Computer Algebra
Independent Integration Tests

Summer 2024

1-Algebraic-functions/1.4-Quartic/144-1.4.1

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May 17, 2024 Compiled on May 17, 2024 at 9:05pm

Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**94**]. This is test number [144].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Mathematica	100.00 (94)	0.00 (0)
Maple	100.00 (94)	0.00 (0)
Rubi	98.94 (93)	1.06 (1)
Fricas	73.40 (69)	26.60 (25)
Mupad	65.96 (62)	34.04 (32)
Sympy	59.57 (56)	40.43 (38)
Giac	57.45 (54)	42.55 (40)
Reduce	44.68 (42)	55.32 (52)
Maxima	41.49 (39)	58.51 (55)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

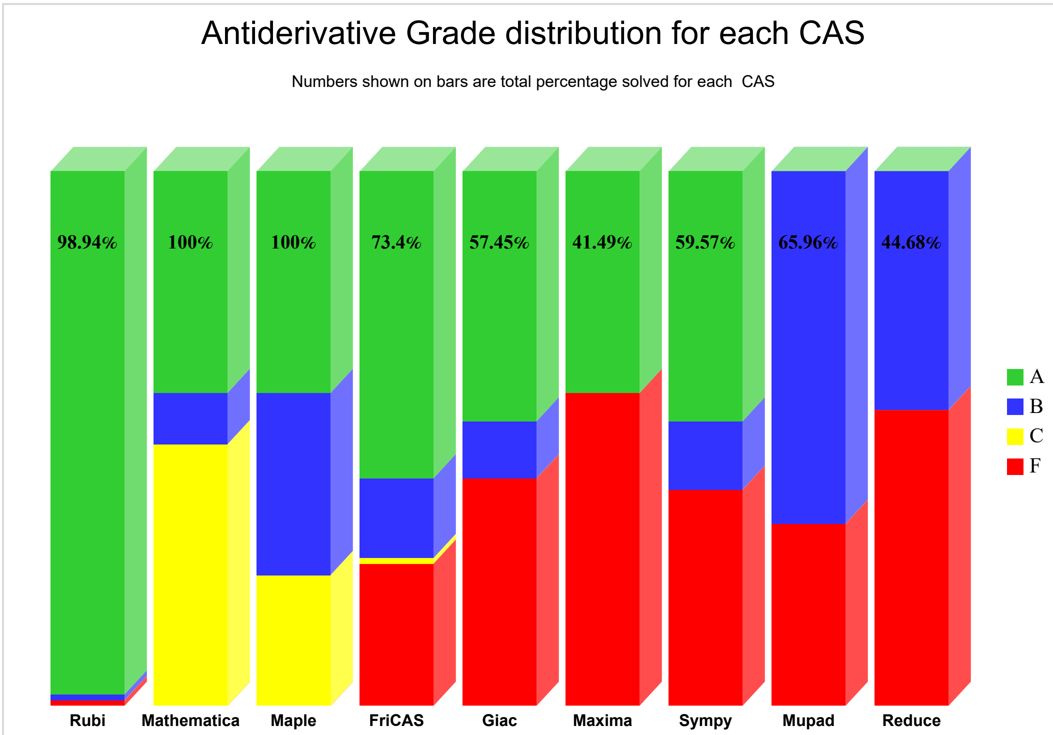
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

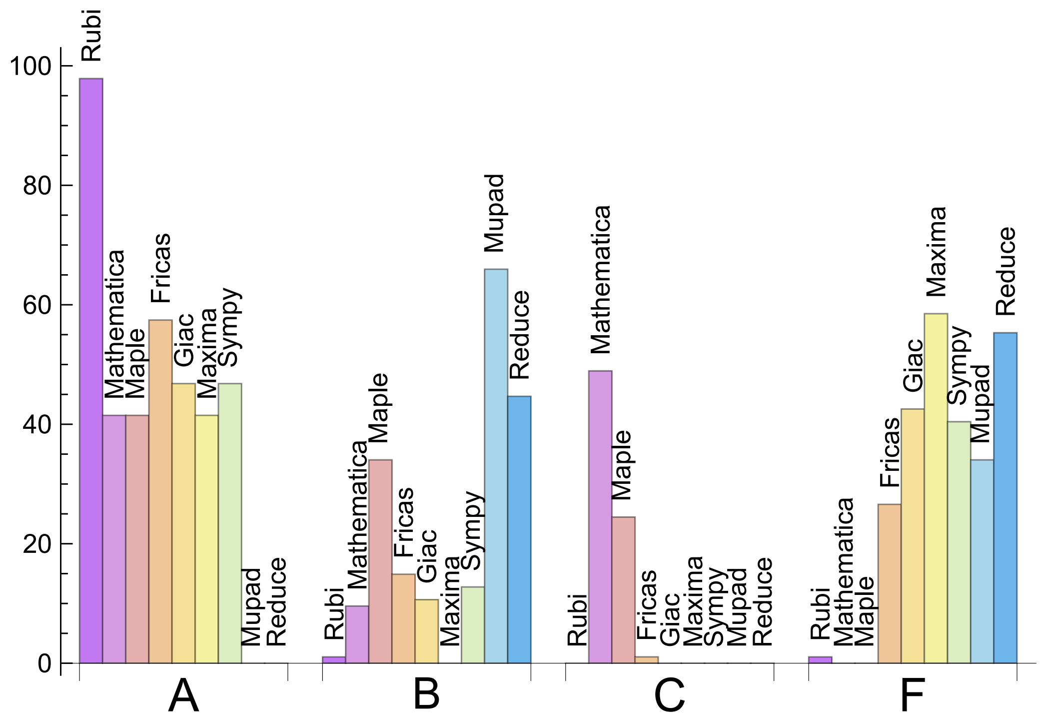
System	% A grade	% B grade	% C grade	% F grade
Rubi	97.872	1.064	0.000	1.064
Fricas	57.447	14.894	1.064	26.596
Giac	46.809	10.638	0.000	42.553
Sympy	46.809	12.766	0.000	40.426
Mathematica	41.489	9.574	48.936	0.000
Maple	41.489	34.043	24.468	0.000
Maxima	41.489	0.000	0.000	58.511
Mupad	0.000	65.957	0.000	34.043
Reduce	0.000	44.681	0.000	55.319

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Mathematica	0	0.00	0.00	0.00
Maple	0	0.00	0.00	0.00
Rubi	1	100.00	0.00	0.00
Fricas	25	88.00	12.00	0.00
Mupad	32	0.00	100.00	0.00
Sympy	38	86.84	13.16	0.00
Giac	40	100.00	0.00	0.00
Reduce	52	100.00	0.00	0.00
Maxima	55	100.00	0.00	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.03
Reduce	0.16
Fricas	0.36
Giac	0.48
Rubi	0.62
Maple	0.95
Sympy	1.99
Mathematica	6.13
Mupad	8.60

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Maxima	96.56	0.93	60.00	0.85
Rubi	262.18	1.12	144.00	1.00
Reduce	300.50	1.81	63.00	0.88
Sympy	514.38	2.63	94.50	0.97
Giac	673.31	3.57	89.50	0.86
Maple	1018.03	3.65	105.50	0.90
Mathematica	1141.89	3.20	116.50	1.00
Mupad	1228.68	3.27	90.00	0.85
Fricas	55474.94	137.20	96.00	0.94

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

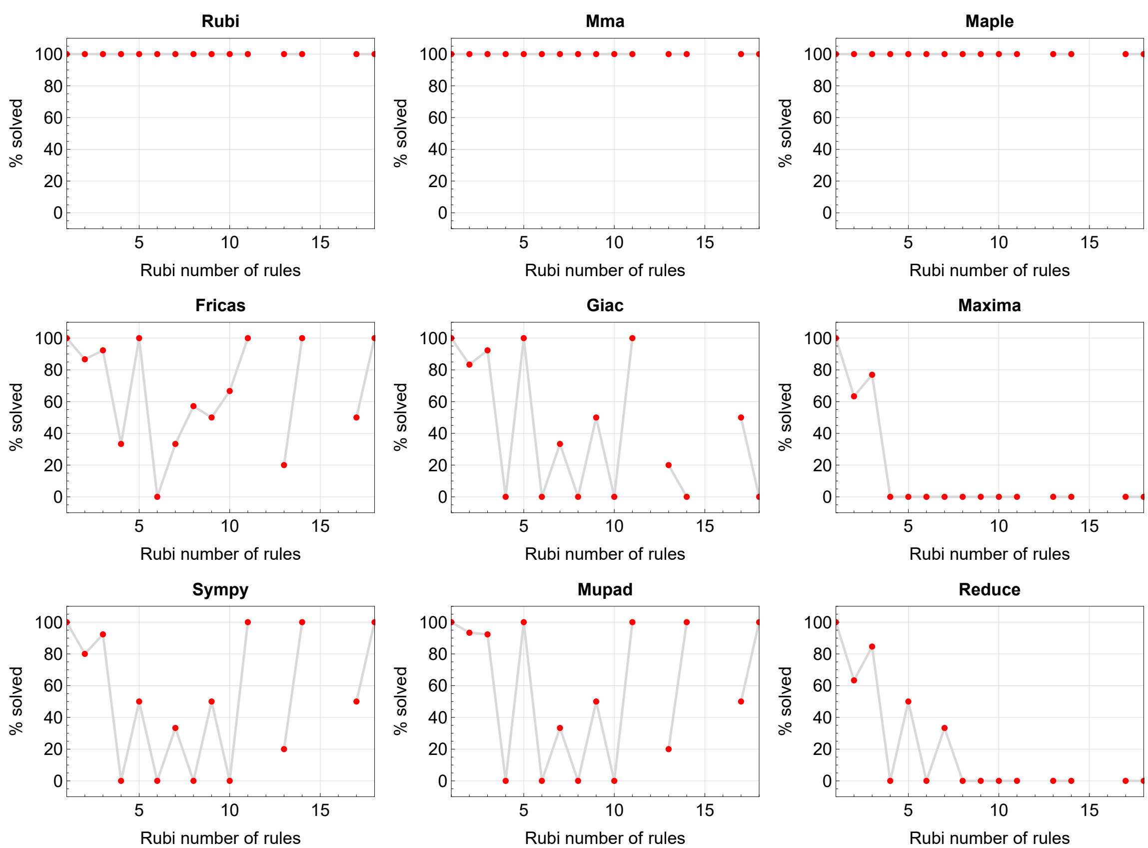


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

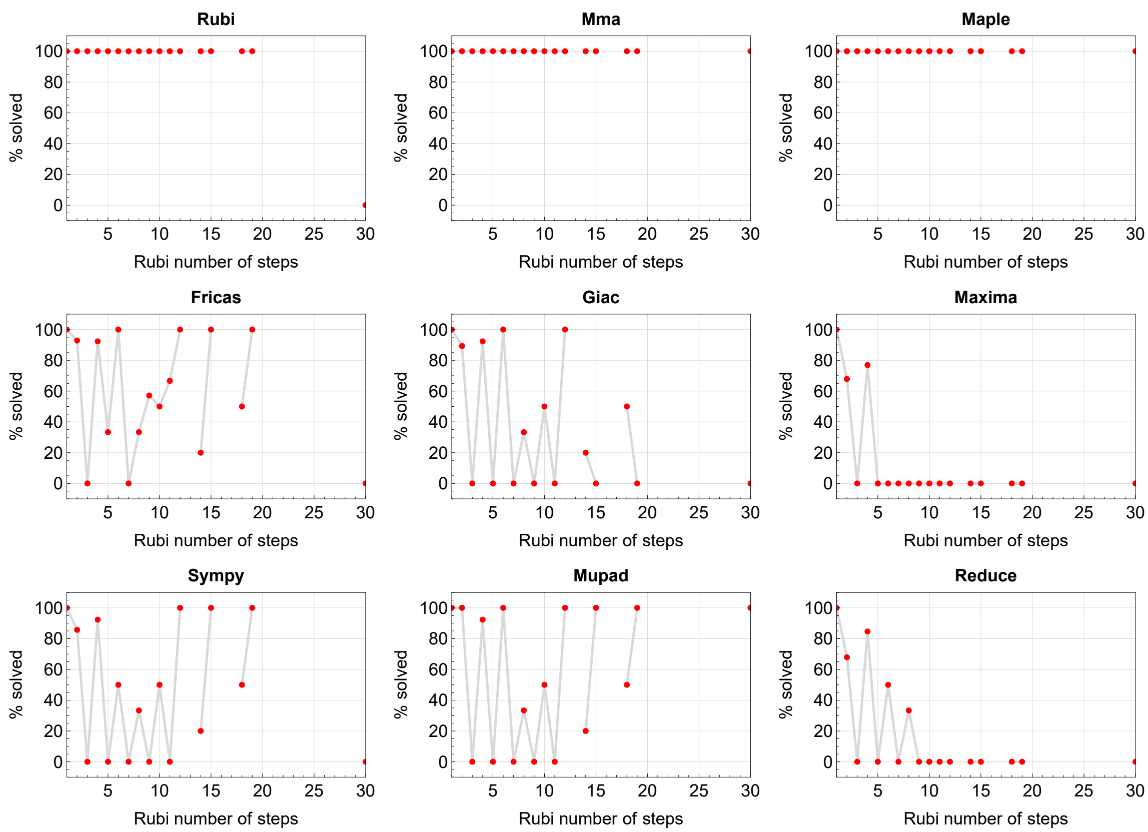


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

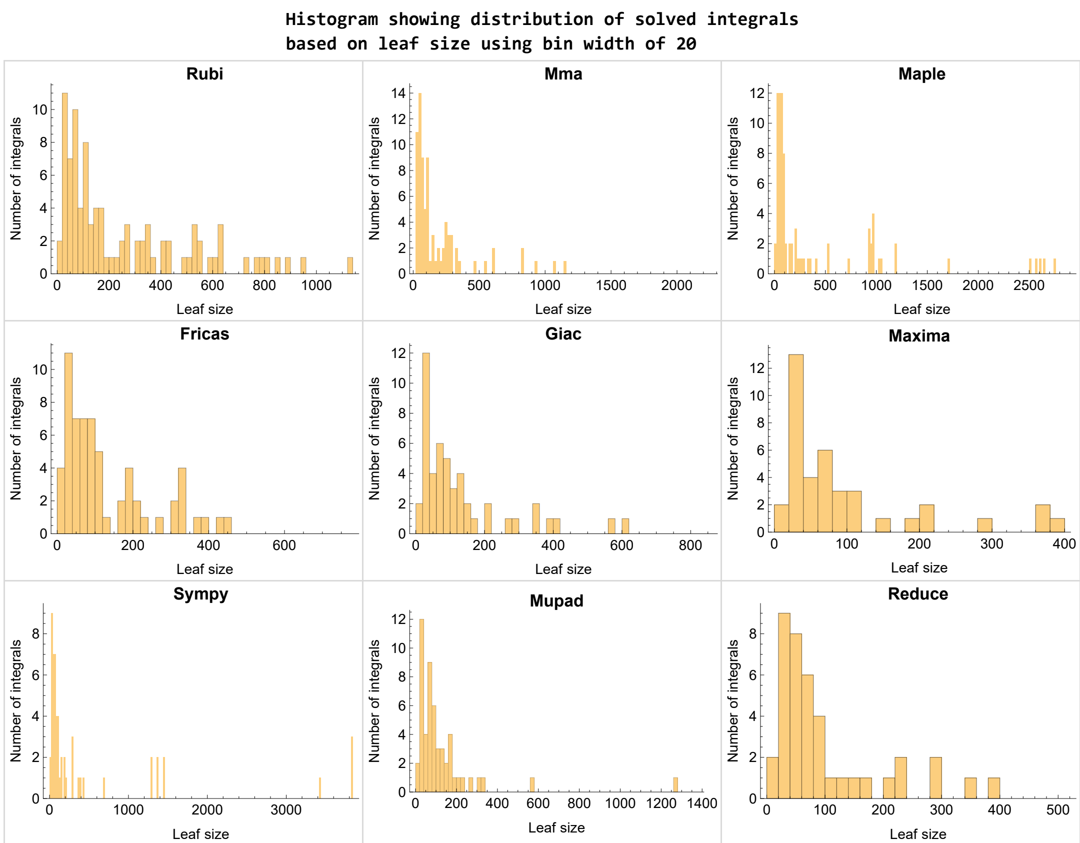


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

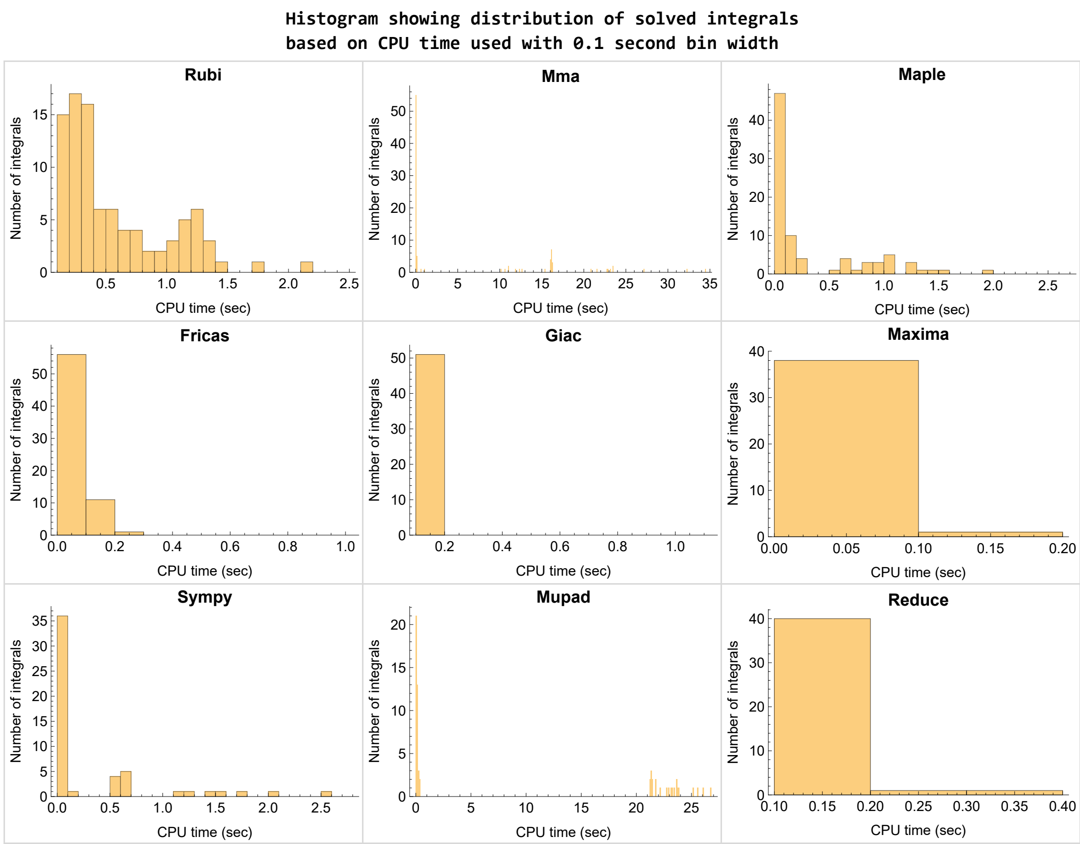


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

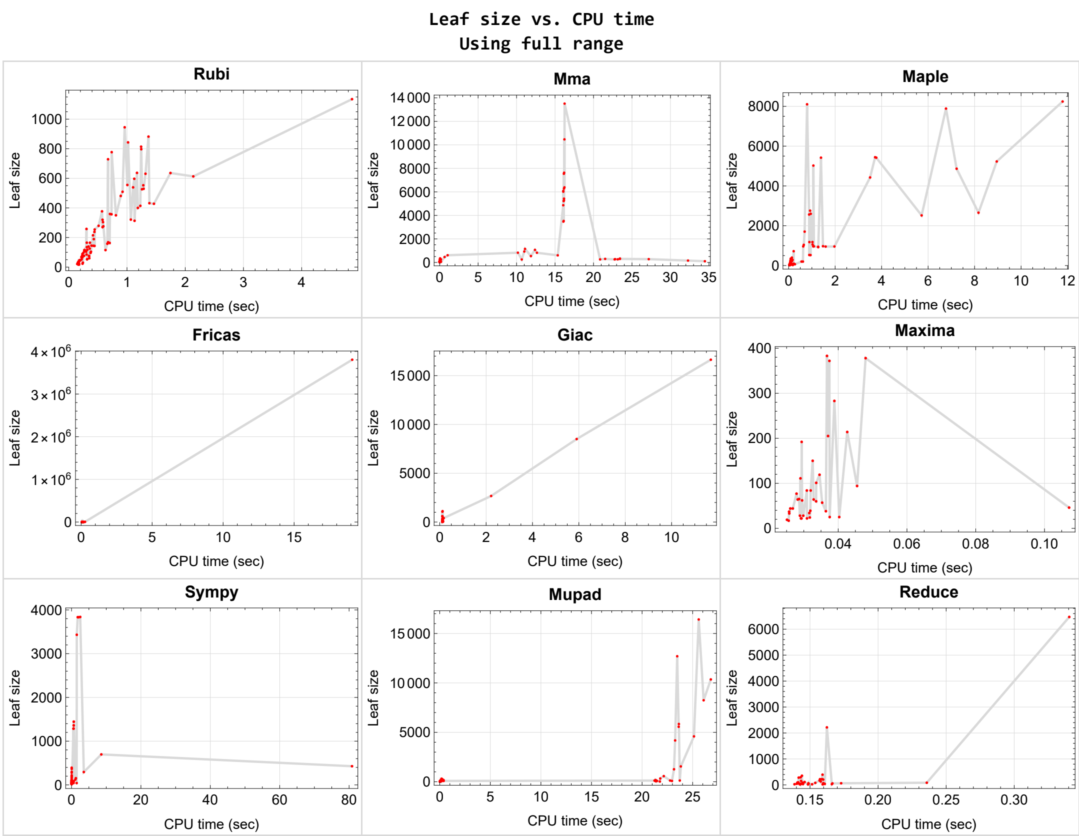


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Reduce {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {8, 88}

Mathematica {50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 67, 85, 86, 87, 88, 89, 90, 91, 92, 93}

Maple $\{60, 61, 63, 64, 65, 67, 68, 69, 71, 72, 85, 87, 89, 92\}$

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

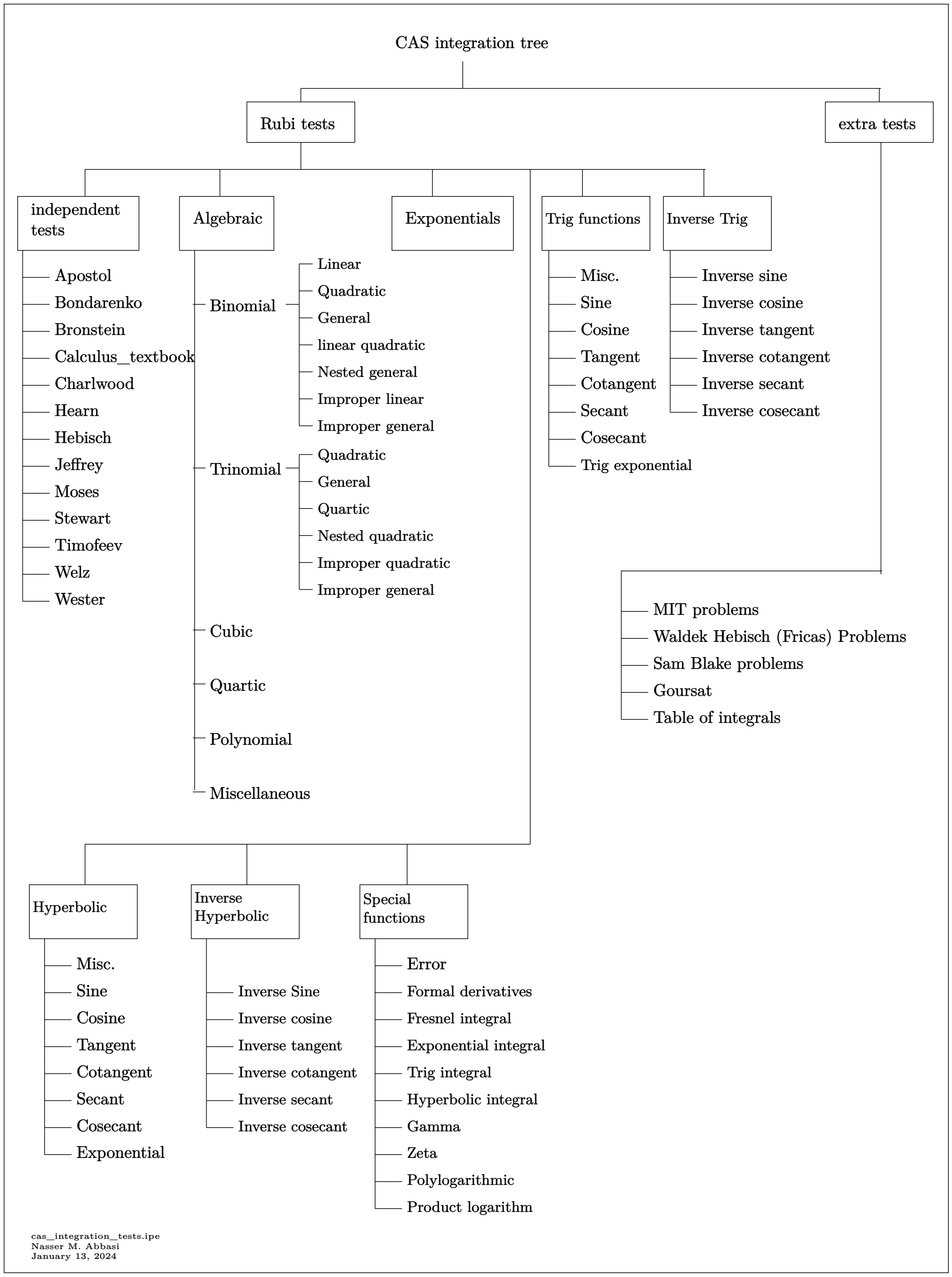
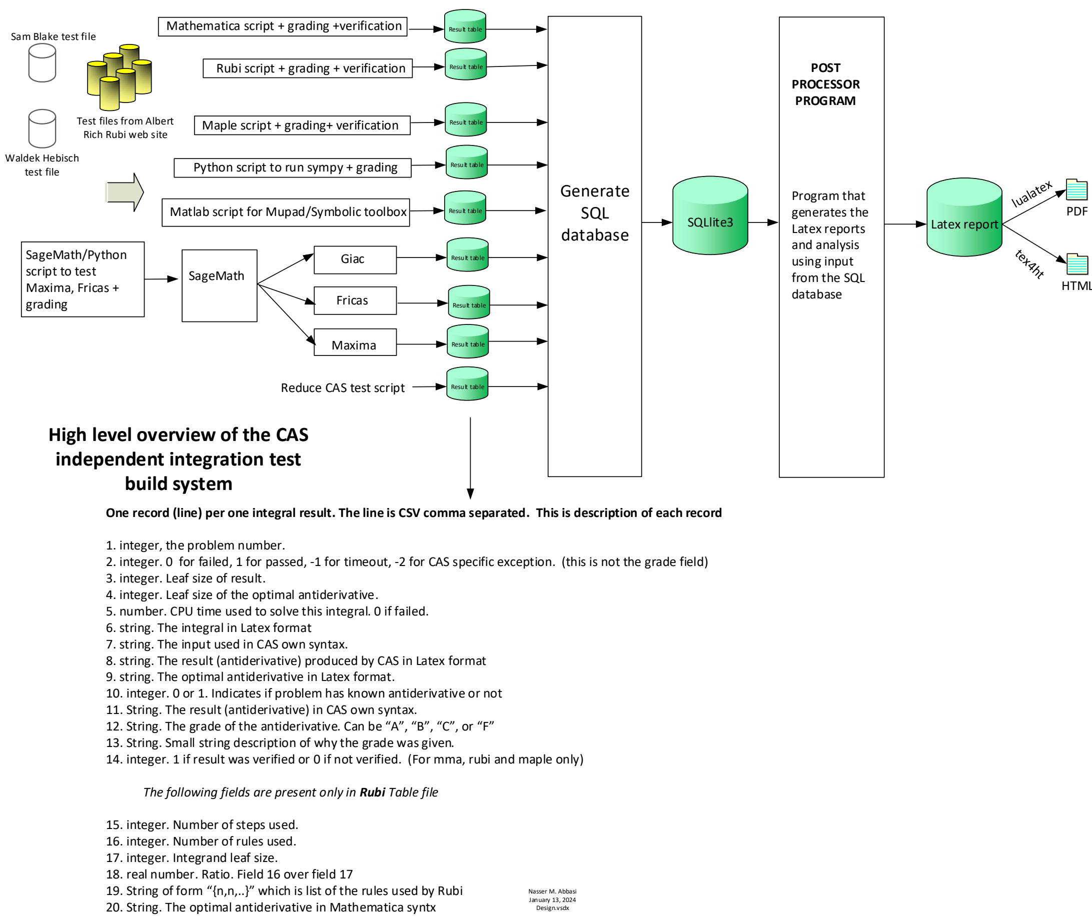


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94 }

B grade { 19 }

C grade { }

F normal fail { 30 }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 9, 10, 11, 15, 16, 17, 21, 22, 23, 26, 27, 28, 31, 32, 33, 34, 37, 38, 39, 40, 43, 44, 45, 46, 73, 74, 75, 76, 79, 80, 81, 82, 94 }

B grade { 64, 65, 66, 67, 68, 69, 70, 71, 72 }

C grade { 7, 8, 12, 13, 14, 18, 19, 20, 24, 25, 29, 30, 35, 36, 41, 42, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 77, 78, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 6, 9, 10, 11, 15, 16, 17, 21, 22, 23, 26, 27, 28, 31, 32, 33, 34, 37, 38, 39, 40, 43, 44, 45, 46, 73, 74, 75, 76, 79, 80, 81, 82, 94 }

B grade { 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 85, 86, 87, 88, 89, 90, 91, 92, 93 }

C grade { 7, 8, 12, 13, 14, 18, 19, 20, 24, 25, 29, 30, 35, 36, 41, 42, 47, 48, 49, 77, 78, 83, 84 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 26, 27, 28, 31, 32, 33, 34, 37, 38, 39, 40, 43, 44, 45, 46, 50, 51, 52, 55, 56, 57, 73, 74, 75, 76, 78, 79, 80, 81, 82, 84, 94 }

B grade { 1, 35, 36, 41, 42, 47, 48, 49, 53, 54, 58, 59, 77, 83 }

C grade { 24 }

F normal fail { 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 85, 86, 87, 88, 89, 90, 91, 92, 93 }

F(-1) timedout fail { 25, 29, 30 }

F(-2) exception fail { }

Maxima

A grade { 1, 2, 3, 4, 5, 6, 9, 10, 11, 15, 16, 17, 21, 22, 23, 26, 27, 28, 31, 32, 33, 34, 37, 38, 39, 40, 43, 44, 45, 46, 73, 74, 75, 76, 79, 80, 81, 82, 94 }

B grade { }

C grade { }

F normal fail { 7, 8, 12, 13, 14, 18, 19, 20, 24, 25, 29, 30, 35, 36, 41, 42, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 77, 78, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93 }

F(-1) timedout fail { }

F(-2) exception fail { }

Giac

A grade { 1, 2, 3, 4, 5, 6, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 26, 27, 28, 31, 32, 33, 34, 37, 38, 39, 40, 44, 45, 46, 73, 74, 75, 76, 79, 80, 81, 82, 94 }

B grade { 7, 8, 35, 36, 41, 42, 43, 47, 48, 49 }

C grade { }

F normal fail { 24, 25, 29, 30, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 77, 78, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93 }

F(-1) timedout fail { }

F(-2) exception fail { }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 94 }

C grade { }

F normal fail { }

F(-1) timedout fail { 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 85, 86, 87, 88, 89, 90, 91, 92, 93 }

F(-2) exception fail { }

Sympy

A grade { 3, 4, 5, 6, 9, 10, 11, 15, 16, 17, 21, 22, 23, 26, 27, 28, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 43, 44, 45, 46, 47, 48, 73, 74, 75, 76, 77, 79, 80, 81, 82, 83, 94 }

B grade { 1, 7, 8, 12, 13, 14, 18, 19, 20, 49, 78, 84 }

C grade { }

F normal fail { 2, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 85, 86, 87, 88, 89, 90, 91, 92, 93 }

F(-1) timedout fail { 24, 25, 29, 30, 42 }

F(-2) exception fail { }

Reduce

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 9, 10, 11, 15, 16, 17, 21, 22, 23, 26, 27, 28, 31, 32, 33, 34, 37, 38, 39, 40, 43, 44, 45, 46, 47, 48, 49, 73, 74, 75, 76, 79, 80, 81, 82, 94 }

C grade { }

F normal fail { 7, 8, 12, 13, 14, 18, 19, 20, 24, 25, 29, 30, 35, 36, 41, 42, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 77, 78, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	26	25	54	153	37	25	25
N.S.	1	1.00	1.00	1.04	1.00	2.16	6.12	1.48	1.00	1.00
time (sec)	N/A	0.159	0.012	0.178	0.040	0.079	1.249	0.133	0.142	21.757

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	25	25	26	25	54	0	95	56	63
N.S.	1	0.45	0.45	0.46	0.45	0.96	0.00	1.70	1.00	1.12
time (sec)	N/A	0.175	0.003	0.059	0.038	0.077	0.000	0.123	0.166	21.402

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	97	78	77	77	94	77	78	77
N.S.	1	1.00	1.00	0.80	0.79	0.79	0.97	0.79	0.80	0.79
time (sec)	N/A	0.252	0.003	0.051	0.028	0.060	0.038	0.108	0.154	0.174

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	69	58	57	57	66	57	58	57
N.S.	1	1.00	1.00	0.84	0.83	0.83	0.96	0.83	0.84	0.83
time (sec)	N/A	0.218	0.002	0.043	0.035	0.058	0.029	0.112	0.142	0.093

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	45	38	37	37	42	37	38	37
N.S.	1	1.00	1.00	0.84	0.82	0.82	0.93	0.82	0.84	0.82
time (sec)	N/A	0.194	0.002	0.042	0.026	0.056	0.026	0.114	0.161	0.028

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	17	17	19	17	18	17
N.S.	1	1.00	1.00	0.86	0.81	0.81	0.90	0.81	0.86	0.81
time (sec)	N/A	0.146	0.000	0.036	0.026	0.061	0.024	0.114	0.149	0.029

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	B	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	270	47	41	0	212	3432	358	19	87
N.S.	1	1.46	0.25	0.22	0.00	1.15	18.55	1.94	0.10	0.47
time (sec)	N/A	0.580	0.021	0.079	0.000	0.112	1.511	0.146	0.141	21.475

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	B	B	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	264	350	108	79	0	366	3834	403	39	174
N.S.	1	1.33	0.41	0.30	0.00	1.39	14.52	1.53	0.15	0.66
time (sec)	N/A	0.810	0.028	0.111	0.000	0.108	2.020	0.172	0.160	21.334

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	76	61	60	60	70	60	63	60
N.S.	1	1.00	1.00	0.80	0.79	0.79	0.92	0.79	0.83	0.79
time (sec)	N/A	0.263	0.002	0.044	0.034	0.058	0.020	0.112	0.160	0.106

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	35	34	34	37	34	43	34
N.S.	1	1.00	1.00	0.83	0.81	0.81	0.88	0.81	1.02	0.81
time (sec)	N/A	0.185	0.002	0.040	0.032	0.063	0.019	0.112	0.145	0.030

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	23	22	22	20	22	23	22
N.S.	1	1.00	1.00	0.77	0.73	0.73	0.67	0.73	0.77	0.73
time (sec)	N/A	0.156	0.000	0.037	0.029	0.063	0.017	0.121	0.159	0.016

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	47	41	0	108	1287	101	16	65
N.S.	1	1.00	0.33	0.29	0.00	0.76	9.00	0.71	0.11	0.45
time (sec)	N/A	0.442	0.011	0.092	0.000	0.084	0.545	0.114	0.146	0.191

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	238	357	87	73	0	186	1360	124	757	115
N.S.	1	1.50	0.37	0.31	0.00	0.78	5.71	0.52	3.18	0.48
time (sec)	N/A	0.734	0.021	0.157	0.000	0.109	0.648	0.137	0.145	21.276

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	371	631	111	86	0	326	1445	136	0	151
N.S.	1	1.70	0.30	0.23	0.00	0.88	3.89	0.37	0.00	0.41
time (sec)	N/A	1.318	0.036	0.228	0.000	0.097	0.641	0.124	0.167	0.132

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	78	63	62	62	70	62	63	62
N.S.	1	1.00	1.00	0.81	0.79	0.79	0.90	0.79	0.81	0.79
time (sec)	N/A	0.232	0.003	0.047	0.030	0.065	0.024	0.114	0.149	0.110

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	39	38	38	37	38	43	38
N.S.	1	1.00	1.00	0.85	0.83	0.83	0.80	0.83	0.93	0.83
time (sec)	N/A	0.198	0.002	0.040	0.036	0.056	0.021	0.111	0.149	0.031

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	23	22	22	20	22	23	22
N.S.	1	1.00	1.00	0.77	0.73	0.73	0.67	0.73	0.77	0.73
time (sec)	N/A	0.162	0.000	0.035	0.031	0.057	0.018	0.114	0.159	0.018

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	145	51	45	0	112	1287	101	20	64
N.S.	1	0.99	0.35	0.31	0.00	0.76	8.76	0.69	0.14	0.44
time (sec)	N/A	0.390	0.011	0.085	0.000	0.076	0.597	0.126	0.144	21.329

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	C	F	A	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	359	96	79	0	210	1360	126	761	118
N.S.	1	2.05	0.55	0.45	0.00	1.20	7.77	0.72	4.35	0.67
time (sec)	N/A	0.707	0.021	0.148	0.000	0.076	0.618	0.121	0.146	21.227

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	389	637	119	94	0	330	1445	140	0	150
N.S.	1	1.64	0.31	0.24	0.00	0.85	3.71	0.36	0.00	0.39
time (sec)	N/A	1.172	0.033	0.218	0.000	0.079	0.650	0.127	0.166	21.365

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	238	238	238	243	283	228	280	287	292	223
N.S.	1	1.00	1.00	1.02	1.19	0.96	1.18	1.21	1.23	0.94
time (sec)	N/A	0.441	0.064	0.081	0.039	0.062	0.038	0.129	0.143	0.154

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	104	101	111	96	107	109	111	93
N.S.	1	1.00	1.00	0.97	1.07	0.92	1.03	1.05	1.07	0.89
time (sec)	N/A	0.298	0.023	0.059	0.029	0.070	0.023	0.129	0.146	0.045

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	36	29	28	28	29	28	29	28
N.S.	1	1.00	1.00	0.81	0.78	0.78	0.81	0.78	0.81	0.78
time (sec)	N/A	0.170	0.000	0.040	0.029	0.064	0.017	0.113	0.166	0.020

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	C	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	407	414	58	52	0	3804183	0	0	24	4180
N.S.	1	1.02	0.14	0.13	0.00	9346.89	0.00	0.00	0.06	10.27
time (sec)	N/A	1.229	0.037	0.181	0.000	19.092	0.000	0.000	0.145	23.268

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1080	1135	460	525	0	0	0	0	0	12699
N.S.	1	1.05	0.43	0.49	0.00	0.00	0.00	0.00	0.00	11.76
time (sec)	N/A	4.864	0.646	0.941	0.000	0.000	0.000	0.000	0.150	23.471

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	320	320	320	345	378	327	393	389	394	311
N.S.	1	1.00	1.00	1.08	1.18	1.02	1.23	1.22	1.23	0.97
time (sec)	N/A	0.580	0.100	0.089	0.048	0.096	0.043	0.125	0.159	0.184

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	141	144	150	135	153	151	153	129
N.S.	1	1.00	1.01	1.04	1.08	0.97	1.10	1.09	1.10	0.93
time (sec)	N/A	0.339	0.029	0.066	0.033	0.085	0.032	0.117	0.143	0.057

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	47	40	39	39	41	39	40	39
N.S.	1	1.00	1.00	0.85	0.83	0.83	0.87	0.83	0.85	0.83
time (sec)	N/A	0.184	0.000	0.044	0.032	0.073	0.020	0.108	0.145	0.020

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	479	613	78	72	0	0	0	0	36	5563
N.S.	1	1.28	0.16	0.15	0.00	0.00	0.00	0.00	0.08	11.61
time (sec)	N/A	2.138	0.041	0.089	0.000	0.000	0.000	0.000	0.165	23.616

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	C	F	F(-1)	F(-1)	F	F	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1379	0	610	722	0	0	0	0	0	16419
N.S.	1	0.00	0.44	0.52	0.00	0.00	0.00	0.00	0.00	11.91
time (sec)	N/A	0.000	1.042	0.216	0.000	0.000	0.000	0.000	0.156	25.598

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	257	276	285	267	372	271	299	277	280	261
N.S.	1	1.07	1.11	1.04	1.45	1.05	1.16	1.08	1.09	1.02
time (sec)	N/A	0.594	0.046	0.083	0.037	0.089	0.057	0.109	0.142	0.245

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	189	171	163	205	163	180	166	169	160
N.S.	1	1.09	0.98	0.94	1.18	0.94	1.03	0.95	0.97	0.92
time (sec)	N/A	0.430	0.022	0.070	0.037	0.099	0.038	0.117	0.157	0.124

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	117	92	83	94	82	95	83	86	82
N.S.	1	1.10	0.87	0.78	0.89	0.77	0.90	0.78	0.81	0.77
time (sec)	N/A	0.318	0.012	0.058	0.045	0.094	0.029	0.111	0.141	0.038

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	29	28	28	31	28	31	28
N.S.	1	1.00	1.00	0.91	0.88	0.88	0.97	0.88	0.97	0.88
time (sec)	N/A	0.170	0.000	0.043	0.030	0.094	0.019	0.113	0.142	0.038

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	386	555	71	64	0	905	88	603	31	1551
N.S.	1	1.44	0.18	0.17	0.00	2.34	0.23	1.56	0.08	4.02
time (sec)	N/A	1.006	0.029	0.152	0.000	0.084	0.633	0.109	0.157	23.829

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	565	797	182	230	0	3222	427	1057	86	5844
N.S.	1	1.41	0.32	0.41	0.00	5.70	0.76	1.87	0.15	10.34
time (sec)	N/A	1.245	0.111	0.149	0.000	0.159	80.843	0.115	0.150	23.637

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	266	304	345	332	383	332	366	353	353	331
N.S.	1	1.14	1.30	1.25	1.44	1.25	1.38	1.33	1.33	1.24
time (sec)	N/A	0.588	0.057	0.104	0.037	0.100	0.054	0.117	0.144	21.797

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	215	207	200	214	198	218	205	205	201
N.S.	1	1.17	1.12	1.09	1.16	1.08	1.18	1.11	1.11	1.09
time (sec)	N/A	0.417	0.032	0.079	0.043	0.068	0.047	0.124	0.160	0.147

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	135	109	99	101	98	112	99	99	98
N.S.	1	1.19	0.96	0.88	0.89	0.87	0.99	0.88	0.88	0.87
time (sec)	N/A	0.310	0.018	0.066	0.034	0.067	0.028	0.110	0.141	0.044

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	34	33	33	36	33	33	33
N.S.	1	1.00	1.00	0.92	0.89	0.89	0.97	0.89	0.89	0.89
time (sec)	N/A	0.167	0.000	0.054	0.026	0.065	0.024	0.118	0.145	0.040

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	165	71	67	0	1115	122	577	34	1264
N.S.	1	1.08	0.46	0.44	0.00	7.29	0.80	3.77	0.22	8.26
time (sec)	N/A	0.364	0.027	0.268	0.000	0.093	1.112	0.119	0.160	23.155

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F(-1)	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	332	377	234	288	0	4285	0	1115	99	10351
N.S.	1	1.14	0.70	0.87	0.00	12.91	0.00	3.36	0.30	31.18
time (sec)	N/A	0.571	0.157	0.170	0.000	0.274	0.000	0.122	0.143	26.795

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	125	195	179	192	179	199	219	220	175
N.S.	1	0.90	1.40	1.29	1.38	1.29	1.43	1.58	1.58	1.26
time (sec)	N/A	0.360	0.033	0.075	0.029	0.068	0.044	0.114	0.157	0.373

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	85	114	110	119	107	114	128	131	108
N.S.	1	0.89	1.20	1.16	1.25	1.13	1.20	1.35	1.38	1.14
time (sec)	N/A	0.306	0.018	0.061	0.035	0.060	0.038	0.110	0.158	0.124

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	52	66	60	65	56	65	65	66	61
N.S.	1	0.90	1.14	1.03	1.12	0.97	1.12	1.12	1.14	1.05
time (sec)	N/A	0.221	0.011	0.052	0.029	0.068	0.028	0.109	0.141	0.039

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	23	22	22	22	22	25	22
N.S.	1	1.00	1.00	0.88	0.85	0.85	0.85	0.85	0.96	0.85
time (sec)	N/A	0.156	0.000	0.040	0.029	0.062	0.021	0.107	0.139	0.020

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	89	57	51	0	457	66	2669	222	571
N.S.	1	0.96	0.61	0.55	0.00	4.91	0.71	28.70	2.39	6.14
time (sec)	N/A	0.234	0.027	0.081	0.000	0.077	0.558	2.217	0.159	22.144

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	165	150	158	0	1948	294	8503	2216	4591
N.S.	1	0.92	0.84	0.88	0.00	10.88	1.64	47.50	12.38	25.65
time (sec)	N/A	0.310	0.081	0.091	0.000	0.110	3.535	5.905	0.162	25.128

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	254	254	400	0	3971	697	16632	6470	8242
N.S.	1	0.94	0.94	1.48	0.00	14.71	2.58	61.60	23.96	30.53
time (sec)	N/A	0.445	0.164	0.139	0.000	0.137	8.582	11.691	0.340	26.083

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	108	278	958	0	90	0	0	255	0
N.S.	1	0.93	2.40	8.26	0.00	0.78	0.00	0.00	2.20	0.00
time (sec)	N/A	0.281	22.808	1.970	0.000	0.077	0.000	0.000	0.288	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	66	256	936	0	69	0	0	163	0
N.S.	1	0.97	3.76	13.76	0.00	1.01	0.00	0.00	2.40	0.00
time (sec)	N/A	0.234	22.742	1.265	0.000	0.081	0.000	0.000	0.245	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	156	200	0	16	0	0	42	0
N.S.	1	1.00	9.18	11.76	0.00	0.94	0.00	0.00	2.47	0.00
time (sec)	N/A	0.178	32.280	0.553	0.000	0.067	0.000	0.000	0.156	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	73	261	932	0	119	0	0	535	0
N.S.	1	0.92	3.30	11.80	0.00	1.51	0.00	0.00	6.77	0.00
time (sec)	N/A	0.236	23.148	1.274	0.000	0.081	0.000	0.000	0.209	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	115	298	972	0	195	0	0	0	0
N.S.	1	0.93	2.42	7.90	0.00	1.59	0.00	0.00	0.00	0.00
time (sec)	N/A	0.274	23.430	1.478	0.000	0.076	0.000	0.000	0.215	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	108	278	954	0	90	0	0	255	0
N.S.	1	0.93	2.40	8.22	0.00	0.78	0.00	0.00	2.20	0.00
time (sec)	N/A	0.268	27.178	1.592	0.000	0.078	0.000	0.000	0.298	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	66	256	932	0	69	0	0	163	0
N.S.	1	0.97	3.76	13.71	0.00	1.01	0.00	0.00	2.40	0.00
time (sec)	N/A	0.221	20.856	1.273	0.000	0.079	0.000	0.000	0.226	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	100	200	0	16	0	0	42	0
N.S.	1	1.00	5.88	11.76	0.00	0.94	0.00	0.00	2.47	0.00
time (sec)	N/A	0.164	34.470	0.613	0.000	0.077	0.000	0.000	0.176	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	73	298	963	0	119	0	0	535	0
N.S.	1	0.92	3.77	12.19	0.00	1.51	0.00	0.00	6.77	0.00
time (sec)	N/A	0.227	21.512	0.628	0.000	0.074	0.000	0.000	0.194	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	115	327	1039	0	195	0	0	0	0
N.S.	1	0.93	2.66	8.45	0.00	1.59	0.00	0.00	0.00	0.00
time (sec)	N/A	0.266	23.430	0.651	0.000	0.074	0.000	0.000	0.223	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	683	843	10468	5229	0	0	0	0	715	0
N.S.	1	1.23	15.33	7.66	0.00	0.00	0.00	0.00	1.05	0.00
time (sec)	N/A	1.018	16.218	8.953	0.000	0.000	0.000	0.000	0.237	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	585	729	5218	4865	0	0	0	0	269	0
N.S.	1	1.25	8.92	8.32	0.00	0.00	0.00	0.00	0.46	0.00
time (sec)	N/A	0.675	16.120	7.227	0.000	0.000	0.000	0.000	0.176	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	258	822	1056	0	0	0	0	60	0
N.S.	1	1.19	3.79	4.87	0.00	0.00	0.00	0.00	0.28	0.00
time (sec)	N/A	0.305	12.646	1.031	0.000	0.000	0.000	0.000	0.157	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	639	777	5276	5024	0	0	0	0	115	0
N.S.	1	1.22	8.26	7.86	0.00	0.00	0.00	0.00	0.18	0.00
time (sec)	N/A	0.737	16.158	1.064	0.000	0.000	0.000	0.000	0.171	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	702	945	13510	8241	0	0	0	0	1113	0
N.S.	1	1.35	19.25	11.74	0.00	0.00	0.00	0.00	1.59	0.00
time (sec)	N/A	0.960	16.240	11.787	0.000	0.000	0.000	0.000	0.284	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	599	814	7543	7887	0	0	0	0	502	0
N.S.	1	1.36	12.59	13.17	0.00	0.00	0.00	0.00	0.84	0.00
time (sec)	N/A	1.242	16.147	6.770	0.000	0.000	0.000	0.000	0.183	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	279	1065	1704	0	0	0	0	66	0
N.S.	1	1.27	4.86	7.78	0.00	0.00	0.00	0.00	0.30	0.00
time (sec)	N/A	0.513	12.391	0.688	0.000	0.000	0.000	0.000	0.227	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	680	882	7629	8103	0	0	0	0	131	0
N.S.	1	1.30	11.22	11.92	0.00	0.00	0.00	0.00	0.19	0.00
time (sec)	N/A	1.369	16.179	0.793	0.000	0.000	0.000	0.000	0.321	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	402	539	6287	2655	0	0	0	0	551	0
N.S.	1	1.34	15.64	6.60	0.00	0.00	0.00	0.00	1.37	0.00
time (sec)	N/A	1.105	16.165	8.168	0.000	0.000	0.000	0.000	0.465	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	330	481	3470	2519	0	0	0	0	245	0
N.S.	1	1.46	10.52	7.63	0.00	0.00	0.00	0.00	0.74	0.00
time (sec)	N/A	0.893	16.088	5.721	0.000	0.000	0.000	0.000	0.323	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	144	540	530	0	0	0	0	46	0
N.S.	1	0.91	3.42	3.35	0.00	0.00	0.00	0.00	0.29	0.00
time (sec)	N/A	0.422	11.864	0.888	0.000	0.000	0.000	0.000	0.280	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	368	509	3526	2601	0	0	0	0	85	0
N.S.	1	1.38	9.58	7.07	0.00	0.00	0.00	0.00	0.23	0.00
time (sec)	N/A	0.924	16.120	0.951	0.000	0.000	0.000	0.000	0.241	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	460	597	6386	2757	0	0	0	0	152	0
N.S.	1	1.30	13.88	5.99	0.00	0.00	0.00	0.00	0.33	0.00
time (sec)	N/A	1.125	16.222	0.923	0.000	0.000	0.000	0.000	0.265	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	96	84	84	84	94	84	83	84
N.S.	1	1.00	1.00	0.88	0.88	0.88	0.98	0.88	0.86	0.88
time (sec)	N/A	0.375	0.003	0.052	0.032	0.087	0.027	0.123	0.236	0.385

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	74	64	64	64	71	64	63	64
N.S.	1	1.00	1.00	0.86	0.86	0.86	0.96	0.86	0.85	0.86
time (sec)	N/A	0.350	0.002	0.048	0.028	0.081	0.027	0.106	0.160	0.107

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	44	44	44	49	44	43	44
N.S.	1	1.00	1.00	0.81	0.81	0.81	0.91	0.81	0.80	0.81
time (sec)	N/A	0.316	0.003	0.040	0.026	0.088	0.026	0.114	0.140	0.032

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	19	19	19	19	19	18	19
N.S.	1	1.00	1.00	0.83	0.83	0.83	0.83	0.83	0.78	0.83
time (sec)	N/A	0.236	0.000	0.036	0.025	0.070	0.017	0.134	0.143	0.031

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	321	45	41	0	303	41	0	19	123
N.S.	1	1.52	0.21	0.19	0.00	1.44	0.19	0.00	0.09	0.58
time (sec)	N/A	1.066	0.019	0.055	0.000	0.119	0.567	0.000	0.171	23.726

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	300	432	113	83	0	398	3834	0	42	176
N.S.	1	1.44	0.38	0.28	0.00	1.33	12.78	0.00	0.14	0.59
time (sec)	N/A	1.385	0.049	0.066	0.000	0.128	1.784	0.000	0.143	0.225

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	104	84	84	84	100	84	83	84
N.S.	1	1.00	1.00	0.81	0.81	0.81	0.96	0.81	0.80	0.81
time (sec)	N/A	0.381	0.004	0.052	0.031	0.094	0.034	0.126	0.145	22.954

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	76	64	64	64	73	64	63	64
N.S.	1	1.00	1.00	0.84	0.84	0.84	0.96	0.84	0.83	0.84
time (sec)	N/A	0.346	0.002	0.048	0.033	0.078	0.031	0.112	0.173	0.107

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	52	44	44	44	49	44	43	44
N.S.	1	1.00	1.00	0.85	0.85	0.85	0.94	0.85	0.83	0.85
time (sec)	N/A	0.314	0.002	0.041	0.027	0.070	0.026	0.144	0.143	0.034

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	24	24	24	27	24	23	24
N.S.	1	1.00	1.00	0.80	0.80	0.80	0.90	0.80	0.77	0.80
time (sec)	N/A	0.243	0.000	0.036	0.032	0.072	0.021	0.108	0.152	0.020

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	313	55	49	0	305	41	0	24	123
N.S.	1	1.50	0.26	0.24	0.00	1.47	0.20	0.00	0.12	0.59
time (sec)	N/A	1.130	0.023	0.056	0.000	0.112	1.477	0.000	0.163	22.766

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	428	128	96	0	422	3839	0	795	181
N.S.	1	1.44	0.43	0.32	0.00	1.42	12.93	0.00	2.68	0.61
time (sec)	N/A	1.463	0.043	0.072	0.000	0.117	2.500	0.000	0.152	0.229

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	131	168	927	965	0	0	0	0	36	0
N.S.	1	1.28	7.08	7.37	0.00	0.00	0.00	0.00	0.27	0.00
time (sec)	N/A	0.676	11.004	1.087	0.000	0.000	0.000	0.000	0.150	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	431	553	4865	4426	0	0	0	0	59	0
N.S.	1	1.28	11.29	10.27	0.00	0.00	0.00	0.00	0.14	0.00
time (sec)	N/A	1.280	16.066	3.504	0.000	0.000	0.000	0.000	0.164	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	108	115	249	961	0	0	0	0	36	0
N.S.	1	1.06	2.31	8.90	0.00	0.00	0.00	0.00	0.33	0.00
time (sec)	N/A	0.631	10.674	1.059	0.000	0.000	0.000	0.000	0.143	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F	F	F	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	367	400	602	2564	0	0	0	0	56	0
N.S.	1	1.09	1.64	6.99	0.00	0.00	0.00	0.00	0.15	0.00
time (sec)	N/A	1.187	15.350	0.881	0.000	0.000	0.000	0.000	0.160	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	125	163	1148	1180	0	0	0	0	24	0
N.S.	1	1.30	9.18	9.44	0.00	0.00	0.00	0.00	0.19	0.00
time (sec)	N/A	0.702	11.099	1.025	0.000	0.000	0.000	0.000	200.029	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	432	529	6019	5421	0	0	0	0	66	0
N.S.	1	1.22	13.93	12.55	0.00	0.00	0.00	0.00	0.15	0.00
time (sec)	N/A	1.286	16.074	3.770	0.000	0.000	0.000	0.000	0.158	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	575	636	6084	5441	0	0	0	0	86	0
N.S.	1	1.11	10.58	9.46	0.00	0.00	0.00	0.00	0.15	0.00
time (sec)	N/A	1.746	16.074	3.720	0.000	0.000	0.000	0.000	0.156	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	126	158	826	1180	0	0	0	0	46	0
N.S.	1	1.25	6.56	9.37	0.00	0.00	0.00	0.00	0.37	0.00
time (sec)	N/A	0.663	10.171	0.893	0.000	0.000	0.000	0.000	32.483	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	434	526	5428	5421	0	0	0	0	66	0
N.S.	1	1.21	12.51	12.49	0.00	0.00	0.00	0.00	0.15	0.00
time (sec)	N/A	1.257	16.165	1.391	0.000	0.000	0.000	0.000	0.164	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	52	47	46	46	56	48	46	52
N.S.	1	1.00	0.87	0.78	0.77	0.77	0.93	0.80	0.77	0.87
time (sec)	N/A	0.353	0.180	0.068	0.107	0.200	0.130	0.114	0.144	0.120

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [78] had the largest ratio of [1.05882000000000009]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	4	3	1.00	9	0.333
2	A	2	2	0.45	40	0.050
3	A	2	2	1.00	17	0.118
4	A	2	2	1.00	17	0.118
5	A	2	2	1.00	17	0.118
6	A	1	1	1.00	15	0.067
7	A	14	13	1.46	17	0.765
8	A	18	17	1.33	17	1.000
9	A	2	2	1.00	14	0.143
10	A	2	2	1.00	14	0.143
11	A	1	1	1.00	12	0.083
12	A	2	2	1.00	14	0.143
13	A	2	2	1.50	14	0.143
14	A	2	2	1.70	14	0.143
15	A	2	2	1.00	18	0.111
16	A	2	2	1.00	18	0.111
17	A	1	1	1.00	16	0.062
18	A	2	2	0.99	18	0.111
19	B	2	2	2.05	18	0.111
20	A	2	2	1.64	18	0.111
21	A	2	2	1.00	22	0.091

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
22	A	2	2	1.00	22	0.091
23	A	1	1	1.00	20	0.050
24	A	2	2	1.02	22	0.091
25	A	2	2	1.05	22	0.091
26	A	2	2	1.00	34	0.059
27	A	2	2	1.00	34	0.059
28	A	1	1	1.00	32	0.031
29	A	2	2	1.28	34	0.059
30	F	0	0	N/A	0.000	N/A
31	A	4	3	1.07	29	0.103
32	A	4	3	1.09	29	0.103
33	A	4	3	1.10	29	0.103
34	A	1	1	1.00	27	0.037
35	A	10	9	1.44	29	0.310
36	A	12	11	1.41	29	0.379
37	A	4	3	1.14	32	0.094
38	A	4	3	1.17	32	0.094
39	A	4	3	1.19	32	0.094
40	A	1	1	1.00	30	0.033
41	A	4	3	1.08	32	0.094
42	A	6	5	1.14	32	0.156
43	A	4	3	0.90	22	0.136
44	A	4	3	0.89	22	0.136
45	A	4	3	0.90	22	0.136
46	A	1	1	1.00	20	0.050
47	A	4	3	0.96	22	0.136
48	A	6	5	0.92	22	0.227
49	A	8	7	0.94	22	0.318
50	A	11	10	0.93	23	0.435
51	A	9	8	0.97	23	0.348
52	A	5	4	1.00	23	0.174
53	A	9	8	0.92	23	0.348

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
54	A	11	10	0.93	23	0.435
55	A	11	10	0.93	19	0.526
56	A	9	8	0.97	19	0.421
57	A	5	4	1.00	19	0.211
58	A	9	8	0.92	19	0.421
59	A	11	10	0.93	19	0.526
60	A	10	9	1.23	31	0.290
61	A	8	7	1.25	31	0.226
62	A	3	2	1.19	31	0.065
63	A	8	7	1.22	31	0.226
64	A	9	8	1.35	34	0.235
65	A	7	6	1.36	34	0.176
66	A	3	2	1.27	34	0.059
67	A	7	6	1.30	34	0.176
68	A	11	10	1.34	24	0.417
69	A	9	8	1.46	24	0.333
70	A	4	3	0.91	24	0.125
71	A	9	8	1.38	24	0.333
72	A	11	10	1.30	24	0.417
73	A	2	2	1.00	17	0.118
74	A	2	2	1.00	17	0.118
75	A	2	2	1.00	17	0.118
76	A	1	1	1.00	15	0.067
77	A	15	14	1.52	17	0.824
78	A	19	18	1.44	17	1.059
79	A	2	2	1.00	22	0.091
80	A	2	2	1.00	22	0.091
81	A	2	2	1.00	22	0.091
82	A	1	1	1.00	20	0.050
83	A	15	14	1.50	22	0.636
84	A	19	18	1.44	22	0.818
85	A	5	4	1.28	19	0.211

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
86	A	14	13	1.28	19	0.684
87	A	5	4	1.06	19	0.211
88	A	14	13	1.09	19	0.684
89	A	5	4	1.30	24	0.167
90	A	14	13	1.22	24	0.542
91	A	18	17	1.11	24	0.708
92	A	5	4	1.25	24	0.167
93	A	14	13	1.21	24	0.542
94	A	2	2	1.00	17	0.118

CHAPTER 3

Listing of integrals

3.1 $\int \left((a+bx)^4\right)^p \, dx$

Optimal result

Integrand size = 9, antiderivative size = 25

$$\int \left((a+bx)^4\right)^p \, dx = \frac{(a+bx)\left((a+bx)^4\right)^p}{b(1+4p)}$$

output
```
(b*x+a)*((b*x+a)^4)^p/b/(1+4*p)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \left((a+bx)^4\right)^p \, dx = \frac{(a+bx)\left((a+bx)^4\right)^p}{b(1+4p)}$$

input
```
Integrate[((a + b*x)^4)^p,x]
```

output
```
((a + b*x)*((a + b*x)^4)^p)/(b*(1 + 4*p))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {239, 20, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left((a+bx)^4\right)^p \, dx$$

$$\Big\downarrow \ 239$$

$$\frac{\int \left((a+bx)^4\right)^p d(a+bx)}{b}$$

$$\Big\downarrow \ 20$$

$$\frac{(a+bx)^{-4p}\left((a+bx)^4\right)^p \int (a+bx)^{4p}d(a+bx)}{b}$$

$$\Big\downarrow \ 15$$

$$\frac{(a+bx)\left((a+bx)^4\right)^p}{b(4p+1)}$$

input
```
Int[((a + b*x)^4)^p,x]
```

output
```
((a + b*x)*((a + b*x)^4)^p)/(b*(1 + 4*p))
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 20
```
Int[((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a*x^n)^p/x^(n*p)   Int[x^(n*
p), x], x] /; FreeQ[{a, n, p}, x] &&  !IntegerQ[p]
```

rule 239
```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Simp[1/Coefficient[v, x, 1
]   Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Lin
earQ[v, x] && NeQ[v, x]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

method	result	size
gosper	$\frac{(bx+a)\left((bx+a)^{4}\right)^{p}}{b(1+4p)}$	26
risch	$\frac{(bx+a)\left((bx+a)^{4}\right)^{p}}{b(1+4p)}$	26
orering	$\frac{(bx+a)\left((bx+a)^{4}\right)^{p}}{b(1+4p)}$	26
parallelrisch	$\frac{x\left((bx+a)^{4}\right)^{p}ab+\left((bx+a)^{4}\right)^{p}a^{2}}{(1+4p)ab}$	42
norman	$\frac{x\,e^{p\ln\left((bx+a)^{4}\right)}}{1+4p}+\frac{a\,e^{p\ln\left((bx+a)^{4}\right)}}{b(1+4p)}$	45

input
```
int(((b*x+a)^4)^p,x,method=_RETURNVERBOSE)
```

output
```
(b*x+a)*((b*x+a)^4)^p/b/(1+4*p)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. 2(25) = 50.

Time = 0.08 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.16

$$\int\left((a+bx)^{4}\right)^{p}\,dx=\frac{(bx+a)(b^{4}x^{4}+4\,ab^{3}x^{3}+6\,a^{2}b^{2}x^{2}+4\,a^{3}bx+a^{4})^{p}}{4\,bp+b}$$

input
```
integrate(((b*x+a)^4)^p,x, algorithm="fricas")
```

output
```
(b*x + a)*(b^4*x^4 + 4*a*b^3*x^3 + 6*a^2*b^2*x^2 + 4*a^3*b*x + a^4)^p/(4*b
*p + b)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(19) = 38$.

Time = 1.25 (sec) , antiderivative size = 153, normalized size of antiderivative = 6.12

$$\int \left((a+bx)^4\right)^p\,dx$$
$$= \begin{cases} \frac{x}{\sqrt[4]{a^4}} & \text{for } b = 0 \wedge p = -\frac{1}{4} \\ x(a^4)^p & \text{for } b = 0 \\ \begin{cases} \frac{\left(\frac{a}{b}+x\right)\log\left(\frac{a}{b}+x\right)}{\sqrt[4]{(a+bx)^4}} & \text{for } b \neq 0 \\ \frac{x}{\sqrt[4]{a^4}} & \text{otherwise} \end{cases} & \text{for } p = -\frac{1}{4} \\ \frac{a(a^4+4a^3bx+6a^2b^2x^2+4ab^3x^3+b^4x^4)^p}{4bp+b} + \frac{bx(a^4+4a^3bx+6a^2b^2x^2+4ab^3x^3+b^4x^4)^p}{4bp+b} & \text{otherwise} \end{cases}$$

input
```
integrate(((b*x+a)**4)**p,x)
```

output
```
Piecewise((x/(a**4)**(1/4), Eq(b, 0) & Eq(p, -1/4)), (x*(a**4)**p, Eq(b, 0
)), (Piecewise(((a/b + x)*log(a/b + x)/((a + b*x)**4)**(1/4), Ne(b, 0)), (
x/(a**4)**(1/4), True)), Eq(p, -1/4)), (a*(a**4 + 4*a**3*b*x + 6*a**2*b**2
*x**2 + 4*a*b**3*x**3 + b**4*x**4)**p/(4*b*p + b) + b*x*(a**4 + 4*a**3*b*x
 + 6*a**2*b**2*x**2 + 4*a*b**3*x**3 + b**4*x**4)**p/(4*b*p + b), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \left((a+bx)^4\right)^p \, dx = \frac{(bx+a)(bx+a)^{4p}}{b(4p+1)}$$

input
```
integrate(((b*x+a)^4)^p,x, algorithm="maxima")
```

output
```
(b*x + a)*(b*x + a)^(4*p)/(b*(4*p + 1))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.48

$$\int \left((a+bx)^4\right)^p \, dx = \frac{((bx+a)\text{sgn}(bx+a))^{4p+1}}{b(4p+1)\text{sgn}\,(bx+a)}$$

input
```
integrate(((b*x+a)^4)^p,x, algorithm="giac")
```

output
```
((b*x + a)*sgn(b*x + a))^(4*p + 1)/(b*(4*p + 1)*sgn(b*x + a))
```

Mupad [B] (verification not implemented)

Time = 21.76 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \left((a+bx)^4\right)^p \, dx = \frac{\left((a+b\,x)^4\right)^p\,(a+b\,x)}{b\,(4\,p+1)}$$

input
```
int(((a + b*x)^4)^p,x)
```

output
```
(((a + b*x)^4)^p*(a + b*x))/(b*(4*p + 1))
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \left((a+bx)^4\right)^p \, dx = \frac{(bx+a)^{4p}\,(bx+a)}{b\,(4p+1)}$$

input
```
int(((b*x+a)^4)^p,x)
```

output
```
((a + b*x)**(4*p)*(a + b*x))/(b*(4*p + 1))
```

3.2 $\int (a^4+4a^3bx+6a^2b^2x^2+4ab^3x^3+b^4x^4)^p\,dx$

Optimal result

Integrand size = 40, antiderivative size = 56

$$\int (a^4+4a^3bx+6a^2b^2x^2+4ab^3x^3+b^4x^4)^p\,dx = \frac{(a+bx)\,(a^4+4a^3bx+6a^2b^2x^2+4ab^3x^3+b^4x^4)^p}{b(1+4p)}$$

output
```
(b*x+a)*(b^4*x^4+4*a*b^3*x^3+6*a^2*b^2*x^2+4*a^3*b*x+a^4)^p/b/(1+4*p)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.45

$$\int (a^4+4a^3bx+6a^2b^2x^2+4ab^3x^3+b^4x^4)^p\,dx = \frac{(a+bx)\,((a+bx)^4)^p}{b(1+4p)}$$

input
```
Integrate[(a^4 + 4*a^3*b*x + 6*a^2*b^2*x^2 + 4*a*b^3*x^3 + b^4*x^4)^p,x]
```

output
```
((a + b*x)*((a + b*x)^4)^p)/(b*(1 + 4*p))
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.45, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {2008, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a^4+4a^3bx+6a^2b^2x^2+4ab^3x^3+b^4x^4\right)^p\,dx$$

$$\Big\downarrow \text{ 2008}$$

$$(a+bx)^{-4p}\left((a+bx)^4\right)^p\int (a+bx)^{4p}dx$$

$$\Big\downarrow \text{ 17}$$

$$\frac{(a+bx)\left((a+bx)^4\right)^p}{b(4p+1)}$$

input
```
Int[(a^4 + 4*a^3*b*x + 6*a^2*b^2*x^2 + 4*a*b^3*x^3 + b^4*x^4)^p,x]
```

output
```
((a + b*x)*((a + b*x)^4)^p)/(b*(1 + 4*p))
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 2008
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{a = Rt[Coeff[Px, x, 0], Expon[Px,
x]], b = Rt[Coeff[Px, x, Expon[Px, x]], Expon[Px, x]]}, Simp[((a + b*x)^Exp
on[Px, x])^p/(a + b*x)^(Expon[Px, x]*p)   Int[u*(a + b*x)^(Expon[Px, x]*p),
 x], x] /; EqQ[Px, (a + b*x)^Expon[Px, x]]] /;  !IntegerQ[p] && PolyQ[Px, x
] && GtQ[Expon[Px, x], 1] && NeQ[Coeff[Px, x, 0], 0]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.46

method	result	size
risch	$\frac{(bx+a)\left(\left(bx+a\right)^{4}\right)^{p}}{b\left(1+4p\right)}$	26
gosper	$\frac{(bx+a)\left(b^{4}x^{4}+4a\,b^{3}x^{3}+6a^{2}b^{2}x^{2}+4b\,a^{3}x+a^{4}\right)^{p}}{b\left(1+4p\right)}$	57
orering	$\frac{(bx+a)\left(b^{4}x^{4}+4a\,b^{3}x^{3}+6a^{2}b^{2}x^{2}+4b\,a^{3}x+a^{4}\right)^{p}}{b\left(1+4p\right)}$	57
parallelrisch	$\frac{x\left(b^{4}x^{4}+4a\,b^{3}x^{3}+6a^{2}b^{2}x^{2}+4b\,a^{3}x+a^{4}\right)^{p}ab+\left(b^{4}x^{4}+4a\,b^{3}x^{3}+6a^{2}b^{2}x^{2}+4b\,a^{3}x+a^{4}\right)^{p}a^{2}}{\left(1+4p\right)ab}$	104
norman	$\frac{x\,{\mathrm e}^{p\ln\left(b^{4}x^{4}+4a\,b^{3}x^{3}+6a^{2}b^{2}x^{2}+4b\,a^{3}x+a^{4}\right)}}{1+4p}+\frac{a\,{\mathrm e}^{p\ln\left(b^{4}x^{4}+4a\,b^{3}x^{3}+6a^{2}b^{2}x^{2}+4b\,a^{3}x+a^{4}\right)}}{b\left(1+4p\right)}$	107

input
```
int((b^4*x^4+4*a*b^3*x^3+6*a^2*b^2*x^2+4*a^3*b*x+a^4)^p,x,method=_RETURNVE
RBOSE)
```

output
```
(b*x+a)*((b*x+a)^4)^p/b/(1+4*p)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.96

$$\int \left(a^4+4a^3bx+6a^2b^2x^2+4ab^3x^3+b^4x^4\right)^p\,dx = \frac{(bx+a)(b^4x^4+4\,ab^3x^3+6\,a^2b^2x^2+4\,a^3bx+a^4)^p}{4\,bp+b}$$

input
```
integrate((b^4*x^4+4*a*b^3*x^3+6*a^2*b^2*x^2+4*a^3*b*x+a^4)^p,x, algorithm
="fricas")
```

output
```
(b*x + a)*(b^4*x^4 + 4*a*b^3*x^3 + 6*a^2*b^2*x^2 + 4*a^3*b*x + a^4)^p/(4*b
*p + b)
```

Sympy [F]

$$\int \left(a^4 + 4a^3bx + 6a^2b^2x^2 + 4ab^3x^3 + b^4x^4\right)^p \, dx$$
$$= \begin{cases} \frac{x}{\sqrt[4]{a^4}} & \text{for } b = 0 \wedge p = -\frac{1}{4} \\ x(a^4)^p & \text{for } b = 0 \\ \int \frac{1}{\sqrt[4]{a^4 + 4a^3bx + 6a^2b^2x^2 + 4ab^3x^3 + b^4x^4}} \, dx & \text{for } p = -\frac{1}{4} \\ \frac{a(a^4+4a^3bx+6a^2b^2x^2+4ab^3x^3+b^4x^4)^p}{4bp+b} + \frac{bx(a^4+4a^3bx+6a^2b^2x^2+4ab^3x^3+b^4x^4)^p}{4bp+b} & \text{otherwise} \end{cases}$$

input
```
integrate((b**4*x**4+4*a*b**3*x**3+6*a**2*b**2*x**2+4*a**3*b*x+a**4)**p,x)
```

output
```
Piecewise((x/(a**4)**(1/4), Eq(b, 0) & Eq(p, -1/4)), (x*(a**4)**p, Eq(b, 0
)), (Integral((a**4 + 4*a**3*b*x + 6*a**2*b**2*x**2 + 4*a*b**3*x**3 + b**4
*x**4)**(-1/4), x), Eq(p, -1/4)), (a*(a**4 + 4*a**3*b*x + 6*a**2*b**2*x**2
 + 4*a*b**3*x**3 + b**4*x**4)**p/(4*b*p + b) + b*x*(a**4 + 4*a**3*b*x + 6*
a**2*b**2*x**2 + 4*a*b**3*x**3 + b**4*x**4)**p/(4*b*p + b), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.45

$$\int \left(a^4 + 4a^3bx + 6a^2b^2x^2 + 4ab^3x^3 + b^4x^4\right)^p \, dx = \frac{(bx + a)(bx + a)^{4p}}{b(4p + 1)}$$

input
```
integrate((b^4*x^4+4*a*b^3*x^3+6*a^2*b^2*x^2+4*a^3*b*x+a^4)^p,x, algorithm
="maxima")
```

output
```
(b*x + a)*(b*x + a)^(4*p)/(b*(4*p + 1))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.70

$$\int \left(a^4+4a^3bx+6a^2b^2x^2+4ab^3x^3+b^4x^4\right)^p\,dx = \frac{(b^4x^4+4\,ab^3x^3+6\,a^2b^2x^2+4\,a^3bx+a^4)^p bx+(b^4x^4+4\,ab^3x^3+6\,a^2b^2x^2+4\,a^3bx+a^4)^p a}{4\,bp+b}$$

input
```
integrate((b^4*x^4+4*a*b^3*x^3+6*a^2*b^2*x^2+4*a^3*b*x+a^4)^p,x, algorithm
="giac")
```

output
```
((b^4*x^4 + 4*a*b^3*x^3 + 6*a^2*b^2*x^2 + 4*a^3*b*x + a^4)^p*b*x + (b^4*x^
4 + 4*a*b^3*x^3 + 6*a^2*b^2*x^2 + 4*a^3*b*x + a^4)^p*a)/(4*b*p + b)
```

Mupad [B] (verification not implemented)

Time = 21.40 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.12

$$\int \left(a^4+4a^3bx+6a^2b^2x^2+4ab^3x^3+b^4x^4\right)^p\,dx = \left(\frac{x}{4\,p+1}+\frac{a}{b\,\left(4\,p+1\right)}\right)\,\left(a^4+4\,a^3\,b\,x+6\,a^2\,b^2\,x^2+4\,a\,b^3\,x^3+b^4\,x^4\right)^p$$

input
```
int((a^4 + b^4*x^4 + 4*a*b^3*x^3 + 6*a^2*b^2*x^2 + 4*a^3*b*x)^p,x)
```

output
```
(x/(4*p + 1) + a/(b*(4*p + 1)))*(a^4 + b^4*x^4 + 4*a*b^3*x^3 + 6*a^2*b^2*x
^2 + 4*a^3*b*x)^p
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00

$$\int \left(a^4+4a^3bx+6a^2b^2x^2+4ab^3x^3+b^4x^4\right)^p\,dx = \frac{\left(b^4x^4+4a\,b^3x^3+6a^2b^2x^2+4a^3bx+a^4\right)^p(bx+a)}{b\,(4p+1)}$$

input
```
int((b^4*x^4+4*a*b^3*x^3+6*a^2*b^2*x^2+4*a^3*b*x+a^4)^p,x)
```

output
```
((a**4 + 4*a**3*b*x + 6*a**2*b**2*x**2 + 4*a*b**3*x**3 + b**4*x**4)**p*(a
+ b*x))/(b*(4*p + 1))
```

3.3 $\int (1+4x+4x^2+4x^4)^4 \, dx$

Optimal result

Integrand size = 17, antiderivative size = 97

$$\int (1+4x+4x^2+4x^4)^4 \, dx = x+8x^2+\frac{112x^3}{3}+112x^4+\frac{1136x^5}{5}+\frac{992x^6}{3}+\frac{2752x^7}{7}+448x^8+\frac{4192x^9}{9}+384x^{10}+\frac{3328x^{11}}{11}+256x^{12}+\frac{1792x^{13}}{13}+\frac{512x^{14}}{7}+\frac{1024x^{15}}{15}+\frac{256x^{17}}{17}$$

output
```
x+8*x^2+112/3*x^3+112*x^4+1136/5*x^5+992/3*x^6+2752/7*x^7+448*x^8+4192/9*x
^9+384*x^10+3328/11*x^11+256*x^12+1792/13*x^13+512/7*x^14+1024/15*x^15+256
/17*x^17
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00

$$\int (1+4x+4x^2+4x^4)^4 \, dx = x+8x^2+\frac{112x^3}{3}+112x^4+\frac{1136x^5}{5}+\frac{992x^6}{3}+\frac{2752x^7}{7}+448x^8+\frac{4192x^9}{9}+384x^{10}+\frac{3328x^{11}}{11}+256x^{12}+\frac{1792x^{13}}{13}+\frac{512x^{14}}{7}+\frac{1024x^{15}}{15}+\frac{256x^{17}}{17}$$

input
```
Integrate[(1 + 4*x + 4*x^2 + 4*x^4)^4,x]
```

output
```
x + 8*x^2 + (112*x^3)/3 + 112*x^4 + (1136*x^5)/5 + (992*x^6)/3 + (2752*x^7
)/7 + 448*x^8 + (4192*x^9)/9 + 384*x^10 + (3328*x^11)/11 + 256*x^12 + (179
2*x^13)/13 + (512*x^14)/7 + (1024*x^15)/15 + (256*x^17)/17
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2465, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(4x^4+4x^2+4x+1\right)^4 dx$$

$$\Big\downarrow \text{ 2465}$$

$$\int (256x^{16}+1024x^{14}+1024x^{13}+1792x^{12}+3072x^{11}+3328x^{10}+3840x^9+4192x^8+3584x^7+2752x^6+1984$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{256x^{17}}{17}+\frac{1024x^{15}}{15}+\frac{512x^{14}}{7}+\frac{1792x^{13}}{13}+256x^{12}+\frac{3328x^{11}}{11}+384x^{10}+\frac{4192x^9}{9}+448x^8+\frac{2752x^7}{7}+\frac{992x^6}{3}+\frac{1136x^5}{5}+112x^4+\frac{112x^3}{3}+8x^2+x$$

input
```
Int[(1 + 4*x + 4*x^2 + 4*x^4)^4,x]
```

output
```
x + 8*x^2 + (112*x^3)/3 + 112*x^4 + (1136*x^5)/5 + (992*x^6)/3 + (2752*x^7
)/7 + 448*x^8 + (4192*x^9)/9 + 384*x^10 + (3328*x^11)/11 + 256*x^12 + (179
2*x^13)/13 + (512*x^14)/7 + (1024*x^15)/15 + (256*x^17)/17
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2465
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> Int[ExpandToSum[u, Px^p, x], x] /; PolyQ
[Px, x] && GtQ[Expon[Px, x], 2] && !BinomialQ[Px, x] && !TrinomialQ[Px, x
] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.80

method	result
gosper	$x+8x^2+\frac{112}{3}x^3+112x^4+\frac{1136}{5}x^5+\frac{992}{3}x^6+\frac{2752}{7}x^7+448x^8+\frac{4192}{9}x^9+384x^{10}+\frac{3328}{11}x^{11}$
default	$x+8x^2+\frac{112}{3}x^3+112x^4+\frac{1136}{5}x^5+\frac{992}{3}x^6+\frac{2752}{7}x^7+448x^8+\frac{4192}{9}x^9+384x^{10}+\frac{3328}{11}x^{11}$
norman	$x+8x^2+\frac{112}{3}x^3+112x^4+\frac{1136}{5}x^5+\frac{992}{3}x^6+\frac{2752}{7}x^7+448x^8+\frac{4192}{9}x^9+384x^{10}+\frac{3328}{11}x^{11}$
risch	$x+8x^2+\frac{112}{3}x^3+112x^4+\frac{1136}{5}x^5+\frac{992}{3}x^6+\frac{2752}{7}x^7+448x^8+\frac{4192}{9}x^9+384x^{10}+\frac{3328}{11}x^{11}$
parallelrisch	$x+8x^2+\frac{112}{3}x^3+112x^4+\frac{1136}{5}x^5+\frac{992}{3}x^6+\frac{2752}{7}x^7+448x^8+\frac{4192}{9}x^9+384x^{10}+\frac{3328}{11}x^{11}$
orering	$\frac{x(11531520x^{16}+52276224x^{14}+56010240x^{13}+105557760x^{12}+196035840x^{11}+231678720x^{10}+294053760x^{9}+356676320x^{8}}{765765}$

input
```
int((4*x^4+4*x^2+4*x+1)^4,x,method=_RETURNVERBOSE)
```

output
```
x+8*x^2+112/3*x^3+112*x^4+1136/5*x^5+992/3*x^6+2752/7*x^7+448*x^8+4192/9*x
^9+384*x^10+3328/11*x^11+256*x^12+1792/13*x^13+512/7*x^14+1024/15*x^15+256
/17*x^17
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.79

$$\int (1+4x+4x^2+4x^4)^4\,dx = \frac{256}{17}\,x^{17} + \frac{1024}{15}\,x^{15} + \frac{512}{7}\,x^{14} + \frac{1792}{13}\,x^{13} + 256\,x^{12} + \frac{3328}{11}\,x^{11} + 384\,x^{10} + \frac{4192}{9}\,x^{9} + 448\,x^{8} + \frac{2752}{7}\,x^{7} + \frac{992}{3}\,x^{6} + \frac{1136}{5}\,x^{5} + 112\,x^{4} + \frac{112}{3}\,x^{3} + 8\,x^{2} + x$$

input
```
integrate((4*x^4+4*x^2+4*x+1)^4,x, algorithm="fricas")
```

output
```
256/17*x^17 + 1024/15*x^15 + 512/7*x^14 + 1792/13*x^13 + 256*x^12 + 3328/1
1*x^11 + 384*x^10 + 4192/9*x^9 + 448*x^8 + 2752/7*x^7 + 992/3*x^6 + 1136/5
*x^5 + 112*x^4 + 112/3*x^3 + 8*x^2 + x
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.97

$$\int (1+4x+4x^2+4x^4)^4\,dx = \frac{256x^{17}}{17} + \frac{1024x^{15}}{15} + \frac{512x^{14}}{7} + \frac{1792x^{13}}{13} + 256x^{12} + \frac{3328x^{11}}{11} + 384x^{10} + \frac{4192x^{9}}{9} + 448x^{8} + \frac{2752x^{7}}{7} + \frac{992x^{6}}{3} + \frac{1136x^{5}}{5} + 112x^{4} + \frac{112x^{3}}{3} + 8x^{2} + x$$

input
```
integrate((4*x**4+4*x**2+4*x+1)**4,x)
```

output
```
256*x**17/17 + 1024*x**15/15 + 512*x**14/7 + 1792*x**13/13 + 256*x**12 + 3
328*x**11/11 + 384*x**10 + 4192*x**9/9 + 448*x**8 + 2752*x**7/7 + 992*x**6
/3 + 1136*x**5/5 + 112*x**4 + 112*x**3/3 + 8*x**2 + x
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.79

$$\int \left(1+4x+4x^2+4x^4\right)^4 dx = \frac{256}{17}x^{17}+\frac{1024}{15}x^{15}+\frac{512}{7}x^{14}+\frac{1792}{13}x^{13}+256x^{12}+\frac{3328}{11}x^{11}+384x^{10}+\frac{4192}{9}x^9+448x^8+\frac{2752}{7}x^7+\frac{992}{3}x^6+\frac{1136}{5}x^5+112x^4+\frac{112}{3}x^3+8x^2+x$$

input
```
integrate((4*x^4+4*x^2+4*x+1)^4,x, algorithm="maxima")
```

output
```
256/17*x^17 + 1024/15*x^15 + 512/7*x^14 + 1792/13*x^13 + 256*x^12 + 3328/1
1*x^11 + 384*x^10 + 4192/9*x^9 + 448*x^8 + 2752/7*x^7 + 992/3*x^6 + 1136/5
*x^5 + 112*x^4 + 112/3*x^3 + 8*x^2 + x
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.79

$$\int \left(1+4x+4x^2+4x^4\right)^4 dx = \frac{256}{17}x^{17}+\frac{1024}{15}x^{15}+\frac{512}{7}x^{14}+\frac{1792}{13}x^{13}+256x^{12}+\frac{3328}{11}x^{11}+384x^{10}+\frac{4192}{9}x^9+448x^8+\frac{2752}{7}x^7+\frac{992}{3}x^6+\frac{1136}{5}x^5+112x^4+\frac{112}{3}x^3+8x^2+x$$

input
```
integrate((4*x^4+4*x^2+4*x+1)^4,x, algorithm="giac")
```

output
```
256/17*x^17 + 1024/15*x^15 + 512/7*x^14 + 1792/13*x^13 + 256*x^12 + 3328/1
1*x^11 + 384*x^10 + 4192/9*x^9 + 448*x^8 + 2752/7*x^7 + 992/3*x^6 + 1136/5
*x^5 + 112*x^4 + 112/3*x^3 + 8*x^2 + x
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.79

$$\int (1+4x+4x^2+4x^4)^4\,dx = \frac{256\,x^{17}}{17}+\frac{1024\,x^{15}}{15}+\frac{512\,x^{14}}{7}+\frac{1792\,x^{13}}{13}+256\,x^{12}+\frac{3328\,x^{11}}{11}+384\,x^{10}+\frac{4192\,x^9}{9}+448\,x^8+\frac{2752\,x^7}{7}+\frac{992\,x^6}{3}+\frac{1136\,x^5}{5}+112\,x^4+\frac{112\,x^3}{3}+8\,x^2+x$$

input

```
int((4*x + 4*x^2 + 4*x^4 + 1)^4,x)
```

output

```
x + 8*x^2 + (112*x^3)/3 + 112*x^4 + (1136*x^5)/5 + (992*x^6)/3 + (2752*x^7
)/7 + 448*x^8 + (4192*x^9)/9 + 384*x^10 + (3328*x^11)/11 + 256*x^12 + (179
2*x^13)/13 + (512*x^14)/7 + (1024*x^15)/15 + (256*x^17)/17
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.80

$$\int (1+4x+4x^2+4x^4)^4\,dx = \frac{x(11531520x^{16}+52276224x^{14}+56010240x^{13}+105557760x^{12}+196035840x^{11}+231678720x^{10}+2940}{}$$

input

```
int((4*x^4+4*x^2+4*x+1)^4,x)
```

output

```
(x*(11531520*x**16 + 52276224*x**14 + 56010240*x**13 + 105557760*x**12 + 1
96035840*x**11 + 231678720*x**10 + 294053760*x**9 + 356676320*x**8 + 34306
2720*x**7 + 301055040*x**6 + 253212960*x**5 + 173981808*x**4 + 85765680*x*
*3 + 28588560*x**2 + 6126120*x + 765765))/765765
```

3.4 $\int (1+4x+4x^2+4x^4)^3 \, dx$

Optimal result

Integrand size = 17, antiderivative size = 69

$$\int (1+4x+4x^2+4x^4)^3 \, dx = x+6x^2+20x^3+40x^4+\frac{252x^5}{5}+48x^6+\frac{352x^7}{7}+48x^8+\frac{80x^9}{3}+\frac{96x^{10}}{5}+\frac{192x^{11}}{11}+\frac{64x^{13}}{13}$$

output
```
x+6*x^2+20*x^3+40*x^4+252/5*x^5+48*x^6+352/7*x^7+48*x^8+80/3*x^9+96/5*x^10
+192/11*x^11+64/13*x^13
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00

$$\int (1+4x+4x^2+4x^4)^3 \, dx = x+6x^2+20x^3+40x^4+\frac{252x^5}{5}+48x^6+\frac{352x^7}{7}+48x^8+\frac{80x^9}{3}+\frac{96x^{10}}{5}+\frac{192x^{11}}{11}+\frac{64x^{13}}{13}$$

input
```
Integrate[(1 + 4*x + 4*x^2 + 4*x^4)^3,x]
```

output
```
x + 6*x^2 + 20*x^3 + 40*x^4 + (252*x^5)/5 + 48*x^6 + (352*x^7)/7 + 48*x^8
+ (80*x^9)/3 + (96*x^10)/5 + (192*x^11)/11 + (64*x^13)/13
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2465, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(4x^4+4x^2+4x+1\right)^3 dx$$

$$\Big\downarrow \text{2465}$$

$$\int \left(64x^{12}+192x^{10}+192x^9+240x^8+384x^7+352x^6+288x^5+252x^4+160x^3+60x^2+12x+1\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{64x^{13}}{13}+\frac{192x^{11}}{11}+\frac{96x^{10}}{5}+\frac{80x^9}{3}+48x^8+\frac{352x^7}{7}+48x^6+\frac{252x^5}{5}+40x^4+20x^3+6x^2+x$$

input
```
Int[(1 + 4*x + 4*x^2 + 4*x^4)^3,x]
```

output
```
x + 6*x^2 + 20*x^3 + 40*x^4 + (252*x^5)/5 + 48*x^6 + (352*x^7)/7 + 48*x^8
+ (80*x^9)/3 + (96*x^10)/5 + (192*x^11)/11 + (64*x^13)/13
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2465
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> Int[ExpandToSum[u, Px^p, x], x] /; PolyQ
[Px, x] && GtQ[Expon[Px, x], 2] && !BinomialQ[Px, x] && !TrinomialQ[Px, x
] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.84

method	result
gosper	$x+6x^2+20x^3+40x^4+\frac{252}{5}x^5+48x^6+\frac{352}{7}x^7+48x^8+\frac{80}{3}x^9+\frac{96}{5}x^{10}+\frac{192}{11}x^{11}+\frac{64}{13}x^{13}$
default	$x+6x^2+20x^3+40x^4+\frac{252}{5}x^5+48x^6+\frac{352}{7}x^7+48x^8+\frac{80}{3}x^9+\frac{96}{5}x^{10}+\frac{192}{11}x^{11}+\frac{64}{13}x^{13}$
norman	$x+6x^2+20x^3+40x^4+\frac{252}{5}x^5+48x^6+\frac{352}{7}x^7+48x^8+\frac{80}{3}x^9+\frac{96}{5}x^{10}+\frac{192}{11}x^{11}+\frac{64}{13}x^{13}$
risch	$x+6x^2+20x^3+40x^4+\frac{252}{5}x^5+48x^6+\frac{352}{7}x^7+48x^8+\frac{80}{3}x^9+\frac{96}{5}x^{10}+\frac{192}{11}x^{11}+\frac{64}{13}x^{13}$
parallelrisch	$x+6x^2+20x^3+40x^4+\frac{252}{5}x^5+48x^6+\frac{352}{7}x^7+48x^8+\frac{80}{3}x^9+\frac{96}{5}x^{10}+\frac{192}{11}x^{11}+\frac{64}{13}x^{13}$
orering	$\frac{x(73920x^{12}+262080x^{10}+288288x^9+400400x^8+720720x^7+755040x^6+720720x^5+756756x^4+600600x^3+300300x^2+90090x}{15015}$

input
```
int((4*x^4+4*x^2+4*x+1)^3,x,method=_RETURNVERBOSE)
```

output
```
x+6*x^2+20*x^3+40*x^4+252/5*x^5+48*x^6+352/7*x^7+48*x^8+80/3*x^9+96/5*x^10
+192/11*x^11+64/13*x^13
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.83

$$\int \left(1+4x+4x^2+4x^4\right)^3\,dx=\frac{64}{13}\,x^{13}+\frac{192}{11}\,x^{11}+\frac{96}{5}\,x^{10}+\frac{80}{3}\,x^9+48\,x^8+\frac{352}{7}\,x^7+48\,x^6+\frac{252}{5}\,x^5+40\,x^4+20\,x^3+6\,x^2+x$$

input
```
integrate((4*x^4+4*x^2+4*x+1)^3,x, algorithm="fricas")
```

output
```
64/13*x^13 + 192/11*x^11 + 96/5*x^10 + 80/3*x^9 + 48*x^8 + 352/7*x^7 + 48*
x^6 + 252/5*x^5 + 40*x^4 + 20*x^3 + 6*x^2 + x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.96

$$\int (1+4x+4x^2+4x^4)^3\,dx = \frac{64x^{13}}{13}+\frac{192x^{11}}{11}+\frac{96x^{10}}{5}+\frac{80x^9}{3}+48x^8+\frac{352x^7}{7}+48x^6+\frac{252x^5}{5}+40x^4+20x^3+6x^2+x$$

input
```
integrate((4*x**4+4*x**2+4*x+1)**3,x)
```

output
```
64*x**13/13 + 192*x**11/11 + 96*x**10/5 + 80*x**9/3 + 48*x**8 + 352*x**7/7
 + 48*x**6 + 252*x**5/5 + 40*x**4 + 20*x**3 + 6*x**2 + x
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.83

$$\int (1+4x+4x^2+4x^4)^3\,dx = \frac{64}{13}\,x^{13}+\frac{192}{11}\,x^{11}+\frac{96}{5}\,x^{10}+\frac{80}{3}\,x^9+48\,x^8+\frac{352}{7}\,x^7+48\,x^6+\frac{252}{5}\,x^5+40\,x^4+20\,x^3+6\,x^2+x$$

input
```
integrate((4*x^4+4*x^2+4*x+1)^3,x, algorithm="maxima")
```

output
```
64/13*x^13 + 192/11*x^11 + 96/5*x^10 + 80/3*x^9 + 48*x^8 + 352/7*x^7 + 48*
x^6 + 252/5*x^5 + 40*x^4 + 20*x^3 + 6*x^2 + x
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.83

$$\int \left(1+4x+4x^2+4x^4\right)^3 \, dx = \frac{64}{13}\,x^{13}+\frac{192}{11}\,x^{11}+\frac{96}{5}\,x^{10}+\frac{80}{3}\,x^9+48\,x^8+\frac{352}{7}\,x^7+48\,x^6+\frac{252}{5}\,x^5+40\,x^4+20\,x^3+6\,x^2+x$$

input
```
integrate((4*x^4+4*x^2+4*x+1)^3,x, algorithm="giac")
```

output
```
64/13*x^13 + 192/11*x^11 + 96/5*x^10 + 80/3*x^9 + 48*x^8 + 352/7*x^7 + 48*
x^6 + 252/5*x^5 + 40*x^4 + 20*x^3 + 6*x^2 + x
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.83

$$\int \left(1+4x+4x^2+4x^4\right)^3 \, dx = \frac{64\,x^{13}}{13}+\frac{192\,x^{11}}{11}+\frac{96\,x^{10}}{5}+\frac{80\,x^9}{3}+48\,x^8+\frac{352\,x^7}{7}+48\,x^6+\frac{252\,x^5}{5}+40\,x^4+20\,x^3+6\,x^2+x$$

input
```
int((4*x + 4*x^2 + 4*x^4 + 1)^3,x)
```

output
```
x + 6*x^2 + 20*x^3 + 40*x^4 + (252*x^5)/5 + 48*x^6 + (352*x^7)/7 + 48*x^8
+ (80*x^9)/3 + (96*x^10)/5 + (192*x^11)/11 + (64*x^13)/13
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.84

$$\int \left(1+4x+4x^2+4x^4\right)^3 \, dx$$
$$= \frac{x(73920x^{12}+262080x^{10}+288288x^9+400400x^8+720720x^7+755040x^6+720720x^5+756756x^4+60}{15015}$$

input
```
int((4*x^4+4*x^2+4*x+1)^3,x)
```

output
```
(x*(73920*x**12 + 262080*x**10 + 288288*x**9 + 400400*x**8 + 720720*x**7 +
 755040*x**6 + 720720*x**5 + 756756*x**4 + 600600*x**3 + 300300*x**2 + 900
90*x + 15015))/15015
```

3.5 $\int (1+4x+4x^2+4x^4)^2 \, dx$

Optimal result

Integrand size = 17, antiderivative size = 45

$$\int (1+4x+4x^2+4x^4)^2 \, dx = x+4x^2+8x^3+8x^4+\frac{24x^5}{5}+\frac{16x^6}{3}+\frac{32x^7}{7}+\frac{16x^9}{9}$$

output
```
x+4*x^2+8*x^3+8*x^4+24/5*x^5+16/3*x^6+32/7*x^7+16/9*x^9
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00

$$\int (1+4x+4x^2+4x^4)^2 \, dx = x+4x^2+8x^3+8x^4+\frac{24x^5}{5}+\frac{16x^6}{3}+\frac{32x^7}{7}+\frac{16x^9}{9}$$

input
```
Integrate[(1 + 4*x + 4*x^2 + 4*x^4)^2,x]
```

output
```
x + 4*x^2 + 8*x^3 + 8*x^4 + (24*x^5)/5 + (16*x^6)/3 + (32*x^7)/7 + (16*x^9
)/9
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2465, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (4x^4 + 4x^2 + 4x + 1)^2 \, dx$$

$$\Big\downarrow \text{2465}$$

$$\int (16x^8 + 32x^6 + 32x^5 + 24x^4 + 32x^3 + 24x^2 + 8x + 1) \, dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{16x^9}{9} + \frac{32x^7}{7} + \frac{16x^6}{3} + \frac{24x^5}{5} + 8x^4 + 8x^3 + 4x^2 + x$$

input
```
Int[(1 + 4*x + 4*x^2 + 4*x^4)^2,x]
```

output
```
x + 4*x^2 + 8*x^3 + 8*x^4 + (24*x^5)/5 + (16*x^6)/3 + (32*x^7)/7 + (16*x^9
)/9
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2465
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> Int[ExpandToSum[u, Px^p, x], x] /; PolyQ
[Px, x] && GtQ[Expon[Px, x], 2] && !BinomialQ[Px, x] && !TrinomialQ[Px, x
] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.84

method	result	size
gosper	$x+4x^2+8x^3+8x^4+\frac{24}{5}x^5+\frac{16}{3}x^6+\frac{32}{7}x^7+\frac{16}{9}x^9$	38
default	$x+4x^2+8x^3+8x^4+\frac{24}{5}x^5+\frac{16}{3}x^6+\frac{32}{7}x^7+\frac{16}{9}x^9$	38
norman	$x+4x^2+8x^3+8x^4+\frac{24}{5}x^5+\frac{16}{3}x^6+\frac{32}{7}x^7+\frac{16}{9}x^9$	38
risch	$x+4x^2+8x^3+8x^4+\frac{24}{5}x^5+\frac{16}{3}x^6+\frac{32}{7}x^7+\frac{16}{9}x^9$	38
parallelrisch	$x+4x^2+8x^3+8x^4+\frac{24}{5}x^5+\frac{16}{3}x^6+\frac{32}{7}x^7+\frac{16}{9}x^9$	38
orering	$\frac{x(560x^8+1440x^6+1680x^5+1512x^4+2520x^3+2520x^2+1260x+315)}{315}$	39

input
```
int((4*x^4+4*x^2+4*x+1)^2,x,method=_RETURNVERBOSE)
```

output
```
x+4*x^2+8*x^3+8*x^4+24/5*x^5+16/3*x^6+32/7*x^7+16/9*x^9
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.82

$$\int (1+4x+4x^2+4x^4)^2\,dx = \frac{16}{9}\,x^9+\frac{32}{7}\,x^7+\frac{16}{3}\,x^6+\frac{24}{5}\,x^5+8\,x^4+8\,x^3+4\,x^2+x$$

input
```
integrate((4*x^4+4*x^2+4*x+1)^2,x, algorithm="fricas")
```

output
```
16/9*x^9 + 32/7*x^7 + 16/3*x^6 + 24/5*x^5 + 8*x^4 + 8*x^3 + 4*x^2 + x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.93

$$\int \left(1+4x+4x^2+4x^4\right)^2\,dx=\frac{16x^9}{9}+\frac{32x^7}{7}+\frac{16x^6}{3}+\frac{24x^5}{5}+8x^4+8x^3+4x^2+x$$

input
```
integrate((4*x**4+4*x**2+4*x+1)**2,x)
```

output
```
16*x**9/9 + 32*x**7/7 + 16*x**6/3 + 24*x**5/5 + 8*x**4 + 8*x**3 + 4*x**2 +
 x
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.82

$$\int \left(1+4x+4x^2+4x^4\right)^2\,dx=\frac{16}{9}\,x^9+\frac{32}{7}\,x^7+\frac{16}{3}\,x^6+\frac{24}{5}\,x^5+8\,x^4+8\,x^3+4\,x^2+x$$

input
```
integrate((4*x^4+4*x^2+4*x+1)^2,x, algorithm="maxima")
```

output
```
16/9*x^9 + 32/7*x^7 + 16/3*x^6 + 24/5*x^5 + 8*x^4 + 8*x^3 + 4*x^2 + x
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.82

$$\int \left(1+4x+4x^2+4x^4\right)^2\,dx=\frac{16}{9}\,x^9+\frac{32}{7}\,x^7+\frac{16}{3}\,x^6+\frac{24}{5}\,x^5+8\,x^4+8\,x^3+4\,x^2+x$$

input
```
integrate((4*x^4+4*x^2+4*x+1)^2,x, algorithm="giac")
```

output
```
16/9*x^9 + 32/7*x^7 + 16/3*x^6 + 24/5*x^5 + 8*x^4 + 8*x^3 + 4*x^2 + x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.82

$$\int \left(1+4x+4x^2+4x^4\right)^2 \, dx = \frac{16\,x^9}{9}+\frac{32\,x^7}{7}+\frac{16\,x^6}{3}+\frac{24\,x^5}{5}+8\,x^4+8\,x^3+4\,x^2+x$$

input
```
int((4*x + 4*x^2 + 4*x^4 + 1)^2,x)
```

output
```
x + 4*x^2 + 8*x^3 + 8*x^4 + (24*x^5)/5 + (16*x^6)/3 + (32*x^7)/7 + (16*x^9
)/9
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.84

$$\int \left(1+4x+4x^2+4x^4\right)^2 \, dx = \frac{x(560x^8+1440x^6+1680x^5+1512x^4+2520x^3+2520x^2+1260x+315)}{315}$$

input
```
int((4*x^4+4*x^2+4*x+1)^2,x)
```

output
```
(x*(560*x**8 + 1440*x**6 + 1680*x**5 + 1512*x**4 + 2520*x**3 + 2520*x**2 +
 1260*x + 315))/315
```

3.6 $\int (1+4x+4x^2+4x^4) \, dx$

Optimal result

Integrand size = 15, antiderivative size = 21

$$\int (1+4x+4x^2+4x^4) \, dx = x+2x^2+\frac{4x^3}{3}+\frac{4x^5}{5}$$

output
```
x+2*x^2+4/3*x^3+4/5*x^5
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int (1+4x+4x^2+4x^4) \, dx = x+2x^2+\frac{4x^3}{3}+\frac{4x^5}{5}$$

input
```
Integrate[1 + 4*x + 4*x^2 + 4*x^4,x]
```

output
```
x + 2*x^2 + (4*x^3)/3 + (4*x^5)/5
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(4x^4+4x^2+4x+1\right) \, dx$$

$$\Big\downarrow \; 2009$$

$$\frac{4x^5}{5}+\frac{4x^3}{3}+2x^2+x$$

input
```
Int[1 + 4*x + 4*x^2 + 4*x^4,x]
```

output
```
x + 2*x^2 + (4*x^3)/3 + (4*x^5)/5
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

method	result	size
gosper	$x+2x^2+\frac{4}{3}x^3+\frac{4}{5}x^5$	18
default	$x+2x^2+\frac{4}{3}x^3+\frac{4}{5}x^5$	18
norman	$x+2x^2+\frac{4}{3}x^3+\frac{4}{5}x^5$	18
risch	$x+2x^2+\frac{4}{3}x^3+\frac{4}{5}x^5$	18
parallelrisch	$x+2x^2+\frac{4}{3}x^3+\frac{4}{5}x^5$	18
parts	$x+2x^2+\frac{4}{3}x^3+\frac{4}{5}x^5$	18
orering	$\frac{x(12x^4+20x^2+30x+15)}{15}$	19

input
```
int(4*x^4+4*x^2+4*x+1,x,method=_RETURNVERBOSE)
```

output
```
x+2*x^2+4/3*x^3+4/5*x^5
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \left(1+4x+4x^2+4x^4\right) dx = \frac{4}{5}x^5 + \frac{4}{3}x^3 + 2x^2 + x$$

input
```
integrate(4*x^4+4*x^2+4*x+1,x, algorithm="fricas")
```

output
```
4/5*x^5 + 4/3*x^3 + 2*x^2 + x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \left(1+4x+4x^2+4x^4\right)\,dx=\frac{4x^5}{5}+\frac{4x^3}{3}+2x^2+x$$

input
```
integrate(4*x**4+4*x**2+4*x+1,x)
```

output
```
4*x**5/5 + 4*x**3/3 + 2*x**2 + x
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \left(1+4x+4x^2+4x^4\right)\,dx=\frac{4}{5}\,x^5+\frac{4}{3}\,x^3+2\,x^2+x$$

input
```
integrate(4*x^4+4*x^2+4*x+1,x, algorithm="maxima")
```

output
```
4/5*x^5 + 4/3*x^3 + 2*x^2 + x
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \left(1+4x+4x^2+4x^4\right)\,dx=\frac{4}{5}\,x^5+\frac{4}{3}\,x^3+2\,x^2+x$$

input
```
integrate(4*x^4+4*x^2+4*x+1,x, algorithm="giac")
```

output
```
4/5*x^5 + 4/3*x^3 + 2*x^2 + x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int (1 + 4x + 4x^2 + 4x^4) \, dx = \frac{4\,x^5}{5} + \frac{4\,x^3}{3} + 2\,x^2 + x$$

input
```
int(4*x + 4*x^2 + 4*x^4 + 1,x)
```

output
```
x + 2*x^2 + (4*x^3)/3 + (4*x^5)/5
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

$$\int (1 + 4x + 4x^2 + 4x^4) \, dx = \frac{x(12x^4 + 20x^2 + 30x + 15)}{15}$$

input
```
int(4*x^4+4*x^2+4*x+1,x)
```

output
```
(x*(12*x**4 + 20*x**2 + 30*x + 15))/15
```

3.7 $\int \frac{1}{1+4x+4x^2+4x^4}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 185

$$\int \frac{1}{1+4x+4x^2+4x^4}\,dx = \frac{1}{2}\arctan\left(\frac{1}{2}\left(-1+\left(1+\frac{1}{x}\right)^2\right)\right) - \frac{\left(1+\sqrt{5}\right)^{3/2}\arctan\left(\frac{2-\sqrt{2\left(1+\sqrt{5}\right)}+\frac{2}{x}}{\sqrt{2\left(-1+\sqrt{5}\right)}}\right)}{4\sqrt{10}} - \frac{\left(1+\sqrt{5}\right)^{3/2}\arctan\left(\frac{2+\sqrt{2\left(1+\sqrt{5}\right)}+\frac{2}{x}}{\sqrt{2\left(-1+\sqrt{5}\right)}}\right)}{4\sqrt{10}} + \frac{1}{2}\sqrt{\frac{1}{5}\left(-2+\sqrt{5}\right)}\,\text{arctanh}\left(\frac{\sqrt{2\left(1+\sqrt{5}\right)}\left(1+\frac{1}{x}\right)}{\sqrt{5}+\left(1+\frac{1}{x}\right)^2}\right)$$

output
```
1/2*arctan(-1/2+1/2*(1+1/x)^2)-1/40*(5^(1/2)+1)^(3/2)*arctan((2-(2+2*5^(1/
2))^(1/2)+2/x)/(-2+2*5^(1/2))^(1/2))*10^(1/2)-1/40*(5^(1/2)+1)^(3/2)*arcta
n((2+(2+2*5^(1/2))^(1/2)+2/x)/(-2+2*5^(1/2))^(1/2))*10^(1/2)+1/10*(-10+5*5
^(1/2))^(1/2)*arctanh((2+2*5^(1/2))^(1/2)*(1+1/x)/(5^(1/2)+(1+1/x)^2))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.25

$$\int \frac{1}{1+4x+4x^2+4x^4}\,dx=\frac{1}{4}\text{RootSum}\left[1+4\#1+4\#1^2+4\#1^4\&,\frac{\log(x-\#1)}{1+2\#1+4\#1^3}\&\right]$$

input
```
Integrate[(1 + 4*x + 4*x^2 + 4*x^4)^(-1),x]
```

output
```
RootSum[1 + 4*#1 + 4*#1^2 + 4*#1^4 & , Log[x - #1]/(1 + 2*#1 + 4*#1^3) & ]
/4
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.46, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.765, Rules used = {2504, 27, 2202, 27, 1432, 1083, 217, 1483, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{4x^4+4x^2+4x+1}dx$$

$$\Big\downarrow \text{2504}$$

$$-16\int \frac{1}{16\left(\left(1+\frac{1}{x}\right)^4-2\left(1+\frac{1}{x}\right)^2+5\right)x^2}d\left(1+\frac{1}{x}\right)$$

$$\Big\downarrow \text{27}$$

$$-\int \frac{1}{\left(\left(1+\frac{1}{x}\right)^4-2\left(1+\frac{1}{x}\right)^2+5\right)x^2}d\left(1+\frac{1}{x}\right)$$

$$\Big\downarrow \text{2202}$$

$$-\int \frac{\left(1+\frac{1}{x}\right)^2+1}{\left(1+\frac{1}{x}\right)^4-2\left(1+\frac{1}{x}\right)^2+5}d\left(1+\frac{1}{x}\right)-\int -\frac{2\left(1+\frac{1}{x}\right)}{\left(1+\frac{1}{x}\right)^4-2\left(1+\frac{1}{x}\right)^2+5}d\left(1+\frac{1}{x}\right)$$

$$\Big\downarrow \ 27$$

$$2\int \frac{1+\frac{1}{x}}{\left(1+\frac{1}{x}\right)^4-2\left(1+\frac{1}{x}\right)^2+5}d\left(1+\frac{1}{x}\right)-\int \frac{\left(1+\frac{1}{x}\right)^2+1}{\left(1+\frac{1}{x}\right)^4-2\left(1+\frac{1}{x}\right)^2+5}d\left(1+\frac{1}{x}\right)$$

$$\Big\downarrow \ 1432$$

$$\int \frac{1}{\left(1+\frac{1}{x}\right)^4-2\left(1+\frac{1}{x}\right)^2+5}d\left(1+\frac{1}{x}\right)^2-\int \frac{\left(1+\frac{1}{x}\right)^2+1}{\left(1+\frac{1}{x}\right)^4-2\left(1+\frac{1}{x}\right)^2+5}d\left(1+\frac{1}{x}\right)$$

$$\Big\downarrow \ 1083$$

$$-2\int \frac{1}{-\left(1+\frac{1}{x}\right)^4-16}d\left(2\left(1+\frac{1}{x}\right)^2-2\right)-\int \frac{\left(1+\frac{1}{x}\right)^2+1}{\left(1+\frac{1}{x}\right)^4-2\left(1+\frac{1}{x}\right)^2+5}d\left(1+\frac{1}{x}\right)$$

$$\Big\downarrow \ 217$$

$$\frac{1}{2}\arctan\left(\frac{1}{4}\left(2\left(\frac{1}{x}+1\right)^2-2\right)\right)-\int \frac{\left(1+\frac{1}{x}\right)^2+1}{\left(1+\frac{1}{x}\right)^4-2\left(1+\frac{1}{x}\right)^2+5}d\left(1+\frac{1}{x}\right)$$

$$\Big\downarrow \ 1483$$

$$-\frac{\int \frac{\sqrt{2\left(1+\sqrt{5}\right)}-\left(1-\sqrt{5}\right)\left(1+\frac{1}{x}\right)}{\left(1+\frac{1}{x}\right)^2-\sqrt{2\left(1+\sqrt{5}\right)}\left(1+\frac{1}{x}\right)+\sqrt{5}}d\left(1+\frac{1}{x}\right)}{2\sqrt{10\left(1+\sqrt{5}\right)}}-\frac{\int \frac{\left(1-\sqrt{5}\right)\left(1+\frac{1}{x}\right)+\sqrt{2\left(1+\sqrt{5}\right)}}{\left(1+\frac{1}{x}\right)^2+\sqrt{2\left(1+\sqrt{5}\right)}\left(1+\frac{1}{x}\right)+\sqrt{5}}d\left(1+\frac{1}{x}\right)}{2\sqrt{10\left(1+\sqrt{5}\right)}}+\frac{1}{2}\arctan\left(\frac{1}{4}\left(2\left(\frac{1}{x}+1\right)^2-2\right)\right)$$

$$\Big\downarrow \ 1142$$

$$-\frac{\frac{\left(1+\sqrt{5}\right)^{3/2}\int \frac{1}{\left(1+\frac{1}{x}\right)^2-\sqrt{2\left(1+\sqrt{5}\right)}\left(1+\frac{1}{x}\right)+\sqrt{5}}d\left(1+\frac{1}{x}\right)}{\sqrt{2}}-\frac{1}{2}\left(1-\sqrt{5}\right)\int -\frac{\sqrt{2\left(1+\sqrt{5}\right)}-2\left(1+\frac{1}{x}\right)}{\left(1+\frac{1}{x}\right)^2-\sqrt{2\left(1+\sqrt{5}\right)}\left(1+\frac{1}{x}\right)+\sqrt{5}}d\left(1+\frac{1}{x}\right)}{2\sqrt{10\left(1+\sqrt{5}\right)}}-\frac{\frac{\left(1+\sqrt{5}\right)^{3/2}\int \frac{1}{\left(1+\frac{1}{x}\right)^2+\sqrt{2\left(1+\sqrt{5}\right)}\left(1+\frac{1}{x}\right)+\sqrt{5}}d\left(1+\frac{1}{x}\right)}{\sqrt{2}}+\frac{1}{2}\left(1-\sqrt{5}\right)\int \frac{2\left(1+\frac{1}{x}\right)+\sqrt{2\left(1+\sqrt{5}\right)}}{\left(1+\frac{1}{x}\right)^2+\sqrt{2\left(1+\sqrt{5}\right)}\left(1+\frac{1}{x}\right)+\sqrt{5}}d\left(1+\frac{1}{x}\right)}{2\sqrt{10\left(1+\sqrt{5}\right)}}+\frac{1}{2}\arctan\left(\frac{1}{4}\left(2\left(\frac{1}{x}+1\right)^2-2\right)\right)$$

$$\Big\downarrow \; 25$$

$$-\frac{\frac{\left(1+\sqrt{5}\right)^{3/2}\int \frac{1}{\left(1+\frac{1}{x}\right)^2-\sqrt{2\left(1+\sqrt{5}\right)}\left(1+\frac{1}{x}\right)+\sqrt{5}}d\left(1+\frac{1}{x}\right)}{\sqrt{2}}+\frac{1}{2}\left(1-\sqrt{5}\right)\int \frac{\sqrt{2\left(1+\sqrt{5}\right)}-2\left(1+\frac{1}{x}\right)}{\left(1+\frac{1}{x}\right)^2-\sqrt{2\left(1+\sqrt{5}\right)}\left(1+\frac{1}{x}\right)+\sqrt{5}}d\left(1+\frac{1}{x}\right)}{2\sqrt{10\left(1+\sqrt{5}\right)}}-\frac{\frac{\left(1+\sqrt{5}\right)^{3/2}\int \frac{1}{\left(1+\frac{1}{x}\right)^2+\sqrt{2\left(1+\sqrt{5}\right)}\left(1+\frac{1}{x}\right)+\sqrt{5}}d\left(1+\frac{1}{x}\right)}{\sqrt{2}}+\frac{1}{2}\left(1-\sqrt{5}\right)\int \frac{2\left(1+\frac{1}{x}\right)+\sqrt{2\left(1+\sqrt{5}\right)}}{\left(1+\frac{1}{x}\right)^2+\sqrt{2\left(1+\sqrt{5}\right)}\left(1+\frac{1}{x}\right)+\sqrt{5}}d\left(1+\frac{1}{x}\right)}{2\sqrt{10\left(1+\sqrt{5}\right)}}+\frac{1}{2}\arctan\left(\frac{1}{4}\left(2\left(\frac{1}{x}+1\right)^2-2\right)\right)$$

$$\Big\downarrow \; 1083$$

$$-\frac{\frac{1}{2}\left(1-\sqrt{5}\right)\int \frac{\sqrt{2\left(1+\sqrt{5}\right)}-2\left(1+\frac{1}{x}\right)}{\left(1+\frac{1}{x}\right)^2-\sqrt{2\left(1+\sqrt{5}\right)}\left(1+\frac{1}{x}\right)+\sqrt{5}}d\left(1+\frac{1}{x}\right)-\sqrt{2}\left(1+\sqrt{5}\right)^{3/2}\int \frac{1}{2\left(1-\sqrt{5}\right)-\left(2\left(1+\frac{1}{x}\right)-\sqrt{2\left(1+\sqrt{5}\right)}\right)^2}d\left(2\left(1+\right.\right.}{2\sqrt{10\left(1+\sqrt{5}\right)}}$$

$$\frac{\frac{1}{2}\left(1-\sqrt{5}\right)\int \frac{2\left(1+\frac{1}{x}\right)+\sqrt{2\left(1+\sqrt{5}\right)}}{\left(1+\frac{1}{x}\right)^2+\sqrt{2\left(1+\sqrt{5}\right)}\left(1+\frac{1}{x}\right)+\sqrt{5}}d\left(1+\frac{1}{x}\right)-\sqrt{2}\left(1+\sqrt{5}\right)^{3/2}\int \frac{1}{2\left(1-\sqrt{5}\right)-\left(2\left(1+\frac{1}{x}\right)+\sqrt{2\left(1+\sqrt{5}\right)}\right)^2}d\left(2\left(1+\frac{1}{x}\right.\right.}{2\sqrt{10\left(1+\sqrt{5}\right)}}$$

$$\frac{1}{2}\arctan\left(\frac{1}{4}\left(2\left(\frac{1}{x}+1\right)^2-2\right)\right)$$

$$\Big\downarrow \; 217$$

$$-\frac{\frac{1}{2}(1-\sqrt{5})\int \frac{\sqrt{2\left(1+\sqrt{5}\right)}-2(1+\frac{1}{x})}{(1+\frac{1}{x})^2-\sqrt{2\left(1+\sqrt{5}\right)}(1+\frac{1}{x})+\sqrt{5}}d(1+\frac{1}{x})+\frac{\left(1+\sqrt{5}\right)^{3/2}\arctan\left(\frac{2\left(\frac{1}{x}+1\right)-\sqrt{2\left(1+\sqrt{5}\right)}}{\sqrt{2\left(\sqrt{5}-1\right)}}\right)}{\sqrt{\sqrt{5}-1}}}{2\sqrt{10\left(1+\sqrt{5}\right)}}-$$

$$\frac{\frac{1}{2}(1-\sqrt{5})\int \frac{2(1+\frac{1}{x})+\sqrt{2\left(1+\sqrt{5}\right)}}{(1+\frac{1}{x})^2+\sqrt{2\left(1+\sqrt{5}\right)}(1+\frac{1}{x})+\sqrt{5}}d(1+\frac{1}{x})+\frac{\left(1+\sqrt{5}\right)^{3/2}\arctan\left(\frac{2\left(\frac{1}{x}+1\right)+\sqrt{2\left(1+\sqrt{5}\right)}}{\sqrt{2\left(\sqrt{5}-1\right)}}\right)}{\sqrt{\sqrt{5}-1}}}{2\sqrt{10\left(1+\sqrt{5}\right)}}+$$

$$\frac{1}{2}\arctan\left(\frac{1}{4}\left(2\left(\frac{1}{x}+1\right)^2-2\right)\right)$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{2}\arctan\left(\frac{1}{4}\left(2\left(\frac{1}{x}+1\right)^2-2\right)\right)-$$

$$\frac{\frac{\left(1+\sqrt{5}\right)^{3/2}\arctan\left(\frac{2\left(\frac{1}{x}+1\right)-\sqrt{2\left(1+\sqrt{5}\right)}}{\sqrt{2\left(\sqrt{5}-1\right)}}\right)}{\sqrt{\sqrt{5}-1}}-\frac{1}{2}(1-\sqrt{5})\log\left(\left(\frac{1}{x}+1\right)^2-\sqrt{2\left(1+\sqrt{5}\right)}\left(\frac{1}{x}+1\right)+\sqrt{5}\right)}{2\sqrt{10\left(1+\sqrt{5}\right)}}-$$

$$\frac{\frac{\left(1+\sqrt{5}\right)^{3/2}\arctan\left(\frac{2\left(\frac{1}{x}+1\right)+\sqrt{2\left(1+\sqrt{5}\right)}}{\sqrt{2\left(\sqrt{5}-1\right)}}\right)}{\sqrt{\sqrt{5}-1}}+\frac{1}{2}(1-\sqrt{5})\log\left(\left(\frac{1}{x}+1\right)^2+\sqrt{2\left(1+\sqrt{5}\right)}\left(\frac{1}{x}+1\right)+\sqrt{5}\right)}{2\sqrt{10\left(1+\sqrt{5}\right)}}$$

input
```
Int[(1 + 4*x + 4*x^2 + 4*x^4)^(-1),x]
```

output
```
ArcTan[(-2 + 2*(1 + x^(-1))^2)/4]/2 - (((1 + Sqrt[5])^(3/2)*ArcTan[(-Sqrt[
2*(1 + Sqrt[5])] + 2*(1 + x^(-1)))/Sqrt[2*(-1 + Sqrt[5])]])/Sqrt[-1 + Sqrt
[5]] - ((1 - Sqrt[5])*Log[Sqrt[5] - Sqrt[2*(1 + Sqrt[5])]*(1 + x^(-1)) + (
1 + x^(-1))^2])/2)/(2*Sqrt[10*(1 + Sqrt[5])]) - (((1 + Sqrt[5])^(3/2)*ArcT
an[(Sqrt[2*(1 + Sqrt[5])] + 2*(1 + x^(-1)))/Sqrt[2*(-1 + Sqrt[5])]])/Sqrt[
-1 + Sqrt[5]] + ((1 - Sqrt[5])*Log[Sqrt[5] + Sqrt[2*(1 + Sqrt[5])]*(1 + x^
(-1)) + (1 + x^(-1))^2])/2)/(2*Sqrt[10*(1 + Sqrt[5])])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1432
```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

rule 1483
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   In
t[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(d*r
 + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

rule 2202
```
Int[(Pn_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{n
 = Expon[Pn, x], k}, Int[Sum[Coeff[Pn, x, 2*k]*x^(2*k), {k, 0, n/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pn, x, 2*k + 1]*x^(2*k), {k, 0, (n -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pn, x]
 &&  !PolyQ[Pn, x^2]
```

rule 2504
```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Simp[-16*
a^2   Subst[Int[(1/(b - 4*a*x)^2)*(a*((-3*b^4 + 16*a*b^2*c - 64*a^2*b*d + 2
56*a^3*e - 32*a^2*(3*b^2 - 8*a*c)*x^2 + 256*a^4*x^4)/(b - 4*a*x)^4))^p, x],
 x, b/(4*a) + 1/x], x] /; NeQ[a, 0] && NeQ[b, 0] && EqQ[b^3 - 4*a*b*c + 8*a
^2*d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && IntegerQ[2*p] &&  !IGtQ[p, 0
]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.22

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(4\_Z^4+4\_Z^2+4\_Z+1\right)}\frac{\ln\left(x-\_R\right)}{4\_R^3+2\_R+1}\right)}{4}$	41
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(4\_Z^4+4\_Z^2+4\_Z+1\right)}\frac{\ln\left(x-\_R\right)}{4\_R^3+2\_R+1}\right)}{4}$	41

input
```
int(1/(4*x^4+4*x^2+4*x+1),x,method=_RETURNVERBOSE)
```

output
```
1/4*sum(1/(4*_R^3+2*_R+1)*ln(x-_R),_R=RootOf(4*_Z^4+4*_Z^2+4*_Z+1))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.15

$$\int \frac{1}{1+4x+4x^2+4x^4}\,dx = \frac{1}{2}\left(\left(\sqrt{5}+2\right)\sqrt{\frac{1}{5}\sqrt{5}-\frac{2}{5}}+1\right)\arctan\left(\frac{1}{2}\sqrt{5}(2x+1)+\frac{1}{2}\left(\sqrt{5}(8x+3)+20x+5\right)\sqrt{\frac{1}{5}\sqrt{5}-\frac{2}{5}}+x+\frac{1}{2}\right)+\frac{1}{2}\left(\left(\sqrt{5}+2\right)\sqrt{\frac{1}{5}\sqrt{5}-\frac{2}{5}}-1\right)\arctan\left(-\frac{1}{2}\sqrt{5}(2x+1)+\frac{1}{2}\left(\sqrt{5}(8x+3)+20x+5\right)\sqrt{\frac{1}{5}\sqrt{5}-\frac{2}{5}}-x-\frac{1}{2}\right)+\frac{1}{4}\sqrt{\frac{1}{5}\sqrt{5}-\frac{2}{5}}\log\left(4x^2+\left(\sqrt{5}(2x-3)+10x-5\right)\sqrt{\frac{1}{5}\sqrt{5}-\frac{2}{5}}+\sqrt{5}+1\right)-\frac{1}{4}\sqrt{\frac{1}{5}\sqrt{5}-\frac{2}{5}}\log\left(4x^2-\left(\sqrt{5}(2x-3)+10x-5\right)\sqrt{\frac{1}{5}\sqrt{5}-\frac{2}{5}}+\sqrt{5}+1\right)$$

input
```
integrate(1/(4*x^4+4*x^2+4*x+1),x, algorithm="fricas")
```

output
```
1/2*((sqrt(5) + 2)*sqrt(1/5*sqrt(5) - 2/5) + 1)*arctan(1/2*sqrt(5)*(2*x +
1) + 1/2*(sqrt(5)*(8*x + 3) + 20*x + 5)*sqrt(1/5*sqrt(5) - 2/5) + x + 1/2)
 + 1/2*((sqrt(5) + 2)*sqrt(1/5*sqrt(5) - 2/5) - 1)*arctan(-1/2*sqrt(5)*(2*
x + 1) + 1/2*(sqrt(5)*(8*x + 3) + 20*x + 5)*sqrt(1/5*sqrt(5) - 2/5) - x -
1/2) + 1/4*sqrt(1/5*sqrt(5) - 2/5)*log(4*x^2 + (sqrt(5)*(2*x - 3) + 10*x -
 5)*sqrt(1/5*sqrt(5) - 2/5) + sqrt(5) + 1) - 1/4*sqrt(1/5*sqrt(5) - 2/5)*l
og(4*x^2 - (sqrt(5)*(2*x - 3) + 10*x - 5)*sqrt(1/5*sqrt(5) - 2/5) + sqrt(5
) + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3432 vs. $2(153) = 306$.

Time = 1.51 (sec) , antiderivative size = 3432, normalized size of antiderivative = 18.55

$$\int \frac{1}{1+4x+4x^2+4x^4}\,dx = \text{Too large to display}$$

input
```
integrate(1/(4*x**4+4*x**2+4*x+1),x)
```

output
```
sqrt(-1/40 + sqrt(5)/80)*log(x**2 + x*(-8 - 21*sqrt(5)*sqrt(-2 + sqrt(5))/
10 - sqrt(-2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19)/2 - sqrt(5)/2 + 12
*sqrt(-2 + sqrt(5)) + 9*sqrt(5)*sqrt(-2 + sqrt(5))*sqrt(-2*sqrt(5)*sqrt(-2
 + sqrt(5)) + sqrt(5) + 19)/5) - 841*sqrt(5)*sqrt(-2*sqrt(5)*sqrt(-2 + sqr
t(5)) + sqrt(5) + 19)/20 - 14351/40 - 441*sqrt(-2 + sqrt(5))/4 - 75*sqrt(5
)*sqrt(-2 + sqrt(5))*sqrt(-2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19)/8
- 3*sqrt(-2 + sqrt(5))*sqrt(-2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19)
+ 301*sqrt(5)*sqrt(-2 + sqrt(5))/10 + 7407*sqrt(5)/40 + 3913*sqrt(-2*sqrt(
5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19)/40) - sqrt(-1/40 + sqrt(5)/80)*log(x
**2 + x*(-8 - 12*sqrt(-2 + sqrt(5)) - sqrt(5)/2 + 21*sqrt(5)*sqrt(-2 + sqr
t(5))/10 + sqrt(2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19)/2 + 9*sqrt(5)
*sqrt(-2 + sqrt(5))*sqrt(2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19)/5) -
 3913*sqrt(2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19)/40 - 14351/40 - 75
*sqrt(5)*sqrt(-2 + sqrt(5))*sqrt(2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) +
19)/8 - 301*sqrt(5)*sqrt(-2 + sqrt(5))/10 - 3*sqrt(-2 + sqrt(5))*sqrt(2*sq
rt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19) + 441*sqrt(-2 + sqrt(5))/4 + 7407
*sqrt(5)/40 + 841*sqrt(5)*sqrt(2*sqrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19
)/20) - 2*sqrt(3/80 + 3*sqrt(5)/80 + sqrt(-2*sqrt(5)*sqrt(-2 + sqrt(5)) +
sqrt(5) + 19)/40)*atan(-20*x/(-27*sqrt(5)*sqrt(3 + 3*sqrt(5) + 2*sqrt(-2*s
qrt(5)*sqrt(-2 + sqrt(5)) + sqrt(5) + 19)) + 5*sqrt(-2 + sqrt(5))*sqrt(...
```

Maxima [F]

$$\int \frac{1}{1+4x+4x^2+4x^4}\,dx = \int \frac{1}{4\,x^4+4\,x^2+4\,x+1}\,dx$$

input
```
integrate(1/(4*x^4+4*x^2+4*x+1),x, algorithm="maxima")
```

output
```
integrate(1/(4*x^4 + 4*x^2 + 4*x + 1), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 358 vs. 2(134) = 268.

Time = 0.15 (sec) , antiderivative size = 358, normalized size of antiderivative = 1.94

$$\int \frac{1}{1+4x+4x^2+4x^4}\,dx$$
$$= \frac{1}{10}\left(\frac{\sqrt{5\,\sqrt{5}-10}}{\sqrt{5}-2}+5\right)\left(\arctan\left(\frac{5}{3}\right)+\arctan\left(x\left(2\,\sqrt{5}\sqrt{17\,\sqrt{5}+38}+\sqrt{5}-4\,\sqrt{17\,\sqrt{5}+38}+1\right)-\right.\right.$$
$$-\frac{1}{10}\left(\frac{\sqrt{5\,\sqrt{5}-10}}{\sqrt{5}-2}-5\right)\left(\arctan\left(\frac{5}{3}\right)+\arctan\left(-x\left(2\,\sqrt{5}\sqrt{17\,\sqrt{5}+38}-\sqrt{5}-4\,\sqrt{17\,\sqrt{5}+38}-1\right.\right.\right.$$
$$+\frac{1}{20}\sqrt{5\,\sqrt{5}-10}\log\left(25\left(170\,\sqrt{5}x+380\,x+51\,\sqrt{5}+\sqrt{17\,\sqrt{5}+38}+114\right)^2\right.$$
$$\left.+25\left(102\,\sqrt{5}x+228\,x+17\,\sqrt{5}\sqrt{17\,\sqrt{5}+38}-85\,\sqrt{5}+38\,\sqrt{17\,\sqrt{5}+38}-190\right)^2\right)$$
$$-\frac{1}{20}\sqrt{5\,\sqrt{5}-10}\log\left(25\left(170\,\sqrt{5}x+380\,x+51\,\sqrt{5}-\sqrt{17\,\sqrt{5}+38}+114\right)^2\right.$$
$$\left.+25\left(102\,\sqrt{5}x+228\,x-17\,\sqrt{5}\sqrt{17\,\sqrt{5}+38}-85\,\sqrt{5}-38\,\sqrt{17\,\sqrt{5}+38}-190\right)^2\right)$$

input
```
integrate(1/(4*x^4+4*x^2+4*x+1),x, algorithm="giac")
```

output
```
1/10*(sqrt(5*sqrt(5) - 10)/(sqrt(5) - 2) + 5)*(arctan(5/3) + arctan(x*(2*s
qrt(5)*sqrt(17*sqrt(5) + 38) + sqrt(5) - 4*sqrt(17*sqrt(5) + 38) + 1) - 3/
2*sqrt(5)*sqrt(17*sqrt(5) + 38) + 1/2*sqrt(5) + 7/2*sqrt(17*sqrt(5) + 38)
+ 1/2)) - 1/10*(sqrt(5*sqrt(5) - 10)/(sqrt(5) - 2) - 5)*(arctan(5/3) + arc
tan(-x*(2*sqrt(5)*sqrt(17*sqrt(5) + 38) - sqrt(5) - 4*sqrt(17*sqrt(5) + 38
) - 1) + 3/2*sqrt(5)*sqrt(17*sqrt(5) + 38) + 1/2*sqrt(5) - 7/2*sqrt(17*sqr
t(5) + 38) + 1/2)) + 1/20*sqrt(5*sqrt(5) - 10)*log(25*(170*sqrt(5)*x + 380
*x + 51*sqrt(5) + sqrt(17*sqrt(5) + 38) + 114)^2 + 25*(102*sqrt(5)*x + 228
*x + 17*sqrt(5)*sqrt(17*sqrt(5) + 38) - 85*sqrt(5) + 38*sqrt(17*sqrt(5) +
38) - 190)^2) - 1/20*sqrt(5*sqrt(5) - 10)*log(25*(170*sqrt(5)*x + 380*x +
51*sqrt(5) - sqrt(17*sqrt(5) + 38) + 114)^2 + 25*(102*sqrt(5)*x + 228*x -
17*sqrt(5)*sqrt(17*sqrt(5) + 38) - 85*sqrt(5) - 38*sqrt(17*sqrt(5) + 38) -
 190)^2)
```

Mupad [B] (verification not implemented)

Time = 21.47 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.47

$$\int \frac{1}{1+4x+4x^2+4x^4}\,dx=\sum_{k=1}^{4}\ln\left(-\mathrm{root}\left(z^4+\frac{9\,z^2}{40}+\frac{z}{40}+\frac{1}{1280},z,k\right)\,\left(\frac{x}{4}+\mathrm{root}\left(z^4+\frac{9\,z^2}{40}+\frac{z}{40}+\frac{1}{1280},z,k\right)\,\left(6\,x+\mathrm{root}\left(z^4+\frac{9\,z^2}{40}+\frac{z}{40}+\frac{1}{1280},z,k\right)\,\left(36\,x+16\right)\right)\right)\right)\,\mathrm{root}\left(z^4+\frac{9\,z^2}{40}+\frac{z}{40}+\frac{1}{1280},z,k\right)$$

input
```
int(1/(4*x + 4*x^2 + 4*x^4 + 1),x)
```

output
```
symsum(log(-root(z^4 + (9*z^2)/40 + z/40 + 1/1280, z, k)*(x/4 + root(z^4 +
 (9*z^2)/40 + z/40 + 1/1280, z, k)*(6*x + root(z^4 + (9*z^2)/40 + z/40 + 1
/1280, z, k)*(36*x + 16))))*root(z^4 + (9*z^2)/40 + z/40 + 1/1280, z, k),
k, 1, 4)
```

Reduce [F]

$$\int \frac{1}{1+4x+4x^2+4x^4}\,dx = \int \frac{1}{4x^4+4x^2+4x+1}dx$$

input
```
int(1/(4*x^4+4*x^2+4*x+1),x)
```

output
```
int(1/(4*x**4 + 4*x**2 + 4*x + 1),x)
```

3.8 $\int \frac{1}{(1+4x+4x^2+4x^4)^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 264

$$\begin{aligned}
\int \frac{1}{(1+4x+4x^2+4x^4)^2}\,dx &= -\frac{17-\left(1+\frac{1}{x}\right)^2}{2\left(5-2\left(1+\frac{1}{x}\right)^2+\left(1+\frac{1}{x}\right)^4\right)}+\frac{\left(59-17\left(1+\frac{1}{x}\right)^2\right)\left(1+\frac{1}{x}\right)}{10\left(5-2\left(1+\frac{1}{x}\right)^2+\left(1+\frac{1}{x}\right)^4\right)} \\
&\quad+\frac{7}{4}\arctan\left(\frac{1}{2}\left(-1+\left(1+\frac{1}{x}\right)^2\right)\right) \\
&\quad-\frac{1}{20}\sqrt{\frac{1}{10}\left(5959+2665\sqrt{5}\right)}\arctan\left(\frac{2-\sqrt{2\left(1+\sqrt{5}\right)}+\frac{2}{x}}{\sqrt{2\left(-1+\sqrt{5}\right)}}\right) \\
&\quad-\frac{1}{20}\sqrt{\frac{1}{10}\left(5959+2665\sqrt{5}\right)}\arctan\left(\frac{2+\sqrt{2\left(1+\sqrt{5}\right)}+\frac{2}{x}}{\sqrt{2\left(-1+\sqrt{5}\right)}}\right) \\
&\quad-\frac{1}{20}\sqrt{\frac{1}{10}\left(-5959+2665\sqrt{5}\right)}\operatorname{arctanh}\left(\frac{\sqrt{2\left(1+\sqrt{5}\right)}\left(1+\frac{1}{x}\right)}{\sqrt{5}+\left(1+\frac{1}{x}\right)^2}\right)
\end{aligned}$$

output
```
-1/2*(17-(1+1/x)^2)/(5-2*(1+1/x)^2+(1+1/x)^4)+(59-17*(1+1/x)^2)*(1+1/x)/(5
0-20*(1+1/x)^2+10*(1+1/x)^4)+7/4*arctan(-1/2+1/2*(1+1/x)^2)-1/200*(59590+2
6650*5^(1/2))^(1/2)*arctan((2-(2+2*5^(1/2))^(1/2)+2/x)/(-2+2*5^(1/2))^(1/2
))-1/200*(59590+26650*5^(1/2))^(1/2)*arctan((2+(2+2*5^(1/2))^(1/2)+2/x)/(-
2+2*5^(1/2))^(1/2))-1/200*(-59590+26650*5^(1/2))^(1/2)*arctanh((2+2*5^(1/2
))^(1/2)*(1+1/x)/(5^(1/2)+(1+1/x)^2))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.41

$$\int \frac{1}{(1+4x+4x^2+4x^4)^2}\,dx$$
$$= \frac{1}{40}\left(\frac{38+84x-32x^2+72x^3}{1+4x+4x^2+4x^4}+\text{RootSum}\left[1+4\#1+4\#1^2 +4\#1^4\&,\frac{27\log(x-\#1)-16\log(x-\#1)\#1+18\log(x-\#1)\#1^2}{1+2\#1+4\#1^3}\&\right]\right)$$

input
```
Integrate[(1 + 4*x + 4*x^2 + 4*x^4)^(-2),x]
```

output
```
((38 + 84*x - 32*x^2 + 72*x^3)/(1 + 4*x + 4*x^2 + 4*x^4) + RootSum[1 + 4*#
1 + 4*#1^2 + 4*#1^4 & , (27*Log[x - #1] - 16*Log[x - #1]*#1 + 18*Log[x - #
1]*#1^2)/(1 + 2*#1 + 4*#1^3) & ])/40
```

Rubi [A] (warning: unable to verify)

Time = 0.81 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.33, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {2504, 27, 2202, 2194, 27, 2191, 27, 1083, 217, 2206, 27, 1483, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(4x^4+4x^2+4x+1)^2}\,dx$$

$$\Big\downarrow \text{ 2504}$$

$$-16\int \frac{1}{16\left(\left(1+\frac{1}{x}\right)^4-2\left(1+\frac{1}{x}\right)^2+5\right)^2 x^6}d\left(1+\frac{1}{x}\right)$$

$$\Big\downarrow \text{ 27}$$

$$-\int \frac{1}{\left(\left(1+\frac{1}{x}\right)^4-2\left(1+\frac{1}{x}\right)^2+5\right)^2 x^6}d\left(1+\frac{1}{x}\right)$$

$$\Big\downarrow \text{ 2202}$$

$$-\int \frac{(1+\frac{1}{x})^6+15(1+\frac{1}{x})^4+15(1+\frac{1}{x})^2+1}{\left(\left(1+\frac{1}{x}\right)^4-2\left(1+\frac{1}{x}\right)^2+5\right)^2}d\left(1+\frac{1}{x}\right)-\int \frac{\left(-6(1+\frac{1}{x})^4-20(1+\frac{1}{x})^2-6\right)\left(1+\frac{1}{x}\right)}{\left(\left(1+\frac{1}{x}\right)^4-2\left(1+\frac{1}{x}\right)^2+5\right)^2}d\left(1+\frac{1}{x}\right)$$

$$\Big\downarrow \text{ 2194}$$

$$-\frac{1}{2}\int -\frac{2\left(3(1+\frac{1}{x})^4+10(1+\frac{1}{x})^2+3\right)}{\left(\left(1+\frac{1}{x}\right)^4-2\left(1+\frac{1}{x}\right)^2+5\right)^2}d\left(1+\frac{1}{x}\right)^2-\int \frac{(1+\frac{1}{x})^6+15(1+\frac{1}{x})^4+15(1+\frac{1}{x})^2+1}{\left(\left(1+\frac{1}{x}\right)^4-2\left(1+\frac{1}{x}\right)^2+5\right)^2}d\left(1+\frac{1}{x}\right)$$

$$\Big\downarrow \text{ 27}$$

$$\int \frac{3(1+\frac{1}{x})^4+10(1+\frac{1}{x})^2+3}{\left(\left(1+\frac{1}{x}\right)^4-2\left(1+\frac{1}{x}\right)^2+5\right)^2}d\left(1+\frac{1}{x}\right)^2-\int \frac{(1+\frac{1}{x})^6+15(1+\frac{1}{x})^4+15(1+\frac{1}{x})^2+1}{\left(\left(1+\frac{1}{x}\right)^4-2\left(1+\frac{1}{x}\right)^2+5\right)^2}d\left(1+\frac{1}{x}\right)$$

$$\Big\downarrow \text{ 2191}$$

$$\frac{1}{16}\int \frac{56}{(1+\frac{1}{x})^4-2\left(1+\frac{1}{x}\right)^2+5}d\left(1+\frac{1}{x}\right)^2-\int \frac{(1+\frac{1}{x})^6+15(1+\frac{1}{x})^4+15(1+\frac{1}{x})^2+1}{\left(\left(1+\frac{1}{x}\right)^4-2\left(1+\frac{1}{x}\right)^2+5\right)^2}d\left(1+\frac{1}{x}\right)-\frac{16-\frac{1}{x}}{2\left(\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{7}{2}\int \frac{1}{\left(1+\frac{1}{x}\right)^4-2\left(1+\frac{1}{x}\right)^2+5}d\left(1+\frac{1}{x}\right)^2-\int \frac{\left(1+\frac{1}{x}\right)^6+15\left(1+\frac{1}{x}\right)^4+15\left(1+\frac{1}{x}\right)^2+1}{\left(\left(1+\frac{1}{x}\right)^4-2\left(1+\frac{1}{x}\right)^2+5\right)^2}d\left(1+\frac{1}{x}\right)-\frac{16-\frac{1}{x}}{2\left(\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5\right)}$$

$$\Big\downarrow \text{1083}$$

$$-7\int \frac{1}{-\left(1+\frac{1}{x}\right)^4-16}d\left(2\left(1+\frac{1}{x}\right)^2-2\right)-\int \frac{\left(1+\frac{1}{x}\right)^6+15\left(1+\frac{1}{x}\right)^4+15\left(1+\frac{1}{x}\right)^2+1}{\left(\left(1+\frac{1}{x}\right)^4-2\left(1+\frac{1}{x}\right)^2+5\right)^2}d\left(1+\frac{1}{x}\right)-\frac{16-\frac{1}{x}}{2\left(\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5\right)}$$

$$\Big\downarrow \text{217}$$

$$-\int \frac{\left(1+\frac{1}{x}\right)^6+15\left(1+\frac{1}{x}\right)^4+15\left(1+\frac{1}{x}\right)^2+1}{\left(\left(1+\frac{1}{x}\right)^4-2\left(1+\frac{1}{x}\right)^2+5\right)^2}d\left(1+\frac{1}{x}\right)+\frac{7}{4}\arctan\left(\frac{1}{4}\left(2\left(\frac{1}{x}+1\right)^2-2\right)\right)-\frac{16-\frac{1}{x}}{2\left(\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5\right)}$$

$$\Big\downarrow \text{2206}$$

$$-\frac{1}{160}\int \frac{16\left(27\left(1+\frac{1}{x}\right)^2+61\right)}{\left(1+\frac{1}{x}\right)^4-2\left(1+\frac{1}{x}\right)^2+5}d\left(1+\frac{1}{x}\right)+\frac{7}{4}\arctan\left(\frac{1}{4}\left(2\left(\frac{1}{x}+1\right)^2-2\right)\right)-\frac{16-\frac{1}{x}}{2\left(\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5\right)}+\frac{\left(59-17\left(\frac{1}{x}+1\right)^2\right)\left(\frac{1}{x}+1\right)}{10\left(\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5\right)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{10}\int \frac{27\left(1+\frac{1}{x}\right)^2+61}{\left(1+\frac{1}{x}\right)^4-2\left(1+\frac{1}{x}\right)^2+5}d\left(1+\frac{1}{x}\right)+\frac{7}{4}\arctan\left(\frac{1}{4}\left(2\left(\frac{1}{x}+1\right)^2-2\right)\right)-\frac{16-\frac{1}{x}}{2\left(\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5\right)}+\frac{\left(59-17\left(\frac{1}{x}+1\right)^2\right)\left(\frac{1}{x}+1\right)}{10\left(\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5\right)}$$

$$\Big\downarrow \text{1483}$$

$$\frac{1}{10}\left(-\frac{\int \frac{61\sqrt{2\left(1+\sqrt{5}\right)}-\left(61-27\sqrt{5}\right)\left(1+\frac{1}{x}\right)}{\left(1+\frac{1}{x}\right)^2-\sqrt{2\left(1+\sqrt{5}\right)}\left(1+\frac{1}{x}\right)+\sqrt{5}}d\left(1+\frac{1}{x}\right)}{2\sqrt{10\left(1+\sqrt{5}\right)}}-\frac{\int \frac{\left(61-27\sqrt{5}\right)\left(1+\frac{1}{x}\right)+61\sqrt{2\left(1+\sqrt{5}\right)}}{\left(1+\frac{1}{x}\right)^2+\sqrt{2\left(1+\sqrt{5}\right)}\left(1+\frac{1}{x}\right)+\sqrt{5}}d\left(1+\frac{1}{x}\right)}{2\sqrt{10\left(1+\sqrt{5}\right)}}\right)+$$
$$\frac{7}{4}\arctan\left(\frac{1}{4}\left(2\left(\frac{1}{x}+1\right)^2-2\right)\right)-\frac{16-\frac{1}{x}}{2\left(\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5\right)}+$$
$$\frac{\left(59-17\left(\frac{1}{x}+1\right)^2\right)\left(\frac{1}{x}+1\right)}{10\left(\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5\right)}$$

$$\Big\downarrow \ 1142$$

$$\frac{1}{10}\left(-\frac{\sqrt{2\left(5959+2665\sqrt{5}\right)}\int \frac{1}{\left(1+\frac{1}{x}\right)^2-\sqrt{2\left(1+\sqrt{5}\right)}\left(1+\frac{1}{x}\right)+\sqrt{5}}d\left(1+\frac{1}{x}\right)-\frac{1}{2}\left(61-27\sqrt{5}\right)\int -\frac{\sqrt{2\left(1+\sqrt{5}\right)}-2\left(1+\frac{1}{x}\right)}{\left(1+\frac{1}{x}\right)^2-\sqrt{2\left(1+\sqrt{5}\right)}\left(1+\frac{1}{x}\right)-}}{2\sqrt{10\left(1+\sqrt{5}\right)}}\right.$$
$$\frac{7}{4}\arctan\left(\frac{1}{4}\left(2\left(\frac{1}{x}+1\right)^2-2\right)\right)-\frac{16-\frac{1}{x}}{2\left(\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5\right)}+$$
$$\frac{\left(59-17\left(\frac{1}{x}+1\right)^2\right)\left(\frac{1}{x}+1\right)}{10\left(\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5\right)}$$

$$\Big\downarrow \ 25$$

$$\frac{1}{10}\left(-\frac{\sqrt{2\left(5959+2665\sqrt{5}\right)}\int \frac{1}{\left(1+\frac{1}{x}\right)^2-\sqrt{2\left(1+\sqrt{5}\right)}\left(1+\frac{1}{x}\right)+\sqrt{5}}d\left(1+\frac{1}{x}\right)+\frac{1}{2}\left(61-27\sqrt{5}\right)\int \frac{\sqrt{2\left(1+\sqrt{5}\right)}-2\left(1+\frac{1}{x}\right)}{\left(1+\frac{1}{x}\right)^2-\sqrt{2\left(1+\sqrt{5}\right)}\left(1+\frac{1}{x}\right)+\sqrt{}}}{2\sqrt{10\left(1+\sqrt{5}\right)}}\right.$$
$$\frac{7}{4}\arctan\left(\frac{1}{4}\left(2\left(\frac{1}{x}+1\right)^2-2\right)\right)-\frac{16-\frac{1}{x}}{2\left(\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5\right)}+$$
$$\frac{\left(59-17\left(\frac{1}{x}+1\right)^2\right)\left(\frac{1}{x}+1\right)}{10\left(\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5\right)}$$

$$\Big\downarrow \ 1083$$

$$\frac{1}{10}\left(-\frac{\frac{1}{2}(61-27\sqrt{5})\int\frac{\sqrt{2\left(1+\sqrt{5}\right)}-2(1+\frac{1}{x})}{(1+\frac{1}{x})^2-\sqrt{2\left(1+\sqrt{5}\right)}(1+\frac{1}{x})+\sqrt{5}}d(1+\frac{1}{x})-2\sqrt{2\left(5959+2665\sqrt{5}\right)}\int\frac{1}{2\left(1-\sqrt{5}\right)-\left(2(1+\frac{1}{x})-\sqrt{2(1+\right.}}{2\sqrt{10\left(1+\sqrt{5}\right)}}\right.$$

$$\frac{7}{4}\arctan\left(\frac{1}{4}\left(2\left(\frac{1}{x}+1\right)^2-2\right)\right)-\frac{16-\frac{1}{x}}{2\left(\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5\right)}+$$

$$\frac{\left(59-17(\frac{1}{x}+1)^2\right)\left(\frac{1}{x}+1\right)}{10\left(\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5\right)}$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{10}\left(-\frac{\frac{1}{2}(61-27\sqrt{5})\int\frac{\sqrt{2\left(1+\sqrt{5}\right)}-2(1+\frac{1}{x})}{(1+\frac{1}{x})^2-\sqrt{2\left(1+\sqrt{5}\right)}(1+\frac{1}{x})+\sqrt{5}}d(1+\frac{1}{x})+2\sqrt{\frac{5959+2665\sqrt{5}}{\sqrt{5}-1}}\arctan\left(\frac{2(\frac{1}{x}+1)-\sqrt{2\left(1+\sqrt{5}\right)}}{\sqrt{2\left(\sqrt{5}-1\right)}}\right)}{2\sqrt{10\left(1+\sqrt{5}\right)}}-\frac{\frac{1}{2}}{}\right.$$

$$\frac{7}{4}\arctan\left(\frac{1}{4}\left(2\left(\frac{1}{x}+1\right)^2-2\right)\right)-\frac{16-\frac{1}{x}}{2\left(\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5\right)}+$$

$$\frac{\left(59-17(\frac{1}{x}+1)^2\right)\left(\frac{1}{x}+1\right)}{10\left(\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5\right)}$$

$$\Big\downarrow \text{1103}$$

$$\frac{7}{4}\arctan\left(\frac{1}{4}\left(2\left(\frac{1}{x}+1\right)^2-2\right)\right)+$$

$$\frac{1}{10}\left(-\frac{2\sqrt{\frac{5959+2665\sqrt{5}}{\sqrt{5}-1}}\arctan\left(\frac{2(\frac{1}{x}+1)-\sqrt{2\left(1+\sqrt{5}\right)}}{\sqrt{2\left(\sqrt{5}-1\right)}}\right)-\frac{1}{2}(61-27\sqrt{5})\log\left(\left(\frac{1}{x}+1\right)^2-\sqrt{2\left(1+\sqrt{5}\right)}\left(\frac{1}{x}+1\right)+\sqrt{\right.}}{2\sqrt{10\left(1+\sqrt{5}\right)}}\right.$$

$$\frac{16-\frac{1}{x}}{2\left(\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5\right)}+\frac{\left(59-17(\frac{1}{x}+1)^2\right)\left(\frac{1}{x}+1\right)}{10\left(\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5\right)}$$

input
```
Int[(1 + 4*x + 4*x^2 + 4*x^4)^(-2),x]
```

output
```
-1/2*(16 - x^(-1))/(5 - 2*(1 + x^(-1))^2 + (1 + x^(-1))^4) + ((59 - 17*(1
+ x^(-1))^2)*(1 + x^(-1)))/(10*(5 - 2*(1 + x^(-1))^2 + (1 + x^(-1))^4)) +
(7*ArcTan[(-2 + 2*(1 + x^(-1))^2)/4])/4 + (-1/2*(2*Sqrt[(5959 + 2665*Sqrt[
5])/(-1 + Sqrt[5])]*ArcTan[(-Sqrt[2*(1 + Sqrt[5])] + 2*(1 + x^(-1)))/Sqrt[
2*(-1 + Sqrt[5])]] - ((61 - 27*Sqrt[5])*Log[Sqrt[5] - Sqrt[2*(1 + Sqrt[5])
]*(1 + x^(-1)) + (1 + x^(-1))^2])/2)/Sqrt[10*(1 + Sqrt[5])] - (2*Sqrt[(595
9 + 2665*Sqrt[5])/(-1 + Sqrt[5])]*ArcTan[(Sqrt[2*(1 + Sqrt[5])] + 2*(1 + x
^(-1)))/Sqrt[2*(-1 + Sqrt[5])]] + ((61 - 27*Sqrt[5])*Log[Sqrt[5] + Sqrt[2*
(1 + Sqrt[5])]*(1 + x^(-1)) + (1 + x^(-1))^2])/2)/(2*Sqrt[10*(1 + Sqrt[5])
]))/10
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1483
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   In
t[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(d*r
 + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 2194
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Simp[1/2   Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)
^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ
[(m - 1)/2]
```

rule 2202
```
Int[(Pn_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{n
 = Expon[Pn, x], k}, Int[Sum[Coeff[Pn, x, 2*k]*x^(2*k), {k, 0, n/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pn, x, 2*k + 1]*x^(2*k), {k, 0, (n -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pn, x]
 &&  !PolyQ[Pn, x^2]
```

rule 2206
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c
*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Px,
a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*
p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x
^2] && Expon[Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 2504
```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Simp[-16*
a^2   Subst[Int[(1/(b - 4*a*x)^2)*(a*((-3*b^4 + 16*a*b^2*c - 64*a^2*b*d + 2
56*a^3*e - 32*a^2*(3*b^2 - 8*a*c)*x^2 + 256*a^4*x^4)/(b - 4*a*x)^4))^p, x],
 x, b/(4*a) + 1/x], x] /; NeQ[a, 0] && NeQ[b, 0] && EqQ[b^3 - 4*a*b*c + 8*a
^2*d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && IntegerQ[2*p] &&  !IGtQ[p, 0
]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.11 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.30

method	result	size
default	$\frac{\frac{9}{20}x^3-\frac{1}{5}x^2+\frac{21}{40}x+\frac{19}{80}}{x^4+x^2+x+\frac{1}{4}}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(4\_Z^4+4\_Z^2+4\_Z+1\right)}\frac{\left(18\_R^2-16\_R+27\right)\ln\left(x-\_R\right)}{4\_R^3+2\_R+1}\right)}{40}$	79
risch	$\frac{\frac{9}{20}x^3-\frac{1}{5}x^2+\frac{21}{40}x+\frac{19}{80}}{x^4+x^2+x+\frac{1}{4}}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(4\_Z^4+4\_Z^2+4\_Z+1\right)}\frac{\left(18\_R^2-16\_R+27\right)\ln\left(x-\_R\right)}{4\_R^3+2\_R+1}\right)}{40}$	79

input
```
int(1/(4*x^4+4*x^2+4*x+1)^2,x,method=_RETURNVERBOSE)
```

output
```
(9/20*x^3-1/5*x^2+21/40*x+19/80)/(x^4+x^2+x+1/4)+1/40*sum((18*_R^2-16*_R+2
7)/(4*_R^3+2*_R+1)*ln(x-_R),_R=RootOf(4*_Z^4+4*_Z^2+4*_Z+1))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 366, normalized size of antiderivative = 1.39

$$\int \frac{1}{(1+4x+4x^2+4x^4)^2}\,dx$$
$$= \frac{1368\,x^3 - 19\,(4\,x^4+4\,x^2+4\,x+1)\sqrt{\frac{533}{2}\sqrt{5}-\frac{5959}{10}}\log\left(76\,x^2+2\left(\sqrt{5}(61\,x-49)+135\,x-110\right)\sqrt{\frac{533}{2}}\right.}{}$$

input
```
integrate(1/(4*x^4+4*x^2+4*x+1)^2,x, algorithm="fricas")
```

output
```
1/760*(1368*x^3 - 19*(4*x^4 + 4*x^2 + 4*x + 1)*sqrt(533/2*sqrt(5) - 5959/1
0)*log(76*x^2 + 2*(sqrt(5)*(61*x - 49) + 135*x - 110)*sqrt(533/2*sqrt(5) -
 5959/10) + 19*sqrt(5) + 19) + 19*(4*x^4 + 4*x^2 + 4*x + 1)*sqrt(533/2*sqr
t(5) - 5959/10)*log(76*x^2 - 2*(sqrt(5)*(61*x - 49) + 135*x - 110)*sqrt(53
3/2*sqrt(5) - 5959/10) + 19*sqrt(5) + 19) - 608*x^2 + (5320*x^4 + 5320*x^2
 + (23836*x^4 + 23836*x^2 + 2665*sqrt(5)*(4*x^4 + 4*x^2 + 4*x + 1) + 23836
*x + 5959)*sqrt(533/2*sqrt(5) - 5959/10) + 5320*x + 1330)*arctan(1/2*sqrt(
5)*(2*x + 1) + 1/19*(sqrt(5)*(159*x + 49) + 355*x + 110)*sqrt(533/2*sqrt(5
) - 5959/10) + x + 1/2) - (5320*x^4 + 5320*x^2 - (23836*x^4 + 23836*x^2 +
2665*sqrt(5)*(4*x^4 + 4*x^2 + 4*x + 1) + 23836*x + 5959)*sqrt(533/2*sqrt(5
) - 5959/10) + 5320*x + 1330)*arctan(-1/2*sqrt(5)*(2*x + 1) + 1/19*(sqrt(5
)*(159*x + 49) + 355*x + 110)*sqrt(533/2*sqrt(5) - 5959/10) - x - 1/2) + 1
596*x + 722)/(4*x^4 + 4*x^2 + 4*x + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3834 vs. 2(211) = 422.

Time = 2.02 (sec) , antiderivative size = 3834, normalized size of antiderivative = 14.52

$$\int \frac{1}{(1+4x+4x^2+4x^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(4*x**4+4*x**2+4*x+1)**2,x)
```

output
```
(36*x**3 - 16*x**2 + 42*x + 19)/(80*x**4 + 80*x**2 + 80*x + 20) - sqrt(-59
59/16000 + 533*sqrt(5)/3200)*log(x**2 + x*(-1601676*sqrt(10)*sqrt(-5959 +
2665*sqrt(5))*sqrt(-665*sqrt(10)*sqrt(-5959 + 2665*sqrt(5)) + 221195*sqrt(
5) + 36004639)/13543383425 - 1067784*sqrt(2)*sqrt(-5959 + 2665*sqrt(5))/10
16389 + 3131659367*sqrt(10)*sqrt(-5959 + 2665*sqrt(5))/13543383425 + 29168
9395/1083470674 + 470215*sqrt(5)/2032778 + 94043*sqrt(-665*sqrt(10)*sqrt(-
5959 + 2665*sqrt(5)) + 221195*sqrt(5) + 36004639)/541735337) - 40634464149
111451*sqrt(5)*sqrt(-665*sqrt(10)*sqrt(-5959 + 2665*sqrt(5)) + 221195*sqrt
(5) + 36004639)/27530691871904650 - 288583554422522791728299 7/146738587677
251784500 - 83803227754187*sqrt(2)*sqrt(-5959 + 2665*sqrt(5))/100111606806
926 - 50208805356*sqrt(2)*sqrt(-5959 + 2665*sqrt(5))*sqrt(-665*sqrt(10)*sq
rt(-5959 + 2665*sqrt(5)) + 221195*sqrt(5) + 36004639)/550613837438093 - 53
8485754891933*sqrt(10)*sqrt(-5959 + 2665*sqrt(5))*sqrt(-665*sqrt(10)*sqrt(
-5959 + 2665*sqrt(5)) + 221195*sqrt(5) + 36004639)/146738587677251784 50 -
92532195509690141 1*sqrt(10)*sqrt(-5959 + 2665*sqrt(5))/293477175354503569 0
0 + 484304611938766076267*sqrt(5)/55061383743809300 + 220130360870147854 03
*sqrt(-665*sqrt(10)*sqrt(-5959 + 2665*sqrt(5)) + 221195*sqrt(5) + 36004639
)/6669935803511444750) + sqrt(-5959/16000 + 533*sqrt(5)/3200)*log(x**2 + x
*(-94043*sqrt(665*sqrt(10)*sqrt(-5959 + 2665*sqrt(5)) + 221195*sqrt(5) + 3
6004639)/541735337 - 1601676*sqrt(10)*sqrt(-5959 + 2665*sqrt(5))*sqrt(6...
```

Maxima [F]

$$\int \frac{1}{(1+4x+4x^2+4x^4)^2}\,dx = \int \frac{1}{(4\,x^4+4\,x^2+4\,x+1)^2}\,dx$$

input
```
integrate(1/(4*x^4+4*x^2+4*x+1)^2,x, algorithm="maxima")
```

output
```
1/20*(36*x^3 - 16*x^2 + 42*x + 19)/(4*x^4 + 4*x^2 + 4*x + 1) + 1/10*integr
ate((18*x^2 - 16*x + 27)/(4*x^4 + 4*x^2 + 4*x + 1), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 403 vs. $2(201) = 402$.

Time = 0.17 (sec) , antiderivative size = 403, normalized size of antiderivative = 1.53

$$\int \frac{1}{(1+4x+4x^2+4x^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(4*x^4+4*x^2+4*x+1)^2,x, algorithm="giac")
```

output
```
-1/100*(19*sqrt(26650*sqrt(5) - 59590)/(2665*sqrt(5) - 5959) + 175)*(arcta
n(19/2980) + arctan(-1/17646838*x*(2999*sqrt(5)*sqrt(17761522*sqrt(5) - 18
213038) + 17646838*sqrt(5) + 8959*sqrt(17761522*sqrt(5) - 18213038) + 1764
6838) - 745/8823419*sqrt(5)*sqrt(17761522*sqrt(5) - 18213038) - 1/2*sqrt(5
) - 1509/17646838*sqrt(17761522*sqrt(5) - 18213038) - 1/2)) + 1/100*(19*sq
rt(26650*sqrt(5) - 59590)/(2665*sqrt(5) - 5959) - 175)*(arctan(19/2980) +
arctan(1/17646838*x*(2999*sqrt(5)*sqrt(17761522*sqrt(5) - 18213038) - 1764
6838*sqrt(5) + 8959*sqrt(17761522*sqrt(5) - 18213038) - 17646838) + 745/88
23419*sqrt(5)*sqrt(17761522*sqrt(5) - 18213038) - 1/2*sqrt(5) + 1509/17646
838*sqrt(17761522*sqrt(5) - 18213038) - 1/2)) - 1/400*sqrt(26650*sqrt(5) -
 59590)*log(577600*(105858671120*sqrt(5)*x - 108549706480*x + 8880761*sqrt
(5)*sqrt(17761522*sqrt(5) - 18213038) + 337468918*sqrt(5) - 9106519*sqrt(1
7761522*sqrt(5) - 18213038) - 346047722)^2 + 2310400*(337468918*sqrt(5)*x
- 346047722*x - 26464667780*sqrt(5) + 8823419*sqrt(17761522*sqrt(5) - 1821
3038) + 27137426620)^2) + 1/400*sqrt(26650*sqrt(5) - 59590)*log(577600*(10
5858671120*sqrt(5)*x - 108549706480*x - 8880761*sqrt(5)*sqrt(17761522*sqrt
(5) - 18213038) + 337468918*sqrt(5) + 9106519*sqrt(17761522*sqrt(5) - 1821
3038) - 346047722)^2 + 2310400*(337468918*sqrt(5)*x - 346047722*x - 264646
67780*sqrt(5) - 8823419*sqrt(17761522*sqrt(5) - 18213038) + 27137426620)^2
) + 1/20*(36*x^3 - 16*x^2 + 42*x + 19)/(4*x^4 + 4*x^2 + 4*x + 1)
```

Mupad [B] (verification not implemented)

Time = 21.33 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.66

$$\int \frac{1}{(1+4x+4x^2+4x^4)^2}\,dx = \left(\sum_{k=1}^{4}\ln\left(-\frac{169\,\mathrm{root}\left(z^4+\frac{3021\,z^2}{1000}-\frac{133\,z}{8000}+\frac{29}{64000},z,k\right)}{100} + \frac{11\,x}{1600} + \frac{\mathrm{root}\left(z^4+\frac{3021\,z^2}{1000}-\frac{133\,z}{8000}+\frac{29}{64000},z,k\right)\,x\,131}{100} - \frac{\mathrm{root}\left(z^4+\frac{3021\,z^2}{1000}-\frac{133\,z}{8000}+\frac{29}{64000},z,k\right)^2\,x\,72}{5} - \mathrm{root}\left(z^4+\frac{3021\,z^2}{1000}-\frac{133\,z}{8000}+\frac{29}{64000},z,k\right)^3\,x\,36 + \frac{59\,\mathrm{root}\left(z^4+\frac{3021\,z^2}{1000}-\frac{133\,z}{8000}+\frac{29}{64000},z,k\right)^2}{20} - 16\,\mathrm{root}\left(z^4+\frac{3021\,z^2}{1000}-\frac{133\,z}{8000}+\frac{29}{64000},z,k\right)^3 + \frac{27}{1600}\right)\mathrm{root}\left(z^4+\frac{3021\,z^2}{1000}-\frac{133\,z}{8000}+\frac{29}{64000},z,k\right)\right) + \frac{\frac{9\,x^3}{20}-\frac{x^2}{5}+\frac{21\,x}{40}+\frac{19}{80}}{x^4+x^2+x+\frac{1}{4}}$$

input
```
int(1/(4*x + 4*x^2 + 4*x^4 + 1)^2,x)
```

output
```
symsum(log((11*x)/1600 - (169*root(z^4 + (3021*z^2)/1000 - (133*z)/8000 +
29/64000, z, k))/100 + (131*root(z^4 + (3021*z^2)/1000 - (133*z)/8000 + 29
/64000, z, k)*x)/100 - (72*root(z^4 + (3021*z^2)/1000 - (133*z)/8000 + 29/
64000, z, k)^2*x)/5 - 36*root(z^4 + (3021*z^2)/1000 - (133*z)/8000 + 29/64
000, z, k)^3*x + (59*root(z^4 + (3021*z^2)/1000 - (133*z)/8000 + 29/64000,
 z, k)^2)/20 - 16*root(z^4 + (3021*z^2)/1000 - (133*z)/8000 + 29/64000, z,
 k)^3 + 27/1600)*root(z^4 + (3021*z^2)/1000 - (133*z)/8000 + 29/64000, z,
k), k, 1, 4) + ((21*x)/40 - x^2/5 + (9*x^3)/20 + 19/80)/(x + x^2 + x^4 + 1
/4)
```

Reduce [F]

$$\int \frac{1}{(1+4x+4x^2+4x^4)^2}\,dx = \int \frac{1}{16x^8+32x^6+32x^5+24x^4+32x^3+24x^2+8x+1}dx$$

input
```
int(1/(4*x^4+4*x^2+4*x+1)^2,x)
```

output
```
int(1/(16*x**8 + 32*x**6 + 32*x**5 + 24*x**4 + 32*x**3 + 24*x**2 + 8*x + 1
),x)
```

3.9 $\int (1+x+x^2+x^3+x^4)^3 \, dx$

Optimal result

Integrand size = 14, antiderivative size = 76

$$\int (1+x+x^2+x^3+x^4)^3 \, dx = x+\frac{3x^2}{2}+2x^3+\frac{5x^4}{2}+3x^5+3x^6+\frac{19x^7}{7}+\frac{9x^8}{4}+\frac{5x^9}{3}+x^{10}+\frac{6x^{11}}{11}+\frac{x^{12}}{4}+\frac{x^{13}}{13}$$

output
```
x+3/2*x^2+2*x^3+5/2*x^4+3*x^5+3*x^6+19/7*x^7+9/4*x^8+5/3*x^9+x^10+6/11*x^1
1+1/4*x^12+1/13*x^13
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00

$$\int (1+x+x^2+x^3+x^4)^3 \, dx = x+\frac{3x^2}{2}+2x^3+\frac{5x^4}{2}+3x^5+3x^6+\frac{19x^7}{7}+\frac{9x^8}{4}+\frac{5x^9}{3}+x^{10}+\frac{6x^{11}}{11}+\frac{x^{12}}{4}+\frac{x^{13}}{13}$$

input
```
Integrate[(1 + x + x^2 + x^3 + x^4)^3,x]
```

output
```
x + (3*x^2)/2 + 2*x^3 + (5*x^4)/2 + 3*x^5 + 3*x^6 + (19*x^7)/7 + (9*x^8)/4
 + (5*x^9)/3 + x^10 + (6*x^11)/11 + x^12/4 + x^13/13
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2465, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(x^4+x^3+x^2+x+1\right)^3 dx$$

$$\Big\downarrow \text{2465}$$

$$\int \left(x^{12}+3x^{11}+6x^{10}+10x^9+15x^8+18x^7+19x^6+18x^5+15x^4+10x^3+6x^2+3x+1\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^{13}}{13}+\frac{x^{12}}{4}+\frac{6x^{11}}{11}+x^{10}+\frac{5x^9}{3}+\frac{9x^8}{4}+\frac{19x^7}{7}+3x^6+3x^5+\frac{5x^4}{2}+2x^3+\frac{3x^2}{2}+x$$

input
```
Int[(1 + x + x^2 + x^3 + x^4)^3,x]
```

output

```
x + (3*x^2)/2 + 2*x^3 + (5*x^4)/2 + 3*x^5 + 3*x^6 + (19*x^7)/7 + (9*x^8)/4
 + (5*x^9)/3 + x^10 + (6*x^11)/11 + x^12/4 + x^13/13
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2465
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> Int[ExpandToSum[u, Px^p, x], x] /; PolyQ
[Px, x] && GtQ[Expon[Px, x], 2] && !BinomialQ[Px, x] && !TrinomialQ[Px, x
] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.80

method	result
gosper	$x+\frac{3}{2}x^{2}+2x^{3}+\frac{5}{2}x^{4}+3x^{5}+3x^{6}+\frac{19}{7}x^{7}+\frac{9}{4}x^{8}+\frac{5}{3}x^{9}+x^{10}+\frac{6}{11}x^{11}+\frac{1}{4}x^{12}+\frac{1}{13}x^{13}$
default	$x+\frac{3}{2}x^{2}+2x^{3}+\frac{5}{2}x^{4}+3x^{5}+3x^{6}+\frac{19}{7}x^{7}+\frac{9}{4}x^{8}+\frac{5}{3}x^{9}+x^{10}+\frac{6}{11}x^{11}+\frac{1}{4}x^{12}+\frac{1}{13}x^{13}$
norman	$x+\frac{3}{2}x^{2}+2x^{3}+\frac{5}{2}x^{4}+3x^{5}+3x^{6}+\frac{19}{7}x^{7}+\frac{9}{4}x^{8}+\frac{5}{3}x^{9}+x^{10}+\frac{6}{11}x^{11}+\frac{1}{4}x^{12}+\frac{1}{13}x^{13}$
risch	$x+\frac{3}{2}x^{2}+2x^{3}+\frac{5}{2}x^{4}+3x^{5}+3x^{6}+\frac{19}{7}x^{7}+\frac{9}{4}x^{8}+\frac{5}{3}x^{9}+x^{10}+\frac{6}{11}x^{11}+\frac{1}{4}x^{12}+\frac{1}{13}x^{13}$
parallelrisch	$x+\frac{3}{2}x^{2}+2x^{3}+\frac{5}{2}x^{4}+3x^{5}+3x^{6}+\frac{19}{7}x^{7}+\frac{9}{4}x^{8}+\frac{5}{3}x^{9}+x^{10}+\frac{6}{11}x^{11}+\frac{1}{4}x^{12}+\frac{1}{13}x^{13}$
orering	$\frac{x(924x^{12}+3003x^{11}+6552x^{10}+12012x^{9}+20020x^{8}+27027x^{7}+32604x^{6}+36036x^{5}+36036x^{4}+30030x^{3}+24024x^{2}+18018x+1}{12012}$

input
```
int((x^4+x^3+x^2+x+1)^3,x,method=_RETURNVERBOSE)
```

output
```
x+3/2*x^2+2*x^3+5/2*x^4+3*x^5+3*x^6+19/7*x^7+9/4*x^8+5/3*x^9+x^10+6/11*x^1
1+1/4*x^12+1/13*x^13
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.79

$$\int \left(1+x+x^2+x^3+x^4\right)^3 \, dx = \frac{1}{13}\,x^{13}+\frac{1}{4}\,x^{12}+\frac{6}{11}\,x^{11}+x^{10}+\frac{5}{3}\,x^{9}+\frac{9}{4}\,x^{8} + \frac{19}{7}\,x^{7}+3\,x^{6}+3\,x^{5}+\frac{5}{2}\,x^{4}+2\,x^{3}+\frac{3}{2}\,x^{2}+x$$

input
```
integrate((x^4+x^3+x^2+x+1)^3,x, algorithm="fricas")
```

output
```
1/13*x^13 + 1/4*x^12 + 6/11*x^11 + x^10 + 5/3*x^9 + 9/4*x^8 + 19/7*x^7 + 3
*x^6 + 3*x^5 + 5/2*x^4 + 2*x^3 + 3/2*x^2 + x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.92

$$\int \left(1+x+x^2+x^3+x^4\right)^3\,dx=\frac{x^{13}}{13}+\frac{x^{12}}{4}+\frac{6x^{11}}{11}+x^{10}+\frac{5x^9}{3}+\frac{9x^8}{4}+\frac{19x^7}{7}+3x^6+3x^5+\frac{5x^4}{2}+2x^3+\frac{3x^2}{2}+x$$

input
```
integrate((x**4+x**3+x**2+x+1)**3,x)
```

output
```
x**13/13 + x**12/4 + 6*x**11/11 + x**10 + 5*x**9/3 + 9*x**8/4 + 19*x**7/7
+ 3*x**6 + 3*x**5 + 5*x**4/2 + 2*x**3 + 3*x**2/2 + x
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.79

$$\int \left(1+x+x^2+x^3+x^4\right)^3\,dx=\frac{1}{13}\,x^{13}+\frac{1}{4}\,x^{12}+\frac{6}{11}\,x^{11}+x^{10}+\frac{5}{3}\,x^9+\frac{9}{4}\,x^8+\frac{19}{7}\,x^7+3\,x^6+3\,x^5+\frac{5}{2}\,x^4+2\,x^3+\frac{3}{2}\,x^2+x$$

input
```
integrate((x^4+x^3+x^2+x+1)^3,x, algorithm="maxima")
```

output
```
1/13*x^13 + 1/4*x^12 + 6/11*x^11 + x^10 + 5/3*x^9 + 9/4*x^8 + 19/7*x^7 + 3
*x^6 + 3*x^5 + 5/2*x^4 + 2*x^3 + 3/2*x^2 + x
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.79

$$\int \left(1+x+x^2+x^3+x^4\right)^3 \, dx = \frac{1}{13}\,x^{13}+\frac{1}{4}\,x^{12}+\frac{6}{11}\,x^{11}+x^{10}+\frac{5}{3}\,x^{9}+\frac{9}{4}\,x^{8}+\frac{19}{7}\,x^{7}+3\,x^{6}+3\,x^{5}+\frac{5}{2}\,x^{4}+2\,x^{3}+\frac{3}{2}\,x^{2}+x$$

input
```
integrate((x^4+x^3+x^2+x+1)^3,x, algorithm="giac")
```

output
```
1/13*x^13 + 1/4*x^12 + 6/11*x^11 + x^10 + 5/3*x^9 + 9/4*x^8 + 19/7*x^7 + 3
*x^6 + 3*x^5 + 5/2*x^4 + 2*x^3 + 3/2*x^2 + x
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.79

$$\int \left(1+x+x^2+x^3+x^4\right)^3 \, dx = \frac{x^{13}}{13}+\frac{x^{12}}{4}+\frac{6\,x^{11}}{11}+x^{10}+\frac{5\,x^{9}}{3}+\frac{9\,x^{8}}{4}+\frac{19\,x^{7}}{7}+3\,x^{6}+3\,x^{5}+\frac{5\,x^{4}}{2}+2\,x^{3}+\frac{3\,x^{2}}{2}+x$$

input
```
int((x + x^2 + x^3 + x^4 + 1)^3,x)
```

output
```
x + (3*x^2)/2 + 2*x^3 + (5*x^4)/2 + 3*x^5 + 3*x^6 + (19*x^7)/7 + (9*x^8)/4
 + (5*x^9)/3 + x^10 + (6*x^11)/11 + x^12/4 + x^13/13
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.83

$$\int (1+x+x^2+x^3+x^4)^3 \, dx$$
$$= \frac{x(924x^{12}+3003x^{11}+6552x^{10}+12012x^9+20020x^8+27027x^7+32604x^6+36036x^5+36036x^4+300}{12012}$$

input
```
int((x^4+x^3+x^2+x+1)^3,x)
```

output
```
(x*(924*x**12 + 3003*x**11 + 6552*x**10 + 12012*x**9 + 20020*x**8 + 27027*
x**7 + 32604*x**6 + 36036*x**5 + 36036*x**4 + 30030*x**3 + 24024*x**2 + 18
018*x + 12012))/12012
```

3.10 $\int (1+x+x^2+x^3+x^4)^2 \, dx$

Optimal result

Integrand size = 14, antiderivative size = 42

$$\int (1+x+x^2+x^3+x^4)^2 \, dx = x+x^2+x^3+x^4+x^5+\frac{2x^6}{3}+\frac{3x^7}{7}+\frac{x^8}{4}+\frac{x^9}{9}$$

output
```
x+x^2+x^3+x^4+x^5+2/3*x^6+3/7*x^7+1/4*x^8+1/9*x^9
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int (1+x+x^2+x^3+x^4)^2 \, dx = x+x^2+x^3+x^4+x^5+\frac{2x^6}{3}+\frac{3x^7}{7}+\frac{x^8}{4}+\frac{x^9}{9}$$

input
```
Integrate[(1 + x + x^2 + x^3 + x^4)^2,x]
```

output
```
x + x^2 + x^3 + x^4 + x^5 + (2*x^6)/3 + (3*x^7)/7 + x^8/4 + x^9/9
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2465, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(x^4+x^3+x^2+x+1\right)^2 dx$$

$$\Big\downarrow \text{2465}$$

$$\int \left(x^8+2x^7+3x^6+4x^5+5x^4+4x^3+3x^2+2x+1\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^9}{9}+\frac{x^8}{4}+\frac{3x^7}{7}+\frac{2x^6}{3}+x^5+x^4+x^3+x^2+x$$

input
```
Int[(1 + x + x^2 + x^3 + x^4)^2,x]
```

output
```
x + x^2 + x^3 + x^4 + x^5 + (2*x^6)/3 + (3*x^7)/7 + x^8/4 + x^9/9
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2465
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> Int[ExpandToSum[u, Px^p, x], x] /; PolyQ
[Px, x] && GtQ[Expon[Px, x], 2] && !BinomialQ[Px, x] && !TrinomialQ[Px, x
] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.83

method	result	size
gosper	$x+x^{2}+x^{3}+x^{4}+x^{5}+\frac{2}{3}x^{6}+\frac{3}{7}x^{7}+\frac{1}{4}x^{8}+\frac{1}{9}x^{9}$	35
default	$x+x^{2}+x^{3}+x^{4}+x^{5}+\frac{2}{3}x^{6}+\frac{3}{7}x^{7}+\frac{1}{4}x^{8}+\frac{1}{9}x^{9}$	35
norman	$x+x^{2}+x^{3}+x^{4}+x^{5}+\frac{2}{3}x^{6}+\frac{3}{7}x^{7}+\frac{1}{4}x^{8}+\frac{1}{9}x^{9}$	35
risch	$x+x^{2}+x^{3}+x^{4}+x^{5}+\frac{2}{3}x^{6}+\frac{3}{7}x^{7}+\frac{1}{4}x^{8}+\frac{1}{9}x^{9}$	35
parallelrisch	$x+x^{2}+x^{3}+x^{4}+x^{5}+\frac{2}{3}x^{6}+\frac{3}{7}x^{7}+\frac{1}{4}x^{8}+\frac{1}{9}x^{9}$	35
orering	$\frac{x\left(28x^{8}+63x^{7}+108x^{6}+168x^{5}+252x^{4}+252x^{3}+252x^{2}+252x+252\right)}{252}$	44

input
```
int((x^4+x^3+x^2+x+1)^2,x,method=_RETURNVERBOSE)
```

output
```
x+x^2+x^3+x^4+x^5+2/3*x^6+3/7*x^7+1/4*x^8+1/9*x^9
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.81

$$\int \left(1+x+x^{2}+x^{3}+x^{4}\right)^{2}\,dx=\frac{1}{9}\,x^{9}+\frac{1}{4}\,x^{8}+\frac{3}{7}\,x^{7}+\frac{2}{3}\,x^{6}+x^{5}+x^{4}+x^{3}+x^{2}+x$$

input
```
integrate((x^4+x^3+x^2+x+1)^2,x, algorithm="fricas")
```

output
```
1/9*x^9 + 1/4*x^8 + 3/7*x^7 + 2/3*x^6 + x^5 + x^4 + x^3 + x^2 + x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.88

$$\int \left(1+x+x^2+x^3+x^4\right)^2\,dx=\frac{x^9}{9}+\frac{x^8}{4}+\frac{3x^7}{7}+\frac{2x^6}{3}+x^5+x^4+x^3+x^2+x$$

input
```
integrate((x**4+x**3+x**2+x+1)**2,x)
```

output
```
x**9/9 + x**8/4 + 3*x**7/7 + 2*x**6/3 + x**5 + x**4 + x**3 + x**2 + x
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.81

$$\int \left(1+x+x^2+x^3+x^4\right)^2\,dx=\frac{1}{9}\,x^9+\frac{1}{4}\,x^8+\frac{3}{7}\,x^7+\frac{2}{3}\,x^6+x^5+x^4+x^3+x^2+x$$

input
```
integrate((x^4+x^3+x^2+x+1)^2,x, algorithm="maxima")
```

output
```
1/9*x^9 + 1/4*x^8 + 3/7*x^7 + 2/3*x^6 + x^5 + x^4 + x^3 + x^2 + x
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.81

$$\int \left(1+x+x^2+x^3+x^4\right)^2\,dx=\frac{1}{9}\,x^9+\frac{1}{4}\,x^8+\frac{3}{7}\,x^7+\frac{2}{3}\,x^6+x^5+x^4+x^3+x^2+x$$

input
```
integrate((x^4+x^3+x^2+x+1)^2,x, algorithm="giac")
```

output
```
1/9*x^9 + 1/4*x^8 + 3/7*x^7 + 2/3*x^6 + x^5 + x^4 + x^3 + x^2 + x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.81

$$\int \left(1+x+x^2+x^3+x^4\right)^2 \, dx = \frac{x^9}{9}+\frac{x^8}{4}+\frac{3\,x^7}{7}+\frac{2\,x^6}{3}+x^5+x^4+x^3+x^2+x$$

input
```
int((x + x^2 + x^3 + x^4 + 1)^2,x)
```

output
```
x + x^2 + x^3 + x^4 + x^5 + (2*x^6)/3 + (3*x^7)/7 + x^8/4 + x^9/9
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.02

$$\int \left(1+x+x^2+x^3+x^4\right)^2 \, dx$$
$$= \frac{x(28x^8+63x^7+108x^6+168x^5+252x^4+252x^3+252x^2+252x+252)}{252}$$

input
```
int((x^4+x^3+x^2+x+1)^2,x)
```

output
```
(x*(28*x**8 + 63*x**7 + 108*x**6 + 168*x**5 + 252*x**4 + 252*x**3 + 252*x*
*2 + 252*x + 252))/252
```

3.11 $\int (1 + x + x^2 + x^3 + x^4) \, dx$

Optimal result

Integrand size = 12, antiderivative size = 30

$$\int \left(1 + x + x^2 + x^3 + x^4\right) \, dx = x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \frac{x^5}{5}$$

output
```
x+1/2*x^2+1/3*x^3+1/4*x^4+1/5*x^5
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \left(1 + x + x^2 + x^3 + x^4\right) \, dx = x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \frac{x^5}{5}$$

input
```
Integrate[1 + x + x^2 + x^3 + x^4,x]
```

output
```
x + x^2/2 + x^3/3 + x^4/4 + x^5/5
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = $\{2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(x^4+x^3+x^2+x+1\right) \, dx$$

$$\Big\downarrow \; 2009$$

$$\frac{x^5}{5}+\frac{x^4}{4}+\frac{x^3}{3}+\frac{x^2}{2}+x$$

input
```
Int[1 + x + x^2 + x^3 + x^4,x]
```

output
```
x + x^2/2 + x^3/3 + x^4/4 + x^5/5
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.77

method	result	size
gosper	$x+\frac{1}{2}x^2+\frac{1}{3}x^3+\frac{1}{4}x^4+\frac{1}{5}x^5$	23
default	$x+\frac{1}{2}x^2+\frac{1}{3}x^3+\frac{1}{4}x^4+\frac{1}{5}x^5$	23
norman	$x+\frac{1}{2}x^2+\frac{1}{3}x^3+\frac{1}{4}x^4+\frac{1}{5}x^5$	23
risch	$x+\frac{1}{2}x^2+\frac{1}{3}x^3+\frac{1}{4}x^4+\frac{1}{5}x^5$	23
parallelrisch	$x+\frac{1}{2}x^2+\frac{1}{3}x^3+\frac{1}{4}x^4+\frac{1}{5}x^5$	23
parts	$x+\frac{1}{2}x^2+\frac{1}{3}x^3+\frac{1}{4}x^4+\frac{1}{5}x^5$	23
orering	$\frac{x(12x^4+15x^3+20x^2+30x+60)}{60}$	24

input
```
int(x^4+x^3+x^2+x+1,x,method=_RETURNVERBOSE)
```

output
```
x+1/2*x^2+1/3*x^3+1/4*x^4+1/5*x^5
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.73

$$\int \left(1+x+x^2+x^3+x^4\right)\,dx=\frac{1}{5}\,x^5+\frac{1}{4}\,x^4+\frac{1}{3}\,x^3+\frac{1}{2}\,x^2+x$$

input
```
integrate(x^4+x^3+x^2+x+1,x, algorithm="fricas")
```

output
```
1/5*x^5 + 1/4*x^4 + 1/3*x^3 + 1/2*x^2 + x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.67

$$\int \left(1+x+x^2+x^3+x^4\right)\,dx=\frac{x^5}{5}+\frac{x^4}{4}+\frac{x^3}{3}+\frac{x^2}{2}+x$$

input
```
integrate(x**4+x**3+x**2+x+1,x)
```

output
```
x**5/5 + x**4/4 + x**3/3 + x**2/2 + x
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.73

$$\int \left(1+x+x^2+x^3+x^4\right)\,dx=\frac{1}{5}\,x^5+\frac{1}{4}\,x^4+\frac{1}{3}\,x^3+\frac{1}{2}\,x^2+x$$

input
```
integrate(x^4+x^3+x^2+x+1,x, algorithm="maxima")
```

output
```
1/5*x^5 + 1/4*x^4 + 1/3*x^3 + 1/2*x^2 + x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.73

$$\int \left(1+x+x^2+x^3+x^4\right)\,dx=\frac{1}{5}\,x^5+\frac{1}{4}\,x^4+\frac{1}{3}\,x^3+\frac{1}{2}\,x^2+x$$

input
```
integrate(x^4+x^3+x^2+x+1,x, algorithm="giac")
```

output
```
1/5*x^5 + 1/4*x^4 + 1/3*x^3 + 1/2*x^2 + x
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.73

$$\int \left(1+x+x^2+x^3+x^4\right)\,dx=\frac{x^5}{5}+\frac{x^4}{4}+\frac{x^3}{3}+\frac{x^2}{2}+x$$

input
```
int(x + x^2 + x^3 + x^4 + 1,x)
```

output
```
x + x^2/2 + x^3/3 + x^4/4 + x^5/5
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.77

$$\int \left(1+x+x^2+x^3+x^4\right)\,dx=\frac{x(12x^4+15x^3+20x^2+30x+60)}{60}$$

input
```
int(x^4+x^3+x^2+x+1,x)
```

output
```
(x*(12*x**4 + 15*x**3 + 20*x**2 + 30*x + 60))/60
```

3.12 $\int \frac{1}{1+x+x^2+x^3+x^4} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 143

$$\int \frac{1}{1+x+x^2+x^3+x^4} \, dx = \frac{1}{5}\sqrt{5-2\sqrt{5}}\arctan\left(\frac{1-\sqrt{5}+4x}{\sqrt{2\left(5+\sqrt{5}\right)}}\right) + \frac{1}{5}\sqrt{5+2\sqrt{5}}\arctan\left(\frac{1}{2}\sqrt{\frac{1}{10}\left(5+\sqrt{5}\right)}\left(1+\sqrt{5}+4x\right)\right) - \frac{\log\left(2+x-\sqrt{5}x+2x^2\right)}{2\sqrt{5}} + \frac{\log\left(2+x+\sqrt{5}x+2x^2\right)}{2\sqrt{5}}$$

output
```
1/5*(5-2*5^(1/2))^(1/2)*arctan((1-5^(1/2)+4*x)/(10+2*5^(1/2))^(1/2))+1/5*(
5+2*5^(1/2))^(1/2)*arctan(1/20*(50+10*5^(1/2))^(1/2)*(1+5^(1/2)+4*x))-1/10
*ln(2+x-x*5^(1/2)+2*x^2)*5^(1/2)+1/10*ln(2+x+x*5^(1/2)+2*x^2)*5^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.33

$$\int \frac{1}{1+x+x^2+x^3+x^4}\,dx = \text{RootSum}\left[1+\#1+\#1^2+\#1^3+\#1^4\&,\frac{\log(x-\#1)}{1+2\#1+3\#1^2+4\#1^3}\&\right]$$

input
```
Integrate[(1 + x + x^2 + x^3 + x^4)^(-1),x]
```

output
```
RootSum[1 + #1 + #1^2 + #1^3 + #1^4 & , Log[x - #1]/(1 + 2*#1 + 3*#1^2 + 4
*#1^3) & ]
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2492, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4+x^3+x^2+x+1}\,dx$$

$$\Big\downarrow \text{2492}$$

$$\int \left(\frac{2x+\sqrt{5}+1}{\sqrt{5}\left(2x^2+\left(1+\sqrt{5}\right)x+2\right)}-\frac{2x-\sqrt{5}+1}{\sqrt{5}\left(2x^2+\left(1-\sqrt{5}\right)x+2\right)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{5}\sqrt{5-2\sqrt{5}}\arctan\left(\frac{4x-\sqrt{5}+1}{\sqrt{2\left(5+\sqrt{5}\right)}}\right)+\frac{1}{5}\sqrt{5+2\sqrt{5}}\arctan\left(\frac{4x+\sqrt{5}+1}{\sqrt{2\left(5-\sqrt{5}\right)}}\right)-\frac{\log\left(2x^2+\left(1-\sqrt{5}\right)x+2\right)}{2\sqrt{5}}+\frac{\log\left(2x^2+\left(1+\sqrt{5}\right)x+2\right)}{2\sqrt{5}}$$

input
```
Int[(1 + x + x^2 + x^3 + x^4)^(-1),x]
```

output
```
(Sqrt[5 - 2*Sqrt[5]]*ArcTan[(1 - Sqrt[5] + 4*x)/Sqrt[2*(5 + Sqrt[5])]])/5
+ (Sqrt[5 + 2*Sqrt[5]]*ArcTan[(1 + Sqrt[5] + 4*x)/Sqrt[2*(5 - Sqrt[5])]])/
5 - Log[2 + (1 - Sqrt[5])*x + 2*x^2]/(2*Sqrt[5]) + Log[2 + (1 + Sqrt[5])*x
 + 2*x^2]/(2*Sqrt[5])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2492
```
Int[(Px_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^3 + (e_.)*(x_)^4)
^(p_), x_Symbol] :> Simp[e^p   Int[ExpandIntegrand[Px*(b/d + ((d + Sqrt[e*(
(b^2 - 4*a*c)/a) + 8*a*d*(e/b)])/(2*e))*x + x^2)^p*(b/d + ((d - Sqrt[e*((b^
2 - 4*a*c)/a) + 8*a*d*(e/b)])/(2*e))*x + x^2)^p, x], x], x] /; FreeQ[{a, b,
 c, d, e}, x] && PolyQ[Px, x] && ILtQ[p, 0] && EqQ[a*d^2 - b^2*e, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.09 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.29

method	result
risch	$\sum_{\_R=\operatorname{RootOf}\left(\_Z^{4}+\_Z^{3}+\_Z^{2}+\_Z+1\right)}\frac{\ln\left(x-\_R\right)}{4\_R^{3}+3\_R^{2}+2\_R+1}$
default	$-\frac{\ln\left(2+x-\sqrt{5}\,x+2x^{2}\right)\sqrt{5}}{10}-\frac{2\left(-\frac{\sqrt{5}\,\left(-\sqrt{5}+1\right)}{2}+\sqrt{5}-5\right)\arctan\left(\frac{1-\sqrt{5}+4x}{\sqrt{10+2\sqrt{5}}}\right)}{5\sqrt{10+2\sqrt{5}}}+\frac{\ln\left(2+x+\sqrt{5}\,x+2x^{2}\right)\sqrt{5}}{10}+\frac{2\left(-\frac{\sqrt{5}\,\left(\sqrt{5}+1\right)}{2}\right.}{}$

input
```
int(1/(x^4+x^3+x^2+x+1),x,method=_RETURNVERBOSE)
```

output
```
sum(1/(4*_R^3+3*_R^2+2*_R+1)*ln(x-_R),_R=RootOf(_Z^4+_Z^3+_Z^2+_Z+1))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.76

$$\int \frac{1}{1+x+x^2+x^3+x^4}\,dx = -\frac{1}{5}\sqrt{2\sqrt{5}+5}\arctan\left(\frac{1}{5}\left(\sqrt{5}(x-1)-5x\right)\sqrt{2\sqrt{5}+5}\right) + \frac{1}{5}\sqrt{-2\sqrt{5}+5}\arctan\left(\frac{1}{5}\left(\sqrt{5}(x-1)+5x\right)\sqrt{-2\sqrt{5}+5}\right) + \frac{1}{10}\sqrt{5}\log\left(2x^2+\sqrt{5}x+x+2\right) - \frac{1}{10}\sqrt{5}\log\left(2x^2-\sqrt{5}x+x+2\right)$$

input
```
integrate(1/(x^4+x^3+x^2+x+1),x, algorithm="fricas")
```

output
```
-1/5*sqrt(2*sqrt(5) + 5)*arctan(1/5*(sqrt(5)*(x - 1) - 5*x)*sqrt(2*sqrt(5)
 + 5)) + 1/5*sqrt(-2*sqrt(5) + 5)*arctan(1/5*(sqrt(5)*(x - 1) + 5*x)*sqrt(
-2*sqrt(5) + 5)) + 1/10*sqrt(5)*log(2*x^2 + sqrt(5)*x + x + 2) - 1/10*sqrt
(5)*log(2*x^2 - sqrt(5)*x + x + 2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1287 vs. 2(122) = 244.

Time = 0.54 (sec) , antiderivative size = 1287, normalized size of antiderivative = 9.00

$$\int \frac{1}{1+x+x^2+x^3+x^4}\,dx = \text{Too large to display}$$

input
```
integrate(1/(x**4+x**3+x**2+x+1),x)
```

output
```
sqrt(5)*log(x**2 + x*(-21*sqrt(5)/11 - 4*sqrt(10)*sqrt(3 - sqrt(5))/11 + 4
5*sqrt(2)*sqrt(3 - sqrt(5))/22 + 48/11) - 2213*sqrt(5)/242 - 1381*sqrt(10)
*sqrt(3 - sqrt(5))/484 + 3045*sqrt(2)*sqrt(3 - sqrt(5))/484 + 5217/242)/10
 - sqrt(5)*log(x**2 + x*(-45*sqrt(2)*sqrt(sqrt(5) + 3)/22 - 4*sqrt(10)*sqr
t(sqrt(5) + 3)/11 + 21*sqrt(5)/11 + 48/11) - 1381*sqrt(10)*sqrt(sqrt(5) +
3)/484 - 3045*sqrt(2)*sqrt(sqrt(5) + 3)/484 + 2213*sqrt(5)/242 + 5217/242)
/10 + 2*sqrt(-sqrt(10)*sqrt(3 - sqrt(5))/50 + 3/20)*atan(44*x/(-8*sqrt(5)*
sqrt(-2*sqrt(10)*sqrt(3 - sqrt(5)) + 15) + 3*sqrt(10)*sqrt(3 - sqrt(5))*sq
rt(-2*sqrt(10)*sqrt(3 - sqrt(5)) + 15) + 18*sqrt(-2*sqrt(10)*sqrt(3 - sqrt
(5)) + 15)) - 42*sqrt(5)/(-8*sqrt(5)*sqrt(-2*sqrt(10)*sqrt(3 - sqrt(5)) +
15) + 3*sqrt(10)*sqrt(3 - sqrt(5))*sqrt(-2*sqrt(10)*sqrt(3 - sqrt(5)) + 15
) + 18*sqrt(-2*sqrt(10)*sqrt(3 - sqrt(5)) + 15)) - 8*sqrt(10)*sqrt(3 - sqr
t(5))/(-8*sqrt(5)*sqrt(-2*sqrt(10)*sqrt(3 - sqrt(5)) + 15) + 3*sqrt(10)*sq
rt(3 - sqrt(5))*sqrt(-2*sqrt(10)*sqrt(3 - sqrt(5)) + 15) + 18*sqrt(-2*sqrt
(10)*sqrt(3 - sqrt(5)) + 15)) + 45*sqrt(2)*sqrt(3 - sqrt(5))/(-8*sqrt(5)*s
qrt(-2*sqrt(10)*sqrt(3 - sqrt(5)) + 15) + 3*sqrt(10)*sqrt(3 - sqrt(5))*sqr
t(-2*sqrt(10)*sqrt(3 - sqrt(5)) + 15) + 18*sqrt(-2*sqrt(10)*sqrt(3 - sqrt(
5)) + 15)) + 96/(-8*sqrt(5)*sqrt(-2*sqrt(10)*sqrt(3 - sqrt(5)) + 15) + 3*s
qrt(10)*sqrt(3 - sqrt(5))*sqrt(-2*sqrt(10)*sqrt(3 - sqrt(5)) + 15) + 18*sq
rt(-2*sqrt(10)*sqrt(3 - sqrt(5)) + 15))) + 2*sqrt(-sqrt(10)*sqrt(sqrt(5...
```

Maxima [F]

$$\int \frac{1}{1+x+x^2+x^3+x^4}\,dx=\int \frac{1}{x^4+x^3+x^2+x+1}\,dx$$

input
```
integrate(1/(x^4+x^3+x^2+x+1),x, algorithm="maxima")
```

output
```
integrate(1/(x^4 + x^3 + x^2 + x + 1), x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.71

$$\int \frac{1}{1+x+x^2+x^3+x^4}\,dx = \frac{1}{5}\sqrt{-2\sqrt{5}+5}\arctan\left(\frac{4x-\sqrt{5}+1}{\sqrt{2\sqrt{5}+10}}\right) + \frac{1}{5}\sqrt{2\sqrt{5}+5}\arctan\left(\frac{4x+\sqrt{5}+1}{\sqrt{-2\sqrt{5}+10}}\right) + \frac{1}{10}\sqrt{5}\log\left(x^2+\frac{1}{2}x\left(\sqrt{5}+1\right)+1\right) - \frac{1}{10}\sqrt{5}\log\left(x^2-\frac{1}{2}x\left(\sqrt{5}-1\right)+1\right)$$

input
```
integrate(1/(x^4+x^3+x^2+x+1),x, algorithm="giac")
```

output
```
1/5*sqrt(-2*sqrt(5) + 5)*arctan((4*x - sqrt(5) + 1)/sqrt(2*sqrt(5) + 10))
+ 1/5*sqrt(2*sqrt(5) + 5)*arctan((4*x + sqrt(5) + 1)/sqrt(-2*sqrt(5) + 10)
) + 1/10*sqrt(5)*log(x^2 + 1/2*x*(sqrt(5) + 1) + 1) - 1/10*sqrt(5)*log(x^2
 - 1/2*x*(sqrt(5) - 1) + 1)
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.45

$$\int \frac{1}{1+x+x^2+x^3+x^4}\,dx = \sum_{k=1}^{4} \ln\left(-\mathrm{root}\left(z^4+\frac{z}{25}+\frac{1}{125},z,k\right)\left(4x + \mathrm{root}\left(z^4+\frac{z}{25}+\frac{1}{125},z,k\right)\left(25\,\mathrm{root}\left(z^4+\frac{z}{25}+\frac{1}{125},z,k\right)+15x+15\right)+1\right)\right)\mathrm{root}\left(z^4+\frac{z}{25}+\frac{1}{125},z,k\right)$$

input
```
int(1/(x + x^2 + x^3 + x^4 + 1),x)
```

output
```
symsum(log(-root(z^4 + z/25 + 1/125, z, k)*(4*x + root(z^4 + z/25 + 1/125,
 z, k)*(25*root(z^4 + z/25 + 1/125, z, k) + 15*x + 15) + 1))*root(z^4 + z/
25 + 1/125, z, k), k, 1, 4)
```

Reduce [F]

$$\int \frac{1}{1+x+x^2+x^3+x^4}\,dx = \int \frac{1}{x^4+x^3+x^2+x+1}dx$$

input
```
int(1/(x^4+x^3+x^2+x+1),x)
```

output
```
int(1/(x**4 + x**3 + x**2 + x + 1),x)
```

3.13 $\int \frac{1}{(1+x+x^2+x^3+x^4)^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 238

$$
\begin{aligned}
&\int \frac{1}{(1+x+x^2+x^3+x^4)^2}\,dx \\
&= -\frac{2}{5\left(2+\left(1+\sqrt{5}\right)x+2x^2\right)} \\
&\quad + \frac{4\left(5+\sqrt{5}+\left(5-\sqrt{5}\right)x\right)}{5\left(5+\sqrt{5}\right)\left(2+\left(1-\sqrt{5}\right)x+2x^2\right)\left(2+\left(1+\sqrt{5}\right)x+2x^2\right)} \\
&\quad - \frac{2}{25}\sqrt{2\left(25-11\sqrt{5}\right)}\arctan\left(\frac{1-\sqrt{5}+4x}{\sqrt{2\left(5+\sqrt{5}\right)}}\right) \\
&\quad + \frac{2}{25}\sqrt{2\left(25+11\sqrt{5}\right)}\arctan\left(\frac{1}{2}\sqrt{\frac{1}{10}\left(5+\sqrt{5}\right)}\left(1+\sqrt{5}+4x\right)\right) \\
&\quad - \frac{2\log\left(2+x-\sqrt{5}x+2x^2\right)}{5\sqrt{5}} + \frac{2\log\left(2+x+\sqrt{5}x+2x^2\right)}{5\sqrt{5}}
\end{aligned}
$$

output

```
-2/(10+5*(5^(1/2)+1)*x+10*x^2)+4/5*(5+5^(1/2)+(5-5^(1/2))*x)/(5+5^(1/2))/(
2+x*(-5^(1/2)+1)+2*x^2)/(2+(5^(1/2)+1)*x+2*x^2)-2/25*(50-22*5^(1/2))^(1/2)
*arctan((1-5^(1/2)+4*x)/(10+2*5^(1/2))^(1/2))+2/25*(50+22*5^(1/2))^(1/2)*a
rctan(1/20*(50+10*5^(1/2))^(1/2)*(1+5^(1/2)+4*x))-2/25*ln(2+x-x*5^(1/2)+2*
x^2)*5^(1/2)+2/25*ln(2+x+x*5^(1/2)+2*x^2)*5^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.37

$$\int \frac{1}{(1+x+x^2+x^3+x^4)^2}\,dx = \frac{1}{5}\left(-\frac{(-1+x)x}{1+x+x^2+x^3+x^4} - 2\text{RootSum}\left[1+\#1+\#1^2+\#1^3+\#1^4\&, \frac{-2\log(x-\#1)+\log(x-\#1)\#1}{1+2\#1+3\#1^2+4\#1^3}\&\right]\right)$$

input
```
Integrate[(1 + x + x^2 + x^3 + x^4)^(-2),x]
```

output
```
(-(((-1 + x)*x)/(1 + x + x^2 + x^3 + x^4)) - 2*RootSum[1 + #1 + #1^2 + #1^
3 + #1^4 & , (-2*Log[x - #1] + Log[x - #1]*#1)/(1 + 2*#1 + 3*#1^2 + 4*#1^3
) & ])/5
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.50, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2492, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x^4+x^3+x^2+x+1)^2}\,dx$$

$$\Big\downarrow \; 2492$$

$$\int \left(\frac{2(1-\sqrt{5})\,(x+1)}{5\left(2x^2+\left(1-\sqrt{5}\right)x+2\right)^2} + \frac{2(1+\sqrt{5})\,(x+1)}{5\left(2x^2+\left(1+\sqrt{5}\right)x+2\right)^2} - \frac{2(4x-2\sqrt{5}+3)}{5\sqrt{5}\left(2x^2+\left(1-\sqrt{5}\right)x+2\right)} + \frac{2(4x+2\sqrt{}}{5\sqrt{5}\left(2x^2+(1+\right.}\right.$$

$$\Big\downarrow \; 2009$$

$$-\frac{1}{25}\sqrt{2\left(5-\sqrt{5}\right)}\arctan\left(\frac{4x-\sqrt{5}+1}{\sqrt{2\left(5+\sqrt{5}\right)}}\right)+\frac{1}{25}\sqrt{2\left(65-29\sqrt{5}\right)}\arctan\left(\frac{4x-\sqrt{5}+1}{\sqrt{2\left(5+\sqrt{5}\right)}}\right)+\frac{1}{25}\sqrt{2\left(65+29\sqrt{5}\right)}\arctan\left(\frac{4x+\sqrt{5}+1}{\sqrt{2\left(5-\sqrt{5}\right)}}\right)+\frac{1}{25}\sqrt{2\left(5+\sqrt{5}\right)}\arctan\left(\frac{4x+\sqrt{5}+1}{\sqrt{2\left(5-\sqrt{5}\right)}}\right)-\frac{\left(1+\sqrt{5}\right)\left(-\left(\left(3-\sqrt{5}\right)x\right)-\sqrt{5}+3\right)}{5\left(5-\sqrt{5}\right)\left(2x^2+\left(1+\sqrt{5}\right)x+2\right)}-\frac{\left(1-\sqrt{5}\right)\left(-\left(\left(3+\sqrt{5}\right)x\right)+\sqrt{5}+3\right)}{5\left(5+\sqrt{5}\right)\left(2x^2+\left(1-\sqrt{5}\right)x+2\right)}-\frac{2\log\left(2x^2+\left(1-\sqrt{5}\right)x+2\right)}{5\sqrt{5}}+\frac{2\log\left(2x^2+\left(1+\sqrt{5}\right)x+2\right)}{5\sqrt{5}}$$

input
```
Int[(1 + x + x^2 + x^3 + x^4)^(-2),x]
```

output
```
-1/5*((1 - Sqrt[5])*(3 + Sqrt[5] - (3 + Sqrt[5])*x))/((5 + Sqrt[5])*(2 + (
1 - Sqrt[5])*x + 2*x^2)) - ((1 + Sqrt[5])*(3 - Sqrt[5] - (3 - Sqrt[5])*x))
/(5*(5 - Sqrt[5])*(2 + (1 + Sqrt[5])*x + 2*x^2)) + (Sqrt[2*(65 - 29*Sqrt[5
])]*ArcTan[(1 - Sqrt[5] + 4*x)/Sqrt[2*(5 + Sqrt[5])]])/25 - (Sqrt[2*(5 - S
qrt[5])]*ArcTan[(1 - Sqrt[5] + 4*x)/Sqrt[2*(5 + Sqrt[5])]])/25 + (Sqrt[2*(
5 + Sqrt[5])]*ArcTan[(1 + Sqrt[5] + 4*x)/Sqrt[2*(5 - Sqrt[5])]])/25 + (Sqr
t[2*(65 + 29*Sqrt[5])]*ArcTan[(1 + Sqrt[5] + 4*x)/Sqrt[2*(5 - Sqrt[5])]])/
25 - (2*Log[2 + (1 - Sqrt[5])*x + 2*x^2])/(5*Sqrt[5]) + (2*Log[2 + (1 + Sq
rt[5])*x + 2*x^2])/(5*Sqrt[5])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2492
```
Int[(Px_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^3 + (e_.)*(x_)^4)
^(p_), x_Symbol] :> Simp[e^p   Int[ExpandIntegrand[Px*(b/d + ((d + Sqrt[e*(
(b^2 - 4*a*c)/a) + 8*a*d*(e/b)])/(2*e))*x + x^2)^p*(b/d + ((d - Sqrt[e*((b^
2 - 4*a*c)/a) + 8*a*d*(e/b)])/(2*e))*x + x^2)^p, x], x], x] /; FreeQ[{a, b,
 c, d, e}, x] && PolyQ[Px, x] && ILtQ[p, 0] && EqQ[a*d^2 - b^2*e, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.16 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.31

method	result
risch	$\frac{-\frac{1}{5}x^2+\frac{1}{5}x}{x^4+x^3+x^2+x+1}+\frac{2\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^4+\_Z^3+\_Z^2+\_Z+1\right)}\frac{\left(2-\_R\right)\ln\left(x-\_R\right)}{4\_R^3+3\_R^2+2\_R+1}\right)}{5}$
default	$-\frac{2\left(\frac{\sqrt{5}\,x}{2}-\frac{\sqrt{5}}{2}\right)}{25\left(x^2+\frac{x}{2}-\frac{\sqrt{5}\,x}{2}+1\right)}-\frac{2\ln\left(2+x-\sqrt{5}\,x+2x^2\right)\sqrt{5}}{25}+\frac{8\left(\frac{\sqrt{5}\left(-\sqrt{5}+1\right)}{2}+5-2\sqrt{5}\right)\arctan\left(\frac{1-\sqrt{5}+4x}{\sqrt{10+2\sqrt{5}}}\right)}{25\sqrt{10+2\sqrt{5}}}+\frac{\frac{\sqrt{5}\,x}{25}-\frac{\sqrt{5}}{25}}{x^2+\frac{x}{2}+\frac{\sqrt{5}\,x}{2}+1}+$

input
```
int(1/(x^4+x^3+x^2+x+1)^2,x,method=_RETURNVERBOSE)
```

output
```
(-1/5*x^2+1/5*x)/(x^4+x^3+x^2+x+1)+2/5*sum((2-_R)/(4*_R^3+3*_R^2+2*_R+1)*l
n(x-_R),_R=RootOf(_Z^4+_Z^3+_Z^2+_Z+1))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.78

$$\int \frac{1}{(1+x+x^2+x^3+x^4)^2}\,dx$$
$$= \frac{2\,(x^4+x^3+x^2+x+1)\sqrt{22\sqrt{5}+50}\arctan\left(\frac{1}{20}\left(\sqrt{5}(6\,x-1)-10\,x+5\right)\sqrt{22\sqrt{5}+50}\right)-2\,(x^4+x^3}{}$$

input
```
integrate(1/(x^4+x^3+x^2+x+1)^2,x, algorithm="fricas")
```

output
```
1/25*(2*(x^4 + x^3 + x^2 + x + 1)*sqrt(22*sqrt(5) + 50)*arctan(1/20*(sqrt(
5)*(6*x - 1) - 10*x + 5)*sqrt(22*sqrt(5) + 50)) - 2*(x^4 + x^3 + x^2 + x +
 1)*sqrt(-22*sqrt(5) + 50)*arctan(1/20*(sqrt(5)*(6*x - 1) + 10*x - 5)*sqrt
(-22*sqrt(5) + 50)) + 2*sqrt(5)*(x^4 + x^3 + x^2 + x + 1)*log(2*x^2 + sqrt
(5)*x + x + 2) - 2*sqrt(5)*(x^4 + x^3 + x^2 + x + 1)*log(2*x^2 - sqrt(5)*x
 + x + 2) - 5*x^2 + 5*x)/(x^4 + x^3 + x^2 + x + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1360 vs. $2(207) = 414$.

Time = 0.65 (sec) , antiderivative size = 1360, normalized size of antiderivative = 5.71

$$\int \frac{1}{(1+x+x^2+x^3+x^4)^2} \, dx = \text{Too large to display}$$

input

```
integrate(1/(x**4+x**3+x**2+x+1)**2,x)
```

output

```
(-x**2 + x)/(5*x**4 + 5*x**3 + 5*x**2 + 5*x + 5) + 2*sqrt(5)*log(x**2 + x*
(-8229/802 - 85*sqrt(5)*sqrt(201 - 88*sqrt(5))/802 + 315*sqrt(201 - 88*sqr
t(5))/401 + 1928*sqrt(5)/401) - 30944507*sqrt(5)/643204 - 3418085*sqrt(201
 - 88*sqrt(5))/643204 + 1570197*sqrt(5)*sqrt(201 - 88*sqrt(5))/643204 + 69
646079/643204)/25 - 2*sqrt(5)*log(x**2 + x*(-1928*sqrt(5)/401 - 8229/802 +
 85*sqrt(5)*sqrt(88*sqrt(5) + 201)/802 + 315*sqrt(88*sqrt(5) + 201)/401) -
 1570197*sqrt(5)*sqrt(88*sqrt(5) + 201)/643204 - 3418085*sqrt(88*sqrt(5) +
 201)/643204 + 30944507*sqrt(5)/643204 + 69646079/643204)/25 - 2*sqrt(-2*s
qrt(5)*sqrt(88*sqrt(5) + 201)/625 + 18/125)*atan(802*sqrt(2)*x/(170*sqrt(5
)*sqrt(-sqrt(5)*sqrt(88*sqrt(5) + 201) + 45) + 382*sqrt(-sqrt(5)*sqrt(88*s
qrt(5) + 201) + 45) + 21*sqrt(5)*sqrt(88*sqrt(5) + 201)*sqrt(-sqrt(5)*sqrt
(88*sqrt(5) + 201) + 45)) - 3856*sqrt(10)/(340*sqrt(5)*sqrt(-sqrt(5)*sqrt(
88*sqrt(5) + 201) + 45) + 764*sqrt(-sqrt(5)*sqrt(88*sqrt(5) + 201) + 45) +
 42*sqrt(5)*sqrt(88*sqrt(5) + 201)*sqrt(-sqrt(5)*sqrt(88*sqrt(5) + 201) +
45)) - 8229*sqrt(2)/(340*sqrt(5)*sqrt(-sqrt(5)*sqrt(88*sqrt(5) + 201) + 45
) + 764*sqrt(-sqrt(5)*sqrt(88*sqrt(5) + 201) + 45) + 42*sqrt(5)*sqrt(88*sq
rt(5) + 201)*sqrt(-sqrt(5)*sqrt(88*sqrt(5) + 201) + 45)) + 85*sqrt(10)*sqr
t(88*sqrt(5) + 201)/(340*sqrt(5)*sqrt(-sqrt(5)*sqrt(88*sqrt(5) + 201) + 45
) + 764*sqrt(-sqrt(5)*sqrt(88*sqrt(5) + 201) + 45) + 42*sqrt(5)*sqrt(88*sq
rt(5) + 201)*sqrt(-sqrt(5)*sqrt(88*sqrt(5) + 201) + 45)) + 630*sqrt(2)*...
```

Maxima [F]

$$\int \frac{1}{(1+x+x^2+x^3+x^4)^2}\,dx = \int \frac{1}{(x^4+x^3+x^2+x+1)^2}\,dx$$

input
```
integrate(1/(x^4+x^3+x^2+x+1)^2,x, algorithm="maxima")
```

output
```
-1/5*(x^2 - x)/(x^4 + x^3 + x^2 + x + 1) - 2/5*integrate((x - 2)/(x^4 + x^
3 + x^2 + x + 1), x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.52

$$\int \frac{1}{(1+x+x^2+x^3+x^4)^2}\,dx = -\frac{2}{25}\sqrt{-22\sqrt{5}+50}\arctan\left(\frac{4x-\sqrt{5}+1}{\sqrt{2\sqrt{5}+10}}\right) + \frac{2}{25}\sqrt{22\sqrt{5}+50}\arctan\left(\frac{4x+\sqrt{5}+1}{\sqrt{-2\sqrt{5}+10}}\right) + \frac{2}{25}\sqrt{5}\log\left(x^2+\frac{1}{2}x\left(\sqrt{5}+1\right)+1\right) - \frac{2}{25}\sqrt{5}\log\left(x^2-\frac{1}{2}x\left(\sqrt{5}-1\right)+1\right) - \frac{x^2-x}{5(x^4+x^3+x^2+x+1)}$$

input
```
integrate(1/(x^4+x^3+x^2+x+1)^2,x, algorithm="giac")
```

output
```
-2/25*sqrt(-22*sqrt(5) + 50)*arctan((4*x - sqrt(5) + 1)/sqrt(2*sqrt(5) + 1
0)) + 2/25*sqrt(22*sqrt(5) + 50)*arctan((4*x + sqrt(5) + 1)/sqrt(-2*sqrt(5
) + 10)) + 2/25*sqrt(5)*log(x^2 + 1/2*x*(sqrt(5) + 1) + 1) - 2/25*sqrt(5)*
log(x^2 - 1/2*x*(sqrt(5) - 1) + 1) - 1/5*(x^2 - x)/(x^4 + x^3 + x^2 + x +
1)
```

Mupad [B] (verification not implemented)

Time = 21.28 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.48

$$\int \frac{1}{(1+x+x^2+x^3+x^4)^2}\,dx = \left(\sum_{k=1}^{4}\ln\left(-\frac{8\,x}{125}\right.\right.$$
$$-\mathrm{root}\left(z^4+\frac{12\,z^2}{125}+\frac{176\,z}{3125}+\frac{496}{78125},z,k\right)\left(\frac{76\,x}{25}+\mathrm{root}\left(z^4+\frac{12\,z^2}{125}+\frac{176\,z}{3125}+\frac{496}{78125},z,k\right)\left(25\,\mathrm{root}\left(z^4+\right.\right.\right.$$
$$\left.\left.+\frac{16}{125}\right)\mathrm{root}\left(z^4+\frac{12\,z^2}{125}+\frac{176\,z}{3125}+\frac{496}{78125},z,k\right)\right)+\frac{\frac{x}{5}-\frac{x^2}{5}}{x^4+x^3+x^2+x+1}$$

input
```
int(1/(x + x^2 + x^3 + x^4 + 1)^2,x)
```

output
```
symsum(log(16/125 - root(z^4 + (12*z^2)/125 + (176*z)/3125 + 496/78125, z,
 k)*((76*x)/25 + root(z^4 + (12*z^2)/125 + (176*z)/3125 + 496/78125, z, k)
*(25*root(z^4 + (12*z^2)/125 + (176*z)/3125 + 496/78125, z, k) + 18*x + 14
) + 44/25) - (8*x)/125)*root(z^4 + (12*z^2)/125 + (176*z)/3125 + 496/78125
, z, k), k, 1, 4) + (x/5 - x^2/5)/(x + x^2 + x^3 + x^4 + 1)
```

Reduce [F]

$$\int \frac{1}{(1+x+x^2+x^3+x^4)^2}\,dx$$
$$= \frac{-4\left(\int \frac{x^3}{x^8+2x^7+3x^6+4x^5+5x^4+4x^3+3x^2+2x+1}dx\right)x^4-4\left(\int \frac{x^3}{x^8+2x^7+3x^6+4x^5+5x^4+4x^3+3x^2+2x+1}dx\right)x^3-4\left(\int \frac{}{x^8+2x^7}\right.}{}$$

input
```
int(1/(x^4+x^3+x^2+x+1)^2,x)
```

output
```
( - 4*int(x**3/(x**8 + 2*x**7 + 3*x**6 + 4*x**5 + 5*x**4 + 4*x**3 + 3*x**2
 + 2*x + 1),x)*x**4 - 4*int(x**3/(x**8 + 2*x**7 + 3*x**6 + 4*x**5 + 5*x**4
 + 4*x**3 + 3*x**2 + 2*x + 1),x)*x**3 - 4*int(x**3/(x**8 + 2*x**7 + 3*x**6
 + 4*x**5 + 5*x**4 + 4*x**3 + 3*x**2 + 2*x + 1),x)*x**2 - 4*int(x**3/(x**8
 + 2*x**7 + 3*x**6 + 4*x**5 + 5*x**4 + 4*x**3 + 3*x**2 + 2*x + 1),x)*x - 4
*int(x**3/(x**8 + 2*x**7 + 3*x**6 + 4*x**5 + 5*x**4 + 4*x**3 + 3*x**2 + 2*
x + 1),x) - 3*int(x**2/(x**8 + 2*x**7 + 3*x**6 + 4*x**5 + 5*x**4 + 4*x**3
+ 3*x**2 + 2*x + 1),x)*x**4 - 3*int(x**2/(x**8 + 2*x**7 + 3*x**6 + 4*x**5
+ 5*x**4 + 4*x**3 + 3*x**2 + 2*x + 1),x)*x**3 - 3*int(x**2/(x**8 + 2*x**7
+ 3*x**6 + 4*x**5 + 5*x**4 + 4*x**3 + 3*x**2 + 2*x + 1),x)*x**2 - 3*int(x*
*2/(x**8 + 2*x**7 + 3*x**6 + 4*x**5 + 5*x**4 + 4*x**3 + 3*x**2 + 2*x + 1),
x)*x - 3*int(x**2/(x**8 + 2*x**7 + 3*x**6 + 4*x**5 + 5*x**4 + 4*x**3 + 3*x
**2 + 2*x + 1),x) - 2*int(x/(x**8 + 2*x**7 + 3*x**6 + 4*x**5 + 5*x**4 + 4*
x**3 + 3*x**2 + 2*x + 1),x)*x**4 - 2*int(x/(x**8 + 2*x**7 + 3*x**6 + 4*x**
5 + 5*x**4 + 4*x**3 + 3*x**2 + 2*x + 1),x)*x**3 - 2*int(x/(x**8 + 2*x**7 +
 3*x**6 + 4*x**5 + 5*x**4 + 4*x**3 + 3*x**2 + 2*x + 1),x)*x**2 - 2*int(x/(
x**8 + 2*x**7 + 3*x**6 + 4*x**5 + 5*x**4 + 4*x**3 + 3*x**2 + 2*x + 1),x)*x
 - 2*int(x/(x**8 + 2*x**7 + 3*x**6 + 4*x**5 + 5*x**4 + 4*x**3 + 3*x**2 + 2
*x + 1),x) - 1)/(x**4 + x**3 + x**2 + x + 1)
```

3.14 $\int \frac{1}{(1+x+x^2+x^3+x^4)^3}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 371

$$\begin{aligned}
\int \frac{1}{(1+x+x^2+x^3+x^4)^3}\,dx
&= -\frac{6\left(2\left(15-8\sqrt{5}\right)-\left(15-11\sqrt{5}\right)x\right)}{25\left(5+\sqrt{5}\right)\left(2+\left(1+\sqrt{5}\right)x+2x^2\right)^2} \\
&\quad+\frac{8\left(5+\sqrt{5}+\left(5-\sqrt{5}\right)x\right)}{5\left(5+\sqrt{5}\right)\left(2+\left(1-\sqrt{5}\right)x+2x^2\right)^2\left(2+\left(1+\sqrt{5}\right)x+2x^2\right)^2} \\
&\quad+\frac{8\left(45-11\sqrt{5}+6\left(15+\sqrt{5}\right)x\right)}{5\left(5+\sqrt{5}\right)^2\left(2+\left(1-\sqrt{5}\right)x+2x^2\right)\left(2+\left(1+\sqrt{5}\right)x+2x^2\right)^2} \\
&\quad-\frac{3\left(6-\sqrt{5}-2x\right)}{25\left(2+\left(1+\sqrt{5}\right)x+2x^2\right)}-\frac{3}{125}\sqrt{1025-422\sqrt{5}}\arctan\left(\frac{1-\sqrt{5}+4x}{\sqrt{2\left(5+\sqrt{5}\right)}}\right) \\
&\quad+\frac{3}{125}\sqrt{1025+422\sqrt{5}}\arctan\left(\frac{1+\sqrt{5}+4x}{\sqrt{2\left(5-\sqrt{5}\right)}}\right) \\
&\quad-\frac{3\log\left(2+\left(1-\sqrt{5}\right)x+2x^2\right)}{10\sqrt{5}}+\frac{3\log\left(2+\left(1+\sqrt{5}\right)x+2x^2\right)}{10\sqrt{5}}
\end{aligned}$$

output
```
1/25*(-180+96*5^(1/2)+6*(15-11*5^(1/2))*x)/(5+5^(1/2))/(2+(5^(1/2)+1)*x+2*
x^2)^2+8/5*(5+5^(1/2)+(5-5^(1/2))*x)/(5+5^(1/2))/(2+x*(-5^(1/2)+1)+2*x^2)^
2/(2+(5^(1/2)+1)*x+2*x^2)^2+8/5*(45-11*5^(1/2)+6*(15+5^(1/2))*x)/(5+5^(1/2
))^2/(2+x*(-5^(1/2)+1)+2*x^2)/(2+(5^(1/2)+1)*x+2*x^2)^2-3*(6-5^(1/2)-2*x)/
(50+25*(5^(1/2)+1)*x+50*x^2)-3/125*(1025-422*5^(1/2))^(1/2)*arctan((1-5^(1
/2)+4*x)/(10+2*5^(1/2))^(1/2))+3/125*(1025+422*5^(1/2))^(1/2)*arctan((1+5^
(1/2)+4*x)/(10-2*5^(1/2))^(1/2))-3/50*ln(2+x*(-5^(1/2)+1)+2*x^2)*5^(1/2)+3
/50*ln(2+(5^(1/2)+1)*x+2*x^2)*5^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.30

$$\int \frac{1}{(1+x+x^2+x^3+x^4)^3}\,dx = \frac{1}{50}\left(\frac{x(14-11x-9x^5+6x^6)}{(1+x+x^2+x^3+x^4)^2}+6\text{RootSum}\left[1+\#1+\#1^2+\#1^3+\#1^4\&,\frac{6\log(x-\#1)-6\log(x-\#1)\#1+\log(x-\#1)\#1^2}{1+2\#1+3\#1^2+4\#1^3}\&\right]\right)$$

input
```
Integrate[(1 + x + x^2 + x^3 + x^4)^(-3),x]
```

output
```
((x*(14 - 11*x - 9*x^5 + 6*x^6))/(1 + x + x^2 + x^3 + x^4)^2 + 6*RootSum[1
 + #1 + #1^2 + #1^3 + #1^4 & , (6*Log[x - #1] - 6*Log[x - #1]*#1 + Log[x -
 #1]*#1^2)/(1 + 2*#1 + 3*#1^2 + 4*#1^3) & ])/50
```

Rubi [A] (verified)

Time = 1.32 (sec) , antiderivative size = 631, normalized size of antiderivative = 1.70, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2492, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x^4+x^3+x^2+x+1)^3}\,dx$$

$$\Big\downarrow \text{2492}$$

$$\int \left(-\frac{3(10x-5\sqrt{5}+11)}{25\sqrt{5}\,(2x^2+(1-\sqrt{5})\,x+2)}+\frac{3(10x+5\sqrt{5}+11)}{25\sqrt{5}\,(2x^2+(1+\sqrt{5})\,x+2)}+\frac{2((9-5\sqrt{5})\,x+2(5-4\sqrt{5}))}{25\,(2x^2+(1-\sqrt{5})\,x+2)^2}+\frac{2((9-}{25\,(2}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{6}{125}\sqrt{5-2\sqrt{5}}\arctan\left(\frac{4x-\sqrt{5}+1}{\sqrt{2\,(5+\sqrt{5})}}\right)-\frac{3}{125}\sqrt{2\left(145-61\sqrt{5}\right)}\arctan\left(\frac{4x-\sqrt{5}+1}{\sqrt{2\,(5+\sqrt{5})}}\right)-\frac{3}{125}\sqrt{365-158\sqrt{5}}\arctan\left(\frac{4x-\sqrt{5}+1}{\sqrt{2\,(5+\sqrt{5})}}\right)+\frac{3}{125}\sqrt{365+158\sqrt{5}}\arctan\left(\frac{4x+\sqrt{5}+1}{\sqrt{2\,(5-\sqrt{5})}}\right)+\frac{3}{125}\sqrt{2\left(145+61\sqrt{5}\right)}\arctan\left(\frac{4x+\sqrt{5}+1}{\sqrt{2\,(5-\sqrt{5})}}\right)-\frac{6}{125}\sqrt{5+2\sqrt{5}}\arctan\left(\frac{4x+\sqrt{5}+1}{\sqrt{2\,(5-\sqrt{5})}}\right)-\frac{3(4x-\sqrt{5}+1)}{25\,(5+\sqrt{5})\,(2x^2+(1-\sqrt{5})\,x+2)}+\frac{2(3(1-3\sqrt{5})\,x+\sqrt{5}+7)}{25\,(5+\sqrt{5})\,(2x^2+(1-\sqrt{5})\,x+2)}-\frac{3(4x+\sqrt{5}+1)}{25\,(5-\sqrt{5})\,(2x^2+(1+\sqrt{5})\,x+2)}+\frac{2(3(1+3\sqrt{5})\,x-\sqrt{5}+7)}{25\,(5-\sqrt{5})\,(2x^2+(1+\sqrt{5})\,x+2)}+\frac{2(-((1+\sqrt{5})\,x)-\sqrt{5}+1)}{5\sqrt{5}\,(5+\sqrt{5})\,(2x^2+(1-\sqrt{5})\,x+2)^2}-\frac{2(-((1-\sqrt{5})\,x)+\sqrt{5}+1)}{5\sqrt{5}\,(5-\sqrt{5})\,(2x^2+(1+\sqrt{5})\,x+2)^2}-\frac{3\log\left(2x^2+(1-\sqrt{5})\,x+2\right)}{10\sqrt{5}}+\frac{3\log\left(2x^2+(1+\sqrt{5})\,x+2\right)}{10\sqrt{5}}$$

input
```
Int[(1 + x + x^2 + x^3 + x^4)^(-3),x]
```

output
```
(2*(1 - Sqrt[5] - (1 + Sqrt[5])*x))/(5*Sqrt[5]*(5 + Sqrt[5])*(2 + (1 - Sqr
t[5])*x + 2*x^2)^2) - (3*(1 - Sqrt[5] + 4*x))/(25*(5 + Sqrt[5])*(2 + (1 -
Sqrt[5])*x + 2*x^2)) + (2*(7 + Sqrt[5] + 3*(1 - 3*Sqrt[5])*x))/(25*(5 + Sq
rt[5])*(2 + (1 - Sqrt[5])*x + 2*x^2)) - (2*(1 + Sqrt[5] - (1 - Sqrt[5])*x)
)/(5*Sqrt[5]*(5 - Sqrt[5])*(2 + (1 + Sqrt[5])*x + 2*x^2)^2) - (3*(1 + Sqrt
[5] + 4*x))/(25*(5 - Sqrt[5])*(2 + (1 + Sqrt[5])*x + 2*x^2)) + (2*(7 - Sqr
t[5] + 3*(1 + 3*Sqrt[5])*x))/(25*(5 - Sqrt[5])*(2 + (1 + Sqrt[5])*x + 2*x^
2)) - (3*Sqrt[365 - 158*Sqrt[5]]*ArcTan[(1 - Sqrt[5] + 4*x)/Sqrt[2*(5 + Sq
rt[5])]])/125 - (3*Sqrt[2*(145 - 61*Sqrt[5])]*ArcTan[(1 - Sqrt[5] + 4*x)/S
qrt[2*(5 + Sqrt[5])]])/125 - (6*Sqrt[5 - 2*Sqrt[5]]*ArcTan[(1 - Sqrt[5] +
4*x)/Sqrt[2*(5 + Sqrt[5])]])/125 - (6*Sqrt[5 + 2*Sqrt[5]]*ArcTan[(1 + Sqrt
[5] + 4*x)/Sqrt[2*(5 - Sqrt[5])]])/125 + (3*Sqrt[2*(145 + 61*Sqrt[5])]*Arc
Tan[(1 + Sqrt[5] + 4*x)/Sqrt[2*(5 - Sqrt[5])]])/125 + (3*Sqrt[365 + 158*Sq
rt[5]]*ArcTan[(1 + Sqrt[5] + 4*x)/Sqrt[2*(5 - Sqrt[5])]])/125 - (3*Log[2 +
 (1 - Sqrt[5])*x + 2*x^2])/(10*Sqrt[5]) + (3*Log[2 + (1 + Sqrt[5])*x + 2*x
^2])/(10*Sqrt[5])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2492
```
Int[(Px_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^3 + (e_.)*(x_)^4)
^(p_), x_Symbol] :> Simp[e^p   Int[ExpandIntegrand[Px*(b/d + ((d + Sqrt[e*(
(b^2 - 4*a*c)/a) + 8*a*d*(e/b)])/(2*e))*x + x^2)^p*(b/d + ((d - Sqrt[e*((b^
2 - 4*a*c)/a) + 8*a*d*(e/b)])/(2*e))*x + x^2)^p, x], x], x] /; FreeQ[{a, b,
 c, d, e}, x] && PolyQ[Px, x] && ILtQ[p, 0] && EqQ[a*d^2 - b^2*e, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.23 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.23

method	result
risch	$\frac{\frac{3}{25}x^7-\frac{9}{50}x^6-\frac{11}{50}x^2+\frac{7}{25}x}{(x^4+x^3+x^2+x+1)^2}+\frac{3\left(\sum_{\_R=\mathrm{RootOf}(\_Z^4+\_Z^3+\_Z^2+\_Z+1)}\frac{(\_R^2-6\_R+6)\ln(x-\_R)}{4\_R^3+3\_R^2+2\_R+1}\right)}{25}$
default	$-\frac{4\left(\left(-\frac{15}{8}+\frac{21\sqrt{5}}{8}\right)x^3+\left(\frac{29\sqrt{5}}{16}-\frac{135}{16}\right)x^2+\left(\frac{23\sqrt{5}}{8}-\frac{5}{8}\right)x-\frac{3\sqrt{5}}{4}\right)}{125\left(x^2+\frac{x}{2}-\frac{\sqrt{5}\,x}{2}+1\right)^2}-\frac{3\ln\left(2+x-\sqrt{5}\,x+2x^2\right)\sqrt{5}}{50}+\frac{12\left(\frac{5\sqrt{5}\,(-\sqrt{5}+1)}{4}-9\sqrt{5}+15\right)\mathrm{a}}{125\sqrt{10+2\sqrt{5}}}$

input
```
int(1/(x^4+x^3+x^2+x+1)^3,x,method=_RETURNVERBOSE)
```

output
```
(3/25*x^7-9/50*x^6-11/50*x^2+7/25*x)/(x^4+x^3+x^2+x+1)^2+3/25*sum((_R^2-6*
_R+6)/(4*_R^3+3*_R^2+2*_R+1)*ln(x-_R),_R=RootOf(_Z^4+_Z^3+_Z^2+_Z+1))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 326, normalized size of antiderivative = 0.88

$$\int \frac{1}{(1+x+x^2+x^3+x^4)^3}\,dx$$
$$=\frac{30\,x^7-45\,x^6+6\,(x^8+2\,x^7+3\,x^6+4\,x^5+5\,x^4+4\,x^3+3\,x^2+2\,x+1)\sqrt{422\,\sqrt{5}+1025}\arctan\left(\frac{1}{895}\,(\sqrt{\right.}$$

input
```
integrate(1/(x^4+x^3+x^2+x+1)^3,x, algorithm="fricas")
```

output
```
1/250*(30*x^7 - 45*x^6 + 6*(x^8 + 2*x^7 + 3*x^6 + 4*x^5 + 5*x^4 + 4*x^3 +
3*x^2 + 2*x + 1)*sqrt(422*sqrt(5) + 1025)*arctan(1/895*(sqrt(5)*(31*x - 1)
 - 35*x + 30)*sqrt(422*sqrt(5) + 1025)) - 2*(x^8 + 2*x^7 + 3*x^6 + 4*x^5 +
 5*x^4 + 4*x^3 + 3*x^2 + 2*x + 1)*sqrt(-3798*sqrt(5) + 9225)*arctan(1/2685
*(sqrt(5)*(31*x - 1) + 35*x - 30)*sqrt(-3798*sqrt(5) + 9225)) + 15*sqrt(5)
*(x^8 + 2*x^7 + 3*x^6 + 4*x^5 + 5*x^4 + 4*x^3 + 3*x^2 + 2*x + 1)*log(2*x^2
 + sqrt(5)*x + x + 2) - 15*sqrt(5)*(x^8 + 2*x^7 + 3*x^6 + 4*x^5 + 5*x^4 +
4*x^3 + 3*x^2 + 2*x + 1)*log(2*x^2 - sqrt(5)*x + x + 2) - 55*x^2 + 70*x)/(
x^8 + 2*x^7 + 3*x^6 + 4*x^5 + 5*x^4 + 4*x^3 + 3*x^2 + 2*x + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1445 vs. $2(332) = 664$.

Time = 0.64 (sec) , antiderivative size = 1445, normalized size of antiderivative = 3.89

$$\int \frac{1}{(1+x+x^2+x^3+x^4)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(x**4+x**3+x**2+x+1)**3,x)
```

output
```
(6*x**7 - 9*x**6 - 11*x**2 + 14*x)/(50*x**8 + 100*x**7 + 150*x**6 + 200*x*
*5 + 250*x**4 + 200*x**3 + 150*x**2 + 100*x + 50) - 3*sqrt(5)*log(x**2 + x
*(-41064583/13115509 - 15796955*sqrt(5)/13115509 + 18333*sqrt(10)*sqrt(527
5*sqrt(5) + 23823)/13115509 + 429675*sqrt(2)*sqrt(5275*sqrt(5) + 23823)/26
231018) - 23080955957875*sqrt(2)*sqrt(5275*sqrt(5) + 23823)/68806630531632
4 - 7519430041655*sqrt(10)*sqrt(5275*sqrt(5) + 23823)/688066305316324 + 54
0905909434020*sqrt(5)/17201657632908l + 1624519191908706/17201657632908l)/
50 + 3*sqrt(5)*log(x**2 + x*(-41064583/13115509 - 18333*sqrt(10)*sqrt(2382
3 - 5275*sqrt(5))/13115509 + 429675*sqrt(2)*sqrt(23823 - 5275*sqrt(5))/262
31018 + 15796955*sqrt(5)/13115509) - 540905909434020*sqrt(5)/1720165763290
81 - 23080955957875*sqrt(2)*sqrt(23823 - 5275*sqrt(5))/688066305316324 + 7
519430041655*sqrt(10)*sqrt(23823 - 5275*sqrt(5))/688066305316324 + 1624519
191908706/172016576329081)/50 - 2*sqrt(-9*sqrt(10)*sqrt(5275*sqrt(5) + 238
23)/31250 + 459/2500)*atan(52462036*x/(183330*sqrt(5)*sqrt(-2*sqrt(10)*sqr
t(5275*sqrt(5) + 23823) + 1275) + 1701818*sqrt(-2*sqrt(10)*sqrt(5275*sqrt(
5) + 23823) + 1275) + 5729*sqrt(10)*sqrt(5275*sqrt(5) + 23823)*sqrt(-2*sqr
t(10)*sqrt(5275*sqrt(5) + 23823) + 1275)) - 82129166/(183330*sqrt(5)*sqrt(
-2*sqrt(10)*sqrt(5275*sqrt(5) + 23823) + 1275) + 1701818*sqrt(-2*sqrt(10)*
sqrt(5275*sqrt(5) + 23823) + 1275) + 5729*sqrt(10)*sqrt(5275*sqrt(5) + 238
23)*sqrt(-2*sqrt(10)*sqrt(5275*sqrt(5) + 23823) + 1275)) - 31593910*sqr...
```

Maxima [F]

$$\int \frac{1}{(1+x+x^2+x^3+x^4)^3}\,dx = \int \frac{1}{(x^4+x^3+x^2+x+1)^3}\,dx$$

input
```
integrate(1/(x^4+x^3+x^2+x+1)^3,x, algorithm="maxima")
```

output
```
1/50*(6*x^7 - 9*x^6 - 11*x^2 + 14*x)/(x^8 + 2*x^7 + 3*x^6 + 4*x^5 + 5*x^4
+ 4*x^3 + 3*x^2 + 2*x + 1) + 3/25*integrate((x^2 - 6*x + 6)/(x^4 + x^3 + x
^2 + x + 1), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.37

$$\int \frac{1}{(1+x+x^2+x^3+x^4)^3}\,dx = -\frac{3}{125}\sqrt{-422\sqrt{5}+1025}\arctan\left(\frac{4x-\sqrt{5}+1}{\sqrt{2\sqrt{5}+10}}\right) + \frac{3}{125}\sqrt{422\sqrt{5}+1025}\arctan\left(\frac{4x+\sqrt{5}+1}{\sqrt{-2\sqrt{5}+10}}\right) + \frac{3}{50}\sqrt{5}\log\left(x^2+\frac{1}{2}x\left(\sqrt{5}+1\right)+1\right) - \frac{3}{50}\sqrt{5}\log\left(x^2-\frac{1}{2}x\left(\sqrt{5}-1\right)+1\right) + \frac{6x^7-9x^6-11x^2+14x}{50\left(x^4+x^3+x^2+x+1\right)^2}$$

input
```
integrate(1/(x^4+x^3+x^2+x+1)^3,x, algorithm="giac")
```

output
```
-3/125*sqrt(-422*sqrt(5) + 1025)*arctan((4*x - sqrt(5) + 1)/sqrt(2*sqrt(5)
 + 10)) + 3/125*sqrt(422*sqrt(5) + 1025)*arctan((4*x + sqrt(5) + 1)/sqrt(-
2*sqrt(5) + 10)) + 3/50*sqrt(5)*log(x^2 + 1/2*x*(sqrt(5) + 1) + 1) - 3/50*
sqrt(5)*log(x^2 - 1/2*x*(sqrt(5) - 1) + 1) + 1/50*(6*x^7 - 9*x^6 - 11*x^2
+ 14*x)/(x^4 + x^3 + x^2 + x + 1)^2
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.41

$$\int \frac{1}{(1+x+x^2+x^3+x^4)^3}\,dx = \left(\sum_{k=1}^{4}\ln\left(-\frac{4887x}{15625} - \mathrm{root}\left(z^4+\frac{162z^2}{625}+\frac{5697z}{78125}+\frac{437481}{48828125},z,k\right)\left(\frac{1026x}{625}+\mathrm{root}\left(z^4+\frac{162z^2}{625}+\frac{5697z}{78125}+\frac{437481}{48828125},z,k\right) + \frac{5967}{15625}\right)\mathrm{root}\left(z^4+\frac{162z^2}{625}+\frac{5697z}{78125}+\frac{437481}{48828125},z,k\right)\right)\right) + \frac{\frac{3x^7}{25}-\frac{9x^6}{50}-\frac{11x^2}{50}+\frac{7x}{25}}{x^8+2x^7+3x^6+4x^5+5x^4+4x^3+3x^2+2x+1}$$

input
```
int(1/(x + x^2 + x^3 + x^4 + 1)^3,x)
```

output
```
symsum(log(5967/15625 - root(z^4 + (162*z^2)/625 + (5697*z)/78125 + 437481
/48828125, z, k)*((1026*x)/625 + root(z^4 + (162*z^2)/625 + (5697*z)/78125
 + 437481/48828125, z, k)*(25*root(z^4 + (162*z^2)/625 + (5697*z)/78125 +
437481/48828125, z, k) + (111*x)/5 + 72/5) + 1314/625) - (4887*x)/15625)*r
oot(z^4 + (162*z^2)/625 + (5697*z)/78125 + 437481/48828125, z, k), k, 1, 4
) + ((7*x)/25 - (11*x^2)/50 - (9*x^6)/50 + (3*x^7)/25)/(2*x + 3*x^2 + 4*x^
3 + 5*x^4 + 4*x^5 + 3*x^6 + 2*x^7 + x^8 + 1)
```

Reduce [F]

$$\int \frac{1}{(1+x+x^2+x^3+x^4)^3}\,dx = \text{too large to display}$$

input
```
int(1/(x^4+x^3+x^2+x+1)^3,x)
```

output

```
(24*int(x**5/(x**12 + 3*x**11 + 6*x**10 + 10*x**9 + 15*x**8 + 18*x**7 + 19
*x**6 + 18*x**5 + 15*x**4 + 10*x**3 + 6*x**2 + 3*x + 1),x)*x**8 + 48*int(x
**5/(x**12 + 3*x**11 + 6*x**10 + 10*x**9 + 15*x**8 + 18*x**7 + 19*x**6 + 1
8*x**5 + 15*x**4 + 10*x**3 + 6*x**2 + 3*x + 1),x)*x**7 + 72*int(x**5/(x**1
2 + 3*x**11 + 6*x**10 + 10*x**9 + 15*x**8 + 18*x**7 + 19*x**6 + 18*x**5 +
15*x**4 + 10*x**3 + 6*x**2 + 3*x + 1),x)*x**6 + 96*int(x**5/(x**12 + 3*x**
11 + 6*x**10 + 10*x**9 + 15*x**8 + 18*x**7 + 19*x**6 + 18*x**5 + 15*x**4 +
 10*x**3 + 6*x**2 + 3*x + 1),x)*x**5 + 120*int(x**5/(x**12 + 3*x**11 + 6*x
**10 + 10*x**9 + 15*x**8 + 18*x**7 + 19*x**6 + 18*x**5 + 15*x**4 + 10*x**3
 + 6*x**2 + 3*x + 1),x)*x**4 + 96*int(x**5/(x**12 + 3*x**11 + 6*x**10 + 10
*x**9 + 15*x**8 + 18*x**7 + 19*x**6 + 18*x**5 + 15*x**4 + 10*x**3 + 6*x**2
 + 3*x + 1),x)*x**3 + 72*int(x**5/(x**12 + 3*x**11 + 6*x**10 + 10*x**9 + 1
5*x**8 + 18*x**7 + 19*x**6 + 18*x**5 + 15*x**4 + 10*x**3 + 6*x**2 + 3*x +
1),x)*x**2 + 48*int(x**5/(x**12 + 3*x**11 + 6*x**10 + 10*x**9 + 15*x**8 +
18*x**7 + 19*x**6 + 18*x**5 + 15*x**4 + 10*x**3 + 6*x**2 + 3*x + 1),x)*x +
 24*int(x**5/(x**12 + 3*x**11 + 6*x**10 + 10*x**9 + 15*x**8 + 18*x**7 + 19
*x**6 + 18*x**5 + 15*x**4 + 10*x**3 + 6*x**2 + 3*x + 1),x) + 44*int(x**4/(
x**12 + 3*x**11 + 6*x**10 + 10*x**9 + 15*x**8 + 18*x**7 + 19*x**6 + 18*x**
5 + 15*x**4 + 10*x**3 + 6*x**2 + 3*x + 1),x)*x**8 + 88*int(x**4/(x**12 + 3
*x**11 + 6*x**10 + 10*x**9 + 15*x**8 + 18*x**7 + 19*x**6 + 18*x**5 + 15...
```

3.15 $\int (1-x+x^2-x^3+x^4)^3 \, dx$

Optimal result

Integrand size = 18, antiderivative size = 78

$$\int (1-x+x^2-x^3+x^4)^3 \, dx = x-\frac{3x^2}{2}+2x^3-\frac{5x^4}{2}+3x^5-3x^6+\frac{19x^7}{7}-\frac{9x^8}{4}+\frac{5x^9}{3}-x^{10}+\frac{6x^{11}}{11}-\frac{x^{12}}{4}+\frac{x^{13}}{13}$$

output
```
x-3/2*x^2+2*x^3-5/2*x^4+3*x^5-3*x^6+19/7*x^7-9/4*x^8+5/3*x^9-x^10+6/11*x^1
1-1/4*x^12+1/13*x^13
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00

$$\int (1-x+x^2-x^3+x^4)^3 \, dx = x-\frac{3x^2}{2}+2x^3-\frac{5x^4}{2}+3x^5-3x^6+\frac{19x^7}{7}-\frac{9x^8}{4}+\frac{5x^9}{3}-x^{10}+\frac{6x^{11}}{11}-\frac{x^{12}}{4}+\frac{x^{13}}{13}$$

input
```
Integrate[(1 - x + x^2 - x^3 + x^4)^3,x]
```

output
```
x - (3*x^2)/2 + 2*x^3 - (5*x^4)/2 + 3*x^5 - 3*x^6 + (19*x^7)/7 - (9*x^8)/4
 + (5*x^9)/3 - x^10 + (6*x^11)/11 - x^12/4 + x^13/13
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2465, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(x^4-x^3+x^2-x+1\right)^3 dx$$

$$\Big\downarrow \text{2465}$$

$$\int \left(x^{12}-3x^{11}+6x^{10}-10x^9+15x^8-18x^7+19x^6-18x^5+15x^4-10x^3+6x^2-3x+1\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^{13}}{13}-\frac{x^{12}}{4}+\frac{6x^{11}}{11}-x^{10}+\frac{5x^9}{3}-\frac{9x^8}{4}+\frac{19x^7}{7}-3x^6+3x^5-\frac{5x^4}{2}+2x^3-\frac{3x^2}{2}+x$$

input
```
Int[(1 - x + x^2 - x^3 + x^4)^3,x]
```

output
```
x - (3*x^2)/2 + 2*x^3 - (5*x^4)/2 + 3*x^5 - 3*x^6 + (19*x^7)/7 - (9*x^8)/4
 + (5*x^9)/3 - x^10 + (6*x^11)/11 - x^12/4 + x^13/13
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2465
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> Int[ExpandToSum[u, Px^p, x], x] /; PolyQ
[Px, x] && GtQ[Expon[Px, x], 2] && !BinomialQ[Px, x] && !TrinomialQ[Px, x
] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.81

method	result
gosper	$x-\frac{3}{2}x^{2}+2x^{3}-\frac{5}{2}x^{4}+3x^{5}-3x^{6}+\frac{19}{7}x^{7}-\frac{9}{4}x^{8}+\frac{5}{3}x^{9}-x^{10}+\frac{6}{11}x^{11}-\frac{1}{4}x^{12}+\frac{1}{13}x^{13}$
default	$x-\frac{3}{2}x^{2}+2x^{3}-\frac{5}{2}x^{4}+3x^{5}-3x^{6}+\frac{19}{7}x^{7}-\frac{9}{4}x^{8}+\frac{5}{3}x^{9}-x^{10}+\frac{6}{11}x^{11}-\frac{1}{4}x^{12}+\frac{1}{13}x^{13}$
norman	$x-\frac{3}{2}x^{2}+2x^{3}-\frac{5}{2}x^{4}+3x^{5}-3x^{6}+\frac{19}{7}x^{7}-\frac{9}{4}x^{8}+\frac{5}{3}x^{9}-x^{10}+\frac{6}{11}x^{11}-\frac{1}{4}x^{12}+\frac{1}{13}x^{13}$
risch	$x-\frac{3}{2}x^{2}+2x^{3}-\frac{5}{2}x^{4}+3x^{5}-3x^{6}+\frac{19}{7}x^{7}-\frac{9}{4}x^{8}+\frac{5}{3}x^{9}-x^{10}+\frac{6}{11}x^{11}-\frac{1}{4}x^{12}+\frac{1}{13}x^{13}$
parallelrisch	$x-\frac{3}{2}x^{2}+2x^{3}-\frac{5}{2}x^{4}+3x^{5}-3x^{6}+\frac{19}{7}x^{7}-\frac{9}{4}x^{8}+\frac{5}{3}x^{9}-x^{10}+\frac{6}{11}x^{11}-\frac{1}{4}x^{12}+\frac{1}{13}x^{13}$
orering	$\frac{x(924x^{12}-3003x^{11}+6552x^{10}-12012x^{9}+20020x^{8}-27027x^{7}+32604x^{6}-36036x^{5}+36036x^{4}-30030x^{3}+24024x^{2}-18018x+12}{12012}$

input
```
int((x^4-x^3+x^2-x+1)^3,x,method=_RETURNVERBOSE)
```

output
```
x-3/2*x^2+2*x^3-5/2*x^4+3*x^5-3*x^6+19/7*x^7-9/4*x^8+5/3*x^9-x^10+6/11*x^1
1-1/4*x^12+1/13*x^13
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.79

$$\int \left(1-x+x^{2}-x^{3}+x^{4}\right)^{3}\,dx=\frac{1}{13}\,x^{13}-\frac{1}{4}\,x^{12}+\frac{6}{11}\,x^{11}-x^{10}+\frac{5}{3}\,x^{9}-\frac{9}{4}\,x^{8}+\frac{19}{7}\,x^{7}-3\,x^{6}+3\,x^{5}-\frac{5}{2}\,x^{4}+2\,x^{3}-\frac{3}{2}\,x^{2}+x$$

input
```
integrate((x^4-x^3+x^2-x+1)^3,x, algorithm="fricas")
```

output
```
1/13*x^13 - 1/4*x^12 + 6/11*x^11 - x^10 + 5/3*x^9 - 9/4*x^8 + 19/7*x^7 - 3
*x^6 + 3*x^5 - 5/2*x^4 + 2*x^3 - 3/2*x^2 + x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.90

$$\int (1-x+x^2-x^3+x^4)^3 \, dx = \frac{x^{13}}{13} - \frac{x^{12}}{4} + \frac{6x^{11}}{11} - x^{10} + \frac{5x^9}{3} - \frac{9x^8}{4} + \frac{19x^7}{7} - 3x^6 + 3x^5 - \frac{5x^4}{2} + 2x^3 - \frac{3x^2}{2} + x$$

input
```
integrate((x**4-x**3+x**2-x+1)**3,x)
```

output
```
x**13/13 - x**12/4 + 6*x**11/11 - x**10 + 5*x**9/3 - 9*x**8/4 + 19*x**7/7
- 3*x**6 + 3*x**5 - 5*x**4/2 + 2*x**3 - 3*x**2/2 + x
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.79

$$\int (1-x+x^2-x^3+x^4)^3 \, dx = \frac{1}{13}x^{13} - \frac{1}{4}x^{12} + \frac{6}{11}x^{11} - x^{10} + \frac{5}{3}x^9 - \frac{9}{4}x^8 + \frac{19}{7}x^7 - 3x^6 + 3x^5 - \frac{5}{2}x^4 + 2x^3 - \frac{3}{2}x^2 + x$$

input
```
integrate((x^4-x^3+x^2-x+1)^3,x, algorithm="maxima")
```

output
```
1/13*x^13 - 1/4*x^12 + 6/11*x^11 - x^10 + 5/3*x^9 - 9/4*x^8 + 19/7*x^7 - 3
*x^6 + 3*x^5 - 5/2*x^4 + 2*x^3 - 3/2*x^2 + x
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.79

$$\int \left(1-x+x^2-x^3+x^4\right)^3 \, dx = \frac{1}{13}\,x^{13}-\frac{1}{4}\,x^{12}+\frac{6}{11}\,x^{11}-x^{10}+\frac{5}{3}\,x^{9}-\frac{9}{4}\,x^{8}+\frac{19}{7}\,x^{7}-3\,x^{6}+3\,x^{5}-\frac{5}{2}\,x^{4}+2\,x^{3}-\frac{3}{2}\,x^{2}+x$$

input
```
integrate((x^4-x^3+x^2-x+1)^3,x, algorithm="giac")
```

output
```
1/13*x^13 - 1/4*x^12 + 6/11*x^11 - x^10 + 5/3*x^9 - 9/4*x^8 + 19/7*x^7 - 3
*x^6 + 3*x^5 - 5/2*x^4 + 2*x^3 - 3/2*x^2 + x
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.79

$$\int \left(1-x+x^2-x^3+x^4\right)^3 \, dx = \frac{x^{13}}{13}-\frac{x^{12}}{4}+\frac{6\,x^{11}}{11}-x^{10}+\frac{5\,x^9}{3}-\frac{9\,x^8}{4}+\frac{19\,x^7}{7}-3\,x^6+3\,x^5-\frac{5\,x^4}{2}+2\,x^3-\frac{3\,x^2}{2}+x$$

input
```
int((x^2 - x - x^3 + x^4 + 1)^3,x)
```

output
```
x - (3*x^2)/2 + 2*x^3 - (5*x^4)/2 + 3*x^5 - 3*x^6 + (19*x^7)/7 - (9*x^8)/4
 + (5*x^9)/3 - x^10 + (6*x^11)/11 - x^12/4 + x^13/13
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.81

$$\int \left(1-x+x^2-x^3+x^4\right)^3 \, dx$$
$$= \frac{x(924x^{12}-3003x^{11}+6552x^{10}-12012x^9+20020x^8-27027x^7+32604x^6-36036x^5+36036x^4-300}{12012}$$

input
```
int((x^4-x^3+x^2-x+1)^3,x)
```

output
```
(x*(924*x**12 - 3003*x**11 + 6552*x**10 - 12012*x**9 + 20020*x**8 - 27027*
x**7 + 32604*x**6 - 36036*x**5 + 36036*x**4 - 30030*x**3 + 24024*x**2 - 18
018*x + 12012))/12012
```

3.16 $\int (1-x+x^2-x^3+x^4)^2 \, dx$

Optimal result

Integrand size = 18, antiderivative size = 46

$$\int \left(1-x+x^2-x^3+x^4\right)^2 \, dx = x-x^2+x^3-x^4+x^5-\frac{2x^6}{3}+\frac{3x^7}{7}-\frac{x^8}{4}+\frac{x^9}{9}$$

output
```
x-x^2+x^3-x^4+x^5-2/3*x^6+3/7*x^7-1/4*x^8+1/9*x^9
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \left(1-x+x^2-x^3+x^4\right)^2 \, dx = x-x^2+x^3-x^4+x^5-\frac{2x^6}{3}+\frac{3x^7}{7}-\frac{x^8}{4}+\frac{x^9}{9}$$

input
```
Integrate[(1 - x + x^2 - x^3 + x^4)^2,x]
```

output
```
x - x^2 + x^3 - x^4 + x^5 - (2*x^6)/3 + (3*x^7)/7 - x^8/4 + x^9/9
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2465, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(x^4-x^3+x^2-x+1\right)^2 dx$$
$$\Big\downarrow \text{2465}$$
$$\int \left(x^8-2x^7+3x^6-4x^5+5x^4-4x^3+3x^2-2x+1\right) dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{x^9}{9}-\frac{x^8}{4}+\frac{3x^7}{7}-\frac{2x^6}{3}+x^5-x^4+x^3-x^2+x$$

input
```
Int[(1 - x + x^2 - x^3 + x^4)^2,x]
```

output
```
x - x^2 + x^3 - x^4 + x^5 - (2*x^6)/3 + (3*x^7)/7 - x^8/4 + x^9/9
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2465
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> Int[ExpandToSum[u, Px^p, x], x] /; PolyQ
[Px, x] && GtQ[Expon[Px, x], 2] && !BinomialQ[Px, x] && !TrinomialQ[Px, x
] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.85

method	result	size
gosper	$x-x^{2}+x^{3}-x^{4}+x^{5}-\frac{2}{3}x^{6}+\frac{3}{7}x^{7}-\frac{1}{4}x^{8}+\frac{1}{9}x^{9}$	39
default	$x-x^{2}+x^{3}-x^{4}+x^{5}-\frac{2}{3}x^{6}+\frac{3}{7}x^{7}-\frac{1}{4}x^{8}+\frac{1}{9}x^{9}$	39
norman	$x-x^{2}+x^{3}-x^{4}+x^{5}-\frac{2}{3}x^{6}+\frac{3}{7}x^{7}-\frac{1}{4}x^{8}+\frac{1}{9}x^{9}$	39
risch	$x-x^{2}+x^{3}-x^{4}+x^{5}-\frac{2}{3}x^{6}+\frac{3}{7}x^{7}-\frac{1}{4}x^{8}+\frac{1}{9}x^{9}$	39
parallelrisch	$x-x^{2}+x^{3}-x^{4}+x^{5}-\frac{2}{3}x^{6}+\frac{3}{7}x^{7}-\frac{1}{4}x^{8}+\frac{1}{9}x^{9}$	39
orering	$\frac{x\left(28x^{8}-63x^{7}+108x^{6}-168x^{5}+252x^{4}-252x^{3}+252x^{2}-252x+252\right)}{252}$	44

input
```
int((x^4-x^3+x^2-x+1)^2,x,method=_RETURNVERBOSE)
```

output
```
x-x^2+x^3-x^4+x^5-2/3*x^6+3/7*x^7-1/4*x^8+1/9*x^9
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.83

$$\int \left(1-x+x^{2}-x^{3}+x^{4}\right)^{2}\,dx=\frac{1}{9}\,x^{9}-\frac{1}{4}\,x^{8}+\frac{3}{7}\,x^{7}-\frac{2}{3}\,x^{6}+x^{5}-x^{4}+x^{3}-x^{2}+x$$

input
```
integrate((x^4-x^3+x^2-x+1)^2,x, algorithm="fricas")
```

output
```
1/9*x^9 - 1/4*x^8 + 3/7*x^7 - 2/3*x^6 + x^5 - x^4 + x^3 - x^2 + x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

$$\int \left(1-x+x^2-x^3+x^4\right)^2\,dx=\frac{x^9}{9}-\frac{x^8}{4}+\frac{3x^7}{7}-\frac{2x^6}{3}+x^5-x^4+x^3-x^2+x$$

input
```
integrate((x**4-x**3+x**2-x+1)**2,x)
```

output
```
x**9/9 - x**8/4 + 3*x**7/7 - 2*x**6/3 + x**5 - x**4 + x**3 - x**2 + x
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.83

$$\int \left(1-x+x^2-x^3+x^4\right)^2\,dx=\frac{1}{9}\,x^9-\frac{1}{4}\,x^8+\frac{3}{7}\,x^7-\frac{2}{3}\,x^6+x^5-x^4+x^3-x^2+x$$

input
```
integrate((x^4-x^3+x^2-x+1)^2,x, algorithm="maxima")
```

output
```
1/9*x^9 - 1/4*x^8 + 3/7*x^7 - 2/3*x^6 + x^5 - x^4 + x^3 - x^2 + x
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.83

$$\int \left(1-x+x^2-x^3+x^4\right)^2\,dx=\frac{1}{9}\,x^9-\frac{1}{4}\,x^8+\frac{3}{7}\,x^7-\frac{2}{3}\,x^6+x^5-x^4+x^3-x^2+x$$

input
```
integrate((x^4-x^3+x^2-x+1)^2,x, algorithm="giac")
```

output
```
1/9*x^9 - 1/4*x^8 + 3/7*x^7 - 2/3*x^6 + x^5 - x^4 + x^3 - x^2 + x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.83

$$\int \left(1-x+x^2-x^3+x^4\right)^2\,dx=\frac{x^9}{9}-\frac{x^8}{4}+\frac{3\,x^7}{7}-\frac{2\,x^6}{3}+x^5-x^4+x^3-x^2+x$$

input
```
int((x^2 - x - x^3 + x^4 + 1)^2,x)
```

output
```
x - x^2 + x^3 - x^4 + x^5 - (2*x^6)/3 + (3*x^7)/7 - x^8/4 + x^9/9
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.93

$$\int \left(1-x+x^2-x^3+x^4\right)^2\,dx$$
$$=\frac{x(28x^8-63x^7+108x^6-168x^5+252x^4-252x^3+252x^2-252x+252)}{252}$$

input
```
int((x^4-x^3+x^2-x+1)^2,x)
```

output
```
(x*(28*x**8 - 63*x**7 + 108*x**6 - 168*x**5 + 252*x**4 - 252*x**3 + 252*x*
*2 - 252*x + 252))/252
```

3.17 $\int (1 - x + x^2 - x^3 + x^4) \, dx$

Optimal result

Integrand size = 16, antiderivative size = 30

$$\int (1 - x + x^2 - x^3 + x^4) \, dx = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5}$$

output
```
x-1/2*x^2+1/3*x^3-1/4*x^4+1/5*x^5
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int (1 - x + x^2 - x^3 + x^4) \, dx = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5}$$

input
```
Integrate[1 - x + x^2 - x^3 + x^4,x]
```

output
```
x - x^2/2 + x^3/3 - x^4/4 + x^5/5
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(x^4-x^3+x^2-x+1\right) \, dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{x^5}{5}-\frac{x^4}{4}+\frac{x^3}{3}-\frac{x^2}{2}+x$$

input
```
Int[1 - x + x^2 - x^3 + x^4,x]
```

output
```
x - x^2/2 + x^3/3 - x^4/4 + x^5/5
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.77

method	result	size
gosper	$x-\frac{1}{2}x^{2}+\frac{1}{3}x^{3}-\frac{1}{4}x^{4}+\frac{1}{5}x^{5}$	23
default	$x-\frac{1}{2}x^{2}+\frac{1}{3}x^{3}-\frac{1}{4}x^{4}+\frac{1}{5}x^{5}$	23
norman	$x-\frac{1}{2}x^{2}+\frac{1}{3}x^{3}-\frac{1}{4}x^{4}+\frac{1}{5}x^{5}$	23
risch	$x-\frac{1}{2}x^{2}+\frac{1}{3}x^{3}-\frac{1}{4}x^{4}+\frac{1}{5}x^{5}$	23
parallelrisch	$x-\frac{1}{2}x^{2}+\frac{1}{3}x^{3}-\frac{1}{4}x^{4}+\frac{1}{5}x^{5}$	23
parts	$x-\frac{1}{2}x^{2}+\frac{1}{3}x^{3}-\frac{1}{4}x^{4}+\frac{1}{5}x^{5}$	23
orering	$\frac{x\left(12x^{4}-15x^{3}+20x^{2}-30x+60\right)}{60}$	24

input
```
int(x^4-x^3+x^2-x+1,x,method=_RETURNVERBOSE)
```

output
```
x-1/2*x^2+1/3*x^3-1/4*x^4+1/5*x^5
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.73

$$\int \left(1-x+x^2-x^3+x^4\right)\,dx=\frac{1}{5}\,x^{5}-\frac{1}{4}\,x^{4}+\frac{1}{3}\,x^{3}-\frac{1}{2}\,x^{2}+x$$

input
```
integrate(x^4-x^3+x^2-x+1,x, algorithm="fricas")
```

output
```
1/5*x^5 - 1/4*x^4 + 1/3*x^3 - 1/2*x^2 + x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.67

$$\int \left(1-x+x^2-x^3+x^4\right) dx = \frac{x^5}{5}-\frac{x^4}{4}+\frac{x^3}{3}-\frac{x^2}{2}+x$$

input
```
integrate(x**4-x**3+x**2-x+1,x)
```

output
```
x**5/5 - x**4/4 + x**3/3 - x**2/2 + x
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.73

$$\int \left(1-x+x^2-x^3+x^4\right) dx = \frac{1}{5}\,x^5-\frac{1}{4}\,x^4+\frac{1}{3}\,x^3-\frac{1}{2}\,x^2+x$$

input
```
integrate(x^4-x^3+x^2-x+1,x, algorithm="maxima")
```

output
```
1/5*x^5 - 1/4*x^4 + 1/3*x^3 - 1/2*x^2 + x
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.73

$$\int \left(1-x+x^2-x^3+x^4\right) dx = \frac{1}{5}\,x^5-\frac{1}{4}\,x^4+\frac{1}{3}\,x^3-\frac{1}{2}\,x^2+x$$

input
```
integrate(x^4-x^3+x^2-x+1,x, algorithm="giac")
```

output
```
1/5*x^5 - 1/4*x^4 + 1/3*x^3 - 1/2*x^2 + x
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.73

$$\int \left(1 - x + x^2 - x^3 + x^4\right) dx = \frac{x^5}{5} - \frac{x^4}{4} + \frac{x^3}{3} - \frac{x^2}{2} + x$$

input
```
int(x^2 - x - x^3 + x^4 + 1,x)
```

output
```
x - x^2/2 + x^3/3 - x^4/4 + x^5/5
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.77

$$\int \left(1 - x + x^2 - x^3 + x^4\right) dx = \frac{x(12x^4 - 15x^3 + 20x^2 - 30x + 60)}{60}$$

input
```
int(x^4-x^3+x^2-x+1,x)
```

output
```
(x*(12*x**4 - 15*x**3 + 20*x**2 - 30*x + 60))/60
```

3.18 $\int \frac{1}{1-x+x^2-x^3+x^4}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 147

$$\int \frac{1}{1-x+x^2-x^3+x^4}\,dx = -\frac{1}{5}\sqrt{5-2\sqrt{5}}\arctan\left(\frac{1-\sqrt{5}-4x}{\sqrt{2\left(5+\sqrt{5}\right)}}\right) - \frac{1}{5}\sqrt{5+2\sqrt{5}}\arctan\left(\frac{1}{2}\sqrt{\frac{1}{10}\left(5+\sqrt{5}\right)}\left(1+\sqrt{5}-4x\right)\right) - \frac{\log\left(2-x-\sqrt{5}x+2x^2\right)}{2\sqrt{5}} + \frac{\log\left(2-x+\sqrt{5}x+2x^2\right)}{2\sqrt{5}}$$

output
```
-1/5*(5-2*5^(1/2))^(1/2)*arctan((1-5^(1/2)-4*x)/(10+2*5^(1/2))^(1/2))-1/5*
(5+2*5^(1/2))^(1/2)*arctan(1/20*(50+10*5^(1/2))^(1/2)*(1+5^(1/2)-4*x))-1/1
0*ln(2-x-x*5^(1/2)+2*x^2)*5^(1/2)+1/10*ln(2-x+x*5^(1/2)+2*x^2)*5^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.35

$$\int \frac{1}{1-x+x^2-x^3+x^4}\,dx = \text{RootSum}\left[1-\#1+\#1^2-\#1^3+\#1^4\&,\frac{\log(x-\#1)}{-1+2\#1-3\#1^2+4\#1^3}\&\right]$$

input
```
Integrate[(1 - x + x^2 - x^3 + x^4)^(-1),x]
```

output
```
RootSum[1 - #1 + #1^2 - #1^3 + #1^4 & , Log[x - #1]/(-1 + 2*#1 - 3*#1^2 +
4*#1^3) & ]
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2492, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4-x^3+x^2-x+1}\,dx$$

$$\Big\downarrow \text{2492}$$

$$\int \left(\frac{-2x+\sqrt{5}+1}{\sqrt{5}\left(2x^2-\left(1+\sqrt{5}\right)x+2\right)}-\frac{-2x-\sqrt{5}+1}{\sqrt{5}\left(2x^2-\left(1-\sqrt{5}\right)x+2\right)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{5}\sqrt{5-2\sqrt{5}}\arctan\left(\frac{-4x-\sqrt{5}+1}{\sqrt{2\left(5+\sqrt{5}\right)}}\right)-\frac{1}{5}\sqrt{5+2\sqrt{5}}\arctan\left(\frac{-4x+\sqrt{5}+1}{\sqrt{2\left(5-\sqrt{5}\right)}}\right)+\frac{\log\left(2x^2-\left(1-\sqrt{5}\right)x+2\right)}{2\sqrt{5}}-\frac{\log\left(2x^2-\left(1+\sqrt{5}\right)x+2\right)}{2\sqrt{5}}$$

input
```
Int[(1 - x + x^2 - x^3 + x^4)^(-1),x]
```

output
```
-1/5*(Sqrt[5 - 2*Sqrt[5]]*ArcTan[(1 - Sqrt[5] - 4*x)/Sqrt[2*(5 + Sqrt[5])]
]) - (Sqrt[5 + 2*Sqrt[5]]*ArcTan[(1 + Sqrt[5] - 4*x)/Sqrt[2*(5 - Sqrt[5])]
])/5 + Log[2 - (1 - Sqrt[5])*x + 2*x^2]/(2*Sqrt[5]) - Log[2 - (1 + Sqrt[5]
)*x + 2*x^2]/(2*Sqrt[5])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2492
```
Int[(Px_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^3 + (e_.)*(x_)^4)
^(p_), x_Symbol] :> Simp[e^p   Int[ExpandIntegrand[Px*(b/d + ((d + Sqrt[e*(
(b^2 - 4*a*c)/a) + 8*a*d*(e/b)])/(2*e))*x + x^2)^p*(b/d + ((d - Sqrt[e*((b^
2 - 4*a*c)/a) + 8*a*d*(e/b)])/(2*e))*x + x^2)^p, x], x], x] /; FreeQ[{a, b,
 c, d, e}, x] && PolyQ[Px, x] && ILtQ[p, 0] && EqQ[a*d^2 - b^2*e, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.31

method	result
risch	$\sum_{\_R=\mathrm{RootOf}(\_Z^4-\_Z^3+\_Z^2-\_Z+1)} \frac{\ln(x-\_R)}{4\_R^3-3\_R^2+2\_R-1}$
default	$\frac{\ln\left(2-x+\sqrt{5}\,x+2x^2\right)\sqrt{5}}{10}+\frac{2\left(-\frac{\sqrt{5}\,(\sqrt{5}-1)}{2}+5-\sqrt{5}\right)\arctan\left(\frac{-1+\sqrt{5}+4x}{\sqrt{10+2\sqrt{5}}}\right)}{5\sqrt{10+2\sqrt{5}}}-\frac{\ln\left(2-x-\sqrt{5}\,x+2x^2\right)\sqrt{5}}{10}-\frac{2\left(-\frac{\sqrt{5}\,(-\sqrt{5}-1)}{2}-\right.}{5}$

input
```
int(1/(x^4-x^3+x^2-x+1),x,method=_RETURNVERBOSE)
```

output
```
sum(1/(4*_R^3-3*_R^2+2*_R-1)*ln(x-_R),_R=RootOf(_Z^4-_Z^3+_Z^2-_Z+1))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.76

$$\int \frac{1}{1-x+x^2-x^3+x^4}\,dx = -\frac{1}{5}\sqrt{2\sqrt{5}+5}\arctan\left(\frac{1}{5}\left(\sqrt{5}(x+1)-5\,x\right)\sqrt{2\sqrt{5}+5}\right) + \frac{1}{5}\sqrt{-2\sqrt{5}+5}\arctan\left(\frac{1}{5}\left(\sqrt{5}(x+1)+5\,x\right)\sqrt{-2\sqrt{5}+5}\right) + \frac{1}{10}\sqrt{5}\log\left(2\,x^2+\sqrt{5}x-x+2\right) - \frac{1}{10}\sqrt{5}\log\left(2\,x^2-\sqrt{5}x-x+2\right)$$

input
```
integrate(1/(x^4-x^3+x^2-x+1),x, algorithm="fricas")
```

output
```
-1/5*sqrt(2*sqrt(5) + 5)*arctan(1/5*(sqrt(5)*(x + 1) - 5*x)*sqrt(2*sqrt(5)
 + 5)) + 1/5*sqrt(-2*sqrt(5) + 5)*arctan(1/5*(sqrt(5)*(x + 1) + 5*x)*sqrt(
-2*sqrt(5) + 5)) + 1/10*sqrt(5)*log(2*x^2 + sqrt(5)*x - x + 2) - 1/10*sqrt
(5)*log(2*x^2 - sqrt(5)*x - x + 2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1287 vs. $2(122) = 244$.

Time = 0.60 (sec) , antiderivative size = 1287, normalized size of antiderivative = 8.76

$$\int \frac{1}{1-x+x^2-x^3+x^4}\,dx = \text{Too large to display}$$

input
```
integrate(1/(x**4-x**3+x**2-x+1),x)
```

output
```
sqrt(5)*log(x**2 + x*(-48/11 - 21*sqrt(5)/11 + 4*sqrt(10)*sqrt(sqrt(5) + 3
)/11 + 45*sqrt(2)*sqrt(sqrt(5) + 3)/22) - 1381*sqrt(10)*sqrt(sqrt(5) + 3)/
484 - 3045*sqrt(2)*sqrt(sqrt(5) + 3)/484 + 2213*sqrt(5)/242 + 5217/242)/10
 - sqrt(5)*log(x**2 + x*(-48/11 - 45*sqrt(2)*sqrt(3 - sqrt(5))/22 + 4*sqrt
(10)*sqrt(3 - sqrt(5))/11 + 21*sqrt(5)/11) - 2213*sqrt(5)/242 - 1381*sqrt(
10)*sqrt(3 - sqrt(5))/484 + 3045*sqrt(2)*sqrt(3 - sqrt(5))/484 + 5217/242)
/10 + 2*sqrt(-sqrt(10)*sqrt(3 - sqrt(5))/50 + 3/20)*atan(44*x/(-8*sqrt(5)*
sqrt(-2*sqrt(10)*sqrt(3 - sqrt(5)) + 15) + 3*sqrt(10)*sqrt(3 - sqrt(5))*sq
rt(-2*sqrt(10)*sqrt(3 - sqrt(5)) + 15) + 18*sqrt(-2*sqrt(10)*sqrt(3 - sqrt
(5)) + 15)) - 96/(-8*sqrt(5)*sqrt(-2*sqrt(10)*sqrt(3 - sqrt(5)) + 15) + 3*
sqrt(10)*sqrt(3 - sqrt(5))*sqrt(-2*sqrt(10)*sqrt(3 - sqrt(5)) + 15) + 18*s
qrt(-2*sqrt(10)*sqrt(3 - sqrt(5)) + 15)) - 45*sqrt(2)*sqrt(3 - sqrt(5))/(-
8*sqrt(5)*sqrt(-2*sqrt(10)*sqrt(3 - sqrt(5)) + 15) + 3*sqrt(10)*sqrt(3 - s
qrt(5))*sqrt(-2*sqrt(10)*sqrt(3 - sqrt(5)) + 15) + 18*sqrt(-2*sqrt(10)*sqr
t(3 - sqrt(5)) + 15)) + 8*sqrt(10)*sqrt(3 - sqrt(5))/(-8*sqrt(5)*sqrt(-2*s
qrt(10)*sqrt(3 - sqrt(5)) + 15) + 3*sqrt(10)*sqrt(3 - sqrt(5))*sqrt(-2*sqr
t(10)*sqrt(3 - sqrt(5)) + 15) + 18*sqrt(-2*sqrt(10)*sqrt(3 - sqrt(5)) + 15
)) + 42*sqrt(5)/(-8*sqrt(5)*sqrt(-2*sqrt(10)*sqrt(3 - sqrt(5)) + 15) + 3*s
qrt(10)*sqrt(3 - sqrt(5))*sqrt(-2*sqrt(10)*sqrt(3 - sqrt(5)) + 15) + 18*sq
rt(-2*sqrt(10)*sqrt(3 - sqrt(5)) + 15))) + 2*sqrt(-sqrt(10)*sqrt(sqrt(5...
```

Maxima [F]

$$\int \frac{1}{1-x+x^2-x^3+x^4}\,dx=\int \frac{1}{x^4-x^3+x^2-x+1}\,dx$$

input
```
integrate(1/(x^4-x^3+x^2-x+1),x, algorithm="maxima")
```

output
```
integrate(1/(x^4 - x^3 + x^2 - x + 1), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.69

$$\int \frac{1}{1-x+x^2-x^3+x^4}\,dx = \frac{1}{5}\sqrt{-2\sqrt{5}+5}\arctan\left(\frac{4x+\sqrt{5}-1}{\sqrt{2\sqrt{5}+10}}\right) + \frac{1}{5}\sqrt{2\sqrt{5}+5}\arctan\left(\frac{4x-\sqrt{5}-1}{\sqrt{-2\sqrt{5}+10}}\right) - \frac{1}{10}\sqrt{5}\log\left(x^2-\frac{1}{2}x\left(\sqrt{5}+1\right)+1\right) + \frac{1}{10}\sqrt{5}\log\left(x^2+\frac{1}{2}x\left(\sqrt{5}-1\right)+1\right)$$

input
```
integrate(1/(x^4-x^3+x^2-x+1),x, algorithm="giac")
```

output
```
1/5*sqrt(-2*sqrt(5) + 5)*arctan((4*x + sqrt(5) - 1)/sqrt(2*sqrt(5) + 10))
+ 1/5*sqrt(2*sqrt(5) + 5)*arctan((4*x - sqrt(5) - 1)/sqrt(-2*sqrt(5) + 10)
) - 1/10*sqrt(5)*log(x^2 - 1/2*x*(sqrt(5) + 1) + 1) + 1/10*sqrt(5)*log(x^2
 + 1/2*x*(sqrt(5) - 1) + 1)
```

Mupad [B] (verification not implemented)

Time = 21.33 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.44

$$\int \frac{1}{1-x+x^2-x^3+x^4}\,dx = \sum_{k=1}^{4} \ln\left(\mathrm{root}\left(z^4-\frac{z}{25}+\frac{1}{125},z,k\right)\left(-4x + \mathrm{root}\left(z^4-\frac{z}{25}+\frac{1}{125},z,k\right)\left(25\,\mathrm{root}\left(z^4-\frac{z}{25}+\frac{1}{125},z,k\right)+15x-15\right) + 1\right)\right)\mathrm{root}\left(z^4-\frac{z}{25}+\frac{1}{125},z,k\right)$$

input
```
int(1/(x^2 - x - x^3 + x^4 + 1),x)
```

output
```
symsum(log(root(z^4 - z/25 + 1/125, z, k)*(root(z^4 - z/25 + 1/125, z, k)*
(25*root(z^4 - z/25 + 1/125, z, k) + 15*x - 15) - 4*x + 1))*root(z^4 - z/2
5 + 1/125, z, k), k, 1, 4)
```

Reduce [F]

$$\int \frac{1}{1-x+x^2-x^3+x^4}\,dx = \int \frac{1}{x^4-x^3+x^2-x+1}dx$$

input
```
int(1/(x^4-x^3+x^2-x+1),x)
```

output
```
int(1/(x**4 - x**3 + x**2 - x + 1),x)
```

3.19 $\int \frac{1}{(1-x+x^2-x^3+x^4)^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 175

$$\int \frac{1}{(1-x+x^2-x^3+x^4)^2}\,dx = \frac{x(1+x)}{5(1-x+x^2-x^3+x^4)} + \frac{2}{25}\sqrt{2\left(25-11\sqrt{5}\right)}\arctan\left(\frac{1-\sqrt{5}-4x}{\sqrt{2\left(5+\sqrt{5}\right)}}\right) - \frac{2}{25}\sqrt{2\left(25+11\sqrt{5}\right)}\arctan\left(\frac{1+\sqrt{5}-4x}{\sqrt{2\left(5-\sqrt{5}\right)}}\right) + \frac{2\log\left(2-\left(1-\sqrt{5}\right)x+2x^2\right)}{5\sqrt{5}} - \frac{2\log\left(2-\left(1+\sqrt{5}\right)x+2x^2\right)}{5\sqrt{5}}$$

output
```
x*(1+x)/(5*x^4-5*x^3+5*x^2-5*x+5)+2/25*(50-22*5^(1/2))^(1/2)*arctan((1-5^(
1/2)-4*x)/(10+2*5^(1/2))^(1/2))-2/25*(50+22*5^(1/2))^(1/2)*arctan((1+5^(1/
2)-4*x)/(10-2*5^(1/2))^(1/2))+2/25*ln(2-x*(-5^(1/2)+1)+2*x^2)*5^(1/2)-2/25
*ln(2-(5^(1/2)+1)*x+2*x^2)*5^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.55

$$\int \frac{1}{(1-x+x^2-x^3+x^4)^2}\,dx = \frac{x+x^2}{5(1-x+x^2-x^3+x^4)} + \frac{2}{5}\text{RootSum}\left[1-\#1+\#1^2 -\#1^3+\#1^4\&, \frac{2\log(x-\#1)+\log(x-\#1)\#1}{-1+2\#1-3\#1^2+4\#1^3}\&\right]$$

input
```
Integrate[(1 - x + x^2 - x^3 + x^4)^(-2),x]
```

output
```
(x + x^2)/(5*(1 - x + x^2 - x^3 + x^4)) + (2*RootSum[1 - #1 + #1^2 - #1^3
+ #1^4 & , (2*Log[x - #1] + Log[x - #1]*#1)/(-1 + 2*#1 - 3*#1^2 + 4*#1^3)
& ])/5
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 359 vs. 2(175) = 350.

Time = 0.71 (sec) , antiderivative size = 359, normalized size of antiderivative = 2.05, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2492, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x^4-x^3+x^2-x+1)^2}dx$$

$$\Big\downarrow \text{ 2492}$$

$$\int \left(-\frac{2(-4x-2\sqrt{5}+3)}{5\sqrt{5}\left(2x^2-\left(1-\sqrt{5}\right)x+2\right)}+\frac{2(-4x+2\sqrt{5}+3)}{5\sqrt{5}\left(2x^2-\left(1+\sqrt{5}\right)x+2\right)}+\frac{2\left(1-\sqrt{5}\right)(1-x)}{5\left(2x^2-\left(1-\sqrt{5}\right)x+2\right)^2}+\frac{2(1+\sqrt{}}{5\left(2x^2-(1\right.}\right.$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{25}\sqrt{2\left(5-\sqrt{5}\right)}\arctan\left(\frac{-4x-\sqrt{5}+1}{\sqrt{2\left(5+\sqrt{5}\right)}}\right)-\frac{1}{25}\sqrt{2\left(65-29\sqrt{5}\right)}\arctan\left(\frac{-4x-\sqrt{5}+1}{\sqrt{2\left(5+\sqrt{5}\right)}}\right)-\frac{1}{25}\sqrt{2\left(65+29\sqrt{5}\right)}\arctan\left(\frac{-4x+\sqrt{5}+1}{\sqrt{2\left(5-\sqrt{5}\right)}}\right)-\frac{1}{25}\sqrt{2\left(5+\sqrt{5}\right)}\arctan\left(\frac{-4x+\sqrt{5}+1}{\sqrt{2\left(5-\sqrt{5}\right)}}\right)+\frac{\left(1+\sqrt{5}\right)\left(\left(3-\sqrt{5}\right)x-\sqrt{5}+3\right)}{5\left(5-\sqrt{5}\right)\left(2x^2-\left(1+\sqrt{5}\right)x+2\right)}+\frac{\left(1-\sqrt{5}\right)\left(\left(3+\sqrt{5}\right)x+\sqrt{5}+3\right)}{5\left(5+\sqrt{5}\right)\left(2x^2-\left(1-\sqrt{5}\right)x+2\right)}+\frac{2\log\left(2x^2-\left(1-\sqrt{5}\right)x+2\right)}{5\sqrt{5}}-\frac{2\log\left(2x^2-\left(1+\sqrt{5}\right)x+2\right)}{5\sqrt{5}}$$

input
```
Int[(1 - x + x^2 - x^3 + x^4)^(-2),x]
```

output
```
((1 - Sqrt[5])*(3 + Sqrt[5] + (3 + Sqrt[5])*x))/(5*(5 + Sqrt[5])*(2 - (1 -
 Sqrt[5])*x + 2*x^2)) + ((1 + Sqrt[5])*(3 - Sqrt[5] + (3 - Sqrt[5])*x))/(5
*(5 - Sqrt[5])*(2 - (1 + Sqrt[5])*x + 2*x^2)) - (Sqrt[2*(65 - 29*Sqrt[5])]
*ArcTan[(1 - Sqrt[5] - 4*x)/Sqrt[2*(5 + Sqrt[5])]])/25 + (Sqrt[2*(5 - Sqrt
[5])]*ArcTan[(1 - Sqrt[5] - 4*x)/Sqrt[2*(5 + Sqrt[5])]])/25 - (Sqrt[2*(5 +
 Sqrt[5])]*ArcTan[(1 + Sqrt[5] - 4*x)/Sqrt[2*(5 - Sqrt[5])]])/25 - (Sqrt[2
*(65 + 29*Sqrt[5])]*ArcTan[(1 + Sqrt[5] - 4*x)/Sqrt[2*(5 - Sqrt[5])]])/25
+ (2*Log[2 - (1 - Sqrt[5])*x + 2*x^2])/(5*Sqrt[5]) - (2*Log[2 - (1 + Sqrt[
5])*x + 2*x^2])/(5*Sqrt[5])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2492
```
Int[(Px_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^3 + (e_.)*(x_)^4)
^(p_), x_Symbol] :> Simp[e^p   Int[ExpandIntegrand[Px*(b/d + ((d + Sqrt[e*(
(b^2 - 4*a*c)/a) + 8*a*d*(e/b)])/(2*e))*x + x^2)^p*(b/d + ((d - Sqrt[e*((b^
2 - 4*a*c)/a) + 8*a*d*(e/b)])/(2*e))*x + x^2)^p, x], x], x] /; FreeQ[{a, b,
 c, d, e}, x] && PolyQ[Px, x] && ILtQ[p, 0] && EqQ[a*d^2 - b^2*e, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.15 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.45

method	result
risch	$\frac{\frac{1}{5}x^2+\frac{1}{5}x}{x^4-x^3+x^2-x+1}+\frac{2\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^4-\_Z^3+\_Z^2-\_Z+1\right)}\frac{(2+\_R)\ln(x-\_R)}{4\_R^3-3\_R^2+2\_R-1}\right)}{5}$
default	$\frac{-\frac{\sqrt{5}\,x}{25}-\frac{\sqrt{5}}{25}}{x^2-\frac{x}{2}+\frac{\sqrt{5}\,x}{2}+1}+\frac{2\ln\left(2-x+\sqrt{5}\,x+2x^2\right)\sqrt{5}}{25}+\frac{8\left(-\frac{\sqrt{5}\,(\sqrt{5}-1)}{2}+5-2\sqrt{5}\right)\arctan\left(\frac{-1+\sqrt{5}+4x}{\sqrt{10+2\sqrt{5}}}\right)}{25\sqrt{10+2\sqrt{5}}}-\frac{2\left(-\frac{\sqrt{5}\,x}{2}-\frac{\sqrt{5}}{2}\right)}{25\left(x^2-\frac{x}{2}-\frac{\sqrt{5}\,x}{2}+1\right)}-$

input
```
int(1/(x^4-x^3+x^2-x+1)^2,x,method=_RETURNVERBOSE)
```

output
```
(1/5*x^2+1/5*x)/(x^4-x^3+x^2-x+1)+2/5*sum((2+_R)/(4*_R^3-3*_R^2+2*_R-1)*ln
(x-_R),_R=RootOf(_Z^4-_Z^3+_Z^2-_Z+1))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.20

$$\int \frac{1}{(1-x+x^2-x^3+x^4)^2}\,dx = \frac{2\,(x^4-x^3+x^2-x+1)\sqrt{22\sqrt{5}+50}\arctan\left(\frac{1}{20}\left(\sqrt{5}(6\,x+1)-10\,x-5\right)\sqrt{22\sqrt{5}+50}\right)-2\,(x^4-x^3}{}$$

input
```
integrate(1/(x^4-x^3+x^2-x+1)^2,x, algorithm="fricas")
```

output
```
1/25*(2*(x^4 - x^3 + x^2 - x + 1)*sqrt(22*sqrt(5) + 50)*arctan(1/20*(sqrt(
5)*(6*x + 1) - 10*x - 5)*sqrt(22*sqrt(5) + 50)) - 2*(x^4 - x^3 + x^2 - x +
 1)*sqrt(-22*sqrt(5) + 50)*arctan(1/20*(sqrt(5)*(6*x + 1) + 10*x + 5)*sqrt
(-22*sqrt(5) + 50)) + 2*sqrt(5)*(x^4 - x^3 + x^2 - x + 1)*log(2*x^2 + sqrt
(5)*x - x + 2) - 2*sqrt(5)*(x^4 - x^3 + x^2 - x + 1)*log(2*x^2 - sqrt(5)*x
 - x + 2) + 5*x^2 + 5*x)/(x^4 - x^3 + x^2 - x + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1360 vs. $2(153) = 306$.

Time = 0.62 (sec) , antiderivative size = 1360, normalized size of antiderivative = 7.77

$$\int \frac{1}{(1 - x + x^2 - x^3 + x^4)^2} \, dx = \text{Too large to display}$$

input
```
integrate(1/(x**4-x**3+x**2-x+1)**2,x)
```

output
```
(x**2 + x)/(5*x**4 - 5*x**3 + 5*x**2 - 5*x + 5) - 2*sqrt(5)*log(x**2 + x*(
-1928*sqrt(5)/401 - 315*sqrt(201 - 88*sqrt(5))/401 + 85*sqrt(5)*sqrt(201 -
 88*sqrt(5))/802 + 8229/802) - 30944507*sqrt(5)/643204 - 3418085*sqrt(201
- 88*sqrt(5))/643204 + 1570197*sqrt(5)*sqrt(201 - 88*sqrt(5))/643204 + 696
46079/643204)/25 + 2*sqrt(5)*log(x**2 + x*(-315*sqrt(88*sqrt(5) + 201)/401
 - 85*sqrt(5)*sqrt(88*sqrt(5) + 201)/802 + 8229/802 + 1928*sqrt(5)/401) -
1570197*sqrt(5)*sqrt(88*sqrt(5) + 201)/643204 - 3418085*sqrt(88*sqrt(5) +
201)/643204 + 30944507*sqrt(5)/643204 + 69646079/643204)/25 - 2*sqrt(-2*sq
rt(5)*sqrt(88*sqrt(5) + 201)/625 + 18/125)*atan(802*sqrt(2)*x/(170*sqrt(5)
*sqrt(-sqrt(5)*sqrt(88*sqrt(5) + 201) + 45) + 382*sqrt(-sqrt(5)*sqrt(88*sq
rt(5) + 201) + 45) + 21*sqrt(5)*sqrt(88*sqrt(5) + 201)*sqrt(-sqrt(5)*sqrt(
88*sqrt(5) + 201) + 45)) - 630*sqrt(2)*sqrt(88*sqrt(5) + 201)/(340*sqrt(5)
*sqrt(-sqrt(5)*sqrt(88*sqrt(5) + 201) + 45) + 764*sqrt(-sqrt(5)*sqrt(88*sq
rt(5) + 201) + 45) + 42*sqrt(5)*sqrt(88*sqrt(5) + 201)*sqrt(-sqrt(5)*sqrt(
88*sqrt(5) + 201) + 45)) - 85*sqrt(10)*sqrt(88*sqrt(5) + 201)/(340*sqrt(5)
*sqrt(-sqrt(5)*sqrt(88*sqrt(5) + 201) + 45) + 764*sqrt(-sqrt(5)*sqrt(88*sq
rt(5) + 201) + 45) + 42*sqrt(5)*sqrt(88*sqrt(5) + 201)*sqrt(-sqrt(5)*sqrt(
88*sqrt(5) + 201) + 45)) + 8229*sqrt(2)/(340*sqrt(5)*sqrt(-sqrt(5)*sqrt(88
*sqrt(5) + 201) + 45) + 764*sqrt(-sqrt(5)*sqrt(88*sqrt(5) + 201) + 45) + 4
2*sqrt(5)*sqrt(88*sqrt(5) + 201)*sqrt(-sqrt(5)*sqrt(88*sqrt(5) + 201) +...
```

Maxima [F]

$$\int \frac{1}{(1-x+x^2-x^3+x^4)^2}\,dx = \int \frac{1}{(x^4-x^3+x^2-x+1)^2}\,dx$$

input
```
integrate(1/(x^4-x^3+x^2-x+1)^2,x, algorithm="maxima")
```

output
```
1/5*(x^2 + x)/(x^4 - x^3 + x^2 - x + 1) + 2/5*integrate((x + 2)/(x^4 - x^3
 + x^2 - x + 1), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.72

$$\int \frac{1}{(1-x+x^2-x^3+x^4)^2}\,dx = -\frac{2}{25}\sqrt{-22\sqrt{5}+50}\arctan\left(\frac{4x+\sqrt{5}-1}{\sqrt{2\sqrt{5}+10}}\right) + \frac{2}{25}\sqrt{22\sqrt{5}+50}\arctan\left(\frac{4x-\sqrt{5}-1}{\sqrt{-2\sqrt{5}+10}}\right) - \frac{2}{25}\sqrt{5}\log\left(x^2-\frac{1}{2}x\left(\sqrt{5}+1\right)+1\right) + \frac{2}{25}\sqrt{5}\log\left(x^2+\frac{1}{2}x\left(\sqrt{5}-1\right)+1\right) + \frac{x^2+x}{5(x^4-x^3+x^2-x+1)}$$

input
```
integrate(1/(x^4-x^3+x^2-x+1)^2,x, algorithm="giac")
```

output
```
-2/25*sqrt(-22*sqrt(5) + 50)*arctan((4*x + sqrt(5) - 1)/sqrt(2*sqrt(5) + 1
0)) + 2/25*sqrt(22*sqrt(5) + 50)*arctan((4*x - sqrt(5) - 1)/sqrt(-2*sqrt(5
) + 10)) - 2/25*sqrt(5)*log(x^2 - 1/2*x*(sqrt(5) + 1) + 1) + 2/25*sqrt(5)*
log(x^2 + 1/2*x*(sqrt(5) - 1) + 1) + 1/5*(x^2 + x)/(x^4 - x^3 + x^2 - x +
1)
```

Mupad [B] (verification not implemented)

Time = 21.23 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.67

$$\int \frac{1}{\left(1-x+x^2-x^3+x^4\right)^2}\,dx = \left(\sum_{k=1}^{4}\ln\left(\frac{8\,x}{125}\right.\right.$$
$$+\mathrm{root}\left(z^4+\frac{12\,z^2}{125}-\frac{176\,z}{3125}+\frac{496}{78125},z,k\right)\left(-\frac{76\,x}{25}+\mathrm{root}\left(z^4+\frac{12\,z^2}{125}-\frac{176\,z}{3125}+\frac{496}{78125},z,k\right)\left(25\,\mathrm{root}\left(z^4\right.\right.\right.$$
$$\left.\left.\left.+\frac{16}{125}\right)\mathrm{root}\left(z^4+\frac{12\,z^2}{125}-\frac{176\,z}{3125}+\frac{496}{78125},z,k\right)\right)+\frac{\frac{x^2}{5}+\frac{x}{5}}{x^4-x^3+x^2-x+1}$$

input
```
int(1/(x^2 - x - x^3 + x^4 + 1)^2,x)
```

output
```
symsum(log((8*x)/125 + root(z^4 + (12*z^2)/125 - (176*z)/3125 + 496/78125,
 z, k)*(root(z^4 + (12*z^2)/125 - (176*z)/3125 + 496/78125, z, k)*(25*root
(z^4 + (12*z^2)/125 - (176*z)/3125 + 496/78125, z, k) + 18*x - 14) - (76*x
)/25 + 44/25) + 16/125)*root(z^4 + (12*z^2)/125 - (176*z)/3125 + 496/78125
, z, k), k, 1, 4) + (x/5 + x^2/5)/(x^2 - x - x^3 + x^4 + 1)
```

Reduce [F]

$$\int \frac{1}{\left(1-x+x^2-x^3+x^4\right)^2}\,dx$$
$$= \frac{4\left(\int \frac{x^3}{x^8-2x^7+3x^6-4x^5+5x^4-4x^3+3x^2-2x+1}dx\right)x^4-4\left(\int \frac{x^3}{x^8-2x^7+3x^6-4x^5+5x^4-4x^3+3x^2-2x+1}dx\right)x^3+4\left(\int \frac{}{x^8-2x^7+}\right.}{}$$

input
```
int(1/(x^4-x^3+x^2-x+1)^2,x)
```

output
```
(4*int(x**3/(x**8 - 2*x**7 + 3*x**6 - 4*x**5 + 5*x**4 - 4*x**3 + 3*x**2 -
2*x + 1),x)*x**4 - 4*int(x**3/(x**8 - 2*x**7 + 3*x**6 - 4*x**5 + 5*x**4 -
4*x**3 + 3*x**2 - 2*x + 1),x)*x**3 + 4*int(x**3/(x**8 - 2*x**7 + 3*x**6 -
4*x**5 + 5*x**4 - 4*x**3 + 3*x**2 - 2*x + 1),x)*x**2 - 4*int(x**3/(x**8 -
2*x**7 + 3*x**6 - 4*x**5 + 5*x**4 - 4*x**3 + 3*x**2 - 2*x + 1),x)*x + 4*in
t(x**3/(x**8 - 2*x**7 + 3*x**6 - 4*x**5 + 5*x**4 - 4*x**3 + 3*x**2 - 2*x +
 1),x) - 3*int(x**2/(x**8 - 2*x**7 + 3*x**6 - 4*x**5 + 5*x**4 - 4*x**3 + 3
*x**2 - 2*x + 1),x)*x**4 + 3*int(x**2/(x**8 - 2*x**7 + 3*x**6 - 4*x**5 + 5
*x**4 - 4*x**3 + 3*x**2 - 2*x + 1),x)*x**3 - 3*int(x**2/(x**8 - 2*x**7 + 3
*x**6 - 4*x**5 + 5*x**4 - 4*x**3 + 3*x**2 - 2*x + 1),x)*x**2 + 3*int(x**2/
(x**8 - 2*x**7 + 3*x**6 - 4*x**5 + 5*x**4 - 4*x**3 + 3*x**2 - 2*x + 1),x)*
x - 3*int(x**2/(x**8 - 2*x**7 + 3*x**6 - 4*x**5 + 5*x**4 - 4*x**3 + 3*x**2
 - 2*x + 1),x) + 2*int(x/(x**8 - 2*x**7 + 3*x**6 - 4*x**5 + 5*x**4 - 4*x**
3 + 3*x**2 - 2*x + 1),x)*x**4 - 2*int(x/(x**8 - 2*x**7 + 3*x**6 - 4*x**5 +
 5*x**4 - 4*x**3 + 3*x**2 - 2*x + 1),x)*x**3 + 2*int(x/(x**8 - 2*x**7 + 3*
x**6 - 4*x**5 + 5*x**4 - 4*x**3 + 3*x**2 - 2*x + 1),x)*x**2 - 2*int(x/(x**
8 - 2*x**7 + 3*x**6 - 4*x**5 + 5*x**4 - 4*x**3 + 3*x**2 - 2*x + 1),x)*x +
2*int(x/(x**8 - 2*x**7 + 3*x**6 - 4*x**5 + 5*x**4 - 4*x**3 + 3*x**2 - 2*x
+ 1),x) + 1)/(x**4 - x**3 + x**2 - x + 1)
```

3.20 $\int \frac{1}{(1-x+x^2-x^3+x^4)^3}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 389

$$\begin{aligned}
\int \frac{1}{(1-x+x^2-x^3+x^4)^3}\,dx
&= \frac{6\left(2\left(15+8\sqrt{5}\right)+\left(15+11\sqrt{5}\right)x\right)}{25\left(5-\sqrt{5}\right)\left(2-\left(1-\sqrt{5}\right)x+2x^2\right)^2}+\frac{3\left(6+\sqrt{5}+2x\right)}{25\left(2-\left(1-\sqrt{5}\right)x+2x^2\right)} \\
&\quad-\frac{8\left(5-\sqrt{5}-\left(5+\sqrt{5}\right)x\right)}{5\left(5-\sqrt{5}\right)\left(2-\left(1-\sqrt{5}\right)x+2x^2\right)^2\left(2-\left(1+\sqrt{5}\right)x+2x^2\right)^2} \\
&\quad-\frac{8\left(45+11\sqrt{5}-6\left(15-\sqrt{5}\right)x\right)}{5\left(5-\sqrt{5}\right)^2\left(2-\left(1-\sqrt{5}\right)x+2x^2\right)^2\left(2-\left(1+\sqrt{5}\right)x+2x^2\right)} \\
&\quad+\frac{3}{125}\sqrt{1025-422\sqrt{5}}\arctan\left(\frac{1-\sqrt{5}-4x}{\sqrt{2\left(5+\sqrt{5}\right)}}\right) \\
&\quad-\frac{3}{125}\sqrt{1025+422\sqrt{5}}\arctan\left(\frac{1+\sqrt{5}-4x}{\sqrt{2\left(5-\sqrt{5}\right)}}\right) \\
&\quad+\frac{3\log\left(2-\left(1-\sqrt{5}\right)x+2x^2\right)}{10\sqrt{5}}-\frac{3\log\left(2-\left(1+\sqrt{5}\right)x+2x^2\right)}{10\sqrt{5}}
\end{aligned}$$

output
```
6/25*(30+16*5^(1/2)+(15+11*5^(1/2))*x)/(5-5^(1/2))/(2-x*(-5^(1/2)+1)+2*x^2
)^2+3*(6+5^(1/2)+2*x)/(50-25*x*(-5^(1/2)+1)+50*x^2)-8/5*(5-5^(1/2)-(5+5^(1
/2))*x)/(5-5^(1/2))/(2-x*(-5^(1/2)+1)+2*x^2)^2/(2-(5^(1/2)+1)*x+2*x^2)^2-8
/5*(45+11*5^(1/2)-6*(15-5^(1/2))*x)/(5-5^(1/2))^2/(2-x*(-5^(1/2)+1)+2*x^2)
^2/(2-(5^(1/2)+1)*x+2*x^2)+3/125*(1025-422*5^(1/2))^(1/2)*arctan((1-5^(1/2
)-4*x)/(10+2*5^(1/2))^(1/2))-3/125*(1025+422*5^(1/2))^(1/2)*arctan((1+5^(1
/2)-4*x)/(10-2*5^(1/2))^(1/2))+3/50*ln(2-x*(-5^(1/2)+1)+2*x^2)*5^(1/2)-3/5
0*ln(2-(5^(1/2)+1)*x+2*x^2)*5^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.31

$$\int \frac{1}{(1-x+x^2-x^3+x^4)^3}\,dx = \frac{1}{50}\left(\frac{x(14+11x+9x^5+6x^6)}{(1-x+x^2-x^3+x^4)^2}+6\text{RootSum}\left[1-\#1+\#1^2-\#1^3+\#1^4\&,\frac{6\log(x-\#1)+6\log(x-\#1)\#1+\log(x-\#1)\#1^2}{-1+2\#1-3\#1^2+4\#1^3}\&\right]\right)$$

input
```
Integrate[(1 - x + x^2 - x^3 + x^4)^(-3),x]
```

output
```
((x*(14 + 11*x + 9*x^5 + 6*x^6))/(1 - x + x^2 - x^3 + x^4)^2 + 6*RootSum[1
 - #1 + #1^2 - #1^3 + #1^4 & , (6*Log[x - #1] + 6*Log[x - #1]*#1 + Log[x -
 #1]*#1^2)/(-1 + 2*#1 - 3*#1^2 + 4*#1^3) & ])/50
```

Rubi [A] (verified)

Time = 1.17 (sec) , antiderivative size = 637, normalized size of antiderivative = 1.64, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2492, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x^4-x^3+x^2-x+1)^3}\,dx$$

$$\Big\downarrow \text{2492}$$

$$\int \left(-\frac{3(-10x-5\sqrt{5}+11)}{25\sqrt{5}\left(2x^2-(1-\sqrt{5})x+2\right)}+\frac{3(-10x+5\sqrt{5}+11)}{25\sqrt{5}\left(2x^2-(1+\sqrt{5})x+2\right)}+\frac{2\left(2(5-4\sqrt{5})-(9-5\sqrt{5})x\right)}{25\left(2x^2-(1-\sqrt{5})x+2\right)^2}+\frac{2(2(5}{25\,(2}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{6}{125}\sqrt{5-2\sqrt{5}}\arctan\left(\frac{-4x-\sqrt{5}+1}{\sqrt{2\left(5+\sqrt{5}\right)}}\right)+\frac{3}{125}\sqrt{2\left(145-61\sqrt{5}\right)}\arctan\left(\frac{-4x-\sqrt{5}+1}{\sqrt{2\left(5+\sqrt{5}\right)}}\right)+\frac{3}{125}\sqrt{365-158\sqrt{5}}\arctan\left(\frac{-4x-\sqrt{5}+1}{\sqrt{2\left(5+\sqrt{5}\right)}}\right)-\frac{3}{125}\sqrt{365+158\sqrt{5}}\arctan\left(\frac{-4x+\sqrt{5}+1}{\sqrt{2\left(5-\sqrt{5}\right)}}\right)-\frac{3}{125}\sqrt{2\left(145+61\sqrt{5}\right)}\arctan\left(\frac{-4x+\sqrt{5}+1}{\sqrt{2\left(5-\sqrt{5}\right)}}\right)+\frac{6}{125}\sqrt{5+2\sqrt{5}}\arctan\left(\frac{-4x+\sqrt{5}+1}{\sqrt{2\left(5-\sqrt{5}\right)}}\right)+\frac{3(-4x-\sqrt{5}+1)}{25\left(5+\sqrt{5}\right)\left(2x^2-(1-\sqrt{5})x+2\right)}-\frac{2\left(-3(1-3\sqrt{5})x+\sqrt{5}+7\right)}{25\left(5+\sqrt{5}\right)\left(2x^2-(1-\sqrt{5})x+2\right)}+\frac{3(-4x+\sqrt{5}+1)}{25\left(5-\sqrt{5}\right)\left(2x^2-(1+\sqrt{5})x+2\right)}-\frac{2\left(-3(1+3\sqrt{5})x-\sqrt{5}+7\right)}{25\left(5-\sqrt{5}\right)\left(2x^2-(1+\sqrt{5})x+2\right)}-\frac{2\left((1+\sqrt{5})x-\sqrt{5}+1\right)}{5\sqrt{5}\left(5+\sqrt{5}\right)\left(2x^2-(1-\sqrt{5})x+2\right)^2}+\frac{2\left((1-\sqrt{5})x+\sqrt{5}+1\right)}{5\sqrt{5}\left(5-\sqrt{5}\right)\left(2x^2-(1+\sqrt{5})x+2\right)^2}+\frac{3\log\left(2x^2-(1-\sqrt{5})x+2\right)}{10\sqrt{5}}-\frac{3\log\left(2x^2-(1+\sqrt{5})x+2\right)}{10\sqrt{5}}$$

input
```
Int[(1 - x + x^2 - x^3 + x^4)^(-3),x]
```

output
```
(-2*(1 - Sqrt[5] + (1 + Sqrt[5])*x))/(5*Sqrt[5]*(5 + Sqrt[5])*(2 - (1 - Sq
rt[5])*x + 2*x^2)^2) + (3*(1 - Sqrt[5] - 4*x))/(25*(5 + Sqrt[5])*(2 - (1 -
 Sqrt[5])*x + 2*x^2)) - (2*(7 + Sqrt[5] - 3*(1 - 3*Sqrt[5])*x))/(25*(5 + S
qrt[5])*(2 - (1 - Sqrt[5])*x + 2*x^2)) + (2*(1 + Sqrt[5] + (1 - Sqrt[5])*x
))/(5*Sqrt[5]*(5 - Sqrt[5])*(2 - (1 + Sqrt[5])*x + 2*x^2)^2) + (3*(1 + Sqr
t[5] - 4*x))/(25*(5 - Sqrt[5])*(2 - (1 + Sqrt[5])*x + 2*x^2)) - (2*(7 - Sq
rt[5] - 3*(1 + 3*Sqrt[5])*x))/(25*(5 - Sqrt[5])*(2 - (1 + Sqrt[5])*x + 2*x
^2)) + (3*Sqrt[365 - 158*Sqrt[5]]*ArcTan[(1 - Sqrt[5] - 4*x)/Sqrt[2*(5 + S
qrt[5])]])/125 + (3*Sqrt[2*(145 - 61*Sqrt[5])]*ArcTan[(1 - Sqrt[5] - 4*x)/
Sqrt[2*(5 + Sqrt[5])]])/125 + (6*Sqrt[5 - 2*Sqrt[5]]*ArcTan[(1 - Sqrt[5] -
 4*x)/Sqrt[2*(5 + Sqrt[5])]])/125 + (6*Sqrt[5 + 2*Sqrt[5]]*ArcTan[(1 + Sqr
t[5] - 4*x)/Sqrt[2*(5 - Sqrt[5])]])/125 - (3*Sqrt[2*(145 + 61*Sqrt[5])]*Ar
cTan[(1 + Sqrt[5] - 4*x)/Sqrt[2*(5 - Sqrt[5])]])/125 - (3*Sqrt[365 + 158*S
qrt[5]]*ArcTan[(1 + Sqrt[5] - 4*x)/Sqrt[2*(5 - Sqrt[5])]])/125 + (3*Log[2
- (1 - Sqrt[5])*x + 2*x^2])/(10*Sqrt[5]) - (3*Log[2 - (1 + Sqrt[5])*x + 2*
x^2])/(10*Sqrt[5])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2492
```
Int[(Px_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^3 + (e_.)*(x_)^4)
^(p_), x_Symbol] :> Simp[e^p   Int[ExpandIntegrand[Px*(b/d + ((d + Sqrt[e*(
(b^2 - 4*a*c)/a) + 8*a*d*(e/b)])/(2*e))*x + x^2)^p*(b/d + ((d - Sqrt[e*((b^
2 - 4*a*c)/a) + 8*a*d*(e/b)])/(2*e))*x + x^2)^p, x], x], x] /; FreeQ[{a, b,
 c, d, e}, x] && PolyQ[Px, x] && ILtQ[p, 0] && EqQ[a*d^2 - b^2*e, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.22 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.24

method	result
risch	$\frac{\frac{3}{25}x^{7}+\frac{9}{50}x^{6}+\frac{11}{50}x^{2}+\frac{7}{25}x}{\left(x^{4}-x^{3}+x^{2}-x+1\right)^{2}}+\frac{3\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{4}-\_Z^{3}+\_Z^{2}-\_Z+1\right)}\frac{\left(\_R^{2}+6\_R+6\right)\ln\left(x-\_R\right)}{4\_R^{3}-3\_R^{2}+2\_R-1}\right)}{25}$
default	$\frac{\frac{4\left(\frac{15}{8}-\frac{21\sqrt{5}}{8}\right)x^{3}}{125}+\frac{4\left(\frac{29\sqrt{5}}{16}-\frac{135}{16}\right)x^{2}}{125}+\frac{4\left(-\frac{23\sqrt{5}}{8}+\frac{5}{8}\right)x}{125}-\frac{3\sqrt{5}}{125}}{\left(x^{2}-\frac{x}{2}+\frac{\sqrt{5}\,x}{2}+1\right)^{2}}+\frac{3\ln\left(2-x+\sqrt{5}\,x+2x^{2}\right)\sqrt{5}}{50}+\frac{12\left(-\frac{5\sqrt{5}\left(\sqrt{5}-1\right)}{4}-9\sqrt{5}+15\right)\mathrm{arct}}{125\sqrt{10+2\sqrt{5}}}$

input
```
int(1/(x^4-x^3+x^2-x+1)^3,x,method=_RETURNVERBOSE)
```

output
```
(3/25*x^7+9/50*x^6+11/50*x^2+7/25*x)/(x^4-x^3+x^2-x+1)^2+3/25*sum((_R^2+6*
_R+6)/(4*_R^3-3*_R^2+2*_R-1)*ln(x-_R),_R=RootOf(_Z^4-_Z^3+_Z^2-_Z+1))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 330, normalized size of antiderivative = 0.85

$$\int\frac{1}{\left(1-x+x^{2}-x^{3}+x^{4}\right)^{3}}\,dx$$
$$=\frac{30\,x^{7}+45\,x^{6}+6\,(x^{8}-2\,x^{7}+3\,x^{6}-4\,x^{5}+5\,x^{4}-4\,x^{3}+3\,x^{2}-2\,x+1)\sqrt{422\,\sqrt{5}+1025}\,\arctan\left(\frac{1}{895}\left(\right.\right.}{}$$

input
```
integrate(1/(x^4-x^3+x^2-x+1)^3,x, algorithm="fricas")
```

output
```
1/250*(30*x^7 + 45*x^6 + 6*(x^8 - 2*x^7 + 3*x^6 - 4*x^5 + 5*x^4 - 4*x^3 +
3*x^2 - 2*x + 1)*sqrt(422*sqrt(5) + 1025)*arctan(1/895*(sqrt(5)*(31*x + 1)
 - 35*x - 30)*sqrt(422*sqrt(5) + 1025)) - 2*(x^8 - 2*x^7 + 3*x^6 - 4*x^5 +
 5*x^4 - 4*x^3 + 3*x^2 - 2*x + 1)*sqrt(-3798*sqrt(5) + 9225)*arctan(1/2685
*(sqrt(5)*(31*x + 1) + 35*x + 30)*sqrt(-3798*sqrt(5) + 9225)) + 15*sqrt(5)
*(x^8 - 2*x^7 + 3*x^6 - 4*x^5 + 5*x^4 - 4*x^3 + 3*x^2 - 2*x + 1)*log(2*x^2
 + sqrt(5)*x - x + 2) - 15*sqrt(5)*(x^8 - 2*x^7 + 3*x^6 - 4*x^5 + 5*x^4 -
4*x^3 + 3*x^2 - 2*x + 1)*log(2*x^2 - sqrt(5)*x - x + 2) + 55*x^2 + 70*x)/(
x^8 - 2*x^7 + 3*x^6 - 4*x^5 + 5*x^4 - 4*x^3 + 3*x^2 - 2*x + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1445 vs. $2(333) = 666$.

Time = 0.65 (sec) , antiderivative size = 1445, normalized size of antiderivative = 3.71

$$\int \frac{1}{(1-x+x^2-x^3+x^4)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(x**4-x**3+x**2-x+1)**3,x)
```

output
```
(6*x**7 + 9*x**6 + 11*x**2 + 14*x)/(50*x**8 - 100*x**7 + 150*x**6 - 200*x*
*5 + 250*x**4 - 200*x**3 + 150*x**2 - 100*x + 50) - 3*sqrt(5)*log(x**2 + x
*(-15796955*sqrt(5)/13115509 - 429675*sqrt(2)*sqrt(23823 - 5275*sqrt(5))/2
6231018 + 18333*sqrt(10)*sqrt(23823 - 5275*sqrt(5))/13115509 + 41064583/13
115509) - 540905909434020*sqrt(5)/172016576329081 - 23080955957875*sqrt(2)
*sqrt(23823 - 5275*sqrt(5))/688066305316324 + 7519430041655*sqrt(10)*sqrt(
23823 - 5275*sqrt(5))/688066305316324 + 1624519191908706/172016576329081)/
50 + 3*sqrt(5)*log(x**2 + x*(-429675*sqrt(2)*sqrt(5275*sqrt(5) + 23823)/26
231018 - 18333*sqrt(10)*sqrt(5275*sqrt(5) + 23823)/13115509 + 15796955*sqr
t(5)/13115509 + 41064583/13115509) - 23080955957875*sqrt(2)*sqrt(5275*sqrt
(5) + 23823)/688066305316324 - 7519430041655*sqrt(10)*sqrt(5275*sqrt(5) +
23823)/688066305316324 + 540905909434020*sqrt(5)/172016576329081 + 1624519
191908706/172016576329081)/50 - 2*sqrt(-9*sqrt(10)*sqrt(5275*sqrt(5) + 238
23)/31250 + 459/2500)*atan(52462036*x/(183330*sqrt(5)*sqrt(-2*sqrt(10)*sqr
t(5275*sqrt(5) + 23823) + 1275) + 1701818*sqrt(-2*sqrt(10)*sqrt(5275*sqrt(
5) + 23823) + 1275) + 5729*sqrt(10)*sqrt(5275*sqrt(5) + 23823)*sqrt(-2*sqr
t(10)*sqrt(5275*sqrt(5) + 23823) + 1275)) - 429675*sqrt(2)*sqrt(5275*sqrt(
5) + 23823)/(183330*sqrt(5)*sqrt(-2*sqrt(10)*sqrt(5275*sqrt(5) + 23823) +
1275) + 1701818*sqrt(-2*sqrt(10)*sqrt(5275*sqrt(5) + 23823) + 1275) + 5729
*sqrt(10)*sqrt(5275*sqrt(5) + 23823)*sqrt(-2*sqrt(10)*sqrt(5275*sqrt(5)...
```

Maxima [F]

$$\int \frac{1}{(1-x+x^2-x^3+x^4)^3}\,dx = \int \frac{1}{(x^4-x^3+x^2-x+1)^3}\,dx$$

input
```
integrate(1/(x^4-x^3+x^2-x+1)^3,x, algorithm="maxima")
```

output
```
1/50*(6*x^7 + 9*x^6 + 11*x^2 + 14*x)/(x^8 - 2*x^7 + 3*x^6 - 4*x^5 + 5*x^4
- 4*x^3 + 3*x^2 - 2*x + 1) + 3/25*integrate((x^2 + 6*x + 6)/(x^4 - x^3 + x
^2 - x + 1), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.36

$$\int \frac{1}{(1-x+x^2-x^3+x^4)^3}\,dx = -\frac{3}{125}\sqrt{-422\sqrt{5}+1025}\arctan\left(\frac{4x+\sqrt{5}-1}{\sqrt{2\sqrt{5}+10}}\right) + \frac{3}{125}\sqrt{422\sqrt{5}+1025}\arctan\left(\frac{4x-\sqrt{5}-1}{\sqrt{-2\sqrt{5}+10}}\right) - \frac{3}{50}\sqrt{5}\log\left(x^2-\frac{1}{2}x\left(\sqrt{5}+1\right)+1\right) + \frac{3}{50}\sqrt{5}\log\left(x^2+\frac{1}{2}x\left(\sqrt{5}-1\right)+1\right) + \frac{6x^7+9x^6+11x^2+14x}{50\left(x^4-x^3+x^2-x+1\right)^2}$$

input
```
integrate(1/(x^4-x^3+x^2-x+1)^3,x, algorithm="giac")
```

output
```
-3/125*sqrt(-422*sqrt(5) + 1025)*arctan((4*x + sqrt(5) - 1)/sqrt(2*sqrt(5)
 + 10)) + 3/125*sqrt(422*sqrt(5) + 1025)*arctan((4*x - sqrt(5) - 1)/sqrt(-
2*sqrt(5) + 10)) - 3/50*sqrt(5)*log(x^2 - 1/2*x*(sqrt(5) + 1) + 1) + 3/50*
sqrt(5)*log(x^2 + 1/2*x*(sqrt(5) - 1) + 1) + 1/50*(6*x^7 + 9*x^6 + 11*x^2
+ 14*x)/(x^4 - x^3 + x^2 - x + 1)^2
```

Mupad [B] (verification not implemented)

Time = 21.36 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.39

$$\int \frac{1}{(1-x+x^2-x^3+x^4)^3}\,dx = \left(\sum_{k=1}^{4}\ln\left(\frac{4887x}{15625} + \mathrm{root}\left(z^4+\frac{162z^2}{625}-\frac{5697z}{78125}+\frac{437481}{48828125},z,k\right)\left(-\frac{1026x}{625}+\mathrm{root}\left(z^4+\frac{162z^2}{625}-\frac{5697z}{78125}+\frac{437481}{48828125},z,k\right) + \frac{5967}{15625}\right)\mathrm{root}\left(z^4+\frac{162z^2}{625}-\frac{5697z}{78125}+\frac{437481}{48828125},z,k\right)\right) + \frac{\frac{3x^7}{25}+\frac{9x^6}{50}+\frac{11x^2}{50}+\frac{7x}{25}}{x^8-2x^7+3x^6-4x^5+5x^4-4x^3+3x^2-2x+1}$$

input
```
int(1/(x^2 - x - x^3 + x^4 + 1)^3,x)
```

output
```
symsum(log((4887*x)/15625 + root(z^4 + (162*z^2)/625 - (5697*z)/78125 + 43
7481/48828125, z, k)*(root(z^4 + (162*z^2)/625 - (5697*z)/78125 + 437481/4
8828125, z, k)*(25*root(z^4 + (162*z^2)/625 - (5697*z)/78125 + 437481/4882
8125, z, k) + (111*x)/5 - 72/5) - (1026*x)/625 + 1314/625) + 5967/15625)*r
oot(z^4 + (162*z^2)/625 - (5697*z)/78125 + 437481/48828125, z, k), k, 1, 4
) + ((7*x)/25 + (11*x^2)/50 + (9*x^6)/50 + (3*x^7)/25)/(3*x^2 - 2*x - 4*x^
3 + 5*x^4 - 4*x^5 + 3*x^6 - 2*x^7 + x^8 + 1)
```

Reduce [F]

$$\int \frac{1}{(1-x+x^2-x^3+x^4)^3}\,dx = \text{too large to display}$$

input
```
int(1/(x^4-x^3+x^2-x+1)^3,x)
```

output

```
( - 24*int(x**5/(x**12 - 3*x**11 + 6*x**10 - 10*x**9 + 15*x**8 - 18*x**7 +
 19*x**6 - 18*x**5 + 15*x**4 - 10*x**3 + 6*x**2 - 3*x + 1),x)*x**8 + 48*in
t(x**5/(x**12 - 3*x**11 + 6*x**10 - 10*x**9 + 15*x**8 - 18*x**7 + 19*x**6
- 18*x**5 + 15*x**4 - 10*x**3 + 6*x**2 - 3*x + 1),x)*x**7 - 72*int(x**5/(x
**12 - 3*x**11 + 6*x**10 - 10*x**9 + 15*x**8 - 18*x**7 + 19*x**6 - 18*x**5
 + 15*x**4 - 10*x**3 + 6*x**2 - 3*x + 1),x)*x**6 + 96*int(x**5/(x**12 - 3*
x**11 + 6*x**10 - 10*x**9 + 15*x**8 - 18*x**7 + 19*x**6 - 18*x**5 + 15*x**
4 - 10*x**3 + 6*x**2 - 3*x + 1),x)*x**5 - 120*int(x**5/(x**12 - 3*x**11 +
6*x**10 - 10*x**9 + 15*x**8 - 18*x**7 + 19*x**6 - 18*x**5 + 15*x**4 - 10*x
**3 + 6*x**2 - 3*x + 1),x)*x**4 + 96*int(x**5/(x**12 - 3*x**11 + 6*x**10 -
 10*x**9 + 15*x**8 - 18*x**7 + 19*x**6 - 18*x**5 + 15*x**4 - 10*x**3 + 6*x
**2 - 3*x + 1),x)*x**3 - 72*int(x**5/(x**12 - 3*x**11 + 6*x**10 - 10*x**9
+ 15*x**8 - 18*x**7 + 19*x**6 - 18*x**5 + 15*x**4 - 10*x**3 + 6*x**2 - 3*x
 + 1),x)*x**2 + 48*int(x**5/(x**12 - 3*x**11 + 6*x**10 - 10*x**9 + 15*x**8
 - 18*x**7 + 19*x**6 - 18*x**5 + 15*x**4 - 10*x**3 + 6*x**2 - 3*x + 1),x)*
x - 24*int(x**5/(x**12 - 3*x**11 + 6*x**10 - 10*x**9 + 15*x**8 - 18*x**7 +
 19*x**6 - 18*x**5 + 15*x**4 - 10*x**3 + 6*x**2 - 3*x + 1),x) + 44*int(x**
4/(x**12 - 3*x**11 + 6*x**10 - 10*x**9 + 15*x**8 - 18*x**7 + 19*x**6 - 18*
x**5 + 15*x**4 - 10*x**3 + 6*x**2 - 3*x + 1),x)*x**8 - 88*int(x**4/(x**12
- 3*x**11 + 6*x**10 - 10*x**9 + 15*x**8 - 18*x**7 + 19*x**6 - 18*x**5 +...
```

3.21 $\int \left(a+bx+cx^2+bx^3+ax^4\right)^3\,dx$

Optimal result

Integrand size = 22, antiderivative size = 238

$$\int \left(a+bx+cx^2+bx^3+ax^4\right)^3\,dx=a^3x+\frac{3}{2}a^2bx^2+a\left(b^2+ac\right)x^3+\frac{1}{4}b\left(3a^2+b^2+6ac\right)x^4+\frac{3}{5}\left(a^3+b^2c+a\left(2b^2+c^2\right)\right)x^5+\frac{1}{2}b\left(2a^2+b^2+2ac+c^2\right)x^6+\frac{1}{7}\left(6ab^2+6a^2c+6b^2c+c^3\right)x^7+\frac{3}{8}b\left(2a^2+b^2+2ac+c^2\right)x^8+\frac{1}{3}\left(a^3+b^2c+a\left(2b^2+c^2\right)\right)x^9+\frac{1}{10}b\left(3a^2+b^2+6ac\right)x^{10}+\frac{3}{11}a\left(b^2+ac\right)x^{11}+\frac{1}{4}a^2bx^{12}+\frac{a^3x^{13}}{13}$$

output
```
a^3*x+3/2*a^2*b*x^2+a*(a*c+b^2)*x^3+1/4*b*(3*a^2+6*a*c+b^2)*x^4+3/5*(a^3+b
^2*c+a*(2*b^2+c^2))*x^5+1/2*b*(2*a^2+2*a*c+b^2+c^2)*x^6+1/7*(6*a^2*c+6*a*b
^2+6*b^2*c+c^3)*x^7+3/8*b*(2*a^2+2*a*c+b^2+c^2)*x^8+1/3*(a^3+b^2*c+a*(2*b^
2+c^2))*x^9+1/10*b*(3*a^2+6*a*c+b^2)*x^10+3/11*a*(a*c+b^2)*x^11+1/4*a^2*b*
x^12+1/13*a^3*x^13
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.00

$$\int \left(a+bx+cx^2+bx^3+ax^4\right)^3 \, dx = a^3x+\frac{3}{2}a^2bx^2+a\left(b^2+ac\right)x^3+\frac{1}{4}b\left(3a^2+b^2+6ac\right)x^4 + \frac{3}{5}\left(a^3+2ab^2+b^2c+ac^2\right)x^5 + \frac{1}{2}b\left(2a^2+b^2+2ac+c^2\right)x^6 + \frac{1}{7}\left(6ab^2+6a^2c+6b^2c+c^3\right)x^7 + \frac{3}{8}b\left(2a^2+b^2+2ac+c^2\right)x^8 + \frac{1}{3}\left(a^3+2ab^2+b^2c+ac^2\right)x^9 + \frac{1}{10}b\left(3a^2+b^2+6ac\right)x^{10} + \frac{3}{11}a\left(b^2+ac\right)x^{11}+\frac{1}{4}a^2bx^{12}+\frac{a^3x^{13}}{13}$$

input
```
Integrate[(a + b*x + c*x^2 + b*x^3 + a*x^4)^3,x]
```

output
```
a^3*x + (3*a^2*b*x^2)/2 + a*(b^2 + a*c)*x^3 + (b*(3*a^2 + b^2 + 6*a*c)*x^4
)/4 + (3*(a^3 + 2*a*b^2 + b^2*c + a*c^2)*x^5)/5 + (b*(2*a^2 + b^2 + 2*a*c
+ c^2)*x^6)/2 + ((6*a*b^2 + 6*a^2*c + 6*b^2*c + c^3)*x^7)/7 + (3*b*(2*a^2
+ b^2 + 2*a*c + c^2)*x^8)/8 + ((a^3 + 2*a*b^2 + b^2*c + a*c^2)*x^9)/3 + (b
*(3*a^2 + b^2 + 6*a*c)*x^10)/10 + (3*a*(b^2 + a*c)*x^11)/11 + (a^2*b*x^12)
/4 + (a^3*x^13)/13
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2465, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(ax^4+a+bx^3+bx+cx^2\right)^3 \, dx$$

$$\Big\downarrow \ 2465$$

$$\int \left(3x^8\left(a^3+a\left(2b^2+c^2\right)+b^2c\right)+3x^4\left(a^3+a\left(2b^2+c^2\right)+b^2c\right)+a^3x^{12}+a^3+x^6\left(6a^2c+6ab^2+6b^2c+c^3\right)+3$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{3}x^9\left(a^3+a\left(2b^2+c^2\right)+b^2c\right)+\frac{3}{5}x^5\left(a^3+a\left(2b^2+c^2\right)+b^2c\right)+\frac{a^3x^{13}}{13}+a^3x+\frac{1}{7}x^7\left(6a^2c+6ab^2+6b^2c+c^3\right)+\frac{3}{8}bx^8\left(2a^2+2ac+b^2+c^2\right)+\frac{1}{2}bx^6\left(2a^2+2ac+b^2+c^2\right)+\frac{1}{10}bx^{10}\left(3a^2+6ac+b^2\right)+\frac{1}{4}bx^4\left(3a^2+6ac+b^2\right)+\frac{1}{4}a^2bx^{12}+\frac{3}{2}a^2bx^2+\frac{3}{11}ax^{11}\left(ac+b^2\right)+ax^3\left(ac+b^2\right)$$

input
```
Int[(a + b*x + c*x^2 + b*x^3 + a*x^4)^3,x]
```

output
```
a^3*x + (3*a^2*b*x^2)/2 + a*(b^2 + a*c)*x^3 + (b*(3*a^2 + b^2 + 6*a*c)*x^4
)/4 + (3*(a^3 + b^2*c + a*(2*b^2 + c^2))*x^5)/5 + (b*(2*a^2 + b^2 + 2*a*c
+ c^2)*x^6)/2 + ((6*a*b^2 + 6*a^2*c + 6*b^2*c + c^3)*x^7)/7 + (3*b*(2*a^2
+ b^2 + 2*a*c + c^2)*x^8)/8 + ((a^3 + b^2*c + a*(2*b^2 + c^2))*x^9)/3 + (b
*(3*a^2 + b^2 + 6*a*c)*x^10)/10 + (3*a*(b^2 + a*c)*x^11)/11 + (a^2*b*x^12)
/4 + (a^3*x^13)/13
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2465
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> Int[ExpandToSum[u, Px^p, x], x] /; PolyQ
[Px, x] && GtQ[Expon[Px, x], 2] && !BinomialQ[Px, x] && !TrinomialQ[Px, x
] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.02

method	result
norman	$\frac{a^3x^{13}}{13}+\frac{a^2b\,x^{12}}{4}+\left(\frac{3}{11}a^2c+\frac{3}{11}b^2a\right)x^{11}+\left(\frac{3}{10}b\,a^2+\frac{3}{5}abc+\frac{1}{10}b^3\right)x^{10}+\left(\frac{1}{3}a^3+\frac{2}{3}b^2a+\frac{1}{3}a\,c^2+\right.$
gosper	$\frac{3}{8}b^3x^8+\frac{3}{4}a^2b\,x^4+\frac{6}{7}a\,b^2x^7+\frac{3}{5}x^{10}abc+\frac{3}{4}x^8abc+x^6abc+\frac{3}{2}x^4abc+\frac{1}{10}b^3x^{10}+\frac{6}{5}b^2a\,x^5+a^3$
risch	$\frac{3}{8}b^3x^8+\frac{3}{4}a^2b\,x^4+\frac{6}{7}a\,b^2x^7+\frac{3}{5}x^{10}abc+\frac{3}{4}x^8abc+x^6abc+\frac{3}{2}x^4abc+\frac{1}{10}b^3x^{10}+\frac{6}{5}b^2a\,x^5+a^3$
parallelrisch	$\frac{3}{8}b^3x^8+\frac{3}{4}a^2b\,x^4+\frac{6}{7}a\,b^2x^7+\frac{3}{5}x^{10}abc+\frac{3}{4}x^8abc+x^6abc+\frac{3}{2}x^4abc+\frac{1}{10}b^3x^{10}+\frac{6}{5}b^2a\,x^5+a^3$
orering	$\frac{x\left(9240a^3x^{12}+30030b\,a^2x^{11}+32760a^2c\,x^{10}+32760a\,b^2x^{10}+36036a^2b\,x^9+72072abc\,x^9+12012x^9b^3+40040a^3x^8+80080a\,b^2x\right.}{}$
default	$\frac{a^3x^{13}}{13}+\frac{a^2b\,x^{12}}{4}+\frac{\left(a\left(2ac+b^2\right)+2b^2a+a^2c\right)x^{11}}{11}+\frac{\left(a(2ab+2bc)+b\left(2ac+b^2\right)+2abc+b\,a^2\right)x^{10}}{10}+\frac{\left(a\left(2a^2+2b^2+c^2\right)+b(2\right.}{}$

input
```
int((a*x^4+b*x^3+c*x^2+b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/13*a^3*x^13+1/4*a^2*b*x^12+(3/11*a^2*c+3/11*b^2*a)*x^11+(3/10*b*a^2+3/5*
a*b*c+1/10*b^3)*x^10+(1/3*a^3+2/3*b^2*a+1/3*a*c^2+1/3*c*b^2)*x^9+(3/4*b*a^
2+3/4*a*b*c+3/8*b^3+3/8*b*c^2)*x^8+(6/7*a^2*c+6/7*b^2*a+6/7*c*b^2+1/7*c^3)
*x^7+(b*a^2+a*b*c+1/2*b^3+1/2*b*c^2)*x^6+(3/5*a^3+6/5*b^2*a+3/5*a*c^2+3/5*
c*b^2)*x^5+(3/4*b*a^2+3/2*a*b*c+1/4*b^3)*x^4+(a^2*c+a*b^2)*x^3+3/2*b*a^2*x
^2+a^3*x
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.96

$$\int \left(a+bx+cx^2+bx^3+ax^4\right)^3\,dx=\frac{1}{13}\,a^3x^{13}+\frac{1}{4}\,a^2bx^{12}+\frac{3}{11}\,\left(ab^2+a^2c\right)x^{11}+\frac{1}{10}\,\left(3\,a^2b+b^3+6\,abc\right)x^{10}+\frac{1}{3}\,\left(a^3+2\,ab^2+b^2c+ac^2\right)x^9+\frac{3}{8}\,\left(2\,a^2b+b^3+2\,abc+bc^2\right)x^8+\frac{1}{7}\,\left(6\,ab^2+c^3+6\,\left(a^2+b^2\right)c\right)x^7+\frac{1}{2}\,\left(2\,a^2b+b^3+2\,abc+bc^2\right)x^6+\frac{3}{5}\,\left(a^3+2\,ab^2+b^2c+ac^2\right)x^5+\frac{3}{2}\,a^2bx^2+\frac{1}{4}\,\left(3\,a^2b+b^3+6\,abc\right)x^4+a^3x+\left(ab^2+a^2c\right)x^3$$

input
```
integrate((a*x^4+b*x^3+c*x^2+b*x+a)^3,x, algorithm="fricas")
```

output
```
1/13*a^3*x^13 + 1/4*a^2*b*x^12 + 3/11*(a*b^2 + a^2*c)*x^11 + 1/10*(3*a^2*b
 + b^3 + 6*a*b*c)*x^10 + 1/3*(a^3 + 2*a*b^2 + b^2*c + a*c^2)*x^9 + 3/8*(2*
a^2*b + b^3 + 2*a*b*c + b*c^2)*x^8 + 1/7*(6*a*b^2 + c^3 + 6*(a^2 + b^2)*c)
*x^7 + 1/2*(2*a^2*b + b^3 + 2*a*b*c + b*c^2)*x^6 + 3/5*(a^3 + 2*a*b^2 + b^
2*c + a*c^2)*x^5 + 3/2*a^2*b*x^2 + 1/4*(3*a^2*b + b^3 + 6*a*b*c)*x^4 + a^3
*x + (a*b^2 + a^2*c)*x^3
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.18

$$\int \left(a+bx+cx^2+bx^3+ax^4\right)^3\,dx=\frac{a^3x^{13}}{13}+a^3x+\frac{a^2bx^{12}}{4}+\frac{3a^2bx^2}{2}+x^{11}\cdot\left(\frac{3a^2c}{11}+\frac{3ab^2}{11}\right)+x^{10}\cdot\left(\frac{3a^2b}{10}+\frac{3abc}{5}+\frac{b^3}{10}\right)+x^9\left(\frac{a^3}{3}+\frac{2ab^2}{3}+\frac{ac^2}{3}+\frac{b^2c}{3}\right)+x^8\cdot\left(\frac{3a^2b}{4}+\frac{3abc}{4}+\frac{3b^3}{8}+\frac{3bc^2}{8}\right)+x^7\cdot\left(\frac{6a^2c}{7}+\frac{6ab^2}{7}+\frac{6b^2c}{7}+\frac{c^3}{7}\right)+x^6\left(a^2b+abc+\frac{b^3}{2}+\frac{bc^2}{2}\right)+x^5\cdot\left(\frac{3a^3}{5}+\frac{6ab^2}{5}+\frac{3ac^2}{5}+\frac{3b^2c}{5}\right)+x^4\cdot\left(\frac{3a^2b}{4}+\frac{3abc}{2}+\frac{b^3}{4}\right)+x^3\left(a^2c+ab^2\right)$$

input
```
integrate((a*x**4+b*x**3+c*x**2+b*x+a)**3,x)
```

output
```
a**3*x**13/13 + a**3*x + a**2*b*x**12/4 + 3*a**2*b*x**2/2 + x**11*(3*a**2*
c/11 + 3*a*b**2/11) + x**10*(3*a**2*b/10 + 3*a*b*c/5 + b**3/10) + x**9*(a*
*3/3 + 2*a*b**2/3 + a*c**2/3 + b**2*c/3) + x**8*(3*a**2*b/4 + 3*a*b*c/4 +
3*b**3/8 + 3*b*c**2/8) + x**7*(6*a**2*c/7 + 6*a*b**2/7 + 6*b**2*c/7 + c**3
/7) + x**6*(a**2*b + a*b*c + b**3/2 + b*c**2/2) + x**5*(3*a**3/5 + 6*a*b**
2/5 + 3*a*c**2/5 + 3*b**2*c/5) + x**4*(3*a**2*b/4 + 3*a*b*c/2 + b**3/4) +
x**3*(a**2*c + a*b**2)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.19

$$\int \left(a+bx+cx^2+bx^3+ax^4\right)^3 \,dx = \frac{1}{13}\,a^3x^{13}+\frac{1}{4}\,a^2bx^{12}+\frac{3}{11}\,ab^2x^{11}+\frac{1}{10}\,b^3x^{10} + \frac{1}{7}\,c^3x^7+\frac{1}{4}\,b^3x^4+a^3x+\frac{1}{20}\left(12\,ax^5+15\,bx^4+20\,cx^3+30\,bx^2\right)a^2 + \frac{1}{70}\left(30\,ax^7+35\,bx^6+42\,cx^5\right)b^2+\frac{1}{24}\left(8\,ax^9+9\,bx^8\right)c^2 + \frac{1}{420}\left(140\,a^2x^9+315\,abx^8+180\,b^2x^7+252\,c^2x^5+420\,b^2x^3+42\left(10\,ax^6+12\,bx^5+15\,cx^4\right)b+60\left(6\,a\right.\right. + \frac{1}{840}\left(252\,a^2x^{10}+560\,abx^9+720\,bcx^7+315\left(b^2+2\,ac\right)x^8+420\,c^2x^6\right)b + \frac{1}{165}\left(45\,a^2x^{11}+99\,abx^{10}+55\,b^2x^9\right)c$$

input
```
integrate((a*x^4+b*x^3+c*x^2+b*x+a)^3,x, algorithm="maxima")
```

output
```
1/13*a^3*x^13 + 1/4*a^2*b*x^12 + 3/11*a*b^2*x^11 + 1/10*b^3*x^10 + 1/7*c^3
*x^7 + 1/4*b^3*x^4 + a^3*x + 1/20*(12*a*x^5 + 15*b*x^4 + 20*c*x^3 + 30*b*x
^2)*a^2 + 1/70*(30*a*x^7 + 35*b*x^6 + 42*c*x^5)*b^2 + 1/24*(8*a*x^9 + 9*b*
x^8)*c^2 + 1/420*(140*a^2*x^9 + 315*a*b*x^8 + 180*b^2*x^7 + 252*c^2*x^5 +
420*b^2*x^3 + 42*(10*a*x^6 + 12*b*x^5 + 15*c*x^4)*b + 60*(6*a*x^7 + 7*b*x^
6)*c)*a + 1/840*(252*a^2*x^10 + 560*a*b*x^9 + 720*b*c*x^7 + 315*(b^2 + 2*a
*c)*x^8 + 420*c^2*x^6)*b + 1/165*(45*a^2*x^11 + 99*a*b*x^10 + 55*b^2*x^9)*
c
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.21

$$\int \left(a+bx+cx^2+bx^3+ax^4\right)^3\,dx=\frac{1}{13}\,a^3x^{13}+\frac{1}{4}\,a^2bx^{12}+\frac{3}{11}\,ab^2x^{11}+\frac{3}{11}\,a^2cx^{11}+\frac{3}{10}\,a^2bx^{10}+\frac{1}{10}\,b^3x^{10}+\frac{3}{5}\,abcx^{10}+\frac{1}{3}\,a^3x^9+\frac{2}{3}\,ab^2x^9+\frac{1}{3}\,b^2cx^9+\frac{1}{3}\,ac^2x^9+\frac{3}{4}\,a^2bx^8+\frac{3}{8}\,b^3x^8+\frac{3}{4}\,abcx^8+\frac{3}{8}\,bc^2x^8+\frac{6}{7}\,ab^2x^7+\frac{6}{7}\,a^2cx^7+\frac{6}{7}\,b^2cx^7+\frac{1}{7}\,c^3x^7+a^2bx^6+\frac{1}{2}\,b^3x^6+abcx^6+\frac{1}{2}\,bc^2x^6+\frac{3}{5}\,a^3x^5+\frac{6}{5}\,ab^2x^5+\frac{3}{5}\,b^2cx^5+\frac{3}{5}\,ac^2x^5+\frac{3}{4}\,a^2bx^4+\frac{1}{4}\,b^3x^4+\frac{3}{2}\,abcx^4+ab^2x^3+a^2cx^3+\frac{3}{2}\,a^2bx^2+a^3x$$

input
```
integrate((a*x^4+b*x^3+c*x^2+b*x+a)^3,x, algorithm="giac")
```

output
```
1/13*a^3*x^13 + 1/4*a^2*b*x^12 + 3/11*a*b^2*x^11 + 3/11*a^2*c*x^11 + 3/10*
a^2*b*x^10 + 1/10*b^3*x^10 + 3/5*a*b*c*x^10 + 1/3*a^3*x^9 + 2/3*a*b^2*x^9
+ 1/3*b^2*c*x^9 + 1/3*a*c^2*x^9 + 3/4*a^2*b*x^8 + 3/8*b^3*x^8 + 3/4*a*b*c*
x^8 + 3/8*b*c^2*x^8 + 6/7*a*b^2*x^7 + 6/7*a^2*c*x^7 + 6/7*b^2*c*x^7 + 1/7*
c^3*x^7 + a^2*b*x^6 + 1/2*b^3*x^6 + a*b*c*x^6 + 1/2*b*c^2*x^6 + 3/5*a^3*x^
5 + 6/5*a*b^2*x^5 + 3/5*b^2*c*x^5 + 3/5*a*c^2*x^5 + 3/4*a^2*b*x^4 + 1/4*b^
3*x^4 + 3/2*a*b*c*x^4 + a*b^2*x^3 + a^2*c*x^3 + 3/2*a^2*b*x^2 + a^3*x
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.94

$$\int \left(a+bx+cx^2+bx^3+ax^4\right)^3\,dx = x^9\left(\frac{a^3}{3}+\frac{2\,a\,b^2}{3}+\frac{a\,c^2}{3}+\frac{b^2\,c}{3}\right)+x^5\left(\frac{3\,a^3}{5}+\frac{6\,a\,b^2}{5}+\frac{3\,a\,c^2}{5}+\frac{3\,b^2\,c}{5}\right)+x^7\left(\frac{6\,a^2\,c}{7}+\frac{6\,a\,b^2}{7}+\frac{6\,b^2\,c}{7}+\frac{c^3}{7}\right)+a^3\,x+\frac{a^3\,x^{13}}{13}+\frac{3\,a^2\,b\,x^2}{2}+\frac{a^2\,b\,x^{12}}{4}+\frac{b\,x^6\,(2\,a^2+2\,a\,c+b^2+c^2)}{2}+\frac{3\,b\,x^8\,(2\,a^2+2\,a\,c+b^2+c^2)}{8}+a\,x^3\,\left(b^2+a\,c\right)+\frac{3\,a\,x^{11}\,(b^2+a\,c)}{11}+\frac{b\,x^4\,(3\,a^2+6\,c\,a+b^2)}{4}+\frac{b\,x^{10}\,(3\,a^2+6\,c\,a+b^2)}{10}$$

input
```
int((a + b*x + a*x^4 + b*x^3 + c*x^2)^3,x)
```

output
```
x^9*((2*a*b^2)/3 + (a*c^2)/3 + (b^2*c)/3 + a^3/3) + x^5*((6*a*b^2)/5 + (3*
a*c^2)/5 + (3*b^2*c)/5 + (3*a^3)/5) + x^7*((6*a*b^2)/7 + (6*a^2*c)/7 + (6*
b^2*c)/7 + c^3/7) + a^3*x + (a^3*x^13)/13 + (3*a^2*b*x^2)/2 + (a^2*b*x^12)
/4 + (b*x^6*(2*a*c + 2*a^2 + b^2 + c^2))/2 + (3*b*x^8*(2*a*c + 2*a^2 + b^2
 + c^2))/8 + a*x^3*(a*c + b^2) + (3*a*x^11*(a*c + b^2))/11 + (b*x^4*(6*a*c
 + 3*a^2 + b^2))/4 + (b*x^10*(6*a*c + 3*a^2 + b^2))/10
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.23

$$\int \left(a+bx+cx^2+bx^3+ax^4\right)^3\,dx = \frac{x(9240a^3x^{12}+30030a^2b\,x^{11}+32760a^2c\,x^{10}+32760a\,b^2x^{10}+36036a^2b\,x^9+72072abc\,x^9+12012b^3x^9+$$

input
```
int((a*x^4+b*x^3+c*x^2+b*x+a)^3,x)
```

output

```
(x*(9240*a**3*x**12 + 40040*a**3*x**8 + 72072*a**3*x**4 + 120120*a**3 + 30
030*a**2*b*x**11 + 36036*a**2*b*x**9 + 90090*a**2*b*x**7 + 120120*a**2*b*x
**5 + 90090*a**2*b*x**3 + 180180*a**2*b*x + 32760*a**2*c*x**10 + 102960*a*
*2*c*x**6 + 120120*a**2*c*x**2 + 32760*a*b**2*x**10 + 80080*a*b**2*x**8 +
102960*a*b**2*x**6 + 144144*a*b**2*x**4 + 120120*a*b**2*x**2 + 72072*a*b*c
*x**9 + 90090*a*b*c*x**7 + 120120*a*b*c*x**5 + 180180*a*b*c*x**3 + 40040*a
*c**2*x**8 + 72072*a*c**2*x**4 + 12012*b**3*x**9 + 45045*b**3*x**7 + 60060
*b**3*x**5 + 30030*b**3*x**3 + 40040*b**2*c*x**8 + 102960*b**2*c*x**6 + 72
072*b**2*c*x**4 + 45045*b*c**2*x**7 + 60060*b*c**2*x**5 + 17160*c**3*x**6)
)/120120
```

3.22 $\int (a+bx+cx^2+bx^3+ax^4)^2 \, dx$

Optimal result

Integrand size = 22, antiderivative size = 104

$$\int (a+bx+cx^2+bx^3+ax^4)^2 \, dx = a^2x+abx^2+\frac{1}{3}(b^2+2ac)\,x^3+\frac{1}{2}b(a+c)x^4 + \frac{1}{5}(2a^2+2b^2+c^2)\,x^5+\frac{1}{3}b(a+c)x^6 + \frac{1}{7}(b^2+2ac)\,x^7+\frac{1}{4}abx^8+\frac{a^2x^9}{9}$$

output
```
a^2*x+a*b*x^2+1/3*(2*a*c+b^2)*x^3+1/2*b*(a+c)*x^4+1/5*(2*a^2+2*b^2+c^2)*x^
5+1/3*b*(a+c)*x^6+1/7*(2*a*c+b^2)*x^7+1/4*a*b*x^8+1/9*a^2*x^9
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00

$$\int (a+bx+cx^2+bx^3+ax^4)^2 \, dx = a^2x+abx^2+\frac{1}{3}(b^2+2ac)\,x^3+\frac{1}{2}b(a+c)x^4 + \frac{1}{5}(2a^2+2b^2+c^2)\,x^5+\frac{1}{3}b(a+c)x^6 + \frac{1}{7}(b^2+2ac)\,x^7+\frac{1}{4}abx^8+\frac{a^2x^9}{9}$$

input
```
Integrate[(a + b*x + c*x^2 + b*x^3 + a*x^4)^2,x]
```

output
```
a^2*x + a*b*x^2 + ((b^2 + 2*a*c)*x^3)/3 + (b*(a + c)*x^4)/2 + ((2*a^2 + 2*
b^2 + c^2)*x^5)/5 + (b*(a + c)*x^6)/3 + ((b^2 + 2*a*c)*x^7)/7 + (a*b*x^8)/
4 + (a^2*x^9)/9
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {2465, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(ax^4+a+bx^3+bx+cx^2\right)^2 \, dx$$

$$\Big\downarrow \text{2465}$$

$$\int \left(x^4\left(2a^2+2b^2+c^2\right)+a^2x^8+a^2+x^6\left(2ac+b^2\right)+x^2\left(2ac+b^2\right)+2bx^5(a+c)+2bx^3(a+c)+2abx^7+2abx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{5}x^5\left(2a^2+2b^2+c^2\right)+\frac{a^2x^9}{9}+a^2x+\frac{1}{7}x^7\left(2ac+b^2\right)+\frac{1}{3}x^3\left(2ac+b^2\right)+\frac{1}{3}bx^6(a+c)+\frac{1}{2}bx^4(a+c)+\frac{1}{4}abx^8+abx^2$$

input
```
Int[(a + b*x + c*x^2 + b*x^3 + a*x^4)^2,x]
```

output
```
a^2*x + a*b*x^2 + ((b^2 + 2*a*c)*x^3)/3 + (b*(a + c)*x^4)/2 + ((2*a^2 + 2*
b^2 + c^2)*x^5)/5 + (b*(a + c)*x^6)/3 + ((b^2 + 2*a*c)*x^7)/7 + (a*b*x^8)/
4 + (a^2*x^9)/9
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2465
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> Int[ExpandToSum[u, Px^p, x], x] /; PolyQ
[Px, x] && GtQ[Expon[Px, x], 2] && !BinomialQ[Px, x] && !TrinomialQ[Px, x
] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.97

method	result
default	$\frac{a^2x^9}{9}+\frac{ab\,x^8}{4}+\frac{(2ac+b^2)x^7}{7}+\frac{(2ab+2bc)x^6}{6}+\frac{(2a^2+2b^2+c^2)x^5}{5}+\frac{(2ab+2bc)x^4}{4}+\frac{(2ac+b^2)x^3}{3}+ab\,x^2+x$
norman	$\frac{a^2x^9}{9}+\frac{ab\,x^8}{4}+\left(\frac{2ac}{7}+\frac{b^2}{7}\right)x^7+\left(\frac{1}{3}ab+\frac{1}{3}bc\right)x^6+\left(\frac{2a^2}{5}+\frac{2b^2}{5}+\frac{c^2}{5}\right)x^5+\left(\frac{1}{2}ab+\frac{1}{2}bc\right)x^4+$
gosper	$\frac{1}{9}a^2x^9+\frac{1}{4}ab\,x^8+\frac{2}{7}x^7ac+\frac{1}{7}b^2x^7+\frac{1}{3}ab\,x^6+\frac{1}{3}bc\,x^6+\frac{2}{5}a^2x^5+\frac{2}{5}b^2x^5+\frac{1}{5}c^2x^5+\frac{1}{2}ab\,x^4+$
risch	$\frac{1}{9}a^2x^9+\frac{1}{4}ab\,x^8+\frac{2}{7}x^7ac+\frac{1}{7}b^2x^7+\frac{1}{3}ab\,x^6+\frac{1}{3}bc\,x^6+\frac{2}{5}a^2x^5+\frac{2}{5}b^2x^5+\frac{1}{5}c^2x^5+\frac{1}{2}ab\,x^4+$
parallelrisch	$\frac{1}{9}a^2x^9+\frac{1}{4}ab\,x^8+\frac{2}{7}x^7ac+\frac{1}{7}b^2x^7+\frac{1}{3}ab\,x^6+\frac{1}{3}bc\,x^6+\frac{2}{5}a^2x^5+\frac{2}{5}b^2x^5+\frac{1}{5}c^2x^5+\frac{1}{2}ab\,x^4+$
orering	$\frac{x(140a^2x^8+315ab\,x^7+360ac\,x^6+180b^2x^6+420ab\,x^5+420bc\,x^5+504a^2x^4+504b^2x^4+252c^2x^4+630a\,x^3b+630bc\,x^3+840ac\,x^2}{1260}$

input
```
int((a*x^4+b*x^3+c*x^2+b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/9*a^2*x^9+1/4*a*b*x^8+1/7*(2*a*c+b^2)*x^7+1/6*(2*a*b+2*b*c)*x^6+1/5*(2*a
^2+2*b^2+c^2)*x^5+1/4*(2*a*b+2*b*c)*x^4+1/3*(2*a*c+b^2)*x^3+a*b*x^2+x*a^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.92

$$\int (a+bx+cx^2+bx^3+ax^4)^2\,dx = \frac{1}{9}\,a^2x^9+\frac{1}{4}\,abx^8+\frac{1}{7}\,(b^2+2\,ac)x^7 + \frac{1}{3}\,(ab+bc)x^6+\frac{1}{5}\,(2\,a^2+2\,b^2+c^2)x^5 + \frac{1}{2}\,(ab+bc)x^4+abx^2+\frac{1}{3}\,(b^2+2\,ac)x^3+a^2x$$

input
```
integrate((a*x^4+b*x^3+c*x^2+b*x+a)^2,x, algorithm="fricas")
```

output
```
1/9*a^2*x^9 + 1/4*a*b*x^8 + 1/7*(b^2 + 2*a*c)*x^7 + 1/3*(a*b + b*c)*x^6 +
1/5*(2*a^2 + 2*b^2 + c^2)*x^5 + 1/2*(a*b + b*c)*x^4 + a*b*x^2 + 1/3*(b^2 +
 2*a*c)*x^3 + a^2*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.03

$$\int (a+bx+cx^2+bx^3+ax^4)^2\,dx = \frac{a^2x^9}{9}+a^2x+\frac{abx^8}{4}+abx^2+x^7\cdot\left(\frac{2ac}{7}+\frac{b^2}{7}\right) + x^6\left(\frac{ab}{3}+\frac{bc}{3}\right)+x^5\cdot\left(\frac{2a^2}{5}+\frac{2b^2}{5}+\frac{c^2}{5}\right) + x^4\left(\frac{ab}{2}+\frac{bc}{2}\right)+x^3\cdot\left(\frac{2ac}{3}+\frac{b^2}{3}\right)$$

input
```
integrate((a*x**4+b*x**3+c*x**2+b*x+a)**2,x)
```

output
```
a**2*x**9/9 + a**2*x + a*b*x**8/4 + a*b*x**2 + x**7*(2*a*c/7 + b**2/7) + x
**6*(a*b/3 + b*c/3) + x**5*(2*a**2/5 + 2*b**2/5 + c**2/5) + x**4*(a*b/2 +
b*c/2) + x**3*(2*a*c/3 + b**2/3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.07

$$\int \left(a+bx+cx^2+bx^3+ax^4\right)^2 \,dx = \frac{1}{9}\,a^2x^9+\frac{1}{4}\,abx^8+\frac{1}{7}\,b^2x^7+\frac{1}{5}\,c^2x^5+\frac{1}{3}\,b^2x^3 + a^2x+\frac{1}{30}\,\left(12\,ax^5+15\,bx^4+20\,cx^3+30\,bx^2\right)a + \frac{1}{30}\,\left(10\,ax^6+12\,bx^5+15\,cx^4\right)b + \frac{1}{21}\,\left(6\,ax^7+7\,bx^6\right)c$$

input
```
integrate((a*x^4+b*x^3+c*x^2+b*x+a)^2,x, algorithm="maxima")
```

output
```
1/9*a^2*x^9 + 1/4*a*b*x^8 + 1/7*b^2*x^7 + 1/5*c^2*x^5 + 1/3*b^2*x^3 + a^2*
x + 1/30*(12*a*x^5 + 15*b*x^4 + 20*c*x^3 + 30*b*x^2)*a + 1/30*(10*a*x^6 +
12*b*x^5 + 15*c*x^4)*b + 1/21*(6*a*x^7 + 7*b*x^6)*c
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.05

$$\int \left(a+bx+cx^2+bx^3+ax^4\right)^2 \,dx = \frac{1}{9}\,a^2x^9+\frac{1}{4}\,abx^8+\frac{1}{7}\,b^2x^7+\frac{2}{7}\,acx^7+\frac{1}{3}\,abx^6 + \frac{1}{3}\,bcx^6+\frac{2}{5}\,a^2x^5+\frac{2}{5}\,b^2x^5+\frac{1}{5}\,c^2x^5+\frac{1}{2}\,abx^4 + \frac{1}{2}\,bcx^4+\frac{1}{3}\,b^2x^3+\frac{2}{3}\,acx^3+abx^2+a^2x$$

input
```
integrate((a*x^4+b*x^3+c*x^2+b*x+a)^2,x, algorithm="giac")
```

output
```
1/9*a^2*x^9 + 1/4*a*b*x^8 + 1/7*b^2*x^7 + 2/7*a*c*x^7 + 1/3*a*b*x^6 + 1/3*
b*c*x^6 + 2/5*a^2*x^5 + 2/5*b^2*x^5 + 1/5*c^2*x^5 + 1/2*a*b*x^4 + 1/2*b*c*
x^4 + 1/3*b^2*x^3 + 2/3*a*c*x^3 + a*b*x^2 + a^2*x
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.89

$$\int \left(a+bx+cx^2+bx^3+ax^4\right)^2\,dx = a^2\,x+x^3\,\left(\frac{b^2}{3}+\frac{2\,a\,c}{3}\right)+x^7\,\left(\frac{b^2}{7}+\frac{2\,a\,c}{7}\right)+\frac{a^2\,x^9}{9}+x^5\,\left(\frac{2\,a^2}{5}+\frac{2\,b^2}{5}+\frac{c^2}{5}\right)+\frac{b\,x^4\,(a+c)}{2}+\frac{b\,x^6\,(a+c)}{3}+a\,b\,x^2+\frac{a\,b\,x^8}{4}$$

input
```
int((a + b*x + a*x^4 + b*x^3 + c*x^2)^2,x)
```

output
```
a^2*x + x^3*((2*a*c)/3 + b^2/3) + x^7*((2*a*c)/7 + b^2/7) + (a^2*x^9)/9 +
x^5*((2*a^2)/5 + (2*b^2)/5 + c^2/5) + (b*x^4*(a + c))/2 + (b*x^6*(a + c))/
3 + a*b*x^2 + (a*b*x^8)/4
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.07

$$\int \left(a+bx+cx^2+bx^3+ax^4\right)^2\,dx$$
$$=\frac{x(140a^2x^8+315ab\,x^7+360ac\,x^6+180b^2x^6+420ab\,x^5+420bc\,x^5+504a^2x^4+504b^2x^4+252c^2x^4+63}{1260}$$

input
```
int((a*x^4+b*x^3+c*x^2+b*x+a)^2,x)
```

output
```
(x*(140*a**2*x**8 + 504*a**2*x**4 + 1260*a**2 + 315*a*b*x**7 + 420*a*b*x**
5 + 630*a*b*x**3 + 1260*a*b*x + 360*a*c*x**6 + 840*a*c*x**2 + 180*b**2*x**
6 + 504*b**2*x**4 + 420*b**2*x**2 + 420*b*c*x**5 + 630*b*c*x**3 + 252*c**2
*x**4))/1260
```

3.23 $\int (a+bx+cx^2+bx^3+ax^4)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 36

$$\int (a+bx+cx^2+bx^3+ax^4)\,dx = ax+\frac{bx^2}{2}+\frac{cx^3}{3}+\frac{bx^4}{4}+\frac{ax^5}{5}$$

output
```
a*x+1/2*b*x^2+1/3*c*x^3+1/4*b*x^4+1/5*a*x^5
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int (a+bx+cx^2+bx^3+ax^4)\,dx = ax+\frac{bx^2}{2}+\frac{cx^3}{3}+\frac{bx^4}{4}+\frac{ax^5}{5}$$

input
```
Integrate[a + b*x + c*x^2 + b*x^3 + a*x^4,x]
```

output
```
a*x + (b*x^2)/2 + (c*x^3)/3 + (b*x^4)/4 + (a*x^5)/5
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(ax^4+a+bx^3+bx+cx^2\right)\,dx$$

$$\Big\downarrow \; 2009$$

$$\frac{ax^5}{5}+ax+\frac{bx^4}{4}+\frac{bx^2}{2}+\frac{cx^3}{3}$$

input
```
Int[a + b*x + c*x^2 + b*x^3 + a*x^4,x]
```

output
```
a*x + (b*x^2)/2 + (c*x^3)/3 + (b*x^4)/4 + (a*x^5)/5
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.81

method	result	size
gosper	$xa+\frac{1}{2}b\,x^{2}+\frac{1}{3}c\,x^{3}+\frac{1}{4}b\,x^{4}+\frac{1}{5}a\,x^{5}$	29
default	$xa+\frac{1}{2}b\,x^{2}+\frac{1}{3}c\,x^{3}+\frac{1}{4}b\,x^{4}+\frac{1}{5}a\,x^{5}$	29
norman	$xa+\frac{1}{2}b\,x^{2}+\frac{1}{3}c\,x^{3}+\frac{1}{4}b\,x^{4}+\frac{1}{5}a\,x^{5}$	29
risch	$xa+\frac{1}{2}b\,x^{2}+\frac{1}{3}c\,x^{3}+\frac{1}{4}b\,x^{4}+\frac{1}{5}a\,x^{5}$	29
parallelrisch	$xa+\frac{1}{2}b\,x^{2}+\frac{1}{3}c\,x^{3}+\frac{1}{4}b\,x^{4}+\frac{1}{5}a\,x^{5}$	29
parts	$xa+\frac{1}{2}b\,x^{2}+\frac{1}{3}c\,x^{3}+\frac{1}{4}b\,x^{4}+\frac{1}{5}a\,x^{5}$	29
orering	$\frac{x\left(12a\,x^{4}+15b\,x^{3}+20c\,x^{2}+30bx+60a\right)}{60}$	30

input
```
int(a*x^4+b*x^3+c*x^2+b*x+a,x,method=_RETURNVERBOSE)
```

output
```
x*a+1/2*b*x^2+1/3*c*x^3+1/4*b*x^4+1/5*a*x^5
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.78

$$\int \left(a+bx+cx^2+bx^3+ax^4\right)\,dx=\frac{1}{5}\,ax^{5}+\frac{1}{4}\,bx^{4}+\frac{1}{3}\,cx^{3}+\frac{1}{2}\,bx^{2}+ax$$

input
```
integrate(a*x^4+b*x^3+c*x^2+b*x+a,x, algorithm="fricas")
```

output
```
1/5*a*x^5 + 1/4*b*x^4 + 1/3*c*x^3 + 1/2*b*x^2 + a*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.81

$$\int \left(a + bx + cx^2 + bx^3 + ax^4\right) \, dx = \frac{ax^5}{5} + ax + \frac{bx^4}{4} + \frac{bx^2}{2} + \frac{cx^3}{3}$$

input
```
integrate(a*x**4+b*x**3+c*x**2+b*x+a,x)
```

output
```
a*x**5/5 + a*x + b*x**4/4 + b*x**2/2 + c*x**3/3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.78

$$\int \left(a + bx + cx^2 + bx^3 + ax^4\right) \, dx = \frac{1}{5} \, ax^5 + \frac{1}{4} \, bx^4 + \frac{1}{3} \, cx^3 + \frac{1}{2} \, bx^2 + ax$$

input
```
integrate(a*x^4+b*x^3+c*x^2+b*x+a,x, algorithm="maxima")
```

output
```
1/5*a*x^5 + 1/4*b*x^4 + 1/3*c*x^3 + 1/2*b*x^2 + a*x
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.78

$$\int \left(a + bx + cx^2 + bx^3 + ax^4\right) \, dx = \frac{1}{5} \, ax^5 + \frac{1}{4} \, bx^4 + \frac{1}{3} \, cx^3 + \frac{1}{2} \, bx^2 + ax$$

input
```
integrate(a*x^4+b*x^3+c*x^2+b*x+a,x, algorithm="giac")
```

output
```
1/5*a*x^5 + 1/4*b*x^4 + 1/3*c*x^3 + 1/2*b*x^2 + a*x
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.78

$$\int \left(a+bx+cx^2+bx^3+ax^4\right)\,dx=\frac{a\,x^5}{5}+\frac{b\,x^4}{4}+\frac{c\,x^3}{3}+\frac{b\,x^2}{2}+a\,x$$

input
```
int(a + b*x + a*x^4 + b*x^3 + c*x^2,x)
```

output
```
a*x + (a*x^5)/5 + (b*x^2)/2 + (b*x^4)/4 + (c*x^3)/3
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.81

$$\int \left(a+bx+cx^2+bx^3+ax^4\right)\,dx=\frac{x(12a\,x^4+15b\,x^3+20c\,x^2+30bx+60a)}{60}$$

input
```
int(a*x^4+b*x^3+c*x^2+b*x+a,x)
```

output
```
(x*(12*a*x**4 + 60*a + 15*b*x**3 + 30*b*x + 20*c*x**2))/60
```

3.24 $\int \frac{1}{a+bx+cx^2+bx^3+ax^4}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 407

$$\begin{aligned}\int \frac{1}{a+bx+cx^2+bx^3+ax^4}\,dx &= -\frac{\left(b-\sqrt{8a^2+b^2-4ac}\right)\arctan\left(\frac{b-\sqrt{8a^2+b^2-4ac}+4ax}{\sqrt{2}\sqrt{4a^2+2ac-b\left(b-\sqrt{8a^2+b^2-4ac}\right)}}\right)}{\sqrt{2}\sqrt{8a^2+b^2-4ac}\sqrt{4a^2+2ac-b\left(b-\sqrt{8a^2+b^2-4ac}\right)}} \\ &\quad+\frac{\left(b+\sqrt{8a^2+b^2-4ac}\right)\arctan\left(\frac{b+\sqrt{8a^2+b^2-4ac}+4ax}{\sqrt{2}\sqrt{4a^2+2ac-b\left(b+\sqrt{8a^2+b^2-4ac}\right)}}\right)}{\sqrt{2}\sqrt{8a^2+b^2-4ac}\sqrt{4a^2+2ac-b\left(b+\sqrt{8a^2+b^2-4ac}\right)}} \\ &\quad-\frac{\log\left(2a+bx-\sqrt{8a^2+b^2-4ac}x+2ax^2\right)}{2\sqrt{8a^2+b^2-4ac}} \\ &\quad+\frac{\log\left(2a+bx+\sqrt{8a^2+b^2-4ac}x+2ax^2\right)}{2\sqrt{8a^2+b^2-4ac}}\end{aligned}$$

output
```
-1/2*(b-(8*a^2-4*a*c+b^2)^(1/2))*arctan(1/2*(b-(8*a^2-4*a*c+b^2)^(1/2)+4*a
*x)*2^(1/2)/(4*a^2+2*a*c-b*(b-(8*a^2-4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)/(8*
a^2-4*a*c+b^2)^(1/2)/(4*a^2+2*a*c-b*(b-(8*a^2-4*a*c+b^2)^(1/2)))^(1/2)+1/2
*(b+(8*a^2-4*a*c+b^2)^(1/2))*arctan(1/2*(b+(8*a^2-4*a*c+b^2)^(1/2)+4*a*x)*
2^(1/2)/(4*a^2+2*a*c-b*(b+(8*a^2-4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)/(8*a^2-
4*a*c+b^2)^(1/2)/(4*a^2+2*a*c-b*(b+(8*a^2-4*a*c+b^2)^(1/2)))^(1/2)-1/2*ln(
2*a+b*x-(8*a^2-4*a*c+b^2)^(1/2)*x+2*a*x^2)/(8*a^2-4*a*c+b^2)^(1/2)+1/2*ln(
2*a+b*x+(8*a^2-4*a*c+b^2)^(1/2)*x+2*a*x^2)/(8*a^2-4*a*c+b^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.14

$$\int \frac{1}{a+bx+cx^2+bx^3+ax^4}\,dx = \text{RootSum}\left[a+b\#1+c\#1^2+b\#1^3+a\#1^4\&,\frac{\log(x-\#1)}{b+2c\#1+3b\#1^2+4a\#1^3}\&\right]$$

input
```
Integrate[(a + b*x + c*x^2 + b*x^3 + a*x^4)^(-1),x]
```

output
```
RootSum[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , Log[x - #1]/(b + 2*c*#1 +
3*b*#1^2 + 4*a*#1^3) & ]
```

Rubi [A] (verified)

Time = 1.23 (sec) , antiderivative size = 414, normalized size of antiderivative = 1.02, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {2492, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{ax^4+a+bx^3+bx+cx^2}\,dx$$

$$\Big\downarrow \text{2492}$$

$$\frac{\int \left(\frac{a\left(b+2ax+\sqrt{8a^2-4ca+b^2}\right)}{\sqrt{8a^2-4ca+b^2}\left(2ax^2+\left(b+\sqrt{8a^2-4ca+b^2}\right)x+2a\right)}-\frac{a\left(b+2ax-\sqrt{8a^2-4ca+b^2}\right)}{\sqrt{8a^2-4ca+b^2}\left(2ax^2+\left(b-\sqrt{8a^2-4ca+b^2}\right)x+2a\right)}\right)dx}{a}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{a\left(b-\sqrt{8a^2-4ac+b^2}\right)\arctan\left(\frac{-\sqrt{8a^2-4ac+b^2}+4ax+b}{\sqrt{2}\sqrt{-b\left(b-\sqrt{8a^2-4ac+b^2}\right)+4a^2+2ac}}\right)}{\sqrt{2}\sqrt{8a^2-4ac+b^2}\sqrt{-b\left(b-\sqrt{8a^2-4ac+b^2}\right)+4a^2+2ac}}+\frac{a\left(\sqrt{8a^2-4ac+b^2}+b\right)\arctan\left(\frac{\sqrt{8a^2-4ac+b^2}+4ax+b}{\sqrt{2}\sqrt{-b\left(\sqrt{8a^2-4ac+b^2}+b\right)+4a^2+2ac}}\right)}{\sqrt{2}\sqrt{8a^2-4ac+b^2}\sqrt{-b\left(\sqrt{8a^2-4ac+b^2}+b\right)+4a^2+2ac}}}{a}$$

input
```
Int[(a + b*x + c*x^2 + b*x^3 + a*x^4)^(-1),x]
```

output
```
(-((a*(b - Sqrt[8*a^2 + b^2 - 4*a*c])*ArcTan[(b - Sqrt[8*a^2 + b^2 - 4*a*c
] + 4*a*x)/(Sqrt[2]*Sqrt[4*a^2 + 2*a*c - b*(b - Sqrt[8*a^2 + b^2 - 4*a*c])
])])/(Sqrt[2]*Sqrt[8*a^2 + b^2 - 4*a*c]*Sqrt[4*a^2 + 2*a*c - b*(b - Sqrt[8
*a^2 + b^2 - 4*a*c])])) + (a*(b + Sqrt[8*a^2 + b^2 - 4*a*c])*ArcTan[(b + S
qrt[8*a^2 + b^2 - 4*a*c] + 4*a*x)/(Sqrt[2]*Sqrt[4*a^2 + 2*a*c - b*(b + Sqr
t[8*a^2 + b^2 - 4*a*c])])])/(Sqrt[2]*Sqrt[8*a^2 + b^2 - 4*a*c]*Sqrt[4*a^2
+ 2*a*c - b*(b + Sqrt[8*a^2 + b^2 - 4*a*c])]) - (a*Log[2*a + (b - Sqrt[8*a
^2 + b^2 - 4*a*c])*x + 2*a*x^2])/(2*Sqrt[8*a^2 + b^2 - 4*a*c]) + (a*Log[2*
a + (b + Sqrt[8*a^2 + b^2 - 4*a*c])*x + 2*a*x^2])/(2*Sqrt[8*a^2 + b^2 - 4*
a*c]))/a
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2492
```
Int[(Px_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^3 + (e_.)*(x_)^4)
^(p_), x_Symbol] :> Simp[e^p   Int[ExpandIntegrand[Px*(b/d + ((d + Sqrt[e*(
(b^2 - 4*a*c)/a) + 8*a*d*(e/b)])/(2*e))*x + x^2)^p*(b/d + ((d - Sqrt[e*((b^
2 - 4*a*c)/a) + 8*a*d*(e/b)])/(2*e))*x + x^2)^p, x], x], x] /; FreeQ[{a, b,
 c, d, e}, x] && PolyQ[Px, x] && ILtQ[p, 0] && EqQ[a*d^2 - b^2*e, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.18 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.13

method	result
risch	$\sum_{\_R=\mathrm{RootOf}\left(a\_Z^4+b\_Z^3+\_Z^2c+b\_Z+a\right)}\frac{\ln\left(x-\_R\right)}{4\_R^3a+3\_R^2b+2\_Rc+b}$
default	$4a\left(\frac{\frac{\left(16a^3-8a^2c+2b^2a\right)\ln\left(2a+bx+\sqrt{8a^2-4ac+b^2}\,x+2a\,x^2\right)}{4a}+\frac{2\left(8\sqrt{8a^2-4ac+b^2}\,a^2-4\sqrt{8a^2-4ac+b^2}\,ac+b^2\sqrt{8a^2-4ac+b^2}+8b\,a^2-4abc+b\right.}{\sqrt{8a^2+4a}}}{4\left(8a^2-4ac+b^2\right)^{\frac{3}{2}}a}\right.$

input
```
int(1/(a*x^4+b*x^3+c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

output
```
sum(1/(4*_R^3*a+3*_R^2*b+2*_R*c+b)*ln(x-_R),_R=RootOf(_Z^4*a+_Z^3*b+_Z^2*c
+_Z*b+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 19.09 (sec) , antiderivative size = 3804183, normalized size of antiderivative = 9346.89

$$\int \frac{1}{a+bx+cx^2+bx^3+ax^4}\,dx=\text{Too large to display}$$

input
```
integrate(1/(a*x^4+b*x^3+c*x^2+b*x+a),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{a+bx+cx^2+bx^3+ax^4}\,dx = \text{Timed out}$$

input
```
integrate(1/(a*x**4+b*x**3+c*x**2+b*x+a),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{a+bx+cx^2+bx^3+ax^4}\,dx = \int \frac{1}{ax^4+bx^3+cx^2+bx+a}\,dx$$

input
```
integrate(1/(a*x^4+b*x^3+c*x^2+b*x+a),x, algorithm="maxima")
```

output
```
integrate(1/(a*x^4 + b*x^3 + c*x^2 + b*x + a), x)
```

Giac [F]

$$\int \frac{1}{a+bx+cx^2+bx^3+ax^4}\,dx = \int \frac{1}{ax^4+bx^3+cx^2+bx+a}\,dx$$

input
```
integrate(1/(a*x^4+b*x^3+c*x^2+b*x+a),x, algorithm="giac")
```

output
```
integrate(1/(a*x^4 + b*x^3 + c*x^2 + b*x + a), x)
```

Mupad [B] (verification not implemented)

Time = 23.27 (sec) , antiderivative size = 4180, normalized size of antiderivative = 10.27

$$\int \frac{1}{a+bx+cx^2+bx^3+ax^4}\,dx = \text{Too large to display}$$

input
```
int(1/(a + b*x + a*x^4 + b*x^3 + c*x^2),x)
```

output
```
symsum(log(-root(36*a*b^4*c*z^4 - 80*a^2*b^2*c^2*z^4 + 288*a^3*b^2*c*z^4 -
 8*a*b^2*c^3*z^4 - 128*a^4*c^2*z^4 + 16*a^2*c^4*z^4 - 192*a^4*b^2*z^4 - 60
*a^2*b^4*z^4 - 4*b^6*z^4 + 256*a^6*z^4 + b^4*c^2*z^4 - 14*a*b^2*c*z^2 - b^
2*c^2*z^2 + 24*a^2*b^2*z^2 - 16*a^3*c*z^2 + 4*a*c^3*z^2 + 3*b^4*z^2 - 4*a*
b*c*z + 8*a^2*b*z + b^3*z + a^2, z, k)*a*(a*b + 16*root(36*a*b^4*c*z^4 - 8
0*a^2*b^2*c^2*z^4 + 288*a^3*b^2*c*z^4 - 8*a*b^2*c^3*z^4 - 128*a^4*c^2*z^4
+ 16*a^2*c^4*z^4 - 192*a^4*b^2*z^4 - 60*a^2*b^4*z^4 - 4*b^6*z^4 + 256*a^6*
z^4 + b^4*c^2*z^4 - 14*a*b^2*c*z^2 - b^2*c^2*z^2 + 24*a^2*b^2*z^2 - 16*a^3
*c*z^2 + 4*a*c^3*z^2 + 3*b^4*z^2 - 4*a*b*c*z + 8*a^2*b*z + b^3*z + a^2, z,
 k)*a^3 + 4*a^2*x + 6*root(36*a*b^4*c*z^4 - 80*a^2*b^2*c^2*z^4 + 288*a^3*b
^2*c*z^4 - 8*a*b^2*c^3*z^4 - 128*a^4*c^2*z^4 + 16*a^2*c^4*z^4 - 192*a^4*b^
2*z^4 - 60*a^2*b^4*z^4 - 4*b^6*z^4 + 256*a^6*z^4 + b^4*c^2*z^4 - 14*a*b^2*
c*z^2 - b^2*c^2*z^2 + 24*a^2*b^2*z^2 - 16*a^3*c*z^2 + 4*a*c^3*z^2 + 3*b^4*
z^2 - 4*a*b*c*z + 8*a^2*b*z + b^3*z + a^2, z, k)^2*a*b^3 + 48*root(36*a*b^
4*c*z^4 - 80*a^2*b^2*c^2*z^4 + 288*a^3*b^2*c*z^4 - 8*a*b^2*c^3*z^4 - 128*a
^4*c^2*z^4 + 16*a^2*c^4*z^4 - 192*a^4*b^2*z^4 - 60*a^2*b^4*z^4 - 4*b^6*z^4
 + 256*a^6*z^4 + b^4*c^2*z^4 - 14*a*b^2*c*z^2 - b^2*c^2*z^2 + 24*a^2*b^2*z
^2 - 16*a^3*c*z^2 + 4*a*c^3*z^2 + 3*b^4*z^2 - 4*a*b*c*z + 8*a^2*b*z + b^3*
z + a^2, z, k)^2*a^3*b - root(36*a*b^4*c*z^4 - 80*a^2*b^2*c^2*z^4 + 288*a^
3*b^2*c*z^4 - 8*a*b^2*c^3*z^4 - 128*a^4*c^2*z^4 + 16*a^2*c^4*z^4 - 192*...
```

Reduce [F]

$$\int \frac{1}{a+bx+cx^2+bx^3+ax^4}\,dx = \int \frac{1}{a\,x^4+b\,x^3+c\,x^2+bx+a}dx$$

input
```
int(1/(a*x^4+b*x^3+c*x^2+b*x+a),x)
```

output
```
int(1/(a*x**4 + a + b*x**3 + b*x + c*x**2),x)
```

3.25 $\int \frac{1}{\left(a+bx+cx^2+bx^3+ax^4\right)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 1080

$$\int \frac{1}{(a+bx+cx^2+bx^3+ax^4)^2}\,dx = \text{Too large to display}$$

output
```
(-12*a^2*b+2*(3*b^2-c^2)*(b+(8*a^2-4*a*c+b^2)^(1/2))-2*a*c*(b+2*(8*a^2-4*a
*c+b^2)^(1/2))+4*a*(3*b^2-c*(2*a+c))*x)/a/(2*a-2*b+c)/(2*a+2*b+c)/(8*a^2-4
*a*c+b^2)/(2+(b+(8*a^2-4*a*c+b^2)^(1/2))*x/a+2*x^2)-4*(b^2-2*a*c-b*(8*a^2-
4*a*c+b^2)^(1/2)+a*(b-(8*a^2-4*a*c+b^2)^(1/2))*x)/a/(8*a^2-4*a*c+b^2)^(1/2
)/(4*a^2+2*a*c-b*(b-(8*a^2-4*a*c+b^2)^(1/2)))/(2+(b-(8*a^2-4*a*c+b^2)^(1/2
))*x/a+2*x^2)/(2+(b+(8*a^2-4*a*c+b^2)^(1/2))*x/a+2*x^2)-4*2^(1/2)*a^2*(c*(
4*b^2-c^2)*(b-(8*a^2-4*a*c+b^2)^(1/2))+4*a^2*c*(2*b-(8*a^2-4*a*c+b^2)^(1/2
))+12*a^3*(3*b-(8*a^2-4*a*c+b^2)^(1/2))-a*(18*b^3-b*c^2-6*b^2*(8*a^2-4*a*c
+b^2)^(1/2)-3*c^2*(8*a^2-4*a*c+b^2)^(1/2)))*arctan(1/2*(b-(8*a^2-4*a*c+b^2
)^(1/2)+4*a*x)*2^(1/2)/(4*a^2+2*a*c-b*(b-(8*a^2-4*a*c+b^2)^(1/2)))^(1/2))/
(8*a^2-4*a*c+b^2)^(3/2)/(4*a^2+2*a*c-b*(b-(8*a^2-4*a*c+b^2)^(1/2)))^(3/2)/
(4*a^2+2*a*c-b*(b+(8*a^2-4*a*c+b^2)^(1/2)))+4*2^(1/2)*a^2*(c*(4*b^2-c^2)*(
b+(8*a^2-4*a*c+b^2)^(1/2))+4*a^2*c*(2*b+(8*a^2-4*a*c+b^2)^(1/2))+12*a^3*(3
*b+(8*a^2-4*a*c+b^2)^(1/2))-a*(18*b^3-b*c^2+6*b^2*(8*a^2-4*a*c+b^2)^(1/2)+
3*c^2*(8*a^2-4*a*c+b^2)^(1/2)))*arctan(1/2*(b+(8*a^2-4*a*c+b^2)^(1/2)+4*a*
x)*2^(1/2)/(4*a^2+2*a*c-b*(b+(8*a^2-4*a*c+b^2)^(1/2)))^(1/2))/(8*a^2-4*a*c
+b^2)^(3/2)/(4*a^2+2*a*c-b*(b-(8*a^2-4*a*c+b^2)^(1/2)))/(4*a^2+2*a*c-b*(b+
(8*a^2-4*a*c+b^2)^(1/2)))^(3/2)-(3*a-c)*ln(2*a+(b-(8*a^2-4*a*c+b^2)^(1/2))
*x+2*a*x^2)/(8*a^2-4*a*c+b^2)^(3/2)+(3*a-c)*ln(2*a+(b+(8*a^2-4*a*c+b^2)^(1
/2))*x+2*a*x^2)/(8*a^2-4*a*c+b^2)^(3/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.65 (sec) , antiderivative size = 460, normalized size of antiderivative = 0.43

$$\int \frac{1}{(a+bx+cx^2+bx^3+ax^4)^2}\,dx$$
$$= \frac{8a^3x+7b^2cx-2c^3x-bc^2(1+2x^2)+b^3(4+6x^2)+a^2(b(2-6x^2)-4cx(-1+x^2))+a(-2c^2x(2+x^2)}{(8a^2+b^2-4ac)\,(4a^2-4b^2+4ac+c^2)\,(x\,(b+cx+bx^2)+a\,(1+x^4))}$$
$$+\frac{2\text{RootSum}\left[a+b\#1+c\#1^2+b\#1^3+a\#1^4\&,\ \frac{12a^3\log(x-\#1)-9ab^2\log(x-\#1)+6a^2c\log(x-\#1)+4b^2c\log(x-\#1}{}\right.}{32a^4-}$$

input
```
Integrate[(a + b*x + c*x^2 + b*x^3 + a*x^4)^(-2),x]
```

output
```
(8*a^3*x + 7*b^2*c*x - 2*c^3*x - b*c^2*(1 + 2*x^2) + b^3*(4 + 6*x^2) + a^2
*(b*(2 - 6*x^2) - 4*c*x*(-1 + x^2)) + a*(-2*c^2*x*(2 + x^2) - b*c*(5 + 3*x
^2) + b^2*(-8*x + 6*x^3)))/((8*a^2 + b^2 - 4*a*c)*(4*a^2 - 4*b^2 + 4*a*c +
 c^2)*(x*(b + c*x + b*x^2) + a*(1 + x^4))) + (2*RootSum[a + b*#1 + c*#1^2
+ b*#1^3 + a*#1^4 & , (12*a^3*Log[x - #1] - 9*a*b^2*Log[x - #1] + 6*a^2*c*
Log[x - #1] + 4*b^2*c*Log[x - #1] - 2*a*c^2*Log[x - #1] - c^3*Log[x - #1]
- 6*a^2*b*Log[x - #1]*#1 + 3*b^3*Log[x - #1]*#1 - a*b*c*Log[x - #1]*#1 - b
*c^2*Log[x - #1]*#1 + 3*a*b^2*Log[x - #1]*#1^2 - 2*a^2*c*Log[x - #1]*#1^2
- a*c^2*Log[x - #1]*#1^2)/(b + 2*c*#1 + 3*b*#1^2 + 4*a*#1^3) & ])/(32*a^4
- 4*b^4 + 16*a^3*c + b^2*c^2 - 4*a^2*(7*b^2 + 2*c^2) + 4*a*(5*b^2*c - c^3)
)
```

Rubi [A] (verified)

Time = 4.86 (sec) , antiderivative size = 1135, normalized size of antiderivative = 1.05, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {2492, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ax^4+a+bx^3+bx+cx^2)^2}\,dx$$

$$\Big\downarrow \text{2492}$$

$$\frac{\int \left(-\frac{2\left(4ab-cb+2a(3a-c)x-(3a-c)\sqrt{8a^2-4ca+b^2}\right)a^2}{(8a^2-4ca+b^2)^{3/2}\left(2ax^2+\left(b-\sqrt{8a^2-4ca+b^2}\right)x+2a\right)}-\frac{2\left(c\left(b+\sqrt{8a^2-4ca+b^2}\right)-a\left(4b+3\sqrt{8a^2-4ca+b^2}\right)-2a(3a-c)x\right)a^2}{(8a^2-4ca+b^2)^{3/2}\left(2ax^2+\left(b+\sqrt{8a^2-4ca+b^2}\right)x+2a\right)}+\frac{2\left(2a^2-\cdots\right.}{(8\cdots}\right.}{a^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{2\sqrt{2}\left(b^2-\sqrt{8a^2-4ca+b^2}b-2ac\right)\arctan\left(\frac{b+4ax-\sqrt{8a^2-4ca+b^2}}{\sqrt{2}\sqrt{4a^2+2ca-b\left(b-\sqrt{8a^2-4ca+b^2}\right)}}\right)a^3}{(8a^2-4ca+b^2)\left(4a^2+2ca-b\left(b-\sqrt{8a^2-4ca+b^2}\right)\right)^{3/2}}+\frac{2\sqrt{2}\left(b^2+\sqrt{8a^2-4ca+b^2}b-2ac\right)\arctan\left(\frac{b+4ax+\sqrt{8a^2\cdots}}{\sqrt{2}\sqrt{4a^2+2ca-b\left(b+\cdots\right.}}\right.}{(8a^2-4ca+b^2)\left(4a^2+2ca-b\left(b+\sqrt{8a^2-4ca+b^2}\cdots\right.\right.}}{}$$

input
```
Int[(a + b*x + c*x^2 + b*x^3 + a*x^4)^(-2),x]
```

output
```
((-2*a^2*((b - Sqrt[8*a^2 + b^2 - 4*a*c])*(2*a^2 + 2*a*c - b*(b - Sqrt[8*a
^2 + b^2 - 4*a*c])) - 2*a*(b^2 - 2*a*c - b*Sqrt[8*a^2 + b^2 - 4*a*c])*x))/
((8*a^2 + b^2 - 4*a*c)*(16*a^2 - (b - Sqrt[8*a^2 + b^2 - 4*a*c])^2)*(2*a +
 (b - Sqrt[8*a^2 + b^2 - 4*a*c])*x + 2*a*x^2)) - (2*a^2*((b + Sqrt[8*a^2 +
 b^2 - 4*a*c])*(2*a^2 + 2*a*c - b*(b + Sqrt[8*a^2 + b^2 - 4*a*c])) - 2*a*(
b^2 - 2*a*c + b*Sqrt[8*a^2 + b^2 - 4*a*c])*x))/((8*a^2 + b^2 - 4*a*c)*(16*
a^2 - (b + Sqrt[8*a^2 + b^2 - 4*a*c])^2)*(2*a + (b + Sqrt[8*a^2 + b^2 - 4*
a*c])*x + 2*a*x^2)) + (2*Sqrt[2]*a^3*(b^2 - 2*a*c - b*Sqrt[8*a^2 + b^2 - 4
*a*c])*ArcTan[(b - Sqrt[8*a^2 + b^2 - 4*a*c] + 4*a*x)/(Sqrt[2]*Sqrt[4*a^2
+ 2*a*c - b*(b - Sqrt[8*a^2 + b^2 - 4*a*c])])])/((8*a^2 + b^2 - 4*a*c)*(4*
a^2 + 2*a*c - b*(b - Sqrt[8*a^2 + b^2 - 4*a*c]))^(3/2)) - (Sqrt[2]*a^2*(5*
a*b - b*c - (3*a - c)*Sqrt[8*a^2 + b^2 - 4*a*c])*ArcTan[(b - Sqrt[8*a^2 +
b^2 - 4*a*c] + 4*a*x)/(Sqrt[2]*Sqrt[4*a^2 + 2*a*c - b*(b - Sqrt[8*a^2 + b^
2 - 4*a*c])])])/((8*a^2 + b^2 - 4*a*c)^(3/2)*Sqrt[4*a^2 + 2*a*c - b*(b - S
qrt[8*a^2 + b^2 - 4*a*c])]) + (2*Sqrt[2]*a^3*(b^2 - 2*a*c + b*Sqrt[8*a^2 +
 b^2 - 4*a*c])*ArcTan[(b + Sqrt[8*a^2 + b^2 - 4*a*c] + 4*a*x)/(Sqrt[2]*Sqr
t[4*a^2 + 2*a*c - b*(b + Sqrt[8*a^2 + b^2 - 4*a*c])])])/((8*a^2 + b^2 - 4*
a*c)*(4*a^2 + 2*a*c - b*(b + Sqrt[8*a^2 + b^2 - 4*a*c]))^(3/2)) - (Sqrt[2]
*a^2*(c*(b + Sqrt[8*a^2 + b^2 - 4*a*c]) - a*(5*b + 3*Sqrt[8*a^2 + b^2 - 4*
a*c]))*ArcTan[(b + Sqrt[8*a^2 + b^2 - 4*a*c] + 4*a*x)/(Sqrt[2]*Sqrt[4*a...
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2492
```
Int[(Px_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^3 + (e_.)*(x_)^4)
^(p_), x_Symbol] :> Simp[e^p   Int[ExpandIntegrand[Px*(b/d + ((d + Sqrt[e*(
(b^2 - 4*a*c)/a) + 8*a*d*(e/b)])/(2*e))*x + x^2)^p*(b/d + ((d - Sqrt[e*((b^
2 - 4*a*c)/a) + 8*a*d*(e/b)])/(2*e))*x + x^2)^p, x], x], x] /; FreeQ[{a, b,
 c, d, e}, x] && PolyQ[Px, x] && ILtQ[p, 0] && EqQ[a*d^2 - b^2*e, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.94 (sec) , antiderivative size = 525, normalized size of antiderivative = 0.49

method	result
risch	$\frac{-\frac{2a\left(2ac-3b^{2}+c^{2}\right)x^{3}}{\left(8a^{2}-4ac+b^{2}\right)\left(4a^{2}+4ac-4b^{2}+c^{2}\right)}-\frac{\left(6a^{2}+3ac-6b^{2}+2c^{2}\right)b\,x^{2}}{\left(4a^{2}+4ac-4b^{2}+c^{2}\right)\left(8a^{2}-4ac+b^{2}\right)}+\frac{\left(8a^{3}+4a^{2}c-8b^{2}a-4a\,c^{2}+7c\,b^{2}-2c^{3}\right)x}{\left(8a^{2}-4ac+b^{2}\right)\left(4a^{2}+4ac-4b^{2}+c^{2}\right)}+\frac{b\left(2a^{2}\right.}{32a^{4}+16a^{3}c-28a^{2}b^{2}-}}{a\,x^{4}+b\,x^{3}+c\,x^{2}+bx+a}$
default	Expression too large to display

input
```
int(1/(a*x^4+b*x^3+c*x^2+b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(-2*a*(2*a*c-3*b^2+c^2)/(8*a^2-4*a*c+b^2)/(4*a^2+4*a*c-4*b^2+c^2)*x^3-(6*a
^2+3*a*c-6*b^2+2*c^2)*b/(4*a^2+4*a*c-4*b^2+c^2)/(8*a^2-4*a*c+b^2)*x^2+(8*a
^3+4*a^2*c-8*a*b^2-4*a*c^2+7*b^2*c-2*c^3)/(8*a^2-4*a*c+b^2)/(4*a^2+4*a*c-4
*b^2+c^2)*x+b*(2*a^2-5*a*c+4*b^2-c^2)/(32*a^4+16*a^3*c-28*a^2*b^2-8*a^2*c^
2+20*a*b^2*c-4*a*c^3-4*b^4+b^2*c^2))/(a*x^4+b*x^3+c*x^2+b*x+a)+2*sum((-a*(
2*a*c-3*b^2+c^2)/(8*a^2-4*a*c+b^2)/(4*a^2+4*a*c-4*b^2+c^2)*_R^2-b*(6*a^2+a
*c-3*b^2+c^2)/(8*a^2-4*a*c+b^2)/(4*a^2+4*a*c-4*b^2+c^2)*_R+(12*a^3+6*a^2*c
-9*a*b^2-2*a*c^2+4*b^2*c-c^3)/(8*a^2-4*a*c+b^2)/(4*a^2+4*a*c-4*b^2+c^2))/(
4*_R^3*a+3*_R^2*b+2*_R*c+b)*ln(x-_R),_R=RootOf(_Z^4*a+_Z^3*b+_Z^2*c+_Z*b+a
))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx+cx^2+bx^3+ax^4)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(a*x^4+b*x^3+c*x^2+b*x+a)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx+cx^2+bx^3+ax^4)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(a*x**4+b*x**3+c*x**2+b*x+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{(a+bx+cx^2+bx^3+ax^4)^2}\,dx = \int \frac{1}{(ax^4+bx^3+cx^2+bx+a)^2}\,dx$$

input
```
integrate(1/(a*x^4+b*x^3+c*x^2+b*x+a)^2,x, algorithm="maxima")
```

output
```
(2*(3*a*b^2 - 2*a^2*c - a*c^2)*x^3 + 2*a^2*b + 4*b^3 - 5*a*b*c - b*c^2 - (
6*a^2*b - 6*b^3 + 3*a*b*c + 2*b*c^2)*x^2 + (8*a^3 - 8*a*b^2 - 4*a*c^2 - 2*
c^3 + (4*a^2 + 7*b^2)*c)*x)/(32*a^5 - 28*a^3*b^2 - 4*a*b^4 - 4*a^2*c^3 + (
32*a^5 - 28*a^3*b^2 - 4*a*b^4 - 4*a^2*c^3 - (8*a^3 - a*b^2)*c^2 + 4*(4*a^4
 + 5*a^2*b^2)*c)*x^4 + (32*a^4*b - 28*a^2*b^3 - 4*b^5 - 4*a*b*c^3 - (8*a^2
*b - b^3)*c^2 + 4*(4*a^3*b + 5*a*b^3)*c)*x^3 - (8*a^3 - a*b^2)*c^2 - (4*a*
c^4 + (8*a^2 - b^2)*c^3 - 4*(4*a^3 + 5*a*b^2)*c^2 - 4*(8*a^4 - 7*a^2*b^2 -
 b^4)*c)*x^2 + 4*(4*a^4 + 5*a^2*b^2)*c + (32*a^4*b - 28*a^2*b^3 - 4*b^5 -
4*a*b*c^3 - (8*a^2*b - b^3)*c^2 + 4*(4*a^3*b + 5*a*b^3)*c)*x) - 2*integrat
e(-(12*a^3 - 9*a*b^2 - 2*a*c^2 - c^3 + (3*a*b^2 - 2*a^2*c - a*c^2)*x^2 + 2
*(3*a^2 + 2*b^2)*c - (6*a^2*b - 3*b^3 + a*b*c + b*c^2)*x)/(a*x^4 + b*x^3 +
 c*x^2 + b*x + a), x)/(32*a^4 - 28*a^2*b^2 - 4*b^4 - 4*a*c^3 - (8*a^2 - b^
2)*c^2 + 4*(4*a^3 + 5*a*b^2)*c)
```

Giac [F]

$$\int \frac{1}{(a+bx+cx^2+bx^3+ax^4)^2}\,dx = \int \frac{1}{(ax^4+bx^3+cx^2+bx+a)^2}\,dx$$

input
```
integrate(1/(a*x^4+b*x^3+c*x^2+b*x+a)^2,x, algorithm="giac")
```

output
```
integrate((a*x^4 + b*x^3 + c*x^2 + b*x + a)^(-2), x)
```

Mupad [B] (verification not implemented)

Time = 23.47 (sec) , antiderivative size = 12699, normalized size of antiderivative = 11.76

$$\int \frac{1}{(a+bx+cx^2+bx^3+ax^4)^2}\,dx = \text{Too large to display}$$

input
```
int(1/(a + b*x + a*x^4 + b*x^3 + c*x^2)^2,x)
```

output
```
symsum(log((576*a^8*c + 216*a^5*b^4 - 40*a^5*c^4 - 80*a^6*c^3 + 288*a^7*c^
2 - 72*a^4*b^4*c - 720*a^6*b^2*c + 16*a^4*b^2*c^3 + 216*a^5*b^2*c^2)/(4096
*a^9*c + 8192*a^10 + 16*b^10 + 352*a^2*b^8 + 2320*a^4*b^6 + 2432*a^6*b^4 -
 13312*a^8*b^2 - 64*a^3*c^7 - 128*a^4*c^6 + 768*a^5*c^5 + 1536*a^6*c^4 - 3
072*a^7*c^3 - 6144*a^8*c^2 + b^6*c^4 - 8*b^8*c^2 - 12*a*b^4*c^5 + 104*a*b^
6*c^3 - 3424*a^3*b^6*c - 11712*a^5*b^4*c + 11264*a^7*b^2*c + 48*a^2*b^2*c^
6 - 456*a^2*b^4*c^4 + 984*a^2*b^6*c^2 + 704*a^3*b^2*c^5 - 1120*a^3*b^4*c^3
 - 1216*a^4*b^2*c^4 + 9408*a^4*b^4*c^2 - 5632*a^5*b^2*c^3 + 7424*a^6*b^2*c
^2 - 224*a*b^8*c) - root(1728*a*b^16*c*z^4 + 85131264*a^12*b^4*c^2*z^4 + 6
4880640*a^9*b^6*c^3*z^4 - 58195968*a^13*b^2*c^3*z^4 - 45121536*a^10*b^4*c^
4*z^4 - 41680896*a^10*b^6*c^2*z^4 - 32870400*a^8*b^8*c^2*z^4 - 32194560*a^
8*b^6*c^4*z^4 + 22806528*a^11*b^2*c^5*z^4 + 22333440*a^7*b^8*c^3*z^4 + 220
20096*a^14*b^2*c^2*z^4 + 14843904*a^9*b^4*c^5*z^4 - 6828288*a^6*b^10*c^2*z
^4 + 5627904*a^8*b^4*c^6*z^4 - 5076480*a^6*b^8*c^4*z^4 - 4128768*a^9*b^2*c
^7*z^4 - 3538944*a^10*b^2*c^6*z^4 - 2998272*a^7*b^4*c^7*z^4 + 2551680*a^5*
b^10*c^3*z^4 + 2359296*a^11*b^4*c^3*z^4 + 2326528*a^6*b^6*c^6*z^4 + 208896
0*a^7*b^6*c^5*z^4 + 1572864*a^12*b^2*c^4*z^4 + 835584*a^8*b^2*c^8*z^4 - 58
8048*a^4*b^12*c^2*z^4 - 536064*a^5*b^8*c^5*z^4 + 319488*a^7*b^2*c^9*z^4 -
294912*a^5*b^6*c^7*z^4 + 218880*a^4*b^8*c^6*z^4 - 112320*a^4*b^10*c^4*z^4
+ 89088*a^5*b^4*c^9*z^4 + 84384*a^3*b^12*c^3*z^4 - 73824*a^3*b^10*c^5*z...
```

Reduce [F]

$$\int \frac{1}{(a+bx+cx^2+bx^3+ax^4)^2}\,dx = \text{too large to display}$$

input
```
int(1/(a*x^4+b*x^3+c*x^2+b*x+a)^2,x)
```

output
```
( - 4*int(x**3/(a**2*x**8 + 2*a**2*x**4 + a**2 + 2*a*b*x**7 + 2*a*b*x**5 +
 2*a*b*x**3 + 2*a*b*x + 2*a*c*x**6 + 2*a*c*x**2 + b**2*x**6 + 2*b**2*x**4
+ b**2*x**2 + 2*b*c*x**5 + 2*b*c*x**3 + c**2*x**4),x)*a**2*x**4 - 4*int(x*
*3/(a**2*x**8 + 2*a**2*x**4 + a**2 + 2*a*b*x**7 + 2*a*b*x**5 + 2*a*b*x**3
+ 2*a*b*x + 2*a*c*x**6 + 2*a*c*x**2 + b**2*x**6 + 2*b**2*x**4 + b**2*x**2
+ 2*b*c*x**5 + 2*b*c*x**3 + c**2*x**4),x)*a**2 - 4*int(x**3/(a**2*x**8 + 2
*a**2*x**4 + a**2 + 2*a*b*x**7 + 2*a*b*x**5 + 2*a*b*x**3 + 2*a*b*x + 2*a*c
*x**6 + 2*a*c*x**2 + b**2*x**6 + 2*b**2*x**4 + b**2*x**2 + 2*b*c*x**5 + 2*
b*c*x**3 + c**2*x**4),x)*a*b*x**3 - 4*int(x**3/(a**2*x**8 + 2*a**2*x**4 +
a**2 + 2*a*b*x**7 + 2*a*b*x**5 + 2*a*b*x**3 + 2*a*b*x + 2*a*c*x**6 + 2*a*c
*x**2 + b**2*x**6 + 2*b**2*x**4 + b**2*x**2 + 2*b*c*x**5 + 2*b*c*x**3 + c*
*2*x**4),x)*a*b*x - 4*int(x**3/(a**2*x**8 + 2*a**2*x**4 + a**2 + 2*a*b*x**
7 + 2*a*b*x**5 + 2*a*b*x**3 + 2*a*b*x + 2*a*c*x**6 + 2*a*c*x**2 + b**2*x**
6 + 2*b**2*x**4 + b**2*x**2 + 2*b*c*x**5 + 2*b*c*x**3 + c**2*x**4),x)*a*c*
x**2 - 3*int(x**2/(a**2*x**8 + 2*a**2*x**4 + a**2 + 2*a*b*x**7 + 2*a*b*x**
5 + 2*a*b*x**3 + 2*a*b*x + 2*a*c*x**6 + 2*a*c*x**2 + b**2*x**6 + 2*b**2*x*
*4 + b**2*x**2 + 2*b*c*x**5 + 2*b*c*x**3 + c**2*x**4),x)*a*b*x**4 - 3*int(
x**2/(a**2*x**8 + 2*a**2*x**4 + a**2 + 2*a*b*x**7 + 2*a*b*x**5 + 2*a*b*x**
3 + 2*a*b*x + 2*a*c*x**6 + 2*a*c*x**2 + b**2*x**6 + 2*b**2*x**4 + b**2*x**
2 + 2*b*c*x**5 + 2*b*c*x**3 + c**2*x**4),x)*a*b - 3*int(x**2/(a**2*x**8...
```

3.26 $\int \left(ab^2+b^3x+cx^2+b^2dx^3+ad^2x^4\right)^3 \, dx$

Optimal result

Integrand size = 34, antiderivative size = 320

$$\begin{aligned}\int \left(ab^2+b^3x+cx^2+b^2dx^3+ad^2x^4\right)^3 \, dx &= a^3b^6x+\frac{3}{2}a^2b^7x^2+ab^4\left(b^4+ac\right)x^3+\frac{1}{4}b^5\left(b^4+6ac+3a^2bd\right)x^4 \\ &+\frac{3}{5}b^2\left(b^4c+ac^2+2ab^5d+a^3b^2d^2\right)x^5+\frac{1}{2}b^3\left(c^2+2abcd+b^2d\left(b^3+2a^2d\right)\right)x^6 \\ &+\frac{1}{7}\left(c^3+6ab^6d^2+6b^2cd\left(b^3+a^2d\right)\right)x^7+\frac{3}{8}b^2d\left(c^2+2abcd+b^2d\left(b^3+2a^2d\right)\right)x^8 \\ &+\frac{1}{3}d^2\left(b^4c+ac^2+2ab^5d+a^3b^2d^2\right)x^9+\frac{1}{10}b^2d^3\left(b^4+6ac+3a^2bd\right)x^{10} \\ &+\frac{3}{11}a\left(b^4+ac\right)d^4x^{11}+\frac{1}{4}a^2b^2d^5x^{12}+\frac{1}{13}a^3d^6x^{13}\end{aligned}$$

output
```
a^3*b^6*x+3/2*a^2*b^7*x^2+a*b^4*(b^4+a*c)*x^3+1/4*b^5*(3*a^2*b*d+b^4+6*a*c
)*x^4+3/5*b^2*(a^3*b^2*d^2+2*a*b^5*d+b^4*c+a*c^2)*x^5+1/2*b^3*(c^2+2*a*b*c
*d+b^2*d*(2*a^2*d+b^3))*x^6+1/7*(c^3+6*a*b^6*d^2+6*b^2*c*d*(a^2*d+b^3))*x^
7+3/8*b^2*d*(c^2+2*a*b*c*d+b^2*d*(2*a^2*d+b^3))*x^8+1/3*d^2*(a^3*b^2*d^2+2
*a*b^5*d+b^4*c+a*c^2)*x^9+1/10*b^2*d^3*(3*a^2*b*d+b^4+6*a*c)*x^10+3/11*a*(
b^4+a*c)*d^4*x^11+1/4*a^2*b^2*d^5*x^12+1/13*a^3*d^6*x^13
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.00

$$\int \left(ab^2+b^3x+cx^2+b^2dx^3+ad^2x^4\right)^3\,dx$$
$$=a^3b^6x+\frac{3}{2}a^2b^7x^2+ab^4\left(b^4+ac\right)x^3+\frac{1}{4}b^5\left(b^4+6ac+3a^2bd\right)x^4$$
$$+\frac{3}{5}b^2\left(b^4c+ac^2+2ab^5d+a^3b^2d^2\right)x^5+\frac{1}{2}b^3\left(c^2+2abcd+b^2d\left(b^3+2a^2d\right)\right)x^6$$
$$+\frac{1}{7}\left(c^3+6ab^6d^2+6b^2cd\left(b^3+a^2d\right)\right)x^7+\frac{3}{8}b^2d\left(c^2+2abcd+b^2d\left(b^3+2a^2d\right)\right)x^8$$
$$+\frac{1}{3}d^2\left(b^4c+ac^2+2ab^5d+a^3b^2d^2\right)x^9+\frac{1}{10}b^2d^3\left(b^4+6ac+3a^2bd\right)x^{10}$$
$$+\frac{3}{11}a\left(b^4+ac\right)d^4x^{11}+\frac{1}{4}a^2b^2d^5x^{12}+\frac{1}{13}a^3d^6x^{13}$$

input
```
Integrate[(a*b^2 + b^3*x + c*x^2 + b^2*d*x^3 + a*d^2*x^4)^3,x]
```

output
```
a^3*b^6*x + (3*a^2*b^7*x^2)/2 + a*b^4*(b^4 + a*c)*x^3 + (b^5*(b^4 + 6*a*c
+ 3*a^2*b*d)*x^4)/4 + (3*b^2*(b^4*c + a*c^2 + 2*a*b^5*d + a^3*b^2*d^2)*x^5
)/5 + (b^3*(c^2 + 2*a*b*c*d + b^2*d*(b^3 + 2*a^2*d))*x^6)/2 + ((c^3 + 6*a*
b^6*d^2 + 6*b^2*c*d*(b^3 + a^2*d))*x^7)/7 + (3*b^2*d*(c^2 + 2*a*b*c*d + b^
2*d*(b^3 + 2*a^2*d))*x^8)/8 + (d^2*(b^4*c + a*c^2 + 2*a*b^5*d + a^3*b^2*d^
2)*x^9)/3 + (b^2*d^3*(b^4 + 6*a*c + 3*a^2*b*d)*x^10)/10 + (3*a*(b^4 + a*c)
*d^4*x^11)/11 + (a^2*b^2*d^5*x^12)/4 + (a^3*d^6*x^13)/13
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}=0.059$, Rules used = {2465, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(ab^2+ad^2x^4+b^3x+b^2dx^3+cx^2\right)^3\,dx$$

$$\Big\downarrow \text{2465}$$

$$\int \left(a^3b^6+3d^2x^8\left(a^3b^2d^2+2ab^5d+ac^2+b^4c\right)+3b^2x^4\left(a^3b^2d^2+2ab^5d+ac^2+b^4c\right)+a^3d^6x^{12}+3a^2b^7x+3a^2b^2\right.$$

$$\Big\downarrow \text{2009}$$

$$a^3b^6x+\frac{1}{3}d^2x^9\left(a^3b^2d^2+2ab^5d+ac^2+b^4c\right)+\frac{3}{5}b^2x^5\left(a^3b^2d^2+2ab^5d+ac^2+b^4c\right)+\frac{1}{13}a^3d^6x^{13}+\frac{3}{2}a^2b^7x^2+\frac{1}{4}a^2b^2d^5x^{12}+\frac{1}{4}b^5x^4\left(3a^2bd+6ac+b^4\right)+\frac{1}{10}b^2d^3x^{10}\left(3a^2bd+6ac+b^4\right)+\frac{3}{8}b^2dx^8\left(b^2d\left(2a^2d+b^3\right)+2abcd+c^2\right)+\frac{1}{2}b^3x^6\left(b^2d\left(2a^2d+b^3\right)+2abcd+c^2\right)+\frac{1}{7}x^7\left(6b^2cd\left(a^2d+b^3\right)+6ab^6d^2+c^3\right)+\frac{3}{11}ad^4x^{11}\left(ac+b^4\right)+ab^4x^3\left(ac+b^4\right)$$

input
```
Int[(a*b^2 + b^3*x + c*x^2 + b^2*d*x^3 + a*d^2*x^4)^3,x]
```

output
```
a^3*b^6*x + (3*a^2*b^7*x^2)/2 + a*b^4*(b^4 + a*c)*x^3 + (b^5*(b^4 + 6*a*c
+ 3*a^2*b*d)*x^4)/4 + (3*b^2*(b^4*c + a*c^2 + 2*a*b^5*d + a^3*b^2*d^2)*x^5
)/5 + (b^3*(c^2 + 2*a*b*c*d + b^2*d*(b^3 + 2*a^2*d))*x^6)/2 + ((c^3 + 6*a*
b^6*d^2 + 6*b^2*c*d*(b^3 + a^2*d))*x^7)/7 + (3*b^2*d*(c^2 + 2*a*b*c*d + b^
2*d*(b^3 + 2*a^2*d))*x^8)/8 + (d^2*(b^4*c + a*c^2 + 2*a*b^5*d + a^3*b^2*d^
2)*x^9)/3 + (b^2*d^3*(b^4 + 6*a*c + 3*a^2*b*d)*x^10)/10 + (3*a*(b^4 + a*c)
*d^4*x^11)/11 + (a^2*b^2*d^5*x^12)/4 + (a^3*d^6*x^13)/13
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2465
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> Int[ExpandToSum[u, Px^p, x], x] /; PolyQ
[Px, x] && GtQ[Expon[Px, x], 2] && !BinomialQ[Px, x] && !TrinomialQ[Px, x
] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.08

method	result
norman	$\frac{a^3d^6x^{13}}{13}+\frac{a^2b^2d^5x^{12}}{4}+\left(\frac{3}{11}a\,b^4d^4+\frac{3}{11}a^2c\,d^4\right)x^{11}+\left(\frac{3}{10}b^3a^2d^4+\frac{1}{10}b^6d^3+\frac{3}{5}a\,b^2c\,d^3\right)x^{10}+\left(\frac{1}{3}a^3\right.$
gosper	$a^3b^6x+\frac{6}{7}x^7b^5cd+x^6a^2d^2b^5+\frac{3}{5}x^5a^3d^2b^4+\frac{6}{5}x^5d\,b^7a+\frac{3}{5}x^5a\,b^2c^2+\frac{3}{4}x^4d\,b^6a^2+\frac{3}{2}x^4a\,b^5c+$
risch	$a^3b^6x+\frac{6}{7}x^7b^5cd+x^6a^2d^2b^5+\frac{3}{5}x^5a^3d^2b^4+\frac{6}{5}x^5d\,b^7a+\frac{3}{5}x^5a\,b^2c^2+\frac{3}{4}x^4d\,b^6a^2+\frac{3}{2}x^4a\,b^5c+$
parallelrisch	$a^3b^6x+\frac{6}{7}x^7b^5cd+x^6a^2d^2b^5+\frac{3}{5}x^5a^3d^2b^4+\frac{6}{5}x^5d\,b^7a+\frac{3}{5}x^5a\,b^2c^2+\frac{3}{4}x^4d\,b^6a^2+\frac{3}{2}x^4a\,b^5c+$
orering	$x(9240a^3d^6x^{12}+30030d^5b^2a^2x^{11}+32760a\,b^4d^4x^{10}+36036a^2b^3d^4x^9+12012b^6d^3x^9+40040a^3b^2d^4x^8+32760a^2c\,d^4x^{10}+8008$
default	$\frac{a^3d^6x^{13}}{13}+\frac{a^2b^2d^5x^{12}}{4}+\frac{\left(a^2c\,d^4+2a\,b^4d^4+a\,d^2\left(b^4d^2+2ac\,d^2\right)\right)x^{11}}{11}+\frac{\left(b^3a^2d^4+2a\,b^2c\,d^3+d\,b^2\left(b^4d^2+2ac\,d^2\right)+a\,d^2\left(2b^3\right.\right.}{10}$

input
```
int((a*d^2*x^4+b^2*d*x^3+b^3*x+a*b^2+c*x^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/13*a^3*d^6*x^13+1/4*a^2*b^2*d^5*x^12+(3/11*a*b^4*d^4+3/11*a^2*c*d^4)*x^1
1+(3/10*b^3*a^2*d^4+1/10*b^6*d^3+3/5*a*b^2*c*d^3)*x^10+(1/3*a^3*b^2*d^4+2/
3*a*b^5*d^3+1/3*b^4*c*d^2+1/3*a*c^2*d^2)*x^9+(3/4*b^4*a^2*d^3+3/8*b^7*d^2+
3/4*a*b^3*c*d^2+3/8*b^2*c^2*d)*x^8+(6/7*a*b^6*d^2+6/7*a^2*b^2*c*d^2+6/7*b^
5*c*d+1/7*c^3)*x^7+(a^2*d^2*b^5+1/2*b^8*d+a*b^4*c*d+1/2*b^3*c^2)*x^6+(3/5*
a^3*d^2*b^4+6/5*d*b^7*a+3/5*b^6*c+3/5*a*b^2*c^2)*x^5+(3/4*d*b^6*a^2+1/4*b^
9+3/2*a*b^5*c)*x^4+(a*b^8+a^2*b^4*c)*x^3+3/2*a^2*b^7*x^2+a^3*b^6*x
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.02

$$\int \left(ab^2+b^3x+cx^2+b^2dx^3+ad^2x^4\right)^3\,dx=\frac{1}{13}\,a^3d^6x^{13}+\frac{1}{4}\,a^2b^2d^5x^{12}+\frac{3}{11}\,\left(ab^4+a^2c\right)d^4x^{11}+\frac{3}{2}\,a^2b^7x^2+\frac{1}{10}\,\left(3\,a^2b^3d^4+\left(b^6+6\,ab^2c\right)d^3\right)x^{10}+a^3b^6x+\frac{1}{3}\,\left(2\,ab^5d^3+a^3b^2d^4+\left(b^4c+ac^2\right)d^2\right)x^9+\frac{3}{8}\,\left(2\,a^2b^4d^3+b^2c^2d+\left(b^7+2\,ab^3c\right)d^2\right)x^8+\frac{1}{7}\,\left(6\,b^5cd+c^3+6\,\left(ab^6+a^2b^2c\right)d^2\right)x^7+\frac{1}{2}\,\left(2\,a^2b^5d^2+b^3c^2+\left(b^8+2\,ab^4c\right)d\right)x^6+\frac{3}{5}\,\left(2\,ab^7d+a^3b^4d^2+b^6c+ab^2c^2\right)x^5+\frac{1}{4}\,\left(b^9+3\,a^2b^6d+6\,ab^5c\right)x^4+\left(ab^8+a^2b^4c\right)x^3$$

input
```
integrate((a*d^2*x^4+b^2*d*x^3+b^3*x+a*b^2+c*x^2)^3,x, algorithm="fricas")
```

output
```
1/13*a^3*d^6*x^13 + 1/4*a^2*b^2*d^5*x^12 + 3/11*(a*b^4 + a^2*c)*d^4*x^11 +
 3/2*a^2*b^7*x^2 + 1/10*(3*a^2*b^3*d^4 + (b^6 + 6*a*b^2*c)*d^3)*x^10 + a^3
*b^6*x + 1/3*(2*a*b^5*d^3 + a^3*b^2*d^4 + (b^4*c + a*c^2)*d^2)*x^9 + 3/8*(
2*a^2*b^4*d^3 + b^2*c^2*d + (b^7 + 2*a*b^3*c)*d^2)*x^8 + 1/7*(6*b^5*c*d +
c^3 + 6*(a*b^6 + a^2*b^2*c)*d^2)*x^7 + 1/2*(2*a^2*b^5*d^2 + b^3*c^2 + (b^8
 + 2*a*b^4*c)*d)*x^6 + 3/5*(2*a*b^7*d + a^3*b^4*d^2 + b^6*c + a*b^2*c^2)*x
^5 + 1/4*(b^9 + 3*a^2*b^6*d + 6*a*b^5*c)*x^4 + (a*b^8 + a^2*b^4*c)*x^3
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 393, normalized size of antiderivative = 1.23

$$\int \left(ab^2+b^3x+cx^2+b^2dx^3+ad^2x^4\right)^3\,dx = a^3b^6x+\frac{a^3d^6x^{13}}{13}+\frac{3a^2b^7x^2}{2}+\frac{a^2b^2d^5x^{12}}{4}+x^{11}\cdot\left(\frac{3a^2cd^4}{11}+\frac{3ab^4d^4}{11}\right)+x^{10}\cdot\left(\frac{3a^2b^3d^4}{10}+\frac{3ab^2cd^3}{5}+\frac{b^6d^3}{10}\right)+x^9\left(\frac{a^3b^2d^4}{3}+\frac{2ab^5d^3}{3}+\frac{ac^2d^2}{3}+\frac{b^4cd^2}{3}\right)+x^8\cdot\left(\frac{3a^2b^4d^3}{4}+\frac{3ab^3cd^2}{4}+\frac{3b^7d^2}{8}+\frac{3b^2c^2d}{8}\right)+x^7\cdot\left(\frac{6a^2b^2cd^2}{7}+\frac{6ab^6d^2}{7}+\frac{6b^5cd}{7}+\frac{c^3}{7}\right)+x^6\left(a^2b^5d^2+ab^4cd+\frac{b^8d}{2}+\frac{b^3c^2}{2}\right)+x^5\cdot\left(\frac{3a^3b^4d^2}{5}+\frac{6ab^7d}{5}+\frac{3ab^2c^2}{5}+\frac{3b^6c}{5}\right)+x^4\cdot\left(\frac{3a^2b^6d}{4}+\frac{3ab^5c}{2}+\frac{b^9}{4}\right)+x^3\left(a^2b^4c+ab^8\right)$$

input
```
integrate((a*d**2*x**4+b**2*d*x**3+b**3*x+a*b**2+c*x**2)**3,x)
```

output
```
a**3*b**6*x + a**3*d**6*x**13/13 + 3*a**2*b**7*x**2/2 + a**2*b**2*d**5*x**
12/4 + x**11*(3*a**2*c*d**4/11 + 3*a*b**4*d**4/11) + x**10*(3*a**2*b**3*d*
*4/10 + 3*a*b**2*c*d**3/5 + b**6*d**3/10) + x**9*(a**3*b**2*d**4/3 + 2*a*b
**5*d**3/3 + a*c**2*d**2/3 + b**4*c*d**2/3) + x**8*(3*a**2*b**4*d**3/4 + 3
*a*b**3*c*d**2/4 + 3*b**7*d**2/8 + 3*b**2*c**2*d/8) + x**7*(6*a**2*b**2*c*
d**2/7 + 6*a*b**6*d**2/7 + 6*b**5*c*d/7 + c**3/7) + x**6*(a**2*b**5*d**2 +
 a*b**4*c*d + b**8*d/2 + b**3*c**2/2) + x**5*(3*a**3*b**4*d**2/5 + 6*a*b**
7*d/5 + 3*a*b**2*c**2/5 + 3*b**6*c/5) + x**4*(3*a**2*b**6*d/4 + 3*a*b**5*c
/2 + b**9/4) + x**3*(a**2*b**4*c + a*b**8)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.18

$$
\begin{aligned}
&\int \left(ab^2+b^3x+cx^2+b^2dx^3+ad^2x^4\right)^3 \, dx \\
&= \frac{1}{13}\,a^3d^6x^{13}+\frac{1}{4}\,a^2b^2d^5x^{12}+\frac{3}{11}\,ab^4d^4x^{11}+\frac{1}{10}\,b^6d^3x^{10}+\frac{1}{4}\,b^9x^4+a^3b^6x \\
&\quad+\frac{1}{7}\,c^3x^7+\frac{1}{20}\,\left(12\,ad^2x^5+15\,b^2dx^4+30\,b^3x^2+20\,cx^3\right)a^2b^4 \\
&\quad+\frac{1}{70}\,\left(30\,ad^2x^7+35\,b^2dx^6+42\,cx^5\right)b^6 \\
&\quad+\frac{1}{420}\,\left(140\,a^2d^4x^9+315\,ab^2d^3x^8+180\,b^4d^2x^7+420\,b^6x^3+252\,c^2x^5+42\,\left(10\,ad^2x^6+12\,b^2dx^5+15\,cx^4\right.\right. \\
&\quad+\frac{1}{840}\,\left(252\,a^2d^4x^{10}+560\,ab^2d^3x^9+720\,b^2cdx^7+315\,\left(b^4+2\,ac\right)d^2x^8+420\,c^2x^6\right)b^3 \\
&\quad+\frac{1}{24}\,\left(8\,ad^2x^9+9\,b^2dx^8\right)c^2+\frac{1}{165}\,\left(45\,a^2d^4x^{11}+99\,ab^2d^3x^{10}+55\,b^4d^2x^9\right)c
\end{aligned}
$$

input
```
integrate((a*d^2*x^4+b^2*d*x^3+b^3*x+a*b^2+c*x^2)^3,x, algorithm="maxima")
```

output
```
1/13*a^3*d^6*x^13 + 1/4*a^2*b^2*d^5*x^12 + 3/11*a*b^4*d^4*x^11 + 1/10*b^6*
d^3*x^10 + 1/4*b^9*x^4 + a^3*b^6*x + 1/7*c^3*x^7 + 1/20*(12*a*d^2*x^5 + 15
*b^2*d*x^4 + 30*b^3*x^2 + 20*c*x^3)*a^2*b^4 + 1/70*(30*a*d^2*x^7 + 35*b^2*
d*x^6 + 42*c*x^5)*b^6 + 1/420*(140*a^2*d^4*x^9 + 315*a*b^2*d^3*x^8 + 180*b
^4*d^2*x^7 + 420*b^6*x^3 + 252*c^2*x^5 + 42*(10*a*d^2*x^6 + 12*b^2*d*x^5 +
 15*c*x^4)*b^3 + 60*(6*a*d^2*x^7 + 7*b^2*d*x^6)*c)*a*b^2 + 1/840*(252*a^2*
d^4*x^10 + 560*a*b^2*d^3*x^9 + 720*b^2*c*d*x^7 + 315*(b^4 + 2*a*c)*d^2*x^8
 + 420*c^2*x^6)*b^3 + 1/24*(8*a*d^2*x^9 + 9*b^2*d*x^8)*c^2 + 1/165*(45*a^2
*d^4*x^11 + 99*a*b^2*d^3*x^10 + 55*b^4*d^2*x^9)*c
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 389, normalized size of antiderivative = 1.22

$$\int \left(ab^2+b^3x+cx^2+b^2dx^3+ad^2x^4\right)^3 \, dx = \frac{1}{13}\,a^3d^6x^{13}+\frac{1}{4}\,a^2b^2d^5x^{12}+\frac{3}{11}\,ab^4d^4x^{11}+\frac{1}{10}\,b^6d^3x^{10}+\frac{3}{10}\,a^2b^3d^4x^{10}+\frac{2}{3}\,ab^5d^3x^9+\frac{1}{3}\,a^3b^2d^4x^9+\frac{3}{11}\,a^2cd^4x^{11}+\frac{3}{8}\,b^7d^2x^8+\frac{3}{4}\,a^2b^4d^3x^8+\frac{3}{5}\,ab^2cd^3x^{10}+\frac{6}{7}\,ab^6d^2x^7+\frac{1}{3}\,b^4cd^2x^9+\frac{1}{2}\,b^8dx^6+a^2b^5d^2x^6+\frac{3}{4}\,ab^3cd^2x^8+\frac{6}{5}\,ab^7dx^5+\frac{3}{5}\,a^3b^4d^2x^5+\frac{6}{7}\,b^5cdx^7+\frac{6}{7}\,a^2b^2cd^2x^7+\frac{1}{3}\,ac^2d^2x^9+\frac{1}{4}\,b^9x^4+\frac{3}{4}\,a^2b^6dx^4+ab^4cdx^6+\frac{3}{8}\,b^2c^2dx^8+ab^8x^3+\frac{3}{5}\,b^6cx^5+\frac{3}{2}\,a^2b^7x^2+\frac{3}{2}\,ab^5cx^4+\frac{1}{2}\,b^3c^2x^6+a^3b^6x+a^2b^4cx^3+\frac{3}{5}\,ab^2c^2x^5+\frac{1}{7}\,c^3x^7$$

input
```
integrate((a*d^2*x^4+b^2*d*x^3+b^3*x+a*b^2+c*x^2)^3,x, algorithm="giac")
```

output
```
1/13*a^3*d^6*x^13 + 1/4*a^2*b^2*d^5*x^12 + 3/11*a*b^4*d^4*x^11 + 1/10*b^6*
d^3*x^10 + 3/10*a^2*b^3*d^4*x^10 + 2/3*a*b^5*d^3*x^9 + 1/3*a^3*b^2*d^4*x^9
 + 3/11*a^2*c*d^4*x^11 + 3/8*b^7*d^2*x^8 + 3/4*a^2*b^4*d^3*x^8 + 3/5*a*b^2
*c*d^3*x^10 + 6/7*a*b^6*d^2*x^7 + 1/3*b^4*c*d^2*x^9 + 1/2*b^8*d*x^6 + a^2*
b^5*d^2*x^6 + 3/4*a*b^3*c*d^2*x^8 + 6/5*a*b^7*d*x^5 + 3/5*a^3*b^4*d^2*x^5
+ 6/7*b^5*c*d*x^7 + 6/7*a^2*b^2*c*d^2*x^7 + 1/3*a*c^2*d^2*x^9 + 1/4*b^9*x^
4 + 3/4*a^2*b^6*d*x^4 + a*b^4*c*d*x^6 + 3/8*b^2*c^2*d*x^8 + a*b^8*x^3 + 3/
5*b^6*c*x^5 + 3/2*a^2*b^7*x^2 + 3/2*a*b^5*c*x^4 + 1/2*b^3*c^2*x^6 + a^3*b^
6*x + a^2*b^4*c*x^3 + 3/5*a*b^2*c^2*x^5 + 1/7*c^3*x^7
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 311, normalized size of antiderivative = 0.97

$$\int \left(ab^2+b^3x+cx^2+b^2dx^3+ad^2x^4\right)^3\,dx = x^4\left(\frac{3\,d\,a^2\,b^6}{4}+\frac{3\,c\,a\,b^5}{2}+\frac{b^9}{4}\right)+x^5\left(\frac{3\,a^3\,b^4\,d^2}{5}+\frac{6\,a\,b^7\,d}{5}+\frac{3\,a\,b^2\,c^2}{5}+\frac{3\,b^6\,c}{5}\right)+x^6\left(a^2\,b^5\,d^2+a\,b^4\,c\,d+\frac{b^8\,d}{2}+\frac{b^3\,c^2}{2}\right)+x^7\left(\frac{6\,a^2\,b^2\,c\,d^2}{7}+\frac{6\,a\,b^6\,d^2}{7}+\frac{6\,b^5\,c\,d}{7}+\frac{c^3}{7}\right)+a^3\,b^6\,x+\frac{3\,a^2\,b^7\,x^2}{2}+\frac{a^3\,d^6\,x^{13}}{13}+\frac{d^2\,x^9\,\left(a^3\,b^2\,d^2+2\,a\,b^5\,d+a\,c^2+b^4\,c\right)}{3}+\frac{a^2\,b^2\,d^5\,x^{12}}{4}+\frac{3\,b^2\,d\,x^8\,\left(2\,a^2\,b^2\,d^2+2\,a\,b\,c\,d+b^5\,d+c^2\right)}{8}+a\,b^4\,x^3\,\left(b^4+a\,c\right)+\frac{3\,a\,d^4\,x^{11}\,\left(b^4+a\,c\right)}{11}+\frac{b^2\,d^3\,x^{10}\,\left(3\,d\,a^2\,b+6\,c\,a+b^4\right)}{10}$$

input

```
int((a*b^2 + b^3*x + c*x^2 + a*d^2*x^4 + b^2*d*x^3)^3,x)
```

output

```
x^4*(b^9/4 + (3*a^2*b^6*d)/4 + (3*a*b^5*c)/2) + x^5*((3*b^6*c)/5 + (3*a*b^
2*c^2)/5 + (3*a^3*b^4*d^2)/5 + (6*a*b^7*d)/5) + x^6*((b^8*d)/2 + (b^3*c^2)
/2 + a^2*b^5*d^2 + a*b^4*c*d) + x^7*(c^3/7 + (6*a*b^6*d^2)/7 + (6*b^5*c*d)
/7 + (6*a^2*b^2*c*d^2)/7) + a^3*b^6*x + (3*a^2*b^7*x^2)/2 + (a^3*d^6*x^13)
/13 + (d^2*x^9*(a*c^2 + b^4*c + a^3*b^2*d^2 + 2*a*b^5*d))/3 + (a^2*b^2*d^5
*x^12)/4 + (3*b^2*d*x^8*(b^5*d + c^2 + 2*a^2*b^2*d^2 + 2*a*b*c*d))/8 + a*b
^4*x^3*(a*c + b^4) + (3*a*d^4*x^11*(a*c + b^4))/11 + (b^2*d^3*x^10*(6*a*c
+ b^4 + 3*a^2*b*d))/10
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 394, normalized size of antiderivative = 1.23

$$\int \left(ab^2 + b^3x + cx^2 + b^2dx^3 + ad^2x^4\right)^3 dx$$
$$= \frac{x(9240a^3d^6x^{12} + 30030a^2b^2d^5x^{11} + 32760a\,b^4d^4x^{10} + 36036a^2b^3d^4x^9 + 12012b^6d^3x^9 + 40040a^3b^2d^4x^8 +}{}$$

input
```
int((a*d^2*x^4+b^2*d*x^3+b^3*x+a*b^2+c*x^2)^3,x)
```

output
```
(x*(120120*a**3*b**6 + 72072*a**3*b**4*d**2*x**4 + 40040*a**3*b**2*d**4*x*
*8 + 9240*a**3*d**6*x**12 + 180180*a**2*b**7*x + 90090*a**2*b**6*d*x**3 +
120120*a**2*b**5*d**2*x**5 + 120120*a**2*b**4*c*x**2 + 90090*a**2*b**4*d**
3*x**7 + 36036*a**2*b**3*d**4*x**9 + 102960*a**2*b**2*c*d**2*x**6 + 30030*
a**2*b**2*d**5*x**11 + 32760*a**2*c*d**4*x**10 + 120120*a*b**8*x**2 + 1441
44*a*b**7*d*x**4 + 102960*a*b**6*d**2*x**6 + 180180*a*b**5*c*x**3 + 80080*
a*b**5*d**3*x**8 + 120120*a*b**4*c*d*x**5 + 32760*a*b**4*d**4*x**10 + 9009
0*a*b**3*c*d**2*x**7 + 72072*a*b**2*c**2*x**4 + 72072*a*b**2*c*d**3*x**9 +
 40040*a*c**2*d**2*x**8 + 30030*b**9*x**3 + 60060*b**8*d*x**5 + 45045*b**7
*d**2*x**7 + 72072*b**6*c*x**4 + 12012*b**6*d**3*x**9 + 102960*b**5*c*d*x*
*6 + 40040*b**4*c*d**2*x**8 + 60060*b**3*c**2*x**5 + 45045*b**2*c**2*d*x**
7 + 17160*c**3*x**6))/120120
```

3.27 $\int \left(ab^2+b^3x+cx^2+b^2dx^3+ad^2x^4\right)^2\,dx$

Optimal result

Integrand size = 34, antiderivative size = 139

$$\int \left(ab^2+b^3x+cx^2+b^2dx^3+ad^2x^4\right)^2\,dx=a^2b^4x+ab^5x^2+\frac{1}{3}b^2\left(b^4+2ac\right)x^3+\frac{1}{2}b^3(c+abd)x^4+\frac{1}{5}\left(c^2+2b^2d\left(b^3+a^2d\right)\right)x^5+\frac{1}{3}b^2d(c+abd)x^6+\frac{1}{7}\left(b^4+2ac\right)d^2x^7+\frac{1}{4}ab^2d^3x^8+\frac{1}{9}a^2d^4x^9$$

output
```
a^2*b^4*x+a*b^5*x^2+1/3*b^2*(b^4+2*a*c)*x^3+1/2*b^3*(a*b*d+c)*x^4+1/5*(c^2
+2*b^2*d*(a^2*d+b^3))*x^5+1/3*b^2*d*(a*b*d+c)*x^6+1/7*(b^4+2*a*c)*d^2*x^7+
1/4*a*b^2*d^3*x^8+1/9*a^2*d^4*x^9
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.01

$$\int \left(ab^2+b^3x+cx^2+b^2dx^3+ad^2x^4\right)^2\,dx=a^2b^4x+ab^5x^2+\frac{1}{3}b^2\left(b^4+2ac\right)x^3+\frac{1}{2}b^3(c+abd)x^4+\frac{1}{5}\left(c^2+2b^5d+2a^2b^2d^2\right)x^5+\frac{1}{3}b^2d(c+abd)x^6+\frac{1}{7}\left(b^4+2ac\right)d^2x^7+\frac{1}{4}ab^2d^3x^8+\frac{1}{9}a^2d^4x^9$$

input
```
Integrate[(a*b^2 + b^3*x + c*x^2 + b^2*d*x^3 + a*d^2*x^4)^2,x]
```

output
```
a^2*b^4*x + a*b^5*x^2 + (b^2*(b^4 + 2*a*c)*x^3)/3 + (b^3*(c + a*b*d)*x^4)/
2 + ((c^2 + 2*b^5*d + 2*a^2*b^2*d^2)*x^5)/5 + (b^2*d*(c + a*b*d)*x^6)/3 +
((b^4 + 2*a*c)*d^2*x^7)/7 + (a*b^2*d^3*x^8)/4 + (a^2*d^4*x^9)/9
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}=0.059$, Rules used = {2465, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(ab^2+ad^2x^4+b^3x+b^2dx^3+cx^2\right)^2\,dx$$

$$\Big\downarrow \text{2465}$$

$$\int \left(a^2b^4+x^4\left(2b^2d\left(a^2d+b^3\right)+c^2\right)+a^2d^4x^8+2ab^5x+d^2x^6\left(2ac+b^4\right)+2b^3x^3(abd+c)+2b^2dx^5(abd+c)+2$$

$$\Big\downarrow \text{2009}$$

$$a^2b^4x + \frac{1}{5}x^5\left(2b^2d\left(a^2d + b^3\right) + c^2\right) + \frac{1}{9}a^2d^4x^9 + ab^5x^2 + \frac{1}{7}d^2x^7\left(2ac + b^4\right) + \frac{1}{2}b^3x^4(abd + c) + \frac{1}{3}b^2dx^6(abd + c) + \frac{1}{4}ab^2d^3x^8 + \frac{1}{3}b^2x^3\left(2ac + b^4\right)$$

input
```
Int[(a*b^2 + b^3*x + c*x^2 + b^2*d*x^3 + a*d^2*x^4)^2,x]
```

output
```
a^2*b^4*x + a*b^5*x^2 + (b^2*(b^4 + 2*a*c)*x^3)/3 + (b^3*(c + a*b*d)*x^4)/
2 + ((c^2 + 2*b^2*d*(b^3 + a^2*d))*x^5)/5 + (b^2*d*(c + a*b*d)*x^6)/3 + ((
b^4 + 2*a*c)*d^2*x^7)/7 + (a*b^2*d^3*x^8)/4 + (a^2*d^4*x^9)/9
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2465
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> Int[ExpandToSum[u, Px^p, x], x] /; PolyQ
[Px, x] && GtQ[Expon[Px, x], 2] && !BinomialQ[Px, x] && !TrinomialQ[Px, x
] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.04

method	result
default	$\frac{a^2d^4x^9}{9} + \frac{a\,b^2d^3x^8}{4} + \frac{\left(b^4d^2+2ac\,d^2\right)x^7}{7} + \frac{\left(2b^3a\,d^2+2b^2cd\right)x^6}{6} + \frac{\left(2a^2b^2d^2+2b^5d+c^2\right)x^5}{5} + \frac{\left(2a\,b^4d+2b^3c\right)x^4}{4} + \frac{(b^6$
norman	$\frac{a^2d^4x^9}{9} + \frac{a\,b^2d^3x^8}{4} + \left(\frac{1}{7}b^4d^2 + \frac{2}{7}ac\,d^2\right)x^7 + \left(\frac{1}{3}b^3a\,d^2 + \frac{1}{3}b^2cd\right)x^6 + \left(\frac{2}{5}a^2b^2d^2 + \frac{2}{5}b^5d + \frac{1}{5}c^2\right)x^5$
gosper	$\frac{1}{9}a^2d^4x^9 + \frac{1}{4}a\,b^2d^3x^8 + \frac{1}{7}x^7b^4d^2 + \frac{2}{7}x^7ac\,d^2 + \frac{1}{3}x^6b^3a\,d^2 + \frac{1}{3}x^6b^2cd + \frac{2}{5}x^5a^2b^2d^2 + \frac{2}{5}x^5b^5d +$
risch	$\frac{1}{9}a^2d^4x^9 + \frac{1}{4}a\,b^2d^3x^8 + \frac{1}{7}x^7b^4d^2 + \frac{2}{7}x^7ac\,d^2 + \frac{1}{3}x^6b^3a\,d^2 + \frac{1}{3}x^6b^2cd + \frac{2}{5}x^5a^2b^2d^2 + \frac{2}{5}x^5b^5d +$
parallelrisch	$\frac{1}{9}a^2d^4x^9 + \frac{1}{4}a\,b^2d^3x^8 + \frac{1}{7}x^7b^4d^2 + \frac{2}{7}x^7ac\,d^2 + \frac{1}{3}x^6b^3a\,d^2 + \frac{1}{3}x^6b^2cd + \frac{2}{5}x^5a^2b^2d^2 + \frac{2}{5}x^5b^5d +$
orering	$\frac{x(140x^8d^4a^2+315b^2d^3x^7a+180b^4d^2x^6+420a\,b^3d^2x^5+504a^2b^2d^2x^4+360ac\,d^2x^6+504b^5d\,x^4+630a\,b^4d\,x^3+420b^2cd\,x^5+420}{1260}$

input
```
int((a*d^2*x^4+b^2*d*x^3+b^3*x+a*b^2+c*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/9*a^2*d^4*x^9+1/4*a*b^2*d^3*x^8+1/7*(b^4*d^2+2*a*c*d^2)*x^7+1/6*(2*a*b^3
*d^2+2*b^2*c*d)*x^6+1/5*(2*a^2*b^2*d^2+2*b^5*d+c^2)*x^5+1/4*(2*a*b^4*d+2*b
^3*c)*x^4+1/3*(b^6+2*a*b^2*c)*x^3+a*b^5*x^2+a^2*b^4*x
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.97

$$\int \left(ab^2+b^3x+cx^2+b^2dx^3+ad^2x^4\right)^2 dx = \frac{1}{9}a^2d^4x^9+\frac{1}{4}ab^2d^3x^8+\frac{1}{7}\left(b^4+2ac\right)d^2x^7 + ab^5x^2+a^2b^4x+\frac{1}{3}\left(ab^3d^2+b^2cd\right)x^6 + \frac{1}{5}\left(2b^5d+2a^2b^2d^2+c^2\right)x^5 + \frac{1}{2}\left(ab^4d+b^3c\right)x^4+\frac{1}{3}\left(b^6+2ab^2c\right)x^3$$

input
```
integrate((a*d^2*x^4+b^2*d*x^3+b^3*x+a*b^2+c*x^2)^2,x, algorithm="fricas")
```

output
```
1/9*a^2*d^4*x^9 + 1/4*a*b^2*d^3*x^8 + 1/7*(b^4 + 2*a*c)*d^2*x^7 + a*b^5*x^
2 + a^2*b^4*x + 1/3*(a*b^3*d^2 + b^2*c*d)*x^6 + 1/5*(2*b^5*d + 2*a^2*b^2*d
^2 + c^2)*x^5 + 1/2*(a*b^4*d + b^3*c)*x^4 + 1/3*(b^6 + 2*a*b^2*c)*x^3
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.10

$$\int \left(ab^2+b^3x+cx^2+b^2dx^3+ad^2x^4\right)^2 dx = a^2b^4x+\frac{a^2d^4x^9}{9}+ab^5x^2+\frac{ab^2d^3x^8}{4}+x^7 \cdot \left(\frac{2acd^2}{7}+\frac{b^4d^2}{7}\right)+x^6\left(\frac{ab^3d^2}{3}+\frac{b^2cd}{3}\right) + x^5\cdot\left(\frac{2a^2b^2d^2}{5}+\frac{2b^5d}{5}+\frac{c^2}{5}\right) + x^4\left(\frac{ab^4d}{2}+\frac{b^3c}{2}\right)+x^3\cdot\left(\frac{2ab^2c}{3}+\frac{b^6}{3}\right)$$

input
```
integrate((a*d**2*x**4+b**2*d*x**3+b**3*x+a*b**2+c*x**2)**2,x)
```

output
```
a**2*b**4*x + a**2*d**4*x**9/9 + a*b**5*x**2 + a*b**2*d**3*x**8/4 + x**7*(
2*a*c*d**2/7 + b**4*d**2/7) + x**6*(a*b**3*d**2/3 + b**2*c*d/3) + x**5*(2*
a**2*b**2*d**2/5 + 2*b**5*d/5 + c**2/5) + x**4*(a*b**4*d/2 + b**3*c/2) + x
**3*(2*a*b**2*c/3 + b**6/3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.08

$$\int \left(ab^2+b^3x+cx^2+b^2dx^3+ad^2x^4\right)^2\,dx = \frac{1}{9}\,a^2d^4x^9+\frac{1}{4}\,ab^2d^3x^8+\frac{1}{7}\,b^4d^2x^7+\frac{1}{3}\,b^6x^3+a^2b^4x + \frac{1}{5}\,c^2x^5+\frac{1}{30}\,\left(12\,ad^2x^5+15\,b^2dx^4+30\,b^3x^2+20\,cx^3\right)ab^2 + \frac{1}{30}\,\left(10\,ad^2x^6+12\,b^2dx^5+15\,cx^4\right)b^3+\frac{1}{21}\,\left(6\,ad^2x^7+7\,b^2dx^6\right)c$$

input
```
integrate((a*d^2*x^4+b^2*d*x^3+b^3*x+a*b^2+c*x^2)^2,x, algorithm="maxima")
```

output
```
1/9*a^2*d^4*x^9 + 1/4*a*b^2*d^3*x^8 + 1/7*b^4*d^2*x^7 + 1/3*b^6*x^3 + a^2*
b^4*x + 1/5*c^2*x^5 + 1/30*(12*a*d^2*x^5 + 15*b^2*d*x^4 + 30*b^3*x^2 + 20*
c*x^3)*a*b^2 + 1/30*(10*a*d^2*x^6 + 12*b^2*d*x^5 + 15*c*x^4)*b^3 + 1/21*(6
*a*d^2*x^7 + 7*b^2*d*x^6)*c
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.09

$$\int \left(ab^2+b^3x+cx^2+b^2dx^3+ad^2x^4\right)^2\,dx = \frac{1}{9}\,a^2d^4x^9+\frac{1}{4}\,ab^2d^3x^8+\frac{1}{7}\,b^4d^2x^7+\frac{1}{3}\,ab^3d^2x^6 + \frac{2}{5}\,b^5dx^5+\frac{2}{5}\,a^2b^2d^2x^5+\frac{2}{7}\,acd^2x^7 + \frac{1}{2}\,ab^4dx^4+\frac{1}{3}\,b^2cdx^6+\frac{1}{3}\,b^6x^3+ab^5x^2 + \frac{1}{2}\,b^3cx^4+a^2b^4x+\frac{2}{3}\,ab^2cx^3+\frac{1}{5}\,c^2x^5$$

input
```
integrate((a*d^2*x^4+b^2*d*x^3+b^3*x+a*b^2+c*x^2)^2,x, algorithm="giac")
```

output
```
1/9*a^2*d^4*x^9 + 1/4*a*b^2*d^3*x^8 + 1/7*b^4*d^2*x^7 + 1/3*a*b^3*d^2*x^6
+ 2/5*b^5*d*x^5 + 2/5*a^2*b^2*d^2*x^5 + 2/7*a*c*d^2*x^7 + 1/2*a*b^4*d*x^4
+ 1/3*b^2*c*d*x^6 + 1/3*b^6*x^3 + a*b^5*x^2 + 1/2*b^3*c*x^4 + a^2*b^4*x +
2/3*a*b^2*c*x^3 + 1/5*c^2*x^5
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.93

$$\int \left(ab^2+b^3x+cx^2+b^2dx^3+ad^2x^4\right)^2\,dx = x^3\left(\frac{b^6}{3}+\frac{2\,a\,c\,b^2}{3}\right) + x^5\left(\frac{2\,a^2\,b^2\,d^2}{5}+\frac{2\,b^5\,d}{5}+\frac{c^2}{5}\right) + \frac{d^2\,x^7\,\left(b^4+2\,a\,c\right)}{7}+\frac{b^3\,x^4\,\left(c+a\,b\,d\right)}{2} + a^2\,b^4\,x+a\,b^5\,x^2+\frac{a^2\,d^4\,x^9}{9} + \frac{a\,b^2\,d^3\,x^8}{4}+\frac{b^2\,d\,x^6\,\left(c+a\,b\,d\right)}{3}$$

input
```
int((a*b^2 + b^3*x + c*x^2 + a*d^2*x^4 + b^2*d*x^3)^2,x)
```

output
```
x^3*(b^6/3 + (2*a*b^2*c)/3) + x^5*((2*b^5*d)/5 + c^2/5 + (2*a^2*b^2*d^2)/5
) + (d^2*x^7*(2*a*c + b^4))/7 + (b^3*x^4*(c + a*b*d))/2 + a^2*b^4*x + a*b^
5*x^2 + (a^2*d^4*x^9)/9 + (a*b^2*d^3*x^8)/4 + (b^2*d*x^6*(c + a*b*d))/3
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.10

$$\int \left(ab^2+b^3x+cx^2+b^2dx^3+ad^2x^4\right)^2\,dx$$
$$=\frac{x(140a^2d^4x^8+315a\,b^2d^3x^7+180b^4d^2x^6+420a\,b^3d^2x^5+504a^2b^2d^2x^4+360ac\,d^2x^6+504b^5d\,x^4+630a}{1260}$$

input
```
int((a*d^2*x^4+b^2*d*x^3+b^3*x+a*b^2+c*x^2)^2,x)
```

output
```
(x*(1260*a**2*b**4 + 504*a**2*b**2*d**2*x**4 + 140*a**2*d**4*x**8 + 1260*a
*b**5*x + 630*a*b**4*d*x**3 + 420*a*b**3*d**2*x**5 + 840*a*b**2*c*x**2 + 3
15*a*b**2*d**3*x**7 + 360*a*c*d**2*x**6 + 420*b**6*x**2 + 504*b**5*d*x**4
+ 180*b**4*d**2*x**6 + 630*b**3*c*x**3 + 420*b**2*c*d*x**5 + 252*c**2*x**4
))/1260
```

3.28 $\int (ab^2+b^3x+cx^2+b^2dx^3+ad^2x^4)\,dx$

Optimal result

Integrand size = 32, antiderivative size = 47

$$\int (ab^2+b^3x+cx^2+b^2dx^3+ad^2x^4)\,dx=ab^2x+\frac{b^3x^2}{2}+\frac{cx^3}{3}+\frac{1}{4}b^2dx^4+\frac{1}{5}ad^2x^5$$

output
```
a*b^2*x+1/2*b^3*x^2+1/3*c*x^3+1/4*b^2*d*x^4+1/5*a*d^2*x^5
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00

$$\int (ab^2+b^3x+cx^2+b^2dx^3+ad^2x^4)\,dx=ab^2x+\frac{b^3x^2}{2}+\frac{cx^3}{3}+\frac{1}{4}b^2dx^4+\frac{1}{5}ad^2x^5$$

input
```
Integrate[a*b^2 + b^3*x + c*x^2 + b^2*d*x^3 + a*d^2*x^4,x]
```

output
```
a*b^2*x + (b^3*x^2)/2 + (c*x^3)/3 + (b^2*d*x^4)/4 + (a*d^2*x^5)/5
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.031$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(ab^2+ad^2x^4+b^3x+b^2dx^3+cx^2\right) \, dx$$

$$\Big\downarrow \text{2009}$$

$$ab^2x+\frac{1}{5}ad^2x^5+\frac{b^3x^2}{2}+\frac{1}{4}b^2dx^4+\frac{cx^3}{3}$$

input
```
Int[a*b^2 + b^3*x + c*x^2 + b^2*d*x^3 + a*d^2*x^4,x]
```

output
```
a*b^2*x + (b^3*x^2)/2 + (c*x^3)/3 + (b^2*d*x^4)/4 + (a*d^2*x^5)/5
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.85

method	result	size
gosper	$x a\,b^{2}+\frac{1}{2} b^{3} x^{2}+\frac{1}{3} c\,x^{3}+\frac{1}{4} b^{2} d\,x^{4}+\frac{1}{5} a\,d^{2} x^{5}$	40
default	$x a\,b^{2}+\frac{1}{2} b^{3} x^{2}+\frac{1}{3} c\,x^{3}+\frac{1}{4} b^{2} d\,x^{4}+\frac{1}{5} a\,d^{2} x^{5}$	40
norman	$x a\,b^{2}+\frac{1}{2} b^{3} x^{2}+\frac{1}{3} c\,x^{3}+\frac{1}{4} b^{2} d\,x^{4}+\frac{1}{5} a\,d^{2} x^{5}$	40
risch	$x a\,b^{2}+\frac{1}{2} b^{3} x^{2}+\frac{1}{3} c\,x^{3}+\frac{1}{4} b^{2} d\,x^{4}+\frac{1}{5} a\,d^{2} x^{5}$	40
parallelrisch	$x a\,b^{2}+\frac{1}{2} b^{3} x^{2}+\frac{1}{3} c\,x^{3}+\frac{1}{4} b^{2} d\,x^{4}+\frac{1}{5} a\,d^{2} x^{5}$	40
parts	$x a\,b^{2}+\frac{1}{2} b^{3} x^{2}+\frac{1}{3} c\,x^{3}+\frac{1}{4} b^{2} d\,x^{4}+\frac{1}{5} a\,d^{2} x^{5}$	40
orering	$\frac{x\left(12 a\,d^{2} x^{4}+15 b^{2} d\,x^{3}+30 b^{3} x+60 b^{2} a+20 c\,x^{2}\right)}{60}$	41

input
```
int(a*d^2*x^4+b^2*d*x^3+b^3*x+a*b^2+c*x^2,x,method=_RETURNVERBOSE)
```

output
```
x*a*b^2+1/2*b^3*x^2+1/3*c*x^3+1/4*b^2*d*x^4+1/5*a*d^2*x^5
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.83

$$\int \left(ab^2+b^3x+cx^2+b^2dx^3+ad^2x^4\right)\,dx=\frac{1}{5}\,ad^2x^5+\frac{1}{4}\,b^2dx^4+\frac{1}{2}\,b^3x^2+ab^2x+\frac{1}{3}\,cx^3$$

input
```
integrate(a*d^2*x^4+b^2*d*x^3+b^3*x+a*b^2+c*x^2,x, algorithm="fricas")
```

output
```
1/5*a*d^2*x^5 + 1/4*b^2*d*x^4 + 1/2*b^3*x^2 + a*b^2*x + 1/3*c*x^3
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.87

$$\int \left(ab^2 + b^3x + cx^2 + b^2dx^3 + ad^2x^4\right) dx = ab^2x + \frac{ad^2x^5}{5} + \frac{b^3x^2}{2} + \frac{b^2dx^4}{4} + \frac{cx^3}{3}$$

input
```
integrate(a*d**2*x**4+b**2*d*x**3+b**3*x+a*b**2+c*x**2,x)
```

output
```
a*b**2*x + a*d**2*x**5/5 + b**3*x**2/2 + b**2*d*x**4/4 + c*x**3/3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.83

$$\int \left(ab^2 + b^3x + cx^2 + b^2dx^3 + ad^2x^4\right) dx = \frac{1}{5}\,ad^2x^5 + \frac{1}{4}\,b^2dx^4 + \frac{1}{2}\,b^3x^2 + ab^2x + \frac{1}{3}\,cx^3$$

input
```
integrate(a*d^2*x^4+b^2*d*x^3+b^3*x+a*b^2+c*x^2,x, algorithm="maxima")
```

output
```
1/5*a*d^2*x^5 + 1/4*b^2*d*x^4 + 1/2*b^3*x^2 + a*b^2*x + 1/3*c*x^3
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.83

$$\int \left(ab^2 + b^3x + cx^2 + b^2dx^3 + ad^2x^4\right) dx = \frac{1}{5}\,ad^2x^5 + \frac{1}{4}\,b^2dx^4 + \frac{1}{2}\,b^3x^2 + ab^2x + \frac{1}{3}\,cx^3$$

input
```
integrate(a*d^2*x^4+b^2*d*x^3+b^3*x+a*b^2+c*x^2,x, algorithm="giac")
```

output
```
1/5*a*d^2*x^5 + 1/4*b^2*d*x^4 + 1/2*b^3*x^2 + a*b^2*x + 1/3*c*x^3
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.83

$$\int \left(ab^2+b^3x+cx^2+b^2dx^3+ad^2x^4\right)\,dx=\frac{b^3\,x^2}{2}+\frac{b^2\,d\,x^4}{4}+a\,b^2\,x+\frac{a\,d^2\,x^5}{5}+\frac{c\,x^3}{3}$$

input
```
int(a*b^2 + b^3*x + c*x^2 + a*d^2*x^4 + b^2*d*x^3,x)
```

output
```
(c*x^3)/3 + (b^3*x^2)/2 + (a*d^2*x^5)/5 + (b^2*d*x^4)/4 + a*b^2*x
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.85

$$\int \left(ab^2+b^3x+cx^2+b^2dx^3+ad^2x^4\right)\,dx$$
$$=\frac{x(12a\,d^2x^4+15b^2d\,x^3+30b^3x+60a\,b^2+20c\,x^2)}{60}$$

input
```
int(a*d^2*x^4+b^2*d*x^3+b^3*x+a*b^2+c*x^2,x)
```

output
```
(x*(60*a*b**2 + 12*a*d**2*x**4 + 30*b**3*x + 15*b**2*d*x**3 + 20*c*x**2))/
60
```

3.29 $\int \frac{1}{ab^2+b^3x+cx^2+b^2dx^3+ad^2x^4}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 479

$$\begin{aligned}\int \frac{1}{ab^2+b^3x+cx^2+b^2dx^3+ad^2x^4}\,dx &= \frac{\left(b^2-\sqrt{b^4-4ac+8a^2bd}\right)\operatorname{arctanh}\left(\frac{b^2-\sqrt{b^4-4ac+8a^2bd}+4adx}{\sqrt{2}\sqrt{b^4-2ac-4a^2bd-b^2\sqrt{b^4-4ac+8a^2bd}}}\right)}{\sqrt{2}b\sqrt{b^4-4ac+8a^2bd}\sqrt{b^4-2ac-4a^2bd-b^2\sqrt{b^4-4ac+8a^2bd}}} \\ &\quad - \frac{\left(b^2+\sqrt{b^4-4ac+8a^2bd}\right)\operatorname{arctanh}\left(\frac{b^2+\sqrt{b^4-4ac+8a^2bd}+4adx}{\sqrt{2}\sqrt{b^4-2ac-4a^2bd+b^2\sqrt{b^4-4ac+8a^2bd}}}\right)}{\sqrt{2}b\sqrt{b^4-4ac+8a^2bd}\sqrt{b^4-2ac-4a^2bd+b^2\sqrt{b^4-4ac+8a^2bd}}} \\ &\quad - \frac{\log\left(2ab+b^2x-\sqrt{b^4-4ac+8a^2bd}x+2adx^2\right)}{2b\sqrt{b^4-4ac+8a^2bd}} \\ &\quad + \frac{\log\left(2ab+b^2x+\sqrt{b^4-4ac+8a^2bd}x+2adx^2\right)}{2b\sqrt{b^4-4ac+8a^2bd}}\end{aligned}$$

output
```
1/2*(b^2-(8*a^2*b*d+b^4-4*a*c)^(1/2))*arctanh(1/2*(b^2-(8*a^2*b*d+b^4-4*a*
c)^(1/2)+4*a*d*x)*2^(1/2)/(b^4-2*a*c-4*a^2*b*d-b^2*(8*a^2*b*d+b^4-4*a*c)^(
1/2))^(1/2))*2^(1/2)/b/(8*a^2*b*d+b^4-4*a*c)^(1/2)/(b^4-2*a*c-4*a^2*b*d-b^
2*(8*a^2*b*d+b^4-4*a*c)^(1/2))^(1/2)-1/2*(b^2+(8*a^2*b*d+b^4-4*a*c)^(1/2))
*arctanh(1/2*(b^2+(8*a^2*b*d+b^4-4*a*c)^(1/2)+4*a*d*x)*2^(1/2)/(b^4-2*a*c-
4*a^2*b*d+b^2*(8*a^2*b*d+b^4-4*a*c)^(1/2))^(1/2))*2^(1/2)/b/(8*a^2*b*d+b^4
-4*a*c)^(1/2)/(b^4-2*a*c-4*a^2*b*d+b^2*(8*a^2*b*d+b^4-4*a*c)^(1/2))^(1/2)-
1/2*ln(2*a*b+b^2*x-(8*a^2*b*d+b^4-4*a*c)^(1/2)*x+2*a*d*x^2)/b/(8*a^2*b*d+b
^4-4*a*c)^(1/2)+1/2*ln(2*a*b+b^2*x+(8*a^2*b*d+b^4-4*a*c)^(1/2)*x+2*a*d*x^2
)/b/(8*a^2*b*d+b^4-4*a*c)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.16

$$\int \frac{1}{ab^2+b^3x+cx^2+b^2dx^3+ad^2x^4}\,dx = \text{RootSum}\left[ab^2+b^3\#1+c\#1^2+b^2d\#1^3+ad^2\#1^4\&,\frac{\log(x-\#1)}{b^3+2c\#1+3b^2d\#1^2+4ad^2\#1^3}\&\right]$$

input
```
Integrate[(a*b^2 + b^3*x + c*x^2 + b^2*d*x^3 + a*d^2*x^4)^(-1),x]
```

output
```
RootSum[a*b^2 + b^3*#1 + c*#1^2 + b^2*d*#1^3 + a*d^2*#1^4 & , Log[x - #1]/
(b^3 + 2*c*#1 + 3*b^2*d*#1^2 + 4*a*d^2*#1^3) & ]
```

Rubi [A] (verified)

Time = 2.14 (sec) , antiderivative size = 613, normalized size of antiderivative = 1.28, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {2492, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{ab^2+ad^2x^4+b^3x+b^2dx^3+cx^2}\,dx$$

$$\Big\downarrow \text{2492}$$

$$\frac{\int \left(\frac{ad^3\left(db^2+2ad^2x+\sqrt{d^2(b^4+8a^2db-4ac)}\right)}{b\sqrt{d^2(b^4+8a^2db-4ac)}\left(2ad^2x^2+\left(db^2+\sqrt{d^2(b^4+8a^2db-4ac)}\right)x+2abd\right)}-\frac{ad^3\left(db^2+2ad^2x-\sqrt{d^2(b^4+8a^2db-4ac)}\right)}{b\sqrt{d^2(b^4+8a^2db-4ac)}\left(2ad^2x^2+\left(b^2d-\sqrt{d^2(b^4+8a^2db-4ac)}\right)\right.}}{ad^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{ad^{5/2}\left(b^2d-\sqrt{d^2(8a^2bd-4ac+b^4)}\right)\text{arctanh}\left(\frac{-\sqrt{d^2\left(8a^2bd-4ac+b^4\right)}+4ad^2x+b^2d}{\sqrt{2}\sqrt{d}\sqrt{-b^2\sqrt{d^2\left(8a^2bd-4ac+b^4\right)}-4a^2bd^2-2acd+b^4d}}\right)}{\sqrt{2}b\sqrt{d^2(8a^2bd-4ac+b^4)}\sqrt{-b^2\sqrt{d^2(8a^2bd-4ac+b^4)}-4a^2bd^2-2acd+b^4d}}-\frac{ad^{5/2}\left(\sqrt{d^2(8a^2bd-4ac+b^4)}+b^2d\right)\text{arct}}{\sqrt{2}b\sqrt{d^2(8a^2bd-4ac+b^4)}\sqrt{}}$$

input
```
Int[(a*b^2 + b^3*x + c*x^2 + b^2*d*x^3 + a*d^2*x^4)^(-1),x]
```

output
```
((a*d^(5/2)*(b^2*d - Sqrt[d^2*(b^4 - 4*a*c + 8*a^2*b*d)])*ArcTanh[(b^2*d -
 Sqrt[d^2*(b^4 - 4*a*c + 8*a^2*b*d)] + 4*a*d^2*x)/(Sqrt[2]*Sqrt[d]*Sqrt[b^
4*d - 2*a*c*d - 4*a^2*b*d^2 - b^2*Sqrt[d^2*(b^4 - 4*a*c + 8*a^2*b*d)]])])/
(Sqrt[2]*b*Sqrt[d^2*(b^4 - 4*a*c + 8*a^2*b*d)]*Sqrt[b^4*d - 2*a*c*d - 4*a^
2*b*d^2 - b^2*Sqrt[d^2*(b^4 - 4*a*c + 8*a^2*b*d)]]) - (a*d^(5/2)*(b^2*d +
Sqrt[d^2*(b^4 - 4*a*c + 8*a^2*b*d)])*ArcTanh[(b^2*d + Sqrt[d^2*(b^4 - 4*a*
c + 8*a^2*b*d)] + 4*a*d^2*x)/(Sqrt[2]*Sqrt[d]*Sqrt[b^4*d - 2*a*c*d - 4*a^2
*b*d^2 + b^2*Sqrt[d^2*(b^4 - 4*a*c + 8*a^2*b*d)]])])/(Sqrt[2]*b*Sqrt[d^2*(
b^4 - 4*a*c + 8*a^2*b*d)]*Sqrt[b^4*d - 2*a*c*d - 4*a^2*b*d^2 + b^2*Sqrt[d^
2*(b^4 - 4*a*c + 8*a^2*b*d)]]) - (a*d^3*Log[2*a*b*d + (b^2*d - Sqrt[d^2*(b
^4 - 4*a*c + 8*a^2*b*d)])*x + 2*a*d^2*x^2])/(2*b*Sqrt[d^2*(b^4 - 4*a*c + 8
*a^2*b*d)]) + (a*d^3*Log[2*a*b*d + (b^2*d + Sqrt[d^2*(b^4 - 4*a*c + 8*a^2*
b*d)])*x + 2*a*d^2*x^2])/(2*b*Sqrt[d^2*(b^4 - 4*a*c + 8*a^2*b*d)]))/(a*d^2
)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2492
```
Int[(Px_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^3 + (e_.)*(x_)^4)
^(p_), x_Symbol] :> Simp[e^p   Int[ExpandIntegrand[Px*(b/d + ((d + Sqrt[e*(
(b^2 - 4*a*c)/a) + 8*a*d*(e/b)])/(2*e))*x + x^2)^p*(b/d + ((d - Sqrt[e*((b^
2 - 4*a*c)/a) + 8*a*d*(e/b)])/(2*e))*x + x^2)^p, x], x], x] /; FreeQ[{a, b,
 c, d, e}, x] && PolyQ[Px, x] && ILtQ[p, 0] && EqQ[a*d^2 - b^2*e, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.09 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.15

method	result	size
default	$\sum_{\_R=\mathrm{RootOf}\left(a\,d^{2}\_Z^{4}+d\,b^{2}\_Z^{3}+b^{3}\_Z+\_Z^{2}c+b^{2}a\right)}\frac{\ln\left(x-\_R\right)}{4\_R^{3}a\,d^{2}+3\_R^{2}b^{2}d+b^{3}+2\_Rc}$	72
risch	$\sum_{\_R=\mathrm{RootOf}\left(a\,d^{2}\_Z^{4}+d\,b^{2}\_Z^{3}+b^{3}\_Z+\_Z^{2}c+b^{2}a\right)}\frac{\ln\left(x-\_R\right)}{4\_R^{3}a\,d^{2}+3\_R^{2}b^{2}d+b^{3}+2\_Rc}$	72

input
```
int(1/(a*d^2*x^4+b^2*d*x^3+b^3*x+a*b^2+c*x^2),x,method=_RETURNVERBOSE)
```

output
```
sum(1/(4*_R^3*a*d^2+3*_R^2*b^2*d+b^3+2*_R*c)*ln(x-_R),_R=RootOf(_Z^4*a*d^2
+_Z^3*b^2*d+_Z*b^3+_Z^2*c+a*b^2))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{ab^2+b^3x+cx^2+b^2dx^3+ad^2x^4}\,dx=\text{Timed out}$$

input
```
integrate(1/(a*d^2*x^4+b^2*d*x^3+b^3*x+a*b^2+c*x^2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{ab^2+b^3x+cx^2+b^2dx^3+ad^2x^4}\,dx=\text{Timed out}$$

input
```
integrate(1/(a*d**2*x**4+b**2*d*x**3+b**3*x+a*b**2+c*x**2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{ab^2 + b^3x + cx^2 + b^2dx^3 + ad^2x^4}\,dx = \int \frac{1}{ad^2x^4 + b^2dx^3 + b^3x + ab^2 + cx^2}\,dx$$

input
```
integrate(1/(a*d^2*x^4+b^2*d*x^3+b^3*x+a*b^2+c*x^2),x, algorithm="maxima")
```

output
```
integrate(1/(a*d^2*x^4 + b^2*d*x^3 + b^3*x + a*b^2 + c*x^2), x)
```

Giac [F]

$$\int \frac{1}{ab^2 + b^3x + cx^2 + b^2dx^3 + ad^2x^4}\,dx = \int \frac{1}{ad^2x^4 + b^2dx^3 + b^3x + ab^2 + cx^2}\,dx$$

input
```
integrate(1/(a*d^2*x^4+b^2*d*x^3+b^3*x+a*b^2+c*x^2),x, algorithm="giac")
```

output
```
integrate(1/(a*d^2*x^4 + b^2*d*x^3 + b^3*x + a*b^2 + c*x^2), x)
```

Mupad [B] (verification not implemented)

Time = 23.62 (sec) , antiderivative size = 5563, normalized size of antiderivative = 11.61

$$\int \frac{1}{ab^2 + b^3x + cx^2 + b^2dx^3 + ad^2x^4}\,dx = \text{Too large to display}$$

input
```
int(1/(a*b^2 + b^3*x + c*x^2 + a*d^2*x^4 + b^2*d*x^3),x)
```

output
```
symsum(log(4*root(288*a^3*b^8*c*d^2*z^4 - 80*a^2*b^7*c^2*d*z^4 - 128*a^4*b
^4*c^2*d^2*z^4 + 36*a*b^11*c*d*z^4 + 256*a^6*b^6*d^4*z^4 - 192*a^4*b^9*d^3
*z^4 - 60*a^2*b^12*d^2*z^4 + 16*a^2*b^2*c^4*z^4 - 8*a*b^6*c^3*z^4 - 4*b^15
*d*z^4 + b^10*c^2*z^4 - 16*a^3*b^2*c*d^2*z^2 - 14*a*b^5*c*d*z^2 + 24*a^2*b
^6*d^2*z^2 - b^4*c^2*z^2 + 3*b^9*d*z^2 + 4*a*c^3*z^2 - 4*a*b^2*c*d*z + 8*a
^2*b^3*d^2*z + b^6*d*z + a^2*d^2, z, k)^2*a^2*c^2*d^4 - 16*root(288*a^3*b^
8*c*d^2*z^4 - 80*a^2*b^7*c^2*d*z^4 - 128*a^4*b^4*c^2*d^2*z^4 + 36*a*b^11*c
*d*z^4 + 256*a^6*b^6*d^4*z^4 - 192*a^4*b^9*d^3*z^4 - 60*a^2*b^12*d^2*z^4 +
 16*a^2*b^2*c^4*z^4 - 8*a*b^6*c^3*z^4 - 4*b^15*d*z^4 + b^10*c^2*z^4 - 16*a
^3*b^2*c*d^2*z^2 - 14*a*b^5*c*d*z^2 + 24*a^2*b^6*d^2*z^2 - b^4*c^2*z^2 + 3
*b^9*d*z^2 + 4*a*c^3*z^2 - 4*a*b^2*c*d*z + 8*a^2*b^3*d^2*z + b^6*d*z + a^2
*d^2, z, k)^2*a^4*b^2*d^6 - 6*root(288*a^3*b^8*c*d^2*z^4 - 80*a^2*b^7*c^2*
d*z^4 - 128*a^4*b^4*c^2*d^2*z^4 + 36*a*b^11*c*d*z^4 + 256*a^6*b^6*d^4*z^4
- 192*a^4*b^9*d^3*z^4 - 60*a^2*b^12*d^2*z^4 + 16*a^2*b^2*c^4*z^4 - 8*a*b^6
*c^3*z^4 - 4*b^15*d*z^4 + b^10*c^2*z^4 - 16*a^3*b^2*c*d^2*z^2 - 14*a*b^5*c
*d*z^2 + 24*a^2*b^6*d^2*z^2 - b^4*c^2*z^2 + 3*b^9*d*z^2 + 4*a*c^3*z^2 - 4*
a*b^2*c*d*z + 8*a^2*b^3*d^2*z + b^6*d*z + a^2*d^2, z, k)^3*a^2*b^8*d^5 - 4
8*root(288*a^3*b^8*c*d^2*z^4 - 80*a^2*b^7*c^2*d*z^4 - 128*a^4*b^4*c^2*d^2*
z^4 + 36*a*b^11*c*d*z^4 + 256*a^6*b^6*d^4*z^4 - 192*a^4*b^9*d^3*z^4 - 60*a
^2*b^12*d^2*z^4 + 16*a^2*b^2*c^4*z^4 - 8*a*b^6*c^3*z^4 - 4*b^15*d*z^4 +...
```

Reduce [F]

$$\int \frac{1}{ab^2 + b^3x + cx^2 + b^2dx^3 + ad^2x^4}\,dx = \int \frac{1}{a\,d^2x^4 + b^2d\,x^3 + b^3x + a\,b^2 + c\,x^2}dx$$

input
```
int(1/(a*d^2*x^4+b^2*d*x^3+b^3*x+a*b^2+c*x^2),x)
```

output
```
int(1/(a*b**2 + a*d**2*x**4 + b**3*x + b**2*d*x**3 + c*x**2),x)
```

3.30 $\int \frac{1}{(ab^2+b^3x+cx^2+b^2dx^3+ad^2x^4)^2}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 1379

$$\int \frac{1}{(ab^2+b^3x+cx^2+b^2dx^3+ad^2x^4)^2}\,dx = \text{Too large to display}$$

output
```
(-2*b^2*c^2+6*b^7*d-2*a*b^3*c*d-12*a^2*b^4*d^2-2*(8*a^2*b*d+b^4-4*a*c)^(1/
2)*(-3*b^5*d+2*a*b*c*d+c^2)-4*a*d*(-3*b^5*d+2*a*b*c*d+c^2)*x)/a/b^2/d/(8*a
^2*b*d+b^4-4*a*c)/(4*a^2*b^2*d^2-4*b^5*d+4*a*b*c*d+c^2)/(2*b/d+(b^2+(8*a^2
*b*d+b^4-4*a*c)^(1/2))*x/a/d+2*x^2)+4*(b^4-2*a*c-b^2*(8*a^2*b*d+b^4-4*a*c)
^(1/2)+a*d*(b^2-(8*a^2*b*d+b^4-4*a*c)^(1/2))*x)/a/b/d^2/(8*a^2*b*d+b^4-4*a
*c)^(1/2)/(b^4-2*a*c-4*a^2*b*d-b^2*(8*a^2*b*d+b^4-4*a*c)^(1/2))/(2*b/d+(b^
2-(8*a^2*b*d+b^4-4*a*c)^(1/2))*x/a/d+2*x^2)/(2*b/d+(b^2+(8*a^2*b*d+b^4-4*a
*c)^(1/2))*x/a/d+2*x^2)+4*2^(1/2)*a^2*(4*b^7*c*d-18*a*b^8*d^2+8*a^2*b^4*c*
d^2+c^3*(8*a^2*b*d+b^4-4*a*c)^(1/2)+3*a*b*c^2*d*(8*a^2*b*d+b^4-4*a*c)^(1/2
)+6*a*b^6*d^2*(8*a^2*b*d+b^4-4*a*c)^(1/2)+4*b^5*(9*a^3*d^3-c*d*(8*a^2*b*d+
b^4-4*a*c)^(1/2))-b^2*(c^3+4*a^2*c*d^2*(8*a^2*b*d+b^4-4*a*c)^(1/2))+b^3*(a
*c^2*d-12*a^3*d^3*(8*a^2*b*d+b^4-4*a*c)^(1/2)))*arctanh(1/2*(b^2-(8*a^2*b*
d+b^4-4*a*c)^(1/2)+4*a*d*x)*2^(1/2)/(b^4-2*a*c-4*a^2*b*d-b^2*(8*a^2*b*d+b^
4-4*a*c)^(1/2))^(1/2))/b^3/(8*a^2*b*d+b^4-4*a*c)^(3/2)/(b^4-2*a*c-4*a^2*b*
d-b^2*(8*a^2*b*d+b^4-4*a*c)^(1/2))^(3/2)/(b^4-2*a*c-4*a^2*b*d+b^2*(8*a^2*b
*d+b^4-4*a*c)^(1/2))-4*2^(1/2)*a^2*(4*b^7*c*d-18*a*b^8*d^2+8*a^2*b^4*c*d^2
-c^3*(8*a^2*b*d+b^4-4*a*c)^(1/2)-3*a*b*c^2*d*(8*a^2*b*d+b^4-4*a*c)^(1/2)-6
*a*b^6*d^2*(8*a^2*b*d+b^4-4*a*c)^(1/2)+4*b^5*(9*a^3*d^3+c*d*(8*a^2*b*d+b^4
-4*a*c)^(1/2))+a*b^3*d*(c^2+12*a^2*d^2*(8*a^2*b*d+b^4-4*a*c)^(1/2))-b^2*(c
^3-4*a^2*c*d^2*(8*a^2*b*d+b^4-4*a*c)^(1/2)))*arctanh(1/2*(b^2+(8*a^2*b*...
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 1.04 (sec) , antiderivative size = 610, normalized size of antiderivative = 0.44

$$\int \frac{1}{(ab^2+b^3x+cx^2+b^2dx^3+ad^2x^4)^2}\,dx$$

$$= \frac{-\frac{-4b^8d+8ab^6d^2x-6b^7d^2x^2+2b^2cdx(-2a^2d+cx)+2c^2x(c+ad^2x^2)+4abcdx(c+ad^2x^2)+ab^4d(5c+6ad^2x^2)+b^3(c^2-8a^3d^3x+3acd^2x^2)-b^5d(\cdots}{(b^4-4ac+8a^2bd)(c^2+4abcd-4b^2d(b^3-a^2d))(x(b^3+cx+b^2dx^2)+a(b^2+d^2x^4))}}{\cdots}$$

input
```
Integrate[(a*b^2 + b^3*x + c*x^2 + b^2*d*x^3 + a*d^2*x^4)^(-2),x]
```

output
```
(-((-4*b^8*d + 8*a*b^6*d^2*x - 6*b^7*d^2*x^2 + 2*b^2*c*d*x*(-2*a^2*d + c*x
) + 2*c^2*x*(c + a*d^2*x^2) + 4*a*b*c*d*x*(c + a*d^2*x^2) + a*b^4*d*(5*c +
 6*a*d^2*x^2) + b^3*(c^2 - 8*a^3*d^3*x + 3*a*c*d^2*x^2) - b^5*d*(2*a^2*d +
 7*c*x + 6*a*d^2*x^3))/((b^4 - 4*a*c + 8*a^2*b*d)*(c^2 + 4*a*b*c*d - 4*b^2
*d*(b^3 - a^2*d))*(x*(b^3 + c*x + b^2*d*x^2) + a*(b^2 + d^2*x^4)))) + (2*R
ootSum[a*b^2 + b^3*#1 + c*#1^2 + b^2*d*#1^3 + a*d^2*#1^4 & , (c^3*Log[x -
#1] - 4*b^5*c*d*Log[x - #1] + 2*a*b*c^2*d*Log[x - #1] + 9*a*b^6*d^2*Log[x
- #1] - 6*a^2*b^2*c*d^2*Log[x - #1] - 12*a^3*b^3*d^3*Log[x - #1] + b^2*c^2
*d*Log[x - #1]*#1 - 3*b^7*d^2*Log[x - #1]*#1 + a*b^3*c*d^2*Log[x - #1]*#1
+ 6*a^2*b^4*d^3*Log[x - #1]*#1 + a*c^2*d^2*Log[x - #1]*#1^2 - 3*a*b^5*d^3*
Log[x - #1]*#1^2 + 2*a^2*b*c*d^3*Log[x - #1]*#1^2)/(b^3 + 2*c*#1 + 3*b^2*d
*#1^2 + 4*a*d^2*#1^3) & ])/(-(b^4*c^2) + 4*a*c^3 + 4*b^9*d - 20*a*b^5*c*d
+ 8*a^2*b*c^2*d + 28*a^2*b^6*d^2 - 16*a^3*b^2*c*d^2 - 32*a^4*b^3*d^3))/b^2
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ab^2+ad^2x^4+b^3x+b^2dx^3+cx^2)^2}dx$$

$$\Big\downarrow \text{2492}$$

$$\frac{\int \left(\frac{2a^2\left(-4ad^2b^3+cdb^2+3ad\sqrt{d^2(b^4+8a^2db-4ac)}b+2ad^2(c-3abd)x-c\sqrt{d^2(b^4+8a^2db-4ac)}\right)d^7}{b^3(d^2(b^4+8a^2db-4ac))^{3/2}\left(2ad^2x^2+\left(b^2d-\sqrt{d^2(b^4+8a^2db-4ac)}\right)x+2abd\right)}-\frac{2a^2\left(-4ad^2b^3+cdb^2-3ad\sqrt{d^2(b^4+8a^2db}\right.}{b^3(d^2(b^4+8a^2db-4ac))^{3/2}\left(2ad^2\right.}\right.}{}$$

$$\Big\downarrow \text{7239}$$

$$\frac{\int \frac{a^2d^4}{(ad^2x^4+b^2dx^3+cx^2+b^3x+ab^2)^2}dx}{a^2d^4}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{1}{(ab^2+ad^2x^4+b^3x+b^2dx^3+cx^2)^2}dx$$

$$\Big\downarrow \text{2492}$$

$$\int \left(\frac{2a^2\left(-4ad^2b^3+cdb^2+3ad\sqrt{d^2(b^4+8a^2db-4ac)}b+2ad^2(c-3abd)x-c\sqrt{d^2(b^4+8a^2db-4ac)}\right)d^7}{b^3(d^2(b^4+8a^2db-4ac))^{3/2}\left(2ad^2x^2+\left(b^2d-\sqrt{d^2(b^4+8a^2db-4ac)}\right)x+2abd\right)} - \frac{2a^2\left(-4ad^2b^3+cdb^2-3ad\sqrt{d^2(b^4+8a^2db\right.}{b^3(d^2(b^4+8a^2db-4ac))^{3/2}\left(2ad^2\right.}$$

$$\downarrow \text{7239}$$

$$\frac{\int \frac{a^2d^4}{(ad^2x^4+b^2dx^3+cx^2+b^3x+ab^2)^2}dx}{a^2d^4}$$

$$\downarrow \text{27}$$

$$\int \frac{1}{(ab^2+ad^2x^4+b^3x+b^2dx^3+cx^2)^2}dx$$

$$\downarrow \text{2492}$$

$$\int \left(\frac{2a^2\left(-4ad^2b^3+cdb^2+3ad\sqrt{d^2(b^4+8a^2db-4ac)}b+2ad^2(c-3abd)x-c\sqrt{d^2(b^4+8a^2db-4ac)}\right)d^7}{b^3(d^2(b^4+8a^2db-4ac))^{3/2}\left(2ad^2x^2+\left(b^2d-\sqrt{d^2(b^4+8a^2db-4ac)}\right)x+2abd\right)} - \frac{2a^2\left(-4ad^2b^3+cdb^2-3ad\sqrt{d^2(b^4+8a^2db\right.}{b^3(d^2(b^4+8a^2db-4ac))^{3/2}\left(2ad^2\right.}$$

$$\downarrow \text{7239}$$

$$\frac{\int \frac{a^2d^4}{(ad^2x^4+b^2dx^3+cx^2+b^3x+ab^2)^2}dx}{a^2d^4}$$

$$\downarrow \text{27}$$

$$\int \frac{1}{(ab^2+ad^2x^4+b^3x+b^2dx^3+cx^2)^2}dx$$

$$\downarrow \text{2492}$$

$$\int \left(\frac{2a^2\left(-4ad^2b^3+cdb^2+3ad\sqrt{d^2(b^4+8a^2db-4ac)}b+2ad^2(c-3abd)x-c\sqrt{d^2(b^4+8a^2db-4ac)}\right)d^7}{b^3(d^2(b^4+8a^2db-4ac))^{3/2}\left(2ad^2x^2+\left(b^2d-\sqrt{d^2(b^4+8a^2db-4ac)}\right)x+2abd\right)} - \frac{2a^2\left(-4ad^2b^3+cdb^2-3ad\sqrt{d^2(b^4+8a^2db\right.}{b^3(d^2(b^4+8a^2db-4ac))^{3/2}\left(2ad^2\right.}$$

$$\downarrow \text{7239}$$

$$\frac{\int \frac{a^2d^4}{(ad^2x^4+b^2dx^3+cx^2+b^3x+ab^2)^2}dx}{a^2d^4}$$

$$\downarrow \text{27}$$

$$\int \frac{1}{(ab^2+ad^2x^4+b^3x+b^2dx^3+cx^2)^2}dx$$

$$\downarrow \text{2492}$$

$$\int \left(\frac{2a^2 \left(-4ad^2b^3+cdb^2+3ad\sqrt{d^2(b^4+8a^2db-4ac)}b+2ad^2(c-3abd)x-c\sqrt{d^2(b^4+8a^2db-4ac)}\right)d^7}{b^3(d^2(b^4+8a^2db-4ac))^{3/2}\left(2ad^2x^2+\left(b^2d-\sqrt{d^2(b^4+8a^2db-4ac)}\right)x+2abd\right)} - \frac{2a^2\left(-4ad^2b^3+cdb^2-3ad\sqrt{d^2(b^4+8a^2db}\right.}{b^3(d^2(b^4+8a^2db-4ac))^{3/2}\left(2ad^2\right.}$$

$\downarrow$ 7239

$$\frac{\int \frac{a^2d^4}{(ad^2x^4+b^2dx^3+cx^2+b^3x+ab^2)^2}dx}{a^2d^4}$$

$\downarrow$ 27

$$\int \frac{1}{(ab^2+ad^2x^4+b^3x+b^2dx^3+cx^2)^2}dx$$

$\downarrow$ 2492

$$\int \left(\frac{2a^2 \left(-4ad^2b^3+cdb^2+3ad\sqrt{d^2(b^4+8a^2db-4ac)}b+2ad^2(c-3abd)x-c\sqrt{d^2(b^4+8a^2db-4ac)}\right)d^7}{b^3(d^2(b^4+8a^2db-4ac))^{3/2}\left(2ad^2x^2+\left(b^2d-\sqrt{d^2(b^4+8a^2db-4ac)}\right)x+2abd\right)} - \frac{2a^2\left(-4ad^2b^3+cdb^2-3ad\sqrt{d^2(b^4+8a^2db}\right.}{b^3(d^2(b^4+8a^2db-4ac))^{3/2}\left(2ad^2\right.}$$

$\downarrow$ 7239

$$\frac{\int \frac{a^2d^4}{(ad^2x^4+b^2dx^3+cx^2+b^3x+ab^2)^2}dx}{a^2d^4}$$

$\downarrow$ 27

$$\int \frac{1}{(ab^2+ad^2x^4+b^3x+b^2dx^3+cx^2)^2}dx$$

$\downarrow$ 2492

$$\int \left(\frac{2a^2 \left(-4ad^2b^3+cdb^2+3ad\sqrt{d^2(b^4+8a^2db-4ac)}b+2ad^2(c-3abd)x-c\sqrt{d^2(b^4+8a^2db-4ac)}\right)d^7}{b^3(d^2(b^4+8a^2db-4ac))^{3/2}\left(2ad^2x^2+\left(b^2d-\sqrt{d^2(b^4+8a^2db-4ac)}\right)x+2abd\right)} - \frac{2a^2\left(-4ad^2b^3+cdb^2-3ad\sqrt{d^2(b^4+8a^2db}\right.}{b^3(d^2(b^4+8a^2db-4ac))^{3/2}\left(2ad^2\right.}$$

$\downarrow$ 7239

$$\frac{\int \frac{a^2d^4}{(ad^2x^4+b^2dx^3+cx^2+b^3x+ab^2)^2}dx}{a^2d^4}$$

$\downarrow$ 27

$$\int \frac{1}{(ab^2+ad^2x^4+b^3x+b^2dx^3+cx^2)^2}dx$$

$\downarrow$ 2492

$$\int \left(\frac{2a^2\left(-4ad^2b^3+cdb^2+3ad\sqrt{d^2(b^4+8a^2db-4ac)}b+2ad^2(c-3abd)x-c\sqrt{d^2(b^4+8a^2db-4ac)}\right)d^7}{b^3(d^2(b^4+8a^2db-4ac))^{3/2}\left(2ad^2x^2+\left(b^2d-\sqrt{d^2(b^4+8a^2db-4ac)}\right)x+2abd\right)}-\frac{2a^2\left(-4ad^2b^3+cdb^2-3ad\sqrt{d^2(b^4+8a^2db}\right.}{b^3(d^2(b^4+8a^2db-4ac))^{3/2}\left(2ad^2\right.}\right.$$

$$\Big\downarrow 7239$$

$$\frac{\int \frac{a^2d^4}{(ad^2x^4+b^2dx^3+cx^2+b^3x+ab^2)^2}dx}{a^2d^4}$$

$$\Big\downarrow 27$$

$$\int \frac{1}{(ab^2+ad^2x^4+b^3x+b^2dx^3+cx^2)^2}dx$$

$$\Big\downarrow 2492$$

$$\int \left(\frac{2a^2\left(-4ad^2b^3+cdb^2+3ad\sqrt{d^2(b^4+8a^2db-4ac)}b+2ad^2(c-3abd)x-c\sqrt{d^2(b^4+8a^2db-4ac)}\right)d^7}{b^3(d^2(b^4+8a^2db-4ac))^{3/2}\left(2ad^2x^2+\left(b^2d-\sqrt{d^2(b^4+8a^2db-4ac)}\right)x+2abd\right)}-\frac{2a^2\left(-4ad^2b^3+cdb^2-3ad\sqrt{d^2(b^4+8a^2db}\right.}{b^3(d^2(b^4+8a^2db-4ac))^{3/2}\left(2ad^2\right.}\right.$$

$$\Big\downarrow 7239$$

$$\frac{\int \frac{a^2d^4}{(ad^2x^4+b^2dx^3+cx^2+b^3x+ab^2)^2}dx}{a^2d^4}$$

$$\Big\downarrow 27$$

$$\int \frac{1}{(ab^2+ad^2x^4+b^3x+b^2dx^3+cx^2)^2}dx$$

$$\Big\downarrow 2492$$

$$\int \left(\frac{2a^2\left(-4ad^2b^3+cdb^2+3ad\sqrt{d^2(b^4+8a^2db-4ac)}b+2ad^2(c-3abd)x-c\sqrt{d^2(b^4+8a^2db-4ac)}\right)d^7}{b^3(d^2(b^4+8a^2db-4ac))^{3/2}\left(2ad^2x^2+\left(b^2d-\sqrt{d^2(b^4+8a^2db-4ac)}\right)x+2abd\right)}-\frac{2a^2\left(-4ad^2b^3+cdb^2-3ad\sqrt{d^2(b^4+8a^2db}\right.}{b^3(d^2(b^4+8a^2db-4ac))^{3/2}\left(2ad^2\right.}\right.$$

$$\Big\downarrow 7239$$

$$\frac{\int \frac{a^2d^4}{(ad^2x^4+b^2dx^3+cx^2+b^3x+ab^2)^2}dx}{a^2d^4}$$

$$\Big\downarrow 27$$

$$\int \frac{1}{(ab^2+ad^2x^4+b^3x+b^2dx^3+cx^2)^2}dx$$

input
```
Int[(a*b^2 + b^3*x + c*x^2 + b^2*d*x^3 + a*d^2*x^4)^(-2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2492
```
Int[(Px_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^3 + (e_.)*(x_)^4)
^(p_), x_Symbol] :> Simp[e^p   Int[ExpandIntegrand[Px*(b/d + ((d + Sqrt[e*(
(b^2 - 4*a*c)/a) + 8*a*d*(e/b)])/(2*e))*x + x^2)^p*(b/d + ((d - Sqrt[e*((b^
2 - 4*a*c)/a) + 8*a*d*(e/b)])/(2*e))*x + x^2)^p, x], x], x] /; FreeQ[{a, b,
 c, d, e}, x] && PolyQ[Px, x] && ILtQ[p, 0] && EqQ[a*d^2 - b^2*e, 0]
```

rule 7239
```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.22 (sec) , antiderivative size = 722, normalized size of antiderivative = 0.52

method	result
default	$\frac{-\frac{2a\,d^{2}\left(-3b^{5}d+2abcd+c^{2}\right)x^{3}}{b^{2}\left(32a^{4}b^{3}d^{3}-28a^{2}b^{6}d^{2}-4b^{9}d+16a^{3}b^{2}c\,d^{2}+20a\,b^{5}cd-8a^{2}b\,c^{2}d+b^{4}c^{2}-4a\,c^{3}\right)}-\frac{\left(6a^{2}b^{2}d^{2}-6b^{5}d+3abcd+2c^{2}\right)d\,x^{2}}{32a^{4}b^{3}d^{3}-28a^{2}b^{6}d^{2}-4b^{9}d+16a^{3}b^{2}c\,d^{2}+20a\,b^{5}cd-8a^{2}b\,c^{2}d}}{a\,d^{2}x^{4}+b^{2}d\,x^{3}+}$
risch	$\frac{-\frac{2a\,d^{2}\left(-3b^{5}d+2abcd+c^{2}\right)x^{3}}{b^{2}\left(32a^{4}b^{3}d^{3}-28a^{2}b^{6}d^{2}-4b^{9}d+16a^{3}b^{2}c\,d^{2}+20a\,b^{5}cd-8a^{2}b\,c^{2}d+b^{4}c^{2}-4a\,c^{3}\right)}-\frac{\left(6a^{2}b^{2}d^{2}-6b^{5}d+3abcd+2c^{2}\right)d\,x^{2}}{32a^{4}b^{3}d^{3}-28a^{2}b^{6}d^{2}-4b^{9}d+16a^{3}b^{2}c\,d^{2}+20a\,b^{5}cd-8a^{2}b\,c^{2}d}}{a\,d^{2}x^{4}+b^{2}d\,x^{3}+}$

input
```
int(1/(a*d^2*x^4+b^2*d*x^3+b^3*x+a*b^2+c*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
(-2*a*d^2*(-3*b^5*d+2*a*b*c*d+c^2)/b^2/(32*a^4*b^3*d^3-28*a^2*b^6*d^2-4*b^
9*d+16*a^3*b^2*c*d^2+20*a*b^5*c*d-8*a^2*b*c^2*d+b^4*c^2-4*a*c^3)*x^3-(6*a^
2*b^2*d^2-6*b^5*d+3*a*b*c*d+2*c^2)*d/(32*a^4*b^3*d^3-28*a^2*b^6*d^2-4*b^9*
d+16*a^3*b^2*c*d^2+20*a*b^5*c*d-8*a^2*b*c^2*d+b^4*c^2-4*a*c^3)*x^2+(8*a^3*
b^3*d^3-8*a*b^6*d^2+4*a^2*b^2*c*d^2+7*b^5*c*d-4*a*b*c^2*d-2*c^3)/b^2/(32*a
^4*b^3*d^3-28*a^2*b^6*d^2-4*b^9*d+16*a^3*b^2*c*d^2+20*a*b^5*c*d-8*a^2*b*c^
2*d+b^4*c^2-4*a*c^3)*x+b*(2*a^2*b^2*d^2+4*b^5*d-5*a*b*c*d-c^2)/(8*a^2*b*d+
b^4-4*a*c)/(4*a^2*b^2*d^2-4*b^5*d+4*a*b*c*d+c^2))/(a*d^2*x^4+b^2*d*x^3+b^3
*x+a*b^2+c*x^2)+2/b^2/(32*a^4*b^3*d^3-28*a^2*b^6*d^2-4*b^9*d+16*a^3*b^2*c*
d^2+20*a*b^5*c*d-8*a^2*b*c^2*d+b^4*c^2-4*a*c^3)*sum((a*d^2*(3*b^5*d-2*a*b*
c*d-c^2)*_R^2+d*b^2*(-6*a^2*b^2*d^2+3*b^5*d-a*b*c*d-c^2)*_R+12*a^3*b^3*d^3
-9*a*b^6*d^2+6*a^2*b^2*c*d^2+4*b^5*c*d-2*d*c^2*b*a-c^3)/(4*_R^3*a*d^2+3*_R
^2*b^2*d+b^3+2*_R*c)*ln(x-_R),_R=RootOf(_Z^4*a*d^2+_Z^3*b^2*d+_Z*b^3+_Z^2*
c+a*b^2))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(ab^2+b^3x+cx^2+b^2dx^3+ad^2x^4)^2}\,dx=\text{Timed out}$$

input
```
integrate(1/(a*d^2*x^4+b^2*d*x^3+b^3*x+a*b^2+c*x^2)^2,x, algorithm="fricas
")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(ab^2+b^3x+cx^2+b^2dx^3+ad^2x^4)^2}\,dx=\text{Timed out}$$

input
```
integrate(1/(a*d**2*x**4+b**2*d*x**3+b**3*x+a*b**2+c*x**2)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{(ab^2 + b^3x + cx^2 + b^2dx^3 + ad^2x^4)^2} dx = \int \frac{1}{(ad^2x^4 + b^2dx^3 + b^3x + ab^2 + cx^2)^2} dx$$

input
```
integrate(1/(a*d^2*x^4+b^2*d*x^3+b^3*x+a*b^2+c*x^2)^2,x, algorithm="maxima
")
```

output
```
(2*a^2*b^5*d^2 - b^3*c^2 - 2*(a*c^2*d^2 - (3*a*b^5 - 2*a^2*b*c)*d^3)*x^3 -
 (6*a^2*b^4*d^3 + 2*b^2*c^2*d - 3*(2*b^7 - a*b^3*c)*d^2)*x^2 + (4*b^8 - 5*
a*b^4*c)*d + (8*a^3*b^3*d^3 - 2*c^3 - 4*(2*a*b^6 - a^2*b^2*c)*d^2 + (7*b^5
*c - 4*a*b*c^2)*d)*x)/(32*a^5*b^7*d^3 + a*b^8*c^2 - 4*a^2*b^4*c^3 + (32*a^
5*b^5*d^5 - 4*(7*a^3*b^8 - 4*a^4*b^4*c)*d^4 - 4*(a*b^11 - 5*a^2*b^7*c + 2*
a^3*b^3*c^2)*d^3 + (a*b^6*c^2 - 4*a^2*b^2*c^3)*d^2)*x^4 + (32*a^4*b^7*d^4
- 4*(7*a^2*b^10 - 4*a^3*b^6*c)*d^3 - 4*(b^13 - 5*a*b^9*c + 2*a^2*b^5*c^2)*
d^2 + (b^8*c^2 - 4*a*b^4*c^3)*d)*x^3 - 4*(7*a^3*b^10 - 4*a^4*b^6*c)*d^2 +
(32*a^4*b^5*c*d^3 + b^6*c^3 - 4*a*b^2*c^4 - 4*(7*a^2*b^8*c - 4*a^3*b^4*c^2
)*d^2 - 4*(b^11*c - 5*a*b^7*c^2 + 2*a^2*b^3*c^3)*d)*x^2 - 4*(a*b^13 - 5*a^
2*b^9*c + 2*a^3*b^5*c^2)*d + (32*a^4*b^8*d^3 + b^9*c^2 - 4*a*b^5*c^3 - 4*(
7*a^2*b^11 - 4*a^3*b^7*c)*d^2 - 4*(b^14 - 5*a*b^10*c + 2*a^2*b^6*c^2)*d)*x
) - 2*integrate(-(12*a^3*b^3*d^3 - c^3 - 3*(3*a*b^6 - 2*a^2*b^2*c)*d^2 - (
a*c^2*d^2 - (3*a*b^5 - 2*a^2*b*c)*d^3)*x^2 + 2*(2*b^5*c - a*b*c^2)*d - (6*
a^2*b^4*d^3 + b^2*c^2*d - (3*b^7 - a*b^3*c)*d^2)*x)/(a*d^2*x^4 + b^2*d*x^3
 + b^3*x + a*b^2 + c*x^2), x)/(32*a^4*b^5*d^3 + b^6*c^2 - 4*a*b^2*c^3 - 4*
(7*a^2*b^8 - 4*a^3*b^4*c)*d^2 - 4*(b^11 - 5*a*b^7*c + 2*a^2*b^3*c^2)*d)
```

Giac [F]

$$\int \frac{1}{(ab^2 + b^3x + cx^2 + b^2dx^3 + ad^2x^4)^2} dx = \int \frac{1}{(ad^2x^4 + b^2dx^3 + b^3x + ab^2 + cx^2)^2} dx$$

input
```
integrate(1/(a*d^2*x^4+b^2*d*x^3+b^3*x+a*b^2+c*x^2)^2,x, algorithm="giac")
```

output
```
integrate((a*d^2*x^4 + b^2*d*x^3 + b^3*x + a*b^2 + c*x^2)^(-2), x)
```

Mupad [B] (verification not implemented)

Time = 25.60 (sec) , antiderivative size = 16419, normalized size of antiderivative = 11.91

$$\int \frac{1}{(ab^2+b^3x+cx^2+b^2dx^3+ad^2x^4)^2}\,dx = \text{Too large to display}$$

input
```
int(1/(a*b^2 + b^3*x + c*x^2 + a*d^2*x^4 + b^2*d*x^3)^2,x)
```

output
```
symsum(log((216*a^5*b^10*d^10 - 40*a^5*c^4*d^8 - 80*a^6*b*c^3*d^9 - 72*a^4
*b^9*c*d^9 - 720*a^6*b^6*c*d^10 + 576*a^8*b^3*c*d^11 + 16*a^4*b^4*c^3*d^8
+ 216*a^5*b^5*c^2*d^9 + 288*a^7*b^2*c^2*d^10)/(b^16*c^4 + 16*b^26*d^2 - 12
*a*b^12*c^5 - 8*b^21*c^2*d - 64*a^3*b^4*c^7 + 48*a^2*b^8*c^6 + 352*a^2*b^2
3*d^3 + 2320*a^4*b^20*d^4 + 2432*a^6*b^17*d^5 - 13312*a^8*b^14*d^6 + 8192*
a^10*b^11*d^7 - 128*a^4*b^5*c^6*d + 704*a^3*b^9*c^5*d - 456*a^2*b^13*c^4*d
 - 3424*a^3*b^19*c*d^3 - 11712*a^5*b^16*c*d^4 + 11264*a^7*b^13*c*d^5 + 409
6*a^9*b^10*c*d^6 + 768*a^5*b^6*c^5*d^2 - 1216*a^4*b^10*c^4*d^2 + 1536*a^6*
b^7*c^4*d^3 - 1120*a^3*b^14*c^3*d^2 - 5632*a^5*b^11*c^3*d^3 - 3072*a^7*b^8
*c^3*d^4 + 984*a^2*b^18*c^2*d^2 + 9408*a^4*b^15*c^2*d^3 + 7424*a^6*b^12*c^
2*d^4 - 6144*a^8*b^9*c^2*d^5 + 104*a*b^17*c^3*d - 224*a*b^22*c*d^2) - root
(77856768*a^13*b^23*c*d^9*z^4 - 56623104*a^15*b^20*c*d^10*z^4 - 18800640*a
^9*b^29*c*d^7*z^4 - 6704640*a^7*b^32*c*d^6*z^4 + 5308416*a^11*b^26*c*d^8*z
^4 - 969408*a^5*b^35*c*d^5*z^4 - 89088*a^5*b^15*c^9*d*z^4 - 65664*a^3*b^38
*c*d^4*z^4 + 61440*a^6*b^11*c^10*d*z^4 + 53760*a^4*b^19*c^8*d*z^4 - 17280*
a^3*b^23*c^7*d*z^4 + 3120*a^2*b^27*c^6*d*z^4 + 1248*a*b^36*c^3*d^2*z^4 - 1
728*a*b^41*c*d^3*z^4 - 300*a*b^31*c^5*d*z^4 - 85131264*a^12*b^22*c^2*d^8*z
^4 - 64880640*a^9*b^24*c^3*d^6*z^4 + 58195968*a^13*b^18*c^3*d^8*z^4 + 4512
1536*a^10*b^20*c^4*d^6*z^4 + 41680896*a^10*b^25*c^2*d^7*z^4 + 32870400*a^8
*b^28*c^2*d^6*z^4 + 32194560*a^8*b^23*c^4*d^5*z^4 + 25165824*a^16*b^16*...
```

Reduce [F]

$$\int \frac{1}{(ab^2+b^3x+cx^2+b^2dx^3+ad^2x^4)^2}\,dx = \text{too large to display}$$

input
```
int(1/(a*d^2*x^4+b^2*d*x^3+b^3*x+a*b^2+c*x^2)^2,x)
```

output

```
( - 4*int(x**3/(a**2*b**4 + 2*a**2*b**2*d**2*x**4 + a**2*d**4*x**8 + 2*a*b
**5*x + 2*a*b**4*d*x**3 + 2*a*b**3*d**2*x**5 + 2*a*b**2*c*x**2 + 2*a*b**2*
d**3*x**7 + 2*a*c*d**2*x**6 + b**6*x**2 + 2*b**5*d*x**4 + b**4*d**2*x**6 +
 2*b**3*c*x**3 + 2*b**2*c*d*x**5 + c**2*x**4),x)*a**2*b**2*d**2 - 4*int(x*
*3/(a**2*b**4 + 2*a**2*b**2*d**2*x**4 + a**2*d**4*x**8 + 2*a*b**5*x + 2*a*
b**4*d*x**3 + 2*a*b**3*d**2*x**5 + 2*a*b**2*c*x**2 + 2*a*b**2*d**3*x**7 +
2*a*c*d**2*x**6 + b**6*x**2 + 2*b**5*d*x**4 + b**4*d**2*x**6 + 2*b**3*c*x*
*3 + 2*b**2*c*d*x**5 + c**2*x**4),x)*a**2*d**4*x**4 - 4*int(x**3/(a**2*b**
4 + 2*a**2*b**2*d**2*x**4 + a**2*d**4*x**8 + 2*a*b**5*x + 2*a*b**4*d*x**3
+ 2*a*b**3*d**2*x**5 + 2*a*b**2*c*x**2 + 2*a*b**2*d**3*x**7 + 2*a*c*d**2*x
**6 + b**6*x**2 + 2*b**5*d*x**4 + b**4*d**2*x**6 + 2*b**3*c*x**3 + 2*b**2*
c*d*x**5 + c**2*x**4),x)*a*b**3*d**2*x - 4*int(x**3/(a**2*b**4 + 2*a**2*b*
*2*d**2*x**4 + a**2*d**4*x**8 + 2*a*b**5*x + 2*a*b**4*d*x**3 + 2*a*b**3*d*
*2*x**5 + 2*a*b**2*c*x**2 + 2*a*b**2*d**3*x**7 + 2*a*c*d**2*x**6 + b**6*x*
*2 + 2*b**5*d*x**4 + b**4*d**2*x**6 + 2*b**3*c*x**3 + 2*b**2*c*d*x**5 + c*
*2*x**4),x)*a*b**2*d**3*x**3 - 4*int(x**3/(a**2*b**4 + 2*a**2*b**2*d**2*x*
*4 + a**2*d**4*x**8 + 2*a*b**5*x + 2*a*b**4*d*x**3 + 2*a*b**3*d**2*x**5 +
2*a*b**2*c*x**2 + 2*a*b**2*d**3*x**7 + 2*a*c*d**2*x**6 + b**6*x**2 + 2*b**
5*d*x**4 + b**4*d**2*x**6 + 2*b**3*c*x**3 + 2*b**2*c*d*x**5 + c**2*x**4),x
)*a*c*d**2*x**2 - 3*int(x**2/(a**2*b**4 + 2*a**2*b**2*d**2*x**4 + a**2*...
```

3.31 $\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^4\,dx$

Optimal result

Integrand size = 29, antiderivative size = 257

$$\begin{aligned}\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^4\,dx &= \frac{c^4(c^3+4ad^2)^4x}{d^8}-\frac{8c^5(c^3+4ad^2)^3(c+dx)^3}{3d^9}\\ &+\frac{4c^3(c^3+4ad^2)^2(7c^3+4ad^2)(c+dx)^5}{5d^9}\\ &-\frac{8c^4(c^3+4ad^2)(7c^3+12ad^2)(c+dx)^7}{7d^9}\\ &+\frac{2c^2(35c^6+120ac^3d^2+48a^2d^4)(c+dx)^9}{9d^9}\\ &-\frac{8c^3(7c^3+12ad^2)(c+dx)^{11}}{11d^9}\\ &+\frac{4c(7c^3+4ad^2)(c+dx)^{13}}{13d^9}\\ &-\frac{8c^2(c+dx)^{15}}{15d^9}+\frac{(c+dx)^{17}}{17d^9}\end{aligned}$$

output
```
c^4*(4*a*d^2+c^3)^4*x/d^8-8/3*c^5*(4*a*d^2+c^3)^3*(d*x+c)^3/d^9+4/5*c^3*(4
*a*d^2+c^3)^2*(4*a*d^2+7*c^3)*(d*x+c)^5/d^9-8/7*c^4*(4*a*d^2+c^3)*(12*a*d^
2+7*c^3)*(d*x+c)^7/d^9+2/9*c^2*(48*a^2*d^4+120*a*c^3*d^2+35*c^6)*(d*x+c)^9
/d^9-8/11*c^3*(12*a*d^2+7*c^3)*(d*x+c)^11/d^9+4/13*c*(4*a*d^2+7*c^3)*(d*x+
c)^13/d^9-8/15*c^2*(d*x+c)^15/d^9+1/17*(d*x+c)^17/d^9
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.11

$$\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^4\,dx=256a^4c^4x+\frac{1024}{3}a^3c^5x^3+256a^3c^4dx^4+\frac{256}{5}a^2c^3\left(6c^3+ad^2\right)x^5+512a^2c^5dx^6+\frac{256}{7}ac^4\left(4c^3+9ad^2\right)x^7+96ac^3d\left(4c^3+ad^2\right)x^8+\frac{32}{9}c^2\left(8c^6+120ac^3d^2+3a^2d^4\right)x^9+\frac{256}{5}c^4d\left(2c^3+5ad^2\right)x^{10}+\frac{64}{11}c^3d^2\left(28c^3+15ad^2\right)x^{11}+\frac{16}{3}c^2d^3\left(28c^3+3ad^2\right)x^{12}+\frac{16}{13}cd^4\left(70c^3+ad^2\right)x^{13}+32c^3d^5x^{14}+\frac{112}{15}c^2d^6x^{15}+cd^7x^{16}+\frac{d^8x^{17}}{17}$$

input
```
Integrate[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^4,x]
```

output
```
256*a^4*c^4*x + (1024*a^3*c^5*x^3)/3 + 256*a^3*c^4*d*x^4 + (256*a^2*c^3*(6
*c^3 + a*d^2)*x^5)/5 + 512*a^2*c^5*d*x^6 + (256*a*c^4*(4*c^3 + 9*a*d^2)*x^
7)/7 + 96*a*c^3*d*(4*c^3 + a*d^2)*x^8 + (32*c^2*(8*c^6 + 120*a*c^3*d^2 + 3
*a^2*d^4)*x^9)/9 + (256*c^4*d*(2*c^3 + 5*a*d^2)*x^10)/5 + (64*c^3*d^2*(28*
c^3 + 15*a*d^2)*x^11)/11 + (16*c^2*d^3*(28*c^3 + 3*a*d^2)*x^12)/3 + (16*c*
d^4*(70*c^3 + a*d^2)*x^13)/13 + 32*c^3*d^5*x^14 + (112*c^2*d^6*x^15)/15 +
c*d^7*x^16 + (d^8*x^17)/17
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {2458, 1403, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^4 dx$$

$$\Big\downarrow \text{2458}$$

$$\int \left(c\left(4a+\frac{c^3}{d^2}\right)-2c^2\left(\frac{c}{d}+x\right)^2+d^2\left(\frac{c}{d}+x\right)^4\right)^4 d\left(\frac{c}{d}+x\right)$$

$$\Big\downarrow \text{1403}$$

$$\int \left(16c^8\left(\frac{6a^2d^4}{c^6}+\frac{15ad^2}{c^3}+\frac{35}{8}\right)\left(\frac{c}{d}+x\right)^8+\frac{\left(4acd^2+c^4\right)^4}{d^8}-\frac{32c^7\left(4ad^2+c^3\right)\left(\frac{3ad^2}{c^3}+\frac{7}{4}\right)\left(\frac{c}{d}+x\right)^6}{d^2}-32c^6d^2\left(\right.\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{9}c^2\left(48a^2d^4+120ac^3d^2+35c^6\right)\left(\frac{c}{d}+x\right)^9-\frac{8}{11}c^3d^2\left(12ad^2+7c^3\right)\left(\frac{c}{d}+x\right)^{11}+\frac{4}{13}cd^4\left(4ad^2+7c^3\right)\left(\frac{c}{d}+x\right)^{13}+\frac{4c^3\left(4ad^2+c^3\right)^2\left(4ad^2+7c^3\right)\left(\frac{c}{d}+x\right)^5}{5d^4}-\frac{8c^5\left(4ad^2+c^3\right)^3\left(\frac{c}{d}+x\right)^3}{3d^6}-\frac{8c^4\left(4ad^2+c^3\right)\left(12ad^2+7c^3\right)\left(\frac{c}{d}+x\right)^7}{7d^2}+\frac{c^4\left(4ad^2+c^3\right)^4\left(\frac{c}{d}+x\right)}{d^8}-\frac{8}{15}c^2d^6\left(\frac{c}{d}+x\right)^{15}+\frac{1}{17}d^8\left(\frac{c}{d}+x\right)^{17}$$

input
```
Int[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^4,x]
```

output
```
(c^4*(c^3 + 4*a*d^2)^4*(c/d + x))/d^8 - (8*c^5*(c^3 + 4*a*d^2)^3*(c/d + x)
^3)/(3*d^6) + (4*c^3*(c^3 + 4*a*d^2)^2*(7*c^3 + 4*a*d^2)*(c/d + x)^5)/(5*d
^4) - (8*c^4*(c^3 + 4*a*d^2)*(7*c^3 + 12*a*d^2)*(c/d + x)^7)/(7*d^2) + (2*
c^2*(35*c^6 + 120*a*c^3*d^2 + 48*a^2*d^4)*(c/d + x)^9)/9 - (8*c^3*d^2*(7*c
^3 + 12*a*d^2)*(c/d + x)^11)/11 + (4*c*d^4*(7*c^3 + 4*a*d^2)*(c/d + x)^13)
/13 - (8*c^2*d^6*(c/d + x)^15)/15 + (d^8*(c/d + x)^17)/17
```

Defintions of rubi rules used

rule 1403
```
Int[((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int[ExpandInte
grand[(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a
*c, 0] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2458
```
Int[(Pn_)^(p_.), x_Symbol] :> With[{S = Coeff[Pn, x, Expon[Pn, x] - 1]/(Exp
on[Pn, x]*Coeff[Pn, x, Expon[Pn, x]])}, Subst[Int[ExpandToSum[Pn /. x -> x
- S, x]^p, x], x, x + S] /; BinomialQ[Pn /. x -> x - S, x] || (IntegerQ[Exp
on[Pn, x]/2] && TrinomialQ[Pn /. x -> x - S, x])] /; FreeQ[p, x] && PolyQ[P
n, x] && GtQ[Expon[Pn, x], 2] && NeQ[Coeff[Pn, x, Expon[Pn, x] - 1], 0]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.04

method	result
norman	$256c^4a^4x+\frac{1024a^3c^5x^3}{3}+256a^3c^4d\,x^4+\left(\frac{256}{5}a^3c^3d^2+\frac{1536}{5}a^2c^6\right)x^5+512a^2c^5d\,x^6+\left(\frac{2304}{7}a^2c^4\right.$
gosper	$\frac{16}{13}x^{13}ac\,d^6+96a^2c^3d^3x^8+384a\,c^6d\,x^8+\frac{256}{5}x^5a^3c^3d^2+\frac{2304}{7}x^7a^2c^4d^2+\frac{32}{3}x^9a^2c^2d^4+\frac{1280}{3}x$
risch	$\frac{16}{13}x^{13}ac\,d^6+96a^2c^3d^3x^8+384a\,c^6d\,x^8+\frac{256}{5}x^5a^3c^3d^2+\frac{2304}{7}x^7a^2c^4d^2+\frac{32}{3}x^9a^2c^2d^4+\frac{1280}{3}x$
parallelrisch	$\frac{16}{13}x^{13}ac\,d^6+96a^2c^3d^3x^8+384a\,c^6d\,x^8+\frac{256}{5}x^5a^3c^3d^2+\frac{2304}{7}x^7a^2c^4d^2+\frac{32}{3}x^9a^2c^2d^4+\frac{1280}{3}x$
orering	$x\left(45045d^8x^{16}+765765c\,d^7x^{15}+5717712c^2d^6x^{14}+24504480c^3d^5x^{13}+942480ac\,d^6x^{12}+65973600c^4d^4x^{12}+12252240a\,c^2d^5x\right.$
default	$\frac{d^8x^{17}}{17}+c\,d^7x^{16}+\frac{112c^2d^6x^{15}}{15}+32c^3d^5x^{14}+\frac{\left(2\left(8ac\,d^2+16c^4\right)d^4+1088c^4d^4\right)x^{13}}{13}+\frac{\left(64a\,c^2d^5+16\left(8ac\,d^2+16c\right.\right.}{12}$

input
```
int((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^4,x,method=_RETURNVERBOSE)
```

output
```
256*c^4*a^4*x+1024/3*a^3*c^5*x^3+256*a^3*c^4*d*x^4+(256/5*a^3*c^3*d^2+1536
/5*a^2*c^6)*x^5+512*a^2*c^5*d*x^6+(2304/7*a^2*c^4*d^2+1024/7*a*c^7)*x^7+(9
6*a^2*c^3*d^3+384*a*c^6*d)*x^8+(32/3*a^2*c^2*d^4+1280/3*a*c^5*d^2+256/9*c^
8)*x^9+(256*a*c^4*d^3+512/5*c^7*d)*x^10+(960/11*a*c^3*d^4+1792/11*c^6*d^2)
*x^11+(16*a*c^2*d^5+448/3*c^5*d^3)*x^12+(16/13*a*c*d^6+1120/13*c^4*d^4)*x^
13+32*c^3*d^5*x^14+112/15*c^2*d^6*x^15+c*d^7*x^16+1/17*d^8*x^17
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.05

$$\int (4ac + 4c^2x^2 + 4cdx^3 + d^2x^4)^4 \, dx = \frac{1}{17} d^8x^{17} + cd^7x^{16} + \frac{112}{15} c^2d^6x^{15} + 32\, c^3d^5x^{14} + 512\, a^2c^5dx^6 + \frac{16}{13} (70\, c^4d^4 + acd^6)x^{13} + \frac{16}{3} (28\, c^5d^3 + 3\, ac^2d^5)x^{12} + 256\, a^3c^4dx^4 + \frac{64}{11} (28\, c^6d^2 + 15\, ac^3d^4)x^{11} + \frac{1024}{3} a^3c^5x^3 + \frac{256}{5} (2\, c^7d + 5\, ac^4d^3)x^{10} + \frac{32}{9} (8\, c^8 + 120\, ac^5d^2 + 3\, a^2c^2d^4)x^9 + 256\, a^4c^4x + 96\, (4\, ac^6d + a^2c^3d^3)x^8 + \frac{256}{7} (4\, ac^7 + 9\, a^2c^4d^2)x^7 + \frac{256}{5} (6\, a^2c^6 + a^3c^3d^2)x^5$$

input
```
integrate((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^4,x, algorithm="fricas")
```

output
```
1/17*d^8*x^17 + c*d^7*x^16 + 112/15*c^2*d^6*x^15 + 32*c^3*d^5*x^14 + 512*a
^2*c^5*d*x^6 + 16/13*(70*c^4*d^4 + a*c*d^6)*x^13 + 16/3*(28*c^5*d^3 + 3*a*
c^2*d^5)*x^12 + 256*a^3*c^4*d*x^4 + 64/11*(28*c^6*d^2 + 15*a*c^3*d^4)*x^11
 + 1024/3*a^3*c^5*x^3 + 256/5*(2*c^7*d + 5*a*c^4*d^3)*x^10 + 32/9*(8*c^8 +
 120*a*c^5*d^2 + 3*a^2*c^2*d^4)*x^9 + 256*a^4*c^4*x + 96*(4*a*c^6*d + a^2*
c^3*d^3)*x^8 + 256/7*(4*a*c^7 + 9*a^2*c^4*d^2)*x^7 + 256/5*(6*a^2*c^6 + a^
3*c^3*d^2)*x^5
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.16

$$\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^4\,dx = 256a^4c^4x + \frac{1024a^3c^5x^3}{3} + 256a^3c^4dx^4 + 512a^2c^5dx^6 + 32c^3d^5x^{14} + \frac{112c^2d^6x^{15}}{15} + cd^7x^{16} + \frac{d^8x^{17}}{17} + x^{13}\cdot\left(\frac{16acd^6}{13}+\frac{1120c^4d^4}{13}\right) + x^{12}\cdot\left(16ac^2d^5+\frac{448c^5d^3}{3}\right) + x^{11}\cdot\left(\frac{960ac^3d^4}{11}+\frac{1792c^6d^2}{11}\right) + x^{10}\cdot\left(256ac^4d^3+\frac{512c^7d}{5}\right) + x^9\cdot\left(\frac{32a^2c^2d^4}{3}+\frac{1280ac^5d^2}{3}+\frac{256c^8}{9}\right) + x^8\cdot\left(96a^2c^3d^3+384ac^6d\right) + x^7\cdot\left(\frac{2304a^2c^4d^2}{7}+\frac{1024ac^7}{7}\right) + x^5\cdot\left(\frac{256a^3c^3d^2}{5}+\frac{1536a^2c^6}{5}\right)$$

input
```
integrate((d**2*x**4+4*c*d*x**3+4*c**2*x**2+4*a*c)**4,x)
```

output
```
256*a**4*c**4*x + 1024*a**3*c**5*x**3/3 + 256*a**3*c**4*d*x**4 + 512*a**2*
c**5*d*x**6 + 32*c**3*d**5*x**14 + 112*c**2*d**6*x**15/15 + c*d**7*x**16 +
 d**8*x**17/17 + x**13*(16*a*c*d**6/13 + 1120*c**4*d**4/13) + x**12*(16*a*
c**2*d**5 + 448*c**5*d**3/3) + x**11*(960*a*c**3*d**4/11 + 1792*c**6*d**2/
11) + x**10*(256*a*c**4*d**3 + 512*c**7*d/5) + x**9*(32*a**2*c**2*d**4/3 +
 1280*a*c**5*d**2/3 + 256*c**8/9) + x**8*(96*a**2*c**3*d**3 + 384*a*c**6*d
) + x**7*(2304*a**2*c**4*d**2/7 + 1024*a*c**7/7) + x**5*(256*a**3*c**3*d**
2/5 + 1536*a**2*c**6/5)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 372, normalized size of antiderivative = 1.45

$$\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^4 \, dx$$
$$= \frac{1}{17}\,d^8x^{17}+cd^7x^{16}+\frac{32}{5}\,c^2d^6x^{15}+\frac{128}{7}\,c^3d^5x^{14}+\frac{256}{13}\,c^4d^4x^{13}+\frac{256}{9}\,c^8x^9$$
$$+256\,a^4c^4x+\frac{256}{15}\,\left(3\,d^2x^5+15\,cdx^4+20\,c^2x^3\right)a^3c^3+\frac{256}{55}\,\left(5\,d^2x^{11}+22\,cdx^{10}\right)c^6$$
$$+\frac{32}{105}\,\left(35\,d^4x^9+315\,cd^3x^8+720\,c^2d^2x^7+1008\,c^4x^5+120\,\left(3\,d^2x^7+14\,cdx^6\right)c^2\right)a^2c^2$$
$$+\frac{32}{143}\,\left(33\,d^4x^{13}+286\,cd^3x^{12}+624\,c^2d^2x^{11}\right)c^4$$
$$+\frac{16}{15015}\,\left(1155\,d^6x^{13}+15015\,cd^5x^{12}+65520\,c^2d^4x^{11}+96096\,c^3d^3x^{10}+137280\,c^6x^7+40040\,\left(2\,d^2x^9+9\right.\right.$$
$$+\frac{16}{1365}\,\left(91\,d^6x^{15}+1170\,cd^5x^{14}+5040\,c^2d^4x^{13}+7280\,c^3d^3x^{12}\right)c^2$$

input
```
integrate((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^4,x, algorithm="maxima")
```

output
```
1/17*d^8*x^17 + c*d^7*x^16 + 32/5*c^2*d^6*x^15 + 128/7*c^3*d^5*x^14 + 256/
13*c^4*d^4*x^13 + 256/9*c^8*x^9 + 256*a^4*c^4*x + 256/15*(3*d^2*x^5 + 15*c
*d*x^4 + 20*c^2*x^3)*a^3*c^3 + 256/55*(5*d^2*x^11 + 22*c*d*x^10)*c^6 + 32/
105*(35*d^4*x^9 + 315*c*d^3*x^8 + 720*c^2*d^2*x^7 + 1008*c^4*x^5 + 120*(3*
d^2*x^7 + 14*c*d*x^6)*c^2)*a^2*c^2 + 32/143*(33*d^4*x^13 + 286*c*d^3*x^12
+ 624*c^2*d^2*x^11)*c^4 + 16/15015*(1155*d^6*x^13 + 15015*c*d^5*x^12 + 655
20*c^2*d^4*x^11 + 96096*c^3*d^3*x^10 + 137280*c^6*x^7 + 40040*(2*d^2*x^9 +
 9*c*d*x^8)*c^4 + 364*(45*d^4*x^11 + 396*c*d^3*x^10 + 880*c^2*d^2*x^9)*c^2
)*a*c + 16/1365*(91*d^6*x^15 + 1170*c*d^5*x^14 + 5040*c^2*d^4*x^13 + 7280*
c^3*d^3*x^12)*c^2
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.08

$$\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^4 \, dx = \frac{1}{17}\,d^8x^{17}+cd^7x^{16}+\frac{112}{15}\,c^2d^6x^{15}+32\,c^3d^5x^{14}+\frac{1120}{13}\,c^4d^4x^{13}+\frac{16}{13}\,acd^6x^{13}+\frac{448}{3}\,c^5d^3x^{12}+16\,ac^2d^5x^{12}+\frac{1792}{11}\,c^6d^2x^{11}+\frac{960}{11}\,ac^3d^4x^{11}+\frac{512}{5}\,c^7dx^{10}+256\,ac^4d^3x^{10}+\frac{256}{9}\,c^8x^9+\frac{1280}{3}\,ac^5d^2x^9+\frac{32}{3}\,a^2c^2d^4x^9+384\,ac^6dx^8+96\,a^2c^3d^3x^8+\frac{1024}{7}\,ac^7x^7+\frac{2304}{7}\,a^2c^4d^2x^7+512\,a^2c^5dx^6+\frac{1536}{5}\,a^2c^6x^5+\frac{256}{5}\,a^3c^3d^2x^5+256\,a^3c^4dx^4+\frac{1024}{3}\,a^3c^5x^3+256\,a^4c^4x$$

input
```
integrate((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^4,x, algorithm="giac")
```

output
```
1/17*d^8*x^17 + c*d^7*x^16 + 112/15*c^2*d^6*x^15 + 32*c^3*d^5*x^14 + 1120/
13*c^4*d^4*x^13 + 16/13*a*c*d^6*x^13 + 448/3*c^5*d^3*x^12 + 16*a*c^2*d^5*x
^12 + 1792/11*c^6*d^2*x^11 + 960/11*a*c^3*d^4*x^11 + 512/5*c^7*d*x^10 + 25
6*a*c^4*d^3*x^10 + 256/9*c^8*x^9 + 1280/3*a*c^5*d^2*x^9 + 32/3*a^2*c^2*d^4
*x^9 + 384*a*c^6*d*x^8 + 96*a^2*c^3*d^3*x^8 + 1024/7*a*c^7*x^7 + 2304/7*a^
2*c^4*d^2*x^7 + 512*a^2*c^5*d*x^6 + 1536/5*a^2*c^6*x^5 + 256/5*a^3*c^3*d^2
*x^5 + 256*a^3*c^4*d*x^4 + 1024/3*a^3*c^5*x^3 + 256*a^4*c^4*x
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.02

$$\int (4ac+4c^2x^2+4cdx^3+d^2x^4)^4 \, dx = x^{10}\left(\frac{512\,c^7\,d}{5}+256\,a\,c^4\,d^3\right)+x^{13}\left(\frac{1120\,c^4\,d^4}{13}+\frac{16\,a\,c\,d^6}{13}\right)+x^9\left(\frac{32\,a^2\,c^2\,d^4}{3}+\frac{1280\,a\,c^5\,d^2}{3}+\frac{256\,c^8}{9}\right)+x^{12}\left(\frac{448\,c^5\,d^3}{3}+16\,a\,c^2\,d^5\right)+x^{11}\left(\frac{1792\,c^6\,d^2}{11}+\frac{960\,a\,c^3\,d^4}{11}\right)+\frac{d^8\,x^{17}}{17}+256\,a^4\,c^4\,x+c\,d^7\,x^{16}+\frac{1024\,a^3\,c^5\,x^3}{3}+32\,c^3\,d^5\,x^{14}+\frac{112\,c^2\,d^6\,x^{15}}{15}+256\,a^3\,c^4\,d\,x^4+512\,a^2\,c^5\,d\,x^6+\frac{256\,a\,c^4\,x^7\,(4\,c^3+9\,a\,d^2)}{7}+\frac{256\,a^2\,c^3\,x^5\,(6\,c^3+a\,d^2)}{5}+96\,a\,c^3\,d\,x^8\,(4\,c^3+a\,d^2)$$

input
```
int((4*a*c + 4*c^2*x^2 + d^2*x^4 + 4*c*d*x^3)^4,x)
```

output
```
x^10*((512*c^7*d)/5 + 256*a*c^4*d^3) + x^13*((1120*c^4*d^4)/13 + (16*a*c*d
^6)/13) + x^9*((256*c^8)/9 + (1280*a*c^5*d^2)/3 + (32*a^2*c^2*d^4)/3) + x^
12*((448*c^5*d^3)/3 + 16*a*c^2*d^5) + x^11*((1792*c^6*d^2)/11 + (960*a*c^3
*d^4)/11) + (d^8*x^17)/17 + 256*a^4*c^4*x + c*d^7*x^16 + (1024*a^3*c^5*x^3
)/3 + 32*c^3*d^5*x^14 + (112*c^2*d^6*x^15)/15 + 256*a^3*c^4*d*x^4 + 512*a^
2*c^5*d*x^6 + (256*a*c^4*x^7*(9*a*d^2 + 4*c^3))/7 + (256*a^2*c^3*x^5*(a*d^
2 + 6*c^3))/5 + 96*a*c^3*d*x^8*(a*d^2 + 4*c^3)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.09

$$\int \left(4ac + 4c^2x^2 + 4cdx^3 + d^2x^4\right)^4 dx$$
$$= \frac{x(45045d^8x^{16} + 765765c\,d^7x^{15} + 5717712c^2d^6x^{14} + 24504480c^3d^5x^{13} + 942480ac\,d^6x^{12} + 65973600c^4d^4x}{}$$

input
```
int((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^4,x)
```

output
```
(x*(196035840*a**4*c**4 + 261381120*a**3*c**5*x**2 + 196035840*a**3*c**4*d
*x**3 + 39207168*a**3*c**3*d**2*x**4 + 235243008*a**2*c**6*x**4 + 39207168
0*a**2*c**5*d*x**5 + 252046080*a**2*c**4*d**2*x**6 + 73513440*a**2*c**3*d*
*3*x**7 + 8168160*a**2*c**2*d**4*x**8 + 112020480*a*c**7*x**6 + 294053760*
a*c**6*d*x**7 + 326726400*a*c**5*d**2*x**8 + 196035840*a*c**4*d**3*x**9 +
66830400*a*c**3*d**4*x**10 + 12252240*a*c**2*d**5*x**11 + 942480*a*c*d**6*
x**12 + 21781760*c**8*x**8 + 78414336*c**7*d*x**9 + 124750080*c**6*d**2*x*
*10 + 114354240*c**5*d**3*x**11 + 65973600*c**4*d**4*x**12 + 24504480*c**3
*d**5*x**13 + 5717712*c**2*d**6*x**14 + 765765*c*d**7*x**15 + 45045*d**8*x
**16))/765765
```

3.32 $\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^3\,dx$

Optimal result

Integrand size = 29, antiderivative size = 174

$$\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^3\,dx = \frac{c^3(c^3+4ad^2)^3\,x}{d^6}-\frac{2c^4(c^3+4ad^2)^2\,(c+dx)^3}{d^7}+\frac{3c^2(c^3+4ad^2)\,(5c^3+4ad^2)\,(c+dx)^5}{5d^7}-\frac{4c^3(5c^3+12ad^2)\,(c+dx)^7}{7d^7}+\frac{c(5c^3+4ad^2)\,(c+dx)^9}{3d^7}-\frac{6c^2(c+dx)^{11}}{11d^7}+\frac{(c+dx)^{13}}{13d^7}$$

output
```
c^3*(4*a*d^2+c^3)^3*x/d^6-2*c^4*(4*a*d^2+c^3)^2*(d*x+c)^3/d^7+3/5*c^2*(4*a
*d^2+c^3)*(4*a*d^2+5*c^3)*(d*x+c)^5/d^7-4/7*c^3*(12*a*d^2+5*c^3)*(d*x+c)^7
/d^7+1/3*c*(4*a*d^2+5*c^3)*(d*x+c)^9/d^7-6/11*c^2*(d*x+c)^11/d^7+1/13*(d*x
+c)^13/d^7
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.98

$$\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^3\,dx = 64a^3c^3x+64a^2c^4x^3+48a^2c^3dx^4 + \frac{48}{5}ac^2\left(4c^3+ad^2\right)x^5+64ac^4dx^6 + \frac{32}{7}c^3\left(2c^3+9ad^2\right)x^7+12c^2d\left(2c^3+ad^2\right)x^8 + \frac{4}{3}cd^2\left(20c^3+ad^2\right)x^9+16c^3d^3x^{10} + \frac{60}{11}c^2d^4x^{11}+cd^5x^{12}+\frac{d^6x^{13}}{13}$$

input
```
Integrate[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^3,x]
```

output
```
64*a^3*c^3*x + 64*a^2*c^4*x^3 + 48*a^2*c^3*d*x^4 + (48*a*c^2*(4*c^3 + a*d^
2)*x^5)/5 + 64*a*c^4*d*x^6 + (32*c^3*(2*c^3 + 9*a*d^2)*x^7)/7 + 12*c^2*d*(
2*c^3 + a*d^2)*x^8 + (4*c*d^2*(20*c^3 + a*d^2)*x^9)/3 + 16*c^3*d^3*x^10 +
(60*c^2*d^4*x^11)/11 + c*d^5*x^12 + (d^6*x^13)/13
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2458, 1403, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^3\,dx$$

$$\Big\downarrow \text{2458}$$

$$\int \left(c\left(4a+\frac{c^3}{d^2}\right)-2c^2\left(\frac{c}{d}+x\right)^2+d^2\left(\frac{c}{d}+x\right)^4\right)^3 d\left(\frac{c}{d}+x\right)$$

$$\Big\downarrow \text{1403}$$

$$\int \left(\frac{(4acd^2+c^4)^3}{d^6} - 8c^6\left(\frac{6ad^2}{c^3}+\frac{5}{2}\right)\left(\frac{c}{d}+x\right)^6 + \frac{12c^5(4ad^2+c^3)\left(\frac{ad^2}{c^3}+\frac{5}{4}\right)\left(\frac{c}{d}+x\right)^4}{d^2} + 12c^4d^2\left(\frac{ad^2}{c^3}+\frac{5}{4}\right)\left(\right.\right.$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{3}cd^2(4ad^2+5c^3)\left(\frac{c}{d}+x\right)^9 - \frac{4}{7}c^3(12ad^2+5c^3)\left(\frac{c}{d}+x\right)^7 + \frac{c^3(4ad^2+c^3)^3\left(\frac{c}{d}+x\right)}{d^6} - \frac{2c^4(4ad^2+c^3)^2\left(\frac{c}{d}+x\right)^3}{d^4} + \frac{3c^2(4ad^2+c^3)(4ad^2+5c^3)\left(\frac{c}{d}+x\right)^5}{5d^2} - \frac{6}{11}c^2d^4\left(\frac{c}{d}+x\right)^{11} + \frac{1}{13}d^6\left(\frac{c}{d}+x\right)^{13}$$

input
```
Int[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^3,x]
```

output
```
(c^3*(c^3 + 4*a*d^2)^3*(c/d + x))/d^6 - (2*c^4*(c^3 + 4*a*d^2)^2*(c/d + x)
^3)/d^4 + (3*c^2*(c^3 + 4*a*d^2)*(5*c^3 + 4*a*d^2)*(c/d + x)^5)/(5*d^2) -
(4*c^3*(5*c^3 + 12*a*d^2)*(c/d + x)^7)/7 + (c*d^2*(5*c^3 + 4*a*d^2)*(c/d +
 x)^9)/3 - (6*c^2*d^4*(c/d + x)^11)/11 + (d^6*(c/d + x)^13)/13
```

Defintions of rubi rules used

rule 1403
```
Int[((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int[ExpandInte
grand[(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a
*c, 0] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2458
```
Int[(Pn_)^(p_.), x_Symbol] :> With[{S = Coeff[Pn, x, Expon[Pn, x] - 1]/(Exp
on[Pn, x]*Coeff[Pn, x, Expon[Pn, x]])}, Subst[Int[ExpandToSum[Pn /. x -> x
- S, x]^p, x], x, x + S] /; BinomialQ[Pn /. x -> x - S, x] || (IntegerQ[Exp
on[Pn, x]/2] && TrinomialQ[Pn /. x -> x - S, x])] /; FreeQ[p, x] && PolyQ[P
n, x] && GtQ[Expon[Pn, x], 2] && NeQ[Coeff[Pn, x, Expon[Pn, x] - 1], 0]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.94

method	result
norman	$\frac{d^6x^{13}}{13}+c\,d^5x^{12}+\frac{60c^2d^4x^{11}}{11}+16c^3d^3x^{10}+\left(\frac{4}{3}d^4ca+\frac{80}{3}c^4d^2\right)x^9+\left(12a\,c^2d^3+24c^5d\right)x^8+\left(\frac{2}{}\right.$
gosper	$\frac{1}{13}d^6x^{13}+c\,d^5x^{12}+\frac{60}{11}c^2d^4x^{11}+16c^3d^3x^{10}+\frac{4}{3}x^9d^4ca+\frac{80}{3}x^9c^4d^2+12a\,c^2d^3x^8+24c^5d\,x^8$
risch	$\frac{1}{13}d^6x^{13}+c\,d^5x^{12}+\frac{60}{11}c^2d^4x^{11}+16c^3d^3x^{10}+\frac{4}{3}x^9d^4ca+\frac{80}{3}x^9c^4d^2+12a\,c^2d^3x^8+24c^5d\,x^8$
parallelrisch	$\frac{1}{13}d^6x^{13}+c\,d^5x^{12}+\frac{60}{11}c^2d^4x^{11}+16c^3d^3x^{10}+\frac{4}{3}x^9d^4ca+\frac{80}{3}x^9c^4d^2+12a\,c^2d^3x^8+24c^5d\,x^8$
orering	$\frac{x\left(1155d^6x^{12}+15015c\,d^5x^{11}+81900c^2d^4x^{10}+240240c^3d^3x^9+20020ac\,d^4x^8+400400c^4d^2x^8+180180a\,c^2d^3x^7+360360c^5d\,x^7\right.}{}$
default	$\frac{d^6x^{13}}{13}+c\,d^5x^{12}+\frac{60c^2d^4x^{11}}{11}+16c^3d^3x^{10}+\frac{\left(4d^4ca+224c^4d^2+d^2\left(8ac\,d^2+16c^4\right)\right)x^9}{9}+\frac{\left(64a\,c^2d^3+128c^5d+4cd\right.}{8}$

input
```
int((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/13*d^6*x^13+c*d^5*x^12+60/11*c^2*d^4*x^11+16*c^3*d^3*x^10+(4/3*d^4*c*a+8
0/3*c^4*d^2)*x^9+(12*a*c^2*d^3+24*c^5*d)*x^8+(288/7*a*c^3*d^2+64/7*c^6)*x^
7+64*a*c^4*d*x^6+(48/5*a^2*c^2*d^2+192/5*a*c^5)*x^5+48*a^2*c^3*d*x^4+64*a^
2*c^4*x^3+64*a^3*c^3*x
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.94

$$\begin{aligned}\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^3\,dx &= \frac{1}{13}\,d^6x^{13}+cd^5x^{12}+\frac{60}{11}\,c^2d^4x^{11}\\&\quad+16\,c^3d^3x^{10}+64\,ac^4dx^6+48\,a^2c^3dx^4\\&\quad+\frac{4}{3}\,\left(20\,c^4d^2+acd^4\right)x^9+64\,a^2c^4x^3\\&\quad+12\,\left(2\,c^5d+ac^2d^3\right)x^8+\frac{32}{7}\,\left(2\,c^6+9\,ac^3d^2\right)x^7\\&\quad+64\,a^3c^3x+\frac{48}{5}\,\left(4\,ac^5+a^2c^2d^2\right)x^5\end{aligned}$$

input
```
integrate((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^3,x, algorithm="fricas")
```

output
```
1/13*d^6*x^13 + c*d^5*x^12 + 60/11*c^2*d^4*x^11 + 16*c^3*d^3*x^10 + 64*a*c
^4*d*x^6 + 48*a^2*c^3*d*x^4 + 4/3*(20*c^4*d^2 + a*c*d^4)*x^9 + 64*a^2*c^4*
x^3 + 12*(2*c^5*d + a*c^2*d^3)*x^8 + 32/7*(2*c^6 + 9*a*c^3*d^2)*x^7 + 64*a
^3*c^3*x + 48/5*(4*a*c^5 + a^2*c^2*d^2)*x^5
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.03

$$\int (4ac+4c^2x^2+4cdx^3+d^2x^4)^3\,dx = 64a^3c^3x + 64a^2c^4x^3 + 48a^2c^3dx^4 + 64ac^4dx^6 + 16c^3d^3x^{10} + \frac{60c^2d^4x^{11}}{11} + cd^5x^{12} + \frac{d^6x^{13}}{13} + x^9 \cdot \left(\frac{4acd^4}{3} + \frac{80c^4d^2}{3}\right) + x^8 \cdot (12ac^2d^3 + 24c^5d) + x^7 \cdot \left(\frac{288ac^3d^2}{7} + \frac{64c^6}{7}\right) + x^5 \cdot \left(\frac{48a^2c^2d^2}{5} + \frac{192ac^5}{5}\right)$$

input
```
integrate((d**2*x**4+4*c*d*x**3+4*c**2*x**2+4*a*c)**3,x)
```

output
```
64*a**3*c**3*x + 64*a**2*c**4*x**3 + 48*a**2*c**3*d*x**4 + 64*a*c**4*d*x**
6 + 16*c**3*d**3*x**10 + 60*c**2*d**4*x**11/11 + c*d**5*x**12 + d**6*x**13
/13 + x**9*(4*a*c*d**4/3 + 80*c**4*d**2/3) + x**8*(12*a*c**2*d**3 + 24*c**
5*d) + x**7*(288*a*c**3*d**2/7 + 64*c**6/7) + x**5*(48*a**2*c**2*d**2/5 +
192*a*c**5/5)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.18

$$\int \left(4ac + 4c^2x^2 + 4cdx^3 + d^2x^4\right)^3 \, dx$$
$$= \frac{1}{13}d^6x^{13} + cd^5x^{12} + \frac{48}{11}c^2d^4x^{11} + \frac{32}{5}c^3d^3x^{10} + \frac{64}{7}c^6x^7 + 64\,a^3c^3x + \frac{16}{5}\left(3\,d^2x^5 + 15\,cdx^4 + 20\,c^2x^3\right)a^2c^2 + \frac{8}{3}\left(2\,d^2x^9 + 9\,cdx^8\right)c^4 + \frac{4}{105}\left(35\,d^4x^9 + 315\,cd^3x^8 + 720\,c^2d^2x^7 + 1008\,c^4x^5 + 120\left(3\,d^2x^7 + 14\,cdx^6\right)c^2\right)ac + \frac{4}{165}\left(45\,d^4x^{11} + 396\,cd^3x^{10} + 880\,c^2d^2x^9\right)c^2$$

input
```
integrate((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^3,x, algorithm="maxima")
```

output
```
1/13*d^6*x^13 + c*d^5*x^12 + 48/11*c^2*d^4*x^11 + 32/5*c^3*d^3*x^10 + 64/7
*c^6*x^7 + 64*a^3*c^3*x + 16/5*(3*d^2*x^5 + 15*c*d*x^4 + 20*c^2*x^3)*a^2*c
^2 + 8/3*(2*d^2*x^9 + 9*c*d*x^8)*c^4 + 4/105*(35*d^4*x^9 + 315*c*d^3*x^8 +
 720*c^2*d^2*x^7 + 1008*c^4*x^5 + 120*(3*d^2*x^7 + 14*c*d*x^6)*c^2)*a*c +
4/165*(45*d^4*x^11 + 396*c*d^3*x^10 + 880*c^2*d^2*x^9)*c^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.95

$$\int \left(4ac + 4c^2x^2 + 4cdx^3 + d^2x^4\right)^3 \, dx = \frac{1}{13}d^6x^{13} + cd^5x^{12} + \frac{60}{11}c^2d^4x^{11} + 16\,c^3d^3x^{10} + \frac{80}{3}c^4d^2x^9 + \frac{4}{3}acd^4x^9 + 24\,c^5dx^8 + 12\,ac^2d^3x^8 + \frac{64}{7}c^6x^7 + \frac{288}{7}ac^3d^2x^7 + 64\,ac^4dx^6 + \frac{192}{5}ac^5x^5 + \frac{48}{5}a^2c^2d^2x^5 + 48\,a^2c^3dx^4 + 64\,a^2c^4x^3 + 64\,a^3c^3x$$

input
```
integrate((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^3,x, algorithm="giac")
```

output
```
1/13*d^6*x^13 + c*d^5*x^12 + 60/11*c^2*d^4*x^11 + 16*c^3*d^3*x^10 + 80/3*c
^4*d^2*x^9 + 4/3*a*c*d^4*x^9 + 24*c^5*d*x^8 + 12*a*c^2*d^3*x^8 + 64/7*c^6*
x^7 + 288/7*a*c^3*d^2*x^7 + 64*a*c^4*d*x^6 + 192/5*a*c^5*x^5 + 48/5*a^2*c^
2*d^2*x^5 + 48*a^2*c^3*d*x^4 + 64*a^2*c^4*x^3 + 64*a^3*c^3*x
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.92

$$\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^3\,dx = x^8\left(24\,c^5\,d+12\,a\,c^2\,d^3\right)+x^9\left(\frac{80\,c^4\,d^2}{3}+\frac{4\,a\,c\,d^4}{3}\right)+\frac{d^6\,x^{13}}{13}+x^7\left(\frac{64\,c^6}{7}+\frac{288\,a\,c^3\,d^2}{7}\right)+64\,a^3\,c^3\,x+c\,d^5\,x^{12}+64\,a^2\,c^4\,x^3+16\,c^3\,d^3\,x^{10}+\frac{60\,c^2\,d^4\,x^{11}}{11}+48\,a^2\,c^3\,d\,x^4+\frac{48\,a\,c^2\,x^5\,\left(4\,c^3+a\,d^2\right)}{5}+64\,a\,c^4\,d\,x^6$$

input
```
int((4*a*c + 4*c^2*x^2 + d^2*x^4 + 4*c*d*x^3)^3,x)
```

output
```
x^8*(24*c^5*d + 12*a*c^2*d^3) + x^9*((80*c^4*d^2)/3 + (4*a*c*d^4)/3) + (d^
6*x^13)/13 + x^7*((64*c^6)/7 + (288*a*c^3*d^2)/7) + 64*a^3*c^3*x + c*d^5*x
^12 + 64*a^2*c^4*x^3 + 16*c^3*d^3*x^10 + (60*c^2*d^4*x^11)/11 + 48*a^2*c^3
*d*x^4 + (48*a*c^2*x^5*(a*d^2 + 4*c^3))/5 + 64*a*c^4*d*x^6
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.97

$$\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^3\,dx = \frac{x(1155d^6x^{12}+15015c\,d^5x^{11}+81900c^2d^4x^{10}+240240c^3d^3x^9+20020ac\,d^4x^8+400400c^4d^2x^8+180180c}{}$$

input
```
int((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^3,x)
```

output
```
(x*(960960*a**3*c**3 + 960960*a**2*c**4*x**2 + 720720*a**2*c**3*d*x**3 + 1
44144*a**2*c**2*d**2*x**4 + 576576*a*c**5*x**4 + 960960*a*c**4*d*x**5 + 61
7760*a*c**3*d**2*x**6 + 180180*a*c**2*d**3*x**7 + 20020*a*c*d**4*x**8 + 13
7280*c**6*x**6 + 360360*c**5*d*x**7 + 400400*c**4*d**2*x**8 + 240240*c**3*
d**3*x**9 + 81900*c**2*d**4*x**10 + 15015*c*d**5*x**11 + 1155*d**6*x**12))
/15015
```

3.33 $\int (4ac+4c^2x^2+4cdx^3+d^2x^4)^2\,dx$

Optimal result

Integrand size = 29, antiderivative size = 106

$$\int (4ac+4c^2x^2+4cdx^3+d^2x^4)^2\,dx = \frac{c^2(c^3+4ad^2)^2x}{d^4} - \frac{4c^3(c^3+4ad^2)(c+dx)^3}{3d^5} + \frac{2c(3c^3+4ad^2)(c+dx)^5}{5d^5} - \frac{4c^2(c+dx)^7}{7d^5} + \frac{(c+dx)^9}{9d^5}$$

output
```
c^2*(4*a*d^2+c^3)^2*x/d^4-4/3*c^3*(4*a*d^2+c^3)*(d*x+c)^3/d^5+2/5*c*(4*a*d
^2+3*c^3)*(d*x+c)^5/d^5-4/7*c^2*(d*x+c)^7/d^5+1/9*(d*x+c)^9/d^5
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.87

$$\int (4ac+4c^2x^2+4cdx^3+d^2x^4)^2\,dx = 16a^2c^2x + \frac{32}{3}ac^3x^3 + 8ac^2dx^4 + \frac{8}{5}c(2c^3+ad^2)x^5 + \frac{16}{3}c^3dx^6 + \frac{24}{7}c^2d^2x^7 + cd^3x^8 + \frac{d^4x^9}{9}$$

input
```
Integrate[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^2,x]
```

output
```
16*a^2*c^2*x + (32*a*c^3*x^3)/3 + 8*a*c^2*d*x^4 + (8*c*(2*c^3 + a*d^2)*x^5
)/5 + (16*c^3*d*x^6)/3 + (24*c^2*d^2*x^7)/7 + c*d^3*x^8 + (d^4*x^9)/9
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {2458, 1403, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^2 dx$$

$$\Big\downarrow \text{ 2458}$$

$$\int \left(c\left(4a+\frac{c^3}{d^2}\right)-2c^2\left(\frac{c}{d}+x\right)^2+d^2\left(\frac{c}{d}+x\right)^4\right)^2 d\left(\frac{c}{d}+x\right)$$

$$\Big\downarrow \text{ 1403}$$

$$\int \left(\frac{\left(4acd^2+c^4\right)^2}{d^4}-\frac{4c^3\left(4ad^2+c^3\right)\left(\frac{c}{d}+x\right)^2}{d^2}+4c^4\left(\frac{2ad^2}{c^3}+\frac{3}{2}\right)\left(\frac{c}{d}+x\right)^4-4c^2d^2\left(\frac{c}{d}+x\right)^6+d^4\left(\frac{c}{d}+x\right)^8\right) d$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2}{5}c\left(4ad^2+3c^3\right)\left(\frac{c}{d}+x\right)^5-\frac{4c^3\left(4ad^2+c^3\right)\left(\frac{c}{d}+x\right)^3}{3d^2}+\frac{c^2\left(4ad^2+c^3\right)^2\left(\frac{c}{d}+x\right)}{d^4}-\frac{4}{7}c^2d^2\left(\frac{c}{d}+x\right)^7+\frac{1}{9}d^4\left(\frac{c}{d}+x\right)^9$$

input
```
Int[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^2,x]
```

output
```
(c^2*(c^3 + 4*a*d^2)^2*(c/d + x))/d^4 - (4*c^3*(c^3 + 4*a*d^2)*(c/d + x)^3
)/(3*d^2) + (2*c*(3*c^3 + 4*a*d^2)*(c/d + x)^5)/5 - (4*c^2*d^2*(c/d + x)^7
)/7 + (d^4*(c/d + x)^9)/9
```

Defintions of rubi rules used

rule 1403
```
Int[((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int[ExpandInte
grand[(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a
*c, 0] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2458
```
Int[(Pn_)^(p_.), x_Symbol] :> With[{S = Coeff[Pn, x, Expon[Pn, x] - 1]/(Exp
on[Pn, x]*Coeff[Pn, x, Expon[Pn, x]])}, Subst[Int[ExpandToSum[Pn /. x -> x
- S, x]^p, x], x, x + S] /; BinomialQ[Pn /. x -> x - S, x] || (IntegerQ[Exp
on[Pn, x]/2] && TrinomialQ[Pn /. x -> x - S, x])] /; FreeQ[p, x] && PolyQ[P
n, x] && GtQ[Expon[Pn, x], 2] && NeQ[Coeff[Pn, x, Expon[Pn, x] - 1], 0]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.78

method	result
norman	$\frac{d^4x^9}{9}+c\,d^3x^8+\frac{24c^2d^2x^7}{7}+\frac{16c^3d\,x^6}{3}+\left(\frac{8}{5}ac\,d^2+\frac{16}{5}c^4\right)x^5+8a\,c^2d\,x^4+\frac{32a\,c^3x^3}{3}+16a^2c^2x$
gosper	$\frac{1}{9}d^4x^9+c\,d^3x^8+\frac{24}{7}c^2d^2x^7+\frac{16}{3}c^3d\,x^6+\frac{8}{5}x^5ac\,d^2+\frac{16}{5}c^4x^5+8a\,c^2d\,x^4+\frac{32}{3}a\,c^3x^3+16a^2c^2$
default	$\frac{d^4x^9}{9}+c\,d^3x^8+\frac{24c^2d^2x^7}{7}+\frac{16c^3d\,x^6}{3}+\frac{(8ac\,d^2+16c^4)x^5}{5}+8a\,c^2d\,x^4+\frac{32a\,c^3x^3}{3}+16a^2c^2x$
risch	$\frac{1}{9}d^4x^9+c\,d^3x^8+\frac{24}{7}c^2d^2x^7+\frac{16}{3}c^3d\,x^6+\frac{8}{5}x^5ac\,d^2+\frac{16}{5}c^4x^5+8a\,c^2d\,x^4+\frac{32}{3}a\,c^3x^3+16a^2c^2$
parallelrisch	$\frac{1}{9}d^4x^9+c\,d^3x^8+\frac{24}{7}c^2d^2x^7+\frac{16}{3}c^3d\,x^6+\frac{8}{5}x^5ac\,d^2+\frac{16}{5}c^4x^5+8a\,c^2d\,x^4+\frac{32}{3}a\,c^3x^3+16a^2c^2$
orering	$\frac{x(35d^4x^8+315c\,d^3x^7+1080c^2d^2x^6+1680c^3d\,x^5+504ac\,d^2x^4+1008c^4x^4+2520a\,c^2d\,x^3+3360a\,c^3x^2+5040a^2c^2)}{315}$

input
```
int((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/9*d^4*x^9+c*d^3*x^8+24/7*c^2*d^2*x^7+16/3*c^3*d*x^6+(8/5*a*c*d^2+16/5*c^
4)*x^5+8*a*c^2*d*x^4+32/3*a*c^3*x^3+16*a^2*c^2*x
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.77

$$\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^2 \, dx = \frac{1}{9}\,d^4x^9+cd^3x^8+\frac{24}{7}\,c^2d^2x^7+\frac{16}{3}\,c^3dx^6+8\,ac^2dx^4 +\frac{32}{3}\,ac^3x^3+\frac{8}{5}\,\left(2\,c^4+acd^2\right)x^5+16\,a^2c^2x$$

input
```
integrate((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^2,x, algorithm="fricas")
```

output
```
1/9*d^4*x^9 + c*d^3*x^8 + 24/7*c^2*d^2*x^7 + 16/3*c^3*d*x^6 + 8*a*c^2*d*x^
4 + 32/3*a*c^3*x^3 + 8/5*(2*c^4 + a*c*d^2)*x^5 + 16*a^2*c^2*x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.90

$$\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^2 \, dx = 16a^2c^2x+\frac{32ac^3x^3}{3}+8ac^2dx^4+\frac{16c^3dx^6}{3} +\frac{24c^2d^2x^7}{7}+cd^3x^8+\frac{d^4x^9}{9}+x^5\cdot\left(\frac{8acd^2}{5}+\frac{16c^4}{5}\right)$$

input
```
integrate((d**2*x**4+4*c*d*x**3+4*c**2*x**2+4*a*c)**2,x)
```

output
```
16*a**2*c**2*x + 32*a*c**3*x**3/3 + 8*a*c**2*d*x**4 + 16*c**3*d*x**6/3 + 2
4*c**2*d**2*x**7/7 + c*d**3*x**8 + d**4*x**9/9 + x**5*(8*a*c*d**2/5 + 16*c
**4/5)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.89

$$\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^2 \,dx = \frac{1}{9}\,d^4x^9+cd^3x^8+\frac{16}{7}\,c^2d^2x^7+\frac{16}{5}\,c^4x^5 +16\,a^2c^2x+\frac{8}{15}\left(3\,d^2x^5+15\,cdx^4+20\,c^2x^3\right)ac +\frac{8}{21}\left(3\,d^2x^7+14\,cdx^6\right)c^2$$

input
```
integrate((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^2,x, algorithm="maxima")
```

output
```
1/9*d^4*x^9 + c*d^3*x^8 + 16/7*c^2*d^2*x^7 + 16/5*c^4*x^5 + 16*a^2*c^2*x +
 8/15*(3*d^2*x^5 + 15*c*d*x^4 + 20*c^2*x^3)*a*c + 8/21*(3*d^2*x^7 + 14*c*d
*x^6)*c^2
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.78

$$\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^2 \,dx = \frac{1}{9}\,d^4x^9+cd^3x^8+\frac{24}{7}\,c^2d^2x^7+\frac{16}{3}\,c^3dx^6+\frac{16}{5}\,c^4x^5 +\frac{8}{5}\,acd^2x^5+8\,ac^2dx^4+\frac{32}{3}\,ac^3x^3+16\,a^2c^2x$$

input
```
integrate((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^2,x, algorithm="giac")
```

output
```
1/9*d^4*x^9 + c*d^3*x^8 + 24/7*c^2*d^2*x^7 + 16/3*c^3*d*x^6 + 16/5*c^4*x^5
 + 8/5*a*c*d^2*x^5 + 8*a*c^2*d*x^4 + 32/3*a*c^3*x^3 + 16*a^2*c^2*x
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.77

$$\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^2\,dx = x^5\left(\frac{16\,c^4}{5}+\frac{8\,a\,c\,d^2}{5}\right)+\frac{d^4\,x^9}{9}+16\,a^2\,c^2\,x+\frac{32\,a\,c^3\,x^3}{3}+\frac{16\,c^3\,d\,x^6}{3}+c\,d^3\,x^8+\frac{24\,c^2\,d^2\,x^7}{7}+8\,a\,c^2\,d\,x^4$$

input
```
int((4*a*c + 4*c^2*x^2 + d^2*x^4 + 4*c*d*x^3)^2,x)
```

output
```
x^5*((16*c^4)/5 + (8*a*c*d^2)/5) + (d^4*x^9)/9 + 16*a^2*c^2*x + (32*a*c^3*
x^3)/3 + (16*c^3*d*x^6)/3 + c*d^3*x^8 + (24*c^2*d^2*x^7)/7 + 8*a*c^2*d*x^4
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.81

$$\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^2\,dx = \frac{x(35d^4x^8+315c\,d^3x^7+1080c^2d^2x^6+1680c^3d\,x^5+504ac\,d^2x^4+1008c^4x^4+2520a\,c^2d\,x^3+3360a\,c^3x^2}{315}$$

input
```
int((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^2,x)
```

output
```
(x*(5040*a**2*c**2 + 3360*a*c**3*x**2 + 2520*a*c**2*d*x**3 + 504*a*c*d**2*
x**4 + 1008*c**4*x**4 + 1680*c**3*d*x**5 + 1080*c**2*d**2*x**6 + 315*c*d**
3*x**7 + 35*d**4*x**8))/315
```

3.34 $\int (4ac+4c^2x^2+4cdx^3+d^2x^4)\ dx$

Optimal result

Integrand size = 27, antiderivative size = 32

$$\int (4ac+4c^2x^2+4cdx^3+d^2x^4)\ dx = 4acx+\frac{4c^2x^3}{3}+cdx^4+\frac{d^2x^5}{5}$$

output
```
4*a*c*x+4/3*c^2*x^3+c*d*x^4+1/5*d^2*x^5
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int (4ac+4c^2x^2+4cdx^3+d^2x^4)\ dx = 4acx+\frac{4c^2x^3}{3}+cdx^4+\frac{d^2x^5}{5}$$

input
```
Integrate[4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4,x]
```

output
```
4*a*c*x + (4*c^2*x^3)/3 + c*d*x^4 + (d^2*x^5)/5
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.037, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right) dx$$

$$\Big\downarrow \ 2009$$

$$4acx+\frac{4c^2x^3}{3}+cdx^4+\frac{d^2x^5}{5}$$

input
```
Int[4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4,x]
```

output
```
4*a*c*x + (4*c^2*x^3)/3 + c*d*x^4 + (d^2*x^5)/5
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.91

method	result	size
gosper	$4xac+\frac{4}{3}c^2x^3+cd\,x^4+\frac{1}{5}d^2x^5$	29
default	$4xac+\frac{4}{3}c^2x^3+cd\,x^4+\frac{1}{5}d^2x^5$	29
norman	$4xac+\frac{4}{3}c^2x^3+cd\,x^4+\frac{1}{5}d^2x^5$	29
risch	$4xac+\frac{4}{3}c^2x^3+cd\,x^4+\frac{1}{5}d^2x^5$	29
parallelrisch	$4xac+\frac{4}{3}c^2x^3+cd\,x^4+\frac{1}{5}d^2x^5$	29
parts	$4xac+\frac{4}{3}c^2x^3+cd\,x^4+\frac{1}{5}d^2x^5$	29
orering	$\frac{x(3d^2x^4+15cd\,x^3+20c^2x^2+60ac)}{15}$	32

input
```
int(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c,x,method=_RETURNVERBOSE)
```

output
```
4*x*a*c+4/3*c^2*x^3+c*d*x^4+1/5*d^2*x^5
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

$$\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)\,dx=\frac{1}{5}\,d^2x^5+cdx^4+\frac{4}{3}\,c^2x^3+4\,acx$$

input
```
integrate(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c,x, algorithm="fricas")
```

output
```
1/5*d^2*x^5 + c*d*x^4 + 4/3*c^2*x^3 + 4*a*c*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.97

$$\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right) dx = 4acx+\frac{4c^2x^3}{3}+cdx^4+\frac{d^2x^5}{5}$$

input
```
integrate(d**2*x**4+4*c*d*x**3+4*c**2*x**2+4*a*c,x)
```

output
```
4*a*c*x + 4*c**2*x**3/3 + c*d*x**4 + d**2*x**5/5
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

$$\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right) dx = \frac{1}{5}d^2x^5+cdx^4+\frac{4}{3}c^2x^3+4acx$$

input
```
integrate(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c,x, algorithm="maxima")
```

output
```
1/5*d^2*x^5 + c*d*x^4 + 4/3*c^2*x^3 + 4*a*c*x
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

$$\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right) dx = \frac{1}{5}d^2x^5+cdx^4+\frac{4}{3}c^2x^3+4acx$$

input
```
integrate(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c,x, algorithm="giac")
```

output
```
1/5*d^2*x^5 + c*d*x^4 + 4/3*c^2*x^3 + 4*a*c*x
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

$$\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)\,dx=\frac{4\,c^2\,x^3}{3}+c\,d\,x^4+4\,a\,c\,x+\frac{d^2\,x^5}{5}$$

input
```
int(4*a*c + 4*c^2*x^2 + d^2*x^4 + 4*c*d*x^3,x)
```

output
```
(4*c^2*x^3)/3 + (d^2*x^5)/5 + 4*a*c*x + c*d*x^4
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.97

$$\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)\,dx=\frac{x(3d^2x^4+15cd\,x^3+20c^2x^2+60ac)}{15}$$

input
```
int(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c,x)
```

output
```
(x*(60*a*c + 20*c**2*x**2 + 15*c*d*x**3 + 3*d**2*x**4))/15
```

3.35 $\int \frac{1}{4ac+4c^2x^2+4cdx^3+d^2x^4}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 386

$$\int \frac{1}{4ac+4c^2x^2+4cdx^3+d^2x^4}\,dx = -\frac{d\arctan\left(\frac{\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}-\sqrt{2}(c+dx)}{\sqrt[4]{c}\sqrt{-c^{3/2}+\sqrt{c^3+4ad^2}}}\right)}{2\sqrt{2}c^{3/4}\sqrt{c^3+4ad^2}\sqrt{-c^{3/2}+\sqrt{c^3+4ad^2}}} + \frac{d\arctan\left(\frac{\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}+\sqrt{2}(c+dx)}{\sqrt[4]{c}\sqrt{-c^{3/2}+\sqrt{c^3+4ad^2}}}\right)}{2\sqrt{2}c^{3/4}\sqrt{c^3+4ad^2}\sqrt{-c^{3/2}+\sqrt{c^3+4ad^2}}} + \frac{d\,\mathrm{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}(c+dx)}{\sqrt{c}\sqrt{c^3+4ad^2}+(c+dx)^2}\right)}{2\sqrt{2}c^{3/4}\sqrt{c^3+4ad^2}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}}$$

output

```
-1/4*d*arctan((c^(1/4)*(c^(3/2)+(4*a*d^2+c^3)^(1/2))^(1/2)-2^(1/2)*(d*x+c)
)/c^(1/4)/(-c^(3/2)+(4*a*d^2+c^3)^(1/2))^(1/2))*2^(1/2)/c^(3/4)/(4*a*d^2+c
^3)^(1/2)/(-c^(3/2)+(4*a*d^2+c^3)^(1/2))^(1/2)+1/4*d*arctan((c^(1/4)*(c^(3
/2)+(4*a*d^2+c^3)^(1/2))^(1/2)+2^(1/2)*(d*x+c))/c^(1/4)/(-c^(3/2)+(4*a*d^2
+c^3)^(1/2))^(1/2))*2^(1/2)/c^(3/4)/(4*a*d^2+c^3)^(1/2)/(-c^(3/2)+(4*a*d^2
+c^3)^(1/2))^(1/2)+1/4*d*arctanh(2^(1/2)*c^(1/4)*(c^(3/2)+(4*a*d^2+c^3)^(1
/2))^(1/2)*(d*x+c)/(c^(1/2)*(4*a*d^2+c^3)^(1/2)+(d*x+c)^2))*2^(1/2)/c^(3/4
)/(4*a*d^2+c^3)^(1/2)/(c^(3/2)+(4*a*d^2+c^3)^(1/2))^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.18

$$\int \frac{1}{4ac+4c^2x^2+4cdx^3+d^2x^4}\,dx = \frac{1}{4}\text{RootSum}\left[4ac+4c^2\#1^2+4cd\#1^3+d^2\#1^4\&,\frac{\log(x-\#1)}{2c^2\#1+3cd\#1^2+d^2\#1^3}\&\right]$$

input
```
Integrate[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^(-1),x]
```

output
```
RootSum[4*a*c + 4*c^2*#1^2 + 4*c*d*#1^3 + d^2*#1^4 & , Log[x - #1]/(2*c^2*
#1 + 3*c*d*#1^2 + d^2*#1^3) & ]/4
```

Rubi [A] (verified)

Time = 1.01 (sec) , antiderivative size = 555, normalized size of antiderivative = 1.44, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.310$, Rules used = {2458, 1407, 27, 1142, 25, 27, 1083, 219, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{4ac+4c^2x^2+4cdx^3+d^2x^4}\,dx$$

$$\Big\downarrow \text{2458}$$

$$\int \frac{1}{c\left(4a+\frac{c^3}{d^2}\right)-2c^2\left(\frac{c}{d}+x\right)^2+d^2\left(\frac{c}{d}+x\right)^4}d\left(\frac{c}{d}+x\right)$$

$$\Big\downarrow \text{1407}$$

$$\frac{d\int \frac{\sqrt{2}\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}-d\left(\frac{c}{d}+x\right)}{d\left(\left(\frac{c}{d}+x\right)^2-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}\left(\frac{c}{d}+x\right)}{d}+\frac{\sqrt{c}\sqrt{c^3+4ad^2}}{d^2}\right)}d\left(\frac{c}{d}+x\right)}{2\sqrt{2}c^{3/4}\sqrt{4ad^2+c^3}\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}}+\frac{d\int \frac{d\left(\frac{c}{d}+x\right)+\sqrt{2}\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}}{d\left(\left(\frac{c}{d}+x\right)^2+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}\left(\frac{c}{d}+x\right)}{d}+\frac{\sqrt{c}\sqrt{c^3+4ad^2}}{d^2}\right)}d\left(\frac{c}{d}+x\right)}{2\sqrt{2}c^{3/4}\sqrt{4ad^2+c^3}\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}}$$

$\downarrow$ 27

$$\frac{\int \frac{\sqrt{2}\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}-d\left(\frac{c}{d}+x\right)}{\left(\frac{c}{d}+x\right)^2-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}\left(\frac{c}{d}+x\right)}{d}+\frac{\sqrt{c}\sqrt{c^3+4ad^2}}{d^2}}d\left(\frac{c}{d}+x\right)}{2\sqrt{2}c^{3/4}\sqrt{4ad^2+c^3}\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}}+\frac{\int \frac{d\left(\frac{c}{d}+x\right)+\sqrt{2}\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}}{\left(\frac{c}{d}+x\right)^2+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}\left(\frac{c}{d}+x\right)}{d}+\frac{\sqrt{c}\sqrt{c^3+4ad^2}}{d^2}}d\left(\frac{c}{d}+x\right)}{2\sqrt{2}c^{3/4}\sqrt{4ad^2+c^3}\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}}$$

$\downarrow$ 1142

$$\frac{\frac{\sqrt[4]{c}\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}\int \frac{1}{\left(\frac{c}{d}+x\right)^2-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}\left(\frac{c}{d}+x\right)}{d}+\frac{\sqrt{c}\sqrt{c^3+4ad^2}}{d^2}}d\left(\frac{c}{d}+x\right)}{\sqrt{2}}-\frac{1}{2}d\int -\frac{\sqrt{2}\left(\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}-\cdots\right.}{d\left(\left(\frac{c}{d}+x\right)^2-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}\left(\cdots\right.}{d}\cdots\right.}}{2\sqrt{2}c^{3/4}\sqrt{4ad^2+c^3}\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}}+\frac{\frac{\sqrt[4]{c}\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}\int \frac{1}{\left(\frac{c}{d}+x\right)^2+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}\left(\frac{c}{d}+x\right)}{d}+\frac{\sqrt{c}\sqrt{c^3+4ad^2}}{d^2}}d\left(\frac{c}{d}+x\right)}{\sqrt{2}}+\frac{1}{2}d\int \frac{\sqrt{2}\left(\sqrt{2}d\left(\frac{c}{d}+x\right)+\sqrt[4]{c}\sqrt{c^{3/2}+\cdots}\right.}{d\left(\left(\frac{c}{d}+x\right)^2+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}\left(\frac{c}{d}+\cdots\right.}{d}\cdots\right.}}{2\sqrt{2}c^{3/4}\sqrt{4ad^2+c^3}\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}}$$

$\downarrow$ 25

$$\frac{\frac{\sqrt[4]{c}\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}\int \frac{1}{\left(\frac{c}{d}+x\right)^2-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}\left(\frac{c}{d}+x\right)}{d}+\frac{\sqrt{c}\sqrt{c^3+4ad^2}}{d^2}}d\left(\frac{c}{d}+x\right)}{\sqrt{2}}+\frac{1}{2}d\int \frac{\sqrt{2}\left(\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}-\sqrt{2}\cdots\right.}{d\left(\left(\frac{c}{d}+x\right)^2-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}\left(\frac{c}{d}+\cdots\right.}{d}\cdots\right.}}{2\sqrt{2}c^{3/4}\sqrt{4ad^2+c^3}\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}}$$

$$\frac{\frac{\sqrt[4]{c}\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}\int \frac{1}{\left(\frac{c}{d}+x\right)^2+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}\left(\frac{c}{d}+x\right)}{d}+\frac{\sqrt{c}\sqrt{c^3+4ad^2}}{d^2}}d\left(\frac{c}{d}+x\right)}{\sqrt{2}}+\frac{1}{2}d\int \frac{\sqrt{2}\left(\sqrt{2}d\left(\frac{c}{d}+x\right)+\sqrt[4]{c}\sqrt{c^{3/2}+\cdots}\right.}{d\left(\left(\frac{c}{d}+x\right)^2+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}\left(\frac{c}{d}+\cdots\right.}{d}\cdots\right.}}{2\sqrt{2}c^{3/4}\sqrt{4ad^2+c^3}\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}}$$

$\big\downarrow$ 27

$$\frac{\frac{\sqrt[4]{c}\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}\int \frac{1}{\left(\frac{c}{d}+x\right)^2-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}\left(\frac{c}{d}+x\right)}{d}+\frac{\sqrt{c}\sqrt{c^3+4ad^2}}{d^2}}d\left(\frac{c}{d}+x\right)}{\sqrt{2}}+\frac{\int \frac{\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}-\sqrt{2}d\left(\frac{c}{d}+x\right)}{\left(\frac{c}{d}+x\right)^2-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}\left(\frac{c}{d}+x\right)}{d}+\sqrt{c}\cdots}}{\sqrt{2}}}{2\sqrt{2}c^{3/4}\sqrt{4ad^2+c^3}\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}}$$

$$\frac{\frac{\sqrt[4]{c}\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}\int \frac{1}{\left(\frac{c}{d}+x\right)^2+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}\left(\frac{c}{d}+x\right)}{d}+\frac{\sqrt{c}\sqrt{c^3+4ad^2}}{d^2}}d\left(\frac{c}{d}+x\right)}{\sqrt{2}}+\frac{\int \frac{\sqrt{2}d\left(\frac{c}{d}+x\right)+\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}}{\left(\frac{c}{d}+x\right)^2+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}\left(\frac{c}{d}+x\right)}{d}+\sqrt{c}\cdots}}{\sqrt{2}}}{2\sqrt{2}c^{3/4}\sqrt{4ad^2+c^3}\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}}$$

$\big\downarrow$ 1083

$$\frac{\frac{\int \frac{\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}-\sqrt{2}d\left(\frac{c}{d}+x\right)}{\left(\frac{c}{d}+x\right)^2-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}\left(\frac{c}{d}+x\right)}{d}+\frac{\sqrt{c}\sqrt{c^3+4ad^2}}{d^2}}d\left(\frac{c}{d}+x\right)}{\sqrt{2}}-\sqrt{2}\sqrt[4]{c}\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}\int \frac{1}{\frac{2\sqrt{c}\left(c^{3/2}-\sqrt{c^3+4ad^2}\right)}{d^2}-\left(2\left(\frac{c}{d}+x\right)-\cdots\right.}}{2\sqrt{2}c^{3/4}\sqrt{4ad^2+c^3}\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}}$$

$$\frac{\frac{\int \frac{\sqrt{2}d\left(\frac{c}{d}+x\right)+\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}}{\left(\frac{c}{d}+x\right)^2+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}\left(\frac{c}{d}+x\right)}{d}+\frac{\sqrt{c}\sqrt{c^3+4ad^2}}{d^2}}d\left(\frac{c}{d}+x\right)}{\sqrt{2}}-\sqrt{2}\sqrt[4]{c}\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}\int \frac{1}{\frac{2\sqrt{c}\left(c^{3/2}-\sqrt{c^3+4ad^2}\right)}{d^2}-\left(2\left(\frac{c}{d}+x\right)-\cdots\right.}}{2\sqrt{2}c^{3/4}\sqrt{4ad^2+c^3}\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}}$$

$\big\downarrow$ 219

$$\frac{\frac{\int \frac{\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}-\sqrt{2}d\left(\frac{c}{d}+x\right)}{\left(\frac{c}{d}+x\right)^2-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}\left(\frac{c}{d}+x\right)}{d}+\frac{\sqrt{c}\sqrt{c^3+4ad^2}}{d^2}}d\left(\frac{c}{d}+x\right)}{\sqrt{2}}-\frac{d\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}\operatorname{arctanh}\left(\frac{d\left(2\left(\frac{c}{d}+x\right)-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}}{d}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt{c^{3/2}-\sqrt{4ad^2+c^3}}}\right)}{\sqrt{c^{3/2}-\sqrt{4ad^2+c^3}}}}{2\sqrt{2}c^{3/4}\sqrt{4ad^2+c^3}\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}}$$

$$\frac{\frac{\int \frac{\sqrt{2}d\left(\frac{c}{d}+x\right)+\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}}{\left(\frac{c}{d}+x\right)^2+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}\left(\frac{c}{d}+x\right)}{d}+\frac{\sqrt{c}\sqrt{c^3+4ad^2}}{d^2}}d\left(\frac{c}{d}+x\right)}{\sqrt{2}}-\frac{d\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}\operatorname{arctanh}\left(\frac{d\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}}{d}+2\left(\frac{c}{d}+x\right)\right)}{\sqrt{2}\sqrt[4]{c}\sqrt{c^{3/2}-\sqrt{4ad^2+c^3}}}\right)}{\sqrt{c^{3/2}-\sqrt{4ad^2+c^3}}}}{2\sqrt{2}c^{3/4}\sqrt{4ad^2+c^3}\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}}$$

$$\Big\downarrow \text{ 1103}$$

$$\frac{-\frac{d\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}\operatorname{arctanh}\left(\frac{d\left(2\left(\frac{c}{d}+x\right)-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}}{d}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt{c^{3/2}-\sqrt{4ad^2+c^3}}}\right)}{\sqrt{c^{3/2}-\sqrt{4ad^2+c^3}}}-\frac{1}{2}d\log\left(\sqrt{c}\sqrt{4ad^2+c^3}-\sqrt{2}\sqrt[4]{c}d\sqrt{\sqrt{4ad^2+c^3}+\right.}{2\sqrt{2}c^{3/4}\sqrt{4ad^2+c^3}\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}}$$

$$\frac{\frac{1}{2}d\log\left(\sqrt{c}\sqrt{4ad^2+c^3}+\sqrt{2}\sqrt[4]{c}d\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}\left(\frac{c}{d}+x\right)+d^2\left(\frac{c}{d}+x\right)^2\right)-\frac{d\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}\operatorname{arctanh}\left(\frac{d\left(\frac{\sqrt{2}\sqrt[4]{}}{}\right.}{\sqrt{}}\right.}{\sqrt{c^{3/2}-\sqrt{4ad}}}}{2\sqrt{2}c^{3/4}\sqrt{4ad^2+c^3}\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}}$$

input
```
Int[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^(-1),x]
```

output
```
(-((d*Sqrt[c^(3/2) + Sqrt[c^3 + 4*a*d^2]]*ArcTanh[(d*(-((Sqrt[2]*c^(1/4)*S
qrt[c^(3/2) + Sqrt[c^3 + 4*a*d^2]])/d) + 2*(c/d + x)))/(Sqrt[2]*c^(1/4)*Sq
rt[c^(3/2) - Sqrt[c^3 + 4*a*d^2]])])/Sqrt[c^(3/2) - Sqrt[c^3 + 4*a*d^2]])
- (d*Log[Sqrt[c]*Sqrt[c^3 + 4*a*d^2] - Sqrt[2]*c^(1/4)*d*Sqrt[c^(3/2) + Sq
rt[c^3 + 4*a*d^2]]*(c/d + x) + d^2*(c/d + x)^2])/2)/(2*Sqrt[2]*c^(3/4)*Sqr
t[c^3 + 4*a*d^2]*Sqrt[c^(3/2) + Sqrt[c^3 + 4*a*d^2]]) + (-((d*Sqrt[c^(3/2)
 + Sqrt[c^3 + 4*a*d^2]]*ArcTanh[(d*((Sqrt[2]*c^(1/4)*Sqrt[c^(3/2) + Sqrt[c
^3 + 4*a*d^2]])/d + 2*(c/d + x)))/(Sqrt[2]*c^(1/4)*Sqrt[c^(3/2) - Sqrt[c^3
 + 4*a*d^2]])])/Sqrt[c^(3/2) - Sqrt[c^3 + 4*a*d^2]]) + (d*Log[Sqrt[c]*Sqrt
[c^3 + 4*a*d^2] + Sqrt[2]*c^(1/4)*d*Sqrt[c^(3/2) + Sqrt[c^3 + 4*a*d^2]]*(c
/d + x) + d^2*(c/d + x)^2])/2)/(2*Sqrt[2]*c^(3/4)*Sqrt[c^3 + 4*a*d^2]*Sqrt
[c^(3/2) + Sqrt[c^3 + 4*a*d^2]])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1407
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   Int[(r - x)/(q - r*
x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(r + x)/(q + r*x + x^2), x], x]]]
 /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

rule 2458
```
Int[(Pn_)^(p_.), x_Symbol] :> With[{S = Coeff[Pn, x, Expon[Pn, x] - 1]/(Exp
on[Pn, x]*Coeff[Pn, x, Expon[Pn, x]])}, Subst[Int[ExpandToSum[Pn /. x -> x
- S, x]^p, x], x, x + S] /; BinomialQ[Pn /. x -> x - S, x] || (IntegerQ[Exp
on[Pn, x]/2] && TrinomialQ[Pn /. x -> x - S, x])] /; FreeQ[p, x] && PolyQ[P
n, x] && GtQ[Expon[Pn, x], 2] && NeQ[Coeff[Pn, x, Expon[Pn, x] - 1], 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.15 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.17

method	result	size
default	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(d^{2}\_Z^{4}+4cd\_Z^{3}+4c^{2}\_Z^{2}+4ac\right)}\frac{\ln\left(x-\_R\right)}{\_R^{3}d^{2}+3\_R^{2}cd+2\_Rc^{2}}\right)}{4}$	64
risch	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(d^{2}\_Z^{4}+4cd\_Z^{3}+4c^{2}\_Z^{2}+4ac\right)}\frac{\ln\left(x-\_R\right)}{\_R^{3}d^{2}+3\_R^{2}cd+2\_Rc^{2}}\right)}{4}$	64

input
```
int(1/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c),x,method=_RETURNVERBOSE)
```

output
```
1/4*sum(1/(_R^3*d^2+3*_R^2*c*d+2*_R*c^2)*ln(x-_R),_R=RootOf(_Z^4*d^2+4*_Z^
3*c*d+4*_Z^2*c^2+4*a*c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 905 vs. $2(295) = 590$.

Time = 0.08 (sec) , antiderivative size = 905, normalized size of antiderivative = 2.34

$$\int \frac{1}{4ac + 4c^2x^2 + 4cdx^3 + d^2x^4}\,dx = \text{Too large to display}$$

input
```
integrate(1/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c),x, algorithm="fricas")
```

output
```
1/8*sqrt(-(2*(a*c^3 + 4*a^2*d^2)*sqrt(-d^2/(a*c^9 + 8*a^2*c^6*d^2 + 16*a^3
*c^3*d^4)) + 1)/(a*c^3 + 4*a^2*d^2))*log(d^2*x + c*d + (2*a*c*d^2 + (a*c^7
 + 4*a^2*c^4*d^2)*sqrt(-d^2/(a*c^9 + 8*a^2*c^6*d^2 + 16*a^3*c^3*d^4)))*sqr
t(-(2*(a*c^3 + 4*a^2*d^2)*sqrt(-d^2/(a*c^9 + 8*a^2*c^6*d^2 + 16*a^3*c^3*d^
4)) + 1)/(a*c^3 + 4*a^2*d^2))) - 1/8*sqrt(-(2*(a*c^3 + 4*a^2*d^2)*sqrt(-d^
2/(a*c^9 + 8*a^2*c^6*d^2 + 16*a^3*c^3*d^4)) + 1)/(a*c^3 + 4*a^2*d^2))*log(
d^2*x + c*d - (2*a*c*d^2 + (a*c^7 + 4*a^2*c^4*d^2)*sqrt(-d^2/(a*c^9 + 8*a^
2*c^6*d^2 + 16*a^3*c^3*d^4)))*sqrt(-(2*(a*c^3 + 4*a^2*d^2)*sqrt(-d^2/(a*c^
9 + 8*a^2*c^6*d^2 + 16*a^3*c^3*d^4)) + 1)/(a*c^3 + 4*a^2*d^2))) + 1/8*sqrt
((2*(a*c^3 + 4*a^2*d^2)*sqrt(-d^2/(a*c^9 + 8*a^2*c^6*d^2 + 16*a^3*c^3*d^4)
) - 1)/(a*c^3 + 4*a^2*d^2))*log(d^2*x + c*d + (2*a*c*d^2 - (a*c^7 + 4*a^2*
c^4*d^2)*sqrt(-d^2/(a*c^9 + 8*a^2*c^6*d^2 + 16*a^3*c^3*d^4)))*sqrt((2*(a*c
^3 + 4*a^2*d^2)*sqrt(-d^2/(a*c^9 + 8*a^2*c^6*d^2 + 16*a^3*c^3*d^4)) - 1)/(
a*c^3 + 4*a^2*d^2))) - 1/8*sqrt((2*(a*c^3 + 4*a^2*d^2)*sqrt(-d^2/(a*c^9 +
8*a^2*c^6*d^2 + 16*a^3*c^3*d^4)) - 1)/(a*c^3 + 4*a^2*d^2))*log(d^2*x + c*d
 - (2*a*c*d^2 - (a*c^7 + 4*a^2*c^4*d^2)*sqrt(-d^2/(a*c^9 + 8*a^2*c^6*d^2 +
 16*a^3*c^3*d^4)))*sqrt((2*(a*c^3 + 4*a^2*d^2)*sqrt(-d^2/(a*c^9 + 8*a^2*c^
6*d^2 + 16*a^3*c^3*d^4)) - 1)/(a*c^3 + 4*a^2*d^2)))
```

Sympy [A] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.23

$$\int \frac{1}{4ac + 4c^2x^2 + 4cdx^3 + d^2x^4}\,dx$$
$$= \operatorname{RootSum}\left(t^4 \cdot \left(16384a^3c^3d^2 + 4096a^2c^6\right) + 128t^2ac^3 + 1, \left(t \mapsto t\log\left(x + \frac{-1024t^3a^2c^4d^2 - 256t^3ac^7 +}{d^2}\right.\right.\right.$$

input
```
integrate(1/(d**2*x**4+4*c*d*x**3+4*c**2*x**2+4*a*c),x)
```

output
```
RootSum(_t**4*(16384*a**3*c**3*d**2 + 4096*a**2*c**6) + 128*_t**2*a*c**3 +
 1, Lambda(_t, _t*log(x + (-1024*_t**3*a**2*c**4*d**2 - 256*_t**3*a*c**7 +
 16*_t*a*c*d**2 - 4*_t*c**4 + c*d)/d**2)))
```

Maxima [F]

$$\int \frac{1}{4ac+4c^2x^2+4cdx^3+d^2x^4}\,dx=\int \frac{1}{d^2x^4+4\,cdx^3+4\,c^2x^2+4\,ac}\,dx$$

input
```
integrate(1/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c),x, algorithm="maxima")
```

output
```
integrate(1/(d^2*x^4 + 4*c*d*x^3 + 4*c^2*x^2 + 4*a*c), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 603 vs. $2(295) = 590$.

Time = 0.11 (sec) , antiderivative size = 603, normalized size of antiderivative = 1.56

$$\int \frac{1}{4ac+4c^2x^2+4cdx^3+d^2x^4}\,dx =$$

$$-\frac{\log\left(x+\sqrt{\frac{c^2d^2+2\sqrt{-ac}d^3}{d^4}}+\frac{c}{d}\right)}{4\left(d^2\left(\sqrt{\frac{c^2d^2+2\sqrt{-ac}d^3}{d^4}}+\frac{c}{d}\right)^3-3\,cd\left(\sqrt{\frac{c^2d^2+2\sqrt{-ac}d^3}{d^4}}+\frac{c}{d}\right)^2+2\,c^2\left(\sqrt{\frac{c^2d^2+2\sqrt{-ac}d^3}{d^4}}+\frac{c}{d}\right)\right)}$$
$$+\frac{\log\left(x-\sqrt{\frac{c^2d^2+2\sqrt{-ac}d^3}{d^4}}+\frac{c}{d}\right)}{4\left(d^2\left(\sqrt{\frac{c^2d^2+2\sqrt{-ac}d^3}{d^4}}-\frac{c}{d}\right)^3+3\,cd\left(\sqrt{\frac{c^2d^2+2\sqrt{-ac}d^3}{d^4}}-\frac{c}{d}\right)^2+2\,c^2\left(\sqrt{\frac{c^2d^2+2\sqrt{-ac}d^3}{d^4}}-\frac{c}{d}\right)\right)}$$
$$-\frac{\log\left(x+\sqrt{\frac{c^2d^2-2\sqrt{-ac}d^3}{d^4}}+\frac{c}{d}\right)}{4\left(d^2\left(\sqrt{\frac{c^2d^2-2\sqrt{-ac}d^3}{d^4}}+\frac{c}{d}\right)^3-3\,cd\left(\sqrt{\frac{c^2d^2-2\sqrt{-ac}d^3}{d^4}}+\frac{c}{d}\right)^2+2\,c^2\left(\sqrt{\frac{c^2d^2-2\sqrt{-ac}d^3}{d^4}}+\frac{c}{d}\right)\right)}$$
$$+\frac{\log\left(x-\sqrt{\frac{c^2d^2-2\sqrt{-ac}d^3}{d^4}}+\frac{c}{d}\right)}{4\left(d^2\left(\sqrt{\frac{c^2d^2-2\sqrt{-ac}d^3}{d^4}}-\frac{c}{d}\right)^3+3\,cd\left(\sqrt{\frac{c^2d^2-2\sqrt{-ac}d^3}{d^4}}-\frac{c}{d}\right)^2+2\,c^2\left(\sqrt{\frac{c^2d^2-2\sqrt{-ac}d^3}{d^4}}-\frac{c}{d}\right)\right)}$$

input
```
integrate(1/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c),x, algorithm="giac")
```

output
```
-1/4*log(x + sqrt((c^2*d^2 + 2*sqrt(-a*c)*d^3)/d^4) + c/d)/(d^2*(sqrt((c^2
*d^2 + 2*sqrt(-a*c)*d^3)/d^4) + c/d)^3 - 3*c*d*(sqrt((c^2*d^2 + 2*sqrt(-a*
c)*d^3)/d^4) + c/d)^2 + 2*c^2*(sqrt((c^2*d^2 + 2*sqrt(-a*c)*d^3)/d^4) + c/
d)) + 1/4*log(x - sqrt((c^2*d^2 + 2*sqrt(-a*c)*d^3)/d^4) + c/d)/(d^2*(sqrt
((c^2*d^2 + 2*sqrt(-a*c)*d^3)/d^4) - c/d)^3 + 3*c*d*(sqrt((c^2*d^2 + 2*sqr
t(-a*c)*d^3)/d^4) - c/d)^2 + 2*c^2*(sqrt((c^2*d^2 + 2*sqrt(-a*c)*d^3)/d^4)
 - c/d)) - 1/4*log(x + sqrt((c^2*d^2 - 2*sqrt(-a*c)*d^3)/d^4) + c/d)/(d^2*
(sqrt((c^2*d^2 - 2*sqrt(-a*c)*d^3)/d^4) + c/d)^3 - 3*c*d*(sqrt((c^2*d^2 -
2*sqrt(-a*c)*d^3)/d^4) + c/d)^2 + 2*c^2*(sqrt((c^2*d^2 - 2*sqrt(-a*c)*d^3)
/d^4) + c/d)) + 1/4*log(x - sqrt((c^2*d^2 - 2*sqrt(-a*c)*d^3)/d^4) + c/d)/
(d^2*(sqrt((c^2*d^2 - 2*sqrt(-a*c)*d^3)/d^4) - c/d)^3 + 3*c*d*(sqrt((c^2*d
^2 - 2*sqrt(-a*c)*d^3)/d^4) - c/d)^2 + 2*c^2*(sqrt((c^2*d^2 - 2*sqrt(-a*c)
*d^3)/d^4) - c/d))
```

Mupad [B] (verification not implemented)

Time = 23.83 (sec) , antiderivative size = 1551, normalized size of antiderivative = 4.02

$$\int \frac{1}{4ac + 4c^2x^2 + 4cdx^3 + d^2x^4} \, dx = \text{Too large to display}$$

input
```
int(1/(4*a*c + 4*c^2*x^2 + d^2*x^4 + 4*c*d*x^3),x)
```

output
```
atan(((-(2*d*(-a^3*c^3)^(1/2) + a*c^3)/(64*(a^2*c^6 + 4*a^3*c^3*d^2)))^(1/
2)*(((256*a*c^4*d^5 + 256*a*c^3*d^6*x)*(-(2*d*(-a^3*c^3)^(1/2) + a*c^3)/(6
4*(a^2*c^6 + 4*a^3*c^3*d^2)))^(1/2) - 64*a*c*d^6)*(-(2*d*(-a^3*c^3)^(1/2)
+ a*c^3)/(64*(a^2*c^6 + 4*a^3*c^3*d^2)))^(1/2) + 4*c*d^5 + 4*d^6*x)*1i + (
-(2*d*(-a^3*c^3)^(1/2) + a*c^3)/(64*(a^2*c^6 + 4*a^3*c^3*d^2)))^(1/2)*(((2
56*a*c^4*d^5 + 256*a*c^3*d^6*x)*(-(2*d*(-a^3*c^3)^(1/2) + a*c^3)/(64*(a^2*
c^6 + 4*a^3*c^3*d^2)))^(1/2) + 64*a*c*d^6)*(-(2*d*(-a^3*c^3)^(1/2) + a*c^3
)/(64*(a^2*c^6 + 4*a^3*c^3*d^2)))^(1/2) + 4*c*d^5 + 4*d^6*x)*1i)/((-(2*d*(
-a^3*c^3)^(1/2) + a*c^3)/(64*(a^2*c^6 + 4*a^3*c^3*d^2)))^(1/2)*(((256*a*c^
4*d^5 + 256*a*c^3*d^6*x)*(-(2*d*(-a^3*c^3)^(1/2) + a*c^3)/(64*(a^2*c^6 + 4
*a^3*c^3*d^2)))^(1/2) - 64*a*c*d^6)*(-(2*d*(-a^3*c^3)^(1/2) + a*c^3)/(64*(
a^2*c^6 + 4*a^3*c^3*d^2)))^(1/2) + 4*c*d^5 + 4*d^6*x) - (-(2*d*(-a^3*c^3)^
(1/2) + a*c^3)/(64*(a^2*c^6 + 4*a^3*c^3*d^2)))^(1/2)*(((256*a*c^4*d^5 + 25
6*a*c^3*d^6*x)*(-(2*d*(-a^3*c^3)^(1/2) + a*c^3)/(64*(a^2*c^6 + 4*a^3*c^3*d
^2)))^(1/2) + 64*a*c*d^6)*(-(2*d*(-a^3*c^3)^(1/2) + a*c^3)/(64*(a^2*c^6 +
4*a^3*c^3*d^2)))^(1/2) + 4*c*d^5 + 4*d^6*x)))*(-(2*d*(-a^3*c^3)^(1/2) + a*
c^3)/(64*(a^2*c^6 + 4*a^3*c^3*d^2)))^(1/2)*2i + atan((((2*d*(-a^3*c^3)^(1/
2) - a*c^3)/(64*(a^2*c^6 + 4*a^3*c^3*d^2)))^(1/2)*(((256*a*c^4*d^5 + 256*a
*c^3*d^6*x)*((2*d*(-a^3*c^3)^(1/2) - a*c^3)/(64*(a^2*c^6 + 4*a^3*c^3*d^2))
)^(1/2) - 64*a*c*d^6)*((2*d*(-a^3*c^3)^(1/2) - a*c^3)/(64*(a^2*c^6 + 4*...
```

Reduce [F]

$$\int \frac{1}{4ac + 4c^2x^2 + 4cdx^3 + d^2x^4} \, dx = \int \frac{1}{d^2x^4 + 4cd\,x^3 + 4c^2x^2 + 4ac} dx$$

input
```
int(1/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c),x)
```

output
```
int(1/(4*a*c + 4*c**2*x**2 + 4*c*d*x**3 + d**2*x**4),x)
```

3.36 $\int \frac{1}{(4ac+4c^2x^2+4cdx^3+d^2x^4)^2}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 565

$$\begin{aligned}
&\int \frac{1}{(4ac+4c^2x^2+4cdx^3+d^2x^4)^2}\,dx \\
&= -\frac{(c+dx)\left(c^3-4ad^2-c(c+dx)^2\right)}{16acd\left(c^3+4ad^2\right)\left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)} \\
&- \frac{d\left(c^3+12ad^2+c^{3/2}\sqrt{c^3+4ad^2}\right)\arctan\left(\frac{\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}-\sqrt{2}(c+dx)}{\sqrt[4]{c}\sqrt{-c^{3/2}+\sqrt{c^3+4ad^2}}}\right)}{32\sqrt{2}ac^{7/4}\left(c^3+4ad^2\right)^{3/2}\sqrt{-c^{3/2}+\sqrt{c^3+4ad^2}}} \\
&+ \frac{d\left(c^3+12ad^2+c^{3/2}\sqrt{c^3+4ad^2}\right)\arctan\left(\frac{\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}+\sqrt{2}(c+dx)}{\sqrt[4]{c}\sqrt{-c^{3/2}+\sqrt{c^3+4ad^2}}}\right)}{32\sqrt{2}ac^{7/4}\left(c^3+4ad^2\right)^{3/2}\sqrt{-c^{3/2}+\sqrt{c^3+4ad^2}}} \\
&+ \frac{d\left(c^3+12ad^2-c^{3/2}\sqrt{c^3+4ad^2}\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}(c+dx)}{\sqrt{c}\sqrt{c^3+4ad^2}+(c+dx)^2}\right)}{32\sqrt{2}ac^{7/4}\left(c^3+4ad^2\right)^{3/2}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}}
\end{aligned}$$

output
```
-1/16*(d*x+c)*(c^3-4*a*d^2-c*(d*x+c)^2)/a/c/d/(4*a*d^2+c^3)/(d^2*x^4+4*c*d
*x^3+4*c^2*x^2+4*a*c)-1/64*d*(c^3+12*a*d^2+c^(3/2)*(4*a*d^2+c^3)^(1/2))*ar
ctan((c^(1/4)*(c^(3/2)+(4*a*d^2+c^3)^(1/2))^(1/2)-2^(1/2)*(d*x+c))/c^(1/4)
/(-c^(3/2)+(4*a*d^2+c^3)^(1/2))^(1/2))*2^(1/2)/a/c^(7/4)/(4*a*d^2+c^3)^(3/
2)/(-c^(3/2)+(4*a*d^2+c^3)^(1/2))^(1/2)+1/64*d*(c^3+12*a*d^2+c^(3/2)*(4*a*
d^2+c^3)^(1/2))*arctan((c^(1/4)*(c^(3/2)+(4*a*d^2+c^3)^(1/2))^(1/2)+2^(1/2
)*(d*x+c))/c^(1/4)/(-c^(3/2)+(4*a*d^2+c^3)^(1/2))^(1/2))*2^(1/2)/a/c^(7/4)
/(4*a*d^2+c^3)^(3/2)/(-c^(3/2)+(4*a*d^2+c^3)^(1/2))^(1/2)+1/64*d*(c^3+12*a
*d^2-c^(3/2)*(4*a*d^2+c^3)^(1/2))*arctanh(2^(1/2)*c^(1/4)*(c^(3/2)+(4*a*d^
2+c^3)^(1/2))^(1/2)*(d*x+c)/(c^(1/2)*(4*a*d^2+c^3)^(1/2)+(d*x+c)^2))*2^(1/
2)/a/c^(7/4)/(4*a*d^2+c^3)^(3/2)/(c^(3/2)+(4*a*d^2+c^3)^(1/2))^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.11 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.32

$$\int \frac{1}{(4ac+4c^2x^2+4cdx^3+d^2x^4)^2}\,dx$$
$$= \frac{\frac{4(c+dx)(4ad+cx(2c+dx))}{4ac+x^2(2c+dx)^2} + \text{RootSum}\left[4ac+4c^2\#1^2+4cd\#1^3+d^2\#1^4\&, \frac{2c^3\log(x-\#1)+12ad^2\log(x-\#1)+2c^2d\log}{2c^2\#1+3cd\#1^2}\right.}{64ac(c^3+4ad^2)}$$

input
```
Integrate[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^(-2),x]
```

output
```
((4*(c + d*x)*(4*a*d + c*x*(2*c + d*x)))/(4*a*c + x^2*(2*c + d*x)^2) + Roo
tSum[4*a*c + 4*c^2*#1^2 + 4*c*d*#1^3 + d^2*#1^4 & , (2*c^3*Log[x - #1] + 1
2*a*d^2*Log[x - #1] + 2*c^2*d*Log[x - #1]*#1 + c*d^2*Log[x - #1]*#1^2)/(2*
c^2*#1 + 3*c*d*#1^2 + d^2*#1^3) & ])/(64*a*c*(c^3 + 4*a*d^2))
```

Rubi [A] (verified)

Time = 1.24 (sec) , antiderivative size = 797, normalized size of antiderivative = 1.41, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.379, Rules used = {2458, 1405, 27, 1483, 27, 1142, 25, 27, 1083, 219, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(4ac+4c^2x^2+4cdx^3+d^2x^4)^2}dx$$

$$\Big\downarrow \text{ 2458}$$

$$\int \frac{1}{\left(c\left(4a+\frac{c^3}{d^2}\right)-2c^2\left(\frac{c}{d}+x\right)^2+d^2\left(\frac{c}{d}+x\right)^4\right)^2}d\left(\frac{c}{d}+x\right)$$

$$\Big\downarrow \text{ 1405}$$

$$\frac{\int \frac{2c\left(c^3+d^2\left(\frac{c}{d}+x\right)^2c+12ad^2\right)}{d^2\left(\frac{c}{d}+x\right)^4-2c^2\left(\frac{c}{d}+x\right)^2+c\left(\frac{c^3}{d^2}+4a\right)}d\left(\frac{c}{d}+x\right)}{32ac^2(4ad^2+c^3)}-\frac{\left(\frac{c}{d}+x\right)\left(-4ad^2+c^3-cd^2\left(\frac{c}{d}+x\right)^2\right)}{16ac(4ad^2+c^3)\left(c\left(4a+\frac{c^3}{d^2}\right)-2c^2\left(\frac{c}{d}+x\right)^2+d^2\left(\frac{c}{d}+x\right)^4\right)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{c^3+d^2\left(\frac{c}{d}+x\right)^2c+12ad^2}{d^2\left(\frac{c}{d}+x\right)^4-2c^2\left(\frac{c}{d}+x\right)^2+c\left(\frac{c^3}{d^2}+4a\right)}d\left(\frac{c}{d}+x\right)}{16ac(4ad^2+c^3)}-\frac{\left(\frac{c}{d}+x\right)\left(-4ad^2+c^3-cd^2\left(\frac{c}{d}+x\right)^2\right)}{16ac(4ad^2+c^3)\left(c\left(4a+\frac{c^3}{d^2}\right)-2c^2\left(\frac{c}{d}+x\right)^2+d^2\left(\frac{c}{d}+x\right)^4\right)}$$

$$\Big\downarrow \text{ 1483}$$

$$\frac{\frac{d\int \frac{\sqrt{2}\sqrt[4]{c}\left(c^3+12ad^2\right)\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}-d\left(c^3-\sqrt{c^3+4ad^2}c^{3/2}+12ad^2\right)\left(\frac{c}{d}+x\right)}{d\left(\left(\frac{c}{d}+x\right)^2-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}\left(\frac{c}{d}+x\right)}{d}+\frac{\sqrt{c}\sqrt{c^3+4ad^2}}{d^2}\right)}d\left(\frac{c}{d}+x\right)}{2\sqrt{2}c^{3/4}\sqrt{4ad^2+c^3}\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}}+\frac{d\int \frac{\sqrt{2}\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}\left(c^3+12ad^2\right)+d\left(c^3-\sqrt{c^3+4a\ldots}\right.}{d\left(\left(\frac{c}{d}+x\right)^2+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}\left(\frac{c}{d}+x\right)}{d}+\ldots\right.}}{2\sqrt{2}c^{3/4}\sqrt{4ad^2+c^3}\sqrt{\sqrt{4ad^2+c^3}\ldots}}}{16ac\left(4ad^2+c^3\right)}$$

$$\frac{\left(\frac{c}{d}+x\right)\left(-4ad^2+c^3-cd^2\left(\frac{c}{d}+x\right)^2\right)}{16ac\left(4ad^2+c^3\right)\left(c\left(4a+\frac{c^3}{d^2}\right)-2c^2\left(\frac{c}{d}+x\right)^2+d^2\left(\frac{c}{d}+x\right)^4\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{c}\left(c^3+12ad^2\right)\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}-d\left(c^3-\sqrt{c^3+4ad^2}c^{3/2}+12ad^2\right)\left(\frac{c}{d}+x\right)}{\left(\frac{c}{d}+x\right)^2-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}\left(\frac{c}{d}+x\right)}{d}+\frac{\sqrt{c}\sqrt{c^3+4ad^2}}{d^2}}d\left(\frac{c}{d}+x\right)}{2\sqrt{2}c^{3/4}\sqrt{4ad^2+c^3}\sqrt{\sqrt{4ad^2+c^3}+c^{3/2}}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}\left(c^3+12ad^2\right)+d\left(c^3-\sqrt{c^3+4ad^2}c\ldots\right.}{\left(\frac{c}{d}+x\right)^2+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}\left(\frac{c}{d}+x\right)}{d}+\sqrt{c}\ldots}}{2\sqrt{2}c^{3/4}\sqrt{4ad^2+c^3}\sqrt{\sqrt{4ad^2+c^3}+\ldots}}}{16ac\left(4ad^2+c^3\right)}$$

$$\frac{\left(\frac{c}{d}+x\right)\left(-4ad^2+c^3-cd^2\left(\frac{c}{d}+x\right)^2\right)}{16ac\left(4ad^2+c^3\right)\left(c\left(4a+\frac{c^3}{d^2}\right)-2c^2\left(\frac{c}{d}+x\right)^2+d^2\left(\frac{c}{d}+x\right)^4\right)}$$

$$\Big\downarrow \; 1142$$

$$\frac{\frac{\frac{\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}\left(c^3+\sqrt{c^3+4ad^2}c^{3/2}+12ad^2\right)\int \frac{1}{\left(\frac{c}{d}+x\right)^2-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}\left(\frac{c}{d}+x\right)}{d}+\frac{\sqrt{c}\sqrt{c^3+4ad^2}}{d^2}}d\left(\frac{c}{d}+x\right)}{\sqrt{2}}-\frac{1}{2}d\left(c^3-\sqrt{c^3+4ad^2}c^{3/2}+1\ldots\right.}{2\sqrt{2}c^{3/4}\sqrt{c^3+4ad^2}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}}}{\ldots}$$

$$\frac{\left(\frac{c}{d}+x\right)\left(c^3-d^2\left(\frac{c}{d}+x\right)^2c-4ad^2\right)}{16ac\left(c^3+4ad^2\right)\left(d^2\left(\frac{c}{d}+x\right)^4-2c^2\left(\frac{c}{d}+x\right)^2+c\left(\frac{c^3}{d^2}+4a\right)\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\frac{\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}\left(c^3+\sqrt{c^3+4ad^2}c^{3/2}+12ad^2\right)\int \frac{1}{\left(\frac{c}{d}+x\right)^2-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}\left(\frac{c}{d}+x\right)}{d}+\frac{\sqrt{c}\sqrt{c^3+4ad^2}}{d^2}}d\left(\frac{c}{d}+x\right)}{\sqrt{2}}+\frac{1}{2}d\left(c^3-\sqrt{c^3+4ad^2}c^{3/2}+1\ldots\right.}{2\sqrt{2}c^{3/4}\sqrt{c^3+4ad^2}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}}}{\ldots}$$

$$\frac{\left(\frac{c}{d}+x\right)\left(c^3-d^2\left(\frac{c}{d}+x\right)^2c-4ad^2\right)}{16ac\left(c^3+4ad^2\right)\left(d^2\left(\frac{c}{d}+x\right)^4-2c^2\left(\frac{c}{d}+x\right)^2+c\left(\frac{c^3}{d^2}+4a\right)\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}\left(c^3+\sqrt{c^3+4ad^2}c^{3/2}+12ad^2\right)\int \frac{1}{\left(\frac{c}{d}+x\right)^2-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}\left(\frac{c}{d}+x\right)}{d}+\frac{\sqrt{c}\sqrt{c^3+4ad^2}}{d^2}}d\left(\frac{c}{d}+x\right)}{\sqrt{2}}+\frac{\left(c^3-\sqrt{c^3+4ad^2}c^{3/2}+12ad^2\cdots\right.}{\cdots}}{2\sqrt{2}c^{3/4}\sqrt{c^3+4ad^2}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}\cdots}$$

$$\frac{\left(\frac{c}{d}+x\right)\left(c^3-d^2\left(\frac{c}{d}+x\right)^2c-4ad^2\right)}{16ac\left(c^3+4ad^2\right)\left(d^2\left(\frac{c}{d}+x\right)^4-2c^2\left(\frac{c}{d}+x\right)^2+c\left(\frac{c^3}{d^2}+4a\right)\right)}$$

$$\Big\downarrow \; 1083$$

$$\frac{\frac{\left(c^3-\sqrt{c^3+4ad^2}c^{3/2}+12ad^2\right)\int \frac{\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}-\sqrt{2}d\left(\frac{c}{d}+x\right)}{\left(\frac{c}{d}+x\right)^2-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}\left(\frac{c}{d}+x\right)}{d}+\frac{\sqrt{c}\sqrt{c^3+4ad^2}}{d^2}}d\left(\frac{c}{d}+x\right)}{\sqrt{2}}-\sqrt{2}\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}\left(c^3+\sqrt{c^3+4ad^2}c^{3/2}+\cdots\right.}{2\sqrt{2}c^{3/4}\sqrt{c^3+4ad^2}\sqrt{c^{3/2}+\sqrt{c^3+\cdots}}}$$

$$\frac{\left(\frac{c}{d}+x\right)\left(c^3-d^2\left(\frac{c}{d}+x\right)^2c-4ad^2\right)}{16ac\left(c^3+4ad^2\right)\left(d^2\left(\frac{c}{d}+x\right)^4-2c^2\left(\frac{c}{d}+x\right)^2+c\left(\frac{c^3}{d^2}+4a\right)\right)}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{\left(c^3-\sqrt{c^3+4ad^2}c^{3/2}+12ad^2\right)\int \frac{\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}-\sqrt{2}d\left(\frac{c}{d}+x\right)}{\left(\frac{c}{d}+x\right)^2-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}\left(\frac{c}{d}+x\right)}{d}+\frac{\sqrt{c}\sqrt{c^3+4ad^2}}{d^2}}d\left(\frac{c}{d}+x\right)}{\sqrt{2}}-\frac{d\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}\left(c^3+\sqrt{c^3+4ad^2}c^{3/2}+12ad^2\right)\mathrm{ar}\cdots}{\sqrt{c^{3/2}-\cdots}}}{2\sqrt{2}c^{3/4}\sqrt{c^3+4ad^2}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}}$$

$$\frac{\left(\frac{c}{d}+x\right)\left(c^3-d^2\left(\frac{c}{d}+x\right)^2c-4ad^2\right)}{16ac\left(c^3+4ad^2\right)\left(d^2\left(\frac{c}{d}+x\right)^4-2c^2\left(\frac{c}{d}+x\right)^2+c\left(\frac{c^3}{d^2}+4a\right)\right)}$$

$$\Big\downarrow \; 1103$$

$$\frac{\frac{-\frac{d\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}\left(c^3+\sqrt{c^3+4ad^2}c^{3/2}+12ad^2\right)\operatorname{arctanh}\left(\frac{d\left(2\left(\frac{c}{d}+x\right)-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}}{d}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt{c^{3/2}-\sqrt{c^3+4ad^2}}}\right)}{\sqrt{c^{3/2}-\sqrt{c^3+4ad^2}}}-\frac{1}{2}d\left(c^3-\sqrt{c^3+4ad^2}c^{3/2}+12ad^2\right)\log\left(d^2(\right.}{2\sqrt{2}c^{3/4}\sqrt{c^3+4ad^2}\sqrt{c^{3/2}+\sqrt{c^3+4ad^2}}}}{}$$

$$\frac{\left(\frac{c}{d}+x\right)\left(c^3-d^2\left(\frac{c}{d}+x\right)^2c-4ad^2\right)}{16ac\left(c^3+4ad^2\right)\left(d^2\left(\frac{c}{d}+x\right)^4-2c^2\left(\frac{c}{d}+x\right)^2+c\left(\frac{c^3}{d^2}+4a\right)\right)}$$

input
```
Int[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^(-2),x]
```

output
```
-1/16*((c/d + x)*(c^3 - 4*a*d^2 - c*d^2*(c/d + x)^2))/(a*c*(c^3 + 4*a*d^2)
*(c*(4*a + c^3/d^2) - 2*c^2*(c/d + x)^2 + d^2*(c/d + x)^4)) + ((-((d*Sqrt[
c^(3/2) + Sqrt[c^3 + 4*a*d^2]]*(c^3 + 12*a*d^2 + c^(3/2)*Sqrt[c^3 + 4*a*d^
2])*ArcTanh[(d*(-((Sqrt[2]*c^(1/4)*Sqrt[c^(3/2) + Sqrt[c^3 + 4*a*d^2]])/d)
 + 2*(c/d + x)))/(Sqrt[2]*c^(1/4)*Sqrt[c^(3/2) - Sqrt[c^3 + 4*a*d^2]])])/S
qrt[c^(3/2) - Sqrt[c^3 + 4*a*d^2]]) - (d*(c^3 + 12*a*d^2 - c^(3/2)*Sqrt[c^
3 + 4*a*d^2])*Log[Sqrt[c]*Sqrt[c^3 + 4*a*d^2] - Sqrt[2]*c^(1/4)*d*Sqrt[c^(
3/2) + Sqrt[c^3 + 4*a*d^2]]*(c/d + x) + d^2*(c/d + x)^2])/2)/(2*Sqrt[2]*c^
(3/4)*Sqrt[c^3 + 4*a*d^2]*Sqrt[c^(3/2) + Sqrt[c^3 + 4*a*d^2]]) + (-((d*Sqr
t[c^(3/2) + Sqrt[c^3 + 4*a*d^2]]*(c^3 + 12*a*d^2 + c^(3/2)*Sqrt[c^3 + 4*a*
d^2])*ArcTanh[(d*((Sqrt[2]*c^(1/4)*Sqrt[c^(3/2) + Sqrt[c^3 + 4*a*d^2]])/d
+ 2*(c/d + x)))/(Sqrt[2]*c^(1/4)*Sqrt[c^(3/2) - Sqrt[c^3 + 4*a*d^2]])])/Sq
rt[c^(3/2) - Sqrt[c^3 + 4*a*d^2]]) + (d*(c^3 + 12*a*d^2 - c^(3/2)*Sqrt[c^3
 + 4*a*d^2])*Log[Sqrt[c]*Sqrt[c^3 + 4*a*d^2] + Sqrt[2]*c^(1/4)*d*Sqrt[c^(3
/2) + Sqrt[c^3 + 4*a*d^2]]*(c/d + x) + d^2*(c/d + x)^2])/2)/(2*Sqrt[2]*c^(
3/4)*Sqrt[c^3 + 4*a*d^2]*Sqrt[c^(3/2) + Sqrt[c^3 + 4*a*d^2]]))/(16*a*c*(c^
3 + 4*a*d^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1483
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   In
t[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(d*r
 + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

rule 2458
```
Int[(Pn_)^(p_.), x_Symbol] :> With[{S = Coeff[Pn, x, Expon[Pn, x] - 1]/(Exp
on[Pn, x]*Coeff[Pn, x, Expon[Pn, x]])}, Subst[Int[ExpandToSum[Pn /. x -> x
- S, x]^p, x], x, x + S] /; BinomialQ[Pn /. x -> x - S, x] || (IntegerQ[Exp
on[Pn, x]/2] && TrinomialQ[Pn /. x -> x - S, x])] /; FreeQ[p, x] && PolyQ[P
n, x] && GtQ[Expon[Pn, x], 2] && NeQ[Coeff[Pn, x, Expon[Pn, x] - 1], 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.15 (sec) , antiderivative size = 230, normalized size of antiderivative = 0.41

method	result
default	$\frac{\frac{d^2x^3}{64a\left(4a\,d^2+c^3\right)}+\frac{3cd\,x^2}{64a\left(4a\,d^2+c^3\right)}+\frac{\left(2a\,d^2+c^3\right)x}{32c\left(4a\,d^2+c^3\right)a}+\frac{d}{64a\,d^2+16c^3}}{\frac{1}{4}d^2x^4+cd\,x^3+c^2x^2+ac}+\frac{\sum_{\_R=\operatorname{RootOf}\left(d^2\_Z^4+4cd\_Z^3+4c^2\_Z^2+4ac\right)}\frac{\left(\_R^2c\,d^2+2\_R\ldots\right.}{\_R^3d^2+\ldots}}{64ac(4a\,d^2+c^3)}$
risch	$\frac{\frac{d^2x^3}{64a\left(4a\,d^2+c^3\right)}+\frac{3cd\,x^2}{64a\left(4a\,d^2+c^3\right)}+\frac{\left(2a\,d^2+c^3\right)x}{32c\left(4a\,d^2+c^3\right)a}+\frac{d}{64a\,d^2+16c^3}}{\frac{1}{4}d^2x^4+cd\,x^3+c^2x^2+ac}+\frac{\sum_{\_R=\operatorname{RootOf}\left(d^2\_Z^4+4cd\_Z^3+4c^2\_Z^2+4ac\right)}\frac{\left(\frac{d^2\_R^2}{4a\,d^2+c^3}+\frac{2cd\ldots}{4a\,d^2+\ldots}\right.}{\_R^3d^2\ldots}}{64a}$

input
```
int(1/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^2,x,method=_RETURNVERBOSE)
```

output
```
(1/64*d^2/a/(4*a*d^2+c^3)*x^3+3/64/a*c*d/(4*a*d^2+c^3)*x^2+1/32/c*(2*a*d^2
+c^3)/(4*a*d^2+c^3)/a*x+1/16*d/(4*a*d^2+c^3))/(1/4*d^2*x^4+c*d*x^3+c^2*x^2
+a*c)+1/64/a/c/(4*a*d^2+c^3)*sum((_R^2*c*d^2+2*_R*c^2*d+12*a*d^2+2*c^3)/(_
R^3*d^2+3*_R^2*c*d+2*_R*c^2)*ln(x-_R),_R=RootOf(_Z^4*d^2+4*_Z^3*c*d+4*_Z^2
*c^2+4*a*c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3222 vs. 2(461) = 922.

Time = 0.16 (sec) , antiderivative size = 3222, normalized size of antiderivative = 5.70

$$\int \frac{1}{(4ac + 4c^2x^2 + 4cdx^3 + d^2x^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [A] (verification not implemented)

Time = 80.84 (sec) , antiderivative size = 427, normalized size of antiderivative = 0.76

$$\int \frac{1}{(4ac + 4c^2x^2 + 4cdx^3 + d^2x^4)^2}\,dx = \frac{4acd + 3c^2dx^2 + cd^2x^3 + x(4ad^2 + 2c^3)}{256a^3c^2d^2 + 64a^2c^5 + x^4 \cdot (64a^2cd^4 + 16ac^4d^2) + x^3 \cdot (256a^2c^2d^3 + 64ac^5d) + x^2 \cdot (256a^2c^3d^2 + 64ac^6)} + \operatorname{RootSum}\Big(t^4 \cdot (1073741824a^9c^7d^6 + 805306368a^8c^{10}d^4 + 201326592a^7c^{13}d^2 + 16777216a^6c^{16}) + t^2 \cdot$$

input
```
integrate(1/(d**2*x**4+4*c*d*x**3+4*c**2*x**2+4*a*c)**2,x)
```

output
```
(4*a*c*d + 3*c**2*d*x**2 + c*d**2*x**3 + x*(4*a*d**2 + 2*c**3))/(256*a**3*
c**2*d**2 + 64*a**2*c**5 + x**4*(64*a**2*c*d**4 + 16*a*c**4*d**2) + x**3*(
256*a**2*c**2*d**3 + 64*a*c**5*d) + x**2*(256*a**2*c**3*d**2 + 64*a*c**6))
 + RootSum(_t**4*(1073741824*a**9*c**7*d**6 + 805306368*a**8*c**10*d**4 +
201326592*a**7*c**13*d**2 + 16777216*a**6*c**16) + _t**2*(491520*a**5*c**5
*d**4 + 122880*a**4*c**8*d**2 + 8192*a**3*c**11) + 81*a**2*d**4 + 18*a*c**
3*d**2 + c**6, Lambda(_t, _t*log(x + (-67108864*_t**3*a**7*c**7*d**8 - 587
20256*_t**3*a**6*c**10*d**6 - 18874368*_t**3*a**5*c**13*d**4 - 2621440*_t*
*3*a**4*c**16*d**2 - 131072*_t**3*a**3*c**19 + 27648*_t*a**4*c**2*d**8 - 9
216*_t*a**3*c**5*d**6 - 5440*_t*a**2*c**8*d**4 - 736*_t*a*c**11*d**2 - 32*
_t*c**14 + 324*a**2*c*d**7 + 81*a*c**4*d**5 + 5*c**7*d**3)/(324*a**2*d**8
+ 81*a*c**3*d**6 + 5*c**6*d**4)))
```

Maxima [F]

$$\int \frac{1}{(4ac+4c^2x^2+4cdx^3+d^2x^4)^2}\,dx = \int \frac{1}{(d^2x^4+4\,cdx^3+4\,c^2x^2+4\,ac)^2}\,dx$$

input
```
integrate(1/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^2,x, algorithm="maxima")
```

output
```
1/16*(c*d^2*x^3 + 3*c^2*d*x^2 + 4*a*c*d + 2*(c^3 + 2*a*d^2)*x)/(4*a^2*c^5
+ 16*a^3*c^2*d^2 + (a*c^4*d^2 + 4*a^2*c*d^4)*x^4 + 4*(a*c^5*d + 4*a^2*c^2*
d^3)*x^3 + 4*(a*c^6 + 4*a^2*c^3*d^2)*x^2) + 1/16*integrate((c*d^2*x^2 + 2*
c^2*d*x + 2*c^3 + 12*a*d^2)/(d^2*x^4 + 4*c*d*x^3 + 4*c^2*x^2 + 4*a*c), x)/
(a*c^4 + 4*a^2*c*d^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1057 vs. $2(461) = 922$.

Time = 0.11 (sec) , antiderivative size = 1057, normalized size of antiderivative = 1.87

$$\int \frac{1}{(4ac+4c^2x^2+4cdx^3+d^2x^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^2,x, algorithm="giac")
```

output
```
-1/64*((c*d^2*(sqrt((c^2*d^2 + 2*sqrt(-a*c)*d^3)/d^4) + c/d)^2 - 2*c^2*d*(
sqrt((c^2*d^2 + 2*sqrt(-a*c)*d^3)/d^4) + c/d) + 2*c^3 + 12*a*d^2)*log(x +
sqrt((c^2*d^2 + 2*sqrt(-a*c)*d^3)/d^4) + c/d)/(d^2*(sqrt((c^2*d^2 + 2*sqrt
(-a*c)*d^3)/d^4) + c/d)^3 - 3*c*d*(sqrt((c^2*d^2 + 2*sqrt(-a*c)*d^3)/d^4)
+ c/d)^2 + 2*c^2*(sqrt((c^2*d^2 + 2*sqrt(-a*c)*d^3)/d^4) + c/d)) - (c*d^2*
(sqrt((c^2*d^2 + 2*sqrt(-a*c)*d^3)/d^4) - c/d)^2 + 2*c^2*d*(sqrt((c^2*d^2
+ 2*sqrt(-a*c)*d^3)/d^4) - c/d) + 2*c^3 + 12*a*d^2)*log(x - sqrt((c^2*d^2
+ 2*sqrt(-a*c)*d^3)/d^4) + c/d)/(d^2*(sqrt((c^2*d^2 + 2*sqrt(-a*c)*d^3)/d^
4) - c/d)^3 + 3*c*d*(sqrt((c^2*d^2 + 2*sqrt(-a*c)*d^3)/d^4) - c/d)^2 + 2*c
^2*(sqrt((c^2*d^2 + 2*sqrt(-a*c)*d^3)/d^4) - c/d)) + (c*d^2*(sqrt((c^2*d^2
 - 2*sqrt(-a*c)*d^3)/d^4) + c/d)^2 - 2*c^2*d*(sqrt((c^2*d^2 - 2*sqrt(-a*c)
*d^3)/d^4) + c/d) + 2*c^3 + 12*a*d^2)*log(x + sqrt((c^2*d^2 - 2*sqrt(-a*c)
*d^3)/d^4) + c/d)/(d^2*(sqrt((c^2*d^2 - 2*sqrt(-a*c)*d^3)/d^4) + c/d)^3 -
3*c*d*(sqrt((c^2*d^2 - 2*sqrt(-a*c)*d^3)/d^4) + c/d)^2 + 2*c^2*(sqrt((c^2*
d^2 - 2*sqrt(-a*c)*d^3)/d^4) + c/d)) - (c*d^2*(sqrt((c^2*d^2 - 2*sqrt(-a*c
)*d^3)/d^4) - c/d)^2 + 2*c^2*d*(sqrt((c^2*d^2 - 2*sqrt(-a*c)*d^3)/d^4) - c
/d) + 2*c^3 + 12*a*d^2)*log(x - sqrt((c^2*d^2 - 2*sqrt(-a*c)*d^3)/d^4) + c
/d)/(d^2*(sqrt((c^2*d^2 - 2*sqrt(-a*c)*d^3)/d^4) - c/d)^3 + 3*c*d*(sqrt((c
^2*d^2 - 2*sqrt(-a*c)*d^3)/d^4) - c/d)^2 + 2*c^2*(sqrt((c^2*d^2 - 2*sqrt(-
a*c)*d^3)/d^4) - c/d)))/(a*c^4 + 4*a^2*c*d^2) + 1/16*(c*d^2*x^3 + 3*c^2...
```

Mupad [B] (verification not implemented)

Time = 23.64 (sec) , antiderivative size = 5844, normalized size of antiderivative = 10.34

$$\int \frac{1}{(4ac+4c^2x^2+4cdx^3+d^2x^4)^2}\,dx = \text{Too large to display}$$

input
```
int(1/(4*a*c + 4*c^2*x^2 + d^2*x^4 + 4*c*d*x^3)^2,x)
```

output
```
(d/(4*(4*a*d^2 + c^3)) + (d^2*x^3)/(16*a*(4*a*d^2 + c^3)) + (x*(2*a*d^2 +
c^3))/(8*a*c*(4*a*d^2 + c^3)) + (3*c*d*x^2)/(16*a*(4*a*d^2 + c^3)))/(4*a*c
 + 4*c^2*x^2 + d^2*x^4 + 4*c*d*x^3) - atan(((-(a^3*c^11 + 10*c^3*d^3*(-a^9
*c^7)^(1/2) + 15*a^4*c^8*d^2 + 60*a^5*c^5*d^4 + 72*a*d^5*(-a^9*c^7)^(1/2))
/(4096*(a^6*c^16 + 12*a^7*c^13*d^2 + 48*a^8*c^10*d^4 + 64*a^9*c^7*d^6)))^(
1/2)*((((262144*a^4*c^12*d^5 + 2097152*a^5*c^9*d^7 + 4194304*a^6*c^6*d^9)/
(1024*(a^3*c^8 + 8*a^4*c^5*d^2 + 16*a^5*c^2*d^4)) + (x*(4096*a^3*c^11*d^6
+ 32768*a^4*c^8*d^8 + 65536*a^5*c^5*d^10))/(16*(a^2*c^8 + 8*a^3*c^5*d^2 +
16*a^4*c^2*d^4)))*(-(a^3*c^11 + 10*c^3*d^3*(-a^9*c^7)^(1/2) + 15*a^4*c^8*d
^2 + 60*a^5*c^5*d^4 + 72*a*d^5*(-a^9*c^7)^(1/2))/(4096*(a^6*c^16 + 12*a^7*
c^13*d^2 + 48*a^8*c^10*d^4 + 64*a^9*c^7*d^6)))^(1/2) - (4096*a^3*c^8*d^6 +
 65536*a^4*c^5*d^8 + 196608*a^5*c^2*d^10)/(1024*(a^3*c^8 + 8*a^4*c^5*d^2 +
 16*a^5*c^2*d^4)))*(-(a^3*c^11 + 10*c^3*d^3*(-a^9*c^7)^(1/2) + 15*a^4*c^8*
d^2 + 60*a^5*c^5*d^4 + 72*a*d^5*(-a^9*c^7)^(1/2))/(4096*(a^6*c^16 + 12*a^7
*c^13*d^2 + 48*a^8*c^10*d^4 + 64*a^9*c^7*d^6)))^(1/2) + (64*a*c^7*d^5 + 23
04*a^3*c*d^9 + 704*a^2*c^4*d^7)/(1024*(a^3*c^8 + 8*a^4*c^5*d^2 + 16*a^5*c^
2*d^4)) + (x*(36*a^2*d^10 + c^6*d^6 + 11*a*c^3*d^8))/(16*(a^2*c^8 + 8*a^3*
c^5*d^2 + 16*a^4*c^2*d^4)))*1i + (-(a^3*c^11 + 10*c^3*d^3*(-a^9*c^7)^(1/2)
 + 15*a^4*c^8*d^2 + 60*a^5*c^5*d^4 + 72*a*d^5*(-a^9*c^7)^(1/2))/(4096*(a^6
*c^16 + 12*a^7*c^13*d^2 + 48*a^8*c^10*d^4 + 64*a^9*c^7*d^6)))^(1/2)*(((...
```

Reduce [F]

$$\int \frac{1}{(4ac+4c^2x^2+4cdx^3+d^2x^4)^2}\,dx = \int \frac{1}{d^4x^8+8c\,d^3x^7+24c^2d^2x^6+32c^3d\,x^5+8ac\,d^2x^4+16c^4x^4+32a\,c^2d\,x^3+32a\,c^3x^2+16a^2c^2}dx$$

input
```
int(1/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^2,x)
```

output
```
int(1/(16*a**2*c**2 + 32*a*c**3*x**2 + 32*a*c**2*d*x**3 + 8*a*c*d**2*x**4
+ 16*c**4*x**4 + 32*c**3*d*x**5 + 24*c**2*d**2*x**6 + 8*c*d**3*x**7 + d**4
*x**8),x)
```

3.37 $\int \left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)^4\,dx$

Optimal result

Integrand size = 32, antiderivative size = 266

$$\begin{aligned}\int \left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)^4\,dx &= \frac{(5d^4+256ae^3)^4 x}{1048576e^4}-\frac{d^2(5d^4+256ae^3)^3(d+4ex)^3}{524288e^5}\\ &\quad+\frac{(5d^4+256ae^3)^2(59d^4+256ae^3)(d+4ex)^5}{5242880e^5}\\ &\quad-\frac{9d^2(5d^4+256ae^3)(17d^4+256ae^3)(d+4ex)^7}{3670016e^5}\\ &\quad+\frac{(601d^8+20992ad^4e^3+65536a^2e^6)(d+4ex)^9}{6291456e^5}-\frac{9d^2(17d^4+256ae^3)(d+4ex)^{11}}{5767168e^5}\\ &\quad+\frac{(59d^4+256ae^3)(d+4ex)^{13}}{13631488e^5}-\frac{d^2(d+4ex)^{15}}{2621440e^5}+\frac{(d+4ex)^{17}}{71303168e^5}\end{aligned}$$

output
```
1/1048576*(256*a*e^3+5*d^4)^4*x/e^4-1/524288*d^2*(256*a*e^3+5*d^4)^3*(4*e*
x+d)^3/e^5+1/5242880*(256*a*e^3+5*d^4)^2*(256*a*e^3+59*d^4)*(4*e*x+d)^5/e^
5-9/3670016*d^2*(256*a*e^3+5*d^4)*(256*a*e^3+17*d^4)*(4*e*x+d)^7/e^5+1/629
1456*(65536*a^2*e^6+20992*a*d^4*e^3+601*d^8)*(4*e*x+d)^9/e^5-9/5767168*d^2
*(256*a*e^3+17*d^4)*(4*e*x+d)^11/e^5+1/13631488*(256*a*e^3+59*d^4)*(4*e*x+
d)^13/e^5-1/2621440*d^2*(4*e*x+d)^15/e^5+1/71303168*(4*e*x+d)^17/e^5
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.30

$$\begin{aligned}&\int (8ae^2-d^3x+8de^2x^3+8e^3x^4)^4\,dx\\&=4096a^4e^8x-1024a^3d^3e^6x^2+128a^2d^6e^4x^3+8ade^2(-d^8+512a^2e^6)\,x^4\\&\quad+\frac{1}{5}(d^{12}-6144a^2d^4e^6+16384a^3e^9)\,x^5-128ad^3e^4(-d^4+8ae^3)\,x^6\\&\quad-\frac{32}{7}d^2e^2(d^8-24ad^4e^3-768a^2e^6)\,x^7-4de^3(d^8+192ad^4e^3-1536a^2e^6)\,x^8\\&\quad+\frac{128}{3}e^4(d^8-32ad^4e^3+64a^2e^6)\,x^9+\frac{128}{5}d^3e^5(3d^4+40ae^3)\,x^{10}\\&\quad+\frac{128}{11}d^2e^6(-13d^4+384ae^3)\,x^{11}-512de^7(d^4-8ae^3)\,x^{12}+\frac{2048}{13}e^8(-d^4+8ae^3)\,x^{13}\\&\quad+1024d^3e^9x^{14}+\frac{8192}{5}d^2e^{10}x^{15}+1024de^{11}x^{16}+\frac{4096e^{12}x^{17}}{17}\end{aligned}$$

input

```
Integrate[(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)^4,x]
```

output

```
4096*a^4*e^8*x - 1024*a^3*d^3*e^6*x^2 + 128*a^2*d^6*e^4*x^3 + 8*a*d*e^2*(-
d^8 + 512*a^2*e^6)*x^4 + ((d^12 - 6144*a^2*d^4*e^6 + 16384*a^3*e^9)*x^5)/5
 - 128*a*d^3*e^4*(-d^4 + 8*a*e^3)*x^6 - (32*d^2*e^2*(d^8 - 24*a*d^4*e^3 -
768*a^2*e^6)*x^7)/7 - 4*d*e^3*(d^8 + 192*a*d^4*e^3 - 1536*a^2*e^6)*x^8 + (
128*e^4*(d^8 - 32*a*d^4*e^3 + 64*a^2*e^6)*x^9)/3 + (128*d^3*e^5*(3*d^4 + 4
0*a*e^3)*x^10)/5 + (128*d^2*e^6*(-13*d^4 + 384*a*e^3)*x^11)/11 - 512*d*e^7
*(d^4 - 8*a*e^3)*x^12 + (2048*e^8*(-d^4 + 8*a*e^3)*x^13)/13 + 1024*d^3*e^9
*x^14 + (8192*d^2*e^10*x^15)/5 + 1024*d*e^11*x^16 + (4096*e^12*x^17)/17
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.14, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {2458, 1403, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4\right)^4 dx$$

$$\Big\downarrow \text{2458}$$

$$\int \left(\frac{1}{32}\left(256ae^2 + \frac{5d^4}{e}\right) - 3d^2e\left(\frac{d}{4e} + x\right)^2 + 8e^3\left(\frac{d}{4e} + x\right)^4\right)^4 d\left(\frac{d}{4e} + x\right)$$

$$\Big\downarrow \text{1403}$$

$$\int \left(\frac{27}{512}d^4\left(256ae^3 + 5d^4\right)^2\left(\frac{1}{54}\left(\frac{256ae^3}{d^4} + 5\right) + 1\right)\left(\frac{d}{4e} + x\right)^4 + 3456d^4e^8\left(\frac{1}{54}\left(\frac{256ae^3}{d^4} + 5\right) + 1\right)\left(\frac{d}{4e} + x\right)\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{24}e^4\left(65536a^2e^6 + 20992ad^4e^3 + 601d^8\right)\left(\frac{d}{4e} + x\right)^9 + \frac{\left(256ae^3 + 5d^4\right)^2\left(256ae^3 + 59d^4\right)\left(\frac{d}{4e} + x\right)^5}{5120} + \frac{64}{13}e^8\left(256ae^3 + 59d^4\right)\left(\frac{d}{4e} + x\right)^{13} + \frac{\left(256ae^3 + 5d^4\right)^4\left(\frac{d}{4e} + x\right)}{1048576e^4} - \frac{72}{11}d^2e^6\left(256ae^3 + 17d^4\right)\left(\frac{d}{4e} + x\right)^{11} - \frac{9}{224}d^2e^2\left(256ae^3 + 5d^4\right)\left(256ae^3 + 17d^4\right)\left(\frac{d}{4e} + x\right)^7 - \frac{d^2\left(256ae^3 + 5d^4\right)^3\left(\frac{d}{4e} + x\right)^3}{8192e^2} - \frac{2048}{5}d^2e^{10}\left(\frac{d}{4e} + x\right)^{15} + \frac{4096}{17}e^{12}\left(\frac{d}{4e} + x\right)^{17}$$

input

```
Int[(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)^4,x]
```

output

```
((5*d^4 + 256*a*e^3)^4*(d/(4*e) + x))/(1048576*e^4) - (d^2*(5*d^4 + 256*a*
e^3)^3*(d/(4*e) + x)^3)/(8192*e^2) + ((5*d^4 + 256*a*e^3)^2*(59*d^4 + 256*
a*e^3)*(d/(4*e) + x)^5)/5120 - (9*d^2*e^2*(5*d^4 + 256*a*e^3)*(17*d^4 + 25
6*a*e^3)*(d/(4*e) + x)^7)/224 + (e^4*(601*d^8 + 20992*a*d^4*e^3 + 65536*a^
2*e^6)*(d/(4*e) + x)^9)/24 - (72*d^2*e^6*(17*d^4 + 256*a*e^3)*(d/(4*e) + x
)^11)/11 + (64*e^8*(59*d^4 + 256*a*e^3)*(d/(4*e) + x)^13)/13 - (2048*d^2*e
^10*(d/(4*e) + x)^15)/5 + (4096*e^12*(d/(4*e) + x)^17)/17
```

Defintions of rubi rules used

rule 1403
```
Int[((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int[ExpandInte
grand[(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a
*c, 0] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2458
```
Int[(Pn_)^(p_.), x_Symbol] :> With[{S = Coeff[Pn, x, Expon[Pn, x] - 1]/(Exp
on[Pn, x]*Coeff[Pn, x, Expon[Pn, x]])}, Subst[Int[ExpandToSum[Pn /. x -> x
- S, x]^p, x], x, x + S] /; BinomialQ[Pn /. x -> x - S, x] || (IntegerQ[Exp
on[Pn, x]/2] && TrinomialQ[Pn /. x -> x - S, x])] /; FreeQ[p, x] && PolyQ[P
n, x] && GtQ[Expon[Pn, x], 2] && NeQ[Coeff[Pn, x, Expon[Pn, x] - 1], 0]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.25

method	result
norman	$4096a^4e^8x - 1024a^3e^6d^3x^2 + 128a^2e^4d^6x^3 + (4096a^3e^8d - 8a\,e^2d^9)\,x^4 + \left(\frac{16384}{5}a^3e^9 - \frac{6144}{5}\right.$
gosper	$4096ad\,e^{10}x^{12} - 768a\,d^5e^6x^8 + 6144a^2d\,e^9x^8 - 1024a^2d^3e^7x^6 + 128a\,d^7e^4x^6 - 8a\,d^9e^2x^4 -$
risch	$4096ad\,e^{10}x^{12} - 768a\,d^5e^6x^8 + 6144a^2d\,e^9x^8 - 1024a^2d^3e^7x^6 + 128a\,d^7e^4x^6 - 8a\,d^9e^2x^4 -$
parallelrisch	$4096ad\,e^{10}x^{12} - 768a\,d^5e^6x^8 + 6144a^2d\,e^9x^8 - 1024a^2d^3e^7x^6 + 128a\,d^7e^4x^6 - 8a\,d^9e^2x^4 -$
orering	$\frac{x(61501440e^{12}x^{16}+261381120d\,e^{11}x^{15}+418209792d^2e^{10}x^{14}+261381120d^3e^9x^{13}+321699840a\,e^{11}x^{12}-40212480d^4e^8x^{12}+}{}$
default	$\frac{4096e^{12}x^{17}}{17} + 1024d\,e^{11}x^{16} + \frac{8192d^2e^{10}x^{15}}{5} + 1024d^3e^9x^{14} + \frac{128(128a\,e^5-16d^4e^2)e^6x^{13}}{13} + \frac{(16384a\,e^{10}d+}{}$

input
```
int((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^4,x,method=_RETURNVERBOSE)
```

output
```
4096*a^4*e^8*x-1024*a^3*e^6*d^3*x^2+128*a^2*e^4*d^6*x^3+(4096*a^3*d*e^8-8*
a*d^9*e^2)*x^4+(16384/5*a^3*e^9-6144/5*a^2*e^6*d^4+1/5*d^12)*x^5+(-1024*a^
2*d^3*e^7+128*a*d^7*e^4)*x^6+(24576/7*a^2*e^8*d^2+768/7*a*e^5*d^6-32/7*d^1
0*e^2)*x^7+(6144*a^2*d*e^9-768*a*d^5*e^6-4*d^9*e^3)*x^8+(8192/3*a^2*e^10-4
096/3*a*e^7*d^4+128/3*d^8*e^4)*x^9+(1024*a*e^8*d^3+384/5*d^7*e^5)*x^10+(49
152/11*a*e^9*d^2-1664/11*d^6*e^6)*x^11+(4096*a*d*e^10-512*d^5*e^7)*x^12+(1
6384/13*a*e^11-2048/13*d^4*e^8)*x^13+1024*d^3*e^9*x^14+8192/5*d^2*e^10*x^1
5+1024*d*e^11*x^16+4096/17*e^12*x^17
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.25

$$\begin{aligned}\int (8ae^2-d^3x+8de^2x^3+8e^3x^4)^4\,dx &= \frac{4096}{17}e^{12}x^{17}+1024\,de^{11}x^{16}+\frac{8192}{5}d^2e^{10}x^{15}+1024\,d^3e^9x^{14}+128\,a^2d^6e^4x^3\\&\quad-1024\,a^3d^3e^6x^2-\frac{2048}{13}\left(d^4e^8-8\,ae^{11}\right)x^{13}+4096\,a^4e^8x-512\left(d^5e^7-8\,ade^{10}\right)x^{12}\\&\quad-\frac{128}{11}\left(13\,d^6e^6-384\,ad^2e^9\right)x^{11}+\frac{128}{5}\left(3\,d^7e^5+40\,ad^3e^8\right)x^{10}\\&\quad+\frac{128}{3}\left(d^8e^4-32\,ad^4e^7+64\,a^2e^{10}\right)x^9-4\left(d^9e^3+192\,ad^5e^6-1536\,a^2de^9\right)x^8\\&\quad-\frac{32}{7}\left(d^{10}e^2-24\,ad^6e^5-768\,a^2d^2e^8\right)x^7+128\left(ad^7e^4-8\,a^2d^3e^7\right)x^6\\&\quad+\frac{1}{5}\left(d^{12}-6144\,a^2d^4e^6+16384\,a^3e^9\right)x^5-8\left(ad^9e^2-512\,a^3de^8\right)x^4\end{aligned}$$

input
```
integrate((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^4,x, algorithm="fricas")
```

output
```
4096/17*e^12*x^17 + 1024*d*e^11*x^16 + 8192/5*d^2*e^10*x^15 + 1024*d^3*e^9
*x^14 + 128*a^2*d^6*e^4*x^3 - 1024*a^3*d^3*e^6*x^2 - 2048/13*(d^4*e^8 - 8*
a*e^11)*x^13 + 4096*a^4*e^8*x - 512*(d^5*e^7 - 8*a*d*e^10)*x^12 - 128/11*(
13*d^6*e^6 - 384*a*d^2*e^9)*x^11 + 128/5*(3*d^7*e^5 + 40*a*d^3*e^8)*x^10 +
 128/3*(d^8*e^4 - 32*a*d^4*e^7 + 64*a^2*e^10)*x^9 - 4*(d^9*e^3 + 192*a*d^5
*e^6 - 1536*a^2*d*e^9)*x^8 - 32/7*(d^10*e^2 - 24*a*d^6*e^5 - 768*a^2*d^2*e
^8)*x^7 + 128*(a*d^7*e^4 - 8*a^2*d^3*e^7)*x^6 + 1/5*(d^12 - 6144*a^2*d^4*e
^6 + 16384*a^3*e^9)*x^5 - 8*(a*d^9*e^2 - 512*a^3*d*e^8)*x^4
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 366, normalized size of antiderivative = 1.38

$$\int \left(8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4\right)^4 \, dx = 4096a^4e^8x - 1024a^3d^3e^6x^2 + 128a^2d^6e^4x^3 + 1024d^3e^9x^{14} + \frac{8192d^2e^{10}x^{15}}{5} + 1024de^{11}x^{16} + \frac{4096e^{12}x^{17}}{17} + x^{13} \cdot \left(\frac{16384ae^{11}}{13} - \frac{2048d^4e^8}{13}\right) + x^{12} \cdot \left(4096ade^{10} - 512d^5e^7\right) + x^{11} \cdot \left(\frac{49152ad^2e^9}{11} - \frac{1664d^6e^6}{11}\right) + x^{10} \cdot \left(1024ad^3e^8 + \frac{384d^7e^5}{5}\right) + x^9 \cdot \left(\frac{8192a^2e^{10}}{3} - \frac{4096ad^4e^7}{3} + \frac{128d^8e^4}{3}\right) + x^8 \cdot \left(6144a^2de^9 - 768ad^5e^6 - 4d^9e^3\right) + x^7 \cdot \left(\frac{24576a^2d^2e^8}{7} + \frac{768ad^6e^5}{7} - \frac{32d^{10}e^2}{7}\right) + x^6\left(-1024a^2d^3e^7 + 128ad^7e^4\right) + x^5 \cdot \left(\frac{16384a^3e^9}{5} - \frac{6144a^2d^4e^6}{5} + \frac{d^{12}}{5}\right) + x^4 \cdot \left(4096a^3de^8 - 8ad^9e^2\right)$$

input
```
integrate((8*e**3*x**4+8*d*e**2*x**3-d**3*x+8*a*e**2)**4,x)
```

output
```
4096*a**4*e**8*x - 1024*a**3*d**3*e**6*x**2 + 128*a**2*d**6*e**4*x**3 + 10
24*d**3*e**9*x**14 + 8192*d**2*e**10*x**15/5 + 1024*d*e**11*x**16 + 4096*e
**12*x**17/17 + x**13*(16384*a*e**11/13 - 2048*d**4*e**8/13) + x**12*(4096
*a*d*e**10 - 512*d**5*e**7) + x**11*(49152*a*d**2*e**9/11 - 1664*d**6*e**6
/11) + x**10*(1024*a*d**3*e**8 + 384*d**7*e**5/5) + x**9*(8192*a**2*e**10/
3 - 4096*a*d**4*e**7/3 + 128*d**8*e**4/3) + x**8*(6144*a**2*d*e**9 - 768*a
*d**5*e**6 - 4*d**9*e**3) + x**7*(24576*a**2*d**2*e**8/7 + 768*a*d**6*e**5
/7 - 32*d**10*e**2/7) + x**6*(-1024*a**2*d**3*e**7 + 128*a*d**7*e**4) + x*
*5*(16384*a**3*e**9/5 - 6144*a**2*d**4*e**6/5 + d**12/5) + x**4*(4096*a**3
*d*e**8 - 8*a*d**9*e**2)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 383, normalized size of antiderivative = 1.44

$$\int \left(8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4\right)^4 \, dx = \frac{4096}{17} e^{12}x^{17} + 1024\, de^{11}x^{16} + \frac{8192}{5} d^2e^{10}x^{15} + \frac{8192}{7} d^3e^9x^{14} + \frac{4096}{13} d^4e^8x^{13} + \frac{1}{5} d^{12}x^5 + 4096\, a^4e^8x - \frac{4}{7} \left(7\, e^3x^8 + 8\, de^2x^7\right) d^9 + \frac{1024}{5} \left(16\, e^3x^5 + 20\, de^2x^4 - 5\, d^3x^2\right) a^3e^6 + \frac{128}{165} \left(45\, e^6x^{11} + 99\, de^5x^{10} + 55\, d^2e^4x^9\right) d^6 + \frac{128}{105} \left(2240\, e^6x^9 + 5040\, de^5x^8 + 2880\, d^2e^4x^7 + 105\, d^6x^3 - 168\left(5\, e^3x^6 + 6\, de^2x^5\right) d^3\right) a^2e^4 - \frac{512}{1001} \left(286\, e^9x^{14} + 924\, de^8x^{13} + 1001\, d^2e^7x^{12} + 364\, d^3e^6x^{11}\right) d^3 + \frac{8}{15015} \left(2365440\, e^9x^{13} + 7687680\, de^8x^{12} + 8386560\, d^2e^7x^{11} + 3075072\, d^3e^6x^{10} - 15015\, d^9x^4 + 34320\, \left(\right.\right.$$

input
```
integrate((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^4,x, algorithm="maxima")
```

output
```
4096/17*e^12*x^17 + 1024*d*e^11*x^16 + 8192/5*d^2*e^10*x^15 + 8192/7*d^3*e
^9*x^14 + 4096/13*d^4*e^8*x^13 + 1/5*d^12*x^5 + 4096*a^4*e^8*x - 4/7*(7*e^
3*x^8 + 8*d*e^2*x^7)*d^9 + 1024/5*(16*e^3*x^5 + 20*d*e^2*x^4 - 5*d^3*x^2)*
a^3*e^6 + 128/165*(45*e^6*x^11 + 99*d*e^5*x^10 + 55*d^2*e^4*x^9)*d^6 + 128
/105*(2240*e^6*x^9 + 5040*d*e^5*x^8 + 2880*d^2*e^4*x^7 + 105*d^6*x^3 - 168
*(5*e^3*x^6 + 6*d*e^2*x^5)*d^3)*a^2*e^4 - 512/1001*(286*e^9*x^14 + 924*d*e
^8*x^13 + 1001*d^2*e^7*x^12 + 364*d^3*e^6*x^11)*d^3 + 8/15015*(2365440*e^9
*x^13 + 7687680*d*e^8*x^12 + 8386560*d^2*e^7*x^11 + 3075072*d^3*e^6*x^10 -
 15015*d^9*x^4 + 34320*(6*e^3*x^7 + 7*d*e^2*x^6)*d^6 - 32032*(36*e^6*x^10
+ 80*d*e^5*x^9 + 45*d^2*e^4*x^8)*d^3)*a*e^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.33

$$\int \left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)^4 \, dx = \frac{4096}{17}\,e^{12}x^{17}+1024\,de^{11}x^{16}+\frac{8192}{5}\,d^2e^{10}x^{15}+1024\,d^3e^9x^{14}-\frac{2048}{13}\,d^4e^8x^{13}+\frac{16384}{13}\,ae^{11}x^{13}-512\,d^5e^7x^{12}+4096\,ade^{10}x^{12}-\frac{1664}{11}\,d^6e^6x^{11}+\frac{49152}{11}\,ad^2e^9x^{11}+\frac{384}{5}\,d^7e^5x^{10}+1024\,ad^3e^8x^{10}+\frac{128}{3}\,d^8e^4x^9-\frac{4096}{3}\,ad^4e^7x^9+\frac{8192}{3}\,a^2e^{10}x^9-4\,d^9e^3x^8-768\,ad^5e^6x^8+6144\,a^2de^9x^8-\frac{32}{7}\,d^{10}e^2x^7+\frac{768}{7}\,ad^6e^5x^7+\frac{24576}{7}\,a^2d^2e^8x^7+128\,ad^7e^4x^6-1024\,a^2d^3e^7x^6+\frac{1}{5}\,d^{12}x^5-\frac{6144}{5}\,a^2d^4e^6x^5+\frac{16384}{5}\,a^3e^9x^5-8\,ad^9e^2x^4+4096\,a^3de^8x^4+128\,a^2d^6e^4x^3-1024\,a^3d^3e^6x^2+4096\,a^4e^8x$$

input
```
integrate((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^4,x, algorithm="giac")
```

output
```
4096/17*e^12*x^17 + 1024*d*e^11*x^16 + 8192/5*d^2*e^10*x^15 + 1024*d^3*e^9
*x^14 - 2048/13*d^4*e^8*x^13 + 16384/13*a*e^11*x^13 - 512*d^5*e^7*x^12 + 4
096*a*d*e^10*x^12 - 1664/11*d^6*e^6*x^11 + 49152/11*a*d^2*e^9*x^11 + 384/5
*d^7*e^5*x^10 + 1024*a*d^3*e^8*x^10 + 128/3*d^8*e^4*x^9 - 4096/3*a*d^4*e^7
*x^9 + 8192/3*a^2*e^10*x^9 - 4*d^9*e^3*x^8 - 768*a*d^5*e^6*x^8 + 6144*a^2*
d*e^9*x^8 - 32/7*d^10*e^2*x^7 + 768/7*a*d^6*e^5*x^7 + 24576/7*a^2*d^2*e^8*
x^7 + 128*a*d^7*e^4*x^6 - 1024*a^2*d^3*e^7*x^6 + 1/5*d^12*x^5 - 6144/5*a^2
*d^4*e^6*x^5 + 16384/5*a^3*e^9*x^5 - 8*a*d^9*e^2*x^4 + 4096*a^3*d*e^8*x^4
+ 128*a^2*d^6*e^4*x^3 - 1024*a^3*d^3*e^6*x^2 + 4096*a^4*e^8*x
```

Mupad [B] (verification not implemented)

Time = 21.80 (sec) , antiderivative size = 331, normalized size of antiderivative = 1.24

$$\int \left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)^4\,dx = x^5\left(\frac{16384\,a^3\,e^9}{5}-\frac{6144\,a^2\,d^4\,e^6}{5}+\frac{d^{12}}{5}\right)+x^{10}\left(\frac{384\,d^7\,e^5}{5}+1024\,a\,d^3\,e^8\right)-x^{11}\left(\frac{1664\,d^6\,e^6}{11}-\frac{49152\,a\,d^2\,e^9}{11}\right)+\frac{4096\,e^{12}\,x^{17}}{17}+\frac{2048\,e^8\,x^{13}\,(8\,a\,e^3-d^4)}{13}+\frac{128\,e^4\,x^9\,(64\,a^2\,e^6-32\,a\,d^4\,e^3+d^8)}{3}+4096\,a^4\,e^8\,x+1024\,d\,e^{11}\,x^{16}+1024\,d^3\,e^9\,x^{14}+\frac{8192\,d^2\,e^{10}\,x^{15}}{5}+512\,d\,e^7\,x^{12}\left(8\,a\,e^3-d^4\right)+\frac{32\,d^2\,e^2\,x^7\,(768\,a^2\,e^6+24\,a\,d^4\,e^3-d^8)}{7}-1024\,a^3\,d^3\,e^6\,x^2+128\,a^2\,d^6\,e^4\,x^3-4\,d\,e^3\,x^8\left(-1536\,a^2\,e^6+192\,a\,d^4\,e^3+d^8\right)-128\,a\,d^3\,e^4\,x^6\left(8\,a\,e^3-d^4\right)-8\,a\,d\,e^2\,x^4\left(d^8-512\,a^2\,e^6\right)$$

input
```
int((8*a*e^2 - d^3*x + 8*e^3*x^4 + 8*d*e^2*x^3)^4,x)
```

output
```
x^5*(d^12/5 + (16384*a^3*e^9)/5 - (6144*a^2*d^4*e^6)/5) + x^10*((384*d^7*e
^5)/5 + 1024*a*d^3*e^8) - x^11*((1664*d^6*e^6)/11 - (49152*a*d^2*e^9)/11)
+ (4096*e^12*x^17)/17 + (2048*e^8*x^13*(8*a*e^3 - d^4))/13 + (128*e^4*x^9*
(d^8 + 64*a^2*e^6 - 32*a*d^4*e^3))/3 + 4096*a^4*e^8*x + 1024*d*e^11*x^16 +
 1024*d^3*e^9*x^14 + (8192*d^2*e^10*x^15)/5 + 512*d*e^7*x^12*(8*a*e^3 - d^
4) + (32*d^2*e^2*x^7*(768*a^2*e^6 - d^8 + 24*a*d^4*e^3))/7 - 1024*a^3*d^3*
e^6*x^2 + 128*a^2*d^6*e^4*x^3 - 4*d*e^3*x^8*(d^8 - 1536*a^2*e^6 + 192*a*d^
4*e^3) - 128*a*d^3*e^4*x^6*(8*a*e^3 - d^4) - 8*a*d*e^2*x^4*(d^8 - 512*a^2*
e^6)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.33

$$\int \left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)^4\,dx$$
$$= \frac{x(61501440e^{12}x^{16}+261381120d\,e^{11}x^{15}+418209792d^2e^{10}x^{14}+261381120d^3e^9x^{13}+321699840a\,e^{11}x^{12}}{}$$

input
```
int((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^4,x)
```

output
```
(x*(1045524480*a**4*e**8 - 261381120*a**3*d**3*e**6*x + 1045524480*a**3*d*
e**8*x**3 + 836419584*a**3*e**9*x**4 + 32672640*a**2*d**6*e**4*x**2 - 3136
57344*a**2*d**4*e**6*x**4 - 261381120*a**2*d**3*e**7*x**5 + 896163840*a**2
*d**2*e**8*x**6 + 1568286720*a**2*d*e**9*x**7 + 697016320*a**2*e**10*x**8
- 2042040*a*d**9*e**2*x**3 + 32672640*a*d**7*e**4*x**5 + 28005120*a*d**6*e
**5*x**6 - 196035840*a*d**5*e**6*x**7 - 348508160*a*d**4*e**7*x**8 + 26138
1120*a*d**3*e**8*x**9 + 1140572160*a*d**2*e**9*x**10 + 1045524480*a*d*e**1
0*x**11 + 321699840*a*e**11*x**12 + 51051*d**12*x**4 - 1166880*d**10*e**2*
x**6 - 1021020*d**9*e**3*x**7 + 10890880*d**8*e**4*x**8 + 19603584*d**7*e*
*5*x**9 - 38613120*d**6*e**6*x**10 - 130690560*d**5*e**7*x**11 - 40212480*
d**4*e**8*x**12 + 261381120*d**3*e**9*x**13 + 418209792*d**2*e**10*x**14 +
 261381120*d*e**11*x**15 + 61501440*e**12*x**16))/255255
```

3.38 $\int (8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4)^3 \, dx$

Optimal result

Integrand size = 32, antiderivative size = 184

$$\int (8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4)^3 \, dx = \frac{(5d^4+256ae^3)^3 x}{32768e^3} - \frac{3d^2(5d^4+256ae^3)^2 (d+4ex)^3}{65536e^4} + \frac{3(5d^4+256ae^3)(41d^4+256ae^3)(d+4ex)^5}{655360e^4} - \frac{9d^2(11d^4+256ae^3)(d+4ex)^7}{229376e^4} + \frac{(41d^4+256ae^3)(d+4ex)^9}{393216e^4} - \frac{9d^2(d+4ex)^{11}}{720896e^4} + \frac{(d+4ex)^{13}}{1703936e^4}$$

output
```
1/32768*(256*a*e^3+5*d^4)^3*x/e^3-3/65536*d^2*(256*a*e^3+5*d^4)^2*(4*e*x+d
)^3/e^4+3/655360*(256*a*e^3+5*d^4)*(256*a*e^3+41*d^4)*(4*e*x+d)^5/e^4-9/22
9376*d^2*(256*a*e^3+11*d^4)*(4*e*x+d)^7/e^4+1/393216*(256*a*e^3+41*d^4)*(4
*e*x+d)^9/e^4-9/720896*d^2*(4*e*x+d)^11/e^4+1/1703936*(4*e*x+d)^13/e^4
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.12

$$\int \left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)^3 \, dx = 512a^3e^6x-96a^2d^3e^4x^2+8ad^6e^2x^3 -\frac{1}{4}d\left(d^8-1536a^2e^6\right)x^4 +\frac{384}{5}ae^4\left(-d^4+4ae^3\right)x^5 +4d^3e^2\left(d^4-16ae^3\right)x^6 +\frac{24}{7}d^2e^3\left(d^4+64ae^3\right)x^7-24de^4\left(d^4-16ae^3\right)x^8 +\frac{128}{3}e^5\left(-d^4+4ae^3\right)x^9+32d^3e^6x^{10} +\frac{1536}{11}d^2e^7x^{11}+128de^8x^{12}+\frac{512e^9x^{13}}{13}$$

input
```
Integrate[(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)^3,x]
```

output
```
512*a^3*e^6*x - 96*a^2*d^3*e^4*x^2 + 8*a*d^6*e^2*x^3 - (d*(d^8 - 1536*a^2*
e^6)*x^4)/4 + (384*a*e^4*(-d^4 + 4*a*e^3)*x^5)/5 + 4*d^3*e^2*(d^4 - 16*a*e
^3)*x^6 + (24*d^2*e^3*(d^4 + 64*a*e^3)*x^7)/7 - 24*d*e^4*(d^4 - 16*a*e^3)*
x^8 + (128*e^5*(-d^4 + 4*a*e^3)*x^9)/3 + 32*d^3*e^6*x^10 + (1536*d^2*e^7*x
^11)/11 + 128*d*e^8*x^12 + (512*e^9*x^13)/13
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.17, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {2458, 1403, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)^3 \, dx$$

$$\Big\downarrow \; 2458$$

$$\int \left(\frac{1}{32}\left(256ae^2+\frac{5d^4}{e}\right)-3d^2e\left(\frac{d}{4e}+x\right)^2+8e^3\left(\frac{d}{4e}+x\right)^4\right)^3 d\left(\frac{d}{4e}+x\right)$$

$\big\downarrow$ 1403

$$\int \left(\frac{27}{32}d^4e\left(256ae^3+5d^4\right)\left(\frac{64ae^3}{9d^4}+\frac{41}{36}\right)\left(\frac{d}{4e}+x\right)^4+\frac{\left(256ae^3+5d^4\right)^3}{32768e^3}+216d^4e^5\left(\frac{64ae^3}{9d^4}+\frac{41}{36}\right)\left(\frac{d}{4e}+x\right)^8$$

$\big\downarrow$ 2009

$$\frac{3}{640}e\left(256ae^3+5d^4\right)\left(256ae^3+41d^4\right)\left(\frac{d}{4e}+x\right)^5+\frac{\left(256ae^3+5d^4\right)^3\left(\frac{d}{4e}+x\right)}{32768e^3}+\frac{2}{3}e^5\left(256ae^3+41d^4\right)\left(\frac{d}{4e}+x\right)^9-\frac{9}{14}d^2e^3\left(256ae^3+11d^4\right)\left(\frac{d}{4e}+x\right)^7-\frac{3d^2\left(256ae^3+5d^4\right)^2\left(\frac{d}{4e}+x\right)^3}{1024e}-\frac{576}{11}d^2e^7\left(\frac{d}{4e}+x\right)^{11}+\frac{512}{13}e^9\left(\frac{d}{4e}+x\right)^{13}$$

input
```
Int[(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)^3,x]
```

output
```
((5*d^4 + 256*a*e^3)^3*(d/(4*e) + x))/(32768*e^3) - (3*d^2*(5*d^4 + 256*a*
e^3)^2*(d/(4*e) + x)^3)/(1024*e) + (3*e*(5*d^4 + 256*a*e^3)*(41*d^4 + 256*
a*e^3)*(d/(4*e) + x)^5)/640 - (9*d^2*e^3*(11*d^4 + 256*a*e^3)*(d/(4*e) + x
)^7)/14 + (2*e^5*(41*d^4 + 256*a*e^3)*(d/(4*e) + x)^9)/3 - (576*d^2*e^7*(d
/(4*e) + x)^11)/11 + (512*e^9*(d/(4*e) + x)^13)/13
```

Defintions of rubi rules used

rule 1403
```
Int[((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int[ExpandInte
grand[(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a
*c, 0] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2458
```
Int[(Pn_)^(p_.), x_Symbol] :> With[{S = Coeff[Pn, x, Expon[Pn, x] - 1]/(Exp
on[Pn, x]*Coeff[Pn, x, Expon[Pn, x]])}, Subst[Int[ExpandToSum[Pn /. x -> x
- S, x]^p, x], x, x + S] /; BinomialQ[Pn /. x -> x - S, x] || (IntegerQ[Exp
on[Pn, x]/2] && TrinomialQ[Pn /. x -> x - S, x])] /; FreeQ[p, x] && PolyQ[P
n, x] && GtQ[Expon[Pn, x], 2] && NeQ[Coeff[Pn, x, Expon[Pn, x] - 1], 0]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.09

method	result
norman	$\frac{512e^9x^{13}}{13}+128d\,e^8x^{12}+\frac{1536d^2e^7x^{11}}{11}+32d^3e^6x^{10}+\left(\frac{512}{3}a\,e^8-\frac{128}{3}d^4e^5\right)x^9+\left(384a\,e^7d-24d^5\right.$
gosper	$\frac{512}{13}e^9x^{13}+128d\,e^8x^{12}+\frac{1536}{11}d^2e^7x^{11}+32d^3e^6x^{10}+\frac{512}{3}x^9a\,e^8-\frac{128}{3}x^9d^4e^5+384ad\,e^7x^8-$
risch	$\frac{512}{13}e^9x^{13}+128d\,e^8x^{12}+\frac{1536}{11}d^2e^7x^{11}+32d^3e^6x^{10}+\frac{512}{3}x^9a\,e^8-\frac{128}{3}x^9d^4e^5+384ad\,e^7x^8-$
parallelrisch	$\frac{512}{13}e^9x^{13}+128d\,e^8x^{12}+\frac{1536}{11}d^2e^7x^{11}+32d^3e^6x^{10}+\frac{512}{3}x^9a\,e^8-\frac{128}{3}x^9d^4e^5+384ad\,e^7x^8-$
orering	$x\left(2365440e^9x^{12}+7687680d\,e^8x^{11}+8386560d^2e^7x^{10}+1921920d^3e^6x^9+10250240a\,e^8x^8-2562560d^4e^5x^8+23063040ad\,e^7x^7\right.$
default	$\frac{512e^9x^{13}}{13}+128d\,e^8x^{12}+\frac{1536d^2e^7x^{11}}{11}+32d^3e^6x^{10}+\frac{\left(512a\,e^8-256d^4e^5+8e^3\left(128a\,e^5-16d^4e^2\right)\right)x^9}{9}+$

input
```
int((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^3,x,method=_RETURNVERBOSE)
```

output
```
512/13*e^9*x^13+128*d*e^8*x^12+1536/11*d^2*e^7*x^11+32*d^3*e^6*x^10+(512/3
*a*e^8-128/3*d^4*e^5)*x^9+(384*a*d*e^7-24*d^5*e^4)*x^8+(1536/7*a*e^6*d^2+2
4/7*d^6*e^3)*x^7+(-64*a*d^3*e^5+4*d^7*e^2)*x^6+(1536/5*e^7*a^2-384/5*d^4*a
*e^4)*x^5+(384*a^2*e^6*d-1/4*d^9)*x^4+8*a*e^2*d^6*x^3-96*a^2*e^4*d^3*x^2+5
12*a^3*e^6*x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.08

$$\int \left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)^3\,dx = \frac{512}{13}\,e^9x^{13}+128\,de^8x^{12}+\frac{1536}{11}\,d^2e^7x^{11}+32\,d^3e^6x^{10}+8\,ad^6e^2x^3-96\,a^2d^3e^4x^2 +512\,a^3e^6x-\frac{128}{3}\,\left(d^4e^5-4\,ae^8\right)x^9-24\,\left(d^5e^4-16\,ade^7\right)x^8+\frac{24}{7}\,\left(d^6e^3+64\,ad^2e^6\right)x^7 +4\,\left(d^7e^2-16\,ad^3e^5\right)x^6-\frac{384}{5}\,\left(ad^4e^4-4\,a^2e^7\right)x^5-\frac{1}{4}\,\left(d^9-1536\,a^2de^6\right)x^4$$

input
```
integrate((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^3,x, algorithm="fricas")
```

output
```
512/13*e^9*x^13 + 128*d*e^8*x^12 + 1536/11*d^2*e^7*x^11 + 32*d^3*e^6*x^10
+ 8*a*d^6*e^2*x^3 - 96*a^2*d^3*e^4*x^2 + 512*a^3*e^6*x - 128/3*(d^4*e^5 -
4*a*e^8)*x^9 - 24*(d^5*e^4 - 16*a*d*e^7)*x^8 + 24/7*(d^6*e^3 + 64*a*d^2*e^
6)*x^7 + 4*(d^7*e^2 - 16*a*d^3*e^5)*x^6 - 384/5*(a*d^4*e^4 - 4*a^2*e^7)*x^
5 - 1/4*(d^9 - 1536*a^2*d*e^6)*x^4
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.18

$$\int \left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)^3\,dx = 512a^3e^6x-96a^2d^3e^4x^2+8ad^6e^2x^3+32d^3e^6x^{10} +\frac{1536d^2e^7x^{11}}{11}+128de^8x^{12}+\frac{512e^9x^{13}}{13}+x^9 \cdot\left(\frac{512ae^8}{3}-\frac{128d^4e^5}{3}\right)+x^8\cdot\left(384ade^7-24d^5e^4\right) +x^7\cdot\left(\frac{1536ad^2e^6}{7}+\frac{24d^6e^3}{7}\right) +x^6\left(-64ad^3e^5+4d^7e^2\right)+x^5 \cdot\left(\frac{1536a^2e^7}{5}-\frac{384ad^4e^4}{5}\right) +x^4\cdot\left(384a^2de^6-\frac{d^9}{4}\right)$$

input
```
integrate((8*e**3*x**4+8*d*e**2*x**3-d**3*x+8*a*e**2)**3,x)
```

output
```
512*a**3*e**6*x - 96*a**2*d**3*e**4*x**2 + 8*a*d**6*e**2*x**3 + 32*d**3*e*
*6*x**10 + 1536*d**2*e**7*x**11/11 + 128*d*e**8*x**12 + 512*e**9*x**13/13
+ x**9*(512*a*e**8/3 - 128*d**4*e**5/3) + x**8*(384*a*d*e**7 - 24*d**5*e**
4) + x**7*(1536*a*d**2*e**6/7 + 24*d**6*e**3/7) + x**6*(-64*a*d**3*e**5 +
4*d**7*e**2) + x**5*(1536*a**2*e**7/5 - 384*a*d**4*e**4/5) + x**4*(384*a**
2*d*e**6 - d**9/4)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.16

$$\int \left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)^3\,dx=\frac{512}{13}\,e^9x^{13}+128\,de^8x^{12}+\frac{1536}{11}\,d^2e^7x^{11}+\frac{256}{5}\,d^3e^6x^{10}-\frac{1}{4}\,d^9x^4+512\,a^3e^6x+\frac{4}{7}\left(6\,e^3x^7+7\,de^2x^6\right)d^6+\frac{96}{5}\left(16\,e^3x^5+20\,de^2x^4-5\,d^3x^2\right)a^2e^4-\frac{8}{15}\left(36\,e^6x^{10}+80\,de^5x^9+45\,d^2e^4x^8\right)d^3+\frac{8}{105}\left(2240\,e^6x^9+5040\,de^5x^8+2880\,d^2e^4x^7+105\,d^6x^3-168\left(5\,e^3x^6+6\,de^2x^5\right)d^3\right)ae^2$$

input
```
integrate((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^3,x, algorithm="maxima")
```

output
```
512/13*e^9*x^13 + 128*d*e^8*x^12 + 1536/11*d^2*e^7*x^11 + 256/5*d^3*e^6*x^
10 - 1/4*d^9*x^4 + 512*a^3*e^6*x + 4/7*(6*e^3*x^7 + 7*d*e^2*x^6)*d^6 + 96/
5*(16*e^3*x^5 + 20*d*e^2*x^4 - 5*d^3*x^2)*a^2*e^4 - 8/15*(36*e^6*x^10 + 80
*d*e^5*x^9 + 45*d^2*e^4*x^8)*d^3 + 8/105*(2240*e^6*x^9 + 5040*d*e^5*x^8 +
2880*d^2*e^4*x^7 + 105*d^6*x^3 - 168*(5*e^3*x^6 + 6*d*e^2*x^5)*d^3)*a*e^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.11

$$\int \left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)^3 dx = \frac{512}{13}e^9x^{13}+128de^8x^{12}+\frac{1536}{11}d^2e^7x^{11}+32d^3e^6x^{10} - \frac{128}{3}d^4e^5x^9+\frac{512}{3}ae^8x^9-24d^5e^4x^8 + 384ade^7x^8+\frac{24}{7}d^6e^3x^7+\frac{1536}{7}ad^2e^6x^7 + 4d^7e^2x^6-64ad^3e^5x^6-\frac{384}{5}ad^4e^4x^5 + \frac{1536}{5}a^2e^7x^5-\frac{1}{4}d^9x^4+384a^2de^6x^4 + 8ad^6e^2x^3-96a^2d^3e^4x^2+512a^3e^6x$$

input
```
integrate((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^3,x, algorithm="giac")
```

output
```
512/13*e^9*x^13 + 128*d*e^8*x^12 + 1536/11*d^2*e^7*x^11 + 32*d^3*e^6*x^10
- 128/3*d^4*e^5*x^9 + 512/3*a*e^8*x^9 - 24*d^5*e^4*x^8 + 384*a*d*e^7*x^8 +
 24/7*d^6*e^3*x^7 + 1536/7*a*d^2*e^6*x^7 + 4*d^7*e^2*x^6 - 64*a*d^3*e^5*x^
6 - 384/5*a*d^4*e^4*x^5 + 1536/5*a^2*e^7*x^5 - 1/4*d^9*x^4 + 384*a^2*d*e^6
*x^4 + 8*a*d^6*e^2*x^3 - 96*a^2*d^3*e^4*x^2 + 512*a^3*e^6*x
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.09

$$\int \left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)^3 dx = \frac{512\,e^9\,x^{13}}{13}-x^4\left(\frac{d^9}{4}-384\,a^2\,d\,e^6\right) + \frac{128\,e^5\,x^9\,\left(4\,a\,e^3-d^4\right)}{3}+512\,a^3\,e^6\,x + 128\,d\,e^8\,x^{12}+32\,d^3\,e^6\,x^{10}+\frac{1536\,d^2\,e^7\,x^{11}}{11} + 8\,a\,d^6\,e^2\,x^3+\frac{384\,a\,e^4\,x^5\,\left(4\,a\,e^3-d^4\right)}{5} + 24\,d\,e^4\,x^8\,\left(16\,a\,e^3-d^4\right) + \frac{24\,d^2\,e^3\,x^7\,\left(d^4+64\,a\,e^3\right)}{7} - 96\,a^2\,d^3\,e^4\,x^2-4\,d^3\,e^2\,x^6\,\left(16\,a\,e^3-d^4\right)$$

input
```
int((8*a*e^2 - d^3*x + 8*e^3*x^4 + 8*d*e^2*x^3)^3,x)
```

output
```
(512*e^9*x^13)/13 - x^4*(d^9/4 - 384*a^2*d*e^6) + (128*e^5*x^9*(4*a*e^3 -
d^4))/3 + 512*a^3*e^6*x + 128*d*e^8*x^12 + 32*d^3*e^6*x^10 + (1536*d^2*e^7
*x^11)/11 + 8*a*d^6*e^2*x^3 + (384*a*e^4*x^5*(4*a*e^3 - d^4))/5 + 24*d*e^4
*x^8*(16*a*e^3 - d^4) + (24*d^2*e^3*x^7*(64*a*e^3 + d^4))/7 - 96*a^2*d^3*e
^4*x^2 - 4*d^3*e^2*x^6*(16*a*e^3 - d^4)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.11

$$\int \left(8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4\right)^3 dx$$
$$= \frac{x(2365440e^9x^{12} + 7687680d\,e^8x^{11} + 8386560d^2e^7x^{10} + 1921920d^3e^6x^9 + 10250240a\,e^8x^8 - 2562560d^4e^5}{}$$

input
```
int((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^3,x)
```

output
```
(x*(30750720*a**3*e**6 - 5765760*a**2*d**3*e**4*x + 23063040*a**2*d*e**6*x
**3 + 18450432*a**2*e**7*x**4 + 480480*a*d**6*e**2*x**2 - 4612608*a*d**4*e
**4*x**4 - 3843840*a*d**3*e**5*x**5 + 13178880*a*d**2*e**6*x**6 + 23063040
*a*d*e**7*x**7 + 10250240*a*e**8*x**8 - 15015*d**9*x**3 + 240240*d**7*e**2
*x**5 + 205920*d**6*e**3*x**6 - 1441440*d**5*e**4*x**7 - 2562560*d**4*e**5
*x**8 + 1921920*d**3*e**6*x**9 + 8386560*d**2*e**7*x**10 + 7687680*d*e**8*
x**11 + 2365440*e**9*x**12))/60060
```

3.39 $\int (8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4)^2 \, dx$

Optimal result

Integrand size = 32, antiderivative size = 113

$$\int (8ae^2-d^3x+8de^2x^3+8e^3x^4)^2 \, dx = \frac{(5d^4+256ae^3)^2 x}{1024e^2} - \frac{d^2(5d^4+256ae^3)(d+4ex)^3}{1024e^3} + \frac{(23d^4+256ae^3)(d+4ex)^5}{10240e^3} - \frac{3d^2(d+4ex)^7}{7168e^3} + \frac{(d+4ex)^9}{36864e^3}$$

output
```
1/1024*(256*a*e^3+5*d^4)^2*x/e^2-1/1024*d^2*(256*a*e^3+5*d^4)*(4*e*x+d)^3/
e^3+1/10240*(256*a*e^3+23*d^4)*(4*e*x+d)^5/e^3-3/7168*d^2*(4*e*x+d)^7/e^3+
1/36864*(4*e*x+d)^9/e^3
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.96

$$\int (8ae^2-d^3x+8de^2x^3+8e^3x^4)^2 \, dx = 64a^2e^4x - 8ad^3e^2x^2 + \frac{d^6x^3}{3} + 32ade^4x^4 + \frac{16}{5}e^2(-d^4+8ae^3)x^5 - \frac{8}{3}d^3e^3x^6 + \frac{64}{7}d^2e^4x^7 + 16de^5x^8 + \frac{64e^6x^9}{9}$$

input
```
Integrate[(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)^2,x]
```

output
```
64*a^2*e^4*x - 8*a*d^3*e^2*x^2 + (d^6*x^3)/3 + 32*a*d*e^4*x^4 + (16*e^2*(-
d^4 + 8*a*e^3)*x^5)/5 - (8*d^3*e^3*x^6)/3 + (64*d^2*e^4*x^7)/7 + 16*d*e^5*
x^8 + (64*e^6*x^9)/9
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.19, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {2458, 1403, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)^2 dx$$

$$\Big\downarrow \ 2458$$

$$\int \left(\frac{1}{32}\left(256ae^2+\frac{5d^4}{e}\right)-3d^2e\left(\frac{d}{4e}+x\right)^2+8e^3\left(\frac{d}{4e}+x\right)^4\right)^2 d\left(\frac{d}{4e}+x\right)$$

$$\Big\downarrow \ 1403$$

$$\int \left(9d^4e^2\left(\frac{128ae^3}{9d^4}+\frac{23}{18}\right)\left(\frac{d}{4e}+x\right)^4+\frac{\left(256ae^3+5d^4\right)^2}{1024e^2}-\frac{3}{16}d^2\left(256ae^3+5d^4\right)\left(\frac{d}{4e}+x\right)^2-48d^2e^4\left(\frac{d}{4e}+x\right.\right.$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{10}e^2\left(256ae^3+23d^4\right)\left(\frac{d}{4e}+x\right)^5+\frac{\left(256ae^3+5d^4\right)^2\left(\frac{d}{4e}+x\right)}{1024e^2}-\frac{1}{16}d^2\left(256ae^3+5d^4\right)\left(\frac{d}{4e}+x\right)^3-\frac{48}{7}d^2e^4\left(\frac{d}{4e}+x\right)^7+\frac{64}{9}e^6\left(\frac{d}{4e}+x\right)^9$$

input
```
Int[(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)^2,x]
```

output
```
((5*d^4 + 256*a*e^3)^2*(d/(4*e) + x))/(1024*e^2) - (d^2*(5*d^4 + 256*a*e^3
)*(d/(4*e) + x)^3)/16 + (e^2*(23*d^4 + 256*a*e^3)*(d/(4*e) + x)^5)/10 - (4
8*d^2*e^4*(d/(4*e) + x)^7)/7 + (64*e^6*(d/(4*e) + x)^9)/9
```

Defintions of rubi rules used

rule 1403
```
Int[((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int[ExpandInte
grand[(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a
*c, 0] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2458
```
Int[(Pn_)^(p_.), x_Symbol] :> With[{S = Coeff[Pn, x, Expon[Pn, x] - 1]/(Exp
on[Pn, x]*Coeff[Pn, x, Expon[Pn, x]])}, Subst[Int[ExpandToSum[Pn /. x -> x
- S, x]^p, x], x, x + S] /; BinomialQ[Pn /. x -> x - S, x] || (IntegerQ[Exp
on[Pn, x]/2] && TrinomialQ[Pn /. x -> x - S, x])] /; FreeQ[p, x] && PolyQ[P
n, x] && GtQ[Expon[Pn, x], 2] && NeQ[Coeff[Pn, x, Expon[Pn, x] - 1], 0]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.88

method	result
norman	$\frac{64e^6x^9}{9}+16d\,e^5x^8+\frac{64d^2e^4x^7}{7}-\frac{8d^3e^3x^6}{3}+\left(\frac{128}{5}a\,e^5-\frac{16}{5}d^4e^2\right)x^5+32a\,e^4d\,x^4+\frac{d^6x^3}{3}-8a\,e^2d^3$
gosper	$\frac{64}{9}e^6x^9+16d\,e^5x^8+\frac{64}{7}d^2e^4x^7-\frac{8}{3}d^3e^3x^6+\frac{128}{5}x^5a\,e^5-\frac{16}{5}x^5d^4e^2+32a\,e^4d\,x^4+\frac{1}{3}d^6x^3-8$
default	$\frac{64e^6x^9}{9}+16d\,e^5x^8+\frac{64d^2e^4x^7}{7}-\frac{8d^3e^3x^6}{3}+\frac{(128a\,e^5-16d^4e^2)x^5}{5}+32a\,e^4d\,x^4+\frac{d^6x^3}{3}-8a\,e^2d^3x^2+$
risch	$\frac{64}{9}e^6x^9+16d\,e^5x^8+\frac{64}{7}d^2e^4x^7-\frac{8}{3}d^3e^3x^6+\frac{128}{5}x^5a\,e^5-\frac{16}{5}x^5d^4e^2+32a\,e^4d\,x^4+\frac{1}{3}d^6x^3-8$
parallelrisch	$\frac{64}{9}e^6x^9+16d\,e^5x^8+\frac{64}{7}d^2e^4x^7-\frac{8}{3}d^3e^3x^6+\frac{128}{5}x^5a\,e^5-\frac{16}{5}x^5d^4e^2+32a\,e^4d\,x^4+\frac{1}{3}d^6x^3-8$
orering	$\frac{x(2240e^6x^8+5040d\,e^5x^7+2880d^2e^4x^6-840d^3e^3x^5+8064a\,e^5x^4-1008d^4e^2x^4+10080a\,e^4d\,x^3+105d^6x^2-2520a\,e^2d^3x+2016}{315}$

input
```
int((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^2,x,method=_RETURNVERBOSE)
```

output
```
64/9*e^6*x^9+16*d*e^5*x^8+64/7*d^2*e^4*x^7-8/3*d^3*e^3*x^6+(128/5*a*e^5-16
/5*d^4*e^2)*x^5+32*a*e^4*d*x^4+1/3*d^6*x^3-8*a*e^2*d^3*x^2+64*a^2*e^4*x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.87

$$\int \left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)^2\,dx=\frac{64}{9}\,e^6x^9+16\,de^5x^8+\frac{64}{7}\,d^2e^4x^7-\frac{8}{3}\,d^3e^3x^6+32\,ade^4x^4+\frac{1}{3}\,d^6x^3-8\,ad^3e^2x^2+64\,a^2e^4x-\frac{16}{5}\left(d^4e^2-8\,ae^5\right)x^5$$

input
```
integrate((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^2,x, algorithm="fricas")
```

output
```
64/9*e^6*x^9 + 16*d*e^5*x^8 + 64/7*d^2*e^4*x^7 - 8/3*d^3*e^3*x^6 + 32*a*d*
e^4*x^4 + 1/3*d^6*x^3 - 8*a*d^3*e^2*x^2 + 64*a^2*e^4*x - 16/5*(d^4*e^2 - 8
*a*e^5)*x^5
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.99

$$\int \left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)^2\,dx=64a^2e^4x-8ad^3e^2x^2+32ade^4x^4+\frac{d^6x^3}{3}-\frac{8d^3e^3x^6}{3}+\frac{64d^2e^4x^7}{7}+16de^5x^8+\frac{64e^6x^9}{9}+x^5\cdot\left(\frac{128ae^5}{5}-\frac{16d^4e^2}{5}\right)$$

input
```
integrate((8*e**3*x**4+8*d*e**2*x**3-d**3*x+8*a*e**2)**2,x)
```

output
```
64*a**2*e**4*x - 8*a*d**3*e**2*x**2 + 32*a*d*e**4*x**4 + d**6*x**3/3 - 8*d
**3*e**3*x**6/3 + 64*d**2*e**4*x**7/7 + 16*d*e**5*x**8 + 64*e**6*x**9/9 +
x**5*(128*a*e**5/5 - 16*d**4*e**2/5)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.89

$$\int \left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)^2\,dx=\frac{64}{9}\,e^6x^9+16\,de^5x^8+\frac{64}{7}\,d^2e^4x^7+\frac{1}{3}\,d^6x^3+64\,a^2e^4x-\frac{8}{15}\left(5\,e^3x^6+6\,de^2x^5\right)d^3+\frac{8}{5}\left(16\,e^3x^5+20\,de^2x^4-5\,d^3x^2\right)ae^2$$

input
```
integrate((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^2,x, algorithm="maxima")
```

output
```
64/9*e^6*x^9 + 16*d*e^5*x^8 + 64/7*d^2*e^4*x^7 + 1/3*d^6*x^3 + 64*a^2*e^4*
x - 8/15*(5*e^3*x^6 + 6*d*e^2*x^5)*d^3 + 8/5*(16*e^3*x^5 + 20*d*e^2*x^4 -
5*d^3*x^2)*a*e^2
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.88

$$\int \left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)^2\,dx=\frac{64}{9}\,e^6x^9+16\,de^5x^8+\frac{64}{7}\,d^2e^4x^7-\frac{8}{3}\,d^3e^3x^6-\frac{16}{5}\,d^4e^2x^5+\frac{128}{5}\,ae^5x^5+32\,ade^4x^4+\frac{1}{3}\,d^6x^3-8\,ad^3e^2x^2+64\,a^2e^4x$$

input
```
integrate((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^2,x, algorithm="giac")
```

output
```
64/9*e^6*x^9 + 16*d*e^5*x^8 + 64/7*d^2*e^4*x^7 - 8/3*d^3*e^3*x^6 - 16/5*d^
4*e^2*x^5 + 128/5*a*e^5*x^5 + 32*a*d*e^4*x^4 + 1/3*d^6*x^3 - 8*a*d^3*e^2*x
^2 + 64*a^2*e^4*x
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.87

$$\int \left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)^2\,dx = x^5\left(\frac{128\,a\,e^5}{5}-\frac{16\,d^4\,e^2}{5}\right)+\frac{d^6\,x^3}{3}+\frac{64\,e^6\,x^9}{9}+64\,a^2\,e^4\,x+16\,d\,e^5\,x^8-\frac{8\,d^3\,e^3\,x^6}{3}+\frac{64\,d^2\,e^4\,x^7}{7}-8\,a\,d^3\,e^2\,x^2+32\,a\,d\,e^4\,x^4$$

input
```
int((8*a*e^2 - d^3*x + 8*e^3*x^4 + 8*d*e^2*x^3)^2,x)
```

output
```
x^5*((128*a*e^5)/5 - (16*d^4*e^2)/5) + (d^6*x^3)/3 + (64*e^6*x^9)/9 + 64*a
^2*e^4*x + 16*d*e^5*x^8 - (8*d^3*e^3*x^6)/3 + (64*d^2*e^4*x^7)/7 - 8*a*d^3
*e^2*x^2 + 32*a*d*e^4*x^4
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.88

$$\int \left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)^2\,dx = \frac{x(2240e^6x^8+5040d\,e^5x^7+2880d^2e^4x^6-840d^3e^3x^5+8064a\,e^5x^4-1008d^4e^2x^4+10080ad\,e^4x^3+105d}{315}$$

input
```
int((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^2,x)
```

output
```
(x*(20160*a**2*e**4 - 2520*a*d**3*e**2*x + 10080*a*d*e**4*x**3 + 8064*a*e*
*5*x**4 + 105*d**6*x**2 - 1008*d**4*e**2*x**4 - 840*d**3*e**3*x**5 + 2880*
d**2*e**4*x**6 + 5040*d*e**5*x**7 + 2240*e**6*x**8))/315
```

3.40 $\int (8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4)\, dx$

Optimal result

Integrand size = 30, antiderivative size = 37

$$\int (8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4)\, dx = 8ae^2x - \frac{d^3x^2}{2} + 2de^2x^4 + \frac{8e^3x^5}{5}$$

output
```
8*a*e^2*x-1/2*d^3*x^2+2*d*e^2*x^4+8/5*e^3*x^5
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int (8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4)\, dx = 8ae^2x - \frac{d^3x^2}{2} + 2de^2x^4 + \frac{8e^3x^5}{5}$$

input
```
Integrate[8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4,x]
```

output
```
8*a*e^2*x - (d^3*x^2)/2 + 2*d*e^2*x^4 + (8*e^3*x^5)/5
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4\right) dx$$

$$\Big\downarrow \text{2009}$$

$$8ae^2x - \frac{d^3x^2}{2} + 2de^2x^4 + \frac{8e^3x^5}{5}$$

input
```
Int[8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4,x]
```

output
```
8*a*e^2*x - (d^3*x^2)/2 + 2*d*e^2*x^4 + (8*e^3*x^5)/5
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.92

method	result	size
gosper	$8a\,e^2x-\frac{1}{2}d^3x^2+2d\,e^2x^4+\frac{8}{5}x^5e^3$	34
default	$8a\,e^2x-\frac{1}{2}d^3x^2+2d\,e^2x^4+\frac{8}{5}x^5e^3$	34
norman	$8a\,e^2x-\frac{1}{2}d^3x^2+2d\,e^2x^4+\frac{8}{5}x^5e^3$	34
risch	$8a\,e^2x-\frac{1}{2}d^3x^2+2d\,e^2x^4+\frac{8}{5}x^5e^3$	34
parallelrisch	$8a\,e^2x-\frac{1}{2}d^3x^2+2d\,e^2x^4+\frac{8}{5}x^5e^3$	34
parts	$8a\,e^2x-\frac{1}{2}d^3x^2+2d\,e^2x^4+\frac{8}{5}x^5e^3$	34
orering	$\frac{x(16x^4e^3+20d\,e^2x^3-5d^3x+80a\,e^2)}{10}$	34

input
```
int(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2,x,method=_RETURNVERBOSE)
```

output
```
8*a*e^2*x-1/2*d^3*x^2+2*d*e^2*x^4+8/5*x^5*e^3
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.89

$$\int \left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)\,dx=\frac{8}{5}\,e^3x^5+2\,de^2x^4-\frac{1}{2}\,d^3x^2+8\,ae^2x$$

input
```
integrate(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2,x, algorithm="fricas")
```

output
```
8/5*e^3*x^5 + 2*d*e^2*x^4 - 1/2*d^3*x^2 + 8*a*e^2*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.97

$$\int \left(8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4\right) dx = 8ae^2x - \frac{d^3x^2}{2} + 2de^2x^4 + \frac{8e^3x^5}{5}$$

input
```
integrate(8*e**3*x**4+8*d*e**2*x**3-d**3*x+8*a*e**2,x)
```

output
```
8*a*e**2*x - d**3*x**2/2 + 2*d*e**2*x**4 + 8*e**3*x**5/5
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.89

$$\int \left(8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4\right) dx = \frac{8}{5}e^3x^5 + 2\,de^2x^4 - \frac{1}{2}d^3x^2 + 8\,ae^2x$$

input
```
integrate(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2,x, algorithm="maxima")
```

output
```
8/5*e^3*x^5 + 2*d*e^2*x^4 - 1/2*d^3*x^2 + 8*a*e^2*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.89

$$\int \left(8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4\right) dx = \frac{8}{5}e^3x^5 + 2\,de^2x^4 - \frac{1}{2}d^3x^2 + 8\,ae^2x$$

input
```
integrate(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2,x, algorithm="giac")
```

output
```
8/5*e^3*x^5 + 2*d*e^2*x^4 - 1/2*d^3*x^2 + 8*a*e^2*x
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.89

$$\int \left(8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4\right) dx = -\frac{d^3 x^2}{2} + 2 d e^2 x^4 + \frac{8 e^3 x^5}{5} + 8 a e^2 x$$

input
```
int(8*a*e^2 - d^3*x + 8*e^3*x^4 + 8*d*e^2*x^3,x)
```

output
```
(8*e^3*x^5)/5 - (d^3*x^2)/2 + 2*d*e^2*x^4 + 8*a*e^2*x
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.89

$$\int \left(8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4\right) dx = \frac{x(16e^3x^4 + 20d e^2x^3 - 5d^3x + 80a e^2)}{10}$$

input
```
int(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2,x)
```

output
```
(x*(80*a*e**2 - 5*d**3*x + 20*d*e**2*x**3 + 16*e**3*x**4))/10
```

3.41 $\int \frac{1}{8ae^2-d^3x+8de^2x^3+8e^3x^4}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 153

$$\int \frac{1}{8ae^2-d^3x+8de^2x^3+8e^3x^4}\,dx=\frac{2\operatorname{arctanh}\left(\frac{d+4ex}{\sqrt{3d^2-2\sqrt{d^4-64ae^3}}}\right)}{\sqrt{d^4-64ae^3}\sqrt{3d^2-2\sqrt{d^4-64ae^3}}}-\frac{2\operatorname{arctanh}\left(\frac{d+4ex}{\sqrt{3d^2+2\sqrt{d^4-64ae^3}}}\right)}{\sqrt{d^4-64ae^3}\sqrt{3d^2+2\sqrt{d^4-64ae^3}}}$$

output
```
2*arctanh((4*e*x+d)/(3*d^2-2*(-64*a*e^3+d^4)^(1/2))^(1/2))/(-64*a*e^3+d^4)
^(1/2)/(3*d^2-2*(-64*a*e^3+d^4)^(1/2))^(1/2)-2*arctanh((4*e*x+d)/(3*d^2+2*
(-64*a*e^3+d^4)^(1/2))^(1/2))/(-64*a*e^3+d^4)^(1/2)/(3*d^2+2*(-64*a*e^3+d^
4)^(1/2))^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.46

$$\int \frac{1}{8ae^2-d^3x+8de^2x^3+8e^3x^4}\,dx=-\text{RootSum}\left[8ae^2-d^3\#1+8de^2\#1^3+8e^3\#1^4\&,\frac{\log(x-\#1)}{d^3-24de^2\#1^2-32e^3\#1^3}\&\right]$$

input
```
Integrate[(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)^(-1),x]
```

output
```
-RootSum[8*a*e^2 - d^3*#1 + 8*d*e^2*#1^3 + 8*e^3*#1^4 & , Log[x - #1]/(d^3
 - 24*d*e^2*#1^2 - 32*e^3*#1^3) & ]
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {2458, 1406, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4} \, dx$$

$$\Big\downarrow \text{2458}$$

$$\int \frac{1}{\frac{1}{32}\left(256ae^2 + \frac{5d^4}{e}\right) - 3d^2e\left(\frac{d}{4e} + x\right)^2 + 8e^3\left(\frac{d}{4e} + x\right)^4} d\left(\frac{d}{4e} + x\right)$$

$$\Big\downarrow \text{1406}$$

$$\frac{4e^2 \int \frac{1}{8e^3\left(\frac{d}{4e}+x\right)^2 - \frac{1}{2}e\left(3d^2+2\sqrt{d^4-64ae^3}\right)} d\left(\frac{d}{4e}+x\right)}{\sqrt{d^4-64ae^3}} - \frac{4e^2 \int \frac{1}{8e^3\left(\frac{d}{4e}+x\right)^2 - \frac{1}{2}e\left(3d^2-2\sqrt{d^4-64ae^3}\right)} d\left(\frac{d}{4e}+x\right)}{\sqrt{d^4-64ae^3}}$$

$$\Big\downarrow \text{221}$$

$$\frac{2\text{arctanh}\left(\frac{4e\left(\frac{d}{4e}+x\right)}{\sqrt{3d^2-2\sqrt{d^4-64ae^3}}}\right)}{\sqrt{d^4-64ae^3}\sqrt{3d^2-2\sqrt{d^4-64ae^3}}} - \frac{2\text{arctanh}\left(\frac{4e\left(\frac{d}{4e}+x\right)}{\sqrt{2\sqrt{d^4-64ae^3}+3d^2}}\right)}{\sqrt{d^4-64ae^3}\sqrt{2\sqrt{d^4-64ae^3}+3d^2}}$$

input
```
Int[(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)^(-1),x]
```

output
```
(2*ArcTanh[(4*e*(d/(4*e) + x))/Sqrt[3*d^2 - 2*Sqrt[d^4 - 64*a*e^3]]])/(Sqr
t[d^4 - 64*a*e^3]*Sqrt[3*d^2 - 2*Sqrt[d^4 - 64*a*e^3]]) - (2*ArcTanh[(4*e*
(d/(4*e) + x))/Sqrt[3*d^2 + 2*Sqrt[d^4 - 64*a*e^3]]])/(Sqrt[d^4 - 64*a*e^3
]*Sqrt[3*d^2 + 2*Sqrt[d^4 - 64*a*e^3]])
```

Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1406
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Simp[c/q   Int[1/(b/2 - q/2 + c*x^2), x], x] - Simp[c/q   I
nt[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c
, 0] && PosQ[b^2 - 4*a*c]
```

rule 2458
```
Int[(Pn_)^(p_.), x_Symbol] :> With[{S = Coeff[Pn, x, Expon[Pn, x] - 1]/(Exp
on[Pn, x]*Coeff[Pn, x, Expon[Pn, x]])}, Subst[Int[ExpandToSum[Pn /. x -> x
- S, x]^p, x], x, x + S] /; BinomialQ[Pn /. x -> x - S, x] || (IntegerQ[Exp
on[Pn, x]/2] && TrinomialQ[Pn /. x -> x - S, x])] /; FreeQ[p, x] && PolyQ[P
n, x] && GtQ[Expon[Pn, x], 2] && NeQ[Coeff[Pn, x, Expon[Pn, x] - 1], 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.27 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.44

method	result	size
default	$\sum_{\_R=\mathrm{RootOf}\left(8e^3\_Z^4+8d\,e^2\_Z^3-d^3\_Z+8a\,e^2\right)}\frac{\ln\left(x-\_R\right)}{32\_R^3e^3+24\_R^2d\,e^2-d^3}$	67
risch	$\sum_{\_R=\mathrm{RootOf}\left(8e^3\_Z^4+8d\,e^2\_Z^3-d^3\_Z+8a\,e^2\right)}\frac{\ln\left(x-\_R\right)}{32\_R^3e^3+24\_R^2d\,e^2-d^3}$	67

input
```
int(1/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2),x,method=_RETURNVERBOSE)
```

output
```
sum(1/(32*_R^3*e^3+24*_R^2*d*e^2-d^3)*ln(x-_R),_R=RootOf(8*_Z^4*e^3+8*_Z^3
*d*e^2-_Z*d^3+8*a*e^2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1115 vs. $2(133) = 266$.

Time = 0.09 (sec) , antiderivative size = 1115, normalized size of antiderivative = 7.29

$$\int \frac{1}{8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4}\,dx = \text{Too large to display}$$

input
```
integrate(1/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2),x, algorithm="fricas")
```

output
```
-sqrt((3*d^2 + 2*(5*d^8 - 64*a*d^4*e^3 - 16384*a^2*e^6)/sqrt(25*d^12 + 960
*a*d^8*e^3 - 98304*a^2*d^4*e^6 - 4194304*a^3*e^9))/(5*d^8 - 64*a*d^4*e^3 -
 16384*a^2*e^6))*log(8*e*x + 2*(2*d^4 - 128*a*e^3 - 3*(5*d^10 - 64*a*d^6*e
^3 - 16384*a^2*d^2*e^6)/sqrt(25*d^12 + 960*a*d^8*e^3 - 98304*a^2*d^4*e^6 -
 4194304*a^3*e^9))*sqrt((3*d^2 + 2*(5*d^8 - 64*a*d^4*e^3 - 16384*a^2*e^6)/
sqrt(25*d^12 + 960*a*d^8*e^3 - 98304*a^2*d^4*e^6 - 4194304*a^3*e^9))/(5*d^
8 - 64*a*d^4*e^3 - 16384*a^2*e^6)) + 2*d) + sqrt((3*d^2 + 2*(5*d^8 - 64*a*
d^4*e^3 - 16384*a^2*e^6)/sqrt(25*d^12 + 960*a*d^8*e^3 - 98304*a^2*d^4*e^6
- 4194304*a^3*e^9))/(5*d^8 - 64*a*d^4*e^3 - 16384*a^2*e^6))*log(8*e*x - 2*
(2*d^4 - 128*a*e^3 - 3*(5*d^10 - 64*a*d^6*e^3 - 16384*a^2*d^2*e^6)/sqrt(25
*d^12 + 960*a*d^8*e^3 - 98304*a^2*d^4*e^6 - 4194304*a^3*e^9))*sqrt((3*d^2
+ 2*(5*d^8 - 64*a*d^4*e^3 - 16384*a^2*e^6)/sqrt(25*d^12 + 960*a*d^8*e^3 -
98304*a^2*d^4*e^6 - 4194304*a^3*e^9))/(5*d^8 - 64*a*d^4*e^3 - 16384*a^2*e^
6)) + 2*d) - sqrt((3*d^2 - 2*(5*d^8 - 64*a*d^4*e^3 - 16384*a^2*e^6)/sqrt(2
5*d^12 + 960*a*d^8*e^3 - 98304*a^2*d^4*e^6 - 4194304*a^3*e^9))/(5*d^8 - 64
*a*d^4*e^3 - 16384*a^2*e^6))*log(8*e*x + 2*(2*d^4 - 128*a*e^3 + 3*(5*d^10
- 64*a*d^6*e^3 - 16384*a^2*d^2*e^6)/sqrt(25*d^12 + 960*a*d^8*e^3 - 98304*a
^2*d^4*e^6 - 4194304*a^3*e^9))*sqrt((3*d^2 - 2*(5*d^8 - 64*a*d^4*e^3 - 163
84*a^2*e^6)/sqrt(25*d^12 + 960*a*d^8*e^3 - 98304*a^2*d^4*e^6 - 4194304*a^3
*e^9))/(5*d^8 - 64*a*d^4*e^3 - 16384*a^2*e^6)) + 2*d) + sqrt((3*d^2 - 2...
```

Sympy [A] (verification not implemented)

Time = 1.11 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.80

$$\int \frac{1}{8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4}\,dx$$
$$= \text{RootSum}\left(t^4\cdot\left(1048576a^3e^9 - 12288a^2d^4e^6 - 384ad^8e^3 + 5d^{12}\right) + t^2\cdot\left(384ad^2e^3 - 6d^6\right) + 1, \left(t \mapsto t\log\right.\right.$$

input
```
integrate(1/(8*e**3*x**4+8*d*e**2*x**3-d**3*x+8*a*e**2),x)
```

output
```
RootSum(_t**4*(1048576*a**3*e**9 - 12288*a**2*d**4*e**6 - 384*a*d**8*e**3
+ 5*d**12) + _t**2*(384*a*d**2*e**3 - 6*d**6) + 1, Lambda(_t, _t*log(x + (
-49152*_t**3*a**2*d**2*e**6 - 192*_t**3*a*d**6*e**3 + 15*_t**3*d**10 + 256
*_t*a*e**3 - 13*_t*d**4 + 2*d)/(8*e))))
```

Maxima [F]

$$\int \frac{1}{8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4}\,dx = \int \frac{1}{8\,e^3x^4 + 8\,de^2x^3 - d^3x + 8\,ae^2}\,dx$$

input
```
integrate(1/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2),x, algorithm="maxima")
```

output
```
integrate(1/(8*e^3*x^4 + 8*d*e^2*x^3 - d^3*x + 8*a*e^2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 577 vs. $2(133) = 266$.

Time = 0.12 (sec) , antiderivative size = 577, normalized size of antiderivative = 3.77

$$\int \frac{1}{8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4}\,dx$$
$$= -\frac{2\log\left(x + \frac{1}{4}\sqrt{\frac{3\,d^2e^2 + 2\,\sqrt{d^4 - 64\,ae^3}e^2}{e^4}} + \frac{d}{4\,e}\right)}{e^3\left(\sqrt{\frac{3\,d^2e^2 + 2\,\sqrt{d^4 - 64\,ae^3}e^2}{e^4}} + \frac{d}{e}\right)^3 - 3\,de^2\left(\sqrt{\frac{3\,d^2e^2 + 2\,\sqrt{d^4 - 64\,ae^3}e^2}{e^4}} + \frac{d}{e}\right)^2 + 2\,d^3}$$
$$+ \frac{2\log\left(x - \frac{1}{4}\sqrt{\frac{3\,d^2e^2 + 2\,\sqrt{d^4 - 64\,ae^3}e^2}{e^4}} + \frac{d}{4\,e}\right)}{e^3\left(\sqrt{\frac{3\,d^2e^2 + 2\,\sqrt{d^4 - 64\,ae^3}e^2}{e^4}} - \frac{d}{e}\right)^3 + 3\,de^2\left(\sqrt{\frac{3\,d^2e^2 + 2\,\sqrt{d^4 - 64\,ae^3}e^2}{e^4}} - \frac{d}{e}\right)^2 - 2\,d^3}$$
$$- \frac{2\log\left(x + \frac{1}{4}\sqrt{\frac{3\,d^2e^2 - 2\,\sqrt{d^4 - 64\,ae^3}e^2}{e^4}} + \frac{d}{4\,e}\right)}{e^3\left(\sqrt{\frac{3\,d^2e^2 - 2\,\sqrt{d^4 - 64\,ae^3}e^2}{e^4}} + \frac{d}{e}\right)^3 - 3\,de^2\left(\sqrt{\frac{3\,d^2e^2 - 2\,\sqrt{d^4 - 64\,ae^3}e^2}{e^4}} + \frac{d}{e}\right)^2 + 2\,d^3}$$
$$+ \frac{2\log\left(x - \frac{1}{4}\sqrt{\frac{3\,d^2e^2 - 2\,\sqrt{d^4 - 64\,ae^3}e^2}{e^4}} + \frac{d}{4\,e}\right)}{e^3\left(\sqrt{\frac{3\,d^2e^2 - 2\,\sqrt{d^4 - 64\,ae^3}e^2}{e^4}} - \frac{d}{e}\right)^3 + 3\,de^2\left(\sqrt{\frac{3\,d^2e^2 - 2\,\sqrt{d^4 - 64\,ae^3}e^2}{e^4}} - \frac{d}{e}\right)^2 - 2\,d^3}$$

input
```
integrate(1/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2),x, algorithm="giac")
```

output
```
-2*log(x + 1/4*sqrt((3*d^2*e^2 + 2*sqrt(d^4 - 64*a*e^3)*e^2)/e^4) + 1/4*d/
e)/(e^3*(sqrt((3*d^2*e^2 + 2*sqrt(d^4 - 64*a*e^3)*e^2)/e^4) + d/e)^3 - 3*d
*e^2*(sqrt((3*d^2*e^2 + 2*sqrt(d^4 - 64*a*e^3)*e^2)/e^4) + d/e)^2 + 2*d^3)
 + 2*log(x - 1/4*sqrt((3*d^2*e^2 + 2*sqrt(d^4 - 64*a*e^3)*e^2)/e^4) + 1/4*
d/e)/(e^3*(sqrt((3*d^2*e^2 + 2*sqrt(d^4 - 64*a*e^3)*e^2)/e^4) - d/e)^3 + 3
*d*e^2*(sqrt((3*d^2*e^2 + 2*sqrt(d^4 - 64*a*e^3)*e^2)/e^4) - d/e)^2 - 2*d^
3) - 2*log(x + 1/4*sqrt((3*d^2*e^2 - 2*sqrt(d^4 - 64*a*e^3)*e^2)/e^4) + 1/
4*d/e)/(e^3*(sqrt((3*d^2*e^2 - 2*sqrt(d^4 - 64*a*e^3)*e^2)/e^4) + d/e)^3 -
 3*d*e^2*(sqrt((3*d^2*e^2 - 2*sqrt(d^4 - 64*a*e^3)*e^2)/e^4) + d/e)^2 + 2*
d^3) + 2*log(x - 1/4*sqrt((3*d^2*e^2 - 2*sqrt(d^4 - 64*a*e^3)*e^2)/e^4) +
1/4*d/e)/(e^3*(sqrt((3*d^2*e^2 - 2*sqrt(d^4 - 64*a*e^3)*e^2)/e^4) - d/e)^3
 + 3*d*e^2*(sqrt((3*d^2*e^2 - 2*sqrt(d^4 - 64*a*e^3)*e^2)/e^4) - d/e)^2 -
2*d^3)
```

Mupad [B] (verification not implemented)

Time = 23.15 (sec) , antiderivative size = 1264, normalized size of antiderivative = 8.26

$$\int \frac{1}{8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4} \, dx = \text{Too large to display}$$

input
```
int(1/(8*a*e^2 - d^3*x + 8*e^3*x^4 + 8*d*e^2*x^3),x)
```

output
```
atan((d^3*(d^12 - 262144*a^3*e^9 - 192*a*d^8*e^3 + 12288*a^2*d^4*e^6)^(1/2
)*3i - d^9*2i + a*d^5*e^3*256i - a^2*d*e^6*8192i - a^2*e^7*x*32768i - d^8*
e*x*8i + d^2*e*x*(d^12 - 262144*a^3*e^9 - 192*a*d^8*e^3 + 12288*a^2*d^4*e^
6)^(1/2)*12i + a*d^4*e^4*x*1024i)/(5*d^12*(-(2*(d^12 - 262144*a^3*e^9 - 19
2*a*d^8*e^3 + 12288*a^2*d^4*e^6)^(1/2) - 3*d^6 + 192*a*d^2*e^3)/(5*d^12 +
1048576*a^3*e^9 - 384*a*d^8*e^3 - 12288*a^2*d^4*e^6))^(1/2) + 1048576*a^3*
e^9*(-(2*(d^12 - 262144*a^3*e^9 - 192*a*d^8*e^3 + 12288*a^2*d^4*e^6)^(1/2)
 - 3*d^6 + 192*a*d^2*e^3)/(5*d^12 + 1048576*a^3*e^9 - 384*a*d^8*e^3 - 1228
8*a^2*d^4*e^6))^(1/2) - 384*a*d^8*e^3*(-(2*(d^12 - 262144*a^3*e^9 - 192*a*
d^8*e^3 + 12288*a^2*d^4*e^6)^(1/2) - 3*d^6 + 192*a*d^2*e^3)/(5*d^12 + 1048
576*a^3*e^9 - 384*a*d^8*e^3 - 12288*a^2*d^4*e^6))^(1/2) - 12288*a^2*d^4*e^
6*(-(2*(d^12 - 262144*a^3*e^9 - 192*a*d^8*e^3 + 12288*a^2*d^4*e^6)^(1/2) -
 3*d^6 + 192*a*d^2*e^3)/(5*d^12 + 1048576*a^3*e^9 - 384*a*d^8*e^3 - 12288*
a^2*d^4*e^6))^(1/2)))*(-(2*(d^12 - 262144*a^3*e^9 - 192*a*d^8*e^3 + 12288*
a^2*d^4*e^6)^(1/2) - 3*d^6 + 192*a*d^2*e^3)/(5*d^12 + 1048576*a^3*e^9 - 38
4*a*d^8*e^3 - 12288*a^2*d^4*e^6))^(1/2)*2i - atan((d^3*(d^12 - 262144*a^3*
e^9 - 192*a*d^8*e^3 + 12288*a^2*d^4*e^6)^(1/2)*3i + d^9*2i - a*d^5*e^3*256
i + a^2*d*e^6*8192i + a^2*e^7*x*32768i + d^8*e*x*8i + d^2*e*x*(d^12 - 2621
44*a^3*e^9 - 192*a*d^8*e^3 + 12288*a^2*d^4*e^6)^(1/2)*12i - a*d^4*e^4*x*10
24i)/(5*d^12*((2*(d^12 - 262144*a^3*e^9 - 192*a*d^8*e^3 + 12288*a^2*d^4...
```

Reduce [F]

$$\int \frac{1}{8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4} \, dx = \int \frac{1}{8e^3x^4 + 8d\,e^2x^3 - d^3x + 8a\,e^2} dx$$

input
```
int(1/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2),x)
```

output
```
int(1/(8*a*e**2 - d**3*x + 8*d*e**2*x**3 + 8*e**3*x**4),x)
```

3.42 $\int \frac{1}{(8ae^2-d^3x+8de^2x^3+8e^3x^4)^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 332

$$\int \frac{1}{(8ae^2-d^3x+8de^2x^3+8e^3x^4)^2}\,dx = \frac{(d+4ex)\left(13d^4-256ae^3-3d^2(d+4ex)^2\right)}{2\left(5d^8-64ad^4e^3-16384a^2e^6\right)(8ae^2-d^3x+8de^2x^3+8e^3x^4)} - \frac{24e\left(d^4+128ae^3-d^2\sqrt{d^4-64ae^3}\right)\operatorname{arctanh}\left(\frac{d+4ex}{\sqrt{3d^2-2\sqrt{d^4-64ae^3}}}\right)}{(d^4-64ae^3)^{3/2}\left(5d^4+256ae^3\right)\sqrt{3d^2-2\sqrt{d^4-64ae^3}}} + \frac{24e\left(d^4+128ae^3+d^2\sqrt{d^4-64ae^3}\right)\operatorname{arctanh}\left(\frac{d+4ex}{\sqrt{3d^2+2\sqrt{d^4-64ae^3}}}\right)}{(d^4-64ae^3)^{3/2}\left(5d^4+256ae^3\right)\sqrt{3d^2+2\sqrt{d^4-64ae^3}}}$$

output
```
1/2*(4*e*x+d)*(13*d^4-256*a*e^3-3*d^2*(4*e*x+d)^2)/(-16384*a^2*e^6-64*a*d^
4*e^3+5*d^8)/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)-24*e*(d^4+128*a*e^3-d^2
*(-64*a*e^3+d^4)^(1/2))*arctanh((4*e*x+d)/(3*d^2-2*(-64*a*e^3+d^4)^(1/2))^
(1/2))/(-64*a*e^3+d^4)^(3/2)/(256*a*e^3+5*d^4)/(3*d^2-2*(-64*a*e^3+d^4)^(1
/2))^(1/2)+24*e*(d^4+128*a*e^3+d^2*(-64*a*e^3+d^4)^(1/2))*arctanh((4*e*x+d
)/(3*d^2+2*(-64*a*e^3+d^4)^(1/2))^(1/2))/(-64*a*e^3+d^4)^(3/2)/(256*a*e^3+
5*d^4)/(3*d^2+2*(-64*a*e^3+d^4)^(1/2))^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.16 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.70

$$\int \frac{1}{(8ae^2-d^3x+8de^2x^3+8e^3x^4)^2}\,dx$$
$$= \frac{(d+4ex)(5d^4-128ae^3-12d^3ex-24d^2e^2x^2)}{(d^4-64ae^3)(5d^4+256ae^3)(8ae^2-d^3x+8de^2x^3+8e^3x^4)}$$
$$+\frac{48e^2\text{RootSum}\left[8ae^2-d^3\#1+8de^2\#1^3+8e^3\#1^4\&,\frac{32ae^2\log(x-\#1)+d^3\log(x-\#1)\#1+2d^2e\log(x-\#1)\#1^2}{-d^3+24de^2\#1^2+32e^3\#1^3}\&\right]}{-5d^8+64ad^4e^3+16384a^2e^6}$$

input
```
Integrate[(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)^(-2),x]
```

output
```
((d + 4*e*x)*(5*d^4 - 128*a*e^3 - 12*d^3*e*x - 24*d^2*e^2*x^2))/((d^4 - 64
*a*e^3)*(5*d^4 + 256*a*e^3)*(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)) +
 (48*e^2*RootSum[8*a*e^2 - d^3*#1 + 8*d*e^2*#1^3 + 8*e^3*#1^4 & , (32*a*e^
2*Log[x - #1] + d^3*Log[x - #1]*#1 + 2*d^2*e*Log[x - #1]*#1^2)/(-d^3 + 24*
d*e^2*#1^2 + 32*e^3*#1^3) & ])/(-5*d^8 + 64*a*d^4*e^3 + 16384*a^2*e^6)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 377, normalized size of antiderivative = 1.14, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {2458, 1405, 27, 1480, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(8ae^2-d^3x+8de^2x^3+8e^3x^4)^2}\,dx$$
$$\Big\downarrow \ 2458$$
$$\int \frac{1}{\left(\frac{1}{32}\left(256ae^2+\frac{5d^4}{e}\right)-3d^2e\left(\frac{d}{4e}+x\right)^2+8e^3\left(\frac{d}{4e}+x\right)^4\right)^2}d\left(\frac{d}{4e}+x\right)$$

$$\Big\downarrow \; 1405$$

$$\frac{64e\left(\frac{d}{4e}+x\right)\left(-256ae^3+13d^4-48d^2e^2\left(\frac{d}{4e}+x\right)^2\right)}{\left(-16384a^2e^6-64ad^4e^3+5d^8\right)\left(256ae^2+\frac{5d^4}{e}-96d^2e\left(\frac{d}{4e}+x\right)^2+256e^3\left(\frac{d}{4e}+x\right)^4\right)}-\frac{4\int -\frac{48e^2\left(d^4-16e^2\left(\frac{d}{4e}+x\right)^2d^2-256ae^3\right)}{\frac{5d^4}{e}-96e\left(\frac{d}{4e}+x\right)^2d^2+256e^3\left(\frac{d}{4e}+x\right)^4+256ae^2}d\left(\frac{d}{4e}+x\right)}{e\left(-16384a^2e^6-64ad^4e^3+5d^8\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{192e\int \frac{d^4-16e^2\left(\frac{d}{4e}+x\right)^2d^2-256ae^3}{\frac{5d^4}{e}-96e\left(\frac{d}{4e}+x\right)^2d^2+256e^3\left(\frac{d}{4e}+x\right)^4+256ae^2}d\left(\frac{d}{4e}+x\right)}{-16384a^2e^6-64ad^4e^3+5d^8}+\frac{64e\left(\frac{d}{4e}+x\right)\left(-256ae^3+13d^4-48d^2e^2\left(\frac{d}{4e}+x\right)^2\right)}{\left(-16384a^2e^6-64ad^4e^3+5d^8\right)\left(256ae^2+\frac{5d^4}{e}-96d^2e\left(\frac{d}{4e}+x\right)^2+256e^3\left(\frac{d}{4e}+x\right)^4\right)}$$

$$\Big\downarrow \; 1480$$

$$\frac{192e\left(\frac{8e^2\left(-d^2\sqrt{d^4-64ae^3}+128ae^3+d^4\right)\int \frac{1}{256e^3\left(\frac{d}{4e}+x\right)^2-16e\left(3d^2-2\sqrt{d^4-64ae^3}\right)}d\left(\frac{d}{4e}+x\right)}{\sqrt{d^4-64ae^3}}-\frac{8e^2\left(d^2\sqrt{d^4-64ae^3}+128ae^3+d^4\right)\int \frac{}{256e^3\left(\frac{d}{4e}\right.}}{\sqrt{d^4-64}}\right.}{-16384a^2e^6-64ad^4e^3+5d^8}+\frac{64e\left(\frac{d}{4e}+x\right)\left(-256ae^3+13d^4-48d^2e^2\left(\frac{d}{4e}+x\right)^2\right)}{\left(-16384a^2e^6-64ad^4e^3+5d^8\right)\left(256ae^2+\frac{5d^4}{e}-96d^2e\left(\frac{d}{4e}+x\right)^2+256e^3\left(\frac{d}{4e}+x\right)^4\right)}$$

$$\Big\downarrow \; 221$$

$$\frac{192e\left(\frac{\left(d^2\sqrt{d^4-64ae^3}+128ae^3+d^4\right)\text{arctanh}\left(\frac{4e\left(\frac{d}{4e}+x\right)}{\sqrt{2\sqrt{d^4-64ae^3}+3d^2}}\right)}{8\sqrt{d^4-64ae^3}\sqrt{2\sqrt{d^4-64ae^3}+3d^2}}-\frac{\left(-d^2\sqrt{d^4-64ae^3}+128ae^3+d^4\right)\text{arctanh}\left(\frac{4e\left(\frac{d}{4e}+x\right)}{\sqrt{3d^2-2\sqrt{d^4-64ae^3}}}\right)}{8\sqrt{d^4-64ae^3}\sqrt{3d^2-2\sqrt{d^4-64ae^3}}}\right)}{-16384a^2e^6-64ad^4e^3+5d^8}+\frac{64e\left(\frac{d}{4e}+x\right)\left(-256ae^3+13d^4-48d^2e^2\left(\frac{d}{4e}+x\right)^2\right)}{\left(-16384a^2e^6-64ad^4e^3+5d^8\right)\left(256ae^2+\frac{5d^4}{e}-96d^2e\left(\frac{d}{4e}+x\right)^2+256e^3\left(\frac{d}{4e}+x\right)^4\right)}$$

input
```
Int[(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)^(-2),x]
```

output
```
(64*e*(d/(4*e) + x)*(13*d^4 - 256*a*e^3 - 48*d^2*e^2*(d/(4*e) + x)^2))/((5
*d^8 - 64*a*d^4*e^3 - 16384*a^2*e^6)*((5*d^4)/e + 256*a*e^2 - 96*d^2*e*(d/
(4*e) + x)^2 + 256*e^3*(d/(4*e) + x)^4)) + (192*e*(-1/8*((d^4 + 128*a*e^3
- d^2*Sqrt[d^4 - 64*a*e^3])*ArcTanh[(4*e*(d/(4*e) + x))/Sqrt[3*d^2 - 2*Sqr
t[d^4 - 64*a*e^3]]])/(Sqrt[d^4 - 64*a*e^3]*Sqrt[3*d^2 - 2*Sqrt[d^4 - 64*a*
e^3]]) + ((d^4 + 128*a*e^3 + d^2*Sqrt[d^4 - 64*a*e^3])*ArcTanh[(4*e*(d/(4*
e) + x))/Sqrt[3*d^2 + 2*Sqrt[d^4 - 64*a*e^3]]])/(8*Sqrt[d^4 - 64*a*e^3]*Sq
rt[3*d^2 + 2*Sqrt[d^4 - 64*a*e^3]])))/(5*d^8 - 64*a*d^4*e^3 - 16384*a^2*e^
6)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 2458
```
Int[(Pn_)^(p_.), x_Symbol] :> With[{S = Coeff[Pn, x, Expon[Pn, x] - 1]/(Exp
on[Pn, x]*Coeff[Pn, x, Expon[Pn, x]])}, Subst[Int[ExpandToSum[Pn /. x -> x
- S, x]^p, x], x, x + S] /; BinomialQ[Pn /. x -> x - S, x] || (IntegerQ[Exp
on[Pn, x]/2] && TrinomialQ[Pn /. x -> x - S, x])] /; FreeQ[p, x] && PolyQ[P
n, x] && GtQ[Expon[Pn, x], 2] && NeQ[Coeff[Pn, x, Expon[Pn, x] - 1], 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.17 (sec) , antiderivative size = 288, normalized size of antiderivative = 0.87

method	result
default	$\frac{\frac{12 d^{2} e^{3} x^{3}}{\left(256 e^{3} a+5 d^{4}\right)\left(64 e^{3} a-d^{4}\right)}+\frac{9 d^{3} e^{2} x^{2}}{\left(256 e^{3} a+5 d^{4}\right)\left(64 e^{3} a-d^{4}\right)}+\frac{e x}{256 e^{3} a+5 d^{4}}+\frac{d\left(128 e^{3} a-5 d^{4}\right)}{131072 a^{2} e^{6}+512 a d^{4} e^{3}-40 d^{8}}}{x^{4} e^{3}+d e^{2} x^{3}-\frac{1}{8} d^{3} x+a e^{2}}+\frac{384 e^{2}\left(\sum_{\_R=\operatorname{RootOf}\left(8 e^{3}\_Z^{4}+\right.}\right.}{}$
risch	$\frac{\frac{12 d^{2} e^{3} x^{3}}{\left(256 e^{3} a+5 d^{4}\right)\left(64 e^{3} a-d^{4}\right)}+\frac{9 d^{3} e^{2} x^{2}}{\left(256 e^{3} a+5 d^{4}\right)\left(64 e^{3} a-d^{4}\right)}+\frac{e x}{256 e^{3} a+5 d^{4}}+\frac{d\left(128 e^{3} a-5 d^{4}\right)}{131072 a^{2} e^{6}+512 a d^{4} e^{3}-40 d^{8}}}{x^{4} e^{3}+d e^{2} x^{3}-\frac{1}{8} d^{3} x+a e^{2}}+48 e^{2}\left(\sum_{\_R=\operatorname{RootOf}\left(8 e^{3}\_Z\right.}\right.$

input
```
int(1/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^2,x,method=_RETURNVERBOSE)
```

output
```
(12*d^2*e^3/(256*a*e^3+5*d^4)/(64*a*e^3-d^4)*x^3+9*d^3*e^2/(256*a*e^3+5*d^
4)/(64*a*e^3-d^4)*x^2+e/(256*a*e^3+5*d^4)*x+1/8*d*(128*a*e^3-5*d^4)/(16384
*a^2*e^6+64*a*d^4*e^3-5*d^8))/(x^4*e^3+d*e^2*x^3-1/8*d^3*x+a*e^2)+384*e^2/
(2048*a*e^3+40*d^4)/(64*a*e^3-d^4)*sum((2*_R^2*d^2*e+_R*d^3+32*a*e^2)/(32*
_R^3*e^3+24*_R^2*d*e^2-d^3)*ln(x-_R),_R=RootOf(8*_Z^4*e^3+8*_Z^3*d*e^2-_Z*
d^3+8*a*e^2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4285 vs. $2(306) = 612$.

Time = 0.27 (sec) , antiderivative size = 4285, normalized size of antiderivative = 12.91

$$\int \frac{1}{(8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(8*e**3*x**4+8*d*e**2*x**3-d**3*x+8*a*e**2)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{(8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4)^2}\,dx = \int \frac{1}{(8\,e^3x^4 + 8\,de^2x^3 - d^3x + 8\,ae^2)^2}\,dx$$

input
```
integrate(1/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^2,x, algorithm="maxima")
```

output
```
-48*e^2*integrate((2*d^2*e*x^2 + d^3*x + 32*a*e^2)/(8*e^3*x^4 + 8*d*e^2*x^
3 - d^3*x + 8*a*e^2), x)/(5*d^8 - 64*a*d^4*e^3 - 16384*a^2*e^6) - (96*d^2*
e^3*x^3 + 72*d^3*e^2*x^2 - 5*d^5 + 128*a*d*e^3 - 8*(d^4*e - 64*a*e^4)*x)/(
40*a*d^8*e^2 - 512*a^2*d^4*e^5 - 131072*a^3*e^8 + 8*(5*d^8*e^3 - 64*a*d^4*
e^6 - 16384*a^2*e^9)*x^4 + 8*(5*d^9*e^2 - 64*a*d^5*e^5 - 16384*a^2*d*e^8)*
x^3 - (5*d^11 - 64*a*d^7*e^3 - 16384*a^2*d^3*e^6)*x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1115 vs. $2(306) = 612$.

Time = 0.12 (sec) , antiderivative size = 1115, normalized size of antiderivative = 3.36

$$\int \frac{1}{(8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^2,x, algorithm="giac")
```

output
```
12*((d^2*e^3*(sqrt((3*d^2*e^2 + 2*sqrt(d^4 - 64*a*e^3)*e^2)/e^4) + d/e)^2
- 2*d^3*e^2*(sqrt((3*d^2*e^2 + 2*sqrt(d^4 - 64*a*e^3)*e^2)/e^4) + d/e) + 2
56*a*e^4)*log(x + 1/4*sqrt((3*d^2*e^2 + 2*sqrt(d^4 - 64*a*e^3)*e^2)/e^4) +
 1/4*d/e)/(e^3*(sqrt((3*d^2*e^2 + 2*sqrt(d^4 - 64*a*e^3)*e^2)/e^4) + d/e)^
3 - 3*d*e^2*(sqrt((3*d^2*e^2 + 2*sqrt(d^4 - 64*a*e^3)*e^2)/e^4) + d/e)^2 +
 2*d^3) - (d^2*e^3*(sqrt((3*d^2*e^2 + 2*sqrt(d^4 - 64*a*e^3)*e^2)/e^4) - d
/e)^2 + 2*d^3*e^2*(sqrt((3*d^2*e^2 + 2*sqrt(d^4 - 64*a*e^3)*e^2)/e^4) - d/
e) + 256*a*e^4)*log(x - 1/4*sqrt((3*d^2*e^2 + 2*sqrt(d^4 - 64*a*e^3)*e^2)/
e^4) + 1/4*d/e)/(e^3*(sqrt((3*d^2*e^2 + 2*sqrt(d^4 - 64*a*e^3)*e^2)/e^4) -
 d/e)^3 + 3*d*e^2*(sqrt((3*d^2*e^2 + 2*sqrt(d^4 - 64*a*e^3)*e^2)/e^4) - d/
e)^2 - 2*d^3) + (d^2*e^3*(sqrt((3*d^2*e^2 - 2*sqrt(d^4 - 64*a*e^3)*e^2)/e^
4) + d/e)^2 - 2*d^3*e^2*(sqrt((3*d^2*e^2 - 2*sqrt(d^4 - 64*a*e^3)*e^2)/e^4
) + d/e) + 256*a*e^4)*log(x + 1/4*sqrt((3*d^2*e^2 - 2*sqrt(d^4 - 64*a*e^3)
*e^2)/e^4) + 1/4*d/e)/(e^3*(sqrt((3*d^2*e^2 - 2*sqrt(d^4 - 64*a*e^3)*e^2)/
e^4) + d/e)^3 - 3*d*e^2*(sqrt((3*d^2*e^2 - 2*sqrt(d^4 - 64*a*e^3)*e^2)/e^4
) + d/e)^2 + 2*d^3) - (d^2*e^3*(sqrt((3*d^2*e^2 - 2*sqrt(d^4 - 64*a*e^3)*e
^2)/e^4) - d/e)^2 + 2*d^3*e^2*(sqrt((3*d^2*e^2 - 2*sqrt(d^4 - 64*a*e^3)*e^
2)/e^4) - d/e) + 256*a*e^4)*log(x - 1/4*sqrt((3*d^2*e^2 - 2*sqrt(d^4 - 64*
a*e^3)*e^2)/e^4) + 1/4*d/e)/(e^3*(sqrt((3*d^2*e^2 - 2*sqrt(d^4 - 64*a*e^3)
*e^2)/e^4) - d/e)^3 + 3*d*e^2*(sqrt((3*d^2*e^2 - 2*sqrt(d^4 - 64*a*e^3)...
```

Mupad [B] (verification not implemented)

Time = 26.79 (sec) , antiderivative size = 10351, normalized size of antiderivative = 31.18

$$\int \frac{1}{(8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4)^2}\,dx = \text{Too large to display}$$

input
```
int(1/(8*a*e^2 - d^3*x + 8*e^3*x^4 + 8*d*e^2*x^3)^2,x)
```

output
```
((8*e*x)/(256*a*e^3 + 5*d^4) - (5*d^5 - 128*a*d*e^3)/((64*a*e^3 - d^4)*(25
6*a*e^3 + 5*d^4)) + (72*d^3*e^2*x^2)/((64*a*e^3 - d^4)*(256*a*e^3 + 5*d^4)
) + (96*d^2*e^3*x^3)/((64*a*e^3 - d^4)*(256*a*e^3 + 5*d^4)))/(8*a*e^2 - d^
3*x + 8*e^3*x^4 + 8*d*e^2*x^3) + atan((((288*(d^22*e^2 + d^4*e^2*(-(64*a*e
^3 - d^4)^9)^(1/2) - 32*a*d^18*e^5 + 22528*a^2*d^14*e^8 - 6160384*a^3*d^10
*e^11 + 461373440*a^4*d^6*e^14 - 10737418240*a^5*d^2*e^17 + 256*a*e^5*(-(6
4*a*e^3 - d^4)^9)^(1/2)))/(125*d^36 + 1152921504606846976*a^9*e^27 - 28800
*a*d^32*e^3 + 1290240*a^2*d^28*e^6 + 163577856*a^3*d^24*e^9 - 15250489344*
a^4*d^20*e^12 - 96636764160*a^5*d^16*e^15 + 44324062494720*a^6*d^12*e^18 -
 791648371998720*a^7*d^8*e^21 - 4053239664633464*a^8*d^4*e^24))^(1/2)*(((
1536*(68719476736*a^5*e^24 + 20*d^20*e^9 - 7936*a*d^16*e^12 + 770048*a^2*d
^12*e^15 - 5242880*a^3*d^8*e^18 - 2147483648*a^4*d^4*e^21))/(25*d^20 - 171
79869184*a^5*e^15 - 2240*a*d^16*e^3 - 118784*a^2*d^12*e^6 + 12320768*a^3*d
^8*e^9 + 134217728*a^4*d^4*e^12) - ((1536*(25*d^27*e^8 - 3840*a*d^23*e^11
+ 24576*a^2*d^19*e^14 + 19922944*a^3*d^15*e^17 - 654311424*a^4*d^11*e^20 -
 25769803776*a^5*d^7*e^23 + 1099511627776*a^6*d^3*e^26))/(25*d^20 - 171798
69184*a^5*e^15 - 2240*a*d^16*e^3 - 118784*a^2*d^12*e^6 + 12320768*a^3*d^8*
e^9 + 134217728*a^4*d^4*e^12) + (6144*x*(25*d^22*e^9 - 2240*a*d^18*e^12 -
118784*a^2*d^14*e^15 + 12320768*a^3*d^10*e^18 + 134217728*a^4*d^6*e^21 - 1
7179869184*a^5*d^2*e^24))/(25*d^16 + 268435456*a^4*e^12 - 640*a*d^12*e^...
```

Reduce [F]

$$\int \frac{1}{(8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4)^2}\,dx$$
$$= \int \frac{1}{64e^6x^8 + 128d\,e^5x^7 + 64d^2e^4x^6 - 16d^3e^3x^5 + 128a\,e^5x^4 - 16d^4e^2x^4 + 128ad\,e^4x^3 + d^6x^2 - 16a\,d^3e^2x}$$

input
```
int(1/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^2,x)
```

output
```
int(1/(64*a**2*e**4 - 16*a*d**3*e**2*x + 128*a*d*e**4*x**3 + 128*a*e**5*x*
*4 + d**6*x**2 - 16*d**4*e**2*x**4 - 16*d**3*e**3*x**5 + 64*d**2*e**4*x**6
 + 128*d*e**5*x**7 + 64*e**6*x**8),x)
```

3.43 $\int \left(a+8x-8x^2+4x^3-x^4\right)^4 dx$

Optimal result

Integrand size = 22, antiderivative size = 139

$$\begin{aligned}\int \left(a+8x-8x^2+4x^3-x^4\right)^4 dx &= \frac{8}{3}(3+a)^3(1-x)^3-\frac{4}{5}(3-a)(3+a)^2(1-x)^5\\ &\quad -\frac{8}{7}(3+a)(5+3a)(1-x)^7+\frac{2}{9}\left(37+6a-3a^2\right)(1-x)^9\\ &\quad +\frac{8}{11}(5+3a)(1-x)^{11}-\frac{4}{13}(3-a)(1-x)^{13}\\ &\quad -\frac{8}{15}(1-x)^{15}-\frac{1}{17}(1-x)^{17}+(3+a)^4x\end{aligned}$$

output
```
8/3*(3+a)^3*(1-x)^3-4/5*(3-a)*(3+a)^2*(1-x)^5-8/7*(3+a)*(5+3*a)*(1-x)^7+2/
9*(-3*a^2+6*a+37)*(1-x)^9+8/11*(5+3*a)*(1-x)^11-4/13*(3-a)*(1-x)^13-8/15*(
1-x)^15-1/17*(1-x)^17+(3+a)^4*x
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.40

$$\int \left(a+8x-8x^2+4x^3-x^4\right)^4 dx = a^4x+16a^3x^2-\frac{32}{3}(-12+a)a^2x^3+4a\left(128-48a+a^2\right)x^4 - \frac{4}{5}\left(-1024+1536a-192a^2+a^3\right)x^5 - \frac{16}{3}\left(512-288a+15a^2\right)x^6 + \frac{64}{7}\left(512-140a+3a^2\right)x^7-6\left(896-128a+a^2\right)x^8 + \frac{2}{9}\left(20480-1536a+3a^2\right)x^9+\frac{16}{5}(-928+35a)x^{10} - \frac{32}{11}(-524+9a)x^{11}+\frac{4}{3}(-464+3a)x^{12} - \frac{4}{13}(-640+a)x^{13}-48x^{14}+\frac{128x^{15}}{15}-x^{16}+\frac{x^{17}}{17}$$

input
```
Integrate[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^4,x]
```

output
```
a^4*x + 16*a^3*x^2 - (32*(-12 + a)*a^2*x^3)/3 + 4*a*(128 - 48*a + a^2)*x^4
 - (4*(-1024 + 1536*a - 192*a^2 + a^3)*x^5)/5 - (16*(512 - 288*a + 15*a^2)
*x^6)/3 + (64*(512 - 140*a + 3*a^2)*x^7)/7 - 6*(896 - 128*a + a^2)*x^8 + (
2*(20480 - 1536*a + 3*a^2)*x^9)/9 + (16*(-928 + 35*a)*x^10)/5 - (32*(-524
+ 9*a)*x^11)/11 + (4*(-464 + 3*a)*x^12)/3 - (4*(-640 + a)*x^13)/13 - 48*x^
14 + (128*x^15)/15 - x^16 + x^17/17
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.90, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2458, 1403, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a-x^4+4x^3-8x^2+8x\right)^4 dx$$

$$\Big\downarrow \text{2458}$$

$$\int \left(a-(x-1)^4-2(x-1)^2+3\right)^4 d(x-1)$$

$$\Big\downarrow \text{1403}$$

$$\int \left(108\left(1-\frac{1}{27}a\left(a^2+3a-9\right)\right)(x-1)^4+81\left(\frac{1}{81}a\left(a^3+12a^2+54a+108\right)+1\right)+12\left(1-\frac{a}{3}\right)(x-1)^{12}-40\right.$$

1)

$$\Big\downarrow \text{2009}$$

$$-\frac{2}{9}\left(-3a^2+6a+37\right)(x-1)^9+\frac{4}{13}(3-a)(x-1)^{13}-\frac{8}{11}(3a+5)(x-1)^{11}+\frac{8}{7}(a+3)(3a+5)(x-1)^7+\frac{4}{5}(3-a)(a+3)^2(x-1)^5-\frac{8}{3}(a+3)^3(x-1)^3+(a+3)^4(x-1)+\frac{1}{17}(x-1)^{17}+\frac{8}{15}(x-1)^{15}$$

input
```
Int[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^4,x]
```

output
```
(3 + a)^4*(-1 + x) - (8*(3 + a)^3*(-1 + x)^3)/3 + (4*(3 - a)*(3 + a)^2*(-1
 + x)^5)/5 + (8*(3 + a)*(5 + 3*a)*(-1 + x)^7)/7 - (2*(37 + 6*a - 3*a^2)*(-
1 + x)^9)/9 - (8*(5 + 3*a)*(-1 + x)^11)/11 + (4*(3 - a)*(-1 + x)^13)/13 +
(8*(-1 + x)^15)/15 + (-1 + x)^17/17
```

Defintions of rubi rules used

rule 1403
```
Int[((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int[ExpandInte
grand[(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a
*c, 0] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2458
```
Int[(Pn_)^(p_.), x_Symbol] :> With[{S = Coeff[Pn, x, Expon[Pn, x] - 1]/(Exp
on[Pn, x]*Coeff[Pn, x, Expon[Pn, x]])}, Subst[Int[ExpandToSum[Pn /. x -> x
- S, x]^p, x], x, x + S] /; BinomialQ[Pn /. x -> x - S, x] || (IntegerQ[Exp
on[Pn, x]/2] && TrinomialQ[Pn /. x -> x - S, x])] /; FreeQ[p, x] && PolyQ[P
n, x] && GtQ[Expon[Pn, x], 2] && NeQ[Coeff[Pn, x, Expon[Pn, x] - 1], 0]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.29

method	result
norman	$a^4x+16a^3x^2+\left(-\frac{32}{3}a^3+128a^2\right)x^3+\left(4a^3-192a^2+512a\right)x^4+\left(-\frac{4}{5}a^3+\frac{768}{5}a^2-\frac{6144}{5}a$
gosper	$-\frac{6144}{5}a\,x^5+128a^2x^3-\frac{1856}{3}x^{12}-48x^{14}-\frac{14848}{5}x^{10}+\frac{40960}{9}x^9-x^{16}+\frac{1}{17}x^{17}+\frac{128}{15}x^{15}+\frac{2560}{13}$
risch	$-\frac{6144}{5}a\,x^5+128a^2x^3-\frac{1856}{3}x^{12}-48x^{14}-\frac{14848}{5}x^{10}+\frac{40960}{9}x^9-x^{16}+\frac{1}{17}x^{17}+\frac{128}{15}x^{15}+\frac{2560}{13}$
parallelrisch	$-\frac{6144}{5}a\,x^5+128a^2x^3-\frac{1856}{3}x^{12}-48x^{14}-\frac{14848}{5}x^{10}+\frac{40960}{9}x^9-x^{16}+\frac{1}{17}x^{17}+\frac{128}{15}x^{15}+\frac{2560}{13}$
orering	$\frac{x(45045x^{16}-765765x^{15}+6534528x^{14}-235620x^{12}a-36756720x^{13}+3063060x^{11}a+150796800x^{12}-20049120x^{10}a-47375328}{}$
default	$\frac{x^{17}}{17}-x^{16}+\frac{128x^{15}}{15}-48x^{14}+\frac{(-4a+2560)x^{13}}{13}+\frac{(48a-7424)x^{12}}{12}+\frac{(-288a+16768)x^{11}}{11}+\frac{(1120a-29696)x^{10}}{10}$

input
```
int((-x^4+4*x^3-8*x^2+a+8*x)^4,x,method=_RETURNVERBOSE)
```

output
```
a^4*x+16*a^3*x^2+(-32/3*a^3+128*a^2)*x^3+(4*a^3-192*a^2+512*a)*x^4+(-4/5*a
^3+768/5*a^2-6144/5*a+4096/5)*x^5+(-80*a^2+1536*a-8192/3)*x^6+(192/7*a^2-1
280*a+32768/7)*x^7+(-6*a^2+768*a-5376)*x^8+(2/3*a^2-1024/3*a+40960/9)*x^9+
(112*a-14848/5)*x^10+(-288/11*a+16768/11)*x^11+(4*a-1856/3)*x^12+(-4/13*a+
2560/13)*x^13-48*x^14+128/15*x^15-x^16+1/17*x^17
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.29

$$\int \left(a+8x-8x^2+4x^3-x^4\right)^4\,dx = \frac{1}{17}\,x^{17}-x^{16}+\frac{128}{15}\,x^{15}-\frac{4}{13}\,(a-640)x^{13} -48\,x^{14}+\frac{4}{3}\,(3\,a-464)x^{12}-\frac{32}{11}\,(9\,a-524)x^{11} +\frac{16}{5}\,(35\,a-928)x^{10}+\frac{2}{9}\,\left(3\,a^2-1536\,a+20480\right)x^9 -6\,\left(a^2-128\,a+896\right)x^8 +\frac{64}{7}\,\left(3\,a^2-140\,a+512\right)x^7 -\frac{16}{3}\,\left(15\,a^2-288\,a+512\right)x^6 -\frac{4}{5}\,\left(a^3-192\,a^2+1536\,a-1024\right)x^5+a^4x +16\,a^3x^2+4\,\left(a^3-48\,a^2+128\,a\right)x^4 -\frac{32}{3}\,\left(a^3-12\,a^2\right)x^3$$

input
```
integrate((-x^4+4*x^3-8*x^2+a+8*x)^4,x, algorithm="fricas")
```

output
```
1/17*x^17 - x^16 + 128/15*x^15 - 4/13*(a - 640)*x^13 - 48*x^14 + 4/3*(3*a
- 464)*x^12 - 32/11*(9*a - 524)*x^11 + 16/5*(35*a - 928)*x^10 + 2/9*(3*a^2
 - 1536*a + 20480)*x^9 - 6*(a^2 - 128*a + 896)*x^8 + 64/7*(3*a^2 - 140*a +
 512)*x^7 - 16/3*(15*a^2 - 288*a + 512)*x^6 - 4/5*(a^3 - 192*a^2 + 1536*a
- 1024)*x^5 + a^4*x + 16*a^3*x^2 + 4*(a^3 - 48*a^2 + 128*a)*x^4 - 32/3*(a^
3 - 12*a^2)*x^3
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.43

$$\int (a+8x-8x^2+4x^3-x^4)^4\,dx = a^4x+16a^3x^2+\frac{x^{17}}{17}-x^{16}+\frac{128x^{15}}{15}-48x^{14} + x^{13}\cdot\left(\frac{2560}{13}-\frac{4a}{13}\right)+x^{12}\cdot\left(4a-\frac{1856}{3}\right)+x^{11} \cdot\left(\frac{16768}{11}-\frac{288a}{11}\right)+x^{10}\cdot\left(112a-\frac{14848}{5}\right)+x^9 \cdot\left(\frac{2a^2}{3}-\frac{1024a}{3}+\frac{40960}{9}\right)+x^8(-6a^2+768a-5376) + x^7\cdot\left(\frac{192a^2}{7}-1280a+\frac{32768}{7}\right) + x^6\left(-80a^2+1536a-\frac{8192}{3}\right) + x^5\left(-\frac{4a^3}{5}+\frac{768a^2}{5}-\frac{6144a}{5}+\frac{4096}{5}\right)+x^4 \cdot(4a^3-192a^2+512a)+x^3\left(-\frac{32a^3}{3}+128a^2\right)$$

input

```
integrate((-x**4+4*x**3-8*x**2+a+8*x)**4,x)
```

output

```
a**4*x + 16*a**3*x**2 + x**17/17 - x**16 + 128*x**15/15 - 48*x**14 + x**13
*(2560/13 - 4*a/13) + x**12*(4*a - 1856/3) + x**11*(16768/11 - 288*a/11) +
 x**10*(112*a - 14848/5) + x**9*(2*a**2/3 - 1024*a/3 + 40960/9) + x**8*(-6
*a**2 + 768*a - 5376) + x**7*(192*a**2/7 - 1280*a + 32768/7) + x**6*(-80*a
**2 + 1536*a - 8192/3) + x**5*(-4*a**3/5 + 768*a**2/5 - 6144*a/5 + 4096/5)
 + x**4*(4*a**3 - 192*a**2 + 512*a) + x**3*(-32*a**3/3 + 128*a**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.38

$$\int \left(a+8x-8x^2+4x^3-x^4\right)^4\,dx = \frac{1}{17}\,x^{17}-x^{16}+\frac{128}{15}\,x^{15}-48\,x^{14}+\frac{2560}{13}\,x^{13}-\frac{1856}{3}\,x^{12}+\frac{16768}{11}\,x^{11}-\frac{14848}{5}\,x^{10}+\frac{40960}{9}\,x^{9}-5376\,x^{8}+\frac{32768}{7}\,x^{7}-\frac{8192}{3}\,x^{6}+a^4x+\frac{4096}{5}\,x^{5}-\frac{4}{15}\,\left(3\,x^5-15\,x^4+40\,x^3-60\,x^2\right)a^3+\frac{2}{105}\,\left(35\,x^9-315\,x^8+1440\,x^7-4200\,x^6+8064\,x^5-10080\,x^4+6720\,x^3\right)a^2-\frac{4}{2145}\,\left(165\,x^{13}-2145\,x^{12}+14040\,x^{11}-60060\,x^{10}+183040\,x^{9}-411840\,x^{8}+686400\,x^{7}-823680\,x^{6}\right.$$

input
```
integrate((-x^4+4*x^3-8*x^2+a+8*x)^4,x, algorithm="maxima")
```

output
```
1/17*x^17 - x^16 + 128/15*x^15 - 48*x^14 + 2560/13*x^13 - 1856/3*x^12 + 16
768/11*x^11 - 14848/5*x^10 + 40960/9*x^9 - 5376*x^8 + 32768/7*x^7 - 8192/3
*x^6 + a^4*x + 4096/5*x^5 - 4/15*(3*x^5 - 15*x^4 + 40*x^3 - 60*x^2)*a^3 +
2/105*(35*x^9 - 315*x^8 + 1440*x^7 - 4200*x^6 + 8064*x^5 - 10080*x^4 + 672
0*x^3)*a^2 - 4/2145*(165*x^13 - 2145*x^12 + 14040*x^11 - 60060*x^10 + 1830
40*x^9 - 411840*x^8 + 686400*x^7 - 823680*x^6 + 658944*x^5 - 274560*x^4)*a
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 219 vs. $2(103) = 206$.

Time = 0.11 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.58

$$\int \left(a+8x-8x^2+4x^3-x^4\right)^4\,dx = \frac{1}{17}\,x^{17}-x^{16}+\frac{128}{15}\,x^{15}-\frac{4}{13}\,ax^{13}-48\,x^{14}+4\,ax^{12}+\frac{2560}{13}\,x^{13}-\frac{288}{11}\,ax^{11}-\frac{1856}{3}\,x^{12}+\frac{2}{3}\,a^2x^9+112\,ax^{10}+\frac{16768}{11}\,x^{11}-6\,a^2x^8-\frac{1024}{3}\,ax^9-\frac{14848}{5}\,x^{10}+\frac{192}{7}\,a^2x^7+768\,ax^8+\frac{40960}{9}\,x^9-\frac{4}{5}\,a^3x^5-80\,a^2x^6-1280\,ax^7-5376\,x^8+4\,a^3x^4+\frac{768}{5}\,a^2x^5+1536\,ax^6+\frac{32768}{7}\,x^7-\frac{32}{3}\,a^3x^3-192\,a^2x^4-\frac{6144}{5}\,ax^5-\frac{8192}{3}\,x^6+a^4x+16\,a^3x^2+128\,a^2x^3+512\,ax^4+\frac{4096}{5}\,x^5$$

input
```
integrate((-x^4+4*x^3-8*x^2+a+8*x)^4,x, algorithm="giac")
```

output
```
1/17*x^17 - x^16 + 128/15*x^15 - 4/13*a*x^13 - 48*x^14 + 4*a*x^12 + 2560/1
3*x^13 - 288/11*a*x^11 - 1856/3*x^12 + 2/3*a^2*x^9 + 112*a*x^10 + 16768/11
*x^11 - 6*a^2*x^8 - 1024/3*a*x^9 - 14848/5*x^10 + 192/7*a^2*x^7 + 768*a*x^
8 + 40960/9*x^9 - 4/5*a^3*x^5 - 80*a^2*x^6 - 1280*a*x^7 - 5376*x^8 + 4*a^3
*x^4 + 768/5*a^2*x^5 + 1536*a*x^6 + 32768/7*x^7 - 32/3*a^3*x^3 - 192*a^2*x
^4 - 6144/5*a*x^5 - 8192/3*x^6 + a^4*x + 16*a^3*x^2 + 128*a^2*x^3 + 512*a*
x^4 + 4096/5*x^5
```

Mupad [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.26

$$\int (a+8x-8x^2+4x^3-x^4)^4 \, dx = x^{12}\left(4\,a-\frac{1856}{3}\right)-x^{13}\left(\frac{4\,a}{13}-\frac{2560}{13}\right)+x^{10}\left(112\,a-\frac{14848}{5}\right)-x^{11}\left(\frac{288\,a}{11}-\frac{16768}{11}\right)-x^8\left(6\,a^2-768\,a+5376\right)-x^6\left(80\,a^2-1536\,a+\frac{8192}{3}\right)+x^7\left(\frac{192\,a^2}{7}-1280\,a+\frac{32768}{7}\right)+x^9\left(\frac{2\,a^2}{3}-\frac{1024\,a}{3}+\frac{40960}{9}\right)-x^5\left(\frac{4\,a^3}{5}-\frac{768\,a^2}{5}+\frac{6144\,a}{5}-\frac{4096}{5}\right)+a^4\,x-48\,x^{14}+\frac{128\,x^{15}}{15}-x^{16}+\frac{x^{17}}{17}+16\,a^3\,x^2+4\,a\,x^4\left(a^2-48\,a+128\right)-\frac{32\,a^2\,x^3\,(a-12)}{3}$$

input
```
int((a + 8*x - 8*x^2 + 4*x^3 - x^4)^4,x)
```

output
```
x^12*(4*a - 1856/3) - x^13*((4*a)/13 - 2560/13) + x^10*(112*a - 14848/5) -
 x^11*((288*a)/11 - 16768/11) - x^8*(6*a^2 - 768*a + 5376) - x^6*(80*a^2 -
 1536*a + 8192/3) + x^7*((192*a^2)/7 - 1280*a + 32768/7) + x^9*((2*a^2)/3
- (1024*a)/3 + 40960/9) - x^5*((6144*a)/5 - (768*a^2)/5 + (4*a^3)/5 - 4096
/5) + a^4*x - 48*x^14 + (128*x^15)/15 - x^16 + x^17/17 + 16*a^3*x^2 + 4*a*
x^4*(a^2 - 48*a + 128) - (32*a^2*x^3*(a - 12))/3
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.58

$$\int \left(a + 8x - 8x^2 + 4x^3 - x^4\right)^4 \, dx$$

$$= \frac{x(45045x^{16} - 765765x^{15} + 6534528x^{14} - 235620a\,x^{12} - 36756720x^{13} + 3063060a\,x^{11} + 150796800x^{12} - }{}$$

input

```
int((-x^4+4*x^3-8*x^2+a+8*x)^4,x)
```

output

```
(x*(765765*a**4 - 612612*a**3*x**4 + 3063060*a**3*x**3 - 8168160*a**3*x**2
 + 12252240*a**3*x + 510510*a**2*x**8 - 4594590*a**2*x**7 + 21003840*a**2*
x**6 - 61261200*a**2*x**5 + 117621504*a**2*x**4 - 147026880*a**2*x**3 + 98
017920*a**2*x**2 - 235620*a*x**12 + 3063060*a*x**11 - 20049120*a*x**10 + 8
5765680*a*x**9 - 261381120*a*x**8 + 588107520*a*x**7 - 980179200*a*x**6 +
1176215040*a*x**5 - 940972032*a*x**4 + 392071680*a*x**3 + 45045*x**16 - 76
5765*x**15 + 6534528*x**14 - 36756720*x**13 + 150796800*x**12 - 473753280*
x**11 + 1167304320*x**10 - 2274015744*x**9 + 3485081600*x**8 - 4116752640*
x**7 + 3584655360*x**6 - 2091048960*x**5 + 627314688*x**4))/765765
```

3.44 $\int (a+8x-8x^2+4x^3-x^4)^3 \, dx$

Optimal result

Integrand size = 22, antiderivative size = 95

$$\int (a+8x-8x^2+4x^3-x^4)^3 \, dx = 2(3+a)^2(1-x)^3-\frac{3}{5}(1-a)(3+a)(1-x)^5 -\frac{4}{7}(7+3a)(1-x)^7+\frac{1}{3}(1-a)(1-x)^9 +\frac{6}{11}(1-x)^{11}+\frac{1}{13}(1-x)^{13}+(3+a)^3x$$

output
```
2*(3+a)^2*(1-x)^3-3/5*(1-a)*(3+a)*(1-x)^5-4/7*(7+3*a)*(1-x)^7+1/3*(1-a)*(1
-x)^9+6/11*(1-x)^11+1/13*(1-x)^13+(3+a)^3*x
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.20

$$\int (a+8x-8x^2+4x^3-x^4)^3 \, dx = a^3x+12a^2x^2-8(-8+a)ax^3+\left(128-96a+3a^2\right)x^4 -\frac{3}{5}\left(512-128a+a^2\right)x^5-8(-48+5a)x^6 +\frac{32}{7}(-70+3a)x^7-3(-64+a)x^8 +\frac{1}{3}(-256+a)x^9+28x^{10}-\frac{72x^{11}}{11}+x^{12}-\frac{x^{13}}{13}$$

input
```
Integrate[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^3,x]
```

output
```
a^3*x + 12*a^2*x^2 - 8*(-8 + a)*a*x^3 + (128 - 96*a + 3*a^2)*x^4 - (3*(512
 - 128*a + a^2)*x^5)/5 - 8*(-48 + 5*a)*x^6 + (32*(-70 + 3*a)*x^7)/7 - 3*(-
64 + a)*x^8 + ((-256 + a)*x^9)/3 + 28*x^10 - (72*x^11)/11 + x^12 - x^13/13
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.89, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {2458, 1403, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a-x^4+4 x^3-8 x^2+8 x\right)^3 dx$$

$$\Big\downarrow \text{ 2458}$$

$$\int \left(a-(x-1)^4-2 (x-1)^2+3\right)^3 d(x-1)$$

$$\Big\downarrow \text{ 1403}$$

$$\int \left(-3 (1-a) (x-1)^8+28 \left(\frac{3 a}{7}+1\right) (x-1)^6+9 \left(1-\frac{1}{3} a (a+2)\right) (x-1)^4-54 \left(\frac{1}{9} a (a+6)+1\right) (x-1)^2+\right.$$
$$1)$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{1}{3} (1-a) (x-1)^9+\frac{4}{7} (3 a+7) (x-1)^7+\frac{3}{5} (1-a) (a+3) (x-1)^5-2 (a+3)^2 (x-1)^3+(a+3)^3 (x-1)-\frac{1}{13} (x-1)^{13}-\frac{6}{11} (x-1)^{11}$$

input
```
Int[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^3,x]
```

output
```
(3 + a)^3*(-1 + x) - 2*(3 + a)^2*(-1 + x)^3 + (3*(1 - a)*(3 + a)*(-1 + x)^
5)/5 + (4*(7 + 3*a)*(-1 + x)^7)/7 - ((1 - a)*(-1 + x)^9)/3 - (6*(-1 + x)^1
1)/11 - (-1 + x)^13/13
```

Defintions of rubi rules used

rule 1403
```
Int[((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int[ExpandInte
grand[(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a
*c, 0] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2458
```
Int[(Pn_)^(p_.), x_Symbol] :> With[{S = Coeff[Pn, x, Expon[Pn, x] - 1]/(Exp
on[Pn, x]*Coeff[Pn, x, Expon[Pn, x]])}, Subst[Int[ExpandToSum[Pn /. x -> x
- S, x]^p, x], x, x + S] /; BinomialQ[Pn /. x -> x - S, x] || (IntegerQ[Exp
on[Pn, x]/2] && TrinomialQ[Pn /. x -> x - S, x])] /; FreeQ[p, x] && PolyQ[P
n, x] && GtQ[Expon[Pn, x], 2] && NeQ[Coeff[Pn, x, Expon[Pn, x] - 1], 0]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.16

method	result
norman	$-\frac{x^{13}}{13}+x^{12}-\frac{72x^{11}}{11}+28x^{10}+\left(\frac{a}{3}-\frac{256}{3}\right)x^9+\left(-3a+192\right)x^8+\left(\frac{96a}{7}-320\right)x^7+\left(-40a+\right.$
gosper	$-\frac{1}{13}x^{13}+x^{12}-\frac{72}{11}x^{11}+28x^{10}+\frac{1}{3}x^9a-\frac{256}{3}x^9-3a\,x^8+192x^8+\frac{96}{7}x^7a-320x^7-40x^6a$
risch	$-\frac{1}{13}x^{13}+x^{12}-\frac{72}{11}x^{11}+28x^{10}+\frac{1}{3}x^9a-\frac{256}{3}x^9-3a\,x^8+192x^8+\frac{96}{7}x^7a-320x^7-40x^6a$
parallelrisch	$-\frac{1}{13}x^{13}+x^{12}-\frac{72}{11}x^{11}+28x^{10}+\frac{1}{3}x^9a-\frac{256}{3}x^9-3a\,x^8+192x^8+\frac{96}{7}x^7a-320x^7-40x^6a$
orering	$\frac{x\left(-1155x^{12}+15015x^{11}-98280x^{10}+5005a\,x^8+420420x^9-45045x^7a-1281280x^8+205920x^6a+2882880x^7-9009a^2x^4-6006\right.}{}$
default	$-\frac{x^{13}}{13}+x^{12}-\frac{72x^{11}}{11}+28x^{10}+\frac{(3a-768)x^9}{9}+\frac{(-24a+1536)x^8}{8}+\frac{(96a-2240)x^7}{7}+\frac{(-240a+2304)x^6}{6}+\frac{(a(}{}$

input
```
int((-x^4+4*x^3-8*x^2+a+8*x)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/13*x^13+x^12-72/11*x^11+28*x^10+(1/3*a-256/3)*x^9+(-3*a+192)*x^8+(96/7*
a-320)*x^7+(-40*a+384)*x^6+(-3/5*a^2+384/5*a-1536/5)*x^5+(3*a^2-96*a+128)*
x^4+(-8*a^2+64*a)*x^3+12*a^2*x^2+a^3*x
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.13

$$\int \left(a+8x-8x^2+4x^3-x^4\right)^3 \, dx = -\frac{1}{13}\,x^{13}+x^{12}-\frac{72}{11}\,x^{11}+\frac{1}{3}\,(a-256)x^9+28\,x^{10} - 3\,(a-64)x^8+\frac{32}{7}\,(3\,a-70)x^7-8\,(5\,a-48)x^6 - \frac{3}{5}\,(a^2-128\,a+512)x^5+\left(3\,a^2-96\,a+128\right)x^4 + a^3x+12\,a^2x^2-8\,\left(a^2-8\,a\right)x^3$$

input
```
integrate((-x^4+4*x^3-8*x^2+a+8*x)^3,x, algorithm="fricas")
```

output
```
-1/13*x^13 + x^12 - 72/11*x^11 + 1/3*(a - 256)*x^9 + 28*x^10 - 3*(a - 64)*
x^8 + 32/7*(3*a - 70)*x^7 - 8*(5*a - 48)*x^6 - 3/5*(a^2 - 128*a + 512)*x^5
 + (3*a^2 - 96*a + 128)*x^4 + a^3*x + 12*a^2*x^2 - 8*(a^2 - 8*a)*x^3
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.20

$$\int \left(a+8x-8x^2+4x^3-x^4\right)^3 \, dx = a^3x+12a^2x^2-\frac{x^{13}}{13}+x^{12}-\frac{72x^{11}}{11}+28x^{10} + x^9\left(\frac{a}{3}-\frac{256}{3}\right)+x^8\cdot(192-3a)+x^7\cdot\left(\frac{96a}{7}-320\right) + x^6\cdot(384-40a)+x^5\left(-\frac{3a^2}{5}+\frac{384a}{5}-\frac{1536}{5}\right) + x^4\cdot\left(3a^2-96a+128\right)+x^3\left(-8a^2+64a\right)$$

input
```
integrate((-x**4+4*x**3-8*x**2+a+8*x)**3,x)
```

output
```
a**3*x + 12*a**2*x**2 - x**13/13 + x**12 - 72*x**11/11 + 28*x**10 + x**9*(
a/3 - 256/3) + x**8*(192 - 3*a) + x**7*(96*a/7 - 320) + x**6*(384 - 40*a)
+ x**5*(-3*a**2/5 + 384*a/5 - 1536/5) + x**4*(3*a**2 - 96*a + 128) + x**3*
(-8*a**2 + 64*a)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.25

$$\int \left(a+8x-8x^2+4x^3-x^4\right)^3\,dx = -\frac{1}{13}\,x^{13}+x^{12}-\frac{72}{11}\,x^{11}+28\,x^{10}-\frac{256}{3}\,x^9+192\,x^8-320\,x^7+384\,x^6-\frac{1536}{5}\,x^5+a^3x+128\,x^4-\frac{1}{5}\left(3\,x^5-15\,x^4+40\,x^3-60\,x^2\right)a^2+\frac{1}{105}\left(35\,x^9-315\,x^8+1440\,x^7-4200\,x^6+8064\,x^5-10080\,x^4+6720\,x^3\right)a$$

input
```
integrate((-x^4+4*x^3-8*x^2+a+8*x)^3,x, algorithm="maxima")
```

output
```
-1/13*x^13 + x^12 - 72/11*x^11 + 28*x^10 - 256/3*x^9 + 192*x^8 - 320*x^7 +
 384*x^6 - 1536/5*x^5 + a^3*x + 128*x^4 - 1/5*(3*x^5 - 15*x^4 + 40*x^3 - 6
0*x^2)*a^2 + 1/105*(35*x^9 - 315*x^8 + 1440*x^7 - 4200*x^6 + 8064*x^5 - 10
080*x^4 + 6720*x^3)*a
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.35

$$\int \left(a+8x-8x^2+4x^3-x^4\right)^3\,dx = -\frac{1}{13}\,x^{13}+x^{12}-\frac{72}{11}\,x^{11}+\frac{1}{3}\,ax^9+28\,x^{10}-3\,ax^8-\frac{256}{3}\,x^9+\frac{96}{7}\,ax^7+192\,x^8-\frac{3}{5}\,a^2x^5-40\,ax^6-320\,x^7+3\,a^2x^4+\frac{384}{5}\,ax^5+384\,x^6-8\,a^2x^3-96\,ax^4-\frac{1536}{5}\,x^5+a^3x+12\,a^2x^2+64\,ax^3+128\,x^4$$

input
```
integrate((-x^4+4*x^3-8*x^2+a+8*x)^3,x, algorithm="giac")
```

output
```
-1/13*x^13 + x^12 - 72/11*x^11 + 1/3*a*x^9 + 28*x^10 - 3*a*x^8 - 256/3*x^9
 + 96/7*a*x^7 + 192*x^8 - 3/5*a^2*x^5 - 40*a*x^6 - 320*x^7 + 3*a^2*x^4 + 3
84/5*a*x^5 + 384*x^6 - 8*a^2*x^3 - 96*a*x^4 - 1536/5*x^5 + a^3*x + 12*a^2*
x^2 + 64*a*x^3 + 128*x^4
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.14

$$\int (a+8x-8x^2+4x^3-x^4)^3 \, dx = x^9 \left(\frac{a}{3}-\frac{256}{3}\right) - x^8 (3a-192) - x^6 (40a-384) + x^7 \left(\frac{96a}{7}-320\right) + x^4 (3a^2-96a+128) - x^5 \left(\frac{3a^2}{5}-\frac{384a}{5}+\frac{1536}{5}\right) + a^3 x + 28x^{10} - \frac{72x^{11}}{11} + x^{12} - \frac{x^{13}}{13} + 12a^2x^2 - 8ax^3(a-8)$$

input
```
int((a + 8*x - 8*x^2 + 4*x^3 - x^4)^3,x)
```

output
```
x^9*(a/3 - 256/3) - x^8*(3*a - 192) - x^6*(40*a - 384) + x^7*((96*a)/7 - 3
20) + x^4*(3*a^2 - 96*a + 128) - x^5*((3*a^2)/5 - (384*a)/5 + 1536/5) + a^
3*x + 28*x^10 - (72*x^11)/11 + x^12 - x^13/13 + 12*a^2*x^2 - 8*a*x^3*(a -
8)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.38

$$\int (a+8x-8x^2+4x^3-x^4)^3 \, dx = \frac{x(-1155x^{12}+15015x^{11}-98280x^{10}+5005a\,x^8+420420x^9-45045a\,x^7-1281280x^8+205920a\,x^6+2$$

input
```
int((-x^4+4*x^3-8*x^2+a+8*x)^3,x)
```

output
```
(x*(15015*a**3 - 9009*a**2*x**4 + 45045*a**2*x**3 - 120120*a**2*x**2 + 180
180*a**2*x + 5005*a*x**8 - 45045*a*x**7 + 205920*a*x**6 - 600600*a*x**5 +
1153152*a*x**4 - 1441440*a*x**3 + 960960*a*x**2 - 1155*x**12 + 15015*x**11
 - 98280*x**10 + 420420*x**9 - 1281280*x**8 + 2882880*x**7 - 4804800*x**6
+ 5765760*x**5 - 4612608*x**4 + 1921920*x**3))/15015
```

3.45 $\int (a+8x-8x^2+4x^3-x^4)^2 \, dx$

Optimal result

Integrand size = 22, antiderivative size = 58

$$\int (a+8x-8x^2+4x^3-x^4)^2 \, dx = \frac{4}{3}(3+a)(1-x)^3+\frac{2}{5}(1+a)(1-x)^5 - \frac{4}{7}(1-x)^7-\frac{1}{9}(1-x)^9+(3+a)^2x$$

output
```
4/3*(3+a)*(1-x)^3+2/5*(1+a)*(1-x)^5-4/7*(1-x)^7-1/9*(1-x)^9+(3+a)^2*x
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.14

$$\int (a+8x-8x^2+4x^3-x^4)^2 \, dx = a^2x+8ax^2-\frac{16}{3}(-4+a)x^3+2(-16+a)x^4 - \frac{2}{5}(-64+a)x^5-\frac{40x^6}{3}+\frac{32x^7}{7}-x^8+\frac{x^9}{9}$$

input
```
Integrate[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x]
```

output
```
a^2*x + 8*a*x^2 - (16*(-4 + a)*x^3)/3 + 2*(-16 + a)*x^4 - (2*(-64 + a)*x^5
)/5 - (40*x^6)/3 + (32*x^7)/7 - x^8 + x^9/9
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.90, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2458, 1403, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a-x^4+4x^3-8x^2+8x\right)^2 dx$$

$$\Big\downarrow \text{2458}$$

$$\int \left(a-(x-1)^4-2(x-1)^2+3\right)^2 d(x-1)$$

$$\Big\downarrow \text{1403}$$

$$\int \left(-2(a+1)(x-1)^4-12\left(\frac{a}{3}+1\right)(x-1)^2+9\left(\frac{1}{9}a(a+6)+1\right)+(x-1)^8+4(x-1)^6\right)d(x-1)$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2}{5}(a+1)(x-1)^5-\frac{4}{3}(a+3)(x-1)^3+(a+3)^2(x-1)+\frac{1}{9}(x-1)^9+\frac{4}{7}(x-1)^7$$

input
```
Int[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x]
```

output
```
(3 + a)^2*(-1 + x) - (4*(3 + a)*(-1 + x)^3)/3 - (2*(1 + a)*(-1 + x)^5)/5 +
 (4*(-1 + x)^7)/7 + (-1 + x)^9/9
```

Defintions of rubi rules used

rule 1403
```
Int[((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int[ExpandInte
grand[(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a
*c, 0] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2458
```
Int[(Pn_)^(p_.), x_Symbol] :> With[{S = Coeff[Pn, x, Expon[Pn, x] - 1]/(Exp
on[Pn, x]*Coeff[Pn, x, Expon[Pn, x]])}, Subst[Int[ExpandToSum[Pn /. x -> x
- S, x]^p, x], x, x + S] /; BinomialQ[Pn /. x -> x - S, x] || (IntegerQ[Exp
on[Pn, x]/2] && TrinomialQ[Pn /. x -> x - S, x])] /; FreeQ[p, x] && PolyQ[P
n, x] && GtQ[Expon[Pn, x], 2] && NeQ[Coeff[Pn, x, Expon[Pn, x] - 1], 0]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.03

method	result
norman	$\frac{x^9}{9}-x^8+\frac{32x^7}{7}-\frac{40x^6}{3}+\left(-\frac{2a}{5}+\frac{128}{5}\right)x^5+\left(2a-32\right)x^4+\left(-\frac{16a}{3}+\frac{64}{3}\right)x^3+8a\,x^2+x\,a^2$
default	$\frac{x^9}{9}-x^8+\frac{32x^7}{7}-\frac{40x^6}{3}+\frac{(-2a+128)x^5}{5}+\frac{(8a-128)x^4}{4}+\frac{(-16a+64)x^3}{3}+8a\,x^2+x\,a^2$
gosper	$\frac{1}{9}x^9-x^8+\frac{32}{7}x^7-\frac{40}{3}x^6-\frac{2}{5}a\,x^5+\frac{128}{5}x^5+2a\,x^4-32x^4-\frac{16}{3}a\,x^3+\frac{64}{3}x^3+8a\,x^2+x\,a^2$
risch	$\frac{1}{9}x^9-x^8+\frac{32}{7}x^7-\frac{40}{3}x^6-\frac{2}{5}a\,x^5+\frac{128}{5}x^5+2a\,x^4-32x^4-\frac{16}{3}a\,x^3+\frac{64}{3}x^3+8a\,x^2+x\,a^2$
parallelrisch	$\frac{1}{9}x^9-x^8+\frac{32}{7}x^7-\frac{40}{3}x^6-\frac{2}{5}a\,x^5+\frac{128}{5}x^5+2a\,x^4-32x^4-\frac{16}{3}a\,x^3+\frac{64}{3}x^3+8a\,x^2+x\,a^2$
orering	$\frac{x\left(35x^8-315x^7+1440x^6-126a\,x^4-4200x^5+630a\,x^3+8064x^4-1680a\,x^2-10080x^3+315a^2+2520xa+6720x^2\right)}{315}$

input
```
int((-x^4+4*x^3-8*x^2+a+8*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/9*x^9-x^8+32/7*x^7-40/3*x^6+(-2/5*a+128/5)*x^5+(2*a-32)*x^4+(-16/3*a+64/
3)*x^3+8*a*x^2+x*a^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.97

$$\int \left(a+8x-8x^2+4x^3-x^4\right)^2\,dx=\frac{1}{9}\,x^9-x^8+\frac{32}{7}\,x^7-\frac{2}{5}\,(a-64)x^5-\frac{40}{3}\,x^6+2\,(a-16)x^4-\frac{16}{3}\,(a-4)x^3+a^2x+8\,ax^2$$

input
```
integrate((-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="fricas")
```

output
```
1/9*x^9 - x^8 + 32/7*x^7 - 2/5*(a - 64)*x^5 - 40/3*x^6 + 2*(a - 16)*x^4 -
16/3*(a - 4)*x^3 + a^2*x + 8*a*x^2
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.12

$$\int \left(a+8x-8x^2+4x^3-x^4\right)^2\,dx = a^2x+8ax^2+\frac{x^9}{9}-x^8+\frac{32x^7}{7}-\frac{40x^6}{3}+x^5\cdot\left(\frac{128}{5}-\frac{2a}{5}\right)+x^4\cdot(2a-32)+x^3\cdot\left(\frac{64}{3}-\frac{16a}{3}\right)$$

input
```
integrate((-x**4+4*x**3-8*x**2+a+8*x)**2,x)
```

output
```
a**2*x + 8*a*x**2 + x**9/9 - x**8 + 32*x**7/7 - 40*x**6/3 + x**5*(128/5 -
2*a/5) + x**4*(2*a - 32) + x**3*(64/3 - 16*a/3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.12

$$\int \left(a+8x-8x^2+4x^3-x^4\right)^2\,dx = \frac{1}{9}x^9-x^8+\frac{32}{7}x^7-\frac{40}{3}x^6+\frac{128}{5}x^5-32x^4+a^2x+\frac{64}{3}x^3-\frac{2}{15}\left(3x^5-15x^4+40x^3-60x^2\right)a$$

input
```
integrate((-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="maxima")
```

output
```
1/9*x^9 - x^8 + 32/7*x^7 - 40/3*x^6 + 128/5*x^5 - 32*x^4 + a^2*x + 64/3*x^
3 - 2/15*(3*x^5 - 15*x^4 + 40*x^3 - 60*x^2)*a
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.12

$$\int \left(a+8x-8x^2+4x^3-x^4\right)^2\,dx=\frac{1}{9}\,x^9-x^8+\frac{32}{7}\,x^7-\frac{2}{5}\,ax^5-\frac{40}{3}\,x^6+2\,ax^4+\frac{128}{5}\,x^5-\frac{16}{3}\,ax^3-32\,x^4+a^2x+8\,ax^2+\frac{64}{3}\,x^3$$

input
```
integrate((-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="giac")
```

output
```
1/9*x^9 - x^8 + 32/7*x^7 - 2/5*a*x^5 - 40/3*x^6 + 2*a*x^4 + 128/5*x^5 - 16
/3*a*x^3 - 32*x^4 + a^2*x + 8*a*x^2 + 64/3*x^3
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.05

$$\int \left(a+8x-8x^2+4x^3-x^4\right)^2\,dx=x^4\,(2\,a-32)-x^3\left(\frac{16\,a}{3}-\frac{64}{3}\right)-x^5\left(\frac{2\,a}{5}-\frac{128}{5}\right)+8\,a\,x^2+a^2\,x-\frac{40\,x^6}{3}+\frac{32\,x^7}{7}-x^8+\frac{x^9}{9}$$

input
```
int((a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x)
```

output
```
x^4*(2*a - 32) - x^3*((16*a)/3 - 64/3) - x^5*((2*a)/5 - 128/5) + 8*a*x^2 +
 a^2*x - (40*x^6)/3 + (32*x^7)/7 - x^8 + x^9/9
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.14

$$\int \left(a + 8x - 8x^2 + 4x^3 - x^4\right)^2 \, dx$$
$$= \frac{x(35x^8 - 315x^7 + 1440x^6 - 126a\,x^4 - 4200x^5 + 630a\,x^3 + 8064x^4 - 1680a\,x^2 - 10080x^3 + 315a^2 + 252}{315}$$

input
```
int((-x^4+4*x^3-8*x^2+a+8*x)^2,x)
```

output
```
(x*(315*a**2 - 126*a*x**4 + 630*a*x**3 - 1680*a*x**2 + 2520*a*x + 35*x**8
- 315*x**7 + 1440*x**6 - 4200*x**5 + 8064*x**4 - 10080*x**3 + 6720*x**2))/
315
```

3.46 $\int (a+8x-8x^2+4x^3-x^4)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 26

$$\int (a+8x-8x^2+4x^3-x^4)\,dx = ax+4x^2-\frac{8x^3}{3}+x^4-\frac{x^5}{5}$$

output
```
a*x+4*x^2-8/3*x^3+x^4-1/5*x^5
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int (a+8x-8x^2+4x^3-x^4)\,dx = ax+4x^2-\frac{8x^3}{3}+x^4-\frac{x^5}{5}$$

input
```
Integrate[a + 8*x - 8*x^2 + 4*x^3 - x^4,x]
```

output
```
a*x + 4*x^2 - (8*x^3)/3 + x^4 - x^5/5
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a - x^4 + 4x^3 - 8x^2 + 8x\right) \, dx$$

$$\Big\downarrow \ 2009$$

$$ax - \frac{x^5}{5} + x^4 - \frac{8x^3}{3} + 4x^2$$

input
```
Int[a + 8*x - 8*x^2 + 4*x^3 - x^4,x]
```

output
```
a*x + 4*x^2 - (8*x^3)/3 + x^4 - x^5/5
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.88

method	result	size
gosper	$xa + 4x^2 - \frac{8}{3}x^3 + x^4 - \frac{1}{5}x^5$	23
default	$xa + 4x^2 - \frac{8}{3}x^3 + x^4 - \frac{1}{5}x^5$	23
norman	$xa + 4x^2 - \frac{8}{3}x^3 + x^4 - \frac{1}{5}x^5$	23
risch	$xa + 4x^2 - \frac{8}{3}x^3 + x^4 - \frac{1}{5}x^5$	23
parallelrisch	$xa + 4x^2 - \frac{8}{3}x^3 + x^4 - \frac{1}{5}x^5$	23
parts	$xa + 4x^2 - \frac{8}{3}x^3 + x^4 - \frac{1}{5}x^5$	23
orering	$\frac{x(-3x^4+15x^3-40x^2+15a+60x)}{15}$	26

input
```
int(-x^4+4*x^3-8*x^2+a+8*x,x,method=_RETURNVERBOSE)
```

output
```
x*a+4*x^2-8/3*x^3+x^4-1/5*x^5
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \left(a + 8x - 8x^2 + 4x^3 - x^4\right) dx = -\frac{1}{5}\,x^5 + x^4 - \frac{8}{3}\,x^3 + ax + 4\,x^2$$

input
```
integrate(-x^4+4*x^3-8*x^2+a+8*x,x, algorithm="fricas")
```

output
```
-1/5*x^5 + x^4 - 8/3*x^3 + a*x + 4*x^2
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \left(a+8x-8x^2+4x^3-x^4\right)\,dx = ax-\frac{x^5}{5}+x^4-\frac{8x^3}{3}+4x^2$$

input
```
integrate(-x**4+4*x**3-8*x**2+a+8*x,x)
```

output
```
a*x - x**5/5 + x**4 - 8*x**3/3 + 4*x**2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \left(a+8x-8x^2+4x^3-x^4\right)\,dx = -\frac{1}{5}\,x^5+x^4-\frac{8}{3}\,x^3+ax+4\,x^2$$

input
```
integrate(-x^4+4*x^3-8*x^2+a+8*x,x, algorithm="maxima")
```

output
```
-1/5*x^5 + x^4 - 8/3*x^3 + a*x + 4*x^2
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \left(a+8x-8x^2+4x^3-x^4\right)\,dx = -\frac{1}{5}\,x^5+x^4-\frac{8}{3}\,x^3+ax+4\,x^2$$

input
```
integrate(-x^4+4*x^3-8*x^2+a+8*x,x, algorithm="giac")
```

output
```
-1/5*x^5 + x^4 - 8/3*x^3 + a*x + 4*x^2
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \left(a+8x-8x^2+4x^3-x^4\right) \, dx = -\frac{x^5}{5}+x^4-\frac{8\,x^3}{3}+4\,x^2+a\,x$$

input
```
int(a + 8*x - 8*x^2 + 4*x^3 - x^4,x)
```

output
```
a*x + 4*x^2 - (8*x^3)/3 + x^4 - x^5/5
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.96

$$\int \left(a+8x-8x^2+4x^3-x^4\right) \, dx = \frac{x(-3x^4+15x^3-40x^2+15a+60x)}{15}$$

input
```
int(-x^4+4*x^3-8*x^2+a+8*x,x)
```

output
```
(x*(15*a - 3*x**4 + 15*x**3 - 40*x**2 + 60*x))/15
```

3.47 $\int \frac{1}{a+8x-8x^2+4x^3-x^4} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 93

$$\int \frac{1}{a+8x-8x^2+4x^3-x^4} \, dx = \frac{\arctan\left(\frac{1-x}{\sqrt{1-\sqrt{4+a}}}\right)}{2\sqrt{4+a}\sqrt{1-\sqrt{4+a}}} - \frac{\arctan\left(\frac{1-x}{\sqrt{1+\sqrt{4+a}}}\right)}{2\sqrt{4+a}\sqrt{1+\sqrt{4+a}}}$$

output
```
1/2*arctan((1-x)/(1-(4+a)^(1/2))^(1/2))/(4+a)^(1/2)/(1-(4+a)^(1/2))^(1/2)-
1/2*arctan((1-x)/(1+(4+a)^(1/2))^(1/2))/(4+a)^(1/2)/(1+(4+a)^(1/2))^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.61

$$\int \frac{1}{a+8x-8x^2+4x^3-x^4} \, dx = -\frac{1}{4}\text{RootSum}\left[a+8\#1-8\#1^2+4\#1^3 - \#1^4\&, \frac{\log(x-\#1)}{-2+4\#1-3\#1^2+\#1^3}\&\right]$$

input
```
Integrate[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(-1),x]
```

output
```
-1/4*RootSum[a + 8*#1 - 8*#1^2 + 4*#1^3 - #1^4 & , Log[x - #1]/(-2 + 4*#1
- 3*#1^2 + #1^3) & ]
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2458, 1406, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a-x^4+4x^3-8x^2+8x}dx$$

$$\Big\downarrow \text{2458}$$

$$\int \frac{1}{a-(x-1)^4-2(x-1)^2+3}d(x-1)$$

$$\Big\downarrow \text{1406}$$

$$\frac{\int \frac{1}{-(x-1)^2+\sqrt{a+4}-1}d(x-1)}{2\sqrt{a+4}}-\frac{\int \frac{1}{-(x-1)^2-\sqrt{a+4}-1}d(x-1)}{2\sqrt{a+4}}$$

$$\Big\downarrow \text{217}$$

$$\frac{\arctan\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)}{2\sqrt{a+4}\sqrt{\sqrt{a+4}+1}}-\frac{\arctan\left(\frac{x-1}{\sqrt{1-\sqrt{a+4}}}\right)}{2\sqrt{a+4}\sqrt{1-\sqrt{a+4}}}$$

input
```
Int[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(-1),x]
```

output
```
-1/2*ArcTan[(-1 + x)/Sqrt[1 - Sqrt[4 + a]]]/(Sqrt[4 + a]*Sqrt[1 - Sqrt[4 +
 a]]) + ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]]/(2*Sqrt[4 + a]*Sqrt[1 + Sqr
t[4 + a]])
```

Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1406
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Simp[c/q   Int[1/(b/2 - q/2 + c*x^2), x], x] - Simp[c/q   I
nt[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c
, 0] && PosQ[b^2 - 4*a*c]
```

rule 2458
```
Int[(Pn_)^(p_.), x_Symbol] :> With[{S = Coeff[Pn, x, Expon[Pn, x] - 1]/(Exp
on[Pn, x]*Coeff[Pn, x, Expon[Pn, x]])}, Subst[Int[ExpandToSum[Pn /. x -> x
- S, x]^p, x], x, x + S] /; BinomialQ[Pn /. x -> x - S, x] || (IntegerQ[Exp
on[Pn, x]/2] && TrinomialQ[Pn /. x -> x - S, x])] /; FreeQ[p, x] && PolyQ[P
n, x] && GtQ[Expon[Pn, x], 2] && NeQ[Coeff[Pn, x, Expon[Pn, x] - 1], 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.55

method	result	size
default	$\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^4-4\_Z^3+8\_Z^2-8\_Z-a\right)}\frac{\ln\left(x-\_R\right)}{-\_R^3+3\_R^2-4\_R+2}}{4}$	51
risch	$\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^4-4\_Z^3+8\_Z^2-8\_Z-a\right)}\frac{\ln\left(x-\_R\right)}{-\_R^3+3\_R^2-4\_R+2}}{4}$	51

input
```
int(1/(-x^4+4*x^3-8*x^2+a+8*x),x,method=_RETURNVERBOSE)
```

output
```
1/4*sum(1/(-_R^3+3*_R^2-4*_R+2)*ln(x-_R),_R=RootOf(_Z^4-4*_Z^3+8*_Z^2-8*_Z
-a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 457 vs. $2(67) = 134$.

Time = 0.08 (sec) , antiderivative size = 457, normalized size of antiderivative = 4.91

$$\int \frac{1}{a+8x-8x^2+4x^3-x^4}\,dx$$
$$= \frac{1}{4}\sqrt{\frac{\frac{a^2+7a+12}{\sqrt{a^3+10a^2+33a+36}}+1}{a^2+7a+12}}\log\left(\left(a-\frac{a^2+7a+12}{\sqrt{a^3+10a^2+33a+36}}+4\right)\sqrt{\frac{\frac{a^2+7a+12}{\sqrt{a^3+10a^2+33a+36}}+1}{a^2+7a+12}}+x-1\right)$$
$$-\frac{1}{4}\sqrt{\frac{\frac{a^2+7a+12}{\sqrt{a^3+10a^2+33a+36}}+1}{a^2+7a+12}}\log\left(-\left(a-\frac{a^2+7a+12}{\sqrt{a^3+10a^2+33a+36}}+4\right)\sqrt{\frac{\frac{a^2+7a+12}{\sqrt{a^3+10a^2+33a+36}}+1}{a^2+7a+12}}+x-1\right)$$
$$+\frac{1}{4}\sqrt{-\frac{\frac{a^2+7a+12}{\sqrt{a^3+10a^2+33a+36}}-1}{a^2+7a+12}}\log\left(\left(a+\frac{a^2+7a+12}{\sqrt{a^3+10a^2+33a+36}}+4\right)\sqrt{-\frac{\frac{a^2+7a+12}{\sqrt{a^3+10a^2+33a+36}}-1}{a^2+7a+12}}+x-1\right)$$
$$-\frac{1}{4}\sqrt{-\frac{\frac{a^2+7a+12}{\sqrt{a^3+10a^2+33a+36}}-1}{a^2+7a+12}}\log\left(-\left(a+\frac{a^2+7a+12}{\sqrt{a^3+10a^2+33a+36}}+4\right)\sqrt{-\frac{\frac{a^2+7a+12}{\sqrt{a^3+10a^2+33a+36}}-1}{a^2+7a+12}}+x-1\right)$$

input
```
integrate(1/(-x^4+4*x^3-8*x^2+a+8*x),x, algorithm="fricas")
```

output
```
1/4*sqrt(((a^2 + 7*a + 12)/sqrt(a^3 + 10*a^2 + 33*a + 36) + 1)/(a^2 + 7*a
+ 12))*log((a - (a^2 + 7*a + 12)/sqrt(a^3 + 10*a^2 + 33*a + 36) + 4)*sqrt(
((a^2 + 7*a + 12)/sqrt(a^3 + 10*a^2 + 33*a + 36) + 1)/(a^2 + 7*a + 12)) +
x - 1) - 1/4*sqrt(((a^2 + 7*a + 12)/sqrt(a^3 + 10*a^2 + 33*a + 36) + 1)/(a
^2 + 7*a + 12))*log(-(a - (a^2 + 7*a + 12)/sqrt(a^3 + 10*a^2 + 33*a + 36)
+ 4)*sqrt(((a^2 + 7*a + 12)/sqrt(a^3 + 10*a^2 + 33*a + 36) + 1)/(a^2 + 7*a
 + 12)) + x - 1) + 1/4*sqrt(-((a^2 + 7*a + 12)/sqrt(a^3 + 10*a^2 + 33*a +
36) - 1)/(a^2 + 7*a + 12))*log((a + (a^2 + 7*a + 12)/sqrt(a^3 + 10*a^2 + 3
3*a + 36) + 4)*sqrt(-((a^2 + 7*a + 12)/sqrt(a^3 + 10*a^2 + 33*a + 36) - 1)
/(a^2 + 7*a + 12)) + x - 1) - 1/4*sqrt(-((a^2 + 7*a + 12)/sqrt(a^3 + 10*a^
2 + 33*a + 36) - 1)/(a^2 + 7*a + 12))*log(-(a + (a^2 + 7*a + 12)/sqrt(a^3
+ 10*a^2 + 33*a + 36) + 4)*sqrt(-((a^2 + 7*a + 12)/sqrt(a^3 + 10*a^2 + 33*
a + 36) - 1)/(a^2 + 7*a + 12)) + x - 1)
```

Sympy [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.71

$$\int \frac{1}{a+8x-8x^2+4x^3-x^4}\,dx = -\operatorname{RootSum}\left(t^4\cdot\left(256a^3+2816a^2+10240a+12288\right)+t^2(-32a-128)-1,\left(t\mapsto t\log\left(64t^3a^2+448t^3\right.\right.\right.$$

input
```
integrate(1/(-x**4+4*x**3-8*x**2+a+8*x),x)
```

output
```
-RootSum(_t**4*(256*a**3 + 2816*a**2 + 10240*a + 12288) + _t**2*(-32*a - 1
28) - 1, Lambda(_t, _t*log(64*_t**3*a**2 + 448*_t**3*a + 768*_t**3 - 4*_t*
a - 20*_t + x - 1)))
```

Maxima [F]

$$\int \frac{1}{a+8x-8x^2+4x^3-x^4}\,dx = \int -\frac{1}{x^4-4\,x^3+8\,x^2-a-8\,x}\,dx$$

input
```
integrate(1/(-x^4+4*x^3-8*x^2+a+8*x),x, algorithm="maxima")
```

output
```
-integrate(1/(x^4 - 4*x^3 + 8*x^2 - a - 8*x), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2669 vs. 2(67) = 134.

Time = 2.22 (sec) , antiderivative size = 2669, normalized size of antiderivative = 28.70

$$\int \frac{1}{a+8x-8x^2+4x^3-x^4}\,dx = \text{Too large to display}$$

input
```
integrate(1/(-x^4+4*x^3-8*x^2+a+8*x),x, algorithm="giac")
```

output
```
-1/4*sqrt(((a + 4)^(3/2) + a + 4)/(a^3 + 11*a^2 + 40*a + 48))*log(abs(sqrt
(a + 4)*a^5 + sqrt(a^2 + (a^2 + 7*a + 12)*sqrt(a + 4) + 7*a + 12)*a^4*x +
a^5 + sqrt(a^2 + (a^2 + 7*a + 12)*sqrt(a + 4) + 7*a + 12)*sqrt(a + 4)*a^3*
x - sqrt(a^2 + (a^2 + 7*a + 12)*sqrt(a + 4) + 7*a + 12)*a^4 + 17*sqrt(a +
4)*a^4 + 14*sqrt(a^2 + (a^2 + 7*a + 12)*sqrt(a + 4) + 7*a + 12)*a^3*x - sq
rt(a^2 + (a^2 + 7*a + 12)*sqrt(a + 4) + 7*a + 12)*sqrt(a + 4)*a^3 + 17*a^4
 + 10*sqrt(a^2 + (a^2 + 7*a + 12)*sqrt(a + 4) + 7*a + 12)*sqrt(a + 4)*a^2*
x - 14*sqrt(a^2 + (a^2 + 7*a + 12)*sqrt(a + 4) + 7*a + 12)*a^3 + 111*sqrt(
a + 4)*a^3 + 69*sqrt(a^2 + (a^2 + 7*a + 12)*sqrt(a + 4) + 7*a + 12)*a^2*x
- 10*sqrt(a^2 + (a^2 + 7*a + 12)*sqrt(a + 4) + 7*a + 12)*sqrt(a + 4)*a^2 +
 111*a^3 + 29*sqrt(a^2 + (a^2 + 7*a + 12)*sqrt(a + 4) + 7*a + 12)*sqrt(a +
 4)*a*x - 69*sqrt(a^2 + (a^2 + 7*a + 12)*sqrt(a + 4) + 7*a + 12)*a^2 + 351
*sqrt(a + 4)*a^2 + 144*sqrt(a^2 + (a^2 + 7*a + 12)*sqrt(a + 4) + 7*a + 12)
*a*x - 29*sqrt(a^2 + (a^2 + 7*a + 12)*sqrt(a + 4) + 7*a + 12)*sqrt(a + 4)*
a + 351*a^2 + 28*sqrt(a^2 + (a^2 + 7*a + 12)*sqrt(a + 4) + 7*a + 12)*sqrt(
a + 4)*x - 144*sqrt(a^2 + (a^2 + 7*a + 12)*sqrt(a + 4) + 7*a + 12)*a + 544
*sqrt(a + 4)*a + 112*sqrt(a^2 + (a^2 + 7*a + 12)*sqrt(a + 4) + 7*a + 12)*x
 - 28*sqrt(a^2 + (a^2 + 7*a + 12)*sqrt(a + 4) + 7*a + 12)*sqrt(a + 4) + 54
4*a - 112*sqrt(a^2 + (a^2 + 7*a + 12)*sqrt(a + 4) + 7*a + 12) + 336*sqrt(a
 + 4) + 336)) + 1/4*sqrt(((a + 4)^(3/2) + a + 4)/(a^3 + 11*a^2 + 40*a +...
```

Mupad [B] (verification not implemented)

Time = 22.14 (sec) , antiderivative size = 571, normalized size of antiderivative = 6.14

$$\int \frac{1}{a+8x-8x^2+4x^3-x^4}\,dx = -\mathrm{atan}\left(-\frac{a\,8\mathrm{i}-x\,16\mathrm{i}+x\,\sqrt{a^3+12\,a^2+48\,a+64}\,1\mathrm{i}-a\,x\,8\mathrm{i}-\sqrt{a^3+12\,a^2+48\,a+64}\,1\mathrm{i}-a}{44\,a^2\,\sqrt{\frac{a-\sqrt{a^3+12\,a^2+48\,a+64}+4}{16\,a^3+176\,a^2+640\,a+768}}+4\,a^3\,\sqrt{\frac{a-\sqrt{a^3+12\,a^2+48\,a+64}+4}{16\,a^3+176\,a^2+640\,a+768}}+160\,a\,\sqrt{\frac{a-\sqrt{a^3+12\,a^2+48\,a+64}+4}{16\,a^3+176\,a^2+640\,a+768}}+}\right. - \mathrm{atan}\left(-\frac{a\,8\mathrm{i}-x\,16\mathrm{i}-x\,\sqrt{a^3+12\,a^2+48\,a+64}\,1\mathrm{i}-a\,x\,8\mathrm{i}+\sqrt{a^3+12\,a^2+48\,a+64}\,1\mathrm{i}-a}{160\,a\,\sqrt{\frac{a+\sqrt{a^3+12\,a^2+48\,a+64}+4}{16\,a^3+176\,a^2+640\,a+768}}+192\,\sqrt{\frac{a+\sqrt{a^3+12\,a^2+48\,a+64}+4}{16\,a^3+176\,a^2+640\,a+768}}+44\,a^2\,\sqrt{\frac{a+\sqrt{a^3+12\,a^2+48\,a+64}+4}{16\,a^3+176\,a^2+640\,a+768}}+}\right.$$

input
```
int(1/(a + 8*x - 8*x^2 + 4*x^3 - x^4),x)
```

output
```
- atan(-(a*8i - x*16i + x*(48*a + 12*a^2 + a^3 + 64)^(1/2)*1i - a*x*8i - (
48*a + 12*a^2 + a^3 + 64)^(1/2)*1i - a^2*x*1i + a^2*1i + 16i)/(44*a^2*((a
- (48*a + 12*a^2 + a^3 + 64)^(1/2) + 4)/(640*a + 176*a^2 + 16*a^3 + 768))^
(1/2) + 4*a^3*((a - (48*a + 12*a^2 + a^3 + 64)^(1/2) + 4)/(640*a + 176*a^2
 + 16*a^3 + 768))^(1/2) + 160*a*((a - (48*a + 12*a^2 + a^3 + 64)^(1/2) + 4
)/(640*a + 176*a^2 + 16*a^3 + 768))^(1/2) + 192*((a - (48*a + 12*a^2 + a^3
 + 64)^(1/2) + 4)/(640*a + 176*a^2 + 16*a^3 + 768))^(1/2)))*((a - (48*a +
12*a^2 + a^3 + 64)^(1/2) + 4)/(640*a + 176*a^2 + 16*a^3 + 768))^(1/2)*2i -
 atan(-(a*8i - x*16i - x*(48*a + 12*a^2 + a^3 + 64)^(1/2)*1i - a*x*8i + (4
8*a + 12*a^2 + a^3 + 64)^(1/2)*1i - a^2*x*1i + a^2*1i + 16i)/(160*a*((a +
(48*a + 12*a^2 + a^3 + 64)^(1/2) + 4)/(640*a + 176*a^2 + 16*a^3 + 768))^(1
/2) + 192*((a + (48*a + 12*a^2 + a^3 + 64)^(1/2) + 4)/(640*a + 176*a^2 + 1
6*a^3 + 768))^(1/2) + 44*a^2*((a + (48*a + 12*a^2 + a^3 + 64)^(1/2) + 4)/(
640*a + 176*a^2 + 16*a^3 + 768))^(1/2) + 4*a^3*((a + (48*a + 12*a^2 + a^3
+ 64)^(1/2) + 4)/(640*a + 176*a^2 + 16*a^3 + 768))^(1/2)))*((a + (48*a + 1
2*a^2 + a^3 + 64)^(1/2) + 4)/(640*a + 176*a^2 + 16*a^3 + 768))^(1/2)*2i
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 222, normalized size of antiderivative = 2.39

$$\int \frac{1}{a+8x-8x^2+4x^3-x^4}\,dx$$
$$= \frac{-2\sqrt{a+4}\,\sqrt{\sqrt{a+4}+1}\,atan\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)+2\sqrt{\sqrt{a+4}+1}\,atan\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)a+8\sqrt{\sqrt{a+4}+1}\,atan\left(\frac{-}{\sqrt{}}\right.}{}$$

input
```
int(1/(-x^4+4*x^3-8*x^2+a+8*x),x)
```

output
```
( - 2*sqrt(a + 4)*sqrt(sqrt(a + 4) + 1)*atan((x - 1)/sqrt(sqrt(a + 4) + 1)
) + 2*sqrt(sqrt(a + 4) + 1)*atan((x - 1)/sqrt(sqrt(a + 4) + 1))*a + 8*sqrt
(sqrt(a + 4) + 1)*atan((x - 1)/sqrt(sqrt(a + 4) + 1)) - sqrt(a + 4)*sqrt(s
qrt(a + 4) - 1)*log(sqrt(sqrt(a + 4) - 1) - x + 1) + sqrt(a + 4)*sqrt(sqrt
(a + 4) - 1)*log(sqrt(sqrt(a + 4) - 1) + x - 1) - sqrt(sqrt(a + 4) - 1)*lo
g(sqrt(sqrt(a + 4) - 1) - x + 1)*a - 4*sqrt(sqrt(a + 4) - 1)*log(sqrt(sqrt
(a + 4) - 1) - x + 1) + sqrt(sqrt(a + 4) - 1)*log(sqrt(sqrt(a + 4) - 1) +
x - 1)*a + 4*sqrt(sqrt(a + 4) - 1)*log(sqrt(sqrt(a + 4) - 1) + x - 1))/(4*
(a**2 + 7*a + 12))
```

3.48 $\int \frac{1}{(a+8x-8x^2+4x^3-x^4)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 179

$$\int \frac{1}{(a+8x-8x^2+4x^3-x^4)^2}\,dx = -\frac{(5+a+(-1+x)^2)\,(1-x)}{4\,(12+7a+a^2)\,(3+a-2(1-x)^2-(1-x)^4)} + \frac{\left(10+3a+\sqrt{4+a}\right)\arctan\left(\frac{1-x}{\sqrt{1-\sqrt{4+a}}}\right)}{8(3+a)(4+a)^{3/2}\sqrt{1-\sqrt{4+a}}} - \frac{\left(10+3a-\sqrt{4+a}\right)\arctan\left(\frac{1-x}{\sqrt{1+\sqrt{4+a}}}\right)}{8(3+a)(4+a)^{3/2}\sqrt{1+\sqrt{4+a}}}$$

output
```
-1/4*(5+a+(-1+x)^2)*(1-x)/(a^2+7*a+12)/(3+a-2*(1-x)^2-(1-x)^4)+1/8*(10+3*a
+(4+a)^(1/2))*arctan((1-x)/(1-(4+a)^(1/2))^(1/2))/(3+a)/(4+a)^(3/2)/(1-(4+
a)^(1/2))^(1/2)-1/8*(10+3*a-(4+a)^(1/2))*arctan((1-x)/(1+(4+a)^(1/2))^(1/2
))/(3+a)/(4+a)^(3/2)/(1+(4+a)^(1/2))^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.08 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.84

$$\int \frac{1}{(a+8x-8x^2+4x^3-x^4)^2}\,dx = \frac{(-1+x)\,(6+a-2x+x^2)}{4(3+a)(4+a)\,(a-x\,(-8+8x-4x^2+x^3))} - \frac{\text{RootSum}\left[a+8\#1-8\#1^2+4\#1^3-\#1^4\&,\frac{12\log(x-\#1)+3a\log(x-\#1)-2\log(x-\#1)\#1+\log(x-\#1)\#1^2}{-2+4\#1-3\#1^2+\#1^3}\&\right]}{16\,(12+7a+a^2)}$$

input
```
Integrate[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(-2),x]
```

output
```
((-1 + x)*(6 + a - 2*x + x^2))/(4*(3 + a)*(4 + a)*(a - x*(-8 + 8*x - 4*x^2
 + x^3))) - RootSum[a + 8*#1 - 8*#1^2 + 4*#1^3 - #1^4 & , (12*Log[x - #1]
+ 3*a*Log[x - #1] - 2*Log[x - #1]*#1 + Log[x - #1]*#1^2)/(-2 + 4*#1 - 3*#1
^2 + #1^3) & ]/(16*(12 + 7*a + a^2))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.92, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {2458, 1405, 27, 1480, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-x^4+4x^3-8x^2+8x)^2}dx$$

$$\Big\downarrow \text{ 2458}$$

$$\int \frac{1}{(a-(x-1)^4-2(x-1)^2+3)^2}d(x-1)$$

$$\Big\downarrow \text{ 1405}$$

$$\frac{(x-1)\,(a+(x-1)^2+5)}{4\,(a^2+7a+12)\,(a-(x-1)^4-2(x-1)^2+3)} - \frac{\int -\frac{2((x-1)^2+3a+11)}{-(x-1)^4-2(x-1)^2+a+3}d(x-1)}{8\,(a^2+7a+12)}$$

$$\Big\downarrow 27$$

$$\frac{\int \frac{(x-1)^2+3a+11}{-(x-1)^4-2(x-1)^2+a+3}d(x-1)}{4\left(a^2+7a+12\right)}+\frac{(x-1)\left(a+(x-1)^2+5\right)}{4\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)}$$

$$\Big\downarrow 1480$$

$$\frac{\frac{1}{2}\left(1-\frac{3a+10}{\sqrt{a+4}}\right)\int \frac{1}{-(x-1)^2-\sqrt{a+4}-1}d(x-1)+\frac{1}{2}\left(\frac{3a+10}{\sqrt{a+4}}+1\right)\int \frac{1}{-(x-1)^2+\sqrt{a+4}-1}d(x-1)}{4\left(a^2+7a+12\right)}+\frac{(x-1)\left(a+(x-1)^2+5\right)}{4\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)}$$

$$\Big\downarrow 217$$

$$\frac{-\frac{\left(\frac{3a+10}{\sqrt{a+4}}+1\right)\arctan\left(\frac{x-1}{\sqrt{1-\sqrt{a+4}}}\right)}{2\sqrt{1-\sqrt{a+4}}}-\frac{\left(1-\frac{3a+10}{\sqrt{a+4}}\right)\arctan\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)}{2\sqrt{\sqrt{a+4}+1}}}{4\left(a^2+7a+12\right)}+\frac{(x-1)\left(a+(x-1)^2+5\right)}{4\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)}$$

input
```
Int[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(-2),x]
```

output
```
((5 + a + (-1 + x)^2)*(-1 + x))/(4*(12 + 7*a + a^2)*(3 + a - 2*(-1 + x)^2
- (-1 + x)^4)) + (-1/2*((1 + (10 + 3*a)/Sqrt[4 + a])*ArcTan[(-1 + x)/Sqrt[
1 - Sqrt[4 + a]]])/Sqrt[1 - Sqrt[4 + a]] - ((1 - (10 + 3*a)/Sqrt[4 + a])*A
rcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]])/(2*Sqrt[1 + Sqrt[4 + a]]))/(4*(12 +
 7*a + a^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 2458
```
Int[(Pn_)^(p_.), x_Symbol] :> With[{S = Coeff[Pn, x, Expon[Pn, x] - 1]/(Exp
on[Pn, x]*Coeff[Pn, x, Expon[Pn, x]])}, Subst[Int[ExpandToSum[Pn /. x -> x
- S, x]^p, x], x, x + S] /; BinomialQ[Pn /. x -> x - S, x] || (IntegerQ[Exp
on[Pn, x]/2] && TrinomialQ[Pn /. x -> x - S, x])] /; FreeQ[p, x] && PolyQ[P
n, x] && GtQ[Expon[Pn, x], 2] && NeQ[Coeff[Pn, x, Expon[Pn, x] - 1], 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.09 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.88

method	result
default	$\frac{\frac{x^3}{4a^2+28a+48}-\frac{3x^2}{4(4+a)(a+3)}+\frac{(a+8)x}{4a^2+28a+48}-\frac{6+a}{4\left(a^2+7a+12\right)}}{-x^4+4x^3-8x^2+a+8x}+\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^4-4\_Z^3+8\_Z^2-8\_Z-a\right)}\frac{\left(\_R^2-2\_R+3a+12\right)\ln\left(\right.}{-\_R^3+3\_R^2-4\_R}}{16(4+a)(a+3)}$
risch	$\frac{\frac{x^3}{4a^2+28a+48}-\frac{3x^2}{4(4+a)(a+3)}+\frac{(a+8)x}{4a^2+28a+48}-\frac{6+a}{4\left(a^2+7a+12\right)}}{-x^4+4x^3-8x^2+a+8x}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^4-4\_Z^3+8\_Z^2-8\_Z-a\right)}\frac{\left(\frac{\_R^2}{a^2+7a+12}-\frac{2\_R}{a^2+7a+12}+\right.}{-\_R^3+3\_R^2-}\right.}{16}$

input
```
int(1/(-x^4+4*x^3-8*x^2+a+8*x)^2,x,method=_RETURNVERBOSE)
```

output
```
(1/4/(a^2+7*a+12)*x^3-3/4/(4+a)/(a+3)*x^2+1/4*(a+8)/(a^2+7*a+12)*x-1/4*(6+
a)/(a^2+7*a+12))/(-x^4+4*x^3-8*x^2+a+8*x)+1/16/(4+a)/(a+3)*sum((_R^2-2*_R+
3*a+12)/(-_R^3+3*_R^2-4*_R+2)*ln(x-_R),_R=RootOf(_Z^4-4*_Z^3+8*_Z^2-8*_Z-a
))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1948 vs. $2(141) = 282$.

Time = 0.11 (sec) , antiderivative size = 1948, normalized size of antiderivative = 10.88

$$\int \frac{1}{(a+8x-8x^2+4x^3-x^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="fricas")
```

output
```
-1/16*(4*x^3 - ((a^2 + 7*a + 12)*x^4 - 4*(a^2 + 7*a + 12)*x^3 - a^3 + 8*(a
^2 + 7*a + 12)*x^2 - 7*a^2 - 8*(a^2 + 7*a + 12)*x - 12*a)*sqrt((15*a^2 + (
a^6 + 21*a^5 + 183*a^4 + 847*a^3 + 2196*a^2 + 3024*a + 1728)*sqrt((81*a^2
+ 558*a + 961)/(a^9 + 30*a^8 + 399*a^7 + 3088*a^6 + 15327*a^5 + 50598*a^4
+ 111105*a^3 + 156492*a^2 + 128304*a + 46656)) + 105*a + 184)/(a^6 + 21*a^
5 + 183*a^4 + 847*a^3 + 2196*a^2 + 3024*a + 1728))*log(-81*a^2 + (81*a^2 +
 567*a + 992)*x + (27*a^4 + 408*a^3 + 2309*a^2 - 2*(2*a^7 + 49*a^6 + 513*a
^5 + 2975*a^4 + 10321*a^3 + 21420*a^2 + 24624*a + 12096)*sqrt((81*a^2 + 55
8*a + 961)/(a^9 + 30*a^8 + 399*a^7 + 3088*a^6 + 15327*a^5 + 50598*a^4 + 11
1105*a^3 + 156492*a^2 + 128304*a + 46656)) + 5800*a + 5456)*sqrt((15*a^2 +
 (a^6 + 21*a^5 + 183*a^4 + 847*a^3 + 2196*a^2 + 3024*a + 1728)*sqrt((81*a^
2 + 558*a + 961)/(a^9 + 30*a^8 + 399*a^7 + 3088*a^6 + 15327*a^5 + 50598*a^
4 + 111105*a^3 + 156492*a^2 + 128304*a + 46656)) + 105*a + 184)/(a^6 + 21*
a^5 + 183*a^4 + 847*a^3 + 2196*a^2 + 3024*a + 1728)) - 567*a - 992) + ((a^
2 + 7*a + 12)*x^4 - 4*(a^2 + 7*a + 12)*x^3 - a^3 + 8*(a^2 + 7*a + 12)*x^2
- 7*a^2 - 8*(a^2 + 7*a + 12)*x - 12*a)*sqrt((15*a^2 + (a^6 + 21*a^5 + 183*
a^4 + 847*a^3 + 2196*a^2 + 3024*a + 1728)*sqrt((81*a^2 + 558*a + 961)/(a^9
 + 30*a^8 + 399*a^7 + 3088*a^6 + 15327*a^5 + 50598*a^4 + 111105*a^3 + 1564
92*a^2 + 128304*a + 46656)) + 105*a + 184)/(a^6 + 21*a^5 + 183*a^4 + 847*a
^3 + 2196*a^2 + 3024*a + 1728))*log(-81*a^2 + (81*a^2 + 567*a + 992)*x ...
```

Sympy [A] (verification not implemented)

Time = 3.54 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.64

$$\int \frac{1}{(a+8x-8x^2+4x^3-x^4)^2}\,dx$$
$$= \frac{a-x^3+3x^2+x(-a-8)+6}{-4a^3-28a^2-48a+x^4\cdot(4a^2+28a+48)+x^3(-16a^2-112a-192)+x^2\cdot(32a^2+224a+384)+x}$$
$$+\mathrm{RootSum}\left(t^4\cdot(65536a^9+2162688a^8+31653888a^7+269680640a^6+1473773568a^5+5357174784a^4\right.$$

input
```
integrate(1/(-x**4+4*x**3-8*x**2+a+8*x)**2,x)
```

output
```
(a - x**3 + 3*x**2 + x*(-a - 8) + 6)/(-4*a**3 - 28*a**2 - 48*a + x**4*(4*a
**2 + 28*a + 48) + x**3*(-16*a**2 - 112*a - 192) + x**2*(32*a**2 + 224*a +
 384) + x*(-32*a**2 - 224*a - 384)) + RootSum(_t**4*(65536*a**9 + 2162688*
a**8 + 31653888*a**7 + 269680640*a**6 + 1473773568*a**5 + 5357174784*a**4
+ 12952010752*a**3 + 20082327552*a**2 + 18119393280*a + 7247757312) + _t**
2*(-7680*a**5 - 145920*a**4 - 1107968*a**3 - 4202496*a**2 - 7962624*a - 60
29312) - 81*a**2 - 576*a - 1024, Lambda(_t, _t*log(x + (-16384*_t**3*a**7
- 401408*_t**3*a**6 - 4202496*_t**3*a**5 - 24371200*_t**3*a**4 - 84549632*
_t**3*a**3 - 175472640*_t**3*a**2 - 201719808*_t**3*a - 99090432*_t**3 + 4
32*_t*a**4 + 7488*_t*a**3 + 47024*_t*a**2 + 128096*_t*a + 128512*_t - 81*a
**2 - 567*a - 992)/(81*a**2 + 567*a + 992))))
```

Maxima [F]

$$\int \frac{1}{(a+8x-8x^2+4x^3-x^4)^2}\,dx = \int \frac{1}{(x^4-4\,x^3+8\,x^2-a-8\,x)^2}\,dx$$

input
```
integrate(1/(-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="maxima")
```

output
```
-1/4*(x^3 + (a + 8)*x - 3*x^2 - a - 6)/((a^2 + 7*a + 12)*x^4 - 4*(a^2 + 7*
a + 12)*x^3 - a^3 + 8*(a^2 + 7*a + 12)*x^2 - 7*a^2 - 8*(a^2 + 7*a + 12)*x
- 12*a) - 1/4*integrate((x^2 + 3*a - 2*x + 12)/(x^4 - 4*x^3 + 8*x^2 - a -
8*x), x)/(a^2 + 7*a + 12)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8503 vs. $2(141) = 282$.

Time = 5.91 (sec) , antiderivative size = 8503, normalized size of antiderivative = 47.50

$$\int \frac{1}{(a + 8x - 8x^2 + 4x^3 - x^4)^2} \, dx = \text{Too large to display}$$

input
```
integrate(1/(-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="giac")
```

output
```
1/16*(sqrt((15*a^3 + 165*a^2 + (9*a^3 + 103*a^2 + 392*a + 496)*sqrt(a + 4)
 + 604*a + 736)/(a^3 + 11*a^2 + 40*a + 48))*log(abs(243*sqrt(a + 4)*a^10 +
 324*a^10 + 8640*sqrt(a + 4)*a^9 + 81*sqrt(15*a^4 + 210*a^3 + 1099*a^2 + (
9*a^4 + 130*a^3 + 701*a^2 + 1672*a + 1488)*sqrt(a + 4) + 2548*a + 2208)*a^
8*x + 11466*a^9 + 81*sqrt(15*a^4 + 210*a^3 + 1099*a^2 + (9*a^4 + 130*a^3 +
 701*a^2 + 1672*a + 1488)*sqrt(a + 4) + 2548*a + 2208)*sqrt(a + 4)*a^7*x -
 81*sqrt(15*a^4 + 210*a^3 + 1099*a^2 + (9*a^4 + 130*a^3 + 701*a^2 + 1672*a
 + 1488)*sqrt(a + 4) + 2548*a + 2208)*a^8 + 138027*sqrt(a + 4)*a^8 + 2340*
sqrt(15*a^4 + 210*a^3 + 1099*a^2 + (9*a^4 + 130*a^3 + 701*a^2 + 1672*a + 1
488)*sqrt(a + 4) + 2548*a + 2208)*a^7*x - 81*sqrt(15*a^4 + 210*a^3 + 1099*
a^2 + (9*a^4 + 130*a^3 + 701*a^2 + 1672*a + 1488)*sqrt(a + 4) + 2548*a + 2
208)*sqrt(a + 4)*a^7 + 182314*a^8 + 2016*sqrt(15*a^4 + 210*a^3 + 1099*a^2
+ (9*a^4 + 130*a^3 + 701*a^2 + 1672*a + 1488)*sqrt(a + 4) + 2548*a + 2208)
*sqrt(a + 4)*a^6*x - 2340*sqrt(15*a^4 + 210*a^3 + 1099*a^2 + (9*a^4 + 130*
a^3 + 701*a^2 + 1672*a + 1488)*sqrt(a + 4) + 2548*a + 2208)*a^7 + 1304648*
sqrt(a + 4)*a^7 + 29518*sqrt(15*a^4 + 210*a^3 + 1099*a^2 + (9*a^4 + 130*a^
3 + 701*a^2 + 1672*a + 1488)*sqrt(a + 4) + 2548*a + 2208)*a^6*x - 2016*sqr
t(15*a^4 + 210*a^3 + 1099*a^2 + (9*a^4 + 130*a^3 + 701*a^2 + 1672*a + 1488
)*sqrt(a + 4) + 2548*a + 2208)*sqrt(a + 4)*a^6 + 1715172*a^7 + 21454*sqrt(
15*a^4 + 210*a^3 + 1099*a^2 + (9*a^4 + 130*a^3 + 701*a^2 + 1672*a + 148...
```

Mupad [B] (verification not implemented)

Time = 25.13 (sec) , antiderivative size = 4591, normalized size of antiderivative = 25.65

$$\int \frac{1}{(a+8x-8x^2+4x^3-x^4)^2}\,dx = \text{Too large to display}$$

input
```
int(1/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x)
```

output
```
atan(-(((15552*a - 9*a*((a + 4)^9)^(1/2) - 31*((a + 4)^9)^(1/2) + 8208*a^2
 + 2164*a^3 + 285*a^4 + 15*a^5 + 11776)/(256*(276480*a + 306432*a^2 + 1976
32*a^3 + 81744*a^4 + 22488*a^5 + 4115*a^6 + 483*a^7 + 33*a^8 + a^9 + 11059
2)))^(1/2)*((((15728640*a + 10878976*a^2 + 3997696*a^3 + 823296*a^4 + 9011
2*a^5 + 4096*a^6 + 9437184)/(64*(816*a + 460*a^2 + 129*a^3 + 18*a^4 + a^5
+ 576)) - (x*(208896*a + 117760*a^2 + 33024*a^3 + 4608*a^4 + 256*a^5 + 147
456))/(4*(168*a + 73*a^2 + 14*a^3 + a^4 + 144)))*((15552*a - 9*a*((a + 4)^
9)^(1/2) - 31*((a + 4)^9)^(1/2) + 8208*a^2 + 2164*a^3 + 285*a^4 + 15*a^5 +
 11776)/(256*(276480*a + 306432*a^2 + 197632*a^3 + 81744*a^4 + 22488*a^5 +
 4115*a^6 + 483*a^7 + 33*a^8 + a^9 + 110592)))^(1/2) - (733184*a + 396288*
a^2 + 106752*a^3 + 14336*a^4 + 768*a^5 + 540672)/(64*(816*a + 460*a^2 + 12
9*a^3 + 18*a^4 + a^5 + 576)))*((15552*a - 9*a*((a + 4)^9)^(1/2) - 31*((a +
 4)^9)^(1/2) + 8208*a^2 + 2164*a^3 + 285*a^4 + 15*a^5 + 11776)/(256*(27648
0*a + 306432*a^2 + 197632*a^3 + 81744*a^4 + 22488*a^5 + 4115*a^6 + 483*a^7
 + 33*a^8 + a^9 + 110592)))^(1/2) + (5568*a + 1552*a^2 + 144*a^3 + 6656)/(
64*(816*a + 460*a^2 + 129*a^3 + 18*a^4 + a^5 + 576)) - (x*(61*a + 9*a^2 +
104))/(4*(168*a + 73*a^2 + 14*a^3 + a^4 + 144)))*1i + ((15552*a - 9*a*((a
+ 4)^9)^(1/2) - 31*((a + 4)^9)^(1/2) + 8208*a^2 + 2164*a^3 + 285*a^4 + 15*
a^5 + 11776)/(256*(276480*a + 306432*a^2 + 197632*a^3 + 81744*a^4 + 22488*
a^5 + 4115*a^6 + 483*a^7 + 33*a^8 + a^9 + 110592)))^(1/2)*(((15728640*...
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 2216, normalized size of antiderivative = 12.38

$$\int \frac{1}{(a+8x-8x^2+4x^3-x^4)^2}\,dx = \text{Too large to display}$$

input
```
int(1/(-x^4+4*x^3-8*x^2+a+8*x)^2,x)
```

output
```
( - 8*sqrt(a + 4)*sqrt(sqrt(a + 4) + 1)*atan((x - 1)/sqrt(sqrt(a + 4) + 1)
)*a**2 + 8*sqrt(a + 4)*sqrt(sqrt(a + 4) + 1)*atan((x - 1)/sqrt(sqrt(a + 4)
 + 1))*a*x**4 - 32*sqrt(a + 4)*sqrt(sqrt(a + 4) + 1)*atan((x - 1)/sqrt(sqr
t(a + 4) + 1))*a*x**3 + 64*sqrt(a + 4)*sqrt(sqrt(a + 4) + 1)*atan((x - 1)/
sqrt(sqrt(a + 4) + 1))*a*x**2 - 64*sqrt(a + 4)*sqrt(sqrt(a + 4) + 1)*atan(
(x - 1)/sqrt(sqrt(a + 4) + 1))*a*x - 28*sqrt(a + 4)*sqrt(sqrt(a + 4) + 1)*
atan((x - 1)/sqrt(sqrt(a + 4) + 1))*a + 28*sqrt(a + 4)*sqrt(sqrt(a + 4) +
1)*atan((x - 1)/sqrt(sqrt(a + 4) + 1))*x**4 - 112*sqrt(a + 4)*sqrt(sqrt(a
+ 4) + 1)*atan((x - 1)/sqrt(sqrt(a + 4) + 1))*x**3 + 224*sqrt(a + 4)*sqrt(
sqrt(a + 4) + 1)*atan((x - 1)/sqrt(sqrt(a + 4) + 1))*x**2 - 224*sqrt(a + 4
)*sqrt(sqrt(a + 4) + 1)*atan((x - 1)/sqrt(sqrt(a + 4) + 1))*x + 6*sqrt(sqr
t(a + 4) + 1)*atan((x - 1)/sqrt(sqrt(a + 4) + 1))*a**3 - 6*sqrt(sqrt(a + 4
) + 1)*atan((x - 1)/sqrt(sqrt(a + 4) + 1))*a**2*x**4 + 24*sqrt(sqrt(a + 4)
 + 1)*atan((x - 1)/sqrt(sqrt(a + 4) + 1))*a**2*x**3 - 48*sqrt(sqrt(a + 4)
+ 1)*atan((x - 1)/sqrt(sqrt(a + 4) + 1))*a**2*x**2 + 48*sqrt(sqrt(a + 4) +
 1)*atan((x - 1)/sqrt(sqrt(a + 4) + 1))*a**2*x + 46*sqrt(sqrt(a + 4) + 1)*
atan((x - 1)/sqrt(sqrt(a + 4) + 1))*a**2 - 46*sqrt(sqrt(a + 4) + 1)*atan((
x - 1)/sqrt(sqrt(a + 4) + 1))*a*x**4 + 184*sqrt(sqrt(a + 4) + 1)*atan((x -
 1)/sqrt(sqrt(a + 4) + 1))*a*x**3 - 368*sqrt(sqrt(a + 4) + 1)*atan((x - 1)
/sqrt(sqrt(a + 4) + 1))*a*x**2 + 368*sqrt(sqrt(a + 4) + 1)*atan((x - 1)...
```

3.49 $\int \frac{1}{(a+8x-8x^2+4x^3-x^4)^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 270

$$\begin{aligned}\int \frac{1}{(a+8x-8x^2+4x^3-x^4)^3}\,dx &= -\frac{((6+a)(25+7a)+6(7+2a)(1-x)^2)(1-x)}{32(3+a)^2(4+a)^2(3+a-2(1-x)^2-(1-x)^4)} \\ &\quad -\frac{(5+a+(-1+x)^2)(1-x)}{8(12+7a+a^2)(3+a-2(1-x)^2-(1-x)^4)^2} \\ &\quad +\frac{3\left(80+7a^2+14\sqrt{4+a}+a(47+4\sqrt{4+a})\right)\arctan\left(\frac{1-x}{\sqrt{1-\sqrt{4+a}}}\right)}{64(3+a)^2(4+a)^{5/2}\sqrt{1-\sqrt{4+a}}} \\ &\quad +\frac{3\left(14+4a-\frac{80+47a+7a^2}{\sqrt{4+a}}\right)\arctan\left(\frac{1-x}{\sqrt{1+\sqrt{4+a}}}\right)}{64(3+a)^2(4+a)^2\sqrt{1+\sqrt{4+a}}}\end{aligned}$$

output
```
-1/32*((6+a)*(25+7*a)+6*(7+2*a)*(1-x)^2)*(1-x)/(3+a)^2/(4+a)^2/(3+a-2*(1-x
)^2-(1-x)^4)-1/8*(5+a+(-1+x)^2)*(1-x)/(a^2+7*a+12)/(3+a-2*(1-x)^2-(1-x)^4)
^2+3/64*(80+7*a^2+14*(4+a)^(1/2)+a*(47+4*(4+a)^(1/2)))*arctan((1-x)/(1-(4+
a)^(1/2))^(1/2))/(3+a)^2/(4+a)^(5/2)/(1-(4+a)^(1/2))^(1/2)+3/64*(14+4*a-(7
*a^2+47*a+80)/(4+a)^(1/2))*arctan((1-x)/(1+(4+a)^(1/2))^(1/2))/(3+a)^2/(4+
a)^2/(1+(4+a)^(1/2))^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.16 (sec) , antiderivative size = 254, normalized size of antiderivative = 0.94

$$\int \frac{1}{(a+8x-8x^2+4x^3-x^4)^3}\,dx = \frac{1}{128}\left(\frac{16(-1+x)\left(6+a-2x+x^2\right)}{(3+a)(4+a)\left(a-x\left(-8+8x-4x^2+x^3\right)\right)^2} + \frac{4(-1+x)\left(7a^2+6(32-14x+7x^2)+a(79-24x+12x^2)\right)}{(3+a)^2(4+a)^2\left(a-x\left(-8+8x-4x^2+x^3\right)\right)} - \frac{3\mathrm{RootSum}\left[a+8\#1-8\#1^2+4\#1^3-\#1^4\&,\frac{108\log(x-\#1)+55a\log(x-\#1)+7a^2\log(x-\#1)-28\log(x-\#1)\#1}{-2+4\#1-3\#1}\right.}{\left(12+7a+a^2\right)^2}$$

input
```
Integrate[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(-3),x]
```

output
```
((16*(-1 + x)*(6 + a - 2*x + x^2))/((3 + a)*(4 + a)*(a - x*(-8 + 8*x - 4*x
^2 + x^3))^2) + (4*(-1 + x)*(7*a^2 + 6*(32 - 14*x + 7*x^2) + a*(79 - 24*x
+ 12*x^2)))/((3 + a)^2*(4 + a)^2*(a - x*(-8 + 8*x - 4*x^2 + x^3))) - (3*Ro
otSum[a + 8*#1 - 8*#1^2 + 4*#1^3 - #1^4 & , (108*Log[x - #1] + 55*a*Log[x
- #1] + 7*a^2*Log[x - #1] - 28*Log[x - #1]*#1 - 8*a*Log[x - #1]*#1 + 14*Lo
g[x - #1]*#1^2 + 4*a*Log[x - #1]*#1^2)/(-2 + 4*#1 - 3*#1^2 + #1^3) & ])/(1
2 + 7*a + a^2)^2)/128
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 254, normalized size of antiderivative = 0.94, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {2458, 1405, 27, 1492, 27, 1480, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-x^4+4x^3-8x^2+8x)^3}\,dx$$

$$\Big\downarrow \text{2458}$$

$$\int \frac{1}{\left(a-(x-1)^4-2(x-1)^2+3\right)^3}d(x-1)$$

$$\Big\downarrow \text{1405}$$

$$\frac{(x-1)\left(a+(x-1)^2+5\right)}{8\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)^2}-\frac{\int -\frac{2\left(5(x-1)^2+7a+27\right)}{\left(-(x-1)^4-2(x-1)^2+a+3\right)^2}d(x-1)}{16\left(a^2+7a+12\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{5(x-1)^2+7a+27}{\left(-(x-1)^4-2(x-1)^2+a+3\right)^2}d(x-1)}{8\left(a^2+7a+12\right)}+\frac{(x-1)\left(a+(x-1)^2+5\right)}{8\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)^2}$$

$$\Big\downarrow \text{1492}$$

$$\frac{\frac{(x-1)\left(6(2a+7)(x-1)^2+(a+6)(7a+25)\right)}{4(a^2+7a+12)(a-(x-1)^4-2(x-1)^2+3)}-\frac{\int -\frac{6\left(7a^2+51a+2(2a+7)(x-1)^2+94\right)}{-(x-1)^4-2(x-1)^2+a+3}d(x-1)}{8(a^2+7a+12)}}{8\left(a^2+7a+12\right)}+\frac{(x-1)\left(a+(x-1)^2+5\right)}{8\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{3\int \frac{7a^2+51a+2(2a+7)(x-1)^2+94}{-(x-1)^4-2(x-1)^2+a+3}d(x-1)}{4(a^2+7a+12)}+\frac{(x-1)\left(6(2a+7)(x-1)^2+(a+6)(7a+25)\right)}{4(a^2+7a+12)(a-(x-1)^4-2(x-1)^2+3)}}{8\left(a^2+7a+12\right)}+\frac{(x-1)\left(a+(x-1)^2+5\right)}{8\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)^2}$$

$$\Big\downarrow \text{1480}$$

$$\frac{\frac{3\left(\frac{1}{2}\left(-\frac{7a^2+47a+80}{\sqrt{a+4}}+4a+14\right)\int \frac{1}{-(x-1)^2-\sqrt{a+4}-1}d(x-1)+\frac{1}{2}\left(\frac{7a^2+47a+80}{\sqrt{a+4}}+4a+14\right)\int \frac{1}{-(x-1)^2+\sqrt{a+4}-1}d(x-1)\right)}{4(a^2+7a+12)}+\frac{(x-1)\left(6(2a+7)(x-1)^2+(\cdots}{4(a^2+7a+12)(a-(x-1)^4\cdots}}{8\left(a^2+7a+12\right)}$$
$$+\frac{(x-1)\left(a+(x-1)^2+5\right)}{8\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)^2}$$

$$\Big\downarrow \text{217}$$

$$\frac{\frac{3\left(-\frac{\left(\frac{7a^2+47a+80}{\sqrt{a+4}}+4a+14\right)\arctan\left(\frac{x-1}{\sqrt{1-\sqrt{a+4}}}\right)}{2\sqrt{1-\sqrt{a+4}}}-\frac{\left(-\frac{7a^2+47a+80}{\sqrt{a+4}}+4a+14\right)\arctan\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)}{2\sqrt{\sqrt{a+4}+1}}\right)}{4(a^2+7a+12)}+\frac{(x-1)(6(2a+7)(x-1)^2+(a+6)(7a+25))}{4(a^2+7a+12)(a-(x-1)^4-2(x-1)^2+3)}}{8\left(a^2+7a+12\right)}+$$
$$\frac{(x-1)\left(a+(x-1)^2+5\right)}{8\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)^2}$$

input
```
Int[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(-3),x]
```

output
```
((5 + a + (-1 + x)^2)*(-1 + x))/(8*(12 + 7*a + a^2)*(3 + a - 2*(-1 + x)^2
- (-1 + x)^4)^2) + ((((6 + a)*(25 + 7*a) + 6*(7 + 2*a)*(-1 + x)^2)*(-1 + x
))/(4*(12 + 7*a + a^2)*(3 + a - 2*(-1 + x)^2 - (-1 + x)^4)) + (3*(-1/2*((1
4 + 4*a + (80 + 47*a + 7*a^2)/Sqrt[4 + a])*ArcTan[(-1 + x)/Sqrt[1 - Sqrt[4
 + a]]])/Sqrt[1 - Sqrt[4 + a]] - ((14 + 4*a - (80 + 47*a + 7*a^2)/Sqrt[4 +
 a])*ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]])/(2*Sqrt[1 + Sqrt[4 + a]])))/(
4*(12 + 7*a + a^2)))/(8*(12 + 7*a + a^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1492
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2
 - 4*a*c))   Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p +
7)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 LtQ[p, -1] && IntegerQ[2*p]
```

rule 2458
```
Int[(Pn_)^(p_.), x_Symbol] :> With[{S = Coeff[Pn, x, Expon[Pn, x] - 1]/(Exp
on[Pn, x]*Coeff[Pn, x, Expon[Pn, x]])}, Subst[Int[ExpandToSum[Pn /. x -> x
- S, x]^p, x], x, x + S] /; BinomialQ[Pn /. x -> x - S, x] || (IntegerQ[Exp
on[Pn, x]/2] && TrinomialQ[Pn /. x -> x - S, x])] /; FreeQ[p, x] && PolyQ[P
n, x] && GtQ[Expon[Pn, x], 2] && NeQ[Coeff[Pn, x, Expon[Pn, x] - 1], 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.14 (sec) , antiderivative size = 400, normalized size of antiderivative = 1.48

method	result
default	$-\frac{\frac{3(7+2a)x^7}{16\left(a^4+14a^3+73a^2+168a+144\right)}-\frac{21(7+2a)x^6}{16\left(a^2+8a+16\right)\left(a^2+6a+9\right)}+\frac{\left(7a^2+343a+1116\right)x^5}{32a^4+448a^3+2336a^2+5376a+4608}-\frac{5\left(7a^2+175a+528\right)x^4}{32\left(a^4+14a^3+73a^2+168a+144\right)}+\frac{}{16a}}{(-x^4+4x^3-8x^2}$
risch	$\frac{-\frac{3(7+2a)x^7}{16\left(a^4+14a^3+73a^2+168a+144\right)}+\frac{21(7+2a)x^6}{16\left(a^2+8a+16\right)\left(a^2+6a+9\right)}-\frac{\left(7a^2+343a+1116\right)x^5}{32\left(a^4+14a^3+73a^2+168a+144\right)}+\frac{5\left(7a^2+175a+528\right)x^4}{32\left(a^4+14a^3+73a^2+168a+144\right)}-\frac{(34}{16(a^4+}}{(-x^4+4x^3-8x^2+a}$

input
```
int(1/(-x^4+4*x^3-8*x^2+a+8*x)^3,x,method=_RETURNVERBOSE)
```

output
```
-(3/16*(7+2*a)/(a^4+14*a^3+73*a^2+168*a+144)*x^7-21/16*(7+2*a)/(a^2+8*a+16
)/(a^2+6*a+9)*x^6+1/32*(7*a^2+343*a+1116)/(a^4+14*a^3+73*a^2+168*a+144)*x^
5-5/32*(7*a^2+175*a+528)/(a^4+14*a^3+73*a^2+168*a+144)*x^4+1/16*(34*a^2+67
9*a+1968)/(a^4+14*a^3+73*a^2+168*a+144)*x^3-1/16*(32*a^2+623*a+1800)/(a^4+
14*a^3+73*a^2+168*a+144)*x^2-1/32*(11*a^3+107*a^2-84*a-1152)/(a^4+14*a^3+7
3*a^2+168*a+144)*x+1/32*(11*a^3+131*a^2+408*a+288)/(a+3)/(a^3+11*a^2+40*a+
48))/(-x^4+4*x^3-8*x^2+a+8*x)^2-3/128/(a^3+10*a^2+33*a+36)/(4+a)*sum((-108
+2*(-2*a-7)*_R^2+4*(7+2*a)*_R-7*a^2-55*a)/(-_R^3+3*_R^2-4*_R+2)*ln(x-_R),_
R=RootOf(_Z^4-4*_Z^3+8*_Z^2-8*_Z-a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3971 vs. $2(222) = 444$.

Time = 0.14 (sec) , antiderivative size = 3971, normalized size of antiderivative = 14.71

$$\int \frac{1}{(a+8x-8x^2+4x^3-x^4)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(-x^4+4*x^3-8*x^2+a+8*x)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 697 vs. $2(236) = 472$.

Time = 8.58 (sec) , antiderivative size = 697, normalized size of antiderivative = 2.58

$$\int \frac{1}{(a+8x-8x^2+4x^3-x^4)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(-x**4+4*x**3-8*x**2+a+8*x)**3,x)
```

output
```
-(11*a**3 + 131*a**2 + 408*a + x**7*(12*a + 42) + x**6*(-84*a - 294) + x**
5*(7*a**2 + 343*a + 1116) + x**4*(-35*a**2 - 875*a - 2640) + x**3*(68*a**2
 + 1358*a + 3936) + x**2*(-64*a**2 - 1246*a - 3600) + x*(-11*a**3 - 107*a*
*2 + 84*a + 1152) + 288)/(32*a**6 + 448*a**5 + 2336*a**4 + 5376*a**3 + 460
8*a**2 + x**8*(32*a**4 + 448*a**3 + 2336*a**2 + 5376*a + 4608) + x**7*(-25
6*a**4 - 3584*a**3 - 18688*a**2 - 43008*a - 36864) + x**6*(1024*a**4 + 143
36*a**3 + 74752*a**2 + 172032*a + 147456) + x**5*(-2560*a**4 - 35840*a**3
- 186880*a**2 - 430080*a - 368640) + x**4*(-64*a**5 + 3200*a**4 + 52672*a*
*3 + 288256*a**2 + 678912*a + 589824) + x**3*(256*a**5 - 512*a**4 - 38656*
a**3 - 256000*a**2 - 651264*a - 589824) + x**2*(-512*a**5 - 5120*a**4 - 87
04*a**3 + 63488*a**2 + 270336*a + 294912) + x*(512*a**5 + 7168*a**4 + 3737
6*a**3 + 86016*a**2 + 73728*a)) - RootSum(_t**4*(268435456*a**15 + 1476395
0080*a**14 + 378493992960*a**13 + 5999532441600*a**12 + 65757291479040*a**
11 + 527875908304896*a**10 + 3206246773555200*a**9 + 15003759578972160*a**
8 + 54537151127224320*a**7 + 153980418717122560*a**6 + 334927734494986240*
a**5 + 551152193655275520*a**4 + 664192984106926080*a**3 + 553362212027105
280*a**2 + 284993413919539200*a + 68398419340689408) + _t**2*(-30965760*a*
*9 - 1052835840*a**8 - 15910207488*a**7 - 140262506496*a**6 - 795007254528
*a**5 - 3004516270080*a**4 - 7571263979520*a**3 - 12268037210112*a**2 - 11
598827618304*a - 4875324751872) - 194481*a**4 - 2762424*a**3 - 14762736...
```

Maxima [F]

$$\int \frac{1}{(a+8x-8x^2+4x^3-x^4)^3}\,dx = \int -\frac{1}{(x^4-4\,x^3+8\,x^2-a-8\,x)^3}\,dx$$

input
```
integrate(1/(-x^4+4*x^3-8*x^2+a+8*x)^3,x, algorithm="maxima")
```

output
```
-1/32*(6*(2*a + 7)*x^7 - 42*(2*a + 7)*x^6 + (7*a^2 + 343*a + 1116)*x^5 - 5
*(7*a^2 + 175*a + 528)*x^4 + 2*(34*a^2 + 679*a + 1968)*x^3 + 11*a^3 - 2*(3
2*a^2 + 623*a + 1800)*x^2 + 131*a^2 - (11*a^3 + 107*a^2 - 84*a - 1152)*x +
 408*a + 288)/((a^4 + 14*a^3 + 73*a^2 + 168*a + 144)*x^8 - 8*(a^4 + 14*a^3
 + 73*a^2 + 168*a + 144)*x^7 + 32*(a^4 + 14*a^3 + 73*a^2 + 168*a + 144)*x^
6 + a^6 - 80*(a^4 + 14*a^3 + 73*a^2 + 168*a + 144)*x^5 + 14*a^5 - 2*(a^5 -
 50*a^4 - 823*a^3 - 4504*a^2 - 10608*a - 9216)*x^4 + 73*a^4 + 8*(a^5 - 2*a
^4 - 151*a^3 - 1000*a^2 - 2544*a - 2304)*x^3 + 168*a^3 - 16*(a^5 + 10*a^4
+ 17*a^3 - 124*a^2 - 528*a - 576)*x^2 + 144*a^2 + 16*(a^5 + 14*a^4 + 73*a^
3 + 168*a^2 + 144*a)*x) - 3/32*integrate((2*(2*a + 7)*x^2 + 7*a^2 - 4*(2*a
 + 7)*x + 55*a + 108)/(x^4 - 4*x^3 + 8*x^2 - a - 8*x), x)/(a^4 + 14*a^3 +
73*a^2 + 168*a + 144)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 16632 vs. $2(222) = 444$.

Time = 11.69 (sec) , antiderivative size = 16632, normalized size of antiderivative = 61.60

$$\int \frac{1}{(a+8x-8x^2+4x^3-x^4)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(-x^4+4*x^3-8*x^2+a+8*x)^3,x, algorithm="giac")
```

output
```
-3/128*(sqrt((105*a^5 + 1890*a^4 + 13629*a^3 + 49224*a^2 + (49*a^5 + 926*a
^4 + 6997*a^3 + 26428*a^2 + 49904*a + 37696)*sqrt(a + 4) + 89056*a + 64576
)/(a^3 + 11*a^2 + 40*a + 48))*log(abs(16807*sqrt(a + 4)*a^15 + 26411*a^15
+ 908950*sqrt(a + 4)*a^14 + 1420804*a^14 + 22929088*sqrt(a + 4)*a^13 + 240
1*sqrt(105*a^6 + 2205*a^5 + 19299*a^4 + 90111*a^3 + 236728*a^2 + (49*a^6 +
 1073*a^5 + 9775*a^4 + 47419*a^3 + 129188*a^2 + 187408*a + 113088)*sqrt(a
+ 4) + 331744*a + 193728)*a^12*x + 35650176*a^13 + 2401*sqrt(105*a^6 + 220
5*a^5 + 19299*a^4 + 90111*a^3 + 236728*a^2 + (49*a^6 + 1073*a^5 + 9775*a^4
 + 47419*a^3 + 129188*a^2 + 187408*a + 113088)*sqrt(a + 4) + 331744*a + 19
3728)*sqrt(a + 4)*a^11*x - 2401*sqrt(105*a^6 + 2205*a^5 + 19299*a^4 + 9011
1*a^3 + 236728*a^2 + (49*a^6 + 1073*a^5 + 9775*a^4 + 47419*a^3 + 129188*a^
2 + 187408*a + 113088)*sqrt(a + 4) + 331744*a + 193728)*a^12 + 357887692*s
qrt(a + 4)*a^12 + 105154*sqrt(105*a^6 + 2205*a^5 + 19299*a^4 + 90111*a^3 +
 236728*a^2 + (49*a^6 + 1073*a^5 + 9775*a^4 + 47419*a^3 + 129188*a^2 + 187
408*a + 113088)*sqrt(a + 4) + 331744*a + 193728)*a^11*x - 2401*sqrt(105*a^
6 + 2205*a^5 + 19299*a^4 + 90111*a^3 + 236728*a^2 + (49*a^6 + 1073*a^5 + 9
775*a^4 + 47419*a^3 + 129188*a^2 + 187408*a + 113088)*sqrt(a + 4) + 331744
*a + 193728)*sqrt(a + 4)*a^11 + 553458148*a^12 + 95550*sqrt(105*a^6 + 2205
*a^5 + 19299*a^4 + 90111*a^3 + 236728*a^2 + (49*a^6 + 1073*a^5 + 9775*a^4
+ 47419*a^3 + 129188*a^2 + 187408*a + 113088)*sqrt(a + 4) + 331744*a + ...
```

Mupad [B] (verification not implemented)

Time = 26.08 (sec) , antiderivative size = 8242, normalized size of antiderivative = 30.53

$$\int \frac{1}{(a+8x-8x^2+4x^3-x^4)^3}\,dx = \text{Too large to display}$$

input
```
int(1/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^3,x)
```

output
```
atan(((((52357496832*a + 57139003392*a^2 + 36322148352*a^3 + 14822473728*a
^4 + 4027170816*a^5 + 728506368*a^6 + 84615168*a^7 + 5726208*a^8 + 172032*
a^9 + 21290287104)/(16384*(940032*a + 1195776*a^2 + 899328*a^3 + 442864*a^
4 + 149208*a^5 + 34833*a^6 + 5564*a^7 + 582*a^8 + 36*a^9 + a^10 + 331776))
 + ((4290672328704*a + 6001143054336*a^2 + 5025917042688*a^3 + 28005200035
84*a^4 + 1090200272896*a^5 + 302556119040*a^6 + 59862155264*a^7 + 82753617
92*a^8 + 761266176*a^9 + 41943040*a^10 + 1048576*a^11 + 1391569403904)/(16
384*(940032*a + 1195776*a^2 + 899328*a^3 + 442864*a^4 + 149208*a^5 + 34833
*a^6 + 5564*a^7 + 582*a^8 + 36*a^9 + a^10 + 331776)) - (x*(3510632448*a +
4020240384*a^2 + 2678587392*a^3 + 1144324096*a^4 + 325074944*a^5 + 6140723
2*a^6 + 7438336*a^7 + 524288*a^8 + 16384*a^9 + 1358954496))/(256*(48384*a
+ 49248*a^2 + 28560*a^3 + 10321*a^4 + 2380*a^5 + 342*a^6 + 28*a^7 + a^8 +
20736)))*((9*(39329792*a - 338*a*((a + 4)^15)^(1/2) - 589*((a + 4)^15)^(1/
2) - 49*a^2*((a + 4)^15)^(1/2) + 41598976*a^2 + 25672960*a^3 + 10187840*a^
4 + 2695744*a^5 + 475608*a^6 + 53949*a^7 + 3570*a^8 + 105*a^9 + 16531456))
/(16384*(1061683200*a + 2061434880*a^2 + 2474311680*a^3 + 2053201920*a^4 +
 1247703040*a^5 + 573621760*a^6 + 203166720*a^7 + 55893360*a^8 + 11944200*
a^9 + 1966491*a^10 + 244965*a^11 + 22350*a^12 + 1410*a^13 + 55*a^14 + a^15
 + 254803968)))^(1/2))*((9*(39329792*a - 338*a*((a + 4)^15)^(1/2) - 589*((
a + 4)^15)^(1/2) - 49*a^2*((a + 4)^15)^(1/2) + 41598976*a^2 + 25672960*...
```

Reduce [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 6470, normalized size of antiderivative = 23.96

$$\int \frac{1}{(a+8x-8x^2+4x^3-x^4)^3}\,dx = \text{Too large to display}$$

input
```
int(1/(-x^4+4*x^3-8*x^2+a+8*x)^3,x)
```

output
```
( - 66*sqrt(a + 4)*sqrt(sqrt(a + 4) + 1)*atan((x - 1)/sqrt(sqrt(a + 4) + 1
))*a**4 + 132*sqrt(a + 4)*sqrt(sqrt(a + 4) + 1)*atan((x - 1)/sqrt(sqrt(a +
 4) + 1))*a**3*x**4 - 528*sqrt(a + 4)*sqrt(sqrt(a + 4) + 1)*atan((x - 1)/s
qrt(sqrt(a + 4) + 1))*a**3*x**3 + 1056*sqrt(a + 4)*sqrt(sqrt(a + 4) + 1)*a
tan((x - 1)/sqrt(sqrt(a + 4) + 1))*a**3*x**2 - 1056*sqrt(a + 4)*sqrt(sqrt(
a + 4) + 1)*atan((x - 1)/sqrt(sqrt(a + 4) + 1))*a**3*x - 462*sqrt(a + 4)*s
qrt(sqrt(a + 4) + 1)*atan((x - 1)/sqrt(sqrt(a + 4) + 1))*a**3 - 66*sqrt(a
+ 4)*sqrt(sqrt(a + 4) + 1)*atan((x - 1)/sqrt(sqrt(a + 4) + 1))*a**2*x**8 +
 528*sqrt(a + 4)*sqrt(sqrt(a + 4) + 1)*atan((x - 1)/sqrt(sqrt(a + 4) + 1))
*a**2*x**7 - 2112*sqrt(a + 4)*sqrt(sqrt(a + 4) + 1)*atan((x - 1)/sqrt(sqrt
(a + 4) + 1))*a**2*x**6 + 5280*sqrt(a + 4)*sqrt(sqrt(a + 4) + 1)*atan((x -
 1)/sqrt(sqrt(a + 4) + 1))*a**2*x**5 - 7524*sqrt(a + 4)*sqrt(sqrt(a + 4) +
 1)*atan((x - 1)/sqrt(sqrt(a + 4) + 1))*a**2*x**4 + 4752*sqrt(a + 4)*sqrt(
sqrt(a + 4) + 1)*atan((x - 1)/sqrt(sqrt(a + 4) + 1))*a**2*x**3 + 3168*sqrt
(a + 4)*sqrt(sqrt(a + 4) + 1)*atan((x - 1)/sqrt(sqrt(a + 4) + 1))*a**2*x**
2 - 7392*sqrt(a + 4)*sqrt(sqrt(a + 4) + 1)*atan((x - 1)/sqrt(sqrt(a + 4) +
 1))*a**2*x - 816*sqrt(a + 4)*sqrt(sqrt(a + 4) + 1)*atan((x - 1)/sqrt(sqrt
(a + 4) + 1))*a**2 - 462*sqrt(a + 4)*sqrt(sqrt(a + 4) + 1)*atan((x - 1)/sq
rt(sqrt(a + 4) + 1))*a*x**8 + 3696*sqrt(a + 4)*sqrt(sqrt(a + 4) + 1)*atan(
(x - 1)/sqrt(sqrt(a + 4) + 1))*a*x**7 - 14784*sqrt(a + 4)*sqrt(sqrt(a +...
```

3.50 $\int \left(8x-8x^2+4x^3-x^4\right)^{3/2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 116

$$\begin{aligned}\int \left(8x-8x^2+4x^3-x^4\right)^{3/2}\,dx= & -\frac{2}{35}\left(13-3(1-x)^2\right)\sqrt{3-2(1-x)^2-(1-x)^4}(1-x) \\ & -\frac{1}{7}\left(3-2(1-x)^2-(1-x)^4\right)^{3/2}(1-x) \\ & +\frac{16}{5}\sqrt{3}E\left(\arcsin(1-x)\left|-\frac{1}{3}\right.\right)-\frac{176}{35}\sqrt{3}\,\text{EllipticF}\left(\arcsin(1-x),-\frac{1}{3}\right)\end{aligned}$$

output
```
-2/35*(13-3*(1-x)^2)*(3-2*(1-x)^2-(1-x)^4)^(1/2)*(1-x)-1/7*(3-2*(1-x)^2-(1
-x)^4)^(3/2)*(1-x)-16/5*3^(1/2)*EllipticE(-1+x,1/3*I*3^(1/2))+176/35*3^(1/
2)*EllipticF(-1+x,1/3*I*3^(1/2))
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 22.81 (sec) , antiderivative size = 278, normalized size of antiderivative = 2.40

$$\int (8x - 8x^2 + 4x^3 - x^4)^{3/2}\, dx = \frac{896 - 1056x + 352x^2 + 848x^3 - 1420x^4 + 1152x^5 - 602x^6 + 206x^7 - 45x^8 + 5x^9 + \frac{112i\sqrt{2}(-}{\cdots}}{35\sqrt{\cdots}}$$

input
```
Integrate[(8*x - 8*x^2 + 4*x^3 - x^4)^(3/2),x]
```

output
```
(896 - 1056*x + 352*x^2 + 848*x^3 - 1420*x^4 + 1152*x^5 - 602*x^6 + 206*x^
7 - 45*x^8 + 5*x^9 + ((112*I)*Sqrt[2]*(-2 + x)*x*Sqrt[(4 - 2*x + x^2)/x^2]
*EllipticE[ArcSin[Sqrt[I + Sqrt[3] - (4*I)/x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[
3])/(-I + Sqrt[3])])/Sqrt[((-I)*(-2 + x))/((-I + Sqrt[3])*x)] - (304*I)*Sq
rt[2]*Sqrt[((-I)*(-2 + x))/((-I + Sqrt[3])*x)]*x^2*Sqrt[(4 - 2*x + x^2)/x^
2]*EllipticF[ArcSin[Sqrt[I + Sqrt[3] - (4*I)/x]/(Sqrt[2]*3^(1/4))], (2*Sqr
t[3])/(-I + Sqrt[3])])/(35*Sqrt[-(x*(-8 + 8*x - 4*x^2 + x^3))])
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.93, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {2458, 1404, 27, 1490, 27, 1494, 27, 399, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(-x^4 + 4x^3 - 8x^2 + 8x\right)^{3/2} dx$$

$$\Big\downarrow \text{ 2458}$$

$$\int \left(-(x-1)^4 - 2(x-1)^2 + 3\right)^{3/2} d(x-1)$$

$$\Big\downarrow \text{1404}$$

$$\frac{3}{7}\int 2\left(3-(x-1)^2\right)\sqrt{-(x-1)^4-2(x-1)^2+3}d(x-1)+\frac{1}{7}(x-1)\left(-(x-1)^4-2(x-1)^2+3\right)^{3/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{6}{7}\int \left(3-(x-1)^2\right)\sqrt{-(x-1)^4-2(x-1)^2+3}d(x-1)+\frac{1}{7}(x-1)\left(-(x-1)^4-2(x-1)^2+3\right)^{3/2}$$

$$\Big\downarrow \text{1490}$$

$$\frac{6}{7}\left(\frac{1}{15}\left(13-3(x-1)^2\right)\sqrt{-(x-1)^4-2(x-1)^2+3}(x-1)-\frac{1}{15}\int -\frac{8\left(12-7(x-1)^2\right)}{\sqrt{-(x-1)^4-2(x-1)^2+3}}d(x-1)\right)+\frac{1}{7}(x-1)\left(-(x-1)^4-2(x-1)^2+3\right)^{3/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{6}{7}\left(\frac{8}{15}\int \frac{12-7(x-1)^2}{\sqrt{-(x-1)^4-2(x-1)^2+3}}d(x-1)+\frac{1}{15}\left(13-3(x-1)^2\right)\sqrt{-(x-1)^4-2(x-1)^2+3}(x-1)\right)+\frac{1}{7}(x-1)\left(-(x-1)^4-2(x-1)^2+3\right)^{3/2}$$

$$\Big\downarrow \text{1494}$$

$$\frac{6}{7}\left(\frac{16}{15}\int \frac{12-7(x-1)^2}{2\sqrt{1-(x-1)^2}\sqrt{(x-1)^2+3}}d(x-1)+\frac{1}{15}\left(13-3(x-1)^2\right)\sqrt{-(x-1)^4-2(x-1)^2+3}(x-1)\right)+\frac{1}{7}(x-1)\left(-(x-1)^4-2(x-1)^2+3\right)^{3/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{6}{7}\left(\frac{8}{15}\int \frac{12-7(x-1)^2}{\sqrt{1-(x-1)^2}\sqrt{(x-1)^2+3}}d(x-1)+\frac{1}{15}\left(13-3(x-1)^2\right)\sqrt{-(x-1)^4-2(x-1)^2+3}(x-1)\right)+\frac{1}{7}(x-1)\left(-(x-1)^4-2(x-1)^2+3\right)^{3/2}$$

$$\Big\downarrow \text{399}$$

$$\frac{6}{7}\left(\frac{8}{15}\left(33\int \frac{1}{\sqrt{1-(x-1)^2}\sqrt{(x-1)^2+3}}d(x-1)-7\int \frac{\sqrt{(x-1)^2+3}}{\sqrt{1-(x-1)^2}}d(x-1)\right)+\frac{1}{15}\left(13-3(x-1)^2\right)\sqrt{-\cdots}\right.$$
$$\frac{1}{7}(x-1)\left(-(x-1)^4-2(x-1)^2+3\right)^{3/2}$$

$\Big\downarrow$ 321

$$\frac{6}{7}\left(\frac{8}{15}\left(-7\int \frac{\sqrt{(x-1)^2+3}}{\sqrt{1-(x-1)^2}}d(x-1)-11\sqrt{3}\,\text{EllipticF}\left(\arcsin(1-x),-\frac{1}{3}\right)\right)+\frac{1}{15}\left(13-3(x-1)^2\right)\sqrt{-(x-\cdots}\right.$$
$$\frac{1}{7}(x-1)\left(-(x-1)^4-2(x-1)^2+3\right)^{3/2}$$

$\Big\downarrow$ 327

$$\frac{6}{7}\left(\frac{8}{15}\left(7\sqrt{3}E\left(\arcsin(1-x)\middle|-\frac{1}{3}\right)-11\sqrt{3}\,\text{EllipticF}\left(\arcsin(1-x),-\frac{1}{3}\right)\right)+\frac{1}{15}\left(13-3(x-1)^2\right)\sqrt{-(x-1)^4\cdots}\right.$$
$$\frac{1}{7}(x-1)\left(-(x-1)^4-2(x-1)^2+3\right)^{3/2}$$

input
```
Int[(8*x - 8*x^2 + 4*x^3 - x^4)^(3/2),x]
```

output
```
((3 - 2*(-1 + x)^2 - (-1 + x)^4)^(3/2)*(-1 + x))/7 + (6*(((13 - 3*(-1 + x)
^2)*Sqrt[3 - 2*(-1 + x)^2 - (-1 + x)^4]*(-1 + x))/15 + (8*(7*Sqrt[3]*Ellip
ticE[ArcSin[1 - x], -1/3] - 11*Sqrt[3]*EllipticF[ArcSin[1 - x], -1/3]))/15
))/7
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 1404
```
Int[((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*((a + b
*x^2 + c*x^4)^p/(4*p + 1)), x] + Simp[2*(p/(4*p + 1))   Int[(2*a + b*x^2)*(
a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*
c, 0] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 1490
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Simp[2*(p/(c*(4*p + 1)*(4*p + 3)))
 Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3)
- b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

rule 1494
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[(d + e*x^2)/(Sqr
t[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e
}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

rule 2458
```
Int[(Pn_)^(p_.), x_Symbol] :> With[{S = Coeff[Pn, x, Expon[Pn, x] - 1]/(Exp
on[Pn, x]*Coeff[Pn, x, Expon[Pn, x]])}, Subst[Int[ExpandToSum[Pn /. x -> x
- S, x]^p, x], x, x + S] /; BinomialQ[Pn /. x -> x - S, x] || (IntegerQ[Exp
on[Pn, x]/2] && TrinomialQ[Pn /. x -> x - S, x])] /; FreeQ[p, x] && PolyQ[P
n, x] && GtQ[Expon[Pn, x], 2] && NeQ[Coeff[Pn, x, Expon[Pn, x] - 1], 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 957 vs. 2(100) = 200.

Time = 1.97 (sec) , antiderivative size = 958, normalized size of antiderivative = 8.26

method	result	size
risch	Expression too large to display	958
default	Expression too large to display	1050
elliptic	Expression too large to display	1050

input
```
int((-x^4+4*x^3-8*x^2+8*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/35*(5*x^5-25*x^4+66*x^3-98*x^2+32*x+20)*x*(x^3-4*x^2+8*x-8)/(-x*(x^3-4*x
^2+8*x-8))^(1/2)+32/7*(-1-I*3^(1/2))*((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2)
)^(1/2)*(x-2)^2*((x-1+I*3^(1/2))/(1-I*3^(1/2))/(x-2))^(1/2)*((x-1-I*3^(1/2
))/(1+I*3^(1/2))/(x-2))^(1/2)/(-1+I*3^(1/2))/(-x*(x-2)*(x-1+I*3^(1/2))*(x-
1-I*3^(1/2)))^(1/2)*EllipticF(((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2)
,((1+I*3^(1/2))*(-1-I*3^(1/2))/(-1+I*3^(1/2))/(1-I*3^(1/2)))^(1/2))+64/5*(
-1-I*3^(1/2))*((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2)*(x-2)^2*((x-1+I
*3^(1/2))/(1-I*3^(1/2))/(x-2))^(1/2)*((x-1-I*3^(1/2))/(1+I*3^(1/2))/(x-2))
^(1/2)/(-1+I*3^(1/2))/(-x*(x-2)*(x-1+I*3^(1/2))*(x-1-I*3^(1/2)))^(1/2)*(2*
EllipticF(((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2),((1+I*3^(1/2))*(-1-
I*3^(1/2))/(-1+I*3^(1/2))/(1-I*3^(1/2)))^(1/2))-2*EllipticPi(((-1+I*3^(1/2
))*x/(1+I*3^(1/2))/(x-2))^(1/2),(1+I*3^(1/2))/(-1+I*3^(1/2)),((1+I*3^(1/2)
)*(-1-I*3^(1/2))/(-1+I*3^(1/2))/(1-I*3^(1/2)))^(1/2)))-16/5*(x*(x-1+I*3^(1
/2))*(x-1-I*3^(1/2))+2*(-1-I*3^(1/2))*((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2
))^(1/2)*(x-2)^2*((x-1+I*3^(1/2))/(1-I*3^(1/2))/(x-2))^(1/2)*((x-1-I*3^(1/
2))/(1+I*3^(1/2))/(x-2))^(1/2)*(1/2*(6+2*I*3^(1/2))/(-1+I*3^(1/2))*Ellipti
cF(((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2),((1+I*3^(1/2))*(-1-I*3^(1/
2))/(-1+I*3^(1/2))/(1-I*3^(1/2)))^(1/2))+1/2*(-1+I*3^(1/2))*EllipticE(((-1
+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2),((1+I*3^(1/2))*(-1-I*3^(1/2))/(-1
+I*3^(1/2))/(1-I*3^(1/2)))^(1/2))-4/(-1+I*3^(1/2))*EllipticPi(((-1+I*3^...
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.78

$$\int \left(8x - 8x^2 + 4x^3 - x^4\right)^{3/2} dx =$$
$$-\frac{112\,(-i\,x + i)E(\arcsin\left(\frac{1}{x-1}\right)|-3) + 80\,(-i\,x + i)F(\arcsin\left(\frac{1}{x-1}\right)|-3) + (5\,x^6 - 30\,x^5 + 91\,x^4 - 164\,x^3}{35\,(x-1)}$$

input
```
integrate((-x^4+4*x^3-8*x^2+8*x)^(3/2),x, algorithm="fricas")
```

output
```
-1/35*(112*(-I*x + I)*elliptic_e(arcsin(1/(x - 1)), -3) + 80*(-I*x + I)*el
liptic_f(arcsin(1/(x - 1)), -3) + (5*x^6 - 30*x^5 + 91*x^4 - 164*x^3 + 130
*x^2 - 12*x - 132)*sqrt(-x^4 + 4*x^3 - 8*x^2 + 8*x))/(x - 1)
```

Sympy [F]

$$\int \left(8x - 8x^2 + 4x^3 - x^4\right)^{3/2} dx = \int \left(-x^4 + 4x^3 - 8x^2 + 8x\right)^{\frac{3}{2}} dx$$

input
```
integrate((-x**4+4*x**3-8*x**2+8*x)**(3/2),x)
```

output
```
Integral((-x**4 + 4*x**3 - 8*x**2 + 8*x)**(3/2), x)
```

Maxima [F]

$$\int \left(8x - 8x^2 + 4x^3 - x^4\right)^{3/2} dx = \int \left(-x^4 + 4\,x^3 - 8\,x^2 + 8\,x\right)^{\frac{3}{2}} dx$$

input
```
integrate((-x^4+4*x^3-8*x^2+8*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-x^4 + 4*x^3 - 8*x^2 + 8*x)^(3/2), x)
```

Giac [F]

$$\int \left(8x-8x^2+4x^3-x^4\right)^{3/2}\,dx=\int \left(-x^4+4\,x^3-8\,x^2+8\,x\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((-x^4+4*x^3-8*x^2+8*x)^(3/2),x, algorithm="giac")
```

output
```
integrate((-x^4 + 4*x^3 - 8*x^2 + 8*x)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(8x-8x^2+4x^3-x^4\right)^{3/2}\,dx=\int \left(-x^4+4\,x^3-8\,x^2+8\,x\right)^{3/2}\,dx$$

input
```
int((8*x - 8*x^2 + 4*x^3 - x^4)^(3/2),x)
```

output
```
int((8*x - 8*x^2 + 4*x^3 - x^4)^(3/2), x)
```

Reduce [F]

$$\begin{aligned}\int \left(8x-8x^2+4x^3-x^4\right)^{3/2}\,dx&=-\frac{\sqrt{x}\,\sqrt{-x^3+4x^2-8x+8}\,x^5}{7}\\&+\frac{5\sqrt{x}\,\sqrt{-x^3+4x^2-8x+8}\,x^4}{7}-\frac{66\sqrt{x}\,\sqrt{-x^3+4x^2-8x+8}\,x^3}{35}\\&+\frac{14\sqrt{x}\,\sqrt{-x^3+4x^2-8x+8}\,x^2}{5}-\frac{32\sqrt{x}\,\sqrt{-x^3+4x^2-8x+8}\,x}{35}\\&-\frac{116\sqrt{x}\,\sqrt{-x^3+4x^2-8x+8}}{105}+\frac{16\left(\int \frac{\sqrt{x}\,\sqrt{-x^3+4x^2-8x+8}\,x^2}{x^3-4x^2+8x-8}dx\right)}{15}\\&-\frac{464\left(\int \frac{\sqrt{x}\,\sqrt{-x^3+4x^2-8x+8}}{x^4-4x^3+8x^2-8x}dx\right)}{105}-\frac{32\left(\int \frac{\sqrt{x}\,\sqrt{-x^3+4x^2-8x+8}}{x^3-4x^2+8x-8}dx\right)}{15}\end{aligned}$$

input
```
int((-x^4+4*x^3-8*x^2+8*x)^(3/2),x)
```

output
```
( - 15*sqrt(x)*sqrt( - x**3 + 4*x**2 - 8*x + 8)*x**5 + 75*sqrt(x)*sqrt( -
x**3 + 4*x**2 - 8*x + 8)*x**4 - 198*sqrt(x)*sqrt( - x**3 + 4*x**2 - 8*x +
8)*x**3 + 294*sqrt(x)*sqrt( - x**3 + 4*x**2 - 8*x + 8)*x**2 - 96*sqrt(x)*s
qrt( - x**3 + 4*x**2 - 8*x + 8)*x - 116*sqrt(x)*sqrt( - x**3 + 4*x**2 - 8*
x + 8) + 112*int((sqrt(x)*sqrt( - x**3 + 4*x**2 - 8*x + 8)*x**2)/(x**3 - 4
*x**2 + 8*x - 8),x) - 464*int((sqrt(x)*sqrt( - x**3 + 4*x**2 - 8*x + 8))/(
x**4 - 4*x**3 + 8*x**2 - 8*x),x) - 224*int((sqrt(x)*sqrt( - x**3 + 4*x**2
- 8*x + 8))/(x**3 - 4*x**2 + 8*x - 8),x))/105
```

3.51 $\int \sqrt{8x-8x^2+4x^3-x^4}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 68

$$\int \sqrt{8x-8x^2+4x^3-x^4}\,dx = -\frac{1}{3}\sqrt{3-2(1-x)^2-(1-x)^4}(1-x) + \frac{2E\left(\arcsin(1-x)\left|-\frac{1}{3}\right.\right)}{\sqrt{3}} - \frac{4\,\text{EllipticF}\left(\arcsin(1-x),-\frac{1}{3}\right)}{\sqrt{3}}$$

output
```
-1/3*(3-2*(1-x)^2-(1-x)^4)^(1/2)*(1-x)-2/3*3^(1/2)*EllipticE(-1+x,1/3*I*3^
(1/2))+4/3*3^(1/2)*EllipticF(-1+x,1/3*I*3^(1/2))
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 22.74 (sec) , antiderivative size = 256, normalized size of antiderivative = 3.76

$$\int \sqrt{8x-8x^2+4x^3-x^4}\,dx =$$

$$-\frac{-16+24x-24x^2+14x^3-5x^4+x^5-\frac{2i\sqrt{2}(-2+x)x\sqrt{\frac{4-2x+x^2}{x^2}}E\left(\arcsin\left(\frac{\sqrt{i+\sqrt{3}-\frac{4i}{x}}}{\sqrt{2}\sqrt[4]{3}}\right)\middle|\frac{2\sqrt{3}}{-i+\sqrt{3}}\right)}{\sqrt{-\frac{i(-2+x)}{\left(-i+\sqrt{3}\right)x}}}+8i\sqrt{2}\sqrt{-\frac{i(-}{(-i}}}{3\sqrt{-x\left(-8+8x-4x^2+x^3\right)}}$$

input
```
Integrate[Sqrt[8*x - 8*x^2 + 4*x^3 - x^4],x]
```

output
```
-1/3*(-16 + 24*x - 24*x^2 + 14*x^3 - 5*x^4 + x^5 - ((2*I)*Sqrt[2]*(-2 + x)
*x*Sqrt[(4 - 2*x + x^2)/x^2]*EllipticE[ArcSin[Sqrt[I + Sqrt[3] - (4*I)/x]/
(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(-I + Sqrt[3])])/Sqrt[((-I)*(-2 + x))/((-I
 + Sqrt[3])*x)] + (8*I)*Sqrt[2]*Sqrt[((-I)*(-2 + x))/((-I + Sqrt[3])*x)]*x
^2*Sqrt[(4 - 2*x + x^2)/x^2]*EllipticF[ArcSin[Sqrt[I + Sqrt[3] - (4*I)/x]/
(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(-I + Sqrt[3])])/Sqrt[-(x*(-8 + 8*x - 4*x^
2 + x^3))]
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.97, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {2458, 1404, 27, 1494, 27, 399, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{-x^4+4x^3-8x^2+8x}\,dx$$

$$\Big\downarrow \text{2458}$$

$$\int \sqrt{-(x-1)^4-2(x-1)^2+3}d(x-1)$$

$$\Big\downarrow \text{1404}$$

$$\frac{1}{3}\int \frac{2\left(3-(x-1)^2\right)}{\sqrt{-(x-1)^4-2(x-1)^2+3}}d(x-1)+\frac{1}{3}\sqrt{-(x-1)^4-2(x-1)^2+3}(x-1)$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{3}\int \frac{3-(x-1)^2}{\sqrt{-(x-1)^4-2(x-1)^2+3}}d(x-1)+\frac{1}{3}\sqrt{-(x-1)^4-2(x-1)^2+3}(x-1)$$

$$\Big\downarrow \text{1494}$$

$$\frac{4}{3}\int \frac{3-(x-1)^2}{2\sqrt{1-(x-1)^2}\sqrt{(x-1)^2+3}}d(x-1)+\frac{1}{3}\sqrt{-(x-1)^4-2(x-1)^2+3}(x-1)$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{3}\int \frac{3-(x-1)^2}{\sqrt{1-(x-1)^2}\sqrt{(x-1)^2+3}}d(x-1)+\frac{1}{3}\sqrt{-(x-1)^4-2(x-1)^2+3}(x-1)$$

$$\Big\downarrow \ 399$$

$$\frac{2}{3}\left(6\int \frac{1}{\sqrt{1-(x-1)^2}\sqrt{(x-1)^2+3}}d(x-1)-\int \frac{\sqrt{(x-1)^2+3}}{\sqrt{1-(x-1)^2}}d(x-1)\right)+\frac{1}{3}\sqrt{-(x-1)^4-2(x-1)^2+3}(x-1)$$

$$\Big\downarrow \ 321$$

$$\frac{2}{3}\left(-\int \frac{\sqrt{(x-1)^2+3}}{\sqrt{1-(x-1)^2}}d(x-1)-2\sqrt{3}\,\text{EllipticF}\left(\arcsin(1-x),-\frac{1}{3}\right)\right)+\frac{1}{3}\sqrt{-(x-1)^4-2(x-1)^2+3}(x-1)$$

$$\Big\downarrow \ 327$$

$$\frac{2}{3}\left(\sqrt{3}E\left(\arcsin(1-x)\left|-\frac{1}{3}\right.\right)-2\sqrt{3}\,\text{EllipticF}\left(\arcsin(1-x),-\frac{1}{3}\right)\right)+\frac{1}{3}\sqrt{-(x-1)^4-2(x-1)^2+3}(x-1)$$

input
```
Int[Sqrt[8*x - 8*x^2 + 4*x^3 - x^4],x]
```

output
```
(Sqrt[3 - 2*(-1 + x)^2 - (-1 + x)^4]*(-1 + x))/3 + (2*(Sqrt[3]*EllipticE[A
rcSin[1 - x], -1/3] - 2*Sqrt[3]*EllipticF[ArcSin[1 - x], -1/3]))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 1404
```
Int[((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*((a + b
*x^2 + c*x^4)^p/(4*p + 1)), x] + Simp[2*(p/(4*p + 1))   Int[(2*a + b*x^2)*(
a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*
c, 0] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 1494
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[(d + e*x^2)/(Sqr
t[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e
}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

rule 2458
```
Int[(Pn_)^(p_.), x_Symbol] :> With[{S = Coeff[Pn, x, Expon[Pn, x] - 1]/(Exp
on[Pn, x]*Coeff[Pn, x, Expon[Pn, x]])}, Subst[Int[ExpandToSum[Pn /. x -> x
- S, x]^p, x], x, x + S] /; BinomialQ[Pn /. x -> x - S, x] || (IntegerQ[Exp
on[Pn, x]/2] && TrinomialQ[Pn /. x -> x - S, x])] /; FreeQ[p, x] && PolyQ[P
n, x] && GtQ[Expon[Pn, x], 2] && NeQ[Coeff[Pn, x, Expon[Pn, x] - 1], 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 935 vs. $2(60) = 120$.

Time = 1.26 (sec) , antiderivative size = 936, normalized size of antiderivative = 13.76

method	result	size
risch	Expression too large to display	936
default	Expression too large to display	946
elliptic	Expression too large to display	946

input
```
int((-x^4+4*x^3-8*x^2+8*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(x-1)*x*(x^3-4*x^2+8*x-8)/(-x*(x^3-4*x^2+8*x-8))^(1/2)-2/3*(x*(x-1+I*
3^(1/2))*(x-1-I*3^(1/2))+2*(-1-I*3^(1/2))*((-1+I*3^(1/2))*x/(1+I*3^(1/2))/
(x-2))^(1/2)*(x-2)^2*((x-1+I*3^(1/2))/(1-I*3^(1/2))/(x-2))^(1/2)*((x-1-I*3
^(1/2))/(1+I*3^(1/2))/(x-2))^(1/2)*(1/2*(6+2*I*3^(1/2))/(-1+I*3^(1/2))*Ell
ipticF(((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2),((1+I*3^(1/2))*(-1-I*3
^(1/2))/(-1+I*3^(1/2))/(1-I*3^(1/2)))^(1/2))+1/2*(-1+I*3^(1/2))*EllipticE(
((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2),((1+I*3^(1/2))*(-1-I*3^(1/2))
/(-1+I*3^(1/2))/(1-I*3^(1/2)))^(1/2))-4/(-1+I*3^(1/2))*EllipticPi(((-1+I*3
^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2),(-1-I*3^(1/2))/(1-I*3^(1/2)),((1+I*3^
(1/2))*(-1-I*3^(1/2))/(-1+I*3^(1/2))/(1-I*3^(1/2)))^(1/2))))/(-x*(x-2)*(x-
1+I*3^(1/2))*(x-1-I*3^(1/2)))^(1/2)+8/3*(-1-I*3^(1/2))*((-1+I*3^(1/2))*x/(
1+I*3^(1/2))/(x-2))^(1/2)*(x-2)^2*((x-1+I*3^(1/2))/(1-I*3^(1/2))/(x-2))^(1
/2)*((x-1-I*3^(1/2))/(1+I*3^(1/2))/(x-2))^(1/2)/(-1+I*3^(1/2))/(-x*(x-2)*(
x-1+I*3^(1/2))*(x-1-I*3^(1/2)))^(1/2)*EllipticF(((-1+I*3^(1/2))*x/(1+I*3^(
1/2))/(x-2))^(1/2),((1+I*3^(1/2))*(-1-I*3^(1/2))/(-1+I*3^(1/2))/(1-I*3^(1/
2)))^(1/2))+8/3*(-1-I*3^(1/2))*((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2
)*(x-2)^2*((x-1+I*3^(1/2))/(1-I*3^(1/2))/(x-2))^(1/2)*((x-1-I*3^(1/2))/(1+
I*3^(1/2))/(x-2))^(1/2)/(-1+I*3^(1/2))/(-x*(x-2)*(x-1+I*3^(1/2))*(x-1-I*3^
(1/2)))^(1/2)*(2*EllipticF(((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2),((
1+I*3^(1/2))*(-1-I*3^(1/2))/(-1+I*3^(1/2))/(1-I*3^(1/2)))^(1/2))-2*Elli...
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.01

$$\int \sqrt{8x-8x^2+4x^3-x^4}\,dx =$$

$$-\frac{2\,(-i\,x+i)E(\arcsin\left(\frac{1}{x-1}\right)\,|-3)+4\,(-i\,x+i)F(\arcsin\left(\frac{1}{x-1}\right)\,|-3)-\sqrt{-x^4+4\,x^3-8\,x^2+8\,x}(x^2-}{3\,(x-1)}$$

input
```
integrate((-x^4+4*x^3-8*x^2+8*x)^(1/2),x, algorithm="fricas")
```

output
```
-1/3*(2*(-I*x + I)*elliptic_e(arcsin(1/(x - 1)), -3) + 4*(-I*x + I)*ellipt
ic_f(arcsin(1/(x - 1)), -3) - sqrt(-x^4 + 4*x^3 - 8*x^2 + 8*x)*(x^2 - 2*x
+ 3))/(x - 1)
```

Sympy [F]

$$\int \sqrt{8x - 8x^2 + 4x^3 - x^4}\, dx = \int \sqrt{-x^4 + 4x^3 - 8x^2 + 8x}\, dx$$

input
```
integrate((-x**4+4*x**3-8*x**2+8*x)**(1/2),x)
```

output
```
Integral(sqrt(-x**4 + 4*x**3 - 8*x**2 + 8*x), x)
```

Maxima [F]

$$\int \sqrt{8x - 8x^2 + 4x^3 - x^4}\, dx = \int \sqrt{-x^4 + 4\,x^3 - 8\,x^2 + 8\,x}\, dx$$

input
```
integrate((-x^4+4*x^3-8*x^2+8*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-x^4 + 4*x^3 - 8*x^2 + 8*x), x)
```

Giac [F]

$$\int \sqrt{8x-8x^2+4x^3-x^4}\,dx=\int \sqrt{-x^4+4\,x^3-8\,x^2+8\,x}\,dx$$

input
```
integrate((-x^4+4*x^3-8*x^2+8*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-x^4 + 4*x^3 - 8*x^2 + 8*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{8x-8x^2+4x^3-x^4}\,dx=\int \sqrt{-x^4+4\,x^3-8\,x^2+8\,x}\,dx$$

input
```
int((8*x - 8*x^2 + 4*x^3 - x^4)^(1/2),x)
```

output
```
int((8*x - 8*x^2 + 4*x^3 - x^4)^(1/2), x)
```

Reduce [F]

$$\begin{aligned}\int \sqrt{8x-8x^2+4x^3-x^4}\,dx &= \frac{\sqrt{x}\,\sqrt{-x^3+4x^2-8x+8}\,x}{3}\\ &-\frac{4\sqrt{x}\,\sqrt{-x^3+4x^2-8x+8}}{9}\\ &+\frac{2\left(\int \frac{\sqrt{x}\,\sqrt{-x^3+4x^2-8x+8}\,x^2}{x^3-4x^2+8x-8}dx\right)}{9}\\ &-\frac{16\left(\int \frac{\sqrt{x}\,\sqrt{-x^3+4x^2-8x+8}}{x^4-4x^3+8x^2-8x}dx\right)}{9}\\ &-\frac{4\left(\int \frac{\sqrt{x}\,\sqrt{-x^3+4x^2-8x+8}}{x^3-4x^2+8x-8}dx\right)}{9}\end{aligned}$$

input
```
int((-x^4+4*x^3-8*x^2+8*x)^(1/2),x)
```

output
```
(3*sqrt(x)*sqrt( - x**3 + 4*x**2 - 8*x + 8)*x - 4*sqrt(x)*sqrt( - x**3 + 4
*x**2 - 8*x + 8) + 2*int((sqrt(x)*sqrt( - x**3 + 4*x**2 - 8*x + 8)*x**2)/(
x**3 - 4*x**2 + 8*x - 8),x) - 16*int((sqrt(x)*sqrt( - x**3 + 4*x**2 - 8*x
+ 8))/(x**4 - 4*x**3 + 8*x**2 - 8*x),x) - 4*int((sqrt(x)*sqrt( - x**3 + 4*
x**2 - 8*x + 8))/(x**3 - 4*x**2 + 8*x - 8),x))/9
```

3.52 $\int \frac{1}{\sqrt{8x-8x^2+4x^3-x^4}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 17

$$\int \frac{1}{\sqrt{8x-8x^2+4x^3-x^4}}\,dx=-\frac{\text{EllipticF}\left(\arcsin(1-x),-\frac{1}{3}\right)}{\sqrt{3}}$$

output
```
1/3*3^(1/2)*EllipticF(-1+x,1/3*I*3^(1/2))
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 32.28 (sec) , antiderivative size = 156, normalized size of antiderivative = 9.18

$$\int \frac{1}{\sqrt{8x-8x^2+4x^3-x^4}}\,dx =\frac{\sqrt{-i+\sqrt{3}+\frac{4i}{x}}\sqrt{-\frac{i(-2+x)}{\left(-i+\sqrt{3}\right)x}}\,x\left(-4+x-i\sqrt{3}x\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{i+\sqrt{3}-\frac{4i}{x}}}{\sqrt{2}\sqrt[4]{3}}\right),\frac{2\sqrt{3}}{-i+\sqrt{3}}\right)}{\sqrt{2}\sqrt{i+\sqrt{3}-\frac{4i}{x}}\sqrt{-x\left(-8+8x-4x^2+x^3\right)}}$$

input
```
Integrate[1/Sqrt[8*x - 8*x^2 + 4*x^3 - x^4],x]
```

output
```
(Sqrt[-I + Sqrt[3] + (4*I)/x]*Sqrt[((-I)*(-2 + x))/((-I + Sqrt[3])*x)]*x*(
-4 + x - I*Sqrt[3]*x)*EllipticF[ArcSin[Sqrt[I + Sqrt[3] - (4*I)/x]/(Sqrt[2
]*3^(1/4))], (2*Sqrt[3])/(-I + Sqrt[3])])/(Sqrt[2]*Sqrt[I + Sqrt[3] - (4*I
)/x]*Sqrt[-(x*(-8 + 8*x - 4*x^2 + x^3))])
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2458, 1408, 27, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-x^4+4x^3-8x^2+8x}}dx$$

$$\Big\downarrow \text{2458}$$

$$\int \frac{1}{\sqrt{-(x-1)^4-2(x-1)^2+3}}d(x-1)$$

$$\Big\downarrow \text{1408}$$

$$2\int \frac{1}{2\sqrt{1-(x-1)^2}\sqrt{(x-1)^2+3}}d(x-1)$$

$$\Big\downarrow \text{27}$$

$$\int \frac{1}{\sqrt{1-(x-1)^2}\sqrt{(x-1)^2+3}}d(x-1)$$

$$\Big\downarrow \text{321}$$

$$-\frac{\text{EllipticF}\left(\arcsin(1-x),-\frac{1}{3}\right)}{\sqrt{3}}$$

input
```
Int[1/Sqrt[8*x - 8*x^2 + 4*x^3 - x^4],x]
```

output
```
-(EllipticF[ArcSin[1 - x], -1/3]/Sqrt[3])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 1408
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
 - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[
c, 0]
```

rule 2458
```
Int[(Pn_)^(p_.), x_Symbol] :> With[{S = Coeff[Pn, x, Expon[Pn, x] - 1]/(Exp
on[Pn, x]*Coeff[Pn, x, Expon[Pn, x]])}, Subst[Int[ExpandToSum[Pn /. x -> x
- S, x]^p, x], x, x + S] /; BinomialQ[Pn /. x -> x - S, x] || (IntegerQ[Exp
on[Pn, x]/2] && TrinomialQ[Pn /. x -> x - S, x])] /; FreeQ[p, x] && PolyQ[P
n, x] && GtQ[Expon[Pn, x], 2] && NeQ[Coeff[Pn, x, Expon[Pn, x] - 1], 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 199 vs. $2(15) = 30$.

Time = 0.55 (sec) , antiderivative size = 200, normalized size of antiderivative = 11.76

method	result	siz
default	$\frac{2\left(-1-i\sqrt{3}\right)\sqrt{\frac{\left(-1+i\sqrt{3}\right)x}{\left(1+i\sqrt{3}\right)\left(x-2\right)}}\,\left(x-2\right)^{2}\sqrt{\frac{x-1+i\sqrt{3}}{\left(1-i\sqrt{3}\right)\left(x-2\right)}}\,\sqrt{\frac{x-1-i\sqrt{3}}{\left(1+i\sqrt{3}\right)\left(x-2\right)}}\,\operatorname{EllipticF}\left(\sqrt{\frac{\left(-1+i\sqrt{3}\right)x}{\left(1+i\sqrt{3}\right)\left(x-2\right)}},\sqrt{\frac{\left(1+i\sqrt{3}\right)\left(-1-i\sqrt{3}\right)}{\left(-1+i\sqrt{3}\right)\left(1-i\sqrt{3}\right)}}\right)}{\left(-1+i\sqrt{3}\right)\sqrt{-x\left(x-2\right)\left(x-1+i\sqrt{3}\right)\left(x-1-i\sqrt{3}\right)}}$	20
elliptic	$\frac{2\left(-1-i\sqrt{3}\right)\sqrt{\frac{\left(-1+i\sqrt{3}\right)x}{\left(1+i\sqrt{3}\right)\left(x-2\right)}}\,\left(x-2\right)^{2}\sqrt{\frac{x-1+i\sqrt{3}}{\left(1-i\sqrt{3}\right)\left(x-2\right)}}\,\sqrt{\frac{x-1-i\sqrt{3}}{\left(1+i\sqrt{3}\right)\left(x-2\right)}}\,\operatorname{EllipticF}\left(\sqrt{\frac{\left(-1+i\sqrt{3}\right)x}{\left(1+i\sqrt{3}\right)\left(x-2\right)}},\sqrt{\frac{\left(1+i\sqrt{3}\right)\left(-1-i\sqrt{3}\right)}{\left(-1+i\sqrt{3}\right)\left(1-i\sqrt{3}\right)}}\right)}{\left(-1+i\sqrt{3}\right)\sqrt{-x\left(x-2\right)\left(x-1+i\sqrt{3}\right)\left(x-1-i\sqrt{3}\right)}}$	20

input
```
int(1/(-x^4+4*x^3-8*x^2+8*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(-1-I*3^(1/2))*((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2)*(x-2)^2*((x-
1+I*3^(1/2))/(1-I*3^(1/2))/(x-2))^(1/2)*((x-1-I*3^(1/2))/(1+I*3^(1/2))/(x-
2))^(1/2)/(-1+I*3^(1/2))/(-x*(x-2)*(x-1+I*3^(1/2))*(x-1-I*3^(1/2)))^(1/2)*
EllipticF(((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2),((1+I*3^(1/2))*(-1-
I*3^(1/2))/(-1+I*3^(1/2))/(1-I*3^(1/2)))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

$$\int \frac{1}{\sqrt{8x-8x^2+4x^3-x^4}}\,dx = -\frac{1}{2}\sqrt{2}\text{weierstrassPInverse}\left(-\frac{2}{3},\frac{7}{54},-\frac{x-3}{3\,x}\right)$$

input
```
integrate(1/(-x^4+4*x^3-8*x^2+8*x)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*sqrt(2)*weierstrassPInverse(-2/3, 7/54, -1/3*(x - 3)/x)
```

Sympy [F]

$$\int \frac{1}{\sqrt{8x-8x^2+4x^3-x^4}}\,dx = \int \frac{1}{\sqrt{-x^4+4x^3-8x^2+8x}}\,dx$$

input
```
integrate(1/(-x**4+4*x**3-8*x**2+8*x)**(1/2),x)
```

output
```
Integral(1/sqrt(-x**4 + 4*x**3 - 8*x**2 + 8*x), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{8x - 8x^2 + 4x^3 - x^4}}\, dx = \int \frac{1}{\sqrt{-x^4 + 4\,x^3 - 8\,x^2 + 8\,x}}\, dx$$

input
```
integrate(1/(-x^4+4*x^3-8*x^2+8*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(-x^4 + 4*x^3 - 8*x^2 + 8*x), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{8x - 8x^2 + 4x^3 - x^4}}\, dx = \int \frac{1}{\sqrt{-x^4 + 4\,x^3 - 8\,x^2 + 8\,x}}\, dx$$

input
```
integrate(1/(-x^4+4*x^3-8*x^2+8*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(-x^4 + 4*x^3 - 8*x^2 + 8*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{8x - 8x^2 + 4x^3 - x^4}}\, dx = \int \frac{1}{\sqrt{-x^4 + 4\,x^3 - 8\,x^2 + 8\,x}}\, dx$$

input
```
int(1/(8*x - 8*x^2 + 4*x^3 - x^4)^(1/2),x)
```

output
```
int(1/(8*x - 8*x^2 + 4*x^3 - x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{8x - 8x^2 + 4x^3 - x^4}}\,dx = -\left(\int \frac{\sqrt{x}\,\sqrt{-x^3 + 4x^2 - 8x + 8}}{x^4 - 4x^3 + 8x^2 - 8x}dx\right)$$

input
```
int(1/(-x^4+4*x^3-8*x^2+8*x)^(1/2),x)
```

output
```
 - int((sqrt(x)*sqrt( - x**3 + 4*x**2 - 8*x + 8))/(x**4 - 4*x**3 + 8*x**2
- 8*x),x)
```

3.53 $\int \frac{1}{(8x-8x^2+4x^3-x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 79

$$\int \frac{1}{(8x-8x^2+4x^3-x^4)^{3/2}}\,dx = -\frac{(5+(-1+x)^2)(1-x)}{24\sqrt{3-2(1-x)^2-(1-x)^4}} + \frac{E\left(\arcsin(1-x)\,\middle|\,-\frac{1}{3}\right)}{8\sqrt{3}} - \frac{\operatorname{EllipticF}\left(\arcsin(1-x),-\frac{1}{3}\right)}{4\sqrt{3}}$$

output
```
-1/24*(5+(-1+x)^2)*(1-x)/(3-2*(1-x)^2-(1-x)^4)^(1/2)-1/24*3^(1/2)*Elliptic
E(-1+x,1/3*I*3^(1/2))+1/12*3^(1/2)*EllipticF(-1+x,1/3*I*3^(1/2))
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 23.15 (sec) , antiderivative size = 261, normalized size of antiderivative = 3.30

$$\int \frac{1}{(8x-8x^2+4x^3-x^4)^{3/2}}\,dx = \frac{\sqrt{-x(-8+8x-4x^2+x^3)}\left(\frac{\sqrt{2}\left(-i+\sqrt{3}\right)\sqrt{-\frac{i(-2+x)}{\left(-i+\sqrt{3}\right)x}}\,E\left(\arcsin\left(\frac{\sqrt{i+\sqrt{3}-\frac{4i}{x}}}{\sqrt{2}\sqrt[4]{3}}\right)\middle|\,\frac{2\cdots}{-i\cdots}\right.}{\sqrt{\frac{4-2x+x^2}{x^2}}}\cdots\right.}{24(-2\cdots}$$

input
```
Integrate[(8*x - 8*x^2 + 4*x^3 - x^4)^(-3/2),x]
```

output
```
(Sqrt[-(x*(-8 + 8*x - 4*x^2 + x^3))]*((Sqrt[2]*(-I + Sqrt[3])*Sqrt[((-I)*(
-2 + x))/((-I + Sqrt[3])*x)]*EllipticE[ArcSin[Sqrt[I + Sqrt[3] - (4*I)/x]/
(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(-I + Sqrt[3])])/Sqrt[(4 - 2*x + x^2)/x^2]
 - (2 + x^2 - (4*I)*Sqrt[2]*Sqrt[((-I)*(-2 + x))/((-I + Sqrt[3])*x)]*x^2*S
qrt[(4 - 2*x + x^2)/x^2]*EllipticF[ArcSin[Sqrt[I + Sqrt[3] - (4*I)/x]/(Sqr
t[2]*3^(1/4))], (2*Sqrt[3])/(-I + Sqrt[3])])/(4 - 2*x + x^2)))/(24*(-2 + x
)*x)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.92, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {2458, 1405, 27, 1494, 27, 399, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(-x^4+4x^3-8x^2+8x)^{3/2}}dx$$

$$\Big\downarrow \text{ 2458}$$

$$\int \frac{1}{(-(x-1)^4-2(x-1)^2+3)^{3/2}}d(x-1)$$

$$\Big\downarrow \text{ 1405}$$

$$\frac{((x-1)^2+5)(x-1)}{24\sqrt{-(x-1)^4-2(x-1)^2+3}}-\frac{1}{48}\int -\frac{2(3-(x-1)^2)}{\sqrt{-(x-1)^4-2(x-1)^2+3}}d(x-1)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{24}\int \frac{3-(x-1)^2}{\sqrt{-(x-1)^4-2(x-1)^2+3}}d(x-1)+\frac{((x-1)^2+5)(x-1)}{24\sqrt{-(x-1)^4-2(x-1)^2+3}}$$

$$\Big\downarrow \text{ 1494}$$

$$\frac{1}{12}\int \frac{3-(x-1)^2}{2\sqrt{1-(x-1)^2}\sqrt{(x-1)^2+3}}d(x-1)+\frac{((x-1)^2+5)(x-1)}{24\sqrt{-(x-1)^4-2(x-1)^2+3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{24}\int \frac{3-(x-1)^2}{\sqrt{1-(x-1)^2}\sqrt{(x-1)^2+3}}d(x-1)+\frac{\left((x-1)^2+5\right)(x-1)}{24\sqrt{-(x-1)^4-2(x-1)^2+3}}$$

$$\Big\downarrow \text{399}$$

$$\frac{1}{24}\left(6\int \frac{1}{\sqrt{1-(x-1)^2}\sqrt{(x-1)^2+3}}d(x-1)-\int \frac{\sqrt{(x-1)^2+3}}{\sqrt{1-(x-1)^2}}d(x-1)\right)+\frac{\left((x-1)^2+5\right)(x-1)}{24\sqrt{-(x-1)^4-2(x-1)^2+3}}$$

$$\Big\downarrow \text{321}$$

$$\frac{1}{24}\left(-\int \frac{\sqrt{(x-1)^2+3}}{\sqrt{1-(x-1)^2}}d(x-1)-2\sqrt{3}\,\text{EllipticF}\left(\arcsin(1-x),-\frac{1}{3}\right)\right)+\frac{\left((x-1)^2+5\right)(x-1)}{24\sqrt{-(x-1)^4-2(x-1)^2+3}}$$

$$\Big\downarrow \text{327}$$

$$\frac{1}{24}\left(\sqrt{3}E\left(\arcsin(1-x)\left|-\frac{1}{3}\right.\right)-2\sqrt{3}\,\text{EllipticF}\left(\arcsin(1-x),-\frac{1}{3}\right)\right)+\frac{\left((x-1)^2+5\right)(x-1)}{24\sqrt{-(x-1)^4-2(x-1)^2+3}}$$

input
```
Int[(8*x - 8*x^2 + 4*x^3 - x^4)^(-3/2),x]
```

output
```
((5 + (-1 + x)^2)*(-1 + x))/(24*Sqrt[3 - 2*(-1 + x)^2 - (-1 + x)^4]) + (Sq
rt[3]*EllipticE[ArcSin[1 - x], -1/3] - 2*Sqrt[3]*EllipticF[ArcSin[1 - x],
-1/3])/24
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1494
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[(d + e*x^2)/(Sqr
t[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e
}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

rule 2458
```
Int[(Pn_)^(p_.), x_Symbol] :> With[{S = Coeff[Pn, x, Expon[Pn, x] - 1]/(Exp
on[Pn, x]*Coeff[Pn, x, Expon[Pn, x]])}, Subst[Int[ExpandToSum[Pn /. x -> x
- S, x]^p, x], x, x + S] /; BinomialQ[Pn /. x -> x - S, x] || (IntegerQ[Exp
on[Pn, x]/2] && TrinomialQ[Pn /. x -> x - S, x])] /; FreeQ[p, x] && PolyQ[P
n, x] && GtQ[Expon[Pn, x], 2] && NeQ[Coeff[Pn, x, Expon[Pn, x] - 1], 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 931 vs. $2(67) = 134$.

Time = 1.27 (sec) , antiderivative size = 932, normalized size of antiderivative = 11.80

method	result	size
risch	Expression too large to display	932
default	Expression too large to display	963
elliptic	Expression too large to display	963

input
```
int(1/(-x^4+4*x^3-8*x^2+8*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/24*(x^3-3*x^2+8*x-6)/(-x*(x^3-4*x^2+8*x-8))^(1/2)-1/24*(x*(x-1+I*3^(1/2)
)*(x-1-I*3^(1/2))+2*(-1-I*3^(1/2))*((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^
(1/2)*(x-2)^2*((x-1+I*3^(1/2))/(1-I*3^(1/2))/(x-2))^(1/2)*((x-1-I*3^(1/2))
/(1+I*3^(1/2))/(x-2))^(1/2)*(1/2*(6+2*I*3^(1/2))/(-1+I*3^(1/2))*EllipticF(
((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2),((1+I*3^(1/2))*(-1-I*3^(1/2))
/(-1+I*3^(1/2))/(1-I*3^(1/2)))^(1/2))+1/2*(-1+I*3^(1/2))*EllipticE(((-1+I*
3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2),((1+I*3^(1/2))*(-1-I*3^(1/2))/(-1+I*
3^(1/2))/(1-I*3^(1/2)))^(1/2))-4/(-1+I*3^(1/2))*EllipticPi(((-1+I*3^(1/2))
*x/(1+I*3^(1/2))/(x-2))^(1/2),(-1-I*3^(1/2))/(1-I*3^(1/2)),((1+I*3^(1/2))*
(-1-I*3^(1/2))/(-1+I*3^(1/2))/(1-I*3^(1/2)))^(1/2))))/(-x*(x-2)*(x-1+I*3^(
1/2))*(x-1-I*3^(1/2)))^(1/2)+1/6*(-1-I*3^(1/2))*((-1+I*3^(1/2))*x/(1+I*3^(
1/2))/(x-2))^(1/2)*(x-2)^2*((x-1+I*3^(1/2))/(1-I*3^(1/2))/(x-2))^(1/2)*((x
-1-I*3^(1/2))/(1+I*3^(1/2))/(x-2))^(1/2)/(-1+I*3^(1/2))/(-x*(x-2)*(x-1+I*3
^(1/2))*(x-1-I*3^(1/2)))^(1/2)*EllipticF(((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(
x-2))^(1/2),((1+I*3^(1/2))*(-1-I*3^(1/2))/(-1+I*3^(1/2))/(1-I*3^(1/2)))^(1
/2))+1/6*(-1-I*3^(1/2))*((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2)*(x-2)
^2*((x-1+I*3^(1/2))/(1-I*3^(1/2))/(x-2))^(1/2)*((x-1-I*3^(1/2))/(1+I*3^(1/
2))/(x-2))^(1/2)/(-1+I*3^(1/2))/(-x*(x-2)*(x-1+I*3^(1/2))*(x-1-I*3^(1/2)))
^(1/2)*(2*EllipticF(((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2),((1+I*3^(
1/2))*(-1-I*3^(1/2))/(-1+I*3^(1/2))/(1-I*3^(1/2)))^(1/2))-2*EllipticPi(...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 119 vs. $2(59) = 118$.

Time = 0.08 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.51

$$\int \frac{1}{(8x - 8x^2 + 4x^3 - x^4)^{3/2}}\,dx = -\frac{5\sqrt{2}(x^4 - 4x^3 + 8x^2 - 8x)\text{weierstrassPInverse}\left(-\frac{2}{3}, \frac{7}{54}, -\frac{x-3}{3x}\right) - 6\sqrt{2}(x^4 - 4x^3 + 8x^2 - 8x)\text{weierstra}}{72\,(x^4 - 4x^3 + 8x^2}$$

input
```
integrate(1/(-x^4+4*x^3-8*x^2+8*x)^(3/2),x, algorithm="fricas")
```

output
```
-1/72*(5*sqrt(2)*(x^4 - 4*x^3 + 8*x^2 - 8*x)*weierstrassPInverse(-2/3, 7/5
4, -1/3*(x - 3)/x) - 6*sqrt(2)*(x^4 - 4*x^3 + 8*x^2 - 8*x)*weierstrassZeta
(-2/3, 7/54, weierstrassPInverse(-2/3, 7/54, -1/3*(x - 3)/x)) + 3*sqrt(-x^
4 + 4*x^3 - 8*x^2 + 8*x)*(x^2 + 2))/(x^4 - 4*x^3 + 8*x^2 - 8*x)
```

Sympy [F]

$$\int \frac{1}{(8x - 8x^2 + 4x^3 - x^4)^{3/2}}\,dx = \int \frac{1}{(-x^4 + 4x^3 - 8x^2 + 8x)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-x**4+4*x**3-8*x**2+8*x)**(3/2),x)
```

output
```
Integral((-x**4 + 4*x**3 - 8*x**2 + 8*x)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(8x-8x^2+4x^3-x^4)^{3/2}}\,dx = \int \frac{1}{(-x^4+4\,x^3-8\,x^2+8\,x)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-x^4+4*x^3-8*x^2+8*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-x^4 + 4*x^3 - 8*x^2 + 8*x)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(8x-8x^2+4x^3-x^4)^{3/2}}\,dx = \int \frac{1}{(-x^4+4\,x^3-8\,x^2+8\,x)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-x^4+4*x^3-8*x^2+8*x)^(3/2),x, algorithm="giac")
```

output
```
integrate((-x^4 + 4*x^3 - 8*x^2 + 8*x)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(8x-8x^2+4x^3-x^4)^{3/2}}\,dx = \int \frac{1}{(-x^4+4\,x^3-8\,x^2+8\,x)^{3/2}}\,dx$$

input
```
int(1/(8*x - 8*x^2 + 4*x^3 - x^4)^(3/2),x)
```

output
```
int(1/(8*x - 8*x^2 + 4*x^3 - x^4)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(8x-8x^2+4x^3-x^4)^{3/2}}\,dx = \frac{\sqrt{-x^3+4x^2-8x+8}\,x^2-2\sqrt{-x^3+4x^2-8x+8}\,x+\sqrt{-x^3+4x^2-8}}{}$$

input
```
int(1/(-x^4+4*x^3-8*x^2+8*x)^(3/2),x)
```

output
```
(sqrt( - x**3 + 4*x**2 - 8*x + 8)*x**2 - 2*sqrt( - x**3 + 4*x**2 - 8*x + 8
)*x + sqrt( - x**3 + 4*x**2 - 8*x + 8) - 2*sqrt(x)*int((sqrt(x)*sqrt( - x*
*3 + 4*x**2 - 8*x + 8)*x)/(x**6 - 8*x**5 + 32*x**4 - 80*x**3 + 128*x**2 -
128*x + 64),x)*x**3 + 8*sqrt(x)*int((sqrt(x)*sqrt( - x**3 + 4*x**2 - 8*x +
 8)*x)/(x**6 - 8*x**5 + 32*x**4 - 80*x**3 + 128*x**2 - 128*x + 64),x)*x**2
 - 16*sqrt(x)*int((sqrt(x)*sqrt( - x**3 + 4*x**2 - 8*x + 8)*x)/(x**6 - 8*x
**5 + 32*x**4 - 80*x**3 + 128*x**2 - 128*x + 64),x)*x + 16*sqrt(x)*int((sq
rt(x)*sqrt( - x**3 + 4*x**2 - 8*x + 8)*x)/(x**6 - 8*x**5 + 32*x**4 - 80*x*
*3 + 128*x**2 - 128*x + 64),x) + 6*sqrt(x)*int((sqrt(x)*sqrt( - x**3 + 4*x
**2 - 8*x + 8))/(x**6 - 8*x**5 + 32*x**4 - 80*x**3 + 128*x**2 - 128*x + 64
),x)*x**3 - 24*sqrt(x)*int((sqrt(x)*sqrt( - x**3 + 4*x**2 - 8*x + 8))/(x**
6 - 8*x**5 + 32*x**4 - 80*x**3 + 128*x**2 - 128*x + 64),x)*x**2 + 48*sqrt(
x)*int((sqrt(x)*sqrt( - x**3 + 4*x**2 - 8*x + 8))/(x**6 - 8*x**5 + 32*x**4
 - 80*x**3 + 128*x**2 - 128*x + 64),x)*x - 48*sqrt(x)*int((sqrt(x)*sqrt( -
 x**3 + 4*x**2 - 8*x + 8))/(x**6 - 8*x**5 + 32*x**4 - 80*x**3 + 128*x**2 -
 128*x + 64),x))/(4*sqrt(x)*(x**3 - 4*x**2 + 8*x - 8))
```

3.54 $\int \frac{1}{(8x-8x^2+4x^3-x^4)^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 123

$$\int \frac{1}{(8x-8x^2+4x^3-x^4)^{5/2}}\,dx = -\frac{(26+7(1-x)^2)\,(1-x)}{432\sqrt{3-2(1-x)^2-(1-x)^4}} - \frac{(5+(-1+x)^2)\,(1-x)}{72\,(3-2(1-x)^2-(1-x)^4)^{3/2}} + \frac{7E\left(\arcsin(1-x)\,\middle|\,-\frac{1}{3}\right)}{144\sqrt{3}} - \frac{11\,\text{EllipticF}\left(\arcsin(1-x),-\frac{1}{3}\right)}{144\sqrt{3}}$$

output

```
-1/432*(26+7*(1-x)^2)*(1-x)/(3-2*(1-x)^2-(1-x)^4)^(1/2)-1/72*(5+(-1+x)^2)*
(1-x)/(3-2*(1-x)^2-(1-x)^4)^(3/2)-7/432*3^(1/2)*EllipticE(-1+x,1/3*I*3^(1/
2))+11/432*3^(1/2)*EllipticF(-1+x,1/3*I*3^(1/2))
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 23.43 (sec) , antiderivative size = 298, normalized size of antiderivative = 2.42

$$\int \frac{1}{(8x-8x^2+4x^3-x^4)^{5/2}}\,dx = \frac{\frac{7i\sqrt{2}(-2+x)x^2\sqrt{\frac{4-2x+x^2}{x^2}}E\left(\arcsin\left(\frac{\sqrt{i+\sqrt{3}-\frac{4i}{x}}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{-i+\sqrt{3}}\right)}{\sqrt{-\frac{i(-2+x)}{\left(-i+\sqrt{3}\right)x}}}+\frac{36-232x+274x^2-226x^3+115x^4}{}}{432x\sqrt{-x\,(}}$$

input
```
Integrate[(8*x - 8*x^2 + 4*x^3 - x^4)^(-5/2),x]
```

output
```
(((7*I)*Sqrt[2]*(-2 + x)*x^2*Sqrt[(4 - 2*x + x^2)/x^2]*EllipticE[ArcSin[Sq
rt[I + Sqrt[3] - (4*I)/x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(-I + Sqrt[3])])
/Sqrt[((-I)*(-2 + x))/((-I + Sqrt[3])*x)] + (36 - 232*x + 274*x^2 - 226*x^
3 + 115*x^4 - 37*x^5 + 7*x^6 - (19*I)*Sqrt[2]*Sqrt[((-I)*(-2 + x))/((-I +
Sqrt[3])*x)]*x^3*Sqrt[(4 - 2*x + x^2)/x^2]*(-8 + 8*x - 4*x^2 + x^3)*Ellipt
icF[ArcSin[Sqrt[I + Sqrt[3] - (4*I)/x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(-I
 + Sqrt[3])])/(-8 + 8*x - 4*x^2 + x^3))/(432*x*Sqrt[-(x*(-8 + 8*x - 4*x^2
+ x^3))])
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.93, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {2458, 1405, 27, 1492, 27, 1494, 27, 399, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(-x^4+4x^3-8x^2+8x)^{5/2}}\,dx$$

$$\Big\downarrow \; 2458$$

$$\int \frac{1}{(-(x-1)^4-2(x-1)^2+3)^{5/2}}d(x-1)$$

$$\Big\downarrow \text{1405}$$

$$\frac{\left((x-1)^2+5\right)(x-1)}{72\left(-(x-1)^4-2(x-1)^2+3\right)^{3/2}}-\frac{1}{144}\int -\frac{2\left(3(x-1)^2+19\right)}{\left(-(x-1)^4-2(x-1)^2+3\right)^{3/2}}d(x-1)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{72}\int \frac{3(x-1)^2+19}{\left(-(x-1)^4-2(x-1)^2+3\right)^{3/2}}d(x-1)+\frac{\left((x-1)^2+5\right)(x-1)}{72\left(-(x-1)^4-2(x-1)^2+3\right)^{3/2}}$$

$$\Big\downarrow \text{1492}$$

$$\frac{1}{72}\left(\frac{\left(7(x-1)^2+26\right)(x-1)}{6\sqrt{-(x-1)^4-2(x-1)^2+3}}-\frac{1}{48}\int -\frac{8\left(12-7(x-1)^2\right)}{\sqrt{-(x-1)^4-2(x-1)^2+3}}d(x-1)\right)+\frac{\left((x-1)^2+5\right)(x-1)}{72\left(-(x-1)^4-2(x-1)^2+3\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{72}\left(\frac{1}{6}\int \frac{12-7(x-1)^2}{\sqrt{-(x-1)^4-2(x-1)^2+3}}d(x-1)+\frac{\left(7(x-1)^2+26\right)(x-1)}{6\sqrt{-(x-1)^4-2(x-1)^2+3}}\right)+\frac{\left((x-1)^2+5\right)(x-1)}{72\left(-(x-1)^4-2(x-1)^2+3\right)^{3/2}}$$

$$\Big\downarrow \text{1494}$$

$$\frac{1}{72}\left(\frac{1}{3}\int \frac{12-7(x-1)^2}{2\sqrt{1-(x-1)^2}\sqrt{(x-1)^2+3}}d(x-1)+\frac{\left(7(x-1)^2+26\right)(x-1)}{6\sqrt{-(x-1)^4-2(x-1)^2+3}}\right)+\frac{\left((x-1)^2+5\right)(x-1)}{72\left(-(x-1)^4-2(x-1)^2+3\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{72}\left(\frac{1}{6}\int \frac{12-7(x-1)^2}{\sqrt{1-(x-1)^2}\sqrt{(x-1)^2+3}}d(x-1)+\frac{\left(7(x-1)^2+26\right)(x-1)}{6\sqrt{-(x-1)^4-2(x-1)^2+3}}\right)+\frac{\left((x-1)^2+5\right)(x-1)}{72\left(-(x-1)^4-2(x-1)^2+3\right)^{3/2}}$$

$$\Big\downarrow \text{399}$$

$$\frac{1}{72}\left(\frac{1}{6}\left(33\int \frac{1}{\sqrt{1-(x-1)^2}\sqrt{(x-1)^2+3}}d(x-1)-7\int \frac{\sqrt{(x-1)^2+3}}{\sqrt{1-(x-1)^2}}d(x-1)\right)+\frac{\left(7(x-1)^2+26\right)(x-}{6\sqrt{-(x-1)^4-2(x-1}}\right.$$

$$\frac{\left((x-1)^2+5\right)(x-1)}{72\left(-(x-1)^4-2(x-1)^2+3\right)^{3/2}}$$

$$\Big\downarrow \text{321}$$

$$\frac{1}{72}\left(\frac{1}{6}\left(-7\int \frac{\sqrt{(x-1)^2+3}}{\sqrt{1-(x-1)^2}}d(x-1)-11\sqrt{3}\,\text{EllipticF}\left(\arcsin(1-x),-\frac{1}{3}\right)\right)+\frac{\left(7(x-1)^2+26\right)(x-1)}{6\sqrt{-(x-1)^4-2(x-1)^2+3}}\right)+\frac{\left((x-1)^2+5\right)(x-1)}{72\left(-(x-1)^4-2(x-1)^2+3\right)^{3/2}}$$

$$\Big\downarrow \text{327}$$

$$\frac{1}{72}\left(\frac{1}{6}\left(7\sqrt{3}E\left(\arcsin(1-x)\left|-\frac{1}{3}\right.\right)-11\sqrt{3}\,\text{EllipticF}\left(\arcsin(1-x),-\frac{1}{3}\right)\right)+\frac{\left(7(x-1)^2+26\right)(x-1)}{6\sqrt{-(x-1)^4-2(x-1)^2+3}}\right)+\frac{\left((x-1)^2+5\right)(x-1)}{72\left(-(x-1)^4-2(x-1)^2+3\right)^{3/2}}$$

input
```
Int[(8*x - 8*x^2 + 4*x^3 - x^4)^(-5/2),x]
```

output
```
((5 + (-1 + x)^2)*(-1 + x))/(72*(3 - 2*(-1 + x)^2 - (-1 + x)^4)^(3/2)) + (
((26 + 7*(-1 + x)^2)*(-1 + x))/(6*Sqrt[3 - 2*(-1 + x)^2 - (-1 + x)^4]) + (
7*Sqrt[3]*EllipticE[ArcSin[1 - x], -1/3] - 11*Sqrt[3]*EllipticF[ArcSin[1 -
 x], -1/3])/6)/72
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1492
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2
 - 4*a*c))   Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p +
7)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 LtQ[p, -1] && IntegerQ[2*p]
```

rule 1494
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[(d + e*x^2)/(Sqr
t[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e
}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

rule 2458
```
Int[(Pn_)^(p_.), x_Symbol] :> With[{S = Coeff[Pn, x, Expon[Pn, x] - 1]/(Exp
on[Pn, x]*Coeff[Pn, x, Expon[Pn, x]])}, Subst[Int[ExpandToSum[Pn /. x -> x
- S, x]^p, x], x, x + S] /; BinomialQ[Pn /. x -> x - S, x] || (IntegerQ[Exp
on[Pn, x]/2] && TrinomialQ[Pn /. x -> x - S, x])] /; FreeQ[p, x] && PolyQ[P
n, x] && GtQ[Expon[Pn, x], 2] && NeQ[Coeff[Pn, x, Expon[Pn, x] - 1], 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 971 vs. $2(107) = 214$.

Time = 1.48 (sec) , antiderivative size = 972, normalized size of antiderivative = 7.90

method	result	size
risch	Expression too large to display	972
default	Expression too large to display	1039
elliptic	Expression too large to display	1039

input
```
int(1/(-x^4+4*x^3-8*x^2+8*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/432*(7*x^7-49*x^6+187*x^5-445*x^4+670*x^3-622*x^2+216*x+36)/(-x*(x^3-4*x
^2+8*x-8))^(1/2)/x/(x^3-4*x^2+8*x-8)+5/216*(-1-I*3^(1/2))*((-1+I*3^(1/2))*
x/(1+I*3^(1/2))/(x-2))^(1/2)*(x-2)^2*((x-1+I*3^(1/2))/(1-I*3^(1/2))/(x-2))
^(1/2)*((x-1-I*3^(1/2))/(1+I*3^(1/2))/(x-2))^(1/2)/(-1+I*3^(1/2))/(-x*(x-2
)*(x-1+I*3^(1/2))*(x-1-I*3^(1/2)))^(1/2)*EllipticF(((-1+I*3^(1/2))*x/(1+I*
3^(1/2))/(x-2))^(1/2),((1+I*3^(1/2))*(-1-I*3^(1/2))/(-1+I*3^(1/2))/(1-I*3^
(1/2)))^(1/2))+7/108*(-1-I*3^(1/2))*((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))
^(1/2)*(x-2)^2*((x-1+I*3^(1/2))/(1-I*3^(1/2))/(x-2))^(1/2)*((x-1-I*3^(1/2)
)/(1+I*3^(1/2))/(x-2))^(1/2)/(-1+I*3^(1/2))/(-x*(x-2)*(x-1+I*3^(1/2))*(x-1
-I*3^(1/2)))^(1/2)*(2*EllipticF(((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/
2),((1+I*3^(1/2))*(-1-I*3^(1/2))/(-1+I*3^(1/2))/(1-I*3^(1/2)))^(1/2))-2*El
lipticPi(((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2),(1+I*3^(1/2))/(-1+I*
3^(1/2)),((1+I*3^(1/2))*(-1-I*3^(1/2))/(-1+I*3^(1/2))/(1-I*3^(1/2)))^(1/2)
))-7/432*(x*(x-1+I*3^(1/2))*(x-1-I*3^(1/2))+2*(-1-I*3^(1/2))*((-1+I*3^(1/2
))*x/(1+I*3^(1/2))/(x-2))^(1/2)*(x-2)^2*((x-1+I*3^(1/2))/(1-I*3^(1/2))/(x-
2))^(1/2)*((x-1-I*3^(1/2))/(1+I*3^(1/2))/(x-2))^(1/2)*(1/2*(6+2*I*3^(1/2))
/(-1+I*3^(1/2))*EllipticF(((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2),((1
+I*3^(1/2))*(-1-I*3^(1/2))/(-1+I*3^(1/2))/(1-I*3^(1/2)))^(1/2))+1/2*(-1+I*
3^(1/2))*EllipticE(((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2),((1+I*3^(1
/2))*(-1-I*3^(1/2))/(-1+I*3^(1/2))/(1-I*3^(1/2)))^(1/2))-4/(-1+I*3^(1/2...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 195 vs. $2(91) = 182$.

Time = 0.08 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.59

$$\int \frac{1}{(8x-8x^2+4x^3-x^4)^{5/2}}\,dx = -\frac{43\sqrt{2}(x^8-8x^7+32x^6-80x^5+128x^4-128x^3+64x^2)\text{weierstrassPInverse}\left(-\frac{2}{3},\frac{7}{54},-\frac{x-3}{3x}\right)-84\sqrt{2}(}{}$$

input
```
integrate(1/(-x^4+4*x^3-8*x^2+8*x)^(5/2),x, algorithm="fricas")
```

output
```
-1/2592*(43*sqrt(2)*(x^8 - 8*x^7 + 32*x^6 - 80*x^5 + 128*x^4 - 128*x^3 + 6
4*x^2)*weierstrassPInverse(-2/3, 7/54, -1/3*(x - 3)/x) - 84*sqrt(2)*(x^8 -
 8*x^7 + 32*x^6 - 80*x^5 + 128*x^4 - 128*x^3 + 64*x^2)*weierstrassZeta(-2/
3, 7/54, weierstrassPInverse(-2/3, 7/54, -1/3*(x - 3)/x)) + 6*(7*x^6 - 37*
x^5 + 115*x^4 - 226*x^3 + 274*x^2 - 232*x + 36)*sqrt(-x^4 + 4*x^3 - 8*x^2
+ 8*x))/(x^8 - 8*x^7 + 32*x^6 - 80*x^5 + 128*x^4 - 128*x^3 + 64*x^2)
```

Sympy [F]

$$\int \frac{1}{(8x-8x^2+4x^3-x^4)^{5/2}}\,dx = \int \frac{1}{(-x^4+4x^3-8x^2+8x)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(-x**4+4*x**3-8*x**2+8*x)**(5/2),x)
```

output
```
Integral((-x**4 + 4*x**3 - 8*x**2 + 8*x)**(-5/2), x)
```

Maxima [F]

$$\int \frac{1}{(8x - 8x^2 + 4x^3 - x^4)^{5/2}}\, dx = \int \frac{1}{(-x^4 + 4\,x^3 - 8\,x^2 + 8\,x)^{\frac{5}{2}}}\, dx$$

input
```
integrate(1/(-x^4+4*x^3-8*x^2+8*x)^(5/2),x, algorithm="maxima")
```

output
```
integrate((-x^4 + 4*x^3 - 8*x^2 + 8*x)^(-5/2), x)
```

Giac [F]

$$\int \frac{1}{(8x - 8x^2 + 4x^3 - x^4)^{5/2}}\, dx = \int \frac{1}{(-x^4 + 4\,x^3 - 8\,x^2 + 8\,x)^{\frac{5}{2}}}\, dx$$

input
```
integrate(1/(-x^4+4*x^3-8*x^2+8*x)^(5/2),x, algorithm="giac")
```

output
```
integrate((-x^4 + 4*x^3 - 8*x^2 + 8*x)^(-5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(8x - 8x^2 + 4x^3 - x^4)^{5/2}}\, dx = \int \frac{1}{(-x^4 + 4\,x^3 - 8\,x^2 + 8\,x)^{5/2}}\, dx$$

input
```
int(1/(8*x - 8*x^2 + 4*x^3 - x^4)^(5/2),x)
```

output
```
int(1/(8*x - 8*x^2 + 4*x^3 - x^4)^(5/2), x)
```

Reduce [F]

$$\int \frac{1}{(8x - 8x^2 + 4x^3 - x^4)^{5/2}} \, dx = \text{too large to display}$$

input
```
int(1/(-x^4+4*x^3-8*x^2+8*x)^(5/2),x)
```

output
```
(sqrt( - x**3 + 4*x**2 - 8*x + 8)*x**6 - 6*sqrt( - x**3 + 4*x**2 - 8*x + 8
)*x**5 + 18*sqrt( - x**3 + 4*x**2 - 8*x + 8)*x**4 - 32*sqrt( - x**3 + 4*x*
*2 - 8*x + 8)*x**3 + 30*sqrt( - x**3 + 4*x**2 - 8*x + 8)*x**2 - 12*sqrt( -
 x**3 + 4*x**2 - 8*x + 8)*x - 2*sqrt( - x**3 + 4*x**2 - 8*x + 8) - 36*sqrt
(x)*int(sqrt( - x**3 + 4*x**2 - 8*x + 8)/(sqrt(x)*x**9 - 12*sqrt(x)*x**8 +
 72*sqrt(x)*x**7 - 280*sqrt(x)*x**6 + 768*sqrt(x)*x**5 - 1536*sqrt(x)*x**4
 + 2240*sqrt(x)*x**3 - 2304*sqrt(x)*x**2 + 1536*sqrt(x)*x - 512*sqrt(x)),x
)*x**7 + 288*sqrt(x)*int(sqrt( - x**3 + 4*x**2 - 8*x + 8)/(sqrt(x)*x**9 -
12*sqrt(x)*x**8 + 72*sqrt(x)*x**7 - 280*sqrt(x)*x**6 + 768*sqrt(x)*x**5 -
1536*sqrt(x)*x**4 + 2240*sqrt(x)*x**3 - 2304*sqrt(x)*x**2 + 1536*sqrt(x)*x
 - 512*sqrt(x)),x)*x**6 - 1152*sqrt(x)*int(sqrt( - x**3 + 4*x**2 - 8*x + 8
)/(sqrt(x)*x**9 - 12*sqrt(x)*x**8 + 72*sqrt(x)*x**7 - 280*sqrt(x)*x**6 + 7
68*sqrt(x)*x**5 - 1536*sqrt(x)*x**4 + 2240*sqrt(x)*x**3 - 2304*sqrt(x)*x**
2 + 1536*sqrt(x)*x - 512*sqrt(x)),x)*x**5 + 2880*sqrt(x)*int(sqrt( - x**3
+ 4*x**2 - 8*x + 8)/(sqrt(x)*x**9 - 12*sqrt(x)*x**8 + 72*sqrt(x)*x**7 - 28
0*sqrt(x)*x**6 + 768*sqrt(x)*x**5 - 1536*sqrt(x)*x**4 + 2240*sqrt(x)*x**3
- 2304*sqrt(x)*x**2 + 1536*sqrt(x)*x - 512*sqrt(x)),x)*x**4 - 4608*sqrt(x)
*int(sqrt( - x**3 + 4*x**2 - 8*x + 8)/(sqrt(x)*x**9 - 12*sqrt(x)*x**8 + 72
*sqrt(x)*x**7 - 280*sqrt(x)*x**6 + 768*sqrt(x)*x**5 - 1536*sqrt(x)*x**4 +
2240*sqrt(x)*x**3 - 2304*sqrt(x)*x**2 + 1536*sqrt(x)*x - 512*sqrt(x)),x...
```

3.55 $\int ((2-x)x(4-2x+x^2))^{3/2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 116

$$\begin{aligned}\int ((2-x)x(4-2x+x^2))^{3/2}\,dx = &-\frac{2}{35}(13-3(1-x)^2)\sqrt{3-2(1-x)^2-(1-x)^4}(1-x)\\ &-\frac{1}{7}\left(3-2(1-x)^2-(1-x)^4\right)^{3/2}(1-x)\\ &+\frac{16}{5}\sqrt{3}E\left(\arcsin(1-x)\left|-\frac{1}{3}\right.\right)-\frac{176}{35}\sqrt{3}\,\text{EllipticF}\left(\arcsin(1-x),-\frac{1}{3}\right)\end{aligned}$$

output
```
-2/35*(13-3*(1-x)^2)*(3-2*(1-x)^2-(1-x)^4)^(1/2)*(1-x)-1/7*(3-2*(1-x)^2-(1
-x)^4)^(3/2)*(1-x)-16/5*3^(1/2)*EllipticE(-1+x,1/3*I*3^(1/2))+176/35*3^(1/
2)*EllipticF(-1+x,1/3*I*3^(1/2))
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 27.18 (sec) , antiderivative size = 278, normalized size of antiderivative = 2.40

$$\int ((2-x)x(4-2x+x^2))^{3/2}\,dx = \frac{\sqrt{-x(-8+8x-4x^2+x^3)}\left(\sqrt{\frac{4-2x+x^2}{x^2}}(-224+152x+44x^2-228x^3+230x^4-116x^5+\cdots\right.}{\cdots}$$

input
```
Integrate[((2 - x)*x*(4 - 2*x + x^2))^(3/2),x]
```

output
```
(Sqrt[-(x*(-8 + 8*x - 4*x^2 + x^3))]*(Sqrt[(4 - 2*x + x^2)/x^2]*(-224 + 15
2*x + 44*x^2 - 228*x^3 + 230*x^4 - 116*x^5 + 35*x^6 - 5*x^7) + 112*Sqrt[2]
*(-I + Sqrt[3])*Sqrt[((-I)*(-2 + x))/((-I + Sqrt[3])*x)]*EllipticE[ArcSin[
Sqrt[I + Sqrt[3] - (4*I)/x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(-I + Sqrt[3])
] + (304*I)*Sqrt[2]*Sqrt[((-I)*(-2 + x))/((-I + Sqrt[3])*x)]*EllipticF[Arc
Sin[Sqrt[I + Sqrt[3] - (4*I)/x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(-I + Sqrt
[3])]))/(35*(-2 + x)*x*Sqrt[(4 - 2*x + x^2)/x^2])
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.93, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {2458, 1404, 27, 1490, 27, 1494, 27, 399, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int ((2-x)x(x^2-2x+4))^{3/2}\,dx$$

$$\Big\downarrow \text{2458}$$

$$\int \left(-(x-1)^4-2(x-1)^2+3\right)^{3/2} d(x-1)$$

$$\Big\downarrow \text{1404}$$

$$\frac{3}{7}\int 2\left(3-(x-1)^2\right)\sqrt{-(x-1)^4-2(x-1)^2+3}d(x-1)+\frac{1}{7}(x-1)\left(-(x-1)^4-2(x-1)^2+3\right)^{3/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{6}{7}\int \left(3-(x-1)^2\right)\sqrt{-(x-1)^4-2(x-1)^2+3}d(x-1)+\frac{1}{7}(x-1)\left(-(x-1)^4-2(x-1)^2+3\right)^{3/2}$$

$$\Big\downarrow \text{1490}$$

$$\frac{6}{7}\left(\frac{1}{15}\left(13-3(x-1)^2\right)\sqrt{-(x-1)^4-2(x-1)^2+3}(x-1)-\frac{1}{15}\int -\frac{8\left(12-7(x-1)^2\right)}{\sqrt{-(x-1)^4-2(x-1)^2+3}}d(x-1)\right)+\frac{1}{7}(x-1)\left(-(x-1)^4-2(x-1)^2+3\right)^{3/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{6}{7}\left(\frac{8}{15}\int \frac{12-7(x-1)^2}{\sqrt{-(x-1)^4-2(x-1)^2+3}}d(x-1)+\frac{1}{15}\left(13-3(x-1)^2\right)\sqrt{-(x-1)^4-2(x-1)^2+3}(x-1)\right)+\frac{1}{7}(x-1)\left(-(x-1)^4-2(x-1)^2+3\right)^{3/2}$$

$$\Big\downarrow \text{1494}$$

$$\frac{6}{7}\left(\frac{16}{15}\int \frac{12-7(x-1)^2}{2\sqrt{1-(x-1)^2}\sqrt{(x-1)^2+3}}d(x-1)+\frac{1}{15}\left(13-3(x-1)^2\right)\sqrt{-(x-1)^4-2(x-1)^2+3}(x-1)\right)+\frac{1}{7}(x-1)\left(-(x-1)^4-2(x-1)^2+3\right)^{3/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{6}{7}\left(\frac{8}{15}\int \frac{12-7(x-1)^2}{\sqrt{1-(x-1)^2}\sqrt{(x-1)^2+3}}d(x-1)+\frac{1}{15}\left(13-3(x-1)^2\right)\sqrt{-(x-1)^4-2(x-1)^2+3}(x-1)\right)+\frac{1}{7}(x-1)\left(-(x-1)^4-2(x-1)^2+3\right)^{3/2}$$

$$\Big\downarrow \text{399}$$

$$\frac{6}{7}\left(\frac{8}{15}\left(33\int \frac{1}{\sqrt{1-(x-1)^2}\sqrt{(x-1)^2+3}}d(x-1)-7\int \frac{\sqrt{(x-1)^2+3}}{\sqrt{1-(x-1)^2}}d(x-1)\right)+\frac{1}{15}\left(13-3(x-1)^2\right)\sqrt{-\right.$$
$$\frac{1}{7}(x-1)\left(-(x-1)^4-2(x-1)^2+3\right)^{3/2}$$

$$\Big\downarrow \ 321$$

$$\frac{6}{7}\left(\frac{8}{15}\left(-7\int \frac{\sqrt{(x-1)^2+3}}{\sqrt{1-(x-1)^2}}d(x-1)-11\sqrt{3}\,\text{EllipticF}\left(\arcsin(1-x),-\frac{1}{3}\right)\right)+\frac{1}{15}\left(13-3(x-1)^2\right)\sqrt{-(x-\right.$$
$$\frac{1}{7}(x-1)\left(-(x-1)^4-2(x-1)^2+3\right)^{3/2}$$

$$\Big\downarrow \ 327$$

$$\frac{6}{7}\left(\frac{8}{15}\left(7\sqrt{3}E\left(\arcsin(1-x)\middle|-\frac{1}{3}\right)-11\sqrt{3}\,\text{EllipticF}\left(\arcsin(1-x),-\frac{1}{3}\right)\right)+\frac{1}{15}\left(13-3(x-1)^2\right)\sqrt{-(x-1)^4\right.$$
$$\frac{1}{7}(x-1)\left(-(x-1)^4-2(x-1)^2+3\right)^{3/2}$$

input
```
Int[((2 - x)*x*(4 - 2*x + x^2))^(3/2),x]
```

output
```
((3 - 2*(-1 + x)^2 - (-1 + x)^4)^(3/2)*(-1 + x))/7 + (6*(((13 - 3*(-1 + x)
^2)*Sqrt[3 - 2*(-1 + x)^2 - (-1 + x)^4]*(-1 + x))/15 + (8*(7*Sqrt[3]*Ellip
ticE[ArcSin[1 - x], -1/3] - 11*Sqrt[3]*EllipticF[ArcSin[1 - x], -1/3]))/15
))/7
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 1404
```
Int[((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*((a + b
*x^2 + c*x^4)^p/(4*p + 1)), x] + Simp[2*(p/(4*p + 1))   Int[(2*a + b*x^2)*(
a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*
c, 0] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 1490
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Simp[2*(p/(c*(4*p + 1)*(4*p + 3)))
 Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3)
- b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

rule 1494
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[(d + e*x^2)/(Sqr
t[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e
}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

rule 2458
```
Int[(Pn_)^(p_.), x_Symbol] :> With[{S = Coeff[Pn, x, Expon[Pn, x] - 1]/(Exp
on[Pn, x]*Coeff[Pn, x, Expon[Pn, x]])}, Subst[Int[ExpandToSum[Pn /. x -> x
- S, x]^p, x], x, x + S] /; BinomialQ[Pn /. x -> x - S, x] || (IntegerQ[Exp
on[Pn, x]/2] && TrinomialQ[Pn /. x -> x - S, x])] /; FreeQ[p, x] && PolyQ[P
n, x] && GtQ[Expon[Pn, x], 2] && NeQ[Coeff[Pn, x, Expon[Pn, x] - 1], 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 953 vs. 2(100) = 200.

Time = 1.59 (sec) , antiderivative size = 954, normalized size of antiderivative = 8.22

method	result	size
risch	Expression too large to display	954
default	Expression too large to display	1050
elliptic	Expression too large to display	1050

input
```
int(((2-x)*x*(x^2-2*x+4))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/35*(5*x^5-25*x^4+66*x^3-98*x^2+32*x+20)*x*(x-2)*(x^2-2*x+4)/(-x*(x-2)*(x
^2-2*x+4))^(1/2)+32/7*(-1-I*3^(1/2))*((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2)
)^(1/2)*(x-2)^2*((x-1+I*3^(1/2))/(1-I*3^(1/2))/(x-2))^(1/2)*((x-1-I*3^(1/2
))/(1+I*3^(1/2))/(x-2))^(1/2)/(-1+I*3^(1/2))/(-x*(x-2)*(x-1+I*3^(1/2))*(x-
1-I*3^(1/2)))^(1/2)*EllipticF(((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2)
,((1+I*3^(1/2))*(-1-I*3^(1/2))/(-1+I*3^(1/2))/(1-I*3^(1/2)))^(1/2))+64/5*(
-1-I*3^(1/2))*((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2)*(x-2)^2*((x-1+I
*3^(1/2))/(1-I*3^(1/2))/(x-2))^(1/2)*((x-1-I*3^(1/2))/(1+I*3^(1/2))/(x-2))
^(1/2)/(-1+I*3^(1/2))/(-x*(x-2)*(x-1+I*3^(1/2))*(x-1-I*3^(1/2)))^(1/2)*(2*
EllipticF(((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2),((1+I*3^(1/2))*(-1-
I*3^(1/2))/(-1+I*3^(1/2))/(1-I*3^(1/2)))^(1/2))-2*EllipticPi(((-1+I*3^(1/2
))*x/(1+I*3^(1/2))/(x-2))^(1/2),(1+I*3^(1/2))/(-1+I*3^(1/2)),((1+I*3^(1/2)
)*(-1-I*3^(1/2))/(-1+I*3^(1/2))/(1-I*3^(1/2)))^(1/2)))-16/5*(x*(x-1+I*3^(1
/2))*(x-1-I*3^(1/2))+2*(-1-I*3^(1/2))*((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2
))^(1/2)*(x-2)^2*((x-1+I*3^(1/2))/(1-I*3^(1/2))/(x-2))^(1/2)*((x-1-I*3^(1/
2))/(1+I*3^(1/2))/(x-2))^(1/2)*(1/2*(6+2*I*3^(1/2))/(-1+I*3^(1/2))*Ellipti
cF(((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2),((1+I*3^(1/2))*(-1-I*3^(1/
2))/(-1+I*3^(1/2))/(1-I*3^(1/2)))^(1/2))+1/2*(-1+I*3^(1/2))*EllipticE(((-1
+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2),((1+I*3^(1/2))*(-1-I*3^(1/2))/(-1
+I*3^(1/2))/(1-I*3^(1/2)))^(1/2))-4/(-1+I*3^(1/2))*EllipticPi(((-1+I*3^...
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.78

$$\int \left((2-x)x\left(4-2x+x^2\right)\right)^{3/2}\,dx =$$
$$-\frac{112\,(-i\,x+i)E(\arcsin\left(\frac{1}{x-1}\right)\,|-3)+80\,(-i\,x+i)F(\arcsin\left(\frac{1}{x-1}\right)\,|-3)+(5\,x^6-30\,x^5+91\,x^4-164\,x^3}{35\,(x-1)}$$

input
```
integrate(((2-x)*x*(x^2-2*x+4))^(3/2),x, algorithm="fricas")
```

output
```
-1/35*(112*(-I*x + I)*elliptic_e(arcsin(1/(x - 1)), -3) + 80*(-I*x + I)*el
liptic_f(arcsin(1/(x - 1)), -3) + (5*x^6 - 30*x^5 + 91*x^4 - 164*x^3 + 130
*x^2 - 12*x - 132)*sqrt(-x^4 + 4*x^3 - 8*x^2 + 8*x))/(x - 1)
```

Sympy [F]

$$\int \left((2-x)x\left(4-2x+x^2\right)\right)^{3/2}\,dx = \int \left(x(2-x)\left(x^2-2x+4\right)\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(((2-x)*x*(x**2-2*x+4))**(3/2),x)
```

output
```
Integral((x*(2 - x)*(x**2 - 2*x + 4))**(3/2), x)
```

Maxima [F]

$$\int \left((2-x)x\left(4-2x+x^2\right)\right)^{3/2}\,dx = \int \left(-\left(x^2-2\,x+4\right)(x-2)x\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(((2-x)*x*(x^2-2*x+4))^(3/2),x, algorithm="maxima")
```

output
```
integrate((-(x^2 - 2*x + 4)*(x - 2)*x)^(3/2), x)
```

Giac [F]

$$\int \left((2-x)x\left(4-2x+x^2\right)\right)^{3/2} dx = \int \left(-\left(x^2-2x+4\right)(x-2)x\right)^{\frac{3}{2}} dx$$

input
```
integrate((((2-x)*x*(x^2-2*x+4))^(3/2),x, algorithm="giac")
```

output
```
integrate((-(x^2 - 2*x + 4)*(x - 2)*x)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left((2-x)x\left(4-2x+x^2\right)\right)^{3/2} dx = \int \left(-x\,(x-2)\,\left(x^2-2x+4\right)\right)^{3/2} dx$$

input
```
int((-x*(x - 2)*(x^2 - 2*x + 4))^(3/2),x)
```

output
```
int((-x*(x - 2)*(x^2 - 2*x + 4))^(3/2), x)
```

Reduce [F]

$$\begin{aligned}\int \left((2-x)x\left(4-2x+x^2\right)\right)^{3/2} dx = & -\frac{\sqrt{x}\,\sqrt{-x^3+4x^2-8x+8}\,x^5}{7} \\ & + \frac{5\sqrt{x}\,\sqrt{-x^3+4x^2-8x+8}\,x^4}{7} - \frac{66\sqrt{x}\,\sqrt{-x^3+4x^2-8x+8}\,x^3}{35} \\ & + \frac{14\sqrt{x}\,\sqrt{-x^3+4x^2-8x+8}\,x^2}{5} - \frac{32\sqrt{x}\,\sqrt{-x^3+4x^2-8x+8}\,x}{35} \\ & - \frac{116\sqrt{x}\,\sqrt{-x^3+4x^2-8x+8}}{105} + \frac{16\left(\int \frac{\sqrt{x}\,\sqrt{-x^3+4x^2-8x+8}\,x^2}{x^3-4x^2+8x-8}dx\right)}{15} \\ & - \frac{464\left(\int \frac{\sqrt{x}\,\sqrt{-x^3+4x^2-8x+8}}{x^4-4x^3+8x^2-8x}dx\right)}{105} - \frac{32\left(\int \frac{\sqrt{x}\,\sqrt{-x^3+4x^2-8x+8}}{x^3-4x^2+8x-8}dx\right)}{15}\end{aligned}$$

input
```
int(((2-x)*x*(x^2-2*x+4))^(3/2),x)
```

output
```
( - 15*sqrt(x)*sqrt( - x**3 + 4*x**2 - 8*x + 8)*x**5 + 75*sqrt(x)*sqrt( -
x**3 + 4*x**2 - 8*x + 8)*x**4 - 198*sqrt(x)*sqrt( - x**3 + 4*x**2 - 8*x +
8)*x**3 + 294*sqrt(x)*sqrt( - x**3 + 4*x**2 - 8*x + 8)*x**2 - 96*sqrt(x)*s
qrt( - x**3 + 4*x**2 - 8*x + 8)*x - 116*sqrt(x)*sqrt( - x**3 + 4*x**2 - 8*
x + 8) + 112*int((sqrt(x)*sqrt( - x**3 + 4*x**2 - 8*x + 8)*x**2)/(x**3 - 4
*x**2 + 8*x - 8),x) - 464*int((sqrt(x)*sqrt( - x**3 + 4*x**2 - 8*x + 8))/(
x**4 - 4*x**3 + 8*x**2 - 8*x),x) - 224*int((sqrt(x)*sqrt( - x**3 + 4*x**2
- 8*x + 8))/(x**3 - 4*x**2 + 8*x - 8),x))/105
```

3.56 $\int \sqrt{(2-x)x\left(4-2x+x^2\right)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 68

$$\int \sqrt{(2-x)x\left(4-2x+x^2\right)}\,dx = -\frac{1}{3}\sqrt{3-2(1-x)^2-(1-x)^4}(1-x) + \frac{2E\left(\arcsin(1-x)\left|-\frac{1}{3}\right.\right)}{\sqrt{3}} - \frac{4\operatorname{EllipticF}\left(\arcsin(1-x),-\frac{1}{3}\right)}{\sqrt{3}}$$

output
```
-1/3*(3-2*(1-x)^2-(1-x)^4)^(1/2)*(1-x)-2/3*3^(1/2)*EllipticE(-1+x,1/3*I*3^
(1/2))+4/3*3^(1/2)*EllipticF(-1+x,1/3*I*3^(1/2))
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 20.86 (sec) , antiderivative size = 256, normalized size of antiderivative = 3.76

$$\int \sqrt{(2-x)x\left(4-2x+x^2\right)}\,dx = \frac{\sqrt{-x\left(-8+8x-4x^2+x^3\right)}\left(\sqrt{\frac{4-2x+x^2}{x^2}}\left(-4+4x-3x^2+x^3\right)+2\sqrt{2}\left(-i+\sqrt{3}\right)\sqrt{-\frac{i(-2+x)}{\left(-i+\sqrt{3}\right)x}}E\left(\arcsin\right.\right.}{3(-2+x)x\sqrt{\frac{4-2x+}{x^2}}}$$

input
```
Integrate[Sqrt[(2 - x)*x*(4 - 2*x + x^2)],x]
```

output
```
(Sqrt[-(x*(-8 + 8*x - 4*x^2 + x^3))]*(Sqrt[(4 - 2*x + x^2)/x^2]*(-4 + 4*x
- 3*x^2 + x^3) + 2*Sqrt[2]*(-I + Sqrt[3])*Sqrt[((-I)*(-2 + x))/((-I + Sqrt
[3])*x)]*EllipticE[ArcSin[Sqrt[I + Sqrt[3] - (4*I)/x]/(Sqrt[2]*3^(1/4))],
(2*Sqrt[3])/(-I + Sqrt[3])] + (8*I)*Sqrt[2]*Sqrt[((-I)*(-2 + x))/((-I + Sq
rt[3])*x)]*EllipticF[ArcSin[Sqrt[I + Sqrt[3] - (4*I)/x]/(Sqrt[2]*3^(1/4))]
, (2*Sqrt[3])/(-I + Sqrt[3])]))/(3*(-2 + x)*x*Sqrt[(4 - 2*x + x^2)/x^2])
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.97, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {2458, 1404, 27, 1494, 27, 399, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{(2-x)x\left(x^2-2x+4\right)}dx$$

$$\Big\downarrow \text{ 2458}$$

$$\int \sqrt{-(x-1)^4-2(x-1)^2+3}d(x-1)$$

$$\Big\downarrow \text{ 1404}$$

$$\frac{1}{3}\int \frac{2\left(3-(x-1)^2\right)}{\sqrt{-(x-1)^4-2(x-1)^2+3}}d(x-1)+\frac{1}{3}\sqrt{-(x-1)^4-2(x-1)^2+3}(x-1)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{2}{3}\int \frac{3-(x-1)^2}{\sqrt{-(x-1)^4-2(x-1)^2+3}}d(x-1)+\frac{1}{3}\sqrt{-(x-1)^4-2(x-1)^2+3}(x-1)$$

$$\Big\downarrow \text{ 1494}$$

$$\frac{4}{3}\int \frac{3-(x-1)^2}{2\sqrt{1-(x-1)^2}\sqrt{(x-1)^2+3}}d(x-1)+\frac{1}{3}\sqrt{-(x-1)^4-2(x-1)^2+3}(x-1)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{2}{3}\int \frac{3-(x-1)^2}{\sqrt{1-(x-1)^2}\sqrt{(x-1)^2+3}}d(x-1)+\frac{1}{3}\sqrt{-(x-1)^4-2(x-1)^2+3}(x-1)$$

$$\Big\downarrow \text{399}$$

$$\frac{2}{3}\left(6\int \frac{1}{\sqrt{1-(x-1)^2}\sqrt{(x-1)^2+3}}d(x-1)-\int \frac{\sqrt{(x-1)^2+3}}{\sqrt{1-(x-1)^2}}d(x-1)\right)+\frac{1}{3}\sqrt{-(x-1)^4-2(x-1)^2+3}(x-1)$$

$$\Big\downarrow \text{321}$$

$$\frac{2}{3}\left(-\int \frac{\sqrt{(x-1)^2+3}}{\sqrt{1-(x-1)^2}}d(x-1)-2\sqrt{3}\,\text{EllipticF}\left(\arcsin(1-x),-\frac{1}{3}\right)\right)+\frac{1}{3}\sqrt{-(x-1)^4-2(x-1)^2+3}(x-1)$$

$$\Big\downarrow \text{327}$$

$$\frac{2}{3}\left(\sqrt{3}E\left(\arcsin(1-x)\left|-\frac{1}{3}\right.\right)-2\sqrt{3}\,\text{EllipticF}\left(\arcsin(1-x),-\frac{1}{3}\right)\right)+\frac{1}{3}\sqrt{-(x-1)^4-2(x-1)^2+3}(x-1)$$

input
```
Int[Sqrt[(2 - x)*x*(4 - 2*x + x^2)],x]
```

output
```
(Sqrt[3 - 2*(-1 + x)^2 - (-1 + x)^4]*(-1 + x))/3 + (2*(Sqrt[3]*EllipticE[A
rcSin[1 - x], -1/3] - 2*Sqrt[3]*EllipticF[ArcSin[1 - x], -1/3]))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 1404
```
Int[((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*((a + b
*x^2 + c*x^4)^p/(4*p + 1)), x] + Simp[2*(p/(4*p + 1))   Int[(2*a + b*x^2)*(
a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*
c, 0] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 1494
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[(d + e*x^2)/(Sqr
t[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e
}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

rule 2458
```
Int[(Pn_)^(p_.), x_Symbol] :> With[{S = Coeff[Pn, x, Expon[Pn, x] - 1]/(Exp
on[Pn, x]*Coeff[Pn, x, Expon[Pn, x]])}, Subst[Int[ExpandToSum[Pn /. x -> x
- S, x]^p, x], x, x + S] /; BinomialQ[Pn /. x -> x - S, x] || (IntegerQ[Exp
on[Pn, x]/2] && TrinomialQ[Pn /. x -> x - S, x])] /; FreeQ[p, x] && PolyQ[P
n, x] && GtQ[Expon[Pn, x], 2] && NeQ[Coeff[Pn, x, Expon[Pn, x] - 1], 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 931 vs. $2(60) = 120$.

Time = 1.27 (sec) , antiderivative size = 932, normalized size of antiderivative = 13.71

method	result	size
risch	Expression too large to display	932
default	Expression too large to display	946
elliptic	Expression too large to display	946

input
```
int(((2-x)*x*(x^2-2*x+4))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(x-1)*x*(x-2)*(x^2-2*x+4)/(-x*(x-2)*(x^2-2*x+4))^(1/2)-2/3*(x*(x-1+I*
3^(1/2))*(x-1-I*3^(1/2))+2*(-1-I*3^(1/2))*((-1+I*3^(1/2))*x/(1+I*3^(1/2))/
(x-2))^(1/2)*(x-2)^2*((x-1+I*3^(1/2))/(1-I*3^(1/2))/(x-2))^(1/2)*((x-1-I*3
^(1/2))/(1+I*3^(1/2))/(x-2))^(1/2)*(1/2*(6+2*I*3^(1/2))/(-1+I*3^(1/2))*Ell
ipticF(((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2),((1+I*3^(1/2))*(-1-I*3
^(1/2))/(-1+I*3^(1/2))/(1-I*3^(1/2)))^(1/2))+1/2*(-1+I*3^(1/2))*EllipticE(
((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2),((1+I*3^(1/2))*(-1-I*3^(1/2))
/(-1+I*3^(1/2))/(1-I*3^(1/2)))^(1/2))-4/(-1+I*3^(1/2))*EllipticPi(((-1+I*3
^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2),(-1-I*3^(1/2))/(1-I*3^(1/2)),((1+I*3^
(1/2))*(-1-I*3^(1/2))/(-1+I*3^(1/2))/(1-I*3^(1/2)))^(1/2))))/(-x*(x-2)*(x-
1+I*3^(1/2))*(x-1-I*3^(1/2)))^(1/2)+8/3*(-1-I*3^(1/2))*((-1+I*3^(1/2))*x/(
1+I*3^(1/2))/(x-2))^(1/2)*(x-2)^2*((x-1+I*3^(1/2))/(1-I*3^(1/2))/(x-2))^(1
/2)*((x-1-I*3^(1/2))/(1+I*3^(1/2))/(x-2))^(1/2)/(-1+I*3^(1/2))/(-x*(x-2)*(
x-1+I*3^(1/2))*(x-1-I*3^(1/2)))^(1/2)*EllipticF(((-1+I*3^(1/2))*x/(1+I*3^(
1/2))/(x-2))^(1/2),((1+I*3^(1/2))*(-1-I*3^(1/2))/(-1+I*3^(1/2))/(1-I*3^(1/
2)))^(1/2))+8/3*(-1-I*3^(1/2))*((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2
)*(x-2)^2*((x-1+I*3^(1/2))/(1-I*3^(1/2))/(x-2))^(1/2)*((x-1-I*3^(1/2))/(1+
I*3^(1/2))/(x-2))^(1/2)/(-1+I*3^(1/2))/(-x*(x-2)*(x-1+I*3^(1/2))*(x-1-I*3^
(1/2)))^(1/2)*(2*EllipticF(((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2),((
1+I*3^(1/2))*(-1-I*3^(1/2))/(-1+I*3^(1/2))/(1-I*3^(1/2)))^(1/2))-2*Elli...
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.01

$$\int \sqrt{(2-x)x\left(4-2x+x^2\right)}\,dx = -\frac{2\,(-i\,x+i)E(\arcsin\left(\frac{1}{x-1}\right)\,|{-3})+4\,(-i\,x+i)F(\arcsin\left(\frac{1}{x-1}\right)\,|{-3})-\sqrt{-x^4+4\,x^3-8\,x^2+8\,x}(x^2-}{3\,(x-1)}$$

input
```
integrate(((2-x)*x*(x^2-2*x+4))^(1/2),x, algorithm="fricas")
```

output
```
-1/3*(2*(-I*x + I)*elliptic_e(arcsin(1/(x - 1)), -3) + 4*(-I*x + I)*ellipt
ic_f(arcsin(1/(x - 1)), -3) - sqrt(-x^4 + 4*x^3 - 8*x^2 + 8*x)*(x^2 - 2*x
+ 3))/(x - 1)
```

Sympy [F]

$$\int \sqrt{(2-x)x\left(4-2x+x^2\right)}\,dx = \int \sqrt{x\left(2-x\right)\left(x^2-2x+4\right)}\,dx$$

input
```
integrate(((2-x)*x*(x**2-2*x+4))**(1/2),x)
```

output
```
Integral(sqrt(x*(2 - x)*(x**2 - 2*x + 4)), x)
```

Maxima [F]

$$\int \sqrt{(2-x)x\left(4-2x+x^2\right)}\,dx = \int \sqrt{-(x^2-2\,x+4)(x-2)x}\,dx$$

input
```
integrate(((2-x)*x*(x^2-2*x+4))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-(x^2 - 2*x + 4)*(x - 2)*x), x)
```

Giac [F]

$$\int \sqrt{(2-x)x\left(4-2x+x^2\right)}\,dx = \int \sqrt{-(x^2-2\,x+4)(x-2)x}\,dx$$

input
```
integrate((((2-x)*x*(x^2-2*x+4))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-(x^2 - 2*x + 4)*(x - 2)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{(2-x)x\left(4-2x+x^2\right)}\,dx = \int \sqrt{-x\,\left(x-2\right)\,\left(x^2-2\,x+4\right)}\,dx$$

input
```
int((-x*(x - 2)*(x^2 - 2*x + 4))^(1/2),x)
```

output
```
int((-x*(x - 2)*(x^2 - 2*x + 4))^(1/2), x)
```

Reduce [F]

$$\begin{aligned}\int \sqrt{(2-x)x\left(4-2x+x^2\right)}\,dx &= \frac{\sqrt{x}\,\sqrt{-x^3+4x^2-8x+8}\,x}{3} \\ &\quad - \frac{4\sqrt{x}\,\sqrt{-x^3+4x^2-8x+8}}{9} \\ &\quad + \frac{2\left(\int \frac{\sqrt{x}\,\sqrt{-x^3+4x^2-8x+8}\,x^2}{x^3-4x^2+8x-8}dx\right)}{9} \\ &\quad - \frac{16\left(\int \frac{\sqrt{x}\,\sqrt{-x^3+4x^2-8x+8}}{x^4-4x^3+8x^2-8x}dx\right)}{9} \\ &\quad - \frac{4\left(\int \frac{\sqrt{x}\,\sqrt{-x^3+4x^2-8x+8}}{x^3-4x^2+8x-8}dx\right)}{9}\end{aligned}$$

input
```
int(((2-x)*x*(x^2-2*x+4))^(1/2),x)
```

output
```
(3*sqrt(x)*sqrt( - x**3 + 4*x**2 - 8*x + 8)*x - 4*sqrt(x)*sqrt( - x**3 + 4
*x**2 - 8*x + 8) + 2*int((sqrt(x)*sqrt( - x**3 + 4*x**2 - 8*x + 8)*x**2)/(
x**3 - 4*x**2 + 8*x - 8),x) - 16*int((sqrt(x)*sqrt( - x**3 + 4*x**2 - 8*x
+ 8))/(x**4 - 4*x**3 + 8*x**2 - 8*x),x) - 4*int((sqrt(x)*sqrt( - x**3 + 4*
x**2 - 8*x + 8))/(x**3 - 4*x**2 + 8*x - 8),x))/9
```

3.57 $\int \frac{1}{\sqrt{(2-x)x(4-2x+x^2)}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 17

$$\int \frac{1}{\sqrt{(2-x)x(4-2x+x^2)}}\,dx=-\frac{\text{EllipticF}\left(\arcsin(1-x),-\frac{1}{3}\right)}{\sqrt{3}}$$

output
```
1/3*3^(1/2)*EllipticF(-1+x,1/3*I*3^(1/2))
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 34.47 (sec) , antiderivative size = 100, normalized size of antiderivative = 5.88

$$\int \frac{1}{\sqrt{(2-x)x(4-2x+x^2)}}\,dx = -\frac{\sqrt[3]{-1}(-2+x)^2\sqrt{\frac{x\left(-1+i\sqrt{3}+x\right)}{(-2+x)^2}}\sqrt{\frac{-2+x-\sqrt[3]{-1}x}{-2+x}}\,\text{EllipticF}\left(\arcsin\left(\sqrt{-\frac{(-1)^{2/3}x}{-2+x}}\right),(-1)^{2/3}\right)}{\sqrt{-x\left(-8+8x-4x^2+x^3\right)}}$$

input
```
Integrate[1/Sqrt[(2 - x)*x*(4 - 2*x + x^2)],x]
```

output
```
-(((-1)^(1/3)*(-2 + x)^2*Sqrt[(x*(-1 + I*Sqrt[3] + x))/(-2 + x)^2]*Sqrt[(-
2 + x - (-1)^(1/3)*x)/(-2 + x)]*EllipticF[ArcSin[Sqrt[-(((-1)^(2/3)*x)/(-2
 + x))]], (-1)^(2/3)])/Sqrt[-(x*(-8 + 8*x - 4*x^2 + x^3))])
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {2458, 1408, 27, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{(2-x)x\left(x^2-2x+4\right)}}dx$$

$$\Big\downarrow \text{ 2458}$$

$$\int \frac{1}{\sqrt{-(x-1)^4-2(x-1)^2+3}}d(x-1)$$

$$\Big\downarrow \text{ 1408}$$

$$2\int \frac{1}{2\sqrt{1-(x-1)^2}\sqrt{(x-1)^2+3}}d(x-1)$$

$$\Big\downarrow \text{ 27}$$

$$\int \frac{1}{\sqrt{1-(x-1)^2}\sqrt{(x-1)^2+3}}d(x-1)$$

$$\Big\downarrow \text{ 321}$$

$$-\frac{\text{EllipticF}\left(\arcsin(1-x),-\frac{1}{3}\right)}{\sqrt{3}}$$

input
```
Int[1/Sqrt[(2 - x)*x*(4 - 2*x + x^2)],x]
```

output
```
-(EllipticF[ArcSin[1 - x], -1/3]/Sqrt[3])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 1408
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
 - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[
c, 0]
```

rule 2458
```
Int[(Pn_)^(p_.), x_Symbol] :> With[{S = Coeff[Pn, x, Expon[Pn, x] - 1]/(Exp
on[Pn, x]*Coeff[Pn, x, Expon[Pn, x]])}, Subst[Int[ExpandToSum[Pn /. x -> x
- S, x]^p, x], x, x + S] /; BinomialQ[Pn /. x -> x - S, x] || (IntegerQ[Exp
on[Pn, x]/2] && TrinomialQ[Pn /. x -> x - S, x])] /; FreeQ[p, x] && PolyQ[P
n, x] && GtQ[Expon[Pn, x], 2] && NeQ[Coeff[Pn, x, Expon[Pn, x] - 1], 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 199 vs. $2(15) = 30$.

Time = 0.61 (sec) , antiderivative size = 200, normalized size of antiderivative = 11.76

method	result	siz
default	$\frac{2\left(-1-i\sqrt{3}\right)\sqrt{\frac{\left(-1+i\sqrt{3}\right)x}{\left(1+i\sqrt{3}\right)(x-2)}}\,(x-2)^{2}\sqrt{\frac{x-1+i\sqrt{3}}{\left(1-i\sqrt{3}\right)(x-2)}}\,\sqrt{\frac{x-1-i\sqrt{3}}{\left(1+i\sqrt{3}\right)(x-2)}}\,\operatorname{EllipticF}\left(\sqrt{\frac{\left(-1+i\sqrt{3}\right)x}{\left(1+i\sqrt{3}\right)(x-2)}},\sqrt{\frac{\left(1+i\sqrt{3}\right)\left(-1-i\sqrt{3}\right)}{\left(-1+i\sqrt{3}\right)\left(1-i\sqrt{3}\right)}}\right)}{\left(-1+i\sqrt{3}\right)\sqrt{-x(x-2)\left(x-1+i\sqrt{3}\right)\left(x-1-i\sqrt{3}\right)}}$	20
elliptic	$\frac{2\left(-1-i\sqrt{3}\right)\sqrt{\frac{\left(-1+i\sqrt{3}\right)x}{\left(1+i\sqrt{3}\right)(x-2)}}\,(x-2)^{2}\sqrt{\frac{x-1+i\sqrt{3}}{\left(1-i\sqrt{3}\right)(x-2)}}\,\sqrt{\frac{x-1-i\sqrt{3}}{\left(1+i\sqrt{3}\right)(x-2)}}\,\operatorname{EllipticF}\left(\sqrt{\frac{\left(-1+i\sqrt{3}\right)x}{\left(1+i\sqrt{3}\right)(x-2)}},\sqrt{\frac{\left(1+i\sqrt{3}\right)\left(-1-i\sqrt{3}\right)}{\left(-1+i\sqrt{3}\right)\left(1-i\sqrt{3}\right)}}\right)}{\left(-1+i\sqrt{3}\right)\sqrt{-x(x-2)\left(x-1+i\sqrt{3}\right)\left(x-1-i\sqrt{3}\right)}}$	20

input
```
int(1/((2-x)*x*(x^2-2*x+4))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(-1-I*3^(1/2))*((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2)*(x-2)^2*((x-
1+I*3^(1/2))/(1-I*3^(1/2))/(x-2))^(1/2)*((x-1-I*3^(1/2))/(1+I*3^(1/2))/(x-
2))^(1/2)/(-1+I*3^(1/2))/(-x*(x-2)*(x-1+I*3^(1/2))*(x-1-I*3^(1/2)))^(1/2)*
EllipticF(((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2),((1+I*3^(1/2))*(-1-
I*3^(1/2))/(-1+I*3^(1/2))/(1-I*3^(1/2)))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

$$\int \frac{1}{\sqrt{(2-x)x\left(4-2x+x^2\right)}}\,dx = -\frac{1}{2}\sqrt{2}\text{weierstrassPInverse}\left(-\frac{2}{3},\frac{7}{54},-\frac{x-3}{3\,x}\right)$$

input
```
integrate(1/((2-x)*x*(x^2-2*x+4))^(1/2),x, algorithm="fricas")
```

output
```
-1/2*sqrt(2)*weierstrassPInverse(-2/3, 7/54, -1/3*(x - 3)/x)
```

Sympy [F]

$$\int \frac{1}{\sqrt{(2-x)x\left(4-2x+x^2\right)}}\,dx = \int \frac{1}{\sqrt{x\left(2-x\right)\left(x^2-2x+4\right)}}\,dx$$

input
```
integrate(1/((2-x)*x*(x**2-2*x+4))**(1/2),x)
```

output
```
Integral(1/sqrt(x*(2 - x)*(x**2 - 2*x + 4)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{(2-x)x\left(4-2x+x^2\right)}}\,dx = \int \frac{1}{\sqrt{-(x^2-2\,x+4)(x-2)x}}\,dx$$

input
```
integrate(1/((2-x)*x*(x^2-2*x+4))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(-(x^2 - 2*x + 4)*(x - 2)*x), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{(2-x)x\left(4-2x+x^2\right)}}\,dx = \int \frac{1}{\sqrt{-(x^2-2\,x+4)(x-2)x}}\,dx$$

input
```
integrate(1/((2-x)*x*(x^2-2*x+4))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(-(x^2 - 2*x + 4)*(x - 2)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{(2-x)x\left(4-2x+x^2\right)}}\,dx = \int \frac{1}{\sqrt{-x\,\left(x-2\right)\,\left(x^2-2\,x+4\right)}}\,dx$$

input
```
int(1/(-x*(x - 2)*(x^2 - 2*x + 4))^(1/2),x)
```

output
```
int(1/(-x*(x - 2)*(x^2 - 2*x + 4))^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{(2-x)x\,(4-2x+x^2)}}\,dx = -\left(\int \frac{\sqrt{x}\,\sqrt{-x^3+4x^2-8x+8}}{x^4-4x^3+8x^2-8x}dx\right)$$

input
```
int(1/((2-x)*x*(x^2-2*x+4))^(1/2),x)
```

output
```
 - int((sqrt(x)*sqrt( - x**3 + 4*x**2 - 8*x + 8))/(x**4 - 4*x**3 + 8*x**2
- 8*x),x)
```

3.58 $\int \frac{1}{\left((2-x)x\left(4-2x+x^2\right)\right)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 79

$$\int \frac{1}{\left((2-x)x\left(4-2x+x^2\right)\right)^{3/2}}\,dx = -\frac{\left(5+(-1+x)^2\right)(1-x)}{24\sqrt{3-2(1-x)^2-(1-x)^4}} + \frac{E\left(\arcsin(1-x)\left|-\frac{1}{3}\right.\right)}{8\sqrt{3}} - \frac{\operatorname{EllipticF}\left(\arcsin(1-x),-\frac{1}{3}\right)}{4\sqrt{3}}$$

output
```
-1/24*(5+(-1+x)^2)*(1-x)/(3-2*(1-x)^2-(1-x)^4)^(1/2)-1/24*3^(1/2)*Elliptic
E(-1+x,1/3*I*3^(1/2))+1/12*3^(1/2)*EllipticF(-1+x,1/3*I*3^(1/2))
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 21.51 (sec) , antiderivative size = 298, normalized size of antiderivative = 3.77

$$\int \frac{1}{\left((2-x)x\left(4-2x+x^2\right)\right)^{3/2}}\,dx = \frac{(-2+x)^2x(4-2x+x^2)\left(2(-1+x)x-3(4-2x+x^2)-\frac{3x(4-2x+x^2}{-2+x}\right.}{}$$

input
```
Integrate[((2 - x)*x*(4 - 2*x + x^2))^(-3/2),x]
```

output
```
((-2 + x)^2*x*(4 - 2*x + x^2)*(2*(-1 + x)*x - 3*(4 - 2*x + x^2) - (3*x*(4
- 2*x + x^2))/(-2 + x) - 4*(2 - x)*Sqrt[(4 - 2*x + x^2)/(-2 + x)^2]*(x*Sqr
t[(4 - 2*x + x^2)/(-2 + x)^2] - Sqrt[2]*(I + Sqrt[3])*Sqrt[(I*x)/((I + Sqr
t[3])*(-2 + x))]*EllipticE[ArcSin[Sqrt[-I + Sqrt[3] - (4*I)/(-2 + x)]/(Sqr
t[2]*3^(1/4))], (2*Sqrt[3])/(I + Sqrt[3])] + (4*I)*Sqrt[2]*Sqrt[(I*x)/((I
+ Sqrt[3])*(-2 + x))]*EllipticF[ArcSin[Sqrt[-I + Sqrt[3] - (4*I)/(-2 + x)]
/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(I + Sqrt[3])])))/(96*(-(x*(-8 + 8*x - 4*
x^2 + x^3)))^(3/2))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.92, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {2458, 1405, 27, 1494, 27, 399, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left((2-x)x\left(x^2-2x+4\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 2458}$$

$$\int \frac{1}{\left(-(x-1)^4-2(x-1)^2+3\right)^{3/2}}d(x-1)$$

$$\Big\downarrow \text{ 1405}$$

$$\frac{\left((x-1)^2+5\right)(x-1)}{24\sqrt{-(x-1)^4-2(x-1)^2+3}}-\frac{1}{48}\int -\frac{2\left(3-(x-1)^2\right)}{\sqrt{-(x-1)^4-2(x-1)^2+3}}d(x-1)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{24}\int \frac{3-(x-1)^2}{\sqrt{-(x-1)^4-2(x-1)^2+3}}d(x-1)+\frac{\left((x-1)^2+5\right)(x-1)}{24\sqrt{-(x-1)^4-2(x-1)^2+3}}$$

$$\Big\downarrow \text{ 1494}$$

$$\frac{1}{12}\int \frac{3-(x-1)^2}{2\sqrt{1-(x-1)^2}\sqrt{(x-1)^2+3}}d(x-1)+\frac{\left((x-1)^2+5\right)(x-1)}{24\sqrt{-(x-1)^4-2(x-1)^2+3}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{24}\int \frac{3-(x-1)^2}{\sqrt{1-(x-1)^2}\sqrt{(x-1)^2+3}}d(x-1)+\frac{\left((x-1)^2+5\right)(x-1)}{24\sqrt{-(x-1)^4-2(x-1)^2+3}}$$

$$\Big\downarrow \text{399}$$

$$\frac{1}{24}\left(6\int \frac{1}{\sqrt{1-(x-1)^2}\sqrt{(x-1)^2+3}}d(x-1)-\int \frac{\sqrt{(x-1)^2+3}}{\sqrt{1-(x-1)^2}}d(x-1)\right)+\frac{\left((x-1)^2+5\right)(x-1)}{24\sqrt{-(x-1)^4-2(x-1)^2+3}}$$

$$\Big\downarrow \text{321}$$

$$\frac{1}{24}\left(-\int \frac{\sqrt{(x-1)^2+3}}{\sqrt{1-(x-1)^2}}d(x-1)-2\sqrt{3}\,\text{EllipticF}\left(\arcsin(1-x),-\frac{1}{3}\right)\right)+\frac{\left((x-1)^2+5\right)(x-1)}{24\sqrt{-(x-1)^4-2(x-1)^2+3}}$$

$$\Big\downarrow \text{327}$$

$$\frac{1}{24}\left(\sqrt{3}E\left(\arcsin(1-x)\left|-\frac{1}{3}\right.\right)-2\sqrt{3}\,\text{EllipticF}\left(\arcsin(1-x),-\frac{1}{3}\right)\right)+\frac{\left((x-1)^2+5\right)(x-1)}{24\sqrt{-(x-1)^4-2(x-1)^2+3}}$$

input
```
Int[((2 - x)*x*(4 - 2*x + x^2))^(-3/2),x]
```

output
```
((5 + (-1 + x)^2)*(-1 + x))/(24*Sqrt[3 - 2*(-1 + x)^2 - (-1 + x)^4]) + (Sq
rt[3]*EllipticE[ArcSin[1 - x], -1/3] - 2*Sqrt[3]*EllipticF[ArcSin[1 - x],
-1/3])/24
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1494
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[(d + e*x^2)/(Sqr
t[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e
}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

rule 2458
```
Int[(Pn_)^(p_.), x_Symbol] :> With[{S = Coeff[Pn, x, Expon[Pn, x] - 1]/(Exp
on[Pn, x]*Coeff[Pn, x, Expon[Pn, x]])}, Subst[Int[ExpandToSum[Pn /. x -> x
- S, x]^p, x], x, x + S] /; BinomialQ[Pn /. x -> x - S, x] || (IntegerQ[Exp
on[Pn, x]/2] && TrinomialQ[Pn /. x -> x - S, x])] /; FreeQ[p, x] && PolyQ[P
n, x] && GtQ[Expon[Pn, x], 2] && NeQ[Coeff[Pn, x, Expon[Pn, x] - 1], 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 962 vs. $2(67) = 134$.

Time = 0.63 (sec) , antiderivative size = 963, normalized size of antiderivative = 12.19

method	result	size
default	Expression too large to display	963
elliptic	Expression too large to display	963

input
```
int(1/((2-x)*x*(x^2-2*x+4))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/32*(-x^3+4*x^2-8*x+8)/(x*(-x^3+4*x^2-8*x+8))^(1/2)+2*x*(1/24+1/192*x^2)
/(-x*(x^3-4*x^2+8*x-8))^(1/2)+1/6*(-1-I*3^(1/2))*((-1+I*3^(1/2))*x/(1+I*3^
(1/2))/(x-2))^(1/2)*(x-2)^2*((x-1+I*3^(1/2))/(1-I*3^(1/2))/(x-2))^(1/2)*((
x-1-I*3^(1/2))/(1+I*3^(1/2))/(x-2))^(1/2)/(-1+I*3^(1/2))/(-x*(x-2)*(x-1+I*
3^(1/2))*(x-1-I*3^(1/2)))^(1/2)*EllipticF(((-1+I*3^(1/2))*x/(1+I*3^(1/2))/
(x-2))^(1/2),((1+I*3^(1/2))*(-1-I*3^(1/2))/(-1+I*3^(1/2))/(1-I*3^(1/2)))^(
1/2))+1/6*(-1-I*3^(1/2))*((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2)*(x-2
)^2*((x-1+I*3^(1/2))/(1-I*3^(1/2))/(x-2))^(1/2)*((x-1-I*3^(1/2))/(1+I*3^(1
/2))/(x-2))^(1/2)/(-1+I*3^(1/2))/(-x*(x-2)*(x-1+I*3^(1/2))*(x-1-I*3^(1/2))
)^(1/2)*(2*EllipticF(((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2),((1+I*3^
(1/2))*(-1-I*3^(1/2))/(-1+I*3^(1/2))/(1-I*3^(1/2)))^(1/2))-2*EllipticPi(((
-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2),(1+I*3^(1/2))/(-1+I*3^(1/2)),((
1+I*3^(1/2))*(-1-I*3^(1/2))/(-1+I*3^(1/2))/(1-I*3^(1/2)))^(1/2)))-1/24*(x*
(x-1+I*3^(1/2))*(x-1-I*3^(1/2))+2*(-1-I*3^(1/2))*((-1+I*3^(1/2))*x/(1+I*3^
(1/2))/(x-2))^(1/2)*(x-2)^2*((x-1+I*3^(1/2))/(1-I*3^(1/2))/(x-2))^(1/2)*((
x-1-I*3^(1/2))/(1+I*3^(1/2))/(x-2))^(1/2)*(1/2*(6+2*I*3^(1/2))/(-1+I*3^(1/
2))*EllipticF(((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2),((1+I*3^(1/2))*
(-1-I*3^(1/2))/(-1+I*3^(1/2))/(1-I*3^(1/2)))^(1/2))+1/2*(-1+I*3^(1/2))*Ell
ipticE(((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2),((1+I*3^(1/2))*(-1-I*3
^(1/2))/(-1+I*3^(1/2))/(1-I*3^(1/2)))^(1/2))-4/(-1+I*3^(1/2))*EllipticP...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 119 vs. $2(59) = 118$.

Time = 0.07 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.51

$$\int \frac{1}{((2-x)x(4-2x+x^2))^{3/2}}\,dx = -\frac{5\sqrt{2}(x^4-4x^3+8x^2-8x)\text{weierstrassPInverse}\left(-\frac{2}{3},\frac{7}{54},-\frac{x-3}{3x}\right)-6\sqrt{2}(x^4-4x^3+8x^2-8x)\text{weierstra}}{72\,(x^4-4x^3+8x^2}$$

input
```
integrate(1/((2-x)*x*(x^2-2*x+4))^(3/2),x, algorithm="fricas")
```

output
```
-1/72*(5*sqrt(2)*(x^4 - 4*x^3 + 8*x^2 - 8*x)*weierstrassPInverse(-2/3, 7/5
4, -1/3*(x - 3)/x) - 6*sqrt(2)*(x^4 - 4*x^3 + 8*x^2 - 8*x)*weierstrassZeta
(-2/3, 7/54, weierstrassPInverse(-2/3, 7/54, -1/3*(x - 3)/x)) + 3*sqrt(-x^
4 + 4*x^3 - 8*x^2 + 8*x)*(x^2 + 2))/(x^4 - 4*x^3 + 8*x^2 - 8*x)
```

Sympy [F]

$$\int \frac{1}{\left((2-x)x\left(4-2x+x^2\right)\right)^{3/2}}\,dx = \int \frac{1}{\left(x\left(2-x\right)\left(x^2-2x+4\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((2-x)*x*(x**2-2*x+4))**(3/2),x)
```

output
```
Integral((x*(2 - x)*(x**2 - 2*x + 4))**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{\left((2-x)x\left(4-2x+x^2\right)\right)^{3/2}}\,dx = \int \frac{1}{\left(-(x^2-2\,x+4)(x-2)x\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((2-x)*x*(x^2-2*x+4))^(3/2),x, algorithm="maxima")
```

output
```
integrate((-(x^2 - 2*x + 4)*(x - 2)*x)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{\left((2-x)x\left(4-2x+x^2\right)\right)^{3/2}}\,dx = \int \frac{1}{\left(-(x^2-2\,x+4)(x-2)x\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((2-x)*x*(x^2-2*x+4))^(3/2),x, algorithm="giac")
```

output
```
integrate((-(x^2 - 2*x + 4)*(x - 2)*x)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{((2-x)x(4-2x+x^2))^{3/2}}\,dx = \int \frac{1}{\left(-x\,\left(x-2\right)\,\left(x^2-2\,x+4\right)\right)^{3/2}}\,dx$$

input
```
int(1/(-x*(x - 2)*(x^2 - 2*x + 4))^(3/2),x)
```

output
```
int(1/(-x*(x - 2)*(x^2 - 2*x + 4))^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{((2-x)x(4-2x+x^2))^{3/2}}\,dx = \frac{\sqrt{-x^3+4x^2-8x+8}\,x^2-2\sqrt{-x^3+4x^2-8x+8}\,x+\sqrt{-x^3+4x^2-}}{}$$

input
```
int(1/((2-x)*x*(x^2-2*x+4))^(3/2),x)
```

output
```
(sqrt( - x**3 + 4*x**2 - 8*x + 8)*x**2 - 2*sqrt( - x**3 + 4*x**2 - 8*x + 8
)*x + sqrt( - x**3 + 4*x**2 - 8*x + 8) - 2*sqrt(x)*int((sqrt(x)*sqrt( - x*
*3 + 4*x**2 - 8*x + 8)*x)/(x**6 - 8*x**5 + 32*x**4 - 80*x**3 + 128*x**2 -
128*x + 64),x)*x**3 + 8*sqrt(x)*int((sqrt(x)*sqrt( - x**3 + 4*x**2 - 8*x +
 8)*x)/(x**6 - 8*x**5 + 32*x**4 - 80*x**3 + 128*x**2 - 128*x + 64),x)*x**2
 - 16*sqrt(x)*int((sqrt(x)*sqrt( - x**3 + 4*x**2 - 8*x + 8)*x)/(x**6 - 8*x
**5 + 32*x**4 - 80*x**3 + 128*x**2 - 128*x + 64),x)*x + 16*sqrt(x)*int((sq
rt(x)*sqrt( - x**3 + 4*x**2 - 8*x + 8)*x)/(x**6 - 8*x**5 + 32*x**4 - 80*x*
*3 + 128*x**2 - 128*x + 64),x) + 6*sqrt(x)*int((sqrt(x)*sqrt( - x**3 + 4*x
**2 - 8*x + 8))/(x**6 - 8*x**5 + 32*x**4 - 80*x**3 + 128*x**2 - 128*x + 64
),x)*x**3 - 24*sqrt(x)*int((sqrt(x)*sqrt( - x**3 + 4*x**2 - 8*x + 8))/(x**
6 - 8*x**5 + 32*x**4 - 80*x**3 + 128*x**2 - 128*x + 64),x)*x**2 + 48*sqrt(
x)*int((sqrt(x)*sqrt( - x**3 + 4*x**2 - 8*x + 8))/(x**6 - 8*x**5 + 32*x**4
 - 80*x**3 + 128*x**2 - 128*x + 64),x)*x - 48*sqrt(x)*int((sqrt(x)*sqrt( -
 x**3 + 4*x**2 - 8*x + 8))/(x**6 - 8*x**5 + 32*x**4 - 80*x**3 + 128*x**2 -
 128*x + 64),x))/(4*sqrt(x)*(x**3 - 4*x**2 + 8*x - 8))
```

3.59 $\int \frac{1}{\left((2-x)x\left(4-2x+x^2\right)\right)^{5/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 123

$$\int \frac{1}{\left((2-x)x\left(4-2x+x^2\right)\right)^{5/2}}\,dx = -\frac{\left(26+7(1-x)^2\right)(1-x)}{432\sqrt{3-2(1-x)^2-(1-x)^4}} - \frac{\left(5+(-1+x)^2\right)(1-x)}{72\left(3-2(1-x)^2-(1-x)^4\right)^{3/2}} + \frac{7E\left(\arcsin(1-x)\left|-\frac{1}{3}\right.\right)}{144\sqrt{3}} - \frac{11\,\text{EllipticF}\left(\arcsin(1-x),-\frac{1}{3}\right)}{144\sqrt{3}}$$

output
```
-1/432*(26+7*(1-x)^2)*(1-x)/(3-2*(1-x)^2-(1-x)^4)^(1/2)-1/72*(5+(-1+x)^2)*
(1-x)/(3-2*(1-x)^2-(1-x)^4)^(3/2)-7/432*3^(1/2)*EllipticE(-1+x,1/3*I*3^(1/
2))+11/432*3^(1/2)*EllipticF(-1+x,1/3*I*3^(1/2))
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 23.43 (sec) , antiderivative size = 327, normalized size of antiderivative = 2.66

$$\int \frac{1}{((2-x)x\,(4-2x+x^2))^{5/2}}\,dx = \frac{(-2+x)^3x^2(4-2x+x^2)^2\left(-\frac{7x(4-2x+x^2)}{-2+x}+\frac{36+216x-622x^2+670x^3-445x^4\cdots}{(-2+x)^2x(4-2x+\cdots}\right.}{\cdots}$$

input
```
Integrate[((2 - x)*x*(4 - 2*x + x^2))^(-5/2),x]
```

output
```
((-2 + x)^3*x^2*(4 - 2*x + x^2)^2*((-7*x*(4 - 2*x + x^2))/(-2 + x) + (36 +
 216*x - 622*x^2 + 670*x^3 - 445*x^4 + 187*x^5 - 49*x^6 + 7*x^7)/((-2 + x)
^2*x*(4 - 2*x + x^2)) + ((7*I)*Sqrt[2]*x*Sqrt[(4 - 2*x + x^2)/(-2 + x)^2]*
EllipticE[ArcSin[Sqrt[-I + Sqrt[3] - (4*I)/(-2 + x)]/(Sqrt[2]*3^(1/4))], (
2*Sqrt[3])/(I + Sqrt[3])])/Sqrt[(I*x)/((I + Sqrt[3])*(-2 + x))] - (19*I)*S
qrt[2]*(-2 + x)*Sqrt[(I*x)/((I + Sqrt[3])*(-2 + x))]*Sqrt[(4 - 2*x + x^2)/
(-2 + x)^2]*EllipticF[ArcSin[Sqrt[-I + Sqrt[3] - (4*I)/(-2 + x)]/(Sqrt[2]*
3^(1/4))], (2*Sqrt[3])/(I + Sqrt[3])]))/(432*(-(x*(-8 + 8*x - 4*x^2 + x^3)
))^(5/2))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.93, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {2458, 1405, 27, 1492, 27, 1494, 27, 399, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{((2-x)x\,(x^2-2x+4))^{5/2}}\,dx$$

$$\Big\downarrow \; 2458$$

$$\int \frac{1}{(-(x-1)^4-2(x-1)^2+3)^{5/2}}d(x-1)$$

$$\Big\downarrow \text{1405}$$

$$\frac{((x-1)^2+5)(x-1)}{72(-(x-1)^4-2(x-1)^2+3)^{3/2}}-\frac{1}{144}\int -\frac{2(3(x-1)^2+19)}{(-(x-1)^4-2(x-1)^2+3)^{3/2}}d(x-1)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{72}\int \frac{3(x-1)^2+19}{(-(x-1)^4-2(x-1)^2+3)^{3/2}}d(x-1)+\frac{((x-1)^2+5)(x-1)}{72(-(x-1)^4-2(x-1)^2+3)^{3/2}}$$

$$\Big\downarrow \text{1492}$$

$$\frac{1}{72}\left(\frac{(7(x-1)^2+26)(x-1)}{6\sqrt{-(x-1)^4-2(x-1)^2+3}}-\frac{1}{48}\int -\frac{8(12-7(x-1)^2)}{\sqrt{-(x-1)^4-2(x-1)^2+3}}d(x-1)\right)+\frac{((x-1)^2+5)(x-1)}{72(-(x-1)^4-2(x-1)^2+3)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{72}\left(\frac{1}{6}\int \frac{12-7(x-1)^2}{\sqrt{-(x-1)^4-2(x-1)^2+3}}d(x-1)+\frac{(7(x-1)^2+26)(x-1)}{6\sqrt{-(x-1)^4-2(x-1)^2+3}}\right)+\frac{((x-1)^2+5)(x-1)}{72(-(x-1)^4-2(x-1)^2+3)^{3/2}}$$

$$\Big\downarrow \text{1494}$$

$$\frac{1}{72}\left(\frac{1}{3}\int \frac{12-7(x-1)^2}{2\sqrt{1-(x-1)^2}\sqrt{(x-1)^2+3}}d(x-1)+\frac{(7(x-1)^2+26)(x-1)}{6\sqrt{-(x-1)^4-2(x-1)^2+3}}\right)+\frac{((x-1)^2+5)(x-1)}{72(-(x-1)^4-2(x-1)^2+3)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{72}\left(\frac{1}{6}\int \frac{12-7(x-1)^2}{\sqrt{1-(x-1)^2}\sqrt{(x-1)^2+3}}d(x-1)+\frac{(7(x-1)^2+26)(x-1)}{6\sqrt{-(x-1)^4-2(x-1)^2+3}}\right)+\frac{((x-1)^2+5)(x-1)}{72(-(x-1)^4-2(x-1)^2+3)^{3/2}}$$

$$\Big\downarrow \text{399}$$

$$\frac{1}{72}\left(\frac{1}{6}\left(33\int \frac{1}{\sqrt{1-(x-1)^2}\sqrt{(x-1)^2+3}}d(x-1)-7\int \frac{\sqrt{(x-1)^2+3}}{\sqrt{1-(x-1)^2}}d(x-1)\right)+\frac{(7(x-1)^2+26)(x-}{6\sqrt{-(x-1)^4-2(x-1}}\right.$$
$$\frac{((x-1)^2+5)(x-1)}{72(-(x-1)^4-2(x-1)^2+3)^{3/2}}$$

$$\Big\downarrow \ 321$$

$$\frac{1}{72}\left(\frac{1}{6}\left(-7\int \frac{\sqrt{(x-1)^2+3}}{\sqrt{1-(x-1)^2}}d(x-1)-11\sqrt{3}\,\text{EllipticF}\left(\arcsin(1-x),-\frac{1}{3}\right)\right)+\frac{\left(7(x-1)^2+26\right)(x-1)}{6\sqrt{-(x-1)^4-2(x-1)^2+3}}\right)$$
$$\frac{\left((x-1)^2+5\right)(x-1)}{72\left(-(x-1)^4-2(x-1)^2+3\right)^{3/2}}$$

$$\Big\downarrow \ 327$$

$$\frac{1}{72}\left(\frac{1}{6}\left(7\sqrt{3}E\left(\arcsin(1-x)\left|-\frac{1}{3}\right.\right)-11\sqrt{3}\,\text{EllipticF}\left(\arcsin(1-x),-\frac{1}{3}\right)\right)+\frac{\left(7(x-1)^2+26\right)(x-1)}{6\sqrt{-(x-1)^4-2(x-1)^2+3}}\right)$$
$$\frac{\left((x-1)^2+5\right)(x-1)}{72\left(-(x-1)^4-2(x-1)^2+3\right)^{3/2}}$$

input
```
Int[((2 - x)*x*(4 - 2*x + x^2))^(-5/2),x]
```

output
```
((5 + (-1 + x)^2)*(-1 + x))/(72*(3 - 2*(-1 + x)^2 - (-1 + x)^4)^(3/2)) + (
((26 + 7*(-1 + x)^2)*(-1 + x))/(6*Sqrt[3 - 2*(-1 + x)^2 - (-1 + x)^4]) + (
7*Sqrt[3]*EllipticE[ArcSin[1 - x], -1/3] - 11*Sqrt[3]*EllipticF[ArcSin[1 -
 x], -1/3])/6)/72
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1492
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2
 - 4*a*c))   Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p +
7)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 LtQ[p, -1] && IntegerQ[2*p]
```

rule 1494
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[(d + e*x^2)/(Sqr
t[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e
}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

rule 2458
```
Int[(Pn_)^(p_.), x_Symbol] :> With[{S = Coeff[Pn, x, Expon[Pn, x] - 1]/(Exp
on[Pn, x]*Coeff[Pn, x, Expon[Pn, x]])}, Subst[Int[ExpandToSum[Pn /. x -> x
- S, x]^p, x], x, x + S] /; BinomialQ[Pn /. x -> x - S, x] || (IntegerQ[Exp
on[Pn, x]/2] && TrinomialQ[Pn /. x -> x - S, x])] /; FreeQ[p, x] && PolyQ[P
n, x] && GtQ[Expon[Pn, x], 2] && NeQ[Coeff[Pn, x, Expon[Pn, x] - 1], 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1038 vs. $2(107) = 214$.

Time = 0.65 (sec) , antiderivative size = 1039, normalized size of antiderivative = 8.45

method	result	size
default	Expression too large to display	1039
elliptic	Expression too large to display	1039

input
```
int(1/((2-x)*x*(x^2-2*x+4))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
(1/36+1/288*x^2-1/96*x)*(-x^4+4*x^3-8*x^2+8*x)^(1/2)/(x^3-4*x^2+8*x-8)^2+2
*x*(53/3456+5/1728*x^2-19/4608*x)/(-x*(x^3-4*x^2+8*x-8))^(1/2)-1/768*(-x^4
+4*x^3-8*x^2+8*x)^(1/2)/x^2-1/96*(-x^3+4*x^2-8*x+8)/(x*(-x^3+4*x^2-8*x+8))
^(1/2)+5/216*(-1-I*3^(1/2))*((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2)*(
x-2)^2*((x-1+I*3^(1/2))/(1-I*3^(1/2))/(x-2))^(1/2)*((x-1-I*3^(1/2))/(1+I*3
^(1/2))/(x-2))^(1/2)/(-1+I*3^(1/2))/(-x*(x-2)*(x-1+I*3^(1/2))*(x-1-I*3^(1/
2)))^(1/2)*EllipticF(((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2),((1+I*3^
(1/2))*(-1-I*3^(1/2))/(-1+I*3^(1/2))/(1-I*3^(1/2)))^(1/2))+7/108*(-1-I*3^(
1/2))*((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2)*(x-2)^2*((x-1+I*3^(1/2)
)/(1-I*3^(1/2))/(x-2))^(1/2)*((x-1-I*3^(1/2))/(1+I*3^(1/2))/(x-2))^(1/2)/(
-1+I*3^(1/2))/(-x*(x-2)*(x-1+I*3^(1/2))*(x-1-I*3^(1/2)))^(1/2)*(2*Elliptic
F(((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2),((1+I*3^(1/2))*(-1-I*3^(1/2
))/(-1+I*3^(1/2))/(1-I*3^(1/2)))^(1/2))-2*EllipticPi(((-1+I*3^(1/2))*x/(1+
I*3^(1/2))/(x-2))^(1/2),(1+I*3^(1/2))/(-1+I*3^(1/2)),((1+I*3^(1/2))*(-1-I*
3^(1/2))/(-1+I*3^(1/2))/(1-I*3^(1/2)))^(1/2)))-7/432*(x*(x-1+I*3^(1/2))*(x
-1-I*3^(1/2))+2*(-1-I*3^(1/2))*((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2
)*(x-2)^2*((x-1+I*3^(1/2))/(1-I*3^(1/2))/(x-2))^(1/2)*((x-1-I*3^(1/2))/(1+
I*3^(1/2))/(x-2))^(1/2)*(1/2*(6+2*I*3^(1/2))/(-1+I*3^(1/2))*EllipticF(((-1
+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2),((1+I*3^(1/2))*(-1-I*3^(1/2))/(-1
+I*3^(1/2))/(1-I*3^(1/2)))^(1/2))+1/2*(-1+I*3^(1/2))*EllipticE(((-1+I*3...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 195 vs. $2(91) = 182$.

Time = 0.07 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.59

$$\int \frac{1}{((2-x)x\,(4-2x+x^2))^{5/2}}\,dx =$$
$$-\frac{43\sqrt{2}(x^8-8\,x^7+32\,x^6-80\,x^5+128\,x^4-128\,x^3+64\,x^2)\text{weierstrassPInverse}\left(-\frac{2}{3},\frac{7}{54},-\frac{x-3}{3\,x}\right)-84\,\sqrt{2}(\cdots}{\cdots}$$

input
```
integrate(1/((2-x)*x*(x^2-2*x+4))^(5/2),x, algorithm="fricas")
```

output
```
-1/2592*(43*sqrt(2)*(x^8 - 8*x^7 + 32*x^6 - 80*x^5 + 128*x^4 - 128*x^3 + 6
4*x^2)*weierstrassPInverse(-2/3, 7/54, -1/3*(x - 3)/x) - 84*sqrt(2)*(x^8 -
 8*x^7 + 32*x^6 - 80*x^5 + 128*x^4 - 128*x^3 + 64*x^2)*weierstrassZeta(-2/
3, 7/54, weierstrassPInverse(-2/3, 7/54, -1/3*(x - 3)/x)) + 6*(7*x^6 - 37*
x^5 + 115*x^4 - 226*x^3 + 274*x^2 - 232*x + 36)*sqrt(-x^4 + 4*x^3 - 8*x^2
+ 8*x))/(x^8 - 8*x^7 + 32*x^6 - 80*x^5 + 128*x^4 - 128*x^3 + 64*x^2)
```

Sympy [F]

$$\int \frac{1}{((2-x)x\,(4-2x+x^2))^{5/2}}\,dx = \int \frac{1}{(x\,(2-x)\,(x^2-2x+4))^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/((2-x)*x*(x**2-2*x+4))**(5/2),x)
```

output
```
Integral((x*(2 - x)*(x**2 - 2*x + 4))**(-5/2), x)
```

Maxima [F]

$$\int \frac{1}{((2-x)x(4-2x+x^2))^{5/2}}\,dx = \int \frac{1}{(-(x^2-2x+4)(x-2)x)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/((2-x)*x*(x^2-2*x+4))^(5/2),x, algorithm="maxima")
```

output
```
integrate((-(x^2 - 2*x + 4)*(x - 2)*x)^(-5/2), x)
```

Giac [F]

$$\int \frac{1}{((2-x)x(4-2x+x^2))^{5/2}}\,dx = \int \frac{1}{(-(x^2-2x+4)(x-2)x)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/((2-x)*x*(x^2-2*x+4))^(5/2),x, algorithm="giac")
```

output
```
integrate((-(x^2 - 2*x + 4)*(x - 2)*x)^(-5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{((2-x)x(4-2x+x^2))^{5/2}}\,dx = \int \frac{1}{(-x\,(x-2)\,(x^2-2x+4))^{5/2}}\,dx$$

input
```
int(1/(-x*(x - 2)*(x^2 - 2*x + 4))^(5/2),x)
```

output
```
int(1/(-x*(x - 2)*(x^2 - 2*x + 4))^(5/2), x)
```

Reduce [F]

$$\int \frac{1}{\left((2-x)x\left(4-2x+x^2\right)\right)^{5/2}}\,dx = \text{too large to display}$$

input

```
int(1/((2-x)*x*(x^2-2*x+4))^(5/2),x)
```

output

```
(sqrt( - x**3 + 4*x**2 - 8*x + 8)*x**6 - 6*sqrt( - x**3 + 4*x**2 - 8*x + 8
)*x**5 + 18*sqrt( - x**3 + 4*x**2 - 8*x + 8)*x**4 - 32*sqrt( - x**3 + 4*x*
*2 - 8*x + 8)*x**3 + 30*sqrt( - x**3 + 4*x**2 - 8*x + 8)*x**2 - 12*sqrt( -
 x**3 + 4*x**2 - 8*x + 8)*x - 2*sqrt( - x**3 + 4*x**2 - 8*x + 8) - 36*sqrt
(x)*int(sqrt( - x**3 + 4*x**2 - 8*x + 8)/(sqrt(x)*x**9 - 12*sqrt(x)*x**8 +
 72*sqrt(x)*x**7 - 280*sqrt(x)*x**6 + 768*sqrt(x)*x**5 - 1536*sqrt(x)*x**4
 + 2240*sqrt(x)*x**3 - 2304*sqrt(x)*x**2 + 1536*sqrt(x)*x - 512*sqrt(x)),x
)*x**7 + 288*sqrt(x)*int(sqrt( - x**3 + 4*x**2 - 8*x + 8)/(sqrt(x)*x**9 -
12*sqrt(x)*x**8 + 72*sqrt(x)*x**7 - 280*sqrt(x)*x**6 + 768*sqrt(x)*x**5 -
1536*sqrt(x)*x**4 + 2240*sqrt(x)*x**3 - 2304*sqrt(x)*x**2 + 1536*sqrt(x)*x
 - 512*sqrt(x)),x)*x**6 - 1152*sqrt(x)*int(sqrt( - x**3 + 4*x**2 - 8*x + 8
)/(sqrt(x)*x**9 - 12*sqrt(x)*x**8 + 72*sqrt(x)*x**7 - 280*sqrt(x)*x**6 + 7
68*sqrt(x)*x**5 - 1536*sqrt(x)*x**4 + 2240*sqrt(x)*x**3 - 2304*sqrt(x)*x**
2 + 1536*sqrt(x)*x - 512*sqrt(x)),x)*x**5 + 2880*sqrt(x)*int(sqrt( - x**3
+ 4*x**2 - 8*x + 8)/(sqrt(x)*x**9 - 12*sqrt(x)*x**8 + 72*sqrt(x)*x**7 - 28
0*sqrt(x)*x**6 + 768*sqrt(x)*x**5 - 1536*sqrt(x)*x**4 + 2240*sqrt(x)*x**3
- 2304*sqrt(x)*x**2 + 1536*sqrt(x)*x - 512*sqrt(x)),x)*x**4 - 4608*sqrt(x)
*int(sqrt( - x**3 + 4*x**2 - 8*x + 8)/(sqrt(x)*x**9 - 12*sqrt(x)*x**8 + 72
*sqrt(x)*x**7 - 280*sqrt(x)*x**6 + 768*sqrt(x)*x**5 - 1536*sqrt(x)*x**4 +
2240*sqrt(x)*x**3 - 2304*sqrt(x)*x**2 + 1536*sqrt(x)*x - 512*sqrt(x)),x...
```

3.60 $\int (4ac+4c^2x^2+4cdx^3+d^2x^4)^{3/2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 683

$$\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^{3/2}\,dx=\frac{(c+dx)\left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^{3/2}}{7d}$$
$$-\frac{16c^3(c^3+8ad^2)(c+dx)\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}}{35d^3\left(\sqrt{c}\sqrt{c^3+4ad^2}+(c+dx)^2\right)}$$
$$+\frac{2c(c+dx)\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}(7c^3+20ad^2-3c(c+dx)^2)}{35d^3}$$
$$+\frac{16c^{13/4}\left(c^3+4ad^2\right)^{3/4}\left(c^3+8ad^2\right)\sqrt{\frac{d^2(4ac+4c^2x^2+4cdx^3+d^2x^4)}{(c^3+4ad^2)\left(\sqrt{c}+\frac{(c+dx)^2}{\sqrt{c^3+4ad^2}}\right)^2}}\left(\sqrt{c}+\frac{(c+dx)^2}{\sqrt{c^3+4ad^2}}\right)E\left(2\arctan\left(\frac{c+dx}{\sqrt[4]{c}\sqrt[4]{c^3+4ad^2}}\right.\right.}{35d^5\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}}$$
$$+\frac{8c^{7/4}\left(c^3+4ad^2\right)^{3/4}\left(\sqrt{c^3+4ad^2}(c^3+5ad^2)-c^{3/2}(c^3+8ad^2)\right)\sqrt{\frac{d^2(4ac+4c^2x^2+4cdx^3+d^2x^4)}{(c^3+4ad^2)\left(\sqrt{c}+\frac{(c+dx)^2}{\sqrt{c^3+4ad^2}}\right)^2}}\left(\sqrt{c}+\frac{(c+dx)^2}{\sqrt{c^3+4ad^2}}\right)}{35d^5\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}}$$

output
```
1/7*(d*x+c)*(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(3/2)/d-16/35*c^3*(8*a*d^2
+c^3)*(d*x+c)*(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(1/2)/d^3/(c^(1/2)*(4*a*
d^2+c^3)^(1/2)+(d*x+c)^2)+2/35*c*(d*x+c)*(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*
c)^(1/2)*(7*c^3+20*a*d^2-3*c*(d*x+c)^2)/d^3+16/35*c^(13/4)*(4*a*d^2+c^3)^(
3/4)*(8*a*d^2+c^3)*(d^2*(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)/(4*a*d^2+c^3)/
(c^(1/2)+(d*x+c)^2/(4*a*d^2+c^3)^(1/2))^2)^(1/2)*(c^(1/2)+(d*x+c)^2/(4*a*d
^2+c^3)^(1/2))*EllipticE(sin(2*arctan((d*x+c)/c^(1/4)/(4*a*d^2+c^3)^(1/4))
),1/2*(2+2*c^(3/2)/(4*a*d^2+c^3)^(1/2))^(1/2))/d^5/(d^2*x^4+4*c*d*x^3+4*c^
2*x^2+4*a*c)^(1/2)+8/35*c^(7/4)*(4*a*d^2+c^3)^(3/4)*((4*a*d^2+c^3)^(1/2)*(
5*a*d^2+c^3)-c^(3/2)*(8*a*d^2+c^3))*(d^2*(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*
c)/(4*a*d^2+c^3)/(c^(1/2)+(d*x+c)^2/(4*a*d^2+c^3)^(1/2))^2)^(1/2)*(c^(1/2)
+(d*x+c)^2/(4*a*d^2+c^3)^(1/2))*InverseJacobiAM(2*arctan((d*x+c)/c^(1/4)/(
4*a*d^2+c^3)^(1/4)),1/2*(2+2*c^(3/2)/(4*a*d^2+c^3)^(1/2))^(1/2))/d^5/(d^2*
x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 16.22 (sec) , antiderivative size = 10468, normalized size of antiderivative = 15.33

$$\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^{3/2}\,dx = \text{Result too large to show}$$

input
```
Integrate[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^(3/2),x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 1.02 (sec) , antiderivative size = 843, normalized size of antiderivative = 1.23, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.290$, Rules used = {2458, 1404, 27, 1490, 27, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^{3/2}dx$$

$\big\downarrow$ 2458

$$\int \left(c\left(4a+\frac{c^3}{d^2}\right)-2c^2\left(\frac{c}{d}+x\right)^2+d^2\left(\frac{c}{d}+x\right)^4\right)^{3/2}d\left(\frac{c}{d}+x\right)$$

$\big\downarrow$ 1404

$$\frac{3}{7}\int 2c\left(\frac{c^3}{d^2}-\left(\frac{c}{d}+x\right)^2c+4a\right)\sqrt{d^2\left(\frac{c}{d}+x\right)^4-2c^2\left(\frac{c}{d}+x\right)^2+c\left(\frac{c^3}{d^2}+4a\right)}d\left(\frac{c}{d}+x\right)+\frac{1}{7}\left(\frac{c}{d}+x\right)\left(c\left(4a+\frac{c^3}{d^2}\right)-2c^2\left(\frac{c}{d}+x\right)^2+d^2\left(\frac{c}{d}+x\right)^4\right)^{3/2}$$

$\big\downarrow$ 27

$$\frac{6}{7}c\int\left(\frac{c^3}{d^2}-\left(\frac{c}{d}+x\right)^2c+4a\right)\sqrt{d^2\left(\frac{c}{d}+x\right)^4-2c^2\left(\frac{c}{d}+x\right)^2+c\left(\frac{c^3}{d^2}+4a\right)}d\left(\frac{c}{d}+x\right)+\frac{1}{7}\left(\frac{c}{d}+x\right)\left(c\left(4a+\frac{c^3}{d^2}\right)-2c^2\left(\frac{c}{d}+x\right)^2+d^2\left(\frac{c}{d}+x\right)^4\right)^{3/2}$$

$\big\downarrow$ 1490

$$\frac{6}{7}c\left(\frac{\int\frac{8c\left(\left(c^3+4ad^2\right)\left(c^3+5ad^2\right)-cd^2\left(c^3+8ad^2\right)\left(\frac{c}{d}+x\right)^2\right)}{d^2\sqrt{d^2\left(\frac{c}{d}+x\right)^4-2c^2\left(\frac{c}{d}+x\right)^2+c\left(\frac{c^3}{d^2}+4a\right)}}d\left(\frac{c}{d}+x\right)}{15d^2}+\frac{\left(\frac{c}{d}+x\right)\sqrt{c\left(4a+\frac{c^3}{d^2}\right)-2c^2\left(\frac{c}{d}+x\right)^2+d^2\left(\frac{c}{d}+x\right)^4}}{15d^2}\right.$$

$$\frac{1}{7}\left(\frac{c}{d}+x\right)\left(c\left(4a+\frac{c^3}{d^2}\right)-2c^2\left(\frac{c}{d}+x\right)^2+d^2\left(\frac{c}{d}+x\right)^4\right)^{3/2}$$

$\big\downarrow$ 27

$$\frac{6}{7}c\left(\frac{8c\int\frac{\left(c^3+4ad^2\right)\left(c^3+5ad^2\right)-cd^2\left(c^3+8ad^2\right)\left(\frac{c}{d}+x\right)^2}{\sqrt{d^2\left(\frac{c}{d}+x\right)^4-2c^2\left(\frac{c}{d}+x\right)^2+c\left(\frac{c^3}{d^2}+4a\right)}}d\left(\frac{c}{d}+x\right)}{15d^4}+\frac{\left(\frac{c}{d}+x\right)\sqrt{c\left(4a+\frac{c^3}{d^2}\right)-2c^2\left(\frac{c}{d}+x\right)^2+d^2\left(\frac{c}{d}+x\right)^4}\left(\right.}{15d^2}\right.$$

$$\frac{1}{7}\left(\frac{c}{d}+x\right)\left(c\left(4a+\frac{c^3}{d^2}\right)-2c^2\left(\frac{c}{d}+x\right)^2+d^2\left(\frac{c}{d}+x\right)^4\right)^{3/2}$$

$\big\downarrow$ 1511

$$\frac{6}{7}c\left(\frac{8c\left(c^{3/2}\sqrt{4ad^2+c^3}\left(8ad^2+c^3\right)\int \frac{\sqrt{c}-\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{c^3+4ad^2}}}{\sqrt{c}\sqrt{d^2\left(\frac{c}{d}+x\right)^4-2c^2\left(\frac{c}{d}+x\right)^2+c\left(\frac{c^3}{d^2}+4a\right)}}d\left(\frac{c}{d}+x\right)-\sqrt{4ad^2+c^3}\left(8ac^{3/2}d^2-\sqrt{4a\right.\right.}{15d^4}\right.$$

$$\frac{1}{7}\left(\frac{c}{d}+x\right)\left(c\left(4a+\frac{c^3}{d^2}\right)-2c^2\left(\frac{c}{d}+x\right)^2+d^2\left(\frac{c}{d}+x\right)^4\right)^{3/2}$$

$\downarrow$ 27

$$\frac{6}{7}c\left(\frac{8c\left(c\sqrt{4ad^2+c^3}\left(8ad^2+c^3\right)\int \frac{\sqrt{c}-\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{c^3+4ad^2}}}{\sqrt{d^2\left(\frac{c}{d}+x\right)^4-2c^2\left(\frac{c}{d}+x\right)^2+c\left(\frac{c^3}{d^2}+4a\right)}}d\left(\frac{c}{d}+x\right)-\sqrt{4ad^2+c^3}\left(8ac^{3/2}d^2-\sqrt{4ad^2+\right.\right.}{15d^4}\right.$$

$$\frac{1}{7}\left(\frac{c}{d}+x\right)\left(c\left(4a+\frac{c^3}{d^2}\right)-2c^2\left(\frac{c}{d}+x\right)^2+d^2\left(\frac{c}{d}+x\right)^4\right)^{3/2}$$

$\downarrow$ 1416

$$\frac{6}{7}c\left(\frac{8c\left(c\sqrt{4ad^2+c^3}\left(8ad^2+c^3\right)\int \frac{\sqrt{c}-\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{c^3+4ad^2}}}{\sqrt{d^2\left(\frac{c}{d}+x\right)^4-2c^2\left(\frac{c}{d}+x\right)^2+c\left(\frac{c^3}{d^2}+4a\right)}}d\left(\frac{c}{d}+x\right)-\frac{\left(4ad^2+c^3\right)^{3/4}\left(8ac^{3/2}d^2-\sqrt{4ad^2+c^3}\left(5ad^2\right.\right.}{}\right.}{}\right.$$

$$\frac{1}{7}\left(\frac{c}{d}+x\right)\left(c\left(4a+\frac{c^3}{d^2}\right)-2c^2\left(\frac{c}{d}+x\right)^2+d^2\left(\frac{c}{d}+x\right)^4\right)^{3/2}$$

$\downarrow$ 1509

$$\frac{1}{7}\left(\frac{c}{d}+x\right)\left(d^2\left(\frac{c}{d}+x\right)^4-2c^2\left(\frac{c}{d}+x\right)^2+c\left(\frac{c^3}{d^2}+4a\right)\right)^{3/2}+$$

$$\frac{6}{7}c\left(\frac{\left(\frac{c}{d}+x\right)\sqrt{d^2\left(\frac{c}{d}+x\right)^4-2c^2\left(\frac{c}{d}+x\right)^2+c\left(\frac{c^3}{d^2}+4a\right)}\left(7c^3-3d^2\left(\frac{c}{d}+x\right)^2c+20ad^2\right)}{15d^2}+\frac{8c\left(c\sqrt{c^3+4ad^2}\left(c^3\right.\right.}{}\right.$$

input
```
Int[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^(3/2),x]
```

output
```
((c/d + x)*(c*(4*a + c^3/d^2) - 2*c^2*(c/d + x)^2 + d^2*(c/d + x)^4)^(3/2)
)/7 + (6*c*(((c/d + x)*(7*c^3 + 20*a*d^2 - 3*c*d^2*(c/d + x)^2)*Sqrt[c*(4*
a + c^3/d^2) - 2*c^2*(c/d + x)^2 + d^2*(c/d + x)^4])/(15*d^2) + (8*c*(c*Sq
rt[c^3 + 4*a*d^2]*(c^3 + 8*a*d^2)*(-(((c/d + x)*Sqrt[c*(4*a + c^3/d^2) - 2
*c^2*(c/d + x)^2 + d^2*(c/d + x)^4])/((4*a + c^3/d^2)*(Sqrt[c] + (d^2*(c/d
 + x)^2)/Sqrt[c^3 + 4*a*d^2]))) + (c^(1/4)*(c^3 + 4*a*d^2)^(1/4)*(Sqrt[c]
+ (d^2*(c/d + x)^2)/Sqrt[c^3 + 4*a*d^2])*Sqrt[(d^2*(c*(4*a + c^3/d^2) - 2*
c^2*(c/d + x)^2 + d^2*(c/d + x)^4))/((c^3 + 4*a*d^2)*(Sqrt[c] + (d^2*(c/d
+ x)^2)/Sqrt[c^3 + 4*a*d^2])^2)]*EllipticE[2*ArcTan[(d*(c/d + x))/(c^(1/4)
*(c^3 + 4*a*d^2)^(1/4))], (1 + c^(3/2)/Sqrt[c^3 + 4*a*d^2])/2])/(d*Sqrt[c*
(4*a + c^3/d^2) - 2*c^2*(c/d + x)^2 + d^2*(c/d + x)^4])) - ((c^3 + 4*a*d^2
)^(3/4)*(c^(9/2) + 8*a*c^(3/2)*d^2 - Sqrt[c^3 + 4*a*d^2]*(c^3 + 5*a*d^2))*
(Sqrt[c] + (d^2*(c/d + x)^2)/Sqrt[c^3 + 4*a*d^2])*Sqrt[(d^2*(c*(4*a + c^3/
d^2) - 2*c^2*(c/d + x)^2 + d^2*(c/d + x)^4))/((c^3 + 4*a*d^2)*(Sqrt[c] + (
d^2*(c/d + x)^2)/Sqrt[c^3 + 4*a*d^2])^2)]*EllipticF[2*ArcTan[(d*(c/d + x))
/(c^(1/4)*(c^3 + 4*a*d^2)^(1/4))], (1 + c^(3/2)/Sqrt[c^3 + 4*a*d^2])/2])/(
2*c^(1/4)*d*Sqrt[c*(4*a + c^3/d^2) - 2*c^2*(c/d + x)^2 + d^2*(c/d + x)^4])
))/(15*d^4)))/7
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1404
```
Int[((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*((a + b
*x^2 + c*x^4)^p/(4*p + 1)), x] + Simp[2*(p/(4*p + 1))   Int[(2*a + b*x^2)*(
a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*
c, 0] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1490
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Simp[2*(p/(c*(4*p + 1)*(4*p + 3)))
 Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3)
- b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 2458
```
Int[(Pn_)^(p_.), x_Symbol] :> With[{S = Coeff[Pn, x, Expon[Pn, x] - 1]/(Exp
on[Pn, x]*Coeff[Pn, x, Expon[Pn, x]])}, Subst[Int[ExpandToSum[Pn /. x -> x
- S, x]^p, x], x, x + S] /; BinomialQ[Pn /. x -> x - S, x] || (IntegerQ[Exp
on[Pn, x]/2] && TrinomialQ[Pn /. x -> x - S, x])] /; FreeQ[p, x] && PolyQ[P
n, x] && GtQ[Expon[Pn, x], 2] && NeQ[Coeff[Pn, x, Expon[Pn, x] - 1], 0]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 5228 vs. $2(614) = 1228$.

Time = 8.95 (sec) , antiderivative size = 5229, normalized size of antiderivative = 7.66

method	result	size
default	Expression too large to display	5229
elliptic	Expression too large to display	5229
risch	Expression too large to display	6015

input
```
int((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [F]

$$\int \left(4ac + 4c^2x^2 + 4cdx^3 + d^2x^4\right)^{3/2} dx = \int \left(d^2x^4 + 4\,cdx^3 + 4\,c^2x^2 + 4\,ac\right)^{\frac{3}{2}} dx$$

input
```
integrate((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(3/2),x, algorithm="fricas")
```

output
```
integral((d^2*x^4 + 4*c*d*x^3 + 4*c^2*x^2 + 4*a*c)^(3/2), x)
```

Sympy [F]

$$\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^{3/2}\,dx=\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((d**2*x**4+4*c*d*x**3+4*c**2*x**2+4*a*c)**(3/2),x)
```

output
```
Integral((4*a*c + 4*c**2*x**2 + 4*c*d*x**3 + d**2*x**4)**(3/2), x)
```

Maxima [F]

$$\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^{3/2}\,dx=\int \left(d^2x^4+4\,cdx^3+4\,c^2x^2+4\,ac\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(3/2),x, algorithm="maxima")
```

output
```
integrate((d^2*x^4 + 4*c*d*x^3 + 4*c^2*x^2 + 4*a*c)^(3/2), x)
```

Giac [F]

$$\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^{3/2}\,dx=\int \left(d^2x^4+4\,cdx^3+4\,c^2x^2+4\,ac\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(3/2),x, algorithm="giac")
```

output
```
integrate((d^2*x^4 + 4*c*d*x^3 + 4*c^2*x^2 + 4*a*c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(4ac + 4c^2x^2 + 4cdx^3 + d^2x^4\right)^{3/2} \, dx = \int \left(4\,c^2\,x^2 + 4\,c\,d\,x^3 + 4\,a\,c + d^2\,x^4\right)^{3/2} dx$$

input
```
int((4*a*c + 4*c^2*x^2 + d^2*x^4 + 4*c*d*x^3)^(3/2),x)
```

output
```
int((4*a*c + 4*c^2*x^2 + d^2*x^4 + 4*c*d*x^3)^(3/2), x)
```

Reduce [F]

$$\int \left(4ac + 4c^2x^2 + 4cdx^3 + d^2x^4\right)^{3/2} \, dx = \frac{116\sqrt{d^2x^4 + 4cd\,x^3 + 4c^2x^2 + 4ac}\,a\,c^2d^2 + 180\sqrt{d^2x^4 + 4cd\,x^3 + 4c^2x^2 + 4ac}\,ac\,d^3x + 16\sqrt{}}{}$$

input
```
int((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(3/2),x)
```

output

```
(116*sqrt(4*a*c + 4*c**2*x**2 + 4*c*d*x**3 + d**2*x**4)*a*c**2*d**2 + 180*
sqrt(4*a*c + 4*c**2*x**2 + 4*c*d*x**3 + d**2*x**4)*a*c*d**3*x + 16*sqrt(4*
a*c + 4*c**2*x**2 + 4*c*d*x**3 + d**2*x**4)*c**5 - 12*sqrt(4*a*c + 4*c**2*
x**2 + 4*c*d*x**3 + d**2*x**4)*c**4*d*x + 6*sqrt(4*a*c + 4*c**2*x**2 + 4*c
*d*x**3 + d**2*x**4)*c**3*d**2*x**2 + 102*sqrt(4*a*c + 4*c**2*x**2 + 4*c*d
*x**3 + d**2*x**4)*c**2*d**3*x**3 + 75*sqrt(4*a*c + 4*c**2*x**2 + 4*c*d*x*
*3 + d**2*x**4)*c*d**4*x**4 + 15*sqrt(4*a*c + 4*c**2*x**2 + 4*c*d*x**3 + d
**2*x**4)*d**5*x**5 + 960*int(sqrt(4*a*c + 4*c**2*x**2 + 4*c*d*x**3 + d**2
*x**4)/(4*a*c + 4*c**2*x**2 + 4*c*d*x**3 + d**2*x**4),x)*a**2*c**2*d**3 +
48*int(sqrt(4*a*c + 4*c**2*x**2 + 4*c*d*x**3 + d**2*x**4)/(4*a*c + 4*c**2*
x**2 + 4*c*d*x**3 + d**2*x**4),x)*a*c**5*d + 128*int((sqrt(4*a*c + 4*c**2*
x**2 + 4*c*d*x**3 + d**2*x**4)*x**3)/(4*a*c + 4*c**2*x**2 + 4*c*d*x**3 + d
**2*x**4),x)*a*c**2*d**4 + 16*int((sqrt(4*a*c + 4*c**2*x**2 + 4*c*d*x**3 +
 d**2*x**4)*x**3)/(4*a*c + 4*c**2*x**2 + 4*c*d*x**3 + d**2*x**4),x)*c**5*d
**2 - 512*int((sqrt(4*a*c + 4*c**2*x**2 + 4*c*d*x**3 + d**2*x**4)*x)/(4*a*
c + 4*c**2*x**2 + 4*c*d*x**3 + d**2*x**4),x)*a*c**4*d**2 - 64*int((sqrt(4*
a*c + 4*c**2*x**2 + 4*c*d*x**3 + d**2*x**4)*x)/(4*a*c + 4*c**2*x**2 + 4*c*
d*x**3 + d**2*x**4),x)*c**7)/(105*d**3)
```

3.61 $\int \sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 585

$$\int \sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}\,dx = \frac{(c+dx)\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}}{3d} - \frac{2c^2(c+dx)\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}}{3d\left(\sqrt{c}\sqrt{c^3+4ad^2}+(c+dx)^2\right)}$$

$$+ \frac{2c^{9/4}(c^3+4ad^2)^{3/4}\sqrt{\frac{d^2(4ac+4c^2x^2+4cdx^3+d^2x^4)}{(c^3+4ad^2)\left(\sqrt{c}+\frac{(c+dx)^2}{\sqrt{c^3+4ad^2}}\right)^2}}\left(\sqrt{c}+\frac{(c+dx)^2}{\sqrt{c^3+4ad^2}}\right)E\left(2\arctan\left(\frac{c+dx}{\sqrt[4]{c}\sqrt[4]{c^3+4ad^2}}\right)|\frac{1}{2}\left(1+\right.\right.}{3d^3\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}}$$

$$+ \frac{c^{3/4}\sqrt[4]{c^3+4ad^2}\left(c^3+4ad^2-c^{3/2}\sqrt{c^3+4ad^2}\right)\sqrt{\frac{d^2(4ac+4c^2x^2+4cdx^3+d^2x^4)}{(c^3+4ad^2)\left(\sqrt{c}+\frac{(c+dx)^2}{\sqrt{c^3+4ad^2}}\right)^2}}\left(\sqrt{c}+\frac{(c+dx)^2}{\sqrt{c^3+4ad^2}}\right)\operatorname{EllipticF}\left(2\right.}{3d^3\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}}$$

output
```
1/3*(d*x+c)*(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(1/2)/d-2/3*c^2*(d*x+c)*(d
^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(1/2)/d/(c^(1/2)*(4*a*d^2+c^3)^(1/2)+(d*
x+c)^2)+2/3*c^(9/4)*(4*a*d^2+c^3)^(3/4)*(d^2*(d^2*x^4+4*c*d*x^3+4*c^2*x^2+
4*a*c)/(4*a*d^2+c^3)/(c^(1/2)+(d*x+c)^2/(4*a*d^2+c^3)^(1/2))^2)^(1/2)*(c^(
1/2)+(d*x+c)^2/(4*a*d^2+c^3)^(1/2))*EllipticE(sin(2*arctan((d*x+c)/c^(1/4)
/(4*a*d^2+c^3)^(1/4))),1/2*(2+2*c^(3/2)/(4*a*d^2+c^3)^(1/2))^(1/2))/d^3/(d
^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(1/2)+1/3*c^(3/4)*(4*a*d^2+c^3)^(1/4)*(c
^3+4*a*d^2-c^(3/2)*(4*a*d^2+c^3)^(1/2))*(d^2*(d^2*x^4+4*c*d*x^3+4*c^2*x^2+
4*a*c)/(4*a*d^2+c^3)/(c^(1/2)+(d*x+c)^2/(4*a*d^2+c^3)^(1/2))^2)^(1/2)*(c^(
1/2)+(d*x+c)^2/(4*a*d^2+c^3)^(1/2))*InverseJacobiAM(2*arctan((d*x+c)/c^(1/
4)/(4*a*d^2+c^3)^(1/4)),1/2*(2+2*c^(3/2)/(4*a*d^2+c^3)^(1/2))^(1/2))/d^3/(
d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 16.12 (sec) , antiderivative size = 5218, normalized size of antiderivative = 8.92

$$\int \sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}\,dx = \text{Result too large to show}$$

input
```
Integrate[Sqrt[4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4],x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 729, normalized size of antiderivative = 1.25, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {2458, 1404, 27, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}\,dx$$

$$\Big\downarrow \ 2458$$

$$\int \sqrt{c\left(4a+\frac{c^3}{d^2}\right)-2c^2\left(\frac{c}{d}+x\right)^2+d^2\left(\frac{c}{d}+x\right)^4}d\left(\frac{c}{d}+x\right)$$

$$\Big\downarrow \ 1404$$

$$\frac{1}{3}\int \frac{2c\left(\frac{c^3}{d^2}-\left(\frac{c}{d}+x\right)^2c+4a\right)}{\sqrt{d^2\left(\frac{c}{d}+x\right)^4-2c^2\left(\frac{c}{d}+x\right)^2+c\left(\frac{c^3}{d^2}+4a\right)}}d\left(\frac{c}{d}+x\right)+\frac{1}{3}\left(\frac{c}{d}+x\right)\sqrt{c\left(4a+\frac{c^3}{d^2}\right)-2c^2\left(\frac{c}{d}+x\right)^2+d^2\left(\frac{c}{d}+x\right)^4}$$

$$\Big\downarrow \ 27$$

$$\frac{2}{3}c\int \frac{\frac{c^3}{d^2}-\left(\frac{c}{d}+x\right)^2c+4a}{\sqrt{d^2\left(\frac{c}{d}+x\right)^4-2c^2\left(\frac{c}{d}+x\right)^2+c\left(\frac{c^3}{d^2}+4a\right)}}d\left(\frac{c}{d}+x\right)+\frac{1}{3}\left(\frac{c}{d}+x\right)\sqrt{c\left(4a+\frac{c^3}{d^2}\right)-2c^2\left(\frac{c}{d}+x\right)^2+d^2\left(\frac{c}{d}+x\right)^4}$$

$$\Big\downarrow \ 1511$$

$$\frac{2}{3}c\left(\frac{c^{3/2}\sqrt{4ad^2+c^3}\int \frac{\sqrt{c}-\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{c^3+4ad^2}}}{\sqrt{c}\sqrt{d^2\left(\frac{c}{d}+x\right)^4-2c^2\left(\frac{c}{d}+x\right)^2+c\left(\frac{c^3}{d^2}+4a\right)}}d\left(\frac{c}{d}+x\right)}{d^2}+\frac{\left(-c^{3/2}\sqrt{4ad^2+c^3}+4ad^2+c^3\right)\int \frac{1}{\sqrt{d^2\left(\frac{c}{d}+x\right)^4 \cdots}}}{d^2}\right.$$

$$\frac{1}{3}\left(\frac{c}{d}+x\right)\sqrt{c\left(4a+\frac{c^3}{d^2}\right)-2c^2\left(\frac{c}{d}+x\right)^2+d^2\left(\frac{c}{d}+x\right)^4}$$

$$\Big\downarrow \ 27$$

$$\frac{2}{3}c\left(\frac{c\sqrt{4ad^2+c^3}\int \frac{\sqrt{c}-\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{c^3+4ad^2}}}{\sqrt{d^2\left(\frac{c}{d}+x\right)^4-2c^2\left(\frac{c}{d}+x\right)^2+c\left(\frac{c^3}{d^2}+4a\right)}}d\left(\frac{c}{d}+x\right)}{d^2}+\frac{\left(-c^{3/2}\sqrt{4ad^2+c^3}+4ad^2+c^3\right)\int \frac{1}{\sqrt{d^2\left(\frac{c}{d}+x\right)^4-2c^2 \cdots}}}{d^2}\right.$$

$$\frac{1}{3}\left(\frac{c}{d}+x\right)\sqrt{c\left(4a+\frac{c^3}{d^2}\right)-2c^2\left(\frac{c}{d}+x\right)^2+d^2\left(\frac{c}{d}+x\right)^4}$$

$\Big\downarrow$ 1416

$$\frac{2}{3}c\left(\frac{c\sqrt{4ad^2+c^3}\int \frac{\sqrt{c}-\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{c^3+4ad^2}}}{\sqrt{d^2\left(\frac{c}{d}+x\right)^4-2c^2\left(\frac{c}{d}+x\right)^2+c\left(\frac{c^3}{d^2}+4a\right)}}d\left(\frac{c}{d}+x\right)}{d^2}+\frac{\sqrt[4]{4ad^2+c^3}\left(-c^{3/2}\sqrt{4ad^2+c^3}+4ad^2+c^3\right)\left(\frac{d^2}{\sqrt{4}}\right.}{}\right.$$

$$\frac{1}{3}\left(\frac{c}{d}+x\right)\sqrt{c\left(4a+\frac{c^3}{d^2}\right)-2c^2\left(\frac{c}{d}+x\right)^2+d^2\left(\frac{c}{d}+x\right)^4}$$

$\Big\downarrow$ 1509

$$\frac{2}{3}c\left(\frac{c\sqrt{4ad^2+c^3}\left(\frac{\sqrt[4]{c}\sqrt[4]{4ad^2+c^3}\left(\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{4ad^2+c^3}}+\sqrt{c}\right)\sqrt{\frac{d^2\left(c\left(4a+\frac{c^3}{d^2}\right)-2c^2\left(\frac{c}{d}+x\right)^2+d^2\left(\frac{c}{d}+x\right)^4\right)}{\left(4ad^2+c^3\right)\left(\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{4ad^2+c^3}}+\sqrt{c}\right)^2}}E\left(2\arctan\left(\frac{d\left(\frac{c}{d}+x\right)}{\sqrt[4]{c}\sqrt[4]{c^3+4ad^2}}\right)\right.}{d\sqrt{c\left(4a+\frac{c^3}{d^2}\right)-2c^2\left(\frac{c}{d}+x\right)^2+d^2\left(\frac{c}{d}+x\right)^4}}\right.}{d^2}\right.$$

$$\frac{1}{3}\left(\frac{c}{d}+x\right)\sqrt{c\left(4a+\frac{c^3}{d^2}\right)-2c^2\left(\frac{c}{d}+x\right)^2+d^2\left(\frac{c}{d}+x\right)^4}$$

input
```
Int[Sqrt[4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4],x]
```

output
```
((c/d + x)*Sqrt[c*(4*a + c^3/d^2) - 2*c^2*(c/d + x)^2 + d^2*(c/d + x)^4])/
3 + (2*c*((c*Sqrt[c^3 + 4*a*d^2]*(-(((c/d + x)*Sqrt[c*(4*a + c^3/d^2) - 2*
c^2*(c/d + x)^2 + d^2*(c/d + x)^4])/((4*a + c^3/d^2)*(Sqrt[c] + (d^2*(c/d
+ x)^2)/Sqrt[c^3 + 4*a*d^2]))) + (c^(1/4)*(c^3 + 4*a*d^2)^(1/4)*(Sqrt[c] +
 (d^2*(c/d + x)^2)/Sqrt[c^3 + 4*a*d^2])*Sqrt[(d^2*(c*(4*a + c^3/d^2) - 2*c
^2*(c/d + x)^2 + d^2*(c/d + x)^4))/((c^3 + 4*a*d^2)*(Sqrt[c] + (d^2*(c/d +
 x)^2)/Sqrt[c^3 + 4*a*d^2])^2)]*EllipticE[2*ArcTan[(d*(c/d + x))/(c^(1/4)*
(c^3 + 4*a*d^2)^(1/4))], (1 + c^(3/2)/Sqrt[c^3 + 4*a*d^2])/2])/(d*Sqrt[c*(
4*a + c^3/d^2) - 2*c^2*(c/d + x)^2 + d^2*(c/d + x)^4])))/d^2 + ((c^3 + 4*a
*d^2)^(1/4)*(c^3 + 4*a*d^2 - c^(3/2)*Sqrt[c^3 + 4*a*d^2])*(Sqrt[c] + (d^2*
(c/d + x)^2)/Sqrt[c^3 + 4*a*d^2])*Sqrt[(d^2*(c*(4*a + c^3/d^2) - 2*c^2*(c/
d + x)^2 + d^2*(c/d + x)^4))/((c^3 + 4*a*d^2)*(Sqrt[c] + (d^2*(c/d + x)^2)
/Sqrt[c^3 + 4*a*d^2])^2)]*EllipticF[2*ArcTan[(d*(c/d + x))/(c^(1/4)*(c^3 +
 4*a*d^2)^(1/4))], (1 + c^(3/2)/Sqrt[c^3 + 4*a*d^2])/2])/(2*c^(1/4)*d^3*Sq
rt[c*(4*a + c^3/d^2) - 2*c^2*(c/d + x)^2 + d^2*(c/d + x)^4])))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1404
```
Int[((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*((a + b
*x^2 + c*x^4)^p/(4*p + 1)), x] + Simp[2*(p/(4*p + 1))   Int[(2*a + b*x^2)*(
a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*
c, 0] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 2458
```
Int[(Pn_)^(p_.), x_Symbol] :> With[{S = Coeff[Pn, x, Expon[Pn, x] - 1]/(Exp
on[Pn, x]*Coeff[Pn, x, Expon[Pn, x]])}, Subst[Int[ExpandToSum[Pn /. x -> x
- S, x]^p, x], x, x + S] /; BinomialQ[Pn /. x -> x - S, x] || (IntegerQ[Exp
on[Pn, x]/2] && TrinomialQ[Pn /. x -> x - S, x])] /; FreeQ[p, x] && PolyQ[P
n, x] && GtQ[Expon[Pn, x], 2] && NeQ[Coeff[Pn, x, Expon[Pn, x] - 1], 0]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 4864 vs. $2(520) = 1040$.

Time = 7.23 (sec) , antiderivative size = 4865, normalized size of antiderivative = 8.32

method	result	size
risch	Expression too large to display	4865
default	Expression too large to display	4890
elliptic	Expression too large to display	4890

input
```
int((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(d*x+c)*(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(1/2)/d+2/3*c/d*(-c*d*((x-
(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*(x-(-c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))
/d)*(x+(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)+((c+(-2*d*(-a*c)^(1/2)+c^2)^(1
/2))/d+(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*((-(c+(-2*d*(-a*c)^(1/2)+c^2)^
(1/2))/d+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*(x-(-c+(2*d*(-a*c)^(1/2)+c^2)
^(1/2))/d)/(-(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(2*d*(-a*c)^(1/2)+c^2
)^(1/2))/d)/(x+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d))^(1/2)*(x+(c+(2*d*(-a*c
)^(1/2)+c^2)^(1/2))/d)^2*((-(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(2*d*(-
a*c)^(1/2)+c^2)^(1/2))/d)*(x-(-c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/((-c+(-
2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(x+(c+
(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d))^(1/2)*((-(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2)
)/d-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*(x+(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/
2))/d)/(-(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1
/2))/d)/(x+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d))^(1/2)*((-(c+(2*d*(-a*c)^(1
/2)+c^2)^(1/2))/d^2*(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))+(c+(-2*d*(-a*c)^(1/2
)+c^2)^(1/2))/d^2*(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))+(c+(-2*d*(-a*c)^(1/2)+
c^2)^(1/2))/d^2*(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))+(c+(2*d*(-a*c)^(1/2)+c^2)
^(1/2))^2/d^2)/(-(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d+(c+(2*d*(-a*c)^(1/2)+
c^2)^(1/2))/d)/(-(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(2*d*(-a*c)^(1/2)+
c^2)^(1/2))/d)*EllipticF(((-(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d+(c+(2*d...
```

Fricas [F]

$$\int \sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}\,dx=\int \sqrt{d^2x^4+4\,cdx^3+4\,c^2x^2+4\,ac}\,dx$$

input
```
integrate((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(d^2*x^4 + 4*c*d*x^3 + 4*c^2*x^2 + 4*a*c), x)
```

Sympy [F]

$$\int \sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}\,dx = \int \sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}\,dx$$

input
```
integrate((d**2*x**4+4*c*d*x**3+4*c**2*x**2+4*a*c)**(1/2),x)
```

output
```
Integral(sqrt(4*a*c + 4*c**2*x**2 + 4*c*d*x**3 + d**2*x**4), x)
```

Maxima [F]

$$\int \sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}\,dx = \int \sqrt{d^2x^4+4\,cdx^3+4\,c^2x^2+4\,ac}\,dx$$

input
```
integrate((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(d^2*x^4 + 4*c*d*x^3 + 4*c^2*x^2 + 4*a*c), x)
```

Giac [F]

$$\int \sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}\,dx = \int \sqrt{d^2x^4+4\,cdx^3+4\,c^2x^2+4\,ac}\,dx$$

input
```
integrate((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(d^2*x^4 + 4*c*d*x^3 + 4*c^2*x^2 + 4*a*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}\,dx = \int \sqrt{4\,c^2\,x^2+4\,c\,d\,x^3+4\,a\,c+d^2\,x^4}\,dx$$

input
```
int((4*a*c + 4*c^2*x^2 + d^2*x^4 + 4*c*d*x^3)^(1/2),x)
```

output
```
int((4*a*c + 4*c^2*x^2 + d^2*x^4 + 4*c*d*x^3)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}\,dx = \frac{2\sqrt{d^2x^4+4cd\,x^3+4c^2x^2+4ac}\,c+3\sqrt{d^2x^4+4cd\,x^3+4c^2x^2+4ac}\,dx+24\left(\int \frac{\sqrt{d^2x^4+4cd\,x^3+4c^2x^2+4ac}}{d^2x^4+4cd\,x^3+4c^2x^2+4ac}dx\right)}{9d}$$

input
```
int((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(1/2),x)
```

output
```
(2*sqrt(4*a*c + 4*c**2*x**2 + 4*c*d*x**3 + d**2*x**4)*c + 3*sqrt(4*a*c + 4
*c**2*x**2 + 4*c*d*x**3 + d**2*x**4)*d*x + 24*int(sqrt(4*a*c + 4*c**2*x**2
 + 4*c*d*x**3 + d**2*x**4)/(4*a*c + 4*c**2*x**2 + 4*c*d*x**3 + d**2*x**4),
x)*a*c*d + 2*int((sqrt(4*a*c + 4*c**2*x**2 + 4*c*d*x**3 + d**2*x**4)*x**3)
/(4*a*c + 4*c**2*x**2 + 4*c*d*x**3 + d**2*x**4),x)*c*d**2 - 8*int((sqrt(4*
a*c + 4*c**2*x**2 + 4*c*d*x**3 + d**2*x**4)*x)/(4*a*c + 4*c**2*x**2 + 4*c*
d*x**3 + d**2*x**4),x)*c**3)/(9*d)
```

3.62 $\int \frac{1}{\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 217

$$\int \frac{1}{\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}}\,dx = \frac{\sqrt[4]{c^3+4ad^2}\sqrt{\frac{d^2(4ac+4c^2x^2+4cdx^3+d^2x^4)}{(c^3+4ad^2)\left(\sqrt{c}+\frac{(c+dx)^2}{\sqrt{c^3+4ad^2}}\right)^2}}\left(\sqrt{c}+\frac{(c+dx)^2}{\sqrt{c^3+4ad^2}}\right)\text{EllipticF}\left(2\arctan\left(\frac{c+dx}{\sqrt[4]{c}\sqrt[4]{c^3+4ad^2}}\right),\frac{1}{2}\left(1+\frac{\cdots}{\sqrt{c\cdots}}\right.\right.}{2\sqrt[4]{c}d\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}}$$

output

```
1/2*(4*a*d^2+c^3)^(1/4)*(d^2*(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)/(4*a*d^2+
c^3)/(c^(1/2)+(d*x+c)^2/(4*a*d^2+c^3)^(1/2))^2)^(1/2)*(c^(1/2)+(d*x+c)^2/(
4*a*d^2+c^3)^(1/2))*InverseJacobiAM(2*arctan((d*x+c)/c^(1/4)/(4*a*d^2+c^3)
^(1/4)),1/2*(2+2*c^(3/2)/(4*a*d^2+c^3)^(1/2))^(1/2))/c^(1/4)/d/(d^2*x^4+4*
c*d*x^3+4*c^2*x^2+4*a*c)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 12.65 (sec) , antiderivative size = 822, normalized size of antiderivative = 3.79

$$\int \frac{1}{\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}}\,dx$$
$$= \frac{2\left(-c+\sqrt{c^2-2i\sqrt{a}\sqrt{c}d}-dx\right)\left(c+\sqrt{c^2-2i\sqrt{a}\sqrt{c}d}+dx\right)\sqrt{-\frac{\sqrt{c^2-2i\sqrt{a}\sqrt{c}d}\left(c-\sqrt{c^2+2i\sqrt{a}\sqrt{c}d}+dx\right.}{\left(\sqrt{c^2-2i\sqrt{a}\sqrt{c}d}+\sqrt{c^2+2i\sqrt{a}\sqrt{c}d}\right)\left(-c+\sqrt{c^2-2i\right.}}}{d\sqrt{c^2-2i\sqrt{a}\sqrt{}}}$$

input
```
Integrate[1/Sqrt[4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4],x]
```

output
```
(2*(-c + Sqrt[c^2 - (2*I)*Sqrt[a]*Sqrt[c]*d] - d*x)*(c + Sqrt[c^2 - (2*I)*
Sqrt[a]*Sqrt[c]*d] + d*x)*Sqrt[-((Sqrt[c^2 - (2*I)*Sqrt[a]*Sqrt[c]*d]*(c -
 Sqrt[c^2 + (2*I)*Sqrt[a]*Sqrt[c]*d] + d*x))/((Sqrt[c^2 - (2*I)*Sqrt[a]*Sq
rt[c]*d] + Sqrt[c^2 + (2*I)*Sqrt[a]*Sqrt[c]*d])*(-c + Sqrt[c^2 - (2*I)*Sqr
t[a]*Sqrt[c]*d] - d*x)))]*Sqrt[-((Sqrt[c^2 - (2*I)*Sqrt[a]*Sqrt[c]*d]*(c +
 Sqrt[c^2 + (2*I)*Sqrt[a]*Sqrt[c]*d] + d*x))/((Sqrt[c^2 - (2*I)*Sqrt[a]*Sq
rt[c]*d] - Sqrt[c^2 + (2*I)*Sqrt[a]*Sqrt[c]*d])*(-c + Sqrt[c^2 - (2*I)*Sqr
t[a]*Sqrt[c]*d] - d*x)))]*EllipticF[ArcSin[Sqrt[((Sqrt[c^2 - (2*I)*Sqrt[a]
*Sqrt[c]*d] - Sqrt[c^2 + (2*I)*Sqrt[a]*Sqrt[c]*d])*(c + Sqrt[c^2 - (2*I)*S
qrt[a]*Sqrt[c]*d] + d*x))/((Sqrt[c^2 - (2*I)*Sqrt[a]*Sqrt[c]*d] + Sqrt[c^2
 + (2*I)*Sqrt[a]*Sqrt[c]*d])*(-c + Sqrt[c^2 - (2*I)*Sqrt[a]*Sqrt[c]*d] - d
*x))]], (Sqrt[c^2 - (2*I)*Sqrt[a]*Sqrt[c]*d] + Sqrt[c^2 + (2*I)*Sqrt[a]*Sq
rt[c]*d])^2/(Sqrt[c^2 - (2*I)*Sqrt[a]*Sqrt[c]*d] - Sqrt[c^2 + (2*I)*Sqrt[a
]*Sqrt[c]*d])^2])/(d*Sqrt[c^2 - (2*I)*Sqrt[a]*Sqrt[c]*d]*Sqrt[((Sqrt[c^2 -
 (2*I)*Sqrt[a]*Sqrt[c]*d] - Sqrt[c^2 + (2*I)*Sqrt[a]*Sqrt[c]*d])*(c + Sqrt
[c^2 - (2*I)*Sqrt[a]*Sqrt[c]*d] + d*x))/((Sqrt[c^2 - (2*I)*Sqrt[a]*Sqrt[c]
*d] + Sqrt[c^2 + (2*I)*Sqrt[a]*Sqrt[c]*d])*(-c + Sqrt[c^2 - (2*I)*Sqrt[a]*
Sqrt[c]*d] - d*x))]*Sqrt[4*a*c + x^2*(2*c + d*x)^2])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.19, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2458, 1416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}}\,dx$$

$$\Big\downarrow \text{2458}$$

$$\int \frac{1}{\sqrt{c\left(4a+\frac{c^3}{d^2}\right)-2c^2\left(\frac{c}{d}+x\right)^2+d^2\left(\frac{c}{d}+x\right)^4}}d\left(\frac{c}{d}+x\right)$$

$$\Big\downarrow \text{1416}$$

$$\frac{\sqrt[4]{4ad^2+c^3}\left(\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{4ad^2+c^3}}+\sqrt{c}\right)\sqrt{\frac{d^2\left(c\left(4a+\frac{c^3}{d^2}\right)-2c^2\left(\frac{c}{d}+x\right)^2+d^2\left(\frac{c}{d}+x\right)^4\right)}{\left(4ad^2+c^3\right)\left(\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{4ad^2+c^3}}+\sqrt{c}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{d\left(\frac{c}{d}+x\right)}{\sqrt[4]{c}\sqrt[4]{c^3+4ad^2}}\right),\frac{1}{2}\left(\frac{}{\sqrt{}}\right.\right.}{2\sqrt[4]{c}d\sqrt{c\left(4a+\frac{c^3}{d^2}\right)-2c^2\left(\frac{c}{d}+x\right)^2+d^2\left(\frac{c}{d}+x\right)^4}}$$

input
```
Int[1/Sqrt[4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4],x]
```

output
```
((c^3 + 4*a*d^2)^(1/4)*(Sqrt[c] + (d^2*(c/d + x)^2)/Sqrt[c^3 + 4*a*d^2])*S
qrt[(d^2*(c*(4*a + c^3/d^2) - 2*c^2*(c/d + x)^2 + d^2*(c/d + x)^4))/((c^3
+ 4*a*d^2)*(Sqrt[c] + (d^2*(c/d + x)^2)/Sqrt[c^3 + 4*a*d^2])^2)]*EllipticF
[2*ArcTan[(d*(c/d + x))/(c^(1/4)*(c^3 + 4*a*d^2)^(1/4))], (1 + c^(3/2)/Sqr
t[c^3 + 4*a*d^2])/2])/(2*c^(1/4)*d*Sqrt[c*(4*a + c^3/d^2) - 2*c^2*(c/d + x
)^2 + d^2*(c/d + x)^4])
```

Defintions of rubi rules used

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 2458
```
Int[(Pn_)^(p_.), x_Symbol] :> With[{S = Coeff[Pn, x, Expon[Pn, x] - 1]/(Exp
on[Pn, x]*Coeff[Pn, x, Expon[Pn, x]])}, Subst[Int[ExpandToSum[Pn /. x -> x
- S, x]^p, x], x, x + S] /; BinomialQ[Pn /. x -> x - S, x] || (IntegerQ[Exp
on[Pn, x]/2] && TrinomialQ[Pn /. x -> x - S, x])] /; FreeQ[p, x] && PolyQ[P
n, x] && GtQ[Expon[Pn, x], 2] && NeQ[Coeff[Pn, x, Expon[Pn, x] - 1], 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1055 vs. $2(192) = 384$.

Time = 1.03 (sec) , antiderivative size = 1056, normalized size of antiderivative = 4.87

method	result	size
default	Expression too large to display	1056
elliptic	Expression too large to display	1056

input
```
int(1/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
2*((c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d+(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d
)*((-(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/
d)*(x-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(-(c+(-2*d*(-a*c)^(1/2)+c^2)^(1
/2))/d-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(x+(c+(2*d*(-a*c)^(1/2)+c^2)^(
1/2))/d))^(1/2)*(x+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)^2*((-(c+(2*d*(-a*c)
^(1/2)+c^2)^(1/2))/d-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*(x-(-c+(-2*d*(-a
*c)^(1/2)+c^2)^(1/2))/d)/((-c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(2*d*(-
a*c)^(1/2)+c^2)^(1/2))/d)/(x+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d))^(1/2)*((
-(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*(
x+(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(-(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))
/d-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(x+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2)
)/d))^(1/2)/(-(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d+(c+(2*d*(-a*c)^(1/2)+c^2
)^(1/2))/d)/(-(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(2*d*(-a*c)^(1/2)+c^2
)^(1/2))/d)/(d^2*(x-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*(x+(c+(2*d*(-a*c)
^(1/2)+c^2)^(1/2))/d)*(x-(-c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*(x+(c+(-2*d
*(-a*c)^(1/2)+c^2)^(1/2))/d))^(1/2)*EllipticF(((-(c+(-2*d*(-a*c)^(1/2)+c^2
)^(1/2))/d+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*(x-(-c+(2*d*(-a*c)^(1/2)+c^
2)^(1/2))/d)/(-(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(2*d*(-a*c)^(1/2)+c
^2)^(1/2))/d)/(x+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d))^(1/2),((-(c+(2*d*(-a
*c)^(1/2)+c^2)^(1/2))/d-(-c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*((c+(-2*d...
```

Fricas [F]

$$\int \frac{1}{\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}}\,dx=\int \frac{1}{\sqrt{d^2x^4+4\,cdx^3+4\,c^2x^2+4\,ac}}\,dx$$

input

```
integrate(1/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(1/2),x, algorithm="fricas
")
```

output

```
integral(1/sqrt(d^2*x^4 + 4*c*d*x^3 + 4*c^2*x^2 + 4*a*c), x)
```

Sympy [F]

$$\int \frac{1}{\sqrt{4ac + 4c^2x^2 + 4cdx^3 + d^2x^4}}\,dx = \int \frac{1}{\sqrt{4ac + 4c^2x^2 + 4cdx^3 + d^2x^4}}\,dx$$

input
```
integrate(1/(d**2*x**4+4*c*d*x**3+4*c**2*x**2+4*a*c)**(1/2),x)
```

output
```
Integral(1/sqrt(4*a*c + 4*c**2*x**2 + 4*c*d*x**3 + d**2*x**4), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{4ac + 4c^2x^2 + 4cdx^3 + d^2x^4}}\,dx = \int \frac{1}{\sqrt{d^2x^4 + 4\,cdx^3 + 4\,c^2x^2 + 4\,ac}}\,dx$$

input
```
integrate(1/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(1/2),x, algorithm="maxima
")
```

output
```
integrate(1/sqrt(d^2*x^4 + 4*c*d*x^3 + 4*c^2*x^2 + 4*a*c), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{4ac + 4c^2x^2 + 4cdx^3 + d^2x^4}}\,dx = \int \frac{1}{\sqrt{d^2x^4 + 4\,cdx^3 + 4\,c^2x^2 + 4\,ac}}\,dx$$

input
```
integrate(1/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(d^2*x^4 + 4*c*d*x^3 + 4*c^2*x^2 + 4*a*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{4ac + 4c^2x^2 + 4cdx^3 + d^2x^4}}\,dx = \int \frac{1}{\sqrt{4\,c^2\,x^2 + 4\,c\,d\,x^3 + 4\,a\,c + d^2\,x^4}}\,dx$$

input
```
int(1/(4*a*c + 4*c^2*x^2 + d^2*x^4 + 4*c*d*x^3)^(1/2),x)
```

output
```
int(1/(4*a*c + 4*c^2*x^2 + d^2*x^4 + 4*c*d*x^3)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{4ac + 4c^2x^2 + 4cdx^3 + d^2x^4}}\,dx = \int \frac{\sqrt{d^2x^4 + 4cd\,x^3 + 4c^2x^2 + 4ac}}{d^2x^4 + 4cd\,x^3 + 4c^2x^2 + 4ac}dx$$

input
```
int(1/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(1/2),x)
```

output
```
int(sqrt(4*a*c + 4*c**2*x**2 + 4*c*d*x**3 + d**2*x**4)/(4*a*c + 4*c**2*x**
2 + 4*c*d*x**3 + d**2*x**4),x)
```

3.63 $\int \frac{1}{(4ac+4c^2x^2+4cdx^3+d^2x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 639

$$\int \frac{1}{(4ac+4c^2x^2+4cdx^3+d^2x^4)^{3/2}}\,dx = -\frac{d(c+dx)\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}}{8a\,(c^3+4ad^2)\left(\sqrt{c}\sqrt{c^3+4ad^2}+(c+dx)^2\right)}$$

$$-\frac{(c+dx)\left(c^3-4ad^2-c(c+dx)^2\right)}{8acd\,(c^3+4ad^2)\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}}$$

$$+\frac{\sqrt[4]{c}\sqrt{\frac{d^2(4ac+4c^2x^2+4cdx^3+d^2x^4)}{(c^3+4ad^2)\left(\sqrt{c}+\frac{(c+dx)^2}{\sqrt{c^3+4ad^2}}\right)^2}}\left(\sqrt{c}+\frac{(c+dx)^2}{\sqrt{c^3+4ad^2}}\right)E\left(2\arctan\left(\frac{c+dx}{\sqrt[4]{c}\sqrt[4]{c^3+4ad^2}}\right)|\frac{1}{2}\left(1+\frac{c^{3/2}}{\sqrt{c^3+4ad^2}}\right)\right)}{8ad\sqrt[4]{c^3+4ad^2}\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}}$$

$$+\frac{\left(c^3+4ad^2-c^{3/2}\sqrt{c^3+4ad^2}\right)\sqrt{\frac{d^2(4ac+4c^2x^2+4cdx^3+d^2x^4)}{(c^3+4ad^2)\left(\sqrt{c}+\frac{(c+dx)^2}{\sqrt{c^3+4ad^2}}\right)^2}}\left(\sqrt{c}+\frac{(c+dx)^2}{\sqrt{c^3+4ad^2}}\right)\text{EllipticF}\left(2\arctan\left(\frac{c+dx}{\sqrt[4]{c}\sqrt[4]{c^3+}}\right.\right.}{16ac^{5/4}d\,(c^3+4ad^2)^{3/4}\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}}$$

output
```
-1/8*d*(d*x+c)*(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(1/2)/a/(4*a*d^2+c^3)/(
c^(1/2)*(4*a*d^2+c^3)^(1/2)+(d*x+c)^2)-1/8*(d*x+c)*(c^3-4*a*d^2-c*(d*x+c)^
2)/a/c/d/(4*a*d^2+c^3)/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(1/2)+1/8*c^(1/
4)*(d^2*(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)/(4*a*d^2+c^3)/(c^(1/2)+(d*x+c)
^2/(4*a*d^2+c^3)^(1/2))^2)^(1/2)*(c^(1/2)+(d*x+c)^2/(4*a*d^2+c^3)^(1/2))*E
llipticE(sin(2*arctan((d*x+c)/c^(1/4)/(4*a*d^2+c^3)^(1/4))),1/2*(2+2*c^(3/
2)/(4*a*d^2+c^3)^(1/2))^(1/2))/a/d/(4*a*d^2+c^3)^(1/4)/(d^2*x^4+4*c*d*x^3+
4*c^2*x^2+4*a*c)^(1/2)+1/16*(c^3+4*a*d^2-c^(3/2)*(4*a*d^2+c^3)^(1/2))*(d^2
*(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)/(4*a*d^2+c^3)/(c^(1/2)+(d*x+c)^2/(4*a
*d^2+c^3)^(1/2))^2)^(1/2)*(c^(1/2)+(d*x+c)^2/(4*a*d^2+c^3)^(1/2))*InverseJ
acobiAM(2*arctan((d*x+c)/c^(1/4)/(4*a*d^2+c^3)^(1/4)),1/2*(2+2*c^(3/2)/(4*
a*d^2+c^3)^(1/2))^(1/2))/a/c^(5/4)/d/(4*a*d^2+c^3)^(3/4)/(d^2*x^4+4*c*d*x^
3+4*c^2*x^2+4*a*c)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 16.16 (sec) , antiderivative size = 5276, normalized size of antiderivative = 8.26

$$\int \frac{1}{\left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^{3/2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^(-3/2),x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 777, normalized size of antiderivative = 1.22, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {2458, 1405, 27, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(4ac+4c^2x^2+4cdx^3+d^2x^4)^{3/2}}dx$$

$$\Big\downarrow \text{2458}$$

$$\int \frac{1}{\left(c\left(4a+\frac{c^3}{d^2}\right)-2c^2\left(\frac{c}{d}+x\right)^2+d^2\left(\frac{c}{d}+x\right)^4\right)^{3/2}}d\left(\frac{c}{d}+x\right)$$

$$\Big\downarrow \text{1405}$$

$$\frac{\int \frac{2c\left(c^3-d^2\left(\frac{c}{d}+x\right)^2c+4ad^2\right)}{\sqrt{d^2\left(\frac{c}{d}+x\right)^4-2c^2\left(\frac{c}{d}+x\right)^2+c\left(\frac{c^3}{d^2}+4a\right)}}d\left(\frac{c}{d}+x\right)}{16ac^2\left(4ad^2+c^3\right)}-\frac{\left(\frac{c}{d}+x\right)\left(-4ad^2+c^3-cd^2\left(\frac{c}{d}+x\right)^2\right)}{8ac\left(4ad^2+c^3\right)\sqrt{c\left(4a+\frac{c^3}{d^2}\right)-2c^2\left(\frac{c}{d}+x\right)^2+d^2\left(\frac{c}{d}+x\right)^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{c^3-d^2\left(\frac{c}{d}+x\right)^2c+4ad^2}{\sqrt{d^2\left(\frac{c}{d}+x\right)^4-2c^2\left(\frac{c}{d}+x\right)^2+c\left(\frac{c^3}{d^2}+4a\right)}}d\left(\frac{c}{d}+x\right)}{8ac\left(4ad^2+c^3\right)}-\frac{\left(\frac{c}{d}+x\right)\left(-4ad^2+c^3-cd^2\left(\frac{c}{d}+x\right)^2\right)}{8ac\left(4ad^2+c^3\right)\sqrt{c\left(4a+\frac{c^3}{d^2}\right)-2c^2\left(\frac{c}{d}+x\right)^2+d^2\left(\frac{c}{d}+x\right)^4}}$$

$$\Big\downarrow \text{1511}$$

$$\frac{c^{3/2}\sqrt{4ad^2+c^3}\int \frac{\sqrt{c}-\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{c^3+4ad^2}}}{\sqrt{c}\sqrt{d^2\left(\frac{c}{d}+x\right)^4-2c^2\left(\frac{c}{d}+x\right)^2+c\left(\frac{c^3}{d^2}+4a\right)}}d\left(\frac{c}{d}+x\right)-\sqrt{4ad^2+c^3}\left(c^{3/2}-\sqrt{4ad^2+c^3}\right)\int \frac{}{\sqrt{d^2\left(\frac{c}{d}+x\right)^4-2c^2}}}{8ac\left(4ad^2+c^3\right)}-\frac{\left(\frac{c}{d}+x\right)\left(-4ad^2+c^3-cd^2\left(\frac{c}{d}+x\right)^2\right)}{8ac\left(4ad^2+c^3\right)\sqrt{c\left(4a+\frac{c^3}{d^2}\right)-2c^2\left(\frac{c}{d}+x\right)^2+d^2\left(\frac{c}{d}+x\right)^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{c\sqrt{4ad^2+c^3}\int \frac{\sqrt{c}-\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{c^3+4ad^2}}}{\sqrt{d^2\left(\frac{c}{d}+x\right)^4-2c^2\left(\frac{c}{d}+x\right)^2+c\left(\frac{c^3}{d^2}+4a\right)}}d\left(\frac{c}{d}+x\right)-\sqrt{4ad^2+c^3}\left(c^{3/2}-\sqrt{4ad^2+c^3}\right)\int \frac{1}{\sqrt{d^2\left(\frac{c}{d}+x\right)^4-2c^2\left(\frac{c}{d}+x\right)}}}{8ac\left(4ad^2+c^3\right)}$$

$$\frac{\left(\frac{c}{d}+x\right)\left(-4ad^2+c^3-cd^2\left(\frac{c}{d}+x\right)^2\right)}{8ac\left(4ad^2+c^3\right)\sqrt{c\left(4a+\frac{c^3}{d^2}\right)-2c^2\left(\frac{c}{d}+x\right)^2+d^2\left(\frac{c}{d}+x\right)^4}}$$

$$\Big\downarrow \text{1416}$$

$$\frac{c\sqrt{4ad^2+c^3}\int \frac{\sqrt{c}-\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{c^3+4ad^2}}}{\sqrt{d^2\left(\frac{c}{d}+x\right)^4-2c^2\left(\frac{c}{d}+x\right)^2+c\left(\frac{c^3}{d^2}+4a\right)}}d\left(\frac{c}{d}+x\right)-\frac{\left(4ad^2+c^3\right)^{3/4}\left(c^{3/2}-\sqrt{4ad^2+c^3}\right)\left(\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{4ad^2+c^3}}+\sqrt{c}\right)\sqrt{\frac{d^2\left(c\left(4a+\frac{c^3}{d^2}\right)\right.}{\left(4ad^2+c^3\right.}}}{2\sqrt[4]{c}d\sqrt{c\left(\right.}}}{8ac\left(4ad^2+c^3\right)}$$

$$\frac{\left(\frac{c}{d}+x\right)\left(-4ad^2+c^3-cd^2\left(\frac{c}{d}+x\right)^2\right)}{8ac\left(4ad^2+c^3\right)\sqrt{c\left(4a+\frac{c^3}{d^2}\right)-2c^2\left(\frac{c}{d}+x\right)^2+d^2\left(\frac{c}{d}+x\right)^4}}$$

$$\Big\downarrow \text{1509}$$

$$\frac{c\sqrt{4ad^2+c^3}\left(\frac{\sqrt[4]{c}\sqrt[4]{4ad^2+c^3}\left(\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{4ad^2+c^3}}+\sqrt{c}\right)\sqrt{\frac{d^2\left(c\left(4a+\frac{c^3}{d^2}\right)-2c^2\left(\frac{c}{d}+x\right)^2+d^2\left(\frac{c}{d}+x\right)^4\right)}{\left(4ad^2+c^3\right)\left(\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{4ad^2+c^3}}+\sqrt{c}\right)^2}}E\left(2\arctan\left(\frac{d\left(\frac{c}{d}+x\right)}{\sqrt[4]{c}\sqrt[4]{c^3+4ad^2}}\right)|\frac{1}{2}\left(\right.\right.}{d\sqrt{c\left(4a+\frac{c^3}{d^2}\right)-2c^2\left(\frac{c}{d}+x\right)^2+d^2\left(\frac{c}{d}+x\right)^4}}\right.}{}$$

$$\frac{\left(\frac{c}{d}+x\right)\left(-4ad^2+c^3-cd^2\left(\frac{c}{d}+x\right)^2\right)}{8ac\left(4ad^2+c^3\right)\sqrt{c\left(4a+\frac{c^3}{d^2}\right)-2c^2\left(\frac{c}{d}+x\right)^2+d^2\left(\frac{c}{d}+x\right)^4}}$$

input
```
Int[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^(-3/2),x]
```

output
```
-1/8*((c/d + x)*(c^3 - 4*a*d^2 - c*d^2*(c/d + x)^2))/(a*c*(c^3 + 4*a*d^2)*
Sqrt[c*(4*a + c^3/d^2) - 2*c^2*(c/d + x)^2 + d^2*(c/d + x)^4]) + (c*Sqrt[c
^3 + 4*a*d^2]*(-(((c/d + x)*Sqrt[c*(4*a + c^3/d^2) - 2*c^2*(c/d + x)^2 + d
^2*(c/d + x)^4])/((4*a + c^3/d^2)*(Sqrt[c] + (d^2*(c/d + x)^2)/Sqrt[c^3 +
4*a*d^2]))) + (c^(1/4)*(c^3 + 4*a*d^2)^(1/4)*(Sqrt[c] + (d^2*(c/d + x)^2)/
Sqrt[c^3 + 4*a*d^2])*Sqrt[(d^2*(c*(4*a + c^3/d^2) - 2*c^2*(c/d + x)^2 + d^
2*(c/d + x)^4))/((c^3 + 4*a*d^2)*(Sqrt[c] + (d^2*(c/d + x)^2)/Sqrt[c^3 + 4
*a*d^2])^2)]*EllipticE[2*ArcTan[(d*(c/d + x))/(c^(1/4)*(c^3 + 4*a*d^2)^(1/
4))], (1 + c^(3/2)/Sqrt[c^3 + 4*a*d^2])/2])/(d*Sqrt[c*(4*a + c^3/d^2) - 2*
c^2*(c/d + x)^2 + d^2*(c/d + x)^4])) - ((c^3 + 4*a*d^2)^(3/4)*(c^(3/2) - S
qrt[c^3 + 4*a*d^2])*(Sqrt[c] + (d^2*(c/d + x)^2)/Sqrt[c^3 + 4*a*d^2])*Sqrt
[(d^2*(c*(4*a + c^3/d^2) - 2*c^2*(c/d + x)^2 + d^2*(c/d + x)^4))/((c^3 + 4
*a*d^2)*(Sqrt[c] + (d^2*(c/d + x)^2)/Sqrt[c^3 + 4*a*d^2])^2)]*EllipticF[2*
ArcTan[(d*(c/d + x))/(c^(1/4)*(c^3 + 4*a*d^2)^(1/4))], (1 + c^(3/2)/Sqrt[c
^3 + 4*a*d^2])/2])/(2*c^(1/4)*d*Sqrt[c*(4*a + c^3/d^2) - 2*c^2*(c/d + x)^2
 + d^2*(c/d + x)^4])))/(8*a*c*(c^3 + 4*a*d^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 2458
```
Int[(Pn_)^(p_.), x_Symbol] :> With[{S = Coeff[Pn, x, Expon[Pn, x] - 1]/(Exp
on[Pn, x]*Coeff[Pn, x, Expon[Pn, x]])}, Subst[Int[ExpandToSum[Pn /. x -> x
- S, x]^p, x], x, x + S] /; BinomialQ[Pn /. x -> x - S, x] || (IntegerQ[Exp
on[Pn, x]/2] && TrinomialQ[Pn /. x -> x - S, x])] /; FreeQ[p, x] && PolyQ[P
n, x] && GtQ[Expon[Pn, x], 2] && NeQ[Coeff[Pn, x, Expon[Pn, x] - 1], 0]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 5023 vs. $2(574) = 1148$.

Time = 1.06 (sec) , antiderivative size = 5024, normalized size of antiderivative = 7.86

method	result	size
default	Expression too large to display	5024
elliptic	Expression too large to display	5024

input
```
int(1/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [F]

$$\int \frac{1}{(4ac+4c^2x^2+4cdx^3+d^2x^4)^{3/2}}\,dx = \int \frac{1}{(d^2x^4+4\,cdx^3+4\,c^2x^2+4\,ac)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(3/2),x, algorithm="fricas
")
```

output
```
integral(sqrt(d^2*x^4 + 4*c*d*x^3 + 4*c^2*x^2 + 4*a*c)/(d^4*x^8 + 8*c*d^3*
x^7 + 24*c^2*d^2*x^6 + 32*c^3*d*x^5 + 32*a*c^2*d*x^3 + 32*a*c^3*x^2 + 8*(2
*c^4 + a*c*d^2)*x^4 + 16*a^2*c^2), x)
```

Sympy [F]

$$\int \frac{1}{(4ac+4c^2x^2+4cdx^3+d^2x^4)^{3/2}}\,dx = \int \frac{1}{(4ac+4c^2x^2+4cdx^3+d^2x^4)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(d**2*x**4+4*c*d*x**3+4*c**2*x**2+4*a*c)**(3/2),x)
```

output
```
Integral((4*a*c + 4*c**2*x**2 + 4*c*d*x**3 + d**2*x**4)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(4ac+4c^2x^2+4cdx^3+d^2x^4)^{3/2}}\,dx = \int \frac{1}{(d^2x^4+4\,cdx^3+4\,c^2x^2+4\,ac)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(3/2),x, algorithm="maxima
")
```

output
```
integrate((d^2*x^4 + 4*c*d*x^3 + 4*c^2*x^2 + 4*a*c)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(4ac+4c^2x^2+4cdx^3+d^2x^4)^{3/2}}\,dx=\int \frac{1}{(d^2x^4+4\,cdx^3+4\,c^2x^2+4\,ac)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(3/2),x, algorithm="giac")
```

output
```
integrate((d^2*x^4 + 4*c*d*x^3 + 4*c^2*x^2 + 4*a*c)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(4ac+4c^2x^2+4cdx^3+d^2x^4)^{3/2}}\,dx=\int \frac{1}{(4\,c^2\,x^2+4\,c\,d\,x^3+4\,a\,c+d^2\,x^4)^{3/2}}\,dx$$

input
```
int(1/(4*a*c + 4*c^2*x^2 + d^2*x^4 + 4*c*d*x^3)^(3/2),x)
```

output
```
int(1/(4*a*c + 4*c^2*x^2 + d^2*x^4 + 4*c*d*x^3)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(4ac+4c^2x^2+4cdx^3+d^2x^4)^{3/2}}\,dx=\int \frac{\sqrt{d^2x^4+4cd\,x^3+4c^2x^2+4ac}}{d^4x^8+8c\,d^3x^7+24c^2d^2x^6+32c^3d\,x^5+8ac\,d^2x^4+16c^4x^4+3}$$

input
```
int(1/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(3/2),x)
```

output
```
int(sqrt(4*a*c + 4*c**2*x**2 + 4*c*d*x**3 + d**2*x**4)/(16*a**2*c**2 + 32*
a*c**3*x**2 + 32*a*c**2*d*x**3 + 8*a*c*d**2*x**4 + 16*c**4*x**4 + 32*c**3*
d*x**5 + 24*c**2*d**2*x**6 + 8*c*d**3*x**7 + d**4*x**8),x)
```

3.64 $\int \left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)^{3/2}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 702

$$\int \left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)^{3/2}\,dx=\frac{(d+4ex)\left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)^{3/2}}{28e}$$

$$-\frac{3d^2(d^4+512ae^3)\,(d+4ex)\sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}}{280e^2\left(\sqrt{5d^4+256ae^3}+(d+4ex)^2\right)}$$

$$+\frac{(d+4ex)\sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}\,(43d^4+1280ae^3-9d^2(d+4ex)^2)}{2240e^2}$$

$$+\frac{3d^2(5d^4+256ae^3)^{3/4}\,(d^4+512ae^3)\left(1+\frac{(d+4ex)^2}{\sqrt{5d^4+256ae^3}}\right)\sqrt{\frac{5d^4+256ae^3-6d^2(d+4ex)^2+(d+4ex)^4}{(5d^4+256ae^3)\left(1+\frac{(d+4ex)^2}{\sqrt{5d^4+256ae^3}}\right)^2}}\,E\left(2\arctan\left(\frac{\ldots}{\sqrt[4]{5d^4\ldots}}\right.\right.}{8960e^3\sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}}$$

$$+\frac{(5d^4+256ae^3)^{3/4}\left(4(d^4+80ae^3)\sqrt{5d^4+256ae^3}-3d^2(d^4+512ae^3)\right)\left(1+\frac{(d+4ex)^2}{\sqrt{5d^4+256ae^3}}\right)\sqrt{\frac{5d^4+256ae^3-6d^2(\ldots}{(5d^4+256ae^3)\left(1\ldots\right.}}}{17920e^3\sqrt{8ae^2-d^3x+8de^2x^3+\ldots}}$$

output
```
1/28*(4*e*x+d)*(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^(3/2)/e-3/280*d^2*(51
2*a*e^3+d^4)*(4*e*x+d)*(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^(1/2)/e^2/((2
56*a*e^3+5*d^4)^(1/2)+(4*e*x+d)^2)+1/2240*(4*e*x+d)*(8*e^3*x^4+8*d*e^2*x^3
-d^3*x+8*a*e^2)^(1/2)*(43*d^4+1280*a*e^3-9*d^2*(4*e*x+d)^2)/e^2+3/8960*d^2
*(256*a*e^3+5*d^4)^(3/4)*(512*a*e^3+d^4)*(1+(4*e*x+d)^2/(256*a*e^3+5*d^4)^
(1/2))*((5*d^4+256*a*e^3-6*d^2*(4*e*x+d)^2+(4*e*x+d)^4)/(256*a*e^3+5*d^4)/
(1+(4*e*x+d)^2/(256*a*e^3+5*d^4)^(1/2))^2)^(1/2)*EllipticE(sin(2*arctan((4
*e*x+d)/(256*a*e^3+5*d^4)^(1/4))),1/2*(2+6*d^2/(256*a*e^3+5*d^4)^(1/2))^(1
/2))/e^3/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^(1/2)+1/17920*(256*a*e^3+5*
d^4)^(3/4)*(4*(80*a*e^3+d^4)*(256*a*e^3+5*d^4)^(1/2)-3*d^2*(512*a*e^3+d^4)
)*(1+(4*e*x+d)^2/(256*a*e^3+5*d^4)^(1/2))*((5*d^4+256*a*e^3-6*d^2*(4*e*x+d
)^2+(4*e*x+d)^4)/(256*a*e^3+5*d^4)/(1+(4*e*x+d)^2/(256*a*e^3+5*d^4)^(1/2))
^2)^(1/2)*InverseJacobiAM(2*arctan((4*e*x+d)/(256*a*e^3+5*d^4)^(1/4)),1/2*
(2+6*d^2/(256*a*e^3+5*d^4)^(1/2))^(1/2))/e^3/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+
8*a*e^2)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 13510 vs. $2(702) = 1404$.

Time = 16.24 (sec) , antiderivative size = 13510, normalized size of antiderivative = 19.25

$$\int \left(8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4\right)^{3/2} dx = \text{Result too large to show}$$

input
```
Integrate[(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)^(3/2),x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 945, normalized size of antiderivative = 1.35, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {2458, 1404, 27, 1490, 27, 1511, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)^{3/2}dx$$

$$\Big\downarrow \text{ 2458}$$

$$\int \left(\frac{1}{32}\left(256ae^2+\frac{5d^4}{e}\right)-3d^2e\left(\frac{d}{4e}+x\right)^2+8e^3\left(\frac{d}{4e}+x\right)^4\right)^{3/2}d\left(\frac{d}{4e}+x\right)$$

$$\Big\downarrow \text{ 1404}$$

$$\frac{3}{7}\int \frac{\left(\frac{5d^4}{e}-48e\left(\frac{d}{4e}+x\right)^2d^2+256ae^2\right)\sqrt{\frac{5d^4}{e}-96e\left(\frac{d}{4e}+x\right)^2d^2+256e^3\left(\frac{d}{4e}+x\right)^4+256ae^2}}{64\sqrt{2}}d\left(\frac{d}{4e}+x\right)+\frac{\left(\frac{d}{4e}+x\right)\left(256ae^2+\frac{5d^4}{e}-96d^2e\left(\frac{d}{4e}+x\right)^2+256e^3\left(\frac{d}{4e}+x\right)^4\right)^{3/2}}{896\sqrt{2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{3\int \left(\frac{5d^4}{e}-48e\left(\frac{d}{4e}+x\right)^2d^2+256ae^2\right)\sqrt{\frac{5d^4}{e}-96e\left(\frac{d}{4e}+x\right)^2d^2+256e^3\left(\frac{d}{4e}+x\right)^4+256ae^2}d\left(\frac{d}{4e}+x\right)}{448\sqrt{2}}+\frac{\left(\frac{d}{4e}+x\right)\left(256ae^2+\frac{5d^4}{e}-96d^2e\left(\frac{d}{4e}+x\right)^2+256e^3\left(\frac{d}{4e}+x\right)^4\right)^{3/2}}{896\sqrt{2}}$$

$$\Big\downarrow \text{ 1490}$$

$$3\left(\frac{\int \frac{8192e\left(\left(d^4+80ae^3\right)\left(5d^4+256ae^3\right)-12d^2e^2\left(d^4+512ae^3\right)\left(\frac{d}{4e}+x\right)^2\right)}{\sqrt{\frac{5d^4}{e}-96e\left(\frac{d}{4e}+x\right)^2d^2+256e^3\left(\frac{d}{4e}+x\right)^4+256ae^2}}d\left(\frac{d}{4e}+x\right)}{3840e^3}+\frac{\left(\frac{d}{4e}+x\right)\sqrt{256ae^2+\frac{5d^4}{e}-96d^2e\left(\frac{d}{4e}+x\right)^2+256e^3\left(\frac{d}{4e}+x\right)^4}\left(1\right.}{15e}\right.$$

$$448\sqrt{2}$$

$$\frac{\left(\frac{d}{4e}+x\right)\left(256ae^2+\frac{5d^4}{e}-96d^2e\left(\frac{d}{4e}+x\right)^2+256e^3\left(\frac{d}{4e}+x\right)^4\right)^{3/2}}{896\sqrt{2}}$$

$\downarrow$ 27

$$3\left(\frac{32\int \frac{\left(d^4+80ae^3\right)\left(5d^4+256ae^3\right)-12d^2e^2\left(d^4+512ae^3\right)\left(\frac{d}{4e}+x\right)^2}{\sqrt{\frac{5d^4}{e}-96e\left(\frac{d}{4e}+x\right)^2d^2+256e^3\left(\frac{d}{4e}+x\right)^4+256ae^2}}d\left(\frac{d}{4e}+x\right)}{15e^2}+\frac{\left(\frac{d}{4e}+x\right)\sqrt{256ae^2+\frac{5d^4}{e}-96d^2e\left(\frac{d}{4e}+x\right)^2+256e^3\left(\frac{d}{4e}+x\right)^4}\left(1280a\right.}{15e}\right.$$

$$448\sqrt{2}$$

$$\frac{\left(\frac{d}{4e}+x\right)\left(256ae^2+\frac{5d^4}{e}-96d^2e\left(\frac{d}{4e}+x\right)^2+256e^3\left(\frac{d}{4e}+x\right)^4\right)^{3/2}}{896\sqrt{2}}$$

$\downarrow$ 1511

$$3\left(\frac{32\left(\frac{3}{4}d^2\sqrt{256ae^3+5d^4}\left(512ae^3+d^4\right)\int \frac{1-\frac{16e^2\left(\frac{d}{4e}+x\right)^2}{\sqrt{5d^4+256ae^3}}}{\sqrt{\frac{5d^4}{e}-96e\left(\frac{d}{4e}+x\right)^2d^2+256e^3\left(\frac{d}{4e}+x\right)^4+256ae^2}}d\left(\frac{d}{4e}+x\right)+\frac{1}{4}\sqrt{256ae^3+5d^4}\left(4\left(80ae^3+d^4\right)\sqrt{256ae^3+5d\right.\right.}{15e^2}\right.$$

$$\frac{\left(\frac{d}{4e}+x\right)\left(256ae^2+\frac{5d^4}{e}-96d^2e\left(\frac{d}{4e}+x\right)^2+256e^3\left(\frac{d}{4e}+x\right)^4\right)^{3/2}}{896\sqrt{2}}$$

$\downarrow$ 1416

$$3\left(\frac{32\left(\frac{3}{4}d^2\sqrt{256ae^3+5d^4}\left(512ae^3+d^4\right)\int \frac{1-\frac{16e^2\left(\frac{d}{4e}+x\right)^2}{\sqrt{5d^4+256ae^3}}}{\sqrt{\frac{5d^4}{e}-96e\left(\frac{d}{4e}+x\right)^2d^2+256e^3\left(\frac{d}{4e}+x\right)^4+256ae^2}}d\left(\frac{d}{4e}+x\right)+\frac{\left(256ae^3+5d^4\right)^{3/4}\left(4\left(80ae^3+d^4\right)\sqrt{256ae^3+5d^4}\cdots\right.}{\cdots}\right.}{\cdots}\right.$$

$$\frac{\left(\frac{d}{4e}+x\right)\left(256ae^2+\frac{5d^4}{e}-96d^2e\left(\frac{d}{4e}+x\right)^2+256e^3\left(\frac{d}{4e}+x\right)^4\right)^{3/2}}{896\sqrt{2}}$$

$\downarrow$ 1509

$$\frac{\left(\frac{d}{4e}+x\right)\left(\frac{5d^4}{e}-96e\left(\frac{d}{4e}+x\right)^2d^2+256e^3\left(\frac{d}{4e}+x\right)^4+256ae^2\right)^{3/2}}{896\sqrt{2}}+$$

$$3\left(\frac{\left(\frac{d}{4e}+x\right)\sqrt{\frac{5d^4}{e}-96e\left(\frac{d}{4e}+x\right)^2d^2+256e^3\left(\frac{d}{4e}+x\right)^4+256ae^2}\left(43d^4-144e^2\left(\frac{d}{4e}+x\right)^2d^2+1280ae^3\right)}{15e}+\frac{32\left(\frac{3}{4}\sqrt{5d^4+256ae^3}\left(d^4+512ae^3\right)\left(\frac{\sqrt[4]{\cdots}}{\cdots}\right.\right.}{\cdots}\right.$$

input
```
Int[(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)^(3/2),x]
```

output

```
((d/(4*e) + x)*((5*d^4)/e + 256*a*e^2 - 96*d^2*e*(d/(4*e) + x)^2 + 256*e^3
*(d/(4*e) + x)^4)^(3/2))/(896*Sqrt[2]) + (3*(((d/(4*e) + x)*(43*d^4 + 1280
*a*e^3 - 144*d^2*e^2*(d/(4*e) + x)^2)*Sqrt[(5*d^4)/e + 256*a*e^2 - 96*d^2*
e*(d/(4*e) + x)^2 + 256*e^3*(d/(4*e) + x)^4])/(15*e) + (32*((3*d^2*Sqrt[5*
d^4 + 256*a*e^3]*(d^4 + 512*a*e^3)*(-((e*(d/(4*e) + x)*Sqrt[(5*d^4)/e + 25
6*a*e^2 - 96*d^2*e*(d/(4*e) + x)^2 + 256*e^3*(d/(4*e) + x)^4])/((5*d^4 + 2
56*a*e^3)*(1 + (16*e^2*(d/(4*e) + x)^2)/Sqrt[5*d^4 + 256*a*e^3]))) + ((5*d
^4 + 256*a*e^3)^(1/4)*(1 + (16*e^2*(d/(4*e) + x)^2)/Sqrt[5*d^4 + 256*a*e^3
])*Sqrt[(5*d^4 + 256*a*e^3 - 96*d^2*e^2*(d/(4*e) + x)^2 + 256*e^4*(d/(4*e)
 + x)^4)/((5*d^4 + 256*a*e^3)*(1 + (16*e^2*(d/(4*e) + x)^2)/Sqrt[5*d^4 + 2
56*a*e^3])^2)]*EllipticE[2*ArcTan[(4*e*(d/(4*e) + x))/(5*d^4 + 256*a*e^3)^
(1/4)], (1 + (3*d^2)/Sqrt[5*d^4 + 256*a*e^3])/2])/(4*e*Sqrt[(5*d^4)/e + 25
6*a*e^2 - 96*d^2*e*(d/(4*e) + x)^2 + 256*e^3*(d/(4*e) + x)^4])))/4 + ((5*d
^4 + 256*a*e^3)^(3/4)*(4*(d^4 + 80*a*e^3)*Sqrt[5*d^4 + 256*a*e^3] - 3*d^2*
(d^4 + 512*a*e^3))*(1 + (16*e^2*(d/(4*e) + x)^2)/Sqrt[5*d^4 + 256*a*e^3])*
Sqrt[(5*d^4 + 256*a*e^3 - 96*d^2*e^2*(d/(4*e) + x)^2 + 256*e^4*(d/(4*e) +
x)^4)/((5*d^4 + 256*a*e^3)*(1 + (16*e^2*(d/(4*e) + x)^2)/Sqrt[5*d^4 + 256*
a*e^3])^2)]*EllipticF[2*ArcTan[(4*e*(d/(4*e) + x))/(5*d^4 + 256*a*e^3)^(1/
4)], (1 + (3*d^2)/Sqrt[5*d^4 + 256*a*e^3])/2])/(32*e*Sqrt[(5*d^4)/e + 256*
a*e^2 - 96*d^2*e*(d/(4*e) + x)^2 + 256*e^3*(d/(4*e) + x)^4])))/(15*e^2)...
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1404

```
Int[((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*((a + b
*x^2 + c*x^4)^p/(4*p + 1)), x] + Simp[2*(p/(4*p + 1))   Int[(2*a + b*x^2)*(
a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*
c, 0] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 1416

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1490
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Simp[2*(p/(c*(4*p + 1)*(4*p + 3)))
 Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3)
- b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 2458
```
Int[(Pn_)^(p_.), x_Symbol] :> With[{S = Coeff[Pn, x, Expon[Pn, x] - 1]/(Exp
on[Pn, x]*Coeff[Pn, x, Expon[Pn, x]])}, Subst[Int[ExpandToSum[Pn /. x -> x
- S, x]^p, x], x, x + S] /; BinomialQ[Pn /. x -> x - S, x] || (IntegerQ[Exp
on[Pn, x]/2] && TrinomialQ[Pn /. x -> x - S, x])] /; FreeQ[p, x] && PolyQ[P
n, x] && GtQ[Expon[Pn, x], 2] && NeQ[Coeff[Pn, x, Expon[Pn, x] - 1], 0]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 8240 vs. 2(655) = 1310.

Time = 11.79 (sec) , antiderivative size = 8241, normalized size of antiderivative = 11.74

method	result	size
default	Expression too large to display	8241
elliptic	Expression too large to display	8241
risch	Expression too large to display	11353

input
```
int((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [F]

$$\int \left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)^{3/2}\,dx=\int \left(8\,e^3x^4+8\,de^2x^3-d^3x+8\,ae^2\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^(3/2),x, algorithm="fricas
")
```

output
```
integral((8*e^3*x^4 + 8*d*e^2*x^3 - d^3*x + 8*a*e^2)^(3/2), x)
```

Sympy [F]

$$\int \left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)^{3/2}\,dx=\int \left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((8*e**3*x**4+8*d*e**2*x**3-d**3*x+8*a*e**2)**(3/2),x)
```

output
```
Integral((8*a*e**2 - d**3*x + 8*d*e**2*x**3 + 8*e**3*x**4)**(3/2), x)
```

Maxima [F]

$$\int \left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)^{3/2}\,dx=\int \left(8\,e^3x^4+8\,de^2x^3-d^3x+8\,ae^2\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^(3/2),x, algorithm="maxima
")
```

output
```
integrate((8*e^3*x^4 + 8*d*e^2*x^3 - d^3*x + 8*a*e^2)^(3/2), x)
```

Giac [F]

$$\int \left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)^{3/2}\,dx=\int \left(8\,e^3x^4+8\,de^2x^3-d^3x+8\,ae^2\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((8*e^3*x^4 + 8*d*e^2*x^3 - d^3*x + 8*a*e^2)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)^{3/2}\,dx=\int \left(-d^3\,x+8\,d\,e^2\,x^3+8\,e^3\,x^4+8\,a\,e^2\right)^{3/2}\,dx$$

input
```
int((8*a*e^2 - d^3*x + 8*e^3*x^4 + 8*d*e^2*x^3)^(3/2),x)
```

output
```
int((8*a*e^2 - d^3*x + 8*e^3*x^4 + 8*d*e^2*x^3)^(3/2), x)
```

Reduce [F]

$$\int \left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)^{3/2}\,dx=\text{Too large to display}$$

input
```
int((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^(3/2),x)
```

output
```
(1920*sqrt(8*a*e**2 - d**3*x + 8*d*e**2*x**3 + 8*e**3*x**4)*a*d*e**4 + 768
0*sqrt(8*a*e**2 - d**3*x + 8*d*e**2*x**3 + 8*e**3*x**4)*a*e**5*x + 34*sqrt
(8*a*e**2 - d**3*x + 8*d*e**2*x**3 + 8*e**3*x**4)*d**5*e - 16*sqrt(8*a*e**
2 - d**3*x + 8*d*e**2*x**3 + 8*e**3*x**4)*d**4*e**2*x - 752*sqrt(8*a*e**2
- d**3*x + 8*d*e**2*x**3 + 8*e**3*x**4)*d**3*e**3*x**2 + 64*sqrt(8*a*e**2
- d**3*x + 8*d*e**2*x**3 + 8*e**3*x**4)*d**2*e**4*x**3 + 3200*sqrt(8*a*e**
2 - d**3*x + 8*d*e**2*x**3 + 8*e**3*x**4)*d*e**5*x**4 + 2560*sqrt(8*a*e**2
 - d**3*x + 8*d*e**2*x**3 + 8*e**3*x**4)*e**6*x**5 - 1536*sqrt(e)*sqrt(2)*
log( - 2*sqrt(e)*sqrt(8*a*e**2 - d**3*x + 8*d*e**2*x**3 + 8*e**3*x**4)*sqr
t(2) + d**2 - 4*d*e*x - 8*e**2*x**2)*a*d**3*e**3 - 3*sqrt(e)*sqrt(2)*log(
- 2*sqrt(e)*sqrt(8*a*e**2 - d**3*x + 8*d*e**2*x**3 + 8*e**3*x**4)*sqrt(2)
+ d**2 - 4*d*e*x - 8*e**2*x**2)*d**7 - 1536*sqrt(e)*sqrt(2)*log(2*sqrt(e)*
sqrt(8*a*e**2 - d**3*x + 8*d*e**2*x**3 + 8*e**3*x**4)*sqrt(2) + d**2 - 4*d
*e*x - 8*e**2*x**2)*a*d**3*e**3 - 3*sqrt(e)*sqrt(2)*log(2*sqrt(e)*sqrt(8*a
*e**2 - d**3*x + 8*d*e**2*x**3 + 8*e**3*x**4)*sqrt(2) + d**2 - 4*d*e*x - 8
*e**2*x**2)*d**7 + 81920*int(sqrt(8*a*e**2 - d**3*x + 8*d*e**2*x**3 + 8*e*
*3*x**4)/(8*a*e**2 - d**3*x + 8*d*e**2*x**3 + 8*e**3*x**4),x)*a**2*e**7 +
1088*int(sqrt(8*a*e**2 - d**3*x + 8*d*e**2*x**3 + 8*e**3*x**4)/(8*a*e**2 -
 d**3*x + 8*d*e**2*x**3 + 8*e**3*x**4),x)*a*d**4*e**4 + 17*int(sqrt(8*a*e*
*2 - d**3*x + 8*d*e**2*x**3 + 8*e**3*x**4)/(8*a*e**2 - d**3*x + 8*d*e**...
```

3.65 $\int \sqrt{8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4}\, dx$

Optimal result

Integrand size = 34, antiderivative size = 599

$$\int \sqrt{8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4}\, dx = \frac{(d+4ex)\sqrt{8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4}}{12e}$$
$$- \frac{d^2(d+4ex)\sqrt{8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4}}{2e\left(\sqrt{5d^4+256ae^3} + (d+4ex)^2\right)}$$
$$+ \frac{d^2(5d^4+256ae^3)^{3/4}\left(1+\frac{(d+4ex)^2}{\sqrt{5d^4+256ae^3}}\right)\sqrt{\frac{5d^4+256ae^3-6d^2(d+4ex)^2+(d+4ex)^4}{(5d^4+256ae^3)\left(1+\frac{(d+4ex)^2}{\sqrt{5d^4+256ae^3}}\right)^2}}\, E\left(2\arctan\left(\frac{d+4ex}{\sqrt[4]{5d^4+256ae^3}}\right)\right.}{64e^2\sqrt{8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4}}$$
$$+ \frac{\sqrt[4]{5d^4+256ae^3}\left(5d^4+256ae^3-3d^2\sqrt{5d^4+256ae^3}\right)\left(1+\frac{(d+4ex)^2}{\sqrt{5d^4+256ae^3}}\right)\sqrt{\frac{5d^4+256ae^3-6d^2(d+4ex)^2+(d+4ex)^4}{(5d^4+256ae^3)\left(1+\frac{(d+4ex)^2}{\sqrt{5d^4+256ae^3}}\right)^2}}}{384e^2\sqrt{8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4}}$$

output
```
1/12*(4*e*x+d)*(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^(1/2)/e-1/2*d^2*(4*e*
x+d)*(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^(1/2)/e/((256*a*e^3+5*d^4)^(1/2
)+(4*e*x+d)^2)+1/64*d^2*(256*a*e^3+5*d^4)^(3/4)*(1+(4*e*x+d)^2/(256*a*e^3+
5*d^4)^(1/2))*((5*d^4+256*a*e^3-6*d^2*(4*e*x+d)^2+(4*e*x+d)^4)/(256*a*e^3+
5*d^4)/(1+(4*e*x+d)^2/(256*a*e^3+5*d^4)^(1/2))^2)^(1/2)*EllipticE(sin(2*ar
ctan((4*e*x+d)/(256*a*e^3+5*d^4)^(1/4))),1/2*(2+6*d^2/(256*a*e^3+5*d^4)^(1
/2))^(1/2))/e^2/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^(1/2)+1/384*(256*a*e
^3+5*d^4)^(1/4)*(5*d^4+256*a*e^3-3*d^2*(256*a*e^3+5*d^4)^(1/2))*(1+(4*e*x+
d)^2/(256*a*e^3+5*d^4)^(1/2))*((5*d^4+256*a*e^3-6*d^2*(4*e*x+d)^2+(4*e*x+d
)^4)/(256*a*e^3+5*d^4)/(1+(4*e*x+d)^2/(256*a*e^3+5*d^4)^(1/2))^2)^(1/2)*In
verseJacobiAM(2*arctan((4*e*x+d)/(256*a*e^3+5*d^4)^(1/4)),1/2*(2+6*d^2/(25
6*a*e^3+5*d^4)^(1/2))^(1/2))/e^2/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^(1/
2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 7543 vs. $2(599) = 1198$.

Time = 16.15 (sec) , antiderivative size = 7543, normalized size of antiderivative = 12.59

$$\int \sqrt{8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4}\,dx = \text{Result too large to show}$$

input
```
Integrate[Sqrt[8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4],x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 1.24 (sec) , antiderivative size = 814, normalized size of antiderivative = 1.36, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {2458, 1404, 27, 1511, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}\,dx$$

$\Big\downarrow$ 2458

$$\int \sqrt{\frac{1}{32}\left(256ae^2+\frac{5d^4}{e}\right)-3d^2e\left(\frac{d}{4e}+x\right)^2+8e^3\left(\frac{d}{4e}+x\right)^4}\,d\left(\frac{d}{4e}+x\right)$$

$\Big\downarrow$ 1404

$$\frac{1}{3}\int \frac{\frac{5d^4}{e}-48e\left(\frac{d}{4e}+x\right)^2d^2+256ae^2}{2\sqrt{2}\sqrt{\frac{5d^4}{e}-96e\left(\frac{d}{4e}+x\right)^2d^2+256e^3\left(\frac{d}{4e}+x\right)^4+256ae^2}}d\left(\frac{d}{4e}+x\right)+\frac{\left(\frac{d}{4e}+x\right)\sqrt{256ae^2+\frac{5d^4}{e}-96d^2e\left(\frac{d}{4e}+x\right)^2+256e^3\left(\frac{d}{4e}+x\right)^4}}{12\sqrt{2}}$$

$\Big\downarrow$ 27

$$\frac{\int \frac{\frac{5d^4}{e}-48e\left(\frac{d}{4e}+x\right)^2d^2+256ae^2}{\sqrt{\frac{5d^4}{e}-96e\left(\frac{d}{4e}+x\right)^2d^2+256e^3\left(\frac{d}{4e}+x\right)^4+256ae^2}}d\left(\frac{d}{4e}+x\right)}{6\sqrt{2}}+\frac{\left(\frac{d}{4e}+x\right)\sqrt{256ae^2+\frac{5d^4}{e}-96d^2e\left(\frac{d}{4e}+x\right)^2+256e^3\left(\frac{d}{4e}+x\right)^4}}{12\sqrt{2}}$$

$\Big\downarrow$ 1511

$$\frac{\frac{3d^2\sqrt{256ae^3+5d^4}\int \frac{1-\frac{16e^2\left(\frac{d}{4e}+x\right)^2}{\sqrt{5d^4+256ae^3}}}{\sqrt{\frac{5d^4}{e}-96e\left(\frac{d}{4e}+x\right)^2d^2+256e^3\left(\frac{d}{4e}+x\right)^4+256ae^2}}d\left(\frac{d}{4e}+x\right)}{e}+\frac{\left(-3d^2\sqrt{256ae^3+5d^4}+256ae^3+5d^4\right)\int \frac{1}{\sqrt{\frac{5d^4}{e}-96e\left(\frac{d}{4e}+x\right)^2d^2+25\ldots}}}{e}}{6\sqrt{2}}+\frac{\left(\frac{d}{4e}+x\right)\sqrt{256ae^2+\frac{5d^4}{e}-96d^2e\left(\frac{d}{4e}+x\right)^2+256e^3\left(\frac{d}{4e}+x\right)^4}}{12\sqrt{2}}$$

$\Big\downarrow$ 1416

$$\frac{\frac{3d^2\sqrt{256ae^3+5d^4}\int \frac{1-\frac{16e^2\left(\frac{d}{4e}+x\right)^2}{\sqrt{5d^4+256ae^3}}}{\sqrt{\frac{5d^4}{e}-96e\left(\frac{d}{4e}+x\right)^2d^2+256e^3\left(\frac{d}{4e}+x\right)^4+256ae^2}}d\left(\frac{d}{4e}+x\right)}{e}+\frac{\sqrt[4]{256ae^3+5d^4}\left(-3d^2\sqrt{256ae^3+5d^4}+256ae^3+5d^4\right)\left(\frac{16e^2}{\sqrt{256\ldots}}\ldots\right.}{\ldots}}{6\sqrt{2}}+\frac{\left(\frac{d}{4e}+x\right)\sqrt{256ae^2+\frac{5d^4}{e}-96d^2e\left(\frac{d}{4e}+x\right)^2+256e^3\left(\frac{d}{4e}+x\right)^4}}{12\sqrt{2}}$$

$$\Big\downarrow \; 1509$$

$$\frac{\sqrt{\frac{5d^4}{e}-96e\left(\frac{d}{4e}+x\right)^2d^2+256e^3\left(\frac{d}{4e}+x\right)^4+256ae^2}\left(\frac{d}{4e}+x\right)}{12\sqrt{2}}+\frac{3\sqrt{5d^4+256ae^3}\left(\frac{\sqrt[4]{5d^4+256ae^3}\left(\frac{16e^2\left(\frac{d}{4e}+x\right)^2}{\sqrt{5d^4+256ae^3}}+1\right)\sqrt{\frac{5d^4-96e^2\left(\frac{d}{4e}+x\right)^2d^2+256e^4\left(\frac{d}{4e}+x\right)^4+256ae^3}{\left(5d^4+256ae^3\right)\left(\frac{16e^2\left(\frac{d}{4e}+x\right)^2}{\sqrt{5d^4+256ae^3}}+1\right)^2}}E\left(2\arctan\left(\frac{4e\left(\frac{d}{4e}+x\right)}{\sqrt[4]{5d^4+256ae^3}}\right)|\frac{1}{2}\left(\cdots\right.\right.}{4e\sqrt{\frac{5d^4}{e}-96e\left(\frac{d}{4e}+x\right)^2d^2+256e^3\left(\frac{d}{4e}+x\right)^4+256ae^2}}\cdots\right)}{e}\cdots$$

input
```
Int[Sqrt[8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4],x]
```

output
```
((d/(4*e) + x)*Sqrt[(5*d^4)/e + 256*a*e^2 - 96*d^2*e*(d/(4*e) + x)^2 + 256
*e^3*(d/(4*e) + x)^4])/(12*Sqrt[2]) + ((3*d^2*Sqrt[5*d^4 + 256*a*e^3]*(-((
e*(d/(4*e) + x)*Sqrt[(5*d^4)/e + 256*a*e^2 - 96*d^2*e*(d/(4*e) + x)^2 + 25
6*e^3*(d/(4*e) + x)^4])/((5*d^4 + 256*a*e^3)*(1 + (16*e^2*(d/(4*e) + x)^2)
/Sqrt[5*d^4 + 256*a*e^3]))) + ((5*d^4 + 256*a*e^3)^(1/4)*(1 + (16*e^2*(d/(
4*e) + x)^2)/Sqrt[5*d^4 + 256*a*e^3])*Sqrt[(5*d^4 + 256*a*e^3 - 96*d^2*e^2
*(d/(4*e) + x)^2 + 256*e^4*(d/(4*e) + x)^4)/((5*d^4 + 256*a*e^3)*(1 + (16*
e^2*(d/(4*e) + x)^2)/Sqrt[5*d^4 + 256*a*e^3])^2)]*EllipticE[2*ArcTan[(4*e*
(d/(4*e) + x))/(5*d^4 + 256*a*e^3)^(1/4)], (1 + (3*d^2)/Sqrt[5*d^4 + 256*a
*e^3])/2])/(4*e*Sqrt[(5*d^4)/e + 256*a*e^2 - 96*d^2*e*(d/(4*e) + x)^2 + 25
6*e^3*(d/(4*e) + x)^4])))/e + ((5*d^4 + 256*a*e^3)^(1/4)*(5*d^4 + 256*a*e^
3 - 3*d^2*Sqrt[5*d^4 + 256*a*e^3])*(1 + (16*e^2*(d/(4*e) + x)^2)/Sqrt[5*d^
4 + 256*a*e^3])*Sqrt[(5*d^4 + 256*a*e^3 - 96*d^2*e^2*(d/(4*e) + x)^2 + 256
*e^4*(d/(4*e) + x)^4)/((5*d^4 + 256*a*e^3)*(1 + (16*e^2*(d/(4*e) + x)^2)/S
qrt[5*d^4 + 256*a*e^3])^2)]*EllipticF[2*ArcTan[(4*e*(d/(4*e) + x))/(5*d^4
+ 256*a*e^3)^(1/4)], (1 + (3*d^2)/Sqrt[5*d^4 + 256*a*e^3])/2])/(8*e^2*Sqrt
[(5*d^4)/e + 256*a*e^2 - 96*d^2*e*(d/(4*e) + x)^2 + 256*e^3*(d/(4*e) + x)^
4]))/(6*Sqrt[2])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1404
```
Int[((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*((a + b
*x^2 + c*x^4)^p/(4*p + 1)), x] + Simp[2*(p/(4*p + 1))   Int[(2*a + b*x^2)*(
a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*
c, 0] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 2458
```
Int[(Pn_)^(p_.), x_Symbol] :> With[{S = Coeff[Pn, x, Expon[Pn, x] - 1]/(Exp
on[Pn, x]*Coeff[Pn, x, Expon[Pn, x]])}, Subst[Int[ExpandToSum[Pn /. x -> x
- S, x]^p, x], x, x + S] /; BinomialQ[Pn /. x -> x - S, x] || (IntegerQ[Exp
on[Pn, x]/2] && TrinomialQ[Pn /. x -> x - S, x])] /; FreeQ[p, x] && PolyQ[P
n, x] && GtQ[Expon[Pn, x], 2] && NeQ[Coeff[Pn, x, Expon[Pn, x] - 1], 0]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 7886 vs. $2(556) = 1112$.

Time = 6.77 (sec) , antiderivative size = 7887, normalized size of antiderivative = 13.17

method	result	size
default	Expression too large to display	7887
elliptic	Expression too large to display	7887
risch	Expression too large to display	9561

input
```
int((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [F]

$$\int \sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}\,dx=\int \sqrt{8\,e^3x^4+8\,de^2x^3-d^3x+8\,ae^2}\,dx$$

input
```
integrate((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^(1/2),x, algorithm="fricas
")
```

output
```
integral(sqrt(8*e^3*x^4 + 8*d*e^2*x^3 - d^3*x + 8*a*e^2), x)
```

Sympy [F]

$$\int \sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}\,dx=\int \sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}\,dx$$

input
```
integrate((8*e**3*x**4+8*d*e**2*x**3-d**3*x+8*a*e**2)**(1/2),x)
```

output
```
Integral(sqrt(8*a*e**2 - d**3*x + 8*d*e**2*x**3 + 8*e**3*x**4), x)
```

Maxima [F]

$$\int \sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}\,dx=\int \sqrt{8\,e^3x^4+8\,de^2x^3-d^3x+8\,ae^2}\,dx$$

input
```
integrate((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^(1/2),x, algorithm="maxima
")
```

output
```
integrate(sqrt(8*e^3*x^4 + 8*d*e^2*x^3 - d^3*x + 8*a*e^2), x)
```

Giac [F]

$$\int \sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}\,dx=\int \sqrt{8\,e^3x^4+8\,de^2x^3-d^3x+8\,ae^2}\,dx$$

input
```
integrate((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(8*e^3*x^4 + 8*d*e^2*x^3 - d^3*x + 8*a*e^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}\,dx=\int \sqrt{-d^3\,x+8\,d\,e^2\,x^3+8\,e^3\,x^4+8\,a\,e^2}\,dx$$

input
```
int((8*a*e^2 - d^3*x + 8*e^3*x^4 + 8*d*e^2*x^3)^(1/2),x)
```

output
```
int((8*a*e^2 - d^3*x + 8*e^3*x^4 + 8*d*e^2*x^3)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}\,dx$$
$$=\frac{4\sqrt{8e^3x^4+8d\,e^2x^3-d^3x+8a\,e^2}\,de+16\sqrt{8e^3x^4+8d\,e^2x^3-d^3x+8a\,e^2}\,e^2x-3\sqrt{e}\,\sqrt{2}\log\left(-2\sqrt{e}\,\sqrt{8e^3}\right.}{}$$

input
```
int((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^(1/2),x)
```

output
```
(4*sqrt(8*a*e**2 - d**3*x + 8*d*e**2*x**3 + 8*e**3*x**4)*d*e + 16*sqrt(8*a
*e**2 - d**3*x + 8*d*e**2*x**3 + 8*e**3*x**4)*e**2*x - 3*sqrt(e)*sqrt(2)*l
og( - 2*sqrt(e)*sqrt(8*a*e**2 - d**3*x + 8*d*e**2*x**3 + 8*e**3*x**4)*sqrt
(2) + d**2 - 4*d*e*x - 8*e**2*x**2)*d**3 - 3*sqrt(e)*sqrt(2)*log(2*sqrt(e)
*sqrt(8*a*e**2 - d**3*x + 8*d*e**2*x**3 + 8*e**3*x**4)*sqrt(2) + d**2 - 4*
d*e*x - 8*e**2*x**2)*d**3 + 256*int(sqrt(8*a*e**2 - d**3*x + 8*d*e**2*x**3
 + 8*e**3*x**4)/(8*a*e**2 - d**3*x + 8*d*e**2*x**3 + 8*e**3*x**4),x)*a*e**
4 + 2*int(sqrt(8*a*e**2 - d**3*x + 8*d*e**2*x**3 + 8*e**3*x**4)/(8*a*e**2
- d**3*x + 8*d*e**2*x**3 + 8*e**3*x**4),x)*d**4*e - 48*int((sqrt(8*a*e**2
- d**3*x + 8*d*e**2*x**3 + 8*e**3*x**4)*x**2)/(8*a*e**2 - d**3*x + 8*d*e**
2*x**3 + 8*e**3*x**4),x)*d**2*e**3 - 24*int((sqrt(8*a*e**2 - d**3*x + 8*d*
e**2*x**3 + 8*e**3*x**4)*x)/(8*a*e**2 - d**3*x + 8*d*e**2*x**3 + 8*e**3*x*
*4),x)*d**3*e**2)/(48*e**2)
```

3.66 $\int \frac{1}{\sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 219

$$\int \frac{1}{\sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}}\,dx = \frac{\sqrt[4]{5d^4+256ae^3}\left(1+\frac{(d+4ex)^2}{\sqrt{5d^4+256ae^3}}\right)\sqrt{\frac{5d^4+256ae^3-6d^2(d+4ex)^2+(d+4ex)^4}{(5d^4+256ae^3)\left(1+\frac{(d+4ex)^2}{\sqrt{5d^4+256ae^3}}\right)^2}}\,\mathrm{EllipticF}\left(2\arctan\left(\frac{d+4ex}{\sqrt[4]{5d^4+256ae^3}}\right)\right.}{8e\sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}}$$

output
```
1/8*(256*a*e^3+5*d^4)^(1/4)*(1+(4*e*x+d)^2/(256*a*e^3+5*d^4)^(1/2))*((5*d^
4+256*a*e^3-6*d^2*(4*e*x+d)^2+(4*e*x+d)^4)/(256*a*e^3+5*d^4)/(1+(4*e*x+d)^
2/(256*a*e^3+5*d^4)^(1/2))^2)^(1/2)*InverseJacobiAM(2*arctan((4*e*x+d)/(25
6*a*e^3+5*d^4)^(1/4)),1/2*(2+6*d^2/(256*a*e^3+5*d^4)^(1/2))^(1/2))/e/(8*e^
3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^(1/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1065 vs. $2(219) = 438$.

Time = 12.39 (sec) , antiderivative size = 1065, normalized size of antiderivative = 4.86

$$\int \frac{1}{\sqrt{8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4}}\,dx =$$

$$-\frac{\left(-d+\sqrt{3d^2-2\sqrt{d^4-64ae^3}}-4ex\right)\left(d-\sqrt{3d^2+2\sqrt{d^4-64ae^3}}+4ex\right)\sqrt{-\frac{\sqrt{3d^2-2\sqrt{d^4-6}}}{\left(\sqrt{3d^2-2\sqrt{d^4-64ae^3}}-\sqrt{3d^2}\right.}}}{}$$

input
```
Integrate[1/Sqrt[8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4],x]
```

output
```
-1/2*((-d + Sqrt[3*d^2 - 2*Sqrt[d^4 - 64*a*e^3]] - 4*e*x)*(d - Sqrt[3*d^2
+ 2*Sqrt[d^4 - 64*a*e^3]] + 4*e*x)*Sqrt[-((Sqrt[3*d^2 - 2*Sqrt[d^4 - 64*a*
e^3]]*(d + Sqrt[3*d^2 + 2*Sqrt[d^4 - 64*a*e^3]] + 4*e*x))/((Sqrt[3*d^2 - 2
*Sqrt[d^4 - 64*a*e^3]] - Sqrt[3*d^2 + 2*Sqrt[d^4 - 64*a*e^3]])*(-d + Sqrt[
3*d^2 - 2*Sqrt[d^4 - 64*a*e^3]] - 4*e*x)))]*Sqrt[(3*d^2 - 2*Sqrt[d^4 - 64*
a*e^3] - Sqrt[3*d^2 - 2*Sqrt[d^4 - 64*a*e^3]]*Sqrt[3*d^2 + 2*Sqrt[d^4 - 64
*a*e^3]] + d*(Sqrt[3*d^2 - 2*Sqrt[d^4 - 64*a*e^3]] - Sqrt[3*d^2 + 2*Sqrt[d
^4 - 64*a*e^3]]) + 4*e*(Sqrt[3*d^2 - 2*Sqrt[d^4 - 64*a*e^3]] - Sqrt[3*d^2
+ 2*Sqrt[d^4 - 64*a*e^3]])*x)/((Sqrt[3*d^2 - 2*Sqrt[d^4 - 64*a*e^3]] + Sqr
t[3*d^2 + 2*Sqrt[d^4 - 64*a*e^3]])*(-d + Sqrt[3*d^2 - 2*Sqrt[d^4 - 64*a*e^
3]] - 4*e*x))]*EllipticF[ArcSin[Sqrt[((Sqrt[3*d^2 - 2*Sqrt[d^4 - 64*a*e^3]
] - Sqrt[3*d^2 + 2*Sqrt[d^4 - 64*a*e^3]])*(d + Sqrt[3*d^2 - 2*Sqrt[d^4 - 6
4*a*e^3]] + 4*e*x))/((Sqrt[3*d^2 - 2*Sqrt[d^4 - 64*a*e^3]] + Sqrt[3*d^2 +
2*Sqrt[d^4 - 64*a*e^3]])*(-d + Sqrt[3*d^2 - 2*Sqrt[d^4 - 64*a*e^3]] - 4*e*
x))]], (Sqrt[3*d^2 - 2*Sqrt[d^4 - 64*a*e^3]] + Sqrt[3*d^2 + 2*Sqrt[d^4 - 6
4*a*e^3]])^2/(Sqrt[3*d^2 - 2*Sqrt[d^4 - 64*a*e^3]] - Sqrt[3*d^2 + 2*Sqrt[d
^4 - 64*a*e^3]])^2])/(e*(Sqrt[3*d^2 - 2*Sqrt[d^4 - 64*a*e^3]] - Sqrt[3*d^2
 + 2*Sqrt[d^4 - 64*a*e^3]])*Sqrt[(Sqrt[3*d^2 - 2*Sqrt[d^4 - 64*a*e^3]]*(-d
 + Sqrt[3*d^2 + 2*Sqrt[d^4 - 64*a*e^3]] - 4*e*x))/((Sqrt[3*d^2 - 2*Sqrt[d^
4 - 64*a*e^3]] + Sqrt[3*d^2 + 2*Sqrt[d^4 - 64*a*e^3]])*(-d + Sqrt[3*d^2...
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.27, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {2458, 1416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}}dx$$

$$\Big\downarrow \text{2458}$$

$$\int \frac{1}{\sqrt{\frac{1}{32}\left(256ae^2+\frac{5d^4}{e}\right)-3d^2e\left(\frac{d}{4e}+x\right)^2+8e^3\left(\frac{d}{4e}+x\right)^4}}d\left(\frac{d}{4e}+x\right)$$

$$\Big\downarrow \text{1416}$$

$$\frac{\sqrt[4]{256ae^3+5d^4}\left(\frac{16e^2\left(\frac{d}{4e}+x\right)^2}{\sqrt{256ae^3+5d^4}}+1\right)\sqrt{\frac{256ae^3+5d^4-96d^2e^2\left(\frac{d}{4e}+x\right)^2+256e^4\left(\frac{d}{4e}+x\right)^4}{(256ae^3+5d^4)\left(\frac{16e^2\left(\frac{d}{4e}+x\right)^2}{\sqrt{256ae^3+5d^4}}+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{4e\left(\frac{d}{4e}+x\right)}{\sqrt[4]{5d^4+256a}\right.\right.}{\sqrt{2}e\sqrt{256ae^2+\frac{5d^4}{e}-96d^2e\left(\frac{d}{4e}+x\right)^2+256e^3\left(\frac{d}{4e}+x\right)^4}}$$

input
```
Int[1/Sqrt[8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4],x]
```

output
```
((5*d^4 + 256*a*e^3)^(1/4)*(1 + (16*e^2*(d/(4*e) + x)^2)/Sqrt[5*d^4 + 256*
a*e^3])*Sqrt[(5*d^4 + 256*a*e^3 - 96*d^2*e^2*(d/(4*e) + x)^2 + 256*e^4*(d/
(4*e) + x)^4)/((5*d^4 + 256*a*e^3)*(1 + (16*e^2*(d/(4*e) + x)^2)/Sqrt[5*d^
4 + 256*a*e^3])^2)]*EllipticF[2*ArcTan[(4*e*(d/(4*e) + x))/(5*d^4 + 256*a*
e^3)^(1/4)], (1 + (3*d^2)/Sqrt[5*d^4 + 256*a*e^3])/2])/(Sqrt[2]*e*Sqrt[(5*
d^4)/e + 256*a*e^2 - 96*d^2*e*(d/(4*e) + x)^2 + 256*e^3*(d/(4*e) + x)^4])
```

Defintions of rubi rules used

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 2458
```
Int[(Pn_)^(p_.), x_Symbol] :> With[{S = Coeff[Pn, x, Expon[Pn, x] - 1]/(Exp
on[Pn, x]*Coeff[Pn, x, Expon[Pn, x]])}, Subst[Int[ExpandToSum[Pn /. x -> x
- S, x]^p, x], x, x + S] /; BinomialQ[Pn /. x -> x - S, x] || (IntegerQ[Exp
on[Pn, x]/2] && TrinomialQ[Pn /. x -> x - S, x])] /; FreeQ[p, x] && PolyQ[P
n, x] && GtQ[Expon[Pn, x], 2] && NeQ[Coeff[Pn, x, Expon[Pn, x] - 1], 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1703 vs. $2(203) = 406$.

Time = 0.69 (sec) , antiderivative size = 1704, normalized size of antiderivative = 7.78

method	result	size
default	Expression too large to display	1704
elliptic	Expression too large to display	1704

input
```
int(1/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
1/2*(1/4*(-d*e+(3*e^2*d^2+2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2+1/4*(d*e
+(3*e^2*d^2-2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2)*((-1/4*(d*e+(3*e^2*d^
2-2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2+1/4*(d*e+(3*e^2*d^2+2*(-64*a*e^3
+d^4)^(1/2)*e^2)^(1/2))/e^2)*(x-1/4*(-d*e+(3*e^2*d^2+2*(-64*a*e^3+d^4)^(1/
2)*e^2)^(1/2))/e^2)/(-1/4*(d*e+(3*e^2*d^2-2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/
2))/e^2-1/4*(-d*e+(3*e^2*d^2+2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2)/(x+1
/4*(d*e+(3*e^2*d^2+2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2))^(1/2)*(x+1/4*
(d*e+(3*e^2*d^2+2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2)^2*((-1/4*(d*e+(3*
e^2*d^2+2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2-1/4*(-d*e+(3*e^2*d^2+2*(-6
4*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2)*(x-1/4*(-d*e+(3*e^2*d^2-2*(-64*a*e^3+d
^4)^(1/2)*e^2)^(1/2))/e^2)/(1/4*(-d*e+(3*e^2*d^2-2*(-64*a*e^3+d^4)^(1/2)*e
^2)^(1/2))/e^2-1/4*(-d*e+(3*e^2*d^2+2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^
2)/(x+1/4*(d*e+(3*e^2*d^2+2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2))^(1/2)*
((-1/4*(d*e+(3*e^2*d^2+2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2-1/4*(-d*e+(
3*e^2*d^2+2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2)*(x+1/4*(d*e+(3*e^2*d^2-
2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2)/(-1/4*(d*e+(3*e^2*d^2-2*(-64*a*e^
3+d^4)^(1/2)*e^2)^(1/2))/e^2-1/4*(-d*e+(3*e^2*d^2+2*(-64*a*e^3+d^4)^(1/2)*
e^2)^(1/2))/e^2)/(x+1/4*(d*e+(3*e^2*d^2+2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2)
)/e^2))^(1/2)/(-1/4*(d*e+(3*e^2*d^2-2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^
2+1/4*(d*e+(3*e^2*d^2+2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2)/(-1/4*(d...
```

Fricas [F]

$$\int \frac{1}{\sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}}\,dx=\int \frac{1}{\sqrt{8\,e^3x^4+8\,de^2x^3-d^3x+8\,ae^2}}\,dx$$

input

```
integrate(1/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^(1/2),x, algorithm="fric
as")
```

output

```
integral(1/sqrt(8*e^3*x^4 + 8*d*e^2*x^3 - d^3*x + 8*a*e^2), x)
```

Sympy [F]

$$\int \frac{1}{\sqrt{8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4}}\,dx = \int \frac{1}{\sqrt{8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4}}\,dx$$

input
```
integrate(1/(8*e**3*x**4+8*d*e**2*x**3-d**3*x+8*a*e**2)**(1/2),x)
```

output
```
Integral(1/sqrt(8*a*e**2 - d**3*x + 8*d*e**2*x**3 + 8*e**3*x**4), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4}}\,dx = \int \frac{1}{\sqrt{8\,e^3x^4 + 8\,de^2x^3 - d^3x + 8\,ae^2}}\,dx$$

input
```
integrate(1/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^(1/2),x, algorithm="maxi
ma")
```

output
```
integrate(1/sqrt(8*e^3*x^4 + 8*d*e^2*x^3 - d^3*x + 8*a*e^2), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4}}\,dx = \int \frac{1}{\sqrt{8\,e^3x^4 + 8\,de^2x^3 - d^3x + 8\,ae^2}}\,dx$$

input
```
integrate(1/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^(1/2),x, algorithm="giac
")
```

output
```
integrate(1/sqrt(8*e^3*x^4 + 8*d*e^2*x^3 - d^3*x + 8*a*e^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4}}\,dx = \int \frac{1}{\sqrt{-d^3\,x + 8\,d\,e^2\,x^3 + 8\,e^3\,x^4 + 8\,a\,e^2}}\,dx$$

input
```
int(1/(8*a*e^2 - d^3*x + 8*e^3*x^4 + 8*d*e^2*x^3)^(1/2),x)
```

output
```
int(1/(8*a*e^2 - d^3*x + 8*e^3*x^4 + 8*d*e^2*x^3)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4}}\,dx = \int \frac{\sqrt{8e^3x^4 + 8d\,e^2x^3 - d^3x + 8a\,e^2}}{8e^3x^4 + 8d\,e^2x^3 - d^3x + 8a\,e^2}dx$$

input
```
int(1/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^(1/2),x)
```

output
```
int(sqrt(8*a*e**2 - d**3*x + 8*d*e**2*x**3 + 8*e**3*x**4)/(8*a*e**2 - d**3
*x + 8*d*e**2*x**3 + 8*e**3*x**4),x)
```

3.67 $\int \frac{1}{\left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)^{3/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 680

$$\int \frac{1}{\left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)^{3/2}}\,dx=\frac{96d^2e(d+4ex)\sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}}{\left(5d^8-64ad^4e^3-16384a^2e^6\right)\left(\sqrt{5d^4+256ae^3}+(d+4ex)^2\right)}$$

$$+\frac{(d+4ex)\left(13d^4-256ae^3-3d^2(d+4ex)^2\right)}{\left(5d^8-64ad^4e^3-16384a^2e^6\right)\sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}}$$

$$-\frac{3d^2\left(1+\frac{(d+4ex)^2}{\sqrt{5d^4+256ae^3}}\right)\sqrt{\frac{5d^4+256ae^3-6d^2(d+4ex)^2+(d+4ex)^4}{\left(5d^4+256ae^3\right)\left(1+\frac{(d+4ex)^2}{\sqrt{5d^4+256ae^3}}\right)^2}}E\left(2\arctan\left(\frac{d+4ex}{\sqrt[4]{5d^4+256ae^3}}\right)|\frac{1}{2}\left(1+\frac{3d^2}{\sqrt{5d^4+256ae^3}}\right.\right.}{\left(d^4-64ae^3\right)\sqrt[4]{5d^4+256ae^3}\sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}}$$

$$-\frac{\left(5d^4+256ae^3-3d^2\sqrt{5d^4+256ae^3}\right)\left(1+\frac{(d+4ex)^2}{\sqrt{5d^4+256ae^3}}\right)\sqrt{\frac{5d^4+256ae^3-6d^2(d+4ex)^2+(d+4ex)^4}{\left(5d^4+256ae^3\right)\left(1+\frac{(d+4ex)^2}{\sqrt{5d^4+256ae^3}}\right)^2}}\text{EllipticF}\left(2\,\text{arct}\right.}{2\left(d^4-64ae^3\right)\left(5d^4+256ae^3\right)^{3/4}\sqrt{8ae^2-d^3x+8de^2x^3+8e^3}}$$

output
```
96*d^2*e*(4*e*x+d)*(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^(1/2)/(-16384*a^2
*e^6-64*a*d^4*e^3+5*d^8)/((256*a*e^3+5*d^4)^(1/2)+(4*e*x+d)^2)+(4*e*x+d)*(
13*d^4-256*a*e^3-3*d^2*(4*e*x+d)^2)/(-16384*a^2*e^6-64*a*d^4*e^3+5*d^8)/(8
*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^(1/2)-3*d^2*(1+(4*e*x+d)^2/(256*a*e^3+
5*d^4)^(1/2))*((5*d^4+256*a*e^3-6*d^2*(4*e*x+d)^2+(4*e*x+d)^4)/(256*a*e^3+
5*d^4)/(1+(4*e*x+d)^2/(256*a*e^3+5*d^4)^(1/2))^2)^(1/2)*EllipticE(sin(2*ar
ctan((4*e*x+d)/(256*a*e^3+5*d^4)^(1/4))),1/2*(2+6*d^2/(256*a*e^3+5*d^4)^(1
/2))^(1/2))/(-64*a*e^3+d^4)/(256*a*e^3+5*d^4)^(1/4)/(8*e^3*x^4+8*d*e^2*x^3
-d^3*x+8*a*e^2)^(1/2)-1/2*(5*d^4+256*a*e^3-3*d^2*(256*a*e^3+5*d^4)^(1/2))*
(1+(4*e*x+d)^2/(256*a*e^3+5*d^4)^(1/2))*((5*d^4+256*a*e^3-6*d^2*(4*e*x+d)^
2+(4*e*x+d)^4)/(256*a*e^3+5*d^4)/(1+(4*e*x+d)^2/(256*a*e^3+5*d^4)^(1/2))^2
)^(1/2)*InverseJacobiAM(2*arctan((4*e*x+d)/(256*a*e^3+5*d^4)^(1/4)),1/2*(2
+6*d^2/(256*a*e^3+5*d^4)^(1/2))^(1/2))/(-64*a*e^3+d^4)/(256*a*e^3+5*d^4)^(
3/4)/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 7629 vs. $2(680) = 1360$.

Time = 16.18 (sec) , antiderivative size = 7629, normalized size of antiderivative = 11.22

$$\int \frac{1}{(8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4)^{3/2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)^(-3/2),x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 1.37 (sec) , antiderivative size = 882, normalized size of antiderivative = 1.30, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {2458, 1405, 27, 1511, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(8ae^2-d^3x+8de^2x^3+8e^3x^4)^{3/2}}dx$$

$$\Big\downarrow \text{2458}$$

$$\int \frac{1}{\left(\frac{1}{32}\left(256ae^2+\frac{5d^4}{e}\right)-3d^2e\left(\frac{d}{4e}+x\right)^2+8e^3\left(\frac{d}{4e}+x\right)^4\right)^{3/2}}d\left(\frac{d}{4e}+x\right)$$

$$\Big\downarrow \text{1405}$$

$$\frac{16\sqrt{2}e\left(\frac{d}{4e}+x\right)\left(-256ae^3+13d^4-48d^2e^2\left(\frac{d}{4e}+x\right)^2\right)}{(-16384a^2e^6-64ad^4e^3+5d^8)\sqrt{256ae^2+\frac{5d^4}{e}-96d^2e\left(\frac{d}{4e}+x\right)^2+256e^3\left(\frac{d}{4e}+x\right)^4}}-\frac{8\int \frac{2\sqrt{2}e^2\left(5d^4-48e^2\left(\frac{d}{4e}+x\right)^2d^2+256ae^3\right)}{\sqrt{\frac{5d^4}{e}-96e\left(\frac{d}{4e}+x\right)^2d^2+256e^3\left(\frac{d}{4e}+x\right)^4+256ae^2}}d\left(\frac{d}{4e}+x\right)}{e\left(-16384a^2e^6-64ad^4e^3+5d^8\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{16\sqrt{2}e\left(\frac{d}{4e}+x\right)\left(-256ae^3+13d^4-48d^2e^2\left(\frac{d}{4e}+x\right)^2\right)}{(-16384a^2e^6-64ad^4e^3+5d^8)\sqrt{256ae^2+\frac{5d^4}{e}-96d^2e\left(\frac{d}{4e}+x\right)^2+256e^3\left(\frac{d}{4e}+x\right)^4}}-\frac{16\sqrt{2}e\int \frac{5d^4-48e^2\left(\frac{d}{4e}+x\right)^2d^2+256ae^3}{\sqrt{\frac{5d^4}{e}-96e\left(\frac{d}{4e}+x\right)^2d^2+256e^3\left(\frac{d}{4e}+x\right)^4+256ae^2}}d\left(\frac{d}{4e}+x\right)}{-16384a^2e^6-64ad^4e^3+5d^8}$$

$$\Big\downarrow \text{1511}$$

$$\frac{16\sqrt{2}e\left(\frac{d}{4e}+x\right)\left(-256ae^3+13d^4-48d^2e^2\left(\frac{d}{4e}+x\right)^2\right)}{(-16384a^2e^6-64ad^4e^3+5d^8)\sqrt{256ae^2+\frac{5d^4}{e}-96d^2e\left(\frac{d}{4e}+x\right)^2+256e^3\left(\frac{d}{4e}+x\right)^4}}-\frac{16\sqrt{2}e\left(3d^2\sqrt{256ae^3+5d^4}\int \frac{1-\frac{16e^2\left(\frac{d}{4e}+x\right)^2}{\sqrt{5d^4+256ae^3}}}{\sqrt{\frac{5d^4}{e}-96e\left(\frac{d}{4e}+x\right)^2d^2+256e^3\left(\frac{d}{4e}+x\right)^4+256ae^2}}d\left(\frac{d}{4e}+x\right)-\sqrt{256ae^3+5d^4}\left(3d^2-\sqrt{256ae\right.\right.}{-16384a^2e^6-64ad^4e^3+5d^8}$$

$$\Big\downarrow \text{1416}$$

$$\frac{16\sqrt{2}e\left(\frac{d}{4e}+x\right)\left(-256ae^3+13d^4-48d^2e^2\left(\frac{d}{4e}+x\right)^2\right)}{\left(-16384a^2e^6-64ad^4e^3+5d^8\right)\sqrt{256ae^2+\frac{5d^4}{e}-96d^2e\left(\frac{d}{4e}+x\right)^2+256e^3\left(\frac{d}{4e}+x\right)^4}}-$$

$$\frac{16\sqrt{2}e\left(3d^2\sqrt{256ae^3+5d^4}\int\frac{1-\frac{16e^2\left(\frac{d}{4e}+x\right)^2}{\sqrt{5d^4+256ae^3}}}{\sqrt{\frac{5d^4}{e}-96e\left(\frac{d}{4e}+x\right)^2d^2+256e^3\left(\frac{d}{4e}+x\right)^4+256ae^2}}d\left(\frac{d}{4e}+x\right)-\frac{\left(256ae^3+5d^4\right)^{3/4}\left(3d^2-\sqrt{256ae^3+5d^4}\right.}{}\right.}{-16384a^2e^6-6}$$

$\downarrow$ 1509

$$\frac{16\sqrt{2}e\left(\frac{d}{4e}+x\right)\left(13d^4-48e^2\left(\frac{d}{4e}+x\right)^2d^2-256ae^3\right)}{\left(5d^8-64ae^3d^4-16384a^2e^6\right)\sqrt{\frac{5d^4}{e}-96e\left(\frac{d}{4e}+x\right)^2d^2+256e^3\left(\frac{d}{4e}+x\right)^4+256ae^2}}-$$

$$\frac{16\sqrt{2}e\left(3d^2\sqrt{5d^4+256ae^3}\left(\frac{\sqrt[4]{5d^4+256ae^3}\left(\frac{16e^2\left(\frac{d}{4e}+x\right)^2}{\sqrt{5d^4+256ae^3}}+1\right)\sqrt{\frac{5d^4-96e^2\left(\frac{d}{4e}+x\right)^2d^2+256e^4\left(\frac{d}{4e}+x\right)^4+256ae^3}{\left(5d^4+256ae^3\right)\left(\frac{16e^2\left(\frac{d}{4e}+x\right)^2}{\sqrt{5d^4+256ae^3}}+1\right)^2}}E\left(2\arctan\left(\frac{}{\sqrt[4]{5}\cdots}\right.\right.}{4e\sqrt{\frac{5d^4}{e}-96e\left(\frac{d}{4e}+x\right)^2d^2+256e^3\left(\frac{d}{4e}+x\right)^4+256ae^2}}\right.\right.}{}$$

input
```
Int[(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)^(-3/2),x]
```

output
```
(16*Sqrt[2]*e*(d/(4*e) + x)*(13*d^4 - 256*a*e^3 - 48*d^2*e^2*(d/(4*e) + x)
^2))/((5*d^8 - 64*a*d^4*e^3 - 16384*a^2*e^6)*Sqrt[(5*d^4)/e + 256*a*e^2 -
96*d^2*e*(d/(4*e) + x)^2 + 256*e^3*(d/(4*e) + x)^4]) - (16*Sqrt[2]*e*(3*d^
2*Sqrt[5*d^4 + 256*a*e^3]*(-((e*(d/(4*e) + x)*Sqrt[(5*d^4)/e + 256*a*e^2 -
 96*d^2*e*(d/(4*e) + x)^2 + 256*e^3*(d/(4*e) + x)^4])/((5*d^4 + 256*a*e^3)
*(1 + (16*e^2*(d/(4*e) + x)^2)/Sqrt[5*d^4 + 256*a*e^3]))) + ((5*d^4 + 256*
a*e^3)^(1/4)*(1 + (16*e^2*(d/(4*e) + x)^2)/Sqrt[5*d^4 + 256*a*e^3])*Sqrt[(
5*d^4 + 256*a*e^3 - 96*d^2*e^2*(d/(4*e) + x)^2 + 256*e^4*(d/(4*e) + x)^4)/
((5*d^4 + 256*a*e^3)*(1 + (16*e^2*(d/(4*e) + x)^2)/Sqrt[5*d^4 + 256*a*e^3]
)^2)]*EllipticE[2*ArcTan[(4*e*(d/(4*e) + x))/(5*d^4 + 256*a*e^3)^(1/4)], (
1 + (3*d^2)/Sqrt[5*d^4 + 256*a*e^3])/2])/(4*e*Sqrt[(5*d^4)/e + 256*a*e^2 -
 96*d^2*e*(d/(4*e) + x)^2 + 256*e^3*(d/(4*e) + x)^4])) - ((5*d^4 + 256*a*e
^3)^(3/4)*(3*d^2 - Sqrt[5*d^4 + 256*a*e^3])*(1 + (16*e^2*(d/(4*e) + x)^2)/
Sqrt[5*d^4 + 256*a*e^3])*Sqrt[(5*d^4 + 256*a*e^3 - 96*d^2*e^2*(d/(4*e) + x
)^2 + 256*e^4*(d/(4*e) + x)^4)/((5*d^4 + 256*a*e^3)*(1 + (16*e^2*(d/(4*e)
+ x)^2)/Sqrt[5*d^4 + 256*a*e^3])^2)]*EllipticF[2*ArcTan[(4*e*(d/(4*e) + x)
)/(5*d^4 + 256*a*e^3)^(1/4)], (1 + (3*d^2)/Sqrt[5*d^4 + 256*a*e^3])/2])/(8
*e*Sqrt[(5*d^4)/e + 256*a*e^2 - 96*d^2*e*(d/(4*e) + x)^2 + 256*e^3*(d/(4*e
) + x)^4])))/(5*d^8 - 64*a*d^4*e^3 - 16384*a^2*e^6)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 2458
```
Int[(Pn_)^(p_.), x_Symbol] :> With[{S = Coeff[Pn, x, Expon[Pn, x] - 1]/(Exp
on[Pn, x]*Coeff[Pn, x, Expon[Pn, x]])}, Subst[Int[ExpandToSum[Pn /. x -> x
- S, x]^p, x], x, x + S] /; BinomialQ[Pn /. x -> x - S, x] || (IntegerQ[Exp
on[Pn, x]/2] && TrinomialQ[Pn /. x -> x - S, x])] /; FreeQ[p, x] && PolyQ[P
n, x] && GtQ[Expon[Pn, x], 2] && NeQ[Coeff[Pn, x, Expon[Pn, x] - 1], 0]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 8102 vs. $2(643) = 1286$.

Time = 0.79 (sec) , antiderivative size = 8103, normalized size of antiderivative = 11.92

method	result	size
default	Expression too large to display	8103
elliptic	Expression too large to display	8103

input
```
int(1/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [F]

$$\int \frac{1}{(8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4)^{3/2}}\,dx = \int \frac{1}{(8\,e^3x^4 + 8\,de^2x^3 - d^3x + 8\,ae^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^(3/2),x, algorithm="fric
as")
```

output
```
integral(sqrt(8*e^3*x^4 + 8*d*e^2*x^3 - d^3*x + 8*a*e^2)/(64*e^6*x^8 + 128
*d*e^5*x^7 + 64*d^2*e^4*x^6 - 16*d^3*e^3*x^5 + 128*a*d*e^4*x^3 + d^6*x^2 -
 16*a*d^3*e^2*x + 64*a^2*e^4 - 16*(d^4*e^2 - 8*a*e^5)*x^4), x)
```

Sympy [F]

$$\int \frac{1}{(8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4)^{3/2}}\,dx = \int \frac{1}{(8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(8*e**3*x**4+8*d*e**2*x**3-d**3*x+8*a*e**2)**(3/2),x)
```

output
```
Integral((8*a*e**2 - d**3*x + 8*d*e**2*x**3 + 8*e**3*x**4)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4)^{3/2}}\,dx = \int \frac{1}{(8\,e^3x^4 + 8\,de^2x^3 - d^3x + 8\,ae^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^(3/2),x, algorithm="maxi
ma")
```

output
```
integrate((8*e^3*x^4 + 8*d*e^2*x^3 - d^3*x + 8*a*e^2)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{\left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)^{3/2}}\,dx=\int \frac{1}{\left(8\,e^3x^4+8\,de^2x^3-d^3x+8\,ae^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^(3/2),x, algorithm="giac
")
```

output
```
integrate((8*e^3*x^4 + 8*d*e^2*x^3 - d^3*x + 8*a*e^2)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)^{3/2}}\,dx=\int \frac{1}{\left(-d^3\,x+8\,d\,e^2\,x^3+8\,e^3\,x^4+8\,a\,e^2\right)^{3/2}}\,dx$$

input
```
int(1/(8*a*e^2 - d^3*x + 8*e^3*x^4 + 8*d*e^2*x^3)^(3/2),x)
```

output
```
int(1/(8*a*e^2 - d^3*x + 8*e^3*x^4 + 8*d*e^2*x^3)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{\left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)^{3/2}}\,dx=\int \frac{\sqrt{8e^3x^4+8d\,e^2x^3-d}}{64e^6x^8+128d\,e^5x^7+64d^2e^4x^6-16d^3e^3x^5+128a\,e^5x^4-16}$$

input
```
int(1/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^(3/2),x)
```

output
```
int(sqrt(8*a*e**2 - d**3*x + 8*d*e**2*x**3 + 8*e**3*x**4)/(64*a**2*e**4 -
16*a*d**3*e**2*x + 128*a*d*e**4*x**3 + 128*a*e**5*x**4 + d**6*x**2 - 16*d*
*4*e**2*x**4 - 16*d**3*e**3*x**5 + 64*d**2*e**4*x**6 + 128*d*e**5*x**7 + 6
4*e**6*x**8),x)
```

3.68 $\int (a+8x-8x^2+4x^3-x^4)^{3/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 402

$$\int (a+8x-8x^2+4x^3-x^4)^{3/2}\,dx =$$
$$-\frac{2}{35}\left(13+5a-3(1-x)^2\right)\sqrt{3+a-2(1-x)^2-(1-x)^4}(1-x)$$
$$-\frac{1}{7}\left(3+a-2(1-x)^2-(1-x)^4\right)^{3/2}(1-x)$$
$$+\frac{16(7+2a)\sqrt{-1+\sqrt{4+a}}(1+\sqrt{4+a})\sqrt{1+\frac{(1-x)^2}{1-\sqrt{4+a}}}\sqrt{1+\frac{(1-x)^2}{1+\sqrt{4+a}}}E\left(\arcsin\left(\frac{1-x}{\sqrt{-1+\sqrt{4+a}}}\right)|\frac{1-\sqrt{4+a}}{1+\sqrt{4+a}}\right)}{35\sqrt{3+a-2(1-x)^2-(1-x)^4}}$$
$$-\frac{4\sqrt{-1+\sqrt{4+a}}\left(76+5a^2+28\sqrt{4+a}+a\left(39+8\sqrt{4+a}\right)\right)\sqrt{1+\frac{(1-x)^2}{1-\sqrt{4+a}}}\sqrt{1+\frac{(1-x)^2}{1+\sqrt{4+a}}}\,\text{EllipticF}\left(\text{arcs}\right.}{35\sqrt{3+a-2(1-x)^2-(1-x)^4}}$$

output
```
-2/35*(13+5*a-3*(1-x)^2)*(3+a-2*(1-x)^2-(1-x)^4)^(1/2)*(1-x)-1/7*(3+a-2*(1
-x)^2-(1-x)^4)^(3/2)*(1-x)+16/35*(7+2*a)*(-1+(4+a)^(1/2))^(1/2)*(1+(4+a)^(
1/2))*(1+(1-x)^2/(1-(4+a)^(1/2)))^(1/2)*(1+(1-x)^2/(1+(4+a)^(1/2)))^(1/2)*
EllipticE((1-x)/(-1+(4+a)^(1/2))^(1/2),((1-(4+a)^(1/2))/(1+(4+a)^(1/2)))^(
1/2))/(3+a-2*(1-x)^2-(1-x)^4)^(1/2)-4/35*(-1+(4+a)^(1/2))^(1/2)*(76+5*a^2+
28*(4+a)^(1/2)+a*(39+8*(4+a)^(1/2)))*(1+(1-x)^2/(1-(4+a)^(1/2)))^(1/2)*(1+
(1-x)^2/(1+(4+a)^(1/2)))^(1/2)*EllipticF((1-x)/(-1+(4+a)^(1/2))^(1/2),((1-
(4+a)^(1/2))/(1+(4+a)^(1/2)))^(1/2))/(3+a-2*(1-x)^2-(1-x)^4)^(1/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 6287 vs. 2(402) = 804.

Time = 16.17 (sec) , antiderivative size = 6287, normalized size of antiderivative = 15.64

$$\int \left(a+8x-8x^2+4x^3-x^4\right)^{3/2} dx = \text{Result too large to show}$$

input
```
Integrate[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2),x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 1.10 (sec) , antiderivative size = 539, normalized size of antiderivative = 1.34, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {2458, 1404, 27, 1490, 27, 1514, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a-x^4+4x^3-8x^2+8x\right)^{3/2} dx$$

$$\Big\downarrow \text{2458}$$

$$\int \left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2} d(x-1)$$

$$\Big\downarrow \text{1404}$$

$$\frac{3}{7}\int 2\left(-(x-1)^2+a+3\right)\sqrt{-(x-1)^4-2(x-1)^2+a+3}d(x-1)+\frac{1}{7}(x-1)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{6}{7}\int \left(-(x-1)^2+a+3\right)\sqrt{-(x-1)^4-2(x-1)^2+a+3}d(x-1)+\frac{1}{7}(x-1)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}$$

$$\downarrow \text{1490}$$

$$\frac{6}{7}\left(\frac{1}{15}(x-1)\left(5a-3(x-1)^2+13\right)\sqrt{a-(x-1)^4-2(x-1)^2+3}-\frac{1}{15}\int -\frac{2\left((a+3)(5a+16)-4(2a+7)(x\right.}{\sqrt{-(x-1)^4-2(x-1)^2+a}}\right.$$
$$\frac{1}{7}(x-1)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}$$

$$\downarrow \text{27}$$

$$\frac{6}{7}\left(\frac{2}{15}\int \frac{(a+3)(5a+16)-4(2a+7)(x-1)^2}{\sqrt{-(x-1)^4-2(x-1)^2+a+3}}d(x-1)+\frac{1}{15}(x-1)\left(5a-3(x-1)^2+13\right)\sqrt{a-(x-1)^4-2(x\right.$$
$$\frac{1}{7}(x-1)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}$$

$$\downarrow \text{1514}$$

$$\frac{6}{7}\left(\frac{2\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}\int \frac{(a+3)(5a+16)-4(2a+7)(x-1)^2}{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}d(x-1)}{15\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{1}{15}(x-1)\left(5a-3(x-1)^2+13\right)\sqrt{a-}\right.$$
$$\frac{1}{7}(x-1)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}$$

$$\downarrow \text{406}$$

$$\frac{6}{7}\left(\frac{2\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}\left((a+3)(5a+16)\int \frac{1}{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}d(x-1)-4(2a+7)\int \frac{(x-1}{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{}}\right.}{15\sqrt{a-(x-1)^4-2(x-1)^2+3}}\right.$$
$$\frac{1}{7}(x-1)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}$$

$$\downarrow \text{320}$$

$$\frac{6}{7}\left(\frac{2\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}\left(\frac{(a+3)(5a+16)\sqrt{\sqrt{a+4}+1}\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\,\text{EllipticF}\left(\arctan\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right),-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}-4(2a+\cdots\right.}{15\sqrt{a-(x-1)^4-2(x-1)^2+3}}\right.$$

$$\frac{1}{7}(x-1)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}$$

$\downarrow$ 388

$$\frac{6}{7}\left(\frac{2\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}\left(\frac{(a+3)(5a+16)\sqrt{\sqrt{a+4}+1}\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\,\text{EllipticF}\left(\arctan\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right),-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}-4(2a+\cdots\right.}{15\sqrt{a-(x-1)^4-2(x-1)^2+3}\cdots}\right.$$

$$\frac{1}{7}(x-1)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}$$

$\downarrow$ 313

$$\frac{6}{7}\left(\frac{2\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}\left(\frac{(a+3)(5a+16)\sqrt{\sqrt{a+4}+1}\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\,\text{EllipticF}\left(\arctan\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right),-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}-4(2a+\cdots\right.}{15\sqrt{a-(x-1)^4-2(x\cdots}}\right.$$

$$\frac{1}{7}(x-1)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}$$

input
```
Int[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2),x]
```

output
```
((3 + a - 2*(-1 + x)^2 - (-1 + x)^4)^(3/2)*(-1 + x))/7 + (6*(((13 + 5*a -
3*(-1 + x)^2)*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]*(-1 + x))/15 + (2*Sq
rt[1 + (-1 + x)^2/(1 - Sqrt[4 + a])]*Sqrt[1 + (-1 + x)^2/(1 + Sqrt[4 + a])
]*(-4*(7 + 2*a)*(((1 - Sqrt[4 + a])*Sqrt[1 + (-1 + x)^2/(1 - Sqrt[4 + a])]
*(-1 + x))/Sqrt[1 + (-1 + x)^2/(1 + Sqrt[4 + a])] - ((1 - Sqrt[4 + a])*Sqr
t[1 + Sqrt[4 + a]]*Sqrt[1 + (-1 + x)^2/(1 - Sqrt[4 + a])]*EllipticE[ArcTan
[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])/(Sq
rt[(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (-1 + x)^2/(1 + Sqrt[4 + a]))]*
Sqrt[1 + (-1 + x)^2/(1 + Sqrt[4 + a])])) + ((3 + a)*(16 + 5*a)*Sqrt[1 + Sq
rt[4 + a]]*Sqrt[1 + (-1 + x)^2/(1 - Sqrt[4 + a])]*EllipticF[ArcTan[(-1 + x
)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])/(Sqrt[(1 +
(-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (-1 + x)^2/(1 + Sqrt[4 + a]))]*Sqrt[1 +
 (-1 + x)^2/(1 + Sqrt[4 + a])])))/(15*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)
^4])))/7
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 1404
```
Int[((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*((a + b
*x^2 + c*x^4)^p/(4*p + 1)), x] + Simp[2*(p/(4*p + 1))   Int[(2*a + b*x^2)*(
a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*
c, 0] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 1490
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Simp[2*(p/(c*(4*p + 1)*(4*p + 3)))
 Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3)
- b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

rule 1514
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[Sqrt[1 + 2*c*(x^2/(b - q))]*(Sqrt
[1 + 2*c*(x^2/(b + q))]/Sqrt[a + b*x^2 + c*x^4])   Int[(d + e*x^2)/(Sqrt[1
+ 2*c*(x^2/(b - q))]*Sqrt[1 + 2*c*(x^2/(b + q))]), x], x]] /; FreeQ[{a, b,
c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

rule 2458
```
Int[(Pn_)^(p_.), x_Symbol] :> With[{S = Coeff[Pn, x, Expon[Pn, x] - 1]/(Exp
on[Pn, x]*Coeff[Pn, x, Expon[Pn, x]])}, Subst[Int[ExpandToSum[Pn /. x -> x
- S, x]^p, x], x, x + S] /; BinomialQ[Pn /. x -> x - S, x] || (IntegerQ[Exp
on[Pn, x]/2] && TrinomialQ[Pn /. x -> x - S, x])] /; FreeQ[p, x] && PolyQ[P
n, x] && GtQ[Expon[Pn, x], 2] && NeQ[Coeff[Pn, x, Expon[Pn, x] - 1], 0]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 2654 vs. $2(342) = 684$.

Time = 8.17 (sec) , antiderivative size = 2655, normalized size of antiderivative = 6.60

method	result	size
default	Expression too large to display	2655
elliptic	Expression too large to display	2655
risch	Expression too large to display	3593

input

```
int((-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x,method=_RETURNVERBOSE)
```

output

```
-1/7*x^5*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)+5/7*x^4*(-x^4+4*x^3-8*x^2+a+8*x)^(
1/2)-66/35*x^3*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)+14/5*x^2*(-x^4+4*x^3-8*x^2+a
+8*x)^(1/2)+(3/7*a-32/35)*x*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)+(-3/7*a-4/7)*(-
x^4+4*x^3-8*x^2+a+8*x)^(1/2)-(a^2-(3/7*a-32/35)*a+12/7*a+16/7)*((-1-(a+4)^
(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(
1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+
4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)*(x-1+(-1+(a+4)^(1/2))
^(1/2))^2*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1-(-1-(a+4)^(1/2))^(1/2))/((-1-(a+
4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2
)*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1+(-1-(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2
))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)/(-(-1
-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-1+(a+4)^(1/2))^(1/2)/(-(x-1-
(-1+(a+4)^(1/2))^(1/2))*(x-1+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1-(a+4)^(1/2))
^(1/2))*(x-1+(-1-(a+4)^(1/2))^(1/2)))^(1/2)*EllipticF(((-(-1-(a+4)^(1/2))^
(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/
2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2),((-(
-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))*((-1-(a+4)^(1/2))^(1/2)+(-1+
(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/((-1-
(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2)))^(1/2))-(64/35*a+32/5)*((-1-(a+
4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*((-(-1-(a+4)^(1/2))^(1/2)+(-1+(...
```

Fricas [F]

$$\int \left(a+8x-8x^2+4x^3-x^4\right)^{3/2}\,dx=\int \left(-x^4+4\,x^3-8\,x^2+a+8\,x\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x, algorithm="fricas")
```

output
```
integral((-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(3/2), x)
```

Sympy [F]

$$\int \left(a+8x-8x^2+4x^3-x^4\right)^{3/2}\,dx=\int \left(a-x^4+4x^3-8x^2+8x\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((-x**4+4*x**3-8*x**2+a+8*x)**(3/2),x)
```

output
```
Integral((a - x**4 + 4*x**3 - 8*x**2 + 8*x)**(3/2), x)
```

Maxima [F]

$$\int \left(a+8x-8x^2+4x^3-x^4\right)^{3/2}\,dx=\int \left(-x^4+4\,x^3-8\,x^2+a+8\,x\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(3/2), x)
```

Giac [F]

$$\int \left(a+8x-8x^2+4x^3-x^4\right)^{3/2} dx = \int \left(-x^4+4x^3-8x^2+a+8x\right)^{\frac{3}{2}} dx$$

input
```
integrate((-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x, algorithm="giac")
```

output
```
integrate((-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+8x-8x^2+4x^3-x^4\right)^{3/2} dx = \int \left(-x^4+4x^3-8x^2+8x+a\right)^{3/2} dx$$

input
```
int((a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2),x)
```

output
```
int((a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2), x)
```

Reduce [F]

$$\int \left(a+8x-8x^2+4x^3-x^4\right)^{3/2} dx = \frac{3\sqrt{-x^4+4x^3-8x^2+a+8x}\,ax}{7} - \frac{61\sqrt{-x^4+4x^3-8x^2+a+8x}\,a}{105} - \frac{\sqrt{-x^4+4x^3-8x^2+a+8x}\,x^5}{7} + \frac{5\sqrt{-x^4+4x^3-8x^2+a+8x}\,x^4}{7} - \frac{66\sqrt{-x^4+4x^3-8x^2+a+8x}\,x^3}{35} + \frac{14\sqrt{-x^4+4x^3-8x^2+a+8x}\,x^2}{5} - \frac{32\sqrt{-x^4+4x^3-8x^2+a+8x}\,x}{35} - \frac{116\sqrt{-x^4+4x^3-8x^2+a+8x}}{105} + \frac{4\left(\int \frac{\sqrt{-x^4+4x^3-8x^2+a+8x}}{-x^4+4x^3-8x^2+a+8x}dx\right)a^2}{7} + \frac{68\left(\int \frac{\sqrt{-x^4+4x^3-8x^2+a+8x}}{-x^4+4x^3-8x^2+a+8x}dx\right)a}{21} + \frac{464\left(\int \frac{\sqrt{-x^4+4x^3-8x^2+a+8x}}{-x^4+4x^3-8x^2+a+8x}dx\right)}{105} - \frac{32\left(\int \frac{\sqrt{-x^4+4x^3-8x^2+a+8x}\,x^3}{-x^4+4x^3-8x^2+a+8x}dx\right)a}{105} - \frac{16\left(\int \frac{\sqrt{-x^4+4x^3-8x^2+a+8x}\,x^3}{-x^4+4x^3-8x^2+a+8x}dx\right)}{15} + \frac{64\left(\int \frac{\sqrt{-x^4+4x^3-8x^2+a+8x}\,x}{-x^4+4x^3-8x^2+a+8x}dx\right)a}{105} + \frac{32\left(\int \frac{\sqrt{-x^4+4x^3-8x^2+a+8x}\,x}{-x^4+4x^3-8x^2+a+8x}dx\right)}{15}$$

input
```
int((-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x)
```

output
```
(45*sqrt(a - x**4 + 4*x**3 - 8*x**2 + 8*x)*a*x - 61*sqrt(a - x**4 + 4*x**3
 - 8*x**2 + 8*x)*a - 15*sqrt(a - x**4 + 4*x**3 - 8*x**2 + 8*x)*x**5 + 75*s
qrt(a - x**4 + 4*x**3 - 8*x**2 + 8*x)*x**4 - 198*sqrt(a - x**4 + 4*x**3 -
8*x**2 + 8*x)*x**3 + 294*sqrt(a - x**4 + 4*x**3 - 8*x**2 + 8*x)*x**2 - 96*
sqrt(a - x**4 + 4*x**3 - 8*x**2 + 8*x)*x - 116*sqrt(a - x**4 + 4*x**3 - 8*
x**2 + 8*x) + 60*int(sqrt(a - x**4 + 4*x**3 - 8*x**2 + 8*x)/(a - x**4 + 4*
x**3 - 8*x**2 + 8*x),x)*a**2 + 340*int(sqrt(a - x**4 + 4*x**3 - 8*x**2 + 8
*x)/(a - x**4 + 4*x**3 - 8*x**2 + 8*x),x)*a + 464*int(sqrt(a - x**4 + 4*x*
*3 - 8*x**2 + 8*x)/(a - x**4 + 4*x**3 - 8*x**2 + 8*x),x) - 32*int((sqrt(a
- x**4 + 4*x**3 - 8*x**2 + 8*x)*x**3)/(a - x**4 + 4*x**3 - 8*x**2 + 8*x),x
)*a - 112*int((sqrt(a - x**4 + 4*x**3 - 8*x**2 + 8*x)*x**3)/(a - x**4 + 4*
x**3 - 8*x**2 + 8*x),x) + 64*int((sqrt(a - x**4 + 4*x**3 - 8*x**2 + 8*x)*x
)/(a - x**4 + 4*x**3 - 8*x**2 + 8*x),x)*a + 224*int((sqrt(a - x**4 + 4*x**
3 - 8*x**2 + 8*x)*x)/(a - x**4 + 4*x**3 - 8*x**2 + 8*x),x))/105
```

3.69 $\int \sqrt{a+8x-8x^2+4x^3-x^4}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 330

$$\int \sqrt{a+8x-8x^2+4x^3-x^4}\,dx = -\frac{1}{3}\sqrt{3+a-2(1-x)^2-(1-x)^4}(1-x)$$
$$+\frac{2\sqrt{-1+\sqrt{4+a}}\left(1+\sqrt{4+a}\right)\sqrt{1+\frac{(1-x)^2}{1-\sqrt{4+a}}}\sqrt{1+\frac{(1-x)^2}{1+\sqrt{4+a}}}E\left(\arcsin\left(\frac{1-x}{\sqrt{-1+\sqrt{4+a}}}\right)|\frac{1-\sqrt{4+a}}{1+\sqrt{4+a}}\right)}{3\sqrt{3+a-2(1-x)^2-(1-x)^4}}$$
$$-\frac{2\sqrt{-1+\sqrt{4+a}}\left(4+a+\sqrt{4+a}\right)\sqrt{1+\frac{(1-x)^2}{1-\sqrt{4+a}}}\sqrt{1+\frac{(1-x)^2}{1+\sqrt{4+a}}}\,\text{EllipticF}\left(\arcsin\left(\frac{1-x}{\sqrt{-1+\sqrt{4+a}}}\right),\frac{1-\sqrt{4+a}}{1+\sqrt{4+a}}\right)}{3\sqrt{3+a-2(1-x)^2-(1-x)^4}}$$

output
```
-1/3*(3+a-2*(1-x)^2-(1-x)^4)^(1/2)*(1-x)+2/3*(-1+(4+a)^(1/2))^(1/2)*(1+(4+
a)^(1/2))*(1+(1-x)^2/(1-(4+a)^(1/2)))^(1/2)*(1+(1-x)^2/(1+(4+a)^(1/2)))^(1
/2)*EllipticE((1-x)/(-1+(4+a)^(1/2))^(1/2),((1-(4+a)^(1/2))/(1+(4+a)^(1/2)
))^(1/2))/(3+a-2*(1-x)^2-(1-x)^4)^(1/2)-2/3*(-1+(4+a)^(1/2))^(1/2)*(4+a+(4
+a)^(1/2))*(1+(1-x)^2/(1-(4+a)^(1/2)))^(1/2)*(1+(1-x)^2/(1+(4+a)^(1/2)))^(
1/2)*EllipticF((1-x)/(-1+(4+a)^(1/2))^(1/2),((1-(4+a)^(1/2))/(1+(4+a)^(1/2
)))^(1/2))/(3+a-2*(1-x)^2-(1-x)^4)^(1/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 3470 vs. 2(330) = 660.

Time = 16.09 (sec) , antiderivative size = 3470, normalized size of antiderivative = 10.52

$$\int \sqrt{a+8x-8x^2+4x^3-x^4}\,dx = \text{Result too large to show}$$

input
```
Integrate[Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4],x]
```

output
```
(-1/3 + x/3)*Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4] + (2*((4*(-Sqrt[-1 - Sqrt
[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x)^2*Sqr
t[((-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt
[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sq
rt[-1 - Sqrt[4 + a]] + x))]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 - Sqrt[-1 + S
qrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 -
 Sqrt[-1 - Sqrt[4 + a]] + x))]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 + Sqrt[-1
+ Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-
1 - Sqrt[-1 - Sqrt[4 + a]] + x))]*EllipticF[ArcSin[Sqrt[((-Sqrt[-1 - Sqrt[
4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqr
t[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]]
 + x))]], ((-Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(Sqrt[-1 - S
qrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]]))/((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1
+ Sqrt[4 + a]])*(-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]]))])/(Sqr
t[-1 - Sqrt[4 + a]]*(-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*Sqr
t[a + 8*x - 8*x^2 + 4*x^3 - x^4]) + (2*a*(-Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-
1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x)^2*Sqrt[((-Sqrt[-1 - Sq
rt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((
Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 +
a]] + x))]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 - Sqrt[-1 + Sqrt[4 + a]] + ...
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 481, normalized size of antiderivative = 1.46, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2458, 1404, 27, 1514, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a-x^4+4x^3-8x^2+8x}\,dx$$

$$\Big\downarrow \text{2458}$$

$$\int \sqrt{a-(x-1)^4-2(x-1)^2+3}d(x-1)$$

$$\Big\downarrow \text{1404}$$

$$\frac{1}{3}\int \frac{2\left(-(x-1)^2+a+3\right)}{\sqrt{-(x-1)^4-2(x-1)^2+a+3}}d(x-1)+\frac{1}{3}(x-1)\sqrt{a-(x-1)^4-2(x-1)^2+3}$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{3}\int \frac{-(x-1)^2+a+3}{\sqrt{-(x-1)^4-2(x-1)^2+a+3}}d(x-1)+\frac{1}{3}(x-1)\sqrt{a-(x-1)^4-2(x-1)^2+3}$$

$$\Big\downarrow \text{1514}$$

$$\frac{2\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}\int \frac{-(x-1)^2+a+3}{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}d(x-1)}{3\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{1}{3}(x-1)\sqrt{a-(x-1)^4-2(x-1)^2+3}$$

$$\Big\downarrow \text{406}$$

$$\frac{2\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}\left((a+3)\int \frac{1}{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}d(x-1)-\int \frac{(x-1)^2}{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}d(x-1)\right)}{3\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{1}{3}(x-1)\sqrt{a-(x-1)^4-2(x-1)^2+3}$$

$$\Big\downarrow \text{320}$$

$$\frac{2\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}\left(\frac{(a+3)\sqrt{\sqrt{a+4}+1}\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\,\text{EllipticF}\left(\arctan\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right),-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}-\int\frac{(x-1)^2}{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-}{\sqrt{a+}}}}\right.}{3\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

$$\frac{1}{3}(x-1)\sqrt{a-(x-1)^4-2(x-1)^2+3}$$

$\big\downarrow$ 388

$$\frac{2\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}\left(\left(1-\sqrt{a+4}\right)\int\frac{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}}{\left(\frac{(x-1)^2}{\sqrt{a+4}+1}+1\right)^{3/2}}d(x-1)+\frac{(a+3)\sqrt{\sqrt{a+4}+1}\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\,\text{EllipticF}\left(\arctan\right.}{\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}}}\right.}{3\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

$$\frac{1}{3}(x-1)\sqrt{a-(x-1)^4-2(x-1)^2+3}$$

$\big\downarrow$ 313

$$\frac{2\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}\left(\frac{(a+3)\sqrt{\sqrt{a+4}+1}\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\,\text{EllipticF}\left(\arctan\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right),-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}+\frac{(1-\sqrt{a+4})\sqrt{\sqrt{a+4}+1}}{}\right.}{3\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

$$\frac{1}{3}(x-1)\sqrt{a-(x-1)^4-2(x-1)^2+3}$$

input

```
Int[Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4],x]
```

output
```
(Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]*(-1 + x))/3 + (2*Sqrt[1 + (-1 + x
)^2/(1 - Sqrt[4 + a])]*Sqrt[1 + (-1 + x)^2/(1 + Sqrt[4 + a])]*(-(((1 - Sqr
t[4 + a])*Sqrt[1 + (-1 + x)^2/(1 - Sqrt[4 + a])]*(-1 + x))/Sqrt[1 + (-1 +
x)^2/(1 + Sqrt[4 + a])]) + ((1 - Sqrt[4 + a])*Sqrt[1 + Sqrt[4 + a]]*Sqrt[1
 + (-1 + x)^2/(1 - Sqrt[4 + a])]*EllipticE[ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4
 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])/(Sqrt[(1 + (-1 + x)^2/(1 - S
qrt[4 + a]))/(1 + (-1 + x)^2/(1 + Sqrt[4 + a]))]*Sqrt[1 + (-1 + x)^2/(1 +
Sqrt[4 + a])]) + ((3 + a)*Sqrt[1 + Sqrt[4 + a]]*Sqrt[1 + (-1 + x)^2/(1 - S
qrt[4 + a])]*EllipticF[ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4
+ a])/(1 - Sqrt[4 + a])])/(Sqrt[(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (-
1 + x)^2/(1 + Sqrt[4 + a]))]*Sqrt[1 + (-1 + x)^2/(1 + Sqrt[4 + a])])))/(3*
Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 1404
```
Int[((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*((a + b
*x^2 + c*x^4)^p/(4*p + 1)), x] + Simp[2*(p/(4*p + 1))   Int[(2*a + b*x^2)*(
a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*
c, 0] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 1514
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[Sqrt[1 + 2*c*(x^2/(b - q))]*(Sqrt
[1 + 2*c*(x^2/(b + q))]/Sqrt[a + b*x^2 + c*x^4])   Int[(d + e*x^2)/(Sqrt[1
+ 2*c*(x^2/(b - q))]*Sqrt[1 + 2*c*(x^2/(b + q))]), x], x]] /; FreeQ[{a, b,
c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

rule 2458
```
Int[(Pn_)^(p_.), x_Symbol] :> With[{S = Coeff[Pn, x, Expon[Pn, x] - 1]/(Exp
on[Pn, x]*Coeff[Pn, x, Expon[Pn, x]])}, Subst[Int[ExpandToSum[Pn /. x -> x
- S, x]^p, x], x, x + S] /; BinomialQ[Pn /. x -> x - S, x] || (IntegerQ[Exp
on[Pn, x]/2] && TrinomialQ[Pn /. x -> x - S, x])] /; FreeQ[p, x] && PolyQ[P
n, x] && GtQ[Expon[Pn, x], 2] && NeQ[Coeff[Pn, x, Expon[Pn, x] - 1], 0]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 2518 vs. $2(276) = 552$.

Time = 5.72 (sec) , antiderivative size = 2519, normalized size of antiderivative = 7.63

method	result	size
default	Expression too large to display	2519
elliptic	Expression too large to display	2519
risch	Expression too large to display	3022

input
```
int((-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*x*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)-1/3*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)-(2
/3*a+4/3)*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*((-(-1-(a+4)^(1/
2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)
^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)*
(x-1+(-1+(a+4)^(1/2))^(1/2))^2*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1-(-1-(a+4)^(
1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4
)^(1/2))^(1/2)))^(1/2)*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1+(-1-(a+4)^(1/2))^(1
/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2)
)^(1/2)))^(1/2)/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-1+(a+4)
^(1/2))^(1/2)/(-(x-1-(-1+(a+4)^(1/2))^(1/2))*(x-1+(-1+(a+4)^(1/2))^(1/2))*
(x-1-(-1-(a+4)^(1/2))^(1/2))*(x-1+(-1-(a+4)^(1/2))^(1/2)))^(1/2)*EllipticF
(((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(
1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2
))^(1/2)))^(1/2),((-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))*((-1-(a
+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4
)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2)))^(1/2))-4/
3*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*((-(-1-(a+4)^(1/2))^(1/2
)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^
(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)*(x-1+(-1
+(a+4)^(1/2))^(1/2))^2*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1-(-1-(a+4)^(1/2))...
```

Fricas [F]

$$\int \sqrt{a+8x-8x^2+4x^3-x^4}\,dx = \int \sqrt{-x^4+4\,x^3-8\,x^2+a+8\,x}\,dx$$

input
```
integrate((-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x), x)
```

Sympy [F]

$$\int \sqrt{a+8x-8x^2+4x^3-x^4}\,dx=\int \sqrt{a-x^4+4x^3-8x^2+8x}\,dx$$

input
```
integrate((-x**4+4*x**3-8*x**2+a+8*x)**(1/2),x)
```

output
```
Integral(sqrt(a - x**4 + 4*x**3 - 8*x**2 + 8*x), x)
```

Maxima [F]

$$\int \sqrt{a+8x-8x^2+4x^3-x^4}\,dx=\int \sqrt{-x^4+4\,x^3-8\,x^2+a+8\,x}\,dx$$

input
```
integrate((-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x), x)
```

Giac [F]

$$\int \sqrt{a+8x-8x^2+4x^3-x^4}\,dx=\int \sqrt{-x^4+4\,x^3-8\,x^2+a+8\,x}\,dx$$

input
```
integrate((-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+8x-8x^2+4x^3-x^4}\,dx=\int \sqrt{-x^4+4\,x^3-8\,x^2+8\,x+a}\,dx$$

input
```
int((a + 8*x - 8*x^2 + 4*x^3 - x^4)^(1/2),x)
```

output
```
int((a + 8*x - 8*x^2 + 4*x^3 - x^4)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{a+8x-8x^2+4x^3-x^4}\,dx=\frac{\sqrt{-x^4+4x^3-8x^2+a+8x}\,x}{3}-\frac{4\sqrt{-x^4+4x^3-8x^2+a+8x}}{9}+\frac{2\left(\int \frac{\sqrt{-x^4+4x^3-8x^2+a+8x}}{-x^4+4x^3-8x^2+a+8x}dx\right)a}{3}+\frac{16\left(\int \frac{\sqrt{-x^4+4x^3-8x^2+a+8x}}{-x^4+4x^3-8x^2+a+8x}dx\right)}{9}-\frac{2\left(\int \frac{\sqrt{-x^4+4x^3-8x^2+a+8x}\,x^3}{-x^4+4x^3-8x^2+a+8x}dx\right)}{9}+\frac{4\left(\int \frac{\sqrt{-x^4+4x^3-8x^2+a+8x}\,x}{-x^4+4x^3-8x^2+a+8x}dx\right)}{9}$$

input
```
int((-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x)
```

output
```
(3*sqrt(a - x**4 + 4*x**3 - 8*x**2 + 8*x)*x - 4*sqrt(a - x**4 + 4*x**3 - 8
*x**2 + 8*x) + 6*int(sqrt(a - x**4 + 4*x**3 - 8*x**2 + 8*x)/(a - x**4 + 4*
x**3 - 8*x**2 + 8*x),x)*a + 16*int(sqrt(a - x**4 + 4*x**3 - 8*x**2 + 8*x)/
(a - x**4 + 4*x**3 - 8*x**2 + 8*x),x) - 2*int((sqrt(a - x**4 + 4*x**3 - 8*
x**2 + 8*x)*x**3)/(a - x**4 + 4*x**3 - 8*x**2 + 8*x),x) + 4*int((sqrt(a -
x**4 + 4*x**3 - 8*x**2 + 8*x)*x)/(a - x**4 + 4*x**3 - 8*x**2 + 8*x),x))/9
```

3.70 $\int \frac{1}{\sqrt{a+8x-8x^2+4x^3-x^4}} dx$

Optimal result

Integrand size = 24, antiderivative size = 158

$$\int \frac{1}{\sqrt{a+8x-8x^2+4x^3-x^4}} dx = -\frac{\sqrt{3+a}\sqrt{1-\frac{(1-\sqrt{4+a})(1-x)^2}{3+a}}\sqrt{1-\frac{(1+\sqrt{4+a})(1-x)^2}{3+a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1+\sqrt{4+a}}(1-x)}{\sqrt{3+a}}\right), \frac{1-\sqrt{4+a}}{1+\sqrt{4+a}}\right)}{\sqrt{1+\sqrt{4+a}}\sqrt{3+a-2(1-x)^2-(1-x)^4}}$$

output
```
-(3+a)^(1/2)*(1-(1-(4+a)^(1/2))*(1-x)^2/(3+a))^(1/2)*(1-(1+(4+a)^(1/2))*(1
-x)^2/(3+a))^(1/2)*EllipticF((1+(4+a)^(1/2))^(1/2)*(1-x)/(3+a)^(1/2),((1-(
4+a)^(1/2))/(1+(4+a)^(1/2)))^(1/2))/(1+(4+a)^(1/2))^(1/2)/(3+a-2*(1-x)^2-(
1-x)^4)^(1/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 540 vs. 2(158) = 316.

Time = 11.86 (sec) , antiderivative size = 540, normalized size of antiderivative = 3.42

$$\int \frac{1}{\sqrt{a+8x-8x^2+4x^3-x^4}}\,dx = \frac{2\left(1+\sqrt{-1-\sqrt{4+a}}-x\right)\sqrt{\frac{\sqrt{-1-\sqrt{4+a}}\left(1+\sqrt{-1+\sqrt{4+a}}-x\right)}{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(1+\sqrt{-1-\sqrt{4+a}}-x\right)}}\left(-1+\sqrt{-1-\sqrt{4+a}}+x\right)\sqrt{\frac{}{(}}}{\sqrt{-1-\sqrt{4+a}}\sqrt{\frac{\left(\sqrt{-1-\sqrt{4+a}}-\right.}{\left(\sqrt{-1-\sqrt{4+a}}-\right.}}}$$

input

```
Integrate[1/Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4],x]
```

output

```
(2*(1 + Sqrt[-1 - Sqrt[4 + a]] - x)*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(1 + Sqrt
[-1 + Sqrt[4 + a]] - x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]]
)*(1 + Sqrt[-1 - Sqrt[4 + a]] - x))]*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x)*Sqr
t[(Sqrt[-1 - Sqrt[4 + a]]*(-1 + Sqrt[-1 + Sqrt[4 + a]] + x))/((-Sqrt[-1 -
Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x))]*
EllipticF[ArcSin[Sqrt[((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(
-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqr
t[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x))]], (Sqrt[-1 - Sqrt[4 + a]] +
Sqrt[-1 + Sqrt[4 + a]])^2/(Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]]
)^2])/(Sqrt[-1 - Sqrt[4 + a]]*Sqrt[((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sq
rt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] +
Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x))]*Sqrt[a - x*(-8
+ 8*x - 4*x^2 + x^3)])
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.91, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2458, 1417, 320}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a-x^4+4x^3-8x^2+8x}}\,dx$$

$$\downarrow \text{2458}$$

$$\int \frac{1}{\sqrt{a-(x-1)^4-2(x-1)^2+3}}d(x-1)$$

$$\downarrow \text{1417}$$

$$\frac{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}\int \frac{1}{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}d(x-1)}{\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

$$\downarrow \text{320}$$

$$\frac{\sqrt{\sqrt{a+4}+1}\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)\text{EllipticF}\left(\arctan\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right),-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

input
```
Int[1/Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4],x]
```

output
```
(Sqrt[1 + Sqrt[4 + a]]*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*EllipticF[ArcTan
[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])/(Sq
rt[(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (-1 + x)^2/(1 + Sqrt[4 + a]))]*
Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4])
```

Defintions of rubi rules used

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] && !SimplerSqrtQ[b/a, d/c]
```

rule 1417
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[Sqrt[1 + 2*c*(x^2/(b - q))]*(Sqrt[1 + 2*c*(x^2/(b + q
))]/Sqrt[a + b*x^2 + c*x^4])   Int[1/(Sqrt[1 + 2*c*(x^2/(b - q))]*Sqrt[1 +
2*c*(x^2/(b + q))]), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
&& NegQ[c/a]
```

rule 2458
```
Int[(Pn_)^(p_.), x_Symbol] :> With[{S = Coeff[Pn, x, Expon[Pn, x] - 1]/(Exp
on[Pn, x]*Coeff[Pn, x, Expon[Pn, x]])}, Subst[Int[ExpandToSum[Pn /. x -> x
- S, x]^p, x], x, x + S] /; BinomialQ[Pn /. x -> x - S, x] || (IntegerQ[Exp
on[Pn, x]/2] && TrinomialQ[Pn /. x -> x - S, x])] /; FreeQ[p, x] && PolyQ[P
n, x] && GtQ[Expon[Pn, x], 2] && NeQ[Coeff[Pn, x, Expon[Pn, x] - 1], 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 529 vs. 2(133) = 266.

Time = 0.89 (sec) , antiderivative size = 530, normalized size of antiderivative = 3.35

method	result
default	$-\frac{\left(\sqrt{-1-\sqrt{a+4}}+\sqrt{-1+\sqrt{a+4}}\right)\sqrt{\frac{\left(-\sqrt{-1-\sqrt{a+4}}+\sqrt{-1+\sqrt{a+4}}\right)\left(x-1-\sqrt{-1+\sqrt{a+4}}\right)}{\left(-\sqrt{-1-\sqrt{a+4}}-\sqrt{-1+\sqrt{a+4}}\right)\left(x-1+\sqrt{-1+\sqrt{a+4}}\right)}}\left(x-1+\sqrt{-1+\sqrt{a+4}}\right)^{2}\sqrt{-\frac{2\sqrt{-1+\sqrt{a}}}{\left(\sqrt{-1-\sqrt{a+4}}-\sqrt{-}\right.}}}{\left(-\sqrt{-1-\sqrt{a+4}}+\sqrt{-1+\sqrt{a+4}}\right.}$
elliptic	$-\frac{\left(\sqrt{-1-\sqrt{a+4}}+\sqrt{-1+\sqrt{a+4}}\right)\sqrt{\frac{\left(-\sqrt{-1-\sqrt{a+4}}+\sqrt{-1+\sqrt{a+4}}\right)\left(x-1-\sqrt{-1+\sqrt{a+4}}\right)}{\left(-\sqrt{-1-\sqrt{a+4}}-\sqrt{-1+\sqrt{a+4}}\right)\left(x-1+\sqrt{-1+\sqrt{a+4}}\right)}}\left(x-1+\sqrt{-1+\sqrt{a+4}}\right)^{2}\sqrt{-\frac{2\sqrt{-1+\sqrt{a}}}{\left(\sqrt{-1-\sqrt{a+4}}-\sqrt{-}\right.}}}{\left(-\sqrt{-1-\sqrt{a+4}}+\sqrt{-1+\sqrt{a+4}}\right.}$

input
```
int(1/(-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*((-(-1-(a+4)^(1/2))^(1/2)
+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(
1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)*(x-1+(-1+
(a+4)^(1/2))^(1/2))^2*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1-(-1-(a+4)^(1/2))^(1/
2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^
(1/2)))^(1/2)*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1+(-1-(a+4)^(1/2))^(1/2))/(-(-
1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))
^(1/2)/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-1+(a+4)^(1/2))^(
1/2)/(-(x-1-(-1+(a+4)^(1/2))^(1/2))*(x-1+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1-
(a+4)^(1/2))^(1/2))*(x-1+(-1-(a+4)^(1/2))^(1/2)))^(1/2)*EllipticF(((-(-1-(
a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(
-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2))
)^(1/2),((-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))*((-1-(a+4)^(1/2)
)^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^
(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2)))^(1/2))
```

Fricas [F]

$$\int \frac{1}{\sqrt{a+8x-8x^2+4x^3-x^4}}\,dx = \int \frac{1}{\sqrt{-x^4+4\,x^3-8\,x^2+a+8\,x}}\,dx$$

input
```
integrate(1/(-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)/(x^4 - 4*x^3 + 8*x^2 - a -
8*x), x)
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+8x-8x^2+4x^3-x^4}}\,dx = \int \frac{1}{\sqrt{a-x^4+4x^3-8x^2+8x}}\,dx$$

input
```
integrate(1/(-x**4+4*x**3-8*x**2+a+8*x)**(1/2),x)
```

output
```
Integral(1/sqrt(a - x**4 + 4*x**3 - 8*x**2 + 8*x), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+8x-8x^2+4x^3-x^4}}\,dx = \int \frac{1}{\sqrt{-x^4+4\,x^3-8\,x^2+a+8\,x}}\,dx$$

input
```
integrate(1/(-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a+8x-8x^2+4x^3-x^4}}\,dx = \int \frac{1}{\sqrt{-x^4+4\,x^3-8\,x^2+a+8\,x}}\,dx$$

input
```
integrate(1/(-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+8x-8x^2+4x^3-x^4}}\,dx = \int \frac{1}{\sqrt{-x^4+4\,x^3-8\,x^2+8\,x+a}}\,dx$$

input
```
int(1/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(1/2),x)
```

output
```
int(1/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{a+8x-8x^2+4x^3-x^4}}\,dx = \int \frac{\sqrt{-x^4+4x^3-8x^2+a+8x}}{-x^4+4x^3-8x^2+a+8x}dx$$

input
```
int(1/(-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x)
```

output
```
int(sqrt(a - x**4 + 4*x**3 - 8*x**2 + 8*x)/(a - x**4 + 4*x**3 - 8*x**2 + 8
*x),x)
```

3.71 $\int \frac{1}{(a+8x-8x^2+4x^3-x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 368

$$\begin{aligned}\int \frac{1}{(a+8x-8x^2+4x^3-x^4)^{3/2}}\,dx &= -\frac{(5+a+(-1+x)^2)(1-x)}{2(12+7a+a^2)\sqrt{3+a-2(1-x)^2-(1-x)^4}} \\ &+ \frac{\sqrt{-1+\sqrt{4+a}}\,(1+\sqrt{4+a})\sqrt{1+\frac{(1-x)^2}{1-\sqrt{4+a}}}\sqrt{1+\frac{(1-x)^2}{1+\sqrt{4+a}}}\,E\left(\arcsin\left(\frac{1-x}{\sqrt{-1+\sqrt{4+a}}}\right)\middle|\frac{1-\sqrt{4+a}}{1+\sqrt{4+a}}\right)}{2(3+a)(4+a)\sqrt{3+a-2(1-x)^2-(1-x)^4}} \\ &- \frac{\sqrt{-1+\sqrt{4+a}}\,(4+a+\sqrt{4+a})\sqrt{1+\frac{(1-x)^2}{1-\sqrt{4+a}}}\sqrt{1+\frac{(1-x)^2}{1+\sqrt{4+a}}}\,\text{EllipticF}\left(\arcsin\left(\frac{1-x}{\sqrt{-1+\sqrt{4+a}}}\right),\frac{1-\sqrt{4+a}}{1+\sqrt{4+a}}\right)}{2(3+a)(4+a)\sqrt{3+a-2(1-x)^2-(1-x)^4}}\end{aligned}$$

output

```
-1/2*(5+a+(-1+x)^2)*(1-x)/(a^2+7*a+12)/(3+a-2*(1-x)^2-(1-x)^4)^(1/2)+1/2*(
-1+(4+a)^(1/2))^(1/2)*(1+(4+a)^(1/2))*(1+(1-x)^2/(1-(4+a)^(1/2)))^(1/2)*(1
+(1-x)^2/(1+(4+a)^(1/2)))^(1/2)*EllipticE((1-x)/(-1+(4+a)^(1/2))^(1/2),((1
-(4+a)^(1/2))/(1+(4+a)^(1/2)))^(1/2))/(3+a)/(4+a)/(3+a-2*(1-x)^2-(1-x)^4)^
(1/2)-1/2*(-1+(4+a)^(1/2))^(1/2)*(4+a+(4+a)^(1/2))*(1+(1-x)^2/(1-(4+a)^(1/
2)))^(1/2)*(1+(1-x)^2/(1+(4+a)^(1/2)))^(1/2)*EllipticF((1-x)/(-1+(4+a)^(1/
2))^(1/2),((1-(4+a)^(1/2))/(1+(4+a)^(1/2)))^(1/2))/(3+a)/(4+a)/(3+a-2*(1-x
)^2-(1-x)^4)^(1/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 3526 vs. $2(368) = 736$.

Time = 16.12 (sec) , antiderivative size = 3526, normalized size of antiderivative = 9.58

$$\int \frac{1}{(a+8x-8x^2+4x^3-x^4)^{3/2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(-3/2),x]
```

output
```
((6 + a - 8*x - a*x + 3*x^2 - x^3)*Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4])/(2
*(3 + a)*(4 + a)*(-a - 8*x + 8*x^2 - 4*x^3 + x^4)) + ((4*(-Sqrt[-1 - Sqrt[
4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x)^2*Sqrt
[((-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[
4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqr
t[-1 - Sqrt[4 + a]] + x))]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 - Sqrt[-1 + Sq
rt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 -
Sqrt[-1 - Sqrt[4 + a]] + x))]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 + Sqrt[-1 +
 Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1
 - Sqrt[-1 - Sqrt[4 + a]] + x))]*EllipticF[ArcSin[Sqrt[((-Sqrt[-1 - Sqrt[4
 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt
[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]]
+ x))]], ((-Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(Sqrt[-1 - Sq
rt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]]))/((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 +
 Sqrt[4 + a]])*(-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]]))])/(Sqrt
[-1 - Sqrt[4 + a]]*(-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*Sqrt
[a + 8*x - 8*x^2 + 4*x^3 - x^4]) + (2*a*(-Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1
 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x)^2*Sqrt[((-Sqrt[-1 - Sqr
t[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((S
qrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 ...
```

Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 509, normalized size of antiderivative = 1.38, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2458, 1405, 27, 1514, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-x^4+4x^3-8x^2+8x)^{3/2}}dx$$

$$\Big\downarrow \text{2458}$$

$$\int \frac{1}{(a-(x-1)^4-2(x-1)^2+3)^{3/2}}d(x-1)$$

$$\Big\downarrow \text{1405}$$

$$\frac{(x-1)\left(a+(x-1)^2+5\right)}{2\left(a^2+7a+12\right)\sqrt{a-(x-1)^4-2(x-1)^2+3}}-\frac{\int -\frac{2(-(x-1)^2+a+3)}{\sqrt{-(x-1)^4-2(x-1)^2+a+3}}d(x-1)}{4\left(a^2+7a+12\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{-(x-1)^2+a+3}{\sqrt{-(x-1)^4-2(x-1)^2+a+3}}d(x-1)}{2\left(a^2+7a+12\right)}+\frac{(x-1)\left(a+(x-1)^2+5\right)}{2\left(a^2+7a+12\right)\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

$$\Big\downarrow \text{1514}$$

$$\frac{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}\int \frac{-(x-1)^2+a+3}{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}d(x-1)}{2\left(a^2+7a+12\right)\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{(x-1)\left(a+(x-1)^2+5\right)}{2\left(a^2+7a+12\right)\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

$$\Big\downarrow \text{406}$$

$$\frac{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}\left((a+3)\int \frac{1}{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}d(x-1)-\int \frac{(x-1)^2}{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}d(x-1)\right)}{2\left(a^2+7a+12\right)\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{(x-1)\left(a+(x-1)^2+5\right)}{2\left(a^2+7a+12\right)\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

$\Big\downarrow$ 320

$$\frac{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}\left(\frac{(a+3)\sqrt{\sqrt{a+4}+1}\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\,\text{EllipticF}\left(\arctan\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right),-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}-\int\frac{(x-1)^2}{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1}{\sqrt{a+4}}}}\right.}{2\left(a^2+7a+12\right)\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$
$$\frac{(x-1)\left(a+(x-1)^2+5\right)}{2\left(a^2+7a+12\right)\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

$\Big\downarrow$ 388

$$\frac{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}\left(\left(1-\sqrt{a+4}\right)\int\frac{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}}{\left(\frac{(x-1)^2}{\sqrt{a+4}+1}+1\right)^{3/2}}d(x-1)+\frac{(a+3)\sqrt{\sqrt{a+4}+1}\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\,\text{EllipticF}\left(\arctan\left(\right.\right.}{\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\right.}{2\left(a^2+7a+12\right)\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$
$$\frac{(x-1)\left(a+(x-1)^2+5\right)}{2\left(a^2+7a+12\right)\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

$\Big\downarrow$ 313

$$\frac{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}\left(\frac{(a+3)\sqrt{\sqrt{a+4}+1}\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\,\text{EllipticF}\left(\arctan\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right),-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}+\frac{\left(1-\sqrt{a+4}\right)\sqrt{\sqrt{a+4}+1}}{}\right.}{2\left(a^2+7a+12\right)\sqrt{a-(x-1)^4-2(x-1)^2+}}$$
$$\frac{(x-1)\left(a+(x-1)^2+5\right)}{2\left(a^2+7a+12\right)\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

input
```
Int[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(-3/2),x]
```

output
```
((5 + a + (-1 + x)^2)*(-1 + x))/(2*(12 + 7*a + a^2)*Sqrt[3 + a - 2*(-1 + x
)^2 - (-1 + x)^4]) + (Sqrt[1 + (-1 + x)^2/(1 - Sqrt[4 + a])]*Sqrt[1 + (-1
+ x)^2/(1 + Sqrt[4 + a])]*(-(((1 - Sqrt[4 + a])*Sqrt[1 + (-1 + x)^2/(1 - S
qrt[4 + a])]*(-1 + x))/Sqrt[1 + (-1 + x)^2/(1 + Sqrt[4 + a])]) + ((1 - Sqr
t[4 + a])*Sqrt[1 + Sqrt[4 + a]]*Sqrt[1 + (-1 + x)^2/(1 - Sqrt[4 + a])]*Ell
ipticE[ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[
4 + a])])/(Sqrt[(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (-1 + x)^2/(1 + Sq
rt[4 + a]))]*Sqrt[1 + (-1 + x)^2/(1 + Sqrt[4 + a])]) + ((3 + a)*Sqrt[1 + S
qrt[4 + a]]*Sqrt[1 + (-1 + x)^2/(1 - Sqrt[4 + a])]*EllipticF[ArcTan[(-1 +
x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])/(Sqrt[(1 +
 (-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (-1 + x)^2/(1 + Sqrt[4 + a]))]*Sqrt[1
+ (-1 + x)^2/(1 + Sqrt[4 + a])])))/(2*(12 + 7*a + a^2)*Sqrt[3 + a - 2*(-1
+ x)^2 - (-1 + x)^4])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1514
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[Sqrt[1 + 2*c*(x^2/(b - q))]*(Sqrt
[1 + 2*c*(x^2/(b + q))]/Sqrt[a + b*x^2 + c*x^4])   Int[(d + e*x^2)/(Sqrt[1
+ 2*c*(x^2/(b - q))]*Sqrt[1 + 2*c*(x^2/(b + q))]), x], x]] /; FreeQ[{a, b,
c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

rule 2458
```
Int[(Pn_)^(p_.), x_Symbol] :> With[{S = Coeff[Pn, x, Expon[Pn, x] - 1]/(Exp
on[Pn, x]*Coeff[Pn, x, Expon[Pn, x]])}, Subst[Int[ExpandToSum[Pn /. x -> x
- S, x]^p, x], x, x + S] /; BinomialQ[Pn /. x -> x - S, x] || (IntegerQ[Exp
on[Pn, x]/2] && TrinomialQ[Pn /. x -> x - S, x])] /; FreeQ[p, x] && PolyQ[P
n, x] && GtQ[Expon[Pn, x], 2] && NeQ[Coeff[Pn, x, Expon[Pn, x] - 1], 0]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 2600 vs. $2(314) = 628$.

Time = 0.95 (sec) , antiderivative size = 2601, normalized size of antiderivative = 7.07

method	result	size
default	Expression too large to display	2601
elliptic	Expression too large to display	2601

input
```
int(1/(-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2*(1/4/(a^2+7*a+12)*x^3-3/4/(a^2+7*a+12)*x^2+1/4*(a+8)/(a^2+7*a+12)*x-1/4*
(6+a)/(a^2+7*a+12))/(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)-((a+5)/(a^2+7*a+12)-1/2
*(a+8)/(a^2+7*a+12))*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*((-(-
1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/
(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/
2)))^(1/2)*(x-1+(-1+(a+4)^(1/2))^(1/2))^2*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1-
(-1-(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x
-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1+(-1-(a+4
)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+
(a+4)^(1/2))^(1/2)))^(1/2)/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2)
)/(-1+(a+4)^(1/2))^(1/2)/(-(x-1-(-1+(a+4)^(1/2))^(1/2))*(x-1+(-1+(a+4)^(1/
2))^(1/2))*(x-1-(-1-(a+4)^(1/2))^(1/2))*(x-1+(-1-(a+4)^(1/2))^(1/2)))^(1/2
)*EllipticF(((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+
4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1
+(a+4)^(1/2))^(1/2)))^(1/2),((-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/
2))*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/
2)+(-1+(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))
)^(1/2))-1/(a^2+7*a+12)*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*((
-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2
))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2...
```

Fricas [F]

$$\int \frac{1}{\left(a+8x-8x^2+4x^3-x^4\right)^{3/2}}\,dx=\int \frac{1}{\left(-x^4+4\,x^3-8\,x^2+a+8\,x\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)/(x^8 - 8*x^7 + 32*x^6 - 2*(a
 - 64)*x^4 - 80*x^5 + 8*(a - 16)*x^3 - 16*(a - 4)*x^2 + a^2 + 16*a*x), x)
```

Sympy [F]

$$\int \frac{1}{(a+8x-8x^2+4x^3-x^4)^{3/2}}\,dx = \int \frac{1}{(a-x^4+4x^3-8x^2+8x)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-x**4+4*x**3-8*x**2+a+8*x)**(3/2),x)
```

output
```
Integral((a - x**4 + 4*x**3 - 8*x**2 + 8*x)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(a+8x-8x^2+4x^3-x^4)^{3/2}}\,dx = \int \frac{1}{(-x^4+4\,x^3-8\,x^2+a+8\,x)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(a+8x-8x^2+4x^3-x^4)^{3/2}}\,dx = \int \frac{1}{(-x^4+4\,x^3-8\,x^2+a+8\,x)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x, algorithm="giac")
```

output
```
integrate((-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+8x-8x^2+4x^3-x^4)^{3/2}}\,dx = \int \frac{1}{(-x^4+4\,x^3-8\,x^2+8\,x+a)^{3/2}}\,dx$$

input
```
int(1/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2),x)
```

output
```
int(1/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(a+8x-8x^2+4x^3-x^4)^{3/2}}\,dx = \int \frac{\sqrt{-x^4+4x^3-8x^2+a+8x}}{x^8-8x^7+32x^6-2a\,x^4-80x^5+8a\,x^3+128x^4-16a\,x^2-128x}$$

input
```
int(1/(-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x)
```

output
```
int(sqrt(a - x**4 + 4*x**3 - 8*x**2 + 8*x)/(a**2 - 2*a*x**4 + 8*a*x**3 - 1
6*a*x**2 + 16*a*x + x**8 - 8*x**7 + 32*x**6 - 80*x**5 + 128*x**4 - 128*x**
3 + 64*x**2),x)
```

3.72 $\int \frac{1}{(a+8x-8x^2+4x^3-x^4)^{5/2}} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 460

$$\int \frac{1}{(a+8x-8x^2+4x^3-x^4)^{5/2}} \, dx = -\frac{(104+47a+5a^2+4(7+2a)(1-x)^2)(1-x)}{12(3+a)^2(4+a)^2\sqrt{3+a-2(1-x)^2-(1-x)^4}} - \frac{(5+a+(-1+x)^2)(1-x)}{6(12+7a+a^2)(3+a-2(1-x)^2-(1-x)^4)^{3/2}} + \frac{(7+2a)\sqrt{-1+\sqrt{4+a}}(1+\sqrt{4+a})\sqrt{1+\frac{(1-x)^2}{1-\sqrt{4+a}}}\sqrt{1+\frac{(1-x)^2}{1+\sqrt{4+a}}}E\left(\arcsin\left(\frac{1-x}{\sqrt{-1+\sqrt{4+a}}}\right)|\frac{1-\sqrt{4+a}}{1+\sqrt{4+a}}\right)}{3(3+a)^2(4+a)^2\sqrt{3+a-2(1-x)^2-(1-x)^4}} - \frac{\sqrt{-1+\sqrt{4+a}}(76+5a^2+28\sqrt{4+a}+a(39+8\sqrt{4+a}))\sqrt{1+\frac{(1-x)^2}{1-\sqrt{4+a}}}\sqrt{1+\frac{(1-x)^2}{1+\sqrt{4+a}}}\text{EllipticF}\left(\text{arcsin}\right.}{12(3+a)^2(4+a)^2\sqrt{3+a-2(1-x)^2-(1-x)^4}}$$

output
```
-1/12*(104+47*a+5*a^2+4*(7+2*a)*(1-x)^2)*(1-x)/(3+a)^2/(4+a)^2/(3+a-2*(1-x
)^2-(1-x)^4)^(1/2)-1/6*(5+a+(-1+x)^2)*(1-x)/(a^2+7*a+12)/(3+a-2*(1-x)^2-(1
-x)^4)^(3/2)+1/3*(7+2*a)*(-1+(4+a)^(1/2))^(1/2)*(1+(4+a)^(1/2))*(1+(1-x)^2
/(1-(4+a)^(1/2)))^(1/2)*(1+(1-x)^2/(1+(4+a)^(1/2)))^(1/2)*EllipticE((1-x)/
(-1+(4+a)^(1/2))^(1/2),((1-(4+a)^(1/2))/(1+(4+a)^(1/2)))^(1/2))/(3+a)^2/(4
+a)^2/(3+a-2*(1-x)^2-(1-x)^4)^(1/2)-1/12*(-1+(4+a)^(1/2))^(1/2)*(76+5*a^2+
28*(4+a)^(1/2)+a*(39+8*(4+a)^(1/2)))*(1+(1-x)^2/(1-(4+a)^(1/2)))^(1/2)*(1+
(1-x)^2/(1+(4+a)^(1/2)))^(1/2)*EllipticF((1-x)/(-1+(4+a)^(1/2))^(1/2),((1-
(4+a)^(1/2))/(1+(4+a)^(1/2)))^(1/2))/(3+a)^2/(4+a)^2/(3+a-2*(1-x)^2-(1-x)^
4)^(1/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 6386 vs. $2(460) = 920$.

Time = 16.22 (sec) , antiderivative size = 6386, normalized size of antiderivative = 13.88

$$\int \frac{1}{(a + 8x - 8x^2 + 4x^3 - x^4)^{5/2}}\, dx = \text{Result too large to show}$$

input
```
Integrate[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(-5/2),x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 1.12 (sec) , antiderivative size = 597, normalized size of antiderivative = 1.30, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {2458, 1405, 27, 1492, 27, 1514, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a - x^4 + 4x^3 - 8x^2 + 8x)^{5/2}}\, dx$$

$$\Big\downarrow \text{2458}$$

$$\int \frac{1}{\left(a-(x-1)^4-2(x-1)^2+3\right)^{5/2}}d(x-1)$$

$$\Big\downarrow \; 1405$$

$$\frac{(x-1)\left(a+(x-1)^2+5\right)}{6\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}}-\frac{\int -\frac{2\left(3(x-1)^2+5a+19\right)}{\left(-(x-1)^4-2(x-1)^2+a+3\right)^{3/2}}d(x-1)}{12\left(a^2+7a+12\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{3(x-1)^2+5a+19}{\left(-(x-1)^4-2(x-1)^2+a+3\right)^{3/2}}d(x-1)}{6\left(a^2+7a+12\right)}+\frac{(x-1)\left(a+(x-1)^2+5\right)}{6\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}}$$

$$\Big\downarrow \; 1492$$

$$\frac{\frac{(x-1)\left(5a^2+4(2a+7)(x-1)^2+47a+104\right)}{2\left(a^2+7a+12\right)\sqrt{a-(x-1)^4-2(x-1)^2+3}}-\frac{\int -\frac{2\left((a+3)(5a+16)-4(2a+7)(x-1)^2\right)}{\sqrt{-(x-1)^4-2(x-1)^2+a+3}}d(x-1)}{4\left(a^2+7a+12\right)}}{6\left(a^2+7a+12\right)}+\frac{(x-1)\left(a+(x-1)^2+5\right)}{6\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int \frac{(a+3)(5a+16)-4(2a+7)(x-1)^2}{\sqrt{-(x-1)^4-2(x-1)^2+a+3}}d(x-1)}{2\left(a^2+7a+12\right)}+\frac{(x-1)\left(5a^2+4(2a+7)(x-1)^2+47a+104\right)}{2\left(a^2+7a+12\right)\sqrt{a-(x-1)^4-2(x-1)^2+3}}}{6\left(a^2+7a+12\right)}+\frac{(x-1)\left(a+(x-1)^2+5\right)}{6\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}}$$

$$\Big\downarrow \; 1514$$

$$\frac{\frac{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}\int \frac{(a+3)(5a+16)-4(2a+7)(x-1)^2}{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}d(x-1)}{2\left(a^2+7a+12\right)\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{(x-1)\left(5a^2+4(2a+7)(x-1)^2+47a+104\right)}{2\left(a^2+7a+12\right)\sqrt{a-(x-1)^4-2(x-1)^2+3}}}{6\left(a^2+7a+12\right)}+\frac{(x-1)\left(a+(x-1)^2+5\right)}{6\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}}$$

$$\Big\downarrow \; 406$$

$$\frac{\frac{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}\left((a+3)(5a+16)\int \frac{1}{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}d(x-1)-4(2a+7)\int \frac{(x-1)^2}{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}d(x-1)\right)}{2(a^2+7a+12)\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{(x-1)(5a}{2(a^2+7a-}}{6\left(a^2+7a+12\right)}$$

$$\frac{(x-1)\left(a+(x-1)^2+5\right)}{6\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}}$$

$$\Big\downarrow \; 320$$

$$\frac{\frac{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}\left(\frac{(a+3)(5a+16)\sqrt{\sqrt{a+4}+1}\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\,\text{EllipticF}\left(\arctan\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right),-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}-4(2a+7)\int \frac{(x-1)^2}{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}}}\right.}{2(a^2+7a+12)\sqrt{a-(x-1)^4-2(x-1)^2+3}}}{6\left(a^2+7a+12\right)}$$

$$\frac{(x-1)\left(a+(x-1)^2+5\right)}{6\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}}$$

$$\Big\downarrow \; 388$$

$$\frac{\frac{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}\left(\frac{(a+3)(5a+16)\sqrt{\sqrt{a+4}+1}\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\,\text{EllipticF}\left(\arctan\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right),-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}-4(2a+7)\left(\frac{(1-\sqrt{a+4})(x-1)\sqrt{\frac{(x-1)^2}{1-\sqrt{a+}}}}{\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\right.\right.}{2(a^2+7a+12)\sqrt{a-(x-1)^4-2(x-1)^2+3}}}{6\left(a^2+7a+12\right)}$$

$$\frac{(x-1)\left(a+(x-1)^2+5\right)}{6\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}}$$

$$\Big\downarrow \; 313$$

$$\frac{\frac{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}\left(\frac{(a+3)(5a+16)\sqrt{\sqrt{a+4}+1}\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\,\text{EllipticF}\left(\arctan\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right),-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}-4(2a+7)\left(\frac{(1-\sqrt{a+4})(x-1)\sqrt{\frac{(x-1)^2}{1-\sqrt{a+}}}}{\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\right.\right.}{2(a^2+7a+12)\sqrt{a-(x-1)^4-2(x-1)^2+3}}}{6\left(a^2+7a+12\right)}$$

$$\frac{(x-1)\left(a+(x-1)^2+5\right)}{6\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}}$$

input
```
Int[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(-5/2),x]
```

output
```
((5 + a + (-1 + x)^2)*(-1 + x))/(6*(12 + 7*a + a^2)*(3 + a - 2*(-1 + x)^2
- (-1 + x)^4)^(3/2)) + (((104 + 47*a + 5*a^2 + 4*(7 + 2*a)*(-1 + x)^2)*(-1
 + x))/(2*(12 + 7*a + a^2)*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) + (Sqr
t[1 + (-1 + x)^2/(1 - Sqrt[4 + a])]*Sqrt[1 + (-1 + x)^2/(1 + Sqrt[4 + a])]
*(-4*(7 + 2*a)*(((1 - Sqrt[4 + a])*Sqrt[1 + (-1 + x)^2/(1 - Sqrt[4 + a])]*
(-1 + x))/Sqrt[1 + (-1 + x)^2/(1 + Sqrt[4 + a])] - ((1 - Sqrt[4 + a])*Sqrt
[1 + Sqrt[4 + a]]*Sqrt[1 + (-1 + x)^2/(1 - Sqrt[4 + a])]*EllipticE[ArcTan[
(-1 + x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])/(Sqr
t[(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (-1 + x)^2/(1 + Sqrt[4 + a]))]*S
qrt[1 + (-1 + x)^2/(1 + Sqrt[4 + a])])) + ((3 + a)*(16 + 5*a)*Sqrt[1 + Sqr
t[4 + a]]*Sqrt[1 + (-1 + x)^2/(1 - Sqrt[4 + a])]*EllipticF[ArcTan[(-1 + x)
/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])/(Sqrt[(1 + (
-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (-1 + x)^2/(1 + Sqrt[4 + a]))]*Sqrt[1 +
(-1 + x)^2/(1 + Sqrt[4 + a])])))/(2*(12 + 7*a + a^2)*Sqrt[3 + a - 2*(-1 +
x)^2 - (-1 + x)^4]))/(6*(12 + 7*a + a^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1492
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2
 - 4*a*c))   Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p +
7)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 LtQ[p, -1] && IntegerQ[2*p]
```

rule 1514
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[Sqrt[1 + 2*c*(x^2/(b - q))]*(Sqrt
[1 + 2*c*(x^2/(b + q))]/Sqrt[a + b*x^2 + c*x^4])   Int[(d + e*x^2)/(Sqrt[1
+ 2*c*(x^2/(b - q))]*Sqrt[1 + 2*c*(x^2/(b + q))]), x], x]] /; FreeQ[{a, b,
c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

rule 2458
```
Int[(Pn_)^(p_.), x_Symbol] :> With[{S = Coeff[Pn, x, Expon[Pn, x] - 1]/(Exp
on[Pn, x]*Coeff[Pn, x, Expon[Pn, x]])}, Subst[Int[ExpandToSum[Pn /. x -> x
- S, x]^p, x], x, x + S] /; BinomialQ[Pn /. x -> x - S, x] || (IntegerQ[Exp
on[Pn, x]/2] && TrinomialQ[Pn /. x -> x - S, x])] /; FreeQ[p, x] && PolyQ[P
n, x] && GtQ[Expon[Pn, x], 2] && NeQ[Coeff[Pn, x, Expon[Pn, x] - 1], 0]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 2756 vs. $2(400) = 800$.

Time = 0.92 (sec) , antiderivative size = 2757, normalized size of antiderivative = 5.99

method	result	size
default	Expression too large to display	2757
elliptic	Expression too large to display	2757

input
```
int(1/(-x^4+4*x^3-8*x^2+a+8*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
(1/6/(a^2+7*a+12)*x^3-1/2/(a^2+7*a+12)*x^2+1/6*(a+8)/(a^2+7*a+12)*x-1/6*(6
+a)/(a^2+7*a+12))*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)/(x^4-4*x^3+8*x^2-a-8*x)^2
+2*(1/6*(7+2*a)/(a^2+7*a+12)^2*x^3-1/2*(7+2*a)/(a^2+7*a+12)^2*x^2+1/24*(5*
a^2+71*a+188)/(a^2+7*a+12)^2*x-1/24*(5*a^2+55*a+132)/(a^2+7*a+12)^2)/(-x^4
+4*x^3-8*x^2+a+8*x)^(1/2)-(1/6*(5*a^2+47*a+104)/(a^2+7*a+12)^2-1/12*(5*a^2
+71*a+188)/(a^2+7*a+12)^2)*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))
*((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(
1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2
))^(1/2)))^(1/2)*(x-1+(-1+(a+4)^(1/2))^(1/2))^2*(-2*(-1+(a+4)^(1/2))^(1/2)
*(x-1-(-1-(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/
2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1+(-
1-(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-
1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))
^(1/2))/(-1+(a+4)^(1/2))^(1/2)/(-(x-1-(-1+(a+4)^(1/2))^(1/2))*(x-1+(-1+(a+
4)^(1/2))^(1/2))*(x-1-(-1-(a+4)^(1/2))^(1/2))*(x-1+(-1-(a+4)^(1/2))^(1/2))
)^(1/2)*EllipticF(((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(
-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x
-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2),((-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2
))^(1/2))*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2
))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2...
```

Fricas [F]

$$\int \frac{1}{(a+8x-8x^2+4x^3-x^4)^{5/2}}\,dx = \int \frac{1}{(-x^4+4\,x^3-8\,x^2+a+8\,x)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(-x^4+4*x^3-8*x^2+a+8*x)^(5/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)/(x^12 - 12*x^11 + 72*x^10 -
 3*(a - 256)*x^8 - 280*x^9 + 24*(a - 64)*x^7 - 32*(3*a - 70)*x^6 + 48*(5*a
 - 48)*x^5 + 3*(a^2 - 128*a + 512)*x^4 - 4*(3*a^2 - 96*a + 128)*x^3 - a^3
- 24*a^2*x + 24*(a^2 - 8*a)*x^2), x)
```

Sympy [F]

$$\int \frac{1}{(a+8x-8x^2+4x^3-x^4)^{5/2}}\,dx = \int \frac{1}{(a-x^4+4x^3-8x^2+8x)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(-x**4+4*x**3-8*x**2+a+8*x)**(5/2),x)
```

output
```
Integral((a - x**4 + 4*x**3 - 8*x**2 + 8*x)**(-5/2), x)
```

Maxima [F]

$$\int \frac{1}{(a+8x-8x^2+4x^3-x^4)^{5/2}}\,dx = \int \frac{1}{(-x^4+4\,x^3-8\,x^2+a+8\,x)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(-x^4+4*x^3-8*x^2+a+8*x)^(5/2),x, algorithm="maxima")
```

output
```
integrate((-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(-5/2), x)
```

Giac [F]

$$\int \frac{1}{(a+8x-8x^2+4x^3-x^4)^{5/2}}\,dx = \int \frac{1}{(-x^4+4\,x^3-8\,x^2+a+8\,x)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(-x^4+4*x^3-8*x^2+a+8*x)^(5/2),x, algorithm="giac")
```

output
```
integrate((-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(-5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+8x-8x^2+4x^3-x^4)^{5/2}}\,dx = \int \frac{1}{(-x^4+4\,x^3-8\,x^2+8\,x+a)^{5/2}}\,dx$$

input
```
int(1/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(5/2),x)
```

output
```
int(1/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(5/2), x)
```

Reduce [F]

$$\int \frac{1}{(a+8x-8x^2+4x^3-x^4)^{5/2}}\,dx = \int \frac{}{-x^{12}+12x^{11}-72x^{10}+3a\,x^8+280x^9-24a\,x^7-768x^8+96a\,x}$$

input
```
int(1/(-x^4+4*x^3-8*x^2+a+8*x)^(5/2),x)
```

output
```
int(sqrt(a - x**4 + 4*x**3 - 8*x**2 + 8*x)/(a**3 - 3*a**2*x**4 + 12*a**2*x
**3 - 24*a**2*x**2 + 24*a**2*x + 3*a*x**8 - 24*a*x**7 + 96*a*x**6 - 240*a*
x**5 + 384*a*x**4 - 384*a*x**3 + 192*a*x**2 - x**12 + 12*x**11 - 72*x**10
+ 280*x**9 - 768*x**8 + 1536*x**7 - 2240*x**6 + 2304*x**5 - 1536*x**4 + 51
2*x**3),x)
```

3.73 $\int (8+8x-x^3+8x^4)^4 \, dx$

Optimal result

Integrand size = 17, antiderivative size = 96

$$
\begin{aligned}
\int (8+8x-x^3+8x^4)^4 \, dx &= 4096x+8192x^2+8192x^3+3584x^4+\frac{14336x^5}{5} \\
&\quad +7168x^6+6784x^7+1376x^8+1408x^9 \\
&\quad +\frac{21488x^{10}}{5}+\frac{25312x^{11}}{11}-448x^{12}+\frac{10241x^{13}}{13} \\
&\quad +1168x^{14}+\frac{128x^{15}}{5}-128x^{16}+\frac{4096x^{17}}{17}
\end{aligned}
$$

output

```
4096*x+8192*x^2+8192*x^3+3584*x^4+14336/5*x^5+7168*x^6+6784*x^7+1376*x^8+1
408*x^9+21488/5*x^10+25312/11*x^11-448*x^12+10241/13*x^13+1168*x^14+128/5*
x^15-128*x^16+4096/17*x^17
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00

$$\int \left(8+8x-x^3+8x^4\right)^4\,dx = 4096x+8192x^2+8192x^3+3584x^4+\frac{14336x^5}{5}+7168x^6+6784x^7+1376x^8+1408x^9+\frac{21488x^{10}}{5}+\frac{25312x^{11}}{11}-448x^{12}+\frac{10241x^{13}}{13}+1168x^{14}+\frac{128x^{15}}{5}-128x^{16}+\frac{4096x^{17}}{17}$$

input
```
Integrate[(8 + 8*x - x^3 + 8*x^4)^4,x]
```

output
```
4096*x + 8192*x^2 + 8192*x^3 + 3584*x^4 + (14336*x^5)/5 + 7168*x^6 + 6784*
x^7 + 1376*x^8 + 1408*x^9 + (21488*x^10)/5 + (25312*x^11)/11 - 448*x^12 +
(10241*x^13)/13 + 1168*x^14 + (128*x^15)/5 - 128*x^16 + (4096*x^17)/17
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2465, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(8x^4-x^3+8x+8\right)^4\,dx$$

$$\Big\downarrow \text{ 2465}$$

$$\int \left(4096x^{16}-2048x^{15}+384x^{14}+16352x^{13}+10241x^{12}-5376x^{11}+25312x^{10}+42976x^9+12672x^8+11008x^7$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{4096x^{17}}{17} - 128x^{16} + \frac{128x^{15}}{5} + 1168x^{14} + \frac{10241x^{13}}{13} - 448x^{12} + \frac{25312x^{11}}{11} + \frac{21488x^{10}}{5} + 1408x^9 + 1376x^8 + 6784x^7 + 7168x^6 + \frac{14336x^5}{5} + 3584x^4 + 8192x^3 + 8192x^2 + 4096x$$

input
```
Int[(8 + 8*x - x^3 + 8*x^4)^4,x]
```

output
```
4096*x + 8192*x^2 + 8192*x^3 + 3584*x^4 + (14336*x^5)/5 + 7168*x^6 + 6784*
x^7 + 1376*x^8 + 1408*x^9 + (21488*x^10)/5 + (25312*x^11)/11 - 448*x^12 +
(10241*x^13)/13 + 1168*x^14 + (128*x^15)/5 - 128*x^16 + (4096*x^17)/17
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2465
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> Int[ExpandToSum[u, Px^p, x], x] /; PolyQ
[Px, x] && GtQ[Expon[Px, x], 2] && !BinomialQ[Px, x] && !TrinomialQ[Px, x
] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.88

method	result
orering	$\frac{x(2928640x^{16}-1555840x^{15}+311168x^{14}+14197040x^{13}+9575335x^{12}-5445440x^{11}+27969760x^{10}+52237328x^{9}+17114240x^{8}}{12155}$
gosper	$4096x+8192x^2+8192x^3+3584x^4+\frac{14336}{5}x^5+7168x^6+6784x^7+1376x^8+1408x^9+$
default	$4096x+8192x^2+8192x^3+3584x^4+\frac{14336}{5}x^5+7168x^6+6784x^7+1376x^8+1408x^9+$
norman	$4096x+8192x^2+8192x^3+3584x^4+\frac{14336}{5}x^5+7168x^6+6784x^7+1376x^8+1408x^9+$
risch	$4096x+8192x^2+8192x^3+3584x^4+\frac{14336}{5}x^5+7168x^6+6784x^7+1376x^8+1408x^9+$
parallelrisch	$4096x+8192x^2+8192x^3+3584x^4+\frac{14336}{5}x^5+7168x^6+6784x^7+1376x^8+1408x^9+$

input
```
int((8*x^4-x^3+8*x+8)^4,x,method=_RETURNVERBOSE)
```

output
```
1/12155*x*(2928640*x^16-1555840*x^15+311168*x^14+14197040*x^13+9575335*x^1
2-5445440*x^11+27969760*x^10+52237328*x^9+17114240*x^8+16725280*x^7+824595
20*x^6+87127040*x^5+34850816*x^4+43563520*x^3+99573760*x^2+99573760*x+4978
6880)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.88

$$\int \left(8+8x-x^3+8x^4\right)^4 \, dx = \frac{4096}{17}\,x^{17}-128\,x^{16}+\frac{128}{5}\,x^{15}+1168\,x^{14}+\frac{10241}{13}\,x^{13}-448\,x^{12}+\frac{25312}{11}\,x^{11}+\frac{21488}{5}\,x^{10}+1408\,x^9+1376\,x^8+6784\,x^7+7168\,x^6+\frac{14336}{5}\,x^5+3584\,x^4+8192\,x^3+8192\,x^2+4096\,x$$

input
```
integrate((8*x^4-x^3+8*x+8)^4,x, algorithm="fricas")
```

output
```
4096/17*x^17 - 128*x^16 + 128/5*x^15 + 1168*x^14 + 10241/13*x^13 - 448*x^1
2 + 25312/11*x^11 + 21488/5*x^10 + 1408*x^9 + 1376*x^8 + 6784*x^7 + 7168*x
^6 + 14336/5*x^5 + 3584*x^4 + 8192*x^3 + 8192*x^2 + 4096*x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.98

$$\int \left(8+8x-x^3+8x^4\right)^4 \, dx = \frac{4096x^{17}}{17}-128x^{16}+\frac{128x^{15}}{5}+1168x^{14}+\frac{10241x^{13}}{13}-448x^{12}+\frac{25312x^{11}}{11}+\frac{21488x^{10}}{5}+1408x^9+1376x^8+6784x^7+7168x^6+\frac{14336x^5}{5}+3584x^4+8192x^3+8192x^2+4096x$$

input
```
integrate((8*x**4-x**3+8*x+8)**4,x)
```

output
```
4096*x**17/17 - 128*x**16 + 128*x**15/5 + 1168*x**14 + 10241*x**13/13 - 44
8*x**12 + 25312*x**11/11 + 21488*x**10/5 + 1408*x**9 + 1376*x**8 + 6784*x*
*7 + 7168*x**6 + 14336*x**5/5 + 3584*x**4 + 8192*x**3 + 8192*x**2 + 4096*x
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.88

$$\int \left(8+8x-x^3+8x^4\right)^4 \, dx = \frac{4096}{17}\,x^{17} - 128\,x^{16} + \frac{128}{5}\,x^{15} + 1168\,x^{14} + \frac{10241}{13}\,x^{13} - 448\,x^{12} + \frac{25312}{11}\,x^{11} + \frac{21488}{5}\,x^{10} + 1408\,x^9 + 1376\,x^8 + 6784\,x^7 + 7168\,x^6 + \frac{14336}{5}\,x^5 + 3584\,x^4 + 8192\,x^3 + 8192\,x^2 + 4096\,x$$

input
```
integrate((8*x^4-x^3+8*x+8)^4,x, algorithm="maxima")
```

output
```
4096/17*x^17 - 128*x^16 + 128/5*x^15 + 1168*x^14 + 10241/13*x^13 - 448*x^1
2 + 25312/11*x^11 + 21488/5*x^10 + 1408*x^9 + 1376*x^8 + 6784*x^7 + 7168*x
^6 + 14336/5*x^5 + 3584*x^4 + 8192*x^3 + 8192*x^2 + 4096*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.88

$$\int \left(8+8x-x^3+8x^4\right)^4 \, dx = \frac{4096}{17}\,x^{17} - 128\,x^{16} + \frac{128}{5}\,x^{15} + 1168\,x^{14} + \frac{10241}{13}\,x^{13} - 448\,x^{12} + \frac{25312}{11}\,x^{11} + \frac{21488}{5}\,x^{10} + 1408\,x^9 + 1376\,x^8 + 6784\,x^7 + 7168\,x^6 + \frac{14336}{5}\,x^5 + 3584\,x^4 + 8192\,x^3 + 8192\,x^2 + 4096\,x$$

input
```
integrate((8*x^4-x^3+8*x+8)^4,x, algorithm="giac")
```

output
```
4096/17*x^17 - 128*x^16 + 128/5*x^15 + 1168*x^14 + 10241/13*x^13 - 448*x^1
2 + 25312/11*x^11 + 21488/5*x^10 + 1408*x^9 + 1376*x^8 + 6784*x^7 + 7168*x
^6 + 14336/5*x^5 + 3584*x^4 + 8192*x^3 + 8192*x^2 + 4096*x
```

Mupad [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.88

$$\int (8+8x-x^3+8x^4)^4\,dx = \frac{4096\,x^{17}}{17}-128\,x^{16}+\frac{128\,x^{15}}{5}+1168\,x^{14}+\frac{10241\,x^{13}}{13}-448\,x^{12}+\frac{25312\,x^{11}}{11}+\frac{21488\,x^{10}}{5}+1408\,x^9+1376\,x^8+6784\,x^7+7168\,x^6+\frac{14336\,x^5}{5}+3584\,x^4+8192\,x^3+8192\,x^2+4096\,x$$

input
```
int((8*x - x^3 + 8*x^4 + 8)^4,x)
```

output
```
4096*x + 8192*x^2 + 8192*x^3 + 3584*x^4 + (14336*x^5)/5 + 7168*x^6 + 6784*
x^7 + 1376*x^8 + 1408*x^9 + (21488*x^10)/5 + (25312*x^11)/11 - 448*x^12 +
(10241*x^13)/13 + 1168*x^14 + (128*x^15)/5 - 128*x^16 + (4096*x^17)/17
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.86

$$\int (8+8x-x^3+8x^4)^4\,dx = \frac{x(2928640x^{16}-1555840x^{15}+311168x^{14}+14197040x^{13}+9575335x^{12}-5445440x^{11}+27969760x^{10}+}{}$$

input
```
int((8*x^4-x^3+8*x+8)^4,x)
```

output
```
(x*(2928640*x**16 - 1555840*x**15 + 311168*x**14 + 14197040*x**13 + 957533
5*x**12 - 5445440*x**11 + 27969760*x**10 + 52237328*x**9 + 17114240*x**8 +
 16725280*x**7 + 82459520*x**6 + 87127040*x**5 + 34850816*x**4 + 43563520*
x**3 + 99573760*x**2 + 99573760*x + 49786880))/12155
```

3.74 $\int (8+8x-x^3+8x^4)^3\,dx$

Optimal result

Integrand size = 17, antiderivative size = 74

$$\int (8+8x-x^3+8x^4)^3\,dx = 512x+768x^2+512x^3+80x^4+\frac{1152x^5}{5}+480x^6+\frac{1560x^7}{7}-45x^8+128x^9+\frac{307x^{10}}{2}+\frac{24x^{11}}{11}-16x^{12}+\frac{512x^{13}}{13}$$

output
```
512*x+768*x^2+512*x^3+80*x^4+1152/5*x^5+480*x^6+1560/7*x^7-45*x^8+128*x^9+
307/2*x^10+24/11*x^11-16*x^12+512/13*x^13
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00

$$\int (8+8x-x^3+8x^4)^3\,dx = 512x+768x^2+512x^3+80x^4+\frac{1152x^5}{5}+480x^6+\frac{1560x^7}{7}-45x^8+128x^9+\frac{307x^{10}}{2}+\frac{24x^{11}}{11}-16x^{12}+\frac{512x^{13}}{13}$$

input
```
Integrate[(8 + 8*x - x^3 + 8*x^4)^3,x]
```

output
```
512*x + 768*x^2 + 512*x^3 + 80*x^4 + (1152*x^5)/5 + 480*x^6 + (1560*x^7)/7
 - 45*x^8 + 128*x^9 + (307*x^10)/2 + (24*x^11)/11 - 16*x^12 + (512*x^13)/1
3
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2465, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(8x^4-x^3+8x+8\right)^3 dx$$

$$\Big\downarrow \text{2465}$$

$$\int \left(512x^{12}-192x^{11}+24x^{10}+1535x^9+1152x^8-360x^7+1560x^6+2880x^5+1152x^4+320x^3+1536x^2+153\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{512x^{13}}{13}-16x^{12}+\frac{24x^{11}}{11}+\frac{307x^{10}}{2}+128x^9-45x^8+\frac{1560x^7}{7}+480x^6+\frac{1152x^5}{5}+80x^4+512x^3+768x^2+512x$$

input
```
Int[(8 + 8*x - x^3 + 8*x^4)^3,x]
```

output
```
512*x + 768*x^2 + 512*x^3 + 80*x^4 + (1152*x^5)/5 + 480*x^6 + (1560*x^7)/7
 - 45*x^8 + 128*x^9 + (307*x^10)/2 + (24*x^11)/11 - 16*x^12 + (512*x^13)/1
3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2465
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> Int[ExpandToSum[u, Px^p, x], x] /; PolyQ
[Px, x] && GtQ[Expon[Px, x], 2] && !BinomialQ[Px, x] && !TrinomialQ[Px, x
] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.86

method	result
orering	$\frac{x(394240x^{12}-160160x^{11}+21840x^{10}+1536535x^{9}+1281280x^{8}-450450x^{7}+2230800x^{6}+4804800x^{5}+2306304x^{4}+800800x^{3}+}{10010}$
gosper	$512x+768x^{2}+512x^{3}+80x^{4}+\frac{1152}{5}x^{5}+480x^{6}+\frac{1560}{7}x^{7}-45x^{8}+128x^{9}+\frac{307}{2}x^{10}+\frac{24}{11}x^{1}$
default	$512x+768x^{2}+512x^{3}+80x^{4}+\frac{1152}{5}x^{5}+480x^{6}+\frac{1560}{7}x^{7}-45x^{8}+128x^{9}+\frac{307}{2}x^{10}+\frac{24}{11}x^{1}$
norman	$512x+768x^{2}+512x^{3}+80x^{4}+\frac{1152}{5}x^{5}+480x^{6}+\frac{1560}{7}x^{7}-45x^{8}+128x^{9}+\frac{307}{2}x^{10}+\frac{24}{11}x^{1}$
risch	$512x+768x^{2}+512x^{3}+80x^{4}+\frac{1152}{5}x^{5}+480x^{6}+\frac{1560}{7}x^{7}-45x^{8}+128x^{9}+\frac{307}{2}x^{10}+\frac{24}{11}x^{1}$
parallelrisch	$512x+768x^{2}+512x^{3}+80x^{4}+\frac{1152}{5}x^{5}+480x^{6}+\frac{1560}{7}x^{7}-45x^{8}+128x^{9}+\frac{307}{2}x^{10}+\frac{24}{11}x^{1}$

input
```
int((8*x^4-x^3+8*x+8)^3,x,method=_RETURNVERBOSE)
```

output
```
1/10010*x*(394240*x^12-160160*x^11+21840*x^10+1536535*x^9+1281280*x^8-4504
50*x^7+2230800*x^6+4804800*x^5+2306304*x^4+800800*x^3+5125120*x^2+7687680*
x+5125120)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.86

$$\int \left(8+8x-x^3+8x^4\right)^3 \, dx = \frac{512}{13}\,x^{13}-16\,x^{12}+\frac{24}{11}\,x^{11}+\frac{307}{2}\,x^{10}+128\,x^9-45\,x^8+\frac{1560}{7}\,x^7 + 480\,x^6+\frac{1152}{5}\,x^5+80\,x^4+512\,x^3+768\,x^2+512\,x$$

input
```
integrate((8*x^4-x^3+8*x+8)^3,x, algorithm="fricas")
```

output
```
512/13*x^13 - 16*x^12 + 24/11*x^11 + 307/2*x^10 + 128*x^9 - 45*x^8 + 1560/
7*x^7 + 480*x^6 + 1152/5*x^5 + 80*x^4 + 512*x^3 + 768*x^2 + 512*x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.96

$$\int \left(8+8x-x^3+8x^4\right)^3 \, dx = \frac{512x^{13}}{13}-16x^{12}+\frac{24x^{11}}{11}+\frac{307x^{10}}{2}+128x^9-45x^8+\frac{1560x^7}{7} + 480x^6+\frac{1152x^5}{5}+80x^4+512x^3+768x^2+512x$$

input
```
integrate((8*x**4-x**3+8*x+8)**3,x)
```

output
```
512*x**13/13 - 16*x**12 + 24*x**11/11 + 307*x**10/2 + 128*x**9 - 45*x**8 +
 1560*x**7/7 + 480*x**6 + 1152*x**5/5 + 80*x**4 + 512*x**3 + 768*x**2 + 51
2*x
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.86

$$\int \left(8+8x-x^3+8x^4\right)^3 \, dx = \frac{512}{13}\,x^{13}-16\,x^{12}+\frac{24}{11}\,x^{11}+\frac{307}{2}\,x^{10}+128\,x^9-45\,x^8+\frac{1560}{7}\,x^7 + 480\,x^6+\frac{1152}{5}\,x^5+80\,x^4+512\,x^3+768\,x^2+512\,x$$

input
```
integrate((8*x^4-x^3+8*x+8)^3,x, algorithm="maxima")
```

output
```
512/13*x^13 - 16*x^12 + 24/11*x^11 + 307/2*x^10 + 128*x^9 - 45*x^8 + 1560/
7*x^7 + 480*x^6 + 1152/5*x^5 + 80*x^4 + 512*x^3 + 768*x^2 + 512*x
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.86

$$\int \left(8+8x-x^3+8x^4\right)^3 \, dx = \frac{512}{13}\,x^{13}-16\,x^{12}+\frac{24}{11}\,x^{11}+\frac{307}{2}\,x^{10}+128\,x^9-45\,x^8+\frac{1560}{7}\,x^7 + 480\,x^6+\frac{1152}{5}\,x^5+80\,x^4+512\,x^3+768\,x^2+512\,x$$

input
```
integrate((8*x^4-x^3+8*x+8)^3,x, algorithm="giac")
```

output
```
512/13*x^13 - 16*x^12 + 24/11*x^11 + 307/2*x^10 + 128*x^9 - 45*x^8 + 1560/
7*x^7 + 480*x^6 + 1152/5*x^5 + 80*x^4 + 512*x^3 + 768*x^2 + 512*x
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.86

$$\int \left(8+8x-x^3+8x^4\right)^3 \, dx = \frac{512\,x^{13}}{13}-16\,x^{12}+\frac{24\,x^{11}}{11}+\frac{307\,x^{10}}{2}+128\,x^9-45\,x^8+\frac{1560\,x^7}{7} + 480\,x^6+\frac{1152\,x^5}{5}+80\,x^4+512\,x^3+768\,x^2+512\,x$$

input
```
int((8*x - x^3 + 8*x^4 + 8)^3,x)
```

output
```
512*x + 768*x^2 + 512*x^3 + 80*x^4 + (1152*x^5)/5 + 480*x^6 + (1560*x^7)/7
 - 45*x^8 + 128*x^9 + (307*x^10)/2 + (24*x^11)/11 - 16*x^12 + (512*x^13)/1
3
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.85

$$\int \left(8+8x-x^3+8x^4\right)^3 \, dx$$
$$= \frac{x(394240x^{12}-160160x^{11}+21840x^{10}+1536535x^9+1281280x^8-450450x^7+2230800x^6+4804800x^5}{10010}$$

input
```
int((8*x^4-x^3+8*x+8)^3,x)
```

output
```
(x*(394240*x**12 - 160160*x**11 + 21840*x**10 + 1536535*x**9 + 1281280*x**
8 - 450450*x**7 + 2230800*x**6 + 4804800*x**5 + 2306304*x**4 + 800800*x**3
 + 5125120*x**2 + 7687680*x + 5125120))/10010
```

3.75 $\int (8+8x-x^3+8x^4)^2\,dx$

Optimal result

Integrand size = 17, antiderivative size = 54

$$\int (8+8x-x^3+8x^4)^2\,dx=64x+64x^2+\frac{64x^3}{3}-4x^4+\frac{112x^5}{5}+\frac{64x^6}{3}+\frac{x^7}{7}-2x^8+\frac{64x^9}{9}$$

output
```
64*x+64*x^2+64/3*x^3-4*x^4+112/5*x^5+64/3*x^6+1/7*x^7-2*x^8+64/9*x^9
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int (8+8x-x^3+8x^4)^2\,dx=64x+64x^2+\frac{64x^3}{3}-4x^4+\frac{112x^5}{5}+\frac{64x^6}{3}+\frac{x^7}{7}-2x^8+\frac{64x^9}{9}$$

input
```
Integrate[(8 + 8*x - x^3 + 8*x^4)^2,x]
```

output
```
64*x + 64*x^2 + (64*x^3)/3 - 4*x^4 + (112*x^5)/5 + (64*x^6)/3 + x^7/7 - 2*
x^8 + (64*x^9)/9
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2465, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(8x^4-x^3+8x+8\right)^2 dx$$

$$\Big\downarrow \text{ 2465}$$

$$\int \left(64x^8-16x^7+x^6+128x^5+112x^4-16x^3+64x^2+128x+64\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{64x^9}{9}-2x^8+\frac{x^7}{7}+\frac{64x^6}{3}+\frac{112x^5}{5}-4x^4+\frac{64x^3}{3}+64x^2+64x$$

input
```
Int[(8 + 8*x - x^3 + 8*x^4)^2,x]
```

output
```
64*x + 64*x^2 + (64*x^3)/3 - 4*x^4 + (112*x^5)/5 + (64*x^6)/3 + x^7/7 - 2*
x^8 + (64*x^9)/9
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2465
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> Int[ExpandToSum[u, Px^p, x], x] /; PolyQ
[Px, x] && GtQ[Expon[Px, x], 2] && !BinomialQ[Px, x] && !TrinomialQ[Px, x
] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

method	result	size
orering	$\frac{x(2240x^8-630x^7+45x^6+6720x^5+7056x^4-1260x^3+6720x^2+20160x+20160)}{315}$	44
gosper	$64x+64x^2+\frac{64}{3}x^3-4x^4+\frac{112}{5}x^5+\frac{64}{3}x^6+\frac{1}{7}x^7-2x^8+\frac{64}{9}x^9$	45
default	$64x+64x^2+\frac{64}{3}x^3-4x^4+\frac{112}{5}x^5+\frac{64}{3}x^6+\frac{1}{7}x^7-2x^8+\frac{64}{9}x^9$	45
norman	$64x+64x^2+\frac{64}{3}x^3-4x^4+\frac{112}{5}x^5+\frac{64}{3}x^6+\frac{1}{7}x^7-2x^8+\frac{64}{9}x^9$	45
risch	$64x+64x^2+\frac{64}{3}x^3-4x^4+\frac{112}{5}x^5+\frac{64}{3}x^6+\frac{1}{7}x^7-2x^8+\frac{64}{9}x^9$	45
parallelrisch	$64x+64x^2+\frac{64}{3}x^3-4x^4+\frac{112}{5}x^5+\frac{64}{3}x^6+\frac{1}{7}x^7-2x^8+\frac{64}{9}x^9$	45

input
```
int((8*x^4-x^3+8*x+8)^2,x,method=_RETURNVERBOSE)
```

output
```
1/315*x*(2240*x^8-630*x^7+45*x^6+6720*x^5+7056*x^4-1260*x^3+6720*x^2+20160
*x+20160)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int (8+8x-x^3+8x^4)^2\,dx=\frac{64}{9}x^9-2x^8+\frac{1}{7}x^7+\frac{64}{3}x^6+\frac{112}{5}x^5-4x^4+\frac{64}{3}x^3+64x^2+64x$$

input
```
integrate((8*x^4-x^3+8*x+8)^2,x, algorithm="fricas")
```

output
```
64/9*x^9 - 2*x^8 + 1/7*x^7 + 64/3*x^6 + 112/5*x^5 - 4*x^4 + 64/3*x^3 + 64*
x^2 + 64*x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.91

$$\int (8+8x-x^3+8x^4)^2\,dx = \frac{64x^9}{9} - 2x^8 + \frac{x^7}{7} + \frac{64x^6}{3} + \frac{112x^5}{5} - 4x^4 + \frac{64x^3}{3} + 64x^2 + 64x$$

input
```
integrate((8*x**4-x**3+8*x+8)**2,x)
```

output
```
64*x**9/9 - 2*x**8 + x**7/7 + 64*x**6/3 + 112*x**5/5 - 4*x**4 + 64*x**3/3
+ 64*x**2 + 64*x
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int (8+8x-x^3+8x^4)^2\,dx = \frac{64}{9}\,x^9 - 2\,x^8 + \frac{1}{7}\,x^7 + \frac{64}{3}\,x^6 + \frac{112}{5}\,x^5 - 4\,x^4 + \frac{64}{3}\,x^3 + 64\,x^2 + 64\,x$$

input
```
integrate((8*x^4-x^3+8*x+8)^2,x, algorithm="maxima")
```

output
```
64/9*x^9 - 2*x^8 + 1/7*x^7 + 64/3*x^6 + 112/5*x^5 - 4*x^4 + 64/3*x^3 + 64*
x^2 + 64*x
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int (8+8x-x^3+8x^4)^2\,dx = \frac{64}{9}\,x^9 - 2\,x^8 + \frac{1}{7}\,x^7 + \frac{64}{3}\,x^6 + \frac{112}{5}\,x^5 - 4\,x^4 + \frac{64}{3}\,x^3 + 64\,x^2 + 64\,x$$

input
```
integrate((8*x^4-x^3+8*x+8)^2,x, algorithm="giac")
```

output
```
64/9*x^9 - 2*x^8 + 1/7*x^7 + 64/3*x^6 + 112/5*x^5 - 4*x^4 + 64/3*x^3 + 64*
x^2 + 64*x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int \left(8+8x-x^3+8x^4\right)^2\,dx=\frac{64\,x^9}{9}-2\,x^8+\frac{x^7}{7}+\frac{64\,x^6}{3}+\frac{112\,x^5}{5}-4\,x^4+\frac{64\,x^3}{3}+64\,x^2+64\,x$$

input
```
int((8*x - x^3 + 8*x^4 + 8)^2,x)
```

output
```
64*x + 64*x^2 + (64*x^3)/3 - 4*x^4 + (112*x^5)/5 + (64*x^6)/3 + x^7/7 - 2*
x^8 + (64*x^9)/9
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.80

$$\int \left(8+8x-x^3+8x^4\right)^2\,dx$$
$$=\frac{x(2240x^8-630x^7+45x^6+6720x^5+7056x^4-1260x^3+6720x^2+20160x+20160)}{315}$$

input
```
int((8*x^4-x^3+8*x+8)^2,x)
```

output
```
(x*(2240*x**8 - 630*x**7 + 45*x**6 + 6720*x**5 + 7056*x**4 - 1260*x**3 + 6
720*x**2 + 20160*x + 20160))/315
```

3.76 $\int (8+8x-x^3+8x^4)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 23

$$\int (8+8x-x^3+8x^4)\,dx = 8x+4x^2-\frac{x^4}{4}+\frac{8x^5}{5}$$

output
```
8*x+4*x^2-1/4*x^4+8/5*x^5
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int (8+8x-x^3+8x^4)\,dx = 8x+4x^2-\frac{x^4}{4}+\frac{8x^5}{5}$$

input
```
Integrate[8 + 8*x - x^3 + 8*x^4,x]
```

output
```
8*x + 4*x^2 - x^4/4 + (8*x^5)/5
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (8x^4 - x^3 + 8x + 8) \, dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{8x^5}{5} - \frac{x^4}{4} + 4x^2 + 8x$$

input
```
Int[8 + 8*x - x^3 + 8*x^4,x]
```

output
```
8*x + 4*x^2 - x^4/4 + (8*x^5)/5
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

method	result	size
orering	$\frac{x(32x^4-5x^3+80x+160)}{20}$	19
gosper	$8x+4x^2-\frac{1}{4}x^4+\frac{8}{5}x^5$	20
default	$8x+4x^2-\frac{1}{4}x^4+\frac{8}{5}x^5$	20
norman	$8x+4x^2-\frac{1}{4}x^4+\frac{8}{5}x^5$	20
risch	$8x+4x^2-\frac{1}{4}x^4+\frac{8}{5}x^5$	20
parallelrisch	$8x+4x^2-\frac{1}{4}x^4+\frac{8}{5}x^5$	20
parts	$8x+4x^2-\frac{1}{4}x^4+\frac{8}{5}x^5$	20

input
```
int(8*x^4-x^3+8*x+8,x,method=_RETURNVERBOSE)
```

output
```
1/20*x*(32*x^4-5*x^3+80*x+160)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int (8+8x-x^3+8x^4)\,dx=\frac{8}{5}\,x^5-\frac{1}{4}\,x^4+4\,x^2+8\,x$$

input
```
integrate(8*x^4-x^3+8*x+8,x, algorithm="fricas")
```

output
```
8/5*x^5 - 1/4*x^4 + 4*x^2 + 8*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \left(8+8x-x^3+8x^4\right)\,dx=\frac{8x^5}{5}-\frac{x^4}{4}+4x^2+8x$$

input
```
integrate(8*x**4-x**3+8*x+8,x)
```

output
```
8*x**5/5 - x**4/4 + 4*x**2 + 8*x
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \left(8+8x-x^3+8x^4\right)\,dx=\frac{8}{5}\,x^5-\frac{1}{4}\,x^4+4\,x^2+8\,x$$

input
```
integrate(8*x^4-x^3+8*x+8,x, algorithm="maxima")
```

output
```
8/5*x^5 - 1/4*x^4 + 4*x^2 + 8*x
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \left(8+8x-x^3+8x^4\right)\,dx=\frac{8}{5}\,x^5-\frac{1}{4}\,x^4+4\,x^2+8\,x$$

input
```
integrate(8*x^4-x^3+8*x+8,x, algorithm="giac")
```

output
```
8/5*x^5 - 1/4*x^4 + 4*x^2 + 8*x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int (8+8x-x^3+8x^4)\ dx = \frac{8\,x^5}{5} - \frac{x^4}{4} + 4\,x^2 + 8\,x$$

input
```
int(8*x - x^3 + 8*x^4 + 8,x)
```

output
```
8*x + 4*x^2 - x^4/4 + (8*x^5)/5
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

$$\int (8+8x-x^3+8x^4)\ dx = \frac{x(32x^4-5x^3+80x+160)}{20}$$

input
```
int(8*x^4-x^3+8*x+8,x)
```

output
```
(x*(32*x**4 - 5*x**3 + 80*x + 160))/20
```

3.77 $\int \frac{1}{8+8x-x^3+8x^4}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 211

$$\begin{aligned}\int \frac{1}{8+8x-x^3+8x^4}\,dx = &-\frac{\arctan\left(\frac{3-\left(1+\frac{4}{x}\right)^2}{6\sqrt{7}}\right)}{12\sqrt{7}} \\ &-\frac{1}{12}\sqrt{\frac{109+67\sqrt{29}}{1218}}\arctan\left(\frac{2+\sqrt{6\left(1+\sqrt{29}\right)}+\frac{8}{x}}{\sqrt{6\left(-1+\sqrt{29}\right)}}\right) \\ &-\frac{1}{12}\sqrt{\frac{109+67\sqrt{29}}{1218}}\arctan\left(\frac{8+\left(2-\sqrt{6\left(1+\sqrt{29}\right)}\right)x}{\sqrt{6\left(-1+\sqrt{29}\right)}x}\right) \\ &+\frac{1}{12}\sqrt{\frac{-109+67\sqrt{29}}{1218}}\operatorname{arctanh}\left(\frac{\sqrt{6\left(1+\sqrt{29}\right)}\left(1+\frac{4}{x}\right)}{3\sqrt{29}+\left(1+\frac{4}{x}\right)^2}\right)\end{aligned}$$

output
```
-1/84*arctan(1/42*(3-(1+4/x)^2)*7^(1/2))*7^(1/2)-1/14616*(132762+81606*29^
(1/2))^(1/2)*arctan((2+(6+6*29^(1/2))^(1/2)+8/x)/(-6+6*29^(1/2))^(1/2))-1/
14616*(132762+81606*29^(1/2))^(1/2)*arctan((8+(2-(6+6*29^(1/2))^(1/2))*x)/
(-6+6*29^(1/2))^(1/2)/x)+1/14616*(-132762+81606*29^(1/2))^(1/2)*arctanh((6
+6*29^(1/2))^(1/2)*(1+4/x)/(3*29^(1/2)+(1+4/x)^2))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.21

$$\int \frac{1}{8+8x-x^3+8x^4}\,dx = \text{RootSum}\left[8+8\#1-\#1^3+8\#1^4\&,\frac{\log(x-\#1)}{8-3\#1^2+32\#1^3}\&\right]$$

input
```
Integrate[(8 + 8*x - x^3 + 8*x^4)^(-1),x]
```

output
```
RootSum[8 + 8*#1 - #1^3 + 8*#1^4 & , Log[x - #1]/(8 - 3*#1^2 + 32*#1^3) &
]
```

Rubi [A] (verified)

Time = 1.07 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.52, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.824$, Rules used = {2504, 27, 2202, 27, 1432, 1083, 217, 1483, 27, 1142, 27, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{8x^4-x^3+8x+8}dx$$

$$\Big\downarrow \text{ 2504}$$

$$-1024\int \frac{\left(1-4\left(\frac{1}{4}+\frac{1}{x}\right)\right)^2}{512\left(256\left(\frac{1}{4}+\frac{1}{x}\right)^4-96\left(\frac{1}{4}+\frac{1}{x}\right)^2+261\right)}d\left(\frac{1}{4}+\frac{1}{x}\right)$$

$$\Big\downarrow \text{ 27}$$

$$-2\int \frac{\left(1-4\left(\frac{1}{4}+\frac{1}{x}\right)\right)^2}{256\left(\frac{1}{4}+\frac{1}{x}\right)^4-96\left(\frac{1}{4}+\frac{1}{x}\right)^2+261}d\left(\frac{1}{4}+\frac{1}{x}\right)$$

$$\Big\downarrow \text{ 2202}$$

$$-2\left(\int \frac{16\left(\frac{1}{4}+\frac{1}{x}\right)^2+1}{256\left(\frac{1}{4}+\frac{1}{x}\right)^4-96\left(\frac{1}{4}+\frac{1}{x}\right)^2+261}d\left(\frac{1}{4}+\frac{1}{x}\right)+\int -\frac{8\left(\frac{1}{4}+\frac{1}{x}\right)}{256\left(\frac{1}{4}+\frac{1}{x}\right)^4-96\left(\frac{1}{4}+\frac{1}{x}\right)^2+261}d\left(\frac{1}{4}+\frac{1}{x}\right)\right)$$

$$\Big\downarrow \text{27}$$

$$-2\left(\int \frac{16\left(\frac{1}{4}+\frac{1}{x}\right)^2+1}{256\left(\frac{1}{4}+\frac{1}{x}\right)^4-96\left(\frac{1}{4}+\frac{1}{x}\right)^2+261}d\left(\frac{1}{4}+\frac{1}{x}\right)-8\int \frac{\frac{1}{4}+\frac{1}{x}}{256\left(\frac{1}{4}+\frac{1}{x}\right)^4-96\left(\frac{1}{4}+\frac{1}{x}\right)^2+261}d\left(\frac{1}{4}+\frac{1}{x}\right)\right)$$

$$\Big\downarrow \text{1432}$$

$$-2\left(\int \frac{16\left(\frac{1}{4}+\frac{1}{x}\right)^2+1}{256\left(\frac{1}{4}+\frac{1}{x}\right)^4-96\left(\frac{1}{4}+\frac{1}{x}\right)^2+261}d\left(\frac{1}{4}+\frac{1}{x}\right)-4\int \frac{1}{256\left(\frac{1}{4}+\frac{1}{x}\right)^4-96\left(\frac{1}{4}+\frac{1}{x}\right)^2+261}d\left(\frac{1}{4}+\frac{1}{x}\right)^2\right)$$

$$\Big\downarrow \text{1083}$$

$$-2\left(8\int \frac{1}{-\left(\frac{1}{4}+\frac{1}{x}\right)^4-258048}d\left(512\left(\frac{1}{4}+\frac{1}{x}\right)^2-96\right)+\int \frac{16\left(\frac{1}{4}+\frac{1}{x}\right)^2+1}{256\left(\frac{1}{4}+\frac{1}{x}\right)^4-96\left(\frac{1}{4}+\frac{1}{x}\right)^2+261}d\left(\frac{1}{4}+\frac{1}{x}\right)\right)$$

$$\Big\downarrow \text{217}$$

$$-2\left(\int \frac{16\left(\frac{1}{4}+\frac{1}{x}\right)^2+1}{256\left(\frac{1}{4}+\frac{1}{x}\right)^4-96\left(\frac{1}{4}+\frac{1}{x}\right)^2+261}d\left(\frac{1}{4}+\frac{1}{x}\right)-\frac{\arctan\left(\frac{512\left(\frac{1}{x}+\frac{1}{4}\right)^2-96}{192\sqrt{7}}\right)}{24\sqrt{7}}\right)$$

$$\Big\downarrow \text{1483}$$

$$-2\left(\frac{\int \frac{8\left(\sqrt{\frac{3}{2}\left(1+\sqrt{29}\right)}-2\left(1-3\sqrt{29}\right)\left(\frac{1}{4}+\frac{1}{x}\right)\right)}{16\left(\frac{1}{4}+\frac{1}{x}\right)^2-4\sqrt{6\left(1+\sqrt{29}\right)}\left(\frac{1}{4}+\frac{1}{x}\right)+3\sqrt{29}}d\left(\frac{1}{4}+\frac{1}{x}\right)}{24\sqrt{174\left(1+\sqrt{29}\right)}}+\frac{\int \frac{8\left(2\left(1-3\sqrt{29}\right)\left(\frac{1}{4}+\frac{1}{x}\right)+\sqrt{\frac{3}{2}\left(1+\sqrt{29}\right)}\right)}{16\left(\frac{1}{4}+\frac{1}{x}\right)^2+4\sqrt{6\left(1+\sqrt{29}\right)}\left(\frac{1}{4}+\frac{1}{x}\right)+3\sqrt{29}}d\left(\frac{1}{4}+\frac{1}{x}\right)}{24\sqrt{174\left(1+\sqrt{29}\right)}}-\frac{\arctan\left(\frac{51}{}\right.}{24}\right.$$

$$\Big\downarrow \text{27}$$

$$-2\left(\frac{\int \frac{\sqrt{\frac{3}{2}\left(1+\sqrt{29}\right)}-2\left(1-3\sqrt{29}\right)\left(\frac{1}{4}+\frac{1}{x}\right)}{16\left(\frac{1}{4}+\frac{1}{x}\right)^2-4\sqrt{6\left(1+\sqrt{29}\right)}\left(\frac{1}{4}+\frac{1}{x}\right)+3\sqrt{29}}d\left(\frac{1}{4}+\frac{1}{x}\right)}{3\sqrt{174\left(1+\sqrt{29}\right)}}+\frac{\int \frac{2\left(1-3\sqrt{29}\right)\left(\frac{1}{4}+\frac{1}{x}\right)+\sqrt{\frac{3}{2}\left(1+\sqrt{29}\right)}}{16\left(\frac{1}{4}+\frac{1}{x}\right)^2+4\sqrt{6\left(1+\sqrt{29}\right)}\left(\frac{1}{4}+\frac{1}{x}\right)+3\sqrt{29}}d\left(\frac{1}{4}+\frac{1}{x}\right)}{3\sqrt{174\left(1+\sqrt{29}\right)}}-\frac{\arctan\left(\frac{51}{}\right.}{24}\right.$$

$$\downarrow 1142$$

$$-2\left(\frac{\sqrt{\frac{3}{2}\left(109+67\sqrt{29}\right)}\int \frac{1}{16\left(\frac{1}{4}+\frac{1}{x}\right)^2-4\sqrt{6\left(1+\sqrt{29}\right)}\left(\frac{1}{4}+\frac{1}{x}\right)+3\sqrt{29}}d\left(\frac{1}{4}+\frac{1}{x}\right)-\frac{1}{16}\left(1-3\sqrt{29}\right)\int -\frac{4\left(\sqrt{6\left(1+\sqrt{29}\right)}-8\left(\right.\right.}{16\left(\frac{1}{4}+\frac{1}{x}\right)^2-4\sqrt{6\left(1+\sqrt{29}\right)}}}{3\sqrt{174\left(1+\sqrt{29}\right)}}\right.$$

$$\downarrow 27$$

$$-2\left(\frac{\sqrt{\frac{3}{2}\left(109+67\sqrt{29}\right)}\int \frac{1}{16\left(\frac{1}{4}+\frac{1}{x}\right)^2-4\sqrt{6\left(1+\sqrt{29}\right)}\left(\frac{1}{4}+\frac{1}{x}\right)+3\sqrt{29}}d\left(\frac{1}{4}+\frac{1}{x}\right)+\frac{1}{4}\left(1-3\sqrt{29}\right)\int \frac{\sqrt{6\left(1+\sqrt{29}\right)}-8\left(\frac{1}{4}+\right.}{16\left(\frac{1}{4}+\frac{1}{x}\right)^2-4\sqrt{6\left(1+\sqrt{29}\right)}\left(\frac{1}{4}\right.}}{3\sqrt{174\left(1+\sqrt{29}\right)}}\right.$$

$$\downarrow 1083$$

$$-2\left(\frac{\frac{1}{4}\left(1-3\sqrt{29}\right)\int \frac{\sqrt{6\left(1+\sqrt{29}\right)}-8\left(\frac{1}{4}+\frac{1}{x}\right)}{16\left(\frac{1}{4}+\frac{1}{x}\right)^2-4\sqrt{6\left(1+\sqrt{29}\right)}\left(\frac{1}{4}+\frac{1}{x}\right)+3\sqrt{29}}d\left(\frac{1}{4}+\frac{1}{x}\right)-\sqrt{6\left(109+67\sqrt{29}\right)}\int \frac{1}{96\left(1-\sqrt{29}\right)-\left(32\left(\frac{1}{4}+\frac{1}{x}\right)-4\sqrt{\right.\right.}}}{3\sqrt{174\left(1+\sqrt{29}\right)}}\right.$$

$$\downarrow 217$$

$$-2\left(\frac{\frac{1}{4}\left(1-3\sqrt{29}\right)\int \frac{\sqrt{6\left(1+\sqrt{29}\right)}-8\left(\frac{1}{4}+\frac{1}{x}\right)}{16\left(\frac{1}{4}+\frac{1}{x}\right)^2-4\sqrt{6\left(1+\sqrt{29}\right)}\left(\frac{1}{4}+\frac{1}{x}\right)+3\sqrt{29}}d\left(\frac{1}{4}+\frac{1}{x}\right)+\frac{1}{4}\sqrt{\frac{109+67\sqrt{29}}{\sqrt{29}-1}}\arctan\left(\frac{32\left(\frac{1}{x}+\frac{1}{4}\right)-4\sqrt{6\left(1+\sqrt{29}\right)}}{4\sqrt{6\left(\sqrt{29}-1\right)}}\right)}{3\sqrt{174\left(1+\sqrt{29}\right)}}\right.$$

$$\downarrow 1103$$

$$-2\left(-\frac{\arctan\left(\frac{512\left(\frac{1}{x}+\frac{1}{4}\right)^2-96}{192\sqrt{7}}\right)}{24\sqrt{7}}+\frac{\frac{1}{4}\sqrt{\frac{109+67\sqrt{29}}{\sqrt{29}-1}}\arctan\left(\frac{32\left(\frac{1}{x}+\frac{1}{4}\right)-4\sqrt{6\left(1+\sqrt{29}\right)}}{4\sqrt{6\left(\sqrt{29}-1\right)}}\right)-\frac{1}{16}\left(1-3\sqrt{29}\right)\log\left(16\left(\frac{1}{x}+\frac{1}{4}\right.\right.}{3\sqrt{174\left(1+\sqrt{29}\right)}}\right.$$

input
```
Int[(8 + 8*x - x^3 + 8*x^4)^(-1),x]
```

output
```
-2*(-1/24*ArcTan[(-96 + 512*(1/4 + x^(-1))^2)/(192*Sqrt[7])]/Sqrt[7] + ((S
qrt[(109 + 67*Sqrt[29])/(-1 + Sqrt[29])]*ArcTan[(-4*Sqrt[6*(1 + Sqrt[29])]
 + 32*(1/4 + x^(-1)))/(4*Sqrt[6*(-1 + Sqrt[29])])])/4 - ((1 - 3*Sqrt[29])*
Log[3*Sqrt[29] - 4*Sqrt[6*(1 + Sqrt[29])]*(1/4 + x^(-1)) + 16*(1/4 + x^(-1
))^2])/16)/(3*Sqrt[174*(1 + Sqrt[29])]) + ((Sqrt[(109 + 67*Sqrt[29])/(-1 +
 Sqrt[29])]*ArcTan[(4*Sqrt[6*(1 + Sqrt[29])] + 32*(1/4 + x^(-1)))/(4*Sqrt[
6*(-1 + Sqrt[29])])])/4 + ((1 - 3*Sqrt[29])*Log[3*Sqrt[29] + 4*Sqrt[6*(1 +
 Sqrt[29])]*(1/4 + x^(-1)) + 16*(1/4 + x^(-1))^2])/16)/(3*Sqrt[174*(1 + Sq
rt[29])]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1432
```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

rule 1483
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   In
t[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(d*r
 + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

rule 2202
```
Int[(Pn_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{n
 = Expon[Pn, x], k}, Int[Sum[Coeff[Pn, x, 2*k]*x^(2*k), {k, 0, n/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pn, x, 2*k + 1]*x^(2*k), {k, 0, (n -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pn, x]
 &&  !PolyQ[Pn, x^2]
```

rule 2504
```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Simp[-16*
a^2   Subst[Int[(1/(b - 4*a*x)^2)*(a*((-3*b^4 + 16*a*b^2*c - 64*a^2*b*d + 2
56*a^3*e - 32*a^2*(3*b^2 - 8*a*c)*x^2 + 256*a^4*x^4)/(b - 4*a*x)^4))^p, x],
 x, b/(4*a) + 1/x], x] /; NeQ[a, 0] && NeQ[b, 0] && EqQ[b^3 - 4*a*b*c + 8*a
^2*d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && IntegerQ[2*p] &&  !IGtQ[p, 0
]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.19

method	result	size
default	$\sum_{\_R=\operatorname{RootOf}(8\_Z^4-\_Z^3+8\_Z+8)}\frac{\ln(x-\_R)}{32\_R^3-3\_R^2+8}$	41
risch	$\sum_{\_R=\operatorname{RootOf}(8\_Z^4-\_Z^3+8\_Z+8)}\frac{\ln(x-\_R)}{32\_R^3-3\_R^2+8}$	41

input
```
int(1/(8*x^4-x^3+8*x+8),x,method=_RETURNVERBOSE)
```

output
```
sum(1/(32*_R^3-3*_R^2+8)*ln(x-_R),_R=RootOf(8*_Z^4-_Z^3+8*_Z+8))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 303 vs. 2(148) = 296.

Time = 0.12 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.44

$$\int \frac{1}{8+8x-x^3+8x^4}\,dx =$$
$$-\frac{1}{12}\sqrt{\frac{1}{455}\left(67\sqrt{29}+109\right)\sqrt{\frac{67}{1218}\sqrt{29}-\frac{109}{1218}}+\frac{67}{1218}\sqrt{29}+\frac{283}{1218}}\arctan\left(-\frac{1}{8187140}\left(65\sqrt{29}(628\right.\right.$$
$$+\frac{1}{12}\sqrt{-\frac{1}{455}\left(67\sqrt{29}+109\right)\sqrt{\frac{67}{1218}\sqrt{29}-\frac{109}{1218}}+\frac{67}{1218}\sqrt{29}+\frac{283}{1218}}\arctan\left(\frac{1}{8187140}\left(65\sqrt{29}(628\right.\right.$$
$$+\frac{1}{24}\sqrt{\frac{67}{1218}\sqrt{29}-\frac{109}{1218}}\log\left(2080\,x^2\right.$$
$$\left.+3\left(\sqrt{29}(65\,x-88)+1885\,x-116\right)\sqrt{\frac{67}{1218}\sqrt{29}-\frac{109}{1218}}-130\,x+390\sqrt{29}+130\right)$$
$$-\frac{1}{24}\sqrt{\frac{67}{1218}\sqrt{29}-\frac{109}{1218}}\log\left(2080\,x^2\right.$$
$$\left.-3\left(\sqrt{29}(65\,x-88)+1885\,x-116\right)\sqrt{\frac{67}{1218}\sqrt{29}-\frac{109}{1218}}-130\,x+390\sqrt{29}+130\right)$$

input
```
integrate(1/(8*x^4-x^3+8*x+8),x, algorithm="fricas")
```

output
```
-1/12*sqrt(1/455*(67*sqrt(29) + 109)*sqrt(67/1218*sqrt(29) - 109/1218) + 6
7/1218*sqrt(29) + 283/1218)*arctan(-1/8187140*(65*sqrt(29)*(62891*x - 4738
) - 174*(sqrt(29)*(15701*x - 33858) + 25102*x - 74206)*sqrt(67/1218*sqrt(2
9) - 109/1218) + 5310045*x - 5553210)*sqrt(1/455*(67*sqrt(29) + 109)*sqrt(
67/1218*sqrt(29) - 109/1218) + 67/1218*sqrt(29) + 283/1218)) + 1/12*sqrt(-
1/455*(67*sqrt(29) + 109)*sqrt(67/1218*sqrt(29) - 109/1218) + 67/1218*sqrt
(29) + 283/1218)*arctan(1/8187140*(65*sqrt(29)*(62891*x - 4738) + 174*(sqr
t(29)*(15701*x - 33858) + 25102*x - 74206)*sqrt(67/1218*sqrt(29) - 109/121
8) + 5310045*x - 5553210)*sqrt(-1/455*(67*sqrt(29) + 109)*sqrt(67/1218*sqr
t(29) - 109/1218) + 67/1218*sqrt(29) + 283/1218)) + 1/24*sqrt(67/1218*sqrt
(29) - 109/1218)*log(2080*x^2 + 3*(sqrt(29)*(65*x - 88) + 1885*x - 116)*sq
rt(67/1218*sqrt(29) - 109/1218) - 130*x + 390*sqrt(29) + 130) - 1/24*sqrt(
67/1218*sqrt(29) - 109/1218)*log(2080*x^2 - 3*(sqrt(29)*(65*x - 88) + 1885
*x - 116)*sqrt(67/1218*sqrt(29) - 109/1218) - 130*x + 390*sqrt(29) + 130)
```

Sympy [A] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.19

$$\int \frac{1}{8+8x-x^3+8x^4}\,dx$$
$$=\text{RootSum}\left(66298176t^4+74088t^2+4095t+64,\left(t\mapsto t\log\left(\frac{359142744424t^3}{2109763}-\frac{1504863360t^2}{2109763}+\frac{102851}{2109}\right.\right.\right.$$

input
```
integrate(1/(8*x**4-x**3+8*x+8),x)
```

output
```
RootSum(66298176*_t**4 + 74088*_t**2 + 4095*_t + 64, Lambda(_t, _t*log(359
14274424*_t**3/2109763 - 1504863360*_t**2/2109763 + 102851343*_t/2109763 +
 x + 6055613/16878104)))
```

Maxima [F]

$$\int \frac{1}{8+8x-x^3+8x^4}\,dx=\int \frac{1}{8\,x^4-x^3+8\,x+8}\,dx$$

input
```
integrate(1/(8*x^4-x^3+8*x+8),x, algorithm="maxima")
```

output
```
integrate(1/(8*x^4 - x^3 + 8*x + 8), x)
```

Giac [F]

$$\int \frac{1}{8+8x-x^3+8x^4}\,dx=\int \frac{1}{8\,x^4-x^3+8\,x+8}\,dx$$

input
```
integrate(1/(8*x^4-x^3+8*x+8),x, algorithm="giac")
```

output
```
integrate(1/(8*x^4 - x^3 + 8*x + 8), x)
```

Mupad [B] (verification not implemented)

Time = 23.73 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.58

$$\int \frac{1}{8+8x-x^3+8x^4}\,dx$$
$$=\sum_{k=1}^{4}\ln\left(-\frac{\operatorname{root}\left(z^4+\frac{7z^2}{6264}+\frac{65z}{1052352}+\frac{1}{1035909},z,k\right)\left(8064\operatorname{root}\left(z^4+\frac{7z^2}{6264}+\frac{65z}{1052352}+\frac{1}{1035909},z,k\right)+256\right.}{\cdots}\right.$$
$$\left.+\frac{7z^2}{6264}+\frac{65z}{1052352}+\frac{1}{1035909},z,k\right)$$

input
```
int(1/(8*x - x^3 + 8*x^4 + 8),x)
```

output
```
symsum(log(-(root(z^4 + (7*z^2)/6264 + (65*z)/1052352 + 1/1035909, z, k)*(
8064*root(z^4 + (7*z^2)/6264 + (65*z)/1052352 + 1/1035909, z, k) + 256*x +
 12285*root(z^4 + (7*z^2)/6264 + (65*z)/1052352 + 1/1035909, z, k)*x + 148
176*root(z^4 + (7*z^2)/6264 + (65*z)/1052352 + 1/1035909, z, k)^2*x + 1980
72*root(z^4 + (7*z^2)/6264 + (65*z)/1052352 + 1/1035909, z, k)^2 - 8))/409
6)*root(z^4 + (7*z^2)/6264 + (65*z)/1052352 + 1/1035909, z, k), k, 1, 4)
```

Reduce [F]

$$\int \frac{1}{8+8x-x^3+8x^4}\,dx=\int \frac{1}{8x^4-x^3+8x+8}dx$$

input
```
int(1/(8*x^4-x^3+8*x+8),x)
```

output
```
int(1/(8*x**4 - x**3 + 8*x + 8),x)
```

3.78 $\int \frac{1}{(8+8x-x^3+8x^4)^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 300

$$\int \frac{1}{(8+8x-x^3+8x^4)^2}\,dx = -\frac{207+29\left(1+\frac{4}{x}\right)^2}{336\left(261-6\left(1+\frac{4}{x}\right)^2+\left(1+\frac{4}{x}\right)^4\right)} + \frac{5\left(5157+199\left(1+\frac{4}{x}\right)^2\right)\left(1+\frac{4}{x}\right)}{87696\left(261-6\left(1+\frac{4}{x}\right)^2+\left(1+\frac{4}{x}\right)^4\right)} - \frac{17\arctan\left(\frac{3-\left(1+\frac{4}{x}\right)^2}{6\sqrt{7}}\right)}{1008\sqrt{7}} - \frac{\sqrt{\frac{180983329+45923327\sqrt{29}}{1218}}\arctan\left(\frac{2+\sqrt{6\left(1+\sqrt{29}\right)}+\frac{8}{x}}{\sqrt{6\left(-1+\sqrt{29}\right)}}\right)}{87696} - \frac{\sqrt{\frac{180983329+45923327\sqrt{29}}{1218}}\arctan\left(\frac{8+\left(2-\sqrt{6\left(1+\sqrt{29}\right)}\right)x}{\sqrt{6\left(-1+\sqrt{29}\right)}x}\right)}{87696} + \frac{\sqrt{\frac{-180983329+45923327\sqrt{29}}{1218}}\operatorname{arctanh}\left(\frac{\sqrt{6\left(1+\sqrt{29}\right)}\left(1+\frac{4}{x}\right)}{3\sqrt{29}+\left(1+\frac{4}{x}\right)^2}\right)}{87696}$$

output
```
-1/336*(207+29*(1+4/x)^2)/(261-6*(1+4/x)^2+(1+4/x)^4)+5*(5157+199*(1+4/x)^
2)*(1+4/x)/(22888656-526176*(1+4/x)^2+87696*(1+4/x)^4)-17/7056*arctan(1/42
*(3-(1+4/x)^2)*7^(1/2))*7^(1/2)-1/106813728*(220437694722+55934612286*29^(
1/2))^(1/2)*arctan((2+(6+6*29^(1/2))^(1/2)+8/x)/(-6+6*29^(1/2))^(1/2))-1/1
06813728*(220437694722+55934612286*29^(1/2))^(1/2)*arctan((8+(2-(6+6*29^(1
/2))^(1/2))*x)/(-6+6*29^(1/2))^(1/2)/x)+1/106813728*(-220437694722+5593461
2286*29^(1/2))^(1/2)*arctanh((6+6*29^(1/2))^(1/2)*(1+4/x)/(3*29^(1/2)+(1+4
/x)^2))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.38

$$\int \frac{1}{(8+8x-x^3+8x^4)^2}\,dx = \frac{544+1539x-1146x^2+784x^3}{43848\,(8+8x-x^3+8x^4)} + \frac{\text{RootSum}\left[8+8\#1-\#1^3+8\#1^4\&,\frac{2243\log(x-\#1)-1097\log(x-\#1)\#1+392\log(x-\#1)\#1^2}{8-3\#1^2+32\#1^3}\&\right]}{21924}$$

input
```
Integrate[(8 + 8*x - x^3 + 8*x^4)^(-2),x]
```

output
```
(544 + 1539*x - 1146*x^2 + 784*x^3)/(43848*(8 + 8*x - x^3 + 8*x^4)) + Root
Sum[8 + 8*#1 - #1^3 + 8*#1^4 & , (2243*Log[x - #1] - 1097*Log[x - #1]*#1 +
 392*Log[x - #1]*#1^2)/(8 - 3*#1^2 + 32*#1^3) & ]/21924
```

Rubi [A] (verified)

Time = 1.38 (sec) , antiderivative size = 432, normalized size of antiderivative = 1.44, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 1.059$, Rules used = {2504, 27, 2202, 2194, 27, 2191, 27, 1083, 217, 2206, 27, 1483, 27, 1142, 27, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(8x^4-x^3+8x+8)^2}dx$$

$$\Big\downarrow \text{2504}$$

$$-1024\int \frac{\left(1-4\left(\frac{1}{4}+\frac{1}{x}\right)\right)^6}{4096\left(256\left(\frac{1}{4}+\frac{1}{x}\right)^4-96\left(\frac{1}{4}+\frac{1}{x}\right)^2+261\right)^2}d\left(\frac{1}{4}+\frac{1}{x}\right)$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{4}\int \frac{\left(1-4\left(\frac{1}{4}+\frac{1}{x}\right)\right)^6}{\left(256\left(\frac{1}{4}+\frac{1}{x}\right)^4-96\left(\frac{1}{4}+\frac{1}{x}\right)^2+261\right)^2}d\left(\frac{1}{4}+\frac{1}{x}\right)$$

$$\Big\downarrow \text{2202}$$

$$\frac{1}{4}\left(-\int \frac{4096\left(\frac{1}{4}+\frac{1}{x}\right)^6+3840\left(\frac{1}{4}+\frac{1}{x}\right)^4+240\left(\frac{1}{4}+\frac{1}{x}\right)^2+1}{\left(256\left(\frac{1}{4}+\frac{1}{x}\right)^4-96\left(\frac{1}{4}+\frac{1}{x}\right)^2+261\right)^2}d\left(\frac{1}{4}+\frac{1}{x}\right)-\int \frac{\left(-6144\left(\frac{1}{4}+\frac{1}{x}\right)^4-1280\left(\frac{1}{4}+\frac{1}{x}\right)^2-\right.}{\left(256\left(\frac{1}{4}+\frac{1}{x}\right)^4-96\left(\frac{1}{4}+\frac{1}{x}\right)^2+\right.}\right.$$

$$\Big\downarrow \text{2194}$$

$$\frac{1}{4}\left(-\frac{1}{2}\int -\frac{8\left(768\left(\frac{1}{4}+\frac{1}{x}\right)^4+160\left(\frac{1}{4}+\frac{1}{x}\right)^2+3\right)}{\left(256\left(\frac{1}{4}+\frac{1}{x}\right)^4-96\left(\frac{1}{4}+\frac{1}{x}\right)^2+261\right)^2}d\left(\frac{1}{4}+\frac{1}{x}\right)^2-\int \frac{4096\left(\frac{1}{4}+\frac{1}{x}\right)^6+3840\left(\frac{1}{4}+\frac{1}{x}\right)^4+240\left(\frac{1}{4}+\frac{1}{x}\right.}{\left(256\left(\frac{1}{4}+\frac{1}{x}\right)^4-96\left(\frac{1}{4}+\frac{1}{x}\right)^2+261\right)^2}\right.$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4}\left(4\int \frac{768\left(\frac{1}{4}+\frac{1}{x}\right)^4+160\left(\frac{1}{4}+\frac{1}{x}\right)^2+3}{\left(256\left(\frac{1}{4}+\frac{1}{x}\right)^4-96\left(\frac{1}{4}+\frac{1}{x}\right)^2+261\right)^2}d\left(\frac{1}{4}+\frac{1}{x}\right)^2-\int \frac{4096\left(\frac{1}{4}+\frac{1}{x}\right)^6+3840\left(\frac{1}{4}+\frac{1}{x}\right)^4+240\left(\frac{1}{4}+\frac{1}{x}\right)^2+}{\left(256\left(\frac{1}{4}+\frac{1}{x}\right)^4-96\left(\frac{1}{4}+\frac{1}{x}\right)^2+261\right)^2}\right.$$

$$\Big\downarrow \text{2191}$$

$$\frac{1}{4}\left(4\left(\frac{\int \frac{417792}{256\left(\frac{1}{4}+\frac{1}{x}\right)^4-96\left(\frac{1}{4}+\frac{1}{x}\right)^2+261}d\left(\frac{1}{4}+\frac{1}{x}\right)^2}{258048}-\frac{464\left(\frac{1}{x}+\frac{1}{4}\right)^2+207}{336\left(256\left(\frac{1}{x}+\frac{1}{4}\right)^4-96\left(\frac{1}{x}+\frac{1}{4}\right)^2+261\right)}\right)-\int \frac{4096\left(\frac{1}{4}+\frac{1}{x}\right)^6+}{\left(256\left(\frac{1}{4}+\right.\right.}\right.$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4}\left(4\left(\frac{34}{21}\int \frac{1}{256\left(\frac{1}{4}+\frac{1}{x}\right)^4-96\left(\frac{1}{4}+\frac{1}{x}\right)^2+261}d\left(\frac{1}{4}+\frac{1}{x}\right)^2-\frac{464\left(\frac{1}{x}+\frac{1}{4}\right)^2+207}{336\left(256\left(\frac{1}{x}+\frac{1}{4}\right)^4-96\left(\frac{1}{x}+\frac{1}{4}\right)^2+261\right)}\right)-\int\right.$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{4}\left(4\left(-\frac{68}{21}\int \frac{1}{-\left(\frac{1}{4}+\frac{1}{x}\right)^4-258048}d\left(512\left(\frac{1}{4}+\frac{1}{x}\right)^2-96\right)-\frac{464\left(\frac{1}{x}+\frac{1}{4}\right)^2+207}{336\left(256\left(\frac{1}{x}+\frac{1}{4}\right)^4-96\left(\frac{1}{x}+\frac{1}{4}\right)^2+261\right)}\right)-\int$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{4}\left(4\left(\frac{17\arctan\left(\frac{512\left(\frac{1}{x}+\frac{1}{4}\right)^2-96}{192\sqrt{7}}\right)}{1008\sqrt{7}}-\frac{464\left(\frac{1}{x}+\frac{1}{4}\right)^2+207}{336\left(256\left(\frac{1}{x}+\frac{1}{4}\right)^4-96\left(\frac{1}{x}+\frac{1}{4}\right)^2+261\right)}\right)-\int\frac{4096\left(\frac{1}{4}+\frac{1}{x}\right)^6+3840\left(\frac{1}{4}+\right.}{\left(256\left(\frac{1}{4}+\frac{1}{x}\right)^4-96\right.}\right.$$

$$\Big\downarrow \text{2206}$$

$$\frac{1}{4}\left(-\frac{\int\frac{49152\left(35888\left(\frac{1}{4}+\frac{1}{x}\right)^2+12903\right)}{256\left(\frac{1}{4}+\frac{1}{x}\right)^4-96\left(\frac{1}{4}+\frac{1}{x}\right)^2+261}d\left(\frac{1}{4}+\frac{1}{x}\right)}{134701056}+4\left(\frac{17\arctan\left(\frac{512\left(\frac{1}{x}+\frac{1}{4}\right)^2-96}{192\sqrt{7}}\right)}{1008\sqrt{7}}-\frac{464\left(\frac{1}{x}+\frac{1}{4}\right)^2+207}{336\left(256\left(\frac{1}{x}+\frac{1}{4}\right)^4-96\left(\frac{1}{x}+\frac{1}{4}\right)^2-\right.}\right.\right.$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4}\left(-\frac{2\int\frac{35888\left(\frac{1}{4}+\frac{1}{x}\right)^2+12903}{256\left(\frac{1}{4}+\frac{1}{x}\right)^4-96\left(\frac{1}{4}+\frac{1}{x}\right)^2+261}d\left(\frac{1}{4}+\frac{1}{x}\right)}{5481}+4\left(\frac{17\arctan\left(\frac{512\left(\frac{1}{x}+\frac{1}{4}\right)^2-96}{192\sqrt{7}}\right)}{1008\sqrt{7}}-\frac{464\left(\frac{1}{x}+\frac{1}{4}\right)^2+207}{336\left(256\left(\frac{1}{x}+\frac{1}{4}\right)^4-96\left(\frac{1}{x}+\frac{1}{4}\right)^2\right.}\right.\right.$$

$$\Big\downarrow \text{1483}$$

$$\frac{1}{4}\left(-\frac{2\left(\frac{\int\frac{24\left(4301\sqrt{\frac{3}{2}\left(1+\sqrt{29}\right)}-2\left(4301-2243\sqrt{29}\right)\left(\frac{1}{4}+\frac{1}{x}\right)\right)}{16\left(\frac{1}{4}+\frac{1}{x}\right)^2-4\sqrt{6\left(1+\sqrt{29}\right)}\left(\frac{1}{4}+\frac{1}{x}\right)+3\sqrt{29}}d\left(\frac{1}{4}+\frac{1}{x}\right)}{24\sqrt{174\left(1+\sqrt{29}\right)}}+\frac{\int\frac{24\left(2\left(4301-2243\sqrt{29}\right)\left(\frac{1}{4}+\frac{1}{x}\right)+4301\sqrt{\frac{3}{2}\left(1+\sqrt{29}\right)}\right)}{16\left(\frac{1}{4}+\frac{1}{x}\right)^2+4\sqrt{6\left(1+\sqrt{29}\right)}\left(\frac{1}{4}+\frac{1}{x}\right)+3\sqrt{29}}d\left(\frac{1}{4}+\frac{1}{x}\right)}{24\sqrt{174\left(1+\sqrt{29}\right)}}\right)}{5481}+4\left(\right.\right.$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4}\left(-\frac{2\left(\frac{\int \frac{4301\sqrt{\frac{3}{2}\left(1+\sqrt{29}\right)}-2\left(4301-2243\sqrt{29}\right)\left(\frac{1}{4}+\frac{1}{x}\right)}{16\left(\frac{1}{4}+\frac{1}{x}\right)^2-4\sqrt{6\left(1+\sqrt{29}\right)}\left(\frac{1}{4}+\frac{1}{x}\right)+3\sqrt{29}}d\left(\frac{1}{4}+\frac{1}{x}\right)}{\sqrt{174\left(1+\sqrt{29}\right)}}+\frac{\int \frac{2\left(4301-2243\sqrt{29}\right)\left(\frac{1}{4}+\frac{1}{x}\right)+4301\sqrt{\frac{3}{2}\left(1+\sqrt{29}\right)}}{16\left(\frac{1}{4}+\frac{1}{x}\right)^2+4\sqrt{6\left(1+\sqrt{29}\right)}\left(\frac{1}{4}+\frac{1}{x}\right)+3\sqrt{29}}d\left(\frac{1}{4}+\frac{1}{x}\right)}{\sqrt{174\left(1+\sqrt{29}\right)}}\right)}{5481}+4\left(\frac{17\,\mathrm{arcta}\cdots}{\cdots}\right.\right.$$

$\big\downarrow$ 1142

$$\frac{1}{4}\left(-\frac{2\left(\frac{\frac{1}{2}\sqrt{\frac{3}{2}\left(1+\sqrt{29}\right)}\left(4301+2243\sqrt{29}\right)\int \frac{1}{16\left(\frac{1}{4}+\frac{1}{x}\right)^2-4\sqrt{6\left(1+\sqrt{29}\right)}\left(\frac{1}{4}+\frac{1}{x}\right)+3\sqrt{29}}d\left(\frac{1}{4}+\frac{1}{x}\right)-\frac{1}{16}\left(4301-2243\sqrt{29}\right)\int -\frac{4\left(\sqrt{6\left(1+\sqrt{29}\right)}\cdots\right.}{16\left(\frac{1}{4}+\frac{1}{x}\right)^2-4\sqrt{6\left(1+\cdots\right.}}\cdots}{\sqrt{174\left(1+\sqrt{29}\right)}}\cdots\right.}{\cdots}\right.$$

$\big\downarrow$ 27

$$\frac{1}{4}\left(-\frac{2\left(\frac{\frac{1}{2}\sqrt{\frac{3}{2}\left(1+\sqrt{29}\right)}\left(4301+2243\sqrt{29}\right)\int \frac{1}{16\left(\frac{1}{4}+\frac{1}{x}\right)^2-4\sqrt{6\left(1+\sqrt{29}\right)}\left(\frac{1}{4}+\frac{1}{x}\right)+3\sqrt{29}}d\left(\frac{1}{4}+\frac{1}{x}\right)+\frac{1}{4}\left(4301-2243\sqrt{29}\right)\int \frac{\sqrt{6\left(1+\sqrt{29}\right)}-8\left(\cdots\right.}{16\left(\frac{1}{4}+\frac{1}{x}\right)^2-4\sqrt{6\left(1+\sqrt{29}\right)\cdots}}\cdots}{\sqrt{174\left(1+\sqrt{29}\right)}}\cdots\right.}{\cdots}\right.$$

$\big\downarrow$ 1083

$$\frac{1}{4}\left(-\frac{2\left(\frac{\frac{1}{4}\left(4301-2243\sqrt{29}\right)\int\frac{\sqrt{6\left(1+\sqrt{29}\right)}-8\left(\frac{1}{4}+\frac{1}{x}\right)}{16\left(\frac{1}{4}+\frac{1}{x}\right)^2-4\sqrt{6\left(1+\sqrt{29}\right)}\left(\frac{1}{4}+\frac{1}{x}\right)+3\sqrt{29}}d\left(\frac{1}{4}+\frac{1}{x}\right)-\sqrt{\frac{3}{2}\left(1+\sqrt{29}\right)}\left(4301+2243\sqrt{29}\right)\int\frac{1}{96\left(1-\sqrt{29}\right)-\left(32\left(\frac{1}{4}+\frac{1}{x}\right)-4\right.}}{\sqrt{174\left(1+\sqrt{29}\right)}}\right.}{}\right.$$

$$\Big\downarrow \; 217$$

$$\frac{1}{4}\left(-\frac{2\left(\frac{\frac{1}{4}\left(4301-2243\sqrt{29}\right)\int\frac{\sqrt{6\left(1+\sqrt{29}\right)}-8\left(\frac{1}{4}+\frac{1}{x}\right)}{16\left(\frac{1}{4}+\frac{1}{x}\right)^2-4\sqrt{6\left(1+\sqrt{29}\right)}\left(\frac{1}{4}+\frac{1}{x}\right)+3\sqrt{29}}d\left(\frac{1}{4}+\frac{1}{x}\right)+\frac{1}{8}\sqrt{\frac{1+\sqrt{29}}{\sqrt{29}-1}}\left(4301+2243\sqrt{29}\right)\arctan\left(\frac{32\left(\frac{1}{x}+\frac{1}{4}\right)-4\sqrt{6\left(1+\sqrt{29}\right.}}{4\sqrt{6\left(\sqrt{29}-1\right)}}\right.}{\sqrt{174\left(1+\sqrt{29}\right)}}\right.}{}\right.$$

$$\Big\downarrow \; 1103$$

$$\frac{1}{4}\left(4\left(\frac{17\arctan\left(\frac{512\left(\frac{1}{x}+\frac{1}{4}\right)^2-96}{192\sqrt{7}}\right)}{1008\sqrt{7}}-\frac{464\left(\frac{1}{x}+\frac{1}{4}\right)^2+207}{336\left(256\left(\frac{1}{x}+\frac{1}{4}\right)^4-96\left(\frac{1}{x}+\frac{1}{4}\right)^2+261\right)}\right)-\frac{2\left(\frac{\frac{1}{8}\sqrt{\frac{1+\sqrt{29}}{\sqrt{29}-1}}\left(4301+2243\sqrt{29}\right)\text{arc}\ldots}{}\right.}{}\right.$$

input
```
Int[(8 + 8*x - x^3 + 8*x^4)^(-2),x]
```

output
```
((5*(5157 + 3184*(1/4 + x^(-1))^2)*(1/4 + x^(-1)))/(5481*(261 - 96*(1/4 +
x^(-1))^2 + 256*(1/4 + x^(-1))^4)) + 4*(-1/336*(207 + 464*(1/4 + x^(-1))^2
)/(261 - 96*(1/4 + x^(-1))^2 + 256*(1/4 + x^(-1))^4) + (17*ArcTan[(-96 + 5
12*(1/4 + x^(-1))^2)/(192*Sqrt[7])])/(1008*Sqrt[7])) - (2*(((Sqrt[(1 + Sqr
t[29])/(-1 + Sqrt[29])]*(4301 + 2243*Sqrt[29])*ArcTan[(-4*Sqrt[6*(1 + Sqrt
[29])] + 32*(1/4 + x^(-1)))/(4*Sqrt[6*(-1 + Sqrt[29])])]])/8 - ((4301 - 224
3*Sqrt[29])*Log[3*Sqrt[29] - 4*Sqrt[6*(1 + Sqrt[29])]*(1/4 + x^(-1)) + 16*
(1/4 + x^(-1))^2])/16)/Sqrt[174*(1 + Sqrt[29])] + ((Sqrt[(1 + Sqrt[29])/(-
1 + Sqrt[29])]*(4301 + 2243*Sqrt[29])*ArcTan[(4*Sqrt[6*(1 + Sqrt[29])] + 3
2*(1/4 + x^(-1)))/(4*Sqrt[6*(-1 + Sqrt[29])])]])/8 + ((4301 - 2243*Sqrt[29]
)*Log[3*Sqrt[29] + 4*Sqrt[6*(1 + Sqrt[29])]*(1/4 + x^(-1)) + 16*(1/4 + x^(
-1))^2])/16)/Sqrt[174*(1 + Sqrt[29])]))/5481)/4
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1483
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   In
t[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(d*r
 + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 2194
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Simp[1/2   Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)
^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ
[(m - 1)/2]
```

rule 2202
```
Int[(Pn_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{n
 = Expon[Pn, x], k}, Int[Sum[Coeff[Pn, x, 2*k]*x^(2*k), {k, 0, n/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pn, x, 2*k + 1]*x^(2*k), {k, 0, (n -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pn, x]
 &&  !PolyQ[Pn, x^2]
```

rule 2206
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c
*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Px,
a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*
p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x
^2] && Expon[Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 2504
```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Simp[-16*
a^2   Subst[Int[(1/(b - 4*a*x)^2)*(a*((-3*b^4 + 16*a*b^2*c - 64*a^2*b*d + 2
56*a^3*e - 32*a^2*(3*b^2 - 8*a*c)*x^2 + 256*a^4*x^4)/(b - 4*a*x)^4))^p, x],
 x, b/(4*a) + 1/x], x] /; NeQ[a, 0] && NeQ[b, 0] && EqQ[b^3 - 4*a*b*c + 8*a
^2*d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && IntegerQ[2*p] &&  !IGtQ[p, 0
]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.07 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.28

method	result	size
default	$\frac{\frac{7}{3132}x^{3}-\frac{191}{58464}x^{2}+\frac{57}{12992}x+\frac{17}{10962}}{x^{4}-\frac{1}{8}x^{3}+x+1}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(8\_Z^{4}-\_Z^{3}+8\_Z+8\right)}\frac{\left(392\_R^{2}-1097\_R+2243\right)\ln\left(x-\_R\right)}{32\_R^{3}-3\_R^{2}+8}\right)}{21924}$	83
risch	$\frac{\frac{7}{3132}x^{3}-\frac{191}{58464}x^{2}+\frac{57}{12992}x+\frac{17}{10962}}{x^{4}-\frac{1}{8}x^{3}+x+1}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(8\_Z^{4}-\_Z^{3}+8\_Z+8\right)}\frac{\left(392\_R^{2}-1097\_R+2243\right)\ln\left(x-\_R\right)}{32\_R^{3}-3\_R^{2}+8}\right)}{21924}$	83

input
```
int(1/(8*x^4-x^3+8*x+8)^2,x,method=_RETURNVERBOSE)
```

output
```
(7/3132*x^3-191/58464*x^2+57/12992*x+17/10962)/(x^4-1/8*x^3+x+1)+1/21924*s
um((392*_R^2-1097*_R+2243)/(32*_R^3-3*_R^2+8)*ln(x-_R),_R=RootOf(8*_Z^4-_Z
^3+8*_Z+8))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 398, normalized size of antiderivative = 1.33

$$\int \frac{1}{(8+8x-x^3+8x^4)^2} \, dx = \text{Too large to display}$$

input
```
integrate(1/(8*x^4-x^3+8*x+8)^2,x, algorithm="fricas")
```

output
```
1/1227744*(21952*x^3 - 2*(8*x^4 - x^3 + 8*x + 8)*sqrt(1479/4550065*(459233
27*sqrt(29) + 180983329)*sqrt(1583563/42*sqrt(29) - 180983329/1218) + 1108
4941/6*sqrt(29) + 3931186441/174)*arctan(-1/47504432368626919842574 60*(159
252275*sqrt(29)*(7172449575781*x - 4063334408258) - 174*(sqrt(29)*(2385676
8674269633*x - 20350906238895184) + 117938631918963916*x - 102889113598186
588)*sqrt(1583563/42*sqrt(29) - 180983329/1218) + 5194358934701729637845*x
 - 4202037046136740466110)*sqrt(1479/4550065*(45923327*sqrt(29) + 18098332
9)*sqrt(1583563/42*sqrt(29) - 180983329/1218) + 11084941/6*sqrt(29) + 3931
186441/174)) + 2*(8*x^4 - x^3 + 8*x + 8)*sqrt(-1479/4550065*(45923327*sqrt
(29) + 180983329)*sqrt(1583563/42*sqrt(29) - 180983329/1218) + 11084941/6*
sqrt(29) + 3931186441/174)*arctan(1/47504432368626919842574 60*(159252275*s
qrt(29)*(7172449575781*x - 4063334408258) + 174*(sqrt(29)*(238567686742696
33*x - 20350906238895184) + 117938631918963916*x - 102889113598186588)*sqr
t(1583563/42*sqrt(29) - 180983329/1218) + 5194358934701729637845*x - 42020
37046136740466110)*sqrt(-1479/4550065*(45923327*sqrt(29) + 180983329)*sqrt
(1583563/42*sqrt(29) - 180983329/1218) + 11084941/6*sqrt(29) + 3931186441/
174)) + 7*(8*x^4 - x^3 + 8*x + 8)*sqrt(1583563/42*sqrt(29) - 180983329/121
8)*log(1019214560*x^2 + 3*(3*sqrt(29)*(41665*x - 23116) + 1460875*x - 1897
76)*sqrt(1583563/42*sqrt(29) - 180983329/1218) - 63700910*x + 191102730*sq
rt(29) + 63700910) - 7*(8*x^4 - x^3 + 8*x + 8)*sqrt(1583563/42*sqrt(29)...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3834 vs. 2(228) = 456.

Time = 1.78 (sec) , antiderivative size = 3834, normalized size of antiderivative = 12.78

$$\int \frac{1}{(8+8x-x^3+8x^4)^2} \, dx = \text{Too large to display}$$

input
```
integrate(1/(8*x**4-x**3+8*x+8)**2,x)
```

output
```
(784*x**3 - 1146*x**2 + 1539*x + 544)/(350784*x**4 - 43848*x**3 + 350784*x
 + 350784) - sqrt(-180983329/37468546762752 + 1583563*sqrt(29)/12920188538
88)*log(x**2 + x*(-6271675673O859*sqrt(1218)*sqrt(-180983329 + 45923327*sq
rt(29))*sqrt(214095423017213*sqrt(29) + 47106822945*sqrt(1218)*sqrt(-18098
3329 + 45923327*sqrt(29)) + 40699873480352667)/227008323264998681573683424
 - 267658292345340*sqrt(214095423017213*sqrt(29) + 47106822945*sqrt(1218)*
sqrt(-180983329 + 45923327*sqrt(29)) + 40699873480352667)/8435208206933660
878927 - 215737452097O352866823*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt
(29))/113504161632499340786841712 + 3881045239007430*sqrt(29)/53267272643б
1229 + 435853770857118353330297/337408328277346435157O8 + 20905585576953*s
qrt(42)*sqrt(-180983329 + 45923327*sqrt(29))/85227636229779664) - 29428140
7410142941508403051018220425006755695З*sqrt(214095423017213*sqrt(29) + 471
06822945*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29)) + 406998734803526
67)/888496186751485201253966401139075287452416534006272 - 142576256328563l
4835831142972765102609010539559351093/277655058359839125391864500355961027
32888016687696 - 751846315028188373888759000608813558Z1*sqrt(1218)*sqrt(-1
80983329 + 45923327*sqrt(29))*sqrt(214095423017213*sqrt(29) + 47106822945*
sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29)) + 40699873480352667)/30637
79954315466211220573797031294094663505289676８ - 963314181796141259748858７6
61065704878094062299*sqrt(1218)*sqrt(-180983329 + 45923327*sqrt(29))/30...
```

Maxima [F]

$$\int \frac{1}{(8+8x-x^3+8x^4)^2}\, dx = \int \frac{1}{(8\,x^4-x^3+8\,x+8)^2}\, dx$$

input
```
integrate(1/(8*x^4-x^3+8*x+8)^2,x, algorithm="maxima")
```

output
```
1/43848*(784*x^3 - 1146*x^2 + 1539*x + 544)/(8*x^4 - x^3 + 8*x + 8) + 1/21
924*integrate((392*x^2 - 1097*x + 2243)/(8*x^4 - x^3 + 8*x + 8), x)
```

Giac [F]

$$\int \frac{1}{(8+8x-x^3+8x^4)^2}\,dx = \int \frac{1}{(8\,x^4-x^3+8\,x+8)^2}\,dx$$

input
```
integrate(1/(8*x^4-x^3+8*x+8)^2,x, algorithm="giac")
```

output
```
integrate((8*x^4 - x^3 + 8*x + 8)^(-2), x)
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.59

$$\int \frac{1}{(8+8x-x^3+8x^4)^2}\,dx = \left(\sum_{k=1}^{4}\ln\left(\left(\frac{2615257\,\mathrm{root}\left(z^4+\frac{6630191\,z^2}{167270298048}+\frac{77351105\,z}{674433841729536}+\frac{1114096}{13723971258377709},z,k\right)}{72918171648}+\frac{4225\,x}{40375589184}-\frac{\mathrm{root}\left(z^4+\frac{6630191\,z^2}{167270298048}+\frac{77351105\,z}{674433841729536}+\frac{1114096}{13723971258377709},z,k\right)\,x\,34885379}{72918171648}-\frac{\mathrm{root}\left(z^4+\frac{6630191\,z^2}{167270298048}+\frac{77351105\,z}{674433841729536}+\frac{1114096}{13723971258377709},z,k\right)^2\,x\,191555}{475136}-\frac{\mathrm{root}\left(z^4+\frac{6630191\,z^2}{167270298048}+\frac{77351105\,z}{674433841729536}+\frac{1114096}{13723971258377709},z,k\right)^3\,x\,9261}{256}-\frac{11205\,\mathrm{root}\left(z^4+\frac{6630191\,z^2}{167270298048}+\frac{77351105\,z}{674433841729536}+\frac{1114096}{13723971258377709},z,k\right)^2}{59392}-\frac{24759\,\mathrm{root}\left(z^4+\frac{6630191\,z^2}{167270298048}+\frac{77351105\,z}{674433841729536}+\frac{1114096}{13723971258377709},z,k\right)^3}{512}+\frac{10901}{107668237824}\right)\mathrm{root}\left(z^4+\frac{6630191\,z^2}{167270298048}+\frac{77351105\,z}{674433841729536}+\frac{1114096}{13723971258377709},z,k\right)\right)\right)+\frac{\frac{7\,x^3}{3132}-\frac{191\,x^2}{58464}+\frac{57\,x}{12992}+\frac{17}{10962}}{x^4-\frac{x^3}{8}+x+1}$$

input
```
int(1/(8*x - x^3 + 8*x^4 + 8)^2,x)
```

output
```
symsum(log((2615257*root(z^4 + (6630191*z^2)/167270298048 + (77351105*z)/6
74433841729536 + 1114096/13723971258377709, z, k))/72918171648 + (4225*x)/
40375589184 - (34885379*root(z^4 + (6630191*z^2)/167270298048 + (77351105*
z)/674433841729536 + 1114096/13723971258377709, z, k)*x)/72918171648 - (19
1555*root(z^4 + (6630191*z^2)/167270298048 + (77351105*z)/674433841729536
+ 1114096/13723971258377709, z, k)^2*x)/475136 - (9261*root(z^4 + (6630191
*z^2)/167270298048 + (77351105*z)/674433841729536 + 1114096/13723971258377
709, z, k)^3*x)/256 - (11205*root(z^4 + (6630191*z^2)/167270298048 + (7735
1105*z)/674433841729536 + 1114096/13723971258377709, z, k)^2)/59392 - (247
59*root(z^4 + (6630191*z^2)/167270298048 + (77351105*z)/674433841729536 +
1114096/13723971258377709, z, k)^3)/512 + 10901/107668237824)*root(z^4 + (
6630191*z^2)/167270298048 + (77351105*z)/674433841729536 + 1114096/1372397
1258377709, z, k), k, 1, 4) + ((57*x)/12992 - (191*x^2)/58464 + (7*x^3)/31
32 + 17/10962)/(x - x^3/8 + x^4 + 1)
```

Reduce [F]

$$\int \frac{1}{(8+8x-x^3+8x^4)^2}\,dx = \int \frac{1}{64x^8-16x^7+x^6+128x^5+112x^4-16x^3+64x^2+128x+64}dx$$

input
```
int(1/(8*x^4-x^3+8*x+8)^2,x)
```

output
```
int(1/(64*x**8 - 16*x**7 + x**6 + 128*x**5 + 112*x**4 - 16*x**3 + 64*x**2
+ 128*x + 64),x)
```

3.79 $\int (8+24x+8x^2-15x^3+8x^4)^4\,dx$

Optimal result

Integrand size = 22, antiderivative size = 104

$$\int (8+24x+8x^2-15x^3+8x^4)^4\,dx=4096x+24576x^2+\frac{237568x^3}{3}+139776x^4+\frac{538624x^5}{5}-30720x^6-\frac{566912x^7}{7}+36384x^8+\frac{641152x^9}{9}-\frac{169584x^{10}}{5}-\frac{331040x^{11}}{11}+31128x^{12}-\frac{12095x^{13}}{13}-\frac{75504x^{14}}{7}+\frac{102784x^{15}}{15}-1920x^{16}+\frac{4096x^{17}}{17}$$

output
```
4096*x+24576*x^2+237568/3*x^3+139776*x^4+538624/5*x^5-30720*x^6-566912/7*x
^7+36384*x^8+641152/9*x^9-169584/5*x^10-331040/11*x^11+31128*x^12-12095/13
*x^13-75504/7*x^14+102784/15*x^15-1920*x^16+4096/17*x^17
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00

$$\int \left(8+24x+8x^2-15x^3+8x^4\right)^4 dx = 4096x+24576x^2+\frac{237568x^3}{3}+139776x^4+\frac{538624x^5}{5} - 30720x^6-\frac{566912x^7}{7}+36384x^8+\frac{641152x^9}{9} - \frac{169584x^{10}}{5}-\frac{331040x^{11}}{11}+31128x^{12}-\frac{12095x^{13}}{13} - \frac{75504x^{14}}{7}+\frac{102784x^{15}}{15}-1920x^{16}+\frac{4096x^{17}}{17}$$

input
```
Integrate[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^4,x]
```

output
```
4096*x + 24576*x^2 + (237568*x^3)/3 + 139776*x^4 + (538624*x^5)/5 - 30720*
x^6 - (566912*x^7)/7 + 36384*x^8 + (641152*x^9)/9 - (169584*x^10)/5 - (331
040*x^11)/11 + 31128*x^12 - (12095*x^13)/13 - (75504*x^14)/7 + (102784*x^1
5)/15 - 1920*x^16 + (4096*x^17)/17
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {2465, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(8x^4-15x^3+8x^2+24x+8\right)^4 dx$$

$$\Big\downarrow \text{2465}$$

$$\int \left(4096x^{16}-30720x^{15}+102784x^{14}-151008x^{13}-12095x^{12}+373536x^{11}-331040x^{10}-339168x^9+641152x\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{4096x^{17}}{17}-1920x^{16}+\frac{102784x^{15}}{15}-\frac{75504x^{14}}{7}-\frac{12095x^{13}}{13}+31128x^{12}-\frac{331040x^{11}}{11}-\frac{169584x^{10}}{5}+\frac{641152x^9}{9}+36384x^8-\frac{566912x^7}{7}-30720x^6+\frac{538624x^5}{5}+139776x^4+\frac{237568x^3}{3}+24576x^2+4096x$$

input
```
Int[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^4,x]
```

output
```
4096*x + 24576*x^2 + (237568*x^3)/3 + 139776*x^4 + (538624*x^5)/5 - 30720*
x^6 - (566912*x^7)/7 + 36384*x^8 + (641152*x^9)/9 - (169584*x^10)/5 - (331
040*x^11)/11 + 31128*x^12 - (12095*x^13)/13 - (75504*x^14)/7 + (102784*x^1
5)/15 - 1920*x^16 + (4096*x^17)/17
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2465
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> Int[ExpandToSum[u, Px^p, x], x] /; PolyQ
[Px, x] && GtQ[Expon[Px, x], 2] && !BinomialQ[Px, x] && !TrinomialQ[Px, x
] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.81

method	result
orering	$\frac{x(184504320x^{16}-1470268800x^{15}+5247225984x^{14}-8259760080x^{13}-712455975x^{12}+23836732920x^{11}-23045349600x^{10}-2}{}$
gosper	$4096x+24576x^2+\frac{237568}{3}x^3+139776x^4+\frac{538624}{5}x^5-30720x^6-\frac{566912}{7}x^7+36384x^8+\frac{641}{9}$
default	$4096x+24576x^2+\frac{237568}{3}x^3+139776x^4+\frac{538624}{5}x^5-30720x^6-\frac{566912}{7}x^7+36384x^8+\frac{641}{9}$
norman	$4096x+24576x^2+\frac{237568}{3}x^3+139776x^4+\frac{538624}{5}x^5-30720x^6-\frac{566912}{7}x^7+36384x^8+\frac{641}{9}$
risch	$4096x+24576x^2+\frac{237568}{3}x^3+139776x^4+\frac{538624}{5}x^5-30720x^6-\frac{566912}{7}x^7+36384x^8+\frac{641}{9}$
parallelrisch	$4096x+24576x^2+\frac{237568}{3}x^3+139776x^4+\frac{538624}{5}x^5-30720x^6-\frac{566912}{7}x^7+36384x^8+\frac{641}{9}$

input
```
int((8*x^4-15*x^3+8*x^2+24*x+8)^4,x,method=_RETURNVERBOSE)
```

output
```
1/765765*x*(184504320*x^16-1470268800*x^15+5247225984*x^14-8259760080*x^13
-712455975*x^12+23836732920*x^11-23045349600*x^10-25972298352*x^9+54552417
920*x^8+27861593760*x^7-62017338240*x^6-23524300800*x^5+82491881472*x^4+10
7035568640*x^3+60640419840*x^2+18819440640*x+3136573440)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.81

$$\int \left(8+24x+8x^2-15x^3+8x^4\right)^4 \, dx = \frac{4096}{17}\,x^{17} - 1920\,x^{16} + \frac{102784}{15}\,x^{15} - \frac{75504}{7}\,x^{14} - \frac{12095}{13}\,x^{13} + 31128\,x^{12} - \frac{331040}{11}\,x^{11} - \frac{169584}{5}\,x^{10} + \frac{641152}{9}\,x^{9} + 36384\,x^{8} - \frac{566912}{7}\,x^{7} - 30720\,x^{6} + \frac{538624}{5}\,x^{5} + 139776\,x^{4} + \frac{237568}{3}\,x^{3} + 24576\,x^{2} + 4096\,x$$

input
```
integrate((8*x^4-15*x^3+8*x^2+24*x+8)^4,x, algorithm="fricas")
```

output
```
4096/17*x^17 - 1920*x^16 + 102784/15*x^15 - 75504/7*x^14 - 12095/13*x^13 +
 31128*x^12 - 331040/11*x^11 - 169584/5*x^10 + 641152/9*x^9 + 36384*x^8 -
566912/7*x^7 - 30720*x^6 + 538624/5*x^5 + 139776*x^4 + 237568/3*x^3 + 2457
6*x^2 + 4096*x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.96

$$\int \left(8+24x+8x^2-15x^3+8x^4\right)^4 dx = \frac{4096x^{17}}{17} - 1920x^{16} + \frac{102784x^{15}}{15} - \frac{75504x^{14}}{7} - \frac{12095x^{13}}{13} + 31128x^{12} - \frac{331040x^{11}}{11} - \frac{169584x^{10}}{5} + \frac{641152x^9}{9} + 36384x^8 - \frac{566912x^7}{7} - 30720x^6 + \frac{538624x^5}{5} + 139776x^4 + \frac{237568x^3}{3} + 24576x^2 + 4096x$$

input
```
integrate((8*x**4-15*x**3+8*x**2+24*x+8)**4,x)
```

output
```
4096*x**17/17 - 1920*x**16 + 102784*x**15/15 - 75504*x**14/7 - 12095*x**13
/13 + 31128*x**12 - 331040*x**11/11 - 169584*x**10/5 + 641152*x**9/9 + 363
84*x**8 - 566912*x**7/7 - 30720*x**6 + 538624*x**5/5 + 139776*x**4 + 23756
8*x**3/3 + 24576*x**2 + 4096*x
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.81

$$\int \left(8+24x+8x^2-15x^3+8x^4\right)^4 dx = \frac{4096}{17}\,x^{17} - 1920\,x^{16} + \frac{102784}{15}\,x^{15} - \frac{75504}{7}\,x^{14} - \frac{12095}{13}\,x^{13} + 31128\,x^{12} - \frac{331040}{11}\,x^{11} - \frac{169584}{5}\,x^{10} + \frac{641152}{9}\,x^9 + 36384\,x^8 - \frac{566912}{7}\,x^7 - 30720\,x^6 + \frac{538624}{5}\,x^5 + 139776\,x^4 + \frac{237568}{3}\,x^3 + 24576\,x^2 + 4096\,x$$

input
```
integrate((8*x^4-15*x^3+8*x^2+24*x+8)^4,x, algorithm="maxima")
```

output
```
4096/17*x^17 - 1920*x^16 + 102784/15*x^15 - 75504/7*x^14 - 12095/13*x^13 +
 31128*x^12 - 331040/11*x^11 - 169584/5*x^10 + 641152/9*x^9 + 36384*x^8 -
566912/7*x^7 - 30720*x^6 + 538624/5*x^5 + 139776*x^4 + 237568/3*x^3 + 2457
6*x^2 + 4096*x
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.81

$$\int (8+24x+8x^2-15x^3+8x^4)^4\,dx = \frac{4096}{17}x^{17} - 1920\,x^{16} + \frac{102784}{15}x^{15} - \frac{75504}{7}x^{14} - \frac{12095}{13}x^{13} + 31128\,x^{12} - \frac{331040}{11}x^{11} - \frac{169584}{5}x^{10} + \frac{641152}{9}x^9 + 36384\,x^8 - \frac{566912}{7}x^7 - 30720\,x^6 + \frac{538624}{5}x^5 + 139776\,x^4 + \frac{237568}{3}x^3 + 24576\,x^2 + 4096\,x$$

input
```
integrate((8*x^4-15*x^3+8*x^2+24*x+8)^4,x, algorithm="giac")
```

output

```
4096/17*x^17 - 1920*x^16 + 102784/15*x^15 - 75504/7*x^14 - 12095/13*x^13 +
 31128*x^12 - 331040/11*x^11 - 169584/5*x^10 + 641152/9*x^9 + 36384*x^8 -
566912/7*x^7 - 30720*x^6 + 538624/5*x^5 + 139776*x^4 + 237568/3*x^3 + 2457
6*x^2 + 4096*x
```

Mupad [B] (verification not implemented)

Time = 22.95 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.81

$$\int (8+24x+8x^2-15x^3+8x^4)^4 \, dx = \frac{4096\,x^{17}}{17} - 1920\,x^{16} + \frac{102784\,x^{15}}{15} - \frac{75504\,x^{14}}{7} - \frac{12095\,x^{13}}{13} + 31128\,x^{12} - \frac{331040\,x^{11}}{11} - \frac{169584\,x^{10}}{5} + \frac{641152\,x^9}{9} + 36384\,x^8 - \frac{566912\,x^7}{7} - 30720\,x^6 + \frac{538624\,x^5}{5} + 139776\,x^4 + \frac{237568\,x^3}{3} + 24576\,x^2 + 4096\,x$$

input
```
int((24*x + 8*x^2 - 15*x^3 + 8*x^4 + 8)^4,x)
```

output
```
4096*x + 24576*x^2 + (237568*x^3)/3 + 139776*x^4 + (538624*x^5)/5 - 30720*
x^6 - (566912*x^7)/7 + 36384*x^8 + (641152*x^9)/9 - (169584*x^10)/5 - (331
040*x^11)/11 + 31128*x^12 - (12095*x^13)/13 - (75504*x^14)/7 + (102784*x^1
5)/15 - 1920*x^16 + (4096*x^17)/17
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.80

$$\int (8+24x+8x^2-15x^3+8x^4)^4 \, dx = \frac{x(184504320x^{16} - 1470268800x^{15} + 5247225984x^{14} - 8259760080x^{13} - 712455975x^{12} + 23836732920x}{}$$

input
```
int((8*x^4-15*x^3+8*x^2+24*x+8)^4,x)
```

output
```
(x*(184504320*x**16 - 1470268800*x**15 + 5247225984*x**14 - 8259760080*x**
13 - 712455975*x**12 + 23836732920*x**11 - 23045349600*x**10 - 25972298352
*x**9 + 54552417920*x**8 + 27861593760*x**7 - 62017338240*x**6 - 235243008
00*x**5 + 82491881472*x**4 + 107035568640*x**3 + 60640419840*x**2 + 188194
40640*x + 3136573440))/765765
```

3.80 $\int (8+24x+8x^2-15x^3+8x^4)^3\,dx$

Optimal result

Integrand size = 22, antiderivative size = 76

$$\int (8+24x+8x^2-15x^3+8x^4)^3\,dx = 512x+2304x^2+5120x^3+5040x^4-\frac{384x^5}{5}-2976x^6+\frac{5528x^7}{7}+2097x^8-\frac{2936x^9}{3}-\frac{4527x^{10}}{10}+\frac{6936x^{11}}{11}-240x^{12}+\frac{512x^{13}}{13}$$

output
```
512*x+2304*x^2+5120*x^3+5040*x^4-384/5*x^5-2976*x^6+5528/7*x^7+2097*x^8-29
36/3*x^9-4527/10*x^10+6936/11*x^11-240*x^12+512/13*x^13
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00

$$\int (8+24x+8x^2-15x^3+8x^4)^3\,dx = 512x+2304x^2+5120x^3+5040x^4-\frac{384x^5}{5}-2976x^6+\frac{5528x^7}{7}+2097x^8-\frac{2936x^9}{3}-\frac{4527x^{10}}{10}+\frac{6936x^{11}}{11}-240x^{12}+\frac{512x^{13}}{13}$$

input
```
Integrate[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^3,x]
```

output
```
512*x + 2304*x^2 + 5120*x^3 + 5040*x^4 - (384*x^5)/5 - 2976*x^6 + (5528*x^
7)/7 + 2097*x^8 - (2936*x^9)/3 - (4527*x^10)/10 + (6936*x^11)/11 - 240*x^1
2 + (512*x^13)/13
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {2465, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(8x^4-15x^3+8x^2+24x+8\right)^3 dx$$

$$\Big\downarrow \text{2465}$$

$$\int \left(512x^{12}-2880x^{11}+6936x^{10}-4527x^9-8808x^8+16776x^7+5528x^6-17856x^5-384x^4+20160x^3+1536\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{512x^{13}}{13}-240x^{12}+\frac{6936x^{11}}{11}-\frac{4527x^{10}}{10}-\frac{2936x^9}{3}+2097x^8+\frac{5528x^7}{7}-2976x^6-\frac{384x^5}{5}+5040x^4+5120x^3+2304x^2+512x$$

input
```
Int[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^3,x]
```

output
```
512*x + 2304*x^2 + 5120*x^3 + 5040*x^4 - (384*x^5)/5 - 2976*x^6 + (5528*x^
7)/7 + 2097*x^8 - (2936*x^9)/3 - (4527*x^10)/10 + (6936*x^11)/11 - 240*x^1
2 + (512*x^13)/13
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2465
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> Int[ExpandToSum[u, Px^p, x], x] /; PolyQ
[Px, x] && GtQ[Expon[Px, x], 2] && !BinomialQ[Px, x] && !TrinomialQ[Px, x
] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.84

method	result
orering	$\frac{x(1182720x^{12}-7207200x^{11}+18935280x^{10}-13594581x^9-29389360x^8+62972910x^7+23715120x^6-89369280x^5-2306304x^4}{30030}$
gosper	$512x+2304x^2+5120x^3+5040x^4-\frac{384}{5}x^5-2976x^6+\frac{5528}{7}x^7+2097x^8-\frac{2936}{3}x^9-\frac{4527}{10}x^1$
default	$512x+2304x^2+5120x^3+5040x^4-\frac{384}{5}x^5-2976x^6+\frac{5528}{7}x^7+2097x^8-\frac{2936}{3}x^9-\frac{4527}{10}x^1$
norman	$512x+2304x^2+5120x^3+5040x^4-\frac{384}{5}x^5-2976x^6+\frac{5528}{7}x^7+2097x^8-\frac{2936}{3}x^9-\frac{4527}{10}x^1$
risch	$512x+2304x^2+5120x^3+5040x^4-\frac{384}{5}x^5-2976x^6+\frac{5528}{7}x^7+2097x^8-\frac{2936}{3}x^9-\frac{4527}{10}x^1$
parallelrisch	$512x+2304x^2+5120x^3+5040x^4-\frac{384}{5}x^5-2976x^6+\frac{5528}{7}x^7+2097x^8-\frac{2936}{3}x^9-\frac{4527}{10}x^1$

input
```
int((8*x^4-15*x^3+8*x^2+24*x+8)^3,x,method=_RETURNVERBOSE)
```

output
```
1/30030*x*(1182720*x^12-7207200*x^11+18935280*x^10-13594581*x^9-29389360*x
^8+62972910*x^7+23715120*x^6-89369280*x^5-2306304*x^4+151351200*x^3+153753
600*x^2+69189120*x+15375360)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.84

$$\int \left(8+24x+8x^2-15x^3+8x^4\right)^3\,dx = \frac{512}{13}\,x^{13} - 240\,x^{12} + \frac{6936}{11}\,x^{11} - \frac{4527}{10}\,x^{10} - \frac{2936}{3}\,x^{9} + 2097\,x^{8} + \frac{5528}{7}\,x^{7} - 2976\,x^{6} - \frac{384}{5}\,x^{5} + 5040\,x^{4} + 5120\,x^{3} + 2304\,x^{2} + 512\,x$$

input
```
integrate((8*x^4-15*x^3+8*x^2+24*x+8)^3,x, algorithm="fricas")
```

output
```
512/13*x^13 - 240*x^12 + 6936/11*x^11 - 4527/10*x^10 - 2936/3*x^9 + 2097*x
^8 + 5528/7*x^7 - 2976*x^6 - 384/5*x^5 + 5040*x^4 + 5120*x^3 + 2304*x^2 +
512*x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.96

$$\int \left(8+24x+8x^2-15x^3+8x^4\right)^3\,dx = \frac{512x^{13}}{13} - 240x^{12} + \frac{6936x^{11}}{11} - \frac{4527x^{10}}{10} - \frac{2936x^{9}}{3} + 2097x^{8} + \frac{5528x^{7}}{7} - 2976x^{6} - \frac{384x^{5}}{5} + 5040x^{4} + 5120x^{3} + 2304x^{2} + 512x$$

input
```
integrate((8*x**4-15*x**3+8*x**2+24*x+8)**3,x)
```

output
```
512*x**13/13 - 240*x**12 + 6936*x**11/11 - 4527*x**10/10 - 2936*x**9/3 + 2
097*x**8 + 5528*x**7/7 - 2976*x**6 - 384*x**5/5 + 5040*x**4 + 5120*x**3 +
2304*x**2 + 512*x
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.84

$$\int \left(8+24x+8x^2-15x^3+8x^4\right)^3\,dx=\frac{512}{13}\,x^{13}-240\,x^{12}+\frac{6936}{11}\,x^{11}-\frac{4527}{10}\,x^{10}-\frac{2936}{3}\,x^{9}+2097\,x^{8}+\frac{5528}{7}\,x^{7}-2976\,x^{6}-\frac{384}{5}\,x^{5}+5040\,x^{4}+5120\,x^{3}+2304\,x^{2}+512\,x$$

input
```
integrate((8*x^4-15*x^3+8*x^2+24*x+8)^3,x, algorithm="maxima")
```

output
```
512/13*x^13 - 240*x^12 + 6936/11*x^11 - 4527/10*x^10 - 2936/3*x^9 + 2097*x
^8 + 5528/7*x^7 - 2976*x^6 - 384/5*x^5 + 5040*x^4 + 5120*x^3 + 2304*x^2 +
512*x
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.84

$$\int \left(8+24x+8x^2-15x^3+8x^4\right)^3\,dx=\frac{512}{13}\,x^{13}-240\,x^{12}+\frac{6936}{11}\,x^{11}-\frac{4527}{10}\,x^{10}-\frac{2936}{3}\,x^{9}+2097\,x^{8}+\frac{5528}{7}\,x^{7}-2976\,x^{6}-\frac{384}{5}\,x^{5}+5040\,x^{4}+5120\,x^{3}+2304\,x^{2}+512\,x$$

input
```
integrate((8*x^4-15*x^3+8*x^2+24*x+8)^3,x, algorithm="giac")
```

output
```
512/13*x^13 - 240*x^12 + 6936/11*x^11 - 4527/10*x^10 - 2936/3*x^9 + 2097*x
^8 + 5528/7*x^7 - 2976*x^6 - 384/5*x^5 + 5040*x^4 + 5120*x^3 + 2304*x^2 +
512*x
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.84

$$\int \left(8+24x+8x^2-15x^3+8x^4\right)^3\,dx = \frac{512\,x^{13}}{13}-240\,x^{12}+\frac{6936\,x^{11}}{11}-\frac{4527\,x^{10}}{10}-\frac{2936\,x^9}{3}+2097\,x^8+\frac{5528\,x^7}{7}-2976\,x^6-\frac{384\,x^5}{5}+5040\,x^4+5120\,x^3+2304\,x^2+512\,x$$

input
```
int((24*x + 8*x^2 - 15*x^3 + 8*x^4 + 8)^3,x)
```

output
```
512*x + 2304*x^2 + 5120*x^3 + 5040*x^4 - (384*x^5)/5 - 2976*x^6 + (5528*x^
7)/7 + 2097*x^8 - (2936*x^9)/3 - (4527*x^10)/10 + (6936*x^11)/11 - 240*x^1
2 + (512*x^13)/13
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.83

$$\int \left(8+24x+8x^2-15x^3+8x^4\right)^3\,dx = \frac{x(1182720x^{12}-7207200x^{11}+18935280x^{10}-13594581x^9-29389360x^8+62972910x^7+23715120x^6-}{30030}$$

input
```
int((8*x^4-15*x^3+8*x^2+24*x+8)^3,x)
```

output
```
(x*(1182720*x**12 - 7207200*x**11 + 18935280*x**10 - 13594581*x**9 - 29389
360*x**8 + 62972910*x**7 + 23715120*x**6 - 89369280*x**5 - 2306304*x**4 +
151351200*x**3 + 153753600*x**2 + 69189120*x + 15375360))/30030
```

3.81 $\int (8+24x+8x^2-15x^3+8x^4)^2\,dx$

Optimal result

Integrand size = 22, antiderivative size = 52

$$\int (8+24x+8x^2-15x^3+8x^4)^2\,dx = 64x+192x^2+\frac{704x^3}{3}+36x^4-\frac{528x^5}{5}+24x^6+\frac{353x^7}{7}-30x^8+\frac{64x^9}{9}$$

output
```
64*x+192*x^2+704/3*x^3+36*x^4-528/5*x^5+24*x^6+353/7*x^7-30*x^8+64/9*x^9
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00

$$\int (8+24x+8x^2-15x^3+8x^4)^2\,dx = 64x+192x^2+\frac{704x^3}{3}+36x^4-\frac{528x^5}{5}+24x^6+\frac{353x^7}{7}-30x^8+\frac{64x^9}{9}$$

input
```
Integrate[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^2,x]
```

output
```
64*x + 192*x^2 + (704*x^3)/3 + 36*x^4 - (528*x^5)/5 + 24*x^6 + (353*x^7)/7
 - 30*x^8 + (64*x^9)/9
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {2465, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(8x^4-15x^3+8x^2+24x+8\right)^2 dx$$

$$\Big\downarrow \text{ 2465}$$

$$\int \left(64x^8-240x^7+353x^6+144x^5-528x^4+144x^3+704x^2+384x+64\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{64x^9}{9}-30x^8+\frac{353x^7}{7}+24x^6-\frac{528x^5}{5}+36x^4+\frac{704x^3}{3}+192x^2+64x$$

input
```
Int[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^2,x]
```

output
```
64*x + 192*x^2 + (704*x^3)/3 + 36*x^4 - (528*x^5)/5 + 24*x^6 + (353*x^7)/7
 - 30*x^8 + (64*x^9)/9
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2465
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> Int[ExpandToSum[u, Px^p, x], x] /; PolyQ
[Px, x] && GtQ[Expon[Px, x], 2] && !BinomialQ[Px, x] && !TrinomialQ[Px, x
] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.85

method	result	size
orering	$\frac{x(2240x^8-9450x^7+15885x^6+7560x^5-33264x^4+11340x^3+73920x^2+60480x+20160)}{315}$	44
gosper	$64x+192x^2+\frac{704}{3}x^3+36x^4-\frac{528}{5}x^5+24x^6+\frac{353}{7}x^7-30x^8+\frac{64}{9}x^9$	45
default	$64x+192x^2+\frac{704}{3}x^3+36x^4-\frac{528}{5}x^5+24x^6+\frac{353}{7}x^7-30x^8+\frac{64}{9}x^9$	45
norman	$64x+192x^2+\frac{704}{3}x^3+36x^4-\frac{528}{5}x^5+24x^6+\frac{353}{7}x^7-30x^8+\frac{64}{9}x^9$	45
risch	$64x+192x^2+\frac{704}{3}x^3+36x^4-\frac{528}{5}x^5+24x^6+\frac{353}{7}x^7-30x^8+\frac{64}{9}x^9$	45
parallelrisch	$64x+192x^2+\frac{704}{3}x^3+36x^4-\frac{528}{5}x^5+24x^6+\frac{353}{7}x^7-30x^8+\frac{64}{9}x^9$	45

input
```
int((8*x^4-15*x^3+8*x^2+24*x+8)^2,x,method=_RETURNVERBOSE)
```

output
```
1/315*x*(2240*x^8-9450*x^7+15885*x^6+7560*x^5-33264*x^4+11340*x^3+73920*x^
2+60480*x+20160)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.85

$$\int \left(8+24x+8x^2-15x^3+8x^4\right)^2 \, dx = \frac{64}{9}\,x^9 - 30\,x^8 + \frac{353}{7}\,x^7 + 24\,x^6 - \frac{528}{5}\,x^5 + 36\,x^4 + \frac{704}{3}\,x^3 + 192\,x^2 + 64\,x$$

input
```
integrate((8*x^4-15*x^3+8*x^2+24*x+8)^2,x, algorithm="fricas")
```

output
```
64/9*x^9 - 30*x^8 + 353/7*x^7 + 24*x^6 - 528/5*x^5 + 36*x^4 + 704/3*x^3 +
192*x^2 + 64*x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.94

$$\int \left(8+24x+8x^2-15x^3+8x^4\right)^2\,dx = \frac{64x^9}{9} - 30x^8 + \frac{353x^7}{7} + 24x^6 - \frac{528x^5}{5} + 36x^4 + \frac{704x^3}{3} + 192x^2 + 64x$$

input
```
integrate((8*x**4-15*x**3+8*x**2+24*x+8)**2,x)
```

output
```
64*x**9/9 - 30*x**8 + 353*x**7/7 + 24*x**6 - 528*x**5/5 + 36*x**4 + 704*x*
*3/3 + 192*x**2 + 64*x
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.85

$$\int \left(8+24x+8x^2-15x^3+8x^4\right)^2\,dx = \frac{64}{9}\,x^9 - 30\,x^8 + \frac{353}{7}\,x^7 + 24\,x^6 - \frac{528}{5}\,x^5 + 36\,x^4 + \frac{704}{3}\,x^3 + 192\,x^2 + 64\,x$$

input
```
integrate((8*x^4-15*x^3+8*x^2+24*x+8)^2,x, algorithm="maxima")
```

output
```
64/9*x^9 - 30*x^8 + 353/7*x^7 + 24*x^6 - 528/5*x^5 + 36*x^4 + 704/3*x^3 +
192*x^2 + 64*x
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.85

$$\int \left(8+24x+8x^2-15x^3+8x^4\right)^2\,dx = \frac{64}{9}\,x^9 - 30\,x^8 + \frac{353}{7}\,x^7 + 24\,x^6 - \frac{528}{5}\,x^5 + 36\,x^4 + \frac{704}{3}\,x^3 + 192\,x^2 + 64\,x$$

input
```
integrate((8*x^4-15*x^3+8*x^2+24*x+8)^2,x, algorithm="giac")
```

output
```
64/9*x^9 - 30*x^8 + 353/7*x^7 + 24*x^6 - 528/5*x^5 + 36*x^4 + 704/3*x^3 +
192*x^2 + 64*x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.85

$$\int \left(8+24x+8x^2-15x^3+8x^4\right)^2\,dx = \frac{64\,x^9}{9} - 30\,x^8 + \frac{353\,x^7}{7} + 24\,x^6 - \frac{528\,x^5}{5} + 36\,x^4 + \frac{704\,x^3}{3} + 192\,x^2 + 64\,x$$

input
```
int((24*x + 8*x^2 - 15*x^3 + 8*x^4 + 8)^2,x)
```

output
```
64*x + 192*x^2 + (704*x^3)/3 + 36*x^4 - (528*x^5)/5 + 24*x^6 + (353*x^7)/7
 - 30*x^8 + (64*x^9)/9
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.83

$$\int \left(8+24x+8x^2-15x^3+8x^4\right)^2 \, dx = \frac{x(2240x^8-9450x^7+15885x^6+7560x^5-33264x^4+11340x^3+73920x^2+60480x+20160)}{315}$$

input
```
int((8*x^4-15*x^3+8*x^2+24*x+8)^2,x)
```

output
```
(x*(2240*x**8 - 9450*x**7 + 15885*x**6 + 7560*x**5 - 33264*x**4 + 11340*x*
*3 + 73920*x**2 + 60480*x + 20160))/315
```

3.82 $\int (8+24x+8x^2-15x^3+8x^4) \, dx$

Optimal result

Integrand size = 20, antiderivative size = 30

$$\int (8+24x+8x^2-15x^3+8x^4) \, dx = 8x+12x^2+\frac{8x^3}{3}-\frac{15x^4}{4}+\frac{8x^5}{5}$$

output
```
8*x+12*x^2+8/3*x^3-15/4*x^4+8/5*x^5
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int (8+24x+8x^2-15x^3+8x^4) \, dx = 8x+12x^2+\frac{8x^3}{3}-\frac{15x^4}{4}+\frac{8x^5}{5}$$

input
```
Integrate[8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4,x]
```

output
```
8*x + 12*x^2 + (8*x^3)/3 - (15*x^4)/4 + (8*x^5)/5
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = $\{2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(8x^4-15x^3+8x^2+24x+8\right)\,dx$$

$$\Big\downarrow \; 2009$$

$$\frac{8x^5}{5}-\frac{15x^4}{4}+\frac{8x^3}{3}+12x^2+8x$$

input
```
Int[8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4,x]
```

output
```
8*x + 12*x^2 + (8*x^3)/3 - (15*x^4)/4 + (8*x^5)/5
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

method	result	size
orering	$\frac{x(96x^4-225x^3+160x^2+720x+480)}{60}$	24
gosper	$8x+12x^2+\frac{8}{3}x^3-\frac{15}{4}x^4+\frac{8}{5}x^5$	25
default	$8x+12x^2+\frac{8}{3}x^3-\frac{15}{4}x^4+\frac{8}{5}x^5$	25
norman	$8x+12x^2+\frac{8}{3}x^3-\frac{15}{4}x^4+\frac{8}{5}x^5$	25
risch	$8x+12x^2+\frac{8}{3}x^3-\frac{15}{4}x^4+\frac{8}{5}x^5$	25
parallelrisch	$8x+12x^2+\frac{8}{3}x^3-\frac{15}{4}x^4+\frac{8}{5}x^5$	25
parts	$8x+12x^2+\frac{8}{3}x^3-\frac{15}{4}x^4+\frac{8}{5}x^5$	25

input
```
int(8*x^4-15*x^3+8*x^2+24*x+8,x,method=_RETURNVERBOSE)
```

output
```
1/60*x*(96*x^4-225*x^3+160*x^2+720*x+480)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \left(8+24x+8x^2-15x^3+8x^4\right)\,dx=\frac{8}{5}\,x^5-\frac{15}{4}\,x^4+\frac{8}{3}\,x^3+12\,x^2+8\,x$$

input
```
integrate(8*x^4-15*x^3+8*x^2+24*x+8,x, algorithm="fricas")
```

output
```
8/5*x^5 - 15/4*x^4 + 8/3*x^3 + 12*x^2 + 8*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.90

$$\int \left(8+24x+8x^2-15x^3+8x^4\right) dx = \frac{8x^5}{5} - \frac{15x^4}{4} + \frac{8x^3}{3} + 12x^2 + 8x$$

input
```
integrate(8*x**4-15*x**3+8*x**2+24*x+8,x)
```

output
```
8*x**5/5 - 15*x**4/4 + 8*x**3/3 + 12*x**2 + 8*x
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \left(8+24x+8x^2-15x^3+8x^4\right) dx = \frac{8}{5}\,x^5 - \frac{15}{4}\,x^4 + \frac{8}{3}\,x^3 + 12\,x^2 + 8\,x$$

input
```
integrate(8*x^4-15*x^3+8*x^2+24*x+8,x, algorithm="maxima")
```

output
```
8/5*x^5 - 15/4*x^4 + 8/3*x^3 + 12*x^2 + 8*x
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \left(8+24x+8x^2-15x^3+8x^4\right) dx = \frac{8}{5}\,x^5 - \frac{15}{4}\,x^4 + \frac{8}{3}\,x^3 + 12\,x^2 + 8\,x$$

input
```
integrate(8*x^4-15*x^3+8*x^2+24*x+8,x, algorithm="giac")
```

output
```
8/5*x^5 - 15/4*x^4 + 8/3*x^3 + 12*x^2 + 8*x
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \left(8+24x+8x^2-15x^3+8x^4\right)\,dx=\frac{8\,x^5}{5}-\frac{15\,x^4}{4}+\frac{8\,x^3}{3}+12\,x^2+8\,x$$

input
```
int(24*x + 8*x^2 - 15*x^3 + 8*x^4 + 8,x)
```

output
```
8*x + 12*x^2 + (8*x^3)/3 - (15*x^4)/4 + (8*x^5)/5
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.77

$$\int \left(8+24x+8x^2-15x^3+8x^4\right)\,dx=\frac{x(96x^4-225x^3+160x^2+720x+480)}{60}$$

input
```
int(8*x^4-15*x^3+8*x^2+24*x+8,x)
```

output
```
(x*(96*x**4 - 225*x**3 + 160*x**2 + 720*x + 480))/60
```

3.83 $\int \frac{1}{8+24x+8x^2-15x^3+8x^4}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 208

$$\begin{aligned}\int \frac{1}{8+24x+8x^2-15x^3+8x^4}\,dx &= -\frac{1}{4}\sqrt{\frac{5167+235\sqrt{517}}{40326}}\arctan\left(\frac{6+\sqrt{2\left(19+\sqrt{517}\right)}+\frac{8}{x}}{\sqrt{2\left(-19+\sqrt{517}\right)}}\right)\\ &\quad -\frac{1}{4}\sqrt{\frac{5167+235\sqrt{517}}{40326}}\arctan\left(\frac{8+\left(6-\sqrt{2\left(19+\sqrt{517}\right)}\right)x}{\sqrt{2\left(-19+\sqrt{517}\right)}x}\right)\\ &\quad +\frac{1}{4}\sqrt{\frac{3}{13}}\arctan\left(\frac{8+12x-5x^2}{\sqrt{39}x^2}\right)\\ &\quad +\frac{1}{4}\sqrt{\frac{-5167+235\sqrt{517}}{40326}}\operatorname{arctanh}\left(\frac{\sqrt{2\left(19+\sqrt{517}\right)}\left(3+\frac{4}{x}\right)}{\sqrt{517}+\left(3+\frac{4}{x}\right)^2}\right)\end{aligned}$$

output
```
-1/161304*(208364442+9476610*517^(1/2))^(1/2)*arctan((6+(38+2*517^(1/2))^(
1/2)+8/x)/(-38+2*517^(1/2))^(1/2))-1/161304*(208364442+9476610*517^(1/2))^
(1/2)*arctan((8+(6-(38+2*517^(1/2))^(1/2))*x)/(-38+2*517^(1/2))^(1/2)/x)+1
/52*39^(1/2)*arctan(1/39*(-5*x^2+12*x+8)*39^(1/2)/x^2)+1/161304*(-20836444
2+9476610*517^(1/2))^(1/2)*arctanh((38+2*517^(1/2))^(1/2)*(3+4/x)/(517^(1/
2)+(3+4/x)^2))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.26

$$\int \frac{1}{8+24x+8x^2-15x^3+8x^4}\,dx = \text{RootSum}\left[8+24\#1+8\#1^2-15\#1^3+8\#1^4\&,\frac{\log(x-\#1)}{24+16\#1-45\#1^2+32\#1^3}\&\right]$$

input
```
Integrate[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^(-1),x]
```

output
```
RootSum[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , Log[x - #1]/(24 + 16*#1
- 45*#1^2 + 32*#1^3) & ]
```

Rubi [A] (verified)

Time = 1.13 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.50, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.636$, Rules used = {2504, 27, 2202, 27, 1432, 1083, 217, 1483, 27, 1142, 27, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{8x^4-15x^3+8x^2+24x+8}dx$$

$$\Big\downarrow \text{2504}$$

$$-1024\int \frac{\left(3-4\left(\frac{3}{4}+\frac{1}{x}\right)\right)^2}{512\left(256\left(\frac{3}{4}+\frac{1}{x}\right)^4-608\left(\frac{3}{4}+\frac{1}{x}\right)^2+517\right)}d\left(\frac{3}{4}+\frac{1}{x}\right)$$

$$\Big\downarrow \text{27}$$

$$-2\int \frac{\left(3-4\left(\frac{3}{4}+\frac{1}{x}\right)\right)^2}{256\left(\frac{3}{4}+\frac{1}{x}\right)^4-608\left(\frac{3}{4}+\frac{1}{x}\right)^2+517}d\left(\frac{3}{4}+\frac{1}{x}\right)$$

$$\Big\downarrow \text{2202}$$

$$-2\left(\int \frac{16\left(\frac{3}{4}+\frac{1}{x}\right)^2+9}{256\left(\frac{3}{4}+\frac{1}{x}\right)^4-608\left(\frac{3}{4}+\frac{1}{x}\right)^2+517}d\left(\frac{3}{4}+\frac{1}{x}\right)+\int -\frac{24\left(\frac{3}{4}+\frac{1}{x}\right)}{256\left(\frac{3}{4}+\frac{1}{x}\right)^4-608\left(\frac{3}{4}+\frac{1}{x}\right)^2+517}d\left(\frac{3}{4}+\frac{1}{x}\right)\right)$$

$$\Big\downarrow \text{27}$$

$$-2\left(\int \frac{16\left(\frac{3}{4}+\frac{1}{x}\right)^2+9}{256\left(\frac{3}{4}+\frac{1}{x}\right)^4-608\left(\frac{3}{4}+\frac{1}{x}\right)^2+517}d\left(\frac{3}{4}+\frac{1}{x}\right)-24\int \frac{\frac{3}{4}+\frac{1}{x}}{256\left(\frac{3}{4}+\frac{1}{x}\right)^4-608\left(\frac{3}{4}+\frac{1}{x}\right)^2+517}d\left(\frac{3}{4}+\frac{1}{x}\right)\right)$$

$$\Big\downarrow \text{1432}$$

$$-2\left(\int \frac{16\left(\frac{3}{4}+\frac{1}{x}\right)^2+9}{256\left(\frac{3}{4}+\frac{1}{x}\right)^4-608\left(\frac{3}{4}+\frac{1}{x}\right)^2+517}d\left(\frac{3}{4}+\frac{1}{x}\right)-12\int \frac{1}{256\left(\frac{3}{4}+\frac{1}{x}\right)^4-608\left(\frac{3}{4}+\frac{1}{x}\right)^2+517}d\left(\frac{3}{4}+\frac{1}{x}\right)^2\right)$$

$$\Big\downarrow \text{1083}$$

$$-2\left(24\int \frac{1}{-\left(\frac{3}{4}+\frac{1}{x}\right)^4-159744}d\left(512\left(\frac{3}{4}+\frac{1}{x}\right)^2-608\right)+\int \frac{16\left(\frac{3}{4}+\frac{1}{x}\right)^2+9}{256\left(\frac{3}{4}+\frac{1}{x}\right)^4-608\left(\frac{3}{4}+\frac{1}{x}\right)^2+517}d\left(\frac{3}{4}+\frac{1}{x}\right)\right)$$

$$\Big\downarrow \text{217}$$

$$-2\left(\int \frac{16\left(\frac{3}{4}+\frac{1}{x}\right)^2+9}{256\left(\frac{3}{4}+\frac{1}{x}\right)^4-608\left(\frac{3}{4}+\frac{1}{x}\right)^2+517}d\left(\frac{3}{4}+\frac{1}{x}\right)-\frac{1}{8}\sqrt{\frac{3}{13}}\arctan\left(\frac{512\left(\frac{1}{x}+\frac{3}{4}\right)^2-608}{64\sqrt{39}}\right)\right)$$

$$\Big\downarrow \text{1483}$$

$$-2\left(\frac{\int \frac{8\left(9\sqrt{\frac{1}{2}\left(19+\sqrt{517}\right)}-2\left(9-\sqrt{517}\right)\left(\frac{3}{4}+\frac{1}{x}\right)\right)}{16\left(\frac{3}{4}+\frac{1}{x}\right)^2-4\sqrt{2\left(19+\sqrt{517}\right)}\left(\frac{3}{4}+\frac{1}{x}\right)+\sqrt{517}}d\left(\frac{3}{4}+\frac{1}{x}\right)}{8\sqrt{1034\left(19+\sqrt{517}\right)}}+\frac{\int \frac{8\left(2\left(9-\sqrt{517}\right)\left(\frac{3}{4}+\frac{1}{x}\right)+9\sqrt{\frac{1}{2}\left(19+\sqrt{517}\right)}\right)}{16\left(\frac{3}{4}+\frac{1}{x}\right)^2+4\sqrt{2\left(19+\sqrt{517}\right)}\left(\frac{3}{4}+\frac{1}{x}\right)+\sqrt{517}}d\left(\frac{3}{4}+\frac{1}{x}\right)}{8\sqrt{1034\left(19+\sqrt{517}\right)}}-\frac{1}{8}\sqrt{\frac{3}{13}}\right.$$

$$\Big\downarrow \text{27}$$

$$-2\left(\frac{\int \frac{9\sqrt{\frac{1}{2}\left(19+\sqrt{517}\right)}-2\left(9-\sqrt{517}\right)\left(\frac{3}{4}+\frac{1}{x}\right)}{16\left(\frac{3}{4}+\frac{1}{x}\right)^2-4\sqrt{2\left(19+\sqrt{517}\right)}\left(\frac{3}{4}+\frac{1}{x}\right)+\sqrt{517}}d\left(\frac{3}{4}+\frac{1}{x}\right)}{\sqrt{1034\left(19+\sqrt{517}\right)}}+\frac{\int \frac{2\left(9-\sqrt{517}\right)\left(\frac{3}{4}+\frac{1}{x}\right)+9\sqrt{\frac{1}{2}\left(19+\sqrt{517}\right)}}{16\left(\frac{3}{4}+\frac{1}{x}\right)^2+4\sqrt{2\left(19+\sqrt{517}\right)}\left(\frac{3}{4}+\frac{1}{x}\right)+\sqrt{517}}d\left(\frac{3}{4}+\frac{1}{x}\right)}{\sqrt{1034\left(19+\sqrt{517}\right)}}-\frac{1}{8}\sqrt{\frac{3}{13}}\right.$$

$$\Big\downarrow \text{1142}$$

$$-2\left(\frac{\sqrt{\frac{1}{2}\left(5167+235\sqrt{517}\right)}\int \frac{1}{16\left(\frac{3}{4}+\frac{1}{x}\right)^2-4\sqrt{2\left(19+\sqrt{517}\right)}\left(\frac{3}{4}+\frac{1}{x}\right)+\sqrt{517}}d\left(\frac{3}{4}+\frac{1}{x}\right)-\frac{1}{16}\left(9-\sqrt{517}\right)\int -\frac{4\left(\sqrt{2\left(19+\sqrt{\right.}\right.}}{16\left(\frac{3}{4}+\frac{1}{x}\right)^2-4\sqrt{2\left(1\right.}}}{\sqrt{1034\left(19+\sqrt{517}\right)}}\right.$$

$$\Big\downarrow \text{27}$$

$$-2\left(\frac{\sqrt{\frac{1}{2}\left(5167+235\sqrt{517}\right)}\int \frac{1}{16\left(\frac{3}{4}+\frac{1}{x}\right)^2-4\sqrt{2\left(19+\sqrt{517}\right)}\left(\frac{3}{4}+\frac{1}{x}\right)+\sqrt{517}}d\left(\frac{3}{4}+\frac{1}{x}\right)+\frac{1}{4}\left(9-\sqrt{517}\right)\int \frac{\sqrt{2\left(19+\sqrt{517}\right.}}{16\left(\frac{3}{4}+\frac{1}{x}\right)^2-4\sqrt{2\left(19+\right.}}}{\sqrt{1034\left(19+\sqrt{517}\right)}}\right.$$

$$\Big\downarrow \text{1083}$$

$$-2\left(\frac{\frac{1}{4}\left(9-\sqrt{517}\right)\int \frac{\sqrt{2\left(19+\sqrt{517}\right)}-8\left(\frac{3}{4}+\frac{1}{x}\right)}{16\left(\frac{3}{4}+\frac{1}{x}\right)^2-4\sqrt{2\left(19+\sqrt{517}\right)}\left(\frac{3}{4}+\frac{1}{x}\right)+\sqrt{517}}d\left(\frac{3}{4}+\frac{1}{x}\right)-\sqrt{2\left(5167+235\sqrt{517}\right)}\int \frac{}{32\left(19-\sqrt{517}\right)-\left(32\left(\frac{3}{4}\right.\right.}}{\sqrt{1034\left(19+\sqrt{517}\right)}}\right.$$

$$\Big\downarrow \text{217}$$

$$-2\left(\frac{\frac{1}{4}\left(9-\sqrt{517}\right)\int \frac{\sqrt{2\left(19+\sqrt{517}\right)}-8\left(\frac{3}{4}+\frac{1}{x}\right)}{16\left(\frac{3}{4}+\frac{1}{x}\right)^2-4\sqrt{2\left(19+\sqrt{517}\right)}\left(\frac{3}{4}+\frac{1}{x}\right)+\sqrt{517}}d\left(\frac{3}{4}+\frac{1}{x}\right)+\frac{1}{4}\sqrt{\frac{5167+235\sqrt{517}}{\sqrt{517}-19}}\arctan\left(\frac{32\left(\frac{1}{x}+\frac{3}{4}\right)-4\sqrt{2\left(19+\right.}}{4\sqrt{2\left(\sqrt{517}-19\right)}}\right.}{\sqrt{1034\left(19+\sqrt{517}\right)}}\right.$$

$$\Big\downarrow \text{1103}$$

$$-2\left(-\frac{1}{8}\sqrt{\frac{3}{13}}\arctan\left(\frac{512\left(\frac{1}{x}+\frac{3}{4}\right)^2-608}{64\sqrt{39}}\right)+\frac{\frac{1}{4}\sqrt{\frac{5167+235\sqrt{517}}{\sqrt{517}-19}}\arctan\left(\frac{32\left(\frac{1}{x}+\frac{3}{4}\right)-4\sqrt{2\left(19+\sqrt{517}\right)}}{4\sqrt{2\left(\sqrt{517}-19\right)}}\right)-\frac{1}{16}\left(9-\sqrt{51\cdots}\right.}{\sqrt{1034\left(19+\cdots\right.}}\right.$$

input
```
Int[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^(-1),x]
```

output
```
-2*(-1/8*(Sqrt[3/13]*ArcTan[(-608 + 512*(3/4 + x^(-1))^2)/(64*Sqrt[39])])
+ ((Sqrt[(5167 + 235*Sqrt[517])/(-19 + Sqrt[517])]*ArcTan[(-4*Sqrt[2*(19 +
 Sqrt[517])] + 32*(3/4 + x^(-1)))/(4*Sqrt[2*(-19 + Sqrt[517])])]])/4 - ((9
- Sqrt[517])*Log[Sqrt[517] - 4*Sqrt[2*(19 + Sqrt[517])]*(3/4 + x^(-1)) + 1
6*(3/4 + x^(-1))^2])/16)/Sqrt[1034*(19 + Sqrt[517])] + ((Sqrt[(5167 + 235*
Sqrt[517])/(-19 + Sqrt[517])]*ArcTan[(4*Sqrt[2*(19 + Sqrt[517])] + 32*(3/4
 + x^(-1)))/(4*Sqrt[2*(-19 + Sqrt[517])])]])/4 + ((9 - Sqrt[517])*Log[Sqrt[
517] + 4*Sqrt[2*(19 + Sqrt[517])]*(3/4 + x^(-1)) + 16*(3/4 + x^(-1))^2])/1
6)/Sqrt[1034*(19 + Sqrt[517])])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1432
```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

rule 1483
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   In
t[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(d*r
 + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

rule 2202
```
Int[(Pn_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{n
 = Expon[Pn, x], k}, Int[Sum[Coeff[Pn, x, 2*k]*x^(2*k), {k, 0, n/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pn, x, 2*k + 1]*x^(2*k), {k, 0, (n -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pn, x]
 &&  !PolyQ[Pn, x^2]
```

rule 2504
```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Simp[-16*
a^2   Subst[Int[(1/(b - 4*a*x)^2)*(a*((-3*b^4 + 16*a*b^2*c - 64*a^2*b*d + 2
56*a^3*e - 32*a^2*(3*b^2 - 8*a*c)*x^2 + 256*a^4*x^4)/(b - 4*a*x)^4))^p, x],
 x, b/(4*a) + 1/x], x] /; NeQ[a, 0] && NeQ[b, 0] && EqQ[b^3 - 4*a*b*c + 8*a
^2*d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && IntegerQ[2*p] &&  !IGtQ[p, 0
]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.24

method	result	size
default	$\sum_{\_R=\mathrm{RootOf}\left(8\_Z^4-15\_Z^3+8\_Z^2+24\_Z+8\right)}\frac{\ln\left(x-\_R\right)}{32\_R^3-45\_R^2+16\_R+24}$	49
risch	$\sum_{\_R=\mathrm{RootOf}\left(8\_Z^4-15\_Z^3+8\_Z^2+24\_Z+8\right)}\frac{\ln\left(x-\_R\right)}{32\_R^3-45\_R^2+16\_R+24}$	49

input

```
int(1/(8*x^4-15*x^3+8*x^2+24*x+8),x,method=_RETURNVERBOSE)
```

output

```
sum(1/(32*_R^3-45*_R^2+16*_R+24)*ln(x-_R),_R=RootOf(8*_Z^4-15*_Z^3+8*_Z^2+
24*_Z+8))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 305 vs. 2(148) = 296.

Time = 0.11 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.47

$$\int \frac{1}{8+24x+8x^2-15x^3+8x^4}\,dx = \text{Too large to display}$$

input

```
integrate(1/(8*x^4-15*x^3+8*x^2+24*x+8),x, algorithm="fricas")
```

output
```
-1/4*sqrt(1/1417*(2585*sqrt(47/11) + 5167)*sqrt(5/78*sqrt(47/11) - 5167/40
326) + 5/78*sqrt(47/11) + 14473/40326)*arctan(-11/414926812*(109*sqrt(47/1
1)*(1650521*x - 1323522) - 94*(11*sqrt(47/11)*(1048281*x - 2936950) + 2272
7406*x - 65658598)*sqrt(5/78*sqrt(47/11) - 5167/40326) + 329311563*x - 314
296050)*sqrt(1/1417*(2585*sqrt(47/11) + 5167)*sqrt(5/78*sqrt(47/11) - 5167
/40326) + 5/78*sqrt(47/11) + 14473/40326)) + 1/4*sqrt(-1/1417*(2585*sqrt(4
7/11) + 5167)*sqrt(5/78*sqrt(47/11) - 5167/40326) + 5/78*sqrt(47/11) + 144
73/40326)*arctan(11/414926812*(109*sqrt(47/11)*(1650521*x - 1323522) + 94*
(11*sqrt(47/11)*(1048281*x - 2936950) + 22727406*x - 65658598)*sqrt(5/78*s
qrt(47/11) - 5167/40326) + 329311563*x - 314296050)*sqrt(-1/1417*(2585*sqr
t(47/11) + 5167)*sqrt(5/78*sqrt(47/11) - 5167/40326) + 5/78*sqrt(47/11) +
14473/40326)) + 1/8*sqrt(5/78*sqrt(47/11) - 5167/40326)*log(3488*x^2 + 11*
(sqrt(47/11)*(2071*x - 2064) + 5123*x - 3948)*sqrt(5/78*sqrt(47/11) - 5167
/40326) - 3270*x + 2398*sqrt(47/11) + 1962) - 1/8*sqrt(5/78*sqrt(47/11) -
5167/40326)*log(3488*x^2 - 11*(sqrt(47/11)*(2071*x - 2064) + 5123*x - 3948
)*sqrt(5/78*sqrt(47/11) - 5167/40326) - 3270*x + 2398*sqrt(47/11) + 1962)
```

Sympy [A] (verification not implemented)

Time = 1.48 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.20

$$\int \frac{1}{8+24x+8x^2-15x^3+8x^4}\,dx$$
$$= \mathrm{RootSum}\left(50326848t^4+765960t^2+12753t+64,\left(t\mapsto t\log\left(\frac{100785893208t^3}{4758335}-\frac{1430512512t^2}{4758335}+\frac{729\ldots}{22\ldots}\right.\right.\right.$$

input
```
integrate(1/(8*x**4-15*x**3+8*x**2+24*x+8),x)
```

output
```
RootSum(50326848*_t**4 + 765960*_t**2 + 12753*_t + 64, Lambda(_t, _t*log(1
00785893208*_t**3/4758335 - 1430512512*_t**2/4758335 + 72982352521*_t/2236
41745 + x + 2270349121/1789133960)))
```

Maxima [F]

$$\int \frac{1}{8+24x+8x^2-15x^3+8x^4}\,dx = \int \frac{1}{8\,x^4-15\,x^3+8\,x^2+24\,x+8}\,dx$$

input
```
integrate(1/(8*x^4-15*x^3+8*x^2+24*x+8),x, algorithm="maxima")
```

output
```
integrate(1/(8*x^4 - 15*x^3 + 8*x^2 + 24*x + 8), x)
```

Giac [F]

$$\int \frac{1}{8+24x+8x^2-15x^3+8x^4}\,dx = \int \frac{1}{8\,x^4-15\,x^3+8\,x^2+24\,x+8}\,dx$$

input
```
integrate(1/(8*x^4-15*x^3+8*x^2+24*x+8),x, algorithm="giac")
```

output
```
integrate(1/(8*x^4 - 15*x^3 + 8*x^2 + 24*x + 8), x)
```

Mupad [B] (verification not implemented)

Time = 22.77 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.59

$$\int \frac{1}{8+24x+8x^2-15x^3+8x^4}\,dx$$
$$= \sum_{k=1}^{4} \ln\left(-\frac{\mathrm{root}\left(z^4+\frac{2455\,z^2}{161304}+\frac{109\,z}{430144}+\frac{1}{786357},z,k\right)\left(2184\,\mathrm{root}\left(z^4+\frac{2455\,z^2}{161304}+\frac{109\,z}{430144}+\frac{1}{786357},z,k\right)+256\right.}{}\right.$$
$$+\frac{2455\,z^2}{161304}+\frac{109\,z}{430144}+\frac{1}{786357},z,k\Big)$$

input
```
int(1/(24*x + 8*x^2 - 15*x^3 + 8*x^4 + 8),x)
```

output
```
symsum(log(-(root(z^4 + (2455*z^2)/161304 + (109*z)/430144 + 1/786357, z,
k)*(2184*root(z^4 + (2455*z^2)/161304 + (109*z)/430144 + 1/786357, z, k) +
 256*x + 38259*root(z^4 + (2455*z^2)/161304 + (109*z)/430144 + 1/786357, z
, k)*x + 1531920*root(z^4 + (2455*z^2)/161304 + (109*z)/430144 + 1/786357,
 z, k)^2*x + 805896*root(z^4 + (2455*z^2)/161304 + (109*z)/430144 + 1/7863
57, z, k)^2 - 120))/4096)*root(z^4 + (2455*z^2)/161304 + (109*z)/430144 +
1/786357, z, k), k, 1, 4)
```

Reduce [F]

$$\int \frac{1}{8+24x+8x^2-15x^3+8x^4}\,dx = \int \frac{1}{8x^4-15x^3+8x^2+24x+8}dx$$

input
```
int(1/(8*x^4-15*x^3+8*x^2+24*x+8),x)
```

output
```
int(1/(8*x**4 - 15*x**3 + 8*x**2 + 24*x + 8),x)
```

3.84 $\int \frac{1}{(8+24x+8x^2-15x^3+8x^4)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 297

$$\begin{aligned}\int \frac{1}{(8+24x+8x^2-15x^3+8x^4)^2}\,dx &= -\frac{3\left(3359-107\left(3+\frac{4}{x}\right)^2\right)}{208\left(517-38\left(3+\frac{4}{x}\right)^2+\left(3+\frac{4}{x}\right)^4\right)} + \frac{\left(3327931-129631\left(3+\frac{4}{x}\right)^2\right)\left(3+\frac{4}{x}\right)}{322608\left(517-38\left(3+\frac{4}{x}\right)^2+\left(3+\frac{4}{x}\right)^4\right)} \\
&\quad - \frac{\sqrt{\frac{59644114671451+2623170438295\sqrt{517}}{40326}}\arctan\left(\frac{6+\sqrt{2\left(19+\sqrt{517}\right)}+\frac{8}{x}}{\sqrt{2\left(-19+\sqrt{517}\right)}}\right)}{322608} \\
&\quad - \frac{\sqrt{\frac{59644114671451+2623170438295\sqrt{517}}{40326}}\arctan\left(\frac{8+\left(6-\sqrt{2\left(19+\sqrt{517}\right)}\right)x}{\sqrt{2\left(-19+\sqrt{517}\right)}x}\right)}{322608} \\
&\quad + \frac{73}{208}\sqrt{\frac{3}{13}}\arctan\left(\frac{8+12x-5x^2}{\sqrt{39}x^2}\right) \\
&\quad + \frac{\sqrt{\frac{-59644114671451+2623170438295\sqrt{517}}{40326}}\operatorname{arctanh}\left(\frac{\sqrt{2\left(19+\sqrt{517}\right)}\left(3+\frac{4}{x}\right)}{\sqrt{517}+\left(3+\frac{4}{x}\right)^2}\right)}{322608}\end{aligned}$$

output
```
(-10077+321*(3+4/x)^2)/(107536-7904*(3+4/x)^2+208*(3+4/x)^4)+(3327931-1296
31*(3+4/x)^2)*(3+4/x)/(166788336-12259104*(3+4/x)^2+322608*(3+4/x)^4)-1/13
009490208*(2405208568240933026+1057819710946841 70*517^(1/2))^(1/2)*arctan(
(6+(38+2*517^(1/2))^(1/2)+8/x)/(-38+2*517^(1/2))^(1/2))-1/13009490208*(240
5208568240933026+105781971094684170*517^(1/2))^(1/2)*arctan((8+(6-(38+2*51
7^(1/2))^(1/2))*x)/(-38+2*517^(1/2))^(1/2)/x)+73/2704*39^(1/2)*arctan(1/39
*(-5*x^2+12*x+8)*39^(1/2)/x^2)+1/13009490208*(-2405208568240933026+1057819
71094684170*517^(1/2))^(1/2)*arctanh((38+2*517^(1/2))^(1/2)*(3+4/x)/(517^(
1/2)+(3+4/x)^2))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.43

$$\int \frac{1}{(8+24x+8x^2-15x^3+8x^4)^2}\,dx = \frac{72888+89033x-94314x^2+39280x^3}{161304\,(8+24x+8x^2-15x^3+8x^4)} + \frac{\text{RootSum}\left[8+24\#1+8\#1^2-15\#1^3+8\#1^4\&,\frac{74897\log(x-\#1)-57489\log(x-\#1)\#1+19640\log(x-\#1)\#1^2}{24+16\#1-45\#1^2+32\#1^3}\&\right]}{80652}$$

input
```
Integrate[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^(-2),x]
```

output
```
(72888 + 89033*x - 94314*x^2 + 39280*x^3)/(161304*(8 + 24*x + 8*x^2 - 15*x
^3 + 8*x^4)) + RootSum[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , (74897*Lo
g[x - #1] - 57489*Log[x - #1]*#1 + 19640*Log[x - #1]*#1^2)/(24 + 16*#1 - 4
5*#1^2 + 32*#1^3) & ]/80652
```

Rubi [A] (verified)

Time = 1.46 (sec) , antiderivative size = 428, normalized size of antiderivative = 1.44, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.818, Rules used = {2504, 27, 2202, 2194, 27, 2191, 27, 1083, 217, 2206, 27, 1483, 27, 1142, 27, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(8x^4-15x^3+8x^2+24x+8)^2}dx$$

$$\Big\downarrow \text{2504}$$

$$-1024\int \frac{\left(3-4\left(\frac{3}{4}+\frac{1}{x}\right)\right)^6}{4096\left(256\left(\frac{3}{4}+\frac{1}{x}\right)^4-608\left(\frac{3}{4}+\frac{1}{x}\right)^2+517\right)^2}d\left(\frac{3}{4}+\frac{1}{x}\right)$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{4}\int \frac{\left(3-4\left(\frac{3}{4}+\frac{1}{x}\right)\right)^6}{\left(256\left(\frac{3}{4}+\frac{1}{x}\right)^4-608\left(\frac{3}{4}+\frac{1}{x}\right)^2+517\right)^2}d\left(\frac{3}{4}+\frac{1}{x}\right)$$

$$\Big\downarrow \text{2202}$$

$$\frac{1}{4}\left(-\int \frac{4096\left(\frac{3}{4}+\frac{1}{x}\right)^6+34560\left(\frac{3}{4}+\frac{1}{x}\right)^4+19440\left(\frac{3}{4}+\frac{1}{x}\right)^2+729}{\left(256\left(\frac{3}{4}+\frac{1}{x}\right)^4-608\left(\frac{3}{4}+\frac{1}{x}\right)^2+517\right)^2}d\left(\frac{3}{4}+\frac{1}{x}\right)-\int \frac{\left(-18432\left(\frac{3}{4}+\frac{1}{x}\right)^4-34560\left(\frac{3}{4}\right.\right.}{\left(256\left(\frac{3}{4}+\frac{1}{x}\right)^4-608\left(\frac{3}{4}\right.\right.}\right.$$

$$\Big\downarrow \text{2194}$$

$$\frac{1}{4}\left(-\frac{1}{2}\int -\frac{72\left(256\left(\frac{3}{4}+\frac{1}{x}\right)^4+480\left(\frac{3}{4}+\frac{1}{x}\right)^2+81\right)}{\left(256\left(\frac{3}{4}+\frac{1}{x}\right)^4-608\left(\frac{3}{4}+\frac{1}{x}\right)^2+517\right)^2}d\left(\frac{3}{4}+\frac{1}{x}\right)^2-\int \frac{4096\left(\frac{3}{4}+\frac{1}{x}\right)^6+34560\left(\frac{3}{4}+\frac{1}{x}\right)^4+19440\left(\frac{3}{4}\right.}{\left(256\left(\frac{3}{4}+\frac{1}{x}\right)^4-608\left(\frac{3}{4}+\frac{1}{x}\right)^2+5\right.}\right.$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4}\left(36\int \frac{256\left(\frac{3}{4}+\frac{1}{x}\right)^4+480\left(\frac{3}{4}+\frac{1}{x}\right)^2+81}{\left(256\left(\frac{3}{4}+\frac{1}{x}\right)^4-608\left(\frac{3}{4}+\frac{1}{x}\right)^2+517\right)^2}d\left(\frac{3}{4}+\frac{1}{x}\right)^2-\int \frac{4096\left(\frac{3}{4}+\frac{1}{x}\right)^6+34560\left(\frac{3}{4}+\frac{1}{x}\right)^4+19440\left(\frac{3}{4}+\right.}{\left(256\left(\frac{3}{4}+\frac{1}{x}\right)^4-608\left(\frac{3}{4}+\frac{1}{x}\right)^2+517\right.}\right.$$

$$\Big\downarrow \text{2191}$$

$$\frac{1}{4}\left(36\left(\frac{\int \frac{598016}{256\left(\frac{3}{4}+\frac{1}{x}\right)^4-608\left(\frac{3}{4}+\frac{1}{x}\right)^2+517}d\left(\frac{3}{4}+\frac{1}{x}\right)^2}{159744}-\frac{3359-1712\left(\frac{1}{x}+\frac{3}{4}\right)^2}{624\left(256\left(\frac{1}{x}+\frac{3}{4}\right)^4-608\left(\frac{1}{x}+\frac{3}{4}\right)^2+517\right)}\right)-\int \frac{4096\left(\frac{3}{4}+\frac{1}{x}\right)^6}{\left(256\right.}$$

$\downarrow$ 27

$$\frac{1}{4}\left(36\left(\frac{146}{39}\int \frac{1}{256\left(\frac{3}{4}+\frac{1}{x}\right)^4-608\left(\frac{3}{4}+\frac{1}{x}\right)^2+517}d\left(\frac{3}{4}+\frac{1}{x}\right)^2-\frac{3359-1712\left(\frac{1}{x}+\frac{3}{4}\right)^2}{624\left(256\left(\frac{1}{x}+\frac{3}{4}\right)^4-608\left(\frac{1}{x}+\frac{3}{4}\right)^2+517\right)}\right)-\right.$$

$\downarrow$ 1083

$$\frac{1}{4}\left(36\left(-\frac{292}{39}\int \frac{1}{-\left(\frac{3}{4}+\frac{1}{x}\right)^4-159744}d\left(512\left(\frac{3}{4}+\frac{1}{x}\right)^2-608\right)-\frac{3359-1712\left(\frac{1}{x}+\frac{3}{4}\right)^2}{624\left(256\left(\frac{1}{x}+\frac{3}{4}\right)^4-608\left(\frac{1}{x}+\frac{3}{4}\right)^2+517\right)}\right)\right.$$

$\downarrow$ 217

$$\frac{1}{4}\left(36\left(\frac{73\arctan\left(\frac{512\left(\frac{1}{x}+\frac{3}{4}\right)^2-608}{64\sqrt{39}}\right)}{624\sqrt{39}}-\frac{3359-1712\left(\frac{1}{x}+\frac{3}{4}\right)^2}{624\left(256\left(\frac{1}{x}+\frac{3}{4}\right)^4-608\left(\frac{1}{x}+\frac{3}{4}\right)^2+517\right)}\right)-\int \frac{4096\left(\frac{3}{4}+\frac{1}{x}\right)^6+34560}{\left(256\left(\frac{3}{4}+\frac{1}{x}\right)^4\right.}\right.$$

$\downarrow$ 2206

$$\frac{1}{4}\left(-\frac{\int \frac{16384\left(1198352\left(\frac{3}{4}+\frac{1}{x}\right)^2+1678181\right)}{256\left(\frac{3}{4}+\frac{1}{x}\right)^4-608\left(\frac{3}{4}+\frac{1}{x}\right)^2+517}d\left(\frac{3}{4}+\frac{1}{x}\right)}{165175296}+36\left(\frac{73\arctan\left(\frac{512\left(\frac{1}{x}+\frac{3}{4}\right)^2-608}{64\sqrt{39}}\right)}{624\sqrt{39}}-\frac{3359-1712\left(\frac{1}{x}+\right.}{624\left(256\left(\frac{1}{x}+\frac{3}{4}\right)^4-608\left(\frac{1}{x}\right.\right.}\right.\right.$$

$\downarrow$ 27

$$\frac{1}{4}\left(-\frac{2\int \frac{1198352\left(\frac{3}{4}+\frac{1}{x}\right)^2+1678181}{256\left(\frac{3}{4}+\frac{1}{x}\right)^4-608\left(\frac{3}{4}+\frac{1}{x}\right)^2+517}d\left(\frac{3}{4}+\frac{1}{x}\right)}{20163}+36\left(\frac{73\arctan\left(\frac{512\left(\frac{1}{x}+\frac{3}{4}\right)^2-608}{64\sqrt{39}}\right)}{624\sqrt{39}}-\frac{3359-1712\left(\frac{1}{x}+\frac{3}{4}\right)}{624\left(256\left(\frac{1}{x}+\frac{3}{4}\right)^4-608\left(\frac{1}{x}+\right.\right.}\right.\right.$$

$\downarrow$ 1483

$$\frac{1}{4}\left(-\frac{2\left(\frac{\int \frac{8\left(1678181\sqrt{\frac{1}{2}\left(19+\sqrt{517}\right)}-2\left(1678181-74897\sqrt{517}\right)\left(\frac{3}{4}+\frac{1}{x}\right)\right)}{16\left(\frac{3}{4}+\frac{1}{x}\right)^2-4\sqrt{2\left(19+\sqrt{517}\right)}\left(\frac{3}{4}+\frac{1}{x}\right)+\sqrt{517}}d\left(\frac{3}{4}+\frac{1}{x}\right)}{8\sqrt{1034\left(19+\sqrt{517}\right)}}+\frac{\int \frac{8\left(2\left(1678181-74897\sqrt{517}\right)\left(\frac{3}{4}+\frac{1}{x}\right)+1678181\sqrt{\frac{1}{2}\left(19+\sqrt{517}\right)}\right)}{16\left(\frac{3}{4}+\frac{1}{x}\right)^2+4\sqrt{2\left(19+\sqrt{517}\right)}\left(\frac{3}{4}+\frac{1}{x}\right)+\sqrt{517}}d\left(\right.}{8\sqrt{1034\left(19+\sqrt{517}\right)}}}{20163}\right.$$

$$\Big\downarrow 27$$

$$\frac{1}{4}\left(-\frac{2\left(\frac{\int \frac{1678181\sqrt{\frac{1}{2}\left(19+\sqrt{517}\right)}-2\left(1678181-74897\sqrt{517}\right)\left(\frac{3}{4}+\frac{1}{x}\right)}{16\left(\frac{3}{4}+\frac{1}{x}\right)^2-4\sqrt{2\left(19+\sqrt{517}\right)}\left(\frac{3}{4}+\frac{1}{x}\right)+\sqrt{517}}d\left(\frac{3}{4}+\frac{1}{x}\right)}{\sqrt{1034\left(19+\sqrt{517}\right)}}+\frac{\int \frac{2\left(1678181-74897\sqrt{517}\right)\left(\frac{3}{4}+\frac{1}{x}\right)+1678181\sqrt{\frac{1}{2}\left(19+\sqrt{517}\right)}}{16\left(\frac{3}{4}+\frac{1}{x}\right)^2+4\sqrt{2\left(19+\sqrt{517}\right)}\left(\frac{3}{4}+\frac{1}{x}\right)+\sqrt{517}}d\left(\frac{3}{4}+\frac{1}{x}\right)}{\sqrt{1034\left(19+\sqrt{517}\right)}}\right)}{20163}\right.$$

$$\Big\downarrow 1142$$

$$\frac{1}{4}\left(-\frac{2\left(\frac{\frac{1}{2}\sqrt{\frac{1}{2}\left(19+\sqrt{517}\right)}\left(1678181+74897\sqrt{517}\right)\int \frac{1}{16\left(\frac{3}{4}+\frac{1}{x}\right)^2-4\sqrt{2\left(19+\sqrt{517}\right)}\left(\frac{3}{4}+\frac{1}{x}\right)+\sqrt{517}}d\left(\frac{3}{4}+\frac{1}{x}\right)-\frac{1}{16}\left(1678181-74897\sqrt{517}\right)\int -\frac{}{16\left(\frac{3}{4}\right.}}{\sqrt{1034\left(19+\sqrt{517}\right)}}\right.}{}\right.$$

$$\Big\downarrow 27$$

$$\frac{1}{4}\left(-\frac{2\left(\frac{\frac{1}{2}\sqrt{\frac{1}{2}\left(19+\sqrt{517}\right)}\left(1678181+74897\sqrt{517}\right)\int\frac{1}{16\left(\frac{3}{4}+\frac{1}{x}\right)^2-4\sqrt{2\left(19+\sqrt{517}\right)}\left(\frac{3}{4}+\frac{1}{x}\right)+\sqrt{517}}d\left(\frac{3}{4}+\frac{1}{x}\right)+\frac{1}{4}\left(1678181-74897\sqrt{517}\right)\int\frac{}{16\left(\frac{3}{4}+\frac{1}{x}\right.}}{\sqrt{1034\left(19+\sqrt{517}\right)}}\right.}{}\right.$$

$$\Big\downarrow \; 1083$$

$$\frac{1}{4}\left(-\frac{2\left(\frac{\frac{1}{4}\left(1678181-74897\sqrt{517}\right)\int\frac{\sqrt{2\left(19+\sqrt{517}\right)}-8\left(\frac{3}{4}+\frac{1}{x}\right)}{16\left(\frac{3}{4}+\frac{1}{x}\right)^2-4\sqrt{2\left(19+\sqrt{517}\right)}\left(\frac{3}{4}+\frac{1}{x}\right)+\sqrt{517}}d\left(\frac{3}{4}+\frac{1}{x}\right)-\sqrt{\frac{1}{2}\left(19+\sqrt{517}\right)}\left(1678181+74897\sqrt{517}\right)\int\frac{}{32\left(19-\sqrt{5}\right.}}{\sqrt{1034\left(19+\sqrt{517}\right)}}\right.}{}\right.$$

$$\Big\downarrow \; 217$$

$$\frac{1}{4}\left(-\frac{2\left(\frac{\frac{1}{4}\left(1678181-74897\sqrt{517}\right)\int\frac{\sqrt{2\left(19+\sqrt{517}\right)}-8\left(\frac{3}{4}+\frac{1}{x}\right)}{16\left(\frac{3}{4}+\frac{1}{x}\right)^2-4\sqrt{2\left(19+\sqrt{517}\right)}\left(\frac{3}{4}+\frac{1}{x}\right)+\sqrt{517}}d\left(\frac{3}{4}+\frac{1}{x}\right)+\frac{1}{8}\sqrt{\frac{19+\sqrt{517}}{\sqrt{517}-19}}\left(1678181+74897\sqrt{517}\right)\arctan\left(\frac{32\left(\frac{1}{x}+\right.}{4}\right.}{\sqrt{1034\left(19+\sqrt{517}\right)}}\right.}{}\right.$$

$$\Big\downarrow \; 1103$$

$$\frac{1}{4}\left(36\left(\frac{73\arctan\left(\frac{512\left(\frac{1}{x}+\frac{3}{4}\right)^2-608}{64\sqrt{39}}\right)}{624\sqrt{39}}-\frac{3359-1712\left(\frac{1}{x}+\frac{3}{4}\right)^2}{624\left(256\left(\frac{1}{x}+\frac{3}{4}\right)^4-608\left(\frac{1}{x}+\frac{3}{4}\right)^2+517\right)}\right)-\frac{2\left(\frac{1}{8}\sqrt{\frac{19+\sqrt{517}}{\sqrt{517}-19}}\left(1678181+7489\right.\right.}{}\right.$$

input
```
Int[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^(-2),x]
```

output
```
(((3327931 - 2074096*(3/4 + x^(-1))^2)*(3/4 + x^(-1)))/(20163*(517 - 608*(
3/4 + x^(-1))^2 + 256*(3/4 + x^(-1))^4)) + 36*(-1/624*(3359 - 1712*(3/4 +
x^(-1))^2)/(517 - 608*(3/4 + x^(-1))^2 + 256*(3/4 + x^(-1))^4) + (73*ArcTa
n[(-608 + 512*(3/4 + x^(-1))^2)/(64*Sqrt[39])])/(624*Sqrt[39])) - (2*(((Sq
rt[(19 + Sqrt[517])/(-19 + Sqrt[517])]*(1678181 + 74897*Sqrt[517])*ArcTan[
(-4*Sqrt[2*(19 + Sqrt[517])] + 32*(3/4 + x^(-1)))/(4*Sqrt[2*(-19 + Sqrt[51
7])])])/8 - ((1678181 - 74897*Sqrt[517])*Log[Sqrt[517] - 4*Sqrt[2*(19 + Sq
rt[517])]*(3/4 + x^(-1)) + 16*(3/4 + x^(-1))^2])/16)/Sqrt[1034*(19 + Sqrt[
517])] + ((Sqrt[(19 + Sqrt[517])/(-19 + Sqrt[517])]*(1678181 + 74897*Sqrt[
517])*ArcTan[(4*Sqrt[2*(19 + Sqrt[517])] + 32*(3/4 + x^(-1)))/(4*Sqrt[2*(-
19 + Sqrt[517])])])/8 + ((1678181 - 74897*Sqrt[517])*Log[Sqrt[517] + 4*Sqr
t[2*(19 + Sqrt[517])]*(3/4 + x^(-1)) + 16*(3/4 + x^(-1))^2])/16)/Sqrt[1034
*(19 + Sqrt[517])]))/20163)/4
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1483
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   In
t[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(d*r
 + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 2194
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Simp[1/2   Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)
^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ
[(m - 1)/2]
```

rule 2202
```
Int[(Pn_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{n
 = Expon[Pn, x], k}, Int[Sum[Coeff[Pn, x, 2*k]*x^(2*k), {k, 0, n/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pn, x, 2*k + 1]*x^(2*k), {k, 0, (n -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pn, x]
 &&  !PolyQ[Pn, x^2]
```

rule 2206
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c
*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Px,
a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*
p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x
^2] && Expon[Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 2504
```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Simp[-16*
a^2   Subst[Int[(1/(b - 4*a*x)^2)*(a*((-3*b^4 + 16*a*b^2*c - 64*a^2*b*d + 2
56*a^3*e - 32*a^2*(3*b^2 - 8*a*c)*x^2 + 256*a^4*x^4)/(b - 4*a*x)^4))^p, x],
 x, b/(4*a) + 1/x], x] /; NeQ[a, 0] && NeQ[b, 0] && EqQ[b^3 - 4*a*b*c + 8*a
^2*d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && IntegerQ[2*p] &&  !IGtQ[p, 0
]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.07 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.32

method	result
default	$\frac{\frac{2455}{80652}x^3-\frac{1429}{19552}x^2+\frac{89033}{1290432}x+\frac{3037}{53768}}{x^4-\frac{15}{8}x^3+x^2+3x+1}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(8\_Z^4-15\_Z^3+8\_Z^2+24\_Z+8\right)}\frac{\left(19640\_R^2-57489\_R+74897\right)\ln\left(x-\_R\right.}{32\_R^3-45\_R^2+16\_R+24}\right.}{80652}$
risch	$\frac{\frac{2455}{80652}x^3-\frac{1429}{19552}x^2+\frac{89033}{1290432}x+\frac{3037}{53768}}{x^4-\frac{15}{8}x^3+x^2+3x+1}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(8\_Z^4-15\_Z^3+8\_Z^2+24\_Z+8\right)}\frac{\left(19640\_R^2-57489\_R+74897\right)\ln\left(x-\_R\right.}{32\_R^3-45\_R^2+16\_R+24}\right.}{80652}$

input
```
int(1/(8*x^4-15*x^3+8*x^2+24*x+8)^2,x,method=_RETURNVERBOSE)
```

output
```
(2455/80652*x^3-1429/19552*x^2+89033/1290432*x+3037/53768)/(x^4-15/8*x^3+x
^2+3*x+1)+1/80652*sum((19640*_R^2-57489*_R+74897)/(32*_R^3-45*_R^2+16*_R+2
4)*ln(x-_R),_R=RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 422, normalized size of antiderivative = 1.42

$$\int \frac{1}{(8+24x+8x^2-15x^3+8x^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(8*x^4-15*x^3+8*x^2+24*x+8)^2,x, algorithm="fricas")
```

output
```
1/8387808*(2042560*x^3 + 2*(8*x^4 - 15*x^3 + 8*x^2 + 24*x + 8)*sqrt(37741/
537508757*(2623170438295*sqrt(517) + 59644114671451)*sqrt(5073830635/78*sq
rt(517) - 59644114671451/40326) + 65959798255/6*sqrt(517) + 23262446380878
25/3102)*arctan(-1/10808697806203856907790898110205148*(6987613841*sqrt(51
7)*(50588327069839882l681*x - 733490208858664386702) - 1034*(7*sqrt(517)*(
4093383458474817335680443*x - 5930580762668771612904880) + 651517188439393
471516833312*x - 943931802280819590076206868)*sqrt(5073830635/78*sqrt(517)
 - 59644114671451/40326) + 80382534602389034825966761194161*x - 1165336966
966271242983789565430l0)*sqrt(37741/537508757*(2623170438295*sqrt(517) + 5
9644114671451)*sqrt(5073830635/78*sqrt(517) - 59644114671451/40326) + 6595
9798255/6*sqrt(517) + 2326244638087825/3102)) - 2*(8*x^4 - 15*x^3 + 8*x^2
+ 24*x + 8)*sqrt(-37741/537508757*(2623170438295*sqrt(517) + 5964411467145
1)*sqrt(5073830635/78*sqrt(517) - 59644114671451/40326) + 65959798255/6*sq
rt(517) + 2326244638087825/3102)*arctan(1/10808697806203856907790898110205
148*(6987613841*sqrt(517)*(50588327069839882l681*x - 733490208858664386702
) + 1034*(7*sqrt(517)*(4093383458474817335680443*x - 593058076266877161290
4880) + 651517188439393471516833312*x - 943931802280819590076206868)*sqrt(
5073830635/78*sqrt(517) - 59644114671451/40326) + 803825346023890348259667
61194161*x - 116533696696627124298378956543010)*sqrt(-37741/537508757*(262
3170438295*sqrt(517) + 59644114671451)*sqrt(5073830635/78*sqrt(517) - 5...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3839 vs. $2(230) = 460$.

Time = 2.50 (sec) , antiderivative size = 3839, normalized size of antiderivative = 12.93

$$\int \frac{1}{(8+24x+8x^2-15x^3+8x^4)^2}\,dx = \text{Too large to display}$$

input

```
integrate(1/(8*x**4-15*x**3+8*x**2+24*x+8)**2,x)
```

output

```
(39280*x**3 - 94314*x**2 + 89033*x + 72888)/(1290432*x**4 - 2419560*x**3 +
 1290432*x**2 + 3871296*x + 1290432) + sqrt(-59644114671451/16787862468089
856 + 5073830635*sqrt(517)/32471687559168)*log(x**2 + x*(-1123969950204685
03306932567484755463/603722125611976319526135612861060 - 29643869829812833
230907750777733957*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(5
17))/1936419398792394461637855141912238396080 - 181533261043120360732*sqrt
(-7120427417275887*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(5
17)) + 6263621568587150042935*sqrt(517) + 3557579971691991294769382675)/15
0930531402994079881533903215265 - 46926347979646613249222*sqrt(517)/297468
60362632912338339 + 994065243322493861977*sqrt(78)*sqrt(-59644114671451 +
2623170438295*sqrt(517))/1427849297406379792240272 + 994065243322493861977
*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517))*sqrt(-7120427
417275887*sqrt(40326)*sqrt(-59644114671451 + 2623170438295*sqrt(517)) + 62
63621568587150042935*sqrt(517) + 3557579971691991294769382675)/12909462658
61596307758570094608158930720) - 45971497067730669689218547912235602388091
89313591735176029*sqrt(517)*sqrt(-7120427417275887*sqrt(40326)*sqrt(-59644
114671451 + 2623170438295*sqrt(517)) + 6263621568587150042935*sqrt(517) +
3557579971691991294769382675)/18432767186998698626450604048374763890148748
05380627572841955840 - 102213276372026717588278042506361313108860193595830
3878081158710949715459967411486447/302201812380681690634631534385892067...
```

Maxima [F]

$$\int \frac{1}{(8+24x+8x^2-15x^3+8x^4)^2}\,dx = \int \frac{1}{(8\,x^4-15\,x^3+8\,x^2+24\,x+8)^2}\,dx$$

input
```
integrate(1/(8*x^4-15*x^3+8*x^2+24*x+8)^2,x, algorithm="maxima")
```

output
```
1/161304*(39280*x^3 - 94314*x^2 + 89033*x + 72888)/(8*x^4 - 15*x^3 + 8*x^2
 + 24*x + 8) + 1/80652*integrate((19640*x^2 - 57489*x + 74897)/(8*x^4 - 15
*x^3 + 8*x^2 + 24*x + 8), x)
```

Giac [F]

$$\int \frac{1}{(8+24x+8x^2-15x^3+8x^4)^2}\,dx = \int \frac{1}{(8\,x^4-15\,x^3+8\,x^2+24\,x+8)^2}\,dx$$

input
```
integrate(1/(8*x^4-15*x^3+8*x^2+24*x+8)^2,x, algorithm="giac")
```

output
```
integrate((8*x^4 - 15*x^3 + 8*x^2 + 24*x + 8)^(-2), x)
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.61

$$\int \frac{1}{(8+24x+8x^2-15x^3+8x^4)^2}\,dx = \frac{\frac{2455\,x^3}{80652}-\frac{1429\,x^2}{19552}+\frac{89033\,x}{1290432}+\frac{3037}{53768}}{x^4-\frac{15\,x^3}{8}+x^2+3\,x+1} + \left(\sum_{k=1}^{4}\ln\left(\frac{2146659825\,\mathrm{root}\left(z^4+\frac{14911625619311\,z^2}{524620702127808}+\frac{39238139261\,z}{3730636104019968}+\frac{43023440}{44204510553294663},z,k\right)}{2960381771776} + \frac{2222183\,x}{338246745408} + \frac{\mathrm{root}\left(z^4+\frac{14911625619311\,z^2}{524620702127808}+\frac{39238139261\,z}{3730636104019968}+\frac{43023440}{44204510553294663},z,k\right)\,x\,924124364159}{26643435945984} - \frac{\mathrm{root}\left(z^4+\frac{14911625619311\,z^2}{524620702127808}+\frac{39238139261\,z}{3730636104019968}+\frac{43023440}{44204510553294663},z,k\right)^2\,x\,72451101}{8470528} - \frac{\mathrm{root}\left(z^4+\frac{14911625619311\,z^2}{524620702127808}+\frac{39238139261\,z}{3730636104019968}+\frac{43023440}{44204510553294663},z,k\right)^3\,x\,95745}{256} + \frac{389551\,\mathrm{root}\left(z^4+\frac{14911625619311\,z^2}{524620702127808}+\frac{39238139261\,z}{3730636104019968}+\frac{43023440}{44204510553294663},z,k\right)^2}{264704} - \frac{100737\,\mathrm{root}\left(z^4+\frac{14911625619311\,z^2}{524620702127808}+\frac{39238139261\,z}{3730636104019968}+\frac{43023440}{44204510553294663},z,k\right)^3}{512} + \frac{271033}{624455529984}\right)\,\mathrm{root}\left(z^4+\frac{14911625619311\,z^2}{524620702127808}+\frac{39238139261\,z}{3730636104019968}+\frac{43023440}{44204510553294663},z,k\right)\right)$$

input
```
int(1/(24*x + 8*x^2 - 15*x^3 + 8*x^4 + 8)^2,x)
```

output
```
((89033*x)/1290432 - (1429*x^2)/19552 + (2455*x^3)/80652 + 3037/53768)/(3*
x + x^2 - (15*x^3)/8 + x^4 + 1) + symsum(log((2146659825*root(z^4 + (14911
625619311*z^2)/524620702127808 + (39238139261*z)/3730636104019968 + 430234
40/44204510553294663, z, k))/2960381771776 + (2222183*x)/338246745408 + (9
24124364159*root(z^4 + (14911625619311*z^2)/524620702127808 + (39238139261
*z)/3730636104019968 + 43023440/44204510553294663, z, k)*x)/26643435945984
 - (72451101*root(z^4 + (14911625619311*z^2)/524620702127808 + (3923813926
1*z)/3730636104019968 + 43023440/44204510553294663, z, k)^2*x)/8470528 - (
95745*root(z^4 + (14911625619311*z^2)/524620702127808 + (39238139261*z)/37
30636104019968 + 43023440/44204510553294663, z, k)^3*x)/256 + (389551*root
(z^4 + (14911625619311*z^2)/524620702127808 + (39238139261*z)/373063610401
9968 + 43023440/44204510553294663, z, k)^2)/264704 - (100737*root(z^4 + (1
4911625619311*z^2)/524620702127808 + (39238139261*z)/3730636104019968 + 43
023440/44204510553294663, z, k)^3)/512 + 271033/624455529984)*root(z^4 + (
14911625619311*z^2)/524620702127808 + (39238139261*z)/3730636104019968 + 4
3023440/44204510553294663, z, k), k, 1, 4)
```

Reduce [F]

$$\int \frac{1}{(8+24x+8x^2-15x^3+8x^4)^2}\,dx = \frac{-256\left(\int \frac{x^3}{64x^8-240x^7+353x^6+144x^5-528x^4+144x^3+704x^2+384x+64}dx\right)x^4+480\left(\int \frac{x^3}{64x^8-240x^7+353x^6+144x^5-528x^4+144x^3}\right.}{}$$

input
```
int(1/(8*x^4-15*x^3+8*x^2+24*x+8)^2,x)
```

output
```
( - 256*int(x**3/(64*x**8 - 240*x**7 + 353*x**6 + 144*x**5 - 528*x**4 + 14
4*x**3 + 704*x**2 + 384*x + 64),x)*x**4 + 480*int(x**3/(64*x**8 - 240*x**7
 + 353*x**6 + 144*x**5 - 528*x**4 + 144*x**3 + 704*x**2 + 384*x + 64),x)*x
**3 - 256*int(x**3/(64*x**8 - 240*x**7 + 353*x**6 + 144*x**5 - 528*x**4 +
144*x**3 + 704*x**2 + 384*x + 64),x)*x**2 - 768*int(x**3/(64*x**8 - 240*x*
*7 + 353*x**6 + 144*x**5 - 528*x**4 + 144*x**3 + 704*x**2 + 384*x + 64),x)
*x - 256*int(x**3/(64*x**8 - 240*x**7 + 353*x**6 + 144*x**5 - 528*x**4 + 1
44*x**3 + 704*x**2 + 384*x + 64),x) + 360*int(x**2/(64*x**8 - 240*x**7 + 3
53*x**6 + 144*x**5 - 528*x**4 + 144*x**3 + 704*x**2 + 384*x + 64),x)*x**4
- 675*int(x**2/(64*x**8 - 240*x**7 + 353*x**6 + 144*x**5 - 528*x**4 + 144*
x**3 + 704*x**2 + 384*x + 64),x)*x**3 + 360*int(x**2/(64*x**8 - 240*x**7 +
 353*x**6 + 144*x**5 - 528*x**4 + 144*x**3 + 704*x**2 + 384*x + 64),x)*x**
2 + 1080*int(x**2/(64*x**8 - 240*x**7 + 353*x**6 + 144*x**5 - 528*x**4 + 1
44*x**3 + 704*x**2 + 384*x + 64),x)*x + 360*int(x**2/(64*x**8 - 240*x**7 +
 353*x**6 + 144*x**5 - 528*x**4 + 144*x**3 + 704*x**2 + 384*x + 64),x) - 1
28*int(x/(64*x**8 - 240*x**7 + 353*x**6 + 144*x**5 - 528*x**4 + 144*x**3 +
 704*x**2 + 384*x + 64),x)*x**4 + 240*int(x/(64*x**8 - 240*x**7 + 353*x**6
 + 144*x**5 - 528*x**4 + 144*x**3 + 704*x**2 + 384*x + 64),x)*x**3 - 128*i
nt(x/(64*x**8 - 240*x**7 + 353*x**6 + 144*x**5 - 528*x**4 + 144*x**3 + 704
*x**2 + 384*x + 64),x)*x**2 - 384*int(x/(64*x**8 - 240*x**7 + 353*x**6 ...
```

3.85 $\int \frac{1}{\sqrt{8+8x-x^3+8x^4}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 131

$$\int \frac{1}{\sqrt{8+8x-x^3+8x^4}}\,dx = -\frac{\left(87+\sqrt{29}\left(1+\frac{4}{x}\right)^2\right)\sqrt{\frac{261-6\left(1+\frac{4}{x}\right)^2+\left(1+\frac{4}{x}\right)^4}{\left(87+\sqrt{29}\left(1+\frac{4}{x}\right)^2\right)^2}}x^2\,\text{EllipticF}\left(2\arctan\left(\frac{4+x}{\sqrt{3}\sqrt[4]{29}x}\right),\frac{1}{58}\left(29+\sqrt{29}\right)\right)}{8\sqrt{3}\sqrt[4]{29}\sqrt{8+8x-x^3+8x^4}}$$

output
```
-1/696*(87+29^(1/2)*(1+4/x)^2)*((261-6*(1+4/x)^2+(1+4/x)^4)/(87+29^(1/2)*(
1+4/x)^2)^2)^(1/2)*x^2*InverseJacobiAM(2*arctan(1/87*(4+x)*3^(1/2)*29^(3/4
)/x),1/58*(1682+58*29^(1/2))^(1/2))*3^(1/2)*29^(3/4)/(8*x^4-x^3+8*x+8)^(1/
2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 11.00 (sec) , antiderivative size = 927, normalized size of antiderivative = 7.08

$$\int \frac{1}{\sqrt{8+8x-x^3+8x^4}}\,dx = \text{Too large to display}$$

input
```
Integrate[1/Sqrt[8 + 8*x - x^3 + 8*x^4],x]
```

output
```
(-2*EllipticF[ArcSin[Sqrt[((x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0])*(
Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 &
, 4, 0]))/((x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0])*(Root[8 + 8*#1 -
#1^3 + 8*#1^4 & , 1, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0]))]], ((R
oot[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & ,
 3, 0])*(Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] - Root[8 + 8*#1 - #1^3 +
8*#1^4 & , 4, 0]))/((Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] - Root[8 + 8*
#1 - #1^3 + 8*#1^4 & , 3, 0])*(Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0] - R
oot[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0]))]*(x - Root[8 + 8*#1 - #1^3 + 8*#1
^4 & , 2, 0])^2*Sqrt[((Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] - Root[8 +
8*#1 - #1^3 + 8*#1^4 & , 2, 0])*(x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 3,
0]))/((x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0])*(Root[8 + 8*#1 - #1^3
+ 8*#1^4 & , 1, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 3, 0]))]*(Root[8 +
8*#1 - #1^3 + 8*#1^4 & , 1, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0])*
Sqrt[((x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0])*(Root[8 + 8*#1 - #1^3
+ 8*#1^4 & , 1, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0])*(x - Root[8
+ 8*#1 - #1^3 + 8*#1^4 & , 4, 0])*(Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0]
 - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0]))/((x - Root[8 + 8*#1 - #1^3 +
8*#1^4 & , 2, 0])^2*(Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] - Root[8 + 8*
#1 - #1^3 + 8*#1^4 & , 4, 0])^2)])/(Sqrt[8 + 8*x - x^3 + 8*x^4]*(-Root[...
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.28, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {2504, 27, 7270, 1416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{8x^4 - x^3 + 8x + 8}} \, dx$$

$$\Big\downarrow \text{2504}$$

$$-1024\int \frac{1}{128\sqrt{2}\left(1-4\left(\frac{1}{4}+\frac{1}{x}\right)\right)^2\sqrt{\frac{256\left(\frac{1}{4}+\frac{1}{x}\right)^4-96\left(\frac{1}{4}+\frac{1}{x}\right)^2+261}{\left(1-4\left(\frac{1}{4}+\frac{1}{x}\right)\right)^4}}}d\left(\frac{1}{4}+\frac{1}{x}\right)$$

$$\Big\downarrow \ 27$$

$$-4\sqrt{2}\int \frac{1}{\left(1-4\left(\frac{1}{4}+\frac{1}{x}\right)\right)^2\sqrt{\frac{256\left(\frac{1}{4}+\frac{1}{x}\right)^4-96\left(\frac{1}{4}+\frac{1}{x}\right)^2+261}{\left(1-4\left(\frac{1}{4}+\frac{1}{x}\right)\right)^4}}}d\left(\frac{1}{4}+\frac{1}{x}\right)$$

$$\Big\downarrow \ 7270$$

$$-\frac{4\sqrt{2}\sqrt{256\left(\frac{1}{x}+\frac{1}{4}\right)^4-96\left(\frac{1}{x}+\frac{1}{4}\right)^2+261}\int \frac{1}{\sqrt{256\left(\frac{1}{4}+\frac{1}{x}\right)^4-96\left(\frac{1}{4}+\frac{1}{x}\right)^2+261}}d\left(\frac{1}{4}+\frac{1}{x}\right)}{\left(1-4\left(\frac{1}{x}+\frac{1}{4}\right)\right)^2\sqrt{\frac{256\left(\frac{1}{x}+\frac{1}{4}\right)^4-96\left(\frac{1}{x}+\frac{1}{4}\right)^2+261}{\left(1-4\left(\frac{1}{x}+\frac{1}{4}\right)\right)^4}}}$$

$$\Big\downarrow \ 1416$$

$$-\frac{\left(16\sqrt{29}\left(\frac{1}{x}+\frac{1}{4}\right)^2+87\right)\sqrt{\frac{256\left(\frac{1}{x}+\frac{1}{4}\right)^4-96\left(\frac{1}{x}+\frac{1}{4}\right)^2+261}{\left(16\sqrt{29}\left(\frac{1}{x}+\frac{1}{4}\right)^2+87\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{4\left(\frac{1}{4}+\frac{1}{x}\right)}{\sqrt{3}\sqrt[4]{29}}\right),\frac{1}{58}\left(29+\sqrt{29}\right)\right)}{\sqrt{6}\sqrt[4]{29}\left(1-4\left(\frac{1}{x}+\frac{1}{4}\right)\right)^2\sqrt{\frac{256\left(\frac{1}{x}+\frac{1}{4}\right)^4-96\left(\frac{1}{x}+\frac{1}{4}\right)^2+261}{\left(1-4\left(\frac{1}{x}+\frac{1}{4}\right)\right)^4}}}$$

input
```
Int[1/Sqrt[8 + 8*x - x^3 + 8*x^4],x]
```

output
```
-(((87 + 16*Sqrt[29]*(1/4 + x^(-1))^2)*Sqrt[(261 - 96*(1/4 + x^(-1))^2 + 2
56*(1/4 + x^(-1))^4)/(87 + 16*Sqrt[29]*(1/4 + x^(-1))^2)^2]*EllipticF[2*Ar
cTan[(4*(1/4 + x^(-1)))/(Sqrt[3]*29^(1/4))], (29 + Sqrt[29])/58])/(Sqrt[6]
*29^(1/4)*(1 - 4*(1/4 + x^(-1)))^2*Sqrt[(261 - 96*(1/4 + x^(-1))^2 + 256*(
1/4 + x^(-1))^4)/(1 - 4*(1/4 + x^(-1)))^4]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 2504
```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Simp[-16*
a^2   Subst[Int[(1/(b - 4*a*x)^2)*(a*((-3*b^4 + 16*a*b^2*c - 64*a^2*b*d + 2
56*a^3*e - 32*a^2*(3*b^2 - 8*a*c)*x^2 + 256*a^4*x^4)/(b - 4*a*x)^4))^p, x],
 x, b/(4*a) + 1/x], x] /; NeQ[a, 0] && NeQ[b, 0] && EqQ[b^3 - 4*a*b*c + 8*a
^2*d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && IntegerQ[2*p] &&  !IGtQ[p, 0
]
```

rule 7270
```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Simp[a^IntPart[p
]*((a*v^m*w^n)^FracPart[p]/(v^(m*FracPart[p])*w^(n*FracPart[p])))   Int[u*v
^(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !Free
Q[v, x] &&  !FreeQ[w, x]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 964 vs. $2(114) = 228$.

Time = 1.09 (sec) , antiderivative size = 965, normalized size of antiderivative = 7.37

method	result	size
default	Expression too large to display	965
elliptic	Expression too large to display	965

input
```
int(1/(8*x^4-x^3+8*x+8)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)
)*((RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2))
*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))/(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4
)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))/(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2
)))^(1/2)*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2))^2*((RootOf(8*_Z^4-_Z^3+8*
_Z+8,index=2)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))*(x-RootOf(8*_Z^4-_Z^3+8*
_Z+8,index=3))/(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=3)-RootOf(8*_Z^4-_Z^3+8*_Z
+8,index=1))/(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)))^(1/2)*((RootOf(8*_Z^4
-_Z^3+8*_Z+8,index=2)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))*(x-RootOf(8*_Z^4
-_Z^3+8*_Z+8,index=4))/(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)-RootOf(8*_Z^4-_
Z^3+8*_Z+8,index=1))/(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)))^(1/2)/(RootOf
(8*_Z^4-_Z^3+8*_Z+8,index=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2))/(RootOf(8
*_Z^4-_Z^3+8*_Z+8,index=2)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))*2^(1/2)/((x
-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)
)*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=3))*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,inde
x=4)))^(1/2)*EllipticF(((RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)-RootOf(8*_Z^4-
_Z^3+8*_Z+8,index=2))*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))/(RootOf(8*_Z^
4-_Z^3+8*_Z+8,index=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))/(x-RootOf(8*_Z^
4-_Z^3+8*_Z+8,index=2)))^(1/2),((RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)-RootOf
(8*_Z^4-_Z^3+8*_Z+8,index=3))*(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1)-RootO...
```

Fricas [F]

$$\int \frac{1}{\sqrt{8+8x-x^3+8x^4}}\,dx=\int \frac{1}{\sqrt{8\,x^4-x^3+8\,x+8}}\,dx$$

input
```
integrate(1/(8*x^4-x^3+8*x+8)^(1/2),x, algorithm="fricas")
```

output
```
integral(1/sqrt(8*x^4 - x^3 + 8*x + 8), x)
```

Sympy [F]

$$\int \frac{1}{\sqrt{8+8x-x^3+8x^4}}\,dx = \int \frac{1}{\sqrt{8x^4-x^3+8x+8}}\,dx$$

input
```
integrate(1/(8*x**4-x**3+8*x+8)**(1/2),x)
```

output
```
Integral(1/sqrt(8*x**4 - x**3 + 8*x + 8), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{8+8x-x^3+8x^4}}\,dx = \int \frac{1}{\sqrt{8\,x^4-x^3+8\,x+8}}\,dx$$

input
```
integrate(1/(8*x^4-x^3+8*x+8)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(8*x^4 - x^3 + 8*x + 8), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{8+8x-x^3+8x^4}}\,dx = \int \frac{1}{\sqrt{8\,x^4-x^3+8\,x+8}}\,dx$$

input
```
integrate(1/(8*x^4-x^3+8*x+8)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(8*x^4 - x^3 + 8*x + 8), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{8+8x-x^3+8x^4}}\,dx = \int \frac{1}{\sqrt{8\,x^4-x^3+8\,x+8}}\,dx$$

input
```
int(1/(8*x - x^3 + 8*x^4 + 8)^(1/2),x)
```

output
```
int(1/(8*x - x^3 + 8*x^4 + 8)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{8+8x-x^3+8x^4}}\,dx = \int \frac{\sqrt{8x^4-x^3+8x+8}}{8x^4-x^3+8x+8}dx$$

input
```
int(1/(8*x^4-x^3+8*x+8)^(1/2),x)
```

output
```
int(sqrt(8*x**4 - x**3 + 8*x + 8)/(8*x**4 - x**3 + 8*x + 8),x)
```

3.86 $\int \frac{1}{(8+8x-x^3+8x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 431

$$\int \frac{1}{(8+8x-x^3+8x^4)^{3/2}}\,dx = -\frac{\left(66-\left(1+\frac{4}{x}\right)^2\right)x^2}{1008\sqrt{8+8x-x^3+8x^4}}$$
$$+\frac{\left(216-7\left(1+\frac{4}{x}\right)^2\right)\left(1+\frac{4}{x}\right)x^2}{12528\sqrt{8+8x-x^3+8x^4}}+\frac{7\left(261-6\left(1+\frac{4}{x}\right)^2+\left(1+\frac{4}{x}\right)^4\right)\left(1+\frac{4}{x}\right)x^2}{12528\left(3\sqrt{29}+\left(1+\frac{4}{x}\right)^2\right)\sqrt{8+8x-x^3+8x^4}}$$
$$-\frac{7\left(87+\sqrt{29}\left(1+\frac{4}{x}\right)^2\right)\sqrt{\frac{261-6\left(1+\frac{4}{x}\right)^2+\left(1+\frac{4}{x}\right)^4}{\left(87+\sqrt{29}\left(1+\frac{4}{x}\right)^2\right)^2}}\,x^2E\left(2\arctan\left(\frac{4+x}{\sqrt{3}\sqrt[4]{29}x}\right)|\frac{1}{58}\left(29+\sqrt{29}\right)\right)}{144\sqrt{3}29^{3/4}\sqrt{8+8x-x^3+8x^4}}$$
$$+\frac{\left(14-5\sqrt{29}\right)\left(87+\sqrt{29}\left(1+\frac{4}{x}\right)^2\right)\sqrt{\frac{261-6\left(1+\frac{4}{x}\right)^2+\left(1+\frac{4}{x}\right)^4}{\left(87+\sqrt{29}\left(1+\frac{4}{x}\right)^2\right)^2}}\,x^2\,\text{EllipticF}\left(2\arctan\left(\frac{4+x}{\sqrt{3}\sqrt[4]{29}x}\right),\frac{1}{58}\left(29+\sqrt{29}\cdots\right.\right.}{576\sqrt{3}29^{3/4}\sqrt{8+8x-x^3+8x^4}}$$

output
```
-1/1008*(66-(1+4/x)^2)*x^2/(8*x^4-x^3+8*x+8)^(1/2)+1/12528*(216-7*(1+4/x)^
2)*(1+4/x)*x^2/(8*x^4-x^3+8*x+8)^(1/2)+7/12528*(261-6*(1+4/x)^2+(1+4/x)^4)
*(1+4/x)*x^2/(3*29^(1/2)+(1+4/x)^2)/(8*x^4-x^3+8*x+8)^(1/2)-7/12528*(87+29
^(1/2)*(1+4/x)^2)*((261-6*(1+4/x)^2+(1+4/x)^4)/(87+29^(1/2)*(1+4/x)^2)^2)^
(1/2)*x^2*EllipticE(sin(2*arctan(1/87*(4+x)*3^(1/2)*29^(3/4)/x)),1/58*(168
2+58*29^(1/2))^(1/2))*3^(1/2)*29^(1/4)/(8*x^4-x^3+8*x+8)^(1/2)+1/50112*(14
-5*29^(1/2))*(87+29^(1/2)*(1+4/x)^2)*((261-6*(1+4/x)^2+(1+4/x)^4)/(87+29^(
1/2)*(1+4/x)^2)^2)^(1/2)*x^2*InverseJacobiAM(2*arctan(1/87*(4+x)*3^(1/2)*2
9^(3/4)/x),1/58*(1682+58*29^(1/2))^(1/2))*3^(1/2)*29^(1/4)/(8*x^4-x^3+8*x+
8)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 16.07 (sec) , antiderivative size = 4865, normalized size of antiderivative = 11.29

$$\int \frac{1}{(8+8x-x^3+8x^4)^{3/2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(8 + 8*x - x^3 + 8*x^4)^(-3/2),x]
```

output
```
(544 + 1539*x - 1146*x^2 + 784*x^3)/(21924*Sqrt[8 + 8*x - x^3 + 8*x^4]) +
((28*(x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0])^2*(-(EllipticF[ArcSin[S
qrt[((x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0])*(Root[8 + 8*#1 - #1^3 +
 8*#1^4 & , 2, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0]))/((x - Root[8
 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0])*(Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0
] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0]))]], -(((Root[8 + 8*#1 - #1^3
+ 8*#1^4 & , 2, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 3, 0])*(Root[8 + 8*
#1 - #1^3 + 8*#1^4 & , 1, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0]))/(
(-Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] + Root[8 + 8*#1 - #1^3 + 8*#1^4
& , 3, 0])*(Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0] - Root[8 + 8*#1 - #1^3
 + 8*#1^4 & , 4, 0])))]*Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0]) + Ellipti
cPi[(-Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] + Root[8 + 8*#1 - #1^3 + 8*#
1^4 & , 4, 0])/(-Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0] + Root[8 + 8*#1 -
 #1^3 + 8*#1^4 & , 4, 0]), ArcSin[Sqrt[((x - Root[8 + 8*#1 - #1^3 + 8*#1^4
 & , 1, 0])*(Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0] - Root[8 + 8*#1 - #1^
3 + 8*#1^4 & , 4, 0]))/((x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0])*(Roo
t[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4
, 0]))]], -(((Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0] - Root[8 + 8*#1 - #1
^3 + 8*#1^4 & , 3, 0])*(Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] - Root[8 +
 8*#1 - #1^3 + 8*#1^4 & , 4, 0]))/((-Root[8 + 8*#1 - #1^3 + 8*#1^4 & , ...
```

Rubi [A] (verified)

Time = 1.28 (sec) , antiderivative size = 553, normalized size of antiderivative = 1.28, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.684$, Rules used = {2504, 27, 7270, 2202, 1576, 27, 1158, 2206, 27, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(8x^4 - x^3 + 8x + 8)^{3/2}} \, dx$$

$$\Big\downarrow \; 2504$$

$$-1024 \int \frac{1}{1024\sqrt{2}\left(1-4\left(\frac{1}{4}+\frac{1}{x}\right)\right)^2 \left(\frac{256\left(\frac{1}{4}+\frac{1}{x}\right)^4-96\left(\frac{1}{4}+\frac{1}{x}\right)^2+261}{\left(1-4\left(\frac{1}{4}+\frac{1}{x}\right)\right)^4}\right)^{3/2}} d\left(\frac{1}{4}+\frac{1}{x}\right)$$

$$\Big\downarrow \; 27$$

$$-\frac{\int \frac{1}{\left(1-4\left(\frac{1}{4}+\frac{1}{x}\right)\right)^2\left(\frac{256\left(\frac{1}{4}+\frac{1}{x}\right)^4-96\left(\frac{1}{4}+\frac{1}{x}\right)^2+261}{\left(1-4\left(\frac{1}{4}+\frac{1}{x}\right)\right)^4}\right)^{3/2}}d\left(\frac{1}{4}+\frac{1}{x}\right)}{\sqrt{2}}$$

$\Big\downarrow$ 7270

$$-\frac{\sqrt{256\left(\frac{1}{x}+\frac{1}{4}\right)^4-96\left(\frac{1}{x}+\frac{1}{4}\right)^2+261}\int \frac{\left(1-4\left(\frac{1}{4}+\frac{1}{x}\right)\right)^4}{\left(256\left(\frac{1}{4}+\frac{1}{x}\right)^4-96\left(\frac{1}{4}+\frac{1}{x}\right)^2+261\right)^{3/2}}d\left(\frac{1}{4}+\frac{1}{x}\right)}{\sqrt{2}\left(1-4\left(\frac{1}{x}+\frac{1}{4}\right)\right)^2\sqrt{\frac{256\left(\frac{1}{x}+\frac{1}{4}\right)^4-96\left(\frac{1}{x}+\frac{1}{4}\right)^2+261}{\left(1-4\left(\frac{1}{x}+\frac{1}{4}\right)\right)^4}}}$$

$\Big\downarrow$ 2202

$$-\frac{\sqrt{256\left(\frac{1}{x}+\frac{1}{4}\right)^4-96\left(\frac{1}{x}+\frac{1}{4}\right)^2+261}\left(\int \frac{256\left(\frac{1}{4}+\frac{1}{x}\right)^4+96\left(\frac{1}{4}+\frac{1}{x}\right)^2+1}{\left(256\left(\frac{1}{4}+\frac{1}{x}\right)^4-96\left(\frac{1}{4}+\frac{1}{x}\right)^2+261\right)^{3/2}}d\left(\frac{1}{4}+\frac{1}{x}\right)+\int \frac{\left(-256\left(\frac{1}{4}+\frac{1}{x}\right)^2-16\right)\left(\frac{1}{4}+\frac{1}{x}\right)}{\left(256\left(\frac{1}{4}+\frac{1}{x}\right)^4-96\left(\frac{1}{4}+\frac{1}{x}\right)^2+261\right)^{3}}\right.}{\sqrt{2}\left(1-4\left(\frac{1}{x}+\frac{1}{4}\right)\right)^2\sqrt{\frac{256\left(\frac{1}{x}+\frac{1}{4}\right)^4-96\left(\frac{1}{x}+\frac{1}{4}\right)^2+261}{\left(1-4\left(\frac{1}{x}+\frac{1}{4}\right)\right)^4}}}$$

$\Big\downarrow$ 1576

$$-\frac{\sqrt{256\left(\frac{1}{x}+\frac{1}{4}\right)^4-96\left(\frac{1}{x}+\frac{1}{4}\right)^2+261}\left(\frac{1}{2}\int -\frac{16\left(16\left(\frac{1}{4}+\frac{1}{x}\right)^2+1\right)}{\left(256\left(\frac{1}{4}+\frac{1}{x}\right)^4-96\left(\frac{1}{4}+\frac{1}{x}\right)^2+261\right)^{3/2}}d\left(\frac{1}{4}+\frac{1}{x}\right)^2+\int \frac{256\left(\frac{1}{4}+\frac{1}{x}\right)^4+96\left(\frac{1}{4}+\frac{1}{x}\right)^2}{\left(256\left(\frac{1}{4}+\frac{1}{x}\right)^4-96\left(\frac{1}{4}+\frac{1}{x}\right)^2+\right.}\right.}{\sqrt{2}\left(1-4\left(\frac{1}{x}+\frac{1}{4}\right)\right)^2\sqrt{\frac{256\left(\frac{1}{x}+\frac{1}{4}\right)^4-96\left(\frac{1}{x}+\frac{1}{4}\right)^2+261}{\left(1-4\left(\frac{1}{x}+\frac{1}{4}\right)\right)^4}}}$$

$\Big\downarrow$ 27

$$-\frac{\sqrt{256\left(\frac{1}{x}+\frac{1}{4}\right)^4-96\left(\frac{1}{x}+\frac{1}{4}\right)^2+261}\left(\int \frac{256\left(\frac{1}{4}+\frac{1}{x}\right)^4+96\left(\frac{1}{4}+\frac{1}{x}\right)^2+1}{\left(256\left(\frac{1}{4}+\frac{1}{x}\right)^4-96\left(\frac{1}{4}+\frac{1}{x}\right)^2+261\right)^{3/2}}d\left(\frac{1}{4}+\frac{1}{x}\right)-8\int \frac{16\left(\frac{1}{4}+\frac{1}{x}\right)^2+1}{\left(256\left(\frac{1}{4}+\frac{1}{x}\right)^4-96\left(\frac{1}{4}+\frac{1}{x}\right)^2+261\right.}\right.}{\sqrt{2}\left(1-4\left(\frac{1}{x}+\frac{1}{4}\right)\right)^2\sqrt{\frac{256\left(\frac{1}{x}+\frac{1}{4}\right)^4-96\left(\frac{1}{x}+\frac{1}{4}\right)^2+261}{\left(1-4\left(\frac{1}{x}+\frac{1}{4}\right)\right)^4}}}$$

$\Big\downarrow$ 1158

$$-\frac{\sqrt{256\left(\frac{1}{x}+\frac{1}{4}\right)^4-96\left(\frac{1}{x}+\frac{1}{4}\right)^2+261}\left(\int \frac{256\left(\frac{1}{4}+\frac{1}{x}\right)^4+96\left(\frac{1}{4}+\frac{1}{x}\right)^2+1}{\left(256\left(\frac{1}{4}+\frac{1}{x}\right)^4-96\left(\frac{1}{4}+\frac{1}{x}\right)^2+261\right)^{3/2}}d\left(\frac{1}{4}+\frac{1}{x}\right)+\frac{33-8\left(\frac{1}{x}+\frac{1}{4}\right)^2}{63\sqrt{256\left(\frac{1}{x}+\frac{1}{4}\right)^4-96\left(\frac{1}{x}+\frac{1}{4}\right)^2+261}}\right)}{\sqrt{2}\left(1-4\left(\frac{1}{x}+\frac{1}{4}\right)\right)^2\sqrt{\frac{256\left(\frac{1}{x}+\frac{1}{4}\right)^4-96\left(\frac{1}{x}+\frac{1}{4}\right)^2+261}{\left(1-4\left(\frac{1}{x}+\frac{1}{4}\right)\right)^4}}}$$

$\Big\downarrow$ 2206

$$-\frac{\sqrt{256\left(\frac{1}{x}+\frac{1}{4}\right)^4-96\left(\frac{1}{x}+\frac{1}{4}\right)^2+261}\left(\frac{\int\frac{86016\left(435-224\left(\frac{1}{4}+\frac{1}{x}\right)^2\right)}{\sqrt{256\left(\frac{1}{4}+\frac{1}{x}\right)^4-96\left(\frac{1}{4}+\frac{1}{x}\right)^2+261}}d\left(\frac{1}{4}+\frac{1}{x}\right)}{67350528}+\frac{33-8\left(\frac{1}{x}+\frac{1}{4}\right)^2}{63\sqrt{256\left(\frac{1}{x}+\frac{1}{4}\right)^4-96\left(\frac{1}{x}+\frac{1}{4}\right)^2+261}}-\frac{16\cdots}{783\sqrt{2\cdots}}\cdots\right.}{\sqrt{2}\left(1-4\left(\frac{1}{x}+\frac{1}{4}\right)\right)^2\sqrt{\frac{256\left(\frac{1}{x}+\frac{1}{4}\right)^4-96\left(\frac{1}{x}+\frac{1}{4}\right)^2+261}{\left(1-4\left(\frac{1}{x}+\frac{1}{4}\right)\right)^4}}\cdots}$$

$\downarrow$ 27

$$-\frac{\sqrt{256\left(\frac{1}{x}+\frac{1}{4}\right)^4-96\left(\frac{1}{x}+\frac{1}{4}\right)^2+261}\left(\frac{1}{783}\int\frac{435-224\left(\frac{1}{4}+\frac{1}{x}\right)^2}{\sqrt{256\left(\frac{1}{4}+\frac{1}{x}\right)^4-96\left(\frac{1}{4}+\frac{1}{x}\right)^2+261}}d\left(\frac{1}{4}+\frac{1}{x}\right)+\frac{33-8\left(\frac{1}{x}+\frac{1}{4}\right)^2}{63\sqrt{256\left(\frac{1}{x}+\frac{1}{4}\right)^4-96\left(\frac{1}{x}+\frac{1}{4}\right)^2+261}}\cdots\right.}{\sqrt{2}\left(1-4\left(\frac{1}{x}+\frac{1}{4}\right)\right)^2\sqrt{\frac{256\left(\frac{1}{x}+\frac{1}{4}\right)^4-96\left(\frac{1}{x}+\frac{1}{4}\right)^2+261}{\left(1-4\left(\frac{1}{x}+\frac{1}{4}\right)\right)^4}}\cdots}$$

$\downarrow$ 1511

$$-\frac{\sqrt{256\left(\frac{1}{x}+\frac{1}{4}\right)^4-96\left(\frac{1}{x}+\frac{1}{4}\right)^2+261}\left(\frac{1}{783}\left(3\left(145-14\sqrt{29}\right)\int\frac{1}{\sqrt{256\left(\frac{1}{4}+\frac{1}{x}\right)^4-96\left(\frac{1}{4}+\frac{1}{x}\right)^2+261}}d\left(\frac{1}{4}+\frac{1}{x}\right)+42\sqrt{29}\int\frac{\cdots}{8\cdots}\cdots\right.\right.}{\sqrt{2}\left(1-4\left(\frac{1}{x}+\frac{1}{4}\right)\right)^2\sqrt{\frac{256(\cdots}{\cdots}}\cdots}$$

$\downarrow$ 27

$$-\frac{\sqrt{256\left(\frac{1}{x}+\frac{1}{4}\right)^4-96\left(\frac{1}{x}+\frac{1}{4}\right)^2+261}\left(\frac{1}{783}\left(3\left(145-14\sqrt{29}\right)\int\frac{1}{\sqrt{256\left(\frac{1}{4}+\frac{1}{x}\right)^4-96\left(\frac{1}{4}+\frac{1}{x}\right)^2+261}}d\left(\frac{1}{4}+\frac{1}{x}\right)+\frac{14\int\frac{8\cdots}{\sqrt{256(\cdots}}\cdots}{\cdots}\right.\right.}{\sqrt{2}\left(1-4\left(\frac{1}{x}+\frac{1}{4}\right)\right)^2\sqrt{\frac{256\left(\frac{1}{x}+\frac{1}{4}\right)^4\cdots}{(1-\cdots}}\cdots}$$

$\downarrow$ 1416

$$-\frac{\sqrt{256\left(\frac{1}{x}+\frac{1}{4}\right)^4-96\left(\frac{1}{x}+\frac{1}{4}\right)^2+261}\left(\frac{1}{783}\left(\frac{14\int\frac{87-16\sqrt{29}\left(\frac{1}{4}+\frac{1}{x}\right)^2}{\sqrt{256\left(\frac{1}{4}+\frac{1}{x}\right)^4-96\left(\frac{1}{4}+\frac{1}{x}\right)^2+261}}d\left(\frac{1}{4}+\frac{1}{x}\right)}{\sqrt{29}}+\frac{\sqrt{3}\left(145-14\sqrt{29}\right)\left(16\sqrt{29}\left(\frac{1}{x}+\frac{1}{4}\right)^2+\cdots\right.}{\cdots}\right.\right.}{\sqrt{2}\left(1-4\left(\frac{1}{x}\cdots\right.\right.}$$

$\downarrow$ 1509

$$-\frac{\sqrt{256\left(\frac{1}{x}+\frac{1}{4}\right)^4-96\left(\frac{1}{x}+\frac{1}{4}\right)^2+261}\left(\frac{1}{783}\left(\frac{\sqrt{3}\left(145-14\sqrt{29}\right)\left(16\sqrt{29}\left(\frac{1}{x}+\frac{1}{4}\right)^2+87\right)\sqrt{\frac{256\left(\frac{1}{x}+\frac{1}{4}\right)^4-96\left(\frac{1}{x}+\frac{1}{4}\right)^2+261}{\left(16\sqrt{29}\left(\frac{1}{x}+\frac{1}{4}\right)^2+87\right)^2}}\,\text{EllipticF}\left(2\,a\right.}{8\sqrt[4]{29}\sqrt{256\left(\frac{1}{x}+\frac{1}{4}\right)^4-96\left(\frac{1}{x}+\frac{1}{4}\right)^2+261}}\right.\right.}{}$$

input
```
Int[(8 + 8*x - x^3 + 8*x^4)^(-3/2),x]
```

output
```
-((Sqrt[261 - 96*(1/4 + x^(-1))^2 + 256*(1/4 + x^(-1))^4]*((33 - 8*(1/4 +
x^(-1))^2)/(63*Sqrt[261 - 96*(1/4 + x^(-1))^2 + 256*(1/4 + x^(-1))^4]) - (
16*(27 - 14*(1/4 + x^(-1))^2)*(1/4 + x^(-1)))/(783*Sqrt[261 - 96*(1/4 + x^
(-1))^2 + 256*(1/4 + x^(-1))^4]) + ((14*((-29*Sqrt[261 - 96*(1/4 + x^(-1))
^2 + 256*(1/4 + x^(-1))^4]*(1/4 + x^(-1)))/(87 + 16*Sqrt[29]*(1/4 + x^(-1)
)^2) + (Sqrt[3]*29^(3/4)*(87 + 16*Sqrt[29]*(1/4 + x^(-1))^2)*Sqrt[(261 - 9
6*(1/4 + x^(-1))^2 + 256*(1/4 + x^(-1))^4)/(87 + 16*Sqrt[29]*(1/4 + x^(-1)
)^2)^2]*EllipticE[2*ArcTan[(4*(1/4 + x^(-1)))/(Sqrt[3]*29^(1/4))], (29 + S
qrt[29])/58])/(4*Sqrt[261 - 96*(1/4 + x^(-1))^2 + 256*(1/4 + x^(-1))^4])))
/Sqrt[29] + (Sqrt[3]*(145 - 14*Sqrt[29])*(87 + 16*Sqrt[29]*(1/4 + x^(-1))^
2)*Sqrt[(261 - 96*(1/4 + x^(-1))^2 + 256*(1/4 + x^(-1))^4)/(87 + 16*Sqrt[2
9]*(1/4 + x^(-1))^2)^2]*EllipticF[2*ArcTan[(4*(1/4 + x^(-1)))/(Sqrt[3]*29^
(1/4))], (29 + Sqrt[29])/58])/(8*29^(1/4)*Sqrt[261 - 96*(1/4 + x^(-1))^2 +
 256*(1/4 + x^(-1))^4]))/783))/(Sqrt[2]*(1 - 4*(1/4 + x^(-1)))^2*Sqrt[(261
 - 96*(1/4 + x^(-1))^2 + 256*(1/4 + x^(-1))^4)/(1 - 4*(1/4 + x^(-1)))^4]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1158
```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[-2*((b*d - 2*a*e + (2*c*d - b*e)*x)/((b^2 - 4*a*c)*Sqrt[a + b*x
+ c*x^2])), x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 1576
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x]
, x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

rule 2202

```
Int[(Pn_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{n
 = Expon[Pn, x], k}, Int[Sum[Coeff[Pn, x, 2*k]*x^(2*k), {k, 0, n/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pn, x, 2*k + 1]*x^(2*k), {k, 0, (n -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pn, x]
 && !PolyQ[Pn, x^2]
```

rule 2206

```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c
*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Px,
a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*
p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x
^2] && Expon[Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 2504

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Simp[-16*
a^2   Subst[Int[(1/(b - 4*a*x)^2)*(a*((-3*b^4 + 16*a*b^2*c - 64*a^2*b*d + 2
56*a^3*e - 32*a^2*(3*b^2 - 8*a*c)*x^2 + 256*a^4*x^4)/(b - 4*a*x)^4))^p, x],
 x, b/(4*a) + 1/x], x] /; NeQ[a, 0] && NeQ[b, 0] && EqQ[b^3 - 4*a*b*c + 8*a
^2*d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && IntegerQ[2*p] && !IGtQ[p, 0
]
```

rule 7270

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Simp[a^IntPart[p
]*((a*v^m*w^n)^FracPart[p]/(v^(m*FracPart[p])*w^(n*FracPart[p])))   Int[u*v
^(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] && !IntegerQ[p] && !Free
Q[v, x] && !FreeQ[w, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 4425 vs. 2(382) = 764.

Time = 3.50 (sec) , antiderivative size = 4426, normalized size of antiderivative = 10.27

method	result	size
default	Expression too large to display	4426
risch	Expression too large to display	4426
elliptic	Expression too large to display	4426

input
```
int(1/(8*x^4-x^3+8*x+8)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-16*(-17/10962-57/12992*x+191/58464*x^2-7/3132*x^3)/(8*x^4-x^3+8*x+8)^(1/2
)+421/12528*(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1)-RootOf(8*_Z^4-_Z^3+8*_Z+8,
index=4))*((RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8,i
ndex=2))*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))/(RootOf(8*_Z^4-_Z^3+8*_Z+8
,index=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))/(x-RootOf(8*_Z^4-_Z^3+8*_Z+8
,index=2)))^(1/2)*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2))^2*((RootOf(8*_Z^4
-_Z^3+8*_Z+8,index=2)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))*(x-RootOf(8*_Z^4
-_Z^3+8*_Z+8,index=3))/(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=3)-RootOf(8*_Z^4-_
Z^3+8*_Z+8,index=1))/(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)))^(1/2)*((RootO
f(8*_Z^4-_Z^3+8*_Z+8,index=2)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))*(x-RootO
f(8*_Z^4-_Z^3+8*_Z+8,index=4))/(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)-RootOf(
8*_Z^4-_Z^3+8*_Z+8,index=1))/(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)))^(1/2)
/(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2))/(
RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))*2^(
1/2)/((x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,
index=2))*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=3))*(x-RootOf(8*_Z^4-_Z^3+8*_
Z+8,index=4)))^(1/2)*EllipticF(((RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)-RootOf
(8*_Z^4-_Z^3+8*_Z+8,index=2))*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))/(Root
Of(8*_Z^4-_Z^3+8*_Z+8,index=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))/(x-Root
Of(8*_Z^4-_Z^3+8*_Z+8,index=2)))^(1/2),((RootOf(8*_Z^4-_Z^3+8*_Z+8,inde...
```

Fricas [F]

$$\int \frac{1}{(8+8x-x^3+8x^4)^{3/2}}\,dx = \int \frac{1}{(8\,x^4-x^3+8\,x+8)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(8*x^4-x^3+8*x+8)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(8*x^4 - x^3 + 8*x + 8)/(64*x^8 - 16*x^7 + x^6 + 128*x^5 + 11
2*x^4 - 16*x^3 + 64*x^2 + 128*x + 64), x)
```

Sympy [F]

$$\int \frac{1}{(8+8x-x^3+8x^4)^{3/2}}\,dx = \int \frac{1}{(8x^4-x^3+8x+8)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(8*x**4-x**3+8*x+8)**(3/2),x)
```

output
```
Integral((8*x**4 - x**3 + 8*x + 8)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(8+8x-x^3+8x^4)^{3/2}}\,dx = \int \frac{1}{(8\,x^4-x^3+8\,x+8)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(8*x^4-x^3+8*x+8)^(3/2),x, algorithm="maxima")
```

output
```
integrate((8*x^4 - x^3 + 8*x + 8)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(8+8x-x^3+8x^4)^{3/2}}\,dx = \int \frac{1}{(8\,x^4-x^3+8\,x+8)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(8*x^4-x^3+8*x+8)^(3/2),x, algorithm="giac")
```

output
```
integrate((8*x^4 - x^3 + 8*x + 8)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(8+8x-x^3+8x^4)^{3/2}}\,dx = \int \frac{1}{(8\,x^4-x^3+8\,x+8)^{3/2}}\,dx$$

input
```
int(1/(8*x - x^3 + 8*x^4 + 8)^(3/2),x)
```

output
```
int(1/(8*x - x^3 + 8*x^4 + 8)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(8+8x-x^3+8x^4)^{3/2}}\,dx = \int \frac{\sqrt{8x^4-x^3+8x+8}}{64x^8-16x^7+x^6+128x^5+112x^4-16x^3+64x^2+128x+64}dx$$

input
```
int(1/(8*x^4-x^3+8*x+8)^(3/2),x)
```

output
```
int(sqrt(8*x**4 - x**3 + 8*x + 8)/(64*x**8 - 16*x**7 + x**6 + 128*x**5 + 1
12*x**4 - 16*x**3 + 64*x**2 + 128*x + 64),x)
```

3.87 $\int \frac{1}{\sqrt{1+4x+4x^2+4x^4}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 108

$$\int \frac{1}{\sqrt{1+4x+4x^2+4x^4}}\,dx = -\frac{\left(\sqrt{5}+\left(1+\frac{1}{x}\right)^2\right)\sqrt{\frac{5-2\left(1+\frac{1}{x}\right)^2+\left(1+\frac{1}{x}\right)^4}{\left(\sqrt{5}+\left(1+\frac{1}{x}\right)^2\right)^2}}x^2\,\text{EllipticF}\left(2\arctan\left(\frac{1+\frac{1}{x}}{\sqrt[4]{5}}\right),\frac{1}{10}\left(5+\sqrt{5}\right)\right)}{2\sqrt[4]{5}\sqrt{1+4x+4x^2+4x^4}}$$

output
```
-1/10*(5^(1/2)+(1+1/x)^2)*((5-2*(1+1/x)^2+(1+1/x)^4)/(5^(1/2)+(1+1/x)^2)^2
)^(1/2)*x^2*InverseJacobiAM(2*arctan(1/5*(1+1/x)*5^(3/4)),1/10*(50+10*5^(1
/2))^(1/2))*5^(3/4)/(4*x^4+4*x^2+4*x+1)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.67 (sec) , antiderivative size = 249, normalized size of antiderivative = 2.31

$$\int \frac{1}{\sqrt{1+4x+4x^2+4x^4}}\,dx = \frac{(2-i)\sqrt{-\frac{1}{10}+\frac{i}{5}}\sqrt{\frac{(2i+\sqrt{-1-2i}-\sqrt{-1+2i})(-i+\sqrt{-1-2i}-2x)}{(-2i+\sqrt{-1-2i}+\sqrt{-1+2i})(i+\sqrt{-1-2i}+2x)}}(1+2x+2ix^2)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{(2i+\sqrt{-1-2i}+\sqrt{-}}{\sqrt{-1+2i}(}}}{}\right.\right.}{\sqrt{\frac{(1+2i)((-1+i)+\sqrt{-1-2i})(1+2x+2ix^2)}{(i+\sqrt{-1-2i}+2x)^2}}\sqrt{1+4x+4x^2+4x^4}}$$

input
```
Integrate[1/Sqrt[1 + 4*x + 4*x^2 + 4*x^4],x]
```

output
```
((2 - I)*Sqrt[-1/10 + I/5]*Sqrt[((2*I + Sqrt[-1 - 2*I] - Sqrt[-1 + 2*I])*(
-I + Sqrt[-1 - 2*I] - 2*x))/((-2*I + Sqrt[-1 - 2*I] + Sqrt[-1 + 2*I])*(I +
 Sqrt[-1 - 2*I] + 2*x))]*(1 + 2*x + (2*I)*x^2)*EllipticF[ArcSin[Sqrt[((2*I
 + Sqrt[-1 - 2*I] + Sqrt[-1 + 2*I])*(-I + Sqrt[-1 + 2*I] + 2*x))/(Sqrt[-1
+ 2*I]*(I + Sqrt[-1 - 2*I] + 2*x))]/Sqrt[2]], (5 - Sqrt[5])/2])/(Sqrt[((1
+ 2*I)*((-1 + I) + Sqrt[-1 - 2*I])*(1 + 2*x + (2*I)*x^2))/(I + Sqrt[-1 - 2
*I] + 2*x)^2]*Sqrt[1 + 4*x + 4*x^2 + 4*x^4])
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {2504, 27, 7270, 1416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{4x^4+4x^2+4x+1}}\,dx$$

$$\Big\downarrow \text{ 2504}$$

$$-16\int \frac{x^2}{16\sqrt{\left(\left(1+\frac{1}{x}\right)^4-2\left(1+\frac{1}{x}\right)^2+5\right)x^4}}\,d\left(1+\frac{1}{x}\right)$$

$$\Big\downarrow \text{ 27}$$

$$-\int \frac{x^2}{\sqrt{\left(\left(1+\frac{1}{x}\right)^4-2\left(1+\frac{1}{x}\right)^2+5\right)x^4}}d\left(1+\frac{1}{x}\right)$$

$$\Big\downarrow \ 7270$$

$$-\frac{\sqrt{\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5}x^2\int \frac{1}{\sqrt{\left(1+\frac{1}{x}\right)^4-2\left(1+\frac{1}{x}\right)^2+5}}d\left(1+\frac{1}{x}\right)}{\sqrt{\left(\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5\right)x^4}}$$

$$\Big\downarrow \ 1416$$

$$-\frac{\left(\left(\frac{1}{x}+1\right)^2+\sqrt{5}\right)\sqrt{\frac{\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5}{\left(\left(\frac{1}{x}+1\right)^2+\sqrt{5}\right)^2}}x^2\operatorname{EllipticF}\left(2\arctan\left(\frac{1+\frac{1}{x}}{\sqrt[4]{5}}\right),\frac{1}{10}\left(5+\sqrt{5}\right)\right)}{2\sqrt[4]{5}\sqrt{\left(\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5\right)x^4}}$$

input
```
Int[1/Sqrt[1 + 4*x + 4*x^2 + 4*x^4],x]
```

output
```
-1/2*((Sqrt[5] + (1 + x^(-1))^2)*Sqrt[(5 - 2*(1 + x^(-1))^2 + (1 + x^(-1))
^4)/(Sqrt[5] + (1 + x^(-1))^2)^2]*x^2*EllipticF[2*ArcTan[(1 + x^(-1))/5^(1
/4)], (5 + Sqrt[5])/10])/(5^(1/4)*Sqrt[(5 - 2*(1 + x^(-1))^2 + (1 + x^(-1)
)^4)*x^4])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 2504
```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Simp[-16*
a^2   Subst[Int[(1/(b - 4*a*x)^2)*(a*((-3*b^4 + 16*a*b^2*c - 64*a^2*b*d + 2
56*a^3*e - 32*a^2*(3*b^2 - 8*a*c)*x^2 + 256*a^4*x^4)/(b - 4*a*x)^4))^p, x],
 x, b/(4*a) + 1/x], x] /; NeQ[a, 0] && NeQ[b, 0] && EqQ[b^3 - 4*a*b*c + 8*a
^2*d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && IntegerQ[2*p] &&  !IGtQ[p, 0
]
```

rule 7270
```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Simp[a^IntPart[p
]*((a*v^m*w^n)^FracPart[p]/(v^(m*FracPart[p])*w^(n*FracPart[p])))   Int[u*v
^(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !Free
Q[v, x] &&  !FreeQ[w, x]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 960 vs. $2(95) = 190$.

Time = 1.06 (sec) , antiderivative size = 961, normalized size of antiderivative = 8.90

method	result	size
default	Expression too large to display	961
elliptic	Expression too large to display	961

input
```
int(1/(4*x^4+4*x^2+4*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
(RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4)
)*((RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index
=2))*(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1))/(RootOf(4*_Z^4+4*_Z^2+4*_Z+1
,index=4)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1))/(x-RootOf(4*_Z^4+4*_Z^2+4*
_Z+1,index=2)))^(1/2)*(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2))^2*((RootOf(
4*_Z^4+4*_Z^2+4*_Z+1,index=2)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1))*(x-Roo
tOf(4*_Z^4+4*_Z^2+4*_Z+1,index=3))/(RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=3)-R
ootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1))/(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=
2)))^(1/2)*((RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2)-RootOf(4*_Z^4+4*_Z^2+4*_
Z+1,index=1))*(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4))/(RootOf(4*_Z^4+4*_Z
^2+4*_Z+1,index=4)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1))/(x-RootOf(4*_Z^4+
4*_Z^2+4*_Z+1,index=2)))^(1/2)/(RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4)-RootO
f(4*_Z^4+4*_Z^2+4*_Z+1,index=2))/(RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2)-Roo
tOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1))/((x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1
))*(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2))*(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1
,index=3))*(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4)))^(1/2)*EllipticF(((Roo
tOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2))*(x
-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1))/(RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=
4)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1))/(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,in
dex=2)))^(1/2),((RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2)-RootOf(4*_Z^4+4*_...
```

Fricas [F]

$$\int \frac{1}{\sqrt{1+4x+4x^2+4x^4}}\,dx = \int \frac{1}{\sqrt{4\,x^4+4\,x^2+4\,x+1}}\,dx$$

input

```
integrate(1/(4*x^4+4*x^2+4*x+1)^(1/2),x, algorithm="fricas")
```

output

```
integral(1/sqrt(4*x^4 + 4*x^2 + 4*x + 1), x)
```

Sympy [F]

$$\int \frac{1}{\sqrt{1+4x+4x^2+4x^4}}\,dx = \int \frac{1}{\sqrt{4x^4+4x^2+4x+1}}\,dx$$

input
```
integrate(1/(4*x**4+4*x**2+4*x+1)**(1/2),x)
```

output
```
Integral(1/sqrt(4*x**4 + 4*x**2 + 4*x + 1), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{1+4x+4x^2+4x^4}}\,dx = \int \frac{1}{\sqrt{4\,x^4+4\,x^2+4\,x+1}}\,dx$$

input
```
integrate(1/(4*x^4+4*x^2+4*x+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(4*x^4 + 4*x^2 + 4*x + 1), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{1+4x+4x^2+4x^4}}\,dx = \int \frac{1}{\sqrt{4\,x^4+4\,x^2+4\,x+1}}\,dx$$

input
```
integrate(1/(4*x^4+4*x^2+4*x+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(4*x^4 + 4*x^2 + 4*x + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{1+4x+4x^2+4x^4}}\,dx=\int \frac{1}{\sqrt{4\,x^4+4\,x^2+4\,x+1}}\,dx$$

input
```
int(1/(4*x + 4*x^2 + 4*x^4 + 1)^(1/2),x)
```

output
```
int(1/(4*x + 4*x^2 + 4*x^4 + 1)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{1+4x+4x^2+4x^4}}\,dx=\int \frac{\sqrt{4x^4+4x^2+4x+1}}{4x^4+4x^2+4x+1}dx$$

input
```
int(1/(4*x^4+4*x^2+4*x+1)^(1/2),x)
```

output
```
int(sqrt(4*x**4 + 4*x**2 + 4*x + 1)/(4*x**4 + 4*x**2 + 4*x + 1),x)
```

3.88 $\int \frac{1}{(1+4x+4x^2+4x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 367

$$\int \frac{1}{(1+4x+4x^2+4x^4)^{3/2}}\,dx = -\frac{\left(3-\left(1+\frac{1}{x}\right)^2\right)x^2}{\sqrt{1+4x+4x^2+4x^4}} + \frac{\left(13-9\left(1+\frac{1}{x}\right)^2\right)\left(1+\frac{1}{x}\right)x^2}{10\sqrt{1+4x+4x^2+4x^4}} + \frac{9\left(5-2\left(1+\frac{1}{x}\right)^2+\left(1+\frac{1}{x}\right)^4\right)\left(1+\frac{1}{x}\right)x^2}{10\left(\sqrt{5}+\left(1+\frac{1}{x}\right)^2\right)\sqrt{1+4x+4x^2+4x^4}} - \frac{9\left(\sqrt{5}+\left(1+\frac{1}{x}\right)^2\right)\sqrt{\frac{5-2\left(1+\frac{1}{x}\right)^2+\left(1+\frac{1}{x}\right)^4}{\left(\sqrt{5}+\left(1+\frac{1}{x}\right)^2\right)^2}}\,x^2 E\left(2\arctan\left(\frac{1+\frac{1}{x}}{\sqrt[4]{5}}\right)|\frac{1}{10}\left(5+\sqrt{5}\right)\right)}{2\,5^{3/4}\sqrt{1+4x+4x^2+4x^4}} + \frac{3\left(3-\sqrt{5}\right)\left(\sqrt{5}+\left(1+\frac{1}{x}\right)^2\right)\sqrt{\frac{5-2\left(1+\frac{1}{x}\right)^2+\left(1+\frac{1}{x}\right)^4}{\left(\sqrt{5}+\left(1+\frac{1}{x}\right)^2\right)^2}}\,x^2\,\text{EllipticF}\left(2\arctan\left(\frac{1+\frac{1}{x}}{\sqrt[4]{5}}\right),\frac{1}{10}\left(5+\sqrt{5}\right)\right)}{4\,5^{3/4}\sqrt{1+4x+4x^2+4x^4}}$$

output
```
-(3-(1+1/x)^2)*x^2/(4*x^4+4*x^2+4*x+1)^(1/2)+1/10*(13-9*(1+1/x)^2)*(1+1/x)
*x^2/(4*x^4+4*x^2+4*x+1)^(1/2)+9/10*(5-2*(1+1/x)^2+(1+1/x)^4)*(1+1/x)*x^2/
(5^(1/2)+(1+1/x)^2)/(4*x^4+4*x^2+4*x+1)^(1/2)-9/10*(5^(1/2)+(1+1/x)^2)*((5
-2*(1+1/x)^2+(1+1/x)^4)/(5^(1/2)+(1+1/x)^2)^2)^(1/2)*x^2*EllipticE(sin(2*a
rctan(1/5*(1+1/x)*5^(3/4))),1/10*(50+10*5^(1/2))^(1/2))*5^(1/4)/(4*x^4+4*x
^2+4*x+1)^(1/2)+3/20*(3-5^(1/2))*(5^(1/2)+(1+1/x)^2)*((5-2*(1+1/x)^2+(1+1/
x)^4)/(5^(1/2)+(1+1/x)^2)^2)^(1/2)*x^2*InverseJacobiAM(2*arctan(1/5*(1+1/x
)*5^(3/4)),1/10*(50+10*5^(1/2))^(1/2))*5^(1/4)/(4*x^4+4*x^2+4*x+1)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 15.35 (sec) , antiderivative size = 602, normalized size of antiderivative = 1.64

$$\int \frac{1}{(1+4x+4x^2+4x^4)^{3/2}}\,dx = \frac{19+42x-16x^2+36x^3+\frac{9}{2}\left(-i+\sqrt{-1-2i}-2x\right)\left(-i-\sqrt{-1+2i}+2\right.}{}$$

input
```
Integrate[(1 + 4*x + 4*x^2 + 4*x^4)^(-3/2),x]
```

output
```
(19 + 42*x - 16*x^2 + 36*x^3 + (9*(-I + Sqrt[-1 - 2*I] - 2*x)*(-I - Sqrt[-
1 + 2*I] + 2*x)*(-I + Sqrt[-1 + 2*I] + 2*x))/2 - ((9*I)*Sqrt[-2/5 + (4*I)/
5]*(-2*I + Sqrt[-1 - 2*I] + Sqrt[-1 + 2*I])*(2*I + Sqrt[-1 - 2*I] + Sqrt[-
1 + 2*I])*((I + Sqrt[-1 - 2*I])/2 + x)^2*Sqrt[((2*I + Sqrt[-1 - 2*I] - Sqr
t[-1 + 2*I])*(-I + Sqrt[-1 - 2*I] - 2*x))/((-2*I + Sqrt[-1 - 2*I] + Sqrt[-
1 + 2*I])*(I + Sqrt[-1 - 2*I] + 2*x))]*Sqrt[((1 + 2*I)*((-1 + I) + Sqrt[-1
 - 2*I])*(1 + 2*x + (2*I)*x^2))/(I + Sqrt[-1 - 2*I] + 2*x)^2]*EllipticE[Ar
cSin[Sqrt[((2*I + Sqrt[-1 - 2*I] + Sqrt[-1 + 2*I])*(-I + Sqrt[-1 + 2*I] +
2*x))/(Sqrt[-1 + 2*I]*(I + Sqrt[-1 - 2*I] + 2*x))]/Sqrt[2]], (5 - Sqrt[5])
/2])/((-1 + I) + Sqrt[-1 - 2*I]) + ((6 - 3*I)*Sqrt[-2/5 + (4*I)/5]*Sqrt[((
2*I + Sqrt[-1 - 2*I] - Sqrt[-1 + 2*I])*(-I + Sqrt[-1 - 2*I] - 2*x))/((-2*I
 + Sqrt[-1 - 2*I] + Sqrt[-1 + 2*I])*(I + Sqrt[-1 - 2*I] + 2*x))]*(1 + 2*x
+ (2*I)*x^2)*EllipticF[ArcSin[Sqrt[((2*I + Sqrt[-1 - 2*I] + Sqrt[-1 + 2*I]
)*(-I + Sqrt[-1 + 2*I] + 2*x))/(Sqrt[-1 + 2*I]*(I + Sqrt[-1 - 2*I] + 2*x))
]/Sqrt[2]], (5 - Sqrt[5])/2])/Sqrt[((1 + 2*I)*((-1 + I) + Sqrt[-1 - 2*I])*
(1 + 2*x + (2*I)*x^2))/(I + Sqrt[-1 - 2*I] + 2*x)^2])/(10*Sqrt[1 + 4*x + 4
*x^2 + 4*x^4])
```

Rubi [A] (warning: unable to verify)

Time = 1.19 (sec) , antiderivative size = 400, normalized size of antiderivative = 1.09, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.684$, Rules used = {2504, 27, 7270, 2202, 1576, 27, 1158, 2206, 27, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(4x^4+4x^2+4x+1)^{3/2}}\,dx$$

$$\Big\downarrow \ 2504$$

$$-16\int \frac{x^2}{16\left(\left(\left(1+\frac{1}{x}\right)^4-2\left(1+\frac{1}{x}\right)^2+5\right)x^4\right)^{3/2}}d\left(1+\frac{1}{x}\right)$$

$$\Big\downarrow \ 27$$

$$-\int \frac{x^2}{\left(\left(\left(1+\frac{1}{x}\right)^4-2\left(1+\frac{1}{x}\right)^2+5\right)x^4\right)^{3/2}}d\left(1+\frac{1}{x}\right)$$

$$\Big\downarrow \text{7270}$$

$$-\frac{\sqrt{\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5}x^2\int \frac{1}{\left(\left(1+\frac{1}{x}\right)^4-2\left(1+\frac{1}{x}\right)^2+5\right)^{3/2}x^4}d\left(1+\frac{1}{x}\right)}{\sqrt{\left(\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5\right)x^4}}$$

$$\Big\downarrow \text{2202}$$

$$-\frac{\sqrt{\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5}x^2\left(\int \frac{\left(1+\frac{1}{x}\right)^4+6\left(1+\frac{1}{x}\right)^2+1}{\left(\left(1+\frac{1}{x}\right)^4-2\left(1+\frac{1}{x}\right)^2+5\right)^{3/2}}d\left(1+\frac{1}{x}\right)+\int \frac{\left(-4\left(1+\frac{1}{x}\right)^2-4\right)\left(1+\frac{1}{x}\right)}{\left(\left(1+\frac{1}{x}\right)^4-2\left(1+\frac{1}{x}\right)^2+5\right)^{3/2}}d\left(1+\frac{1}{x}\right)\right)}{\sqrt{\left(\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5\right)x^4}}$$

$$\Big\downarrow \text{1576}$$

$$-\frac{\sqrt{\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5}x^2\left(\int \frac{\left(1+\frac{1}{x}\right)^4+6\left(1+\frac{1}{x}\right)^2+1}{\left(\left(1+\frac{1}{x}\right)^4-2\left(1+\frac{1}{x}\right)^2+5\right)^{3/2}}d\left(1+\frac{1}{x}\right)+\frac{1}{2}\int -\frac{4\left(2+\frac{1}{x}\right)}{\left(\left(1+\frac{1}{x}\right)^4-2\left(1+\frac{1}{x}\right)^2+5\right)^{3/2}}d\left(1+\frac{1}{x}\right)^2\right)}{\sqrt{\left(\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5\right)x^4}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\sqrt{\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5}x^2\left(\int \frac{\left(1+\frac{1}{x}\right)^4+6\left(1+\frac{1}{x}\right)^2+1}{\left(\left(1+\frac{1}{x}\right)^4-2\left(1+\frac{1}{x}\right)^2+5\right)^{3/2}}d\left(1+\frac{1}{x}\right)-2\int \frac{2+\frac{1}{x}}{\left(\left(1+\frac{1}{x}\right)^4-2\left(1+\frac{1}{x}\right)^2+5\right)^{3/2}}d\left(1+\frac{1}{x}\right)^2\right)}{\sqrt{\left(\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5\right)x^4}}$$

$$\Big\downarrow \text{1158}$$

$$-\frac{\sqrt{\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5}x^2\left(\int \frac{\left(1+\frac{1}{x}\right)^4+6\left(1+\frac{1}{x}\right)^2+1}{\left(\left(1+\frac{1}{x}\right)^4-2\left(1+\frac{1}{x}\right)^2+5\right)^{3/2}}d\left(1+\frac{1}{x}\right)+\frac{2-\frac{1}{x}}{\sqrt{\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5}}\right)}{\sqrt{\left(\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5\right)x^4}}$$

$$\Big\downarrow \text{2206}$$

$$-\frac{\sqrt{\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5}x^2\left(\frac{1}{80}\int \frac{24\left(5-3\left(1+\frac{1}{x}\right)^2\right)}{\sqrt{\left(1+\frac{1}{x}\right)^4-2\left(1+\frac{1}{x}\right)^2+5}}d\left(1+\frac{1}{x}\right)+\frac{2-\frac{1}{x}}{\sqrt{\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5}}-\frac{\left(13-9\left(\frac{1}{x}+1\right)^2\right)\left(\frac{1}{x}+1\right.}{10\sqrt{\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2}}\right.}{\sqrt{\left(\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5\right)x^4}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\sqrt{\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5}\,x^2\left(\frac{3}{10}\int \frac{5-3\left(1+\frac{1}{x}\right)^2}{\sqrt{\left(1+\frac{1}{x}\right)^4-2\left(1+\frac{1}{x}\right)^2+5}}d\left(1+\frac{1}{x}\right)+\frac{2-\frac{1}{x}}{\sqrt{\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5}}-\frac{\left(13-9\left(\frac{1}{x}+1\right)^2\right)\left(\frac{1}{x}+1\right.}{10\sqrt{\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2}}\right.}{\sqrt{\left(\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5\right)x^4}}$$

$$\Big\downarrow \text{1511}$$

$$-\frac{\sqrt{\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5}\,x^2\left(\frac{3}{10}\left(\left(5-3\sqrt{5}\right)\int \frac{1}{\sqrt{\left(1+\frac{1}{x}\right)^4-2\left(1+\frac{1}{x}\right)^2+5}}d\left(1+\frac{1}{x}\right)+3\sqrt{5}\int \frac{\sqrt{5}-\left(1+\frac{1}{x}\right)^2}{\sqrt{5}\sqrt{\left(1+\frac{1}{x}\right)^4-2\left(1+\frac{1}{x}\right)^2+5}}\right.\right.}{\sqrt{\left(\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5\right)x^4}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\sqrt{\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5}\,x^2\left(\frac{3}{10}\left(\left(5-3\sqrt{5}\right)\int \frac{1}{\sqrt{\left(1+\frac{1}{x}\right)^4-2\left(1+\frac{1}{x}\right)^2+5}}d\left(1+\frac{1}{x}\right)+3\int \frac{\sqrt{5}-\left(1+\frac{1}{x}\right)^2}{\sqrt{\left(1+\frac{1}{x}\right)^4-2\left(1+\frac{1}{x}\right)^2+5}}d\left(1+\right.\right.\right.}{\sqrt{\left(\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5\right)x^4}}$$

$$\Big\downarrow \text{1416}$$

$$-\frac{\sqrt{\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5}\,x^2\left(\frac{3}{10}\left(3\int \frac{\sqrt{5}-\left(1+\frac{1}{x}\right)^2}{\sqrt{\left(1+\frac{1}{x}\right)^4-2\left(1+\frac{1}{x}\right)^2+5}}d\left(1+\frac{1}{x}\right)+\frac{\left(5-3\sqrt{5}\right)\left(\left(\frac{1}{x}+1\right)^2+\sqrt{5}\right)\sqrt{\frac{\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+}{\left(\left(\frac{1}{x}+1\right)^2+\sqrt{5}\right)^2}}}{2\sqrt[4]{5}\sqrt{\left(\frac{1}{x}+1\right)^4-}}\right.\right.}{\sqrt{\left(\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5\right.}}$$

$$\Big\downarrow \text{1509}$$

$$-\frac{\sqrt{\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5}\,x^2\left(\frac{3}{10}\left(\frac{\left(5-3\sqrt{5}\right)\left(\left(\frac{1}{x}+1\right)^2+\sqrt{5}\right)\sqrt{\frac{\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5}{\left(\left(\frac{1}{x}+1\right)^2+\sqrt{5}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{1+\frac{1}{x}}{\sqrt[4]{5}}\right),\frac{1}{10}\left(5+\sqrt{5}\right)\right)}{2\sqrt[4]{5}\sqrt{\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5}}\right.\right.}{}$$

input
```
Int[(1 + 4*x + 4*x^2 + 4*x^4)^(-3/2),x]
```

output
```
-((Sqrt[5 - 2*(1 + x^(-1))^2 + (1 + x^(-1))^4]*x^2*((2 - x^(-1))/Sqrt[5 -
2*(1 + x^(-1))^2 + (1 + x^(-1))^4] - ((13 - 9*(1 + x^(-1))^2)*(1 + x^(-1))
)/(10*Sqrt[5 - 2*(1 + x^(-1))^2 + (1 + x^(-1))^4]) + (3*(3*(-((Sqrt[5 - 2*
(1 + x^(-1))^2 + (1 + x^(-1))^4]*(1 + x^(-1)))/(Sqrt[5] + (1 + x^(-1))^2))
 + (5^(1/4)*(Sqrt[5] + (1 + x^(-1))^2)*Sqrt[(5 - 2*(1 + x^(-1))^2 + (1 + x
^(-1))^4)/(Sqrt[5] + (1 + x^(-1))^2)^2]*EllipticE[2*ArcTan[(1 + x^(-1))/5^
(1/4)], (5 + Sqrt[5])/10])/Sqrt[5 - 2*(1 + x^(-1))^2 + (1 + x^(-1))^4]) +
((5 - 3*Sqrt[5])*(Sqrt[5] + (1 + x^(-1))^2)*Sqrt[(5 - 2*(1 + x^(-1))^2 + (
1 + x^(-1))^4)/(Sqrt[5] + (1 + x^(-1))^2)^2]*EllipticF[2*ArcTan[(1 + x^(-1
))/5^(1/4)], (5 + Sqrt[5])/10])/(2*5^(1/4)*Sqrt[5 - 2*(1 + x^(-1))^2 + (1
+ x^(-1))^4])))/10))/Sqrt[(5 - 2*(1 + x^(-1))^2 + (1 + x^(-1))^4)*x^4])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1158
```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[-2*((b*d - 2*a*e + (2*c*d - b*e)*x)/((b^2 - 4*a*c)*Sqrt[a + b*x
+ c*x^2])), x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 1576
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x]
, x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

rule 2202
```
Int[(Pn_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{n
 = Expon[Pn, x], k}, Int[Sum[Coeff[Pn, x, 2*k]*x^(2*k), {k, 0, n/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pn, x, 2*k + 1]*x^(2*k), {k, 0, (n -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pn, x]
 &&  !PolyQ[Pn, x^2]
```

rule 2206
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c
*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Px,
a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*
p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x
^2] && Expon[Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 2504
```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Simp[-16*
a^2   Subst[Int[(1/(b - 4*a*x)^2)*(a*((-3*b^4 + 16*a*b^2*c - 64*a^2*b*d + 2
56*a^3*e - 32*a^2*(3*b^2 - 8*a*c)*x^2 + 256*a^4*x^4)/(b - 4*a*x)^4))^p, x],
 x, b/(4*a) + 1/x], x] /; NeQ[a, 0] && NeQ[b, 0] && EqQ[b^3 - 4*a*b*c + 8*a
^2*d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && IntegerQ[2*p] &&  !IGtQ[p, 0
]
```

rule 7270
```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Simp[a^IntPart[p
]*((a*v^m*w^n)^FracPart[p]/(v^(m*FracPart[p])*w^(n*FracPart[p])))   Int[u*v
^(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !Free
Q[v, x] &&  !FreeQ[w, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2563 vs. $2(328) = 656$.

Time = 0.88 (sec) , antiderivative size = 2564, normalized size of antiderivative = 6.99

method	result	size
default	Expression too large to display	2564
risch	Expression too large to display	2564
elliptic	Expression too large to display	2564

input
```
int(1/(4*x^4+4*x^2+4*x+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-8*(-9/20*x^3+1/5*x^2-21/40*x-19/80)/(4*x^4+4*x^2+4*x+1)^(1/2)+3/5*(RootOf
(4*_Z^4+4*_Z^2+4*_Z+1,index=1)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4))*((Roo
tOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2))*(x
-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1))/(RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=
4)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1))/(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,in
dex=2)))^(1/2)*(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2))^2*((RootOf(4*_Z^4+
4*_Z^2+4*_Z+1,index=2)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1))*(x-RootOf(4*_
Z^4+4*_Z^2+4*_Z+1,index=3))/(RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=3)-RootOf(4
*_Z^4+4*_Z^2+4*_Z+1,index=1))/(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2)))^(1
/2)*((RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,ind
ex=1))*(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4))/(RootOf(4*_Z^4+4*_Z^2+4*_Z
+1,index=4)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1))/(x-RootOf(4*_Z^4+4*_Z^2+
4*_Z+1,index=2)))^(1/2)/(RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4)-RootOf(4*_Z^
4+4*_Z^2+4*_Z+1,index=2))/(RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2)-RootOf(4*_
Z^4+4*_Z^2+4*_Z+1,index=1))/((x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1))*(x-R
ootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2))*(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=
3))*(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4)))^(1/2)*EllipticF(((RootOf(4*_
Z^4+4*_Z^2+4*_Z+1,index=4)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2))*(x-RootOf
(4*_Z^4+4*_Z^2+4*_Z+1,index=1))/(RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4)-Root
Of(4*_Z^4+4*_Z^2+4*_Z+1,index=1))/(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=...
```

Fricas [F]

$$\int \frac{1}{(1+4x+4x^2+4x^4)^{3/2}}\,dx = \int \frac{1}{(4\,x^4+4\,x^2+4\,x+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(4*x^4+4*x^2+4*x+1)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(4*x^4 + 4*x^2 + 4*x + 1)/(16*x^8 + 32*x^6 + 32*x^5 + 24*x^4
+ 32*x^3 + 24*x^2 + 8*x + 1), x)
```

Sympy [F]

$$\int \frac{1}{(1+4x+4x^2+4x^4)^{3/2}}\,dx = \int \frac{1}{(4x^4+4x^2+4x+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(4*x**4+4*x**2+4*x+1)**(3/2),x)
```

output
```
Integral((4*x**4 + 4*x**2 + 4*x + 1)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(1+4x+4x^2+4x^4)^{3/2}}\,dx = \int \frac{1}{(4\,x^4+4\,x^2+4\,x+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(4*x^4+4*x^2+4*x+1)^(3/2),x, algorithm="maxima")
```

output
```
integrate((4*x^4 + 4*x^2 + 4*x + 1)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(1+4x+4x^2+4x^4)^{3/2}}\,dx = \int \frac{1}{(4\,x^4+4\,x^2+4\,x+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(4*x^4+4*x^2+4*x+1)^(3/2),x, algorithm="giac")
```

output
```
integrate((4*x^4 + 4*x^2 + 4*x + 1)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(1+4x+4x^2+4x^4)^{3/2}}\,dx = \int \frac{1}{(4\,x^4+4\,x^2+4\,x+1)^{3/2}}\,dx$$

input
```
int(1/(4*x + 4*x^2 + 4*x^4 + 1)^(3/2),x)
```

output
```
int(1/(4*x + 4*x^2 + 4*x^4 + 1)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(1+4x+4x^2+4x^4)^{3/2}}\,dx = \int \frac{\sqrt{4x^4+4x^2+4x+1}}{16x^8+32x^6+32x^5+24x^4+32x^3+24x^2+8x+1}dx$$

input
```
int(1/(4*x^4+4*x^2+4*x+1)^(3/2),x)
```

output
```
int(sqrt(4*x**4 + 4*x**2 + 4*x + 1)/(16*x**8 + 32*x**6 + 32*x**5 + 24*x**4
 + 32*x**3 + 24*x**2 + 8*x + 1),x)
```

3.89 $\int \frac{1}{\sqrt{8+24x+8x^2-15x^3+8x^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 125

$$\int \frac{1}{\sqrt{8+24x+8x^2-15x^3+8x^4}}\,dx = -\frac{\left(\sqrt{517}+\left(3+\frac{4}{x}\right)^2\right)\sqrt{\frac{517-38\left(3+\frac{4}{x}\right)^2+\left(3+\frac{4}{x}\right)^4}{\left(\sqrt{517}+\left(3+\frac{4}{x}\right)^2\right)^2}}\,x^2\,\text{EllipticF}\left(2\arctan\left(\frac{3+\frac{4}{x}}{\sqrt[4]{517}}\right),\frac{517+19\sqrt{517}}{1034}\right)}{8\sqrt[4]{517}\sqrt{8+24x+8x^2-15x^3+8x^4}}$$

output
```
-1/4136*(517^(1/2)+(3+4/x)^2)*((517-38*(3+4/x)^2+(3+4/x)^4)/(517^(1/2)+(3+
4/x)^2)^2)^(1/2)*x^2*InverseJacobiAM(2*arctan(1/517*(3+4/x)*517^(3/4)),1/1
034*(534578+19646*517^(1/2))^(1/2))*517^(3/4)/(8*x^4-15*x^3+8*x^2+24*x+8)^
(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 11.10 (sec) , antiderivative size = 1148, normalized size of antiderivative = 9.18

$$\int \frac{1}{\sqrt{8+24x+8x^2-15x^3+8x^4}}\,dx = \text{Too large to display}$$

input
```
Integrate[1/Sqrt[8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4],x]
```

output
```
(-2*EllipticF[ArcSin[Sqrt[((x - Root[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4
 & , 1, 0])*(Root[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , 2, 0] - Root[8
 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , 4, 0]))/((x - Root[8 + 24*#1 + 8*
#1^2 - 15*#1^3 + 8*#1^4 & , 2, 0])*(Root[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*
#1^4 & , 1, 0] - Root[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , 4, 0]))]],
 ((Root[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , 2, 0] - Root[8 + 24*#1 +
 8*#1^2 - 15*#1^3 + 8*#1^4 & , 3, 0])*(Root[8 + 24*#1 + 8*#1^2 - 15*#1^3 +
 8*#1^4 & , 1, 0] - Root[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , 4, 0]))
/((Root[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , 1, 0] - Root[8 + 24*#1 +
 8*#1^2 - 15*#1^3 + 8*#1^4 & , 3, 0])*(Root[8 + 24*#1 + 8*#1^2 - 15*#1^3 +
 8*#1^4 & , 2, 0] - Root[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , 4, 0]))
]*(x - Root[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , 2, 0])^2*Sqrt[((Root
[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , 1, 0] - Root[8 + 24*#1 + 8*#1^2
 - 15*#1^3 + 8*#1^4 & , 2, 0])*(x - Root[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*
#1^4 & , 3, 0]))/((x - Root[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , 2, 0
])*(Root[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , 1, 0] - Root[8 + 24*#1
+ 8*#1^2 - 15*#1^3 + 8*#1^4 & , 3, 0]))]*(Root[8 + 24*#1 + 8*#1^2 - 15*#1^
3 + 8*#1^4 & , 1, 0] - Root[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , 4, 0
])*Sqrt[((x - Root[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , 1, 0])*(Root[
8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , 1, 0] - Root[8 + 24*#1 + 8*#1...
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.30, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2504, 27, 7270, 1416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{8x^4 - 15x^3 + 8x^2 + 24x + 8}} dx$$

$$\Big\downarrow \; 2504$$

$$-1024\int \frac{1}{128\sqrt{2}\left(3-4\left(\frac{3}{4}+\frac{1}{x}\right)\right)^2\sqrt{\frac{256\left(\frac{3}{4}+\frac{1}{x}\right)^4-608\left(\frac{3}{4}+\frac{1}{x}\right)^2+517}{\left(3-4\left(\frac{3}{4}+\frac{1}{x}\right)\right)^4}}}d\left(\frac{3}{4}+\frac{1}{x}\right)$$

$$\Big\downarrow \text{27}$$

$$-4\sqrt{2}\int \frac{1}{\left(3-4\left(\frac{3}{4}+\frac{1}{x}\right)\right)^2\sqrt{\frac{256\left(\frac{3}{4}+\frac{1}{x}\right)^4-608\left(\frac{3}{4}+\frac{1}{x}\right)^2+517}{\left(3-4\left(\frac{3}{4}+\frac{1}{x}\right)\right)^4}}}d\left(\frac{3}{4}+\frac{1}{x}\right)$$

$$\Big\downarrow \text{7270}$$

$$-\frac{4\sqrt{2}\sqrt{256\left(\frac{1}{x}+\frac{3}{4}\right)^4-608\left(\frac{1}{x}+\frac{3}{4}\right)^2+517}\int \frac{1}{\sqrt{256\left(\frac{3}{4}+\frac{1}{x}\right)^4-608\left(\frac{3}{4}+\frac{1}{x}\right)^2+517}}d\left(\frac{3}{4}+\frac{1}{x}\right)}{\left(3-4\left(\frac{1}{x}+\frac{3}{4}\right)\right)^2\sqrt{\frac{256\left(\frac{1}{x}+\frac{3}{4}\right)^4-608\left(\frac{1}{x}+\frac{3}{4}\right)^2+517}{\left(3-4\left(\frac{1}{x}+\frac{3}{4}\right)\right)^4}}}$$

$$\Big\downarrow \text{1416}$$

$$-\frac{\left(16\left(\frac{1}{x}+\frac{3}{4}\right)^2+\sqrt{517}\right)\sqrt{\frac{256\left(\frac{1}{x}+\frac{3}{4}\right)^4-608\left(\frac{1}{x}+\frac{3}{4}\right)^2+517}{\left(16\left(\frac{1}{x}+\frac{3}{4}\right)^2+\sqrt{517}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{4\left(\frac{3}{4}+\frac{1}{x}\right)}{\sqrt[4]{517}}\right),\frac{517+19\sqrt{517}}{1034}\right)}{\sqrt{2}\sqrt[4]{517}\left(3-4\left(\frac{1}{x}+\frac{3}{4}\right)\right)^2\sqrt{\frac{256\left(\frac{1}{x}+\frac{3}{4}\right)^4-608\left(\frac{1}{x}+\frac{3}{4}\right)^2+517}{\left(3-4\left(\frac{1}{x}+\frac{3}{4}\right)\right)^4}}}$$

input
```
Int[1/Sqrt[8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4],x]
```

output
```
-(((Sqrt[517] + 16*(3/4 + x^(-1))^2)*Sqrt[(517 - 608*(3/4 + x^(-1))^2 + 25
6*(3/4 + x^(-1))^4)/(Sqrt[517] + 16*(3/4 + x^(-1))^2)^2]*EllipticF[2*ArcTa
n[(4*(3/4 + x^(-1)))/517^(1/4)], (517 + 19*Sqrt[517])/1034])/(Sqrt[2]*517^
(1/4)*(3 - 4*(3/4 + x^(-1)))^2*Sqrt[(517 - 608*(3/4 + x^(-1))^2 + 256*(3/4
 + x^(-1))^4)/(3 - 4*(3/4 + x^(-1)))^4]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 2504
```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Simp[-16*
a^2   Subst[Int[(1/(b - 4*a*x)^2)*(a*((-3*b^4 + 16*a*b^2*c - 64*a^2*b*d + 2
56*a^3*e - 32*a^2*(3*b^2 - 8*a*c)*x^2 + 256*a^4*x^4)/(b - 4*a*x)^4))^p, x],
 x, b/(4*a) + 1/x], x] /; NeQ[a, 0] && NeQ[b, 0] && EqQ[b^3 - 4*a*b*c + 8*a
^2*d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && IntegerQ[2*p] &&  !IGtQ[p, 0
]
```

rule 7270
```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Simp[a^IntPart[p
]*((a*v^m*w^n)^FracPart[p]/(v^(m*FracPart[p])*w^(n*FracPart[p])))   Int[u*v
^(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !Free
Q[v, x] &&  !FreeQ[w, x]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 1179 vs. $2(110) = 220$.

Time = 1.02 (sec) , antiderivative size = 1180, normalized size of antiderivative = 9.44

method	result	size
default	Expression too large to display	1180
elliptic	Expression too large to display	1180

input
```
int(1/(8*x^4-15*x^3+8*x^2+24*x+8)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=1)-RootOf(8*_Z^4-15*_Z^3+8
*_Z^2+24*_Z+8,index=4))*((RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=4)-Ro
otOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=2))*(x-RootOf(8*_Z^4-15*_Z^3+8*_Z
^2+24*_Z+8,index=1))/(RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=4)-RootOf
(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=1))/(x-RootOf(8*_Z^4-15*_Z^3+8*_Z^2+2
4*_Z+8,index=2)))^(1/2)*(x-RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=2))^
2*((RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=2)-RootOf(8*_Z^4-15*_Z^3+8*
_Z^2+24*_Z+8,index=1))*(x-RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=3))/(
RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=3)-RootOf(8*_Z^4-15*_Z^3+8*_Z^2
+24*_Z+8,index=1))/(x-RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=2)))^(1/2
)*((RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=2)-RootOf(8*_Z^4-15*_Z^3+8*
_Z^2+24*_Z+8,index=1))*(x-RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=4))/(
RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=4)-RootOf(8*_Z^4-15*_Z^3+8*_Z^2
+24*_Z+8,index=1))/(x-RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=2)))^(1/2
)/(RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=4)-RootOf(8*_Z^4-15*_Z^3+8*_
Z^2+24*_Z+8,index=2))/(RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=2)-RootO
f(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=1))*2^(1/2)/((x-RootOf(8*_Z^4-15*_Z^
3+8*_Z^2+24*_Z+8,index=1))*(x-RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=2
))*(x-RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=3))*(x-RootOf(8*_Z^4-15*_
Z^3+8*_Z^2+24*_Z+8,index=4)))^(1/2)*EllipticF(((RootOf(8*_Z^4-15*_Z^3+8...
```

Fricas [F]

$$\int \frac{1}{\sqrt{8+24x+8x^2-15x^3+8x^4}}\,dx = \int \frac{1}{\sqrt{8\,x^4-15\,x^3+8\,x^2+24\,x+8}}\,dx$$

input
```
integrate(1/(8*x^4-15*x^3+8*x^2+24*x+8)^(1/2),x, algorithm="fricas")
```

output
```
integral(1/sqrt(8*x^4 - 15*x^3 + 8*x^2 + 24*x + 8), x)
```

Sympy [F]

$$\int \frac{1}{\sqrt{8+24x+8x^2-15x^3+8x^4}}\,dx = \int \frac{1}{\sqrt{8x^4-15x^3+8x^2+24x+8}}\,dx$$

input
```
integrate(1/(8*x**4-15*x**3+8*x**2+24*x+8)**(1/2),x)
```

output
```
Integral(1/sqrt(8*x**4 - 15*x**3 + 8*x**2 + 24*x + 8), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{8+24x+8x^2-15x^3+8x^4}}\,dx = \int \frac{1}{\sqrt{8\,x^4-15\,x^3+8\,x^2+24\,x+8}}\,dx$$

input
```
integrate(1/(8*x^4-15*x^3+8*x^2+24*x+8)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(8*x^4 - 15*x^3 + 8*x^2 + 24*x + 8), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{8+24x+8x^2-15x^3+8x^4}}\,dx = \int \frac{1}{\sqrt{8\,x^4-15\,x^3+8\,x^2+24\,x+8}}\,dx$$

input
```
integrate(1/(8*x^4-15*x^3+8*x^2+24*x+8)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(8*x^4 - 15*x^3 + 8*x^2 + 24*x + 8), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{8+24x+8x^2-15x^3+8x^4}}\,dx = \int \frac{1}{\sqrt{8\,x^4-15\,x^3+8\,x^2+24\,x+8}}\,dx$$

input
```
int(1/(24*x + 8*x^2 - 15*x^3 + 8*x^4 + 8)^(1/2),x)
```

output
```
int(1/(24*x + 8*x^2 - 15*x^3 + 8*x^4 + 8)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{8+24x+8x^2-15x^3+8x^4}}\,dx = \int \frac{1}{\sqrt{8x^4-15x^3+8x^2+24x+8}}dx$$

input
```
int(1/(8*x^4-15*x^3+8*x^2+24*x+8)^(1/2),x)
```

output
```
int(1/(8*x^4-15*x^3+8*x^2+24*x+8)^(1/2),x)
```

3.90 $\int \frac{1}{(8+24x+8x^2-15x^3+8x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 432

$$\int \frac{1}{(8+24x+8x^2-15x^3+8x^4)^{3/2}}\,dx = -\frac{\left(172-7\left(3+\frac{4}{x}\right)^2\right)x^2}{208\sqrt{8+24x+8x^2-15x^3+8x^4}}+\frac{\left(50896-2455\left(3+\frac{4}{x}\right)^2\right)\left(3+\frac{4}{x}\right)x^2}{322608\sqrt{8+24x+8x^2-15x^3+8x^4}}$$
$$+\frac{2455\left(517-38\left(3+\frac{4}{x}\right)^2+\left(3+\frac{4}{x}\right)^4\right)\left(3+\frac{4}{x}\right)x^2}{322608\left(\sqrt{517}+\left(3+\frac{4}{x}\right)^2\right)\sqrt{8+24x+8x^2-15x^3+8x^4}}$$
$$-\frac{2455\left(\sqrt{517}+\left(3+\frac{4}{x}\right)^2\right)\sqrt{\frac{517-38\left(3+\frac{4}{x}\right)^2+\left(3+\frac{4}{x}\right)^4}{\left(\sqrt{517}+\left(3+\frac{4}{x}\right)^2\right)^2}}\,x^2E\left(2\arctan\left(\frac{3+\frac{4}{x}}{\sqrt[4]{517}}\right)\middle|\frac{517+19\sqrt{517}}{1034}\right)}{624\ 517^{3/4}\sqrt{8+24x+8x^2-15x^3+8x^4}}$$
$$+\frac{\left(4910-203\sqrt{517}\right)\left(\sqrt{517}+\left(3+\frac{4}{x}\right)^2\right)\sqrt{\frac{517-38\left(3+\frac{4}{x}\right)^2+\left(3+\frac{4}{x}\right)^4}{\left(\sqrt{517}+\left(3+\frac{4}{x}\right)^2\right)^2}}\,x^2\,\text{EllipticF}\left(2\arctan\left(\frac{3+\frac{4}{x}}{\sqrt[4]{517}}\right),\frac{517+19\sqrt{51}}{1034}\right.}{2496\ 517^{3/4}\sqrt{8+24x+8x^2-15x^3+8x^4}}$$

output
```
-1/208*(172-7*(3+4/x)^2)*x^2/(8*x^4-15*x^3+8*x^2+24*x+8)^(1/2)+1/322608*(5
0896-2455*(3+4/x)^2)*(3+4/x)*x^2/(8*x^4-15*x^3+8*x^2+24*x+8)^(1/2)+2455/32
2608*(517-38*(3+4/x)^2+(3+4/x)^4)*(3+4/x)*x^2/(517^(1/2)+(3+4/x)^2)/(8*x^4
-15*x^3+8*x^2+24*x+8)^(1/2)-2455/322608*(517^(1/2)+(3+4/x)^2)*((517-38*(3+
4/x)^2+(3+4/x)^4)/(517^(1/2)+(3+4/x)^2)^2)^(1/2)*x^2*EllipticE(sin(2*arcta
n(1/517*(3+4/x)*517^(3/4))),1/1034*(534578+19646*517^(1/2))^(1/2))*517^(1/
4)/(8*x^4-15*x^3+8*x^2+24*x+8)^(1/2)+1/1290432*(4910-203*517^(1/2))*(517^(
1/2)+(3+4/x)^2)*((517-38*(3+4/x)^2+(3+4/x)^4)/(517^(1/2)+(3+4/x)^2)^2)^(1/
2)*x^2*InverseJacobiAM(2*arctan(1/517*(3+4/x)*517^(3/4)),1/1034*(534578+19
646*517^(1/2))^(1/2))*517^(1/4)/(8*x^4-15*x^3+8*x^2+24*x+8)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 16.07 (sec) , antiderivative size = 6019, normalized size of antiderivative = 13.93

$$\int \frac{1}{(8+24x+8x^2-15x^3+8x^4)^{3/2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^(-3/2),x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 1.29 (sec) , antiderivative size = 529, normalized size of antiderivative = 1.22, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.542$, Rules used = {2504, 27, 7270, 2202, 1576, 27, 1158, 2206, 27, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(8x^4-15x^3+8x^2+24x+8)^{3/2}}dx$$

$$\Big\downarrow \text{2504}$$

$$-1024\int \frac{1}{1024\sqrt{2}\left(3-4\left(\frac{3}{4}+\frac{1}{x}\right)\right)^2\left(\frac{256\left(\frac{3}{4}+\frac{1}{x}\right)^4-608\left(\frac{3}{4}+\frac{1}{x}\right)^2+517}{\left(3-4\left(\frac{3}{4}+\frac{1}{x}\right)\right)^4}\right)^{3/2}}d\left(\frac{3}{4}+\frac{1}{x}\right)$$

$\downarrow$ 27

$$-\frac{\int \frac{1}{\left(3-4\left(\frac{3}{4}+\frac{1}{x}\right)\right)^2\left(\frac{256\left(\frac{3}{4}+\frac{1}{x}\right)^4-608\left(\frac{3}{4}+\frac{1}{x}\right)^2+517}{\left(3-4\left(\frac{3}{4}+\frac{1}{x}\right)\right)^4}\right)^{3/2}}d\left(\frac{3}{4}+\frac{1}{x}\right)}{\sqrt{2}}$$

$\downarrow$ 7270

$$-\frac{\sqrt{256\left(\frac{1}{x}+\frac{3}{4}\right)^4-608\left(\frac{1}{x}+\frac{3}{4}\right)^2+517}\int \frac{\left(3-4\left(\frac{3}{4}+\frac{1}{x}\right)\right)^4}{\left(256\left(\frac{3}{4}+\frac{1}{x}\right)^4-608\left(\frac{3}{4}+\frac{1}{x}\right)^2+517\right)^{3/2}}d\left(\frac{3}{4}+\frac{1}{x}\right)}{\sqrt{2}\left(3-4\left(\frac{1}{x}+\frac{3}{4}\right)\right)^2\sqrt{\frac{256\left(\frac{1}{x}+\frac{3}{4}\right)^4-608\left(\frac{1}{x}+\frac{3}{4}\right)^2+517}{\left(3-4\left(\frac{1}{x}+\frac{3}{4}\right)\right)^4}}}$$

$\downarrow$ 2202

$$-\frac{\sqrt{256\left(\frac{1}{x}+\frac{3}{4}\right)^4-608\left(\frac{1}{x}+\frac{3}{4}\right)^2+517}\left(\int \frac{256\left(\frac{3}{4}+\frac{1}{x}\right)^4+864\left(\frac{3}{4}+\frac{1}{x}\right)^2+81}{\left(256\left(\frac{3}{4}+\frac{1}{x}\right)^4-608\left(\frac{3}{4}+\frac{1}{x}\right)^2+517\right)^{3/2}}d\left(\frac{3}{4}+\frac{1}{x}\right)+\int \frac{\left(-768\left(\frac{3}{4}+\frac{1}{x}\right)^2-432\right)\left(\frac{3}{4}+\frac{1}{x}\right.}{\left(256\left(\frac{3}{4}+\frac{1}{x}\right)^4-608\left(\frac{3}{4}+\frac{1}{x}\right)^2+51\right.}\right.}{\sqrt{2}\left(3-4\left(\frac{1}{x}+\frac{3}{4}\right)\right)^2\sqrt{\frac{256\left(\frac{1}{x}+\frac{3}{4}\right)^4-608\left(\frac{1}{x}+\frac{3}{4}\right)^2+517}{\left(3-4\left(\frac{1}{x}+\frac{3}{4}\right)\right)^4}}}$$

$\downarrow$ 1576

$$-\frac{\sqrt{256\left(\frac{1}{x}+\frac{3}{4}\right)^4-608\left(\frac{1}{x}+\frac{3}{4}\right)^2+517}\left(\frac{1}{2}\int -\frac{48\left(16\left(\frac{3}{4}+\frac{1}{x}\right)^2+9\right)}{\left(256\left(\frac{3}{4}+\frac{1}{x}\right)^4-608\left(\frac{3}{4}+\frac{1}{x}\right)^2+517\right)^{3/2}}d\left(\frac{3}{4}+\frac{1}{x}\right)^2+\int \frac{256\left(\frac{3}{4}+\frac{1}{x}\right)^4+864\left(\frac{3}{4}+\right.}{\left(256\left(\frac{3}{4}+\frac{1}{x}\right)^4-608\left(\frac{3}{4}+\frac{1}{x}\right.\right.}\right.}{\sqrt{2}\left(3-4\left(\frac{1}{x}+\frac{3}{4}\right)\right)^2\sqrt{\frac{256\left(\frac{1}{x}+\frac{3}{4}\right)^4-608\left(\frac{1}{x}+\frac{3}{4}\right)^2+517}{\left(3-4\left(\frac{1}{x}+\frac{3}{4}\right)\right)^4}}}$$

$\downarrow$ 27

$$-\frac{\sqrt{256\left(\frac{1}{x}+\frac{3}{4}\right)^4-608\left(\frac{1}{x}+\frac{3}{4}\right)^2+517}\left(\int \frac{256\left(\frac{3}{4}+\frac{1}{x}\right)^4+864\left(\frac{3}{4}+\frac{1}{x}\right)^2+81}{\left(256\left(\frac{3}{4}+\frac{1}{x}\right)^4-608\left(\frac{3}{4}+\frac{1}{x}\right)^2+517\right)^{3/2}}d\left(\frac{3}{4}+\frac{1}{x}\right)-24\int \frac{16\left(\frac{3}{4}+\frac{1}{x}\right)^2+9}{\left(256\left(\frac{3}{4}+\frac{1}{x}\right)^4-608\left(\frac{3}{4}+\frac{1}{x}\right)^2-\right.}\right.}{\sqrt{2}\left(3-4\left(\frac{1}{x}+\frac{3}{4}\right)\right)^2\sqrt{\frac{256\left(\frac{1}{x}+\frac{3}{4}\right)^4-608\left(\frac{1}{x}+\frac{3}{4}\right)^2+517}{\left(3-4\left(\frac{1}{x}+\frac{3}{4}\right)\right)^4}}}$$

$\downarrow$ 1158

$$-\frac{\sqrt{256\left(\frac{1}{x}+\frac{3}{4}\right)^4-608\left(\frac{1}{x}+\frac{3}{4}\right)^2+517}\left(\int \frac{256\left(\frac{3}{4}+\frac{1}{x}\right)^4+864\left(\frac{3}{4}+\frac{1}{x}\right)^2+81}{\left(256\left(\frac{3}{4}+\frac{1}{x}\right)^4-608\left(\frac{3}{4}+\frac{1}{x}\right)^2+517\right)^{3/2}}d\left(\frac{3}{4}+\frac{1}{x}\right)+\frac{2\left(43-28\left(\frac{1}{x}+\frac{3}{4}\right)^2\right)}{13\sqrt{256\left(\frac{1}{x}+\frac{3}{4}\right)^4-608\left(\frac{1}{x}+\frac{3}{4}\right)^2+51}}\right.}{\sqrt{2}\left(3-4\left(\frac{1}{x}+\frac{3}{4}\right)\right)^2\sqrt{\frac{256\left(\frac{1}{x}+\frac{3}{4}\right)^4-608\left(\frac{1}{x}+\frac{3}{4}\right)^2+517}{\left(3-4\left(\frac{1}{x}+\frac{3}{4}\right)\right)^4}}}$$

$\downarrow$ 2206

$$-\frac{\sqrt{256\left(\frac{1}{x}+\frac{3}{4}\right)^4-608\left(\frac{1}{x}+\frac{3}{4}\right)^2+517}\left(\frac{\int \frac{4096\left(104951-78560\left(\frac{3}{4}+\frac{1}{x}\right)^2\right)}{\sqrt{256\left(\frac{3}{4}+\frac{1}{x}\right)^4-608\left(\frac{3}{4}+\frac{1}{x}\right)^2+517}}d\left(\frac{3}{4}+\frac{1}{x}\right)}{82587648}+\frac{2\left(43-28\left(\frac{1}{x}+\frac{3}{4}\right)^2\right)}{13\sqrt{256\left(\frac{1}{x}+\frac{3}{4}\right)^4-608\left(\frac{1}{x}+\frac{3}{4}\right)^2+517}}-\frac{}{201}\right.}{\sqrt{2}\left(3-4\left(\frac{1}{x}+\frac{3}{4}\right)\right)^2\sqrt{\frac{256\left(\frac{1}{x}+\frac{3}{4}\right)^4-608\left(\frac{1}{x}+\frac{3}{4}\right)^2+517}{\left(3-4\left(\frac{1}{x}+\frac{3}{4}\right)\right)^4}}}$$

$\downarrow$ 27

$$-\frac{\sqrt{256\left(\frac{1}{x}+\frac{3}{4}\right)^4-608\left(\frac{1}{x}+\frac{3}{4}\right)^2+517}\left(\frac{\int \frac{104951-78560\left(\frac{3}{4}+\frac{1}{x}\right)^2}{\sqrt{256\left(\frac{3}{4}+\frac{1}{x}\right)^4-608\left(\frac{3}{4}+\frac{1}{x}\right)^2+517}}d\left(\frac{3}{4}+\frac{1}{x}\right)}{20163}+\frac{2\left(43-28\left(\frac{1}{x}+\frac{3}{4}\right)^2\right)}{13\sqrt{256\left(\frac{1}{x}+\frac{3}{4}\right)^4-608\left(\frac{1}{x}+\frac{3}{4}\right)^2+517}}-\frac{}{201}\right.}{\sqrt{2}\left(3-4\left(\frac{1}{x}+\frac{3}{4}\right)\right)^2\sqrt{\frac{256\left(\frac{1}{x}+\frac{3}{4}\right)^4-608\left(\frac{1}{x}+\frac{3}{4}\right)^2+517}{\left(3-4\left(\frac{1}{x}+\frac{3}{4}\right)\right)^4}}}$$

$\downarrow$ 1511

$$-\frac{\sqrt{256\left(\frac{1}{x}+\frac{3}{4}\right)^4-608\left(\frac{1}{x}+\frac{3}{4}\right)^2+517}\left(\frac{\left(104951-4910\sqrt{517}\right)\int \frac{1}{\sqrt{256\left(\frac{3}{4}+\frac{1}{x}\right)^4-608\left(\frac{3}{4}+\frac{1}{x}\right)^2+517}}d\left(\frac{3}{4}+\frac{1}{x}\right)+4910\sqrt{517}\int \frac{\sqrt{}}{\sqrt{517}\sqrt{256(}}}{20163}\right.}{\sqrt{2}\left(3-4\left(\frac{1}{x}+\frac{3}{4}\right)\right)^2\sqrt{\frac{256\left(\frac{1}{x}+\frac{3}{4}\right)}{(3}}}$$

$\downarrow$ 27

$$-\frac{\sqrt{256\left(\frac{1}{x}+\frac{3}{4}\right)^4-608\left(\frac{1}{x}+\frac{3}{4}\right)^2+517}\left(\frac{\left(104951-4910\sqrt{517}\right)\int \frac{1}{\sqrt{256\left(\frac{3}{4}+\frac{1}{x}\right)^4-608\left(\frac{3}{4}+\frac{1}{x}\right)^2+517}}d\left(\frac{3}{4}+\frac{1}{x}\right)+4910\int \frac{\sqrt{517}-16(}{\sqrt{256\left(\frac{3}{4}+\frac{1}{x}\right)^4-60}}}{20163}\right.}{\sqrt{2}\left(3-4\left(\frac{1}{x}+\frac{3}{4}\right)\right)^2\sqrt{\frac{256\left(\frac{1}{x}+\frac{3}{4}\right)^4-60}{(3-4(\frac{1}{x}}}}$$

$\downarrow$ 1416

$$-\frac{\sqrt{256\left(\frac{1}{x}+\frac{3}{4}\right)^4-608\left(\frac{1}{x}+\frac{3}{4}\right)^2+517}\left(\frac{4910\int \frac{\sqrt{517}-16\left(\frac{3}{4}+\frac{1}{x}\right)^2}{\sqrt{256\left(\frac{3}{4}+\frac{1}{x}\right)^4-608\left(\frac{3}{4}+\frac{1}{x}\right)^2+517}}d\left(\frac{3}{4}+\frac{1}{x}\right)+\frac{\left(104951-4910\sqrt{517}\right)\left(16\left(\frac{1}{x}+\frac{3}{4}\right)^2+\sqrt{517}\right)\sqrt{\cdots}}{8\sqrt[4]{\cdots}}}{20163}\cdots\right.}{\sqrt{2}\left(3-4\left(\frac{1}{x}+\frac{3}{4}\cdots\right.\right.}$$

$\Big\downarrow$ 1509

$$-\frac{\sqrt{256\left(\frac{1}{x}+\frac{3}{4}\right)^4-608\left(\frac{1}{x}+\frac{3}{4}\right)^2+517}\left(\frac{\frac{\left(104951-4910\sqrt{517}\right)\left(16\left(\frac{1}{x}+\frac{3}{4}\right)^2+\sqrt{517}\right)\sqrt{\frac{256\left(\frac{1}{x}+\frac{3}{4}\right)^4-608\left(\frac{1}{x}+\frac{3}{4}\right)^2+517}{\left(16\left(\frac{1}{x}+\frac{3}{4}\right)^2+\sqrt{517}\right)^2}}\text{EllipticF}\left(2\arctan\left(\cdots\right.\right.}{8\sqrt[4]{517}\sqrt{256\left(\frac{1}{x}+\frac{3}{4}\right)^4-608\left(\frac{1}{x}+\frac{3}{4}\right)^2+517}}\cdots}{\cdots}\right.}{\cdots}$$

input
```
Int[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^(-3/2),x]
```

output
```
-((Sqrt[517 - 608*(3/4 + x^(-1))^2 + 256*(3/4 + x^(-1))^4]*((2*(43 - 28*(3
/4 + x^(-1))^2))/(13*Sqrt[517 - 608*(3/4 + x^(-1))^2 + 256*(3/4 + x^(-1))^
4]) - (32*(3181 - 2455*(3/4 + x^(-1))^2)*(3/4 + x^(-1)))/(20163*Sqrt[517 -
 608*(3/4 + x^(-1))^2 + 256*(3/4 + x^(-1))^4]) + (4910*((-517*Sqrt[517 - 6
08*(3/4 + x^(-1))^2 + 256*(3/4 + x^(-1))^4]*(3/4 + x^(-1)))/(517*Sqrt[517]
 + 8272*(3/4 + x^(-1))^2) + (517^(1/4)*(Sqrt[517] + 16*(3/4 + x^(-1))^2)*S
qrt[(517 - 608*(3/4 + x^(-1))^2 + 256*(3/4 + x^(-1))^4)/(Sqrt[517] + 16*(3
/4 + x^(-1))^2)^2]*EllipticE[2*ArcTan[(4*(3/4 + x^(-1)))/517^(1/4)], (517
+ 19*Sqrt[517])/1034])/(4*Sqrt[517 - 608*(3/4 + x^(-1))^2 + 256*(3/4 + x^(
-1))^4])) + ((104951 - 4910*Sqrt[517])*(Sqrt[517] + 16*(3/4 + x^(-1))^2)*S
qrt[(517 - 608*(3/4 + x^(-1))^2 + 256*(3/4 + x^(-1))^4)/(Sqrt[517] + 16*(3
/4 + x^(-1))^2)^2]*EllipticF[2*ArcTan[(4*(3/4 + x^(-1)))/517^(1/4)], (517
+ 19*Sqrt[517])/1034])/(8*517^(1/4)*Sqrt[517 - 608*(3/4 + x^(-1))^2 + 256*
(3/4 + x^(-1))^4]))/20163))/(Sqrt[2]*(3 - 4*(3/4 + x^(-1)))^2*Sqrt[(517 -
608*(3/4 + x^(-1))^2 + 256*(3/4 + x^(-1))^4)/(3 - 4*(3/4 + x^(-1)))^4]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1158
```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[-2*((b*d - 2*a*e + (2*c*d - b*e)*x)/((b^2 - 4*a*c)*Sqrt[a + b*x
+ c*x^2])), x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 1576
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x]
, x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

rule 2202
```
Int[(Pn_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{n
 = Expon[Pn, x], k}, Int[Sum[Coeff[Pn, x, 2*k]*x^(2*k), {k, 0, n/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pn, x, 2*k + 1]*x^(2*k), {k, 0, (n -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pn, x]
 &&  !PolyQ[Pn, x^2]
```

rule 2206
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c
*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Px,
a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*
p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x
^2] && Expon[Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 2504
```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Simp[-16*
a^2   Subst[Int[(1/(b - 4*a*x)^2)*(a*((-3*b^4 + 16*a*b^2*c - 64*a^2*b*d + 2
56*a^3*e - 32*a^2*(3*b^2 - 8*a*c)*x^2 + 256*a^4*x^4)/(b - 4*a*x)^4))^p, x],
 x, b/(4*a) + 1/x], x] /; NeQ[a, 0] && NeQ[b, 0] && EqQ[b^3 - 4*a*b*c + 8*a
^2*d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && IntegerQ[2*p] &&  !IGtQ[p, 0
]
```

rule 7270
```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Simp[a^IntPart[p
]*((a*v^m*w^n)^FracPart[p]/(v^(m*FracPart[p])*w^(n*FracPart[p])))   Int[u*v
^(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !Free
Q[v, x] &&  !FreeQ[w, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 5420 vs. $2(387) = 774$.

Time = 3.77 (sec) , antiderivative size = 5421, normalized size of antiderivative = 12.55

method	result	size
default	Expression too large to display	5421
risch	Expression too large to display	5421
elliptic	Expression too large to display	5421

input
```
int(1/(8*x^4-15*x^3+8*x^2+24*x+8)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [F]

$$\int \frac{1}{(8+24x+8x^2-15x^3+8x^4)^{3/2}}\,dx = \int \frac{1}{(8\,x^4-15\,x^3+8\,x^2+24\,x+8)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(8*x^4-15*x^3+8*x^2+24*x+8)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(8*x^4 - 15*x^3 + 8*x^2 + 24*x + 8)/(64*x^8 - 240*x^7 + 353*x
^6 + 144*x^5 - 528*x^4 + 144*x^3 + 704*x^2 + 384*x + 64), x)
```

Sympy [F]

$$\int \frac{1}{(8+24x+8x^2-15x^3+8x^4)^{3/2}}\,dx = \int \frac{1}{(8x^4-15x^3+8x^2+24x+8)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(8*x**4-15*x**3+8*x**2+24*x+8)**(3/2),x)
```

output
```
Integral((8*x**4 - 15*x**3 + 8*x**2 + 24*x + 8)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(8+24x+8x^2-15x^3+8x^4)^{3/2}}\,dx = \int \frac{1}{(8\,x^4-15\,x^3+8\,x^2+24\,x+8)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(8*x^4-15*x^3+8*x^2+24*x+8)^(3/2),x, algorithm="maxima")
```

output
```
integrate((8*x^4 - 15*x^3 + 8*x^2 + 24*x + 8)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(8+24x+8x^2-15x^3+8x^4)^{3/2}}\,dx = \int \frac{1}{(8\,x^4-15\,x^3+8\,x^2+24\,x+8)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(8*x^4-15*x^3+8*x^2+24*x+8)^(3/2),x, algorithm="giac")
```

output
```
integrate((8*x^4 - 15*x^3 + 8*x^2 + 24*x + 8)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(8+24x+8x^2-15x^3+8x^4)^{3/2}}\,dx = \int \frac{1}{(8\,x^4-15\,x^3+8\,x^2+24\,x+8)^{3/2}}\,dx$$

input
```
int(1/(24*x + 8*x^2 - 15*x^3 + 8*x^4 + 8)^(3/2),x)
```

output
```
int(1/(24*x + 8*x^2 - 15*x^3 + 8*x^4 + 8)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(8+24x+8x^2-15x^3+8x^4)^{3/2}}\,dx = \int \frac{\sqrt{8x^4-15x^3+8x^2+24x+8}}{64x^8-240x^7+353x^6+144x^5-528x^4+144x^3+704x^2+3}$$

input
```
int(1/(8*x^4-15*x^3+8*x^2+24*x+8)^(3/2),x)
```

output
```
int(sqrt(8*x**4 - 15*x**3 + 8*x**2 + 24*x + 8)/(64*x**8 - 240*x**7 + 353*x
**6 + 144*x**5 - 528*x**4 + 144*x**3 + 704*x**2 + 384*x + 64),x)
```

3.91 $\int \frac{1}{\left(8+24x+8x^2-15x^3+8x^4\right)^{5/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 575

$$\int \frac{1}{(8+24x+8x^2-15x^3+8x^4)^{5/2}}\,dx = -\frac{\left(124415-6308\left(3+\frac{4}{x}\right)^2\right)x^2}{97344\sqrt{8+24x+8x^2-15x^3+8x^4}} - \frac{\left(64489-1399\left(3+\frac{4}{x}\right)^2\right)x^2}{624\left(517-38\left(3+\frac{4}{x}\right)^2+\left(3+\frac{4}{x}\right)^4\right)\sqrt{8+24x+8x^2-15x^3+8x^4}} + \frac{\left(18932921731-1086525994\left(3+\frac{4}{x}\right)^2\right)\left(3+\frac{4}{x}\right)x^2}{78056941248\sqrt{8+24x+8x^2-15x^3+8x^4}} + \frac{\left(11921698-359497\left(3+\frac{4}{x}\right)^2\right)\left(3+\frac{4}{x}\right)x^2}{483912\left(517-38\left(3+\frac{4}{x}\right)^2+\left(3+\frac{4}{x}\right)^4\right)\sqrt{8+24x+8x^2-15x^3+8x^4}} + \frac{543262997\left(517-38\left(3+\frac{4}{x}\right)^2+\left(3+\frac{4}{x}\right)^4\right)\left(3+\frac{4}{x}\right)x^2}{39028470624\left(\sqrt{517}+\left(3+\frac{4}{x}\right)^2\right)\sqrt{8+24x+8x^2-15x^3+8x^4}} - \frac{543262997\left(\sqrt{517}+\left(3+\frac{4}{x}\right)^2\right)\sqrt{\frac{517-38\left(3+\frac{4}{x}\right)^2+\left(3+\frac{4}{x}\right)^4}{\left(\sqrt{517}+\left(3+\frac{4}{x}\right)^2\right)^2}}\,x^2 E\left(2\arctan\left(\frac{3+\frac{4}{x}}{\sqrt[4]{517}}\right)\middle|\frac{517+19\sqrt{517}}{1034}\right)}{75490272\ 517^{3/4}\sqrt{8+24x+8x^2-15x^3+8x^4}} + \frac{\left(4346103976-175318963\sqrt{517}\right)\left(\sqrt{517}+\left(3+\frac{4}{x}\right)^2\right)\sqrt{\frac{517-38\left(3+\frac{4}{x}\right)^2+\left(3+\frac{4}{x}\right)^4}{\left(\sqrt{517}+\left(3+\frac{4}{x}\right)^2\right)^2}}\,x^2\,\text{EllipticF}\left(2\arctan\left(\frac{3+\frac{4}{x}}{\sqrt[4]{51}}\right.\right.}{1207844352\ 517^{3/4}\sqrt{8+24x+8x^2-15x^3+8x^4}}$$

output
```
-1/97344*(124415-6308*(3+4/x)^2)*x^2/(8*x^4-15*x^3+8*x^2+24*x+8)^(1/2)-1/6
24*(64489-1399*(3+4/x)^2)*x^2/(517-38*(3+4/x)^2+(3+4/x)^4)/(8*x^4-15*x^3+8
*x^2+24*x+8)^(1/2)+1/78056941248*(18932921731-1086525994*(3+4/x)^2)*(3+4/x
)*x^2/(8*x^4-15*x^3+8*x^2+24*x+8)^(1/2)+1/483912*(11921698-359497*(3+4/x)^
2)*(3+4/x)*x^2/(517-38*(3+4/x)^2+(3+4/x)^4)/(8*x^4-15*x^3+8*x^2+24*x+8)^(1
/2)+543262997/39028470624*(517-38*(3+4/x)^2+(3+4/x)^4)*(3+4/x)*x^2/(517^(1
/2)+(3+4/x)^2)/(8*x^4-15*x^3+8*x^2+24*x+8)^(1/2)-543262997/39028470624*(51
7^(1/2)+(3+4/x)^2)*((517-38*(3+4/x)^2+(3+4/x)^4)/(517^(1/2)+(3+4/x)^2)^2)^
(1/2)*x^2*EllipticE(sin(2*arctan(1/517*(3+4/x)*517^(3/4))),1/1034*(534578+
19646*517^(1/2))^(1/2))*517^(1/4)/(8*x^4-15*x^3+8*x^2+24*x+8)^(1/2)+1/6244
55529984*(4346103976-175318963*517^(1/2))*(517^(1/2)+(3+4/x)^2)*((517-38*(
3+4/x)^2+(3+4/x)^4)/(517^(1/2)+(3+4/x)^2)^2)^(1/2)*x^2*InverseJacobiAM(2*a
rctan(1/517*(3+4/x)*517^(3/4)),1/1034*(534578+19646*517^(1/2))^(1/2))*517^
(1/4)/(8*x^4-15*x^3+8*x^2+24*x+8)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 16.07 (sec) , antiderivative size = 6084, normalized size of antiderivative = 10.58

$$\int \frac{1}{(8+24x+8x^2-15x^3+8x^4)^{5/2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^(-5/2),x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 1.75 (sec) , antiderivative size = 636, normalized size of antiderivative = 1.11, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.708$, Rules used = {2504, 27, 7270, 2202, 2194, 27, 2191, 27, 1158, 2206, 27, 2206, 27, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(8x^4-15x^3+8x^2+24x+8)^{5/2}}\,dx$$

$\big\downarrow$ 2504

$$-1024\int \frac{1}{8192\sqrt{2}\left(3-4\left(\frac{3}{4}+\frac{1}{x}\right)\right)^2\left(\frac{256\left(\frac{3}{4}+\frac{1}{x}\right)^4-608\left(\frac{3}{4}+\frac{1}{x}\right)^2+517}{\left(3-4\left(\frac{3}{4}+\frac{1}{x}\right)\right)^4}\right)^{5/2}}d\left(\frac{3}{4}+\frac{1}{x}\right)$$

$\big\downarrow$ 27

$$-\frac{\int \frac{1}{\left(3-4\left(\frac{3}{4}+\frac{1}{x}\right)\right)^2\left(\frac{256\left(\frac{3}{4}+\frac{1}{x}\right)^4-608\left(\frac{3}{4}+\frac{1}{x}\right)^2+517}{\left(3-4\left(\frac{3}{4}+\frac{1}{x}\right)\right)^4}\right)^{5/2}}d\left(\frac{3}{4}+\frac{1}{x}\right)}{8\sqrt{2}}$$

$\big\downarrow$ 7270

$$-\frac{\sqrt{256\left(\frac{1}{x}+\frac{3}{4}\right)^4-608\left(\frac{1}{x}+\frac{3}{4}\right)^2+517}\int \frac{\left(3-4\left(\frac{3}{4}+\frac{1}{x}\right)\right)^8}{\left(256\left(\frac{3}{4}+\frac{1}{x}\right)^4-608\left(\frac{3}{4}+\frac{1}{x}\right)^2+517\right)^{5/2}}d\left(\frac{3}{4}+\frac{1}{x}\right)}{8\sqrt{2}\left(3-4\left(\frac{1}{x}+\frac{3}{4}\right)\right)^2\sqrt{\frac{256\left(\frac{1}{x}+\frac{3}{4}\right)^4-608\left(\frac{1}{x}+\frac{3}{4}\right)^2+517}{\left(3-4\left(\frac{1}{x}+\frac{3}{4}\right)\right)^4}}}$$

$\big\downarrow$ 2202

$$-\frac{\sqrt{256\left(\frac{1}{x}+\frac{3}{4}\right)^4-608\left(\frac{1}{x}+\frac{3}{4}\right)^2+517}\left(\int \frac{65536\left(\frac{3}{4}+\frac{1}{x}\right)^8+1032192\left(\frac{3}{4}+\frac{1}{x}\right)^6+1451520\left(\frac{3}{4}+\frac{1}{x}\right)^4+326592\left(\frac{3}{4}+\frac{1}{x}\right)^2+6561}{\left(256\left(\frac{3}{4}+\frac{1}{x}\right)^4-608\left(\frac{3}{4}+\frac{1}{x}\right)^2+517\right)^{5/2}}d\left(\frac{3}{4}+\frac{1}{x}\right)\cdots\right.}{8\sqrt{2}\left(3-4\left(\frac{1}{x}+\frac{3}{4}\right)\right)^2\sqrt{\frac{256\left(\frac{1}{x}+\frac{3}{4}\right)^4-608\left(\frac{1}{x}+\frac{3}{4}\right)\cdots}{\left(3-4\left(\frac{1}{x}+\frac{3}{4}\right)\right)^4}}}$$

$\big\downarrow$ 2194

$$-\frac{\sqrt{256\left(\frac{1}{x}+\frac{3}{4}\right)^4-608\left(\frac{1}{x}+\frac{3}{4}\right)^2+517}\left(\frac{1}{2}\int -\frac{96\left(4096\left(\frac{3}{4}+\frac{1}{x}\right)^6+16128\left(\frac{3}{4}+\frac{1}{x}\right)^4+9072\left(\frac{3}{4}+\frac{1}{x}\right)^2+729\right)}{\left(256\left(\frac{3}{4}+\frac{1}{x}\right)^4-608\left(\frac{3}{4}+\frac{1}{x}\right)^2+517\right)^{5/2}}d\left(\frac{3}{4}+\frac{1}{x}\right)^2+\int \frac{65536\cdots}{\cdots}\right.}{8\sqrt{2}\left(3-4\left(\frac{1}{x}+\frac{3}{4}\right)\right)^2\sqrt{\frac{256\left(\frac{1}{x}+\frac{3}{4}\right)^4-608\left(\frac{1}{x}+\frac{3}{4}\right)^2+5\cdots}{\left(3-4\left(\frac{1}{x}+\frac{3}{4}\right)\right)^4}}}$$

$\big\downarrow$ 27

$$-\frac{\sqrt{256\left(\frac{1}{x}+\frac{3}{4}\right)^4-608\left(\frac{1}{x}+\frac{3}{4}\right)^2+517}\left(\int \frac{65536\left(\frac{3}{4}+\frac{1}{x}\right)^8+1032192\left(\frac{3}{4}+\frac{1}{x}\right)^6+1451520\left(\frac{3}{4}+\frac{1}{x}\right)^4+326592\left(\frac{3}{4}+\frac{1}{x}\right)^2+6561}{\left(256\left(\frac{3}{4}+\frac{1}{x}\right)^4-608\left(\frac{3}{4}+\frac{1}{x}\right)^2+517\right)^{5/2}}d\left(\frac{3}{4}+\frac{1}{x}\right)\cdots\right.}{8\sqrt{2}\left(3-4\left(\frac{1}{x}+\frac{3}{4}\right)\right)^2\sqrt{\frac{256\left(\frac{1}{x}+\frac{3}{4}\right)^4-608\left(\frac{1}{x}+\frac{3}{4}\right)^2+517}{\left(3-4\left(\frac{1}{x}+\frac{3}{4}\right)\right)^4}}}$$

$\big\downarrow$ 2191

$$-\frac{\sqrt{256\left(\frac{1}{x}+\frac{3}{4}\right)^4-608\left(\frac{1}{x}+\frac{3}{4}\right)^2+517}\left(\int \frac{65536\left(\frac{3}{4}+\frac{1}{x}\right)^8+1032192\left(\frac{3}{4}+\frac{1}{x}\right)^6+1451520\left(\frac{3}{4}+\frac{1}{x}\right)^4+326592\left(\frac{3}{4}+\frac{1}{x}\right)^2+6561}{\left(256\left(\frac{3}{4}+\frac{1}{x}\right)^4-608\left(\frac{3}{4}+\frac{1}{x}\right)^2+517\right)^{5/2}}d\left(\frac{3}{4}+\frac{1}{x}\right)}{8\sqrt{2}\left(3-4\left(\frac{1}{x}+\frac{3}{4}\right)\right)^2\sqrt{\frac{256\left(\frac{1}{x}+\frac{3}{4}\right)^4-60}{\left(3-4\left(\frac{1}{x}\right.\right.}}}$$

$\downarrow$ 27

$$-\frac{\sqrt{256\left(\frac{1}{x}+\frac{3}{4}\right)^4-608\left(\frac{1}{x}+\frac{3}{4}\right)^2+517}\left(\int \frac{65536\left(\frac{3}{4}+\frac{1}{x}\right)^8+1032192\left(\frac{3}{4}+\frac{1}{x}\right)^6+1451520\left(\frac{3}{4}+\frac{1}{x}\right)^4+326592\left(\frac{3}{4}+\frac{1}{x}\right)^2+6561}{\left(256\left(\frac{3}{4}+\frac{1}{x}\right)^4-608\left(\frac{3}{4}+\frac{1}{x}\right)^2+517\right)^{5/2}}d\left(\frac{3}{4}+\frac{1}{x}\right)}{8\sqrt{2}\left(3-4\left(\frac{1}{x}+\frac{3}{4}\right)\right)^2\sqrt{\frac{256\left(\frac{1}{x}+\frac{3}{4}\right)^4}{(3-}}}$$

$\downarrow$ 1158

$$-\frac{\sqrt{256\left(\frac{1}{x}+\frac{3}{4}\right)^4-608\left(\frac{1}{x}+\frac{3}{4}\right)^2+517}\left(\int \frac{65536\left(\frac{3}{4}+\frac{1}{x}\right)^8+1032192\left(\frac{3}{4}+\frac{1}{x}\right)^6+1451520\left(\frac{3}{4}+\frac{1}{x}\right)^4+326592\left(\frac{3}{4}+\frac{1}{x}\right)^2+6561}{\left(256\left(\frac{3}{4}+\frac{1}{x}\right)^4-608\left(\frac{3}{4}+\frac{1}{x}\right)^2+517\right)^{5/2}}d\left(\frac{3}{4}+\frac{1}{x}\right)}{8\sqrt{2}\left(3-4\left(\frac{1}{x}+\frac{3}{4}\right)\right)^2\sqrt{\frac{256\left(\frac{1}{x}+\frac{3}{4}\right)^4-608\left(\frac{1}{x}\right.}{\left(3-4\left(\frac{1}{x}+\frac{3}{4}\right.\right.}}}$$

$\downarrow$ 2206

$$-\frac{\sqrt{256\left(\frac{1}{x}+\frac{3}{4}\right)^4-608\left(\frac{1}{x}+\frac{3}{4}\right)^2+517}\left(\frac{\int \frac{4096\left(15485184\left(\frac{3}{4}+\frac{1}{x}\right)^4+832856352\left(\frac{3}{4}+\frac{1}{x}\right)^2+382261973\right)}{\left(256\left(\frac{3}{4}+\frac{1}{x}\right)^4-608\left(\frac{3}{4}+\frac{1}{x}\right)^2+517\right)^{3/2}}d\left(\frac{3}{4}+\frac{1}{x}\right)}{247762944}-48\left(-\frac{12441}{73008\sqrt{256\left(\frac{1}{x}\right.}}\right.\right.}{8\sqrt{2}\left(3-4\left(\frac{1}{x}+\frac{3}{4}\right)\right)^2\sqrt{\frac{256\left(\frac{1}{x}+\right.}{}}}$$

$\downarrow$ 27

$$-\frac{\sqrt{256\left(\frac{1}{x}+\frac{3}{4}\right)^4-608\left(\frac{1}{x}+\frac{3}{4}\right)^2+517}\left(\frac{\int \frac{15485184\left(\frac{3}{4}+\frac{1}{x}\right)^4+832856352\left(\frac{3}{4}+\frac{1}{x}\right)^2+382261973}{\left(256\left(\frac{3}{4}+\frac{1}{x}\right)^4-608\left(\frac{3}{4}+\frac{1}{x}\right)^2+517\right)^{3/2}}d\left(\frac{3}{4}+\frac{1}{x}\right)}{60489}-48\left(-\frac{124415-1009}{73008\sqrt{256\left(\frac{1}{x}+\frac{3}{4}\right)^4}}\right.\right.}{8\sqrt{2}\left(3-4\left(\frac{1}{x}+\frac{3}{4}\right)\right)^2\sqrt{\frac{256\left(\frac{1}{x}+\frac{3}{4}\right)^4}{(3-}}}$$

$\downarrow$ 2206

$$-\frac{\sqrt{256\left(\frac{1}{x}+\frac{3}{4}\right)^4-608\left(\frac{1}{x}+\frac{3}{4}\right)^2+517}\left(\frac{\frac{\int \frac{4096\left(90639903871-69537663616\left(\frac{3}{4}+\frac{1}{x}\right)^2\right)}{\sqrt{256\left(\frac{3}{4}+\frac{1}{x}\right)^4-608\left(\frac{3}{4}+\frac{1}{x}\right)^2+517}}d\left(\frac{3}{4}+\frac{1}{x}\right)}{82587648}-\frac{4\left(18932921731-17384415904\left(\frac{1}{x}+\frac{3}{4}\right)^2\right)(}{20163\sqrt{256\left(\frac{1}{x}+\frac{3}{4}\right)^4-608\left(\frac{1}{x}+\frac{3}{4}\right)^2+5}}}{60489}\right.}{8\sqrt{2}\left(3-4\left(\frac{1}{x}+\frac{3}{4}\right.\right.}$$

$$\downarrow \; 27$$

$$-\frac{\sqrt{256\left(\frac{1}{x}+\frac{3}{4}\right)^4-608\left(\frac{1}{x}+\frac{3}{4}\right)^2+517}\left(\frac{\frac{\int \frac{90639903871-69537663616\left(\frac{3}{4}+\frac{1}{x}\right)^2}{\sqrt{256\left(\frac{3}{4}+\frac{1}{x}\right)^4-608\left(\frac{3}{4}+\frac{1}{x}\right)^2+517}}d\left(\frac{3}{4}+\frac{1}{x}\right)}{20163}-\frac{4\left(18932921731-17384415904\left(\frac{1}{x}+\frac{3}{4}\right)^2\right)\left(\frac{1}{x}+\frac{3}{4}\right)}{20163\sqrt{256\left(\frac{1}{x}+\frac{3}{4}\right)^4-608\left(\frac{1}{x}+\frac{3}{4}\right)^2+517}}}{60489}\right.}{8\sqrt{2}\left(3-4\left(\frac{1}{x}+\frac{3}{4}\right)\right)^2}$$

$$\downarrow \; 1511$$

$$-\frac{\sqrt{256\left(\frac{1}{x}+\frac{3}{4}\right)^4-608\left(\frac{1}{x}+\frac{3}{4}\right)^2+517}\left(\frac{\frac{\left(90639903871-4346103976\sqrt{517}\right)\int \frac{1}{\sqrt{256\left(\frac{3}{4}+\frac{1}{x}\right)^4-608\left(\frac{3}{4}+\frac{1}{x}\right)^2+517}}d\left(\frac{3}{4}+\frac{1}{x}\right)+4346103976\sqrt{517}\int}{20163}}{60}\right.}{}$$

$$\downarrow \; 27$$

$$-\frac{\sqrt{256\left(\frac{1}{x}+\frac{3}{4}\right)^4-608\left(\frac{1}{x}+\frac{3}{4}\right)^2+517}\left(\frac{\frac{\left(90639903871-4346103976\sqrt{517}\right)\int \frac{1}{\sqrt{256\left(\frac{3}{4}+\frac{1}{x}\right)^4-608\left(\frac{3}{4}+\frac{1}{x}\right)^2+517}}d\left(\frac{3}{4}+\frac{1}{x}\right)+4346103976\int \frac{}{\sqrt{25}}}{20163}}{60489}\right.}{}$$

$\Big\downarrow$ 1416

$$-\frac{\sqrt{256\left(\frac{1}{x}+\frac{3}{4}\right)^4-608\left(\frac{1}{x}+\frac{3}{4}\right)^2+517}\left(\frac{4346103976\int\frac{\sqrt{517}-16\left(\frac{3}{4}+\frac{1}{x}\right)^2}{\sqrt{256\left(\frac{3}{4}+\frac{1}{x}\right)^4-608\left(\frac{3}{4}+\frac{1}{x}\right)^2+517}}d\left(\frac{3}{4}+\frac{1}{x}\right)+\frac{\left(90639903871-4346103976\sqrt{517}\right)\left(16\left(\frac{1}{x}\right.\right.}{}}{201}\right.}{}$$

$\Big\downarrow$ 1509

$$-\frac{\sqrt{256\left(\frac{1}{x}+\frac{3}{4}\right)^4-608\left(\frac{1}{x}+\frac{3}{4}\right)^2+517}\left(\frac{\left(90639903871-4346103976\sqrt{517}\right)\left(16\left(\frac{1}{x}+\frac{3}{4}\right)^2+\sqrt{517}\right)\sqrt{\frac{256\left(\frac{1}{x}+\frac{3}{4}\right)^4-608\left(\frac{1}{x}+\frac{3}{4}\right)^2+517}{\left(16\left(\frac{1}{x}+\frac{3}{4}\right)^2+\sqrt{517}\right)^2}}\text{EllipticF}}{8\sqrt[4]{517}\sqrt{256\left(\frac{1}{x}+\frac{3}{4}\right)^4-608\left(\frac{1}{x}+\frac{3}{4}\right)^2+517}}\right.}{}$$

input
```
Int[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^(-5/2),x]
```

output
```
-1/8*(Sqrt[517 - 608*(3/4 + x^(-1))^2 + 256*(3/4 + x^(-1))^4]*(-48*(-1/468
*(64489 - 22384*(3/4 + x^(-1))^2)/(517 - 608*(3/4 + x^(-1))^2 + 256*(3/4 +
 x^(-1))^4)^(3/2) - (124415 - 100928*(3/4 + x^(-1))^2)/(73008*Sqrt[517 - 6
08*(3/4 + x^(-1))^2 + 256*(3/4 + x^(-1))^4])) - (64*(5960849 - 2875976*(3/
4 + x^(-1))^2)*(3/4 + x^(-1)))/(60489*(517 - 608*(3/4 + x^(-1))^2 + 256*(3
/4 + x^(-1))^4)^(3/2)) + ((-4*(18932921731 - 17384415904*(3/4 + x^(-1))^2)
*(3/4 + x^(-1)))/(20163*Sqrt[517 - 608*(3/4 + x^(-1))^2 + 256*(3/4 + x^(-1
))^4]) + (4346103976*((-517*Sqrt[517 - 608*(3/4 + x^(-1))^2 + 256*(3/4 + x
^(-1))^4]*(3/4 + x^(-1)))/(517*Sqrt[517] + 8272*(3/4 + x^(-1))^2) + (517^(
1/4)*(Sqrt[517] + 16*(3/4 + x^(-1))^2)*Sqrt[(517 - 608*(3/4 + x^(-1))^2 +
256*(3/4 + x^(-1))^4)/(Sqrt[517] + 16*(3/4 + x^(-1))^2)^2]*EllipticE[2*Arc
Tan[(4*(3/4 + x^(-1)))/517^(1/4)], (517 + 19*Sqrt[517])/1034])/(4*Sqrt[517
 - 608*(3/4 + x^(-1))^2 + 256*(3/4 + x^(-1))^4])) + ((90639903871 - 434610
3976*Sqrt[517])*(Sqrt[517] + 16*(3/4 + x^(-1))^2)*Sqrt[(517 - 608*(3/4 + x
^(-1))^2 + 256*(3/4 + x^(-1))^4)/(Sqrt[517] + 16*(3/4 + x^(-1))^2)^2]*Elli
pticF[2*ArcTan[(4*(3/4 + x^(-1)))/517^(1/4)], (517 + 19*Sqrt[517])/1034])/
(8*517^(1/4)*Sqrt[517 - 608*(3/4 + x^(-1))^2 + 256*(3/4 + x^(-1))^4]))/201
63)/60489))/(Sqrt[2]*(3 - 4*(3/4 + x^(-1)))^2*Sqrt[(517 - 608*(3/4 + x^(-1
))^2 + 256*(3/4 + x^(-1))^4)/(3 - 4*(3/4 + x^(-1)))^4])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1158
```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[-2*((b*d - 2*a*e + (2*c*d - b*e)*x)/((b^2 - 4*a*c)*Sqrt[a + b*x
+ c*x^2])), x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 2194
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Simp[1/2   Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)
^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ
[(m - 1)/2]
```

rule 2202
```
Int[(Pn_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{n
 = Expon[Pn, x], k}, Int[Sum[Coeff[Pn, x, 2*k]*x^(2*k), {k, 0, n/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pn, x, 2*k + 1]*x^(2*k), {k, 0, (n -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pn, x]
 &&  !PolyQ[Pn, x^2]
```

rule 2206
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c
*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Px,
a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*
p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x
^2] && Expon[Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 2504
```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Simp[-16*
a^2   Subst[Int[(1/(b - 4*a*x)^2)*(a*((-3*b^4 + 16*a*b^2*c - 64*a^2*b*d + 2
56*a^3*e - 32*a^2*(3*b^2 - 8*a*c)*x^2 + 256*a^4*x^4)/(b - 4*a*x)^4))^p, x],
 x, b/(4*a) + 1/x], x] /; NeQ[a, 0] && NeQ[b, 0] && EqQ[b^3 - 4*a*b*c + 8*a
^2*d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && IntegerQ[2*p] &&  !IGtQ[p, 0
]
```

rule 7270
```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Simp[a^IntPart[p
]*((a*v^m*w^n)^FracPart[p]/(v^(m*FracPart[p])*w^(n*FracPart[p])))   Int[u*v
^(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !Free
Q[v, x] &&  !FreeQ[w, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 5440 vs. $2(522) = 1044$.

Time = 3.72 (sec) , antiderivative size = 5441, normalized size of antiderivative = 9.46

method	result	size
risch	Expression too large to display	5441
default	Expression too large to display	5477
elliptic	Expression too large to display	5477

input
```
int(1/(8*x^4-15*x^3+8*x^2+24*x+8)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [F]

$$\int \frac{1}{(8+24x+8x^2-15x^3+8x^4)^{5/2}}\,dx = \int \frac{1}{(8\,x^4-15\,x^3+8\,x^2+24\,x+8)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(8*x^4-15*x^3+8*x^2+24*x+8)^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(8*x^4 - 15*x^3 + 8*x^2 + 24*x + 8)/(512*x^12 - 2880*x^11 + 6
936*x^10 - 4527*x^9 - 8808*x^8 + 16776*x^7 + 5528*x^6 - 17856*x^5 - 384*x^
4 + 20160*x^3 + 15360*x^2 + 4608*x + 512), x)
```

Sympy [F]

$$\int \frac{1}{(8+24x+8x^2-15x^3+8x^4)^{5/2}}\,dx = \int \frac{1}{(8x^4-15x^3+8x^2+24x+8)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(8*x**4-15*x**3+8*x**2+24*x+8)**(5/2),x)
```

output
```
Integral((8*x**4 - 15*x**3 + 8*x**2 + 24*x + 8)**(-5/2), x)
```

Maxima [F]

$$\int \frac{1}{(8+24x+8x^2-15x^3+8x^4)^{5/2}}\,dx = \int \frac{1}{(8\,x^4-15\,x^3+8\,x^2+24\,x+8)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(8*x^4-15*x^3+8*x^2+24*x+8)^(5/2),x, algorithm="maxima")
```

output
```
integrate((8*x^4 - 15*x^3 + 8*x^2 + 24*x + 8)^(-5/2), x)
```

Giac [F]

$$\int \frac{1}{(8+24x+8x^2-15x^3+8x^4)^{5/2}}\,dx = \int \frac{1}{(8\,x^4-15\,x^3+8\,x^2+24\,x+8)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(8*x^4-15*x^3+8*x^2+24*x+8)^(5/2),x, algorithm="giac")
```

output
```
integrate((8*x^4 - 15*x^3 + 8*x^2 + 24*x + 8)^(-5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(8+24x+8x^2-15x^3+8x^4)^{5/2}}\,dx = \int \frac{1}{(8\,x^4-15\,x^3+8\,x^2+24\,x+8)^{5/2}}\,dx$$

input
```
int(1/(24*x + 8*x^2 - 15*x^3 + 8*x^4 + 8)^(5/2),x)
```

output
```
int(1/(24*x + 8*x^2 - 15*x^3 + 8*x^4 + 8)^(5/2), x)
```

Reduce [F]

$$\int \frac{1}{(8+24x+8x^2-15x^3+8x^4)^{5/2}}\,dx = \int \frac{\sqrt{8x^4-15x^3}}{512x^{12}-2880x^{11}+6936x^{10}-4527x^9-8808x^8+16776x^7+}$$

input
```
int(1/(8*x^4-15*x^3+8*x^2+24*x+8)^(5/2),x)
```

output
```
int(sqrt(8*x**4 - 15*x**3 + 8*x**2 + 24*x + 8)/(512*x**12 - 2880*x**11 + 6
936*x**10 - 4527*x**9 - 8808*x**8 + 16776*x**7 + 5528*x**6 - 17856*x**5 -
384*x**4 + 20160*x**3 + 15360*x**2 + 4608*x + 512),x)
```

3.92 $\int \frac{1}{\sqrt{9-6x-44x^2+15x^3+3x^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 126

$$\int \frac{1}{\sqrt{9-6x-44x^2+15x^3+3x^4}}\,dx = -\frac{\left(\sqrt{613}+\left(-1+\frac{6}{x}\right)^2\right)\sqrt{\frac{613-182\left(1-\frac{6}{x}\right)^2+\left(-1+\frac{6}{x}\right)^4}{\left(\sqrt{613}+\left(-1+\frac{6}{x}\right)^2\right)^2}}\,x^2\,\text{EllipticF}\left(2\arctan\left(\frac{6-x}{\sqrt[4]{613}x}\right),\frac{613+91\sqrt{613}}{1226}\right)}{12\sqrt[4]{613}\sqrt{9-6x-44x^2+15x^3+3x^4}}$$

output
```
-1/7356*(613^(1/2)+(-1+6/x)^2)*((613-182*(1-6/x)^2+(-1+6/x)^4)/(613^(1/2)+
(-1+6/x)^2)^2)^(1/2)*x^2*InverseJacobiAM(2*arctan(1/613*(6-x)*613^(3/4)/x)
,1/1226*(751538+111566*613^(1/2))^(1/2))*613^(3/4)/(3*x^4+15*x^3-44*x^2-6*
x+9)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 10.17 (sec) , antiderivative size = 826, normalized size of antiderivative = 6.56

$$\int \frac{1}{\sqrt{9-6x-44x^2+15x^3+3x^4}}\,dx = \text{Too large to display}$$

input
```
Integrate[1/Sqrt[9 - 6*x - 44*x^2 + 15*x^3 + 3*x^4],x]
```

output
```
(-2*EllipticF[ArcSin[Sqrt[((x - Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4
 & , 1, 0])*(Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 2, 0] - Root[9
 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 4, 0]))/((x - Root[9 - 6*#1 - 44*
#1^2 + 15*#1^3 + 3*#1^4 & , 2, 0])*(Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*
#1^4 & , 1, 0] - Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 4, 0]))]],
 ((Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 2, 0] - Root[9 - 6*#1 -
44*#1^2 + 15*#1^3 + 3*#1^4 & , 3, 0])*(Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 +
 3*#1^4 & , 1, 0] - Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 4, 0]))
/((Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 1, 0] - Root[9 - 6*#1 -
44*#1^2 + 15*#1^3 + 3*#1^4 & , 3, 0])*(Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 +
 3*#1^4 & , 2, 0] - Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 4, 0]))
]*Sqrt[(x - Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 1, 0])/(x - Roo
t[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 2, 0])]*(x - Root[9 - 6*#1 - 4
4*#1^2 + 15*#1^3 + 3*#1^4 & , 2, 0])^2*Sqrt[(x - Root[9 - 6*#1 - 44*#1^2 +
 15*#1^3 + 3*#1^4 & , 3, 0])/(x - Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1
^4 & , 2, 0])]*Sqrt[(x - Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 4,
 0])/(x - Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 2, 0])])/Sqrt[(9
- 6*x - 44*x^2 + 15*x^3 + 3*x^4)*(Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1
^4 & , 1, 0] - Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 3, 0])*(Root
[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 2, 0] - Root[9 - 6*#1 - 44*#...
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.25, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2504, 27, 7270, 1409}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{3x^4+15x^3-44x^2-6x+9}}\,dx$$

$$\Big\downarrow \text{ 2504}$$

$$-1296\int \frac{1}{108\left(6\left(\frac{1}{x}-\frac{1}{6}\right)+1\right)^2\sqrt{\frac{1296\left(\frac{1}{x}-\frac{1}{6}\right)^4-6552\left(\frac{1}{x}-\frac{1}{6}\right)^2+613}{\left(6\left(\frac{1}{x}-\frac{1}{6}\right)+1\right)^4}}}d\left(\frac{1}{x}-\frac{1}{6}\right)$$

$$\Big\downarrow \text{27}$$

$$-12\int \frac{1}{\left(6\left(\frac{1}{x}-\frac{1}{6}\right)+1\right)^2\sqrt{\frac{1296\left(\frac{1}{x}-\frac{1}{6}\right)^4-6552\left(\frac{1}{x}-\frac{1}{6}\right)^2+613}{\left(6\left(\frac{1}{x}-\frac{1}{6}\right)+1\right)^4}}}d\left(\frac{1}{x}-\frac{1}{6}\right)$$

$$\Big\downarrow \text{7270}$$

$$-\frac{12\sqrt{1296\left(\frac{1}{x}-\frac{1}{6}\right)^4-6552\left(\frac{1}{x}-\frac{1}{6}\right)^2+613}\int \frac{1}{\sqrt{1296\left(\frac{1}{x}-\frac{1}{6}\right)^4-6552\left(\frac{1}{x}-\frac{1}{6}\right)^2+613}}d\left(\frac{1}{x}-\frac{1}{6}\right)}{\left(6\left(\frac{1}{x}-\frac{1}{6}\right)+1\right)^2\sqrt{\frac{1296\left(\frac{1}{x}-\frac{1}{6}\right)^4-6552\left(\frac{1}{x}-\frac{1}{6}\right)^2+613}{\left(6\left(\frac{1}{x}-\frac{1}{6}\right)+1\right)^4}}}$$

$$\Big\downarrow \text{1409}$$

$$-\frac{\left(36\left(\frac{1}{x}-\frac{1}{6}\right)^2+\sqrt{613}\right)\sqrt{\frac{1296\left(\frac{1}{x}-\frac{1}{6}\right)^4-6552\left(\frac{1}{x}-\frac{1}{6}\right)^2+613}{\left(36\left(\frac{1}{x}-\frac{1}{6}\right)^2+\sqrt{613}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{6\left(\frac{1}{x}-\frac{1}{6}\right)}{\sqrt[4]{613}}\right),\frac{613+91\sqrt{613}}{1226}\right)}{\sqrt[4]{613}\left(6\left(\frac{1}{x}-\frac{1}{6}\right)+1\right)^2\sqrt{\frac{1296\left(\frac{1}{x}-\frac{1}{6}\right)^4-6552\left(\frac{1}{x}-\frac{1}{6}\right)^2+613}{\left(6\left(\frac{1}{x}-\frac{1}{6}\right)+1\right)^4}}}$$

input

```
Int[1/Sqrt[9 - 6*x - 44*x^2 + 15*x^3 + 3*x^4],x]
```

output

```
-(((Sqrt[613] + 36*(-1/6 + x^(-1))^2)*Sqrt[(613 - 6552*(-1/6 + x^(-1))^2 +
 1296*(-1/6 + x^(-1))^4)/(Sqrt[613] + 36*(-1/6 + x^(-1))^2)^2]*EllipticF[2
*ArcTan[(6*(-1/6 + x^(-1)))/613^(1/4)], (613 + 91*Sqrt[613])/1226])/(613^(
1/4)*(1 + 6*(-1/6 + x^(-1)))^2*Sqrt[(613 - 6552*(-1/6 + x^(-1))^2 + 1296*(
-1/6 + x^(-1))^4)/(1 + 6*(-1/6 + x^(-1)))^4]))
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1409
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && GtQ[c/a, 0] && LtQ[
b/a, 0]
```

rule 2504
```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Simp[-16*
a^2   Subst[Int[(1/(b - 4*a*x)^2)*(a*((-3*b^4 + 16*a*b^2*c - 64*a^2*b*d + 2
56*a^3*e - 32*a^2*(3*b^2 - 8*a*c)*x^2 + 256*a^4*x^4)/(b - 4*a*x)^4))^p, x],
 x, b/(4*a) + 1/x], x] /; NeQ[a, 0] && NeQ[b, 0] && EqQ[b^3 - 4*a*b*c + 8*a
^2*d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && IntegerQ[2*p] &&  !IGtQ[p, 0
]
```

rule 7270
```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Simp[a^IntPart[p
]*((a*v^m*w^n)^FracPart[p]/(v^(m*FracPart[p])*w^(n*FracPart[p])))   Int[u*v
^(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !Free
Q[v, x] &&  !FreeQ[w, x]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 1179 vs. $2(111) = 222$.

Time = 0.89 (sec) , antiderivative size = 1180, normalized size of antiderivative = 9.37

method	result	size
default	Expression too large to display	1180
elliptic	Expression too large to display	1180

input
```
int(1/(3*x^4+15*x^3-44*x^2-6*x+9)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
2/3*(RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=1)-RootOf(3*_Z^4+15*_Z^3-4
4*_Z^2-6*_Z+9,index=4))*((RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=4)-Ro
otOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=2))*(x-RootOf(3*_Z^4+15*_Z^3-44*_
Z^2-6*_Z+9,index=1))/(RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=4)-RootOf
(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=1))/(x-RootOf(3*_Z^4+15*_Z^3-44*_Z^2-
6*_Z+9,index=2)))^(1/2)*(x-RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=2))^
2*((RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=2)-RootOf(3*_Z^4+15*_Z^3-44
*_Z^2-6*_Z+9,index=1))*(x-RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=3))/(
RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=3)-RootOf(3*_Z^4+15*_Z^3-44*_Z^
2-6*_Z+9,index=1))/(x-RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=2)))^(1/2
)*((RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=2)-RootOf(3*_Z^4+15*_Z^3-44
*_Z^2-6*_Z+9,index=1))*(x-RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=4))/(
RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=4)-RootOf(3*_Z^4+15*_Z^3-44*_Z^
2-6*_Z+9,index=1))/(x-RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=2)))^(1/2
)/(RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=4)-RootOf(3*_Z^4+15*_Z^3-44*
_Z^2-6*_Z+9,index=2))/(RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=2)-RootO
f(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=1))*3^(1/2)/((x-RootOf(3*_Z^4+15*_Z^
3-44*_Z^2-6*_Z+9,index=1))*(x-RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=2
))*(x-RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=3))*(x-RootOf(3*_Z^4+15*_
Z^3-44*_Z^2-6*_Z+9,index=4)))^(1/2)*EllipticF(((RootOf(3*_Z^4+15*_Z^3-4...
```

Fricas [F]

$$\int \frac{1}{\sqrt{9-6x-44x^2+15x^3+3x^4}}\,dx = \int \frac{1}{\sqrt{3\,x^4+15\,x^3-44\,x^2-6\,x+9}}\,dx$$

input

```
integrate(1/(3*x^4+15*x^3-44*x^2-6*x+9)^(1/2),x, algorithm="fricas")
```

output

```
integral(1/sqrt(3*x^4 + 15*x^3 - 44*x^2 - 6*x + 9), x)
```

Sympy [F]

$$\int \frac{1}{\sqrt{9-6x-44x^2+15x^3+3x^4}}\,dx=\int \frac{1}{\sqrt{3x^4+15x^3-44x^2-6x+9}}\,dx$$

input
```
integrate(1/(3*x**4+15*x**3-44*x**2-6*x+9)**(1/2),x)
```

output
```
Integral(1/sqrt(3*x**4 + 15*x**3 - 44*x**2 - 6*x + 9), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{9-6x-44x^2+15x^3+3x^4}}\,dx=\int \frac{1}{\sqrt{3\,x^4+15\,x^3-44\,x^2-6\,x+9}}\,dx$$

input
```
integrate(1/(3*x^4+15*x^3-44*x^2-6*x+9)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(3*x^4 + 15*x^3 - 44*x^2 - 6*x + 9), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{9-6x-44x^2+15x^3+3x^4}}\,dx=\int \frac{1}{\sqrt{3\,x^4+15\,x^3-44\,x^2-6\,x+9}}\,dx$$

input
```
integrate(1/(3*x^4+15*x^3-44*x^2-6*x+9)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(3*x^4 + 15*x^3 - 44*x^2 - 6*x + 9), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{9-6x-44x^2+15x^3+3x^4}}\,dx = \int \frac{1}{\sqrt{3\,x^4+15\,x^3-44\,x^2-6\,x+9}}\,dx$$

input
```
int(1/(15*x^3 - 44*x^2 - 6*x + 3*x^4 + 9)^(1/2),x)
```

output
```
int(1/(15*x^3 - 44*x^2 - 6*x + 3*x^4 + 9)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{9-6x-44x^2+15x^3+3x^4}}\,dx = \int \frac{\sqrt{3x^4+15x^3-44x^2-6x+9}}{3x^4+15x^3-44x^2-6x+9}dx$$

input
```
int(1/(3*x^4+15*x^3-44*x^2-6*x+9)^(1/2),x)
```

output
```
int(sqrt(3*x**4 + 15*x**3 - 44*x**2 - 6*x + 9)/(3*x**4 + 15*x**3 - 44*x**2
 - 6*x + 9),x)
```

3.93 $\int \frac{1}{(9-6x-44x^2+15x^3+3x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 434

$$\int \frac{1}{(9-6x-44x^2+15x^3+3x^4)^{3/2}}\,dx = -\frac{\left(176-23\left(1-\frac{6}{x}\right)^2\right)x^2}{51759\sqrt{9-6x-44x^2+15x^3+3x^4}}$$
$$+\frac{\left(45401-3722\left(1-\frac{6}{x}\right)^2\right)\left(1-\frac{6}{x}\right)x^2}{31728267\sqrt{9-6x-44x^2+15x^3+3x^4}}$$
$$+\frac{3722\left(613-182\left(1-\frac{6}{x}\right)^2+\left(-1+\frac{6}{x}\right)^4\right)\left(1-\frac{6}{x}\right)x^2}{31728267\left(\sqrt{613}+\left(-1+\frac{6}{x}\right)^2\right)\sqrt{9-6x-44x^2+15x^3+3x^4}}$$
$$+\frac{3722\left(\sqrt{613}+\left(-1+\frac{6}{x}\right)^2\right)\sqrt{\frac{613-182\left(1-\frac{6}{x}\right)^2+\left(-1+\frac{6}{x}\right)^4}{\left(\sqrt{613}+\left(-1+\frac{6}{x}\right)^2\right)^2}}\,x^2 E\left(2\arctan\left(\frac{6-x}{\sqrt[4]{613}x}\right)\middle|\frac{613+91\sqrt{613}}{1226}\right)}{51759\ 613^{3/4}\sqrt{9-6x-44x^2+15x^3+3x^4}}$$
$$-\frac{\left(7444-145\sqrt{613}\right)\left(\sqrt{613}+\left(-1+\frac{6}{x}\right)^2\right)\sqrt{\frac{613-182\left(1-\frac{6}{x}\right)^2+\left(-1+\frac{6}{x}\right)^4}{\left(\sqrt{613}+\left(-1+\frac{6}{x}\right)^2\right)^2}}\,x^2\ \text{EllipticF}\left(2\arctan\left(\frac{6-x}{\sqrt[4]{613}x}\right),\frac{613+\cdots}{\cdots}\right)}{207036\ 613^{3/4}\sqrt{9-6x-44x^2+15x^3+3x^4}}$$

output
```
-1/51759*(176-23*(1-6/x)^2)*x^2/(3*x^4+15*x^3-44*x^2-6*x+9)^(1/2)+1/317282
67*(45401-3722*(1-6/x)^2)*(1-6/x)*x^2/(3*x^4+15*x^3-44*x^2-6*x+9)^(1/2)+37
22/31728267*(613-182*(1-6/x)^2+(-1+6/x)^4)*(1-6/x)*x^2/(613^(1/2)+(-1+6/x)
^2)/(3*x^4+15*x^3-44*x^2-6*x+9)^(1/2)+3722/31728267*(613^(1/2)+(-1+6/x)^2)
*((613-182*(1-6/x)^2+(-1+6/x)^4)/(613^(1/2)+(-1+6/x)^2)^2)^(1/2)*x^2*Ellip
ticE(sin(2*arctan(1/613*(6-x)*613^(3/4)/x)),1/1226*(751538+111566*613^(1/2
))^(1/2))*613^(1/4)/(3*x^4+15*x^3-44*x^2-6*x+9)^(1/2)-1/126913068*(7444-14
5*613^(1/2))*(613^(1/2)+(-1+6/x)^2)*((613-182*(1-6/x)^2+(-1+6/x)^4)/(613^(
1/2)+(-1+6/x)^2)^2)^(1/2)*x^2*InverseJacobiAM(2*arctan(1/613*(6-x)*613^(3/
4)/x),1/1226*(751538+111566*613^(1/2))^(1/2))*613^(1/4)/(3*x^4+15*x^3-44*x
^2-6*x+9)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 16.16 (sec) , antiderivative size = 5428, normalized size of antiderivative = 12.51

$$\int \frac{1}{(9-6x-44x^2+15x^3+3x^4)^{3/2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(9 - 6*x - 44*x^2 + 15*x^3 + 3*x^4)^(-3/2),x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 1.26 (sec) , antiderivative size = 526, normalized size of antiderivative = 1.21, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.542$, Rules used = {2504, 27, 7270, 2202, 1576, 27, 1158, 2206, 27, 1497, 27, 1409, 1496}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(3x^4+15x^3-44x^2-6x+9)^{3/2}}\,dx$$

$$\Big\downarrow \text{2504}$$

$$-1296\int \frac{1}{972\left(6\left(\frac{1}{x}-\frac{1}{6}\right)+1\right)^2\left(\frac{1296\left(\frac{1}{x}-\frac{1}{6}\right)^4-6552\left(\frac{1}{x}-\frac{1}{6}\right)^2+613}{\left(6\left(\frac{1}{x}-\frac{1}{6}\right)+1\right)^4}\right)^{3/2}}d\left(\frac{1}{x}-\frac{1}{6}\right)$$

$$\Big\downarrow \text{27}$$

$$-\frac{4}{3}\int \frac{1}{\left(6\left(\frac{1}{x}-\frac{1}{6}\right)+1\right)^2\left(\frac{1296\left(\frac{1}{x}-\frac{1}{6}\right)^4-6552\left(\frac{1}{x}-\frac{1}{6}\right)^2+613}{\left(6\left(\frac{1}{x}-\frac{1}{6}\right)+1\right)^4}\right)^{3/2}}d\left(\frac{1}{x}-\frac{1}{6}\right)$$

$$\Big\downarrow \text{7270}$$

$$-\frac{4\sqrt{1296\left(\frac{1}{x}-\frac{1}{6}\right)^4-6552\left(\frac{1}{x}-\frac{1}{6}\right)^2+613}\int \frac{\left(6\left(\frac{1}{x}-\frac{1}{6}\right)+1\right)^4}{\left(1296\left(\frac{1}{x}-\frac{1}{6}\right)^4-6552\left(\frac{1}{x}-\frac{1}{6}\right)^2+613\right)^{3/2}}d\left(\frac{1}{x}-\frac{1}{6}\right)}{3\left(6\left(\frac{1}{x}-\frac{1}{6}\right)+1\right)^2\sqrt{\frac{1296\left(\frac{1}{x}-\frac{1}{6}\right)^4-6552\left(\frac{1}{x}-\frac{1}{6}\right)^2+613}{\left(6\left(\frac{1}{x}-\frac{1}{6}\right)+1\right)^4}}}$$

$$\Big\downarrow \text{2202}$$

$$-\frac{4\sqrt{1296\left(\frac{1}{x}-\frac{1}{6}\right)^4-6552\left(\frac{1}{x}-\frac{1}{6}\right)^2+613}\left(\int \frac{1296\left(\frac{1}{x}-\frac{1}{6}\right)^4+216\left(\frac{1}{x}-\frac{1}{6}\right)^2+1}{\left(1296\left(\frac{1}{x}-\frac{1}{6}\right)^4-6552\left(\frac{1}{x}-\frac{1}{6}\right)^2+613\right)^{3/2}}d\left(\frac{1}{x}-\frac{1}{6}\right)+\int \frac{\left(864\left(\frac{1}{x}-\frac{1}{6}\right)^2+24\right.}{\left(1296\left(\frac{1}{x}-\frac{1}{6}\right)^4-6552\left(\frac{1}{x}-\right.\right.}\right.}{3\left(6\left(\frac{1}{x}-\frac{1}{6}\right)+1\right)^2\sqrt{\frac{1296\left(\frac{1}{x}-\frac{1}{6}\right)^4-6552\left(\frac{1}{x}-\frac{1}{6}\right)^2+613}{\left(6\left(\frac{1}{x}-\frac{1}{6}\right)+1\right)^4}}}$$

$$\Big\downarrow \text{1576}$$

$$-\frac{4\sqrt{1296\left(\frac{1}{x}-\frac{1}{6}\right)^4-6552\left(\frac{1}{x}-\frac{1}{6}\right)^2+613}\left(\frac{1}{2}\int \frac{24\left(36\left(\frac{1}{x}-\frac{1}{6}\right)^2+1\right)}{\left(1296\left(\frac{1}{x}-\frac{1}{6}\right)^4-6552\left(\frac{1}{x}-\frac{1}{6}\right)^2+613\right)^{3/2}}d\left(\frac{1}{x}-\frac{1}{6}\right)^2+\int \frac{1296\left(\frac{1}{x}-\frac{1}{6}\right)^4+2}{\left(1296\left(\frac{1}{x}-\frac{1}{6}\right)^4-6552\right.}\right.}{3\left(6\left(\frac{1}{x}-\frac{1}{6}\right)+1\right)^2\sqrt{\frac{1296\left(\frac{1}{x}-\frac{1}{6}\right)^4-6552\left(\frac{1}{x}-\frac{1}{6}\right)^2+613}{\left(6\left(\frac{1}{x}-\frac{1}{6}\right)+1\right)^4}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{4\sqrt{1296\left(\frac{1}{x}-\frac{1}{6}\right)^4-6552\left(\frac{1}{x}-\frac{1}{6}\right)^2+613}\left(12\int \frac{36\left(\frac{1}{x}-\frac{1}{6}\right)^2+1}{\left(1296\left(\frac{1}{x}-\frac{1}{6}\right)^4-6552\left(\frac{1}{x}-\frac{1}{6}\right)^2+613\right)^{3/2}}d\left(\frac{1}{x}-\frac{1}{6}\right)^2+\int \frac{1296\left(\frac{1}{x}-\frac{1}{6}\right)^4+}{\left(1296\left(\frac{1}{x}-\frac{1}{6}\right)^4-655\right.}\right.}{3\left(6\left(\frac{1}{x}-\frac{1}{6}\right)+1\right)^2\sqrt{\frac{1296\left(\frac{1}{x}-\frac{1}{6}\right)^4-6552\left(\frac{1}{x}-\frac{1}{6}\right)^2+613}{\left(6\left(\frac{1}{x}-\frac{1}{6}\right)+1\right)^4}}}$$

$$\Big\downarrow \text{1158}$$

$$-\frac{4\sqrt{1296\left(\frac{1}{x}-\frac{1}{6}\right)^4-6552\left(\frac{1}{x}-\frac{1}{6}\right)^2+613}\left(\int \frac{1296\left(\frac{1}{x}-\frac{1}{6}\right)^4+216\left(\frac{1}{x}-\frac{1}{6}\right)^2+1}{\left(1296\left(\frac{1}{x}-\frac{1}{6}\right)^4-6552\left(\frac{1}{x}-\frac{1}{6}\right)^2+613\right)^{3/2}}d\left(\frac{1}{x}-\frac{1}{6}\right)+\frac{4\left(44-207\left(\frac{1}{x}-\frac{1}{6}\right)\right.}{5751\sqrt{1296\left(\frac{1}{x}-\frac{1}{6}\right)^4-6552}}\right.}{3\left(6\left(\frac{1}{x}-\frac{1}{6}\right)+1\right)^2\sqrt{\frac{1296\left(\frac{1}{x}-\frac{1}{6}\right)^4-6552\left(\frac{1}{x}-\frac{1}{6}\right)^2+613}{\left(6\left(\frac{1}{x}-\frac{1}{6}\right)+1\right)^4}}}$$

$\downarrow$ 2206

$$-\frac{4\sqrt{1296\left(\frac{1}{x}-\frac{1}{6}\right)^4-6552\left(\frac{1}{x}-\frac{1}{6}\right)^2+613}\left(-\frac{\int \frac{20736\left(88885-267984\left(\frac{1}{x}-\frac{1}{6}\right)^2\right)}{\sqrt{1296\left(\frac{1}{x}-\frac{1}{6}\right)^4-6552\left(\frac{1}{x}-\frac{1}{6}\right)^2+613}}d\left(\frac{1}{x}-\frac{1}{6}\right)}{24367309056}+\frac{4\left(44-207\left(\frac{1}{x}-\frac{1}{6}\right)^2\right)}{5751\sqrt{1296\left(\frac{1}{x}-\frac{1}{6}\right)^4-6552\left(\frac{1}{x}-\frac{1}{6}\right)}}\right.}{3\left(6\left(\frac{1}{x}-\frac{1}{6}\right)+1\right)^2\sqrt{\frac{1296\left(\frac{1}{x}-\frac{1}{6}\right)^4-6552\left(\frac{1}{x}-\frac{1}{6}\right)^2+613}{\left(6\left(\frac{1}{x}-\frac{1}{6}\right)+1\right)^4}}}$$

$\downarrow$ 27

$$-\frac{4\sqrt{1296\left(\frac{1}{x}-\frac{1}{6}\right)^4-6552\left(\frac{1}{x}-\frac{1}{6}\right)^2+613}\left(-\frac{\int \frac{88885-267984\left(\frac{1}{x}-\frac{1}{6}\right)^2}{\sqrt{1296\left(\frac{1}{x}-\frac{1}{6}\right)^4-6552\left(\frac{1}{x}-\frac{1}{6}\right)^2+613}}d\left(\frac{1}{x}-\frac{1}{6}\right)}{1175121}+\frac{4\left(44-207\left(\frac{1}{x}-\frac{1}{6}\right)^2\right)}{5751\sqrt{1296\left(\frac{1}{x}-\frac{1}{6}\right)^4-6552\left(\frac{1}{x}-\frac{1}{6}\right)}}\right.}{3\left(6\left(\frac{1}{x}-\frac{1}{6}\right)+1\right)^2\sqrt{\frac{1296\left(\frac{1}{x}-\frac{1}{6}\right)^4-6552\left(\frac{1}{x}-\frac{1}{6}\right)^2+613}{\left(6\left(\frac{1}{x}-\frac{1}{6}\right)+1\right)^4}}}$$

$\downarrow$ 1497

$$-\frac{4\sqrt{1296\left(\frac{1}{x}-\frac{1}{6}\right)^4-6552\left(\frac{1}{x}-\frac{1}{6}\right)^2+613}\left(\frac{-\left(\left(88885-7444\sqrt{613}\right)\int \frac{1}{\sqrt{1296\left(\frac{1}{x}-\frac{1}{6}\right)^4-6552\left(\frac{1}{x}-\frac{1}{6}\right)^2+613}}d\left(\frac{1}{x}-\frac{1}{6}\right)\right)-7444\sqrt{613}\int}{1175121}\right.}{3\left(6\left(\frac{1}{x}-\frac{1}{6}\right)+1\right)^2\sqrt{12}}$$

$\downarrow$ 27

$$-\frac{4\sqrt{1296\left(\frac{1}{x}-\frac{1}{6}\right)^4-6552\left(\frac{1}{x}-\frac{1}{6}\right)^2+613}\left(\frac{-\left(\left(88885-7444\sqrt{613}\right)\int \frac{1}{\sqrt{1296\left(\frac{1}{x}-\frac{1}{6}\right)^4-6552\left(\frac{1}{x}-\frac{1}{6}\right)^2+613}}d\left(\frac{1}{x}-\frac{1}{6}\right)\right)-7444\int \frac{}{\sqrt{129}}}{1175121}\right.}{3\left(6\left(\frac{1}{x}-\frac{1}{6}\right)+1\right)^2\sqrt{\frac{1296(\frac{1}{x}}{}}}$$

$\Big\downarrow$ 1409

$$-\frac{4\sqrt{1296\left(\frac{1}{x}-\frac{1}{6}\right)^4-6552\left(\frac{1}{x}-\frac{1}{6}\right)^2+613}\left(\frac{-7444\int\frac{\sqrt{613}-36\left(\frac{1}{x}-\frac{1}{6}\right)^2}{\sqrt{1296\left(\frac{1}{x}-\frac{1}{6}\right)^4-6552\left(\frac{1}{x}-\frac{1}{6}\right)^2+613}}d\left(\frac{1}{x}-\frac{1}{6}\right)-\frac{\left(88885-7444\sqrt{613}\right)\left(36\left(\frac{1}{x}-\frac{1}{6}\right)^2+\sqrt{\ldots}\right.}{\ldots}}{11751\ldots}\right.}{3\left(6\left(\frac{1}{x}-\ldots\right.\right.}$$

$\Big\downarrow$ 1496

$$-\frac{4\sqrt{1296\left(\frac{1}{x}-\frac{1}{6}\right)^4-6552\left(\frac{1}{x}-\frac{1}{6}\right)^2+613}\left(-\frac{\left(88885-7444\sqrt{613}\right)\left(36\left(\frac{1}{x}-\frac{1}{6}\right)^2+\sqrt{613}\right)\sqrt{\frac{1296\left(\frac{1}{x}-\frac{1}{6}\right)^4-6552\left(\frac{1}{x}-\frac{1}{6}\right)^2+613}{\left(36\left(\frac{1}{x}-\frac{1}{6}\right)^2+\sqrt{613}\right)^2}}\text{EllipticF}\left(2\,\text{a}\ldots\right.}{12\sqrt[4]{613}\sqrt{1296\left(\frac{1}{x}-\frac{1}{6}\right)^4-6552\left(\frac{1}{x}-\frac{1}{6}\right)^2+613}}\ldots\right.}{\ldots}$$

input

```
Int[(9 - 6*x - 44*x^2 + 15*x^3 + 3*x^4)^(-3/2),x]
```

output

```
(-4*Sqrt[613 - 6552*(-1/6 + x^(-1))^2 + 1296*(-1/6 + x^(-1))^4]*((4*(44 -
207*(-1/6 + x^(-1))^2))/(5751*Sqrt[613 - 6552*(-1/6 + x^(-1))^2 + 1296*(-1
/6 + x^(-1))^4]) + (2*(45401 - 133992*(-1/6 + x^(-1))^2)*(-1/6 + x^(-1)))/
(1175121*Sqrt[613 - 6552*(-1/6 + x^(-1))^2 + 1296*(-1/6 + x^(-1))^4]) + (-
7444*((-613*Sqrt[613 - 6552*(-1/6 + x^(-1))^2 + 1296*(-1/6 + x^(-1))^4]*(-
1/6 + x^(-1)))/(613*Sqrt[613] + 22068*(-1/6 + x^(-1))^2) + (613^(1/4)*(Sqr
t[613] + 36*(-1/6 + x^(-1))^2)*Sqrt[(613 - 6552*(-1/6 + x^(-1))^2 + 1296*(
-1/6 + x^(-1))^4)/(Sqrt[613] + 36*(-1/6 + x^(-1))^2)^2]*EllipticE[2*ArcTan
[(6*(-1/6 + x^(-1)))/613^(1/4)], (613 + 91*Sqrt[613])/1226])/(6*Sqrt[613 -
 6552*(-1/6 + x^(-1))^2 + 1296*(-1/6 + x^(-1))^4])) - ((88885 - 7444*Sqrt[
613])*(Sqrt[613] + 36*(-1/6 + x^(-1))^2)*Sqrt[(613 - 6552*(-1/6 + x^(-1))^
2 + 1296*(-1/6 + x^(-1))^4)/(Sqrt[613] + 36*(-1/6 + x^(-1))^2)^2]*Elliptic
F[2*ArcTan[(6*(-1/6 + x^(-1)))/613^(1/4)], (613 + 91*Sqrt[613])/1226])/(12
*613^(1/4)*Sqrt[613 - 6552*(-1/6 + x^(-1))^2 + 1296*(-1/6 + x^(-1))^4]))/1
175121))/(3*(1 + 6*(-1/6 + x^(-1)))^2*Sqrt[(613 - 6552*(-1/6 + x^(-1))^2 +
 1296*(-1/6 + x^(-1))^4)/(1 + 6*(-1/6 + x^(-1)))^4])
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1158

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[-2*((b*d - 2*a*e + (2*c*d - b*e)*x)/((b^2 - 4*a*c)*Sqrt[a + b*x
+ c*x^2])), x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1409

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && GtQ[c/a, 0] && LtQ[
b/a, 0]
```

rule 1496
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2
- 4*a*c, 0] && GtQ[c/a, 0] && LtQ[b/a, 0]
```

rule 1497
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] && GtQ
[c/a, 0] && LtQ[b/a, 0]
```

rule 1576
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x]
, x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

rule 2202
```
Int[(Pn_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{n
 = Expon[Pn, x], k}, Int[Sum[Coeff[Pn, x, 2*k]*x^(2*k), {k, 0, n/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pn, x, 2*k + 1]*x^(2*k), {k, 0, (n -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pn, x]
 &&  !PolyQ[Pn, x^2]
```

rule 2206
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c
*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Px,
a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*
p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x
^2] && Expon[Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 2504
```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Simp[-16*
a^2   Subst[Int[(1/(b - 4*a*x)^2)*(a*((-3*b^4 + 16*a*b^2*c - 64*a^2*b*d + 2
56*a^3*e - 32*a^2*(3*b^2 - 8*a*c)*x^2 + 256*a^4*x^4)/(b - 4*a*x)^4))^p, x],
 x, b/(4*a) + 1/x], x] /; NeQ[a, 0] && NeQ[b, 0] && EqQ[b^3 - 4*a*b*c + 8*a
^2*d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && IntegerQ[2*p] &&  !IGtQ[p, 0
]
```

rule 7270
```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Simp[a^IntPart[p
]*((a*v^m*w^n)^FracPart[p]/(v^(m*FracPart[p])*w^(n*FracPart[p])))   Int[u*v
^(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !Free
Q[v, x] &&  !FreeQ[w, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 5420 vs. $2(389) = 778$.

Time = 1.39 (sec) , antiderivative size = 5421, normalized size of antiderivative = 12.49

method	result	size
default	Expression too large to display	5421
risch	Expression too large to display	5421
elliptic	Expression too large to display	5421

input
```
int(1/(3*x^4+15*x^3-44*x^2-6*x+9)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [F]

$$\int \frac{1}{(9-6x-44x^2+15x^3+3x^4)^{3/2}}\,dx = \int \frac{1}{(3\,x^4+15\,x^3-44\,x^2-6\,x+9)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(3*x^4+15*x^3-44*x^2-6*x+9)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(3*x^4 + 15*x^3 - 44*x^2 - 6*x + 9)/(9*x^8 + 90*x^7 - 39*x^6
- 1356*x^5 + 1810*x^4 + 798*x^3 - 756*x^2 - 108*x + 81), x)
```

Sympy [F]

$$\int \frac{1}{(9-6x-44x^2+15x^3+3x^4)^{3/2}}\,dx = \int \frac{1}{(3x^4+15x^3-44x^2-6x+9)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(3*x**4+15*x**3-44*x**2-6*x+9)**(3/2),x)
```

output
```
Integral((3*x**4 + 15*x**3 - 44*x**2 - 6*x + 9)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(9-6x-44x^2+15x^3+3x^4)^{3/2}}\,dx = \int \frac{1}{(3\,x^4+15\,x^3-44\,x^2-6\,x+9)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(3*x^4+15*x^3-44*x^2-6*x+9)^(3/2),x, algorithm="maxima")
```

output
```
integrate((3*x^4 + 15*x^3 - 44*x^2 - 6*x + 9)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(9-6x-44x^2+15x^3+3x^4)^{3/2}}\,dx = \int \frac{1}{(3\,x^4+15\,x^3-44\,x^2-6\,x+9)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(3*x^4+15*x^3-44*x^2-6*x+9)^(3/2),x, algorithm="giac")
```

output
```
integrate((3*x^4 + 15*x^3 - 44*x^2 - 6*x + 9)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(9-6x-44x^2+15x^3+3x^4)^{3/2}}\,dx = \int \frac{1}{(3\,x^4+15\,x^3-44\,x^2-6\,x+9)^{3/2}}\,dx$$

input
```
int(1/(15*x^3 - 44*x^2 - 6*x + 3*x^4 + 9)^(3/2),x)
```

output
```
int(1/(15*x^3 - 44*x^2 - 6*x + 3*x^4 + 9)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(9-6x-44x^2+15x^3+3x^4)^{3/2}}\,dx = \int \frac{\sqrt{3x^4+15x^3-44x^2-6x+9}}{9x^8+90x^7-39x^6-1356x^5+1810x^4+798x^3-756x^2-10$$

input
```
int(1/(3*x^4+15*x^3-44*x^2-6*x+9)^(3/2),x)
```

output
```
int(sqrt(3*x**4 + 15*x**3 - 44*x**2 - 6*x + 9)/(9*x**8 + 90*x**7 - 39*x**6
 - 1356*x**5 + 1810*x**4 + 798*x**3 - 756*x**2 - 108*x + 81),x)
```

3.94 $\int \frac{1}{81-54x+24x^3-16x^4}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 60

$$\int \frac{1}{81-54x+24x^3-16x^4}\,dx = -\frac{\arctan\left(\frac{3-4x}{3\sqrt{3}}\right)}{54\sqrt{3}} - \frac{1}{108}\log(3-2x) + \frac{1}{324}\log(3+2x) + \frac{1}{324}\log\left(9-6x+4x^2\right)$$

output
```
-1/162*arctan(1/9*(3-4*x)*3^(1/2))*3^(1/2)-1/108*ln(3-2*x)+1/324*ln(3+2*x)
+1/324*ln(4*x^2-6*x+9)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.87

$$\int \frac{1}{81-54x+24x^3-16x^4}\,dx = \frac{1}{324}\left(2\sqrt{3}\arctan\left(\frac{-3+4x}{3\sqrt{3}}\right) - 3\log(3-2x) + \log(3+2x) + \log\left(9-6x+4x^2\right)\right)$$

input
```
Integrate[(81 - 54*x + 24*x^3 - 16*x^4)^(-1),x]
```

output
```
(2*Sqrt[3]*ArcTan[(-3 + 4*x)/(3*Sqrt[3])] - 3*Log[3 - 2*x] + Log[3 + 2*x]
+ Log[9 - 6*x + 4*x^2])/324
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2462, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{-16x^4+24x^3-54x+81}dx$$

$$\Big\downarrow \text{2462}$$

$$\int \left(\frac{2x+3}{81\left(4x^2-6x+9\right)}-\frac{1}{54(2x-3)}+\frac{1}{162(2x+3)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\arctan\left(\frac{3-4x}{3\sqrt{3}}\right)}{54\sqrt{3}}+\frac{1}{324}\log\left(4x^2-6x+9\right)-\frac{1}{108}\log(3-2x)+\frac{1}{324}\log(2x+3)$$

input
```
Int[(81 - 54*x + 24*x^3 - 16*x^4)^(-1),x]
```

output
```
-1/54*ArcTan[(3 - 4*x)/(3*Sqrt[3])]/Sqrt[3] - Log[3 - 2*x]/108 + Log[3 + 2
*x]/324 + Log[9 - 6*x + 4*x^2]/324
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.78

method	result	size
default	$\frac{\ln(2x+3)}{324}+\frac{\ln(4x^2-6x+9)}{324}+\frac{\sqrt{3}\,\arctan\left(\frac{(8x-6)\sqrt{3}}{18}\right)}{162}-\frac{\ln(2x-3)}{108}$	47
risch	$\frac{\ln(16x^2-24x+36)}{324}+\frac{\sqrt{3}\,\arctan\left(\frac{(4x-3)\sqrt{3}}{9}\right)}{162}+\frac{\ln(2x+3)}{324}-\frac{\ln(2x-3)}{108}$	47

input
```
int(1/(-16*x^4+24*x^3-54*x+81),x,method=_RETURNVERBOSE)
```

output
```
1/324*ln(2*x+3)+1/324*ln(4*x^2-6*x+9)+1/162*3^(1/2)*arctan(1/18*(8*x-6)*3^
(1/2))-1/108*ln(2*x-3)
```

Fricas [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.77

$$\int \frac{1}{81-54x+24x^3-16x^4}\,dx=\frac{1}{162}\,\sqrt{3}\arctan\left(\frac{1}{9}\,\sqrt{3}(4\,x-3)\right)+\frac{1}{324}\,\log\left(4\,x^2-6\,x+9\right)+\frac{1}{324}\,\log\left(2\,x+3\right)-\frac{1}{108}\,\log\left(2\,x-3\right)$$

input
```
integrate(1/(-16*x^4+24*x^3-54*x+81),x, algorithm="fricas")
```

output
```
1/162*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3)) + 1/324*log(4*x^2 - 6*x + 9) +
 1/324*log(2*x + 3) - 1/108*log(2*x - 3)
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.93

$$\int \frac{1}{81-54x+24x^3-16x^4}\,dx = -\frac{\log\left(x-\frac{3}{2}\right)}{108}+\frac{\log\left(x+\frac{3}{2}\right)}{324} + \frac{\log\left(x^2-\frac{3x}{2}+\frac{9}{4}\right)}{324}+\frac{\sqrt{3}\operatorname{atan}\left(\frac{4\sqrt{3}x}{9}-\frac{\sqrt{3}}{3}\right)}{162}$$

input
```
integrate(1/(-16*x**4+24*x**3-54*x+81),x)
```

output
```
-log(x - 3/2)/108 + log(x + 3/2)/324 + log(x**2 - 3*x/2 + 9/4)/324 + sqrt(
3)*atan(4*sqrt(3)*x/9 - sqrt(3)/3)/162
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.77

$$\int \frac{1}{81-54x+24x^3-16x^4}\,dx = \frac{1}{162}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}(4x-3)\right) + \frac{1}{324}\log\left(4x^2-6x+9\right) + \frac{1}{324}\log\left(2x+3\right)-\frac{1}{108}\log\left(2x-3\right)$$

input
```
integrate(1/(-16*x^4+24*x^3-54*x+81),x, algorithm="maxima")
```

output
```
1/162*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3)) + 1/324*log(4*x^2 - 6*x + 9) +
 1/324*log(2*x + 3) - 1/108*log(2*x - 3)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.80

$$\int \frac{1}{81-54x+24x^3-16x^4}\,dx = \frac{1}{162}\,\sqrt{3}\arctan\left(\frac{1}{9}\,\sqrt{3}(4\,x-3)\right) + \frac{1}{324}\,\log\left(4\,x^2-6\,x+9\right) + \frac{1}{324}\,\log\left(|2\,x+3|\right) - \frac{1}{108}\,\log\left(|2\,x-3|\right)$$

input
```
integrate(1/(-16*x^4+24*x^3-54*x+81),x, algorithm="giac")
```

output
```
1/162*sqrt(3)*arctan(1/9*sqrt(3)*(4*x - 3)) + 1/324*log(4*x^2 - 6*x + 9) +
 1/324*log(abs(2*x + 3)) - 1/108*log(abs(2*x - 3))
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.87

$$\int \frac{1}{81-54x+24x^3-16x^4}\,dx = \frac{\ln\left(x+\frac{3}{2}\right)}{324} - \frac{\ln\left(x-\frac{3}{2}\right)}{108} - \ln\left(x-\frac{3}{4}-\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(-\frac{1}{324}+\frac{\sqrt{3}\,1\mathrm{i}}{324}\right) + \ln\left(x-\frac{3}{4}+\frac{\sqrt{3}\,3\mathrm{i}}{4}\right)\left(\frac{1}{324}+\frac{\sqrt{3}\,1\mathrm{i}}{324}\right)$$

input
```
int(-1/(54*x - 24*x^3 + 16*x^4 - 81),x)
```

output
```
log(x + 3/2)/324 - log(x - 3/2)/108 - log(x - (3^(1/2)*3i)/4 - 3/4)*((3^(1
/2)*1i)/324 - 1/324) + log(x + (3^(1/2)*3i)/4 - 3/4)*((3^(1/2)*1i)/324 + 1
/324)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.77

$$\int \frac{1}{81-54x+24x^3-16x^4}\,dx = \frac{\sqrt{3}\,atan\left(\frac{4x-3}{3\sqrt{3}}\right)}{162} + \frac{\log(4x^2-6x+9)}{324} - \frac{\log(2x-3)}{108} + \frac{\log(2x+3)}{324}$$

input
```
int(1/(-16*x^4+24*x^3-54*x+81),x)
```

output
```
(2*sqrt(3)*atan((4*x - 3)/(3*sqrt(3))) + log(4*x**2 - 6*x + 9) - 3*log(2*x
 - 3) + log(2*x + 3))/324
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                   convert(leaf_count_result,string)," vs. $2 (",
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                  convert(leaf_count_result,string),"$ vs. $2(",
                                  convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False

   except AttributeError as error:
      return False

def expnType(expn):
   debug=False
   if debug:
      print("expn=",expn,"type(expn)=",type(expn))

   if is_atom(expn):
     return 1
   elif isinstance(expn,list):
      return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
         return 1
      else:
         return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
   elif isinstance(expn,Pow):   #type(expn,'`^`')
      if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
         return expnType(expn.args[0])   #ExpnType(op(1,expn))
      elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
         if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
         else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
      else:
         return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),Expn
   elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)  #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
else:
    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
            else: #result contains complex but optimal is not
                grade = "C"
                grade_annotation ="Result contains complex when optimal does not."
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =""
            else:
                grade = "B"
                grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
    else:
        grade = "C"
        grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType


#print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#             Albert Rich to use with Sagemath. This is used to
#             grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#             'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#             issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral',
          'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
          'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
        print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
```

```
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expr
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):



    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file