30e^2\left(1-\sqrt{1-6b}-18x^2+27x^3+3b\left(-3+2\sqrt{1-6b}+9x\right)\right) + 6efx\left(5-5\sqrt{1-6b}-135x^2+216x^3+15b\left(-3+2\sqrt{1-6b}+12x\right)\right) + f^2x^2\left(10-10\sqrt{1-6b}-324x^2+540x^3+15b\left(-6+4\sqrt{1-6b}+27x\right)\right)\right)$$

input
```
Integrate[(e + f*x)^2*(1 - (1 - 6*b)^(3/2) - 9*b + 54*b*x - 54*x^2 + 108*x
^3),x]
```

output
```
(x*(30*e^2*(1 - Sqrt[1 - 6*b] - 18*x^2 + 27*x^3 + 3*b*(-3 + 2*Sqrt[1 - 6*b
] + 9*x)) + 6*e*f*x*(5 - 5*Sqrt[1 - 6*b] - 135*x^2 + 216*x^3 + 15*b*(-3 +
2*Sqrt[1 - 6*b] + 12*x)) + f^2*x^2*(10 - 10*Sqrt[1 - 6*b] - 324*x^2 + 540*
x^3 + 15*b*(-6 + 4*Sqrt[1 - 6*b] + 27*x))))/30
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.07, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(54bx-(1-6b)^{3/2}-9b+108x^3-54x^2+1\right)(e+fx)^2dx$$

$$\Big\downarrow \text{2389}$$

$$\int \left(\frac{54\left(bf^2+6e^2+2ef\right)(e+fx)^3}{f^3}+\frac{\left(-54bef^2-(1-6b)^{3/2}f^3-9bf^3-108e^3-54e^2f+f^3\right)(e+fx)^2}{f^3}+10\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{27\left(bf^2+6e^2+2ef\right)(e+fx)^4}{2f^4}-\frac{\left(54bef^2+(1-6b)^{3/2}f^3+9bf^3+108e^3+54e^2f-f^3\right)(e+fx)^3}{3f^4}+\frac{18(e+fx)^6}{f^4}-\frac{54(6e+f)(e+fx)^5}{5f^4}$$

input
```
Int[(e + f*x)^2*(1 - (1 - 6*b)^(3/2) - 9*b + 54*b*x - 54*x^2 + 108*x^3),x]
```

output
```
-1/3*((108*e^3 + 54*e^2*f + 54*b*e*f^2 - f^3 + (1 - 6*b)^(3/2)*f^3 + 9*b*f
^3)*(e + f*x)^3)/f^4 + (27*(6*e^2 + 2*e*f + b*f^2)*(e + f*x)^4)/(2*f^4) -
(54*(6*e + f)*(e + f*x)^5)/(5*f^4) + (18*(e + f*x)^6)/f^4
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2389
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.17

method	result
default	$18f^2x^6+\frac{(216ef-54f^2)x^5}{5}+\frac{(54b f^2+108e^2-108ef)x^4}{4}+\frac{\left(-54e^2+108bef+f^2\left(1-(1-6b)^{\frac{3}{2}}-9b\right)\right)x^3}{3}+\frac{\left(54b e^2+\right.}{}$
norman	$\left(\frac{216}{5}ef-\frac{54}{5}f^2\right)x^5+\left(\frac{27}{2}b f^2+27e^2-27ef\right)x^4+\left(2\sqrt{1-6b}\, b f^2+36bef-18e^2-\frac{\sqrt{1-6b}}{3}\right.$
parallelrisch	$18f^2x^6+\frac{216x^5ef}{5}-\frac{54f^2x^5}{5}+\frac{27x^4b f^2}{2}+27e^2x^4-27x^4ef+2x^3\sqrt{1-6b}\, b f^2+36bef x^3-18$
gosper	$-\frac{x\left(540f^2x^5+60\sqrt{1-6b}\, b f^2x^2+405b f^2x^3+1296x^4ef-324x^4f^2+180\sqrt{1-6b}\, befx-10\sqrt{1-6b}\, f^2x^2+1080bef x^2-90b f^2x^2\right.}{3}$

input
```
int((f*x+e)^2*(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3),x,method=_RETURN
VERBOSE)
```

output
```
18*f^2*x^6+1/5*(216*e*f-54*f^2)*x^5+1/4*(54*b*f^2+108*e^2-108*e*f)*x^4+1/3
*(-54*e^2+108*b*e*f+f^2*(1-(1-6*b)^(3/2)-9*b))*x^3+1/2*(54*b*e^2+2*e*f*(1-
(1-6*b)^(3/2)-9*b))*x^2+x*e^2*(1-(1-6*b)^(3/2)-9*b)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.33

$$\int (e+fx)^2 \left(1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3\right) \, dx = 18\,f^2x^6 + \frac{54}{5}\,\left(4\,ef-f^2\right)x^5 + \frac{27}{2}\,\left(bf^2+2\,e^2-2\,ef\right)x^4 - (9\,b-1)e^2x + \frac{1}{3}\,\left(108\,bef-(9\,b-1)f^2-54\,e^2\right)x^3 + \left(27\,be^2-(9\,b-1)ef\right)x^2 + \frac{1}{3}\,\left((6\,b-1)f^2x^3+3\,(6\,b-1)efx^2+3\,(6\,b-1)e^2x\right)\sqrt{-6\,b+1}$$

input
```
integrate((f*x+e)^2*(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3),x, algorit
hm="fricas")
```

output
```
18*f^2*x^6 + 54/5*(4*e*f - f^2)*x^5 + 27/2*(b*f^2 + 2*e^2 - 2*e*f)*x^4 - (
9*b - 1)*e^2*x + 1/3*(108*b*e*f - (9*b - 1)*f^2 - 54*e^2)*x^3 + (27*b*e^2
- (9*b - 1)*e*f)*x^2 + 1/3*((6*b - 1)*f^2*x^3 + 3*(6*b - 1)*e*f*x^2 + 3*(6
*b - 1)*e^2*x)*sqrt(-6*b + 1)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.72

$$\int (e+fx)^2 \left(1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3\right) \, dx = 18f^2x^6 + x^5 \cdot \left(\frac{216ef}{5}-\frac{54f^2}{5}\right) + x^4 \cdot \left(\frac{27bf^2}{2}+27e^2-27ef\right) + x^3 \cdot \left(36bef+2bf^2\sqrt{1-6b}-3bf^2-18e^2-\frac{f^2\sqrt{1-6b}}{3}+\frac{f^2}{3}\right) + x^2 \cdot \left(27be^2+6bef\sqrt{1-6b}-9bef-ef\sqrt{1-6b}+ef\right) + x\left(6be^2\sqrt{1-6b}-9be^2-e^2\sqrt{1-6b}+e^2\right)$$

input
```
integrate((f*x+e)**2*(1-(1-6*b)**(3/2)-9*b+54*b*x-54*x**2+108*x**3),x)
```

output
```
18*f**2*x**6 + x**5*(216*e*f/5 - 54*f**2/5) + x**4*(27*b*f**2/2 + 27*e**2
- 27*e*f) + x**3*(36*b*e*f + 2*b*f**2*sqrt(1 - 6*b) - 3*b*f**2 - 18*e**2 -
 f**2*sqrt(1 - 6*b)/3 + f**2/3) + x**2*(27*b*e**2 + 6*b*e*f*sqrt(1 - 6*b)
- 9*b*e*f - e*f*sqrt(1 - 6*b) + e*f) + x*(6*b*e**2*sqrt(1 - 6*b) - 9*b*e**
2 - e**2*sqrt(1 - 6*b) + e**2)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.11

$$\int (e+fx)^2\left(1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3\right)\,dx=18\,f^2x^6+\frac{54}{5}\left(4\,ef-f^2\right)x^5+\frac{27}{2}\left(bf^2+2\,e^2-2\,ef\right)x^4-\left(\left(-6\,b+1\right)^{\frac{3}{2}}+9\,b-1\right)e^2x+\frac{1}{3}\left(108\,bef-\left(\left(-6\,b+1\right)^{\frac{3}{2}}+9\,b-1\right)f^2-54\,e^2\right)x^3+\left(27\,be^2-\left(\left(-6\,b+1\right)^{\frac{3}{2}}+9\,b-1\right)ef\right)x^2$$

input
```
integrate((f*x+e)^2*(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3),x, algorit
hm="maxima")
```

output
```
18*f^2*x^6 + 54/5*(4*e*f - f^2)*x^5 + 27/2*(b*f^2 + 2*e^2 - 2*e*f)*x^4 - (
(-6*b + 1)^(3/2) + 9*b - 1)*e^2*x + 1/3*(108*b*e*f - ((-6*b + 1)^(3/2) + 9
*b - 1)*f^2 - 54*e^2)*x^3 + (27*b*e^2 - ((-6*b + 1)^(3/2) + 9*b - 1)*e*f)*
x^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.38

$$\int (e+fx)^2\left(1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3\right)\,dx=18\,f^2x^6+\frac{27}{2}\,bf^2x^4+\frac{216}{5}\,efx^5-\frac{54}{5}\,f^2x^5+36\,befx^3+\frac{1}{3}\left(6\,b\sqrt{-6\,b+1}-9\,b-\sqrt{-6\,b+1}+1\right)f^2x^3+27\,e^2x^4-27\,efx^4+27\,be^2x^2+\left(\left(6\,b-1\right)\sqrt{-6\,b+1}-9\,b+1\right)efx^2-18\,e^2x^3+\left(\left(6\,b-1\right)\sqrt{-6\,b+1}-9\,b+1\right)e^2x$$

input
```
integrate((f*x+e)^2*(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3),x, algorit
hm="giac")
```

output
```
18*f^2*x^6 + 27/2*b*f^2*x^4 + 216/5*e*f*x^5 - 54/5*f^2*x^5 + 36*b*e*f*x^3
+ 1/3*(6*b*sqrt(-6*b + 1) - 9*b - sqrt(-6*b + 1) + 1)*f^2*x^3 + 27*e^2*x^4
 - 27*e*f*x^4 + 27*b*e^2*x^2 + ((6*b - 1)*sqrt(-6*b + 1) - 9*b + 1)*e*f*x^
2 - 18*e^2*x^3 + ((6*b - 1)*sqrt(-6*b + 1) - 9*b + 1)*e^2*x
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.13

$$\int (e+fx)^2\left(1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3\right)\,dx=x^4\left(27\,e^2-27\,e\,f+\frac{27\,b\,f^2}{2}\right)-x^3\left(3\,b\,f^2+\frac{f^2\,{\left(1-6\,b\right)}^{3/2}}{3}+18\,e^2-\frac{f^2}{3}-36\,b\,e\,f\right)+18\,f^2\,x^6-e^2\,x\left(9\,b+{\left(1-6\,b\right)}^{3/2}-1\right)+\frac{54\,f\,x^5\,\left(4\,e-f\right)}{5}+e\,x^2\left(f+27\,b\,e-9\,b\,f-f\,{\left(1-6\,b\right)}^{3/2}\right)$$

input
```
int(-(e + f*x)^2*(9*b - 54*b*x + (1 - 6*b)^(3/2) + 54*x^2 - 108*x^3 - 1),x
)
```

output
```
x^4*((27*b*f^2)/2 - 27*e*f + 27*e^2) - x^3*(3*b*f^2 + (f^2*(1 - 6*b)^(3/2)
)/3 + 18*e^2 - f^2/3 - 36*b*e*f) + 18*f^2*x^6 - e^2*x*(9*b + (1 - 6*b)^(3/
2) - 1) + (54*f*x^5*(4*e - f))/5 + e*x^2*(f + 27*b*e - 9*b*f - f*(1 - 6*b)
^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.71

$$\int (e+fx)^2 \left(1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3\right) dx = \frac{x\left(180\sqrt{-6b+1}\,b\,e^2+180\sqrt{-6b+1}\,befx+60\sqrt{-6b+1}\,b\,f^2x^2-30\sqrt{-6b+1}\,e^2-30\sqrt{-6\right.}{}$$

input
```
int((f*x+e)^2*(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3),x)
```

output
```
(x*(180*sqrt( - 6*b + 1)*b*e**2 + 180*sqrt( - 6*b + 1)*b*e*f*x + 60*sqrt(
- 6*b + 1)*b*f**2*x**2 - 30*sqrt( - 6*b + 1)*e**2 - 30*sqrt( - 6*b + 1)*e*
f*x - 10*sqrt( - 6*b + 1)*f**2*x**2 + 810*b*e**2*x - 270*b*e**2 + 1080*b*e
*f*x**2 - 270*b*e*f*x + 405*b*f**2*x**3 - 90*b*f**2*x**2 + 810*e**2*x**3 -
 540*e**2*x**2 + 30*e**2 + 1296*e*f*x**4 - 810*e*f*x**3 + 30*e*f*x + 540*f
**2*x**5 - 324*f**2*x**4 + 10*f**2*x**2))/30
```

3.135 $\int (e+fx) \left(1-(1-6b)^{3/2}-9b+54bx-54x^2+10\right.$

Optimal result

Integrand size = 36, antiderivative size = 108

$$\int (e+fx) \left(1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3\right) dx = \left(1-\sqrt{1-6b}-\left(9-6\sqrt{1-6b}\right)b\right)ex + \frac{1}{2}\left(f-\sqrt{1-6b}f+b\left(54e-9f+6\sqrt{1-6b}f\right)\right)x^2 - 18(e-bf)x^3 + \frac{27}{2}(2e-f)x^4 + \frac{108fx^5}{5}$$

output
```
(1-(1-6*b)^(1/2)-(9-6*(1-6*b)^(1/2))*b)*e*x+1/2*(f-(1-6*b)^(1/2)*f+b*(54*e
-9*f+6*(1-6*b)^(1/2)*f))*x^2-18*(-b*f+e)*x^3+27/2*(2*e-f)*x^4+108/5*f*x^5
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.89

$$\int (e+fx) \left(1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3\right) dx = ex\left(1-\sqrt{1-6b}-18x^2+27x^3+3b\left(-3+2\sqrt{1-6b}+9x\right)\right) + \frac{1}{10}fx^2\left(5-5\sqrt{1-6b}-135x^2+216x^3+15b\left(-3+2\sqrt{1-6b}+12x\right)\right)$$

input
```
Integrate[(e + f*x)*(1 - (1 - 6*b)^(3/2) - 9*b + 54*b*x - 54*x^2 + 108*x^3
),x]
```

output
```
e*x*(1 - Sqrt[1 - 6*b] - 18*x^2 + 27*x^3 + 3*b*(-3 + 2*Sqrt[1 - 6*b] + 9*x
)) + (f*x^2*(5 - 5*Sqrt[1 - 6*b] - 135*x^2 + 216*x^3 + 15*b*(-3 + 2*Sqrt[1
 - 6*b] + 12*x)))/10
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.76, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {2389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(54bx-(1-6b)^{3/2}-9b+108x^3-54x^2+1\right)(e+fx)\,dx$$

$$\Big\downarrow \; 2389$$

$$\int \left(-54x^2(e-bf)+x\left(54be+(1-6b)^{3/2}(-f)-9bf+f\right)-\left(\left((1-6b)^{3/2}+9b-1\right)e+54x^3(2e-f)+108fx\right.\right.$$

$$\Big\downarrow \; 2009$$

$$-18x^3(e-bf)+\frac{1}{2}x^2\left(54be+(1-6b)^{3/2}(-f)-9bf+f\right)+\left(-(1-6b)^{3/2}-9b+1\right)ex+\frac{27}{2}x^4(2e-f)+\frac{108fx^5}{5}$$

input
```
Int[(e + f*x)*(1 - (1 - 6*b)^(3/2) - 9*b + 54*b*x - 54*x^2 + 108*x^3),x]
```

output
```
(1 - (1 - 6*b)^(3/2) - 9*b)*e*x + ((54*b*e + f - (1 - 6*b)^(3/2)*f - 9*b*f
)*x^2)/2 - 18*(e - b*f)*x^3 + (27*(2*e - f)*x^4)/2 + (108*f*x^5)/5
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2389
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.70

method	result
default	$\frac{108 f \,x^{5}}{5}+\frac{\left(108 e -54 f \right) x^{4}}{4}+\frac{\left(54 b f -54 e \right) x^{3}}{3}+\frac{\left(54 e b +f \left(1-\left(1-6 b \right)^{\frac{3}{2}}-9 b \right)\right) x^{2}}{2}+x e \left(1-\left(1-6 b \right)^{\frac{3}{2}}-9 b \right)$
norman	$\left(27 e -\frac{27 f}{2}\right) x^{4}+\left(3 \sqrt{1-6 b}\, f b +27 e b -\frac{\sqrt{1-6 b}\, f}{2}-\frac{9 b f}{2}+\frac{f}{2}\right) x^{2}+\left(18 b f -18 e \right) x^{3}+e \left(6 \sqrt{\right.}$
parallelrisch	$\frac{108 f \,x^{5}}{5}+27 x^{4} e -\frac{27 f \,x^{4}}{2}+18 x^{3} b f -18 e \,x^{3}+3 x^{2} \sqrt{1-6 b}\, f b +27 x^{2} e b -\frac{\sqrt{1-6 b}\, f \,x^{2}}{2}-\frac{9 b f \,x^{2}}{2}$
gosper	$-\frac{\left(\left(1-6 b \right)^{\frac{3}{2}}-108 x^{3}-54 b x +54 x^{2}+9 b -1\right) x \left(216 f \,x^{4}+30 \sqrt{1-6 b}\, b f x +180 b f \,x^{2}+270 e \,x^{3}-135 f \,x^{3}+60 \sqrt{1-6 b}\, b e -5 \sqrt{1-6 b}\, f\right.}{10 \left(108 x^{3}+54 b x +6 \sqrt{1-6 b}\, b -54 x^{2}-9 b -\sqrt{1-6 b}+1\right)}$

input
```
int((f*x+e)*(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3),x,method=_RETURNVE
RBOSE)
```

output
```
108/5*f*x^5+1/4*(108*e-54*f)*x^4+1/3*(54*b*f-54*e)*x^3+1/2*(54*e*b+f*(1-(1
-6*b)^(3/2)-9*b))*x^2+x*e*(1-(1-6*b)^(3/2)-9*b)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.81

$$\int (e+fx) \left(1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3\right) \, dx = \frac{108}{5} \, f x^{5} + \frac{27}{2} \, {\left(2 \, e - f\right)} x^{4} + 18 \, {\left(b f - e\right)} x^{3} - {\left(9 \, b - 1\right)} e x + \frac{1}{2} \, {\left(54 \, b e - {\left(9 \, b - 1\right)} f\right)} x^{2} + \frac{1}{2} \, {\left({\left(6 \, b - 1\right)} f x^{2} + 2 \, {\left(6 \, b - 1\right)} e x\right)} \sqrt{-6 \, b + 1}$$

input
```
integrate((f*x+e)*(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3),x, algorithm
="fricas")
```

output
```
108/5*f*x^5 + 27/2*(2*e - f)*x^4 + 18*(b*f - e)*x^3 - (9*b - 1)*e*x + 1/2*
(54*b*e - (9*b - 1)*f)*x^2 + 1/2*((6*b - 1)*f*x^2 + 2*(6*b - 1)*e*x)*sqrt(
-6*b + 1)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.99

$$\int (e+fx)\left(1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3\right)\,dx=\frac{108fx^5}{5}+x^4\cdot\left(27e-\frac{27f}{2}\right)+x^3\cdot(18bf-18e)+x^2\cdot\left(27be+3bf\sqrt{1-6b}-\frac{9bf}{2}-\frac{f\sqrt{1-6b}}{2}+\frac{f}{2}\right)+x\left(6be\sqrt{1-6b}-9be-e\sqrt{1-6b}+e\right)$$

input
```
integrate((f*x+e)*(1-(1-6*b)**(3/2)-9*b+54*b*x-54*x**2+108*x**3),x)
```

output
```
108*f*x**5/5 + x**4*(27*e - 27*f/2) + x**3*(18*b*f - 18*e) + x**2*(27*b*e
+ 3*b*f*sqrt(1 - 6*b) - 9*b*f/2 - f*sqrt(1 - 6*b)/2 + f/2) + x*(6*b*e*sqrt
(1 - 6*b) - 9*b*e - e*sqrt(1 - 6*b) + e)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.67

$$\int (e+fx)\left(1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3\right)\,dx=\frac{108}{5}\,fx^5+\frac{27}{2}\,(2\,e-f)x^4+18\,(bf-e)x^3-\left((-6\,b+1)^{\frac{3}{2}}+9\,b-1\right)ex+\frac{1}{2}\,\left(54\,be-\left((-6\,b+1)^{\frac{3}{2}}+9\,b-1\right)f\right)x^2$$

input
```
integrate((f*x+e)*(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3),x, algorithm
="maxima")
```

output
```
108/5*f*x^5 + 27/2*(2*e - f)*x^4 + 18*(b*f - e)*x^3 - ((-6*b + 1)^(3/2) +
9*b - 1)*e*x + 1/2*(54*b*e - ((-6*b + 1)^(3/2) + 9*b - 1)*f)*x^2
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.83

$$\int (e+fx)\left(1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3\right)\,dx=\frac{108}{5}fx^5+18\,bfx^3+27\,ex^4-\frac{27}{2}fx^4+27\,bex^2+\frac{1}{2}\left(6\,b\sqrt{-6\,b+1}-9\,b-\sqrt{-6\,b+1}+1\right)fx^2-18\,ex^3+\left((6\,b-1)\sqrt{-6\,b+1}-9\,b+1\right)ex$$

input
```
integrate((f*x+e)*(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3),x, algorithm
="giac")
```

output
```
108/5*f*x^5 + 18*b*f*x^3 + 27*e*x^4 - 27/2*f*x^4 + 27*b*e*x^2 + 1/2*(6*b*s
qrt(-6*b + 1) - 9*b - sqrt(-6*b + 1) + 1)*f*x^2 - 18*e*x^3 + ((6*b - 1)*sq
rt(-6*b + 1) - 9*b + 1)*e*x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.68

$$\int (e+fx)\left(1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3\right)\,dx=\frac{108\,f\,x^5}{5}+\left(27\,e-\frac{27\,f}{2}\right)\,x^4+\left(18\,b\,f-18\,e\right)\,x^3+\left(\frac{f}{2}+27\,b\,e-\frac{9\,b\,f}{2}-\frac{f\,{\left(1-6\,b\right)}^{3/2}}{2}\right)\,x^2-e\,\left(9\,b+{\left(1-6\,b\right)}^{3/2}-1\right)\,x$$

input
```
int(-(e + f*x)*(9*b - 54*b*x + (1 - 6*b)^(3/2) + 54*x^2 - 108*x^3 - 1),x)
```

output
```
x^4*(27*e - (27*f)/2) - x^3*(18*e - 18*b*f) + (108*f*x^5)/5 + x^2*(f/2 + 2
7*b*e - (9*b*f)/2 - (f*(1 - 6*b)^(3/2))/2) - e*x*(9*b + (1 - 6*b)^(3/2) -
1)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.89

$$\int (e+fx)\left(1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3\right)\,dx=\frac{x\left(60\sqrt{-6b+1}\,be+30\sqrt{-6b+1}\,bfx-10\sqrt{-6b+1}\,e-5\sqrt{-6b+1}\,fx+270bex-90be+\right.}{10}$$

input
```
int((f*x+e)*(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3),x)
```

output
```
(x*(60*sqrt( - 6*b + 1)*b*e + 30*sqrt( - 6*b + 1)*b*f*x - 10*sqrt( - 6*b +
 1)*e - 5*sqrt( - 6*b + 1)*f*x + 270*b*e*x - 90*b*e + 180*b*f*x**2 - 45*b*
f*x + 270*e*x**3 - 180*e*x**2 + 10*e + 216*f*x**4 - 135*f*x**3 + 5*f*x))/1
0
```

3.136 $\int \left(1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3\right) dx$

Optimal result

Integrand size = 30, antiderivative size = 35

$$\int \left(1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3\right) dx = \left(1-(1-6b)^{3/2}-9b\right)x+27bx^2-18x^3+27x^4$$

output
```
(1-(1-6*b)^(3/2)-9*b)*x+27*b*x^2-18*x^3+27*x^4
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.97

$$\int \left(1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3\right) dx = x-(1-6b)^{3/2}x-9bx+27bx^2-18x^3+27x^4$$

input
```
Integrate[1 - (1 - 6*b)^(3/2) - 9*b + 54*b*x - 54*x^2 + 108*x^3,x]
```

output
```
x - (1 - 6*b)^(3/2)*x - 9*b*x + 27*b*x^2 - 18*x^3 + 27*x^4
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.033$, Rules used = $\{2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(54bx-(1-6b)^{3/2}-9b+108x^3-54x^2+1\right) \, dx$$

$$\Big\downarrow \text{2009}$$

$$27bx^2+\left(-(1-6b)^{3/2}-9b+1\right)x+27x^4-18x^3$$

input
```
Int[1 - (1 - 6*b)^(3/2) - 9*b + 54*b*x - 54*x^2 + 108*x^3,x]
```

output
```
(1 - (1 - 6*b)^(3/2) - 9*b)*x + 27*b*x^2 - 18*x^3 + 27*x^4
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94

method	result	size
default	$x-x(1-6b)^{\frac{3}{2}}-9bx+27b\,x^2-18x^3+27x^4$	33
parts	$x-x(1-6b)^{\frac{3}{2}}-9bx+27b\,x^2-18x^3+27x^4$	33
parallelrisch	$\left(1-(1-6b)^{\frac{3}{2}}-9b\right)x+27b\,x^2-18x^3+27x^4$	34
norman	$\left(6\sqrt{1-6b}\,b-\sqrt{1-6b}-9b+1\right)x-18x^3+27x^4+27b\,x^2$	44
gosper	$-\frac{x\left(27x^3+6\sqrt{1-6b}\,b+27bx-18x^2-\sqrt{1-6b}-9b+1\right)\left((1-6b)^{\frac{3}{2}}-108x^3-54bx+54x^2+9b-1\right)}{108x^3+54bx+6\sqrt{1-6b}\,b-54x^2-9b-\sqrt{1-6b}+1}$	108

input
```
int(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3,x,method=_RETURNVERBOSE)
```

output
```
x-x*(1-6*b)^(3/2)-9*b*x+27*b*x^2-18*x^3+27*x^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.11

$$\int \left(1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3\right)\,dx=27\,x^4+27\,bx^2-18\,x^3+(6\,b-1)\sqrt{-6\,b+1}x-(9\,b-1)x$$

input
```
integrate(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3,x, algorithm="fricas")
```

output
```
27*x^4 + 27*b*x^2 - 18*x^3 + (6*b - 1)*sqrt(-6*b + 1)*x - (9*b - 1)*x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.20

$$\int \left(1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3\right)\,dx=27bx^2+27x^4-18x^3+x\left(6b\sqrt{1-6b}-9b-\sqrt{1-6b}+1\right)$$

input
```
integrate(1-(1-6*b)**(3/2)-9*b+54*b*x-54*x**2+108*x**3,x)
```

output
```
27*b*x**2 + 27*x**4 - 18*x**3 + x*(6*b*sqrt(1 - 6*b) - 9*b - sqrt(1 - 6*b)
 + 1)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

$$\int \left(1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3\right)\,dx=27\,x^4+27\,bx^2-18\,x^3-(-6\,b+1)^{\frac{3}{2}}x-9\,bx+x$$

input
```
integrate(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3,x, algorithm="maxima")
```

output
```
27*x^4 + 27*b*x^2 - 18*x^3 - (-6*b + 1)^(3/2)*x - 9*b*x + x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

$$\int \left(1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3\right)\,dx=27\,x^4+27\,bx^2-18\,x^3-(-6\,b+1)^{\frac{3}{2}}x-9\,bx+x$$

input
```
integrate(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3,x, algorithm="giac")
```

output
```
27*x^4 + 27*b*x^2 - 18*x^3 - (-6*b + 1)^(3/2)*x - 9*b*x + x
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

$$\int \left(1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3\right)\,dx=27\,x^4-18\,x^3+27\,b\,x^2+\left(1-(1-6\,b)^{3/2}-9\,b\right)\,x$$

input
```
int(54*b*x - 9*b - (1 - 6*b)^(3/2) - 54*x^2 + 108*x^3 + 1,x)
```

output
```
27*b*x^2 - x*(9*b + (1 - 6*b)^(3/2) - 1) - 18*x^3 + 27*x^4
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.09

$$\int \left(1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3\right)\,dx=x\left(6\sqrt{-6b+1}\,b-\sqrt{-6b+1}+27bx-9b+27x^3-18x^2+1\right)$$

input
```
int(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3,x)
```

output
```
x*(6*sqrt( - 6*b + 1)*b - sqrt( - 6*b + 1) + 27*b*x - 9*b + 27*x**3 - 18*x
**2 + 1)
```

3.137 $\int \frac{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}{e+fx} dx$

Optimal result

Integrand size = 38, antiderivative size = 106

$$\int \frac{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}{e+fx} dx = \frac{54(2e^2+ef+bf^2)x}{f^3} - \frac{27(2e+f)x^2}{f^2} + \frac{36x^3}{f} - \frac{\left(108e^3+54e^2f+54bef^2-\left(1-\sqrt{1-6b}-\left(9-6\sqrt{1-6b}\right)b\right)f^3\right)\log(e+fx)}{f^4}$$

output
```
54*(b*f^2+2*e^2+e*f)*x/f^3-27*(2*e+f)*x^2/f^2+36*x^3/f-(108*e^3+54*e^2*f+5
4*b*e*f^2-(1-(1-6*b)^(1/2)-(9-6*(1-6*b)^(1/2))*b)*f^3)*ln(f*x+e)/f^4
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.89

$$\int \frac{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}{e+fx} dx = \frac{9fx(12e^2-6ef(-1+x)+f^2(6b+x(-3+4x))) -}{}$$

input
```
Integrate[(1 - (1 - 6*b)^(3/2) - 9*b + 54*b*x - 54*x^2 + 108*x^3)/(e + f*x
),x]
```

output
```
(9*f*x*(12*e^2 - 6*e*f*(-1 + x) + f^2*(6*b + x*(-3 + 4*x))) - (108*e^3 + 5
4*e^2*f + 54*b*e*f^2 + (-1 + Sqrt[1 - 6*b] + 9*b - 6*Sqrt[1 - 6*b]*b)*f^3)
*Log[e + f*x])/f^4
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.91, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {2389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{54bx-(1-6b)^{3/2}-9b+108x^3-54x^2+1}{e+fx}\,dx$$

$$\Big\downarrow \text{2389}$$

$$\int \left(\frac{54\left(bf^2+2e^2+ef\right)}{f^3}+\frac{-54bef^2-(1-6b)^{3/2}f^3-9bf^3-108e^3-54e^2f+f^3}{f^3(e+fx)}-\frac{54x(2e+f)}{f^2}+\frac{108x^2}{f}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{54x\left(bf^2+2e^2+ef\right)}{f^3}-\frac{\left(54bef^2+(1-6b)^{3/2}f^3+9bf^3+108e^3+54e^2f-f^3\right)\log(e+fx)}{f^4}-\frac{27x^2(2e+f)}{f^2}+\frac{36x^3}{f}$$

input
```
Int[(1 - (1 - 6*b)^(3/2) - 9*b + 54*b*x - 54*x^2 + 108*x^3)/(e + f*x),x]
```

output
```
(54*(2*e^2 + e*f + b*f^2)*x)/f^3 - (27*(2*e + f)*x^2)/f^2 + (36*x^3)/f - (
(108*e^3 + 54*e^2*f + 54*b*e*f^2 - f^3 + (1 - 6*b)^(3/2)*f^3 + 9*b*f^3)*Lo
g[e + f*x])/f^4
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2389
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.92

method	result
default	$\frac{36 f^{2} x^{3}+54 b x f^{2}-54 e f x^{2}-27 f^{2} x^{2}+108 e^{2} x+54 e f x}{f^{3}}+\frac{\left(-(1-6 b)^{\frac{3}{2}} f^{3}-54 b e f^{2}-9 b f^{3}-108 e^{3}-54 e^{2} f+f^{3}\right) \ln (f x+e)}{f^{4}}$
norman	$\frac{36 x^{3}}{f}-\frac{27(2 e+f) x^{2}}{f^{2}}+\frac{54\left(b f^{2}+2 e^{2}+e f\right) x}{f^{3}}-\frac{\left(-6 \sqrt{1-6 b}\, f^{3} b+54 b e f^{2}+9 b f^{3}+\sqrt{1-6 b}\, f^{3}+108 e^{3}+54 e^{2} f-f^{3}\right) \ln (f x+e}{f^{4}}$
parallelrisch	$\frac{6 \ln (f x+e) \sqrt{1-6 b}\, b f^{3}+36 x^{3} f^{3}-\ln (f x+e) \sqrt{1-6 b}\, f^{3}-54 \ln (f x+e) b e f^{2}-9 \ln (f x+e) b f^{3}-54 x^{2} f^{2} e-27 x^{2} f^{3}+54 x b f^{3}-108}{f^{4}}$

input
```
int((1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3)/(f*x+e),x,method=_RETURNVE
RBOSE)
```

output
```
54/f^3*(2/3*f^2*x^3+b*x*f^2-e*f*x^2-1/2*f^2*x^2+2*e^2*x+e*f*x)+1/f^4*(-(1-
6*b)^(3/2)*f^3-54*b*e*f^2-9*b*f^3-108*e^3-54*e^2*f+f^3)*ln(f*x+e)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00

$$\int \frac{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}{e+fx}\,dx = \frac{36\, f^3x^3+(6\,b-1)\sqrt{-6\,b+1}f^3\log\left(fx+e\right)-27\,(2\,e$$

input
```
integrate((1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3)/(f*x+e),x, algorithm
="fricas")
```

output
```
(36*f^3*x^3 + (6*b - 1)*sqrt(-6*b + 1)*f^3*log(f*x + e) - 27*(2*e*f^2 + f^
3)*x^2 + 54*(b*f^3 + 2*e^2*f + e*f^2)*x - (54*b*e*f^2 + (9*b - 1)*f^3 + 10
8*e^3 + 54*e^2*f)*log(f*x + e))/f^4
```

Sympy [A] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.01

$$\int \frac{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}{e+fx}\,dx = x^2\left(-\frac{54e}{f^2}-\frac{27}{f}\right) + x\left(\frac{54b}{f}+\frac{108e^2}{f^3}+\frac{54e}{f^2}\right) + \left(-\frac{54be}{f^2}+\frac{6b\sqrt{1-6b}}{f}-\frac{9b}{f}-\frac{108e^3}{f^4}-\frac{54e^2}{f^3}-\frac{\sqrt{1-6b}}{f}+\frac{1}{f}\right)\log(e+fx)+\frac{36x^3}{f}$$

input
```
integrate((1-(1-6*b)**(3/2)-9*b+54*b*x-54*x**2+108*x**3)/(f*x+e),x)
```

output
```
x**2*(-54*e/f**2 - 27/f) + x*(54*b/f + 108*e**2/f**3 + 54*e/f**2) + (-54*b
*e/f**2 + 6*b*sqrt(1 - 6*b)/f - 9*b/f - 108*e**3/f**4 - 54*e**2/f**3 - sqr
t(1 - 6*b)/f + 1/f)*log(e + f*x) + 36*x**3/f
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.86

$$\int \frac{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}{e+fx}\,dx = \frac{9\left(4f^2x^3-3\left(2ef+f^2\right)x^2+6\left(bf^2+2e^2+ef\right)x\right)}{f^3} - \frac{\left(54bef^2+\left((-6b+1)^{\frac{3}{2}}+9b-1\right)f^3+108e^3+54e^2f\right)\log(fx+e)}{f^4}$$

input
```
integrate((1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3)/(f*x+e),x, algorithm
="maxima")
```

output
```
9*(4*f^2*x^3 - 3*(2*e*f + f^2)*x^2 + 6*(b*f^2 + 2*e^2 + e*f)*x)/f^3 - (54*
b*e*f^2 + ((-6*b + 1)^(3/2) + 9*b - 1)*f^3 + 108*e^3 + 54*e^2*f)*log(f*x +
 e)/f^4
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.96

$$\int \frac{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}{e+fx}\,dx = \frac{9\left(4\,f^2x^3+6\,bf^2x-6\,efx^2-3\,f^2x^2+12\,e^2x+6\,ef}{f^3}$$
$$-\frac{\left(54\,bef^2-\left((6\,b-1)\sqrt{-6\,b+1}-9\,b+1\right)f^3+108\,e^3+54\,e^2f\right)\log\left(|fx+e|\right)}{f^4}$$

input
```
integrate((1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3)/(f*x+e),x, algorithm
="giac")
```

output
```
9*(4*f^2*x^3 + 6*b*f^2*x - 6*e*f*x^2 - 3*f^2*x^2 + 12*e^2*x + 6*e*f*x)/f^3
 - (54*b*e*f^2 - ((6*b - 1)*sqrt(-6*b + 1) - 9*b + 1)*f^3 + 108*e^3 + 54*e
^2*f)*log(abs(f*x + e))/f^4
```

Mupad [B] (verification not implemented)

Time = 11.97 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00

$$\int \frac{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}{e+fx}\,dx = x\left(\frac{54\,b}{f}+\frac{e\left(\frac{108\,e}{f^2}+\frac{54}{f}\right)}{f}\right)$$
$$-x^2\left(\frac{54\,e}{f^2}+\frac{27}{f}\right)+\frac{36\,x^3}{f}$$
$$-\frac{\ln\left(x+\frac{e}{f}\right)\,\left(9\,b\,f^3+54\,e^2\,f+f^3\,(1-6\,b)^{3/2}+108\,e^3-f^3+54\,b\,e\,f^2\right)}{f^4}$$

input
```
int(-(9*b - 54*b*x + (1 - 6*b)^(3/2) + 54*x^2 - 108*x^3 - 1)/(e + f*x),x)
```

output
```
x*((54*b)/f + (e*((108*e)/f^2 + 54/f))/f) - x^2*((54*e)/f^2 + 27/f) + (36*
x^3)/f - (log(x + e/f)*(9*b*f^3 + 54*e^2*f + f^3*(1 - 6*b)^(3/2) + 108*e^3
 - f^3 + 54*b*e*f^2))/f^4
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.36

$$\int \frac{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}{e+fx}\,dx = \frac{6\sqrt{-6b+1}\,\log(fx+e)\,b\,f^3-\sqrt{-6b+1}\,\log(fx+e)}{}$$

input
```
int((1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3)/(f*x+e),x)
```

output
```
(6*sqrt( - 6*b + 1)*log(e + f*x)*b*f**3 - sqrt( - 6*b + 1)*log(e + f*x)*f*
*3 - 54*log(e + f*x)*b*e*f**2 - 9*log(e + f*x)*b*f**3 - 108*log(e + f*x)*e
**3 - 54*log(e + f*x)*e**2*f + log(e + f*x)*f**3 + 54*b*f**3*x + 108*e**2*
f*x - 54*e*f**2*x**2 + 54*e*f**2*x + 36*f**3*x**3 - 27*f**3*x**2)/f**4
```

3.138 $\int \frac{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}{(e+fx)^2} dx$

Optimal result

Integrand size = 38, antiderivative size = 110

$$\int \frac{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}{(e+fx)^2} dx = -\frac{54(4e+f)x}{f^3}+\frac{54x^2}{f^2} + \frac{108e^3+54e^2f+54bef^2-\left(1-\sqrt{1-6b}-\left(9-6\sqrt{1-6b}\right)b\right)f^3}{f^4(e+fx)} + \frac{54(6e^2+2ef+bf^2)\log(e+fx)}{f^4}$$

output
```
-54*(4*e+f)*x/f^3+54*x^2/f^2+(108*e^3+54*e^2*f+54*b*e*f^2-(1-(1-6*b)^(1/2)
-(9-6*(1-6*b)^(1/2))*b)*f^3)/f^4/(f*x+e)+54*(b*f^2+6*e^2+2*e*f)*ln(f*x+e)/
f^4
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.02

$$\int \frac{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}{(e+fx)^2} dx = \frac{108e^3-54e^2f(-1+4x)+f^3\left(-1+\sqrt{1-6b}+(9-\right.}{}$$

input
```
Integrate[(1 - (1 - 6*b)^(3/2) - 9*b + 54*b*x - 54*x^2 + 108*x^3)/(e + f*x
)^2,x]
```

output
```
(108*e^3 - 54*e^2*f*(-1 + 4*x) + f^3*(-1 + Sqrt[1 - 6*b] + (9 - 6*Sqrt[1 -
 6*b])*b - 54*x^2 + 54*x^3) + 54*e*f^2*(b - x*(1 + 3*x)) + 54*(6*e^2 + 2*e
*f + b*f^2)*(e + f*x)*Log[e + f*x])/(f^4*(e + f*x))
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.91, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {2389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{54bx-(1-6b)^{3/2}-9b+108x^3-54x^2+1}{(e+fx)^2}dx$$

$$\Big\downarrow \text{2389}$$

$$\int \left(\frac{54(bf^2+6e^2+2ef)}{f^3(e+fx)}+\frac{-54bef^2-(1-6b)^{3/2}f^3-9bf^3-108e^3-54e^2f+f^3}{f^3(e+fx)^2}-\frac{54(4e+f)}{f^3}+\frac{108x}{f^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{54(bf^2+6e^2+2ef)\log(e+fx)}{f^4}+\frac{54bef^2+(1-6b)^{3/2}f^3+9bf^3+108e^3+54e^2f-f^3}{f^4(e+fx)}-\frac{54x(4e+f)}{f^3}+\frac{54x^2}{f^2}$$

input
```
Int[(1 - (1 - 6*b)^(3/2) - 9*b + 54*b*x - 54*x^2 + 108*x^3)/(e + f*x)^2,x]
```

output
```
(-54*(4*e + f)*x)/f^3 + (54*x^2)/f^2 + (108*e^3 + 54*e^2*f + 54*b*e*f^2 -
f^3 + (1 - 6*b)^(3/2)*f^3 + 9*b*f^3)/(f^4*(e + f*x)) + (54*(6*e^2 + 2*e*f
+ b*f^2)*Log[e + f*x])/f^4
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2389
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.90

method	result
default	$-\frac{54\left(-f\,x^{2}+4ex+fx\right)}{f^{3}}-\frac{-\left(1-6b\right)^{\frac{3}{2}}f^{3}-54be\,f^{2}-9b\,f^{3}-108e^{3}-54e^{2}f+f^{3}}{f^{4}\left(fx+e\right)}+\frac{\left(54b\,f^{2}+324e^{2}+108ef\right)\ln\left(fx+e\right)}{f^{4}}$
norman	$\frac{\frac{54x^{3}}{f}-\frac{54\left(3e+f\right)x^{2}}{f^{2}}-\frac{\left(-6\sqrt{1-6b}\,f^{3}b+\sqrt{1-6b}\,f^{3}+54be\,f^{2}+9b\,f^{3}+324e^{3}+108e^{2}f-f^{3}\right)x}{e\,f^{3}}}{fx+e}+\frac{54\left(b\,f^{2}+6e^{2}+2ef\right)\ln\left(fx+e\right)}{f^{4}}$
parallelrisch	$\frac{54\ln\left(fx+e\right)xb\,f^{3}+54x^{3}f^{3}+324\ln\left(fx+e\right)x\,e^{2}f+108\ln\left(fx+e\right)xe\,f^{2}+54\ln\left(fx+e\right)be\,f^{2}-6\sqrt{1-6b}\,f^{3}b-162x^{2}f^{2}e-54x^{2}f^{3}+\cdots}{f^{4}\left(fx+e\right)}$

input
```
int((1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3)/(f*x+e)^2,x,method=_RETURN
VERBOSE)
```

output
```
-54/f^3*(-f*x^2+4*e*x+f*x)-1/f^4*(-(1-6*b)^(3/2)*f^3-54*b*e*f^2-9*b*f^3-10
8*e^3-54*e^2*f+f^3)/(f*x+e)+(54*b*f^2+324*e^2+108*e*f)/f^4*ln(f*x+e)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.30

$$\int \frac{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}{(e+fx)^2}\,dx=\frac{54\,f^3x^3-(6\,b-1)\sqrt{-6\,b+1}f^3+54\,bef^2+(9\,b-1)\cdots}{\cdots}$$

input
```
integrate((1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3)/(f*x+e)^2,x, algorit
hm="fricas")
```

output
```
(54*f^3*x^3 - (6*b - 1)*sqrt(-6*b + 1)*f^3 + 54*b*e*f^2 + (9*b - 1)*f^3 +
108*e^3 + 54*e^2*f - 54*(3*e*f^2 + f^3)*x^2 - 54*(4*e^2*f + e*f^2)*x + 54*
(b*e*f^2 + 6*e^3 + 2*e^2*f + (b*f^3 + 6*e^2*f + 2*e*f^2)*x)*log(f*x + e))/
(f^5*x + e*f^4)
```

Sympy [A] (verification not implemented)

Time = 0.89 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.11

$$\int \frac{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}{(e+fx)^2}\,dx = x\left(-\frac{216e}{f^3}-\frac{54}{f^2}\right) + \frac{54bef^6-6bf^7\sqrt{1-6b}+9bf^7+108e^3f^4+54e^2f^5+f^7\sqrt{1-6b}-f^7}{ef^8+f^9x} + \frac{54x^2}{f^2} + \frac{54(bf^2+6e^2+2ef)\log(e+fx)}{f^4}$$

input
```
integrate((1-(1-6*b)**(3/2)-9*b+54*b*x-54*x**2+108*x**3)/(f*x+e)**2,x)
```

output
```
x*(-216*e/f**3 - 54/f**2) + (54*b*e*f**6 - 6*b*f**7*sqrt(1 - 6*b) + 9*b*f*
*7 + 108*e**3*f**4 + 54*e**2*f**5 + f**7*sqrt(1 - 6*b) - f**7)/(e*f**8 + f
**9*x) + 54*x**2/f**2 + 54*(b*f**2 + 6*e**2 + 2*e*f)*log(e + f*x)/f**4
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.86

$$\int \frac{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}{(e+fx)^2}\,dx = \frac{54\,bef^2+\left((-6\,b+1)^{\frac{3}{2}}+9\,b-1\right)f^3+108\,e^3+54\,e^2}{f^5x+ef^4} + \frac{54\,(fx^2-(4\,e+f)x)}{f^3} + \frac{54\,(bf^2+6\,e^2+2\,ef)\log(fx+e)}{f^4}$$

input
```
integrate((1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3)/(f*x+e)^2,x, algorit
hm="maxima")
```

output
```
(54*b*e*f^2 + ((-6*b + 1)^(3/2) + 9*b - 1)*f^3 + 108*e^3 + 54*e^2*f)/(f^5*
x + e*f^4) + 54*(f*x^2 - (4*e + f)*x)/f^3 + 54*(b*f^2 + 6*e^2 + 2*e*f)*log
(f*x + e)/f^4
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.85

$$\int \frac{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}{(e+fx)^2}\,dx = -\frac{54\,b\left(\frac{\log\left(\frac{|fx+e|}{(fx+e)^2|f|}\right)}{f}-\frac{e}{(fx+e)f}\right)}{f}+\frac{(-6\,b+1)^{\frac{3}{2}}}{(fx+e)f}+\frac{9\,b}{(fx+e)f} - \frac{54\,(fx+e)^2\left(\frac{6\,e}{fx+e}-1\right)}{f^4}-\frac{324\,e^2\log\left(\frac{|fx+e|}{(fx+e)^2|f|}\right)}{f^4}-\frac{108\,e\log\left(\frac{|fx+e|}{(fx+e)^2|f|}\right)}{f^3} + \frac{108\,e^3}{(fx+e)f^4}-\frac{54\,(fx+e)}{f^3}+\frac{54\,e^2}{(fx+e)f^3}-\frac{1}{(fx+e)f}$$

input
```
integrate((1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3)/(f*x+e)^2,x, algorit
hm="giac")
```

output
```
-54*b*(log(abs(f*x + e)/((f*x + e)^2*abs(f)))/f - e/((f*x + e)*f))/f + (-6
*b + 1)^(3/2)/((f*x + e)*f) + 9*b/((f*x + e)*f) - 54*(f*x + e)^2*(6*e/(f*x
 + e) - 1)/f^4 - 324*e^2*log(abs(f*x + e)/((f*x + e)^2*abs(f)))/f^4 - 108*
e*log(abs(f*x + e)/((f*x + e)^2*abs(f)))/f^3 + 108*e^3/((f*x + e)*f^4) - 5
4*(f*x + e)/f^3 + 54*e^2/((f*x + e)*f^3) - 1/((f*x + e)*f)
```

Mupad [B] (verification not implemented)

Time = 12.23 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.98

$$\int \frac{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}{(e+fx)^2}\,dx = \frac{\frac{(1-6\,b)^{3/2}}{f^2}+\frac{9\,b\,f^3+54\,e^2\,f+108\,e^3-f^3+54\,b\,e\,f^2}{f^5}}{x+\frac{e}{f}} - x\left(\frac{216\,e}{f^3}+\frac{54}{f^2}\right)+\frac{54\,x^2}{f^2}+\frac{\ln\left(x+\frac{e}{f}\right)\,(324\,e^2+108\,e\,f+54\,b\,f^2)}{f^4}$$

input
```
int(-(9*b - 54*b*x + (1 - 6*b)^(3/2) + 54*x^2 - 108*x^3 - 1)/(e + f*x)^2,x
)
```

output
```
((1 - 6*b)^(3/2)/f^2 + (9*b*f^3 + 54*e^2*f + 108*e^3 - f^3 + 54*b*e*f^2)/f
^5)/(x + e/f) - x*((216*e)/f^3 + 54/f^2) + (54*x^2)/f^2 + (log(x + e/f)*(1
08*e*f + 54*b*f^2 + 324*e^2))/f^4
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.68

$$\int \frac{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}{(e+fx)^2}\,dx = \frac{6\sqrt{-6b+1}\,b\,f^4x-\sqrt{-6b+1}\,f^4x+54\log(fx+e)\,b$$

input
```
int((1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3)/(f*x+e)^2,x)
```

output
```
(6*sqrt( - 6*b + 1)*b*f**4*x - sqrt( - 6*b + 1)*f**4*x + 54*log(e + f*x)*b
*e**2*f**2 + 54*log(e + f*x)*b*e*f**3*x + 324*log(e + f*x)*e**4 + 324*log(
e + f*x)*e**3*f*x + 108*log(e + f*x)*e**3*f + 108*log(e + f*x)*e**2*f**2*x
 - 54*b*e*f**3*x - 9*b*f**4*x - 324*e**3*f*x - 162*e**2*f**2*x**2 - 108*e*
*2*f**2*x + 54*e*f**3*x**3 - 54*e*f**3*x**2 + f**4*x)/(e*f**4*(e + f*x))
```

3.139 $\int \frac{A+Bx+Cx^2}{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 194

$$\begin{aligned}\int \frac{A+Bx+Cx^2}{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}\,dx &= -\frac{18A+3\left(1-\sqrt{1-6b}\right)B+\left(1-\sqrt{1-6b}-3b\right)C}{162\sqrt{1-6b}\left(1-\sqrt{1-6b}-6x\right)} \\ &\quad -\frac{\left(18A+\left(3+6\sqrt{1-6b}\right)B-\left(2-2\sqrt{1-6b}-15b\right)C\right)\log\left(1-\sqrt{1-6b}-6x\right)}{486(1-6b)} \\ &\quad +\frac{\left(36A+6B+5C-24bC+4\sqrt{1-6b}(3B+C)\right)\log\left(1+2\sqrt{1-6b}-6x\right)}{972(1-6b)}\end{aligned}$$

output
```
-1/162*(18*A+3*(1-(1-6*b)^(1/2))*B+(1-(1-6*b)^(1/2)-3*b)*C)/(1-6*b)^(1/2)/
(1-(1-6*b)^(1/2)-6*x)-(18*A+(3+6*(1-6*b)^(1/2))*B-(2-2*(1-6*b)^(1/2)-15*b)
*C)*ln(1-(1-6*b)^(1/2)-6*x)/(486-2916*b)+(36*A+6*B+5*C-24*C*b+4*(1-6*b)^(1
/2)*(3*B+C))*ln(1+2*(1-6*b)^(1/2)-6*x)/(972-5832*b)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 423 vs. $2(194) = 388$.

Time = 1.07 (sec) , antiderivative size = 423, normalized size of antiderivative = 2.18

$$\int \frac{A+Bx+Cx^2}{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}\,dx = \frac{\frac{6\left(6b^2C+6bB\left(\sqrt{1-6b}-6x\right)-2\left(-1+\sqrt{1-6b}\right)(3B+C)x+bC\left(-1+\sqrt{1-6b}-\right.\right.}{(-1+6b)(b+2x(-1+3}}{}$$

input
```
Integrate[(A + B*x + C*x^2)/(1 - (1 - 6*b)^(3/2) - 9*b + 54*b*x - 54*x^2 +
 108*x^3),x]
```

output
```
((6*(6*b^2*C + 6*b*B*(Sqrt[1 - 6*b] - 6*x) - 2*(-1 + Sqrt[1 - 6*b])*(3*B +
 C)*x + b*C*(-1 + Sqrt[1 - 6*b] + 6*(-2 + Sqrt[1 - 6*b])*x) - 6*A*(-1 - Sq
rt[1 - 6*b] + 6*b + 6*Sqrt[1 - 6*b]*x)))/((-1 + 6*b)*(b + 2*x*(-1 + 3*x)))
 + (2*(-36*A*Sqrt[1 - 6*b] - 6*(2 + Sqrt[1 - 6*b] - 12*b)*B + (-4 - 5*Sqrt
[1 - 6*b] + 24*(1 + Sqrt[1 - 6*b])*b)*C)*ArcTan[(-1 + 6*x)/(2*Sqrt[-1 + 6*
b])])/(-1 + 6*b)^(3/2) - (4*(18*A*Sqrt[1 - 6*b] + 3*(2 + Sqrt[1 - 6*b] - 1
2*b)*B + (2 - 2*Sqrt[1 - 6*b] + 3*(-4 + 5*Sqrt[1 - 6*b])*b)*C)*ArcTan[(-1
+ 6*x)/Sqrt[-1 + 6*b]])/(-1 + 6*b)^(3/2) - ((36*A + 6*(1 + 2*Sqrt[1 - 6*b]
)*B + (5 + 4*Sqrt[1 - 6*b] - 24*b)*C)*Log[1 - 8*b + 4*x - 12*x^2])/(-1 + 6
*b) + (2*(18*A + (3 + 6*Sqrt[1 - 6*b])*B + (-2 + 2*Sqrt[1 - 6*b] + 15*b)*C
)*Log[b - 2*x + 6*x^2])/(-1 + 6*b))/1944
```

Rubi [A] (verified)

Time = 1.23 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.19, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.163$, Rules used = {2525, 27, 2488, 27, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{54bx-(1-6b)^{3/2}-9b+108x^3-54x^2+1}dx$$

$$\Big\downarrow \ 2525$$

$$\frac{1}{324}\int \frac{54(6A-bC+2(3B+C)x)}{108x^3-54x^2+54bx-(1-6b)^{3/2}-9b+1}dx+\frac{1}{324}C\log\left(54bx-(1-6b)^{3/2}-9b+108x^3-54x^2+1\right)$$

$$\Big\downarrow \ 27$$

$$\frac{1}{6}\int \frac{6A-bC+2(3B+C)x}{108x^3-54x^2+54bx-(1-6b)^{3/2}-9b+1}dx+\frac{1}{324}C\log\left(54bx-(1-6b)^{3/2}-9b+108x^3-54x^2+1\right)$$

$$\Big\downarrow \ 2488$$

$$16529940864(1-6b)^3\int -\frac{6A-bC+2(3B+C)x}{49589822592\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)^2\left(\left(2\sqrt{1-6b}+1\right)(1-6b)-6(1-6b)x\right)}dx+\frac{1}{324}C\log\left(54bx-(1-6b)^{3/2}-9b+108x^3-54x^2+1\right)$$

$$\Big\downarrow \ 27$$

$$\frac{1}{324}C\log\left(54bx-(1-6b)^{3/2}-9b+108x^3-54x^2+1\right)-\frac{1}{3}(1-6b)^3\int \frac{6A-bC+2(3B+C)x}{(1-6b)\left(-6x+2\sqrt{1-6b}+1\right)\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)^2}dx$$

$$\Big\downarrow \ 27$$

$$\frac{1}{324}C\log\left(54bx-(1-6b)^{3/2}-9b+108x^3-54x^2+1\right)-\frac{1}{3}(1-6b)^2\int \frac{6A-bC+2(3B+C)x}{\left(-6x+2\sqrt{1-6b}+1\right)\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)^2}dx$$

$$\Big\downarrow \ 86$$

$$\frac{1}{324}C\log\left(54bx-(1-6b)^{3/2}-9b+108x^3-54x^2+1\right)-\frac{1}{3}(1-6b)^2\int \left(\frac{18A+3\left(1-\sqrt{1-6b}\right)B+\left(-3b-\sqrt{1-6b}+1\right)C}{9(1-6b)^{5/2}\left(-6x-\sqrt{1-6b}+1\right)^2}+\frac{-18A-3\left(2\sqrt{1-6b}+1\right)B-\left(-3b+2\sqrt{1-6b}\right.}{27(1-6b)^3\left(-6x-\sqrt{1-6b}+1\right)}\right.$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{324}C\log\left(54bx-(1-6b)^{3/2}-9b+108x^3-54x^2+1\right)-\frac{1}{3}(1-6b)^2\left(\frac{18A+3\left(1-\sqrt{1-6b}\right)B+\left(-3b-\sqrt{1-6b}+1\right)C}{54(1-6b)^{5/2}\left(-\sqrt{1-6b}-6x+1\right)}+\frac{\log\left(-\sqrt{1-6b}-6x+1\right)\left(18A+2\sqrt{1-6b}(3B+C\right.}{162(1-6b)^3}\right.$$

input
```
Int[(A + B*x + C*x^2)/(1 - (1 - 6*b)^(3/2) - 9*b + 54*b*x - 54*x^2 + 108*x
^3),x]
```

output
```
-1/3*((1 - 6*b)^2*((18*A + 3*(1 - Sqrt[1 - 6*b])*B + (1 - Sqrt[1 - 6*b] -
3*b)*C)/(54*(1 - 6*b)^(5/2)*(1 - Sqrt[1 - 6*b] - 6*x)) + ((18*A + 3*B + C
- 3*b*C + 2*Sqrt[1 - 6*b]*(3*B + C))*Log[1 - Sqrt[1 - 6*b] - 6*x])/(162*(1
 - 6*b)^3) - ((18*A + 3*B + C - 3*b*C + 2*Sqrt[1 - 6*b]*(3*B + C))*Log[1 +
 2*Sqrt[1 - 6*b] - 6*x])/(162*(1 - 6*b)^3))) + (C*Log[1 - (1 - 6*b)^(3/2)
- 9*b + 54*b*x - 54*x^2 + 108*x^3])/324
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2488
```
Int[((e_.) + (f_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(
x_)^3)^(p_), x_Symbol] :> Simp[1/(4^p*(c^2 - 3*b*d)^(3*p))   Int[(e + f*x)^
m*(c^3 - 4*b*c*d + 9*a*d^2 + d*(c^2 - 3*b*d)*x)^p*(b*c - 9*a*d + 2*(c^2 - 3
*b*d)*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && NeQ[c^2 - 3
*b*d, 0] && EqQ[b^2*c^2 - 4*a*c^3 - 4*b^3*d + 18*a*b*c*d - 27*a^2*d^2, 0] &
& ILtQ[p, 0]
```

rule 2525
```
Int[(Pm_)/(Qn_), x_Symbol] :> With[{m = Expon[Pm, x], n = Expon[Qn, x]}, Si
mp[Coeff[Pm, x, m]*(Log[Qn]/(n*Coeff[Qn, x, n])), x] + Simp[1/(n*Coeff[Qn,
x, n])   Int[ExpandToSum[n*Coeff[Qn, x, n]*Pm - Coeff[Pm, x, m]*D[Qn, x], x
]/Qn, x], x] /; EqQ[m, n - 1]] /; PolyQ[Pm, x] && PolyQ[Qn, x]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.56 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.36

method	result
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(-(1-6b)^{\frac{3}{2}}+108\_Z^3+54b\_Z-54\_Z^2-9b+1\right)}\frac{\left(-C\_R^2-B\_R-A\right)\ln\left(x-\_R\right)}{-6\_R^2+2\_R-b}\right)}{54}$
parallelrisch	$\frac{6C+18B-36bC-108Bb-6B\sqrt{1-6b}\,\ln\left(-\frac{1}{6}+x+\frac{\sqrt{1-6b}}{6}\right)+6B\sqrt{1-6b}\,\ln\left(x-\frac{1}{6}-\frac{\sqrt{1-6b}}{3}\right)+36B\ln\left(-\frac{1}{6}+x+\frac{\sqrt{1-6b}}{6}\right)x-36B\,\mathrm{l}}{}$

input
```
int((C*x^2+B*x+A)/(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3),x,method=_RE
TURNVERBOSE)
```

output
```
1/54*sum((-C*_R^2-B*_R-A)/(-6*_R^2+2*_R-b)*ln(x-_R),_R=RootOf(-(1-6*b)^(3/
2)+108*_Z^3+54*b*_Z-54*_Z^2-9*b+1))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.63

$$\int \frac{A+Bx+Cx^2}{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}\,dx = \frac{18\,Cb^2-3\,(36\,A+C)b-6\,(6\,(3\,B+C)b-3\,B-C}{}$$

input
```
integrate((C*x^2+B*x+A)/(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3),x, alg
orithm="fricas")
```

output
```
1/972*(18*C*b^2 - 3*(36*A + C)*b - 6*(6*(3*B + C)*b - 3*B - C)*x + 2*(15*C
*b^2 + 6*(15*C*b + 18*A + 3*B - 2*C)*x^2 + (18*A + 3*B - 2*C)*b - 2*(15*C*
b + 18*A + 3*B - 2*C)*x + 2*(6*(3*B + C)*x^2 + (3*B + C)*b - 2*(3*B + C)*x
)*sqrt(-6*b + 1))*log(6*x + sqrt(-6*b + 1) - 1) + (24*C*b^2 + 6*(24*C*b -
36*A - 6*B - 5*C)*x^2 - (36*A + 6*B + 5*C)*b - 2*(24*C*b - 36*A - 6*B - 5*
C)*x - 4*(6*(3*B + C)*x^2 + (3*B + C)*b - 2*(3*B + C)*x)*sqrt(-6*b + 1))*l
og(6*x - 2*sqrt(-6*b + 1) - 1) + 3*((6*B + C)*b + 2*(3*C*b - 18*A - 3*B -
C)*x + 6*A)*sqrt(-6*b + 1) + 18*A)/(6*(6*b - 1)*x^2 + 6*b^2 - 2*(6*b - 1)*
x - b)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 247850 vs. $2(165) = 330$.

Time = 10.77 (sec) , antiderivative size = 247850, normalized size of antiderivative = 1277.58

$$\int \frac{A + Bx + Cx^2}{1 - (1 - 6b)^{3/2} - 9b + 54bx - 54x^2 + 108x^3}\,dx = \text{Too large to display}$$

input
```
integrate((C*x**2+B*x+A)/(1-(1-6*b)**(3/2)-9*b+54*b*x-54*x**2+108*x**3),x)
```

output
```
(C/216 - sqrt((36*b**2 - 12*b + 1)*(-31104*A**2*b + 5184*A**2 - 10368*A*B*
b + 1728*A*B - 5184*A*C*b**2 + 144*A*C + 20736*B**2*b**2 - 7776*B**2*b + 7
20*B**2 + 12960*B*C*b**2 - 4608*B*C*b + 408*B*C + 17280*C**2*b**3 - 6480*C
**2*b**2 + 696*C**2*b + 81*C**2*(1 - 6*b)*(36*b**2 - 12*b + 1) - 16*C**2 +
 16*sqrt(1 - 6*b)*(-1296*A*B*b + 216*A*B - 432*A*C*b + 72*A*C - 216*B**2*b
 + 36*B**2 - 108*B*C*b**2 - 72*B*C*b + 15*B*C - 36*C**2*b**2 + C**2)))/(19
44*sqrt(1 - 6*b)*(36*b**2 - 12*b + 1)))*log(-80621568*A**4*b/(-161243136*A
**4*sqrt(1 - 6*b) + 1289945088*A**3*B*b - 107495424*A**3*B*sqrt(1 - 6*b) -
 214990848*A**3*B - 53747712*A**3*C*b*sqrt(1 - 6*b) + 429981696*A**3*C*b -
 8957952*A**3*C*sqrt(1 - 6*b) - 71663616*A**3*C + 644972544*A**2*B**2*b*sq
rt(1 - 6*b) + 644972544*A**2*B**2*b - 134369280*A**2*B**2*sqrt(1 - 6*b) -
107495424*A**2*B**2 + 322486272*A**2*B*C*b**2 + 403107840*A**2*B*C*b*sqrt(
1 - 6*b) + 214990848*A**2*B*C*b - 76142592*A**2*B*C*sqrt(1 - 6*b) - 447897
60*A**2*B*C - 6718464*A**2*C**2*b**2*sqrt(1 - 6*b) + 107495424*A**2*C**2*b
**2 + 69424128*A**2*C**2*b*sqrt(1 - 6*b) - 12130560*A**2*C**2*sqrt(1 - 6*b
) - 2985984*A**2*C**2 - 859963392*A*B**3*b**2 + 214990848*A*B**3*b*sqrt(1
- 6*b) + 394149888*A*B**3*b - 38817792*A*B**3*sqrt(1 - 6*b) - 41803776*A*B
**3 + 107495424*A*B**2*C*b**2*sqrt(1 - 6*b) - 752467968*A*B**2*C*b**2 + 13
8848256*A*B**2*C*b*sqrt(1 - 6*b) + 322486272*A*B**2*C*b - 27620352*A*B**2*
C*sqrt(1 - 6*b) - 32845824*A*B**2*C + 26873856*A*B*C**2*b**3 + 69424128...
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}\,dx=\int \frac{Cx^2+Bx+A}{108\,x^3+54\,bx-54\,x^2-(-6\,b+1)^{\frac{3}{2}}-9\,b+1}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3),x, alg
orithm="maxima")
```

output
```
integrate((C*x^2 + B*x + A)/(108*x^3 + 54*b*x - 54*x^2 - (-6*b + 1)^(3/2)
- 9*b + 1), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A+Bx+Cx^2}{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((C*x^2+B*x+A)/(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3),x, alg
orithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{-972,[1]%%%}+%%%{162,[0]%%%},[2]%%%}+%%%{%%{[%%%{-324,
[1]%%%}+%
```

Mupad [B] (verification not implemented)

Time = 13.20 (sec) , antiderivative size = 1077, normalized size of antiderivative = 5.55

$$\int \frac{A+Bx+Cx^2}{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}\,dx = \text{Too large to display}$$

input
```
int(-(A + B*x + C*x^2)/(9*b - 54*b*x + (1 - 6*b)^(3/2) + 54*x^2 - 108*x^3
- 1),x)
```

output
```
(B/324 + C/972 - ((1 - 6*b)^(3/2)*(A/1944 + B/11664 + C/34992 - (C*b)/1166
4))/(b^2 - b/3 + 1/36))/((1 - 6*b)^(3/2)/(36*(b - 1/6)) - x + 1/6) + (log(
(1 - 6*b)^(3/2)*(-(6*b - 1)^3)^(1/2) - 12*b*(-(6*b - 1)^3)^(1/2) - 12*x*(-
(6*b - 1)^3)^(1/2) - 54*b + 2*(-(6*b - 1)^3)^(1/2) + 324*b^2 - 648*b^3 + 7
2*b*x*(-(6*b - 1)^3)^(1/2) + 3)*(24*C - 576*C*b - 3*C*(6*b - 1)^3 + 24*B*(
864*b^2 - (6*b - 1)^3 - 144*b - 1728*b^3 + 8)^(1/2) + 8*C*(864*b^2 - (6*b
- 1)^3 - 144*b - 1728*b^3 + 8)^(1/2) + 5184*C*b^2 - 20736*C*b^3 + 31104*C*
b^4 + 864*B*b^2*(864*b^2 - (6*b - 1)^3 - 144*b - 1728*b^3 + 8)^(1/2) + 288
*C*b^2*(864*b^2 - (6*b - 1)^3 - 144*b - 1728*b^3 + 8)^(1/2) + 18*C*b*(6*b
- 1)^3 - 288*B*b*(864*b^2 - (6*b - 1)^3 - 144*b - 1728*b^3 + 8)^(1/2) - 96
*C*b*(864*b^2 - (6*b - 1)^3 - 144*b - 1728*b^3 + 8)^(1/2) + 72*A*(1 - 6*b)
^(3/2)*(864*b^2 - (6*b - 1)^3 - 144*b - 1728*b^3 + 8)^(1/2) + 12*B*(1 - 6*
b)^(3/2)*(864*b^2 - (6*b - 1)^3 - 144*b - 1728*b^3 + 8)^(1/2) + C*(1 - 6*b
)^(3/2)*(864*b^2 - (6*b - 1)^3 - 144*b - 1728*b^3 + 8)^(1/2) + 6*C*b*(1 -
6*b)^(3/2)*(864*b^2 - (6*b - 1)^3 - 144*b - 1728*b^3 + 8)^(1/2)))/(648*(6*
b*(6*b - 1)^3 - 192*b - (6*b - 1)^3 + 1728*b^2 - 6912*b^3 + 10368*b^4 + 8)
) - (log(54*b - 12*b*(-(6*b - 1)^3)^(1/2) - 12*x*(-(6*b - 1)^3)^(1/2) + (1
 - 6*b)^(3/2)*(-(6*b - 1)^3)^(1/2) + 2*(-(6*b - 1)^3)^(1/2) - 324*b^2 + 64
8*b^3 + 72*b*x*(-(6*b - 1)^3)^(1/2) - 3)*(576*C*b - 24*C + 3*C*(6*b - 1)^3
 + 24*B*(864*b^2 - (6*b - 1)^3 - 144*b - 1728*b^3 + 8)^(1/2) + 8*C*(864...
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 3061, normalized size of antiderivative = 15.78

$$\int \frac{A+Bx+Cx^2}{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}\,dx = \text{Too large to display}$$

input
```
int((C*x^2+B*x+A)/(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3),x)
```

output

```
( - 72*sqrt(6*b - 1)*sqrt( - 6*b + 1)*atan((6*x - 1)/sqrt(6*b - 1))*a*b -
432*sqrt(6*b - 1)*sqrt( - 6*b + 1)*atan((6*x - 1)/sqrt(6*b - 1))*a*x**2 +
144*sqrt(6*b - 1)*sqrt( - 6*b + 1)*atan((6*x - 1)/sqrt(6*b - 1))*a*x - 60*
sqrt(6*b - 1)*sqrt( - 6*b + 1)*atan((6*x - 1)/sqrt(6*b - 1))*b**2*c - 12*s
qrt(6*b - 1)*sqrt( - 6*b + 1)*atan((6*x - 1)/sqrt(6*b - 1))*b**2 - 360*sqr
t(6*b - 1)*sqrt( - 6*b + 1)*atan((6*x - 1)/sqrt(6*b - 1))*b*c*x**2 + 120*s
qrt(6*b - 1)*sqrt( - 6*b + 1)*atan((6*x - 1)/sqrt(6*b - 1))*b*c*x + 8*sqrt
(6*b - 1)*sqrt( - 6*b + 1)*atan((6*x - 1)/sqrt(6*b - 1))*b*c - 72*sqrt(6*b
 - 1)*sqrt( - 6*b + 1)*atan((6*x - 1)/sqrt(6*b - 1))*b*x**2 + 24*sqrt(6*b
- 1)*sqrt( - 6*b + 1)*atan((6*x - 1)/sqrt(6*b - 1))*b*x + 48*sqrt(6*b - 1)
*sqrt( - 6*b + 1)*atan((6*x - 1)/sqrt(6*b - 1))*c*x**2 - 16*sqrt(6*b - 1)*
sqrt( - 6*b + 1)*atan((6*x - 1)/sqrt(6*b - 1))*c*x + 144*sqrt(6*b - 1)*ata
n((6*x - 1)/sqrt(6*b - 1))*b**3 + 48*sqrt(6*b - 1)*atan((6*x - 1)/sqrt(6*b
 - 1))*b**2*c + 864*sqrt(6*b - 1)*atan((6*x - 1)/sqrt(6*b - 1))*b**2*x**2
- 288*sqrt(6*b - 1)*atan((6*x - 1)/sqrt(6*b - 1))*b**2*x - 24*sqrt(6*b - 1
)*atan((6*x - 1)/sqrt(6*b - 1))*b**2 + 288*sqrt(6*b - 1)*atan((6*x - 1)/sq
rt(6*b - 1))*b*c*x**2 - 96*sqrt(6*b - 1)*atan((6*x - 1)/sqrt(6*b - 1))*b*c
*x - 8*sqrt(6*b - 1)*atan((6*x - 1)/sqrt(6*b - 1))*b*c - 144*sqrt(6*b - 1)
*atan((6*x - 1)/sqrt(6*b - 1))*b*x**2 + 48*sqrt(6*b - 1)*atan((6*x - 1)/sq
rt(6*b - 1))*b*x - 48*sqrt(6*b - 1)*atan((6*x - 1)/sqrt(6*b - 1))*c*x**...
```

3.140 $\int \frac{(e+fx)^2}{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 218

$$\int \frac{(e+fx)^2}{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}\,dx = -\frac{18e^2+6\left(1-\sqrt{1-6b}\right)ef+\left(1-\sqrt{1-6b}-3b\right)f^2}{162\sqrt{1-6b}\left(1-\sqrt{1-6b}-6x\right)} - \frac{\left(18e^2-\left(2-2\sqrt{1-6b}-15b\right)f^2+6e\left(f+2\sqrt{1-6b}f\right)\right)\log\left(1-\sqrt{1-6b}-6x\right)}{486(1-6b)} + \frac{\left(36e^2+\left(5+4\sqrt{1-6b}-24b\right)f^2+12e\left(f+2\sqrt{1-6b}f\right)\right)\log\left(1+2\sqrt{1-6b}-6x\right)}{972(1-6b)}$$

output

```
-1/162*(18*e^2+6*(1-(1-6*b)^(1/2))*e*f+(1-(1-6*b)^(1/2)-3*b)*f^2)/(1-6*b)^
(1/2)/(1-(1-6*b)^(1/2)-6*x)-(18*e^2-(2-2*(1-6*b)^(1/2)-15*b)*f^2+6*e*(f+2*
(1-6*b)^(1/2)*f))*ln(1-(1-6*b)^(1/2)-6*x)/(486-2916*b)+(36*e^2+(5+4*(1-6*b
)^(1/2)-24*b)*f^2+12*e*(f+2*(1-6*b)^(1/2)*f))*ln(1+2*(1-6*b)^(1/2)-6*x)/(9
72-5832*b)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 460 vs. $2(218) = 436$.

Time = 1.11 (sec) , antiderivative size = 460, normalized size of antiderivative = 2.11

$$\int \frac{(e+fx)^2}{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}\,dx = \frac{\frac{6\left(12ef\left(b\left(\sqrt{1-6b}-6x\right)+x-\sqrt{1-6b}x\right)-6e^2\left(-1-\sqrt{1-6b}+6b+6\sqrt{1-6b}x\right)\cdots\right.}{(-1+6b)(b+2x(\cdots}}{}$$

input
```
Integrate[(e + f*x)^2/(1 - (1 - 6*b)^(3/2) - 9*b + 54*b*x - 54*x^2 + 108*x
^3),x]
```

output
```
((6*(12*e*f*(b*(Sqrt[1 - 6*b] - 6*x) + x - Sqrt[1 - 6*b]*x) - 6*e^2*(-1 -
Sqrt[1 - 6*b] + 6*b + 6*Sqrt[1 - 6*b]*x) + f^2*(6*b^2 - 2*(-1 + Sqrt[1 - 6
*b])*x + b*(-1 + Sqrt[1 - 6*b] + 6*(-2 + Sqrt[1 - 6*b])*x))))/((-1 + 6*b)*
(b + 2*x*(-1 + 3*x))) + (2*(-36*Sqrt[1 - 6*b]*e^2 - 12*(2 + Sqrt[1 - 6*b]
- 12*b)*e*f + (-4 - 5*Sqrt[1 - 6*b] + 24*(1 + Sqrt[1 - 6*b])*b)*f^2)*ArcTa
n[(-1 + 6*x)/(2*Sqrt[-1 + 6*b])])/(-1 + 6*b)^(3/2) - (4*(18*Sqrt[1 - 6*b]*
e^2 + 6*(2 + Sqrt[1 - 6*b] - 12*b)*e*f + (2 - 2*Sqrt[1 - 6*b] + 3*(-4 + 5*
Sqrt[1 - 6*b])*b)*f^2)*ArcTan[(-1 + 6*x)/Sqrt[-1 + 6*b]])/(-1 + 6*b)^(3/2)
 - ((36*e^2 + (5 + 4*Sqrt[1 - 6*b] - 24*b)*f^2 + 12*e*(f + 2*Sqrt[1 - 6*b]
*f))*Log[1 - 8*b + 4*x - 12*x^2])/(-1 + 6*b) + (2*(18*e^2 + (-2 + 2*Sqrt[1
 - 6*b] + 15*b)*f^2 + 6*e*(f + 2*Sqrt[1 - 6*b]*f))*Log[b - 2*x + 6*x^2])/(
-1 + 6*b))/1944
```

Rubi [A] (verified)

Time = 1.04 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2488, 27, 27, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2}{54bx-(1-6b)^{3/2}-9b+108x^3-54x^2+1}\,dx$$

$$\Big\downarrow \; 2488$$

$$99179645184(1-6b)^3\int -\frac{(e+fx)^2}{49589822592\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)^2\left(\left(2\sqrt{1-6b}+1\right)(1-6b)-6(1-6b)x\right)}dx$$

$$\Big\downarrow \text{27}$$

$$-2(1-6b)^3\int \frac{(e+fx)^2}{(1-6b)\left(-6x+2\sqrt{1-6b}+1\right)\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)^2}dx$$

$$\Big\downarrow \text{27}$$

$$-2(1-6b)^2\int \frac{(e+fx)^2}{\left(-6x+2\sqrt{1-6b}+1\right)\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)^2}dx$$

$$\Big\downarrow \text{99}$$

$$-2(1-6b)^2\int \left(\frac{18e^2+6\left(1-\sqrt{1-6b}\right)fe+\left(-3b-\sqrt{1-6b}+1\right)f^2}{54(1-6b)^{5/2}\left(-6x-\sqrt{1-6b}+1\right)^2}+\frac{-18e^2-6\left(2\sqrt{1-6b}f+f\right)e+\left(-15b-2\sqrt{1-\right.}}{162(1-6b)^3\left(-6x-\sqrt{1-6b}+1\right)}\right.$$

$$\Big\downarrow \text{2009}$$

$$-2(1-6b)^2\left(\frac{6\left(1-\sqrt{1-6b}\right)ef+\left(-3b-\sqrt{1-6b}+1\right)f^2+18e^2}{324(1-6b)^{5/2}\left(-\sqrt{1-6b}-6x+1\right)}+\frac{\left(6e\left(2\sqrt{1-6b}f+f\right)-\left(-15b-2\sqrt{1-6b}+2\right)f^2\right.}{972(1-6b)^3}\right.$$

input
```
Int[(e + f*x)^2/(1 - (1 - 6*b)^(3/2) - 9*b + 54*b*x - 54*x^2 + 108*x^3),x]
```

output
```
-2*(1 - 6*b)^2*((18*e^2 + 6*(1 - Sqrt[1 - 6*b])*e*f + (1 - Sqrt[1 - 6*b] -
 3*b)*f^2)/(324*(1 - 6*b)^(5/2)*(1 - Sqrt[1 - 6*b] - 6*x)) + ((18*e^2 - (2
 - 2*Sqrt[1 - 6*b] - 15*b)*f^2 + 6*e*(f + 2*Sqrt[1 - 6*b]*f))*Log[1 - Sqrt
[1 - 6*b] - 6*x])/(972*(1 - 6*b)^3) - ((36*e^2 + (5 + 4*Sqrt[1 - 6*b] - 24
*b)*f^2 + 12*e*(f + 2*Sqrt[1 - 6*b]*f))*Log[1 + 2*Sqrt[1 - 6*b] - 6*x])/(1
944*(1 - 6*b)^3))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2488
```
Int[((e_.) + (f_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(
x_)^3)^(p_), x_Symbol] :> Simp[1/(4^p*(c^2 - 3*b*d)^(3*p))   Int[(e + f*x)^
m*(c^3 - 4*b*c*d + 9*a*d^2 + d*(c^2 - 3*b*d)*x)^p*(b*c - 9*a*d + 2*(c^2 - 3
*b*d)*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && NeQ[c^2 - 3
*b*d, 0] && EqQ[b^2*c^2 - 4*a*c^3 - 4*b^3*d + 18*a*b*c*d - 27*a^2*d^2, 0] &
& ILtQ[p, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.56 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.34

method	result
default	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(-(1-6b)^{\frac{3}{2}}+108\_Z^{3}+54b\_Z-54\_Z^{2}-9b+1\right)}\frac{\left(-\_R^{2}f^{2}-2\_Ref-e^{2}\right)\ln\left(x-\_R\right)}{-6\_R^{2}+2\_R-b}\right)}{54}$
parallelrisch	$\frac{6f^{2}+24\sqrt{1-6b}\,\ln\left(-\frac{1}{6}+x+\frac{\sqrt{1-6b}}{6}\right)x\,f^{2}+30\sqrt{1-6b}\,\ln\left(-\frac{1}{6}+x+\frac{\sqrt{1-6b}}{6}\right)b\,f^{2}+144\ln\left(x-\frac{1}{6}-\frac{\sqrt{1-6b}}{3}\right)efb-24\sqrt{1-6b}\,\ln\left(x-\frac{1}{6}\right.}{}$

input
```
int((f*x+e)^2/(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3),x,method=_RETURN
VERBOSE)
```

output
```
1/54*sum((-_R^2*f^2-2*_R*e*f-e^2)/(-6*_R^2+2*_R-b)*ln(x-_R),_R=RootOf(-(1-
6*b)^(3/2)+108*_Z^3+54*b*_Z-54*_Z^2-9*b+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 387 vs. $2(185) = 370$.

Time = 0.10 (sec) , antiderivative size = 387, normalized size of antiderivative = 1.78

$$\int \frac{(e+fx)^2}{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}\,dx = -\frac{18\,(6\,b-1)e^2-3\,(6\,b^2-b)f^2+6\,(6\,(6\,b-1)ef+(6\,b-1)f^2)x-2\,\big(18\,be^2+6\,bef+(15\,b^2-2\,b)f^2+6}{}$$

input
```
integrate((f*x+e)^2/(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3),x, algorit
hm="fricas")
```

output
```
-1/972*(18*(6*b - 1)*e^2 - 3*(6*b^2 - b)*f^2 + 6*(6*(6*b - 1)*e*f + (6*b -
 1)*f^2)*x - 2*(18*b*e^2 + 6*b*e*f + (15*b^2 - 2*b)*f^2 + 6*((15*b - 2)*f^
2 + 18*e^2 + 6*e*f)*x^2 - 2*((15*b - 2)*f^2 + 18*e^2 + 6*e*f)*x + 2*(6*b*e
*f + b*f^2 + 6*(6*e*f + f^2)*x^2 - 2*(6*e*f + f^2)*x)*sqrt(-6*b + 1))*log(
6*x + sqrt(-6*b + 1) - 1) + (36*b*e^2 + 12*b*e*f - (24*b^2 - 5*b)*f^2 - 6*
((24*b - 5)*f^2 - 36*e^2 - 12*e*f)*x^2 + 2*((24*b - 5)*f^2 - 36*e^2 - 12*e
*f)*x + 4*(6*b*e*f + b*f^2 + 6*(6*e*f + f^2)*x^2 - 2*(6*e*f + f^2)*x)*sqrt
(-6*b + 1))*log(6*x - 2*sqrt(-6*b + 1) - 1) - 3*(12*b*e*f + b*f^2 + 6*e^2
+ 2*((3*b - 1)*f^2 - 18*e^2 - 6*e*f)*x)*sqrt(-6*b + 1))/(6*(6*b - 1)*x^2 +
 6*b^2 - 2*(6*b - 1)*x - b)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 95302 vs. $2(187) = 374$.

Time = 7.55 (sec) , antiderivative size = 95302, normalized size of antiderivative = 437.17

$$\int \frac{(e+fx)^2}{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)**2/(1-(1-6*b)**(3/2)-9*b+54*b*x-54*x**2+108*x**3),x)
```

output
```
(f**2/216 - sqrt((36*b**2 - 12*b + 1)*(17280*b**3*f**4 + 77760*b**2*e**2*f
**2 + 25920*b**2*e*f**3 - 6480*b**2*f**4 - 31104*b*e**4 - 20736*b*e**3*f -
 31104*b*e**2*f**2 - 9216*b*e*f**3 + 696*b*f**4 + 5184*e**4 + 3456*e**3*f
+ 3024*e**2*f**2 + 816*e*f**3 + 81*f**4*(1 - 6*b)*(36*b**2 - 12*b + 1) - 1
6*f**4 + 16*f*sqrt(1 - 6*b)*(-216*b**2*e*f**2 - 36*b**2*f**3 - 2592*b*e**3
 - 1296*b*e**2*f - 144*b*e*f**2 + 432*e**3 + 216*e**2*f + 30*e*f**2 + f**3
)))/(1944*sqrt(1 - 6*b)*(36*b**2 - 12*b + 1)))*log(-108864*b**5*f**8/(1492
992*b**4*e*f**7 - 7776*b**4*f**8*sqrt(1 - 6*b) + 248832*b**4*f**8 - 519561
216*b**3*e**3*f**5 + 17542656*b**3*e**2*f**6*sqrt(1 - 6*b) - 259780608*b**
3*e**2*f**6 + 5847552*b**3*e*f**7*sqrt(1 - 6*b) - 44292096*b**3*e*f**7 + 4
92480*b**3*f**8*sqrt(1 - 6*b) - 2571264*b**3*f**8 - 6234734592*b**2*e**5*f
**3 - 723354624*b**2*e**4*f**4*sqrt(1 - 6*b) - 5195612160*b**2*e**4*f**4 -
 482236416*b**2*e**3*f**5*sqrt(1 - 6*b) - 1472090112*b**2*e**3*f**5 - 1293
30432*b**2*e**2*f**6*sqrt(1 - 6*b) - 158754816*b**2*e**2*f**6 - 16319232*b
**2*e*f**7*sqrt(1 - 6*b) - 2156544*b**2*e*f**7 - 803088*b**2*f**8*sqrt(1 -
 6*b) + 442368*b**2*f**8 + 2579890176*b*e**7*f + 2526142464*b*e**6*f**2*sq
rt(1 - 6*b) + 3009871872*b*e**6*f**2 + 2526142464*b*e**5*f**3*sqrt(1 - 6*b
) + 3583180800*b*e**5*f**3 + 1293677568*b*e**4*f**4*sqrt(1 - 6*b) + 214990
8480*b*e**4*f**4 + 394647552*b*e**3*f**5*sqrt(1 - 6*b) + 603666432*b*e**3*
f**5 + 70886016*b*e**2*f**6*sqrt(1 - 6*b) + 81533952*b*e**2*f**6 + 6901...
```

Maxima [F]

$$\int \frac{(e+fx)^2}{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}\,dx=\int \frac{(fx+e)^2}{108\,x^3+54\,bx-54\,x^2-(-6\,b+1)^{\frac{3}{2}}-9\,b+1}\,dx$$

input
```
integrate((f*x+e)^2/(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3),x, algorit
hm="maxima")
```

output
```
integrate((f*x + e)^2/(108*x^3 + 54*b*x - 54*x^2 - (-6*b + 1)^(3/2) - 9*b
+ 1), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(e+fx)^2}{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((f*x+e)^2/(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3),x, algorit
hm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{-972,[1]%%%}+%%%{162,[0]%%%},[2]%%%}+%%%{%%{[%%%{-324,
[1]%%%}+%
```

Mupad [B] (verification not implemented)

Time = 13.07 (sec) , antiderivative size = 1147, normalized size of antiderivative = 5.26

$$\int \frac{(e+fx)^2}{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}\,dx = \text{Too large to display}$$

input
```
int(-(e + f*x)^2/(9*b - 54*b*x + (1 - 6*b)^(3/2) + 54*x^2 - 108*x^3 - 1),x
)
```

output
```
((e*f)/162 + f^2/972 - ((1 - 6*b)^(3/2)*((e*f)/5832 - (b*f^2)/11664 + e^2/
1944 + f^2/34992))/(b^2 - b/3 + 1/36))/((1 - 6*b)^(3/2)/(36*(b - 1/6)) - x
 + 1/6) + (log((1 - 6*b)^(3/2)*(-(6*b - 1)^3)^(1/2) - 12*b*(-(6*b - 1)^3)^
(1/2) - 12*x*(-(6*b - 1)^3)^(1/2) - 54*b + 2*(-(6*b - 1)^3)^(1/2) + 324*b^
2 - 648*b^3 + 72*b*x*(-(6*b - 1)^3)^(1/2) + 3)*(24*f^2 - 3*f^2*(6*b - 1)^3
 - 576*b*f^2 + 8*f^2*(864*b^2 - (6*b - 1)^3 - 144*b - 1728*b^3 + 8)^(1/2)
+ 5184*b^2*f^2 - 20736*b^3*f^2 + 31104*b^4*f^2 + 18*b*f^2*(6*b - 1)^3 - 96
*b*f^2*(864*b^2 - (6*b - 1)^3 - 144*b - 1728*b^3 + 8)^(1/2) + 72*e^2*(1 -
6*b)^(3/2)*(864*b^2 - (6*b - 1)^3 - 144*b - 1728*b^3 + 8)^(1/2) + f^2*(1 -
 6*b)^(3/2)*(864*b^2 - (6*b - 1)^3 - 144*b - 1728*b^3 + 8)^(1/2) + 288*b^2
*f^2*(864*b^2 - (6*b - 1)^3 - 144*b - 1728*b^3 + 8)^(1/2) + 48*e*f*(864*b^
2 - (6*b - 1)^3 - 144*b - 1728*b^3 + 8)^(1/2) - 576*b*e*f*(864*b^2 - (6*b
- 1)^3 - 144*b - 1728*b^3 + 8)^(1/2) + 24*e*f*(1 - 6*b)^(3/2)*(864*b^2 - (
6*b - 1)^3 - 144*b - 1728*b^3 + 8)^(1/2) + 1728*b^2*e*f*(864*b^2 - (6*b -
1)^3 - 144*b - 1728*b^3 + 8)^(1/2) + 6*b*f^2*(1 - 6*b)^(3/2)*(864*b^2 - (6
*b - 1)^3 - 144*b - 1728*b^3 + 8)^(1/2)))/(648*(6*b*(6*b - 1)^3 - 192*b -
(6*b - 1)^3 + 1728*b^2 - 6912*b^3 + 10368*b^4 + 8)) - (log(54*b - 12*b*(-(
6*b - 1)^3)^(1/2) - 12*x*(-(6*b - 1)^3)^(1/2) + (1 - 6*b)^(3/2)*(-(6*b - 1
)^3)^(1/2) + 2*(-(6*b - 1)^3)^(1/2) - 324*b^2 + 648*b^3 + 72*b*x*(-(6*b -
1)^3)^(1/2) - 3)*(576*b*f^2 + 3*f^2*(6*b - 1)^3 - 24*f^2 + 8*f^2*(864*b...
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 3275, normalized size of antiderivative = 15.02

$$\int \frac{(e+fx)^2}{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}\,dx = \text{Too large to display}$$

input
```
int((f*x+e)^2/(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3),x)
```

output

```
( - 60*sqrt(6*b - 1)*sqrt( - 6*b + 1)*atan((6*x - 1)/sqrt(6*b - 1))*b**2*f
**2 - 72*sqrt(6*b - 1)*sqrt( - 6*b + 1)*atan((6*x - 1)/sqrt(6*b - 1))*b*e*
*2 - 24*sqrt(6*b - 1)*sqrt( - 6*b + 1)*atan((6*x - 1)/sqrt(6*b - 1))*b*e*f
 - 360*sqrt(6*b - 1)*sqrt( - 6*b + 1)*atan((6*x - 1)/sqrt(6*b - 1))*b*f**2
*x**2 + 120*sqrt(6*b - 1)*sqrt( - 6*b + 1)*atan((6*x - 1)/sqrt(6*b - 1))*b
*f**2*x + 8*sqrt(6*b - 1)*sqrt( - 6*b + 1)*atan((6*x - 1)/sqrt(6*b - 1))*b
*f**2 - 432*sqrt(6*b - 1)*sqrt( - 6*b + 1)*atan((6*x - 1)/sqrt(6*b - 1))*e
**2*x**2 + 144*sqrt(6*b - 1)*sqrt( - 6*b + 1)*atan((6*x - 1)/sqrt(6*b - 1)
)*e**2*x - 144*sqrt(6*b - 1)*sqrt( - 6*b + 1)*atan((6*x - 1)/sqrt(6*b - 1)
)*e*f*x**2 + 48*sqrt(6*b - 1)*sqrt( - 6*b + 1)*atan((6*x - 1)/sqrt(6*b - 1
))*e*f*x + 48*sqrt(6*b - 1)*sqrt( - 6*b + 1)*atan((6*x - 1)/sqrt(6*b - 1))
*f**2*x**2 - 16*sqrt(6*b - 1)*sqrt( - 6*b + 1)*atan((6*x - 1)/sqrt(6*b - 1
))*f**2*x + 288*sqrt(6*b - 1)*atan((6*x - 1)/sqrt(6*b - 1))*b**2*e*f + 48*
sqrt(6*b - 1)*atan((6*x - 1)/sqrt(6*b - 1))*b**2*f**2 + 1728*sqrt(6*b - 1)
*atan((6*x - 1)/sqrt(6*b - 1))*b*e*f*x**2 - 576*sqrt(6*b - 1)*atan((6*x -
1)/sqrt(6*b - 1))*b*e*f*x - 48*sqrt(6*b - 1)*atan((6*x - 1)/sqrt(6*b - 1))
*b*e*f + 288*sqrt(6*b - 1)*atan((6*x - 1)/sqrt(6*b - 1))*b*f**2*x**2 - 96*
sqrt(6*b - 1)*atan((6*x - 1)/sqrt(6*b - 1))*b*f**2*x - 8*sqrt(6*b - 1)*ata
n((6*x - 1)/sqrt(6*b - 1))*b*f**2 - 288*sqrt(6*b - 1)*atan((6*x - 1)/sqrt(
6*b - 1))*e*f*x**2 + 96*sqrt(6*b - 1)*atan((6*x - 1)/sqrt(6*b - 1))*e*f...
```

3.141 $\int \frac{e+fx}{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 139

$$\int \frac{e+fx}{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}\,dx = -\frac{6e+f-\sqrt{1-6b}f}{54\sqrt{1-6b}\left(1-\sqrt{1-6b}-6x\right)} - \frac{\left(6e+f+2\sqrt{1-6b}f\right)\log\left(1-\sqrt{1-6b}-6x\right)}{162(1-6b)} + \frac{\left(6e+f+2\sqrt{1-6b}f\right)\log\left(1+2\sqrt{1-6b}-6x\right)}{162(1-6b)}$$

output
```
-1/54*(6*e+f-(1-6*b)^(1/2)*f)/(1-6*b)^(1/2)/(1-(1-6*b)^(1/2)-6*x)-(6*e+f+2
*(1-6*b)^(1/2)*f)*ln(1-(1-6*b)^(1/2)-6*x)/(162-972*b)+(6*e+f+2*(1-6*b)^(1/
2)*f)*ln(1+2*(1-6*b)^(1/2)-6*x)/(162-972*b)
```

Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.99

$$\int \frac{e+fx}{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}\,dx = \frac{1}{324}\left(\frac{6\left(e\left(1+\sqrt{1-6b}-6b-6\sqrt{1-6b}x\right)+f\left(b(\sqrt{\right.\right.}{(-1+6b)(b+2x(-1}\right.$$
$$-\frac{2\left(6\sqrt{1-6b}e+\left(2+\sqrt{1-6b}-12b\right)f\right)\arctan\left(\frac{-1+6x}{2\sqrt{-1+6b}}\right)}{(-1+6b)^{3/2}}$$
$$-\frac{2\left(6\sqrt{1-6b}e+\left(2+\sqrt{1-6b}-12b\right)f\right)\arctan\left(\frac{-1+6x}{\sqrt{-1+6b}}\right)}{(-1+6b)^{3/2}}$$
$$-\frac{\left(6e+f+2\sqrt{1-6b}f\right)\log\left(1-8b+4x-12x^2\right)}{-1+6b}$$
$$\left.+\frac{\left(6e+f+2\sqrt{1-6b}f\right)\log\left(b-2x+6x^2\right)}{-1+6b}\right)$$

input
```
Integrate[(e + f*x)/(1 - (1 - 6*b)^(3/2) - 9*b + 54*b*x - 54*x^2 + 108*x^3
),x]
```

output
```
((6*(e*(1 + Sqrt[1 - 6*b] - 6*b - 6*Sqrt[1 - 6*b]*x) + f*(b*(Sqrt[1 - 6*b]
 - 6*x) + x - Sqrt[1 - 6*b]*x)))/((-1 + 6*b)*(b + 2*x*(-1 + 3*x))) - (2*(6
*Sqrt[1 - 6*b]*e + (2 + Sqrt[1 - 6*b] - 12*b)*f)*ArcTan[(-1 + 6*x)/(2*Sqrt
[-1 + 6*b])])/(-1 + 6*b)^(3/2) - (2*(6*Sqrt[1 - 6*b]*e + (2 + Sqrt[1 - 6*b
] - 12*b)*f)*ArcTan[(-1 + 6*x)/Sqrt[-1 + 6*b]])/(-1 + 6*b)^(3/2) - ((6*e +
 f + 2*Sqrt[1 - 6*b]*f)*Log[1 - 8*b + 4*x - 12*x^2])/(-1 + 6*b) + ((6*e +
f + 2*Sqrt[1 - 6*b]*f)*Log[b - 2*x + 6*x^2])/(-1 + 6*b))/324
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$, Rules used = {2488, 27, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e+fx}{54bx-(1-6b)^{3/2}-9b+108x^3-54x^2+1}dx$$

$$\Big\downarrow \text{2488}$$

$$99179645184(1-6b)^3\int -\frac{e+fx}{49589822592\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)^2\left(\left(2\sqrt{1-6b}+1\right)(1-6b)-6(1-6b)x\right)}dx$$

$$\Big\downarrow \text{27}$$

$$-2(1-6b)^3\int \frac{e+fx}{(1-6b)\left(-6x+2\sqrt{1-6b}+1\right)\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)^2}dx$$

$$\Big\downarrow \text{27}$$

$$-2(1-6b)^2\int \frac{e+fx}{\left(-6x+2\sqrt{1-6b}+1\right)\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)^2}dx$$

$$\Big\downarrow \text{86}$$

$$-2(1-6b)^2\int \left(\frac{-6e-2\sqrt{1-6b}f-f}{54(6b-1)^3\left(-6x+2\sqrt{1-6b}+1\right)}+\frac{-6e-2\sqrt{1-6b}f-f}{54(6b-1)^3\left(6x+\sqrt{1-6b}-1\right)}+\frac{6e-\sqrt{1-6b}f+f}{18\sqrt{1-6b}(6b-1)^2\left(6x+\sqrt{1\right.}}$$

$$\Big\downarrow \text{2009}$$

$$-2(1-6b)^2\left(\frac{-\sqrt{1-6b}f+6e+f}{108(1-6b)^{5/2}\left(-\sqrt{1-6b}-6x+1\right)}+\frac{\left(2\sqrt{1-6b}f+6e+f\right)\log\left(-\sqrt{1-6b}-6x+1\right)}{324(1-6b)^3}-\frac{\left(2\sqrt{1-6b}f+\right.}{}\right.$$

input
```
Int[(e + f*x)/(1 - (1 - 6*b)^(3/2) - 9*b + 54*b*x - 54*x^2 + 108*x^3),x]
```

output
```
-2*(1 - 6*b)^2*((6*e + f - Sqrt[1 - 6*b]*f)/(108*(1 - 6*b)^(5/2)*(1 - Sqrt
[1 - 6*b] - 6*x)) + ((6*e + f + 2*Sqrt[1 - 6*b]*f)*Log[1 - Sqrt[1 - 6*b] -
 6*x])/(324*(1 - 6*b)^3) - ((6*e + f + 2*Sqrt[1 - 6*b]*f)*Log[1 + 2*Sqrt[1
 - 6*b] - 6*x])/(324*(1 - 6*b)^3))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2488
```
Int[((e_.) + (f_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(
x_)^3)^(p_), x_Symbol] :> Simp[1/(4^p*(c^2 - 3*b*d)^(3*p))   Int[(e + f*x)^
m*(c^3 - 4*b*c*d + 9*a*d^2 + d*(c^2 - 3*b*d)*x)^p*(b*c - 9*a*d + 2*(c^2 - 3
*b*d)*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && NeQ[c^2 - 3
*b*d, 0] && EqQ[b^2*c^2 - 4*a*c^3 - 4*b^3*d + 18*a*b*c*d - 27*a^2*d^2, 0] &
& ILtQ[p, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.48 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.46

method	result
default	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(-(1-6b)^{\frac{3}{2}}+108\_Z^{3}+54b\_Z-54\_Z^{2}-9b+1\right)}\frac{\left(-\_R_{f-e}\right)\ln\left(x-\_R\right)}{-6\_R^{2}+2\_R_{-b}}\right)}{54}$
parallelrisch	$\frac{3f-12\sqrt{1-6b}\,\ln\left(x-\frac{1}{6}-\frac{\sqrt{1-6b}}{3}\right)xf+12\sqrt{1-6b}\,\ln\left(-\frac{1}{6}+x+\frac{\sqrt{1-6b}}{6}\right)xf-18bf-18\sqrt{1-6b}\,e-6\ln\left(-\frac{1}{6}+x+\frac{\sqrt{1-6b}}{6}\right)e+\ln\left(-\frac{1}{6}\right.}{}$

input
```
int((f*x+e)/(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3),x,method=_RETURNVE
RBOSE)
```

output
```
1/54*sum((-_R*f-e)/(-6*_R^2+2*_R-b)*ln(x-_R),_R=RootOf(-(1-6*b)^(3/2)+108*
_Z^3+54*b*_Z-54*_Z^2-9*b+1))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.44

$$\int \frac{e+fx}{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}\,dx =$$
$$-\frac{3\,(6\,b-1)fx+3\,(6\,b-1)e-\left(6\,(6\,e+f)x^2+6\,be+bf-2\,(6\,e+f)x+2\,(6\,fx^2+bf-2\,fx)\sqrt{-6\,b+}\right.}{}$$

input
```
integrate((f*x+e)/(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3),x, algorithm
="fricas")
```

output
```
-1/162*(3*(6*b - 1)*f*x + 3*(6*b - 1)*e - (6*(6*e + f)*x^2 + 6*b*e + b*f -
 2*(6*e + f)*x + 2*(6*f*x^2 + b*f - 2*f*x)*sqrt(-6*b + 1))*log(6*x + sqrt(
-6*b + 1) - 1) + (6*(6*e + f)*x^2 + 6*b*e + b*f - 2*(6*e + f)*x + 2*(6*f*x
^2 + b*f - 2*f*x)*sqrt(-6*b + 1))*log(6*x - 2*sqrt(-6*b + 1) - 1) - 3*(b*f
 - (6*e + f)*x + e)*sqrt(-6*b + 1))/(6*(6*b - 1)*x^2 + 6*b^2 - 2*(6*b - 1)
*x - b)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3281 vs. $2(116) = 232$.

Time = 1.46 (sec) , antiderivative size = 3281, normalized size of antiderivative = 23.60

$$\int \frac{e+fx}{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)/(1-(1-6*b)**(3/2)-9*b+54*b*x-54*x**2+108*x**3),x)
```

output
```
-sqrt((-24*b*f**2 + 36*e**2 + 24*e*f*sqrt(1 - 6*b) + 12*e*f + 4*f**2*sqrt(
1 - 6*b) + 5*f**2)/(36*b**2 - 12*b + 1))*log(144*b**2*f**2/(1728*b*e*f + 2
88*b*f**2*sqrt(1 - 6*b) + 288*b*f**2 - 432*e**2*sqrt(1 - 6*b) - 144*e*f*sq
rt(1 - 6*b) - 288*e*f - 60*f**2*sqrt(1 - 6*b) - 48*f**2) - 216*b*e**2/(172
8*b*e*f + 288*b*f**2*sqrt(1 - 6*b) + 288*b*f**2 - 432*e**2*sqrt(1 - 6*b) -
 144*e*f*sqrt(1 - 6*b) - 288*e*f - 60*f**2*sqrt(1 - 6*b) - 48*f**2) - 144*
b*e*f*sqrt(1 - 6*b)/(1728*b*e*f + 288*b*f**2*sqrt(1 - 6*b) + 288*b*f**2 -
432*e**2*sqrt(1 - 6*b) - 144*e*f*sqrt(1 - 6*b) - 288*e*f - 60*f**2*sqrt(1
- 6*b) - 48*f**2) - 360*b*e*f/(1728*b*e*f + 288*b*f**2*sqrt(1 - 6*b) + 288
*b*f**2 - 432*e**2*sqrt(1 - 6*b) - 144*e*f*sqrt(1 - 6*b) - 288*e*f - 60*f*
*2*sqrt(1 - 6*b) - 48*f**2) - 72*b*f**2*sqrt(1 - 6*b)/(1728*b*e*f + 288*b*
f**2*sqrt(1 - 6*b) + 288*b*f**2 - 432*e**2*sqrt(1 - 6*b) - 144*e*f*sqrt(1
- 6*b) - 288*e*f - 60*f**2*sqrt(1 - 6*b) - 48*f**2) - 102*b*f**2/(1728*b*e
*f + 288*b*f**2*sqrt(1 - 6*b) + 288*b*f**2 - 432*e**2*sqrt(1 - 6*b) - 144*
e*f*sqrt(1 - 6*b) - 288*e*f - 60*f**2*sqrt(1 - 6*b) - 48*f**2) + 72*e**2*s
qrt(1 - 6*b)/(1728*b*e*f + 288*b*f**2*sqrt(1 - 6*b) + 288*b*f**2 - 432*e**
2*sqrt(1 - 6*b) - 144*e*f*sqrt(1 - 6*b) - 288*e*f - 60*f**2*sqrt(1 - 6*b)
- 48*f**2) + 36*e**2/(1728*b*e*f + 288*b*f**2*sqrt(1 - 6*b) + 288*b*f**2 -
 432*e**2*sqrt(1 - 6*b) - 144*e*f*sqrt(1 - 6*b) - 288*e*f - 60*f**2*sqrt(1
 - 6*b) - 48*f**2) + 48*e*f*sqrt(1 - 6*b)/(1728*b*e*f + 288*b*f**2*sqrt...
```

Maxima [F]

$$\int \frac{e+fx}{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}\,dx=\int \frac{fx+e}{108\,x^3+54\,bx-54\,x^2-(-6\,b+1)^{\frac{3}{2}}-9\,b+1}\,dx$$

input
```
integrate((f*x+e)/(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3),x, algorithm
="maxima")
```

output
```
integrate((f*x + e)/(108*x^3 + 54*b*x - 54*x^2 - (-6*b + 1)^(3/2) - 9*b +
1), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e+fx}{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((f*x+e)/(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3),x, algorithm
="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{-972,[1]%%%}+%%%{162,[0]%%%},[2]%%%}+%%%{%%{[%%%{-324,
[1]%%%}+%
```

Mupad [B] (verification not implemented)

Time = 12.26 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.27

$$\int \frac{e+fx}{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}\,dx = \frac{\frac{f}{324}-\frac{(1-6\,b)^{3/2}\left(\frac{e}{1944}+\frac{f}{11664}\right)}{b^2-\frac{b}{3}+\frac{1}{36}}}{\frac{(1-6\,b)^{3/2}}{36\,\left(b-\frac{1}{6}\right)}-x+\frac{1}{6}} + \frac{\mathrm{atan}\left(\frac{-b\,12\mathrm{i}-x\,12\mathrm{i}+b\,x\,72\mathrm{i}+(1-6\,b)^{3/2}\,1\mathrm{i}+2\mathrm{i}}{\sqrt{864\,b^2-(6\,b-1)^3-144\,b-1728\,b^3+8}}\right)\left(2\,f-24\,b\,f+6\,e\,(1-6\,b)^{3/2}+f\,(1-6\,b)^{3/2}+72\,b^2\,f\right)\,1\mathrm{i}}{27\,(6\,b-1)\,\sqrt{864\,b^2-(6\,b-1)^3-144\,b-1728\,b^3+8}}$$

input
```
int(-(e + f*x)/(9*b - 54*b*x + (1 - 6*b)^(3/2) + 54*x^2 - 108*x^3 - 1),x)
```

output
```
(f/324 - ((1 - 6*b)^(3/2)*(e/1944 + f/11664))/(b^2 - b/3 + 1/36))/((1 - 6*
b)^(3/2)/(36*(b - 1/6)) - x + 1/6) + (atan((b*x*72i - x*12i - b*12i + (1 -
 6*b)^(3/2)*1i + 2i)/(864*b^2 - (6*b - 1)^3 - 144*b - 1728*b^3 + 8)^(1/2))
*(2*f - 24*b*f + 6*e*(1 - 6*b)^(3/2) + f*(1 - 6*b)^(3/2) + 72*b^2*f)*1i)/(
27*(6*b - 1)*(864*b^2 - (6*b - 1)^3 - 144*b - 1728*b^3 + 8)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 1625, normalized size of antiderivative = 11.69

$$\int \frac{e+fx}{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}\,dx = \text{Too large to display}$$

input
```
int((f*x+e)/(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3),x)
```

output
```
( - 12*sqrt(6*b - 1)*sqrt( - 6*b + 1)*atan((6*x - 1)/sqrt(6*b - 1))*b*e -
2*sqrt(6*b - 1)*sqrt( - 6*b + 1)*atan((6*x - 1)/sqrt(6*b - 1))*b*f - 72*sq
rt(6*b - 1)*sqrt( - 6*b + 1)*atan((6*x - 1)/sqrt(6*b - 1))*e*x**2 + 24*sqr
t(6*b - 1)*sqrt( - 6*b + 1)*atan((6*x - 1)/sqrt(6*b - 1))*e*x - 12*sqrt(6*
b - 1)*sqrt( - 6*b + 1)*atan((6*x - 1)/sqrt(6*b - 1))*f*x**2 + 4*sqrt(6*b
- 1)*sqrt( - 6*b + 1)*atan((6*x - 1)/sqrt(6*b - 1))*f*x + 24*sqrt(6*b - 1)
*atan((6*x - 1)/sqrt(6*b - 1))*b**2*f + 144*sqrt(6*b - 1)*atan((6*x - 1)/s
qrt(6*b - 1))*b*f*x**2 - 48*sqrt(6*b - 1)*atan((6*x - 1)/sqrt(6*b - 1))*b*
f*x - 4*sqrt(6*b - 1)*atan((6*x - 1)/sqrt(6*b - 1))*b*f - 24*sqrt(6*b - 1)
*atan((6*x - 1)/sqrt(6*b - 1))*f*x**2 + 8*sqrt(6*b - 1)*atan((6*x - 1)/sqr
t(6*b - 1))*f*x - 12*sqrt(6*b - 1)*sqrt( - 6*b + 1)*atan((6*x - 1)/(2*sqrt
(6*b - 1)))*b*e - 2*sqrt(6*b - 1)*sqrt( - 6*b + 1)*atan((6*x - 1)/(2*sqrt(
6*b - 1)))*b*f - 72*sqrt(6*b - 1)*sqrt( - 6*b + 1)*atan((6*x - 1)/(2*sqrt(
6*b - 1)))*e*x**2 + 24*sqrt(6*b - 1)*sqrt( - 6*b + 1)*atan((6*x - 1)/(2*sq
rt(6*b - 1)))*e*x - 12*sqrt(6*b - 1)*sqrt( - 6*b + 1)*atan((6*x - 1)/(2*sq
rt(6*b - 1)))*f*x**2 + 4*sqrt(6*b - 1)*sqrt( - 6*b + 1)*atan((6*x - 1)/(2*
sqrt(6*b - 1)))*f*x + 24*sqrt(6*b - 1)*atan((6*x - 1)/(2*sqrt(6*b - 1)))*b
**2*f + 144*sqrt(6*b - 1)*atan((6*x - 1)/(2*sqrt(6*b - 1)))*b*f*x**2 - 48*
sqrt(6*b - 1)*atan((6*x - 1)/(2*sqrt(6*b - 1)))*b*f*x - 4*sqrt(6*b - 1)*at
an((6*x - 1)/(2*sqrt(6*b - 1)))*b*f - 24*sqrt(6*b - 1)*atan((6*x - 1)/(...
```

3.142 $\int \frac{1}{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 88

$$\int \frac{1}{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}\,dx = -\frac{1}{9\sqrt{1-6b}\left(1-\sqrt{1-6b}-6x\right)} - \frac{\log\left(1-\sqrt{1-6b}-6x\right)}{27(1-6b)} + \frac{\log\left(1+2\sqrt{1-6b}-6x\right)}{27(1-6b)}$$

output
```
-1/9/(1-6*b)^(1/2)/(1-(1-6*b)^(1/2)-6*x)-ln(1-(1-6*b)^(1/2)-6*x)/(27-162*b
)+ln(1+2*(1-6*b)^(1/2)-6*x)/(27-162*b)
```

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.90

$$\int \frac{1}{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}\,dx = \frac{1}{54}\left(\frac{1+\sqrt{1-6b}-6b-6\sqrt{1-6b}x}{(-1+6b)(b+2x(-1+3x))} - \frac{2\sqrt{1-6b}\arctan\left(\frac{-1+6x}{2\sqrt{-1+6b}}\right)}{(-1+6b)^{3/2}} - \frac{2\sqrt{1-6b}\arctan\left(\frac{-1+6x}{\sqrt{-1+6b}}\right)}{(-1+6b)^{3/2}} + \frac{\log(1-8b+4x-12x^2)}{1-6b} + \frac{\log(b-2x+6x^2)}{-1+6b}\right)$$

input
```
Integrate[(1 - (1 - 6*b)^(3/2) - 9*b + 54*b*x - 54*x^2 + 108*x^3)^(-1),x]
```

output
```
((1 + Sqrt[1 - 6*b] - 6*b - 6*Sqrt[1 - 6*b]*x)/((-1 + 6*b)*(b + 2*x*(-1 +
3*x))) - (2*Sqrt[1 - 6*b]*ArcTan[(-1 + 6*x)/(2*Sqrt[-1 + 6*b])])/(-1 + 6*b
)^(3/2) - (2*Sqrt[1 - 6*b]*ArcTan[(-1 + 6*x)/Sqrt[-1 + 6*b]])/(-1 + 6*b)^(
3/2) + Log[1 - 8*b + 4*x - 12*x^2]/(1 - 6*b) + Log[b - 2*x + 6*x^2]/(-1 +
6*b))/54
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.10, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {2479, 27, 27, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{54bx-(1-6b)^{3/2}-9b+108x^3-54x^2+1}\,dx$$

$$\Big\downarrow \text{2479}$$

$$99179645184(1-6b)^3 \int -\frac{1}{49589822592\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)^2\left(\left(2\sqrt{1-6b}+1\right)(1-6b)-6(1-6b)x\right)}dx$$

$$\Big\downarrow \text{27}$$

$$-2(1-6b)^3\int \frac{1}{(1-6b)\left(-6x+2\sqrt{1-6b}+1\right)\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)^2}dx$$

$$\Big\downarrow \text{27}$$

$$-2(1-6b)^2\int \frac{1}{\left(-6x+2\sqrt{1-6b}+1\right)\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)^2}dx$$

$$\Big\downarrow \text{54}$$

$$-2(1-6b)^2\int \left(-\frac{1}{9(6b-1)^3\left(-6x+2\sqrt{1-6b}+1\right)}-\frac{1}{9(6b-1)^3\left(6x+\sqrt{1-6b}-1\right)}+\frac{1}{3(6b-1)^2\left(6x+\sqrt{1-6b}-1\right)\right.$$

$$\Big\downarrow \text{2009}$$

$$-2(1-6b)^2\left(\frac{1}{18(1-6b)^{5/2}\left(-\sqrt{1-6b}-6x+1\right)}+\frac{\log\left(-\sqrt{1-6b}-6x+1\right)}{54(1-6b)^3}-\frac{\log\left(2\sqrt{1-6b}-6x+1\right)}{54(1-6b)^3}\right)$$

input
```
Int[(1 - (1 - 6*b)^(3/2) - 9*b + 54*b*x - 54*x^2 + 108*x^3)^(-1),x]
```

output
```
-2*(1 - 6*b)^2*(1/(18*(1 - 6*b)^(5/2)*(1 - Sqrt[1 - 6*b] - 6*x)) + Log[1 -
 Sqrt[1 - 6*b] - 6*x]/(54*(1 - 6*b)^3) - Log[1 + 2*Sqrt[1 - 6*b] - 6*x]/(5
4*(1 - 6*b)^3))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2479
```
Int[(Px_)^(p_), x_Symbol] :> With[{a = Coeff[Px, x, 0], b = Coeff[Px, x, 1]
, c = Coeff[Px, x, 2], d = Coeff[Px, x, 3]}, Simp[1/(4^p*(c^2 - 3*b*d)^(3*p
))   Int[(c^3 - 4*b*c*d + 9*a*d^2 + d*(c^2 - 3*b*d)*x)^p*(b*c - 9*a*d + 2*(
c^2 - 3*b*d)*x)^(2*p), x], x] /; EqQ[b^2*c^2 - 4*a*c^3 - 4*b^3*d + 18*a*b*c
*d - 27*a^2*d^2, 0] && NeQ[c^2 - 3*b*d, 0]] /; FreeQ[p, x] && PolyQ[Px, x,
3] && IntegerQ[p]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.42 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.64

method	result
default	$-\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(-(1-6b)^{\frac{3}{2}}+108\_Z^3+54b\_Z-54\_Z^2-9b+1\right)}\frac{\ln\left(x-\_R\right)}{-6\_R^2+2\_R-b}\right)}{54}$
parallelrisch	$\frac{\sqrt{1-6b}\,\ln\left(-\frac{1}{6}+x+\frac{\sqrt{1-6b}}{6}\right)+6\ln\left(-\frac{1}{6}+x+\frac{\sqrt{1-6b}}{6}\right)x-\sqrt{1-6b}\,\ln\left(x-\frac{1}{6}-\frac{\sqrt{1-6b}}{3}\right)-6\ln\left(x-\frac{1}{6}-\frac{\sqrt{1-6b}}{3}\right)x-\ln\left(-\frac{1}{6}+x+\frac{\sqrt{1-6}}{6}\right.}{27(-1+6b)\left(-1+6x+\sqrt{1-6b}\right)}$

input
```
int(1/(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3),x,method=_RETURNVERBOSE)
```

output
```
-1/54*sum(1/(-6*_R^2+2*_R-b)*ln(x-_R),_R=RootOf(-(1-6*b)^(3/2)+108*_Z^3+54
*b*_Z-54*_Z^2-9*b+1))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.16

$$\int \frac{1}{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}\,dx=\frac{2\left(6\,x^2+b-2\,x\right)\log\left(6\,x+\sqrt{-6\,b+1}-1\right)-2\left(6\,x^2\right.}{54\left(6\left(6\,b-\right.\right.}$$

input
```
integrate(1/(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3),x, algorithm="fric
as")
```

output
```
1/54*(2*(6*x^2 + b - 2*x)*log(6*x + sqrt(-6*b + 1) - 1) - 2*(6*x^2 + b - 2
*x)*log(6*x - 2*sqrt(-6*b + 1) - 1) - sqrt(-6*b + 1)*(6*x - 1) - 6*b + 1)/
(6*(6*b - 1)*x^2 + 6*b^2 - 2*(6*b - 1)*x - b)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 184 vs. $2(66) = 132$.

Time = 0.60 (sec) , antiderivative size = 184, normalized size of antiderivative = 2.09

$$\int \frac{1}{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}\,dx = \frac{6}{-324b+324x\sqrt{1-6b}-54\sqrt{1-6b}+54} - \frac{\log\left(\frac{b}{2\sqrt{1-6b}}+x-\frac{1}{6}-\frac{-\frac{243b^2}{\sqrt{1-6b}}+\frac{81b}{\sqrt{1-6b}}-\frac{27}{4\sqrt{1-6b}}}{27\cdot(6b-1)}-\frac{1}{12\sqrt{1-6b}}\right)}{27\cdot(6b-1)} + \frac{\log\left(\frac{b}{2\sqrt{1-6b}}+x-\frac{1}{6}+\frac{-\frac{243b^2}{\sqrt{1-6b}}+\frac{81b}{\sqrt{1-6b}}-\frac{27}{4\sqrt{1-6b}}}{27\cdot(6b-1)}-\frac{1}{12\sqrt{1-6b}}\right)}{27\cdot(6b-1)}$$

input
```
integrate(1/(1-(1-6*b)**(3/2)-9*b+54*b*x-54*x**2+108*x**3),x)
```

output
```
6/(-324*b + 324*x*sqrt(1 - 6*b) - 54*sqrt(1 - 6*b) + 54) - log(b/(2*sqrt(1
 - 6*b)) + x - 1/6 - (-243*b**2/sqrt(1 - 6*b) + 81*b/sqrt(1 - 6*b) - 27/(4
*sqrt(1 - 6*b)))/(27*(6*b - 1)) - 1/(12*sqrt(1 - 6*b)))/(27*(6*b - 1)) + l
og(b/(2*sqrt(1 - 6*b)) + x - 1/6 + (-243*b**2/sqrt(1 - 6*b) + 81*b/sqrt(1
- 6*b) - 27/(4*sqrt(1 - 6*b)))/(27*(6*b - 1)) - 1/(12*sqrt(1 - 6*b)))/(27*
(6*b - 1))
```

Maxima [F]

$$\int \frac{1}{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}\,dx = \int \frac{1}{108\,x^3+54\,bx-54\,x^2-(-6\,b+1)^{\frac{3}{2}}-9\,b+1}\,dx$$

input
```
integrate(1/(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3),x, algorithm="maxi
ma")
```

output
```
integrate(1/(108*x^3 + 54*b*x - 54*x^2 - (-6*b + 1)^(3/2) - 9*b + 1), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3),x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{-972,[1]%%%}+%%%{162,[0]%%%},[2]%%%}+%%%{%%{[%%%{-324,
[1]%%%}+%
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.50

$$\int \frac{1}{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}\,dx = -\frac{(1-6\,b)^{3/2}}{1944\,\left(\frac{(1-6\,b)^{3/2}}{36\,\left(b-\frac{1}{6}\right)}-x+\frac{1}{6}\right)\,\left(b^2-\frac{b}{3}+\frac{1}{36}\right)} - \frac{\mathrm{atan}\left(\frac{-b\,12\mathrm{i}-x\,12\mathrm{i}+b\,x\,72\mathrm{i}+(1-6\,b)^{3/2}\,1\mathrm{i}+2\mathrm{i}}{\sqrt{864\,b^2-(6\,b-1)^3-144\,b-1728\,b^3+8}}\right)\,\sqrt{1-6\,b}\,2\mathrm{i}}{9\,\sqrt{864\,b^2-(6\,b-1)^3-144\,b-1728\,b^3+8}}$$

input
```
int(-1/(9*b - 54*b*x + (1 - 6*b)^(3/2) + 54*x^2 - 108*x^3 - 1),x)
```

output
```
- (1 - 6*b)^(3/2)/(1944*((1 - 6*b)^(3/2)/(36*(b - 1/6)) - x + 1/6)*(b^2 -
b/3 + 1/36)) - (atan((b*x*72i - x*12i - b*12i + (1 - 6*b)^(3/2)*1i + 2i)/(
864*b^2 - (6*b - 1)^3 - 144*b - 1728*b^3 + 8)^(1/2))*(1 - 6*b)^(1/2)*2i)/(
9*(864*b^2 - (6*b - 1)^3 - 144*b - 1728*b^3 + 8)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 498, normalized size of antiderivative = 5.66

$$\int \frac{1}{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}\,dx = \frac{-1+12b-108\sqrt{-6b+1}\,b\,x^2+\log(12x^2+8b-4x}{}$$

input
```
int(1/(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3),x)
```

output
```
( - 2*sqrt(6*b - 1)*sqrt( - 6*b + 1)*atan((6*x - 1)/sqrt(6*b - 1))*b - 12*
sqrt(6*b - 1)*sqrt( - 6*b + 1)*atan((6*x - 1)/sqrt(6*b - 1))*x**2 + 4*sqrt
(6*b - 1)*sqrt( - 6*b + 1)*atan((6*x - 1)/sqrt(6*b - 1))*x - 2*sqrt(6*b -
1)*sqrt( - 6*b + 1)*atan((6*x - 1)/(2*sqrt(6*b - 1)))*b - 12*sqrt(6*b - 1)
*sqrt( - 6*b + 1)*atan((6*x - 1)/(2*sqrt(6*b - 1)))*x**2 + 4*sqrt(6*b - 1)
*sqrt( - 6*b + 1)*atan((6*x - 1)/(2*sqrt(6*b - 1)))*x - 18*sqrt( - 6*b + 1
)*b**2 - 108*sqrt( - 6*b + 1)*b*x**2 + 9*sqrt( - 6*b + 1)*b + 18*sqrt( - 6
*b + 1)*x**2 - sqrt( - 6*b + 1) - 6*log(8*b + 12*x**2 - 4*x - 1)*b**2 - 36
*log(8*b + 12*x**2 - 4*x - 1)*b*x**2 + 12*log(8*b + 12*x**2 - 4*x - 1)*b*x
 + log(8*b + 12*x**2 - 4*x - 1)*b + 6*log(8*b + 12*x**2 - 4*x - 1)*x**2 -
2*log(8*b + 12*x**2 - 4*x - 1)*x + 6*log(b + 6*x**2 - 2*x)*b**2 + 36*log(b
 + 6*x**2 - 2*x)*b*x**2 - 12*log(b + 6*x**2 - 2*x)*b*x - log(b + 6*x**2 -
2*x)*b - 6*log(b + 6*x**2 - 2*x)*x**2 + 2*log(b + 6*x**2 - 2*x)*x - 36*b**
2 + 12*b - 1)/(54*(36*b**3 + 216*b**2*x**2 - 72*b**2*x - 12*b**2 - 72*b*x*
*2 + 24*b*x + b + 6*x**2 - 2*x))
```

3.143 $\int \frac{1}{(e+fx)\left(1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3\right)}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 211

$$\begin{aligned}\int \frac{1}{(e+fx)\left(1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3\right)}\,dx &= \\ -\frac{2}{3\sqrt{1-6b}\left(6e+f-\sqrt{1-6b}f\right)\left(1-\sqrt{1-6b}-6x\right)} \\ -\frac{2\left(6e+f-4\sqrt{1-6b}f\right)\log\left(1-\sqrt{1-6b}-6x\right)}{9(1-6b)\left(6e+f-\sqrt{1-6b}f\right)^2} \\ +\frac{2\log\left(1+2\sqrt{1-6b}-6x\right)}{9(1-6b)\left(6e+f+2\sqrt{1-6b}f\right)} \\ -\frac{2f^2\log(e+fx)}{\left(6e+f-\sqrt{1-6b}f\right)^2\left(6e+f+2\sqrt{1-6b}f\right)}\end{aligned}$$

output

```
-2/3/(1-6*b)^(1/2)/(6*e+f-(1-6*b)^(1/2)*f)/(1-(1-6*b)^(1/2)-6*x)-2/9*(6*e+
f-4*(1-6*b)^(1/2)*f)*ln(1-(1-6*b)^(1/2)-6*x)/(1-6*b)/(6*e+f-(1-6*b)^(1/2)*
f)^2+2/9*ln(1+2*(1-6*b)^(1/2)-6*x)/(1-6*b)/(6*e+f+2*(1-6*b)^(1/2)*f)-2*f^2
*ln(f*x+e)/(6*e+f-(1-6*b)^(1/2)*f)^2/(6*e+f+2*(1-6*b)^(1/2)*f)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 621 vs. 2(211) = 422.

Time = 4.53 (sec) , antiderivative size = 621, normalized size of antiderivative = 2.94

$$\int \frac{1}{(e+fx)\left(1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3\right)}\,dx=\frac{1}{54}\left(-\frac{6bf\left(2+\sqrt{1-6b}-6x\right)+2\left(1+\sqrt{\vphantom{1}}\right.}{(-1+6b)\left(\right.}\right.$$

$$-\frac{4\left(6\sqrt{1-6b}e+\left(-2+\sqrt{1-6b}+12b\right)f\right)\arctan\left(\frac{-1+6x}{2\sqrt{-1+6b}}\right)}{(-1+6b)^{3/2}\left(12e^2+4ef+(-1+8b)f^2\right)}$$

$$+\frac{2\left(-36\sqrt{1-6b}e^3-6\left(-2+3\sqrt{1-6b}+12b\right)e^2f+2\left(2\left(1+\sqrt{1-6b}\right)-3\left(4+7\sqrt{1-6b}\right)b\right)ef^2+(1+\right.}{(-1+6b)^{3/2}\left(6e^2+2ef+bf^2\right)^2}$$

$$-\frac{2f^2\left(108e^3+54e^2f+54bef^2+\left(-1-\sqrt{1-6b}+\left(9+6\sqrt{1-6b}\right)b\right)f^3\right)\log(e+fx)}{\left(6e^2+2ef+bf^2\right)^2\left(12e^2+4ef+(-1+8b)f^2\right)}$$

$$-\frac{2\left(6e+f-2\sqrt{1-6b}f\right)\log\left(1-8b+4x-12x^2\right)}{(-1+6b)\left(12e^2+4ef+(-1+8b)f^2\right)}$$

$$+\frac{\left(36e^3-6\left(-3+2\sqrt{1-6b}\right)e^2f-2\left(2+2\sqrt{1-6b}-21b\right)ef^2+\left(-1-\sqrt{1-6b}+\left(7+4\sqrt{1-6b}\right)b\right)f^3\right)}{(-1+6b)\left(6e^2+2ef+bf^2\right)^2}$$

input
```
Integrate[1/((e + f*x)*(1 - (1 - 6*b)^(3/2) - 9*b + 54*b*x - 54*x^2 + 108*
x^3)),x]
```

output
```
(-((6*b*f*(2 + Sqrt[1 - 6*b] - 6*x) + 2*(1 + Sqrt[1 - 6*b])*f*(-1 + 3*x) +
 6*e*(-1 - Sqrt[1 - 6*b] + 6*b + 6*Sqrt[1 - 6*b]*x))/((-1 + 6*b)*(6*e^2 +
2*e*f + b*f^2)*(b + 2*x*(-1 + 3*x)))) - (4*(6*Sqrt[1 - 6*b]*e + (-2 + Sqrt
[1 - 6*b] + 12*b)*f)*ArcTan[(-1 + 6*x)/(2*Sqrt[-1 + 6*b])])/((-1 + 6*b)^(3
/2)*(12*e^2 + 4*e*f + (-1 + 8*b)*f^2)) + (2*(-36*Sqrt[1 - 6*b]*e^3 - 6*(-2
 + 3*Sqrt[1 - 6*b] + 12*b)*e^2*f + 2*(2*(1 + Sqrt[1 - 6*b]) - 3*(4 + 7*Sqr
t[1 - 6*b])*b)*e*f^2 + (1 + Sqrt[1 - 6*b] - (10 + 7*Sqrt[1 - 6*b])*b + 24*
b^2)*f^3)*ArcTan[(-1 + 6*x)/Sqrt[-1 + 6*b]])/((-1 + 6*b)^(3/2)*(6*e^2 + 2*
e*f + b*f^2)^2) - (2*f^2*(108*e^3 + 54*e^2*f + 54*b*e*f^2 + (-1 - Sqrt[1 -
 6*b] + (9 + 6*Sqrt[1 - 6*b])*b)*f^3)*Log[e + f*x])/((6*e^2 + 2*e*f + b*f^
2)^2*(12*e^2 + 4*e*f + (-1 + 8*b)*f^2)) - (2*(6*e + f - 2*Sqrt[1 - 6*b]*f)
*Log[1 - 8*b + 4*x - 12*x^2])/((-1 + 6*b)*(12*e^2 + 4*e*f + (-1 + 8*b)*f^2
)) + ((36*e^3 - 6*(-3 + 2*Sqrt[1 - 6*b])*e^2*f - 2*(2 + 2*Sqrt[1 - 6*b] -
21*b)*e*f^2 + (-1 - Sqrt[1 - 6*b] + (7 + 4*Sqrt[1 - 6*b])*b)*f^3)*Log[b -
2*x + 6*x^2])/((-1 + 6*b)*(6*e^2 + 2*e*f + b*f^2)^2))/54
```

Rubi [A] (verified)

Time = 1.17 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2488, 27, 27, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(54bx-(1-6b)^{3/2}-9b+108x^3-54x^2+1\right)(e+fx)}dx$$

$$\Big\downarrow \text{2488}$$

$$99179645184(1-6b)^3\int -\frac{1}{49589822592\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)^2\left(\left(2\sqrt{1-6b}+1\right)(1-6b)-6(1-6b)x\right)(e+fx)}d$$

$$\Big\downarrow \text{27}$$

$$-2(1-6b)^3\int \frac{1}{(1-6b)\left(-6x+2\sqrt{1-6b}+1\right)\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)^2(e+fx)}dx$$

$$\Big\downarrow \text{27}$$

$$-2(1-6b)^2\int \frac{1}{\left(-6x+2\sqrt{1-6b}+1\right)\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)^2(e+fx)}dx$$

$$\Big\downarrow \; 93$$

$$-2(1-6b)^2\int \left(\frac{f^3}{(1-6b)^2\left(6e+\left(1-\sqrt{1-6b}\right)f\right)^2\left(6e+2\sqrt{1-6b}f+f\right)(e+fx)}-\frac{2}{3(6b-1)^3\left(6e+2\sqrt{1-6b}f+f\right)\cdots}\right.$$

$$\Big\downarrow \; 2009$$

$$-2(1-6b)^2\left(\frac{f^2\log(e+fx)}{(1-6b)^2\left(-\sqrt{1-6b}f+6e+f\right)^2\left(2\sqrt{1-6b}f+6e+f\right)}+\frac{1}{3(1-6b)^{5/2}\left(-\sqrt{1-6b}-6x+1\right)\left(-\sqrt{1-6b}\cdots\right.}\right.$$

input
```
Int[1/((e + f*x)*(1 - (1 - 6*b)^(3/2) - 9*b + 54*b*x - 54*x^2 + 108*x^3)),
x]
```

output
```
-2*(1 - 6*b)^2*(1/(3*(1 - 6*b)^(5/2)*(6*e + f - Sqrt[1 - 6*b]*f)*(1 - Sqrt
[1 - 6*b] - 6*x)) + ((6*e + f - 4*Sqrt[1 - 6*b]*f)*Log[1 - Sqrt[1 - 6*b] -
 6*x])/(9*(1 - 6*b)^3*(6*e + f - Sqrt[1 - 6*b]*f)^2) - Log[1 + 2*Sqrt[1 -
6*b] - 6*x]/(9*(1 - 6*b)^3*(6*e + f + 2*Sqrt[1 - 6*b]*f)) + (f^2*Log[e + f
*x])/((1 - 6*b)^2*(6*e + f - Sqrt[1 - 6*b]*f)^2*(6*e + f + 2*Sqrt[1 - 6*b]
*f)))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2488
```
Int[((e_.) + (f_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(
x_)^3)^(p_), x_Symbol] :> Simp[1/(4^p*(c^2 - 3*b*d)^(3*p))   Int[(e + f*x)^
m*(c^3 - 4*b*c*d + 9*a*d^2 + d*(c^2 - 3*b*d)*x)^p*(b*c - 9*a*d + 2*(c^2 - 3
*b*d)*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && NeQ[c^2 - 3
*b*d, 0] && EqQ[b^2*c^2 - 4*a*c^3 - 4*b^3*d + 18*a*b*c*d - 27*a^2*d^2, 0] &
& ILtQ[p, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.30 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.89

method	result
default	$-\frac{f^{2} \ln \left(f x+e\right)}{\left(1-6 b\right)^{\frac{3}{2}} f^{3}+54 b e \,f^{2}+9 b \,f^{3}+108 e^{3}+54 e^{2} f-f^{3}}+\frac{\sum_{\_R=\operatorname{RootOf}\left(-\left(1-6 b\right)^{\frac{3}{2}}+108\_Z^{3}+54 b\_Z-54\_Z^{2}-9 b+1\right)}\frac{\left(-2\_R^{2} f^{2}-\right.}{}}{\left(1-6 b\right)^{\frac{3}{2}} f^{3}+54 b e \,f^{2}+9 b \,f^{3}+108 e^{3}}$
parallelrisch	Expression too large to display

input
```
int(1/(f*x+e)/(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3),x,method=_RETURN
VERBOSE)
```

output
```
-f^2/((1-6*b)^(3/2)*f^3+54*b*e*f^2+9*b*f^3+108*e^3+54*e^2*f-f^3)*ln(f*x+e)
+sum((-2*_R^2*f^2+2*_R*e*f+_R*f^2-b*f^2-2*e^2-e*f)/(-6*_R^2+2*_R-b)*ln(x-_
R),_R=RootOf(-(1-6*b)^(3/2)+108*_Z^3+54*b*_Z-54*_Z^2-9*b+1))/((1-6*b)^(3/2
)*f^3+54*b*e*f^2+9*b*f^3+108*e^3+54*e^2*f-f^3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1738 vs. $2(188) = 376$.

Time = 6.91 (sec) , antiderivative size = 1738, normalized size of antiderivative = 8.24

$$\int \frac{1}{(e+fx)\left(1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3\right)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(f*x+e)/(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3),x, algorit
hm="fricas")
```

output
```
-1/27*(216*(6*b - 1)*e^5 + 216*(6*b - 1)*e^4*f + 18*(60*b^2 + 8*b - 3)*e^3
*f^2 + 4*(180*b^2 - 36*b + 1)*e^2*f^3 + (144*b^3 + 78*b^2 - 29*b + 2)*e*f^
4 + (48*b^3 - 14*b^2 + b)*f^5 - 3*(72*(6*b - 1)*e^4*f + 48*(6*b - 1)*e^3*f
^2 + 2*(180*b^2 - 24*b - 1)*e^2*f^3 + 2*(60*b^2 - 16*b + 1)*e*f^4 + (48*b^
3 - 14*b^2 + b)*f^5)*x + (108*(6*b^2 - b)*e^3*f^2 + 54*(6*b^2 - b)*e^2*f^3
 + 54*(6*b^3 - b^2)*e*f^4 + (54*b^3 - 15*b^2 + b)*f^5 + 6*(108*(6*b - 1)*e
^3*f^2 + 54*(6*b - 1)*e^2*f^3 + 54*(6*b^2 - b)*e*f^4 + (54*b^2 - 15*b + 1)
*f^5)*x^2 - 2*(108*(6*b - 1)*e^3*f^2 + 54*(6*b - 1)*e^2*f^3 + 54*(6*b^2 -
b)*e*f^4 + (54*b^2 - 15*b + 1)*f^5)*x)*log(f*x + e) - (432*b*e^5 + 360*b*e
^4*f + 12*(66*b^2 - b)*e^3*f^2 + 2*(198*b^2 - 23*b)*e^2*f^3 + 2*(168*b^3 -
 23*b^2)*e*f^4 + (56*b^3 - 15*b^2 + b)*f^5 + 6*(12*(66*b - 1)*e^3*f^2 + 2*
(198*b - 23)*e^2*f^3 + 2*(168*b^2 - 23*b)*e*f^4 + (56*b^2 - 15*b + 1)*f^5
+ 432*e^5 + 360*e^4*f)*x^2 - 2*(12*(66*b - 1)*e^3*f^2 + 2*(198*b - 23)*e^2
*f^3 + 2*(168*b^2 - 23*b)*e*f^4 + (56*b^2 - 15*b + 1)*f^5 + 432*e^5 + 360*
e^4*f)*x - (16*b^2*e*f^4 + 144*b*e^4*f + 96*b*e^3*f^2 + 16*(3*b^2 + b)*e^2
*f^3 - (32*b^3 - 12*b^2 + b)*f^5 + 6*(16*(3*b + 1)*e^2*f^3 + 16*b*e*f^4 -
(32*b^2 - 12*b + 1)*f^5 + 144*e^4*f + 96*e^3*f^2)*x^2 - 2*(16*(3*b + 1)*e^
2*f^3 + 16*b*e*f^4 - (32*b^2 - 12*b + 1)*f^5 + 144*e^4*f + 96*e^3*f^2)*x)*
sqrt(-6*b + 1))*log(6*x + sqrt(-6*b + 1) - 1) + 2*(b^3*f^5 + 216*b*e^5 + 1
80*b*e^4*f + 24*(3*b^2 + 2*b)*e^3*f^2 + 4*(9*b^2 + b)*e^2*f^3 + 2*(3*b^...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(e+fx)\left(1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3\right)}\,dx = \text{Timed out}$$

input
```
integrate(1/(f*x+e)/(1-(1-6*b)**(3/2)-9*b+54*b*x-54*x**2+108*x**3),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{(e+fx)\left(1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3\right)}\,dx = \int \frac{1}{\left(108\,x^3+54\,bx-54\,x^2-(-6\,b+1)^{\frac{3}{2}}\right.}$$

input
```
integrate(1/(f*x+e)/(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3),x, algorit
hm="maxima")
```

output
```
-f^2*log(f*x + e)/(54*b*e*f^2 + ((-6*b + 1)^(3/2) + 9*b - 1)*f^3 + 108*e^3
 + 54*e^2*f) + 54*integrate((2*f^2*x^2 + b*f^2 + 2*e^2 + e*f - (2*e*f + f^
2)*x)/(108*x^3 + 54*b*x - 54*x^2 - (-6*b + 1)^(3/2) - 9*b + 1), x)/(54*b*e
*f^2 + ((-6*b + 1)^(3/2) + 9*b - 1)*f^3 + 108*e^3 + 54*e^2*f)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(e+fx)\left(1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3\right)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(f*x+e)/(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3),x, algorit
hm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{-972,[1]%%%}+%%%{162,[0]%%%},[2]%%%}+%%%{%%{[%%%{-324,
[1]%%%}+%
```

Mupad [B] (verification not implemented)

Time = 48.64 (sec) , antiderivative size = 12009, normalized size of antiderivative = 56.91

$$\int \frac{1}{(e+fx)\left(1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3\right)}\,dx = \text{Too large to display}$$

input
```
int(-1/((e + f*x)*(9*b - 54*b*x + (1 - 6*b)^(3/2) + 54*x^2 - 108*x^3 - 1))
,x)
```

output
```
(log((x*(6*e*f - 108*b*e^2 - 9*b*f^2 + f^2*(1 - 6*b)^(3/2) + 18*e^2 + f^2
+ 18*b^2*f^2 - 36*b*e*f + 6*e*f*(1 - 6*b)^(3/2)))/(1458*(6*b - 1)^2*(2*e*f
 + b*f^2 + 6*e^2)^2) - (216*b*e^3 - 12*b*f^3 + 4*e*f^2 - 12*e^2*f + f^3*(1
 - 6*b)^(3/2) - 36*e^3 + f^3 + 36*b^2*f^3 - 3*b*f^3*(1 - 6*b)^(3/2) + 4*e*
f^2*(1 - 6*b)^(3/2) + 6*e^2*f*(1 - 6*b)^(3/2) + 180*b^2*e*f^2 - 54*b*e*f^2
 + 72*b*e^2*f)/(2916*f*(6*b - 1)^2*(2*e*f + b*f^2 + 6*e^2)^2) + ((54*e*f^2
 - 216*b*f^3 - 9*f^3*(-(6*b - 1)^3)^(1/2) + 8*f^3*(1 - 6*b)^(3/2) + f^3*(1
 - 6*b)^(9/2) + 9*f^3 + 1944*b^2*f^3 - 7776*b^3*f^3 + 11664*b^4*f^3 - 144*
b*f^3*(1 - 6*b)^(3/2) - 828*b^2*f^3*(-(6*b - 1)^3)^(1/2) + 1512*b^3*f^3*(-
(6*b - 1)^3)^(1/2) + 11664*b^2*e*f^2 - 46656*b^3*e*f^2 + 69984*b^4*e*f^2 +
 864*b^2*f^3*(1 - 6*b)^(3/2) - 1728*b^3*f^3*(1 - 6*b)^(3/2) + 150*b*f^3*(-
(6*b - 1)^3)^(1/2) - 24*e*f^2*(-(6*b - 1)^3)^(1/2) - 72*e^2*f*(-(6*b - 1)^
3)^(1/2) + 432*e^3*(1 - 6*b)^(3/2)*(-(6*b - 1)^3)^(1/2) - 9*f^3*(1 - 6*b)^
(3/2)*(-(6*b - 1)^3)^(1/2) - 1296*b*e*f^2 + 288*b*e*f^2*(-(6*b - 1)^3)^(1/
2) + 864*b*e^2*f*(-(6*b - 1)^3)^(1/2) + 66*b*f^3*(1 - 6*b)^(3/2)*(-(6*b -
1)^3)^(1/2) - 30*e*f^2*(1 - 6*b)^(3/2)*(-(6*b - 1)^3)^(1/2) + 216*e^2*f*(1
 - 6*b)^(3/2)*(-(6*b - 1)^3)^(1/2) - 864*b^2*e*f^2*(-(6*b - 1)^3)^(1/2) -
2592*b^2*e^2*f*(-(6*b - 1)^3)^(1/2) + 396*b*e*f^2*(1 - 6*b)^(3/2)*(-(6*b -
 1)^3)^(1/2))*(15552*e^5*(-(6*b - 1)^3)^(1/2) + 684*b*f^5 + 396*e*f^4 - 77
76*e^4*f - 972*f^5*x + 46656*e^5*(1 - 6*b)^(3/2) - 5*f^5*(1 - 6*b)^(3/2...
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 11241, normalized size of antiderivative = 53.27

$$\int \frac{1}{(e+fx)\left(1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3\right)}\,dx = \text{Too large to display}$$

input
```
int(1/(f*x+e)/(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3),x)
```

output
```
( - 672*sqrt(6*b - 1)*sqrt( - 6*b + 1)*atan((6*x - 1)/sqrt(6*b - 1))*b**3*
e*f**4 - 112*sqrt(6*b - 1)*sqrt( - 6*b + 1)*atan((6*x - 1)/sqrt(6*b - 1))*
b**3*f**5 - 1584*sqrt(6*b - 1)*sqrt( - 6*b + 1)*atan((6*x - 1)/sqrt(6*b -
1))*b**2*e**3*f**2 - 792*sqrt(6*b - 1)*sqrt( - 6*b + 1)*atan((6*x - 1)/sqr
t(6*b - 1))*b**2*e**2*f**3 - 4032*sqrt(6*b - 1)*sqrt( - 6*b + 1)*atan((6*x
 - 1)/sqrt(6*b - 1))*b**2*e*f**4*x**2 + 1344*sqrt(6*b - 1)*sqrt( - 6*b + 1
)*atan((6*x - 1)/sqrt(6*b - 1))*b**2*e*f**4*x + 92*sqrt(6*b - 1)*sqrt( - 6
*b + 1)*atan((6*x - 1)/sqrt(6*b - 1))*b**2*e*f**4 - 672*sqrt(6*b - 1)*sqrt
( - 6*b + 1)*atan((6*x - 1)/sqrt(6*b - 1))*b**2*f**5*x**2 + 224*sqrt(6*b -
 1)*sqrt( - 6*b + 1)*atan((6*x - 1)/sqrt(6*b - 1))*b**2*f**5*x + 30*sqrt(6
*b - 1)*sqrt( - 6*b + 1)*atan((6*x - 1)/sqrt(6*b - 1))*b**2*f**5 - 864*sqr
t(6*b - 1)*sqrt( - 6*b + 1)*atan((6*x - 1)/sqrt(6*b - 1))*b*e**5 - 720*sqr
t(6*b - 1)*sqrt( - 6*b + 1)*atan((6*x - 1)/sqrt(6*b - 1))*b*e**4*f - 9504*
sqrt(6*b - 1)*sqrt( - 6*b + 1)*atan((6*x - 1)/sqrt(6*b - 1))*b*e**3*f**2*x
**2 + 3168*sqrt(6*b - 1)*sqrt( - 6*b + 1)*atan((6*x - 1)/sqrt(6*b - 1))*b*
e**3*f**2*x + 24*sqrt(6*b - 1)*sqrt( - 6*b + 1)*atan((6*x - 1)/sqrt(6*b -
1))*b*e**3*f**2 - 4752*sqrt(6*b - 1)*sqrt( - 6*b + 1)*atan((6*x - 1)/sqrt(
6*b - 1))*b*e**2*f**3*x**2 + 1584*sqrt(6*b - 1)*sqrt( - 6*b + 1)*atan((6*x
 - 1)/sqrt(6*b - 1))*b*e**2*f**3*x + 92*sqrt(6*b - 1)*sqrt( - 6*b + 1)*ata
n((6*x - 1)/sqrt(6*b - 1))*b*e**2*f**3 + 552*sqrt(6*b - 1)*sqrt( - 6*b ...
```

3.144 $\int \frac{1}{(e+fx)^2\left(1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3\right)}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 275

$$\int \frac{1}{(e+fx)^2\,(1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3)}\,dx = -\frac{4}{\sqrt{1-6b}\left(6e+f-\sqrt{1-6b}f\right)^2\left(1-\sqrt{1-6b}-6x\right)}+\frac{2f^2}{\left(6e+f-\sqrt{1-6b}f\right)^2\left(6e+f+2\sqrt{1-6b}f\right)(e+fx)}-\frac{4\left(6e+f-7\sqrt{1-6b}f\right)\log\left(1-\sqrt{1-6b}-6x\right)}{3(1-6b)\left(6e+f-\sqrt{1-6b}f\right)^3}+\frac{4\log\left(1+2\sqrt{1-6b}-6x\right)}{3(1-6b)\left(6e+f+2\sqrt{1-6b}f\right)^2}-\frac{36f^2\left(6e+f+\sqrt{1-6b}f\right)\log(e+fx)}{\left(6e+f-\sqrt{1-6b}f\right)^3\left(6e+f+2\sqrt{1-6b}f\right)^2}$$

output
```
-4/(1-6*b)^(1/2)/(6*e+f-(1-6*b)^(1/2)*f)^2/(1-(1-6*b)^(1/2)-6*x)+2*f^2/(6*
e+f-(1-6*b)^(1/2)*f)^2/(6*e+f+2*(1-6*b)^(1/2)*f)/(f*x+e)-4/3*(6*e+f-7*(1-6
*b)^(1/2)*f)*ln(1-(1-6*b)^(1/2)-6*x)/(1-6*b)/(6*e+f-(1-6*b)^(1/2)*f)^3+4/3
*ln(1+2*(1-6*b)^(1/2)-6*x)/(1-6*b)/(6*e+f+2*(1-6*b)^(1/2)*f)^2-36*f^2*(6*e
+f+(1-6*b)^(1/2)*f)*ln(f*x+e)/(6*e+f-(1-6*b)^(1/2)*f)^3/(6*e+f+2*(1-6*b)^(
1/2)*f)^2
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1347 vs. 2(275) = 550.

Time = 7.43 (sec) , antiderivative size = 1347, normalized size of antiderivative = 4.90

$$\int \frac{1}{(e+fx)^2 \left(1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3\right)} \, dx = \text{Too large to display}$$

input
```
Integrate[1/((e + f*x)^2*(1 - (1 - 6*b)^(3/2) - 9*b + 54*b*x - 54*x^2 + 10
8*x^3)),x]
```

output
```
(f^2*(108*e^3 + 54*e^2*f + 54*b*e*f^2 - f^3 - Sqrt[1 - 6*b]*f^3 + 9*b*f^3
+ 6*Sqrt[1 - 6*b]*b*f^3))/(27*(6*e^2 + 2*e*f + b*f^2)^2*(12*e^2 + 4*e*f -
f^2 + 8*b*f^2)*(e + f*x)) + (18*e^2 + 18*Sqrt[1 - 6*b]*e^2 - 108*b*e^2 + 1
2*e*f + 12*Sqrt[1 - 6*b]*e*f - 72*b*e*f - 36*Sqrt[1 - 6*b]*b*e*f + 2*f^2 +
 2*Sqrt[1 - 6*b]*f^2 - 15*b*f^2 - 9*Sqrt[1 - 6*b]*b*f^2 + 18*b^2*f^2 - 108
*Sqrt[1 - 6*b]*e^2*x - 36*e*f*x - 36*Sqrt[1 - 6*b]*e*f*x + 216*b*e*f*x - 6
*f^2*x - 6*Sqrt[1 - 6*b]*f^2*x + 36*b*f^2*x + 18*Sqrt[1 - 6*b]*b*f^2*x)/(2
7*(-1 + 6*b)*(6*e^2 + 2*e*f + b*f^2)^2*(b - 2*x + 6*x^2)) - (4*(36*Sqrt[1
- 6*b]*e^2 - 24*e*f + 12*Sqrt[1 - 6*b]*e*f + 144*b*e*f - 4*f^2 + 5*Sqrt[1
- 6*b]*f^2 + 24*b*f^2 - 24*Sqrt[1 - 6*b]*b*f^2)*ArcTan[(-1 + 6*x)/(2*Sqrt[
-1 + 6*b])])/(27*(-1 + 6*b)^(3/2)*(12*e^2 + 4*e*f - f^2 + 8*b*f^2)^2) + (2
*(-108*Sqrt[1 - 6*b]*e^4 + 72*e^3*f - 72*Sqrt[1 - 6*b]*e^3*f - 432*b*e^3*f
 + 36*e^2*f^2 + 36*Sqrt[1 - 6*b]*e^2*f^2 - 216*b*e^2*f^2 - 324*Sqrt[1 - 6*
b]*b*e^2*f^2 + 16*e*f^3 + 16*Sqrt[1 - 6*b]*e*f^3 - 156*b*e*f^3 - 108*Sqrt[
1 - 6*b]*b*e*f^3 + 360*b^2*e*f^3 + 2*f^4 + 2*Sqrt[1 - 6*b]*f^4 - 22*b*f^4
- 16*Sqrt[1 - 6*b]*b*f^4 + 60*b^2*f^4 + 21*Sqrt[1 - 6*b]*b^2*f^4)*ArcTan[(
-1 + 6*x)/Sqrt[-1 + 6*b]])/(27*(-1 + 6*b)^(3/2)*(6*e^2 + 2*e*f + b*f^2)^3)
 + (2*f^2*(-11664*e^6 - 11664*e^5*f - 2916*e^4*f^2 - 11664*b*e^4*f^2 + 216
*e^3*f^3 + 216*Sqrt[1 - 6*b]*e^3*f^3 - 7776*b*e^3*f^3 - 1296*Sqrt[1 - 6*b]
*b*e^3*f^3 + 108*e^2*f^4 + 108*Sqrt[1 - 6*b]*e^2*f^4 - 972*b*e^2*f^4 - ...
```

Rubi [A] (verified)

Time = 1.56 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2488, 27, 27, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(54bx-(1-6b)^{3/2}-9b+108x^3-54x^2+1\right)(e+fx)^2}dx$$

$$\Big\downarrow \text{2488}$$

$$99179645184(1-6b)^3\int -\frac{1}{49589822592\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)^2\left(\left(2\sqrt{1-6b}+1\right)(1-6b)-6(1-6b)x\right)(e+fx)^2}$$

$$\Big\downarrow \text{27}$$

$$-2(1-6b)^3\int \frac{1}{(1-6b)\left(-6x+2\sqrt{1-6b}+1\right)\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)^2(e+fx)^2}dx$$

$$\Big\downarrow \text{ 27}$$

$$-2(1-6b)^2\int \frac{1}{\left(-6x+2\sqrt{1-6b}+1\right)\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)^2(e+fx)^2}dx$$

$$\Big\downarrow \text{ 99}$$

$$-2(1-6b)^2\int \left(\frac{18\left(6e+\sqrt{1-6b}f+f\right)f^3}{(1-6b)^2\left(6e+\left(1-\sqrt{1-6b}\right)f\right)^3\left(6e+2\sqrt{1-6b}f+f\right)^2(e+fx)}+\frac{}{(1-6b)^2\left(6e+\left(1-\sqrt{1-6b}\right)f\right)}\right.$$

$$\Big\downarrow \text{ 2009}$$

$$-2(1-6b)^2\left(-\frac{f^2}{(1-6b)^2\left(-\sqrt{1-6b}f+6e+f\right)^2\left(2\sqrt{1-6b}f+6e+f\right)(e+fx)}+\frac{18f^2\left(\sqrt{1-6b}f+6e+f\right)}{(1-6b)^2\left(-\sqrt{1-6b}f+6e+f\right)^3(}\right.$$

input
```
Int[1/((e + f*x)^2*(1 - (1 - 6*b)^(3/2) - 9*b + 54*b*x - 54*x^2 + 108*x^3)
),x]
```

output
```
-2*(1 - 6*b)^2*(2/((1 - 6*b)^(5/2)*(6*e + f - Sqrt[1 - 6*b]*f)^2*(1 - Sqrt
[1 - 6*b] - 6*x)) - f^2/((1 - 6*b)^2*(6*e + f - Sqrt[1 - 6*b]*f)^2*(6*e +
f + 2*Sqrt[1 - 6*b]*f)*(e + f*x)) + (2*(6*e + f - 7*Sqrt[1 - 6*b]*f)*Log[1
 - Sqrt[1 - 6*b] - 6*x])/(3*(1 - 6*b)^3*(6*e + f - Sqrt[1 - 6*b]*f)^3) - (
2*Log[1 + 2*Sqrt[1 - 6*b] - 6*x])/(3*(1 - 6*b)^3*(6*e + f + 2*Sqrt[1 - 6*b
]*f)^2) + (18*f^2*(6*e + f + Sqrt[1 - 6*b]*f)*Log[e + f*x])/((1 - 6*b)^2*(
6*e + f - Sqrt[1 - 6*b]*f)^3*(6*e + f + 2*Sqrt[1 - 6*b]*f)^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2488
```
Int[((e_.) + (f_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(
x_)^3)^(p_), x_Symbol] :> Simp[1/(4^p*(c^2 - 3*b*d)^(3*p))   Int[(e + f*x)^
m*(c^3 - 4*b*c*d + 9*a*d^2 + d*(c^2 - 3*b*d)*x)^p*(b*c - 9*a*d + 2*(c^2 - 3
*b*d)*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && NeQ[c^2 - 3
*b*d, 0] && EqQ[b^2*c^2 - 4*a*c^3 - 4*b^3*d + 18*a*b*c*d - 27*a^2*d^2, 0] &
& ILtQ[p, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 3.33 (sec) , antiderivative size = 383, normalized size of antiderivative = 1.39

method	result
default	$\frac{f^{2}}{\left(\left(1-6b\right)^{\frac{3}{2}} f^{3}+54be f^{2}+9b f^{3}+108e^{3}+54e^{2}f-f^{3}\right)\left(fx+e\right)}-\frac{54f^{2}\left(b f^{2}+6e^{2}+2ef\right)\ln\left(fx+e\right)}{\left(\left(1-6b\right)^{\frac{3}{2}} f^{3}+54be f^{2}+9b f^{3}+108e^{3}+54e^{2}f-f^{3}\right)^{2}}+\frac{\_R=}{}$
parallelrisch	Expression too large to display

input
```
int(1/(f*x+e)^2/(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3),x,method=_RETU
RNVERBOSE)
```

output
```
f^2/((1-6*b)^(3/2)*f^3+54*b*e*f^2+9*b*f^3+108*e^3+54*e^2*f-f^3)/(f*x+e)-54
*f^2*(b*f^2+6*e^2+2*e*f)/((1-6*b)^(3/2)*f^3+54*b*e*f^2+9*b*f^3+108*e^3+54*
e^2*f-f^3)^2*ln(f*x+e)+sum(1/(-6*_R^2+2*_R-b)*(-108*b*f^4*_R^2-648*e^2*f^2
*_R^2-216*e*f^3*_R^2+36*b*f^4*_R-54*b^2*f^4+432*e^3*f*_R+432*e^2*f^2*_R+10
8*f^3*e*_R+2*f^4*_R-216*b*e^2*f^2-72*b*e*f^3+9*b*f^4-216*e^4-216*e^3*f-54*
e^2*f^2-4*e*f^3-f^4+f^3*(-2*f*_R*(1-6*b)^(3/2)+4*e*(1-6*b)^(3/2)+(1-6*b)^(
3/2)*f))*ln(x-_R),_R=RootOf(-(1-6*b)^(3/2)+108*_Z^3+54*b*_Z-54*_Z^2-9*b+1)
)/((1-6*b)^(3/2)*f^3+54*b*e*f^2+9*b*f^3+108*e^3+54*e^2*f-f^3)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5609 vs. $2(249) = 498$.

Time = 93.96 (sec) , antiderivative size = 5609, normalized size of antiderivative = 20.40

$$\int \frac{1}{(e+fx)^2\left(1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3\right)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(f*x+e)^2/(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3),x, algor
ithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(e+fx)^2\left(1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3\right)}\,dx = \text{Timed out}$$

input
```
integrate(1/(f*x+e)**2/(1-(1-6*b)**(3/2)-9*b+54*b*x-54*x**2+108*x**3),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{(e+fx)^2\left(1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3\right)}\,dx = \int \frac{1}{\left(108\,x^3+54\,bx-54\,x^2-(-6\,b+1)^{\frac{3}{2}}\right.}$$

input
```
integrate(1/(f*x+e)^2/(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3),x, algor
ithm="maxima")
```

output
```
f^2/(54*b*e^2*f^2 + ((-6*b + 1)^(3/2) + 9*b - 1)*e*f^3 + 108*e^4 + 54*e^3*
f + (54*b*e*f^3 + ((-6*b + 1)^(3/2) + 9*b - 1)*f^4 + 108*e^3*f + 54*e^2*f^
2)*x) - 54*(b*f^4 + 6*e^2*f^2 + 2*e*f^3)*log(f*x + e)/(2916*(4*b + 1)*e^4*
f^2 + 216*((-6*b + 1)^(3/2) + 36*b - 1)*e^3*f^3 + 108*(27*b^2 + (-6*b + 1)
^(3/2) + 9*b - 1)*e^2*f^4 + 108*(((-6*b + 1)^(3/2) - 1)*b + 9*b^2)*e*f^5 -
 ((6*b - 1)^3 - 18*((-6*b + 1)^(3/2) - 1)*b - 81*b^2 + 2*(-6*b + 1)^(3/2)
- 1)*f^6 + 11664*e^6 + 11664*e^5*f) + 54*integrate((54*(4*b + 1)*e^2*f^2 -
 4*((-6*b + 1)^(3/2) - 18*b - 1)*e*f^3 + (54*b^2 - (-6*b + 1)^(3/2) - 9*b
+ 1)*f^4 + 216*e^4 + 216*e^3*f + 108*(b*f^4 + 6*e^2*f^2 + 2*e*f^3)*x^2 + 2
*(((-6*b + 1)^(3/2) - 18*b - 1)*f^4 - 216*e^3*f - 216*e^2*f^2 - 54*e*f^3)*
x)/(108*x^3 + 54*b*x - 54*x^2 - (-6*b + 1)^(3/2) - 9*b + 1), x)/(2916*(4*b
 + 1)*e^4*f^2 + 216*((-6*b + 1)^(3/2) + 36*b - 1)*e^3*f^3 + 108*(27*b^2 +
(-6*b + 1)^(3/2) + 9*b - 1)*e^2*f^4 + 108*(((-6*b + 1)^(3/2) - 1)*b + 9*b^
2)*e*f^5 - ((6*b - 1)^3 - 18*((-6*b + 1)^(3/2) - 1)*b - 81*b^2 + 2*(-6*b +
 1)^(3/2) - 1)*f^6 + 11664*e^6 + 11664*e^5*f)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(e+fx)^2\left(1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3\right)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(f*x+e)^2/(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3),x, algor
ithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{-972,[1]%%%}+%%%{162,[0]%%%},[2]%%%}+%%%{%%{[%%%{-324,
[1]%%%}+%
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(e+fx)^2 \left(1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3\right)} \, dx = \text{Hanged}$$

input
```
int(-1/((e + f*x)^2*(9*b - 54*b*x + (1 - 6*b)^(3/2) + 54*x^2 - 108*x^3 - 1
)),x)
```

output
```
\text{Hanged}
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 44312, normalized size of antiderivative = 161.13

$$\int \frac{1}{(e+fx)^2 \left(1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3\right)} \, dx = \text{Too large to display}$$

input
```
int(1/(f*x+e)^2/(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3),x)
```

output

```
(8064*sqrt(6*b - 1)*sqrt( - 6*b + 1)*atan((6*x - 1)/sqrt(6*b - 1))*b**5*e*
*2*f**8 + 8064*sqrt(6*b - 1)*sqrt( - 6*b + 1)*atan((6*x - 1)/sqrt(6*b - 1)
)*b**5*e*f**9*x - 2688*sqrt(6*b - 1)*sqrt( - 6*b + 1)*atan((6*x - 1)/sqrt(
6*b - 1))*b**5*e*f**9 - 2688*sqrt(6*b - 1)*sqrt( - 6*b + 1)*atan((6*x - 1)
/sqrt(6*b - 1))*b**5*f**10*x - 100224*sqrt(6*b - 1)*sqrt( - 6*b + 1)*atan(
(6*x - 1)/sqrt(6*b - 1))*b**4*e**4*f**6 - 100224*sqrt(6*b - 1)*sqrt( - 6*b
 + 1)*atan((6*x - 1)/sqrt(6*b - 1))*b**4*e**3*f**7*x + 48384*sqrt(6*b - 1)
*sqrt( - 6*b + 1)*atan((6*x - 1)/sqrt(6*b - 1))*b**4*e**2*f**8*x**2 - 1612
8*sqrt(6*b - 1)*sqrt( - 6*b + 1)*atan((6*x - 1)/sqrt(6*b - 1))*b**4*e**2*f
**8*x + 2976*sqrt(6*b - 1)*sqrt( - 6*b + 1)*atan((6*x - 1)/sqrt(6*b - 1))*
b**4*e**2*f**8 + 48384*sqrt(6*b - 1)*sqrt( - 6*b + 1)*atan((6*x - 1)/sqrt(
6*b - 1))*b**4*e*f**9*x**3 - 32256*sqrt(6*b - 1)*sqrt( - 6*b + 1)*atan((6*
x - 1)/sqrt(6*b - 1))*b**4*e*f**9*x**2 + 8352*sqrt(6*b - 1)*sqrt( - 6*b +
1)*atan((6*x - 1)/sqrt(6*b - 1))*b**4*e*f**9*x + 2720*sqrt(6*b - 1)*sqrt(
- 6*b + 1)*atan((6*x - 1)/sqrt(6*b - 1))*b**4*e*f**9 - 16128*sqrt(6*b - 1)
*sqrt( - 6*b + 1)*atan((6*x - 1)/sqrt(6*b - 1))*b**4*f**10*x**3 + 5376*sqr
t(6*b - 1)*sqrt( - 6*b + 1)*atan((6*x - 1)/sqrt(6*b - 1))*b**4*f**10*x**2
+ 2720*sqrt(6*b - 1)*sqrt( - 6*b + 1)*atan((6*x - 1)/sqrt(6*b - 1))*b**4*f
**10*x - 396576*sqrt(6*b - 1)*sqrt( - 6*b + 1)*atan((6*x - 1)/sqrt(6*b - 1
))*b**3*e**6*f**4 - 396576*sqrt(6*b - 1)*sqrt( - 6*b + 1)*atan((6*x - 1...
```

3.145 $\int \frac{(e+fx)^2}{\left(1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3\right)^2}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 404

$$\begin{aligned}
&\int \frac{(e+fx)^2}{\left(1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3\right)^2}\,dx=\frac{18e^2+6ef+f^2-3bf^2-\sqrt{1-6b}f(6e+f)}{729(1-6b)\left(1-\sqrt{1-6b}-6x\right)^3}\\
&-\frac{36e^2+6\left(2+\sqrt{1-6b}\right)ef-\left(1-\sqrt{1-6b}-12b\right)f^2}{1458(1-6b)^{3/2}\left(1-\sqrt{1-6b}-6x\right)^2}\\
&+\frac{(6e+f)\left(6e+f+2\sqrt{1-6b}f\right)}{1458(1-6b)^2\left(1-\sqrt{1-6b}-6x\right)}\\
&+\frac{36e^2+12ef+5f^2-24bf^2+\sqrt{1-6b}f(24e+4f)}{4374(1-6b)^2\left(1+2\sqrt{1-6b}-6x\right)}\\
&+\frac{\left(72e^2+6\left(4+5\sqrt{1-6b}\right)ef+\left(4+5\sqrt{1-6b}-12b\right)f^2\right)\log\left(1-\sqrt{1-6b}-6x\right)}{6561(1-6b)^{5/2}}\\
&-\frac{\left(72e^2+6\left(4+5\sqrt{1-6b}\right)ef+\left(4+5\sqrt{1-6b}-12b\right)f^2\right)\log\left(1+2\sqrt{1-6b}-6x\right)}{6561(1-6b)^{5/2}}
\end{aligned}$$

output
```
1/729*(18*e^2+6*e*f+f^2-3*b*f^2-(1-6*b)^(1/2)*f*(6*e+f))/(1-6*b)/(1-(1-6*b
)^(1/2)-6*x)^3-1/1458*(36*e^2+6*(2+(1-6*b)^(1/2))*e*f-(1-(1-6*b)^(1/2)-12*
b)*f^2)/(1-6*b)^(3/2)/(1-(1-6*b)^(1/2)-6*x)^2+1/1458*(6*e+f)*(6*e+f+2*(1-6
*b)^(1/2)*f)/(1-6*b)^2/(1-(1-6*b)^(1/2)-6*x)+1/4374*(36*e^2+12*e*f+5*f^2-2
4*b*f^2+(1-6*b)^(1/2)*f*(24*e+4*f))/(1-6*b)^2/(1+2*(1-6*b)^(1/2)-6*x)+1/65
61*(72*e^2+6*(4+5*(1-6*b)^(1/2))*e*f+(4+5*(1-6*b)^(1/2)-12*b)*f^2)*ln(1-(1
-6*b)^(1/2)-6*x)/(1-6*b)^(5/2)-1/6561*(72*e^2+6*(4+5*(1-6*b)^(1/2))*e*f+(4
+5*(1-6*b)^(1/2)-12*b)*f^2)*ln(1+2*(1-6*b)^(1/2)-6*x)/(1-6*b)^(5/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 914 vs. $2(404) = 808$.

Time = 6.91 (sec) , antiderivative size = 914, normalized size of antiderivative = 2.26

$$\int \frac{(e+fx)^2}{\left(1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3\right)^2}\,dx = \frac{6e^2+6\sqrt{1-6b}e^2+12bef+bf^2-\sqrt{1-6b}bf^2-}{131}$$
$$+\frac{18e^2-18\sqrt{1-6b}e^2+30ef+6\sqrt{1-6b}ef-144bef+2f^2-2\sqrt{1-6b}f^2-9bf^2+21\sqrt{1-6b}bf^2-108}{26244(-1+6b)\left(b-2x+6x^2\right)^2}$$
$$+\frac{18e^2+9ef+6\sqrt{1-6b}ef-18bef+f^2+2\sqrt{1-6b}f^2-3bf^2-6\sqrt{1-6b}bf^2-108e^2x-36efx-36\sqrt{}}{4374(-1+6b)^2\left(b-2x+6x^2\right)}$$
$$+\frac{12e^2-24\sqrt{1-6b}e^2-12ef+96bef-f^2-2\sqrt{1-6b}f^2+8bf^2+16\sqrt{1-6b}bf^2-72e^2x-24efx-48}{4374(-1+6b)^2\left(-1+8b-4x+12x^2\right)}$$
$$+\frac{\left(-72e^2-24ef-30\sqrt{1-6b}ef-4f^2-5\sqrt{1-6b}f^2+12bf^2\right)\arctan\left(\frac{-1+6x}{2\sqrt{-1+6b}}\right)}{6561(-1+6b)^{5/2}}$$
$$+\frac{\left(-72e^2-24ef-30\sqrt{1-6b}ef-4f^2-5\sqrt{1-6b}f^2+12bf^2\right)\arctan\left(\frac{-1+6x}{\sqrt{-1+6b}}\right)}{6561(-1+6b)^{5/2}}$$
$$+\frac{\left(72\sqrt{1-6b}e^2+30ef+24\sqrt{1-6b}ef-180bef+5f^2+4\sqrt{1-6b}f^2-30bf^2-12\sqrt{1-6b}bf^2\right)\log(1-}{13122(-1+6b)^3}$$
$$+\frac{\left(-72\sqrt{1-6b}e^2-30ef-24\sqrt{1-6b}ef+180bef-5f^2-4\sqrt{1-6b}f^2+30bf^2+12\sqrt{1-6b}bf^2\right)\log(b}{13122(-1+6b)^3}$$

input
```
Integrate[(e + f*x)^2/(1 - (1 - 6*b)^(3/2) - 9*b + 54*b*x - 54*x^2 + 108*x
^3)^2,x]
```

output
```
(6*e^2 + 6*Sqrt[1 - 6*b]*e^2 + 12*b*e*f + b*f^2 - Sqrt[1 - 6*b]*b*f^2 - 36
*e^2*x - 12*e*f*x + 12*Sqrt[1 - 6*b]*e*f*x - 2*f^2*x + 2*Sqrt[1 - 6*b]*f^2
*x + 6*b*f^2*x)/(13122*(b - 2*x + 6*x^2)^3) + (18*e^2 - 18*Sqrt[1 - 6*b]*e
^2 + 30*e*f + 6*Sqrt[1 - 6*b]*e*f - 144*b*e*f + 2*f^2 - 2*Sqrt[1 - 6*b]*f^
2 - 9*b*f^2 + 21*Sqrt[1 - 6*b]*b*f^2 - 108*e^2*x - 36*e*f*x - 72*Sqrt[1 -
6*b]*e*f*x + 12*f^2*x - 12*Sqrt[1 - 6*b]*f^2*x - 90*b*f^2*x)/(26244*(-1 +
6*b)*(b - 2*x + 6*x^2)^2) + (18*e^2 + 9*e*f + 6*Sqrt[1 - 6*b]*e*f - 18*b*e
*f + f^2 + 2*Sqrt[1 - 6*b]*f^2 - 3*b*f^2 - 6*Sqrt[1 - 6*b]*b*f^2 - 108*e^2
*x - 36*e*f*x - 36*Sqrt[1 - 6*b]*e*f*x - 3*f^2*x - 6*Sqrt[1 - 6*b]*f^2*x)/
(4374*(-1 + 6*b)^2*(b - 2*x + 6*x^2)) + (12*e^2 - 24*Sqrt[1 - 6*b]*e^2 - 1
2*e*f + 96*b*e*f - f^2 - 2*Sqrt[1 - 6*b]*f^2 + 8*b*f^2 + 16*Sqrt[1 - 6*b]*
b*f^2 - 72*e^2*x - 24*e*f*x - 48*Sqrt[1 - 6*b]*e*f*x - 10*f^2*x - 8*Sqrt[1
 - 6*b]*f^2*x + 48*b*f^2*x)/(4374*(-1 + 6*b)^2*(-1 + 8*b - 4*x + 12*x^2))
+ ((-72*e^2 - 24*e*f - 30*Sqrt[1 - 6*b]*e*f - 4*f^2 - 5*Sqrt[1 - 6*b]*f^2
+ 12*b*f^2)*ArcTan[(-1 + 6*x)/(2*Sqrt[-1 + 6*b])])/(6561*(-1 + 6*b)^(5/2))
 + ((-72*e^2 - 24*e*f - 30*Sqrt[1 - 6*b]*e*f - 4*f^2 - 5*Sqrt[1 - 6*b]*f^2
 + 12*b*f^2)*ArcTan[(-1 + 6*x)/Sqrt[-1 + 6*b]])/(6561*(-1 + 6*b)^(5/2)) +
((72*Sqrt[1 - 6*b]*e^2 + 30*e*f + 24*Sqrt[1 - 6*b]*e*f - 180*b*e*f + 5*f^2
 + 4*Sqrt[1 - 6*b]*f^2 - 30*b*f^2 - 12*Sqrt[1 - 6*b]*b*f^2)*Log[1 - 8*b +
4*x - 12*x^2])/(13122*(-1 + 6*b)^3) + ((-72*Sqrt[1 - 6*b]*e^2 - 30*e*f ...
```

Rubi [A] (verified)

Time = 2.03 (sec) , antiderivative size = 424, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2488, 27, 27, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2}{\left(54bx-(1-6b)^{3/2}-9b+108x^3-54x^2+1\right)^2}dx$$

$$\Big\downarrow \text{2488}$$

$$983660201882413439385 6(1-6b)^6\int \frac{(e+fx)^2}{2459150504706033598464\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)^4\left(\left(2\sqrt{1-6b}+1\right)(1-6b)-6(1-6b)x\right.}$$

$$\Big\downarrow \text{27}$$

$$4(1-6b)^6\int \frac{(e+fx)^2}{(1-6b)^2\left(-6x+2\sqrt{1-6b}+1\right)^2\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)^4}dx$$

$$\Big\downarrow \; 27$$

$$4(1-6b)^4\int \frac{(e+fx)^2}{\left(-6x+2\sqrt{1-6b}+1\right)^2\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)^4}dx$$

$$\Big\downarrow \; 99$$

$$4(1-6b)^4\int \left(\frac{(6e+f)\left(6e+2\sqrt{1-6b}f+f\right)}{972(6b-1)^6\left(6x+\sqrt{1-6b}-1\right)^2}+\frac{-72e^2-6\left(5\sqrt{1-6b}+4\right)fe-\left(-12b+5\sqrt{1-6b}+4\right)f^2}{4374(1-6b)^{13/2}\left(-6x-\sqrt{1-6b}+1\right)}+\frac{72e^2\ldots}{\ldots}\right.$$

$$\Big\downarrow \; 2009$$

$$4(1-6b)^4\left(\frac{12e\left(2\sqrt{1-6b}f+f\right)+\left(-24b+4\sqrt{1-6b}+5\right)f^2+36e^2}{17496(1-6b)^6\left(2\sqrt{1-6b}-6x+1\right)}-\frac{6\left(\sqrt{1-6b}+2\right)ef-\left(-12b-\sqrt{1-6b}+1\right)\ldots}{5832(1-6b)^{11/2}\left(-\sqrt{1-6b}-6x+1\ldots\right.}\right.$$

input
```
Int[(e + f*x)^2/(1 - (1 - 6*b)^(3/2) - 9*b + 54*b*x - 54*x^2 + 108*x^3)^2,
x]
```

output
```
4*(1 - 6*b)^4*((18*e^2 + 6*(1 - Sqrt[1 - 6*b])*e*f + (1 - Sqrt[1 - 6*b] -
3*b)*f^2)/(2916*(1 - 6*b)^5*(1 - Sqrt[1 - 6*b] - 6*x)^3) - (36*e^2 + 6*(2
+ Sqrt[1 - 6*b])*e*f - (1 - Sqrt[1 - 6*b] - 12*b)*f^2)/(5832*(1 - 6*b)^(11
/2)*(1 - Sqrt[1 - 6*b] - 6*x)^2) + ((6*e + f)*(6*e + f + 2*Sqrt[1 - 6*b]*f
))/(5832*(1 - 6*b)^6*(1 - Sqrt[1 - 6*b] - 6*x)) + (36*e^2 + (5 + 4*Sqrt[1
- 6*b] - 24*b)*f^2 + 12*e*(f + 2*Sqrt[1 - 6*b]*f))/(17496*(1 - 6*b)^6*(1 +
 2*Sqrt[1 - 6*b] - 6*x)) + ((72*e^2 + 6*(4 + 5*Sqrt[1 - 6*b])*e*f + (4 + 5
*Sqrt[1 - 6*b] - 12*b)*f^2)*Log[1 - Sqrt[1 - 6*b] - 6*x])/(26244*(1 - 6*b)
^(13/2)) - ((72*e^2 + 6*(4 + 5*Sqrt[1 - 6*b])*e*f + (4 + 5*Sqrt[1 - 6*b] -
 12*b)*f^2)*Log[1 + 2*Sqrt[1 - 6*b] - 6*x])/(26244*(1 - 6*b)^(13/2)))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2488
```
Int[((e_.) + (f_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(
x_)^3)^(p_), x_Symbol] :> Simp[1/(4^p*(c^2 - 3*b*d)^(3*p))   Int[(e + f*x)^
m*(c^3 - 4*b*c*d + 9*a*d^2 + d*(c^2 - 3*b*d)*x)^p*(b*c - 9*a*d + 2*(c^2 - 3
*b*d)*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && NeQ[c^2 - 3
*b*d, 0] && EqQ[b^2*c^2 - 4*a*c^3 - 4*b^3*d + 18*a*b*c*d - 27*a^2*d^2, 0] &
& ILtQ[p, 0]
```

Maple [A] (verified)

Time = 2.58 (sec) , antiderivative size = 703, normalized size of antiderivative = 1.74

method	result
default	$-\frac{\left(30\sqrt{1-6b}\,ef+5\sqrt{1-6b}\,f^{2}-12b\,f^{2}+72e^{2}+24ef+4f^{2}\right)x^{3}}{13122\left(36b^{2}-12b+1\right)}-\frac{\left(30\sqrt{1-6b}\,ef+5\sqrt{1-6b}\,f^{2}-12b\,f^{2}+72e^{2}+24ef+4f^{2}\right)\left(\sqrt{1-6b}-2\right)x^{2}}{52488\left(36b^{2}-12b+1\right)}-\frac{(6b+\cdots}{\cdots}$
parallelrisch	Expression too large to display

input
```
int((f*x+e)^2/(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3)^2,x,method=_RETU
RNVERBOSE)
```

output
```
(-1/13122*(30*(1-6*b)^(1/2)*e*f+5*(1-6*b)^(1/2)*f^2-12*b*f^2+72*e^2+24*e*f
+4*f^2)/(36*b^2-12*b+1)*x^3-1/52488*(30*(1-6*b)^(1/2)*e*f+5*(1-6*b)^(1/2)*
f^2-12*b*f^2+72*e^2+24*e*f+4*f^2)/(36*b^2-12*b+1)*((1-6*b)^(1/2)-2)*x^2-1/
314928*(6*b+1-2*(1-6*b)^(1/2))*(864*b^2*e^2-72*e^2-f^2+360*(1-6*b)^(1/2)*b
*e*f+360*(1-6*b)^(1/2)*b^2*e*f+864*b*e^2+288*b^2*e*f+276*(1-6*b)^(1/2)*b^2
*f^2+504*b^3*f^2+42*b*f^2-204*b^2*f^2-24*e*f+(1-6*b)^(1/2)*f^2+288*b*e*f-3
0*(1-6*b)^(1/2)*e*f-12*(1-6*b)^(1/2)*b*f^2)/(12*b^2+12*b-1)/(36*b^2-12*b+1
)*x-1/2834352*(102*(1-6*b)^(1/2)*b-11*(1-6*b)^(1/2)-36*b+2)*(2448*(1-6*b)^
(1/2)*b^3*e*f+3972*(1-6*b)^(1/2)*b^3*f^2+12240*b^4*f^2+3060*(1-6*b)^(1/2)*
b^2*e*f-1164*(1-6*b)^(1/2)*b^2*f^2+124848*b^3*e^2+24120*b^3*e*f-6600*b^3*f
^2-804*(1-6*b)^(1/2)*b*e*f+127*(1-6*b)^(1/2)*b*f^2-45144*b^2*e^2-8244*b^2*
e*f+1464*b^2*f^2+51*(1-6*b)^(1/2)*e*f-5*(1-6*b)^(1/2)*f^2+5652*b*e^2+1074*
b*e*f-142*b*f^2-234*e^2-51*e*f+5*f^2)/(6936*b^3-2508*b^2+314*b-13)/(36*b^2
-12*b+1))/(x^3+1/18*(1-6*b)^(1/2)*b+1/2*b*x-1/2*x^2-1/108*(1-6*b)^(1/2)-1/
12*b+1/108)/(-1/6+x+1/6*(1-6*b)^(1/2))+8/81*(30*(1-6*b)^(1/2)*e*f+5*(1-6*b
)^(1/2)*f^2-12*b*f^2+72*e^2+24*e*f+4*f^2)/(11664*b^2-3888*b+324)/(-1+6*b)^
(1/2)*arctan(1/18*(-72*x+6*(1-6*b)^(1/2)+12)/(-1+6*b)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2724 vs. $2(347) = 694$.

Time = 0.16 (sec) , antiderivative size = 2724, normalized size of antiderivative = 6.74

$$\int \frac{(e+fx)^2}{\left(1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2/(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3)^2,x, algor
ithm="fricas")
```

output
```
-1/26244*(20736*(18*(6*b - 1)*e^2 + 6*(6*b - 1)*e*f - (18*b^2 - 9*b + 1)*f
^2)*x^7 - 1296*(336*(6*b - 1)*e^2 + 6*(60*b^2 + 92*b - 17)*e*f - (276*b^2
- 148*b + 17)*f^2)*x^6 + 648*(48*(72*b^2 + 18*b - 5)*e^2 + 12*(156*b^2 + 4
*b - 5)*e*f + (72*b^3 - 156*b^2 + 102*b - 13)*f^2)*x^5 - 24*(360*(216*b^2
- 30*b - 1)*e^2 - 3*(3888*b^3 - 13284*b^2 + 2424*b - 53)*e*f - (324*b^3 -
1152*b^2 - 63*b + 41)*f^2)*x^4 + 12*(6*(12312*b^3 + 2484*b^2 - 1014*b + 43
)*e^2 + 6*(1512*b^3 + 2724*b^2 - 754*b + 43)*e*f + (11016*b^4 - 6804*b^3 +
 2766*b^2 - 419*b + 16)*f^2)*x^3 - 6*(5256*b^4 - 2820*b^3 + 582*b^2 - 55*b
 + 2)*e^2 + 6*(288*b^5 - 804*b^4 + 264*b^3 - 29*b^2 + b)*e*f - 3*(96*b^5 +
 20*b^4 - 12*b^3 + b^2)*f^2 - 18*(2*(12312*b^3 - 3276*b^2 + 234*b - 5)*e^2
 - 2*(7128*b^4 - 7560*b^3 + 1482*b^2 - 52*b - 3)*e*f + (1296*b^4 - 492*b^3
 + 184*b^2 - 35*b + 2)*f^2)*x^2 + 6*(2*(15768*b^4 - 4356*b^3 + 270*b^2 + 9
*b - 1)*e^2 - 2*(1872*b^4 - 2868*b^3 + 852*b^2 - 89*b + 3)*e*f + (576*b^5
- 144*b^4 + 164*b^3 - 44*b^2 + 3*b)*f^2)*x - 4*(12960*(6*(6*b - 1)*e*f + (
6*b - 1)*f^2)*x^8 - 17280*(6*(6*b - 1)*e*f + (6*b - 1)*f^2)*x^7 + 7560*(6*
(12*b^2 + 4*b - 1)*e*f + (12*b^2 + 4*b - 1)*f^2)*x^6 - 840*(6*(108*b^2 - 1
2*b - 1)*e*f + (108*b^2 - 12*b - 1)*f^2)*x^5 + 100*(6*(324*b^3 + 216*b^2 -
 57*b + 2)*e*f + (324*b^3 + 216*b^2 - 57*b + 2)*f^2)*x^4 - 40*(6*(540*b^3
- 60*b^2 - 11*b + 1)*e*f + (540*b^3 - 60*b^2 - 11*b + 1)*f^2)*x^3 + 30*(48
*b^5 - 14*b^4 + b^3)*e*f + 5*(48*b^5 - 14*b^4 + b^3)*f^2 + 30*(6*(156*b...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2}{\left(1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3\right)^2}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**2/(1-(1-6*b)**(3/2)-9*b+54*b*x-54*x**2+108*x**3)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(e+fx)^2}{\left(1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3\right)^2}\,dx = \int \frac{(fx+e)^2}{\left(108\,x^3+54\,bx-54\,x^2-(-6\,b+1)^{\frac{3}{2}}-9\,b+1\right.}$$

input
```
integrate((f*x+e)^2/(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3)^2,x, algor
ithm="maxima")
```

output
```
1/324*(18*(72*b^2 - (-6*b + 1)^(3/2) - 18*b + 1)*e^2 + 12*(3*(2*(-6*b + 1)
^(3/2) - 5)*b + 54*b^2 - (-6*b + 1)^(3/2) + 1)*e*f - ((6*b - 1)^3 - 15*((-
6*b + 1)^(3/2) - 1)*b - 54*b^2 + 2*(-6*b + 1)^(3/2) - 1)*f^2 + 36*(18*(6*b
 - 1)*e^2 + 6*((-6*b + 1)^(3/2) + 6*b - 1)*e*f - (18*b^2 - (-6*b + 1)^(3/2
) - 9*b + 1)*f^2)*x^2 + 6*(18*((-6*b + 1)^(3/2) - 12*b + 2)*e^2 - 6*(36*b^
2 + (-6*b + 1)^(3/2) - 1)*e*f - (3*((-6*b + 1)^(3/2) + 2)*b + (-6*b + 1)^(
3/2) - 1)*f^2)*x)/((-6*b + 1)^(9/2) + 108*(2*(-6*b + 1)^(3/2) - 11)*b^3 +
1944*b^4 + 108*((6*b - 1)^3 - 216*b^3 + 108*b^2 - 18*b + 1)*x^3 + (6*b - 1
)^3 - 54*(2*(-6*b + 1)^(3/2) - 5)*b^2 - 54*((6*b - 1)^3 - 216*b^3 + 108*b^
2 - 18*b + 1)*x^2 - 9*((6*b - 1)^3 - 2*(-6*b + 1)^(3/2) + 3)*b - 54*(216*b
^4 - 108*b^3 - ((6*b - 1)^3 + 1)*b + 18*b^2)*x - (-6*b + 1)^(3/2) + 1) - 1
/54*integrate(-(18*(2*(-6*b + 1)^(3/2) - 6*b + 1)*e^2 - 6*(72*b^2 - (-6*b
+ 1)^(3/2) - 18*b + 1)*e*f - (3*(2*(-6*b + 1)^(3/2) - 5)*b + 54*b^2 - (-6*
b + 1)^(3/2) + 1)*f^2 + 6*(18*(6*b - 1)*e^2 + 6*((-6*b + 1)^(3/2) + 6*b -
1)*e*f - (18*b^2 - (-6*b + 1)^(3/2) - 9*b + 1)*f^2)*x)/(108*x^3 + 54*b*x -
 54*x^2 - (-6*b + 1)^(3/2) - 9*b + 1), x)/((6*b - 1)^3 - 216*b^3 + 108*b^2
 - 18*b + 1)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(e+fx)^2}{\left(1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3\right)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((f*x+e)^2/(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3)^2,x, algor
ithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{17006112,[3]%%%}+%%%{-8503056,[2]%%%}+%%%{1417176,[1]%
%%}+%%%{-
```

Mupad [B] (verification not implemented)

Time = 1.95 (sec) , antiderivative size = 645, normalized size of antiderivative = 1.60

$$\int \frac{(e+fx)^2}{\left(1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3\right)^2}\,dx = \text{Too large to display}$$

input
```
int((e + f*x)^2/(9*b - 54*b*x + (1 - 6*b)^(3/2) + 54*x^2 - 108*x^3 - 1)^2,
x)
```

output
```
(x^2*(((e*f)/104976 + (b*f^2)/314928 + e^2/13122 + f^2/629856 + (5*b*e*f)/
52488)/(b^2 - b/3 + 1/36) - ((1 - 6*b)^(3/2)*((e*f)/314928 + (b*f^2)/94478
4 - e^2/157464 + f^2/1889568))/(b/12 - b^2/2 + b^3 - 1/216)) - x^3*(((e*f)
/19683 - (b*f^2)/39366 + e^2/6561 + f^2/118098)/(b^2 - b/3 + 1/36) - ((1 -
 6*b)^(3/2)*((5*e*f)/472392 + (5*f^2)/2834352))/(b/12 - b^2/2 + b^3 - 1/21
6)) - x*(((b*e^2)/26244 - (e*f)/314928 - (b*f^2)/314928 + e^2/157464 + f^2
/3779136 + (7*b^2*f^2)/314928 + (7*b*e*f)/157464)/(b^2 - b/3 + 1/36) + ((1
 - 6*b)^(3/2)*((e*f)/3779136 - (b*f^2)/1259712 + e^2/472392 + f^2/22674816
 - (5*b*e*f)/1889568))/(b/12 - b^2/2 + b^3 - 1/216)) + ((b*e^2)/157464 - (
17*e*f)/34012224 - (29*b*f^2)/34012224 - e^2/2834352 + (5*f^2)/102036672 +
 (23*b^2*f^2)/5668704 + (23*b*e*f)/5668704 + (b^2*e*f)/472392)/(b^2 - b/3
+ 1/36) + ((1 - 6*b)^(3/2)*((17*b*e^2)/5668704 - (17*e*f)/204073344 - (b*f
^2)/8503056 - (11*e^2)/34012224 + (5*f^2)/612220032 + (5*b^2*f^2)/17006112
 + (19*b*e*f)/34012224))/(b/12 - b^2/2 + b^3 - 1/216))/((7*b)/216 - x*(b/6
 + ((1 - 6*b)^(3/2)*((5*b)/216 - 1/648))/(b - 1/6) - 1/108) + x^2*(b/2 + (
1 - 6*b)^(3/2)/(72*(b - 1/6)) + 1/12) - x^3*((1 - 6*b)^(3/2)/(36*(b - 1/6)
) + 2/3) - b^2/18 + x^4 + ((1 - 6*b)^(3/2)*((5*b)/1296 - 1/1944))/(b - 1/6
) - 1/324) + (atan((b*x*72i - x*12i - b*12i + (1 - 6*b)^(3/2)*1i + 2i)/(86
4*b^2 - (6*b - 1)^3 - 144*b - 1728*b^3 + 8)^(1/2))*(24*e*f - 432*b*e^2 - 3
6*b*f^2 + 5*f^2*(1 - 6*b)^(3/2) + 72*e^2 + 4*f^2 + 72*b^2*f^2 - 144*b*e...
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 15132, normalized size of antiderivative = 37.46

$$\int \frac{(e+fx)^2}{\left(1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3\right)^2}\,dx = \text{Too large to display}$$

input
```
int((f*x+e)^2/(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3)^2,x)
```

output
```
( - 1920*sqrt(6*b - 1)*sqrt( - 6*b + 1)*atan((6*x - 1)/sqrt(6*b - 1))*b**4
*e*f - 320*sqrt(6*b - 1)*sqrt( - 6*b + 1)*atan((6*x - 1)/sqrt(6*b - 1))*b*
*4*f**2 - 37440*sqrt(6*b - 1)*sqrt( - 6*b + 1)*atan((6*x - 1)/sqrt(6*b - 1
))*b**3*e*f*x**2 + 12480*sqrt(6*b - 1)*sqrt( - 6*b + 1)*atan((6*x - 1)/sqr
t(6*b - 1))*b**3*e*f*x + 240*sqrt(6*b - 1)*sqrt( - 6*b + 1)*atan((6*x - 1)
/sqrt(6*b - 1))*b**3*e*f - 6240*sqrt(6*b - 1)*sqrt( - 6*b + 1)*atan((6*x -
 1)/sqrt(6*b - 1))*b**3*f**2*x**2 + 2080*sqrt(6*b - 1)*sqrt( - 6*b + 1)*at
an((6*x - 1)/sqrt(6*b - 1))*b**3*f**2*x + 40*sqrt(6*b - 1)*sqrt( - 6*b + 1
)*atan((6*x - 1)/sqrt(6*b - 1))*b**3*f**2 - 259200*sqrt(6*b - 1)*sqrt( - 6
*b + 1)*atan((6*x - 1)/sqrt(6*b - 1))*b**2*e*f*x**4 + 172800*sqrt(6*b - 1)
*sqrt( - 6*b + 1)*atan((6*x - 1)/sqrt(6*b - 1))*b**2*e*f*x**3 - 24480*sqrt
(6*b - 1)*sqrt( - 6*b + 1)*atan((6*x - 1)/sqrt(6*b - 1))*b**2*e*f*x**2 - 1
440*sqrt(6*b - 1)*sqrt( - 6*b + 1)*atan((6*x - 1)/sqrt(6*b - 1))*b**2*e*f*
x - 43200*sqrt(6*b - 1)*sqrt( - 6*b + 1)*atan((6*x - 1)/sqrt(6*b - 1))*b**
2*f**2*x**4 + 28800*sqrt(6*b - 1)*sqrt( - 6*b + 1)*atan((6*x - 1)/sqrt(6*b
 - 1))*b**2*f**2*x**3 - 4080*sqrt(6*b - 1)*sqrt( - 6*b + 1)*atan((6*x - 1)
/sqrt(6*b - 1))*b**2*f**2*x**2 - 240*sqrt(6*b - 1)*sqrt( - 6*b + 1)*atan((
6*x - 1)/sqrt(6*b - 1))*b**2*f**2*x - 725760*sqrt(6*b - 1)*sqrt( - 6*b + 1
)*atan((6*x - 1)/sqrt(6*b - 1))*b*e*f*x**6 + 725760*sqrt(6*b - 1)*sqrt( -
6*b + 1)*atan((6*x - 1)/sqrt(6*b - 1))*b*e*f*x**5 - 216000*sqrt(6*b - 1...
```

3.146 $\int \frac{e+fx}{\left(1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3\right)^2}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 284

$$\int \frac{e+fx}{(1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3)^2}\,dx=\frac{6e+f-\sqrt{1-6b}f}{243(1-6b)\left(1-\sqrt{1-6b}-6x\right)^3}-\frac{12e+\left(2+\sqrt{1-6b}\right)f}{486(1-6b)^{3/2}\left(1-\sqrt{1-6b}-6x\right)^2}+\frac{6e+f+\sqrt{1-6b}f}{243(1-6b)^2\left(1-\sqrt{1-6b}-6x\right)}+\frac{6e+f+2\sqrt{1-6b}f}{729(1-6b)^2\left(1+2\sqrt{1-6b}-6x\right)}+\frac{\left(24e+\left(4+5\sqrt{1-6b}\right)f\right)\log\left(1-\sqrt{1-6b}-6x\right)}{2187(1-6b)^{5/2}}-\frac{\left(24e+\left(4+5\sqrt{1-6b}\right)f\right)\log\left(1+2\sqrt{1-6b}-6x\right)}{2187(1-6b)^{5/2}}$$

output

```
1/243*(6*e+f-(1-6*b)^(1/2)*f)/(1-6*b)/(1-(1-6*b)^(1/2)-6*x)^3-1/486*(12*e+
(2+(1-6*b)^(1/2))*f)/(1-6*b)^(3/2)/(1-(1-6*b)^(1/2)-6*x)^2+1/243*(6*e+f+(1
-6*b)^(1/2)*f)/(1-6*b)^2/(1-(1-6*b)^(1/2)-6*x)+1/729*(6*e+f+2*(1-6*b)^(1/2
)*f)/(1-6*b)^2/(1+2*(1-6*b)^(1/2)-6*x)+1/2187*(24*e+(4+5*(1-6*b)^(1/2))*f)
*ln(1-(1-6*b)^(1/2)-6*x)/(1-6*b)^(5/2)-1/2187*(24*e+(4+5*(1-6*b)^(1/2))*f)
*ln(1+2*(1-6*b)^(1/2)-6*x)/(1-6*b)^(5/2)
```

Mathematica [A] (verified)

Time = 5.97 (sec) , antiderivative size = 451, normalized size of antiderivative = 1.59

$$\int \frac{e+fx}{\left(1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3\right)^2}\,dx = \frac{\frac{4\left(e\left(1+\sqrt{1-6b}-6x\right)+f\left(b+\left(-1+\sqrt{1-6b}\right)x\right)\right)}{(b+2x(-1+3x))^3} - \frac{3(12e(-1+6x)+f(}{(1-}}{}$$

input
```
Integrate[(e + f*x)/(1 - (1 - 6*b)^(3/2) - 9*b + 54*b*x - 54*x^2 + 108*x^3
)^2,x]
```

output
```
((4*(e*(1 + Sqrt[1 - 6*b] - 6*x) + f*(b + (-1 + Sqrt[1 - 6*b])*x)))/(b + 2
*x*(-1 + 3*x))^3 - (3*(12*e*(-1 + 6*x) + f*(-3 - 2*Sqrt[1 - 6*b] + 6*b + 1
2*(1 + Sqrt[1 - 6*b])*x)))/((1 - 6*b)^2*(b + 2*x*(-1 + 3*x))) + (-6*e*(-1
+ Sqrt[1 - 6*b] + 6*x) + f*(5 + Sqrt[1 - 6*b] - 24*b - 6*(1 + 2*Sqrt[1 - 6
*b])*x))/((-1 + 6*b)*(b + 2*x*(-1 + 3*x))^2) - (12*(2*e*(-1 + 2*Sqrt[1 - 6
*b] + 6*x) + f*(1 - 8*b + 2*x + 4*Sqrt[1 - 6*b]*x)))/((1 - 6*b)^2*(-1 + 8*
b - 4*x + 12*x^2)) - (4*(24*e + (4 + 5*Sqrt[1 - 6*b])*f)*ArcTan[(-1 + 6*x)
/(2*Sqrt[-1 + 6*b])])/(-1 + 6*b)^(5/2) - (4*(24*e + (4 + 5*Sqrt[1 - 6*b])*
f)*ArcTan[(-1 + 6*x)/Sqrt[-1 + 6*b]])/(-1 + 6*b)^(5/2) + (2*(24*Sqrt[1 - 6
*b]*e + (5 + 4*Sqrt[1 - 6*b] - 30*b)*f)*Log[1 - 8*b + 4*x - 12*x^2])/(-1 +
 6*b)^3 - (2*(24*Sqrt[1 - 6*b]*e + (5 + 4*Sqrt[1 - 6*b] - 30*b)*f)*Log[b -
 2*x + 6*x^2])/(-1 + 6*b)^3)/8748
```

Rubi [A] (verified)

Time = 1.30 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$, Rules used = {2488, 27, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e+fx}{\left(54bx-(1-6b)^{3/2}-9b+108x^3-54x^2+1\right)^2}dx$$

$$\Big\downarrow \text{ 2488}$$

$$983660201882413439385 6(1-6b)^6\int \frac{e+fx}{2459150504706033598464\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)^4\left(\left(2\sqrt{1-6b}+1\right)(1-6b)-6(1-6b)x\right.}$$

$$\Big\downarrow \; 27$$

$$4(1-6b)^6\int \frac{e+fx}{(1-6b)^2\left(-6x+2\sqrt{1-6b}+1\right)^2\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)^4}dx$$

$$\Big\downarrow \; 27$$

$$4(1-6b)^4\int \frac{e+fx}{\left(-6x+2\sqrt{1-6b}+1\right)^2\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)^4}dx$$

$$\Big\downarrow \; 86$$

$$4(1-6b)^4\int \left(\frac{6e+\left(1-\sqrt{1-6b}\right)f}{54(1-6b)^5\left(-6x-\sqrt{1-6b}+1\right)^4}+\frac{-24e-\left(5\sqrt{1-6b}+4\right)f}{1458(1-6b)^{13/2}\left(-6x-\sqrt{1-6b}+1\right)}+\frac{24e+\left(5\sqrt{1-\right.}{1458(1-6b)^{13/2}\left(-6x\right.}\right.$$

$$\Big\downarrow \; 2009$$

$$4(1-6b)^4\left(-\frac{\left(\sqrt{1-6b}+2\right)f+12e}{1944(1-6b)^{11/2}\left(-\sqrt{1-6b}-6x+1\right)^2}+\frac{\sqrt{1-6b}f+6e+f}{972(1-6b)^6\left(-\sqrt{1-6b}-6x+1\right)}+\frac{2\sqrt{1-6b}f+6e}{2916(1-6b)^6\left(2\sqrt{1-\right.}\right.$$

input
```
Int[(e + f*x)/(1 - (1 - 6*b)^(3/2) - 9*b + 54*b*x - 54*x^2 + 108*x^3)^2,x]
```

output
```
4*(1 - 6*b)^4*((6*e + f - Sqrt[1 - 6*b]*f)/(972*(1 - 6*b)^5*(1 - Sqrt[1 -
6*b] - 6*x)^3) - (12*e + (2 + Sqrt[1 - 6*b])*f)/(1944*(1 - 6*b)^(11/2)*(1
- Sqrt[1 - 6*b] - 6*x)^2) + (6*e + f + Sqrt[1 - 6*b]*f)/(972*(1 - 6*b)^6*(
1 - Sqrt[1 - 6*b] - 6*x)) + (6*e + f + 2*Sqrt[1 - 6*b]*f)/(2916*(1 - 6*b)^
6*(1 + 2*Sqrt[1 - 6*b] - 6*x)) + ((24*e + (4 + 5*Sqrt[1 - 6*b])*f)*Log[1 -
 Sqrt[1 - 6*b] - 6*x])/(8748*(1 - 6*b)^(13/2)) - ((24*e + (4 + 5*Sqrt[1 -
6*b])*f)*Log[1 + 2*Sqrt[1 - 6*b] - 6*x])/(8748*(1 - 6*b)^(13/2)))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2488
```
Int[((e_.) + (f_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(
x_)^3)^(p_), x_Symbol] :> Simp[1/(4^p*(c^2 - 3*b*d)^(3*p))   Int[(e + f*x)^
m*(c^3 - 4*b*c*d + 9*a*d^2 + d*(c^2 - 3*b*d)*x)^p*(b*c - 9*a*d + 2*(c^2 - 3
*b*d)*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && NeQ[c^2 - 3
*b*d, 0] && EqQ[b^2*c^2 - 4*a*c^3 - 4*b^3*d + 18*a*b*c*d - 27*a^2*d^2, 0] &
& ILtQ[p, 0]
```

Maple [A] (verified)

Time = 2.40 (sec) , antiderivative size = 382, normalized size of antiderivative = 1.35

method	result
default	$\frac{-\frac{\left(5\sqrt{1-6b}\,f+24e+4f\right)x^{3}}{4374\left(36b^{2}-12b+1\right)}-\frac{\left(5\sqrt{1-6b}\,f+24e+4f\right)\left(\sqrt{1-6b}-2\right)x^{2}}{17496\left(36b^{2}-12b+1\right)}-\frac{\left(5\sqrt{1-6b}\,f+24e+4f\right)\left(6b+1-2\sqrt{1-6b}\right)x}{104976\left(36b^{2}-12b+1\right)}-\frac{\left(102\sqrt{1-6b}\,b-11\sqrt{1-6b}-36b\right.}{}}{\left(x^{3}+\frac{\sqrt{1-6b}\,b}{18}+\frac{bx}{2}-\frac{x^{2}}{2}-\frac{\sqrt{1-6b}}{108}\right.}$
parallelrisch	Expression too large to display

input
```
int((f*x+e)/(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3)^2,x,method=_RETURN
VERBOSE)
```

output
```
(-1/4374*(5*(1-6*b)^(1/2)*f+24*e+4*f)/(36*b^2-12*b+1)*x^3-1/17496*(5*(1-6*
b)^(1/2)*f+24*e+4*f)/(36*b^2-12*b+1)*((1-6*b)^(1/2)-2)*x^2-1/104976*(5*(1-
6*b)^(1/2)*f+24*e+4*f)/(36*b^2-12*b+1)*(6*b+1-2*(1-6*b)^(1/2))*x-1/1889568
*(102*(1-6*b)^(1/2)*b-11*(1-6*b)^(1/2)-36*b+2)*(816*(1-6*b)^(1/2)*b^3*f+10
20*(1-6*b)^(1/2)*b^2*f+83232*b^3*e+8040*b^3*f-268*(1-6*b)^(1/2)*f*b-30096*
b^2*e-2748*b^2*f+17*(1-6*b)^(1/2)*f+3768*e*b+358*b*f-156*e-17*f)/(6936*b^3
-2508*b^2+314*b-13)/(36*b^2-12*b+1))/(x^3+1/18*(1-6*b)^(1/2)*b+1/2*b*x-1/2
*x^2-1/108*(1-6*b)^(1/2)-1/12*b+1/108)/(-1/6+x+1/6*(1-6*b)^(1/2))+8/27*(5*
(1-6*b)^(1/2)*f+24*e+4*f)/(11664*b^2-3888*b+324)/(-1+6*b)^(1/2)*arctan(1/1
8*(-72*x+6*(1-6*b)^(1/2)+12)/(-1+6*b)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1615 vs. $2(237) = 474$.

Time = 0.13 (sec) , antiderivative size = 1615, normalized size of antiderivative = 5.69

$$\int \frac{e+fx}{\left(1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)/(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3)^2,x, algorit
hm="fricas")
```

output
```
-1/8748*(20736*(6*(6*b - 1)*e + (6*b - 1)*f)*x^7 - 1296*(112*(6*b - 1)*e +
 (60*b^2 + 92*b - 17)*f)*x^6 + 1296*(8*(72*b^2 + 18*b - 5)*e + (156*b^2 +
4*b - 5)*f)*x^5 - 12*(240*(216*b^2 - 30*b - 1)*e - (3888*b^3 - 13284*b^2 +
 2424*b - 53)*f)*x^4 + 12*(2*(12312*b^3 + 2484*b^2 - 1014*b + 43)*e + (151
2*b^3 + 2724*b^2 - 754*b + 43)*f)*x^3 - 6*(2*(12312*b^3 - 3276*b^2 + 234*b
 - 5)*e - (7128*b^4 - 7560*b^3 + 1482*b^2 - 52*b - 3)*f)*x^2 - 2*(5256*b^4
 - 2820*b^3 + 582*b^2 - 55*b + 2)*e + (288*b^5 - 804*b^4 + 264*b^3 - 29*b^
2 + b)*f + 2*(2*(15768*b^4 - 4356*b^3 + 270*b^2 + 9*b - 1)*e - (1872*b^4 -
 2868*b^3 + 852*b^2 - 89*b + 3)*f)*x - 4*(12960*(6*b - 1)*f*x^8 - 17280*(6
*b - 1)*f*x^7 + 7560*(12*b^2 + 4*b - 1)*f*x^6 - 840*(108*b^2 - 12*b - 1)*f
*x^5 + 100*(324*b^3 + 216*b^2 - 57*b + 2)*f*x^4 - 40*(540*b^3 - 60*b^2 - 1
1*b + 1)*f*x^3 + 30*(156*b^4 + 76*b^3 - 29*b^2 + 2*b)*f*x^2 - 10*(156*b^4
- 44*b^3 + 3*b^2)*f*x + 5*(48*b^5 - 14*b^4 + b^3)*f - 4*(2592*(6*e + f)*x^
8 - 3456*(6*e + f)*x^7 + 1512*(6*(2*b + 1)*e + (2*b + 1)*f)*x^6 - 168*(6*(
18*b + 1)*e + (18*b + 1)*f)*x^5 + 20*(6*(54*b^2 + 45*b - 2)*e + (54*b^2 +
45*b - 2)*f)*x^4 - 8*(6*(90*b^2 + 5*b - 1)*e + (90*b^2 + 5*b - 1)*f)*x^3 +
 6*(6*(26*b^3 + 17*b^2 - 2*b)*e + (26*b^3 + 17*b^2 - 2*b)*f)*x^2 + 6*(8*b^
4 - b^3)*e + (8*b^4 - b^3)*f - 2*(6*(26*b^3 - 3*b^2)*e + (26*b^3 - 3*b^2)*
f)*x)*sqrt(-6*b + 1))*log(6*x + sqrt(-6*b + 1) - 1) + 4*(12960*(6*b - 1)*f
*x^8 - 17280*(6*b - 1)*f*x^7 + 7560*(12*b^2 + 4*b - 1)*f*x^6 - 840*(108...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2183 vs. $2(257) = 514$.

Time = 162.57 (sec) , antiderivative size = 2183, normalized size of antiderivative = 7.69

$$\int \frac{e + fx}{\left(1 - (1 - 6b)^{3/2} - 9b + 54bx - 54x^2 + 108x^3\right)^2}\, dx = \text{Too large to display}$$

input
```
integrate((f*x+e)/(1-(1-6*b)**(3/2)-9*b+54*b*x-54*x**2+108*x**3)**2,x)
```

output
```
-sqrt((150*b*f**2 - 576*e**2 - 240*e*f*sqrt(1 - 6*b) - 192*e*f - 40*f**2*s
qrt(1 - 6*b) - 41*f**2)/(7776*b**5 - 6480*b**4 + 2160*b**3 - 360*b**2 + 30
*b - 1))*log(x - sqrt((150*b*f**2 - 576*e**2 - 240*e*f*sqrt(1 - 6*b) - 192
*e*f - 40*f**2*sqrt(1 - 6*b) - 41*f**2)/(7776*b**5 - 6480*b**4 + 2160*b**3
 - 360*b**2 + 30*b - 1))*(11337408*b**3*e/(600*b*f**2 - 2304*e**2 - 960*e*
f*sqrt(1 - 6*b) - 768*e*f - 160*f**2*sqrt(1 - 6*b) - 164*f**2) + 2361960*b
**3*f*sqrt(1 - 6*b)/(600*b*f**2 - 2304*e**2 - 960*e*f*sqrt(1 - 6*b) - 768*
e*f - 160*f**2*sqrt(1 - 6*b) - 164*f**2) + 1889568*b**3*f/(600*b*f**2 - 23
04*e**2 - 960*e*f*sqrt(1 - 6*b) - 768*e*f - 160*f**2*sqrt(1 - 6*b) - 164*f
**2) - 5668704*b**2*e/(600*b*f**2 - 2304*e**2 - 960*e*f*sqrt(1 - 6*b) - 76
8*e*f - 160*f**2*sqrt(1 - 6*b) - 164*f**2) - 1180980*b**2*f*sqrt(1 - 6*b)/
(600*b*f**2 - 2304*e**2 - 960*e*f*sqrt(1 - 6*b) - 768*e*f - 160*f**2*sqrt(
1 - 6*b) - 164*f**2) - 944784*b**2*f/(600*b*f**2 - 2304*e**2 - 960*e*f*sqr
t(1 - 6*b) - 768*e*f - 160*f**2*sqrt(1 - 6*b) - 164*f**2) + 944784*b*e/(60
0*b*f**2 - 2304*e**2 - 960*e*f*sqrt(1 - 6*b) - 768*e*f - 160*f**2*sqrt(1 -
 6*b) - 164*f**2) + 196830*b*f*sqrt(1 - 6*b)/(600*b*f**2 - 2304*e**2 - 960
*e*f*sqrt(1 - 6*b) - 768*e*f - 160*f**2*sqrt(1 - 6*b) - 164*f**2) + 157464
*b*f/(600*b*f**2 - 2304*e**2 - 960*e*f*sqrt(1 - 6*b) - 768*e*f - 160*f**2*
sqrt(1 - 6*b) - 164*f**2) - 52488*e/(600*b*f**2 - 2304*e**2 - 960*e*f*sqrt
(1 - 6*b) - 768*e*f - 160*f**2*sqrt(1 - 6*b) - 164*f**2) - 10935*f*sqrt...
```

Maxima [F]

$$\int \frac{e+fx}{\left(1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3\right)^2}\,dx = \int \frac{fx+e}{\left(108\,x^3+54\,bx-54\,x^2-(-6\,b+1)^{\frac{3}{2}}-9\,b+1\right)^2}$$

input
```
integrate((f*x+e)/(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3)^2,x, algorit
hm="maxima")
```

output
```
1/54*(18*(6*(6*b - 1)*e + ((-6*b + 1)^(3/2) + 6*b - 1)*f)*x^2 + 3*(72*b^2
- (-6*b + 1)^(3/2) - 18*b + 1)*e + (3*(2*(-6*b + 1)^(3/2) - 5)*b + 54*b^2
- (-6*b + 1)^(3/2) + 1)*f + 3*(6*((-6*b + 1)^(3/2) - 12*b + 2)*e - (36*b^2
 + (-6*b + 1)^(3/2) - 1)*f)*x)/((-6*b + 1)^(9/2) + 108*(2*(-6*b + 1)^(3/2)
 - 11)*b^3 + 1944*b^4 + 108*((6*b - 1)^3 - 216*b^3 + 108*b^2 - 18*b + 1)*x
^3 + (6*b - 1)^3 - 54*(2*(-6*b + 1)^(3/2) - 5)*b^2 - 54*((6*b - 1)^3 - 216
*b^3 + 108*b^2 - 18*b + 1)*x^2 - 9*((6*b - 1)^3 - 2*(-6*b + 1)^(3/2) + 3)*
b - 54*(216*b^4 - 108*b^3 - ((6*b - 1)^3 + 1)*b + 18*b^2)*x - (-6*b + 1)^(
3/2) + 1) + 1/18*integrate((6*(2*(-6*b + 1)^(3/2) - 6*b + 1)*e - (72*b^2 -
 (-6*b + 1)^(3/2) - 18*b + 1)*f + 6*(6*(6*b - 1)*e + ((-6*b + 1)^(3/2) + 6
*b - 1)*f)*x)/(108*x^3 + 54*b*x - 54*x^2 - (-6*b + 1)^(3/2) - 9*b + 1), x)
/((6*b - 1)^3 - 216*b^3 + 108*b^2 - 18*b + 1)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e+fx}{\left(1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3\right)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((f*x+e)/(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3)^2,x, algorit
hm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{17006112,[3]%%%}+%%%{-8503056,[2]%%%}+%%%{1417176,[1]%
%%}+%%%{-
```

Mupad [B] (verification not implemented)

Time = 1.01 (sec) , antiderivative size = 466, normalized size of antiderivative = 1.64

$$\int \frac{e+fx}{\left(1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3\right)^2}\,dx =$$

$$-\frac{x\left(\frac{\frac{e}{157464}-\frac{f}{629856}+\frac{b\,e}{26244}+\frac{7\,b\,f}{314928}}{b^2-\frac{b}{3}+\frac{1}{36}}+\frac{(1-6\,b)^{3/2}\left(\frac{e}{472392}+\frac{f}{7558272}-\frac{5\,b\,f}{3779136}\right)}{b^3-\frac{b^2}{2}+\frac{b}{12}-\frac{1}{216}}\right)-\frac{\frac{b\,e}{157464}-\frac{17\,f}{68024448}-\frac{e}{2834352}+\frac{23\,b\,f}{11337408}+\frac{b^2\,f}{944784}}{b^2-\frac{b}{3}+\frac{1}{36}}-x^2}{\frac{7\,b}{216}-x\left(\frac{b}{6}+\frac{(1-6\,b)^{3/2}\left(\frac{5\,b}{216}-\frac{1}{648}\right)}{b-\frac{1}{6}}-\frac{1}{108}\right)+x^2\left(\frac{b}{2}+\right.}$$

$$+\frac{\mathrm{atan}\left(\frac{-b\,12\mathrm{i}-x\,12\mathrm{i}+b\,x\,72\mathrm{i}+(1-6\,b)^{3/2}\,1\mathrm{i}+2\mathrm{i}}{\sqrt{864\,b^2-(6\,b-1)^3-144\,b-1728\,b^3+8}}\right)\left(24\,e+4\,f-144\,b\,e-24\,b\,f+5\,f\,(1-6\,b)^{3/2}\right)\,2\mathrm{i}}{729\,(6\,b-1)^2\,\sqrt{864\,b^2-(6\,b-1)^3-144\,b-1728\,b^3+8}}$$

input
```
int((e + f*x)/(9*b - 54*b*x + (1 - 6*b)^(3/2) + 54*x^2 - 108*x^3 - 1)^2,x)
```

output
```
(atan((b*x*72i - x*12i - b*12i + (1 - 6*b)^(3/2)*1i + 2i)/(864*b^2 - (6*b
- 1)^3 - 144*b - 1728*b^3 + 8)^(1/2))*(24*e + 4*f - 144*b*e - 24*b*f + 5*f
*(1 - 6*b)^(3/2))*2i)/(729*(6*b - 1)^2*(864*b^2 - (6*b - 1)^3 - 144*b - 17
28*b^3 + 8)^(1/2)) - (x*((e/157464 - f/629856 + (b*e)/26244 + (7*b*f)/3149
28)/(b^2 - b/3 + 1/36) + ((1 - 6*b)^(3/2)*(e/472392 + f/7558272 - (5*b*f)/
3779136))/(b/12 - b^2/2 + b^3 - 1/216)) - ((b*e)/157464 - (17*f)/68024448
- e/2834352 + (23*b*f)/11337408 + (b^2*f)/944784)/(b^2 - b/3 + 1/36) - x^2
*((e/13122 + f/209952 + (5*b*f)/104976)/(b^2 - b/3 + 1/36) + ((1 - 6*b)^(3
/2)*(e/157464 - f/629856))/(b/12 - b^2/2 + b^3 - 1/216)) + x^3*((e/6561 +
f/39366)/(b^2 - b/3 + 1/36) - (5*f*(1 - 6*b)^(3/2))/(944784*(b/12 - b^2/2
+ b^3 - 1/216))) + ((1 - 6*b)^(3/2)*((11*e)/34012224 + (17*f)/408146688 -
(17*b*e)/5668704 - (19*b*f)/68024448))/(b/12 - b^2/2 + b^3 - 1/216))/((7*b
)/216 - x*(b/6 + ((1 - 6*b)^(3/2)*((5*b)/216 - 1/648))/(b - 1/6) - 1/108)
+ x^2*(b/2 + (1 - 6*b)^(3/2)/(72*(b - 1/6)) + 1/12) - x^3*((1 - 6*b)^(3/2)
/(36*(b - 1/6)) + 2/3) - b^2/18 + x^4 + ((1 - 6*b)^(3/2)*((5*b)/1296 - 1/1
944))/(b - 1/6) - 1/324)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 7954, normalized size of antiderivative = 28.01

$$\int \frac{e+fx}{\left(1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3\right)^2}\,dx = \text{Too large to display}$$

input
```
int((f*x+e)/(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3)^2,x)
```

output
```
( - 320*sqrt(6*b - 1)*sqrt( - 6*b + 1)*atan((6*x - 1)/sqrt(6*b - 1))*b**4*
f - 6240*sqrt(6*b - 1)*sqrt( - 6*b + 1)*atan((6*x - 1)/sqrt(6*b - 1))*b**3
*f*x**2 + 2080*sqrt(6*b - 1)*sqrt( - 6*b + 1)*atan((6*x - 1)/sqrt(6*b - 1)
)*b**3*f*x + 40*sqrt(6*b - 1)*sqrt( - 6*b + 1)*atan((6*x - 1)/sqrt(6*b - 1
))*b**3*f - 43200*sqrt(6*b - 1)*sqrt( - 6*b + 1)*atan((6*x - 1)/sqrt(6*b -
 1))*b**2*f*x**4 + 28800*sqrt(6*b - 1)*sqrt( - 6*b + 1)*atan((6*x - 1)/sqr
t(6*b - 1))*b**2*f*x**3 - 4080*sqrt(6*b - 1)*sqrt( - 6*b + 1)*atan((6*x -
1)/sqrt(6*b - 1))*b**2*f*x**2 - 240*sqrt(6*b - 1)*sqrt( - 6*b + 1)*atan((6
*x - 1)/sqrt(6*b - 1))*b**2*f*x - 120960*sqrt(6*b - 1)*sqrt( - 6*b + 1)*at
an((6*x - 1)/sqrt(6*b - 1))*b*f*x**6 + 120960*sqrt(6*b - 1)*sqrt( - 6*b +
1)*atan((6*x - 1)/sqrt(6*b - 1))*b*f*x**5 - 36000*sqrt(6*b - 1)*sqrt( - 6*
b + 1)*atan((6*x - 1)/sqrt(6*b - 1))*b*f*x**4 + 1600*sqrt(6*b - 1)*sqrt( -
 6*b + 1)*atan((6*x - 1)/sqrt(6*b - 1))*b*f*x**3 + 480*sqrt(6*b - 1)*sqrt(
 - 6*b + 1)*atan((6*x - 1)/sqrt(6*b - 1))*b*f*x**2 - 103680*sqrt(6*b - 1)*
sqrt( - 6*b + 1)*atan((6*x - 1)/sqrt(6*b - 1))*f*x**8 + 138240*sqrt(6*b -
1)*sqrt( - 6*b + 1)*atan((6*x - 1)/sqrt(6*b - 1))*f*x**7 - 60480*sqrt(6*b
- 1)*sqrt( - 6*b + 1)*atan((6*x - 1)/sqrt(6*b - 1))*f*x**6 + 6720*sqrt(6*b
 - 1)*sqrt( - 6*b + 1)*atan((6*x - 1)/sqrt(6*b - 1))*f*x**5 + 1600*sqrt(6*
b - 1)*sqrt( - 6*b + 1)*atan((6*x - 1)/sqrt(6*b - 1))*f*x**4 - 320*sqrt(6*
b - 1)*sqrt( - 6*b + 1)*atan((6*x - 1)/sqrt(6*b - 1))*f*x**3 - 1536*sqr...
```

3.147 $\int \frac{1}{\left(1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3\right)^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 179

$$\int \frac{1}{(1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3)^2}\,dx = \frac{2}{81(1-6b)\left(1-\sqrt{1-6b}-6x\right)^3} - \frac{2}{81(1-6b)^{3/2}\left(1-\sqrt{1-6b}-6x\right)^2} + \frac{2}{81(1-6b)^2\left(1-\sqrt{1-6b}-6x\right)} + \frac{2}{243(1-6b)^2\left(1+2\sqrt{1-6b}-6x\right)} + \frac{8\log\left(1-\sqrt{1-6b}-6x\right)}{729(1-6b)^{5/2}} - \frac{8\log\left(1+2\sqrt{1-6b}-6x\right)}{729(1-6b)^{5/2}}$$

output

```
2/81/(1-6*b)/(1-(1-6*b)^(1/2)-6*x)^3-2/81/(1-6*b)^(3/2)/(1-(1-6*b)^(1/2)-6
*x)^2+2/81/(1-6*b)^2/(1-(1-6*b)^(1/2)-6*x)+2/243/(1-6*b)^2/(1+2*(1-6*b)^(1
/2)-6*x)+8/729*ln(1-(1-6*b)^(1/2)-6*x)/(1-6*b)^(5/2)-8/729*ln(1+2*(1-6*b)^
(1/2)-6*x)/(1-6*b)^(5/2)
```

Mathematica [A] (verified)

Time = 3.70 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.33

$$\int \frac{1}{\left(1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3\right)^2}\,dx = \frac{-\frac{12\left(-1+2\sqrt{1-6b}+6x\right)}{(1-6b)^2(-1+8b-4x+12x^2)}+\frac{2\left(1+\sqrt{1-6b}-6x\right)}{(b+2x(-1+3x))^3}-\frac{3(-1+}{(-1+6b)(b}}{}$$

input
```
Integrate[(1 - (1 - 6*b)^(3/2) - 9*b + 54*b*x - 54*x^2 + 108*x^3)^(-2),x]
```

output
```
((-12*(-1 + 2*Sqrt[1 - 6*b] + 6*x))/((1 - 6*b)^2*(-1 + 8*b - 4*x + 12*x^2)
) + (2*(1 + Sqrt[1 - 6*b] - 6*x))/(b + 2*x*(-1 + 3*x))^3 - (3*(-1 + Sqrt[1
 - 6*b] + 6*x))/((-1 + 6*b)*(b + 2*x*(-1 + 3*x))^2) + (18*(1 - 6*x))/((1 -
 6*b)^2*(b + 2*x*(-1 + 3*x))) - (48*ArcTan[(-1 + 6*x)/(2*Sqrt[-1 + 6*b])])
/(-1 + 6*b)^(5/2) - (48*ArcTan[(-1 + 6*x)/Sqrt[-1 + 6*b]])/(-1 + 6*b)^(5/2
) - (24*Log[1 - 8*b + 4*x - 12*x^2])/(1 - 6*b)^(5/2) + (24*Log[b - 2*x + 6
*x^2])/(1 - 6*b)^(5/2))/4374
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {2479, 27, 27, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(54bx-(1-6b)^{3/2}-9b+108x^3-54x^2+1\right)^2}dx$$

$$\Big\downarrow \text{2479}$$

$$983660201882413439385 6(1-6b)^6\int \frac{1}{2459150504706033598464\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)^4\left(\left(2\sqrt{1-6b}+1\right)(1-6b)-6(1-6b)x\right.}$$

$$\Big\downarrow \text{27}$$

$$4(1-6b)^6\int \frac{1}{(1-6b)^2\left(-6x+2\sqrt{1-6b}+1\right)^2\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)^4}dx$$

$$\Big\downarrow \; 27$$

$$4(1-6b)^4\int \frac{1}{\left(-6x+2\sqrt{1-6b}+1\right)^2\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)^4}dx$$

$$\Big\downarrow \; 54$$

$$4(1-6b)^4\int \left(\frac{1}{81(6b-1)^6\left(-6x+2\sqrt{1-6b}+1\right)^2}+\frac{1}{27(6b-1)^6\left(6x+\sqrt{1-6b}-1\right)^2}-\frac{1}{9(6b-1)^5\left(6x+\sqrt{1-6b}-\right.}\right.$$

$$\Big\downarrow \; 2009$$

$$4(1-6b)^4\left(\frac{1}{162(1-6b)^6\left(-\sqrt{1-6b}-6x+1\right)}+\frac{1}{486(1-6b)^6\left(2\sqrt{1-6b}-6x+1\right)}-\frac{1}{162(1-6b)^{11/2}\left(-\sqrt{1-6b}-\right.}\right.$$

input
```
Int[(1 - (1 - 6*b)^(3/2) - 9*b + 54*b*x - 54*x^2 + 108*x^3)^(-2),x]
```

output
```
4*(1 - 6*b)^4*(1/(162*(1 - 6*b)^5*(1 - Sqrt[1 - 6*b] - 6*x)^3) - 1/(162*(1
 - 6*b)^(11/2)*(1 - Sqrt[1 - 6*b] - 6*x)^2) + 1/(162*(1 - 6*b)^6*(1 - Sqrt
[1 - 6*b] - 6*x)) + 1/(486*(1 - 6*b)^6*(1 + 2*Sqrt[1 - 6*b] - 6*x)) + (2*L
og[1 - Sqrt[1 - 6*b] - 6*x])/(729*(1 - 6*b)^(13/2)) - (2*Log[1 + 2*Sqrt[1
- 6*b] - 6*x])/(729*(1 - 6*b)^(13/2)))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2479
```
Int[(Px_)^(p_), x_Symbol] :> With[{a = Coeff[Px, x, 0], b = Coeff[Px, x, 1]
, c = Coeff[Px, x, 2], d = Coeff[Px, x, 3]}, Simp[1/(4^p*(c^2 - 3*b*d)^(3*p
))   Int[(c^3 - 4*b*c*d + 9*a*d^2 + d*(c^2 - 3*b*d)*x)^p*(b*c - 9*a*d + 2*(
c^2 - 3*b*d)*x)^(2*p), x], x] /; EqQ[b^2*c^2 - 4*a*c^3 - 4*b^3*d + 18*a*b*c
*d - 27*a^2*d^2, 0] && NeQ[c^2 - 3*b*d, 0]] /; FreeQ[p, x] && PolyQ[Px, x,
3] && IntegerQ[p]
```

Maple [A] (verified)

Time = 2.40 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.18

method	result
default	$\frac{-\frac{4x^3}{729\left(36b^2-12b+1\right)}-\frac{\left(\sqrt{1-6b}-2\right)x^2}{729\left(36b^2-12b+1\right)}-\frac{\left(6b+1-2\sqrt{1-6b}\right)x}{4374\left(36b^2-12b+1\right)}-\frac{102\sqrt{1-6b}\,b-11\sqrt{1-6b}-36b+2}{157464\left(36b^2-12b+1\right)}}{\left(x^3+\frac{\sqrt{1-6b}\,b}{18}+\frac{bx}{2}-\frac{x^2}{2}-\frac{\sqrt{1-6b}}{108}-\frac{b}{12}+\frac{1}{108}\right)\left(-\frac{1}{6}+x+\frac{\sqrt{1-6b}}{6}\right)}+\frac{64\arctan\left(\frac{-72x+6\sqrt{1-6b}+12}{18\sqrt{-1+6b}}\right.}{9(11664b^2-3888b+324)\sqrt{-1+}}$
parallelrisch	Expression too large to display

input
```
int(1/(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3)^2,x,method=_RETURNVERBOS
E)
```

output
```
(-4/729/(36*b^2-12*b+1)*x^3-1/729/(36*b^2-12*b+1)*((1-6*b)^(1/2)-2)*x^2-1/
4374/(36*b^2-12*b+1)*(6*b+1-2*(1-6*b)^(1/2))*x-1/157464/(36*b^2-12*b+1)*(1
02*(1-6*b)^(1/2)*b-11*(1-6*b)^(1/2)-36*b+2))/(x^3+1/18*(1-6*b)^(1/2)*b+1/2
*b*x-1/2*x^2-1/108*(1-6*b)^(1/2)-1/12*b+1/108)/(-1/6+x+1/6*(1-6*b)^(1/2))+
64/9/(11664*b^2-3888*b+324)/(-1+6*b)^(1/2)*arctan(1/18*(-72*x+6*(1-6*b)^(1
/2)+12)/(-1+6*b)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 638 vs. 2(143) = 286.

Time = 0.09 (sec) , antiderivative size = 638, normalized size of antiderivative = 3.56

$$\int \frac{1}{\left(1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3)^2,x, algorithm="fr
icas")
```

output
```
-1/4374*(62208*(6*b - 1)*x^7 - 72576*(6*b - 1)*x^6 + 5184*(72*b^2 + 18*b -
 5)*x^5 - 1440*(216*b^2 - 30*b - 1)*x^4 - 5256*b^4 + 12*(12312*b^3 + 2484*
b^2 - 1014*b + 43)*x^3 + 2820*b^3 - 6*(12312*b^3 - 3276*b^2 + 234*b - 5)*x
^2 + 48*(2592*x^8 + 1512*(2*b + 1)*x^6 - 3456*x^7 - 168*(18*b + 1)*x^5 + 2
0*(54*b^2 + 45*b - 2)*x^4 + 8*b^4 - 8*(90*b^2 + 5*b - 1)*x^3 - b^3 + 6*(26
*b^3 + 17*b^2 - 2*b)*x^2 - 2*(26*b^3 - 3*b^2)*x)*sqrt(-6*b + 1)*log((12*x^
2 + sqrt(-6*b + 1)*(6*x - 1) - 4*b - 4*x + 1)/(12*x^2 + 8*b - 4*x - 1)) -
582*b^2 + 2*(15768*b^4 - 4356*b^3 + 270*b^2 + 9*b - 1)*x + (5184*(6*b - 1)
*x^6 - 5184*(6*b - 1)*x^5 + 216*(108*b^2 + 24*b - 7)*x^4 - 2448*b^4 - 48*(
324*b^2 - 48*b - 1)*x^3 + 1740*b^3 + 6*(648*b^3 + 324*b^2 - 102*b + 5)*x^2
 - 444*b^2 - 2*(648*b^3 - 108*b^2 - 6*b + 1)*x + 49*b - 2)*sqrt(-6*b + 1)
+ 55*b - 2)/(2592*(216*b^3 - 108*b^2 + 18*b - 1)*x^8 - 3456*(216*b^3 - 108
*b^2 + 18*b - 1)*x^7 + 1728*b^7 + 1512*(432*b^4 - 72*b^2 + 16*b - 1)*x^6 -
 1080*b^6 - 168*(3888*b^4 - 1728*b^3 + 216*b^2 - 1)*x^5 + 252*b^5 + 20*(11
664*b^5 + 3888*b^4 - 4320*b^3 + 972*b^2 - 81*b + 2)*x^4 - 26*b^4 - 8*(1944
0*b^5 - 8640*b^4 + 864*b^3 + 108*b^2 - 23*b + 1)*x^3 + b^3 + 6*(5616*b^6 +
 864*b^5 - 1800*b^4 + 496*b^3 - 53*b^2 + 2*b)*x^2 - 2*(5616*b^6 - 3456*b^5
 + 792*b^4 - 80*b^3 + 3*b^2)*x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 462 vs. $2(155) = 310$.

Time = 2.41 (sec) , antiderivative size = 462, normalized size of antiderivative = 2.58

$$\int \frac{1}{\left(1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3\right)^2}\,dx =$$

$$-\frac{8\sqrt{-\frac{1}{(6b-1)^5}}\log\left(x-\frac{8\sqrt{-\frac{1}{(6b-1)^5}}\left(-\frac{19683b^3}{4}+\frac{19683b^2}{8}-\frac{6561b}{16}+\frac{729}{32}\right)}{729}-\frac{\sqrt{1-6b}}{12}-\frac{1}{6}\right)}{729}$$

$$+\frac{8\sqrt{-\frac{1}{(6b-1)^5}}\log\left(x+\frac{8\sqrt{-\frac{1}{(6b-1)^5}}\left(-\frac{19683b^3}{4}+\frac{19683b^2}{8}-\frac{6561b}{16}+\frac{729}{32}\right)}{729}-\frac{\sqrt{1-6b}}{12}-\frac{1}{6}\right)}{729}$$

$$+\frac{}{-204073344b^4-85030560b^3\sqrt{1-6b}+187067232b^3+39680928b^2\sqrt{1-6b}-56687040b^2-6141096b\sqrt{1}}$$

input
```
integrate(1/(1-(1-6*b)**(3/2)-9*b+54*b*x-54*x**2+108*x**3)**2,x)
```

output
```
-8*sqrt(-1/(6*b - 1)**5)*log(x - 8*sqrt(-1/(6*b - 1)**5)*(-19683*b**3/4 +
19683*b**2/8 - 6561*b/16 + 729/32)/729 - sqrt(1 - 6*b)/12 - 1/6)/729 + 8*s
qrt(-1/(6*b - 1)**5)*log(x + 8*sqrt(-1/(6*b - 1)**5)*(-19683*b**3/4 + 1968
3*b**2/8 - 6561*b/16 + 729/32)/729 - sqrt(1 - 6*b)/12 - 1/6)/729 + (-66096
*b*sqrt(1 - 6*b) + 23328*b - 559872*x**3 + x**2*(279936 - 139968*sqrt(1 -
6*b)) + x*(-139968*b + 46656*sqrt(1 - 6*b) - 23328) + 7128*sqrt(1 - 6*b) -
 1296)/(-204073344*b**4 - 85030560*b**3*sqrt(1 - 6*b) + 187067232*b**3 + 3
9680928*b**2*sqrt(1 - 6*b) - 56687040*b**2 - 6141096*b*sqrt(1 - 6*b) + 708
5880*b + x**4*(3673320192*b**2 - 1224440064*b + 102036672) + x**3*(6122200
32*b**2*sqrt(1 - 6*b) - 2448880128*b**2 - 204073344*b*sqrt(1 - 6*b) + 8162
93376*b + 17006112*sqrt(1 - 6*b) - 68024448) + x**2*(1836660096*b**3 - 306
110016*b**2*sqrt(1 - 6*b) - 306110016*b**2 + 102036672*b*sqrt(1 - 6*b) - 5
1018336*b - 8503056*sqrt(1 - 6*b) + 8503056) + x*(510183360*b**3*sqrt(1 -
6*b) - 612220032*b**3 - 204073344*b**2*sqrt(1 - 6*b) + 238085568*b**2 + 25
509168*b*sqrt(1 - 6*b) - 28343520*b - 944784*sqrt(1 - 6*b) + 944784) + 314
928*sqrt(1 - 6*b) - 314928)
```

Maxima [F]

$$\int \frac{1}{\left(1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3\right)^2}\,dx=\int \frac{1}{\left(108\,x^3+54\,bx-54\,x^2-(-6\,b+1)^{\frac{3}{2}}-9\,b+1\right.}$$

input
```
integrate(1/(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3)^2,x, algorithm="ma
xima")
```

output
```
1/18*(36*(6*b - 1)*x^2 + 72*b^2 + 6*((-6*b + 1)^(3/2) - 12*b + 2)*x - (-6*
b + 1)^(3/2) - 18*b + 1)/((-6*b + 1)^(9/2) + 108*(2*(-6*b + 1)^(3/2) - 11)
*b^3 + 1944*b^4 + 108*((6*b - 1)^3 - 216*b^3 + 108*b^2 - 18*b + 1)*x^3 + (
6*b - 1)^3 - 54*(2*(-6*b + 1)^(3/2) - 5)*b^2 - 54*((6*b - 1)^3 - 216*b^3 +
 108*b^2 - 18*b + 1)*x^2 - 9*((6*b - 1)^3 - 2*(-6*b + 1)^(3/2) + 3)*b - 54
*(216*b^4 - 108*b^3 - ((6*b - 1)^3 + 1)*b + 18*b^2)*x - (-6*b + 1)^(3/2) +
 1) + 1/3*integrate((6*(6*b - 1)*x + 2*(-6*b + 1)^(3/2) - 6*b + 1)/(108*x^
3 + 54*b*x - 54*x^2 - (-6*b + 1)^(3/2) - 9*b + 1), x)/((6*b - 1)^3 - 216*b
^3 + 108*b^2 - 18*b + 1)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3)^2,x, algorithm="gi
ac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{17006112,[3]%%%}+%%%{-8503056,[2]%%%}+%%%{1417176,[1]%
%%}+%%%{-
```

Mupad [B] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 346, normalized size of antiderivative = 1.93

$$\int \frac{1}{(1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3)^2}\,dx = \frac{x^2\left(\frac{1}{13122\left(b^2-\frac{b}{3}+\frac{1}{36}\right)}+\frac{(1-6\,b)^{3/2}}{157464\left(b^3-\frac{b^2}{2}+\frac{b}{12}-\frac{1}{216}\right)}\right)-\frac{\cdots}{6561\cdots}}{\frac{7\,b}{216}-x\left(\frac{b}{6}+\frac{(1-6\,b)^{3/2}\left(\frac{5\,b}{216}-\frac{1}{648}\right)}{b-\frac{1}{6}}-\frac{1}{108}\right)-\cdots}$$
$$-\frac{\mathrm{atan}\left(\frac{-b\,12\mathrm{i}-x\,12\mathrm{i}+b\,x\,72\mathrm{i}+(1-6\,b)^{3/2}\,1\mathrm{i}+2\mathrm{i}}{\sqrt{864\,b^2-(6\,b-1)^3-144\,b-1728\,b^3+8}}\right)\,16\mathrm{i}}{243\,(6\,b-1)\,\sqrt{864\,b^2-(6\,b-1)^3-144\,b-1728\,b^3+8}}$$

input
```
int(1/(9*b - 54*b*x + (1 - 6*b)^(3/2) + 54*x^2 - 108*x^3 - 1)^2,x)
```

output
```
(x^2*(1/(13122*(b^2 - b/3 + 1/36)) + (1 - 6*b)^(3/2)/(157464*(b/12 - b^2/2
 + b^3 - 1/216))) - x^3/(6561*(b^2 - b/3 + 1/36)) - x*((1 - 6*b)^(3/2)/(47
2392*(b/12 - b^2/2 + b^3 - 1/216)) + (b/26244 + 1/157464)/(b^2 - b/3 + 1/3
6)) + (b/157464 - 1/2834352)/(b^2 - b/3 + 1/36) + ((1 - 6*b)^(3/2)*((17*b)
/5668704 - 11/34012224))/(b/12 - b^2/2 + b^3 - 1/216))/((7*b)/216 - x*(b/6
 + ((1 - 6*b)^(3/2)*((5*b)/216 - 1/648))/(b - 1/6) - 1/108) + x^2*(b/2 + (
1 - 6*b)^(3/2)/(72*(b - 1/6)) + 1/12) - x^3*((1 - 6*b)^(3/2)/(36*(b - 1/6)
) + 2/3) - b^2/18 + x^4 + ((1 - 6*b)^(3/2)*((5*b)/1296 - 1/1944))/(b - 1/6
) - 1/324) - (atan((b*x*72i - x*12i - b*12i + (1 - 6*b)^(3/2)*1i + 2i)/(86
4*b^2 - (6*b - 1)^3 - 144*b - 1728*b^3 + 8)^(1/2))*16i)/(243*(6*b - 1)*(86
4*b^2 - (6*b - 1)^3 - 144*b - 1728*b^3 + 8)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 2729, normalized size of antiderivative = 15.25

$$\int \frac{1}{\left(1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3\right)^2}\,dx = \text{Too large to display}$$

input
```
int(1/(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3)^2,x)
```

output

```
( - 384*sqrt(6*b - 1)*atan((6*x - 1)/sqrt(6*b - 1))*b**4 - 7488*sqrt(6*b -
 1)*atan((6*x - 1)/sqrt(6*b - 1))*b**3*x**2 + 2496*sqrt(6*b - 1)*atan((6*x
 - 1)/sqrt(6*b - 1))*b**3*x + 48*sqrt(6*b - 1)*atan((6*x - 1)/sqrt(6*b - 1
))*b**3 - 51840*sqrt(6*b - 1)*atan((6*x - 1)/sqrt(6*b - 1))*b**2*x**4 + 34
560*sqrt(6*b - 1)*atan((6*x - 1)/sqrt(6*b - 1))*b**2*x**3 - 4896*sqrt(6*b
- 1)*atan((6*x - 1)/sqrt(6*b - 1))*b**2*x**2 - 288*sqrt(6*b - 1)*atan((6*x
 - 1)/sqrt(6*b - 1))*b**2*x - 145152*sqrt(6*b - 1)*atan((6*x - 1)/sqrt(6*b
 - 1))*b*x**6 + 145152*sqrt(6*b - 1)*atan((6*x - 1)/sqrt(6*b - 1))*b*x**5
- 43200*sqrt(6*b - 1)*atan((6*x - 1)/sqrt(6*b - 1))*b*x**4 + 1920*sqrt(6*b
 - 1)*atan((6*x - 1)/sqrt(6*b - 1))*b*x**3 + 576*sqrt(6*b - 1)*atan((6*x -
 1)/sqrt(6*b - 1))*b*x**2 - 124416*sqrt(6*b - 1)*atan((6*x - 1)/sqrt(6*b -
 1))*x**8 + 165888*sqrt(6*b - 1)*atan((6*x - 1)/sqrt(6*b - 1))*x**7 - 7257
6*sqrt(6*b - 1)*atan((6*x - 1)/sqrt(6*b - 1))*x**6 + 8064*sqrt(6*b - 1)*at
an((6*x - 1)/sqrt(6*b - 1))*x**5 + 1920*sqrt(6*b - 1)*atan((6*x - 1)/sqrt(
6*b - 1))*x**4 - 384*sqrt(6*b - 1)*atan((6*x - 1)/sqrt(6*b - 1))*x**3 - 38
4*sqrt(6*b - 1)*atan((6*x - 1)/(2*sqrt(6*b - 1)))*b**4 - 7488*sqrt(6*b - 1
)*atan((6*x - 1)/(2*sqrt(6*b - 1)))*b**3*x**2 + 2496*sqrt(6*b - 1)*atan((6
*x - 1)/(2*sqrt(6*b - 1)))*b**3*x + 48*sqrt(6*b - 1)*atan((6*x - 1)/(2*sqr
t(6*b - 1)))*b**3 - 51840*sqrt(6*b - 1)*atan((6*x - 1)/(2*sqrt(6*b - 1)))*
b**2*x**4 + 34560*sqrt(6*b - 1)*atan((6*x - 1)/(2*sqrt(6*b - 1)))*b**2*...
```

3.148 $\int \frac{1}{(e+fx)\left(1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3\right)^2}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 500

$$\int \frac{1}{(e+fx)\left(1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3\right)^2}\,dx = \frac{4}{27(1-6b)\left(6e+f-\sqrt{1-6b}f\right)\left(1-\sqrt{1\right.}}$$

$$-\frac{2\left(12e+\left(2-5\sqrt{1-6b}\right)f\right)}{27(1-6b)^{3/2}\left(6e+f-\sqrt{1-6b}f\right)^2\left(1-\sqrt{1-6b}-6x\right)^2}$$

$$+\frac{4\left(36e^2+12ef+7f^2-36bf^2-\sqrt{1-6b}f(24e+4f)\right)}{27(1-6b)^2\left(6e+f-\sqrt{1-6b}f\right)^3\left(1-\sqrt{1-6b}-6x\right)}$$

$$+\frac{4}{81(1-6b)^2\left(6e+f+2\sqrt{1-6b}f\right)\left(1+2\sqrt{1-6b}-6x\right)}$$

$$+\frac{4\left(864e^3+108\left(4-7\sqrt{1-6b}\right)e^2f+36\left(10-7\sqrt{1-6b}-48b\right)ef^2+\left(52-79\sqrt{1-6b}-12\left(24-29\sqrt{1\right.\right.}}{243(1-6b)^{5/2}\left(6e+f-\sqrt{1-6b}f\right)^4}$$

$$-\frac{4\left(24e+\left(4+11\sqrt{1-6b}\right)f\right)\log\left(1+2\sqrt{1-6b}-6x\right)}{243(1-6b)^{5/2}\left(6e+f+2\sqrt{1-6b}f\right)^2}$$

$$+\frac{4f^5\log(e+fx)}{\left(6e+f-\sqrt{1-6b}f\right)^4\left(6e+f+2\sqrt{1-6b}f\right)^2}$$

output
```
4/27/(1-6*b)/(6*e+f-(1-6*b)^(1/2)*f)/(1-(1-6*b)^(1/2)-6*x)^3-2/27*(12*e+(2
-5*(1-6*b)^(1/2))*f)/(1-6*b)^(3/2)/(6*e+f-(1-6*b)^(1/2)*f)^2/(1-(1-6*b)^(1
/2)-6*x)^2+4/27*(36*e^2+12*e*f+7*f^2-36*b*f^2-(1-6*b)^(1/2)*f*(24*e+4*f))/
(1-6*b)^2/(6*e+f-(1-6*b)^(1/2)*f)^3/(1-(1-6*b)^(1/2)-6*x)+4/81/(1-6*b)^2/(
6*e+f+2*(1-6*b)^(1/2)*f)/(1+2*(1-6*b)^(1/2)-6*x)+4/243*(864*e^3+108*(4-7*(
1-6*b)^(1/2))*e^2*f+36*(10-7*(1-6*b)^(1/2)-48*b)*e*f^2+(52-79*(1-6*b)^(1/2
)-12*(24-29*(1-6*b)^(1/2))*b)*f^3)*ln(1-(1-6*b)^(1/2)-6*x)/(1-6*b)^(5/2)/(
6*e+f-(1-6*b)^(1/2)*f)^4-4/243*(24*e+(4+11*(1-6*b)^(1/2))*f)*ln(1+2*(1-6*b
)^(1/2)-6*x)/(1-6*b)^(5/2)/(6*e+f+2*(1-6*b)^(1/2)*f)^2+4*f^5*ln(f*x+e)/(6*
e+f-(1-6*b)^(1/2)*f)^4/(6*e+f+2*(1-6*b)^(1/2)*f)^2
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 2761 vs. 2(500) = 1000.

Time = 10.47 (sec) , antiderivative size = 2761, normalized size of antiderivative = 5.52

$$\int \frac{1}{(e+fx)\left(1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3\right)^2}\,dx = \text{Result too large to show}$$

input
```
Integrate[1/((e + f*x)*(1 - (1 - 6*b)^(3/2) - 9*b + 54*b*x - 54*x^2 + 108*
x^3)^2),x]
```

output
```
(-2*(-3*e - 3*Sqrt[1 - 6*b]*e - f - Sqrt[1 - 6*b]*f + 3*b*f + 18*e*x + 3*f
*x + 3*Sqrt[1 - 6*b]*f*x))/(2187*(6*e^2 + 2*e*f + b*f^2)*(b - 2*x + 6*x^2)
^3) + (36*e^3 - 36*Sqrt[1 - 6*b]*e^3 - 6*e^2*f - 30*Sqrt[1 - 6*b]*e^2*f +
144*b*e^2*f - 12*e*f^2 - 12*Sqrt[1 - 6*b]*e*f^2 + 90*b*e*f^2 + 30*Sqrt[1 -
 6*b]*b*e*f^2 - 2*f^3 - 2*Sqrt[1 - 6*b]*f^3 + 15*b*f^3 + 9*Sqrt[1 - 6*b]*b
*f^3 - 12*b^2*f^3 - 216*e^3*x - 108*e^2*f*x + 72*Sqrt[1 - 6*b]*e^2*f*x + 2
4*e*f^2*x + 24*Sqrt[1 - 6*b]*e*f^2*x - 252*b*e*f^2*x + 6*f^3*x + 6*Sqrt[1
- 6*b]*f^3*x - 42*b*f^3*x - 24*Sqrt[1 - 6*b]*b*f^3*x)/(1458*(-1 + 6*b)*(6*
e^2 + 2*e*f + b*f^2)^2*(b - 2*x + 6*x^2)^2) + (1296*e^5 + 972*e^4*f - 216*
Sqrt[1 - 6*b]*e^4*f + 648*b*e^4*f + 180*e^3*f^2 - 72*Sqrt[1 - 6*b]*e^3*f^2
 + 1080*b*e^3*f^2 - 432*Sqrt[1 - 6*b]*b*e^3*f^2 + 42*e^2*f^3 + 42*Sqrt[1 -
 6*b]*e^2*f^3 - 216*b*e^2*f^3 - 468*Sqrt[1 - 6*b]*b*e^2*f^3 + 1944*b^2*e^2
*f^3 + 26*e*f^4 + 26*Sqrt[1 - 6*b]*e*f^4 - 318*b*e*f^4 - 240*Sqrt[1 - 6*b]
*b*e*f^4 + 1152*b^2*e*f^4 + 360*Sqrt[1 - 6*b]*b^2*e*f^4 + 4*f^5 + 4*Sqrt[1
 - 6*b]*f^5 - 53*b*f^5 - 41*Sqrt[1 - 6*b]*b*f^5 + 201*b^2*f^5 + 96*Sqrt[1
- 6*b]*b^2*f^5 - 126*b^3*f^5 - 7776*e^5*x - 6480*e^4*f*x + 1296*Sqrt[1 - 6
*b]*e^4*f*x - 1512*e^3*f^2*x + 864*Sqrt[1 - 6*b]*e^3*f^2*x - 3888*b*e^3*f^
2*x - 36*e^2*f^3*x - 36*Sqrt[1 - 6*b]*e^2*f^3*x - 1944*b*e^2*f^3*x + 1512*
Sqrt[1 - 6*b]*b*e^2*f^3*x - 60*e*f^4*x - 60*Sqrt[1 - 6*b]*e*f^4*x + 684*b*
e*f^4*x + 504*Sqrt[1 - 6*b]*b*e*f^4*x - 3024*b^2*e*f^4*x - 12*f^5*x - 1...
```

Rubi [A] (verified)

Time = 4.26 (sec) , antiderivative size = 518, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2488, 27, 27, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(54bx-(1-6b)^{3/2}-9b+108x^3-54x^2+1\right)^2(e+fx)}dx$$

$$\Big\downarrow \text{ 2488}$$

$$98366020188241343938560(1-6b)^6\int \frac{1}{245915050470603359846 4\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)^4\left(\left(2\sqrt{1-6b}+1\right)(1-6b)-6(1-6b)x\right.}$$

$$\Big\downarrow \text{ 27}$$

$$4(1-6b)^6\int \frac{1}{(1-6b)^2\left(-6x+2\sqrt{1-6b}+1\right)^2\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)^4(e+fx)}dx$$

$$\Big\downarrow \; 27$$

$$4(1-6b)^4\int \frac{1}{\left(-6x+2\sqrt{1-6b}+1\right)^2\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)^4(e+fx)}dx$$

$$\Big\downarrow \; 99$$

$$4(1-6b)^4\int \left(\frac{f^6}{(1-6b)^4\left(6e+\left(1-\sqrt{1-6b}\right)f\right)^4\left(6e+2\sqrt{1-6b}f+f\right)^2(e+fx)}+\frac{2\left(-864e^3-108\left(4-7\sqrt{1-6b}\right)\ldots\right.}{81(\ldots}\right.$$

$$\Big\downarrow \; 2009$$

$$4(1-6b)^4\left(\frac{12\left(1-2\sqrt{1-6b}\right)ef+\left(-36b-4\sqrt{1-6b}+7\right)f^2+36e^2}{27(1-6b)^6\left(-\sqrt{1-6b}-6x+1\right)\left(-\sqrt{1-6b}f+6e+f\right)^3}+\frac{\left(108\left(4-7\sqrt{1-6b}\right)e^2f+36\left(-48b-7\sqrt{1-\ldots}\right.\right.}{\ldots}\right.$$

input
```
Int[1/((e + f*x)*(1 - (1 - 6*b)^(3/2) - 9*b + 54*b*x - 54*x^2 + 108*x^3)^2
),x]
```

output
```
4*(1 - 6*b)^4*(1/(27*(1 - 6*b)^5*(6*e + f - Sqrt[1 - 6*b]*f)*(1 - Sqrt[1 -
 6*b] - 6*x)^3) - (12*e + (2 - 5*Sqrt[1 - 6*b])*f)/(54*(1 - 6*b)^(11/2)*(6
*e + f - Sqrt[1 - 6*b]*f)^2*(1 - Sqrt[1 - 6*b] - 6*x)^2) + (36*e^2 + 12*(1
 - 2*Sqrt[1 - 6*b])*e*f + (7 - 4*Sqrt[1 - 6*b] - 36*b)*f^2)/(27*(1 - 6*b)^
6*(6*e + f - Sqrt[1 - 6*b]*f)^3*(1 - Sqrt[1 - 6*b] - 6*x)) + 1/(81*(1 - 6*
b)^6*(6*e + f + 2*Sqrt[1 - 6*b]*f)*(1 + 2*Sqrt[1 - 6*b] - 6*x)) + ((864*e^
3 + 108*(4 - 7*Sqrt[1 - 6*b])*e^2*f + 36*(10 - 7*Sqrt[1 - 6*b] - 48*b)*e*f
^2 + (52 - 79*Sqrt[1 - 6*b] - 12*(24 - 29*Sqrt[1 - 6*b])*b)*f^3)*Log[1 - S
qrt[1 - 6*b] - 6*x])/(243*(1 - 6*b)^(13/2)*(6*e + f - Sqrt[1 - 6*b]*f)^4)
- ((24*e + (4 + 11*Sqrt[1 - 6*b])*f)*Log[1 + 2*Sqrt[1 - 6*b] - 6*x])/(243*
(1 - 6*b)^(13/2)*(6*e + f + 2*Sqrt[1 - 6*b]*f)^2) + (f^5*Log[e + f*x])/((1
 - 6*b)^4*(6*e + f - Sqrt[1 - 6*b]*f)^4*(6*e + f + 2*Sqrt[1 - 6*b]*f)^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2488
```
Int[((e_.) + (f_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(
x_)^3)^(p_), x_Symbol] :> Simp[1/(4^p*(c^2 - 3*b*d)^(3*p))   Int[(e + f*x)^
m*(c^3 - 4*b*c*d + 9*a*d^2 + d*(c^2 - 3*b*d)*x)^p*(b*c - 9*a*d + 2*(c^2 - 3
*b*d)*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && NeQ[c^2 - 3
*b*d, 0] && EqQ[b^2*c^2 - 4*a*c^3 - 4*b^3*d + 18*a*b*c*d - 27*a^2*d^2, 0] &
& ILtQ[p, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2311 vs. $2(440) = 880$.

Time = 18.92 (sec) , antiderivative size = 2312, normalized size of antiderivative = 4.62

method	result	size
default	Expression too large to display	2312
parallelrisch	Expression too large to display	383514

input
```
int(1/(f*x+e)/(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3)^2,x,method=_RETU
RNVERBOSE)
```

output
```
f^5/((1-6*b)^(3/2)*f^3+54*b*e*f^2+9*b*f^3+108*e^3+54*e^2*f-f^3)^2*ln(f*x+e
)+54/((1-6*b)^(3/2)*f^3+54*b*e*f^2+9*b*f^3+108*e^3+54*e^2*f-f^3)^2*((-1/26
244*(-2520*(1-6*b)^(1/2)*b^2*f^5-10800*(1-6*b)^(1/2)*b*e^2*f^3-3600*(1-6*b
)^(1/2)*b*e*f^4+540*(1-6*b)^(1/2)*b*f^5+8856*f^4*e*b^2+1476*b^2*f^5-6480*(
1-6*b)^(1/2)*e^4*f-4320*(1-6*b)^(1/2)*e^3*f^2+720*(1-6*b)^(1/2)*e^2*f^3+48
0*(1-6*b)^(1/2)*e*f^4-25*(1-6*b)^(1/2)*f^5+25920*b*e^3*f^2+12960*b*e^2*f^3
-792*b*e*f^4-372*b*f^5+31104*e^5+25920*e^4*f+4320*e^3*f^2-720*e^2*f^3+6*e*
f^4+25*f^5)/(36*b^2-12*b+1)*x^3-1/52488*((1-6*b)^(1/2)-2)*(-1206*(1-6*b)^(
1/2)*b^2*f^5+245*(1-6*b)^(1/2)*b*f^5+15552*e^5+3024*b^3*f^4*e+25920*b^2*e^
3*f^2-4968*b^2*(1-6*b)^(1/2)*e^2*f^3-6480*b*(1-6*b)^(1/2)*e^4*f+288*(1-6*b
)^(1/2)*f^4*e*b^2-4320*(1-6*b)^(1/2)*b*e^3*f^2+24624*b^2*e^2*f^3+25920*e^4
*f*b-8*(1-6*b)^(1/2)*f^5-576*b^3*(1-6*b)^(1/2)*f^5+12960*e^4*f+4392*b^3*f^
5-3240*(1-6*b)^(1/2)*e^4*f-2160*(1-6*b)^(1/2)*e^3*f^2+522*(1-6*b)^(1/2)*e^
2*f^3+348*(1-6*b)^(1/2)*e*f^4+2160*e^3*f^2-36*e^2*f^3+138*e*f^4-768*b^2*f^
5-26*b*f^5-2172*b*e*f^4+17280*b*e^3*f^2+10440*f^4*e*b^2-6624*(1-6*b)^(1/2)
*b*e^2*f^3-2616*(1-6*b)^(1/2)*b*e*f^4+1872*b*e^2*f^3+8*f^5+31104*b*e^5)/(2
*b+1)/(36*b^2-12*b+1)*x^2-1/314928*(6*b+1-2*(1-6*b)^(1/2))*(-1752*(1-6*b)^
(1/2)*b^2*f^5+219*(1-6*b)^(1/2)*b*f^5-15552*e^5+48384*b^3*f^4*e+158112*b^2
*e^3*f^2+287712*b^3*e^2*f^3+155520*e^4*f*b^2-99792*(1-6*b)^(1/2)*b^3*e^2*f
^3-38880*(1-6*b)^(1/2)*b^2*e^4*f-7992*b^2*(1-6*b)^(1/2)*e^2*f^3-38880*b...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(e+fx)\left(1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3\right)^2}\,dx=\text{Timed out}$$

input
```
integrate(1/(f*x+e)/(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3)^2,x, algor
ithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(e+fx)\left(1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(f*x+e)/(1-(1-6*b)**(3/2)-9*b+54*b*x-54*x**2+108*x**3)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{(e+fx)\left(1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3\right)^2}\,dx = \int \frac{1}{\left(108\,x^3+54\,bx-54\,x^2-(-6\,b+1)^{\frac{3}{2}}\right.}$$

input
```
integrate(1/(f*x+e)/(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3)^2,x, algor
ithm="maxima")
```

output
```
f^5*log(f*x + e)/(2916*(4*b + 1)*e^4*f^2 + 216*((-6*b + 1)^(3/2) + 36*b -
1)*e^3*f^3 + 108*(27*b^2 + (-6*b + 1)^(3/2) + 9*b - 1)*e^2*f^4 + 108*(((-6
*b + 1)^(3/2) - 1)*b + 9*b^2)*e*f^5 - ((6*b - 1)^3 - 18*((-6*b + 1)^(3/2)
- 1)*b - 81*b^2 + 2*(-6*b + 1)^(3/2) - 1)*f^6 + 11664*e^6 + 11664*e^5*f) +
 1/3*(18*(72*b^2 - (-6*b + 1)^(3/2) - 18*b + 1)*e^2 - 3*(12*((-6*b + 1)^(3
/2) + 2)*b - 108*b^2 + (-6*b + 1)^(3/2) - 1)*e*f - ((6*b - 1)^3 - 648*b^3
+ 3*(4*(-6*b + 1)^(3/2) - 13)*b + 270*b^2 - (-6*b + 1)^(3/2) + 2)*f^2 + 18
*(36*(6*b - 1)*e^2 - 6*((-6*b + 1)^(3/2) - 12*b + 2)*e*f + (72*b^2 - (-6*b
 + 1)^(3/2) - 18*b + 1)*f^2)*x^2 + 3*(36*((-6*b + 1)^(3/2) - 12*b + 2)*e^2
 + 6*(36*b^2 + 4*(-6*b + 1)^(3/2) - 36*b + 5)*e*f + (12*((-6*b + 1)^(3/2)
+ 2)*b - 108*b^2 + (-6*b + 1)^(3/2) - 1)*f^2)*x)/(108*((-6*b + 1)^(9/2) +
108*(2*(-6*b + 1)^(3/2) - 11)*b^3 + 1944*b^4 + (6*b - 1)^3 - 54*(2*(-6*b +
 1)^(3/2) - 5)*b^2 - 9*((6*b - 1)^3 - 2*(-6*b + 1)^(3/2) + 3)*b - (-6*b +
1)^(3/2) + 1)*e^3 + 54*((-6*b + 1)^(9/2) + 108*(2*(-6*b + 1)^(3/2) - 11)*b
^3 + 1944*b^4 + (6*b - 1)^3 - 54*(2*(-6*b + 1)^(3/2) - 5)*b^2 - 9*((6*b -
1)^3 - 2*(-6*b + 1)^(3/2) + 3)*b - (-6*b + 1)^(3/2) + 1)*e^2*f + 54*(108*(
2*(-6*b + 1)^(3/2) - 11)*b^4 + 1944*b^5 - 54*(2*(-6*b + 1)^(3/2) - 5)*b^3
- 9*((6*b - 1)^3 - 2*(-6*b + 1)^(3/2) + 3)*b^2 + ((-6*b + 1)^(9/2) + (6*b
- 1)^3 - (-6*b + 1)^(3/2) + 1)*b)*e*f^2 + ((6*b - 1)^6 + 972*(4*(-6*b + 1)
^(3/2) - 13)*b^4 + 17496*b^5 - 2*(-6*b + 1)^(9/2) - 54*(4*(6*b - 1)^3 +...
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(e+fx)\left(1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3\right)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(f*x+e)/(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3)^2,x, algor
ithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{17006112,[3]%%%}+%%%{-8503056,[2]%%%}+%%%{1417176,[1]%
%%}+%%%{-
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(e+fx)\left(1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3\right)^2}\,dx = \text{Hanged}$$

input
```
int(1/((e + f*x)*(9*b - 54*b*x + (1 - 6*b)^(3/2) + 54*x^2 - 108*x^3 - 1)^2
),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{1}{(e+fx)\left(1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3\right)^2}\,dx = \int \frac{1}{(fx+e)\left(1-(-6b+1)^{\frac{3}{2}}-9b+54bx\right.}$$

input
```
int(1/(f*x+e)/(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3)^2,x)
```

output
```
int(1/(f*x+e)/(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3)^2,x)
```

3.149 $\int \frac{1}{(e+fx)^2\left(1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3\right)^2}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 564

$$\int \frac{1}{(e+fx)^2\left(1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3\right)^2}\,dx = \frac{8}{9(1-6b)\left(6e+f-\sqrt{1-6b}f\right)^2\left(1-\sqrt{\right.}}$$

$$-\frac{8\left(6e+f-4\sqrt{1-6b}f\right)}{9(1-6b)^{3/2}\left(6e+f-\sqrt{1-6b}f\right)^3\left(1-\sqrt{1-6b}-6x\right)^2}$$

$$+\frac{8\left(12e^2+4ef+5f^2-28bf^2-\sqrt{1-6b}f(12e+2f)\right)}{3(1-6b)^2\left(6e+f-\sqrt{1-6b}f\right)^4\left(1-\sqrt{1-6b}-6x\right)}$$

$$+\frac{8}{27(1-6b)^2\left(6e+f+2\sqrt{1-6b}f\right)^2\left(1+2\sqrt{1-6b}-6x\right)}$$

$$-\frac{4f^5}{\left(6e+f-\sqrt{1-6b}f\right)^4\left(6e+f+2\sqrt{1-6b}f\right)^2(e+fx)}$$

$$+\frac{16\left(432e^3+108\left(2-5\sqrt{1-6b}\right)e^2f+18\left(19-10\sqrt{1-6b}-102b\right)ef^2+\left(53-107\sqrt{1-6b}-6(51-92\right.\right.}{81(1-6b)^{5/2}\left(6e+f-\sqrt{1-6b}f\right)^5}$$

$$-\frac{16\left(12e+\left(2+7\sqrt{1-6b}\right)f\right)\log\left(1+2\sqrt{1-6b}-6x\right)}{81(1-6b)^{5/2}\left(6e+f+2\sqrt{1-6b}f\right)^3}$$

$$+\frac{144f^5\left(6e+f+\sqrt{1-6b}f\right)\log(e+fx)}{\left(6e+f-\sqrt{1-6b}f\right)^5\left(6e+f+2\sqrt{1-6b}f\right)^3}$$

output
```
8/9/(1-6*b)/(6*e+f-(1-6*b)^(1/2)*f)^2/(1-(1-6*b)^(1/2)-6*x)^3-8/9*(6*e+f-4
*(1-6*b)^(1/2)*f)/(1-6*b)^(3/2)/(6*e+f-(1-6*b)^(1/2)*f)^3/(1-(1-6*b)^(1/2)
-6*x)^2+8/3*(12*e^2+4*e*f+5*f^2-28*b*f^2-(1-6*b)^(1/2)*f*(12*e+2*f))/(1-6*
b)^2/(6*e+f-(1-6*b)^(1/2)*f)^4/(1-(1-6*b)^(1/2)-6*x)+8/27/(1-6*b)^2/(6*e+f
+2*(1-6*b)^(1/2)*f)^2/(1+2*(1-6*b)^(1/2)-6*x)-4*f^5/(6*e+f-(1-6*b)^(1/2)*f
)^4/(6*e+f+2*(1-6*b)^(1/2)*f)^2/(f*x+e)+16/81*(432*e^3+108*(2-5*(1-6*b)^(1
/2))*e^2*f+18*(19-10*(1-6*b)^(1/2)-102*b)*e*f^2+(53-107*(1-6*b)^(1/2)-6*(5
1-92*(1-6*b)^(1/2))*b)*f^3)*ln(1-(1-6*b)^(1/2)-6*x)/(1-6*b)^(5/2)/(6*e+f-(
1-6*b)^(1/2)*f)^5-16/81*(12*e+(2+7*(1-6*b)^(1/2))*f)*ln(1+2*(1-6*b)^(1/2)-
6*x)/(1-6*b)^(5/2)/(6*e+f+2*(1-6*b)^(1/2)*f)^3+144*f^5*(6*e+f+(1-6*b)^(1/2
)*f)*ln(f*x+e)/(6*e+f-(1-6*b)^(1/2)*f)^5/(6*e+f+2*(1-6*b)^(1/2)*f)^3
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 4351 vs. $2(564) = 1128$.

Time = 15.92 (sec) , antiderivative size = 4351, normalized size of antiderivative = 7.71

$$\int \frac{1}{(e+fx)^2\left(1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3\right)^2}\,dx = \text{Result too large to show}$$

input
```
Integrate[1/((e + f*x)^2*(1 - (1 - 6*b)^(3/2) - 9*b + 54*b*x - 54*x^2 + 10
8*x^3)^2),x]
```

output
```
(f^5*(-11664*e^6 - 11664*e^5*f - 2916*e^4*f^2 - 11664*b*e^4*f^2 + 216*e^3*
f^3 + 216*Sqrt[1 - 6*b]*e^3*f^3 - 7776*b*e^3*f^3 - 1296*Sqrt[1 - 6*b]*b*e^
3*f^3 + 108*e^2*f^4 + 108*Sqrt[1 - 6*b]*e^2*f^4 - 972*b*e^2*f^4 - 648*Sqrt
[1 - 6*b]*b*e^2*f^4 - 2916*b^2*e^2*f^4 + 108*b*e*f^5 + 108*Sqrt[1 - 6*b]*b
*e*f^5 - 972*b^2*e*f^5 - 648*Sqrt[1 - 6*b]*b^2*e*f^5 - 2*f^6 - 2*Sqrt[1 -
6*b]*f^6 + 36*b*f^6 + 30*Sqrt[1 - 6*b]*b*f^6 - 189*b^2*f^6 - 108*Sqrt[1 -
6*b]*b^2*f^6 + 216*b^3*f^6))/(729*(6*e^2 + 2*e*f + b*f^2)^4*(12*e^2 + 4*e*
f - f^2 + 8*b*f^2)^2*(e + f*x)) - (2*(-18*e^2 - 18*Sqrt[1 - 6*b]*e^2 - 12*
e*f - 12*Sqrt[1 - 6*b]*e*f + 36*b*e*f - 2*f^2 - 2*Sqrt[1 - 6*b]*f^2 + 9*b*
f^2 + 3*Sqrt[1 - 6*b]*b*f^2 + 108*e^2*x + 36*e*f*x + 36*Sqrt[1 - 6*b]*e*f*
x + 6*f^2*x + 6*Sqrt[1 - 6*b]*f^2*x - 18*b*f^2*x))/(2187*(6*e^2 + 2*e*f +
b*f^2)^2*(b - 2*x + 6*x^2)^3) + (108*e^4 - 108*Sqrt[1 - 6*b]*e^4 - 72*e^3*
f - 144*Sqrt[1 - 6*b]*e^3*f + 864*b*e^3*f - 108*e^2*f^2 - 108*Sqrt[1 - 6*b
]*e^2*f^2 + 756*b*e^2*f^2 + 324*Sqrt[1 - 6*b]*b*e^2*f^2 - 36*e*f^3 - 36*Sq
rt[1 - 6*b]*e*f^3 + 276*b*e*f^3 + 168*Sqrt[1 - 6*b]*b*e*f^3 - 288*b^2*e*f^
3 - 4*f^4 - 4*Sqrt[1 - 6*b]*f^4 + 36*b*f^4 + 24*Sqrt[1 - 6*b]*b*f^4 - 69*b
^2*f^4 - 15*Sqrt[1 - 6*b]*b^2*f^4 - 648*e^4*x - 432*e^3*f*x + 432*Sqrt[1 -
 6*b]*e^3*f*x + 216*e^2*f^2*x + 216*Sqrt[1 - 6*b]*e^2*f^2*x - 1944*b*e^2*f
^2*x + 96*e*f^3*x + 96*Sqrt[1 - 6*b]*e*f^3*x - 648*b*e*f^3*x - 360*Sqrt[1
- 6*b]*b*e*f^3*x + 12*f^4*x + 12*Sqrt[1 - 6*b]*f^4*x - 96*b*f^4*x - 60*...
```

Rubi [A] (verified)

Time = 6.16 (sec) , antiderivative size = 590, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2488, 27, 27, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(54bx-(1-6b)^{3/2}-9b+108x^3-54x^2+1\right)^2(e+fx)^2}dx$$

$$\Big\downarrow \text{2488}$$

$$98366020188241343938 56(1-6b)^6\int \frac{1}{2459150504706033598464\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)^4\left(\left(2\sqrt{1-6b}+1\right)(1-6b)-6(1-6b)x\right.}$$

$$\Big\downarrow \text{27}$$

$$4(1-6b)^6\int \frac{1}{(1-6b)^2\left(-6x+2\sqrt{1-6b}+1\right)^2\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)^4(e+fx)^2}dx$$

$\Big\downarrow$ 27

$$4(1-6b)^4\int \frac{1}{\left(-6x+2\sqrt{1-6b}+1\right)^2\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)^4(e+fx)^2}dx$$

$\Big\downarrow$ 99

$$4(1-6b)^4\int \left(\frac{36\left(6e+\sqrt{1-6b}f+f\right)f^6}{(1-6b)^4\left(6e+\left(1-\sqrt{1-6b}\right)f\right)^5\left(6e+2\sqrt{1-6b}f+f\right)^3(e+fx)}+\frac{}{(1-6b)^4\left(6e+\left(1-\sqrt{1-6b}\right)f\right)}\right.$$

$\Big\downarrow$ 2009

$$4(1-6b)^4\left(\frac{2\left(4\left(1-3\sqrt{1-6b}\right)ef+\left(-28b-2\sqrt{1-6b}+5\right)f^2+12e^2\right)}{3(1-6b)^6\left(-\sqrt{1-6b}-6x+1\right)\left(-\sqrt{1-6b}f+6e+f\right)^4}+\frac{4\left(108\left(2-5\sqrt{1-6b}\right)e^2f+18(-102b-10\right.}{}\right.$$

input
```
Int[1/((e + f*x)^2*(1 - (1 - 6*b)^(3/2) - 9*b + 54*b*x - 54*x^2 + 108*x^3)
^2),x]
```

output
```
4*(1 - 6*b)^4*(2/(9*(1 - 6*b)^5*(6*e + f - Sqrt[1 - 6*b]*f)^2*(1 - Sqrt[1
 - 6*b] - 6*x)^3) - (2*(6*e + f - 4*Sqrt[1 - 6*b]*f))/(9*(1 - 6*b)^(11/2)*(
6*e + f - Sqrt[1 - 6*b]*f)^3*(1 - Sqrt[1 - 6*b] - 6*x)^2) + (2*(12*e^2 + 4
*(1 - 3*Sqrt[1 - 6*b])*e*f + (5 - 2*Sqrt[1 - 6*b] - 28*b)*f^2))/(3*(1 - 6*
b)^6*(6*e + f - Sqrt[1 - 6*b]*f)^4*(1 - Sqrt[1 - 6*b] - 6*x)) + 2/(27*(1 -
 6*b)^6*(6*e + f + 2*Sqrt[1 - 6*b]*f)^2*(1 + 2*Sqrt[1 - 6*b] - 6*x)) - f^5
/((1 - 6*b)^4*(6*e + f - Sqrt[1 - 6*b]*f)^4*(6*e + f + 2*Sqrt[1 - 6*b]*f)^
2*(e + f*x)) + (4*(432*e^3 + 108*(2 - 5*Sqrt[1 - 6*b])*e^2*f + 18*(19 - 10
*Sqrt[1 - 6*b] - 102*b)*e*f^2 + (53 - 107*Sqrt[1 - 6*b] - 6*(51 - 92*Sqrt[
1 - 6*b])*b)*f^3)*Log[1 - Sqrt[1 - 6*b] - 6*x])/(81*(1 - 6*b)^(13/2)*(6*e
+ f - Sqrt[1 - 6*b]*f)^5) - (4*(12*e + (2 + 7*Sqrt[1 - 6*b])*f)*Log[1 + 2*
Sqrt[1 - 6*b] - 6*x])/(81*(1 - 6*b)^(13/2)*(6*e + f + 2*Sqrt[1 - 6*b]*f)^3
) + (36*f^5*(6*e + f + Sqrt[1 - 6*b]*f)*Log[e + f*x])/((1 - 6*b)^4*(6*e +
f - Sqrt[1 - 6*b]*f)^5*(6*e + f + 2*Sqrt[1 - 6*b]*f)^3))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2488
```
Int[((e_.) + (f_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(
x_)^3)^(p_), x_Symbol] :> Simp[1/(4^p*(c^2 - 3*b*d)^(3*p))   Int[(e + f*x)^
m*(c^3 - 4*b*c*d + 9*a*d^2 + d*(c^2 - 3*b*d)*x)^p*(b*c - 9*a*d + 2*(c^2 - 3
*b*d)*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && NeQ[c^2 - 3
*b*d, 0] && EqQ[b^2*c^2 - 4*a*c^3 - 4*b^3*d + 18*a*b*c*d - 27*a^2*d^2, 0] &
& ILtQ[p, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3756 vs. $2(498) = 996$.

Time = 46.14 (sec) , antiderivative size = 3757, normalized size of antiderivative = 6.66

method	result	size
default	Expression too large to display	3757
parallelrisch	Expression too large to display	551226

input
```
int(1/(f*x+e)^2/(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3)^2,x,method=_RE
TURNVERBOSE)
```

output
```
-f^5/((1-6*b)^(3/2)*f^3+54*b*e*f^2+9*b*f^3+108*e^3+54*e^2*f-f^3)^2/(f*x+e)
+108*f^5/((1-6*b)^(3/2)*f^3+54*b*e*f^2+9*b*f^3+108*e^3+54*e^2*f-f^3)^3*(b*
f^2+6*e^2+2*e*f)*ln(f*x+e)+54/((1-6*b)^(3/2)*f^3+54*b*e*f^2+9*b*f^3+108*e^
3+54*e^2*f-f^3)^3*((-1/4374*(-79*f^7+559872*e^7-194400*(1-6*b)^(1/2)*e^2*f
^5*b^2-19440*(1-6*b)^(1/2)*e^2*f^5*b-64800*(1-6*b)^(1/2)*e*f^6*b^2-336960*
(1-6*b)^(1/2)*e^3*f^4*b+12240*(1-6*b)^(1/2)*e*f^6*b-505440*(1-6*b)^(1/2)*e
^4*f^3*b-233280*(1-6*b)^(1/2)*e^6*f+12852*(1-6*b)^(1/2)*f^7*b^2-420*(1-6*b
)^(1/2)*e*f^6+79*(1-6*b)^(1/2)*f^7+9180*e^3*f^4+7182*e^2*f^5+420*e*f^6-141
12*b^2*f^7+1869*b*f^7+34884*b^3*f^7+653184*e^6*f+233280*e^5*f^2+12960*e^4*
f^3+34560*(1-6*b)^(1/2)*e^3*f^4-44712*b^2*e*f^6-1632*(1-6*b)^(1/2)*f^7*b-3
6504*(1-6*b)^(1/2)*f^7*b^3+559872*b*e^5*f^2+466560*b*e^4*f^3-84240*b*e^3*f
^4-93960*b*e^2*f^5-378*b*e*f^6-233280*(1-6*b)^(1/2)*e^5*f^2+5940*(1-6*b)^(
1/2)*e^2*f^5+209304*b^3*e*f^6+719280*b^2*e^3*f^4+359640*b^2*e^2*f^5-12960*
(1-6*b)^(1/2)*e^4*f^3)/(36*b^2-12*b+1)*x^3-1/8748*((1-6*b)^(1/2)-2)*(-35*f
^7+653184*b*e^6*f+517104*b^3*e^2*f^5+1306368*b^2*e^4*f^3+86832*b^4*e*f^6+1
94400*b^3*e^3*f^4+559872*b^2*e^5*f^2+279936*e^7-64152*(1-6*b)^(1/2)*e^2*f^
5*b^2-38772*(1-6*b)^(1/2)*e^2*f^5*b-53208*(1-6*b)^(1/2)*e*f^6*b^2-285552*(
1-6*b)^(1/2)*e^3*f^4*b+7482*(1-6*b)^(1/2)*e*f^6*b-405648*(1-6*b)^(1/2)*e^4
*f^3*b-116640*(1-6*b)^(1/2)*e^6*f+4470*(1-6*b)^(1/2)*f^7*b^2-102*(1-6*b)^(
1/2)*e*f^6+35*(1-6*b)^(1/2)*f^7+22572*e^3*f^4+6750*e^2*f^5+102*e*f^6-28...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(e+fx)^2\left(1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(f*x+e)^2/(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3)^2,x, alg
orithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(e+fx)^2\left(1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3\right)^2}\,dx=\text{Timed out}$$

input
```
integrate(1/(f*x+e)**2/(1-(1-6*b)**(3/2)-9*b+54*b*x-54*x**2+108*x**3)**2,x
)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{(e+fx)^2\left(1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3\right)^2}\,dx=\int \frac{1}{\left(108\,x^3+54\,bx-54\,x^2-(-6\,b+1)\right.}$$

input
```
integrate(1/(f*x+e)^2/(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3)^2,x, alg
orithm="maxima")
```

output
```
108*(b*f^7 + 6*e^2*f^5 + 2*e*f^6)*log(f*x + e)/(944784*(2*b + 1)*e^7*f^2 +
 17496*(2*(-6*b + 1)^(3/2) + 126*b + 7)*e^6*f^3 + 17496*(54*b^2 + 2*(-6*b
+ 1)^(3/2) + 45*b - 2)*e^5*f^4 + 8748*((4*(-6*b + 1)^(3/2) + 5)*b + 90*b^2
 + (-6*b + 1)^(3/2) - 1)*e^4*f^5 - 324*((6*b - 1)^3 - 486*b^3 - 72*((-6*b
+ 1)^(3/2) - 1)*b - 567*b^2 + 2*(-6*b + 1)^(3/2) - 1)*e^3*f^6 - 162*((6*b
- 1)^3 - 27*(2*(-6*b + 1)^(3/2) + 1)*b^2 - 486*b^3 - 18*((-6*b + 1)^(3/2)
- 1)*b + 2*(-6*b + 1)^(3/2) - 1)*e^2*f^7 + 162*(18*((-6*b + 1)^(3/2) - 1)*
b^2 + 81*b^3 - ((6*b - 1)^3 + 2*(-6*b + 1)^(3/2) - 1)*b)*e*f^8 + ((-6*b +
1)^(9/2) + 3*(6*b - 1)^3 + 243*((-6*b + 1)^(3/2) - 1)*b^2 + 729*b^3 - 27*(
(6*b - 1)^3 + 2*(-6*b + 1)^(3/2) - 1)*b + 3*(-6*b + 1)^(3/2) - 1)*f^9 + 12
59712*e^9 + 1889568*e^8*f) + (648*(72*b^2 - (-6*b + 1)^(3/2) - 18*b + 1)*e
^5 - 216*(12*((-6*b + 1)^(3/2) + 2)*b - 108*b^2 + (-6*b + 1)^(3/2) - 1)*e^
4*f - 54*(2*(6*b - 1)^3 - 864*b^3 + 6*(5*(-6*b + 1)^(3/2) - 8)*b + 324*b^2
 - (-6*b + 1)^(3/2) + 3)*e^3*f^2 - 12*(4*(6*b - 1)^3 + 72*((-6*b + 1)^(3/2
) + 8)*b^2 - 1296*b^3 + 9*((-6*b + 1)^(3/2) - 10)*b - (-6*b + 1)^(3/2) + 5
)*e^2*f^3 + (11664*b^4 - (6*b - 1)^3 - 54*(4*(-6*b + 1)^(3/2) - 19)*b^2 -
5832*b^3 - 3*(10*(6*b - 1)^3 - 13*(-6*b + 1)^(3/2) + 23)*b - 2*(-6*b + 1)^
(3/2) + 1)*e*f^4 - ((-6*b + 1)^(9/2) + 108*(2*(-6*b + 1)^(3/2) - 11)*b^3 +
 1944*b^4 + (6*b - 1)^3 - 54*(2*(-6*b + 1)^(3/2) - 5)*b^2 - 9*((6*b - 1)^3
 - 2*(-6*b + 1)^(3/2) + 3)*b - (-6*b + 1)^(3/2) + 1)*f^5 + 36*(648*(6*b...
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(e+fx)^2\left(1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3\right)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(f*x+e)^2/(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3)^2,x, alg
orithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{17006112,[3]%%%}+%%%{-8503056,[2]%%%}+%%%{1417176,[1]%
%%}+%%%{-
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(e+fx)^2 \left(1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3\right)^2}\,dx = \text{Hanged}$$

input
```
int(1/((e + f*x)^2*(9*b - 54*b*x + (1 - 6*b)^(3/2) + 54*x^2 - 108*x^3 - 1)
^2),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{1}{(e+fx)^2 \left(1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3\right)^2}\,dx = \int \frac{1}{(fx+e)^2 \left(1-(-6b+1)^{\frac{3}{2}}-9b+54\right.}$$

input
```
int(1/(f*x+e)^2/(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3)^2,x)
```

output
```
int(1/(f*x+e)^2/(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3)^2,x)
```

3.150 $\int (e+fx)^2\sqrt{1-(1-6b)^{3/2}-9b+54bx-54x^2+1$

Optimal result

Integrand size = 42, antiderivative size = 496

$$\int (e+fx)^2\sqrt{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}\,dx =$$

$$-\frac{\sqrt{1-6b}\left(36e^2+\left(5+4\sqrt{1-6b}-24b\right)f^2+12e\left(f+2\sqrt{1-6b}f\right)\right)\left(2+\frac{1-6x}{\sqrt{1-6b}}\right)\sqrt{-2(1-6b)^{3/2}+3(1-}{108\sqrt{2}\left(1-\frac{1-6x}{\sqrt{1-6b}}\right)}$$

$$+\frac{\sqrt{1-6b}\left(36e^2+\left(17+10\sqrt{1-6b}-96b\right)f^2+12e\left(f+5\sqrt{1-6b}f\right)\right)\left(2+\frac{1-6x}{\sqrt{1-6b}}\right)^2\sqrt{-2(1-6b)^{3/2}+3(}{540\sqrt{2}\left(1-\frac{1-6x}{\sqrt{1-6b}}\right)}$$

$$-\frac{\sqrt{1-6b}f\left(12\sqrt{1-6b}e+\left(7+2\sqrt{1-6b}-42b\right)f\right)\left(2+\frac{1-6x}{\sqrt{1-6b}}\right)^3\sqrt{-2(1-6b)^{3/2}+3(1-6b)(1-6x)-}{756\sqrt{2}\left(1-\frac{1-6x}{\sqrt{1-6b}}\right)}$$

$$+\frac{(1-6b)^{3/2}f^2\left(2+\frac{1-6x}{\sqrt{1-6b}}\right)^4\sqrt{-2(1-6b)^{3/2}+3(1-6b)(1-6x)-(1-6x)^3}}{972\sqrt{2}\left(1-\frac{1-6x}{\sqrt{1-6b}}\right)}$$

output
```
-1/216*(1-6*b)^(1/2)*(36*e^2+(5+4*(1-6*b)^(1/2)-24*b)*f^2+12*e*(f+2*(1-6*b
)^(1/2)*f))*(2+(1-6*x)/(1-6*b)^(1/2))*(-2*(1-6*b)^(3/2)+3*(1-6*b)*(1-6*x)-
(1-6*x)^3)^(1/2)*2^(1/2)/(1-(1-6*x)/(1-6*b)^(1/2))+1/1080*(1-6*b)^(1/2)*(3
6*e^2+(17+10*(1-6*b)^(1/2)-96*b)*f^2+12*e*(f+5*(1-6*b)^(1/2)*f))*(2+(1-6*x
)/(1-6*b)^(1/2))^2*(-2*(1-6*b)^(3/2)+3*(1-6*b)*(1-6*x)-(1-6*x)^3)^(1/2)*2^
(1/2)/(1-(1-6*x)/(1-6*b)^(1/2))-1/1512*(1-6*b)^(1/2)*f*(12*(1-6*b)^(1/2)*e
+(7+2*(1-6*b)^(1/2)-42*b)*f)*(2+(1-6*x)/(1-6*b)^(1/2))^3*(-2*(1-6*b)^(3/2)
+3*(1-6*b)*(1-6*x)-(1-6*x)^3)^(1/2)*2^(1/2)/(1-(1-6*x)/(1-6*b)^(1/2))+1/19
44*(1-6*b)^(3/2)*f^2*(2+(1-6*x)/(1-6*b)^(1/2))^4*(-2*(1-6*b)^(3/2)+3*(1-6*
b)*(1-6*x)-(1-6*x)^3)^(1/2)*2^(1/2)/(1-(1-6*x)/(1-6*b)^(1/2))
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.

Time = 17.22 (sec) , antiderivative size = 17362, normalized size of antiderivative = 35.00

$$\int (e+fx)^2\sqrt{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}\,dx = \text{Result too large to show}$$

input
```
Integrate[(e + f*x)^2*Sqrt[1 - (1 - 6*b)^(3/2) - 9*b + 54*b*x - 54*x^2 + 1
08*x^3],x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 1.25 (sec) , antiderivative size = 386, normalized size of antiderivative = 0.78, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2489, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{54bx-(1-6b)^{3/2}-9b+108x^3-54x^2+1}(e+fx)^2\,dx$$

$\downarrow$ 2489

$$-\frac{\sqrt{54bx-(1-6b)^{3/2}-9b+108x^3-54x^2+1}\int -157464\sqrt{2}\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)\sqrt{6(1-6b}}{157464\sqrt{2}\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)\sqrt{6(1-6b)x-\left(2\sqrt{1-6b}+\right.}}$$

$\downarrow$ 27

$$\frac{\sqrt{54bx-(1-6b)^{3/2}-9b+108x^3-54x^2+1}\int \left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)\sqrt{6(1-6b)x-\left(2\sqrt{1-\right.}}}{\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)\sqrt{6(1-6b)x-\left(2\sqrt{1-6b}+1\right)(1-6b)}}$$

$\downarrow$ 86

$$\frac{\sqrt{54bx-(1-6b)^{3/2}-9b+108x^3-54x^2+1}\int \left(-\frac{f^2\left(6(1-6b)x-\left(2\sqrt{1-6b}+1\right)(1-6b)\right)^{7/2}}{36(6b-1)^2}+\frac{f\left(-12e-\left(7\sqrt{1-6b}+2\right)f\right)\left(6(1-6\right.}{36(1-}\right.}{}$$

$\downarrow$ 2009

$$\frac{\sqrt{54bx-(1-6b)^{3/2}-9b+108x^3-54x^2+1}\left(-\frac{\left(6(1-6b)x-\left(2\sqrt{1-6b}+1\right)(1-6b)\right)^{5/2}\left(12e\left(5\sqrt{1-6b}f+f\right)+\left(-96b+10\sqrt{1-6b}+1\right.\right.}{540(1-6b)}\right.}{}$$

input
```
Int[(e + f*x)^2*Sqrt[1 - (1 - 6*b)^(3/2) - 9*b + 54*b*x - 54*x^2 + 108*x^3
],x]
```

output
```
(Sqrt[1 - (1 - 6*b)^(3/2) - 9*b + 54*b*x - 54*x^2 + 108*x^3]*(-1/108*(Sqrt
[1 - 6*b]*(36*e^2 + (5 + 4*Sqrt[1 - 6*b] - 24*b)*f^2 + 12*e*(f + 2*Sqrt[1
- 6*b]*f))*(-((1 + 2*Sqrt[1 - 6*b])*(1 - 6*b)) + 6*(1 - 6*b)*x)^(3/2)) - (
(36*e^2 + (17 + 10*Sqrt[1 - 6*b] - 96*b)*f^2 + 12*e*(f + 5*Sqrt[1 - 6*b]*f
))*(-((1 + 2*Sqrt[1 - 6*b])*(1 - 6*b)) + 6*(1 - 6*b)*x)^(5/2))/(540*(1 - 6
*b)) - (f*(12*e + (2 + 7*Sqrt[1 - 6*b])*f)*(-((1 + 2*Sqrt[1 - 6*b])*(1 - 6
*b)) + 6*(1 - 6*b)*x)^(7/2))/(756*(1 - 6*b)^2) - (f^2*(-((1 + 2*Sqrt[1 - 6
*b])*(1 - 6*b)) + 6*(1 - 6*b)*x)^(9/2))/(972*(1 - 6*b)^3)))/(((1 - Sqrt[1
- 6*b])*(1 - 6*b) - 6*(1 - 6*b)*x)*Sqrt[-((1 + 2*Sqrt[1 - 6*b])*(1 - 6*b))
 + 6*(1 - 6*b)*x])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2489
```
Int[((e_.) + (f_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(
x_)^3)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2 + d*x^3)^p/((c^3 - 4*b*c*d
+ 9*a*d^2 + d*(c^2 - 3*b*d)*x)^p*(b*c - 9*a*d + 2*(c^2 - 3*b*d)*x)^(2*p))
 Int[(e + f*x)^m*(c^3 - 4*b*c*d + 9*a*d^2 + d*(c^2 - 3*b*d)*x)^p*(b*c - 9*a
*d + 2*(c^2 - 3*b*d)*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x]
 && NeQ[c^2 - 3*b*d, 0] && EqQ[b^2*c^2 - 4*a*c^3 - 4*b^3*d + 18*a*b*c*d - 2
7*a^2*d^2, 0] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 350, normalized size of antiderivative = 0.71

method	result
risch	$\frac{(-2835e^2-486\sqrt{1-6b}\,efx+9720befx+864bx\,f^2+6480\sqrt{1-6b}\,bef-6804e^2x-170f^2-7776ef\,x^2+20412e^2x^2+4860\sqrt{1-6b}\,x^2ef+2}{}$

input
```
int((f*x+e)^2*(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3)^(1/2),x,method=_
RETURNVERBOSE)
```

output
```
1/17010*(-2835*e^2-486*(1-6*b)^(1/2)*e*f*x+9720*b*e*f*x+864*b*x*f^2+6480*(
1-6*b)^(1/2)*b*e*f-6804*e^2*x-170*f^2-7776*e*f*x^2+20412*e^2*x^2+4860*(1-6
*b)^(1/2)*x^2*e*f+20412*b*e^2+11340*x^4*f^2-351*f^2*x^2-154*(1-6*b)^(1/2)*
f^2+29160*e*f*x^3+1890*x^3*(1-6*b)^(1/2)*f^2+3402*(1-6*b)^(1/2)*x*e^2-567*
(1-6*b)^(1/2)*e^2+1704*b*f^2-4032*b^2*f^2-810*e*f+2268*b*f^2*x^2-1458*e*f*
x-138*f^2*x+5184*b*e*f-135*(1-6*b)^(1/2)*f^2*x^2-1134*(1-6*b)^(1/2)*e*f-18
6*(1-6*b)^(1/2)*x*f^2-2700*f^2*x^3+912*(1-6*b)^(1/2)*b*f^2+1008*(1-6*b)^(1
/2)*x*b*f^2)*(-1-2*(1-6*b)^(1/2)+6*x)*(-1+6*x+(1-6*b)^(1/2))/(108*x^3+54*b
*x+6*(1-6*b)^(1/2)*b-54*x^2-9*b-(1-6*b)^(1/2)+1)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 281, normalized size of antiderivative = 0.57

$$\int (e+fx)^2\sqrt{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}\,dx=\frac{(11340\,f^2x^5+270\,(108\,ef-17\,f^2)x^4+54\,((77$$

input
```
integrate((f*x+e)^2*(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3)^(1/2),x, a
lgorithm="fricas")
```

output
```
1/8505*(11340*f^2*x^5 + 270*(108*e*f - 17*f^2)*x^4 + 54*((77*b - 4)*f^2 +
378*e^2 - 234*e*f)*x^3 - 567*(7*b - 1)*e^2 + 162*(40*b^2 - 19*b + 2)*e*f +
 2*(792*b^2 - 295*b + 27)*f^2 + 3*(324*(15*b - 1)*e*f + (117*b - 19)*f^2 -
 3402*e^2)*x^2 + 2*(567*(21*b - 2)*e^2 + 81*(19*b - 3)*e*f - (1512*b^2 - 6
03*b + 58)*f^2)*x + (105*(6*b - 1)*f^2*x^2 - 567*(6*b - 1)*e^2 - 324*(6*b
- 1)*e*f + 2*(336*b^2 - 218*b + 27)*f^2 + 10*(81*(6*b - 1)*e*f + 10*(6*b -
 1)*f^2)*x)*sqrt(-6*b + 1))*sqrt(108*x^3 + 54*b*x - 54*x^2 + (6*b - 1)*sqr
t(-6*b + 1) - 9*b + 1)/(6*x^2 + b - 2*x)
```

Sympy [F]

$$\int (e+fx)^2\sqrt{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}\,dx=\int (e+fx)^2\sqrt{54bx+6b\sqrt{1-6b}-9b+108x^3-54x^2-\sqrt{1-6b}+1}\,dx$$

input
```
integrate((f*x+e)**2*(1-(1-6*b)**(3/2)-9*b+54*b*x-54*x**2+108*x**3)**(1/2)
,x)
```

output
```
Integral((e + f*x)**2*sqrt(54*b*x + 6*b*sqrt(1 - 6*b) - 9*b + 108*x**3 - 5
4*x**2 - sqrt(1 - 6*b) + 1), x)
```

Maxima [F]

$$\int (e+fx)^2\sqrt{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}\,dx=\int \sqrt{108\,x^3+54\,bx-54\,x^2-(-6\,b+1)^{\frac{3}{2}}-9\,b$$

input
```
integrate((f*x+e)^2*(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3)^(1/2),x, a
lgorithm="maxima")
```

output
```
integrate(sqrt(108*x^3 + 54*b*x - 54*x^2 - (-6*b + 1)^(3/2) - 9*b + 1)*(f*
x + e)^2, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1711 vs. $2(416) = 832$.

Time = 0.17 (sec) , antiderivative size = 1711, normalized size of antiderivative = 3.45

$$\int (e+fx)^2\sqrt{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3)^(1/2),x, a
lgorithm="giac")
```

output
```
1/34020*sqrt(1/2)*(15120*((6*x - 2*sqrt(-6*b + 1) - 1)^(3/2) + 6*sqrt(-6*b
 + 1)*sqrt(6*x - 2*sqrt(-6*b + 1) - 1) + 3*sqrt(6*x - 2*sqrt(-6*b + 1) - 1
))*b*e*f*sgn(6*x + sqrt(-6*b + 1) - 1) + 252*(3*(6*x - 2*sqrt(-6*b + 1) -
1)^(5/2) + 10*(6*x - 2*sqrt(-6*b + 1) - 1)^(3/2)*(2*sqrt(-6*b + 1) + 1) -
15*(24*b - 4*sqrt(-6*b + 1) - 5)*sqrt(6*x - 2*sqrt(-6*b + 1) - 1))*b*f^2*s
gn(6*x + sqrt(-6*b + 1) - 1) - 3780*((6*x - 2*sqrt(-6*b + 1) - 1)^(3/2) +
6*sqrt(-6*b + 1)*sqrt(6*x - 2*sqrt(-6*b + 1) - 1) + 3*sqrt(6*x - 2*sqrt(-6
*b + 1) - 1))*sqrt(-6*b + 1)*e^2*sgn(6*x + sqrt(-6*b + 1) - 1) - 252*(3*(6
*x - 2*sqrt(-6*b + 1) - 1)^(5/2) + 10*(6*x - 2*sqrt(-6*b + 1) - 1)^(3/2)*(
2*sqrt(-6*b + 1) + 1) - 15*(24*b - 4*sqrt(-6*b + 1) - 5)*sqrt(6*x - 2*sqrt
(-6*b + 1) - 1))*sqrt(-6*b + 1)*e*f*sgn(6*x + sqrt(-6*b + 1) - 1) + 1260*(
(6*x - 2*sqrt(-6*b + 1) - 1)^(3/2) + 6*sqrt(-6*b + 1)*sqrt(6*x - 2*sqrt(-6
*b + 1) - 1) + 3*sqrt(6*x - 2*sqrt(-6*b + 1) - 1))*sqrt(-6*b + 1)*e*f*sgn(
6*x + sqrt(-6*b + 1) - 1) - 9*(5*(6*x - 2*sqrt(-6*b + 1) - 1)^(7/2) + 21*(
6*x - 2*sqrt(-6*b + 1) - 1)^(5/2)*(2*sqrt(-6*b + 1) + 1) - 35*(24*b - 4*sq
rt(-6*b + 1) - 5)*(6*x - 2*sqrt(-6*b + 1) - 1)^(3/2) - 35*(2*(24*b - 7)*sq
rt(-6*b + 1) + 72*b - 13)*sqrt(6*x - 2*sqrt(-6*b + 1) - 1))*sqrt(-6*b + 1)
*f^2*sgn(6*x + sqrt(-6*b + 1) - 1) + 21*(3*(6*x - 2*sqrt(-6*b + 1) - 1)^(5
/2) + 10*(6*x - 2*sqrt(-6*b + 1) - 1)^(3/2)*(2*sqrt(-6*b + 1) + 1) - 15*(2
4*b - 4*sqrt(-6*b + 1) - 5)*sqrt(6*x - 2*sqrt(-6*b + 1) - 1))*sqrt(-6*b...
```

Mupad [F(-1)]

Timed out.

$$\int (e+fx)^2\sqrt{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}\,dx=\int (e+f\,x)^2\,\sqrt{54\,b\,x-9\,b-(1-6\,b)^{3/2}-54\,x$$

input
```
int((e + f*x)^2*(54*b*x - 9*b - (1 - 6*b)^(3/2) - 54*x^2 + 108*x^3 + 1)^(1
/2),x)
```

output
```
int((e + f*x)^2*(54*b*x - 9*b - (1 - 6*b)^(3/2) - 54*x^2 + 108*x^3 + 1)^(1
/2), x)
```

Reduce [F]

$$\int (e+fx)^2\sqrt{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}\,dx=\text{too large to display}$$

input
```
int((f*x+e)^2*(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3)^(1/2),x)
```

output
```
( - 1080*sqrt(6*sqrt( - 6*b + 1)*b - sqrt( - 6*b + 1) + 54*b*x - 9*b + 108
*x**3 - 54*x**2 + 1)*sqrt( - 6*b + 1)*b*e*f + 30*sqrt(6*sqrt( - 6*b + 1)*b
 - sqrt( - 6*b + 1) + 54*b*x - 9*b + 108*x**3 - 54*x**2 + 1)*sqrt( - 6*b +
 1)*b*f**2 + 180*sqrt(6*sqrt( - 6*b + 1)*b - sqrt( - 6*b + 1) + 54*b*x - 9
*b + 108*x**3 - 54*x**2 + 1)*sqrt( - 6*b + 1)*e*f - 5*sqrt(6*sqrt( - 6*b +
 1)*b - sqrt( - 6*b + 1) + 54*b*x - 9*b + 108*x**3 - 54*x**2 + 1)*sqrt( -
6*b + 1)*f**2 + 4536*sqrt( - 6*b + 1)*int(sqrt(6*sqrt( - 6*b + 1)*b - sqrt
( - 6*b + 1) + 54*b*x - 9*b + 108*x**3 - 54*x**2 + 1)/(8*b**2 + 60*b*x**2
- 20*b*x - b + 72*x**4 - 48*x**3 + 2*x**2 + 2*x),x)*b**3*f**2 - 27216*sqrt
( - 6*b + 1)*int(sqrt(6*sqrt( - 6*b + 1)*b - sqrt( - 6*b + 1) + 54*b*x - 9
*b + 108*x**3 - 54*x**2 + 1)/(8*b**2 + 60*b*x**2 - 20*b*x - b + 72*x**4 -
48*x**3 + 2*x**2 + 2*x),x)*b**2*e**2 + 648*sqrt( - 6*b + 1)*int(sqrt(6*sqr
t( - 6*b + 1)*b - sqrt( - 6*b + 1) + 54*b*x - 9*b + 108*x**3 - 54*x**2 + 1
)/(8*b**2 + 60*b*x**2 - 20*b*x - b + 72*x**4 - 48*x**3 + 2*x**2 + 2*x),x)*
b**2*e*f - 270*sqrt( - 6*b + 1)*int(sqrt(6*sqrt( - 6*b + 1)*b - sqrt( - 6*
b + 1) + 54*b*x - 9*b + 108*x**3 - 54*x**2 + 1)/(8*b**2 + 60*b*x**2 - 20*b
*x - b + 72*x**4 - 48*x**3 + 2*x**2 + 2*x),x)*b**2*f**2 + 2268*sqrt( - 6*b
 + 1)*int(sqrt(6*sqrt( - 6*b + 1)*b - sqrt( - 6*b + 1) + 54*b*x - 9*b + 10
8*x**3 - 54*x**2 + 1)/(8*b**2 + 60*b*x**2 - 20*b*x - b + 72*x**4 - 48*x**3
 + 2*x**2 + 2*x),x)*b*e**2 - 1404*sqrt( - 6*b + 1)*int(sqrt(6*sqrt( - 6...
```

3.151 $\int (e+fx)\sqrt{1-(1-6b)^{3/2}-9b+54bx-54x^2+10$

Optimal result

Integrand size = 40, antiderivative size = 312

$$\int (e+fx)\sqrt{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}\,dx =
-\frac{\sqrt{1-6b}\left(6e+f+2\sqrt{1-6b}f\right)\left(2+\frac{1-6x}{\sqrt{1-6b}}\right)\sqrt{-2(1-6b)^{3/2}+3(1-6b)(1-6x)-(1-6x)^3}}{18\sqrt{2}\left(1-\frac{1-6x}{\sqrt{1-6b}}\right)}
+\frac{\sqrt{1-6b}\left(6e+f+5\sqrt{1-6b}f\right)\left(2+\frac{1-6x}{\sqrt{1-6b}}\right)^2\sqrt{-2(1-6b)^{3/2}+3(1-6b)(1-6x)-(1-6x)^3}}{90\sqrt{2}\left(1-\frac{1-6x}{\sqrt{1-6b}}\right)}
-\frac{(1-6b)f\left(2+\frac{1-6x}{\sqrt{1-6b}}\right)^3\sqrt{-2(1-6b)^{3/2}+3(1-6b)(1-6x)-(1-6x)^3}}{126\sqrt{2}\left(1-\frac{1-6x}{\sqrt{1-6b}}\right)}$$

output

```
-1/36*(1-6*b)^(1/2)*(6*e+f+2*(1-6*b)^(1/2)*f)*(2+(1-6*x)/(1-6*b)^(1/2))*(-
2*(1-6*b)^(3/2)+3*(1-6*b)*(1-6*x)-(1-6*x)^3)^(1/2)*2^(1/2)/(1-(1-6*x)/(1-6
*b)^(1/2))+1/180*(1-6*b)^(1/2)*(6*e+f+5*(1-6*b)^(1/2)*f)*(2+(1-6*x)/(1-6*b
)^(1/2))^2*(-2*(1-6*b)^(3/2)+3*(1-6*b)*(1-6*x)-(1-6*x)^3)^(1/2)*2^(1/2)/(1
-(1-6*x)/(1-6*b)^(1/2))-1/252*(1-6*b)*f*(2+(1-6*x)/(1-6*b)^(1/2))^3*(-2*(1
-6*b)^(3/2)+3*(1-6*b)*(1-6*x)-(1-6*x)^3)^(1/2)*2^(1/2)/(1-(1-6*x)/(1-6*b)^
(1/2))
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.

Time = 16.74 (sec) , antiderivative size = 9146, normalized size of antiderivative = 29.31

$$\int (e+fx)\sqrt{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}\,dx = \text{Result too large to show}$$

input
```
Integrate[(e + f*x)*Sqrt[1 - (1 - 6*b)^(3/2) - 9*b + 54*b*x - 54*x^2 + 108
*x^3],x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 268, normalized size of antiderivative = 0.86, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2489, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{54bx-(1-6b)^{3/2}-9b+108x^3-54x^2+1}(e+fx)\,dx$$

$$\Big\downarrow \text{2489}$$

$$-\frac{\sqrt{54bx-(1-6b)^{3/2}-9b+108x^3-54x^2+1}\int -157464\sqrt{2}\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)\sqrt{6(1-6b}}{157464\sqrt{2}\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)\sqrt{6(1-6b)x-(2\sqrt{1-6b}+1}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{54bx-(1-6b)^{3/2}-9b+108x^3-54x^2+1}\int \left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)\sqrt{6(1-6b)x-(2\sqrt{1-}}}{\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)\sqrt{6(1-6b)x-\left(2\sqrt{1-6b}+1\right)(1-6b)}}$$

$\Big\downarrow$ 86

$$\frac{\sqrt{54bx-(1-6b)^{3/2}-9b+108x^3-54x^2+1}\int\left(\frac{f\left(6(1-6b)x-(2\sqrt{1-6b}+1)(1-6b)\right)^{5/2}}{6(6b-1)}+\frac{1}{6}\left(-6e-5\sqrt{1-6b}f-f\right)\left(6\right.\right.}{\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1\right.}$$

$\Big\downarrow$ 2009

$$\frac{\sqrt{54bx-(1-6b)^{3/2}-9b+108x^3-54x^2+1}\left(-\frac{\left(6(1-6b)x-(2\sqrt{1-6b}+1)(1-6b)\right)^{5/2}\left(5\sqrt{1-6b}f+6e+f\right)}{90(1-6b)}-\frac{1}{18}\sqrt{1-6b}\left(6(1\right.\right.}{\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)\sqrt{6(1-}}$$

input
```
Int[(e + f*x)*Sqrt[1 - (1 - 6*b)^(3/2) - 9*b + 54*b*x - 54*x^2 + 108*x^3],
x]
```

output
```
(Sqrt[1 - (1 - 6*b)^(3/2) - 9*b + 54*b*x - 54*x^2 + 108*x^3]*(-1/18*(Sqrt[
1 - 6*b]*(6*e + f + 2*Sqrt[1 - 6*b]*f)*(-((1 + 2*Sqrt[1 - 6*b])*(1 - 6*b))
 + 6*(1 - 6*b)*x)^(3/2)) - ((6*e + f + 5*Sqrt[1 - 6*b]*f)*(-((1 + 2*Sqrt[1
 - 6*b])*(1 - 6*b)) + 6*(1 - 6*b)*x)^(5/2))/(90*(1 - 6*b)) - (f*(-((1 + 2*
Sqrt[1 - 6*b])*(1 - 6*b)) + 6*(1 - 6*b)*x)^(7/2))/(126*(1 - 6*b)^2)))/(((1
 - Sqrt[1 - 6*b])*(1 - 6*b) - 6*(1 - 6*b)*x)*Sqrt[-((1 + 2*Sqrt[1 - 6*b])*
(1 - 6*b)) + 6*(1 - 6*b)*x])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2489
```
Int[((e_.) + (f_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(
x_)^3)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2 + d*x^3)^p/((c^3 - 4*b*c*d
+ 9*a*d^2 + d*(c^2 - 3*b*d)*x)^p*(b*c - 9*a*d + 2*(c^2 - 3*b*d)*x)^(2*p))
 Int[(e + f*x)^m*(c^3 - 4*b*c*d + 9*a*d^2 + d*(c^2 - 3*b*d)*x)^p*(b*c - 9*a
*d + 2*(c^2 - 3*b*d)*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x]
 && NeQ[c^2 - 3*b*d, 0] && EqQ[b^2*c^2 - 4*a*c^3 - 4*b^3*d + 18*a*b*c*d - 2
7*a^2*d^2, 0] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.37

method	result
gosper	$-\frac{\sqrt{1-(1-6b)^{\frac{3}{2}}-9b+54bx-54x^{2}+108x^{3}}\,(15\sqrt{1-6b}\,fx+30f\,x^{2}+21\sqrt{1-6b}\,e+\sqrt{1-6b}\,f-20bf+42ex-3fx-7e+3f)\,(1+2\sqrt{1-6b}-}{105(-1+6x+\sqrt{1-6b})}$
risch	$\frac{(30\sqrt{1-6b}\,f\,x^{2}+180f\,x^{3}+40\sqrt{1-6b}\,fb+42\sqrt{1-6b}\,ex-3\sqrt{1-6b}\,fx+60bfx+252e\,x^{2}-48f\,x^{2}-7\sqrt{1-6b}\,e-7\sqrt{1-6b}\,f+252eb+32bf}{210\sqrt{108x^{3}+54bx+6\sqrt{1-6b}\,b-54x^{2}-9b-\sqrt{1-6b}+1}}$

input
```
int((f*x+e)*(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3)^(1/2),x,method=_RE
TURNVERBOSE)
```

output
```
-1/105*(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3)^(1/2)*(15*(1-6*b)^(1/2)
*f*x+30*f*x^2+21*(1-6*b)^(1/2)*e+(1-6*b)^(1/2)*f-20*b*f+42*e*x-3*f*x-7*e+3
*f)*(1+2*(1-6*b)^(1/2)-6*x)/(-1+6*x+(1-6*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.50

$$\int (e+fx)\sqrt{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}\,dx=\frac{(180\,fx^4+6\,(42\,e-13\,f)x^3+6\,((15\,b-1)f-2$$

input
```
integrate((f*x+e)*(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3)^(1/2),x, alg
orithm="fricas")
```

output
```
1/105*(180*f*x^4 + 6*(42*e - 13*f)*x^3 + 6*((15*b - 1)*f - 21*e)*x^2 - 7*(
7*b - 1)*e + (40*b^2 - 19*b + 2)*f + (14*(21*b - 2)*e + (19*b - 3)*f)*x +
(5*(6*b - 1)*f*x - 7*(6*b - 1)*e - 2*(6*b - 1)*f)*sqrt(-6*b + 1))*sqrt(108
*x^3 + 54*b*x - 54*x^2 + (6*b - 1)*sqrt(-6*b + 1) - 9*b + 1)/(6*x^2 + b -
2*x)
```

Sympy [F]

$$\int (e+fx)\sqrt{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}\,dx = \int (e+fx)\sqrt{54bx+6b\sqrt{1-6b}-9b+108x^3-54x^2-\sqrt{1-6b}+1}\,dx$$

input
```
integrate((f*x+e)*(1-(1-6*b)**(3/2)-9*b+54*b*x-54*x**2+108*x**3)**(1/2),x)
```

output
```
Integral((e + f*x)*sqrt(54*b*x + 6*b*sqrt(1 - 6*b) - 9*b + 108*x**3 - 54*x
**2 - sqrt(1 - 6*b) + 1), x)
```

Maxima [F]

$$\int (e+fx)\sqrt{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}\,dx = \int \sqrt{108\,x^3+54\,bx-54\,x^2-(-6\,b+1)^{\frac{3}{2}}-9\,b-$$

input
```
integrate((f*x+e)*(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3)^(1/2),x, alg
orithm="maxima")
```

output
```
integrate(sqrt(108*x^3 + 54*b*x - 54*x^2 - (-6*b + 1)^(3/2) - 9*b + 1)*(f*
x + e), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 933 vs. $2(259) = 518$.

Time = 0.13 (sec) , antiderivative size = 933, normalized size of antiderivative = 2.99

$$\int (e+fx)\sqrt{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)*(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3)^(1/2),x, alg
orithm="giac")
```

output
```
1/1890*sqrt(1/2)*(420*((6*x - 2*sqrt(-6*b + 1) - 1)^(3/2) + 6*sqrt(-6*b +
1)*sqrt(6*x - 2*sqrt(-6*b + 1) - 1) + 3*sqrt(6*x - 2*sqrt(-6*b + 1) - 1))*
b*f*sgn(6*x + sqrt(-6*b + 1) - 1) - 210*((6*x - 2*sqrt(-6*b + 1) - 1)^(3/2
) + 6*sqrt(-6*b + 1)*sqrt(6*x - 2*sqrt(-6*b + 1) - 1) + 3*sqrt(6*x - 2*sqr
t(-6*b + 1) - 1))*sqrt(-6*b + 1)*e*sgn(6*x + sqrt(-6*b + 1) - 1) - 7*(3*(6
*x - 2*sqrt(-6*b + 1) - 1)^(5/2) + 10*(6*x - 2*sqrt(-6*b + 1) - 1)^(3/2)*(
2*sqrt(-6*b + 1) + 1) - 15*(24*b - 4*sqrt(-6*b + 1) - 5)*sqrt(6*x - 2*sqrt
(-6*b + 1) - 1))*sqrt(-6*b + 1)*f*sgn(6*x + sqrt(-6*b + 1) - 1) + 35*((6*x
 - 2*sqrt(-6*b + 1) - 1)^(3/2) + 6*sqrt(-6*b + 1)*sqrt(6*x - 2*sqrt(-6*b +
 1) - 1) + 3*sqrt(6*x - 2*sqrt(-6*b + 1) - 1))*sqrt(-6*b + 1)*f*sgn(6*x +
sqrt(-6*b + 1) - 1) + 7560*b*e*sqrt(6*x - 2*sqrt(-6*b + 1) - 1)*sgn(6*x +
sqrt(-6*b + 1) - 1) + 42*(3*(6*x - 2*sqrt(-6*b + 1) - 1)^(5/2) + 10*(6*x -
 2*sqrt(-6*b + 1) - 1)^(3/2)*(2*sqrt(-6*b + 1) + 1) - 15*(24*b - 4*sqrt(-6
*b + 1) - 5)*sqrt(6*x - 2*sqrt(-6*b + 1) - 1))*e*sgn(6*x + sqrt(-6*b + 1)
- 1) - 420*((6*x - 2*sqrt(-6*b + 1) - 1)^(3/2) + 6*sqrt(-6*b + 1)*sqrt(6*x
 - 2*sqrt(-6*b + 1) - 1) + 3*sqrt(6*x - 2*sqrt(-6*b + 1) - 1))*e*sgn(6*x +
 sqrt(-6*b + 1) - 1) + 3*(5*(6*x - 2*sqrt(-6*b + 1) - 1)^(7/2) + 21*(6*x -
 2*sqrt(-6*b + 1) - 1)^(5/2)*(2*sqrt(-6*b + 1) + 1) - 35*(24*b - 4*sqrt(-6
*b + 1) - 5)*(6*x - 2*sqrt(-6*b + 1) - 1)^(3/2) - 35*(2*(24*b - 7)*sqrt(-6
*b + 1) + 72*b - 13)*sqrt(6*x - 2*sqrt(-6*b + 1) - 1))*f*sgn(6*x + sqrt...
```

Mupad [F(-1)]

Timed out.

$$\int (e+fx)\sqrt{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}\,dx = \int (e+f\,x)\,\sqrt{54\,b\,x-9\,b-{(1-6\,b)}^{3/2}-54\,x^2+108\,x^3+1}\,dx$$

input
```
int((e + f*x)*(54*b*x - 9*b - (1 - 6*b)^(3/2) - 54*x^2 + 108*x^3 + 1)^(1/2
),x)
```

output
```
int((e + f*x)*(54*b*x - 9*b - (1 - 6*b)^(3/2) - 54*x^2 + 108*x^3 + 1)^(1/2
), x)
```

Reduce [F]

$$\int (e+fx)\sqrt{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}\,dx = \text{too large to display}$$

input
```
int((f*x+e)*(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3)^(1/2),x)
```

output

```
( - 30*sqrt(6*sqrt( - 6*b + 1)*b - sqrt( - 6*b + 1) + 54*b*x - 9*b + 108*x
**3 - 54*x**2 + 1)*sqrt( - 6*b + 1)*b*f + 5*sqrt(6*sqrt( - 6*b + 1)*b - sq
rt( - 6*b + 1) + 54*b*x - 9*b + 108*x**3 - 54*x**2 + 1)*sqrt( - 6*b + 1)*f
 - 1512*sqrt( - 6*b + 1)*int(sqrt(6*sqrt( - 6*b + 1)*b - sqrt( - 6*b + 1)
+ 54*b*x - 9*b + 108*x**3 - 54*x**2 + 1)/(8*b**2 + 60*b*x**2 - 20*b*x - b
+ 72*x**4 - 48*x**3 + 2*x**2 + 2*x),x)*b**2*e + 18*sqrt( - 6*b + 1)*int(sq
rt(6*sqrt( - 6*b + 1)*b - sqrt( - 6*b + 1) + 54*b*x - 9*b + 108*x**3 - 54*
x**2 + 1)/(8*b**2 + 60*b*x**2 - 20*b*x - b + 72*x**4 - 48*x**3 + 2*x**2 +
2*x),x)*b**2*f + 126*sqrt( - 6*b + 1)*int(sqrt(6*sqrt( - 6*b + 1)*b - sqrt
( - 6*b + 1) + 54*b*x - 9*b + 108*x**3 - 54*x**2 + 1)/(8*b**2 + 60*b*x**2
- 20*b*x - b + 72*x**4 - 48*x**3 + 2*x**2 + 2*x),x)*b*e - 39*sqrt( - 6*b +
 1)*int(sqrt(6*sqrt( - 6*b + 1)*b - sqrt( - 6*b + 1) + 54*b*x - 9*b + 108*
x**3 - 54*x**2 + 1)/(8*b**2 + 60*b*x**2 - 20*b*x - b + 72*x**4 - 48*x**3 +
 2*x**2 + 2*x),x)*b*f + 21*sqrt( - 6*b + 1)*int(sqrt(6*sqrt( - 6*b + 1)*b
- sqrt( - 6*b + 1) + 54*b*x - 9*b + 108*x**3 - 54*x**2 + 1)/(8*b**2 + 60*b
*x**2 - 20*b*x - b + 72*x**4 - 48*x**3 + 2*x**2 + 2*x),x)*e + 6*sqrt( - 6*
b + 1)*int(sqrt(6*sqrt( - 6*b + 1)*b - sqrt( - 6*b + 1) + 54*b*x - 9*b + 1
08*x**3 - 54*x**2 + 1)/(8*b**2 + 60*b*x**2 - 20*b*x - b + 72*x**4 - 48*x**
3 + 2*x**2 + 2*x),x)*f + 3240*sqrt( - 6*b + 1)*int((sqrt(6*sqrt( - 6*b + 1
)*b - sqrt( - 6*b + 1) + 54*b*x - 9*b + 108*x**3 - 54*x**2 + 1)*x**3)/(...
```

3.152 $\int \sqrt{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 187

$$\int \sqrt{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}\,dx = -\frac{\sqrt{1-6b}\left(2+\frac{1-6x}{\sqrt{1-6b}}\right)\sqrt{-2(1-6b)^{3/2}+3(1-6b)(1-6x)-(1-6x)^3}}{3\sqrt{2}\left(1-\frac{1-6x}{\sqrt{1-6b}}\right)} + \frac{\sqrt{1-6b}\left(2+\frac{1-6x}{\sqrt{1-6b}}\right)^2\sqrt{-2(1-6b)^{3/2}+3(1-6b)(1-6x)-(1-6x)^3}}{15\sqrt{2}\left(1-\frac{1-6x}{\sqrt{1-6b}}\right)}$$

output

```
-1/6*(1-6*b)^(1/2)*(2+(1-6*x)/(1-6*b)^(1/2))*(-2*(1-6*b)^(3/2)+3*(1-6*b)*(
1-6*x)-(1-6*x)^3)^(1/2)*2^(1/2)/(1-(1-6*x)/(1-6*b)^(1/2))+1/30*(1-6*b)^(1/
2)*(2+(1-6*x)/(1-6*b)^(1/2))^2*(-2*(1-6*b)^(3/2)+3*(1-6*b)*(1-6*x)-(1-6*x)
^3)^(1/2)*2^(1/2)/(1-(1-6*x)/(1-6*b)^(1/2))
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2 in optimal.

Time = 2.85 (sec) , antiderivative size = 1461, normalized size of antiderivative = 7.81

$$\int \sqrt{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}\,dx = \text{Too large to display}$$

input

```
Integrate[Sqrt[1 - (1 - 6*b)^(3/2) - 9*b + 54*b*x - 54*x^2 + 108*x^3],x]
```

output

```
(((-1 + 6*x)*(1 - Sqrt[1 - 6*b] - 54*x^2 + 108*x^3 + b*(-9 + 6*Sqrt[1 - 6*
b] + 54*x)))/3 - 6*(1 - 6*b)*Sqrt[(-x + Root[1 - Sqrt[1 - 6*b] - 9*b + 6*S
qrt[1 - 6*b]*b + 54*b*#1 - 54*#1^2 + 108*#1^3 & , 1])/(Root[1 - Sqrt[1 - 6
*b] - 9*b + 6*Sqrt[1 - 6*b]*b + 54*b*#1 - 54*#1^2 + 108*#1^3 & , 1] - Root
[1 - Sqrt[1 - 6*b] - 9*b + 6*Sqrt[1 - 6*b]*b + 54*b*#1 - 54*#1^2 + 108*#1^
3 & , 3])]*Sqrt[-(((x - Root[1 - Sqrt[1 - 6*b] - 9*b + 6*Sqrt[1 - 6*b]*b +
 54*b*#1 - 54*#1^2 + 108*#1^3 & , 2])*(x - Root[1 - Sqrt[1 - 6*b] - 9*b +
6*Sqrt[1 - 6*b]*b + 54*b*#1 - 54*#1^2 + 108*#1^3 & , 3]))/(Root[1 - Sqrt[1
 - 6*b] - 9*b + 6*Sqrt[1 - 6*b]*b + 54*b*#1 - 54*#1^2 + 108*#1^3 & , 2] -
Root[1 - Sqrt[1 - 6*b] - 9*b + 6*Sqrt[1 - 6*b]*b + 54*b*#1 - 54*#1^2 + 108
*#1^3 & , 3])^2)]*(Root[1 - Sqrt[1 - 6*b] - 9*b + 6*Sqrt[1 - 6*b]*b + 54*b
*#1 - 54*#1^2 + 108*#1^3 & , 2] - Root[1 - Sqrt[1 - 6*b] - 9*b + 6*Sqrt[1
- 6*b]*b + 54*b*#1 - 54*#1^2 + 108*#1^3 & , 3])*(EllipticF[ArcSin[Sqrt[(-x
 + Root[1 - Sqrt[1 - 6*b] - 9*b + 6*Sqrt[1 - 6*b]*b + 54*b*#1 - 54*#1^2 +
108*#1^3 & , 3])/(-Root[1 - Sqrt[1 - 6*b] - 9*b + 6*Sqrt[1 - 6*b]*b + 54*b
*#1 - 54*#1^2 + 108*#1^3 & , 2] + Root[1 - Sqrt[1 - 6*b] - 9*b + 6*Sqrt[1
- 6*b]*b + 54*b*#1 - 54*#1^2 + 108*#1^3 & , 3])]], (Root[1 - Sqrt[1 - 6*b]
 - 9*b + 6*Sqrt[1 - 6*b]*b + 54*b*#1 - 54*#1^2 + 108*#1^3 & , 2] - Root[1
- Sqrt[1 - 6*b] - 9*b + 6*Sqrt[1 - 6*b]*b + 54*b*#1 - 54*#1^2 + 108*#1^3 &
 , 3])/(Root[1 - Sqrt[1 - 6*b] - 9*b + 6*Sqrt[1 - 6*b]*b + 54*b*#1 - 54...
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2480, 27, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{54bx-(1-6b)^{3/2}-9b+108x^3-54x^2+1}\,dx$$

$$\Big\downarrow \text{2480}$$

$$-\frac{\sqrt{54bx-(1-6b)^{3/2}-9b+108x^3-54x^2+1}\int -157464\sqrt{2}\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)\sqrt{6(1-6b}}{157464\sqrt{2}\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)\sqrt{6(1-6b)x-\left(2\sqrt{1-6b}+1\right)(1}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{54bx-(1-6b)^{3/2}-9b+108x^3-54x^2+1}\int \left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)\sqrt{6(1-6b)x-\left(2\sqrt{1-}\right.}}{\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)\sqrt{6(1-6b)x-\left(2\sqrt{1-6b}+1\right)(1-6b)}}$$

$$\Big\downarrow \text{53}$$

$$\frac{\sqrt{54bx-(1-6b)^{3/2}-9b+108x^3-54x^2+1}\int \left(-3\sqrt{6(1-6b)x-\left(2\sqrt{1-6b}+1\right)(1-6b)}(1-6b)^{3/2}-(6(1\right.}{\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)\sqrt{6(1-6b)x-\left(2\sqrt{1-6b}+1\right)(1-}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{54bx-(1-6b)^{3/2}-9b+108x^3-54x^2+1}\left(-\frac{\left(6(1-6b)x-\left(2\sqrt{1-6b}+1\right)(1-6b)\right)^{5/2}}{15(1-6b)}-\frac{1}{3}\sqrt{1-6b}(6(1-6b)x-(2\sqrt{1}\right.}{\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)\sqrt{6(1-6b)x-\left(2\sqrt{1-6b}+1\right)(1-6b)}}$$

input
```
Int[Sqrt[1 - (1 - 6*b)^(3/2) - 9*b + 54*b*x - 54*x^2 + 108*x^3],x]
```

output
```
(Sqrt[1 - (1 - 6*b)^(3/2) - 9*b + 54*b*x - 54*x^2 + 108*x^3]*(-1/3*(Sqrt[1
 - 6*b]*(-((1 + 2*Sqrt[1 - 6*b])*(1 - 6*b)) + 6*(1 - 6*b)*x)^(3/2)) - (-((
1 + 2*Sqrt[1 - 6*b])*(1 - 6*b)) + 6*(1 - 6*b)*x)^(5/2)/(15*(1 - 6*b))))/((
(1 - Sqrt[1 - 6*b])*(1 - 6*b) - 6*(1 - 6*b)*x)*Sqrt[-((1 + 2*Sqrt[1 - 6*b]
)*(1 - 6*b)) + 6*(1 - 6*b)*x])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2480
```
Int[(Px_)^(p_), x_Symbol] :> With[{a = Coeff[Px, x, 0], b = Coeff[Px, x, 1]
, c = Coeff[Px, x, 2], d = Coeff[Px, x, 3]}, Simp[Px^p/((c^3 - 4*b*c*d + 9*
a*d^2 + d*(c^2 - 3*b*d)*x)^p*(b*c - 9*a*d + 2*(c^2 - 3*b*d)*x)^(2*p))   Int
[(c^3 - 4*b*c*d + 9*a*d^2 + d*(c^2 - 3*b*d)*x)^p*(b*c - 9*a*d + 2*(c^2 - 3*
b*d)*x)^(2*p), x], x] /; EqQ[b^2*c^2 - 4*a*c^3 - 4*b^3*d + 18*a*b*c*d - 27*
a^2*d^2, 0] && NeQ[c^2 - 3*b*d, 0]] /; FreeQ[p, x] && PolyQ[Px, x, 3] &&  !
IntegerQ[p]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.40

method	result	size
gosper	$-\frac{\left(1+2\sqrt{1-6b}-6x\right)\left(6x+3\sqrt{1-6b}-1\right)\sqrt{1-\left(1-6b\right)^{\frac{3}{2}}-9b+54bx-54x^{2}+108x^{3}}}{15\left(-1+6x+\sqrt{1-6b}\right)}$	75
risch	$\frac{\left(6x\sqrt{1-6b}+36x^{2}-\sqrt{1-6b}+36b-12x-5\right)\left(-1-2\sqrt{1-6b}+6x\right)\left(-1+6x+\sqrt{1-6b}\right)}{30\sqrt{108x^{3}+54bx+6\sqrt{1-6b}\,b-54x^{2}-9b-\sqrt{1-6b}+1}}$	101

input
```
int((1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3)^(1/2),x,method=_RETURNVERB
OSE)
```

output
```
-1/15*(1+2*(1-6*b)^(1/2)-6*x)*(6*x+3*(1-6*b)^(1/2)-1)*(1-(1-6*b)^(3/2)-9*b
+54*b*x-54*x^2+108*x^3)^(1/2)/(-1+6*x+(1-6*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.42

$$\int \sqrt{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}\,dx = \frac{\sqrt{108\,x^3+54\,bx-54\,x^2+(6\,b-1)\sqrt{-6\,b+1}-9}}{15\,(}$$

input
```
integrate((1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3)^(1/2),x, algorithm="
fricas")
```

output
```
1/15*sqrt(108*x^3 + 54*b*x - 54*x^2 + (6*b - 1)*sqrt(-6*b + 1) - 9*b + 1)*
(36*x^3 + 2*(21*b - 2)*x - 18*x^2 + (-6*b + 1)^(3/2) - 7*b + 1)/(6*x^2 + b
 - 2*x)
```

Sympy [F]

$$\int \sqrt{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}\,dx = \int \sqrt{54bx-9b+108x^3-54x^2-(1-6b)^{\frac{3}{2}}+1}\,dx$$

input
```
integrate((1-(1-6*b)**(3/2)-9*b+54*b*x-54*x**2+108*x**3)**(1/2),x)
```

output
```
Integral(sqrt(54*b*x - 9*b + 108*x**3 - 54*x**2 - (1 - 6*b)**(3/2) + 1), x
)
```

Maxima [F]

$$\int \sqrt{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}\,dx=\int \sqrt{108\,x^3+54\,bx-54\,x^2-(-6\,b+1)^{\frac{3}{2}}-9\,b+1}$$

input
```
integrate((1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3)^(1/2),x, algorithm="
maxima")
```

output
```
integrate(sqrt(108*x^3 + 54*b*x - 54*x^2 - (-6*b + 1)^(3/2) - 9*b + 1), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 359 vs. $2(153) = 306$.

Time = 0.12 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.92

$$\int \sqrt{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}\,dx=$$
$$-\frac{1}{45}\sqrt{\frac{1}{2}}\left(5\left(\left(6\,x-2\,\sqrt{-6\,b+1}-1\right)^{\frac{3}{2}}+6\,\sqrt{-6\,b+1}\sqrt{6\,x-2\,\sqrt{-6\,b+1}-1}+3\,\sqrt{6\,x-2\,\sqrt{-6\,b+1}\right.\right.$$

input
```
integrate((1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3)^(1/2),x, algorithm="
giac")
```

output
```
-1/45*sqrt(1/2)*(5*((6*x - 2*sqrt(-6*b + 1) - 1)^(3/2) + 6*sqrt(-6*b + 1)*
sqrt(6*x - 2*sqrt(-6*b + 1) - 1) + 3*sqrt(6*x - 2*sqrt(-6*b + 1) - 1))*sqr
t(-6*b + 1)*sgn(6*x + sqrt(-6*b + 1) - 1) - 180*b*sqrt(6*x - 2*sqrt(-6*b +
 1) - 1)*sgn(6*x + sqrt(-6*b + 1) - 1) - (3*(6*x - 2*sqrt(-6*b + 1) - 1)^(
5/2) + 10*(6*x - 2*sqrt(-6*b + 1) - 1)^(3/2)*(2*sqrt(-6*b + 1) + 1) - 15*(
24*b - 4*sqrt(-6*b + 1) - 5)*sqrt(6*x - 2*sqrt(-6*b + 1) - 1))*sgn(6*x + s
qrt(-6*b + 1) - 1) + 10*((6*x - 2*sqrt(-6*b + 1) - 1)^(3/2) + 6*sqrt(-6*b
+ 1)*sqrt(6*x - 2*sqrt(-6*b + 1) - 1) + 3*sqrt(6*x - 2*sqrt(-6*b + 1) - 1)
)*sgn(6*x + sqrt(-6*b + 1) - 1) - 15*sqrt(-6*b + 1)*sqrt(6*x - 2*sqrt(-6*b
 + 1) - 1)*sgn(6*x + sqrt(-6*b + 1) - 1) + 15*sqrt(6*x - 2*sqrt(-6*b + 1)
- 1)*sgn(6*x + sqrt(-6*b + 1) - 1))
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}\,dx = \int \sqrt{54\,b\,x-9\,b-{(1-6\,b)}^{3/2}-54\,x^2+108\,x^3+1}$$

input
```
int((54*b*x - 9*b - (1 - 6*b)^(3/2) - 54*x^2 + 108*x^3 + 1)^(1/2),x)
```

output
```
int((54*b*x - 9*b - (1 - 6*b)^(3/2) - 54*x^2 + 108*x^3 + 1)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}\,dx = \text{Too large to display}$$

input
```
int((1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3)^(1/2),x)
```

output

```
( - 72*sqrt( - 6*b + 1)*int(sqrt(6*sqrt( - 6*b + 1)*b - sqrt( - 6*b + 1) +
 54*b*x - 9*b + 108*x**3 - 54*x**2 + 1)/(8*b**2 + 60*b*x**2 - 20*b*x - b +
 72*x**4 - 48*x**3 + 2*x**2 + 2*x),x)*b**2 + 6*sqrt( - 6*b + 1)*int(sqrt(6
*sqrt( - 6*b + 1)*b - sqrt( - 6*b + 1) + 54*b*x - 9*b + 108*x**3 - 54*x**2
 + 1)/(8*b**2 + 60*b*x**2 - 20*b*x - b + 72*x**4 - 48*x**3 + 2*x**2 + 2*x)
,x)*b + sqrt( - 6*b + 1)*int(sqrt(6*sqrt( - 6*b + 1)*b - sqrt( - 6*b + 1)
+ 54*b*x - 9*b + 108*x**3 - 54*x**2 + 1)/(8*b**2 + 60*b*x**2 - 20*b*x - b
+ 72*x**4 - 48*x**3 + 2*x**2 + 2*x),x) + 108*sqrt( - 6*b + 1)*int((sqrt(6*
sqrt( - 6*b + 1)*b - sqrt( - 6*b + 1) + 54*b*x - 9*b + 108*x**3 - 54*x**2
+ 1)*x)/(8*b**2 + 60*b*x**2 - 20*b*x - b + 72*x**4 - 48*x**3 + 2*x**2 + 2*
x),x)*b - 18*sqrt( - 6*b + 1)*int((sqrt(6*sqrt( - 6*b + 1)*b - sqrt( - 6*b
 + 1) + 54*b*x - 9*b + 108*x**3 - 54*x**2 + 1)*x)/(8*b**2 + 60*b*x**2 - 20
*b*x - b + 72*x**4 - 48*x**3 + 2*x**2 + 2*x),x) - 12*sqrt(6*sqrt( - 6*b +
1)*b - sqrt( - 6*b + 1) + 54*b*x - 9*b + 108*x**3 - 54*x**2 + 1)*b + 18*sq
rt(6*sqrt( - 6*b + 1)*b - sqrt( - 6*b + 1) + 54*b*x - 9*b + 108*x**3 - 54*
x**2 + 1)*x - sqrt(6*sqrt( - 6*b + 1)*b - sqrt( - 6*b + 1) + 54*b*x - 9*b
+ 108*x**3 - 54*x**2 + 1) + 108*int(sqrt(6*sqrt( - 6*b + 1)*b - sqrt( - 6*
b + 1) + 54*b*x - 9*b + 108*x**3 - 54*x**2 + 1)/(8*b**2 + 60*b*x**2 - 20*b
*x - b + 72*x**4 - 48*x**3 + 2*x**2 + 2*x),x)*b**2 - 24*int(sqrt(6*sqrt( -
 6*b + 1)*b - sqrt( - 6*b + 1) + 54*b*x - 9*b + 108*x**3 - 54*x**2 + 1)...
```

3.153 $\int \frac{\sqrt{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}}{e+fx}\,dx$

Optimal result

Integrand size = 42, antiderivative size = 365

$$\int \frac{\sqrt{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}}{e+fx}\,dx = \frac{\sqrt{2}\left(f-\frac{6e+f}{\sqrt{1-6b}}\right)\sqrt{-2(1-6b)^{3/2}+3(1-6b)(1-6x)-(1-6x)^3}}{f^2\left(1-\frac{1-6x}{\sqrt{1-6b}}\right)}$$

$$-\frac{\sqrt{2}\left(2+\frac{1-6x}{\sqrt{1-6b}}\right)\sqrt{-2(1-6b)^{3/2}+3(1-6b)(1-6x)-(1-6x)^3}}{3f\left(1-\frac{1-6x}{\sqrt{1-6b}}\right)}$$

$$+\frac{\sqrt{2}(6e+f-\sqrt{1-6b}f)\sqrt{6e+f+2\sqrt{1-6b}f}\sqrt{-2(1-6b)^{3/2}+3(1-6b)(1-6x)-(1-6x)^3}\,\mathrm{arctanh}\ldots}{(1-6b)^{3/4}f^{5/2}\left(1-\frac{1-6x}{\sqrt{1-6b}}\right)\sqrt{2+\frac{1-6x}{\sqrt{1-6b}}}}$$

output
```
2^(1/2)*(f-(6*e+f)/(1-6*b)^(1/2))*(-2*(1-6*b)^(3/2)+3*(1-6*b)*(1-6*x)-(1-6
*x)^3)^(1/2)/f^2/(1-(1-6*x)/(1-6*b)^(1/2))-1/3*2^(1/2)*(2+(1-6*x)/(1-6*b)^
(1/2))*(-2*(1-6*b)^(3/2)+3*(1-6*b)*(1-6*x)-(1-6*x)^3)^(1/2)/f/(1-(1-6*x)/(
1-6*b)^(1/2))+2^(1/2)*(6*e+f-(1-6*b)^(1/2)*f)*(6*e+f+2*(1-6*b)^(1/2)*f)^(1
/2)*(-2*(1-6*b)^(3/2)+3*(1-6*b)*(1-6*x)-(1-6*x)^3)^(1/2)*arctanh((1-6*b)^(
1/4)*f^(1/2)*(2+(1-6*x)/(1-6*b)^(1/2))^(1/2)/(6*e+f+2*(1-6*b)^(1/2)*f)^(1/
2))/(1-6*b)^(3/4)/f^(5/2)/(1-(1-6*x)/(1-6*b)^(1/2))/(2+(1-6*x)/(1-6*b)^(1/
2))^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 16.69 (sec) , antiderivative size = 13435, normalized size of antiderivative = 36.81

$$\int \frac{\sqrt{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}}{e+fx}\,dx = \text{Result too large to show}$$

input
```
Integrate[Sqrt[1 - (1 - 6*b)^(3/2) - 9*b + 54*b*x - 54*x^2 + 108*x^3]/(e +
 f*x),x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 313, normalized size of antiderivative = 0.86, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2489, 27, 90, 60, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{54bx-(1-6b)^{3/2}-9b+108x^3-54x^2+1}}{e+fx}dx$$

$$\Big\downarrow \text{ 2489}$$

$$-\frac{\sqrt{54bx-(1-6b)^{3/2}-9b+108x^3-54x^2+1}\int -\frac{157464\sqrt{2}\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)\sqrt{6(1-6b)x-\left(2\sqrt{1-6b}+1\right)(1-6b}}{e+fx}}{157464\sqrt{2}\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)\sqrt{6(1-6b)x-\left(2\sqrt{1-6b}+1\right)(1-6b)}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{54bx-(1-6b)^{3/2}-9b+108x^3-54x^2+1}\int \frac{\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)\sqrt{6(1-6b)x-\left(2\sqrt{1-6b}+1\right)(1-6b)}}{e+fx}dx}{\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)\sqrt{6(1-6b)x-\left(2\sqrt{1-6b}+1\right)(1-6b)}}$$

$$\Big\downarrow \; 90$$

$$\frac{\sqrt{54bx-(1-6b)^{3/2}-9b+108x^3-54x^2+1}\left(\frac{(1-6b)\left(\left(1-\sqrt{1-6b}\right)f+6e\right)\int\frac{\sqrt{6(1-6b)x-\left(2\sqrt{1-6b}+1\right)(1-6b)}}{e+fx}dx}{f}-\frac{2(6(1-6b)x-(2\cdots}{\cdots}\right.}{\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)\sqrt{6(1-6b)x-\left(2\sqrt{1-6b}+1\right)(1-6b)}}$$

$$\Big\downarrow \; 60$$

$$\frac{\sqrt{54bx-(1-6b)^{3/2}-9b+108x^3-54x^2+1}\left(\frac{(1-6b)\left(\left(1-\sqrt{1-6b}\right)f+6e\right)\left(\frac{2\sqrt{6(1-6b)x-\left(2\sqrt{1-6b}+1\right)(1-6b)}}{f}-\frac{(1-6b)\left(2\sqrt{1-6b}f+6e\cdots\right.}{\cdots}\right.}{f}\right.}{\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)\sqrt{6(1-6b)x-\left(2\sqrt{\cdots}\right.}}$$

$$\Big\downarrow \; 73$$

$$\frac{\sqrt{54bx-(1-6b)^{3/2}-9b+108x^3-54x^2+1}\left(\frac{(1-6b)\left(\left(1-\sqrt{1-6b}\right)f+6e\right)\left(\frac{2\sqrt{6(1-6b)x-\left(2\sqrt{1-6b}+1\right)(1-6b)}}{f}-\frac{\left(2\sqrt{1-6b}f+6e+f\right)\int\cdots}{\cdots}\right.}{\cdots}\right.}{\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)\sqrt{6(1\cdots}}$$

$$\Big\downarrow \; 218$$

$$\frac{\sqrt{54bx-(1-6b)^{3/2}-9b+108x^3-54x^2+1}\left(\frac{(1-6b)\left(\left(1-\sqrt{1-6b}\right)f+6e\right)\left(\frac{2\sqrt{6(1-6b)x-\left(2\sqrt{1-6b}+1\right)(1-6b)}}{f}-\frac{2\sqrt{1-6b}\sqrt{2\sqrt{1-6b}f+\cdots}}{\cdots}\right.}{f}\right.}{\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)\sqrt{6(1-6b)x-\left(2\sqrt{\cdots}\right.}}$$

input

```
Int[Sqrt[1 - (1 - 6*b)^(3/2) - 9*b + 54*b*x - 54*x^2 + 108*x^3]/(e + f*x),
x]
```

output
```
(Sqrt[1 - (1 - 6*b)^(3/2) - 9*b + 54*b*x - 54*x^2 + 108*x^3]*((-2*(-((1 +
2*Sqrt[1 - 6*b])*(1 - 6*b)) + 6*(1 - 6*b)*x)^(3/2))/(3*f) + ((1 - 6*b)*(6*
e + (1 - Sqrt[1 - 6*b])*f)*((2*Sqrt[-((1 + 2*Sqrt[1 - 6*b])*(1 - 6*b)) + 6
*(1 - 6*b)*x])/f - (2*Sqrt[1 - 6*b]*Sqrt[6*e + f + 2*Sqrt[1 - 6*b]*f]*ArcT
an[(Sqrt[f]*Sqrt[-((1 + 2*Sqrt[1 - 6*b])*(1 - 6*b)) + 6*(1 - 6*b)*x])/(Sqr
t[1 - 6*b]*Sqrt[6*e + f + 2*Sqrt[1 - 6*b]*f])])/f^(3/2)))/f))/(((1 - Sqrt[
1 - 6*b])*(1 - 6*b) - 6*(1 - 6*b)*x)*Sqrt[-((1 + 2*Sqrt[1 - 6*b])*(1 - 6*b
)) + 6*(1 - 6*b)*x])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 2489
```
Int[((e_.) + (f_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(
x_)^3)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2 + d*x^3)^p/((c^3 - 4*b*c*d
+ 9*a*d^2 + d*(c^2 - 3*b*d)*x)^p*(b*c - 9*a*d + 2*(c^2 - 3*b*d)*x)^(2*p))
 Int[(e + f*x)^m*(c^3 - 4*b*c*d + 9*a*d^2 + d*(c^2 - 3*b*d)*x)^p*(b*c - 9*a
*d + 2*(c^2 - 3*b*d)*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x]
 && NeQ[c^2 - 3*b*d, 0] && EqQ[b^2*c^2 - 4*a*c^3 - 4*b^3*d + 18*a*b*c*d - 2
7*a^2*d^2, 0] &&  !IntegerQ[p]
```

Maple [F]

$$\int \frac{\sqrt{1-(1-6b)^{\frac{3}{2}}-9b+54bx-54x^2+108x^3}}{fx+e}dx$$

input
```
int((1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3)^(1/2)/(f*x+e),x)
```

output
```
int((1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3)^(1/2)/(f*x+e),x)
```

Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 1033, normalized size of antiderivative = 2.83

$$\int \frac{\sqrt{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}}{e+fx}\,dx = \text{Too large to display}$$

input
```
integrate((1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3)^(1/2)/(f*x+e),x, alg
orithm="fricas")
```

output
```
[1/3*(3*(6*x^2 + b - 2*x)*sqrt(((6*b - 1)*sqrt(-6*b + 1)*f^3 - 54*b*e*f^2
- (9*b - 1)*f^3 - 108*e^3 - 54*e^2*f)/f)*log((9*b^2*f^3 + 162*b*e^3 + 108*
b*e^2*f + 9*(3*b^2 + 2*b)*e*f^2 - 162*(b*f^3 + 6*e^2*f + 2*e*f^2)*x^3 + 54
*((3*b + 4)*e*f^2 + 2*b*f^3 + 18*e^3 + 18*e^2*f)*x^2 - 9*(6*(3*b + 4)*e^2*
f + 4*(3*b + 1)*e*f^2 + (3*b^2 + 2*b)*f^3 + 36*e^3)*x - ((3*b - 1)*f^2 - 3
*e*f + 3*(6*e*f + f^2)*x + (3*f^2*x - 3*e*f - f^2)*sqrt(-6*b + 1))*sqrt(10
8*x^3 + 54*b*x - 54*x^2 + (6*b - 1)*sqrt(-6*b + 1) - 9*b + 1)*sqrt(((6*b -
 1)*sqrt(-6*b + 1)*f^3 - 54*b*e*f^2 - (9*b - 1)*f^3 - 108*e^3 - 54*e^2*f)/
f) + 18*(b^2*f^3 + 6*b*e^2*f + 2*b*e*f^2 + 6*(b*f^3 + 6*e^2*f + 2*e*f^2)*x
^2 - 2*(b*f^3 + 6*e^2*f + 2*e*f^2)*x)*sqrt(-6*b + 1))/(6*f*x^3 + 2*(3*e -
f)*x^2 + b*e + (b*f - 2*e)*x)) + (12*f*x^2 + (2*b + 1)*f - 2*(18*e + 5*f)*
x + sqrt(-6*b + 1)*(6*e + f) + 6*e)*sqrt(108*x^3 + 54*b*x - 54*x^2 + (6*b
- 1)*sqrt(-6*b + 1) - 9*b + 1))/(6*f^2*x^2 + b*f^2 - 2*f^2*x), -1/3*(6*(6*
x^2 + b - 2*x)*sqrt(-((6*b - 1)*sqrt(-6*b + 1)*f^3 - 54*b*e*f^2 - (9*b - 1
)*f^3 - 108*e^3 - 54*e^2*f)/f)*arctan(1/27*(18*(6*e + f)*x^2 + 3*(12*b - 1
)*e + (9*b - 1)*f - 3*((6*b + 1)*f + 12*e)*x + (18*f*x^2 + (6*b - 1)*f + 3
*(6*e - f)*x - 3*e)*sqrt(-6*b + 1))*sqrt(108*x^3 + 54*b*x - 54*x^2 + (6*b
- 1)*sqrt(-6*b + 1) - 9*b + 1)*sqrt(-((6*b - 1)*sqrt(-6*b + 1)*f^3 - 54*b*
e*f^2 - (9*b - 1)*f^3 - 108*e^3 - 54*e^2*f)/f)/(72*(b*f^2 + 6*e^2 + 2*e*f)
*x^4 - 48*(b*f^2 + 6*e^2 + 2*e*f)*x^3 + 6*(8*b^2 - b)*e^2 + 2*(8*b^2 - ...
```

Sympy [F]

$$\int \frac{\sqrt{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}}{e+fx}\,dx = \int \frac{\sqrt{54bx+6b\sqrt{1-6b}-9b+108x^3-54x^2-\sqrt{1}}}{e+fx}$$

input
```
integrate((1-(1-6*b)**(3/2)-9*b+54*b*x-54*x**2+108*x**3)**(1/2)/(f*x+e),x)
```

output
```
Integral(sqrt(54*b*x + 6*b*sqrt(1 - 6*b) - 9*b + 108*x**3 - 54*x**2 - sqrt
(1 - 6*b) + 1)/(e + f*x), x)
```

Maxima [F]

$$\int \frac{\sqrt{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}}{e+fx}\,dx = \int \frac{\sqrt{108\,x^3+54\,bx-54\,x^2-(-6\,b+1)^{\frac{3}{2}}-9\,b+}}{fx+e}$$

input
```
integrate((1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3)^(1/2)/(f*x+e),x, alg
orithm="maxima")
```

output
```
integrate(sqrt(108*x^3 + 54*b*x - 54*x^2 - (-6*b + 1)^(3/2) - 9*b + 1)/(f*
x + e), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}}{e+fx}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3)^(1/2)/(f*x+e),x, alg
orithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}}{e+fx}\,dx = \int \frac{\sqrt{54\,b\,x-9\,b-(1-6\,b)^{3/2}-54\,x^2+108\,x^3+}}{e+f\,x}$$

input
```
int((54*b*x - 9*b - (1 - 6*b)^(3/2) - 54*x^2 + 108*x^3 + 1)^(1/2)/(e + f*x
),x)
```

output
```
int((54*b*x - 9*b - (1 - 6*b)^(3/2) - 54*x^2 + 108*x^3 + 1)^(1/2)/(e + f*x
), x)
```

Reduce [F]

$$\int \frac{\sqrt{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}}{e+fx}\,dx=\int \frac{\sqrt{1-(-6b+1)^{\frac{3}{2}}-9b+54bx-54x^2+108x^3}}{fx+e}\,dx$$

input
```
int((1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3)^(1/2)/(f*x+e),x)
```

output
```
int((1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3)^(1/2)/(f*x+e),x)
```

3.154 $\int \frac{\sqrt{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}}{(e+fx)^2}\,dx$

Optimal result

Integrand size = 42, antiderivative size = 435

$$\int \frac{\sqrt{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}}{(e+fx)^2}\,dx = \frac{9\sqrt{2}\left(6e+f+\sqrt{1-6b}f\right)\sqrt{-2(1-6b)^{3/2}+3(1-}}{\sqrt{1-6b}f^2\left(6e+f+2\sqrt{1-6b}f\right)\left(\right.}$$

$$+\frac{\left(6e+f-\sqrt{1-6b}f\right)\left(2+\frac{1-6x}{\sqrt{1-6b}}\right)\sqrt{-2(1-6b)^{3/2}+3(1-6b)(1-6x)-(1-6x)^3}}{\sqrt{2}f\left(6e+f+2\sqrt{1-6b}f\right)\left(1-\frac{1-6x}{\sqrt{1-6b}}\right)(e+fx)}$$

$$-\frac{9\sqrt{2}\left(6e+f+\sqrt{1-6b}f\right)\sqrt{-2(1-6b)^{3/2}+3(1-6b)(1-6x)-(1-6x)^3}\operatorname{arctanh}\left(\frac{\sqrt[4]{1-6b}\sqrt{f}\sqrt{2+\frac{1-6}{\sqrt{1-}}}}{\sqrt{6e+f+2\sqrt{1-6b}f}}\right.}{(1-6b)^{3/4}f^{5/2}\sqrt{6e+2\left(\frac{1}{2}+\sqrt{1-6b}\right)f}\left(1-\frac{1-6x}{\sqrt{1-6b}}\right)\sqrt{2+\frac{1-6x}{\sqrt{1-6b}}}}$$

output
```
9*2^(1/2)*(6*e+f+(1-6*b)^(1/2)*f)*(-2*(1-6*b)^(3/2)+3*(1-6*b)*(1-6*x)-(1-6
*x)^3)^(1/2)/(1-6*b)^(1/2)/f^2/(6*e+f+2*(1-6*b)^(1/2)*f)/(1-(1-6*x)/(1-6*b
)^(1/2))+1/2*(6*e+f-(1-6*b)^(1/2)*f)*(2+(1-6*x)/(1-6*b)^(1/2))*(-2*(1-6*b)
^(3/2)+3*(1-6*b)*(1-6*x)-(1-6*x)^3)^(1/2)*2^(1/2)/f/(6*e+f+2*(1-6*b)^(1/2)
*f)/(1-(1-6*x)/(1-6*b)^(1/2))/(f*x+e)-9*2^(1/2)*(6*e+f+(1-6*b)^(1/2)*f)*(-
2*(1-6*b)^(3/2)+3*(1-6*b)*(1-6*x)-(1-6*x)^3)^(1/2)*arctanh((1-6*b)^(1/4)*f
^(1/2)*(2+(1-6*x)/(1-6*b)^(1/2))^(1/2)/(6*e+f+2*(1-6*b)^(1/2)*f)^(1/2))/(1
-6*b)^(3/4)/f^(5/2)/(6*e+2*(1/2+(1-6*b)^(1/2))*f)^(1/2)/(1-(1-6*x)/(1-6*b)
^(1/2))/(2+(1-6*x)/(1-6*b)^(1/2))^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 16.25 (sec) , antiderivative size = 5628, normalized size of antiderivative = 12.94

$$\int \frac{\sqrt{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}}{(e+fx)^2}\,dx = \text{Result too large to show}$$

input

```
Integrate[Sqrt[1 - (1 - 6*b)^(3/2) - 9*b + 54*b*x - 54*x^2 + 108*x^3]/(e +
 f*x)^2,x]
```

output

```
Result too large to show
```

Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 372, normalized size of antiderivative = 0.86, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2489, 27, 87, 60, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{54bx-(1-6b)^{3/2}-9b+108x^3-54x^2+1}}{(e+fx)^2}dx$$

$$\Big\downarrow \ 2489$$

$$-\frac{\sqrt{54bx-(1-6b)^{3/2}-9b+108x^3-54x^2+1}\int -\frac{157464\sqrt{2}\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)\sqrt{6(1-6b)x-\left(2\sqrt{1-6b}+1\right)(1-6b}}{(e+fx)^2}}{157464\sqrt{2}\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)\sqrt{6(1-6b)x-\left(2\sqrt{1-6b}+1\right)(1-6b)}}$$

$$\Big\downarrow \ 27$$

$$\frac{\sqrt{54bx-(1-6b)^{3/2}-9b+108x^3-54x^2+1}\int \frac{\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)\sqrt{6(1-6b)x-\left(2\sqrt{1-6b}+1\right)(1-6b)}}{(e+fx)^2}dx}{\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)\sqrt{6(1-6b)x-\left(2\sqrt{1-6b}+1\right)(1-6b)}}$$

$\Big\downarrow$ 87

$$\frac{\sqrt{54bx-(1-6b)^{3/2}-9b+108x^3-54x^2+1}\left(\frac{\left(6(1-6b)x-\left(2\sqrt{1-6b}+1\right)(1-6b)\right)^{3/2}\left(\left(1-\sqrt{1-6b}\right)f+6e\right)}{f\left(2\sqrt{1-6b}f+6e+f\right)(e+fx)}-\frac{9(1-6b)\left(\sqrt{1-6b}f+\cdots\right.}{\cdots}\right.}{\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)\sqrt{6(1-6b)x-\left(2\sqrt{1-6b}+1\right)(1-6b)\cdots}}$$

$\Big\downarrow$ 60

$$\frac{\sqrt{54bx-(1-6b)^{3/2}-9b+108x^3-54x^2+1}\left(\frac{\left(6(1-6b)x-\left(2\sqrt{1-6b}+1\right)(1-6b)\right)^{3/2}\left(\left(1-\sqrt{1-6b}\right)f+6e\right)}{f\left(2\sqrt{1-6b}f+6e+f\right)(e+fx)}-\frac{9(1-6b)\left(\sqrt{1-6b}f+\cdots\right.}{\cdots}\right.}{\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)\sqrt{6(1-6b)x\cdots}}$$

$\Big\downarrow$ 73

$$\frac{\sqrt{54bx-(1-6b)^{3/2}-9b+108x^3-54x^2+1}\left(\frac{\left(6(1-6b)x-\left(2\sqrt{1-6b}+1\right)(1-6b)\right)^{3/2}\left(\left(1-\sqrt{1-6b}\right)f+6e\right)}{f\left(2\sqrt{1-6b}f+6e+f\right)(e+fx)}-\frac{9(1-6b)\left(\sqrt{1-6b}f+\cdots\right.}{\cdots}\right.}{\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)\cdots}$$

$\Big\downarrow$ 218

$$\frac{\sqrt{54bx-(1-6b)^{3/2}-9b+108x^3-54x^2+1}\left(\frac{\left(6(1-6b)x-\left(2\sqrt{1-6b}+1\right)(1-6b)\right)^{3/2}\left(\left(1-\sqrt{1-6b}\right)f+6e\right)}{f\left(2\sqrt{1-6b}f+6e+f\right)(e+fx)}-\frac{9(1-6b)\left(\sqrt{1-6b}f+\cdots\right.}{\cdots}\right.}{\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)\sqrt{6(1-6b)x\cdots}}$$

input
```
Int[Sqrt[1 - (1 - 6*b)^(3/2) - 9*b + 54*b*x - 54*x^2 + 108*x^3]/(e + f*x)^
2,x]
```

output
```
(Sqrt[1 - (1 - 6*b)^(3/2) - 9*b + 54*b*x - 54*x^2 + 108*x^3]*(((6*e + (1 -
 Sqrt[1 - 6*b])*f)*(-((1 + 2*Sqrt[1 - 6*b])*(1 - 6*b)) + 6*(1 - 6*b)*x)^(3
/2))/(f*(6*e + f + 2*Sqrt[1 - 6*b]*f)*(e + f*x)) - (9*(1 - 6*b)*(6*e + f +
 Sqrt[1 - 6*b]*f)*((2*Sqrt[-((1 + 2*Sqrt[1 - 6*b])*(1 - 6*b)) + 6*(1 - 6*b
)*x])/f - (2*Sqrt[1 - 6*b]*Sqrt[6*e + f + 2*Sqrt[1 - 6*b]*f]*ArcTan[(Sqrt[
f]*Sqrt[-((1 + 2*Sqrt[1 - 6*b])*(1 - 6*b)) + 6*(1 - 6*b)*x])/(Sqrt[1 - 6*b
]*Sqrt[6*e + f + 2*Sqrt[1 - 6*b]*f])])/f^(3/2)))/(f*(6*e + f + 2*Sqrt[1 -
6*b]*f))))/(((1 - Sqrt[1 - 6*b])*(1 - 6*b) - 6*(1 - 6*b)*x)*Sqrt[-((1 + 2*
Sqrt[1 - 6*b])*(1 - 6*b)) + 6*(1 - 6*b)*x])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 2489
```
Int[((e_.) + (f_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(
x_)^3)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2 + d*x^3)^p/((c^3 - 4*b*c*d
+ 9*a*d^2 + d*(c^2 - 3*b*d)*x)^p*(b*c - 9*a*d + 2*(c^2 - 3*b*d)*x)^(2*p))
 Int[(e + f*x)^m*(c^3 - 4*b*c*d + 9*a*d^2 + d*(c^2 - 3*b*d)*x)^p*(b*c - 9*a
*d + 2*(c^2 - 3*b*d)*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x]
 && NeQ[c^2 - 3*b*d, 0] && EqQ[b^2*c^2 - 4*a*c^3 - 4*b^3*d + 18*a*b*c*d - 2
7*a^2*d^2, 0] &&  !IntegerQ[p]
```

Maple [F]

$$\int \frac{\sqrt{1-(1-6b)^{\frac{3}{2}}-9b+54bx-54x^2+108x^3}}{(fx+e)^2}dx$$

input
```
int((1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3)^(1/2)/(f*x+e)^2,x)
```

output
```
int((1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3)^(1/2)/(f*x+e)^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 1309, normalized size of antiderivative = 3.01

$$\int \frac{\sqrt{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}}{(e+fx)^2}\,dx = \text{Too large to display}$$

input
```
integrate((1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3)^(1/2)/(f*x+e)^2,x, a
lgorithm="fricas")
```

output
```
[(3*sqrt(3)*(6*f*x^3 + 2*(3*e - f)*x^2 + b*e + (b*f - 2*e)*x)*sqrt(-((6*b
- 1)*sqrt(-6*b + 1)*f^3 + 54*b*e*f^2 + (9*b - 1)*f^3 + 108*e^3 + 54*e^2*f)
/((8*b - 1)*f^3 + 12*e^2*f + 4*e*f^2))*log((9*b^2*f^3 + 162*b*e^3 + 108*b*
e^2*f + 9*(3*b^2 + 2*b)*e*f^2 - 162*(b*f^3 + 6*e^2*f + 2*e*f^2)*x^3 + 54*(
(3*b + 4)*e*f^2 + 2*b*f^3 + 18*e^3 + 18*e^2*f)*x^2 + sqrt(3)*(3*(2*b - 1)*
e*f^2 - b*f^3 - 6*e^2*f + ((12*b - 1)*f^3 + 36*e^2*f + 12*e*f^2)*x - (2*b*
f^3 + 6*e^2*f + 3*e*f^2 - (6*e*f^2 + f^3)*x)*sqrt(-6*b + 1))*sqrt(108*x^3
+ 54*b*x - 54*x^2 + (6*b - 1)*sqrt(-6*b + 1) - 9*b + 1)*sqrt(-((6*b - 1)*s
qrt(-6*b + 1)*f^3 + 54*b*e*f^2 + (9*b - 1)*f^3 + 108*e^3 + 54*e^2*f)/((8*b
 - 1)*f^3 + 12*e^2*f + 4*e*f^2)) - 9*(6*(3*b + 4)*e^2*f + 4*(3*b + 1)*e*f^
2 + (3*b^2 + 2*b)*f^3 + 36*e^3)*x + 18*(b^2*f^3 + 6*b*e^2*f + 2*b*e*f^2 +
6*(b*f^3 + 6*e^2*f + 2*e*f^2)*x^2 - 2*(b*f^3 + 6*e^2*f + 2*e*f^2)*x)*sqrt(
-6*b + 1))/(6*f*x^3 + 2*(3*e - f)*x^2 + b*e + (b*f - 2*e)*x)) + (12*f*x^2
- b*f + (18*e - f)*x - 3*(f*x + e)*sqrt(-6*b + 1) - 3*e)*sqrt(108*x^3 + 54
*b*x - 54*x^2 + (6*b - 1)*sqrt(-6*b + 1) - 9*b + 1))/(6*f^3*x^3 + b*e*f^2
+ 2*(3*e*f^2 - f^3)*x^2 + (b*f^3 - 2*e*f^2)*x), (6*sqrt(3)*(6*f*x^3 + 2*(3
*e - f)*x^2 + b*e + (b*f - 2*e)*x)*sqrt(((6*b - 1)*sqrt(-6*b + 1)*f^3 + 54
*b*e*f^2 + (9*b - 1)*f^3 + 108*e^3 + 54*e^2*f)/((8*b - 1)*f^3 + 12*e^2*f +
 4*e*f^2))*arctan(1/9*sqrt(3)*(2*(12*b - 1)*e^2 + (10*b - 1)*e*f + (8*b^2
- b)*f^2 + 2*((12*b - 1)*f^2 + 36*e^2 + 12*e*f)*x^2 - (6*(2*b + 1)*e*f ...
```

Sympy [F]

$$\int \frac{\sqrt{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}}{(e+fx)^2}\,dx = \int \frac{\sqrt{54bx+6b\sqrt{1-6b}-9b+108x^3-54x^2-\sqrt{1}}}{(e+fx)^2}$$

input
```
integrate((1-(1-6*b)**(3/2)-9*b+54*b*x-54*x**2+108*x**3)**(1/2)/(f*x+e)**2
,x)
```

output
```
Integral(sqrt(54*b*x + 6*b*sqrt(1 - 6*b) - 9*b + 108*x**3 - 54*x**2 - sqrt
(1 - 6*b) + 1)/(e + f*x)**2, x)
```

Maxima [F]

$$\int \frac{\sqrt{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}}{(e+fx)^2}\,dx = \int \frac{\sqrt{108\,x^3+54\,bx-54\,x^2-(-6\,b+1)^{\frac{3}{2}}-9\,b+}}{(fx+e)^2}$$

input
```
integrate((1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3)^(1/2)/(f*x+e)^2,x, a
lgorithm="maxima")
```

output
```
integrate(sqrt(108*x^3 + 54*b*x - 54*x^2 - (-6*b + 1)^(3/2) - 9*b + 1)/(f*
x + e)^2, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}}{(e+fx)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3)^(1/2)/(f*x+e)^2,x, a
lgorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}}{(e+fx)^2}\,dx = \int \frac{\sqrt{54\,b\,x-9\,b-(1-6\,b)^{3/2}-54\,x^2+108\,x^3+}}{(e+f\,x)^2}$$

input
```
int((54*b*x - 9*b - (1 - 6*b)^(3/2) - 54*x^2 + 108*x^3 + 1)^(1/2)/(e + f*x
)^2,x)
```

output
```
int((54*b*x - 9*b - (1 - 6*b)^(3/2) - 54*x^2 + 108*x^3 + 1)^(1/2)/(e + f*x
)^2, x)
```

Reduce [F]

$$\int \frac{\sqrt{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}}{(e+fx)^2}\,dx = \int \frac{\sqrt{1-(-6b+1)^{\frac{3}{2}}-9b+54bx-54x^2+108x^3}}{(fx+e)^2}\,dx$$

input
```
int((1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3)^(1/2)/(f*x+e)^2,x)
```

output
```
int((1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3)^(1/2)/(f*x+e)^2,x)
```

3.155 $\int \frac{(e+fx)^2}{\sqrt{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}}\,dx$

Optimal result

Integrand size = 42, antiderivative size = 374

$$\int \frac{(e+fx)^2}{\sqrt{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}}\,dx = -\frac{\sqrt{1-6b}f\left(f-6bf+2\sqrt{1-6b}(6e+f)\right)\left(1-\frac{1-6x}{\sqrt{1-6b}}\right)\left(2+\frac{1-6x}{\sqrt{1-6b}}\right)}{54\sqrt{2}\sqrt{-2(1-6b)^{3/2}+3(1-6b)(1-6x)-(1-6x)^3}} + \frac{(1-6b)^{3/2}f^2\left(1-\frac{1-6x}{\sqrt{1-6b}}\right)\left(2+\frac{1-6x}{\sqrt{1-6b}}\right)^2}{162\sqrt{2}\sqrt{-2(1-6b)^{3/2}+3(1-6b)(1-6x)-(1-6x)^3}} - \frac{\sqrt{1-6b}\left(18e^2+6\left(1-\sqrt{1-6b}\right)ef+\left(1-\sqrt{1-6b}-3b\right)f^2\right)\left(1-\frac{1-6x}{\sqrt{1-6b}}\right)\sqrt{2+\frac{1-6x}{\sqrt{1-6b}}}\operatorname{arctanh}\left(\frac{\sqrt{2+\frac{1-6x}{\sqrt{1-6b}}}}{\sqrt{3}}\right)}{27\sqrt{6}\sqrt{-2(1-6b)^{3/2}+3(1-6b)(1-6x)-(1-6x)^3}}$$

output
```
-1/108*(1-6*b)^(1/2)*f*(f-6*b*f+2*(1-6*b)^(1/2)*(6*e+f))*(1-(1-6*x)/(1-6*b
)^(1/2))*(2+(1-6*x)/(1-6*b)^(1/2))*2^(1/2)/(-2*(1-6*b)^(3/2)+3*(1-6*b)*(1-
6*x)-(1-6*x)^3)^(1/2)+1/324*(1-6*b)^(3/2)*f^2*(1-(1-6*x)/(1-6*b)^(1/2))*(2
+(1-6*x)/(1-6*b)^(1/2))^2*2^(1/2)/(-2*(1-6*b)^(3/2)+3*(1-6*b)*(1-6*x)-(1-6
*x)^3)^(1/2)-1/162*(1-6*b)^(1/2)*(18*e^2+6*(1-(1-6*b)^(1/2))*e*f+(1-(1-6*b
)^(1/2)-3*b)*f^2)*(1-(1-6*x)/(1-6*b)^(1/2))*(2+(1-6*x)/(1-6*b)^(1/2))^(1/2
)*arctanh(1/3*(2+(1-6*x)/(1-6*b)^(1/2))^(1/2)*3^(1/2))*6^(1/2)/(-2*(1-6*b)
^(3/2)+3*(1-6*b)*(1-6*x)-(1-6*x)^3)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 17.49 (sec) , antiderivative size = 1754, normalized size of antiderivative = 4.69

$$\int \frac{(e+fx)^2}{\sqrt{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}}\,dx = \text{Too large to display}$$

input
```
Integrate[(e + f*x)^2/Sqrt[1 - (1 - 6*b)^(3/2) - 9*b + 54*b*x - 54*x^2 + 1
08*x^3],x]
```

output
```
(((1 - Sqrt[1 - 6*b] - 54*x^2 + 108*x^3 + b*(-9 + 6*Sqrt[1 - 6*b] + 54*x))
*(6*e^2*(1 - Sqrt[1 - 6*b] - 6*b + 6*Sqrt[1 - 6*b]*x) + 12*e*f*((1 + Sqrt[
1 - 6*b])*x - b*(Sqrt[1 - 6*b] + 6*x)) - f^2*(-6*b^2 - 2*(1 + Sqrt[1 - 6*b
])*x + b*(1 + Sqrt[1 - 6*b] + 6*(2 + Sqrt[1 - 6*b])*x))))/((1 - 6*b)*(b +
2*x*(-1 + 3*x))) + ((1 - Sqrt[1 - 6*b] - 54*x^2 + 108*x^3 + b*(-9 + 6*Sqrt
[1 - 6*b] + 54*x))*(-6*e^2*(-1 + Sqrt[1 - 6*b] + 6*x) - 12*e*f*(-b + x + S
qrt[1 - 6*b]*x) + f^2*(b*(1 + 5*Sqrt[1 - 6*b] + 6*x) + 2*x*(-1 - 5*Sqrt[1
- 6*b] + 12*Sqrt[1 - 6*b]*x))))/(Sqrt[1 - 6*b]*(b + 2*x*(-1 + 3*x))) + 216
*(EllipticF[ArcSin[Sqrt[(-x + Root[1 - Sqrt[1 - 6*b] - 9*b + 6*Sqrt[1 - 6*
b]*b + 54*b*#1 - 54*#1^2 + 108*#1^3 & , 3])/(-Root[1 - Sqrt[1 - 6*b] - 9*b
 + 6*Sqrt[1 - 6*b]*b + 54*b*#1 - 54*#1^2 + 108*#1^3 & , 2] + Root[1 - Sqrt
[1 - 6*b] - 9*b + 6*Sqrt[1 - 6*b]*b + 54*b*#1 - 54*#1^2 + 108*#1^3 & , 3])
]], (Root[1 - Sqrt[1 - 6*b] - 9*b + 6*Sqrt[1 - 6*b]*b + 54*b*#1 - 54*#1^2
+ 108*#1^3 & , 2] - Root[1 - Sqrt[1 - 6*b] - 9*b + 6*Sqrt[1 - 6*b]*b + 54*
b*#1 - 54*#1^2 + 108*#1^3 & , 3])/(Root[1 - Sqrt[1 - 6*b] - 9*b + 6*Sqrt[1
 - 6*b]*b + 54*b*#1 - 54*#1^2 + 108*#1^3 & , 1] - Root[1 - Sqrt[1 - 6*b] -
 9*b + 6*Sqrt[1 - 6*b]*b + 54*b*#1 - 54*#1^2 + 108*#1^3 & , 3])]*(6*e^2 -
b*f^2 + 2*f*(6*e + f)*Root[1 - Sqrt[1 - 6*b] - 9*b + 6*Sqrt[1 - 6*b]*b + 5
4*b*#1 - 54*#1^2 + 108*#1^3 & , 1]) - 2*f*(6*e + f)*EllipticE[ArcSin[Sqrt[
(-x + Root[1 - Sqrt[1 - 6*b] - 9*b + 6*Sqrt[1 - 6*b]*b + 54*b*#1 - 54*#...
```

Rubi [A] (verified)

Time = 1.65 (sec) , antiderivative size = 349, normalized size of antiderivative = 0.93, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2489, 27, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2}{\sqrt{54bx-(1-6b)^{3/2}-9b+108x^3-54x^2+1}}dx$$

$$\Big\downarrow \text{2489}$$

$$-\frac{157464\sqrt{2}\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)\sqrt{6(1-6b)x-\left(2\sqrt{1-6b}+1\right)(1-6b)}\int -\frac{}{157464\sqrt{2}\left(\left(1-\sqrt{1-6\right.\right.}}}{\sqrt{54bx-(1-6b)^{3/2}-9b+108x^3-54x^2+1}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)\sqrt{6(1-6b)x-\left(2\sqrt{1-6b}+1\right)(1-6b)}\int \frac{(e+fx)^2}{\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)\sqrt{6(1-}}}{\sqrt{54bx-(1-6b)^{3/2}-9b+108x^3-54x^2+1}}$$

$$\Big\downarrow \text{99}$$

$$\frac{\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)\sqrt{6(1-6b)x-\left(2\sqrt{1-6b}+1\right)(1-6b)}\int \left(-\frac{\sqrt{6(1-6b)x-\left(2\sqrt{1-6b}+1\right)(1-6b)}}{36(6b-1)^2}\right.}{\sqrt{54bx-(1-6b)^{3/2}-9}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)\sqrt{6(1-6b)x-\left(2\sqrt{1-6b}+1\right)(1-6b)}\left(-\frac{\arctan\left(\frac{\sqrt{6(1-6b)x-\left(2\sqrt{1-6b}+1\right)(1-6b}}{\sqrt{3}(1-6b)^{3/4}}\right.}{}\right.}{\sqrt{54b}}$$

input
```
Int[(e + f*x)^2/Sqrt[1 - (1 - 6*b)^(3/2) - 9*b + 54*b*x - 54*x^2 + 108*x^3
],x]
```

output
```
(((1 - Sqrt[1 - 6*b])*(1 - 6*b) - 6*(1 - 6*b)*x)*Sqrt[-((1 + 2*Sqrt[1 - 6*
b])*(1 - 6*b)) + 6*(1 - 6*b)*x]*(-1/108*(f*(f - 6*b*f + 2*Sqrt[1 - 6*b]*(6
*e + f))*Sqrt[-((1 + 2*Sqrt[1 - 6*b])*(1 - 6*b)) + 6*(1 - 6*b)*x])/(1 - 6*
b)^(5/2) - (f^2*(-((1 + 2*Sqrt[1 - 6*b])*(1 - 6*b)) + 6*(1 - 6*b)*x)^(3/2)
)/(324*(1 - 6*b)^3) - ((18*Sqrt[1 - 6*b]*e^2 - 6*(1 - Sqrt[1 - 6*b] - 6*b)
*e*f - (1 - Sqrt[1 - 6*b] - 3*(2 - Sqrt[1 - 6*b])*b)*f^2)*ArcTan[Sqrt[-((1
 + 2*Sqrt[1 - 6*b])*(1 - 6*b)) + 6*(1 - 6*b)*x]/(Sqrt[3]*(1 - 6*b)^(3/4))]
)/(54*Sqrt[3]*(1 - 6*b)^(9/4))))/Sqrt[1 - (1 - 6*b)^(3/2) - 9*b + 54*b*x -
 54*x^2 + 108*x^3]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2489
```
Int[((e_.) + (f_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(
x_)^3)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2 + d*x^3)^p/((c^3 - 4*b*c*d
+ 9*a*d^2 + d*(c^2 - 3*b*d)*x)^p*(b*c - 9*a*d + 2*(c^2 - 3*b*d)*x)^(2*p))
 Int[(e + f*x)^m*(c^3 - 4*b*c*d + 9*a*d^2 + d*(c^2 - 3*b*d)*x)^p*(b*c - 9*a
*d + 2*(c^2 - 3*b*d)*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x]
 && NeQ[c^2 - 3*b*d, 0] && EqQ[b^2*c^2 - 4*a*c^3 - 4*b^3*d + 18*a*b*c*d - 2
7*a^2*d^2, 0] &&  !IntegerQ[p]
```

Maple [F]

$$\int \frac{(fx+e)^2}{\sqrt{1-(1-6b)^{\frac{3}{2}}-9b+54bx-54x^2+108x^3}}dx$$

input
```
int((f*x+e)^2/(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3)^(1/2),x)
```

output
```
int((f*x+e)^2/(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3)^(1/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 915 vs. $2(310) = 620$.

Time = 0.21 (sec) , antiderivative size = 2078, normalized size of antiderivative = 5.56

$$\int \frac{(e+fx)^2}{\sqrt{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2/(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3)^(1/2),x, a
lgorithm="fricas")
```

output
```
[1/162*(3*sqrt(1/6)*(6*x^2 + b - 2*x)*sqrt((216*(6*b - 1)*e^3*f + 108*(6*b
 - 1)*e^2*f^2 - 12*(18*b^2 - 15*b + 2)*e*f^3 - 2*(18*b^2 - 9*b + 1)*f^4 -
(108*(3*b - 1)*e^2*f^2 + 12*(9*b - 2)*e*f^3 - (9*b^2 - 12*b + 2)*f^4 - 324
*e^4 - 216*e^3*f)*sqrt(-6*b + 1))/(6*b - 1))*log((972*(5*b^2 - b)*e^4 + 64
8*(5*b^2 - b)*e^3*f + 108*(15*b^3 + 2*b^2 - b)*e^2*f^2 + 108*(5*b^3 - b^2)
*e*f^3 + 27*(5*b^4 - b^3)*f^4 - 972*(b^2*f^4 + 4*(3*b + 1)*e^2*f^2 + 4*b*e
*f^3 + 36*e^4 + 24*e^3*f)*x^4 + 648*(b^2*f^4 + 4*(3*b + 1)*e^2*f^2 + 4*b*e
*f^3 + 36*e^4 + 24*e^3*f)*x^3 + 54*(36*(12*b - 5)*e^4 + 24*(12*b - 5)*e^3*
f + 4*(36*b^2 - 3*b - 5)*e^2*f^2 + 4*(12*b^2 - 5*b)*e*f^3 + (12*b^3 - 5*b^
2)*f^4)*x^2 + 2*sqrt(1/6)*(18*(6*b - 1)*e^2 - 6*(18*b^2 - 15*b + 2)*e*f -
2*(18*b^2 - 9*b + 1)*f^2 + 18*(6*(6*b - 1)*e*f + (6*b - 1)*f^2)*x^2 - 6*(1
8*(6*b - 1)*e^2 + 12*(6*b - 1)*e*f - (18*b^2 - 15*b + 2)*f^2)*x + (18*(3*b
 - 1)*e^2 + 6*(9*b - 2)*e*f - (9*b^2 - 12*b + 2)*f^2 + 18*((3*b - 1)*f^2 -
 18*e^2 - 6*e*f)*x^2 - 6*(12*(3*b - 1)*e*f + (9*b - 2)*f^2 - 18*e^2)*x)*sq
rt(-6*b + 1))*sqrt(108*x^3 + 54*b*x - 54*x^2 + (6*b - 1)*sqrt(-6*b + 1) -
9*b + 1)*sqrt((216*(6*b - 1)*e^3*f + 108*(6*b - 1)*e^2*f^2 - 12*(18*b^2 -
15*b + 2)*e*f^3 - 2*(18*b^2 - 9*b + 1)*f^4 - (108*(3*b - 1)*e^2*f^2 + 12*(
9*b - 2)*e*f^3 - (9*b^2 - 12*b + 2)*f^4 - 324*e^4 - 216*e^3*f)*sqrt(-6*b +
 1))/(6*b - 1)) - 54*(36*(4*b - 1)*e^4 + 24*(4*b - 1)*e^3*f + 4*(12*b^2 +
b - 1)*e^2*f^2 + 4*(4*b^2 - b)*e*f^3 + (4*b^3 - b^2)*f^4)*x - 27*(b^3*f...
```

Sympy [F]

$$\int \frac{(e+fx)^2}{\sqrt{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}}\,dx=\int \frac{(e+fx)^2}{\sqrt{54bx+6b\sqrt{1-6b}-9b+108x^3-54x^2-\sqrt{1}}}$$

input
```
integrate((f*x+e)**2/(1-(1-6*b)**(3/2)-9*b+54*b*x-54*x**2+108*x**3)**(1/2)
,x)
```

output
```
Integral((e + f*x)**2/sqrt(54*b*x + 6*b*sqrt(1 - 6*b) - 9*b + 108*x**3 - 5
4*x**2 - sqrt(1 - 6*b) + 1), x)
```

Maxima [F]

$$\int \frac{(e+fx)^2}{\sqrt{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}}\,dx = \int \frac{(fx+e)^2}{\sqrt{108\,x^3+54\,bx-54\,x^2-(-6\,b+1)^{\frac{3}{2}}-9\,b+}}$$

input
```
integrate((f*x+e)^2/(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3)^(1/2),x, a
lgorithm="maxima")
```

output
```
integrate((f*x + e)^2/sqrt(108*x^3 + 54*b*x - 54*x^2 - (-6*b + 1)^(3/2) -
9*b + 1), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(e+fx)^2}{\sqrt{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((f*x+e)^2/(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3)^(1/2),x, a
lgorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2}{\sqrt{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}}\,dx = \int \frac{(e+f\,x)^2}{\sqrt{54\,b\,x-9\,b-(1-6\,b)^{3/2}-54\,x^2+108\,x^3+}}$$

input
```
int((e + f*x)^2/(54*b*x - 9*b - (1 - 6*b)^(3/2) - 54*x^2 + 108*x^3 + 1)^(1
/2),x)
```

output
```
int((e + f*x)^2/(54*b*x - 9*b - (1 - 6*b)^(3/2) - 54*x^2 + 108*x^3 + 1)^(1
/2), x)
```

Reduce [F]

$$\int \frac{(e+fx)^2}{\sqrt{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}}\,dx = \text{too large to display}$$

input
```
int((f*x+e)^2/(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3)^(1/2),x)
```

output
```
( - 36*sqrt(6*sqrt( - 6*b + 1)*b - sqrt( - 6*b + 1) + 54*b*x - 9*b + 108*x
**3 - 54*x**2 + 1)*sqrt( - 6*b + 1)*b*e*f - 6*sqrt(6*sqrt( - 6*b + 1)*b -
sqrt( - 6*b + 1) + 54*b*x - 9*b + 108*x**3 - 54*x**2 + 1)*sqrt( - 6*b + 1)
*b*f**2 + 6*sqrt(6*sqrt( - 6*b + 1)*b - sqrt( - 6*b + 1) + 54*b*x - 9*b +
108*x**3 - 54*x**2 + 1)*sqrt( - 6*b + 1)*e*f + sqrt(6*sqrt( - 6*b + 1)*b -
 sqrt( - 6*b + 1) + 54*b*x - 9*b + 108*x**3 - 54*x**2 + 1)*sqrt( - 6*b + 1
)*f**2 - 141087744*sqrt( - 6*b + 1)*int(sqrt(6*sqrt( - 6*b + 1)*b - sqrt(
- 6*b + 1) + 54*b*x - 9*b + 108*x**3 - 54*x**2 + 1)/(23079168*sqrt( - 6*b
+ 1)*b**8 + 311568768*sqrt( - 6*b + 1)*b**7*x**2 - 103856256*sqrt( - 6*b +
 1)*b**7*x - 4588992*sqrt( - 6*b + 1)*b**7 + 1246275072*sqrt( - 6*b + 1)*b
**6*x**4 - 830850048*sqrt( - 6*b + 1)*b**6*x**3 + 80850960*sqrt( - 6*b + 1
)*b**6*x**2 + 19208016*sqrt( - 6*b + 1)*b**6*x - 2027100*sqrt( - 6*b + 1)*
b**6 + 1246275072*sqrt( - 6*b + 1)*b**5*x**6 - 1246275072*sqrt( - 6*b + 1)
*b**5*x**5 + 219547584*sqrt( - 6*b + 1)*b**5*x**4 + 84426624*sqrt( - 6*b +
 1)*b**5*x**3 - 49449528*sqrt( - 6*b + 1)*b**5*x**2 + 9228456*sqrt( - 6*b
+ 1)*b**5*x + 814462*sqrt( - 6*b + 1)*b**5 - 92021184*sqrt( - 6*b + 1)*b**
4*x**6 + 92021184*sqrt( - 6*b + 1)*b**4*x**5 - 143971344*sqrt( - 6*b + 1)*
b**4*x**4 + 78939936*sqrt( - 6*b + 1)*b**4*x**3 - 2013408*sqrt( - 6*b + 1)
*b**4*x**2 - 3525072*sqrt( - 6*b + 1)*b**4*x - 111254*sqrt( - 6*b + 1)*b**
4 - 120966048*sqrt( - 6*b + 1)*b**3*x**6 + 120966048*sqrt( - 6*b + 1)*b...
```

3.156 $\int \frac{e+fx}{\sqrt{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 224

$$\int \frac{e+fx}{\sqrt{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}}\,dx = -\frac{(1-6b)f\left(1-\frac{1-6x}{\sqrt{1-6b}}\right)\left(2+\frac{1-6x}{\sqrt{1-6b}}\right)}{9\sqrt{2}\sqrt{-2(1-6b)^{3/2}+3(1-6b)(1-6x)-(1-6x)^3}} + \frac{(1-6b)\left(f-\frac{6e+f}{\sqrt{1-6b}}\right)\left(1-\frac{1-6x}{\sqrt{1-6b}}\right)\sqrt{2+\frac{1-6x}{\sqrt{1-6b}}}\,\text{arctanh}\left(\frac{\sqrt{2+\frac{1-6x}{\sqrt{1-6b}}}}{\sqrt{3}}\right)}{9\sqrt{6}\sqrt{-2(1-6b)^{3/2}+3(1-6b)(1-6x)-(1-6x)^3}}$$

output
```
-1/18*(1-6*b)*f*(1-(1-6*x)/(1-6*b)^(1/2))*(2+(1-6*x)/(1-6*b)^(1/2))*2^(1/2
)/(-2*(1-6*b)^(3/2)+3*(1-6*b)*(1-6*x)-(1-6*x)^3)^(1/2)+1/54*(1-6*b)*(f-(6*
e+f)/(1-6*b)^(1/2))*(1-(1-6*x)/(1-6*b)^(1/2))*(2+(1-6*x)/(1-6*b)^(1/2))^(1
/2)*arctanh(1/3*(2+(1-6*x)/(1-6*b)^(1/2))^(1/2)*3^(1/2))*6^(1/2)/(-2*(1-6*
b)^(3/2)+3*(1-6*b)*(1-6*x)-(1-6*x)^3)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 11.91 (sec) , antiderivative size = 1497, normalized size of antiderivative = 6.68

$$\int \frac{e+fx}{\sqrt{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}}\,dx = \text{Too large to display}$$

input

```
Integrate[(e + f*x)/Sqrt[1 - (1 - 6*b)^(3/2) - 9*b + 54*b*x - 54*x^2 + 108
*x^3],x]
```

output

```
(2*(x - Root[1 - Sqrt[1 - 6*b] - 9*b + 6*Sqrt[1 - 6*b]*b + 54*b*#1 - 54*#1
^2 + 108*#1^3 & , 2])*(x - Root[1 - Sqrt[1 - 6*b] - 9*b + 6*Sqrt[1 - 6*b]*
b + 54*b*#1 - 54*#1^2 + 108*#1^3 & , 3])*Sqrt[(-x + Root[1 - Sqrt[1 - 6*b]
 - 9*b + 6*Sqrt[1 - 6*b]*b + 54*b*#1 - 54*#1^2 + 108*#1^3 & , 1])/(Root[1
- Sqrt[1 - 6*b] - 9*b + 6*Sqrt[1 - 6*b]*b + 54*b*#1 - 54*#1^2 + 108*#1^3 &
 , 1] - Root[1 - Sqrt[1 - 6*b] - 9*b + 6*Sqrt[1 - 6*b]*b + 54*b*#1 - 54*#1
^2 + 108*#1^3 & , 3])]*(EllipticF[ArcSin[Sqrt[(-x + Root[1 - Sqrt[1 - 6*b]
 - 9*b + 6*Sqrt[1 - 6*b]*b + 54*b*#1 - 54*#1^2 + 108*#1^3 & , 3])/(-Root[1
 - Sqrt[1 - 6*b] - 9*b + 6*Sqrt[1 - 6*b]*b + 54*b*#1 - 54*#1^2 + 108*#1^3
& , 2] + Root[1 - Sqrt[1 - 6*b] - 9*b + 6*Sqrt[1 - 6*b]*b + 54*b*#1 - 54*#
1^2 + 108*#1^3 & , 3])]], (Root[1 - Sqrt[1 - 6*b] - 9*b + 6*Sqrt[1 - 6*b]*
b + 54*b*#1 - 54*#1^2 + 108*#1^3 & , 2] - Root[1 - Sqrt[1 - 6*b] - 9*b + 6
*Sqrt[1 - 6*b]*b + 54*b*#1 - 54*#1^2 + 108*#1^3 & , 3])/(Root[1 - Sqrt[1 -
 6*b] - 9*b + 6*Sqrt[1 - 6*b]*b + 54*b*#1 - 54*#1^2 + 108*#1^3 & , 1] - Ro
ot[1 - Sqrt[1 - 6*b] - 9*b + 6*Sqrt[1 - 6*b]*b + 54*b*#1 - 54*#1^2 + 108*#
1^3 & , 3])]*(e + f*Root[1 - Sqrt[1 - 6*b] - 9*b + 6*Sqrt[1 - 6*b]*b + 54*
b*#1 - 54*#1^2 + 108*#1^3 & , 1]) + f*EllipticE[ArcSin[Sqrt[(-x + Root[1 -
 Sqrt[1 - 6*b] - 9*b + 6*Sqrt[1 - 6*b]*b + 54*b*#1 - 54*#1^2 + 108*#1^3 &
, 3])/(-Root[1 - Sqrt[1 - 6*b] - 9*b + 6*Sqrt[1 - 6*b]*b + 54*b*#1 - 54*#1
^2 + 108*#1^3 & , 2] + Root[1 - Sqrt[1 - 6*b] - 9*b + 6*Sqrt[1 - 6*b]*b...
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2489, 27, 90, 73, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e+fx}{\sqrt{54bx-(1-6b)^{3/2}-9b+108x^3-54x^2+1}}dx$$

$$\Big\downarrow \; 2489$$

$$-\frac{157464\sqrt{2}\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)\sqrt{6(1-6b)x-\left(2\sqrt{1-6b}+1\right)(1-6b)}\int -\frac{}{157464\sqrt{2}\left(\left(1-\sqrt{1-6\right.\right.}}}{\sqrt{54bx-(1-6b)^{3/2}-9b+108x^3-54x^2+1}}$$

$$\Big\downarrow \; 27$$

$$\frac{\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)\sqrt{6(1-6b)x-\left(2\sqrt{1-6b}+1\right)(1-6b)}\int \frac{e+fx}{\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)\sqrt{6(1-}}}{\sqrt{54bx-(1-6b)^{3/2}-9b+108x^3-54x^2+1}}$$

$$\Big\downarrow \; 90$$

$$\frac{\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)\sqrt{6(1-6b)x-\left(2\sqrt{1-6b}+1\right)(1-6b)}\left(\frac{1}{6}\left(-\sqrt{1-6b}f+6e+f\right)\int \frac{}{((1-}\right.}{\sqrt{54bx-(1-6b)^{3/2}-9b+108x^3-}}$$

$$\Big\downarrow \; 73$$

$$\frac{\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)\sqrt{6(1-6b)x-\left(2\sqrt{1-6b}+1\right)(1-6b)}\left(\frac{\left(-\sqrt{1-6b}f+6e+f\right)\int \frac{}{-3(1-6b)^{3/2}+(2\sqrt{}}}{}\right.}{\sqrt{54bx-(1-6b)^{3/2}-9b+108x^3-54}}$$

$$\Big\downarrow \; 217$$

$$\frac{\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)\sqrt{6(1-6b)x-\left(2\sqrt{1-6b}+1\right)(1-6b)}\left(-\frac{\arctan\left(\frac{\sqrt{6(1-6b)x-\left(2\sqrt{1-6b}+1\right)(1-6b}}{\sqrt{3}(1-6b)^{3/4}}\right.}{18\sqrt{3}(1-6b)^{7/4}}\right.}{\sqrt{54bx-(1-6b)^{3/2}-9b+108x^3-54x^2+1}}$$

input
```
Int[(e + f*x)/Sqrt[1 - (1 - 6*b)^(3/2) - 9*b + 54*b*x - 54*x^2 + 108*x^3],
x]
```

output
```
(((1 - Sqrt[1 - 6*b])*(1 - 6*b) - 6*(1 - 6*b)*x)*Sqrt[-((1 + 2*Sqrt[1 - 6*
b])*(1 - 6*b)) + 6*(1 - 6*b)*x]*(-1/18*(f*Sqrt[-((1 + 2*Sqrt[1 - 6*b])*(1
- 6*b)) + 6*(1 - 6*b)*x])/(1 - 6*b)^2 - ((6*e + f - Sqrt[1 - 6*b]*f)*ArcTa
n[Sqrt[-((1 + 2*Sqrt[1 - 6*b])*(1 - 6*b)) + 6*(1 - 6*b)*x]/(Sqrt[3]*(1 - 6
*b)^(3/4))])/(18*Sqrt[3]*(1 - 6*b)^(7/4))))/Sqrt[1 - (1 - 6*b)^(3/2) - 9*b
 + 54*b*x - 54*x^2 + 108*x^3]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 2489
```
Int[((e_.) + (f_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(
x_)^3)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2 + d*x^3)^p/((c^3 - 4*b*c*d
+ 9*a*d^2 + d*(c^2 - 3*b*d)*x)^p*(b*c - 9*a*d + 2*(c^2 - 3*b*d)*x)^(2*p))
 Int[(e + f*x)^m*(c^3 - 4*b*c*d + 9*a*d^2 + d*(c^2 - 3*b*d)*x)^p*(b*c - 9*a
*d + 2*(c^2 - 3*b*d)*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x]
 && NeQ[c^2 - 3*b*d, 0] && EqQ[b^2*c^2 - 4*a*c^3 - 4*b^3*d + 18*a*b*c*d - 2
7*a^2*d^2, 0] &&  !IntegerQ[p]
```

Maple [F]

$$\int \frac{fx+e}{\sqrt{1-(1-6b)^{\frac{3}{2}}-9b+54bx-54x^2+108x^3}}dx$$

input
```
int((f*x+e)/(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3)^(1/2),x)
```

output
```
int((f*x+e)/(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3)^(1/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 498 vs. $2(187) = 374$.

Time = 0.16 (sec) , antiderivative size = 1130, normalized size of antiderivative = 5.04

$$\int \frac{e+fx}{\sqrt{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)/(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3)^(1/2),x, alg
orithm="fricas")
```

output
```
[1/54*(3*sqrt(1/3)*(6*x^2 + b - 2*x)*sqrt((6*(6*b - 1)*e*f + (6*b - 1)*f^2
 - ((3*b - 1)*f^2 - 18*e^2 - 6*e*f)*sqrt(-6*b + 1))/(6*b - 1))*log(-(324*(
b*f^2 + 6*e^2 + 2*e*f)*x^4 - 216*(b*f^2 + 6*e^2 + 2*e*f)*x^3 - 54*(5*b^2 -
 b)*e^2 - 18*(5*b^2 - b)*e*f - 9*(5*b^3 - b^2)*f^2 - 18*(6*(12*b - 5)*e^2
+ 2*(12*b - 5)*e*f + (12*b^2 - 5*b)*f^2)*x^2 - sqrt(1/3)*(18*(6*b - 1)*f*x
^2 + 6*(6*b - 1)*e - (18*b^2 - 15*b + 2)*f - 12*(3*(6*b - 1)*e + (6*b - 1)
*f)*x - (18*(6*e + f)*x^2 - 6*(3*b - 1)*e - (9*b - 2)*f + 12*((3*b - 1)*f
- 3*e)*x)*sqrt(-6*b + 1))*sqrt(108*x^3 + 54*b*x - 54*x^2 + (6*b - 1)*sqrt(
-6*b + 1) - 9*b + 1)*sqrt((6*(6*b - 1)*e*f + (6*b - 1)*f^2 - ((3*b - 1)*f^
2 - 18*e^2 - 6*e*f)*sqrt(-6*b + 1))/(6*b - 1)) + 18*(6*(4*b - 1)*e^2 + 2*(
4*b - 1)*e*f + (4*b^2 - b)*f^2)*x + 9*(b^2*f^2 - 36*(b*f^2 + 6*e^2 + 2*e*f
)*x^3 + 6*b*e^2 + 2*b*e*f + 18*(b*f^2 + 6*e^2 + 2*e*f)*x^2 - 2*(6*(3*b + 1
)*e^2 + 2*(3*b + 1)*e*f + (3*b^2 + b)*f^2)*x)*sqrt(-6*b + 1))/(36*x^4 + 4*
(3*b + 1)*x^2 - 24*x^3 + b^2 - 4*b*x)) + sqrt(108*x^3 + 54*b*x - 54*x^2 +
(6*b - 1)*sqrt(-6*b + 1) - 9*b + 1)*(6*f*x - sqrt(-6*b + 1)*f - f))/(6*x^2
 + b - 2*x), -1/54*(6*sqrt(1/3)*(6*x^2 + b - 2*x)*sqrt(-(6*(6*b - 1)*e*f +
 (6*b - 1)*f^2 - ((3*b - 1)*f^2 - 18*e^2 - 6*e*f)*sqrt(-6*b + 1))/(6*b - 1
))*arctan(-1/9*sqrt(1/3)*(18*(6*b - 1)*f*x^2 - 3*(6*b - 1)*e + (36*b^2 - 1
2*b + 1)*f + 3*(6*(6*b - 1)*e - (6*b - 1)*f)*x - (18*(6*e + f)*x^2 + 3*(12
*b - 1)*e + (9*b - 1)*f - 3*((6*b + 1)*f + 12*e)*x)*sqrt(-6*b + 1))*sqr...
```

Sympy [F]

$$\int \frac{e+fx}{\sqrt{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}}\,dx = \int \frac{e+fx}{\sqrt{54bx+6b\sqrt{1-6b}-9b+108x^3-54x^2-\sqrt{1}$$

input
```
integrate((f*x+e)/(1-(1-6*b)**(3/2)-9*b+54*b*x-54*x**2+108*x**3)**(1/2),x)
```

output
```
Integral((e + f*x)/sqrt(54*b*x + 6*b*sqrt(1 - 6*b) - 9*b + 108*x**3 - 54*x
**2 - sqrt(1 - 6*b) + 1), x)
```

Maxima [F]

$$\int \frac{e+fx}{\sqrt{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}}\,dx = \int \frac{fx+e}{\sqrt{108\,x^3+54\,bx-54\,x^2-(-6\,b+1)^{\frac{3}{2}}-9\,b+}}$$

input
```
integrate((f*x+e)/(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3)^(1/2),x, alg
orithm="maxima")
```

output
```
integrate((f*x + e)/sqrt(108*x^3 + 54*b*x - 54*x^2 - (-6*b + 1)^(3/2) - 9*
b + 1), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e+fx}{\sqrt{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((f*x+e)/(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3)^(1/2),x, alg
orithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e+fx}{\sqrt{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}}\,dx = \int \frac{e+f\,x}{\sqrt{54\,b\,x-9\,b-(1-6\,b)^{3/2}-54\,x^2+108\,x^3+}}$$

input
```
int((e + f*x)/(54*b*x - 9*b - (1 - 6*b)^(3/2) - 54*x^2 + 108*x^3 + 1)^(1/2
),x)
```

output
```
int((e + f*x)/(54*b*x - 9*b - (1 - 6*b)^(3/2) - 54*x^2 + 108*x^3 + 1)^(1/2
), x)
```

Reduce [F]

$$\int \frac{e+fx}{\sqrt{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}}\,dx = \text{too large to display}$$

input
```
int((f*x+e)/(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3)^(1/2),x)
```

output
```
( - 6*sqrt(6*sqrt( - 6*b + 1)*b - sqrt( - 6*b + 1) + 54*b*x - 9*b + 108*x*
*3 - 54*x**2 + 1)*sqrt( - 6*b + 1)*b*f + sqrt(6*sqrt( - 6*b + 1)*b - sqrt(
 - 6*b + 1) + 54*b*x - 9*b + 108*x**3 - 54*x**2 + 1)*sqrt( - 6*b + 1)*f +
282175488*sqrt( - 6*b + 1)*int(sqrt(6*sqrt( - 6*b + 1)*b - sqrt( - 6*b + 1
) + 54*b*x - 9*b + 108*x**3 - 54*x**2 + 1)/(23079168*sqrt( - 6*b + 1)*b**8
 + 311568768*sqrt( - 6*b + 1)*b**7*x**2 - 103856256*sqrt( - 6*b + 1)*b**7*
x - 4588992*sqrt( - 6*b + 1)*b**7 + 1246275072*sqrt( - 6*b + 1)*b**6*x**4
- 830850048*sqrt( - 6*b + 1)*b**6*x**3 + 80850960*sqrt( - 6*b + 1)*b**6*x*
*2 + 19208016*sqrt( - 6*b + 1)*b**6*x - 2027100*sqrt( - 6*b + 1)*b**6 + 12
46275072*sqrt( - 6*b + 1)*b**5*x**6 - 1246275072*sqrt( - 6*b + 1)*b**5*x**
5 + 219547584*sqrt( - 6*b + 1)*b**5*x**4 + 84426624*sqrt( - 6*b + 1)*b**5*
x**3 - 49449528*sqrt( - 6*b + 1)*b**5*x**2 + 9228456*sqrt( - 6*b + 1)*b**5
*x + 814462*sqrt( - 6*b + 1)*b**5 - 92021184*sqrt( - 6*b + 1)*b**4*x**6 +
92021184*sqrt( - 6*b + 1)*b**4*x**5 - 143971344*sqrt( - 6*b + 1)*b**4*x**4
 + 78939936*sqrt( - 6*b + 1)*b**4*x**3 - 2013408*sqrt( - 6*b + 1)*b**4*x**
2 - 3525072*sqrt( - 6*b + 1)*b**4*x - 111254*sqrt( - 6*b + 1)*b**4 - 12096
6048*sqrt( - 6*b + 1)*b**3*x**6 + 120966048*sqrt( - 6*b + 1)*b**3*x**5 - 1
381320*sqrt( - 6*b + 1)*b**3*x**4 - 21480240*sqrt( - 6*b + 1)*b**3*x**3 +
2925024*sqrt( - 6*b + 1)*b**3*x**2 + 467240*sqrt( - 6*b + 1)*b**3*x + 6836
*sqrt( - 6*b + 1)*b**3 + 28860192*sqrt( - 6*b + 1)*b**2*x**6 - 28860192...
```

3.157 $\int \frac{1}{\sqrt{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 124

$$\int \frac{1}{\sqrt{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}}\,dx = -\frac{\sqrt{\frac{2}{3}}\sqrt{1-6b}\left(1-\frac{1-6x}{\sqrt{1-6b}}\right)\sqrt{2+\frac{1-6x}{\sqrt{1-6b}}}\operatorname{arctanh}\left(\frac{\sqrt{2+\frac{1-6x}{\sqrt{1-6b}}}}{\sqrt{3}}\right)}{3\sqrt{-2(1-6b)^{3/2}+3(1-6b)(1-6x)-(1-6x)^3}}$$

output
```
-1/9*6^(1/2)*(1-6*b)^(1/2)*(1-(1-6*x)/(1-6*b)^(1/2))*(2+(1-6*x)/(1-6*b)^(1
/2))^(1/2)*arctanh(1/3*(2+(1-6*x)/(1-6*b)^(1/2))^(1/2)*3^(1/2))/(-2*(1-6*b
)^(3/2)+3*(1-6*b)*(1-6*x)-(1-6*x)^3)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.42 (sec) , antiderivative size = 934, normalized size of antiderivative = 7.53

$$\int \frac{1}{\sqrt{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}}\,dx = \text{Too large to display}$$

input
```
Integrate[1/Sqrt[1 - (1 - 6*b)^(3/2) - 9*b + 54*b*x - 54*x^2 + 108*x^3],x]
```

output
```
(2*EllipticF[ArcSin[Sqrt[(-x + Root[1 - Sqrt[1 - 6*b] - 9*b + 6*Sqrt[1 - 6
*b]*b + 54*b*#1 - 54*#1^2 + 108*#1^3 & , 3])/(-Root[1 - Sqrt[1 - 6*b] - 9*
b + 6*Sqrt[1 - 6*b]*b + 54*b*#1 - 54*#1^2 + 108*#1^3 & , 2] + Root[1 - Sqr
t[1 - 6*b] - 9*b + 6*Sqrt[1 - 6*b]*b + 54*b*#1 - 54*#1^2 + 108*#1^3 & , 3]
)]], (Root[1 - Sqrt[1 - 6*b] - 9*b + 6*Sqrt[1 - 6*b]*b + 54*b*#1 - 54*#1^2
 + 108*#1^3 & , 2] - Root[1 - Sqrt[1 - 6*b] - 9*b + 6*Sqrt[1 - 6*b]*b + 54
*b*#1 - 54*#1^2 + 108*#1^3 & , 3])/(Root[1 - Sqrt[1 - 6*b] - 9*b + 6*Sqrt[
1 - 6*b]*b + 54*b*#1 - 54*#1^2 + 108*#1^3 & , 1] - Root[1 - Sqrt[1 - 6*b]
- 9*b + 6*Sqrt[1 - 6*b]*b + 54*b*#1 - 54*#1^2 + 108*#1^3 & , 3])]*(x - Roo
t[1 - Sqrt[1 - 6*b] - 9*b + 6*Sqrt[1 - 6*b]*b + 54*b*#1 - 54*#1^2 + 108*#1
^3 & , 3])*Sqrt[(-x + Root[1 - Sqrt[1 - 6*b] - 9*b + 6*Sqrt[1 - 6*b]*b + 5
4*b*#1 - 54*#1^2 + 108*#1^3 & , 1])/(Root[1 - Sqrt[1 - 6*b] - 9*b + 6*Sqrt
[1 - 6*b]*b + 54*b*#1 - 54*#1^2 + 108*#1^3 & , 1] - Root[1 - Sqrt[1 - 6*b]
 - 9*b + 6*Sqrt[1 - 6*b]*b + 54*b*#1 - 54*#1^2 + 108*#1^3 & , 3])]*Sqrt[(-
x + Root[1 - Sqrt[1 - 6*b] - 9*b + 6*Sqrt[1 - 6*b]*b + 54*b*#1 - 54*#1^2 +
 108*#1^3 & , 2])/(Root[1 - Sqrt[1 - 6*b] - 9*b + 6*Sqrt[1 - 6*b]*b + 54*b
*#1 - 54*#1^2 + 108*#1^3 & , 2] - Root[1 - Sqrt[1 - 6*b] - 9*b + 6*Sqrt[1
- 6*b]*b + 54*b*#1 - 54*#1^2 + 108*#1^3 & , 3])])/(Sqrt[1 - Sqrt[1 - 6*b]
- 54*x^2 + 108*x^3 + b*(-9 + 6*Sqrt[1 - 6*b] + 54*x)]*Sqrt[(x - Root[1 - S
qrt[1 - 6*b] - 9*b + 6*Sqrt[1 - 6*b]*b + 54*b*#1 - 54*#1^2 + 108*#1^3 &...
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.31, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2480, 27, 73, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{54bx-(1-6b)^{3/2}-9b+108x^3-54x^2+1}}dx$$

$$\Big\downarrow \text{2480}$$

$$-\frac{157464\sqrt{2}\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)\sqrt{6(1-6b)x-\left(2\sqrt{1-6b}+1\right)(1-6b)}\int -\frac{}{157464\sqrt{2}\left(\left(1-\sqrt{1-6\right.\right.}}}{\sqrt{54bx-(1-6b)^{3/2}-9b+108x^3-54x^2+1}}$$

$\Big\downarrow$ 27

$$\frac{\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)\sqrt{6(1-6b)x-\left(2\sqrt{1-6b}+1\right)(1-6b)}\int \frac{1}{\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)\sqrt{6(1-}}}{\sqrt{54bx-(1-6b)^{3/2}-9b+108x^3-54x^2+1}}$$

$\Big\downarrow$ 73

$$\frac{\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)\sqrt{6(1-6b)x-\left(2\sqrt{1-6b}+1\right)(1-6b)}\int \frac{1}{-3(1-6b)^{3/2}+\left(2\sqrt{1-6b}+1\right)(1-6b)-6x}}{3(1-6b)\sqrt{54bx-(1-6b)^{3/2}-9b+108x^3-54x^2+1}}$$

$\Big\downarrow$ 217

$$-\frac{\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)\sqrt{6(1-6b)x-\left(2\sqrt{1-6b}+1\right)(1-6b)}\arctan\left(\frac{\sqrt{6(1-6b)x-\left(2\sqrt{1-6b}+1\right)(}}{\sqrt{3}(1-6b)^{3/4}}\right.}{3\sqrt{3}(1-6b)^{7/4}\sqrt{54bx-(1-6b)^{3/2}-9b+108x^3-54x^2+1}}$$

input
```
Int[1/Sqrt[1 - (1 - 6*b)^(3/2) - 9*b + 54*b*x - 54*x^2 + 108*x^3],x]
```

output
```
-1/3*(((1 - Sqrt[1 - 6*b])*(1 - 6*b) - 6*(1 - 6*b)*x)*Sqrt[-((1 + 2*Sqrt[1
 - 6*b])*(1 - 6*b)) + 6*(1 - 6*b)*x]*ArcTan[Sqrt[-((1 + 2*Sqrt[1 - 6*b])*(
1 - 6*b)) + 6*(1 - 6*b)*x]/(Sqrt[3]*(1 - 6*b)^(3/4))])/(Sqrt[3]*(1 - 6*b)^
(7/4)*Sqrt[1 - (1 - 6*b)^(3/2) - 9*b + 54*b*x - 54*x^2 + 108*x^3])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 2480
```
Int[(Px_)^(p_), x_Symbol] :> With[{a = Coeff[Px, x, 0], b = Coeff[Px, x, 1]
, c = Coeff[Px, x, 2], d = Coeff[Px, x, 3]}, Simp[Px^p/((c^3 - 4*b*c*d + 9*
a*d^2 + d*(c^2 - 3*b*d)*x)^p*(b*c - 9*a*d + 2*(c^2 - 3*b*d)*x)^(2*p))   Int
[(c^3 - 4*b*c*d + 9*a*d^2 + d*(c^2 - 3*b*d)*x)^p*(b*c - 9*a*d + 2*(c^2 - 3*
b*d)*x)^(2*p), x], x] /; EqQ[b^2*c^2 - 4*a*c^3 - 4*b^3*d + 18*a*b*c*d - 27*
a^2*d^2, 0] && NeQ[c^2 - 3*b*d, 0]] /; FreeQ[p, x] && PolyQ[Px, x, 3] &&  !
IntegerQ[p]
```

Maple [F]

$$\int \frac{1}{\sqrt{1-(1-6b)^{\frac{3}{2}}-9b+54bx-54x^2+108x^3}}dx$$

input
```
int(1/(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3)^(1/2),x)
```

output
```
int(1/(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 368, normalized size of antiderivative = 2.97

$$\int \frac{1}{\sqrt{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}}\,dx = \left[\frac{1}{6}\sqrt{\frac{2}{3}}\sqrt{\frac{\sqrt{-6\,b+1}}{6\,b-1}}\log\left(-\frac{108\,x^4-6\,(12\,b-5)x}{\quad}\right.\right.$$

$$-\frac{1}{3}\sqrt{\frac{2}{3}}\sqrt{-\frac{\sqrt{-6\,b+1}}{6\,b-1}}\arctan\left(-\frac{\sqrt{\frac{2}{3}}\sqrt{108\,x^3+54\,bx-54\,x^2+(6\,b-1)\sqrt{-6\,b+1}-9\,b+1}\,(6\,(6\,b-1)}{6\,(72\,x^4+2\,(30\,b+1)x^2-48\,x^3+}\right.$$

input
```
integrate(1/(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3)^(1/2),x, algorithm
="fricas")
```

output
```
[1/6*sqrt(2/3)*sqrt(sqrt(-6*b + 1)/(6*b - 1))*log(-(108*x^4 - 6*(12*b - 5)
*x^2 - 72*x^3 + sqrt(2/3)*sqrt(108*x^3 + 54*b*x - 54*x^2 + (6*b - 1)*sqrt(
-6*b + 1) - 9*b + 1)*(6*(6*b - 1)*x + (18*x^2 - 3*b - 6*x + 1)*sqrt(-6*b +
 1) - 6*b + 1)*sqrt(sqrt(-6*b + 1)/(6*b - 1)) - 15*b^2 + 6*(4*b - 1)*x - 3
*(36*x^3 + 2*(3*b + 1)*x - 18*x^2 - b)*sqrt(-6*b + 1) + 3*b)/(36*x^4 + 4*(
3*b + 1)*x^2 - 24*x^3 + b^2 - 4*b*x)), -1/3*sqrt(2/3)*sqrt(-sqrt(-6*b + 1)
/(6*b - 1))*arctan(-1/6*sqrt(2/3)*sqrt(108*x^3 + 54*b*x - 54*x^2 + (6*b -
1)*sqrt(-6*b + 1) - 9*b + 1)*(6*(6*b - 1)*x - (36*x^2 + 12*b - 12*x - 1)*s
qrt(-6*b + 1) - 6*b + 1)*sqrt(-sqrt(-6*b + 1)/(6*b - 1))/(72*x^4 + 2*(30*b
 + 1)*x^2 - 48*x^3 + 8*b^2 - 2*(10*b - 1)*x - b))]
```

Sympy [F]

$$\int \frac{1}{\sqrt{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}}\,dx = \int \frac{1}{\sqrt{54bx-9b+108x^3-54x^2-(1-6b)^{\frac{3}{2}}+1}}\,dx$$

input
```
integrate(1/(1-(1-6*b)**(3/2)-9*b+54*b*x-54*x**2+108*x**3)**(1/2),x)
```

output
```
Integral(1/sqrt(54*b*x - 9*b + 108*x**3 - 54*x**2 - (1 - 6*b)**(3/2) + 1),
 x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}}\,dx=\int \frac{1}{\sqrt{108\,x^3+54\,bx-54\,x^2-(-6\,b+1)^{\frac{3}{2}}-9\,b+}}$$

input
```
integrate(1/(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3)^(1/2),x, algorithm
="maxima")
```

output
```
integrate(1/sqrt(108*x^3 + 54*b*x - 54*x^2 - (-6*b + 1)^(3/2) - 9*b + 1),
x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.48

$$\int \frac{1}{\sqrt{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}}\,dx=\frac{\sqrt{3}\sqrt{2}\arctan\left(\frac{\sqrt{3}\sqrt{6\,x-2\,\sqrt{-6\,b+1}-1}}{3\,(-6\,b+1)^{\frac{1}{4}}}\right)}{9\,(-6\,b+1)^{\frac{1}{4}}\operatorname{sgn}\left(6\,x+\sqrt{-6\,b+1}-1\right)}$$

input
```
integrate(1/(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3)^(1/2),x, algorithm
="giac")
```

output
```
1/9*sqrt(3)*sqrt(2)*arctan(1/3*sqrt(3)*sqrt(6*x - 2*sqrt(-6*b + 1) - 1)/(-
6*b + 1)^(1/4))/((-6*b + 1)^(1/4)*sgn(6*x + sqrt(-6*b + 1) - 1))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}}\,dx=\int \frac{1}{\sqrt{54\,b\,x-9\,b-(1-6\,b)^{3/2}-54\,x^2+108\,x^3+}}$$

input
```
int(1/(54*b*x - 9*b - (1 - 6*b)^(3/2) - 54*x^2 + 108*x^3 + 1)^(1/2),x)
```

output
```
int(1/(54*b*x - 9*b - (1 - 6*b)^(3/2) - 54*x^2 + 108*x^3 + 1)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}}\,dx = \text{too large to display}$$

input
```
int(1/(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3)^(1/2),x)
```

output
```
(282175488*sqrt( - 6*b + 1)*int(sqrt(6*sqrt( - 6*b + 1)*b - sqrt( - 6*b +
1) + 54*b*x - 9*b + 108*x**3 - 54*x**2 + 1)/(23079168*sqrt( - 6*b + 1)*b**
8 + 311568768*sqrt( - 6*b + 1)*b**7*x**2 - 103856256*sqrt( - 6*b + 1)*b**7
*x - 4588992*sqrt( - 6*b + 1)*b**7 + 1246275072*sqrt( - 6*b + 1)*b**6*x**4
 - 830850048*sqrt( - 6*b + 1)*b**6*x**3 + 80850960*sqrt( - 6*b + 1)*b**6*x
**2 + 19208016*sqrt( - 6*b + 1)*b**6*x - 2027100*sqrt( - 6*b + 1)*b**6 + 1
246275072*sqrt( - 6*b + 1)*b**5*x**6 - 1246275072*sqrt( - 6*b + 1)*b**5*x*
*5 + 219547584*sqrt( - 6*b + 1)*b**5*x**4 + 84426624*sqrt( - 6*b + 1)*b**5
*x**3 - 49449528*sqrt( - 6*b + 1)*b**5*x**2 + 9228456*sqrt( - 6*b + 1)*b**
5*x + 814462*sqrt( - 6*b + 1)*b**5 - 92021184*sqrt( - 6*b + 1)*b**4*x**6 +
 92021184*sqrt( - 6*b + 1)*b**4*x**5 - 143971344*sqrt( - 6*b + 1)*b**4*x**
4 + 78939936*sqrt( - 6*b + 1)*b**4*x**3 - 2013408*sqrt( - 6*b + 1)*b**4*x*
*2 - 3525072*sqrt( - 6*b + 1)*b**4*x - 111254*sqrt( - 6*b + 1)*b**4 - 1209
66048*sqrt( - 6*b + 1)*b**3*x**6 + 120966048*sqrt( - 6*b + 1)*b**3*x**5 -
1381320*sqrt( - 6*b + 1)*b**3*x**4 - 21480240*sqrt( - 6*b + 1)*b**3*x**3 +
 2925024*sqrt( - 6*b + 1)*b**3*x**2 + 467240*sqrt( - 6*b + 1)*b**3*x + 683
6*sqrt( - 6*b + 1)*b**3 + 28860192*sqrt( - 6*b + 1)*b**2*x**6 - 28860192*s
qrt( - 6*b + 1)*b**2*x**5 + 4814856*sqrt( - 6*b + 1)*b**2*x**4 + 2134576*s
qrt( - 6*b + 1)*b**2*x**3 - 449960*sqrt( - 6*b + 1)*b**2*x**2 - 27984*sqrt
( - 6*b + 1)*b**2*x - 160*sqrt( - 6*b + 1)*b**2 - 2400192*sqrt( - 6*b +...
```

3.158 $\int \frac{1}{(e+fx)\sqrt{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}}\,dx$

Optimal result

Integrand size = 42, antiderivative size = 329

$$\int \frac{1}{(e+fx)\sqrt{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}}\,dx = \frac{2\sqrt{\frac{2}{3}}\left(1-\frac{1-6x}{\sqrt{1-6b}}\right)\sqrt{2+\frac{1-6x}{\sqrt{1-6b}}}\,\text{arctar}}{\left(f-\frac{6e+f}{\sqrt{1-6b}}\right)\sqrt{-2(1-6b)^{3/2}+3(1-6b)}}$$

$$+\frac{2\sqrt{2}(1-6b)^{3/4}\sqrt{f}\left(1-\frac{1-6x}{\sqrt{1-6b}}\right)\sqrt{2+\frac{1-6x}{\sqrt{1-6b}}}\,\text{arctanh}\left(\frac{\sqrt[4]{1-6b}\sqrt{f}\sqrt{2+\frac{1-6x}{\sqrt{1-6b}}}}{\sqrt{6e+f+2\sqrt{1-6b}f}}\right)}{\left(6e+f-\sqrt{1-6b}f\right)\sqrt{6e+f+2\sqrt{1-6b}f}\sqrt{-2(1-6b)^{3/2}+3(1-6b)(1-6x)-(1-6x)^3}}$$

output

```
2/3*6^(1/2)*(1-(1-6*x)/(1-6*b)^(1/2))*(2+(1-6*x)/(1-6*b)^(1/2))^(1/2)*arct
anh(1/3*(2+(1-6*x)/(1-6*b)^(1/2))^(1/2)*3^(1/2))/(f-(6*e+f)/(1-6*b)^(1/2))
/(-2*(1-6*b)^(3/2)+3*(1-6*b)*(1-6*x)-(1-6*x)^3)^(1/2)+2*2^(1/2)*(1-6*b)^(3
/4)*f^(1/2)*(1-(1-6*x)/(1-6*b)^(1/2))*(2+(1-6*x)/(1-6*b)^(1/2))^(1/2)*arct
anh((1-6*b)^(1/4)*f^(1/2)*(2+(1-6*x)/(1-6*b)^(1/2))^(1/2)/(6*e+f+2*(1-6*b)
^(1/2)*f)^(1/2))/(6*e+f-(1-6*b)^(1/2)*f)/(6*e+f+2*(1-6*b)^(1/2)*f)^(1/2)/(
-2*(1-6*b)^(3/2)+3*(1-6*b)*(1-6*x)-(1-6*x)^3)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 20.36 (sec) , antiderivative size = 1352, normalized size of antiderivative = 4.11

$$\int \frac{1}{(e+fx)\sqrt{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}}\,dx = \text{Too large to display}$$

input
```
Integrate[1/((e + f*x)*Sqrt[1 - (1 - 6*b)^(3/2) - 9*b + 54*b*x - 54*x^2 +
108*x^3]),x]
```

output
```
(16*(-1 + 6*b)^3*(5832*e^6 + 5832*(1 + Sqrt[1 - 6*b])*e^5*f + 4860*(1 + Sq
rt[1 - 6*b] - 3*b)*e^4*f^2 - 1080*(-2*(1 + Sqrt[1 - 6*b]) + 3*(3 + Sqrt[1
- 6*b])*b)*e^3*f^3 - 270*(-2*(1 + Sqrt[1 - 6*b]) + 6*(2 + Sqrt[1 - 6*b])*b
 - 9*b^2)*e^2*f^4 + 18*(4*(1 + Sqrt[1 - 6*b]) - 6*(5 + 3*Sqrt[1 - 6*b])*b
+ 9*(5 + Sqrt[1 - 6*b])*b^2)*e*f^5 + (4*(1 + Sqrt[1 - 6*b]) - 12*(3 + 2*Sq
rt[1 - 6*b])*b + 27*(3 + Sqrt[1 - 6*b])*b^2 - 27*b^3)*f^6)*EllipticPi[(f*(
-Root[1 - Sqrt[1 - 6*b] - 9*b + 6*Sqrt[1 - 6*b]*b + 54*b*#1 - 54*#1^2 + 10
8*#1^3 & , 2] + Root[1 - Sqrt[1 - 6*b] - 9*b + 6*Sqrt[1 - 6*b]*b + 54*b*#1
 - 54*#1^2 + 108*#1^3 & , 3]))/(e + f*Root[1 - Sqrt[1 - 6*b] - 9*b + 6*Sqr
t[1 - 6*b]*b + 54*b*#1 - 54*#1^2 + 108*#1^3 & , 3]), ArcSin[Sqrt[(-x + Roo
t[1 - Sqrt[1 - 6*b] - 9*b + 6*Sqrt[1 - 6*b]*b + 54*b*#1 - 54*#1^2 + 108*#1
^3 & , 3])/(-Root[1 - Sqrt[1 - 6*b] - 9*b + 6*Sqrt[1 - 6*b]*b + 54*b*#1 -
54*#1^2 + 108*#1^3 & , 2] + Root[1 - Sqrt[1 - 6*b] - 9*b + 6*Sqrt[1 - 6*b]
*b + 54*b*#1 - 54*#1^2 + 108*#1^3 & , 3])]], (Root[1 - Sqrt[1 - 6*b] - 9*b
 + 6*Sqrt[1 - 6*b]*b + 54*b*#1 - 54*#1^2 + 108*#1^3 & , 2] - Root[1 - Sqrt
[1 - 6*b] - 9*b + 6*Sqrt[1 - 6*b]*b + 54*b*#1 - 54*#1^2 + 108*#1^3 & , 3])
/(Root[1 - Sqrt[1 - 6*b] - 9*b + 6*Sqrt[1 - 6*b]*b + 54*b*#1 - 54*#1^2 + 1
08*#1^3 & , 1] - Root[1 - Sqrt[1 - 6*b] - 9*b + 6*Sqrt[1 - 6*b]*b + 54*b*#
1 - 54*#1^2 + 108*#1^3 & , 3])]*Sqrt[(-x + Root[1 - Sqrt[1 - 6*b] - 9*b +
6*Sqrt[1 - 6*b]*b + 54*b*#1 - 54*#1^2 + 108*#1^3 & , 1])/(Root[1 - Sqrt...
```

Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 307, normalized size of antiderivative = 0.93, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2489, 27, 97, 73, 217, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{54bx-(1-6b)^{3/2}-9b+108x^3-54x^2+1}(e+fx)}dx$$

$$\Big\downarrow \text{2489}$$

$$-\frac{157464\sqrt{2}\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)\sqrt{6(1-6b)x-\left(2\sqrt{1-6b}+1\right)(1-6b)}\int -\frac{}{157464\sqrt{2}\left(\left(1-\sqrt{1-6\right.\right.}}{\sqrt{54bx-(1-6b)^{3/2}-9b+108x^3-54x^2+1}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)\sqrt{6(1-6b)x-\left(2\sqrt{1-6b}+1\right)(1-6b)}\int \frac{1}{\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)\sqrt{6(1-}}}{\sqrt{54bx-(1-6b)^{3/2}-9b+108x^3-54x^2+1}}$$

$$\Big\downarrow \text{97}$$

$$\frac{\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)\sqrt{6(1-6b)x-\left(2\sqrt{1-6b}+1\right)(1-6b)}\left(\frac{6\int \frac{1}{\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)\sqrt{6(1-6}}}{-\sqrt{1-6b}f+6e+}\right.}{\sqrt{54bx-(1-6b)^{3/2}-9b+108x^3-54x^2+1}}$$

$$\Big\downarrow \text{73}$$

$$\frac{\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)\sqrt{6(1-6b)x-\left(2\sqrt{1-6b}+1\right)(1-6b)}\left(\frac{2\int \frac{1}{-3(1-6b)^{3/2}+\left(2\sqrt{1-6b}+1\right)(1-6b)-6x(}}{(1-6b)\left(-\sqrt{1\right.}}\right.}{\sqrt{54bx-(1-6b)^{3/2}-9}}$$

$$\Big\downarrow \text{217}$$

$$\frac{\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)\sqrt{6(1-6b)x-\left(2\sqrt{1-6b}+1\right)(1-6b)}\left(\frac{f\int\frac{1}{\frac{1}{6}\left(6e+2\sqrt{1-6b}f+f\right)+\frac{f\left(6(1-6b)x-(2\sqrt{\cdots}\right)}{6(1-\cdots}}}{3(1-6b)^2\cdots}\right.}{\sqrt{54bx-(1-6b)^{3/2}-9b+108x^3-54\cdots}}$$

$\big\downarrow$ 218

$$\frac{\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)\sqrt{6(1-6b)x-\left(2\sqrt{1-6b}+1\right)(1-6b)}\left(\frac{2\sqrt{f}\arctan\left(\frac{\sqrt{f}\sqrt{6(1-6b)x-(2\sqrt{1-6b}+1\cdots}}{\sqrt{1-6b}\sqrt{2\sqrt{1-6b}f+6e\cdots}}\right.}{(1-6b)^{3/2}\left(-\sqrt{1-6b}f+6e+f\right)\sqrt{2\sqrt{1-6}\cdots}}\right.}{\sqrt{54bx-(1-6b)^{3/2}-9b+108x^3-54x^2+1}}$$

input
```
Int[1/((e + f*x)*Sqrt[1 - (1 - 6*b)^(3/2) - 9*b + 54*b*x - 54*x^2 + 108*x^
3]),x]
```

output
```
(((1 - Sqrt[1 - 6*b])*(1 - 6*b) - 6*(1 - 6*b)*x)*Sqrt[-((1 + 2*Sqrt[1 - 6*
b])*(1 - 6*b)) + 6*(1 - 6*b)*x]*((-2*ArcTan[Sqrt[-((1 + 2*Sqrt[1 - 6*b])*(
1 - 6*b)) + 6*(1 - 6*b)*x]/(Sqrt[3]*(1 - 6*b)^(3/4))])/(Sqrt[3]*(1 - 6*b)^
(7/4)*(6*e + f - Sqrt[1 - 6*b]*f)) + (2*Sqrt[f]*ArcTan[(Sqrt[f]*Sqrt[-((1
+ 2*Sqrt[1 - 6*b])*(1 - 6*b)) + 6*(1 - 6*b)*x])/(Sqrt[1 - 6*b]*Sqrt[6*e +
f + 2*Sqrt[1 - 6*b]*f])])/((1 - 6*b)^(3/2)*(6*e + f - Sqrt[1 - 6*b]*f)*Sqr
t[6*e + f + 2*Sqrt[1 - 6*b]*f])))/Sqrt[1 - (1 - 6*b)^(3/2) - 9*b + 54*b*x
- 54*x^2 + 108*x^3]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 97
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[b/(b*c - a*d)   Int[(e + f*x)^p/(a + b*x), x], x] - Simp[d/(b*c
 - a*d)   Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p},
 x] &&  !IntegerQ[p]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 2489
```
Int[((e_.) + (f_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(
x_)^3)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2 + d*x^3)^p/((c^3 - 4*b*c*d
+ 9*a*d^2 + d*(c^2 - 3*b*d)*x)^p*(b*c - 9*a*d + 2*(c^2 - 3*b*d)*x)^(2*p))
 Int[(e + f*x)^m*(c^3 - 4*b*c*d + 9*a*d^2 + d*(c^2 - 3*b*d)*x)^p*(b*c - 9*a
*d + 2*(c^2 - 3*b*d)*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x]
 && NeQ[c^2 - 3*b*d, 0] && EqQ[b^2*c^2 - 4*a*c^3 - 4*b^3*d + 18*a*b*c*d - 2
7*a^2*d^2, 0] &&  !IntegerQ[p]
```

Maple [F]

$$\int \frac{1}{(fx+e)\sqrt{1-(1-6b)^{\frac{3}{2}}-9b+54bx-54x^2+108x^3}}dx$$

input
```
int(1/(f*x+e)/(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3)^(1/2),x)
```

output
```
int(1/(f*x+e)/(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3)^(1/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 920 vs. $2(273) = 546$.

Time = 0.50 (sec) , antiderivative size = 3961, normalized size of antiderivative = 12.04

$$\int \frac{1}{(e+fx)\sqrt{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(f*x+e)/(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3)^(1/2),x, a
lgorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{1}{(e+fx)\sqrt{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}}\,dx = \int \frac{1}{(e+fx)\sqrt{54bx+6b\sqrt{1-6b}-9b+10}}$$

input
```
integrate(1/(f*x+e)/(1-(1-6*b)**(3/2)-9*b+54*b*x-54*x**2+108*x**3)**(1/2),
x)
```

output
```
Integral(1/((e + f*x)*sqrt(54*b*x + 6*b*sqrt(1 - 6*b) - 9*b + 108*x**3 - 5
4*x**2 - sqrt(1 - 6*b) + 1)), x)
```

Maxima [F]

$$\int \frac{1}{(e+fx)\sqrt{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}}\,dx = \int \frac{1}{\sqrt{108\,x^3+54\,bx-54\,x^2-(-6\,b+1)^{\frac{3}{2}}}}$$

input
```
integrate(1/(f*x+e)/(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3)^(1/2),x, a
lgorithm="maxima")
```

output
```
integrate(1/(sqrt(108*x^3 + 54*b*x - 54*x^2 - (-6*b + 1)^(3/2) - 9*b + 1)*
(f*x + e)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(e+fx)\sqrt{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(f*x+e)/(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3)^(1/2),x, a
lgorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(e+fx)\sqrt{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}}\,dx = \text{Hanged}$$

input
```
int(1/((e + f*x)*(54*b*x - 9*b - (1 - 6*b)^(3/2) - 54*x^2 + 108*x^3 + 1)^(
1/2)),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{1}{(e+fx)\sqrt{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}}\,dx = \int \frac{1}{(fx+e)\sqrt{1-(-6b+1)^{\frac{3}{2}}-9b+54bx}}$$

input
```
int(1/(f*x+e)/(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3)^(1/2),x)
```

output
```
int(1/(f*x+e)/(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3)^(1/2),x)
```

3.159 $\int \frac{1}{(e+fx)^2\sqrt{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}}\,dx$

Optimal result

Integrand size = 42, antiderivative size = 493

$$\int \frac{1}{(e+fx)^2\sqrt{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}}\,dx = \frac{\sqrt{2}(1-6\cdots}{\left(6e+f-\sqrt{1-6b}f\right)\left(6e+f+2\sqrt{1-6\cdots}\right.}$$

$$-\frac{4\sqrt{6}\sqrt{1-6b}\left(1-\frac{1-6x}{\sqrt{1-6b}}\right)\sqrt{2+\frac{1-6x}{\sqrt{1-6b}}}\operatorname{arctanh}\left(\frac{\sqrt{2+\frac{1-6x}{\sqrt{1-6b}}}}{\sqrt{3}}\right)}{\left(6e+f-\sqrt{1-6b}f\right)^2\sqrt{-2(1-6b)^{3/2}+3(1-6b)(1-6x)-(1-6x)^3}}$$

$$+\frac{18\sqrt{2}\sqrt[4]{1-6b}\sqrt{f}\left(6\sqrt{1-6b}e+\left(1+\sqrt{1-6b}-6b\right)f\right)\left(1-\frac{1-6x}{\sqrt{1-6b}}\right)\sqrt{2+\frac{1-6x}{\sqrt{1-6b}}}\operatorname{arctanh}\left(\frac{\sqrt[4]{1-6b}\sqrt{f}\sqrt{2\cdots}}{\sqrt{6e+f+2\sqrt{1-\cdots}}}\right.}{\left(6e+f-\sqrt{1-6b}f\right)^2\left(6e+f+2\sqrt{1-6b}f\right)^{3/2}\sqrt{-2(1-6b)^{3/2}+3(1-6b)(1-6x)-(1-6x)\cdots}}$$

output
```
2^(1/2)*(1-6*b)*f*(1-(1-6*x)/(1-6*b)^(1/2))*(2+(1-6*x)/(1-6*b)^(1/2))/(6*e
+f-(1-6*b)^(1/2)*f)/(6*e+f+2*(1-6*b)^(1/2)*f)/(-2*(1-6*b)^(3/2)+3*(1-6*b)*
(1-6*x)-(1-6*x)^3)^(1/2)/(f*x+e)-4*6^(1/2)*(1-6*b)^(1/2)*(1-(1-6*x)/(1-6*b
)^(1/2))*(2+(1-6*x)/(1-6*b)^(1/2))^(1/2)*arctanh(1/3*(2+(1-6*x)/(1-6*b)^(1
/2))^(1/2)*3^(1/2))/(6*e+f-(1-6*b)^(1/2)*f)^2/(-2*(1-6*b)^(3/2)+3*(1-6*b)*
(1-6*x)-(1-6*x)^3)^(1/2)+18*2^(1/2)*(1-6*b)^(1/4)*f^(1/2)*(6*(1-6*b)^(1/2)
*e+(1+(1-6*b)^(1/2)-6*b)*f)*(1-(1-6*x)/(1-6*b)^(1/2))*(2+(1-6*x)/(1-6*b)^(
1/2))^(1/2)*arctanh((1-6*b)^(1/4)*f^(1/2)*(2+(1-6*x)/(1-6*b)^(1/2))^(1/2)/
(6*e+f+2*(1-6*b)^(1/2)*f)^(1/2))/(6*e+f-(1-6*b)^(1/2)*f)^2/(6*e+f+2*(1-6*b
)^(1/2)*f)^(3/2)/(-2*(1-6*b)^(3/2)+3*(1-6*b)*(1-6*x)-(1-6*x)^3)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 88.26 (sec) , antiderivative size = 451671, normalized size of antiderivative = 916.17

$$\int \frac{1}{(e+fx)^2\sqrt{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}}\,dx = \text{Result too large to show}$$

input
```
Integrate[1/((e + f*x)^2*Sqrt[1 - (1 - 6*b)^(3/2) - 9*b + 54*b*x - 54*x^2
+ 108*x^3]),x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 1.31 (sec) , antiderivative size = 470, normalized size of antiderivative = 0.95, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2489, 27, 114, 27, 174, 73, 217, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{54bx-(1-6b)^{3/2}-9b+108x^3-54x^2+1}(e+fx)^2}\,dx$$

$\downarrow$ 2489

$$-\frac{157464\sqrt{2}\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)\sqrt{6(1-6b)x-\left(2\sqrt{1-6b}+1\right)(1-6b)}\int -\frac{}{157464\sqrt{2}\left(\left(1-\sqrt{1-6\right.\right.}}}{\sqrt{54bx-(1-6b)^{3/2}-9b+108x^3-54x^2+1}}$$

$\downarrow$ 27

$$\frac{\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)\sqrt{6(1-6b)x-\left(2\sqrt{1-6b}+1\right)(1-6b)}\int \frac{1}{\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)\sqrt{6(1-}}}{\sqrt{54bx-(1-6b)^{3/2}-9b+108x^3-54x^2+1}}$$

$\downarrow$ 114

$$\frac{\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)\sqrt{6(1-6b)x-\left(2\sqrt{1-6b}+1\right)(1-6b)}\left(\frac{\int \frac{9(1-6b)^2\left(4e+\sqrt{1-6b}\right.}{\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)\sqrt{6(1-6b)}}}{(1-6b)^2\left(-\sqrt{1-6b}f+6e+f\right)}\right.}{\sqrt{54bx-(1-6b)^{3/2}-9b+108x^3-54x^2}}$$

$\downarrow$ 27

$$\frac{\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)\sqrt{6(1-6b)x-\left(2\sqrt{1-6b}+1\right)(1-6b)}\left(\frac{9\int \frac{4e+\sqrt{1-6b}f+}{\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)\sqrt{6(1-6}}}{\left(-\sqrt{1-6b}f+6e+f\right)\left(2\sqrt{}\right.}\right.}{\sqrt{54bx-(1-6b)^{3/2}-9b+108x^3-54x^2}}$$

$\downarrow$ 174

$$\frac{\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)\sqrt{6(1-6b)x-\left(2\sqrt{1-6b}+1\right)(1-6b)}\left(\frac{9\left(\frac{4\left(2\sqrt{1-6b}f+6e+f\right)\int \frac{}{\left(\left(1-\sqrt{1-6b}\right)(1-6\right.}}{-\sqrt{}}\right.}{}\right.}{\sqrt{54bx-(1-}}$$

$\downarrow$ 73

$$\frac{\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)\sqrt{6(1-6b)x-\left(2\sqrt{1-6b}+1\right)(1-6b)}\left(\frac{9\left(\frac{4\left(2\sqrt{1-6b}f+6e+f\right)\int \frac{}{-3(1-6b)^{3/2}+(2\sqrt{\cdots}}}{3(\cdots}\right.}{}\right.}{}$$

$$\Big\downarrow \text{217}$$

$$\frac{\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)\sqrt{6(1-6b)x-\left(2\sqrt{1-6b}+1\right)(1-6b)}\left(\frac{9\left(\frac{f\left(\sqrt{1-6b}f+6e+f\right)\int \frac{}{\frac{1}{6}\left(6e+2\sqrt{1-6b}f+f\cdots\right.}}{}\right.}{}\right.}{\sqrt{54b\cdots}}$$

$$\Big\downarrow \text{218}$$

$$\frac{\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)\sqrt{6(1-6b)x-\left(2\sqrt{1-6b}+1\right)(1-6b)}\left(\frac{9\left(\frac{2\sqrt{f}\left(\sqrt{1-6b}f+6e+f\right)\arctan\left(\frac{\sqrt{f}\sqrt{6(1-\cdots}}{\sqrt{1-\cdots}}\right.}{(1-6b)^{3/2}\left(-\sqrt{1-6b}f+6e+f\right)}\right.}{}\right.}{\sqrt{54bx-(1-6b)^{3/2}\cdots}}$$

input
```
Int[1/((e + f*x)^2*Sqrt[1 - (1 - 6*b)^(3/2) - 9*b + 54*b*x - 54*x^2 + 108*
x^3]),x]
```

output
```
(((1 - Sqrt[1 - 6*b])*(1 - 6*b) - 6*(1 - 6*b)*x)*Sqrt[-((1 + 2*Sqrt[1 - 6*
b])*(1 - 6*b)) + 6*(1 - 6*b)*x]*((f*Sqrt[-((1 + 2*Sqrt[1 - 6*b])*(1 - 6*b)
) + 6*(1 - 6*b)*x])/((1 - 6*b)^2*(6*e + f - Sqrt[1 - 6*b]*f)*(6*e + f + 2*
Sqrt[1 - 6*b]*f)*(e + f*x)) + (9*((-4*(6*e + f + 2*Sqrt[1 - 6*b]*f)*ArcTan
[Sqrt[-((1 + 2*Sqrt[1 - 6*b])*(1 - 6*b)) + 6*(1 - 6*b)*x]/(Sqrt[3]*(1 - 6*
b)^(3/4))])/(3*Sqrt[3]*(1 - 6*b)^(7/4)*(6*e + f - Sqrt[1 - 6*b]*f)) + (2*S
qrt[f]*(6*e + f + Sqrt[1 - 6*b]*f)*ArcTan[(Sqrt[f]*Sqrt[-((1 + 2*Sqrt[1 -
6*b])*(1 - 6*b)) + 6*(1 - 6*b)*x])/(Sqrt[1 - 6*b]*Sqrt[6*e + f + 2*Sqrt[1
- 6*b]*f])])/((1 - 6*b)^(3/2)*(6*e + f - Sqrt[1 - 6*b]*f)*Sqrt[6*e + f + 2
*Sqrt[1 - 6*b]*f])))/((6*e + f - Sqrt[1 - 6*b]*f)*(6*e + f + 2*Sqrt[1 - 6*
b]*f))))/Sqrt[1 - (1 - 6*b)^(3/2) - 9*b + 54*b*x - 54*x^2 + 108*x^3]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 2489
```
Int[((e_.) + (f_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(
x_)^3)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2 + d*x^3)^p/((c^3 - 4*b*c*d
+ 9*a*d^2 + d*(c^2 - 3*b*d)*x)^p*(b*c - 9*a*d + 2*(c^2 - 3*b*d)*x)^(2*p))
 Int[(e + f*x)^m*(c^3 - 4*b*c*d + 9*a*d^2 + d*(c^2 - 3*b*d)*x)^p*(b*c - 9*a
*d + 2*(c^2 - 3*b*d)*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x]
 && NeQ[c^2 - 3*b*d, 0] && EqQ[b^2*c^2 - 4*a*c^3 - 4*b^3*d + 18*a*b*c*d - 2
7*a^2*d^2, 0] &&  !IntegerQ[p]
```

Maple [F]

$$\int \frac{1}{(fx+e)^2\sqrt{1-(1-6b)^{\frac{3}{2}}-9b+54bx-54x^2+108x^3}}dx$$

input
```
int(1/(f*x+e)^2/(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3)^(1/2),x)
```

output
```
int(1/(f*x+e)^2/(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3)^(1/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3641 vs. $2(421) = 842$.

Time = 4.51 (sec) , antiderivative size = 15167, normalized size of antiderivative = 30.76

$$\int \frac{1}{(e+fx)^2\sqrt{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(f*x+e)^2/(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3)^(1/2),x,
 algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{1}{(e+fx)^2\sqrt{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}}\,dx = \int \frac{1}{(e+fx)^2\,\sqrt{54bx+6b\sqrt{1-6b}-9b+}}$$

input
```
integrate(1/(f*x+e)**2/(1-(1-6*b)**(3/2)-9*b+54*b*x-54*x**2+108*x**3)**(1/
2),x)
```

output
```
Integral(1/((e + f*x)**2*sqrt(54*b*x + 6*b*sqrt(1 - 6*b) - 9*b + 108*x**3
- 54*x**2 - sqrt(1 - 6*b) + 1)), x)
```

Maxima [F]

$$\int \frac{1}{(e+fx)^2\sqrt{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}}\,dx = \int \frac{1}{\sqrt{108\,x^3+54\,bx-54\,x^2-(-6\,b+1)}}$$

input
```
integrate(1/(f*x+e)^2/(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3)^(1/2),x,
 algorithm="maxima")
```

output
```
integrate(1/(sqrt(108*x^3 + 54*b*x - 54*x^2 - (-6*b + 1)^(3/2) - 9*b + 1)*
(f*x + e)^2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(e+fx)^2\sqrt{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(f*x+e)^2/(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3)^(1/2),x,
 algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(e+fx)^2\sqrt{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}}\,dx = \int \frac{1}{(e+f\,x)^2\,\sqrt{54\,b\,x-9\,b-(1-6\,b)^{3/2}$$

input
```
int(1/((e + f*x)^2*(54*b*x - 9*b - (1 - 6*b)^(3/2) - 54*x^2 + 108*x^3 + 1)
^(1/2)),x)
```

output
```
int(1/((e + f*x)^2*(54*b*x - 9*b - (1 - 6*b)^(3/2) - 54*x^2 + 108*x^3 + 1)
^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{(e+fx)^2\sqrt{1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3}}\,dx = \int \frac{1}{(fx+e)^2\,\sqrt{1-(-6b+1)^{\frac{3}{2}}-9b+54$$

input
```
int(1/(f*x+e)^2/(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3)^(1/2),x)
```

output
```
int(1/(f*x+e)^2/(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3)^(1/2),x)
```

3.160 $\int \frac{(e+fx)^2}{\left(1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3\right)^{3/2}}\,dx$

Optimal result

Integrand size = 42, antiderivative size = 549

$$\int \frac{(e+fx)^2}{(1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3)^{3/2}}\,dx = \frac{\sqrt{1-6b}\left(6e+f+2\sqrt{1-6b}f\right)^2\left(1-\frac{1-6x}{\sqrt{1-6b}}\right)\left(2\right.}{81\sqrt{2}\left(-2(1-6b)^{3/2}+3(1-6b)(1-6x)-(1-\right.}$$
$$-\frac{\sqrt{1-6b}\left(90e^2+6\left(5+7\sqrt{1-6b}\right)ef+\left(11+7\sqrt{1-6b}-51b\right)f^2\right)\left(1-\frac{1-6x}{\sqrt{1-6b}}\right)\left(2+\frac{1-6x}{\sqrt{1-6b}}\right)^2}{486\sqrt{2}\left(-2(1-6b)^{3/2}+3(1-6b)(1-6x)-(1-6x)^3\right)^{3/2}}$$
$$-\frac{\sqrt{1-6b}\left(90e^2+6\left(5+7\sqrt{1-6b}\right)ef+\left(5+7\sqrt{1-6b}-15b\right)f^2\right)\left(1-\frac{1-6x}{\sqrt{1-6b}}\right)^2\left(2+\frac{1-6x}{\sqrt{1-6b}}\right)^2}{972\sqrt{2}\left(-2(1-6b)^{3/2}+3(1-6b)(1-6x)-(1-6x)^3\right)^{3/2}}$$
$$-\frac{\sqrt{1-6b}\left(90e^2+6\left(5+7\sqrt{1-6b}\right)ef+\left(5+7\sqrt{1-6b}-15b\right)f^2\right)\left(1-\frac{1-6x}{\sqrt{1-6b}}\right)^3\left(2+\frac{1-6x}{\sqrt{1-6b}}\right)^{3/2}\operatorname{arctanh}}{972\sqrt{6}\left(-2(1-6b)^{3/2}+3(1-6b)(1-6x)-(1-6x)^3\right)^{3/2}}$$

output
```
1/162*(1-6*b)^(1/2)*(6*e+f+2*(1-6*b)^(1/2)*f)^2*(1-(1-6*x)/(1-6*b)^(1/2))*
(2+(1-6*x)/(1-6*b)^(1/2))*2^(1/2)/(-2*(1-6*b)^(3/2)+3*(1-6*b)*(1-6*x)-(1-6
*x)^3)^(3/2)-1/972*(1-6*b)^(1/2)*(90*e^2+6*(5+7*(1-6*b)^(1/2))*e*f+(11+7*(
1-6*b)^(1/2)-51*b)*f^2)*(1-(1-6*x)/(1-6*b)^(1/2))*(2+(1-6*x)/(1-6*b)^(1/2)
)^2*2^(1/2)/(-2*(1-6*b)^(3/2)+3*(1-6*b)*(1-6*x)-(1-6*x)^3)^(3/2)-1/1944*(1
-6*b)^(1/2)*(90*e^2+6*(5+7*(1-6*b)^(1/2))*e*f+(5+7*(1-6*b)^(1/2)-15*b)*f^2
)*(1-(1-6*x)/(1-6*b)^(1/2))^2*(2+(1-6*x)/(1-6*b)^(1/2))^2*2^(1/2)/(-2*(1-6
*b)^(3/2)+3*(1-6*b)*(1-6*x)-(1-6*x)^3)^(3/2)-1/5832*(1-6*b)^(1/2)*(90*e^2+
6*(5+7*(1-6*b)^(1/2))*e*f+(5+7*(1-6*b)^(1/2)-15*b)*f^2)*(1-(1-6*x)/(1-6*b)
^(1/2))^3*(2+(1-6*x)/(1-6*b)^(1/2))^(3/2)*arctanh(1/3*(2+(1-6*x)/(1-6*b)^(
1/2))^(1/2)*3^(1/2))*6^(1/2)/(-2*(1-6*b)^(3/2)+3*(1-6*b)*(1-6*x)-(1-6*x)^3
)^(3/2)
```

Mathematica [F]

$$\int \frac{(e+fx)^2}{\left(1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3\right)^{3/2}}\,dx=\int \frac{(e+fx)^2}{(1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3)}$$

input
```
Integrate[(e + f*x)^2/(1 - (1 - 6*b)^(3/2) - 9*b + 54*b*x - 54*x^2 + 108*x
^3)^(3/2),x]
```

output
```
Integrate[(e + f*x)^2/(1 - (1 - 6*b)^(3/2) - 9*b + 54*b*x - 54*x^2 + 108*x
^3)^(3/2), x]
```

Rubi [A] (verified)

Time = 1.48 (sec) , antiderivative size = 498, normalized size of antiderivative = 0.91, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2489, 27, 100, 27, 87, 61, 73, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2}{\left(54bx-(1-6b)^{3/2}-9b+108x^3-54x^2+1\right)^{3/2}}\,dx$$

$\Big\downarrow$ 2489

$$-\frac{7808611824626688\sqrt{2}\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)^3\left(6(1-6b)x-\left(2\sqrt{1-6b}+1\right)(1-6b)\right)^{3/2}\int -\frac{\ }{78}}{\left(54bx-(1-6b)^{3/2}-9b+108x^3-54x^2+\right.}$$

$\Big\downarrow$ 27

$$\frac{\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)^3\left(6(1-6b)x-\left(2\sqrt{1-6b}+1\right)(1-6b)\right)^{3/2}\int \frac{(e+}{\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)^3(}}{\left(54bx-(1-6b)^{3/2}-9b+108x^3-54x^2+1\right)^{3/2}}$$

$\Big\downarrow$ 100

$$\frac{\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)^3\left(6(1-6b)x-\left(2\sqrt{1-6b}+1\right)(1-6b)\right)^{3/2}\left(\frac{\int -\frac{6(1-6b)^3\left(90e^2+6\left(7\sqrt{1-6b}+5\right)fe\right.}{\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)^2}}{1296(}\right.}{\left(54bx-(1-6b)^{3/2}-9b+108\right.}$$

$\Big\downarrow$ 27

$$\frac{\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)^3\left(6(1-6b)x-\left(2\sqrt{1-6b}+1\right)(1-6b)\right)^{3/2}\left(-\frac{\int \frac{90e^2+6\left(7\sqrt{1-6b}+5\right)fe-(-2}{\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)^2}}{216(1}\right.}{\left(54bx-(1-6b)^{3/2}-9b+108x\right.}$$

$\Big\downarrow$ 87

$$\frac{\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)^3\left(6(1-6b)x-\left(2\sqrt{1-6b}+1\right)(1-6b)\right)^{3/2}\left(-\frac{-\frac{\left(6\left(7\sqrt{1-6b}+5\right)ef+\left(-15b+7\sqrt{1-}\right.\right.}{}}{}\right.}{}$$

$\Big\downarrow$ 61

$$\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)^3\left(6(1-6b)x-\left(2\sqrt{1-6b}+1\right)(1-6b)\right)^{3/2}\left(-\frac{-\frac{\left(6\left(7\sqrt{1-6b}+5\right)ef+\left(-15b+7\sqrt{1-\right.}}{}}{}\right.$$

$\downarrow$ 73

$$\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)^3\left(6(1-6b)x-\left(2\sqrt{1-6b}+1\right)(1-6b)\right)^{3/2}\left(-\frac{-\frac{\left(6\left(7\sqrt{1-6b}+5\right)ef+\left(-15b+7\sqrt{1-\right.}}{}}{}\right.$$

$\downarrow$ 217

$$\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)^3\left(6(1-6b)x-\left(2\sqrt{1-6b}+1\right)(1-6b)\right)^{3/2}\left(-\frac{-\frac{\left(\frac{\arctan\left(\frac{\sqrt{6(1-6b)x-(2\sqrt{1-6b}}}{\sqrt{3}(1-6b)^{3/4}}\right.}{9\sqrt{3}(1-6b)^{13/4}}\right.}{}}{}\right.$$

input
```
Int[(e + f*x)^2/(1 - (1 - 6*b)^(3/2) - 9*b + 54*b*x - 54*x^2 + 108*x^3)^(3
/2),x]
```

output
```
(((1 - Sqrt[1 - 6*b])*(1 - 6*b) - 6*(1 - 6*b)*x)^3*(-((1 + 2*Sqrt[1 - 6*b]
)*(1 - 6*b)) + 6*(1 - 6*b)*x)^(3/2)*(-1/1296*(6*e + f - Sqrt[1 - 6*b]*f)^2
/((1 - 6*b)^(5/2)*((1 - Sqrt[1 - 6*b])*(1 - 6*b) - 6*(1 - 6*b)*x)^2*Sqrt[-
((1 + 2*Sqrt[1 - 6*b])*(1 - 6*b)) + 6*(1 - 6*b)*x]) - (-1/18*(90*e^2 + 6*(
5 + 7*Sqrt[1 - 6*b])*e*f - (7 - 7*Sqrt[1 - 6*b] - 57*b)*f^2)/((1 - 6*b)^(5
/2)*((1 - Sqrt[1 - 6*b])*(1 - 6*b) - 6*(1 - 6*b)*x)*Sqrt[-((1 + 2*Sqrt[1 -
 6*b])*(1 - 6*b)) + 6*(1 - 6*b)*x]) - ((90*e^2 + 6*(5 + 7*Sqrt[1 - 6*b])*e
*f + (5 + 7*Sqrt[1 - 6*b] - 15*b)*f^2)*(1/(9*(1 - 6*b)^(5/2)*Sqrt[-((1 + 2
*Sqrt[1 - 6*b])*(1 - 6*b)) + 6*(1 - 6*b)*x]) + ArcTan[Sqrt[-((1 + 2*Sqrt[1
 - 6*b])*(1 - 6*b)) + 6*(1 - 6*b)*x]/(Sqrt[3]*(1 - 6*b)^(3/4))]/(9*Sqrt[3]
*(1 - 6*b)^(13/4))))/(2*(1 - 6*b)^(3/2)))/(216*(1 - 6*b)^(3/2))))/(1 - (1
- 6*b)^(3/2) - 9*b + 54*b*x - 54*x^2 + 108*x^3)^(3/2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 2489
```
Int[((e_.) + (f_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(
x_)^3)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2 + d*x^3)^p/((c^3 - 4*b*c*d
+ 9*a*d^2 + d*(c^2 - 3*b*d)*x)^p*(b*c - 9*a*d + 2*(c^2 - 3*b*d)*x)^(2*p))
 Int[(e + f*x)^m*(c^3 - 4*b*c*d + 9*a*d^2 + d*(c^2 - 3*b*d)*x)^p*(b*c - 9*a
*d + 2*(c^2 - 3*b*d)*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x]
 && NeQ[c^2 - 3*b*d, 0] && EqQ[b^2*c^2 - 4*a*c^3 - 4*b^3*d + 18*a*b*c*d - 2
7*a^2*d^2, 0] &&  !IntegerQ[p]
```

Maple [F]

$$\int \frac{(fx+e)^2}{\left(1-(1-6b)^{\frac{3}{2}}-9b+54bx-54x^2+108x^3\right)^{\frac{3}{2}}}dx$$

input
```
int((f*x+e)^2/(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3)^(3/2),x)
```

output
```
int((f*x+e)^2/(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3)^(3/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1774 vs. 2(462) = 924.

Time = 0.36 (sec) , antiderivative size = 3837, normalized size of antiderivative = 6.99

$$\int \frac{(e+fx)^2}{\left(1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2/(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3)^(3/2),x, a
lgorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2}{\left(1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3\right)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**2/(1-(1-6*b)**(3/2)-9*b+54*b*x-54*x**2+108*x**3)**(3/2)
,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e+fx)^2}{\left(1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3\right)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((f*x+e)^2/(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3)^(3/2),x, a
lgorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(e+fx)^2}{\left(1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3\right)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((f*x+e)^2/(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3)^(3/2),x, a
lgorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{%%{[17794363111902506549001421627856069105221941539635
2,0]:[1,0
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2}{\left(1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3\right)^{3/2}}\,dx=\int \frac{(e+f\,x)^2}{\left(54\,b\,x-9\,b-(1-6\,b)^{3/2}-54\,x^2+108\,x^3+\right.}$$

input
```
int((e + f*x)^2/(54*b*x - 9*b - (1 - 6*b)^(3/2) - 54*x^2 + 108*x^3 + 1)^(3
/2),x)
```

output
```
int((e + f*x)^2/(54*b*x - 9*b - (1 - 6*b)^(3/2) - 54*x^2 + 108*x^3 + 1)^(3
/2), x)
```

Reduce [F]

$$\int \frac{(e+fx)^2}{\left(1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3\right)^{3/2}}\,dx=\int \frac{(fx+e)^2}{\left(1-(-6b+1)^{\frac{3}{2}}-9b+54bx-54x^2+108x^3\right.}$$

input
```
int((f*x+e)^2/(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3)^(3/2),x)
```

output
```
int((f*x+e)^2/(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3)^(3/2),x)
```

3.161 $\int \frac{e+fx}{\left(1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3\right)^{3/2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 461

$$\int \frac{e+fx}{\left(1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3\right)^{3/2}}\,dx = \frac{(1-6b)\left(f-\frac{6e+f}{\sqrt{1-6b}}\right)\left(1-\frac{1-6x}{\sqrt{1-6b}}\right)\left(2+\frac{1-\cdots}{\sqrt{1-\cdots}}\right.}{54\sqrt{2}\left(-2(1-6b)^{3/2}+3(1-6b)(1-6x)-(1-\cdots\right.}$$

$$-\frac{(1-6b)\left(7f+\frac{5(6e+f)}{\sqrt{1-6b}}\right)\left(1-\frac{1-6x}{\sqrt{1-6b}}\right)^2\left(2+\frac{1-6x}{\sqrt{1-6b}}\right)}{324\sqrt{2}\left(-2(1-6b)^{3/2}+3(1-6b)(1-6x)-(1-6x)^3\right)^{3/2}}$$

$$+\frac{(1-6b)\left(7f+\frac{5(6e+f)}{\sqrt{1-6b}}\right)\left(1-\frac{1-6x}{\sqrt{1-6b}}\right)^3\left(2+\frac{1-6x}{\sqrt{1-6b}}\right)}{324\sqrt{2}\left(-2(1-6b)^{3/2}+3(1-6b)(1-6x)-(1-6x)^3\right)^{3/2}}$$

$$-\frac{(1-6b)\left(7f+\frac{5(6e+f)}{\sqrt{1-6b}}\right)\left(1-\frac{1-6x}{\sqrt{1-6b}}\right)^3\left(2+\frac{1-6x}{\sqrt{1-6b}}\right)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{2+\frac{1-6x}{\sqrt{1-6b}}}}{\sqrt{3}}\right)}{324\sqrt{6}\left(-2(1-6b)^{3/2}+3(1-6b)(1-6x)-(1-6x)^3\right)^{3/2}}$$

output
```
1/108*(1-6*b)*(f-(6*e+f)/(1-6*b)^(1/2))*(1-(1-6*x)/(1-6*b)^(1/2))*(2+(1-6*
x)/(1-6*b)^(1/2))*2^(1/2)/(-2*(1-6*b)^(3/2)+3*(1-6*b)*(1-6*x)-(1-6*x)^3)^(
3/2)-1/648*(1-6*b)*(7*f+5*(6*e+f)/(1-6*b)^(1/2))*(1-(1-6*x)/(1-6*b)^(1/2))
^2*(2+(1-6*x)/(1-6*b)^(1/2))*2^(1/2)/(-2*(1-6*b)^(3/2)+3*(1-6*b)*(1-6*x)-(
1-6*x)^3)^(3/2)+1/648*(1-6*b)*(7*f+5*(6*e+f)/(1-6*b)^(1/2))*(1-(1-6*x)/(1-
6*b)^(1/2))^3*(2+(1-6*x)/(1-6*b)^(1/2))*2^(1/2)/(-2*(1-6*b)^(3/2)+3*(1-6*b
)*(1-6*x)-(1-6*x)^3)^(3/2)-1/1944*(1-6*b)*(7*f+5*(6*e+f)/(1-6*b)^(1/2))*(1
-(1-6*x)/(1-6*b)^(1/2))^3*(2+(1-6*x)/(1-6*b)^(1/2))^(3/2)*arctanh(1/3*(2+(
1-6*x)/(1-6*b)^(1/2))^(1/2)*3^(1/2))*6^(1/2)/(-2*(1-6*b)^(3/2)+3*(1-6*b)*(
1-6*x)-(1-6*x)^3)^(3/2)
```

Mathematica [F]

$$\int \frac{e+fx}{\left(1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3\right)^{3/2}}\,dx=\int \frac{e+fx}{\left(1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3\right)^{3/2}}\,dx$$

input
```
Integrate[(e + f*x)/(1 - (1 - 6*b)^(3/2) - 9*b + 54*b*x - 54*x^2 + 108*x^3
)^(3/2),x]
```

output
```
Integrate[(e + f*x)/(1 - (1 - 6*b)^(3/2) - 9*b + 54*b*x - 54*x^2 + 108*x^3
)^(3/2), x]
```

Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 428, normalized size of antiderivative = 0.93, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {2489, 27, 87, 52, 61, 73, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e+fx}{\left(54bx-(1-6b)^{3/2}-9b+108x^3-54x^2+1\right)^{3/2}}dx$$

$$\Big\downarrow \text{2489}$$

$$-\frac{7808611824626688\sqrt{2}\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)^3\left(6(1-6b)x-\left(2\sqrt{1-6b}+1\right)(1-6b)\right)^{3/2}\int -\frac{}{78\ldots}}{\left(54bx-(1-6b)^{3/2}-9b+108x^3-54x^2+\ldots\right.}$$

$\downarrow$ 27

$$\frac{\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)^3\left(6(1-6b)x-\left(2\sqrt{1-6b}+1\right)(1-6b)\right)^{3/2}\int \frac{e\ldots}{\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)^3\ldots}}{\left(54bx-(1-6b)^{3/2}-9b+108x^3-54x^2+1\right)^{3/2}}$$

$\downarrow$ 87

$$\frac{\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)^3\left(6(1-6b)x-\left(2\sqrt{1-6b}+1\right)(1-6b)\right)^{3/2}\left(-\frac{\left(\left(7\sqrt{1-6b}+5\right)f+30e\right)\int \frac{}{\left(\left(1-\sqrt{1\ldots}\right.\right.}}{}\ldots\right.}{\left(54bx-(1-6b)^{3/2}-9\ldots\right.}$$

$\downarrow$ 52

$$\frac{\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)^3\left(6(1-6b)x-\left(2\sqrt{1-6b}+1\right)(1-6b)\right)^{3/2}\left(-\frac{\left(\left(7\sqrt{1-6b}+5\right)f+30e\right)\left(-\frac{\int \frac{}{\left(\left(1-\ldots\right.\right.}}{}\ldots\right.}{}\ldots\right.}{\ldots}$$

$\downarrow$ 61

$$\frac{\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)^3\left(6(1-6b)x-\left(2\sqrt{1-6b}+1\right)(1-6b)\right)^{3/2}\left(-\frac{\left(\left(7\sqrt{1-6b}+5\right)f+30e\right)\left(-\frac{}{9(1-6\ldots}\ldots\right.}{}\ldots\right.}{\ldots}$$

$\downarrow$ 73

$$\frac{\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)^3\left(6(1-6b)x-\left(2\sqrt{1-6b}+1\right)(1-6b)\right)^{3/2}\left(-\frac{\left(\left(7\sqrt{1-6b}+5\right)f+30e\right)\left(-\frac{9(1-6\cdots}{\cdots}\cdots\right.}{\cdots}\cdots\right.}{\cdots}$$

$\big\downarrow$ 217

$$\frac{\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)^3\left(6(1-6b)x-\left(2\sqrt{1-6b}+1\right)(1-6b)\right)^{3/2}\left(-\frac{\left(-\frac{\arctan\left(\frac{\sqrt{6(1-6b)x-(2\sqrt{1-6b}\cdots}}{\sqrt{3}(1-6b)^{3/\cdots}}\right)}{9\sqrt{3}(1-6b)^{13/4}}\cdots\right.}{\cdots}\cdots\right.}{\cdots}$$

input

```
Int[(e + f*x)/(1 - (1 - 6*b)^(3/2) - 9*b + 54*b*x - 54*x^2 + 108*x^3)^(3/2
),x]
```

output

```
(((1 - Sqrt[1 - 6*b])*(1 - 6*b) - 6*(1 - 6*b)*x)^3*(-((1 + 2*Sqrt[1 - 6*b]
)*(1 - 6*b)) + 6*(1 - 6*b)*x)^(3/2)*(-1/216*(6*e + f - Sqrt[1 - 6*b]*f)/((
1 - 6*b)^(5/2)*((1 - Sqrt[1 - 6*b])*(1 - 6*b) - 6*(1 - 6*b)*x)^2*Sqrt[-((1
 + 2*Sqrt[1 - 6*b])*(1 - 6*b)) + 6*(1 - 6*b)*x]) - ((30*e + (5 + 7*Sqrt[1
- 6*b])*f)*(-1/18*1/((1 - 6*b)^(5/2)*((1 - Sqrt[1 - 6*b])*(1 - 6*b) - 6*(1
 - 6*b)*x)*Sqrt[-((1 + 2*Sqrt[1 - 6*b])*(1 - 6*b)) + 6*(1 - 6*b)*x]) - (1/
(9*(1 - 6*b)^(5/2)*Sqrt[-((1 + 2*Sqrt[1 - 6*b])*(1 - 6*b)) + 6*(1 - 6*b)*x
]) + ArcTan[Sqrt[-((1 + 2*Sqrt[1 - 6*b])*(1 - 6*b)) + 6*(1 - 6*b)*x]/(Sqrt
[3]*(1 - 6*b)^(3/4))]/(9*Sqrt[3]*(1 - 6*b)^(13/4)))/(2*(1 - 6*b)^(3/2))))/
(72*(1 - 6*b)^(3/2))))/(1 - (1 - 6*b)^(3/2) - 9*b + 54*b*x - 54*x^2 + 108*
x^3)^(3/2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 2489
```
Int[((e_.) + (f_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(
x_)^3)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2 + d*x^3)^p/((c^3 - 4*b*c*d
+ 9*a*d^2 + d*(c^2 - 3*b*d)*x)^p*(b*c - 9*a*d + 2*(c^2 - 3*b*d)*x)^(2*p))
 Int[(e + f*x)^m*(c^3 - 4*b*c*d + 9*a*d^2 + d*(c^2 - 3*b*d)*x)^p*(b*c - 9*a
*d + 2*(c^2 - 3*b*d)*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x]
 && NeQ[c^2 - 3*b*d, 0] && EqQ[b^2*c^2 - 4*a*c^3 - 4*b^3*d + 18*a*b*c*d - 2
7*a^2*d^2, 0] &&  !IntegerQ[p]
```

Maple [F]

$$\int \frac{fx+e}{\left(1-(1-6b)^{\frac{3}{2}}-9b+54bx-54x^2+108x^3\right)^{\frac{3}{2}}}dx$$

input
```
int((f*x+e)/(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3)^(3/2),x)
```

output
```
int((f*x+e)/(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3)^(3/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1087 vs. $2(388) = 776$.

Time = 0.28 (sec) , antiderivative size = 2326, normalized size of antiderivative = 5.05

$$\int \frac{e+fx}{(1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)/(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3)^(3/2),x, alg
orithm="fricas")
```

output
```
[1/5832*(9*(2592*x^8 + 1512*(2*b + 1)*x^6 - 3456*x^7 - 168*(18*b + 1)*x^5
+ 20*(54*b^2 + 45*b - 2)*x^4 + 8*b^4 - 8*(90*b^2 + 5*b - 1)*x^3 - b^3 + 6*
(26*b^3 + 17*b^2 - 2*b)*x^2 - 2*(26*b^3 - 3*b^2)*x)*sqrt(70*(6*b - 1)*e*f
+ 35/3*(6*b - 1)*f^2 + 1/3*((147*b - 37)*f^2 - 450*e^2 - 150*e*f)*sqrt(-6*
b + 1))*log(-(324*((49*b - 4)*f^2 + 150*e^2 + 50*e*f)*x^4 - 216*((49*b - 4
)*f^2 + 150*e^2 + 50*e*f)*x^3 - 1350*(5*b^2 - b)*e^2 - 450*(5*b^2 - b)*e*f
 - 9*(245*b^3 - 69*b^2 + 4*b)*f^2 - 18*(150*(12*b - 5)*e^2 + 50*(12*b - 5)
*e*f + (588*b^2 - 293*b + 20)*f^2)*x^2 - sqrt(70*(6*b - 1)*e*f + 35/3*(6*b
 - 1)*f^2 + 1/3*((147*b - 37)*f^2 - 450*e^2 - 150*e*f)*sqrt(-6*b + 1))*(90
*(6*e + f)*x^2 - 30*(3*b - 1)*e + (27*b - 2)*f - 12*((21*b - 1)*f + 15*e)*
x - (126*f*x^2 - (21*b - 2)*f + 12*(15*e - f)*x - 30*e)*sqrt(-6*b + 1))*sq
rt(108*x^3 + 54*b*x - 54*x^2 + (6*b - 1)*sqrt(-6*b + 1) - 9*b + 1) + 18*(1
50*(4*b - 1)*e^2 + 50*(4*b - 1)*e*f + (196*b^2 - 65*b + 4)*f^2)*x - 9*(36*
((49*b - 4)*f^2 + 150*e^2 + 50*e*f)*x^3 - 150*b*e^2 - 50*b*e*f - (49*b^2 -
 4*b)*f^2 - 18*((49*b - 4)*f^2 + 150*e^2 + 50*e*f)*x^2 + 2*(150*(3*b + 1)*
e^2 + 50*(3*b + 1)*e*f + (147*b^2 + 37*b - 4)*f^2)*x)*sqrt(-6*b + 1))/(36*
x^4 + 4*(3*b + 1)*x^2 - 24*x^3 + b^2 - 4*b*x)) + (9072*(6*b - 1)*f*x^6 - 9
072*(6*b - 1)*f*x^5 + 36*(792*b^2 + 366*b - 83)*f*x^4 - 72*(8*(36*b^2 - 12
*b + 1)*e + (312*b^2 - 34*b - 3)*f)*x^3 + 36*(8*(36*b^2 - 12*b + 1)*e + (1
8*b^3 + 171*b^2 - 41*b + 2)*f)*x^2 + 4*(360*b^3 - 156*b^2 + 22*b - 1)*e...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e+fx}{\left(1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3\right)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)/(1-(1-6*b)**(3/2)-9*b+54*b*x-54*x**2+108*x**3)**(3/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{e+fx}{\left(1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3\right)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((f*x+e)/(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3)^(3/2),x, alg
orithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F(-2)]

Exception generated.

$$\int \frac{e+fx}{\left(1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3\right)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((f*x+e)/(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3)^(3/2),x, alg
orithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{%%{[17794363111902506549001421627856069105221941539635
2,0]:[1,0
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e+fx}{\left(1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3\right)^{3/2}}\,dx=\int \frac{e+f\,x}{\left(54\,b\,x-9\,b-(1-6\,b)^{3/2}-54\,x^2+108\,x^3+\right.}$$

input
```
int((e + f*x)/(54*b*x - 9*b - (1 - 6*b)^(3/2) - 54*x^2 + 108*x^3 + 1)^(3/2
),x)
```

output
```
int((e + f*x)/(54*b*x - 9*b - (1 - 6*b)^(3/2) - 54*x^2 + 108*x^3 + 1)^(3/2
), x)
```

Reduce [F]

$$\int \frac{e+fx}{\left(1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3\right)^{3/2}}\,dx=\int \frac{fx+e}{\left(1-(-6b+1)^{\frac{3}{2}}-9b+54bx-54x^2+108x^3\right.}$$

input
```
int((f*x+e)/(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3)^(3/2),x)
```

output
```
int((f*x+e)/(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3)^(3/2),x)
```

3.162 $\int \frac{1}{\left(1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3\right)^{3/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 399

$$\int \frac{1}{\left(1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3\right)^{3/2}}\,dx = -\frac{\sqrt{1-6b}\left(1-\frac{1-6x}{\sqrt{1-6b}}\right)\left(2+\frac{1-6x}{\sqrt{1-6b}}\right)}{9\sqrt{2}\left(-2(1-6b)^{3/2}+3(1-6b)(1-6x)-(1-6x)^3\right)^{3/2}} - \frac{5\sqrt{1-6b}\left(1-\frac{1-6x}{\sqrt{1-6b}}\right)^2\left(2+\frac{1-6x}{\sqrt{1-6b}}\right)}{54\sqrt{2}\left(-2(1-6b)^{3/2}+3(1-6b)(1-6x)-(1-6x)^3\right)^{3/2}} + \frac{5\sqrt{1-6b}\left(1-\frac{1-6x}{\sqrt{1-6b}}\right)^3\left(2+\frac{1-6x}{\sqrt{1-6b}}\right)}{54\sqrt{2}\left(-2(1-6b)^{3/2}+3(1-6b)(1-6x)-(1-6x)^3\right)^{3/2}} - \frac{5\sqrt{1-6b}\left(1-\frac{1-6x}{\sqrt{1-6b}}\right)^3\left(2+\frac{1-6x}{\sqrt{1-6b}}\right)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{2+\frac{1-6x}{\sqrt{1-6b}}}}{\sqrt{3}}\right)}{54\sqrt{6}\left(-2(1-6b)^{3/2}+3(1-6b)(1-6x)-(1-6x)^3\right)^{3/2}}$$

output
```
-1/18*(1-6*b)^(1/2)*(1-(1-6*x)/(1-6*b)^(1/2))*(2+(1-6*x)/(1-6*b)^(1/2))*2^
(1/2)/(-2*(1-6*b)^(3/2)+3*(1-6*b)*(1-6*x)-(1-6*x)^3)^(3/2)-5/108*(1-6*b)^(
1/2)*(1-(1-6*x)/(1-6*b)^(1/2))^2*(2+(1-6*x)/(1-6*b)^(1/2))*2^(1/2)/(-2*(1-
6*b)^(3/2)+3*(1-6*b)*(1-6*x)-(1-6*x)^3)^(3/2)+5/108*(1-6*b)^(1/2)*(1-(1-6*
x)/(1-6*b)^(1/2))^3*(2+(1-6*x)/(1-6*b)^(1/2))*2^(1/2)/(-2*(1-6*b)^(3/2)+3*
(1-6*b)*(1-6*x)-(1-6*x)^3)^(3/2)-5/324*(1-6*b)^(1/2)*(1-(1-6*x)/(1-6*b)^(1
/2))^3*(2+(1-6*x)/(1-6*b)^(1/2))^(3/2)*arctanh(1/3*(2+(1-6*x)/(1-6*b)^(1/2
))^(1/2)*3^(1/2))*6^(1/2)/(-2*(1-6*b)^(3/2)+3*(1-6*b)*(1-6*x)-(1-6*x)^3)^(
3/2)
```

Mathematica [F]

$$\int \frac{1}{\left(1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3\right)^{3/2}}\,dx=\int \frac{1}{\left(1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3\right)^{3/2}}\,dx$$

input
```
Integrate[(1 - (1 - 6*b)^(3/2) - 9*b + 54*b*x - 54*x^2 + 108*x^3)^(-3/2),x
]
```

output
```
Integrate[(1 - (1 - 6*b)^(3/2) - 9*b + 54*b*x - 54*x^2 + 108*x^3)^(-3/2),
x]
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 392, normalized size of antiderivative = 0.98, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.206, Rules used = {2480, 27, 52, 52, 61, 73, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(54bx-(1-6b)^{3/2}-9b+108x^3-54x^2+1\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 2480}$$

$$-\frac{7808611824626688\sqrt{2}\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)^3\left(6(1-6b)x-\left(2\sqrt{1-6b}+1\right)(1-6b)\right)^{3/2}\int -\frac{\ldots}{78\ldots}}{\left(54bx-(1-6b)^{3/2}-9b+108x^3-54x^2+\ldots\right.}$$

$\downarrow$ 27

$$\frac{\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)^3\left(6(1-6b)x-\left(2\sqrt{1-6b}+1\right)(1-6b)\right)^{3/2}\int \frac{\ldots}{\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)^3\ldots}}{\left(54bx-(1-6b)^{3/2}-9b+108x^3-54x^2+1\right)^{3/2}}$$

$\downarrow$ 52

$$\frac{\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)^3\left(6(1-6b)x-\left(2\sqrt{1-6b}+1\right)(1-6b)\right)^{3/2}\left(-\frac{5\int \frac{\ldots}{\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)\ldots}}{12(1\ldots}\ldots\right.}{\left(54bx-(1-6b)^{3/2}-9b+108x\ldots\right.}$$

$\downarrow$ 52

$$\frac{\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)^3\left(6(1-6b)x-\left(2\sqrt{1-6b}+1\right)(1-6b)\right)^{3/2}\left(-\frac{5\left(-\frac{\int \ldots}{\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6\ldots\right.}\ldots\right.}{\ldots}\ldots\right.}{\left(5\ldots\right.}$$

$\downarrow$ 61

$$\frac{\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)^3\left(6(1-6b)x-\left(2\sqrt{1-6b}+1\right)(1-6b)\right)^{3/2}\left(-\frac{5\left(-\frac{\frac{1}{9(1-6b)^{5/2}\sqrt{6(1-6b)x-\left(2\sqrt{1\ldots}\right.}}}{\ldots}\ldots\right.}{\ldots}\ldots\right.}{\ldots}$$

$\downarrow$ 73

$$\frac{\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)^3\left(6(1-6b)x-\left(2\sqrt{1-6b}+1\right)(1-6b)\right)^{3/2}\left(-\frac{5\left(-\frac{1}{9(1-6b)^{5/2}\sqrt{6(1-6b)x-(2\sqrt{1}}}\cdots\right.}{\cdots}\right.}{\cdots}$$

$$\Big\downarrow \; 217$$

$$\frac{\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)^3\left(6(1-6b)x-\left(2\sqrt{1-6b}+1\right)(1-6b)\right)^{3/2}\left(-\frac{5\left(-\frac{\arctan\left(\frac{\sqrt{6(1-6b)x-(2\sqrt{1-6}}}{\sqrt{3}(1-6b)^{3}}\cdots\right)}{9\sqrt{3}(1-6b)^{13/4}}\cdots\right.}{\cdots}\right.}{\cdots}$$

input
```
Int[(1 - (1 - 6*b)^(3/2) - 9*b + 54*b*x - 54*x^2 + 108*x^3)^(-3/2),x]
```

output
```
(((1 - Sqrt[1 - 6*b])*(1 - 6*b) - 6*(1 - 6*b)*x)^3*(-((1 + 2*Sqrt[1 - 6*b]
)*(1 - 6*b)) + 6*(1 - 6*b)*x)^(3/2)*(-1/36*1/((1 - 6*b)^(5/2)*((1 - Sqrt[1
 - 6*b])*(1 - 6*b) - 6*(1 - 6*b)*x)^2*Sqrt[-((1 + 2*Sqrt[1 - 6*b])*(1 - 6*
b)) + 6*(1 - 6*b)*x]) - (5*(-1/18*1/((1 - 6*b)^(5/2)*((1 - Sqrt[1 - 6*b])*
(1 - 6*b) - 6*(1 - 6*b)*x)*Sqrt[-((1 + 2*Sqrt[1 - 6*b])*(1 - 6*b)) + 6*(1
- 6*b)*x]) - (1/(9*(1 - 6*b)^(5/2)*Sqrt[-((1 + 2*Sqrt[1 - 6*b])*(1 - 6*b))
 + 6*(1 - 6*b)*x]) + ArcTan[Sqrt[-((1 + 2*Sqrt[1 - 6*b])*(1 - 6*b)) + 6*(1
 - 6*b)*x]/(Sqrt[3]*(1 - 6*b)^(3/4))]/(9*Sqrt[3]*(1 - 6*b)^(13/4)))/(2*(1
- 6*b)^(3/2))))/(12*(1 - 6*b)^(3/2))))/(1 - (1 - 6*b)^(3/2) - 9*b + 54*b*x
 - 54*x^2 + 108*x^3)^(3/2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 2480
```
Int[(Px_)^(p_), x_Symbol] :> With[{a = Coeff[Px, x, 0], b = Coeff[Px, x, 1]
, c = Coeff[Px, x, 2], d = Coeff[Px, x, 3]}, Simp[Px^p/((c^3 - 4*b*c*d + 9*
a*d^2 + d*(c^2 - 3*b*d)*x)^p*(b*c - 9*a*d + 2*(c^2 - 3*b*d)*x)^(2*p))   Int
[(c^3 - 4*b*c*d + 9*a*d^2 + d*(c^2 - 3*b*d)*x)^p*(b*c - 9*a*d + 2*(c^2 - 3*
b*d)*x)^(2*p), x], x] /; EqQ[b^2*c^2 - 4*a*c^3 - 4*b^3*d + 18*a*b*c*d - 27*
a^2*d^2, 0] && NeQ[c^2 - 3*b*d, 0]] /; FreeQ[p, x] && PolyQ[Px, x, 3] &&  !
IntegerQ[p]
```

Maple [F]

$$\int \frac{1}{\left(1-(1-6b)^{\frac{3}{2}}-9b+54bx-54x^2+108x^3\right)^{\frac{3}{2}}}dx$$

input
```
int(1/(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3)^(3/2),x)
```

output
```
int(1/(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3)^(3/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 607, normalized size of antiderivative = 1.52

$$\int \frac{1}{(1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3)^(3/2),x, algorithm
="fricas")
```

output
```
1/2916*(270*sqrt(1/6)*(2592*x^8 + 1512*(2*b + 1)*x^6 - 3456*x^7 - 168*(18*
b + 1)*x^5 + 20*(54*b^2 + 45*b - 2)*x^4 + 8*b^4 - 8*(90*b^2 + 5*b - 1)*x^3
 - b^3 + 6*(26*b^3 + 17*b^2 - 2*b)*x^2 - 2*(26*b^3 - 3*b^2)*x)*(-6*b + 1)^
(1/4)*arctan(1/3*sqrt(1/6)*sqrt(108*x^3 + 54*b*x - 54*x^2 + (6*b - 1)*sqrt
(-6*b + 1) - 9*b + 1)*(36*x^2 + sqrt(-6*b + 1)*(6*x - 1) + 12*b - 12*x - 1
)*(-6*b + 1)^(1/4)/(72*x^4 + 2*(30*b + 1)*x^2 - 48*x^3 + 8*b^2 - 2*(10*b -
 1)*x - b)) - (288*(36*b^2 - 12*b + 1)*x^3 - 720*b^3 - 144*(36*b^2 - 12*b
+ 1)*x^2 + 312*b^2 + 4*(1080*b^3 - 324*b^2 + 18*b + 1)*x + (19440*x^6 + 10
8*(156*b + 49)*x^4 - 19440*x^5 - 72*(156*b - 1)*x^3 - 216*b^3 + 36*(99*b^2
 + 45*b - 4)*x^2 + 207*b^2 - 4*(297*b^2 - 21*b - 1)*x - 38*b + 2)*sqrt(-6*
b + 1) - 44*b + 2)*sqrt(108*x^3 + 54*b*x - 54*x^2 + (6*b - 1)*sqrt(-6*b +
1) - 9*b + 1))/(2592*(36*b^2 - 12*b + 1)*x^8 - 3456*(36*b^2 - 12*b + 1)*x^
7 + 1512*(72*b^3 + 12*b^2 - 10*b + 1)*x^6 + 288*b^6 - 168*(648*b^3 - 180*b
^2 + 6*b + 1)*x^5 - 132*b^5 + 20*(1944*b^4 + 972*b^3 - 558*b^2 + 69*b - 2)
*x^4 + 20*b^4 - 8*(3240*b^4 - 900*b^3 - 6*b^2 + 17*b - 1)*x^3 - b^3 + 6*(9
36*b^5 + 300*b^4 - 250*b^3 + 41*b^2 - 2*b)*x^2 - 2*(936*b^5 - 420*b^4 + 62
*b^3 - 3*b^2)*x)
```

Sympy [F]

$$\int \frac{1}{(1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3)^{3/2}}\,dx = \int \frac{1}{\left(54bx-9b+108x^3-54x^2-(1-6b)^{\frac{3}{2}}+1\right)$$

input
```
integrate(1/(1-(1-6*b)**(3/2)-9*b+54*b*x-54*x**2+108*x**3)**(3/2),x)
```

output
```
Integral((54*b*x - 9*b + 108*x**3 - 54*x**2 - (1 - 6*b)**(3/2) + 1)**(-3/2
), x)
```

Maxima [F]

$$\int \frac{1}{(1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3)^{3/2}}\,dx = \int \frac{1}{\left(108\,x^3+54\,bx-54\,x^2-(-6\,b+1)^{\frac{3}{2}}-9\,b+$$

input
```
integrate(1/(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3)^(3/2),x, algorithm
="maxima")
```

output
```
integrate((108*x^3 + 54*b*x - 54*x^2 - (-6*b + 1)^(3/2) - 9*b + 1)^(-3/2),
 x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3)^(3/2),x, algorithm
="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{%%{[177943631119025065490014216278560691052219415396352,0]:[1,0
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3)^{3/2}}\,dx = \int \frac{1}{\left(54\,b\,x-9\,b-(1-6\,b)^{3/2}-54\,x^2+108\,x^3+\right.}$$

input
```
int(1/(54*b*x - 9*b - (1 - 6*b)^(3/2) - 54*x^2 + 108*x^3 + 1)^(3/2),x)
```

output
```
int(1/(54*b*x - 9*b - (1 - 6*b)^(3/2) - 54*x^2 + 108*x^3 + 1)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3)^{3/2}}\,dx = \int \frac{1}{\left(1-(-6b+1)^{\frac{3}{2}}-9b+54bx-54x^2+108x^3\right.}$$

input
```
int(1/(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3)^(3/2),x)
```

output
```
int(1/(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3)^(3/2),x)
```

3.163 $\int \frac{1}{(e+fx)\left(1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3\right)^{3/2}}\,dx$

Optimal result

Integrand size = 42, antiderivative size = 846

$$\int \frac{1}{(e+fx)\left(1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3\right)^{3/2}}\,dx = \frac{\sqrt{2}\left(1-\frac{1-6x}{\sqrt{1-6b}}\right)\left(2+\right.}{3\left(f-\frac{6e+f}{\sqrt{1-6b}}\right)\left(-2(1-6b)^{3/2}+3(1-\right.}$$

$$-\frac{\sqrt{1-6b}\left(30e+\left(5-17\sqrt{1-6b}\right)f\right)\left(1-\frac{1-6x}{\sqrt{1-6b}}\right)^2\left(2+\frac{1-6x}{\sqrt{1-6b}}\right)}{9\sqrt{2}\left(6e+f-\sqrt{1-6b}f\right)^2\left(-2(1-6b)^{3/2}+3(1-6b)(1-6x)-(1-6x)^3\right)^{3/2}}$$

$$+\frac{\sqrt{1-6b}\left(180e^2+6\left(10-7\sqrt{1-6b}\right)ef-\left(5+7\sqrt{1-6b}-60b\right)f^2\right)\left(1-\frac{1-6x}{\sqrt{1-6b}}\right)^3\left(2+\frac{1-6x}{\sqrt{1-6b}}\right)}{9\sqrt{2}\left(6e+f-\sqrt{1-6b}f\right)^2\left(6e+f+2\sqrt{1-6b}f\right)\left(-2(1-6b)^{3/2}+3(1-6b)(1-6x)-(1-6x)^3\right)^{3/2}}$$

$$-\frac{\sqrt{\frac{2}{3}}\sqrt{1-6b}\left(540e^3+54\left(5-4\sqrt{1-6b}\right)e^2f+18\left(2-4\sqrt{1-6b}+3b\right)ef^2+\left(1+35\sqrt{1-6b}+\left(9-24\right.\right.\right.}{9\left(6e+f-\sqrt{1-6b}f\right)^3\left(6e+f+2\sqrt{1-6b}f\right)\left(-2(1-6b)^{3/2}+3(\right.}$$

$$+\frac{4\sqrt{2}(1-6b)^{9/4}f^{7/2}\left(1-\frac{1-6x}{\sqrt{1-6b}}\right)^3\left(2+\frac{1-6x}{\sqrt{1-6b}}\right)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt[4]{1-6b}\sqrt{f}\sqrt{2+\frac{1-6x}{\sqrt{1-6b}}}}{\sqrt{6e+f+2\sqrt{1-6b}f}}\right)}{\left(6e+f-\sqrt{1-6b}f\right)^3\left(6e+f+2\sqrt{1-6b}f\right)^{3/2}\left(-2(1-6b)^{3/2}+3(1-6b)(1-6x)-(1-6x)^3\right)^{3/2}}$$

output
```
1/3*2^(1/2)*(1-(1-6*x)/(1-6*b)^(1/2))*(2+(1-6*x)/(1-6*b)^(1/2))/(f-(6*e+f)
/(1-6*b)^(1/2))/(-2*(1-6*b)^(3/2)+3*(1-6*b)*(1-6*x)-(1-6*x)^3)^(3/2)-1/18*
(1-6*b)^(1/2)*(30*e+(5-17*(1-6*b)^(1/2))*f)*(1-(1-6*x)/(1-6*b)^(1/2))^2*(2
+(1-6*x)/(1-6*b)^(1/2))*2^(1/2)/(6*e+f-(1-6*b)^(1/2)*f)^2/(-2*(1-6*b)^(3/2
)+3*(1-6*b)*(1-6*x)-(1-6*x)^3)^(3/2)+1/18*(1-6*b)^(1/2)*(180*e^2+6*(10-7*(
1-6*b)^(1/2))*e*f-(5+7*(1-6*b)^(1/2)-60*b)*f^2)*(1-(1-6*x)/(1-6*b)^(1/2))^
3*(2+(1-6*x)/(1-6*b)^(1/2))*2^(1/2)/(6*e+f-(1-6*b)^(1/2)*f)^2/(6*e+f+2*(1-
6*b)^(1/2)*f)/(-2*(1-6*b)^(3/2)+3*(1-6*b)*(1-6*x)-(1-6*x)^3)^(3/2)-1/27*6^
(1/2)*(1-6*b)^(1/2)*(540*e^3+54*(5-4*(1-6*b)^(1/2))*e^2*f+18*(2-4*(1-6*b)^
(1/2)+3*b)*e*f^2+(1+35*(1-6*b)^(1/2)+(9-246*(1-6*b)^(1/2))*b)*f^3)*(1-(1-6
*x)/(1-6*b)^(1/2))^3*(2+(1-6*x)/(1-6*b)^(1/2))^(3/2)*arctanh(1/3*(2+(1-6*x
)/(1-6*b)^(1/2))^(1/2)*3^(1/2))/(6*e+f-(1-6*b)^(1/2)*f)^3/(6*e+f+2*(1-6*b)
^(1/2)*f)/(-2*(1-6*b)^(3/2)+3*(1-6*b)*(1-6*x)-(1-6*x)^3)^(3/2)+4*2^(1/2)*(
1-6*b)^(9/4)*f^(7/2)*(1-(1-6*x)/(1-6*b)^(1/2))^3*(2+(1-6*x)/(1-6*b)^(1/2))
^(3/2)*arctanh((1-6*b)^(1/4)*f^(1/2)*(2+(1-6*x)/(1-6*b)^(1/2))^(1/2)/(6*e+
f+2*(1-6*b)^(1/2)*f)^(1/2))/(6*e+f-(1-6*b)^(1/2)*f)^3/(6*e+f+2*(1-6*b)^(1/
2)*f)^(3/2)/(-2*(1-6*b)^(3/2)+3*(1-6*b)*(1-6*x)-(1-6*x)^3)^(3/2)
```

Mathematica [F(-1)]

Timed out.

$$\int \frac{1}{(e+fx)\left(1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3\right)^{3/2}}\,dx=\$\text{Aborted}$$

input
```
Integrate[1/((e + f*x)*(1 - (1 - 6*b)^(3/2) - 9*b + 54*b*x - 54*x^2 + 108*
x^3)^(3/2)),x]
```

output
```
$Aborted
```

Rubi [A] (verified)

Time = 2.38 (sec) , antiderivative size = 806, normalized size of antiderivative = 0.95, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2489, 27, 114, 27, 168, 27, 169, 27, 174, 73, 217, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(54bx-(1-6b)^{3/2}-9b+108x^3-54x^2+1\right)^{3/2}(e+fx)}dx$$

$$\Big\downarrow \text{2489}$$

$$-\frac{7808611824626688\sqrt{2}\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)^3\left(6(1-6b)x-\left(2\sqrt{1-6b}+1\right)(1-6b)\right)^{3/2}\int -\frac{}{78\cdots}}{\left(54bx-(1-6b)^{3/2}-9b+108x^3-54x\cdots\right.}$$

$$\Big\downarrow \text{27}$$

$$\frac{\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)^3\left(6(1-6b)x-\left(2\sqrt{1-6b}+1\right)(1-6b)\right)^{3/2}\int \frac{}{\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)^3\cdots}}{\left(54bx-(1-6b)^{3/2}-9b+108x^3-54x^2+1\right)^{3/2}}$$

$$\Big\downarrow \text{114}$$

$$\frac{\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)^3\left(6(1-6b)x-\left(2\sqrt{1-6b}+1\right)(1-6b)\right)^{3/2}\left(\frac{\int -\frac{18(1-6b)^2\cdots}{\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)^2\cdots}}{36(1-6b)^{7/2}}\cdots\right.}{\left(54bx-(1-6b)^{3/2}-\cdots\right.}$$

$$\Big\downarrow \text{27}$$

$$\frac{\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)^3\left(6(1-6b)x-\left(2\sqrt{1-6b}+1\right)(1-6b)\right)^{3/2}\left(-\frac{\int \frac{5e-\cdots}{\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)^2\cdots}}{2(1-6b)^{3/2}\cdots}\cdots\right.}{\left(54bx-(1-6b)^{3/2}-\cdots\right.}$$

$$\Big\downarrow \text{168}$$

$$\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)^3\left(6(1-6b)x-\left(2\sqrt{1-6b}+1\right)(1-6b)\right)^{3/2}\left(-\frac{\frac{\int -\frac{9(1-6b)^2\left(30e^2+(5-17\sqrt{1-6b})\right.}{\left((1-\sqrt{1-6b})(1-6b)-6(1-6b)x\right)}}{18(1-6b)^7}}{}\right.$$

$$\downarrow 27$$

$$\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)^3\left(6(1-6b)x-\left(2\sqrt{1-6b}+1\right)(1-6b)\right)^{3/2}\left(-\frac{-\frac{\int \frac{30e^2+(5-17\sqrt{1-6b})fe+}{\left((1-\sqrt{1-6b})(1-6b)-6(1-6b)x\right.}}{2(1-6b)}}{}\right.$$

$$\downarrow 169$$

$$\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)^3\left(6(1-6b)x-\left(2\sqrt{1-6b}+1\right)(1-6b)\right)^{3/2}\left(-\frac{-\frac{\int -\frac{3(1-6b)^2\left(180e^3+6(10-7\sqrt{1-}\right.}{}}{}}{}\right.$$

$$\downarrow 27$$

$$\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)^3\left(6(1-6b)x-\left(2\sqrt{1-6b}+1\right)(1-6b)\right)^{3/2}\left(-\frac{-\frac{6(10-7\sqrt{1-6b})ef-(-60b-}{9(1-6b)^{5/2}\sqrt{6(1-6b)x-(2\sqrt{1-6}}}}{}\right.$$

$$\downarrow 174$$

$$\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)^3\left(6(1-6b)x-\left(2\sqrt{1-6b}+1\right)(1-6b)\right)^{3/2}\left(-\frac{-\frac{6\left(10-7\sqrt{1-6b}\right)ef-(-60b-\cdots}{9(1-6b)^{5/2}\sqrt{6(1-6b)x-\left(2\sqrt{1-6\cdots}\right.}}}{\cdots}\right.$$

$\downarrow$ 73

$$\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)^3\left(6(1-6b)x-\left(2\sqrt{1-6b}+1\right)(1-6b)\right)^{3/2}\left(-\frac{-\frac{\cdots}{18(1-6b)^{5/2}\left(6e+\left(1-\sqrt{1-6b}\right)f\right)(\cdots}}{\cdots}\right.$$

$\downarrow$ 217

$$\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)^3\left(6(1-6b)x-\left(2\sqrt{1-6b}+1\right)(1-6b)\right)^{3/2}\left(-\frac{-\frac{\cdots}{18(1-6b)^{5/2}\left(6e+\left(1-\sqrt{1-6b}\right)f\right)(\cdots}}{\cdots}\right.$$

$\downarrow$ 218

$$\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)^3\left(6(1-6b)x-\left(2\sqrt{1-6b}+1\right)(1-6b)\right)^{3/2}\left(-\frac{-}{18(1-6b)^{5/2}\left(6e+\left(1-\sqrt{1-6b}\right)f\right)(\cdots}\right.$$

input
```
Int[1/((e + f*x)*(1 - (1 - 6*b)^(3/2) - 9*b + 54*b*x - 54*x^2 + 108*x^3)^(
3/2)),x]
```

output
```
(((1 - Sqrt[1 - 6*b])*(1 - 6*b) - 6*(1 - 6*b)*x)^3*(-((1 + 2*Sqrt[1 - 6*b]
)*(1 - 6*b)) + 6*(1 - 6*b)*x)^(3/2)*(-1/6*1/((1 - 6*b)^(5/2)*(6*e + (1 - S
qrt[1 - 6*b])*f)*((1 - Sqrt[1 - 6*b])*(1 - 6*b) - 6*(1 - 6*b)*x)^2*Sqrt[-(
(1 + 2*Sqrt[1 - 6*b])*(1 - 6*b)) + 6*(1 - 6*b)*x]) - (-1/18*(30*e + (5 - 1
7*Sqrt[1 - 6*b])*f)/((1 - 6*b)^(5/2)*(6*e + (1 - Sqrt[1 - 6*b])*f)*((1 - S
qrt[1 - 6*b])*(1 - 6*b) - 6*(1 - 6*b)*x)*Sqrt[-((1 + 2*Sqrt[1 - 6*b])*(1 -
 6*b)) + 6*(1 - 6*b)*x]) - ((180*e^2 + 6*(10 - 7*Sqrt[1 - 6*b])*e*f - (5 +
 7*Sqrt[1 - 6*b] - 60*b)*f^2)/(9*(1 - 6*b)^(5/2)*(6*e + f + 2*Sqrt[1 - 6*b
]*f)*Sqrt[-((1 + 2*Sqrt[1 - 6*b])*(1 - 6*b)) + 6*(1 - 6*b)*x]) - ((-2*(540
*e^3 + 54*(5 - 4*Sqrt[1 - 6*b])*e^2*f + 18*(2 - 4*Sqrt[1 - 6*b] + 3*b)*e*f
^2 + (1 + 35*Sqrt[1 - 6*b] + (9 - 246*Sqrt[1 - 6*b])*b)*f^3)*ArcTan[Sqrt[-
((1 + 2*Sqrt[1 - 6*b])*(1 - 6*b)) + 6*(1 - 6*b)*x]/(Sqrt[3]*(1 - 6*b)^(3/4
))])/(3*Sqrt[3]*(1 - 6*b)^(7/4)*(6*e + f - Sqrt[1 - 6*b]*f)) + (24*f^(7/2)
*ArcTan[(Sqrt[f]*Sqrt[-((1 + 2*Sqrt[1 - 6*b])*(1 - 6*b)) + 6*(1 - 6*b)*x])
/(Sqrt[1 - 6*b]*Sqrt[6*e + f + 2*Sqrt[1 - 6*b]*f])])/((6*e + f - Sqrt[1 -
6*b]*f)*Sqrt[6*e + f + 2*Sqrt[1 - 6*b]*f]))/(3*(1 - 6*b)^(3/2)*(6*e + f +
2*Sqrt[1 - 6*b]*f)))/(2*(1 - 6*b)^(3/2)*(6*e + (1 - Sqrt[1 - 6*b])*f)))/(2
*(1 - 6*b)^(3/2)*(6*e + (1 - Sqrt[1 - 6*b])*f))))/(1 - (1 - 6*b)^(3/2) - 9
*b + 54*b*x - 54*x^2 + 108*x^3)^(3/2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 2489
```
Int[((e_.) + (f_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(
x_)^3)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2 + d*x^3)^p/((c^3 - 4*b*c*d
+ 9*a*d^2 + d*(c^2 - 3*b*d)*x)^p*(b*c - 9*a*d + 2*(c^2 - 3*b*d)*x)^(2*p))
 Int[(e + f*x)^m*(c^3 - 4*b*c*d + 9*a*d^2 + d*(c^2 - 3*b*d)*x)^p*(b*c - 9*a
*d + 2*(c^2 - 3*b*d)*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x]
 && NeQ[c^2 - 3*b*d, 0] && EqQ[b^2*c^2 - 4*a*c^3 - 4*b^3*d + 18*a*b*c*d - 2
7*a^2*d^2, 0] &&  !IntegerQ[p]
```

Maple [F]

$$\int \frac{1}{(fx+e)\left(1-(1-6b)^{\frac{3}{2}}-9b+54bx-54x^2+108x^3\right)^{\frac{3}{2}}}dx$$

input
```
int(1/(f*x+e)/(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3)^(3/2),x)
```

output
```
int(1/(f*x+e)/(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3)^(3/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10360 vs. 2(717) = 1434.

Time = 24.61 (sec) , antiderivative size = 43371, normalized size of antiderivative = 51.27

$$\int \frac{1}{(e+fx)\left(1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(f*x+e)/(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3)^(3/2),x, a
lgorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{1}{(e+fx)\left(1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3\right)^{3/2}}\,dx = \int \frac{1}{(e+fx)\left(54bx+6b\sqrt{1-6b}-9b+\right.}$$

input
```
integrate(1/(f*x+e)/(1-(1-6*b)**(3/2)-9*b+54*b*x-54*x**2+108*x**3)**(3/2),
x)
```

output
```
Integral(1/((e + f*x)*(54*b*x + 6*b*sqrt(1 - 6*b) - 9*b + 108*x**3 - 54*x*
*2 - sqrt(1 - 6*b) + 1)**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{(e+fx)\left(1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3\right)^{3/2}}\,dx = \int \frac{1}{\left(108\,x^3+54\,bx-54\,x^2-(-6\,b+1\right.}$$

input
```
integrate(1/(f*x+e)/(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3)^(3/2),x, a
lgorithm="maxima")
```

output
```
integrate(1/((108*x^3 + 54*b*x - 54*x^2 - (-6*b + 1)^(3/2) - 9*b + 1)^(3/2
)*(f*x + e)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(e+fx)\left(1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3\right)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(f*x+e)/(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3)^(3/2),x, a
lgorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(e+fx)\left(1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3\right)^{3/2}}\,dx = \int \frac{1}{(e+f\,x)\,\left(54\,b\,x-9\,b-(1-6\,b)^{3/2}\right.}$$

input
```
int(1/((e + f*x)*(54*b*x - 9*b - (1 - 6*b)^(3/2) - 54*x^2 + 108*x^3 + 1)^(
3/2)),x)
```

output
```
int(1/((e + f*x)*(54*b*x - 9*b - (1 - 6*b)^(3/2) - 54*x^2 + 108*x^3 + 1)^(
3/2)), x)
```

Reduce [F]

$$\int \frac{1}{(e+fx)\left(1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3\right)^{3/2}}\,dx = \int \frac{1}{(fx+e)\left(1-(-6b+1)^{\frac{3}{2}}-9b+54b\right.}$$

input
```
int(1/(f*x+e)/(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3)^(3/2),x)
```

output
```
int(1/(f*x+e)/(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3)^(3/2),x)
```

3.164 $\int \frac{1}{(e+fx)^2\left(1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3\right)^{3/2}}\,dx$

Optimal result

Integrand size = 42, antiderivative size = 1153

$$\int \frac{1}{(e+fx)^2\left(1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3\right)^{3/2}}\,dx = \text{Too large to display}$$

output
```
1/3*2^(1/2)*(1080*(1-6*b)^(1/2)*e^3-108*(3-5*(1-6*b)^(1/2)-18*b)*e^2*f-54*
(2+(1-6*b)^(1/2)-4*(3+4*(1-6*b)^(1/2))*b)*e*f^2-(89+19*(1-6*b)^(1/2)-6*(16
9+24*(1-6*b)^(1/2))*b+2880*b^2)*f^3)*(1-(1-6*x)/(1-6*b)^(1/2))^3*(2+(1-6*x
)/(1-6*b)^(1/2))/(6*e+f-(1-6*b)^(1/2)*f)^3/(6*e+f+2*(1-6*b)^(1/2)*f)^2/(-2
*(1-6*b)^(3/2)+3*(1-6*b)*(1-6*x)-(1-6*x)^3)^(3/2)+1/3*2^(1/2)*(1-(1-6*x)/(
1-6*b)^(1/2))*(2+(1-6*x)/(1-6*b)^(1/2))/(f-(6*e+f)/(1-6*b)^(1/2))/(-2*(1-6
*b)^(3/2)+3*(1-6*b)*(1-6*x)-(1-6*x)^3)^(3/2)/(f*x+e)-1/18*(1-6*b)^(1/2)*(3
0*e+(5-23*(1-6*b)^(1/2))*f)*(1-(1-6*x)/(1-6*b)^(1/2))^2*(2+(1-6*x)/(1-6*b)
^(1/2))*2^(1/2)/(6*e+f-(1-6*b)^(1/2)*f)^2/(-2*(1-6*b)^(3/2)+3*(1-6*b)*(1-6
*x)-(1-6*x)^3)^(3/2)/(f*x+e)-1/18*(1-6*b)*f*(180*e^2+6*(10-19*(1-6*b)^(1/2
))*e*f-(89+19*(1-6*b)^(1/2)-564*b)*f^2)*(1-(1-6*x)/(1-6*b)^(1/2))^3*(2+(1-
6*x)/(1-6*b)^(1/2))*2^(1/2)/(6*e+f-(1-6*b)^(1/2)*f)^3/(6*e+f+2*(1-6*b)^(1/
2)*f)/(-2*(1-6*b)^(3/2)+3*(1-6*b)*(1-6*x)-(1-6*x)^3)^(3/2)/(f*x+e)-2/9*6^(
1/2)*(1-6*b)^(1/2)*(3240*e^4+216*(10-7*(1-6*b)^(1/2))*e^3*f+54*(5-14*(1-6*
b)^(1/2)+30*b)*e^2*f^2-6*(5-113*(1-6*b)^(1/2)-(90-804*(1-6*b)^(1/2))*b)*e*
f^3+(197+127*(1-6*b)^(1/2)-3*(793+268*(1-6*b)^(1/2))*b+7272*b^2)*f^4)*(1-(
1-6*x)/(1-6*b)^(1/2))^3*(2+(1-6*x)/(1-6*b)^(1/2))^(3/2)*arctanh(1/3*(2+(1-
6*x)/(1-6*b)^(1/2))^(1/2)*3^(1/2))/(6*e+f-(1-6*b)^(1/2)*f)^4/(6*e+f+2*(1-6
*b)^(1/2)*f)^2/(-2*(1-6*b)^(3/2)+3*(1-6*b)*(1-6*x)-(1-6*x)^3)^(3/2)+108*2^
(1/2)*(1-6*b)^(9/4)*f^(7/2)*(6*e+f+(1-6*b)^(1/2)*f)*(1-(1-6*x)/(1-6*b)^...
```

Mathematica [F]

$$\int \frac{1}{(e+fx)^2\left(1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3\right)^{3/2}}\,dx=\int \frac{1}{(e+fx)^2\left(1-(1-6b)^{3/2}-9b+54\right.}$$

input
```
Integrate[1/((e + f*x)^2*(1 - (1 - 6*b)^(3/2) - 9*b + 54*b*x - 54*x^2 + 10
8*x^3)^(3/2)),x]
```

output
```
Integrate[1/((e + f*x)^2*(1 - (1 - 6*b)^(3/2) - 9*b + 54*b*x - 54*x^2 + 10
8*x^3)^(3/2)), x]
```

Rubi [A] (warning: unable to verify)

Time = 3.45 (sec) , antiderivative size = 1113, normalized size of antiderivative = 0.97, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2489, 27, 114, 27, 168, 27, 168, 27, 169, 27, 174, 73, 217, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(54bx-(1-6b)^{3/2}-9b+108x^3-54x^2+1\right)^{3/2}(e+fx)^2}dx$$

$$\Big\downarrow \; 2489$$

$$-\frac{7808611824626688\sqrt{2}\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)^3\left(6(1-6b)x-\left(2\sqrt{1-6b}+1\right)(1-6b)\right)^{3/2}\int -\frac{}{78\cdots}}{\left(54bx-(1-6b)^{3/2}-9b+108x^3-54x^2\cdots\right.}$$

$$\Big\downarrow \; 27$$

$$\frac{\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)^3\left(6(1-6b)x-\left(2\sqrt{1-6b}+1\right)(1-6b)\right)^{3/2}\int \frac{}{\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)^3\cdots}}{\left(54bx-(1-6b)^{3/2}-9b+108x^3-54x^2+1\right)^{3/2}}$$

$$\Big\downarrow \; 114$$

$$\frac{\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)^3\left(6(1-6b)x-\left(2\sqrt{1-6b}+1\right)(1-6b)\right)^{3/2}\left(\frac{\int -\frac{6\left(\left(15e-8\sqrt{1-6b}f\cdots\right.\right.}{\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)^2\cdots}}{36(1-6b)^{7/2}}\cdots\right.}{\left(54bx-(1-6b)^{3/2}\cdots\right.}$$

$$\Big\downarrow \; 27$$

$$\frac{\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)^3\left(6(1-6b)x-\left(2\sqrt{1-6b}+1\right)(1-6b)\right)^{3/2}\left(-\frac{\int \frac{\left(15e-8\sqrt{1-6b}f\cdots\right.}{\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)^2\cdots}}{6(1-6b)^{7/\cdots}}\cdots\right.}{\left(54bx-(1-6b)^{3/2}\cdots\right.}$$

$$\Big\downarrow \; 168$$

$$\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)^3\left(6(1-6b)x-\left(2\sqrt{1-6b}+1\right)(1-6b)\right)^{3/2}\left(-\frac{\int -\frac{3(1-6b)^4\left(270e^2+3(5-77\sqrt{1-6\cdots}\right.}{\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6\cdots\right.}}{1\cdots}\right.$$

$\downarrow$ 27

$$\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)^3\left(6(1-6b)x-\left(2\sqrt{1-6b}+1\right)(1-6b)\right)^{3/2}\left(-\frac{-\frac{\sqrt{1-6b}\int \frac{270e^2+3(5-77\sqrt{1-6b})f\cdots}{\left(\left(1-\sqrt{1-6b}\right)(1-6b)-\cdots\right.}}{\cdots}}{\cdots}\right.$$

$\downarrow$ 168

$$\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)^3\left(6(1-6b)x-\left(2\sqrt{1-6b}+1\right)(1-6b)\right)^{3/2}\left(-\frac{-\frac{\sqrt{1-6b}\left(\int \frac{54(1-6b)^2\left(180e^3+6(1\cdots\right.}{\cdots}\right.}{\cdots}}{\cdots}\right.$$

$\downarrow$ 27

$$\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)^3\left(6(1-6b)x-\left(2\sqrt{1-6b}+1\right)(1-6b)\right)^{3/2}\left(-\frac{-\frac{\sqrt{1-6b}\left(54\int \frac{180e^3+6(10-19\sqrt{1\cdots}}{\cdots}\right.}{\cdots}}{\cdots}\right.$$

$\downarrow$ 169

$$\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)^3\left(6(1-6b)x-\left(2\sqrt{1-6b}+1\right)(1-6b)\right)^{3/2}\left(-\frac{-\overline{6\sqrt{1-6b}\left(6e+\left(1-\sqrt{1-6b}\right)f\right)\left(\left(1-\sqrt{\right.\right.}}}{}\right.$$

$$\Big\downarrow \ 27$$

$$\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)^3\left(6(1-6b)x-\left(2\sqrt{1-6b}+1\right)(1-6b)\right)^{3/2}\left(-\frac{-\overline{6\sqrt{1-6b}\left(6e+\left(1-\sqrt{1-6b}\right)f\right)\left(\left(1-\sqrt{\right.\right.}}}{}\right.$$

$$\Big\downarrow \ 174$$

$$\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)^3\left(6(1-6b)x-\left(2\sqrt{1-6b}+1\right)(1-6b)\right)^{3/2}\left(-\frac{-\frac{}{6\sqrt{1-6b}\left(6e+\left(1-\sqrt{1-6b}\right)f\right)\left(\left(1-\sqrt{\right.\right.}}}{}\right.$$

$$\Big\downarrow 73$$

$$\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)^3\left(6(1-6b)x-\left(2\sqrt{1-6b}+1\right)(1-6b)\right)^{3/2}\left(-\frac{-\frac{}{6\sqrt{1-6b}\left(6e+\left(1-\sqrt{1-6b}\right)f\right)\left(\left(1-\sqrt{\right.\right.}}}{}\right.$$

$$\Big\downarrow 217$$

$$\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)^3\left(6(1-6b)x-\left(2\sqrt{1-6b}+1\right)(1-6b)\right)^{3/2}\left(-\frac{-\;\overline{}}{6\sqrt{1-6b}\left(6e+\left(1-\sqrt{1-6b}\right)f\right)\left(\left(1-\sqrt{\right.\right.}}\right.$$

↓ 218

$$\left(\left(1-\sqrt{1-6b}\right)(1-6b)-6(1-6b)x\right)^3\left(6(1-6b)x-\left(2\sqrt{1-6b}+1\right)(1-6b)\right)^{3/2}\left(-\frac{-}{6\sqrt{1-6b}\left(6e+\left(1-\sqrt{1-6b}\right)f\right)\left(\left(1-\sqrt{\right.\right.}}\right.$$

input

```
Int[1/((e + f*x)^2*(1 - (1 - 6*b)^(3/2) - 9*b + 54*b*x - 54*x^2 + 108*x^3)
^(3/2)),x]
```

output
```
(((1 - Sqrt[1 - 6*b])*(1 - 6*b) - 6*(1 - 6*b)*x)^3*(-((1 + 2*Sqrt[1 - 6*b]
)*(1 - 6*b)) + 6*(1 - 6*b)*x)^(3/2)*(-1/6*1/((1 - 6*b)^(5/2)*(6*e + (1 - S
qrt[1 - 6*b])*f)*((1 - Sqrt[1 - 6*b])*(1 - 6*b) - 6*(1 - 6*b)*x)^2*Sqrt[-(
(1 + 2*Sqrt[1 - 6*b])*(1 - 6*b)) + 6*(1 - 6*b)*x]*(e + f*x)) - (-1/6*(30*e
 + (5 - 23*Sqrt[1 - 6*b])*f)/(Sqrt[1 - 6*b]*(6*e + (1 - Sqrt[1 - 6*b])*f)*
((1 - Sqrt[1 - 6*b])*(1 - 6*b) - 6*(1 - 6*b)*x)*Sqrt[-((1 + 2*Sqrt[1 - 6*b
])*(1 - 6*b)) + 6*(1 - 6*b)*x]*(e + f*x)) - (Sqrt[1 - 6*b]*(-((f*(180*e^2
+ 6*(10 - 19*Sqrt[1 - 6*b])*e*f - (89 + 19*Sqrt[1 - 6*b] - 564*b)*f^2))/((
1 - 6*b)^2*(6*e + f - Sqrt[1 - 6*b]*f)*(6*e + f + 2*Sqrt[1 - 6*b]*f)*Sqrt[
-((1 + 2*Sqrt[1 - 6*b])*(1 - 6*b)) + 6*(1 - 6*b)*x]*(e + f*x))) + (54*((10
80*e^3 + 108*(5 - 3*Sqrt[1 - 6*b])*e^2*f - 54*(1 + 2*Sqrt[1 - 6*b] - 16*b)
*e*f^2 - (19 + 89*Sqrt[1 - 6*b] - 48*(3 + 10*Sqrt[1 - 6*b])*b)*f^3)/(9*(1
- 6*b)^(5/2)*(6*e + f + 2*Sqrt[1 - 6*b]*f)*Sqrt[-((1 + 2*Sqrt[1 - 6*b])*(1
 - 6*b)) + 6*(1 - 6*b)*x]) - ((-2*(3240*e^4 + 216*(10 - 7*Sqrt[1 - 6*b])*e
^3*f + 54*(5 - 14*Sqrt[1 - 6*b] + 30*b)*e^2*f^2 - 6*(5 - 113*Sqrt[1 - 6*b]
 - (90 - 804*Sqrt[1 - 6*b])*b)*e*f^3 + (197 + 127*Sqrt[1 - 6*b] - 3*(793 +
 268*Sqrt[1 - 6*b])*b + 7272*b^2)*f^4)*ArcTan[Sqrt[-((1 + 2*Sqrt[1 - 6*b])
*(1 - 6*b)) + 6*(1 - 6*b)*x]/(Sqrt[3]*(1 - 6*b)^(3/4))])/(3*Sqrt[3]*(1 - 6
*b)^(7/4)*(6*e + f - Sqrt[1 - 6*b]*f)) + (108*f^(7/2)*(6*Sqrt[1 - 6*b]*e +
 (1 + Sqrt[1 - 6*b] - 6*b)*f)*ArcTan[(Sqrt[f]*Sqrt[-((1 + 2*Sqrt[1 - 6*...
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 168

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 169

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 174

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 217

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 2489
```
Int[((e_.) + (f_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(
x_)^3)^(p_), x_Symbol] :> Simp[(a + b*x + c*x^2 + d*x^3)^p/((c^3 - 4*b*c*d
+ 9*a*d^2 + d*(c^2 - 3*b*d)*x)^p*(b*c - 9*a*d + 2*(c^2 - 3*b*d)*x)^(2*p))
 Int[(e + f*x)^m*(c^3 - 4*b*c*d + 9*a*d^2 + d*(c^2 - 3*b*d)*x)^p*(b*c - 9*a
*d + 2*(c^2 - 3*b*d)*x)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x]
 && NeQ[c^2 - 3*b*d, 0] && EqQ[b^2*c^2 - 4*a*c^3 - 4*b^3*d + 18*a*b*c*d - 2
7*a^2*d^2, 0] &&  !IntegerQ[p]
```

Maple [F]

$$\int \frac{1}{(fx+e)^2 \left(1-(1-6b)^{\frac{3}{2}}-9b+54bx-54x^2+108x^3\right)^{\frac{3}{2}}}dx$$

input
```
int(1/(f*x+e)^2/(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3)^(3/2),x)
```

output
```
int(1/(f*x+e)^2/(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3)^(3/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19567 vs. 2(990) = 1980.

Time = 129.87 (sec) , antiderivative size = 81003, normalized size of antiderivative = 70.25

$$\int \frac{1}{(e+fx)^2 (1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(f*x+e)^2/(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3)^(3/2),x,
 algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{1}{(e+fx)^2 \left(1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3\right)^{3/2}}\,dx = \int \frac{1}{(e+fx)^2 \left(54bx+6b\sqrt{1-6b}-9b\right.}$$

input
```
integrate(1/(f*x+e)**2/(1-(1-6*b)**(3/2)-9*b+54*b*x-54*x**2+108*x**3)**(3/
2),x)
```

output
```
Integral(1/((e + f*x)**2*(54*b*x + 6*b*sqrt(1 - 6*b) - 9*b + 108*x**3 - 54
*x**2 - sqrt(1 - 6*b) + 1)**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{(e+fx)^2 \left(1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3\right)^{3/2}}\,dx = \int \frac{1}{\left(108\,x^3+54\,bx-54\,x^2-(-6\,b+1\right.}$$

input
```
integrate(1/(f*x+e)^2/(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3)^(3/2),x,
 algorithm="maxima")
```

output
```
integrate(1/((108*x^3 + 54*b*x - 54*x^2 - (-6*b + 1)^(3/2) - 9*b + 1)^(3/2
)*(f*x + e)^2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(e+fx)^2\left(1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3\right)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(f*x+e)^2/(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3)^(3/2),x,
 algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(e+fx)^2\left(1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3\right)^{3/2}}\,dx = \text{Hanged}$$

input
```
int(1/((e + f*x)^2*(54*b*x - 9*b - (1 - 6*b)^(3/2) - 54*x^2 + 108*x^3 + 1)
^(3/2)),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{1}{(e+fx)^2\left(1-(1-6b)^{3/2}-9b+54bx-54x^2+108x^3\right)^{3/2}}\,dx = \int \frac{1}{(fx+e)^2\left(1-(-6b+1)^{\frac{3}{2}}-9b+5$$

input
```
int(1/(f*x+e)^2/(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3)^(3/2),x)
```

output
```
int(1/(f*x+e)^2/(1-(1-6*b)^(3/2)-9*b+54*b*x-54*x^2+108*x^3)^(3/2),x)
```

3.165 $\int (A+Bx+Cx^2)(2+3x-5x^2+x^3)^3\,dx$

Optimal result

Integrand size = 26, antiderivative size = 171

$$\begin{aligned}\int (A+Bx+Cx^2)\left(2+3x-5x^2+x^3\right)^3\,dx &= 8Ax+2(9A+2B)x^2-\frac{2}{3}(3A-18B-4C)x^3-\frac{3}{4}(47A+2B-12C)x^4\\&\quad+\frac{3}{5}(17A-47B-2C)x^5+\frac{1}{2}(64A+17B-47C)x^6-\frac{1}{7}(209A-192B-51C)x^7\\&\quad+\frac{1}{8}(84A-209B+192C)x^8-\frac{1}{9}(15A-84B+209C)x^9\\&\quad+\frac{1}{10}(A-15B+84C)x^{10}+\frac{1}{11}(B-15C)x^{11}+\frac{Cx^{12}}{12}\end{aligned}$$

output
```
8*A*x+2*(9*A+2*B)*x^2-2/3*(3*A-18*B-4*C)*x^3-3/4*(47*A+2*B-12*C)*x^4+3/5*(
17*A-47*B-2*C)*x^5+1/2*(64*A+17*B-47*C)*x^6-1/7*(209*A-192*B-51*C)*x^7+1/8
*(84*A-209*B+192*C)*x^8-1/9*(15*A-84*B+209*C)*x^9+1/10*(A-15*B+84*C)*x^10+
1/11*(B-15*C)*x^11+1/12*C*x^12
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.00

$$\begin{aligned}&\int (A+Bx+Cx^2)\left(2+3x-5x^2+x^3\right)^3\,dx\\&=8Ax+2(9A+2B)x^2-\frac{2}{3}(3A-18B-4C)x^3-\frac{3}{4}(47A+2B-12C)x^4\\&+\frac{3}{5}(17A-47B-2C)x^5+\frac{1}{2}(64A+17B-47C)x^6+\frac{1}{7}(-209A+192B+51C)x^7\\&+\frac{1}{8}(84A-209B+192C)x^8+\frac{1}{9}(-15A+84B-209C)x^9\\&+\frac{1}{10}(A-15B+84C)x^{10}+\frac{1}{11}(B-15C)x^{11}+\frac{Cx^{12}}{12}\end{aligned}$$

input
```
Integrate[(A + B*x + C*x^2)*(2 + 3*x - 5*x^2 + x^3)^3,x]
```

output
```
8*A*x + 2*(9*A + 2*B)*x^2 - (2*(3*A - 18*B - 4*C)*x^3)/3 - (3*(47*A + 2*B
- 12*C)*x^4)/4 + (3*(17*A - 47*B - 2*C)*x^5)/5 + ((64*A + 17*B - 47*C)*x^6
)/2 + ((-209*A + 192*B + 51*C)*x^7)/7 + ((84*A - 209*B + 192*C)*x^8)/8 + (
(-15*A + 84*B - 209*C)*x^9)/9 + ((A - 15*B + 84*C)*x^10)/10 + ((B - 15*C)*
x^11)/11 + (C*x^12)/12
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}=0.077$, Rules used = {2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(x^3-5x^2+3x+2\right)^3\left(A+Bx+Cx^2\right)\,dx$$

$$\Big\downarrow \text{ 2188}$$

$$\int \left(x^9(A-15B+84C)-x^8(15A-84B+209C)+x^7(84A-209B+192C)-x^6(209A-192B-51C)+3x^5\right.$$

$$\downarrow \text{2009}$$

$$\frac{1}{10}x^{10}(A-15B+84C)-\frac{1}{9}x^9(15A-84B+209C)+\frac{1}{8}x^8(84A-209B+192C)-\frac{1}{7}x^7(209A-192B-51C)+\frac{1}{2}x^6(64A+17B-47C)+\frac{3}{5}x^5(17A-47B-2C)-\frac{3}{4}x^4(47A+2B-12C)-\frac{2}{3}x^3(3A-18B-4C)+2x^2(9A+2B)+8Ax+\frac{1}{11}x^{11}(B-15C)+\frac{Cx^{12}}{12}$$

input

```
Int[(A + B*x + C*x^2)*(2 + 3*x - 5*x^2 + x^3)^3,x]
```

output

```
8*A*x + 2*(9*A + 2*B)*x^2 - (2*(3*A - 18*B - 4*C)*x^3)/3 - (3*(47*A + 2*B
 - 12*C)*x^4)/4 + (3*(17*A - 47*B - 2*C)*x^5)/5 + ((64*A + 17*B - 47*C)*x^6
)/2 - ((209*A - 192*B - 51*C)*x^7)/7 + ((84*A - 209*B + 192*C)*x^8)/8 - ((
15*A - 84*B + 209*C)*x^9)/9 + ((A - 15*B + 84*C)*x^10)/10 + ((B - 15*C)*x^
11)/11 + (C*x^12)/12
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.85

method	result
norman	$\frac{C\,x^{12}}{12}+\left(\frac{B}{11}-\frac{15C}{11}\right)x^{11}+\left(\frac{A}{10}-\frac{3B}{2}+\frac{42C}{5}\right)x^{10}+\left(-\frac{5A}{3}+\frac{28B}{3}-\frac{209C}{9}\right)x^{9}+\left(\frac{21A}{2}-\frac{209B}{8}+2\right.$
default	$\frac{C\,x^{12}}{12}+\frac{(B-15C)x^{11}}{11}+\frac{(A-15B+84C)x^{10}}{10}+\frac{(-15A+84B-209C)x^{9}}{9}+\frac{(84A-209B+192C)x^{8}}{8}+\frac{(-209A+192B}{7}$
orering	$x(2310C\,x^{11}+2520B\,x^{10}-37800x^{10}C+2772A\,x^{9}-41580x^{9}B+232848C\,x^{9}-46200x^{8}A+258720B\,x^{8}-643720x^{8}C+291060x$
gosper	$9C\,x^{4}+\frac{8}{3}C\,x^{3}+4B\,x^{2}+18A\,x^{2}-\frac{141}{4}x^{4}A+\frac{51}{5}x^{5}A-\frac{5}{3}A\,x^{9}-\frac{3}{2}B\,x^{10}+24x^{8}C-\frac{209}{7}x^{7}A$
risch	$9C\,x^{4}+\frac{8}{3}C\,x^{3}+4B\,x^{2}+18A\,x^{2}-\frac{141}{4}x^{4}A+\frac{51}{5}x^{5}A-\frac{5}{3}A\,x^{9}-\frac{3}{2}B\,x^{10}+24x^{8}C-\frac{209}{7}x^{7}A$
parallelrisch	$9C\,x^{4}+\frac{8}{3}C\,x^{3}+4B\,x^{2}+18A\,x^{2}-\frac{141}{4}x^{4}A+\frac{51}{5}x^{5}A-\frac{5}{3}A\,x^{9}-\frac{3}{2}B\,x^{10}+24x^{8}C-\frac{209}{7}x^{7}A$

input
```
int((C*x^2+B*x+A)*(x^3-5*x^2+3*x+2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/12*C*x^12+(1/11*B-15/11*C)*x^11+(1/10*A-3/2*B+42/5*C)*x^10+(-5/3*A+28/3*
B-209/9*C)*x^9+(21/2*A-209/8*B+24*C)*x^8+(-209/7*A+192/7*B+51/7*C)*x^7+(32
*A+17/2*B-47/2*C)*x^6+(51/5*A-141/5*B-6/5*C)*x^5+(-141/4*A-3/2*B+9*C)*x^4+
(-2*A+12*B+8/3*C)*x^3+(18*A+4*B)*x^2+8*A*x
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.88

$$\int (A+Bx+Cx^2)\left(2+3x-5x^2+x^3\right)^3\,dx$$
$$=\frac{1}{12}\,Cx^{12}+\frac{1}{11}\,(B-15\,C)x^{11}+\frac{1}{10}\,(A-15\,B+84\,C)x^{10}-\frac{1}{9}\,(15\,A-84\,B+209\,C)x^{9}$$
$$+\frac{1}{8}\,(84\,A-209\,B+192\,C)x^{8}-\frac{1}{7}\,(209\,A-192\,B-51\,C)x^{7}$$
$$+\frac{1}{2}\,(64\,A+17\,B-47\,C)x^{6}+\frac{3}{5}\,(17\,A-47\,B-2\,C)x^{5}$$
$$-\frac{3}{4}\,(47\,A+2\,B-12\,C)x^{4}-\frac{2}{3}\,(3\,A-18\,B-4\,C)x^{3}+2\,(9\,A+2\,B)x^{2}+8\,Ax$$

input
```
integrate((C*x^2+B*x+A)*(x^3-5*x^2+3*x+2)^3,x, algorithm="fricas")
```

output
```
1/12*C*x^12 + 1/11*(B - 15*C)*x^11 + 1/10*(A - 15*B + 84*C)*x^10 - 1/9*(15
*A - 84*B + 209*C)*x^9 + 1/8*(84*A - 209*B + 192*C)*x^8 - 1/7*(209*A - 192
*B - 51*C)*x^7 + 1/2*(64*A + 17*B - 47*C)*x^6 + 3/5*(17*A - 47*B - 2*C)*x^
5 - 3/4*(47*A + 2*B - 12*C)*x^4 - 2/3*(3*A - 18*B - 4*C)*x^3 + 2*(9*A + 2*
B)*x^2 + 8*A*x
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.01

$$\begin{aligned}\int (A+Bx+Cx^2)\left(2+3x-5x^2+x^3\right)^3\,dx &= 8Ax+\frac{Cx^{12}}{12}+x^{11}\left(\frac{B}{11}-\frac{15C}{11}\right)+x^{10}\left(\frac{A}{10}-\frac{3B}{2}+\frac{42C}{5}\right)+x^9\left(-\frac{5A}{3}+\frac{28B}{3}-\frac{209C}{9}\right)\\&\quad+x^8\cdot\left(\frac{21A}{2}-\frac{209B}{8}+24C\right)+x^7\left(-\frac{209A}{7}+\frac{192B}{7}+\frac{51C}{7}\right)\\&\quad+x^6\cdot\left(32A+\frac{17B}{2}-\frac{47C}{2}\right)+x^5\cdot\left(\frac{51A}{5}-\frac{141B}{5}-\frac{6C}{5}\right)\\&\quad+x^4\left(-\frac{141A}{4}-\frac{3B}{2}+9C\right)+x^3\left(-2A+12B+\frac{8C}{3}\right)+x^2\cdot(18A+4B)\end{aligned}$$

input
```
integrate((C*x**2+B*x+A)*(x**3-5*x**2+3*x+2)**3,x)
```

output
```
8*A*x + C*x**12/12 + x**11*(B/11 - 15*C/11) + x**10*(A/10 - 3*B/2 + 42*C/5
) + x**9*(-5*A/3 + 28*B/3 - 209*C/9) + x**8*(21*A/2 - 209*B/8 + 24*C) + x*
*7*(-209*A/7 + 192*B/7 + 51*C/7) + x**6*(32*A + 17*B/2 - 47*C/2) + x**5*(5
1*A/5 - 141*B/5 - 6*C/5) + x**4*(-141*A/4 - 3*B/2 + 9*C) + x**3*(-2*A + 12
*B + 8*C/3) + x**2*(18*A + 4*B)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.88

$$\int (A+Bx+Cx^2)(2+3x-5x^2+x^3)^3\,dx$$
$$=\frac{1}{12}Cx^{12}+\frac{1}{11}(B-15C)x^{11}+\frac{1}{10}(A-15B+84C)x^{10}-\frac{1}{9}(15A-84B+209C)x^9+\frac{1}{8}(84A-209B+192C)x^8-\frac{1}{7}(209A-192B-51C)x^7+\frac{1}{2}(64A+17B-47C)x^6+\frac{3}{5}(17A-47B-2C)x^5-\frac{3}{4}(47A+2B-12C)x^4-\frac{2}{3}(3A-18B-4C)x^3+2(9A+2B)x^2+8Ax$$

input
```
integrate((C*x^2+B*x+A)*(x^3-5*x^2+3*x+2)^3,x, algorithm="maxima")
```

output
```
1/12*C*x^12 + 1/11*(B - 15*C)*x^11 + 1/10*(A - 15*B + 84*C)*x^10 - 1/9*(15
*A - 84*B + 209*C)*x^9 + 1/8*(84*A - 209*B + 192*C)*x^8 - 1/7*(209*A - 192
*B - 51*C)*x^7 + 1/2*(64*A + 17*B - 47*C)*x^6 + 3/5*(17*A - 47*B - 2*C)*x^
5 - 3/4*(47*A + 2*B - 12*C)*x^4 - 2/3*(3*A - 18*B - 4*C)*x^3 + 2*(9*A + 2*
B)*x^2 + 8*A*x
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.05

$$\int (A+Bx+Cx^2)(2+3x-5x^2+x^3)^3\,dx$$
$$=\frac{1}{12}Cx^{12}+\frac{1}{11}Bx^{11}-\frac{15}{11}Cx^{11}+\frac{1}{10}Ax^{10}-\frac{3}{2}Bx^{10}+\frac{42}{5}Cx^{10}-\frac{5}{3}Ax^9+\frac{28}{3}Bx^9-\frac{209}{9}Cx^9+\frac{21}{2}Ax^8-\frac{209}{8}Bx^8+24Cx^8-\frac{209}{7}Ax^7+\frac{192}{7}Bx^7+\frac{51}{7}Cx^7+32Ax^6+\frac{17}{2}Bx^6-\frac{47}{2}Cx^6+\frac{51}{5}Ax^5-\frac{141}{5}Bx^5-\frac{6}{5}Cx^5-\frac{141}{4}Ax^4-\frac{3}{2}Bx^4+9Cx^4-2Ax^3+12Bx^3+\frac{8}{3}Cx^3+18Ax^2+4Bx^2+8Ax$$

input
```
integrate((C*x^2+B*x+A)*(x^3-5*x^2+3*x+2)^3,x, algorithm="giac")
```

output
```
1/12*C*x^12 + 1/11*B*x^11 - 15/11*C*x^11 + 1/10*A*x^10 - 3/2*B*x^10 + 42/5
*C*x^10 - 5/3*A*x^9 + 28/3*B*x^9 - 209/9*C*x^9 + 21/2*A*x^8 - 209/8*B*x^8
+ 24*C*x^8 - 209/7*A*x^7 + 192/7*B*x^7 + 51/7*C*x^7 + 32*A*x^6 + 17/2*B*x^
6 - 47/2*C*x^6 + 51/5*A*x^5 - 141/5*B*x^5 - 6/5*C*x^5 - 141/4*A*x^4 - 3/2*
B*x^4 + 9*C*x^4 - 2*A*x^3 + 12*B*x^3 + 8/3*C*x^3 + 18*A*x^2 + 4*B*x^2 + 8*
A*x
```

Mupad [B] (verification not implemented)

Time = 21.63 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.87

$$\int (A+Bx+Cx^2)\left(2+3x-5x^2+x^3\right)^3\,dx = \frac{C\,x^{12}}{12}+\left(\frac{B}{11}-\frac{15\,C}{11}\right)x^{11}+\left(\frac{A}{10}-\frac{3\,B}{2}+\frac{42\,C}{5}\right)x^{10}+\left(\frac{28\,B}{3}-\frac{5\,A}{3}-\frac{209\,C}{9}\right)x^9 + \left(\frac{21\,A}{2}-\frac{209\,B}{8}+24\,C\right)x^8+\left(\frac{192\,B}{7}-\frac{209\,A}{7}+\frac{51\,C}{7}\right)x^7 + \left(32\,A+\frac{17\,B}{2}-\frac{47\,C}{2}\right)x^6+\left(\frac{51\,A}{5}-\frac{141\,B}{5}-\frac{6\,C}{5}\right)x^5 + \left(9\,C-\frac{3\,B}{2}-\frac{141\,A}{4}\right)x^4+\left(12\,B-2\,A+\frac{8\,C}{3}\right)x^3+(18\,A+4\,B)\,x^2+8\,A\,x$$

input
```
int((A + B*x + C*x^2)*(3*x - 5*x^2 + x^3 + 2)^3,x)
```

output
```
8*A*x + (C*x^12)/12 + x^3*(12*B - 2*A + (8*C)/3) + x^10*(A/10 - (3*B)/2 +
(42*C)/5) + x^6*(32*A + (17*B)/2 - (47*C)/2) - x^4*((141*A)/4 + (3*B)/2 -
9*C) - x^5*((141*B)/5 - (51*A)/5 + (6*C)/5) - x^9*((5*A)/3 - (28*B)/3 + (2
09*C)/9) + x^8*((21*A)/2 - (209*B)/8 + 24*C) + x^7*((192*B)/7 - (209*A)/7
+ (51*C)/7) + x^2*(18*A + 4*B) + x^11*(B/11 - (15*C)/11)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.04

$$\int (A + Bx + Cx^2) (2 + 3x - 5x^2 + x^3)^3 \, dx$$
$$= \frac{x(2310c\,x^{11} + 2520b\,x^{10} - 37800c\,x^{10} + 2772a\,x^9 - 41580b\,x^9 + 232848c\,x^9 - 46200a\,x^8 + 258720b\,x^8 - }{}$$

input
```
int((C*x^2+B*x+A)*(x^3-5*x^2+3*x+2)^3,x)
```

output
```
(x*(2772*a*x**9 - 46200*a*x**8 + 291060*a*x**7 - 827640*a*x**6 + 887040*a*
x**5 + 282744*a*x**4 - 977130*a*x**3 - 55440*a*x**2 + 498960*a*x + 221760*
a + 2520*b*x**10 - 41580*b*x**9 + 258720*b*x**8 - 724185*b*x**7 + 760320*b
*x**6 + 235620*b*x**5 - 781704*b*x**4 - 41580*b*x**3 + 332640*b*x**2 + 110
880*b*x + 2310*c*x**11 - 37800*c*x**10 + 232848*c*x**9 - 643720*c*x**8 + 6
65280*c*x**7 + 201960*c*x**6 - 651420*c*x**5 - 33264*c*x**4 + 249480*c*x**
3 + 73920*c*x**2))/27720
```

3.166 $\int (A+Bx+Cx^2)(2+3x-5x^2+x^3)^2\,dx$

Optimal result

Integrand size = 26, antiderivative size = 119

$$\begin{aligned}\int (A+Bx+Cx^2)\left(2+3x-5x^2+x^3\right)^2\,dx &= 4Ax+2(3A+B)x^2-\frac{1}{3}(11A-4(3B+C))x^3-\frac{1}{4}(26A+11B-12C)x^4\\&\quad+\frac{1}{5}(31A-26B-11C)x^5-\frac{1}{6}(10A-31B+26C)x^6\\&\quad+\frac{1}{7}(A-10B+31C)x^7+\frac{1}{8}(B-10C)x^8+\frac{Cx^9}{9}\end{aligned}$$

output
```
4*A*x+2*(3*A+B)*x^2-1/3*(11*A-12*B-4*C)*x^3-1/4*(26*A+11*B-12*C)*x^4+1/5*(
31*A-26*B-11*C)*x^5-1/6*(10*A-31*B+26*C)*x^6+1/7*(A-10*B+31*C)*x^7+1/8*(B-
10*C)*x^8+1/9*C*x^9
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.99

$$\int (A+Bx+Cx^2)(2+3x-5x^2+x^3)^2\,dx$$
$$=4Ax+2(3A+B)x^2+\frac{1}{3}(-11A+12B+4C)x^3+\frac{1}{4}(-26A-11B+12C)x^4+\frac{1}{5}(31A-26B-11C)x^5+\frac{1}{6}(-10A+31B-26C)x^6+\frac{1}{7}(A-10B+31C)x^7+\frac{1}{8}(B-10C)x^8+\frac{Cx^9}{9}$$

input
```
Integrate[(A + B*x + C*x^2)*(2 + 3*x - 5*x^2 + x^3)^2,x]
```

output
```
4*A*x + 2*(3*A + B)*x^2 + ((-11*A + 12*B + 4*C)*x^3)/3 + ((-26*A - 11*B +
12*C)*x^4)/4 + ((31*A - 26*B - 11*C)*x^5)/5 + ((-10*A + 31*B - 26*C)*x^6)/
6 + ((A - 10*B + 31*C)*x^7)/7 + ((B - 10*C)*x^8)/8 + (C*x^9)/9
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (x^3-5x^2+3x+2)^2(A+Bx+Cx^2)\,dx$$

$$\Big\downarrow \text{2188}$$

$$\int (x^6(A-10B+31C)-x^5(10A-31B+26C)+x^4(31A-26B-11C)-x^3(26A+11B-12C)-x^2(11A-$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{7}x^7(A-10B+31C)-\frac{1}{6}x^6(10A-31B+26C)+\frac{1}{5}x^5(31A-26B-11C)-\frac{1}{4}x^4(26A+11B-12C)-\frac{1}{3}x^3(11A-4(3B+C))+2x^2(3A+B)+4Ax+\frac{1}{8}x^8(B-10C)+\frac{Cx^9}{9}$$

input
```
Int[(A + B*x + C*x^2)*(2 + 3*x - 5*x^2 + x^3)^2,x]
```

output
```
4*A*x + 2*(3*A + B)*x^2 - ((11*A - 4*(3*B + C))*x^3)/3 - ((26*A + 11*B - 1
2*C)*x^4)/4 + ((31*A - 26*B - 11*C)*x^5)/5 - ((10*A - 31*B + 26*C)*x^6)/6
+ ((A - 10*B + 31*C)*x^7)/7 + ((B - 10*C)*x^8)/8 + (C*x^9)/9
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.87

method	result
norman	$\frac{C\,x^9}{9}+\left(\frac{B}{8}-\frac{5C}{4}\right)x^8+\left(\frac{A}{7}-\frac{10B}{7}+\frac{31C}{7}\right)x^7+\left(-\frac{5A}{3}+\frac{31B}{6}-\frac{13C}{3}\right)x^6+\left(\frac{31A}{5}-\frac{26B}{5}-\frac{11C}{5}\right)x$
default	$\frac{C\,x^9}{9}+\frac{(B-10C)x^8}{8}+\frac{(A-10B+31C)x^7}{7}+\frac{(-10A+31B-26C)x^6}{6}+\frac{(31A-26B-11C)x^5}{5}+\frac{(-26A-11B+12C)x^4}{4}$
orering	$\frac{x(280x^8C+315x^7B-3150x^7C+360x^6A-3600x^6B+11160C\,x^6-4200x^5A+13020B\,x^5-10920x^5C+15624x^4A-13104x^4B}{2520}$
gosper	$\frac{1}{9}C\,x^9+\frac{1}{8}B\,x^8-\frac{5}{4}x^8C+\frac{1}{7}x^7A-\frac{10}{7}x^7B+\frac{31}{7}x^7C-\frac{5}{3}x^6A+\frac{31}{6}x^6B-\frac{13}{3}C\,x^6+\frac{31}{5}x^5A-$
risch	$\frac{1}{9}C\,x^9+\frac{1}{8}B\,x^8-\frac{5}{4}x^8C+\frac{1}{7}x^7A-\frac{10}{7}x^7B+\frac{31}{7}x^7C-\frac{5}{3}x^6A+\frac{31}{6}x^6B-\frac{13}{3}C\,x^6+\frac{31}{5}x^5A-$
parallelrisch	$\frac{1}{9}C\,x^9+\frac{1}{8}B\,x^8-\frac{5}{4}x^8C+\frac{1}{7}x^7A-\frac{10}{7}x^7B+\frac{31}{7}x^7C-\frac{5}{3}x^6A+\frac{31}{6}x^6B-\frac{13}{3}C\,x^6+\frac{31}{5}x^5A-$

input
```
int((C*x^2+B*x+A)*(x^3-5*x^2+3*x+2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/9*C*x^9+(1/8*B-5/4*C)*x^8+(1/7*A-10/7*B+31/7*C)*x^7+(-5/3*A+31/6*B-13/3*
C)*x^6+(31/5*A-26/5*B-11/5*C)*x^5+(-13/2*A-11/4*B+3*C)*x^4+(-11/3*A+4*B+4/
3*C)*x^3+(6*A+2*B)*x^2+4*A*x
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.87

$$
\begin{aligned}
\int (A+Bx+Cx^2)\left(2+3x-5x^2+x^3\right)^2\,dx
&= \frac{1}{9}\,Cx^9+\frac{1}{8}\,(B-10\,C)x^8+\frac{1}{7}\,(A-10\,B+31\,C)x^7-\frac{1}{6}\,(10\,A-31\,B+26\,C)x^6\\
&\quad+\frac{1}{5}\,(31\,A-26\,B-11\,C)x^5-\frac{1}{4}\,(26\,A+11\,B-12\,C)x^4\\
&\quad-\frac{1}{3}\,(11\,A-12\,B-4\,C)x^3+2\,(3\,A+B)x^2+4\,Ax
\end{aligned}
$$

input
```
integrate((C*x^2+B*x+A)*(x^3-5*x^2+3*x+2)^2,x, algorithm="fricas")
```

output
```
1/9*C*x^9 + 1/8*(B - 10*C)*x^8 + 1/7*(A - 10*B + 31*C)*x^7 - 1/6*(10*A - 3
1*B + 26*C)*x^6 + 1/5*(31*A - 26*B - 11*C)*x^5 - 1/4*(26*A + 11*B - 12*C)*
x^4 - 1/3*(11*A - 12*B - 4*C)*x^3 + 2*(3*A + B)*x^2 + 4*A*x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.02

$$
\begin{aligned}
\int (A+Bx+Cx^2)\left(2+3x-5x^2+x^3\right)^2\,dx
&= 4Ax+\frac{Cx^9}{9}+x^8\left(\frac{B}{8}-\frac{5C}{4}\right)+x^7\left(\frac{A}{7}-\frac{10B}{7}+\frac{31C}{7}\right)\\
&\quad+x^6\left(-\frac{5A}{3}+\frac{31B}{6}-\frac{13C}{3}\right)+x^5\cdot\left(\frac{31A}{5}-\frac{26B}{5}-\frac{11C}{5}\right)\\
&\quad+x^4\left(-\frac{13A}{2}-\frac{11B}{4}+3C\right)+x^3\left(-\frac{11A}{3}+4B+\frac{4C}{3}\right)+x^2\cdot(6A+2B)
\end{aligned}
$$

input
```
integrate((C*x**2+B*x+A)*(x**3-5*x**2+3*x+2)**2,x)
```

output
```
4*A*x + C*x**9/9 + x**8*(B/8 - 5*C/4) + x**7*(A/7 - 10*B/7 + 31*C/7) + x**
6*(-5*A/3 + 31*B/6 - 13*C/3) + x**5*(31*A/5 - 26*B/5 - 11*C/5) + x**4*(-13
*A/2 - 11*B/4 + 3*C) + x**3*(-11*A/3 + 4*B + 4*C/3) + x**2*(6*A + 2*B)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.87

$$\int (A+Bx+Cx^2)\left(2+3x-5x^2+x^3\right)^2\,dx = \frac{1}{9}\,Cx^9+\frac{1}{8}\,(B-10\,C)x^8+\frac{1}{7}\,(A-10\,B+31\,C)x^7-\frac{1}{6}\,(10\,A-31\,B+26\,C)x^6 + \frac{1}{5}\,(31\,A-26\,B-11\,C)x^5-\frac{1}{4}\,(26\,A+11\,B-12\,C)x^4 - \frac{1}{3}\,(11\,A-12\,B-4\,C)x^3+2\,(3\,A+B)x^2+4\,Ax$$

input
```
integrate((C*x^2+B*x+A)*(x^3-5*x^2+3*x+2)^2,x, algorithm="maxima")
```

output
```
1/9*C*x^9 + 1/8*(B - 10*C)*x^8 + 1/7*(A - 10*B + 31*C)*x^7 - 1/6*(10*A - 3
1*B + 26*C)*x^6 + 1/5*(31*A - 26*B - 11*C)*x^5 - 1/4*(26*A + 11*B - 12*C)*
x^4 - 1/3*(11*A - 12*B - 4*C)*x^3 + 2*(3*A + B)*x^2 + 4*A*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.05

$$\int (A+Bx+Cx^2)\left(2+3x-5x^2+x^3\right)^2\,dx = \frac{1}{9}\,Cx^9+\frac{1}{8}\,Bx^8-\frac{5}{4}\,Cx^8+\frac{1}{7}\,Ax^7-\frac{10}{7}\,Bx^7+\frac{31}{7}\,Cx^7-\frac{5}{3}\,Ax^6 + \frac{31}{6}\,Bx^6-\frac{13}{3}\,Cx^6+\frac{31}{5}\,Ax^5-\frac{26}{5}\,Bx^5-\frac{11}{5}\,Cx^5-\frac{13}{2}\,Ax^4-\frac{11}{4}\,Bx^4 + 3\,Cx^4-\frac{11}{3}\,Ax^3+4\,Bx^3+\frac{4}{3}\,Cx^3+6\,Ax^2+2\,Bx^2+4\,Ax$$

input
```
integrate((C*x^2+B*x+A)*(x^3-5*x^2+3*x+2)^2,x, algorithm="giac")
```

output
```
1/9*C*x^9 + 1/8*B*x^8 - 5/4*C*x^8 + 1/7*A*x^7 - 10/7*B*x^7 + 31/7*C*x^7 -
5/3*A*x^6 + 31/6*B*x^6 - 13/3*C*x^6 + 31/5*A*x^5 - 26/5*B*x^5 - 11/5*C*x^5
 - 13/2*A*x^4 - 11/4*B*x^4 + 3*C*x^4 - 11/3*A*x^3 + 4*B*x^3 + 4/3*C*x^3 +
6*A*x^2 + 2*B*x^2 + 4*A*x
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.89

$$\int (A+Bx+Cx^2)\left(2+3x-5x^2+x^3\right)^2\,dx = \frac{C\,x^9}{9}+\left(\frac{B}{8}-\frac{5\,C}{4}\right)x^8+\left(\frac{A}{7}-\frac{10\,B}{7}+\frac{31\,C}{7}\right)x^7+\left(\frac{31\,B}{6}-\frac{5\,A}{3}-\frac{13\,C}{3}\right)x^6+\left(\frac{31\,A}{5}-\frac{26\,B}{5}-\frac{11\,C}{5}\right)x^5+\left(3\,C-\frac{11\,B}{4}-\frac{13\,A}{2}\right)x^4+\left(4\,B-\frac{11\,A}{3}+\frac{4\,C}{3}\right)x^3+(6\,A+2\,B)\,x^2+4\,A\,x$$

input
```
int((A + B*x + C*x^2)*(3*x - 5*x^2 + x^3 + 2)^2,x)
```

output
```
4*A*x + (C*x^9)/9 + x^3*(4*B - (11*A)/3 + (4*C)/3) - x^4*((13*A)/2 + (11*B
)/4 - 3*C) - x^6*((5*A)/3 - (31*B)/6 + (13*C)/3) + x^7*(A/7 - (10*B)/7 + (
31*C)/7) - x^5*((26*B)/5 - (31*A)/5 + (11*C)/5) + x^2*(6*A + 2*B) + x^8*(B
/8 - (5*C)/4)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.03

$$\int (A+Bx+Cx^2)\left(2+3x-5x^2+x^3\right)^2\,dx = \frac{x(280c\,x^8+315b\,x^7-3150c\,x^7+360a\,x^6-3600b\,x^6+11160c\,x^6-4200a\,x^5+13020b\,x^5-10920c\,x^5+\cdots}{}$$

input
```
int((C*x^2+B*x+A)*(x^3-5*x^2+3*x+2)^2,x)
```

output
```
(x*(360*a*x**6 - 4200*a*x**5 + 15624*a*x**4 - 16380*a*x**3 - 9240*a*x**2 +
 15120*a*x + 10080*a + 315*b*x**7 - 3600*b*x**6 + 13020*b*x**5 - 13104*b*x
**4 - 6930*b*x**3 + 10080*b*x**2 + 5040*b*x + 280*c*x**8 - 3150*c*x**7 + 1
1160*c*x**6 - 10920*c*x**5 - 5544*c*x**4 + 7560*c*x**3 + 3360*c*x**2))/252
0
```

3.167 $\int (A+Bx+Cx^2) (2+3x-5x^2+x^3) \, dx$

Optimal result

Integrand size = 24, antiderivative size = 71

$$\int (A+Bx+Cx^2) (2+3x-5x^2+x^3) \, dx = 2Ax+\frac{1}{2}(3A+2B)x^2-\frac{1}{3}(5A-3B-2C)x^3+\frac{1}{4}(A-5B+3C)x^4+\frac{1}{5}(B-5C)x^5+\frac{Cx^6}{6}$$

output
```
2*A*x+1/2*(3*A+2*B)*x^2-1/3*(5*A-3*B-2*C)*x^3+1/4*(A-5*B+3*C)*x^4+1/5*(B-5
*C)*x^5+1/6*C*x^6
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00

$$\int (A+Bx+Cx^2) (2+3x-5x^2+x^3) \, dx = 2Ax+\frac{1}{2}(3A+2B)x^2+\frac{1}{3}(-5A+3B+2C)x^3 + \frac{1}{4}(A-5B+3C)x^4+\frac{1}{5}(B-5C)x^5+\frac{Cx^6}{6}$$

input
```
Integrate[(A + B*x + C*x^2)*(2 + 3*x - 5*x^2 + x^3),x]
```

output
```
2*A*x + ((3*A + 2*B)*x^2)/2 + ((-5*A + 3*B + 2*C)*x^3)/3 + ((A - 5*B + 3*C
)*x^4)/4 + ((B - 5*C)*x^5)/5 + (C*x^6)/6
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (x^3-5x^2+3x+2)(A+Bx+Cx^2)\,dx$$

$$\Big\downarrow \text{2188}$$

$$\int \left(x^3(A-5B+3C)-x^2(5A-3B-2C)+x(3A+2B)+2A+x^4(B-5C)+Cx^5\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}x^4(A-5B+3C)-\frac{1}{3}x^3(5A-3B-2C)+\frac{1}{2}x^2(3A+2B)+2Ax+\frac{1}{5}x^5(B-5C)+\frac{Cx^6}{6}$$

input
```
Int[(A + B*x + C*x^2)*(2 + 3*x - 5*x^2 + x^3),x]
```

output
```
2*A*x + ((3*A + 2*B)*x^2)/2 - ((5*A - 3*B - 2*C)*x^3)/3 + ((A - 5*B + 3*C)
*x^4)/4 + ((B - 5*C)*x^5)/5 + (C*x^6)/6
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.82

method	result
norman	$\frac{C\,x^{6}}{6}+\left(\frac{B}{5}-C\right)x^{5}+\left(\frac{A}{4}-\frac{5B}{4}+\frac{3C}{4}\right)x^{4}+\left(-\frac{5A}{3}+B+\frac{2C}{3}\right)x^{3}+\left(\frac{3A}{2}+B\right)x^{2}+2Ax$
default	$\frac{C\,x^{6}}{6}+\frac{(B-5C)x^{5}}{5}+\frac{(A-5B+3C)x^{4}}{4}+\frac{(-5A+3B+2C)x^{3}}{3}+\frac{(3A+2B)x^{2}}{2}+2Ax$
gosper	$\frac{1}{6}C\,x^{6}+\frac{1}{5}B\,x^{5}-x^{5}C+\frac{1}{4}x^{4}A-\frac{5}{4}x^{4}B+\frac{3}{4}C\,x^{4}-\frac{5}{3}x^{3}A+B\,x^{3}+\frac{2}{3}C\,x^{3}+\frac{3}{2}A\,x^{2}+B\,x^{2}+$
risch	$\frac{1}{6}C\,x^{6}+\frac{1}{5}B\,x^{5}-x^{5}C+\frac{1}{4}x^{4}A-\frac{5}{4}x^{4}B+\frac{3}{4}C\,x^{4}-\frac{5}{3}x^{3}A+B\,x^{3}+\frac{2}{3}C\,x^{3}+\frac{3}{2}A\,x^{2}+B\,x^{2}+$
parallelrisch	$\frac{1}{6}C\,x^{6}+\frac{1}{5}B\,x^{5}-x^{5}C+\frac{1}{4}x^{4}A-\frac{5}{4}x^{4}B+\frac{3}{4}C\,x^{4}-\frac{5}{3}x^{3}A+B\,x^{3}+\frac{2}{3}C\,x^{3}+\frac{3}{2}A\,x^{2}+B\,x^{2}+$
orering	$\frac{x\left(10x^{5}C+12x^{4}B-60C\,x^{4}+15x^{3}A-75B\,x^{3}+45C\,x^{3}-100A\,x^{2}+60B\,x^{2}+40C\,x^{2}+90Ax+60Bx+120A\right)}{60}$

input
```
int((C*x^2+B*x+A)*(x^3-5*x^2+3*x+2),x,method=_RETURNVERBOSE)
```

output
```
1/6*C*x^6+(1/5*B-C)*x^5+(1/4*A-5/4*B+3/4*C)*x^4+(-5/3*A+B+2/3*C)*x^3+(3/2*
A+B)*x^2+2*A*x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.86

$$\int\left(A+Bx+Cx^{2}\right)\left(2+3x-5x^{2}+x^{3}\right)\,dx=\frac{1}{6}\,Cx^{6}+\frac{1}{5}\,(B-5\,C)x^{5}+\frac{1}{4}\,(A-5\,B+3\,C)x^{4}-\frac{1}{3}\,(5\,A-3\,B-2\,C)x^{3}+\frac{1}{2}\,(3\,A+2\,B)x^{2}+2\,Ax$$

input
```
integrate((C*x^2+B*x+A)*(x^3-5*x^2+3*x+2),x, algorithm="fricas")
```

output
```
1/6*C*x^6 + 1/5*(B - 5*C)*x^5 + 1/4*(A - 5*B + 3*C)*x^4 - 1/3*(5*A - 3*B -
 2*C)*x^3 + 1/2*(3*A + 2*B)*x^2 + 2*A*x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.86

$$\int (A+Bx+Cx^2)(2+3x-5x^2+x^3)\,dx = 2Ax+\frac{Cx^6}{6}+x^5\left(\frac{B}{5}-C\right)+x^4\left(\frac{A}{4}-\frac{5B}{4}+\frac{3C}{4}\right) + x^3\left(-\frac{5A}{3}+B+\frac{2C}{3}\right)+x^2\cdot\left(\frac{3A}{2}+B\right)$$

input
```
integrate((C*x**2+B*x+A)*(x**3-5*x**2+3*x+2),x)
```

output
```
2*A*x + C*x**6/6 + x**5*(B/5 - C) + x**4*(A/4 - 5*B/4 + 3*C/4) + x**3*(-5*
A/3 + B + 2*C/3) + x**2*(3*A/2 + B)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.86

$$\int (A+Bx+Cx^2)(2+3x-5x^2+x^3)\,dx = \frac{1}{6}Cx^6+\frac{1}{5}(B-5C)x^5+\frac{1}{4}(A-5B+3C)x^4 - \frac{1}{3}(5A-3B-2C)x^3+\frac{1}{2}(3A+2B)x^2+2Ax$$

input
```
integrate((C*x^2+B*x+A)*(x^3-5*x^2+3*x+2),x, algorithm="maxima")
```

output
```
1/6*C*x^6 + 1/5*(B - 5*C)*x^5 + 1/4*(A - 5*B + 3*C)*x^4 - 1/3*(5*A - 3*B -
 2*C)*x^3 + 1/2*(3*A + 2*B)*x^2 + 2*A*x
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.97

$$\int (A+Bx+Cx^2)\left(2+3x-5x^2+x^3\right)\,dx = \frac{1}{6}Cx^6+\frac{1}{5}Bx^5-Cx^5+\frac{1}{4}Ax^4-\frac{5}{4}Bx^4+\frac{3}{4}Cx^4 - \frac{5}{3}Ax^3+Bx^3+\frac{2}{3}Cx^3+\frac{3}{2}Ax^2+Bx^2+2\,Ax$$

input
```
integrate((C*x^2+B*x+A)*(x^3-5*x^2+3*x+2),x, algorithm="giac")
```

output
```
1/6*C*x^6 + 1/5*B*x^5 - C*x^5 + 1/4*A*x^4 - 5/4*B*x^4 + 3/4*C*x^4 - 5/3*A*
x^3 + B*x^3 + 2/3*C*x^3 + 3/2*A*x^2 + B*x^2 + 2*A*x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.80

$$\int (A+Bx+Cx^2)\left(2+3x-5x^2+x^3\right)\,dx = \frac{C\,x^6}{6}+\left(\frac{B}{5}-C\right)x^5+\left(\frac{A}{4}-\frac{5\,B}{4}+\frac{3\,C}{4}\right)x^4 + \left(B-\frac{5\,A}{3}+\frac{2\,C}{3}\right)x^3+\left(\frac{3\,A}{2}+B\right)x^2+2\,A\,x$$

input
```
int((A + B*x + C*x^2)*(3*x - 5*x^2 + x^3 + 2),x)
```

output
```
2*A*x + x^3*(B - (5*A)/3 + (2*C)/3) + x^2*((3*A)/2 + B) + (C*x^6)/6 + x^4*
(A/4 - (5*B)/4 + (3*C)/4) + x^5*(B/5 - C)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.97

$$\int (A+Bx+Cx^2)\,(2+3x-5x^2+x^3)\,dx$$
$$=\frac{x(10c\,x^5+12b\,x^4-60c\,x^4+15a\,x^3-75b\,x^3+45c\,x^3-100a\,x^2+60b\,x^2+40c\,x^2+90ax+60bx+120}{60}$$

input
```
int((C*x^2+B*x+A)*(x^3-5*x^2+3*x+2),x)
```

output
```
(x*(15*a*x**3 - 100*a*x**2 + 90*a*x + 120*a + 12*b*x**4 - 75*b*x**3 + 60*b
*x**2 + 60*b*x + 10*c*x**5 - 60*c*x**4 + 45*c*x**3 + 40*c*x**2))/60
```

3.168 $\int \frac{A+Bx+Cx^2}{2+3x-5x^2+x^3}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 297

$$\int \frac{A+Bx+Cx^2}{2+3x-5x^2+x^3}\,dx = \frac{1}{3}C\log\left(2+3x-5x^2+x^3\right)$$
$$+\frac{\log\left(5-3x-8\sin\left(\frac{1}{3}\arcsin\left(\frac{61}{128}\right)\right)\right)\left(9A+C\left(41-80\sin\left(\frac{1}{3}\arcsin\left(\frac{61}{128}\right)\right)\right)+3B\left(5-8\sin\left(\frac{1}{3}\arcsin\left(\right.\right.\right.\right.}{48\left(1-2\cos\left(\frac{2}{3}\arcsin\left(\frac{61}{128}\right)\right)\right)}$$
$$+\frac{\left(9A+15B+41C-24B\cos\left(\frac{1}{6}\left(\pi+2\arcsin\left(\frac{61}{128}\right)\right)\right)-80C\cos\left(\frac{1}{6}\left(\pi+2\arcsin\left(\frac{61}{128}\right)\right)\right)\right)\log\left(5-3x\right.}{64\sqrt{3}\left(\cos\left(\frac{1}{6}\left(\pi+2\arcsin\left(\frac{61}{128}\right)\right)\right)-\sin\left(\frac{1}{3}\arcsin\left(\frac{61}{128}\right.\right.\right.}$$
$$+\frac{\log\left(5-3x+8\sin\left(\frac{1}{3}\left(\pi+\arcsin\left(\frac{61}{128}\right)\right)\right)\right)\sec\left(\frac{1}{3}\arcsin\left(\frac{61}{128}\right)\right)\left(9A+15B+41C+24B\sin\left(\frac{1}{3}\left(\pi+\right.\right.\right.}{64\sqrt{3}\left(\sin\left(\frac{1}{3}\arcsin\left(\frac{61}{128}\right)\right)+\sin\left(\frac{1}{3}\left(\pi+\arcsin\left(\frac{61}{128}\right)\right)\right)\right)}$$

output
```
1/3*C*ln(x^3-5*x^2+3*x+2)+ln(5-3*x-8*sin(1/3*arcsin(61/128)))*(9*A+C*(41-8
0*sin(1/3*arcsin(61/128)))+3*B*(5-8*sin(1/3*arcsin(61/128))))/(48-96*cos(2
/3*arcsin(61/128)))+1/192*(9*A+15*B+41*C-24*B*cos(1/6*Pi+1/3*arcsin(61/128
))-80*C*cos(1/6*Pi+1/3*arcsin(61/128)))*ln(5-3*x-8*cos(1/6*Pi+1/3*arcsin(6
1/128)))*sec(1/3*arcsin(61/128))*3^(1/2)/(cos(1/6*Pi+1/3*arcsin(61/128))-s
in(1/3*arcsin(61/128)))+1/192*ln(5-3*x+8*sin(1/3*Pi+1/3*arcsin(61/128)))*s
ec(1/3*arcsin(61/128))*(9*A+15*B+41*C+24*B*sin(1/3*Pi+1/3*arcsin(61/128))+
80*C*sin(1/3*Pi+1/3*arcsin(61/128)))*3^(1/2)/(sin(1/3*arcsin(61/128))+sin(
1/3*Pi+1/3*arcsin(61/128)))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.23

$$\int \frac{A+Bx+Cx^2}{2+3x-5x^2+x^3}\,dx = \text{RootSum}\left[2+3\#1-5\#1^2+\#1^3\&,\frac{A\log(x-\#1)+B\log(x-\#1)\#1+C\log(x-\#1)\#1^2}{3-10\#1+3\#1^2}\&\right]$$

input
```
Integrate[(A + B*x + C*x^2)/(2 + 3*x - 5*x^2 + x^3),x]
```

output
```
RootSum[2 + 3*#1 - 5*#1^2 + #1^3 & , (A*Log[x - #1] + B*Log[x - #1]*#1 + C
*Log[x - #1]*#1^2)/(3 - 10*#1 + 3*#1^2) & ]
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 14.03 (sec) , antiderivative size = 920, normalized size of antiderivative = 3.10, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2525, 2490, 2485, 27, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{x^3-5x^2+3x+2}\,dx$$

$$\Big\downarrow \text{ 2525}$$

$$\frac{1}{3}\int \frac{3(A-C)+(3B+10C)x}{x^3-5x^2+3x+2}dx+\frac{1}{3}C\log\left(x^3-5x^2+3x+2\right)$$

$$\Big\downarrow \text{ 2490}$$

$$\frac{1}{3}\int \frac{\frac{1}{3}(9(A-C)+5(3B+10C))+(3B+10C)\left(x-\frac{5}{3}\right)}{\left(x-\frac{5}{3}\right)^3-\frac{16}{3}\left(x-\frac{5}{3}\right)-\frac{61}{27}}d\left(x-\frac{5}{3}\right)+$$
$$\frac{1}{3}C\log\left(x^3-5x^2+3x+2\right)$$

$\downarrow$ 2485

$$\frac{1}{3}C\log\left(x^3-5x^2+3x+2\right)+$$
$$\frac{1}{3}\int \frac{36\left(9A+15B+41C+3(3B+10C)\left(x-\frac{5}{3}\right.\right.}{\left(\frac{32\sqrt[3]{2}+\left(122+6i\sqrt{1407}\right)^{2/3}}{\sqrt[3]{61+3i\sqrt{1407}}}-6\left(x-\frac{5}{3}\right)\right)\left(-18\left(x-\frac{5}{3}\right)^2-6\left(\frac{16}{\sqrt[3]{\frac{1}{2}\left(61+3i\sqrt{1407}\right)}}+\sqrt[3]{\frac{1}{2}\left(61+3i\sqrt{1407}\right)}\right.\right.}$$

$\downarrow$ 27

$$\frac{1}{3}C\log\left(x^3-5x^2+3x+2\right)+$$
$$12\int \frac{9A+15B+41C+3(3B+10C)\left(x-\frac{5}{3}\right)}{\left(\frac{32\sqrt[3]{2}+\left(122+6i\sqrt{1407}\right)^{2/3}}{\sqrt[3]{61+3i\sqrt{1407}}}-6\left(x-\frac{5}{3}\right)\right)\left(-18\left(x-\frac{5}{3}\right)^2-6\left(\frac{16}{\sqrt[3]{\frac{1}{2}\left(61+3i\sqrt{1407}\right)}}+\sqrt[3]{\frac{1}{2}\left(61+3i\sqrt{1407}\right.}\right.\right.}$$

$\downarrow$ 1200

$$\frac{1}{3}C\log\left(x^3-5x^2+3x+2\right)+$$
$$12\int\left(\frac{\left(61+3i\sqrt{1407}\right)^{2/3}\left(-18\sqrt[3]{61+3i\sqrt{1407}}A-3\left(32\sqrt[3]{2}+10\sqrt[3]{61+3i\sqrt{1407}}+\left(122+6i\sqrt{1407}\right)^{2/3}\right)B\right.}{6\left(512\ 2^{2/3}+32\left(61+3i\sqrt{1407}\right)^{2/3}+\sqrt[3]{2}\left(61+3i\sqrt{1407}\right)^{4/3}\right)\left(-6\sqrt[3]{61+3i\sqrt{1407}}\right.}\right.$$

$\downarrow$ 2009

$$12\left(-\frac{\left(3i\left(61i-3\sqrt{1407}\right)\left(32+\sqrt[3]{2}\left(61+3i\sqrt{1407}\right)^{2/3}\right)A-3\left(6234+38i\sqrt{1407}-\sqrt[3]{2}\left(61+3i\sqrt{1407}\right)^{2/3}\left(69-\right.\right.\right.}{2\ 2^{2/3}\left(512\ 2^{2/3}+32\left(61+3i\sqrt{1407}\right)^{2/3}+\sqrt[3]{2}\right.}\right.$$
$$\frac{1}{3}C\log\left(x^3-5x^2+3x+2\right)$$

input
```
Int[(A + B*x + C*x^2)/(2 + 3*x - 5*x^2 + x^3),x]
```

output
```
12*(-1/2*(((3*I)*(61*I - 3*Sqrt[1407])*(32 + 2^(1/3)*(61 + (3*I)*Sqrt[1407
])^(2/3))*A - 3*(6234 + (38*I)*Sqrt[1407] - 2^(1/3)*(61 + (3*I)*Sqrt[1407]
)^(2/3)*(69 - (5*I)*Sqrt[1407]))*B - (56484 + (92*I)*Sqrt[1407] - 2^(1/3)*
(61 + (3*I)*Sqrt[1407])^(2/3)*(873 - (41*I)*Sqrt[1407]))*C)*ArcTanh[(2^(2/
3)*(61 + (3*I)*Sqrt[1407]) + 32*(122 + (6*I)*Sqrt[1407])^(1/3) + 12*(61 +
(3*I)*Sqrt[1407])^(2/3)*(-5/3 + x))/(2*Sqrt[3*(32*(61 + (3*I)*Sqrt[1407])^
(4/3) + 2^(1/3)*(4471 - (183*I)*Sqrt[1407] - 256*2^(1/3)*(61 + (3*I)*Sqrt[
1407])^(2/3)))])])/(2^(2/3)*(512*2^(2/3) + 32*(61 + (3*I)*Sqrt[1407])^(2/3
) + 2^(1/3)*(61 + (3*I)*Sqrt[1407])^(4/3))*Sqrt[3*(32*(61 + (3*I)*Sqrt[140
7])^(4/3) + 2^(1/3)*(4471 - (183*I)*Sqrt[1407] - 256*2^(1/3)*(61 + (3*I)*S
qrt[1407])^(2/3)))]) + ((61 + (3*I)*Sqrt[1407])^(1/3)*(18*(61 + (3*I)*Sqrt
[1407])^(1/3)*A + 3*(32*2^(1/3) + 10*(61 + (3*I)*Sqrt[1407])^(1/3) + (122
+ (6*I)*Sqrt[1407])^(2/3))*B + 2*(160*2^(1/3) + 41*(61 + (3*I)*Sqrt[1407])
^(1/3) + 5*(122 + (6*I)*Sqrt[1407])^(2/3))*C)*Log[32*2^(1/3) + (122 + (6*I
)*Sqrt[1407])^(2/3) - 6*(61 + (3*I)*Sqrt[1407])^(1/3)*(-5/3 + x)])/(36*(51
2*2^(2/3) + 32*(61 + (3*I)*Sqrt[1407])^(2/3) + 2^(1/3)*(61 + (3*I)*Sqrt[14
07])^(4/3))) - ((18*(61 + (3*I)*Sqrt[1407])^(2/3)*A + 3*(10*(61 + (3*I)*Sq
rt[1407])^(2/3) + 2^(2/3)*(61 + (3*I)*Sqrt[1407]) + 32*(122 + (6*I)*Sqrt[1
407])^(1/3))*B + 2*(41*(61 + (3*I)*Sqrt[1407])^(2/3) + 5*2^(2/3)*(61 + (3*
I)*Sqrt[1407]) + 160*(122 + (6*I)*Sqrt[1407])^(1/3))*C)*Log[512*2^(2/3)...
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2485
```
Int[((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (d_.)*(x_)^3)^(p_), x_S
ymbol] :> With[{r = Rt[-9*a*d^2 + Sqrt[3]*d*Sqrt[4*b^3*d + 27*a^2*d^2], 3]}
, Simp[1/d^(2*p)   Int[(e + f*x)^m*Simp[18^(1/3)*b*(d/(3*r)) - r/18^(1/3) +
 d*x, x]^p*Simp[b*(d/3) + 12^(1/3)*b^2*(d^2/(3*r^2)) + r^2/(3*12^(1/3)) - d
*(2^(1/3)*b*(d/(3^(1/3)*r)) - r/18^(1/3))*x + d^2*x^2, x]^p, x], x]] /; Fre
eQ[{a, b, d, e, f, m}, x] && NeQ[4*b^3 + 27*a^2*d, 0] && ILtQ[p, 0]
```

rule 2490
```
Int[(P3_)^(p_.)*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] :> With[{a = Coeff[P3
, x, 0], b = Coeff[P3, x, 1], c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Su
bst[Int[((3*d*e - c*f)/(3*d) + f*x)^m*Simp[(2*c^3 - 9*b*c*d + 27*a*d^2)/(27
*d^2) - (c^2 - 3*b*d)*(x/(3*d)) + d*x^3, x]^p, x], x, x + c/(3*d)] /; NeQ[c
, 0]] /; FreeQ[{e, f, m, p}, x] && PolyQ[P3, x, 3]
```

rule 2525
```
Int[(Pm_)/(Qn_), x_Symbol] :> With[{m = Expon[Pm, x], n = Expon[Qn, x]}, Si
mp[Coeff[Pm, x, m]*(Log[Qn]/(n*Coeff[Qn, x, n])), x] + Simp[1/(n*Coeff[Qn,
x, n])   Int[ExpandToSum[n*Coeff[Qn, x, n]*Pm - Coeff[Pm, x, m]*D[Qn, x], x
]/Qn, x], x] /; EqQ[m, n - 1]] /; PolyQ[Pm, x] && PolyQ[Qn, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.07 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.16

method	result	size
default	$\sum_{\_R=\mathrm{RootOf}(\_Z^3-5\_Z^2+3\_Z+2)} \frac{\left(C\_R^2+B\_R+A\right)\ln\left(x-\_R\right)}{3\_R^2-10\_R+3}$	47
risch	$\sum_{\_R=\mathrm{RootOf}(\_Z^3-5\_Z^2+3\_Z+2)} \frac{\left(C\_R^2+B\_R+A\right)\ln\left(x-\_R\right)}{3\_R^2-10\_R+3}$	47

input
```
int((C*x^2+B*x+A)/(x^3-5*x^2+3*x+2),x,method=_RETURNVERBOSE)
```

output
```
sum((C*_R^2+B*_R+A)/(3*_R^2-10*_R+3)*ln(x-_R),_R=RootOf(_Z^3-5*_Z^2+3*_Z+2
))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.55 (sec) , antiderivative size = 7641, normalized size of antiderivative = 25.73

$$\int \frac{A+Bx+Cx^2}{2+3x-5x^2+x^3}\,dx = \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)/(x^3-5*x^2+3*x+2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [A] (verification not implemented)

Time = 8.27 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.00

$$\int \frac{A+Bx+Cx^2}{2+3x-5x^2+x^3}\,dx$$
$$= \text{RootSum}\left(469t^3 - 469t^2C + t(-16A^2 - 33AB - 78AC - 39B^2 - 227BC - 183C^2) + A^3 + 5A^2B + 1\right.$$

input
```
integrate((C*x**2+B*x+A)/(x**3-5*x**2+3*x+2),x)
```

output
```
RootSum(469*_t**3 - 469*_t**2*C + _t*(-16*A**2 - 33*A*B - 78*A*C - 39*B**2
 - 227*B*C - 183*C**2) + A**3 + 5*A**2*B + 19*A**2*C + 3*A*B**2 + 21*A*B*C
 + 29*A*C**2 - 2*B**3 - 10*B**2*C - 6*B*C**2 + 4*C**3, Lambda(_t, _t*log(x
 + (15008*_t**2*A + 15477*_t**2*B + 36582*_t**2*C + 1407*_t*A**2 + 4690*_t
*A*B + 2814*_t*A*C + 1407*_t*B**2 - 5628*_t*B*C - 22981*_t*C**2 - 443*A**3
 - 711*A**2*B - 1510*A**2*C - 103*A*B**2 - 828*A*B*C - 339*A*C**2 - 138*B*
*3 - 1746*B**2*C - 4468*B*C**2 - 2298*C**3)/(61*A**3 - 207*A**2*B - 873*A*
*2*C - 873*A*B**2 - 5406*A*B*C - 8137*A*C**2 - 432*B**3 - 3447*B**2*C - 87
87*B*C**2 - 7051*C**3))))
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{2+3x-5x^2+x^3}\,dx = \int \frac{Cx^2+Bx+A}{x^3-5\,x^2+3\,x+2}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(x^3-5*x^2+3*x+2),x, algorithm="maxima")
```

output
```
integrate((C*x^2 + B*x + A)/(x^3 - 5*x^2 + 3*x + 2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A+Bx+Cx^2}{2+3x-5x^2+x^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((C*x^2+B*x+A)/(x^3-5*x^2+3*x+2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 0.66 (sec) , antiderivative size = 369, normalized size of antiderivative = 1.24

$$\int \frac{A+Bx+Cx^2}{2+3x-5x^2+x^3}\,dx = \sum_{k=1}^{3} \ln\Bigg(x\left(B^2+5\,B\,C+3\,C^2-A\,C\right)+2\,C^2+A\,B+5\,A\,C$$
$$-\operatorname{root}\Bigg(z^3-C\,z^2-\frac{z\left(16\,A^2+39\,B^2+183\,C^2+33\,A\,B+78\,A\,C+227\,B\,C\right)}{469}$$
$$+\frac{3\,A\,B\,C}{67}-\frac{10\,B^2\,C}{469}-\frac{6\,B\,C^2}{469}+\frac{29\,A\,C^2}{469}+\frac{19\,A^2\,C}{469}+\frac{5\,A^2\,B}{469}+\frac{3\,A\,B^2}{469}+\frac{A^3}{469}$$
$$+\frac{4\,C^3}{469}-\frac{2\,B^3}{469},z,k\Bigg)\Bigg(5\,A+3\,B+27\,C-x\left(3\,A+5\,B+35\,C\right)$$
$$+\operatorname{root}\Bigg(z^3-C\,z^2-\frac{z\left(16\,A^2+39\,B^2+183\,C^2+33\,A\,B+78\,A\,C+227\,B\,C\right)}{469}+\frac{3\,A\,B\,C}{67}-\frac{10\,B^2\,C}{469}-\frac{6\,B}{4}\cdots$$
$$-C\,z^2-\frac{z\left(16\,A^2+39\,B^2+183\,C^2+33\,A\,B+78\,A\,C+227\,B\,C\right)}{469}+\frac{3\,A\,B\,C}{67}$$
$$-\frac{10\,B^2\,C}{469}-\frac{6\,B\,C^2}{469}+\frac{29\,A\,C^2}{469}+\frac{19\,A^2\,C}{469}+\frac{5\,A^2\,B}{469}+\frac{3\,A\,B^2}{469}+\frac{A^3}{469}+\frac{4\,C^3}{469}-\frac{2\,B^3}{469},z,k\Bigg)$$

input

```
int((A + B*x + C*x^2)/(3*x - 5*x^2 + x^3 + 2),x)
```

output

```
symsum(log(x*(B^2 + 3*C^2 - A*C + 5*B*C) + 2*C^2 + A*B + 5*A*C - root(z^3
- C*z^2 - (z*(16*A^2 + 39*B^2 + 183*C^2 + 33*A*B + 78*A*C + 227*B*C))/469
+ (3*A*B*C)/67 - (10*B^2*C)/469 - (6*B*C^2)/469 + (29*A*C^2)/469 + (19*A^2
*C)/469 + (5*A^2*B)/469 + (3*A*B^2)/469 + A^3/469 + (4*C^3)/469 - (2*B^3)/
469, z, k)*(5*A + 3*B + 27*C - x*(3*A + 5*B + 35*C) + root(z^3 - C*z^2 - (
z*(16*A^2 + 39*B^2 + 183*C^2 + 33*A*B + 78*A*C + 227*B*C))/469 + (3*A*B*C)
/67 - (10*B^2*C)/469 - (6*B*C^2)/469 + (29*A*C^2)/469 + (19*A^2*C)/469 + (
5*A^2*B)/469 + (3*A*B^2)/469 + A^3/469 + (4*C^3)/469 - (2*B^3)/469, z, k)*
(32*x - 33)))*root(z^3 - C*z^2 - (z*(16*A^2 + 39*B^2 + 183*C^2 + 33*A*B +
78*A*C + 227*B*C))/469 + (3*A*B*C)/67 - (10*B^2*C)/469 - (6*B*C^2)/469 + (
29*A*C^2)/469 + (19*A^2*C)/469 + (5*A^2*B)/469 + (3*A*B^2)/469 + A^3/469 +
 (4*C^3)/469 - (2*B^3)/469, z, k), k, 1, 3)
```

Reduce [F]

$$\int \frac{A+Bx+Cx^2}{2+3x-5x^2+x^3}\,dx = \left(\int \frac{x}{x^3-5x^2+3x+2}dx\right)b+\frac{10\left(\int \frac{x}{x^3-5x^2+3x+2}dx\right)c}{3} + \left(\int \frac{1}{x^3-5x^2+3x+2}dx\right)a - \left(\int \frac{1}{x^3-5x^2+3x+2}dx\right)c+\frac{\log(x^3-5x^2+3x+2)\,c}{3}$$

input

```
int((C*x^2+B*x+A)/(x^3-5*x^2+3*x+2),x)
```

output

```
(3*int(x/(x**3 - 5*x**2 + 3*x + 2),x)*b + 10*int(x/(x**3 - 5*x**2 + 3*x +
2),x)*c + 3*int(1/(x**3 - 5*x**2 + 3*x + 2),x)*a - 3*int(1/(x**3 - 5*x**2
+ 3*x + 2),x)*c + log(x**3 - 5*x**2 + 3*x + 2)*c)/3
```

3.169 $\int \frac{A+Bx+Cx^2}{(2+3x-5x^2+x^3)^2} \, dx$

Optimal result

Integrand size = 26, antiderivative size = 816

$$\int \frac{A+Bx+Cx^2}{(2+3x-5x^2+x^3)^2} \, dx = \text{Too large to display}$$

output
```
-1/3*C/(x^3-5*x^2+3*x+2)+9/4096*(9*A+15*B+41*C-24*B*cos(1/6*Pi+1/3*arcsin(
61/128))-80*C*cos(1/6*Pi+1/3*arcsin(61/128)))*sec(1/3*arcsin(61/128))^2/(5
-3*x-8*cos(1/6*Pi+1/3*arcsin(61/128)))/(cos(1/6*Pi+1/3*arcsin(61/128))-sin
(1/3*arcsin(61/128)))^2-3/256*ln(5-3*x-8*sin(1/3*arcsin(61/128)))*(C*(30-2
0*cos(2/3*arcsin(61/128))-41*sin(1/3*arcsin(61/128)))+3*B*(3-2*cos(2/3*arc
sin(61/128))-5*sin(1/3*arcsin(61/128)))-9*A*sin(1/3*arcsin(61/128)))/(1-2*
cos(2/3*arcsin(61/128)))^3+3/256*(9*A+15*B+41*C-24*B*sin(1/3*arcsin(61/128
))-80*C*sin(1/3*arcsin(61/128)))/(1-2*cos(2/3*arcsin(61/128)))^2/(5-3*x-8*
sin(1/3*arcsin(61/128)))+3/4096*ln(5-3*x-8*cos(1/6*Pi+1/3*arcsin(61/128)))
*sec(1/3*arcsin(61/128))^3*(9*A*(3^(1/2)*cos(1/3*arcsin(61/128))-sin(1/3*a
rcsin(61/128)))-C*(60-41*3^(1/2)*cos(1/3*arcsin(61/128))+20*cos(2/3*arcsin
(61/128))+41*sin(1/3*arcsin(61/128))-20*3^(1/2)*sin(2/3*arcsin(61/128)))-3
*B*(6-5*3^(1/2)*cos(1/3*arcsin(61/128))+2*cos(2/3*arcsin(61/128))+5*sin(1/
3*arcsin(61/128))-2*3^(1/2)*sin(2/3*arcsin(61/128))))/(cos(1/3*arcsin(61/1
28))-3^(1/2)*sin(1/3*arcsin(61/128)))^3-3/4096*ln(5-3*x+8*sin(1/3*Pi+1/3*a
rcsin(61/128)))*sec(1/3*arcsin(61/128))^3*(9*A*(3^(1/2)*cos(1/3*arcsin(61/
128))+sin(1/3*arcsin(61/128)))+3*B*(6+5*3^(1/2)*cos(1/3*arcsin(61/128))+2*
cos(2/3*arcsin(61/128))+5*sin(1/3*arcsin(61/128))+2*3^(1/2)*sin(2/3*arcsin
(61/128)))+C*(60+41*3^(1/2)*cos(1/3*arcsin(61/128))+20*cos(2/3*arcsin(61/1
28))+41*sin(1/3*arcsin(61/128))+20*3^(1/2)*sin(2/3*arcsin(61/128))))/(c...
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.20

$$\int \frac{A+Bx+Cx^2}{(2+3x-5x^2+x^3)^2}\,dx$$
$$= \frac{1}{469}\left(\frac{C(66+163x-78x^2)+B(64+87x-33x^2)+A(-9+127x-32x^2)}{2+3x-5x^2+x^3}\right.$$
$$-\operatorname{RootSum}\Big[2+3\#1-5\#1^2$$
$$+\#1^3\&,\frac{-94A\log(x-\#1)-9B\log(x-\#1)+64C\log(x-\#1)+32A\log(x-\#1)\#1+33B\log(x}{3-10\#1+3\#1^2}$$

input
```
Integrate[(A + B*x + C*x^2)/(2 + 3*x - 5*x^2 + x^3)^2,x]
```

output
```
((C*(66 + 163*x - 78*x^2) + B*(64 + 87*x - 33*x^2) + A*(-9 + 127*x - 32*x^
2))/(2 + 3*x - 5*x^2 + x^3) - RootSum[2 + 3*#1 - 5*#1^2 + #1^3 & , (-94*A*
Log[x - #1] - 9*B*Log[x - #1] + 64*C*Log[x - #1] + 32*A*Log[x - #1]*#1 + 3
3*B*Log[x - #1]*#1 + 78*C*Log[x - #1]*#1)/(3 - 10*#1 + 3*#1^2) & ])/469
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 54.90 (sec) , antiderivative size = 1777, normalized size of antiderivative = 2.18, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {2526, 2490, 2485, 27, 1235, 27, 1237, 27, 1200, 7239, 27, 25, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{(x^3-5x^2+3x+2)^2}dx$$

$$\Big\downarrow \text{2526}$$

$$\frac{1}{3}\int \frac{3(A-C)+(3B+10C)x}{(x^3-5x^2+3x+2)^2}dx-\frac{C}{3(x^3-5x^2+3x+2)}$$

$$\Big\downarrow \text{2490}$$

$$\frac{1}{3}\int \frac{\frac{1}{3}(9(A-C)+5(3B+10C))+(3B+10C)\left(x-\frac{5}{3}\right)}{\left(\left(x-\frac{5}{3}\right)^3-\frac{16}{3}\left(x-\frac{5}{3}\right)-\frac{61}{27}\right)^2}d\left(x-\frac{5}{3}\right)-\frac{C}{3(x^3-5x^2+3x+2)}$$

$$\Big\downarrow \text{2485}$$

$$-\frac{C}{3(x^3-5x^2+3x+2)}+$$

$$\frac{1}{3}\int \frac{3888\big(9A+15B+41C+3(3B+10C)\left(x-\right.}{\left(\frac{32\sqrt[3]{2}+\left(122+6i\sqrt{1407}\right)^{2/3}}{\sqrt[3]{61+3i\sqrt{1407}}}-6\left(x-\frac{5}{3}\right)\right)^2\left(-18\left(x-\frac{5}{3}\right)^2-6\left(\frac{16}{\sqrt[3]{\frac{1}{2}\left(61+3i\sqrt{1407}\right)}}+\sqrt[3]{\frac{1}{2}\left(61+3i\sqrt{1407}\right.}\right.\right.}$$

$$\Big\downarrow \text{27}$$

$$1296 \int \frac{-\frac{C}{3\left(x^3-5x^2+3x+2\right)}+ \qquad 9A+15B+41C+3(3B+10C)\left(x-\right.}{\left(\frac{32\sqrt[3]{2}+\left(122+6i\sqrt{1407}\right)^{2/3}}{\sqrt[3]{61+3i\sqrt{1407}}}-6\left(x-\frac{5}{3}\right)\right)^2\left(-18\left(x-\frac{5}{3}\right)^2-6\left(\frac{16}{\sqrt[3]{\frac{1}{2}\left(61+3i\sqrt{1407}\right)}}+\sqrt[3]{\frac{1}{2}\left(61+3i\sqrt{14\right.}}\right.\right.}$$

$\downarrow$ 1235

$$1296\left(-\frac{\left(61i-3\sqrt{1407}\right)\sqrt[3]{61+3i\sqrt{1407}}\left(2^{2/3}(32A+33B+78C)+\frac{\left(3\sqrt[3]{2}\left(61+3i\sqrt{1407}\right)A+96\sqrt[3]{61+3i\sqrt{14}}\right.}{}\right.}{12\left(4221i+61\sqrt{1407}\right)\left(32-\sqrt[3]{2}\left(61+3i\sqrt{1407}\right)^{2/3}\right)\left(\frac{32\sqrt[3]{2}+\left(122+6i\sqrt{1407}\right)^{2/3}}{\sqrt[3]{61+3i\sqrt{1407}}}-6\left(x-\frac{5}{3}\right)\right)\left(-18\left(x-\right.\right.}\right.$$

$$\frac{C}{3\left(x^3-5x^2+3x+2\right)}$$

$\downarrow$ 27

$$1296\left(\frac{\left(61i-3\sqrt{1407}\right)\sqrt[3]{\frac{1}{2}\left(61+3i\sqrt{1407}\right)}\int\frac{\frac{3\left(1952i-96\sqrt{1407}-512i2^{2/3}\sqrt[3]{61+3i\sqrt{1407}}-\sqrt[3]{2}\left(61i-3\sqrt{1407}\right)\left(61+3i\sqrt{1407}\right)^{2/3}\cdots\right.}{\cdots}}{\cdots}}{\cdots}\right.$$

$$\frac{C}{3\left(x^3-5x^2+3x+2\right)}$$

$\downarrow$ 1237

$$1296\left(\frac{\left(61i-3\sqrt{1407}\right)\sqrt[3]{\frac{1}{2}\left(61+3i\sqrt{1407}\right)}\left(\frac{\left(61+3i\sqrt{1407}\right)^{2/3}\int-\frac{\cdots}{\left(\frac{32\sqrt[3]{2}+\left(122+6i\sqrt{1407}\right)^{2/3}}{\sqrt[3]{61+3i\sqrt{1407}}}-6\left(x-\frac{5}{3}\right)\right)\left(-18\left(x-\frac{5}{3}\right)^2-6\left(\frac{\cdots}{\sqrt[3]{\cdots}}\cdots\right.\right.}}{108\left(512\ 2^{2/3}\cdots\right.}\cdots\right.}{\cdots}\right.$$

$$\frac{C}{3\left(x^3-5x^2+3x+2\right)}$$

$\downarrow$ 27

$$1296 \left(\frac{\left(61i - 3\sqrt{1407}\right) \sqrt[3]{\frac{1}{2}\left(61 + 3i\sqrt{1407}\right)} \left(-\frac{\left(61+3i\sqrt{1407}\right)^{2/3}(32A+33B+78C)}{2\left(512\ 2^{2/3}+32\left(61+3i\sqrt{1407}\right)^{2/3}+\sqrt[3]{2}\left(61+3i\sqrt{1407}\right)^{4/3}\right)\left(\frac{32\sqrt[3]{2}+\left(122+6i\sqrt{\cdot}\right.}{\sqrt[3]{61+3i\sqrt{\cdot}}}\right.} \right.}{} \right.$$

$$\frac{C}{3\left(x^3-5x^2+3x+2\right)}$$

$$\Big\downarrow 1200$$

$$1296 \left(\frac{\left(61i - 3\sqrt{1407}\right) \sqrt[3]{\frac{1}{2}\left(61 + 3i\sqrt{1407}\right)} \left(-\frac{\left(61+3i\sqrt{1407}\right)^{2/3}(32A+33B+78C)}{2\left(512\ 2^{2/3}+32\left(61+3i\sqrt{1407}\right)^{2/3}+\sqrt[3]{2}\left(61+3i\sqrt{1407}\right)^{4/3}\right)\left(\frac{32\sqrt[3]{2}+\left(122+6i\sqrt{\cdot}\right.}{\sqrt[3]{61+3i\sqrt{\cdot}}}\right.} \right.}{} \right.$$

$$\frac{C}{3\left(x^3-5x^2+3x+2\right)}$$

$\downarrow$ 7239

$$1296 \left(\frac{\left(61i - 3\sqrt{1407}\right)\sqrt[3]{\frac{1}{2}\left(61+3i\sqrt{1407}\right)}\left(-\frac{\left(61+3i\sqrt{1407}\right)^{2/3}(32A+33B+78C)}{2\left(512\ 2^{2/3}+32\left(61+3i\sqrt{1407}\right)^{2/3}+\sqrt[3]{2}\left(61+3i\sqrt{1407}\right)^{4/3}\right)\left(\frac{32\sqrt[3]{2}+\left(122+6i\sqrt{}\right.}{\sqrt[3]{61+3i\sqrt{}}}\right.} \right.}{} \right.$$

$$\frac{C}{3\left(x^3-5x^2+3x+2\right)}$$

$\downarrow$ 27

$$1296 \left(\frac{\left(61i - 3\sqrt{1407}\right)\sqrt[3]{\frac{1}{2}\left(61+3i\sqrt{1407}\right)}\left(-\frac{\left(61+3i\sqrt{1407}\right)^{2/3}(32A+33B+78C)}{2\left(512\ 2^{2/3}+32\left(61+3i\sqrt{1407}\right)^{2/3}+\sqrt[3]{2}\left(61+3i\sqrt{1407}\right)^{4/3}\right)\left(\frac{32\sqrt[3]{2}+\left(122+6i\sqrt{}\right.}{\sqrt[3]{61+3i\sqrt{}}}\right.} \right.}{} \right.$$

$$\frac{C}{3\left(x^3-5x^2+3x+2\right)}$$

$\downarrow$ 25

$$1296\left(\frac{\left(61i-3\sqrt{1407}\right)\sqrt[3]{\frac{1}{2}\left(61+3i\sqrt{1407}\right)}\left(\frac{3\left(61+3i\sqrt{1407}\right)^{4/3}\left(1952+96i\sqrt{1407}+512\ 2^{2/3}\sqrt[3]{61+3i\sqrt{1407}}+\sqrt[3]{2}\left(61+3i\right.\right.}{}\right.}{}\right.$$

$$\frac{C}{3\left(x^3-5x^2+3x+2\right)}$$

$\downarrow$ 1200

$$1296\left(\frac{\left(61i-3\sqrt{1407}\right)\sqrt[3]{\frac{1}{2}\left(61+3i\sqrt{1407}\right)}\left(\frac{3\left(61+3i\sqrt{1407}\right)^{4/3}\left(1952+96i\sqrt{1407}+512\ 2^{2/3}\sqrt[3]{61+3i\sqrt{1407}}+\sqrt[3]{2}\left(61+3i\right.\right.}{}\right.}{}\right.$$

$$\frac{C}{3\left(x^3-5x^2+3x+2\right)}$$

$\downarrow$ 2009

$$1296 \left(\frac{\left(61i-3\sqrt{1407}\right)\sqrt[3]{\frac{1}{2}\left(61+3i\sqrt{1407}\right)}\left(\frac{3\left(61+3i\sqrt{1407}\right)^{4/3}\left(1952+96i\sqrt{1407}+512\ 2^{2/3}\sqrt[3]{61+3i\sqrt{1407}}+\sqrt[3]{2}\left(61+3i\right.\right.}{}\right.}{} \right.$$

$$\frac{C}{3\left(x^3-5x^2+3x+2\right)}$$

input
```
Int[(A + B*x + C*x^2)/(2 + 3*x - 5*x^2 + x^3)^2,x]
```

output
```
-1/3*C/(2 + 3*x - 5*x^2 + x^3) + 1296*(-1/12*((61*I - 3*Sqrt[1407])*(61 +
(3*I)*Sqrt[1407])^(1/3)*(2^(2/3)*(32*A + 33*B + 78*C) + ((96*(61 + (3*I)*S
qrt[1407])^(1/3)*A + 3*2^(1/3)*(61 + (3*I)*Sqrt[1407])*A + 3*(33 - I*Sqrt[
1407])*(61 + (3*I)*Sqrt[1407])^(1/3)*B - 3*2^(1/3)*(69 - (5*I)*Sqrt[1407])
*B + 2*(61 + (3*I)*Sqrt[1407])^(1/3)*(117 - (5*I)*Sqrt[1407])*C - 2^(1/3)*
(873 - (41*I)*Sqrt[1407])*C)*(-5/3 + x))/(61 + (3*I)*Sqrt[1407])^(2/3)))/(
(4221*I + 61*Sqrt[1407])*(32 - 2^(1/3)*(61 + (3*I)*Sqrt[1407])^(2/3))*((32
*2^(1/3) + (122 + (6*I)*Sqrt[1407])^(2/3))/(61 + (3*I)*Sqrt[1407])^(1/3) -
 6*(-5/3 + x))*(32 - 512/((61 + (3*I)*Sqrt[1407])/2)^(2/3) - 2^(1/3)*(61 +
 (3*I)*Sqrt[1407])^(2/3) - 6*(16/((61 + (3*I)*Sqrt[1407])/2)^(1/3) + ((61
+ (3*I)*Sqrt[1407])/2)^(1/3))*(-5/3 + x) - 18*(-5/3 + x)^2)) + ((61*I - 3*
Sqrt[1407])*((61 + (3*I)*Sqrt[1407])/2)^(1/3)*(-1/2*((61 + (3*I)*Sqrt[1407
])^(2/3)*(32*A + 33*B + 78*C))/((512*2^(2/3) + 32*(61 + (3*I)*Sqrt[1407])^
(2/3) + 2^(1/3)*(61 + (3*I)*Sqrt[1407])^(4/3))*((32*2^(1/3) + (122 + (6*I)
*Sqrt[1407])^(2/3))/(61 + (3*I)*Sqrt[1407])^(1/3) - 6*(-5/3 + x))) + (3*(6
1 + (3*I)*Sqrt[1407])^(4/3)*(1952 + (96*I)*Sqrt[1407] + 512*2^(2/3)*(61 +
(3*I)*Sqrt[1407])^(1/3) + 2^(1/3)*(61 + (3*I)*Sqrt[1407])^(5/3))*(((64*(61
 + (3*I)*Sqrt[1407])^(1/3)*(125 - (61*I)*Sqrt[1407])*A + 2*2^(1/3)*(7625 -
 (7817*I)*Sqrt[1407])*A - 6*2^(1/3)*(62907 + (5*I)*Sqrt[1407])*B + 3*(61 +
 (3*I)*Sqrt[1407])^(1/3)*(31359 + (65*I)*Sqrt[1407])*B + 2*(61 + (3*I)*...
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1235
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^m
*(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*
m + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) -
f*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
)
```

rule 1237
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[
(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && LtQ[m, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2485
```
Int[((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (d_.)*(x_)^3)^(p_), x_S
ymbol] :> With[{r = Rt[-9*a*d^2 + Sqrt[3]*d*Sqrt[4*b^3*d + 27*a^2*d^2], 3]}
, Simp[1/d^(2*p)   Int[(e + f*x)^m*Simp[18^(1/3)*b*(d/(3*r)) - r/18^(1/3) +
 d*x, x]^p*Simp[b*(d/3) + 12^(1/3)*b^2*(d^2/(3*r^2)) + r^2/(3*12^(1/3)) - d
*(2^(1/3)*b*(d/(3^(1/3)*r)) - r/18^(1/3))*x + d^2*x^2, x]^p, x], x]] /; Fre
eQ[{a, b, d, e, f, m}, x] && NeQ[4*b^3 + 27*a^2*d, 0] && ILtQ[p, 0]
```

rule 2490
```
Int[(P3_)^(p_.)*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] :> With[{a = Coeff[P3
, x, 0], b = Coeff[P3, x, 1], c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Su
bst[Int[((3*d*e - c*f)/(3*d) + f*x)^m*Simp[(2*c^3 - 9*b*c*d + 27*a*d^2)/(27
*d^2) - (c^2 - 3*b*d)*(x/(3*d)) + d*x^3, x]^p, x], x, x + c/(3*d)] /; NeQ[c
, 0]] /; FreeQ[{e, f, m, p}, x] && PolyQ[P3, x, 3]
```

rule 2526
```
Int[(Pm_)*(Qn_)^(p_), x_Symbol] :> With[{m = Expon[Pm, x], n = Expon[Qn, x]
}, Simp[Coeff[Pm, x, m]*(Qn^(p + 1)/(n*(p + 1)*Coeff[Qn, x, n])), x] + Simp
[1/(n*Coeff[Qn, x, n])   Int[ExpandToSum[n*Coeff[Qn, x, n]*Pm - Coeff[Pm, x
, m]*D[Qn, x], x]*Qn^p, x], x] /; EqQ[m, n - 1]] /; FreeQ[p, x] && PolyQ[Pm
, x] && PolyQ[Qn, x] && NeQ[p, -1]
```

rule 7239
```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.09 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.14

method	result
default	$\frac{\left(-\frac{33B}{469}-\frac{32A}{469}-\frac{78C}{469}\right)x^2+\left(\frac{87B}{469}+\frac{127A}{469}+\frac{163C}{469}\right)x+\frac{64B}{469}-\frac{9A}{469}+\frac{66C}{469}}{x^3-5x^2+3x+2}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^3-5\_Z^2+3\_Z+2\right)}\frac{\left(-32A\_R-33B\_R-\right.}{3\_}\right.}{469}$
risch	$\frac{\left(-\frac{33B}{469}-\frac{32A}{469}-\frac{78C}{469}\right)x^2+\left(\frac{87B}{469}+\frac{127A}{469}+\frac{163C}{469}\right)x+\frac{64B}{469}-\frac{9A}{469}+\frac{66C}{469}}{x^3-5x^2+3x+2}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^3-5\_Z^2+3\_Z+2\right)}\frac{\left((-32A-33B-78C)\_R\right.}{3\_R}\right.}{469}$

input
```
int((C*x^2+B*x+A)/(x^3-5*x^2+3*x+2)^2,x,method=_RETURNVERBOSE)
```

output
```
((-33/469*B-32/469*A-78/469*C)*x^2+(87/469*B+127/469*A+163/469*C)*x+64/469
*B-9/469*A+66/469*C)/(x^3-5*x^2+3*x+2)+1/469*sum((-32*A*_R-33*B*_R-78*C*_R
+94*A+9*B-64*C)/(3*_R^2-10*_R+3)*ln(x-_R),_R=RootOf(_Z^3-5*_Z^2+3*_Z+2))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.50 (sec) , antiderivative size = 5331, normalized size of antiderivative = 6.53

$$\int \frac{A+Bx+Cx^2}{(2+3x-5x^2+x^3)^2}\,dx = \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)/(x^3-5*x^2+3*x+2)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [A] (verification not implemented)

Time = 12.04 (sec) , antiderivative size = 343, normalized size of antiderivative = 0.42

$$\int \frac{A+Bx+Cx^2}{(2+3x-5x^2+x^3)^2}\,dx$$
$$= \text{RootSum}\Big(103161709t^3 + t\big(-82048A^2 + 2430AB + 172196AC - 33966B^2 - 228870BC - 467548C^2\big)$$
$$+ \frac{-9A+64B+66C+x^2(-32A-33B-78C)+x(127A+87B+163C)}{469x^3-2345x^2+1407x+938}$$

input
```
integrate((C*x**2+B*x+A)/(x**3-5*x**2+3*x+2)**2,x)
```

output
```
RootSum(103161709*_t**3 + _t*(-82048*A**2 + 2430*A*B + 172196*A*C - 33966*
B**2 - 228870*B*C - 467548*C**2) - 488*A**3 - 1416*A**2*B - 3256*A**2*C +
648*A*B**2 + 7152*A*B*C + 15176*A*C**2 + 189*B**3 + 1242*B**2*C + 564*B*C*
*2 - 4432*C**3, Lambda(_t, _t*log(x + (201371655968*_t**2*A - 82632528909*
_t**2*B - 476813418998*_t**2*C - 109980500*_t*A**2 - 4946482968*_t*A*B - 1
6268315560*_t*A*C + 118778940*_t*B**2 + 5738342568*_t*B*C + 17698062060*_t
*C**2 - 172304424*A**3 + 132720312*A**2*B + 959314312*A**2*C + 33348852*A*
B**2 - 43114944*A*B*C - 1031172552*A*C**2 + 5527359*B**3 + 21924738*B**2*C
 + 94639932*B*C**2 + 448879664*C**3)/(39319576*A**3 - 51446664*A**2*B - 28
9447608*A**2*C - 47308536*A*B**2 - 212496912*A*B*C - 64713912*A*C**2 + 756
6291*B**3 + 122971446*B**2*C + 516153276*B*C**2 + 595074944*C**3)))) + (-9
*A + 64*B + 66*C + x**2*(-32*A - 33*B - 78*C) + x*(127*A + 87*B + 163*C))/
(469*x**3 - 2345*x**2 + 1407*x + 938)
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{(2+3x-5x^2+x^3)^2}\,dx = \int \frac{Cx^2+Bx+A}{(x^3-5\,x^2+3\,x+2)^2}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(x^3-5*x^2+3*x+2)^2,x, algorithm="maxima")
```

output
```
-1/469*((32*A + 33*B + 78*C)*x^2 - (127*A + 87*B + 163*C)*x + 9*A - 64*B -
 66*C)/(x^3 - 5*x^2 + 3*x + 2) - 1/469*integrate(((32*A + 33*B + 78*C)*x -
 94*A - 9*B + 64*C)/(x^3 - 5*x^2 + 3*x + 2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A+Bx+Cx^2}{(2+3x-5x^2+x^3)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((C*x^2+B*x+A)/(x^3-5*x^2+3*x+2)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 997, normalized size of antiderivative = 1.22

$$\int \frac{A+Bx+Cx^2}{(2+3x-5x^2+x^3)^2}\,dx = \text{Too large to display}$$

input
```
int((A + B*x + C*x^2)/(3*x - 5*x^2 + x^3 + 2)^2,x)
```

output
```
symsum(log((1024*A^2*x)/219961 + (1089*B^2*x)/219961 + (6084*C^2*x)/219961
 - 32*root(z^3 - z*((82048*A^2)/103161709 + (33966*B^2)/103161709 + (46754
8*C^2)/103161709 - (2430*A*B)/103161709 - (172196*A*C)/103161709 + (228870
*B*C)/103161709) + (7152*A*B*C)/103161709 + (1242*B^2*C)/103161709 + (564*
B*C^2)/103161709 - (3256*A^2*C)/103161709 - (1416*A^2*B)/103161709 + (648*
A*B^2)/103161709 + (2168*A*C^2)/14737387 - (4432*C^3)/103161709 - (488*A^3
)/103161709 + (27*B^3)/14737387, z, k)^2*x - (3008*A^2)/219961 - (297*B^2)
/219961 + (4992*C^2)/219961 + 33*root(z^3 - z*((82048*A^2)/103161709 + (33
966*B^2)/103161709 + (467548*C^2)/103161709 - (2430*A*B)/103161709 - (1721
96*A*C)/103161709 + (228870*B*C)/103161709) + (7152*A*B*C)/103161709 + (12
42*B^2*C)/103161709 + (564*B*C^2)/103161709 - (3256*A^2*C)/103161709 - (14
16*A^2*B)/103161709 + (648*A*B^2)/103161709 + (2168*A*C^2)/14737387 - (443
2*C^3)/103161709 - (488*A^3)/103161709 + (27*B^3)/14737387, z, k)^2 - (339
0*A*B)/219961 - (5284*A*C)/219961 + (1410*B*C)/219961 - (374*A*root(z^3 -
z*((82048*A^2)/103161709 + (33966*B^2)/103161709 + (467548*C^2)/103161709
- (2430*A*B)/103161709 - (172196*A*C)/103161709 + (228870*B*C)/103161709)
+ (7152*A*B*C)/103161709 + (1242*B^2*C)/103161709 + (564*B*C^2)/103161709
- (3256*A^2*C)/103161709 - (1416*A^2*B)/103161709 + (648*A*B^2)/103161709
+ (2168*A*C^2)/14737387 - (4432*C^3)/103161709 - (488*A^3)/103161709 + (27
*B^3)/14737387, z, k))/469 + (54*B*root(z^3 - z*((82048*A^2)/103161709 ...
```

Reduce [F]

$$\int \frac{A+Bx+Cx^2}{(2+3x-5x^2+x^3)^2}\,dx$$
$$= \frac{6\left(\int \frac{x^3}{x^6-10x^5+31x^4-26x^3-11x^2+12x+4}dx\right)b\,x^3-30\left(\int \frac{x^3}{x^6-10x^5+31x^4-26x^3-11x^2+12x+4}dx\right)b\,x^2+18\left(\int \frac{}{x^6-10x^5+31}\right.}{}$$

input
```
int((C*x^2+B*x+A)/(x^3-5*x^2+3*x+2)^2,x)
```

output
```
(6*int(x**3/(x**6 - 10*x**5 + 31*x**4 - 26*x**3 - 11*x**2 + 12*x + 4),x)*b
*x**3 - 30*int(x**3/(x**6 - 10*x**5 + 31*x**4 - 26*x**3 - 11*x**2 + 12*x +
 4),x)*b*x**2 + 18*int(x**3/(x**6 - 10*x**5 + 31*x**4 - 26*x**3 - 11*x**2
+ 12*x + 4),x)*b*x + 12*int(x**3/(x**6 - 10*x**5 + 31*x**4 - 26*x**3 - 11*
x**2 + 12*x + 4),x)*b + 20*int(x**3/(x**6 - 10*x**5 + 31*x**4 - 26*x**3 -
11*x**2 + 12*x + 4),x)*c*x**3 - 100*int(x**3/(x**6 - 10*x**5 + 31*x**4 - 2
6*x**3 - 11*x**2 + 12*x + 4),x)*c*x**2 + 60*int(x**3/(x**6 - 10*x**5 + 31*
x**4 - 26*x**3 - 11*x**2 + 12*x + 4),x)*c*x + 40*int(x**3/(x**6 - 10*x**5
+ 31*x**4 - 26*x**3 - 11*x**2 + 12*x + 4),x)*c + 50*int(1/(x**6 - 10*x**5
+ 31*x**4 - 26*x**3 - 11*x**2 + 12*x + 4),x)*a*x**3 - 250*int(1/(x**6 - 10
*x**5 + 31*x**4 - 26*x**3 - 11*x**2 + 12*x + 4),x)*a*x**2 + 150*int(1/(x**
6 - 10*x**5 + 31*x**4 - 26*x**3 - 11*x**2 + 12*x + 4),x)*a*x + 100*int(1/(
x**6 - 10*x**5 + 31*x**4 - 26*x**3 - 11*x**2 + 12*x + 4),x)*a + 9*int(1/(x
**6 - 10*x**5 + 31*x**4 - 26*x**3 - 11*x**2 + 12*x + 4),x)*b*x**3 - 45*int
(1/(x**6 - 10*x**5 + 31*x**4 - 26*x**3 - 11*x**2 + 12*x + 4),x)*b*x**2 + 2
7*int(1/(x**6 - 10*x**5 + 31*x**4 - 26*x**3 - 11*x**2 + 12*x + 4),x)*b*x +
 18*int(1/(x**6 - 10*x**5 + 31*x**4 - 26*x**3 - 11*x**2 + 12*x + 4),x)*b -
 20*int(1/(x**6 - 10*x**5 + 31*x**4 - 26*x**3 - 11*x**2 + 12*x + 4),x)*c*x
**3 + 100*int(1/(x**6 - 10*x**5 + 31*x**4 - 26*x**3 - 11*x**2 + 12*x + 4),
x)*c*x**2 - 60*int(1/(x**6 - 10*x**5 + 31*x**4 - 26*x**3 - 11*x**2 + 12...
```

3.170 $\int (A+Bx+Cx^2)(2+3x-5x^2+x^3)^p\,dx$

Optimal result

Integrand size = 26, antiderivative size = 520

$$\int (A+Bx+Cx^2)(2+3x-5x^2+x^3)^p\,dx = \text{Too large to display}$$

output
```
C*(x^3-5*x^2+3*x+2)^(p+1)/(3*p+3)+3^(-3+1/2*p)*(3*B+10*C)*(x^3-5*x^2+3*x+2
)^p*AppellF1(2+p,-p,-p,3+p,(5-3*x+8*sin(1/3*Pi+1/3*arcsin(61/128)))/(8*sin
(1/3*arcsin(61/128))+8*sin(1/3*Pi+1/3*arcsin(61/128))),1/24*sec(1/3*arcsin
(61/128))*(5-3*x+8*sin(1/3*Pi+1/3*arcsin(61/128)))*3^(1/2))*(32*cos(1/3*ar
csin(61/128))*(3^(1/2)*cos(1/3*arcsin(61/128))+3*sin(1/3*arcsin(61/128))))
^p*(5-3*x+8*sin(1/3*Pi+1/3*arcsin(61/128)))^2/(2+p)/((-5+3*x+8*cos(1/6*Pi+
1/3*arcsin(61/128)))^p)/((-5+3*x+8*sin(1/3*arcsin(61/128)))^p)-3^(-3+1/2*p
)*(x^3-5*x^2+3*x+2)^p*AppellF1(p+1,-p,-p,2+p,(5-3*x+8*sin(1/3*Pi+1/3*arcsi
n(61/128)))/(8*sin(1/3*arcsin(61/128))+8*sin(1/3*Pi+1/3*arcsin(61/128))),1
/24*sec(1/3*arcsin(61/128))*(5-3*x+8*sin(1/3*Pi+1/3*arcsin(61/128)))*3^(1/
2))*(32*cos(1/3*arcsin(61/128))*(3^(1/2)*cos(1/3*arcsin(61/128))+3*sin(1/3
*arcsin(61/128))))^p*(5-3*x+8*sin(1/3*Pi+1/3*arcsin(61/128)))*(9*A+15*B+41
*C+8*(3*B+10*C)*sin(1/3*Pi+1/3*arcsin(61/128)))/(p+1)/((-5+3*x+8*cos(1/6*P
i+1/3*arcsin(61/128)))^p)/((-5+3*x+8*sin(1/3*arcsin(61/128)))^p)
```

Mathematica [F]

$$\int (A+Bx+Cx^2)\left(2+3x-5x^2+x^3\right)^p\,dx = \int (A+Bx+Cx^2)\left(2+3x-5x^2+x^3\right)^p\,dx$$

input
```
Integrate[(A + B*x + C*x^2)*(2 + 3*x - 5*x^2 + x^3)^p,x]
```

output
```
Integrate[(A + B*x + C*x^2)*(2 + 3*x - 5*x^2 + x^3)^p, x]
```

Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 4.15 (sec) , antiderivative size = 1899, normalized size of antiderivative = 3.65, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {2526, 2490, 2486, 27, 1269, 1179, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(x^3-5x^2+3x+2\right)^p\left(A+Bx+Cx^2\right)dx$$

$$\Big\downarrow \text{2526}$$

$$\frac{1}{3}\int (3(A-C)+(3B+10C)x)\left(x^3-5x^2+3x+2\right)^p dx+\frac{C\left(x^3-5x^2+3x+2\right)^{p+1}}{3(p+1)}$$

$$\Big\downarrow \text{2490}$$

$$\frac{1}{3}\int \left(\frac{1}{3}(9(A-C)+5(3B+10C))+(3B+10C)\left(x-\frac{5}{3}\right)\right)\left(\left(x-\frac{5}{3}\right)^3-\frac{16}{3}\left(x-\frac{5}{3}\right)-\frac{61}{27}\right)^p d\left(x-\frac{5}{3}\right)+\frac{C\left(x^3-5x^2+3x+2\right)^{p+1}}{3(p+1)}$$

$$\Big\downarrow \text{2486}$$

$$\frac{C\left(x^3-5x^2+3x+2\right)^{p+1}}{3(p+1)}+$$

$$\frac{1}{3}\left(\left(x-\frac{5}{3}\right)^2+\frac{1}{3}\left(\frac{16}{\sqrt[3]{\frac{1}{2}\left(61+3i\sqrt{1407}\right)}}+\sqrt[3]{\frac{1}{2}\left(61+3i\sqrt{1407}\right)}\right)\left(x-\frac{5}{3}\right)+\frac{1}{18}\left(-32+\frac{512}{\left(\frac{1}{2}\left(61+3i\sqrt{1407}\right)\right.}\right.\right.$$

$\downarrow$ 27

$$\frac{C\left(x^3-5x^2+3x+2\right)^{p+1}}{3(p+1)}+$$

$$\frac{1}{9}\left(\left(x-\frac{5}{3}\right)^2+\frac{1}{3}\left(\frac{16}{\sqrt[3]{\frac{1}{2}\left(61+3i\sqrt{1407}\right)}}+\sqrt[3]{\frac{1}{2}\left(61+3i\sqrt{1407}\right)}\right)\left(x-\frac{5}{3}\right)+\frac{1}{18}\left(-32+\frac{512}{\left(\frac{1}{2}\left(61+3i\sqrt{1407}\right)\right.}\right.\right.$$

$\downarrow$ 1269

$$\frac{C\left(x^3-5x^2+3x+2\right)^{p+1}}{3(p+1)}+$$

$$\frac{1}{9}\left(\left(x-\frac{5}{3}\right)^2+\frac{1}{3}\left(\frac{16}{\sqrt[3]{\frac{1}{2}\left(61+3i\sqrt{1407}\right)}}+\sqrt[3]{\frac{1}{2}\left(61+3i\sqrt{1407}\right)}\right)\left(x-\frac{5}{3}\right)+\frac{1}{18}\left(-32+\frac{512}{\left(\frac{1}{2}\left(61+3i\sqrt{1407}\right)\right.}\right.\right.$$

$\downarrow$ 1179

$$\frac{1}{9}\left(\left(x-\frac{5}{3}\right)^2+\frac{1}{3}\left(\frac{16}{\sqrt[3]{\frac{1}{2}\left(61+3i\sqrt{1407}\right)}}+\sqrt[3]{\frac{1}{2}\left(61+3i\sqrt{1407}\right)}\right)\left(x-\frac{5}{3}\right)+\frac{1}{18}\left(-32+\frac{512}{\left(\frac{1}{2}\left(61+3i\sqrt{1407}\right)\right.}\right.\right.$$

$$\frac{C\left(x^3-5x^2+3x+2\right)^{p+1}}{3(p+1)}$$

$\downarrow$ 150

$$\frac{1}{9}\left(\left(x-\frac{5}{3}\right)^2+\frac{1}{3}\left(\frac{16}{\sqrt[3]{\frac{1}{2}\left(61+3i\sqrt{1407}\right)}}+\sqrt[3]{\frac{1}{2}\left(61+3i\sqrt{1407}\right)}\right)\left(x-\frac{5}{3}\right)+\frac{1}{18}\left(-32+\frac{512}{\left(\frac{1}{2}\left(61+3i\sqrt{1407}\right)\right.}\right.\right.$$

$$\frac{C(x^3-5x^2+3x+2)^{p+1}}{3(p+1)}$$

input
```
Int[(A + B*x + C*x^2)*(2 + 3*x - 5*x^2 + x^3)^p,x]
```

output
```
(C*(2 + 3*x - 5*x^2 + x^3)^(1 + p))/(3*(1 + p)) + ((-61/27 - (16*(-5/3 + x
))/3 + (-5/3 + x)^3)^p*(((18*A + 30*B + 82*C + ((32*2^(1/3) + (122 + (6*I)
*Sqrt[1407])^(2/3))*(3*B + 10*C))/(61 + (3*I)*Sqrt[1407])^(1/3))*((-32 + 5
12/((61 + (3*I)*Sqrt[1407])/2)^(2/3) + 2^(1/3)*(61 + (3*I)*Sqrt[1407])^(2/
3))/18 + ((16/((61 + (3*I)*Sqrt[1407])/2)^(1/3) + ((61 + (3*I)*Sqrt[1407])
/2)^(1/3))*(-5/3 + x))/3 + (-5/3 + x)^2)^p*(-5/3 - (32*2^(1/3) + (122 + (6
*I)*Sqrt[1407])^(2/3))/(6*(61 + (3*I)*Sqrt[1407])^(1/3)) + x)^(1 + p)*Appe
llF1[1 + p, -p, -p, 2 + p, (-6*2^(2/3)*(61 + (3*I)*Sqrt[1407])^(1/3)*(-5/3
 - (32*2^(1/3) + (122 + (6*I)*Sqrt[1407])^(2/3))/(6*(61 + (3*I)*Sqrt[1407]
)^(1/3)) + x))/(96 + 3*2^(1/3)*(61 + (3*I)*Sqrt[1407])^(2/3) - 2^(1/6)*Sqr
t[3*(-512*2^(2/3) + 64*(61 + (3*I)*Sqrt[1407])^(2/3) - 2^(1/3)*(61 + (3*I)
*Sqrt[1407])^(4/3))]), (-6*2^(2/3)*(61 + (3*I)*Sqrt[1407])^(1/3)*(-5/3 - (
32*2^(1/3) + (122 + (6*I)*Sqrt[1407])^(2/3))/(6*(61 + (3*I)*Sqrt[1407])^(1
/3)) + x))/(96 + 3*2^(1/3)*(61 + (3*I)*Sqrt[1407])^(2/3) + 2^(1/6)*Sqrt[3*
(-512*2^(2/3) + 64*(61 + (3*I)*Sqrt[1407])^(2/3) - 2^(1/3)*(61 + (3*I)*Sqr
t[1407])^(4/3))])])/(2*(1 + p)*(1 - (2^(2/3)*(61 + (3*I)*Sqrt[1407])^(1/3)
*((32*2^(1/3) + (122 + (6*I)*Sqrt[1407])^(2/3))/(61 + (3*I)*Sqrt[1407])^(1
/3) - 6*(-5/3 + x)))/(96 + 3*2^(1/3)*(61 + (3*I)*Sqrt[1407])^(2/3) - 2^(1/
6)*Sqrt[3*(-512*2^(2/3) + 64*(61 + (3*I)*Sqrt[1407])^(2/3) - 2^(1/3)*(61 +
 (3*I)*Sqrt[1407])^(4/3))]))^p*(1 - (2^(2/3)*(61 + (3*I)*Sqrt[1407])^(1...
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 1179
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(a + b*x + c*x^2)^p/(e*(1 - (
d + e*x)/(d - e*((b - q)/(2*c))))^p*(1 - (d + e*x)/(d - e*((b + q)/(2*c))))
^p)   Subst[Int[x^m*Simp[1 - x/(d - e*((b - q)/(2*c))), x]^p*Simp[1 - x/(d
- e*((b + q)/(2*c))), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, b, c, d, e, m
, p}, x]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2486
```
Int[((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (d_.)*(x_)^3)^(p_), x_S
ymbol] :> With[{r = Rt[-9*a*d^2 + Sqrt[3]*d*Sqrt[4*b^3*d + 27*a^2*d^2], 3]}
, Simp[(a + b*x + d*x^3)^p/(Simp[18^(1/3)*b*(d/(3*r)) - r/18^(1/3) + d*x, x
]^p*Simp[b*(d/3) + 12^(1/3)*b^2*(d^2/(3*r^2)) + r^2/(3*12^(1/3)) - d*(2^(1/
3)*b*(d/(3^(1/3)*r)) - r/18^(1/3))*x + d^2*x^2, x]^p)   Int[(e + f*x)^m*Sim
p[18^(1/3)*b*(d/(3*r)) - r/18^(1/3) + d*x, x]^p*Simp[b*(d/3) + 12^(1/3)*b^2
*(d^2/(3*r^2)) + r^2/(3*12^(1/3)) - d*(2^(1/3)*b*(d/(3^(1/3)*r)) - r/18^(1/
3))*x + d^2*x^2, x]^p, x], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && NeQ[4*
b^3 + 27*a^2*d, 0] &&  !IntegerQ[p]
```

rule 2490
```
Int[(P3_)^(p_.)*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] :> With[{a = Coeff[P3
, x, 0], b = Coeff[P3, x, 1], c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Su
bst[Int[((3*d*e - c*f)/(3*d) + f*x)^m*Simp[(2*c^3 - 9*b*c*d + 27*a*d^2)/(27
*d^2) - (c^2 - 3*b*d)*(x/(3*d)) + d*x^3, x]^p, x], x, x + c/(3*d)] /; NeQ[c
, 0]] /; FreeQ[{e, f, m, p}, x] && PolyQ[P3, x, 3]
```

rule 2526
```
Int[(Pm_)*(Qn_)^(p_), x_Symbol] :> With[{m = Expon[Pm, x], n = Expon[Qn, x]
}, Simp[Coeff[Pm, x, m]*(Qn^(p + 1)/(n*(p + 1)*Coeff[Qn, x, n])), x] + Simp
[1/(n*Coeff[Qn, x, n])   Int[ExpandToSum[n*Coeff[Qn, x, n]*Pm - Coeff[Pm, x
, m]*D[Qn, x], x]*Qn^p, x], x] /; EqQ[m, n - 1]] /; FreeQ[p, x] && PolyQ[Pm
, x] && PolyQ[Qn, x] && NeQ[p, -1]
```

Maple [F]

$$\int \left(C\,x^{2}+Bx+A\right)\left(x^{3}-5x^{2}+3x+2\right)^{p}dx$$

input
```
int((C*x^2+B*x+A)*(x^3-5*x^2+3*x+2)^p,x)
```

output
```
int((C*x^2+B*x+A)*(x^3-5*x^2+3*x+2)^p,x)
```

Fricas [F]

$$\int \left(A+Bx+Cx^2\right)\left(2+3x-5x^2+x^3\right)^p\,dx$$
$$=\int \left(Cx^{2}+Bx+A\right)\left(x^{3}-5\,x^{2}+3\,x+2\right)^{p}dx$$

input
```
integrate((C*x^2+B*x+A)*(x^3-5*x^2+3*x+2)^p,x, algorithm="fricas")
```

output
```
integral((C*x^2 + B*x + A)*(x^3 - 5*x^2 + 3*x + 2)^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int (A+Bx+Cx^2)\left(2+3x-5x^2+x^3\right)^p\,dx = \text{Timed out}$$

input
```
integrate((C*x**2+B*x+A)*(x**3-5*x**2+3*x+2)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (A+Bx+Cx^2)\left(2+3x-5x^2+x^3\right)^p\,dx = \int (Cx^2+Bx+A)\left(x^3-5\,x^2+3\,x+2\right)^p\,dx$$

input
```
integrate((C*x^2+B*x+A)*(x^3-5*x^2+3*x+2)^p,x, algorithm="maxima")
```

output
```
integrate((C*x^2 + B*x + A)*(x^3 - 5*x^2 + 3*x + 2)^p, x)
```

Giac [F]

$$\int (A+Bx+Cx^2)\left(2+3x-5x^2+x^3\right)^p\,dx = \int (Cx^2+Bx+A)\left(x^3-5\,x^2+3\,x+2\right)^p\,dx$$

input
```
integrate((C*x^2+B*x+A)*(x^3-5*x^2+3*x+2)^p,x, algorithm="giac")
```

output
```
integrate((C*x^2 + B*x + A)*(x^3 - 5*x^2 + 3*x + 2)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (A+Bx+Cx^2)\left(2+3x-5x^2+x^3\right)^p\,dx = \int (C\,x^2+B\,x+A)\,\left(x^3-5\,x^2+3\,x+2\right)^p\,dx$$

input
```
int((A + B*x + C*x^2)*(3*x - 5*x^2 + x^3 + 2)^p,x)
```

output
```
int((A + B*x + C*x^2)*(3*x - 5*x^2 + x^3 + 2)^p, x)
```

Reduce [F]

$$\int (A+Bx+Cx^2)\left(2+3x-5x^2+x^3\right)^p\,dx = \text{too large to display}$$

input
```
int((C*x^2+B*x+A)*(x^3-5*x^2+3*x+2)^p,x)
```

output

```
(90*(x**3 - 5*x**2 + 3*x + 2)**p*a*p**2*x - 54*(x**3 - 5*x**2 + 3*x + 2)**
p*a*p**2 + 150*(x**3 - 5*x**2 + 3*x + 2)**p*a*p*x - 90*(x**3 - 5*x**2 + 3*
x + 2)**p*a*p + 60*(x**3 - 5*x**2 + 3*x + 2)**p*a*x - 36*(x**3 - 5*x**2 +
3*x + 2)**p*a + 90*(x**3 - 5*x**2 + 3*x + 2)**p*b*p**2*x**2 - 150*(x**3 -
5*x**2 + 3*x + 2)**p*b*p**2*x - 99*(x**3 - 5*x**2 + 3*x + 2)**p*b*p**2 + 1
20*(x**3 - 5*x**2 + 3*x + 2)**p*b*p*x**2 - 150*(x**3 - 5*x**2 + 3*x + 2)**
p*b*p*x - 162*(x**3 - 5*x**2 + 3*x + 2)**p*b*p + 30*(x**3 - 5*x**2 + 3*x +
 2)**p*b*x**2 - 63*(x**3 - 5*x**2 + 3*x + 2)**p*b + 90*(x**3 - 5*x**2 + 3*
x + 2)**p*c*p**2*x**3 - 150*(x**3 - 5*x**2 + 3*x + 2)**p*c*p**2*x**2 - 320
*(x**3 - 5*x**2 + 3*x + 2)**p*c*p**2*x - 96*(x**3 - 5*x**2 + 3*x + 2)**p*c
*p**2 + 90*(x**3 - 5*x**2 + 3*x + 2)**p*c*p*x**3 - 50*(x**3 - 5*x**2 + 3*x
 + 2)**p*c*p*x**2 - 380*(x**3 - 5*x**2 + 3*x + 2)**p*c*p*x - 270*(x**3 - 5
*x**2 + 3*x + 2)**p*c*p + 20*(x**3 - 5*x**2 + 3*x + 2)**p*c*x**3 - 134*(x*
*3 - 5*x**2 + 3*x + 2)**p*c + 6318*int((x**3 - 5*x**2 + 3*x + 2)**p/(9*p**
2*x**3 - 45*p**2*x**2 + 27*p**2*x + 18*p**2 + 9*p*x**3 - 45*p*x**2 + 27*p*
x + 18*p + 2*x**3 - 10*x**2 + 6*x + 4),x)*a*p**5 + 16848*int((x**3 - 5*x**
2 + 3*x + 2)**p/(9*p**2*x**3 - 45*p**2*x**2 + 27*p**2*x + 18*p**2 + 9*p*x*
*3 - 45*p*x**2 + 27*p*x + 18*p + 2*x**3 - 10*x**2 + 6*x + 4),x)*a*p**4 + 1
6146*int((x**3 - 5*x**2 + 3*x + 2)**p/(9*p**2*x**3 - 45*p**2*x**2 + 27*p**
2*x + 18*p**2 + 9*p*x**3 - 45*p*x**2 + 27*p*x + 18*p + 2*x**3 - 10*x**2...
```

3.171 $\int (A+Bx+Cx^2) (2+3x+4x^2+x^3)^3 \, dx$

Optimal result

Integrand size = 26, antiderivative size = 171

$$\begin{aligned}\int (A+Bx+Cx^2) (2+3x+4x^2+x^3)^3 \, dx &= 8Ax+2(9A+2B)x^2+\frac{2}{3}(51A+18B+4C)x^3+\frac{3}{4}(61A+34B+12C)x^4\\&\quad+\frac{3}{5}(80A+61B+34C)x^5+\frac{1}{2}(73A+80B+61C)x^6\\&\quad+\frac{1}{7}(142A+219B+240C)x^7+\frac{1}{8}(57A+142B+219C)x^8\\&\quad+\frac{1}{9}(12A+57B+142C)x^9+\frac{1}{10}(A+12B+57C)x^{10}+\frac{1}{11}(B+12C)x^{11}+\frac{Cx^{12}}{12}\end{aligned}$$

output
```
8*A*x+2*(9*A+2*B)*x^2+2/3*(51*A+18*B+4*C)*x^3+3/4*(61*A+34*B+12*C)*x^4+3/5
*(80*A+61*B+34*C)*x^5+1/2*(73*A+80*B+61*C)*x^6+1/7*(142*A+219*B+240*C)*x^7
+1/8*(57*A+142*B+219*C)*x^8+1/9*(12*A+57*B+142*C)*x^9+1/10*(A+12*B+57*C)*x
^10+1/11*(B+12*C)*x^11+1/12*C*x^12
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.00

$$\int \left(A+Bx+Cx^2\right)\left(2+3x+4x^2+x^3\right)^3\,dx = 8Ax+2(9A+2B)x^2+\frac{2}{3}(51A+18B+4C)x^3+\frac{3}{4}(61A+34B+12C)x^4+\frac{3}{5}(80A+61B+34C)x^5+\frac{1}{2}(73A+80B+61C)x^6+\frac{1}{7}(142A+219B+240C)x^7+\frac{1}{8}(57A+142B+219C)x^8+\frac{1}{9}(12A+57B+142C)x^9+\frac{1}{10}(A+12B+57C)x^{10}+\frac{1}{11}(B+12C)x^{11}+\frac{Cx^{12}}{12}$$

input
```
Integrate[(A + B*x + C*x^2)*(2 + 3*x + 4*x^2 + x^3)^3,x]
```

output
```
8*A*x + 2*(9*A + 2*B)*x^2 + (2*(51*A + 18*B + 4*C)*x^3)/3 + (3*(61*A + 34*
B + 12*C)*x^4)/4 + (3*(80*A + 61*B + 34*C)*x^5)/5 + ((73*A + 80*B + 61*C)*
x^6)/2 + ((142*A + 219*B + 240*C)*x^7)/7 + ((57*A + 142*B + 219*C)*x^8)/8
+ ((12*A + 57*B + 142*C)*x^9)/9 + ((A + 12*B + 57*C)*x^10)/10 + ((B + 12*C
)*x^11)/11 + (C*x^12)/12
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(x^3+4x^2+3x+2\right)^3\left(A+Bx+Cx^2\right)dx$$

$$\Big\downarrow \text{ 2188}$$

$$\int \left(x^9(A+12B+57C)+x^8(12A+57B+142C)+x^7(57A+142B+219C)+x^6(142A+219B+240C)+3x\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{10}x^{10}(A+12B+57C)+\frac{1}{9}x^9(12A+57B+142C)+\frac{1}{8}x^8(57A+142B+219C)+\frac{1}{7}x^7(142A+219B+240C)+\frac{1}{2}x^6(73A+80B+61C)+\frac{3}{5}x^5(80A+61B+34C)+\frac{3}{4}x^4(61A+34B+12C)+\frac{2}{3}x^3(51A+18B+4C)+2x^2(9A+2B)+8Ax+\frac{1}{11}x^{11}(B+12C)+\frac{Cx^{12}}{12}$$

input
```
Int[(A + B*x + C*x^2)*(2 + 3*x + 4*x^2 + x^3)^3,x]
```

output
```
8*A*x + 2*(9*A + 2*B)*x^2 + (2*(51*A + 18*B + 4*C)*x^3)/3 + (3*(61*A + 34*
B + 12*C)*x^4)/4 + (3*(80*A + 61*B + 34*C)*x^5)/5 + ((73*A + 80*B + 61*C)*
x^6)/2 + ((142*A + 219*B + 240*C)*x^7)/7 + ((57*A + 142*B + 219*C)*x^8)/8
+ ((12*A + 57*B + 142*C)*x^9)/9 + ((A + 12*B + 57*C)*x^10)/10 + ((B + 12*C
)*x^11)/11 + (C*x^12)/12
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.85

method	result
norman	$\frac{C x^{12}}{12}+\left(\frac{B}{11}+\frac{12C}{11}\right) x^{11}+\left(\frac{A}{10}+\frac{6B}{5}+\frac{57C}{10}\right) x^{10}+\left(\frac{4A}{3}+\frac{19B}{3}+\frac{142C}{9}\right) x^{9}+\left(\frac{57A}{8}+\frac{71B}{4}+\frac{219C}{8}\right.$
default	$\frac{C x^{12}}{12}+\frac{(B+12C)x^{11}}{11}+\frac{(A+12B+57C)x^{10}}{10}+\frac{(12A+57B+142C)x^{9}}{9}+\frac{(57A+142B+219C)x^{8}}{8}+\frac{(142A+219B+24}{7}$
orering	$\frac{x(2310C x^{11}+2520B x^{10}+30240x^{10}C+2772A x^{9}+33264x^{9}B+158004C x^{9}+36960x^{8}A+175560B x^{8}+437360x^{8}C+197505x}{}$
gosper	$9C x^{4}+\frac{8}{3}C x^{3}+4B x^{2}+18A x^{2}+\frac{183}{4}x^{4}A+48x^{5}A+\frac{4}{3}A x^{9}+\frac{6}{5}B x^{10}+\frac{219}{8}x^{8}C+\frac{142}{7}x^{7}A$
risch	$9C x^{4}+\frac{8}{3}C x^{3}+4B x^{2}+18A x^{2}+\frac{183}{4}x^{4}A+48x^{5}A+\frac{4}{3}A x^{9}+\frac{6}{5}B x^{10}+\frac{219}{8}x^{8}C+\frac{142}{7}x^{7}A$
parallelrisch	$9C x^{4}+\frac{8}{3}C x^{3}+4B x^{2}+18A x^{2}+\frac{183}{4}x^{4}A+48x^{5}A+\frac{4}{3}A x^{9}+\frac{6}{5}B x^{10}+\frac{219}{8}x^{8}C+\frac{142}{7}x^{7}A$

input
```
int((C*x^2+B*x+A)*(x^3+4*x^2+3*x+2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/12*C*x^12+(1/11*B+12/11*C)*x^11+(1/10*A+6/5*B+57/10*C)*x^10+(4/3*A+19/3*
B+142/9*C)*x^9+(57/8*A+71/4*B+219/8*C)*x^8+(142/7*A+219/7*B+240/7*C)*x^7+(
73/2*A+40*B+61/2*C)*x^6+(48*A+183/5*B+102/5*C)*x^5+(183/4*A+51/2*B+9*C)*x^
4+(34*A+12*B+8/3*C)*x^3+(18*A+4*B)*x^2+8*A*x
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.88

$$\begin{aligned}\int \left(A+Bx+Cx^2\right)\left(2+3x+4x^2+x^3\right)^3\,dx &= \frac{1}{12}\,Cx^{12}+\frac{1}{11}\,(B+12\,C)x^{11}+\frac{1}{10}\,(A+12\,B+57\,C)x^{10}+\frac{1}{9}\,(12\,A+57\,B+142\,C)x^{9}\\ &\quad+\frac{1}{8}\,(57\,A+142\,B+219\,C)x^{8}+\frac{1}{7}\,(142\,A+219\,B+240\,C)x^{7}\\ &\quad+\frac{1}{2}\,(73\,A+80\,B+61\,C)x^{6}+\frac{3}{5}\,(80\,A+61\,B+34\,C)x^{5}\\ &\quad+\frac{3}{4}\,(61\,A+34\,B+12\,C)x^{4}+\frac{2}{3}\,(51\,A+18\,B+4\,C)x^{3}+2\,(9\,A+2\,B)x^{2}+8\,Ax\end{aligned}$$

input
```
integrate((C*x^2+B*x+A)*(x^3+4*x^2+3*x+2)^3,x, algorithm="fricas")
```

output
```
1/12*C*x^12 + 1/11*(B + 12*C)*x^11 + 1/10*(A + 12*B + 57*C)*x^10 + 1/9*(12
*A + 57*B + 142*C)*x^9 + 1/8*(57*A + 142*B + 219*C)*x^8 + 1/7*(142*A + 219
*B + 240*C)*x^7 + 1/2*(73*A + 80*B + 61*C)*x^6 + 3/5*(80*A + 61*B + 34*C)*
x^5 + 3/4*(61*A + 34*B + 12*C)*x^4 + 2/3*(51*A + 18*B + 4*C)*x^3 + 2*(9*A
+ 2*B)*x^2 + 8*A*x
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.01

$$\int (A+Bx+Cx^2)\left(2+3x+4x^2+x^3\right)^3\,dx$$
$$= 8Ax+\frac{Cx^{12}}{12}+x^{11}\left(\frac{B}{11}+\frac{12C}{11}\right)+x^{10}\left(\frac{A}{10}+\frac{6B}{5}+\frac{57C}{10}\right)+x^9 \cdot\left(\frac{4A}{3}+\frac{19B}{3}+\frac{142C}{9}\right)+x^8\cdot\left(\frac{57A}{8}+\frac{71B}{4}+\frac{219C}{8}\right)+x^7\cdot\left(\frac{142A}{7}+\frac{219B}{7}+\frac{240C}{7}\right) +x^6\cdot\left(\frac{73A}{2}+40B+\frac{61C}{2}\right)+x^5\cdot\left(48A+\frac{183B}{5}+\frac{102C}{5}\right)+x^4 \cdot\left(\frac{183A}{4}+\frac{51B}{2}+9C\right)+x^3\cdot\left(34A+12B+\frac{8C}{3}\right)+x^2\cdot(18A+4B)$$

input
```
integrate((C*x**2+B*x+A)*(x**3+4*x**2+3*x+2)**3,x)
```

output
```
8*A*x + C*x**12/12 + x**11*(B/11 + 12*C/11) + x**10*(A/10 + 6*B/5 + 57*C/1
0) + x**9*(4*A/3 + 19*B/3 + 142*C/9) + x**8*(57*A/8 + 71*B/4 + 219*C/8) +
x**7*(142*A/7 + 219*B/7 + 240*C/7) + x**6*(73*A/2 + 40*B + 61*C/2) + x**5*
(48*A + 183*B/5 + 102*C/5) + x**4*(183*A/4 + 51*B/2 + 9*C) + x**3*(34*A +
12*B + 8*C/3) + x**2*(18*A + 4*B)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.88

$$\int (A+Bx+Cx^2)\left(2+3x+4x^2+x^3\right)^3\,dx$$
$$=\frac{1}{12}Cx^{12}+\frac{1}{11}(B+12C)x^{11}+\frac{1}{10}(A+12B+57C)x^{10}+\frac{1}{9}(12A+57B+142C)x^9+\frac{1}{8}(57A+142B+219C)x^8+\frac{1}{7}(142A+219B+240C)x^7+\frac{1}{2}(73A+80B+61C)x^6+\frac{3}{5}(80A+61B+34C)x^5+\frac{3}{4}(61A+34B+12C)x^4+\frac{2}{3}(51A+18B+4C)x^3+2(9A+2B)x^2+8Ax$$

input
```
integrate((C*x^2+B*x+A)*(x^3+4*x^2+3*x+2)^3,x, algorithm="maxima")
```

output
```
1/12*C*x^12 + 1/11*(B + 12*C)*x^11 + 1/10*(A + 12*B + 57*C)*x^10 + 1/9*(12
*A + 57*B + 142*C)*x^9 + 1/8*(57*A + 142*B + 219*C)*x^8 + 1/7*(142*A + 219
*B + 240*C)*x^7 + 1/2*(73*A + 80*B + 61*C)*x^6 + 3/5*(80*A + 61*B + 34*C)*
x^5 + 3/4*(61*A + 34*B + 12*C)*x^4 + 2/3*(51*A + 18*B + 4*C)*x^3 + 2*(9*A
+ 2*B)*x^2 + 8*A*x
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.05

$$\int (A+Bx+Cx^2)\left(2+3x+4x^2+x^3\right)^3\,dx$$
$$=\frac{1}{12}Cx^{12}+\frac{1}{11}Bx^{11}+\frac{12}{11}Cx^{11}+\frac{1}{10}Ax^{10}+\frac{6}{5}Bx^{10}+\frac{57}{10}Cx^{10}+\frac{4}{3}Ax^9+\frac{19}{3}Bx^9+\frac{142}{9}Cx^9+\frac{57}{8}Ax^8+\frac{71}{4}Bx^8+\frac{219}{8}Cx^8+\frac{142}{7}Ax^7+\frac{219}{7}Bx^7+\frac{240}{7}Cx^7+\frac{73}{2}Ax^6+40Bx^6+\frac{61}{2}Cx^6+48Ax^5+\frac{183}{5}Bx^5+\frac{102}{5}Cx^5+\frac{183}{4}Ax^4+\frac{51}{2}Bx^4+9Cx^4+34Ax^3+12Bx^3+\frac{8}{3}Cx^3+18Ax^2+4Bx^2+8Ax$$

input
```
integrate((C*x^2+B*x+A)*(x^3+4*x^2+3*x+2)^3,x, algorithm="giac")
```

output
```
1/12*C*x^12 + 1/11*B*x^11 + 12/11*C*x^11 + 1/10*A*x^10 + 6/5*B*x^10 + 57/1
0*C*x^10 + 4/3*A*x^9 + 19/3*B*x^9 + 142/9*C*x^9 + 57/8*A*x^8 + 71/4*B*x^8
+ 219/8*C*x^8 + 142/7*A*x^7 + 219/7*B*x^7 + 240/7*C*x^7 + 73/2*A*x^6 + 40*
B*x^6 + 61/2*C*x^6 + 48*A*x^5 + 183/5*B*x^5 + 102/5*C*x^5 + 183/4*A*x^4 +
51/2*B*x^4 + 9*C*x^4 + 34*A*x^3 + 12*B*x^3 + 8/3*C*x^3 + 18*A*x^2 + 4*B*x^
2 + 8*A*x
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.85

$$\int (A+Bx+Cx^2)\left(2+3x+4x^2+x^3\right)^3\,dx = \frac{C\,x^{12}}{12}+\left(\frac{B}{11}+\frac{12\,C}{11}\right)\,x^{11}+\left(\frac{A}{10}+\frac{6\,B}{5}+\frac{57\,C}{10}\right)\,x^{10}+\left(\frac{4\,A}{3}+\frac{19\,B}{3}+\frac{142\,C}{9}\right)\,x^9 + \left(\frac{57\,A}{8}+\frac{71\,B}{4}+\frac{219\,C}{8}\right)\,x^8+\left(\frac{142\,A}{7}+\frac{219\,B}{7}+\frac{240\,C}{7}\right)\,x^7 + \left(\frac{73\,A}{2}+40\,B+\frac{61\,C}{2}\right)\,x^6+\left(48\,A+\frac{183\,B}{5}+\frac{102\,C}{5}\right)\,x^5 + \left(\frac{183\,A}{4}+\frac{51\,B}{2}+9\,C\right)\,x^4+\left(34\,A+12\,B+\frac{8\,C}{3}\right)\,x^3+(18\,A+4\,B)\,x^2+8\,A\,x$$

input
```
int((A + B*x + C*x^2)*(3*x + 4*x^2 + x^3 + 2)^3,x)
```

output
```
8*A*x + (C*x^12)/12 + x^3*(34*A + 12*B + (8*C)/3) + x^10*(A/10 + (6*B)/5 +
 (57*C)/10) + x^6*((73*A)/2 + 40*B + (61*C)/2) + x^9*((4*A)/3 + (19*B)/3 +
 (142*C)/9) + x^4*((183*A)/4 + (51*B)/2 + 9*C) + x^5*(48*A + (183*B)/5 + (
102*C)/5) + x^8*((57*A)/8 + (71*B)/4 + (219*C)/8) + x^7*((142*A)/7 + (219*
B)/7 + (240*C)/7) + x^2*(18*A + 4*B) + x^11*(B/11 + (12*C)/11)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.04

$$\int (A + Bx + Cx^2) (2 + 3x + 4x^2 + x^3)^3 \, dx$$
$$= \frac{x(2310c\,x^{11} + 2520b\,x^{10} + 30240c\,x^{10} + 2772a\,x^9 + 33264b\,x^9 + 158004c\,x^9 + 36960a\,x^8 + 175560b\,x^8 +}{}$$

input
```
int((C*x^2+B*x+A)*(x^3+4*x^2+3*x+2)^3,x)
```

output
```
(x*(2772*a*x**9 + 36960*a*x**8 + 197505*a*x**7 + 562320*a*x**6 + 1011780*a
*x**5 + 1330560*a*x**4 + 1268190*a*x**3 + 942480*a*x**2 + 498960*a*x + 221
760*a + 2520*b*x**10 + 33264*b*x**9 + 175560*b*x**8 + 492030*b*x**7 + 8672
40*b*x**6 + 1108800*b*x**5 + 1014552*b*x**4 + 706860*b*x**3 + 332640*b*x**
2 + 110880*b*x + 2310*c*x**11 + 30240*c*x**10 + 158004*c*x**9 + 437360*c*x
**8 + 758835*c*x**7 + 950400*c*x**6 + 845460*c*x**5 + 565488*c*x**4 + 2494
80*c*x**3 + 73920*c*x**2))/27720
```

3.172 $\int (A+Bx+Cx^2)(2+3x+4x^2+x^3)^2\,dx$

Optimal result

Integrand size = 26, antiderivative size = 118

$$\begin{aligned}\int (A+Bx+Cx^2)(2+3x+4x^2+x^3)^2\,dx &= 4Ax+2(3A+B)x^2+\frac{1}{3}(25A+12B+4C)x^3+\frac{1}{4}(28A+25B+12C)x^4\\ &\quad+\frac{1}{5}(22A+28B+25C)x^5+\frac{1}{3}(4A+11B+14C)x^6\\ &\quad+\frac{1}{7}(A+8B+22C)x^7+\frac{1}{8}(B+8C)x^8+\frac{Cx^9}{9}\end{aligned}$$

output
```
4*A*x+2*(3*A+B)*x^2+1/3*(25*A+12*B+4*C)*x^3+1/4*(28*A+25*B+12*C)*x^4+1/5*(
22*A+28*B+25*C)*x^5+1/3*(4*A+11*B+14*C)*x^6+1/7*(A+8*B+22*C)*x^7+1/8*(B+8*
C)*x^8+1/9*C*x^9
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00

$$\int (A+Bx+Cx^2)(2+3x+4x^2+x^3)^2\,dx$$
$$=4Ax+2(3A+B)x^2+\frac{1}{3}(25A+12B+4C)x^3+\frac{1}{4}(28A+25B+12C)x^4$$
$$+\frac{1}{5}(22A+28B+25C)x^5+\frac{1}{3}(4A+11B+14C)x^6$$
$$+\frac{1}{7}(A+8B+22C)x^7+\frac{1}{8}(B+8C)x^8+\frac{Cx^9}{9}$$

input
```
Integrate[(A + B*x + C*x^2)*(2 + 3*x + 4*x^2 + x^3)^2,x]
```

output
```
4*A*x + 2*(3*A + B)*x^2 + ((25*A + 12*B + 4*C)*x^3)/3 + ((28*A + 25*B + 12
*C)*x^4)/4 + ((22*A + 28*B + 25*C)*x^5)/5 + ((4*A + 11*B + 14*C)*x^6)/3 +
((A + 8*B + 22*C)*x^7)/7 + ((B + 8*C)*x^8)/8 + (C*x^9)/9
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (x^3+4x^2+3x+2)^2(A+Bx+Cx^2)\,dx$$

$\Big\downarrow$ 2188

$$\int \left(x^6(A+8B+22C)+2x^5(4A+11B+14C)+x^4(22A+28B+25C)+x^3(28A+25B+12C)+x^2(25A+\right.$$

$\Big\downarrow$ 2009

$$\frac{1}{7}x^7(A+8B+22C)+\frac{1}{3}x^6(4A+11B+14C)+\frac{1}{5}x^5(22A+28B+25C)+\frac{1}{4}x^4(28A+25B+12C)+\frac{1}{3}x^3(25A+12B+4C)+2x^2(3A+B)+4Ax+\frac{1}{8}x^8(B+8C)+\frac{Cx^9}{9}$$

input
```
Int[(A + B*x + C*x^2)*(2 + 3*x + 4*x^2 + x^3)^2,x]
```

output
```
4*A*x + 2*(3*A + B)*x^2 + ((25*A + 12*B + 4*C)*x^3)/3 + ((28*A + 25*B + 12
*C)*x^4)/4 + ((22*A + 28*B + 25*C)*x^5)/5 + ((4*A + 11*B + 14*C)*x^6)/3 +
((A + 8*B + 22*C)*x^7)/7 + ((B + 8*C)*x^8)/8 + (C*x^9)/9
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.86

method	result
norman	$\frac{C\,x^9}{9}+\left(\frac{B}{8}+C\right)x^8+\left(\frac{A}{7}+\frac{8B}{7}+\frac{22C}{7}\right)x^7+\left(\frac{4A}{3}+\frac{11B}{3}+\frac{14C}{3}\right)x^6+\left(\frac{22A}{5}+\frac{28B}{5}+5C\right)x^5+$
default	$\frac{C\,x^9}{9}+\frac{(B+8C)x^8}{8}+\frac{(A+8B+22C)x^7}{7}+\frac{(8A+22B+28C)x^6}{6}+\frac{(22A+28B+25C)x^5}{5}+\frac{(28A+25B+12C)x^4}{4}+$ (2
orering	$\frac{x(280x^8C+315x^7B+2520x^7C+360x^6A+2880x^6B+7920C\,x^6+3360x^5A+9240B\,x^5+11760x^5C+11088x^4A+14112x^4B+1}{2520}$
gosper	$\frac{1}{9}C\,x^9+\frac{1}{8}B\,x^8+x^8C+\frac{1}{7}x^7A+\frac{8}{7}x^7B+\frac{22}{7}x^7C+\frac{4}{3}x^6A+\frac{11}{3}x^6B+\frac{14}{3}C\,x^6+\frac{22}{5}x^5A+\frac{28}{5}$
risch	$\frac{1}{9}C\,x^9+\frac{1}{8}B\,x^8+x^8C+\frac{1}{7}x^7A+\frac{8}{7}x^7B+\frac{22}{7}x^7C+\frac{4}{3}x^6A+\frac{11}{3}x^6B+\frac{14}{3}C\,x^6+\frac{22}{5}x^5A+\frac{28}{5}$
parallelrisch	$\frac{1}{9}C\,x^9+\frac{1}{8}B\,x^8+x^8C+\frac{1}{7}x^7A+\frac{8}{7}x^7B+\frac{22}{7}x^7C+\frac{4}{3}x^6A+\frac{11}{3}x^6B+\frac{14}{3}C\,x^6+\frac{22}{5}x^5A+\frac{28}{5}$

input
```
int((C*x^2+B*x+A)*(x^3+4*x^2+3*x+2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/9*C*x^9+(1/8*B+C)*x^8+(1/7*A+8/7*B+22/7*C)*x^7+(4/3*A+11/3*B+14/3*C)*x^6
+(22/5*A+28/5*B+5*C)*x^5+(7*A+25/4*B+3*C)*x^4+(25/3*A+4*B+4/3*C)*x^3+(6*A+
2*B)*x^2+4*A*x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.88

$$\begin{aligned}\int (A+Bx+Cx^2)\left(2+3x+4x^2+x^3\right)^2\,dx &= \frac{1}{9}\,Cx^9+\frac{1}{8}\,(B+8\,C)x^8+\frac{1}{7}\,(A+8\,B+22\,C)x^7+\frac{1}{3}\,(4\,A+11\,B+14\,C)x^6\\&\quad+\frac{1}{5}\,(22\,A+28\,B+25\,C)x^5+\frac{1}{4}\,(28\,A+25\,B+12\,C)x^4\\&\quad+\frac{1}{3}\,(25\,A+12\,B+4\,C)x^3+2\,(3\,A+B)x^2+4\,Ax\end{aligned}$$

input
```
integrate((C*x^2+B*x+A)*(x^3+4*x^2+3*x+2)^2,x, algorithm="fricas")
```

output
```
1/9*C*x^9 + 1/8*(B + 8*C)*x^8 + 1/7*(A + 8*B + 22*C)*x^7 + 1/3*(4*A + 11*B
 + 14*C)*x^6 + 1/5*(22*A + 28*B + 25*C)*x^5 + 1/4*(28*A + 25*B + 12*C)*x^4
 + 1/3*(25*A + 12*B + 4*C)*x^3 + 2*(3*A + B)*x^2 + 4*A*x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.97

$$\begin{aligned}&\int (A+Bx+Cx^2)\left(2+3x+4x^2+x^3\right)^2\,dx\\&= 4Ax+\frac{Cx^9}{9}+x^8\left(\frac{B}{8}+C\right)+x^7\left(\frac{A}{7}+\frac{8B}{7}+\frac{22C}{7}\right)+x^6\\&\quad\cdot\left(\frac{4A}{3}+\frac{11B}{3}+\frac{14C}{3}\right)+x^5\cdot\left(\frac{22A}{5}+\frac{28B}{5}+5C\right)+x^4\\&\quad\cdot\left(7A+\frac{25B}{4}+3C\right)+x^3\cdot\left(\frac{25A}{3}+4B+\frac{4C}{3}\right)+x^2\cdot(6A+2B)\end{aligned}$$

input
```
integrate((C*x**2+B*x+A)*(x**3+4*x**2+3*x+2)**2,x)
```

output
```
4*A*x + C*x**9/9 + x**8*(B/8 + C) + x**7*(A/7 + 8*B/7 + 22*C/7) + x**6*(4*
A/3 + 11*B/3 + 14*C/3) + x**5*(22*A/5 + 28*B/5 + 5*C) + x**4*(7*A + 25*B/4
 + 3*C) + x**3*(25*A/3 + 4*B + 4*C/3) + x**2*(6*A + 2*B)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.88

$$\int (A+Bx+Cx^2)\left(2+3x+4x^2+x^3\right)^2 dx = \frac{1}{9}Cx^9+\frac{1}{8}(B+8C)x^8+\frac{1}{7}(A+8B+22C)x^7+\frac{1}{3}(4A+11B+14C)x^6 + \frac{1}{5}(22A+28B+25C)x^5+\frac{1}{4}(28A+25B+12C)x^4 + \frac{1}{3}(25A+12B+4C)x^3+2(3A+B)x^2+4Ax$$

input
```
integrate((C*x^2+B*x+A)*(x^3+4*x^2+3*x+2)^2,x, algorithm="maxima")
```

output
```
1/9*C*x^9 + 1/8*(B + 8*C)*x^8 + 1/7*(A + 8*B + 22*C)*x^7 + 1/3*(4*A + 11*B
 + 14*C)*x^6 + 1/5*(22*A + 28*B + 25*C)*x^5 + 1/4*(28*A + 25*B + 12*C)*x^4
 + 1/3*(25*A + 12*B + 4*C)*x^3 + 2*(3*A + B)*x^2 + 4*A*x
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.05

$$\int (A+Bx+Cx^2)\left(2+3x+4x^2+x^3\right)^2 dx = \frac{1}{9}Cx^9+\frac{1}{8}Bx^8+Cx^8+\frac{1}{7}Ax^7+\frac{8}{7}Bx^7+\frac{22}{7}Cx^7+\frac{4}{3}Ax^6 + \frac{11}{3}Bx^6+\frac{14}{3}Cx^6+\frac{22}{5}Ax^5+\frac{28}{5}Bx^5+5Cx^5+7Ax^4+\frac{25}{4}Bx^4 + 3Cx^4+\frac{25}{3}Ax^3+4Bx^3+\frac{4}{3}Cx^3+6Ax^2+2Bx^2+4Ax$$

input
```
integrate((C*x^2+B*x+A)*(x^3+4*x^2+3*x+2)^2,x, algorithm="giac")
```

output
```
1/9*C*x^9 + 1/8*B*x^8 + C*x^8 + 1/7*A*x^7 + 8/7*B*x^7 + 22/7*C*x^7 + 4/3*A
*x^6 + 11/3*B*x^6 + 14/3*C*x^6 + 22/5*A*x^5 + 28/5*B*x^5 + 5*C*x^5 + 7*A*x
^4 + 25/4*B*x^4 + 3*C*x^4 + 25/3*A*x^3 + 4*B*x^3 + 4/3*C*x^3 + 6*A*x^2 + 2
*B*x^2 + 4*A*x
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.86

$$\int (A+Bx+Cx^2) (2+3x+4x^2+x^3)^2 \, dx = \frac{C\,x^9}{9}+\left(\frac{B}{8}+C\right)\,x^8+\left(\frac{A}{7}+\frac{8\,B}{7}+\frac{22\,C}{7}\right)\,x^7+\left(\frac{4\,A}{3}+\frac{11\,B}{3}+\frac{14\,C}{3}\right)\,x^6 +\left(\frac{22\,A}{5}+\frac{28\,B}{5}+5\,C\right)\,x^5+\left(7\,A+\frac{25\,B}{4}+3\,C\right)\,x^4 +\left(\frac{25\,A}{3}+4\,B+\frac{4\,C}{3}\right)\,x^3+\left(6\,A+2\,B\right)\,x^2+4\,A\,x$$

input
```
int((A + B*x + C*x^2)*(3*x + 4*x^2 + x^3 + 2)^2,x)
```

output
```
4*A*x + (C*x^9)/9 + x^3*((25*A)/3 + 4*B + (4*C)/3) + x^4*(7*A + (25*B)/4 +
 3*C) + x^6*((4*A)/3 + (11*B)/3 + (14*C)/3) + x^7*(A/7 + (8*B)/7 + (22*C)/
7) + x^5*((22*A)/5 + (28*B)/5 + 5*C) + x^8*(B/8 + C) + x^2*(6*A + 2*B)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.04

$$\int (A+Bx+Cx^2) (2+3x+4x^2+x^3)^2 \, dx = \frac{x(280c\,x^8+315b\,x^7+2520c\,x^7+360a\,x^6+2880b\,x^6+7920c\,x^6+3360a\,x^5+9240b\,x^5+11760c\,x^5+1}{}$$

input
```
int((C*x^2+B*x+A)*(x^3+4*x^2+3*x+2)^2,x)
```

output

```
(x*(360*a*x**6 + 3360*a*x**5 + 11088*a*x**4 + 17640*a*x**3 + 21000*a*x**2
+ 15120*a*x + 10080*a + 315*b*x**7 + 2880*b*x**6 + 9240*b*x**5 + 14112*b*x
**4 + 15750*b*x**3 + 10080*b*x**2 + 5040*b*x + 280*c*x**8 + 2520*c*x**7 +
7920*c*x**6 + 11760*c*x**5 + 12600*c*x**4 + 7560*c*x**3 + 3360*c*x**2))/25
20
```

3.173 $\int (A+Bx+Cx^2)(2+3x+4x^2+x^3)\,dx$

Optimal result

Integrand size = 24, antiderivative size = 71

$$\int (A+Bx+Cx^2)(2+3x+4x^2+x^3)\,dx = 2Ax+\frac{1}{2}(3A+2B)x^2+\frac{1}{3}(4A+3B+2C)x^3+\frac{1}{4}(A+4B+3C)x^4+\frac{1}{5}(B+4C)x^5+\frac{Cx^6}{6}$$

output
```
2*A*x+1/2*(3*A+2*B)*x^2+1/3*(4*A+3*B+2*C)*x^3+1/4*(A+4*B+3*C)*x^4+1/5*(B+4
*C)*x^5+1/6*C*x^6
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00

$$\int (A+Bx+Cx^2)(2+3x+4x^2+x^3)\,dx = 2Ax+\frac{1}{2}(3A+2B)x^2+\frac{1}{3}(4A+3B+2C)x^3+\frac{1}{4}(A+4B+3C)x^4+\frac{1}{5}(B+4C)x^5+\frac{Cx^6}{6}$$

input
```
Integrate[(A + B*x + C*x^2)*(2 + 3*x + 4*x^2 + x^3),x]
```

output
```
2*A*x + ((3*A + 2*B)*x^2)/2 + ((4*A + 3*B + 2*C)*x^3)/3 + ((A + 4*B + 3*C)
*x^4)/4 + ((B + 4*C)*x^5)/5 + (C*x^6)/6
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (x^3+4x^2+3x+2)\left(A+Bx+Cx^2\right)dx$$

$$\Big\downarrow \text{ 2188}$$

$$\int \left(x^3(A+4B+3C)+x^2(4A+3B+2C)+x(3A+2B)+2A+x^4(B+4C)+Cx^5\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{4}x^4(A+4B+3C)+\frac{1}{3}x^3(4A+3B+2C)+\frac{1}{2}x^2(3A+2B)+2Ax+\frac{1}{5}x^5(B+4C)+\frac{Cx^6}{6}$$

input
```
Int[(A + B*x + C*x^2)*(2 + 3*x + 4*x^2 + x^3),x]
```

output

```
2*A*x + ((3*A + 2*B)*x^2)/2 + ((4*A + 3*B + 2*C)*x^3)/3 + ((A + 4*B + 3*C)
*x^4)/4 + ((B + 4*C)*x^5)/5 + (C*x^6)/6
```


Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.79

method	result
norman	$\frac{C\,x^{6}}{6}+\left(\frac{B}{5}+\frac{4C}{5}\right)x^{5}+\left(\frac{A}{4}+B+\frac{3C}{4}\right)x^{4}+\left(\frac{4A}{3}+B+\frac{2C}{3}\right)x^{3}+\left(\frac{3A}{2}+B\right)x^{2}+2Ax$
default	$2Ax+\frac{(3A+2B)x^{2}}{2}+\frac{(4A+3B+2C)x^{3}}{3}+\frac{(A+4B+3C)x^{4}}{4}+\frac{(B+4C)x^{5}}{5}+\frac{C\,x^{6}}{6}$
gosper	$\frac{1}{6}C\,x^{6}+\frac{1}{5}B\,x^{5}+\frac{4}{5}x^{5}C+\frac{1}{4}x^{4}A+x^{4}B+\frac{3}{4}C\,x^{4}+\frac{4}{3}x^{3}A+B\,x^{3}+\frac{2}{3}C\,x^{3}+\frac{3}{2}A\,x^{2}+B\,x^{2}+$
risch	$\frac{1}{6}C\,x^{6}+\frac{1}{5}B\,x^{5}+\frac{4}{5}x^{5}C+\frac{1}{4}x^{4}A+x^{4}B+\frac{3}{4}C\,x^{4}+\frac{4}{3}x^{3}A+B\,x^{3}+\frac{2}{3}C\,x^{3}+\frac{3}{2}A\,x^{2}+B\,x^{2}+$
parallelrisch	$\frac{1}{6}C\,x^{6}+\frac{1}{5}B\,x^{5}+\frac{4}{5}x^{5}C+\frac{1}{4}x^{4}A+x^{4}B+\frac{3}{4}C\,x^{4}+\frac{4}{3}x^{3}A+B\,x^{3}+\frac{2}{3}C\,x^{3}+\frac{3}{2}A\,x^{2}+B\,x^{2}+$
orering	$\frac{x\left(10x^{5}C+12x^{4}B+48C\,x^{4}+15x^{3}A+60B\,x^{3}+45C\,x^{3}+80A\,x^{2}+60B\,x^{2}+40C\,x^{2}+90Ax+60Bx+120A\right)}{60}$

input
```
int((C*x^2+B*x+A)*(x^3+4*x^2+3*x+2),x,method=_RETURNVERBOSE)
```

output
```
1/6*C*x^6+(1/5*B+4/5*C)*x^5+(1/4*A+B+3/4*C)*x^4+(4/3*A+B+2/3*C)*x^3+(3/2*A
+B)*x^2+2*A*x
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.86

$$\int \left(A+Bx+Cx^2\right)\left(2+3x+4x^2+x^3\right)\,dx$$
$$=\frac{1}{6}\,Cx^{6}+\frac{1}{5}\,(B+4\,C)x^{5}+\frac{1}{4}\,(A+4\,B+3\,C)x^{4}$$
$$+\frac{1}{3}\,(4\,A+3\,B+2\,C)x^{3}+\frac{1}{2}\,(3\,A+2\,B)x^{2}+2\,Ax$$

input
```
integrate((C*x^2+B*x+A)*(x^3+4*x^2+3*x+2),x, algorithm="fricas")
```

output
```
1/6*C*x^6 + 1/5*(B + 4*C)*x^5 + 1/4*(A + 4*B + 3*C)*x^4 + 1/3*(4*A + 3*B +
 2*C)*x^3 + 1/2*(3*A + 2*B)*x^2 + 2*A*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.86

$$\int (A+Bx+Cx^2)\,(2+3x+4x^2+x^3)\,dx$$
$$=2Ax+\frac{Cx^6}{6}+x^5\left(\frac{B}{5}+\frac{4C}{5}\right)+x^4\left(\frac{A}{4}+B+\frac{3C}{4}\right)$$
$$+x^3\cdot\left(\frac{4A}{3}+B+\frac{2C}{3}\right)+x^2\cdot\left(\frac{3A}{2}+B\right)$$

input
```
integrate((C*x**2+B*x+A)*(x**3+4*x**2+3*x+2),x)
```

output
```
2*A*x + C*x**6/6 + x**5*(B/5 + 4*C/5) + x**4*(A/4 + B + 3*C/4) + x**3*(4*A
/3 + B + 2*C/3) + x**2*(3*A/2 + B)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.86

$$\int (A+Bx+Cx^2)\,(2+3x+4x^2+x^3)\,dx$$
$$=\frac{1}{6}\,Cx^6+\frac{1}{5}\,(B+4\,C)x^5+\frac{1}{4}\,(A+4\,B+3\,C)x^4$$
$$+\frac{1}{3}\,(4\,A+3\,B+2\,C)x^3+\frac{1}{2}\,(3\,A+2\,B)x^2+2\,Ax$$

input
```
integrate((C*x^2+B*x+A)*(x^3+4*x^2+3*x+2),x, algorithm="maxima")
```

output
```
1/6*C*x^6 + 1/5*(B + 4*C)*x^5 + 1/4*(A + 4*B + 3*C)*x^4 + 1/3*(4*A + 3*B +
 2*C)*x^3 + 1/2*(3*A + 2*B)*x^2 + 2*A*x
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.96

$$\int (A+Bx+Cx^2)\left(2+3x+4x^2+x^3\right)\,dx = \frac{1}{6}Cx^6+\frac{1}{5}Bx^5+\frac{4}{5}Cx^5+\frac{1}{4}Ax^4+Bx^4+\frac{3}{4}Cx^4 + \frac{4}{3}Ax^3+Bx^3+\frac{2}{3}Cx^3+\frac{3}{2}Ax^2+Bx^2+2\,Ax$$

input
```
integrate((C*x^2+B*x+A)*(x^3+4*x^2+3*x+2),x, algorithm="giac")
```

output
```
1/6*C*x^6 + 1/5*B*x^5 + 4/5*C*x^5 + 1/4*A*x^4 + B*x^4 + 3/4*C*x^4 + 4/3*A*
x^3 + B*x^3 + 2/3*C*x^3 + 3/2*A*x^2 + B*x^2 + 2*A*x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.77

$$\int (A+Bx+Cx^2)\left(2+3x+4x^2+x^3\right)\,dx = \frac{C\,x^6}{6}+\left(\frac{B}{5}+\frac{4\,C}{5}\right)\,x^5+\left(\frac{A}{4}+B+\frac{3\,C}{4}\right)\,x^4 + \left(\frac{4\,A}{3}+B+\frac{2\,C}{3}\right)\,x^3+\left(\frac{3\,A}{2}+B\right)\,x^2+2\,A\,x$$

input
```
int((A + B*x + C*x^2)*(3*x + 4*x^2 + x^3 + 2),x)
```

output
```
2*A*x + x^3*((4*A)/3 + B + (2*C)/3) + x^4*(A/4 + B + (3*C)/4) + x^2*((3*A)
/2 + B) + (C*x^6)/6 + x^5*(B/5 + (4*C)/5)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.97

$$\int (A+Bx+Cx^2)(2+3x+4x^2+x^3)\,dx$$
$$= \frac{x(10c\,x^5+12b\,x^4+48c\,x^4+15a\,x^3+60b\,x^3+45c\,x^3+80a\,x^2+60b\,x^2+40c\,x^2+90ax+60bx+120a}{60}$$

input
```
int((C*x^2+B*x+A)*(x^3+4*x^2+3*x+2),x)
```

output
```
(x*(15*a*x**3 + 80*a*x**2 + 90*a*x + 120*a + 12*b*x**4 + 60*b*x**3 + 60*b*
x**2 + 60*b*x + 10*c*x**5 + 48*c*x**4 + 45*c*x**3 + 40*c*x**2))/60
```

3.174 $\int \frac{A+Bx+Cx^2}{2+3x+4x^2+x^3} \, dx$

Optimal result

Integrand size = 26, antiderivative size = 706

$$\int \frac{A+Bx+Cx^2}{2+3x+4x^2+x^3} \, dx =$$

$$-\frac{\sqrt{\frac{3}{2395-222\sqrt{114}+49\left(37-3\sqrt{114}\right)^{2/3}-14\left(37-3\sqrt{114}\right)^{4/3}}}\left(3\left(37-3\sqrt{114}\right)\left(7+\left(37-3\sqrt{114}\right)^{2/3}\right)A+3\left(453-4\right.\right.}{}$$

$$-\frac{\left(9\left(37-3\sqrt{114}\right)^{2/3}A-3\left(37-3\sqrt{114}+7\sqrt[3]{37-3\sqrt{114}}+4\left(37-3\sqrt{114}\right)^{2/3}\right)B+\left(296-24\sqrt{11}\right.\right.}{}$$

$$+\frac{1}{3}C\log\left(2+3x+4x^2+x^3\right)+\frac{\sqrt[3]{37-3\sqrt{114}}\left(9\sqrt[3]{37-3\sqrt{114}}A-3\left(7+4\sqrt[3]{37-3\sqrt{114}}+\left(37-3\sqrt{11}\right.\right.\right.}{3\left(\right.}$$

output
```
-3^(1/2)/(2395-222*114^(1/2)+49*(37-3*114^(1/2))^(2/3)-14*(37-3*114^(1/2))
^(4/3))^(1/2)*(3*(37-3*114^(1/2))*(7+(37-3*114^(1/2))^(2/3))*A+3*(453-46*1
14^(1/2)-(33-4*114^(1/2))*(37-3*114^(1/2))^(2/3))*B-(4401-431*114^(1/2)-(1
53-23*114^(1/2))*(37-3*114^(1/2))^(2/3))*C)*arctan((37-3*114^(1/2)+7*(37-3
*114^(1/2))^(1/3)-2*(37-3*114^(1/2))^(2/3)*(4+3*x))/(7185-666*114^(1/2)+14
7*(37-3*114^(1/2))^(2/3)-42*(37-3*114^(1/2))^(4/3))^(1/2))/(49+7*(37-3*114
^(1/2))^(2/3)+(37-3*114^(1/2))^(4/3))-(9*(37-3*114^(1/2))^(2/3)*A-3*(37-3*
114^(1/2)+7*(37-3*114^(1/2))^(1/3)+4*(37-3*114^(1/2))^(2/3))*B+(296-24*114
^(1/2)+56*(37-3*114^(1/2))^(1/3)+23*(37-3*114^(1/2))^(2/3))*C)*ln(33-4*114
^(1/2)-3*(37-3*114^(1/2))^(2/3)-(37-3*114^(1/2))^(1/3)*(3-114^(1/2))+37*x-
3*114^(1/2)*x+7*(37-3*114^(1/2))^(1/3)*x-8*(37-3*114^(1/2))^(2/3)*x-3*(37-
3*114^(1/2))^(2/3)*x^2)/(294+42*(37-3*114^(1/2))^(2/3)+6*(37-3*114^(1/2))^
(4/3))+1/3*C*ln(x^3+4*x^2+3*x+2)+(37-3*114^(1/2))^(1/3)*(9*(37-3*114^(1/2)
)^(1/3)*A-3*((37-3*114^(1/2))^(2/3)+4*(37-3*114^(1/2))^(1/3)+7)*B+(56+23*(
37-3*114^(1/2))^(1/3)+8*(37-3*114^(1/2))^(2/3))*C)*ln(7+(37-3*114^(1/2))^(
2/3)+(37-3*114^(1/2))^(1/3)*(4+3*x))/(147+21*(37-3*114^(1/2))^(2/3)+3*(37-
3*114^(1/2))^(4/3))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.10

$$\int \frac{A+Bx+Cx^2}{2+3x+4x^2+x^3}\,dx = \text{RootSum}\left[2+3\#1+4\#1^2+\#1^3\&,\frac{A\log(x-\#1)+B\log(x-\#1)\#1+C\log(x-\#1)\#1^2}{3+8\#1+3\#1^2}\&\right]$$

input
```
Integrate[(A + B*x + C*x^2)/(2 + 3*x + 4*x^2 + x^3),x]
```

output
```
RootSum[2 + 3*#1 + 4*#1^2 + #1^3 & , (A*Log[x - #1] + B*Log[x - #1]*#1 + C
*Log[x - #1]*#1^2)/(3 + 8*#1 + 3*#1^2) & ]
```

Rubi [A] (verified)

Time = 3.65 (sec) , antiderivative size = 681, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2525, 2490, 2485, 27, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{x^3+4x^2+3x+2}dx$$

$$\Big\downarrow \text{ 2525}$$

$$\frac{1}{3}\int \frac{3(A-C)+(3B-8C)x}{x^3+4x^2+3x+2}dx+\frac{1}{3}C\log\left(x^3+4x^2+3x+2\right)$$

$$\Big\downarrow \text{ 2490}$$

$$\frac{1}{3}\int \frac{\frac{1}{3}(9(A-C)-4(3B-8C))+(3B-8C)\left(x+\frac{4}{3}\right)}{\left(x+\frac{4}{3}\right)^3-\frac{7}{3}\left(x+\frac{4}{3}\right)+\frac{74}{27}}d\left(x+\frac{4}{3}\right)+\frac{1}{3}C\log\left(x^3+4x^2+3x+2\right)$$

$$\Big\downarrow \text{ 2485}$$

$$\frac{1}{3}\int -\frac{9\left(9A-12B+23C+3(3B-8C)\left(x+\frac{4}{3}\right)\right)}{\left(3\left(x+\frac{4}{3}\right)+\frac{7+\left(37-3\sqrt{114}\right)^{2/3}}{\sqrt[3]{37-3\sqrt{114}}}\right)\left(-9\left(x+\frac{4}{3}\right)^2+\frac{3\left(7+\left(37-3\sqrt{114}\right)^{2/3}\right)\left(x+\frac{4}{3}\right)}{\sqrt[3]{37-3\sqrt{114}}}-\left(37-3\sqrt{114}\right)^{2/3}-\frac{49}{\left(37-3\sqrt{1\ldots}\right.}\ldots\right.}\ldots$$
$$\frac{1}{3}C\log\left(x^3+4x^2+3x+2\right)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{3}C\log\left(x^3+4x^2+3x+2\right)-$$
$$3\int \frac{9A-12B+23C+3(3B-8C)\left(x+\frac{4}{3}\right)}{\left(3\left(x+\frac{4}{3}\right)+\frac{7+\left(37-3\sqrt{114}\right)^{2/3}}{\sqrt[3]{37-3\sqrt{114}}}\right)\left(-9\left(x+\frac{4}{3}\right)^2+\frac{3\left(7+\left(37-3\sqrt{114}\right)^{2/3}\right)\left(x+\frac{4}{3}\right)}{\sqrt[3]{37-3\sqrt{114}}}-\left(37-3\sqrt{114}\right)^{2/3}-\frac{49}{\left(37-3\sqrt{114}\right.}\ldots\right.}\ldots$$

$$\Big\downarrow \text{ 1200}$$

$$\frac{1}{3}C\log\left(x^3+4x^2+3x+2\right)-$$
$$3\int\left(\frac{\left(37-3\sqrt{114}\right)^{2/3}\left(-9\sqrt[3]{37-3\sqrt{114}}A+3\left(7+4\sqrt[3]{37-3\sqrt{114}}+\left(37-3\sqrt{114}\right)^{2/3}\right)B-\left(56+23\sqrt[3]{37-\right.\right.}{3\left(49+7\left(37-3\sqrt{114}\right)^{2/3}+\left(37-3\sqrt{114}\right)^{4/3}\right)\left(3\sqrt[3]{37-3\sqrt{114}}\left(x+\frac{4}{3}\right)+\left(37-3\right.\right.}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}C\log\left(x^3+4x^2+3x+2\right)-$$
$$3\left(\frac{\arctan\left(\frac{-6\left(37-3\sqrt{114}\right)^{2/3}\left(x+\frac{4}{3}\right)+7\sqrt[3]{37-3\sqrt{114}}-3\sqrt{114}+37}{\sqrt{3\left(2395-222\sqrt{114}+49\left(37-3\sqrt{114}\right)^{2/3}-14\left(37-3\sqrt{114}\right)^{4/3}\right)}}\right)\left(3\left(37-3\sqrt{114}\right)\left(7+\left(37-3\sqrt{114}\right)^{2/3}\right)A+3\right.}{\sqrt{3\left(2395-222\sqrt{114}+49\left(37-3\sqrt{114}\right)^{2/3}-14\left(37\right.\right.}}\right.$$

input
```
Int[(A + B*x + C*x^2)/(2 + 3*x + 4*x^2 + x^3),x]
```

output
```
(C*Log[2 + 3*x + 4*x^2 + x^3])/3 - 3*(((3*(37 - 3*Sqrt[114])*(7 + (37 - 3*
Sqrt[114])^(2/3))*A + 3*(453 - 46*Sqrt[114] - (33 - 4*Sqrt[114])*(37 - 3*S
qrt[114])^(2/3))*B - (4401 - 431*Sqrt[114] - (153 - 23*Sqrt[114])*(37 - 3*
Sqrt[114])^(2/3))*C)*ArcTan[(37 - 3*Sqrt[114] + 7*(37 - 3*Sqrt[114])^(1/3)
 - 6*(37 - 3*Sqrt[114])^(2/3)*(4/3 + x))/Sqrt[3*(2395 - 222*Sqrt[114] + 49
*(37 - 3*Sqrt[114])^(2/3) - 14*(37 - 3*Sqrt[114])^(4/3))]])/(Sqrt[3*(2395
- 222*Sqrt[114] + 49*(37 - 3*Sqrt[114])^(2/3) - 14*(37 - 3*Sqrt[114])^(4/3
))]*(49 + 7*(37 - 3*Sqrt[114])^(2/3) + (37 - 3*Sqrt[114])^(4/3))) - ((37 -
 3*Sqrt[114])^(1/3)*(9*(37 - 3*Sqrt[114])^(1/3)*A - 3*(7 + 4*(37 - 3*Sqrt[
114])^(1/3) + (37 - 3*Sqrt[114])^(2/3))*B + (56 + 23*(37 - 3*Sqrt[114])^(1
/3) + 8*(37 - 3*Sqrt[114])^(2/3))*C)*Log[7 + (37 - 3*Sqrt[114])^(2/3) + 3*
(37 - 3*Sqrt[114])^(1/3)*(4/3 + x)])/(9*(49 + 7*(37 - 3*Sqrt[114])^(2/3) +
 (37 - 3*Sqrt[114])^(4/3))) + ((9*(37 - 3*Sqrt[114])^(2/3)*A - 3*(37 - 3*S
qrt[114] + 7*(37 - 3*Sqrt[114])^(1/3) + 4*(37 - 3*Sqrt[114])^(2/3))*B + (2
96 - 24*Sqrt[114] + 56*(37 - 3*Sqrt[114])^(1/3) + 23*(37 - 3*Sqrt[114])^(2
/3))*C)*Log[49 - 7*(37 - 3*Sqrt[114])^(2/3) + (37 - 3*Sqrt[114])^(4/3) - 3
*(37 - 3*Sqrt[114] + 7*(37 - 3*Sqrt[114])^(1/3))*(4/3 + x) + 9*(37 - 3*Sqr
t[114])^(2/3)*(4/3 + x)^2])/(18*(49 + 7*(37 - 3*Sqrt[114])^(2/3) + (37 - 3
*Sqrt[114])^(4/3))))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2485
```
Int[((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (d_.)*(x_)^3)^(p_), x_S
ymbol] :> With[{r = Rt[-9*a*d^2 + Sqrt[3]*d*Sqrt[4*b^3*d + 27*a^2*d^2], 3]}
, Simp[1/d^(2*p)   Int[(e + f*x)^m*Simp[18^(1/3)*b*(d/(3*r)) - r/18^(1/3) +
 d*x, x]^p*Simp[b*(d/3) + 12^(1/3)*b^2*(d^2/(3*r^2)) + r^2/(3*12^(1/3)) - d
*(2^(1/3)*b*(d/(3^(1/3)*r)) - r/18^(1/3))*x + d^2*x^2, x]^p, x], x]] /; Fre
eQ[{a, b, d, e, f, m}, x] && NeQ[4*b^3 + 27*a^2*d, 0] && ILtQ[p, 0]
```

rule 2490
```
Int[(P3_)^(p_.)*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] :> With[{a = Coeff[P3
, x, 0], b = Coeff[P3, x, 1], c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Su
bst[Int[((3*d*e - c*f)/(3*d) + f*x)^m*Simp[(2*c^3 - 9*b*c*d + 27*a*d^2)/(27
*d^2) - (c^2 - 3*b*d)*(x/(3*d)) + d*x^3, x]^p, x], x, x + c/(3*d)] /; NeQ[c
, 0]] /; FreeQ[{e, f, m, p}, x] && PolyQ[P3, x, 3]
```

rule 2525
```
Int[(Pm_)/(Qn_), x_Symbol] :> With[{m = Expon[Pm, x], n = Expon[Qn, x]}, Si
mp[Coeff[Pm, x, m]*(Log[Qn]/(n*Coeff[Qn, x, n])), x] + Simp[1/(n*Coeff[Qn,
x, n])   Int[ExpandToSum[n*Coeff[Qn, x, n]*Pm - Coeff[Pm, x, m]*D[Qn, x], x
]/Qn, x], x] /; EqQ[m, n - 1]] /; PolyQ[Pm, x] && PolyQ[Qn, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.07

method	result	size
default	$\sum_{\_R=\mathrm{RootOf}(\_Z^3+4\_Z^2+3\_Z+2)} \frac{(C\_R^2+B\_R+A)\ln(x-\_R)}{3\_R^2+8\_R+3}$	47
risch	$\sum_{\_R=\mathrm{RootOf}(\_Z^3+4\_Z^2+3\_Z+2)} \frac{(C\_R^2+B\_R+A)\ln(x-\_R)}{3\_R^2+8\_R+3}$	47

input
```
int((C*x^2+B*x+A)/(x^3+4*x^2+3*x+2),x,method=_RETURNVERBOSE)
```

output
```
sum((C*_R^2+B*_R+A)/(3*_R^2+8*_R+3)*ln(x-_R),_R=RootOf(_Z^3+4*_Z^2+3*_Z+2)
)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.50 (sec) , antiderivative size = 7620, normalized size of antiderivative = 10.79

$$\int \frac{A+Bx+Cx^2}{2+3x+4x^2+x^3}\,dx = \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)/(x^3+4*x^2+3*x+2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [A] (verification not implemented)

Time = 8.06 (sec) , antiderivative size = 296, normalized size of antiderivative = 0.42

$$\int \frac{A+Bx+Cx^2}{2+3x+4x^2+x^3}\,dx$$
$$=\text{RootSum}\left(152t^3-152t^2C+t\left(7A^2+6AB-30AC-15B^2+74BC-33C^2\right)-A^3+4A^2B-10A^2C\right.$$

input
```
integrate((C*x**2+B*x+A)/(x**3+4*x**2+3*x+2),x)
```

output
```
RootSum(152*_t**3 - 152*_t**2*C + _t*(7*A**2 + 6*A*B - 30*A*C - 15*B**2 +
74*B*C - 33*C**2) - A**3 + 4*A**2*B - 10*A**2*C - 3*A*B**2 + 6*A*B*C + 7*A
*C**2 + 2*B**3 - 8*B**2*C + 6*B*C**2 - 4*C**3, Lambda(_t, _t*log(x + (1064
*_t**2*A + 456*_t**2*B - 2280*_t**2*C + 228*_t*A**2 - 608*_t*A*B + 456*_t*
A*C + 228*_t*B**2 - 912*_t*B*C + 1748*_t*C**2 + 82*A**3 - 90*A**2*B - 82*A
**2*C + 146*A*B**2 - 396*A*B*C + 534*A*C**2 - 66*B**3 + 306*B**2*C - 466*B
*C**2 + 42*C**3)/(37*A**3 - 99*A**2*B + 153*A**2*C + 153*A*B**2 - 618*A*B*
C + 671*A*C**2 - 27*B**3 + 63*B**2*C + 141*B*C**2 - 349*C**3))))
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{2+3x+4x^2+x^3}\,dx=\int \frac{Cx^2+Bx+A}{x^3+4\,x^2+3\,x+2}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(x^3+4*x^2+3*x+2),x, algorithm="maxima")
```

output
```
integrate((C*x^2 + B*x + A)/(x^3 + 4*x^2 + 3*x + 2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A+Bx+Cx^2}{2+3x+4x^2+x^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((C*x^2+B*x+A)/(x^3+4*x^2+3*x+2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 21.93 (sec) , antiderivative size = 365, normalized size of antiderivative = 0.52

$$\int \frac{A+Bx+Cx^2}{2+3x+4x^2+x^3}\,dx$$

$$= \sum_{k=1}^{3} \ln\left(\mathrm{root}\left(z^3 - C z^2 + \frac{z\,(7A^2-15B^2-33C^2+6AB-30AC+74BC)}{152} + \frac{3ABC}{76} + \frac{3BC^2}{76} - \frac{B^2C}{19} + \frac{7AC^2}{152} - \frac{5A^2C}{76} - \frac{3AB^2}{152} + \frac{A^2B}{38} - \frac{A^3}{152} - \frac{C^3}{38} + \frac{B^3}{76}, z, k\right)\left(4A-3B+x\,(3A-4B+17C) - \mathrm{root}\left(z^3-Cz^2+\frac{z\,(7A^2-15B^2-33C^2+6AB-30AC+74BC)}{152}+\frac{3ABC}{76}+\frac{3BC^2}{76}-\frac{B^2C}{19}+\frac{7\right.\right.\right.$$

$$\left.+x\,(B^2-4BC+3C^2-AC)+2C^2+AB-4AC\right)\mathrm{root}\left(z^3 - C z^2 + \frac{z\,(7A^2-15B^2-33C^2+6AB-30AC+74BC)}{152} + \frac{3ABC}{76} + \frac{3BC^2}{76} - \frac{B^2C}{19} + \frac{7AC^2}{152} - \frac{5A^2C}{76} - \frac{3AB^2}{152} + \frac{A^2B}{38} - \frac{A^3}{152} - \frac{C^3}{38} + \frac{B^3}{76}, z, k\right)$$

input
```
int((A + B*x + C*x^2)/(3*x + 4*x^2 + x^3 + 2),x)
```

output
```
symsum(log(root(z^3 - C*z^2 + (z*(7*A^2 - 15*B^2 - 33*C^2 + 6*A*B - 30*A*C
 + 74*B*C))/152 + (3*A*B*C)/76 + (3*B*C^2)/76 - (B^2*C)/19 + (7*A*C^2)/152
 - (5*A^2*C)/76 - (3*A*B^2)/152 + (A^2*B)/38 - A^3/152 - C^3/38 + B^3/76,
z, k)*(4*A - 3*B + x*(3*A - 4*B + 17*C) - root(z^3 - C*z^2 + (z*(7*A^2 - 1
5*B^2 - 33*C^2 + 6*A*B - 30*A*C + 74*B*C))/152 + (3*A*B*C)/76 + (3*B*C^2)/
76 - (B^2*C)/19 + (7*A*C^2)/152 - (5*A^2*C)/76 - (3*A*B^2)/152 + (A^2*B)/3
8 - A^3/152 - C^3/38 + B^3/76, z, k)*(14*x - 6)) + x*(B^2 + 3*C^2 - A*C -
4*B*C) + 2*C^2 + A*B - 4*A*C)*root(z^3 - C*z^2 + (z*(7*A^2 - 15*B^2 - 33*C
^2 + 6*A*B - 30*A*C + 74*B*C))/152 + (3*A*B*C)/76 + (3*B*C^2)/76 - (B^2*C)
/19 + (7*A*C^2)/152 - (5*A^2*C)/76 - (3*A*B^2)/152 + (A^2*B)/38 - A^3/152
- C^3/38 + B^3/76, z, k), k, 1, 3)
```

Reduce [F]

$$\int \frac{A+Bx+Cx^2}{2+3x+4x^2+x^3}\,dx = \left(\int \frac{x}{x^3+4x^2+3x+2}dx\right)b - \frac{8\left(\int \frac{x}{x^3+4x^2+3x+2}dx\right)c}{3} + \left(\int \frac{1}{x^3+4x^2+3x+2}dx\right)a - \left(\int \frac{1}{x^3+4x^2+3x+2}dx\right)c + \frac{\log(x^3+4x^2+3x+2)\,c}{3}$$

input
```
int((C*x^2+B*x+A)/(x^3+4*x^2+3*x+2),x)
```

output
```
(3*int(x/(x**3 + 4*x**2 + 3*x + 2),x)*b - 8*int(x/(x**3 + 4*x**2 + 3*x + 2
),x)*c + 3*int(1/(x**3 + 4*x**2 + 3*x + 2),x)*a - 3*int(1/(x**3 + 4*x**2 +
 3*x + 2),x)*c + log(x**3 + 4*x**2 + 3*x + 2)*c)/3
```

3.175 $\int \frac{A+Bx+Cx^2}{(2+3x+4x^2+x^3)^2} \, dx$

Optimal result

Integrand size = 26, antiderivative size = 1237

$$\int \frac{A+Bx+Cx^2}{(2+3x+4x^2+x^3)^2} \, dx = \text{Too large to display}$$

output
```
-1/3*C/(x^3+4*x^2+3*x+2)+3*(37-3*114^(1/2))^(1/3)*(7*A+3*B-15*C)/(532+76*(
37-3*114^(1/2))^(2/3)+76*(37-3*114^(1/2))^(1/3)*(4+3*x))+3/76*(-1026+111*1
14^(1/2))^(1/3)*(42*A+18*B-90*C-(3*(37-3*114^(1/2)+7*(37-3*114^(1/2))^(1/3
))*A-3*(33-4*114^(1/2)-(37-3*114^(1/2))^(1/3)*(3-114^(1/2)))*B+(153-23*114
^(1/2)-(45-8*114^(1/2))*(37-3*114^(1/2))^(1/3))*C)*(4+3*x)/(37-3*114^(1/2)
)^(2/3))*38^(1/3)/(7-(37-3*114^(1/2))^(2/3))/(4+7/(37-3*114^(1/2))^(1/3)+(
37-3*114^(1/2))^(1/3)+3*x)/(7-49/(37-3*114^(1/2))^(2/3)-(37-3*114^(1/2))^(
2/3)+(7+(37-3*114^(1/2))^(2/3))*(4+3*x)/(37-3*114^(1/2))^(1/3)-(4+3*x)^2)-
162*3^(1/2)/(2395-222*114^(1/2)+49*(37-3*114^(1/2))^(2/3)-14*(37-3*114^(1/
2))^(4/3))^(1/2)*((54904003-5140114*114^(1/2)+(4332457-405224*114^(1/2))*(
37-3*114^(1/2))^(2/3))*A-3*(4661535-436022*114^(1/2)+(1167501-109300*114^(
1/2))*(37-3*114^(1/2))^(2/3))*B-(17611723-1651938*114^(1/2)-(5007551-46917
6*114^(1/2))*(37-3*114^(1/2))^(2/3))*C)*arctan((37-3*114^(1/2)+7*(37-3*114
^(1/2))^(1/3)-2*(37-3*114^(1/2))^(2/3)*(4+3*x))/(7185-666*114^(1/2)+147*(3
7-3*114^(1/2))^(2/3)-42*(37-3*114^(1/2))^(4/3))^(1/2))/(7-(37-3*114^(1/2))
^(2/3))^2/(49+7*(37-3*114^(1/2))^(2/3)+(37-3*114^(1/2))^(4/3))^3+27*((1151
773-107793*114^(1/2)+49*(2395-222*114^(1/2))*(37-3*114^(1/2))^(1/3)-74*(23
95-222*114^(1/2))*(37-3*114^(1/2))^(2/3))*A+3*(164539-15399*114^(1/2)+7*(2
395-222*114^(1/2))*(37-3*114^(1/2))^(1/3)+22*(2395-222*114^(1/2))*(37-3*11
4^(1/2))^(2/3))*B-3*(822695-76995*114^(1/2)+(83825-7770*114^(1/2))*(37-...
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.13

$$\int \frac{A+Bx+Cx^2}{(2+3x+4x^2+x^3)^2}\,dx$$
$$= \frac{1}{76}\left(\frac{B(-14-3x+3x^2)+A(18+31x+7x^2)-C(6+23x+15x^2)}{2+3x+4x^2+x^3}\right.$$
$$+\text{RootSum}\left[2+3\#1+4\#1^2\right.$$
$$+\#1^3\&,\frac{34A\log(x-\#1)-18B\log(x-\#1)+14C\log(x-\#1)+7A\log(x-\#1)\#1+3B\log(x-\#1)}{3+8\#1+3\#1^2}$$

input
```
Integrate[(A + B*x + C*x^2)/(2 + 3*x + 4*x^2 + x^3)^2,x]
```

output
```
((B*(-14 - 3*x + 3*x^2) + A*(18 + 31*x + 7*x^2) - C*(6 + 23*x + 15*x^2))/(
2 + 3*x + 4*x^2 + x^3) + RootSum[2 + 3*#1 + 4*#1^2 + #1^3 & , (34*A*Log[x
- #1] - 18*B*Log[x - #1] + 14*C*Log[x - #1] + 7*A*Log[x - #1]*#1 + 3*B*Log
[x - #1]*#1 - 15*C*Log[x - #1]*#1)/(3 + 8*#1 + 3*#1^2) & ])/76
```

Rubi [A] (warning: unable to verify)

Time = 8.60 (sec) , antiderivative size = 1268, normalized size of antiderivative = 1.03, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2526, 2490, 2485, 27, 1235, 27, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{\left(x^3+4x^2+3x+2\right)^2}dx$$

$$\Big\downarrow \text{ 2526}$$

$$\frac{1}{3}\int \frac{3(A-C)+(3B-8C)x}{\left(x^3+4x^2+3x+2\right)^2}dx-\frac{C}{3\left(x^3+4x^2+3x+2\right)}$$

$$\Big\downarrow \text{ 2490}$$

$$\frac{1}{3}\int \frac{\frac{1}{3}(9(A-C)-4(3B-8C))+(3B-8C)\left(x+\frac{4}{3}\right)}{\left(\left(x+\frac{4}{3}\right)^3-\frac{7}{3}\left(x+\frac{4}{3}\right)+\frac{74}{27}\right)^2}d\left(x+\frac{4}{3}\right)-\frac{C}{3\left(x^3+4x^2+3x+2\right)}$$

$$\Big\downarrow \text{ 2485}$$

$$\frac{1}{3}\int \frac{243\left(9A-12B+23C+3(3B-8C)\left(x+\frac{4}{3}\right)\right)}{\left(3\left(x+\frac{4}{3}\right)+\frac{7+\left(37-3\sqrt{114}\right)^{2/3}}{\sqrt[3]{37-3\sqrt{114}}}\right)^2\left(-9\left(x+\frac{4}{3}\right)^2+\frac{3\left(7+\left(37-3\sqrt{114}\right)^{2/3}\right)\left(x+\frac{4}{3}\right)}{\sqrt[3]{37-3\sqrt{114}}}-\left(37-3\sqrt{114}\right)^{2/3}-\frac{49}{\left(37-3\sqrt{11\ldots}\right.}\right.}$$
$$\frac{C}{3\left(x^3+4x^2+3x+2\right)}$$

$$\Big\downarrow \text{ 27}$$

$$81\int \frac{9A-12B+23C+3(3B-8C)\left(x+\frac{4}{3}\right)}{\left(3\left(x+\frac{4}{3}\right)+\frac{7+\left(37-3\sqrt{114}\right)^{2/3}}{\sqrt[3]{37-3\sqrt{114}}}\right)^2\left(-9\left(x+\frac{4}{3}\right)^2+\frac{3\left(7+\left(37-3\sqrt{114}\right)^{2/3}\right)\left(x+\frac{4}{3}\right)}{\sqrt[3]{37-3\sqrt{114}}}-\left(37-3\sqrt{114}\right)^{2/3}-\frac{49}{\left(37-3\sqrt{1\ldots}\right.}\right.}$$

$$\frac{C}{3\left(x^3+4x^2+3x+2\right)}$$

$\downarrow$ 1235

$$81\left(\frac{\sqrt[3]{\frac{37}{114\sqrt{114}}-\frac{1}{38}}\left(2(7A+3B-15C)-\frac{\left(x+\frac{4}{3}\right)\left(3\left(37-3\sqrt{114}+7\sqrt[3]{37-3\sqrt{114}}\right)A-3\left(33-4\sqrt{114}-\sqrt[3]{37-3\sqrt{114}}\ldots\right.\right.}{\left(37-3\sqrt{114}\right)^{\ldots}}\right.}{6\left(7-\left(37-3\sqrt{114}\right)^{2/3}\right)\left(3\left(x+\frac{4}{3}\right)+\frac{7+\left(37-3\sqrt{114}\right)^{2/3}}{\sqrt[3]{37-3\sqrt{114}}}\right)\left(-9\left(x+\frac{4}{3}\right)^2+\frac{3\left(7+\left(37-3\sqrt{114}\right)^{2/3}\right)\left(\ldots\right.}{\sqrt[3]{37-3\sqrt{114}}}\right.}\right.$$

$$\frac{C}{3\left(x^3+4x^2+3x+2\right)}$$

$\downarrow$ 27

$$81\left(\frac{\sqrt[3]{\frac{37}{114\sqrt{114}}-\frac{1}{38}}\int \frac{\frac{\left(7+\left(37-3\sqrt{114}\right)^{2/3}\right)\left(49-21\left(37-3\sqrt{114}\right)^{2/3}+\left(37-3\sqrt{114}\right)^{4/3}\right)(3B-8C)-2\sqrt[3]{37-3\sqrt{114}}\left(49-7\left(37-3\sqrt{114}\right)^{2/3}+\ldots\right.}{37-3\sqrt{114}}}{\left(3\left(x+\frac{4}{3}\right)+\frac{7+\left(37-3\sqrt{114}\ldots\right.}{\sqrt[3]{37-3\sqrt{\ldots}}}\right.}}{\ldots}\right.$$

$$\frac{C}{3\left(x^3+4x^2+3x+2\right)}$$

$\big\downarrow$ 1200

$$81\left(\frac{\sqrt[3]{-\frac{1}{38}+\frac{37}{114\sqrt{114}}}\left(2(7A+3B-15C)-\frac{\left(3\left(37-3\sqrt{114}+7\sqrt[3]{37-3\sqrt{114}}\right)A-3\left(33-4\sqrt{114}-\sqrt[3]{37-3\sqrt{114}}(3-\right.\right.}{\left(37-3\sqrt{114}\right.}\right.}{6\left(7-\left(37-3\sqrt{114}\right)^{2/3}\right)\left(3\left(x+\frac{4}{3}\right)+\frac{7+\left(37-3\sqrt{114}\right)^{2/3}}{\sqrt[3]{37-3\sqrt{114}}}\right)\left(-9\left(x+\frac{4}{3}\right)^2+\frac{3\left(7+\left(37-3\sqrt{114}\right)^{2/3}\right)}{\sqrt[3]{37-3\sqrt{11}}}\right.}\right.$$

$$\frac{C}{3\left(x^3+4x^2+3x+2\right)}$$

$\big\downarrow$ 2009

$$81\left(\frac{\sqrt[3]{-\frac{1}{38}+\frac{37}{114\sqrt{114}}}\left(2(7A+3B-15C)-\frac{\left(3\left(37-3\sqrt{114}+7\sqrt[3]{37-3\sqrt{114}}\right)A-3\left(33-4\sqrt{114}-\sqrt[3]{37-3\sqrt{114}}(3-\right.\right.}{\left(37-3\sqrt{11}\right.}\right.}{6\left(7-\left(37-3\sqrt{114}\right)^{2/3}\right)\left(3\left(x+\frac{4}{3}\right)+\frac{7+\left(37-3\sqrt{114}\right)^{2/3}}{\sqrt[3]{37-3\sqrt{114}}}\right)\left(-9\left(x+\frac{4}{3}\right)^2+\frac{3\left(7+\left(37-3\sqrt{114}\right)^{2/3}\right)}{\sqrt[3]{37-3\sqrt{11}}}\right.}\right.$$

$$\frac{C}{3\left(x^3+4x^2+3x+2\right)}$$

input
```
Int[(A + B*x + C*x^2)/(2 + 3*x + 4*x^2 + x^3)^2,x]
```

output

```
-1/3*C/(2 + 3*x + 4*x^2 + x^3) + 81*(((-1/38 + 37/(114*Sqrt[114]))^(1/3)*(
2*(7*A + 3*B - 15*C) - ((3*(37 - 3*Sqrt[114] + 7*(37 - 3*Sqrt[114])^(1/3))
*A - 3*(33 - 4*Sqrt[114] - (37 - 3*Sqrt[114])^(1/3)*(3 - Sqrt[114]))*B + (
153 - 23*Sqrt[114] - (45 - 8*Sqrt[114])*(37 - 3*Sqrt[114])^(1/3))*C)*(4/3
+ x))/(37 - 3*Sqrt[114])^(2/3)))/(6*(7 - (37 - 3*Sqrt[114])^(2/3))*((7 + (
37 - 3*Sqrt[114])^(2/3))/(37 - 3*Sqrt[114])^(1/3) + 3*(4/3 + x))*(7 - 49/(
37 - 3*Sqrt[114])^(2/3) - (37 - 3*Sqrt[114])^(2/3) + (3*(7 + (37 - 3*Sqrt[
114])^(2/3))*(4/3 + x))/(37 - 3*Sqrt[114])^(1/3) - 9*(4/3 + x)^2)) + ((-1/
38 + 37/(114*Sqrt[114]))^(1/3)*((6*(2395 - 222*Sqrt[114])*(7*A + 3*B - 15*
C))/((37 - 3*Sqrt[114])*(49 + 7*(37 - 3*Sqrt[114])^(2/3) + (37 - 3*Sqrt[11
4])^(4/3))*(7 + (37 - 3*Sqrt[114])^(2/3) + 3*(37 - 3*Sqrt[114])^(1/3)*(4/3
 + x))) - (6*Sqrt[3/(2395 - 222*Sqrt[114] + 49*(37 - 3*Sqrt[114])^(2/3) -
14*(37 - 3*Sqrt[114])^(4/3))]*((7*(7843429 - 734302*Sqrt[114]) + (4332457
- 405224*Sqrt[114])*(37 - 3*Sqrt[114])^(2/3))*A - 3*(4661535 - 436022*Sqrt
[114] + (1167501 - 109300*Sqrt[114])*(37 - 3*Sqrt[114])^(2/3))*B - (176117
23 - 1651938*Sqrt[114] - (5007551 - 469176*Sqrt[114])*(37 - 3*Sqrt[114])^(
2/3))*C)*ArcTan[(37 - 3*Sqrt[114] + 7*(37 - 3*Sqrt[114])^(1/3) - 6*(37 - 3
*Sqrt[114])^(2/3)*(4/3 + x))/Sqrt[3*(2395 - 222*Sqrt[114] + 49*(37 - 3*Sqr
t[114])^(2/3) - 14*(37 - 3*Sqrt[114])^(4/3))]])/((37 - 3*Sqrt[114])^(4/3)*
(49 + 7*(37 - 3*Sqrt[114])^(2/3) + (37 - 3*Sqrt[114])^(4/3))^2) - (2*((...
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1200

```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1235
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^m
*(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*
m + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) -
f*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
)
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2485
```
Int[((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (d_.)*(x_)^3)^(p_), x_S
ymbol] :> With[{r = Rt[-9*a*d^2 + Sqrt[3]*d*Sqrt[4*b^3*d + 27*a^2*d^2], 3]}
, Simp[1/d^(2*p)   Int[(e + f*x)^m*Simp[18^(1/3)*b*(d/(3*r)) - r/18^(1/3) +
 d*x, x]^p*Simp[b*(d/3) + 12^(1/3)*b^2*(d^2/(3*r^2)) + r^2/(3*12^(1/3)) - d
*(2^(1/3)*b*(d/(3^(1/3)*r)) - r/18^(1/3))*x + d^2*x^2, x]^p, x], x]] /; Fre
eQ[{a, b, d, e, f, m}, x] && NeQ[4*b^3 + 27*a^2*d, 0] && ILtQ[p, 0]
```

rule 2490
```
Int[(P3_)^(p_.)*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] :> With[{a = Coeff[P3
, x, 0], b = Coeff[P3, x, 1], c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Su
bst[Int[((3*d*e - c*f)/(3*d) + f*x)^m*Simp[(2*c^3 - 9*b*c*d + 27*a*d^2)/(27
*d^2) - (c^2 - 3*b*d)*(x/(3*d)) + d*x^3, x]^p, x], x, x + c/(3*d)] /; NeQ[c
, 0]] /; FreeQ[{e, f, m, p}, x] && PolyQ[P3, x, 3]
```

rule 2526
```
Int[(Pm_)*(Qn_)^(p_), x_Symbol] :> With[{m = Expon[Pm, x], n = Expon[Qn, x]
}, Simp[Coeff[Pm, x, m]*(Qn^(p + 1)/(n*(p + 1)*Coeff[Qn, x, n])), x] + Simp
[1/(n*Coeff[Qn, x, n])   Int[ExpandToSum[n*Coeff[Qn, x, n]*Pm - Coeff[Pm, x
, m]*D[Qn, x], x]*Qn^p, x], x] /; EqQ[m, n - 1]] /; FreeQ[p, x] && PolyQ[Pm
, x] && PolyQ[Qn, x] && NeQ[p, -1]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.09 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.09

method	result
default	$\frac{\left(\frac{7A}{76}+\frac{3B}{76}-\frac{15C}{76}\right)x^2+\left(\frac{31A}{76}-\frac{3B}{76}-\frac{23C}{76}\right)x+\frac{9A}{38}-\frac{7B}{38}-\frac{3C}{38}}{x^3+4x^2+3x+2}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^3+4\_Z^2+3\_Z+2\right)}\frac{\left(7A\_R+3B\_R-15C\_R+34A\right.}{3\_R^2+8\_}\right.}{76}$
risch	$\frac{\left(\frac{7A}{76}+\frac{3B}{76}-\frac{15C}{76}\right)x^2+\left(\frac{31A}{76}-\frac{3B}{76}-\frac{23C}{76}\right)x+\frac{9A}{38}-\frac{7B}{38}-\frac{3C}{38}}{x^3+4x^2+3x+2}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^3+4\_Z^2+3\_Z+2\right)}\frac{\left((7A+3B-15C)\_R+34A-18B+1\right.}{3\_R^2+8\_R+}\right.}{76}$

input
```
int((C*x^2+B*x+A)/(x^3+4*x^2+3*x+2)^2,x,method=_RETURNVERBOSE)
```

output
```
((7/76*A+3/76*B-15/76*C)*x^2+(31/76*A-3/76*B-23/76*C)*x+9/38*A-7/38*B-3/38
*C)/(x^3+4*x^2+3*x+2)+1/76*sum((7*A*_R+3*B*_R-15*C*_R+34*A-18*B+14*C)/(3*_
R^2+8*_R+3)*ln(x-_R),_R=RootOf(_Z^3+4*_Z^2+3*_Z+2))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.54 (sec) , antiderivative size = 5342, normalized size of antiderivative = 4.32

$$\int \frac{A+Bx+Cx^2}{\left(2+3x+4x^2+x^3\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)/(x^3+4*x^2+3*x+2)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [A] (verification not implemented)

Time = 8.46 (sec) , antiderivative size = 342, normalized size of antiderivative = 0.28

$$\int \frac{A+Bx+Cx^2}{(2+3x+4x^2+x^3)^2}\,dx$$
$$=\text{RootSum}\left(877952t^3+t(8785A^2-9342AB+7342AC+1809B^2-306BC-3263C^2)-148A^3+543\right.$$
$$+\frac{18A-14B-6C+x^2\cdot(7A+3B-15C)+x(31A-3B-23C)}{76x^3+304x^2+228x+152}$$

input
```
integrate((C*x**2+B*x+A)/(x**3+4*x**2+3*x+2)**2,x)
```

output
```
RootSum(877952*_t**3 + _t*(8785*A**2 - 9342*A*B + 7342*A*C + 1809*B**2 - 3
06*B*C - 3263*C**2) - 148*A**3 + 543*A**2*B - 1004*A**2*C - 486*A*B**2 + 1
506*A*B*C - 1004*A*C**2 + 135*B**3 - 594*B**2*C + 831*B*C**2 - 404*C**3, L
ambda(_t, _t*log(x + (227389568*_t**2*A - 102720384*_t**2*B + 46531456*_t*
*2*C + 9882736*_t*A**2 - 19372704*_t*A*B + 31895072*_t*A*C + 8265456*_t*B*
*2 - 24709728*_t*B*C + 16998768*_t*C**2 + 2715246*A**3 - 4609158*A**2*B +
4145350*A**2*C + 2758122*A*B**2 - 5491668*A*B*C + 3176874*A*C**2 - 591138*
B**3 + 1970982*B**2*C - 2510118*B*C**2 + 1172258*C**3)/(898777*A**3 - 1733
157*A**2*B + 1925421*A**2*C + 1287819*A*B**2 - 3402054*A*B*C + 2610651*A*C
**2 - 337527*B**3 + 1412397*B**2*C - 2065365*B*C**2 + 965663*C**3)))) + (1
8*A - 14*B - 6*C + x**2*(7*A + 3*B - 15*C) + x*(31*A - 3*B - 23*C))/(76*x*
*3 + 304*x**2 + 228*x + 152)
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{(2+3x+4x^2+x^3)^2}\,dx=\int \frac{Cx^2+Bx+A}{(x^3+4\,x^2+3\,x+2)^2}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(x^3+4*x^2+3*x+2)^2,x, algorithm="maxima")
```

output
```
1/76*((7*A + 3*B - 15*C)*x^2 + (31*A - 3*B - 23*C)*x + 18*A - 14*B - 6*C)/
(x^3 + 4*x^2 + 3*x + 2) - 1/76*integrate(-((7*A + 3*B - 15*C)*x + 34*A - 1
8*B + 14*C)/(x^3 + 4*x^2 + 3*x + 2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A+Bx+Cx^2}{(2+3x+4x^2+x^3)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((C*x^2+B*x+A)/(x^3+4*x^2+3*x+2)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 22.12 (sec) , antiderivative size = 989, normalized size of antiderivative = 0.80

$$\int \frac{A+Bx+Cx^2}{(2+3x+4x^2+x^3)^2}\,dx = \text{Too large to display}$$

input
```
int((A + B*x + C*x^2)/(3*x + 4*x^2 + x^3 + 2)^2,x)
```

output
```
symsum(log((49*A^2*x)/5776 + (9*B^2*x)/5776 + (225*C^2*x)/5776 - 14*root(z
^3 + z*((8785*A^2)/877952 + (1809*B^2)/877952 - (3263*C^2)/877952 - (4671*
A*B)/438976 + (3671*A*C)/438976 - (153*B*C)/438976) + (753*A*B*C)/438976 +
 (831*B*C^2)/877952 - (297*B^2*C)/438976 - (251*A^2*C)/219488 - (251*A*C^2
)/219488 + (543*A^2*B)/877952 - (243*A*B^2)/438976 - (37*A^3)/219488 - (10
1*C^3)/219488 + (135*B^3)/877952, z, k)^2*x + (119*A^2)/2888 - (27*B^2)/28
88 - (105*C^2)/2888 + 6*root(z^3 + z*((8785*A^2)/877952 + (1809*B^2)/87795
2 - (3263*C^2)/877952 - (4671*A*B)/438976 + (3671*A*C)/438976 - (153*B*C)/
438976) + (753*A*B*C)/438976 + (831*B*C^2)/877952 - (297*B^2*C)/438976 - (
251*A^2*C)/219488 - (251*A*C^2)/219488 + (543*A^2*B)/877952 - (243*A*B^2)/
438976 - (37*A^3)/219488 - (101*C^3)/219488 + (135*B^3)/877952, z, k)^2 -
(3*A*B)/722 - (103*A*C)/1444 + (39*B*C)/722 + (115*A*root(z^3 + z*((8785*A
^2)/877952 + (1809*B^2)/877952 - (3263*C^2)/877952 - (4671*A*B)/438976 + (
3671*A*C)/438976 - (153*B*C)/438976) + (753*A*B*C)/438976 + (831*B*C^2)/87
7952 - (297*B^2*C)/438976 - (251*A^2*C)/219488 - (251*A*C^2)/219488 + (543
*A^2*B)/877952 - (243*A*B^2)/438976 - (37*A^3)/219488 - (101*C^3)/219488 +
 (135*B^3)/877952, z, k))/76 - (81*B*root(z^3 + z*((8785*A^2)/877952 + (18
09*B^2)/877952 - (3263*C^2)/877952 - (4671*A*B)/438976 + (3671*A*C)/438976
 - (153*B*C)/438976) + (753*A*B*C)/438976 + (831*B*C^2)/877952 - (297*B^2*
C)/438976 - (251*A^2*C)/219488 - (251*A*C^2)/219488 + (543*A^2*B)/87795...
```

Reduce [F]

$$\int \frac{A+Bx+Cx^2}{(2+3x+4x^2+x^3)^2}\,dx$$
$$= \frac{6\left(\int \frac{x^3}{x^6+8x^5+22x^4+28x^3+25x^2+12x+4}dx\right) b\,x^3 + 24\left(\int \frac{x^3}{x^6+8x^5+22x^4+28x^3+25x^2+12x+4}dx\right) b\,x^2 + 18\left(\int \frac{}{x^6+8x^5+22x^4\cdots}\right.}{}$$

input
```
int((C*x^2+B*x+A)/(x^3+4*x^2+3*x+2)^2,x)
```

output

```
(6*int(x**3/(x**6 + 8*x**5 + 22*x**4 + 28*x**3 + 25*x**2 + 12*x + 4),x)*b*
x**3 + 24*int(x**3/(x**6 + 8*x**5 + 22*x**4 + 28*x**3 + 25*x**2 + 12*x + 4
),x)*b*x**2 + 18*int(x**3/(x**6 + 8*x**5 + 22*x**4 + 28*x**3 + 25*x**2 + 1
2*x + 4),x)*b*x + 12*int(x**3/(x**6 + 8*x**5 + 22*x**4 + 28*x**3 + 25*x**2
 + 12*x + 4),x)*b - 16*int(x**3/(x**6 + 8*x**5 + 22*x**4 + 28*x**3 + 25*x*
*2 + 12*x + 4),x)*c*x**3 - 64*int(x**3/(x**6 + 8*x**5 + 22*x**4 + 28*x**3
+ 25*x**2 + 12*x + 4),x)*c*x**2 - 48*int(x**3/(x**6 + 8*x**5 + 22*x**4 + 2
8*x**3 + 25*x**2 + 12*x + 4),x)*c*x - 32*int(x**3/(x**6 + 8*x**5 + 22*x**4
 + 28*x**3 + 25*x**2 + 12*x + 4),x)*c + 32*int(1/(x**6 + 8*x**5 + 22*x**4
+ 28*x**3 + 25*x**2 + 12*x + 4),x)*a*x**3 + 128*int(1/(x**6 + 8*x**5 + 22*
x**4 + 28*x**3 + 25*x**2 + 12*x + 4),x)*a*x**2 + 96*int(1/(x**6 + 8*x**5 +
 22*x**4 + 28*x**3 + 25*x**2 + 12*x + 4),x)*a*x + 64*int(1/(x**6 + 8*x**5
+ 22*x**4 + 28*x**3 + 25*x**2 + 12*x + 4),x)*a - 18*int(1/(x**6 + 8*x**5 +
 22*x**4 + 28*x**3 + 25*x**2 + 12*x + 4),x)*b*x**3 - 72*int(1/(x**6 + 8*x*
*5 + 22*x**4 + 28*x**3 + 25*x**2 + 12*x + 4),x)*b*x**2 - 54*int(1/(x**6 +
8*x**5 + 22*x**4 + 28*x**3 + 25*x**2 + 12*x + 4),x)*b*x - 36*int(1/(x**6 +
 8*x**5 + 22*x**4 + 28*x**3 + 25*x**2 + 12*x + 4),x)*b + 16*int(1/(x**6 +
8*x**5 + 22*x**4 + 28*x**3 + 25*x**2 + 12*x + 4),x)*c*x**3 + 64*int(1/(x**
6 + 8*x**5 + 22*x**4 + 28*x**3 + 25*x**2 + 12*x + 4),x)*c*x**2 + 48*int(1/
(x**6 + 8*x**5 + 22*x**4 + 28*x**3 + 25*x**2 + 12*x + 4),x)*c*x + 32*in...
```

3.176 $\int (A+Bx+Cx^2)(2+3x+4x^2+x^3)^p\,dx$

Optimal result

Integrand size = 26, antiderivative size = 960

$$\int (A+Bx+Cx^2)\left(2+3x+4x^2+x^3\right)^p\,dx = \text{Too large to display}$$

output
```
C*(x^3+4*x^2+3*x+2)^(p+1)/(3*p+3)+1/27*(9*A-12*B-(7+(37-3*114^(1/2))^(2/3)
)*(3*B-8*C)/(37-3*114^(1/2))^(1/3)+23*C)*(4+7/(37-3*114^(1/2))^(1/3)+(37-3
*114^(1/2))^(1/3)+3*x)*(x^3+4*x^2+3*x+2)^p*AppellF1(p+1,-p,-p,2+p,2*I*(37-
3*114^(1/2))^(1/3)*(4+7/(37-3*114^(1/2))^(1/3)+(37-3*114^(1/2))^(1/3)+3*x)
/(21*I-7*3^(1/2)+(3*I+3^(1/2))*(37-3*114^(1/2))^(2/3)),2*I*(37-3*114^(1/2)
)^(1/3)*(4+7/(37-3*114^(1/2))^(1/3)+(37-3*114^(1/2))^(1/3)+3*x)/(21*I+7*3^
(1/2)+(3*I-3^(1/2))*(37-3*114^(1/2))^(2/3)))/(p+1)/((1-2*I*(37-3*114^(1/2)
)^(1/3)*(4+7/(37-3*114^(1/2))^(1/3)+(37-3*114^(1/2))^(1/3)+3*x)/(21*I+7*3^
(1/2)+(3*I-3^(1/2))*(37-3*114^(1/2))^(2/3)))^p)/((1-2*I*(37-3*114^(1/2))^(
1/3)*(4+7/(37-3*114^(1/2))^(1/3)+(37-3*114^(1/2))^(1/3)+3*x)/(21*I-7*3^(1/
2)+(3*I+3^(1/2))*(37-3*114^(1/2))^(2/3)))^p)+1/27*(3*B-8*C)*(4+7/(37-3*114
^(1/2))^(1/3)+(37-3*114^(1/2))^(1/3)+3*x)^2*(x^3+4*x^2+3*x+2)^p*AppellF1(2
+p,-p,-p,3+p,2*I*(37-3*114^(1/2))^(1/3)*(4+7/(37-3*114^(1/2))^(1/3)+(37-3*
114^(1/2))^(1/3)+3*x)/(21*I-7*3^(1/2)+(3*I+3^(1/2))*(37-3*114^(1/2))^(2/3)
),2*I*(37-3*114^(1/2))^(1/3)*(4+7/(37-3*114^(1/2))^(1/3)+(37-3*114^(1/2))^
(1/3)+3*x)/(21*I+7*3^(1/2)+(3*I-3^(1/2))*(37-3*114^(1/2))^(2/3)))/(2+p)/((
1-2*I*(37-3*114^(1/2))^(1/3)*(4+7/(37-3*114^(1/2))^(1/3)+(37-3*114^(1/2))^
(1/3)+3*x)/(21*I+7*3^(1/2)+(3*I-3^(1/2))*(37-3*114^(1/2))^(2/3)))^p)/((1-2
*I*(37-3*114^(1/2))^(1/3)*(4+7/(37-3*114^(1/2))^(1/3)+(37-3*114^(1/2))^(1/
3)+3*x)/(21*I-7*3^(1/2)+(3*I+3^(1/2))*(37-3*114^(1/2))^(2/3)))^p)
```

Mathematica [F]

$$\int (A+Bx+Cx^2)\left(2+3x+4x^2+x^3\right)^p\,dx = \int (A+Bx+Cx^2)\left(2+3x+4x^2+x^3\right)^p\,dx$$

input
```
Integrate[(A + B*x + C*x^2)*(2 + 3*x + 4*x^2 + x^3)^p,x]
```

output
```
Integrate[(A + B*x + C*x^2)*(2 + 3*x + 4*x^2 + x^3)^p, x]
```

Rubi [A] (warning: unable to verify)

Time = 1.89 (sec) , antiderivative size = 1284, normalized size of antiderivative = 1.34, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {2526, 2490, 2486, 27, 1269, 1179, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (x^3+4x^2+3x+2)^p\,(A+Bx+Cx^2)\,dx$$

$$\Big\downarrow \text{2526}$$

$$\frac{1}{3}\int (3(A-C)+(3B-8C)x)\,(x^3+4x^2+3x+2)^p\,dx+\frac{C(x^3+4x^2+3x+2)^{p+1}}{3(p+1)}$$

$$\Big\downarrow \text{2490}$$

$$\frac{1}{3}\int \left(\frac{1}{3}(9(A-C)-4(3B-8C))+(3B-8C)\left(x+\frac{4}{3}\right)\right)\left(\left(x+\frac{4}{3}\right)^3-\frac{7}{3}\left(x+\frac{4}{3}\right)+\frac{74}{27}\right)^p d\left(x+\frac{4}{3}\right)+\frac{C(x^3+4x^2+3x+2)^{p+1}}{3(p+1)}$$

$$\Big\downarrow \text{2486}$$

$$\frac{1}{3}\left(x+\frac{7+(37-3\sqrt{114})^{2/3}}{3\sqrt[3]{37-3\sqrt{114}}}+\frac{4}{3}\right)^{-p}\left(\left(x+\frac{4}{3}\right)^3-\frac{7}{3}\left(x+\frac{4}{3}\right)+\frac{74}{27}\right)^p\left(\left(x+\frac{4}{3}\right)^2-\frac{(7+(37-3\sqrt{114})^{2/3}}{3\sqrt[3]{37-3\sqrt{11}}}\right.$$
$$\frac{C(x^3+4x^2+3x+2)^{p+1}}{3(p+1)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{9}\left(x+\frac{7+(37-3\sqrt{114})^{2/3}}{3\sqrt[3]{37-3\sqrt{114}}}+\frac{4}{3}\right)^{-p}\left(\left(x+\frac{4}{3}\right)^3-\frac{7}{3}\left(x+\frac{4}{3}\right)+\frac{74}{27}\right)^p\left(\left(x+\frac{4}{3}\right)^2-\frac{(7+(37-3\sqrt{114})^{2/3}}{3\sqrt[3]{37-3\sqrt{11}}}\right.$$
$$\frac{C(x^3+4x^2+3x+2)^{p+1}}{3(p+1)}$$

$$\Big\downarrow \text{1269}$$

$$\frac{1}{9}\left(x+\frac{7+\left(37-3\sqrt{114}\right)^{2/3}}{3\sqrt[3]{37-3\sqrt{114}}}+\frac{4}{3}\right)^{-p}\left(\left(x+\frac{4}{3}\right)^3-\frac{7}{3}\left(x+\frac{4}{3}\right)+\frac{74}{27}\right)^p\left(\left(x+\frac{4}{3}\right)^2-\frac{\left(7+\left(37-3\sqrt{114}\right)^{2/3}\right.}{3\sqrt[3]{37-3\sqrt{11}}}\right.$$

$$\frac{C\left(x^3+4x^2+3x+2\right)^{p+1}}{3(p+1)}$$

$$\Big\downarrow \; 1179$$

$$\frac{1}{9}\left(x+\frac{7+\left(37-3\sqrt{114}\right)^{2/3}}{3\sqrt[3]{37-3\sqrt{114}}}+\frac{4}{3}\right)^{-p}\left(\left(x+\frac{4}{3}\right)^3-\frac{7}{3}\left(x+\frac{4}{3}\right)+\frac{74}{27}\right)^p\left(\left(9A-12B-\frac{\left(7+\left(37-3\sqrt{114}\right)^{2}\right.}{\sqrt[3]{37-3\sqrt{}}}\right.\right.$$

$$\frac{C\left(x^3+4x^2+3x+2\right)^{p+1}}{3(p+1)}$$

$$\Big\downarrow \; 150$$

$$\frac{1}{9}\left(x+\frac{7+\left(37-3\sqrt{114}\right)^{2/3}}{3\sqrt[3]{37-3\sqrt{114}}}+\frac{4}{3}\right)^{-p}\left(\left(x+\frac{4}{3}\right)^3-\frac{7}{3}\left(x+\frac{4}{3}\right)+\frac{74}{27}\right)^p\left(\frac{\left(9A-12B-\frac{\left(7+\left(37-3\sqrt{114}\right)^{2/3}\right)(3}{\sqrt[3]{37-3\sqrt{114}}}\right.}{}\right.$$

$$\frac{C\left(x^3+4x^2+3x+2\right)^{p+1}}{3(p+1)}$$

input
```
Int[(A + B*x + C*x^2)*(2 + 3*x + 4*x^2 + x^3)^p,x]
```

output
```
(C*(2 + 3*x + 4*x^2 + x^3)^(1 + p))/(3*(1 + p)) + ((74/27 - (7*(4/3 + x))/
3 + (4/3 + x)^3)^p*(((9*A - 12*B - ((7 + (37 - 3*Sqrt[114])^(2/3))*(3*B -
8*C))/(37 - 3*Sqrt[114])^(1/3) + 23*C)*(4/3 + (7 + (37 - 3*Sqrt[114])^(2/3
))/(3*(37 - 3*Sqrt[114])^(1/3)) + x)^(1 + p)*((-7 + 49/(37 - 3*Sqrt[114])^
(2/3) + (37 - 3*Sqrt[114])^(2/3))/9 - ((7 + (37 - 3*Sqrt[114])^(2/3))*(4/3
 + x))/(3*(37 - 3*Sqrt[114])^(1/3)) + (4/3 + x)^2)^p*AppellF1[1 + p, -p, -
p, 2 + p, ((6*I)*(37 - 3*Sqrt[114])^(1/3)*(4/3 + (7 + (37 - 3*Sqrt[114])^(
2/3))/(3*(37 - 3*Sqrt[114])^(1/3)) + x))/(7*(3*I + Sqrt[3]) + (3*I - Sqrt[
3])*(37 - 3*Sqrt[114])^(2/3)), ((6*I)*(37 - 3*Sqrt[114])^(1/3)*(4/3 + (7 +
 (37 - 3*Sqrt[114])^(2/3))/(3*(37 - 3*Sqrt[114])^(1/3)) + x))/(7*(3*I - Sq
rt[3]) + (3*I + Sqrt[3])*(37 - 3*Sqrt[114])^(2/3))])/((1 + p)*(1 - ((2*I)*
(37 - 3*Sqrt[114])^(1/3)*((7 + (37 - 3*Sqrt[114])^(2/3))/(37 - 3*Sqrt[114]
)^(1/3) + 3*(4/3 + x)))/(7*(3*I + Sqrt[3]) + (3*I - Sqrt[3])*(37 - 3*Sqrt[
114])^(2/3)))^p*(1 - ((2*I)*(37 - 3*Sqrt[114])^(1/3)*((7 + (37 - 3*Sqrt[11
4])^(2/3))/(37 - 3*Sqrt[114])^(1/3) + 3*(4/3 + x)))/(7*(3*I - Sqrt[3]) + (
3*I + Sqrt[3])*(37 - 3*Sqrt[114])^(2/3)))^p) + (3*(3*B - 8*C)*(4/3 + (7 +
(37 - 3*Sqrt[114])^(2/3))/(3*(37 - 3*Sqrt[114])^(1/3)) + x)^(2 + p)*((-7 +
 49/(37 - 3*Sqrt[114])^(2/3) + (37 - 3*Sqrt[114])^(2/3))/9 - ((7 + (37 - 3
*Sqrt[114])^(2/3))*(4/3 + x))/(3*(37 - 3*Sqrt[114])^(1/3)) + (4/3 + x)^2)^
p*AppellF1[2 + p, -p, -p, 3 + p, ((6*I)*(37 - 3*Sqrt[114])^(1/3)*(4/3 +...
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 1179
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(a + b*x + c*x^2)^p/(e*(1 - (
d + e*x)/(d - e*((b - q)/(2*c))))^p*(1 - (d + e*x)/(d - e*((b + q)/(2*c))))
^p)   Subst[Int[x^m*Simp[1 - x/(d - e*((b - q)/(2*c))), x]^p*Simp[1 - x/(d
 - e*((b + q)/(2*c))), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, b, c, d, e, m
, p}, x]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2486
```
Int[((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (d_.)*(x_)^3)^(p_), x_S
ymbol] :> With[{r = Rt[-9*a*d^2 + Sqrt[3]*d*Sqrt[4*b^3*d + 27*a^2*d^2], 3]}
, Simp[(a + b*x + d*x^3)^p/(Simp[18^(1/3)*b*(d/(3*r)) - r/18^(1/3) + d*x, x
]^p*Simp[b*(d/3) + 12^(1/3)*b^2*(d^2/(3*r^2)) + r^2/(3*12^(1/3)) - d*(2^(1/
3)*b*(d/(3^(1/3)*r)) - r/18^(1/3))*x + d^2*x^2, x]^p)   Int[(e + f*x)^m*Sim
p[18^(1/3)*b*(d/(3*r)) - r/18^(1/3) + d*x, x]^p*Simp[b*(d/3) + 12^(1/3)*b^2
*(d^2/(3*r^2)) + r^2/(3*12^(1/3)) - d*(2^(1/3)*b*(d/(3^(1/3)*r)) - r/18^(1/
3))*x + d^2*x^2, x]^p, x], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && NeQ[4*
b^3 + 27*a^2*d, 0] &&  !IntegerQ[p]
```

rule 2490
```
Int[(P3_)^(p_.)*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] :> With[{a = Coeff[P3
, x, 0], b = Coeff[P3, x, 1], c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Su
bst[Int[((3*d*e - c*f)/(3*d) + f*x)^m*Simp[(2*c^3 - 9*b*c*d + 27*a*d^2)/(27
*d^2) - (c^2 - 3*b*d)*(x/(3*d)) + d*x^3, x]^p, x], x, x + c/(3*d)] /; NeQ[c
, 0]] /; FreeQ[{e, f, m, p}, x] && PolyQ[P3, x, 3]
```

rule 2526
```
Int[(Pm_)*(Qn_)^(p_), x_Symbol] :> With[{m = Expon[Pm, x], n = Expon[Qn, x]
}, Simp[Coeff[Pm, x, m]*(Qn^(p + 1)/(n*(p + 1)*Coeff[Qn, x, n])), x] + Simp
[1/(n*Coeff[Qn, x, n])   Int[ExpandToSum[n*Coeff[Qn, x, n]*Pm - Coeff[Pm, x
, m]*D[Qn, x], x]*Qn^p, x], x] /; EqQ[m, n - 1]] /; FreeQ[p, x] && PolyQ[Pm
, x] && PolyQ[Qn, x] && NeQ[p, -1]
```

Maple [F]

$$\int \left(C\,x^{2}+Bx+A\right)\left(x^{3}+4x^{2}+3x+2\right)^{p}dx$$

input
```
int((C*x^2+B*x+A)*(x^3+4*x^2+3*x+2)^p,x)
```

output
```
int((C*x^2+B*x+A)*(x^3+4*x^2+3*x+2)^p,x)
```

Fricas [F]

$$\int \left(A+Bx+Cx^2\right)\left(2+3x+4x^2+x^3\right)^p\,dx = \int \left(Cx^2+Bx+A\right)\left(x^3+4\,x^2+3\,x+2\right)^p\,dx$$

input
```
integrate((C*x^2+B*x+A)*(x^3+4*x^2+3*x+2)^p,x, algorithm="fricas")
```

output
```
integral((C*x^2 + B*x + A)*(x^3 + 4*x^2 + 3*x + 2)^p, x)
```

Sympy [F]

$$\int \left(A+Bx+Cx^2\right)\left(2+3x+4x^2+x^3\right)^p\,dx = \int \left(A+Bx+Cx^2\right)\left(x^3+4x^2+3x+2\right)^p\,dx$$

input
```
integrate((C*x**2+B*x+A)*(x**3+4*x**2+3*x+2)**p,x)
```

output
```
Integral((A + B*x + C*x**2)*(x**3 + 4*x**2 + 3*x + 2)**p, x)
```

Maxima [F]

$$\int \left(A+Bx+Cx^2\right)\left(2+3x+4x^2+x^3\right)^p\,dx = \int \left(Cx^2+Bx+A\right)\left(x^3+4\,x^2+3\,x+2\right)^p\,dx$$

input
```
integrate((C*x^2+B*x+A)*(x^3+4*x^2+3*x+2)^p,x, algorithm="maxima")
```

output
```
integrate((C*x^2 + B*x + A)*(x^3 + 4*x^2 + 3*x + 2)^p, x)
```

Giac [F]

$$\int (A+Bx+Cx^2) \left(2+3x+4x^2+x^3\right)^p \, dx = \int (Cx^2+Bx+A)(x^3+4\,x^2+3\,x+2)^p \, dx$$

input
```
integrate((C*x^2+B*x+A)*(x^3+4*x^2+3*x+2)^p,x, algorithm="giac")
```

output
```
integrate((C*x^2 + B*x + A)*(x^3 + 4*x^2 + 3*x + 2)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (A+Bx+Cx^2) \left(2+3x+4x^2+x^3\right)^p \, dx = \int \left(C\,x^2+B\,x+A\right) \left(x^3+4\,x^2+3\,x+2\right)^p \, dx$$

input
```
int((A + B*x + C*x^2)*(3*x + 4*x^2 + x^3 + 2)^p,x)
```

output
```
int((A + B*x + C*x^2)*(3*x + 4*x^2 + x^3 + 2)^p, x)
```

Reduce [F]

$$\int (A+Bx+Cx^2) \left(2+3x+4x^2+x^3\right)^p \, dx = \text{too large to display}$$

input
```
int((C*x^2+B*x+A)*(x^3+4*x^2+3*x+2)^p,x)
```

output

```
(36*(x**3 + 4*x**2 + 3*x + 2)**p*a*p**2*x + 27*(x**3 + 4*x**2 + 3*x + 2)**
p*a*p**2 + 60*(x**3 + 4*x**2 + 3*x + 2)**p*a*p*x + 45*(x**3 + 4*x**2 + 3*x
 + 2)**p*a*p + 24*(x**3 + 4*x**2 + 3*x + 2)**p*a*x + 18*(x**3 + 4*x**2 + 3
*x + 2)**p*a + 36*(x**3 + 4*x**2 + 3*x + 2)**p*b*p**2*x**2 + 48*(x**3 + 4*
x**2 + 3*x + 2)**p*b*p**2*x + 9*(x**3 + 4*x**2 + 3*x + 2)**p*b*p**2 + 48*(
x**3 + 4*x**2 + 3*x + 2)**p*b*p*x**2 + 48*(x**3 + 4*x**2 + 3*x + 2)**p*b*p
*x + 12*(x**3 + 4*x**2 + 3*x + 2)**p*b*x**2 - 9*(x**3 + 4*x**2 + 3*x + 2)*
*p*b + 36*(x**3 + 4*x**2 + 3*x + 2)**p*c*p**2*x**3 + 48*(x**3 + 4*x**2 + 3
*x + 2)**p*c*p**2*x**2 - 56*(x**3 + 4*x**2 + 3*x + 2)**p*c*p**2*x + 21*(x*
*3 + 4*x**2 + 3*x + 2)**p*c*p**2 + 36*(x**3 + 4*x**2 + 3*x + 2)**p*c*p*x**
3 + 16*(x**3 + 4*x**2 + 3*x + 2)**p*c*p*x**2 - 80*(x**3 + 4*x**2 + 3*x + 2
)**p*c*p*x + 27*(x**3 + 4*x**2 + 3*x + 2)**p*c*p + 8*(x**3 + 4*x**2 + 3*x
+ 2)**p*c*x**3 + 22*(x**3 + 4*x**2 + 3*x + 2)**p*c + 1215*int((x**3 + 4*x*
*2 + 3*x + 2)**p/(9*p**2*x**3 + 36*p**2*x**2 + 27*p**2*x + 18*p**2 + 9*p*x
**3 + 36*p*x**2 + 27*p*x + 18*p + 2*x**3 + 8*x**2 + 6*x + 4),x)*a*p**5 + 3
240*int((x**3 + 4*x**2 + 3*x + 2)**p/(9*p**2*x**3 + 36*p**2*x**2 + 27*p**2
*x + 18*p**2 + 9*p*x**3 + 36*p*x**2 + 27*p*x + 18*p + 2*x**3 + 8*x**2 + 6*
x + 4),x)*a*p**4 + 3105*int((x**3 + 4*x**2 + 3*x + 2)**p/(9*p**2*x**3 + 36
*p**2*x**2 + 27*p**2*x + 18*p**2 + 9*p*x**3 + 36*p*x**2 + 27*p*x + 18*p +
2*x**3 + 8*x**2 + 6*x + 4),x)*a*p**3 + 1260*int((x**3 + 4*x**2 + 3*x + ...
```

3.177 $\int \frac{18-2x-4x^2}{-6+x+4x^2+x^3} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 19

$$\int \frac{18-2x-4x^2}{-6+x+4x^2+x^3} \, dx = \log(1-x) - 2\log(2+x) - 3\log(3+x)$$

output
```
ln(1-x)-2*ln(2+x)-3*ln(3+x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.32

$$\int \frac{18-2x-4x^2}{-6+x+4x^2+x^3} \, dx = -2\left(-\frac{1}{2}\log(1-x) + \log(2+x) + \frac{3}{2}\log(3+x)\right)$$

input
```
Integrate[(18 - 2*x - 4*x^2)/(-6 + x + 4*x^2 + x^3),x]
```

output
```
-2*(-1/2*Log[1 - x] + Log[2 + x] + (3*Log[3 + x])/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2462, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{-4x^2-2x+18}{x^3+4x^2+x-6} \, dx$$
$$\Big\downarrow \text{2462}$$
$$\int \left(-\frac{2}{x+2}-\frac{3}{x+3}+\frac{1}{x-1}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\log(1-x)-2\log(x+2)-3\log(x+3)$$

input
```
Int[(18 - 2*x - 4*x^2)/(-6 + x + 4*x^2 + x^3),x]
```

output
```
Log[1 - x] - 2*Log[2 + x] - 3*Log[3 + x]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

method	result	size
default	$\ln(x-1)-3\ln(3+x)-2\ln(2+x)$	18
norman	$\ln(x-1)-3\ln(3+x)-2\ln(2+x)$	18
risch	$\ln(x-1)-3\ln(3+x)-2\ln(2+x)$	18
parallelrisch	$\ln(x-1)-3\ln(3+x)-2\ln(2+x)$	18

input
```
int((-4*x^2-2*x+18)/(x^3+4*x^2+x-6),x,method=_RETURNVERBOSE)
```

output
```
ln(x-1)-3*ln(3+x)-2*ln(2+x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{18-2x-4x^2}{-6+x+4x^2+x^3}\,dx = -3\,\log(x+3)-2\,\log(x+2)+\log(x-1)$$

input
```
integrate((-4*x^2-2*x+18)/(x^3+4*x^2+x-6),x, algorithm="fricas")
```

output
```
-3*log(x + 3) - 2*log(x + 2) + log(x - 1)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{18-2x-4x^2}{-6+x+4x^2+x^3}\,dx = \log(x-1)-2\log(x+2)-3\log(x+3)$$

input
```
integrate((-4*x**2-2*x+18)/(x**3+4*x**2+x-6),x)
```

output
```
log(x - 1) - 2*log(x + 2) - 3*log(x + 3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{18-2x-4x^2}{-6+x+4x^2+x^3}\,dx = -3\,\log\left(x+3\right)-2\,\log\left(x+2\right)+\log\left(x-1\right)$$

input
```
integrate((-4*x^2-2*x+18)/(x^3+4*x^2+x-6),x, algorithm="maxima")
```

output
```
-3*log(x + 3) - 2*log(x + 2) + log(x - 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{18-2x-4x^2}{-6+x+4x^2+x^3}\,dx = -3\,\log\left(|x+3|\right)-2\,\log\left(|x+2|\right)+\log\left(|x-1|\right)$$

input
```
integrate((-4*x^2-2*x+18)/(x^3+4*x^2+x-6),x, algorithm="giac")
```

output
```
-3*log(abs(x + 3)) - 2*log(abs(x + 2)) + log(abs(x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{18-2x-4x^2}{-6+x+4x^2+x^3}\,dx = \ln\left(x-1\right)-2\,\ln\left(x+2\right)-3\,\ln\left(x+3\right)$$

input
```
int(-(2*x + 4*x^2 - 18)/(x + 4*x^2 + x^3 - 6),x)
```

output
```
log(x - 1) - 2*log(x + 2) - 3*log(x + 3)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{18-2x-4x^2}{-6+x+4x^2+x^3}\,dx = \log(x-1)-3\log(x+3)-2\log(x+2)$$

input
```
int((-4*x^2-2*x+18)/(x^3+4*x^2+x-6),x)
```

output
```
log(x - 1) - 3*log(x + 3) - 2*log(x + 2)
```

3.178 $\int \frac{be-af+2cex+(3de+cf)x^2+2dfx^3}{(a+bx+cx^2+dx^3)^2}\,dx$

Optimal result

Integrand size = 50, antiderivative size = 42

$$\int \frac{be-af+2cex+(3de+cf)x^2+2dfx^3}{(a+bx+cx^2+dx^3)^2}\,dx = -\frac{e}{a+bx+cx^2+dx^3}-\frac{fx}{a+bx+cx^2+dx^3}$$

output
```
-e/(d*x^3+c*x^2+b*x+a)-f*x/(d*x^3+c*x^2+b*x+a)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.52

$$\int \frac{be-af+2cex+(3de+cf)x^2+2dfx^3}{(a+bx+cx^2+dx^3)^2}\,dx = -\frac{e+fx}{a+x(b+x(c+dx))}$$

input
```
Integrate[(b*e - a*f + 2*c*e*x + (3*d*e + c*f)*x^2 + 2*d*f*x^3)/(a + b*x +
 c*x^2 + d*x^3)^2,x]
```

output
```
-((e + f*x)/(a + x*(b + x*(c + d*x))))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.060$, Rules used = {2527, 27, 2021}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{-af+be+x^2(cf+3de)+2cex+2dfx^3}{(a+bx+cx^2+dx^3)^2}dx$$

$$\Big\downarrow \text{2527}$$

$$-\frac{\int -\frac{2(3d^2ex^2+2cdex+bde)}{(dx^3+cx^2+bx+a)^2}dx}{2d}-\frac{fx}{a+bx+cx^2+dx^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{3d^2ex^2+2cdex+bde}{(dx^3+cx^2+bx+a)^2}dx}{d}-\frac{fx}{a+bx+cx^2+dx^3}$$

$$\Big\downarrow \text{2021}$$

$$-\frac{e}{a+bx+cx^2+dx^3}-\frac{fx}{a+bx+cx^2+dx^3}$$

input
```
Int[(b*e - a*f + 2*c*e*x + (3*d*e + c*f)*x^2 + 2*d*f*x^3)/(a + b*x + c*x^2
 + d*x^3)^2,x]
```

output
```
-(e/(a + b*x + c*x^2 + d*x^3)) - (f*x)/(a + b*x + c*x^2 + d*x^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2021
```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

rule 2527
```
Int[(Pm_)*(Qn_)^(p_.), x_Symbol] :> With[{m = Expon[Pm, x], n = Expon[Qn, x
]}, Simp[Coeff[Pm, x, m]*x^(m - n + 1)*(Qn^(p + 1)/((m + n*p + 1)*Coeff[Qn,
 x, n])), x] + Simp[1/((m + n*p + 1)*Coeff[Qn, x, n])   Int[ExpandToSum[(m
+ n*p + 1)*Coeff[Qn, x, n]*Pm - Coeff[Pm, x, m]*x^(m - n)*((m - n + 1)*Qn +
 (p + 1)*x*D[Qn, x]), x]*Qn^p, x], x] /; LtQ[1, n, m + 1] && m + n*p + 1 <
0] /; FreeQ[p, x] && PolyQ[Pm, x] && PolyQ[Qn, x] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.60

method	result	size
gosper	$-\frac{fx+e}{d\,x^{3}+c\,x^{2}+bx+a}$	25
default	$-\frac{fx+e}{d\,x^{3}+c\,x^{2}+bx+a}$	25
norman	$\frac{-fx-e}{d\,x^{3}+c\,x^{2}+bx+a}$	27
risch	$\frac{-fx-e}{d\,x^{3}+c\,x^{2}+bx+a}$	27
parallelrisch	$\frac{-cfx-ce}{c\left(d\,x^{3}+c\,x^{2}+bx+a\right)}$	32
orering	$\frac{(fx+e)\left(eb-af+2cex+(cf+3de)x^{2}+2df\,x^{3}\right)}{\left(d\,x^{3}+c\,x^{2}+bx+a\right)\left(-2df\,x^{3}-cf\,x^{2}-3de\,x^{2}-2cex+af-eb\right)}$	92

input
```
int((e*b-a*f+2*c*e*x+(c*f+3*d*e)*x^2+2*d*f*x^3)/(d*x^3+c*x^2+b*x+a)^2,x,me
thod=_RETURNVERBOSE)
```

output
```
-(f*x+e)/(d*x^3+c*x^2+b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.57

$$\int \frac{be - af + 2cex + (3de + cf)x^2 + 2dfx^3}{(a + bx + cx^2 + dx^3)^2}\,dx = -\frac{fx + e}{dx^3 + cx^2 + bx + a}$$

input
```
integrate((b*e-a*f+2*c*e*x+(c*f+3*d*e)*x^2+2*d*f*x^3)/(d*x^3+c*x^2+b*x+a)^
2,x, algorithm="fricas")
```

output
```
-(f*x + e)/(d*x^3 + c*x^2 + b*x + a)
```

Sympy [A] (verification not implemented)

Time = 13.48 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.48

$$\int \frac{be - af + 2cex + (3de + cf)x^2 + 2dfx^3}{(a + bx + cx^2 + dx^3)^2}\,dx = \frac{-e - fx}{a + bx + cx^2 + dx^3}$$

input
```
integrate((b*e-a*f+2*c*e*x+(c*f+3*d*e)*x**2+2*d*f*x**3)/(d*x**3+c*x**2+b*x
+a)**2,x)
```

output
```
(-e - f*x)/(a + b*x + c*x**2 + d*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.57

$$\int \frac{be - af + 2cex + (3de + cf)x^2 + 2dfx^3}{(a + bx + cx^2 + dx^3)^2}\,dx = -\frac{fx + e}{dx^3 + cx^2 + bx + a}$$

input
```
integrate((b*e-a*f+2*c*e*x+(c*f+3*d*e)*x^2+2*d*f*x^3)/(d*x^3+c*x^2+b*x+a)^
2,x, algorithm="maxima")
```

output
```
-(f*x + e)/(d*x^3 + c*x^2 + b*x + a)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.57

$$\int \frac{be-af+2cex+(3de+cf)x^2+2dfx^3}{(a+bx+cx^2+dx^3)^2}\,dx=-\frac{fx+e}{dx^3+cx^2+bx+a}$$

input
```
integrate((b*e-a*f+2*c*e*x+(c*f+3*d*e)*x^2+2*d*f*x^3)/(d*x^3+c*x^2+b*x+a)^
2,x, algorithm="giac")
```

output
```
-(f*x + e)/(d*x^3 + c*x^2 + b*x + a)
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.57

$$\int \frac{be-af+2cex+(3de+cf)x^2+2dfx^3}{(a+bx+cx^2+dx^3)^2}\,dx=-\frac{e+f\,x}{d\,x^3+c\,x^2+b\,x+a}$$

input
```
int((b*e - a*f + x^2*(c*f + 3*d*e) + 2*d*f*x^3 + 2*c*e*x)/(a + b*x + c*x^2
 + d*x^3)^2,x)
```

output
```
-(e + f*x)/(a + b*x + c*x^2 + d*x^3)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.98

$$\int \frac{be-af+2cex+(3de+cf)x^2+2dfx^3}{(a+bx+cx^2+dx^3)^2}\,dx=\frac{df\,x^3+cf\,x^2+af-be}{b\,(d\,x^3+c\,x^2+bx+a)}$$

input
```
int((b*e-a*f+2*c*e*x+(c*f+3*d*e)*x^2+2*d*f*x^3)/(d*x^3+c*x^2+b*x+a)^2,x)
```

output
```
(a*f - b*e + c*f*x**2 + d*f*x**3)/(b*(a + b*x + c*x**2 + d*x**3))
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                     return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                   convert(leaf_count_result,string)," vs. $2 (",
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                  print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False

   except AttributeError as error:
      return False

def expnType(expn):
   debug=False
   if debug:
      print("expn=",expn,"type(expn)=",type(expn))

   if is_atom(expn):
     return 1
   elif isinstance(expn,list):
      return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
         return 1
      else:
         return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
   elif isinstance(expn,Pow):   #type(expn,'`^`')
      if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
         return expnType(expn.args[0])   #ExpnType(op(1,expn))
      elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
         if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
         else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
      else:
         return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),Expr
   elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
else:
    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
            else: #result contains complex but optimal is not
                grade = "C"
                grade_annotation ="Result contains complex when optimal does not."
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =""
            else:
                grade = "B"
                grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
    else:
        grade = "C"
        grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType


#print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
           'elliptic_pi','exp_integral_e','log_integral',
           'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
           'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
         print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
```

```
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file