Computer Algebra
Independent Integration Tests

Summer 2024

1-Algebraic-functions/1.4-Quartic/145-1.4.2

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Contents

CHAPTER 1

INTRODUCTION

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**67**]. This is test number [145].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (67)	0.00 (0)
Mathematica	100.00 (67)	0.00 (0)
Maple	100.00 (67)	0.00 (0)
Fricas	77.61 (52)	22.39 (15)
Mupad	77.61 (52)	22.39 (15)
Sympy	71.64 (48)	28.36 (19)
Reduce	56.72 (38)	43.28 (29)
Giac	49.25 (33)	50.75 (34)
Maxima	49.25 (33)	50.75 (34)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

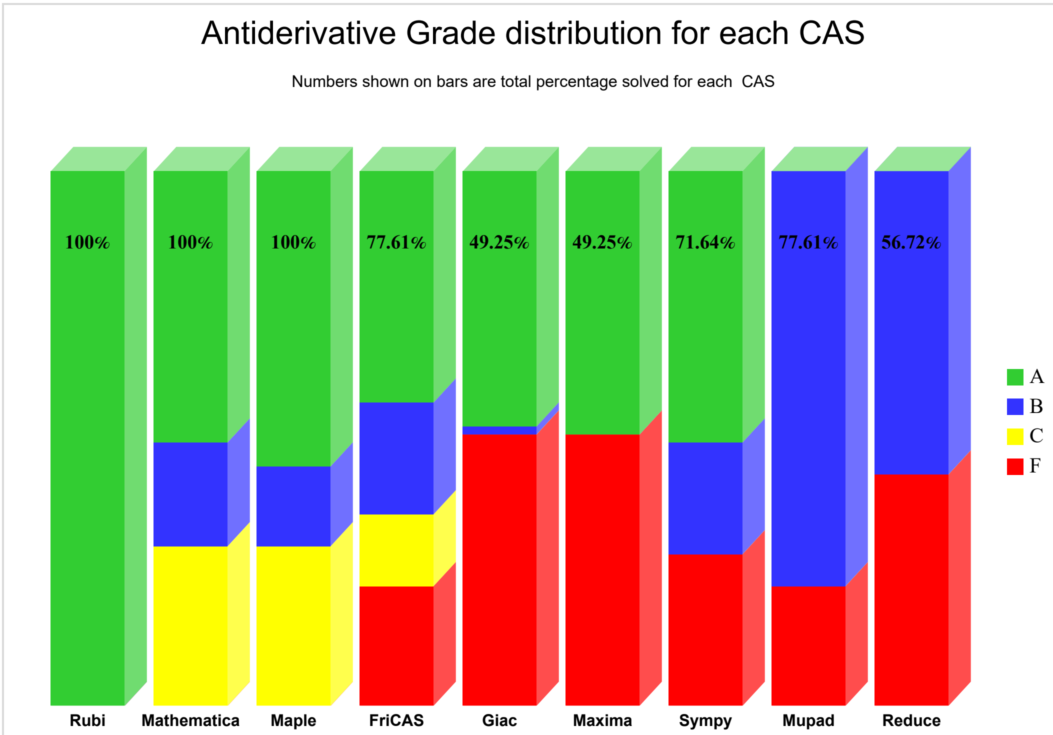
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

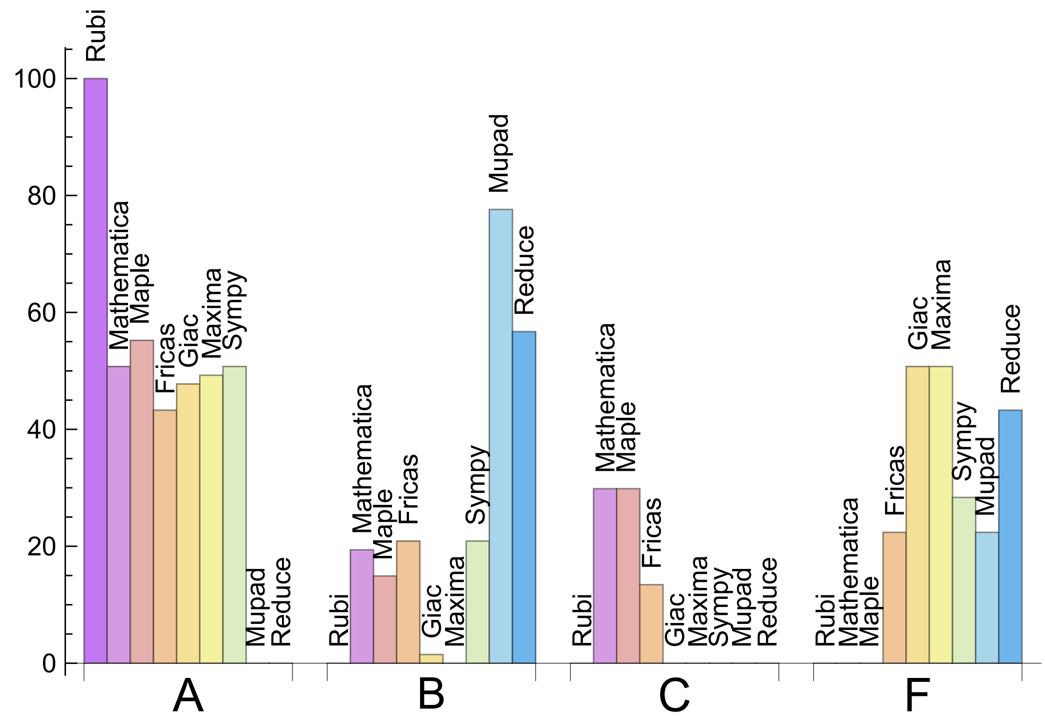
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.000	0.000	0.000	0.000
Maple	55.224	14.925	29.851	0.000
Mathematica	50.746	19.403	29.851	0.000
Sympy	50.746	20.896	0.000	28.358
Maxima	49.254	0.000	0.000	50.746
Giac	47.761	1.493	0.000	50.746
Fricas	43.284	20.896	13.433	22.388
Mupad	0.000	77.612	0.000	22.388
Reduce	0.000	56.716	0.000	43.284

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	0	0.00	0.00	0.00
Maple	0	0.00	0.00	0.00
Fricas	15	66.67	33.33	0.00
Mupad	15	0.00	100.00	0.00
Sympy	19	63.16	36.84	0.00
Reduce	29	100.00	0.00	0.00
Giac	34	85.29	5.88	8.82
Maxima	34	100.00	0.00	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.05
Reduce	0.19
Giac	0.63
Maple	0.67
Rubi	0.93
Mathematica	2.63
Fricas	4.05
Sympy	5.33
Mupad	12.50

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Maxima	53.48	0.92	38.00	0.87
Giac	62.36	0.98	58.00	0.92
Rubi	208.63	1.09	121.00	1.00
Mupad	287.31	2.02	92.00	1.06
Maple	417.69	1.43	65.00	0.83
Reduce	510.97	3.18	68.00	1.03
Sympy	742.42	4.27	78.00	1.03
Mathematica	775.21	2.26	94.00	0.90
Fricas	91158.65	469.15	83.50	1.42

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

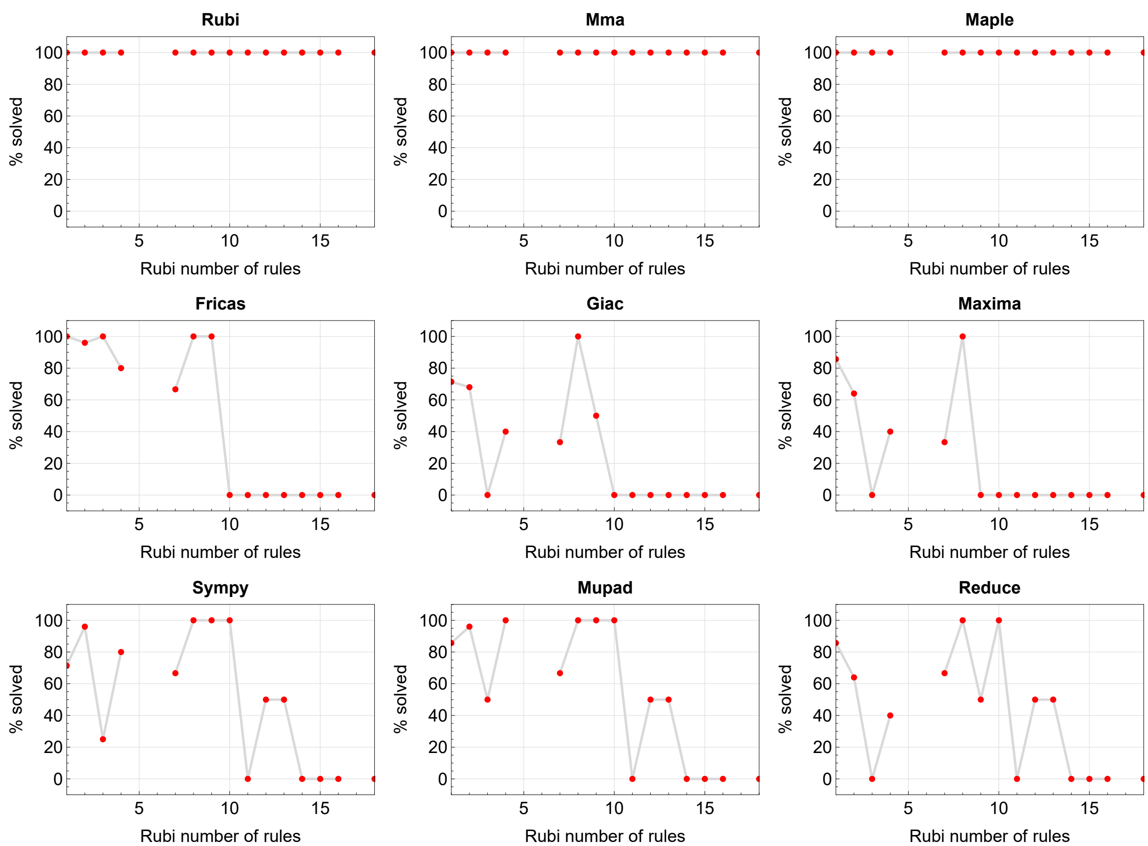


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

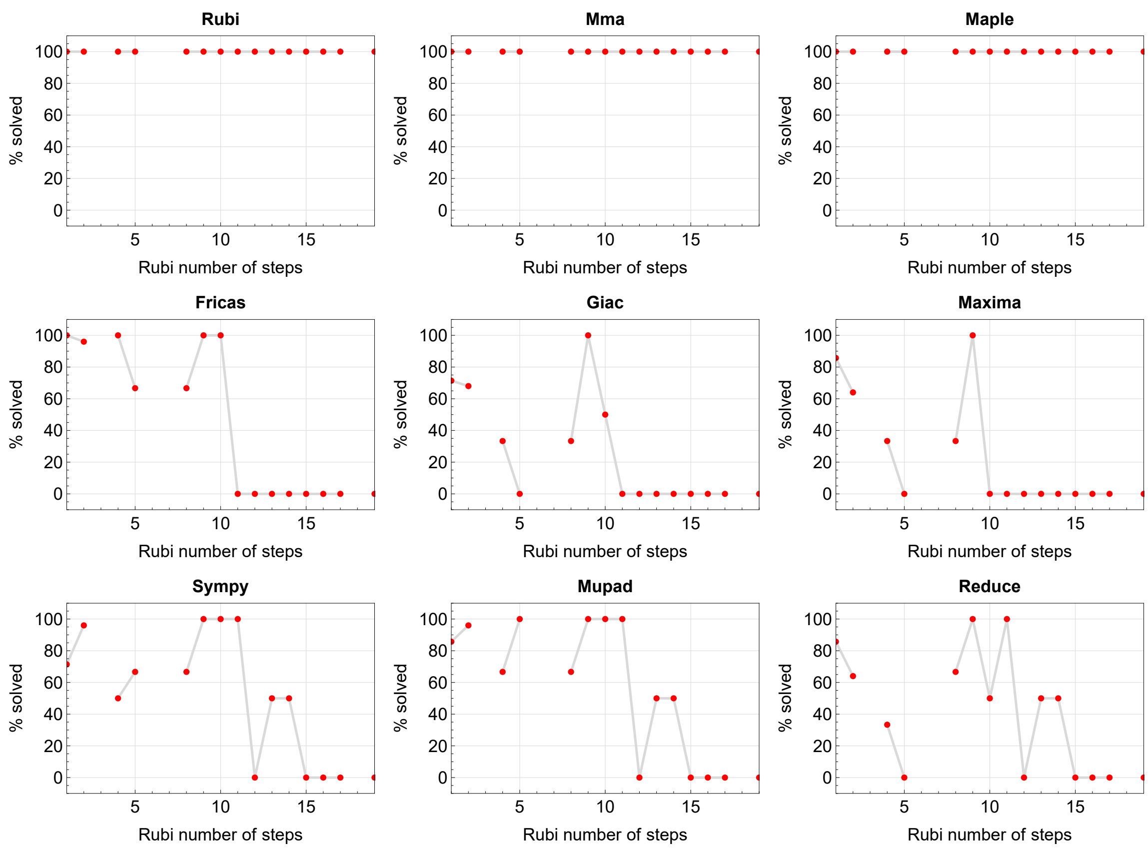


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

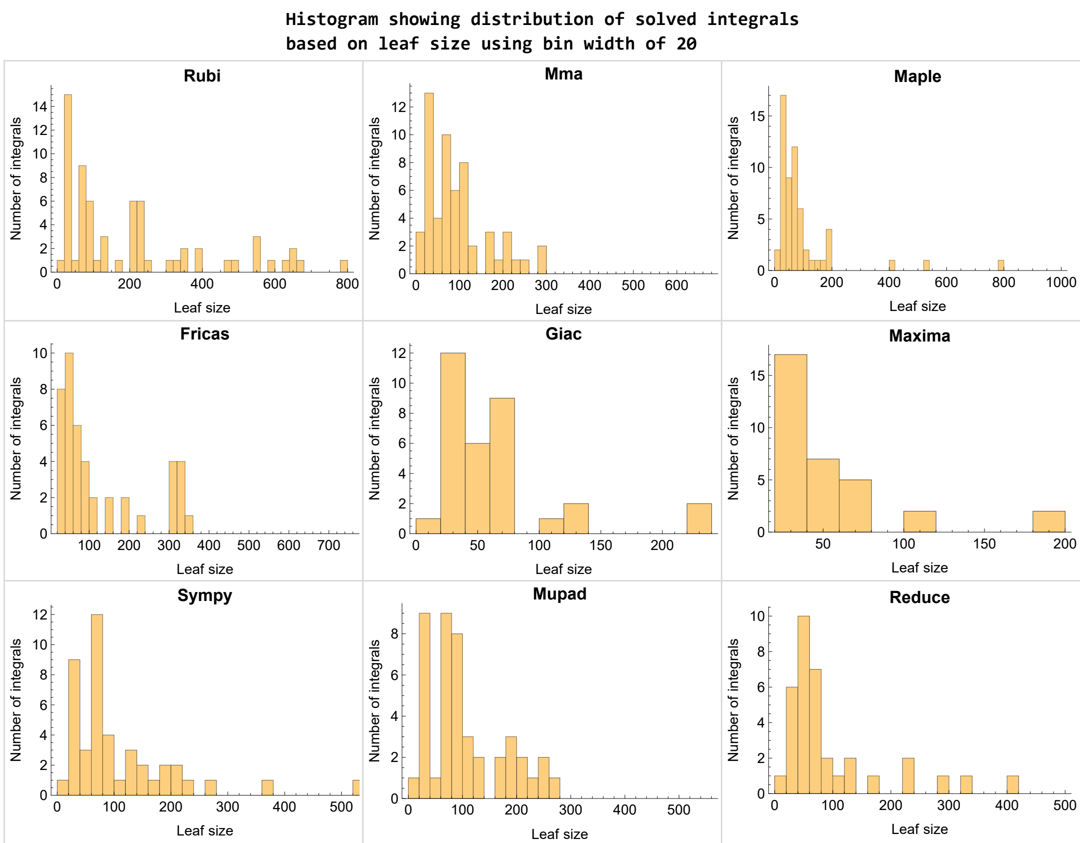


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

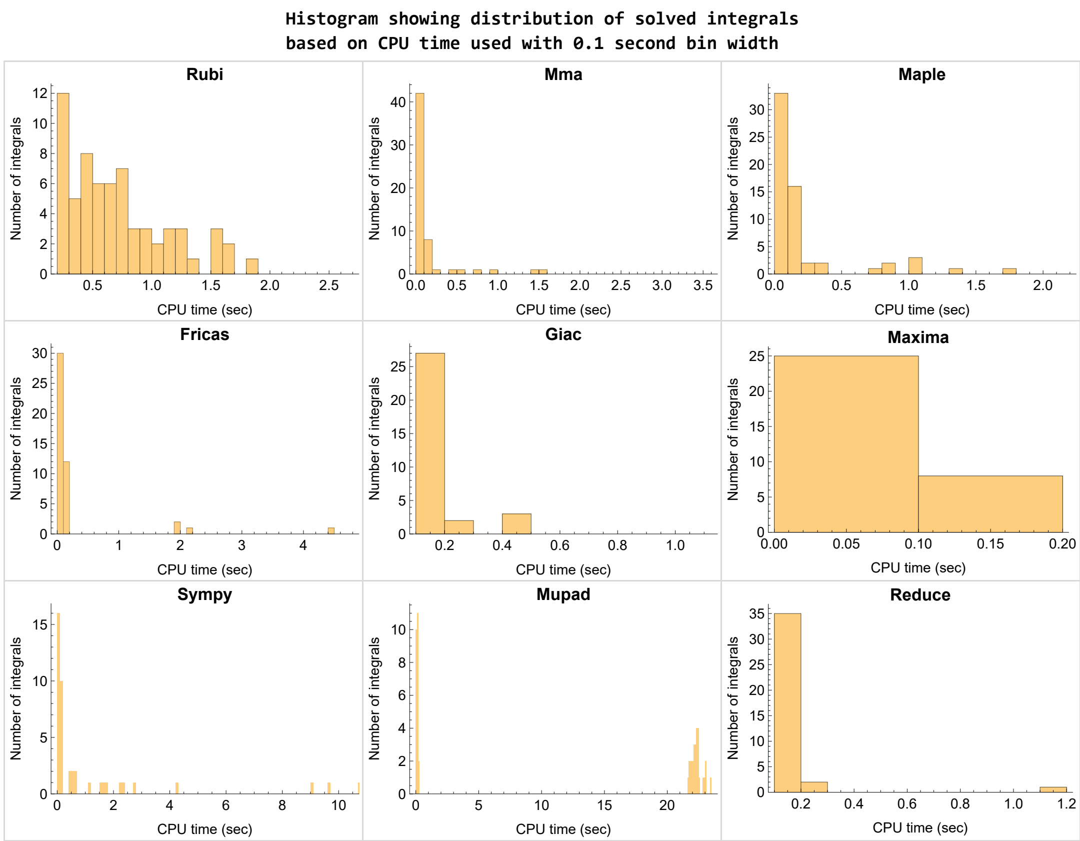


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

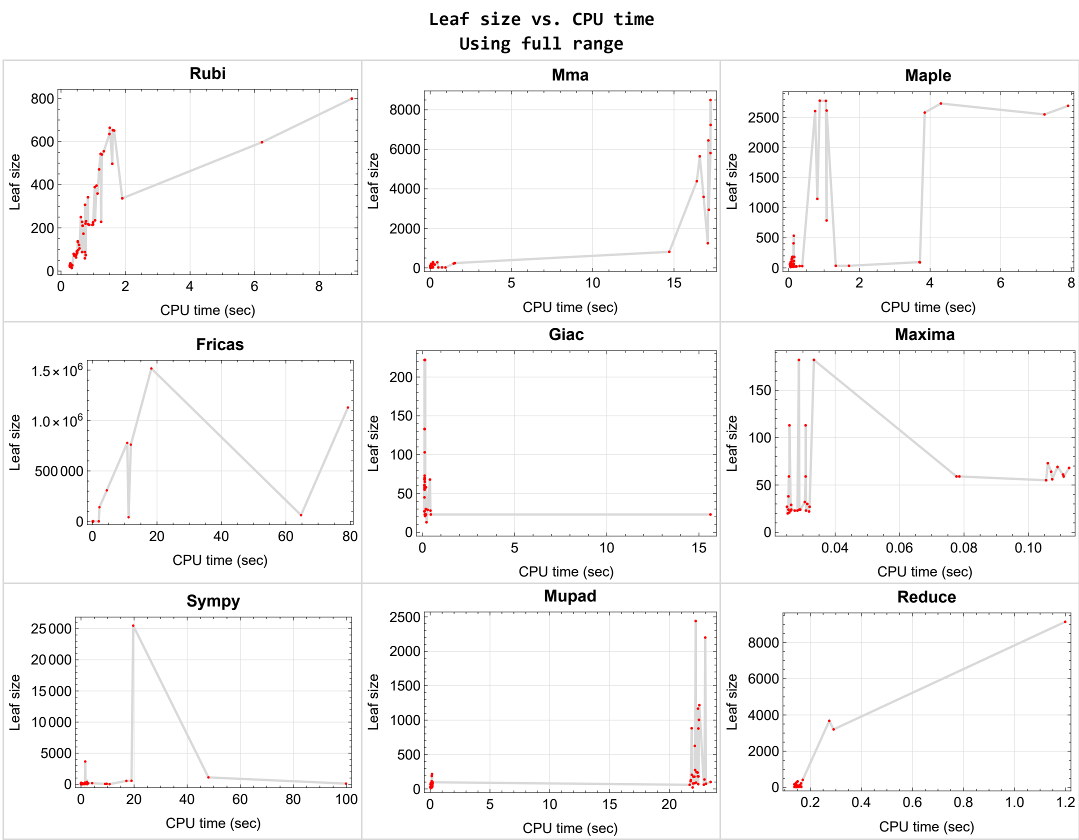


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Reduce {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi $\{25, 26, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42\}$

Mathematica $\{38, 66\}$

Maple $\{33, 34, 36, 37, 38, 39, 41, 42\}$

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

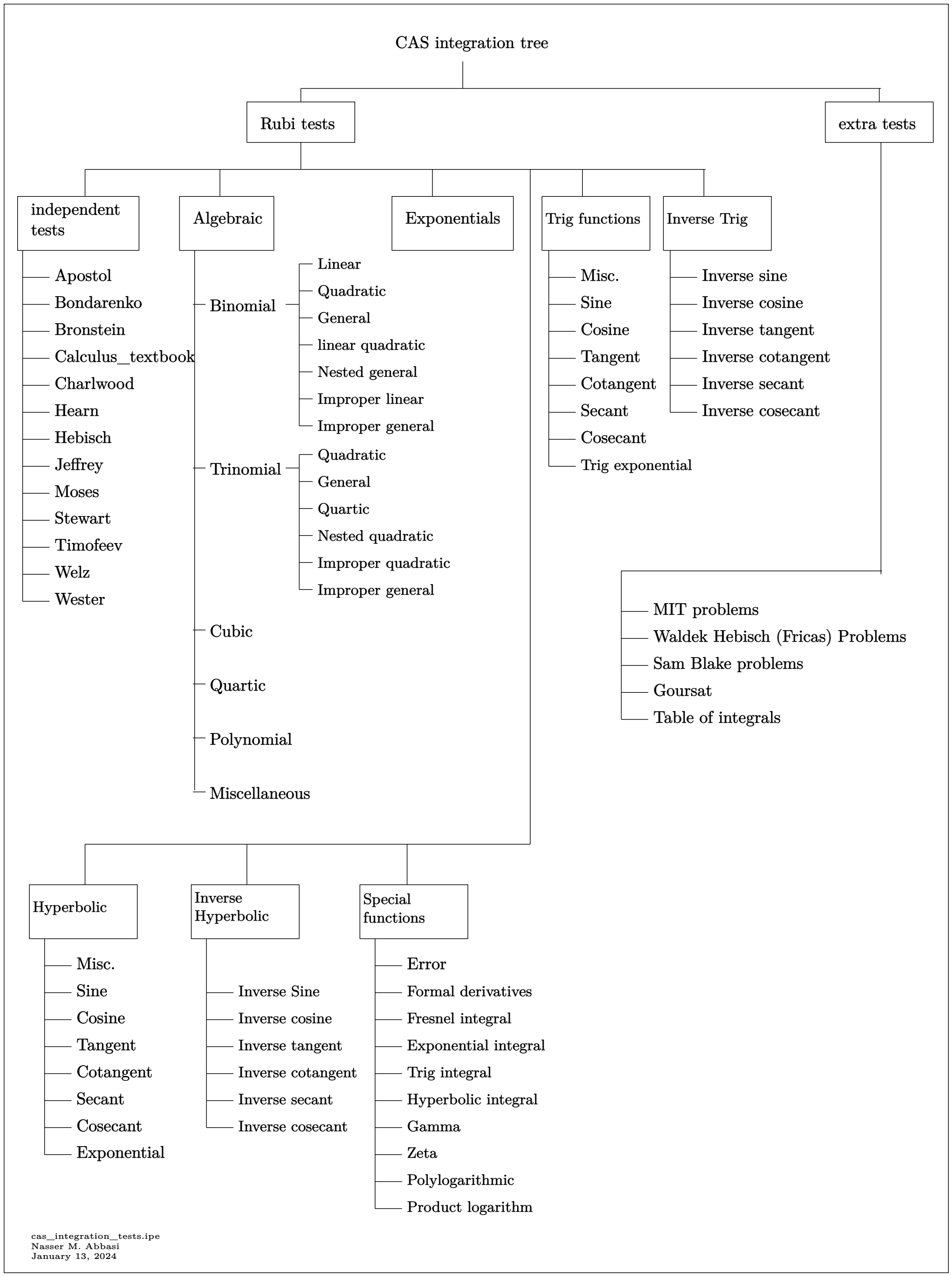
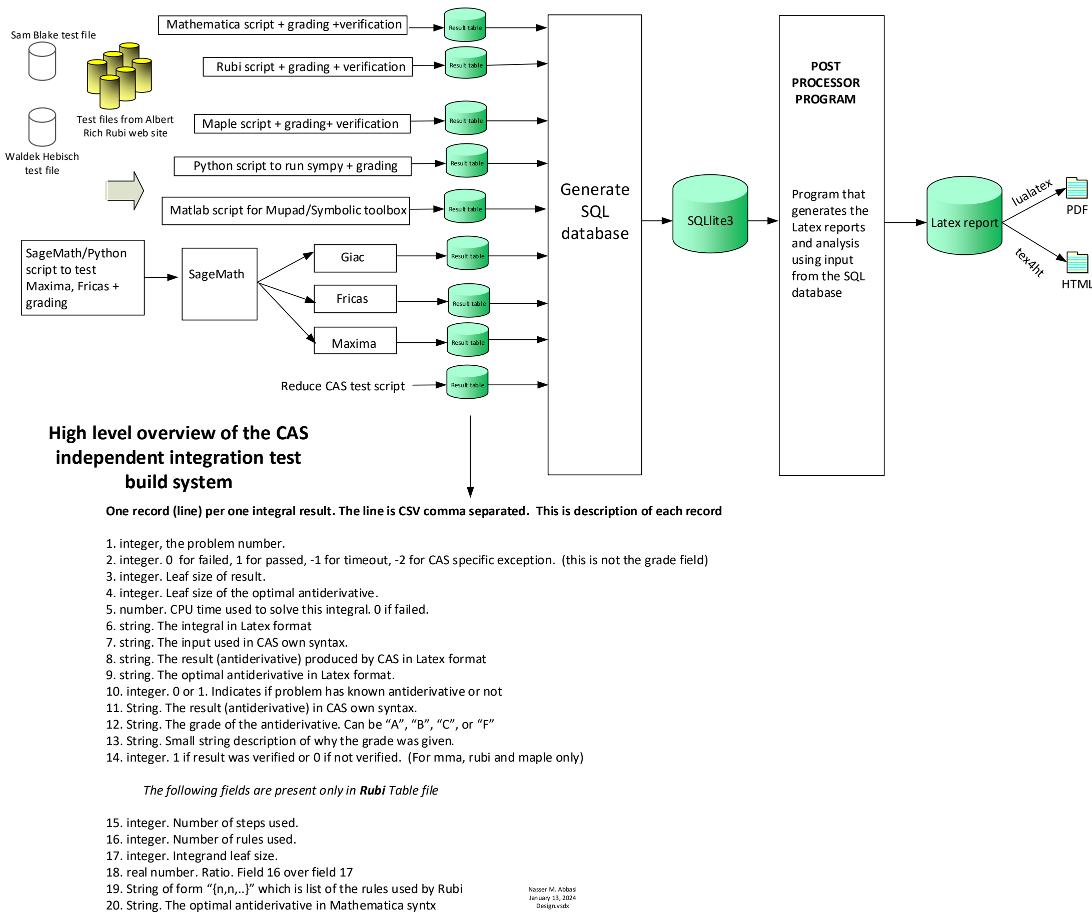


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 12, 13, 17, 18, 19, 20, 21, 22, 23, 27, 28, 29, 30, 46, 49, 50, 51, 52, 53, 54, 55, 56, 64, 65 }

B grade { 6, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 66, 67 }

C grade { 14, 15, 16, 24, 25, 26, 31, 32, 43, 44, 45, 47, 48, 57, 58, 59, 60, 61, 62, 63 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 17, 18, 19, 20, 21, 22, 23, 27, 28, 29, 30, 49, 50, 51, 52, 53, 54, 55, 56, 64, 65, 66, 67 }

B grade { 33, 34, 35, 36, 37, 38, 39, 40, 41, 42 }

C grade { 15, 16, 24, 25, 26, 31, 32, 43, 44, 45, 46, 47, 48, 57, 58, 59, 60, 61, 62, 63 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 3, 4, 8, 9, 11, 12, 13, 18, 19, 20, 21, 22, 23, 27, 28, 29, 30, 49, 50, 51, 52, 53, 54, 55, 56, 64, 65, 66, 67 }

B grade { 1, 2, 5, 6, 7, 10, 17, 57, 58, 59, 60, 61, 62, 63 }

C grade { 15, 16, 24, 31, 44, 45, 46, 47, 48 }

F normal fail { 33, 34, 35, 36, 37, 38, 39, 40, 41, 42 }

F(-1) timedout fail { 14, 25, 26, 32, 43 }

F(-2) exception fail { }

Maxima

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 17, 18, 19, 20, 21, 22, 23, 27, 28, 29, 30, 49, 50, 51, 52, 53, 54, 55, 56, 64, 65 }

B grade { }

C grade { }

F normal fail { 13, 14, 15, 16, 24, 25, 26, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 57, 58, 59, 60, 61, 62, 63, 66, 67 }

F(-1) timedout fail { }

F(-2) exception fail { }

Giac

A grade { 1, 2, 3, 4, 5, 7, 8, 11, 12, 13, 17, 18, 19, 20, 21, 22, 23, 27, 28, 29, 30, 46, 49, 50, 51, 52, 53, 54, 55, 56, 64, 65 }

B grade { 6 }

C grade { }

F normal fail { 24, 25, 26, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 47, 48, 57, 58, 59, 60, 61, 62, 63, 66, 67 }

F(-1) timedout fail { 9, 10 }

F(-2) exception fail { 14, 15, 16 }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65 }

C grade { }

F normal fail { }

F(-1) timedout fail { 14, 15, 16, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 66, 67 }

F(-2) exception fail { }

Sympy

A grade { 3, 4, 8, 9, 13, 17, 18, 19, 20, 21, 22, 23, 24, 27, 28, 29, 30, 43, 44, 45, 46, 49, 50, 51, 52, 53, 54, 55, 56, 57, 59, 60, 61, 63 }

B grade { 1, 2, 5, 6, 7, 10, 25, 26, 31, 32, 58, 62, 64, 65 }

C grade { }

F normal fail { 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 66, 67 }

F(-1) timedout fail { 11, 12, 14, 15, 16, 47, 48 }

F(-2) exception fail { }

Reduce

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 49, 50, 51, 52, 53, 54, 55, 56, 64, 65 }

C grade { }

F normal fail { 13, 14, 15, 16, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 57, 58, 59, 60, 61, 62, 63, 66, 67 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	22	23	23	143	141	23	168	137
N.S.	1	1.00	0.88	0.92	0.92	5.72	5.64	0.92	6.72	5.48
time (sec)	N/A	0.319	0.122	0.219	0.028	0.084	0.042	0.135	0.137	22.934

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	22	23	23	66	63	23	70	66
N.S.	1	1.00	0.88	0.92	0.92	2.64	2.52	0.92	2.80	2.64
time (sec)	N/A	0.271	0.039	0.157	0.026	0.089	0.039	0.124	0.155	0.043

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	19	19	20	20	19	21	19	20
N.S.	1	1.00	0.95	0.95	1.00	1.00	0.95	1.05	0.95	1.00
time (sec)	N/A	0.280	0.094	0.086	0.025	0.077	0.488	0.139	0.143	0.087

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	20	23	23	23	20	23	22	23
N.S.	1	1.00	0.87	1.00	1.00	1.00	0.87	1.00	0.96	1.00
time (sec)	N/A	0.276	0.943	0.121	0.031	0.088	2.364	0.452	0.146	0.127

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	22	23	23	69	76	23	72	75
N.S.	1	1.00	0.88	0.92	0.92	2.76	3.04	0.92	2.88	3.00
time (sec)	N/A	0.276	0.501	0.196	0.027	0.084	9.686	15.625	0.153	23.054

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	60	25	24	224	228	70	285	217
N.S.	1	1.00	2.31	0.96	0.92	8.62	8.77	2.69	10.96	8.35
time (sec)	N/A	0.353	0.095	0.145	0.025	0.105	0.056	0.125	0.147	0.156

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	37	25	24	84	78	45	95	83
N.S.	1	1.00	1.42	0.96	0.92	3.23	3.00	1.73	3.65	3.19
time (sec)	N/A	0.286	0.018	0.119	0.026	0.089	0.032	0.109	0.152	0.046

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	18	22	21	21	20	22	21	21
N.S.	1	1.00	0.86	1.05	1.00	1.00	0.95	1.05	1.00	1.00
time (sec)	N/A	0.278	0.011	0.085	0.026	0.177	1.131	0.161	0.165	0.091

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	21	25	24	24	22	0	24	24
N.S.	1	1.00	0.88	1.04	1.00	1.00	0.92	0.00	1.00	1.00
time (sec)	N/A	0.276	0.018	0.122	0.029	0.158	10.767	0.000	0.152	0.154

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	23	25	24	91	104	0	110	101
N.S.	1	1.00	0.88	0.96	0.92	3.50	4.00	0.00	4.23	3.88
time (sec)	N/A	0.281	0.020	0.206	0.029	0.127	99.836	0.000	0.144	23.451

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	25	30	29	46	0	29	43	61
N.S.	1	1.00	0.86	1.03	1.00	1.59	0.00	1.00	1.48	2.10
time (sec)	N/A	0.288	0.730	0.381	0.026	0.146	0.000	0.282	0.163	22.881

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	27	31	30	48	0	30	48	69
N.S.	1	1.00	0.90	1.03	1.00	1.60	0.00	1.00	1.60	2.30
time (sec)	N/A	0.284	0.170	0.308	0.031	0.174	0.000	0.178	0.149	22.415

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	55	53	0	83	58	58	68	75
N.S.	1	1.00	0.87	0.84	0.00	1.32	0.92	0.92	1.08	1.19
time (sec)	N/A	0.467	0.085	0.093	0.000	0.101	0.065	0.188	0.141	0.200

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	605	597	98	535	0	0	0	0	210	0
N.S.	1	0.99	0.16	0.88	0.00	0.00	0.00	0.00	0.35	0.00
time (sec)	N/A	6.229	0.094	0.143	0.000	0.000	0.000	0.000	0.159	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	C	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	289	337	99	66	0	778293	0	0	196	0
N.S.	1	1.17	0.34	0.23	0.00	2693.06	0.00	0.00	0.68	0.00
time (sec)	N/A	1.900	0.128	0.141	0.000	10.776	0.000	0.000	0.150	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	C	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	544	799	296	182	0	761015	0	0	0	0
N.S.	1	1.47	0.54	0.33	0.00	1398.92	0.00	0.00	0.00	0.00
time (sec)	N/A	9.013	0.175	0.123	0.000	11.891	0.000	0.000	0.151	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	22	38	20	13	51	12
N.S.	1	1.00	1.00	0.93	1.57	2.71	1.43	0.93	3.64	0.86
time (sec)	N/A	0.333	0.024	0.051	0.032	0.065	0.038	0.218	0.157	0.047

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	24	22	32	46	29	23	56	21
N.S.	1	1.00	0.86	0.79	1.14	1.64	1.04	0.82	2.00	0.75
time (sec)	N/A	0.343	0.038	0.053	0.031	0.120	0.051	0.187	0.154	21.943

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	28	29	38	38	36	28	38	28
N.S.	1	1.00	0.47	0.49	0.64	0.64	0.61	0.47	0.64	0.47
time (sec)	N/A	0.741	0.040	0.086	0.025	0.069	0.066	0.432	0.141	0.054

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	204	182	182	182	212	222	222	178
N.S.	1	1.00	0.97	0.87	0.87	0.87	1.01	1.06	1.06	0.85
time (sec)	N/A	0.667	0.076	0.153	0.029	0.066	0.045	0.138	0.163	22.099

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	130	115	113	113	128	133	133	113
N.S.	1	1.00	0.97	0.86	0.84	0.84	0.96	0.99	0.99	0.84
time (sec)	N/A	0.524	0.033	0.138	0.031	0.059	0.035	0.112	0.145	21.779

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	75	63	59	59	70	68	68	64
N.S.	1	1.00	0.95	0.80	0.75	0.75	0.89	0.86	0.86	0.81
time (sec)	N/A	0.400	0.015	0.133	0.031	0.067	0.026	0.118	0.141	0.040

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	28	27	27	29	27	27	27
N.S.	1	1.00	1.00	0.80	0.77	0.77	0.83	0.77	0.77	0.77
time (sec)	N/A	0.292	0.005	0.040	0.032	0.064	0.018	0.108	0.153	0.023

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	C	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	121	59	52	0	140500	155	0	334	275
N.S.	1	1.01	0.49	0.43	0.00	1170.83	1.29	0.00	2.78	2.29
time (sec)	N/A	0.559	0.053	0.075	0.000	2.143	2.703	0.000	0.150	22.151

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	B	F	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	231	166	158	0	0	539	0	3204	1167
N.S.	1	0.94	0.68	0.64	0.00	0.00	2.20	0.00	13.08	4.76
time (sec)	N/A	0.775	0.285	0.089	0.000	0.000	17.096	0.000	0.291	22.391

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	B	F	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	375	359	284	409	0	0	1102	0	9147	2200
N.S.	1	0.96	0.76	1.09	0.00	0.00	2.94	0.00	24.39	5.87
time (sec)	N/A	1.132	0.442	0.135	0.000	0.000	48.013	0.000	1.198	23.004

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	204	182	182	182	219	222	222	178
N.S.	1	1.00	0.97	0.87	0.87	0.87	1.04	1.06	1.06	0.85
time (sec)	N/A	0.676	0.063	0.155	0.033	0.092	0.046	0.112	0.143	22.009

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	132	115	113	113	134	133	133	113
N.S.	1	1.00	0.96	0.83	0.82	0.82	0.97	0.96	0.96	0.82
time (sec)	N/A	0.519	0.031	0.146	0.026	0.115	0.036	0.119	0.160	0.126

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	73	63	59	59	73	68	68	64
N.S.	1	1.00	0.92	0.80	0.75	0.75	0.92	0.86	0.86	0.81
time (sec)	N/A	0.393	0.017	0.134	0.026	0.082	0.025	0.114	0.141	0.041

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	28	27	27	29	27	27	27
N.S.	1	1.00	1.00	0.80	0.77	0.77	0.83	0.77	0.77	0.77
time (sec)	N/A	0.288	0.005	0.043	0.025	0.085	0.019	0.112	0.138	0.022

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	C	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	105	61	54	0	1515766	172	0	415	878
N.S.	1	1.02	0.59	0.52	0.00	14716.17	1.67	0.00	4.03	8.52
time (sec)	N/A	0.575	0.037	0.057	0.000	18.263	4.209	0.000	0.170	22.426

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	B	F	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	220	182	160	0	0	561	0	3672	1218
N.S.	1	0.95	0.79	0.69	0.00	0.00	2.43	0.00	15.90	5.27
time (sec)	N/A	0.759	0.129	0.092	0.000	0.000	19.034	0.000	0.274	22.508

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	520	635	7235	2694	0	0	0	0	656	0
N.S.	1	1.22	13.91	5.18	0.00	0.00	0.00	0.00	1.26	0.00
time (sec)	N/A	1.501	17.254	7.904	0.000	0.000	0.000	0.000	0.341	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	407	543	4389	2551	0	0	0	0	321	0
N.S.	1	1.33	10.78	6.27	0.00	0.00	0.00	0.00	0.79	0.00
time (sec)	N/A	1.228	16.406	7.237	0.000	0.000	0.000	0.000	0.236	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	173	813	788	0	0	0	0	47	0
N.S.	1	0.88	4.13	4.00	0.00	0.00	0.00	0.00	0.24	0.00
time (sec)	N/A	0.695	14.712	1.070	0.000	0.000	0.000	0.000	0.186	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	409	540	3593	2616	0	0	0	0	86	0
N.S.	1	1.32	8.78	6.40	0.00	0.00	0.00	0.00	0.21	0.00
time (sec)	N/A	1.266	16.822	1.072	0.000	0.000	0.000	0.000	0.158	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	542	664	6452	2777	0	0	0	0	153	0
N.S.	1	1.23	11.90	5.12	0.00	0.00	0.00	0.00	0.28	0.00
time (sec)	N/A	1.512	17.117	1.052	0.000	0.000	0.000	0.000	0.176	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	F	F	F	F	F(-1)
verified	N/A	No	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	544	651	8500	2733	0	0	0	0	789	0
N.S.	1	1.20	15.62	5.02	0.00	0.00	0.00	0.00	1.45	0.00
time (sec)	N/A	1.647	17.247	4.305	0.000	0.000	0.000	0.000	0.360	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	430	555	5647	2582	0	0	0	0	423	0
N.S.	1	1.29	13.13	6.00	0.00	0.00	0.00	0.00	0.98	0.00
time (sec)	N/A	1.327	16.587	3.848	0.000	0.000	0.000	0.000	0.261	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	322	471	1247	1147	0	0	0	0	49	0
N.S.	1	1.46	3.87	3.56	0.00	0.00	0.00	0.00	0.15	0.00
time (sec)	N/A	1.182	17.081	0.809	0.000	0.000	0.000	0.000	0.184	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	333	389	2941	2607	0	0	0	0	88	0
N.S.	1	1.17	8.83	7.83	0.00	0.00	0.00	0.00	0.26	0.00
time (sec)	N/A	1.046	17.143	0.746	0.000	0.000	0.000	0.000	0.187	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	525	653	5812	2780	0	0	0	0	155	0
N.S.	1	1.24	11.07	5.30	0.00	0.00	0.00	0.00	0.30	0.00
time (sec)	N/A	1.605	17.246	0.878	0.000	0.000	0.000	0.000	0.207	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	275	342	106	97	0	0	374	0	231	1003
N.S.	1	1.24	0.39	0.35	0.00	0.00	1.36	0.00	0.84	3.65
time (sec)	N/A	0.836	0.060	0.079	0.000	0.000	2.253	0.000	0.157	22.470

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	C	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	240	307	106	97	0	61993	274	0	53	625
N.S.	1	1.28	0.44	0.40	0.00	258.30	1.14	0.00	0.22	2.60
time (sec)	N/A	0.746	0.043	0.056	0.000	64.701	1.524	0.000	0.155	22.128

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	C	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	250	104	95	0	40785	131	0	51	205
N.S.	1	1.26	0.53	0.48	0.00	205.98	0.66	0.00	0.26	1.04
time (sec)	N/A	0.616	0.036	0.058	0.000	11.180	0.485	0.000	0.143	21.883

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	228	161	94	0	152	26	103	49	60
N.S.	1	1.43	1.01	0.59	0.00	0.96	0.16	0.65	0.31	0.38
time (sec)	N/A	0.655	0.100	0.056	0.000	0.103	0.141	0.119	0.162	21.697

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	C	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	308	396	163	139	0	307773	0	0	54	882
N.S.	1	1.29	0.53	0.45	0.00	999.26	0.00	0.00	0.18	2.86
time (sec)	N/A	1.101	0.085	0.081	0.000	4.429	0.000	0.000	0.150	21.863

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	C	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	392	497	238	184	0	1128605	0	0	58	2440
N.S.	1	1.27	0.61	0.47	0.00	2879.09	0.00	0.00	0.15	6.22
time (sec)	N/A	1.590	0.182	0.099	0.000	79.262	0.000	0.000	0.141	22.201

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	83	74	73	73	97	73	71	97
N.S.	1	1.00	0.86	0.76	0.75	0.75	1.00	0.75	0.73	1.00
time (sec)	N/A	0.532	0.050	0.063	0.106	0.108	0.125	0.116	0.149	0.207

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	78	69	68	68	92	68	66	92
N.S.	1	1.00	0.87	0.77	0.76	0.76	1.02	0.76	0.73	1.02
time (sec)	N/A	0.492	0.036	0.054	0.113	0.083	0.129	0.405	0.151	0.183

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	72	62	61	61	78	61	59	85
N.S.	1	1.00	0.94	0.81	0.79	0.79	1.01	0.79	0.77	1.10
time (sec)	N/A	0.473	0.042	0.055	0.111	0.095	0.110	0.110	0.143	0.177

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	69	57	56	56	75	56	54	80
N.S.	1	1.00	0.96	0.79	0.78	0.78	1.04	0.78	0.75	1.11
time (sec)	N/A	0.443	0.031	0.048	0.107	0.075	0.113	0.112	0.155	0.150

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	65	56	55	55	75	55	53	79
N.S.	1	1.00	0.92	0.79	0.77	0.77	1.06	0.77	0.75	1.11
time (sec)	N/A	0.413	0.022	0.049	0.105	0.081	0.111	0.114	0.159	21.986

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	69	58	59	59	78	60	57	83
N.S.	1	1.00	0.92	0.77	0.79	0.79	1.04	0.80	0.76	1.11
time (sec)	N/A	0.473	0.026	0.056	0.111	0.080	0.158	0.109	0.150	22.110

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	78	65	64	70	87	65	68	88
N.S.	1	1.00	0.93	0.77	0.76	0.83	1.04	0.77	0.81	1.05
time (sec)	N/A	0.503	0.041	0.057	0.107	0.077	0.170	0.130	0.152	0.161

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	82	70	69	83	94	70	81	92
N.S.	1	1.00	0.90	0.77	0.76	0.91	1.03	0.77	0.89	1.01
time (sec)	N/A	0.490	0.076	0.064	0.109	0.083	0.184	0.111	0.148	22.231

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	230	228	109	74	0	322	61	0	105	128
N.S.	1	0.99	0.47	0.32	0.00	1.40	0.27	0.00	0.46	0.56
time (sec)	N/A	1.244	0.030	0.049	0.000	0.095	0.610	0.000	0.140	21.797

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	216	101	67	0	315	3662	0	98	188
N.S.	1	0.99	0.46	0.31	0.00	1.44	16.72	0.00	0.45	0.86
time (sec)	N/A	0.994	0.025	0.037	0.000	0.097	1.607	0.000	0.157	0.134

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	217	94	62	0	309	48	0	93	183
N.S.	1	1.05	0.45	0.30	0.00	1.49	0.23	0.00	0.45	0.88
time (sec)	N/A	0.840	0.023	0.039	0.000	0.099	0.564	0.000	0.150	22.379

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	214	90	58	0	310	46	0	92	181
N.S.	1	1.04	0.44	0.28	0.00	1.50	0.22	0.00	0.45	0.88
time (sec)	N/A	0.874	0.020	0.035	0.000	0.094	0.503	0.000	0.147	22.436

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	214	101	51	0	315	60	0	96	237
N.S.	1	0.99	0.47	0.24	0.00	1.45	0.28	0.00	0.44	1.09
time (sec)	N/A	0.974	0.027	0.046	0.000	0.113	9.089	0.000	0.158	22.362

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	226	109	58	0	329	25507	0	111	242
N.S.	1	1.01	0.49	0.26	0.00	1.47	113.87	0.00	0.50	1.08
time (sec)	N/A	0.993	0.024	0.048	0.000	0.097	19.683	0.000	0.147	22.343

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	235	116	62	0	346	70	0	130	246
N.S.	1	1.02	0.50	0.27	0.00	1.50	0.30	0.00	0.56	1.06
time (sec)	N/A	1.055	0.024	0.052	0.000	0.091	1.747	0.000	0.141	22.263

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	74	42	35	59	58	182	58	57	38
N.S.	1	1.76	1.00	0.83	1.40	1.38	4.33	1.38	1.36	0.90
time (sec)	N/A	0.766	0.173	1.704	0.078	0.083	0.157	0.120	0.156	0.182

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	74	42	35	59	58	182	58	57	51
N.S.	1	1.76	1.00	0.83	1.40	1.38	4.33	1.38	1.36	1.21
time (sec)	N/A	0.769	0.138	1.330	0.079	0.072	0.607	0.118	0.155	0.179

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	244	92	0	324	0	0	52	0
N.S.	1	1.00	2.77	1.05	0.00	3.68	0.00	0.00	0.59	0.00
time (sec)	N/A	0.653	1.522	3.711	0.000	1.940	0.000	0.000	200.011	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	218	96	0	331	0	0	226	0
N.S.	1	1.00	2.48	1.09	0.00	3.76	0.00	0.00	2.57	0.00
time (sec)	N/A	0.741	1.447	3.699	0.000	1.929	0.000	0.000	0.696	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [46] had the largest ratio of [.692308000000000034]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	1	1	1.00	40	0.025
2	A	1	1	1.00	38	0.026
3	A	1	1	1.00	40	0.025
4	A	1	1	1.00	40	0.025
5	A	1	1	1.00	40	0.025
6	A	1	1	1.00	41	0.024
7	A	1	1	1.00	39	0.026
8	A	1	1	1.00	41	0.024
9	A	1	1	1.00	41	0.024
10	A	1	1	1.00	41	0.024
11	A	1	1	1.00	40	0.025
12	A	1	1	1.00	41	0.024
13	A	2	2	1.00	32	0.062
14	A	2	2	0.99	38	0.053
15	A	4	3	1.17	38	0.079
16	A	4	3	1.47	38	0.079
17	A	4	4	1.00	33	0.121
18	A	4	4	1.00	34	0.118
19	A	9	8	1.00	43	0.186
20	A	2	2	1.00	24	0.083
21	A	2	2	1.00	24	0.083

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
22	A	2	2	1.00	24	0.083
23	A	2	2	1.00	22	0.091
24	A	8	7	1.01	24	0.292
25	A	11	10	0.94	24	0.417
26	A	14	13	0.96	24	0.542
27	A	2	2	1.00	26	0.077
28	A	2	2	1.00	26	0.077
29	A	2	2	1.00	26	0.077
30	A	2	2	1.00	24	0.083
31	A	10	9	1.02	26	0.346
32	A	13	12	0.95	26	0.462
33	A	17	16	1.22	26	0.615
34	A	14	13	1.33	26	0.500
35	A	8	7	0.88	26	0.269
36	A	12	11	1.32	26	0.423
37	A	15	14	1.23	26	0.538
38	A	19	18	1.20	28	0.643
39	A	16	15	1.29	28	0.536
40	A	13	12	1.46	28	0.429
41	A	12	11	1.17	28	0.393
42	A	17	16	1.24	28	0.571
43	A	5	4	1.24	17	0.235
44	A	4	3	1.28	17	0.176
45	A	5	4	1.26	15	0.267
46	A	10	9	1.43	13	0.692
47	A	5	4	1.29	17	0.235
48	A	4	3	1.27	17	0.176
49	A	2	2	1.00	35	0.057
50	A	2	2	1.00	35	0.057
51	A	2	2	1.00	35	0.057
52	A	2	2	1.00	33	0.061
53	A	2	2	1.00	32	0.062

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
54	A	2	2	1.00	35	0.057
55	A	2	2	1.00	35	0.057
56	A	2	2	1.00	35	0.057
57	A	2	2	0.99	35	0.057
58	A	2	2	0.99	35	0.057
59	A	2	2	1.05	33	0.061
60	A	2	2	1.04	32	0.062
61	A	2	2	0.99	35	0.057
62	A	2	2	1.01	35	0.057
63	A	2	2	1.02	35	0.057
64	A	8	7	1.76	35	0.200
65	A	9	8	1.76	28	0.286
66	A	1	1	1.00	52	0.019
67	A	1	1	1.00	57	0.018

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int (b+2cx+3dx^2+4ex^3) (bx+cx^2+dx^3+ex^4)^2 \, dx$

Optimal result

Integrand size = 40, antiderivative size = 25

$$\int (b+2cx+3dx^2+4ex^3) (bx+cx^2+dx^3+ex^4)^2 \, dx = \frac{1}{3}(bx+cx^2+dx^3+ex^4)^3$$

output
```
1/3*(e*x^4+d*x^3+c*x^2+b*x)^3
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int (b+2cx+3dx^2+4ex^3) (bx+cx^2+dx^3+ex^4)^2 \, dx = \frac{1}{3}x^3(b+x(c+x(d+ex)))^3$$

input
```
Integrate[(b + 2*c*x + 3*d*x^2 + 4*e*x^3)*(b*x + c*x^2 + d*x^3 + e*x^4)^2,
x]
```

output
```
(x^3*(b + x*(c + x*(d + e*x)))^3)/3
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.025, Rules used = {2021}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b+2cx+3dx^2+4ex^3)(bx+cx^2+dx^3+ex^4)^2 \, dx$$

$$\Big\downarrow \text{2021}$$

$$\frac{1}{3}(bx+cx^2+dx^3+ex^4)^3$$

input
```
Int[(b + 2*c*x + 3*d*x^2 + 4*e*x^3)*(b*x + c*x^2 + d*x^3 + e*x^4)^2,x]
```

output
```
(b*x + c*x^2 + d*x^3 + e*x^4)^3/3
```

Defintions of rubi rules used

rule 2021
```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

method	result
gosper	$\frac{(e\,x^3+d\,x^2+cx+b)^3 x^3}{3}$
default	$\frac{(x^4 e+d\,x^3+c\,x^2+bx)^3}{3}$
orering	$\frac{(e\,x^3+d\,x^2+cx+b)x(x^4 e+d\,x^3+c\,x^2+bx)^2}{3}$
norman	$\frac{e^3x^{12}}{3}+d\,e^2x^{11}+(c\,e^2+d^2e)\,x^{10}+\left(b\,e^2+2cde+\frac{1}{3}d^3\right)x^9+(2bde+c^2e+c\,d^2)\,x^8+(2bce$
risch	$\frac{1}{3}e^3x^{12}+d\,e^2x^{11}+c\,e^2x^{10}+d^2e\,x^{10}+x^9b\,e^2+2x^9cde+\frac{1}{3}x^9d^3+2bde\,x^8+c^2e\,x^8+c\,d^2x^8$
parallelrisch	$\frac{1}{3}e^3x^{12}+d\,e^2x^{11}+c\,e^2x^{10}+d^2e\,x^{10}+x^9b\,e^2+2x^9cde+\frac{1}{3}x^9d^3+2bde\,x^8+c^2e\,x^8+c\,d^2x^8$

input
```
int((4*e*x^3+3*d*x^2+2*c*x+b)*(e*x^4+d*x^3+c*x^2+b*x)^2,x,method=_RETURNVE
RBOSE)
```

output
```
1/3*(e*x^3+d*x^2+c*x+b)^3*x^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 143 vs. 2(23) = 46.

Time = 0.08 (sec) , antiderivative size = 143, normalized size of antiderivative = 5.72

$$\int (b+2cx+3dx^2+4ex^3)\left(bx+cx^2+dx^3+ex^4\right)^2\,dx$$
$$=\frac{1}{3}e^3x^{12}+de^2x^{11}+(d^2e+ce^2)x^{10}+\frac{1}{3}\left(d^3+6\,cde+3\,be^2\right)x^9+\left(cd^2+\left(c^2+2\,bd\right)e\right)x^8$$
$$+\left(c^2d+bd^2+2\,bce\right)x^7+b^2cx^4+\frac{1}{3}\left(c^3+6\,bcd+3\,b^2e\right)x^6+\frac{1}{3}b^3x^3+\left(bc^2+b^2d\right)x^5$$

input
```
integrate((4*e*x^3+3*d*x^2+2*c*x+b)*(e*x^4+d*x^3+c*x^2+b*x)^2,x, algorithm
="fricas")
```

output
```
1/3*e^3*x^12 + d*e^2*x^11 + (d^2*e + c*e^2)*x^10 + 1/3*(d^3 + 6*c*d*e + 3*
b*e^2)*x^9 + (c*d^2 + (c^2 + 2*b*d)*e)*x^8 + (c^2*d + b*d^2 + 2*b*c*e)*x^7
 + b^2*c*x^4 + 1/3*(c^3 + 6*b*c*d + 3*b^2*e)*x^6 + 1/3*b^3*x^3 + (b*c^2 +
b^2*d)*x^5
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 141 vs. $2(20) = 40$.

Time = 0.04 (sec) , antiderivative size = 141, normalized size of antiderivative = 5.64

$$\int \left(b + 2cx + 3dx^2 + 4ex^3\right)\left(bx + cx^2 + dx^3 + ex^4\right)^2 \, dx$$
$$= \frac{b^3x^3}{3} + b^2cx^4 + de^2x^{11} + \frac{e^3x^{12}}{3} + x^{10}\left(ce^2 + d^2e\right) + x^9\left(be^2 + 2cde + \frac{d^3}{3}\right) + x^8 \cdot \left(2bde + c^2e + cd^2\right) + x^7 \cdot \left(2bce + bd^2 + c^2d\right) + x^6\left(b^2e + 2bcd + \frac{c^3}{3}\right) + x^5\left(b^2d + bc^2\right)$$

input
```
integrate((4*e*x**3+3*d*x**2+2*c*x+b)*(e*x**4+d*x**3+c*x**2+b*x)**2,x)
```

output
```
b**3*x**3/3 + b**2*c*x**4 + d*e**2*x**11 + e**3*x**12/3 + x**10*(c*e**2 +
d**2*e) + x**9*(b*e**2 + 2*c*d*e + d**3/3) + x**8*(2*b*d*e + c**2*e + c*d*
*2) + x**7*(2*b*c*e + b*d**2 + c**2*d) + x**6*(b**2*e + 2*b*c*d + c**3/3)
+ x**5*(b**2*d + b*c**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \left(b + 2cx + 3dx^2 + 4ex^3\right)\left(bx + cx^2 + dx^3 + ex^4\right)^2 \, dx = \frac{1}{3}\left(ex^4 + dx^3 + cx^2 + bx\right)^3$$

input
```
integrate((4*e*x^3+3*d*x^2+2*c*x+b)*(e*x^4+d*x^3+c*x^2+b*x)^2,x, algorithm
="maxima")
```

output
```
1/3*(e*x^4 + d*x^3 + c*x^2 + b*x)^3
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int (b+2cx+3dx^2+4ex^3) (bx+cx^2+dx^3+ex^4)^2 \, dx = \frac{1}{3} (ex^4+dx^3+cx^2+bx)^3$$

input
```
integrate((4*e*x^3+3*d*x^2+2*c*x+b)*(e*x^4+d*x^3+c*x^2+b*x)^2,x, algorithm
="giac")
```

output
```
1/3*(e*x^4 + d*x^3 + c*x^2 + b*x)^3
```

Mupad [B] (verification not implemented)

Time = 22.93 (sec) , antiderivative size = 137, normalized size of antiderivative = 5.48

$$\begin{aligned}\int (b+2cx+3dx^2+4ex^3) (bx+cx^2+dx^3+ex^4)^2 \, dx &= x^7 (c^2 d+2 b e c+b d^2)+x^8 (e c^2+c d^2+2 b e d) \\ &+ x^6 \left(e b^2+2 d b c+\frac{c^3}{3}\right)+x^9 \left(\frac{d^3}{3}+2 c d e+b e^2\right)+\frac{b^3 x^3}{3} \\ &+ \frac{e^3 x^{12}}{3}+b^2 c x^4+d e^2 x^{11}+b x^5 (c^2+b d)+e x^{10} (d^2+c e)\end{aligned}$$

input
```
int((b + 2*c*x + 3*d*x^2 + 4*e*x^3)*(b*x + c*x^2 + d*x^3 + e*x^4)^2,x)
```

output
```
x^7*(b*d^2 + c^2*d + 2*b*c*e) + x^8*(c*d^2 + c^2*e + 2*b*d*e) + x^6*(b^2*e
 + c^3/3 + 2*b*c*d) + x^9*(b*e^2 + d^3/3 + 2*c*d*e) + (b^3*x^3)/3 + (e^3*x
^12)/3 + b^2*c*x^4 + d*e^2*x^11 + b*x^5*(b*d + c^2) + e*x^10*(c*e + d^2)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 168, normalized size of antiderivative = 6.72

$$\int \left(b+2cx+3dx^2+4ex^3\right)\left(bx+cx^2+dx^3+ex^4\right)^2\,dx$$
$$=\frac{x^3(e^3x^9+3d\,e^2x^8+3c\,e^2x^7+3d^2e\,x^7+3b\,e^2x^6+6cde\,x^6+d^3x^6+6bde\,x^5+3c^2e\,x^5+3c\,d^2x^5+6bce\,x}{3}$$

input
```
int((4*e*x^3+3*d*x^2+2*c*x+b)*(e*x^4+d*x^3+c*x^2+b*x)^2,x)
```

output
```
(x**3*(b**3 + 3*b**2*c*x + 3*b**2*d*x**2 + 3*b**2*e*x**3 + 3*b*c**2*x**2 +
 6*b*c*d*x**3 + 6*b*c*e*x**4 + 3*b*d**2*x**4 + 6*b*d*e*x**5 + 3*b*e**2*x**
6 + c**3*x**3 + 3*c**2*d*x**4 + 3*c**2*e*x**5 + 3*c*d**2*x**5 + 6*c*d*e*x*
*6 + 3*c*e**2*x**7 + d**3*x**6 + 3*d**2*e*x**7 + 3*d*e**2*x**8 + e**3*x**9
))/3
```

3.2 $\int (b+2cx+3dx^2+4ex^3)(bx+cx^2+dx^3+ex^4)\,dx$

Optimal result

Integrand size = 38, antiderivative size = 25

$$\int (b+2cx+3dx^2+4ex^3)(bx+cx^2+dx^3+ex^4)\,dx=\frac{1}{2}(bx+cx^2+dx^3+ex^4)^2$$

output
```
1/2*(e*x^4+d*x^3+c*x^2+b*x)^2
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int (b+2cx+3dx^2+4ex^3)(bx+cx^2+dx^3+ex^4)\,dx=\frac{1}{2}x^2(b+x(c+x(d+ex)))^2$$

input
```
Integrate[(b + 2*c*x + 3*d*x^2 + 4*e*x^3)*(b*x + c*x^2 + d*x^3 + e*x^4),x]
```

output
```
(x^2*(b + x*(c + x*(d + e*x)))^2)/2
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.026, Rules used = {2021}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b+2cx+3dx^2+4ex^3)(bx+cx^2+dx^3+ex^4)\,dx$$

$$\Big\downarrow \text{2021}$$

$$\frac{1}{2}(bx+cx^2+dx^3+ex^4)^2$$

input
```
Int[(b + 2*c*x + 3*d*x^2 + 4*e*x^3)*(b*x + c*x^2 + d*x^3 + e*x^4),x]
```

output
```
(b*x + c*x^2 + d*x^3 + e*x^4)^2/2
```

Defintions of rubi rules used

rule 2021
```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

method	result	size
gosper	$\frac{x^2(e\,x^3+d\,x^2+cx+b)^2}{2}$	23
default	$\frac{(x^4e+d\,x^3+c\,x^2+bx)^2}{2}$	24
orering	$\frac{(e\,x^3+d\,x^2+cx+b)x(x^4e+d\,x^3+c\,x^2+bx)}{2}$	38
norman	$\frac{e^2x^8}{2}+de\,x^7+\left(ce+\frac{d^2}{2}\right)x^6+(eb+cd)\,x^5+\left(bd+\frac{c^2}{2}\right)x^4+bc\,x^3+\frac{b^2x^2}{2}$	67
risch	$\frac{1}{2}e^2x^8+de\,x^7+ce\,x^6+\frac{1}{2}x^6d^2+b\,x^5e+cd\,x^5+bd\,x^4+\frac{1}{2}c^2x^4+bc\,x^3+\frac{1}{2}b^2x^2$	70
parallelrisch	$\frac{1}{2}e^2x^8+de\,x^7+ce\,x^6+\frac{1}{2}x^6d^2+b\,x^5e+cd\,x^5+bd\,x^4+\frac{1}{2}c^2x^4+bc\,x^3+\frac{1}{2}b^2x^2$	70

input
```
int((4*e*x^3+3*d*x^2+2*c*x+b)*(e*x^4+d*x^3+c*x^2+b*x),x,method=_RETURNVERB
OSE)
```

output
```
1/2*x^2*(e*x^3+d*x^2+c*x+b)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(23) = 46$.

Time = 0.09 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.64

$$\int (b+2cx+3dx^2+4ex^3)(bx+cx^2+dx^3+ex^4)\,dx$$
$$=\frac{1}{2}e^2x^8+dex^7+\frac{1}{2}(d^2+2\,ce)x^6+(cd+be)x^5+bcx^3+\frac{1}{2}(c^2+2\,bd)x^4+\frac{1}{2}b^2x^2$$

input
```
integrate((4*e*x^3+3*d*x^2+2*c*x+b)*(e*x^4+d*x^3+c*x^2+b*x),x, algorithm="
fricas")
```

output
```
1/2*e^2*x^8 + d*e*x^7 + 1/2*(d^2 + 2*c*e)*x^6 + (c*d + b*e)*x^5 + b*c*x^3
+ 1/2*(c^2 + 2*b*d)*x^4 + 1/2*b^2*x^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(20) = 40$.

Time = 0.04 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.52

$$\int (b + 2cx + 3dx^2 + 4ex^3) (bx + cx^2 + dx^3 + ex^4) \, dx$$
$$= \frac{b^2x^2}{2} + bcx^3 + dex^7 + \frac{e^2x^8}{2} + x^6\left(ce + \frac{d^2}{2}\right) + x^5(be + cd) + x^4\left(bd + \frac{c^2}{2}\right)$$

input
```
integrate((4*e*x**3+3*d*x**2+2*c*x+b)*(e*x**4+d*x**3+c*x**2+b*x),x)
```

output
```
b**2*x**2/2 + b*c*x**3 + d*e*x**7 + e**2*x**8/2 + x**6*(c*e + d**2/2) + x*
*5*(b*e + c*d) + x**4*(b*d + c**2/2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int (b + 2cx + 3dx^2 + 4ex^3) (bx + cx^2 + dx^3 + ex^4) \, dx = \frac{1}{2} (ex^4 + dx^3 + cx^2 + bx)^2$$

input
```
integrate((4*e*x^3+3*d*x^2+2*c*x+b)*(e*x^4+d*x^3+c*x^2+b*x),x, algorithm="
maxima")
```

output
```
1/2*(e*x^4 + d*x^3 + c*x^2 + b*x)^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int (b+2cx+3dx^2+4ex^3) (bx+cx^2+dx^3+ex^4) \, dx = \frac{1}{2} (ex^4+dx^3+cx^2+bx)^2$$

input
```
integrate((4*e*x^3+3*d*x^2+2*c*x+b)*(e*x^4+d*x^3+c*x^2+b*x),x, algorithm="
giac")
```

output
```
1/2*(e*x^4 + d*x^3 + c*x^2 + b*x)^2
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.64

$$\int (b+2cx+3dx^2+4ex^3) (bx+cx^2+dx^3+ex^4) \, dx$$
$$= x^4 \left(\frac{c^2}{2}+b\,d\right)+x^6 \left(\frac{d^2}{2}+c\,e\right)+x^5 (b\,e+c\,d)+\frac{b^2\,x^2}{2}+\frac{e^2\,x^8}{2}+b\,c\,x^3+d\,e\,x^7$$

input
```
int((b + 2*c*x + 3*d*x^2 + 4*e*x^3)*(b*x + c*x^2 + d*x^3 + e*x^4),x)
```

output
```
x^4*(b*d + c^2/2) + x^6*(c*e + d^2/2) + x^5*(b*e + c*d) + (b^2*x^2)/2 + (e
^2*x^8)/2 + b*c*x^3 + d*e*x^7
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 70, normalized size of antiderivative = 2.80

$$\int (b+2cx+3dx^2+4ex^3) (bx+cx^2+dx^3+ex^4) \, dx$$
$$= \frac{x^2(e^2x^6+2de\,x^5+2ce\,x^4+d^2x^4+2be\,x^3+2cd\,x^3+2bd\,x^2+c^2x^2+2bcx+b^2)}{2}$$

input
```
int((4*e*x^3+3*d*x^2+2*c*x+b)*(e*x^4+d*x^3+c*x^2+b*x),x)
```

output
```
(x**2*(b**2 + 2*b*c*x + 2*b*d*x**2 + 2*b*e*x**3 + c**2*x**2 + 2*c*d*x**3 +
 2*c*e*x**4 + d**2*x**4 + 2*d*e*x**5 + e**2*x**6))/2
```

3.3 $\int \frac{b+2cx+3dx^2+4ex^3}{bx+cx^2+dx^3+ex^4} \, dx$

Optimal result

Integrand size = 40, antiderivative size = 20

$$\int \frac{b+2cx+3dx^2+4ex^3}{bx+cx^2+dx^3+ex^4} \, dx = \log \left(bx+cx^2+dx^3+ex^4\right)$$

output
```
ln(e*x^4+d*x^3+c*x^2+b*x)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{b+2cx+3dx^2+4ex^3}{bx+cx^2+dx^3+ex^4} \, dx = \log(x)+\log \left(b+cx+dx^2+ex^3\right)$$

input
```
Integrate[(b + 2*c*x + 3*d*x^2 + 4*e*x^3)/(b*x + c*x^2 + d*x^3 + e*x^4),x]
```

output
```
Log[x] + Log[b + c*x + d*x^2 + e*x^3]
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.025$, Rules used = {2020}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{b+2cx+3dx^2+4ex^3}{bx+cx^2+dx^3+ex^4}\,dx$$

$$\Big\downarrow \ 2020$$

$$\log\left(bx+cx^2+dx^3+ex^4\right)$$

input
```
Int[(b + 2*c*x + 3*d*x^2 + 4*e*x^3)/(b*x + c*x^2 + d*x^3 + e*x^4),x]
```

output
```
Log[b*x + c*x^2 + d*x^3 + e*x^4]
```

Defintions of rubi rules used

rule 2020
```
Int[(Pp_)/(Qq_), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Si
mp[Coeff[Pp, x, p]*(Log[RemoveContent[Qq, x]]/(q*Coeff[Qq, x, q])), x] /; E
qQ[p, q - 1] && EqQ[Pp, Simplify[(Coeff[Pp, x, p]/(q*Coeff[Qq, x, q]))*D[Qq
, x]]]] /; PolyQ[Pp, x] && PolyQ[Qq, x]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

method	result	size
default	$\ln\left(x(e\,x^3+d\,x^2+cx+b)\right)$	19
norman	$\ln\left(x\right)+\ln\left(e\,x^3+d\,x^2+cx+b\right)$	20
parallelrisch	$\ln\left(x\right)+\ln\left(e\,x^3+d\,x^2+cx+b\right)$	20
derivativedivides	$\ln\left(x^4e+d\,x^3+c\,x^2+bx\right)$	21
risch	$\ln\left(x^4e+d\,x^3+c\,x^2+bx\right)$	21

input
```
int((4*e*x^3+3*d*x^2+2*c*x+b)/(e*x^4+d*x^3+c*x^2+b*x),x,method=_RETURNVERB
OSE)
```

output
```
ln(x*(e*x^3+d*x^2+c*x+b))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{b+2cx+3dx^2+4ex^3}{bx+cx^2+dx^3+ex^4}\,dx=\log\left(ex^4+dx^3+cx^2+bx\right)$$

input
```
integrate((4*e*x^3+3*d*x^2+2*c*x+b)/(e*x^4+d*x^3+c*x^2+b*x),x, algorithm="
fricas")
```

output
```
log(e*x^4 + d*x^3 + c*x^2 + b*x)
```

Sympy [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{b+2cx+3dx^2+4ex^3}{bx+cx^2+dx^3+ex^4}\,dx=\log\left(bx+cx^2+dx^3+ex^4\right)$$

input
```
integrate((4*e*x**3+3*d*x**2+2*c*x+b)/(e*x**4+d*x**3+c*x**2+b*x),x)
```

output
```
log(b*x + c*x**2 + d*x**3 + e*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{b + 2cx + 3dx^2 + 4ex^3}{bx + cx^2 + dx^3 + ex^4} \, dx = \log \left(ex^4 + dx^3 + cx^2 + bx\right)$$

input
```
integrate((4*e*x^3+3*d*x^2+2*c*x+b)/(e*x^4+d*x^3+c*x^2+b*x),x, algorithm="
maxima")
```

output
```
log(e*x^4 + d*x^3 + c*x^2 + b*x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.05

$$\int \frac{b + 2cx + 3dx^2 + 4ex^3}{bx + cx^2 + dx^3 + ex^4} \, dx = \log \left(\left|ex^4 + dx^3 + cx^2 + bx\right|\right)$$

input
```
integrate((4*e*x^3+3*d*x^2+2*c*x+b)/(e*x^4+d*x^3+c*x^2+b*x),x, algorithm="
giac")
```

output
```
log(abs(e*x^4 + d*x^3 + c*x^2 + b*x))
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{b+2cx+3dx^2+4ex^3}{bx+cx^2+dx^3+ex^4}\,dx = \ln\left(e\,x^4+d\,x^3+c\,x^2+b\,x\right)$$

input
```
int((b + 2*c*x + 3*d*x^2 + 4*e*x^3)/(b*x + c*x^2 + d*x^3 + e*x^4),x)
```

output
```
log(b*x + c*x^2 + d*x^3 + e*x^4)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{b+2cx+3dx^2+4ex^3}{bx+cx^2+dx^3+ex^4}\,dx = \log\left(e\,x^3+d\,x^2+cx+b\right)+\log(x)$$

input
```
int((4*e*x^3+3*d*x^2+2*c*x+b)/(e*x^4+d*x^3+c*x^2+b*x),x)
```

output
```
log(b + c*x + d*x**2 + e*x**3) + log(x)
```

3.4 $\int \frac{b+2cx+3dx^2+4ex^3}{(bx+cx^2+dx^3+ex^4)^2}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 23

$$\int \frac{b+2cx+3dx^2+4ex^3}{(bx+cx^2+dx^3+ex^4)^2}\,dx=-\frac{1}{bx+cx^2+dx^3+ex^4}$$

output
```
-1/(e*x^4+d*x^3+c*x^2+b*x)
```

Mathematica [A] (verified)

Time = 0.94 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{b+2cx+3dx^2+4ex^3}{(bx+cx^2+dx^3+ex^4)^2}\,dx=-\frac{1}{x(b+x(c+x(d+ex)))}$$

input
```
Integrate[(b + 2*c*x + 3*d*x^2 + 4*e*x^3)/(b*x + c*x^2 + d*x^3 + e*x^4)^2,
x]
```

output
```
-(1/(x*(b + x*(c + x*(d + e*x)))))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.025$, Rules used = $\{2021\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{b+2cx+3dx^2+4ex^3}{(bx+cx^2+dx^3+ex^4)^2}\,dx$$

$$\Big\downarrow \text{2021}$$

$$-\frac{1}{bx+cx^2+dx^3+ex^4}$$

input
```
Int[(b + 2*c*x + 3*d*x^2 + 4*e*x^3)/(b*x + c*x^2 + d*x^3 + e*x^4)^2,x]
```

output
```
-(b*x + c*x^2 + d*x^3 + e*x^4)^(-1)
```

Defintions of rubi rules used

rule 2021
```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

method	result	size
gosper	$-\frac{1}{x(e\,x^3+d\,x^2+cx+b)}$	23
norman	$-\frac{1}{x(e\,x^3+d\,x^2+cx+b)}$	23
risch	$-\frac{1}{x(e\,x^3+d\,x^2+cx+b)}$	23
parallelrisch	$-\frac{1}{x(e\,x^3+d\,x^2+cx+b)}$	23
derivativedivides	$-\frac{1}{x^4e+d\,x^3+c\,x^2+bx}$	24
orering	$-\frac{x(e\,x^3+d\,x^2+cx+b)}{(x^4e+d\,x^3+c\,x^2+bx)^2}$	40
default	$-\frac{1}{xb}+\frac{e\,x^2+dx+c}{b(e\,x^3+d\,x^2+cx+b)}$	41

input
```
int((4*e*x^3+3*d*x^2+2*c*x+b)/(e*x^4+d*x^3+c*x^2+b*x)^2,x,method=_RETURNVE
RBOSE)
```

output
```
-1/x/(e*x^3+d*x^2+c*x+b)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{b+2cx+3dx^2+4ex^3}{(bx+cx^2+dx^3+ex^4)^2}\,dx=-\frac{1}{ex^4+dx^3+cx^2+bx}$$

input
```
integrate((4*e*x^3+3*d*x^2+2*c*x+b)/(e*x^4+d*x^3+c*x^2+b*x)^2,x, algorithm
="fricas")
```

output
```
-1/(e*x^4 + d*x^3 + c*x^2 + b*x)
```

Sympy [A] (verification not implemented)

Time = 2.36 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{b+2cx+3dx^2+4ex^3}{(bx+cx^2+dx^3+ex^4)^2}\,dx=-\frac{1}{bx+cx^2+dx^3+ex^4}$$

input
```
integrate((4*e*x**3+3*d*x**2+2*c*x+b)/(e*x**4+d*x**3+c*x**2+b*x)**2,x)
```

output
```
-1/(b*x + c*x**2 + d*x**3 + e*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{b+2cx+3dx^2+4ex^3}{(bx+cx^2+dx^3+ex^4)^2}\,dx=-\frac{1}{ex^4+dx^3+cx^2+bx}$$

input
```
integrate((4*e*x^3+3*d*x^2+2*c*x+b)/(e*x^4+d*x^3+c*x^2+b*x)^2,x, algorithm
="maxima")
```

output
```
-1/(e*x^4 + d*x^3 + c*x^2 + b*x)
```

Giac [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{b+2cx+3dx^2+4ex^3}{(bx+cx^2+dx^3+ex^4)^2}\,dx=-\frac{1}{ex^4+dx^3+cx^2+bx}$$

input
```
integrate((4*e*x^3+3*d*x^2+2*c*x+b)/(e*x^4+d*x^3+c*x^2+b*x)^2,x, algorithm
="giac")
```

output
```
-1/(e*x^4 + d*x^3 + c*x^2 + b*x)
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{b+2cx+3dx^2+4ex^3}{(bx+cx^2+dx^3+ex^4)^2}\,dx = -\frac{1}{e\,x^4+d\,x^3+c\,x^2+b\,x}$$

input
```
int((b + 2*c*x + 3*d*x^2 + 4*e*x^3)/(b*x + c*x^2 + d*x^3 + e*x^4)^2,x)
```

output
```
-1/(b*x + c*x^2 + d*x^3 + e*x^4)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{b+2cx+3dx^2+4ex^3}{(bx+cx^2+dx^3+ex^4)^2}\,dx = -\frac{1}{x\,(e\,x^3+d\,x^2+cx+b)}$$

input
```
int((4*e*x^3+3*d*x^2+2*c*x+b)/(e*x^4+d*x^3+c*x^2+b*x)^2,x)
```

output
```
( - 1)/(x*(b + c*x + d*x**2 + e*x**3))
```

3.5 $\int \frac{b+2cx+3dx^2+4ex^3}{(bx+cx^2+dx^3+ex^4)^3}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 25

$$\int \frac{b+2cx+3dx^2+4ex^3}{(bx+cx^2+dx^3+ex^4)^3}\,dx = -\frac{1}{2\,(bx+cx^2+dx^3+ex^4)^2}$$

output
```
-1/2/(e*x^4+d*x^3+c*x^2+b*x)^2
```

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \frac{b+2cx+3dx^2+4ex^3}{(bx+cx^2+dx^3+ex^4)^3}\,dx = -\frac{1}{2x^2(b+x(c+x(d+ex)))^2}$$

input
```
Integrate[(b + 2*c*x + 3*d*x^2 + 4*e*x^3)/(b*x + c*x^2 + d*x^3 + e*x^4)^3,
x]
```

output
```
-1/2*1/(x^2*(b + x*(c + x*(d + e*x)))^2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.025$, Rules used = {2021}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{b+2cx+3dx^2+4ex^3}{(bx+cx^2+dx^3+ex^4)^3} \, dx$$
$$\Big\downarrow \text{2021}$$
$$-\frac{1}{2(bx+cx^2+dx^3+ex^4)^2}$$

input
```
Int[(b + 2*c*x + 3*d*x^2 + 4*e*x^3)/(b*x + c*x^2 + d*x^3 + e*x^4)^3,x]
```

output
```
-1/2*1/(b*x + c*x^2 + d*x^3 + e*x^4)^2
```

Defintions of rubi rules used

rule 2021
```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

method	result
gosper	$-\frac{1}{2x^{2}\left(e\,x^{3}+d\,x^{2}+cx+b\right)^{2}}$
norman	$-\frac{1}{2x^{2}\left(e\,x^{3}+d\,x^{2}+cx+b\right)^{2}}$
risch	$-\frac{1}{2x^{2}\left(e\,x^{3}+d\,x^{2}+cx+b\right)^{2}}$
parallelrisch	$-\frac{1}{2x^{2}\left(e\,x^{3}+d\,x^{2}+cx+b\right)^{2}}$
derivativedivides	$-\frac{1}{2\left(x^{4}e+d\,x^{3}+c\,x^{2}+bx\right)^{2}}$
orering	$-\frac{x\left(e\,x^{3}+d\,x^{2}+cx+b\right)}{2\left(x^{4}e+d\,x^{3}+c\,x^{2}+bx\right)^{3}}$
default	$-\frac{1}{2b^{2}x^{2}}+\frac{c}{b^{3}x}+\frac{-c\,e^{2}x^{5}+\frac{(eb-4cd)e\,x^{4}}{2}+\left(bde-2c^{2}e-c\,d^{2}\right)x^{3}+\left(-bce+\frac{1}{2}b\,d^{2}-2c^{2}d\right)x^{2}+\left(b^{2}e-cbd-c^{3}\right)x+\frac{\left(2bd-3\right.}{2}}{b^{3}\left(e\,x^{3}+d\,x^{2}+cx+b\right)^{2}}$

input
```
int((4*e*x^3+3*d*x^2+2*c*x+b)/(e*x^4+d*x^3+c*x^2+b*x)^3,x,method=_RETURNVE
RBOSE)
```

output
```
-1/2/x^2/(e*x^3+d*x^2+c*x+b)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 69 vs. $2(23) = 46$.

Time = 0.08 (sec) , antiderivative size = 69, normalized size of antiderivative = 2.76

$$\int \frac{b+2cx+3dx^2+4ex^3}{\left(bx+cx^2+dx^3+ex^4\right)^3}\,dx$$
$$= -\frac{1}{2\left(e^2x^8+2\,dex^7+(d^2+2\,ce)x^6+2\,(cd+be)x^5+2\,bcx^3+(c^2+2\,bd)x^4+b^2x^2\right)}$$

input
```
integrate((4*e*x^3+3*d*x^2+2*c*x+b)/(e*x^4+d*x^3+c*x^2+b*x)^3,x, algorithm
="fricas")
```

output
```
-1/2/(e^2*x^8 + 2*d*e*x^7 + (d^2 + 2*c*e)*x^6 + 2*(c*d + b*e)*x^5 + 2*b*c*
x^3 + (c^2 + 2*b*d)*x^4 + b^2*x^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(24) = 48$.

Time = 9.69 (sec) , antiderivative size = 76, normalized size of antiderivative = 3.04

$$\int \frac{b + 2cx + 3dx^2 + 4ex^3}{(bx + cx^2 + dx^3 + ex^4)^3}\,dx = -\frac{1}{2b^2x^2 + 4bcx^3 + 4dex^7 + 2e^2x^8 + x^6 \cdot (4ce + 2d^2) + x^5 \cdot (4be + 4cd) + x^4 \cdot (4bd + 2c^2)}$$

input
```
integrate((4*e*x**3+3*d*x**2+2*c*x+b)/(e*x**4+d*x**3+c*x**2+b*x)**3,x)
```

output
```
-1/(2*b**2*x**2 + 4*b*c*x**3 + 4*d*e*x**7 + 2*e**2*x**8 + x**6*(4*c*e + 2*
d**2) + x**5*(4*b*e + 4*c*d) + x**4*(4*b*d + 2*c**2))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{b + 2cx + 3dx^2 + 4ex^3}{(bx + cx^2 + dx^3 + ex^4)^3}\,dx = -\frac{1}{2\,(ex^4 + dx^3 + cx^2 + bx)^2}$$

input
```
integrate((4*e*x^3+3*d*x^2+2*c*x+b)/(e*x^4+d*x^3+c*x^2+b*x)^3,x, algorithm
="maxima")
```

output
```
-1/2/(e*x^4 + d*x^3 + c*x^2 + b*x)^2
```

Giac [A] (verification not implemented)

Time = 15.62 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{b+2cx+3dx^2+4ex^3}{(bx+cx^2+dx^3+ex^4)^3}\,dx = -\frac{1}{2\,(ex^4+dx^3+cx^2+bx)^2}$$

input
```
integrate((4*e*x^3+3*d*x^2+2*c*x+b)/(e*x^4+d*x^3+c*x^2+b*x)^3,x, algorithm
="giac")
```

output
```
-1/2/(e*x^4 + d*x^3 + c*x^2 + b*x)^2
```

Mupad [B] (verification not implemented)

Time = 23.05 (sec) , antiderivative size = 75, normalized size of antiderivative = 3.00

$$\int \frac{b+2cx+3dx^2+4ex^3}{(bx+cx^2+dx^3+ex^4)^3}\,dx = -\frac{1}{2\,\left(x^4\,\left(c^2+2\,b\,d\right)+x^6\,\left(d^2+2\,c\,e\right)+x^5\,\left(2\,b\,e+2\,c\,d\right)+b^2\,x^2+e^2\,x^8+2\,b\,c\,x^3+2\,d\,e\,x^7\right)}$$

input
```
int((b + 2*c*x + 3*d*x^2 + 4*e*x^3)/(b*x + c*x^2 + d*x^3 + e*x^4)^3,x)
```

output
```
-1/(2*(x^4*(2*b*d + c^2) + x^6*(2*c*e + d^2) + x^5*(2*b*e + 2*c*d) + b^2*x
^2 + e^2*x^8 + 2*b*c*x^3 + 2*d*e*x^7))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 72, normalized size of antiderivative = 2.88

$$\int \frac{b+2cx+3dx^2+4ex^3}{(bx+cx^2+dx^3+ex^4)^3}\,dx = -\frac{1}{2x^2\,(e^2x^6+2de\,x^5+2ce\,x^4+d^2x^4+2be\,x^3+2cd\,x^3+2bd\,x^2+c^2x^2+2bcx+b^2)}$$

input
```
int((4*e*x^3+3*d*x^2+2*c*x+b)/(e*x^4+d*x^3+c*x^2+b*x)^3,x)
```

output
```
( - 1)/(2*x**2*(b**2 + 2*b*c*x + 2*b*d*x**2 + 2*b*e*x**3 + c**2*x**2 + 2*c
*d*x**3 + 2*c*e*x**4 + d**2*x**4 + 2*d*e*x**5 + e**2*x**6))
```

3.6 $\int (b+2cx+3dx^2+4ex^3)(a+bx+cx^2+dx^3+ex^4)$

Optimal result

Integrand size = 41, antiderivative size = 26

$$\int (b+2cx+3dx^2+4ex^3)\left(a+bx+cx^2+dx^3+ex^4\right)^2\,dx = \frac{1}{3}\left(a+bx+cx^2+dx^3+ex^4\right)^3$$

output
```
1/3*(e*x^4+d*x^3+c*x^2+b*x+a)^3
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 60 vs. 2(26) = 52.

Time = 0.10 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.31

$$\int (b+2cx+3dx^2+4ex^3)\left(a+bx+cx^2+dx^3+ex^4\right)^2\,dx = \frac{1}{3}x(b+x(c+x(d+ex)))\left(3a^2+3ax(b+x(c+x(d+ex)))+x^2(b+x(c+x(d+ex)))^2\right)$$

input
```
Integrate[(b + 2*c*x + 3*d*x^2 + 4*e*x^3)*(a + b*x + c*x^2 + d*x^3 + e*x^4
)^2,x]
```

output
```
(x*(b + x*(c + x*(d + e*x)))*(3*a^2 + 3*a*x*(b + x*(c + x*(d + e*x))) + x^
2*(b + x*(c + x*(d + e*x)))^2))/3
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.024$, Rules used = {2021}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(b + 2cx + 3dx^2 + 4ex^3\right)\left(a + bx + cx^2 + dx^3 + ex^4\right)^2 dx$$

$$\Big\downarrow \; 2021$$

$$\frac{1}{3}\left(a + bx + cx^2 + dx^3 + ex^4\right)^3$$

input
```
Int[(b + 2*c*x + 3*d*x^2 + 4*e*x^3)*(a + b*x + c*x^2 + d*x^3 + e*x^4)^2,x]
```

output
```
(a + b*x + c*x^2 + d*x^3 + e*x^4)^3/3
```

Defintions of rubi rules used

rule 2021
```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.96

method	result
default	$\frac{(x^4e+d\,x^3+c\,x^2+bx+a)^3}{3}$
norman	$\frac{e^3x^{12}}{3}+d\,e^2x^{11}+(c\,e^2+d^2e)\,x^{10}+\left(b\,e^2+2cde+\frac{1}{3}d^3\right)x^9+(a\,e^2+2bde+c^2e+c\,d^2)\,x^8$ -
gosper	$\frac{1}{3}b^3x^3+2acd\,x^5+2abd\,x^4+2ade\,x^7+2abe\,x^5+2x^6ace+2abc\,x^3+\frac{1}{3}c^3x^6+x^6b^2e+\frac{1}{3}e^3$
parallelrisch	$\frac{1}{3}b^3x^3+2acd\,x^5+2abd\,x^4+2ade\,x^7+2abe\,x^5+2x^6ace+2abc\,x^3+\frac{1}{3}c^3x^6+x^6b^2e+\frac{1}{3}e^3$
risch	$\frac{1}{3}a^3+\frac{1}{3}b^3x^3+2acd\,x^5+2abd\,x^4+2ade\,x^7+2abe\,x^5+2x^6ace+2abc\,x^3+\frac{1}{3}c^3x^6+x^6b^2e$
orering	$\frac{x(e^3x^{11}+3d\,e^2x^{10}+3c\,e^2x^9+3d^2e\,x^9+3b\,e^2x^8+6cde\,x^8+d^3x^8+3a\,e^2x^7+6bde\,x^7+3c^2e\,x^7+3c\,d^2x^7+6ade\,x^6+6bce\,x^6+3b\,d^2}{}$

input
```
int((4*e*x^3+3*d*x^2+2*c*x+b)*(e*x^4+d*x^3+c*x^2+b*x+a)^2,x,method=_RETURN
VERBOSE)
```

output
```
1/3*(e*x^4+d*x^3+c*x^2+b*x+a)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 224 vs. 2(24) = 48.

Time = 0.10 (sec) , antiderivative size = 224, normalized size of antiderivative = 8.62

$$\begin{aligned}
&\int (b+2cx+3dx^2+4ex^3)\,(a+bx+cx^2+dx^3+ex^4)^2\,dx\\
&=\frac{1}{3}e^3x^{12}+de^2x^{11}+(d^2e+ce^2)x^{10}+\frac{1}{3}\,(d^3+6\,cde+3\,be^2)x^9\\
&\quad+\left(cd^2+ae^2+(c^2+2\,bd)e\right)x^8+\left(c^2d+bd^2+2\,(bc+ad)e\right)x^7\\
&\quad+\frac{1}{3}\left(c^3+6\,bcd+3\,ad^2+3\,(b^2+2\,ac)e\right)x^6+\left(bc^2+2\,abe+(b^2+2\,ac)d\right)x^5\\
&\quad+\left(b^2c+ac^2+2\,abd+a^2e\right)x^4+a^2bx+\frac{1}{3}\,(b^3+6\,abc+3\,a^2d)x^3+\left(ab^2+a^2c\right)x^2
\end{aligned}$$

input
```
integrate((4*e*x^3+3*d*x^2+2*c*x+b)*(e*x^4+d*x^3+c*x^2+b*x+a)^2,x, algorit
hm="fricas")
```

output
```
1/3*e^3*x^12 + d*e^2*x^11 + (d^2*e + c*e^2)*x^10 + 1/3*(d^3 + 6*c*d*e + 3*
b*e^2)*x^9 + (c*d^2 + a*e^2 + (c^2 + 2*b*d)*e)*x^8 + (c^2*d + b*d^2 + 2*(b
*c + a*d)*e)*x^7 + 1/3*(c^3 + 6*b*c*d + 3*a*d^2 + 3*(b^2 + 2*a*c)*e)*x^6 +
 (b*c^2 + 2*a*b*e + (b^2 + 2*a*c)*d)*x^5 + (b^2*c + a*c^2 + 2*a*b*d + a^2*
e)*x^4 + a^2*b*x + 1/3*(b^3 + 6*a*b*c + 3*a^2*d)*x^3 + (a*b^2 + a^2*c)*x^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 228 vs. $2(22) = 44$.

Time = 0.06 (sec) , antiderivative size = 228, normalized size of antiderivative = 8.77

$$\int (b + 2cx + 3dx^2 + 4ex^3) (a + bx + cx^2 + dx^3 + ex^4)^2 \, dx$$
$$= a^2bx + de^2x^{11} + \frac{e^3x^{12}}{3} + x^{10}(ce^2 + d^2e) + x^9\left(be^2 + 2cde + \frac{d^3}{3}\right) + x^8(ae^2 + 2bde + c^2e + cd^2) + x^7 \cdot (2ade + 2bce + bd^2 + c^2d) + x^6 \cdot \left(2ace + ad^2 + b^2e + 2bcd + \frac{c^3}{3}\right) + x^5 \cdot (2abe + 2acd + b^2d + bc^2) + x^4(a^2e + 2abd + ac^2 + b^2c) + x^3\left(a^2d + 2abc + \frac{b^3}{3}\right) + x^2(a^2c + ab^2)$$

input
```
integrate((4*e*x**3+3*d*x**2+2*c*x+b)*(e*x**4+d*x**3+c*x**2+b*x+a)**2,x)
```

output
```
a**2*b*x + d*e**2*x**11 + e**3*x**12/3 + x**10*(c*e**2 + d**2*e) + x**9*(b
*e**2 + 2*c*d*e + d**3/3) + x**8*(a*e**2 + 2*b*d*e + c**2*e + c*d**2) + x*
*7*(2*a*d*e + 2*b*c*e + b*d**2 + c**2*d) + x**6*(2*a*c*e + a*d**2 + b**2*e
 + 2*b*c*d + c**3/3) + x**5*(2*a*b*e + 2*a*c*d + b**2*d + b*c**2) + x**4*(
a**2*e + 2*a*b*d + a*c**2 + b**2*c) + x**3*(a**2*d + 2*a*b*c + b**3/3) + x
**2*(a**2*c + a*b**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int (b + 2cx + 3dx^2 + 4ex^3) (a + bx + cx^2 + dx^3 + ex^4)^2 dx$$
$$= \frac{1}{3} (ex^4 + dx^3 + cx^2 + bx + a)^3$$

input
```
integrate((4*e*x^3+3*d*x^2+2*c*x+b)*(e*x^4+d*x^3+c*x^2+b*x+a)^2,x, algorit
hm="maxima")
```

output
```
1/3*(e*x^4 + d*x^3 + c*x^2 + b*x + a)^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. 2(24) = 48.

Time = 0.12 (sec) , antiderivative size = 70, normalized size of antiderivative = 2.69

$$\int (b + 2cx + 3dx^2 + 4ex^3) (a + bx + cx^2 + dx^3 + ex^4)^2 dx$$
$$= \frac{1}{3} (ex^4 + dx^3 + cx^2 + bx)^3 + (ex^4 + dx^3 + cx^2 + bx)^2 a + (ex^4 + dx^3 + cx^2 + bx) a^2$$

input
```
integrate((4*e*x^3+3*d*x^2+2*c*x+b)*(e*x^4+d*x^3+c*x^2+b*x+a)^2,x, algorit
hm="giac")
```

output
```
1/3*(e*x^4 + d*x^3 + c*x^2 + b*x)^3 + (e*x^4 + d*x^3 + c*x^2 + b*x)^2*a +
(e*x^4 + d*x^3 + c*x^2 + b*x)*a^2
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 217, normalized size of antiderivative = 8.35

$$\int (b + 2cx + 3dx^2 + 4ex^3) (a + bx + cx^2 + dx^3 + ex^4)^2 \, dx$$
$$= x^4 (e\,a^2 + 2\,d\,a\,b + a\,c^2 + b^2\,c) + x^5 (d\,b^2 + b\,c^2 + 2\,a\,e\,b + 2\,a\,d\,c) + x^8 (c^2\,e + c\,d^2 + 2\,b\,d\,e + a\,e^2) + x^7 (c^2\,d + 2\,b\,e\,c + b\,d^2 + 2\,a\,e\,d) + x^3 \left(d\,a^2 + 2\,c\,a\,b + \frac{b^3}{3}\right) + x^9 \left(\frac{d^3}{3} + 2\,c\,d\,e + b\,e^2\right) + x^6 \left(e\,b^2 + 2\,b\,c\,d + \frac{c^3}{3} + 2\,a\,e\,c + a\,d^2\right) + \frac{e^3\,x^{12}}{3} + d\,e^2\,x^{11} + a\,x^2 (b^2 + a\,c) + e\,x^{10} (d^2 + c\,e) + a^2\,b\,x$$

input
```
int((b + 2*c*x + 3*d*x^2 + 4*e*x^3)*(a + b*x + c*x^2 + d*x^3 + e*x^4)^2,x)
```

output
```
x^4*(a*c^2 + b^2*c + a^2*e + 2*a*b*d) + x^5*(b*c^2 + b^2*d + 2*a*b*e + 2*a
*c*d) + x^8*(a*e^2 + c*d^2 + c^2*e + 2*b*d*e) + x^7*(b*d^2 + c^2*d + 2*a*d
*e + 2*b*c*e) + x^3*(a^2*d + b^3/3 + 2*a*b*c) + x^9*(b*e^2 + d^3/3 + 2*c*d
*e) + x^6*(a*d^2 + b^2*e + c^3/3 + 2*a*c*e + 2*b*c*d) + (e^3*x^12)/3 + d*e
^2*x^11 + a*x^2*(a*c + b^2) + e*x^10*(c*e + d^2) + a^2*b*x
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 285, normalized size of antiderivative = 10.96

$$\int (b + 2cx + 3dx^2 + 4ex^3) (a + bx + cx^2 + dx^3 + ex^4)^2 \, dx$$
$$= \frac{x(e^3x^{11} + 3d\,e^2x^{10} + 3c\,e^2x^9 + 3d^2e\,x^9 + 3b\,e^2x^8 + 6cde\,x^8 + d^3x^8 + 3a\,e^2x^7 + 6bde\,x^7 + 3c^2e\,x^7 + 3c\,d^2}{}$$

input
```
int((4*e*x^3+3*d*x^2+2*c*x+b)*(e*x^4+d*x^3+c*x^2+b*x+a)^2,x)
```

output

```
(x*(3*a**2*b + 3*a**2*c*x + 3*a**2*d*x**2 + 3*a**2*e*x**3 + 3*a*b**2*x + 6
*a*b*c*x**2 + 6*a*b*d*x**3 + 6*a*b*e*x**4 + 3*a*c**2*x**3 + 6*a*c*d*x**4 +
 6*a*c*e*x**5 + 3*a*d**2*x**5 + 6*a*d*e*x**6 + 3*a*e**2*x**7 + b**3*x**2 +
 3*b**2*c*x**3 + 3*b**2*d*x**4 + 3*b**2*e*x**5 + 3*b*c**2*x**4 + 6*b*c*d*x
**5 + 6*b*c*e*x**6 + 3*b*d**2*x**6 + 6*b*d*e*x**7 + 3*b*e**2*x**8 + c**3*x
**5 + 3*c**2*d*x**6 + 3*c**2*e*x**7 + 3*c*d**2*x**7 + 6*c*d*e*x**8 + 3*c*e
**2*x**9 + d**3*x**8 + 3*d**2*e*x**9 + 3*d*e**2*x**10 + e**3*x**11))/3
```

3.7 $\int (b+2cx+3dx^2+4ex^3)(a+bx+cx^2+dx^3+ex^4)$

Optimal result

Integrand size = 39, antiderivative size = 26

$$\int (b+2cx+3dx^2+4ex^3)\left(a+bx+cx^2+dx^3+ex^4\right)\,dx=\frac{1}{2}\left(a+bx+cx^2+dx^3+ex^4\right)^2$$

output
```
1/2*(e*x^4+d*x^3+c*x^2+b*x+a)^2
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.42

$$\int (b+2cx+3dx^2+4ex^3)\left(a+bx+cx^2+dx^3+ex^4\right)\,dx$$
$$=\frac{1}{2}x(b+x(c+x(d+ex)))(2a+x(b+x(c+x(d+ex))))$$

input
```
Integrate[(b + 2*c*x + 3*d*x^2 + 4*e*x^3)*(a + b*x + c*x^2 + d*x^3 + e*x^4
),x]
```

output
```
(x*(b + x*(c + x*(d + e*x)))*(2*a + x*(b + x*(c + x*(d + e*x)))))/2
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.026, Rules used = {2021}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b+2cx+3dx^2+4ex^3) (a+bx+cx^2+dx^3+ex^4) \, dx$$

$$\Big\downarrow \text{2021}$$

$$\frac{1}{2}(a+bx+cx^2+dx^3+ex^4)^2$$

input
```
Int[(b + 2*c*x + 3*d*x^2 + 4*e*x^3)*(a + b*x + c*x^2 + d*x^3 + e*x^4),x]
```

output
```
(a + b*x + c*x^2 + d*x^3 + e*x^4)^2/2
```

Defintions of rubi rules used

rule 2021
```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.96

method	result
default	$\frac{\left(x^{4} e+d \,x^{3}+c \,x^{2}+b x+a \right)^{2}}{2}$
norman	$\frac{e^{2} x^{8}}{2}+d e \,x^{7}+\left(c e+\frac{d^{2}}{2}\right) x^{6}+\left(e b+c d \right) x^{5}+\left(a e+b d+\frac{c^{2}}{2}\right) x^{4}+\left(a d+b c \right) x^{3}+\left(a c+\frac{b^{2}}{2}\right.$
gosper	$\frac{1}{2} e^{2} x^{8}+d e \,x^{7}+c e \,x^{6}+\frac{1}{2} x^{6} d^{2}+b \,x^{5} e+c d \,x^{5}+a e \,x^{4}+b d \,x^{4}+\frac{1}{2} c^{2} x^{4}+a d \,x^{3}+b c \,x^{3}+x$
parallelrisch	$\frac{1}{2} e^{2} x^{8}+d e \,x^{7}+c e \,x^{6}+\frac{1}{2} x^{6} d^{2}+b \,x^{5} e+c d \,x^{5}+a e \,x^{4}+b d \,x^{4}+\frac{1}{2} c^{2} x^{4}+a d \,x^{3}+b c \,x^{3}+x$
orering	$\frac{x \left(e^{2} x^{7}+2 d e \,x^{6}+2 c e \,x^{5}+d^{2} x^{5}+2 b e \,x^{4}+2 c d \,x^{4}+2 a e \,x^{3}+2 b d \,x^{3}+c^{2} x^{3}+2 a d \,x^{2}+2 b c \,x^{2}+2 x a c+b^{2} x+2 a b \right)}{2}$
risch	$\frac{1}{2} e^{2} x^{8}+d e \,x^{7}+c e \,x^{6}+\frac{1}{2} x^{6} d^{2}+b \,x^{5} e+c d \,x^{5}+a e \,x^{4}+b d \,x^{4}+\frac{1}{2} c^{2} x^{4}+a d \,x^{3}+b c \,x^{3}+x$

input
```
int((4*e*x^3+3*d*x^2+2*c*x+b)*(e*x^4+d*x^3+c*x^2+b*x+a),x,method=_RETURNVE
RBOSE)
```

output
```
1/2*(e*x^4+d*x^3+c*x^2+b*x+a)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 84 vs. $2(24) = 48$.

Time = 0.09 (sec) , antiderivative size = 84, normalized size of antiderivative = 3.23

$$\int \left(b+2cx+3dx^2+4ex^3\right)\left(a+bx+cx^2+dx^3+ex^4\right)\,dx$$
$$=\frac{1}{2}\,e^2x^8+dex^7+\frac{1}{2}\,(d^2+2\,ce)x^6+(cd+be)x^5+\frac{1}{2}\,(c^2+2\,bd+2\,ae)x^4+(bc+ad)x^3+abx+\frac{1}{2}\,(b^2+2\,ac)x^2$$

input
```
integrate((4*e*x^3+3*d*x^2+2*c*x+b)*(e*x^4+d*x^3+c*x^2+b*x+a),x, algorithm
="fricas")
```

output
```
1/2*e^2*x^8 + d*e*x^7 + 1/2*(d^2 + 2*c*e)*x^6 + (c*d + b*e)*x^5 + 1/2*(c^2
 + 2*b*d + 2*a*e)*x^4 + (b*c + a*d)*x^3 + a*b*x + 1/2*(b^2 + 2*a*c)*x^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 78 vs. $2(22) = 44$.

Time = 0.03 (sec) , antiderivative size = 78, normalized size of antiderivative = 3.00

$$\int (b+2cx+3dx^2+4ex^3)(a+bx+cx^2+dx^3+ex^4)\,dx = abx+dex^7+\frac{e^2x^8}{2}+x^6\left(ce+\frac{d^2}{2}\right)+x^5(be+cd) + x^4\left(ae+bd+\frac{c^2}{2}\right)+x^3(ad+bc)+x^2\left(ac+\frac{b^2}{2}\right)$$

input
```
integrate((4*e*x**3+3*d*x**2+2*c*x+b)*(e*x**4+d*x**3+c*x**2+b*x+a),x)
```

output
```
a*b*x + d*e*x**7 + e**2*x**8/2 + x**6*(c*e + d**2/2) + x**5*(b*e + c*d) +
x**4*(a*e + b*d + c**2/2) + x**3*(a*d + b*c) + x**2*(a*c + b**2/2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int (b+2cx+3dx^2+4ex^3)(a+bx+cx^2+dx^3+ex^4)\,dx = \frac{1}{2}(ex^4+dx^3+cx^2+bx+a)^2$$

input
```
integrate((4*e*x^3+3*d*x^2+2*c*x+b)*(e*x^4+d*x^3+c*x^2+b*x+a),x, algorithm
="maxima")
```

output
```
1/2*(e*x^4 + d*x^3 + c*x^2 + b*x + a)^2
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.73

$$\int (b + 2cx + 3dx^2 + 4ex^3) (a + bx + cx^2 + dx^3 + ex^4) \, dx$$
$$= \frac{1}{2} (ex^4 + dx^3 + cx^2 + bx)^2 + (ex^4 + dx^3 + cx^2 + bx)a$$

input
```
integrate((4*e*x^3+3*d*x^2+2*c*x+b)*(e*x^4+d*x^3+c*x^2+b*x+a),x, algorithm
="giac")
```

output
```
1/2*(e*x^4 + d*x^3 + c*x^2 + b*x)^2 + (e*x^4 + d*x^3 + c*x^2 + b*x)*a
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 83, normalized size of antiderivative = 3.19

$$\int (b + 2cx + 3dx^2 + 4ex^3) (a + bx + cx^2 + dx^3 + ex^4) \, dx$$
$$= x^4 \left(\frac{c^2}{2} + a\,e + b\,d\right) + x^2 \left(\frac{b^2}{2} + a\,c\right) + x^3 (a\,d + b\,c)$$
$$+ x^6 \left(\frac{d^2}{2} + c\,e\right) + x^5 (b\,e + c\,d) + \frac{e^2\,x^8}{2} + a\,b\,x + d\,e\,x^7$$

input
```
int((b + 2*c*x + 3*d*x^2 + 4*e*x^3)*(a + b*x + c*x^2 + d*x^3 + e*x^4),x)
```

output
```
x^4*(a*e + b*d + c^2/2) + x^2*(a*c + b^2/2) + x^3*(a*d + b*c) + x^6*(c*e +
 d^2/2) + x^5*(b*e + c*d) + (e^2*x^8)/2 + a*b*x + d*e*x^7
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 95, normalized size of antiderivative = 3.65

$$\int (b + 2cx + 3dx^2 + 4ex^3) (a + bx + cx^2 + dx^3 + ex^4) \, dx$$
$$= \frac{x(e^2x^7 + 2de\,x^6 + 2ce\,x^5 + d^2x^5 + 2be\,x^4 + 2cd\,x^4 + 2ae\,x^3 + 2bd\,x^3 + c^2x^3 + 2ad\,x^2 + 2bc\,x^2 + 2acx +}{2}$$

input
```
int((4*e*x^3+3*d*x^2+2*c*x+b)*(e*x^4+d*x^3+c*x^2+b*x+a),x)
```

output
```
(x*(2*a*b + 2*a*c*x + 2*a*d*x**2 + 2*a*e*x**3 + b**2*x + 2*b*c*x**2 + 2*b*
d*x**3 + 2*b*e*x**4 + c**2*x**3 + 2*c*d*x**4 + 2*c*e*x**5 + d**2*x**5 + 2*
d*e*x**6 + e**2*x**7))/2
```

3.8 $\int \frac{b+2cx+3dx^2+4ex^3}{a+bx+cx^2+dx^3+ex^4} \, dx$

Optimal result

Integrand size = 41, antiderivative size = 21

$$\int \frac{b+2cx+3dx^2+4ex^3}{a+bx+cx^2+dx^3+ex^4} \, dx = \log \left(a+bx+cx^2+dx^3+ex^4\right)$$

output
```
ln(e*x^4+d*x^3+c*x^2+b*x+a)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

$$\int \frac{b+2cx+3dx^2+4ex^3}{a+bx+cx^2+dx^3+ex^4} \, dx = \log(a+x(b+x(c+x(d+ex))))$$

input
```
Integrate[(b + 2*c*x + 3*d*x^2 + 4*e*x^3)/(a + b*x + c*x^2 + d*x^3 + e*x^4
),x]
```

output
```
Log[a + x*(b + x*(c + x*(d + e*x)))]
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.024$, Rules used = {2020}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{b+2cx+3dx^2+4ex^3}{a+bx+cx^2+dx^3+ex^4}\,dx$$

$$\Big\downarrow \; 2020$$

$$\log\left(a+bx+cx^2+dx^3+ex^4\right)$$

input
```
Int[(b + 2*c*x + 3*d*x^2 + 4*e*x^3)/(a + b*x + c*x^2 + d*x^3 + e*x^4),x]
```

output
```
Log[a + b*x + c*x^2 + d*x^3 + e*x^4]
```

Defintions of rubi rules used

rule 2020
```
Int[(Pp_)/(Qq_), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Si
mp[Coeff[Pp, x, p]*(Log[RemoveContent[Qq, x]]/(q*Coeff[Qq, x, q])), x] /; E
qQ[p, q - 1] && EqQ[Pp, Simplify[(Coeff[Pp, x, p]/(q*Coeff[Qq, x, q]))*D[Qq
, x]]]] /; PolyQ[Pp, x] && PolyQ[Qq, x]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.05

method	result	size
derivativedivides	$\ln\left(x^4e+d\,x^3+c\,x^2+bx+a\right)$	22
default	$\ln\left(x^4e+d\,x^3+c\,x^2+bx+a\right)$	22
norman	$\ln\left(x^4e+d\,x^3+c\,x^2+bx+a\right)$	22
risch	$\ln\left(x^4e+d\,x^3+c\,x^2+bx+a\right)$	22
parallelrisch	$\ln\left(x^4e+d\,x^3+c\,x^2+bx+a\right)$	22

input
```
int((4*e*x^3+3*d*x^2+2*c*x+b)/(e*x^4+d*x^3+c*x^2+b*x+a),x,method=_RETURNVE
RBOSE)
```

output
```
ln(e*x^4+d*x^3+c*x^2+b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{b+2cx+3dx^2+4ex^3}{a+bx+cx^2+dx^3+ex^4}\,dx=\log\left(ex^4+dx^3+cx^2+bx+a\right)$$

input
```
integrate((4*e*x^3+3*d*x^2+2*c*x+b)/(e*x^4+d*x^3+c*x^2+b*x+a),x, algorithm
="fricas")
```

output
```
log(e*x^4 + d*x^3 + c*x^2 + b*x + a)
```

Sympy [A] (verification not implemented)

Time = 1.13 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int \frac{b+2cx+3dx^2+4ex^3}{a+bx+cx^2+dx^3+ex^4}\,dx=\log\left(a+bx+cx^2+dx^3+ex^4\right)$$

input
```
integrate((4*e*x**3+3*d*x**2+2*c*x+b)/(e*x**4+d*x**3+c*x**2+b*x+a),x)
```

output
```
log(a + b*x + c*x**2 + d*x**3 + e*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{b+2cx+3dx^2+4ex^3}{a+bx+cx^2+dx^3+ex^4}\,dx = \log\left(ex^4+dx^3+cx^2+bx+a\right)$$

input
```
integrate((4*e*x^3+3*d*x^2+2*c*x+b)/(e*x^4+d*x^3+c*x^2+b*x+a),x, algorithm
="maxima")
```

output
```
log(e*x^4 + d*x^3 + c*x^2 + b*x + a)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.05

$$\int \frac{b+2cx+3dx^2+4ex^3}{a+bx+cx^2+dx^3+ex^4}\,dx = \log\left(\left|ex^4+dx^3+cx^2+bx+a\right|\right)$$

input
```
integrate((4*e*x^3+3*d*x^2+2*c*x+b)/(e*x^4+d*x^3+c*x^2+b*x+a),x, algorithm
="giac")
```

output
```
log(abs(e*x^4 + d*x^3 + c*x^2 + b*x + a))
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{b+2cx+3dx^2+4ex^3}{a+bx+cx^2+dx^3+ex^4}\,dx = \ln\left(e\,x^4+d\,x^3+c\,x^2+b\,x+a\right)$$

input
```
int((b + 2*c*x + 3*d*x^2 + 4*e*x^3)/(a + b*x + c*x^2 + d*x^3 + e*x^4),x)
```

output
```
log(a + b*x + c*x^2 + d*x^3 + e*x^4)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{b+2cx+3dx^2+4ex^3}{a+bx+cx^2+dx^3+ex^4}\,dx = \log\left(e\,x^4+d\,x^3+c\,x^2+bx+a\right)$$

input
```
int((4*e*x^3+3*d*x^2+2*c*x+b)/(e*x^4+d*x^3+c*x^2+b*x+a),x)
```

output
```
log(a + b*x + c*x**2 + d*x**3 + e*x**4)
```

3.9 $\int \frac{b+2cx+3dx^2+4ex^3}{(a+bx+cx^2+dx^3+ex^4)^2}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 24

$$\int \frac{b+2cx+3dx^2+4ex^3}{(a+bx+cx^2+dx^3+ex^4)^2}\,dx=-\frac{1}{a+bx+cx^2+dx^3+ex^4}$$

output
```
-1/(e*x^4+d*x^3+c*x^2+b*x+a)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.88

$$\int \frac{b+2cx+3dx^2+4ex^3}{(a+bx+cx^2+dx^3+ex^4)^2}\,dx=-\frac{1}{a+x(b+x(c+x(d+ex)))}$$

input
```
Integrate[(b + 2*c*x + 3*d*x^2 + 4*e*x^3)/(a + b*x + c*x^2 + d*x^3 + e*x^4
)^2,x]
```

output
```
-(a + x*(b + x*(c + x*(d + e*x))))^(-1)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.024$, Rules used = $\{2021\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{b+2cx+3dx^2+4ex^3}{(a+bx+cx^2+dx^3+ex^4)^2}\,dx$$

$$\Big\downarrow \ 2021$$

$$-\frac{1}{a+bx+cx^2+dx^3+ex^4}$$

input
```
Int[(b + 2*c*x + 3*d*x^2 + 4*e*x^3)/(a + b*x + c*x^2 + d*x^3 + e*x^4)^2,x]
```

output
```
-(a + b*x + c*x^2 + d*x^3 + e*x^4)^(-1)
```

Defintions of rubi rules used

rule 2021
```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.04

method	result	size
gosper	$-\frac{1}{x^4e+d\,x^3+c\,x^2+bx+a}$	25
derivativedivides	$-\frac{1}{x^4e+d\,x^3+c\,x^2+bx+a}$	25
default	$-\frac{1}{x^4e+d\,x^3+c\,x^2+bx+a}$	25
risch	$-\frac{1}{x^4e+d\,x^3+c\,x^2+bx+a}$	25
orering	$-\frac{1}{x^4e+d\,x^3+c\,x^2+bx+a}$	25
parallelrisch	$\frac{x^4e+d\,x^3+c\,x^2+bx}{a(x^4e+d\,x^3+c\,x^2+bx+a)}$	46
norman	$\frac{\frac{bx}{a}+\frac{c\,x^2}{a}+\frac{d\,x^3}{a}+\frac{e\,x^4}{a}}{x^4e+d\,x^3+c\,x^2+bx+a}$	55

input
```
int((4*e*x^3+3*d*x^2+2*c*x+b)/(e*x^4+d*x^3+c*x^2+b*x+a)^2,x,method=_RETURN
VERBOSE)
```

output
```
-1/(e*x^4+d*x^3+c*x^2+b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{b+2cx+3dx^2+4ex^3}{(a+bx+cx^2+dx^3+ex^4)^2}\,dx = -\frac{1}{ex^4+dx^3+cx^2+bx+a}$$

input
```
integrate((4*e*x^3+3*d*x^2+2*c*x+b)/(e*x^4+d*x^3+c*x^2+b*x+a)^2,x, algorit
hm="fricas")
```

output
```
-1/(e*x^4 + d*x^3 + c*x^2 + b*x + a)
```

Sympy [A] (verification not implemented)

Time = 10.77 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{b+2cx+3dx^2+4ex^3}{(a+bx+cx^2+dx^3+ex^4)^2}\,dx=-\frac{1}{a+bx+cx^2+dx^3+ex^4}$$

input
```
integrate((4*e*x**3+3*d*x**2+2*c*x+b)/(e*x**4+d*x**3+c*x**2+b*x+a)**2,x)
```

output
```
-1/(a + b*x + c*x**2 + d*x**3 + e*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{b+2cx+3dx^2+4ex^3}{(a+bx+cx^2+dx^3+ex^4)^2}\,dx=-\frac{1}{ex^4+dx^3+cx^2+bx+a}$$

input
```
integrate((4*e*x^3+3*d*x^2+2*c*x+b)/(e*x^4+d*x^3+c*x^2+b*x+a)^2,x, algorit
hm="maxima")
```

output
```
-1/(e*x^4 + d*x^3 + c*x^2 + b*x + a)
```

Giac [F(-1)]

Timed out.

$$\int \frac{b+2cx+3dx^2+4ex^3}{(a+bx+cx^2+dx^3+ex^4)^2}\,dx=\text{Timed out}$$

input
```
integrate((4*e*x^3+3*d*x^2+2*c*x+b)/(e*x^4+d*x^3+c*x^2+b*x+a)^2,x, algorit
hm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{b+2cx+3dx^2+4ex^3}{(a+bx+cx^2+dx^3+ex^4)^2}\,dx=-\frac{1}{e\,x^4+d\,x^3+c\,x^2+b\,x+a}$$

input
```
int((b + 2*c*x + 3*d*x^2 + 4*e*x^3)/(a + b*x + c*x^2 + d*x^3 + e*x^4)^2,x)
```

output
```
-1/(a + b*x + c*x^2 + d*x^3 + e*x^4)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{b+2cx+3dx^2+4ex^3}{(a+bx+cx^2+dx^3+ex^4)^2}\,dx=-\frac{1}{e\,x^4+d\,x^3+c\,x^2+bx+a}$$

input
```
int((4*e*x^3+3*d*x^2+2*c*x+b)/(e*x^4+d*x^3+c*x^2+b*x+a)^2,x)
```

output
```
( - 1)/(a + b*x + c*x**2 + d*x**3 + e*x**4)
```

3.10 $\int \frac{b+2cx+3dx^2+4ex^3}{\left(a+bx+cx^2+dx^3+ex^4\right)^3} dx$

Optimal result

Integrand size = 41, antiderivative size = 26

$$\int \frac{b+2cx+3dx^2+4ex^3}{\left(a+bx+cx^2+dx^3+ex^4\right)^3} dx = -\frac{1}{2\left(a+bx+cx^2+dx^3+ex^4\right)^2}$$

output
```
-1/2/(e*x^4+d*x^3+c*x^2+b*x+a)^2
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.88

$$\int \frac{b+2cx+3dx^2+4ex^3}{\left(a+bx+cx^2+dx^3+ex^4\right)^3} dx = -\frac{1}{2(a+x(b+x(c+x(d+ex))))^2}$$

input
```
Integrate[(b + 2*c*x + 3*d*x^2 + 4*e*x^3)/(a + b*x + c*x^2 + d*x^3 + e*x^4
)^3,x]
```

output
```
-1/2*1/(a + x*(b + x*(c + x*(d + e*x))))^2
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.024, Rules used = {2021}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{b+2cx+3dx^2+4ex^3}{(a+bx+cx^2+dx^3+ex^4)^3}\,dx$$

$$\Big\downarrow \text{2021}$$

$$-\frac{1}{2(a+bx+cx^2+dx^3+ex^4)^2}$$

input
```
Int[(b + 2*c*x + 3*d*x^2 + 4*e*x^3)/(a + b*x + c*x^2 + d*x^3 + e*x^4)^3,x]
```

output
```
-1/2*1/(a + b*x + c*x^2 + d*x^3 + e*x^4)^2
```

Defintions of rubi rules used

rule 2021
```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.96

method	result	size
gosper	$-\frac{1}{2(x^4e+d\,x^3+c\,x^2+bx+a)^2}$	25
derivativedivides	$-\frac{1}{2(x^4e+d\,x^3+c\,x^2+bx+a)^2}$	25
default	$-\frac{1}{2(x^4e+d\,x^3+c\,x^2+bx+a)^2}$	25
norman	$-\frac{1}{2(x^4e+d\,x^3+c\,x^2+bx+a)^2}$	25
risch	$-\frac{1}{2(x^4e+d\,x^3+c\,x^2+bx+a)^2}$	25
parallelrisch	$-\frac{1}{2(x^4e+d\,x^3+c\,x^2+bx+a)^2}$	25
orering	$-\frac{1}{2(x^4e+d\,x^3+c\,x^2+bx+a)^2}$	25

input
```
int((4*e*x^3+3*d*x^2+2*c*x+b)/(e*x^4+d*x^3+c*x^2+b*x+a)^3,x,method=_RETURN
VERBOSE)
```

output
```
-1/2/(e*x^4+d*x^3+c*x^2+b*x+a)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 91 vs. 2(24) = 48.

Time = 0.13 (sec) , antiderivative size = 91, normalized size of antiderivative = 3.50

$$\int \frac{b+2cx+3dx^2+4ex^3}{(a+bx+cx^2+dx^3+ex^4)^3}\,dx = -\frac{1}{2\left(e^2x^8+2\,dex^7+(d^2+2\,ce)x^6+2\,(cd+be)x^5+(c^2+2\,bd+2\,ae)x^4+2\,(bc+ad)x^3+2\,abx+(b^2}$$

input
```
integrate((4*e*x^3+3*d*x^2+2*c*x+b)/(e*x^4+d*x^3+c*x^2+b*x+a)^3,x, algorit
hm="fricas")
```

output
```
-1/2/(e^2*x^8 + 2*d*e*x^7 + (d^2 + 2*c*e)*x^6 + 2*(c*d + b*e)*x^5 + (c^2 +
 2*b*d + 2*a*e)*x^4 + 2*(b*c + a*d)*x^3 + 2*a*b*x + (b^2 + 2*a*c)*x^2 + a^
2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 104 vs. $2(26) = 52$.

Time = 99.84 (sec) , antiderivative size = 104, normalized size of antiderivative = 4.00

$$\int \frac{b + 2cx + 3dx^2 + 4ex^3}{(a + bx + cx^2 + dx^3 + ex^4)^3}\,dx =$$
$$-\frac{1}{2a^2 + 4abx + 4dex^7 + 2e^2x^8 + x^6 \cdot (4ce + 2d^2) + x^5 \cdot (4be + 4cd) + x^4 \cdot (4ae + 4bd + 2c^2) + x^3 \cdot (4ad}$$

input
```
integrate((4*e*x**3+3*d*x**2+2*c*x+b)/(e*x**4+d*x**3+c*x**2+b*x+a)**3,x)
```

output
```
-1/(2*a**2 + 4*a*b*x + 4*d*e*x**7 + 2*e**2*x**8 + x**6*(4*c*e + 2*d**2) +
x**5*(4*b*e + 4*c*d) + x**4*(4*a*e + 4*b*d + 2*c**2) + x**3*(4*a*d + 4*b*c
) + x**2*(4*a*c + 2*b**2))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{b + 2cx + 3dx^2 + 4ex^3}{(a + bx + cx^2 + dx^3 + ex^4)^3}\,dx = -\frac{1}{2\,(ex^4 + dx^3 + cx^2 + bx + a)^2}$$

input
```
integrate((4*e*x^3+3*d*x^2+2*c*x+b)/(e*x^4+d*x^3+c*x^2+b*x+a)^3,x, algorit
hm="maxima")
```

output
```
-1/2/(e*x^4 + d*x^3 + c*x^2 + b*x + a)^2
```

Giac [F(-1)]

Timed out.

$$\int \frac{b+2cx+3dx^2+4ex^3}{(a+bx+cx^2+dx^3+ex^4)^3}\,dx = \text{Timed out}$$

input
```
integrate((4*e*x^3+3*d*x^2+2*c*x+b)/(e*x^4+d*x^3+c*x^2+b*x+a)^3,x, algorit
hm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 23.45 (sec) , antiderivative size = 101, normalized size of antiderivative = 3.88

$$\int \frac{b+2cx+3dx^2+4ex^3}{(a+bx+cx^2+dx^3+ex^4)^3}\,dx = -\frac{1}{2\left(x^2\left(b^2+2\,a\,c\right)+x^6\left(d^2+2\,c\,e\right)+x^4\left(c^2+2\,a\,e+2\,b\,d\right)+a^2+x^3\left(2\,a\,d+2\,b\,c\right)+x^5\left(2\,b\,e+2\,c\,d\right)\right.}$$

input
```
int((b + 2*c*x + 3*d*x^2 + 4*e*x^3)/(a + b*x + c*x^2 + d*x^3 + e*x^4)^3,x)
```

output
```
-1/(2*(x^2*(2*a*c + b^2) + x^6*(2*c*e + d^2) + x^4*(2*a*e + 2*b*d + c^2) +
 a^2 + x^3*(2*a*d + 2*b*c) + x^5*(2*b*e + 2*c*d) + e^2*x^8 + 2*a*b*x + 2*d
*e*x^7))
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 110, normalized size of antiderivative = 4.23

$$\int \frac{b+2cx+3dx^2+4ex^3}{(a+bx+cx^2+dx^3+ex^4)^3}\,dx = -\frac{1}{2e^2x^8+4de\,x^7+4ce\,x^6+2d^2x^6+4be\,x^5+4cd\,x^5+4ae\,x^4+4bd\,x^4+2c^2x^4+4ad\,x^3+4bc\,x^3+4ac}$$

input
```
int((4*e*x^3+3*d*x^2+2*c*x+b)/(e*x^4+d*x^3+c*x^2+b*x+a)^3,x)
```

output
```
( - 1)/(2*(a**2 + 2*a*b*x + 2*a*c*x**2 + 2*a*d*x**3 + 2*a*e*x**4 + b**2*x*
*2 + 2*b*c*x**3 + 2*b*d*x**4 + 2*b*e*x**5 + c**2*x**4 + 2*c*d*x**5 + 2*c*e
*x**6 + d**2*x**6 + 2*d*e*x**7 + e**2*x**8))
```

3.11 $\int (b+2cx+3dx^2+4ex^3)(bx+cx^2+dx^3+ex^4)^p\,dx$

Optimal result

Integrand size = 40, antiderivative size = 29

$$\int (b+2cx+3dx^2+4ex^3)(bx+cx^2+dx^3+ex^4)^p\,dx=\frac{(bx+cx^2+dx^3+ex^4)^{1+p}}{1+p}$$

output
```
(e*x^4+d*x^3+c*x^2+b*x)^(p+1)/(p+1)
```

Mathematica [A] (verified)

Time = 0.73 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int (b+2cx+3dx^2+4ex^3)(bx+cx^2+dx^3+ex^4)^p\,dx=\frac{(x(b+x(c+x(d+ex))))^{1+p}}{1+p}$$

input
```
Integrate[(b + 2*c*x + 3*d*x^2 + 4*e*x^3)*(b*x + c*x^2 + d*x^3 + e*x^4)^p,
x]
```

output
```
(x*(b + x*(c + x*(d + e*x))))^(1 + p)/(1 + p)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.025, Rules used = {2021}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b + 2cx + 3dx^2 + 4ex^3) (bx + cx^2 + dx^3 + ex^4)^p \, dx$$

$$\Big\downarrow \text{2021}$$

$$\frac{(bx + cx^2 + dx^3 + ex^4)^{p+1}}{p+1}$$

input
```
Int[(b + 2*c*x + 3*d*x^2 + 4*e*x^3)*(b*x + c*x^2 + d*x^3 + e*x^4)^p,x]
```

output
```
(b*x + c*x^2 + d*x^3 + e*x^4)^(1 + p)/(1 + p)
```

Defintions of rubi rules used

rule 2021
```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.03

method	result
derivativedivides	$\frac{\left(x^{4} e+d\,x^{3}+c\,x^{2}+b x\right)^{p+1}}{p+1}$
default	$\frac{\left(x^{4} e+d\,x^{3}+c\,x^{2}+b x\right)^{p+1}}{p+1}$
risch	$\frac{\left(e\,x^{3}+d\,x^{2}+c x+b\right) x \left(x \left(e\,x^{3}+d\,x^{2}+c x+b\right)\right)^{p}}{p+1}$
gosper	$\frac{x \left(e\,x^{3}+d\,x^{2}+c x+b\right) \left(x^{4} e+d\,x^{3}+c\,x^{2}+b x\right)^{p}}{p+1}$
orering	$\frac{x \left(e\,x^{3}+d\,x^{2}+c x+b\right) \left(x^{4} e+d\,x^{3}+c\,x^{2}+b x\right)^{p}}{p+1}$
parallelrisch	$\frac{x^{4} \left(x \left(e\,x^{3}+d\,x^{2}+c x+b\right)\right)^{p} e^{2}+x^{3} \left(x \left(e\,x^{3}+d\,x^{2}+c x+b\right)\right)^{p} d e+x^{2} \left(x \left(e\,x^{3}+d\,x^{2}+c x+b\right)\right)^{p} c e+x \left(x \left(e\,x^{3}+d\,x^{2}+c x+b\right)\right)^{p} b}{e \left(p+1\right)}$
norman	$\frac{b x\,{\mathrm e}^{p \ln \left(x^{4} e+d\,x^{3}+c\,x^{2}+b x\right)}}{p+1}+\frac{c\,x^{2} {\mathrm e}^{p \ln \left(x^{4} e+d\,x^{3}+c\,x^{2}+b x\right)}}{p+1}+\frac{d\,x^{3} {\mathrm e}^{p \ln \left(x^{4} e+d\,x^{3}+c\,x^{2}+b x\right)}}{p+1}+\frac{e\,x^{4} {\mathrm e}^{p \ln \left(x^{4} e+d\,x^{3}+c \right.}}{p+1}$

input
```
int((4*e*x^3+3*d*x^2+2*c*x+b)*(e*x^4+d*x^3+c*x^2+b*x)^p,x,method=_RETURNVE
RBOSE)
```

output
```
(e*x^4+d*x^3+c*x^2+b*x)^(p+1)/(p+1)
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.59

$$\int \left(b+2cx+3dx^2+4ex^3\right)\left(bx+cx^2+dx^3+ex^4\right)^p\,dx = \frac{(ex^4+dx^3+cx^2+bx)(ex^4+dx^3+cx^2+bx)^p}{p+1}$$

input
```
integrate((4*e*x^3+3*d*x^2+2*c*x+b)*(e*x^4+d*x^3+c*x^2+b*x)^p,x, algorithm
="fricas")
```

output
```
(e*x^4 + d*x^3 + c*x^2 + b*x)*(e*x^4 + d*x^3 + c*x^2 + b*x)^p/(p + 1)
```

Sympy [F(-1)]

Timed out.

$$\int (b + 2cx + 3dx^2 + 4ex^3) (bx + cx^2 + dx^3 + ex^4)^p \, dx = \text{Timed out}$$

input
```
integrate((4*e*x**3+3*d*x**2+2*c*x+b)*(e*x**4+d*x**3+c*x**2+b*x)**p,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int (b + 2cx + 3dx^2 + 4ex^3) (bx + cx^2 + dx^3 + ex^4)^p \, dx = \frac{(ex^4 + dx^3 + cx^2 + bx)^{p+1}}{p + 1}$$

input
```
integrate((4*e*x^3+3*d*x^2+2*c*x+b)*(e*x^4+d*x^3+c*x^2+b*x)^p,x, algorithm
="maxima")
```

output
```
(e*x^4 + d*x^3 + c*x^2 + b*x)^(p + 1)/(p + 1)
```

Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int (b + 2cx + 3dx^2 + 4ex^3) (bx + cx^2 + dx^3 + ex^4)^p \, dx = \frac{(ex^4 + dx^3 + cx^2 + bx)^{p+1}}{p + 1}$$

input
```
integrate((4*e*x^3+3*d*x^2+2*c*x+b)*(e*x^4+d*x^3+c*x^2+b*x)^p,x, algorithm
="giac")
```

output
```
(e*x^4 + d*x^3 + c*x^2 + b*x)^(p + 1)/(p + 1)
```

Mupad [B] (verification not implemented)

Time = 22.88 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.10

$$\int \left(b+2cx+3dx^2+4ex^3\right)\left(bx+cx^2+dx^3+ex^4\right)^p\,dx = \left(\frac{b\,x}{p+1}+\frac{c\,x^2}{p+1}+\frac{d\,x^3}{p+1}+\frac{e\,x^4}{p+1}\right)\left(e\,x^4+d\,x^3+c\,x^2+b\,x\right)^p$$

input
```
int((b + 2*c*x + 3*d*x^2 + 4*e*x^3)*(b*x + c*x^2 + d*x^3 + e*x^4)^p,x)
```

output
```
((b*x)/(p + 1) + (c*x^2)/(p + 1) + (d*x^3)/(p + 1) + (e*x^4)/(p + 1))*(b*x
 + c*x^2 + d*x^3 + e*x^4)^p
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.48

$$\int \left(b+2cx+3dx^2+4ex^3\right)\left(bx+cx^2+dx^3+ex^4\right)^p\,dx = \frac{\left(e\,x^4+d\,x^3+c\,x^2+bx\right)^p x\left(e\,x^3+d\,x^2+cx+b\right)}{p+1}$$

input
```
int((4*e*x^3+3*d*x^2+2*c*x+b)*(e*x^4+d*x^3+c*x^2+b*x)^p,x)
```

output
```
((b*x + c*x**2 + d*x**3 + e*x**4)**p*x*(b + c*x + d*x**2 + e*x**3))/(p + 1
)
```

3.12 $\int (b+2cx+3dx^2+4ex^3)(a+bx+cx^2+dx^3+ex^4$

Optimal result

Integrand size = 41, antiderivative size = 30

$$\int (b+2cx+3dx^2+4ex^3)(a+bx+cx^2+dx^3+ex^4)^p\,dx = \frac{(a+bx+cx^2+dx^3+ex^4)^{1+p}}{1+p}$$

output
```
(e*x^4+d*x^3+c*x^2+b*x+a)^(p+1)/(p+1)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.90

$$\int (b+2cx+3dx^2+4ex^3)(a+bx+cx^2+dx^3+ex^4)^p\,dx = \frac{(a+x(b+x(c+x(d+ex))))^{1+p}}{1+p}$$

input
```
Integrate[(b + 2*c*x + 3*d*x^2 + 4*e*x^3)*(a + b*x + c*x^2 + d*x^3 + e*x^4
)^p,x]
```

output
```
(a + x*(b + x*(c + x*(d + e*x))))^(1 + p)/(1 + p)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.024$, Rules used = {2021}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(b+2cx+3dx^2+4ex^3\right)\left(a+bx+cx^2+dx^3+ex^4\right)^p\,dx$$

$$\Big\downarrow \ 2021$$

$$\frac{\left(a+bx+cx^2+dx^3+ex^4\right)^{p+1}}{p+1}$$

input
```
Int[(b + 2*c*x + 3*d*x^2 + 4*e*x^3)*(a + b*x + c*x^2 + d*x^3 + e*x^4)^p,x]
```

output
```
(a + b*x + c*x^2 + d*x^3 + e*x^4)^(1 + p)/(1 + p)
```

Defintions of rubi rules used

rule 2021
```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.03

method	result
gosper	$\frac{\left(x^{4} e+d\,x^{3}+c\,x^{2}+b x+a\right)^{p+1}}{p+1}$
derivativedivides	$\frac{\left(x^{4} e+d\,x^{3}+c\,x^{2}+b x+a\right)^{p+1}}{p+1}$
default	$\frac{\left(x^{4} e+d\,x^{3}+c\,x^{2}+b x+a\right)^{p+1}}{p+1}$
risch	$\frac{\left(x^{4} e+d\,x^{3}+c\,x^{2}+b x+a\right)\left(x^{4} e+d\,x^{3}+c\,x^{2}+b x+a\right)^{p}}{p+1}$
orering	$\frac{\left(x^{4} e+d\,x^{3}+c\,x^{2}+b x+a\right)\left(x^{4} e+d\,x^{3}+c\,x^{2}+b x+a\right)^{p}}{p+1}$
parallelrisch	$\frac{x^{4}\left(x^{4} e+d\,x^{3}+c\,x^{2}+b x+a\right)^{p} e^{2}+x^{3}\left(x^{4} e+d\,x^{3}+c\,x^{2}+b x+a\right)^{p} d e+x^{2}\left(x^{4} e+d\,x^{3}+c\,x^{2}+b x+a\right)^{p} c e+x\left(x^{4} e+d\,x^{3}+c\,x^{2}+\right.}{e(p+1)}$
norman	$\frac{a\,{\mathrm e}^{p \ln\left(x^{4} e+d\,x^{3}+c\,x^{2}+b x+a\right)}}{p+1}+\frac{b x\,{\mathrm e}^{p \ln\left(x^{4} e+d\,x^{3}+c\,x^{2}+b x+a\right)}}{p+1}+\frac{c\,x^{2} {\mathrm e}^{p \ln\left(x^{4} e+d\,x^{3}+c\,x^{2}+b x+a\right)}}{p+1}+\frac{d\,x^{3} {\mathrm e}^{p \ln\left(x^{4} e+\right.}}{p}$

input
```
int((4*e*x^3+3*d*x^2+2*c*x+b)*(e*x^4+d*x^3+c*x^2+b*x+a)^p,x,method=_RETURN
VERBOSE)
```

output
```
(e*x^4+d*x^3+c*x^2+b*x+a)^(p+1)/(p+1)
```

Fricas [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.60

$$\int (b+2cx+3dx^2+4ex^3)\left(a+bx+cx^2+dx^3+ex^4\right)^p\,dx = \frac{(ex^4+dx^3+cx^2+bx+a)(ex^4+dx^3+cx^2+bx+a)^p}{p+1}$$

input
```
integrate((4*e*x^3+3*d*x^2+2*c*x+b)*(e*x^4+d*x^3+c*x^2+b*x+a)^p,x, algorit
hm="fricas")
```

output
```
(e*x^4 + d*x^3 + c*x^2 + b*x + a)*(e*x^4 + d*x^3 + c*x^2 + b*x + a)^p/(p +
 1)
```

Sympy [F(-1)]

Timed out.

$$\int \left(b+2cx+3dx^2+4ex^3\right)\left(a+bx+cx^2+dx^3+ex^4\right)^p\,dx = \text{Timed out}$$

input
```
integrate((4*e*x**3+3*d*x**2+2*c*x+b)*(e*x**4+d*x**3+c*x**2+b*x+a)**p,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \left(b+2cx+3dx^2+4ex^3\right)\left(a+bx+cx^2+dx^3+ex^4\right)^p\,dx = \frac{\left(ex^4+dx^3+cx^2+bx+a\right)^{p+1}}{p+1}$$

input
```
integrate((4*e*x^3+3*d*x^2+2*c*x+b)*(e*x^4+d*x^3+c*x^2+b*x+a)^p,x, algorit
hm="maxima")
```

output
```
(e*x^4 + d*x^3 + c*x^2 + b*x + a)^(p + 1)/(p + 1)
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \left(b+2cx+3dx^2+4ex^3\right)\left(a+bx+cx^2+dx^3+ex^4\right)^p\,dx = \frac{\left(ex^4+dx^3+cx^2+bx+a\right)^{p+1}}{p+1}$$

input
```
integrate((4*e*x^3+3*d*x^2+2*c*x+b)*(e*x^4+d*x^3+c*x^2+b*x+a)^p,x, algorit
hm="giac")
```

output
```
(e*x^4 + d*x^3 + c*x^2 + b*x + a)^(p + 1)/(p + 1)
```

Mupad [B] (verification not implemented)

Time = 22.41 (sec) , antiderivative size = 69, normalized size of antiderivative = 2.30

$$\int (b+2cx+3dx^2+4ex^3)\left(a+bx+cx^2+dx^3+ex^4\right)^p\,dx$$
$$=\left(\frac{a}{p+1}+\frac{b\,x}{p+1}+\frac{c\,x^2}{p+1}+\frac{d\,x^3}{p+1}+\frac{e\,x^4}{p+1}\right)\left(e\,x^4+d\,x^3+c\,x^2+b\,x+a\right)^p$$

input
```
int((b + 2*c*x + 3*d*x^2 + 4*e*x^3)*(a + b*x + c*x^2 + d*x^3 + e*x^4)^p,x)
```

output
```
(a/(p + 1) + (b*x)/(p + 1) + (c*x^2)/(p + 1) + (d*x^3)/(p + 1) + (e*x^4)/(
p + 1))*(a + b*x + c*x^2 + d*x^3 + e*x^4)^p
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.60

$$\int (b+2cx+3dx^2+4ex^3)\left(a+bx+cx^2+dx^3+ex^4\right)^p\,dx$$
$$=\frac{\left(e\,x^4+d\,x^3+c\,x^2+bx+a\right)^p\left(e\,x^4+d\,x^3+c\,x^2+bx+a\right)}{p+1}$$

input
```
int((4*e*x^3+3*d*x^2+2*c*x+b)*(e*x^4+d*x^3+c*x^2+b*x+a)^p,x)
```

output
```
((a + b*x + c*x**2 + d*x**3 + e*x**4)**p*(a + b*x + c*x**2 + d*x**3 + e*x*
*4))/(p + 1)
```

3.13 $\int \frac{2+x-4x^2+2x^3}{1-x+x^2-x^3+x^4} \, dx$

Optimal result

Integrand size = 32, antiderivative size = 63

$$\int \frac{2+x-4x^2+2x^3}{1-x+x^2-x^3+x^4} \, dx = -\frac{2\log \left(2-\left(1-\sqrt{5}\right)x+2x^2\right)}{1-\sqrt{5}} - \frac{2\log \left(2-\left(1+\sqrt{5}\right)x+2x^2\right)}{1+\sqrt{5}}$$

output
```
-2*ln(2-x*(-5^(1/2)+1)+2*x^2)/(-5^(1/2)+1)-2*ln(2-(5^(1/2)+1)*x+2*x^2)/(5^
(1/2)+1)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.87

$$\int \frac{2+x-4x^2+2x^3}{1-x+x^2-x^3+x^4} \, dx = \frac{1}{2}\left(-\left(\left(-1+\sqrt{5}\right)\log \left(-2+x+\sqrt{5}x-2x^2\right)\right) + \left(1+\sqrt{5}\right)\log \left(2+\left(-1+\sqrt{5}\right)x+2x^2\right)\right)$$

input
```
Integrate[(2 + x - 4*x^2 + 2*x^3)/(1 - x + x^2 - x^3 + x^4),x]
```

output
```
(-((-1 + Sqrt[5])*Log[-2 + x + Sqrt[5]*x - 2*x^2]) + (1 + Sqrt[5])*Log[2 +
 (-1 + Sqrt[5])*x + 2*x^2])/2
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {2492, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2x^3-4x^2+x+2}{x^4-x^3+x^2-x+1} \, dx$$

$$\Big\downarrow \text{2492}$$

$$\int \left(\frac{2\left(\left(1-\sqrt{5}\right)x+1\right)}{2x^2-\left(1+\sqrt{5}\right)x+2}+\frac{2\left(\left(1+\sqrt{5}\right)x+1\right)}{2x^2-\left(1-\sqrt{5}\right)x+2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2\log\left(2x^2-\left(1-\sqrt{5}\right)x+2\right)}{1-\sqrt{5}}-\frac{2\log\left(2x^2-\left(1+\sqrt{5}\right)x+2\right)}{1+\sqrt{5}}$$

input
```
Int[(2 + x - 4*x^2 + 2*x^3)/(1 - x + x^2 - x^3 + x^4),x]
```

output
```
(-2*Log[2 - (1 - Sqrt[5])*x + 2*x^2])/(1 - Sqrt[5]) - (2*Log[2 - (1 + Sqrt
[5])*x + 2*x^2])/(1 + Sqrt[5])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2492
```
Int[(Px_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^3 + (e_.)*(x_)^4)
^(p_), x_Symbol] :> Simp[e^p   Int[ExpandIntegrand[Px*(b/d + ((d + Sqrt[e*(
(b^2 - 4*a*c)/a) + 8*a*d*(e/b)])/(2*e))*x + x^2)^p*(b/d + ((d - Sqrt[e*((b^
2 - 4*a*c)/a) + 8*a*d*(e/b)])/(2*e))*x + x^2)^p, x], x], x] /; FreeQ[{a, b,
 c, d, e}, x] && PolyQ[Px, x] && ILtQ[p, 0] && EqQ[a*d^2 - b^2*e, 0]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.84

method	result	size
default	$2\left(\frac{\sqrt{5}}{4}+\frac{1}{4}\right)\ln\left(2-x+\sqrt{5}\,x+2x^{2}\right)-2\left(\frac{\sqrt{5}}{4}-\frac{1}{4}\right)\ln\left(2-x-\sqrt{5}\,x+2x^{2}\right)$	53
risch	$\frac{\ln\left(2+2x^{2}+\left(\sqrt{5}-1\right)x\right)}{2}+\frac{\ln\left(2+2x^{2}+\left(\sqrt{5}-1\right)x\right)\sqrt{5}}{2}+\frac{\ln\left(2+2x^{2}+\left(-\sqrt{5}-1\right)x\right)}{2}-\frac{\ln\left(2+2x^{2}+\left(-\sqrt{5}-1\right)x\right)\sqrt{5}}{2}$	80

input
```
int((2*x^3-4*x^2+x+2)/(x^4-x^3+x^2-x+1),x,method=_RETURNVERBOSE)
```

output
```
2*(1/4*5^(1/2)+1/4)*ln(2-x+5^(1/2)*x+2*x^2)-2*(1/4*5^(1/2)-1/4)*ln(2-x-5^(
1/2)*x+2*x^2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.32

$$\int\frac{2+x-4x^{2}+2x^{3}}{1-x+x^{2}-x^{3}+x^{4}}\,dx =\frac{1}{2}\sqrt{5}\log\left(\frac{2\,x^{4}-2\,x^{3}+7\,x^{2}+\sqrt{5}(2\,x^{3}-x^{2}+2\,x)-2\,x+2}{x^{4}-x^{3}+x^{2}-x+1}\right)+\frac{1}{2}\log\left(x^{4}-x^{3}+x^{2}-x+1\right)$$

input
```
integrate((2*x^3-4*x^2+x+2)/(x^4-x^3+x^2-x+1),x, algorithm="fricas")
```

output
```
1/2*sqrt(5)*log((2*x^4 - 2*x^3 + 7*x^2 + sqrt(5)*(2*x^3 - x^2 + 2*x) - 2*x
 + 2)/(x^4 - x^3 + x^2 - x + 1)) + 1/2*log(x^4 - x^3 + x^2 - x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.92

$$\int \frac{2+x-4x^2+2x^3}{1-x+x^2-x^3+x^4}\,dx = \left(\frac{1}{2}+\frac{\sqrt{5}}{2}\right)\log\left(x^2+x\left(-\frac{1}{2}+\frac{\sqrt{5}}{2}\right)+1\right) + \left(\frac{1}{2}-\frac{\sqrt{5}}{2}\right)\log\left(x^2+x\left(-\frac{\sqrt{5}}{2}-\frac{1}{2}\right)+1\right)$$

input
```
integrate((2*x**3-4*x**2+x+2)/(x**4-x**3+x**2-x+1),x)
```

output
```
(1/2 + sqrt(5)/2)*log(x**2 + x*(-1/2 + sqrt(5)/2) + 1) + (1/2 - sqrt(5)/2)
*log(x**2 + x*(-sqrt(5)/2 - 1/2) + 1)
```

Maxima [F]

$$\int \frac{2+x-4x^2+2x^3}{1-x+x^2-x^3+x^4}\,dx = \int \frac{2\,x^3-4\,x^2+x+2}{x^4-x^3+x^2-x+1}\,dx$$

input
```
integrate((2*x^3-4*x^2+x+2)/(x^4-x^3+x^2-x+1),x, algorithm="maxima")
```

output
```
integrate((2*x^3 - 4*x^2 + x + 2)/(x^4 - x^3 + x^2 - x + 1), x)
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.92

$$\int \frac{2+x-4x^2+2x^3}{1-x+x^2-x^3+x^4}\,dx = -\frac{1}{2}\sqrt{5}\log\left(x^2-\frac{1}{2}x\left(\sqrt{5}+1\right)+1\right)+\frac{1}{2}\sqrt{5}\log\left(x^2+\frac{1}{2}x\left(\sqrt{5}-1\right)+1\right)+\frac{1}{2}\log\left(x^4-x^3+x^2-x+1\right)$$

input
```
integrate((2*x^3-4*x^2+x+2)/(x^4-x^3+x^2-x+1),x, algorithm="giac")
```

output
```
-1/2*sqrt(5)*log(x^2 - 1/2*x*(sqrt(5) + 1) + 1) + 1/2*sqrt(5)*log(x^2 + 1/
2*x*(sqrt(5) - 1) + 1) + 1/2*log(x^4 - x^3 + x^2 - x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.19

$$\int \frac{2+x-4x^2+2x^3}{1-x+x^2-x^3+x^4}\,dx = \frac{\ln\left(x^2-\frac{\sqrt{5}\,x}{2}-\frac{x}{2}+1\right)}{2}+\frac{\ln\left(\frac{\sqrt{5}\,x}{2}-\frac{x}{2}+x^2+1\right)}{2}-\frac{\sqrt{5}\,\ln\left(x^2-\frac{\sqrt{5}\,x}{2}-\frac{x}{2}+1\right)}{2}+\frac{\sqrt{5}\,\ln\left(\frac{\sqrt{5}\,x}{2}-\frac{x}{2}+x^2+1\right)}{2}$$

input
```
int((x - 4*x^2 + 2*x^3 + 2)/(x^2 - x - x^3 + x^4 + 1),x)
```

output
```
log(x^2 - (5^(1/2)*x)/2 - x/2 + 1)/2 + log((5^(1/2)*x)/2 - x/2 + x^2 + 1)/
2 - (5^(1/2)*log(x^2 - (5^(1/2)*x)/2 - x/2 + 1))/2 + (5^(1/2)*log((5^(1/2)
*x)/2 - x/2 + x^2 + 1))/2
```

Reduce [F]

$$\int \frac{2+x-4x^2+2x^3}{1-x+x^2-x^3+x^4}\,dx = -\frac{5\left(\int \frac{x^2}{x^4-x^3+x^2-x+1}dx\right)}{2} + \frac{5\left(\int \frac{1}{x^4-x^3+x^2-x+1}dx\right)}{2} + \frac{\log(x^4-x^3+x^2-x+1)}{2}$$

input
```
int((2*x^3-4*x^2+x+2)/(x^4-x^3+x^2-x+1),x)
```

output
```
( - 5*int(x**2/(x**4 - x**3 + x**2 - x + 1),x) + 5*int(1/(x**4 - x**3 + x*
*2 - x + 1),x) + log(x**4 - x**3 + x**2 - x + 1))/2
```

3.14 $\int \frac{A+Bx+Cx^2+Dx^3}{a+bx+cx^2+bx^3+ax^4} \, dx$

Optimal result

Integrand size = 38, antiderivative size = 605

$$\int \frac{A+Bx+Cx^2+Dx^3}{a+bx+cx^2+bx^3+ax^4} \, dx$$

$$= \frac{\left(4a^2B+b\left(b-\sqrt{8a^2+b^2-4ac}\right)D-a\left(A\left(b-\sqrt{8a^2+b^2-4ac}\right)+bC-\sqrt{8a^2+b^2-4ac}C+2cD\right)\right) \text{a}}{\sqrt{2}a\sqrt{8a^2+b^2-4ac}\sqrt{4a^2+2ac-b\left(b-\sqrt{8a^2+b^2-4ac}\right)}}$$

$$- \frac{\left(4a^2B+b\left(b+\sqrt{8a^2+b^2-4ac}\right)D-a\left(A\left(b+\sqrt{8a^2+b^2-4ac}\right)+bC+\sqrt{8a^2+b^2-4ac}C+2cD\right)\right)}{\sqrt{2}a\sqrt{8a^2+b^2-4ac}\sqrt{4a^2+2ac-b\left(b+\sqrt{8a^2+b^2-4ac}\right)}}$$

$$- \frac{\left(2a(A-C)+\left(b-\sqrt{8a^2+b^2-4ac}\right)D\right)\log\left(2a+\left(b-\sqrt{8a^2+b^2-4ac}\right)x+2ax^2\right)}{4a\sqrt{8a^2+b^2-4ac}}$$

$$+ \frac{\left(2a(A-C)+\left(b+\sqrt{8a^2+b^2-4ac}\right)D\right)\log\left(2a+\left(b+\sqrt{8a^2+b^2-4ac}\right)x+2ax^2\right)}{4a\sqrt{8a^2+b^2-4ac}}$$

output
```
1/2*(4*a^2*B+b*(b-(8*a^2-4*a*c+b^2)^(1/2))*D-a*(A*(b-(8*a^2-4*a*c+b^2)^(1/
2))+b*C-(8*a^2-4*a*c+b^2)^(1/2)*C+2*c*D))*arctan(1/2*(b-(8*a^2-4*a*c+b^2)^
(1/2)+4*a*x)*2^(1/2)/(4*a^2+2*a*c-b*(b-(8*a^2-4*a*c+b^2)^(1/2)))^(1/2))*2^
(1/2)/a/(8*a^2-4*a*c+b^2)^(1/2)/(4*a^2+2*a*c-b*(b-(8*a^2-4*a*c+b^2)^(1/2))
)^(1/2)-1/2*(4*a^2*B+b*(b+(8*a^2-4*a*c+b^2)^(1/2))*D-a*(A*(b+(8*a^2-4*a*c+
b^2)^(1/2))+b*C+(8*a^2-4*a*c+b^2)^(1/2)*C+2*c*D))*arctan(1/2*(b+(8*a^2-4*a
*c+b^2)^(1/2)+4*a*x)*2^(1/2)/(4*a^2+2*a*c-b*(b+(8*a^2-4*a*c+b^2)^(1/2)))^(
1/2))*2^(1/2)/a/(8*a^2-4*a*c+b^2)^(1/2)/(4*a^2+2*a*c-b*(b+(8*a^2-4*a*c+b^2
)^(1/2)))^(1/2)-1/4*(2*a*(A-C)+(b-(8*a^2-4*a*c+b^2)^(1/2))*D)*ln(2*a+(b-(8
*a^2-4*a*c+b^2)^(1/2))*x+2*a*x^2)/a/(8*a^2-4*a*c+b^2)^(1/2)+1/4*(2*a*(A-C)
+(b+(8*a^2-4*a*c+b^2)^(1/2))*D)*ln(2*a+(b+(8*a^2-4*a*c+b^2)^(1/2))*x+2*a*x
^2)/a/(8*a^2-4*a*c+b^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.09 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.16

$$\int \frac{A+Bx+Cx^2+Dx^3}{a+bx+cx^2+bx^3+ax^4}\,dx = \text{RootSum}\left[a+b\#1+c\#1^2+b\#1^3 +a\#1^4\&,\frac{A\log(x-\#1)+B\log(x-\#1)\#1+C\log(x-\#1)\#1^2+D\log(x-\#1)\#1^3}{b+2c\#1+3b\#1^2+4a\#1^3}\&\right]$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/(a + b*x + c*x^2 + b*x^3 + a*x^4),x]
```

output
```
RootSum[a + b*#1 + c*#1^2 + b*#1^3 + a*#1^4 & , (A*Log[x - #1] + B*Log[x -
 #1]*#1 + C*Log[x - #1]*#1^2 + D*Log[x - #1]*#1^3)/(b + 2*c*#1 + 3*b*#1^2
+ 4*a*#1^3) & ]
```

Rubi [A] (verified)

Time = 6.23 (sec) , antiderivative size = 597, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {2492, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{ax^4+a+bx^3+bx+cx^2}dx$$

$$\Big\downarrow \text{2492}$$

$$\frac{\int \left(\frac{a\left(A\left(b+\sqrt{8a^2-4ca+b^2}\right)-2a(B-D)+\left(2a(A-C)+\left(b+\sqrt{8a^2-4ca+b^2}\right)D\right)x\right)}{\sqrt{8a^2-4ca+b^2}\left(2ax^2+\left(b+\sqrt{8a^2-4ca+b^2}\right)x+2a\right)}-\frac{a\left(A\left(b-\sqrt{8a^2-4ca+b^2}\right)-2a(B-D)+\left(2aA-2aC+bD-\right.\right.}{\sqrt{8a^2-4ca+b^2}\left(2ax^2+\left(b-\sqrt{8a^2-4ca+b^2}\right)\right.}\right.}{a}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\arctan\left(\frac{-\sqrt{8a^2-4ac+b^2}+4ax+b}{\sqrt{2}\sqrt{-b\left(b-\sqrt{8a^2-4ac+b^2}\right)+4a^2+2ac}}\right)\left(-a\left(A\left(b-\sqrt{8a^2-4ac+b^2}\right)-C\sqrt{8a^2-4ac+b^2}+bC+2cD\right)+bD\left(b-\sqrt{8a^2-4ac+b^2}\right)+4a^2B\right)}{\sqrt{2}\sqrt{8a^2-4ac+b^2}\sqrt{-b\left(b-\sqrt{8a^2-4ac+b^2}\right)+4a^2+2ac}} - \cdots$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/(a + b*x + c*x^2 + b*x^3 + a*x^4),x]
```

output
```
(((4*a^2*B + b*(b - Sqrt[8*a^2 + b^2 - 4*a*c])*D - a*(A*(b - Sqrt[8*a^2 +
b^2 - 4*a*c]) + b*C - Sqrt[8*a^2 + b^2 - 4*a*c]*C + 2*c*D))*ArcTan[(b - Sq
rt[8*a^2 + b^2 - 4*a*c] + 4*a*x)/(Sqrt[2]*Sqrt[4*a^2 + 2*a*c - b*(b - Sqrt
[8*a^2 + b^2 - 4*a*c])])])/(Sqrt[2]*Sqrt[8*a^2 + b^2 - 4*a*c]*Sqrt[4*a^2 +
 2*a*c - b*(b - Sqrt[8*a^2 + b^2 - 4*a*c])]) - ((4*a^2*B + b*(b + Sqrt[8*a
^2 + b^2 - 4*a*c])*D - a*(A*(b + Sqrt[8*a^2 + b^2 - 4*a*c]) + b*C + Sqrt[8
*a^2 + b^2 - 4*a*c]*C + 2*c*D))*ArcTan[(b + Sqrt[8*a^2 + b^2 - 4*a*c] + 4*
a*x)/(Sqrt[2]*Sqrt[4*a^2 + 2*a*c - b*(b + Sqrt[8*a^2 + b^2 - 4*a*c])])])/(
Sqrt[2]*Sqrt[8*a^2 + b^2 - 4*a*c]*Sqrt[4*a^2 + 2*a*c - b*(b + Sqrt[8*a^2 +
 b^2 - 4*a*c])]) - ((2*a*(A - C) + (b - Sqrt[8*a^2 + b^2 - 4*a*c])*D)*Log[
2*a + (b - Sqrt[8*a^2 + b^2 - 4*a*c])*x + 2*a*x^2])/(4*Sqrt[8*a^2 + b^2 -
4*a*c]) + ((2*a*(A - C) + (b + Sqrt[8*a^2 + b^2 - 4*a*c])*D)*Log[2*a + (b
+ Sqrt[8*a^2 + b^2 - 4*a*c])*x + 2*a*x^2])/(4*Sqrt[8*a^2 + b^2 - 4*a*c]))/
a
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2492
```
Int[(Px_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^3 + (e_.)*(x_)^4)
^(p_), x_Symbol] :> Simp[e^p   Int[ExpandIntegrand[Px*(b/d + ((d + Sqrt[e*(
(b^2 - 4*a*c)/a) + 8*a*d*(e/b)])/(2*e))*x + x^2)^p*(b/d + ((d - Sqrt[e*((b^
2 - 4*a*c)/a) + 8*a*d*(e/b)])/(2*e))*x + x^2)^p, x], x], x] /; FreeQ[{a, b,
 c, d, e}, x] && PolyQ[Px, x] && ILtQ[p, 0] && EqQ[a*d^2 - b^2*e, 0]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 535, normalized size of antiderivative = 0.88

method	result
default	$4a\left(\frac{\frac{\left(2Aa-2Ca+\sqrt{8a^{2}-4ac+b^{2}}\,D+Db\right)\ln\left(2a+bx+\sqrt{8a^{2}-4ac+b^{2}}\,x+2a\,x^{2}\right)}{4a}+\frac{2\left(-\frac{\left(2Aa-2Ca+\sqrt{8a^{2}-4ac+b^{2}}\,D+Db\right)\left(b+\sqrt{8a^{2}-4ac+b^{2}}\right)}{4a}\right.}{\sqrt{8a^{2}-}}}{4a\sqrt{8a^{2}-4ac+b^{2}}}\right.$

input
```
int((D*x^3+C*x^2+B*x+A)/(a*x^4+b*x^3+c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

output
```
4*a*(1/4/a/(8*a^2-4*a*c+b^2)^(1/2)*(1/4*(2*A*a-2*C*a+(8*a^2-4*a*c+b^2)^(1/
2)*D+D*b)/a*ln(2*a+b*x+(8*a^2-4*a*c+b^2)^(1/2)*x+2*a*x^2)+2*(-1/4*(2*A*a-2
*C*a+(8*a^2-4*a*c+b^2)^(1/2)*D+D*b)/a*(b+(8*a^2-4*a*c+b^2)^(1/2))+(8*a^2-4
*a*c+b^2)^(1/2)*A+A*b-2*B*a+2*a*D)/(8*a^2+4*a*c-2*b^2-2*b*(8*a^2-4*a*c+b^2
)^(1/2))^(1/2)*arctan((b+(8*a^2-4*a*c+b^2)^(1/2)+4*x*a)/(8*a^2+4*a*c-2*b^2
-2*b*(8*a^2-4*a*c+b^2)^(1/2))^(1/2)))+1/4/a/(8*a^2-4*a*c+b^2)^(1/2)*(-1/4*
(2*A*a-2*C*a-(8*a^2-4*a*c+b^2)^(1/2)*D+D*b)/a*ln(-2*a*x^2+(8*a^2-4*a*c+b^2
)^(1/2)*x-b*x-2*a)+2*(1/4*(2*A*a-2*C*a-(8*a^2-4*a*c+b^2)^(1/2)*D+D*b)/a*((
8*a^2-4*a*c+b^2)^(1/2)-b)-(8*a^2-4*a*c+b^2)^(1/2)*A+A*b-2*B*a+2*a*D)/(8*a^
2+4*a*c-2*b^2+2*b*(8*a^2-4*a*c+b^2)^(1/2))^(1/2)*arctan((-4*x*a+(8*a^2-4*a
*c+b^2)^(1/2)-b)/(8*a^2+4*a*c-2*b^2+2*b*(8*a^2-4*a*c+b^2)^(1/2))^(1/2))))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{a+bx+cx^2+bx^3+ax^4}\,dx = \text{Timed out}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(a*x^4+b*x^3+c*x^2+b*x+a),x, algorithm="fric
as")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{a+bx+cx^2+bx^3+ax^4}\,dx = \text{Timed out}$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/(a*x**4+b*x**3+c*x**2+b*x+a),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2+Dx^3}{a+bx+cx^2+bx^3+ax^4}\,dx=\int \frac{Dx^3+Cx^2+Bx+A}{ax^4+bx^3+cx^2+bx+a}\,dx$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(a*x^4+b*x^3+c*x^2+b*x+a),x, algorithm="maxi
ma")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)/(a*x^4 + b*x^3 + c*x^2 + b*x + a), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A+Bx+Cx^2+Dx^3}{a+bx+cx^2+bx^3+ax^4}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(a*x^4+b*x^3+c*x^2+b*x+a),x, algorithm="giac
")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Not invertible Error: Bad Argument
Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{a+bx+cx^2+bx^3+ax^4}\,dx=\int \frac{A+B\,x+C\,x^2+x^3\,D}{a\,x^4+b\,x^3+c\,x^2+b\,x+a}\,dx$$

input
```
int((A + B*x + C*x^2 + x^3*D)/(a + b*x + a*x^4 + b*x^3 + c*x^2),x)
```

output
```
int((A + B*x + C*x^2 + x^3*D)/(a + b*x + a*x^4 + b*x^3 + c*x^2), x)
```

Reduce [F]

$$\int \frac{A+Bx+Cx^2+Dx^3}{a+bx+cx^2+bx^3+ax^4}\,dx$$
$$= \frac{4\left(\int \frac{x^2}{a\,x^4+b\,x^3+c\,x^2+bx+a}dx\right)ac-3\left(\int \frac{x^2}{a\,x^4+b\,x^3+c\,x^2+bx+a}dx\right)bd+4\left(\int \frac{x}{a\,x^4+b\,x^3+c\,x^2+bx+a}dx\right)ab-2\left(\int \frac{}{a\,x^4+b\,x}\right.}{}$$

input
```
int((D*x^3+C*x^2+B*x+A)/(a*x^4+b*x^3+c*x^2+b*x+a),x)
```

output
```
(4*int(x**2/(a*x**4 + a + b*x**3 + b*x + c*x**2),x)*a*c - 3*int(x**2/(a*x*
*4 + a + b*x**3 + b*x + c*x**2),x)*b*d + 4*int(x/(a*x**4 + a + b*x**3 + b*
x + c*x**2),x)*a*b - 2*int(x/(a*x**4 + a + b*x**3 + b*x + c*x**2),x)*c*d +
 4*int(1/(a*x**4 + a + b*x**3 + b*x + c*x**2),x)*a**2 - int(1/(a*x**4 + a
+ b*x**3 + b*x + c*x**2),x)*b*d + log(a*x**4 + a + b*x**3 + b*x + c*x**2)*
d)/(4*a)
```

3.15 $\int \frac{A+Bx+Cx^2+Dx^3}{88-402x+855x^2-837x^3+324x^4}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 289

$$\int \frac{A+Bx+Cx^2+Dx^3}{88-402x+855x^2-837x^3+324x^4}\,dx =$$

$$-\frac{\left(54\left(1+\sqrt{177}\right)A-36\left(15-\sqrt{177}\right)B-732C+36\sqrt{177}C-689D+39\sqrt{177}D\right)\arctan\left(\frac{31-\sqrt{177}-4\ldots}{\sqrt{2\left(167-\sqrt{1\ldots}}}\ldots\right.}{324\sqrt{354\left(167-\sqrt{177}\right)}}$$

$$+\frac{\left(54\left(1-\sqrt{177}\right)A-36\left(15+\sqrt{177}\right)B-732C-36\sqrt{177}C-689D-39\sqrt{177}D\right)\arctan\left(\frac{31+\sqrt{177}-4\ldots}{\sqrt{2\left(167+\sqrt{1\ldots}}}\ldots\right.}{324\sqrt{354\left(167+\sqrt{177}\right)}}$$

$$-\frac{\left(\sqrt{177}(36A+24B+8C-5D)-59D\right)\log\left(16-\left(1-\sqrt{177}\right)(2-3x)+8(2-3x)^2\right)}{76464}$$

$$+\frac{\left(\sqrt{177}(36A+24B+8C-5D)+59D\right)\log\left(16-\left(1+\sqrt{177}\right)(2-3x)+8(2-3x)^2\right)}{76464}$$

output
```
-1/324*(54*(1+177^(1/2))*A-36*(15-177^(1/2))*B-732*C+36*177^(1/2)*C-689*D+
39*177^(1/2)*D)*arctan((31-177^(1/2)-48*x)/(334-2*177^(1/2))^(1/2))/(59118
-354*177^(1/2))^(1/2)+1/324*(54*(1-177^(1/2))*A-36*(15+177^(1/2))*B-732*C-
36*177^(1/2)*C-689*D-39*177^(1/2)*D)*arctan((31+177^(1/2)-48*x)/(334+2*177
^(1/2))^(1/2))/(59118+354*177^(1/2))^(1/2)-1/76464*(177^(1/2)*(36*A+24*B+8
*C-5*D)-59*D)*ln(16-(1-177^(1/2))*(2-3*x)+8*(2-3*x)^2)+1/76464*(177^(1/2)*
(36*A+24*B+8*C-5*D)+59*D)*ln(16-(1+177^(1/2))*(2-3*x)+8*(2-3*x)^2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.13 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.34

$$\int \frac{A+Bx+Cx^2+Dx^3}{88-402x+855x^2-837x^3+324x^4}\,dx=\frac{1}{3}\text{RootSum}\left[88-402\#1+855\#1^2-837\#1^3+324\#1^4\&,\frac{A\log(x-\#1)+B\log(x-\#1)\#1+C\log(x-\#1)\#1^2+D\log(x-\#1)\#1^3}{-134+570\#1-837\#1^2+432\#1^3}\&\right]$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/(88 - 402*x + 855*x^2 - 837*x^3 + 324*
x^4),x]
```

output
```
RootSum[88 - 402*#1 + 855*#1^2 - 837*#1^3 + 324*#1^4 & , (A*Log[x - #1] +
B*Log[x - #1]*#1 + C*Log[x - #1]*#1^2 + D*Log[x - #1]*#1^3)/(-134 + 570*#1
 - 837*#1^2 + 432*#1^3) & ]/3
```

Rubi [A] (verified)

Time = 1.90 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.17, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.079, Rules used = {2494, 2492, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{324x^4-837x^3+855x^2-402x+88}\,dx$$

$$\Big\downarrow \; 2494$$

$$\int \frac{A+Bx+Cx^2+Dx^3}{324\left(x-\frac{2}{3}\right)^4+27\left(x-\frac{2}{3}\right)^3+45\left(x-\frac{2}{3}\right)^2+6\left(x-\frac{2}{3}\right)+16}d\left(x-\frac{2}{3}\right)$$

$$\Big\downarrow \; 2492$$

$$\frac{1}{324}\int \left(\frac{2\sqrt{\frac{3}{59}}\left(27\left(1+\sqrt{177}\right)A-18\left(7-\sqrt{177}\right)B-12\left(15-\sqrt{177}\right)C-8\left(19-\sqrt{177}\right)D+6(108A+72B+\right.}{72\left(x-\frac{2}{3}\right)^2+3\left(1+\sqrt{177}\right)\left(x-\frac{2}{3}\right)+16}\right.$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{324}\left(-\frac{\arctan\left(\frac{48\left(x-\frac{2}{3}\right)-\sqrt{177}+1}{\sqrt{2\left(167+\sqrt{177}\right)}}\right)\left(54\left(1-\sqrt{177}\right)A-36\left(15+\sqrt{177}\right)B-36\sqrt{177}C-732C-39\sqrt{177}D-68\right.}{\sqrt{354\left(167+\sqrt{177}\right)}}\right.$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/(88 - 402*x + 855*x^2 - 837*x^3 + 324*x^4),x
]
```

output
```
(-(((54*(1 - Sqrt[177])*A - 36*(15 + Sqrt[177])*B - 732*C - 36*Sqrt[177]*C
 - 689*D - 39*Sqrt[177]*D)*ArcTan[(1 - Sqrt[177] + 48*(-2/3 + x))/Sqrt[2*(
167 + Sqrt[177])]])/Sqrt[354*(167 + Sqrt[177])]) + ((54*(1 + Sqrt[177])*A
- 36*(15 - Sqrt[177])*B - 732*C + 36*Sqrt[177]*C - 689*D + 39*Sqrt[177]*D)
*ArcTan[(1 + Sqrt[177] + 48*(-2/3 + x))/Sqrt[2*(167 - Sqrt[177])]])/Sqrt[3
54*(167 - Sqrt[177])] + ((59*D - Sqrt[59]*(36*Sqrt[3]*A + 24*Sqrt[3]*B + 8
*Sqrt[3]*C - 5*Sqrt[3]*D))*Log[16 + 3*(1 - Sqrt[177])*(-2/3 + x) + 72*(-2/
3 + x)^2])/236 + ((59*D + Sqrt[59]*(36*Sqrt[3]*A + 24*Sqrt[3]*B + 8*Sqrt[3
]*C - 5*Sqrt[3]*D))*Log[16 + 3*(1 + Sqrt[177])*(-2/3 + x) + 72*(-2/3 + x)^
2])/236)/324
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2492
```
Int[(Px_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^3 + (e_.)*(x_)^4)
^(p_), x_Symbol] :> Simp[e^p   Int[ExpandIntegrand[Px*(b/d + ((d + Sqrt[e*(
(b^2 - 4*a*c)/a) + 8*a*d*(e/b)])/(2*e))*x + x^2)^p*(b/d + ((d - Sqrt[e*((b^
2 - 4*a*c)/a) + 8*a*d*(e/b)])/(2*e))*x + x^2)^p, x], x], x] /; FreeQ[{a, b,
 c, d, e}, x] && PolyQ[Px, x] && ILtQ[p, 0] && EqQ[a*d^2 - b^2*e, 0]
```

rule 2494
```
Int[(Px_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^3 + (e_.)*(x_)^4)
^(p_), x_Symbol] :> With[{S = Root[a*d^2 - b^2*e + (b*d^2 - 4*b*c*e + 8*a*d
*e)*x + (c*d^2 - 4*c^2*e + 2*b*d*e + 16*a*e^2)*x^2 + (d^3 - 4*c*d*e + 8*b*e
^2)*x^3, 3]}, Subst[Int[(Px /. x -> x + S)*ExpandToSum[a + b*(x + S) + c*(x
 + S)^2 + d*(x + S)^3 + e*(x + S)^4, x]^p, x], x, x - S] /; RationalQ[S]] /
; FreeQ[{a, b, c, d, e}, x] && PolyQ[Px, x] && ILtQ[p, 0] && RationalQ[a, b
, c, d, e] && NeQ[a*d^2 - b^2*e, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.14 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.23

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(324\_Z^{4}-837\_Z^{3}+855\_Z^{2}-402\_Z+88\right)}\frac{\left(D\_R^{3}+C\_R^{2}+B\_R+A\right)\ln\left(x-\_R\right)}{432\_R^{3}-837\_R^{2}+570\_R-134}\right)}{3}$	66

input
```
int((D*x^3+C*x^2+B*x+A)/(324*x^4-837*x^3+855*x^2-402*x+88),x,method=_RETUR
NVERBOSE)
```

output
```
1/3*sum((D*_R^3+C*_R^2+B*_R+A)/(432*_R^3-837*_R^2+570*_R-134)*ln(x-_R),_R=
RootOf(324*_Z^4-837*_Z^3+855*_Z^2-402*_Z+88))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 10.78 (sec) , antiderivative size = 778293, normalized size of antiderivative = 2693.06

$$\int \frac{A+Bx+Cx^2+Dx^3}{88-402x+855x^2-837x^3+324x^4}\,dx = \text{Too large to display}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(324*x^4-837*x^3+855*x^2-402*x+88),x, algori
thm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{88-402x+855x^2-837x^3+324x^4}\,dx = \text{Timed out}$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/(324*x**4-837*x**3+855*x**2-402*x+88),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2+Dx^3}{88-402x+855x^2-837x^3+324x^4}\,dx = \int \frac{Dx^3+Cx^2+Bx+A}{324\,x^4-837\,x^3+855\,x^2-402\,x+88}\,dx$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(324*x^4-837*x^3+855*x^2-402*x+88),x, algori
thm="maxima")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)/(324*x^4 - 837*x^3 + 855*x^2 - 402*x +
 88), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A+Bx+Cx^2+Dx^3}{88-402x+855x^2-837x^3+324x^4}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(324*x^4-837*x^3+855*x^2-402*x+88),x, algori
thm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{88-402x+855x^2-837x^3+324x^4}\,dx = \int \frac{A+B\,x+C\,x^2+x^3\,D}{324\,x^4-837\,x^3+855\,x^2-402\,x+88}\,dx$$

input
```
int((A + B*x + C*x^2 + x^3*D)/(855*x^2 - 402*x - 837*x^3 + 324*x^4 + 88),x
)
```

output
```
int((A + B*x + C*x^2 + x^3*D)/(855*x^2 - 402*x - 837*x^3 + 324*x^4 + 88),
x)
```

Reduce [F]

$$\int \frac{A+Bx+Cx^2+Dx^3}{88-402x+855x^2-837x^3+324x^4}\,dx$$
$$= \left(\int \frac{x^2}{324x^4-837x^3+855x^2-402x+88}dx\right)c + \frac{31\left(\int \frac{x^2}{324x^4-837x^3+855x^2-402x+88}dx\right)d}{16} + \left(\int \frac{x}{324x^4-837x^3+855x^2-402x+88}dx\right)b - \frac{95\left(\int \frac{x}{324x^4-837x^3+855x^2-402x+88}dx\right)d}{72} + \left(\int \frac{1}{324x^4-837x^3+855x^2-402x+88}dx\right)a + \frac{67\left(\int \frac{1}{324x^4-837x^3+855x^2-402x+88}dx\right)d}{216} + \frac{\log(324x^4-837x^3+855x^2-402x+88)\,d}{1296}$$

input
```
int((D*x^3+C*x^2+B*x+A)/(324*x^4-837*x^3+855*x^2-402*x+88),x)
```

output
```
(1296*int(x**2/(324*x**4 - 837*x**3 + 855*x**2 - 402*x + 88),x)*c + 2511*i
nt(x**2/(324*x**4 - 837*x**3 + 855*x**2 - 402*x + 88),x)*d + 1296*int(x/(3
24*x**4 - 837*x**3 + 855*x**2 - 402*x + 88),x)*b - 1710*int(x/(324*x**4 -
837*x**3 + 855*x**2 - 402*x + 88),x)*d + 1296*int(1/(324*x**4 - 837*x**3 +
 855*x**2 - 402*x + 88),x)*a + 402*int(1/(324*x**4 - 837*x**3 + 855*x**2 -
 402*x + 88),x)*d + log(324*x**4 - 837*x**3 + 855*x**2 - 402*x + 88)*d)/12
96
```

3.16 $\int \frac{A+Bx+Cx^2+Dx^3}{(88-402x+855x^2-837x^3+324x^4)^2}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 544

$$\int \frac{A+Bx+Cx^2+Dx^3}{(88-402x+855x^2-837x^3+324x^4)^2}\,dx =$$

$$-\frac{11961A+2673\sqrt{177}A-39735B+2169\sqrt{177}B-58236C-300\sqrt{177}C-56694D-2614\sqrt{177}D-}{12415842\left(16-\left(1+\sqrt{177}\right)(2-3x)+8(2-3x)^2\right)}$$

$$+\frac{8\left(4779A+1053\sqrt{177}A-9558B+774\sqrt{177}B-14868C-204\sqrt{177}C-18408D-1112\sqrt{177}D-\right.}{14337\left(167+\sqrt{177}\right)\left(16-\left(1-\sqrt{177}\right)(2-3x)+8(2\right.}$$

$$-\frac{\left(187371A+59787\sqrt{177}A-384747B+39093\sqrt{177}B-591872C+29312\sqrt{177}C-580980D+237\right.}{8277228\sqrt{354\left(167-\sqrt{177}\right)}}$$

$$+\frac{\left(187371A-59787\sqrt{177}A-384747B-39093\sqrt{177}B-591872C-29312\sqrt{177}C-580980D-237\right.}{8277228\sqrt{354\left(167+\sqrt{177}\right)}}$$

$$-\frac{(513A+351B+192C+68D)\log\left(16-\left(1-\sqrt{177}\right)(2-3x)+8(2-3x)^2\right)}{114696\sqrt{177}}$$

$$+\frac{(513A+351B+192C+68D)\log\left(16-\left(1+\sqrt{177}\right)(2-3x)+8(2-3x)^2\right)}{114696\sqrt{177}}$$

output
```
-1/12415842*(11961*A+2673*177^(1/2)*A-39735*B+2169*177^(1/2)*B-58236*C-300
*177^(1/2)*C-56694*D-2614*177^(1/2)*D-8*(2673*A+2169*B-300*C-2614*D)*(2-3*
x))/(16-(1+177^(1/2))*(2-3*x)+8*(2-3*x)^2)+8/14337*(4779*A+1053*177^(1/2)*
A-9558*B+774*177^(1/2)*B-14868*C-204*177^(1/2)*C-18408*D-1112*177^(1/2)*D-
2*(6372*C-177^(1/2)*(54*(1-177^(1/2))*A-36*(15+177^(1/2))*B-732*C-689*D-39
*177^(1/2)*D))*(2-3*x))/(167+177^(1/2))/(16-(1-177^(1/2))*(2-3*x)+8*(2-3*x
)^2)/(16-(1+177^(1/2))*(2-3*x)+8*(2-3*x)^2)-1/8277228*(187371*A+59787*177^
(1/2)*A-384747*B+39093*177^(1/2)*B-591872*C+29312*177^(1/2)*C-580980*D+237
56*177^(1/2)*D)*arctan((31-177^(1/2)-48*x)/(334-2*177^(1/2))^(1/2))/(59118
-354*177^(1/2))^(1/2)+1/8277228*(187371*A-59787*177^(1/2)*A-384747*B-39093
*177^(1/2)*B-591872*C-29312*177^(1/2)*C-580980*D-23756*177^(1/2)*D)*arctan
((31+177^(1/2)-48*x)/(334+2*177^(1/2))^(1/2))/(59118+354*177^(1/2))^(1/2)-
1/20301192*(513*A+351*B+192*C+68*D)*ln(16-(1-177^(1/2))*(2-3*x)+8*(2-3*x)^
2)*177^(1/2)+1/20301192*(513*A+351*B+192*C+68*D)*ln(16-(1+177^(1/2))*(2-3*
x)+8*(2-3*x)^2)*177^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.18 (sec) , antiderivative size = 296, normalized size of antiderivative = 0.54

$$\int \frac{A+Bx+Cx^2+Dx^3}{(88-402x+855x^2-837x^3+324x^4)^2}\,dx$$

$$= \frac{\frac{6C(10604-35373x+33102x^2+5400x^3)-27B(-2904+15539x-23613x^2+8676x^3)-27A(2273+4602x-18945x^2+10692x^3)+2D(-4400+5191}{88-402x+855x^2-837x^3+324x^4}}{}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/(88 - 402*x + 855*x^2 - 837*x^3 + 324*
x^4)^2,x]
```

output
```
((6*C*(10604 - 35373*x + 33102*x^2 + 5400*x^3) - 27*B*(-2904 + 15539*x - 2
3613*x^2 + 8676*x^3) - 27*A*(2273 + 4602*x - 18945*x^2 + 10692*x^3) + 2*D*
(-4400 + 51912*x - 148869*x^2 + 141156*x^3))/(88 - 402*x + 855*x^2 - 837*x
^3 + 324*x^4) - RootSum[88 - 402*#1 + 855*#1^2 - 837*#1^3 + 324*#1^4 & , (
-181899*A*Log[x - #1] - 20457*B*Log[x - #1] + 26136*C*Log[x - #1] + 21208*
D*Log[x - #1] - 92421*A*Log[x - #1]*#1 - 223317*B*Log[x - #1]*#1 - 160308*
C*Log[x - #1]*#1 - 44610*D*Log[x - #1]*#1 + 96228*A*Log[x - #1]*#1^2 + 780
84*B*Log[x - #1]*#1^2 - 10800*C*Log[x - #1]*#1^2 - 94104*D*Log[x - #1]*#1^
2)/(-134 + 570*#1 - 837*#1^2 + 432*#1^3) & ])/12415842
```

Rubi [A] (verified)

Time = 9.01 (sec) , antiderivative size = 799, normalized size of antiderivative = 1.47, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.079, Rules used = {2494, 2492, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(324x^4-837x^3+855x^2-402x+88)^2}dx$$

$$\Big\downarrow \text{ 2494}$$

$$\int \frac{A+Bx+Cx^2+Dx^3}{\left(324\left(x-\frac{2}{3}\right)^4+27\left(x-\frac{2}{3}\right)^3+45\left(x-\frac{2}{3}\right)^2+6\left(x-\frac{2}{3}\right)+16\right)^2}d\left(x-\frac{2}{3}\right)$$

$$\Big\downarrow \text{ 2492}$$

$$\int \left(-\frac{108\sqrt{\frac{3}{59}}\left(-513\sqrt{177}A+729A-351\sqrt{177}B-657B-192\sqrt{177}C-1248C-68\sqrt{177}D-1148D+24(513A+351B+192C+68D)\left(x-\frac{2}{3}\right)\right)}{59\left(72\left(x-\frac{2}{3}\right)^2+3\left(1-\sqrt{177}\right)\left(x-\frac{2}{3}\right)+16\right)}+\frac{1}{}\right.$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{648\left(459\sqrt{177}A+333A+330\sqrt{177}B+1158B+172\sqrt{177}C+1460C+40\sqrt{177}D+792D-4\left(27\sqrt{177}A+1053A-54\sqrt{177}B+774B-84\sqrt{177}C-204C\right.\right.}{59\left(167+\sqrt{177}\right)\left(72\left(x-\frac{2}{3}\right)^2+3\left(1-\sqrt{177}\right)\left(x-\frac{2}{3}\right)+16\right)}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/(88 - 402*x + 855*x^2 - 837*x^3 + 324*x^4)^2
,x]
```

output
```
((648*(333*A + 459*Sqrt[177]*A + 1158*B + 330*Sqrt[177]*B + 1460*C + 172*S
qrt[177]*C + 792*D + 40*Sqrt[177]*D - 4*(1053*A + 27*Sqrt[177]*A + 774*B -
 54*Sqrt[177]*B - 204*C - 84*Sqrt[177]*C - 1112*D - 104*Sqrt[177]*D)*(-2/3
 + x)))/(59*(167 + Sqrt[177])*(16 + 3*(1 - Sqrt[177])*(-2/3 + x) + 72*(-2/
3 + x)^2)) + (648*(333*A - 459*Sqrt[177]*A + 1158*B - 330*Sqrt[177]*B + 14
60*C - 172*Sqrt[177]*C + 792*D - 40*Sqrt[177]*D - 4*(1053*A - 27*Sqrt[177]
*A + 774*B + 54*Sqrt[177]*B - 204*C + 84*Sqrt[177]*C - 1112*D + 104*Sqrt[1
77]*D)*(-2/3 + x)))/(59*(167 - Sqrt[177])*(16 + 3*(1 + Sqrt[177])*(-2/3 +
x) + 72*(-2/3 + x)^2)) - (54*Sqrt[2/(59*(167 + Sqrt[177]))]*(315*Sqrt[3]*A
 - 513*Sqrt[59]*A - 555*Sqrt[3]*B - 351*Sqrt[59]*B - 896*Sqrt[3]*C - 192*S
qrt[59]*C - 788*Sqrt[3]*D - 68*Sqrt[59]*D)*ArcTan[(1 - Sqrt[177] + 48*(-2/
3 + x))/Sqrt[2*(167 + Sqrt[177])]])/59 - (864*Sqrt[2]*(1053*A + 27*Sqrt[17
7]*A + 774*B - 54*Sqrt[177]*B - 204*C - 84*Sqrt[177]*C - 1112*D - 104*Sqrt
[177]*D)*ArcTan[(1 - Sqrt[177] + 48*(-2/3 + x))/Sqrt[2*(167 + Sqrt[177])]]
)/(59*(167 + Sqrt[177])^(3/2)) - (864*Sqrt[2]*(1053*A - 27*Sqrt[177]*A + 7
74*B + 54*Sqrt[177]*B - 204*C + 84*Sqrt[177]*C - 1112*D + 104*Sqrt[177]*D)
*ArcTan[(1 + Sqrt[177] + 48*(-2/3 + x))/Sqrt[2*(167 - Sqrt[177])]])/(59*(1
67 - Sqrt[177])^(3/2)) + (18*Sqrt[6/(59*(167 - Sqrt[177]))]*(27*(35 + 19*S
qrt[177])*A - 9*(185 - 39*Sqrt[177])*B - 192*(14 - Sqrt[177])*C - 4*(591 -
 17*Sqrt[177])*D)*ArcTan[(1 + Sqrt[177] + 48*(-2/3 + x))/Sqrt[2*(167 - ...
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2492
```
Int[(Px_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^3 + (e_.)*(x_)^4)
^(p_), x_Symbol] :> Simp[e^p   Int[ExpandIntegrand[Px*(b/d + ((d + Sqrt[e*(
(b^2 - 4*a*c)/a) + 8*a*d*(e/b)])/(2*e))*x + x^2)^p*(b/d + ((d - Sqrt[e*((b^
2 - 4*a*c)/a) + 8*a*d*(e/b)])/(2*e))*x + x^2)^p, x], x], x] /; FreeQ[{a, b,
 c, d, e}, x] && PolyQ[Px, x] && ILtQ[p, 0] && EqQ[a*d^2 - b^2*e, 0]
```

rule 2494
```
Int[(Px_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^3 + (e_.)*(x_)^4)
^(p_), x_Symbol] :> With[{S = Root[a*d^2 - b^2*e + (b*d^2 - 4*b*c*e + 8*a*d
*e)*x + (c*d^2 - 4*c^2*e + 2*b*d*e + 16*a*e^2)*x^2 + (d^3 - 4*c*d*e + 8*b*e
^2)*x^3, 3]}, Subst[Int[(Px /. x -> x + S)*ExpandToSum[a + b*(x + S) + c*(x
 + S)^2 + d*(x + S)^3 + e*(x + S)^4, x]^p, x], x, x - S] /; RationalQ[S]] /
; FreeQ[{a, b, c, d, e}, x] && PolyQ[Px, x] && ILtQ[p, 0] && RationalQ[a, b
, c, d, e] && NeQ[a*d^2 - b^2*e, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.12 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.33

method	result
default	$\frac{\left(\frac{1307D}{18623763}-\frac{11A}{153282}-\frac{241B}{4138614}+\frac{50C}{6207921}\right)x^3+\left(-\frac{16541D}{223485156}+\frac{2105A}{16554456}+\frac{7871B}{49663368}+\frac{613C}{12415842}\right)x^2+\left(\frac{1442D}{55871289}-\frac{13A}{420876}-\frac{15539B}{148990104}-\frac{11}{223}\right.}{x^4-\frac{31}{12}x^3+\frac{95}{36}x^2-\frac{67}{54}x+\frac{22}{81}}$

input
```
int((D*x^3+C*x^2+B*x+A)/(324*x^4-837*x^3+855*x^2-402*x+88)^2,x,method=_RET
URNVERBOSE)
```

output
```
((1307/18623763*D-11/153282*A-241/4138614*B+50/6207921*C)*x^3+(-16541/2234
85156*D+2105/16554456*A+7871/49663368*B+613/12415842*C)*x^2+(1442/55871289
*D-13/420876*A-15539/148990104*B-11791/223485156*C)*x-1100/502841601*D-227
3/148990104*A+121/6207921*B+2651/167613867*C)/(x^4-31/12*x^3+95/36*x^2-67/
54*x+22/81)+1/12415842*sum((36*(-2673*A-2169*B+300*C+2614*D)*_R^2+3*(30807
*A+74439*B+53436*C+14870*D)*_R+181899*A+20457*B-26136*C-21208*D)/(432*_R^3
-837*_R^2+570*_R-134)*ln(x-_R),_R=RootOf(324*_Z^4-837*_Z^3+855*_Z^2-402*_Z
+88))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 11.89 (sec) , antiderivative size = 761015, normalized size of antiderivative = 1398.92

$$\int \frac{A+Bx+Cx^2+Dx^3}{(88-402x+855x^2-837x^3+324x^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(324*x^4-837*x^3+855*x^2-402*x+88)^2,x, algo
rithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(88-402x+855x^2-837x^3+324x^4)^2}\,dx = \text{Timed out}$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/(324*x**4-837*x**3+855*x**2-402*x+88)**2,x
)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2+Dx^3}{(88-402x+855x^2-837x^3+324x^4)^2}\,dx = \int \frac{Dx^3+Cx^2+Bx+A}{(324\,x^4-837\,x^3+855\,x^2-402\,x+88)^2}\,dx$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(324*x^4-837*x^3+855*x^2-402*x+88)^2,x, algo
rithm="maxima")
```

output
```
-1/12415842*(108*(2673*A + 2169*B - 300*C - 2614*D)*x^3 - 9*(56835*A + 708
39*B + 22068*C - 33082*D)*x^2 + 9*(13806*A + 46617*B + 23582*C - 11536*D)*
x + 61371*A - 78408*B - 63624*C + 8800*D)/(324*x^4 - 837*x^3 + 855*x^2 - 4
02*x + 88) + 1/4138614*integrate(-(36*(2673*A + 2169*B - 300*C - 2614*D)*x
^2 - 3*(30807*A + 74439*B + 53436*C + 14870*D)*x - 181899*A - 20457*B + 26
136*C + 21208*D)/(324*x^4 - 837*x^3 + 855*x^2 - 402*x + 88), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(88-402x+855x^2-837x^3+324x^4)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(324*x^4-837*x^3+855*x^2-402*x+88)^2,x, algo
rithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(88-402x+855x^2-837x^3+324x^4)^2}\,dx = \int \frac{A+B\,x+C\,x^2+x^3\,D}{(324\,x^4-837\,x^3+855\,x^2-402\,x+88)^2}\,dx$$

input
```
int((A + B*x + C*x^2 + x^3*D)/(855*x^2 - 402*x - 837*x^3 + 324*x^4 + 88)^2
,x)
```

output
```
int((A + B*x + C*x^2 + x^3*D)/(855*x^2 - 402*x - 837*x^3 + 324*x^4 + 88)^2
, x)
```

Reduce [F]

$$\int \frac{A+Bx+Cx^2+Dx^3}{(88-402x+855x^2-837x^3+324x^4)^2}\,dx = \text{too large to display}$$

input
```
int((D*x^3+C*x^2+B*x+A)/(324*x^4-837*x^3+855*x^2-402*x+88)^2,x)
```

output
```
(419904*int(x**3/(104976*x**8 - 542376*x**7 + 1254609*x**6 - 1691766*x**5
+ 1460997*x**4 - 834732*x**3 + 312084*x**2 - 70752*x + 7744),x)*a*x**4 - 1
084752*int(x**3/(104976*x**8 - 542376*x**7 + 1254609*x**6 - 1691766*x**5 +
 1460997*x**4 - 834732*x**3 + 312084*x**2 - 70752*x + 7744),x)*a*x**3 + 11
08080*int(x**3/(104976*x**8 - 542376*x**7 + 1254609*x**6 - 1691766*x**5 +
1460997*x**4 - 834732*x**3 + 312084*x**2 - 70752*x + 7744),x)*a*x**2 - 520
992*int(x**3/(104976*x**8 - 542376*x**7 + 1254609*x**6 - 1691766*x**5 + 14
60997*x**4 - 834732*x**3 + 312084*x**2 - 70752*x + 7744),x)*a*x + 114048*i
nt(x**3/(104976*x**8 - 542376*x**7 + 1254609*x**6 - 1691766*x**5 + 1460997
*x**4 - 834732*x**3 + 312084*x**2 - 70752*x + 7744),x)*a + 130248*int(x**3
/(104976*x**8 - 542376*x**7 + 1254609*x**6 - 1691766*x**5 + 1460997*x**4 -
 834732*x**3 + 312084*x**2 - 70752*x + 7744),x)*d*x**4 - 336474*int(x**3/(
104976*x**8 - 542376*x**7 + 1254609*x**6 - 1691766*x**5 + 1460997*x**4 - 8
34732*x**3 + 312084*x**2 - 70752*x + 7744),x)*d*x**3 + 343710*int(x**3/(10
4976*x**8 - 542376*x**7 + 1254609*x**6 - 1691766*x**5 + 1460997*x**4 - 834
732*x**3 + 312084*x**2 - 70752*x + 7744),x)*d*x**2 - 161604*int(x**3/(1049
76*x**8 - 542376*x**7 + 1254609*x**6 - 1691766*x**5 + 1460997*x**4 - 83473
2*x**3 + 312084*x**2 - 70752*x + 7744),x)*d*x + 35376*int(x**3/(104976*x**
8 - 542376*x**7 + 1254609*x**6 - 1691766*x**5 + 1460997*x**4 - 834732*x**3
 + 312084*x**2 - 70752*x + 7744),x)*d - 813564*int(x**2/(104976*x**8 - ...
```

3.17 $\int \frac{3x+3x^2+x^3}{1+4x+6x^2+4x^3+x^4} \, dx$

Optimal result

Integrand size = 33, antiderivative size = 14

$$\int \frac{3x+3x^2+x^3}{1+4x+6x^2+4x^3+x^4} \, dx = \frac{1}{3(1+x)^3} + \log(1+x)$$

output
```
1/3/(1+x)^3+ln(1+x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{3x+3x^2+x^3}{1+4x+6x^2+4x^3+x^4} \, dx = \frac{1}{3(1+x)^3} + \log(1+x)$$

input
```
Integrate[(3*x + 3*x^2 + x^3)/(1 + 4*x + 6*x^2 + 4*x^3 + x^4),x]
```

output
```
1/(3*(1 + x)^3) + Log[1 + x]
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {2007, 2028, 1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3+3x^2+3x}{x^4+4x^3+6x^2+4x+1}dx$$

$$\Big\downarrow \text{ 2007}$$

$$\int \frac{x^3+3x^2+3x}{(x+1)^4}dx$$

$$\Big\downarrow \text{ 2028}$$

$$\int \frac{x(x^2+3x+3)}{(x+1)^4}dx$$

$$\Big\downarrow \text{ 1195}$$

$$\int \left(\frac{1}{x+1}-\frac{1}{(x+1)^4}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{3(x+1)^3}+\log(x+1)$$

input
```
Int[(3*x + 3*x^2 + x^3)/(1 + 4*x + 6*x^2 + 4*x^3 + x^4),x]
```

output
```
1/(3*(1 + x)^3) + Log[1 + x]
```

Defintions of rubi rules used

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2007
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{a = Rt[Coeff[Px, x, 0], Expon[Px,
x]], b = Rt[Coeff[Px, x, Expon[Px, x]], Expon[Px, x]]}, Int[u*(a + b*x)^(Ex
pon[Px, x]*p), x] /; EqQ[Px, (a + b*x)^Expon[Px, x]]] /; IntegerQ[p] && Pol
yQ[Px, x] && GtQ[Expon[Px, x], 1] && NeQ[Coeff[Px, x, 0], 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2028
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.) + (c_.)*(x_)^(t_.))^(p_.),
x_Symbol] :> Int[x^(p*r)*(a + b*x^(s - r) + c*x^(t - r))^p*Fx, x] /; FreeQ[
{a, b, c, r, s, t}, x] && IntegerQ[p] && PosQ[s - r] && PosQ[t - r] &&  !(E
qQ[p, 1] && EqQ[u, 1])
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result	size
default	$\frac{1}{3(x+1)^3}+\ln(x+1)$	13
norman	$\frac{1}{3(x+1)^3}+\ln(x+1)$	13
risch	$\frac{1}{3x^3+9x^2+9x+3}+\ln(x+1)$	23
parallelrisch	$\frac{3\ln(x+1)x^3+1+9\ln(x+1)x^2+9\ln(x+1)x+3\ln(x+1)}{3x^3+9x^2+9x+3}$	51

input
```
int((x^3+3*x^2+3*x)/(x^4+4*x^3+6*x^2+4*x+1),x,method=_RETURNVERBOSE)
```

output
```
1/3/(x+1)^3+ln(x+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 38 vs. $2(12) = 24$.

Time = 0.06 (sec) , antiderivative size = 38, normalized size of antiderivative = 2.71

$$\int \frac{3x + 3x^2 + x^3}{1 + 4x + 6x^2 + 4x^3 + x^4} \, dx = \frac{3\,(x^3 + 3\,x^2 + 3\,x + 1)\log{(x + 1)} + 1}{3\,(x^3 + 3\,x^2 + 3\,x + 1)}$$

input
```
integrate((x^3+3*x^2+3*x)/(x^4+4*x^3+6*x^2+4*x+1),x, algorithm="fricas")
```

output
```
1/3*(3*(x^3 + 3*x^2 + 3*x + 1)*log(x + 1) + 1)/(x^3 + 3*x^2 + 3*x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.43

$$\int \frac{3x + 3x^2 + x^3}{1 + 4x + 6x^2 + 4x^3 + x^4} \, dx = \log{(x + 1)} + \frac{1}{3x^3 + 9x^2 + 9x + 3}$$

input
```
integrate((x**3+3*x**2+3*x)/(x**4+4*x**3+6*x**2+4*x+1),x)
```

output
```
log(x + 1) + 1/(3*x**3 + 9*x**2 + 9*x + 3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.57

$$\int \frac{3x + 3x^2 + x^3}{1 + 4x + 6x^2 + 4x^3 + x^4} \, dx = \frac{1}{3\,(x^3 + 3\,x^2 + 3\,x + 1)} + \log{(x + 1)}$$

input
```
integrate((x^3+3*x^2+3*x)/(x^4+4*x^3+6*x^2+4*x+1),x, algorithm="maxima")
```

output
```
1/3/(x^3 + 3*x^2 + 3*x + 1) + log(x + 1)
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

$$\int \frac{3x+3x^2+x^3}{1+4x+6x^2+4x^3+x^4}\,dx = \frac{1}{3\,(x+1)^3}+\log\left(|x+1|\right)$$

input
```
integrate((x^3+3*x^2+3*x)/(x^4+4*x^3+6*x^2+4*x+1),x, algorithm="giac")
```

output
```
1/3/(x + 1)^3 + log(abs(x + 1))
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{3x+3x^2+x^3}{1+4x+6x^2+4x^3+x^4}\,dx = \ln\left(x+1\right)+\frac{1}{3\,(x+1)^3}$$

input
```
int((3*x + 3*x^2 + x^3)/(4*x + 6*x^2 + 4*x^3 + x^4 + 1),x)
```

output
```
log(x + 1) + 1/(3*(x + 1)^3)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 51, normalized size of antiderivative = 3.64

$$\int \frac{3x+3x^2+x^3}{1+4x+6x^2+4x^3+x^4}\,dx = \frac{3\log(x+1)\,x^3+9\log(x+1)\,x^2+9\log(x+1)\,x+3\log(x+1)+1}{3x^3+9x^2+9x+3}$$

input
```
int((x^3+3*x^2+3*x)/(x^4+4*x^3+6*x^2+4*x+1),x)
```

output
```
(3*log(x + 1)*x**3 + 9*log(x + 1)*x**2 + 9*log(x + 1)*x + 3*log(x + 1) + 1
)/(3*(x**3 + 3*x**2 + 3*x + 1))
```

3.18 $\int \frac{-1+3x-3x^2+x^3}{1+4x+6x^2+4x^3+x^4} \, dx$

Optimal result

Integrand size = 34, antiderivative size = 28

$$\int \frac{-1+3x-3x^2+x^3}{1+4x+6x^2+4x^3+x^4} \, dx = \frac{8}{3(1+x)^3} - \frac{6}{(1+x)^2} + \frac{6}{1+x} + \log(1+x)$$

output
```
8/3/(1+x)^3-6/(1+x)^2+6/(1+x)+ln(1+x)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \frac{-1+3x-3x^2+x^3}{1+4x+6x^2+4x^3+x^4} \, dx = \frac{2(4+9x+9x^2)}{3(1+x)^3} + \log(1+x)$$

input
```
Integrate[(-1 + 3*x - 3*x^2 + x^3)/(1 + 4*x + 6*x^2 + 4*x^3 + x^4),x]
```

output
```
(2*(4 + 9*x + 9*x^2))/(3*(1 + x)^3) + Log[1 + x]
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2006, 2007, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3-3x^2+3x-1}{x^4+4x^3+6x^2+4x+1}dx$$
$$\Big\downarrow \text{ 2006}$$
$$\int \frac{(x-1)^3}{x^4+4x^3+6x^2+4x+1}dx$$
$$\Big\downarrow \text{ 2007}$$
$$\int \frac{(x-1)^3}{(x+1)^4}dx$$
$$\Big\downarrow \text{ 49}$$
$$\int \left(\frac{1}{x+1}-\frac{6}{(x+1)^2}+\frac{12}{(x+1)^3}-\frac{8}{(x+1)^4}\right)dx$$
$$\Big\downarrow \text{ 2009}$$
$$\frac{6}{x+1}-\frac{6}{(x+1)^2}+\frac{8}{3(x+1)^3}+\log(x+1)$$

input
```
Int[(-1 + 3*x - 3*x^2 + x^3)/(1 + 4*x + 6*x^2 + 4*x^3 + x^4),x]
```

output
```
8/(3*(1 + x)^3) - 6/(1 + x)^2 + 6/(1 + x) + Log[1 + x]
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2006
```
Int[(u_.)*(Px_), x_Symbol] :> With[{a = Rt[Coeff[Px, x, 0], Expon[Px, x]],
b = Rt[Coeff[Px, x, Expon[Px, x]], Expon[Px, x]]}, Int[u*(a + b*x)^Expon[Px
, x], x] /; EqQ[Px, (a + b*x)^Expon[Px, x]]] /; PolyQ[Px, x] && GtQ[Expon[P
x, x], 1] && NeQ[Coeff[Px, x, 0], 0] &&  !MatchQ[Px, (a_.)*(v_)^Expon[Px, x
] /; FreeQ[a, x] && LinearQ[v, x]]
```

rule 2007
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{a = Rt[Coeff[Px, x, 0], Expon[Px,
x]], b = Rt[Coeff[Px, x, Expon[Px, x]], Expon[Px, x]]}, Int[u*(a + b*x)^(Ex
pon[Px, x]*p), x] /; EqQ[Px, (a + b*x)^Expon[Px, x]]] /; IntegerQ[p] && Pol
yQ[Px, x] && GtQ[Expon[Px, x], 1] && NeQ[Coeff[Px, x, 0], 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.79

method	result	size
norman	$\frac{6x+6x^2+\frac{8}{3}}{(x+1)^3}+\ln(x+1)$	22
default	$\frac{8}{3(x+1)^3}-\frac{6}{(x+1)^2}+\frac{6}{x+1}+\ln(x+1)$	27
risch	$\frac{6x+6x^2+\frac{8}{3}}{x^3+3x^2+3x+1}+\ln(x+1)$	32
parallelrisch	$\frac{3\ln(x+1)x^3+8+9\ln(x+1)x^2+9\ln(x+1)x+18x^2+3\ln(x+1)+18x}{3x^3+9x^2+9x+3}$	59

input
```
int((x^3-3*x^2+3*x-1)/(x^4+4*x^3+6*x^2+4*x+1),x,method=_RETURNVERBOSE)
```

output
```
(6*x+6*x^2+8/3)/(x+1)^3+ln(x+1)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.64

$$\int \frac{-1+3x-3x^2+x^3}{1+4x+6x^2+4x^3+x^4}\,dx = \frac{18\,x^2+3\,(x^3+3\,x^2+3\,x+1)\log(x+1)+18\,x+8}{3\,(x^3+3\,x^2+3\,x+1)}$$

input
```
integrate((x^3-3*x^2+3*x-1)/(x^4+4*x^3+6*x^2+4*x+1),x, algorithm="fricas")
```

output
```
1/3*(18*x^2 + 3*(x^3 + 3*x^2 + 3*x + 1)*log(x + 1) + 18*x + 8)/(x^3 + 3*x^
2 + 3*x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

$$\int \frac{-1+3x-3x^2+x^3}{1+4x+6x^2+4x^3+x^4}\,dx = \frac{18x^2+18x+8}{3x^3+9x^2+9x+3}+\log(x+1)$$

input
```
integrate((x**3-3*x**2+3*x-1)/(x**4+4*x**3+6*x**2+4*x+1),x)
```

output
```
(18*x**2 + 18*x + 8)/(3*x**3 + 9*x**2 + 9*x + 3) + log(x + 1)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.14

$$\int \frac{-1+3x-3x^2+x^3}{1+4x+6x^2+4x^3+x^4}\,dx = \frac{2\,(9\,x^2+9\,x+4)}{3\,(x^3+3\,x^2+3\,x+1)}+\log(x+1)$$

input
```
integrate((x^3-3*x^2+3*x-1)/(x^4+4*x^3+6*x^2+4*x+1),x, algorithm="maxima")
```

output
```
2/3*(9*x^2 + 9*x + 4)/(x^3 + 3*x^2 + 3*x + 1) + log(x + 1)
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.82

$$\int \frac{-1+3x-3x^2+x^3}{1+4x+6x^2+4x^3+x^4}\,dx = \frac{2\,(9\,x^2+9\,x+4)}{3\,(x+1)^3} + \log\left(|x+1|\right)$$

input
```
integrate((x^3-3*x^2+3*x-1)/(x^4+4*x^3+6*x^2+4*x+1),x, algorithm="giac")
```

output
```
2/3*(9*x^2 + 9*x + 4)/(x + 1)^3 + log(abs(x + 1))
```

Mupad [B] (verification not implemented)

Time = 21.94 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.75

$$\int \frac{-1+3x-3x^2+x^3}{1+4x+6x^2+4x^3+x^4}\,dx = \ln\left(x+1\right) + \frac{6\,x^2+6\,x+\frac{8}{3}}{(x+1)^3}$$

input
```
int((3*x - 3*x^2 + x^3 - 1)/(4*x + 6*x^2 + 4*x^3 + x^4 + 1),x)
```

output
```
log(x + 1) + (6*x + 6*x^2 + 8/3)/(x + 1)^3
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.00

$$\int \frac{-1+3x-3x^2+x^3}{1+4x+6x^2+4x^3+x^4}\,dx = \frac{3\log(x+1)\,x^3+9\log(x+1)\,x^2+9\log(x+1)\,x+3\log(x+1)-6x^3+2}{3x^3+9x^2+9x+3}$$

input
```
int((x^3-3*x^2+3*x-1)/(x^4+4*x^3+6*x^2+4*x+1),x)
```

output
```
(3*log(x + 1)*x**3 + 9*log(x + 1)*x**2 + 9*log(x + 1)*x + 3*log(x + 1) - 6
*x**3 + 2)/(3*(x**3 + 3*x**2 + 3*x + 1))
```

3.19 $\int \frac{9-40x-18x^2+174x^4+24x^5+26x^6-39x^8}{(3+2x^2+x^4)^3}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 59

$$\int \frac{9-40x-18x^2+174x^4+24x^5+26x^6-39x^8}{(3+2x^2+x^4)^3}\,dx = \frac{2(1-2x^2)}{(3+2x^2+x^4)^2} - \frac{2x(18+13x^2)}{(3+2x^2+x^4)^2} + \frac{13x}{3+2x^2+x^4}$$

output
```
2*(-2*x^2+1)/(x^4+2*x^2+3)^2-2*x*(13*x^2+18)/(x^4+2*x^2+3)^2+13*x/(x^4+2*x
^2+3)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.47

$$\int \frac{9-40x-18x^2+174x^4+24x^5+26x^6-39x^8}{(3+2x^2+x^4)^3}\,dx = \frac{2+3x-4x^2+13x^5}{(3+2x^2+x^4)^2}$$

input
```
Integrate[(9 - 40*x - 18*x^2 + 174*x^4 + 24*x^5 + 26*x^6 - 39*x^8)/(3 + 2*
x^2 + x^4)^3,x]
```

output
```
(2 + 3*x - 4*x^2 + 13*x^5)/(3 + 2*x^2 + x^4)^2
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.186, Rules used = {2202, 2194, 27, 2191, 24, 2206, 27, 2021}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{-39x^8+26x^6+24x^5+174x^4-18x^2-40x+9}{(x^4+2x^2+3)^3}dx$$

$$\Big\downarrow \text{2202}$$

$$\int \frac{x(24x^4-40)}{(x^4+2x^2+3)^3}dx+\int \frac{-39x^8+26x^6+174x^4-18x^2+9}{(x^4+2x^2+3)^3}dx$$

$$\Big\downarrow \text{2194}$$

$$\frac{1}{2}\int -\frac{8(5-3x^4)}{(x^4+2x^2+3)^3}dx^2+\int \frac{-39x^8+26x^6+174x^4-18x^2+9}{(x^4+2x^2+3)^3}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{-39x^8+26x^6+174x^4-18x^2+9}{(x^4+2x^2+3)^3}dx-4\int \frac{5-3x^4}{(x^4+2x^2+3)^3}dx^2$$

$$\Big\downarrow \text{2191}$$

$$\int \frac{-39x^8+26x^6+174x^4-18x^2+9}{(x^4+2x^2+3)^3}dx-4\left(\frac{\int 0dx^2}{16}-\frac{1-2x^2}{2(x^4+2x^2+3)^2}\right)$$

$$\Big\downarrow \text{24}$$

$$\int \frac{-39x^8+26x^6+174x^4-18x^2+9}{(x^4+2x^2+3)^3}dx+\frac{2(1-2x^2)}{(x^4+2x^2+3)^2}$$

$$\Big\downarrow \text{2206}$$

$$\frac{1}{96}\int \frac{1248(-3x^4-2x^2+3)}{(x^4+2x^2+3)^2}dx+\frac{2(1-2x^2)}{(x^4+2x^2+3)^2}-\frac{2x(13x^2+18)}{(x^4+2x^2+3)^2}$$

$$\Big\downarrow \text{27}$$

$$13\int \frac{-3x^4-2x^2+3}{(x^4+2x^2+3)^2}dx+\frac{2(1-2x^2)}{(x^4+2x^2+3)^2}-\frac{2x(13x^2+18)}{(x^4+2x^2+3)^2}$$

$$\Big\downarrow \; 2021$$

$$\frac{13x}{x^4+2x^2+3}-\frac{2(13x^2+18)x}{(x^4+2x^2+3)^2}+\frac{2(1-2x^2)}{(x^4+2x^2+3)^2}$$

input
```
Int[(9 - 40*x - 18*x^2 + 174*x^4 + 24*x^5 + 26*x^6 - 39*x^8)/(3 + 2*x^2 +
x^4)^3,x]
```

output
```
(2*(1 - 2*x^2))/(3 + 2*x^2 + x^4)^2 - (2*x*(18 + 13*x^2))/(3 + 2*x^2 + x^4
)^2 + (13*x)/(3 + 2*x^2 + x^4)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2021
```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 2194
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Simp[1/2   Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)
^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ
[(m - 1)/2]
```

rule 2202
```
Int[(Pn_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{n
 = Expon[Pn, x], k}, Int[Sum[Coeff[Pn, x, 2*k]*x^(2*k), {k, 0, n/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pn, x, 2*k + 1]*x^(2*k), {k, 0, (n -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pn, x]
 &&  !PolyQ[Pn, x^2]
```

rule 2206
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c
*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Px,
a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*
p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x
^2] && Expon[Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.49

method	result	size
gosper	$\frac{13x^{5}-4x^{2}+3x+2}{\left(x^{4}+2x^{2}+3\right)^{2}}$	29
norman	$\frac{13x^{5}-4x^{2}+3x+2}{\left(x^{4}+2x^{2}+3\right)^{2}}$	29
risch	$\frac{13x^{5}-4x^{2}+3x+2}{\left(x^{4}+2x^{2}+3\right)^{2}}$	29
parallelrisch	$\frac{13x^{5}-4x^{2}+3x+2}{\left(x^{4}+2x^{2}+3\right)^{2}}$	29
default	$-\frac{-13x^{5}+4x^{2}-3x-2}{\left(x^{4}+2x^{2}+3\right)^{2}}$	30
orering	$-\frac{\left(13x^{5}-4x^{2}+3x+2\right)\left(-39x^{8}+26x^{6}+24x^{5}+174x^{4}-18x^{2}-40x+9\right)}{\left(x^{4}+2x^{2}+3\right)^{2}\left(39x^{8}-26x^{6}-24x^{5}-174x^{4}+18x^{2}+40x-9\right)}$	92

input
```
int((-39*x^8+26*x^6+24*x^5+174*x^4-18*x^2-40*x+9)/(x^4+2*x^2+3)^3,x,method
=_RETURNVERBOSE)
```

output
```
(13*x^5-4*x^2+3*x+2)/(x^4+2*x^2+3)^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.64

$$\int \frac{9-40x-18x^2+174x^4+24x^5+26x^6-39x^8}{(3+2x^2+x^4)^3}\,dx = \frac{13\,x^5-4\,x^2+3\,x+2}{x^8+4\,x^6+10\,x^4+12\,x^2+9}$$

input
```
integrate((-39*x^8+26*x^6+24*x^5+174*x^4-18*x^2-40*x+9)/(x^4+2*x^2+3)^3,x,
 algorithm="fricas")
```

output
```
(13*x^5 - 4*x^2 + 3*x + 2)/(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.61

$$\int \frac{9-40x-18x^2+174x^4+24x^5+26x^6-39x^8}{(3+2x^2+x^4)^3}\,dx = -\frac{-13x^5+4x^2-3x-2}{x^8+4x^6+10x^4+12x^2+9}$$

input
```
integrate((-39*x**8+26*x**6+24*x**5+174*x**4-18*x**2-40*x+9)/(x**4+2*x**2+
3)**3,x)
```

output
```
-(-13*x**5 + 4*x**2 - 3*x - 2)/(x**8 + 4*x**6 + 10*x**4 + 12*x**2 + 9)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.64

$$\int \frac{9-40x-18x^2+174x^4+24x^5+26x^6-39x^8}{(3+2x^2+x^4)^3}\,dx = \frac{13\,x^5-4\,x^2+3\,x+2}{x^8+4\,x^6+10\,x^4+12\,x^2+9}$$

input
```
integrate((-39*x^8+26*x^6+24*x^5+174*x^4-18*x^2-40*x+9)/(x^4+2*x^2+3)^3,x,
 algorithm="maxima")
```

output
```
(13*x^5 - 4*x^2 + 3*x + 2)/(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9)
```

Giac [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.47

$$\int \frac{9-40x-18x^2+174x^4+24x^5+26x^6-39x^8}{(3+2x^2+x^4)^3}\,dx = \frac{13\,x^5-4\,x^2+3\,x+2}{(x^4+2\,x^2+3)^2}$$

input
```
integrate((-39*x^8+26*x^6+24*x^5+174*x^4-18*x^2-40*x+9)/(x^4+2*x^2+3)^3,x,
 algorithm="giac")
```

output
```
(13*x^5 - 4*x^2 + 3*x + 2)/(x^4 + 2*x^2 + 3)^2
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.47

$$\int \frac{9-40x-18x^2+174x^4+24x^5+26x^6-39x^8}{(3+2x^2+x^4)^3}\,dx = \frac{13\,x^5-4\,x^2+3\,x+2}{(x^4+2\,x^2+3)^2}$$

input
```
int((174*x^4 - 18*x^2 - 40*x + 24*x^5 + 26*x^6 - 39*x^8 + 9)/(2*x^2 + x^4
+ 3)^3,x)
```

output
```
(3*x - 4*x^2 + 13*x^5 + 2)/(2*x^2 + x^4 + 3)^2
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.64

$$\int \frac{9-40x-18x^2+174x^4+24x^5+26x^6-39x^8}{(3+2x^2+x^4)^3}\,dx = \frac{13x^5-4x^2+3x+2}{x^8+4x^6+10x^4+12x^2+9}$$

input
```
int((-39*x^8+26*x^6+24*x^5+174*x^4-18*x^2-40*x+9)/(x^4+2*x^2+3)^3,x)
```

output
```
(13*x**5 - 4*x**2 + 3*x + 2)/(x**8 + 4*x**6 + 10*x**4 + 12*x**2 + 9)
```

3.20 $\int x(a+8x-8x^2+4x^3-x^4)^4\,dx$

Optimal result

Integrand size = 24, antiderivative size = 210

$$\begin{aligned}\int x\left(a+8x-8x^2+4x^3-x^4\right)^4\,dx =& \frac{a^4x^2}{2}+\frac{32a^3x^3}{3}+8(12-a)a^2x^4\\&+\frac{16}{5}a\left(128-48a+a^2\right)x^5\\&+\frac{2}{3}\left(1024-1536a+192a^2-a^3\right)x^6\\&-\frac{32}{7}\left(512-288a+15a^2\right)x^7\\&+8(128-3a)(4-a)x^8-\frac{16}{3}\left(896-128a+a^2\right)x^9\\&+\frac{1}{5}\left(20480-1536a+3a^2\right)x^{10}-\frac{32}{11}(928-35a)x^{11}\\&+\frac{8}{3}(524-9a)x^{12}-\frac{16}{13}(464-3a)x^{13}\\&+\frac{2}{7}(640-a)x^{14}-\frac{224x^{15}}{5}+8x^{16}-\frac{16x^{17}}{17}+\frac{x^{18}}{18}\end{aligned}$$

output
```
1/2*a^4*x^2+32/3*a^3*x^3+8*(12-a)*a^2*x^4+16/5*a*(a^2-48*a+128)*x^5+2/3*(-
a^3+192*a^2-1536*a+1024)*x^6-32/7*(15*a^2-288*a+512)*x^7+8*(128-3*a)*(4-a)
*x^8-16/3*(a^2-128*a+896)*x^9+1/5*(3*a^2-1536*a+20480)*x^10-32/11*(928-35*
a)*x^11+8/3*(524-9*a)*x^12-16/13*(464-3*a)*x^13+2/7*(640-a)*x^14-224/5*x^1
5+8*x^16-16/17*x^17+1/18*x^18
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.97

$$\int x\left(a+8x-8x^2+4x^3-x^4\right)^4\,dx=\frac{a^4x^2}{2}+\frac{32a^3x^3}{3}-8(-12+a)a^2x^4+\frac{16}{5}a\left(128-48a+a^2\right)x^5-\frac{2}{3}\left(-1024+1536a-192a^2+a^3\right)x^6-\frac{32}{7}\left(512-288a+15a^2\right)x^7+8\left(512-140a+3a^2\right)x^8-\frac{16}{3}\left(896-128a+a^2\right)x^9+\frac{1}{5}\left(20480-1536a+3a^2\right)x^{10}+\frac{32}{11}(-928+35a)x^{11}-\frac{8}{3}(-524+9a)x^{12}+\frac{16}{13}(-464+3a)x^{13}-\frac{2}{7}(-640+a)x^{14}-\frac{224x^{15}}{5}+8x^{16}-\frac{16x^{17}}{17}+\frac{x^{18}}{18}$$

input
```
Integrate[x*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^4,x]
```

output
```
(a^4*x^2)/2 + (32*a^3*x^3)/3 - 8*(-12 + a)*a^2*x^4 + (16*a*(128 - 48*a + a
^2)*x^5)/5 - (2*(-1024 + 1536*a - 192*a^2 + a^3)*x^6)/3 - (32*(512 - 288*a
 + 15*a^2)*x^7)/7 + 8*(512 - 140*a + 3*a^2)*x^8 - (16*(896 - 128*a + a^2)*
x^9)/3 + ((20480 - 1536*a + 3*a^2)*x^10)/5 + (32*(-928 + 35*a)*x^11)/11 -
(8*(-524 + 9*a)*x^12)/3 + (16*(-464 + 3*a)*x^13)/13 - (2*(-640 + a)*x^14)/
7 - (224*x^15)/5 + 8*x^16 - (16*x^17)/17 + x^18/18
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2465, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a-x^4+4x^3-8x^2+8x\right)^4\,dx$$

$$\Big\downarrow \; 2465$$

$$\int \left(a^4x+32a^3x^2+2\left(3a^2-1536a+20480\right)x^9-48\left(a^2-128a+896\right)x^8-32\left(15a^2-288a+512\right)x^6+16a(a^2$$

$$\Big\downarrow \; 2009$$

$$\frac{a^4x^2}{2}+\frac{32a^3x^3}{3}+\frac{1}{5}\left(3a^2-1536a+20480\right)x^{10}-\frac{16}{3}\left(a^2-128a+896\right)x^9-\frac{32}{7}\left(15a^2-288a+512\right)x^7+\frac{16}{5}a\left(a^2-48a+128\right)x^5+8(12-a)a^2x^4+\frac{2}{3}\left(-a^3+192a^2-1536a+1024\right)x^6+\frac{2}{7}(640-a)x^{14}-\frac{16}{13}(464-3a)x^{13}+\frac{8}{3}(524-9a)x^{12}-\frac{32}{11}(928-35a)x^{11}+8(128-3a)(4-a)x^8+\frac{x^{18}}{18}-\frac{16x^{17}}{17}+8x^{16}-\frac{224x^{15}}{5}$$

input
```
Int[x*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^4,x]
```

output
```
(a^4*x^2)/2 + (32*a^3*x^3)/3 + 8*(12 - a)*a^2*x^4 + (16*a*(128 - 48*a + a^
2)*x^5)/5 + (2*(1024 - 1536*a + 192*a^2 - a^3)*x^6)/3 - (32*(512 - 288*a +
 15*a^2)*x^7)/7 + 8*(128 - 3*a)*(4 - a)*x^8 - (16*(896 - 128*a + a^2)*x^9)
/3 + ((20480 - 1536*a + 3*a^2)*x^10)/5 - (32*(928 - 35*a)*x^11)/11 + (8*(5
24 - 9*a)*x^12)/3 - (16*(464 - 3*a)*x^13)/13 + (2*(640 - a)*x^14)/7 - (224
*x^15)/5 + 8*x^16 - (16*x^17)/17 + x^18/18
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2465
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> Int[ExpandToSum[u, Px^p, x], x] /; PolyQ
[Px, x] && GtQ[Expon[Px, x], 2] && !BinomialQ[Px, x] && !TrinomialQ[Px, x
] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.87

method	result
norman	$\frac{a^4x^2}{2}+\frac{32a^3x^3}{3}+\left(-8a^3+96a^2\right)x^4+\left(\frac{16}{5}a^3-\frac{768}{5}a^2+\frac{2048}{5}a\right)x^5+\left(-\frac{2}{3}a^3+128a^2-1024a+\right.$
gosper	$\frac{2048}{5}a\,x^5+\frac{4192}{3}x^{12}+\frac{1280}{7}x^{14}+4096x^{10}-\frac{14336}{3}x^9+8x^{16}-\frac{16}{17}x^{17}-\frac{224}{5}x^{15}-\frac{7424}{13}x^{13}-\frac{1638}{7}$
risch	$\frac{2048}{5}a\,x^5+\frac{4192}{3}x^{12}+\frac{1280}{7}x^{14}+4096x^{10}-\frac{14336}{3}x^9+8x^{16}-\frac{16}{17}x^{17}-\frac{224}{5}x^{15}-\frac{7424}{13}x^{13}-\frac{1638}{7}$
parallelrisch	$\frac{2048}{5}a\,x^5+\frac{4192}{3}x^{12}+\frac{1280}{7}x^{14}+4096x^{10}-\frac{14336}{3}x^9+8x^{16}-\frac{16}{17}x^{17}-\frac{224}{5}x^{15}-\frac{7424}{13}x^{13}-\frac{1638}{7}$
orering	$\frac{x^2\left(85085x^{16}-1441440x^{15}+12252240x^{14}-437580x^{12}a-68612544x^{13}+5654880x^{11}a+280051200x^{12}-36756720x^{10}a-87462\right.}{}$
default	$\frac{x^{18}}{18}-\frac{16x^{17}}{17}+8x^{16}-\frac{224x^{15}}{5}+\frac{(-4a+2560)x^{14}}{14}+\frac{(48a-7424)x^{13}}{13}+\frac{(-288a+16768)x^{12}}{12}+\frac{(1120a-29696)x^{1}}{11}$

input

```
int(x*(-x^4+4*x^3-8*x^2+a+8*x)^4,x,method=_RETURNVERBOSE)
```

output

```
1/2*a^4*x^2+32/3*a^3*x^3+(-8*a^3+96*a^2)*x^4+(16/5*a^3-768/5*a^2+2048/5*a)
*x^5+(-2/3*a^3+128*a^2-1024*a+2048/3)*x^6+(-480/7*a^2+9216/7*a-16384/7)*x^
7+(24*a^2-1120*a+4096)*x^8+(-16/3*a^2+2048/3*a-14336/3)*x^9+(3/5*a^2-1536/
5*a+4096)*x^10+(1120/11*a-29696/11)*x^11+(-24*a+4192/3)*x^12+(48/13*a-7424
/13)*x^13+(-2/7*a+1280/7)*x^14-224/5*x^15+8*x^16-16/17*x^17+1/18*x^18
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.87

$$\int x(a+8x-8x^2+4x^3-x^4)^4\,dx = \frac{1}{18}\,x^{18} - \frac{16}{17}\,x^{17} + 8\,x^{16} - \frac{2}{7}\,(a-640)x^{14} - \frac{224}{5}\,x^{15} + \frac{16}{13}\,(3\,a-464)x^{13} - \frac{8}{3}\,(9\,a-524)x^{12} + \frac{32}{11}\,(35\,a-928)x^{11} + \frac{1}{5}\,\left(3\,a^2-1536\,a+20480\right)x^{10} - \frac{16}{3}\,\left(a^2-128\,a+896\right)x^9 + 8\,\left(3\,a^2-140\,a+512\right)x^8 - \frac{32}{7}\,\left(15\,a^2-288\,a+512\right)x^7 - \frac{2}{3}\,\left(a^3-192\,a^2+1536\,a-1024\right)x^6 + \frac{1}{2}\,a^4x^2 + \frac{32}{3}\,a^3x^3 + \frac{16}{5}\,\left(a^3-48\,a^2+128\,a\right)x^5 - 8\,\left(a^3-12\,a^2\right)x^4$$

input
```
integrate(x*(-x^4+4*x^3-8*x^2+a+8*x)^4,x, algorithm="fricas")
```

output
```
1/18*x^18 - 16/17*x^17 + 8*x^16 - 2/7*(a - 640)*x^14 - 224/5*x^15 + 16/13*
(3*a - 464)*x^13 - 8/3*(9*a - 524)*x^12 + 32/11*(35*a - 928)*x^11 + 1/5*(3
*a^2 - 1536*a + 20480)*x^10 - 16/3*(a^2 - 128*a + 896)*x^9 + 8*(3*a^2 - 14
0*a + 512)*x^8 - 32/7*(15*a^2 - 288*a + 512)*x^7 - 2/3*(a^3 - 192*a^2 + 15
36*a - 1024)*x^6 + 1/2*a^4*x^2 + 32/3*a^3*x^3 + 16/5*(a^3 - 48*a^2 + 128*a
)*x^5 - 8*(a^3 - 12*a^2)*x^4
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.01

$$\int x(a+8x-8x^2+4x^3-x^4)^4\,dx = \frac{a^4x^2}{2}+\frac{32a^3x^3}{3}+\frac{x^{18}}{18}-\frac{16x^{17}}{17}+8x^{16}-\frac{224x^{15}}{5}+x^{14}\cdot\left(\frac{1280}{7}-\frac{2a}{7}\right)+x^{13}\cdot\left(\frac{48a}{13}-\frac{7424}{13}\right)+x^{12}\cdot\left(\frac{4192}{3}-24a\right)+x^{11}\cdot\left(\frac{1120a}{11}-\frac{29696}{11}\right)+x^{10}\cdot\left(\frac{3a^2}{5}-\frac{1536a}{5}+4096\right)+x^9\left(-\frac{16a^2}{3}+\frac{2048a}{3}-\frac{14336}{3}\right)+x^8\cdot(24a^2-1120a+4096)+x^7\left(-\frac{480a^2}{7}+\frac{9216a}{7}-\frac{16384}{7}\right)+x^6\left(-\frac{2a^3}{3}+128a^2-1024a+\frac{2048}{3}\right)+x^5\cdot\left(\frac{16a^3}{5}-\frac{768a^2}{5}+\frac{2048a}{5}\right)+x^4(-8a^3+96a^2)$$

input
```
integrate(x*(-x**4+4*x**3-8*x**2+a+8*x)**4,x)
```

output
```
a**4*x**2/2 + 32*a**3*x**3/3 + x**18/18 - 16*x**17/17 + 8*x**16 - 224*x**1
5/5 + x**14*(1280/7 - 2*a/7) + x**13*(48*a/13 - 7424/13) + x**12*(4192/3 -
 24*a) + x**11*(1120*a/11 - 29696/11) + x**10*(3*a**2/5 - 1536*a/5 + 4096)
 + x**9*(-16*a**2/3 + 2048*a/3 - 14336/3) + x**8*(24*a**2 - 1120*a + 4096)
 + x**7*(-480*a**2/7 + 9216*a/7 - 16384/7) + x**6*(-2*a**3/3 + 128*a**2 -
1024*a + 2048/3) + x**5*(16*a**3/5 - 768*a**2/5 + 2048*a/5) + x**4*(-8*a**
3 + 96*a**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.87

$$\int x(a+8x-8x^2+4x^3-x^4)^4\,dx = \frac{1}{18}x^{18} - \frac{16}{17}x^{17} + 8x^{16} - \frac{2}{7}(a-640)x^{14} - \frac{224}{5}x^{15} + \frac{16}{13}(3a-464)x^{13} - \frac{8}{3}(9a-524)x^{12} + \frac{32}{11}(35a-928)x^{11} + \frac{1}{5}(3a^2-1536a+20480)x^{10} - \frac{16}{3}(a^2-128a+896)x^9 + 8(3a^2-140a+512)x^8 - \frac{32}{7}(15a^2-288a+512)x^7 - \frac{2}{3}(a^3-192a^2+1536a-1024)x^6 + \frac{1}{2}a^4x^2 + \frac{32}{3}a^3x^3 + \frac{16}{5}(a^3-48a^2+128a)x^5 - 8(a^3-12a^2)x^4$$

input
```
integrate(x*(-x^4+4*x^3-8*x^2+a+8*x)^4,x, algorithm="maxima")
```

output
```
1/18*x^18 - 16/17*x^17 + 8*x^16 - 2/7*(a - 640)*x^14 - 224/5*x^15 + 16/13*
(3*a - 464)*x^13 - 8/3*(9*a - 524)*x^12 + 32/11*(35*a - 928)*x^11 + 1/5*(3
*a^2 - 1536*a + 20480)*x^10 - 16/3*(a^2 - 128*a + 896)*x^9 + 8*(3*a^2 - 14
0*a + 512)*x^8 - 32/7*(15*a^2 - 288*a + 512)*x^7 - 2/3*(a^3 - 192*a^2 + 15
36*a - 1024)*x^6 + 1/2*a^4*x^2 + 32/3*a^3*x^3 + 16/5*(a^3 - 48*a^2 + 128*a
)*x^5 - 8*(a^3 - 12*a^2)*x^4
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.06

$$\int x(a+8x-8x^2+4x^3-x^4)^4\,dx = \frac{1}{18}x^{18}-\frac{16}{17}x^{17}+8x^{16}-\frac{2}{7}ax^{14}-\frac{224}{5}x^{15}+\frac{48}{13}ax^{13}+\frac{1280}{7}x^{14}-24ax^{12}-\frac{7424}{13}x^{13}+\frac{3}{5}a^2x^{10}+\frac{1120}{11}ax^{11}+\frac{4192}{3}x^{12}-\frac{16}{3}a^2x^9-\frac{1536}{5}ax^{10}-\frac{29696}{11}x^{11}+24a^2x^8+\frac{2048}{3}ax^9+4096x^{10}-\frac{2}{3}a^3x^6-\frac{480}{7}a^2x^7-1120ax^8-\frac{14336}{3}x^9+\frac{16}{5}a^3x^5+128a^2x^6+\frac{9216}{7}ax^7+4096x^8-8a^3x^4-\frac{768}{5}a^2x^5-1024ax^6-\frac{16384}{7}x^7+\frac{1}{2}a^4x^2+\frac{32}{3}a^3x^3+96a^2x^4+\frac{2048}{5}ax^5+\frac{2048}{3}x^6$$

input
```
integrate(x*(-x^4+4*x^3-8*x^2+a+8*x)^4,x, algorithm="giac")
```

output
```
1/18*x^18 - 16/17*x^17 + 8*x^16 - 2/7*a*x^14 - 224/5*x^15 + 48/13*a*x^13 +
 1280/7*x^14 - 24*a*x^12 - 7424/13*x^13 + 3/5*a^2*x^10 + 1120/11*a*x^11 +
4192/3*x^12 - 16/3*a^2*x^9 - 1536/5*a*x^10 - 29696/11*x^11 + 24*a^2*x^8 +
2048/3*a*x^9 + 4096*x^10 - 2/3*a^3*x^6 - 480/7*a^2*x^7 - 1120*a*x^8 - 1433
6/3*x^9 + 16/5*a^3*x^5 + 128*a^2*x^6 + 9216/7*a*x^7 + 4096*x^8 - 8*a^3*x^4
 - 768/5*a^2*x^5 - 1024*a*x^6 - 16384/7*x^7 + 1/2*a^4*x^2 + 32/3*a^3*x^3 +
 96*a^2*x^4 + 2048/5*a*x^5 + 2048/3*x^6
```

Mupad [B] (verification not implemented)

Time = 22.10 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.85

$$\int x\left(a+8x-8x^2+4x^3-x^4\right)^4\,dx = x^{13}\left(\frac{48\,a}{13}-\frac{7424}{13}\right)-x^{12}\left(24\,a-\frac{4192}{3}\right) - x^{14}\left(\frac{2\,a}{7}-\frac{1280}{7}\right)+x^{11}\left(\frac{1120\,a}{11}-\frac{29696}{11}\right) + x^8\left(24\,a^2-1120\,a+4096\right) + x^{10}\left(\frac{3\,a^2}{5}-\frac{1536\,a}{5}+4096\right) - x^9\left(\frac{16\,a^2}{3}-\frac{2048\,a}{3}+\frac{14336}{3}\right) - x^7\left(\frac{480\,a^2}{7}-\frac{9216\,a}{7}+\frac{16384}{7}\right) - x^6\left(\frac{2\,a^3}{3}-128\,a^2+1024\,a-\frac{2048}{3}\right)-\frac{224\,x^{15}}{5} + 8\,x^{16}-\frac{16\,x^{17}}{17}+\frac{x^{18}}{18}+\frac{32\,a^3\,x^3}{3}+\frac{a^4\,x^2}{2} + \frac{16\,a\,x^5\left(a^2-48\,a+128\right)}{5}-8\,a^2\,x^4\,(a-12)$$

input

```
int(x*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^4,x)
```

output

```
x^13*((48*a)/13 - 7424/13) - x^12*(24*a - 4192/3) - x^14*((2*a)/7 - 1280/7
) + x^11*((1120*a)/11 - 29696/11) + x^8*(24*a^2 - 1120*a + 4096) + x^10*((
3*a^2)/5 - (1536*a)/5 + 4096) - x^9*((16*a^2)/3 - (2048*a)/3 + 14336/3) -
x^7*((480*a^2)/7 - (9216*a)/7 + 16384/7) - x^6*(1024*a - 128*a^2 + (2*a^3)
/3 - 2048/3) - (224*x^15)/5 + 8*x^16 - (16*x^17)/17 + x^18/18 + (32*a^3*x^
3)/3 + (a^4*x^2)/2 + (16*a*x^5*(a^2 - 48*a + 128))/5 - 8*a^2*x^4*(a - 12)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.06

$$\int x\left(a+8x-8x^2+4x^3-x^4\right)^4\,dx$$
$$=\frac{x^2(85085x^{16}-1441440x^{15}+12252240x^{14}-437580a\,x^{12}-68612544x^{13}+5654880a\,x^{11}+280051200x^{12}}{}$$

input
```
int(x*(-x^4+4*x^3-8*x^2+a+8*x)^4,x)
```

output
```
(x**2*(765765*a**4 - 1021020*a**3*x**4 + 4900896*a**3*x**3 - 12252240*a**3
*x**2 + 16336320*a**3*x + 918918*a**2*x**8 - 8168160*a**2*x**7 + 36756720*
a**2*x**6 - 105019200*a**2*x**5 + 196035840*a**2*x**4 - 235243008*a**2*x**
3 + 147026880*a**2*x**2 - 437580*a*x**12 + 5654880*a*x**11 - 36756720*a*x*
*10 + 155937600*a*x**9 - 470486016*a*x**8 + 1045524480*a*x**7 - 1715313600
*a*x**6 + 2016368640*a*x**5 - 1568286720*a*x**4 + 627314688*a*x**3 + 85085
*x**16 - 1441440*x**15 + 12252240*x**14 - 68612544*x**13 + 280051200*x**12
 - 874621440*x**11 + 2140057920*x**10 - 4134574080*x**9 + 6273146880*x**8
- 7318671360*x**7 + 6273146880*x**6 - 3584655360*x**5 + 1045524480*x**4))/
1531530
```

3.21 $\int x(a+8x-8x^2+4x^3-x^4)^3\,dx$

Optimal result

Integrand size = 24, antiderivative size = 134

$$\int x(a+8x-8x^2+4x^3-x^4)^3\,dx=\frac{a^3x^2}{2}+8a^2x^3+6(8-a)ax^4+\frac{4}{5}(128-96a+3a^2)\,x^5-\frac{1}{2}(512-128a+a^2)\,x^6+\frac{48}{7}(48-5a)x^7-4(70-3a)x^8+\frac{8}{3}(64-a)x^9-\frac{3}{10}(256-a)x^{10}+\frac{280x^{11}}{11}-6x^{12}+\frac{12x^{13}}{13}-\frac{x^{14}}{14}$$

output
```
1/2*a^3*x^2+8*a^2*x^3+6*(8-a)*a*x^4+4/5*(3*a^2-96*a+128)*x^5-1/2*(a^2-128*
a+512)*x^6+48/7*(48-5*a)*x^7-4*(70-3*a)*x^8+8/3*(64-a)*x^9-3/10*(256-a)*x^
10+280/11*x^11-6*x^12+12/13*x^13-1/14*x^14
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.97

$$\int x\left(a+8x-8x^2+4x^3-x^4\right)^3\,dx=\frac{a^3x^2}{2}+8a^2x^3-6(-8+a)ax^4+\frac{4}{5}\left(128-96a+3a^2\right)x^5+\frac{1}{2}\left(-512+128a-a^2\right)x^6-\frac{48}{7}(-48+5a)x^7+4(-70+3a)x^8-\frac{8}{3}(-64+a)x^9+\frac{3}{10}(-256+a)x^{10}+\frac{280x^{11}}{11}-6x^{12}+\frac{12x^{13}}{13}-\frac{x^{14}}{14}$$

input
```
Integrate[x*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^3,x]
```

output
```
(a^3*x^2)/2 + 8*a^2*x^3 - 6*(-8 + a)*a*x^4 + (4*(128 - 96*a + 3*a^2)*x^5)/
5 + ((-512 + 128*a - a^2)*x^6)/2 - (48*(-48 + 5*a)*x^7)/7 + 4*(-70 + 3*a)*
x^8 - (8*(-64 + a)*x^9)/3 + (3*(-256 + a)*x^10)/10 + (280*x^11)/11 - 6*x^1
2 + (12*x^13)/13 - x^14/14
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2465, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a-x^4+4x^3-8x^2+8x\right)^3\,dx$$

$$\Big\downarrow \text{2465}$$

$$\int \left(a^3x-3\left(a^2-128a+512\right)x^5+4\left(3a^2-96a+128\right)x^4+24a^2x^2-3(256-a)x^9+24(64-a)x^8-32(70-3$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^3x^2}{2} - \frac{1}{2}\left(a^2 - 128a + 512\right)x^6 + \frac{4}{5}\left(3a^2 - 96a + 128\right)x^5 + 8a^2x^3 - \frac{3}{10}(256-a)x^{10} + \frac{8}{3}(64-a)x^9 - 4(70-3a)x^8 + \frac{48}{7}(48-5a)x^7 + 6(8-a)ax^4 - \frac{x^{14}}{14} + \frac{12x^{13}}{13} - 6x^{12} + \frac{280x^{11}}{11}$$

input
```
Int[x*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^3,x]
```

output
```
(a^3*x^2)/2 + 8*a^2*x^3 + 6*(8 - a)*a*x^4 + (4*(128 - 96*a + 3*a^2)*x^5)/5
 - ((512 - 128*a + a^2)*x^6)/2 + (48*(48 - 5*a)*x^7)/7 - 4*(70 - 3*a)*x^8
+ (8*(64 - a)*x^9)/3 - (3*(256 - a)*x^10)/10 + (280*x^11)/11 - 6*x^12 + (1
2*x^13)/13 - x^14/14
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2465
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> Int[ExpandToSum[u, Px^p, x], x] /; PolyQ
[Px, x] && GtQ[Expon[Px, x], 2] && !BinomialQ[Px, x] && !TrinomialQ[Px, x
] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.86

method	result
norman	$\frac{a^3x^2}{2} + 8a^2x^3 + \left(-6a^2 + 48a\right)x^4 + \left(\frac{12}{5}a^2 - \frac{384}{5}a + \frac{512}{5}\right)x^5 + \left(-\frac{1}{2}a^2 + 64a - 256\right)x^6 + (-$
gosper	$\frac{1}{2}a^3x^2 + 8a^2x^3 - 6a^2x^4 + 48a\,x^4 + \frac{12}{5}a^2x^5 - \frac{384}{5}a\,x^5 + \frac{512}{5}x^5 - \frac{1}{2}a^2x^6 + 64x^6a - 256x^6 -$
risch	$\frac{1}{2}a^3x^2 + 8a^2x^3 - 6a^2x^4 + 48a\,x^4 + \frac{12}{5}a^2x^5 - \frac{384}{5}a\,x^5 + \frac{512}{5}x^5 - \frac{1}{2}a^2x^6 + 64x^6a - 256x^6 -$
parallelrisch	$\frac{1}{2}a^3x^2 + 8a^2x^3 - 6a^2x^4 + 48a\,x^4 + \frac{12}{5}a^2x^5 - \frac{384}{5}a\,x^5 + \frac{512}{5}x^5 - \frac{1}{2}a^2x^6 + 64x^6a - 256x^6 -$
orering	$x^2(-2145x^{12}+27720x^{11}-180180x^{10}+9009a\,x^8+764400x^9-80080x^7a-2306304x^8+360360x^6a+5125120x^7-15015a^2x^4-$
default	$-\frac{x^{14}}{14} + \frac{12x^{13}}{13} - 6x^{12} + \frac{280x^{11}}{11} + \frac{(3a-768)x^{10}}{10} + \frac{(-24a+1536)x^9}{9} + \frac{(96a-2240)x^8}{8} + \frac{(-240a+2304)x^7}{7} +$

input
```
int(x*(-x^4+4*x^3-8*x^2+a+8*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*a^3*x^2+8*a^2*x^3+(-6*a^2+48*a)*x^4+(12/5*a^2-384/5*a+512/5)*x^5+(-1/2
*a^2+64*a-256)*x^6+(-240/7*a+2304/7)*x^7+(12*a-280)*x^8+(-8/3*a+512/3)*x^9
+(3/10*a-384/5)*x^10+280/11*x^11-6*x^12+12/13*x^13-1/14*x^14
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.84

$$\int x\left(a+8x-8x^2+4x^3-x^4\right)^3\,dx = -\frac{1}{14}\,x^{14}+\frac{12}{13}\,x^{13}-6\,x^{12}+\frac{3}{10}\,(a-256)x^{10}+\frac{280}{11}\,x^{11}-\frac{8}{3}\,(a-64)x^9+4\,(3\,a-70)x^8-\frac{48}{7}\,(5\,a-48)x^7-\frac{1}{2}\,\left(a^2-128\,a+512\right)x^6+\frac{4}{5}\,\left(3\,a^2-96\,a+128\right)x^5+\frac{1}{2}\,a^3x^2+8\,a^2x^3-6\,\left(a^2-8\,a\right)x^4$$

input
```
integrate(x*(-x^4+4*x^3-8*x^2+a+8*x)^3,x, algorithm="fricas")
```

output
```
-1/14*x^14 + 12/13*x^13 - 6*x^12 + 3/10*(a - 256)*x^10 + 280/11*x^11 - 8/3
*(a - 64)*x^9 + 4*(3*a - 70)*x^8 - 48/7*(5*a - 48)*x^7 - 1/2*(a^2 - 128*a
+ 512)*x^6 + 4/5*(3*a^2 - 96*a + 128)*x^5 + 1/2*a^3*x^2 + 8*a^2*x^3 - 6*(a
^2 - 8*a)*x^4
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.96

$$\int x\left(a+8x-8x^2+4x^3-x^4\right)^3\,dx=\frac{a^3x^2}{2}+8a^2x^3-\frac{x^{14}}{14}+\frac{12x^{13}}{13}-6x^{12}+\frac{280x^{11}}{11}+x^{10}\cdot\left(\frac{3a}{10}-\frac{384}{5}\right)+x^9\cdot\left(\frac{512}{3}-\frac{8a}{3}\right)+x^8\cdot(12a-280)+x^7\cdot\left(\frac{2304}{7}-\frac{240a}{7}\right)+x^6\left(-\frac{a^2}{2}+64a-256\right)+x^5\cdot\left(\frac{12a^2}{5}-\frac{384a}{5}+\frac{512}{5}\right)+x^4\left(-6a^2+48a\right)$$

input
```
integrate(x*(-x**4+4*x**3-8*x**2+a+8*x)**3,x)
```

output
```
a**3*x**2/2 + 8*a**2*x**3 - x**14/14 + 12*x**13/13 - 6*x**12 + 280*x**11/1
1 + x**10*(3*a/10 - 384/5) + x**9*(512/3 - 8*a/3) + x**8*(12*a - 280) + x*
*7*(2304/7 - 240*a/7) + x**6*(-a**2/2 + 64*a - 256) + x**5*(12*a**2/5 - 38
4*a/5 + 512/5) + x**4*(-6*a**2 + 48*a)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.84

$$\int x\left(a+8x-8x^2+4x^3-x^4\right)^3\,dx=-\frac{1}{14}\,x^{14}+\frac{12}{13}\,x^{13}-6\,x^{12}+\frac{3}{10}\,(a-256)x^{10}+\frac{280}{11}\,x^{11}-\frac{8}{3}\,(a-64)x^9+4\,(3\,a-70)x^8-\frac{48}{7}\,(5\,a-48)x^7-\frac{1}{2}\,\left(a^2-128\,a+512\right)x^6+\frac{4}{5}\,\left(3\,a^2-96\,a+128\right)x^5+\frac{1}{2}\,a^3x^2+8\,a^2x^3-6\,\left(a^2-8\,a\right)x^4$$

input
```
integrate(x*(-x^4+4*x^3-8*x^2+a+8*x)^3,x, algorithm="maxima")
```

output
```
-1/14*x^14 + 12/13*x^13 - 6*x^12 + 3/10*(a - 256)*x^10 + 280/11*x^11 - 8/3
*(a - 64)*x^9 + 4*(3*a - 70)*x^8 - 48/7*(5*a - 48)*x^7 - 1/2*(a^2 - 128*a
+ 512)*x^6 + 4/5*(3*a^2 - 96*a + 128)*x^5 + 1/2*a^3*x^2 + 8*a^2*x^3 - 6*(a
^2 - 8*a)*x^4
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.99

$$\int x(a+8x-8x^2+4x^3-x^4)^3\,dx = -\frac{1}{14}x^{14}+\frac{12}{13}x^{13}-6x^{12}+\frac{3}{10}ax^{10}+\frac{280}{11}x^{11} -\frac{8}{3}ax^9-\frac{384}{5}x^{10}+12ax^8+\frac{512}{3}x^9 -\frac{1}{2}a^2x^6-\frac{240}{7}ax^7-280x^8+\frac{12}{5}a^2x^5 +64ax^6+\frac{2304}{7}x^7-6a^2x^4-\frac{384}{5}ax^5 -256x^6+\frac{1}{2}a^3x^2+8a^2x^3+48ax^4+\frac{512}{5}x^5$$

input
```
integrate(x*(-x^4+4*x^3-8*x^2+a+8*x)^3,x, algorithm="giac")
```

output
```
-1/14*x^14 + 12/13*x^13 - 6*x^12 + 3/10*a*x^10 + 280/11*x^11 - 8/3*a*x^9 -
 384/5*x^10 + 12*a*x^8 + 512/3*x^9 - 1/2*a^2*x^6 - 240/7*a*x^7 - 280*x^8 +
 12/5*a^2*x^5 + 64*a*x^6 + 2304/7*x^7 - 6*a^2*x^4 - 384/5*a*x^5 - 256*x^6
+ 1/2*a^3*x^2 + 8*a^2*x^3 + 48*a*x^4 + 512/5*x^5
```

Mupad [B] (verification not implemented)

Time = 21.78 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.84

$$\int x(a+8x-8x^2+4x^3-x^4)^3\,dx = x^8\,(12\,a-280)+x^{10}\left(\frac{3\,a}{10}-\frac{384}{5}\right)-x^9\left(\frac{8\,a}{3}-\frac{512}{3}\right) -x^7\left(\frac{240\,a}{7}-\frac{2304}{7}\right)-x^6\left(\frac{a^2}{2}-64\,a+256\right) +x^5\left(\frac{12\,a^2}{5}-\frac{384\,a}{5}+\frac{512}{5}\right)+\frac{280\,x^{11}}{11}-6\,x^{12} +\frac{12\,x^{13}}{13}-\frac{x^{14}}{14}+8\,a^2\,x^3+\frac{a^3\,x^2}{2}-6\,a\,x^4\,(a-8)$$

input
```
int(x*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^3,x)
```

output
```
x^8*(12*a - 280) + x^10*((3*a)/10 - 384/5) - x^9*((8*a)/3 - 512/3) - x^7*(
(240*a)/7 - 2304/7) - x^6*(a^2/2 - 64*a + 256) + x^5*((12*a^2)/5 - (384*a)
/5 + 512/5) + (280*x^11)/11 - 6*x^12 + (12*x^13)/13 - x^14/14 + 8*a^2*x^3
+ (a^3*x^2)/2 - 6*a*x^4*(a - 8)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.99

$$\int x\left(a+8x-8x^2+4x^3-x^4\right)^3\,dx$$
$$= \frac{x^2(-2145x^{12}+27720x^{11}-180180x^{10}+9009a\,x^8+764400x^9-80080a\,x^7-2306304x^8+360360a\,x^6-}{}$$

input
```
int(x*(-x^4+4*x^3-8*x^2+a+8*x)^3,x)
```

output
```
(x**2*(15015*a**3 - 15015*a**2*x**4 + 72072*a**2*x**3 - 180180*a**2*x**2 +
 240240*a**2*x + 9009*a*x**8 - 80080*a*x**7 + 360360*a*x**6 - 1029600*a*x*
*5 + 1921920*a*x**4 - 2306304*a*x**3 + 1441440*a*x**2 - 2145*x**12 + 27720
*x**11 - 180180*x**10 + 764400*x**9 - 2306304*x**8 + 5125120*x**7 - 840840
0*x**6 + 9884160*x**5 - 7687680*x**4 + 3075072*x**3))/30030
```

3.22 $\int x(a+8x-8x^2+4x^3-x^4)^2\,dx$

Optimal result

Integrand size = 24, antiderivative size = 79

$$\int x\left(a+8x-8x^2+4x^3-x^4\right)^2\,dx=\frac{a^2x^2}{2}+\frac{16ax^3}{3}+4(4-a)x^4-\frac{8}{5}(16-a)x^5+\frac{1}{3}(64-a)x^6-\frac{80x^7}{7}+4x^8-\frac{8x^9}{9}+\frac{x^{10}}{10}$$

output
```
1/2*a^2*x^2+16/3*a*x^3+4*(4-a)*x^4-8/5*(16-a)*x^5+1/3*(64-a)*x^6-80/7*x^7+
4*x^8-8/9*x^9+1/10*x^10
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.95

$$\int x\left(a+8x-8x^2+4x^3-x^4\right)^2\,dx=\frac{a^2x^2}{2}+\frac{16ax^3}{3}-4(-4+a)x^4+\frac{8}{5}(-16+a)x^5+\frac{1}{3}(64-a)x^6-\frac{80x^7}{7}+4x^8-\frac{8x^9}{9}+\frac{x^{10}}{10}$$

input
```
Integrate[x*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x]
```

output
```
(a^2*x^2)/2 + (16*a*x^3)/3 - 4*(-4 + a)*x^4 + (8*(-16 + a)*x^5)/5 + ((64 -
 a)*x^6)/3 - (80*x^7)/7 + 4*x^8 - (8*x^9)/9 + x^10/10
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {2465, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a-x^4+4x^3-8x^2+8x\right)^2 dx$$

$$\Big\downarrow \; 2465$$

$$\int \left(a^2x+2(64-a)x^5-8(16-a)x^4+16(4-a)x^3+16ax^2+x^9-8x^8+32x^7-80x^6\right) dx$$

$$\Big\downarrow \; 2009$$

$$\frac{a^2x^2}{2}+\frac{1}{3}(64-a)x^6-\frac{8}{5}(16-a)x^5+4(4-a)x^4+\frac{16ax^3}{3}+\frac{x^{10}}{10}-\frac{8x^9}{9}+4x^8-\frac{80x^7}{7}$$

input
```
Int[x*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x]
```

output
```
(a^2*x^2)/2 + (16*a*x^3)/3 + 4*(4 - a)*x^4 - (8*(16 - a)*x^5)/5 + ((64 - a
)*x^6)/3 - (80*x^7)/7 + 4*x^8 - (8*x^9)/9 + x^10/10
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2465
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> Int[ExpandToSum[u, Px^p, x], x] /; PolyQ
[Px, x] && GtQ[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x
] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.80

method	result
norman	$\frac{x^{10}}{10}-\frac{8x^9}{9}+4x^8-\frac{80x^7}{7}+\left(\frac{64}{3}-\frac{a}{3}\right)x^6+\left(\frac{8a}{5}-\frac{128}{5}\right)x^5+\left(-4a+16\right)x^4+\frac{16a\,x^3}{3}+\frac{a^2x^2}{2}$
default	$\frac{x^{10}}{10}-\frac{8x^9}{9}+4x^8-\frac{80x^7}{7}+\frac{(-2a+128)x^6}{6}+\frac{(8a-128)x^5}{5}+\frac{(-16a+64)x^4}{4}+\frac{16a\,x^3}{3}+\frac{a^2x^2}{2}$
gosper	$\frac{1}{10}x^{10}-\frac{8}{9}x^9+4x^8-\frac{80}{7}x^7+\frac{64}{3}x^6-\frac{1}{3}x^6a+\frac{8}{5}a\,x^5-\frac{128}{5}x^5-4a\,x^4+16x^4+\frac{16}{3}a\,x^3+\frac{1}{2}a^2x$
risch	$\frac{1}{10}x^{10}-\frac{8}{9}x^9+4x^8-\frac{80}{7}x^7+\frac{64}{3}x^6-\frac{1}{3}x^6a+\frac{8}{5}a\,x^5-\frac{128}{5}x^5-4a\,x^4+16x^4+\frac{16}{3}a\,x^3+\frac{1}{2}a^2x$
parallelrisch	$\frac{1}{10}x^{10}-\frac{8}{9}x^9+4x^8-\frac{80}{7}x^7+\frac{64}{3}x^6-\frac{1}{3}x^6a+\frac{8}{5}a\,x^5-\frac{128}{5}x^5-4a\,x^4+16x^4+\frac{16}{3}a\,x^3+\frac{1}{2}a^2x$
orering	$\frac{x^2\left(63x^8-560x^7+2520x^6-210a\,x^4-7200x^5+1008a\,x^3+13440x^4-2520a\,x^2-16128x^3+315a^2+3360xa+10080x^2\right)}{630}$

input
```
int(x*(-x^4+4*x^3-8*x^2+a+8*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/10*x^10-8/9*x^9+4*x^8-80/7*x^7+(64/3-1/3*a)*x^6+(8/5*a-128/5)*x^5+(-4*a+
16)*x^4+16/3*a*x^3+1/2*a^2*x^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.75

$$\int x\left(a+8x-8x^2+4x^3-x^4\right)^2\,dx=\frac{1}{10}\,x^{10}-\frac{8}{9}\,x^9+4\,x^8-\frac{1}{3}\,(a-64)x^6-\frac{80}{7}\,x^7+\frac{8}{5}\,(a-16)x^5-4\,(a-4)x^4+\frac{1}{2}\,a^2x^2+\frac{16}{3}\,ax^3$$

input
```
integrate(x*(-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="fricas")
```

output
```
1/10*x^10 - 8/9*x^9 + 4*x^8 - 1/3*(a - 64)*x^6 - 80/7*x^7 + 8/5*(a - 16)*x
^5 - 4*(a - 4)*x^4 + 1/2*a^2*x^2 + 16/3*a*x^3
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.89

$$\int x\left(a+8x-8x^2+4x^3-x^4\right)^2\,dx=\frac{a^2x^2}{2}+\frac{16ax^3}{3}+\frac{x^{10}}{10}-\frac{8x^9}{9}+4x^8-\frac{80x^7}{7}+x^6\cdot\left(\frac{64}{3}-\frac{a}{3}\right)+x^5\cdot\left(\frac{8a}{5}-\frac{128}{5}\right)+x^4\cdot(16-4a)$$

input
```
integrate(x*(-x**4+4*x**3-8*x**2+a+8*x)**2,x)
```

output
```
a**2*x**2/2 + 16*a*x**3/3 + x**10/10 - 8*x**9/9 + 4*x**8 - 80*x**7/7 + x**
6*(64/3 - a/3) + x**5*(8*a/5 - 128/5) + x**4*(16 - 4*a)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.75

$$\int x\left(a+8x-8x^2+4x^3-x^4\right)^2\,dx=\frac{1}{10}\,x^{10}-\frac{8}{9}\,x^9+4\,x^8-\frac{1}{3}\,(a-64)x^6-\frac{80}{7}\,x^7+\frac{8}{5}\,(a-16)x^5-4\,(a-4)x^4+\frac{1}{2}\,a^2x^2+\frac{16}{3}\,ax^3$$

input
```
integrate(x*(-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="maxima")
```

output
```
1/10*x^10 - 8/9*x^9 + 4*x^8 - 1/3*(a - 64)*x^6 - 80/7*x^7 + 8/5*(a - 16)*x
^5 - 4*(a - 4)*x^4 + 1/2*a^2*x^2 + 16/3*a*x^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.86

$$\int x\left(a+8x-8x^2+4x^3-x^4\right)^2\,dx=\frac{1}{10}\,x^{10}-\frac{8}{9}\,x^9+4\,x^8-\frac{1}{3}\,ax^6-\frac{80}{7}\,x^7+\frac{8}{5}\,ax^5+\frac{64}{3}\,x^6-4\,ax^4-\frac{128}{5}\,x^5+\frac{1}{2}\,a^2x^2+\frac{16}{3}\,ax^3+16\,x^4$$

input
```
integrate(x*(-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="giac")
```

output
```
1/10*x^10 - 8/9*x^9 + 4*x^8 - 1/3*a*x^6 - 80/7*x^7 + 8/5*a*x^5 + 64/3*x^6
- 4*a*x^4 - 128/5*x^5 + 1/2*a^2*x^2 + 16/3*a*x^3 + 16*x^4
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.81

$$\int x\left(a+8x-8x^2+4x^3-x^4\right)^2\,dx=x^5\left(\frac{8\,a}{5}-\frac{128}{5}\right)-x^6\left(\frac{a}{3}-\frac{64}{3}\right)-x^4\,(4\,a-16)+\frac{16\,a\,x^3}{3}-\frac{80\,x^7}{7}+4\,x^8-\frac{8\,x^9}{9}+\frac{x^{10}}{10}+\frac{a^2\,x^2}{2}$$

input
```
int(x*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x)
```

output
```
x^5*((8*a)/5 - 128/5) - x^6*(a/3 - 64/3) - x^4*(4*a - 16) + (16*a*x^3)/3 -
 (80*x^7)/7 + 4*x^8 - (8*x^9)/9 + x^10/10 + (a^2*x^2)/2
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.86

$$\int x\left(a+8x-8x^2+4x^3-x^4\right)^2\,dx$$
$$=\frac{x^2(63x^8-560x^7+2520x^6-210a\,x^4-7200x^5+1008a\,x^3+13440x^4-2520a\,x^2-16128x^3+315a^2+}{630}$$

input
```
int(x*(-x^4+4*x^3-8*x^2+a+8*x)^2,x)
```

output
```
(x**2*(315*a**2 - 210*a*x**4 + 1008*a*x**3 - 2520*a*x**2 + 3360*a*x + 63*x
**8 - 560*x**7 + 2520*x**6 - 7200*x**5 + 13440*x**4 - 16128*x**3 + 10080*x
**2))/630
```

3.23 $\int x(a+8x-8x^2+4x^3-x^4)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 35

$$\int x\left(a+8x-8x^2+4x^3-x^4\right)\,dx=\frac{ax^2}{2}+\frac{8x^3}{3}-2x^4+\frac{4x^5}{5}-\frac{x^6}{6}$$

output
```
1/2*a*x^2+8/3*x^3-2*x^4+4/5*x^5-1/6*x^6
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int x\left(a+8x-8x^2+4x^3-x^4\right)\,dx=\frac{ax^2}{2}+\frac{8x^3}{3}-2x^4+\frac{4x^5}{5}-\frac{x^6}{6}$$

input
```
Integrate[x*(a + 8*x - 8*x^2 + 4*x^3 - x^4),x]
```

output
```
(a*x^2)/2 + (8*x^3)/3 - 2*x^4 + (4*x^5)/5 - x^6/6
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a-x^4+4x^3-8x^2+8x\right)\,dx$$

$$\Big\downarrow \text{2010}$$

$$\int \left(ax-x^5+4x^4-8x^3+8x^2\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{ax^2}{2}-\frac{x^6}{6}+\frac{4x^5}{5}-2x^4+\frac{8x^3}{3}$$

input
```
Int[x*(a + 8*x - 8*x^2 + 4*x^3 - x^4),x]
```

output
```
(a*x^2)/2 + (8*x^3)/3 - 2*x^4 + (4*x^5)/5 - x^6/6
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.80

method	result	size
gosper	$\frac{1}{2}a\,x^{2}+\frac{8}{3}x^{3}-2x^{4}+\frac{4}{5}x^{5}-\frac{1}{6}x^{6}$	28
default	$\frac{1}{2}a\,x^{2}+\frac{8}{3}x^{3}-2x^{4}+\frac{4}{5}x^{5}-\frac{1}{6}x^{6}$	28
norman	$\frac{1}{2}a\,x^{2}+\frac{8}{3}x^{3}-2x^{4}+\frac{4}{5}x^{5}-\frac{1}{6}x^{6}$	28
risch	$\frac{1}{2}a\,x^{2}+\frac{8}{3}x^{3}-2x^{4}+\frac{4}{5}x^{5}-\frac{1}{6}x^{6}$	28
parallelrisch	$\frac{1}{2}a\,x^{2}+\frac{8}{3}x^{3}-2x^{4}+\frac{4}{5}x^{5}-\frac{1}{6}x^{6}$	28
orering	$\frac{x^{2}\left(-5x^{4}+24x^{3}-60x^{2}+15a+80x\right)}{30}$	28

input
```
int(x*(-x^4+4*x^3-8*x^2+a+8*x),x,method=_RETURNVERBOSE)
```

output
```
1/2*a*x^2+8/3*x^3-2*x^4+4/5*x^5-1/6*x^6
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.77

$$\int x\left(a+8x-8x^{2}+4x^{3}-x^{4}\right)\,dx=-\frac{1}{6}\,x^{6}+\frac{4}{5}\,x^{5}-2\,x^{4}+\frac{1}{2}\,ax^{2}+\frac{8}{3}\,x^{3}$$

input
```
integrate(x*(-x^4+4*x^3-8*x^2+a+8*x),x, algorithm="fricas")
```

output
```
-1/6*x^6 + 4/5*x^5 - 2*x^4 + 1/2*a*x^2 + 8/3*x^3
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

$$\int x(a+8x-8x^2+4x^3-x^4)\ dx = \frac{ax^2}{2} - \frac{x^6}{6} + \frac{4x^5}{5} - 2x^4 + \frac{8x^3}{3}$$

input
```
integrate(x*(-x**4+4*x**3-8*x**2+a+8*x),x)
```

output
```
a*x**2/2 - x**6/6 + 4*x**5/5 - 2*x**4 + 8*x**3/3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.77

$$\int x(a+8x-8x^2+4x^3-x^4)\ dx = -\frac{1}{6}x^6 + \frac{4}{5}x^5 - 2x^4 + \frac{1}{2}ax^2 + \frac{8}{3}x^3$$

input
```
integrate(x*(-x^4+4*x^3-8*x^2+a+8*x),x, algorithm="maxima")
```

output
```
-1/6*x^6 + 4/5*x^5 - 2*x^4 + 1/2*a*x^2 + 8/3*x^3
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.77

$$\int x(a+8x-8x^2+4x^3-x^4)\ dx = -\frac{1}{6}x^6 + \frac{4}{5}x^5 - 2x^4 + \frac{1}{2}ax^2 + \frac{8}{3}x^3$$

input
```
integrate(x*(-x^4+4*x^3-8*x^2+a+8*x),x, algorithm="giac")
```

output
```
-1/6*x^6 + 4/5*x^5 - 2*x^4 + 1/2*a*x^2 + 8/3*x^3
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.77

$$\int x(a+8x-8x^2+4x^3-x^4)\,dx=-\frac{x^6}{6}+\frac{4\,x^5}{5}-2\,x^4+\frac{8\,x^3}{3}+\frac{a\,x^2}{2}$$

input
```
int(x*(a + 8*x - 8*x^2 + 4*x^3 - x^4),x)
```

output
```
(a*x^2)/2 + (8*x^3)/3 - 2*x^4 + (4*x^5)/5 - x^6/6
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.77

$$\int x(a+8x-8x^2+4x^3-x^4)\,dx=\frac{x^2(-5x^4+24x^3-60x^2+15a+80x)}{30}$$

input
```
int(x*(-x^4+4*x^3-8*x^2+a+8*x),x)
```

output
```
(x**2*(15*a - 5*x**4 + 24*x**3 - 60*x**2 + 80*x))/30
```

3.24 $\int \frac{x}{a+8x-8x^2+4x^3-x^4} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 120

$$\int \frac{x}{a+8x-8x^2+4x^3-x^4} \, dx = \frac{\arctan\left(\frac{1-x}{\sqrt{1-\sqrt{4+a}}}\right)}{2\sqrt{4+a}\sqrt{1-\sqrt{4+a}}} - \frac{\arctan\left(\frac{1-x}{\sqrt{1+\sqrt{4+a}}}\right)}{2\sqrt{4+a}\sqrt{1+\sqrt{4+a}}} + \frac{\operatorname{arctanh}\left(\frac{1+(-1+x)^2}{\sqrt{4+a}}\right)}{2\sqrt{4+a}}$$

output
```
1/2*arctan((1-x)/(1-(4+a)^(1/2))^(1/2))/(4+a)^(1/2)/(1-(4+a)^(1/2))^(1/2)-
1/2*arctan((1-x)/(1+(4+a)^(1/2))^(1/2))/(4+a)^(1/2)/(1+(4+a)^(1/2))^(1/2)+
1/2*arctanh((1+(-1+x)^2)/(4+a)^(1/2))/(4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.49

$$\int \frac{x}{a+8x-8x^2+4x^3-x^4} \, dx = -\frac{1}{4}\text{RootSum}\left[a+8\#1-8\#1^2+4\#1^3 - \#1^4\&, \frac{\log(x-\#1)\#1}{-2+4\#1-3\#1^2+\#1^3}\&\right]$$

input
```
Integrate[x/(a + 8*x - 8*x^2 + 4*x^3 - x^4),x]
```

output
```
-1/4*RootSum[a + 8*#1 - 8*#1^2 + 4*#1^3 - #1^4 & , (Log[x - #1]*#1)/(-2 +
4*#1 - 3*#1^2 + #1^3) & ]
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {2459, 2202, 1406, 217, 1432, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{a-x^4+4x^3-8x^2+8x}dx$$

$$\Big\downarrow \text{2459}$$

$$\int \frac{x}{a-(x-1)^4-2(x-1)^2+3}d(x-1)$$

$$\Big\downarrow \text{2202}$$

$$\int \frac{1}{-(x-1)^4-2(x-1)^2+a+3}d(x-1)+\int \frac{x-1}{-(x-1)^4-2(x-1)^2+a+3}d(x-1)$$

$$\Big\downarrow \text{1406}$$

$$-\frac{\int \frac{1}{-(x-1)^2-\sqrt{a+4}-1}d(x-1)}{2\sqrt{a+4}}+\frac{\int \frac{1}{-(x-1)^2+\sqrt{a+4}-1}d(x-1)}{2\sqrt{a+4}}+\int \frac{x-1}{-(x-1)^4-2(x-1)^2+a+3}d(x-1)$$

$$\Big\downarrow \text{217}$$

$$\int \frac{x-1}{-(x-1)^4-2(x-1)^2+a+3}d(x-1)-\frac{\arctan\left(\frac{x-1}{\sqrt{1-\sqrt{a+4}}}\right)}{2\sqrt{a+4}\sqrt{1-\sqrt{a+4}}}+\frac{\arctan\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)}{2\sqrt{a+4}\sqrt{\sqrt{a+4}+1}}$$

$$\Big\downarrow \text{1432}$$

$$\frac{1}{2}\int \frac{1}{-(x-1)^4-2(x-1)^2+a+3}d(x-1)^2-\frac{\arctan\left(\frac{x-1}{\sqrt{1-\sqrt{a+4}}}\right)}{2\sqrt{a+4}\sqrt{1-\sqrt{a+4}}}+\frac{\arctan\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)}{2\sqrt{a+4}\sqrt{\sqrt{a+4}+1}}$$

$$\Big\downarrow \text{1083}$$

$$-\int \frac{1}{4(a+4)-(x-1)^4}d(-2(x-1)^2-2)-\frac{\arctan\left(\frac{x-1}{\sqrt{1-\sqrt{a+4}}}\right)}{2\sqrt{a+4}\sqrt{1-\sqrt{a+4}}}+\frac{\arctan\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)}{2\sqrt{a+4}\sqrt{\sqrt{a+4}+1}}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\arctan\left(\frac{x-1}{\sqrt{1-\sqrt{a+4}}}\right)}{2\sqrt{a+4}\sqrt{1-\sqrt{a+4}}}+\frac{\arctan\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)}{2\sqrt{a+4}\sqrt{\sqrt{a+4}+1}}-\frac{\text{arctanh}\left(\frac{-2(x-1)^2-2}{2\sqrt{a+4}}\right)}{2\sqrt{a+4}}$$

input
```
Int[x/(a + 8*x - 8*x^2 + 4*x^3 - x^4),x]
```

output
```
-1/2*ArcTan[(-1 + x)/Sqrt[1 - Sqrt[4 + a]]]/(Sqrt[4 + a]*Sqrt[1 - Sqrt[4 +
 a]]) + ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]]/(2*Sqrt[4 + a]*Sqrt[1 + Sqr
t[4 + a]]) - ArcTanh[(-2 - 2*(-1 + x)^2)/(2*Sqrt[4 + a])]/(2*Sqrt[4 + a])
```

Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1406
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Simp[c/q   Int[1/(b/2 - q/2 + c*x^2), x], x] - Simp[c/q   I
nt[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c
, 0] && PosQ[b^2 - 4*a*c]
```

rule 1432
```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

rule 2202
```
Int[(Pn_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{n
 = Expon[Pn, x], k}, Int[Sum[Coeff[Pn, x, 2*k]*x^(2*k), {k, 0, n/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pn, x, 2*k + 1]*x^(2*k), {k, 0, (n -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pn, x]
 &&  !PolyQ[Pn, x^2]
```

rule 2459
```
Int[(Pn_)^(p_.)*(Qx_), x_Symbol] :> With[{S = Coeff[Pn, x, Expon[Pn, x] - 1
]/(Expon[Pn, x]*Coeff[Pn, x, Expon[Pn, x]])}, Subst[Int[ExpandToSum[Pn /. x
 -> x - S, x]^p*ExpandToSum[Qx /. x -> x - S, x], x], x, x + S] /; Binomial
Q[Pn /. x -> x - S, x] || (IntegerQ[Expon[Pn, x]/2] && TrinomialQ[Pn /. x -
> x - S, x])] /; FreeQ[p, x] && PolyQ[Pn, x] && GtQ[Expon[Pn, x], 2] && NeQ
[Coeff[Pn, x, Expon[Pn, x] - 1], 0] && PolyQ[Qx, x] &&  !(MonomialQ[Qx, x]
&& IGtQ[p, 0])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.43

method	result	size
default	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\_Z^4-4\_Z^3+8\_Z^2-8\_Z-a\right)}\frac{\_R\ln\left(x-\_R\right)}{-\_R^3+3\_R^2-4\_R+2}\right)}{4}$	52
risch	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\_Z^4-4\_Z^3+8\_Z^2-8\_Z-a\right)}\frac{\_R\ln\left(x-\_R\right)}{-\_R^3+3\_R^2-4\_R+2}\right)}{4}$	52

input
```
int(x/(-x^4+4*x^3-8*x^2+a+8*x),x,method=_RETURNVERBOSE)
```

output
```
1/4*sum(_R/(-_R^3+3*_R^2-4*_R+2)*ln(x-_R),_R=RootOf(_Z^4-4*_Z^3+8*_Z^2-8*_
Z-a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.14 (sec) , antiderivative size = 140500, normalized size of antiderivative = 1170.83

$$\int \frac{x}{a+8x-8x^2+4x^3-x^4}\,dx = \text{Too large to display}$$

input
```
integrate(x/(-x^4+4*x^3-8*x^2+a+8*x),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [A] (verification not implemented)

Time = 2.70 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.29

$$\int \frac{x}{a+8x-8x^2+4x^3-x^4}\,dx = -\operatorname{RootSum}\left(t^4\cdot\left(256a^3+2816a^2+10240a+12288\right)+t^2\left(-32a^2-256a-512\right)+t(-16a-64)+a,\right.$$

input
```
integrate(x/(-x**4+4*x**3-8*x**2+a+8*x),x)
```

output
```
-RootSum(_t**4*(256*a**3 + 2816*a**2 + 10240*a + 12288) + _t**2*(-32*a**2
- 256*a - 512) + _t*(-16*a - 64) + a, Lambda(_t, _t*log(x + (-128*_t**3*a*
*4 - 1728*_t**3*a**3 - 8640*_t**3*a**2 - 18944*_t**3*a - 15360*_t**3 + 48*
_t**2*a**3 + 464*_t**2*a**2 + 1472*_t**2*a + 1536*_t**2 + 8*_t*a**3 + 88*_
t*a**2 + 312*_t*a + 352*_t - a**2 - 2*a)/(4*a**2 + 21*a + 28))))
```

Maxima [F]

$$\int \frac{x}{a+8x-8x^2+4x^3-x^4}\,dx = \int -\frac{x}{x^4-4\,x^3+8\,x^2-a-8\,x}\,dx$$

input
```
integrate(x/(-x^4+4*x^3-8*x^2+a+8*x),x, algorithm="maxima")
```

output
```
-integrate(x/(x^4 - 4*x^3 + 8*x^2 - a - 8*x), x)
```

Giac [F]

$$\int \frac{x}{a+8x-8x^2+4x^3-x^4}\,dx = \int -\frac{x}{x^4-4\,x^3+8\,x^2-a-8\,x}\,dx$$

input
```
integrate(x/(-x^4+4*x^3-8*x^2+a+8*x),x, algorithm="giac")
```

output
```
integrate(-x/(x^4 - 4*x^3 + 8*x^2 - a - 8*x), x)
```

Mupad [B] (verification not implemented)

Time = 22.15 (sec) , antiderivative size = 275, normalized size of antiderivative = 2.29

$$\int \frac{x}{a+8x-8x^2+4x^3-x^4}\,dx = \sum_{k=1}^{4} \ln\left(-x-\mathrm{root}\left(2816\,a^2\,z^4+256\,a^3\,z^4+10240\,a\,z^4+12288\,z^4-32\,a^2\,z^2-256\,a\,z^2-512\,z^2+16\,a\,z+64\,z+a,z,k\right)\left(\mathrm{root}\left(2816\,a^2\,z^4+256\,a^3\,z^4+10240\,a\,z^4+12288\,z^4-32\,a^2\,z^2-256\,a\,z^2-512\,z^2+16\,a\,z+64\,z+ -8\right)\right)\mathrm{root}\left(2816\,a^2\,z^4+256\,a^3\,z^4+10240\,a\,z^4+12288\,z^4-32\,a^2\,z^2-256\,a\,z^2-512\,z^2+16\,a\,z+64\,z+a,z,k\right)$$

input
```
int(x/(a + 8*x - 8*x^2 + 4*x^3 - x^4),x)
```

output
```
symsum(log(- x - root(2816*a^2*z^4 + 256*a^3*z^4 + 10240*a*z^4 + 12288*z^4
 - 32*a^2*z^2 - 256*a*z^2 - 512*z^2 + 16*a*z + 64*z + a, z, k)*(root(2816*
a^2*z^4 + 256*a^3*z^4 + 10240*a*z^4 + 12288*z^4 - 32*a^2*z^2 - 256*a*z^2 -
 512*z^2 + 16*a*z + 64*z + a, z, k)*(32*a - root(2816*a^2*z^4 + 256*a^3*z^
4 + 10240*a*z^4 + 12288*z^4 - 32*a^2*z^2 - 256*a*z^2 - 512*z^2 + 16*a*z +
64*z + a, z, k)*(64*a - x*(64*a + 256) + 256) - x*(16*a + 64) + 128) - 8))
*root(2816*a^2*z^4 + 256*a^3*z^4 + 10240*a*z^4 + 12288*z^4 - 32*a^2*z^2 -
256*a*z^2 - 512*z^2 + 16*a*z + 64*z + a, z, k), k, 1, 4)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 334, normalized size of antiderivative = 2.78

$$\int \frac{x}{a+8x-8x^2+4x^3-x^4}\,dx = \frac{-2\sqrt{a+4}\,\sqrt{\sqrt{a+4}+1}\,atan\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)+2\sqrt{\sqrt{a+4}+1}\,atan\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)a+8\sqrt{\sqrt{a+4}+1}\,atan\left(\frac{}{\sqrt{}}\right.}{}$$

input
```
int(x/(-x^4+4*x^3-8*x^2+a+8*x),x)
```

output
```
( - 2*sqrt(a + 4)*sqrt(sqrt(a + 4) + 1)*atan((x - 1)/sqrt(sqrt(a + 4) + 1)
) + 2*sqrt(sqrt(a + 4) + 1)*atan((x - 1)/sqrt(sqrt(a + 4) + 1))*a + 8*sqrt
(sqrt(a + 4) + 1)*atan((x - 1)/sqrt(sqrt(a + 4) + 1)) - sqrt(a + 4)*sqrt(s
qrt(a + 4) - 1)*log(sqrt(sqrt(a + 4) - 1) - x + 1) + sqrt(a + 4)*sqrt(sqrt
(a + 4) - 1)*log(sqrt(sqrt(a + 4) - 1) + x - 1) - sqrt(sqrt(a + 4) - 1)*lo
g(sqrt(sqrt(a + 4) - 1) - x + 1)*a - 4*sqrt(sqrt(a + 4) - 1)*log(sqrt(sqrt
(a + 4) - 1) - x + 1) + sqrt(sqrt(a + 4) - 1)*log(sqrt(sqrt(a + 4) - 1) +
x - 1)*a + 4*sqrt(sqrt(a + 4) - 1)*log(sqrt(sqrt(a + 4) - 1) + x - 1) - sq
rt(a + 4)*log(sqrt(sqrt(a + 4) - 1) - x + 1)*a - 3*sqrt(a + 4)*log(sqrt(sq
rt(a + 4) - 1) - x + 1) - sqrt(a + 4)*log(sqrt(sqrt(a + 4) - 1) + x - 1)*a
 - 3*sqrt(a + 4)*log(sqrt(sqrt(a + 4) - 1) + x - 1) + sqrt(a + 4)*log(sqrt
(a + 4) + x**2 - 2*x + 2)*a + 3*sqrt(a + 4)*log(sqrt(a + 4) + x**2 - 2*x +
 2))/(4*(a**2 + 7*a + 12))
```

3.25 $\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 245

$$\begin{aligned}\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^2}\,dx &= \frac{1+(-1+x)^2}{4(4+a)\left(3+a-2(1-x)^2-(1-x)^4\right)} \\ &\quad - \frac{(5+a+(-1+x)^2)(1-x)}{4(12+7a+a^2)\left(3+a-2(1-x)^2-(1-x)^4\right)} \\ &\quad + \frac{\left(10+3a+\sqrt{4+a}\right)\arctan\left(\frac{1-x}{\sqrt{1-\sqrt{4+a}}}\right)}{8(3+a)(4+a)^{3/2}\sqrt{1-\sqrt{4+a}}} \\ &\quad - \frac{\left(10+3a-\sqrt{4+a}\right)\arctan\left(\frac{1-x}{\sqrt{1+\sqrt{4+a}}}\right)}{8(3+a)(4+a)^{3/2}\sqrt{1+\sqrt{4+a}}} \\ &\quad + \frac{\operatorname{arctanh}\left(\frac{1+(-1+x)^2}{\sqrt{4+a}}\right)}{4(4+a)^{3/2}}\end{aligned}$$

output
```
1/4*(1+(-1+x)^2)/(4+a)/(3+a-2*(1-x)^2-(1-x)^4)-1/4*(5+a+(-1+x)^2)*(1-x)/(a
^2+7*a+12)/(3+a-2*(1-x)^2-(1-x)^4)+1/8*(10+3*a+(4+a)^(1/2))*arctan((1-x)/(
1-(4+a)^(1/2))^(1/2))/(3+a)/(4+a)^(3/2)/(1-(4+a)^(1/2))^(1/2)-1/8*(10+3*a-
(4+a)^(1/2))*arctan((1-x)/(1+(4+a)^(1/2))^(1/2))/(3+a)/(4+a)^(3/2)/(1+(4+a
)^(1/2))^(1/2)+1/4*arctanh((1+(-1+x)^2)/(4+a)^(1/2))/(4+a)^(3/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.28 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.68

$$\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^2}\,dx = \frac{a+2x-ax+ax^2+x^3}{4(3+a)(4+a)\left(a-x\left(-8+8x-4x^2+x^3\right)\right)} - \frac{\text{RootSum}\left[a+8\#1-8\#1^2+4\#1^3-\#1^4\&,\frac{6\log(x-\#1)+a\log(x-\#1)+4\log(x-\#1)\#1+2a\log(x-\#1)\#1+\log}{-2+4\#1-3\#1^2+\#1^3}\right.}{16\left(12+7a+a^2\right)}$$

input
```
Integrate[x/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x]
```

output
```
(a + 2*x - a*x + a*x^2 + x^3)/(4*(3 + a)*(4 + a)*(a - x*(-8 + 8*x - 4*x^2
+ x^3))) - RootSum[a + 8*#1 - 8*#1^2 + 4*#1^3 - #1^4 & , (6*Log[x - #1] +
a*Log[x - #1] + 4*Log[x - #1]*#1 + 2*a*Log[x - #1]*#1 + Log[x - #1]*#1^2)/
(-2 + 4*#1 - 3*#1^2 + #1^3) & ]/(16*(12 + 7*a + a^2))
```

Rubi [A] (warning: unable to verify)

Time = 0.78 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.94, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {2459, 2202, 1405, 27, 1432, 1086, 1083, 219, 1480, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a-x^4+4x^3-8x^2+8x)^2}\,dx$$

$$\Big\downarrow \; 2459$$

$$\int \frac{x}{(a-(x-1)^4-2(x-1)^2+3)^2}d(x-1)$$

$$\Big\downarrow \; 2202$$

$$\int \frac{1}{(-(x-1)^4-2(x-1)^2+a+3)^2}d(x-1)+\int \frac{x-1}{(-(x-1)^4-2(x-1)^2+a+3)^2}d(x-1)$$

$$\Big\downarrow \text{1405}$$

$$-\frac{\int -\frac{2\left((x-1)^2+3a+11\right)}{-(x-1)^4-2(x-1)^2+a+3}d(x-1)}{8\left(a^2+7a+12\right)}+\int \frac{x-1}{\left(-(x-1)^4-2(x-1)^2+a+3\right)^2}d(x-1)+\frac{(x-1)\left(a+(x-1)^2+5\right)}{4\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(x-1)^2+3a+11}{-(x-1)^4-2(x-1)^2+a+3}d(x-1)}{4\left(a^2+7a+12\right)}+\int \frac{x-1}{\left(-(x-1)^4-2(x-1)^2+a+3\right)^2}d(x-1)+\frac{(x-1)\left(a+(x-1)^2+5\right)}{4\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)}$$

$$\Big\downarrow \text{1432}$$

$$\frac{\int \frac{(x-1)^2+3a+11}{-(x-1)^4-2(x-1)^2+a+3}d(x-1)}{4\left(a^2+7a+12\right)}+\frac{1}{2}\int \frac{1}{\left(-(x-1)^4-2(x-1)^2+a+3\right)^2}d(x-1)^2+\frac{(x-1)\left(a+(x-1)^2+5\right)}{4\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)}$$

$$\Big\downarrow \text{1086}$$

$$\frac{\int \frac{(x-1)^2+3a+11}{-(x-1)^4-2(x-1)^2+a+3}d(x-1)}{4\left(a^2+7a+12\right)}+\frac{1}{2}\left(\frac{\int \frac{1}{-(x-1)^4-2(x-1)^2+a+3}d(x-1)^2}{2(a+4)}+\frac{x}{2(a+4)\left(a-(x-1)^4-2(x-1)^2+3\right)}\right)+\frac{(x-1)\left(a+(x-1)^2+5\right)}{4\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)}$$

$$\Big\downarrow \text{1083}$$

$$\frac{\int \frac{(x-1)^2+3a+11}{-(x-1)^4-2(x-1)^2+a+3}d(x-1)}{4\left(a^2+7a+12\right)}+\frac{1}{2}\left(\frac{x}{2(a+4)\left(a-(x-1)^4-2(x-1)^2+3\right)}-\frac{\int \frac{1}{4(a+4)-(x-1)^4}d\left(-2(x-1)^2-2\right)}{a+4}\right)+\frac{(x-1)\left(a+(x-1)^2+5\right)}{4\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)}$$

$$\Big\downarrow \text{219}$$

$$\frac{\int \frac{(x-1)^2+3a+11}{-(x-1)^4-2(x-1)^2+a+3}d(x-1)}{4\left(a^2+7a+12\right)}+\frac{(x-1)\left(a+(x-1)^2+5\right)}{4\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)}+\frac{1}{2}\left(\frac{x}{2(a+4)\left(a-(x-1)^4-2(x-1)^2+3\right)}-\frac{\text{arctanh}\left(\frac{-2(x-1)^2-2}{2\sqrt{a+4}}\right)}{2(a+4)^{3/2}}\right)$$

$$\Big\downarrow \text{1480}$$

$$\frac{\frac{1}{2}\left(1-\frac{3a+10}{\sqrt{a+4}}\right)\int \frac{1}{-(x-1)^2-\sqrt{a+4}-1}d(x-1)+\frac{1}{2}\left(\frac{3a+10}{\sqrt{a+4}}+1\right)\int \frac{1}{-(x-1)^2+\sqrt{a+4}-1}d(x-1)}{4\left(a^2+7a+12\right)}+\frac{(x-1)\left(a+(x-1)^2+5\right)}{4\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)}+\frac{1}{2}\left(\frac{x}{2(a+4)\left(a-(x-1)^4-2(x-1)^2+3\right)}-\frac{\text{arctanh}\left(\frac{-2(x-1)^2-2}{2\sqrt{a+4}}\right)}{2(a+4)^{3/2}}\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{-\frac{\left(\frac{3a+10}{\sqrt{a+4}}+1\right)\arctan\left(\frac{x-1}{\sqrt{1-\sqrt{a+4}}}\right)}{2\sqrt{1-\sqrt{a+4}}}-\frac{\left(1-\frac{3a+10}{\sqrt{a+4}}\right)\arctan\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)}{2\sqrt{\sqrt{a+4}+1}}}{4\left(a^2+7a+12\right)}+\frac{(x-1)\left(a+(x-1)^2+5\right)}{4\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)}+\frac{1}{2}\left(\frac{x}{2(a+4)\left(a-(x-1)^4-2(x-1)^2+3\right)}-\frac{\text{arctanh}\left(\frac{-2(x-1)^2-2}{2\sqrt{a+4}}\right)}{2(a+4)^{3/2}}\right)$$

input

```
Int[x/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x]
```

output

```
((5 + a + (-1 + x)^2)*(-1 + x))/(4*(12 + 7*a + a^2)*(3 + a - 2*(-1 + x)^2
- (-1 + x)^4)) + (-1/2*((1 + (10 + 3*a)/Sqrt[4 + a])*ArcTan[(-1 + x)/Sqrt[
1 - Sqrt[4 + a]]])/Sqrt[1 - Sqrt[4 + a]] - ((1 - (10 + 3*a)/Sqrt[4 + a])*A
rcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]])/(2*Sqrt[1 + Sqrt[4 + a]]))/(4*(12 +
 7*a + a^2)) + (x/(2*(4 + a)*(3 + a - 2*(-1 + x)^2 - (-1 + x)^4)) - ArcTan
h[(-2 - 2*(-1 + x)^2)/(2*Sqrt[4 + a])]/(2*(4 + a)^(3/2)))/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1086
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && ILtQ[p, -1]
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1432
```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 2202
```
Int[(Pn_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{n
 = Expon[Pn, x], k}, Int[Sum[Coeff[Pn, x, 2*k]*x^(2*k), {k, 0, n/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pn, x, 2*k + 1]*x^(2*k), {k, 0, (n -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pn, x]
 &&  !PolyQ[Pn, x^2]
```

rule 2459
```
Int[(Pn_)^(p_.)*(Qx_), x_Symbol] :> With[{S = Coeff[Pn, x, Expon[Pn, x] - 1
]/(Expon[Pn, x]*Coeff[Pn, x, Expon[Pn, x]])}, Subst[Int[ExpandToSum[Pn /. x
 -> x - S, x]^p*ExpandToSum[Qx /. x -> x - S, x], x], x, x + S] /; Binomial
Q[Pn /. x -> x - S, x] || (IntegerQ[Expon[Pn, x]/2] && TrinomialQ[Pn /. x -
> x - S, x])] /; FreeQ[p, x] && PolyQ[Pn, x] && GtQ[Expon[Pn, x], 2] && NeQ
[Coeff[Pn, x, Expon[Pn, x] - 1], 0] && PolyQ[Qx, x] &&  !(MonomialQ[Qx, x]
&& IGtQ[p, 0])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.09 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.64

method	result
default	$\frac{\frac{x^3}{4a^2+28a+48}+\frac{a\,x^2}{4(a+4)(a+3)}-\frac{(a-2)x}{4\left(a^2+7a+12\right)}+\frac{a}{4a^2+28a+48}}{-x^4+4x^3-8x^2+a+8x}+\frac{\sum_{\_R=\text{RootOf}\left(\_Z^4-4\_Z^3+8\_Z^2-8\_Z-a\right)}\frac{\left(6+\_R^2+2(a+2)\_R+a\right)\ldots}{-\_R^3+3\_R^2-4\ldots}}{16a^2+112a+192}$
risch	$\frac{\frac{x^3}{4a^2+28a+48}+\frac{a\,x^2}{4(a+4)(a+3)}-\frac{(a-2)x}{4\left(a^2+7a+12\right)}+\frac{a}{4a^2+28a+48}}{-x^4+4x^3-8x^2+a+8x}+\frac{\left(\sum_{\_R=\text{RootOf}\left(\_Z^4-4\_Z^3+8\_Z^2-8\_Z-a\right)}\frac{\left(\frac{\_R^2}{a^2+7a+12}+\frac{2(a+2)\_R}{a^2+7a+12}\ldots\right.}{-\_R^3+3\ldots}\right.}{16}$

input
```
int(x/(-x^4+4*x^3-8*x^2+a+8*x)^2,x,method=_RETURNVERBOSE)
```

output
```
(1/4/(a^2+7*a+12)*x^3+1/4*a/(a+4)/(a+3)*x^2-1/4*(a-2)/(a^2+7*a+12)*x+1/4/(
a^2+7*a+12)*a)/(-x^4+4*x^3-8*x^2+a+8*x)+1/16/(a^2+7*a+12)*sum((6+_R^2+2*(a
+2)*_R+a)/(-_R^3+3*_R^2-4*_R+2)*ln(x-_R),_R=RootOf(_Z^4-4*_Z^3+8*_Z^2-8*_Z
-a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^2}\,dx = \text{Timed out}$$

input
```
integrate(x/(-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 539 vs. $2(204) = 408$.

Time = 17.10 (sec) , antiderivative size = 539, normalized size of antiderivative = 2.20

$$\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^2}\,dx$$
$$= \frac{-ax^2-a-x^3+x(a-2)}{-4a^3-28a^2-48a+x^4\cdot(4a^2+28a+48)+x^3(-16a^2-112a-192)+x^2\cdot(32a^2+224a+384)+x}$$
$$+\text{RootSum}\Big(t^4\cdot(65536a^9+2162688a^8+31653888a^7+269680640a^6+1473773568a^5+5357174784a^4$$

input
```
integrate(x/(-x**4+4*x**3-8*x**2+a+8*x)**2,x)
```

output
```
(-a*x**2 - a - x**3 + x*(a - 2))/(-4*a**3 - 28*a**2 - 48*a + x**4*(4*a**2
+ 28*a + 48) + x**3*(-16*a**2 - 112*a - 192) + x**2*(32*a**2 + 224*a + 384
) + x*(-32*a**2 - 224*a - 384)) + RootSum(_t**4*(65536*a**9 + 2162688*a**8
 + 31653888*a**7 + 269680640*a**6 + 1473773568*a**5 + 5357174784*a**4 + 12
952010752*a**3 + 20082327552*a**2 + 18119393280*a + 7247757312) + _t**2*(-
2048*a**6 - 50688*a**5 - 520704*a**4 - 2842624*a**3 - 8699904*a**2 - 14155
776*a - 9568256) + _t*(1152*a**4 + 17792*a**3 + 102912*a**2 + 264192*a + 2
53952) + 16*a**3 - 57*a**2 - 984*a - 2064, Lambda(_t, _t*log(x + (98304*_t
**3*a**12 + 3948544*_t**3*a**11 + 72196096*_t**3*a**10 + 793837568*_t**3*a
**9 + 5839372288*_t**3*a**8 + 30226464768*_t**3*a**7 + 112668450816*_t**3*
a**6 + 303864643584*_t**3*a**5 + 586157391872*_t**3*a**4 + 784017129472*_t
**3*a**3 + 683648483328*_t**3*a**2 + 343136010240*_t**3*a + 72477573120*_t
**3 + 30208*_t**2*a**10 + 986624*_t**2*a**9 + 14420992*_t**2*a**8 + 124156
928*_t**2*a**7 + 696815104*_t**2*a**6 + 2661758464*_t**2*a**5 + 7001485312
*_t**2*a**4 + 12506562560*_t**2*a**3 + 14494924800*_t**2*a**2 + 9820569600
*_t**2*a + 2944401408*_t**2 - 1536*_t*a**9 - 52048*_t*a**8 - 757040*_t*a**
7 - 6200656*_t*a**6 - 31380496*_t*a**5 - 100736416*_t*a**4 - 200813696*_t*
a**3 - 228144640*_t*a**2 - 114632704*_t*a - 2490368*_t + 248*a**7 + 6797*a
**6 + 71132*a**5 + 369745*a**4 + 987758*a**3 + 1128896*a**2 - 129568*a - 9
56416)/(576*a**7 + 10985*a**6 + 88746*a**5 + 396609*a**4 + 1076268*a**3...
```

Maxima [F]

$$\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^2}\,dx = \int \frac{x}{(x^4-4\,x^3+8\,x^2-a-8\,x)^2}\,dx$$

input
```
integrate(x/(-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="maxima")
```

output
```
-1/4*(a*x^2 + x^3 - (a - 2)*x + a)/((a^2 + 7*a + 12)*x^4 - 4*(a^2 + 7*a +
12)*x^3 - a^3 + 8*(a^2 + 7*a + 12)*x^2 - 7*a^2 - 8*(a^2 + 7*a + 12)*x - 12
*a) - 1/4*integrate((2*(a + 2)*x + x^2 + a + 6)/(x^4 - 4*x^3 + 8*x^2 - a -
 8*x), x)/(a^2 + 7*a + 12)
```

Giac [F]

$$\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^2}\,dx=\int \frac{x}{(x^4-4\,x^3+8\,x^2-a-8\,x)^2}\,dx$$

input
```
integrate(x/(-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="giac")
```

output
```
integrate(x/(x^4 - 4*x^3 + 8*x^2 - a - 8*x)^2, x)
```

Mupad [B] (verification not implemented)

Time = 22.39 (sec) , antiderivative size = 1167, normalized size of antiderivative = 4.76

$$\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^2}\,dx=\text{Too large to display}$$

input
```
int(x/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x)
```

output
```
symsum(log((35*a + 4*a^2 + 68)/(64*(816*a + 460*a^2 + 129*a^3 + 18*a^4 + a
^5 + 576)) - root(12952010752*a^3*z^4 + 31653888*a^7*z^4 + 2162688*a^8*z^4
 + 65536*a^9*z^4 + 18119393280*a*z^4 + 20082327552*a^2*z^4 + 1473773568*a^
5*z^4 + 5357174784*a^4*z^4 + 269680640*a^6*z^4 + 7247757312*z^4 - 8699904*
a^2*z^2 - 2842624*a^3*z^2 - 520704*a^4*z^2 - 50688*a^5*z^2 - 2048*a^6*z^2
- 14155776*a*z^2 - 9568256*z^2 + 102912*a^2*z + 17792*a^3*z + 1152*a^4*z +
 264192*a*z + 253952*z - 984*a - 57*a^2 + 16*a^3 - 2064, z, k)*((12800*a +
 3600*a^2 + 336*a^3 + 15104)/(64*(816*a + 460*a^2 + 129*a^3 + 18*a^4 + a^5
 + 576)) + root(12952010752*a^3*z^4 + 31653888*a^7*z^4 + 2162688*a^8*z^4 +
 65536*a^9*z^4 + 18119393280*a*z^4 + 20082327552*a^2*z^4 + 1473773568*a^5*
z^4 + 5357174784*a^4*z^4 + 269680640*a^6*z^4 + 7247757312*z^4 - 8699904*a^
2*z^2 - 2842624*a^3*z^2 - 520704*a^4*z^2 - 50688*a^5*z^2 - 2048*a^6*z^2 -
14155776*a*z^2 - 9568256*z^2 + 102912*a^2*z + 17792*a^3*z + 1152*a^4*z + 2
64192*a*z + 253952*z - 984*a - 57*a^2 + 16*a^3 - 2064, z, k)*(root(1295201
0752*a^3*z^4 + 31653888*a^7*z^4 + 2162688*a^8*z^4 + 65536*a^9*z^4 + 181193
93280*a*z^4 + 20082327552*a^2*z^4 + 1473773568*a^5*z^4 + 5357174784*a^4*z^
4 + 269680640*a^6*z^4 + 7247757312*z^4 - 8699904*a^2*z^2 - 2842624*a^3*z^2
 - 520704*a^4*z^2 - 50688*a^5*z^2 - 2048*a^6*z^2 - 14155776*a*z^2 - 956825
6*z^2 + 102912*a^2*z + 17792*a^3*z + 1152*a^4*z + 264192*a*z + 253952*z -
984*a - 57*a^2 + 16*a^3 - 2064, z, k)*((15728640*a + 10878976*a^2 + 399...
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 3204, normalized size of antiderivative = 13.08

$$\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^2}\,dx = \text{Too large to display}$$

input
```
int(x/(-x^4+4*x^3-8*x^2+a+8*x)^2,x)
```

output

```
( - 8*sqrt(a + 4)*sqrt(sqrt(a + 4) + 1)*atan((x - 1)/sqrt(sqrt(a + 4) + 1)
)*a**2 + 8*sqrt(a + 4)*sqrt(sqrt(a + 4) + 1)*atan((x - 1)/sqrt(sqrt(a + 4)
 + 1))*a*x**4 - 32*sqrt(a + 4)*sqrt(sqrt(a + 4) + 1)*atan((x - 1)/sqrt(sqr
t(a + 4) + 1))*a*x**3 + 64*sqrt(a + 4)*sqrt(sqrt(a + 4) + 1)*atan((x - 1)/
sqrt(sqrt(a + 4) + 1))*a*x**2 - 64*sqrt(a + 4)*sqrt(sqrt(a + 4) + 1)*atan(
(x - 1)/sqrt(sqrt(a + 4) + 1))*a*x - 28*sqrt(a + 4)*sqrt(sqrt(a + 4) + 1)*
atan((x - 1)/sqrt(sqrt(a + 4) + 1))*a + 28*sqrt(a + 4)*sqrt(sqrt(a + 4) +
1)*atan((x - 1)/sqrt(sqrt(a + 4) + 1))*x**4 - 112*sqrt(a + 4)*sqrt(sqrt(a
+ 4) + 1)*atan((x - 1)/sqrt(sqrt(a + 4) + 1))*x**3 + 224*sqrt(a + 4)*sqrt(
sqrt(a + 4) + 1)*atan((x - 1)/sqrt(sqrt(a + 4) + 1))*x**2 - 224*sqrt(a + 4
)*sqrt(sqrt(a + 4) + 1)*atan((x - 1)/sqrt(sqrt(a + 4) + 1))*x + 6*sqrt(sqr
t(a + 4) + 1)*atan((x - 1)/sqrt(sqrt(a + 4) + 1))*a**3 - 6*sqrt(sqrt(a + 4
) + 1)*atan((x - 1)/sqrt(sqrt(a + 4) + 1))*a**2*x**4 + 24*sqrt(sqrt(a + 4)
 + 1)*atan((x - 1)/sqrt(sqrt(a + 4) + 1))*a**2*x**3 - 48*sqrt(sqrt(a + 4)
+ 1)*atan((x - 1)/sqrt(sqrt(a + 4) + 1))*a**2*x**2 + 48*sqrt(sqrt(a + 4) +
 1)*atan((x - 1)/sqrt(sqrt(a + 4) + 1))*a**2*x + 46*sqrt(sqrt(a + 4) + 1)*
atan((x - 1)/sqrt(sqrt(a + 4) + 1))*a**2 - 46*sqrt(sqrt(a + 4) + 1)*atan((
x - 1)/sqrt(sqrt(a + 4) + 1))*a*x**4 + 184*sqrt(sqrt(a + 4) + 1)*atan((x -
 1)/sqrt(sqrt(a + 4) + 1))*a*x**3 - 368*sqrt(sqrt(a + 4) + 1)*atan((x - 1)
/sqrt(sqrt(a + 4) + 1))*a*x**2 + 368*sqrt(sqrt(a + 4) + 1)*atan((x - 1)...
```

3.26 $\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 375

$$\begin{aligned}
&\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^3}\,dx \\
&= \frac{1+(-1+x)^2}{8(4+a)\left(3+a-2(1-x)^2-(1-x)^4\right)^2} \\
&\quad + \frac{3(1+(-1+x)^2)}{16(4+a)^2\left(3+a-2(1-x)^2-(1-x)^4\right)} \\
&\quad - \frac{\left((6+a)(25+7a)+6(7+2a)(1-x)^2\right)(1-x)}{32(3+a)^2(4+a)^2\left(3+a-2(1-x)^2-(1-x)^4\right)} \\
&\quad - \frac{\left(5+a+(-1+x)^2\right)(1-x)}{8\left(12+7a+a^2\right)\left(3+a-2(1-x)^2-(1-x)^4\right)^2} \\
&\quad + \frac{3\left(80+7a^2+14\sqrt{4+a}+a\left(47+4\sqrt{4+a}\right)\right)\arctan\left(\frac{1-x}{\sqrt{1-\sqrt{4+a}}}\right)}{64(3+a)^2(4+a)^{5/2}\sqrt{1-\sqrt{4+a}}} \\
&\quad + \frac{3\left(14+4a-\frac{80+47a+7a^2}{\sqrt{4+a}}\right)\arctan\left(\frac{1-x}{\sqrt{1+\sqrt{4+a}}}\right)}{64(3+a)^2(4+a)^2\sqrt{1+\sqrt{4+a}}} + \frac{3\operatorname{arctanh}\left(\frac{1+(-1+x)^2}{\sqrt{4+a}}\right)}{16(4+a)^{5/2}}
\end{aligned}$$

output
```
1/8*(1+(-1+x)^2)/(4+a)/(3+a-2*(1-x)^2-(1-x)^4)^2+3/16*(1+(-1+x)^2)/(4+a)^2
/(3+a-2*(1-x)^2-(1-x)^4)-1/32*((6+a)*(25+7*a)+6*(7+2*a)*(1-x)^2)*(1-x)/(3+
a)^2/(4+a)^2/(3+a-2*(1-x)^2-(1-x)^4)-1/8*(5+a+(-1+x)^2)*(1-x)/(a^2+7*a+12)
/(3+a-2*(1-x)^2-(1-x)^4)^2+3/64*(80+7*a^2+14*(4+a)^(1/2)+a*(47+4*(4+a)^(1/
2)))*arctan((1-x)/(1-(4+a)^(1/2))^(1/2))/(3+a)^2/(4+a)^(5/2)/(1-(4+a)^(1/2
))^(1/2)+3/64*(14+4*a-(7*a^2+47*a+80)/(4+a)^(1/2))*arctan((1-x)/(1+(4+a)^(
1/2))^(1/2))/(3+a)^2/(4+a)^2/(1+(4+a)^(1/2))^(1/2)+3/16*arctanh((1+(-1+x)^
2)/(4+a)^(1/2))/(4+a)^(5/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.44 (sec) , antiderivative size = 284, normalized size of antiderivative = 0.76

$$\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^3}\,dx = \frac{1}{128}\left(\frac{16(a+2x-ax+ax^2+x^3)}{(3+a)(4+a)\left(a-x\left(-8+8x-4x^2+x^3\right)\right)^2} + \frac{4(a^2(5-5x+6x^2)+6(-14+28x-12x^2+7x^3)+a(-7+31x+12x^3))}{(3+a)^2(4+a)^2\left(a-x\left(-8+8x-4x^2+x^3\right)\right)} - \frac{3\text{RootSum}\left[a+8\#1-8\#1^2+4\#1^3-\#1^4\&, \frac{72\log(x-\#1)+31a\log(x-\#1)+3a^2\log(x-\#1)+8\log(x-\#1)\#1+1}{-2+4}\right.}{(12+7a+a^2)^2}\right.$$

input
```
Integrate[x/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^3,x]
```

output
```
((16*(a + 2*x - a*x + a*x^2 + x^3))/((3 + a)*(4 + a)*(a - x*(-8 + 8*x - 4*
x^2 + x^3))^2) + (4*(a^2*(5 - 5*x + 6*x^2) + 6*(-14 + 28*x - 12*x^2 + 7*x^
3) + a*(-7 + 31*x + 12*x^3)))/((3 + a)^2*(4 + a)^2*(a - x*(-8 + 8*x - 4*x^
2 + x^3))) - (3*RootSum[a + 8*#1 - 8*#1^2 + 4*#1^3 - #1^4 & , (72*Log[x -
#1] + 31*a*Log[x - #1] + 3*a^2*Log[x - #1] + 8*Log[x - #1]*#1 + 16*a*Log[x
 - #1]*#1 + 4*a^2*Log[x - #1]*#1 + 14*Log[x - #1]*#1^2 + 4*a*Log[x - #1]*#
1^2)/(-2 + 4*#1 - 3*#1^2 + #1^3) & ])/(12 + 7*a + a^2)^2)/128
```

Rubi [A] (warning: unable to verify)

Time = 1.13 (sec) , antiderivative size = 359, normalized size of antiderivative = 0.96, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.542, Rules used = {2459, 2202, 1405, 27, 1432, 1086, 1086, 1083, 219, 1492, 27, 1480, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a-x^4+4x^3-8x^2+8x)^3}dx$$

$$\Big\downarrow \text{2459}$$

$$\int \frac{x}{(a-(x-1)^4-2(x-1)^2+3)^3}d(x-1)$$

$$\Big\downarrow \text{2202}$$

$$\int \frac{1}{(-(x-1)^4-2(x-1)^2+a+3)^3}d(x-1)+\int \frac{x-1}{(-(x-1)^4-2(x-1)^2+a+3)^3}d(x-1)$$

$$\Big\downarrow \text{1405}$$

$$-\frac{\int -\frac{2(5(x-1)^2+7a+27)}{(-(x-1)^4-2(x-1)^2+a+3)^2}d(x-1)}{16(a^2+7a+12)}+\int \frac{x-1}{(-(x-1)^4-2(x-1)^2+a+3)^3}d(x-1)+\frac{(x-1)(a+(x-1)^2+5)}{8(a^2+7a+12)(a-(x-1)^4-2(x-1)^2+3)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{5(x-1)^2+7a+27}{(-(x-1)^4-2(x-1)^2+a+3)^2}d(x-1)}{8(a^2+7a+12)}+\int \frac{x-1}{(-(x-1)^4-2(x-1)^2+a+3)^3}d(x-1)+\frac{(x-1)(a+(x-1)^2+5)}{8(a^2+7a+12)(a-(x-1)^4-2(x-1)^2+3)^2}$$

$$\Big\downarrow \text{1432}$$

$$\frac{\int \frac{5(x-1)^2+7a+27}{(-(x-1)^4-2(x-1)^2+a+3)^2}d(x-1)}{8(a^2+7a+12)}+\frac{1}{2}\int \frac{1}{(-(x-1)^4-2(x-1)^2+a+3)^3}d(x-1)^2+\frac{(x-1)(a+(x-1)^2+5)}{8(a^2+7a+12)(a-(x-1)^4-2(x-1)^2+3)^2}$$

$$\Big\downarrow \text{1086}$$

$$\frac{\int \frac{5(x-1)^2+7a+27}{\left(-(x-1)^4-2(x-1)^2+a+3\right)^2}d(x-1)}{8\left(a^2+7a+12\right)}+\frac{1}{2}\left(\frac{3\int \frac{1}{\left(-(x-1)^4-2(x-1)^2+a+3\right)^2}d(x-1)^2}{4(a+4)}+\frac{x}{4(a+4)\left(a-(x-1)^4-2(x-1)^2+3\right)^2}\right)+\frac{(x-1)\left(a+(x-1)^2+5\right)}{8\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)^2}$$

$$\Big\downarrow \ 1086$$

$$\frac{\int \frac{5(x-1)^2+7a+27}{\left(-(x-1)^4-2(x-1)^2+a+3\right)^2}d(x-1)}{8\left(a^2+7a+12\right)}+\frac{1}{2}\left(\frac{3\left(\frac{\int \frac{1}{-(x-1)^4-2(x-1)^2+a+3}d(x-1)^2}{2(a+4)}+\frac{x}{2(a+4)\left(a-(x-1)^4-2(x-1)^2+3\right)}\right)}{4(a+4)}+\frac{x}{4(a+4)\left(a-(x-1)^4-2(x-1)^2+3\right)^2}\right)+\frac{(x-1)\left(a+(x-1)^2+5\right)}{8\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)^2}$$

$$\Big\downarrow \ 1083$$

$$\frac{\int \frac{5(x-1)^2+7a+27}{\left(-(x-1)^4-2(x-1)^2+a+3\right)^2}d(x-1)}{8\left(a^2+7a+12\right)}+\frac{1}{2}\left(\frac{3\left(\frac{x}{2(a+4)\left(a-(x-1)^4-2(x-1)^2+3\right)}-\frac{\int \frac{1}{4(a+4)-(x-1)^4}d\left(-2(x-1)^2-2\right)}{a+4}\right)}{4(a+4)}+\frac{x}{4(a+4)\left(a-(x-1)^4-2(x-1)^2+3\right)^2}\right)+\frac{(x-1)\left(a+(x-1)^2+5\right)}{8\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)^2}$$

$$\Big\downarrow \ 219$$

$$\frac{\int \frac{5(x-1)^2+7a+27}{\left(-(x-1)^4-2(x-1)^2+a+3\right)^2}d(x-1)}{8\left(a^2+7a+12\right)}+\frac{(x-1)\left(a+(x-1)^2+5\right)}{8\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)^2}+\frac{1}{2}\left(\frac{3\left(\frac{x}{2(a+4)\left(a-(x-1)^4-2(x-1)^2+3\right)}-\frac{\text{arctanh}\left(\frac{-2(x-1)^2-2}{2\sqrt{a+4}}\right)}{2(a+4)^{3/2}}\right)}{4(a+4)}+\frac{x}{4(a+4)\left(a-(x-1)^4-2(x-1)^2+3\right)^2}\right)$$

$$\Big\downarrow \ 1492$$

$$\frac{\frac{(x-1)(6(2a+7)(x-1)^2+(a+6)(7a+25))}{4(a^2+7a+12)(a-(x-1)^4-2(x-1)^2+3)}-\frac{\int -\frac{6\left(7a^2+51a+2(2a+7)(x-1)^2+94\right)}{-(x-1)^4-2(x-1)^2+a+3}d(x-1)}{8(a^2+7a+12)}}{8\left(a^2+7a+12\right)}+\frac{(x-1)\left(a+(x-1)^2+5\right)}{8\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)^2}+\frac{1}{2}\left(\frac{3\left(\frac{x}{2(a+4)(a-(x-1)^4-2(x-1)^2+3)}-\frac{\text{arctanh}\left(\frac{-2(x-1)^2-2}{2\sqrt{a+4}}\right)}{2(a+4)^{3/2}}\right)}{4(a+4)}+\frac{x}{4(a+4)\left(a-(x-1)^4-2(x-1)^2+3\right)^2}\right)$$

$\downarrow$ 27

$$\frac{\frac{3\int \frac{7a^2+51a+2(2a+7)(x-1)^2+94}{-(x-1)^4-2(x-1)^2+a+3}d(x-1)}{4(a^2+7a+12)}+\frac{(x-1)(6(2a+7)(x-1)^2+(a+6)(7a+25))}{4(a^2+7a+12)(a-(x-1)^4-2(x-1)^2+3)}}{8\left(a^2+7a+12\right)}+\frac{(x-1)\left(a+(x-1)^2+5\right)}{8\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)^2}+\frac{1}{2}\left(\frac{3\left(\frac{x}{2(a+4)(a-(x-1)^4-2(x-1)^2+3)}-\frac{\text{arctanh}\left(\frac{-2(x-1)^2-2}{2\sqrt{a+4}}\right)}{2(a+4)^{3/2}}\right)}{4(a+4)}+\frac{x}{4(a+4)\left(a-(x-1)^4-2(x-1)^2+3\right)^2}\right)$$

$\downarrow$ 1480

$$\frac{\frac{3\left(\frac{1}{2}\left(-\frac{7a^2+47a+80}{\sqrt{a+4}}+4a+14\right)\int \frac{1}{-(x-1)^2-\sqrt{a+4}-1}d(x-1)+\frac{1}{2}\left(\frac{7a^2+47a+80}{\sqrt{a+4}}+4a+14\right)\int \frac{1}{-(x-1)^2+\sqrt{a+4}-1}d(x-1)\right)}{4(a^2+7a+12)}+\frac{(x-1)(6(2a+7)(x-1)^2+(}{4(a^2+7a+12)(a-(x-1)^4}}{8\left(a^2+7a+12\right)}+\frac{(x-1)\left(a+(x-1)^2+5\right)}{8\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)^2}+\frac{1}{2}\left(\frac{3\left(\frac{x}{2(a+4)(a-(x-1)^4-2(x-1)^2+3)}-\frac{\text{arctanh}\left(\frac{-2(x-1)^2-2}{2\sqrt{a+4}}\right)}{2(a+4)^{3/2}}\right)}{4(a+4)}+\frac{x}{4(a+4)\left(a-(x-1)^4-2(x-1)^2+3\right)^2}\right)$$

$\downarrow$ 217

$$\frac{\frac{3\left(-\frac{\left(\frac{7a^2+47a+80}{\sqrt{a+4}}+4a+14\right)\arctan\left(\frac{x-1}{\sqrt{1-\sqrt{a+4}}}\right)}{2\sqrt{1-\sqrt{a+4}}}-\frac{\left(-\frac{7a^2+47a+80}{\sqrt{a+4}}+4a+14\right)\arctan\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)}{2\sqrt{\sqrt{a+4}+1}}\right)}{4(a^2+7a+12)}+\frac{(x-1)(6(2a+7)(x-1)^2+(a+6)(7a+25))}{4(a^2+7a+12)(a-(x-1)^4-2(x-1)^2+3)}}{8\left(a^2+7a+12\right)}+\frac{(x-1)\left(a+(x-1)^2+5\right)}{8\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)^2}+\frac{1}{2}\left(\frac{3\left(\frac{x}{2(a+4)(a-(x-1)^4-2(x-1)^2+3)}-\frac{\text{arctanh}\left(\frac{-2(x-1)^2-2}{2\sqrt{a+4}}\right)}{2(a+4)^{3/2}}\right)}{4(a+4)}+\frac{x}{4(a+4)\left(a-(x-1)^4-2(x-1)^2+3\right)^2}\right)$$

input
```
Int[x/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^3,x]
```

output
```
((5 + a + (-1 + x)^2)*(-1 + x))/(8*(12 + 7*a + a^2)*(3 + a - 2*(-1 + x)^2
- (-1 + x)^4)^2) + ((((6 + a)*(25 + 7*a) + 6*(7 + 2*a)*(-1 + x)^2)*(-1 + x
))/(4*(12 + 7*a + a^2)*(3 + a - 2*(-1 + x)^2 - (-1 + x)^4)) + (3*(-1/2*((1
4 + 4*a + (80 + 47*a + 7*a^2)/Sqrt[4 + a])*ArcTan[(-1 + x)/Sqrt[1 - Sqrt[4
 + a]]])/Sqrt[1 - Sqrt[4 + a]] - ((14 + 4*a - (80 + 47*a + 7*a^2)/Sqrt[4 +
 a])*ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]])/(2*Sqrt[1 + Sqrt[4 + a]])))/(
4*(12 + 7*a + a^2)))/(8*(12 + 7*a + a^2)) + (x/(4*(4 + a)*(3 + a - 2*(-1 +
 x)^2 - (-1 + x)^4)^2) + (3*(x/(2*(4 + a)*(3 + a - 2*(-1 + x)^2 - (-1 + x)
^4)) - ArcTanh[(-2 - 2*(-1 + x)^2)/(2*Sqrt[4 + a])]/(2*(4 + a)^(3/2))))/(4
*(4 + a)))/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1086
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && ILtQ[p, -1]
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1432
```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1492
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2
 - 4*a*c))   Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p +
7)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 LtQ[p, -1] && IntegerQ[2*p]
```

rule 2202
```
Int[(Pn_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{n
 = Expon[Pn, x], k}, Int[Sum[Coeff[Pn, x, 2*k]*x^(2*k), {k, 0, n/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pn, x, 2*k + 1]*x^(2*k), {k, 0, (n -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pn, x]
 &&  !PolyQ[Pn, x^2]
```

rule 2459
```
Int[(Pn_)^(p_.)*(Qx_), x_Symbol] :> With[{S = Coeff[Pn, x, Expon[Pn, x] - 1
]/(Expon[Pn, x]*Coeff[Pn, x, Expon[Pn, x]])}, Subst[Int[ExpandToSum[Pn /. x
 -> x - S, x]^p*ExpandToSum[Qx /. x -> x - S, x], x], x, x + S] /; Binomial
Q[Pn /. x -> x - S, x] || (IntegerQ[Expon[Pn, x]/2] && TrinomialQ[Pn /. x -
> x - S, x])] /; FreeQ[p, x] && PolyQ[Pn, x] && GtQ[Expon[Pn, x], 2] && NeQ
[Coeff[Pn, x, Expon[Pn, x] - 1], 0] && PolyQ[Qx, x] &&  !(MonomialQ[Qx, x]
&& IGtQ[p, 0])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.14 (sec) , antiderivative size = 409, normalized size of antiderivative = 1.09

method	result
default	$-\frac{\frac{3(7+2a)x^7}{16\left(a^4+14a^3+73a^2+168a+144\right)}+\frac{3\left(a^2-8a-40\right)x^6}{16\left(a^4+14a^3+73a^2+168a+144\right)}-\frac{\left(29a^2-127a-792\right)x^5}{32\left(a^4+14a^3+73a^2+168a+144\right)}+\frac{\left(73a^2-227a-1668\right)x^4}{32a^4+448a^3+2336a^2+5376a+4608}-}{(-x^4+4x^3-8x}$
risch	$\frac{-\frac{3(7+2a)x^7}{16\left(a^4+14a^3+73a^2+168a+144\right)}-\frac{3\left(a^2-8a-40\right)x^6}{16\left(a^4+14a^3+73a^2+168a+144\right)}+\frac{\left(29a^2-127a-792\right)x^5}{32a^4+448a^3+2336a^2+5376a+4608}-\frac{\left(73a^2-227a-1668\right)x^4}{32\left(a^4+14a^3+73a^2+168a+144\right)}+}{(-x^4+4x^3-8}$

input
```
int(x/(-x^4+4*x^3-8*x^2+a+8*x)^3,x,method=_RETURNVERBOSE)
```

output
```
-(3/16*(7+2*a)/(a^4+14*a^3+73*a^2+168*a+144)*x^7+3/16*(a^2-8*a-40)/(a^4+14
*a^3+73*a^2+168*a+144)*x^6-1/32*(29*a^2-127*a-792)/(a^4+14*a^3+73*a^2+168*
a+144)*x^5+1/32*(73*a^2-227*a-1668)/(a^4+14*a^3+73*a^2+168*a+144)*x^4-1/16
*(62*a^2-103*a-1104)/(a^4+14*a^3+73*a^2+168*a+144)*x^3-1/16*(5*a^3-26*a^2+
140*a+1008)/(a^4+14*a^3+73*a^2+168*a+144)*x^2+3/32*(3*a^3-17*a^2-40*a+192)
/(a^4+14*a^3+73*a^2+168*a+144)*x-3/32*a*(3*a^2+7*a-12)/(a^4+14*a^3+73*a^2+
168*a+144))/(-x^4+4*x^3-8*x^2+a+8*x)^2-3/128/(a^4+14*a^3+73*a^2+168*a+144)
*sum((-72+2*(-2*a-7)*_R^2+4*(-a^2-4*a-2)*_R-3*a^2-31*a)/(-_R^3+3*_R^2-4*_R
+2)*ln(x-_R),_R=RootOf(_Z^4-4*_Z^3+8*_Z^2-8*_Z-a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^3}\,dx = \text{Timed out}$$

input
```
integrate(x/(-x^4+4*x^3-8*x^2+a+8*x)^3,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1102 vs. $2(323) = 646$.

Time = 48.01 (sec) , antiderivative size = 1102, normalized size of antiderivative = 2.94

$$\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x/(-x**4+4*x**3-8*x**2+a+8*x)**3,x)
```

output
```
-(-9*a**3 - 21*a**2 + 36*a + x**7*(12*a + 42) + x**6*(6*a**2 - 48*a - 240)
 + x**5*(-29*a**2 + 127*a + 792) + x**4*(73*a**2 - 227*a - 1668) + x**3*(-
124*a**2 + 206*a + 2208) + x**2*(-10*a**3 + 52*a**2 - 280*a - 2016) + x*(9
*a**3 - 51*a**2 - 120*a + 576))/(32*a**6 + 448*a**5 + 2336*a**4 + 5376*a**
3 + 4608*a**2 + x**8*(32*a**4 + 448*a**3 + 2336*a**2 + 5376*a + 4608) + x*
*7*(-256*a**4 - 3584*a**3 - 18688*a**2 - 43008*a - 36864) + x**6*(1024*a**
4 + 14336*a**3 + 74752*a**2 + 172032*a + 147456) + x**5*(-2560*a**4 - 3584
0*a**3 - 186880*a**2 - 430080*a - 368640) + x**4*(-64*a**5 + 3200*a**4 + 5
2672*a**3 + 288256*a**2 + 678912*a + 589824) + x**3*(256*a**5 - 512*a**4 -
 38656*a**3 - 256000*a**2 - 651264*a - 589824) + x**2*(-512*a**5 - 5120*a*
*4 - 8704*a**3 + 63488*a**2 + 270336*a + 294912) + x*(512*a**5 + 7168*a**4
 + 37376*a**3 + 86016*a**2 + 73728*a)) - RootSum(_t**4*(268435456*a**15 +
14763950080*a**14 + 378493992960*a**13 + 5999532441600*a**12 + 65757291479
040*a**11 + 527875908304896*a**10 + 3206246773555200*a**9 + 15003759578972
160*a**8 + 54537151127224320*a**7 + 153980418717122560*a**6 + 334927734494
986240*a**5 + 551152193655275520*a**4 + 664192984106926080*a**3 + 55336221
2027105280*a**2 + 284993413919539200*a + 68398419340689408) + _t**2*(-4718
592*a**10 - 196116480*a**9 - 3648061440*a**8 - 40022212608*a**7 - 28693993
8816*a**6 - 1405437345792*a**5 - 4764645457920*a**4 - 11043392716800*a**3
- 16752587046912*a**2 - 15023392948224*a - 6049461436416) + _t*(-270950...
```

Maxima [F]

$$\int \frac{x}{\left(a+8x-8x^2+4x^3-x^4\right)^3}\,dx = \int -\frac{x}{\left(x^4-4\,x^3+8\,x^2-a-8\,x\right)^3}\,dx$$

input
```
integrate(x/(-x^4+4*x^3-8*x^2+a+8*x)^3,x, algorithm="maxima")
```

output
```
-1/32*(6*(2*a + 7)*x^7 + 6*(a^2 - 8*a - 40)*x^6 - (29*a^2 - 127*a - 792)*x
^5 + (73*a^2 - 227*a - 1668)*x^4 - 2*(62*a^2 - 103*a - 1104)*x^3 - 9*a^3 -
 2*(5*a^3 - 26*a^2 + 140*a + 1008)*x^2 - 21*a^2 + 3*(3*a^3 - 17*a^2 - 40*a
 + 192)*x + 36*a)/((a^4 + 14*a^3 + 73*a^2 + 168*a + 144)*x^8 - 8*(a^4 + 14
*a^3 + 73*a^2 + 168*a + 144)*x^7 + 32*(a^4 + 14*a^3 + 73*a^2 + 168*a + 144
)*x^6 + a^6 - 80*(a^4 + 14*a^3 + 73*a^2 + 168*a + 144)*x^5 + 14*a^5 - 2*(a
^5 - 50*a^4 - 823*a^3 - 4504*a^2 - 10608*a - 9216)*x^4 + 73*a^4 + 8*(a^5 -
 2*a^4 - 151*a^3 - 1000*a^2 - 2544*a - 2304)*x^3 + 168*a^3 - 16*(a^5 + 10*
a^4 + 17*a^3 - 124*a^2 - 528*a - 576)*x^2 + 144*a^2 + 16*(a^5 + 14*a^4 + 7
3*a^3 + 168*a^2 + 144*a)*x) - 3/32*integrate((2*(2*a + 7)*x^2 + 3*a^2 + 4*
(a^2 + 4*a + 2)*x + 31*a + 72)/(x^4 - 4*x^3 + 8*x^2 - a - 8*x), x)/(a^4 +
14*a^3 + 73*a^2 + 168*a + 144)
```

Giac [F]

$$\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^3}\,dx=\int -\frac{x}{(x^4-4\,x^3+8\,x^2-a-8\,x)^3}\,dx$$

input
```
integrate(x/(-x^4+4*x^3-8*x^2+a+8*x)^3,x, algorithm="giac")
```

output
```
integrate(-x/(x^4 - 4*x^3 + 8*x^2 - a - 8*x)^3, x)
```

Mupad [B] (verification not implemented)

Time = 23.00 (sec) , antiderivative size = 2200, normalized size of antiderivative = 5.87

$$\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^3}\,dx=\text{Too large to display}$$

input
```
int(x/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^3,x)
```

output
```
symsum(log(root(15003759578972160*a^8*z^4 + 54537151127224320*a^7*z^4 + 15
3980418717122560*a^6*z^4 + 334927734494986240*a^5*z^4 + 551152193655275520
*a^4*z^4 + 664192984106926080*a^3*z^4 + 553362212027105280*a^2*z^4 + 59995
32441600*a^12*z^4 + 527875908304896*a^10*z^4 + 284993413919539200*a*z^4 +
3206246773555200*a^9*z^4 + 14763950080*a^14*z^4 + 65757291479040*a^11*z^4
+ 378493992960*a^13*z^4 + 268435456*a^15*z^4 + 68398419340689408*z^4 - 471
8592*a^10*z^2 - 3648061440*a^8*z^2 - 286939938816*a^6*z^2 - 15023392948224
*a*z^2 - 16752587046912*a^2*z^2 - 4764645457920*a^4*z^2 - 40022212608*a^7*
z^2 - 11043392716800*a^3*z^2 - 1405437345792*a^5*z^2 - 196116480*a^9*z^2 -
 6049461436416*z^2 + 5375877120*a^4*z + 839890944*a^5*z + 47542173696*a^2*
z + 72880128*a^6*z + 2709504*a^7*z + 20640890880*a^3*z + 60827369472*a*z +
 33351008256*z - 74027520*a - 29249424*a^2 - 4706424*a^3 - 155601*a^4 + 20
736*a^5 - 68345856, z, k)*((242823168*a + 170044416*a^2 + 63509760*a^3 + 1
3340736*a^4 + 1494144*a^5 + 69696*a^6 + 144506880)/(16384*(940032*a + 1195
776*a^2 + 899328*a^3 + 442864*a^4 + 149208*a^5 + 34833*a^6 + 5564*a^7 + 58
2*a^8 + 36*a^9 + a^10 + 331776)) + root(15003759578972160*a^8*z^4 + 545371
51127224320*a^7*z^4 + 153980418717122560*a^6*z^4 + 334927734494986240*a^5*
z^4 + 551152193655275520*a^4*z^4 + 664192984106926080*a^3*z^4 + 5533622120
27105280*a^2*z^4 + 5999532441600*a^12*z^4 + 527875908304896*a^10*z^4 + 284
993413919539200*a*z^4 + 3206246773555200*a^9*z^4 + 14763950080*a^14*z^4...
```

Reduce [B] (verification not implemented)

Time = 1.20 (sec) , antiderivative size = 9147, normalized size of antiderivative = 24.39

$$\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^3}\,dx = \text{Too large to display}$$

input
```
int(x/(-x^4+4*x^3-8*x^2+a+8*x)^3,x)
```

output

```
( - 66*sqrt(a + 4)*sqrt(sqrt(a + 4) + 1)*atan((x - 1)/sqrt(sqrt(a + 4) + 1
))*a**4 + 132*sqrt(a + 4)*sqrt(sqrt(a + 4) + 1)*atan((x - 1)/sqrt(sqrt(a +
 4) + 1))*a**3*x**4 - 528*sqrt(a + 4)*sqrt(sqrt(a + 4) + 1)*atan((x - 1)/s
qrt(sqrt(a + 4) + 1))*a**3*x**3 + 1056*sqrt(a + 4)*sqrt(sqrt(a + 4) + 1)*a
tan((x - 1)/sqrt(sqrt(a + 4) + 1))*a**3*x**2 - 1056*sqrt(a + 4)*sqrt(sqrt(
a + 4) + 1)*atan((x - 1)/sqrt(sqrt(a + 4) + 1))*a**3*x - 462*sqrt(a + 4)*s
qrt(sqrt(a + 4) + 1)*atan((x - 1)/sqrt(sqrt(a + 4) + 1))*a**3 - 66*sqrt(a
+ 4)*sqrt(sqrt(a + 4) + 1)*atan((x - 1)/sqrt(sqrt(a + 4) + 1))*a**2*x**8 +
 528*sqrt(a + 4)*sqrt(sqrt(a + 4) + 1)*atan((x - 1)/sqrt(sqrt(a + 4) + 1))
*a**2*x**7 - 2112*sqrt(a + 4)*sqrt(sqrt(a + 4) + 1)*atan((x - 1)/sqrt(sqrt
(a + 4) + 1))*a**2*x**6 + 5280*sqrt(a + 4)*sqrt(sqrt(a + 4) + 1)*atan((x -
 1)/sqrt(sqrt(a + 4) + 1))*a**2*x**5 - 7524*sqrt(a + 4)*sqrt(sqrt(a + 4) +
 1)*atan((x - 1)/sqrt(sqrt(a + 4) + 1))*a**2*x**4 + 4752*sqrt(a + 4)*sqrt(
sqrt(a + 4) + 1)*atan((x - 1)/sqrt(sqrt(a + 4) + 1))*a**2*x**3 + 3168*sqrt
(a + 4)*sqrt(sqrt(a + 4) + 1)*atan((x - 1)/sqrt(sqrt(a + 4) + 1))*a**2*x**
2 - 7392*sqrt(a + 4)*sqrt(sqrt(a + 4) + 1)*atan((x - 1)/sqrt(sqrt(a + 4) +
 1))*a**2*x - 816*sqrt(a + 4)*sqrt(sqrt(a + 4) + 1)*atan((x - 1)/sqrt(sqrt
(a + 4) + 1))*a**2 - 462*sqrt(a + 4)*sqrt(sqrt(a + 4) + 1)*atan((x - 1)/sq
rt(sqrt(a + 4) + 1))*a*x**8 + 3696*sqrt(a + 4)*sqrt(sqrt(a + 4) + 1)*atan(
(x - 1)/sqrt(sqrt(a + 4) + 1))*a*x**7 - 14784*sqrt(a + 4)*sqrt(sqrt(a +...
```

3.27 $\int x^2(a+8x-8x^2+4x^3-x^4)^4\,dx$

Optimal result

Integrand size = 26, antiderivative size = 210

$$\begin{aligned}\int x^2\left(a+8x-8x^2+4x^3-x^4\right)^4\,dx &= \frac{a^4x^3}{3}+8a^3x^4+\frac{32}{5}(12-a)a^2x^5 \\ &+\frac{8}{3}a\left(128-48a+a^2\right)x^6 \\ &+\frac{4}{7}\left(1024-1536a+192a^2-a^3\right)x^7 \\ &-4\left(512-288a+15a^2\right)x^8 \\ &+\frac{64}{9}(128-3a)(4-a)x^9-\frac{24}{5}\left(896-128a+a^2\right)x^{10} \\ &+\frac{2}{11}\left(20480-1536a+3a^2\right)x^{11}-\frac{8}{3}(928-35a)x^{12} \\ &+\frac{32}{13}(524-9a)x^{13}-\frac{8}{7}(464-3a)x^{14} \\ &+\frac{4}{15}(640-a)x^{15}-42x^{16}+\frac{128x^{17}}{17}-\frac{8x^{18}}{9}+\frac{x^{19}}{19}\end{aligned}$$

output
```
1/3*a^4*x^3+8*a^3*x^4+32/5*(12-a)*a^2*x^5+8/3*a*(a^2-48*a+128)*x^6+4/7*(-a
^3+192*a^2-1536*a+1024)*x^7-4*(15*a^2-288*a+512)*x^8+64/9*(128-3*a)*(4-a)*
x^9-24/5*(a^2-128*a+896)*x^10+2/11*(3*a^2-1536*a+20480)*x^11-8/3*(928-35*a
)*x^12+32/13*(524-9*a)*x^13-8/7*(464-3*a)*x^14+4/15*(640-a)*x^15-42*x^16+1
28/17*x^17-8/9*x^18+1/19*x^19
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.97

$$\int x^2\left(a+8x-8x^2+4x^3-x^4\right)^4\,dx=\frac{a^4x^3}{3}+8a^3x^4-\frac{32}{5}(-12+a)a^2x^5+\frac{8}{3}a\left(128-48a+a^2\right)x^6-\frac{4}{7}\left(-1024+1536a-192a^2+a^3\right)x^7-4\left(512-288a+15a^2\right)x^8+\frac{64}{9}\left(512-140a+3a^2\right)x^9-\frac{24}{5}\left(896-128a+a^2\right)x^{10}+\frac{2}{11}\left(20480-1536a+3a^2\right)x^{11}+\frac{8}{3}(-928+35a)x^{12}-\frac{32}{13}(-524+9a)x^{13}+\frac{8}{7}(-464+3a)x^{14}-\frac{4}{15}(-640+a)x^{15}-42x^{16}+\frac{128x^{17}}{17}-\frac{8x^{18}}{9}+\frac{x^{19}}{19}$$

input
```
Integrate[x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^4,x]
```

output
```
(a^4*x^3)/3 + 8*a^3*x^4 - (32*(-12 + a)*a^2*x^5)/5 + (8*a*(128 - 48*a + a^
2)*x^6)/3 - (4*(-1024 + 1536*a - 192*a^2 + a^3)*x^7)/7 - 4*(512 - 288*a +
15*a^2)*x^8 + (64*(512 - 140*a + 3*a^2)*x^9)/9 - (24*(896 - 128*a + a^2)*x
^10)/5 + (2*(20480 - 1536*a + 3*a^2)*x^11)/11 + (8*(-928 + 35*a)*x^12)/3 -
 (32*(-524 + 9*a)*x^13)/13 + (8*(-464 + 3*a)*x^14)/7 - (4*(-640 + a)*x^15)
/15 - 42*x^16 + (128*x^17)/17 - (8*x^18)/9 + x^19/19
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2465, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a - x^4 + 4x^3 - 8x^2 + 8x\right)^4 dx$$

$$\Big\downarrow \text{ 2465}$$

$$\int \left(a^4x^2 + 32a^3x^3 + 2\left(3a^2 - 1536a + 20480\right)x^{10} - 48\left(a^2 - 128a + 896\right)x^9 - 32\left(15a^2 - 288a + 512\right)x^7 + 16a(\right.$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{a^4x^3}{3} + 8a^3x^4 + \frac{2}{11}\left(3a^2 - 1536a + 20480\right)x^{11} - \frac{24}{5}\left(a^2 - 128a + 896\right)x^{10} - 4\left(15a^2 - 288a + 512\right)x^8 + \frac{8}{3}a\left(a^2 - 48a + 128\right)x^6 + \frac{32}{5}(12 - a)a^2x^5 + \frac{4}{7}\left(-a^3 + 192a^2 - 1536a + 1024\right)x^7 + \frac{4}{15}(640 - a)x^{15} - \frac{8}{7}(464 - 3a)x^{14} + \frac{32}{13}(524 - 9a)x^{13} - \frac{8}{3}(928 - 35a)x^{12} + \frac{64}{9}(128 - 3a)(4 - a)x^9 + \frac{x^{19}}{19} - \frac{8x^{18}}{9} + \frac{128x^{17}}{17} - 42x^{16}$$

input
```
Int[x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^4,x]
```

output
```
(a^4*x^3)/3 + 8*a^3*x^4 + (32*(12 - a)*a^2*x^5)/5 + (8*a*(128 - 48*a + a^2
)*x^6)/3 + (4*(1024 - 1536*a + 192*a^2 - a^3)*x^7)/7 - 4*(512 - 288*a + 15
*a^2)*x^8 + (64*(128 - 3*a)*(4 - a)*x^9)/9 - (24*(896 - 128*a + a^2)*x^10)
/5 + (2*(20480 - 1536*a + 3*a^2)*x^11)/11 - (8*(928 - 35*a)*x^12)/3 + (32*
(524 - 9*a)*x^13)/13 - (8*(464 - 3*a)*x^14)/7 + (4*(640 - a)*x^15)/15 - 42
*x^16 + (128*x^17)/17 - (8*x^18)/9 + x^19/19
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2465
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> Int[ExpandToSum[u, Px^p, x], x] /; PolyQ
[Px, x] && GtQ[Expon[Px, x], 2] && !BinomialQ[Px, x] && !TrinomialQ[Px, x
] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.87

method	result
norman	$\frac{a^4x^3}{3}+8a^3x^4+\left(-\frac{32}{5}a^3+\frac{384}{5}a^2\right)x^5+\left(\frac{8}{3}a^3-128a^2+\frac{1024}{3}a\right)x^6+\left(-\frac{4}{7}a^3+\frac{768}{7}a^2-\frac{6144}{7}a-\right.$
gosper	$-\frac{7424}{3}x^{12}-\frac{3712}{7}x^{14}-\frac{21504}{5}x^{10}+\frac{32768}{9}x^9-42x^{16}+\frac{1}{3}a^4x^3+\frac{1}{19}x^{19}+\frac{128}{17}x^{17}+\frac{512}{3}x^{15}+\frac{1676}{13}$
risch	$-\frac{7424}{3}x^{12}-\frac{3712}{7}x^{14}-\frac{21504}{5}x^{10}+\frac{32768}{9}x^9-42x^{16}+\frac{1}{3}a^4x^3+\frac{1}{19}x^{19}+\frac{128}{17}x^{17}+\frac{512}{3}x^{15}+\frac{1676}{13}$
parallelrisch	$-\frac{7424}{3}x^{12}-\frac{3712}{7}x^{14}-\frac{21504}{5}x^{10}+\frac{32768}{9}x^9-42x^{16}+\frac{1}{3}a^4x^3+\frac{1}{19}x^{19}+\frac{128}{17}x^{17}+\frac{512}{3}x^{15}+\frac{1676}{13}$
orering	$\frac{x^3(765765x^{16}-12932920x^{15}+109549440x^{14}-3879876x^{12}a-611080470x^{13}+49884120x^{11}a+2483120640x^{12}-322328160x^{1}}{}$
default	$\frac{x^{19}}{19}-\frac{8x^{18}}{9}+\frac{128x^{17}}{17}-42x^{16}+\frac{(-4a+2560)x^{15}}{15}+\frac{(48a-7424)x^{14}}{14}+\frac{(-288a+16768)x^{13}}{13}+\frac{(1120a-29696)x^{1}}{12}$

input
```
int(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^4,x,method=_RETURNVERBOSE)
```

output
```
1/3*a^4*x^3+8*a^3*x^4+(-32/5*a^3+384/5*a^2)*x^5+(8/3*a^3-128*a^2+1024/3*a)
*x^6+(-4/7*a^3+768/7*a^2-6144/7*a+4096/7)*x^7+(-60*a^2+1152*a-2048)*x^8+(6
4/3*a^2-8960/9*a+32768/9)*x^9+(-24/5*a^2+3072/5*a-21504/5)*x^10+(6/11*a^2-
3072/11*a+40960/11)*x^11+(280/3*a-7424/3)*x^12+(-288/13*a+16768/13)*x^13+(
24/7*a-3712/7)*x^14+(-4/15*a+512/3)*x^15-42*x^16+128/17*x^17-8/9*x^18+1/19
*x^19
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.87

$$
\begin{aligned}
\int x^2\left(a+8x-8x^2+4x^3-x^4\right)^4\,dx &= \frac{1}{19}\,x^{19}-\frac{8}{9}\,x^{18}+\frac{128}{17}\,x^{17}-\frac{4}{15}\,(a-640)x^{15} \\
&\quad -42\,x^{16}+\frac{8}{7}\,(3\,a-464)x^{14} \\
&\quad -\frac{32}{13}\,(9\,a-524)x^{13}+\frac{8}{3}\,(35\,a-928)x^{12} \\
&\quad +\frac{2}{11}\,\left(3\,a^2-1536\,a+20480\right)x^{11} \\
&\quad -\frac{24}{5}\,\left(a^2-128\,a+896\right)x^{10} \\
&\quad +\frac{64}{9}\,\left(3\,a^2-140\,a+512\right)x^{9} \\
&\quad -4\left(15\,a^2-288\,a+512\right)x^{8} \\
&\quad -\frac{4}{7}\,\left(a^3-192\,a^2+1536\,a-1024\right)x^{7} \\
&\quad +\frac{1}{3}\,a^4x^3+8\,a^3x^4+\frac{8}{3}\,\left(a^3-48\,a^2+128\,a\right)x^{6} \\
&\quad -\frac{32}{5}\,\left(a^3-12\,a^2\right)x^{5}
\end{aligned}
$$

input

```
integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^4,x, algorithm="fricas")
```

output

```
1/19*x^19 - 8/9*x^18 + 128/17*x^17 - 4/15*(a - 640)*x^15 - 42*x^16 + 8/7*(
3*a - 464)*x^14 - 32/13*(9*a - 524)*x^13 + 8/3*(35*a - 928)*x^12 + 2/11*(3
*a^2 - 1536*a + 20480)*x^11 - 24/5*(a^2 - 128*a + 896)*x^10 + 64/9*(3*a^2
- 140*a + 512)*x^9 - 4*(15*a^2 - 288*a + 512)*x^8 - 4/7*(a^3 - 192*a^2 + 1
536*a - 1024)*x^7 + 1/3*a^4*x^3 + 8*a^3*x^4 + 8/3*(a^3 - 48*a^2 + 128*a)*x
^6 - 32/5*(a^3 - 12*a^2)*x^5
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.04

$$\int x^2(a+8x-8x^2+4x^3-x^4)^4\,dx = \frac{a^4x^3}{3}+8a^3x^4+\frac{x^{19}}{19}-\frac{8x^{18}}{9}+\frac{128x^{17}}{17}-42x^{16} + x^{15}\cdot\left(\frac{512}{3}-\frac{4a}{15}\right)+x^{14}\cdot\left(\frac{24a}{7}-\frac{3712}{7}\right) + x^{13}\cdot\left(\frac{16768}{13}-\frac{288a}{13}\right)+x^{12}\cdot\left(\frac{280a}{3}-\frac{7424}{3}\right) + x^{11}\cdot\left(\frac{6a^2}{11}-\frac{3072a}{11}+\frac{40960}{11}\right) + x^{10}\left(-\frac{24a^2}{5}+\frac{3072a}{5}-\frac{21504}{5}\right) + x^9\cdot\left(\frac{64a^2}{3}-\frac{8960a}{9}+\frac{32768}{9}\right) + x^8(-60a^2+1152a-2048) + x^7\left(-\frac{4a^3}{7}+\frac{768a^2}{7}-\frac{6144a}{7}+\frac{4096}{7}\right)+x^6 \cdot\left(\frac{8a^3}{3}-128a^2+\frac{1024a}{3}\right)+x^5\left(-\frac{32a^3}{5}+\frac{384a^2}{5}\right)$$

input

```
integrate(x**2*(-x**4+4*x**3-8*x**2+a+8*x)**4,x)
```

output

```
a**4*x**3/3 + 8*a**3*x**4 + x**19/19 - 8*x**18/9 + 128*x**17/17 - 42*x**16
 + x**15*(512/3 - 4*a/15) + x**14*(24*a/7 - 3712/7) + x**13*(16768/13 - 28
8*a/13) + x**12*(280*a/3 - 7424/3) + x**11*(6*a**2/11 - 3072*a/11 + 40960/
11) + x**10*(-24*a**2/5 + 3072*a/5 - 21504/5) + x**9*(64*a**2/3 - 8960*a/9
 + 32768/9) + x**8*(-60*a**2 + 1152*a - 2048) + x**7*(-4*a**3/7 + 768*a**2
/7 - 6144*a/7 + 4096/7) + x**6*(8*a**3/3 - 128*a**2 + 1024*a/3) + x**5*(-3
2*a**3/5 + 384*a**2/5)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.87

$$\int x^2(a+8x-8x^2+4x^3-x^4)^4\,dx = \frac{1}{19}x^{19} - \frac{8}{9}x^{18} + \frac{128}{17}x^{17} - \frac{4}{15}(a-640)x^{15} - 42x^{16} + \frac{8}{7}(3a-464)x^{14} - \frac{32}{13}(9a-524)x^{13} + \frac{8}{3}(35a-928)x^{12} + \frac{2}{11}(3a^2-1536a+20480)x^{11} - \frac{24}{5}(a^2-128a+896)x^{10} + \frac{64}{9}(3a^2-140a+512)x^9 - 4(15a^2-288a+512)x^8 - \frac{4}{7}(a^3-192a^2+1536a-1024)x^7 + \frac{1}{3}a^4x^3 + 8a^3x^4 + \frac{8}{3}(a^3-48a^2+128a)x^6 - \frac{32}{5}(a^3-12a^2)x^5$$

input
```
integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^4,x, algorithm="maxima")
```

output
```
1/19*x^19 - 8/9*x^18 + 128/17*x^17 - 4/15*(a - 640)*x^15 - 42*x^16 + 8/7*(
3*a - 464)*x^14 - 32/13*(9*a - 524)*x^13 + 8/3*(35*a - 928)*x^12 + 2/11*(3
*a^2 - 1536*a + 20480)*x^11 - 24/5*(a^2 - 128*a + 896)*x^10 + 64/9*(3*a^2
- 140*a + 512)*x^9 - 4*(15*a^2 - 288*a + 512)*x^8 - 4/7*(a^3 - 192*a^2 + 1
536*a - 1024)*x^7 + 1/3*a^4*x^3 + 8*a^3*x^4 + 8/3*(a^3 - 48*a^2 + 128*a)*x
^6 - 32/5*(a^3 - 12*a^2)*x^5
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.06

$$\int x^2(a+8x-8x^2+4x^3-x^4)^4\,dx = \frac{1}{19}x^{19}-\frac{8}{9}x^{18}+\frac{128}{17}x^{17}-\frac{4}{15}ax^{15}-42x^{16}+\frac{24}{7}ax^{14}+\frac{512}{3}x^{15}-\frac{288}{13}ax^{13}-\frac{3712}{7}x^{14}+\frac{6}{11}a^2x^{11}+\frac{280}{3}ax^{12}+\frac{16768}{13}x^{13}-\frac{24}{5}a^2x^{10}-\frac{3072}{11}ax^{11}-\frac{7424}{3}x^{12}+\frac{64}{3}a^2x^9+\frac{3072}{5}ax^{10}+\frac{40960}{11}x^{11}-\frac{4}{7}a^3x^7-60a^2x^8-\frac{8960}{9}ax^9-\frac{21504}{5}x^{10}+\frac{8}{3}a^3x^6+\frac{768}{7}a^2x^7+1152ax^8+\frac{32768}{9}x^9-\frac{32}{5}a^3x^5-128a^2x^6-\frac{6144}{7}ax^7-2048x^8+\frac{1}{3}a^4x^3+8a^3x^4+\frac{384}{5}a^2x^5+\frac{1024}{3}ax^6+\frac{4096}{7}x^7$$

input
```
integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^4,x, algorithm="giac")
```

output
```
1/19*x^19 - 8/9*x^18 + 128/17*x^17 - 4/15*a*x^15 - 42*x^16 + 24/7*a*x^14 +
 512/3*x^15 - 288/13*a*x^13 - 3712/7*x^14 + 6/11*a^2*x^11 + 280/3*a*x^12 +
 16768/13*x^13 - 24/5*a^2*x^10 - 3072/11*a*x^11 - 7424/3*x^12 + 64/3*a^2*x
^9 + 3072/5*a*x^10 + 40960/11*x^11 - 4/7*a^3*x^7 - 60*a^2*x^8 - 8960/9*a*x
^9 - 21504/5*x^10 + 8/3*a^3*x^6 + 768/7*a^2*x^7 + 1152*a*x^8 + 32768/9*x^9
 - 32/5*a^3*x^5 - 128*a^2*x^6 - 6144/7*a*x^7 - 2048*x^8 + 1/3*a^4*x^3 + 8*
a^3*x^4 + 384/5*a^2*x^5 + 1024/3*a*x^6 + 4096/7*x^7
```

Mupad [B] (verification not implemented)

Time = 22.01 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.85

$$\int x^2(a+8x-8x^2+4x^3-x^4)^4\,dx = x^{14}\left(\frac{24\,a}{7}-\frac{3712}{7}\right)-x^{15}\left(\frac{4\,a}{15}-\frac{512}{3}\right)+x^{12}\left(\frac{280\,a}{3}-\frac{7424}{3}\right)-x^{13}\left(\frac{288\,a}{13}-\frac{16768}{13}\right)-x^8\left(60\,a^2-1152\,a+2048\right)-x^{10}\left(\frac{24\,a^2}{5}-\frac{3072\,a}{5}+\frac{21504}{5}\right)+x^9\left(\frac{64\,a^2}{3}-\frac{8960\,a}{9}+\frac{32768}{9}\right)+x^{11}\left(\frac{6\,a^2}{11}-\frac{3072\,a}{11}+\frac{40960}{11}\right)-x^7\left(\frac{4\,a^3}{7}-\frac{768\,a^2}{7}+\frac{6144\,a}{7}-\frac{4096}{7}\right)-42\,x^{16}+\frac{128\,x^{17}}{17}-\frac{8\,x^{18}}{9}+\frac{x^{19}}{19}+8\,a^3\,x^4+\frac{a^4\,x^3}{3}+\frac{8\,a\,x^6\,(a^2-48\,a+128)}{3}-\frac{32\,a^2\,x^5\,(a-12)}{5}$$

input
```
int(x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^4,x)
```

output
```
x^14*((24*a)/7 - 3712/7) - x^15*((4*a)/15 - 512/3) + x^12*((280*a)/3 - 742
4/3) - x^13*((288*a)/13 - 16768/13) - x^8*(60*a^2 - 1152*a + 2048) - x^10*
((24*a^2)/5 - (3072*a)/5 + 21504/5) + x^9*((64*a^2)/3 - (8960*a)/9 + 32768
/9) + x^11*((6*a^2)/11 - (3072*a)/11 + 40960/11) - x^7*((6144*a)/7 - (768*
a^2)/7 + (4*a^3)/7 - 4096/7) - 42*x^16 + (128*x^17)/17 - (8*x^18)/9 + x^19
/19 + 8*a^3*x^4 + (a^4*x^3)/3 + (8*a*x^6*(a^2 - 48*a + 128))/3 - (32*a^2*x
^5*(a - 12))/5
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.06

$$\int x^2 \left(a + 8x - 8x^2 + 4x^3 - x^4\right)^4 \, dx$$
$$= \frac{x^3(765765x^{16} - 12932920x^{15} + 109549440x^{14} - 3879876a\,x^{12} - 611080470x^{13} + 49884120a\,x^{11} + 2483\ldots}{}$$

input
```
int(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^4,x)
```

output
```
(x**3*(4849845*a**4 - 8314020*a**3*x**4 + 38798760*a**3*x**3 - 93117024*a*
*3*x**2 + 116396280*a**3*x + 7936110*a**2*x**8 - 69837768*a**2*x**7 + 3103
90080*a**2*x**6 - 872972100*a**2*x**5 + 1596291840*a**2*x**4 - 1862340480*
a**2*x**3 + 1117404288*a**2*x**2 - 3879876*a*x**12 + 49884120*a*x**11 - 32
2328160*a*x**10 + 1357956600*a*x**9 - 4063288320*a*x**8 + 8939234304*a*x**
7 - 14484870400*a*x**6 + 16761064320*a*x**5 - 12770334720*a*x**4 + 4966241
280*a*x**3 + 765765*x**16 - 12932920*x**15 + 109549440*x**14 - 611080470*x
**13 + 2483120640*x**12 - 7715410560*x**11 + 18766661760*x**10 - 360052492
80*x**9 + 54177177600*x**8 - 62574640128*x**7 + 52973240320*x**6 - 2979744
7680*x**5 + 8513556480*x**4))/14549535
```

3.28 $\int x^2(a+8x-8x^2+4x^3-x^4)^3\,dx$

Optimal result

Integrand size = 26, antiderivative size = 138

$$\int x^2(a+8x-8x^2+4x^3-x^4)^3\,dx=\frac{a^3x^3}{3}+6a^2x^4+\frac{24}{5}(8-a)ax^5+\frac{2}{3}\left(128-96a+3a^2\right)x^6-\frac{3}{7}\left(512-128a+a^2\right)x^7+6(48-5a)x^8-\frac{32}{9}(70-3a)x^9+\frac{12}{5}(64-a)x^{10}-\frac{3}{11}(256-a)x^{11}+\frac{70x^{12}}{3}-\frac{72x^{13}}{13}+\frac{6x^{14}}{7}-\frac{x^{15}}{15}$$

output
```
1/3*a^3*x^3+6*a^2*x^4+24/5*(8-a)*a*x^5+2/3*(3*a^2-96*a+128)*x^6-3/7*(a^2-1
28*a+512)*x^7+6*(48-5*a)*x^8-32/9*(70-3*a)*x^9+12/5*(64-a)*x^10-3/11*(256-
a)*x^11+70/3*x^12-72/13*x^13+6/7*x^14-1/15*x^15
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.96

$$\int x^2(a+8x-8x^2+4x^3-x^4)^3\,dx = \frac{a^3x^3}{3}+6a^2x^4-\frac{24}{5}(-8+a)ax^5+\frac{2}{3}(128-96a+3a^2)\,x^6-\frac{3}{7}(512-128a+a^2)\,x^7-6(-48+5a)x^8+\frac{32}{9}(-70+3a)x^9-\frac{12}{5}(-64+a)x^{10}+\frac{3}{11}(-256+a)x^{11}+\frac{70x^{12}}{3}-\frac{72x^{13}}{13}+\frac{6x^{14}}{7}-\frac{x^{15}}{15}$$

input
```
Integrate[x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^3,x]
```

output
```
(a^3*x^3)/3 + 6*a^2*x^4 - (24*(-8 + a)*a*x^5)/5 + (2*(128 - 96*a + 3*a^2)*
x^6)/3 - (3*(512 - 128*a + a^2)*x^7)/7 - 6*(-48 + 5*a)*x^8 + (32*(-70 + 3*
a)*x^9)/9 - (12*(-64 + a)*x^10)/5 + (3*(-256 + a)*x^11)/11 + (70*x^12)/3 -
 (72*x^13)/13 + (6*x^14)/7 - x^15/15
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2465, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(a-x^4+4x^3-8x^2+8x\right)^3dx$$

$$\Big\downarrow \text{2465}$$

$$\int \left(a^3x^2-3\left(a^2-128a+512\right)x^6+4\left(3a^2-96a+128\right)x^5+24a^2x^3-3(256-a)x^{10}+24(64-a)x^9-32(70-$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^3x^3}{3} - \frac{3}{7}\left(a^2 - 128a + 512\right)x^7 + \frac{2}{3}\left(3a^2 - 96a + 128\right)x^6 + 6a^2x^4 - \frac{3}{11}(256 - a)x^{11} + \frac{12}{5}(64 - a)x^{10} - \frac{32}{9}(70 - 3a)x^9 + 6(48 - 5a)x^8 + \frac{24}{5}(8 - a)ax^5 - \frac{x^{15}}{15} + \frac{6x^{14}}{7} - \frac{72x^{13}}{13} + \frac{70x^{12}}{3}$$

input
```
Int[x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^3,x]
```

output
```
(a^3*x^3)/3 + 6*a^2*x^4 + (24*(8 - a)*a*x^5)/5 + (2*(128 - 96*a + 3*a^2)*x
^6)/3 - (3*(512 - 128*a + a^2)*x^7)/7 + 6*(48 - 5*a)*x^8 - (32*(70 - 3*a)*
x^9)/9 + (12*(64 - a)*x^10)/5 - (3*(256 - a)*x^11)/11 + (70*x^12)/3 - (72*
x^13)/13 + (6*x^14)/7 - x^15/15
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2465
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> Int[ExpandToSum[u, Px^p, x], x] /; PolyQ
[Px, x] && GtQ[Expon[Px, x], 2] && !BinomialQ[Px, x] && !TrinomialQ[Px, x
] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.83

method	result
norman	$\frac{a^3x^3}{3} + 6a^2x^4 + \left(-\frac{24}{5}a^2 + \frac{192}{5}a\right)x^5 + \left(2a^2 - 64a + \frac{256}{3}\right)x^6 + \left(-\frac{3}{7}a^2 + \frac{384}{7}a - \frac{1536}{7}\right)x^7 + (-$
gosper	$\frac{1}{3}a^3x^3 + 6a^2x^4 - \frac{24}{5}a^2x^5 + \frac{192}{5}a\,x^5 + 2a^2x^6 - 64x^6a + \frac{256}{3}x^6 - \frac{3}{7}a^2x^7 + \frac{384}{7}x^7a - \frac{1536}{7}x^7 -$
risch	$\frac{1}{3}a^3x^3 + 6a^2x^4 - \frac{24}{5}a^2x^5 + \frac{192}{5}a\,x^5 + 2a^2x^6 - 64x^6a + \frac{256}{3}x^6 - \frac{3}{7}a^2x^7 + \frac{384}{7}x^7a - \frac{1536}{7}x^7 -$
parallelrisch	$\frac{1}{3}a^3x^3 + 6a^2x^4 - \frac{24}{5}a^2x^5 + \frac{192}{5}a\,x^5 + 2a^2x^6 - 64x^6a + \frac{256}{3}x^6 - \frac{3}{7}a^2x^7 + \frac{384}{7}x^7a - \frac{1536}{7}x^7 -$
orering	$\frac{x^3(-3003x^{12}+38610x^{11}-249480x^{10}+12285a\,x^8+1051050x^9-108108x^7a-3144960x^8+480480x^6a+6918912x^7-19305a^2x}{}$
default	$-\frac{x^{15}}{15} + \frac{6x^{14}}{7} - \frac{72x^{13}}{13} + \frac{70x^{12}}{3} + \frac{(3a-768)x^{11}}{11} + \frac{(-24a+1536)x^{10}}{10} + \frac{(96a-2240)x^9}{9} + \frac{(-240a+2304)x^8}{8} + \frac{(}{}$

input
```
int(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/3*a^3*x^3+6*a^2*x^4+(-24/5*a^2+192/5*a)*x^5+(2*a^2-64*a+256/3)*x^6+(-3/7
*a^2+384/7*a-1536/7)*x^7+(-30*a+288)*x^8+(32/3*a-2240/9)*x^9+(-12/5*a+768/
5)*x^10+(3/11*a-768/11)*x^11+70/3*x^12-72/13*x^13+6/7*x^14-1/15*x^15
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.82

$$\int x^2(a+8x-8x^2+4x^3-x^4)^3\,dx = -\frac{1}{15}x^{15}+\frac{6}{7}x^{14}-\frac{72}{13}x^{13}+\frac{3}{11}(a-256)x^{11}+\frac{70}{3}x^{12}-\frac{12}{5}(a-64)x^{10}+\frac{32}{9}(3a-70)x^9-6(5a-48)x^8-\frac{3}{7}(a^2-128a+512)x^7+\frac{2}{3}(3a^2-96a+128)x^6+\frac{1}{3}a^3x^3+6a^2x^4-\frac{24}{5}(a^2-8a)x^5$$

input
```
integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^3,x, algorithm="fricas")
```

output
```
-1/15*x^15 + 6/7*x^14 - 72/13*x^13 + 3/11*(a - 256)*x^11 + 70/3*x^12 - 12/
5*(a - 64)*x^10 + 32/9*(3*a - 70)*x^9 - 6*(5*a - 48)*x^8 - 3/7*(a^2 - 128*
a + 512)*x^7 + 2/3*(3*a^2 - 96*a + 128)*x^6 + 1/3*a^3*x^3 + 6*a^2*x^4 - 24
/5*(a^2 - 8*a)*x^5
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.97

$$\int x^2 \left(a+8x-8x^2+4x^3-x^4\right)^3 \, dx = \frac{a^3x^3}{3} + 6a^2x^4 - \frac{x^{15}}{15} + \frac{6x^{14}}{7} - \frac{72x^{13}}{13} + \frac{70x^{12}}{3} + x^{11}\cdot\left(\frac{3a}{11} - \frac{768}{11}\right) + x^{10}\cdot\left(\frac{768}{5} - \frac{12a}{5}\right) + x^9\cdot\left(\frac{32a}{3} - \frac{2240}{9}\right) + x^8\cdot(288-30a) + x^7\left(-\frac{3a^2}{7} + \frac{384a}{7} - \frac{1536}{7}\right) + x^6 \cdot\left(2a^2 - 64a + \frac{256}{3}\right) + x^5\left(-\frac{24a^2}{5} + \frac{192a}{5}\right)$$

input
```
integrate(x**2*(-x**4+4*x**3-8*x**2+a+8*x)**3,x)
```

output
```
a**3*x**3/3 + 6*a**2*x**4 - x**15/15 + 6*x**14/7 - 72*x**13/13 + 70*x**12/
3 + x**11*(3*a/11 - 768/11) + x**10*(768/5 - 12*a/5) + x**9*(32*a/3 - 2240
/9) + x**8*(288 - 30*a) + x**7*(-3*a**2/7 + 384*a/7 - 1536/7) + x**6*(2*a*
*2 - 64*a + 256/3) + x**5*(-24*a**2/5 + 192*a/5)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.82

$$\int x^2 \left(a+8x-8x^2+4x^3-x^4\right)^3 \, dx = -\frac{1}{15}x^{15} + \frac{6}{7}x^{14} - \frac{72}{13}x^{13} + \frac{3}{11}(a-256)x^{11} + \frac{70}{3}x^{12} - \frac{12}{5}(a-64)x^{10} + \frac{32}{9}(3\,a-70)x^9 - 6\,(5\,a-48)x^8 - \frac{3}{7}\left(a^2-128\,a+512\right)x^7 + \frac{2}{3}\left(3\,a^2-96\,a+128\right)x^6 + \frac{1}{3}a^3x^3 + 6\,a^2x^4 - \frac{24}{5}\left(a^2-8\,a\right)x^5$$

input
```
integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^3,x, algorithm="maxima")
```

output
```
-1/15*x^15 + 6/7*x^14 - 72/13*x^13 + 3/11*(a - 256)*x^11 + 70/3*x^12 - 12/
5*(a - 64)*x^10 + 32/9*(3*a - 70)*x^9 - 6*(5*a - 48)*x^8 - 3/7*(a^2 - 128*
a + 512)*x^7 + 2/3*(3*a^2 - 96*a + 128)*x^6 + 1/3*a^3*x^3 + 6*a^2*x^4 - 24
/5*(a^2 - 8*a)*x^5
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.96

$$\int x^2\left(a+8x-8x^2+4x^3-x^4\right)^3\,dx = -\frac{1}{15}\,x^{15}+\frac{6}{7}\,x^{14}-\frac{72}{13}\,x^{13}+\frac{3}{11}\,ax^{11}+\frac{70}{3}\,x^{12} -\frac{12}{5}\,ax^{10}-\frac{768}{11}\,x^{11}+\frac{32}{3}\,ax^{9}+\frac{768}{5}\,x^{10} -\frac{3}{7}\,a^2x^7-30\,ax^8-\frac{2240}{9}\,x^9+2\,a^2x^6+\frac{384}{7}\,ax^7 +288\,x^8-\frac{24}{5}\,a^2x^5-64\,ax^6-\frac{1536}{7}\,x^7 +\frac{1}{3}\,a^3x^3+6\,a^2x^4+\frac{192}{5}\,ax^5+\frac{256}{3}\,x^6$$

input
```
integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^3,x, algorithm="giac")
```

output
```
-1/15*x^15 + 6/7*x^14 - 72/13*x^13 + 3/11*a*x^11 + 70/3*x^12 - 12/5*a*x^10
 - 768/11*x^11 + 32/3*a*x^9 + 768/5*x^10 - 3/7*a^2*x^7 - 30*a*x^8 - 2240/9
*x^9 + 2*a^2*x^6 + 384/7*a*x^7 + 288*x^8 - 24/5*a^2*x^5 - 64*a*x^6 - 1536/
7*x^7 + 1/3*a^3*x^3 + 6*a^2*x^4 + 192/5*a*x^5 + 256/3*x^6
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.82

$$\int x^2 (a+8x-8x^2+4x^3-x^4)^3 \, dx = x^{11}\left(\frac{3\,a}{11}-\frac{768}{11}\right)-x^{10}\left(\frac{12\,a}{5}-\frac{768}{5}\right)-x^8\,(30\,a-288)+x^9\left(\frac{32\,a}{3}-\frac{2240}{9}\right)+x^6\left(2\,a^2-64\,a+\frac{256}{3}\right)-x^7\left(\frac{3\,a^2}{7}-\frac{384\,a}{7}+\frac{1536}{7}\right)+\frac{70\,x^{12}}{3}-\frac{72\,x^{13}}{13}+\frac{6\,x^{14}}{7}-\frac{x^{15}}{15}+6\,a^2\,x^4+\frac{a^3\,x^3}{3}-\frac{24\,a\,x^5\,(a-8)}{5}$$

input
```
int(x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^3,x)
```

output
```
x^11*((3*a)/11 - 768/11) - x^10*((12*a)/5 - 768/5) - x^8*(30*a - 288) + x^
9*((32*a)/3 - 2240/9) + x^6*(2*a^2 - 64*a + 256/3) - x^7*((3*a^2)/7 - (384
*a)/7 + 1536/7) + (70*x^12)/3 - (72*x^13)/13 + (6*x^14)/7 - x^15/15 + 6*a^
2*x^4 + (a^3*x^3)/3 - (24*a*x^5*(a - 8))/5
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.96

$$\int x^2 (a+8x-8x^2+4x^3-x^4)^3 \, dx = \frac{x^3(-3003x^{12}+38610x^{11}-249480x^{10}+12285a\,x^8+1051050x^9-108108a\,x^7-3144960x^8+480480a}{}$$

input
```
int(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^3,x)
```

output
```
(x**3*(15015*a**3 - 19305*a**2*x**4 + 90090*a**2*x**3 - 216216*a**2*x**2 +
 270270*a**2*x + 12285*a*x**8 - 108108*a*x**7 + 480480*a*x**6 - 1351350*a*
x**5 + 2471040*a*x**4 - 2882880*a*x**3 + 1729728*a*x**2 - 3003*x**12 + 386
10*x**11 - 249480*x**10 + 1051050*x**9 - 3144960*x**8 + 6918912*x**7 - 112
11200*x**6 + 12972960*x**5 - 9884160*x**4 + 3843840*x**3))/45045
```

3.29 $\int x^2(a+8x-8x^2+4x^3-x^4)^2\,dx$

Optimal result

Integrand size = 26, antiderivative size = 79

$$\int x^2\left(a+8x-8x^2+4x^3-x^4\right)^2\,dx=\frac{a^2x^3}{3}+4ax^4+\frac{16}{5}(4-a)x^5-\frac{4}{3}(16-a)x^6+\frac{2}{7}(64-a)x^7-10x^8+\frac{32x^9}{9}-\frac{4x^{10}}{5}+\frac{x^{11}}{11}$$

output
```
1/3*a^2*x^3+4*a*x^4+16/5*(4-a)*x^5-4/3*(16-a)*x^6+2/7*(64-a)*x^7-10*x^8+32
/9*x^9-4/5*x^10+1/11*x^11
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.92

$$\int x^2\left(a+8x-8x^2+4x^3-x^4\right)^2\,dx=\frac{a^2x^3}{3}+4ax^4-\frac{16}{5}(-4+a)x^5+\frac{4}{3}(-16+a)x^6-\frac{2}{7}(-64+a)x^7-10x^8+\frac{32x^9}{9}-\frac{4x^{10}}{5}+\frac{x^{11}}{11}$$

input
```
Integrate[x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x]
```

output
```
(a^2*x^3)/3 + 4*a*x^4 - (16*(-4 + a)*x^5)/5 + (4*(-16 + a)*x^6)/3 - (2*(-6
4 + a)*x^7)/7 - 10*x^8 + (32*x^9)/9 - (4*x^10)/5 + x^11/11
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2465, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(a - x^4 + 4x^3 - 8x^2 + 8x)^2 \, dx$$

$$\Big\downarrow \text{ 2465}$$

$$\int (a^2x^2 + 2(64 - a)x^6 - 8(16 - a)x^5 + 16(4 - a)x^4 + 16ax^3 + x^{10} - 8x^9 + 32x^8 - 80x^7) \, dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{a^2x^3}{3} + \frac{2}{7}(64 - a)x^7 - \frac{4}{3}(16 - a)x^6 + \frac{16}{5}(4 - a)x^5 + 4ax^4 + \frac{x^{11}}{11} - \frac{4x^{10}}{5} + \frac{32x^9}{9} - 10x^8$$

input
```
Int[x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x]
```

output
```
(a^2*x^3)/3 + 4*a*x^4 + (16*(4 - a)*x^5)/5 - (4*(16 - a)*x^6)/3 + (2*(64 -
 a)*x^7)/7 - 10*x^8 + (32*x^9)/9 - (4*x^10)/5 + x^11/11
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2465
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> Int[ExpandToSum[u, Px^p, x], x] /; PolyQ
[Px, x] && GtQ[Expon[Px, x], 2] && !BinomialQ[Px, x] && !TrinomialQ[Px, x
] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.80

method	result
norman	$\frac{x^{11}}{11}-\frac{4x^{10}}{5}+\frac{32x^{9}}{9}-10x^{8}+\left(-\frac{2a}{7}+\frac{128}{7}\right)x^{7}+\left(\frac{4a}{3}-\frac{64}{3}\right)x^{6}+\left(-\frac{16a}{5}+\frac{64}{5}\right)x^{5}+4a\,x^{4}+\frac{a^{2}x^{3}}{3}$
default	$\frac{x^{11}}{11}-\frac{4x^{10}}{5}+\frac{32x^{9}}{9}-10x^{8}+\frac{(-2a+128)x^{7}}{7}+\frac{(8a-128)x^{6}}{6}+\frac{(-16a+64)x^{5}}{5}+4a\,x^{4}+\frac{a^{2}x^{3}}{3}$
gosper	$\frac{1}{11}x^{11}-\frac{4}{5}x^{10}+\frac{32}{9}x^{9}-10x^{8}-\frac{2}{7}x^{7}a+\frac{128}{7}x^{7}+\frac{4}{3}x^{6}a-\frac{64}{3}x^{6}-\frac{16}{5}a\,x^{5}+\frac{64}{5}x^{5}+4a\,x^{4}+\frac{1}{3}a^{2}$
risch	$\frac{1}{11}x^{11}-\frac{4}{5}x^{10}+\frac{32}{9}x^{9}-10x^{8}-\frac{2}{7}x^{7}a+\frac{128}{7}x^{7}+\frac{4}{3}x^{6}a-\frac{64}{3}x^{6}-\frac{16}{5}a\,x^{5}+\frac{64}{5}x^{5}+4a\,x^{4}+\frac{1}{3}a^{2}$
parallelrisch	$\frac{1}{11}x^{11}-\frac{4}{5}x^{10}+\frac{32}{9}x^{9}-10x^{8}-\frac{2}{7}x^{7}a+\frac{128}{7}x^{7}+\frac{4}{3}x^{6}a-\frac{64}{3}x^{6}-\frac{16}{5}a\,x^{5}+\frac{64}{5}x^{5}+4a\,x^{4}+\frac{1}{3}a^{2}$
orering	$\frac{x^{3}\left(315x^{8}-2772x^{7}+12320x^{6}-990a\,x^{4}-34650x^{5}+4620a\,x^{3}+63360x^{4}-11088a\,x^{2}-73920x^{3}+1155a^{2}+13860xa+44352x^{2}\right)}{3465}$

input
```
int(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/11*x^11-4/5*x^10+32/9*x^9-10*x^8+(-2/7*a+128/7)*x^7+(4/3*a-64/3)*x^6+(-1
6/5*a+64/5)*x^5+4*a*x^4+1/3*a^2*x^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.75

$$\int x^{2}\left(a+8x-8x^{2}+4x^{3}-x^{4}\right)^{2}\,dx=\frac{1}{11}\,x^{11}-\frac{4}{5}\,x^{10}+\frac{32}{9}\,x^{9}-\frac{2}{7}\,(a-64)x^{7}-10\,x^{8}+\frac{4}{3}\,(a-16)x^{6}-\frac{16}{5}\,(a-4)x^{5}+\frac{1}{3}\,a^{2}x^{3}+4\,ax^{4}$$

input
```
integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="fricas")
```

output
```
1/11*x^11 - 4/5*x^10 + 32/9*x^9 - 2/7*(a - 64)*x^7 - 10*x^8 + 4/3*(a - 16)
*x^6 - 16/5*(a - 4)*x^5 + 1/3*a^2*x^3 + 4*a*x^4
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.92

$$\int x^2(a+8x-8x^2+4x^3-x^4)^2\,dx = \frac{a^2x^3}{3}+4ax^4+\frac{x^{11}}{11}-\frac{4x^{10}}{5}+\frac{32x^9}{9}-10x^8+x^7\cdot\left(\frac{128}{7}-\frac{2a}{7}\right)+x^6\cdot\left(\frac{4a}{3}-\frac{64}{3}\right)+x^5\cdot\left(\frac{64}{5}-\frac{16a}{5}\right)$$

input
```
integrate(x**2*(-x**4+4*x**3-8*x**2+a+8*x)**2,x)
```

output
```
a**2*x**3/3 + 4*a*x**4 + x**11/11 - 4*x**10/5 + 32*x**9/9 - 10*x**8 + x**7
*(128/7 - 2*a/7) + x**6*(4*a/3 - 64/3) + x**5*(64/5 - 16*a/5)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.75

$$\int x^2(a+8x-8x^2+4x^3-x^4)^2\,dx = \frac{1}{11}x^{11}-\frac{4}{5}x^{10}+\frac{32}{9}x^9-\frac{2}{7}(a-64)x^7-10x^8+\frac{4}{3}(a-16)x^6-\frac{16}{5}(a-4)x^5+\frac{1}{3}a^2x^3+4ax^4$$

input
```
integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="maxima")
```

output
```
1/11*x^11 - 4/5*x^10 + 32/9*x^9 - 2/7*(a - 64)*x^7 - 10*x^8 + 4/3*(a - 16)
*x^6 - 16/5*(a - 4)*x^5 + 1/3*a^2*x^3 + 4*a*x^4
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.86

$$\int x^2(a+8x-8x^2+4x^3-x^4)^2\,dx = \frac{1}{11}x^{11} - \frac{4}{5}x^{10} + \frac{32}{9}x^9 - \frac{2}{7}ax^7 - 10x^8 + \frac{4}{3}ax^6 + \frac{128}{7}x^7 - \frac{16}{5}ax^5 - \frac{64}{3}x^6 + \frac{1}{3}a^2x^3 + 4ax^4 + \frac{64}{5}x^5$$

input
```
integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="giac")
```

output
```
1/11*x^11 - 4/5*x^10 + 32/9*x^9 - 2/7*a*x^7 - 10*x^8 + 4/3*a*x^6 + 128/7*x
^7 - 16/5*a*x^5 - 64/3*x^6 + 1/3*a^2*x^3 + 4*a*x^4 + 64/5*x^5
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.81

$$\int x^2(a+8x-8x^2+4x^3-x^4)^2\,dx = x^6\left(\frac{4a}{3}-\frac{64}{3}\right) - x^5\left(\frac{16a}{5}-\frac{64}{5}\right) - x^7\left(\frac{2a}{7}-\frac{128}{7}\right) + 4ax^4 - 10x^8 + \frac{32x^9}{9} - \frac{4x^{10}}{5} + \frac{x^{11}}{11} + \frac{a^2x^3}{3}$$

input
```
int(x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x)
```

output
```
x^6*((4*a)/3 - 64/3) - x^5*((16*a)/5 - 64/5) - x^7*((2*a)/7 - 128/7) + 4*a
*x^4 - 10*x^8 + (32*x^9)/9 - (4*x^10)/5 + x^11/11 + (a^2*x^3)/3
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.86

$$\int x^2(a+8x-8x^2+4x^3-x^4)^2\,dx$$
$$=\frac{x^3(315x^8-2772x^7+12320x^6-990a\,x^4-34650x^5+4620a\,x^3+63360x^4-11088a\,x^2-73920x^3+11}{3465}$$

input
```
int(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^2,x)
```

output
```
(x**3*(1155*a**2 - 990*a*x**4 + 4620*a*x**3 - 11088*a*x**2 + 13860*a*x + 3
15*x**8 - 2772*x**7 + 12320*x**6 - 34650*x**5 + 63360*x**4 - 73920*x**3 +
44352*x**2))/3465
```

3.30 $\int x^2(a+8x-8x^2+4x^3-x^4)\,dx$

Optimal result

Integrand size = 24, antiderivative size = 35

$$\int x^2\left(a+8x-8x^2+4x^3-x^4\right)\,dx=\frac{ax^3}{3}+2x^4-\frac{8x^5}{5}+\frac{2x^6}{3}-\frac{x^7}{7}$$

output
```
1/3*a*x^3+2*x^4-8/5*x^5+2/3*x^6-1/7*x^7
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int x^2\left(a+8x-8x^2+4x^3-x^4\right)\,dx=\frac{ax^3}{3}+2x^4-\frac{8x^5}{5}+\frac{2x^6}{3}-\frac{x^7}{7}$$

input
```
Integrate[x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4),x]
```

output
```
(a*x^3)/3 + 2*x^4 - (8*x^5)/5 + (2*x^6)/3 - x^7/7
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(a-x^4+4x^3-8x^2+8x\right)dx$$

$$\Big\downarrow \text{ 2010}$$

$$\int \left(ax^2-x^6+4x^5-8x^4+8x^3\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{ax^3}{3}-\frac{x^7}{7}+\frac{2x^6}{3}-\frac{8x^5}{5}+2x^4$$

input
```
Int[x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4),x]
```

output
```
(a*x^3)/3 + 2*x^4 - (8*x^5)/5 + (2*x^6)/3 - x^7/7
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.80

method	result	size
gosper	$\frac{1}{3}a\,x^3+2x^4-\frac{8}{5}x^5+\frac{2}{3}x^6-\frac{1}{7}x^7$	28
default	$\frac{1}{3}a\,x^3+2x^4-\frac{8}{5}x^5+\frac{2}{3}x^6-\frac{1}{7}x^7$	28
norman	$\frac{1}{3}a\,x^3+2x^4-\frac{8}{5}x^5+\frac{2}{3}x^6-\frac{1}{7}x^7$	28
risch	$\frac{1}{3}a\,x^3+2x^4-\frac{8}{5}x^5+\frac{2}{3}x^6-\frac{1}{7}x^7$	28
parallelrisch	$\frac{1}{3}a\,x^3+2x^4-\frac{8}{5}x^5+\frac{2}{3}x^6-\frac{1}{7}x^7$	28
orering	$\frac{x^3(-15x^4+70x^3-168x^2+35a+210x)}{105}$	28

input
```
int(x^2*(-x^4+4*x^3-8*x^2+a+8*x),x,method=_RETURNVERBOSE)
```

output
```
1/3*a*x^3+2*x^4-8/5*x^5+2/3*x^6-1/7*x^7
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.77

$$\int x^2\left(a+8x-8x^2+4x^3-x^4\right)\,dx=-\frac{1}{7}\,x^7+\frac{2}{3}\,x^6-\frac{8}{5}\,x^5+\frac{1}{3}\,ax^3+2\,x^4$$

input
```
integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x),x, algorithm="fricas")
```

output
```
-1/7*x^7 + 2/3*x^6 - 8/5*x^5 + 1/3*a*x^3 + 2*x^4
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

$$\int x^2(a+8x-8x^2+4x^3-x^4)\ dx = \frac{ax^3}{3} - \frac{x^7}{7} + \frac{2x^6}{3} - \frac{8x^5}{5} + 2x^4$$

input
```
integrate(x**2*(-x**4+4*x**3-8*x**2+a+8*x),x)
```

output
```
a*x**3/3 - x**7/7 + 2*x**6/3 - 8*x**5/5 + 2*x**4
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.77

$$\int x^2(a+8x-8x^2+4x^3-x^4)\ dx = -\frac{1}{7}\,x^7 + \frac{2}{3}\,x^6 - \frac{8}{5}\,x^5 + \frac{1}{3}\,ax^3 + 2\,x^4$$

input
```
integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x),x, algorithm="maxima")
```

output
```
-1/7*x^7 + 2/3*x^6 - 8/5*x^5 + 1/3*a*x^3 + 2*x^4
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.77

$$\int x^2(a+8x-8x^2+4x^3-x^4)\ dx = -\frac{1}{7}\,x^7 + \frac{2}{3}\,x^6 - \frac{8}{5}\,x^5 + \frac{1}{3}\,ax^3 + 2\,x^4$$

input
```
integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x),x, algorithm="giac")
```

output
```
-1/7*x^7 + 2/3*x^6 - 8/5*x^5 + 1/3*a*x^3 + 2*x^4
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.77

$$\int x^2(a+8x-8x^2+4x^3-x^4)\,dx=-\frac{x^7}{7}+\frac{2\,x^6}{3}-\frac{8\,x^5}{5}+2\,x^4+\frac{a\,x^3}{3}$$

input
```
int(x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4),x)
```

output
```
(a*x^3)/3 + 2*x^4 - (8*x^5)/5 + (2*x^6)/3 - x^7/7
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.77

$$\int x^2(a+8x-8x^2+4x^3-x^4)\,dx=\frac{x^3(-15x^4+70x^3-168x^2+35a+210x)}{105}$$

input
```
int(x^2*(-x^4+4*x^3-8*x^2+a+8*x),x)
```

output
```
(x**3*(35*a - 15*x**4 + 70*x**3 - 168*x**2 + 210*x))/105
```

3.31 $\int \frac{x^2}{a+8x-8x^2+4x^3-x^4}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 103

$$\int \frac{x^2}{a+8x-8x^2+4x^3-x^4}\,dx=\frac{\arctan\left(\frac{1-x}{\sqrt{1-\sqrt{4+a}}}\right)}{2\sqrt{1-\sqrt{4+a}}}+\frac{\arctan\left(\frac{1-x}{\sqrt{1+\sqrt{4+a}}}\right)}{2\sqrt{1+\sqrt{4+a}}}+\frac{\operatorname{arctanh}\left(\frac{1+(-1+x)^2}{\sqrt{4+a}}\right)}{\sqrt{4+a}}$$

output
```
1/2*arctan((1-x)/(1-(4+a)^(1/2))^(1/2))/(1-(4+a)^(1/2))^(1/2)+1/2*arctan((
1-x)/(1+(4+a)^(1/2))^(1/2))/(1+(4+a)^(1/2))^(1/2)+arctanh((1+(-1+x)^2)/(4+
a)^(1/2))/(4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.59

$$\int \frac{x^2}{a+8x-8x^2+4x^3-x^4}\,dx=-\frac{1}{4}\text{RootSum}\left[a+8\#1-8\#1^2+4\#1^3-\#1^4\&,\frac{\log(x-\#1)\#1^2}{-2+4\#1-3\#1^2+\#1^3}\&\right]$$

input
```
Integrate[x^2/(a + 8*x - 8*x^2 + 4*x^3 - x^4),x]
```

output
```
-1/4*RootSum[a + 8*#1 - 8*#1^2 + 4*#1^3 - #1^4 & , (Log[x - #1]*#1^2)/(-2
+ 4*#1 - 3*#1^2 + #1^3) & ]
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.02, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {2459, 2006, 2202, 27, 1432, 1083, 219, 1480, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{a-x^4+4x^3-8x^2+8x}dx$$

$$\Big\downarrow \text{2459}$$

$$\int \frac{(x-1)^2+2(x-1)+1}{a-(x-1)^4-2(x-1)^2+3}d(x-1)$$

$$\Big\downarrow \text{2006}$$

$$\int \frac{x^2}{a-(x-1)^4-2(x-1)^2+3}d(x-1)$$

$$\Big\downarrow \text{2202}$$

$$\int \frac{(x-1)^2+1}{-(x-1)^4-2(x-1)^2+a+3}d(x-1)+\int \frac{2(x-1)}{-(x-1)^4-2(x-1)^2+a+3}d(x-1)$$

$$\Big\downarrow \text{27}$$

$$\int \frac{(x-1)^2+1}{-(x-1)^4-2(x-1)^2+a+3}d(x-1)+2\int \frac{x-1}{-(x-1)^4-2(x-1)^2+a+3}d(x-1)$$

$$\Big\downarrow \text{1432}$$

$$\int \frac{1}{-(x-1)^4-2(x-1)^2+a+3}d(x-1)^2+\int \frac{(x-1)^2+1}{-(x-1)^4-2(x-1)^2+a+3}d(x-1)$$

$$\Big\downarrow \text{1083}$$

$$\int \frac{(x-1)^2+1}{-(x-1)^4-2(x-1)^2+a+3}d(x-1)-2\int \frac{1}{4(a+4)-(x-1)^4}d\left(-2(x-1)^2-2\right)$$

$$\Big\downarrow \textbf{219}$$

$$\int \frac{(x-1)^2+1}{-(x-1)^4-2(x-1)^2+a+3}d(x-1)-\frac{\text{arctanh}\left(\frac{-2(x-1)^2-2}{2\sqrt{a+4}}\right)}{\sqrt{a+4}}$$

$$\Big\downarrow \textbf{1480}$$

$$\frac{1}{2}\int \frac{1}{-(x-1)^2-\sqrt{a+4}-1}d(x-1)+\frac{1}{2}\int \frac{1}{-(x-1)^2+\sqrt{a+4}-1}d(x-1)-\frac{\text{arctanh}\left(\frac{-2(x-1)^2-2}{2\sqrt{a+4}}\right)}{\sqrt{a+4}}$$

$$\Big\downarrow \textbf{217}$$

$$-\frac{\arctan\left(\frac{x-1}{\sqrt{1-\sqrt{a+4}}}\right)}{2\sqrt{1-\sqrt{a+4}}}-\frac{\arctan\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)}{2\sqrt{\sqrt{a+4}+1}}-\frac{\text{arctanh}\left(\frac{-2(x-1)^2-2}{2\sqrt{a+4}}\right)}{\sqrt{a+4}}$$

input
```
Int[x^2/(a + 8*x - 8*x^2 + 4*x^3 - x^4),x]
```

output
```
-1/2*ArcTan[(-1 + x)/Sqrt[1 - Sqrt[4 + a]]]/Sqrt[1 - Sqrt[4 + a]] - ArcTan
[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]]/(2*Sqrt[1 + Sqrt[4 + a]]) - ArcTanh[(-2 -
 2*(-1 + x)^2)/(2*Sqrt[4 + a])]/Sqrt[4 + a]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1432
```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 2006
```
Int[(u_.)*(Px_), x_Symbol] :> With[{a = Rt[Coeff[Px, x, 0], Expon[Px, x]],
b = Rt[Coeff[Px, x, Expon[Px, x]], Expon[Px, x]]}, Int[u*(a + b*x)^Expon[Px
, x], x] /; EqQ[Px, (a + b*x)^Expon[Px, x]]] /; PolyQ[Px, x] && GtQ[Expon[P
x, x], 1] && NeQ[Coeff[Px, x, 0], 0] &&  !MatchQ[Px, (a_.)*(v_)^Expon[Px, x
] /; FreeQ[a, x] && LinearQ[v, x]]
```

rule 2202
```
Int[(Pn_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{n
 = Expon[Pn, x], k}, Int[Sum[Coeff[Pn, x, 2*k]*x^(2*k), {k, 0, n/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pn, x, 2*k + 1]*x^(2*k), {k, 0, (n -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pn, x]
 &&  !PolyQ[Pn, x^2]
```

rule 2459
```
Int[(Pn_)^(p_.)*(Qx_), x_Symbol] :> With[{S = Coeff[Pn, x, Expon[Pn, x] - 1
]/(Expon[Pn, x]*Coeff[Pn, x, Expon[Pn, x]])}, Subst[Int[ExpandToSum[Pn /. x
 -> x - S, x]^p*ExpandToSum[Qx /. x -> x - S, x], x], x, x + S] /; Binomial
Q[Pn /. x -> x - S, x] || (IntegerQ[Expon[Pn, x]/2] && TrinomialQ[Pn /. x -
> x - S, x])] /; FreeQ[p, x] && PolyQ[Pn, x] && GtQ[Expon[Pn, x], 2] && NeQ
[Coeff[Pn, x, Expon[Pn, x] - 1], 0] && PolyQ[Qx, x] && !(MonomialQ[Qx, x]
&& IGtQ[p, 0])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.52

method	result	size
default	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\_Z^4-4\_Z^3+8\_Z^2-8\_Z-a\right)}\frac{\_R^2\ln\left(x-\_R\right)}{-\_R^3+3\_R^2-4\_R+2}\right)}{4}$	54
risch	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\_Z^4-4\_Z^3+8\_Z^2-8\_Z-a\right)}\frac{\_R^2\ln\left(x-\_R\right)}{-\_R^3+3\_R^2-4\_R+2}\right)}{4}$	54

input
```
int(x^2/(-x^4+4*x^3-8*x^2+a+8*x),x,method=_RETURNVERBOSE)
```

output
```
1/4*sum(_R^2/(-_R^3+3*_R^2-4*_R+2)*ln(x-_R),_R=RootOf(_Z^4-4*_Z^3+8*_Z^2-8
*_Z-a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 18.26 (sec) , antiderivative size = 1515766, normalized size of antiderivative = 14716.17

$$\int \frac{x^2}{a+8x-8x^2+4x^3-x^4}\,dx = \text{Too large to display}$$

input
```
integrate(x^2/(-x^4+4*x^3-8*x^2+a+8*x),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 172 vs. $2(82) = 164$.

Time = 4.21 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.67

$$\int \frac{x^2}{a+8x-8x^2+4x^3-x^4}\,dx = -\operatorname{RootSum}\Big(t^4\cdot\left(256a^3+2816a^2+10240a+12288\right)+t^2\left(-160a^2-1152a-2048\right)+t\left(-32a^2-256\right.$$

input
```
integrate(x**2/(-x**4+4*x**3-8*x**2+a+8*x),x)
```

output
```
-RootSum(_t**4*(256*a**3 + 2816*a**2 + 10240*a + 12288) + _t**2*(-160*a**2
 - 1152*a - 2048) + _t*(-32*a**2 - 256*a - 512) - a**2, Lambda(_t, _t*log(
x + (-64*_t**3*a**4 - 448*_t**3*a**3 - 256*_t**3*a**2 + 3584*_t**3*a + 614
4*_t**3 - 224*_t**2*a**3 - 2208*_t**2*a**2 - 7168*_t**2*a - 7680*_t**2 + 5
6*_t*a**3 + 400*_t*a**2 + 864*_t*a + 512*_t + 5*a**3 + 34*a**2 + 56*a)/(a*
*3 + 60*a**2 + 320*a + 448)))
```

Maxima [F]

$$\int \frac{x^2}{a+8x-8x^2+4x^3-x^4}\,dx = \int -\frac{x^2}{x^4-4\,x^3+8\,x^2-a-8\,x}\,dx$$

input
```
integrate(x^2/(-x^4+4*x^3-8*x^2+a+8*x),x, algorithm="maxima")
```

output
```
-integrate(x^2/(x^4 - 4*x^3 + 8*x^2 - a - 8*x), x)
```

Giac [F]

$$\int \frac{x^2}{a+8x-8x^2+4x^3-x^4}\,dx = \int -\frac{x^2}{x^4-4\,x^3+8\,x^2-a-8\,x}\,dx$$

input
```
integrate(x^2/(-x^4+4*x^3-8*x^2+a+8*x),x, algorithm="giac")
```

output
```
integrate(-x^2/(x^4 - 4*x^3 + 8*x^2 - a - 8*x), x)
```

Mupad [B] (verification not implemented)

Time = 22.43 (sec) , antiderivative size = 878, normalized size of antiderivative = 8.52

$$\int \frac{x^2}{a+8x-8x^2+4x^3-x^4}\,dx = \sum_{k=1}^{4}\ln\Big(64\,\mathrm{root}\big(2816\,a^2\,z^4+256\,a^3\,z^4+10240\,a\,z^4 +12288\,z^4-160\,a^2\,z^2-1152\,a\,z^2-2048\,z^2+32\,a^2\,z+256\,a\,z+512\,z-a^2,z,k\big) -a-8\,x+\mathrm{root}\big(2816\,a^2\,z^4+256\,a^3\,z^4+10240\,a\,z^4+12288\,z^4-160\,a^2\,z^2 -1152\,a\,z^2-2048\,z^2+32\,a^2\,z+256\,a\,z+512\,z-a^2,z,k\big)\;a\,20$$
$$-\mathrm{root}\big(2816\,a^2\,z^4+256\,a^3\,z^4+10240\,a\,z^4+12288\,z^4-160\,a^2\,z^2-1152\,a\,z^2-2048\,z^2+32\,a^2\,z+256$$
$$+\mathrm{root}\big(2816\,a^2\,z^4+256\,a^3\,z^4+10240\,a\,z^4+12288\,z^4-160\,a^2\,z^2-1152\,a\,z^2-2048\,z^2+32\,a^2\,z+256$$
$$+\mathrm{root}\big(2816\,a^2\,z^4+256\,a^3\,z^4+10240\,a\,z^4+12288\,z^4-160\,a^2\,z^2-1152\,a\,z^2-2048\,z^2+32\,a^2\,z+256$$
$$-\mathrm{root}\big(2816\,a^2\,z^4+256\,a^3\,z^4+10240\,a\,z^4+12288\,z^4-160\,a^2\,z^2-1152\,a\,z^2-2048\,z^2+32\,a^2\,z+256$$
$$-192\,\mathrm{root}\big(2816\,a^2\,z^4+256\,a^3\,z^4+10240\,a\,z^4+12288\,z^4-160\,a^2\,z^2-1152\,a\,z^2-2048\,z^2+32\,a^2\,z+$$
$$+256\,\mathrm{root}\big(2816\,a^2\,z^4+256\,a^3\,z^4+10240\,a\,z^4+12288\,z^4-160\,a^2\,z^2-1152\,a\,z^2-2048\,z^2+32\,a^2\,z+$$
$$-\mathrm{root}\big(2816\,a^2\,z^4+256\,a^3\,z^4+10240\,a\,z^4+12288\,z^4-160\,a^2\,z^2-1152\,a\,z^2 -2048\,z^2+32\,a^2\,z+256\,a\,z+512\,z-a^2,z,k\big)\;a\,x\,4$$
$$+\mathrm{root}\big(2816\,a^2\,z^4+256\,a^3\,z^4+10240\,a\,z^4+12288\,z^4-160\,a^2\,z^2-1152\,a\,z^2-2048\,z^2+32\,a^2\,z+256$$
$$-\mathrm{root}\big(2816\,a^2\,z^4+256\,a^3\,z^4+10240\,a\,z^4+12288\,z^4-160\,a^2\,z^2-1152\,a\,z^2-2048\,z^2+32\,a^2\,z+256$$
$$+256\,a^3\,z^4+10240\,a\,z^4+12288\,z^4-160\,a^2\,z^2-1152\,a\,z^2-2048\,z^2+32\,a^2\,z +256\,a\,z+512\,z-a^2,z,k\big)$$

input
```
int(x^2/(a + 8*x - 8*x^2 + 4*x^3 - x^4),x)
```

output
```
symsum(log(64*root(2816*a^2*z^4 + 256*a^3*z^4 + 10240*a*z^4 + 12288*z^4 -
160*a^2*z^2 - 1152*a*z^2 - 2048*z^2 + 32*a^2*z + 256*a*z + 512*z - a^2, z,
 k) - a - 8*x + 20*root(2816*a^2*z^4 + 256*a^3*z^4 + 10240*a*z^4 + 12288*z
^4 - 160*a^2*z^2 - 1152*a*z^2 - 2048*z^2 + 32*a^2*z + 256*a*z + 512*z - a^
2, z, k)*a - 48*root(2816*a^2*z^4 + 256*a^3*z^4 + 10240*a*z^4 + 12288*z^4
- 160*a^2*z^2 - 1152*a*z^2 - 2048*z^2 + 32*a^2*z + 256*a*z + 512*z - a^2,
z, k)^2*a + 64*root(2816*a^2*z^4 + 256*a^3*z^4 + 10240*a*z^4 + 12288*z^4 -
 160*a^2*z^2 - 1152*a*z^2 - 2048*z^2 + 32*a^2*z + 256*a*z + 512*z - a^2, z
, k)^3*a + 128*root(2816*a^2*z^4 + 256*a^3*z^4 + 10240*a*z^4 + 12288*z^4 -
 160*a^2*z^2 - 1152*a*z^2 - 2048*z^2 + 32*a^2*z + 256*a*z + 512*z - a^2, z
, k)^2*x - 256*root(2816*a^2*z^4 + 256*a^3*z^4 + 10240*a*z^4 + 12288*z^4 -
 160*a^2*z^2 - 1152*a*z^2 - 2048*z^2 + 32*a^2*z + 256*a*z + 512*z - a^2, z
, k)^3*x - 192*root(2816*a^2*z^4 + 256*a^3*z^4 + 10240*a*z^4 + 12288*z^4 -
 160*a^2*z^2 - 1152*a*z^2 - 2048*z^2 + 32*a^2*z + 256*a*z + 512*z - a^2, z
, k)^2 + 256*root(2816*a^2*z^4 + 256*a^3*z^4 + 10240*a*z^4 + 12288*z^4 - 1
60*a^2*z^2 - 1152*a*z^2 - 2048*z^2 + 32*a^2*z + 256*a*z + 512*z - a^2, z,
k)^3 - 4*root(2816*a^2*z^4 + 256*a^3*z^4 + 10240*a*z^4 + 12288*z^4 - 160*a
^2*z^2 - 1152*a*z^2 - 2048*z^2 + 32*a^2*z + 256*a*z + 512*z - a^2, z, k)*a
*x + 32*root(2816*a^2*z^4 + 256*a^3*z^4 + 10240*a*z^4 + 12288*z^4 - 160*a^
2*z^2 - 1152*a*z^2 - 2048*z^2 + 32*a^2*z + 256*a*z + 512*z - a^2, z, k)...
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 415, normalized size of antiderivative = 4.03

$$\int \frac{x^2}{a+8x-8x^2+4x^3-x^4}\,dx$$
$$= \frac{-2\sqrt{a+4}\,\sqrt{\sqrt{a+4}+1}\,atan\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)a-8\sqrt{a+4}\,\sqrt{\sqrt{a+4}+1}\,atan\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)+2\sqrt{\sqrt{a+4}+1}}{}$$

input
```
int(x^2/(-x^4+4*x^3-8*x^2+a+8*x),x)
```

output
```
( - 2*sqrt(a + 4)*sqrt(sqrt(a + 4) + 1)*atan((x - 1)/sqrt(sqrt(a + 4) + 1)
)*a - 8*sqrt(a + 4)*sqrt(sqrt(a + 4) + 1)*atan((x - 1)/sqrt(sqrt(a + 4) +
1)) + 2*sqrt(sqrt(a + 4) + 1)*atan((x - 1)/sqrt(sqrt(a + 4) + 1))*a + 8*sq
rt(sqrt(a + 4) + 1)*atan((x - 1)/sqrt(sqrt(a + 4) + 1)) - sqrt(a + 4)*sqrt
(sqrt(a + 4) - 1)*log(sqrt(sqrt(a + 4) - 1) - x + 1)*a - 4*sqrt(a + 4)*sqr
t(sqrt(a + 4) - 1)*log(sqrt(sqrt(a + 4) - 1) - x + 1) + sqrt(a + 4)*sqrt(s
qrt(a + 4) - 1)*log(sqrt(sqrt(a + 4) - 1) + x - 1)*a + 4*sqrt(a + 4)*sqrt(
sqrt(a + 4) - 1)*log(sqrt(sqrt(a + 4) - 1) + x - 1) - sqrt(sqrt(a + 4) - 1
)*log(sqrt(sqrt(a + 4) - 1) - x + 1)*a - 4*sqrt(sqrt(a + 4) - 1)*log(sqrt(
sqrt(a + 4) - 1) - x + 1) + sqrt(sqrt(a + 4) - 1)*log(sqrt(sqrt(a + 4) - 1
) + x - 1)*a + 4*sqrt(sqrt(a + 4) - 1)*log(sqrt(sqrt(a + 4) - 1) + x - 1)
- 2*sqrt(a + 4)*log(sqrt(sqrt(a + 4) - 1) - x + 1)*a - 6*sqrt(a + 4)*log(s
qrt(sqrt(a + 4) - 1) - x + 1) - 2*sqrt(a + 4)*log(sqrt(sqrt(a + 4) - 1) +
x - 1)*a - 6*sqrt(a + 4)*log(sqrt(sqrt(a + 4) - 1) + x - 1) + 2*sqrt(a + 4
)*log(sqrt(a + 4) + x**2 - 2*x + 2)*a + 6*sqrt(a + 4)*log(sqrt(a + 4) + x*
*2 - 2*x + 2))/(4*(a**2 + 7*a + 12))
```

3.32 $\int \frac{x^2}{(a+8x-8x^2+4x^3-x^4)^2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 231

$$\begin{aligned}\int \frac{x^2}{(a+8x-8x^2+4x^3-x^4)^2}\,dx &= \frac{1+(-1+x)^2}{2(4+a)\,(3+a-2(1-x)^2-(1-x)^4)} \\ &\quad-\frac{(2+(-1+x)^2)\,(1-x)}{4(3+a)\,(3+a-2(1-x)^2-(1-x)^4)} \\ &\quad+\frac{\left(4+a+\sqrt{4+a}\right)\arctan\left(\frac{1-x}{\sqrt{1-\sqrt{4+a}}}\right)}{8(3+a)(4+a)\sqrt{1-\sqrt{4+a}}} \\ &\quad+\frac{\left(4+a-\sqrt{4+a}\right)\arctan\left(\frac{1-x}{\sqrt{1+\sqrt{4+a}}}\right)}{8(3+a)(4+a)\sqrt{1+\sqrt{4+a}}} \\ &\quad+\frac{\operatorname{arctanh}\left(\frac{1+(-1+x)^2}{\sqrt{4+a}}\right)}{2(4+a)^{3/2}}\end{aligned}$$

output
```
1/2*(1+(-1+x)^2)/(4+a)/(3+a-2*(1-x)^2-(1-x)^4)-1/4*(2+(-1+x)^2)*(1-x)/(3+a
)/(3+a-2*(1-x)^2-(1-x)^4)+1/8*(4+a+(4+a)^(1/2))*arctan((1-x)/(1-(4+a)^(1/2
))^(1/2))/(3+a)/(4+a)/(1-(4+a)^(1/2))^(1/2)+1/8*(4+a-(4+a)^(1/2))*arctan((
1-x)/(1+(4+a)^(1/2))^(1/2))/(3+a)/(4+a)/(1+(4+a)^(1/2))^(1/2)+1/2*arctanh(
(1+(-1+x)^2)/(4+a)^(1/2))/(4+a)^(3/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.13 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.79

$$\int \frac{x^2}{(a+8x-8x^2+4x^3-x^4)^2}\,dx = \frac{2x(4-3x+2x^2)+a(1+x-x^2+x^3)}{4(3+a)(4+a)\left(a-x\left(-8+8x-4x^2+x^3\right)\right)} - \frac{\text{RootSum}\left[a+8\#1-8\#1^2+4\#1^3-\#1^4\&, \frac{-a\log(x-\#1)+4\log(x-\#1)\#1+2a\log(x-\#1)\#1+4\log(x-\#1)\#}{-2+4\#1-3\#1^2+\#1^3}\right.}{16\left(12+7a+a^2\right)}$$

input
```
Integrate[x^2/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x]
```

output
```
(2*x*(4 - 3*x + 2*x^2) + a*(1 + x - x^2 + x^3))/(4*(3 + a)*(4 + a)*(a - x*
(-8 + 8*x - 4*x^2 + x^3))) - RootSum[a + 8*#1 - 8*#1^2 + 4*#1^3 - #1^4 & ,
 (-(a*Log[x - #1]) + 4*Log[x - #1]*#1 + 2*a*Log[x - #1]*#1 + 4*Log[x - #1]
*#1^2 + a*Log[x - #1]*#1^2)/(-2 + 4*#1 - 3*#1^2 + #1^3) & ]/(16*(12 + 7*a
+ a^2))
```

Rubi [A] (warning: unable to verify)

Time = 0.76 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.95, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {2459, 2006, 2202, 27, 1432, 1086, 1083, 219, 1492, 27, 1480, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a-x^4+4x^3-8x^2+8x)^2}dx$$

$$\Big\downarrow \text{ 2459}$$

$$\int \frac{(x-1)^2+2(x-1)+1}{(a-(x-1)^4-2(x-1)^2+3)^2}d(x-1)$$

$$\Big\downarrow \text{ 2006}$$

$$\int \frac{x^2}{\left(a-(x-1)^4-2(x-1)^2+3\right)^2}d(x-1)$$

$$\Big\downarrow \; 2202$$

$$\int \frac{(x-1)^2+1}{\left(-(x-1)^4-2(x-1)^2+a+3\right)^2}d(x-1)+\int \frac{2(x-1)}{\left(-(x-1)^4-2(x-1)^2+a+3\right)^2}d(x-1)$$

$$\Big\downarrow \; 27$$

$$\int \frac{(x-1)^2+1}{\left(-(x-1)^4-2(x-1)^2+a+3\right)^2}d(x-1)+2\int \frac{x-1}{\left(-(x-1)^4-2(x-1)^2+a+3\right)^2}d(x-1)$$

$$\Big\downarrow \; 1432$$

$$\int \frac{1}{\left(-(x-1)^4-2(x-1)^2+a+3\right)^2}d(x-1)^2+\int \frac{(x-1)^2+1}{\left(-(x-1)^4-2(x-1)^2+a+3\right)^2}d(x-1)$$

$$\Big\downarrow \; 1086$$

$$\int \frac{(x-1)^2+1}{\left(-(x-1)^4-2(x-1)^2+a+3\right)^2}d(x-1)+\frac{\int \frac{1}{-(x-1)^4-2(x-1)^2+a+3}d(x-1)^2}{2(a+4)}+\frac{x}{2(a+4)\left(a-(x-1)^4-2(x-1)^2+3\right)}$$

$$\Big\downarrow \; 1083$$

$$-\frac{\int \frac{1}{4(a+4)-(x-1)^4}d\left(-2(x-1)^2-2\right)}{a+4}+\int \frac{(x-1)^2+1}{\left(-(x-1)^4-2(x-1)^2+a+3\right)^2}d(x-1)+\frac{x}{2(a+4)\left(a-(x-1)^4-2(x-1)^2+3\right)}$$

$$\Big\downarrow \; 219$$

$$\int \frac{(x-1)^2+1}{\left(-(x-1)^4-2(x-1)^2+a+3\right)^2}d(x-1)-\frac{\text{arctanh}\left(\frac{-2(x-1)^2-2}{2\sqrt{a+4}}\right)}{2(a+4)^{3/2}}+\frac{x}{2(a+4)\left(a-(x-1)^4-2(x-1)^2+3\right)}$$

$$\Big\downarrow \; 1492$$

$$-\frac{\int -\frac{2(a+4)\left((x-1)^2+2\right)}{-(x-1)^4-2(x-1)^2+a+3}d(x-1)}{8\left(a^2+7a+12\right)}+\frac{(a+4)\left((x-1)^2+2\right)(x-1)}{4\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)}-\frac{\text{arctanh}\left(\frac{-2(x-1)^2-2}{2\sqrt{a+4}}\right)}{2(a+4)^{3/2}}+\frac{x}{2(a+4)\left(a-(x-1)^4-2(x-1)^2+3\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{(a+4)\int \frac{(x-1)^2+2}{-(x-1)^4-2(x-1)^2+a+3}d(x-1)}{4\left(a^2+7a+12\right)}+\frac{(a+4)\left((x-1)^2+2\right)(x-1)}{4\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)}-\frac{\text{arctanh}\left(\frac{-2(x-1)^2-2}{2\sqrt{a+4}}\right)}{2(a+4)^{3/2}}+\frac{x}{2(a+4)\left(a-(x-1)^4-2(x-1)^2+3\right)}$$

$$\Big\downarrow \text{1480}$$

$$\frac{(a+4)\left(\frac{1}{2}\left(1-\frac{1}{\sqrt{a+4}}\right)\int \frac{1}{-(x-1)^2-\sqrt{a+4}-1}d(x-1)+\frac{1}{2}\left(\frac{1}{\sqrt{a+4}}+1\right)\int \frac{1}{-(x-1)^2+\sqrt{a+4}-1}d(x-1)\right)}{4\left(a^2+7a+12\right)}+\frac{(a+4)\left((x-1)^2+2\right)(x-1)}{4\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)}-\frac{\text{arctanh}\left(\frac{-2(x-1)^2-2}{2\sqrt{a+4}}\right)}{2(a+4)^{3/2}}+\frac{x}{2(a+4)\left(a-(x-1)^4-2(x-1)^2+3\right)}$$

$$\Big\downarrow \text{217}$$

$$\frac{(a+4)\left(-\frac{\left(\frac{1}{\sqrt{a+4}}+1\right)\arctan\left(\frac{x-1}{\sqrt{1-\sqrt{a+4}}}\right)}{2\sqrt{1-\sqrt{a+4}}}-\frac{\left(1-\frac{1}{\sqrt{a+4}}\right)\arctan\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)}{2\sqrt{\sqrt{a+4}+1}}\right)}{4\left(a^2+7a+12\right)}+\frac{(a+4)\left((x-1)^2+2\right)(x-1)}{4\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)}-\frac{\text{arctanh}\left(\frac{-2(x-1)^2-2}{2\sqrt{a+4}}\right)}{2(a+4)^{3/2}}+\frac{x}{2(a+4)\left(a-(x-1)^4-2(x-1)^2+3\right)}$$

input
```
Int[x^2/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x]
```

output
```
((4 + a)*(2 + (-1 + x)^2)*(-1 + x))/(4*(12 + 7*a + a^2)*(3 + a - 2*(-1 + x
)^2 - (-1 + x)^4)) + x/(2*(4 + a)*(3 + a - 2*(-1 + x)^2 - (-1 + x)^4)) + (
(4 + a)*(-1/2*((1 + 1/Sqrt[4 + a])*ArcTan[(-1 + x)/Sqrt[1 - Sqrt[4 + a]]])
/Sqrt[1 - Sqrt[4 + a]] - ((1 - 1/Sqrt[4 + a])*ArcTan[(-1 + x)/Sqrt[1 + Sqr
t[4 + a]]])/(2*Sqrt[1 + Sqrt[4 + a]])))/(4*(12 + 7*a + a^2)) - ArcTanh[(-2
 - 2*(-1 + x)^2)/(2*Sqrt[4 + a])]/(2*(4 + a)^(3/2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1086
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && ILtQ[p, -1]
```

rule 1432
```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1492
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2
 - 4*a*c))   Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p +
7)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 LtQ[p, -1] && IntegerQ[2*p]
```

rule 2006
```
Int[(u_.)*(Px_), x_Symbol] :> With[{a = Rt[Coeff[Px, x, 0], Expon[Px, x]],
b = Rt[Coeff[Px, x, Expon[Px, x]], Expon[Px, x]]}, Int[u*(a + b*x)^Expon[Px
, x], x] /; EqQ[Px, (a + b*x)^Expon[Px, x]]] /; PolyQ[Px, x] && GtQ[Expon[P
x, x], 1] && NeQ[Coeff[Px, x, 0], 0] &&  !MatchQ[Px, (a_.)*(v_)^Expon[Px, x
] /; FreeQ[a, x] && LinearQ[v, x]]
```

rule 2202
```
Int[(Pn_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{n
 = Expon[Pn, x], k}, Int[Sum[Coeff[Pn, x, 2*k]*x^(2*k), {k, 0, n/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pn, x, 2*k + 1]*x^(2*k), {k, 0, (n -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pn, x]
 &&  !PolyQ[Pn, x^2]
```

rule 2459
```
Int[(Pn_)^(p_.)*(Qx_), x_Symbol] :> With[{S = Coeff[Pn, x, Expon[Pn, x] - 1
]/(Expon[Pn, x]*Coeff[Pn, x, Expon[Pn, x]])}, Subst[Int[ExpandToSum[Pn /. x
 -> x - S, x]^p*ExpandToSum[Qx /. x -> x - S, x], x], x, x + S] /; Binomial
Q[Pn /. x -> x - S, x] || (IntegerQ[Expon[Pn, x]/2] && TrinomialQ[Pn /. x -
> x - S, x])] /; FreeQ[p, x] && PolyQ[Pn, x] && GtQ[Expon[Pn, x], 2] && NeQ
[Coeff[Pn, x, Expon[Pn, x] - 1], 0] && PolyQ[Qx, x] &&  !(MonomialQ[Qx, x]
&& IGtQ[p, 0])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.09 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.69

method	result
default	$\frac{\frac{x^3}{4a+12}-\frac{(6+a)x^2}{4(a+3)(a+4)}+\frac{(a+8)x}{4(a+3)(a+4)}+\frac{a}{4(a+4)(a+3)}}{-x^4+4x^3-8x^2+a+8x}+\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^4-4\_Z^3+8\_Z^2-8\_Z-a\right)}\frac{\left(\_R^2(a+4)+2(a+2)\_R-a\right)\ln\left(x-\right.}{-\_R^3+3\_R^2-4\_R+2}}{16(a+3)(a+4)}$
risch	$\frac{\frac{x^3}{4a+12}-\frac{(6+a)x^2}{4(a+3)(a+4)}+\frac{(a+8)x}{4(a+3)(a+4)}+\frac{a}{4(a+4)(a+3)}}{-x^4+4x^3-8x^2+a+8x}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^4-4\_Z^3+8\_Z^2-8\_Z-a\right)}\frac{\left(\frac{\_R^2}{a+3}+\frac{2(a+2)\_R}{(a+3)(a+4)}-\frac{a}{(a+4)(a+3}\right.}{-\_R^3+3\_R^2-4\_}\right.}{16}$

input
```
int(x^2/(-x^4+4*x^3-8*x^2+a+8*x)^2,x,method=_RETURNVERBOSE)
```

output
```
(1/4/(a+3)*x^3-1/4*(6+a)/(a+3)/(a+4)*x^2+1/4*(a+8)/(a+3)/(a+4)*x+1/4*a/(a+
4)/(a+3))/(-x^4+4*x^3-8*x^2+a+8*x)+1/16/(a+3)/(a+4)*sum((_R^2*(a+4)+2*(a+2
)*_R-a)/(-_R^3+3*_R^2-4*_R+2)*ln(x-_R),_R=RootOf(_Z^4-4*_Z^3+8*_Z^2-8*_Z-a
))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^2}{(a+8x-8x^2+4x^3-x^4)^2}\,dx=\text{Timed out}$$

input
```
integrate(x^2/(-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 561 vs. 2(185) = 370.

Time = 19.03 (sec) , antiderivative size = 561, normalized size of antiderivative = 2.43

$$\int \frac{x^2}{(a+8x-8x^2+4x^3-x^4)^2}\,dx$$
$$= \frac{-a+x^3(-a-4)+x^2(a+6)+x(-a-8)}{-4a^3-28a^2-48a+x^4\cdot(4a^2+28a+48)+x^3\,(-16a^2-112a-192)+x^2\cdot(32a^2+224a+384)+x}$$
$$+\mathrm{RootSum}\Big(t^4\cdot\big(65536a^9+2162688a^8+31653888a^7+269680640a^6+1473773568a^5+5357174784a^4$$

input
```
integrate(x**2/(-x**4+4*x**3-8*x**2+a+8*x)**2,x)
```

output
```
(-a + x**3*(-a - 4) + x**2*(a + 6) + x*(-a - 8))/(-4*a**3 - 28*a**2 - 48*a
 + x**4*(4*a**2 + 28*a + 48) + x**3*(-16*a**2 - 112*a - 192) + x**2*(32*a*
*2 + 224*a + 384) + x*(-32*a**2 - 224*a - 384)) + RootSum(_t**4*(65536*a**
9 + 2162688*a**8 + 31653888*a**7 + 269680640*a**6 + 1473773568*a**5 + 5357
174784*a**4 + 12952010752*a**3 + 20082327552*a**2 + 18119393280*a + 724775
7312) + _t**2*(-9728*a**6 - 209408*a**5 - 1878016*a**4 - 8986624*a**3 - 24
215552*a**2 - 34865152*a - 20971520) + _t*(256*a**5 + 5888*a**4 + 53248*a*
*3 + 237568*a**2 + 524288*a + 458752) - a**4 + 144*a**3 + 1024*a**2 + 1792
*a, Lambda(_t, _t*log(x + (4096*_t**3*a**12 - 61440*_t**3*a**11 - 5480448*
_t**3*a**10 - 111403008*_t**3*a**9 - 1227173888*_t**3*a**8 - 8682876928*_t
**3*a**7 - 42187440128*_t**3*a**6 - 144630284288*_t**3*a**5 - 350972280832
*_t**3*a**4 - 591750234112*_t**3*a**3 - 660716126208*_t**3*a**2 - 43984827
1872*_t**3*a - 132271570944*_t**3 - 28672*_t**2*a**10 - 993280*_t**2*a**9
- 15400960*_t**2*a**8 - 140742656*_t**2*a**7 - 839462912*_t**2*a**6 - 3414
427648*_t**2*a**5 - 9590087680*_t**2*a**4 - 18363547648*_t**2*a**3 - 22938
255360*_t**2*a**2 - 16873684992*_t**2*a - 5549064192*_t**2 - 848*_t*a**9 -
 6096*_t*a**8 + 174608*_t*a**7 + 3323792*_t*a**6 + 26276224*_t*a**5 + 1190
09280*_t*a**4 + 332017664*_t*a**3 + 566497280*_t*a**2 + 544112640*_t*a + 2
25837056*_t + 11*a**8 + 958*a**7 + 17419*a**6 + 142964*a**5 + 632632*a**4
+ 1567552*a**3 + 2049792*a**2 + 1100800*a)/(a**8 + 870*a**7 + 18289*a**...
```

Maxima [F]

$$\int \frac{x^2}{(a+8x-8x^2+4x^3-x^4)^2}\,dx = \int \frac{x^2}{(x^4-4\,x^3+8\,x^2-a-8\,x)^2}\,dx$$

input
```
integrate(x^2/(-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="maxima")
```

output
```
-1/4*((a + 4)*x^3 - (a + 6)*x^2 + (a + 8)*x + a)/((a^2 + 7*a + 12)*x^4 - 4
*(a^2 + 7*a + 12)*x^3 - a^3 + 8*(a^2 + 7*a + 12)*x^2 - 7*a^2 - 8*(a^2 + 7*
a + 12)*x - 12*a) - 1/4*integrate(((a + 4)*x^2 + 2*(a + 2)*x - a)/(x^4 - 4
*x^3 + 8*x^2 - a - 8*x), x)/(a^2 + 7*a + 12)
```

Giac [F]

$$\int \frac{x^2}{(a+8x-8x^2+4x^3-x^4)^2}\,dx = \int \frac{x^2}{(x^4-4\,x^3+8\,x^2-a-8\,x)^2}\,dx$$

input
```
integrate(x^2/(-x^4+4*x^3-8*x^2+a+8*x)^2,x, algorithm="giac")
```

output
```
integrate(x^2/(x^4 - 4*x^3 + 8*x^2 - a - 8*x)^2, x)
```

Mupad [B] (verification not implemented)

Time = 22.51 (sec) , antiderivative size = 1218, normalized size of antiderivative = 5.27

$$\int \frac{x^2}{(a+8x-8x^2+4x^3-x^4)^2}\,dx = \text{Too large to display}$$

input
```
int(x^2/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2,x)
```

output
```
symsum(log((x*(40*a + 7*a^2 + 56))/(8*(816*a + 460*a^2 + 129*a^3 + 18*a^4
+ a^5 + 576)) - (48*a + 12*a^2 - a^3)/(64*(816*a + 460*a^2 + 129*a^3 + 18*
a^4 + a^5 + 576)) - root(12952010752*a^3*z^4 + 31653888*a^7*z^4 + 2162688*
a^8*z^4 + 65536*a^9*z^4 + 18119393280*a*z^4 + 20082327552*a^2*z^4 + 147377
3568*a^5*z^4 + 5357174784*a^4*z^4 + 269680640*a^6*z^4 + 7247757312*z^4 - 2
4215552*a^2*z^2 - 8986624*a^3*z^2 - 1878016*a^4*z^2 - 209408*a^5*z^2 - 972
8*a^6*z^2 - 34865152*a*z^2 - 20971520*z^2 + 237568*a^2*z + 53248*a^3*z + 5
888*a^4*z + 256*a^5*z + 524288*a*z + 458752*z + 1792*a + 1024*a^2 + 144*a^
3 - a^4, z, k)*((28160*a + 11328*a^2 + 2064*a^3 + 144*a^4 + 26624)/(64*(81
6*a + 460*a^2 + 129*a^3 + 18*a^4 + a^5 + 576)) + root(12952010752*a^3*z^4
+ 31653888*a^7*z^4 + 2162688*a^8*z^4 + 65536*a^9*z^4 + 18119393280*a*z^4 +
 20082327552*a^2*z^4 + 1473773568*a^5*z^4 + 5357174784*a^4*z^4 + 269680640
*a^6*z^4 + 7247757312*z^4 - 24215552*a^2*z^2 - 8986624*a^3*z^2 - 1878016*a
^4*z^2 - 209408*a^5*z^2 - 9728*a^6*z^2 - 34865152*a*z^2 - 20971520*z^2 + 2
37568*a^2*z + 53248*a^3*z + 5888*a^4*z + 256*a^5*z + 524288*a*z + 458752*z
 + 1792*a + 1024*a^2 + 144*a^3 - a^4, z, k)*(root(12952010752*a^3*z^4 + 31
653888*a^7*z^4 + 2162688*a^8*z^4 + 65536*a^9*z^4 + 18119393280*a*z^4 + 200
82327552*a^2*z^4 + 1473773568*a^5*z^4 + 5357174784*a^4*z^4 + 269680640*a^6
*z^4 + 7247757312*z^4 - 24215552*a^2*z^2 - 8986624*a^3*z^2 - 1878016*a^4*z
^2 - 209408*a^5*z^2 - 9728*a^6*z^2 - 34865152*a*z^2 - 20971520*z^2 + 23...
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 3672, normalized size of antiderivative = 15.90

$$\int \frac{x^2}{(a+8x-8x^2+4x^3-x^4)^2}\,dx = \text{Too large to display}$$

input
```
int(x^2/(-x^4+4*x^3-8*x^2+a+8*x)^2,x)
```

output
```
( - 2*sqrt(a + 4)*sqrt(sqrt(a + 4) + 1)*atan((x - 1)/sqrt(sqrt(a + 4) + 1)
)*a**3 + 2*sqrt(a + 4)*sqrt(sqrt(a + 4) + 1)*atan((x - 1)/sqrt(sqrt(a + 4)
 + 1))*a**2*x**4 - 8*sqrt(a + 4)*sqrt(sqrt(a + 4) + 1)*atan((x - 1)/sqrt(s
qrt(a + 4) + 1))*a**2*x**3 + 16*sqrt(a + 4)*sqrt(sqrt(a + 4) + 1)*atan((x
- 1)/sqrt(sqrt(a + 4) + 1))*a**2*x**2 - 16*sqrt(a + 4)*sqrt(sqrt(a + 4) +
1)*atan((x - 1)/sqrt(sqrt(a + 4) + 1))*a**2*x - 18*sqrt(a + 4)*sqrt(sqrt(a
 + 4) + 1)*atan((x - 1)/sqrt(sqrt(a + 4) + 1))*a**2 + 18*sqrt(a + 4)*sqrt(
sqrt(a + 4) + 1)*atan((x - 1)/sqrt(sqrt(a + 4) + 1))*a*x**4 - 72*sqrt(a +
4)*sqrt(sqrt(a + 4) + 1)*atan((x - 1)/sqrt(sqrt(a + 4) + 1))*a*x**3 + 144*
sqrt(a + 4)*sqrt(sqrt(a + 4) + 1)*atan((x - 1)/sqrt(sqrt(a + 4) + 1))*a*x*
*2 - 144*sqrt(a + 4)*sqrt(sqrt(a + 4) + 1)*atan((x - 1)/sqrt(sqrt(a + 4) +
 1))*a*x - 40*sqrt(a + 4)*sqrt(sqrt(a + 4) + 1)*atan((x - 1)/sqrt(sqrt(a +
 4) + 1))*a + 40*sqrt(a + 4)*sqrt(sqrt(a + 4) + 1)*atan((x - 1)/sqrt(sqrt(
a + 4) + 1))*x**4 - 160*sqrt(a + 4)*sqrt(sqrt(a + 4) + 1)*atan((x - 1)/sqr
t(sqrt(a + 4) + 1))*x**3 + 320*sqrt(a + 4)*sqrt(sqrt(a + 4) + 1)*atan((x -
 1)/sqrt(sqrt(a + 4) + 1))*x**2 - 320*sqrt(a + 4)*sqrt(sqrt(a + 4) + 1)*at
an((x - 1)/sqrt(sqrt(a + 4) + 1))*x + 4*sqrt(sqrt(a + 4) + 1)*atan((x - 1)
/sqrt(sqrt(a + 4) + 1))*a**3 - 4*sqrt(sqrt(a + 4) + 1)*atan((x - 1)/sqrt(s
qrt(a + 4) + 1))*a**2*x**4 + 16*sqrt(sqrt(a + 4) + 1)*atan((x - 1)/sqrt(sq
rt(a + 4) + 1))*a**2*x**3 - 32*sqrt(sqrt(a + 4) + 1)*atan((x - 1)/sqrt(...
```

3.33 $\int x(a+8x-8x^2+4x^3-x^4)^{3/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 520

$$\begin{aligned}\int x\big(a+8x-8x^2+4x^3 &- x^4\big)^{3/2}\,dx = \frac{3}{16}(4+a)\sqrt{3+a-2(1-x)^2-(1-x)^4}\big(1+(-1+x)^2\big)\\ &+\frac{1}{8}\big(3+a-2(1-x)^2-(1-x)^4\big)^{3/2}\big(1+(-1+x)^2\big)\\ &-\frac{2}{35}\big(13+5a-3(1-x)^2\big)\sqrt{3+a-2(1-x)^2-(1-x)^4}(1-x)\\ &-\frac{1}{7}\big(3+a-2(1-x)^2-(1-x)^4\big)^{3/2}(1-x)+\frac{3}{16}(4+a)^2\arctan\left(\frac{1+(-1+x)^2}{\sqrt{3+a-2(1-x)^2-(1-x)^4}}\right)+\frac{16(7+2\end{aligned}$$

output
```
3/16*(4+a)*(3+a-2*(1-x)^2-(1-x)^4)^(1/2)*(1+(-1+x)^2)+1/8*(3+a-2*(1-x)^2-(
1-x)^4)^(3/2)*(1+(-1+x)^2)-2/35*(13+5*a-3*(1-x)^2)*(3+a-2*(1-x)^2-(1-x)^4)
^(1/2)*(1-x)-1/7*(3+a-2*(1-x)^2-(1-x)^4)^(3/2)*(1-x)+3/16*(4+a)^2*arctan((
1+(-1+x)^2)/(3+a-2*(1-x)^2-(1-x)^4)^(1/2))+16/35*(7+2*a)*(-1+(4+a)^(1/2))^
(1/2)*(1+(4+a)^(1/2))*(1+(1-x)^2/(1-(4+a)^(1/2)))^(1/2)*(1+(1-x)^2/(1+(4+a
)^(1/2)))^(1/2)*EllipticE((1-x)/(-1+(4+a)^(1/2))^(1/2),((1-(4+a)^(1/2))/(1
+(4+a)^(1/2)))^(1/2))/(3+a-2*(1-x)^2-(1-x)^4)^(1/2)-4/35*(-1+(4+a)^(1/2))^
(1/2)*(76+5*a^2+28*(4+a)^(1/2)+a*(39+8*(4+a)^(1/2)))*(1+(1-x)^2/(1-(4+a)^(
1/2)))^(1/2)*(1+(1-x)^2/(1+(4+a)^(1/2)))^(1/2)*EllipticF((1-x)/(-1+(4+a)^(
1/2))^(1/2),((1-(4+a)^(1/2))/(1+(4+a)^(1/2)))^(1/2))/(3+a-2*(1-x)^2-(1-x)^
4)^(1/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 7235 vs. $2(520) = 1040$.

Time = 17.25 (sec) , antiderivative size = 7235, normalized size of antiderivative = 13.91

$$\int x\left(a+8x-8x^2+4x^3-x^4\right)^{3/2}\,dx=\text{Result too large to show}$$

input
```
Integrate[x*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2),x]
```

output
```
Result too large to show
```

Rubi [A] (warning: unable to verify)

Time = 1.50 (sec) , antiderivative size = 635, normalized size of antiderivative = 1.22, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.615$, Rules used = {2459, 2202, 1404, 27, 1432, 1087, 1087, 1092, 217, 1490, 27, 1514, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a-x^4+4x^3-8x^2+8x\right)^{3/2}dx$$

$$\Big\downarrow \; 2459$$

$$\int x\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}d(x-1)$$

$$\Big\downarrow \; 2202$$

$$\int \left(-(x-1)^4-2(x-1)^2+a+3\right)^{3/2}d(x-1)+\int \left(-(x-1)^4-2(x-1)^2+a+3\right)^{3/2}(x-1)d(x-1)$$

$$\Big\downarrow \; 1404$$

$$\frac{3}{7}\int 2\left(-(x-1)^2+a+3\right)\sqrt{-(x-1)^4-2(x-1)^2+a+3}d(x-1)+\int \left(-(x-1)^4-2(x-1)^2+a+3\right)^{3/2}(x-1)d(x-1)+\frac{1}{7}(x-1)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}$$

$$\Big\downarrow \; 27$$

$$\frac{6}{7}\int \left(-(x-1)^2+a+3\right)\sqrt{-(x-1)^4-2(x-1)^2+a+3}d(x-1)+\int \left(-(x-1)^4-2(x-1)^2+a+3\right)^{3/2}(x-1)d(x-1)+\frac{1}{7}(x-1)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}$$

$$\Big\downarrow \; 1432$$

$$\frac{6}{7}\int \left(-(x-1)^2+a+3\right)\sqrt{-(x-1)^4-2(x-1)^2+a+3}d(x-1)+\frac{1}{2}\int \left(-(x-1)^4-2(x-1)^2+a+3\right)^{3/2}d(x-1)^2+\frac{1}{7}(x-1)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}$$

$$\Big\downarrow \; 1087$$

$$\frac{1}{2}\left(\frac{3}{4}(a+4)\int \sqrt{-(x-1)^4-2(x-1)^2+a+3}d(x-1)^2+\frac{1}{4}x\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}\right)+\frac{6}{7}\int \left(-(x-1)^2+a+3\right)\sqrt{-(x-1)^4-2(x-1)^2+a+3}d(x-1)+\frac{1}{7}(x-1)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}$$

$$\Big\downarrow \; 1087$$

$$\frac{1}{2}\left(\frac{3}{4}(a+4)\left(\frac{1}{2}(a+4)\int \frac{1}{\sqrt{-(x-1)^4-2(x-1)^2+a+3}}d(x-1)^2+\frac{1}{2}x\sqrt{a-(x-1)^4-2(x-1)^2+3}\right)+\right.$$
$$\frac{6}{7}\int \left(-(x-1)^2+a+3\right)\sqrt{-(x-1)^4-2(x-1)^2+a+3}d(x-1)+\frac{1}{7}(x-1)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}$$

$$\Big\downarrow \ 1092$$

$$\frac{1}{2}\left(\frac{3}{4}(a+4)\left((a+4)\int \frac{1}{-(x-1)^4-4}d\left(-\frac{2x}{\sqrt{-(x-1)^4-2(x-1)^2+a+3}}\right)+\frac{1}{2}x\sqrt{a-(x-1)^4-2(x-1)^2}\right.\right.$$
$$\frac{6}{7}\int \left(-(x-1)^2+a+3\right)\sqrt{-(x-1)^4-2(x-1)^2+a+3}d(x-1)+\frac{1}{7}(x-1)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}$$

$$\Big\downarrow \ 217$$

$$\frac{6}{7}\int \left(-(x-1)^2+a+3\right)\sqrt{-(x-1)^4-2(x-1)^2+a+3}d(x-1)+$$
$$\frac{1}{2}\left(\frac{3}{4}(a+4)\left(\frac{1}{2}(a+4)\arctan\left(\frac{x}{\sqrt{a-(x-1)^4-2(x-1)^2+3}}\right)+\frac{1}{2}x\sqrt{a-(x-1)^4-2(x-1)^2+3}\right)+\frac{1}{4}x(\right.$$
$$\frac{1}{7}(x-1)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}$$

$$\Big\downarrow \ 1490$$

$$\frac{6}{7}\left(\frac{1}{15}(x-1)\left(5a-3(x-1)^2+13\right)\sqrt{a-(x-1)^4-2(x-1)^2+3}-\frac{1}{15}\int -\frac{2\left((a+3)(5a+16)-4(2a+7)(x\right.}{\sqrt{-(x-1)^4-2(x-1)^2+a}}\right.$$
$$\frac{1}{2}\left(\frac{3}{4}(a+4)\left(\frac{1}{2}(a+4)\arctan\left(\frac{x}{\sqrt{a-(x-1)^4-2(x-1)^2+3}}\right)+\frac{1}{2}x\sqrt{a-(x-1)^4-2(x-1)^2+3}\right)+\frac{1}{4}x(\right.$$
$$\frac{1}{7}(x-1)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}$$

$$\Big\downarrow \ 27$$

$$\frac{6}{7}\left(\frac{2}{15}\int \frac{(a+3)(5a+16)-4(2a+7)(x-1)^2}{\sqrt{-(x-1)^4-2(x-1)^2+a+3}}d(x-1)+\frac{1}{15}(x-1)\left(5a-3(x-1)^2+13\right)\sqrt{a-(x-1)^4-2(x\right.$$
$$\frac{1}{2}\left(\frac{3}{4}(a+4)\left(\frac{1}{2}(a+4)\arctan\left(\frac{x}{\sqrt{a-(x-1)^4-2(x-1)^2+3}}\right)+\frac{1}{2}x\sqrt{a-(x-1)^4-2(x-1)^2+3}\right)+\frac{1}{4}x(\right.$$
$$\frac{1}{7}(x-1)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}$$

$$\Big\downarrow \ 1514$$

$$\frac{6}{7}\left(\frac{2\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}\int \frac{(a+3)(5a+16)-4(2a+7)(x-1)^2}{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}d(x-1)}{15\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{1}{15}(x-1)\left(5a-3(x-1)^2+13\right)\sqrt{a-\right.$$

$$\frac{1}{2}\left(\frac{3}{4}(a+4)\left(\frac{1}{2}(a+4)\arctan\left(\frac{x}{\sqrt{a-(x-1)^4-2(x-1)^2+3}}\right)+\frac{1}{2}x\sqrt{a-(x-1)^4-2(x-1)^2+3}\right)+\frac{1}{4}x(\right.$$

$$\frac{1}{7}(x-1)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}$$

$\downarrow$ 406

$$\frac{6}{7}\left(\frac{2\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}\left((a+3)(5a+16)\int \frac{1}{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}d(x-1)-4(2a+7)\int \frac{(x-1}{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{}}\right.}{15\sqrt{a-(x-1)^4-2(x-1)^2+3}}\right.$$

$$\frac{1}{2}\left(\frac{3}{4}(a+4)\left(\frac{1}{2}(a+4)\arctan\left(\frac{x}{\sqrt{a-(x-1)^4-2(x-1)^2+3}}\right)+\frac{1}{2}x\sqrt{a-(x-1)^4-2(x-1)^2+3}\right)+\frac{1}{4}x(\right.$$

$$\frac{1}{7}(x-1)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}$$

$\downarrow$ 320

$$\frac{6}{7}\left(\frac{2\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}\left(\frac{(a+3)(5a+16)\sqrt{\sqrt{a+4}+1}\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\,\text{EllipticF}\left(\arctan\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right),-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}-4(2a+\right.}{15\sqrt{a-(x-1)^4-2(x-1)^2+3}}\right.$$

$$\frac{1}{2}\left(\frac{3}{4}(a+4)\left(\frac{1}{2}(a+4)\arctan\left(\frac{x}{\sqrt{a-(x-1)^4-2(x-1)^2+3}}\right)+\frac{1}{2}x\sqrt{a-(x-1)^4-2(x-1)^2+3}\right)+\frac{1}{4}x(\right.$$

$$\frac{1}{7}(x-1)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}$$

$\downarrow$ 388

$$\frac{6}{7}\left(\frac{2\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}\left(\frac{(a+3)(5a+16)\sqrt{\sqrt{a+4}+1}\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\,\text{EllipticF}\left(\arctan\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right),-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}-4(2a+\right.}{15\sqrt{a-(x-1)^4-2(x-1)^2+3}}\right.$$

$$\frac{1}{2}\left(\frac{3}{4}(a+4)\left(\frac{1}{2}(a+4)\arctan\left(\frac{x}{\sqrt{a-(x-1)^4-2(x-1)^2+3}}\right)+\frac{1}{2}x\sqrt{a-(x-1)^4-2(x-1)^2+3}\right)+\frac{1}{4}x(\right.$$

$$\frac{1}{7}(x-1)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}$$

$\Big\downarrow$ 313

$$\frac{1}{2}\left(\frac{3}{4}(a+4)\left(\frac{1}{2}(a+4)\arctan\left(\frac{x}{\sqrt{a-(x-1)^4-2(x-1)^2+3}}\right)+\frac{1}{2}x\sqrt{a-(x-1)^4-2(x-1)^2+3}\right)+\frac{1}{4}x(\right.$$

$$\frac{6}{7}\left(\frac{2\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}\left(\frac{(a+3)(5a+16)\sqrt{\sqrt{a+4}+1}\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\,\text{EllipticF}\left(\arctan\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right),-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}-4(2a+\right.}{15\sqrt{a-(x-1)^4-2(x}}\right.$$

$$\frac{1}{7}(x-1)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}$$

input
```
Int[x*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2),x]
```

output
```
((3 + a - 2*(-1 + x)^2 - (-1 + x)^4)^(3/2)*(-1 + x))/7 + (((3 + a - 2*(-1
+ x)^2 - (-1 + x)^4)^(3/2)*x)/4 + (3*(4 + a)*((Sqrt[3 + a - 2*(-1 + x)^2 -
 (-1 + x)^4]*x)/2 + ((4 + a)*ArcTan[x/Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)
^4]])/2))/4)/2 + (6*(((13 + 5*a - 3*(-1 + x)^2)*Sqrt[3 + a - 2*(-1 + x)^2
- (-1 + x)^4]*(-1 + x))/15 + (2*Sqrt[1 + (-1 + x)^2/(1 - Sqrt[4 + a])]*Sqr
t[1 + (-1 + x)^2/(1 + Sqrt[4 + a])]*(-4*(7 + 2*a)*(((1 - Sqrt[4 + a])*Sqrt
[1 + (-1 + x)^2/(1 - Sqrt[4 + a])]*(-1 + x))/Sqrt[1 + (-1 + x)^2/(1 + Sqrt
[4 + a])] - ((1 - Sqrt[4 + a])*Sqrt[1 + Sqrt[4 + a]]*Sqrt[1 + (-1 + x)^2/(
1 - Sqrt[4 + a])]*EllipticE[ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sq
rt[4 + a])/(1 - Sqrt[4 + a])])/(Sqrt[(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1
 + (-1 + x)^2/(1 + Sqrt[4 + a]))]*Sqrt[1 + (-1 + x)^2/(1 + Sqrt[4 + a])]))
 + ((3 + a)*(16 + 5*a)*Sqrt[1 + Sqrt[4 + a]]*Sqrt[1 + (-1 + x)^2/(1 - Sqrt
[4 + a])]*EllipticF[ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a
])/(1 - Sqrt[4 + a])])/(Sqrt[(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (-1 +
 x)^2/(1 + Sqrt[4 + a]))]*Sqrt[1 + (-1 + x)^2/(1 + Sqrt[4 + a])])))/(15*Sq
rt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4])))/7
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1404
```
Int[((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*((a + b
*x^2 + c*x^4)^p/(4*p + 1)), x] + Simp[2*(p/(4*p + 1))   Int[(2*a + b*x^2)*(
a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*
c, 0] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 1432
```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

rule 1490
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Simp[2*(p/(c*(4*p + 1)*(4*p + 3)))
 Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3)
- b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

rule 1514
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[Sqrt[1 + 2*c*(x^2/(b - q))]*(Sqrt
[1 + 2*c*(x^2/(b + q))]/Sqrt[a + b*x^2 + c*x^4])   Int[(d + e*x^2)/(Sqrt[1
+ 2*c*(x^2/(b - q))]*Sqrt[1 + 2*c*(x^2/(b + q))]), x], x]] /; FreeQ[{a, b,
c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

rule 2202
```
Int[(Pn_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{n
 = Expon[Pn, x], k}, Int[Sum[Coeff[Pn, x, 2*k]*x^(2*k), {k, 0, n/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pn, x, 2*k + 1]*x^(2*k), {k, 0, (n -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pn, x]
 &&  !PolyQ[Pn, x^2]
```

rule 2459
```
Int[(Pn_)^(p_.)*(Qx_), x_Symbol] :> With[{S = Coeff[Pn, x, Expon[Pn, x] - 1
]/(Expon[Pn, x]*Coeff[Pn, x, Expon[Pn, x]])}, Subst[Int[ExpandToSum[Pn /. x
 -> x - S, x]^p*ExpandToSum[Qx /. x -> x - S, x], x], x, x + S] /; Binomial
Q[Pn /. x -> x - S, x] || (IntegerQ[Expon[Pn, x]/2] && TrinomialQ[Pn /. x -
> x - S, x])] /; FreeQ[p, x] && PolyQ[Pn, x] && GtQ[Expon[Pn, x], 2] && NeQ
[Coeff[Pn, x, Expon[Pn, x] - 1], 0] && PolyQ[Qx, x] &&  !(MonomialQ[Qx, x]
&& IGtQ[p, 0])
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 2693 vs. $2(448) = 896$.

Time = 7.90 (sec) , antiderivative size = 2694, normalized size of antiderivative = 5.18

method	result	size
default	Expression too large to display	2694
elliptic	Expression too large to display	2694
risch	Expression too large to display	3609

input
```
int(x*(-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8*x^6*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)+17/28*x^5*(-x^4+4*x^3-8*x^2+a+8*x)
^(1/2)-43/28*x^4*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)+74/35*x^3*(-x^4+4*x^3-8*x^
2+a+8*x)^(1/2)+(5/16*a-9/20)*x^2*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)+(-11/56*a-
29/70)*x*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)+(11/56*a+13/14)*(-x^4+4*x^3-8*x^2+
a+8*x)^(1/2)-(-(-11/56*a-29/70)*a-11/14*a-26/7)*((-1-(a+4)^(1/2))^(1/2)+(-
1+(a+4)^(1/2))^(1/2))*((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x
-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2)
)/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)*(x-1+(-1+(a+4)^(1/2))^(1/2))^2*(-2*(
-1+(a+4)^(1/2))^(1/2)*(x-1-(-1-(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)
-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)*(-2*(-1+(a+4)
^(1/2))^(1/2)*(x-1+(-1-(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a
+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)/(-(-1-(a+4)^(1/2))^(
1/2)+(-1+(a+4)^(1/2))^(1/2))/(-1+(a+4)^(1/2))^(1/2)/(-(x-1-(-1+(a+4)^(1/2)
)^(1/2))*(x-1+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1-(a+4)^(1/2))^(1/2))*(x-1+(-
1-(a+4)^(1/2))^(1/2)))^(1/2)*EllipticF(((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)
^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(
a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2),((-(-1-(a+4)^(1/2))
^(1/2)-(-1+(a+4)^(1/2))^(1/2))*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1
/2))/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1
/2)-(-1+(a+4)^(1/2))^(1/2)))^(1/2))-(a^2-2*(5/16*a-9/20)*a+55/14*a+62/5...
```

Fricas [F]

$$\int x\left(a+8x-8x^2+4x^3-x^4\right)^{3/2}\,dx=\int\left(-x^4+4\,x^3-8\,x^2+a+8\,x\right)^{\frac{3}{2}}x\,dx$$

input
```
integrate(x*(-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x, algorithm="fricas")
```

output
```
integral(-(x^5 - 4*x^4 + 8*x^3 - a*x - 8*x^2)*sqrt(-x^4 + 4*x^3 - 8*x^2 +
a + 8*x), x)
```

Sympy [F]

$$\int x\left(a+8x-8x^2+4x^3-x^4\right)^{3/2}\,dx=\int x\left(a-x^4+4x^3-8x^2+8x\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(x*(-x**4+4*x**3-8*x**2+a+8*x)**(3/2),x)
```

output
```
Integral(x*(a - x**4 + 4*x**3 - 8*x**2 + 8*x)**(3/2), x)
```

Maxima [F]

$$\int x\left(a+8x-8x^2+4x^3-x^4\right)^{3/2}\,dx=\int\left(-x^4+4\,x^3-8\,x^2+a+8\,x\right)^{\frac{3}{2}}x\,dx$$

input
```
integrate(x*(-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(3/2)*x, x)
```

Giac [F]

$$\int x\left(a+8x-8x^2+4x^3-x^4\right)^{3/2}\,dx=\int\left(-x^4+4\,x^3-8\,x^2+a+8\,x\right)^{\frac{3}{2}}x\,dx$$

input
```
integrate(x*(-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x, algorithm="giac")
```

output
```
integrate((-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(3/2)*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\left(a+8x-8x^2+4x^3-x^4\right)^{3/2}\,dx=\int x\,\left(-x^4+4\,x^3-8\,x^2+8\,x+a\right)^{3/2}\,dx$$

input
```
int(x*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2),x)
```

output
```
int(x*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2), x)
```

Reduce [F]

$$\begin{aligned}\int x\left(a+8x-8x^2+4x^3-x^4\right)^{3/2}\,dx=&\ \frac{5\sqrt{-x^4+4x^3-8x^2+a+8x}\,a\,x^2}{16}\\&-\frac{11\sqrt{-x^4+4x^3-8x^2+a+8x}\,ax}{56}\\&+\frac{37\sqrt{-x^4+4x^3-8x^2+a+8x}\,a}{840}-\frac{\sqrt{-x^4+4x^3-8x^2+a+8x}\,x^6}{8}\\&+\frac{17\sqrt{-x^4+4x^3-8x^2+a+8x}\,x^5}{28}-\frac{43\sqrt{-x^4+4x^3-8x^2+a+8x}\,x^4}{28}\\&+\frac{74\sqrt{-x^4+4x^3-8x^2+a+8x}\,x^3}{35}-\frac{9\sqrt{-x^4+4x^3-8x^2+a+8x}\,x^2}{20}\\&-\frac{29\sqrt{-x^4+4x^3-8x^2+a+8x}\,x}{70}+\frac{83\sqrt{-x^4+4x^3-8x^2+a+8x}}{210}\\&+\frac{11\left(\int\frac{\sqrt{-x^4+4x^3-8x^2+a+8x}}{-x^4+4x^3-8x^2+a+8x}dx\right)a^2}{56}+\frac{5\left(\int\frac{\sqrt{-x^4+4x^3-8x^2+a+8x}}{-x^4+4x^3-8x^2+a+8x}dx\right)a}{21}\\&-\frac{166\left(\int\frac{\sqrt{-x^4+4x^3-8x^2+a+8x}}{-x^4+4x^3-8x^2+a+8x}dx\right)}{105}-\frac{32\left(\int\frac{\sqrt{-x^4+4x^3-8x^2+a+8x}\,x^3}{-x^4+4x^3-8x^2+a+8x}dx\right)a}{105}\\&-\frac{16\left(\int\frac{\sqrt{-x^4+4x^3-8x^2+a+8x}\,x^3}{-x^4+4x^3-8x^2+a+8x}dx\right)}{15}+\frac{3\left(\int\frac{\sqrt{-x^4+4x^3-8x^2+a+8x}\,x}{-x^4+4x^3-8x^2+a+8x}dx\right)a^2}{8}\\&+\frac{379\left(\int\frac{\sqrt{-x^4+4x^3-8x^2+a+8x}\,x}{-x^4+4x^3-8x^2+a+8x}dx\right)a}{105}+\frac{122\left(\int\frac{\sqrt{-x^4+4x^3-8x^2+a+8x}\,x}{-x^4+4x^3-8x^2+a+8x}dx\right)}{15}\end{aligned}$$

input
```
int(x*(-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x)
```

output
```
(525*sqrt(a - x**4 + 4*x**3 - 8*x**2 + 8*x)*a*x**2 - 330*sqrt(a - x**4 + 4
*x**3 - 8*x**2 + 8*x)*a*x + 74*sqrt(a - x**4 + 4*x**3 - 8*x**2 + 8*x)*a -
210*sqrt(a - x**4 + 4*x**3 - 8*x**2 + 8*x)*x**6 + 1020*sqrt(a - x**4 + 4*x
**3 - 8*x**2 + 8*x)*x**5 - 2580*sqrt(a - x**4 + 4*x**3 - 8*x**2 + 8*x)*x**
4 + 3552*sqrt(a - x**4 + 4*x**3 - 8*x**2 + 8*x)*x**3 - 756*sqrt(a - x**4 +
 4*x**3 - 8*x**2 + 8*x)*x**2 - 696*sqrt(a - x**4 + 4*x**3 - 8*x**2 + 8*x)*
x + 664*sqrt(a - x**4 + 4*x**3 - 8*x**2 + 8*x) + 330*int(sqrt(a - x**4 + 4
*x**3 - 8*x**2 + 8*x)/(a - x**4 + 4*x**3 - 8*x**2 + 8*x),x)*a**2 + 400*int
(sqrt(a - x**4 + 4*x**3 - 8*x**2 + 8*x)/(a - x**4 + 4*x**3 - 8*x**2 + 8*x)
,x)*a - 2656*int(sqrt(a - x**4 + 4*x**3 - 8*x**2 + 8*x)/(a - x**4 + 4*x**3
 - 8*x**2 + 8*x),x) - 512*int((sqrt(a - x**4 + 4*x**3 - 8*x**2 + 8*x)*x**3
)/(a - x**4 + 4*x**3 - 8*x**2 + 8*x),x)*a - 1792*int((sqrt(a - x**4 + 4*x*
*3 - 8*x**2 + 8*x)*x**3)/(a - x**4 + 4*x**3 - 8*x**2 + 8*x),x) + 630*int((
sqrt(a - x**4 + 4*x**3 - 8*x**2 + 8*x)*x)/(a - x**4 + 4*x**3 - 8*x**2 + 8*
x),x)*a**2 + 6064*int((sqrt(a - x**4 + 4*x**3 - 8*x**2 + 8*x)*x)/(a - x**4
 + 4*x**3 - 8*x**2 + 8*x),x)*a + 13664*int((sqrt(a - x**4 + 4*x**3 - 8*x**
2 + 8*x)*x)/(a - x**4 + 4*x**3 - 8*x**2 + 8*x),x))/1680
```

3.34 $\int x\sqrt{a+8x-8x^2+4x^3-x^4}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 407

$$\int x\sqrt{a+8x-8x^2+4x^3-x^4}\,dx = \frac{1}{4}\sqrt{3+a-2(1-x)^2-(1-x)^4}\left(1+(-1+x)^2\right) - \frac{1}{3}\sqrt{3+a-2(1-x)^2-(1-x)^4}(1-x) + \frac{1}{4}(4+a)\arctan\left(\frac{1+(-1+x)^2}{\sqrt{3+a-2(1-x)^2-(1-x)^4}}\right) + \frac{2\sqrt{-1+\sqrt{4+a}}\left(1+\sqrt{4+a}\right)\sqrt{1+\frac{(1-x)^2}{1-\sqrt{4+a}}}\sqrt{1+\frac{(1-x)^2}{1+\sqrt{4+a}}}E\left(\arcsin\left(\frac{1-x}{\sqrt{-1+\sqrt{4+a}}}\right)|\frac{1-\sqrt{4+a}}{1+\sqrt{4+a}}\right)}{3\sqrt{3+a-2(1-x)^2-(1-x)^4}} - \frac{2\sqrt{-1+\sqrt{4+a}}\left(4+a+\sqrt{4+a}\right)\sqrt{1+\frac{(1-x)^2}{1-\sqrt{4+a}}}\sqrt{1+\frac{(1-x)^2}{1+\sqrt{4+a}}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1-x}{\sqrt{-1+\sqrt{4+a}}}\right),\frac{1-\sqrt{4+a}}{1+\sqrt{4+a}}\right)}{3\sqrt{3+a-2(1-x)^2-(1-x)^4}}$$

output
```
1/4*(3+a-2*(1-x)^2-(1-x)^4)^(1/2)*(1+(-1+x)^2)-1/3*(3+a-2*(1-x)^2-(1-x)^4)
^(1/2)*(1-x)+1/4*(4+a)*arctan((1+(-1+x)^2)/(3+a-2*(1-x)^2-(1-x)^4)^(1/2))+
2/3*(-1+(4+a)^(1/2))^(1/2)*(1+(4+a)^(1/2))*(1+(1-x)^2/(1-(4+a)^(1/2)))^(1/
2)*(1+(1-x)^2/(1+(4+a)^(1/2)))^(1/2)*EllipticE((1-x)/(-1+(4+a)^(1/2))^(1/2
),((1-(4+a)^(1/2))/(1+(4+a)^(1/2)))^(1/2))/(3+a-2*(1-x)^2-(1-x)^4)^(1/2)-2
/3*(-1+(4+a)^(1/2))^(1/2)*(4+a+(4+a)^(1/2))*(1+(1-x)^2/(1-(4+a)^(1/2)))^(1
/2)*(1+(1-x)^2/(1+(4+a)^(1/2)))^(1/2)*EllipticF((1-x)/(-1+(4+a)^(1/2))^(1/
2),((1-(4+a)^(1/2))/(1+(4+a)^(1/2)))^(1/2))/(3+a-2*(1-x)^2-(1-x)^4)^(1/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 4389 vs. $2(407) = 814$.

Time = 16.41 (sec) , antiderivative size = 4389, normalized size of antiderivative = 10.78

$$\int x\sqrt{a+8x-8x^2+4x^3-x^4}\,dx = \text{Result too large to show}$$

input

```
Integrate[x*Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4],x]
```

output

```
(1/6 - x/6 + x^2/4)*Sqrt[a - x*(-8 + 8*x - 4*x^2 + x^3)] + (Sqrt[a - x*(-8
 + 8*x - 4*x^2 + x^3)]*((-8*(-Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 +
a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x)^2*Sqrt[((-Sqrt[-1 - Sqrt[4 + a]] +
Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqr
t[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]*Sq
rt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 - Sqrt[-1 + Sqrt[4 + a]] + x))/((Sqrt[-1 -
Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]
*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 + Sqrt[-1 + Sqrt[4 + a]] + x))/((Sqrt[-1
 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x
))]*EllipticF[ArcSin[Sqrt[((-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a
]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1
+ Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]], ((-Sqrt[-1 - Sqrt[4
+ a]] - Sqrt[-1 + Sqrt[4 + a]])*(Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4
 + a]]))/((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-Sqrt[-1 - Sq
rt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]]))])/(Sqrt[-1 - Sqrt[4 + a]]*(-Sqrt[-1
- Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^
4]) + (2*a*(-Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-
1 - Sqrt[4 + a]] + x)^2*Sqrt[((-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4
+ a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[
-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]*Sqrt[(Sqrt[-1 - ...
```

Rubi [A] (warning: unable to verify)

Time = 1.23 (sec) , antiderivative size = 543, normalized size of antiderivative = 1.33, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {2459, 2202, 1404, 27, 1432, 1087, 1092, 217, 1514, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{a-x^4+4x^3-8x^2+8x}\,dx$$

$$\Big\downarrow \text{ 2459}$$

$$\int x\sqrt{a-(x-1)^4-2(x-1)^2+3}d(x-1)$$

$$\Big\downarrow \text{ 2202}$$

$$\int \sqrt{-(x-1)^4-2(x-1)^2+a+3}d(x-1)+\int \sqrt{-(x-1)^4-2(x-1)^2+a+3}(x-1)d(x-1)$$

$$\Big\downarrow \text{ 1404}$$

$$\frac{1}{3}\int \frac{2\left(-(x-1)^2+a+3\right)}{\sqrt{-(x-1)^4-2(x-1)^2+a+3}}d(x-1)+\int \sqrt{-(x-1)^4-2(x-1)^2+a+3}(x-1)d(x-1)+\frac{1}{3}(x-1)\sqrt{a-(x-1)^4-2(x-1)^2+3}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{2}{3}\int \frac{-(x-1)^2+a+3}{\sqrt{-(x-1)^4-2(x-1)^2+a+3}}d(x-1)+\int \sqrt{-(x-1)^4-2(x-1)^2+a+3}(x-1)d(x-1)+\frac{1}{3}(x-1)\sqrt{a-(x-1)^4-2(x-1)^2+3}$$

$$\Big\downarrow \text{ 1432}$$

$$\frac{2}{3}\int \frac{-(x-1)^2+a+3}{\sqrt{-(x-1)^4-2(x-1)^2+a+3}}d(x-1)+\frac{1}{2}\int \sqrt{-(x-1)^4-2(x-1)^2+a+3}d(x-1)^2+\frac{1}{3}(x-1)\sqrt{a-(x-1)^4-2(x-1)^2+3}$$

$$\Big\downarrow \text{ 1087}$$

$$\frac{1}{2}\left(\frac{1}{2}(a+4)\int\frac{1}{\sqrt{-(x-1)^4-2(x-1)^2+a+3}}d(x-1)^2+\frac{1}{2}x\sqrt{a-(x-1)^4-2(x-1)^2+3}\right)+\frac{2}{3}\int\frac{-(x-1)^2+a+3}{\sqrt{-(x-1)^4-2(x-1)^2+a+3}}d(x-1)+\frac{1}{3}(x-1)\sqrt{a-(x-1)^4-2(x-1)^2+3}$$

$$\Big\downarrow \text{1092}$$

$$\frac{1}{2}\left((a+4)\int\frac{1}{-(x-1)^4-4}d\left(-\frac{2x}{\sqrt{-(x-1)^4-2(x-1)^2+a+3}}\right)+\frac{1}{2}x\sqrt{a-(x-1)^4-2(x-1)^2+3}\right)+\frac{2}{3}\int\frac{-(x-1)^2+a+3}{\sqrt{-(x-1)^4-2(x-1)^2+a+3}}d(x-1)+\frac{1}{3}(x-1)\sqrt{a-(x-1)^4-2(x-1)^2+3}$$

$$\Big\downarrow \text{217}$$

$$\frac{2}{3}\int\frac{-(x-1)^2+a+3}{\sqrt{-(x-1)^4-2(x-1)^2+a+3}}d(x-1)+\frac{1}{2}\left(\frac{1}{2}(a+4)\arctan\left(\frac{x}{\sqrt{a-(x-1)^4-2(x-1)^2+3}}\right)+\frac{1}{2}x\sqrt{a-(x-1)^4-2(x-1)^2+3}\right)+\frac{1}{3}(x-1)\sqrt{a-(x-1)^4-2(x-1)^2+3}$$

$$\Big\downarrow \text{1514}$$

$$\frac{2\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}\int\frac{-(x-1)^2+a+3}{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}d(x-1)}{3\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{1}{2}\left(\frac{1}{2}(a+4)\arctan\left(\frac{x}{\sqrt{a-(x-1)^4-2(x-1)^2+3}}\right)+\frac{1}{2}x\sqrt{a-(x-1)^4-2(x-1)^2+3}\right)+\frac{1}{3}(x-1)\sqrt{a-(x-1)^4-2(x-1)^2+3}$$

$$\Big\downarrow \text{406}$$

$$\frac{2\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}\left((a+3)\int\frac{1}{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}d(x-1)-\int\frac{(x-1)^2}{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}d(x-1)\right)}{3\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{1}{2}\left(\frac{1}{2}(a+4)\arctan\left(\frac{x}{\sqrt{a-(x-1)^4-2(x-1)^2+3}}\right)+\frac{1}{2}x\sqrt{a-(x-1)^4-2(x-1)^2+3}\right)+\frac{1}{3}(x-1)\sqrt{a-(x-1)^4-2(x-1)^2+3}$$

$$\Big\downarrow \text{320}$$

$$\frac{2\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}\left(\frac{(a+3)\sqrt{\sqrt{a+4}+1}\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\,\text{EllipticF}\left(\arctan\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right),-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}-\int\frac{(x-1)^2}{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-}{\sqrt{a+}}}}\right.}{3\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

$$\frac{1}{2}\left(\frac{1}{2}(a+4)\arctan\left(\frac{x}{\sqrt{a-(x-1)^4-2(x-1)^2+3}}\right)+\frac{1}{2}x\sqrt{a-(x-1)^4-2(x-1)^2+3}\right)+$$
$$\frac{1}{3}(x-1)\sqrt{a-(x-1)^4-2(x-1)^2+3}$$

$$\Big\downarrow \text{388}$$

$$\frac{2\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}\left((1-\sqrt{a+4})\int\frac{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}}{\left(\frac{(x-1)^2}{\sqrt{a+4}+1}+1\right)^{3/2}}d(x-1)+\frac{(a+3)\sqrt{\sqrt{a+4}+1}\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\,\text{EllipticF}\left(\arctan\right.}{\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}-}}\right.}{3\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

$$\frac{1}{2}\left(\frac{1}{2}(a+4)\arctan\left(\frac{x}{\sqrt{a-(x-1)^4-2(x-1)^2+3}}\right)+\frac{1}{2}x\sqrt{a-(x-1)^4-2(x-1)^2+3}\right)+$$
$$\frac{1}{3}(x-1)\sqrt{a-(x-1)^4-2(x-1)^2+3}$$

$$\Big\downarrow \text{313}$$

$$\frac{1}{2}\left(\frac{1}{2}(a+4)\arctan\left(\frac{x}{\sqrt{a-(x-1)^4-2(x-1)^2+3}}\right)+\frac{1}{2}x\sqrt{a-(x-1)^4-2(x-1)^2+3}\right)+$$

$$\frac{2\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}\left(\frac{(a+3)\sqrt{\sqrt{a+4}+1}\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\,\text{EllipticF}\left(\arctan\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right),-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}+\frac{(1-\sqrt{a+4})\sqrt{\sqrt{a+4}+1}}{}\right.}{3\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

$$\frac{1}{3}(x-1)\sqrt{a-(x-1)^4-2(x-1)^2+3}$$

input
```
Int[x*Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4],x]
```

output
```
(Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]*(-1 + x))/3 + ((Sqrt[3 + a - 2*(-
1 + x)^2 - (-1 + x)^4]*x)/2 + ((4 + a)*ArcTan[x/Sqrt[3 + a - 2*(-1 + x)^2
- (-1 + x)^4]])/2)/2 + (2*Sqrt[1 + (-1 + x)^2/(1 - Sqrt[4 + a])]*Sqrt[1 +
(-1 + x)^2/(1 + Sqrt[4 + a])]*(-(((1 - Sqrt[4 + a])*Sqrt[1 + (-1 + x)^2/(1
 - Sqrt[4 + a])]*(-1 + x))/Sqrt[1 + (-1 + x)^2/(1 + Sqrt[4 + a])]) + ((1 -
 Sqrt[4 + a])*Sqrt[1 + Sqrt[4 + a]]*Sqrt[1 + (-1 + x)^2/(1 - Sqrt[4 + a])]
*EllipticE[ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - S
qrt[4 + a])])/(Sqrt[(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (-1 + x)^2/(1
+ Sqrt[4 + a]))]*Sqrt[1 + (-1 + x)^2/(1 + Sqrt[4 + a])]) + ((3 + a)*Sqrt[1
 + Sqrt[4 + a]]*Sqrt[1 + (-1 + x)^2/(1 - Sqrt[4 + a])]*EllipticF[ArcTan[(-
1 + x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])/(Sqrt[
(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (-1 + x)^2/(1 + Sqrt[4 + a]))]*Sqr
t[1 + (-1 + x)^2/(1 + Sqrt[4 + a])])))/(3*Sqrt[3 + a - 2*(-1 + x)^2 - (-1
+ x)^4])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1404
```
Int[((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*((a + b
*x^2 + c*x^4)^p/(4*p + 1)), x] + Simp[2*(p/(4*p + 1))   Int[(2*a + b*x^2)*(
a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*
c, 0] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 1432
```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

rule 1514
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[Sqrt[1 + 2*c*(x^2/(b - q))]*(Sqrt
[1 + 2*c*(x^2/(b + q))]/Sqrt[a + b*x^2 + c*x^4])   Int[(d + e*x^2)/(Sqrt[1
+ 2*c*(x^2/(b - q))]*Sqrt[1 + 2*c*(x^2/(b + q))]), x], x]] /; FreeQ[{a, b,
c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

rule 2202
```
Int[(Pn_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{n
 = Expon[Pn, x], k}, Int[Sum[Coeff[Pn, x, 2*k]*x^(2*k), {k, 0, n/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pn, x, 2*k + 1]*x^(2*k), {k, 0, (n -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pn, x]
 &&  !PolyQ[Pn, x^2]
```

rule 2459
```
Int[(Pn_)^(p_.)*(Qx_), x_Symbol] :> With[{S = Coeff[Pn, x, Expon[Pn, x] - 1
]/(Expon[Pn, x]*Coeff[Pn, x, Expon[Pn, x]])}, Subst[Int[ExpandToSum[Pn /. x
 -> x - S, x]^p*ExpandToSum[Qx /. x -> x - S, x], x], x, x + S] /; Binomial
Q[Pn /. x -> x - S, x] || (IntegerQ[Expon[Pn, x]/2] && TrinomialQ[Pn /. x -
> x - S, x])] /; FreeQ[p, x] && PolyQ[Pn, x] && GtQ[Expon[Pn, x], 2] && NeQ
[Coeff[Pn, x, Expon[Pn, x] - 1], 0] && PolyQ[Qx, x] &&  !(MonomialQ[Qx, x]
&& IGtQ[p, 0])
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 2550 vs. $2(345) = 690$.

Time = 7.24 (sec) , antiderivative size = 2551, normalized size of antiderivative = 6.27

method	result	size
default	Expression too large to display	2551
elliptic	Expression too large to display	2551
risch	Expression too large to display	3034

input
```
int(x*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*x^2*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)-1/6*x*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2
)+1/6*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)-(1/6*a-2/3)*((-1-(a+4)^(1/2))^(1/2)+(
-1+(a+4)^(1/2))^(1/2))*((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(
x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2
))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)*(x-1+(-1+(a+4)^(1/2))^(1/2))^2*(-2*
(-1+(a+4)^(1/2))^(1/2)*(x-1-(-1-(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2
)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)*(-2*(-1+(a+4
)^(1/2))^(1/2)*(x-1+(-1-(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(
a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)/(-(-1-(a+4)^(1/2))^
(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-1+(a+4)^(1/2))^(1/2)/(-(x-1-(-1+(a+4)^(1/2
))^(1/2))*(x-1+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1-(a+4)^(1/2))^(1/2))*(x-1+(
-1-(a+4)^(1/2))^(1/2)))^(1/2)*EllipticF(((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4
)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+
(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2),((-(-1-(a+4)^(1/2)
)^(1/2)-(-1+(a+4)^(1/2))^(1/2))*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(
1/2))/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(
1/2)-(-1+(a+4)^(1/2))^(1/2)))^(1/2))-(1/2*a+10/3)*((-1-(a+4)^(1/2))^(1/2)+
(-1+(a+4)^(1/2))^(1/2))*((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*
(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/
2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)*(x-1+(-1+(a+4)^(1/2))^(1/2))^2*...
```

Fricas [F]

$$\int x\sqrt{a+8x-8x^2+4x^3-x^4}\,dx=\int \sqrt{-x^4+4\,x^3-8\,x^2+a+8\,x}x\,dx$$

input
```
integrate(x*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)*x, x)
```

Sympy [F]

$$\int x\sqrt{a+8x-8x^2+4x^3-x^4}\,dx=\int x\sqrt{a-x^4+4x^3-8x^2+8x}\,dx$$

input
```
integrate(x*(-x**4+4*x**3-8*x**2+a+8*x)**(1/2),x)
```

output
```
Integral(x*sqrt(a - x**4 + 4*x**3 - 8*x**2 + 8*x), x)
```

Maxima [F]

$$\int x\sqrt{a+8x-8x^2+4x^3-x^4}\,dx=\int \sqrt{-x^4+4\,x^3-8\,x^2+a+8\,x}x\,dx$$

input
```
integrate(x*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)*x, x)
```

Giac [F]

$$\int x\sqrt{a+8x-8x^2+4x^3-x^4}\,dx=\int \sqrt{-x^4+4\,x^3-8\,x^2+a+8\,x}x\,dx$$

input
```
integrate(x*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\sqrt{a+8x-8x^2+4x^3-x^4}\,dx=\int x\,\sqrt{-x^4+4\,x^3-8\,x^2+8\,x+a}\,dx$$

input
```
int(x*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(1/2),x)
```

output
```
int(x*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(1/2), x)
```

Reduce [F]

$$\begin{aligned}\int x\sqrt{a+8x-8x^2+4x^3-x^4}\,dx &= \frac{\sqrt{-x^4+4x^3-8x^2+a+8x}\,x^2}{4} \\ &\quad-\frac{\sqrt{-x^4+4x^3-8x^2+a+8x}\,x}{6} \\ &\quad+\frac{\sqrt{-x^4+4x^3-8x^2+a+8x}}{18} \\ &\quad+\frac{\left(\int\frac{\sqrt{-x^4+4x^3-8x^2+a+8x}}{-x^4+4x^3-8x^2+a+8x}dx\right)a}{6} \\ &\quad-\frac{2\left(\int\frac{\sqrt{-x^4+4x^3-8x^2+a+8x}}{-x^4+4x^3-8x^2+a+8x}dx\right)}{9} \\ &\quad-\frac{2\left(\int\frac{\sqrt{-x^4+4x^3-8x^2+a+8x}\,x^3}{-x^4+4x^3-8x^2+a+8x}dx\right)}{9} \\ &\quad+\frac{\left(\int\frac{\sqrt{-x^4+4x^3-8x^2+a+8x}\,x}{-x^4+4x^3-8x^2+a+8x}dx\right)a}{2} \\ &\quad+\frac{22\left(\int\frac{\sqrt{-x^4+4x^3-8x^2+a+8x}\,x}{-x^4+4x^3-8x^2+a+8x}dx\right)}{9}\end{aligned}$$

input
```
int(x*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x)
```

output
```
(9*sqrt(a - x**4 + 4*x**3 - 8*x**2 + 8*x)*x**2 - 6*sqrt(a - x**4 + 4*x**3
- 8*x**2 + 8*x)*x + 2*sqrt(a - x**4 + 4*x**3 - 8*x**2 + 8*x) + 6*int(sqrt(
a - x**4 + 4*x**3 - 8*x**2 + 8*x)/(a - x**4 + 4*x**3 - 8*x**2 + 8*x),x)*a
- 8*int(sqrt(a - x**4 + 4*x**3 - 8*x**2 + 8*x)/(a - x**4 + 4*x**3 - 8*x**2
 + 8*x),x) - 8*int((sqrt(a - x**4 + 4*x**3 - 8*x**2 + 8*x)*x**3)/(a - x**4
 + 4*x**3 - 8*x**2 + 8*x),x) + 18*int((sqrt(a - x**4 + 4*x**3 - 8*x**2 + 8
*x)*x)/(a - x**4 + 4*x**3 - 8*x**2 + 8*x),x)*a + 88*int((sqrt(a - x**4 + 4
*x**3 - 8*x**2 + 8*x)*x)/(a - x**4 + 4*x**3 - 8*x**2 + 8*x),x))/36
```

3.35 $\int \frac{x}{\sqrt{a+8x-8x^2+4x^3-x^4}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 197

$$\int \frac{x}{\sqrt{a+8x-8x^2+4x^3-x^4}}\,dx = \frac{1}{2}\arctan\left(\frac{1+(-1+x)^2}{\sqrt{3+a-2(1-x)^2-(1-x)^4}}\right) - \frac{\sqrt{3+a}\sqrt{1-\frac{\left(1-\sqrt{4+a}\right)(1-x)^2}{3+a}}\sqrt{1-\frac{\left(1+\sqrt{4+a}\right)(1-x)^2}{3+a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1+\sqrt{4+a}}(1-x)}{\sqrt{3+a}}\right),\frac{1-\sqrt{4+a}}{1+\sqrt{4+a}}\right)}{\sqrt{1+\sqrt{4+a}}\sqrt{3+a-2(1-x)^2-(1-x)^4}}$$

output
```
1/2*arctan((1+(-1+x)^2)/(3+a-2*(1-x)^2-(1-x)^4)^(1/2))-(3+a)^(1/2)*(1-(1-(
4+a)^(1/2))*(1-x)^2/(3+a))^(1/2)*(1-(1+(4+a)^(1/2))*(1-x)^2/(3+a))^(1/2)*E
llipticF((1+(4+a)^(1/2))^(1/2)*(1-x)/(3+a)^(1/2),((1-(4+a)^(1/2))/(1+(4+a)
^(1/2)))^(1/2))/(1+(4+a)^(1/2))^(1/2)/(3+a-2*(1-x)^2-(1-x)^4)^(1/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 813 vs. 2(197) = 394.

Time = 14.71 (sec) , antiderivative size = 813, normalized size of antiderivative = 4.13

$$\int \frac{x}{\sqrt{a+8x-8x^2+4x^3-x^4}}\,dx$$
$$= \frac{2\left(1+\sqrt{-1-\sqrt{4+a}}-x\right)\sqrt{\frac{\sqrt{-1-\sqrt{4+a}}\left(1+\sqrt{-1+\sqrt{4+a}}-x\right)}{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(1+\sqrt{-1-\sqrt{4+a}}-x\right)}}\left(-1+\sqrt{-1-\sqrt{4+a}}+x\right)\sqrt{\frac{(}{}}}{}$$

input

```
Integrate[x/Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4],x]
```

output

```
(2*(1 + Sqrt[-1 - Sqrt[4 + a]] - x)*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(1 + Sqrt
[-1 + Sqrt[4 + a]] - x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]]
)*(1 + Sqrt[-1 - Sqrt[4 + a]] - x))]*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x)*Sqr
t[(Sqrt[-1 - Sqrt[4 + a]]*(-1 + Sqrt[-1 + Sqrt[4 + a]] + x))/((-Sqrt[-1 -
Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x))]*
((1 + Sqrt[-1 - Sqrt[4 + a]])*EllipticF[ArcSin[Sqrt[((Sqrt[-1 - Sqrt[4 + a
]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1
- Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x))
]], (Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])^2/(Sqrt[-1 - Sqrt[4
+ a]] - Sqrt[-1 + Sqrt[4 + a]])^2] - 2*Sqrt[-1 - Sqrt[4 + a]]*EllipticPi[(
Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])/(-Sqrt[-1 - Sqrt[4 + a]]
+ Sqrt[-1 + Sqrt[4 + a]]), ArcSin[Sqrt[((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1
+ Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]
] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x))]], (Sqrt[-1
- Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])^2/(Sqrt[-1 - Sqrt[4 + a]] - Sqrt[
-1 + Sqrt[4 + a]])^2]))/(Sqrt[-1 - Sqrt[4 + a]]*Sqrt[((Sqrt[-1 - Sqrt[4 +
a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1
 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x)
)]*Sqrt[a - x*(-8 + 8*x - 4*x^2 + x^3)])
```

Rubi [A] (warning: unable to verify)

Time = 0.70 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.88, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {2459, 2202, 1417, 320, 1432, 1092, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{a-x^4+4x^3-8x^2+8x}}dx$$

$$\Big\downarrow \text{2459}$$

$$\int \frac{x}{\sqrt{a-(x-1)^4-2(x-1)^2+3}}d(x-1)$$

$$\Big\downarrow \text{2202}$$

$$\int \frac{1}{\sqrt{-(x-1)^4-2(x-1)^2+a+3}}d(x-1)+\int \frac{x-1}{\sqrt{-(x-1)^4-2(x-1)^2+a+3}}d(x-1)$$

$$\Big\downarrow \text{1417}$$

$$\frac{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}\int \frac{1}{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}d(x-1)}{\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\int \frac{x-1}{\sqrt{-(x-1)^4-2(x-1)^2+a+3}}d(x-1)$$

$$\Big\downarrow \text{320}$$

$$\int \frac{x-1}{\sqrt{-(x-1)^4-2(x-1)^2+a+3}}d(x-1)+\frac{\sqrt{\sqrt{a+4}+1}\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)\text{EllipticF}\left(\arctan\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right),-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

$$\Big\downarrow \text{1432}$$

$$\frac{1}{2}\int \frac{1}{\sqrt{-(x-1)^4-2(x-1)^2+a+3}}d(x-1)^2+\frac{\sqrt{\sqrt{a+4}+1}\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)\text{EllipticF}\left(\arctan\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right),-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

$$\Big\downarrow \text{1092}$$

$$\int \frac{1}{-(x-1)^4-4}d\left(-\frac{2x}{\sqrt{-(x-1)^4-2(x-1)^2+a+3}}\right)+\frac{\sqrt{\sqrt{a+4}+1}\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)\text{EllipticF}\left(\arctan\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right),-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{2}\arctan\left(\frac{x}{\sqrt{a-(x-1)^4-2(x-1)^2+3}}\right)+\frac{\sqrt{\sqrt{a+4}+1}\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)\text{EllipticF}\left(\arctan\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right),-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

input
```
Int[x/Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4],x]
```

output
```
ArcTan[x/Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]]/2 + (Sqrt[1 + Sqrt[4 + a
]]*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*EllipticF[ArcTan[(-1 + x)/Sqrt[1 + S
qrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])/(Sqrt[(1 + (-1 + x)^2/(
1 - Sqrt[4 + a]))/(1 + (-1 + x)^2/(1 + Sqrt[4 + a]))]*Sqrt[3 + a - 2*(-1 +
 x)^2 - (-1 + x)^4])
```

Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1417
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[Sqrt[1 + 2*c*(x^2/(b - q))]*(Sqrt[1 + 2*c*(x^2/(b + q
))]/Sqrt[a + b*x^2 + c*x^4])   Int[1/(Sqrt[1 + 2*c*(x^2/(b - q))]*Sqrt[1 +
2*c*(x^2/(b + q))]), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
&& NegQ[c/a]
```

rule 1432
```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

rule 2202
```
Int[(Pn_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{n
 = Expon[Pn, x], k}, Int[Sum[Coeff[Pn, x, 2*k]*x^(2*k), {k, 0, n/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pn, x, 2*k + 1]*x^(2*k), {k, 0, (n -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pn, x]
 &&  !PolyQ[Pn, x^2]
```

rule 2459
```
Int[(Pn_)^(p_.)*(Qx_), x_Symbol] :> With[{S = Coeff[Pn, x, Expon[Pn, x] - 1
]/(Expon[Pn, x]*Coeff[Pn, x, Expon[Pn, x]])}, Subst[Int[ExpandToSum[Pn /. x
 -> x - S, x]^p*ExpandToSum[Qx /. x -> x - S, x], x], x, x + S] /; Binomial
Q[Pn /. x -> x - S, x] || (IntegerQ[Expon[Pn, x]/2] && TrinomialQ[Pn /. x -
> x - S, x])] /; FreeQ[p, x] && PolyQ[Pn, x] && GtQ[Expon[Pn, x], 2] && NeQ
[Coeff[Pn, x, Expon[Pn, x] - 1], 0] && PolyQ[Qx, x] &&  !(MonomialQ[Qx, x]
&& IGtQ[p, 0])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 787 vs. $2(168) = 336$.

Time = 1.07 (sec) , antiderivative size = 788, normalized size of antiderivative = 4.00

method	result
default	$-\frac{\left(\sqrt{-1-\sqrt{a+4}}+\sqrt{-1+\sqrt{a+4}}\right)\sqrt{\frac{\left(-\sqrt{-1-\sqrt{a+4}}+\sqrt{-1+\sqrt{a+4}}\right)\left(x-1-\sqrt{-1+\sqrt{a+4}}\right)}{\left(-\sqrt{-1-\sqrt{a+4}}-\sqrt{-1+\sqrt{a+4}}\right)\left(x-1+\sqrt{-1+\sqrt{a+4}}\right)}}\left(x-1+\sqrt{-1+\sqrt{a+4}}\right)^{2}\sqrt{-\frac{2\sqrt{-1+\sqrt{a}}}{\left(\sqrt{-1-\sqrt{a+4}}-\sqrt{-}\right.}}}{}$
elliptic	$-\frac{\left(\sqrt{-1-\sqrt{a+4}}+\sqrt{-1+\sqrt{a+4}}\right)\sqrt{\frac{\left(-\sqrt{-1-\sqrt{a+4}}+\sqrt{-1+\sqrt{a+4}}\right)\left(x-1-\sqrt{-1+\sqrt{a+4}}\right)}{\left(-\sqrt{-1-\sqrt{a+4}}-\sqrt{-1+\sqrt{a+4}}\right)\left(x-1+\sqrt{-1+\sqrt{a+4}}\right)}}\left(x-1+\sqrt{-1+\sqrt{a+4}}\right)^{2}\sqrt{-\frac{2\sqrt{-1+\sqrt{a}}}{\left(\sqrt{-1-\sqrt{a+4}}-\sqrt{-}\right.}}}{}$

input
```
int(x/(-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*((-(-1-(a+4)^(1/2))^(1/2)
+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(
1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)*(x-1+(-1+
(a+4)^(1/2))^(1/2))^2*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1-(-1-(a+4)^(1/2))^(1/
2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^
(1/2)))^(1/2)*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1+(-1-(a+4)^(1/2))^(1/2))/(-(-
1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))
^(1/2)/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-1+(a+4)^(1/2))^(
1/2)/(-(x-1-(-1+(a+4)^(1/2))^(1/2))*(x-1+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1-
(a+4)^(1/2))^(1/2))*(x-1+(-1-(a+4)^(1/2))^(1/2)))^(1/2)*((1-(-1+(a+4)^(1/2
))^(1/2))*EllipticF(((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1
-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/
(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2),((-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1
/2))^(1/2))*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1
/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2)
)^(1/2)))^(1/2))+2*(-1+(a+4)^(1/2))^(1/2)*EllipticPi(((-(-1-(a+4)^(1/2))^(
1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2
))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2),(-(-1
-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(
a+4)^(1/2))^(1/2)),((-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))*((...
```

Fricas [F]

$$\int \frac{x}{\sqrt{a+8x-8x^2+4x^3-x^4}}\,dx=\int \frac{x}{\sqrt{-x^4+4\,x^3-8\,x^2+a+8\,x}}\,dx$$

input
```
integrate(x/(-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)*x/(x^4 - 4*x^3 + 8*x^2 - a
- 8*x), x)
```

Sympy [F]

$$\int \frac{x}{\sqrt{a+8x-8x^2+4x^3-x^4}}\,dx = \int \frac{x}{\sqrt{a-x^4+4x^3-8x^2+8x}}\,dx$$

input
```
integrate(x/(-x**4+4*x**3-8*x**2+a+8*x)**(1/2),x)
```

output
```
Integral(x/sqrt(a - x**4 + 4*x**3 - 8*x**2 + 8*x), x)
```

Maxima [F]

$$\int \frac{x}{\sqrt{a+8x-8x^2+4x^3-x^4}}\,dx = \int \frac{x}{\sqrt{-x^4+4\,x^3-8\,x^2+a+8\,x}}\,dx$$

input
```
integrate(x/(-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x), x)
```

Giac [F]

$$\int \frac{x}{\sqrt{a+8x-8x^2+4x^3-x^4}}\,dx = \int \frac{x}{\sqrt{-x^4+4\,x^3-8\,x^2+a+8\,x}}\,dx$$

input
```
integrate(x/(-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(x/sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{a+8x-8x^2+4x^3-x^4}}\,dx=\int \frac{x}{\sqrt{-x^4+4\,x^3-8\,x^2+8\,x+a}}\,dx$$

input
```
int(x/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(1/2),x)
```

output
```
int(x/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{x}{\sqrt{a+8x-8x^2+4x^3-x^4}}\,dx=\int \frac{\sqrt{-x^4+4x^3-8x^2+a+8x}\,x}{-x^4+4x^3-8x^2+a+8x}dx$$

input
```
int(x/(-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x)
```

output
```
int((sqrt(a - x**4 + 4*x**3 - 8*x**2 + 8*x)*x)/(a - x**4 + 4*x**3 - 8*x**2
 + 8*x),x)
```

3.36 $\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 409

$$\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^{3/2}}\,dx = \frac{1+(-1+x)^2}{2(4+a)\sqrt{3+a-2(1-x)^2-(1-x)^4}} - \frac{(5+a+(-1+x)^2)\,(1-x)}{2\,(12+7a+a^2)\,\sqrt{3+a-2(1-x)^2-(1-x)^4}} + \frac{\sqrt{-1+\sqrt{4+a}}(1+\sqrt{4+a})\,\sqrt{1+\frac{(1-x)^2}{1-\sqrt{4+a}}}\sqrt{1+\frac{(1-x)^2}{1+\sqrt{4+a}}}E\left(\arcsin\left(\frac{1-x}{\sqrt{-1+\sqrt{4+a}}}\right)|\frac{1-\sqrt{4+a}}{1+\sqrt{4+a}}\right)}{2(3+a)(4+a)\sqrt{3+a-2(1-x)^2-(1-x)^4}} - \frac{\sqrt{-1+\sqrt{4+a}}(4+a+\sqrt{4+a})\,\sqrt{1+\frac{(1-x)^2}{1-\sqrt{4+a}}}\sqrt{1+\frac{(1-x)^2}{1+\sqrt{4+a}}}\,\text{EllipticF}\left(\arcsin\left(\frac{1-x}{\sqrt{-1+\sqrt{4+a}}}\right),\frac{1-\sqrt{4+a}}{1+\sqrt{4+a}}\right)}{2(3+a)(4+a)\sqrt{3+a-2(1-x)^2-(1-x)^4}}$$

output
```
1/2*(1+(-1+x)^2)/(4+a)/(3+a-2*(1-x)^2-(1-x)^4)^(1/2)-1/2*(5+a+(-1+x)^2)*(1
-x)/(a^2+7*a+12)/(3+a-2*(1-x)^2-(1-x)^4)^(1/2)+1/2*(-1+(4+a)^(1/2))^(1/2)*
(1+(4+a)^(1/2))*(1+(1-x)^2/(1-(4+a)^(1/2)))^(1/2)*(1+(1-x)^2/(1+(4+a)^(1/2
)))^(1/2)*EllipticE((1-x)/(-1+(4+a)^(1/2))^(1/2),((1-(4+a)^(1/2))/(1+(4+a)
^(1/2)))^(1/2))/(3+a)/(4+a)/(3+a-2*(1-x)^2-(1-x)^4)^(1/2)-1/2*(-1+(4+a)^(1
/2))^(1/2)*(4+a+(4+a)^(1/2))*(1+(1-x)^2/(1-(4+a)^(1/2)))^(1/2)*(1+(1-x)^2/
(1+(4+a)^(1/2)))^(1/2)*EllipticF((1-x)/(-1+(4+a)^(1/2))^(1/2),((1-(4+a)^(1
/2))/(1+(4+a)^(1/2)))^(1/2))/(3+a)/(4+a)/(3+a-2*(1-x)^2-(1-x)^4)^(1/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 3593 vs. $2(409) = 818$.

Time = 16.82 (sec) , antiderivative size = 3593, normalized size of antiderivative = 8.78

$$\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^{3/2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[x/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2),x]
```

output
```
((-a - 2*x + a*x - a*x^2 - x^3)*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2)/(2*(3 +
 a)*(4 + a)*(-a - 8*x + 8*x^2 - 4*x^3 + x^4)*(a - x*(-8 + 8*x - 4*x^2 + x^
3))^(3/2)) + ((a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2)*((4*(-Sqrt[-1 - Sqrt[4
 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x)^2*Sqrt[
((-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4
 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt
[-1 - Sqrt[4 + a]] + x))]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 - Sqrt[-1 + Sqr
t[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - S
qrt[-1 - Sqrt[4 + a]] + x))]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 + Sqrt[-1 +
Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1
- Sqrt[-1 - Sqrt[4 + a]] + x))]*EllipticF[ArcSin[Sqrt[((-Sqrt[-1 - Sqrt[4
+ a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[
-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] +
 x))]], ((-Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(Sqrt[-1 - Sqr
t[4 + a]] + Sqrt[-1 + Sqrt[4 + a]]))/((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 +
Sqrt[4 + a]])*(-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]]))])/(Sqrt[
-1 - Sqrt[4 + a]]*(-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*Sqrt[
a + 8*x - 8*x^2 + 4*x^3 - x^4]) + (2*a*(-Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1
+ Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x)^2*Sqrt[((-Sqrt[-1 - Sqrt
[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/(...
```

Rubi [A] (warning: unable to verify)

Time = 1.27 (sec) , antiderivative size = 540, normalized size of antiderivative = 1.32, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.423$, Rules used = {2459, 2202, 1405, 27, 1432, 1088, 1514, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a-x^4+4x^3-8x^2+8x)^{3/2}}\,dx$$

$$\Big\downarrow \text{2459}$$

$$\int \frac{x}{(a-(x-1)^4-2(x-1)^2+3)^{3/2}}d(x-1)$$

$$\Big\downarrow \text{2202}$$

$$\int \frac{1}{(-(x-1)^4-2(x-1)^2+a+3)^{3/2}}d(x-1)+\int \frac{x-1}{(-(x-1)^4-2(x-1)^2+a+3)^{3/2}}d(x-1)$$

$$\Big\downarrow \text{1405}$$

$$-\frac{\int -\frac{2(-(x-1)^2+a+3)}{\sqrt{-(x-1)^4-2(x-1)^2+a+3}}d(x-1)}{4(a^2+7a+12)}+\int \frac{x-1}{(-(x-1)^4-2(x-1)^2+a+3)^{3/2}}d(x-1)+\frac{(x-1)(a+(x-1)^2+5)}{2(a^2+7a+12)\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{-(x-1)^2+a+3}{\sqrt{-(x-1)^4-2(x-1)^2+a+3}}d(x-1)}{2(a^2+7a+12)}+\int \frac{x-1}{(-(x-1)^4-2(x-1)^2+a+3)^{3/2}}d(x-1)+\frac{(x-1)(a+(x-1)^2+5)}{2(a^2+7a+12)\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

$$\Big\downarrow \text{1432}$$

$$\frac{\int \frac{-(x-1)^2+a+3}{\sqrt{-(x-1)^4-2(x-1)^2+a+3}}d(x-1)}{2(a^2+7a+12)}+\frac{1}{2}\int \frac{1}{(-(x-1)^4-2(x-1)^2+a+3)^{3/2}}d(x-1)^2+\frac{(x-1)(a+(x-1)^2+5)}{2(a^2+7a+12)\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

$$\Big\downarrow \text{1088}$$

$$\frac{\int \frac{-(x-1)^2+a+3}{\sqrt{-(x-1)^4-2(x-1)^2+a+3}}d(x-1)}{2\left(a^2+7a+12\right)}+\frac{(x-1)\left(a+(x-1)^2+5\right)}{2\left(a^2+7a+12\right)\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{x}{2(a+4)\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

$\downarrow$ 1514

$$\frac{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}\int \frac{-(x-1)^2+a+3}{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}d(x-1)}{2\left(a^2+7a+12\right)\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{(x-1)\left(a+(x-1)^2+5\right)}{2\left(a^2+7a+12\right)\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{x}{2(a+4)\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

$\downarrow$ 406

$$\frac{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}\left((a+3)\int \frac{1}{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}d(x-1)-\int \frac{(x-1)^2}{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}d(x-1)\right)}{2\left(a^2+7a+12\right)\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{(x-1)\left(a+(x-1)^2+5\right)}{2\left(a^2+7a+12\right)\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{x}{2(a+4)\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

$\downarrow$ 320

$$\frac{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}\left(\frac{(a+3)\sqrt{\sqrt{a+4}+1}\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\text{EllipticF}\left(\arctan\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right),-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}-\int \frac{(x-1)^2}{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1}{\sqrt{a+4}}}}\right.}{2\left(a^2+7a+12\right)\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{(x-1)\left(a+(x-1)^2+5\right)}{2\left(a^2+7a+12\right)\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{x}{2(a+4)\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

$\downarrow$ 388

$$\frac{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}\left(\left(1-\sqrt{a+4}\right)\int \frac{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}}{\left(\frac{(x-1)^2}{\sqrt{a+4}+1}+1\right)^{3/2}}d(x-1)+\frac{(a+3)\sqrt{\sqrt{a+4}+1}\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\text{EllipticF}\left(\arctan\left(\right.\right.}{\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\right.}{2\left(a^2+7a+12\right)\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{(x-1)\left(a+(x-1)^2+5\right)}{2\left(a^2+7a+12\right)\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{x}{2(a+4)\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

$$\Big\downarrow \text{313}$$

$$\frac{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}\left(\frac{(a+3)\sqrt{\sqrt{a+4}+1}\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\,\text{EllipticF}\left(\arctan\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right),-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}+\frac{(1-\sqrt{a+4})\sqrt{\sqrt{a+4}+1}\sqrt{}}{\sqrt{}}\right.}{2\left(a^2+7a+12\right)\sqrt{a-(x-1)^4-2(x-1)^2+}}$$

$$\frac{(x-1)\left(a+(x-1)^2+5\right)}{2\left(a^2+7a+12\right)\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{x}{2(a+4)\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

input

```
Int[x/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2),x]
```

output

```
((5 + a + (-1 + x)^2)*(-1 + x))/(2*(12 + 7*a + a^2)*Sqrt[3 + a - 2*(-1 + x
)^2 - (-1 + x)^4]) + x/(2*(4 + a)*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4])
 + (Sqrt[1 + (-1 + x)^2/(1 - Sqrt[4 + a])]*Sqrt[1 + (-1 + x)^2/(1 + Sqrt[4
 + a])]*(-(((1 - Sqrt[4 + a])*Sqrt[1 + (-1 + x)^2/(1 - Sqrt[4 + a])]*(-1 +
 x))/Sqrt[1 + (-1 + x)^2/(1 + Sqrt[4 + a])]) + ((1 - Sqrt[4 + a])*Sqrt[1 +
 Sqrt[4 + a]]*Sqrt[1 + (-1 + x)^2/(1 - Sqrt[4 + a])]*EllipticE[ArcTan[(-1
+ x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])/(Sqrt[(1
 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (-1 + x)^2/(1 + Sqrt[4 + a]))]*Sqrt[
1 + (-1 + x)^2/(1 + Sqrt[4 + a])]) + ((3 + a)*Sqrt[1 + Sqrt[4 + a]]*Sqrt[1
 + (-1 + x)^2/(1 - Sqrt[4 + a])]*EllipticF[ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4
 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])/(Sqrt[(1 + (-1 + x)^2/(1 - S
qrt[4 + a]))/(1 + (-1 + x)^2/(1 + Sqrt[4 + a]))]*Sqrt[1 + (-1 + x)^2/(1 +
Sqrt[4 + a])])))/(2*(12 + 7*a + a^2)*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^
4])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1432
```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

rule 1514
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[Sqrt[1 + 2*c*(x^2/(b - q))]*(Sqrt
[1 + 2*c*(x^2/(b + q))]/Sqrt[a + b*x^2 + c*x^4])   Int[(d + e*x^2)/(Sqrt[1
+ 2*c*(x^2/(b - q))]*Sqrt[1 + 2*c*(x^2/(b + q))]), x], x]] /; FreeQ[{a, b,
c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

rule 2202
```
Int[(Pn_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{n
 = Expon[Pn, x], k}, Int[Sum[Coeff[Pn, x, 2*k]*x^(2*k), {k, 0, n/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pn, x, 2*k + 1]*x^(2*k), {k, 0, (n -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pn, x]
 &&  !PolyQ[Pn, x^2]
```

rule 2459
```
Int[(Pn_)^(p_.)*(Qx_), x_Symbol] :> With[{S = Coeff[Pn, x, Expon[Pn, x] - 1
]/(Expon[Pn, x]*Coeff[Pn, x, Expon[Pn, x]])}, Subst[Int[ExpandToSum[Pn /. x
 -> x - S, x]^p*ExpandToSum[Qx /. x -> x - S, x], x], x, x + S] /; Binomial
Q[Pn /. x -> x - S, x] || (IntegerQ[Expon[Pn, x]/2] && TrinomialQ[Pn /. x -
> x - S, x])] /; FreeQ[p, x] && PolyQ[Pn, x] && GtQ[Expon[Pn, x], 2] && NeQ
[Coeff[Pn, x, Expon[Pn, x] - 1], 0] && PolyQ[Qx, x] &&  !(MonomialQ[Qx, x]
&& IGtQ[p, 0])
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 2615 vs. $2(351) = 702$.

Time = 1.07 (sec) , antiderivative size = 2616, normalized size of antiderivative = 6.40

method	result	size
default	Expression too large to display	2616
elliptic	Expression too large to display	2616

input
```
int(x/(-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2*(1/4/(a^2+7*a+12)*x^3+1/4/(a^2+7*a+12)*a*x^2-1/4*(a-2)/(a^2+7*a+12)*x+1/
4/(a^2+7*a+12)*a)/(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)-(2/(a^2+7*a+12)+1/2*(a-2)
/(a^2+7*a+12))*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*((-(-1-(a+4
)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-
(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(
1/2)*(x-1+(-1+(a+4)^(1/2))^(1/2))^2*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1-(-1-(a
+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1
+(a+4)^(1/2))^(1/2)))^(1/2)*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1+(-1-(a+4)^(1/2
))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^
(1/2))^(1/2)))^(1/2)/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-1+
(a+4)^(1/2))^(1/2)/(-(x-1-(-1+(a+4)^(1/2))^(1/2))*(x-1+(-1+(a+4)^(1/2))^(1
/2))*(x-1-(-1-(a+4)^(1/2))^(1/2))*(x-1+(-1-(a+4)^(1/2))^(1/2)))^(1/2)*Elli
pticF(((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/
2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)
^(1/2))^(1/2)))^(1/2),((-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))*((
-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)+(-1
+(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2)))^(1/2
))-((1+a)/(a^2+7*a+12)-1/(a^2+7*a+12)*a)*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)
^(1/2))^(1/2))*((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+
(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x...
```

Fricas [F]

$$\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^{3/2}}\,dx=\int \frac{x}{(-x^4+4\,x^3-8\,x^2+a+8\,x)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)*x/(x^8 - 8*x^7 + 32*x^6 - 2*
(a - 64)*x^4 - 80*x^5 + 8*(a - 16)*x^3 - 16*(a - 4)*x^2 + a^2 + 16*a*x), x
)
```

Sympy [F]

$$\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^{3/2}}\,dx=\int \frac{x}{(a-x^4+4x^3-8x^2+8x)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(-x**4+4*x**3-8*x**2+a+8*x)**(3/2),x)
```

output
```
Integral(x/(a - x**4 + 4*x**3 - 8*x**2 + 8*x)**(3/2), x)
```

Maxima [F]

$$\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^{3/2}}\,dx=\int \frac{x}{(-x^4+4\,x^3-8\,x^2+a+8\,x)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x/(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(3/2), x)
```

Giac [F]

$$\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^{3/2}}\,dx=\int \frac{x}{(-x^4+4\,x^3-8\,x^2+a+8\,x)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(x/(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^{3/2}}\,dx = \int \frac{x}{\left(-x^4+4\,x^3-8\,x^2+8\,x+a\right)^{3/2}}\,dx$$

input
```
int(x/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2),x)
```

output
```
int(x/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2), x)
```

Reduce [F]

$$\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^{3/2}}\,dx = \int \frac{\sqrt{-x^4+4x^3-8x^2+a+8x}\,x}{x^8-8x^7+32x^6-2a\,x^4-80x^5+8a\,x^3+128x^4-16a\,x^2-128x}$$

input
```
int(x/(-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x)
```

output
```
int((sqrt(a - x**4 + 4*x**3 - 8*x**2 + 8*x)*x)/(a**2 - 2*a*x**4 + 8*a*x**3
 - 16*a*x**2 + 16*a*x + x**8 - 8*x**7 + 32*x**6 - 80*x**5 + 128*x**4 - 128
*x**3 + 64*x**2),x)
```

3.37 $\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^{5/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 542

$$\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^{5/2}}\,dx = \frac{1+(-1+x)^2}{6(4+a)\left(3+a-2(1-x)^2-(1-x)^4\right)^{3/2}}$$
$$+\frac{1+(-1+x)^2}{3(4+a)^2\sqrt{3+a-2(1-x)^2-(1-x)^4}}$$
$$-\frac{\left(104+47a+5a^2+4(7+2a)(1-x)^2\right)(1-x)}{12(3+a)^2(4+a)^2\sqrt{3+a-2(1-x)^2-(1-x)^4}}$$
$$-\frac{\left(5+a+(-1+x)^2\right)(1-x)}{6\left(12+7a+a^2\right)\left(3+a-2(1-x)^2-(1-x)^4\right)^{3/2}}$$
$$+\frac{(7+2a)\sqrt{-1+\sqrt{4+a}}\left(1+\sqrt{4+a}\right)\sqrt{1+\frac{(1-x)^2}{1-\sqrt{4+a}}}\sqrt{1+\frac{(1-x)^2}{1+\sqrt{4+a}}}E\left(\arcsin\left(\frac{1-x}{\sqrt{-1+\sqrt{4+a}}}\right)|\frac{1-\sqrt{4+a}}{1+\sqrt{4+a}}\right)}{3(3+a)^2(4+a)^2\sqrt{3+a-2(1-x)^2-(1-x)^4}}$$
$$-\frac{\sqrt{-1+\sqrt{4+a}}\left(76+5a^2+28\sqrt{4+a}+a\left(39+8\sqrt{4+a}\right)\right)\sqrt{1+\frac{(1-x)^2}{1-\sqrt{4+a}}}\sqrt{1+\frac{(1-x)^2}{1+\sqrt{4+a}}}\text{EllipticF}\left(\text{arcsin}\right.}{12(3+a)^2(4+a)^2\sqrt{3+a-2(1-x)^2-(1-x)^4}}$$

output
```
1/6*(1+(-1+x)^2)/(4+a)/(3+a-2*(1-x)^2-(1-x)^4)^(3/2)+1/3*(1+(-1+x)^2)/(4+a
)^2/(3+a-2*(1-x)^2-(1-x)^4)^(1/2)-1/12*(104+47*a+5*a^2+4*(7+2*a)*(1-x)^2)*
(1-x)/(3+a)^2/(4+a)^2/(3+a-2*(1-x)^2-(1-x)^4)^(1/2)-1/6*(5+a+(-1+x)^2)*(1-
x)/(a^2+7*a+12)/(3+a-2*(1-x)^2-(1-x)^4)^(3/2)+1/3*(7+2*a)*(-1+(4+a)^(1/2))
^(1/2)*(1+(4+a)^(1/2))*(1+(1-x)^2/(1-(4+a)^(1/2)))^(1/2)*(1+(1-x)^2/(1+(4+
a)^(1/2)))^(1/2)*EllipticE((1-x)/(-1+(4+a)^(1/2))^(1/2),((1-(4+a)^(1/2))/(
1+(4+a)^(1/2)))^(1/2))/(3+a)^2/(4+a)^2/(3+a-2*(1-x)^2-(1-x)^4)^(1/2)-1/12*
(-1+(4+a)^(1/2))^(1/2)*(76+5*a^2+28*(4+a)^(1/2)+a*(39+8*(4+a)^(1/2)))*(1+(
1-x)^2/(1-(4+a)^(1/2)))^(1/2)*(1+(1-x)^2/(1+(4+a)^(1/2)))^(1/2)*EllipticF(
(1-x)/(-1+(4+a)^(1/2))^(1/2),((1-(4+a)^(1/2))/(1+(4+a)^(1/2)))^(1/2))/(3+a
)^2/(4+a)^2/(3+a-2*(1-x)^2-(1-x)^4)^(1/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 6452 vs. $2(542) = 1084$.

Time = 17.12 (sec) , antiderivative size = 6452, normalized size of antiderivative = 11.90

$$\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^{5/2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[x/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(5/2),x]
```

output
```
Result too large to show
```

Rubi [A] (warning: unable to verify)

Time = 1.51 (sec) , antiderivative size = 664, normalized size of antiderivative = 1.23, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {2459, 2202, 1405, 27, 1432, 1089, 1088, 1492, 27, 1514, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a-x^4+4x^3-8x^2+8x)^{5/2}}\,dx$$

$$\Big\downarrow \text{2459}$$

$$\int \frac{x}{\left(a-(x-1)^4-2(x-1)^2+3\right)^{5/2}}d(x-1)$$

$$\Big\downarrow \text{2202}$$

$$\int \frac{1}{\left(-(x-1)^4-2(x-1)^2+a+3\right)^{5/2}}d(x-1)+\int \frac{x-1}{\left(-(x-1)^4-2(x-1)^2+a+3\right)^{5/2}}d(x-1)$$

$$\Big\downarrow \text{1405}$$

$$-\frac{\int -\frac{2\left(3(x-1)^2+5a+19\right)}{\left(-(x-1)^4-2(x-1)^2+a+3\right)^{3/2}}d(x-1)}{12\left(a^2+7a+12\right)}+\int \frac{x-1}{\left(-(x-1)^4-2(x-1)^2+a+3\right)^{5/2}}d(x-1)+\frac{(x-1)\left(a+(x-1)^2+5\right)}{6\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{3(x-1)^2+5a+19}{\left(-(x-1)^4-2(x-1)^2+a+3\right)^{3/2}}d(x-1)}{6\left(a^2+7a+12\right)}+\int \frac{x-1}{\left(-(x-1)^4-2(x-1)^2+a+3\right)^{5/2}}d(x-1)+\frac{(x-1)\left(a+(x-1)^2+5\right)}{6\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}}$$

$$\Big\downarrow \text{1432}$$

$$\frac{\int \frac{3(x-1)^2+5a+19}{\left(-(x-1)^4-2(x-1)^2+a+3\right)^{3/2}}d(x-1)}{6\left(a^2+7a+12\right)}+\frac{1}{2}\int \frac{1}{\left(-(x-1)^4-2(x-1)^2+a+3\right)^{5/2}}d(x-1)^2+\frac{(x-1)\left(a+(x-1)^2+5\right)}{6\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}}$$

$$\Big\downarrow \text{1089}$$

$$\frac{\int \frac{3(x-1)^2+5a+19}{\left(-(x-1)^4-2(x-1)^2+a+3\right)^{3/2}}d(x-1)}{6\left(a^2+7a+12\right)}+\frac{1}{2}\left(\frac{2\int \frac{1}{\left(-(x-1)^4-2(x-1)^2+a+3\right)^{3/2}}d(x-1)^2}{3(a+4)}+\frac{x}{3(a+4)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}}\right)+\frac{(x-1)\left(a+(x-1)^2+5\right)}{6\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}}$$

$$\Big\downarrow \text{1088}$$

$$\frac{\int \frac{3(x-1)^2+5a+19}{\left(-(x-1)^4-2(x-1)^2+a+3\right)^{3/2}}d(x-1)}{6\left(a^2+7a+12\right)}+\frac{(x-1)\left(a+(x-1)^2+5\right)}{6\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}}+\frac{1}{2}\left(\frac{2x}{3(a+4)^2\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{x}{3(a+4)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}}\right)$$

$$\Big\downarrow \text{1492}$$

$$\frac{\frac{(x-1)\left(5a^2+4(2a+7)(x-1)^2+47a+104\right)}{2(a^2+7a+12)\sqrt{a-(x-1)^4-2(x-1)^2+3}}-\frac{\int -\frac{2\left((a+3)(5a+16)-4(2a+7)(x-1)^2\right)}{\sqrt{-(x-1)^4-2(x-1)^2+a+3}}d(x-1)}{4(a^2+7a+12)}}{6\left(a^2+7a+12\right)}+\frac{(x-1)\left(a+(x-1)^2+5\right)}{6\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}}+\frac{1}{2}\left(\frac{2x}{3(a+4)^2\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{x}{3(a+4)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{(a+3)(5a+16)-4(2a+7)(x-1)^2}{\sqrt{-(x-1)^4-2(x-1)^2+a+3}}d(x-1)}{2(a^2+7a+12)}+\frac{(x-1)\left(5a^2+4(2a+7)(x-1)^2+47a+104\right)}{2(a^2+7a+12)\sqrt{a-(x-1)^4-2(x-1)^2+3}}}{6\left(a^2+7a+12\right)}+\frac{(x-1)\left(a+(x-1)^2+5\right)}{6\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}}+\frac{1}{2}\left(\frac{2x}{3(a+4)^2\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{x}{3(a+4)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}}\right)$$

$$\Big\downarrow \text{1514}$$

$$\frac{\frac{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}\int \frac{(a+3)(5a+16)-4(2a+7)(x-1)^2}{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}d(x-1)}{2(a^2+7a+12)\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{(x-1)\left(5a^2+4(2a+7)(x-1)^2+47a+104\right)}{2(a^2+7a+12)\sqrt{a-(x-1)^4-2(x-1)^2+3}}}{6\left(a^2+7a+12\right)}+\frac{(x-1)\left(a+(x-1)^2+5\right)}{6\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}}+\frac{1}{2}\left(\frac{2x}{3(a+4)^2\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{x}{3(a+4)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}}\right)$$

$$\Big\downarrow \text{406}$$

$$\frac{\frac{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}\left((a+3)(5a+16)\int\frac{1}{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}d(x-1)-4(2a+7)\int\frac{(x-1)^2}{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}d(x-1)\right)}{2(a^2+7a+12)\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{(x-1)(5a\cdots}{2(a^2+7a\cdots}}{6\left(a^2+7a+12\right)}$$

$$\frac{(x-1)\left(a+(x-1)^2+5\right)}{6\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}}+$$

$$\frac{1}{2}\left(\frac{2x}{3(a+4)^2\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{x}{3(a+4)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}}\right)$$

$$\Big\downarrow \; 320$$

$$\frac{\frac{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}\left(\frac{(a+3)(5a+16)\sqrt{\sqrt{a+4}+1}\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\,\text{EllipticF}\left(\arctan\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right),-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}-4(2a+7)\int\frac{(x-1)^2}{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}\cdots}}\cdots\right.}{2(a^2+7a+12)\sqrt{a-(x-1)^4-2(x-1)^2+3}}\cdots}{6\left(a^2+7a+12\right)}$$

$$\frac{(x-1)\left(a+(x-1)^2+5\right)}{6\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}}+$$

$$\frac{1}{2}\left(\frac{2x}{3(a+4)^2\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{x}{3(a+4)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}}\right)$$

$$\Big\downarrow \; 388$$

$$\frac{\frac{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}\left(\frac{(a+3)(5a+16)\sqrt{\sqrt{a+4}+1}\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\,\text{EllipticF}\left(\arctan\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right),-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}-4(2a+7)\left(\frac{(1-\sqrt{a+4})(x-1)\sqrt{\frac{(x-1)^2}{1-\sqrt{a+\cdots}}\cdots}}{\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\cdots\right.\right.}{2(a^2+7a+12)\sqrt{a-(x-1)^4-2(x-1)^2+3}}\cdots}{6\left(a^2+7a+12\right)}$$

$$\frac{(x-1)\left(a+(x-1)^2+5\right)}{6\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}}+$$

$$\frac{1}{2}\left(\frac{2x}{3(a+4)^2\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{x}{3(a+4)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}}\right)$$

$$\Big\downarrow \; 313$$

$$\frac{\frac{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}\left(\frac{(a+3)(5a+16)\sqrt{\sqrt{a+4}+1}\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\,\text{EllipticF}\left(\arctan\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right),-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}-4(2a+7)\left(\frac{(1-\sqrt{a+4})(x-1)\sqrt{\frac{(x-1)^2}{1-\sqrt{a+}}}}{\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\right.\right.}{2(a^2+7a+12)\sqrt{a-(x-1)^4-2(x-1)^2+3}}}{6\left(a^2+7a+12\right)}$$

$$\frac{(x-1)\left(a+(x-1)^2+5\right)}{6\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}}+$$

$$\frac{1}{2}\left(\frac{2x}{3(a+4)^2\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{x}{3(a+4)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}}\right)$$

input
```
Int[x/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(5/2),x]
```

output
```
((5 + a + (-1 + x)^2)*(-1 + x))/(6*(12 + 7*a + a^2)*(3 + a - 2*(-1 + x)^2
 - (-1 + x)^4)^(3/2)) + (x/(3*(4 + a)*(3 + a - 2*(-1 + x)^2 - (-1 + x)^4)^(
3/2)) + (2*x)/(3*(4 + a)^2*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]))/2 + (
((104 + 47*a + 5*a^2 + 4*(7 + 2*a)*(-1 + x)^2)*(-1 + x))/(2*(12 + 7*a + a^
2)*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) + (Sqrt[1 + (-1 + x)^2/(1 - Sq
rt[4 + a])]*Sqrt[1 + (-1 + x)^2/(1 + Sqrt[4 + a])]*(-4*(7 + 2*a)*(((1 - Sq
rt[4 + a])*Sqrt[1 + (-1 + x)^2/(1 - Sqrt[4 + a])]*(-1 + x))/Sqrt[1 + (-1 +
 x)^2/(1 + Sqrt[4 + a])] - ((1 - Sqrt[4 + a])*Sqrt[1 + Sqrt[4 + a]]*Sqrt[1
 + (-1 + x)^2/(1 - Sqrt[4 + a])]*EllipticE[ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4
 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])/(Sqrt[(1 + (-1 + x)^2/(1 - S
qrt[4 + a]))/(1 + (-1 + x)^2/(1 + Sqrt[4 + a]))]*Sqrt[1 + (-1 + x)^2/(1 +
Sqrt[4 + a])])) + ((3 + a)*(16 + 5*a)*Sqrt[1 + Sqrt[4 + a]]*Sqrt[1 + (-1 +
 x)^2/(1 - Sqrt[4 + a])]*EllipticF[ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]],
 (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])/(Sqrt[(1 + (-1 + x)^2/(1 - Sqrt[4 +
a]))/(1 + (-1 + x)^2/(1 + Sqrt[4 + a]))]*Sqrt[1 + (-1 + x)^2/(1 + Sqrt[4 +
 a])])))/(2*(12 + 7*a + a^2)*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]))/(6*
(12 + 7*a + a^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

rule 1089
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1432
```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

rule 1492
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2
 - 4*a*c))   Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p +
7)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 LtQ[p, -1] && IntegerQ[2*p]
```

rule 1514
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[Sqrt[1 + 2*c*(x^2/(b - q))]*(Sqrt
[1 + 2*c*(x^2/(b + q))]/Sqrt[a + b*x^2 + c*x^4])   Int[(d + e*x^2)/(Sqrt[1
+ 2*c*(x^2/(b - q))]*Sqrt[1 + 2*c*(x^2/(b + q))]), x], x]] /; FreeQ[{a, b,
c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

rule 2202
```
Int[(Pn_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{n
 = Expon[Pn, x], k}, Int[Sum[Coeff[Pn, x, 2*k]*x^(2*k), {k, 0, n/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pn, x, 2*k + 1]*x^(2*k), {k, 0, (n -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pn, x]
 &&  !PolyQ[Pn, x^2]
```

rule 2459
```
Int[(Pn_)^(p_.)*(Qx_), x_Symbol] :> With[{S = Coeff[Pn, x, Expon[Pn, x] - 1
]/(Expon[Pn, x]*Coeff[Pn, x, Expon[Pn, x]])}, Subst[Int[ExpandToSum[Pn /. x
 -> x - S, x]^p*ExpandToSum[Qx /. x -> x - S, x], x], x, x + S] /; Binomial
Q[Pn /. x -> x - S, x] || (IntegerQ[Expon[Pn, x]/2] && TrinomialQ[Pn /. x -
> x - S, x])] /; FreeQ[p, x] && PolyQ[Pn, x] && GtQ[Expon[Pn, x], 2] && NeQ
[Coeff[Pn, x, Expon[Pn, x] - 1], 0] && PolyQ[Qx, x] &&  !(MonomialQ[Qx, x]
&& IGtQ[p, 0])
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 2776 vs. $2(474) = 948$.

Time = 1.05 (sec) , antiderivative size = 2777, normalized size of antiderivative = 5.12

method	result	size
default	Expression too large to display	2777
elliptic	Expression too large to display	2777

input
```
int(x/(-x^4+4*x^3-8*x^2+a+8*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
(1/6/(a^2+7*a+12)*x^3+1/6/(a^2+7*a+12)*a*x^2-1/6*(a-2)/(a^2+7*a+12)*x+1/6/
(a^2+7*a+12)*a)*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)/(x^4-4*x^3+8*x^2-a-8*x)^2+2
*(1/6*(7+2*a)/(a^2+7*a+12)^2*x^3+1/6*(a^2-12)/(a^2+7*a+12)^2*x^2-1/24*(3*a
^2-23*a-116)/(a^2+7*a+12)^2*x+1/24*(3*a^2-7*a-60)/(a^2+7*a+12)^2)/(-x^4+4*
x^3-8*x^2+a+8*x)^(1/2)-(1/6*(a^2+23*a+68)/(a^2+7*a+12)^2+1/12*(3*a^2-23*a-
116)/(a^2+7*a+12)^2)*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*((-(-
1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/
(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/
2)))^(1/2)*(x-1+(-1+(a+4)^(1/2))^(1/2))^2*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1-
(-1-(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x
-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1+(-1-(a+4
)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+
(a+4)^(1/2))^(1/2)))^(1/2)/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2)
)/(-1+(a+4)^(1/2))^(1/2)/(-(x-1-(-1+(a+4)^(1/2))^(1/2))*(x-1+(-1+(a+4)^(1/
2))^(1/2))*(x-1-(-1-(a+4)^(1/2))^(1/2))*(x-1+(-1-(a+4)^(1/2))^(1/2)))^(1/2
)*EllipticF(((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+
4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1
+(a+4)^(1/2))^(1/2)))^(1/2),((-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/
2))*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/
2)+(-1+(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/...
```

Fricas [F]

$$\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^{5/2}}\,dx = \int \frac{x}{(-x^4+4\,x^3-8\,x^2+a+8\,x)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x/(-x^4+4*x^3-8*x^2+a+8*x)^(5/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)*x/(x^12 - 12*x^11 + 72*x^10
 - 3*(a - 256)*x^8 - 280*x^9 + 24*(a - 64)*x^7 - 32*(3*a - 70)*x^6 + 48*(5
*a - 48)*x^5 + 3*(a^2 - 128*a + 512)*x^4 - 4*(3*a^2 - 96*a + 128)*x^3 - a^
3 - 24*a^2*x + 24*(a^2 - 8*a)*x^2), x)
```

Sympy [F]

$$\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^{5/2}}\,dx = \int \frac{x}{(a-x^4+4x^3-8x^2+8x)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x/(-x**4+4*x**3-8*x**2+a+8*x)**(5/2),x)
```

output
```
Integral(x/(a - x**4 + 4*x**3 - 8*x**2 + 8*x)**(5/2), x)
```

Maxima [F]

$$\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^{5/2}}\,dx = \int \frac{x}{(-x^4+4\,x^3-8\,x^2+a+8\,x)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x/(-x^4+4*x^3-8*x^2+a+8*x)^(5/2),x, algorithm="maxima")
```

output
```
integrate(x/(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(5/2), x)
```

Giac [F]

$$\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^{5/2}}\,dx = \int \frac{x}{(-x^4+4\,x^3-8\,x^2+a+8\,x)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x/(-x^4+4*x^3-8*x^2+a+8*x)^(5/2),x, algorithm="giac")
```

output
```
integrate(x/(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^{5/2}}\,dx = \int \frac{x}{(-x^4+4\,x^3-8\,x^2+8\,x+a)^{5/2}}\,dx$$

input
```
int(x/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(5/2),x)
```

output
```
int(x/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(5/2), x)
```

Reduce [F]

$$\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^{5/2}}\,dx = \int \frac{}{-x^{12}+12x^{11}-72x^{10}+3a\,x^8+280x^9-24a\,x^7-768x^8+96a\,x}$$

input
```
int(x/(-x^4+4*x^3-8*x^2+a+8*x)^(5/2),x)
```

output
```
int((sqrt(a - x**4 + 4*x**3 - 8*x**2 + 8*x)*x)/(a**3 - 3*a**2*x**4 + 12*a*
*2*x**3 - 24*a**2*x**2 + 24*a**2*x + 3*a*x**8 - 24*a*x**7 + 96*a*x**6 - 24
0*a*x**5 + 384*a*x**4 - 384*a*x**3 + 192*a*x**2 - x**12 + 12*x**11 - 72*x*
*10 + 280*x**9 - 768*x**8 + 1536*x**7 - 2240*x**6 + 2304*x**5 - 1536*x**4
+ 512*x**3),x)
```

3.38 $\int x^2(a+8x-8x^2+4x^3-x^4)^{3/2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 544

$$\int x^2(a+8x-8x^2+4x^3-x^4)^{3/2}\,dx = \frac{3}{8}(4+a)\sqrt{3+a-2(1-x)^2-(1-x)^4}\left(1+(-1+x)^2\right)$$
$$+\frac{1}{4}\left(3+a-2(1-x)^2-(1-x)^4\right)^{3/2}\left(1+(-1+x)^2\right)$$
$$-\frac{2}{315}\left(2(80+27a)+3(20+7a)(1-x)^2\right)\sqrt{3+a-2(1-x)^2-(1-x)^4}(1-x)$$
$$-\frac{1}{63}\left(15+7(1-x)^2\right)\left(3+a-2(1-x)^2-(1-x)^4\right)^{3/2}(1-x)+\frac{3}{8}(4+a)^2\arctan\left(\frac{1+(-1+x)^2}{\sqrt{3+a-2(1-x)^2-(1-

output
```
3/8*(4+a)*(3+a-2*(1-x)^2-(1-x)^4)^(1/2)*(1+(-1+x)^2)+1/4*(3+a-2*(1-x)^2-(1
-x)^4)^(3/2)*(1+(-1+x)^2)-2/315*(160+54*a+3*(20+7*a)*(1-x)^2)*(3+a-2*(1-x)
^2-(1-x)^4)^(1/2)*(1-x)-1/63*(15+7*(1-x)^2)*(3+a-2*(1-x)^2-(1-x)^4)^(3/2)*
(1-x)+3/8*(4+a)^2*arctan((1+(-1+x)^2)/(3+a-2*(1-x)^2-(1-x)^4)^(1/2))-4/315
*(21*a^2+111*a+140)*(-1+(4+a)^(1/2))^(1/2)*(1+(4+a)^(1/2))*(1+(1-x)^2/(1-(
4+a)^(1/2)))^(1/2)*(1+(1-x)^2/(1+(4+a)^(1/2)))^(1/2)*EllipticE((1-x)/(-1+(
4+a)^(1/2))^(1/2),((1-(4+a)^(1/2))/(1+(4+a)^(1/2)))^(1/2))/(3+a-2*(1-x)^2-
(1-x)^4)^(1/2)-4/315*(-1+(4+a)^(1/2))^(1/2)*(160+88*a+12*a^2-(4+a)^(1/2)*(
21*a^2+111*a+140))*(1+(1-x)^2/(1-(4+a)^(1/2)))^(1/2)*(1+(1-x)^2/(1+(4+a)^(
1/2)))^(1/2)*EllipticF((1-x)/(-1+(4+a)^(1/2))^(1/2),((1-(4+a)^(1/2))/(1+(4
+a)^(1/2)))^(1/2))/(3+a-2*(1-x)^2-(1-x)^4)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 8500 vs. $2(544) = 1088$.

Time = 17.25 (sec) , antiderivative size = 8500, normalized size of antiderivative = 15.62

$$\int x^2 \left(a + 8x - 8x^2 + 4x^3 - x^4\right)^{3/2} dx = \text{Result too large to show}$$

input
```
Integrate[x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2),x]
```

output
```
Result too large to show
```

Rubi [A] (warning: unable to verify)

Time = 1.65 (sec) , antiderivative size = 651, normalized size of antiderivative = 1.20, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.643$, Rules used = {2459, 2006, 2202, 27, 1432, 1087, 1087, 1092, 217, 1490, 27, 1490, 27, 1514, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a - x^4 + 4x^3 - 8x^2 + 8x\right)^{3/2} dx$$

$$\Big\downarrow \; 2459$$

$$\int \left((x-1)^2 + 2(x-1) + 1\right) \left(a - (x-1)^4 - 2(x-1)^2 + 3\right)^{3/2} d(x-1)$$

$$\Big\downarrow \; 2006$$

$$\int x^2 \left(a - (x-1)^4 - 2(x-1)^2 + 3\right)^{3/2} d(x-1)$$

$$\Big\downarrow \; 2202$$

$$\int \left((x-1)^2 + 1\right) \left(-(x-1)^4 - 2(x-1)^2 + a + 3\right)^{3/2} d(x-1) + \int 2\left(-(x-1)^4 - 2(x-1)^2 + a + 3\right)^{3/2} (x-1) d(x-1)$$

$$\Big\downarrow \; 27$$

$$\int \left((x-1)^2 + 1\right) \left(-(x-1)^4 - 2(x-1)^2 + a + 3\right)^{3/2} d(x-1) + 2\int \left(-(x-1)^4 - 2(x-1)^2 + a + 3\right)^{3/2} (x-1) d(x-1)$$

$$\Big\downarrow \; 1432$$

$$\int \left(-(x-1)^4 - 2(x-1)^2 + a + 3\right)^{3/2} d(x-1)^2 + \int \left((x-1)^2 + 1\right) \left(-(x-1)^4 - 2(x-1)^2 + a + 3\right)^{3/2} d(x-1)$$

$$\Big\downarrow \; 1087$$

$$\frac{3}{4}(a+4) \int \sqrt{-(x-1)^4 - 2(x-1)^2 + a + 3} d(x-1)^2 + \int \left((x-1)^2 + 1\right) \left(-(x-1)^4 - 2(x-1)^2 + a + 3\right)^{3/2} d(x-1) + \frac{1}{4} x \left(a - (x-1)^4 - 2(x-1)^2 + 3\right)^{3/2}$$

$$\Big\downarrow \; 1087$$

$$\frac{3}{4}(a+4) \left(\frac{1}{2}(a+4) \int \frac{1}{\sqrt{-(x-1)^4 - 2(x-1)^2 + a + 3}} d(x-1)^2 + \frac{1}{2} x \sqrt{a - (x-1)^4 - 2(x-1)^2 + 3}\right) + \int \left((x-1)^2 + 1\right) \left(-(x-1)^4 - 2(x-1)^2 + a + 3\right)^{3/2} d(x-1) + \frac{1}{4} x \left(a - (x-1)^4 - 2(x-1)^2 + 3\right)^{3/2}$$

$$\Big\downarrow \text{1092}$$

$$\frac{3}{4}(a+4)\left((a+4)\int \frac{1}{-(x-1)^4-4}d\left(-\frac{2x}{\sqrt{-(x-1)^4-2(x-1)^2+a+3}}\right)+\frac{1}{2}x\sqrt{a-(x-1)^4-2(x-1)^2+3}\right)+\int \left((x-1)^2+1\right)\left(-(x-1)^4-2(x-1)^2+a+3\right)^{3/2}d(x-1)+\frac{1}{4}x\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}$$

$$\Big\downarrow \text{217}$$

$$\int \left((x-1)^2+1\right)\left(-(x-1)^4-2(x-1)^2+a+3\right)^{3/2}d(x-1)+\frac{3}{4}(a+4)\left(\frac{1}{2}(a+4)\arctan\left(\frac{x}{\sqrt{a-(x-1)^4-2(x-1)^2+3}}\right)+\frac{1}{2}x\sqrt{a-(x-1)^4-2(x-1)^2+3}\right)+\frac{1}{4}x\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}$$

$$\Big\downarrow \text{1490}$$

$$-\frac{1}{21}\int -2\left((7a+20)(x-1)^2+8(a+3)\right)\sqrt{-(x-1)^4-2(x-1)^2+a+3}d(x-1)+\frac{3}{4}(a+4)\left(\frac{1}{2}(a+4)\arctan\left(\frac{x}{\sqrt{a-(x-1)^4-2(x-1)^2+3}}\right)+\frac{1}{2}x\sqrt{a-(x-1)^4-2(x-1)^2+3}\right)+\frac{1}{63}\left(7(x-1)^2+15\right)(x-1)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}+\frac{1}{4}x\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{21}\int \left((7a+20)(x-1)^2+8(a+3)\right)\sqrt{-(x-1)^4-2(x-1)^2+a+3}d(x-1)+\frac{3}{4}(a+4)\left(\frac{1}{2}(a+4)\arctan\left(\frac{x}{\sqrt{a-(x-1)^4-2(x-1)^2+3}}\right)+\frac{1}{2}x\sqrt{a-(x-1)^4-2(x-1)^2+3}\right)+\frac{1}{63}\left(7(x-1)^2+15\right)(x-1)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}+\frac{1}{4}x\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}$$

$$\Big\downarrow \text{1490}$$

$$\frac{2}{21}\left(\frac{1}{15}(x-1)\left(3(7a+20)(x-1)^2+2(27a+80)\right)\sqrt{a-(x-1)^4-2(x-1)^2+3}-\frac{1}{15}\int -\frac{2\left(\left(21a^2+111a+\right.\right.}{\sqrt{-(x-}}\right.$$

$$\frac{3}{4}(a+4)\left(\frac{1}{2}(a+4)\arctan\left(\frac{x}{\sqrt{a-(x-1)^4-2(x-1)^2+3}}\right)+\frac{1}{2}x\sqrt{a-(x-1)^4-2(x-1)^2+3}\right)+$$

$$\frac{1}{63}\left(7(x-1)^2+15\right)(x-1)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}+$$

$$\frac{1}{4}x\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}$$

$$\Big\downarrow \; 27$$

$$\frac{2}{21}\left(\frac{2}{15}\int\frac{\left(21a^2+111a+140\right)(x-1)^2+(a+3)(33a+100)}{\sqrt{-(x-1)^4-2(x-1)^2+a+3}}d(x-1)+\frac{1}{15}(x-1)\left(3(7a+20)(x-1)^2+2(27a\right.\right.$$

$$\frac{3}{4}(a+4)\left(\frac{1}{2}(a+4)\arctan\left(\frac{x}{\sqrt{a-(x-1)^4-2(x-1)^2+3}}\right)+\frac{1}{2}x\sqrt{a-(x-1)^4-2(x-1)^2+3}\right)+$$

$$\frac{1}{63}\left(7(x-1)^2+15\right)(x-1)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}+$$

$$\frac{1}{4}x\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}$$

$$\Big\downarrow \; 1514$$

$$\frac{2}{21}\left(\frac{2\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}\int\frac{\left(21a^2+111a+140\right)(x-1)^2+(a+3)(33a+100)}{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}d(x-1)}{15\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{1}{15}(x-1)\left(3(7a+20)(x-1\right.\right.$$

$$\frac{3}{4}(a+4)\left(\frac{1}{2}(a+4)\arctan\left(\frac{x}{\sqrt{a-(x-1)^4-2(x-1)^2+3}}\right)+\frac{1}{2}x\sqrt{a-(x-1)^4-2(x-1)^2+3}\right)+$$

$$\frac{1}{63}\left(7(x-1)^2+15\right)(x-1)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}+$$

$$\frac{1}{4}x\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}$$

$$\Big\downarrow \; 406$$

$$\frac{2}{21}\left(\frac{2\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}\left((21a^2+111a+140)\int\frac{(x-1)^2}{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}d(x-1)+(a+3)(33a+100)\right.}{15\sqrt{a-(x-1)^4-2(x-1)^2+3}}\right.$$

$$\frac{3}{4}(a+4)\left(\frac{1}{2}(a+4)\arctan\left(\frac{x}{\sqrt{a-(x-1)^4-2(x-1)^2+3}}\right)+\frac{1}{2}x\sqrt{a-(x-1)^4-2(x-1)^2+3}\right)+$$

$$\frac{1}{63}\left(7(x-1)^2+15\right)(x-1)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}+$$

$$\frac{1}{4}x\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}$$

$$\Big\downarrow \text{ 320}$$

$$\frac{2}{21}\left(\frac{2\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}\left((21a^2+111a+140)\int\frac{(x-1)^2}{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}d(x-1)+\frac{(a+3)(33a+100)\sqrt{\sqrt{a+}}}{}\right.}{15\sqrt{a-(x-1)^4-2(x-1)^2+3}}\right.$$

$$\frac{3}{4}(a+4)\left(\frac{1}{2}(a+4)\arctan\left(\frac{x}{\sqrt{a-(x-1)^4-2(x-1)^2+3}}\right)+\frac{1}{2}x\sqrt{a-(x-1)^4-2(x-1)^2+3}\right)+$$

$$\frac{1}{63}\left(7(x-1)^2+15\right)(x-1)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}+$$

$$\frac{1}{4}x\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}$$

$$\Big\downarrow \text{ 388}$$

$$\frac{2}{21}\left(\frac{2\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}\left((21a^2+111a+140)\left(\frac{(1-\sqrt{a+4})(x-1)\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}}{\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}-(1-\sqrt{a+4})\int\frac{\sqrt{\frac{(x-1)}{1-\sqrt{a+}}}}{\left(\frac{(x-1)^2}{\sqrt{a+4+1}}-\right.}\right.\right.}{15\sqrt{a-(x-1)^4-2(x-}}\right.$$

$$\frac{3}{4}(a+4)\left(\frac{1}{2}(a+4)\arctan\left(\frac{x}{\sqrt{a-(x-1)^4-2(x-1)^2+3}}\right)+\frac{1}{2}x\sqrt{a-(x-1)^4-2(x-1)^2+3}\right)+\frac{1}{63}\left(7(x-1)^2+15\right)(x-1)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}+\frac{1}{4}x\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}$$

$$\Big\downarrow \text{313}$$

$$\frac{2}{21}\left(\frac{2\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}\left((21a^2+111a+140)\left(\frac{(1-\sqrt{a+4})(x-1)\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}}{\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}-\frac{(1-\sqrt{a+4})\sqrt{\sqrt{a+4}+1}\sqrt{\frac{(x-1}{1-\sqrt{a}}}}{\sqrt{\frac{\frac{(x-1}{1-\sqrt{a}}}{\frac{(x-1}{\sqrt{a+4}}}}}\right.\right.}{15\sqrt{a-(x-1)}}\right.$$

$$\frac{3}{4}(a+4)\left(\frac{1}{2}(a+4)\arctan\left(\frac{x}{\sqrt{a-(x-1)^4-2(x-1)^2+3}}\right)+\frac{1}{2}x\sqrt{a-(x-1)^4-2(x-1)^2+3}\right)+\frac{1}{63}\left(7(x-1)^2+15\right)(x-1)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}+\frac{1}{4}x\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}$$

input
```
Int[x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2),x]
```

output
```
((15 + 7*(-1 + x)^2)*(3 + a - 2*(-1 + x)^2 - (-1 + x)^4)^(3/2)*(-1 + x))/6
3 + ((3 + a - 2*(-1 + x)^2 - (-1 + x)^4)^(3/2)*x)/4 + (3*(4 + a)*((Sqrt[3
+ a - 2*(-1 + x)^2 - (-1 + x)^4]*x)/2 + ((4 + a)*ArcTan[x/Sqrt[3 + a - 2*(
-1 + x)^2 - (-1 + x)^4]])/2))/4 + (2*(((2*(80 + 27*a) + 3*(20 + 7*a)*(-1 +
 x)^2)*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]*(-1 + x))/15 + (2*Sqrt[1 +
(-1 + x)^2/(1 - Sqrt[4 + a])]*Sqrt[1 + (-1 + x)^2/(1 + Sqrt[4 + a])]*((140
 + 111*a + 21*a^2)*(((1 - Sqrt[4 + a])*Sqrt[1 + (-1 + x)^2/(1 - Sqrt[4 + a
])]*(-1 + x))/Sqrt[1 + (-1 + x)^2/(1 + Sqrt[4 + a])] - ((1 - Sqrt[4 + a])*
Sqrt[1 + Sqrt[4 + a]]*Sqrt[1 + (-1 + x)^2/(1 - Sqrt[4 + a])]*EllipticE[Arc
Tan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])/
(Sqrt[(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (-1 + x)^2/(1 + Sqrt[4 + a])
)]*Sqrt[1 + (-1 + x)^2/(1 + Sqrt[4 + a])])) + ((3 + a)*(100 + 33*a)*Sqrt[1
 + Sqrt[4 + a]]*Sqrt[1 + (-1 + x)^2/(1 - Sqrt[4 + a])]*EllipticF[ArcTan[(-
1 + x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])/(Sqrt[
(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (-1 + x)^2/(1 + Sqrt[4 + a]))]*Sqr
t[1 + (-1 + x)^2/(1 + Sqrt[4 + a])])))/(15*Sqrt[3 + a - 2*(-1 + x)^2 - (-1
 + x)^4])))/21
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406

```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 1087

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1432

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

rule 1490

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Simp[2*(p/(c*(4*p + 1)*(4*p + 3)))
 Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3)
- b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

rule 1514
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[Sqrt[1 + 2*c*(x^2/(b - q))]*(Sqrt
[1 + 2*c*(x^2/(b + q))]/Sqrt[a + b*x^2 + c*x^4])   Int[(d + e*x^2)/(Sqrt[1
+ 2*c*(x^2/(b - q))]*Sqrt[1 + 2*c*(x^2/(b + q))]), x], x]] /; FreeQ[{a, b,
c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

rule 2006
```
Int[(u_.)*(Px_), x_Symbol] :> With[{a = Rt[Coeff[Px, x, 0], Expon[Px, x]],
b = Rt[Coeff[Px, x, Expon[Px, x]], Expon[Px, x]]}, Int[u*(a + b*x)^Expon[Px
, x], x] /; EqQ[Px, (a + b*x)^Expon[Px, x]]] /; PolyQ[Px, x] && GtQ[Expon[P
x, x], 1] && NeQ[Coeff[Px, x, 0], 0] &&  !MatchQ[Px, (a_.)*(v_)^Expon[Px, x
] /; FreeQ[a, x] && LinearQ[v, x]]
```

rule 2202
```
Int[(Pn_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{n
 = Expon[Pn, x], k}, Int[Sum[Coeff[Pn, x, 2*k]*x^(2*k), {k, 0, n/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pn, x, 2*k + 1]*x^(2*k), {k, 0, (n -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pn, x]
 &&  !PolyQ[Pn, x^2]
```

rule 2459
```
Int[(Pn_)^(p_.)*(Qx_), x_Symbol] :> With[{S = Coeff[Pn, x, Expon[Pn, x] - 1
]/(Expon[Pn, x]*Coeff[Pn, x, Expon[Pn, x]])}, Subst[Int[ExpandToSum[Pn /. x
 -> x - S, x]^p*ExpandToSum[Qx /. x -> x - S, x], x], x, x + S] /; Binomial
Q[Pn /. x -> x - S, x] || (IntegerQ[Expon[Pn, x]/2] && TrinomialQ[Pn /. x -
> x - S, x])] /; FreeQ[p, x] && PolyQ[Pn, x] && GtQ[Expon[Pn, x], 2] && NeQ
[Coeff[Pn, x, Expon[Pn, x] - 1], 0] && PolyQ[Qx, x] &&  !(MonomialQ[Qx, x]
&& IGtQ[p, 0])
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 2732 vs. $2(471) = 942$.

Time = 4.30 (sec) , antiderivative size = 2733, normalized size of antiderivative = 5.02

method	result	size
default	Expression too large to display	2733
elliptic	Expression too large to display	2733
risch	Expression too large to display	3625

input
```
int(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/9*x^7*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)+19/36*x^6*(-x^4+4*x^3-8*x^2+a+8*x)
^(1/2)-163/126*x^5*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)+71/42*x^4*(-x^4+4*x^3-8*
x^2+a+8*x)^(1/2)+(11/45*a-16/63)*x^3*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)+(-13/1
20*a-5/18)*x^2*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)+(9/140*a+23/63)*x*(-x^4+4*x^
3-8*x^2+a+8*x)^(1/2)+(107/252*a+101/63)*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)-(-(
9/140*a+23/63)*a-107/63*a-404/63)*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))
^(1/2))*((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(
1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+
4)^(1/2))^(1/2)))^(1/2)*(x-1+(-1+(a+4)^(1/2))^(1/2))^2*(-2*(-1+(a+4)^(1/2)
)^(1/2)*(x-1-(-1-(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/
2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)*(-2*(-1+(a+4)^(1/2))^(1/2)*
(x-1+(-1-(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/
2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)
^(1/2))^(1/2))/(-1+(a+4)^(1/2))^(1/2)/(-(x-1-(-1+(a+4)^(1/2))^(1/2))*(x-1+
(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1-(a+4)^(1/2))^(1/2))*(x-1+(-1-(a+4)^(1/2))
^(1/2)))^(1/2)*EllipticF(((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))
*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1
/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2),((-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+
4)^(1/2))^(1/2))*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+
4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+...
```

Fricas [F]

$$\int x^2\left(a+8x-8x^2+4x^3-x^4\right)^{3/2}\,dx=\int \left(-x^4+4\,x^3-8\,x^2+a+8\,x\right)^{\frac{3}{2}}x^2\,dx$$

input
```
integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x, algorithm="fricas")
```

output
```
integral(-(x^6 - 4*x^5 + 8*x^4 - a*x^2 - 8*x^3)*sqrt(-x^4 + 4*x^3 - 8*x^2
+ a + 8*x), x)
```

Sympy [F]

$$\int x^2\left(a+8x-8x^2+4x^3-x^4\right)^{3/2}\,dx=\int x^2\left(a-x^4+4x^3-8x^2+8x\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(x**2*(-x**4+4*x**3-8*x**2+a+8*x)**(3/2),x)
```

output
```
Integral(x**2*(a - x**4 + 4*x**3 - 8*x**2 + 8*x)**(3/2), x)
```

Maxima [F]

$$\int x^2\left(a+8x-8x^2+4x^3-x^4\right)^{3/2}\,dx=\int\left(-x^4+4\,x^3-8\,x^2+a+8\,x\right)^{\frac{3}{2}}x^2\,dx$$

input
```
integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(3/2)*x^2, x)
```

Giac [F]

$$\int x^2\left(a+8x-8x^2+4x^3-x^4\right)^{3/2}\,dx=\int\left(-x^4+4\,x^3-8\,x^2+a+8\,x\right)^{\frac{3}{2}}x^2\,dx$$

input
```
integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x, algorithm="giac")
```

output
```
integrate((-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(3/2)*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \left(a+8x-8x^2+4x^3-x^4\right)^{3/2} \, dx = \int x^2 \left(-x^4+4\,x^3-8\,x^2+8\,x+a\right)^{3/2} dx$$

input

```
int(x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2),x)
```

output

```
int(x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2), x)
```

Reduce [F]

$$\int x^2(a+8x-8x^2+4x^3-x^4)^{3/2}\,dx = \frac{2\sqrt{-x^4+4x^3-8x^2+a+8x}\,a^2}{45} + \frac{11\sqrt{-x^4+4x^3-8x^2+a+8x}\,a\,x^3}{45} - \frac{13\sqrt{-x^4+4x^3-8x^2+a+8x}\,a\,x^2}{120} + \frac{9\sqrt{-x^4+4x^3-8x^2+a+8x}\,ax}{140} + \frac{277\sqrt{-x^4+4x^3-8x^2+a+8x}\,a}{420} - \frac{\sqrt{-x^4+4x^3-8x^2+a+8x}\,x^7}{9} + \frac{19\sqrt{-x^4+4x^3-8x^2+a+8x}\,x^6}{36} - \frac{163\sqrt{-x^4+4x^3-8x^2+a+8x}\,x^5}{126} + \frac{71\sqrt{-x^4+4x^3-8x^2+a+8x}\,x^4}{42} - \frac{16\sqrt{-x^4+4x^3-8x^2+a+8x}\,x^3}{63} - \frac{5\sqrt{-x^4+4x^3-8x^2+a+8x}\,x^2}{18} + \frac{23\sqrt{-x^4+4x^3-8x^2+a+8x}\,x}{63} + \frac{359\sqrt{-x^4+4x^3-8x^2+a+8x}}{189} - \frac{61\left(\int \frac{\sqrt{-x^4+4x^3-8x^2+a+8x}}{-x^4+4x^3-8x^2+a+8x}dx\right)a^2}{252} - \frac{946\left(\int \frac{\sqrt{-x^4+4x^3-8x^2+a+8x}}{-x^4+4x^3-8x^2+a+8x}dx\right)a}{315} - \frac{1436\left(\int \frac{\sqrt{-x^4+4x^3-8x^2+a+8x}}{-x^4+4x^3-8x^2+a+8x}dx\right)}{189} + \frac{4\left(\int \frac{\sqrt{-x^4+4x^3-8x^2+a+8x}\,x^3}{-x^4+4x^3-8x^2+a+8x}dx\right)a^2}{45} + \frac{148\left(\int \frac{\sqrt{-x^4+4x^3-8x^2+a+8x}\,x^3}{-x^4+4x^3-8x^2+a+8x}dx\right)a}{315} + \frac{16\left(\int \frac{\sqrt{-x^4+4x^3-8x^2+a+8x}\,x^3}{-x^4+4x^3-8x^2+a+8x}dx\right)}{27} + \frac{103\left(\int \frac{\sqrt{-x^4+4x^3-8x^2+a+8x}\,x}{-x^4+4x^3-8x^2+a+8x}dx\right)a^2}{180} + \frac{1594\left(\int \frac{\sqrt{-x^4+4x^3-8x^2+a+8x}\,x}{-x^4+4x^3-8x^2+a+8x}dx\right)a}{315} + \frac{292\left(\int \frac{\sqrt{-x^4+4x^3-8x^2+a+8x}\,x}{-x^4+4x^3-8x^2+a+8x}dx\right)}{27}$$

input
```
int(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x)
```

output
```
(336*sqrt(a - x**4 + 4*x**3 - 8*x**2 + 8*x)*a**2 + 1848*sqrt(a - x**4 + 4*
x**3 - 8*x**2 + 8*x)*a*x**3 - 819*sqrt(a - x**4 + 4*x**3 - 8*x**2 + 8*x)*a
*x**2 + 486*sqrt(a - x**4 + 4*x**3 - 8*x**2 + 8*x)*a*x + 4986*sqrt(a - x**
4 + 4*x**3 - 8*x**2 + 8*x)*a - 840*sqrt(a - x**4 + 4*x**3 - 8*x**2 + 8*x)*
x**7 + 3990*sqrt(a - x**4 + 4*x**3 - 8*x**2 + 8*x)*x**6 - 9780*sqrt(a - x*
*4 + 4*x**3 - 8*x**2 + 8*x)*x**5 + 12780*sqrt(a - x**4 + 4*x**3 - 8*x**2 +
 8*x)*x**4 - 1920*sqrt(a - x**4 + 4*x**3 - 8*x**2 + 8*x)*x**3 - 2100*sqrt(
a - x**4 + 4*x**3 - 8*x**2 + 8*x)*x**2 + 2760*sqrt(a - x**4 + 4*x**3 - 8*x
**2 + 8*x)*x + 14360*sqrt(a - x**4 + 4*x**3 - 8*x**2 + 8*x) - 1830*int(sqr
t(a - x**4 + 4*x**3 - 8*x**2 + 8*x)/(a - x**4 + 4*x**3 - 8*x**2 + 8*x),x)*
a**2 - 22704*int(sqrt(a - x**4 + 4*x**3 - 8*x**2 + 8*x)/(a - x**4 + 4*x**3
 - 8*x**2 + 8*x),x)*a - 57440*int(sqrt(a - x**4 + 4*x**3 - 8*x**2 + 8*x)/(
a - x**4 + 4*x**3 - 8*x**2 + 8*x),x) + 672*int((sqrt(a - x**4 + 4*x**3 - 8
*x**2 + 8*x)*x**3)/(a - x**4 + 4*x**3 - 8*x**2 + 8*x),x)*a**2 + 3552*int((
sqrt(a - x**4 + 4*x**3 - 8*x**2 + 8*x)*x**3)/(a - x**4 + 4*x**3 - 8*x**2 +
 8*x),x)*a + 4480*int((sqrt(a - x**4 + 4*x**3 - 8*x**2 + 8*x)*x**3)/(a - x
**4 + 4*x**3 - 8*x**2 + 8*x),x) + 4326*int((sqrt(a - x**4 + 4*x**3 - 8*x**
2 + 8*x)*x)/(a - x**4 + 4*x**3 - 8*x**2 + 8*x),x)*a**2 + 38256*int((sqrt(a
 - x**4 + 4*x**3 - 8*x**2 + 8*x)*x)/(a - x**4 + 4*x**3 - 8*x**2 + 8*x),x)*
a + 81760*int((sqrt(a - x**4 + 4*x**3 - 8*x**2 + 8*x)*x)/(a - x**4 + 4*...
```

3.39 $\int x^2\sqrt{a+8x-8x^2+4x^3-x^4}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 430

$$\begin{aligned}\int x^2\sqrt{a+8x-8x^2+4x^3-x^4}\,dx &= \frac{1}{2}\sqrt{3+a-2(1-x)^2-(1-x)^4}\left(1+(-1+x)^2\right)\\
&-\frac{1}{15}\left(7+3(1-x)^2\right)\sqrt{3+a-2(1-x)^2-(1-x)^4}(1-x)\\
&+\frac{1}{2}(4+a)\arctan\left(\frac{1+(-1+x)^2}{\sqrt{3+a-2(1-x)^2-(1-x)^4}}\right)\\
&-\frac{2(8+3a)\sqrt{-1+\sqrt{4+a}}\left(1+\sqrt{4+a}\right)\sqrt{1+\frac{(1-x)^2}{1-\sqrt{4+a}}}\sqrt{1+\frac{(1-x)^2}{1+\sqrt{4+a}}}E\left(\arcsin\left(\frac{1-x}{\sqrt{-1+\sqrt{4+a}}}\right)\middle|\frac{1-\sqrt{4+a}}{1+\sqrt{4+a}}\right)}{15\sqrt{3+a-2(1-x)^2-(1-x)^4}}\\
&-\frac{2\sqrt{-1+\sqrt{4+a}}\left(4+a-\sqrt{4+a}(8+3a)\right)\sqrt{1+\frac{(1-x)^2}{1-\sqrt{4+a}}}\sqrt{1+\frac{(1-x)^2}{1+\sqrt{4+a}}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1-x}{\sqrt{-1+\sqrt{4+a}}}\right)\right.}{15\sqrt{3+a-2(1-x)^2-(1-x)^4}}\end{aligned}$$

output
```
1/2*(3+a-2*(1-x)^2-(1-x)^4)^(1/2)*(1+(-1+x)^2)-1/15*(7+3*(1-x)^2)*(3+a-2*(
1-x)^2-(1-x)^4)^(1/2)*(1-x)+1/2*(4+a)*arctan((1+(-1+x)^2)/(3+a-2*(1-x)^2-(
1-x)^4)^(1/2))-2/15*(8+3*a)*(-1+(4+a)^(1/2))^(1/2)*(1+(4+a)^(1/2))*(1+(1-x
)^2/(1-(4+a)^(1/2)))^(1/2)*(1+(1-x)^2/(1+(4+a)^(1/2)))^(1/2)*EllipticE((1-
x)/(-1+(4+a)^(1/2))^(1/2),((1-(4+a)^(1/2))/(1+(4+a)^(1/2)))^(1/2))/(3+a-2*
(1-x)^2-(1-x)^4)^(1/2)-2/15*(-1+(4+a)^(1/2))^(1/2)*(4+a-(4+a)^(1/2)*(8+3*a
))*(1+(1-x)^2/(1-(4+a)^(1/2)))^(1/2)*(1+(1-x)^2/(1+(4+a)^(1/2)))^(1/2)*Ell
ipticF((1-x)/(-1+(4+a)^(1/2))^(1/2),((1-(4+a)^(1/2))/(1+(4+a)^(1/2)))^(1/2
))/(3+a-2*(1-x)^2-(1-x)^4)^(1/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 5647 vs. $2(430) = 860$.

Time = 16.59 (sec) , antiderivative size = 5647, normalized size of antiderivative = 13.13

$$\int x^2\sqrt{a+8x-8x^2+4x^3-x^4}\,dx = \text{Result too large to show}$$

input
```
Integrate[x^2*Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4],x]
```

output
```
Result too large to show
```

Rubi [A] (warning: unable to verify)

Time = 1.33 (sec) , antiderivative size = 555, normalized size of antiderivative = 1.29, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.536$, Rules used = {2459, 2006, 2202, 27, 1432, 1087, 1092, 217, 1490, 27, 1514, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{a-x^4+4x^3-8x^2+8x}\,dx$$

$$\Big\downarrow \; 2459$$

$$\int \left((x-1)^2+2(x-1)+1\right)\sqrt{a-(x-1)^4-2(x-1)^2+3}d(x-1)$$

$$\Big\downarrow \text{ 2006}$$

$$\int x^2\sqrt{a-(x-1)^4-2(x-1)^2+3}d(x-1)$$

$$\Big\downarrow \text{ 2202}$$

$$\int \left((x-1)^2+1\right)\sqrt{-(x-1)^4-2(x-1)^2+a+3}d(x-1)+\int 2\sqrt{-(x-1)^4-2(x-1)^2+a+3}(x-1)d(x-1)$$

$$\Big\downarrow \text{ 27}$$

$$\int \left((x-1)^2+1\right)\sqrt{-(x-1)^4-2(x-1)^2+a+3}d(x-1)+2\int \sqrt{-(x-1)^4-2(x-1)^2+a+3}(x-1)d(x-1)$$

$$\Big\downarrow \text{ 1432}$$

$$\int \sqrt{-(x-1)^4-2(x-1)^2+a+3}d(x-1)^2+\int \left((x-1)^2+1\right)\sqrt{-(x-1)^4-2(x-1)^2+a+3}d(x-1)$$

$$\Big\downarrow \text{ 1087}$$

$$\frac{1}{2}(a+4)\int \frac{1}{\sqrt{-(x-1)^4-2(x-1)^2+a+3}}d(x-1)^2+\int \left((x-1)^2+1\right)\sqrt{-(x-1)^4-2(x-1)^2+a+3}d(x-1)+\frac{1}{2}x\sqrt{a-(x-1)^4-2(x-1)^2+3}$$

$$\Big\downarrow \text{ 1092}$$

$$(a+4)\int \frac{1}{-(x-1)^4-4}d\left(-\frac{2x}{\sqrt{-(x-1)^4-2(x-1)^2+a+3}}\right)+\int \left((x-1)^2+1\right)\sqrt{-(x-1)^4-2(x-1)^2+a+3}d(x-1)+\frac{1}{2}x\sqrt{a-(x-1)^4-2(x-1)^2+3}$$

$$\Big\downarrow \text{ 217}$$

$$\int \left((x-1)^2+1\right)\sqrt{-(x-1)^4-2(x-1)^2+a+3}d(x-1)+\frac{1}{2}(a+4)\arctan\left(\frac{x}{\sqrt{a-(x-1)^4-2(x-1)^2+3}}\right)+\frac{1}{2}x\sqrt{a-(x-1)^4-2(x-1)^2+3}$$

$$\Big\downarrow \text{ 1490}$$

$$-\frac{1}{15}\int -\frac{2\left((3a+8)(x-1)^2+4(a+3)\right)}{\sqrt{-(x-1)^4-2(x-1)^2+a+3}}d(x-1)+\frac{1}{2}(a+4)\arctan\left(\frac{x}{\sqrt{a-(x-1)^4-2(x-1)^2+3}}\right)+\frac{1}{15}\left(3(x-1)^2+7\right)(x-1)\sqrt{a-(x-1)^4-2(x-1)^2+3}+\frac{1}{2}x\sqrt{a-(x-1)^4-2(x-1)^2+3}$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{15}\int \frac{(3a+8)(x-1)^2+4(a+3)}{\sqrt{-(x-1)^4-2(x-1)^2+a+3}}d(x-1)+\frac{1}{2}(a+4)\arctan\left(\frac{x}{\sqrt{a-(x-1)^4-2(x-1)^2+3}}\right)+\frac{1}{15}\left(3(x-1)^2+7\right)(x-1)\sqrt{a-(x-1)^4-2(x-1)^2+3}+\frac{1}{2}x\sqrt{a-(x-1)^4-2(x-1)^2+3}$$

$$\Big\downarrow \text{1514}$$

$$\frac{2\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}\int \frac{(3a+8)(x-1)^2+4(a+3)}{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}d(x-1)}{15\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{1}{2}(a+4)\arctan\left(\frac{x}{\sqrt{a-(x-1)^4-2(x-1)^2+3}}\right)+\frac{1}{15}\left(3(x-1)^2+7\right)(x-1)\sqrt{a-(x-1)^4-2(x-1)^2+3}+\frac{1}{2}x\sqrt{a-(x-1)^4-2(x-1)^2+3}$$

$$\Big\downarrow \text{406}$$

$$\frac{2\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}\left(4(a+3)\int \frac{1}{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}d(x-1)+(3a+8)\int \frac{(x-1)^2}{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}d(x\right.}{15\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

$$\frac{1}{2}(a+4)\arctan\left(\frac{x}{\sqrt{a-(x-1)^4-2(x-1)^2+3}}\right)+\frac{1}{15}\left(3(x-1)^2+7\right)(x-1)\sqrt{a-(x-1)^4-2(x-1)^2+3}+\frac{1}{2}x\sqrt{a-(x-1)^4-2(x-1)^2+3}$$

$$\Big\downarrow \text{320}$$

$$\frac{2\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}\left((3a+8)\int \frac{(x-1)^2}{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}d(x-1)+\frac{4(a+3)\sqrt{\sqrt{a+4}+1}\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\,\text{EllipticF}\left(\text{arc}\right.}{\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{\frac{(x-1}{\sqrt{a+4}}}}\right.}{15\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

$$\frac{1}{2}(a+4)\arctan\left(\frac{x}{\sqrt{a-(x-1)^4-2(x-1)^2+3}}\right)+\frac{1}{15}\left(3(x-1)^2+7\right)(x-1)\sqrt{a-(x-1)^4-2(x-1)^2+3}+\frac{1}{2}x\sqrt{a-(x-1)^4-2(x-1)^2+3}$$

$\Big\downarrow$ 388

$$\frac{2\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}\left((3a+8)\left(\frac{(1-\sqrt{a+4})(x-1)\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}}{\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}-(1-\sqrt{a+4})\int \frac{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}}{\left(\frac{(x-1)^2}{\sqrt{a+4}+1}+1\right)^{3/2}}d(x-1)\right)-\right.}{15\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

$$\frac{1}{2}(a+4)\arctan\left(\frac{x}{\sqrt{a-(x-1)^4-2(x-1)^2+3}}\right)+\frac{1}{15}\left(3(x-1)^2+7\right)(x-1)\sqrt{a-(x-1)^4-2(x-1)^2+3}+\frac{1}{2}x\sqrt{a-(x-1)^4-2(x-1)^2+3}$$

$\Big\downarrow$ 313

$$\frac{1}{2}(a+4)\arctan\left(\frac{x}{\sqrt{a-(x-1)^4-2(x-1)^2+3}}\right)+$$

$$\frac{2\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}\left(\frac{4(a+3)\sqrt{\sqrt{a+4}+1}\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\,\text{EllipticF}\left(\arctan\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right),-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}+(3a+8)\left(\frac{(1-\sqrt{a-}}{}\right.\right.}{15\sqrt{a-(x-1)^4-2(x-1)^2+}}$$

$$\frac{1}{15}\left(3(x-1)^2+7\right)(x-1)\sqrt{a-(x-1)^4-2(x-1)^2+3}+\frac{1}{2}x\sqrt{a-(x-1)^4-2(x-1)^2+3}$$

input
```
Int[x^2*Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4],x]
```

output
```
((7 + 3*(-1 + x)^2)*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]*(-1 + x))/15 +
 (Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]*x)/2 + ((4 + a)*ArcTan[x/Sqrt[3
+ a - 2*(-1 + x)^2 - (-1 + x)^4]])/2 + (2*Sqrt[1 + (-1 + x)^2/(1 - Sqrt[4
+ a])]*Sqrt[1 + (-1 + x)^2/(1 + Sqrt[4 + a])]*((8 + 3*a)*(((1 - Sqrt[4 + a
])*Sqrt[1 + (-1 + x)^2/(1 - Sqrt[4 + a])]*(-1 + x))/Sqrt[1 + (-1 + x)^2/(1
 + Sqrt[4 + a])] - ((1 - Sqrt[4 + a])*Sqrt[1 + Sqrt[4 + a]]*Sqrt[1 + (-1 +
 x)^2/(1 - Sqrt[4 + a])]*EllipticE[ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]],
 (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])/(Sqrt[(1 + (-1 + x)^2/(1 - Sqrt[4 +
a]))/(1 + (-1 + x)^2/(1 + Sqrt[4 + a]))]*Sqrt[1 + (-1 + x)^2/(1 + Sqrt[4 +
 a])])) + (4*(3 + a)*Sqrt[1 + Sqrt[4 + a]]*Sqrt[1 + (-1 + x)^2/(1 - Sqrt[4
 + a])]*EllipticF[ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])
/(1 - Sqrt[4 + a])])/(Sqrt[(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (-1 + x
)^2/(1 + Sqrt[4 + a]))]*Sqrt[1 + (-1 + x)^2/(1 + Sqrt[4 + a])])))/(15*Sqrt
[3 + a - 2*(-1 + x)^2 - (-1 + x)^4])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1432
```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

rule 1490
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Simp[2*(p/(c*(4*p + 1)*(4*p + 3)))
 Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3)
- b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

rule 1514
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[Sqrt[1 + 2*c*(x^2/(b - q))]*(Sqrt
[1 + 2*c*(x^2/(b + q))]/Sqrt[a + b*x^2 + c*x^4])   Int[(d + e*x^2)/(Sqrt[1
+ 2*c*(x^2/(b - q))]*Sqrt[1 + 2*c*(x^2/(b + q))]), x], x]] /; FreeQ[{a, b,
c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

rule 2006
```
Int[(u_.)*(Px_), x_Symbol] :> With[{a = Rt[Coeff[Px, x, 0], Expon[Px, x]],
b = Rt[Coeff[Px, x, Expon[Px, x]], Expon[Px, x]]}, Int[u*(a + b*x)^Expon[Px
, x], x] /; EqQ[Px, (a + b*x)^Expon[Px, x]]] /; PolyQ[Px, x] && GtQ[Expon[P
x, x], 1] && NeQ[Coeff[Px, x, 0], 0] &&  !MatchQ[Px, (a_.)*(v_)^Expon[Px, x
] /; FreeQ[a, x] && LinearQ[v, x]]
```

rule 2202
```
Int[(Pn_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{n
 = Expon[Pn, x], k}, Int[Sum[Coeff[Pn, x, 2*k]*x^(2*k), {k, 0, n/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pn, x, 2*k + 1]*x^(2*k), {k, 0, (n -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pn, x]
 &&  !PolyQ[Pn, x^2]
```

rule 2459
```
Int[(Pn_)^(p_.)*(Qx_), x_Symbol] :> With[{S = Coeff[Pn, x, Expon[Pn, x] - 1
]/(Expon[Pn, x]*Coeff[Pn, x, Expon[Pn, x]])}, Subst[Int[ExpandToSum[Pn /. x
 -> x - S, x]^p*ExpandToSum[Qx /. x -> x - S, x], x], x, x + S] /; Binomial
Q[Pn /. x -> x - S, x] || (IntegerQ[Expon[Pn, x]/2] && TrinomialQ[Pn /. x -
> x - S, x])] /; FreeQ[p, x] && PolyQ[Pn, x] && GtQ[Expon[Pn, x], 2] && NeQ
[Coeff[Pn, x, Expon[Pn, x] - 1], 0] && PolyQ[Qx, x] &&  !(MonomialQ[Qx, x]
&& IGtQ[p, 0])
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 2581 vs. $2(368) = 736$.

Time = 3.85 (sec) , antiderivative size = 2582, normalized size of antiderivative = 6.00

method	result	size
default	Expression too large to display	2582
elliptic	Expression too large to display	2582
risch	Expression too large to display	3044

input
```
int(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/5*x^3*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)-1/10*x^2*(-x^4+4*x^3-8*x^2+a+8*x)^(
1/2)+1/15*x*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)+1/3*(-x^4+4*x^3-8*x^2+a+8*x)^(1
/2)-(-1/15*a-4/3)*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*((-(-1-(
a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(
-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2))
)^(1/2)*(x-1+(-1+(a+4)^(1/2))^(1/2))^2*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1-(-1
-(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+
(-1+(a+4)^(1/2))^(1/2)))^(1/2)*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1+(-1-(a+4)^(
1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+
4)^(1/2))^(1/2)))^(1/2)/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(
-1+(a+4)^(1/2))^(1/2)/(-(x-1-(-1+(a+4)^(1/2))^(1/2))*(x-1+(-1+(a+4)^(1/2))
^(1/2))*(x-1-(-1-(a+4)^(1/2))^(1/2))*(x-1+(-1-(a+4)^(1/2))^(1/2)))^(1/2)*E
llipticF(((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^
(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a
+4)^(1/2))^(1/2)))^(1/2),((-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))
*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)+
(-1+(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2)))^(
1/2))-(1/5*a+28/15)*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*((-(-1
-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(
-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(...
```

Fricas [F]

$$\int x^2\sqrt{a+8x-8x^2+4x^3-x^4}\,dx=\int \sqrt{-x^4+4\,x^3-8\,x^2+a+8\,x}x^2\,dx$$

input
```
integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)*x^2, x)
```

Sympy [F]

$$\int x^2\sqrt{a+8x-8x^2+4x^3-x^4}\,dx = \int x^2\sqrt{a-x^4+4x^3-8x^2+8x}\,dx$$

input
```
integrate(x**2*(-x**4+4*x**3-8*x**2+a+8*x)**(1/2),x)
```

output
```
Integral(x**2*sqrt(a - x**4 + 4*x**3 - 8*x**2 + 8*x), x)
```

Maxima [F]

$$\int x^2\sqrt{a+8x-8x^2+4x^3-x^4}\,dx = \int \sqrt{-x^4+4\,x^3-8\,x^2+a+8\,x}x^2\,dx$$

input
```
integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)*x^2, x)
```

Giac [F]

$$\int x^2\sqrt{a+8x-8x^2+4x^3-x^4}\,dx = \int \sqrt{-x^4+4\,x^3-8\,x^2+a+8\,x}x^2\,dx$$

input
```
integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2\sqrt{a+8x-8x^2+4x^3-x^4}\,dx=\int x^2\,\sqrt{-x^4+4\,x^3-8\,x^2+8\,x+a}\,dx$$

input
```
int(x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(1/2),x)
```

output
```
int(x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(1/2), x)
```

Reduce [F]

$$\begin{aligned}\int x^2\sqrt{a+8x-8x^2+4x^3-x^4}\,dx&=\frac{\sqrt{-x^4+4x^3-8x^2+a+8x}\,a}{15}\\&+\frac{\sqrt{-x^4+4x^3-8x^2+a+8x}\,x^3}{5}\\&-\frac{\sqrt{-x^4+4x^3-8x^2+a+8x}\,x^2}{10}\\&+\frac{\sqrt{-x^4+4x^3-8x^2+a+8x}\,x}{15}\\&+\frac{23\sqrt{-x^4+4x^3-8x^2+a+8x}}{45}\\&-\frac{\left(\int\frac{\sqrt{-x^4+4x^3-8x^2+a+8x}}{-x^4+4x^3-8x^2+a+8x}dx\right)a}{3}\\&-\frac{92\left(\int\frac{\sqrt{-x^4+4x^3-8x^2+a+8x}}{-x^4+4x^3-8x^2+a+8x}dx\right)}{45}\\&+\frac{2\left(\int\frac{\sqrt{-x^4+4x^3-8x^2+a+8x}\,x^3}{-x^4+4x^3-8x^2+a+8x}dx\right)a}{15}\\&+\frac{16\left(\int\frac{\sqrt{-x^4+4x^3-8x^2+a+8x}\,x^3}{-x^4+4x^3-8x^2+a+8x}dx\right)}{45}\\&+\frac{11\left(\int\frac{\sqrt{-x^4+4x^3-8x^2+a+8x}\,x}{-x^4+4x^3-8x^2+a+8x}dx\right)a}{15}\\&+\frac{148\left(\int\frac{\sqrt{-x^4+4x^3-8x^2+a+8x}\,x}{-x^4+4x^3-8x^2+a+8x}dx\right)}{45}\end{aligned}$$

input
```
int(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x)
```

output
```
(6*sqrt(a - x**4 + 4*x**3 - 8*x**2 + 8*x)*a + 18*sqrt(a - x**4 + 4*x**3 -
8*x**2 + 8*x)*x**3 - 9*sqrt(a - x**4 + 4*x**3 - 8*x**2 + 8*x)*x**2 + 6*sqr
t(a - x**4 + 4*x**3 - 8*x**2 + 8*x)*x + 46*sqrt(a - x**4 + 4*x**3 - 8*x**2
 + 8*x) - 30*int(sqrt(a - x**4 + 4*x**3 - 8*x**2 + 8*x)/(a - x**4 + 4*x**3
 - 8*x**2 + 8*x),x)*a - 184*int(sqrt(a - x**4 + 4*x**3 - 8*x**2 + 8*x)/(a
- x**4 + 4*x**3 - 8*x**2 + 8*x),x) + 12*int((sqrt(a - x**4 + 4*x**3 - 8*x*
*2 + 8*x)*x**3)/(a - x**4 + 4*x**3 - 8*x**2 + 8*x),x)*a + 32*int((sqrt(a -
 x**4 + 4*x**3 - 8*x**2 + 8*x)*x**3)/(a - x**4 + 4*x**3 - 8*x**2 + 8*x),x)
 + 66*int((sqrt(a - x**4 + 4*x**3 - 8*x**2 + 8*x)*x)/(a - x**4 + 4*x**3 -
8*x**2 + 8*x),x)*a + 296*int((sqrt(a - x**4 + 4*x**3 - 8*x**2 + 8*x)*x)/(a
 - x**4 + 4*x**3 - 8*x**2 + 8*x),x))/90
```

3.40 $\int \frac{x^2}{\sqrt{a+8x-8x^2+4x^3-x^4}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 322

$$\int \frac{x^2}{\sqrt{a+8x-8x^2+4x^3-x^4}}\,dx = \arctan\left(\frac{1+(-1+x)^2}{\sqrt{3+a-2(1-x)^2-(1-x)^4}}\right)$$
$$-\frac{\sqrt{-1+\sqrt{4+a}}\left(1+\sqrt{4+a}\right)\sqrt{1+\frac{(1-x)^2}{1-\sqrt{4+a}}}\sqrt{1+\frac{(1-x)^2}{1+\sqrt{4+a}}}\,E\left(\arcsin\left(\frac{1-x}{\sqrt{-1+\sqrt{4+a}}}\right)\middle|\frac{1-\sqrt{4+a}}{1+\sqrt{4+a}}\right)}{\sqrt{3+a-2(1-x)^2-(1-x)^4}}$$
$$+\frac{\sqrt{4+a}\sqrt{-1+\sqrt{4+a}}\sqrt{1+\frac{(1-x)^2}{1-\sqrt{4+a}}}\sqrt{1+\frac{(1-x)^2}{1+\sqrt{4+a}}}\,\text{EllipticF}\left(\arcsin\left(\frac{1-x}{\sqrt{-1+\sqrt{4+a}}}\right),\frac{1-\sqrt{4+a}}{1+\sqrt{4+a}}\right)}{\sqrt{3+a-2(1-x)^2-(1-x)^4}}$$

output
```
arctan((1+(-1+x)^2)/(3+a-2*(1-x)^2-(1-x)^4)^(1/2))-(-1+(4+a)^(1/2))^(1/2)*
(1+(4+a)^(1/2))*(1+(1-x)^2/(1-(4+a)^(1/2)))^(1/2)*(1+(1-x)^2/(1+(4+a)^(1/2
)))^(1/2)*EllipticE((1-x)/(-1+(4+a)^(1/2))^(1/2),((1-(4+a)^(1/2))/(1+(4+a)
^(1/2)))^(1/2))/(3+a-2*(1-x)^2-(1-x)^4)^(1/2)+(4+a)^(1/2)*(-1+(4+a)^(1/2))
^(1/2)*(1+(1-x)^2/(1-(4+a)^(1/2)))^(1/2)*(1+(1-x)^2/(1+(4+a)^(1/2)))^(1/2)
*EllipticF((1-x)/(-1+(4+a)^(1/2))^(1/2),((1-(4+a)^(1/2))/(1+(4+a)^(1/2)))^
(1/2))/(3+a-2*(1-x)^2-(1-x)^4)^(1/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1247 vs. $2(322) = 644$.

Time = 17.08 (sec) , antiderivative size = 1247, normalized size of antiderivative = 3.87

$$\int \frac{x^2}{\sqrt{a+8x-8x^2+4x^3-x^4}}\,dx = \text{Too large to display}$$

input
```
Integrate[x^2/Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4],x]
```

output
```
((-1 + Sqrt[-1 - Sqrt[4 + a]] + x)*(-1 - Sqrt[-1 + Sqrt[4 + a]] + x)*(-1 +
 Sqrt[-1 + Sqrt[4 + a]] + x) + 2*(Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[
4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x)^2*Sqrt[((Sqrt[-1 - Sqrt[4 + a]]
 - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 -
Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x))]*
Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 - Sqrt[-1 + Sqrt[4 + a]] + x))/((Sqrt[-1
- Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x)
)]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 + Sqrt[-1 + Sqrt[4 + a]] + x))/((Sqrt[
-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] +
 x))]*(((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*EllipticE[ArcSin
[Sqrt[((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - S
qrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 +
Sqrt[-1 - Sqrt[4 + a]] - x))]], (Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4
 + a]])^2/(Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])^2])/(2*Sqrt[-1
 - Sqrt[4 + a]]) + ((-((-1 - Sqrt[-1 - Sqrt[4 + a]])*(-2 - Sqrt[-1 - Sqrt[
4 + a]] - Sqrt[-1 + Sqrt[4 + a]])) + (-1 + Sqrt[-1 - Sqrt[4 + a]])*(Sqrt[-
1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]]))*EllipticF[ArcSin[Sqrt[((Sqrt[-
1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] +
x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt
[4 + a]] - x))]], (Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])^2/(...
```

Rubi [A] (warning: unable to verify)

Time = 1.18 (sec) , antiderivative size = 471, normalized size of antiderivative = 1.46, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {2459, 2006, 2202, 27, 1432, 1092, 217, 1514, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{a-x^4+4x^3-8x^2+8x}}dx$$

$$\Big\downarrow \text{2459}$$

$$\int \frac{(x-1)^2+2(x-1)+1}{\sqrt{a-(x-1)^4-2(x-1)^2+3}}d(x-1)$$

$$\Big\downarrow \text{2006}$$

$$\int \frac{x^2}{\sqrt{a-(x-1)^4-2(x-1)^2+3}}d(x-1)$$

$$\Big\downarrow \text{2202}$$

$$\int \frac{(x-1)^2+1}{\sqrt{-(x-1)^4-2(x-1)^2+a+3}}d(x-1)+\int \frac{2(x-1)}{\sqrt{-(x-1)^4-2(x-1)^2+a+3}}d(x-1)$$

$$\Big\downarrow \text{27}$$

$$\int \frac{(x-1)^2+1}{\sqrt{-(x-1)^4-2(x-1)^2+a+3}}d(x-1)+2\int \frac{x-1}{\sqrt{-(x-1)^4-2(x-1)^2+a+3}}d(x-1)$$

$$\Big\downarrow \text{1432}$$

$$\int \frac{1}{\sqrt{-(x-1)^4-2(x-1)^2+a+3}}d(x-1)^2+\int \frac{(x-1)^2+1}{\sqrt{-(x-1)^4-2(x-1)^2+a+3}}d(x-1)$$

$$\Big\downarrow \text{1092}$$

$$2\int \frac{1}{-(x-1)^4-4}d\left(-\frac{2x}{\sqrt{-(x-1)^4-2(x-1)^2+a+3}}\right)+\int \frac{(x-1)^2+1}{\sqrt{-(x-1)^4-2(x-1)^2+a+3}}d(x-1)$$

$$\Big\downarrow \text{217}$$

$$\int \frac{(x-1)^2+1}{\sqrt{-(x-1)^4-2(x-1)^2+a+3}}d(x-1)+\arctan\left(\frac{x}{\sqrt{a-(x-1)^4-2(x-1)^2+3}}\right)$$

$$\Big\downarrow \text{ 1514}$$

$$\frac{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}\int \frac{(x-1)^2+1}{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}d(x-1)}{\sqrt{a-(x-1)^4-2(x-1)^2+3}}+$$
$$\arctan\left(\frac{x}{\sqrt{a-(x-1)^4-2(x-1)^2+3}}\right)$$

$$\Big\downarrow \text{ 406}$$

$$\frac{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}\left(\int \frac{1}{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}d(x-1)+\int \frac{(x-1)^2}{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}d(x-1)\right)}{\sqrt{a-(x-1)^4-2(x-1)^2+3}}+$$
$$\arctan\left(\frac{x}{\sqrt{a-(x-1)^4-2(x-1)^2+3}}\right)$$

$$\Big\downarrow \text{ 320}$$

$$\frac{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}\left(\int \frac{(x-1)^2}{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}d(x-1)+\frac{\sqrt{\sqrt{a+4}+1}\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\,\text{EllipticF}\left(\arctan\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right),-\right.}{\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\right.}{\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$
$$\arctan\left(\frac{x}{\sqrt{a-(x-1)^4-2(x-1)^2+3}}\right)$$

$$\Big\downarrow \text{ 388}$$

$$\frac{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}\left(-\left(1-\sqrt{a+4}\right)\int \frac{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}}{\left(\frac{(x-1)^2}{\sqrt{a+4}+1}+1\right)^{3/2}}d(x-1)+\frac{\sqrt{\sqrt{a+4}+1}\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\,\text{EllipticF}\left(\arctan\left(\frac{x}{\sqrt{\sqrt{}}}\right.\right.}{\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\right.}{\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$
$$\arctan\left(\frac{x}{\sqrt{a-(x-1)^4-2(x-1)^2+3}}\right)$$

$$\Big\downarrow \; 313$$

$$\arctan\left(\frac{x}{\sqrt{a-(x-1)^4-2(x-1)^2+3}}\right)+$$
$$\frac{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}\left(\frac{\sqrt{\sqrt{a+4}+1}\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\,\text{EllipticF}\left(\arctan\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right),-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}-\frac{\left(1-\sqrt{a+4}\right)\sqrt{\sqrt{a+4}+1}\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}\cdots}}{\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}\cdots}{\frac{(x-1)^2}{\sqrt{a+4}+1}\cdots}}}\cdots\right.}{\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

input
```
Int[x^2/Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4],x]
```

output
```
ArcTan[x/Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]] + (Sqrt[1 + (-1 + x)^2/(
1 - Sqrt[4 + a])]*Sqrt[1 + (-1 + x)^2/(1 + Sqrt[4 + a])]*(((1 - Sqrt[4 + a
])*Sqrt[1 + (-1 + x)^2/(1 - Sqrt[4 + a])]*(-1 + x))/Sqrt[1 + (-1 + x)^2/(1
 + Sqrt[4 + a])] - ((1 - Sqrt[4 + a])*Sqrt[1 + Sqrt[4 + a]]*Sqrt[1 + (-1 +
 x)^2/(1 - Sqrt[4 + a])]*EllipticE[ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]],
 (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])/(Sqrt[(1 + (-1 + x)^2/(1 - Sqrt[4 +
a]))/(1 + (-1 + x)^2/(1 + Sqrt[4 + a]))]*Sqrt[1 + (-1 + x)^2/(1 + Sqrt[4 +
 a])]) + (Sqrt[1 + Sqrt[4 + a]]*Sqrt[1 + (-1 + x)^2/(1 - Sqrt[4 + a])]*Ell
ipticF[ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[
4 + a])])/(Sqrt[(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (-1 + x)^2/(1 + Sq
rt[4 + a]))]*Sqrt[1 + (-1 + x)^2/(1 + Sqrt[4 + a])])))/Sqrt[3 + a - 2*(-1
+ x)^2 - (-1 + x)^4]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1432
```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

rule 1514
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[Sqrt[1 + 2*c*(x^2/(b - q))]*(Sqrt
[1 + 2*c*(x^2/(b + q))]/Sqrt[a + b*x^2 + c*x^4])   Int[(d + e*x^2)/(Sqrt[1
+ 2*c*(x^2/(b - q))]*Sqrt[1 + 2*c*(x^2/(b + q))]), x], x]] /; FreeQ[{a, b,
c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

rule 2006
```
Int[(u_.)*(Px_), x_Symbol] :> With[{a = Rt[Coeff[Px, x, 0], Expon[Px, x]],
b = Rt[Coeff[Px, x, Expon[Px, x]], Expon[Px, x]]}, Int[u*(a + b*x)^Expon[Px
, x], x] /; EqQ[Px, (a + b*x)^Expon[Px, x]]] /; PolyQ[Px, x] && GtQ[Expon[P
x, x], 1] && NeQ[Coeff[Px, x, 0], 0] &&  !MatchQ[Px, (a_.)*(v_)^Expon[Px, x
] /; FreeQ[a, x] && LinearQ[v, x]]
```

rule 2202
```
Int[(Pn_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{n
 = Expon[Pn, x], k}, Int[Sum[Coeff[Pn, x, 2*k]*x^(2*k), {k, 0, n/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pn, x, 2*k + 1]*x^(2*k), {k, 0, (n -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pn, x]
 &&  !PolyQ[Pn, x^2]
```

rule 2459
```
Int[(Pn_)^(p_.)*(Qx_), x_Symbol] :> With[{S = Coeff[Pn, x, Expon[Pn, x] - 1
]/(Expon[Pn, x]*Coeff[Pn, x, Expon[Pn, x]])}, Subst[Int[ExpandToSum[Pn /. x
 -> x - S, x]^p*ExpandToSum[Qx /. x -> x - S, x], x], x, x + S] /; Binomial
Q[Pn /. x -> x - S, x] || (IntegerQ[Expon[Pn, x]/2] && TrinomialQ[Pn /. x -
> x - S, x])] /; FreeQ[p, x] && PolyQ[Pn, x] && GtQ[Expon[Pn, x], 2] && NeQ
[Coeff[Pn, x, Expon[Pn, x] - 1], 0] && PolyQ[Qx, x] &&  !(MonomialQ[Qx, x]
&& IGtQ[p, 0])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1146 vs. $2(274) = 548$.

Time = 0.81 (sec) , antiderivative size = 1147, normalized size of antiderivative = 3.56

method	result	size
default	Expression too large to display	1147
elliptic	Expression too large to display	1147

input
```
int(x^2/(-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((x-1-(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1-(a+4)^(1/2))^(1/2))*(x-1+(-1-(a+4)^
(1/2))^(1/2))+((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*((-(-1-(a+4)
^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(
a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1
/2)*(x-1+(-1+(a+4)^(1/2))^(1/2))^2*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1-(-1-(a+
4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+
(a+4)^(1/2))^(1/2)))^(1/2)*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1+(-1-(a+4)^(1/2)
)^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(
1/2))^(1/2)))^(1/2)*(-1/2*((1-(-1+(a+4)^(1/2))^(1/2))*(1+(-1+(a+4)^(1/2))^
(1/2))-(1-(-1-(a+4)^(1/2))^(1/2))*(1+(-1+(a+4)^(1/2))^(1/2))+(1-(-1-(a+4)^
(1/2))^(1/2))*(1-(-1+(a+4)^(1/2))^(1/2))+(1-(-1+(a+4)^(1/2))^(1/2))^2)/(-(
-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-1+(a+4)^(1/2))^(1/2)*Ellip
ticF(((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2
))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^
(1/2))^(1/2)))^(1/2),((-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))*((-
1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)+(-1+
(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2)))^(1/2)
)-1/2*(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*EllipticE(((-(-1-(a
+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-
1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2...
```

Fricas [F]

$$\int \frac{x^2}{\sqrt{a+8x-8x^2+4x^3-x^4}}\,dx = \int \frac{x^2}{\sqrt{-x^4+4\,x^3-8\,x^2+a+8\,x}}\,dx$$

input
```
integrate(x^2/(-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)*x^2/(x^4 - 4*x^3 + 8*x^2 -
a - 8*x), x)
```

Sympy [F]

$$\int \frac{x^2}{\sqrt{a+8x-8x^2+4x^3-x^4}}\,dx = \int \frac{x^2}{\sqrt{a-x^4+4x^3-8x^2+8x}}\,dx$$

input
```
integrate(x**2/(-x**4+4*x**3-8*x**2+a+8*x)**(1/2),x)
```

output
```
Integral(x**2/sqrt(a - x**4 + 4*x**3 - 8*x**2 + 8*x), x)
```

Maxima [F]

$$\int \frac{x^2}{\sqrt{a+8x-8x^2+4x^3-x^4}}\,dx = \int \frac{x^2}{\sqrt{-x^4+4\,x^3-8\,x^2+a+8\,x}}\,dx$$

input
```
integrate(x^2/(-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2/sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x), x)
```

Giac [F]

$$\int \frac{x^2}{\sqrt{a+8x-8x^2+4x^3-x^4}}\,dx = \int \frac{x^2}{\sqrt{-x^4+4\,x^3-8\,x^2+a+8\,x}}\,dx$$

input
```
integrate(x^2/(-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^2/sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{a+8x-8x^2+4x^3-x^4}}\,dx = \int \frac{x^2}{\sqrt{-x^4+4\,x^3-8\,x^2+8\,x+a}}\,dx$$

input
```
int(x^2/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(1/2),x)
```

output
```
int(x^2/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^2}{\sqrt{a+8x-8x^2+4x^3-x^4}}\,dx = \int \frac{\sqrt{-x^4+4x^3-8x^2+a+8x}\,x^2}{-x^4+4x^3-8x^2+a+8x}dx$$

input
```
int(x^2/(-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x)
```

output
```
int((sqrt(a - x**4 + 4*x**3 - 8*x**2 + 8*x)*x**2)/(a - x**4 + 4*x**3 - 8*x
**2 + 8*x),x)
```

3.41 $\int \frac{x^2}{\left(a+8x-8x^2+4x^3-x^4\right)^{3/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 333

$$\int \frac{x^2}{(a+8x-8x^2+4x^3-x^4)^{3/2}}\,dx = \frac{1+(-1+x)^2}{(4+a)\sqrt{3+a-2(1-x)^2-(1-x)^4}} + \frac{\left(1-\sqrt{4+a}\right)\left(1+\frac{(1-x)^2}{1-\sqrt{4+a}}\right)(1-x)}{2(3+a)\sqrt{3+a-2(1-x)^2-(1-x)^4}} - \frac{\left(2+(-1+x)^2\right)(1-x)}{2(3+a)\sqrt{3+a-2(1-x)^2-(1-x)^4}} - \frac{\left(1-\sqrt{4+a}\right)\sqrt{1+\sqrt{4+a}}\left(1+\frac{(1-x)^2}{1-\sqrt{4+a}}\right)E\left(\arctan\left(\frac{1-x}{\sqrt{1+\sqrt{4+a}}}\right)\middle|-\frac{2\sqrt{4+a}}{1-\sqrt{4+a}}\right)}{2(3+a)\sqrt{\frac{1+\frac{(1-x)^2}{1-\sqrt{4+a}}}{1+\frac{(1-x)^2}{1+\sqrt{4+a}}}}\sqrt{3+a-2(1-x)^2-(1-x)^4}}$$

output

```
(1+(-1+x)^2)/(4+a)/(3+a-2*(1-x)^2-(1-x)^4)^(1/2)+1/2*(1-(4+a)^(1/2))*(1+(1
-x)^2/(1-(4+a)^(1/2)))*(1-x)/(3+a)/(3+a-2*(1-x)^2-(1-x)^4)^(1/2)-1/2*(2+(-
1+x)^2)*(1-x)/(3+a)/(3+a-2*(1-x)^2-(1-x)^4)^(1/2)-1/2*(1-(4+a)^(1/2))*(1+(
4+a)^(1/2))^(1/2)*(1+(1-x)^2/(1-(4+a)^(1/2)))*EllipticE((1-x)/(1+(4+a)^(1/
2))^(1/2)/(1+(1-x)^2/(1+(4+a)^(1/2)))^(1/2),(-2*(4+a)^(1/2)/(1-(4+a)^(1/2)
))^(1/2))/(3+a)/((1+(1-x)^2/(1-(4+a)^(1/2)))/(1+(1-x)^2/(1+(4+a)^(1/2))))^
(1/2)/(3+a-2*(1-x)^2-(1-x)^4)^(1/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 2941 vs. $2(333) = 666$.

Time = 17.14 (sec) , antiderivative size = 2941, normalized size of antiderivative = 8.83

$$\int \frac{x^2}{(a+8x-8x^2+4x^3-x^4)^{3/2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[x^2/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2),x]
```

output
```
((-a - 8*x - a*x + 6*x^2 + a*x^2 - 4*x^3 - a*x^3)*(a + 8*x - 8*x^2 + 4*x^3
 - x^4)^2)/(2*(3 + a)*(4 + a)*(-a - 8*x + 8*x^2 - 4*x^3 + x^4)*(a - x*(-8
+ 8*x - 4*x^2 + x^3))^(3/2)) - ((a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2)*((2*
(-Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4
+ a]] + x)^2*Sqrt[((-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1
+ Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4
 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(
-1 - Sqrt[-1 + Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqr
t[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]
]*(-1 + Sqrt[-1 + Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 +
Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]*EllipticF[ArcSin[Sqrt[((
-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 +
 a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-
1 - Sqrt[4 + a]] + x))]], ((-Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a
]])*(Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]]))/((Sqrt[-1 - Sqrt[4
+ a]] - Sqrt[-1 + Sqrt[4 + a]])*(-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[
4 + a]]))])/(Sqrt[-1 - Sqrt[4 + a]]*(-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + S
qrt[4 + a]])*Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4]) - (4*(-Sqrt[-1 - Sqrt[4
+ a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x)^2*Sqrt[(
(Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[...
```

Rubi [A] (warning: unable to verify)

Time = 1.05 (sec) , antiderivative size = 389, normalized size of antiderivative = 1.17, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.393, Rules used = {2459, 2006, 2202, 27, 1432, 1088, 1492, 27, 1460, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a-x^4+4x^3-8x^2+8x)^{3/2}}\,dx$$

$$\Big\downarrow \text{2459}$$

$$\int \frac{(x-1)^2+2(x-1)+1}{(a-(x-1)^4-2(x-1)^2+3)^{3/2}}d(x-1)$$

$$\Big\downarrow \text{2006}$$

$$\int \frac{x^2}{(a-(x-1)^4-2(x-1)^2+3)^{3/2}}d(x-1)$$

$$\Big\downarrow \text{2202}$$

$$\int \frac{(x-1)^2+1}{(-(x-1)^4-2(x-1)^2+a+3)^{3/2}}d(x-1)+\int \frac{2(x-1)}{(-(x-1)^4-2(x-1)^2+a+3)^{3/2}}d(x-1)$$

$$\Big\downarrow \text{27}$$

$$\int \frac{(x-1)^2+1}{(-(x-1)^4-2(x-1)^2+a+3)^{3/2}}d(x-1)+2\int \frac{x-1}{(-(x-1)^4-2(x-1)^2+a+3)^{3/2}}d(x-1)$$

$$\Big\downarrow \text{1432}$$

$$\int \frac{1}{(-(x-1)^4-2(x-1)^2+a+3)^{3/2}}d(x-1)^2+\int \frac{(x-1)^2+1}{(-(x-1)^4-2(x-1)^2+a+3)^{3/2}}d(x-1)$$

$$\Big\downarrow \text{1088}$$

$$\int \frac{(x-1)^2+1}{(-(x-1)^4-2(x-1)^2+a+3)^{3/2}}d(x-1)+\frac{x}{(a+4)\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

$$\Big\downarrow \text{1492}$$

$$-\frac{\int \frac{2(a+4)(x-1)^2}{\sqrt{-(x-1)^4-2(x-1)^2+a+3}}d(x-1)}{4\left(a^2+7a+12\right)}+\frac{(a+4)\left((x-1)^2+2\right)(x-1)}{2\left(a^2+7a+12\right)\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{x}{(a+4)\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

$$\Big\downarrow \; 27$$

$$-\frac{(a+4)\int \frac{(x-1)^2}{\sqrt{-(x-1)^4-2(x-1)^2+a+3}}d(x-1)}{2\left(a^2+7a+12\right)}+\frac{(a+4)\left((x-1)^2+2\right)(x-1)}{2\left(a^2+7a+12\right)\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{x}{(a+4)\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

$$\Big\downarrow \; 1460$$

$$-\frac{(a+4)\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}\int \frac{(x-1)^2}{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}d(x-1)}{2\left(a^2+7a+12\right)\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{(a+4)\left((x-1)^2+2\right)(x-1)}{2\left(a^2+7a+12\right)\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{x}{(a+4)\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

$$\Big\downarrow \; 388$$

$$-\frac{(a+4)\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}\left(\frac{\left(1-\sqrt{a+4}\right)(x-1)\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}}{\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}-\left(1-\sqrt{a+4}\right)\int \frac{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}}{\left(\frac{(x-1)^2}{\sqrt{a+4}+1}+1\right)^{3/2}}d(x-1)\right)}{2\left(a^2+7a+12\right)\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{(a+4)\left((x-1)^2+2\right)(x-1)}{2\left(a^2+7a+12\right)\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{x}{(a+4)\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

$$\Big\downarrow \; 313$$

$$-\frac{(a+4)\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}\left(\frac{\left(1-\sqrt{a+4}\right)(x-1)\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}}{\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}-\frac{\left(1-\sqrt{a+4}\right)\sqrt{\sqrt{a+4}+1}\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}E\left(\arctan\left(\frac{x-1}{\sqrt{\sqrt{a+4}+}}\right.\right.}{\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\right.}{2\left(a^2+7a+12\right)\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{(a+4)\left((x-1)^2+2\right)(x-1)}{2\left(a^2+7a+12\right)\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{x}{(a+4)\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

input
```
Int[x^2/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2),x]
```

output
```
((4 + a)*(2 + (-1 + x)^2)*(-1 + x))/(2*(12 + 7*a + a^2)*Sqrt[3 + a - 2*(-1
 + x)^2 - (-1 + x)^4]) + x/((4 + a)*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4
]) - ((4 + a)*Sqrt[1 + (-1 + x)^2/(1 - Sqrt[4 + a])]*Sqrt[1 + (-1 + x)^2/(
1 + Sqrt[4 + a])]*(((1 - Sqrt[4 + a])*Sqrt[1 + (-1 + x)^2/(1 - Sqrt[4 + a]
)]*(-1 + x))/Sqrt[1 + (-1 + x)^2/(1 + Sqrt[4 + a])] - ((1 - Sqrt[4 + a])*S
qrt[1 + Sqrt[4 + a]]*Sqrt[1 + (-1 + x)^2/(1 - Sqrt[4 + a])]*EllipticE[ArcT
an[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])/(
Sqrt[(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (-1 + x)^2/(1 + Sqrt[4 + a]))
]*Sqrt[1 + (-1 + x)^2/(1 + Sqrt[4 + a])])))/(2*(12 + 7*a + a^2)*Sqrt[3 + a
 - 2*(-1 + x)^2 - (-1 + x)^4])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

rule 1432
```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

rule 1460

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[Sqrt[1 + 2*c*(x^2/(b - q))]*(Sqrt[1 + 2*c*(x^2/(
b + q))]/Sqrt[a + b*x^2 + c*x^4])   Int[x^2/(Sqrt[1 + 2*c*(x^2/(b - q))]*Sq
rt[1 + 2*c*(x^2/(b + q))]), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a
*c, 0] && NegQ[c/a]
```

rule 1492

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2
 - 4*a*c))   Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p +
7)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 LtQ[p, -1] && IntegerQ[2*p]
```

rule 2006

```
Int[(u_.)*(Px_), x_Symbol] :> With[{a = Rt[Coeff[Px, x, 0], Expon[Px, x]],
b = Rt[Coeff[Px, x, Expon[Px, x]], Expon[Px, x]]}, Int[u*(a + b*x)^Expon[Px
, x], x] /; EqQ[Px, (a + b*x)^Expon[Px, x]]] /; PolyQ[Px, x] && GtQ[Expon[P
x, x], 1] && NeQ[Coeff[Px, x, 0], 0] &&  !MatchQ[Px, (a_.)*(v_)^Expon[Px, x
] /; FreeQ[a, x] && LinearQ[v, x]]
```

rule 2202

```
Int[(Pn_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{n
 = Expon[Pn, x], k}, Int[Sum[Coeff[Pn, x, 2*k]*x^(2*k), {k, 0, n/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pn, x, 2*k + 1]*x^(2*k), {k, 0, (n -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pn, x]
 &&  !PolyQ[Pn, x^2]
```

rule 2459

```
Int[(Pn_)^(p_.)*(Qx_), x_Symbol] :> With[{S = Coeff[Pn, x, Expon[Pn, x] - 1
]/(Expon[Pn, x]*Coeff[Pn, x, Expon[Pn, x]])}, Subst[Int[ExpandToSum[Pn /. x
 -> x - S, x]^p*ExpandToSum[Qx /. x -> x - S, x], x], x, x + S] /; Binomial
Q[Pn /. x -> x - S, x] || (IntegerQ[Expon[Pn, x]/2] && TrinomialQ[Pn /. x -
> x - S, x])] /; FreeQ[p, x] && PolyQ[Pn, x] && GtQ[Expon[Pn, x], 2] && NeQ
[Coeff[Pn, x, Expon[Pn, x] - 1], 0] && PolyQ[Qx, x] &&  !(MonomialQ[Qx, x]
&& IGtQ[p, 0])
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 2606 vs. $2(315) = 630$.

Time = 0.75 (sec) , antiderivative size = 2607, normalized size of antiderivative = 7.83

method	result	size
default	Expression too large to display	2607
elliptic	Expression too large to display	2607

input
```
int(x^2/(-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2*(1/4/(a+3)*x^3-1/4*(6+a)/(a^2+7*a+12)*x^2+1/4*(a+8)/(a^2+7*a+12)*x+1/4/(
a^2+7*a+12)*a)/(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)-(2/(a^2+7*a+12)-1/2*(a+8)/(a
^2+7*a+12))*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*((-(-1-(a+4)^(
1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+
4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2
)*(x-1+(-1+(a+4)^(1/2))^(1/2))^2*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1-(-1-(a+4)
^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a
+4)^(1/2))^(1/2)))^(1/2)*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1+(-1-(a+4)^(1/2))^
(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/
2))^(1/2)))^(1/2)/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-1+(a+
4)^(1/2))^(1/2)/(-(x-1-(-1+(a+4)^(1/2))^(1/2))*(x-1+(-1+(a+4)^(1/2))^(1/2)
)*(x-1-(-1-(a+4)^(1/2))^(1/2))*(x-1+(-1-(a+4)^(1/2))^(1/2)))^(1/2)*Ellipti
cF(((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))
^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1
/2))^(1/2)))^(1/2),((-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))*((-1-
(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a
+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2)))^(1/2))-
(-2/(a^2+7*a+12)+(6+a)/(a^2+7*a+12))*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/
2))^(1/2))*((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4
)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(...
```

Fricas [F]

$$\int \frac{x^2}{\left(a + 8x - 8x^2 + 4x^3 - x^4\right)^{3/2}}\,dx = \int \frac{x^2}{\left(-x^4 + 4\,x^3 - 8\,x^2 + a + 8\,x\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2/(-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)*x^2/(x^8 - 8*x^7 + 32*x^6 -
2*(a - 64)*x^4 - 80*x^5 + 8*(a - 16)*x^3 - 16*(a - 4)*x^2 + a^2 + 16*a*x),
 x)
```

Sympy [F]

$$\int \frac{x^2}{\left(a + 8x - 8x^2 + 4x^3 - x^4\right)^{3/2}}\,dx = \int \frac{x^2}{\left(a - x^4 + 4x^3 - 8x^2 + 8x\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**2/(-x**4+4*x**3-8*x**2+a+8*x)**(3/2),x)
```

output
```
Integral(x**2/(a - x**4 + 4*x**3 - 8*x**2 + 8*x)**(3/2), x)
```

Maxima [F]

$$\int \frac{x^2}{\left(a + 8x - 8x^2 + 4x^3 - x^4\right)^{3/2}}\,dx = \int \frac{x^2}{\left(-x^4 + 4\,x^3 - 8\,x^2 + a + 8\,x\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2/(-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^2/(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(3/2), x)
```

Giac [F]

$$\int \frac{x^2}{(a+8x-8x^2+4x^3-x^4)^{3/2}}\,dx = \int \frac{x^2}{(-x^4+4\,x^3-8\,x^2+a+8\,x)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2/(-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^2/(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(a+8x-8x^2+4x^3-x^4)^{3/2}}\,dx = \int \frac{x^2}{(-x^4+4\,x^3-8\,x^2+8\,x+a)^{3/2}}\,dx$$

input
```
int(x^2/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2),x)
```

output
```
int(x^2/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2), x)
```

Reduce [F]

$$\int \frac{x^2}{(a+8x-8x^2+4x^3-x^4)^{3/2}}\,dx = \int \frac{\sqrt{-x^4+4x^3-8x^2+a+8x}\,x^2}{x^8-8x^7+32x^6-2a\,x^4-80x^5+8a\,x^3+128x^4-16a\,x^2-128}$$

input
```
int(x^2/(-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x)
```

output
```
int((sqrt(a - x**4 + 4*x**3 - 8*x**2 + 8*x)*x**2)/(a**2 - 2*a*x**4 + 8*a*x
**3 - 16*a*x**2 + 16*a*x + x**8 - 8*x**7 + 32*x**6 - 80*x**5 + 128*x**4 -
128*x**3 + 64*x**2),x)
```

3.42 $\int \frac{x^2}{(a+8x-8x^2+4x^3-x^4)^{5/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 525

$$\int \frac{x^2}{(a+8x-8x^2+4x^3-x^4)^{5/2}}\,dx = \frac{1+(-1+x)^2}{3(4+a)\left(3+a-2(1-x)^2-(1-x)^4\right)^{3/2}}$$
$$+\frac{2(1+(-1+x)^2)}{3(4+a)^2\sqrt{3+a-2(1-x)^2-(1-x)^4}}$$
$$-\frac{\left(29+7a+(13+3a)(1-x)^2\right)(1-x)}{12(3+a)^2(4+a)\sqrt{3+a-2(1-x)^2-(1-x)^4}}$$
$$-\frac{\left(2+(-1+x)^2\right)(1-x)}{6(3+a)\left(3+a-2(1-x)^2-(1-x)^4\right)^{3/2}}$$
$$+\frac{(13+3a)\sqrt{-1+\sqrt{4+a}}\left(1+\sqrt{4+a}\right)\sqrt{1+\frac{(1-x)^2}{1-\sqrt{4+a}}}\sqrt{1+\frac{(1-x)^2}{1+\sqrt{4+a}}}E\left(\arcsin\left(\frac{1-x}{\sqrt{-1+\sqrt{4+a}}}\right)\middle|\frac{1-\sqrt{4+a}}{1+\sqrt{4+a}}\right)}{12(3+a)^2(4+a)\sqrt{3+a-2(1-x)^2-(1-x)^4}}$$
$$-\frac{\sqrt{-1+\sqrt{4+a}}\left(16+13\sqrt{4+a}+a\left(4+3\sqrt{4+a}\right)\right)\sqrt{1+\frac{(1-x)^2}{1-\sqrt{4+a}}}\sqrt{1+\frac{(1-x)^2}{1+\sqrt{4+a}}}\text{EllipticF}\left(\arcsin\left(\frac{1-}{\sqrt{-1+}}\right.\right.}{12(3+a)^2(4+a)\sqrt{3+a-2(1-x)^2-(1-x)^4}}$$

output
```
1/3*(1+(-1+x)^2)/(4+a)/(3+a-2*(1-x)^2-(1-x)^4)^(3/2)+2/3*(1+(-1+x)^2)/(4+a
)^2/(3+a-2*(1-x)^2-(1-x)^4)^(1/2)-1/12*(29+7*a+(13+3*a)*(1-x)^2)*(1-x)/(3+
a)^2/(4+a)/(3+a-2*(1-x)^2-(1-x)^4)^(1/2)-1/6*(2+(-1+x)^2)*(1-x)/(3+a)/(3+a
-2*(1-x)^2-(1-x)^4)^(3/2)+1/12*(13+3*a)*(-1+(4+a)^(1/2))^(1/2)*(1+(4+a)^(1
/2))*(1+(1-x)^2/(1-(4+a)^(1/2)))^(1/2)*(1+(1-x)^2/(1+(4+a)^(1/2)))^(1/2)*E
llipticE((1-x)/(-1+(4+a)^(1/2))^(1/2),((1-(4+a)^(1/2))/(1+(4+a)^(1/2)))^(1
/2))/(3+a)^2/(4+a)/(3+a-2*(1-x)^2-(1-x)^4)^(1/2)-1/12*(-1+(4+a)^(1/2))^(1/
2)*(16+13*(4+a)^(1/2)+a*(4+3*(4+a)^(1/2)))*(1+(1-x)^2/(1-(4+a)^(1/2)))^(1/
2)*(1+(1-x)^2/(1+(4+a)^(1/2)))^(1/2)*EllipticF((1-x)/(-1+(4+a)^(1/2))^(1/2
),((1-(4+a)^(1/2))/(1+(4+a)^(1/2)))^(1/2))/(3+a)^2/(4+a)/(3+a-2*(1-x)^2-(1
-x)^4)^(1/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 5812 vs. $2(525) = 1050$.

Time = 17.25 (sec) , antiderivative size = 5812, normalized size of antiderivative = 11.07

$$\int \frac{x^2}{(a+8x-8x^2+4x^3-x^4)^{5/2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[x^2/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(5/2),x]
```

output
```
Result too large to show
```

Rubi [A] (warning: unable to verify)

Time = 1.61 (sec) , antiderivative size = 653, normalized size of antiderivative = 1.24, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {2459, 2006, 2202, 27, 1432, 1089, 1088, 1492, 27, 1492, 27, 1514, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a-x^4+4x^3-8x^2+8x)^{5/2}}\,dx$$

$$\Big\downarrow \; 2459$$

$$\int \frac{(x-1)^2+2(x-1)+1}{(a-(x-1)^4-2(x-1)^2+3)^{5/2}}d(x-1)$$

$$\Big\downarrow \; 2006$$

$$\int \frac{x^2}{(a-(x-1)^4-2(x-1)^2+3)^{5/2}}d(x-1)$$

$$\Big\downarrow \; 2202$$

$$\int \frac{(x-1)^2+1}{(-(x-1)^4-2(x-1)^2+a+3)^{5/2}}d(x-1)+\int \frac{2(x-1)}{(-(x-1)^4-2(x-1)^2+a+3)^{5/2}}d(x-1)$$

$$\Big\downarrow \; 27$$

$$\int \frac{(x-1)^2+1}{(-(x-1)^4-2(x-1)^2+a+3)^{5/2}}d(x-1)+ 2\int \frac{x-1}{(-(x-1)^4-2(x-1)^2+a+3)^{5/2}}d(x-1)$$

$$\Big\downarrow \; 1432$$

$$\int \frac{1}{(-(x-1)^4-2(x-1)^2+a+3)^{5/2}}d(x-1)^2+\int \frac{(x-1)^2+1}{(-(x-1)^4-2(x-1)^2+a+3)^{5/2}}d(x-1)$$

$$\Big\downarrow \; 1089$$

$$\int \frac{(x-1)^2+1}{(-(x-1)^4-2(x-1)^2+a+3)^{5/2}}d(x-1)+\frac{2\int \frac{1}{(-(x-1)^4-2(x-1)^2+a+3)^{3/2}}d(x-1)^2}{3(a+4)}+ \frac{x}{3(a+4)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}}$$

$$\Big\downarrow \; 1088$$

$$\int \frac{(x-1)^2+1}{(-(x-1)^4-2(x-1)^2+a+3)^{5/2}}d(x-1)+\frac{2x}{3(a+4)^2\sqrt{a-(x-1)^4-2(x-1)^2+3}}+ \frac{x}{3(a+4)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}}$$

$$\Big\downarrow \; 1492$$

$$-\frac{\int -\frac{2(a+4)\left(3(x-1)^2+4\right)}{\left(-(x-1)^4-2(x-1)^2+a+3\right)^{3/2}}d(x-1)}{12\left(a^2+7a+12\right)}+\frac{(a+4)\left((x-1)^2+2\right)(x-1)}{6\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}}+\frac{2x}{3(a+4)^2\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{x}{3(a+4)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{(a+4)\int \frac{3(x-1)^2+4}{\left(-(x-1)^4-2(x-1)^2+a+3\right)^{3/2}}d(x-1)}{6\left(a^2+7a+12\right)}+\frac{(a+4)\left((x-1)^2+2\right)(x-1)}{6\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}}+\frac{2x}{3(a+4)^2\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{x}{3(a+4)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}}$$

$$\Big\downarrow \text{1492}$$

$$\frac{(a+4)\left(\frac{(x-1)\left((3a+13)(x-1)^2+7a+29\right)}{2\left(a^2+7a+12\right)\sqrt{a-(x-1)^4-2(x-1)^2+3}}-\frac{\int -\frac{2\left(-\left((3a+13)(x-1)^2\right)+a+3\right)}{\sqrt{-(x-1)^4-2(x-1)^2+a+3}}d(x-1)}{4\left(a^2+7a+12\right)}\right)}{6\left(a^2+7a+12\right)}+\frac{(a+4)\left((x-1)^2+2\right)(x-1)}{6\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}}+\frac{2x}{3(a+4)^2\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{x}{3(a+4)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{(a+4)\left(\frac{\int \frac{-\left((3a+13)(x-1)^2\right)+a+3}{\sqrt{-(x-1)^4-2(x-1)^2+a+3}}d(x-1)}{2\left(a^2+7a+12\right)}+\frac{(x-1)\left((3a+13)(x-1)^2+7a+29\right)}{2\left(a^2+7a+12\right)\sqrt{a-(x-1)^4-2(x-1)^2+3}}\right)}{6\left(a^2+7a+12\right)}+\frac{(a+4)\left((x-1)^2+2\right)(x-1)}{6\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}}+\frac{2x}{3(a+4)^2\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{x}{3(a+4)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}}$$

$$\Big\downarrow \text{1514}$$

$$\frac{(a+4)\left(\frac{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}\int \frac{-\left((3a+13)(x-1)^2\right)+a+3}{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}d(x-1)}{2(a^2+7a+12)\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{(x-1)\left((3a+13)(x-1)^2+7a+29\right)}{2(a^2+7a+12)\sqrt{a-(x-1)^4-2(x-1)^2+3}}\right)}{6\left(a^2+7a+12\right)}+\frac{(a+4)\left((x-1)^2+2\right)(x-1)}{6\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}}+\frac{2x}{3(a+4)^2\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{x}{3(a+4)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}}$$

$\Big\downarrow$ 406

$$\frac{(a+4)\left(\frac{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}\left((a+3)\int \frac{1}{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}d(x-1)-(3a+13)\int \frac{(x-1)^2}{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}d(x-1)\right)}{2(a^2+7a+12)\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{(x}{2(a^2+}\right.}{6\left(a^2+7a+12\right)}+\frac{(a+4)\left((x-1)^2+2\right)(x-1)}{6\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}}+\frac{2x}{3(a+4)^2\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{x}{3(a+4)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}}$$

$\Big\downarrow$ 320

$$\frac{(a+4)\left(\frac{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}\left(\frac{(a+3)\sqrt{\sqrt{a+4}+1}\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\,\text{EllipticF}\left(\arctan\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right),-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}-(3a+13)\int \frac{(x-1)^2}{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-}{\sqrt{a+}}}}\right.}{2(a^2+7a+12)\sqrt{a-(x-1)^4-2(x-1)^2+3}}\right.}{6\left(a^2+7a+12\right)}+\frac{(a+4)\left((x-1)^2+2\right)(x-1)}{6\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}}+\frac{2x}{3(a+4)^2\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{x}{3(a+4)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}}$$

$\Big\downarrow$ 388

$$\frac{(a+4)\left(\frac{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}\left(\frac{(a+3)\sqrt{\sqrt{a+4}+1}\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\operatorname{EllipticF}\left(\arctan\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right),-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}-(3a+13)\left(\frac{(1-\sqrt{a+4})(x-1)\sqrt{\frac{(\ldots}{1-\ldots}}}{\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+\ldots}}\right.\right.}{2(a^2+7a+12)\sqrt{a-(x-1)^4-2(x-1)^2+3}}\right.}{6\left(a^2+7a+12\right)}$$

$$\frac{(a+4)\left((x-1)^2+2\right)(x-1)}{6\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}}+$$

$$\frac{2x}{3(a+4)^2\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{x}{3(a+4)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}}$$

$$\Big\downarrow \; 313$$

$$\frac{(a+4)\left(\frac{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}\left(\frac{(a+3)\sqrt{\sqrt{a+4}+1}\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\operatorname{EllipticF}\left(\arctan\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right),-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}-(3a+13)\left(\frac{(1-\sqrt{a+4})(x-1)\sqrt{\frac{(\ldots}{1-\ldots}}}{\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+\ldots}}\right.\right.}{2(a^2+7a+12)\sqrt{a-(x-1)^4-2(x-1)^2+3}}\right.}{6\left(a^2+7a+1\ldots\right.}$$

$$\frac{(a+4)\left((x-1)^2+2\right)(x-1)}{6\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}}+$$

$$\frac{2x}{3(a+4)^2\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{x}{3(a+4)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}}$$

input
```
Int[x^2/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(5/2),x]
```

output
```
((4 + a)*(2 + (-1 + x)^2)*(-1 + x))/(6*(12 + 7*a + a^2)*(3 + a - 2*(-1 + x
)^2 - (-1 + x)^4)^(3/2)) + x/(3*(4 + a)*(3 + a - 2*(-1 + x)^2 - (-1 + x)^4
)^(3/2)) + (2*x)/(3*(4 + a)^2*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) + (
(4 + a)*(((29 + 7*a + (13 + 3*a)*(-1 + x)^2)*(-1 + x))/(2*(12 + 7*a + a^2)
*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) + (Sqrt[1 + (-1 + x)^2/(1 - Sqrt
[4 + a])]*Sqrt[1 + (-1 + x)^2/(1 + Sqrt[4 + a])]*(-((13 + 3*a)*(((1 - Sqrt
[4 + a])*Sqrt[1 + (-1 + x)^2/(1 - Sqrt[4 + a])]*(-1 + x))/Sqrt[1 + (-1 + x
)^2/(1 + Sqrt[4 + a])] - ((1 - Sqrt[4 + a])*Sqrt[1 + Sqrt[4 + a]]*Sqrt[1 +
 (-1 + x)^2/(1 - Sqrt[4 + a])]*EllipticE[ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 +
 a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])/(Sqrt[(1 + (-1 + x)^2/(1 - Sqr
t[4 + a]))/(1 + (-1 + x)^2/(1 + Sqrt[4 + a]))]*Sqrt[1 + (-1 + x)^2/(1 + Sq
rt[4 + a])]))) + ((3 + a)*Sqrt[1 + Sqrt[4 + a]]*Sqrt[1 + (-1 + x)^2/(1 - S
qrt[4 + a])]*EllipticF[ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4
+ a])/(1 - Sqrt[4 + a])])/(Sqrt[(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (-
1 + x)^2/(1 + Sqrt[4 + a]))]*Sqrt[1 + (-1 + x)^2/(1 + Sqrt[4 + a])])))/(2*
(12 + 7*a + a^2)*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4])))/(6*(12 + 7*a +
 a^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

rule 1089
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1432
```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

rule 1492
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2
 - 4*a*c))   Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p +
7)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 LtQ[p, -1] && IntegerQ[2*p]
```

rule 1514
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[Sqrt[1 + 2*c*(x^2/(b - q))]*(Sqrt
[1 + 2*c*(x^2/(b + q))]/Sqrt[a + b*x^2 + c*x^4])   Int[(d + e*x^2)/(Sqrt[1
+ 2*c*(x^2/(b - q))]*Sqrt[1 + 2*c*(x^2/(b + q))]), x], x]] /; FreeQ[{a, b,
c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

rule 2006
```
Int[(u_.)*(Px_), x_Symbol] :> With[{a = Rt[Coeff[Px, x, 0], Expon[Px, x]],
b = Rt[Coeff[Px, x, Expon[Px, x]], Expon[Px, x]]}, Int[u*(a + b*x)^Expon[Px
, x], x] /; EqQ[Px, (a + b*x)^Expon[Px, x]]] /; PolyQ[Px, x] && GtQ[Expon[P
x, x], 1] && NeQ[Coeff[Px, x, 0], 0] &&  !MatchQ[Px, (a_.)*(v_)^Expon[Px, x
] /; FreeQ[a, x] && LinearQ[v, x]]
```

rule 2202
```
Int[(Pn_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{n
 = Expon[Pn, x], k}, Int[Sum[Coeff[Pn, x, 2*k]*x^(2*k), {k, 0, n/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pn, x, 2*k + 1]*x^(2*k), {k, 0, (n -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pn, x]
 &&  !PolyQ[Pn, x^2]
```

rule 2459
```
Int[(Pn_)^(p_.)*(Qx_), x_Symbol] :> With[{S = Coeff[Pn, x, Expon[Pn, x] - 1
]/(Expon[Pn, x]*Coeff[Pn, x, Expon[Pn, x]])}, Subst[Int[ExpandToSum[Pn /. x
 -> x - S, x]^p*ExpandToSum[Qx /. x -> x - S, x], x], x, x + S] /; Binomial
Q[Pn /. x -> x - S, x] || (IntegerQ[Expon[Pn, x]/2] && TrinomialQ[Pn /. x -
> x - S, x])] /; FreeQ[p, x] && PolyQ[Pn, x] && GtQ[Expon[Pn, x], 2] && NeQ
[Coeff[Pn, x, Expon[Pn, x] - 1], 0] && PolyQ[Qx, x] &&  !(MonomialQ[Qx, x]
&& IGtQ[p, 0])
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 2779 vs. $2(457) = 914$.

Time = 0.88 (sec) , antiderivative size = 2780, normalized size of antiderivative = 5.30

method	result	size
default	Expression too large to display	2780
elliptic	Expression too large to display	2780

input
```
int(x^2/(-x^4+4*x^3-8*x^2+a+8*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
(1/6/(a+3)*x^3-1/6*(6+a)/(a^2+7*a+12)*x^2+1/6*(a+8)/(a^2+7*a+12)*x+1/6/(a^
2+7*a+12)*a)*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)/(x^4-4*x^3+8*x^2-a-8*x)^2+2*(1
/24*(13+3*a)/(a+3)/(a^2+7*a+12)*x^3-1/24*(a^2+27*a+84)/(a^2+7*a+12)^2*x^2+
1/6*(9*a+32)/(a^2+7*a+12)^2*x+1/12*(3*a^2+7*a-12)/(a^2+7*a+12)^2)/(-x^4+4*
x^3-8*x^2+a+8*x)^(1/2)-(-1/6*(a^2-9*a-44)/(a^2+7*a+12)^2-1/3*(9*a+32)/(a^2
+7*a+12)^2)*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*((-(-1-(a+4)^(
1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+
4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2
)*(x-1+(-1+(a+4)^(1/2))^(1/2))^2*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1-(-1-(a+4)
^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a
+4)^(1/2))^(1/2)))^(1/2)*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1+(-1-(a+4)^(1/2))^
(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/
2))^(1/2)))^(1/2)/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-1+(a+
4)^(1/2))^(1/2)/(-(x-1-(-1+(a+4)^(1/2))^(1/2))*(x-1+(-1+(a+4)^(1/2))^(1/2)
)*(x-1-(-1-(a+4)^(1/2))^(1/2))*(x-1+(-1-(a+4)^(1/2))^(1/2)))^(1/2)*Ellipti
cF(((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))
^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1
/2))^(1/2)))^(1/2),((-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))*((-1-
(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a
+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2)))^(1/2...
```

Fricas [F]

$$\int \frac{x^2}{\left(a+8x-8x^2+4x^3-x^4\right)^{5/2}}\,dx = \int \frac{x^{2}}{{\left(-x^{4} + 4\,x^{3} - 8\,x^{2} + a + 8\,x\right)}^{\frac{5}{2}}}\,{d x}$$

input
```
integrate(x^2/(-x^4+4*x^3-8*x^2+a+8*x)^(5/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)*x^2/(x^12 - 12*x^11 + 72*x^
10 - 3*(a - 256)*x^8 - 280*x^9 + 24*(a - 64)*x^7 - 32*(3*a - 70)*x^6 + 48*
(5*a - 48)*x^5 + 3*(a^2 - 128*a + 512)*x^4 - 4*(3*a^2 - 96*a + 128)*x^3 -
a^3 - 24*a^2*x + 24*(a^2 - 8*a)*x^2), x)
```

Sympy [F]

$$\int \frac{x^2}{(a+8x-8x^2+4x^3-x^4)^{5/2}}\,dx = \int \frac{x^2}{(a-x^4+4x^3-8x^2+8x)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**2/(-x**4+4*x**3-8*x**2+a+8*x)**(5/2),x)
```

output
```
Integral(x**2/(a - x**4 + 4*x**3 - 8*x**2 + 8*x)**(5/2), x)
```

Maxima [F]

$$\int \frac{x^2}{(a+8x-8x^2+4x^3-x^4)^{5/2}}\,dx = \int \frac{x^2}{(-x^4+4\,x^3-8\,x^2+a+8\,x)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^2/(-x^4+4*x^3-8*x^2+a+8*x)^(5/2),x, algorithm="maxima")
```

output
```
integrate(x^2/(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(5/2), x)
```

Giac [F]

$$\int \frac{x^2}{(a+8x-8x^2+4x^3-x^4)^{5/2}}\,dx = \int \frac{x^2}{(-x^4+4\,x^3-8\,x^2+a+8\,x)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^2/(-x^4+4*x^3-8*x^2+a+8*x)^(5/2),x, algorithm="giac")
```

output
```
integrate(x^2/(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(a+8x-8x^2+4x^3-x^4)^{5/2}}\,dx = \int \frac{x^2}{{\left(-x^4+4\,x^3-8\,x^2+8\,x+a\right)}^{5/2}}\,dx$$

input
```
int(x^2/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(5/2),x)
```

output
```
int(x^2/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(5/2), x)
```

Reduce [F]

$$\int \frac{x^2}{(a+8x-8x^2+4x^3-x^4)^{5/2}}\,dx = \int \frac{}{-x^{12}+12x^{11}-72x^{10}+3a\,x^8+280x^9-24a\,x^7-768x^8+96a\,x}$$

input
```
int(x^2/(-x^4+4*x^3-8*x^2+a+8*x)^(5/2),x)
```

output
```
int((sqrt(a - x**4 + 4*x**3 - 8*x**2 + 8*x)*x**2)/(a**3 - 3*a**2*x**4 + 12
*a**2*x**3 - 24*a**2*x**2 + 24*a**2*x + 3*a*x**8 - 24*a*x**7 + 96*a*x**6 -
 240*a*x**5 + 384*a*x**4 - 384*a*x**3 + 192*a*x**2 - x**12 + 12*x**11 - 72
*x**10 + 280*x**9 - 768*x**8 + 1536*x**7 - 2240*x**6 + 2304*x**5 - 1536*x*
*4 + 512*x**3),x)
```

3.43 $\int \frac{x^3}{a+b(c+dx)^4}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 275

$$\int \frac{x^3}{a+b(c+dx)^4}\,dx = \frac{3c^2\arctan\left(\frac{\sqrt{b}(c+dx)^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}d^4} + \frac{c\left(3\sqrt{a}+\sqrt{b}c^2\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}d^4} - \frac{c\left(3\sqrt{a}+\sqrt{b}c^2\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}d^4} + \frac{c\left(3\sqrt{a}-\sqrt{b}c^2\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)}{\sqrt{a}+\sqrt{b}(c+dx)^2}\right)}{2\sqrt{2}a^{3/4}b^{3/4}d^4} + \frac{\log\left(a+b(c+dx)^4\right)}{4bd^4}$$

output
```
3/2*c^2*arctan(b^(1/2)*(d*x+c)^2/a^(1/2))/a^(1/2)/b^(1/2)/d^4-1/4*c*(3*a^(
1/2)+b^(1/2)*c^2)*arctan(-1+2^(1/2)*b^(1/4)*(d*x+c)/a^(1/4))*2^(1/2)/a^(3/
4)/b^(3/4)/d^4-1/4*c*(3*a^(1/2)+b^(1/2)*c^2)*arctan(1+2^(1/2)*b^(1/4)*(d*x
+c)/a^(1/4))*2^(1/2)/a^(3/4)/b^(3/4)/d^4+1/4*c*(3*a^(1/2)-b^(1/2)*c^2)*arc
tanh(2^(1/2)*a^(1/4)*b^(1/4)*(d*x+c)/(a^(1/2)+b^(1/2)*(d*x+c)^2))*2^(1/2)/
a^(3/4)/b^(3/4)/d^4+1/4*ln(a+b*(d*x+c)^4)/b/d^4
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.06 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.39

$$\int \frac{x^3}{a+b(c+dx)^4}\,dx = \frac{\text{RootSum}\left[a+bc^4+4bc^3d\#1+6bc^2d^2\#1^2+4bcd^3\#1^3+bd^4\#1^4\&,\frac{\log(x-\#1)\#1^3}{c^3+3c^2d\#1+3cd^2\#1^2+d^3\#1^3}\&\right]}{4bd}$$

input
```
Integrate[x^3/(a + b*(c + d*x)^4),x]
```

output
```
RootSum[a + b*c^4 + 4*b*c^3*d*#1 + 6*b*c^2*d^2*#1^2 + 4*b*c*d^3*#1^3 + b*d
^4*#1^4 & , (Log[x - #1]*#1^3)/(c^3 + 3*c^2*d*#1 + 3*c*d^2*#1^2 + d^3*#1^3
) & ]/(4*b*d)
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 342, normalized size of antiderivative = 1.24, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {896, 25, 2415, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{a+b(c+dx)^4}\,dx$$

$$\Big\downarrow \text{896}$$

$$\frac{\int \frac{d^3x^3}{b(c+dx)^4+a}d(c+dx)}{d^4}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int -\frac{d^3x^3}{b(c+dx)^4+a}d(c+dx)}{d^4}$$

$$\Big\downarrow \text{2415}$$

$$-\frac{\int \left(\frac{(c+dx)(-3c^2-(c+dx)^2)}{b(c+dx)^4+a}+\frac{c^3+3(c+dx)^2c}{b(c+dx)^4+a}\right)d(c+dx)}{d^4}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{c\left(3\sqrt{a}+\sqrt{b}c^2\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}}-\frac{c\left(3\sqrt{a}+\sqrt{b}c^2\right)\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}b^{3/4}}-\frac{c\left(3\sqrt{a}-\sqrt{b}c^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{a}+\sqrt{}\right.}{4\sqrt{2}a^{3/4}b^{3/4}}}{d^4}$$

input
```
Int[x^3/(a + b*(c + d*x)^4),x]
```

output
```
((3*c^2*ArcTan[(Sqrt[b]*(c + d*x)^2)/Sqrt[a]])/(2*Sqrt[a]*Sqrt[b]) + (c*(3
*Sqrt[a] + Sqrt[b]*c^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*(c + d*x))/a^(1/4)])/(
2*Sqrt[2]*a^(3/4)*b^(3/4)) - (c*(3*Sqrt[a] + Sqrt[b]*c^2)*ArcTan[1 + (Sqrt
[2]*b^(1/4)*(c + d*x))/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(3/4)) - (c*(3*Sqrt[
a] - Sqrt[b]*c^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*(c + d*x) + Sqrt[b
]*(c + d*x)^2])/(4*Sqrt[2]*a^(3/4)*b^(3/4)) + (c*(3*Sqrt[a] - Sqrt[b]*c^2)
*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*(c + d*x) + Sqrt[b]*(c + d*x)^2])/(
4*Sqrt[2]*a^(3/4)*b^(3/4)) + Log[a + b*(c + d*x)^4]/(4*b))/d^4
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2415
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.35

method	result	size
default	$\frac{\sum_{\_R=\operatorname{RootOf}\left(b\,d^4\_Z^4+4bc\,d^3\_Z^3+6b\,c^2d^2\_Z^2+4b\,c^3d\_Z+b\,c^4+a\right)}\frac{\_R^3\ln\left(x-\_R\right)}{d^3\_R^3+3c\,d^2\_R^2+3c^2d\_R+c^3}}{4bd}$	97
risch	$\frac{\sum_{\_R=\operatorname{RootOf}\left(b\,d^4\_Z^4+4bc\,d^3\_Z^3+6b\,c^2d^2\_Z^2+4b\,c^3d\_Z+b\,c^4+a\right)}\frac{\_R^3\ln\left(x-\_R\right)}{d^3\_R^3+3c\,d^2\_R^2+3c^2d\_R+c^3}}{4bd}$	97

input
```
int(x^3/(a+b*(d*x+c)^4),x,method=_RETURNVERBOSE)
```

output
```
1/4/b/d*sum(_R^3/(_R^3*d^3+3*_R^2*c*d^2+3*_R*c^2*d+c^3)*ln(x-_R),_R=RootOf
(_Z^4*b*d^4+4*_Z^3*b*c*d^3+6*_Z^2*b*c^2*d^2+4*_Z*b*c^3*d+b*c^4+a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^3}{a+b(c+dx)^4}\,dx = \text{Timed out}$$

input
```
integrate(x^3/(a+b*(d*x+c)^4),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [A] (verification not implemented)

Time = 2.25 (sec) , antiderivative size = 374, normalized size of antiderivative = 1.36

$$\int \frac{x^3}{a+b(c+dx)^4}\,dx$$
$$=\text{RootSum}\Big(256t^4a^3b^4d^{16}-256t^3a^3b^3d^{12}+t^2\cdot\left(96a^3b^2d^8+480a^2b^3c^4d^8\right)+t\left(-16a^3bd^4+192a^2b^2c^4d^4-\right.$$

input
```
integrate(x**3/(a+b*(d*x+c)**4),x)
```

output
```
RootSum(256*_t**4*a**3*b**4*d**16 - 256*_t**3*a**3*b**3*d**12 + _t**2*(96*
a**3*b**2*d**8 + 480*a**2*b**3*c**4*d**8) + _t*(-16*a**3*b*d**4 + 192*a**2
*b**2*c**4*d**4 - 48*a*b**3*c**8*d**4) + a**3 + 3*a**2*b*c**4 + 3*a*b**2*c
**8 + b**3*c**12, Lambda(_t, _t*log(x + (-1728*_t**3*a**4*b**3*d**12 - 960
*_t**3*a**3*b**4*c**4*d**12 + 1296*_t**2*a**4*b**2*d**8 + 2016*_t**2*a**3*
b**3*c**4*d**8 - 48*_t**2*a**2*b**4*c**8*d**8 - 324*_t*a**4*b*d**4 - 4716*
_t*a**3*b**2*c**4*d**4 - 1452*_t*a**2*b**3*c**8*d**4 - 4*_t*a*b**4*c**12*d
**4 + 27*a**4 - 390*a**3*b*c**4 - 444*a**2*b**2*c**8 - 26*a*b**3*c**12 + b
**4*c**16)/(729*a**3*b*c**3*d - 1053*a**2*b**2*c**7*d - 117*a*b**3*c**11*d
 + b**4*c**15*d)))
```

Maxima [F]

$$\int \frac{x^3}{a+b(c+dx)^4}\,dx=\int \frac{x^3}{(dx+c)^4b+a}\,dx$$

input
```
integrate(x^3/(a+b*(d*x+c)^4),x, algorithm="maxima")
```

output
```
integrate(x^3/((d*x + c)^4*b + a), x)
```

Giac [F]

$$\int \frac{x^3}{a+b(c+dx)^4}\,dx=\int \frac{x^3}{(dx+c)^4b+a}\,dx$$

input
```
integrate(x^3/(a+b*(d*x+c)^4),x, algorithm="giac")
```

output
```
integrate(x^3/((d*x + c)^4*b + a), x)
```

Mupad [B] (verification not implemented)

Time = 22.47 (sec) , antiderivative size = 1003, normalized size of antiderivative = 3.65

$$\int \frac{x^3}{a+b(c+dx)^4}\,dx=\sum_{k=1}^{4}\ln\Big(b\,c^2\,d\,\Big(2\,a\,c+2\,b\,c^5-3\,a\,d\,x+5\,b\,c^4\,d\,x$$
$$-\mathrm{root}\big(256\,a^3\,b^4\,d^{16}\,z^4-256\,a^3\,b^3\,d^{12}\,z^3+480\,a^2\,b^3\,c^4\,d^8\,z^2+96\,a^3\,b^2\,d^8\,z^2+192\,a^2\,b^2\,c^4\,d^4\,z-48\,a\,b^3\,c^8\,d^4\,z$$
$$+\mathrm{root}\big(256\,a^3\,b^4\,d^{16}\,z^4-256\,a^3\,b^3\,d^{12}\,z^3+480\,a^2\,b^3\,c^4\,d^8\,z^2+96\,a^3\,b^2\,d^8\,z^2+192\,a^2\,b^2\,c^4\,d^4\,z-48\,a\,b^3\,c^8$$
$$+\mathrm{root}\big(256\,a^3\,b^4\,d^{16}\,z^4-256\,a^3\,b^3\,d^{12}\,z^3+480\,a^2\,b^3\,c^4\,d^8\,z^2+96\,a^3\,b^2\,d^8\,z^2+192\,a^2\,b^2\,c^4\,d^4\,z-48\,a\,b^3\,c^8$$
$$-\mathrm{root}\big(256\,a^3\,b^4\,d^{16}\,z^4-256\,a^3\,b^3\,d^{12}\,z^3+480\,a^2\,b^3\,c^4\,d^8\,z^2+96\,a^3\,b^2\,d^8\,z^2+192\,a^2\,b^2\,c^4\,d^4\,z-48\,a\,b^3\,c^8\,d^4\,z$$
$$+\mathrm{root}\big(256\,a^3\,b^4\,d^{16}\,z^4-256\,a^3\,b^3\,d^{12}\,z^3+480\,a^2\,b^3\,c^4\,d^8\,z^2+96\,a^3\,b^2\,d^8\,z^2+192\,a^2\,b^2\,c^4\,d^4\,z-48\,a\,b^3\,c^8\,d^4\,z$$
$$+\mathrm{root}\big(256\,a^3\,b^4\,d^{16}\,z^4-256\,a^3\,b^3\,d^{12}\,z^3+480\,a^2\,b^3\,c^4\,d^8\,z^2+96\,a^3\,b^2\,d^8\,z^2+192\,a^2\,b^2\,c^4\,d^4\,z-48\,a\,b^3\,c^8\,d^4\,z$$
$$-256\,a^3\,b^3\,d^{12}\,z^3+480\,a^2\,b^3\,c^4\,d^8\,z^2+96\,a^3\,b^2\,d^8\,z^2+192\,a^2\,b^2\,c^4\,d^4\,z-48\,a\,b^3\,c^8\,d^4\,z$$
$$-16\,a^3\,b\,d^4\,z+3\,a\,b^2\,c^8+3\,a^2\,b\,c^4+b^3\,c^{12}+a^3,z,k\big)$$

input
```
int(x^3/(a + b*(c + d*x)^4),x)
```

output
```
symsum(log(2*b*c^2*d*(2*a*c + 2*b*c^5 - 3*a*d*x + 5*b*c^4*d*x - 2*root(256
*a^3*b^4*d^16*z^4 - 256*a^3*b^3*d^12*z^3 + 480*a^2*b^3*c^4*d^8*z^2 + 96*a^
3*b^2*d^8*z^2 + 192*a^2*b^2*c^4*d^4*z - 48*a*b^3*c^8*d^4*z - 16*a^3*b*d^4*
z + 3*a*b^2*c^8 + 3*a^2*b*c^4 + b^3*c^12 + a^3, z, k)*b^2*c^5*d^4 + 32*roo
t(256*a^3*b^4*d^16*z^4 - 256*a^3*b^3*d^12*z^3 + 480*a^2*b^3*c^4*d^8*z^2 +
96*a^3*b^2*d^8*z^2 + 192*a^2*b^2*c^4*d^4*z - 48*a*b^3*c^8*d^4*z - 16*a^3*b
*d^4*z + 3*a*b^2*c^8 + 3*a^2*b*c^4 + b^3*c^12 + a^3, z, k)^2*a*b^2*c*d^8 +
 24*root(256*a^3*b^4*d^16*z^4 - 256*a^3*b^3*d^12*z^3 + 480*a^2*b^3*c^4*d^8
*z^2 + 96*a^3*b^2*d^8*z^2 + 192*a^2*b^2*c^4*d^4*z - 48*a*b^3*c^8*d^4*z - 1
6*a^3*b*d^4*z + 3*a*b^2*c^8 + 3*a^2*b*c^4 + b^3*c^12 + a^3, z, k)^2*a*b^2*
d^9*x - 2*root(256*a^3*b^4*d^16*z^4 - 256*a^3*b^3*d^12*z^3 + 480*a^2*b^3*c
^4*d^8*z^2 + 96*a^3*b^2*d^8*z^2 + 192*a^2*b^2*c^4*d^4*z - 48*a*b^3*c^8*d^4
*z - 16*a^3*b*d^4*z + 3*a*b^2*c^8 + 3*a^2*b*c^4 + b^3*c^12 + a^3, z, k)*b^
2*c^4*d^5*x + 38*root(256*a^3*b^4*d^16*z^4 - 256*a^3*b^3*d^12*z^3 + 480*a^
2*b^3*c^4*d^8*z^2 + 96*a^3*b^2*d^8*z^2 + 192*a^2*b^2*c^4*d^4*z - 48*a*b^3*
c^8*d^4*z - 16*a^3*b*d^4*z + 3*a*b^2*c^8 + 3*a^2*b*c^4 + b^3*c^12 + a^3, z
, k)*a*b*c*d^4 + 6*root(256*a^3*b^4*d^16*z^4 - 256*a^3*b^3*d^12*z^3 + 480*
a^2*b^3*c^4*d^8*z^2 + 96*a^3*b^2*d^8*z^2 + 192*a^2*b^2*c^4*d^4*z - 48*a*b^
3*c^8*d^4*z - 16*a^3*b*d^4*z + 3*a*b^2*c^8 + 3*a^2*b*c^4 + b^3*c^12 + a^3,
 z, k)*a*b*d^5*x))*root(256*a^3*b^4*d^16*z^4 - 256*a^3*b^3*d^12*z^3 + 4...
```

Reduce [F]

$$\int \frac{x^3}{a+b(c+dx)^4}\,dx = \frac{-12\left(\int \frac{x^2}{b\,d^4x^4+4bc\,d^3x^3+6b\,c^2d^2x^2+4b\,c^3dx+b\,c^4+a}dx\right)bc\,d^3-12\left(\int \frac{x}{b\,d^4x^4+4bc\,d^3x^3+6b\,c^2d^2x^2+4b\,c^3dx+b\,c^4+a}dx\right)b\,c^2d^2 -}{4b}$$

input
```
int(x^3/(a+b*(d*x+c)^4),x)
```

output
```
( - 12*int(x**2/(a + b*c**4 + 4*b*c**3*d*x + 6*b*c**2*d**2*x**2 + 4*b*c*d*
*3*x**3 + b*d**4*x**4),x)*b*c*d**3 - 12*int(x/(a + b*c**4 + 4*b*c**3*d*x +
 6*b*c**2*d**2*x**2 + 4*b*c*d**3*x**3 + b*d**4*x**4),x)*b*c**2*d**2 - 4*in
t(1/(a + b*c**4 + 4*b*c**3*d*x + 6*b*c**2*d**2*x**2 + 4*b*c*d**3*x**3 + b*
d**4*x**4),x)*b*c**3*d + log(a + b*c**4 + 4*b*c**3*d*x + 6*b*c**2*d**2*x**
2 + 4*b*c*d**3*x**3 + b*d**4*x**4))/(4*b*d**4)
```

3.44 $\int \frac{x^2}{a+b(c+dx)^4}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 240

$$\int \frac{x^2}{a+b(c+dx)^4}\,dx = -\frac{c\arctan\left(\frac{\sqrt{b}(c+dx)^2}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}d^3} - \frac{\left(\sqrt{a}+\sqrt{b}c^2\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}d^3} + \frac{\left(\sqrt{a}+\sqrt{b}c^2\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}d^3} - \frac{\left(\sqrt{a}-\sqrt{b}c^2\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)}{\sqrt{a}+\sqrt{b}(c+dx)^2}\right)}{2\sqrt{2}a^{3/4}b^{3/4}d^3}$$

output
```
-c*arctan(b^(1/2)*(d*x+c)^2/a^(1/2))/a^(1/2)/b^(1/2)/d^3+1/4*(a^(1/2)+b^(1
/2)*c^2)*arctan(-1+2^(1/2)*b^(1/4)*(d*x+c)/a^(1/4))*2^(1/2)/a^(3/4)/b^(3/4
)/d^3+1/4*(a^(1/2)+b^(1/2)*c^2)*arctan(1+2^(1/2)*b^(1/4)*(d*x+c)/a^(1/4))*
2^(1/2)/a^(3/4)/b^(3/4)/d^3-1/4*(a^(1/2)-b^(1/2)*c^2)*arctanh(2^(1/2)*a^(1
/4)*b^(1/4)*(d*x+c)/(a^(1/2)+b^(1/2)*(d*x+c)^2))*2^(1/2)/a^(3/4)/b^(3/4)/d
^3
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.44

$$\int \frac{x^2}{a+b(c+dx)^4}\,dx = \frac{\text{RootSum}\left[a+bc^4+4bc^3d\#1+6bc^2d^2\#1^2+4bcd^3\#1^3+bd^4\#1^4\&,\frac{\log(x-\#1)\#1^2}{c^3+3c^2d\#1+3cd^2\#1^2+d^3\#1^3}\&\right]}{4bd}$$

input
```
Integrate[x^2/(a + b*(c + d*x)^4),x]
```

output
```
RootSum[a + b*c^4 + 4*b*c^3*d*#1 + 6*b*c^2*d^2*#1^2 + 4*b*c*d^3*#1^3 + b*d
^4*#1^4 & , (Log[x - #1]*#1^2)/(c^3 + 3*c^2*d*#1 + 3*c*d^2*#1^2 + d^3*#1^3
) & ]/(4*b*d)
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.28, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {896, 2415, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{a+b(c+dx)^4}\,dx$$

$$\Big\downarrow \; 896$$

$$\frac{\int \frac{d^2x^2}{b(c+dx)^4+a}d(c+dx)}{d^3}$$

$$\Big\downarrow \; 2415$$

$$\frac{\int \left(\frac{c^2+(c+dx)^2}{b(c+dx)^4+a}-\frac{2c(c+dx)}{b(c+dx)^4+a}\right)d(c+dx)}{d^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{\left(\sqrt{a}+\sqrt{b}c^2\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{3/4}}+\frac{\left(\sqrt{a}+\sqrt{b}c^2\right)\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}b^{3/4}}+\frac{\left(\sqrt{a}-\sqrt{b}c^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{a}+\sqrt{b}(c-\right.}{4\sqrt{2}a^{3/4}b^{3/4}}}{d^3}$$

input

```
Int[x^2/(a + b*(c + d*x)^4),x]
```

output

```
(-((c*ArcTan[(Sqrt[b]*(c + d*x)^2)/Sqrt[a]])/(Sqrt[a]*Sqrt[b])) - ((Sqrt[a
] + Sqrt[b]*c^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*(c + d*x))/a^(1/4)])/(2*Sqrt[
2]*a^(3/4)*b^(3/4)) + ((Sqrt[a] + Sqrt[b]*c^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)
*(c + d*x))/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(3/4)) + ((Sqrt[a] - Sqrt[b]*c^
2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*(c + d*x) + Sqrt[b]*(c + d*x)^2])
/(4*Sqrt[2]*a^(3/4)*b^(3/4)) - ((Sqrt[a] - Sqrt[b]*c^2)*Log[Sqrt[a] + Sqrt
[2]*a^(1/4)*b^(1/4)*(c + d*x) + Sqrt[b]*(c + d*x)^2])/(4*Sqrt[2]*a^(3/4)*b
^(3/4)))/d^3
```

Defintions of rubi rules used

rule 896

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2415

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.40

method	result	size
default	$\frac{\sum_{\_R=\mathrm{RootOf}\left(b\,d^4\_Z^4+4bc\,d^3\_Z^3+6b\,c^2d^2\_Z^2+4b\,c^3d\_Z+b\,c^4+a\right)}\frac{\_R^2\ln\left(x-\_R\right)}{d^3\_R^3+3c\,d^2\_R^2+3c^2d\_R+c^3}}{4bd}$	97
risch	$\frac{\sum_{\_R=\mathrm{RootOf}\left(b\,d^4\_Z^4+4bc\,d^3\_Z^3+6b\,c^2d^2\_Z^2+4b\,c^3d\_Z+b\,c^4+a\right)}\frac{\_R^2\ln\left(x-\_R\right)}{d^3\_R^3+3c\,d^2\_R^2+3c^2d\_R+c^3}}{4bd}$	97

input
```
int(x^2/(a+b*(d*x+c)^4),x,method=_RETURNVERBOSE)
```

output
```
1/4/b/d*sum(_R^2/(_R^3*d^3+3*_R^2*c*d^2+3*_R*c^2*d+c^3)*ln(x-_R),_R=RootOf
(_Z^4*b*d^4+4*_Z^3*b*c*d^3+6*_Z^2*b*c^2*d^2+4*_Z*b*c^3*d+b*c^4+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 64.70 (sec) , antiderivative size = 61993, normalized size of antiderivative = 258.30

$$\int \frac{x^2}{a+b(c+dx)^4}\,dx = \text{Too large to display}$$

input
```
integrate(x^2/(a+b*(d*x+c)^4),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [A] (verification not implemented)

Time = 1.52 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.14

$$\int \frac{x^2}{a+b(c+dx)^4}\,dx$$
$$= \operatorname{RootSum}\left(256t^4a^3b^3d^{12}+192t^2a^2b^2c^2d^6+t\left(-32a^2bcd^3+32ab^2c^5d^3\right)+a^2+2abc^4+b^2c^8,\left(t\mapsto t\log\left(\right.\right.\right.$$

input
```
integrate(x**2/(a+b*(d*x+c)**4),x)
```

output
```
RootSum(256*_t**4*a**3*b**3*d**12 + 192*_t**2*a**2*b**2*c**2*d**6 + _t*(-3
2*a**2*b*c*d**3 + 32*a*b**2*c**5*d**3) + a**2 + 2*a*b*c**4 + b**2*c**8, La
mbda(_t, _t*log(x + (64*_t**3*a**4*b**2*d**9 + 448*_t**3*a**3*b**3*c**4*d*
*9 + 160*_t**2*a**3*b**2*c**3*d**6 - 32*_t**2*a**2*b**3*c**7*d**6 + 60*_t*
a**3*b*c**2*d**3 + 256*_t*a**2*b**2*c**6*d**3 + 4*_t*a*b**3*c**10*d**3 - 5
*a**3*c - 9*a**2*b*c**5 - 3*a*b**2*c**9 + b**3*c**13)/(a**3*d - 33*a**2*b*
c**4*d - 33*a*b**2*c**8*d + b**3*c**12*d))))
```

Maxima [F]

$$\int \frac{x^2}{a+b(c+dx)^4}\,dx=\int \frac{x^2}{(dx+c)^4b+a}\,dx$$

input
```
integrate(x^2/(a+b*(d*x+c)^4),x, algorithm="maxima")
```

output
```
integrate(x^2/((d*x + c)^4*b + a), x)
```

Giac [F]

$$\int \frac{x^2}{a+b(c+dx)^4}\,dx=\int \frac{x^2}{(dx+c)^4b+a}\,dx$$

input
```
integrate(x^2/(a+b*(d*x+c)^4),x, algorithm="giac")
```

output
```
integrate(x^2/((d*x + c)^4*b + a), x)
```

Mupad [B] (verification not implemented)

Time = 22.13 (sec) , antiderivative size = 625, normalized size of antiderivative = 2.60

$$\int \frac{x^2}{a+b(c+dx)^4}\,dx=\sum_{k=1}^{4}\ln\Big(-b\,d^4\,\Big(a+b\,c^4+4\,b\,c^3\,d\,x$$
$$+\mathrm{root}\left(256\,a^3\,b^3\,d^{12}\,z^4+192\,a^2\,b^2\,c^2\,d^6\,z^2+32\,a\,b^2\,c^5\,d^3\,z-32\,a^2\,b\,c\,d^3\,z+2\,a\,b\,c^4+b^2\,c^8+a^2,z,k\right)\,b^2\,c^5\,d^3\,4$$
$$+\mathrm{root}\left(256\,a^3\,b^3\,d^{12}\,z^4+192\,a^2\,b^2\,c^2\,d^6\,z^2+32\,a\,b^2\,c^5\,d^3\,z-32\,a^2\,b\,c\,d^3\,z+2\,a\,b\,c^4+b^2\,c^8+a^2,z,k\right)\,b^2\,c^4\,d^4\,x$$
$$-\mathrm{root}\left(256\,a^3\,b^3\,d^{12}\,z^4+192\,a^2\,b^2\,c^2\,d^6\,z^2+32\,a\,b^2\,c^5\,d^3\,z-32\,a^2\,b\,c\,d^3\,z+2\,a\,b\,c^4+b^2\,c^8+a^2,z,k\right)\,a\,b\,c\,d^3\,2$$
$$-\mathrm{root}\left(256\,a^3\,b^3\,d^{12}\,z^4+192\,a^2\,b^2\,c^2\,d^6\,z^2+32\,a\,b^2\,c^5\,d^3\,z-32\,a^2\,b\,c\,d^3\,z+2\,a\,b\,c^4+b^2\,c^8+a^2,z,k\right)\,a\,b\,d^4\,x\,4$$
$$+\mathrm{root}\left(256\,a^3\,b^3\,d^{12}\,z^4+192\,a^2\,b^2\,c^2\,d^6\,z^2+32\,a\,b^2\,c^5\,d^3\,z-32\,a^2\,b\,c\,d^3\,z+2\,a\,b\,c^4+b^2\,c^8+a^2,z,k\right)^2\,a\,b$$
$$+\mathrm{root}\left(256\,a^3\,b^3\,d^{12}\,z^4+192\,a^2\,b^2\,c^2\,d^6\,z^2+32\,a\,b^2\,c^5\,d^3\,z-32\,a^2\,b\,c\,d^3\,z+2\,a\,b\,c^4+b^2\,c^8+a^2,z,k\right)^2\,a\,b$$
$$+192\,a^2\,b^2\,c^2\,d^6\,z^2+32\,a\,b^2\,c^5\,d^3\,z-32\,a^2\,b\,c\,d^3\,z+2\,a\,b\,c^4+b^2\,c^8+a^2,z,k\Big)$$

input
```
int(x^2/(a + b*(c + d*x)^4),x)
```

output

```
symsum(log(-b*d^4*(a + b*c^4 + 4*b*c^3*d*x + 4*root(256*a^3*b^3*d^12*z^4 +
 192*a^2*b^2*c^2*d^6*z^2 + 32*a*b^2*c^5*d^3*z - 32*a^2*b*c*d^3*z + 2*a*b*c
^4 + b^2*c^8 + a^2, z, k)*b^2*c^5*d^3 + 4*root(256*a^3*b^3*d^12*z^4 + 192*
a^2*b^2*c^2*d^6*z^2 + 32*a*b^2*c^5*d^3*z - 32*a^2*b*c*d^3*z + 2*a*b*c^4 +
b^2*c^8 + a^2, z, k)*b^2*c^4*d^4*x - 20*root(256*a^3*b^3*d^12*z^4 + 192*a^
2*b^2*c^2*d^6*z^2 + 32*a*b^2*c^5*d^3*z - 32*a^2*b*c*d^3*z + 2*a*b*c^4 + b^
2*c^8 + a^2, z, k)*a*b*c*d^3 - 4*root(256*a^3*b^3*d^12*z^4 + 192*a^2*b^2*c
^2*d^6*z^2 + 32*a*b^2*c^5*d^3*z - 32*a^2*b*c*d^3*z + 2*a*b*c^4 + b^2*c^8 +
 a^2, z, k)*a*b*d^4*x + 48*root(256*a^3*b^3*d^12*z^4 + 192*a^2*b^2*c^2*d^6
*z^2 + 32*a*b^2*c^5*d^3*z - 32*a^2*b*c*d^3*z + 2*a*b*c^4 + b^2*c^8 + a^2,
z, k)^2*a*b^2*c^2*d^6 + 32*root(256*a^3*b^3*d^12*z^4 + 192*a^2*b^2*c^2*d^6
*z^2 + 32*a*b^2*c^5*d^3*z - 32*a^2*b*c*d^3*z + 2*a*b*c^4 + b^2*c^8 + a^2,
z, k)^2*a*b^2*c*d^7*x))*root(256*a^3*b^3*d^12*z^4 + 192*a^2*b^2*c^2*d^6*z^
2 + 32*a*b^2*c^5*d^3*z - 32*a^2*b*c*d^3*z + 2*a*b*c^4 + b^2*c^8 + a^2, z,
k), k, 1, 4)
```

Reduce [F]

$$\int \frac{x^2}{a+b(c+dx)^4}\,dx=\int \frac{x^2}{b\,d^4x^4+4bc\,d^3x^3+6b\,c^2d^2x^2+4b\,c^3dx+b\,c^4+a}dx$$

input

```
int(x^2/(a+b*(d*x+c)^4),x)
```

output

```
int(x**2/(a + b*c**4 + 4*b*c**3*d*x + 6*b*c**2*d**2*x**2 + 4*b*c*d**3*x**3
 + b*d**4*x**4),x)
```

3.45 $\int \frac{x}{a+b(c+dx)^4}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 198

$$\int \frac{x}{a+b(c+dx)^4}\,dx=\frac{\arctan\left(\frac{\sqrt{b}(c+dx)^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}d^2}+\frac{c\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}d^2}-\frac{c\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}d^2}-\frac{c\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)}{\sqrt{a}+\sqrt{b}(c+dx)^2}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}d^2}$$

output
```
1/2*arctan(b^(1/2)*(d*x+c)^2/a^(1/2))/a^(1/2)/b^(1/2)/d^2-1/4*c*arctan(-1+
2^(1/2)*b^(1/4)*(d*x+c)/a^(1/4))*2^(1/2)/a^(3/4)/b^(1/4)/d^2-1/4*c*arctan(
1+2^(1/2)*b^(1/4)*(d*x+c)/a^(1/4))*2^(1/2)/a^(3/4)/b^(1/4)/d^2-1/4*c*arcta
nh(2^(1/2)*a^(1/4)*b^(1/4)*(d*x+c)/(a^(1/2)+b^(1/2)*(d*x+c)^2))*2^(1/2)/a^
(3/4)/b^(1/4)/d^2
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.53

$$\int \frac{x}{a+b(c+dx)^4}\,dx = \frac{\text{RootSum}\left[a+bc^4+4bc^3d\#1+6bc^2d^2\#1^2+4bcd^3\#1^3+bd^4\#1^4\&,\frac{\log(x-\#1)\#1}{c^3+3c^2d\#1+3cd^2\#1^2+d^3\#1^3}\&\right]}{4bd}$$

input
```
Integrate[x/(a + b*(c + d*x)^4),x]
```

output
```
RootSum[a + b*c^4 + 4*b*c^3*d*#1 + 6*b*c^2*d^2*#1^2 + 4*b*c*d^3*#1^3 + b*d
^4*#1^4 & , (Log[x - #1]*#1)/(c^3 + 3*c^2*d*#1 + 3*c*d^2*#1^2 + d^3*#1^3)
& ]/(4*b*d)
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.26, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {896, 25, 2415, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{a+b(c+dx)^4}\,dx$$

$$\Big\downarrow \text{896}$$

$$\frac{\int \frac{dx}{b(c+dx)^4+a}d(c+dx)}{d^2}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int -\frac{dx}{b(c+dx)^4+a}d(c+dx)}{d^2}$$

$$\Big\downarrow \text{2415}$$

$$-\frac{\int \left(\frac{c}{b(c+dx)^4+a}-\frac{c+dx}{b(c+dx)^4+a}\right)d(c+dx)}{d^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{c\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}}-\frac{c\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}}+\frac{c\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{a}+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{b}}-\frac{c\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\right.}{4\sqrt{2}a^{3/4}\sqrt[4]{}}}{d^2}$$

input
```
Int[x/(a + b*(c + d*x)^4),x]
```

output
```
(ArcTan[(Sqrt[b]*(c + d*x)^2)/Sqrt[a]]/(2*Sqrt[a]*Sqrt[b]) + (c*ArcTan[1 -
 (Sqrt[2]*b^(1/4)*(c + d*x))/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(1/4)) - (c*Ar
cTan[1 + (Sqrt[2]*b^(1/4)*(c + d*x))/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(1/4))
 + (c*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*(c + d*x) + Sqrt[b]*(c + d*x)^
2])/(4*Sqrt[2]*a^(3/4)*b^(1/4)) - (c*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)
*(c + d*x) + Sqrt[b]*(c + d*x)^2])/(4*Sqrt[2]*a^(3/4)*b^(1/4)))/d^2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2415
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.48

method	result	size
default	$\frac{\sum_{\_R=\mathrm{RootOf}\left(b\,d^4\_Z^4+4bc\,d^3\_Z^3+6b\,c^2d^2\_Z^2+4b\,c^3d\_Z+b\,c^4+a\right)}\frac{\_R\ln\left(x-\_R\right)}{d^3\_R^3+3c\,d^2\_R^2+3c^2d\_R+c^3}}{4bd}$	95
risch	$\frac{\sum_{\_R=\mathrm{RootOf}\left(b\,d^4\_Z^4+4bc\,d^3\_Z^3+6b\,c^2d^2\_Z^2+4b\,c^3d\_Z+b\,c^4+a\right)}\frac{\_R\ln\left(x-\_R\right)}{d^3\_R^3+3c\,d^2\_R^2+3c^2d\_R+c^3}}{4bd}$	95

input
```
int(x/(a+b*(d*x+c)^4),x,method=_RETURNVERBOSE)
```

output
```
1/4/b/d*sum(_R/(_R^3*d^3+3*_R^2*c*d^2+3*_R*c^2*d+c^3)*ln(x-_R),_R=RootOf(_
Z^4*b*d^4+4*_Z^3*b*c*d^3+6*_Z^2*b*c^2*d^2+4*_Z*b*c^3*d+b*c^4+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 11.18 (sec) , antiderivative size = 40785, normalized size of antiderivative = 205.98

$$\int \frac{x}{a+b(c+dx)^4}\,dx = \text{Too large to display}$$

input
```
integrate(x/(a+b*(d*x+c)^4),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.66

$$\int \frac{x}{a+b(c+dx)^4}\,dx$$
$$=\text{RootSum}\left(256t^4a^3b^2d^8+32t^2a^2bd^4-16tabc^2d^2+a+bc^4,\left(t\mapsto t\log\left(x+\frac{128t^3a^3bd^6+16t^2a^2bc^2d^4+\cdots}{4ac\cdots}\right.\right.\right.$$

input
```
integrate(x/(a+b*(d*x+c)**4),x)
```

output
```
RootSum(256*_t**4*a**3*b**2*d**8 + 32*_t**2*a**2*b*d**4 - 16*_t*a*b*c**2*d
**2 + a + b*c**4, Lambda(_t, _t*log(x + (128*_t**3*a**3*b*d**6 + 16*_t**2*
a**2*b*c**2*d**4 + 8*_t*a**2*d**2 + 4*_t*a*b*c**4*d**2 - a*c**2 - b*c**6)/
(4*a*c*d - b*c**5*d))))
```

Maxima [F]

$$\int \frac{x}{a+b(c+dx)^4}\,dx = \int \frac{x}{(dx+c)^4b+a}\,dx$$

input
```
integrate(x/(a+b*(d*x+c)^4),x, algorithm="maxima")
```

output
```
integrate(x/((d*x + c)^4*b + a), x)
```

Giac [F]

$$\int \frac{x}{a+b(c+dx)^4}\,dx = \int \frac{x}{(dx+c)^4b+a}\,dx$$

input
```
integrate(x/(a+b*(d*x+c)^4),x, algorithm="giac")
```

output
```
integrate(x/((d*x + c)^4*b + a), x)
```

Mupad [B] (verification not implemented)

Time = 21.88 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.04

$$\int \frac{x}{a+b(c+dx)^4}\,dx=\sum_{k=1}^{4}\ln\Big(-\mathrm{root}\big(256\,a^3\,b^2\,d^8\,z^4+32\,a^2\,b\,d^4\,z^2-16\,a\,b\,c^2\,d^2\,z+b\,c^4+a,z,k\big)\,\big(-\mathrm{root}\big(256\,a^3\,b^2\,d^8\,z^4+32\,a^2\,b\,d^4\,z^2-16\,a\,b\,c^2\,d^2\,z+b\,c^4+a,z,k\big)\,\big(16\,a\,x\,b^3\,d^{12}+32\,a\,c\,b^3\,d^{11}\big)+4\,b^3\,c^3\,d^9+4\,b^3\,c^2\,d^{10}\,x\big)+b^2\,d^8\,x\Big)\,\mathrm{root}\big(256\,a^3\,b^2\,d^8\,z^4+32\,a^2\,b\,d^4\,z^2-16\,a\,b\,c^2\,d^2\,z+b\,c^4+a,z,k\big)$$

input
```
int(x/(a + b*(c + d*x)^4),x)
```

output
```
symsum(log(b^2*d^8*x - root(256*a^3*b^2*d^8*z^4 + 32*a^2*b*d^4*z^2 - 16*a*
b*c^2*d^2*z + b*c^4 + a, z, k)*(4*b^3*c^3*d^9 - root(256*a^3*b^2*d^8*z^4 +
 32*a^2*b*d^4*z^2 - 16*a*b*c^2*d^2*z + b*c^4 + a, z, k)*(32*a*b^3*c*d^11 +
 16*a*b^3*d^12*x) + 4*b^3*c^2*d^10*x))*root(256*a^3*b^2*d^8*z^4 + 32*a^2*b
*d^4*z^2 - 16*a*b*c^2*d^2*z + b*c^4 + a, z, k), k, 1, 4)
```

Reduce [F]

$$\int \frac{x}{a+b(c+dx)^4}\,dx=\int \frac{x}{b\,d^4x^4+4bc\,d^3x^3+6b\,c^2d^2x^2+4b\,c^3dx+b\,c^4+a}dx$$

input
```
int(x/(a+b*(d*x+c)^4),x)
```

output
```
int(x/(a + b*c**4 + 4*b*c**3*d*x + 6*b*c**2*d**2*x**2 + 4*b*c*d**3*x**3 +
b*d**4*x**4),x)
```

3.46 $\int \frac{1}{a+b(c+dx)^4}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 159

$$\int \frac{1}{a+b(c+dx)^4}\,dx = -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}d} + \frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}d} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)}{\sqrt{a}+\sqrt{b}(c+dx)^2}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}d}$$

output
```
1/4*arctan(-1+2^(1/2)*b^(1/4)*(d*x+c)/a^(1/4))*2^(1/2)/a^(3/4)/b^(1/4)/d+1
/4*arctan(1+2^(1/2)*b^(1/4)*(d*x+c)/a^(1/4))*2^(1/2)/a^(3/4)/b^(1/4)/d+1/4
*arctanh(2^(1/2)*a^(1/4)*b^(1/4)*(d*x+c)/(a^(1/2)+b^(1/2)*(d*x+c)^2))*2^(1
/2)/a^(3/4)/b^(1/4)/d
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.01

$$\int \frac{1}{a+b(c+dx)^4}\,dx$$
$$=\frac{-2\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)+2\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)-\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{b}(c+d\right.}{4\sqrt{2}a^{3/4}\sqrt[4]{b}d}$$

input
```
Integrate[(a + b*(c + d*x)^4)^(-1),x]
```

output
```
(-2*ArcTan[1 - (Sqrt[2]*b^(1/4)*(c + d*x))/a^(1/4)] + 2*ArcTan[1 + (Sqrt[2
]*b^(1/4)*(c + d*x))/a^(1/4)] - Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*(c +
 d*x) + Sqrt[b]*(c + d*x)^2] + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*(c +
d*x) + Sqrt[b]*(c + d*x)^2])/(4*Sqrt[2]*a^(3/4)*b^(1/4)*d)
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.43, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.692, Rules used = {239, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+b(c+dx)^4}\,dx$$
$$\Big\downarrow \; 239$$
$$\frac{\int \frac{1}{b(c+dx)^4+a}d(c+dx)}{d}$$
$$\Big\downarrow \; 755$$
$$\frac{\frac{\int \frac{\sqrt{a}-\sqrt{b}(c+dx)^2}{b(c+dx)^4+a}d(c+dx)}{2\sqrt{a}}+\frac{\int \frac{\sqrt{b}(c+dx)^2+\sqrt{a}}{b(c+dx)^4+a}d(c+dx)}{2\sqrt{a}}}{d}$$

$$\Big\downarrow \; 1476$$

$$\frac{\frac{\frac{\int \frac{1}{(c+dx)^2-\frac{\sqrt{2}\sqrt[4]{a}(c+dx)}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d(c+dx)}{2\sqrt{b}}+\frac{\int \frac{1}{(c+dx)^2+\frac{\sqrt{2}\sqrt[4]{a}(c+dx)}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d(c+dx)}{2\sqrt{b}}}{2\sqrt{a}}+\frac{\int \frac{\sqrt{a}-\sqrt{b}(c+dx)^2}{b(c+dx)^4+a}d(c+dx)}{2\sqrt{a}}}{d}$$

$$\Big\downarrow \; 1082$$

$$\frac{\frac{\int \frac{\sqrt{a}-\sqrt{b}(c+dx)^2}{b(c+dx)^4+a}d(c+dx)}{2\sqrt{a}}+\frac{\frac{\int \frac{1}{-\left(1-\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)^2-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int \frac{1}{-\left(\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}+1\right)^2-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}}{d}$$

$$\Big\downarrow \; 217$$

$$\frac{\frac{\int \frac{\sqrt{a}-\sqrt{b}(c+dx)^2}{b(c+dx)^4+a}d(c+dx)}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}}{d}$$

$$\Big\downarrow \; 1479$$

$$\frac{\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}\left((c+dx)^2-\frac{\sqrt{2}\sqrt[4]{a}(c+dx)}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d(c+dx)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}(c+dx)+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left((c+dx)^2+\frac{\sqrt{2}\sqrt[4]{a}(c+dx)}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d(c+dx)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\cdots\right)}{\sqrt{2}\cdots}}{2\sqrt{a}}}{d}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}\left((c+dx)^2-\frac{\sqrt{2}\sqrt[4]{a}(c+dx)}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d(c+dx)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}(c+dx)+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left((c+dx)^2+\frac{\sqrt{2}\sqrt[4]{a}(c+dx)}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d(c+dx)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\cdots}{\sqrt[4]{\cdots}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{\cdots}}}{2\sqrt{a}}}{d}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}(c+dx)}{(c+dx)^2-\frac{\sqrt{2}\sqrt[4]{a}(c+dx)}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d(c+dx)}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{b}(c+dx)+\sqrt[4]{a}}{(c+dx)^2+\frac{\sqrt{2}\sqrt[4]{a}(c+dx)}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d(c+dx)}{2\sqrt[4]{a}\sqrt{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}}{d}$$

$$\Big\downarrow \text{1103}$$

$$\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{a}+\sqrt{b}(c+dx)^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{a}+\sqrt{b}(c+dx)^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}}{d}$$

input

```
Int[(a + b*(c + d*x)^4)^(-1),x]
```

output

```
((-(ArcTan[1 - (Sqrt[2]*b^(1/4)*(c + d*x))/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/
4))) + ArcTan[1 + (Sqrt[2]*b^(1/4)*(c + d*x))/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^
(1/4)))/(2*Sqrt[a]) + (-1/2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*(c + d*x
) + Sqrt[b]*(c + d*x)^2]/(Sqrt[2]*a^(1/4)*b^(1/4)) + Log[Sqrt[a] + Sqrt[2]
*a^(1/4)*b^(1/4)*(c + d*x) + Sqrt[b]*(c + d*x)^2]/(2*Sqrt[2]*a^(1/4)*b^(1/
4)))/(2*Sqrt[a]))/d
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 239
```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Simp[1/Coefficient[v, x, 1
]   Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Lin
earQ[v, x] && NeQ[v, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.59

method	result	size
default	$\frac{\sum_{\_R=\mathrm{RootOf}\left(b\,d^{4}\_Z^{4}+4bc\,d^{3}\_Z^{3}+6b\,c^{2}d^{2}\_Z^{2}+4b\,c^{3}d\_Z+b\,c^{4}+a\right)}\frac{\ln\left(x-\_R\right)}{d^{3}\_R^{3}+3c\,d^{2}\_R^{2}+3c^{2}d\_R+c^{3}}}{4bd}$	94
risch	$\frac{\sum_{\_R=\mathrm{RootOf}\left(b\,d^{4}\_Z^{4}+4bc\,d^{3}\_Z^{3}+6b\,c^{2}d^{2}\_Z^{2}+4b\,c^{3}d\_Z+b\,c^{4}+a\right)}\frac{\ln\left(x-\_R\right)}{d^{3}\_R^{3}+3c\,d^{2}\_R^{2}+3c^{2}d\_R+c^{3}}}{4bd}$	94

input
```
int(1/(a+b*(d*x+c)^4),x,method=_RETURNVERBOSE)
```

output
```
1/4/b/d*sum(1/(_R^3*d^3+3*_R^2*c*d^2+3*_R*c^2*d+c^3)*ln(x-_R),_R=RootOf(_Z
^4*b*d^4+4*_Z^3*b*c*d^3+6*_Z^2*b*c^2*d^2+4*_Z*b*c^3*d+b*c^4+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.96

$$\int \frac{1}{a+b(c+dx)^4}\,dx = \frac{1}{4}\left(-\frac{1}{a^3bd^4}\right)^{\frac{1}{4}}\log\left(ad\left(-\frac{1}{a^3bd^4}\right)^{\frac{1}{4}}+dx+c\right) + \frac{1}{4}i\left(-\frac{1}{a^3bd^4}\right)^{\frac{1}{4}}\log\left(i\,ad\left(-\frac{1}{a^3bd^4}\right)^{\frac{1}{4}}+dx+c\right) - \frac{1}{4}i\left(-\frac{1}{a^3bd^4}\right)^{\frac{1}{4}}\log\left(-i\,ad\left(-\frac{1}{a^3bd^4}\right)^{\frac{1}{4}}+dx+c\right) - \frac{1}{4}\left(-\frac{1}{a^3bd^4}\right)^{\frac{1}{4}}\log\left(-ad\left(-\frac{1}{a^3bd^4}\right)^{\frac{1}{4}}+dx+c\right)$$

input
```
integrate(1/(a+b*(d*x+c)^4),x, algorithm="fricas")
```

output
```
1/4*(-1/(a^3*b*d^4))^(1/4)*log(a*d*(-1/(a^3*b*d^4))^(1/4) + d*x + c) + 1/4
*I*(-1/(a^3*b*d^4))^(1/4)*log(I*a*d*(-1/(a^3*b*d^4))^(1/4) + d*x + c) - 1/
4*I*(-1/(a^3*b*d^4))^(1/4)*log(-I*a*d*(-1/(a^3*b*d^4))^(1/4) + d*x + c) -
1/4*(-1/(a^3*b*d^4))^(1/4)*log(-a*d*(-1/(a^3*b*d^4))^(1/4) + d*x + c)
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.16

$$\int \frac{1}{a+b(c+dx)^4}\,dx = \frac{\operatorname{RootSum}\left(256t^4a^3b+1,\left(t\mapsto t\log\left(x+\frac{4ta+c}{d}\right)\right)\right)}{d}$$

input
```
integrate(1/(a+b*(d*x+c)**4),x)
```

output
```
RootSum(256*_t**4*a**3*b + 1, Lambda(_t, _t*log(x + (4*_t*a + c)/d)))/d
```

Maxima [F]

$$\int \frac{1}{a+b(c+dx)^4}\,dx = \int \frac{1}{(dx+c)^4b+a}\,dx$$

input
```
integrate(1/(a+b*(d*x+c)^4),x, algorithm="maxima")
```

output
```
integrate(1/((d*x + c)^4*b + a), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.65

$$\int \frac{1}{a+b(c+dx)^4}\,dx = -\frac{1}{2}\left(-\frac{1}{a^3bd^4}\right)^{\frac{1}{4}}\arctan\left(-\frac{bdx+bc}{(-ab^3)^{\frac{1}{4}}}\right) + \frac{1}{4}\left(-\frac{1}{a^3bd^4}\right)^{\frac{1}{4}}\log\left(\left|bdx+bc+\left(-ab^3\right)^{\frac{1}{4}}\right|\right) - \frac{1}{4}\left(-\frac{1}{a^3bd^4}\right)^{\frac{1}{4}}\log\left(\left|-bdx-bc+\left(-ab^3\right)^{\frac{1}{4}}\right|\right)$$

input
```
integrate(1/(a+b*(d*x+c)^4),x, algorithm="giac")
```

output
```
-1/2*(-1/(a^3*b*d^4))^(1/4)*arctan(-(b*d*x + b*c)/(-a*b^3)^(1/4)) + 1/4*(-
1/(a^3*b*d^4))^(1/4)*log(abs(b*d*x + b*c + (-a*b^3)^(1/4))) - 1/4*(-1/(a^3
*b*d^4))^(1/4)*log(abs(-b*d*x - b*c + (-a*b^3)^(1/4)))
```

Mupad [B] (verification not implemented)

Time = 21.70 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.38

$$\int \frac{1}{a+b(c+dx)^4}\,dx = -\frac{\operatorname{atan}\left(\frac{b^{1/4}\,c}{(-a)^{1/4}}+\frac{b^{1/4}\,d\,x}{(-a)^{1/4}}\right)+\operatorname{atanh}\left(\frac{b^{1/4}\,c}{(-a)^{1/4}}+\frac{b^{1/4}\,d\,x}{(-a)^{1/4}}\right)}{2\,(-a)^{3/4}\,b^{1/4}\,d}$$

input
```
int(1/(a + b*(c + d*x)^4),x)
```

output
```
-(atan((b^(1/4)*c)/(-a)^(1/4) + (b^(1/4)*d*x)/(-a)^(1/4)) + atanh((b^(1/4)
*c)/(-a)^(1/4) + (b^(1/4)*d*x)/(-a)^(1/4)))/(2*(-a)^(3/4)*b^(1/4)*d)
```

Reduce [F]

$$\int \frac{1}{a+b(c+dx)^4}\,dx=\int \frac{1}{b\,d^4x^4+4bc\,d^3x^3+6b\,c^2d^2x^2+4b\,c^3dx+b\,c^4+a}dx$$

input

```
int(1/(a+b*(d*x+c)^4),x)
```

output

```
int(1/(a + b*c**4 + 4*b*c**3*d*x + 6*b*c**2*d**2*x**2 + 4*b*c*d**3*x**3 +
b*d**4*x**4),x)
```

3.47 $\int \frac{1}{x(a+b(c+dx)^4)}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 308

$$\int \frac{1}{x\left(a+b(c+dx)^4\right)}\,dx = -\frac{\sqrt{b}c^2\arctan\left(\frac{\sqrt{b}(c+dx)^2}{\sqrt{a}}\right)}{2\sqrt{a}\left(a+bc^4\right)} + \frac{\sqrt[4]{b}c\left(\sqrt{a}+\sqrt{b}c^2\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\left(a+bc^4\right)} - \frac{\sqrt[4]{b}c\left(\sqrt{a}+\sqrt{b}c^2\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\left(a+bc^4\right)} + \frac{\sqrt[4]{b}c\left(\sqrt{a}-\sqrt{b}c^2\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)}{\sqrt{a}+\sqrt{b}(c+dx)^2}\right)}{2\sqrt{2}a^{3/4}\left(a+bc^4\right)} + \frac{\log(x)}{a+bc^4} - \frac{\log\left(a+b(c+dx)^4\right)}{4\left(a+bc^4\right)}$$

output
```
-1/2*b^(1/2)*c^2*arctan(b^(1/2)*(d*x+c)^2/a^(1/2))/a^(1/2)/(b*c^4+a)-1/4*b
^(1/4)*c*(a^(1/2)+b^(1/2)*c^2)*arctan(-1+2^(1/2)*b^(1/4)*(d*x+c)/a^(1/4))*
2^(1/2)/a^(3/4)/(b*c^4+a)-1/4*b^(1/4)*c*(a^(1/2)+b^(1/2)*c^2)*arctan(1+2^(
1/2)*b^(1/4)*(d*x+c)/a^(1/4))*2^(1/2)/a^(3/4)/(b*c^4+a)+1/4*b^(1/4)*c*(a^(
1/2)-b^(1/2)*c^2)*arctanh(2^(1/2)*a^(1/4)*b^(1/4)*(d*x+c)/(a^(1/2)+b^(1/2)
*(d*x+c)^2))*2^(1/2)/a^(3/4)/(b*c^4+a)+ln(x)/(b*c^4+a)-ln(a+b*(d*x+c)^4)/(
4*b*c^4+4*a)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.09 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.53

$$\int \frac{1}{x\left(a+b(c+dx)^4\right)}\,dx = -\frac{-4\log(x)+\text{RootSum}\left[a+bc^4+4bc^3d\#1+6bc^2d^2\#1^2+4bcd^3\#1^3+bd^4\#1^4\&,\frac{4c^3\log(x-\#1)+6c^2d\log(\ldots}{c^{\ldots}}\right.}{4\left(a+bc^4\right)}$$

input
```
Integrate[1/(x*(a + b*(c + d*x)^4)),x]
```

output
```
-1/4*(-4*Log[x] + RootSum[a + b*c^4 + 4*b*c^3*d*#1 + 6*b*c^2*d^2*#1^2 + 4*
b*c*d^3*#1^3 + b*d^4*#1^4 & , (4*c^3*Log[x - #1] + 6*c^2*d*Log[x - #1]*#1
+ 4*c*d^2*Log[x - #1]*#1^2 + d^3*Log[x - #1]*#1^3)/(c^3 + 3*c^2*d*#1 + 3*c
*d^2*#1^2 + d^3*#1^3) & ])/(a + b*c^4)
```

Rubi [A] (verified)

Time = 1.10 (sec) , antiderivative size = 396, normalized size of antiderivative = 1.29, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {896, 25, 7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(a+b(c+dx)^4\right)}dx$$

$$\Big\downarrow \; 896$$

$$\int \frac{1}{dx\left(a+b(c+dx)^4\right)}d(c+dx)$$

$$\Big\downarrow \; 25$$

$$-\int -\frac{1}{dx\left(b(c+dx)^4+a\right)}d(c+dx)$$

$$\Big\downarrow \; 7276$$

$$-\int \left(\frac{b\left(c^3+(c+dx)c^2+(c+dx)^2c+(c+dx)^3\right)}{(bc^4+a)\left(b(c+dx)^4+a\right)}-\frac{1}{(bc^4+a)\,dx}\right)d(c+dx)$$

$$\Big\downarrow \; 2009$$

$$\frac{\sqrt[4]{b}c\left(\sqrt{a}+\sqrt{b}c^2\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\left(a+bc^4\right)}-\frac{\sqrt[4]{b}c\left(\sqrt{a}+\sqrt{b}c^2\right)\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}\left(a+bc^4\right)}-\frac{\sqrt[4]{b}c\left(\sqrt{a}-\sqrt{b}c^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{a}+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}a^{3/4}\left(a+bc^4\right)}+\frac{\sqrt[4]{b}c\left(\sqrt{a}-\sqrt{b}c^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)+\sqrt{a}+\sqrt{b}(c+dx)^2\right)}{4\sqrt{2}a^{3/4}\left(a+bc^4\right)}-\frac{\sqrt{b}c^2\arctan\left(\frac{\sqrt{b}(c+dx)^2}{\sqrt{a}}\right)}{2\sqrt{a}\left(a+bc^4\right)}+\frac{\log(-dx)}{a+bc^4}-\frac{\log\left(a+b(c+dx)^4\right)}{4\left(a+bc^4\right)}$$

input
```
Int[1/(x*(a + b*(c + d*x)^4)),x]
```

output
```
-1/2*(Sqrt[b]*c^2*ArcTan[(Sqrt[b]*(c + d*x)^2)/Sqrt[a]])/(Sqrt[a]*(a + b*c
^4)) + (b^(1/4)*c*(Sqrt[a] + Sqrt[b]*c^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*(c +
 d*x))/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(a + b*c^4)) - (b^(1/4)*c*(Sqrt[a] + S
qrt[b]*c^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*(c + d*x))/a^(1/4)])/(2*Sqrt[2]*a^
(3/4)*(a + b*c^4)) + Log[-(d*x)]/(a + b*c^4) - (b^(1/4)*c*(Sqrt[a] - Sqrt[
b]*c^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*(c + d*x) + Sqrt[b]*(c + d*x
)^2])/(4*Sqrt[2]*a^(3/4)*(a + b*c^4)) + (b^(1/4)*c*(Sqrt[a] - Sqrt[b]*c^2)
*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*(c + d*x) + Sqrt[b]*(c + d*x)^2])/(
4*Sqrt[2]*a^(3/4)*(a + b*c^4)) - Log[a + b*(c + d*x)^4]/(4*(a + b*c^4))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.45

method	result
default	$-\frac{\sum_{\_R=\mathrm{RootOf}\left(b\,d^4\,\_Z^4+4bc\,d^3\,\_Z^3+6b\,c^2d^2\,\_Z^2+4b\,c^3d\,\_Z+b\,c^4+a\right)}\frac{\left(d^3\,\_R^3+4c\,d^2\,\_R^2+6c^2d\,\_R+4c^3\right)\ln\left(x-\_R\right)}{d^3\,\_R^3+3c\,d^2\,\_R^2+3c^2d\,\_R+c^3}}{4(b\,c^4+a)}+\frac{\ln(x)}{b\,c^4+a}$
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(1+\left(a^3b\,c^4+a^4\right)\_Z^4+4a^3\,\_Z^3+6a^2\,\_Z^2+4a\,\_Z\right)}\_R\ln\left(\left(\left(-3a^2b\,c^4d+5a^3d\right)\_R^3+\left(-3ab\,c^4d+15d\,a^2\right)\_R^2+\left(-b\,c\right.\right.\right.\right.}{4}$

input
```
int(1/x/(a+b*(d*x+c)^4),x,method=_RETURNVERBOSE)
```

output
```
-1/4/(b*c^4+a)*sum((_R^3*d^3+4*_R^2*c*d^2+6*_R*c^2*d+4*c^3)/(_R^3*d^3+3*_R
^2*c*d^2+3*_R*c^2*d+c^3)*ln(x-_R),_R=RootOf(_Z^4*b*d^4+4*_Z^3*b*c*d^3+6*_Z
^2*b*c^2*d^2+4*_Z*b*c^3*d+b*c^4+a))+ln(x)/(b*c^4+a)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.43 (sec) , antiderivative size = 307773, normalized size of antiderivative = 999.26

$$\int \frac{1}{x\left(a+b(c+dx)^4\right)}\,dx = \text{Too large to display}$$

input
```
integrate(1/x/(a+b*(d*x+c)^4),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x\left(a+b(c+dx)^4\right)}\,dx = \text{Timed out}$$

input
```
integrate(1/x/(a+b*(d*x+c)**4),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{x\left(a+b(c+dx)^4\right)}\,dx = \int \frac{1}{\left((dx+c)^4b+a\right)x}\,dx$$

input
```
integrate(1/x/(a+b*(d*x+c)^4),x, algorithm="maxima")
```

output
```
-b*d*integrate((d^3*x^3 + 4*c*d^2*x^2 + 6*c^2*d*x + 4*c^3)/(b*d^4*x^4 + 4*
b*c*d^3*x^3 + 6*b*c^2*d^2*x^2 + 4*b*c^3*d*x + b*c^4 + a), x)/(b*c^4 + a) +
 log(x)/(b*c^4 + a)
```

Giac [F]

$$\int \frac{1}{x\left(a+b(c+dx)^4\right)}\,dx=\int \frac{1}{\left((dx+c)^4b+a\right)x}\,dx$$

input
```
integrate(1/x/(a+b*(d*x+c)^4),x, algorithm="giac")
```

output
```
integrate(1/(((d*x + c)^4*b + a)*x), x)
```

Mupad [B] (verification not implemented)

Time = 21.86 (sec) , antiderivative size = 882, normalized size of antiderivative = 2.86

$$\int \frac{1}{x\left(a+b(c+dx)^4\right)}\,dx=\text{Too large to display}$$

input
```
int(1/(x*(a + b*(c + d*x)^4)),x)
```

output
```
log(x)/(a + b*c^4) + symsum(log(4*root(256*a^3*b*c^4*z^4 + 256*a^4*z^4 + 2
56*a^3*z^3 + 96*a^2*z^2 + 16*a*z + 1, z, k)*b^4*c*d^15 - 4*root(256*a^3*b*
c^4*z^4 + 256*a^4*z^4 + 256*a^3*z^3 + 96*a^2*z^2 + 16*a*z + 1, z, k)^2*b^5
*c^5*d^15 + 5*root(256*a^3*b*c^4*z^4 + 256*a^4*z^4 + 256*a^3*z^3 + 96*a^2*
z^2 + 16*a*z + 1, z, k)*b^4*d^16*x - 64*root(256*a^3*b*c^4*z^4 + 256*a^4*z
^4 + 256*a^3*z^3 + 96*a^2*z^2 + 16*a*z + 1, z, k)^4*a^2*b^5*c^5*d^15 + 28*
root(256*a^3*b*c^4*z^4 + 256*a^4*z^4 + 256*a^3*z^3 + 96*a^2*z^2 + 16*a*z +
 1, z, k)^2*a*b^4*c*d^15 + 60*root(256*a^3*b*c^4*z^4 + 256*a^4*z^4 + 256*a
^3*z^3 + 96*a^2*z^2 + 16*a*z + 1, z, k)^2*a*b^4*d^16*x + 32*root(256*a^3*b
*c^4*z^4 + 256*a^4*z^4 + 256*a^3*z^3 + 96*a^2*z^2 + 16*a*z + 1, z, k)^3*a^
2*b^4*c*d^15 - 64*root(256*a^3*b*c^4*z^4 + 256*a^4*z^4 + 256*a^3*z^3 + 96*
a^2*z^2 + 16*a*z + 1, z, k)^4*a^3*b^4*c*d^15 - 32*root(256*a^3*b*c^4*z^4 +
 256*a^4*z^4 + 256*a^3*z^3 + 96*a^2*z^2 + 16*a*z + 1, z, k)^3*a*b^5*c^5*d^
15 + 240*root(256*a^3*b*c^4*z^4 + 256*a^4*z^4 + 256*a^3*z^3 + 96*a^2*z^2 +
 16*a*z + 1, z, k)^3*a^2*b^4*d^16*x + 320*root(256*a^3*b*c^4*z^4 + 256*a^4
*z^4 + 256*a^3*z^3 + 96*a^2*z^2 + 16*a*z + 1, z, k)^4*a^3*b^4*d^16*x - 4*r
oot(256*a^3*b*c^4*z^4 + 256*a^4*z^4 + 256*a^3*z^3 + 96*a^2*z^2 + 16*a*z +
1, z, k)^2*b^5*c^4*d^16*x - 48*root(256*a^3*b*c^4*z^4 + 256*a^4*z^4 + 256*
a^3*z^3 + 96*a^2*z^2 + 16*a*z + 1, z, k)^3*a*b^5*c^4*d^16*x - 192*root(256
*a^3*b*c^4*z^4 + 256*a^4*z^4 + 256*a^3*z^3 + 96*a^2*z^2 + 16*a*z + 1, z...
```

Reduce [F]

$$\int \frac{1}{x\left(a+b(c+dx)^4\right)}\,dx=\int \frac{1}{b\,d^4x^5+4bc\,d^3x^4+6b\,c^2d^2x^3+4b\,c^3d\,x^2+b\,c^4x+ax}dx$$

input
```
int(1/x/(a+b*(d*x+c)^4),x)
```

output
```
int(1/(a*x + b*c**4*x + 4*b*c**3*d*x**2 + 6*b*c**2*d**2*x**3 + 4*b*c*d**3*
x**4 + b*d**4*x**5),x)
```

3.48 $\int \frac{1}{x^2(a+b(c+dx)^4)}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 392

$$\begin{aligned}
&\int \frac{1}{x^2\left(a+b(c+dx)^4\right)}\,dx \\
&= -\frac{1}{(a+bc^4)\,x} - \frac{\sqrt{b}c(a-bc^4)\,d\arctan\left(\frac{\sqrt{b}(c+dx)^2}{\sqrt{a}}\right)}{\sqrt{a}\,(a+bc^4)^2} \\
&\quad + \frac{\sqrt[4]{b}\left(\sqrt{a}(a-3bc^4)+\sqrt{b}c^2(3a-bc^4)\right)d\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\,(a+bc^4)^2} \\
&\quad - \frac{\sqrt[4]{b}\left(\sqrt{a}(a-3bc^4)+\sqrt{b}c^2(3a-bc^4)\right)d\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\,(a+bc^4)^2} \\
&\quad + \frac{\sqrt[4]{b}\left(\sqrt{a}(a-3bc^4)-\sqrt{b}c^2(3a-bc^4)\right)d\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}(c+dx)}{\sqrt{a}+\sqrt{b}(c+dx)^2}\right)}{2\sqrt{2}a^{3/4}\,(a+bc^4)^2} \\
&\quad - \frac{4bc^3d\log(x)}{(a+bc^4)^2} + \frac{bc^3d\log\left(a+b(c+dx)^4\right)}{(a+bc^4)^2}
\end{aligned}$$

output
```
-1/(b*c^4+a)/x-b^(1/2)*c*(-b*c^4+a)*d*arctan(b^(1/2)*(d*x+c)^2/a^(1/2))/a^
(1/2)/(b*c^4+a)^2-1/4*b^(1/4)*(a^(1/2)*(-3*b*c^4+a)+b^(1/2)*c^2*(-b*c^4+3*
a))*d*arctan(-1+2^(1/2)*b^(1/4)*(d*x+c)/a^(1/4))*2^(1/2)/a^(3/4)/(b*c^4+a)
^2-1/4*b^(1/4)*(a^(1/2)*(-3*b*c^4+a)+b^(1/2)*c^2*(-b*c^4+3*a))*d*arctan(1+
2^(1/2)*b^(1/4)*(d*x+c)/a^(1/4))*2^(1/2)/a^(3/4)/(b*c^4+a)^2+1/4*b^(1/4)*(
a^(1/2)*(-3*b*c^4+a)-b^(1/2)*c^2*(-b*c^4+3*a))*d*arctanh(2^(1/2)*a^(1/4)*b
^(1/4)*(d*x+c)/(a^(1/2)+b^(1/2)*(d*x+c)^2))*2^(1/2)/a^(3/4)/(b*c^4+a)^2-4*
b*c^3*d*ln(x)/(b*c^4+a)^2+b*c^3*d*ln(a+b*(d*x+c)^4)/(b*c^4+a)^2
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.18 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.61

$$\int \frac{1}{x^2 (a + b(c + dx)^4)} dx$$
$$= \frac{-4(a + bc^4 + 4bc^3dx\log(x)) + dx\text{RootSum}\Big[a + bc^4 + 4bc^3d\#1 + 6bc^2d^2\#1^2 + 4bcd^3\#1^3 + bd^4\#1^4\&, \ldots}{\ldots}$$

input
```
Integrate[1/(x^2*(a + b*(c + d*x)^4)),x]
```

output
```
(-4*(a + b*c^4 + 4*b*c^3*d*x*Log[x]) + d*x*RootSum[a + b*c^4 + 4*b*c^3*d*#
1 + 6*b*c^2*d^2*#1^2 + 4*b*c*d^3*#1^3 + b*d^4*#1^4 & , (-6*a*c^2*Log[x - #
1] + 10*b*c^6*Log[x - #1] - 4*a*c*d*Log[x - #1]*#1 + 20*b*c^5*d*Log[x - #1
]*#1 - a*d^2*Log[x - #1]*#1^2 + 15*b*c^4*d^2*Log[x - #1]*#1^2 + 4*b*c^3*d^
3*Log[x - #1]*#1^3)/(c^3 + 3*c^2*d*#1 + 3*c*d^2*#1^2 + d^3*#1^3) & ])/(4*(
a + b*c^4)^2*x)
```

Rubi [A] (verified)

Time = 1.59 (sec) , antiderivative size = 497, normalized size of antiderivative = 1.27, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {896, 7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (a+b(c+dx)^4)} \, dx$$

$$\Big\downarrow \ 896$$

$$d\int \frac{1}{d^2x^2 (b(c+dx)^4+a)}d(c+dx)$$

$$\Big\downarrow \ 7276$$

$$d\int \left(-\frac{4bc^3}{(bc^4+a)^2 dx}+\frac{b(4bc^3(c+dx)^3-(a-3bc^4)(c+dx)^2-2c(a-bc^4)(c+dx)-c^2(3a-bc^4))}{(bc^4+a)^2 (b(c+dx)^4+a)}+\frac{}{(bc^4+\dots}\right.$$

$$\left. dx\right)$$

$$\Big\downarrow \ 2009$$

$$d\left(\frac{\sqrt[4]{b}\left(\sqrt{a}(a-3bc^4)+\sqrt{b}c^2(3a-bc^4)\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}(a+bc^4)^2}-\frac{\sqrt[4]{b}\left(\sqrt{a}(a-3bc^4)+\sqrt{b}c^2(3a-bc^4)\right)\text{ar}\dots}{2\sqrt{2}a^{3/4}(a+bc^4)\dots}\right.$$

input
```
Int[1/(x^2*(a + b*(c + d*x)^4)),x]
```

output
```
d*(-(1/((a + b*c^4)*d*x)) - (Sqrt[b]*c*(a - b*c^4)*ArcTan[(Sqrt[b]*(c + d*
x)^2)/Sqrt[a]])/(Sqrt[a]*(a + b*c^4)^2) + (b^(1/4)*(Sqrt[a]*(a - 3*b*c^4)
+ Sqrt[b]*c^2*(3*a - b*c^4))*ArcTan[1 - (Sqrt[2]*b^(1/4)*(c + d*x))/a^(1/4
)]])/(2*Sqrt[2]*a^(3/4)*(a + b*c^4)^2) - (b^(1/4)*(Sqrt[a]*(a - 3*b*c^4) +
Sqrt[b]*c^2*(3*a - b*c^4))*ArcTan[1 + (Sqrt[2]*b^(1/4)*(c + d*x))/a^(1/4)]
)/(2*Sqrt[2]*a^(3/4)*(a + b*c^4)^2) - (4*b*c^3*Log[-(d*x)])/(a + b*c^4)^2
- (b^(1/4)*(Sqrt[a]*(a - 3*b*c^4) - Sqrt[b]*c^2*(3*a - b*c^4))*Log[Sqrt[a]
 - Sqrt[2]*a^(1/4)*b^(1/4)*(c + d*x) + Sqrt[b]*(c + d*x)^2])/(4*Sqrt[2]*a^
(3/4)*(a + b*c^4)^2) + (b^(1/4)*(Sqrt[a]*(a - 3*b*c^4) - Sqrt[b]*c^2*(3*a
- b*c^4))*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*(c + d*x) + Sqrt[b]*(c + d
*x)^2])/(4*Sqrt[2]*a^(3/4)*(a + b*c^4)^2) + (b*c^3*Log[a + b*(c + d*x)^4])
/(a + b*c^4)^2)
```

Defintions of rubi rules used

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.10 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.47

method	result
default	$-\frac{d\left(\sum_{\_R=\mathrm{RootOf}\left(b\,d^4\_Z^4+4bc\,d^3\_Z^3+6b\,c^2d^2\_Z^2+4b\,c^3d\_Z+b\,c^4+a\right)}\frac{\left(-4b\,d^3c^3\_R^3+d^2\left(-15b\,c^4+a\right)\_R^2+4cd\left(-5b\,c^4+a\right)\_R-\cdots\right.}{d^3\_R^3+3c\,d^2\_R^2+3c^2d\_R+c^3\cdots}\cdots\right.}{4\left(b\,c^4+a\right)^2}$
risch	$-\frac{1}{\left(b\,c^4+a\right)x}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(a^3b^2c^8+2a^4b\,c^4+a^5\right)\_Z^4-16a^3b\,c^3d\_Z^3+20a^2b\,c^2d^2\_Z^2-8abc\,d^3\_Z+b\,d^4\right)}\_R\ln\left(\left(\left(-3a^2b^3c^{12}d-\cdots\right.\right.\right.\right.}{\cdots}$

input
```
int(1/x^2/(a+b*(d*x+c)^4),x,method=_RETURNVERBOSE)
```

output
```
-1/4*d/(b*c^4+a)^2*sum((-4*b*d^3*c^3*_R^3+d^2*(-15*b*c^4+a)*_R^2+4*c*d*(-5
*b*c^4+a)*_R-10*b*c^6+6*a*c^2)/(_R^3*d^3+3*_R^2*c*d^2+3*_R*c^2*d+c^3)*ln(x
-_R),_R=RootOf(_Z^4*b*d^4+4*_Z^3*b*c*d^3+6*_Z^2*b*c^2*d^2+4*_Z*b*c^3*d+b*c
^4+a))-1/(b*c^4+a)/x-4*b*c^3*d*ln(x)/(b*c^4+a)^2
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 79.26 (sec) , antiderivative size = 1128605, normalized size of antiderivative = 2879.09

$$\int \frac{1}{x^2\left(a+b(c+dx)^4\right)}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^2/(a+b*(d*x+c)^4),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (a + b(c + dx)^4)} dx = \text{Timed out}$$

input
```
integrate(1/x**2/(a+b*(d*x+c)**4),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{x^2 (a + b(c + dx)^4)} dx = \int \frac{1}{\left((dx + c)^4 b + a\right) x^2} dx$$

input
```
integrate(1/x^2/(a+b*(d*x+c)^4),x, algorithm="maxima")
```

output
```
-4*b*c^3*d*log(x)/(b^2*c^8 + 2*a*b*c^4 + a^2) + b*d^2*integrate((4*b*c^3*d
^3*x^3 + 10*b*c^6 + (15*b*c^4 - a)*d^2*x^2 - 6*a*c^2 + 4*(5*b*c^5 - a*c)*d
*x)/(b*d^4*x^4 + 4*b*c*d^3*x^3 + 6*b*c^2*d^2*x^2 + 4*b*c^3*d*x + b*c^4 + a
), x)/(b^2*c^8 + 2*a*b*c^4 + a^2) - 1/((b*c^4 + a)*x)
```

Giac [F]

$$\int \frac{1}{x^2 (a + b(c + dx)^4)} dx = \int \frac{1}{\left((dx + c)^4 b + a\right) x^2} dx$$

input
```
integrate(1/x^2/(a+b*(d*x+c)^4),x, algorithm="giac")
```

output
```
integrate(1/(((d*x + c)^4*b + a)*x^2), x)
```

Mupad [B] (verification not implemented)

Time = 22.20 (sec) , antiderivative size = 2440, normalized size of antiderivative = 6.22

$$\int \frac{1}{x^2 (a+b(c+dx)^4)} \, dx = \text{Too large to display}$$

input
```
int(1/(x^2*(a + b*(c + d*x)^4)),x)
```

output
```
symsum(log(-(4*root(256*a^3*b^2*c^8*z^4 + 512*a^4*b*c^4*z^4 + 256*a^5*z^4
- 1024*a^3*b*c^3*d*z^3 + 320*a^2*b*c^2*d^2*z^2 - 32*a*b*c*d^3*z + b*d^4, z
, k)^2*b^7*c^11*d^17 - 16*root(256*a^3*b^2*c^8*z^4 + 512*a^4*b*c^4*z^4 + 2
56*a^5*z^4 - 1024*a^3*b*c^3*d*z^3 + 320*a^2*b*c^2*d^2*z^2 - 32*a*b*c*d^3*z
 + b*d^4, z, k)^3*a^4*b^4*d^16 - b^5*d^20*x + 16*root(256*a^3*b^2*c^8*z^4
+ 512*a^4*b*c^4*z^4 + 256*a^5*z^4 - 1024*a^3*b*c^3*d*z^3 + 320*a^2*b*c^2*d
^2*z^2 - 32*a*b*c*d^3*z + b*d^4, z, k)*b^6*c^6*d^18 - 60*root(256*a^3*b^2*
c^8*z^4 + 512*a^4*b*c^4*z^4 + 256*a^5*z^4 - 1024*a^3*b*c^3*d*z^3 + 320*a^2
*b*c^2*d^2*z^2 - 32*a*b*c*d^3*z + b*d^4, z, k)^2*a^2*b^5*c^3*d^17 + 176*ro
ot(256*a^3*b^2*c^8*z^4 + 512*a^4*b*c^4*z^4 + 256*a^5*z^4 - 1024*a^3*b*c^3*
d*z^3 + 320*a^2*b*c^2*d^2*z^2 - 32*a*b*c*d^3*z + b*d^4, z, k)^3*a^3*b^5*c^
4*d^16 + 192*root(256*a^3*b^2*c^8*z^4 + 512*a^4*b*c^4*z^4 + 256*a^5*z^4 -
1024*a^3*b*c^3*d*z^3 + 320*a^2*b*c^2*d^2*z^2 - 32*a*b*c*d^3*z + b*d^4, z,
k)^4*a^4*b^5*c^5*d^15 + 144*root(256*a^3*b^2*c^8*z^4 + 512*a^4*b*c^4*z^4 +
 256*a^5*z^4 - 1024*a^3*b*c^3*d*z^3 + 320*a^2*b*c^2*d^2*z^2 - 32*a*b*c*d^3
*z + b*d^4, z, k)^3*a^2*b^6*c^8*d^16 + 192*root(256*a^3*b^2*c^8*z^4 + 512*
a^4*b*c^4*z^4 + 256*a^5*z^4 - 1024*a^3*b*c^3*d*z^3 + 320*a^2*b*c^2*d^2*z^2
 - 32*a*b*c*d^3*z + b*d^4, z, k)^4*a^3*b^6*c^9*d^15 + 64*root(256*a^3*b^2*
c^8*z^4 + 512*a^4*b*c^4*z^4 + 256*a^5*z^4 - 1024*a^3*b*c^3*d*z^3 + 320*a^2
*b*c^2*d^2*z^2 - 32*a*b*c*d^3*z + b*d^4, z, k)^4*a^2*b^7*c^13*d^15 + 16...
```

Reduce [F]

$$\int \frac{1}{x^2 (a+b(c+dx)^4)} \, dx = \int \frac{1}{b\, d^4 x^6 + 4bc\, d^3 x^5 + 6b\, c^2 d^2 x^4 + 4b\, c^3 d\, x^3 + b\, c^4 x^2 + a\, x^2} dx$$

input
```
int(1/x^2/(a+b*(d*x+c)^4),x)
```

output

```
int(1/(a*x**2 + b*c**4*x**2 + 4*b*c**3*d*x**3 + 6*b*c**2*d**2*x**4 + 4*b*c
*d**3*x**5 + b*d**4*x**6),x)
```

3.49 $\int \frac{x^4(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 97

$$\int \frac{x^4(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4}\,dx = \frac{5x}{4}-\frac{3x^2}{4}+\frac{x^3}{3}+\frac{x^4}{4}+\frac{1}{24}\sqrt{\frac{5}{3}}\arctan\left(\frac{1-4x}{\sqrt{15}}\right) - \frac{10\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{1}{3}\log\left(1+x+x^2\right) - \frac{13}{48}\log\left(2-x+2x^2\right)$$

output
```
5/4*x-3/4*x^2+1/3*x^3+1/4*x^4+1/72*15^(1/2)*arctan(1/15*(1-4*x)*15^(1/2))-
10/9*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)+1/3*ln(x^2+x+1)-13/48*ln(2*x^2-x+
2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.86

$$\int \frac{x^4(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4}\,dx = \frac{1}{144}\Bigg(180x-108x^2+48x^3+36x^4 - 160\sqrt{3}\arctan\left(\frac{1+2x}{\sqrt{3}}\right)-2\sqrt{15}\arctan\left(\frac{-1+4x}{\sqrt{15}}\right) + 48\log\left(1+x+x^2\right)-39\log\left(2-x+2x^2\right)\Bigg)$$

input
```
Integrate[(x^4*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 3*x^2 + x^3 + 2*x^4),x]
```

output
```
(180*x - 108*x^2 + 48*x^3 + 36*x^4 - 160*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]]
 - 2*Sqrt[15]*ArcTan[(-1 + 4*x)/Sqrt[15]] + 48*Log[1 + x + x^2] - 39*Log[2
 - x + 2*x^2])/144
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = {2462, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4 (2x^3+3x^2+x+5)}{2x^4+x^3+3x^2+x+2} \, dx$$

$$\Big\downarrow \text{ 2462}$$

$$\int \left(x^3+x^2+\frac{2(x-2)}{3(x^2+x+1)}+\frac{2-13x}{12(2x^2-x+2)}-\frac{3x}{2}+\frac{5}{4}\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{24}\sqrt{\frac{5}{3}}\arctan\left(\frac{1-4x}{\sqrt{15}}\right)-\frac{10\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{x^4}{4}+\frac{x^3}{3}-\frac{3x^2}{4}+\frac{1}{3}\log\left(x^2+x+1\right)-\frac{13}{48}\log\left(2x^2-x+2\right)+\frac{5x}{4}$$

input
```
Int[(x^4*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 3*x^2 + x^3 + 2*x^4),x]
```

output
```
(5*x)/4 - (3*x^2)/4 + x^3/3 + x^4/4 + (Sqrt[5/3]*ArcTan[(1 - 4*x)/Sqrt[15]
])/24 - (10*ArcTan[(1 + 2*x)/Sqrt[3]])/(3*Sqrt[3]) + Log[1 + x + x^2]/3 -
(13*Log[2 - x + 2*x^2])/48
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.76

method	result
default	$\frac{x^4}{4}+\frac{x^3}{3}-\frac{3x^2}{4}+\frac{5x}{4}+\frac{\ln(x^2+x+1)}{3}-\frac{10\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)\sqrt{3}}{9}-\frac{13\ln(2x^2-x+2)}{48}-\frac{\sqrt{15}\,\arctan\left(\frac{(4x-1)\sqrt{15}}{15}\right)}{72}$
risch	$\frac{x^4}{4}+\frac{x^3}{3}-\frac{3x^2}{4}+\frac{5x}{4}-\frac{13\ln(16x^2-8x+16)}{48}-\frac{\sqrt{15}\,\arctan\left(\frac{(4x-1)\sqrt{15}}{15}\right)}{72}+\frac{\ln(4x^2+4x+4)}{3}-\frac{10\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)\sqrt{3}}{9}$

input
```
int(x^4*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x,method=_RETURNVERBOSE)
```

output
```
1/4*x^4+1/3*x^3-3/4*x^2+5/4*x+1/3*ln(x^2+x+1)-10/9*arctan(1/3*(2*x+1)*3^(1
/2))*3^(1/2)-13/48*ln(2*x^2-x+2)-1/72*15^(1/2)*arctan(1/15*(4*x-1)*15^(1/2
))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.75

$$\int \frac{x^4(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4}\,dx = \frac{1}{4}\,x^4+\frac{1}{3}\,x^3-\frac{3}{4}\,x^2-\frac{1}{24}\,\sqrt{\frac{5}{3}}\arctan\left(\frac{1}{5}\,\sqrt{\frac{5}{3}}(4\,x-1)\right) - \frac{10}{9}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right)+\frac{5}{4}\,x - \frac{13}{48}\,\log\left(2\,x^2-x+2\right)+\frac{1}{3}\,\log\left(x^2+x+1\right)$$

input
```
integrate(x^4*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x, algorithm="fricas
")
```

output
```
1/4*x^4 + 1/3*x^3 - 3/4*x^2 - 1/24*sqrt(5/3)*arctan(1/5*sqrt(5/3)*(4*x - 1
)) - 10/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 5/4*x - 13/48*log(2*x^2
- x + 2) + 1/3*log(x^2 + x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00

$$\int \frac{x^4(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4}\,dx = \frac{x^4}{4}+\frac{x^3}{3}-\frac{3x^2}{4}+\frac{5x}{4}-\frac{13\log\left(x^2-\frac{x}{2}+1\right)}{48} + \frac{\log\left(x^2+x+1\right)}{3}-\frac{\sqrt{15}\operatorname{atan}\left(\frac{4\sqrt{15}x}{15}-\frac{\sqrt{15}}{15}\right)}{72} - \frac{10\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{9}$$

input
```
integrate(x**4*(2*x**3+3*x**2+x+5)/(2*x**4+x**3+3*x**2+x+2),x)
```

output
```
x**4/4 + x**3/3 - 3*x**2/4 + 5*x/4 - 13*log(x**2 - x/2 + 1)/48 + log(x**2
+ x + 1)/3 - sqrt(15)*atan(4*sqrt(15)*x/15 - sqrt(15)/15)/72 - 10*sqrt(3)*
atan(2*sqrt(3)*x/3 + sqrt(3)/3)/9
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.75

$$\int \frac{x^4(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4}\,dx = \frac{1}{4}\,x^4+\frac{1}{3}\,x^3-\frac{3}{4}\,x^2-\frac{1}{72}\,\sqrt{15}\arctan\left(\frac{1}{15}\,\sqrt{15}(4\,x-1)\right) - \frac{10}{9}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right)+\frac{5}{4}\,x - \frac{13}{48}\,\log\left(2\,x^2-x+2\right)+\frac{1}{3}\,\log\left(x^2+x+1\right)$$

input
```
integrate(x^4*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x, algorithm="maxima
")
```

output
```
1/4*x^4 + 1/3*x^3 - 3/4*x^2 - 1/72*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1)
) - 10/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 5/4*x - 13/48*log(2*x^2 -
 x + 2) + 1/3*log(x^2 + x + 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.75

$$\int \frac{x^4(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4}\,dx = \frac{1}{4}\,x^4+\frac{1}{3}\,x^3-\frac{3}{4}\,x^2-\frac{1}{72}\,\sqrt{15}\arctan\left(\frac{1}{15}\,\sqrt{15}(4\,x-1)\right) - \frac{10}{9}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right)+\frac{5}{4}\,x - \frac{13}{48}\,\log\left(2\,x^2-x+2\right)+\frac{1}{3}\,\log\left(x^2+x+1\right)$$

input
```
integrate(x^4*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x, algorithm="giac")
```

output
```
1/4*x^4 + 1/3*x^3 - 3/4*x^2 - 1/72*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1)
) - 10/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 5/4*x - 13/48*log(2*x^2 -
 x + 2) + 1/3*log(x^2 + x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00

$$\int \frac{x^4(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4}\,dx = \frac{5\,x}{4}+\ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(\frac{1}{3}+\frac{\sqrt{3}\,5\mathrm{i}}{9}\right) -\ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-\frac{1}{3}+\frac{\sqrt{3}\,5\mathrm{i}}{9}\right) +\ln\left(x-\frac{1}{4}-\frac{\sqrt{15}\,1\mathrm{i}}{4}\right)\,\left(-\frac{13}{48}+\frac{\sqrt{15}\,1\mathrm{i}}{144}\right) -\ln\left(x-\frac{1}{4}+\frac{\sqrt{15}\,1\mathrm{i}}{4}\right)\,\left(\frac{13}{48}+\frac{\sqrt{15}\,1\mathrm{i}}{144}\right)-\frac{3\,x^2}{4}+\frac{x^3}{3}+\frac{x^4}{4}$$

input
```
int((x^4*(x + 3*x^2 + 2*x^3 + 5))/(x + 3*x^2 + x^3 + 2*x^4 + 2),x)
```

output
```
(5*x)/4 + log(x - (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*5i)/9 + 1/3) - log(x + (
3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*5i)/9 - 1/3) + log(x - (15^(1/2)*1i)/4 - 1/
4)*((15^(1/2)*1i)/144 - 13/48) - log(x + (15^(1/2)*1i)/4 - 1/4)*((15^(1/2)
*1i)/144 + 13/48) - (3*x^2)/4 + x^3/3 + x^4/4
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.73

$$\int \frac{x^4(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4}\,dx = -\frac{\sqrt{15}\,atan\left(\frac{4x-1}{\sqrt{15}}\right)}{72}-\frac{10\sqrt{3}\,atan\left(\frac{2x+1}{\sqrt{3}}\right)}{9}+\frac{\log(x^2+x+1)}{3}-\frac{13\log(2x^2-x+2)}{48}+\frac{x^4}{4}+\frac{x^3}{3}-\frac{3x^2}{4}+\frac{5x}{4}$$

input
```
int(x^4*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x)
```

output
```
( - 2*sqrt(15)*atan((4*x - 1)/sqrt(15)) - 160*sqrt(3)*atan((2*x + 1)/sqrt(
3)) + 48*log(x**2 + x + 1) - 39*log(2*x**2 - x + 2) + 36*x**4 + 48*x**3 -
108*x**2 + 180*x)/144
```

3.50 $\int \frac{x^3(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 90

$$\int \frac{x^3(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4}\,dx = -\frac{3x}{2}+\frac{x^2}{2}+\frac{x^3}{3}+\frac{5}{12}\sqrt{\frac{5}{3}}\arctan\left(\frac{1-4x}{\sqrt{15}}\right)+\frac{8\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{2}{3}\log\left(1+x+x^2\right)-\frac{1}{24}\log\left(2-x+2x^2\right)$$

output
```
-3/2*x+1/2*x^2+1/3*x^3+5/36*15^(1/2)*arctan(1/15*(1-4*x)*15^(1/2))+8/9*arc
tan(1/3*(1+2*x)*3^(1/2))*3^(1/2)+2/3*ln(x^2+x+1)-1/24*ln(2*x^2-x+2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.87

$$\int \frac{x^3(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4}\,dx = \frac{1}{72}\left(-108x+36x^2+24x^3+64\sqrt{3}\arctan\left(\frac{1+2x}{\sqrt{3}}\right)-10\sqrt{15}\arctan\left(\frac{-1+4x}{\sqrt{15}}\right)+48\log\left(1+x+x^2\right)-3\log\left(2-x+2x^2\right)\right)$$

input
```
Integrate[(x^3*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 3*x^2 + x^3 + 2*x^4),x]
```

output
```
(-108*x + 36*x^2 + 24*x^3 + 64*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] - 10*Sqrt
[15]*ArcTan[(-1 + 4*x)/Sqrt[15]] + 48*Log[1 + x + x^2] - 3*Log[2 - x + 2*x
^2])/72
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = {2462, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \left(2 x^3+3 x^2+x+5\right)}{2 x^4+x^3+3 x^2+x+2} \, dx$$

$$\Big\downarrow \text{2462}$$

$$\int \left(x^2+\frac{2(2 x+3)}{3 \left(x^2+x+1\right)}+\frac{-x-6}{6 \left(2 x^2-x+2\right)}+x-\frac{3}{2}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{5}{12} \sqrt{\frac{5}{3}} \arctan \left(\frac{1-4 x}{\sqrt{15}}\right)+\frac{8 \arctan \left(\frac{2 x+1}{\sqrt{3}}\right)}{3 \sqrt{3}}+\frac{x^3}{3}+\frac{x^2}{2}+\frac{2}{3} \log \left(x^2+x+1\right)-\frac{1}{24} \log \left(2 x^2-x+2\right)-\frac{3 x}{2}$$

input
```
Int[(x^3*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 3*x^2 + x^3 + 2*x^4),x]
```

output
```
(-3*x)/2 + x^2/2 + x^3/3 + (5*Sqrt[5/3]*ArcTan[(1 - 4*x)/Sqrt[15]])/12 + (
8*ArcTan[(1 + 2*x)/Sqrt[3]])/(3*Sqrt[3]) + (2*Log[1 + x + x^2])/3 - Log[2
- x + 2*x^2]/24
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.77

method	result	size
default	$\frac{x^3}{3}+\frac{x^2}{2}-\frac{3x}{2}+\frac{2\ln(x^2+x+1)}{3}+\frac{8\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)\sqrt{3}}{9}-\frac{\ln(2x^2-x+2)}{24}-\frac{5\sqrt{15}\,\arctan\left(\frac{(4x-1)\sqrt{15}}{15}\right)}{36}$	69
risch	$\frac{x^3}{3}+\frac{x^2}{2}-\frac{3x}{2}+\frac{2\ln(4x^2+4x+4)}{3}+\frac{8\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)\sqrt{3}}{9}-\frac{\ln(16x^2-8x+16)}{24}-\frac{5\sqrt{15}\,\arctan\left(\frac{(4x-1)\sqrt{15}}{15}\right)}{36}$	73

input
```
int(x^3*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x,method=_RETURNVERBOSE)
```

output
```
1/3*x^3+1/2*x^2-3/2*x+2/3*ln(x^2+x+1)+8/9*arctan(1/3*(2*x+1)*3^(1/2))*3^(1
/2)-1/24*ln(2*x^2-x+2)-5/36*15^(1/2)*arctan(1/15*(4*x-1)*15^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.76

$$\int \frac{x^3(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4}\,dx = \frac{1}{3}\,x^3+\frac{1}{2}\,x^2-\frac{5}{12}\,\sqrt{\frac{5}{3}}\arctan\left(\frac{1}{5}\,\sqrt{\frac{5}{3}}(4\,x-1)\right) + \frac{8}{9}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right)-\frac{3}{2}\,x - \frac{1}{24}\,\log\left(2\,x^2-x+2\right)+\frac{2}{3}\,\log\left(x^2+x+1\right)$$

input
```
integrate(x^3*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x, algorithm="fricas
")
```

output
```
1/3*x^3 + 1/2*x^2 - 5/12*sqrt(5/3)*arctan(1/5*sqrt(5/3)*(4*x - 1)) + 8/9*s
qrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 3/2*x - 1/24*log(2*x^2 - x + 2) + 2
/3*log(x^2 + x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.02

$$\int \frac{x^3(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4}\,dx = \frac{x^3}{3}+\frac{x^2}{2}-\frac{3x}{2}-\frac{\log\left(x^2-\frac{x}{2}+1\right)}{24}+\frac{2\log\left(x^2+x+1\right)}{3} - \frac{5\sqrt{15}\operatorname{atan}\left(\frac{4\sqrt{15}x}{15}-\frac{\sqrt{15}}{15}\right)}{36}+\frac{8\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{9}$$

input
```
integrate(x**3*(2*x**3+3*x**2+x+5)/(2*x**4+x**3+3*x**2+x+2),x)
```

output
```
x**3/3 + x**2/2 - 3*x/2 - log(x**2 - x/2 + 1)/24 + 2*log(x**2 + x + 1)/3 -
 5*sqrt(15)*atan(4*sqrt(15)*x/15 - sqrt(15)/15)/36 + 8*sqrt(3)*atan(2*sqrt
(3)*x/3 + sqrt(3)/3)/9
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.76

$$\int \frac{x^3(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4}\,dx = \frac{1}{3}\,x^3+\frac{1}{2}\,x^2-\frac{5}{36}\,\sqrt{15}\arctan\left(\frac{1}{15}\,\sqrt{15}(4\,x-1)\right) + \frac{8}{9}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right)-\frac{3}{2}\,x - \frac{1}{24}\,\log\left(2\,x^2-x+2\right)+\frac{2}{3}\,\log\left(x^2+x+1\right)$$

input
```
integrate(x^3*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x, algorithm="maxima
")
```

output
```
1/3*x^3 + 1/2*x^2 - 5/36*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1)) + 8/9*sq
rt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 3/2*x - 1/24*log(2*x^2 - x + 2) + 2/
3*log(x^2 + x + 1)
```

Giac [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.76

$$\int \frac{x^3(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4}\,dx = \frac{1}{3}\,x^3+\frac{1}{2}\,x^2-\frac{5}{36}\,\sqrt{15}\arctan\left(\frac{1}{15}\,\sqrt{15}(4\,x-1)\right)+\frac{8}{9}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right)-\frac{3}{2}\,x-\frac{1}{24}\,\log\left(2\,x^2-x+2\right)+\frac{2}{3}\,\log\left(x^2+x+1\right)$$

input
```
integrate(x^3*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x, algorithm="giac")
```

output
```
1/3*x^3 + 1/2*x^2 - 5/36*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1)) + 8/9*sq
rt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 3/2*x - 1/24*log(2*x^2 - x + 2) + 2/
3*log(x^2 + x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.02

$$\int \frac{x^3(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4}\,dx = \frac{x^2}{2}-\ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{2}{3}+\frac{\sqrt{3}\,4\mathrm{i}}{9}\right)+\ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{2}{3}+\frac{\sqrt{3}\,4\mathrm{i}}{9}\right)+\ln\left(x-\frac{1}{4}-\frac{\sqrt{15}\,1\mathrm{i}}{4}\right)\left(-\frac{1}{24}+\frac{\sqrt{15}\,5\mathrm{i}}{72}\right)-\ln\left(x-\frac{1}{4}+\frac{\sqrt{15}\,1\mathrm{i}}{4}\right)\left(\frac{1}{24}+\frac{\sqrt{15}\,5\mathrm{i}}{72}\right)-\frac{3\,x}{2}+\frac{x^3}{3}$$

input
```
int((x^3*(x + 3*x^2 + 2*x^3 + 5))/(x + 3*x^2 + x^3 + 2*x^4 + 2),x)
```

output
```
log(x + (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*4i)/9 + 2/3) - log(x - (3^(1/2)*1i
)/2 + 1/2)*((3^(1/2)*4i)/9 - 2/3) - (3*x)/2 + log(x - (15^(1/2)*1i)/4 - 1/
4)*((15^(1/2)*5i)/72 - 1/24) - log(x + (15^(1/2)*1i)/4 - 1/4)*((15^(1/2)*5
i)/72 + 1/24) + x^2/2 + x^3/3
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.73

$$\int \frac{x^3(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4}\,dx = -\frac{5\sqrt{15}\,atan\left(\frac{4x-1}{\sqrt{15}}\right)}{36}+\frac{8\sqrt{3}\,atan\left(\frac{2x+1}{\sqrt{3}}\right)}{9}+\frac{2\log(x^2+x+1)}{3}-\frac{\log(2x^2-x+2)}{24}+\frac{x^3}{3}+\frac{x^2}{2}-\frac{3x}{2}$$

input
```
int(x^3*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x)
```

output
```
( - 10*sqrt(15)*atan((4*x - 1)/sqrt(15)) + 64*sqrt(3)*atan((2*x + 1)/sqrt(
3)) + 48*log(x**2 + x + 1) - 3*log(2*x**2 - x + 2) + 24*x**3 + 36*x**2 - 1
08*x)/72
```

3.51 $\int \frac{x^2(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4} \, dx$

Optimal result

Integrand size = 35, antiderivative size = 77

$$\int \frac{x^2(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4} \, dx = x+\frac{x^2}{2}+\frac{1}{6}\sqrt{\frac{5}{3}}\arctan\left(\frac{1-4x}{\sqrt{15}}\right)+\frac{2\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{3\sqrt{3}} - \log\left(1+x+x^2\right)+\frac{1}{4}\log\left(2-x+2x^2\right)$$

output
```
x+1/2*x^2+1/18*15^(1/2)*arctan(1/15*(1-4*x)*15^(1/2))+2/9*arctan(1/3*(1+2*
x)*3^(1/2))*3^(1/2)-ln(x^2+x+1)+1/4*ln(2*x^2-x+2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.94

$$\int \frac{x^2(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4} \, dx = \frac{1}{36}\left(8\sqrt{3}\arctan\left(\frac{1+2x}{\sqrt{3}}\right)-2\sqrt{15}\arctan\left(\frac{-1+4x}{\sqrt{15}}\right) +9\left(2x(2+x)-4\log\left(1+x+x^2\right)+\log\left(2-x+2x^2\right)\right)\right)$$

input
```
Integrate[(x^2*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 3*x^2 + x^3 + 2*x^4),x]
```

output
```
(8*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] - 2*Sqrt[15]*ArcTan[(-1 + 4*x)/Sqrt[1
5]] + 9*(2*x*(2 + x) - 4*Log[1 + x + x^2] + Log[2 - x + 2*x^2]))/36
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {2462, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \left(2 x^3+3 x^2+x+5\right)}{2 x^4+x^3+3 x^2+x+2} \, dx$$

$$\Big\downarrow \text{ 2462}$$

$$\int \left(-\frac{2(3 x+1)}{3 \left(x^2+x+1\right)}+\frac{3 x-2}{3 \left(2 x^2-x+2\right)}+x+1\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{6} \sqrt{\frac{5}{3}} \arctan \left(\frac{1-4 x}{\sqrt{15}}\right)+\frac{2 \arctan \left(\frac{2 x+1}{\sqrt{3}}\right)}{3 \sqrt{3}}+\frac{x^2}{2}-\log \left(x^2+x+1\right)+\frac{1}{4} \log \left(2 x^2-x+2\right)+x$$

input
```
Int[(x^2*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 3*x^2 + x^3 + 2*x^4),x]
```

output
```
x + x^2/2 + (Sqrt[5/3]*ArcTan[(1 - 4*x)/Sqrt[15]])/6 + (2*ArcTan[(1 + 2*x)
/Sqrt[3]])/(3*Sqrt[3]) - Log[1 + x + x^2] + Log[2 - x + 2*x^2]/4
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.81

method	result	size
default	$\frac{x^2}{2}+x-\ln\left(x^2+x+1\right)+\frac{2\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)\sqrt{3}}{9}+\frac{\ln(2x^2-x+2)}{4}-\frac{\sqrt{15}\,\arctan\left(\frac{(4x-1)\sqrt{15}}{15}\right)}{18}$	62
risch	$\frac{x^2}{2}+x+\frac{\ln(16x^2-8x+16)}{4}-\frac{\sqrt{15}\,\arctan\left(\frac{(4x-1)\sqrt{15}}{15}\right)}{18}-\ln\left(4x^2+4x+4\right)+\frac{2\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)\sqrt{3}}{9}$	66

input
```
int(x^2*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2+x-ln(x^2+x+1)+2/9*arctan(1/3*(2*x+1)*3^(1/2))*3^(1/2)+1/4*ln(2*x^2
-x+2)-1/18*15^(1/2)*arctan(1/15*(4*x-1)*15^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.79

$$\int \frac{x^2(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4}\,dx = \frac{1}{2}\,x^2-\frac{1}{6}\,\sqrt{\frac{5}{3}}\arctan\left(\frac{1}{5}\,\sqrt{\frac{5}{3}}(4\,x-1)\right) + \frac{2}{9}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right)+x + \frac{1}{4}\,\log\left(2\,x^2-x+2\right)-\log\left(x^2+x+1\right)$$

input
```
integrate(x^2*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x, algorithm="fricas
")
```

output
```
1/2*x^2 - 1/6*sqrt(5/3)*arctan(1/5*sqrt(5/3)*(4*x - 1)) + 2/9*sqrt(3)*arct
an(1/3*sqrt(3)*(2*x + 1)) + x + 1/4*log(2*x^2 - x + 2) - log(x^2 + x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.01

$$\int \frac{x^2(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4}\,dx = \frac{x^2}{2}+x+\frac{\log\left(x^2-\frac{x}{2}+1\right)}{4}-\log\left(x^2+x+1\right) - \frac{\sqrt{15}\operatorname{atan}\left(\frac{4\sqrt{15}x}{15}-\frac{\sqrt{15}}{15}\right)}{18}+\frac{2\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{9}$$

input
```
integrate(x**2*(2*x**3+3*x**2+x+5)/(2*x**4+x**3+3*x**2+x+2),x)
```

output
```
x**2/2 + x + log(x**2 - x/2 + 1)/4 - log(x**2 + x + 1) - sqrt(15)*atan(4*s
qrt(15)*x/15 - sqrt(15)/15)/18 + 2*sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3)
/9
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.79

$$\int \frac{x^2(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4}\,dx = \frac{1}{2}\,x^2-\frac{1}{18}\,\sqrt{15}\arctan\left(\frac{1}{15}\,\sqrt{15}(4\,x-1)\right)+\frac{2}{9}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right)+x+\frac{1}{4}\,\log\left(2\,x^2-x+2\right)-\log\left(x^2+x+1\right)$$

input
```
integrate(x^2*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x, algorithm="maxima
")
```

output
```
1/2*x^2 - 1/18*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1)) + 2/9*sqrt(3)*arct
an(1/3*sqrt(3)*(2*x + 1)) + x + 1/4*log(2*x^2 - x + 2) - log(x^2 + x + 1)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.79

$$\int \frac{x^2(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4}\,dx = \frac{1}{2}\,x^2 - \frac{1}{18}\,\sqrt{15}\arctan\left(\frac{1}{15}\,\sqrt{15}(4\,x-1)\right) + \frac{2}{9}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right) + x + \frac{1}{4}\,\log\left(2\,x^2-x+2\right) - \log\left(x^2+x+1\right)$$

input
```
integrate(x^2*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x, algorithm="giac")
```

output
```
1/2*x^2 - 1/18*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1)) + 2/9*sqrt(3)*arct
an(1/3*sqrt(3)*(2*x + 1)) + x + 1/4*log(2*x^2 - x + 2) - log(x^2 + x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.10

$$\int \frac{x^2(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4}\,dx = x - \ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(1+\frac{\sqrt{3}\,1\mathrm{i}}{9}\right) + \ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-1+\frac{\sqrt{3}\,1\mathrm{i}}{9}\right) + \ln\left(x-\frac{1}{4}-\frac{\sqrt{15}\,1\mathrm{i}}{4}\right)\left(\frac{1}{4}+\frac{\sqrt{15}\,1\mathrm{i}}{36}\right) - \ln\left(x-\frac{1}{4}+\frac{\sqrt{15}\,1\mathrm{i}}{4}\right)\left(-\frac{1}{4}+\frac{\sqrt{15}\,1\mathrm{i}}{36}\right) + \frac{x^2}{2}$$

input
```
int((x^2*(x + 3*x^2 + 2*x^3 + 5))/(x + 3*x^2 + x^3 + 2*x^4 + 2),x)
```

output
```
x - log(x - (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*1i)/9 + 1) + log(x + (3^(1/2)*
1i)/2 + 1/2)*((3^(1/2)*1i)/9 - 1) + log(x - (15^(1/2)*1i)/4 - 1/4)*((15^(1
/2)*1i)/36 + 1/4) - log(x + (15^(1/2)*1i)/4 - 1/4)*((15^(1/2)*1i)/36 - 1/4
) + x^2/2
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.77

$$\int \frac{x^2(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4}\,dx = -\frac{\sqrt{15}\,atan\left(\frac{4x-1}{\sqrt{15}}\right)}{18} + \frac{2\sqrt{3}\,atan\left(\frac{2x+1}{\sqrt{3}}\right)}{9} - \log(x^2+x+1) + \frac{\log(2x^2-x+2)}{4} + \frac{x^2}{2} + x$$

input
```
int(x^2*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x)
```

output
```
( - 2*sqrt(15)*atan((4*x - 1)/sqrt(15)) + 8*sqrt(3)*atan((2*x + 1)/sqrt(3)
) - 36*log(x**2 + x + 1) + 9*log(2*x**2 - x + 2) + 18*x**2 + 36*x)/36
```

3.52 $\int \frac{x(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4} dx$

Optimal result

Integrand size = 33, antiderivative size = 72

$$\int \frac{x(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4} \, dx = x-\frac{1}{3}\sqrt{\frac{5}{3}}\arctan\left(\frac{1-4x}{\sqrt{15}}\right)-\frac{10\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{3\sqrt{3}} +\frac{1}{3}\log\left(1+x+x^2\right)+\frac{1}{6}\log\left(2-x+2x^2\right)$$

output
```
x-1/9*15^(1/2)*arctan(1/15*(1-4*x)*15^(1/2))-10/9*arctan(1/3*(1+2*x)*3^(1/
2))*3^(1/2)+1/3*ln(x^2+x+1)+1/6*ln(2*x^2-x+2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.96

$$\int \frac{x(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4} \, dx = \frac{1}{18}\left(-20\sqrt{3}\arctan\left(\frac{1+2x}{\sqrt{3}}\right) +2\sqrt{15}\arctan\left(\frac{-1+4x}{\sqrt{15}}\right) +3\left(6x+2\log\left(1+x+x^2\right)+\log\left(2-x+2x^2\right)\right)\right)$$

input
```
Integrate[(x*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 3*x^2 + x^3 + 2*x^4),x]
```

output
```
(-20*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] + 2*Sqrt[15]*ArcTan[(-1 + 4*x)/Sqrt
[15]] + 3*(6*x + 2*Log[1 + x + x^2] + Log[2 - x + 2*x^2]))/18
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.061, Rules used = {2462, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(2x^3+3x^2+x+5)}{2x^4+x^3+3x^2+x+2}dx$$

$$\Big\downarrow \text{ 2462}$$

$$\int \left(\frac{2(x-2)}{3(x^2+x+1)}+\frac{2(x+1)}{3(2x^2-x+2)}+1\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{1}{3}\sqrt{\frac{5}{3}}\arctan\left(\frac{1-4x}{\sqrt{15}}\right)-\frac{10\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{1}{3}\log\left(x^2+x+1\right)+\frac{1}{6}\log\left(2x^2-x+2\right)+x$$

input
```
Int[(x*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 3*x^2 + x^3 + 2*x^4),x]
```

output
```
x - (Sqrt[5/3]*ArcTan[(1 - 4*x)/Sqrt[15]])/3 - (10*ArcTan[(1 + 2*x)/Sqrt[3
]])/(3*Sqrt[3]) + Log[1 + x + x^2]/3 + Log[2 - x + 2*x^2]/6
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.79

method	result	size
default	$x+\frac{\ln\left(x^{2}+x+1\right)}{3}-\frac{10\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)\sqrt{3}}{9}+\frac{\ln\left(2x^{2}-x+2\right)}{6}+\frac{\sqrt{15}\,\arctan\left(\frac{(4x-1)\sqrt{15}}{15}\right)}{9}$	57
risch	$x+\frac{\ln\left(16x^{2}-8x+16\right)}{6}+\frac{\sqrt{15}\,\arctan\left(\frac{(4x-1)\sqrt{15}}{15}\right)}{9}+\frac{\ln\left(4x^{2}+4x+4\right)}{3}-\frac{10\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)\sqrt{3}}{9}$	61

input
```
int(x*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x,method=_RETURNVERBOSE)
```

output
```
x+1/3*ln(x^2+x+1)-10/9*arctan(1/3*(2*x+1)*3^(1/2))*3^(1/2)+1/6*ln(2*x^2-x+
2)+1/9*15^(1/2)*arctan(1/15*(4*x-1)*15^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.78

$$\int \frac{x(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4}\,dx = \frac{1}{3}\sqrt{\frac{5}{3}}\arctan\left(\frac{1}{5}\sqrt{\frac{5}{3}}(4x-1)\right) - \frac{10}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+x + \frac{1}{6}\log\left(2x^2-x+2\right)+\frac{1}{3}\log\left(x^2+x+1\right)$$

input
```
integrate(x*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x, algorithm="fricas")
```

output
```
1/3*sqrt(5/3)*arctan(1/5*sqrt(5/3)*(4*x - 1)) - 10/9*sqrt(3)*arctan(1/3*sq
rt(3)*(2*x + 1)) + x + 1/6*log(2*x^2 - x + 2) + 1/3*log(x^2 + x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.04

$$\int \frac{x(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4}\,dx = x+\frac{\log\left(x^2-\frac{x}{2}+1\right)}{6}+\frac{\log\left(x^2+x+1\right)}{3}+\frac{\sqrt{15}\,\mathrm{atan}\left(\frac{4\sqrt{15}x}{15}-\frac{\sqrt{15}}{15}\right)}{9}-\frac{10\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{9}$$

input
```
integrate(x*(2*x**3+3*x**2+x+5)/(2*x**4+x**3+3*x**2+x+2),x)
```

output
```
x + log(x**2 - x/2 + 1)/6 + log(x**2 + x + 1)/3 + sqrt(15)*atan(4*sqrt(15)
*x/15 - sqrt(15)/15)/9 - 10*sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3)/9
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.78

$$\int \frac{x(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4}\,dx = \frac{1}{9}\sqrt{15}\arctan\left(\frac{1}{15}\sqrt{15}(4x-1)\right)-\frac{10}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+x+\frac{1}{6}\log\left(2x^2-x+2\right)+\frac{1}{3}\log\left(x^2+x+1\right)$$

input
```
integrate(x*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x, algorithm="maxima")
```

output
```
1/9*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1)) - 10/9*sqrt(3)*arctan(1/3*sqr
t(3)*(2*x + 1)) + x + 1/6*log(2*x^2 - x + 2) + 1/3*log(x^2 + x + 1)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.78

$$\int \frac{x(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4}\,dx = \frac{1}{9}\sqrt{15}\arctan\left(\frac{1}{15}\sqrt{15}(4x-1)\right) - \frac{10}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) + x + \frac{1}{6}\log\left(2x^2-x+2\right) + \frac{1}{3}\log\left(x^2+x+1\right)$$

input
```
integrate(x*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x, algorithm="giac")
```

output
```
1/9*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1)) - 10/9*sqrt(3)*arctan(1/3*sqr
t(3)*(2*x + 1)) + x + 1/6*log(2*x^2 - x + 2) + 1/3*log(x^2 + x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.11

$$\int \frac{x(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4}\,dx = x + \ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{3}+\frac{\sqrt{3}\,5\mathrm{i}}{9}\right) - \ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{1}{3}+\frac{\sqrt{3}\,5\mathrm{i}}{9}\right) - \ln\left(x-\frac{1}{4}-\frac{\sqrt{15}\,1\mathrm{i}}{4}\right)\left(-\frac{1}{6}+\frac{\sqrt{15}\,1\mathrm{i}}{18}\right) + \ln\left(x-\frac{1}{4}+\frac{\sqrt{15}\,1\mathrm{i}}{4}\right)\left(\frac{1}{6}+\frac{\sqrt{15}\,1\mathrm{i}}{18}\right)$$

input
```
int((x*(x + 3*x^2 + 2*x^3 + 5))/(x + 3*x^2 + x^3 + 2*x^4 + 2),x)
```

output
```
x + log(x - (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*5i)/9 + 1/3) - log(x + (3^(1/2
)*1i)/2 + 1/2)*((3^(1/2)*5i)/9 - 1/3) - log(x - (15^(1/2)*1i)/4 - 1/4)*((1
5^(1/2)*1i)/18 - 1/6) + log(x + (15^(1/2)*1i)/4 - 1/4)*((15^(1/2)*1i)/18 +
 1/6)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.75

$$\int \frac{x(5+x+3x^2+2x^3)}{2+x+3x^2+x^3+2x^4}\,dx = \frac{\sqrt{15}\,atan\left(\frac{4x-1}{\sqrt{15}}\right)}{9} - \frac{10\sqrt{3}\,atan\left(\frac{2x+1}{\sqrt{3}}\right)}{9} + \frac{\log(x^2+x+1)}{3} + \frac{\log(2x^2-x+2)}{6} + x$$

input
```
int(x*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x)
```

output
```
(2*sqrt(15)*atan((4*x - 1)/sqrt(15)) - 20*sqrt(3)*atan((2*x + 1)/sqrt(3))
+ 6*log(x**2 + x + 1) + 3*log(2*x**2 - x + 2) + 18*x)/18
```

3.53 $\int \frac{5+x+3x^2+2x^3}{2+x+3x^2+x^3+2x^4} \, dx$

Optimal result

Integrand size = 32, antiderivative size = 71

$$\int \frac{5+x+3x^2+2x^3}{2+x+3x^2+x^3+2x^4} \, dx = -\frac{1}{3}\sqrt{\frac{5}{3}}\arctan\left(\frac{1-4x}{\sqrt{15}}\right)+\frac{8\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{3\sqrt{3}} + \frac{2}{3}\log\left(1+x+x^2\right)-\frac{1}{6}\log\left(2-x+2x^2\right)$$

output
```
-1/9*15^(1/2)*arctan(1/15*(1-4*x)*15^(1/2))+8/9*arctan(1/3*(1+2*x)*3^(1/2)
)*3^(1/2)+2/3*ln(x^2+x+1)-1/6*ln(2*x^2-x+2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.92

$$\int \frac{5+x+3x^2+2x^3}{2+x+3x^2+x^3+2x^4} \, dx = \frac{1}{18}\left(16\sqrt{3}\arctan\left(\frac{1+2x}{\sqrt{3}}\right)+2\sqrt{15}\arctan\left(\frac{-1+4x}{\sqrt{15}}\right) + 12\log\left(1+x+x^2\right)-3\log\left(2-x+2x^2\right)\right)$$

input
```
Integrate[(5 + x + 3*x^2 + 2*x^3)/(2 + x + 3*x^2 + x^3 + 2*x^4),x]
```

output
```
(16*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] + 2*Sqrt[15]*ArcTan[(-1 + 4*x)/Sqrt[
15]] + 12*Log[1 + x + x^2] - 3*Log[2 - x + 2*x^2])/18
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {2462, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2x^3+3x^2+x+5}{2x^4+x^3+3x^2+x+2}\,dx$$
$$\Big\downarrow \text{2462}$$
$$\int \left(\frac{3-2x}{3\left(2x^2-x+2\right)}+\frac{2(2x+3)}{3\left(x^2+x+1\right)}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$-\frac{1}{3}\sqrt{\frac{5}{3}}\arctan\left(\frac{1-4x}{\sqrt{15}}\right)+\frac{8\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{2}{3}\log\left(x^2+x+1\right)-\frac{1}{6}\log\left(2x^2-x+2\right)$$

input
```
Int[(5 + x + 3*x^2 + 2*x^3)/(2 + x + 3*x^2 + x^3 + 2*x^4),x]
```

output
```
-1/3*(Sqrt[5/3]*ArcTan[(1 - 4*x)/Sqrt[15]]) + (8*ArcTan[(1 + 2*x)/Sqrt[3]]
)/(3*Sqrt[3]) + (2*Log[1 + x + x^2])/3 - Log[2 - x + 2*x^2]/6
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.79

method	result	size
default	$\frac{2\ln\left(x^{2}+x+1\right)}{3}+\frac{8\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)\sqrt{3}}{9}-\frac{\ln\left(2x^{2}-x+2\right)}{6}+\frac{\sqrt{15}\,\arctan\left(\frac{(4x-1)\sqrt{15}}{15}\right)}{9}$	56
risch	$\frac{2\ln\left(4x^{2}+4x+4\right)}{3}+\frac{8\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)\sqrt{3}}{9}-\frac{\ln\left(16x^{2}-8x+16\right)}{6}+\frac{\sqrt{15}\,\arctan\left(\frac{(4x-1)\sqrt{15}}{15}\right)}{9}$	60

input
```
int((2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x,method=_RETURNVERBOSE)
```

output
```
2/3*ln(x^2+x+1)+8/9*arctan(1/3*(2*x+1)*3^(1/2))*3^(1/2)-1/6*ln(2*x^2-x+2)+
1/9*15^(1/2)*arctan(1/15*(4*x-1)*15^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.77

$$\int \frac{5+x+3x^2+2x^3}{2+x+3x^2+x^3+2x^4}\,dx = \frac{1}{3}\sqrt{\frac{5}{3}}\arctan\left(\frac{1}{5}\sqrt{\frac{5}{3}}(4\,x-1)\right) + \frac{8}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2\,x+1)\right) - \frac{1}{6}\log\left(2\,x^2-x+2\right)+\frac{2}{3}\log\left(x^2+x+1\right)$$

input
```
integrate((2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x, algorithm="fricas")
```

output
```
1/3*sqrt(5/3)*arctan(1/5*sqrt(5/3)*(4*x - 1)) + 8/9*sqrt(3)*arctan(1/3*sqr
t(3)*(2*x + 1)) - 1/6*log(2*x^2 - x + 2) + 2/3*log(x^2 + x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.06

$$\int \frac{5+x+3x^2+2x^3}{2+x+3x^2+x^3+2x^4}\,dx = -\frac{\log\left(x^2-\frac{x}{2}+1\right)}{6}+\frac{2\log\left(x^2+x+1\right)}{3}+\frac{\sqrt{15}\operatorname{atan}\left(\frac{4\sqrt{15}x}{15}-\frac{\sqrt{15}}{15}\right)}{9}+\frac{8\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{9}$$

input
```
integrate((2*x**3+3*x**2+x+5)/(2*x**4+x**3+3*x**2+x+2),x)
```

output
```
-log(x**2 - x/2 + 1)/6 + 2*log(x**2 + x + 1)/3 + sqrt(15)*atan(4*sqrt(15)*
x/15 - sqrt(15)/15)/9 + 8*sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3)/9
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.77

$$\int \frac{5+x+3x^2+2x^3}{2+x+3x^2+x^3+2x^4}\,dx = \frac{1}{9}\sqrt{15}\arctan\left(\frac{1}{15}\sqrt{15}(4x-1)\right)+\frac{8}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{6}\log\left(2x^2-x+2\right)+\frac{2}{3}\log\left(x^2+x+1\right)$$

input
```
integrate((2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x, algorithm="maxima")
```

output
```
1/9*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1)) + 8/9*sqrt(3)*arctan(1/3*sqrt
(3)*(2*x + 1)) - 1/6*log(2*x^2 - x + 2) + 2/3*log(x^2 + x + 1)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.77

$$\int \frac{5+x+3x^2+2x^3}{2+x+3x^2+x^3+2x^4}\,dx = \frac{1}{9}\sqrt{15}\arctan\left(\frac{1}{15}\sqrt{15}(4x-1)\right) + \frac{8}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - \frac{1}{6}\log\left(2x^2-x+2\right) + \frac{2}{3}\log\left(x^2+x+1\right)$$

input
```
integrate((2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x, algorithm="giac")
```

output
```
1/9*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1)) + 8/9*sqrt(3)*arctan(1/3*sqrt
(3)*(2*x + 1)) - 1/6*log(2*x^2 - x + 2) + 2/3*log(x^2 + x + 1)
```

Mupad [B] (verification not implemented)

Time = 21.99 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.11

$$\int \frac{5+x+3x^2+2x^3}{2+x+3x^2+x^3+2x^4}\,dx = -\ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1i}{2}\right)\left(-\frac{2}{3}+\frac{\sqrt{3}\,4i}{9}\right) + \ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{2}{3}+\frac{\sqrt{3}\,4i}{9}\right) - \ln\left(x-\frac{1}{4}-\frac{\sqrt{15}\,1i}{4}\right)\left(\frac{1}{6}+\frac{\sqrt{15}\,1i}{18}\right) + \ln\left(x-\frac{1}{4}+\frac{\sqrt{15}\,1i}{4}\right)\left(-\frac{1}{6}+\frac{\sqrt{15}\,1i}{18}\right)$$

input
```
int((x + 3*x^2 + 2*x^3 + 5)/(x + 3*x^2 + x^3 + 2*x^4 + 2),x)
```

output
```
log(x + (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*4i)/9 + 2/3) - log(x - (3^(1/2)*1i
)/2 + 1/2)*((3^(1/2)*4i)/9 - 2/3) - log(x - (15^(1/2)*1i)/4 - 1/4)*((15^(1
/2)*1i)/18 + 1/6) + log(x + (15^(1/2)*1i)/4 - 1/4)*((15^(1/2)*1i)/18 - 1/6
)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.75

$$\int \frac{5+x+3x^2+2x^3}{2+x+3x^2+x^3+2x^4}\,dx = \frac{\sqrt{15}\,atan\left(\frac{4x-1}{\sqrt{15}}\right)}{9} + \frac{8\sqrt{3}\,atan\left(\frac{2x+1}{\sqrt{3}}\right)}{9} + \frac{2\log(x^2+x+1)}{3} - \frac{\log(2x^2-x+2)}{6}$$

input

```
int((2*x^3+3*x^2+x+5)/(2*x^4+x^3+3*x^2+x+2),x)
```

output

```
(2*sqrt(15)*atan((4*x - 1)/sqrt(15)) + 16*sqrt(3)*atan((2*x + 1)/sqrt(3))
+ 12*log(x**2 + x + 1) - 3*log(2*x**2 - x + 2))/18
```

3.54 $\int \frac{5+x+3x^2+2x^3}{x(2+x+3x^2+x^3+2x^4)} \, dx$

Optimal result

Integrand size = 35, antiderivative size = 75

$$\int \frac{5+x+3x^2+2x^3}{x(2+x+3x^2+x^3+2x^4)} \, dx = \frac{1}{6}\sqrt{\frac{5}{3}}\arctan\left(\frac{1-4x}{\sqrt{15}}\right)+\frac{2\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{3\sqrt{3}} + \frac{5\log(x)}{2}-\log\left(1+x+x^2\right)-\frac{1}{4}\log\left(2-x+2x^2\right)$$

output
```
1/18*15^(1/2)*arctan(1/15*(1-4*x)*15^(1/2))+2/9*arctan(1/3*(1+2*x)*3^(1/2)
)*3^(1/2)+5/2*ln(x)-ln(x^2+x+1)-1/4*ln(2*x^2-x+2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.92

$$\int \frac{5+x+3x^2+2x^3}{x(2+x+3x^2+x^3+2x^4)} \, dx = \frac{1}{36}\left(8\sqrt{3}\arctan\left(\frac{1+2x}{\sqrt{3}}\right) - 2\sqrt{15}\arctan\left(\frac{-1+4x}{\sqrt{15}}\right)+90\log(x) - 36\log\left(1+x+x^2\right)-9\log\left(2-x+2x^2\right)\right)$$

input
```
Integrate[(5 + x + 3*x^2 + 2*x^3)/(x*(2 + x + 3*x^2 + x^3 + 2*x^4)),x]
```

output
```
(8*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] - 2*Sqrt[15]*ArcTan[(-1 + 4*x)/Sqrt[1
5]] + 90*Log[x] - 36*Log[1 + x + x^2] - 9*Log[2 - x + 2*x^2])/36
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {2462, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2x^3+3x^2+x+5}{x\left(2x^4+x^3+3x^2+x+2\right)}dx$$

$$\Big\downarrow \text{2462}$$

$$\int \left(\frac{-6x-1}{6\left(2x^2-x+2\right)}-\frac{2(3x+1)}{3\left(x^2+x+1\right)}+\frac{5}{2x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{6}\sqrt{\frac{5}{3}}\arctan\left(\frac{1-4x}{\sqrt{15}}\right)+\frac{2\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}-\log\left(x^2+x+1\right)-\frac{1}{4}\log\left(2x^2-x+2\right)+\frac{5\log(x)}{2}$$

input
```
Int[(5 + x + 3*x^2 + 2*x^3)/(x*(2 + x + 3*x^2 + x^3 + 2*x^4)),x]
```

output
```
(Sqrt[5/3]*ArcTan[(1 - 4*x)/Sqrt[15]])/6 + (2*ArcTan[(1 + 2*x)/Sqrt[3]])/(
3*Sqrt[3]) + (5*Log[x])/2 - Log[1 + x + x^2] - Log[2 - x + 2*x^2]/4
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.77

method	result	size
risch	$\frac{5\ln(x)}{2}-\ln\left(x^{2}+x+1\right)+\frac{2\sqrt{3}\,\arctan\left(\frac{2\sqrt{3}\,\left(x+\frac{1}{2}\right)}{3}\right)}{9}-\frac{\ln(4x^{2}-2x+4)}{4}-\frac{\sqrt{15}\,\arctan\left(\frac{2\left(2x-\frac{1}{2}\right)\sqrt{15}}{15}\right)}{18}$	58
default	$-\ln\left(x^{2}+x+1\right)+\frac{2\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)\sqrt{3}}{9}-\frac{\ln(2x^{2}-x+2)}{4}-\frac{\sqrt{15}\,\arctan\left(\frac{(4x-1)\sqrt{15}}{15}\right)}{18}+\frac{5\ln(x)}{2}$	60

input
```
int((2*x^3+3*x^2+x+5)/x/(2*x^4+x^3+3*x^2+x+2),x,method=_RETURNVERBOSE)
```

output
```
5/2*ln(x)-ln(x^2+x+1)+2/9*3^(1/2)*arctan(2/3*3^(1/2)*(x+1/2))-1/4*ln(4*x^2
-2*x+4)-1/18*15^(1/2)*arctan(2/15*(2*x-1/2)*15^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.79

$$\int \frac{5+x+3x^2+2x^3}{x\left(2+x+3x^2+x^3+2x^4\right)}\,dx = -\frac{1}{6}\,\sqrt{\frac{5}{3}}\arctan\left(\frac{1}{5}\,\sqrt{\frac{5}{3}}(4\,x-1)\right) + \frac{2}{9}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right) - \frac{1}{4}\,\log\left(2\,x^2-x+2\right)-\log\left(x^2+x+1\right)+\frac{5}{2}\,\log\left(x\right)$$

input
```
integrate((2*x^3+3*x^2+x+5)/x/(2*x^4+x^3+3*x^2+x+2),x, algorithm="fricas")
```

output
```
-1/6*sqrt(5/3)*arctan(1/5*sqrt(5/3)*(4*x - 1)) + 2/9*sqrt(3)*arctan(1/3*sq
rt(3)*(2*x + 1)) - 1/4*log(2*x^2 - x + 2) - log(x^2 + x + 1) + 5/2*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.04

$$\int \frac{5+x+3x^2+2x^3}{x(2+x+3x^2+x^3+2x^4)}\,dx = \frac{5\log(x)}{2} - \frac{\log\left(x^2-\frac{x}{2}+1\right)}{4} - \log\left(x^2+x+1\right) - \frac{\sqrt{15}\,\mathrm{atan}\left(\frac{4\sqrt{15}x}{15}-\frac{\sqrt{15}}{15}\right)}{18} + \frac{2\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{9}$$

input
```
integrate((2*x**3+3*x**2+x+5)/x/(2*x**4+x**3+3*x**2+x+2),x)
```

output
```
5*log(x)/2 - log(x**2 - x/2 + 1)/4 - log(x**2 + x + 1) - sqrt(15)*atan(4*s
qrt(15)*x/15 - sqrt(15)/15)/18 + 2*sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3)
/9
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.79

$$\int \frac{5+x+3x^2+2x^3}{x(2+x+3x^2+x^3+2x^4)}\,dx = -\frac{1}{18}\sqrt{15}\arctan\left(\frac{1}{15}\sqrt{15}(4x-1)\right) + \frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - \frac{1}{4}\log\left(2x^2-x+2\right) - \log\left(x^2+x+1\right) + \frac{5}{2}\log(x)$$

input
```
integrate((2*x^3+3*x^2+x+5)/x/(2*x^4+x^3+3*x^2+x+2),x, algorithm="maxima")
```

output
```
-1/18*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1)) + 2/9*sqrt(3)*arctan(1/3*sq
rt(3)*(2*x + 1)) - 1/4*log(2*x^2 - x + 2) - log(x^2 + x + 1) + 5/2*log(x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.80

$$\int \frac{5+x+3x^2+2x^3}{x\,(2+x+3x^2+x^3+2x^4)}\,dx = -\frac{1}{18}\,\sqrt{15}\arctan\left(\frac{1}{15}\,\sqrt{15}(4\,x-1)\right) + \frac{2}{9}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right) - \frac{1}{4}\,\log\left(2\,x^2-x+2\right)-\log\left(x^2+x+1\right)+\frac{5}{2}\,\log\left(|x|\right)$$

input
```
integrate((2*x^3+3*x^2+x+5)/x/(2*x^4+x^3+3*x^2+x+2),x, algorithm="giac")
```

output
```
-1/18*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1)) + 2/9*sqrt(3)*arctan(1/3*sq
rt(3)*(2*x + 1)) - 1/4*log(2*x^2 - x + 2) - log(x^2 + x + 1) + 5/2*log(abs
(x))
```

Mupad [B] (verification not implemented)

Time = 22.11 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.11

$$\int \frac{5+x+3x^2+2x^3}{x\,(2+x+3x^2+x^3+2x^4)}\,dx = \frac{5\,\ln\left(x\right)}{2} - \ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(1+\frac{\sqrt{3}\,1\mathrm{i}}{9}\right) + \ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-1+\frac{\sqrt{3}\,1\mathrm{i}}{9}\right) + \ln\left(x-\frac{1}{4}-\frac{\sqrt{15}\,1\mathrm{i}}{4}\right)\left(-\frac{1}{4}+\frac{\sqrt{15}\,1\mathrm{i}}{36}\right) - \ln\left(x-\frac{1}{4}+\frac{\sqrt{15}\,1\mathrm{i}}{4}\right)\left(\frac{1}{4}+\frac{\sqrt{15}\,1\mathrm{i}}{36}\right)$$

input
```
int((x + 3*x^2 + 2*x^3 + 5)/(x*(x + 3*x^2 + x^3 + 2*x^4 + 2)),x)
```

output
```
(5*log(x))/2 - log(x - (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*1i)/9 + 1) + log(x
+ (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*1i)/9 - 1) + log(x - (15^(1/2)*1i)/4 - 1
/4)*((15^(1/2)*1i)/36 - 1/4) - log(x + (15^(1/2)*1i)/4 - 1/4)*((15^(1/2)*1
i)/36 + 1/4)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.76

$$\int \frac{5+x+3x^2+2x^3}{x(2+x+3x^2+x^3+2x^4)}\,dx = -\frac{\sqrt{15}\,atan\left(\frac{4x-1}{\sqrt{15}}\right)}{18}+\frac{2\sqrt{3}\,atan\left(\frac{2x+1}{\sqrt{3}}\right)}{9} - \log\left(x^2+x+1\right)-\frac{\log(2x^2-x+2)}{4}+\frac{5\log(x)}{2}$$

input
```
int((2*x^3+3*x^2+x+5)/x/(2*x^4+x^3+3*x^2+x+2),x)
```

output
```
( - 2*sqrt(15)*atan((4*x - 1)/sqrt(15)) + 8*sqrt(3)*atan((2*x + 1)/sqrt(3)
) - 36*log(x**2 + x + 1) - 9*log(2*x**2 - x + 2) + 90*log(x))/36
```

3.55 $\int \frac{5+x+3x^2+2x^3}{x^2(2+x+3x^2+x^3+2x^4)} dx$

Optimal result

Integrand size = 35, antiderivative size = 84

$$\int \frac{5+x+3x^2+2x^3}{x^2(2+x+3x^2+x^3+2x^4)} dx = -\frac{5}{2x} + \frac{5}{12}\sqrt{\frac{5}{3}}\arctan\left(\frac{1-4x}{\sqrt{15}}\right) - \frac{10\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{3\sqrt{3}} - \frac{3\log(x)}{4} + \frac{1}{3}\log\left(1+x+x^2\right) + \frac{1}{24}\log\left(2-x+2x^2\right)$$

output
```
-5/2/x+5/36*15^(1/2)*arctan(1/15*(1-4*x)*15^(1/2))-10/9*arctan(1/3*(1+2*x)
*3^(1/2))*3^(1/2)-3/4*ln(x)+1/3*ln(x^2+x+1)+1/24*ln(2*x^2-x+2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.93

$$\int \frac{5+x+3x^2+2x^3}{x^2(2+x+3x^2+x^3+2x^4)} dx = -\frac{180+80\sqrt{3}x\arctan\left(\frac{1+2x}{\sqrt{3}}\right)+10\sqrt{15}x\arctan\left(\frac{-1+4x}{\sqrt{15}}\right)+54x\log(x)-24x\log\left(1+x+x^2\right)-3x\log(}{72x}$$

input
```
Integrate[(5 + x + 3*x^2 + 2*x^3)/(x^2*(2 + x + 3*x^2 + x^3 + 2*x^4)),x]
```

output
```
-1/72*(180 + 80*Sqrt[3]*x*ArcTan[(1 + 2*x)/Sqrt[3]] + 10*Sqrt[15]*x*ArcTan
[(-1 + 4*x)/Sqrt[15]] + 54*x*Log[x] - 24*x*Log[1 + x + x^2] - 3*x*Log[2 -
x + 2*x^2])/x
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = {2462, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2x^3+3x^2+x+5}{x^2(2x^4+x^3+3x^2+x+2)}\,dx$$

$$\Big\downarrow \; 2462$$

$$\int \left(\frac{2(x-2)}{3(x^2+x+1)}+\frac{2x-13}{12(2x^2-x+2)}+\frac{5}{2x^2}-\frac{3}{4x}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{5}{12}\sqrt{\frac{5}{3}}\arctan\left(\frac{1-4x}{\sqrt{15}}\right)-\frac{10\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{1}{3}\log\left(x^2+x+1\right)+\frac{1}{24}\log\left(2x^2-x+2\right)-\frac{5}{2x}-\frac{3\log(x)}{4}$$

input
```
Int[(5 + x + 3*x^2 + 2*x^3)/(x^2*(2 + x + 3*x^2 + x^3 + 2*x^4)),x]
```

output
```
-5/(2*x) + (5*Sqrt[5/3]*ArcTan[(1 - 4*x)/Sqrt[15]])/12 - (10*ArcTan[(1 + 2
*x)/Sqrt[3]])/(3*Sqrt[3]) - (3*Log[x])/4 + Log[1 + x + x^2]/3 + Log[2 - x
+ 2*x^2]/24
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.77

method	result
default	$\frac{\ln(x^2+x+1)}{3}-\frac{10\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)\sqrt{3}}{9}+\frac{\ln(2x^2-x+2)}{24}-\frac{5\sqrt{15}\,\arctan\left(\frac{(4x-1)\sqrt{15}}{15}\right)}{36}-\frac{5}{2x}-\frac{3\ln(x)}{4}$
risch	$-\frac{5}{2x}-\frac{3\ln(x)}{4}+\frac{\ln(25x^2+25x+25)}{3}-\frac{10\sqrt{3}\,\arctan\left(\frac{2\left(5x+\frac{5}{2}\right)\sqrt{3}}{15}\right)}{9}+\frac{\ln(100x^2-50x+100)}{24}-\frac{5\sqrt{15}\,\arctan\left(\frac{2\left(10x-\frac{5}{2}\right)\sqrt{}}{75}\right.}{36}$

input
```
int((2*x^3+3*x^2+x+5)/x^2/(2*x^4+x^3+3*x^2+x+2),x,method=_RETURNVERBOSE)
```

output
```
1/3*ln(x^2+x+1)-10/9*arctan(1/3*(2*x+1)*3^(1/2))*3^(1/2)+1/24*ln(2*x^2-x+2
)-5/36*15^(1/2)*arctan(1/15*(4*x-1)*15^(1/2))-5/2/x-3/4*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.83

$$\int \frac{5+x+3x^2+2x^3}{x^2\,(2+x+3x^2+x^3+2x^4)}\,dx =$$
$$-\frac{30\,\sqrt{\frac{5}{3}}x\arctan\left(\frac{1}{5}\,\sqrt{\frac{5}{3}}(4\,x-1)\right)+80\,\sqrt{3}x\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right)-3\,x\log\left(2\,x^2-x+2\right)-24\,x\log\left(\right.}{72\,x}$$

input
```
integrate((2*x^3+3*x^2+x+5)/x^2/(2*x^4+x^3+3*x^2+x+2),x, algorithm="fricas
")
```

output
```
-1/72*(30*sqrt(5/3)*x*arctan(1/5*sqrt(5/3)*(4*x - 1)) + 80*sqrt(3)*x*arcta
n(1/3*sqrt(3)*(2*x + 1)) - 3*x*log(2*x^2 - x + 2) - 24*x*log(x^2 + x + 1)
+ 54*x*log(x) + 180)/x
```

Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.04

$$\int \frac{5+x+3x^2+2x^3}{x^2(2+x+3x^2+x^3+2x^4)}\,dx = -\frac{3\log(x)}{4} + \frac{\log\left(x^2-\frac{x}{2}+1\right)}{24} + \frac{\log(x^2+x+1)}{3} - \frac{5\sqrt{15}\,\mathrm{atan}\left(\frac{4\sqrt{15}x}{15}-\frac{\sqrt{15}}{15}\right)}{36} - \frac{10\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{9} - \frac{5}{2x}$$

input
```
integrate((2*x**3+3*x**2+x+5)/x**2/(2*x**4+x**3+3*x**2+x+2),x)
```

output
```
-3*log(x)/4 + log(x**2 - x/2 + 1)/24 + log(x**2 + x + 1)/3 - 5*sqrt(15)*at
an(4*sqrt(15)*x/15 - sqrt(15)/15)/36 - 10*sqrt(3)*atan(2*sqrt(3)*x/3 + sqr
t(3)/3)/9 - 5/(2*x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.76

$$\int \frac{5+x+3x^2+2x^3}{x^2(2+x+3x^2+x^3+2x^4)}\,dx = -\frac{5}{36}\sqrt{15}\arctan\left(\frac{1}{15}\sqrt{15}(4x-1)\right) - \frac{10}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - \frac{5}{2x} + \frac{1}{24}\log\left(2x^2-x+2\right) + \frac{1}{3}\log\left(x^2+x+1\right) - \frac{3}{4}\log(x)$$

input
```
integrate((2*x^3+3*x^2+x+5)/x^2/(2*x^4+x^3+3*x^2+x+2),x, algorithm="maxima
")
```

output
```
-5/36*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1)) - 10/9*sqrt(3)*arctan(1/3*s
qrt(3)*(2*x + 1)) - 5/2/x + 1/24*log(2*x^2 - x + 2) + 1/3*log(x^2 + x + 1)
 - 3/4*log(x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.77

$$\int \frac{5+x+3x^2+2x^3}{x^2\left(2+x+3x^2+x^3+2x^4\right)}\,dx = -\frac{5}{36}\,\sqrt{15}\arctan\left(\frac{1}{15}\,\sqrt{15}(4\,x-1)\right) - \frac{10}{9}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right) - \frac{5}{2\,x} + \frac{1}{24}\,\log\left(2\,x^2-x+2\right) + \frac{1}{3}\,\log\left(x^2+x+1\right) - \frac{3}{4}\,\log\left(|x|\right)$$

input
```
integrate((2*x^3+3*x^2+x+5)/x^2/(2*x^4+x^3+3*x^2+x+2),x, algorithm="giac")
```

output
```
-5/36*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1)) - 10/9*sqrt(3)*arctan(1/3*s
qrt(3)*(2*x + 1)) - 5/2/x + 1/24*log(2*x^2 - x + 2) + 1/3*log(x^2 + x + 1)
 - 3/4*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.05

$$\int \frac{5+x+3x^2+2x^3}{x^2\left(2+x+3x^2+x^3+2x^4\right)}\,dx = -\frac{3\,\ln\left(x\right)}{4} + \ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(\frac{1}{3}+\frac{\sqrt{3}\,5\mathrm{i}}{9}\right) - \ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-\frac{1}{3}+\frac{\sqrt{3}\,5\mathrm{i}}{9}\right) + \ln\left(x-\frac{1}{4}-\frac{\sqrt{15}\,1\mathrm{i}}{4}\right)\,\left(\frac{1}{24}+\frac{\sqrt{15}\,5\mathrm{i}}{72}\right) - \ln\left(x-\frac{1}{4}+\frac{\sqrt{15}\,1\mathrm{i}}{4}\right)\,\left(-\frac{1}{24}+\frac{\sqrt{15}\,5\mathrm{i}}{72}\right) - \frac{5}{2\,x}$$

input
```
int((x + 3*x^2 + 2*x^3 + 5)/(x^2*(x + 3*x^2 + x^3 + 2*x^4 + 2)),x)
```

output
```
log(x - (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*5i)/9 + 1/3) - (3*log(x))/4 - log(
x + (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*5i)/9 - 1/3) + log(x - (15^(1/2)*1i)/4
 - 1/4)*((15^(1/2)*5i)/72 + 1/24) - log(x + (15^(1/2)*1i)/4 - 1/4)*((15^(1
/2)*5i)/72 - 1/24) - 5/(2*x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.81

$$\int \frac{5+x+3x^2+2x^3}{x^2\left(2+x+3x^2+x^3+2x^4\right)}\,dx = \frac{-10\sqrt{15}\,atan\left(\frac{4x-1}{\sqrt{15}}\right)x-80\sqrt{3}\,atan\left(\frac{2x+1}{\sqrt{3}}\right)x+24\log(x^2+x+1)\,x+3\log(2x^2-x+2)\,x-54\log(x)}{72x}$$

input
```
int((2*x^3+3*x^2+x+5)/x^2/(2*x^4+x^3+3*x^2+x+2),x)
```

output
```
( - 10*sqrt(15)*atan((4*x - 1)/sqrt(15))*x - 80*sqrt(3)*atan((2*x + 1)/sqr
t(3))*x + 24*log(x**2 + x + 1)*x + 3*log(2*x**2 - x + 2)*x - 54*log(x)*x -
 180)/(72*x)
```

3.56 $\int \frac{5+x+3x^2+2x^3}{x^3(2+x+3x^2+x^3+2x^4)} \, dx$

Optimal result

Integrand size = 35, antiderivative size = 91

$$\int \frac{5+x+3x^2+2x^3}{x^3(2+x+3x^2+x^3+2x^4)} \, dx = -\frac{5}{4x^2}+\frac{3}{4x}+\frac{1}{24}\sqrt{\frac{5}{3}}\arctan\left(\frac{1-4x}{\sqrt{15}}\right)+\frac{8\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{15\log(x)}{8}+\frac{2}{3}\log\left(1+x+x^2\right)+\frac{13}{48}\log\left(2-x+2x^2\right)$$

output
```
-5/4/x^2+3/4/x+1/72*15^(1/2)*arctan(1/15*(1-4*x)*15^(1/2))+8/9*arctan(1/3*
(1+2*x)*3^(1/2))*3^(1/2)-15/8*ln(x)+2/3*ln(x^2+x+1)+13/48*ln(2*x^2-x+2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.90

$$\int \frac{5+x+3x^2+2x^3}{x^3\left(2+x+3x^2+x^3+2x^4\right)}\,dx = \frac{1}{144}\left(128\sqrt{3}\arctan\left(\frac{1+2x}{\sqrt{3}}\right) - 2\sqrt{15}\arctan\left(\frac{-1+4x}{\sqrt{15}}\right) + 3\left(-\frac{60}{x^2}+\frac{36}{x}-90\log(x)+32\log\left(1+x+x^2\right) + 13\log\left(2-x+2x^2\right)\right)\right)$$

input
```
Integrate[(5 + x + 3*x^2 + 2*x^3)/(x^3*(2 + x + 3*x^2 + x^3 + 2*x^4)),x]
```

output
```
(128*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] - 2*Sqrt[15]*ArcTan[(-1 + 4*x)/Sqrt
[15]] + 3*(-60/x^2 + 36/x - 90*Log[x] + 32*Log[1 + x + x^2] + 13*Log[2 - x
 + 2*x^2]))/144
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {2462, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2x^3+3x^2+x+5}{x^3\left(2x^4+x^3+3x^2+x+2\right)}dx$$

$$\Big\downarrow \text{ 2462}$$

$$\int \left(\frac{5}{2x^3}+\frac{2(2x+3)}{3\left(x^2+x+1\right)}+\frac{26x-9}{24\left(2x^2-x+2\right)}-\frac{3}{4x^2}-\frac{15}{8x}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{24}\sqrt{\frac{5}{3}}\arctan\left(\frac{1-4x}{\sqrt{15}}\right)+\frac{8\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{5}{4x^2}+\frac{2}{3}\log\left(x^2+x+1\right)+\frac{13}{48}\log\left(2x^2-x+2\right)+\frac{3}{4x}-\frac{15\log(x)}{8}$$

input
```
Int[(5 + x + 3*x^2 + 2*x^3)/(x^3*(2 + x + 3*x^2 + x^3 + 2*x^4)),x]
```

output
```
-5/(4*x^2) + 3/(4*x) + (Sqrt[5/3]*ArcTan[(1 - 4*x)/Sqrt[15]])/24 + (8*ArcT
an[(1 + 2*x)/Sqrt[3]])/(3*Sqrt[3]) - (15*Log[x])/8 + (2*Log[1 + x + x^2])/
3 + (13*Log[2 - x + 2*x^2])/48
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.77

method	result	si
default	$\frac{2\ln\left(x^2+x+1\right)}{3}+\frac{8\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)\sqrt{3}}{9}+\frac{13\ln\left(2x^2-x+2\right)}{48}-\frac{\sqrt{15}\,\arctan\left(\frac{(4x-1)\sqrt{15}}{15}\right)}{72}-\frac{5}{4x^2}+\frac{3}{4x}-\frac{15\ln(x)}{8}$	7
risch	$\frac{\frac{3x}{4}-\frac{5}{4}}{x^2}+\frac{2\ln\left(4x^2+4x+4\right)}{3}+\frac{8\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)\sqrt{3}}{9}-\frac{15\ln(x)}{8}+\frac{13\ln\left(4x^2-2x+4\right)}{48}-\frac{\sqrt{15}\,\arctan\left(\frac{2\left(2x-\frac{1}{2}\right)\sqrt{15}}{15}\right)}{72}$	7

input
```
int((2*x^3+3*x^2+x+5)/x^3/(2*x^4+x^3+3*x^2+x+2),x,method=_RETURNVERBOSE)
```

output
```
2/3*ln(x^2+x+1)+8/9*arctan(1/3*(2*x+1)*3^(1/2))*3^(1/2)+13/48*ln(2*x^2-x+2
)-1/72*15^(1/2)*arctan(1/15*(4*x-1)*15^(1/2))-5/4/x^2+3/4/x-15/8*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.91

$$\int \frac{5+x+3x^2+2x^3}{x^3\left(2+x+3x^2+x^3+2x^4\right)}\,dx = -\frac{6\sqrt{\frac{5}{3}}x^2\arctan\left(\frac{1}{5}\sqrt{\frac{5}{3}}(4\,x-1)\right)-128\sqrt{3}x^2\arctan\left(\frac{1}{3}\sqrt{3}(2\,x+1)\right)-39\,x^2\log\left(2\,x^2-x+2\right)-96\,x}{144\,x^2}$$

input
```
integrate((2*x^3+3*x^2+x+5)/x^3/(2*x^4+x^3+3*x^2+x+2),x, algorithm="fricas
")
```

output
```
-1/144*(6*sqrt(5/3)*x^2*arctan(1/5*sqrt(5/3)*(4*x - 1)) - 128*sqrt(3)*x^2*
arctan(1/3*sqrt(3)*(2*x + 1)) - 39*x^2*log(2*x^2 - x + 2) - 96*x^2*log(x^2
 + x + 1) + 270*x^2*log(x) - 108*x + 180)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.03

$$\int \frac{5+x+3x^2+2x^3}{x^3\left(2+x+3x^2+x^3+2x^4\right)}\,dx = -\frac{15\log(x)}{8}+\frac{13\log\left(x^2-\frac{x}{2}+1\right)}{48} + \frac{2\log\left(x^2+x+1\right)}{3}-\frac{\sqrt{15}\operatorname{atan}\left(\frac{4\sqrt{15}x}{15}-\frac{\sqrt{15}}{15}\right)}{72} + \frac{8\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{9}+\frac{3x-5}{4x^2}$$

input
```
integrate((2*x**3+3*x**2+x+5)/x**3/(2*x**4+x**3+3*x**2+x+2),x)
```

output
```
-15*log(x)/8 + 13*log(x**2 - x/2 + 1)/48 + 2*log(x**2 + x + 1)/3 - sqrt(15
)*atan(4*sqrt(15)*x/15 - sqrt(15)/15)/72 + 8*sqrt(3)*atan(2*sqrt(3)*x/3 +
sqrt(3)/3)/9 + (3*x - 5)/(4*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.76

$$\int \frac{5+x+3x^2+2x^3}{x^3\left(2+x+3x^2+x^3+2x^4\right)}\,dx = -\frac{1}{72}\,\sqrt{15}\arctan\left(\frac{1}{15}\,\sqrt{15}(4\,x-1)\right) + \frac{8}{9}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right) + \frac{3\,x-5}{4\,x^2} + \frac{13}{48}\,\log\left(2\,x^2-x+2\right) + \frac{2}{3}\,\log\left(x^2+x+1\right) - \frac{15}{8}\,\log\left(x\right)$$

input
```
integrate((2*x^3+3*x^2+x+5)/x^3/(2*x^4+x^3+3*x^2+x+2),x, algorithm="maxima
")
```

output
```
-1/72*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1)) + 8/9*sqrt(3)*arctan(1/3*sq
rt(3)*(2*x + 1)) + 1/4*(3*x - 5)/x^2 + 13/48*log(2*x^2 - x + 2) + 2/3*log(
x^2 + x + 1) - 15/8*log(x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.77

$$\int \frac{5+x+3x^2+2x^3}{x^3\left(2+x+3x^2+x^3+2x^4\right)}\,dx = -\frac{1}{72}\,\sqrt{15}\arctan\left(\frac{1}{15}\,\sqrt{15}(4\,x-1)\right) + \frac{8}{9}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right) + \frac{3\,x-5}{4\,x^2} + \frac{13}{48}\,\log\left(2\,x^2-x+2\right) + \frac{2}{3}\,\log\left(x^2+x+1\right) - \frac{15}{8}\,\log\left(|x|\right)$$

input
```
integrate((2*x^3+3*x^2+x+5)/x^3/(2*x^4+x^3+3*x^2+x+2),x, algorithm="giac")
```

output
```
-1/72*sqrt(15)*arctan(1/15*sqrt(15)*(4*x - 1)) + 8/9*sqrt(3)*arctan(1/3*sq
rt(3)*(2*x + 1)) + 1/4*(3*x - 5)/x^2 + 13/48*log(2*x^2 - x + 2) + 2/3*log(
x^2 + x + 1) - 15/8*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 22.23 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.01

$$\int \frac{5+x+3x^2+2x^3}{x^3(2+x+3x^2+x^3+2x^4)}\,dx = \frac{\frac{3x}{4}-\frac{5}{4}}{x^2} - \frac{15\ln(x)}{8} - \ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{2}{3}+\frac{\sqrt{3}\,4\mathrm{i}}{9}\right) + \ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{2}{3}+\frac{\sqrt{3}\,4\mathrm{i}}{9}\right) + \ln\left(x-\frac{1}{4}-\frac{\sqrt{15}\,1\mathrm{i}}{4}\right)\left(\frac{13}{48}+\frac{\sqrt{15}\,1\mathrm{i}}{144}\right) - \ln\left(x-\frac{1}{4}+\frac{\sqrt{15}\,1\mathrm{i}}{4}\right)\left(-\frac{13}{48}+\frac{\sqrt{15}\,1\mathrm{i}}{144}\right)$$

input
```
int((x + 3*x^2 + 2*x^3 + 5)/(x^3*(x + 3*x^2 + x^3 + 2*x^4 + 2)),x)
```

output
```
((3*x)/4 - 5/4)/x^2 - (15*log(x))/8 - log(x - (3^(1/2)*1i)/2 + 1/2)*((3^(1
/2)*4i)/9 - 2/3) + log(x + (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*4i)/9 + 2/3) +
log(x - (15^(1/2)*1i)/4 - 1/4)*((15^(1/2)*1i)/144 + 13/48) - log(x + (15^(
1/2)*1i)/4 - 1/4)*((15^(1/2)*1i)/144 - 13/48)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.89

$$\int \frac{5+x+3x^2+2x^3}{x^3(2+x+3x^2+x^3+2x^4)}\,dx$$
$$= \frac{-2\sqrt{15}\,atan\left(\frac{4x-1}{\sqrt{15}}\right)x^2+128\sqrt{3}\,atan\left(\frac{2x+1}{\sqrt{3}}\right)x^2+96\log(x^2+x+1)\,x^2+39\log(2x^2-x+2)\,x^2-270\log}{144x^2}$$

input
```
int((2*x^3+3*x^2+x+5)/x^3/(2*x^4+x^3+3*x^2+x+2),x)
```

output
```
( - 2*sqrt(15)*atan((4*x - 1)/sqrt(15))*x**2 + 128*sqrt(3)*atan((2*x + 1)/
sqrt(3))*x**2 + 96*log(x**2 + x + 1)*x**2 + 39*log(2*x**2 - x + 2)*x**2 -
270*log(x)*x**2 + 108*x - 180)/(144*x**2)
```

3.57 $\int \frac{x^3(5+x+3x^2+2x^3)}{2+x+5x^2+x^3+2x^4}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 230

$$\begin{aligned}\int \frac{x^3(5+x+3x^2+2x^3)}{2+x+5x^2+x^3+2x^4}\,dx &= -\frac{5x}{2}+\frac{x^2}{2}+\frac{x^3}{3}-\frac{11\left(9+5i\sqrt{7}\right)\operatorname{arctanh}\left(\frac{i-\sqrt{7}+8ix}{\sqrt{2\left(35-i\sqrt{7}\right)}}\right)}{4\sqrt{14\left(35-i\sqrt{7}\right)}}\\
&+\frac{11\left(9-5i\sqrt{7}\right)\operatorname{arctanh}\left(\frac{i+\sqrt{7}+8ix}{\sqrt{2\left(35+i\sqrt{7}\right)}}\right)}{4\sqrt{14\left(35+i\sqrt{7}\right)}}\\
&+\frac{3}{112}\left(7+11i\sqrt{7}\right)\log\left(4i+\left(i-\sqrt{7}\right)x+4ix^2\right)\\
&+\frac{3}{112}\left(7-11i\sqrt{7}\right)\log\left(4i+\left(i+\sqrt{7}\right)x+4ix^2\right)\end{aligned}$$

output
```
-5/2*x+1/2*x^2+1/3*x^3-11/4*(9+5*I*7^(1/2))*arctanh((I-7^(1/2)+8*I*x)/(70-
2*I*7^(1/2))^(1/2))/(490-14*I*7^(1/2))^(1/2)+11/4*(9-5*I*7^(1/2))*arctanh(
(I+7^(1/2)+8*I*x)/(70+2*I*7^(1/2))^(1/2))/(490+14*I*7^(1/2))^(1/2)+3/112*(
7+11*I*7^(1/2))*ln(4*I+(I-7^(1/2))*x+4*I*x^2)+3/112*(7-11*I*7^(1/2))*ln(4*
I+(I+7^(1/2))*x+4*I*x^2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.47

$$\int \frac{x^3(5+x+3x^2+2x^3)}{2+x+5x^2+x^3+2x^4}\,dx = \frac{1}{6}\left(x(-15+3x+2x^2)+3\text{RootSum}\left[2+\#1+5\#1^2+\#1^3+2\#1^4\&,\frac{10\log(x-\#1)+\log(x-\#1)\#1+19\log(x-\#1)\#1^2+3\log(x-\#1)\#1^3}{1+10\#1+3\#1^2+8\#1^3}\&\right]\right)$$

input
```
Integrate[(x^3*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 5*x^2 + x^3 + 2*x^4),x]
```

output
```
(x*(-15 + 3*x + 2*x^2) + 3*RootSum[2 + #1 + 5*#1^2 + #1^3 + 2*#1^4 & , (10
*Log[x - #1] + Log[x - #1]*#1 + 19*Log[x - #1]*#1^2 + 3*Log[x - #1]*#1^3)/
(1 + 10*#1 + 3*#1^2 + 8*#1^3) & ])/6
```

Rubi [A] (verified)

Time = 1.24 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = {2492, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(2x^3+3x^2+x+5)}{2x^4+x^3+5x^2+x+2}\,dx$$

$$\Big\downarrow \text{ 2492}$$

$$\frac{1}{2}\int \left(2x^2+2x+\frac{3(7i-11\sqrt{7})x+2(35i+9\sqrt{7})}{7(4ix^2+(i-\sqrt{7})x+4i)}+\frac{3(7i+11\sqrt{7})x+2(35i-9\sqrt{7})}{7(4ix^2+(i+\sqrt{7})x+4i)}-5\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(-\frac{11\left(9+5i\sqrt{7}\right)\operatorname{arctanh}\left(\frac{8ix-\sqrt{7}+i}{\sqrt{2\left(35-i\sqrt{7}\right)}}\right)}{2\sqrt{14\left(35-i\sqrt{7}\right)}}+\frac{11\left(9-5i\sqrt{7}\right)\operatorname{arctanh}\left(\frac{8ix+\sqrt{7}+i}{\sqrt{2\left(35+i\sqrt{7}\right)}}\right)}{2\sqrt{14\left(35+i\sqrt{7}\right)}}+\frac{2x^3}{3}+x^2+\frac{3}{56}\left(7+11i\right.\right.$$

input
```
Int[(x^3*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 5*x^2 + x^3 + 2*x^4),x]
```

output
```
(-5*x + x^2 + (2*x^3)/3 - (11*(9 + (5*I)*Sqrt[7])*ArcTanh[(I - Sqrt[7] + (
8*I)*x)/Sqrt[2*(35 - I*Sqrt[7])]])/(2*Sqrt[14*(35 - I*Sqrt[7])]) + (11*(9
- (5*I)*Sqrt[7])*ArcTanh[(I + Sqrt[7] + (8*I)*x)/Sqrt[2*(35 + I*Sqrt[7])]]
)/(2*Sqrt[14*(35 + I*Sqrt[7])]) + (3*(7 + (11*I)*Sqrt[7])*Log[4*I + (I - S
qrt[7])*x + (4*I)*x^2])/56 + (3*(7 - (11*I)*Sqrt[7])*Log[4*I + (I + Sqrt[7
])*x + (4*I)*x^2])/56)/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2492
```
Int[(Px_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^3 + (e_.)*(x_)^4)
^(p_), x_Symbol] :> Simp[e^p   Int[ExpandIntegrand[Px*(b/d + ((d + Sqrt[e*(
(b^2 - 4*a*c)/a) + 8*a*d*(e/b)])/(2*e))*x + x^2)^p*(b/d + ((d - Sqrt[e*((b^
2 - 4*a*c)/a) + 8*a*d*(e/b)])/(2*e))*x + x^2)^p, x], x], x] /; FreeQ[{a, b,
 c, d, e}, x] && PolyQ[Px, x] && ILtQ[p, 0] && EqQ[a*d^2 - b^2*e, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.05 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.32

method	result	size
default	$\frac{x^3}{3}+\frac{x^2}{2}-\frac{5x}{2}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(2\_Z^4+\_Z^3+5\_Z^2+\_Z+2\right)}\frac{\left(3\_R^3+19\_R^2+\_R+10\right)\ln\left(x-\_R\right)}{8\_R^3+3\_R^2+10\_R+1}\right)}{2}$	74
risch	$\frac{x^3}{3}+\frac{x^2}{2}-\frac{5x}{2}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(2\_Z^4+\_Z^3+5\_Z^2+\_Z+2\right)}\frac{\left(3\_R^3+19\_R^2+\_R+10\right)\ln\left(x-\_R\right)}{8\_R^3+3\_R^2+10\_R+1}\right)}{2}$	74

input
```
int(x^3*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x,method=_RETURNVERBOSE)
```

output
```
1/3*x^3+1/2*x^2-5/2*x+1/2*sum((3*_R^3+19*_R^2+_R+10)/(8*_R^3+3*_R^2+10*_R+
1)*ln(x-_R),_R=RootOf(2*_Z^4+_Z^3+5*_Z^2+_Z+2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 322 vs. $2(139) = 278$.

Time = 0.10 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.40

$$\int \frac{x^3(5+x+3x^2+2x^3)}{2+x+5x^2+x^3+2x^4}\,dx$$

$$= \frac{1}{3}\,x^3 + \frac{1}{2}\,x^2 - \frac{1}{16}\left(\sqrt{\frac{704}{7}\,\sqrt{\frac{11}{7}} - 55} - 3\right)\log\left(16808\,x^2 + 7\left(7\,\sqrt{\frac{11}{7}}(20\,x-9) - 198\,x - 121\right)\sqrt{\frac{704}{7}\,\sqrt{\frac{11}{7}} - 55} + 4202\,x + 14707\,\sqrt{\frac{11}{7}} + 2101\right) + \frac{1}{16}\left(\sqrt{\frac{704}{7}\,\sqrt{\frac{11}{7}} - 55} + 3\right)\log\left(16808\,x^2 - 7\left(7\,\sqrt{\frac{11}{7}}(20\,x-9) - 198\,x - 121\right)\sqrt{\frac{704}{7}\,\sqrt{\frac{11}{7}} - 55} + 4202\,x + 14707\,\sqrt{\frac{11}{7}} + 2101\right)$$

$$- \frac{1}{4}\,\sqrt{\frac{33}{382}\,\sqrt{\frac{704}{7}\,\sqrt{\frac{11}{7}} - 55}\left(64\,\sqrt{\frac{11}{7}} + 35\right) + \frac{176}{7}\,\sqrt{\frac{11}{7}} + \frac{737}{14}}\,\arctan\left(-\frac{7}{268928}\left(191\,\sqrt{\frac{11}{7}}(92\,x +\right.\right.$$

$$+ \frac{1}{4}\,\sqrt{-\frac{33}{382}\,\sqrt{\frac{704}{7}\,\sqrt{\frac{11}{7}} - 55}\left(64\,\sqrt{\frac{11}{7}} + 35\right) + \frac{176}{7}\,\sqrt{\frac{11}{7}} + \frac{737}{14}}\,\arctan\left(\frac{7}{268928}\left(191\,\sqrt{\frac{11}{7}}(92\,x +\right.\right.$$

$$- \frac{5}{2}\,x$$

input
```
integrate(x^3*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x, algorithm="fricas
")
```

output
```
1/3*x^3 + 1/2*x^2 - 1/16*(sqrt(704/7*sqrt(11/7) - 55) - 3)*log(16808*x^2 +
 7*(7*sqrt(11/7)*(20*x - 9) - 198*x - 121)*sqrt(704/7*sqrt(11/7) - 55) + 4
202*x + 14707*sqrt(11/7) + 2101) + 1/16*(sqrt(704/7*sqrt(11/7) - 55) + 3)*
log(16808*x^2 - 7*(7*sqrt(11/7)*(20*x - 9) - 198*x - 121)*sqrt(704/7*sqrt(
11/7) - 55) + 4202*x + 14707*sqrt(11/7) + 2101) - 1/4*sqrt(33/382*sqrt(704
/7*sqrt(11/7) - 55)*(64*sqrt(11/7) + 35) + 176/7*sqrt(11/7) + 737/14)*arct
an(-7/268928*(191*sqrt(11/7)*(92*x + 29) - 7*(sqrt(11/7)*(404*x + 87) + 40
0*x - 33)*sqrt(704/7*sqrt(11/7) - 55) + 33616*x - 2101)*sqrt(33/382*sqrt(7
04/7*sqrt(11/7) - 55)*(64*sqrt(11/7) + 35) + 176/7*sqrt(11/7) + 737/14)) +
 1/4*sqrt(-33/382*sqrt(704/7*sqrt(11/7) - 55)*(64*sqrt(11/7) + 35) + 176/7
*sqrt(11/7) + 737/14)*arctan(7/268928*(191*sqrt(11/7)*(92*x + 29) + 7*(sqr
t(11/7)*(404*x + 87) + 400*x - 33)*sqrt(704/7*sqrt(11/7) - 55) + 33616*x -
 2101)*sqrt(-33/382*sqrt(704/7*sqrt(11/7) - 55)*(64*sqrt(11/7) + 35) + 176
/7*sqrt(11/7) + 737/14)) - 5/2*x
```

Sympy [A] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.27

$$\int \frac{x^3(5+x+3x^2+2x^3)}{2+x+5x^2+x^3+2x^4}\,dx = \frac{x^3}{3}+\frac{x^2}{2}-\frac{5x}{2}$$
$$+\operatorname{RootSum}\left(1372t^4-1029t^3+3136t^2+2688t+512,\left(t\mapsto t\log\left(\frac{5831t^3}{1936}-\frac{23765t^2}{7744}+\frac{2065t}{242}+x+\frac{4}{1}\right.\right.\right.$$

input
```
integrate(x**3*(2*x**3+3*x**2+x+5)/(2*x**4+x**3+5*x**2+x+2),x)
```

output
```
x**3/3 + x**2/2 - 5*x/2 + RootSum(1372*_t**4 - 1029*_t**3 + 3136*_t**2 + 2
688*_t + 512, Lambda(_t, _t*log(5831*_t**3/1936 - 23765*_t**2/7744 + 2065*
_t/242 + x + 415/121)))
```

Maxima [F]

$$\int \frac{x^3(5+x+3x^2+2x^3)}{2+x+5x^2+x^3+2x^4}\,dx = \int \frac{(2\,x^3+3\,x^2+x+5)x^3}{2\,x^4+x^3+5\,x^2+x+2}\,dx$$

input
```
integrate(x^3*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x, algorithm="maxima
")
```

output
```
1/3*x^3 + 1/2*x^2 - 5/2*x + 1/2*integrate((3*x^3 + 19*x^2 + x + 10)/(2*x^4
 + x^3 + 5*x^2 + x + 2), x)
```

Giac [F]

$$\int \frac{x^3(5+x+3x^2+2x^3)}{2+x+5x^2+x^3+2x^4}\,dx = \int \frac{(2\,x^3+3\,x^2+x+5)x^3}{2\,x^4+x^3+5\,x^2+x+2}\,dx$$

input
```
integrate(x^3*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x, algorithm="giac")
```

output
```
integrate((2*x^3 + 3*x^2 + x + 5)*x^3/(2*x^4 + x^3 + 5*x^2 + x + 2), x)
```

Mupad [B] (verification not implemented)

Time = 21.80 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.56

$$\int \frac{x^3(5+x+3x^2+2x^3)}{2+x+5x^2+x^3+2x^4}\,dx = \left(\sum_{k=1}^{4}\ln\left(-29\,x +\mathrm{root}\left(z^4-\frac{3\,z^3}{4}+\frac{16\,z^2}{7}+\frac{96\,z}{49}+\frac{128}{343},z,k\right)\left(-\frac{289\,x}{4}+\mathrm{root}\left(z^4-\frac{3\,z^3}{4}+\frac{16\,z^2}{7}+\frac{96\,z}{49}+\frac{128}{343},z,k\right)\left(\frac{581}{16}\right.\right.\right.\right.$$
$$\left.\left.\left.+7\right)\mathrm{root}\left(z^4-\frac{3\,z^3}{4}+\frac{16\,z^2}{7}+\frac{96\,z}{49}+\frac{128}{343},z,k\right)\right)\right)-\frac{5\,x}{2}+\frac{x^2}{2}+\frac{x^3}{3}$$

input
```
int((x^3*(x + 3*x^2 + 2*x^3 + 5))/(x + 5*x^2 + x^3 + 2*x^4 + 2),x)
```

output
```
symsum(log(root(z^4 - (3*z^3)/4 + (16*z^2)/7 + (96*z)/49 + 128/343, z, k)*
(root(z^4 - (3*z^3)/4 + (16*z^2)/7 + (96*z)/49 + 128/343, z, k)*((581*x)/1
6 - root(z^4 - (3*z^3)/4 + (16*z^2)/7 + (96*z)/49 + 128/343, z, k)*((147*x
)/4 - 49/16) + 1141/64) - (289*x)/4 + 47/4) - 29*x + 7)*root(z^4 - (3*z^3)
/4 + (16*z^2)/7 + (96*z)/49 + 128/343, z, k), k, 1, 4) - (5*x)/2 + x^2/2 +
 x^3/3
```

Reduce [F]

$$\int \frac{x^3(5+x+3x^2+2x^3)}{2+x+5x^2+x^3+2x^4}\,dx = \frac{143\left(\int \frac{x^2}{2x^4+x^3+5x^2+x+2}dx\right)}{16} - \frac{11\left(\int \frac{x}{2x^4+x^3+5x^2+x+2}dx\right)}{8} + \frac{77\left(\int \frac{1}{2x^4+x^3+5x^2+x+2}dx\right)}{16} + \frac{3\log(2x^4+x^3+5x^2+x+2)}{16} + \frac{x^3}{3} + \frac{x^2}{2} - \frac{5x}{2}$$

input
```
int(x^3*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x)
```

output
```
(429*int(x**2/(2*x**4 + x**3 + 5*x**2 + x + 2),x) - 66*int(x/(2*x**4 + x**
3 + 5*x**2 + x + 2),x) + 231*int(1/(2*x**4 + x**3 + 5*x**2 + x + 2),x) + 9
*log(2*x**4 + x**3 + 5*x**2 + x + 2) + 16*x**3 + 24*x**2 - 120*x)/48
```

3.58 $\int \frac{x^2(5+x+3x^2+2x^3)}{2+x+5x^2+x^3+2x^4}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 219

$$\int \frac{x^2(5+x+3x^2+2x^3)}{2+x+5x^2+x^3+2x^4}\,dx = x+\frac{x^2}{2}+\frac{\left(53+i\sqrt{7}\right)\operatorname{arctanh}\left(\frac{i-\sqrt{7}+8ix}{\sqrt{2\left(35-i\sqrt{7}\right)}}\right)}{2\sqrt{14\left(35-i\sqrt{7}\right)}} - \frac{\left(53-i\sqrt{7}\right)\operatorname{arctanh}\left(\frac{i+\sqrt{7}+8ix}{\sqrt{2\left(35+i\sqrt{7}\right)}}\right)}{2\sqrt{14\left(35+i\sqrt{7}\right)}} - \frac{1}{56}\left(35-9i\sqrt{7}\right)\log\left(4i+\left(i-\sqrt{7}\right)x+4ix^2\right) - \frac{1}{56}\left(35+9i\sqrt{7}\right)\log\left(4i+\left(i+\sqrt{7}\right)x+4ix^2\right)$$

output
```
x+1/2*x^2+1/2*(53+I*7^(1/2))*arctanh((I-7^(1/2)+8*I*x)/(70-2*I*7^(1/2))^(1
/2))/(490-14*I*7^(1/2))^(1/2)-1/2*(53-I*7^(1/2))*arctanh((I+7^(1/2)+8*I*x)
/(70+2*I*7^(1/2))^(1/2))/(490+14*I*7^(1/2))^(1/2)-1/56*(35-9*I*7^(1/2))*ln
(4*I+(I-7^(1/2))*x+4*I*x^2)-1/56*(35+9*I*7^(1/2))*ln(4*I+(I+7^(1/2))*x+4*I
*x^2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.46

$$\int \frac{x^2(5+x+3x^2+2x^3)}{2+x+5x^2+x^3+2x^4}\,dx = x+\frac{x^2}{2}-\text{RootSum}\Big[2+\#1+5\#1^2+\#1^3+2\#1^4\&,\frac{2\log(x-\#1)+3\log(x-\#1)\#1+\log(x-\#1)\#1^2+5\log(x-\#1)\#1^3}{1+10\#1+3\#1^2+8\#1^3}\&\Big]$$

input
```
Integrate[(x^2*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 5*x^2 + x^3 + 2*x^4),x]
```

output
```
x + x^2/2 - RootSum[2 + #1 + 5*#1^2 + #1^3 + 2*#1^4 & , (2*Log[x - #1] + 3
*Log[x - #1]*#1 + Log[x - #1]*#1^2 + 5*Log[x - #1]*#1^3)/(1 + 10*#1 + 3*#1
^2 + 8*#1^3) & ]
```

Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = {2492, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(2x^3+3x^2+x+5)}{2x^4+x^3+5x^2+x+2}\,dx$$

$$\Big\downarrow \ 2492$$

$$\frac{1}{2}\int \left(2x-\frac{2\left(\left(9+5i\sqrt{7}\right)x+2\left(5+i\sqrt{7}\right)\right)}{\sqrt{7}\left(4ix^2+\left(i-\sqrt{7}\right)x+4i\right)}-\frac{2\left(\left(35i-9\sqrt{7}\right)x+2\left(7i-5\sqrt{7}\right)\right)}{7\left(4ix^2+\left(i+\sqrt{7}\right)x+4i\right)}+2\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{2}\left(\frac{(53+i\sqrt{7})\operatorname{arctanh}\left(\frac{8ix-\sqrt{7}+i}{\sqrt{2(35-i\sqrt{7})}}\right)}{\sqrt{14(35-i\sqrt{7})}}-\frac{(53-i\sqrt{7})\operatorname{arctanh}\left(\frac{8ix+\sqrt{7}+i}{\sqrt{2(35+i\sqrt{7})}}\right)}{\sqrt{14(35+i\sqrt{7})}}+x^2-\frac{1}{28}\left(35-9i\sqrt{7}\right)\log\left(4ix^2\right.\right.$$

input
```
Int[(x^2*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 5*x^2 + x^3 + 2*x^4),x]
```

output
```
(2*x + x^2 + ((53 + I*Sqrt[7])*ArcTanh[(I - Sqrt[7] + (8*I)*x)/Sqrt[2*(35
- I*Sqrt[7])]])/Sqrt[14*(35 - I*Sqrt[7])] - ((53 - I*Sqrt[7])*ArcTanh[(I +
 Sqrt[7] + (8*I)*x)/Sqrt[2*(35 + I*Sqrt[7])]])/Sqrt[14*(35 + I*Sqrt[7])] -
 ((35 - (9*I)*Sqrt[7])*Log[4*I + (I - Sqrt[7])*x + (4*I)*x^2])/28 - ((35 +
 (9*I)*Sqrt[7])*Log[4*I + (I + Sqrt[7])*x + (4*I)*x^2])/28)/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2492
```
Int[(Px_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^3 + (e_.)*(x_)^4)
^(p_), x_Symbol] :> Simp[e^p   Int[ExpandIntegrand[Px*(b/d + ((d + Sqrt[e*(
(b^2 - 4*a*c)/a) + 8*a*d*(e/b)])/(2*e))*x + x^2)^p*(b/d + ((d - Sqrt[e*((b^
2 - 4*a*c)/a) + 8*a*d*(e/b)])/(2*e))*x + x^2)^p, x], x], x] /; FreeQ[{a, b,
 c, d, e}, x] && PolyQ[Px, x] && ILtQ[p, 0] && EqQ[a*d^2 - b^2*e, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.04 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.31

method	result	size
default	$\frac{x^2}{2}+x+\left(\sum_{\_R=\mathrm{RootOf}(2\_Z^4+\_Z^3+5\_Z^2+\_Z+2)}\frac{(-5\_R^3-\_R^2-3\_R-2)\ln(x-\_R)}{8\_R^3+3\_R^2+10\_R+1}\right)$	67
risch	$\frac{x^2}{2}+x+\left(\sum_{\_R=\mathrm{RootOf}(2\_Z^4+\_Z^3+5\_Z^2+\_Z+2)}\frac{(-5\_R^3-\_R^2-3\_R-2)\ln(x-\_R)}{8\_R^3+3\_R^2+10\_R+1}\right)$	67

input
```
int(x^2*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2+x+sum((-5*_R^3-_R^2-3*_R-2)/(8*_R^3+3*_R^2+10*_R+1)*ln(x-_R),_R=Ro
otOf(2*_Z^4+_Z^3+5*_Z^2+_Z+2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 315 vs. $2(132) = 264$.

Time = 0.10 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.44

$$\int \frac{x^2(5+x+3x^2+2x^3)}{2+x+5x^2+x^3+2x^4}\,dx = \frac{1}{2}x^2+\frac{1}{8}\left(\sqrt{\frac{64}{7}\sqrt{\frac{11}{7}}+\frac{79}{7}}-5\right)\log\left(296x^2 + 7\left(7\sqrt{\frac{11}{7}}(12x+1)-106x-15\right)\sqrt{\frac{64}{7}\sqrt{\frac{11}{7}}+\frac{79}{7}}+74x+259\sqrt{\frac{11}{7}}+37\right) - \frac{1}{8}\left(\sqrt{\frac{64}{7}\sqrt{\frac{11}{7}}+\frac{79}{7}}+5\right)\log\left(296x^2 - 7\left(7\sqrt{\frac{11}{7}}(12x+1)-106x-15\right)\sqrt{\frac{64}{7}\sqrt{\frac{11}{7}}+\frac{79}{7}}+74x+259\sqrt{\frac{11}{7}}+37\right) - \frac{1}{2}\sqrt{\frac{9}{74}\left(64\sqrt{\frac{11}{7}}-79\right)\sqrt{\frac{64}{7}\sqrt{\frac{11}{7}}+\frac{79}{7}}+\frac{16}{7}\sqrt{\frac{11}{7}}+\frac{1}{14}}\arctan\left(\frac{7}{77552}\left(37\sqrt{\frac{11}{7}}(97x+104)+7\right.\right. + \frac{1}{2}\sqrt{-\frac{9}{74}\left(64\sqrt{\frac{11}{7}}-79\right)\sqrt{\frac{64}{7}\sqrt{\frac{11}{7}}+\frac{79}{7}}+\frac{16}{7}\sqrt{\frac{11}{7}}+\frac{1}{14}}\arctan\left(-\frac{7}{77552}\left(37\sqrt{\frac{11}{7}}(97x+104)\right.\right. + x$$

input
```
integrate(x^2*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x, algorithm="fricas
")
```

output
```
1/2*x^2 + 1/8*(sqrt(64/7*sqrt(11/7) + 79/7) - 5)*log(296*x^2 + 7*(7*sqrt(1
1/7)*(12*x + 1) - 106*x - 15)*sqrt(64/7*sqrt(11/7) + 79/7) + 74*x + 259*sq
rt(11/7) + 37) - 1/8*(sqrt(64/7*sqrt(11/7) + 79/7) + 5)*log(296*x^2 - 7*(7
*sqrt(11/7)*(12*x + 1) - 106*x - 15)*sqrt(64/7*sqrt(11/7) + 79/7) + 74*x +
 259*sqrt(11/7) + 37) - 1/2*sqrt(9/74*(64*sqrt(11/7) - 79)*sqrt(64/7*sqrt(
11/7) + 79/7) + 16/7*sqrt(11/7) + 1/14)*arctan(7/77552*(37*sqrt(11/7)*(97*
x + 104) + 7*(sqrt(11/7)*(1223*x - 936) - 1331*x + 1197)*sqrt(64/7*sqrt(11
/7) + 79/7) - 3145*x - 4921)*sqrt(9/74*(64*sqrt(11/7) - 79)*sqrt(64/7*sqrt
(11/7) + 79/7) + 16/7*sqrt(11/7) + 1/14)) + 1/2*sqrt(-9/74*(64*sqrt(11/7)
- 79)*sqrt(64/7*sqrt(11/7) + 79/7) + 16/7*sqrt(11/7) + 1/14)*arctan(-7/775
52*(37*sqrt(11/7)*(97*x + 104) - 7*(sqrt(11/7)*(1223*x - 936) - 1331*x + 1
197)*sqrt(64/7*sqrt(11/7) + 79/7) - 3145*x - 4921)*sqrt(-9/74*(64*sqrt(11/
7) - 79)*sqrt(64/7*sqrt(11/7) + 79/7) + 16/7*sqrt(11/7) + 1/14)) + x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3662 vs. $2(178) = 356$.

Time = 1.61 (sec) , antiderivative size = 3662, normalized size of antiderivative = 16.72

$$\int \frac{x^2(5 + x + 3x^2 + 2x^3)}{2 + x + 5x^2 + x^3 + 2x^4}\,dx = \text{Too large to display}$$

input
```
integrate(x**2*(2*x**3+3*x**2+x+5)/(2*x**4+x**3+5*x**2+x+2),x)
```

output
```
x**2/2 + x + (-5/8 + sqrt(79/448 + sqrt(77)/49))*log(x**2 + x*(-1459*sqrt(
14)*sqrt(-333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77))/536576 - 15
*sqrt(77)*sqrt(553 + 64*sqrt(77))/2096 - 10391*sqrt(553 + 64*sqrt(77))/268
288 + 1459*sqrt(77)/8384 + 522933/268288 + 45*sqrt(14)*sqrt(553 + 64*sqrt(
77))*sqrt(-333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77))/536576) -
510895297*sqrt(14)*sqrt(-333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(7
7))/71978450944 - 6009493*sqrt(22)*sqrt(-333*sqrt(553 + 64*sqrt(77)) + 219
75 + 7648*sqrt(77))/1124663296 - 38714551*sqrt(77)*sqrt(553 + 64*sqrt(77))
/2249326592 - 4417610843*sqrt(553 + 64*sqrt(77))/35989225472 + 153195*sqrt
(22)*sqrt(553 + 64*sqrt(77))*sqrt(-333*sqrt(553 + 64*sqrt(77)) + 21975 + 7
648*sqrt(77))/2249326592 + 8313499*sqrt(14)*sqrt(553 + 64*sqrt(77))*sqrt(-
333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77))/71978450944 + 2908324
44193/35989225472 + 2303470247*sqrt(77)/2249326592) + (-5/8 - sqrt(79/448
+ sqrt(77)/49))*log(x**2 + x*(-45*sqrt(14)*sqrt(553 + 64*sqrt(77))*sqrt(33
3*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77))/536576 - 1459*sqrt(14)*
sqrt(333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77))/536576 + 10391*s
qrt(553 + 64*sqrt(77))/268288 + 1459*sqrt(77)/8384 + 522933/268288 + 15*sq
rt(77)*sqrt(553 + 64*sqrt(77))/2096) - 510895297*sqrt(14)*sqrt(333*sqrt(55
3 + 64*sqrt(77)) + 21975 + 7648*sqrt(77))/71978450944 - 6009493*sqrt(22)*s
qrt(333*sqrt(553 + 64*sqrt(77)) + 21975 + 7648*sqrt(77))/1124663296 - 8...
```

Maxima [F]

$$\int \frac{x^2(5+x+3x^2+2x^3)}{2+x+5x^2+x^3+2x^4}\,dx = \int \frac{(2\,x^3+3\,x^2+x+5)x^2}{2\,x^4+x^3+5\,x^2+x+2}\,dx$$

input
```
integrate(x^2*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x, algorithm="maxima
")
```

output
```
1/2*x^2 + x - integrate((5*x^3 + x^2 + 3*x + 2)/(2*x^4 + x^3 + 5*x^2 + x +
 2), x)
```

Giac [F]

$$\int \frac{x^2(5+x+3x^2+2x^3)}{2+x+5x^2+x^3+2x^4}\,dx = \int \frac{(2\,x^3+3\,x^2+x+5)x^2}{2\,x^4+x^3+5\,x^2+x+2}\,dx$$

input
```
integrate(x^2*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x, algorithm="giac")
```

output
```
integrate((2*x^3 + 3*x^2 + x + 5)*x^2/(2*x^4 + x^3 + 5*x^2 + x + 2), x)
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.86

$$\int \frac{x^2(5+x+3x^2+2x^3)}{2+x+5x^2+x^3+2x^4}\,dx = x+\frac{x^2}{2}+\left(\sum_{k=1}^{4}\ln\left(-\frac{179\,\mathrm{root}\left(z^4+\frac{5\,z^3}{2}+2\,z^2+\frac{32\,z}{49}+\frac{128}{343},z,k\right)}{8}-7\,x-\frac{\mathrm{root}\left(z^4+\frac{5\,z^3}{2}+2\,z^2+\frac{32\,z}{49}+\frac{128}{343},z,k\right)\,x\,459}{8}-\frac{\mathrm{root}\left(z^4+\frac{5\,z^3}{2}+2\,z^2+\frac{32\,z}{49}+\frac{128}{343},z,k\right)^2\,x\,665}{8}-\frac{\mathrm{root}\left(z^4+\frac{5\,z^3}{2}+2\,z^2+\frac{32\,z}{49}+\frac{128}{343},z,k\right)^3\,x\,147}{4}-\frac{35\,\mathrm{root}\left(z^4+\frac{5\,z^3}{2}+2\,z^2+\frac{32\,z}{49}+\frac{128}{343},z,k\right)^2}{32}+\frac{49\,\mathrm{root}\left(z^4+\frac{5\,z^3}{2}+2\,z^2+\frac{32\,z}{49}+\frac{128}{343},z,k\right)^3}{16}-15\right)\,\mathrm{root}\left(z^4+\frac{5\,z^3}{2}+2\,z^2+\frac{32\,z}{49}+\frac{128}{343},z,k\right)\right)$$

input
```
int((x^2*(x + 3*x^2 + 2*x^3 + 5))/(x + 5*x^2 + x^3 + 2*x^4 + 2),x)
```

output
```
x + x^2/2 + symsum(log((49*root(z^4 + (5*z^3)/2 + 2*z^2 + (32*z)/49 + 128/
343, z, k)^3)/16 - 7*x - (459*root(z^4 + (5*z^3)/2 + 2*z^2 + (32*z)/49 + 1
28/343, z, k)*x)/8 - (665*root(z^4 + (5*z^3)/2 + 2*z^2 + (32*z)/49 + 128/3
43, z, k)^2*x)/8 - (147*root(z^4 + (5*z^3)/2 + 2*z^2 + (32*z)/49 + 128/343
, z, k)^3*x)/4 - (35*root(z^4 + (5*z^3)/2 + 2*z^2 + (32*z)/49 + 128/343, z
, k)^2)/32 - (179*root(z^4 + (5*z^3)/2 + 2*z^2 + (32*z)/49 + 128/343, z, k
))/8 - 15)*root(z^4 + (5*z^3)/2 + 2*z^2 + (32*z)/49 + 128/343, z, k), k, 1
, 4)
```

Reduce [F]

$$\int \frac{x^2(5+x+3x^2+2x^3)}{2+x+5x^2+x^3+2x^4}\,dx = \frac{7\left(\int \frac{x^2}{2x^4+x^3+5x^2+x+2}dx\right)}{8} + \frac{13\left(\int \frac{x}{2x^4+x^3+5x^2+x+2}dx\right)}{4} - \frac{11\left(\int \frac{1}{2x^4+x^3+5x^2+x+2}dx\right)}{8} - \frac{5\log(2x^4+x^3+5x^2+x+2)}{8} + \frac{x^2}{2} + x$$

input
```
int(x^2*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x)
```

output
```
(7*int(x**2/(2*x**4 + x**3 + 5*x**2 + x + 2),x) + 26*int(x/(2*x**4 + x**3
+ 5*x**2 + x + 2),x) - 11*int(1/(2*x**4 + x**3 + 5*x**2 + x + 2),x) - 5*lo
g(2*x**4 + x**3 + 5*x**2 + x + 2) + 4*x**2 + 8*x)/8
```

3.59 $\int \frac{x(5+x+3x^2+2x^3)}{2+x+5x^2+x^3+2x^4}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 207

$$\int \frac{x(5+x+3x^2+2x^3)}{2+x+5x^2+x^3+2x^4}\,dx = x + \frac{(19+7i\sqrt{7})\,\text{arctanh}\left(\frac{i-\sqrt{7}+8ix}{\sqrt{2(35-i\sqrt{7})}}\right)}{\sqrt{14\,(35-i\sqrt{7})}} - \frac{(19-7i\sqrt{7})\,\text{arctanh}\left(\frac{i+\sqrt{7}+8ix}{\sqrt{2(35+i\sqrt{7})}}\right)}{\sqrt{14\,(35+i\sqrt{7})}} + \frac{1}{28}\left(7-5i\sqrt{7}\right)\log\left(4i+\left(i-\sqrt{7}\right)x+4ix^2\right) + \frac{1}{28}\left(7+5i\sqrt{7}\right)\log\left(4i+\left(i+\sqrt{7}\right)x+4ix^2\right)$$

output
```
x+(19+7*I*7^(1/2))*arctanh((I-7^(1/2)+8*I*x)/(70-2*I*7^(1/2))^(1/2))/(490-
14*I*7^(1/2))^(1/2)-(19-7*I*7^(1/2))*arctanh((I+7^(1/2)+8*I*x)/(70+2*I*7^(
1/2))^(1/2))/(490+14*I*7^(1/2))^(1/2)+1/28*(7-5*I*7^(1/2))*ln(4*I+(I-7^(1/
2))*x+4*I*x^2)+1/28*(7+5*I*7^(1/2))*ln(4*I+(I+7^(1/2))*x+4*I*x^2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.45

$$\int \frac{x(5+x+3x^2+2x^3)}{2+x+5x^2+x^3+2x^4}\,dx = x+2\text{RootSum}\Big[2+\#1+5\#1^2+\#1^3+2\#1^4\&, \frac{-\log(x-\#1)+2\log(x-\#1)\#1-2\log(x-\#1)\#1^2+\log(x-\#1)\#1^3}{1+10\#1+3\#1^2+8\#1^3}\&\Big]$$

input
```
Integrate[(x*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 5*x^2 + x^3 + 2*x^4),x]
```

output
```
x + 2*RootSum[2 + #1 + 5*#1^2 + #1^3 + 2*#1^4 & , (-Log[x - #1] + 2*Log[x
- #1]*#1 - 2*Log[x - #1]*#1^2 + Log[x - #1]*#1^3)/(1 + 10*#1 + 3*#1^2 + 8*
#1^3) & ]
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.05, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.061$, Rules used = {2492, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(2x^3+3x^2+x+5)}{2x^4+x^3+5x^2+x+2}\,dx$$

$$\Big\downarrow \; 2492$$

$$\frac{1}{2}\int \left(-\frac{4(7i+5\sqrt{7})(1-x)}{7\left(4ix^2+(i-\sqrt{7})x+4i\right)}-\frac{4(7i-5\sqrt{7})(1-x)}{7\left(4ix^2+(i+\sqrt{7})x+4i\right)}+2\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{2}\left(\frac{(19+7i\sqrt{7})\operatorname{arctanh}\left(\frac{8ix-\sqrt{7}+i}{\sqrt{2(35-i\sqrt{7})}}\right)}{\sqrt{\frac{7}{2}(35-i\sqrt{7})}}-\frac{(19-7i\sqrt{7})\operatorname{arctanh}\left(\frac{8ix+\sqrt{7}+i}{\sqrt{2(35+i\sqrt{7})}}\right)}{\sqrt{\frac{7}{2}(35+i\sqrt{7})}}+\frac{1}{14}\left(7-5i\sqrt{7}\right)\log\left(4ix^2+\left(\right.\right.\right.$$

input
```
Int[(x*(5 + x + 3*x^2 + 2*x^3))/(2 + x + 5*x^2 + x^3 + 2*x^4),x]
```

output
```
(2*x + ((19 + (7*I)*Sqrt[7])*ArcTanh[(I - Sqrt[7] + (8*I)*x)/Sqrt[2*(35 -
I*Sqrt[7])]])/Sqrt[(7*(35 - I*Sqrt[7]))/2] - ((19 - (7*I)*Sqrt[7])*ArcTanh
[(I + Sqrt[7] + (8*I)*x)/Sqrt[2*(35 + I*Sqrt[7])]])/Sqrt[(7*(35 + I*Sqrt[7
]))/2] + ((7 - (5*I)*Sqrt[7])*Log[4*I + (I - Sqrt[7])*x + (4*I)*x^2])/14 +
 ((7 + (5*I)*Sqrt[7])*Log[4*I + (I + Sqrt[7])*x + (4*I)*x^2])/14)/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2492
```
Int[(Px_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^3 + (e_.)*(x_)^4)
^(p_), x_Symbol] :> Simp[e^p   Int[ExpandIntegrand[Px*(b/d + ((d + Sqrt[e*(
(b^2 - 4*a*c)/a) + 8*a*d*(e/b)])/(2*e))*x + x^2)^p*(b/d + ((d - Sqrt[e*((b^
2 - 4*a*c)/a) + 8*a*d*(e/b)])/(2*e))*x + x^2)^p, x], x], x] /; FreeQ[{a, b,
 c, d, e}, x] && PolyQ[Px, x] && ILtQ[p, 0] && EqQ[a*d^2 - b^2*e, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.04 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.30

method	result	size
default	$x+2\left(\sum_{\_R=\mathrm{RootOf}(2\_Z^4+\_Z^3+5\_Z^2+\_Z+2)}\frac{(\_R^3-2\_R^2+2\_R-1)\ln(x-\_R)}{8\_R^3+3\_R^2+10\_R+1}\right)$	62
risch	$x+2\left(\sum_{\_R=\mathrm{RootOf}(2\_Z^4+\_Z^3+5\_Z^2+\_Z+2)}\frac{(\_R^3-2\_R^2+2\_R-1)\ln(x-\_R)}{8\_R^3+3\_R^2+10\_R+1}\right)$	62

input
```
int(x*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x,method=_RETURNVERBOSE)
```

output
```
x+2*sum((_R^3-2*_R^2+2*_R-1)/(8*_R^3+3*_R^2+10*_R+1)*ln(x-_R),_R=RootOf(2*
_Z^4+_Z^3+5*_Z^2+_Z+2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 309 vs. 2(126) = 252.

Time = 0.10 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.49

$$\int\frac{x(5+x+3x^2+2x^3)}{2+x+5x^2+x^3+2x^4}\,dx=\frac{1}{4}\left(\sqrt{\frac{16}{7}\sqrt{\frac{11}{7}}-\frac{1}{7}}+1\right)\log\left(424\,x^2+7\left(7\sqrt{\frac{11}{7}}(4\,x-1)-38\,x-17\right)\sqrt{\frac{16}{7}\sqrt{\frac{11}{7}}-\frac{1}{7}}+106\,x+371\sqrt{\frac{11}{7}}+53\right)-\frac{1}{4}\left(\sqrt{\frac{16}{7}\sqrt{\frac{11}{7}}-\frac{1}{7}}-1\right)\log\left(424\,x^2-7\left(7\sqrt{\frac{11}{7}}(4\,x-1)-38\,x-17\right)\sqrt{\frac{16}{7}\sqrt{\frac{11}{7}}-\frac{1}{7}}+106\,x+371\sqrt{\frac{11}{7}}+53\right)-\sqrt{\frac{5}{106}\left(16\sqrt{\frac{11}{7}}+1\right)\sqrt{\frac{16}{7}\sqrt{\frac{11}{7}}-\frac{1}{7}}+\frac{4}{7}\sqrt{\frac{11}{7}}+\frac{13}{14}}\arctan\left(-\frac{7}{4028}\left(53\sqrt{\frac{11}{7}}(45\,x-4)-7\left(\sqrt{\frac{1}{7}}\right.\right.\right.+\sqrt{-\frac{5}{106}\left(16\sqrt{\frac{11}{7}}+1\right)\sqrt{\frac{16}{7}\sqrt{\frac{11}{7}}-\frac{1}{7}}+\frac{4}{7}\sqrt{\frac{11}{7}}+\frac{13}{14}}\arctan\left(\frac{7}{4028}\left(53\sqrt{\frac{11}{7}}(45\,x-4)+7\left(\sqrt{\frac{1}{7}}\right.\right.\right.+x$$

input
```
integrate(x*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x, algorithm="fricas")
```

output
```
1/4*(sqrt(16/7*sqrt(11/7) - 1/7) + 1)*log(424*x^2 + 7*(7*sqrt(11/7)*(4*x -
 1) - 38*x - 17)*sqrt(16/7*sqrt(11/7) - 1/7) + 106*x + 371*sqrt(11/7) + 53
) - 1/4*(sqrt(16/7*sqrt(11/7) - 1/7) - 1)*log(424*x^2 - 7*(7*sqrt(11/7)*(4
*x - 1) - 38*x - 17)*sqrt(16/7*sqrt(11/7) - 1/7) + 106*x + 371*sqrt(11/7)
+ 53) - sqrt(5/106*(16*sqrt(11/7) + 1)*sqrt(16/7*sqrt(11/7) - 1/7) + 4/7*s
qrt(11/7) + 13/14)*arctan(-7/4028*(53*sqrt(11/7)*(45*x - 4) - 7*(sqrt(11/7
)*(149*x - 20) + 271*x + 65)*sqrt(16/7*sqrt(11/7) - 1/7) + 2067*x + 689)*s
qrt(5/106*(16*sqrt(11/7) + 1)*sqrt(16/7*sqrt(11/7) - 1/7) + 4/7*sqrt(11/7)
 + 13/14)) + sqrt(-5/106*(16*sqrt(11/7) + 1)*sqrt(16/7*sqrt(11/7) - 1/7) +
 4/7*sqrt(11/7) + 13/14)*arctan(7/4028*(53*sqrt(11/7)*(45*x - 4) + 7*(sqrt
(11/7)*(149*x - 20) + 271*x + 65)*sqrt(16/7*sqrt(11/7) - 1/7) + 2067*x + 6
89)*sqrt(-5/106*(16*sqrt(11/7) + 1)*sqrt(16/7*sqrt(11/7) - 1/7) + 4/7*sqrt
(11/7) + 13/14)) + x
```

Sympy [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.23

$$\int \frac{x(5+x+3x^2+2x^3)}{2+x+5x^2+x^3+2x^4}\,dx = x + \text{RootSum}\left(343t^4-343t^3+294t^2-336t+128, \left(t\mapsto t\log\left(\frac{3773t^3}{304}-\frac{1029t^2}{304}+\frac{1001t}{152}+x-\frac{121}{19}\right)\right)\right)$$

input
```
integrate(x*(2*x**3+3*x**2+x+5)/(2*x**4+x**3+5*x**2+x+2),x)
```

output
```
x + RootSum(343*_t**4 - 343*_t**3 + 294*_t**2 - 336*_t + 128, Lambda(_t, _
t*log(3773*_t**3/304 - 1029*_t**2/304 + 1001*_t/152 + x - 121/19)))
```

Maxima [F]

$$\int \frac{x(5+x+3x^2+2x^3)}{2+x+5x^2+x^3+2x^4}\,dx = \int \frac{(2\,x^3+3\,x^2+x+5)x}{2\,x^4+x^3+5\,x^2+x+2}\,dx$$

input
```
integrate(x*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x, algorithm="maxima")
```

output
```
x + 2*integrate((x^3 - 2*x^2 + 2*x - 1)/(2*x^4 + x^3 + 5*x^2 + x + 2), x)
```

Giac [F]

$$\int \frac{x(5+x+3x^2+2x^3)}{2+x+5x^2+x^3+2x^4}\,dx = \int \frac{(2\,x^3+3\,x^2+x+5)x}{2\,x^4+x^3+5\,x^2+x+2}\,dx$$

input
```
integrate(x*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x, algorithm="giac")
```

output
```
integrate((2*x^3 + 3*x^2 + x + 5)*x/(2*x^4 + x^3 + 5*x^2 + x + 2), x)
```

Mupad [B] (verification not implemented)

Time = 22.38 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.88

$$\int \frac{x(5+x+3x^2+2x^3)}{2+x+5x^2+x^3+2x^4}\,dx = x+\left(\sum_{k=1}^{4}\ln\left(\frac{115\,\mathrm{root}\left(z^4-z^3+\frac{6\,z^2}{7}-\frac{48\,z}{49}+\frac{128}{343},z,k\right)}{8} + 15\,x - \frac{\mathrm{root}\left(z^4-z^3+\frac{6\,z^2}{7}-\frac{48\,z}{49}+\frac{128}{343},z,k\right)\,x\,137}{8} + \frac{\mathrm{root}\left(z^4-z^3+\frac{6\,z^2}{7}-\frac{48\,z}{49}+\frac{128}{343},z,k\right)^2\,x\,133}{8} - \frac{\mathrm{root}\left(z^4-z^3+\frac{6\,z^2}{7}-\frac{48\,z}{49}+\frac{128}{343},z,k\right)^3\,x\,147}{4} - \frac{189\,\mathrm{root}\left(z^4-z^3+\frac{6\,z^2}{7}-\frac{48\,z}{49}+\frac{128}{343},z,k\right)^2}{16} + \frac{49\,\mathrm{root}\left(z^4-z^3+\frac{6\,z^2}{7}-\frac{48\,z}{49}+\frac{128}{343},z,k\right)^3}{16} - 4\right)\mathrm{root}\left(z^4-z^3+\frac{6\,z^2}{7}-\frac{48\,z}{49}+\frac{128}{343},z,k\right)\right)$$

input
```
int((x*(x + 3*x^2 + 2*x^3 + 5))/(x + 5*x^2 + x^3 + 2*x^4 + 2),x)
```

output
```
x + symsum(log((115*root(z^4 - z^3 + (6*z^2)/7 - (48*z)/49 + 128/343, z, k
))/8 + 15*x - (137*root(z^4 - z^3 + (6*z^2)/7 - (48*z)/49 + 128/343, z, k)
*x)/8 + (133*root(z^4 - z^3 + (6*z^2)/7 - (48*z)/49 + 128/343, z, k)^2*x)/
8 - (147*root(z^4 - z^3 + (6*z^2)/7 - (48*z)/49 + 128/343, z, k)^3*x)/4 -
(189*root(z^4 - z^3 + (6*z^2)/7 - (48*z)/49 + 128/343, z, k)^2)/16 + (49*r
oot(z^4 - z^3 + (6*z^2)/7 - (48*z)/49 + 128/343, z, k)^3)/16 - 4)*root(z^4
 - z^3 + (6*z^2)/7 - (48*z)/49 + 128/343, z, k), k, 1, 4)
```

Reduce [F]

$$\int \frac{x(5+x+3x^2+2x^3)}{2+x+5x^2+x^3+2x^4}\,dx = -\frac{19\left(\int \frac{x^2}{2x^4+x^3+5x^2+x+2}dx\right)}{4} + \frac{3\left(\int \frac{x}{2x^4+x^3+5x^2+x+2}dx\right)}{2} - \frac{9\left(\int \frac{1}{2x^4+x^3+5x^2+x+2}dx\right)}{4} + \frac{\log(2x^4+x^3+5x^2+x+2)}{4} + x$$

input
```
int(x*(2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x)
```

output
```
( - 19*int(x**2/(2*x**4 + x**3 + 5*x**2 + x + 2),x) + 6*int(x/(2*x**4 + x*
*3 + 5*x**2 + x + 2),x) - 9*int(1/(2*x**4 + x**3 + 5*x**2 + x + 2),x) + lo
g(2*x**4 + x**3 + 5*x**2 + x + 2) + 4*x)/4
```

3.60 $\int \frac{5+x+3x^2+2x^3}{2+x+5x^2+x^3+2x^4}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 206

$$\int \frac{5+x+3x^2+2x^3}{2+x+5x^2+x^3+2x^4}\,dx = -\frac{\left(19+7i\sqrt{7}\right)\operatorname{arctanh}\left(\frac{i-\sqrt{7}+8ix}{\sqrt{2\left(35-i\sqrt{7}\right)}}\right)}{\sqrt{14\left(35-i\sqrt{7}\right)}} + \frac{\left(19-7i\sqrt{7}\right)\operatorname{arctanh}\left(\frac{i+\sqrt{7}+8ix}{\sqrt{2\left(35+i\sqrt{7}\right)}}\right)}{\sqrt{14\left(35+i\sqrt{7}\right)}} + \frac{1}{28}\left(7-5i\sqrt{7}\right)\log\left(4i+\left(i-\sqrt{7}\right)x+4ix^2\right) + \frac{1}{28}\left(7+5i\sqrt{7}\right)\log\left(4i+\left(i+\sqrt{7}\right)x+4ix^2\right)$$

output

```
-(19+7*I*7^(1/2))*arctanh((I-7^(1/2)+8*I*x)/(70-2*I*7^(1/2))^(1/2))/(490-1
4*I*7^(1/2))^(1/2)+(19-7*I*7^(1/2))*arctanh((I+7^(1/2)+8*I*x)/(70+2*I*7^(1
/2))^(1/2))/(490+14*I*7^(1/2))^(1/2)+1/28*(7-5*I*7^(1/2))*ln(4*I+(I-7^(1/2
))*x+4*I*x^2)+1/28*(7+5*I*7^(1/2))*ln(4*I+(I+7^(1/2))*x+4*I*x^2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.44

$$\int \frac{5+x+3x^2+2x^3}{2+x+5x^2+x^3+2x^4}\,dx = \text{RootSum}\Big[2+\#1+5\#1^2+\#1^3 +2\#1^4\&, \frac{5\log(x-\#1)+\log(x-\#1)\#1+3\log(x-\#1)\#1^2+2\log(x-\#1)\#1^3}{1+10\#1+3\#1^2+8\#1^3}\&\Big]$$

input
```
Integrate[(5 + x + 3*x^2 + 2*x^3)/(2 + x + 5*x^2 + x^3 + 2*x^4),x]
```

output
```
RootSum[2 + #1 + 5*#1^2 + #1^3 + 2*#1^4 & , (5*Log[x - #1] + Log[x - #1]*#
1 + 3*Log[x - #1]*#1^2 + 2*Log[x - #1]*#1^3)/(1 + 10*#1 + 3*#1^2 + 8*#1^3)
 & ]
```

Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.04, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {2492, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2x^3+3x^2+x+5}{2x^4+x^3+5x^2+x+2}\,dx$$

$$\Big\downarrow \text{2492}$$

$$\frac{1}{2}\int \left(\frac{2i\left(2\left(5i+\sqrt{7}\right)x+5\sqrt{7}+9i\right)}{\sqrt{7}\left(4ix^2+\left(i+\sqrt{7}\right)x+4i\right)}-\frac{2i\left(2\left(5i-\sqrt{7}\right)x-5\sqrt{7}+9i\right)}{\sqrt{7}\left(4ix^2+\left(i-\sqrt{7}\right)x+4i\right)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{(19+7i\sqrt{7})\,\mathrm{arctanh}\left(\frac{8ix-\sqrt{7}+i}{\sqrt{2\left(35-i\sqrt{7}\right)}}\right)}{\sqrt{\frac{7}{2}\,(35-i\sqrt{7})}}+\frac{(19-7i\sqrt{7})\,\mathrm{arctanh}\left(\frac{8ix+\sqrt{7}+i}{\sqrt{2\left(35+i\sqrt{7}\right)}}\right)}{\sqrt{\frac{7}{2}\,(35+i\sqrt{7})}}+\frac{1}{14}\left(7-5i\sqrt{7}\right)\log\left(4ix^2+\right.\right.$$

input
```
Int[(5 + x + 3*x^2 + 2*x^3)/(2 + x + 5*x^2 + x^3 + 2*x^4),x]
```

output
```
(-(((19 + (7*I)*Sqrt[7])*ArcTanh[(I - Sqrt[7] + (8*I)*x)/Sqrt[2*(35 - I*Sq
rt[7])]])/Sqrt[(7*(35 - I*Sqrt[7]))/2]) + ((19 - (7*I)*Sqrt[7])*ArcTanh[(I
 + Sqrt[7] + (8*I)*x)/Sqrt[2*(35 + I*Sqrt[7])]])/Sqrt[(7*(35 + I*Sqrt[7]))
/2] + ((7 - (5*I)*Sqrt[7])*Log[4*I + (I - Sqrt[7])*x + (4*I)*x^2])/14 + ((
7 + (5*I)*Sqrt[7])*Log[4*I + (I + Sqrt[7])*x + (4*I)*x^2])/14)/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2492
```
Int[(Px_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^3 + (e_.)*(x_)^4)
^(p_), x_Symbol] :> Simp[e^p   Int[ExpandIntegrand[Px*(b/d + ((d + Sqrt[e*(
(b^2 - 4*a*c)/a) + 8*a*d*(e/b)])/(2*e))*x + x^2)^p*(b/d + ((d - Sqrt[e*((b^
2 - 4*a*c)/a) + 8*a*d*(e/b)])/(2*e))*x + x^2)^p, x], x], x] /; FreeQ[{a, b,
 c, d, e}, x] && PolyQ[Px, x] && ILtQ[p, 0] && EqQ[a*d^2 - b^2*e, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.04 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.28

method	result	size
default	$\sum_{\_R=\mathrm{RootOf}(2\_Z^4+\_Z^3+5\_Z^2+\_Z+2)} \frac{(2\_R^3+3\_R^2+\_R+5)\ln(x-\_R)}{8\_R^3+3\_R^2+10\_R+1}$	58
risch	$\sum_{\_R=\mathrm{RootOf}(2\_Z^4+\_Z^3+5\_Z^2+\_Z+2)} \frac{(2\_R^3+3\_R^2+\_R+5)\ln(x-\_R)}{8\_R^3+3\_R^2+10\_R+1}$	58

input
```
int((2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x,method=_RETURNVERBOSE)
```

output
```
sum((2*_R^3+3*_R^2+_R+5)/(8*_R^3+3*_R^2+10*_R+1)*ln(x-_R),_R=RootOf(2*_Z^4
+_Z^3+5*_Z^2+_Z+2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 310 vs. $2(124) = 248$.

Time = 0.09 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.50

$$\int \frac{5+x+3x^2+2x^3}{2+x+5x^2+x^3+2x^4}\,dx = -\frac{1}{4}\left(\sqrt{\frac{16}{7}\sqrt{\frac{11}{7}}-\frac{1}{7}}-1\right)\log\left(424\,x^2 + 7\left(7\sqrt{\frac{11}{7}}(4\,x-1)-38\,x-17\right)\sqrt{\frac{16}{7}\sqrt{\frac{11}{7}}-\frac{1}{7}}+106\,x+371\sqrt{\frac{11}{7}}+53\right) + \frac{1}{4}\left(\sqrt{\frac{16}{7}\sqrt{\frac{11}{7}}-\frac{1}{7}}+1\right)\log\left(424\,x^2 - 7\left(7\sqrt{\frac{11}{7}}(4\,x-1)-38\,x-17\right)\sqrt{\frac{16}{7}\sqrt{\frac{11}{7}}-\frac{1}{7}}+106\,x+371\sqrt{\frac{11}{7}}+53\right) + \sqrt{\frac{5}{106}\left(16\sqrt{\frac{11}{7}}+1\right)\sqrt{\frac{16}{7}\sqrt{\frac{11}{7}}-\frac{1}{7}}+\frac{4}{7}\sqrt{\frac{11}{7}}+\frac{13}{14}}\arctan\left(-\frac{7}{4028}\left(106\sqrt{\frac{11}{7}}(5\,x-2)-7\left(2\sqrt{\right.\right.\right. - \sqrt{-\frac{5}{106}\left(16\sqrt{\frac{11}{7}}+1\right)\sqrt{\frac{16}{7}\sqrt{\frac{11}{7}}-\frac{1}{7}}+\frac{4}{7}\sqrt{\frac{11}{7}}+\frac{13}{14}}\arctan\left(\frac{7}{4028}\left(106\sqrt{\frac{11}{7}}(5\,x-2)+7\left(2\sqrt{\right.\right.\right.$$

input
```
integrate((2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x, algorithm="fricas")
```

output
```
-1/4*(sqrt(16/7*sqrt(11/7) - 1/7) - 1)*log(424*x^2 + 7*(7*sqrt(11/7)*(4*x
- 1) - 38*x - 17)*sqrt(16/7*sqrt(11/7) - 1/7) + 106*x + 371*sqrt(11/7) + 5
3) + 1/4*(sqrt(16/7*sqrt(11/7) - 1/7) + 1)*log(424*x^2 - 7*(7*sqrt(11/7)*(
4*x - 1) - 38*x - 17)*sqrt(16/7*sqrt(11/7) - 1/7) + 106*x + 371*sqrt(11/7)
 + 53) + sqrt(5/106*(16*sqrt(11/7) + 1)*sqrt(16/7*sqrt(11/7) - 1/7) + 4/7*
sqrt(11/7) + 13/14)*arctan(-7/4028*(106*sqrt(11/7)*(5*x - 2) - 7*(2*sqrt(1
1/7)*(63*x - 10) + 94*x + 65)*sqrt(16/7*sqrt(11/7) - 1/7) + 1802*x + 689)*
sqrt(5/106*(16*sqrt(11/7) + 1)*sqrt(16/7*sqrt(11/7) - 1/7) + 4/7*sqrt(11/7
) + 13/14)) - sqrt(-5/106*(16*sqrt(11/7) + 1)*sqrt(16/7*sqrt(11/7) - 1/7)
+ 4/7*sqrt(11/7) + 13/14)*arctan(7/4028*(106*sqrt(11/7)*(5*x - 2) + 7*(2*s
qrt(11/7)*(63*x - 10) + 94*x + 65)*sqrt(16/7*sqrt(11/7) - 1/7) + 1802*x +
689)*sqrt(-5/106*(16*sqrt(11/7) + 1)*sqrt(16/7*sqrt(11/7) - 1/7) + 4/7*sqr
t(11/7) + 13/14))
```

Sympy [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.22

$$\int \frac{5+x+3x^2+2x^3}{2+x+5x^2+x^3+2x^4}\,dx = \operatorname{RootSum}\left(343t^4-343t^3+294t^2-336t+128,\left(t\mapsto t\log\left(-\frac{7203t^3}{304}+\frac{2303t^2}{304}-\frac{2177t}{152}+x+\frac{250}{19}\right)\right)\right)$$

input
```
integrate((2*x**3+3*x**2+x+5)/(2*x**4+x**3+5*x**2+x+2),x)
```

output
```
RootSum(343*_t**4 - 343*_t**3 + 294*_t**2 - 336*_t + 128, Lambda(_t, _t*lo
g(-7203*_t**3/304 + 2303*_t**2/304 - 2177*_t/152 + x + 250/19)))
```

Maxima [F]

$$\int \frac{5+x+3x^2+2x^3}{2+x+5x^2+x^3+2x^4}\,dx=\int \frac{2\,x^3+3\,x^2+x+5}{2\,x^4+x^3+5\,x^2+x+2}\,dx$$

input
```
integrate((2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x, algorithm="maxima")
```

output
```
integrate((2*x^3 + 3*x^2 + x + 5)/(2*x^4 + x^3 + 5*x^2 + x + 2), x)
```

Giac [F]

$$\int \frac{5+x+3x^2+2x^3}{2+x+5x^2+x^3+2x^4}\,dx=\int \frac{2\,x^3+3\,x^2+x+5}{2\,x^4+x^3+5\,x^2+x+2}\,dx$$

input
```
integrate((2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x, algorithm="giac")
```

output
```
integrate((2*x^3 + 3*x^2 + x + 5)/(2*x^4 + x^3 + 5*x^2 + x + 2), x)
```

Mupad [B] (verification not implemented)

Time = 22.44 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.88

$$\int \frac{5+x+3x^2+2x^3}{2+x+5x^2+x^3+2x^4}\,dx = \sum_{k=1}^{4} \ln\left(-\frac{193\,\mathrm{root}\left(z^4-z^3+\frac{6\,z^2}{7}-\frac{48\,z}{49}+\frac{128}{343},z,k\right)}{8} + 4\,x - \frac{\mathrm{root}\left(z^4-z^3+\frac{6\,z^2}{7}-\frac{48\,z}{49}+\frac{128}{343},z,k\right)\,x\,137}{8} + \frac{\mathrm{root}\left(z^4-z^3+\frac{6\,z^2}{7}-\frac{48\,z}{49}+\frac{128}{343},z,k\right)^2\,x\,651}{16} - \frac{\mathrm{root}\left(z^4-z^3+\frac{6\,z^2}{7}-\frac{48\,z}{49}+\frac{128}{343},z,k\right)^3\,x\,147}{4} + \frac{273\,\mathrm{root}\left(z^4-z^3+\frac{6\,z^2}{7}-\frac{48\,z}{49}+\frac{128}{343},z,k\right)^2}{16} + \frac{49\,\mathrm{root}\left(z^4-z^3+\frac{6\,z^2}{7}-\frac{48\,z}{49}+\frac{128}{343},z,k\right)^3}{16} + 7\right)\,\mathrm{root}\left(z^4-z^3+\frac{6\,z^2}{7}-\frac{48\,z}{49}+\frac{128}{343},z,k\right)$$

input
```
int((x + 3*x^2 + 2*x^3 + 5)/(x + 5*x^2 + x^3 + 2*x^4 + 2),x)
```

output
```
symsum(log(4*x - (193*root(z^4 - z^3 + (6*z^2)/7 - (48*z)/49 + 128/343, z,
 k))/8 - (137*root(z^4 - z^3 + (6*z^2)/7 - (48*z)/49 + 128/343, z, k)*x)/8
 + (651*root(z^4 - z^3 + (6*z^2)/7 - (48*z)/49 + 128/343, z, k)^2*x)/16 -
(147*root(z^4 - z^3 + (6*z^2)/7 - (48*z)/49 + 128/343, z, k)^3*x)/4 + (273
*root(z^4 - z^3 + (6*z^2)/7 - (48*z)/49 + 128/343, z, k)^2)/16 + (49*root(
z^4 - z^3 + (6*z^2)/7 - (48*z)/49 + 128/343, z, k)^3)/16 + 7)*root(z^4 - z
^3 + (6*z^2)/7 - (48*z)/49 + 128/343, z, k), k, 1, 4)
```

Reduce [F]

$$\int \frac{5+x+3x^2+2x^3}{2+x+5x^2+x^3+2x^4}\,dx = \frac{9\left(\int \frac{x^2}{2x^4+x^3+5x^2+x+2}dx\right)}{4} - \frac{3\left(\int \frac{x}{2x^4+x^3+5x^2+x+2}dx\right)}{2} + \frac{19\left(\int \frac{1}{2x^4+x^3+5x^2+x+2}dx\right)}{4} + \frac{\log(2x^4+x^3+5x^2+x+2)}{4}$$

input
```
int((2*x^3+3*x^2+x+5)/(2*x^4+x^3+5*x^2+x+2),x)
```

output
```
(9*int(x**2/(2*x**4 + x**3 + 5*x**2 + x + 2),x) - 6*int(x/(2*x**4 + x**3 +
 5*x**2 + x + 2),x) + 19*int(1/(2*x**4 + x**3 + 5*x**2 + x + 2),x) + log(2
*x**4 + x**3 + 5*x**2 + x + 2))/4
```

3.61 $\int \frac{5+x+3x^2+2x^3}{x(2+x+5x^2+x^3+2x^4)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 217

$$\int \frac{5+x+3x^2+2x^3}{x\left(2+x+5x^2+x^3+2x^4\right)}\,dx = -\frac{\left(53+i\sqrt{7}\right)\operatorname{arctanh}\left(\frac{i-\sqrt{7}+8ix}{\sqrt{2\left(35-i\sqrt{7}\right)}}\right)}{2\sqrt{14\left(35-i\sqrt{7}\right)}} + \frac{\left(53-i\sqrt{7}\right)\operatorname{arctanh}\left(\frac{i+\sqrt{7}+8ix}{\sqrt{2\left(35+i\sqrt{7}\right)}}\right)}{2\sqrt{14\left(35+i\sqrt{7}\right)}} + \frac{5\log(x)}{2} - \frac{1}{56}\left(35-9i\sqrt{7}\right)\log\left(4i+\left(i-\sqrt{7}\right)x+4ix^2\right) - \frac{1}{56}\left(35+9i\sqrt{7}\right)\log\left(4i+\left(i+\sqrt{7}\right)x+4ix^2\right)$$

output
```
-1/2*(53+I*7^(1/2))*arctanh((I-7^(1/2)+8*I*x)/(70-2*I*7^(1/2))^(1/2))/(490
-14*I*7^(1/2))^(1/2)+1/2*(53-I*7^(1/2))*arctanh((I+7^(1/2)+8*I*x)/(70+2*I*
7^(1/2))^(1/2))/(490+14*I*7^(1/2))^(1/2)+5/2*ln(x)-1/56*(35-9*I*7^(1/2))*l
n(4*I+(I-7^(1/2))*x+4*I*x^2)-1/56*(35+9*I*7^(1/2))*ln(4*I+(I+7^(1/2))*x+4*
I*x^2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.47

$$\int \frac{5+x+3x^2+2x^3}{x(2+x+5x^2+x^3+2x^4)}\,dx = \frac{5\log(x)}{2}-\frac{1}{2}\text{RootSum}\Big[2+\#1+5\#1^2+\#1^3+2\#1^4\&,\frac{3\log(x-\#1)+19\log(x-\#1)\#1+\log(x-\#1)\#1^2+10\log(x-\#1)\#1^3}{1+10\#1+3\#1^2+8\#1^3}\&\Big]$$

input
```
Integrate[(5 + x + 3*x^2 + 2*x^3)/(x*(2 + x + 5*x^2 + x^3 + 2*x^4)),x]
```

output
```
(5*Log[x])/2 - RootSum[2 + #1 + 5*#1^2 + #1^3 + 2*#1^4 & , (3*Log[x - #1]
+ 19*Log[x - #1]*#1 + Log[x - #1]*#1^2 + 10*Log[x - #1]*#1^3)/(1 + 10*#1 +
 3*#1^2 + 8*#1^3) & ]/2
```

Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = {2496, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2x^3+3x^2+x+5}{x(2x^4+x^3+5x^2+x+2)}\,dx$$

$$\Big\downarrow \text{2496}$$

$$\frac{1}{2}\int \left(-\frac{2(35i-9\sqrt{7})x+3(7i+11\sqrt{7})}{7(4ix^2+(i+\sqrt{7})x+4i)}+\frac{5}{x}-\frac{2(35i+9\sqrt{7})x+3(7i-11\sqrt{7})}{7(4ix^2+(i-\sqrt{7})x+4i)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{\left(53+i\sqrt{7}\right)\operatorname{arctanh}\left(\frac{8ix-\sqrt{7}+i}{\sqrt{2\left(35-i\sqrt{7}\right)}}\right)}{\sqrt{14\left(35-i\sqrt{7}\right)}}+\frac{\left(53-i\sqrt{7}\right)\operatorname{arctanh}\left(\frac{8ix+\sqrt{7}+i}{\sqrt{2\left(35+i\sqrt{7}\right)}}\right)}{\sqrt{14\left(35+i\sqrt{7}\right)}}-\frac{1}{28}\left(35-9i\sqrt{7}\right)\log\left(4ix^2+\right.\right.$$

input
```
Int[(5 + x + 3*x^2 + 2*x^3)/(x*(2 + x + 5*x^2 + x^3 + 2*x^4)),x]
```

output
```
(-(((53 + I*Sqrt[7])*ArcTanh[(I - Sqrt[7] + (8*I)*x)/Sqrt[2*(35 - I*Sqrt[7
])]])/Sqrt[14*(35 - I*Sqrt[7])]) + ((53 - I*Sqrt[7])*ArcTanh[(I + Sqrt[7]
+ (8*I)*x)/Sqrt[2*(35 + I*Sqrt[7])]])/Sqrt[14*(35 + I*Sqrt[7])] + 5*Log[x]
 - ((35 - (9*I)*Sqrt[7])*Log[4*I + (I - Sqrt[7])*x + (4*I)*x^2])/28 - ((35
 + (9*I)*Sqrt[7])*Log[4*I + (I + Sqrt[7])*x + (4*I)*x^2])/28)/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2496
```
Int[(Px_.)*(x_)^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^3 + (e
_.)*(x_)^4)^(p_), x_Symbol] :> Simp[e^p   Int[ExpandIntegrand[x^m*Px*(b/d +
 ((d + Sqrt[e*((b^2 - 4*a*c)/a) + 8*a*d*(e/b)])/(2*e))*x + x^2)^p*(b/d + ((
d - Sqrt[e*((b^2 - 4*a*c)/a) + 8*a*d*(e/b)])/(2*e))*x + x^2)^p, x], x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Px, x] && ILtQ[p, 0] && EqQ[a*d^2
- b^2*e, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.24

method	result
risch	$\frac{5\ln(x)}{2}+\left(\sum_{\_R=\mathrm{RootOf}(686\_Z^4+1715\_Z^3+1372\_Z^2+448\_Z+256)} \_R\ln\left(2058\_R^3+20825\_R^2+25844\_R\right.\right.$
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(2\_Z^4+\_Z^3+5\_Z^2+\_Z+2\right)}\frac{\left(-10\_R^3-\_R^2-19\_R-3\right)\ln\left(x-\_R\right)}{8\_R^3+3\_R^2+10\_R+1}\right)}{2}+\frac{5\ln(x)}{2}$

input
```
int((2*x^3+3*x^2+x+5)/x/(2*x^4+x^3+5*x^2+x+2),x,method=_RETURNVERBOSE)
```

output
```
5/2*ln(x)+sum(_R*ln(2058*_R^3+20825*_R^2+25844*_R+8384*x+6816),_R=RootOf(6
86*_Z^4+1715*_Z^3+1372*_Z^2+448*_Z+256))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 315 vs. $2(130) = 260$.

Time = 0.11 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.45

$$\int \frac{5+x+3x^2+2x^3}{x(2+x+5x^2+x^3+2x^4)}\,dx = -\frac{1}{8}\left(\sqrt{\frac{64}{7}\sqrt{\frac{11}{7}}+\frac{79}{7}}+5\right)\log\left(296\,x^2 + 7\left(7\sqrt{\frac{11}{7}}(12\,x+1)-106\,x-15\right)\sqrt{\frac{64}{7}\sqrt{\frac{11}{7}}+\frac{79}{7}}+74\,x+259\sqrt{\frac{11}{7}}+37\right) + \frac{1}{8}\left(\sqrt{\frac{64}{7}\sqrt{\frac{11}{7}}+\frac{79}{7}}-5\right)\log\left(296\,x^2 - 7\left(7\sqrt{\frac{11}{7}}(12\,x+1)-106\,x-15\right)\sqrt{\frac{64}{7}\sqrt{\frac{11}{7}}+\frac{79}{7}}+74\,x+259\sqrt{\frac{11}{7}}+37\right) - \frac{1}{2}\sqrt{\frac{9}{74}\left(64\sqrt{\frac{11}{7}}-79\right)\sqrt{\frac{64}{7}\sqrt{\frac{11}{7}}+\frac{79}{7}}+\frac{16}{7}\sqrt{\frac{11}{7}}+\frac{1}{14}}\arctan\left(\frac{7}{77552}\left(148\sqrt{\frac{11}{7}}(23\,x-26)+7\right.\right. + \frac{1}{2}\sqrt{-\frac{9}{74}\left(64\sqrt{\frac{11}{7}}-79\right)\sqrt{\frac{64}{7}\sqrt{\frac{11}{7}}+\frac{79}{7}}+\frac{16}{7}\sqrt{\frac{11}{7}}+\frac{1}{14}}\arctan\left(-\frac{7}{77552}\left(148\sqrt{\frac{11}{7}}(23\,x-26)\right.\right. + \frac{5}{2}\log(x)$$

input
```
integrate((2*x^3+3*x^2+x+5)/x/(2*x^4+x^3+5*x^2+x+2),x, algorithm="fricas")
```

output
```
-1/8*(sqrt(64/7*sqrt(11/7) + 79/7) + 5)*log(296*x^2 + 7*(7*sqrt(11/7)*(12*
x + 1) - 106*x - 15)*sqrt(64/7*sqrt(11/7) + 79/7) + 74*x + 259*sqrt(11/7)
+ 37) + 1/8*(sqrt(64/7*sqrt(11/7) + 79/7) - 5)*log(296*x^2 - 7*(7*sqrt(11/
7)*(12*x + 1) - 106*x - 15)*sqrt(64/7*sqrt(11/7) + 79/7) + 74*x + 259*sqrt
(11/7) + 37) - 1/2*sqrt(9/74*(64*sqrt(11/7) - 79)*sqrt(64/7*sqrt(11/7) + 7
9/7) + 16/7*sqrt(11/7) + 1/14)*arctan(7/77552*(148*sqrt(11/7)*(23*x - 26)
+ 7*(4*sqrt(11/7)*(317*x + 234) - 1808*x - 1197)*sqrt(64/7*sqrt(11/7) + 79
/7) - 1184*x + 4921)*sqrt(9/74*(64*sqrt(11/7) - 79)*sqrt(64/7*sqrt(11/7) +
 79/7) + 16/7*sqrt(11/7) + 1/14)) + 1/2*sqrt(-9/74*(64*sqrt(11/7) - 79)*sq
rt(64/7*sqrt(11/7) + 79/7) + 16/7*sqrt(11/7) + 1/14)*arctan(-7/77552*(148*
sqrt(11/7)*(23*x - 26) - 7*(4*sqrt(11/7)*(317*x + 234) - 1808*x - 1197)*sq
rt(64/7*sqrt(11/7) + 79/7) - 1184*x + 4921)*sqrt(-9/74*(64*sqrt(11/7) - 79
)*sqrt(64/7*sqrt(11/7) + 79/7) + 16/7*sqrt(11/7) + 1/14)) + 5/2*log(x)
```

Sympy [A] (verification not implemented)

Time = 9.09 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.28

$$\int \frac{5+x+3x^2+2x^3}{x(2+x+5x^2+x^3+2x^4)}\,dx = \frac{5\log{(x)}}{2} + \operatorname{RootSum}{\left(686t^4+1715t^3+1372t^2+448t+256, \left(t \mapsto t\log{\left(-\frac{160344611t^4}{532759184} - \frac{16880402t^3}{33297449} + \frac{40105\ldots}{2131\ldots}\right.}\right.}$$

input
```
integrate((2*x**3+3*x**2+x+5)/x/(2*x**4+x**3+5*x**2+x+2),x)
```

output
```
5*log(x)/2 + RootSum(686*_t**4 + 1715*_t**3 + 1372*_t**2 + 448*_t + 256, L
ambda(_t, _t*log(-160344611*_t**4/532759184 - 16880402*_t**3/33297449 + 40
10520787*_t**2/2131036736 + 1537535671*_t/532759184 + x + 46660495/6659489
8)))
```

Maxima [F]

$$\int \frac{5+x+3x^2+2x^3}{x(2+x+5x^2+x^3+2x^4)}\,dx = \int \frac{2\,x^3+3\,x^2+x+5}{(2\,x^4+x^3+5\,x^2+x+2)x}\,dx$$

input
```
integrate((2*x^3+3*x^2+x+5)/x/(2*x^4+x^3+5*x^2+x+2),x, algorithm="maxima")
```

output
```
-1/2*integrate((10*x^3 + x^2 + 19*x + 3)/(2*x^4 + x^3 + 5*x^2 + x + 2), x)
 + 5/2*log(x)
```

Giac [F]

$$\int \frac{5+x+3x^2+2x^3}{x(2+x+5x^2+x^3+2x^4)}\,dx = \int \frac{2\,x^3+3\,x^2+x+5}{(2\,x^4+x^3+5\,x^2+x+2)x}\,dx$$

input
```
integrate((2*x^3+3*x^2+x+5)/x/(2*x^4+x^3+5*x^2+x+2),x, algorithm="giac")
```

output
```
integrate((2*x^3 + 3*x^2 + x + 5)/((2*x^4 + x^3 + 5*x^2 + x + 2)*x), x)
```

Mupad [B] (verification not implemented)

Time = 22.36 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.09

$$\int \frac{5+x+3x^2+2x^3}{x(2+x+5x^2+x^3+2x^4)}\,dx$$
$$= \frac{5\ln(x)}{2} + \left(\sum_{k=1}^{4} \ln\left(\frac{223\,\mathrm{root}\left(z^4+\frac{5z^3}{2}+2z^2+\frac{32z}{49}+\frac{128}{343},z,k\right)}{8} - \frac{31x}{2} + \frac{\mathrm{root}\left(z^4+\frac{5z^3}{2}+2z^2+\frac{32z}{49}+\frac{128}{343},z,k\right)x\,71}{16} - \frac{\mathrm{root}\left(z^4+\frac{5z^3}{2}+2z^2+\frac{32z}{49}+\frac{128}{343},z,k\right)^2 x\,4463}{64} + \frac{\mathrm{root}\left(z^4+\frac{5z^3}{2}+2z^2+\frac{32z}{49}+\frac{128}{343},z,k\right)^3 x\,1449}{16} + \frac{\mathrm{root}\left(z^4+\frac{5z^3}{2}+2z^2+\frac{32z}{49}+\frac{128}{343},z,k\right)^4 x\,3675}{32} + \frac{257\,\mathrm{root}\left(z^4+\frac{5z^3}{2}+2z^2+\frac{32z}{49}+\frac{128}{343},z,k\right)^2}{32} + \frac{1673\,\mathrm{root}\left(z^4+\frac{5z^3}{2}+2z^2+\frac{32z}{49}+\frac{128}{343},z,k\right)^3}{64} - \frac{441\,\mathrm{root}\left(z^4+\frac{5z^3}{2}+2z^2+\frac{32z}{49}+\frac{128}{343},z,k\right)^4}{32} + 10\right)\mathrm{root}\left(z^4+\frac{5z^3}{2}+2z^2+\frac{32z}{49}+\frac{128}{343},z,k\right)\right)$$

input
```
int((x + 3*x^2 + 2*x^3 + 5)/(x*(x + 5*x^2 + x^3 + 2*x^4 + 2)),x)
```

output
```
(5*log(x))/2 + symsum(log((223*root(z^4 + (5*z^3)/2 + 2*z^2 + (32*z)/49 +
128/343, z, k))/8 - (31*x)/2 + (71*root(z^4 + (5*z^3)/2 + 2*z^2 + (32*z)/4
9 + 128/343, z, k)*x)/16 - (4463*root(z^4 + (5*z^3)/2 + 2*z^2 + (32*z)/49
+ 128/343, z, k)^2*x)/64 + (1449*root(z^4 + (5*z^3)/2 + 2*z^2 + (32*z)/49
+ 128/343, z, k)^3*x)/16 + (3675*root(z^4 + (5*z^3)/2 + 2*z^2 + (32*z)/49
+ 128/343, z, k)^4*x)/32 + (257*root(z^4 + (5*z^3)/2 + 2*z^2 + (32*z)/49 +
 128/343, z, k)^2)/32 + (1673*root(z^4 + (5*z^3)/2 + 2*z^2 + (32*z)/49 + 1
28/343, z, k)^3)/64 - (441*root(z^4 + (5*z^3)/2 + 2*z^2 + (32*z)/49 + 128/
343, z, k)^4)/32 + 10)*root(z^4 + (5*z^3)/2 + 2*z^2 + (32*z)/49 + 128/343,
 z, k), k, 1, 4)
```

Reduce [F]

$$\int \frac{5+x+3x^2+2x^3}{x(2+x+5x^2+x^3+2x^4)}\,dx = -17\left(\int \frac{x}{2x^4+x^3+5x^2+x+2}dx\right) - 11\left(\int \frac{1}{2x^5+x^4+5x^3+x^2+2x}dx\right) - 5\left(\int \frac{1}{2x^4+x^3+5x^2+x+2}dx\right) - 2\log\left(2x^4+x^3+5x^2+x+2\right) + 8\log(x)$$

input
```
int((2*x^3+3*x^2+x+5)/x/(2*x^4+x^3+5*x^2+x+2),x)
```

output
```
 - 17*int(x/(2*x**4 + x**3 + 5*x**2 + x + 2),x) - 11*int(1/(2*x**5 + x**4
+ 5*x**3 + x**2 + 2*x),x) - 5*int(1/(2*x**4 + x**3 + 5*x**2 + x + 2),x) -
2*log(2*x**4 + x**3 + 5*x**2 + x + 2) + 8*log(x)
```

3.62 $\int \frac{5+x+3x^2+2x^3}{x^2(2+x+5x^2+x^3+2x^4)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 224

$$\int \frac{5+x+3x^2+2x^3}{x^2(2+x+5x^2+x^3+2x^4)}\,dx = -\frac{5}{2x}+\frac{11(9+5i\sqrt{7})\operatorname{arctanh}\left(\frac{i-\sqrt{7}+8ix}{\sqrt{2(35-i\sqrt{7})}}\right)}{4\sqrt{14(35-i\sqrt{7})}} - \frac{11(9-5i\sqrt{7})\operatorname{arctanh}\left(\frac{i+\sqrt{7}+8ix}{\sqrt{2(35+i\sqrt{7})}}\right)}{4\sqrt{14(35+i\sqrt{7})}} - \frac{3\log(x)}{4} + \frac{3}{112}\left(7+11i\sqrt{7}\right)\log\left(4i+\left(i-\sqrt{7}\right)x+4ix^2\right) + \frac{3}{112}\left(7-11i\sqrt{7}\right)\log\left(4i+\left(i+\sqrt{7}\right)x+4ix^2\right)$$

output
```
-5/2/x+11/4*(9+5*I*7^(1/2))*arctanh((I-7^(1/2)+8*I*x)/(70-2*I*7^(1/2))^(1/
2))/(490-14*I*7^(1/2))^(1/2)-11/4*(9-5*I*7^(1/2))*arctanh((I+7^(1/2)+8*I*x
)/(70+2*I*7^(1/2))^(1/2))/(490+14*I*7^(1/2))^(1/2)-3/4*ln(x)+3/112*(7+11*I
*7^(1/2))*ln(4*I+(I-7^(1/2))*x+4*I*x^2)+3/112*(7-11*I*7^(1/2))*ln(4*I+(I+7
^(1/2))*x+4*I*x^2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.49

$$\int \frac{5+x+3x^2+2x^3}{x^2(2+x+5x^2+x^3+2x^4)}\,dx = -\frac{5}{2x}-\frac{3\log(x)}{4}+\frac{1}{4}\text{RootSum}\left[2+\#1+5\#1^2+\#1^3+2\#1^4\&,\frac{-35\log(x-\#1)+13\log(x-\#1)\#1-17\log(x-\#1)\#1^2+6\log(x-\#1)\#1^3}{1+10\#1+3\#1^2+8\#1^3}\&\right]$$

input
```
Integrate[(5 + x + 3*x^2 + 2*x^3)/(x^2*(2 + x + 5*x^2 + x^3 + 2*x^4)),x]
```

output
```
-5/(2*x) - (3*Log[x])/4 + RootSum[2 + #1 + 5*#1^2 + #1^3 + 2*#1^4 & , (-35
*Log[x - #1] + 13*Log[x - #1]*#1 - 17*Log[x - #1]*#1^2 + 6*Log[x - #1]*#1^
3)/(1 + 10*#1 + 3*#1^2 + 8*#1^3) & ]/4
```

Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.01, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = {2496, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2x^3+3x^2+x+5}{x^2(2x^4+x^3+5x^2+x+2)}\,dx$$

$$\Big\downarrow \text{ 2496}$$

$$\frac{1}{2}\int \left(-\frac{6(11-i\sqrt{7})x+7(9+5i\sqrt{7})}{2\sqrt{7}(4ix^2+(i-\sqrt{7})x+4i)}-\frac{3}{2x}-\frac{7(35i-9\sqrt{7})-6(7i+11\sqrt{7})x}{14(4ix^2+(i+\sqrt{7})x+4i)}+\frac{5}{x^2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(\frac{11(9+5i\sqrt{7})\operatorname{arctanh}\left(\frac{8ix-\sqrt{7}+i}{\sqrt{2\left(35-i\sqrt{7}\right)}}\right)}{2\sqrt{14\left(35-i\sqrt{7}\right)}}-\frac{11(9-5i\sqrt{7})\operatorname{arctanh}\left(\frac{8ix+\sqrt{7}+i}{\sqrt{2\left(35+i\sqrt{7}\right)}}\right)}{2\sqrt{14\left(35+i\sqrt{7}\right)}}+\frac{3}{56}\left(7+11i\sqrt{7}\right)\log\left(4ix^2\right.\right.$$

input
```
Int[(5 + x + 3*x^2 + 2*x^3)/(x^2*(2 + x + 5*x^2 + x^3 + 2*x^4)),x]
```

output
```
(-5/x + (11*(9 + (5*I)*Sqrt[7])*ArcTanh[(I - Sqrt[7] + (8*I)*x)/Sqrt[2*(35
 - I*Sqrt[7])]])/(2*Sqrt[14*(35 - I*Sqrt[7])]) - (11*(9 - (5*I)*Sqrt[7])*A
rcTanh[(I + Sqrt[7] + (8*I)*x)/Sqrt[2*(35 + I*Sqrt[7])]])/(2*Sqrt[14*(35 +
 I*Sqrt[7])]) - (3*Log[x])/2 + (3*(7 + (11*I)*Sqrt[7])*Log[4*I + (I - Sqrt
[7])*x + (4*I)*x^2])/56 + (3*(7 - (11*I)*Sqrt[7])*Log[4*I + (I + Sqrt[7])*
x + (4*I)*x^2])/56)/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2496
```
Int[(Px_.)*(x_)^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^3 + (e
_.)*(x_)^4)^(p_), x_Symbol] :> Simp[e^p   Int[ExpandIntegrand[x^m*Px*(b/d +
 ((d + Sqrt[e*((b^2 - 4*a*c)/a) + 8*a*d*(e/b)])/(2*e))*x + x^2)^p*(b/d + ((
d - Sqrt[e*((b^2 - 4*a*c)/a) + 8*a*d*(e/b)])/(2*e))*x + x^2)^p, x], x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Px, x] && ILtQ[p, 0] && EqQ[a*d^2
- b^2*e, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.05 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.26

method	result
risch	$-\frac{5}{2x}-\frac{3\ln(x)}{4}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(686\_Z^4-1029\_Z^3+6272\_Z^2+10752\_Z+4096\right)}\_R\ln\left(-45962\_R^3+98735\_R^2-497168\_R\right.\right.}{2}$
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(2\_Z^4+\_Z^3+5\_Z^2+\_Z+2\right)}\frac{\left(6\_R^3-17\_R^2+13\_R-35\right)\ln\left(x-\_R\right)}{8\_R^3+3\_R^2+10\_R+1}\right)}{4}-\frac{5}{2x}-\frac{3\ln(x)}{4}$

input
```
int((2*x^3+3*x^2+x+5)/x^2/(2*x^4+x^3+5*x^2+x+2),x,method=_RETURNVERBOSE)
```

output
```
-5/2/x-3/4*ln(x)+1/2*sum(_R*ln(-45962*_R^3+98735*_R^2-497168*_R+61952*x-38
4256),_R=RootOf(686*_Z^4-1029*_Z^3+6272*_Z^2+10752*_Z+4096))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 329 vs. $2(135) = 270$.

Time = 0.10 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.47

$$\int \frac{5+x+3x^2+2x^3}{x^2\left(2+x+5x^2+x^3+2x^4\right)}\,dx$$
$$=\frac{4\sqrt{\frac{33}{382}\sqrt{\frac{704}{7}\sqrt{\frac{11}{7}}-55}\left(64\sqrt{\frac{11}{7}}+35\right)+\frac{176}{7}\sqrt{\frac{11}{7}}+\frac{737}{14}}\,x\arctan\left(-\frac{7}{268928}\left(191\sqrt{\frac{11}{7}}(92x+29)-7\left(\right.\right.\right.}{}$$

input
```
integrate((2*x^3+3*x^2+x+5)/x^2/(2*x^4+x^3+5*x^2+x+2),x, algorithm="fricas
")
```

output
```
1/16*(4*sqrt(33/382*sqrt(704/7*sqrt(11/7) - 55)*(64*sqrt(11/7) + 35) + 176
/7*sqrt(11/7) + 737/14)*x*arctan(-7/268928*(191*sqrt(11/7)*(92*x + 29) - 7
*(sqrt(11/7)*(148*x + 87) + 260*x - 33)*sqrt(704/7*sqrt(11/7) - 55) + 8404
*x - 2101)*sqrt(33/382*sqrt(704/7*sqrt(11/7) - 55)*(64*sqrt(11/7) + 35) +
176/7*sqrt(11/7) + 737/14)) - 4*sqrt(-33/382*sqrt(704/7*sqrt(11/7) - 55)*(
64*sqrt(11/7) + 35) + 176/7*sqrt(11/7) + 737/14)*x*arctan(7/268928*(191*sq
rt(11/7)*(92*x + 29) + 7*(sqrt(11/7)*(148*x + 87) + 260*x - 33)*sqrt(704/7
*sqrt(11/7) - 55) + 8404*x - 2101)*sqrt(-33/382*sqrt(704/7*sqrt(11/7) - 55
)*(64*sqrt(11/7) + 35) + 176/7*sqrt(11/7) + 737/14)) + (x*sqrt(704/7*sqrt(
11/7) - 55) + 3*x)*log(16808*x^2 + 7*(7*sqrt(11/7)*(20*x - 9) - 198*x - 12
1)*sqrt(704/7*sqrt(11/7) - 55) + 4202*x + 14707*sqrt(11/7) + 2101) - (x*sq
rt(704/7*sqrt(11/7) - 55) - 3*x)*log(16808*x^2 - 7*(7*sqrt(11/7)*(20*x - 9
) - 198*x - 121)*sqrt(704/7*sqrt(11/7) - 55) + 4202*x + 14707*sqrt(11/7) +
 2101) - 12*x*log(x) - 40)/x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 25507 vs. $2(187) = 374$.

Time = 19.68 (sec) , antiderivative size = 25507, normalized size of antiderivative = 113.87

$$\int \frac{5 + x + 3x^2 + 2x^3}{x^2 (2 + x + 5x^2 + x^3 + 2x^4)} \, dx = \text{Too large to display}$$

input
```
integrate((2*x**3+3*x**2+x+5)/x**2/(2*x**4+x**3+5*x**2+x+2),x)
```

output
```
-3*log(x)/4 + (3/16 - sqrt(-55/256 + 11*sqrt(77)/196))*log(x**2 + x*(10896
479943156192*sqrt(77)/(-39365093785600*sqrt(7)*sqrt(-245 + 64*sqrt(77)) -
815992034457600 + 6974290892800*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 225454
628044800*sqrt(77)) + 1720992726634016*sqrt(7)*sqrt(-245 + 64*sqrt(77))/(-
39365093785600*sqrt(7)*sqrt(-245 + 64*sqrt(77)) - 815992034457600 + 697429
0892800*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 225454628044800*sqrt(77)) + 39
6034568160*sqrt(14)*sqrt(-245 + 64*sqrt(77))*sqrt(-62589*sqrt(11)*sqrt(-24
5 + 64*sqrt(77)) - 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77)) - 103712*sqrt(77
) + 5983777)/(-39365093785600*sqrt(7)*sqrt(-245 + 64*sqrt(77)) - 815992034
457600 + 6974290892800*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 225454628044800
*sqrt(77)) + 1300300581888*sqrt(154)*sqrt(-62589*sqrt(11)*sqrt(-245 + 64*s
qrt(77)) - 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77)) - 103712*sqrt(77) + 5983
777)/(-39365093785600*sqrt(7)*sqrt(-245 + 64*sqrt(77)) - 815992034457600 +
 6974290892800*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 225454628044800*sqrt(77
)) - 278094051039*sqrt(22)*sqrt(-245 + 64*sqrt(77))*sqrt(-62589*sqrt(11)*s
qrt(-245 + 64*sqrt(77)) - 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77)) - 103712*
sqrt(77) + 5983777)/(-39365093785600*sqrt(7)*sqrt(-245 + 64*sqrt(77)) - 81
5992034457600 + 6974290892800*sqrt(11)*sqrt(-245 + 64*sqrt(77)) + 22545462
8044800*sqrt(77)) - 29480043023893*sqrt(2)*sqrt(-62589*sqrt(11)*sqrt(-245
+ 64*sqrt(77)) - 21120*sqrt(7)*sqrt(-245 + 64*sqrt(77)) - 103712*sqrt(7...
```

Maxima [F]

$$\int \frac{5+x+3x^2+2x^3}{x^2(2+x+5x^2+x^3+2x^4)}\,dx = \int \frac{2\,x^3+3\,x^2+x+5}{(2\,x^4+x^3+5\,x^2+x+2)x^2}\,dx$$

input
```
integrate((2*x^3+3*x^2+x+5)/x^2/(2*x^4+x^3+5*x^2+x+2),x, algorithm="maxima
")
```

output
```
-5/2/x + 1/4*integrate((6*x^3 - 17*x^2 + 13*x - 35)/(2*x^4 + x^3 + 5*x^2 +
 x + 2), x) - 3/4*log(x)
```

Giac [F]

$$\int \frac{5+x+3x^2+2x^3}{x^2\left(2+x+5x^2+x^3+2x^4\right)}\,dx=\int \frac{2\,x^3+3\,x^2+x+5}{(2\,x^4+x^3+5\,x^2+x+2)x^2}\,dx$$

input
```
integrate((2*x^3+3*x^2+x+5)/x^2/(2*x^4+x^3+5*x^2+x+2),x, algorithm="giac")
```

output
```
integrate((2*x^3 + 3*x^2 + x + 5)/((2*x^4 + x^3 + 5*x^2 + x + 2)*x^2), x)
```

Mupad [B] (verification not implemented)

Time = 22.34 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.08

$$\int \frac{5+x+3x^2+2x^3}{x^2(2+x+5x^2+x^3+2x^4)}\,dx = \left(\sum_{k=1}^{4}\ln\left(\frac{1199\,\mathrm{root}\left(z^4-\frac{3z^3}{4}+\frac{16z^2}{7}+\frac{96z}{49}+\frac{128}{343},z,k\right)}{32}+25\,x+\frac{\mathrm{root}\left(z^4-\frac{3z^3}{4}+\frac{16z^2}{7}+\frac{96z}{49}+\frac{128}{343},z,k\right)\,x\,4169}{32}+\frac{\mathrm{root}\left(z^4-\frac{3z^3}{4}+\frac{16z^2}{7}+\frac{96z}{49}+\frac{128}{343},z,k\right)^2\,x\,43993}{256}+\mathrm{root}\left(z^4-\frac{3z^3}{4}+\frac{16z^2}{7}+\frac{96z}{49}+\frac{128}{343},z,k\right)^3\,x\,28+\frac{\mathrm{root}\left(z^4-\frac{3z^3}{4}+\frac{16z^2}{7}+\frac{96z}{49}+\frac{128}{343},z,k\right)^4\,x\,3675}{32}+\frac{11647\,\mathrm{root}\left(z^4-\frac{3z^3}{4}+\frac{16z^2}{7}+\frac{96z}{49}+\frac{128}{343},z,k\right)^2}{128}+\frac{7273\,\mathrm{root}\left(z^4-\frac{3z^3}{4}+\frac{16z^2}{7}+\frac{96z}{49}+\frac{128}{343},z,k\right)^3}{128}-\frac{441\,\mathrm{root}\left(z^4-\frac{3z^3}{4}+\frac{16z^2}{7}+\frac{96z}{49}+\frac{128}{343},z,k\right)^4}{32}+\frac{21}{4}\right)\mathrm{root}\left(z^4-\frac{3z^3}{4}+\frac{16z^2}{7}+\frac{96z}{49}+\frac{128}{343},z,k\right)\right)-\frac{3\ln(x)}{4}-\frac{5}{2\,x}$$

input
```
int((x + 3*x^2 + 2*x^3 + 5)/(x^2*(x + 5*x^2 + x^3 + 2*x^4 + 2)),x)
```

output

```
symsum(log((1199*root(z^4 - (3*z^3)/4 + (16*z^2)/7 + (96*z)/49 + 128/343,
z, k))/32 + 25*x + (4169*root(z^4 - (3*z^3)/4 + (16*z^2)/7 + (96*z)/49 + 1
28/343, z, k)*x)/32 + (43993*root(z^4 - (3*z^3)/4 + (16*z^2)/7 + (96*z)/49
 + 128/343, z, k)^2*x)/256 + 28*root(z^4 - (3*z^3)/4 + (16*z^2)/7 + (96*z)
/49 + 128/343, z, k)^3*x + (3675*root(z^4 - (3*z^3)/4 + (16*z^2)/7 + (96*z
)/49 + 128/343, z, k)^4*x)/32 + (11647*root(z^4 - (3*z^3)/4 + (16*z^2)/7 +
 (96*z)/49 + 128/343, z, k)^2)/128 + (7273*root(z^4 - (3*z^3)/4 + (16*z^2)
/7 + (96*z)/49 + 128/343, z, k)^3)/128 - (441*root(z^4 - (3*z^3)/4 + (16*z
^2)/7 + (96*z)/49 + 128/343, z, k)^4)/32 + 21/4)*root(z^4 - (3*z^3)/4 + (1
6*z^2)/7 + (96*z)/49 + 128/343, z, k), k, 1, 4) - (3*log(x))/4 - 5/(2*x)
```

Reduce [F]

$$\int \frac{5+x+3x^2+2x^3}{x^2\left(2+x+5x^2+x^3+2x^4\right)}\,dx$$
$$= \frac{99\left(\int \frac{1}{2x^6+x^5+5x^4+x^3+2x^2}dx\right)x - 11\left(\int \frac{1}{2x^5+x^4+5x^3+x^2+2x}dx\right)x + 55\left(\int \frac{1}{2x^4+x^3+5x^2+x+2}dx\right)x - 4\log(2x^4+x}{19x}$$

input

```
int((2*x^3+3*x^2+x+5)/x^2/(2*x^4+x^3+5*x^2+x+2),x)
```

output

```
(99*int(1/(2*x**6 + x**5 + 5*x**4 + x**3 + 2*x**2),x)*x - 11*int(1/(2*x**5
 + x**4 + 5*x**3 + x**2 + 2*x),x)*x + 55*int(1/(2*x**4 + x**3 + 5*x**2 + x
 + 2),x)*x - 4*log(2*x**4 + x**3 + 5*x**2 + x + 2)*x + 16*log(x)*x + 2)/(1
9*x)
```

3.63 $\int \frac{5+x+3x^2+2x^3}{x^3\left(2+x+5x^2+x^3+2x^4\right)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 231

$$\int \frac{5+x+3x^2+2x^3}{x^3\left(2+x+5x^2+x^3+2x^4\right)}\,dx = -\frac{5}{4x^2}+\frac{3}{4x} + \frac{\left(355-73i\sqrt{7}\right)\operatorname{arctanh}\left(\frac{i-\sqrt{7}+8ix}{\sqrt{2\left(35-i\sqrt{7}\right)}}\right)}{8\sqrt{14\left(35-i\sqrt{7}\right)}} - \frac{\left(355+73i\sqrt{7}\right)\operatorname{arctanh}\left(\frac{i+\sqrt{7}+8ix}{\sqrt{2\left(35+i\sqrt{7}\right)}}\right)}{8\sqrt{14\left(35+i\sqrt{7}\right)}} - \frac{35\log(x)}{8} + \frac{1}{32}\left(35-9i\sqrt{7}\right)\log\left(4i+\left(i-\sqrt{7}\right)x+4ix^2\right) + \frac{1}{32}\left(35+9i\sqrt{7}\right)\log\left(4i+\left(i+\sqrt{7}\right)x+4ix^2\right)$$

output
```
-5/4/x^2+3/4/x+1/8*(355-73*I*7^(1/2))*arctanh((I-7^(1/2)+8*I*x)/(70-2*I*7^
(1/2))^(1/2))/(490-14*I*7^(1/2))^(1/2)-1/8*(355+73*I*7^(1/2))*arctanh((I+7
^(1/2)+8*I*x)/(70+2*I*7^(1/2))^(1/2))/(490+14*I*7^(1/2))^(1/2)-35/8*ln(x)+
1/32*(35-9*I*7^(1/2))*ln(4*I+(I-7^(1/2))*x+4*I*x^2)+1/32*(35+9*I*7^(1/2))*
ln(4*I+(I+7^(1/2))*x+4*I*x^2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.50

$$\int \frac{5+x+3x^2+2x^3}{x^3(2+x+5x^2+x^3+2x^4)}\,dx$$
$$= -\frac{5}{4x^2}+\frac{3}{4x}-\frac{35\log(x)}{8}+\frac{1}{8}\text{RootSum}\Big[2+\#1+5\#1^2+\#1^3 +2\#1^4\&, \frac{61\log(x-\#1)+141\log(x-\#1)\#1+47\log(x-\#1)\#1^2+70\log(x-\#1)\#1^3}{1+10\#1+3\#1^2+8\#1^3}\&\Big]$$

input
```
Integrate[(5 + x + 3*x^2 + 2*x^3)/(x^3*(2 + x + 5*x^2 + x^3 + 2*x^4)),x]
```

output
```
-5/(4*x^2) + 3/(4*x) - (35*Log[x])/8 + RootSum[2 + #1 + 5*#1^2 + #1^3 + 2*
#1^4 & , (61*Log[x - #1] + 141*Log[x - #1]*#1 + 47*Log[x - #1]*#1^2 + 70*L
og[x - #1]*#1^3)/(1 + 10*#1 + 3*#1^2 + 8*#1^3) & ]/8
```

Rubi [A] (verified)

Time = 1.05 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.02, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = {2496, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2x^3+3x^2+x+5}{x^3(2x^4+x^3+5x^2+x+2)}\,dx$$

$$\Big\downarrow \ 2496$$

$$\frac{1}{2}\int \left(\frac{14\left(35i-9\sqrt{7}\right)x+223\sqrt{7}+427i}{28\left(4ix^2+\left(i+\sqrt{7}\right)x+4i\right)}-\frac{35}{4x}-\frac{-14\left(9+5i\sqrt{7}\right)x-61i\sqrt{7}+223}{4\sqrt{7}\left(4ix^2+\left(i-\sqrt{7}\right)x+4i\right)}-\frac{3}{2x^2}+\frac{5}{x^3}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{2}\left(\frac{\left(355-73i\sqrt{7}\right)\text{arctanh}\left(\frac{8ix-\sqrt{7}+i}{\sqrt{2\left(35-i\sqrt{7}\right)}}\right)}{4\sqrt{14\left(35-i\sqrt{7}\right)}}-\frac{\left(355+73i\sqrt{7}\right)\text{arctanh}\left(\frac{8ix+\sqrt{7}+i}{\sqrt{2\left(35+i\sqrt{7}\right)}}\right)}{4\sqrt{14\left(35+i\sqrt{7}\right)}}-\frac{5}{2x^2}+\frac{1}{16}\left(35-9i\sqrt{7}\right)\right.$$

input
```
Int[(5 + x + 3*x^2 + 2*x^3)/(x^3*(2 + x + 5*x^2 + x^3 + 2*x^4)),x]
```

output
```
(-5/(2*x^2) + 3/(2*x) + ((355 - (73*I)*Sqrt[7])*ArcTanh[(I - Sqrt[7] + (8*
I)*x)/Sqrt[2*(35 - I*Sqrt[7])]])/(4*Sqrt[14*(35 - I*Sqrt[7])]) - ((355 + (
73*I)*Sqrt[7])*ArcTanh[(I + Sqrt[7] + (8*I)*x)/Sqrt[2*(35 + I*Sqrt[7])]])/
(4*Sqrt[14*(35 + I*Sqrt[7])]) - (35*Log[x])/4 + ((35 - (9*I)*Sqrt[7])*Log[
4*I + (I - Sqrt[7])*x + (4*I)*x^2])/16 + ((35 + (9*I)*Sqrt[7])*Log[4*I + (
I + Sqrt[7])*x + (4*I)*x^2])/16)/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2496
```
Int[(Px_.)*(x_)^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^3 + (e
_.)*(x_)^4)^(p_), x_Symbol] :> Simp[e^p   Int[ExpandIntegrand[x^m*Px*(b/d +
 ((d + Sqrt[e*((b^2 - 4*a*c)/a) + 8*a*d*(e/b)])/(2*e))*x + x^2)^p*(b/d + ((
d - Sqrt[e*((b^2 - 4*a*c)/a) + 8*a*d*(e/b)])/(2*e))*x + x^2)^p, x], x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Px, x] && ILtQ[p, 0] && EqQ[a*d^2
- b^2*e, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.05 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.27

method	result
risch	$\frac{\frac{3x}{4}-\frac{5}{4}}{x^2}-\frac{35\ln(x)}{8}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(686\_Z^4-12005\_Z^3+73696\_Z^2-50176\_Z+65536\right)}\_R\ln\left(-2261742\_R^3+41411909\_R^2-\right.\right.}{4}$
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(2\_Z^4+\_Z^3+5\_Z^2+\_Z+2\right)}\frac{\left(70\_R^3+47\_R^2+141\_R+61\right)\ln\left(x-\_R\right)}{8\_R^3+3\_R^2+10\_R+1}\right)}{8}-\frac{5}{4x^2}+\frac{3}{4x}-\frac{35\ln(x)}{8}$

input
```
int((2*x^3+3*x^2+x+5)/x^3/(2*x^4+x^3+5*x^2+x+2),x,method=_RETURNVERBOSE)
```

output
```
(3/4*x-5/4)/x^2-35/8*ln(x)+1/4*sum(_R*ln(-2261742*_R^3+41411909*_R^2-24959
3568*_R+154597376*x+130505728),_R=RootOf(686*_Z^4-12005*_Z^3+73696*_Z^2-50
176*_Z+65536))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 346 vs. 2(140) = 280.

Time = 0.09 (sec) , antiderivative size = 346, normalized size of antiderivative = 1.50

$$\int \frac{5+x+3x^2+2x^3}{x^3\left(2+x+5x^2+x^3+2x^4\right)}\,dx = \text{Too large to display}$$

input
```
integrate((2*x^3+3*x^2+x+5)/x^3/(2*x^4+x^3+5*x^2+x+2),x, algorithm="fricas
")
```

output
```
1/32*(4*sqrt(63/19606*(3712*sqrt(11/7) - 2815)*sqrt(3712/7*sqrt(11/7) + 28
15/7) + 928/7*sqrt(11/7) + 577/14)*x^2*arctan(7/6532405504*(9803*sqrt(11/7
)*(460*x - 5287) + 7*(sqrt(11/7)*(637388*x + 333081) - 266948*x - 341271)*
sqrt(3712/7*sqrt(11/7) + 2815/7) - 62151020*x + 53102851)*sqrt(63/19606*(3
712*sqrt(11/7) - 2815)*sqrt(3712/7*sqrt(11/7) + 2815/7) + 928/7*sqrt(11/7)
 + 577/14)) - 4*sqrt(-63/19606*(3712*sqrt(11/7) - 2815)*sqrt(3712/7*sqrt(1
1/7) + 2815/7) + 928/7*sqrt(11/7) + 577/14)*x^2*arctan(-7/6532405504*(9803
*sqrt(11/7)*(460*x - 5287) - 7*(sqrt(11/7)*(637388*x + 333081) - 266948*x
- 341271)*sqrt(3712/7*sqrt(11/7) + 2815/7) - 62151020*x + 53102851)*sqrt(-
63/19606*(3712*sqrt(11/7) - 2815)*sqrt(3712/7*sqrt(11/7) + 2815/7) + 928/7
*sqrt(11/7) + 577/14)) - 140*x^2*log(x) - (x^2*sqrt(3712/7*sqrt(11/7) + 28
15/7) - 35*x^2)*log(78424*x^2 + 7*(7*sqrt(11/7)*(84*x + 23) - 710*x + 39)*
sqrt(3712/7*sqrt(11/7) + 2815/7) + 19606*x + 68621*sqrt(11/7) + 9803) + (x
^2*sqrt(3712/7*sqrt(11/7) + 2815/7) + 35*x^2)*log(78424*x^2 - 7*(7*sqrt(11
/7)*(84*x + 23) - 710*x + 39)*sqrt(3712/7*sqrt(11/7) + 2815/7) + 19606*x +
 68621*sqrt(11/7) + 9803) + 24*x - 40)/x^2
```

Sympy [A] (verification not implemented)

Time = 1.75 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.30

$$\int \frac{5+x+3x^2+2x^3}{x^3(2+x+5x^2+x^3+2x^4)}\,dx = -\frac{35\log(x)}{8} + \operatorname{RootSum}\left(2744t^4-12005t^3+18424t^2-3136t+1024, \left(t\mapsto t\log\left(-\frac{20101387287723t^4}{91907904361586}+\frac{9445152}{4595395}\right.\right.\right. + \frac{3x-5}{4x^2}$$

input
```
integrate((2*x**3+3*x**2+x+5)/x**3/(2*x**4+x**3+5*x**2+x+2),x)
```

output
```
-35*log(x)/8 + RootSum(2744*_t**4 - 12005*_t**3 + 18424*_t**2 - 3136*_t +
1024, Lambda(_t, _t*log(-20101387287723*_t**4/91907904361586 + 94451521449
6*_t**3/45953952180793 + 16572327093911939*_t**2/5882105879141504 - 456447
1749800865*_t/735263234892688 + x + 70084064010625/91907904361586))) + (3*
x - 5)/(4*x**2)
```

Maxima [F]

$$\int \frac{5+x+3x^2+2x^3}{x^3(2+x+5x^2+x^3+2x^4)}\,dx=\int \frac{2\,x^3+3\,x^2+x+5}{(2\,x^4+x^3+5\,x^2+x+2)x^3}\,dx$$

input
```
integrate((2*x^3+3*x^2+x+5)/x^3/(2*x^4+x^3+5*x^2+x+2),x, algorithm="maxima
")
```

output
```
1/4*(3*x - 5)/x^2 + 1/8*integrate((70*x^3 + 47*x^2 + 141*x + 61)/(2*x^4 +
x^3 + 5*x^2 + x + 2), x) - 35/8*log(x)
```

Giac [F]

$$\int \frac{5+x+3x^2+2x^3}{x^3(2+x+5x^2+x^3+2x^4)}\,dx=\int \frac{2\,x^3+3\,x^2+x+5}{(2\,x^4+x^3+5\,x^2+x+2)x^3}\,dx$$

input
```
integrate((2*x^3+3*x^2+x+5)/x^3/(2*x^4+x^3+5*x^2+x+2),x, algorithm="giac")
```

output
```
integrate((2*x^3 + 3*x^2 + x + 5)/((2*x^4 + x^3 + 5*x^2 + x + 2)*x^3), x)
```

Mupad [B] (verification not implemented)

Time = 22.26 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.06

$$\int \frac{5+x+3x^2+2x^3}{x^3(2+x+5x^2+x^3+2x^4)}\,dx = \left(\sum_{k=1}^{4}\ln\left(-\frac{8939\,\mathrm{root}\left(z^4-\frac{35\,z^3}{8}+\frac{47\,z^2}{7}-\frac{8\,z}{7}+\frac{128}{343},z,k\right)}{128}-\frac{69\,x}{8}+\frac{\mathrm{root}\left(z^4-\frac{35\,z^3}{8}+\frac{47\,z^2}{7}-\frac{8\,z}{7}+\frac{128}{343},z,k\right)\,x\,14945}{128}-\frac{\mathrm{root}\left(z^4-\frac{35\,z^3}{8}+\frac{47\,z^2}{7}-\frac{8\,z}{7}+\frac{128}{343},z,k\right)^2\,x\,269991}{1024}-\frac{\mathrm{root}\left(z^4-\frac{35\,z^3}{8}+\frac{47\,z^2}{7}-\frac{8\,z}{7}+\frac{128}{343},z,k\right)^3\,x\,1393}{8}+\frac{\mathrm{root}\left(z^4-\frac{35\,z^3}{8}+\frac{47\,z^2}{7}-\frac{8\,z}{7}+\frac{128}{343},z,k\right)^4\,x\,3675}{32}-\frac{35697\,\mathrm{root}\left(z^4-\frac{35\,z^3}{8}+\frac{47\,z^2}{7}-\frac{8\,z}{7}+\frac{128}{343},z,k\right)^2}{512}-\frac{18487\,\mathrm{root}\left(z^4-\frac{35\,z^3}{8}+\frac{47\,z^2}{7}-\frac{8\,z}{7}+\frac{128}{343},z,k\right)^3}{256}-\frac{441\,\mathrm{root}\left(z^4-\frac{35\,z^3}{8}+\frac{47\,z^2}{7}-\frac{8\,z}{7}+\frac{128}{343},z,k\right)^4}{32}+\frac{245}{8}\right)\mathrm{root}\left(z^4-\frac{35\,z^3}{8}+\frac{47\,z^2}{7}-\frac{8\,z}{7}+\frac{128}{343},z,k\right)\right)-\frac{35\,\ln(x)}{8}+\frac{\frac{3\,x}{4}-\frac{5}{4}}{x^2}$$

input
```
int((x + 3*x^2 + 2*x^3 + 5)/(x^3*(x + 5*x^2 + x^3 + 2*x^4 + 2)),x)
```

output
```
symsum(log((14945*root(z^4 - (35*z^3)/8 + (47*z^2)/7 - (8*z)/7 + 128/343,
z, k)*x)/128 - (69*x)/8 - (8939*root(z^4 - (35*z^3)/8 + (47*z^2)/7 - (8*z)
/7 + 128/343, z, k))/128 - (269991*root(z^4 - (35*z^3)/8 + (47*z^2)/7 - (8
*z)/7 + 128/343, z, k)^2*x)/1024 - (1393*root(z^4 - (35*z^3)/8 + (47*z^2)/
7 - (8*z)/7 + 128/343, z, k)^3*x)/8 + (3675*root(z^4 - (35*z^3)/8 + (47*z^
2)/7 - (8*z)/7 + 128/343, z, k)^4*x)/32 - (35697*root(z^4 - (35*z^3)/8 + (
47*z^2)/7 - (8*z)/7 + 128/343, z, k)^2)/512 - (18487*root(z^4 - (35*z^3)/8
 + (47*z^2)/7 - (8*z)/7 + 128/343, z, k)^3)/256 - (441*root(z^4 - (35*z^3)
/8 + (47*z^2)/7 - (8*z)/7 + 128/343, z, k)^4)/32 + 245/8)*root(z^4 - (35*z
^3)/8 + (47*z^2)/7 - (8*z)/7 + 128/343, z, k), k, 1, 4) - (35*log(x))/8 +
((3*x)/4 - 5/4)/x^2
```

Reduce [F]

$$\int \frac{5+x+3x^2+2x^3}{x^3\left(2+x+5x^2+x^3+2x^4\right)}\,dx = \frac{9\left(\int \frac{1}{2x^7+x^6+5x^5+x^4+2x^3}dx\right)x^2-29\left(\int \frac{1}{2x^6+x^5+5x^4+x^3+2x^2}dx\right)x^2-79\left(\int \frac{1}{2x^5+x^4+5x^3+x^2+2x}dx\right)x^2+8\log(2x^4}{17x^2}$$

input
```
int((2*x^3+3*x^2+x+5)/x^3/(2*x^4+x^3+5*x^2+x+2),x)
```

output
```
(9*int(1/(2*x**7 + x**6 + 5*x**5 + x**4 + 2*x**3),x)*x**2 - 29*int(1/(2*x*
*6 + x**5 + 5*x**4 + x**3 + 2*x**2),x)*x**2 - 79*int(1/(2*x**5 + x**4 + 5*
x**3 + x**2 + 2*x),x)*x**2 + 8*log(2*x**4 + x**3 + 5*x**2 + x + 2)*x**2 -
32*log(x)*x**2 - 4*x - 19)/(17*x**2)
```

3.64 $\int x^3(1+x)^3(1+2x)\sqrt{1-x^2-2x^3-x^4}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 42

$$\int x^3(1+x)^3(1+2x)\sqrt{1-x^2-2x^3-x^4}\,dx = -\frac{1}{15}\left(1-x^2-2x^3-x^4\right)^{3/2}\left(2+3x^2+6x^3+3x^4\right)$$

output
```
-1/15*(-x^4-2*x^3-x^2+1)^(3/2)*(3*x^4+6*x^3+3*x^2+2)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int x^3(1+x)^3(1+2x)\sqrt{1-x^2-2x^3-x^4}\,dx = \frac{1}{15}\left(-2-3x^2-6x^3-3x^4\right)\left(1-x^2-2x^3-x^4\right)^{3/2}$$

input
```
Integrate[x^3*(1 + x)^3*(1 + 2*x)*Sqrt[1 - x^2 - 2*x^3 - x^4],x]
```

output
```
((-2 - 3*x^2 - 6*x^3 - 3*x^4)*(1 - x^2 - 2*x^3 - x^4)^(3/2))/15
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.76, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2459, 2029, 2069, 1576, 27, 1116, 1104}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3(x+1)^3(2x+1)\sqrt{-x^4-2x^3-x^2+1}\,dx$$

$$\Big\downarrow \text{2459}$$

$$\int \sqrt{-\left(x+\frac{1}{2}\right)^4+\frac{1}{2}\left(x+\frac{1}{2}\right)^2+\frac{15}{16}}\left(2\left(x+\frac{1}{2}\right)^7-\frac{3}{2}\left(x+\frac{1}{2}\right)^5+\frac{3}{8}\left(x+\frac{1}{2}\right)^3+\frac{1}{32}\left(-x-\frac{1}{2}\right)\right)d\left(x+\frac{1}{2}\right)$$

$$\Big\downarrow \text{2029}$$

$$\int \left(x+\frac{1}{2}\right)\sqrt{-\left(x+\frac{1}{2}\right)^4+\frac{1}{2}\left(x+\frac{1}{2}\right)^2+\frac{15}{16}}\left(2\left(x+\frac{1}{2}\right)^6-\frac{3}{2}\left(x+\frac{1}{2}\right)^4+\frac{3}{8}\left(x+\frac{1}{2}\right)^2-\frac{1}{32}\right)d\left(x+\frac{1}{2}\right)$$

$$\Big\downarrow \text{2069}$$

$$\int \left(x+\frac{1}{2}\right)\left(\sqrt[3]{2}\left(x+\frac{1}{2}\right)^2-\frac{1}{2\,2^{2/3}}\right)^3\sqrt{-\left(x+\frac{1}{2}\right)^4+\frac{1}{2}\left(x+\frac{1}{2}\right)^2+\frac{15}{16}}d\left(x+\frac{1}{2}\right)$$

$$\Big\downarrow \text{1576}$$

$$\frac{1}{2}\int -\frac{1}{128}\left(1-4\left(x+\frac{1}{2}\right)^2\right)^3\sqrt{-16\left(x+\frac{1}{2}\right)^4+8\left(x+\frac{1}{2}\right)^2+15}d\left(x+\frac{1}{2}\right)^2$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{256}\int \left(1-4\left(x+\frac{1}{2}\right)^2\right)^3\sqrt{-16\left(x+\frac{1}{2}\right)^4+8\left(x+\frac{1}{2}\right)^2+15}d\left(x+\frac{1}{2}\right)^2$$

$$\Big\downarrow \text{1116}$$

$$\frac{1}{256}\left(-\frac{32}{5}\int \left(1-4\left(x+\frac{1}{2}\right)^2\right)\sqrt{-16\left(x+\frac{1}{2}\right)^4+8\left(x+\frac{1}{2}\right)^2+15}d\left(x+\frac{1}{2}\right)^2-\frac{1}{20}\left(-16\left(x+\frac{1}{2}\right)^4+8\left(x\right.\right.\right.$$

$$\Big\downarrow \text{1104}$$

$$\frac{1}{256}\left(-\frac{1}{20}\left(-16\left(x+\frac{1}{2}\right)^4+8\left(x+\frac{1}{2}\right)^2+15\right)^{3/2}\left(1-4\left(x+\frac{1}{2}\right)^2\right)^2-\frac{8}{15}\left(-16\left(x+\frac{1}{2}\right)^4+8\left(x+\frac{1}{2}\right)^2+\right.\right.$$

input
```
Int[x^3*(1 + x)^3*(1 + 2*x)*Sqrt[1 - x^2 - 2*x^3 - x^4],x]
```

output
```
((-8*(15 + 8*(1/2 + x)^2 - 16*(1/2 + x)^4)^(3/2))/15 - ((1 - 4*(1/2 + x)^2
)^2*(15 + 8*(1/2 + x)^2 - 16*(1/2 + x)^4)^(3/2))/20)/256
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1104
```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1116
```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[2*d*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(b*(m + 2*p
 + 1))), x] + Simp[d^2*(m - 1)*((b^2 - 4*a*c)/(b^2*(m + 2*p + 1)))   Int[(d
 + e*x)^(m - 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ[m, 1] && NeQ[m + 2*p
+ 1, 0] && (IntegerQ[2*p] || (IntegerQ[m] && RationalQ[p]) || OddQ[m])
```

rule 1576
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x]
, x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

rule 2029
```
Int[(Fx_.)*((d_.)*(x_)^(q_.) + (a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.) + (c_.)*
(x_)^(t_.))^(p_.), x_Symbol] :> Int[x^(p*r)*(a + b*x^(s - r) + c*x^(t - r)
+ d*x^(q - r))^p*Fx, x] /; FreeQ[{a, b, c, d, r, s, t, q}, x] && IntegerQ[p
] && PosQ[s - r] && PosQ[t - r] && PosQ[q - r] &&  !(EqQ[p, 1] && EqQ[u, 1]
)
```

rule 2069
```
Int[(u_.)*(Px_), x_Symbol] :> With[{a = Rt[Coeff[Px, x^2, 0], Expon[Px, x^2
]], b = Rt[Coeff[Px, x^2, Expon[Px, x^2]], Expon[Px, x^2]]}, Int[u*(a + b*x
^2)^Expon[Px, x^2], x] /; EqQ[Px, (a + b*x^2)^Expon[Px, x^2]]] /; PolyQ[Px,
 x^2] && GtQ[Expon[Px, x^2], 1] && NeQ[Coeff[Px, x^2, 0], 0] &&  !MatchQ[Px
, (a_.)*(v_)^Expon[Px, x^2] /; FreeQ[a, x] && BinomialQ[v, x, 2]]
```

rule 2459
```
Int[(Pn_)^(p_.)*(Qx_), x_Symbol] :> With[{S = Coeff[Pn, x, Expon[Pn, x] - 1
]/(Expon[Pn, x]*Coeff[Pn, x, Expon[Pn, x]])}, Subst[Int[ExpandToSum[Pn /. x
 -> x - S, x]^p*ExpandToSum[Qx /. x -> x - S, x], x], x, x + S] /; Binomial
Q[Pn /. x -> x - S, x] || (IntegerQ[Expon[Pn, x]/2] && TrinomialQ[Pn /. x -
> x - S, x])] /; FreeQ[p, x] && PolyQ[Pn, x] && GtQ[Expon[Pn, x], 2] && NeQ
[Coeff[Pn, x, Expon[Pn, x] - 1], 0] && PolyQ[Qx, x] &&  !(MonomialQ[Qx, x]
&& IGtQ[p, 0])
```

Maple [A] (verified)

Time = 1.70 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.83

method	result
pseudoelliptic	$-\frac{\left(-x^{4}-2 x^{3}-x^{2}+1\right)^{\frac{3}{2}} \left(x^{4}+2 x^{3}+x^{2}+\frac{2}{3}\right)}{5}$
gosper	$\frac{\left(x^{2}+x+1\right) \left(x^{2}+x-1\right) \left(3 x^{4}+6 x^{3}+3 x^{2}+2\right) \sqrt{-x^{4}-2 x^{3}-x^{2}+1}}{15}$
orering	$\frac{\left(x^{2}+x+1\right) \left(x^{2}+x-1\right) \left(3 x^{4}+6 x^{3}+3 x^{2}+2\right) \sqrt{-x^{4}-2 x^{3}-x^{2}+1}}{15}$
trager	$\left(\frac{1}{5} x^{8}+\frac{4}{5} x^{7}+\frac{6}{5} x^{6}+\frac{4}{5} x^{5}+\frac{2}{15} x^{4}-\frac{2}{15} x^{3}-\frac{1}{15} x^{2}-\frac{2}{15}\right) \sqrt{-x^{4}-2 x^{3}-x^{2}+1}$
risch	$-\frac{\left(3 x^{8}+12 x^{7}+18 x^{6}+12 x^{5}+2 x^{4}-2 x^{3}-x^{2}-2\right) \left(x^{4}+2 x^{3}+x^{2}-1\right)}{15 \sqrt{-x^{4}-2 x^{3}-x^{2}+1}}$
default	$\frac{2 x^{4} \sqrt{-x^{4}-2 x^{3}-x^{2}+1}}{15}-\frac{2 x^{3} \sqrt{-x^{4}-2 x^{3}-x^{2}+1}}{15}-\frac{x^{2} \sqrt{-x^{4}-2 x^{3}-x^{2}+1}}{15}-\frac{2 \sqrt{-x^{4}-2 x^{3}-x^{2}+1}}{15}+\frac{4 x^{5} \sqrt{-x^{4}-2x^{3}}}{5}$
elliptic	$\frac{2 x^{4} \sqrt{-x^{4}-2 x^{3}-x^{2}+1}}{15}-\frac{2 x^{3} \sqrt{-x^{4}-2 x^{3}-x^{2}+1}}{15}-\frac{x^{2} \sqrt{-x^{4}-2 x^{3}-x^{2}+1}}{15}-\frac{2 \sqrt{-x^{4}-2 x^{3}-x^{2}+1}}{15}+\frac{4 x^{5} \sqrt{-x^{4}-2x^{3}}}{5}$

input
```
int(x^3*(x+1)^3*(2*x+1)*(-x^4-2*x^3-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/5*(-x^4-2*x^3-x^2+1)^(3/2)*(x^4+2*x^3+x^2+2/3)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.38

$$\int x^3(1+x)^3(1+2x)\sqrt{1-x^2-2x^3-x^4}\,dx = \frac{1}{15}\left(3\,x^8+12\,x^7+18\,x^6+12\,x^5+2\,x^4-2\,x^3-x^2-2\right)\sqrt{-x^4-2\,x^3-x^2+1}$$

input
```
integrate(x^3*(1+x)^3*(1+2*x)*(-x^4-2*x^3-x^2+1)^(1/2),x, algorithm="frica
s")
```

output
```
1/15*(3*x^8 + 12*x^7 + 18*x^6 + 12*x^5 + 2*x^4 - 2*x^3 - x^2 - 2)*sqrt(-x^
4 - 2*x^3 - x^2 + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 182 vs. $2(36) = 72$.

Time = 0.16 (sec) , antiderivative size = 182, normalized size of antiderivative = 4.33

$$\begin{aligned}\int x^3(1+x)^3(1+2x)\sqrt{1-x^2-2x^3-x^4}\,dx &= \frac{x^8\sqrt{-x^4-2x^3-x^2+1}}{5}+\frac{4x^7\sqrt{-x^4-2x^3-x^2+1}}{5}+\frac{6x^6\sqrt{-x^4-2x^3-x^2+1}}{5}\\&+\frac{4x^5\sqrt{-x^4-2x^3-x^2+1}}{5}+\frac{2x^4\sqrt{-x^4-2x^3-x^2+1}}{15}\\&-\frac{2x^3\sqrt{-x^4-2x^3-x^2+1}}{15}-\frac{x^2\sqrt{-x^4-2x^3-x^2+1}}{15}-\frac{2\sqrt{-x^4-2x^3-x^2+1}}{15}\end{aligned}$$

input
```
integrate(x**3*(1+x)**3*(1+2*x)*(-x**4-2*x**3-x**2+1)**(1/2),x)
```

output
```
x**8*sqrt(-x**4 - 2*x**3 - x**2 + 1)/5 + 4*x**7*sqrt(-x**4 - 2*x**3 - x**2
 + 1)/5 + 6*x**6*sqrt(-x**4 - 2*x**3 - x**2 + 1)/5 + 4*x**5*sqrt(-x**4 - 2
*x**3 - x**2 + 1)/5 + 2*x**4*sqrt(-x**4 - 2*x**3 - x**2 + 1)/15 - 2*x**3*s
qrt(-x**4 - 2*x**3 - x**2 + 1)/15 - x**2*sqrt(-x**4 - 2*x**3 - x**2 + 1)/1
5 - 2*sqrt(-x**4 - 2*x**3 - x**2 + 1)/15
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.40

$$\int x^3(1+x)^3(1+2x)\sqrt{1-x^2-2x^3-x^4}\,dx$$
$$=\frac{1}{15}\left(3\,x^8+12\,x^7+18\,x^6+12\,x^5+2\,x^4-2\,x^3-x^2-2\right)\sqrt{x^2+x+1}\sqrt{-x^2-x+1}$$

input
```
integrate(x^3*(1+x)^3*(1+2*x)*(-x^4-2*x^3-x^2+1)^(1/2),x, algorithm="maxim
a")
```

output
```
1/15*(3*x^8 + 12*x^7 + 18*x^6 + 12*x^5 + 2*x^4 - 2*x^3 - x^2 - 2)*sqrt(x^2
 + x + 1)*sqrt(-x^2 - x + 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.38

$$\int x^3(1+x)^3(1+2x)\sqrt{1-x^2-2x^3-x^4}\,dx$$
$$=\frac{1}{5}\left(x^4+2\,x^3+x^2-1\right)^2\sqrt{-x^4-2\,x^3-x^2+1}-\frac{1}{3}\left(-x^4-2\,x^3-x^2+1\right)^{\frac{3}{2}}$$

input
```
integrate(x^3*(1+x)^3*(1+2*x)*(-x^4-2*x^3-x^2+1)^(1/2),x, algorithm="giac"
)
```

output
```
1/5*(x^4 + 2*x^3 + x^2 - 1)^2*sqrt(-x^4 - 2*x^3 - x^2 + 1) - 1/3*(-x^4 - 2
*x^3 - x^2 + 1)^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.90

$$\int x^3(1+x)^3(1+2x)\sqrt{1-x^2-2x^3-x^4}\,dx$$
$$= -\frac{(3\,x^4+6\,x^3+3\,x^2+2)\,\left(-x^4-2\,x^3-x^2+1\right)^{3/2}}{15}$$

input
```
int(x^3*(2*x + 1)*(x + 1)^3*(1 - 2*x^3 - x^4 - x^2)^(1/2),x)
```

output
```
-((3*x^2 + 6*x^3 + 3*x^4 + 2)*(1 - 2*x^3 - x^4 - x^2)^(3/2))/15
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.36

$$\int x^3(1+x)^3(1+2x)\sqrt{1-x^2-2x^3-x^4}\,dx$$
$$= \frac{\sqrt{-x^4-2x^3-x^2+1}\,(3x^8+12x^7+18x^6+12x^5+2x^4-2x^3-x^2-2)}{15}$$

input
```
int(x^3*(1+x)^3*(1+2*x)*(-x^4-2*x^3-x^2+1)^(1/2),x)
```

output
```
(sqrt( - x**4 - 2*x**3 - x**2 + 1)*(3*x**8 + 12*x**7 + 18*x**6 + 12*x**5 +
 2*x**4 - 2*x**3 - x**2 - 2))/15
```

3.65 $\int (1+2x)\left(x+x^2\right)^3\sqrt{1-\left(x+x^2\right)^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 42

$$\int (1+2x)\left(x+x^2\right)^3\sqrt{1-\left(x+x^2\right)^2}\,dx=-\frac{1}{15}\left(1-x^2-2x^3-x^4\right)^{3/2}\left(2+3x^2+6x^3+3x^4\right)$$

output
```
-1/15*(-x^4-2*x^3-x^2+1)^(3/2)*(3*x^4+6*x^3+3*x^2+2)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int (1+2x)\left(x+x^2\right)^3\sqrt{1-\left(x+x^2\right)^2}\,dx=\frac{1}{15}\left(-2-3x^2-6x^3-3x^4\right)\left(1-x^2-2x^3-x^4\right)^{3/2}$$

input
```
Integrate[(1 + 2*x)*(x + x^2)^3*Sqrt[1 - (x + x^2)^2],x]
```

output
```
((-2 - 3*x^2 - 6*x^3 - 3*x^4)*(1 - x^2 - 2*x^3 - x^4)^(3/2))/15
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.76, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2027, 2459, 2029, 2069, 1576, 27, 1116, 1104}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (2x+1)\left(x^2+x\right)^3\sqrt{1-\left(x^2+x\right)^2}dx$$

$$\Big\downarrow \text{2027}$$

$$\int x^3(x+1)^3(2x+1)\sqrt{1-\left(x^2+x\right)^2}dx$$

$$\Big\downarrow \text{2459}$$

$$\int \sqrt{-\left(x+\frac{1}{2}\right)^4+\frac{1}{2}\left(x+\frac{1}{2}\right)^2+\frac{15}{16}}\left(2\left(x+\frac{1}{2}\right)^7-\frac{3}{2}\left(x+\frac{1}{2}\right)^5+\frac{3}{8}\left(x+\frac{1}{2}\right)^3+\frac{1}{32}\left(-x-\frac{1}{2}\right)\right)d\left(x+\frac{1}{2}\right)$$

$$\Big\downarrow \text{2029}$$

$$\int \left(x+\frac{1}{2}\right)\sqrt{-\left(x+\frac{1}{2}\right)^4+\frac{1}{2}\left(x+\frac{1}{2}\right)^2+\frac{15}{16}}\left(2\left(x+\frac{1}{2}\right)^6-\frac{3}{2}\left(x+\frac{1}{2}\right)^4+\frac{3}{8}\left(x+\frac{1}{2}\right)^2-\frac{1}{32}\right)d\left(x+\frac{1}{2}\right)$$

$$\Big\downarrow \text{2069}$$

$$\int \left(x+\frac{1}{2}\right)\left(\sqrt[3]{2}\left(x+\frac{1}{2}\right)^2-\frac{1}{2\ 2^{2/3}}\right)^3\sqrt{-\left(x+\frac{1}{2}\right)^4+\frac{1}{2}\left(x+\frac{1}{2}\right)^2+\frac{15}{16}}d\left(x+\frac{1}{2}\right)$$

$$\Big\downarrow \text{1576}$$

$$\frac{1}{2}\int -\frac{1}{128}\left(1-4\left(x+\frac{1}{2}\right)^2\right)^3\sqrt{-16\left(x+\frac{1}{2}\right)^4+8\left(x+\frac{1}{2}\right)^2+15}d\left(x+\frac{1}{2}\right)^2$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{256}\int \left(1-4\left(x+\frac{1}{2}\right)^2\right)^3\sqrt{-16\left(x+\frac{1}{2}\right)^4+8\left(x+\frac{1}{2}\right)^2+15}d\left(x+\frac{1}{2}\right)^2$$

$$\Big\downarrow \text{1116}$$

$$\frac{1}{256}\left(-\frac{32}{5}\int \left(1-4\left(x+\frac{1}{2}\right)^2\right)\sqrt{-16\left(x+\frac{1}{2}\right)^4+8\left(x+\frac{1}{2}\right)^2+15}d\left(x+\frac{1}{2}\right)^2-\frac{1}{20}\left(-16\left(x+\frac{1}{2}\right)^4+8\left(x\right.\right.\right.$$

$$\Big\downarrow \text{1104}$$

$$\frac{1}{256}\left(-\frac{1}{20}\left(-16\left(x+\frac{1}{2}\right)^4+8\left(x+\frac{1}{2}\right)^2+15\right)^{3/2}\left(1-4\left(x+\frac{1}{2}\right)^2\right)^2-\frac{8}{15}\left(-16\left(x+\frac{1}{2}\right)^4+8\left(x+\frac{1}{2}\right)^2+\right.\right.$$

input
```
Int[(1 + 2*x)*(x + x^2)^3*Sqrt[1 - (x + x^2)^2],x]
```

output
```
((-8*(15 + 8*(1/2 + x)^2 - 16*(1/2 + x)^4)^(3/2))/15 - ((1 - 4*(1/2 + x)^2
)^2*(15 + 8*(1/2 + x)^2 - 16*(1/2 + x)^4)^(3/2))/20)/256
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1104
```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1116
```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[2*d*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(b*(m + 2*p
 + 1))), x] + Simp[d^2*(m - 1)*((b^2 - 4*a*c)/(b^2*(m + 2*p + 1)))   Int[(d
 + e*x)^(m - 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ[m, 1] && NeQ[m + 2*p
+ 1, 0] && (IntegerQ[2*p] || (IntegerQ[m] && RationalQ[p]) || OddQ[m])
```

rule 1576
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x]
, x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

rule 2029
```
Int[(Fx_.)*((d_.)*(x_)^(q_.) + (a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.) + (c_.)*
(x_)^(t_.))^(p_.), x_Symbol] :> Int[x^(p*r)*(a + b*x^(s - r) + c*x^(t - r)
+ d*x^(q - r))^p*Fx, x] /; FreeQ[{a, b, c, d, r, s, t, q}, x] && IntegerQ[p
] && PosQ[s - r] && PosQ[t - r] && PosQ[q - r] &&  !(EqQ[p, 1] && EqQ[u, 1]
)
```

rule 2069
```
Int[(u_.)*(Px_), x_Symbol] :> With[{a = Rt[Coeff[Px, x^2, 0], Expon[Px, x^2
]], b = Rt[Coeff[Px, x^2, Expon[Px, x^2]], Expon[Px, x^2]]}, Int[u*(a + b*x
^2)^Expon[Px, x^2], x] /; EqQ[Px, (a + b*x^2)^Expon[Px, x^2]]] /; PolyQ[Px,
 x^2] && GtQ[Expon[Px, x^2], 1] && NeQ[Coeff[Px, x^2, 0], 0] &&  !MatchQ[Px
, (a_.)*(v_)^Expon[Px, x^2] /; FreeQ[a, x] && BinomialQ[v, x, 2]]
```

rule 2459
```
Int[(Pn_)^(p_.)*(Qx_), x_Symbol] :> With[{S = Coeff[Pn, x, Expon[Pn, x] - 1
]/(Expon[Pn, x]*Coeff[Pn, x, Expon[Pn, x]])}, Subst[Int[ExpandToSum[Pn /. x
 -> x - S, x]^p*ExpandToSum[Qx /. x -> x - S, x], x], x, x + S] /; Binomial
Q[Pn /. x -> x - S, x] || (IntegerQ[Expon[Pn, x]/2] && TrinomialQ[Pn /. x -
> x - S, x])] /; FreeQ[p, x] && PolyQ[Pn, x] && GtQ[Expon[Pn, x], 2] && NeQ
[Coeff[Pn, x, Expon[Pn, x] - 1], 0] && PolyQ[Qx, x] &&  !(MonomialQ[Qx, x]
&& IGtQ[p, 0])
```

Maple [A] (verified)

Time = 1.33 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.83

method	result
pseudoelliptic	$-\frac{\left(-x^{4}-2x^{3}-x^{2}+1\right)^{\frac{3}{2}}\left(x^{4}+2x^{3}+x^{2}+\frac{2}{3}\right)}{5}$
gosper	$\frac{\left(x^{2}+x+1\right)\left(x^{2}+x-1\right)\left(3x^{4}+6x^{3}+3x^{2}+2\right)\sqrt{-x^{4}-2x^{3}-x^{2}+1}}{15}$
trager	$\left(\frac{1}{5}x^{8}+\frac{4}{5}x^{7}+\frac{6}{5}x^{6}+\frac{4}{5}x^{5}+\frac{2}{15}x^{4}-\frac{2}{15}x^{3}-\frac{1}{15}x^{2}-\frac{2}{15}\right)\sqrt{-x^{4}-2x^{3}-x^{2}+1}$
orering	$\frac{\left(3x^{4}+6x^{3}+3x^{2}+2\right)\left(x^{2}+x+1\right)\left(x^{2}+x-1\right)\left(x^{2}+x\right)^{3}\sqrt{1-\left(x^{2}+x\right)^{2}}}{15x^{3}\left(x+1\right)^{3}}$
risch	$-\frac{\left(3x^{8}+12x^{7}+18x^{6}+12x^{5}+2x^{4}-2x^{3}-x^{2}-2\right)\left(x^{4}+2x^{3}+x^{2}-1\right)}{15\sqrt{-x^{4}-2x^{3}-x^{2}+1}}$
default	$\frac{2x^{4}\sqrt{-x^{4}-2x^{3}-x^{2}+1}}{15}-\frac{2x^{3}\sqrt{-x^{4}-2x^{3}-x^{2}+1}}{15}-\frac{x^{2}\sqrt{-x^{4}-2x^{3}-x^{2}+1}}{15}-\frac{2\sqrt{-x^{4}-2x^{3}-x^{2}+1}}{15}+\frac{4x^{5}\sqrt{-x^{4}-2x^{3}}}{5}$
elliptic	$\frac{2x^{4}\sqrt{-x^{4}-2x^{3}-x^{2}+1}}{15}-\frac{2x^{3}\sqrt{-x^{4}-2x^{3}-x^{2}+1}}{15}-\frac{x^{2}\sqrt{-x^{4}-2x^{3}-x^{2}+1}}{15}-\frac{2\sqrt{-x^{4}-2x^{3}-x^{2}+1}}{15}+\frac{4x^{5}\sqrt{-x^{4}-2x^{3}}}{5}$

input
```
int((2*x+1)*(x^2+x)^3*(1-(x^2+x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/5*(-x^4-2*x^3-x^2+1)^(3/2)*(x^4+2*x^3+x^2+2/3)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.38

$$\int (1+2x)\left(x+x^2\right)^3\sqrt{1-\left(x+x^2\right)^2}\,dx$$
$$=\frac{1}{15}\left(3\,x^{8}+12\,x^{7}+18\,x^{6}+12\,x^{5}+2\,x^{4}-2\,x^{3}-x^{2}-2\right)\sqrt{-x^{4}-2\,x^{3}-x^{2}+1}$$

input
```
integrate((1+2*x)*(x^2+x)^3*(1-(x^2+x)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/15*(3*x^8 + 12*x^7 + 18*x^6 + 12*x^5 + 2*x^4 - 2*x^3 - x^2 - 2)*sqrt(-x^
4 - 2*x^3 - x^2 + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 182 vs. $2(36) = 72$.

Time = 0.61 (sec) , antiderivative size = 182, normalized size of antiderivative = 4.33

$$\int (1+2x)\left(x+x^2\right)^3\sqrt{1-\left(x+x^2\right)^2}\,dx = \frac{x^8\sqrt{-x^4-2x^3-x^2+1}}{5} + \frac{4x^7\sqrt{-x^4-2x^3-x^2+1}}{5} + \frac{6x^6\sqrt{-x^4-2x^3-x^2+1}}{5} + \frac{4x^5\sqrt{-x^4-2x^3-x^2+1}}{5} + \frac{2x^4\sqrt{-x^4-2x^3-x^2+1}}{15} - \frac{2x^3\sqrt{-x^4-2x^3-x^2+1}}{15} - \frac{x^2\sqrt{-x^4-2x^3-x^2+1}}{15} - \frac{2\sqrt{-x^4-2x^3-x^2+1}}{15}$$

input
```
integrate((1+2*x)*(x**2+x)**3*(1-(x**2+x)**2)**(1/2),x)
```

output
```
x**8*sqrt(-x**4 - 2*x**3 - x**2 + 1)/5 + 4*x**7*sqrt(-x**4 - 2*x**3 - x**2
 + 1)/5 + 6*x**6*sqrt(-x**4 - 2*x**3 - x**2 + 1)/5 + 4*x**5*sqrt(-x**4 - 2
*x**3 - x**2 + 1)/5 + 2*x**4*sqrt(-x**4 - 2*x**3 - x**2 + 1)/15 - 2*x**3*s
qrt(-x**4 - 2*x**3 - x**2 + 1)/15 - x**2*sqrt(-x**4 - 2*x**3 - x**2 + 1)/1
5 - 2*sqrt(-x**4 - 2*x**3 - x**2 + 1)/15
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.40

$$\int (1+2x)\left(x+x^2\right)^3\sqrt{1-\left(x+x^2\right)^2}\,dx$$
$$=\frac{1}{15}\left(3\,x^8+12\,x^7+18\,x^6+12\,x^5+2\,x^4-2\,x^3-x^2-2\right)\sqrt{x^2+x+1}\sqrt{-x^2-x+1}$$

input
```
integrate((1+2*x)*(x^2+x)^3*(1-(x^2+x)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/15*(3*x^8 + 12*x^7 + 18*x^6 + 12*x^5 + 2*x^4 - 2*x^3 - x^2 - 2)*sqrt(x^2
 + x + 1)*sqrt(-x^2 - x + 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.38

$$\int (1+2x)\left(x+x^2\right)^3\sqrt{1-\left(x+x^2\right)^2}\,dx=\frac{1}{5}\left(x^4+2\,x^3+x^2-1\right)^2\sqrt{-x^4-2\,x^3-x^2+1}-\frac{1}{3}\left(-x^4-2\,x^3-x^2+1\right)^{\frac{3}{2}}$$

input
```
integrate((1+2*x)*(x^2+x)^3*(1-(x^2+x)^2)^(1/2),x, algorithm="giac")
```

output
```
1/5*(x^4 + 2*x^3 + x^2 - 1)^2*sqrt(-x^4 - 2*x^3 - x^2 + 1) - 1/3*(-x^4 - 2
*x^3 - x^2 + 1)^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.21

$$\int (1+2x)\left(x+x^2\right)^3\sqrt{1-\left(x+x^2\right)^2}\,dx=\sqrt{1-\left(x^2+x\right)^2}\left(\frac{x^8}{5}+\frac{4\,x^7}{5}+\frac{6\,x^6}{5}+\frac{4\,x^5}{5}+\frac{2\,x^4}{15}-\frac{2\,x^3}{15}-\frac{x^2}{15}-\frac{2}{15}\right)$$

input
```
int((2*x + 1)*(1 - (x + x^2)^2)^(1/2)*(x + x^2)^3,x)
```

output
```
(1 - (x + x^2)^2)^(1/2)*((2*x^4)/15 - (2*x^3)/15 - x^2/15 + (4*x^5)/5 + (6
*x^6)/5 + (4*x^7)/5 + x^8/5 - 2/15)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.36

$$\int (1+2x)\left(x+x^2\right)^3\sqrt{1-\left(x+x^2\right)^2}\,dx = \frac{\sqrt{-x^4-2x^3-x^2+1}\,(3x^8+12x^7+18x^6+12x^5+2x^4-2x^3-x^2-2)}{15}$$

input
```
int((1+2*x)*(x^2+x)^3*(1-(x^2+x)^2)^(1/2),x)
```

output
```
(sqrt( - x**4 - 2*x**3 - x**2 + 1)*(3*x**8 + 12*x**7 + 18*x**6 + 12*x**5 +
 2*x**4 - 2*x**3 - x**2 - 2))/15
```

3.66 $\int \frac{ef-efx^2}{(ad+bdx+adx^2)\sqrt{a+bx+cx^2+bx^3+ax^4}}\,dx$

Optimal result

Integrand size = 52, antiderivative size = 88

$$\int \frac{ef-efx^2}{(ad+bdx+adx^2)\sqrt{a+bx+cx^2+bx^3+ax^4}}\,dx = \frac{ef\arctan\left(\frac{ab+(4a^2+b^2-2ac)x+abx^2}{2a\sqrt{2a-c}\sqrt{a+bx+cx^2+bx^3+ax^4}}\right)}{a\sqrt{2a-c}\,d}$$

output
```
e*f*arctan(1/2*(a*b+(4*a^2-2*a*c+b^2)*x+a*b*x^2)/a/(2*a-c)^(1/2)/(a*x^4+b*
x^3+c*x^2+b*x+a)^(1/2))/a/(2*a-c)^(1/2)/d
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 244 vs. 2(88) = 176.

Time = 1.52 (sec) , antiderivative size = 244, normalized size of antiderivative = 2.77

$$\int \frac{ef-efx^2}{(ad+bdx+adx^2)\sqrt{a+bx+cx^2+bx^3+ax^4}}\,dx = -\frac{ef\left(2\sqrt{-2a+c}\arctan\left(\frac{2a\sqrt{2a-c}(1+x^2)}{b\left(-\sqrt{-2a+c}x+\sqrt{a+bx+cx^2+bx^3+ax^4}\right)}\right)+\sqrt{2a-c}\left(2\log\left(-\sqrt{-2a+c}x+\sqrt{a+bx\right.\right.\right.}{}$$

input
```
Integrate[(e*f - e*f*x^2)/((a*d + b*d*x + a*d*x^2)*Sqrt[a + b*x + c*x^2 +
b*x^3 + a*x^4]),x]
```

output
```
-1/2*(e*f*(2*Sqrt[-2*a + c]*ArcTan[(2*a*Sqrt[2*a - c]*(1 + x^2))/(b*(-(Sqr
t[-2*a + c]*x) + Sqrt[a + b*x + c*x^2 + b*x^3 + a*x^4]))] + Sqrt[2*a - c]*
(2*Log[-(Sqrt[-2*a + c]*x) + Sqrt[a + b*x + c*x^2 + b*x^3 + a*x^4]] - Log[
a*b^2*(-1 + x^2)^2 + 8*a^3*(1 + x^2)^2 - 4*a^2*c*(1 + x^2)^2 + b^2*x*(b +
2*c*x + b*x^2 - 2*Sqrt[-2*a + c]*Sqrt[a + b*x + c*x^2 + b*x^3 + a*x^4])]))
)/(a*Sqrt[-(-2*a + c)^2]*d)
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.019$, Rules used = {2507}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ef - efx^2}{\sqrt{ax^4 + a + bx^3 + bx + cx^2}\,(adx^2 + ad + bdx)}dx$$

$$\Big\downarrow \; 2507$$

$$\frac{ef \arctan\left(\frac{x(4a^2-2ac+b^2)+abx^2+ab}{2a\sqrt{2a-c}\sqrt{ax^4+a+bx^3+bx+cx^2}}\right)}{ad\sqrt{2a-c}}$$

input
```
Int[(e*f - e*f*x^2)/((a*d + b*d*x + a*d*x^2)*Sqrt[a + b*x + c*x^2 + b*x^3
+ a*x^4]),x]
```

output
```
(e*f*ArcTan[(a*b + (4*a^2 + b^2 - 2*a*c)*x + a*b*x^2)/(2*a*Sqrt[2*a - c]*S
qrt[a + b*x + c*x^2 + b*x^3 + a*x^4])])/(a*Sqrt[2*a - c]*d)
```

Defintions of rubi rules used

rule 2507
```
Int[((f_) + (g_.)*(x_)^2)/(((d_) + (e_.)*(x_) + (d_.)*(x_)^2)*Sqrt[(a_) + (
b_.)*(x_) + (c_.)*(x_)^2 + (b_.)*(x_)^3 + (a_.)*(x_)^4]), x_Symbol] :> Simp
[a*(f/(d*Rt[a^2*(2*a - c), 2]))*ArcTan[(a*b + (4*a^2 + b^2 - 2*a*c)*x + a*b
*x^2)/(2*Rt[a^2*(2*a - c), 2]*Sqrt[a + b*x + c*x^2 + b*x^3 + a*x^4])], x] /
; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[b*d - a*e, 0] && EqQ[f + g, 0] &&
PosQ[a^2*(2*a - c)]
```

Maple [A] (verified)

Time = 3.71 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.05

method	result	size
default	$\frac{ef \ln\left(\frac{2\sqrt{-2a+c}\,\sqrt{a\,x^4+b\,x^3+c\,x^2+bx+a}\,a-4x\,a^2+\left(-b\,x^2+2cx-b\right)a-b^2x}{a\,x^2+bx+a}\right)}{d\sqrt{-2a+c}\,a}$	92
pseudoelliptic	$\frac{ef \ln\left(\frac{2\sqrt{-2a+c}\,\sqrt{a\,x^4+b\,x^3+c\,x^2+bx+a}\,a-4x\,a^2+\left(-b\,x^2+2cx-b\right)a-b^2x}{a\,x^2+bx+a}\right)}{d\sqrt{-2a+c}\,a}$	92
elliptic	Expression too large to display	254498

input
```
int((-e*f*x^2+e*f)/(a*d*x^2+b*d*x+a*d)/(a*x^4+b*x^3+c*x^2+b*x+a)^(1/2),x,m
ethod=_RETURNVERBOSE)
```

output
```
e*f/d/(-2*a+c)^(1/2)*ln((2*(-2*a+c)^(1/2)*(a*x^4+b*x^3+c*x^2+b*x+a)^(1/2)*
a-4*x*a^2+(-b*x^2+2*c*x-b)*a-b^2*x)/(a*x^2+b*x+a))/a
```

Fricas [A] (verification not implemented)

Time = 1.94 (sec) , antiderivative size = 324, normalized size of antiderivative = 3.68

$$\int \frac{ef - efx^2}{(ad + bdx + adx^2)\sqrt{a + bx + cx^2 + bx^3 + ax^4}}\,dx$$
$$= \left[-\frac{\sqrt{-2\,a + c}ef\log\left(\frac{2\,ab^3x^3+2\,ab^3x-(8\,a^4-a^2b^2-4\,a^3c)x^4-8\,a^4+a^2b^2+4\,a^3c+(16\,a^4+10\,a^2b^2+b^4+8\,a^2c^2-4\,(6\,a^3+ab^2)c)x^2-4\,(a}{a^2x^4+2\,abx^3+2\,abx+(2\,a^2+b^2)x^2+a^2}\right.}{2\,(2\,a^2 - ac)d} \right.$$
$$\left. -\frac{\sqrt{2\,a - c}ef\arctan\left(\frac{2\,\sqrt{ax^4+bx^3+cx^2+bx+a}\sqrt{2\,a-c}a}{abx^2+ab+(4\,a^2+b^2-2\,ac)x}\right)}{(2\,a^2 - ac)d}\right]$$

input
```
integrate((-e*f*x^2+e*f)/(a*d*x^2+b*d*x+a*d)/(a*x^4+b*x^3+c*x^2+b*x+a)^(1/
2),x, algorithm="fricas")
```

output
```
[-1/2*sqrt(-2*a + c)*e*f*log((2*a*b^3*x^3 + 2*a*b^3*x - (8*a^4 - a^2*b^2 -
 4*a^3*c)*x^4 - 8*a^4 + a^2*b^2 + 4*a^3*c + (16*a^4 + 10*a^2*b^2 + b^4 + 8
*a^2*c^2 - 4*(6*a^3 + a*b^2)*c)*x^2 - 4*(a^2*b*x^2 + a^2*b + (4*a^3 + a*b^
2 - 2*a^2*c)*x)*sqrt(a*x^4 + b*x^3 + c*x^2 + b*x + a)*sqrt(-2*a + c))/(a^2
*x^4 + 2*a*b*x^3 + 2*a*b*x + (2*a^2 + b^2)*x^2 + a^2))/((2*a^2 - a*c)*d),
-sqrt(2*a - c)*e*f*arctan(2*sqrt(a*x^4 + b*x^3 + c*x^2 + b*x + a)*sqrt(2*a
 - c)*a/(a*b*x^2 + a*b + (4*a^2 + b^2 - 2*a*c)*x))/((2*a^2 - a*c)*d)]
```

Sympy [F]

$$\int \frac{ef - efx^2}{(ad + bdx + adx^2)\sqrt{a + bx + cx^2 + bx^3 + ax^4}}\,dx =$$
$$-\frac{ef\left(\int \frac{x^2}{ax^2\sqrt{ax^4+a+bx^3+bx+cx^2}+a\sqrt{ax^4+a+bx^3+bx+cx^2}+bx\sqrt{ax^4+a+bx^3+bx+cx^2}}\,dx + \int\left(-\frac{}{ax^2\sqrt{ax^4+a+bx^3+bx+cx^2}+a\sqrt{a}}\right.\right.}{d}$$

input
```
integrate((-e*f*x**2+e*f)/(a*d*x**2+b*d*x+a*d)/(a*x**4+b*x**3+c*x**2+b*x+a
)**(1/2),x)
```

output
```
-e*f*(Integral(x**2/(a*x**2*sqrt(a*x**4 + a + b*x**3 + b*x + c*x**2) + a*s
qrt(a*x**4 + a + b*x**3 + b*x + c*x**2) + b*x*sqrt(a*x**4 + a + b*x**3 + b
*x + c*x**2)), x) + Integral(-1/(a*x**2*sqrt(a*x**4 + a + b*x**3 + b*x + c
*x**2) + a*sqrt(a*x**4 + a + b*x**3 + b*x + c*x**2) + b*x*sqrt(a*x**4 + a
+ b*x**3 + b*x + c*x**2)), x))/d
```

Maxima [F]

$$\int \frac{ef - efx^2}{(ad + bdx + adx^2)\sqrt{a + bx + cx^2 + bx^3 + ax^4}}\,dx = \int -\frac{efx^2 - ef}{\sqrt{ax^4 + bx^3 + cx^2 + bx + a}(adx^2 + bdx + ad)}\,dx$$

input
```
integrate((-e*f*x^2+e*f)/(a*d*x^2+b*d*x+a*d)/(a*x^4+b*x^3+c*x^2+b*x+a)^(1/
2),x, algorithm="maxima")
```

output
```
-integrate((e*f*x^2 - e*f)/(sqrt(a*x^4 + b*x^3 + c*x^2 + b*x + a)*(a*d*x^2
 + b*d*x + a*d)), x)
```

Giac [F]

$$\int \frac{ef - efx^2}{(ad + bdx + adx^2)\sqrt{a + bx + cx^2 + bx^3 + ax^4}}\,dx = \int -\frac{efx^2 - ef}{\sqrt{ax^4 + bx^3 + cx^2 + bx + a}(adx^2 + bdx + ad)}\,dx$$

input
```
integrate((-e*f*x^2+e*f)/(a*d*x^2+b*d*x+a*d)/(a*x^4+b*x^3+c*x^2+b*x+a)^(1/
2),x, algorithm="giac")
```

output
```
integrate(-(e*f*x^2 - e*f)/(sqrt(a*x^4 + b*x^3 + c*x^2 + b*x + a)*(a*d*x^2
 + b*d*x + a*d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{ef-efx^2}{(ad+bdx+adx^2)\sqrt{a+bx+cx^2+bx^3+ax^4}}\,dx = \int \frac{e\,f-e\,f\,x^2}{(a\,d\,x^2+b\,d\,x+a\,d)\,\sqrt{a\,x^4+b\,x^3+c\,x^2+b\,x+a}}\,dx$$

input
```
int((e*f - e*f*x^2)/((a*d + b*d*x + a*d*x^2)*(a + b*x + a*x^4 + b*x^3 + c*
x^2)^(1/2)),x)
```

output
```
int((e*f - e*f*x^2)/((a*d + b*d*x + a*d*x^2)*(a + b*x + a*x^4 + b*x^3 + c*
x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{ef-efx^2}{(ad+bdx+adx^2)\sqrt{a+bx+cx^2+bx^3+ax^4}}\,dx = \int \frac{-ef\,x^2+ef}{(ad\,x^2+bdx+ad)\,\sqrt{a\,x^4+b\,x^3+c\,x^2+bx+a}}dx$$

input
```
int((-e*f*x^2+e*f)/(a*d*x^2+b*d*x+a*d)/(a*x^4+b*x^3+c*x^2+b*x+a)^(1/2),x)
```

output
```
int((-e*f*x^2+e*f)/(a*d*x^2+b*d*x+a*d)/(a*x^4+b*x^3+c*x^2+b*x+a)^(1/2),x)
```

3.67 $\int \frac{ef-efx^2}{(-ad+bdx-adx^2)\sqrt{-a+bx+cx^2+bx^3-ax^4}}\,dx$

Optimal result

Integrand size = 57, antiderivative size = 88

$$\int \frac{ef-efx^2}{(-ad+bdx-adx^2)\sqrt{-a+bx+cx^2+bx^3-ax^4}}\,dx = \frac{ef\,\text{arctanh}\left(\frac{ab-(4a^2+b^2+2ac)x+abx^2}{2a\sqrt{2a+c}\sqrt{-a+bx+cx^2+bx^3-ax^4}}\right)}{a\sqrt{2a+c}\,d}$$

output
```
e*f*arctanh(1/2*(a*b-(4*a^2+2*a*c+b^2)*x+a*b*x^2)/a/(2*a+c)^(1/2)/(-a*x^4+
b*x^3+c*x^2+b*x-a)^(1/2))/a/(2*a+c)^(1/2)/d
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 218 vs. 2(88) = 176.

Time = 1.45 (sec) , antiderivative size = 218, normalized size of antiderivative = 2.48

$$\int \frac{ef-efx^2}{(-ad+bdx-adx^2)\sqrt{-a+bx+cx^2+bx^3-ax^4}}\,dx = -\frac{ef\left(2\text{arctanh}\left(\frac{2a\sqrt{2a+c}(1+x^2)}{b\left(-\sqrt{2a+c}x+\sqrt{x(b+cx+bx^2)-a(1+x^4)}\right)}\right)-2\log\left(-\sqrt{2a+c}x+\sqrt{x(b+cx+bx^2)-a(1+x\right.\right.}{}$$

input
```
Integrate[(e*f - e*f*x^2)/((-(a*d) + b*d*x - a*d*x^2)*Sqrt[-a + b*x + c*x^
2 + b*x^3 - a*x^4]),x]
```

output
```
-1/2*(e*f*(2*ArcTanh[(2*a*Sqrt[2*a + c]*(1 + x^2))/(b*(-(Sqrt[2*a + c]*x)
+ Sqrt[x*(b + c*x + b*x^2) - a*(1 + x^4)]))] - 2*Log[-(Sqrt[2*a + c]*x) +
Sqrt[x*(b + c*x + b*x^2) - a*(1 + x^4)]] + Log[a*b^2*(-1 + x^2)^2 + 8*a^3*
(1 + x^2)^2 + 4*a^2*c*(1 + x^2)^2 - b^2*x*(b + 2*c*x + b*x^2 - 2*Sqrt[2*a
+ c]*Sqrt[x*(b + c*x + b*x^2) - a*(1 + x^4)])]))/(a*Sqrt[2*a + c]*d)
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.018$, Rules used = {2508}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ef - efx^2}{\sqrt{-ax^4 - a + bx^3 + bx + cx^2}\,(-adx^2 - ad + bdx)}dx$$
$$\Big\downarrow \; 2508$$
$$\frac{ef\text{arctanh}\left(\frac{-x(4a^2+2ac+b^2)+abx^2+ab}{2a\sqrt{2a+c}\sqrt{-ax^4-a+bx^3+bx+cx^2}}\right)}{ad\sqrt{2a+c}}$$

input
```
Int[(e*f - e*f*x^2)/((-(a*d) + b*d*x - a*d*x^2)*Sqrt[-a + b*x + c*x^2 + b*
x^3 - a*x^4]),x]
```

output
```
(e*f*ArcTanh[(a*b - (4*a^2 + b^2 + 2*a*c)*x + a*b*x^2)/(2*a*Sqrt[2*a + c]*
Sqrt[-a + b*x + c*x^2 + b*x^3 - a*x^4]])])/(a*Sqrt[2*a + c]*d)
```

Defintions of rubi rules used

rule 2508
```
Int[((f_) + (g_.)*(x_)^2)/(((d_) + (e_.)*(x_) + (d_.)*(x_)^2)*Sqrt[(a_) + (
b_.)*(x_) + (c_.)*(x_)^2 + (b_.)*(x_)^3 + (a_.)*(x_)^4]), x_Symbol] :> Simp
[(-a)*(f/(d*Rt[(-a^2)*(2*a - c), 2]))*ArcTanh[(a*b + (4*a^2 + b^2 - 2*a*c)*
x + a*b*x^2)/(2*Rt[(-a^2)*(2*a - c), 2]*Sqrt[a + b*x + c*x^2 + b*x^3 + a*x^
4])], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[b*d - a*e, 0] && EqQ[f +
 g, 0] && NegQ[a^2*(2*a - c)]
```

Maple [A] (verified)

Time = 3.70 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.09

method	result	size
default	$-\frac{ef \ln\left(\frac{2\sqrt{2a+c}\,\sqrt{-a\,x^{4}+b\,x^{3}+c\,x^{2}+bx-a}\,a+4x\,a^{2}+\left(-b\,x^{2}+2cx-b\right)a+b^{2}x}{a\,x^{2}-bx+a}\right)}{d\sqrt{2a+c}\,a}$	96
pseudoelliptic	$-\frac{ef \ln\left(\frac{2\sqrt{2a+c}\,\sqrt{-a\,x^{4}+b\,x^{3}+c\,x^{2}+bx-a}\,a+4x\,a^{2}+\left(-b\,x^{2}+2cx-b\right)a+b^{2}x}{a\,x^{2}-bx+a}\right)}{d\sqrt{2a+c}\,a}$	96
elliptic	Expression too large to display	281960

input
```
int((-e*f*x^2+e*f)/(-a*d*x^2+b*d*x-a*d)/(-a*x^4+b*x^3+c*x^2+b*x-a)^(1/2),x
,method=_RETURNVERBOSE)
```

output
```
-e*f/d/(2*a+c)^(1/2)*ln((2*(2*a+c)^(1/2)*(-a*x^4+b*x^3+c*x^2+b*x-a)^(1/2)*
a+4*x*a^2+(-b*x^2+2*c*x-b)*a+b^2*x)/(a*x^2-b*x+a))/a
```

Fricas [A] (verification not implemented)

Time = 1.93 (sec) , antiderivative size = 331, normalized size of antiderivative = 3.76

$$\int \frac{ef - efx^2}{(-ad + bdx - adx^2)\sqrt{-a + bx + cx^2 + bx^3 - ax^4}}\,dx = \left[\frac{\sqrt{2a+c}ef\log\left(\frac{2ab^3x^3+2ab^3x+(8a^4-a^2b^2+4a^3c)x^4+8a^4-a^2b^2+4a^3c-(16a^4+10a^2b^2+b^4+8a^2c^2+4(6a^3+ab^2)c)x^2-4(a^2bx^2 \cdots}{a^2x^4-2abx^3-2abx+(2a^2+b^2)x^2+a^2}\right)}{2(2a^2+ac)d} - \frac{\sqrt{-2a-c}ef\arctan\left(\frac{2\sqrt{-ax^4+bx^3+cx^2+bx-a}a\sqrt{-2a-c}}{abx^2+ab-(4a^2+b^2+2ac)x}\right)}{(2a^2+ac)d}\right]$$

input
```
integrate((-e*f*x^2+e*f)/(-a*d*x^2+b*d*x-a*d)/(-a*x^4+b*x^3+c*x^2+b*x-a)^(
1/2),x, algorithm="fricas")
```

output
```
[1/2*sqrt(2*a + c)*e*f*log((2*a*b^3*x^3 + 2*a*b^3*x + (8*a^4 - a^2*b^2 + 4
*a^3*c)*x^4 + 8*a^4 - a^2*b^2 + 4*a^3*c - (16*a^4 + 10*a^2*b^2 + b^4 + 8*a
^2*c^2 + 4*(6*a^3 + a*b^2)*c)*x^2 - 4*(a^2*b*x^2 + a^2*b - (4*a^3 + a*b^2
+ 2*a^2*c)*x)*sqrt(-a*x^4 + b*x^3 + c*x^2 + b*x - a)*sqrt(2*a + c))/(a^2*x
^4 - 2*a*b*x^3 - 2*a*b*x + (2*a^2 + b^2)*x^2 + a^2))/((2*a^2 + a*c)*d), -s
qrt(-2*a - c)*e*f*arctan(2*sqrt(-a*x^4 + b*x^3 + c*x^2 + b*x - a)*a*sqrt(-
2*a - c)/(a*b*x^2 + a*b - (4*a^2 + b^2 + 2*a*c)*x))/((2*a^2 + a*c)*d)]
```

Sympy [F]

$$\int \frac{ef - efx^2}{(-ad + bdx - adx^2)\sqrt{-a + bx + cx^2 + bx^3 - ax^4}}\,dx = \frac{ef\left(\int \frac{x^2}{ax^2\sqrt{-ax^4-a+bx^3+bx+cx^2}+a\sqrt{-ax^4-a+bx^3+bx+cx^2}-bx\sqrt{-ax^4-a+bx^3+bx+cx^2}}\,dx + \int\left(-\frac{\cdots}{ax^2\sqrt{-ax^4-a+bx^3+bx+cx^2}+\cdots}\right.\right.}{d}$$

input
```
integrate((-e*f*x**2+e*f)/(-a*d*x**2+b*d*x-a*d)/(-a*x**4+b*x**3+c*x**2+b*x
-a)**(1/2),x)
```

output
```
e*f*(Integral(x**2/(a*x**2*sqrt(-a*x**4 - a + b*x**3 + b*x + c*x**2) + a*s
qrt(-a*x**4 - a + b*x**3 + b*x + c*x**2) - b*x*sqrt(-a*x**4 - a + b*x**3 +
 b*x + c*x**2)), x) + Integral(-1/(a*x**2*sqrt(-a*x**4 - a + b*x**3 + b*x
+ c*x**2) + a*sqrt(-a*x**4 - a + b*x**3 + b*x + c*x**2) - b*x*sqrt(-a*x**4
 - a + b*x**3 + b*x + c*x**2)), x))/d
```

Maxima [F]

$$\int \frac{ef - efx^2}{(-ad + bdx - adx^2)\sqrt{-a + bx + cx^2 + bx^3 - ax^4}}\,dx = \int \frac{efx^2 - ef}{\sqrt{-ax^4 + bx^3 + cx^2 + bx - a}(adx^2 - bdx + ad)}\,dx$$

input
```
integrate((-e*f*x^2+e*f)/(-a*d*x^2+b*d*x-a*d)/(-a*x^4+b*x^3+c*x^2+b*x-a)^(
1/2),x, algorithm="maxima")
```

output
```
integrate((e*f*x^2 - e*f)/(sqrt(-a*x^4 + b*x^3 + c*x^2 + b*x - a)*(a*d*x^2
 - b*d*x + a*d)), x)
```

Giac [F]

$$\int \frac{ef - efx^2}{(-ad + bdx - adx^2)\sqrt{-a + bx + cx^2 + bx^3 - ax^4}}\,dx = \int \frac{efx^2 - ef}{\sqrt{-ax^4 + bx^3 + cx^2 + bx - a}(adx^2 - bdx + ad)}\,dx$$

input
```
integrate((-e*f*x^2+e*f)/(-a*d*x^2+b*d*x-a*d)/(-a*x^4+b*x^3+c*x^2+b*x-a)^(
1/2),x, algorithm="giac")
```

output
```
integrate((e*f*x^2 - e*f)/(sqrt(-a*x^4 + b*x^3 + c*x^2 + b*x - a)*(a*d*x^2
 - b*d*x + a*d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{ef-efx^2}{(-ad+bdx-adx^2)\sqrt{-a+bx+cx^2+bx^3-ax^4}}\,dx = \int -\frac{e\,f-e\,f\,x^2}{\left(a\,d\,x^2-b\,d\,x+a\,d\right)\,\sqrt{-a\,x^4+b\,x^3+c\,x^2+b\,x-a}}\,dx$$

input
```
int(-(e*f - e*f*x^2)/((a*d - b*d*x + a*d*x^2)*(b*x - a - a*x^4 + b*x^3 + c
*x^2)^(1/2)),x)
```

output
```
int(-(e*f - e*f*x^2)/((a*d - b*d*x + a*d*x^2)*(b*x - a - a*x^4 + b*x^3 + c
*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{ef-efx^2}{(-ad+bdx-adx^2)\sqrt{-a+bx+cx^2+bx^3-ax^4}}\,dx = \frac{ef\left(\int \frac{\sqrt{-a\,x^4+b\,x^3+c\,x^2+bx-a}}{a^2x^6-2ab\,x^5+a^2x^4-ac\,x^4+b^2x^4-2ab\,x^3+bc\,x^3+a^2x^2-ac\,x^2+b^2x^2-2abx+a^2}dx-\left(\int \frac{\sqrt{-a\,x^4+b\,x^3+c\,x}}{a^2x^6-2ab\,x^5+a^2x^4-ac\,x^4+b^2x^4-2ab\,x^3-}\right.\right.}{d}$$

input
```
int((-e*f*x^2+e*f)/(-a*d*x^2+b*d*x-a*d)/(-a*x^4+b*x^3+c*x^2+b*x-a)^(1/2),x
)
```

output
```
(e*f*(int(sqrt( - a*x**4 - a + b*x**3 + b*x + c*x**2)/(a**2*x**6 + a**2*x*
*4 + a**2*x**2 + a**2 - 2*a*b*x**5 - 2*a*b*x**3 - 2*a*b*x - a*c*x**4 - a*c
*x**2 + b**2*x**4 + b**2*x**2 + b*c*x**3),x) - int((sqrt( - a*x**4 - a + b
*x**3 + b*x + c*x**2)*x**2)/(a**2*x**6 + a**2*x**4 + a**2*x**2 + a**2 - 2*
a*b*x**5 - 2*a*b*x**3 - 2*a*b*x - a*c*x**4 - a*c*x**2 + b**2*x**4 + b**2*x
**2 + b*c*x**3),x)))/d
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                   convert(leaf_count_result,string)," vs. $2 (",
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
  return func in [hyper]

def is_appell_function(func):
  return func in [appellf1]

def is_atom(expn):
  try:
    if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
      return True
    else:
      return False

  except AttributeError as error:
    return False

def expnType(expn):
  debug=False
  if debug:
    print("expn=",expn,"type(expn)=",type(expn))

  if is_atom(expn):
    return 1
  elif isinstance(expn,list):
    return max(map(expnType, expn))  #apply(max,map(ExpnType,expn))
  elif  is_sqrt(expn):
    if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
      return 1
    else:
      return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
  elif isinstance(expn,Pow):  #type(expn,'`^`')
    if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
      return expnType(expn.args[0])  #ExpnType(op(1,expn))
    elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
      if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
        return 1
      else:
        return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    else:
      return max(3,expnType(expn.args[0]),expnType(expn.args[1]))  #max(3,ExpnType(op(1,expn)),Exp
  elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)  #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
else:
    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
            else: #result contains complex but optimal is not
                grade = "C"
                grade_annotation ="Result contains complex when optimal does not."
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =""
            else:
                grade = "B"
                grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
    else:
        grade = "C"
        grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType(


#print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#           Albert Rich to use with Sagemath. This is used to
#           grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#           'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#           issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
   r"""
   Return the tree size of this expression.
   """
   #print("Enter tree_size, expr is ",expr)

   if expr not in SR:
      # deal with lists, tuples, vectors
      return 1 + sum(tree_size(a) for a in expr)
   expr = SR(expr)
   x, aa = expr.operator(), expr.operands()
   if x is None:
      return 1
   else:
      return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
   if expr.operator() == operator.pow:   #isinstance(expr,Pow):
      if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
         if debug: print ("expr is sqrt")
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   #debug=False
   m = func.name() in ['exp','log','ln',
         'sin','cos','tan','cot','sec','csc',
         'arcsin','arccos','arctan','arccot','arcsec','arccsc',
         'sinh','cosh','tanh','coth','sech','csch',
         'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
      'arctan2','floor','abs'
      ]
   if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral',
          'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
          'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
         print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
```

```
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expr
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file