Computer Algebra
Independent Integration Tests

Summer 2024

1-Algebraic-functions/1.5-Polynomial/147-1.7.2

Nasser M. Abbasi

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Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**320**]. This is test number [147].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	93.12 (298)	6.88 (22)
Mathematica	93.12 (298)	6.88 (22)
Maple	80.94 (259)	19.06 (61)
Mupad	60.00 (192)	40.00 (128)
Fricas	59.38 (190)	40.62 (130)
Sympy	58.12 (186)	41.88 (134)
Giac	47.81 (153)	52.19 (167)
Reduce	45.31 (145)	54.69 (175)
Maxima	39.06 (125)	60.94 (195)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

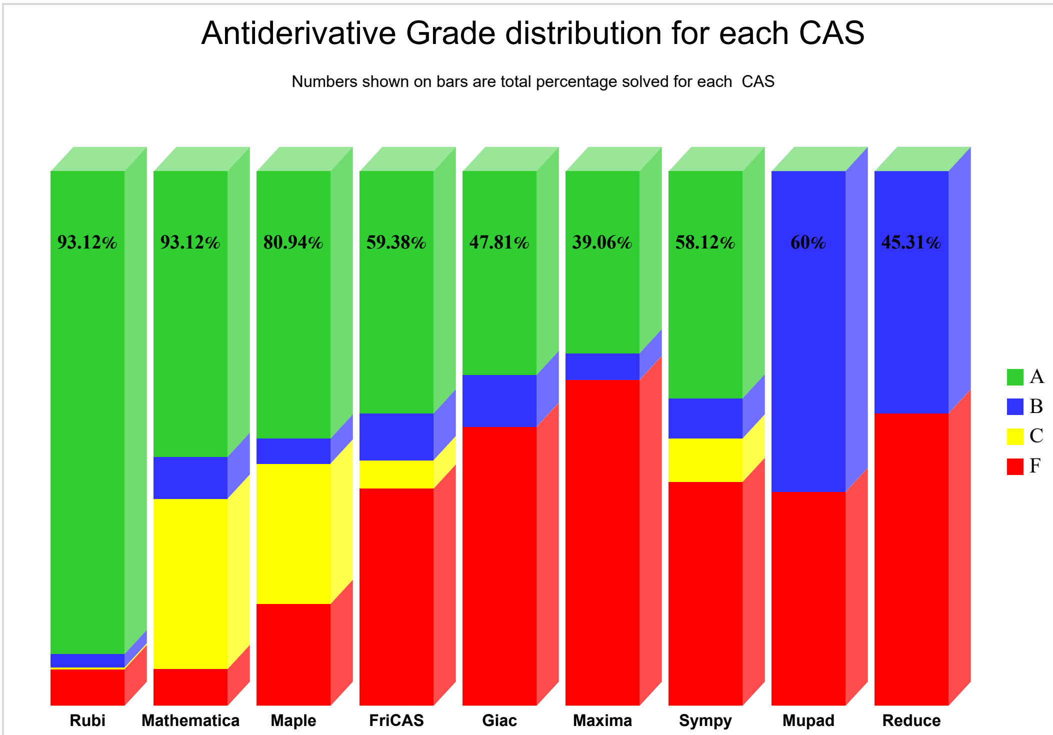
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

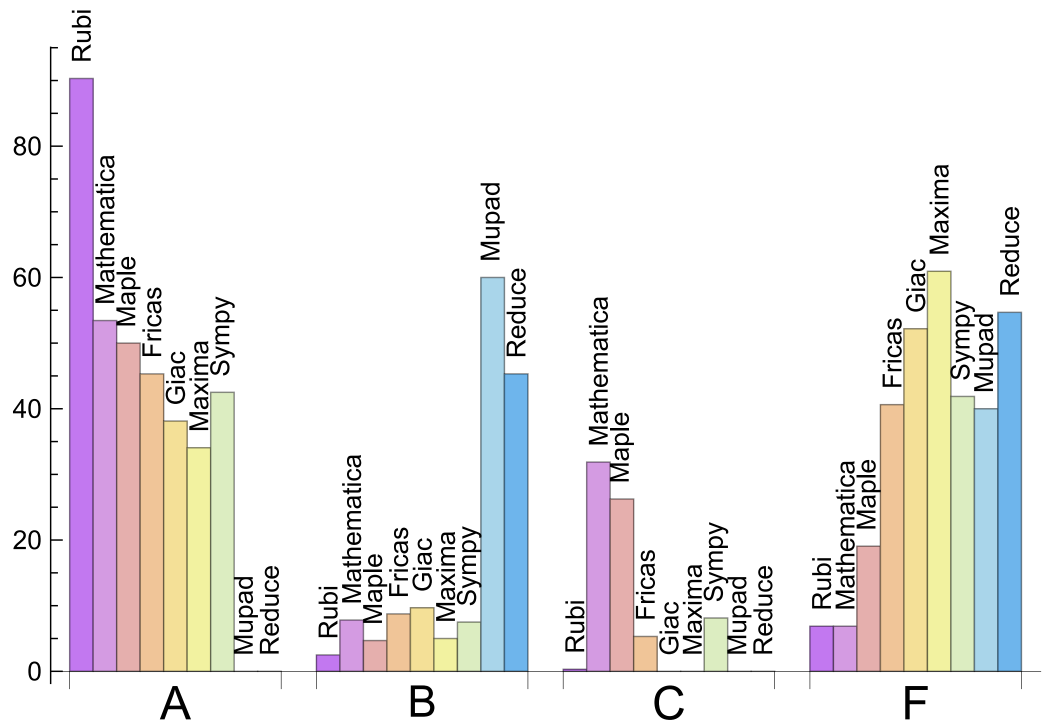
System	% A grade	% B grade	% C grade	% F grade
Rubi	90.312	2.500	0.312	6.875
Mathematica	53.438	7.812	31.875	6.875
Maple	50.000	4.688	26.250	19.062
Fricas	45.312	8.750	5.312	40.625
Sympy	42.500	7.500	8.125	41.875
Giac	38.125	9.688	0.000	52.188
Maxima	34.062	5.000	0.000	60.938
Mupad	0.000	60.000	0.000	40.000
Reduce	0.000	45.312	0.000	54.688

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	22	100.00	0.00	0.00
Mathematica	22	90.91	9.09	0.00
Maple	61	100.00	0.00	0.00
Mupad	128	0.00	100.00	0.00
Fricas	130	36.15	59.23	4.62
Sympy	134	58.21	37.31	4.48
Giac	167	91.02	4.79	4.19
Reduce	175	100.00	0.00	0.00
Maxima	195	96.92	0.00	3.08

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.06
Reduce	0.25
Giac	0.51
Rubi	1.15
Maple	1.17
Fricas	1.49
Mathematica	3.49
Sympy	7.65
Mupad	11.98

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Sympy	109.53	1.31	75.00	0.84
Maxima	133.85	1.43	65.00	0.98
Maple	222.72	1.07	91.00	0.87
Rubi	259.58	1.12	154.00	1.00
Mathematica	391.98	1.27	121.50	1.00
Reduce	557.82	2.32	85.00	1.12
Mupad	844.25	2.29	78.50	0.96
Giac	1266.93	2.91	50.00	1.06
Fricas	10955.22	46.94	86.50	1.09

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

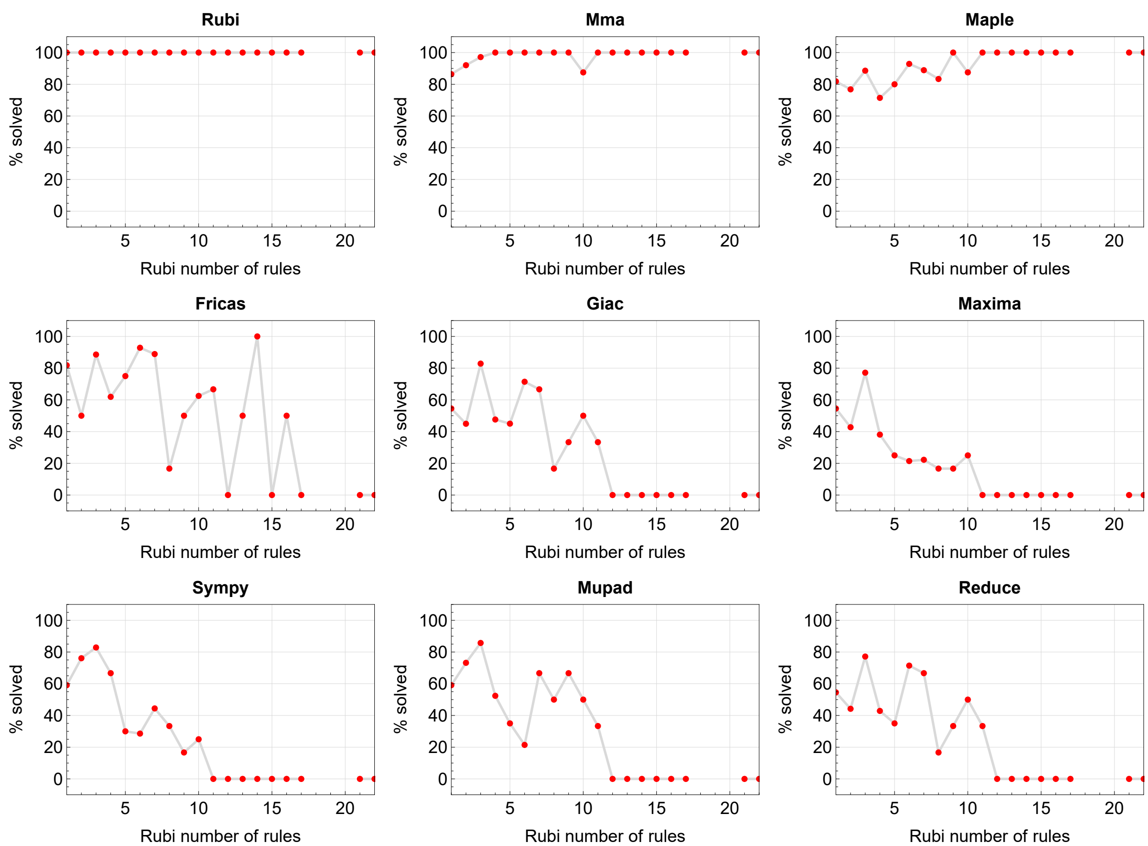


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

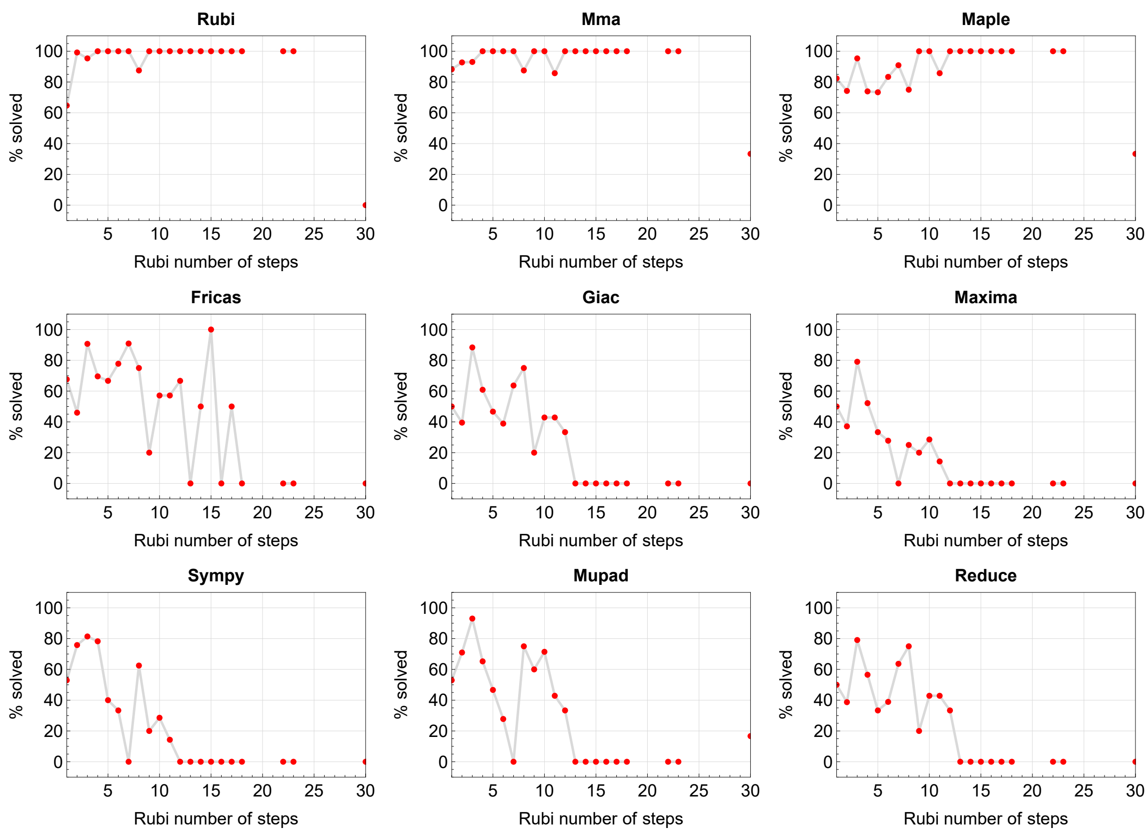


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

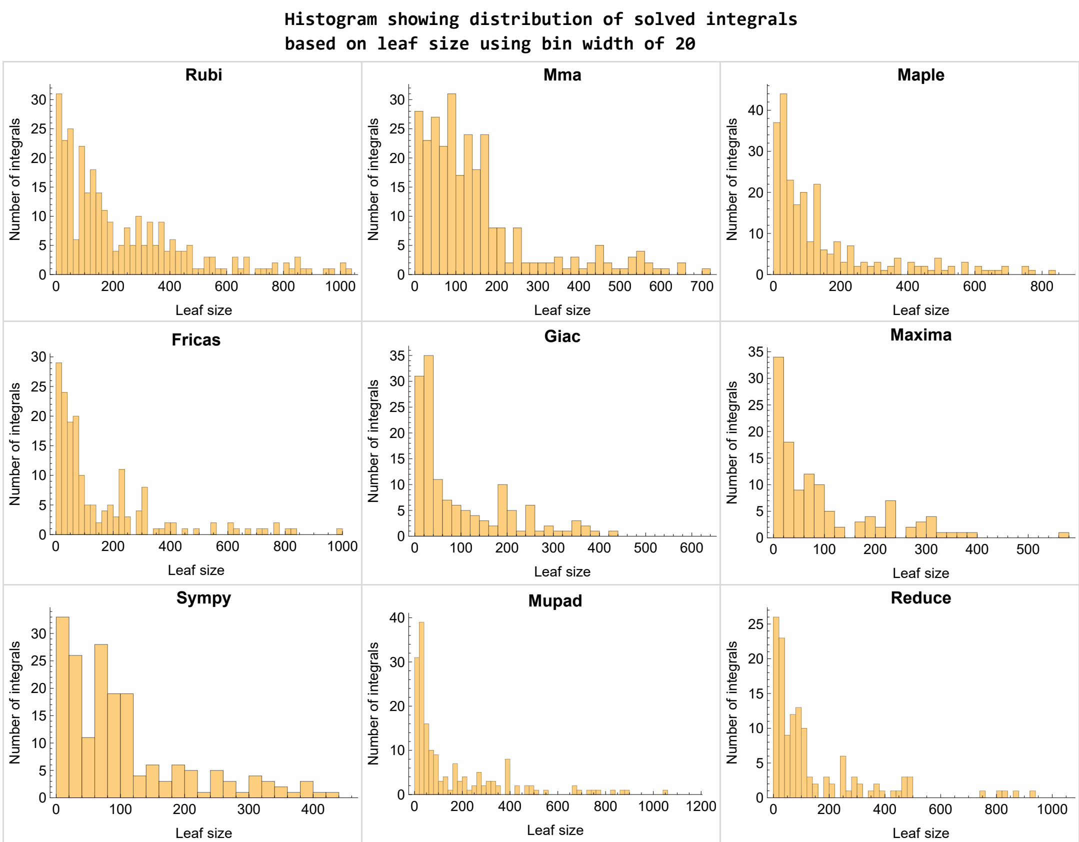


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

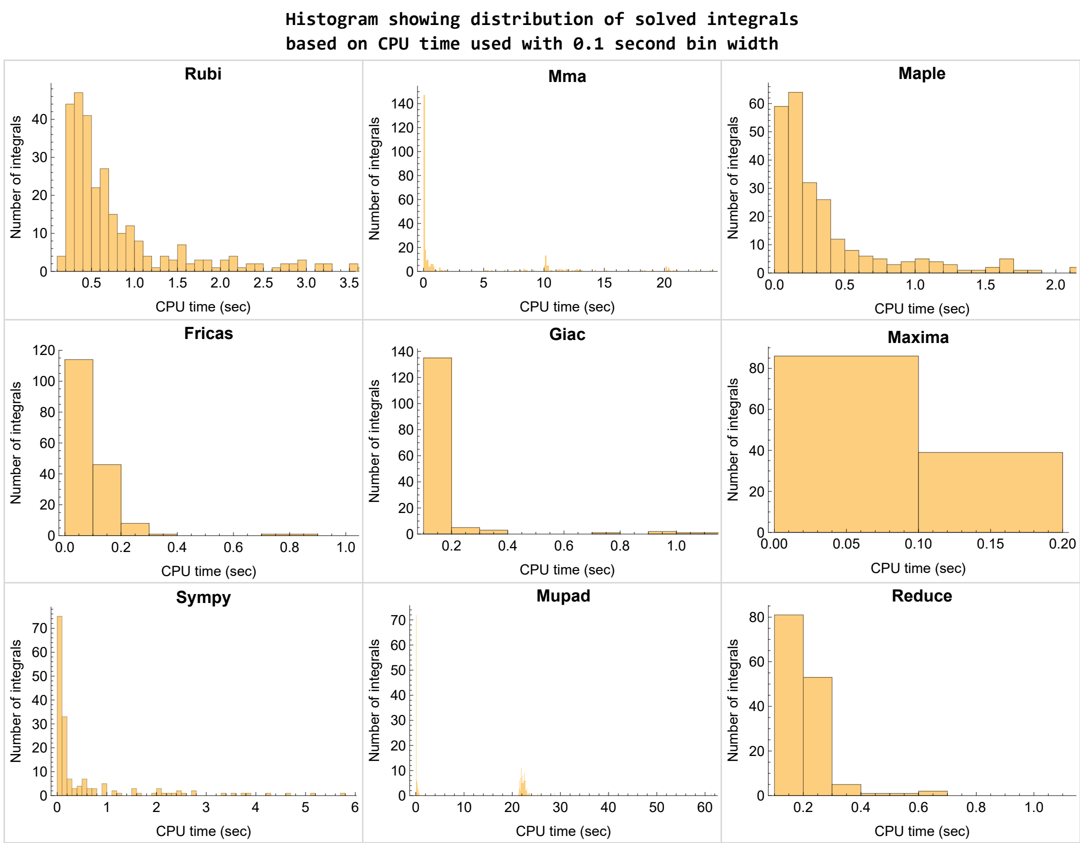


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

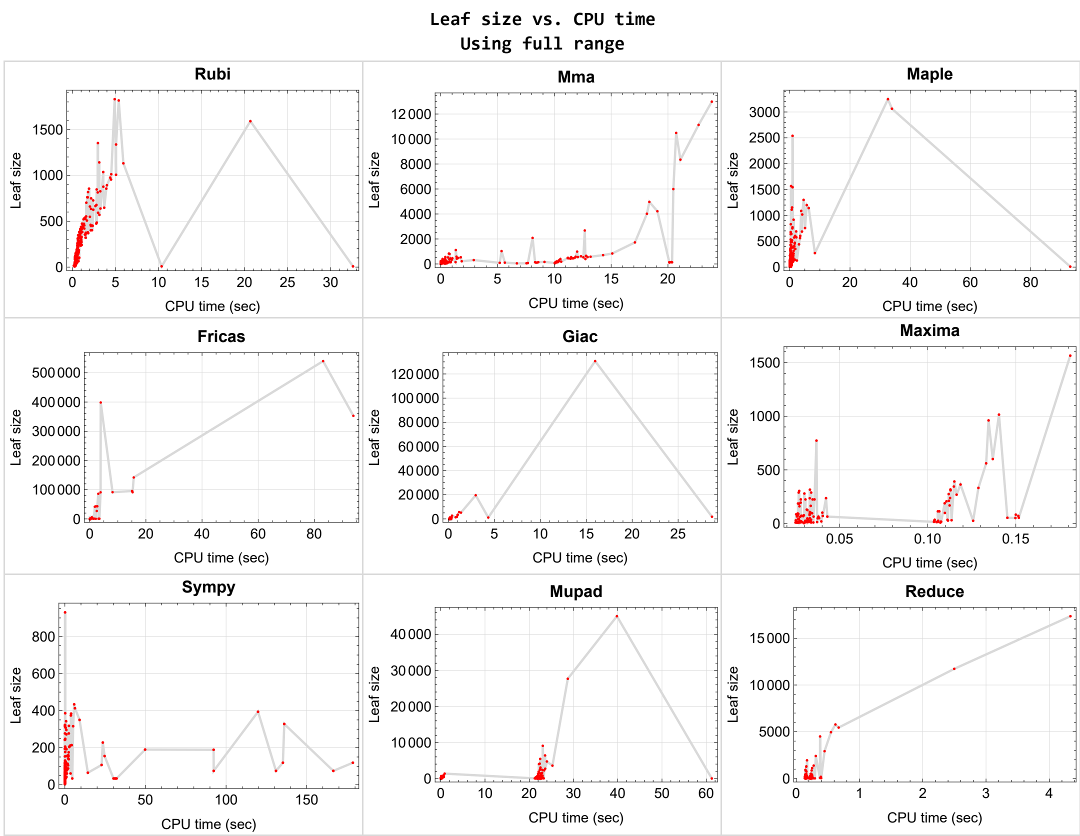


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Reduce {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {25, 26, 27, 28, 131, 132, 133, 134, 258, 281}

Mathematica {118, 119, 120, 121, 126, 127, 128, 129, 130, 131, 132, 133, 134, 196, 197, 202, 203, 208, 209, 210, 216, 253, 258, 259, 270, 271, 281, 299, 300, 301}

Maple {138}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

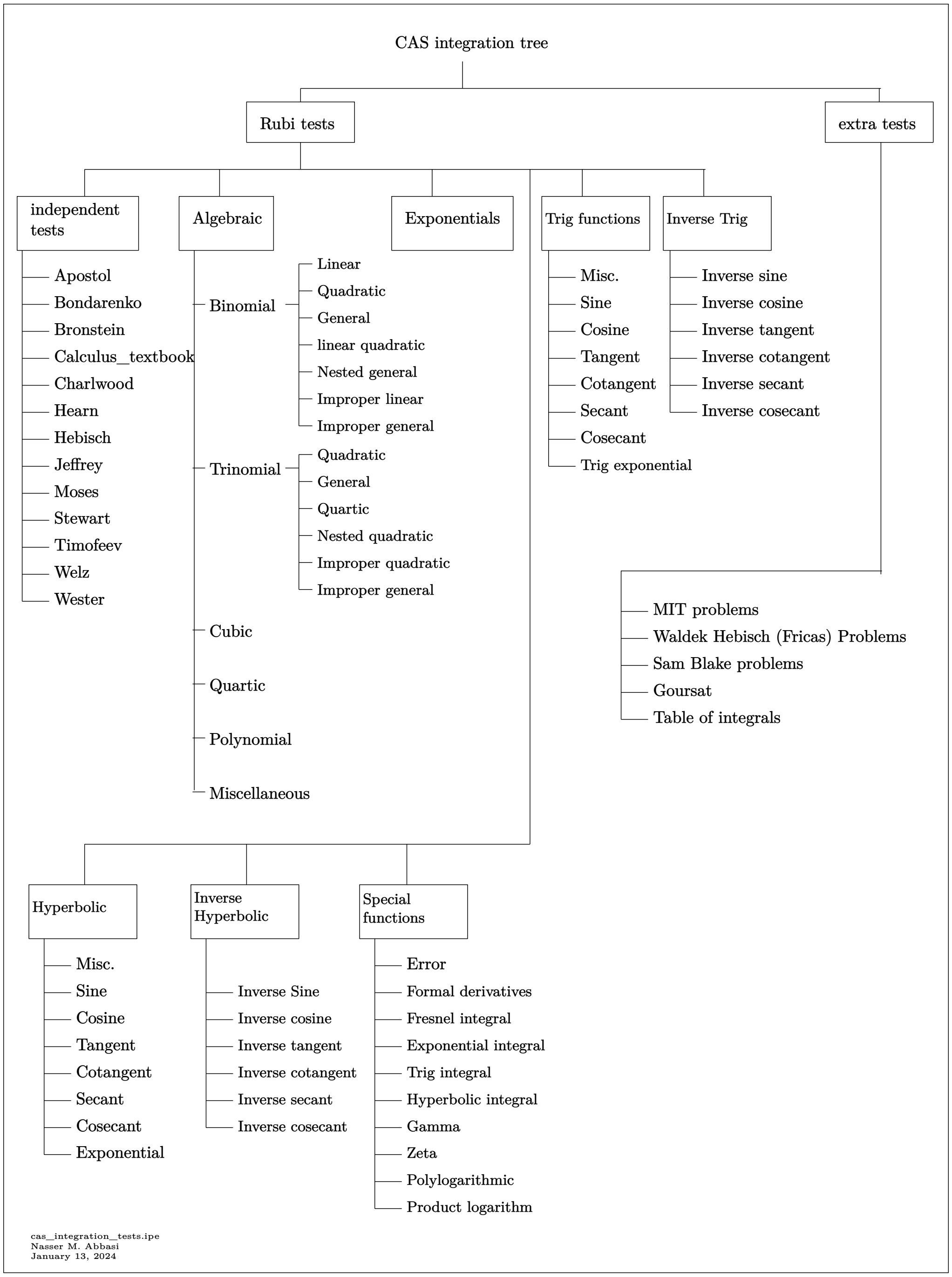
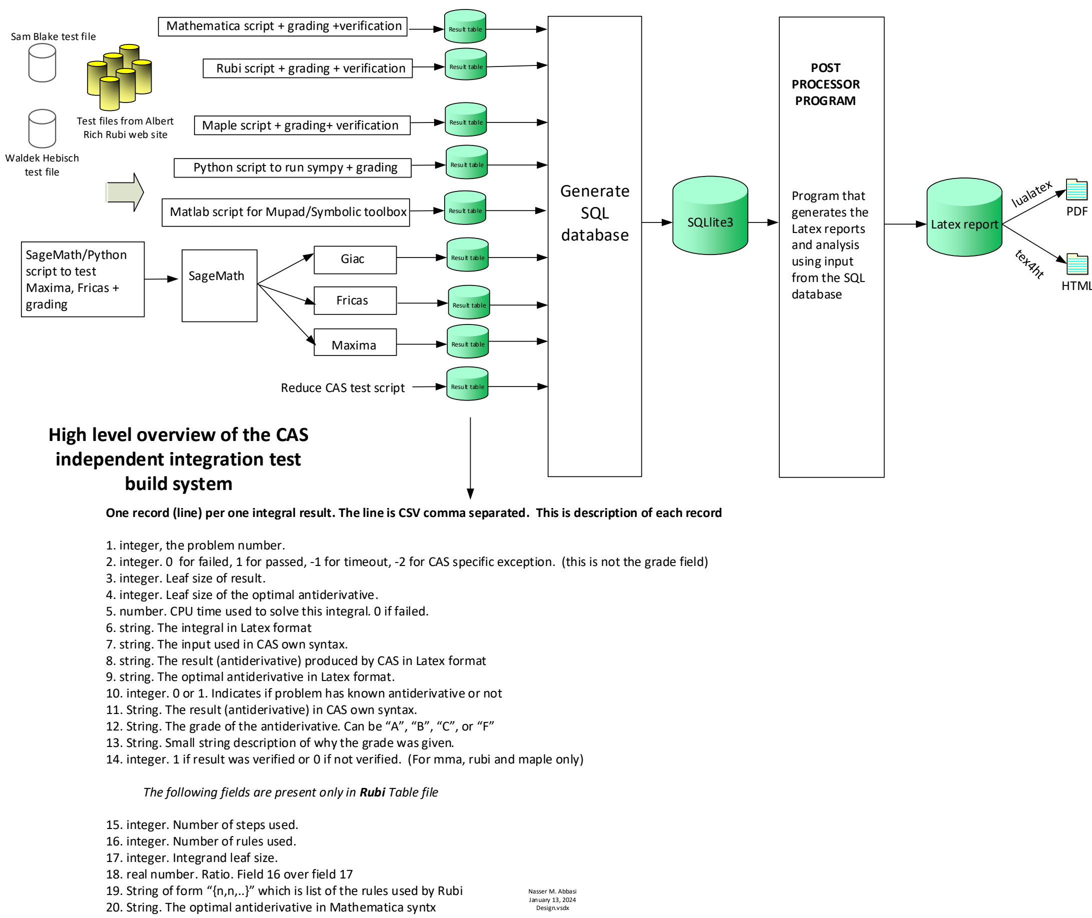


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 66, 67, 68, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 88, 89, 90, 91, 92, 93, 94, 95, 96, 102, 103, 104, 105, 106, 107, 108, 109, 110, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 198, 199, 200, 201, 202, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 247, 248, 249, 250, 251, 254, 255, 256, 257, 258, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 295, 296, 297, 298, 299, 300, 301, 304, 305, 306, 307, 308, 309, 310, 311, 312, 315, 316, 317, 318 }

B grade { 69, 70, 86, 87, 97, 99, 101, 203 }

C grade { 293 }

F normal fail { 3, 64, 65, 98, 100, 111, 112, 113, 197, 222, 246, 252, 253, 259, 271, 294, 302, 303, 313, 314, 319, 320 }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 4, 5, 6, 7, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 63, 64, 65, 66, 67, 68, 69, 73, 81, 82, 83, 84, 85, 88, 89, 90, 91, 92, 93, 94, 95, 96, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 135, 137, 138, 143, 144, 145, 146, 149, 150, 151, 152, 155, 156, 157, 158, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 217, 218, 219, 220, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 272, 273, 274, 275, 276, 277, 278, 279, 280, 282, 284, 285, 295, 296, 297, 298, 302, 303, 304, 305, 309, 310, 311, 312, 315, 316, 317, 318 }

B grade { 70, 71, 72, 74, 75, 76, 77, 78, 79, 80, 86, 87, 97, 98, 99, 100, 101, 114, 115, 116, 117, 264, 299, 300, 301 }

C grade { 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 62, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 136, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 265, 266, 267, 268, 269, 270, 271, 281, 283, 286, 287, 288, 289, 290, 291, 292, 293, 294 }

F normal fail { 3, 139, 140, 141, 142, 147, 148, 153, 154, 159, 160, 221, 222, 306, 307, 308, 313, 314, 319, 320 }

F(-1) timedout fail { 1, 2 }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 63, 64, 65, 66, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 118, 119, 120, 121, 127, 128, 129, 130, 131, 132, 133, 134, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 178, 183, 184, 189, 190, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 239, 240, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 282, 283 }

B grade { 87, 114, 115, 116, 117, 126, 272, 273, 274, 275, 276, 277, 278, 279, 280 }

C grade { 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 62, 67, 138, 141, 142, 173, 174, 175, 176, 177, 179, 180, 181, 182, 185, 186, 187, 188, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 235, 236, 237, 238, 241, 242, 243, 244, 281, 286, 287, 288, 289, 290, 291, 292, 293, 294 }

F normal fail { 122, 123, 124, 125, 135, 136, 137, 139, 140, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 217, 218, 219, 220, 221, 222, 284, 285, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 4, 5, 6, 7, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 66, 67, 70, 73, 74, 81, 82, 83, 84, 85, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 118, 119, 126, 127, 128, 129, 130, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 191, 192, 193, 194, 198, 199, 200, 201, 204, 205, 206, 207, 211, 212, 213, 214, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 247, 248, 249, 250, 254, 255, 256, 257, 260, 261, 262, 263, 266, 267, 268, 269, 272, 273, 274, 275, 276, 277, 278, 279, 280 }

B grade { 18, 26, 64, 65, 68, 69, 71, 72, 75, 76, 77, 78, 79, 80, 86, 87, 88, 114, 115, 116, 117, 138, 141, 142, 238, 244, 282, 283 }

C grade { 10, 173, 174, 175, 176, 177, 179, 180, 181, 182, 185, 186, 187, 188, 236, 237, 294 }

F normal fail { 131, 132, 133, 134, 144, 145, 146, 151, 152, 156, 157, 158, 195, 217, 218, 219, 220, 221, 222, 284, 285, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320 }

F(-1) timedout fail { 1, 2, 3, 13, 14, 15, 16, 17, 19, 20, 21, 22, 23, 24, 25, 27, 28, 29, 122, 123, 124, 125, 135, 136, 137, 139, 140, 143, 147, 148, 149, 150, 153, 154, 155, 159, 160, 178, 183, 184, 189, 190, 196, 197, 202, 203, 208, 209, 210, 215, 216, 235, 239, 240, 241, 242, 243, 245, 246, 251, 252, 253, 258, 259, 264, 265, 270, 271, 281, 286, 287, 288, 289, 290, 291, 292, 293 }

F(-2) exception fail { 8, 9, 11, 12, 120, 121 }

Maxima

A grade { 4, 5, 6, 7, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 63, 66, 67, 68, 69, 73, 74, 81, 82, 83, 84, 85, 88, 89, 90, 91, 92, 93, 94, 95, 96, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234 }

B grade { 64, 65, 71, 72, 75, 76, 77, 78, 79, 80, 86, 87, 114, 115, 116, 117 }

C grade { }

F normal fail { 1, 2, 3, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 60, 61, 62, 70, 97, 98, 99, 100, 101, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320 }

F(-1) timedout fail { }

F(-2) exception fail { 275, 276, 277, 278, 279, 280 }

Giac

A grade { 4, 5, 6, 7, 30, 31, 32, 33, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 66, 67, 68, 69, 70, 71, 73, 74, 75, 77, 79, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234 }

B grade { 34, 64, 65, 72, 76, 78, 80, 114, 115, 116, 117, 176, 235, 236, 237, 238, 239, 241, 242, 243, 244, 245, 272, 273, 274, 275, 276, 277, 278, 279, 280 }

C grade { }

F normal fail { 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 118, 126, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 240, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320 }

F(-1) timedout fail { 1, 2, 3, 122, 123, 124, 125, 246 }

F(-2) exception fail { 119, 120, 121, 127, 128, 129, 130 }

Mupad

A grade { }

B grade { 1, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 131, 132, 133, 134, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 194, 201, 207, 213, 214, 220, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 286, 287, 288, 289, 290, 291, 292, 293, 294, 298, 305, 312, 318 }

C grade { }

F normal fail { }

F(-1) timedout fail { 2, 3, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 191, 192, 193, 195, 196, 197, 198, 199, 200, 202, 203, 204, 205, 206, 208, 209, 210, 211, 212, 215, 216, 217, 218, 219, 221, 222, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 295, 296, 297, 299, 300, 301, 302, 303, 304, 306, 307, 308, 309, 310, 311, 313, 314, 315, 316, 317, 319, 320 }

F(-2) exception fail { }

Sympy

A grade { 4, 5, 6, 7, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 66, 67, 84, 85, 89, 90, 91, 92, 93, 94, 95, 96, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 143, 144, 145, 149, 150, 151, 155, 156, 157, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 179, 180, 181, 182, 185, 186, 187, 188, 191, 198, 204, 217, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 238, 244, 286, 287, 288, 289, 290, 291, 292, 293, 294 }

B grade { 38, 64, 65, 68, 69, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 86, 87, 88, 114, 115, 116, 117 }

C grade { 146, 152, 158, 192, 193, 194, 199, 200, 201, 205, 206, 207, 213, 214, 218, 219, 220, 298, 304, 305, 310, 311, 312, 316, 317, 318 }

F normal fail { 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 147, 148, 153, 154, 159, 160, 195, 196, 197, 202, 203, 208, 209, 210, 211, 212, 215, 216, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 277, 279, 281, 282, 283, 285 }

F(-1) timedout fail { 1, 2, 3, 8, 9, 10, 11, 12, 13, 14, 83, 177, 178, 183, 184, 189, 190, 221, 222, 235, 236, 237, 239, 240, 241, 242, 243, 245, 246, 276, 278, 280, 284, 295, 296, 297, 299, 300, 301, 302, 303, 306, 307, 308, 309, 313, 314, 315, 319, 320 }

F(-2) exception fail { 70, 97, 98, 99, 100, 101 }

Reduce

A grade { }

B grade { 4, 5, 6, 7, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 63, 64, 65, 66, 67, 68, 69, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 241, 242, 243, 244, 245, 272, 273, 274, 275, 276, 277, 278, 279, 280 }

C grade { }

F normal fail { 1, 2, 3, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 62, 70, 97, 98, 99, 100, 101, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 178, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 240, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F(-1)	A	F	F(-1)	F(-1)	F(-1)	F	B
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	0	9	0	0	0	0	47	8
N.S.	1	1.00	0.00	1.12	0.00	0.00	0.00	0.00	5.88	1.00
time (sec)	N/A	10.356	0.000	93.123	0.000	0.000	0.000	0.000	0.809	61.263

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F(-1)	A	F	F(-1)	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	0	9	0	0	0	0	47	0
N.S.	1	1.00	0.00	1.12	0.00	0.00	0.00	0.00	5.88	0.00
time (sec)	N/A	32.604	0.000	0.004	0.000	0.000	0.000	0.000	0.865	0.000

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	A	F	F(-1)	F(-1)	F(-1)	F	F(-1)
verified	N/A	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	0	0	9	0	0	0	0	47	0
N.S.	1	0.00	0.00	1.12	0.00	0.00	0.00	0.00	5.88	0.00
time (sec)	N/A	0.000	0.000	0.004	0.000	0.000	0.000	0.000	0.978	0.000

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	14	15	10	14	15	15
N.S.	1	1.00	1.00	0.94	0.88	0.94	0.62	0.88	0.94	0.94
time (sec)	N/A	0.166	0.002	0.031	0.037	0.092	0.027	0.286	0.235	0.028

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	15	19	14	16	19	13
N.S.	1	1.00	1.00	0.93	1.00	1.27	0.93	1.07	1.27	0.87
time (sec)	N/A	0.168	0.007	0.029	0.031	0.086	0.047	0.132	0.209	0.028

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	15	10	14	15	13
N.S.	1	1.00	1.00	0.93	0.87	1.00	0.67	0.93	1.00	0.87
time (sec)	N/A	0.164	0.002	0.030	0.032	0.095	0.035	0.116	0.237	0.024

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	17	16	20	15	17	20	16
N.S.	1	1.00	1.00	0.94	0.89	1.11	0.83	0.94	1.11	0.89
time (sec)	N/A	0.169	0.002	0.030	0.033	0.084	0.035	0.113	0.218	0.033

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-2)	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	545	530	99	93	0	0	0	0	48	1563
N.S.	1	0.97	0.18	0.17	0.00	0.00	0.00	0.00	0.09	2.87
time (sec)	N/A	1.349	0.085	0.072	0.000	0.000	0.000	0.000	0.244	22.916

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-2)	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	487	472	99	93	0	0	0	0	48	1354
N.S.	1	0.97	0.20	0.19	0.00	0.00	0.00	0.00	0.10	2.78
time (sec)	N/A	1.323	0.058	0.063	0.000	0.000	0.000	0.000	0.216	0.880

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	C	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	334	319	97	93	0	27094	0	0	48	825
N.S.	1	0.96	0.29	0.28	0.00	81.12	0.00	0.00	0.14	2.47
time (sec)	N/A	0.996	0.046	0.063	0.000	2.481	0.000	0.000	0.236	23.007

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-2)	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	469	454	95	91	0	0	0	0	46	1057
N.S.	1	0.97	0.20	0.19	0.00	0.00	0.00	0.00	0.10	2.25
time (sec)	N/A	1.371	0.049	0.059	0.000	0.000	0.000	0.000	0.215	22.610

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-2)	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	522	507	99	90	0	0	0	0	44	1394
N.S.	1	0.97	0.19	0.17	0.00	0.00	0.00	0.00	0.08	2.67
time (sec)	N/A	1.671	0.084	0.059	0.000	0.000	0.000	0.000	0.259	22.684

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	563	548	157	134	0	0	0	0	45	4002
N.S.	1	0.97	0.28	0.24	0.00	0.00	0.00	0.00	0.08	7.11
time (sec)	N/A	2.018	0.115	0.082	0.000	0.000	0.000	0.000	0.152	22.199

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	645	625	163	133	0	0	0	0	223	2663
N.S.	1	0.97	0.25	0.21	0.00	0.00	0.00	0.00	0.35	4.13
time (sec)	N/A	2.415	0.141	0.089	0.000	0.000	0.000	0.000	0.163	22.509

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	395	402	61	56	0	0	70	0	28	427
N.S.	1	1.02	0.15	0.14	0.00	0.00	0.18	0.00	0.07	1.08
time (sec)	N/A	2.263	0.022	0.053	0.000	0.000	0.130	0.000	0.147	0.698

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	383	379	61	56	0	0	65	0	28	390
N.S.	1	0.99	0.16	0.15	0.00	0.00	0.17	0.00	0.07	1.02
time (sec)	N/A	1.782	0.019	0.044	0.000	0.000	0.135	0.000	0.143	22.523

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	361	354	61	56	0	0	61	0	28	276
N.S.	1	0.98	0.17	0.16	0.00	0.00	0.17	0.00	0.08	0.76
time (sec)	N/A	1.427	0.019	0.040	0.000	0.000	0.116	0.000	0.155	22.475

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	248	241	59	56	0	1173	48	0	28	247
N.S.	1	0.97	0.24	0.23	0.00	4.73	0.19	0.00	0.11	1.00
time (sec)	N/A	1.068	0.017	0.042	0.000	0.827	0.100	0.000	0.149	22.656

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	361	354	57	54	0	0	61	0	26	176
N.S.	1	0.98	0.16	0.15	0.00	0.00	0.17	0.00	0.07	0.49
time (sec)	N/A	1.504	0.016	0.036	0.000	0.000	0.121	0.000	0.159	22.260

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	383	370	62	53	0	0	65	0	24	306
N.S.	1	0.97	0.16	0.14	0.00	0.00	0.17	0.00	0.06	0.80
time (sec)	N/A	1.811	0.016	0.037	0.000	0.000	0.134	0.000	0.161	22.554

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	415	401	103	73	0	0	82	0	26	432
N.S.	1	0.97	0.25	0.18	0.00	0.00	0.20	0.00	0.06	1.04
time (sec)	N/A	2.124	0.027	0.053	0.000	0.000	0.272	0.000	0.145	22.141

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	448	453	109	69	0	0	70	0	125	340
N.S.	1	1.01	0.24	0.15	0.00	0.00	0.16	0.00	0.28	0.76
time (sec)	N/A	2.303	0.025	0.053	0.000	0.000	0.179	0.000	0.142	0.261

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1063	1012	167	122	0	0	112	0	0	388
N.S.	1	0.95	0.16	0.11	0.00	0.00	0.11	0.00	0.00	0.37
time (sec)	N/A	4.504	0.045	0.063	0.000	0.000	0.205	0.000	0.185	0.289

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1001	954	167	122	0	0	112	0	0	387
N.S.	1	0.95	0.17	0.12	0.00	0.00	0.11	0.00	0.00	0.39
time (sec)	N/A	4.477	0.038	0.056	0.000	0.000	0.227	0.000	0.259	22.183

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	A	F	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	677	638	167	122	0	0	112	0	0	388
N.S.	1	0.94	0.25	0.18	0.00	0.00	0.17	0.00	0.00	0.57
time (sec)	N/A	3.248	0.041	0.056	0.000	0.000	0.198	0.000	0.268	22.294

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	A	F	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	682	666	167	122	0	1340	104	0	0	299
N.S.	1	0.98	0.24	0.18	0.00	1.96	0.15	0.00	0.00	0.44
time (sec)	N/A	2.660	0.030	0.053	0.000	1.452	0.143	0.000	0.228	22.753

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	A	F	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	850	826	167	122	0	0	112	0	0	388
N.S.	1	0.97	0.20	0.14	0.00	0.00	0.13	0.00	0.00	0.46
time (sec)	N/A	3.228	0.041	0.051	0.000	0.000	0.189	0.000	0.239	0.372

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	A	F	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	884	855	167	122	0	0	112	0	0	387
N.S.	1	0.97	0.19	0.14	0.00	0.00	0.13	0.00	0.00	0.44
time (sec)	N/A	3.927	0.033	0.051	0.000	0.000	0.222	0.000	0.241	23.013

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	986	892	167	122	0	0	112	0	0	388
N.S.	1	0.90	0.17	0.12	0.00	0.00	0.11	0.00	0.00	0.39
time (sec)	N/A	3.997	0.041	0.051	0.000	0.000	0.197	0.000	0.242	22.582

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	31	21	20	20	20	21	20	20
N.S.	1	1.00	1.29	0.88	0.83	0.83	0.83	0.88	0.83	0.83
time (sec)	N/A	0.276	0.011	0.141	0.040	0.059	0.032	0.120	0.234	0.037

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	25	21	20	20	19	21	20	20
N.S.	1	1.00	0.96	0.81	0.77	0.77	0.73	0.81	0.77	0.77
time (sec)	N/A	0.282	0.011	0.145	0.031	0.060	0.039	0.145	0.216	0.035

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	12	12	10	12	13	13
N.S.	1	1.00	1.00	0.81	0.75	0.75	0.62	0.75	0.81	0.81
time (sec)	N/A	0.261	0.001	0.131	0.027	0.057	0.024	0.123	0.224	0.022

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	22	21	21	22	21	24	21
N.S.	1	1.00	1.00	0.88	0.84	0.84	0.88	0.84	0.96	0.84
time (sec)	N/A	0.344	0.005	0.242	0.032	0.059	0.031	0.114	0.199	0.039

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	79	96	105	105	88	365	116	106
N.S.	1	1.00	0.84	1.02	1.12	1.12	0.94	3.88	1.23	1.13
time (sec)	N/A	0.494	0.041	0.181	0.033	0.065	0.158	0.134	0.163	22.515

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	7	9	9	5	9	6	6
N.S.	1	1.00	1.00	0.64	0.82	0.82	0.45	0.82	0.55	0.55
time (sec)	N/A	0.227	0.001	0.423	0.027	0.057	0.020	0.128	0.150	0.016

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	32	32	27	26	26	29	26	27	26
N.S.	1	1.03	1.03	0.87	0.84	0.84	0.94	0.84	0.87	0.84
time (sec)	N/A	0.321	0.005	0.168	0.030	0.065	0.033	0.139	0.141	0.048

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	12	11	10	10	8	10	10	10
N.S.	1	1.00	0.86	0.79	0.71	0.71	0.57	0.71	0.71	0.71
time (sec)	N/A	0.321	0.003	0.140	0.026	0.066	0.033	0.127	0.154	0.019

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	32	31	98	124	31	39	32
N.S.	1	1.00	1.00	0.76	0.74	2.33	2.95	0.74	0.93	0.76
time (sec)	N/A	0.354	0.020	0.158	0.113	0.078	0.166	0.136	0.151	0.060

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	22	22	19	18	18	17	19	18	18
N.S.	1	1.47	1.47	1.27	1.20	1.20	1.13	1.27	1.20	1.20
time (sec)	N/A	0.267	0.022	0.136	0.107	0.061	0.046	0.126	0.141	0.139

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	10	9	9	8	10	9	17
N.S.	1	1.00	1.00	0.91	0.82	0.82	0.73	0.91	0.82	1.55
time (sec)	N/A	0.322	0.010	0.248	0.107	0.070	0.066	0.128	0.150	0.046

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	10	13	8	11	13	10
N.S.	1	1.00	1.00	0.92	0.83	1.08	0.67	0.92	1.08	0.83
time (sec)	N/A	0.288	0.008	0.243	0.035	0.060	0.044	0.125	0.158	22.012

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	12	12	12	14	12	12
N.S.	1	1.00	1.00	1.08	1.00	1.00	1.00	1.17	1.00	1.00
time (sec)	N/A	0.277	0.008	0.164	0.029	0.061	0.058	0.108	0.224	22.405

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	15	15	15	16	15	23
N.S.	1	1.00	1.00	0.94	0.88	0.88	0.88	0.94	0.88	1.35
time (sec)	N/A	0.333	0.010	0.240	0.106	0.067	0.066	0.123	0.296	0.056

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	10	10	8	12	10	10
N.S.	1	1.00	1.00	0.92	0.83	0.83	0.67	1.00	0.83	0.83
time (sec)	N/A	0.326	0.009	0.178	0.026	0.071	0.051	0.126	0.219	0.045

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	15	21	14	17	21	15
N.S.	1	1.00	1.00	0.94	0.88	1.24	0.82	1.00	1.24	0.88
time (sec)	N/A	0.325	0.009	0.194	0.033	0.063	0.051	0.117	0.226	0.030

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	15	15	14	17	15	15
N.S.	1	1.00	1.00	0.94	0.88	0.88	0.82	1.00	0.88	0.88
time (sec)	N/A	0.324	0.009	0.236	0.026	0.061	0.051	0.130	0.238	0.034

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	24	18	17	16	25	15	23	25	16
N.S.	1	1.33	1.00	0.94	0.89	1.39	0.83	1.28	1.39	0.89
time (sec)	N/A	0.310	0.010	0.148	0.104	0.060	0.055	0.134	0.219	0.057

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	25	19	14	13	13	12	16	21	13
N.S.	1	1.32	1.00	0.74	0.68	0.68	0.63	0.84	1.11	0.68
time (sec)	N/A	0.278	0.008	0.161	0.028	0.062	0.053	0.128	0.265	22.005

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	17	11	10	10	8	13	13	10
N.S.	1	1.00	1.42	0.92	0.83	0.83	0.67	1.08	1.08	0.83
time (sec)	N/A	0.228	0.009	0.140	0.026	0.065	0.041	0.112	0.219	22.220

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	11	11	10	10	8	11	11	10
N.S.	1	1.00	1.10	1.10	1.00	1.00	0.80	1.10	1.10	1.00
time (sec)	N/A	0.233	0.010	0.192	0.026	0.060	0.075	0.118	0.237	0.057

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	15	15	15	18	15	15
N.S.	1	1.00	1.00	0.94	0.88	0.88	0.88	1.06	0.88	0.88
time (sec)	N/A	0.274	0.015	0.171	0.031	0.071	0.076	0.134	0.231	0.048

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	18	17	17	17	18	17	21
N.S.	1	1.00	1.00	0.78	0.74	0.74	0.74	0.78	0.74	0.91
time (sec)	N/A	0.274	0.009	0.182	0.108	0.069	0.068	0.114	0.161	22.491

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	15	14	14	12	14	14	14
N.S.	1	1.00	1.00	1.07	1.00	1.00	0.86	1.00	1.00	1.00
time (sec)	N/A	0.273	0.007	0.144	0.105	0.068	0.053	0.111	0.142	0.036

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	25	26	26	25	20	27	28	25
N.S.	1	1.00	0.96	1.00	1.00	0.96	0.77	1.04	1.08	0.96
time (sec)	N/A	0.310	0.014	0.145	0.033	0.090	0.097	0.112	0.142	22.393

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	4	5	4	4	3	5	4	4
N.S.	1	1.00	0.67	0.83	0.67	0.67	0.50	0.83	0.67	0.67
time (sec)	N/A	0.227	0.002	0.019	0.030	0.064	0.022	0.131	0.157	0.018

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	10	14	7	11	16	10
N.S.	1	1.00	1.00	1.10	1.00	1.40	0.70	1.10	1.60	1.00
time (sec)	N/A	0.293	0.017	0.033	0.034	0.062	0.041	0.142	0.148	0.033

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	31	30	30	36	33	30	30
N.S.	1	1.00	1.00	0.74	0.71	0.71	0.86	0.79	0.71	0.71
time (sec)	N/A	0.355	0.011	0.049	0.025	0.071	0.090	0.128	0.144	22.380

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	10	13	26	13
N.S.	1	1.00	1.00	0.93	0.87	0.87	0.67	0.87	1.73	0.87
time (sec)	N/A	0.244	0.008	0.034	0.025	0.065	0.049	0.132	0.158	0.042

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	14	13	11	16	17	11	17	20
N.S.	1	1.00	1.08	1.00	0.85	1.23	1.31	0.85	1.31	1.54
time (sec)	N/A	0.239	0.018	0.174	0.027	0.081	0.070	0.118	0.145	22.466

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	54	45	38	0	47	44	38	256	42
N.S.	1	1.20	1.00	0.84	0.00	1.04	0.98	0.84	5.69	0.93
time (sec)	N/A	0.427	0.037	0.059	0.000	0.091	0.085	0.296	0.143	0.060

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	66	44	41	0	75	46	42	366	47
N.S.	1	1.12	0.75	0.69	0.00	1.27	0.78	0.71	6.20	0.80
time (sec)	N/A	0.435	0.030	0.104	0.000	0.063	0.094	0.295	0.162	22.078

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	85	99	70	0	66	100	64	108	76
N.S.	1	1.09	1.27	0.90	0.00	0.85	1.28	0.82	1.38	0.97
time (sec)	N/A	0.446	0.025	0.064	0.000	0.066	0.103	0.294	0.146	0.107

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	64	27	28	65	65	60	30	62	27
N.S.	1	1.07	0.45	0.47	1.08	1.08	1.00	0.50	1.03	0.45
time (sec)	N/A	0.697	0.018	0.062	0.035	0.061	0.089	0.116	0.375	22.497

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	A	B	B	B	B	B	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	27	28	65	65	60	197	62	27
N.S.	1	0.00	1.00	1.04	2.41	2.41	2.22	7.30	2.30	1.00
time (sec)	N/A	0.000	0.014	0.082	0.043	0.062	0.143	0.115	0.227	0.045

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	A	B	B	B	B	B	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	27	28	65	65	60	111	62	27
N.S.	1	0.00	1.00	1.04	2.41	2.41	2.22	4.11	2.30	1.00
time (sec)	N/A	0.000	0.012	0.078	0.038	0.061	0.114	0.122	0.249	0.041

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	11	11	8	11	14	11
N.S.	1	1.00	1.00	1.09	1.00	1.00	0.73	1.00	1.27	1.00
time (sec)	N/A	0.248	0.011	0.049	0.028	0.054	0.053	0.118	0.220	0.064

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	112	101	54	99	75	61	92	122	170
N.S.	1	1.23	1.11	0.59	1.09	0.82	0.67	1.01	1.34	1.87
time (sec)	N/A	0.522	0.073	0.185	0.110	0.076	0.668	0.114	0.235	22.490

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	245	132	116	114	223	296	134	492	126
N.S.	1	1.98	1.06	0.94	0.92	1.80	2.39	1.08	3.97	1.02
time (sec)	N/A	0.678	0.123	0.084	0.107	0.072	0.903	0.116	0.228	21.871

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	245	132	116	114	223	272	134	492	124
N.S.	1	2.21	1.19	1.05	1.03	2.01	2.45	1.21	4.43	1.12
time (sec)	N/A	0.715	0.097	0.081	0.106	0.082	0.933	0.144	0.232	0.131

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	B	A	F	A	F(-2)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	103	139	34	0	33	0	35	207	34
N.S.	1	4.68	6.32	1.55	0.00	1.50	0.00	1.59	9.41	1.55
time (sec)	N/A	0.786	0.202	0.310	0.000	0.066	0.000	0.152	0.229	23.276

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	30	80	25	66	66	70	24	66	66
N.S.	1	1.07	2.86	0.89	2.36	2.36	2.50	0.86	2.36	2.36
time (sec)	N/A	0.321	0.011	0.246	0.026	0.059	0.035	0.123	0.227	0.055

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	32	108	27	187	187	194	88	205	180
N.S.	1	1.03	3.48	0.87	6.03	6.03	6.26	2.84	6.61	5.81
time (sec)	N/A	0.358	0.049	0.362	0.026	0.061	0.049	0.112	0.254	21.812

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	31	52	48	228	30	70	31
N.S.	1	1.00	1.00	0.91	1.53	1.41	6.71	0.88	2.06	0.91
time (sec)	N/A	0.272	0.389	0.342	0.150	0.071	23.531	0.139	0.158	21.889

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	36	73	33	54	52	328	32	102	58
N.S.	1	1.03	2.09	0.94	1.54	1.49	9.37	0.91	2.91	1.66
time (sec)	N/A	0.272	0.352	0.407	0.145	0.075	136.095	0.130	0.145	21.884

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	30	93	25	77	77	87	24	79	77
N.S.	1	1.07	3.32	0.89	2.75	2.75	3.11	0.86	2.82	2.75
time (sec)	N/A	0.346	0.010	0.224	0.030	0.060	0.042	0.134	0.153	21.811

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	32	140	27	280	280	314	105	292	266
N.S.	1	1.03	4.52	0.87	9.03	9.03	10.13	3.39	9.42	8.58
time (sec)	N/A	0.417	0.056	0.232	0.030	0.075	0.070	0.116	0.152	0.405

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	40	98	31	80	80	90	30	80	80
N.S.	1	1.18	2.88	0.91	2.35	2.35	2.65	0.88	2.35	2.35
time (sec)	N/A	0.331	0.012	0.196	0.033	0.070	0.046	0.119	0.146	21.798

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	42	146	33	289	289	321	126	293	273
N.S.	1	1.08	3.74	0.85	7.41	7.41	8.23	3.23	7.51	7.00
time (sec)	N/A	0.359	0.073	0.148	0.027	0.067	0.076	0.132	0.152	22.366

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	46	244	37	289	289	323	37	310	270
N.S.	1	1.07	5.67	0.86	6.72	6.72	7.51	0.86	7.21	6.28
time (sec)	N/A	0.389	0.081	0.348	0.034	0.090	0.101	0.114	0.162	21.962

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	48	248	39	773	773	930	153	926	753
N.S.	1	1.04	5.39	0.85	16.80	16.80	20.22	3.33	20.13	16.37
time (sec)	N/A	0.536	0.133	0.648	0.037	0.080	0.133	0.117	0.145	22.290

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	36	31	54	48	190	30	76	33
N.S.	1	1.00	1.06	0.91	1.59	1.41	5.59	0.88	2.24	0.97
time (sec)	N/A	0.270	0.327	0.313	0.152	0.096	49.771	0.116	0.155	21.849

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	42	37	71	57	189	36	81	37
N.S.	1	1.00	0.95	0.84	1.61	1.30	4.30	0.82	1.84	0.84
time (sec)	N/A	0.274	0.393	0.282	0.151	0.094	92.266	0.115	0.150	21.934

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	49	43	83	72	0	42	131	73
N.S.	1	1.00	0.98	0.86	1.66	1.44	0.00	0.84	2.62	1.46
time (sec)	N/A	0.277	0.511	0.227	0.150	0.079	0.000	0.125	0.142	21.843

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	33	18	17	29	29	30	26	29
N.S.	1	1.00	1.74	0.95	0.89	1.53	1.53	1.58	1.37	1.53
time (sec)	N/A	0.253	0.003	0.050	0.030	0.061	0.022	0.117	0.154	0.026

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	21	15	14	17	17	22	20	17
N.S.	1	1.00	1.31	0.94	0.88	1.06	1.06	1.38	1.25	1.06
time (sec)	N/A	0.238	0.003	0.088	0.026	0.063	0.026	0.132	0.256	0.034

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	B	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	96	96	29	86	86	94	28	85	86
N.S.	1	2.91	2.91	0.88	2.61	2.61	2.85	0.85	2.58	2.61
time (sec)	N/A	0.753	0.011	0.155	0.025	0.064	0.030	0.104	0.262	22.080

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	B	B	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	96	96	86	86	86	94	28	85	86
N.S.	1	2.91	2.91	2.61	2.61	2.61	2.85	0.85	2.58	2.61
time (sec)	N/A	0.681	0.008	0.188	0.026	0.058	0.038	0.132	0.214	0.336

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	12	57	56	12	57	12
N.S.	1	1.00	1.00	0.93	0.86	4.07	4.00	0.86	4.07	0.86
time (sec)	N/A	0.244	0.009	0.054	0.031	0.065	0.106	0.118	0.238	21.843

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	12	14	13	13
N.S.	1	1.00	1.00	0.93	0.87	0.87	0.80	0.93	0.87	0.87
time (sec)	N/A	0.246	0.007	0.031	0.030	0.063	0.048	0.122	0.219	0.054

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	20	14	15	15	14	18	16	13
N.S.	1	1.00	1.18	0.82	0.88	0.88	0.82	1.06	0.94	0.76
time (sec)	N/A	0.246	0.009	0.032	0.027	0.069	0.058	0.118	0.242	21.978

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	33	33	29	28	28	34	28	27	30
N.S.	1	1.03	1.03	0.91	0.88	0.88	1.06	0.88	0.84	0.94
time (sec)	N/A	0.286	0.016	0.444	0.104	0.091	0.073	0.116	0.223	0.045

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	33	33	29	28	28	34	28	27	30
N.S.	1	1.03	1.03	0.91	0.88	0.88	1.06	0.88	0.84	0.94
time (sec)	N/A	0.332	0.009	0.162	0.111	0.072	0.074	0.106	0.241	0.035

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	36	31	27	26	26	36	26	25	30
N.S.	1	1.12	0.97	0.84	0.81	0.81	1.12	0.81	0.78	0.94
time (sec)	N/A	0.288	0.013	0.314	0.126	0.070	0.065	0.108	0.222	21.757

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	36	31	27	26	26	36	26	25	30
N.S.	1	1.12	0.97	0.84	0.81	0.81	1.12	0.81	0.78	0.94
time (sec)	N/A	0.337	0.006	0.165	0.106	0.102	0.073	0.133	0.262	0.033

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	42	29	35	45	39	44	41	34
N.S.	1	1.00	0.93	0.64	0.78	1.00	0.87	0.98	0.91	0.76
time (sec)	N/A	0.310	0.026	0.258	0.104	0.095	0.061	0.132	0.166	21.875

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	42	29	35	45	39	44	41	34
N.S.	1	1.00	0.93	0.64	0.78	1.00	0.87	0.98	0.91	0.76
time (sec)	N/A	0.334	0.006	0.025	0.111	0.108	0.067	0.135	0.161	0.047

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	B	A	F	A	F(-2)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	103	45	34	0	33	0	35	207	34
N.S.	1	4.68	2.05	1.55	0.00	1.50	0.00	1.59	9.41	1.55
time (sec)	N/A	0.436	0.085	0.160	0.000	0.089	0.000	0.145	0.145	22.715

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	B	A	F	A	F(-2)	A	F	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	0	45	34	0	33	0	35	207	34
N.S.	1	0.00	2.05	1.55	0.00	1.50	0.00	1.59	9.41	1.55
time (sec)	N/A	0.000	0.109	0.204	0.000	0.084	0.000	0.166	0.150	22.755

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	B	A	F	A	F(-2)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	103	45	34	0	33	0	35	207	34
N.S.	1	4.68	2.05	1.55	0.00	1.50	0.00	1.59	9.41	1.55
time (sec)	N/A	0.522	0.069	0.169	0.000	0.106	0.000	0.172	0.166	22.643

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	B	A	F	A	F(-2)	A	F	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	0	179	34	0	33	0	35	207	34
N.S.	1	0.00	8.14	1.55	0.00	1.50	0.00	1.59	9.41	1.55
time (sec)	N/A	0.000	0.153	0.202	0.000	0.085	0.000	0.174	0.147	22.842

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	B	A	F	A	F(-2)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	103	139	34	0	33	0	35	207	34
N.S.	1	4.68	6.32	1.55	0.00	1.50	0.00	1.59	9.41	1.55
time (sec)	N/A	0.685	0.087	0.227	0.000	0.086	0.000	0.176	0.147	22.649

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	102	70	71	71	75	83	73	74	71
N.S.	1	1.04	0.71	0.72	0.72	0.77	0.85	0.74	0.76	0.72
time (sec)	N/A	0.419	0.097	0.336	0.040	0.079	0.576	0.132	0.157	0.046

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	65	49	49	48	52	63	50	48	49
N.S.	1	1.08	0.82	0.82	0.80	0.87	1.05	0.83	0.80	0.82
time (sec)	N/A	0.359	0.059	0.182	0.032	0.075	0.110	0.117	0.153	0.028

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	41	32	27	26	30	36	26	26	26
N.S.	1	1.08	0.84	0.71	0.68	0.79	0.95	0.68	0.68	0.68
time (sec)	N/A	0.301	0.030	0.103	0.027	0.108	0.525	0.130	0.143	0.045

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	190	171	176	165	169	248	186	194	150
N.S.	1	1.14	1.03	1.06	0.99	1.02	1.49	1.12	1.17	0.90
time (sec)	N/A	0.643	0.299	1.158	0.034	0.086	0.210	0.122	0.159	22.058

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	120	97	91	88	92	131	99	98	88
N.S.	1	1.14	0.92	0.87	0.84	0.88	1.25	0.94	0.93	0.84
time (sec)	N/A	0.488	0.128	0.438	0.033	0.078	0.140	0.114	0.145	21.672

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	56	45	39	38	42	53	40	38	39
N.S.	1	1.12	0.90	0.78	0.76	0.84	1.06	0.80	0.76	0.78
time (sec)	N/A	0.323	0.043	0.181	0.032	0.082	0.114	0.113	0.395	0.026

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	118	103	103	102	106	117	111	109	101
N.S.	1	1.02	0.89	0.89	0.88	0.91	1.01	0.96	0.94	0.87
time (sec)	N/A	0.485	0.146	0.892	0.040	0.086	0.624	0.133	0.228	0.053

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	82	73	70	69	73	99	75	71	69
N.S.	1	1.06	0.95	0.91	0.90	0.95	1.29	0.97	0.92	0.90
time (sec)	N/A	0.414	0.079	0.581	0.029	0.093	0.115	0.139	0.229	0.039

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	51	47	37	36	40	42	40	37	38
N.S.	1	1.11	1.02	0.80	0.78	0.87	0.91	0.87	0.80	0.83
time (sec)	N/A	0.343	0.051	0.241	0.034	0.110	0.524	0.114	0.228	0.047

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	A	A	A	A	A	B	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	242	0	271	271	237	241	386	282	293	233
N.S.	1	0.00	1.12	1.12	0.98	1.00	1.60	1.17	1.21	0.96
time (sec)	N/A	0.000	0.441	8.342	0.042	0.120	0.211	0.140	0.236	21.874

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	A	A	A	A	A	B	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	0	158	127	126	130	206	149	148	126
N.S.	1	0.00	1.13	0.91	0.90	0.93	1.47	1.06	1.06	0.90
time (sec)	N/A	0.000	0.227	2.352	0.032	0.112	0.159	0.128	0.231	0.053

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	A	A	A	A	A	B	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	0	68	52	51	55	85	61	56	55
N.S.	1	0.00	1.08	0.83	0.81	0.87	1.35	0.97	0.89	0.87
time (sec)	N/A	0.000	0.086	0.486	0.038	0.093	0.103	0.125	0.228	21.713

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	217	222	225	225	243	245	247	208
N.S.	1	1.00	2.36	2.41	2.45	2.45	2.64	2.66	2.68	2.26
time (sec)	N/A	0.482	0.039	0.135	0.028	0.084	0.048	0.112	0.231	21.777

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	217	222	225	225	243	245	247	208
N.S.	1	1.00	2.36	2.41	2.45	2.45	2.64	2.66	2.68	2.26
time (sec)	N/A	0.433	0.009	0.137	0.035	0.100	0.046	0.127	0.224	0.059

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	217	222	225	225	243	245	247	208
N.S.	1	1.00	2.36	2.41	2.45	2.45	2.64	2.66	2.68	2.26
time (sec)	N/A	0.417	0.009	0.134	0.031	0.113	0.037	0.135	0.215	0.061

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	217	222	225	225	243	245	247	208
N.S.	1	1.00	2.36	2.41	2.45	2.45	2.64	2.66	2.68	2.26
time (sec)	N/A	0.471	0.008	0.088	0.036	0.075	0.038	0.136	0.167	0.056

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	148	139	0	74	0	0	23	0
N.S.	1	1.00	1.02	0.96	0.00	0.51	0.00	0.00	0.16	0.00
time (sec)	N/A	0.653	20.282	1.672	0.000	0.167	0.000	0.000	0.142	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	148	143	0	79	0	0	28	0
N.S.	1	1.00	0.92	0.89	0.00	0.49	0.00	0.00	0.18	0.00
time (sec)	N/A	0.701	10.182	1.645	0.000	0.171	0.000	0.000	0.161	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-2)	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	146	143	0	0	0	0	24	0
N.S.	1	1.00	0.90	0.88	0.00	0.00	0.00	0.00	0.15	0.00
time (sec)	N/A	0.696	20.306	1.645	0.000	0.000	0.000	0.000	0.152	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-2)	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	150	139	0	0	0	0	26	0
N.S.	1	1.00	0.96	0.89	0.00	0.00	0.00	0.00	0.17	0.00
time (sec)	N/A	0.678	10.136	1.581	0.000	0.000	0.000	0.000	0.144	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	280	280	164	0	0	0	0	0	33	0
N.S.	1	1.00	0.59	0.00	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	0.987	10.314	0.000	0.000	0.000	0.000	0.000	0.157	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	288	288	166	0	0	0	0	0	36	0
N.S.	1	1.00	0.58	0.00	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	0.987	10.289	0.000	0.000	0.000	0.000	0.000	0.154	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	297	297	167	0	0	0	0	0	38	0
N.S.	1	1.00	0.56	0.00	0.00	0.00	0.00	0.00	0.13	0.00
time (sec)	N/A	1.005	10.272	0.000	0.000	0.000	0.000	0.000	0.156	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	293	293	167	0	0	0	0	0	36	0
N.S.	1	1.00	0.57	0.00	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	1.004	10.285	0.000	0.000	0.000	0.000	0.000	0.162	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	249	249	169	495	0	349	0	0	45	0
N.S.	1	1.00	0.68	1.99	0.00	1.40	0.00	0.00	0.18	0.00
time (sec)	N/A	0.848	10.318	0.506	0.000	0.224	0.000	0.000	0.147	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	136	132	0	57	0	0	102	0
N.S.	1	1.00	0.93	0.90	0.00	0.39	0.00	0.00	0.70	0.00
time (sec)	N/A	0.670	20.319	1.076	0.000	0.160	0.000	0.000	0.211	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	136	143	0	66	0	0	115	0
N.S.	1	1.00	0.83	0.87	0.00	0.40	0.00	0.00	0.70	0.00
time (sec)	N/A	0.698	10.168	1.004	0.000	0.166	0.000	0.000	0.216	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	134	132	0	211	0	0	110	0
N.S.	1	1.00	0.80	0.79	0.00	1.26	0.00	0.00	0.66	0.00
time (sec)	N/A	0.677	20.362	1.034	0.000	0.171	0.000	0.000	0.274	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	138	139	0	213	0	0	107	0
N.S.	1	1.00	0.88	0.89	0.00	1.36	0.00	0.00	0.68	0.00
time (sec)	N/A	0.663	10.116	0.951	0.000	0.159	0.000	0.000	0.226	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	B
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	329	321	128	123	0	0	0	0	15	164
N.S.	1	0.98	0.39	0.37	0.00	0.00	0.00	0.00	0.05	0.50
time (sec)	N/A	1.511	20.126	0.396	0.000	0.000	0.000	0.000	200.024	0.247

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	B
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	380	371	128	133	0	0	0	0	28	180
N.S.	1	0.98	0.34	0.35	0.00	0.00	0.00	0.00	0.07	0.47
time (sec)	N/A	1.550	20.127	0.441	0.000	0.000	0.000	0.000	0.147	0.201

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	B
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	374	374	126	124	0	0	0	0	24	164
N.S.	1	1.00	0.34	0.33	0.00	0.00	0.00	0.00	0.06	0.44
time (sec)	N/A	1.554	20.125	0.360	0.000	0.000	0.000	0.000	22.785	21.744

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	B
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	340	321	130	133	0	0	0	0	17	179
N.S.	1	0.94	0.38	0.39	0.00	0.00	0.00	0.00	0.05	0.53
time (sec)	N/A	1.540	20.121	0.383	0.000	0.000	0.000	0.000	200.023	0.047

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	215	0	0	0	0	0	38	0
N.S.	1	1.00	1.57	0.00	0.00	0.00	0.00	0.00	0.28	0.00
time (sec)	N/A	0.441	1.847	0.000	0.000	0.000	0.000	0.000	0.231	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	311	0	0	0	0	0	40	0
N.S.	1	1.00	2.24	0.00	0.00	0.00	0.00	0.00	0.29	0.00
time (sec)	N/A	0.421	2.906	0.000	0.000	0.000	0.000	0.000	0.279	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	249	0	0	0	0	0	42	0
N.S.	1	1.00	1.48	0.00	0.00	0.00	0.00	0.00	0.25	0.00
time (sec)	N/A	0.485	0.147	0.000	0.000	0.000	0.000	0.000	0.235	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	148	1142	0	298	0	0	25	0
N.S.	1	1.00	1.53	11.77	0.00	3.07	0.00	0.00	0.26	0.00
time (sec)	N/A	0.348	1.304	6.293	0.000	2.127	0.000	0.000	0.219	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	186	180	0	0	0	0	0	0	40	0
N.S.	1	0.97	0.00	0.00	0.00	0.00	0.00	0.00	0.22	0.00
time (sec)	N/A	0.705	0.000	0.000	0.000	0.000	0.000	0.000	0.233	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	0	0	0	0	0	0	40	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.21	0.00
time (sec)	N/A	0.474	0.000	0.000	0.000	0.000	0.000	0.000	0.214	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	0	3063	0	712	0	0	24	0
N.S.	1	1.00	0.00	20.84	0.00	4.84	0.00	0.00	0.16	0.00
time (sec)	N/A	0.395	0.000	33.916	0.000	3.424	0.000	0.000	0.228	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	0	3248	0	720	0	0	32	0
N.S.	1	1.00	0.00	20.43	0.00	4.53	0.00	0.00	0.20	0.00
time (sec)	N/A	0.420	0.000	32.595	0.000	3.100	0.000	0.000	0.216	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-1)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	304	373	163	0	0	0	212	0	232	0
N.S.	1	1.23	0.54	0.00	0.00	0.00	0.70	0.00	0.76	0.00
time (sec)	N/A	0.912	9.131	0.000	0.000	0.000	3.372	0.000	0.312	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	242	234	142	0	0	0	160	0	168	0
N.S.	1	0.97	0.59	0.00	0.00	0.00	0.66	0.00	0.69	0.00
time (sec)	N/A	0.966	8.589	0.000	0.000	0.000	2.104	0.000	0.291	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	183	186	111	0	0	0	114	0	109	0
N.S.	1	1.02	0.61	0.00	0.00	0.00	0.62	0.00	0.60	0.00
time (sec)	N/A	0.684	8.370	0.000	0.000	0.000	1.662	0.000	0.177	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	155	152	75	0	0	0	82	0	61	0
N.S.	1	0.98	0.48	0.00	0.00	0.00	0.53	0.00	0.39	0.00
time (sec)	N/A	0.501	7.669	0.000	0.000	0.000	1.515	0.000	0.157	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	435	435	0	0	0	0	0	0	92	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.21	0.00
time (sec)	N/A	1.197	0.000	0.000	0.000	0.000	0.000	0.000	0.188	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	818	818	0	0	0	0	0	0	30	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	1.803	0.000	0.000	0.000	0.000	0.000	0.000	0.149	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-1)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	238	310	392	0	0	0	206	0	98	0
N.S.	1	1.30	1.65	0.00	0.00	0.00	0.87	0.00	0.41	0.00
time (sec)	N/A	0.665	10.530	0.000	0.000	0.000	2.725	0.000	0.181	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-1)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	198	255	287	0	0	0	155	0	75	0
N.S.	1	1.29	1.45	0.00	0.00	0.00	0.78	0.00	0.38	0.00
time (sec)	N/A	0.573	10.396	0.000	0.000	0.000	2.266	0.000	0.190	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	201	0	0	0	110	0	52	0
N.S.	1	1.00	1.37	0.00	0.00	0.00	0.75	0.00	0.35	0.00
time (sec)	N/A	0.526	10.220	0.000	0.000	0.000	1.541	0.000	0.160	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	163	0	0	0	78	0	29	0
N.S.	1	1.00	1.31	0.00	0.00	0.00	0.63	0.00	0.23	0.00
time (sec)	N/A	0.389	10.172	0.000	0.000	0.000	1.128	0.000	0.164	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	333	333	0	0	0	0	0	0	28	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	0.909	0.000	0.000	0.000	0.000	0.000	0.000	0.152	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	761	761	0	0	0	0	0	0	48	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.06	0.00
time (sec)	N/A	1.715	0.000	0.000	0.000	0.000	0.000	0.000	0.148	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-1)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	231	306	166	0	0	0	204	0	100	0
N.S.	1	1.32	0.72	0.00	0.00	0.00	0.88	0.00	0.43	0.00
time (sec)	N/A	0.669	10.162	0.000	0.000	0.000	2.761	0.000	0.162	0.000

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	187	184	145	0	0	0	153	0	76	0
N.S.	1	0.98	0.78	0.00	0.00	0.00	0.82	0.00	0.41	0.00
time (sec)	N/A	0.734	10.125	0.000	0.000	0.000	2.067	0.000	0.147	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	95	0	0	0	109	0	52	0
N.S.	1	1.00	0.67	0.00	0.00	0.00	0.77	0.00	0.37	0.00
time (sec)	N/A	0.521	10.057	0.000	0.000	0.000	1.592	0.000	0.139	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	78	0	0	0	78	0	29	0
N.S.	1	1.00	0.64	0.00	0.00	0.00	0.64	0.00	0.24	0.00
time (sec)	N/A	0.391	10.044	0.000	0.000	0.000	1.139	0.000	0.149	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	332	332	0	0	0	0	0	0	28	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	0.895	0.000	0.000	0.000	0.000	0.000	0.000	0.148	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	760	760	0	0	0	0	0	0	48	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.06	0.00
time (sec)	N/A	1.770	0.000	0.000	0.000	0.000	0.000	0.000	0.136	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	90	85	74	73	73	85	73	75	73
N.S.	1	1.45	1.37	1.19	1.18	1.18	1.37	1.18	1.21	1.18
time (sec)	N/A	0.437	0.005	0.142	0.028	0.082	0.024	0.111	0.143	0.041

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	57	57	50	49	49	54	49	51	49
N.S.	1	1.19	1.19	1.04	1.02	1.02	1.12	1.02	1.06	1.02
time (sec)	N/A	0.354	0.004	0.132	0.037	0.062	0.026	0.140	0.156	0.027

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	32	32	27	26	26	29	26	27	26
N.S.	1	0.94	0.94	0.79	0.76	0.76	0.85	0.76	0.79	0.76
time (sec)	N/A	0.307	0.003	0.032	0.031	0.058	0.023	0.113	0.138	0.045

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	10	10	8	10	12	10
N.S.	1	1.00	1.00	0.92	0.83	0.83	0.67	0.83	1.00	0.83
time (sec)	N/A	0.225	0.000	0.020	0.025	0.079	0.019	0.113	0.141	0.018

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	61	61	64	64	63	65	69	63
N.S.	1	1.00	0.88	0.88	0.93	0.93	0.91	0.94	1.00	0.91
time (sec)	N/A	0.381	0.030	0.171	0.028	0.072	0.105	0.124	0.150	0.042

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	66	67	71	87	70	95	90	74
N.S.	1	1.00	0.94	0.96	1.01	1.24	1.00	1.36	1.29	1.06
time (sec)	N/A	0.389	0.047	0.151	0.028	0.080	0.156	0.113	0.149	0.046

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	129	148	131	130	130	153	130	133	130
N.S.	1	1.08	1.24	1.10	1.09	1.09	1.29	1.09	1.12	1.09
time (sec)	N/A	0.582	0.008	0.187	0.031	0.066	0.027	0.109	0.142	21.501

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	104	104	91	90	90	105	90	92	90
N.S.	1	1.14	1.14	1.00	0.99	0.99	1.15	0.99	1.01	0.99
time (sec)	N/A	0.478	0.005	0.149	0.026	0.064	0.030	0.128	0.152	0.054

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	60	51	50	50	58	50	51	50
N.S.	1	1.00	1.00	0.85	0.83	0.83	0.97	0.83	0.85	0.83
time (sec)	N/A	0.379	0.003	0.220	0.033	0.080	0.025	0.109	0.139	0.025

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	22	21	21	22	21	24	21
N.S.	1	1.00	1.00	0.88	0.84	0.84	0.88	0.84	0.96	0.84
time (sec)	N/A	0.262	0.001	0.124	0.028	0.068	0.027	0.109	0.138	0.029

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	146	166	185	185	177	188	198	193
N.S.	1	1.00	0.82	0.93	1.04	1.04	0.99	1.06	1.11	1.08
time (sec)	N/A	0.613	0.073	0.233	0.027	0.080	0.217	0.127	0.159	0.059

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	178	172	195	231	199	252	242	262
N.S.	1	1.00	1.00	0.97	1.10	1.30	1.12	1.42	1.36	1.47
time (sec)	N/A	0.663	0.092	0.171	0.028	0.083	0.364	0.133	0.143	21.462

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	C	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	248	320	322	54	310	141845	384	314	433	894
N.S.	1	1.29	1.30	0.22	1.25	571.96	1.55	1.27	1.75	3.60
time (sec)	N/A	0.774	0.247	0.173	0.113	15.659	3.867	0.124	0.143	0.653

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	C	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	291	243	43	275	86139	277	287	371	556
N.S.	1	1.32	1.10	0.20	1.25	391.54	1.26	1.30	1.69	2.53
time (sec)	N/A	0.687	0.108	0.263	0.112	3.037	2.444	0.116	0.156	21.724

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	C	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	219	184	32	207	41851	124	213	213	160
N.S.	1	1.31	1.10	0.19	1.24	250.60	0.74	1.28	1.28	0.96
time (sec)	N/A	0.572	0.061	0.161	0.111	1.792	0.529	0.117	0.137	21.461

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	C	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	200	134	27	169	112	20	179	112	33
N.S.	1	1.49	1.00	0.20	1.26	0.84	0.15	1.34	0.84	0.25
time (sec)	N/A	0.594	0.023	0.180	0.112	0.094	0.099	0.113	0.142	0.086

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	C	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	331	416	404	211	345	352864	0	377	454	874
N.S.	1	1.26	1.22	0.64	1.04	1066.05	0.00	1.14	1.37	2.64
time (sec)	N/A	1.077	0.165	0.305	0.115	93.920	0.000	0.147	0.155	22.046

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F(-1)	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	443	552	524	354	561	0	0	668	19	2436
N.S.	1	1.25	1.18	0.80	1.27	0.00	0.00	1.51	0.04	5.50
time (sec)	N/A	1.581	0.638	0.237	0.133	0.000	0.000	0.297	200.017	22.419

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	C	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	284	354	347	100	332	91191	350	345	836	670
N.S.	1	1.25	1.22	0.35	1.17	321.10	1.23	1.21	2.94	2.36
time (sec)	N/A	0.889	0.387	0.204	0.129	15.245	9.112	0.119	0.153	22.355

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	C	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	328	321	90	318	90963	318	325	805	391
N.S.	1	1.31	1.28	0.36	1.27	363.85	1.27	1.30	3.22	1.56
time (sec)	N/A	0.812	0.360	0.174	0.113	3.845	2.528	0.119	0.153	21.746

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	C	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	246	224	66	238	43065	155	238	468	282
N.S.	1	1.30	1.19	0.35	1.26	227.86	0.82	1.26	2.48	1.49
time (sec)	N/A	0.643	0.217	0.249	0.113	2.155	0.905	0.119	0.159	0.281

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	C	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	225	183	46	189	183	39	194	305	58
N.S.	1	1.49	1.21	0.30	1.25	1.21	0.26	1.28	2.02	0.38
time (sec)	N/A	0.630	0.114	0.158	0.110	0.096	0.189	0.116	0.147	0.094

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F(-1)	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	703	855	558	430	601	0	0	795	1946	1591
N.S.	1	1.22	0.79	0.61	0.85	0.00	0.00	1.13	2.77	2.26
time (sec)	N/A	1.918	0.403	0.240	0.137	0.000	0.000	0.166	0.171	22.278

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F(-1)	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	937	1141	807	534	961	0	0	1145	5777	2246
N.S.	1	1.22	0.86	0.57	1.03	0.00	0.00	1.22	6.17	2.40
time (sec)	N/A	3.107	0.775	0.287	0.135	0.000	0.000	4.339	0.620	23.376

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	C	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	328	408	388	141	392	95566	413	392	1303	721
N.S.	1	1.24	1.18	0.43	1.20	291.36	1.26	1.20	3.97	2.20
time (sec)	N/A	1.031	0.389	0.178	0.115	15.068	6.251	0.321	0.161	0.495

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	C	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	288	374	358	126	364	91420	374	358	1265	676
N.S.	1	1.30	1.24	0.44	1.26	317.43	1.30	1.24	4.39	2.35
time (sec)	N/A	0.986	0.344	0.268	0.119	8.111	3.725	0.125	0.159	21.859

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	C	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	280	249	86	269	43180	192	256	740	315
N.S.	1	1.31	1.16	0.40	1.26	201.78	0.90	1.20	3.46	1.47
time (sec)	N/A	0.759	0.213	0.250	0.116	2.674	0.979	0.118	0.162	0.305

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	C	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	250	200	57	212	255	63	204	474	80
N.S.	1	1.49	1.19	0.34	1.26	1.52	0.38	1.21	2.82	0.48
time (sec)	N/A	0.701	0.093	0.162	0.112	0.088	0.304	0.128	0.159	0.098

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F(-1)	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1133	1352	835	677	1015	0	0	1311	4499	2720
N.S.	1	1.19	0.74	0.60	0.90	0.00	0.00	1.16	3.97	2.40
time (sec)	N/A	2.951	0.716	0.301	0.140	0.000	0.000	0.180	0.378	23.610

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F(-1)	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1533	1830	1115	829	1564	0	0	1809	11727	3572
N.S.	1	1.19	0.73	0.54	1.02	0.00	0.00	1.18	7.65	2.33
time (sec)	N/A	4.898	1.326	0.336	0.181	0.000	0.000	28.688	2.498	25.243

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	355	355	186	261	0	189	175	0	197	0
N.S.	1	1.00	0.52	0.74	0.00	0.53	0.49	0.00	0.55	0.00
time (sec)	N/A	0.785	10.182	1.257	0.000	0.154	2.373	0.000	0.183	0.000

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	326	326	146	234	0	165	138	0	158	0
N.S.	1	1.00	0.45	0.72	0.00	0.51	0.42	0.00	0.48	0.00
time (sec)	N/A	0.721	10.157	1.063	0.000	0.161	2.414	0.000	0.185	0.000

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	109	119	0	94	88	0	104	0
N.S.	1	1.00	0.69	0.75	0.00	0.59	0.56	0.00	0.66	0.00
time (sec)	N/A	0.459	8.471	0.631	0.000	0.147	2.004	0.000	0.164	0.000

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	89	85	0	41	37	0	35	37
N.S.	1	1.00	0.85	0.81	0.00	0.39	0.35	0.00	0.33	0.35
time (sec)	N/A	0.296	5.191	0.365	0.000	0.084	0.553	0.000	0.161	21.431

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	605	660	421	404	0	0	0	0	19	0
N.S.	1	1.09	0.70	0.67	0.00	0.00	0.00	0.00	0.03	0.00
time (sec)	N/A	2.018	11.414	1.527	0.000	0.000	0.000	0.000	200.030	0.000

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	648	975	394	402	0	0	0	0	19	0
N.S.	1	1.50	0.61	0.62	0.00	0.00	0.00	0.00	0.03	0.00
time (sec)	N/A	4.374	12.735	0.510	0.000	0.000	0.000	0.000	200.031	0.000

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	C	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	811	0	989	509	0	0	0	0	19	0
N.S.	1	0.00	1.22	0.63	0.00	0.00	0.00	0.00	0.02	0.00
time (sec)	N/A	0.000	12.005	0.647	0.000	0.000	0.000	0.000	200.064	0.000

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	414	414	185	315	0	250	434	0	289	0
N.S.	1	1.00	0.45	0.76	0.00	0.60	1.05	0.00	0.70	0.00
time (sec)	N/A	0.893	10.603	1.279	0.000	0.177	5.779	0.000	0.256	0.000

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	382	382	158	265	0	209	316	0	225	0
N.S.	1	1.00	0.41	0.69	0.00	0.55	0.83	0.00	0.59	0.00
time (sec)	N/A	0.831	10.209	1.135	0.000	0.174	5.126	0.000	0.223	0.000

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	121	139	0	117	214	0	146	0
N.S.	1	1.00	0.61	0.70	0.00	0.59	1.08	0.00	0.74	0.00
time (sec)	N/A	0.519	10.183	0.653	0.000	0.212	4.237	0.000	0.177	0.000

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	127	47	96	0	51	37	0	52	37
N.S.	1	1.04	0.39	0.79	0.00	0.42	0.30	0.00	0.43	0.30
time (sec)	N/A	0.323	6.704	0.359	0.000	0.143	0.559	0.000	0.182	21.444

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	800	876	530	576	0	0	0	0	19	0
N.S.	1	1.10	0.66	0.72	0.00	0.00	0.00	0.00	0.02	0.00
time (sec)	N/A	3.582	12.555	2.192	0.000	0.000	0.000	0.000	200.029	0.000

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	C	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	831	1816	549	604	0	0	0	0	19	0
N.S.	1	2.19	0.66	0.73	0.00	0.00	0.00	0.00	0.02	0.00
time (sec)	N/A	5.370	12.927	3.478	0.000	0.000	0.000	0.000	200.047	0.000

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	295	295	157	218	0	152	141	0	312	0
N.S.	1	1.00	0.53	0.74	0.00	0.52	0.48	0.00	1.06	0.00
time (sec)	N/A	0.645	10.180	1.178	0.000	0.110	2.071	0.000	0.263	0.000

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	264	263	133	197	0	137	105	0	105	0
N.S.	1	1.00	0.50	0.75	0.00	0.52	0.40	0.00	0.40	0.00
time (sec)	N/A	0.587	10.177	0.735	0.000	0.112	1.992	0.000	0.246	0.000

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	79	96	0	72	61	0	72	0
N.S.	1	1.00	0.65	0.79	0.00	0.60	0.50	0.00	0.60	0.00
time (sec)	N/A	0.401	10.049	0.481	0.000	0.104	1.287	0.000	0.247	0.000

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	74	70	0	29	36	0	20	37
N.S.	1	1.00	0.84	0.80	0.00	0.33	0.41	0.00	0.23	0.42
time (sec)	N/A	0.272	0.035	0.321	0.000	0.075	0.499	0.000	0.229	21.249

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	401	431	200	169	0	0	0	0	33	0
N.S.	1	1.07	0.50	0.42	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	1.102	10.377	0.408	0.000	0.000	0.000	0.000	0.266	0.000

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	605	634	448	421	0	0	0	0	19	0
N.S.	1	1.05	0.74	0.70	0.00	0.00	0.00	0.00	0.03	0.00
time (sec)	N/A	2.148	11.573	0.484	0.000	0.000	0.000	0.000	200.070	0.000

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	655	677	614	483	0	0	0	0	19	0
N.S.	1	1.03	0.94	0.74	0.00	0.00	0.00	0.00	0.03	0.00
time (sec)	N/A	2.778	12.346	0.642	0.000	0.000	0.000	0.000	200.034	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	307	295	126	245	0	160	0	0	704	0
N.S.	1	0.96	0.41	0.80	0.00	0.52	0.00	0.00	2.29	0.00
time (sec)	N/A	0.642	10.110	0.827	0.000	0.123	0.000	0.000	0.198	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	268	108	233	0	143	0	0	330	0
N.S.	1	0.99	0.40	0.86	0.00	0.53	0.00	0.00	1.22	0.00
time (sec)	N/A	0.602	10.098	0.769	0.000	0.136	0.000	0.000	0.194	0.000

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	59	115	0	76	61	0	186	57
N.S.	1	1.00	0.52	1.01	0.00	0.67	0.54	0.00	1.63	0.50
time (sec)	N/A	0.357	10.047	0.485	0.000	0.107	3.519	0.000	0.172	21.611

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	55	94	0	64	36	0	31	37
N.S.	1	1.00	0.51	0.87	0.00	0.59	0.33	0.00	0.29	0.34
time (sec)	N/A	0.300	7.547	0.320	0.000	0.107	0.560	0.000	0.171	21.475

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	676	727	455	496	0	0	0	0	57	0
N.S.	1	1.08	0.67	0.73	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	2.401	11.244	0.431	0.000	0.000	0.000	0.000	0.170	0.000

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	882	1336	578	642	0	0	0	0	98	0
N.S.	1	1.51	0.66	0.73	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	5.060	13.190	0.524	0.000	0.000	0.000	0.000	0.185	0.000

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	152	0	0	0	155	0	0	0
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.88	0.00	0.00	0.00
time (sec)	N/A	0.573	0.717	0.000	0.000	0.000	24.636	0.000	0.187	0.000

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	101	0	0	0	107	0	1518	0
N.S.	1	1.00	0.68	0.00	0.00	0.00	0.72	0.00	10.26	0.00
time (sec)	N/A	0.499	0.598	0.000	0.000	0.000	22.774	0.000	0.177	0.000

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	73	0	0	0	65	0	503	0
N.S.	1	1.00	0.76	0.00	0.00	0.00	0.68	0.00	5.24	0.00
time (sec)	N/A	0.394	0.381	0.000	0.000	0.000	14.258	0.000	0.183	0.000

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	44	0	0	0	34	0	94	41
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.77	0.00	2.14	0.93
time (sec)	N/A	0.259	0.005	0.000	0.000	0.000	4.606	0.000	0.157	21.552

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	257	262	0	0	0	0	0	0	99	0
N.S.	1	1.02	0.00	0.00	0.00	0.00	0.00	0.00	0.39	0.00
time (sec)	N/A	0.943	0.000	0.000	0.000	0.000	0.000	0.000	0.175	0.000

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	452	0	0	0	0	0	0	0	1029	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	2.28	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.242	0.000

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	118	122	108	108	108	119	111	113	107
N.S.	1	1.16	1.20	1.06	1.06	1.06	1.17	1.09	1.11	1.05
time (sec)	N/A	0.523	0.030	0.158	0.033	0.076	0.036	0.113	0.160	0.054

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	81	81	72	71	71	78	75	77	73
N.S.	1	1.07	1.07	0.95	0.93	0.93	1.03	0.99	1.01	0.96
time (sec)	N/A	0.428	0.030	0.209	0.026	0.064	0.031	0.133	0.170	0.034

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	50	50	41	40	40	46	40	41	40
N.S.	1	0.96	0.96	0.79	0.77	0.77	0.88	0.77	0.79	0.77
time (sec)	N/A	0.334	0.006	0.037	0.026	0.073	0.024	0.123	0.154	0.025

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	16	16	15	16	19	16
N.S.	1	1.00	1.00	0.85	0.80	0.80	0.75	0.80	0.95	0.80
time (sec)	N/A	0.236	0.000	0.024	0.034	0.067	0.020	0.116	0.152	0.026

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	83	87	87	87	85	90	101	90
N.S.	1	1.00	0.90	0.95	0.95	0.95	0.92	0.98	1.10	0.98
time (sec)	N/A	0.434	0.040	0.182	0.033	0.068	0.146	0.112	0.174	0.045

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	91	92	96	131	92	147	141	99
N.S.	1	1.00	0.97	0.98	1.02	1.39	0.98	1.56	1.50	1.05
time (sec)	N/A	0.444	0.143	0.161	0.034	0.173	0.328	0.105	0.150	0.058

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	235	328	258	237	236	236	277	260	263	241
N.S.	1	1.40	1.10	1.01	1.00	1.00	1.18	1.11	1.12	1.03
time (sec)	N/A	1.054	0.070	0.163	0.033	0.073	0.047	0.112	0.166	0.157

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	177	162	161	161	194	179	182	165
N.S.	1	1.00	1.00	0.92	0.91	0.91	1.10	1.01	1.03	0.93
time (sec)	N/A	0.703	0.036	0.164	0.027	0.083	0.033	0.129	0.391	0.124

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	97	95	94	94	116	100	101	94
N.S.	1	1.00	0.87	0.85	0.84	0.84	1.04	0.89	0.90	0.84
time (sec)	N/A	0.495	0.067	0.093	0.026	0.168	0.030	0.116	0.241	21.411

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	42	45	41	48	43	46	42
N.S.	1	1.00	1.00	0.86	0.92	0.84	0.98	0.88	0.94	0.86
time (sec)	N/A	0.315	0.006	0.032	0.027	0.063	0.027	0.126	0.216	0.021

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	270	268	319	304	304	308	357	386	347
N.S.	1	1.00	0.99	1.18	1.13	1.13	1.14	1.32	1.43	1.29
time (sec)	N/A	0.896	0.182	0.260	0.027	0.144	0.457	0.132	0.246	21.512

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	286	286	286	334	316	409	343	428	491	690
N.S.	1	1.00	1.00	1.17	1.10	1.43	1.20	1.50	1.72	2.41
time (sec)	N/A	0.894	0.186	0.159	0.033	0.071	0.749	0.122	0.240	0.106

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F(-1)	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	273	302	65	0	0	0	3317	1120	9076
N.S.	1	1.01	1.12	0.24	0.00	0.00	0.00	12.29	4.15	33.61
time (sec)	N/A	1.041	0.349	0.148	0.000	0.000	0.000	0.992	0.252	23.060

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	224	245	54	0	540080	0	1723	863	3046
N.S.	1	1.00	1.09	0.24	0.00	2411.07	0.00	7.69	3.85	13.60
time (sec)	N/A	0.681	0.225	0.214	0.000	83.146	0.000	0.928	0.244	22.538

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	190	194	43	0	398481	0	1342	467	1308
N.S.	1	1.01	1.03	0.23	0.00	2108.37	0.00	7.10	2.47	6.92
time (sec)	N/A	0.548	0.258	0.077	0.000	3.891	0.000	0.787	0.240	21.998

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	129	38	0	613	87	1026	351	763
N.S.	1	1.00	0.86	0.25	0.00	4.09	0.58	6.84	2.34	5.09
time (sec)	N/A	0.357	0.082	0.052	0.000	0.117	0.694	0.351	0.252	0.544

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F(-1)	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	372	372	366	307	0	0	0	19589	1383	1881
N.S.	1	1.00	0.98	0.83	0.00	0.00	0.00	52.66	3.72	5.06
time (sec)	N/A	1.582	0.755	0.292	0.000	0.000	0.000	2.970	0.276	22.129

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F(-1)	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	569	569	585	637	0	0	0	0	24	4722
N.S.	1	1.00	1.03	1.12	0.00	0.00	0.00	0.00	0.04	8.30
time (sec)	N/A	3.125	1.388	0.293	0.000	0.000	0.000	0.000	200.039	23.989

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F(-1)	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	446	420	477	293	0	0	0	5647	5447	5441
N.S.	1	0.94	1.07	0.66	0.00	0.00	0.00	12.66	12.21	12.20
time (sec)	N/A	1.370	1.538	0.253	0.000	0.000	0.000	1.166	0.671	22.479

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F(-1)	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	410	382	423	255	0	0	0	5199	4951	4118
N.S.	1	0.93	1.03	0.62	0.00	0.00	0.00	12.68	12.08	10.04
time (sec)	N/A	1.131	1.417	0.221	0.000	0.000	0.000	1.368	0.550	22.377

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F(-1)	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	330	316	341	209	0	0	0	3429	2923	2382
N.S.	1	0.96	1.03	0.63	0.00	0.00	0.00	10.39	8.86	7.22
time (sec)	N/A	0.978	0.973	0.194	0.000	0.000	0.000	1.017	0.448	22.963

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	236	243	151	0	2309	394	2682	2409	6404
N.S.	1	0.94	0.96	0.60	0.00	9.16	1.56	10.64	9.56	25.41
time (sec)	N/A	0.556	0.503	0.112	0.000	0.229	119.815	0.396	0.310	23.578

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F(-1)	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1006	1006	1031	1568	0	0	0	130635	17358	27673
N.S.	1	1.00	1.02	1.56	0.00	0.00	0.00	129.86	17.25	27.51
time (sec)	N/A	5.056	5.352	0.423	0.000	0.000	0.000	15.971	4.334	28.711

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	A	F	F(-1)	F(-1)	F(-1)	F	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1689	0	2086	2539	0	0	0	0	24	45031
N.S.	1	0.00	1.24	1.50	0.00	0.00	0.00	0.00	0.01	26.66
time (sec)	N/A	0.000	8.084	0.918	0.000	0.000	0.000	0.000	200.024	39.814

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	546	526	707	687	0	625	0	0	1216	0
N.S.	1	0.96	1.29	1.26	0.00	1.14	0.00	0.00	2.23	0.00
time (sec)	N/A	1.497	14.291	4.049	0.000	0.281	0.000	0.000	1.581	0.000

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	491	474	640	561	0	551	0	0	836	0
N.S.	1	0.97	1.30	1.14	0.00	1.12	0.00	0.00	1.70	0.00
time (sec)	N/A	1.099	12.801	3.393	0.000	0.241	0.000	0.000	1.202	0.000

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	397	403	579	453	0	391	0	0	473	0
N.S.	1	1.02	1.46	1.14	0.00	0.98	0.00	0.00	1.19	0.00
time (sec)	N/A	0.932	11.873	1.625	0.000	0.195	0.000	0.000	0.780	0.000

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	309	312	445	379	0	301	0	0	86	0
N.S.	1	1.01	1.44	1.23	0.00	0.97	0.00	0.00	0.28	0.00
time (sec)	N/A	0.599	0.849	1.148	0.000	0.110	0.000	0.000	0.264	0.000

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	794	846	5994	755	0	0	0	0	24	0
N.S.	1	1.07	7.55	0.95	0.00	0.00	0.00	0.00	0.03	0.00
time (sec)	N/A	2.804	20.474	5.060	0.000	0.000	0.000	0.000	200.035	0.000

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	855	0	4221	749	0	0	0	0	24	0
N.S.	1	0.00	4.94	0.88	0.00	0.00	0.00	0.00	0.03	0.00
time (sec)	N/A	0.000	19.070	0.867	0.000	0.000	0.000	0.000	200.057	0.000

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1140	0	8346	917	0	0	0	0	24	0
N.S.	1	0.00	7.32	0.80	0.00	0.00	0.00	0.00	0.02	0.00
time (sec)	N/A	0.000	21.097	1.260	0.000	0.000	0.000	0.000	200.047	0.000

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	799	749	2681	1300	0	981	0	0	24	0
N.S.	1	0.94	3.36	1.63	0.00	1.23	0.00	0.00	0.03	0.00
time (sec)	N/A	2.162	12.678	4.585	0.000	0.320	0.000	0.000	200.067	0.000

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	733	702	838	1087	0	834	0	0	0	0
N.S.	1	0.96	1.14	1.48	0.00	1.14	0.00	0.00	0.00	0.00
time (sec)	N/A	1.605	15.101	3.780	0.000	0.241	0.000	0.000	112.976	0.000

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	512	521	645	690	0	541	0	0	952	0
N.S.	1	1.02	1.26	1.35	0.00	1.06	0.00	0.00	1.86	0.00
time (sec)	N/A	1.281	12.787	2.122	0.000	0.177	0.000	0.000	84.176	0.000

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	381	383	533	471	0	396	0	0	231	0
N.S.	1	1.01	1.40	1.24	0.00	1.04	0.00	0.00	0.61	0.00
time (sec)	N/A	0.781	1.764	1.649	0.000	0.089	0.000	0.000	0.213	0.000

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1150	1131	12989	1017	0	0	0	0	24	0
N.S.	1	0.98	11.29	0.88	0.00	0.00	0.00	0.00	0.02	0.00
time (sec)	N/A	5.911	23.861	4.131	0.000	0.000	0.000	0.000	200.032	0.000

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1195	0	11129	1199	0	0	0	0	24	0
N.S.	1	0.00	9.31	1.00	0.00	0.00	0.00	0.00	0.02	0.00
time (sec)	N/A	0.000	22.697	5.631	0.000	0.000	0.000	0.000	200.027	0.000

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	380	379	541	440	0	403	0	0	0	0
N.S.	1	1.00	1.42	1.16	0.00	1.06	0.00	0.00	0.00	0.00
time (sec)	N/A	0.997	11.481	3.061	0.000	0.272	0.000	0.000	0.609	0.000

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	334	333	490	401	0	371	0	0	339	0
N.S.	1	1.00	1.47	1.20	0.00	1.11	0.00	0.00	1.01	0.00
time (sec)	N/A	0.747	11.202	1.382	0.000	0.210	0.000	0.000	0.355	0.000

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	230	181	0	189	0	0	221	0
N.S.	1	1.00	1.44	1.13	0.00	1.18	0.00	0.00	1.38	0.00
time (sec)	N/A	0.484	10.396	0.736	0.000	0.146	0.000	0.000	0.274	0.000

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	186	144	0	121	0	0	30	0
N.S.	1	1.00	1.63	1.26	0.00	1.06	0.00	0.00	0.26	0.00
time (sec)	N/A	0.292	0.082	0.356	0.000	0.106	0.000	0.000	0.156	0.000

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	512	540	1725	281	0	0	0	0	24	0
N.S.	1	1.05	3.37	0.55	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	1.467	17.092	0.595	0.000	0.000	0.000	0.000	200.038	0.000

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	816	814	4019	771	0	0	0	0	24	0
N.S.	1	1.00	4.93	0.94	0.00	0.00	0.00	0.00	0.03	0.00
time (sec)	N/A	2.929	18.149	0.910	0.000	0.000	0.000	0.000	200.044	0.000

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	481	461	551	603	0	805	0	0	0	0
N.S.	1	0.96	1.15	1.25	0.00	1.67	0.00	0.00	0.00	0.00
time (sec)	N/A	1.175	12.070	1.779	0.000	0.114	0.000	0.000	0.449	0.000

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	456	438	517	572	0	765	0	0	0	0
N.S.	1	0.96	1.13	1.25	0.00	1.68	0.00	0.00	0.00	0.00
time (sec)	N/A	0.990	11.836	1.451	0.000	0.113	0.000	0.000	0.365	0.000

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	394	388	470	520	0	487	0	0	481	0
N.S.	1	0.98	1.19	1.32	0.00	1.24	0.00	0.00	1.22	0.00
time (sec)	N/A	0.873	11.172	0.759	0.000	0.097	0.000	0.000	0.304	0.000

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	353	348	456	481	0	452	0	0	57	0
N.S.	1	0.99	1.29	1.36	0.00	1.28	0.00	0.00	0.16	0.00
time (sec)	N/A	0.651	1.026	0.377	0.000	0.090	0.000	0.000	0.260	0.000

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1010	1036	4965	1111	0	0	0	0	24	0
N.S.	1	1.03	4.92	1.10	0.00	0.00	0.00	0.00	0.02	0.00
time (sec)	N/A	3.569	18.363	0.645	0.000	0.000	0.000	0.000	200.033	0.000

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1490	0	10488	1545	0	0	0	0	24	0
N.S.	1	0.00	7.04	1.04	0.00	0.00	0.00	0.00	0.02	0.00
time (sec)	N/A	0.000	20.723	0.915	0.000	0.000	0.000	0.000	200.032	0.000

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	128	105	364	0	221	0	203	100	0
N.S.	1	1.15	0.95	3.28	0.00	1.99	0.00	1.83	0.90	0.00
time (sec)	N/A	0.457	0.136	0.505	0.000	0.122	0.000	0.135	0.178	0.000

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	128	105	363	0	221	0	203	100	0
N.S.	1	1.15	0.95	3.27	0.00	1.99	0.00	1.83	0.90	0.00
time (sec)	N/A	0.462	0.004	0.309	0.000	0.098	0.000	0.143	0.181	0.000

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	128	105	364	0	221	0	203	100	0
N.S.	1	1.15	0.95	3.28	0.00	1.99	0.00	1.83	0.90	0.00
time (sec)	N/A	0.483	0.004	0.310	0.000	0.095	0.000	0.147	0.165	0.000

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	120	97	195	0	313	0	182	93	0
N.S.	1	1.14	0.92	1.86	0.00	2.98	0.00	1.73	0.89	0.00
time (sec)	N/A	0.449	0.137	0.524	0.000	0.139	0.000	0.149	0.177	0.000

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	120	97	195	0	313	0	182	93	0
N.S.	1	1.14	0.92	1.86	0.00	2.98	0.00	1.73	0.89	0.00
time (sec)	N/A	0.461	0.003	0.324	0.000	0.173	0.000	0.130	0.175	0.000

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	120	97	195	0	313	0	182	93	0
N.S.	1	1.14	0.92	1.86	0.00	2.98	0.00	1.73	0.89	0.00
time (sec)	N/A	0.457	0.003	0.318	0.000	0.167	0.000	0.145	0.161	0.000

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	120	97	195	0	313	0	182	93	0
N.S.	1	1.14	0.92	1.86	0.00	2.98	0.00	1.73	0.89	0.00
time (sec)	N/A	0.472	0.003	0.323	0.000	0.140	0.000	0.145	0.179	0.000

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	120	97	195	0	313	0	182	93	0
N.S.	1	1.14	0.92	1.86	0.00	2.98	0.00	1.73	0.89	0.00
time (sec)	N/A	0.464	0.003	0.331	0.000	0.153	0.000	0.135	0.178	0.000

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	120	97	195	0	313	0	182	93	0
N.S.	1	1.14	0.92	1.86	0.00	2.98	0.00	1.73	0.89	0.00
time (sec)	N/A	0.468	0.003	0.326	0.000	0.157	0.000	0.140	0.168	0.000

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1613	1590	455	1153	0	0	0	0	45	0
N.S.	1	0.99	0.28	0.71	0.00	0.00	0.00	0.00	0.03	0.00
time (sec)	N/A	20.675	11.689	0.766	0.000	0.000	0.000	0.000	0.192	0.000

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	191	139	105	0	603	0	0	30	0
N.S.	1	1.32	0.96	0.72	0.00	4.16	0.00	0.00	0.21	0.00
time (sec)	N/A	0.592	0.882	1.093	0.000	1.383	0.000	0.000	0.164	0.000

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	151	351	0	669	0	0	30	0
N.S.	1	1.00	0.63	1.47	0.00	2.80	0.00	0.00	0.13	0.00
time (sec)	N/A	0.611	0.838	1.836	0.000	1.431	0.000	0.000	0.180	0.000

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	82	0	0	0	0	0	0	0
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.321	0.215	0.000	0.000	0.000	0.000	0.000	0.428	0.000

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	82	0	0	0	0	0	0	0
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.530	0.006	0.000	0.000	0.000	0.000	0.000	0.405	0.000

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	437	449	86	68	0	0	65	0	115	338
N.S.	1	1.03	0.20	0.16	0.00	0.00	0.15	0.00	0.26	0.77
time (sec)	N/A	2.336	0.034	0.166	0.000	0.000	0.133	0.000	0.182	22.445

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	455	460	86	63	0	0	65	0	85	338
N.S.	1	1.01	0.19	0.14	0.00	0.00	0.14	0.00	0.19	0.74
time (sec)	N/A	2.099	0.031	0.139	0.000	0.000	0.115	0.000	0.162	22.098

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	441	448	76	58	0	0	65	0	55	338
N.S.	1	1.02	0.17	0.13	0.00	0.00	0.15	0.00	0.12	0.77
time (sec)	N/A	2.142	0.024	0.042	0.000	0.000	0.114	0.000	0.176	22.326

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	383	370	62	53	0	0	65	0	24	306
N.S.	1	0.97	0.16	0.14	0.00	0.00	0.17	0.00	0.06	0.80
time (sec)	N/A	1.821	0.012	0.033	0.000	0.000	0.108	0.000	0.181	22.708

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	610	592	173	77	0	0	87	0	39	467
N.S.	1	0.97	0.28	0.13	0.00	0.00	0.14	0.00	0.06	0.77
time (sec)	N/A	2.968	0.037	0.142	0.000	0.000	0.431	0.000	0.167	22.151

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	610	474	139	77	0	0	87	0	39	467
N.S.	1	0.78	0.23	0.13	0.00	0.00	0.14	0.00	0.06	0.77
time (sec)	N/A	2.757	0.027	0.056	0.000	0.000	0.422	0.000	0.177	22.022

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	651	647	185	84	0	0	94	0	44	480
N.S.	1	0.99	0.28	0.13	0.00	0.00	0.14	0.00	0.07	0.74
time (sec)	N/A	3.684	0.049	0.151	0.000	0.000	0.774	0.000	0.176	0.098

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	C	F	F(-1)	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	651	485	185	84	0	0	94	0	44	480
N.S.	1	0.75	0.28	0.13	0.00	0.00	0.14	0.00	0.07	0.74
time (sec)	N/A	2.820	0.028	0.059	0.000	0.000	0.772	0.000	0.172	21.941

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	C	F	C	A	F	F	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	0	95	67	0	5653	42	0	95	504
N.S.	1	0.00	0.57	0.40	0.00	33.65	0.25	0.00	0.57	3.00
time (sec)	N/A	0.000	0.063	0.576	0.000	0.754	0.980	0.000	0.180	22.845

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	287	288	137	0	0	0	0	0	0	0
N.S.	1	1.00	0.48	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.886	8.303	0.000	0.000	0.000	0.000	0.000	0.543	0.000

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	171	175	106	0	0	0	0	0	1442	0
N.S.	1	1.02	0.62	0.00	0.00	0.00	0.00	0.00	8.43	0.00
time (sec)	N/A	0.551	5.641	0.000	0.000	0.000	0.000	0.000	0.373	0.000

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	93	85	90	0	0	0	0	0	498	0
N.S.	1	0.91	0.97	0.00	0.00	0.00	0.00	0.00	5.35	0.00
time (sec)	N/A	0.354	0.485	0.000	0.000	0.000	0.000	0.000	0.274	0.000

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	44	0	0	0	34	0	94	41
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.77	0.00	2.14	0.93
time (sec)	N/A	0.260	0.156	0.000	0.000	0.000	32.077	0.000	0.234	22.698

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	162	0	0	0	0	0	21	0
N.S.	1	1.00	2.84	0.00	0.00	0.00	0.00	0.00	0.37	0.00
time (sec)	N/A	0.298	0.740	0.000	0.000	0.000	0.000	0.000	2.566	0.000

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	162	0	0	0	0	0	32	0
N.S.	1	1.00	2.84	0.00	0.00	0.00	0.00	0.00	0.56	0.00
time (sec)	N/A	0.290	0.788	0.000	0.000	0.000	0.000	0.000	34.773	0.000

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	162	0	0	0	0	0	43	0
N.S.	1	1.00	2.84	0.00	0.00	0.00	0.00	0.00	0.75	0.00
time (sec)	N/A	0.290	0.888	0.000	0.000	0.000	0.000	0.000	57.449	0.000

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	209	0	137	0	0	0	0	0	0	0
N.S.	1	0.00	0.66	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	0.680	0.000	0.000	0.000	0.000	0.000	0.678	0.000

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	150	0	106	0	0	0	0	0	0	0
N.S.	1	0.00	0.71	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	0.019	0.000	0.000	0.000	0.000	0.000	0.427	0.000

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	75	0	0	0	75	0	535	0
N.S.	1	1.00	0.78	0.00	0.00	0.00	0.78	0.00	5.57	0.00
time (sec)	N/A	0.356	0.010	0.000	0.000	0.000	166.517	0.000	0.291	0.000

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	44	0	0	0	34	0	94	41
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.77	0.00	2.14	0.93
time (sec)	N/A	0.252	0.002	0.000	0.000	0.000	31.165	0.000	0.222	0.002

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	0	0	0	0	0	0	21	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.17	0.00
time (sec)	N/A	0.462	0.000	0.000	0.000	0.000	0.000	0.000	0.756	0.000

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	0	0	0	0	0	0	32	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.17	0.00
time (sec)	N/A	0.605	0.000	0.000	0.000	0.000	0.000	0.000	12.504	0.000

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	0	0	0	0	0	0	43	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.17	0.00
time (sec)	N/A	0.730	0.000	0.000	0.000	0.000	0.000	0.000	64.295	0.000

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	136	0	0	0	0	0	0	0
N.S.	1	1.00	0.67	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.597	0.636	0.000	0.000	0.000	0.000	0.000	0.752	0.000

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	106	0	0	0	119	0	0	0
N.S.	1	1.00	0.70	0.00	0.00	0.00	0.79	0.00	0.00	0.00
time (sec)	N/A	0.482	0.605	0.000	0.000	0.000	178.631	0.000	0.479	0.000

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	75	0	0	0	75	0	535	0
N.S.	1	1.00	0.78	0.00	0.00	0.00	0.78	0.00	5.57	0.00
time (sec)	N/A	0.371	0.561	0.000	0.000	0.000	130.947	0.000	0.313	0.000

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	44	0	0	0	34	0	94	41
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.77	0.00	2.14	0.93
time (sec)	N/A	0.255	0.001	0.000	0.000	0.000	31.098	0.000	0.233	0.002

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	255	0	0	0	0	0	0	0	21	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.433	0.000

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	453	0	0	0	0	0	0	0	32	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.07	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	5.027	0.000

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	129	0	0	0	0	0	0	0
N.S.	1	1.00	0.64	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.572	0.640	0.000	0.000	0.000	0.000	0.000	1.115	0.000

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	101	0	0	0	119	0	1518	0
N.S.	1	1.00	0.68	0.00	0.00	0.00	0.80	0.00	10.26	0.00
time (sec)	N/A	0.460	0.608	0.000	0.000	0.000	135.241	0.000	0.657	0.000

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	73	0	0	0	75	0	503	0
N.S.	1	1.00	0.76	0.00	0.00	0.00	0.78	0.00	5.24	0.00
time (sec)	N/A	0.388	0.449	0.000	0.000	0.000	92.307	0.000	0.440	0.000

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	44	0	0	0	34	0	94	41
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.77	0.00	2.14	0.93
time (sec)	N/A	0.255	0.001	0.000	0.000	0.000	30.166	0.000	0.310	0.002

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	521	0	0	0	0	0	0	0	99	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.19	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.403	0.000

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	970	0	0	0	0	0	0	0	1029	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.06	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	11.755	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [188] had the largest ratio of [1.1111100000000004]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	3	2	1.00	22	0.091
2	A	3	2	1.00	22	0.091
3	F	0	0	N/A	0.000	N/A
4	A	2	2	1.00	14	0.143
5	A	2	2	1.00	12	0.167
6	A	2	2	1.00	10	0.200
7	A	3	3	1.00	17	0.176
8	A	2	2	0.97	46	0.043
9	A	2	2	0.97	46	0.043
10	A	2	2	0.96	46	0.043
11	A	2	2	0.97	44	0.045
12	A	2	2	0.97	42	0.048
13	A	2	2	0.97	46	0.043
14	A	2	2	0.97	46	0.043
15	A	2	2	1.02	26	0.077
16	A	2	2	0.99	26	0.077
17	A	2	2	0.98	26	0.077
18	A	2	2	0.97	26	0.077
19	A	2	2	0.98	24	0.083
20	A	2	2	0.97	22	0.091
21	A	2	2	0.97	26	0.077

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
22	A	2	2	1.01	26	0.077
23	A	2	2	0.95	26	0.077
24	A	2	2	0.95	26	0.077
25	A	2	2	0.94	26	0.077
26	A	2	2	0.98	26	0.077
27	A	2	2	0.97	26	0.077
28	A	2	2	0.97	26	0.077
29	A	2	2	0.90	26	0.077
30	A	3	3	1.00	15	0.200
31	A	3	3	1.00	11	0.273
32	A	2	2	1.00	11	0.182
33	A	2	2	1.00	52	0.038
34	A	5	5	1.00	52	0.096
35	A	2	2	1.00	14	0.143
36	A	2	2	1.03	54	0.037
37	A	3	3	1.00	54	0.056
38	A	5	5	1.00	54	0.093
39	A	3	3	1.47	13	0.231
40	A	3	3	1.00	18	0.167
41	A	3	3	1.00	22	0.136
42	A	3	3	1.00	16	0.188
43	A	3	3	1.00	16	0.188
44	A	3	3	1.00	15	0.200
45	A	3	3	1.00	17	0.176
46	A	3	3	1.00	15	0.200
47	A	5	4	1.33	15	0.267
48	A	5	4	1.32	15	0.267
49	A	1	1	1.00	15	0.067
50	A	1	1	1.00	20	0.050
51	A	3	3	1.00	15	0.200
52	A	3	3	1.00	13	0.231
53	A	4	4	1.00	17	0.235

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
54	A	4	4	1.00	25	0.160
55	A	2	2	1.00	20	0.100
56	A	4	4	1.00	18	0.222
57	A	3	3	1.00	20	0.150
58	A	1	1	1.00	22	0.045
59	A	1	1	1.00	17	0.059
60	A	8	7	1.20	27	0.259
61	A	8	7	1.12	20	0.350
62	A	3	3	1.09	38	0.079
63	A	10	10	1.07	50	0.200
64	F	0	0	N/A	0.000	N/A
65	F	0	0	N/A	0.000	N/A
66	A	1	1	1.00	16	0.062
67	A	3	3	1.23	16	0.188
68	A	2	2	1.98	17	0.118
69	B	2	2	2.21	25	0.080
70	B	5	5	4.68	81	0.062
71	A	3	2	1.07	22	0.091
72	A	3	2	1.03	23	0.087
73	A	3	2	1.00	22	0.091
74	A	3	2	1.03	23	0.087
75	A	3	2	1.07	24	0.083
76	A	3	2	1.03	25	0.080
77	A	3	2	1.18	31	0.065
78	A	3	2	1.08	32	0.062
79	A	3	2	1.07	35	0.057
80	A	3	2	1.04	36	0.056
81	A	3	2	1.00	24	0.083
82	A	3	2	1.00	31	0.065
83	A	3	2	1.00	35	0.057
84	A	1	1	1.00	22	0.045
85	A	1	1	1.00	18	0.056

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
86	B	3	3	2.91	26	0.115
87	B	2	2	2.91	28	0.071
88	A	1	1	1.00	18	0.056
89	A	1	1	1.00	20	0.050
90	A	1	1	1.00	21	0.048
91	A	6	5	1.03	14	0.357
92	A	8	7	1.03	13	0.538
93	A	6	5	1.12	16	0.312
94	A	8	7	1.12	18	0.389
95	A	2	2	1.00	14	0.143
96	A	3	3	1.00	16	0.188
97	B	3	3	4.68	40	0.075
98	F	0	0	N/A	0.000	N/A
99	B	4	4	4.68	69	0.058
100	F	0	0	N/A	0.000	N/A
101	B	5	5	4.68	80	0.062
102	A	4	3	1.04	17	0.176
103	A	4	3	1.08	17	0.176
104	A	4	3	1.08	15	0.200
105	A	2	2	1.14	20	0.100
106	A	2	2	1.14	20	0.100
107	A	2	2	1.12	18	0.111
108	A	4	3	1.02	24	0.125
109	A	4	3	1.06	24	0.125
110	A	4	3	1.11	22	0.136
111	F	0	0	N/A	0.000	N/A
112	F	0	0	N/A	0.000	N/A
113	F	0	0	N/A	0.000	N/A
114	A	2	2	1.00	15	0.133
115	A	3	3	1.00	35	0.086
116	A	3	3	1.00	46	0.065
117	A	4	4	1.00	66	0.061

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
118	A	5	4	1.00	19	0.211
119	A	6	5	1.00	23	0.217
120	A	6	5	1.00	21	0.238
121	A	5	4	1.00	21	0.190
122	A	5	4	1.00	33	0.121
123	A	6	5	1.00	35	0.143
124	A	6	5	1.00	36	0.139
125	A	5	4	1.00	36	0.111
126	A	5	4	1.00	24	0.167
127	A	6	5	1.00	20	0.250
128	A	6	5	1.00	24	0.208
129	A	6	5	1.00	22	0.227
130	A	6	5	1.00	22	0.227
131	A	10	9	0.98	15	0.600
132	A	9	8	0.98	17	0.471
133	A	9	8	1.00	15	0.533
134	A	10	9	0.94	17	0.529
135	A	1	1	1.00	24	0.042
136	A	1	1	1.00	25	0.040
137	A	1	1	1.00	27	0.037
138	A	1	1	1.00	17	0.059
139	A	3	3	0.97	25	0.120
140	A	1	1	1.00	25	0.040
141	A	1	1	1.00	21	0.048
142	A	1	1	1.00	24	0.042
143	A	4	4	1.23	19	0.211
144	A	8	8	0.97	19	0.421
145	A	6	6	1.02	19	0.316
146	A	4	4	0.98	17	0.235
147	A	2	2	1.00	19	0.105
148	A	2	2	1.00	19	0.105
149	A	2	2	1.30	19	0.105

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
150	A	2	2	1.29	19	0.105
151	A	4	4	1.00	19	0.211
152	A	2	2	1.00	17	0.118
153	A	2	2	1.00	19	0.105
154	A	2	2	1.00	19	0.105
155	A	2	2	1.32	19	0.105
156	A	5	5	0.98	19	0.263
157	A	4	4	1.00	19	0.211
158	A	2	2	1.00	17	0.118
159	A	2	2	1.00	19	0.105
160	A	2	2	1.00	19	0.105
161	A	2	2	1.45	15	0.133
162	A	2	2	1.19	15	0.133
163	A	2	2	0.94	13	0.154
164	A	1	1	1.00	7	0.143
165	A	2	2	1.00	15	0.133
166	A	2	2	1.00	15	0.133
167	A	3	3	1.08	17	0.176
168	A	2	2	1.14	17	0.118
169	A	2	2	1.00	15	0.133
170	A	2	2	1.00	9	0.222
171	A	2	2	1.00	17	0.118
172	A	2	2	1.00	17	0.118
173	A	2	2	1.29	17	0.118
174	A	2	2	1.32	17	0.118
175	A	2	2	1.31	15	0.133
176	A	9	8	1.49	9	0.889
177	A	2	2	1.26	17	0.118
178	A	2	2	1.25	17	0.118
179	A	5	5	1.25	17	0.294
180	A	4	4	1.31	17	0.235
181	A	4	4	1.30	15	0.267

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
182	A	10	9	1.49	9	1.000
183	A	2	2	1.22	17	0.118
184	A	2	2	1.22	17	0.118
185	A	6	6	1.24	17	0.353
186	A	6	6	1.30	17	0.353
187	A	6	6	1.31	15	0.400
188	A	11	10	1.49	9	1.111
189	A	2	2	1.19	17	0.118
190	A	2	2	1.19	17	0.118
191	A	2	2	1.00	19	0.105
192	A	2	2	1.00	19	0.105
193	A	2	2	1.00	17	0.118
194	A	2	2	1.00	11	0.182
195	A	17	16	1.09	19	0.842
196	A	13	12	1.50	19	0.632
197	F	0	0	N/A	0.000	N/A
198	A	2	2	1.00	19	0.105
199	A	2	2	1.00	19	0.105
200	A	2	2	1.00	17	0.118
201	A	3	3	1.04	11	0.273
202	A	22	21	1.10	19	1.105
203	B	14	13	2.19	19	0.684
204	A	2	2	1.00	19	0.105
205	A	2	2	1.00	19	0.105
206	A	2	2	1.00	17	0.118
207	A	1	1	1.00	11	0.091
208	A	9	8	1.07	19	0.421
209	A	16	15	1.05	19	0.789
210	A	18	17	1.03	19	0.895
211	A	7	7	0.96	19	0.368
212	A	6	6	0.99	19	0.316
213	A	4	4	1.00	17	0.235

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
214	A	2	2	1.00	11	0.182
215	A	18	17	1.08	19	0.895
216	A	13	12	1.51	19	0.632
217	A	2	2	1.00	17	0.118
218	A	2	2	1.00	17	0.118
219	A	2	2	1.00	15	0.133
220	A	2	2	1.00	9	0.222
221	A	11	10	1.02	17	0.588
222	F	0	0	N/A	0.000	N/A
223	A	2	2	1.16	20	0.100
224	A	2	2	1.07	20	0.100
225	A	2	2	0.96	18	0.111
226	A	1	1	1.00	12	0.083
227	A	2	2	1.00	20	0.100
228	A	2	2	1.00	20	0.100
229	A	2	2	1.40	22	0.091
230	A	2	2	1.00	22	0.091
231	A	2	2	1.00	20	0.100
232	A	2	2	1.00	14	0.143
233	A	2	2	1.00	22	0.091
234	A	2	2	1.00	22	0.091
235	A	10	9	1.01	22	0.409
236	A	8	7	1.00	22	0.318
237	A	8	7	1.01	20	0.350
238	A	2	2	1.00	14	0.143
239	A	2	2	1.00	22	0.091
240	A	2	2	1.00	22	0.091
241	A	12	11	0.94	22	0.500
242	A	11	10	0.93	22	0.455
243	A	11	10	0.96	20	0.500
244	A	4	4	0.94	14	0.286
245	A	2	2	1.00	22	0.091

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
246	F	0	0	N/A	0.000	N/A
247	A	14	13	0.96	24	0.542
248	A	12	11	0.97	24	0.458
249	A	12	11	1.02	22	0.500
250	A	5	5	1.01	16	0.312
251	A	16	15	1.07	24	0.625
252	F	0	0	N/A	0.000	N/A
253	F	0	0	N/A	0.000	N/A
254	A	17	16	0.94	24	0.667
255	A	15	14	0.96	24	0.583
256	A	15	14	1.02	22	0.636
257	A	7	7	1.01	16	0.438
258	A	23	22	0.98	24	0.917
259	F	0	0	N/A	0.000	N/A
260	A	11	10	1.00	24	0.417
261	A	10	9	1.00	24	0.375
262	A	7	6	1.00	22	0.273
263	A	1	1	1.00	16	0.062
264	A	9	8	1.05	24	0.333
265	A	16	15	1.00	24	0.625
266	A	11	10	0.96	24	0.417
267	A	10	9	0.96	24	0.375
268	A	11	10	0.98	22	0.455
269	A	6	6	0.99	16	0.375
270	A	16	15	1.03	24	0.625
271	F	0	0	N/A	0.000	N/A
272	A	6	5	1.15	29	0.172
273	A	7	6	1.15	28	0.214
274	A	7	6	1.15	29	0.207
275	A	6	5	1.14	27	0.185
276	A	7	6	1.14	26	0.231
277	A	7	6	1.14	28	0.214

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
278	A	7	6	1.14	26	0.231
279	A	7	6	1.14	27	0.222
280	A	7	6	1.14	27	0.222
281	A	4	4	0.99	24	0.167
282	A	4	3	1.32	30	0.100
283	A	1	1	1.00	30	0.033
284	A	4	3	1.00	21	0.143
285	A	4	3	1.00	67	0.045
286	A	2	2	1.03	30	0.067
287	A	2	2	1.01	30	0.067
288	A	2	2	1.02	28	0.071
289	A	2	2	0.97	22	0.091
290	A	2	2	0.97	30	0.067
291	A	2	2	0.78	37	0.054
292	A	2	2	0.99	30	0.067
293	C	2	2	0.75	42	0.048
294	F	0	0	N/A	0.000	N/A
295	A	7	7	1.00	19	0.368
296	A	5	5	1.02	19	0.263
297	A	3	3	0.91	17	0.176
298	A	2	2	1.00	9	0.222
299	A	2	2	1.00	19	0.105
300	A	2	2	1.00	19	0.105
301	A	2	2	1.00	19	0.105
302	F	0	0	N/A	0.000	N/A
303	F	0	0	N/A	0.000	N/A
304	A	2	2	1.00	17	0.118
305	A	2	2	1.00	9	0.222
306	A	2	2	1.00	19	0.105
307	A	2	2	1.00	19	0.105
308	A	2	2	1.00	19	0.105
309	A	2	2	1.00	19	0.105

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
310	A	2	2	1.00	19	0.105
311	A	2	2	1.00	17	0.118
312	A	2	2	1.00	9	0.222
313	F	0	0	N/A	0.000	N/A
314	F	0	0	N/A	0.000	N/A
315	A	2	2	1.00	17	0.118
316	A	2	2	1.00	17	0.118
317	A	2	2	1.00	15	0.133
318	A	2	2	1.00	9	0.222
319	F	0	0	N/A	0.000	N/A
320	F	0	0	N/A	0.000	N/A

CHAPTER 3

Listing of integrals

3.1 $\int \frac{x^{2016}(2017+2018x)}{1+\left(x^{2017}+x^{2018}\right)^2} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 8

$$\int \frac{x^{2016}(2017+2018x)}{1+\left(x^{2017}+x^{2018}\right)^2} \, dx = \arctan\left(x^{2017}(1+x)\right)$$

output
```
arctan(x^2017*(1+x))
```

Mathematica [F(-1)]

Timed out.

$$\int \frac{x^{2016}(2017+2018x)}{1+\left(x^{2017}+x^{2018}\right)^2} \, dx = \$\text{Aborted}$$

input
```
Integrate[(x^2016*(2017 + 2018*x))/(1 + (x^2017 + x^2018)^2),x]
```

output
```
$Aborted
```

Rubi [A] (verified)

Time = 10.36 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2024, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{2016}(2018x+2017)}{(x^{2018}+x^{2017})^2+1} \, dx$$

$$\Big\downarrow \text{2024}$$

$$\int \frac{1}{(x^{2018}+x^{2017})^2+1} d(x^{2018}+x^{2017})$$

$$\Big\downarrow \text{216}$$

$$\arctan\left(x^{2018}+x^{2017}\right)$$

input
```
Int[(x^2016*(2017 + 2018*x))/(1 + (x^2017 + x^2018)^2),x]
```

output
```
ArcTan[x^2017 + x^2018]
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 2024
```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Simp[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q])   Subst[
Int[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D
[Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &
& PolyQ[Qr, x]
```

Maple [A] (verified)

Time = 93.12 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.12

$$\arctan\left(x^{2018}+x^{2017}\right)$$

input
```
int(x^2016*(2017+2018*x)/(1+(x^2018+x^2017)^2),x)
```

output
```
arctan(x^2018+x^2017)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^{2016}(2017+2018x)}{1+\left(x^{2017}+x^{2018}\right)^2}\,dx=\text{Timed out}$$

input
```
integrate(x^2016*(2017+2018*x)/(1+(x^2018+x^2017)^2),x, algorithm="fricas"
)
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{2016}(2017+2018x)}{1+\left(x^{2017}+x^{2018}\right)^2}\,dx=\text{Timed out}$$

input
```
integrate(x**2016*(2017+2018*x)/(1+(x**2018+x**2017)**2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^{2016}(2017 + 2018x)}{1 + (x^{2017} + x^{2018})^2} \, dx = \int \frac{(2018\,x + 2017)x^{2016}}{(x^{2018} + x^{2017})^2 + 1} \, dx$$

input
```
integrate(x^2016*(2017+2018*x)/(1+(x^2018+x^2017)^2),x, algorithm="maxima"
)
```

output
```
integrate((2018*x + 2017)*x^2016/((x^2018 + x^2017)^2 + 1), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{x^{2016}(2017 + 2018x)}{1 + (x^{2017} + x^{2018})^2} \, dx = \text{Timed out}$$

input
```
integrate(x^2016*(2017+2018*x)/(1+(x^2018+x^2017)^2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 61.26 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{x^{2016}(2017 + 2018x)}{1 + (x^{2017} + x^{2018})^2} \, dx = \operatorname{atan}\left(x^{2017}\,(x + 1)\right)$$

input
```
int((x^2016*(2018*x + 2017))/((x^2017 + x^2018)^2 + 1),x)
```

output
```
atan(x^2017*(x + 1))
```

Reduce [F]

$$\int \frac{x^{2016}(2017+2018x)}{1+(x^{2017}+x^{2018})^2}\,dx = 2018\left(\int \frac{x^{2017}}{x^{4036}+2x^{4035}+x^{4034}+1}dx\right) + 2017\left(\int \frac{x^{2016}}{x^{4036}+2x^{4035}+x^{4034}+1}dx\right)$$

input
```
int(x^2016*(2017+2018*x)/(1+(x^2018+x^2017)^2),x)
```

output
```
2018*int(x**2017/(x**4036 + 2*x**4035 + x**4034 + 1),x) + 2017*int(x**2016
/(x**4036 + 2*x**4035 + x**4034 + 1),x)
```

3.2 $\int \frac{x^{2016}(2017+2018x)}{1+x^{4034}(1+x)^2} dx$

Optimal result

Integrand size = 22, antiderivative size = 8

$$\int \frac{x^{2016}(2017+2018x)}{1+x^{4034}(1+x)^2} dx = \arctan\left(x^{2017}(1+x)\right)$$

output
```
arctan(x^2017*(1+x))
```

Mathematica [F(-1)]

Timed out.

$$\int \frac{x^{2016}(2017+2018x)}{1+x^{4034}(1+x)^2} dx = \$\text{Aborted}$$

input
```
Integrate[(x^2016*(2017 + 2018*x))/(1 + x^4034*(1 + x)^2),x]
```

output
```
$Aborted
```

Rubi [A] (verified)

Time = 32.60 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {7260, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{2016}(2018x+2017)}{(x+1)^2x^{4034}+1}dx$$
$$\Big\downarrow \text{7260}$$
$$\int \frac{1}{(x+1)^2x^{4034}+1}d\left(x^{2017}(x+1)\right)$$
$$\Big\downarrow \text{216}$$
$$\arctan\left(x^{2017}(x+1)\right)$$

input
```
Int[(x^2016*(2017 + 2018*x))/(1 + x^4034*(1 + x)^2),x]
```

output
```
ArcTan[x^2017*(1 + x)]
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 7260
```
Int[(u_)*(v_)^(r_.)*((a_) + (b_.)*(v_)^(p_.)*(w_)^(q_.))^(m_.), x_Symbol] :
> With[{c = Simplify[u/(p*w*D[v, x] + q*v*D[w, x])]}, Simp[c*(p/(r + 1))
Subst[Int[(a + b*x^(p/(r + 1)))^m, x], x, v^(r + 1)*w], x] /; FreeQ[c, x]]
/; FreeQ[{a, b, m, p, q, r}, x] && EqQ[p, q*(r + 1)] && NeQ[r, -1] && Integ
erQ[p/(r + 1)]
```

Maple [A] (verified)

Time = 0.00 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.12

$$\arctan\left(x^{2018}+x^{2017}\right)$$

input
```
int(x^2016*(2017+2018*x)/(1+x^4034*(x+1)^2),x)
```

output
```
arctan(x^2018+x^2017)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^{2016}(2017+2018x)}{1+x^{4034}(1+x)^2}\,dx = \text{Timed out}$$

input
```
integrate(x^2016*(2017+2018*x)/(1+x^4034*(1+x)^2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{2016}(2017+2018x)}{1+x^{4034}(1+x)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**2016*(2017+2018*x)/(1+x**4034*(1+x)**2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^{2016}(2017 + 2018x)}{1 + x^{4034}(1 + x)^2}\, dx = \int \frac{(2018\, x + 2017)x^{2016}}{(x + 1)^2 x^{4034} + 1}\, dx$$

input
```
integrate(x^2016*(2017+2018*x)/(1+x^4034*(1+x)^2),x, algorithm="maxima")
```

output
```
integrate((2018*x + 2017)*x^2016/((x + 1)^2*x^4034 + 1), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{x^{2016}(2017 + 2018x)}{1 + x^{4034}(1 + x)^2}\, dx = \text{Timed out}$$

input
```
integrate(x^2016*(2017+2018*x)/(1+x^4034*(1+x)^2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{2016}(2017 + 2018x)}{1 + x^{4034}(1 + x)^2}\, dx = \int \frac{x^{2016}\, (2018\, x + 2017)}{x^{4034}\, (x + 1)^2 + 1}\, dx$$

input
```
int((x^2016*(2018*x + 2017))/(x^4034*(x + 1)^2 + 1),x)
```

output
```
int((x^2016*(2018*x + 2017))/(x^4034*(x + 1)^2 + 1), x)
```

Reduce [F]

$$\int \frac{x^{2016}(2017 + 2018x)}{1 + x^{4034}(1 + x)^2} \, dx = 2018\left(\int \frac{x^{2017}}{x^{4036} + 2x^{4035} + x^{4034} + 1} dx\right) + 2017\left(\int \frac{x^{2016}}{x^{4036} + 2x^{4035} + x^{4034} + 1} dx\right)$$

input

```
int(x^2016*(2017+2018*x)/(1+x^4034*(1+x)^2),x)
```

output

```
2018*int(x**2017/(x**4036 + 2*x**4035 + x**4034 + 1),x) + 2017*int(x**2016
/(x**4036 + 2*x**4035 + x**4034 + 1),x)
```

3.3 $\int \frac{x^{2016}(2017+2018x)}{1+x^{4034}+2x^{4035}+x^{4036}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 8

$$\int \frac{x^{2016}(2017+2018x)}{1+x^{4034}+2x^{4035}+x^{4036}}\,dx = \arctan\left(x^{2017}(1+x)\right)$$

output
```
arctan(x^2017*(1+x))
```

Mathematica [F]

$$\int \frac{x^{2016}(2017+2018x)}{1+x^{4034}+2x^{4035}+x^{4036}}\,dx = \int \frac{x^{2016}(2017+2018x)}{1+x^{4034}+2x^{4035}+x^{4036}}\,dx$$

input
```
Integrate[(x^2016*(2017 + 2018*x))/(1 + x^4034 + 2*x^4035 + x^4036),x]
```

output
```
Integrate[(x^2016*(2017 + 2018*x))/(1 + x^4034 + 2*x^4035 + x^4036), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{2016}(2018x + 2017)}{x^{4036} + 2x^{4035} + x^{4034} + 1} dx$$
$$\Big\downarrow \text{ 7299}$$
$$\int \frac{x^{2016}(2018x + 2017)}{x^{4036} + 2x^{4035} + x^{4034} + 1} dx$$

input
```
Int[(x^2016*(2017 + 2018*x))/(1 + x^4034 + 2*x^4035 + x^4036),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [A] (verified)

Time = 0.00 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.12

$$\arctan\left(x^{2018} + x^{2017}\right)$$

input
```
int(x^2016*(2017+2018*x)/(x^4036+2*x^4035+x^4034+1),x)
```

output
```
arctan(x^2018+x^2017)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^{2016}(2017 + 2018x)}{1 + x^{4034} + 2x^{4035} + x^{4036}}\,dx = \text{Timed out}$$

input
```
integrate(x^2016*(2017+2018*x)/(x^4036+2*x^4035+x^4034+1),x, algorithm="fr
icas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{2016}(2017 + 2018x)}{1 + x^{4034} + 2x^{4035} + x^{4036}}\,dx = \text{Timed out}$$

input
```
integrate(x**2016*(2017+2018*x)/(x**4036+2*x**4035+x**4034+1),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^{2016}(2017 + 2018x)}{1 + x^{4034} + 2x^{4035} + x^{4036}}\,dx = \int \frac{(2018\,x + 2017)x^{2016}}{x^{4036} + 2\,x^{4035} + x^{4034} + 1}\,dx$$

input
```
integrate(x^2016*(2017+2018*x)/(x^4036+2*x^4035+x^4034+1),x, algorithm="ma
xima")
```

output
```
integrate((2018*x + 2017)*x^2016/(x^4036 + 2*x^4035 + x^4034 + 1), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{x^{2016}(2017+2018x)}{1+x^{4034}+2x^{4035}+x^{4036}}\,dx = \text{Timed out}$$

input
```
integrate(x^2016*(2017+2018*x)/(x^4036+2*x^4035+x^4034+1),x, algorithm="gi
ac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{2016}(2017+2018x)}{1+x^{4034}+2x^{4035}+x^{4036}}\,dx = \int \frac{x^{2016}\,(2018\,x+2017)}{x^{4036}+2\,x^{4035}+x^{4034}+1}\,dx$$

input
```
int((x^2016*(2018*x + 2017))/(x^4034 + 2*x^4035 + x^4036 + 1),x)
```

output
```
int((x^2016*(2018*x + 2017))/(x^4034 + 2*x^4035 + x^4036 + 1), x)
```

Reduce [F]

$$\int \frac{x^{2016}(2017+2018x)}{1+x^{4034}+2x^{4035}+x^{4036}}\,dx = 2018\left(\int \frac{x^{2017}}{x^{4036}+2x^{4035}+x^{4034}+1}dx\right) + 2017\left(\int \frac{x^{2016}}{x^{4036}+2x^{4035}+x^{4034}+1}dx\right)$$

input
```
int(x^2016*(2017+2018*x)/(x^4036+2*x^4035+x^4034+1),x)
```

output
```
2018*int(x**2017/(x**4036 + 2*x**4035 + x**4034 + 1),x) + 2017*int(x**2016
/(x**4036 + 2*x**4035 + x**4034 + 1),x)
```

3.4 $\int \frac{-4+5x^2+x^3}{x^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 16

$$\int \frac{-4+5x^2+x^3}{x^2}\,dx = \frac{4}{x}+5x+\frac{x^2}{2}$$

output
```
4/x+5*x+1/2*x^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{-4+5x^2+x^3}{x^2}\,dx = \frac{4}{x}+5x+\frac{x^2}{2}$$

input
```
Integrate[(-4 + 5*x^2 + x^3)/x^2,x]
```

output
```
4/x + 5*x + x^2/2
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3+5x^2-4}{x^2} \, dx$$

$$\Big\downarrow \text{ 2010}$$

$$\int \left(-\frac{4}{x^2}+x+5\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{x^2}{2}+5x+\frac{4}{x}$$

input
```
Int[(-4 + 5*x^2 + x^3)/x^2,x]
```

output
```
4/x + 5*x + x^2/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result	size
default	$\frac{4}{x}+5x+\frac{x^2}{2}$	15
risch	$\frac{4}{x}+5x+\frac{x^2}{2}$	15
gosper	$\frac{x^3+10x^2+8}{2x}$	16
parallelrisch	$\frac{x^3+10x^2+8}{2x}$	16
norman	$\frac{\frac{1}{2}x^3+5x^2+4}{x}$	17
orering	$\frac{(x^3+10x^2+8)(x^3+5x^2-4)}{2x(x+1)(x^2+4x-4)}$	41

input
```
int((x^3+5*x^2-4)/x^2,x,method=_RETURNVERBOSE)
```

output
```
4/x+5*x+1/2*x^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{-4+5x^2+x^3}{x^2}\,dx=\frac{x^3+10\,x^2+8}{2\,x}$$

input
```
integrate((x^3+5*x^2-4)/x^2,x, algorithm="fricas")
```

output
```
1/2*(x^3 + 10*x^2 + 8)/x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

$$\int \frac{-4+5x^2+x^3}{x^2}\,dx = \frac{x^2}{2}+5x+\frac{4}{x}$$

input
```
integrate((x**3+5*x**2-4)/x**2,x)
```

output
```
x**2/2 + 5*x + 4/x
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{-4+5x^2+x^3}{x^2}\,dx = \frac{1}{2}\,x^2+5\,x+\frac{4}{x}$$

input
```
integrate((x^3+5*x^2-4)/x^2,x, algorithm="maxima")
```

output
```
1/2*x^2 + 5*x + 4/x
```

Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{-4+5x^2+x^3}{x^2}\,dx = \frac{1}{2}\,x^2+5\,x+\frac{4}{x}$$

input
```
integrate((x^3+5*x^2-4)/x^2,x, algorithm="giac")
```

output
```
1/2*x^2 + 5*x + 4/x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{-4+5x^2+x^3}{x^2}\,dx=\frac{x^3+10\,x^2+8}{2\,x}$$

input
```
int((5*x^2 + x^3 - 4)/x^2,x)
```

output
```
(10*x^2 + x^3 + 8)/(2*x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{-4+5x^2+x^3}{x^2}\,dx=\frac{x^3+10x^2+8}{2x}$$

input
```
int((x^3+5*x^2-4)/x^2,x)
```

output
```
(x**3 + 10*x**2 + 8)/(2*x)
```

3.5 $\int \frac{-2+x^2+x^3}{x^4} \, dx$

Optimal result

Integrand size = 12, antiderivative size = 15

$$\int \frac{-2+x^2+x^3}{x^4} \, dx = \frac{2}{3x^3} - \frac{1}{x} + \log(x)$$

output
```
2/3/x^3-1/x+ln(x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{-2+x^2+x^3}{x^4} \, dx = \frac{2}{3x^3} - \frac{1}{x} + \log(x)$$

input
```
Integrate[(-2 + x^2 + x^3)/x^4,x]
```

output
```
2/(3*x^3) - x^(-1) + Log[x]
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3+x^2-2}{x^4} \, dx$$

$$\Big\downarrow \text{2010}$$

$$\int \left(-\frac{2}{x^4}+\frac{1}{x^2}+\frac{1}{x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{3x^3}-\frac{1}{x}+\log(x)$$

input
```
Int[(-2 + x^2 + x^3)/x^4,x]
```

output
```
2/(3*x^3) - x^(-1) + Log[x]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
default	$\frac{2}{3x^3}-\frac{1}{x}+\ln\left(x\right)$	14
norman	$\frac{\frac{2}{3}-x^2}{x^3}+\ln\left(x\right)$	15
risch	$\frac{\frac{2}{3}-x^2}{x^3}+\ln\left(x\right)$	15
parallelrisch	$\frac{3\ln\left(x\right)x^3+2-3x^2}{3x^3}$	20

input
```
int((x^3+x^2-2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
2/3/x^3-1/x+ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.27

$$\int \frac{-2+x^2+x^3}{x^4}\,dx = \frac{3\,x^3\log\left(x\right)-3\,x^2+2}{3\,x^3}$$

input
```
integrate((x^3+x^2-2)/x^4,x, algorithm="fricas")
```

output
```
1/3*(3*x^3*log(x) - 3*x^2 + 2)/x^3
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

$$\int \frac{-2+x^2+x^3}{x^4}\,dx = \log\left(x\right)+\frac{2-3x^2}{3x^3}$$

input
```
integrate((x**3+x**2-2)/x**4,x)
```

output
```
log(x) + (2 - 3*x**2)/(3*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{-2+x^2+x^3}{x^4}\,dx = -\frac{3\,x^2-2}{3\,x^3}+\log\left(x\right)$$

input
```
integrate((x^3+x^2-2)/x^4,x, algorithm="maxima")
```

output
```
-1/3*(3*x^2 - 2)/x^3 + log(x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.07

$$\int \frac{-2+x^2+x^3}{x^4}\,dx = -\frac{3\,x^2-2}{3\,x^3}+\log\left(|x|\right)$$

input
```
integrate((x^3+x^2-2)/x^4,x, algorithm="giac")
```

output
```
-1/3*(3*x^2 - 2)/x^3 + log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{-2+x^2+x^3}{x^4}\,dx = \ln\left(x\right)-\frac{x^2-\frac{2}{3}}{x^3}$$

input
```
int((x^2 + x^3 - 2)/x^4,x)
```

output
```
log(x) - (x^2 - 2/3)/x^3
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.27

$$\int \frac{-2+x^2+x^3}{x^4}\,dx = \frac{3\log(x)\,x^3-3x^2+2}{3x^3}$$

input
```
int((x^3+x^2-2)/x^4,x)
```

output
```
(3*log(x)*x**3 - 3*x**2 + 2)/(3*x**3)
```

3.6 $\int \frac{1+x+x^3}{x^2} dx$

Optimal result

Integrand size = 10, antiderivative size = 15

$$\int \frac{1+x+x^3}{x^2} dx = -\frac{1}{x} + \frac{x^2}{2} + \log(x)$$

output
```
-1/x+1/2*x^2+ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{1+x+x^3}{x^2} dx = -\frac{1}{x} + \frac{x^2}{2} + \log(x)$$

input
```
Integrate[(1 + x + x^3)/x^2,x]
```

output
```
-x^(-1) + x^2/2 + Log[x]
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3+x+1}{x^2} \, dx$$

$$\Big\downarrow \text{2010}$$

$$\int \left(\frac{1}{x^2}+x+\frac{1}{x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^2}{2}-\frac{1}{x}+\log(x)$$

input
```
Int[(1 + x + x^3)/x^2,x]
```

output
```
-x^(-1) + x^2/2 + Log[x]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
default	$-\frac{1}{x}+\frac{x^{2}}{2}+\ln\left(x\right)$	14
risch	$-\frac{1}{x}+\frac{x^{2}}{2}+\ln\left(x\right)$	14
norman	$\frac{-1+\frac{x^{3}}{2}}{x}+\ln\left(x\right)$	15
parallelrisch	$\frac{x^{3}+2\ln\left(x\right)x-2}{2x}$	16

input
```
int((x^3+x+1)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/x+1/2*x^2+ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{1+x+x^3}{x^2}\,dx = \frac{x^3+2\,x\log\left(x\right)-2}{2\,x}$$

input
```
integrate((x^3+x+1)/x^2,x, algorithm="fricas")
```

output
```
1/2*(x^3 + 2*x*log(x) - 2)/x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.67

$$\int \frac{1+x+x^3}{x^2}\,dx = \frac{x^2}{2}+\log\left(x\right)-\frac{1}{x}$$

input
```
integrate((x**3+x+1)/x**2,x)
```

output
```
x**2/2 + log(x) - 1/x
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{1+x+x^3}{x^2}\,dx = \frac{1}{2}\,x^2 - \frac{1}{x} + \log\left(x\right)$$

input
```
integrate((x^3+x+1)/x^2,x, algorithm="maxima")
```

output
```
1/2*x^2 - 1/x + log(x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

$$\int \frac{1+x+x^3}{x^2}\,dx = \frac{1}{2}\,x^2 - \frac{1}{x} + \log\left(|x|\right)$$

input
```
integrate((x^3+x+1)/x^2,x, algorithm="giac")
```

output
```
1/2*x^2 - 1/x + log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{1+x+x^3}{x^2}\,dx = \ln\left(x\right) - \frac{1}{x} + \frac{x^2}{2}$$

input
```
int((x + x^3 + 1)/x^2,x)
```

output
```
log(x) - 1/x + x^2/2
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{1+x+x^3}{x^2}\,dx = \frac{2\log(x)\,x+x^3-2}{2x}$$

input
```
int((x^3+x+1)/x^2,x)
```

output
```
(2*log(x)*x + x**3 - 2)/(2*x)
```

3.7 $\int \frac{-1+3x-3x^2+x^3}{x^2} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 18

$$\int \frac{-1+3x-3x^2+x^3}{x^2} \, dx = \frac{1}{x} - 3x + \frac{x^2}{2} + 3\log(x)$$

output
```
1/x-3*x+1/2*x^2+3*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{-1+3x-3x^2+x^3}{x^2} \, dx = \frac{1}{x} - 3x + \frac{x^2}{2} + 3\log(x)$$

input
```
Integrate[(-1 + 3*x - 3*x^2 + x^3)/x^2,x]
```

output
```
x^(-1) - 3*x + x^2/2 + 3*Log[x]
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {2006, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3-3x^2+3x-1}{x^2} \, dx$$

$$\Big\downarrow \; 2006$$

$$\int \frac{(x-1)^3}{x^2}dx$$

$$\Big\downarrow \; 49$$

$$\int \left(-\frac{1}{x^2}+x+\frac{3}{x}-3\right) dx$$

$$\Big\downarrow \; 2009$$

$$\frac{x^2}{2}-3x+\frac{1}{x}+3\log(x)$$

input
```
Int[(-1 + 3*x - 3*x^2 + x^3)/x^2,x]
```

output
```
x^(-1) - 3*x + x^2/2 + 3*Log[x]
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2006
```
Int[(u_.)*(Px_), x_Symbol] :> With[{a = Rt[Coeff[Px, x, 0], Expon[Px, x]],
b = Rt[Coeff[Px, x, Expon[Px, x]], Expon[Px, x]]}, Int[u*(a + b*x)^Expon[Px
, x], x] /; EqQ[Px, (a + b*x)^Expon[Px, x]]] /; PolyQ[Px, x] && GtQ[Expon[P
x, x], 1] && NeQ[Coeff[Px, x, 0], 0] &&  !MatchQ[Px, (a_.)*(v_)^Expon[Px, x
] /; FreeQ[a, x] && LinearQ[v, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

method	result	size
default	$\frac{1}{x}-3x+\frac{x^{2}}{2}+3\ln\left(x\right)$	17
risch	$\frac{1}{x}-3x+\frac{x^{2}}{2}+3\ln\left(x\right)$	17
parallelrisch	$\frac{x^{3}+6\ln\left(x\right)x-6x^{2}+2}{2x}$	21
norman	$\frac{1-3x^{2}+\frac{1}{2}x^{3}}{x}+3\ln\left(x\right)$	22

input
```
int((x^3-3*x^2+3*x-1)/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/x-3*x+1/2*x^2+3*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{-1+3x-3x^2+x^3}{x^2}\,dx=\frac{x^{3}-6\,x^{2}+6\,x\log\left(x\right)+2}{2\,x}$$

input
```
integrate((x^3-3*x^2+3*x-1)/x^2,x, algorithm="fricas")
```

output
```
1/2*(x^3 - 6*x^2 + 6*x*log(x) + 2)/x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{-1+3x-3x^2+x^3}{x^2}\,dx = \frac{x^2}{2} - 3x + 3\log{(x)} + \frac{1}{x}$$

input
```
integrate((x**3-3*x**2+3*x-1)/x**2,x)
```

output
```
x**2/2 - 3*x + 3*log(x) + 1/x
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{-1+3x-3x^2+x^3}{x^2}\,dx = \frac{1}{2}\,x^2 - 3\,x + \frac{1}{x} + 3\,\log\left(x\right)$$

input
```
integrate((x^3-3*x^2+3*x-1)/x^2,x, algorithm="maxima")
```

output
```
1/2*x^2 - 3*x + 1/x + 3*log(x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{-1+3x-3x^2+x^3}{x^2}\,dx = \frac{1}{2}\,x^2 - 3\,x + \frac{1}{x} + 3\,\log\left(|x|\right)$$

input
```
integrate((x^3-3*x^2+3*x-1)/x^2,x, algorithm="giac")
```

output
```
1/2*x^2 - 3*x + 1/x + 3*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{-1+3x-3x^2+x^3}{x^2}\,dx = 3\,\ln(x) - 3\,x + \frac{1}{x} + \frac{x^2}{2}$$

input
```
int((3*x - 3*x^2 + x^3 - 1)/x^2,x)
```

output
```
3*log(x) - 3*x + 1/x + x^2/2
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{-1+3x-3x^2+x^3}{x^2}\,dx = \frac{6\log(x)\,x + x^3 - 6x^2 + 2}{2x}$$

input
```
int((x^3-3*x^2+3*x-1)/x^2,x)
```

output
```
(6*log(x)*x + x**3 - 6*x**2 + 2)/(2*x)
```

3.8 $\int \frac{x^4}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 545

$$\begin{aligned}
&\int \frac{x^4}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx \\
&= -\frac{\sqrt[3]{-1}\left(2\sqrt[3]{-1}b+3\sqrt[3]{a}c^{2/3}\right)\arctan\left(\frac{3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}\right)}{3\sqrt{3}\left(1+\sqrt[3]{-1}\right)^2a^{5/6}b^2\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}\,c^{2/3}} \\
&\quad-\frac{\left(2b-3\sqrt[3]{a}c^{2/3}\right)\arctan\left(\frac{3a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3\sqrt[3]{a}c^{2/3}}}\right)}{9\sqrt{3}a^{5/6}b^2\sqrt{4b-3\sqrt[3]{a}c^{2/3}}\,c^{2/3}} \\
&\quad-\frac{(-1)^{2/3}\left(2b+3\sqrt[3]{-1}\sqrt[3]{a}c^{2/3}\right)\arctan\left(\frac{3(-1)^{2/3}a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\sqrt{a}\sqrt{4b+3\sqrt[3]{-1}\sqrt[3]{a}c^{2/3}}}\right)}{3\sqrt{3}\left(1-\sqrt[3]{-1}\right)\left(1+\sqrt[3]{-1}\right)^2a^{5/6}b^2\sqrt{4b+3\sqrt[3]{-1}\sqrt[3]{a}c^{2/3}}\,c^{2/3}} \\
&\quad-\frac{\log\left(3a+3a^{2/3}\sqrt[3]{c}x+bx^2\right)}{18a^{2/3}b^2\sqrt[3]{c}}+\frac{\log\left(3a-3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}x+bx^2\right)}{6\left(1+\sqrt[3]{-1}\right)^2a^{2/3}b^2\sqrt[3]{c}} \\
&\quad+\frac{\sqrt[3]{-1}\log\left(3a+3(-1)^{2/3}a^{2/3}\sqrt[3]{c}x+bx^2\right)}{18a^{2/3}b^2\sqrt[3]{c}}
\end{aligned}$$

output
```
-1/9*(-1)^(1/3)*(2*(-1)^(1/3)*b+3*a^(1/3)*c^(2/3))*arctan(1/3*(3*(-1)^(1/3
)*a^(2/3)*c^(1/3)-2*b*x)*3^(1/2)/a^(1/2)/(4*b-3*(-1)^(2/3)*a^(1/3)*c^(2/3)
)^(1/2))*3^(1/2)/(1+(-1)^(1/3))^2/a^(5/6)/b^2/(4*b-3*(-1)^(2/3)*a^(1/3)*c^
(2/3))^(1/2)/c^(2/3)-1/27*(2*b-3*a^(1/3)*c^(2/3))*arctan(1/3*(3*a^(2/3)*c^
(1/3)+2*b*x)*3^(1/2)/a^(1/2)/(4*b-3*a^(1/3)*c^(2/3))^(1/2))*3^(1/2)/a^(5/6
)/b^2/(4*b-3*a^(1/3)*c^(2/3))^(1/2)/c^(2/3)-1/9*(-1)^(2/3)*(2*b+3*(-1)^(1/
3)*a^(1/3)*c^(2/3))*arctan(1/3*(3*(-1)^(2/3)*a^(2/3)*c^(1/3)+2*b*x)*3^(1/2
)/a^(1/2)/(4*b+3*(-1)^(1/3)*a^(1/3)*c^(2/3))^(1/2))*3^(1/2)/(1-(-1)^(1/3))
/(1+(-1)^(1/3))^2/a^(5/6)/b^2/(4*b+3*(-1)^(1/3)*a^(1/3)*c^(2/3))^(1/2)/c^(
2/3)-1/18*ln(3*a+3*a^(2/3)*c^(1/3)*x+b*x^2)/a^(2/3)/b^2/c^(1/3)+1/6*ln(3*a
-3*(-1)^(1/3)*a^(2/3)*c^(1/3)*x+b*x^2)/(1+(-1)^(1/3))^2/a^(2/3)/b^2/c^(1/3
)+1/18*(-1)^(1/3)*ln(3*a+3*(-1)^(2/3)*a^(2/3)*c^(1/3)*x+b*x^2)/a^(2/3)/b^2
/c^(1/3)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.08 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.18

$$\int \frac{x^4}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx$$
$$=\frac{1}{3}\text{RootSum}\Big[27a^3+27a^2b\#1^2+27a^2c\#1^3+9ab^2\#1^4$$
$$+b^3\#1^6\&,\frac{\log(x-\#1)\#1^3}{18a^2b+27a^2c\#1+12ab^2\#1^2+2b^3\#1^4}\&\Big]$$

input
```
Integrate[x^4/(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^
6),x]
```

output
```
RootSum[27*a^3 + 27*a^2*b*#1^2 + 27*a^2*c*#1^3 + 9*a*b^2*#1^4 + b^3*#1^6 &
 , (Log[x - #1]*#1^3)/(18*a^2*b + 27*a^2*c*#1 + 12*a*b^2*#1^2 + 2*b^3*#1^4
) & ]/3
```

Rubi [A] (verified)

Time = 1.35 (sec) , antiderivative size = 530, normalized size of antiderivative = 0.97, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {2466, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}dx$$

$$\Big\downarrow \text{2466}$$

$$19683a^6\int \left(-\frac{\sqrt[3]{c}x+\sqrt[3]{a}}{177147a^{20/3}bc^{2/3}\left(bx^2+3a^{2/3}\sqrt[3]{c}x+3a\right)}+\frac{\sqrt[3]{c}x+(-1)^{2/3}\sqrt[3]{a}}{59049\left(1+\sqrt[3]{-1}\right)^2a^{20/3}bc^{2/3}\left(bx^2-3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}x+\right.}\right.$$

$$\Big\downarrow \text{2009}$$

$$19683a^6\left(-\frac{\sqrt[3]{-1}\left(3\sqrt[3]{a}c^{2/3}+2\sqrt[3]{-1}b\right)\arctan\left(\frac{3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}\right)}{59049\sqrt{3}\left(1+\sqrt[3]{-1}\right)^2a^{41/6}b^2c^{2/3}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}-\frac{\left(2b-3\sqrt[3]{a}c^{2/3}\right)\arctan\left(\frac{3a^{2/3}\sqrt[3]{c}}{\sqrt{3}\sqrt{a}\sqrt{4b-3}}\right.}{177147\sqrt{3}a^{41/6}b^2c^{2/3}\sqrt{4b-3\sqrt[3]{}}}\right.$$

input
```
Int[x^4/(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^6),x]
```

output
```
19683*a^6*(-1/59049*((-1)^(1/3)*(2*(-1)^(1/3)*b + 3*a^(1/3)*c^(2/3))*ArcTa
n[(3*(-1)^(1/3)*a^(2/3)*c^(1/3) - 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b - 3*(-1
)^(2/3)*a^(1/3)*c^(2/3)])])/(Sqrt[3]*(1 + (-1)^(1/3))^2*a^(41/6)*b^2*Sqrt[
4*b - 3*(-1)^(2/3)*a^(1/3)*c^(2/3)]*c^(2/3)) - ((2*b - 3*a^(1/3)*c^(2/3))*
ArcTan[(3*a^(2/3)*c^(1/3) + 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b - 3*a^(1/3)*c
^(2/3)])])/(177147*Sqrt[3]*a^(41/6)*b^2*Sqrt[4*b - 3*a^(1/3)*c^(2/3)]*c^(2
/3)) - ((-1)^(2/3)*(2*b + 3*(-1)^(1/3)*a^(1/3)*c^(2/3))*ArcTan[(3*(-1)^(2/
3)*a^(2/3)*c^(1/3) + 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b + 3*(-1)^(1/3)*a^(1/
3)*c^(2/3)])])/(177147*Sqrt[3]*a^(41/6)*b^2*Sqrt[4*b + 3*(-1)^(1/3)*a^(1/3
)*c^(2/3)]*c^(2/3)) - Log[3*a + 3*a^(2/3)*c^(1/3)*x + b*x^2]/(354294*a^(20
/3)*b^2*c^(1/3)) + Log[3*a - 3*(-1)^(1/3)*a^(2/3)*c^(1/3)*x + b*x^2]/(1180
98*(1 + (-1)^(1/3))^2*a^(20/3)*b^2*c^(1/3)) + ((-1)^(1/3)*Log[3*a + 3*(-1)
^(2/3)*a^(2/3)*c^(1/3)*x + b*x^2])/(354294*a^(20/3)*b^2*c^(1/3)))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2466
```
Int[(u_.)*(Q6_)^(p_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
 x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Sim
p[1/(3^(3*p)*a^(2*p))   Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*
x + b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*
(-1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d,
 0] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coef
f[Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.07 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.17

method	result	size
default	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(b^{3}\_Z^{6}+9b^{2}a\_Z^{4}+27a^{2}c\_Z^{3}+27b\,a^{2}\_Z^{2}+27a^{3}\right)}\frac{\_R^{4}\ln\left(x-\_R\right)}{2\_R^{5}b^{3}+12\_R^{3}a\,b^{2}+27\_R^{2}a^{2}c+18a^{2}b\_R}\right)}{3}$	93
risch	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(b^{3}\_Z^{6}+9b^{2}a\_Z^{4}+27a^{2}c\_Z^{3}+27b\,a^{2}\_Z^{2}+27a^{3}\right)}\frac{\_R^{4}\ln\left(x-\_R\right)}{2\_R^{5}b^{3}+12\_R^{3}a\,b^{2}+27\_R^{2}a^{2}c+18a^{2}b\_R}\right)}{3}$	93

input
```
int(x^4/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x,method=_R
ETURNVERBOSE)
```

output
```
1/3*sum(_R^4/(2*_R^5*b^3+12*_R^3*a*b^2+27*_R^2*a^2*c+18*_R*a^2*b)*ln(x-_R)
,_R=RootOf(_Z^6*b^3+9*_Z^4*a*b^2+27*_Z^3*a^2*c+27*_Z^2*a^2*b+27*a^3))
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^4}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate(x^4/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, al
gorithm="fricas")
```

output
```
Exception raised: RuntimeError >> no explicit roots found
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^4}{27a^3 + 27a^2bx^2 + 27a^2cx^3 + 9ab^2x^4 + b^3x^6}\,dx = \text{Timed out}$$

input
```
integrate(x**4/(b**3*x**6+9*a*b**2*x**4+27*a**2*c*x**3+27*a**2*b*x**2+27*a
**3),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^4}{27a^3 + 27a^2bx^2 + 27a^2cx^3 + 9ab^2x^4 + b^3x^6}\,dx = \int \frac{x^4}{b^3x^6 + 9\,ab^2x^4 + 27\,a^2cx^3 + 27\,a^2bx^2 + 27\,a^3}\,dx$$

input
```
integrate(x^4/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, al
gorithm="maxima")
```

output
```
integrate(x^4/(b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2 + 27*a^
3), x)
```

Giac [F]

$$\int \frac{x^4}{27a^3 + 27a^2bx^2 + 27a^2cx^3 + 9ab^2x^4 + b^3x^6}\,dx = \int \frac{x^4}{b^3x^6 + 9\,ab^2x^4 + 27\,a^2cx^3 + 27\,a^2bx^2 + 27\,a^3}\,dx$$

input
```
integrate(x^4/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, al
gorithm="giac")
```

output
```
integrate(x^4/(b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2 + 27*a^
3), x)
```

Mupad [B] (verification not implemented)

Time = 22.92 (sec) , antiderivative size = 1563, normalized size of antiderivative = 2.87

$$\int \frac{x^4}{27a^3 + 27a^2bx^2 + 27a^2cx^3 + 9ab^2x^4 + b^3x^6}\,dx = \text{Too large to display}$$

input
```
int(x^4/(27*a^3 + b^3*x^6 + 27*a^2*b*x^2 + 9*a*b^2*x^4 + 27*a^2*c*x^3),x)
```

output
```
symsum(log(-19683*a^8*b^3*(c*x - b + 6561*root(918330048*a^5*b^9*c^4*z^6 -
 387420489*a^6*b^6*c^6*z^6 + 1594323*a^4*b^4*c^4*z^4 + 1023516*a^3*b^3*c^3
*z^3 - 531441*a^4*c^5*z^3 + 32805*a^2*b^2*c^2*z^2 + 324*a*b*c*z + 1, z, k)
^2*a^3*c^4 + 2*root(918330048*a^5*b^9*c^4*z^6 - 387420489*a^6*b^6*c^6*z^6
+ 1594323*a^4*b^4*c^4*z^4 + 1023516*a^3*b^3*c^3*z^3 - 531441*a^4*c^5*z^3 +
 32805*a^2*b^2*c^2*z^2 + 324*a*b*c*z + 1, z, k)*b^4*x - 198*root(918330048
*a^5*b^9*c^4*z^6 - 387420489*a^6*b^6*c^6*z^6 + 1594323*a^4*b^4*c^4*z^4 + 1
023516*a^3*b^3*c^3*z^3 - 531441*a^4*c^5*z^3 + 32805*a^2*b^2*c^2*z^2 + 324*
a*b*c*z + 1, z, k)*a*b^2*c - 8991*root(918330048*a^5*b^9*c^4*z^6 - 3874204
89*a^6*b^6*c^6*z^6 + 1594323*a^4*b^4*c^4*z^4 + 1023516*a^3*b^3*c^3*z^3 - 5
31441*a^4*c^5*z^3 + 32805*a^2*b^2*c^2*z^2 + 324*a*b*c*z + 1, z, k)^2*a^2*b
^3*c^2 - 19683*root(918330048*a^5*b^9*c^4*z^6 - 387420489*a^6*b^6*c^6*z^6
+ 1594323*a^4*b^4*c^4*z^4 + 1023516*a^3*b^3*c^3*z^3 - 531441*a^4*c^5*z^3 +
 32805*a^2*b^2*c^2*z^2 + 324*a*b*c*z + 1, z, k)^3*a^3*b^4*c^3 + 104976*roo
t(918330048*a^5*b^9*c^4*z^6 - 387420489*a^6*b^6*c^6*z^6 + 1594323*a^4*b^4*
c^4*z^4 + 1023516*a^3*b^3*c^3*z^3 - 531441*a^4*c^5*z^3 + 32805*a^2*b^2*c^2
*z^2 + 324*a*b*c*z + 1, z, k)^4*a^3*b^8*c^2 - 8503056*root(918330048*a^5*b
^9*c^4*z^6 - 387420489*a^6*b^6*c^6*z^6 + 1594323*a^4*b^4*c^4*z^4 + 1023516
*a^3*b^3*c^3*z^3 - 531441*a^4*c^5*z^3 + 32805*a^2*b^2*c^2*z^2 + 324*a*b*c*
z + 1, z, k)^5*a^4*b^9*c^3 + 4782969*root(918330048*a^5*b^9*c^4*z^6 - 3...
```

Reduce [F]

$$\int \frac{x^4}{27a^3 + 27a^2bx^2 + 27a^2cx^3 + 9ab^2x^4 + b^3x^6}\,dx$$
$$= \int \frac{x^4}{b^3x^6 + 9a\,b^2x^4 + 27a^2c\,x^3 + 27a^2b\,x^2 + 27a^3}dx$$

input
```
int(x^4/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x)
```

output
```
int(x**4/(27*a**3 + 27*a**2*b*x**2 + 27*a**2*c*x**3 + 9*a*b**2*x**4 + b**3
*x**6),x)
```

3.9 $\int \frac{x^3}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 487

$$\begin{aligned}\int \frac{x^3}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx &= -\frac{\arctan\left(\frac{3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}\right)}{3\sqrt{3}\left(1+\sqrt[3]{-1}\right)^2a^{7/6}b\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}\sqrt[3]{c}}-\frac{\arctan\left(\frac{3a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3\sqrt[3]{a}c^{2/3}}}\right)}{9\sqrt{3}a^{7/6}b\sqrt{4b-3\sqrt[3]{a}c^{2/3}}\sqrt[3]{c}}\\&\quad+\frac{\sqrt[3]{-1}\arctan\left(\frac{3(-1)^{2/3}a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\sqrt{a}\sqrt{4b+3\sqrt[3]{-1}\sqrt[3]{a}c^{2/3}}}\right)}{3\sqrt{3}\left(1-\sqrt[3]{-1}\right)\left(1+\sqrt[3]{-1}\right)^2a^{7/6}b\sqrt{4b+3\sqrt[3]{-1}\sqrt[3]{a}c^{2/3}}\sqrt[3]{c}}\\&\quad+\frac{\log\left(3a+3a^{2/3}\sqrt[3]{c}x+bx^2\right)}{54a^{4/3}bc^{2/3}}-\frac{(-1)^{2/3}\log\left(3a-3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}x+bx^2\right)}{18\left(1+\sqrt[3]{-1}\right)^2a^{4/3}bc^{2/3}}\\&\quad+\frac{(-1)^{2/3}\log\left(3a+3(-1)^{2/3}a^{2/3}\sqrt[3]{c}x+bx^2\right)}{54a^{4/3}bc^{2/3}}\end{aligned}$$

output
```
-1/9*arctan(1/3*(3*(-1)^(1/3)*a^(2/3)*c^(1/3)-2*b*x)*3^(1/2)/a^(1/2)/(4*b-
3*(-1)^(2/3)*a^(1/3)*c^(2/3))^(1/2))*3^(1/2)/(1+(-1)^(1/3))^2/a^(7/6)/b/(4
*b-3*(-1)^(2/3)*a^(1/3)*c^(2/3))^(1/2)/c^(1/3)-1/27*arctan(1/3*(3*a^(2/3)*
c^(1/3)+2*b*x)*3^(1/2)/a^(1/2)/(4*b-3*a^(1/3)*c^(2/3))^(1/2))*3^(1/2)/a^(7
/6)/b/(4*b-3*a^(1/3)*c^(2/3))^(1/2)/c^(1/3)+1/9*(-1)^(1/3)*arctan(1/3*(3*(
-1)^(2/3)*a^(2/3)*c^(1/3)+2*b*x)*3^(1/2)/a^(1/2)/(4*b+3*(-1)^(1/3)*a^(1/3)
*c^(2/3))^(1/2))*3^(1/2)/(1-(-1)^(1/3))/(1+(-1)^(1/3))^2/a^(7/6)/b/(4*b+3*
(-1)^(1/3)*a^(1/3)*c^(2/3))^(1/2)/c^(1/3)+1/54*ln(3*a+3*a^(2/3)*c^(1/3)*x+
b*x^2)/a^(4/3)/b/c^(2/3)-1/18*(-1)^(2/3)*ln(3*a-3*(-1)^(1/3)*a^(2/3)*c^(1/
3)*x+b*x^2)/(1+(-1)^(1/3))^2/a^(4/3)/b/c^(2/3)+1/54*(-1)^(2/3)*ln(3*a+3*(-
1)^(2/3)*a^(2/3)*c^(1/3)*x+b*x^2)/a^(4/3)/b/c^(2/3)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.06 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.20

$$\int \frac{x^3}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx = \frac{1}{3}\text{RootSum}\left[27a^3+27a^2b\#1^2+27a^2c\#1^3+9ab^2\#1^4 + b^3\#1^6\&,\frac{\log(x-\#1)\#1^2}{18a^2b+27a^2c\#1+12ab^2\#1^2+2b^3\#1^4}\&\right]$$

input
```
Integrate[x^3/(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^
6),x]
```

output
```
RootSum[27*a^3 + 27*a^2*b*#1^2 + 27*a^2*c*#1^3 + 9*a*b^2*#1^4 + b^3*#1^6 &
 , (Log[x - #1]*#1^2)/(18*a^2*b + 27*a^2*c*#1 + 12*a*b^2*#1^2 + 2*b^3*#1^4
) & ]/3
```

Rubi [A] (verified)

Time = 1.32 (sec) , antiderivative size = 472, normalized size of antiderivative = 0.97, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {2466, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx$$

$$\Big\downarrow \text{2466}$$

$$19683a^6\int\left(\frac{x}{531441a^{22/3}c^{2/3}\left(bx^2+3a^{2/3}\sqrt[3]{c}x+3a\right)}-\frac{(-1)^{2/3}x}{177147\left(1+\sqrt[3]{-1}\right)^2a^{22/3}c^{2/3}\left(bx^2-3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}x+3\right.}\right.$$

$$\Big\downarrow \text{2009}$$

$$19683a^6\left(-\frac{\arctan\left(\frac{3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}\right)}{59049\sqrt{3}\left(1+\sqrt[3]{-1}\right)^2a^{43/6}b\sqrt[3]{c}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}-\frac{\arctan\left(\frac{3a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3\sqrt[3]{a}c^{2/3}}}\right)}{177147\sqrt{3}a^{43/6}b\sqrt[3]{c}\sqrt{4b-3\sqrt[3]{a}c^{2/3}}}+\frac{\sqrt[3]{-}}{177}\right.$$

input
```
Int[x^3/(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^6),x]
```

output
```
19683*a^6*(-1/59049*ArcTan[(3*(-1)^(1/3)*a^(2/3)*c^(1/3) - 2*b*x)/(Sqrt[3]
*Sqrt[a]*Sqrt[4*b - 3*(-1)^(2/3)*a^(1/3)*c^(2/3)])]/(Sqrt[3]*(1 + (-1)^(1/
3))^2*a^(43/6)*b*Sqrt[4*b - 3*(-1)^(2/3)*a^(1/3)*c^(2/3)]*c^(1/3)) - ArcTa
n[(3*a^(2/3)*c^(1/3) + 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b - 3*a^(1/3)*c^(2/3
)])]/(177147*Sqrt[3]*a^(43/6)*b*Sqrt[4*b - 3*a^(1/3)*c^(2/3)]*c^(1/3)) + (
(-1)^(1/3)*ArcTan[(3*(-1)^(2/3)*a^(2/3)*c^(1/3) + 2*b*x)/(Sqrt[3]*Sqrt[a]*
Sqrt[4*b + 3*(-1)^(1/3)*a^(1/3)*c^(2/3)])])/(177147*Sqrt[3]*a^(43/6)*b*Sqr
t[4*b + 3*(-1)^(1/3)*a^(1/3)*c^(2/3)]*c^(1/3)) + Log[3*a + 3*a^(2/3)*c^(1/
3)*x + b*x^2]/(1062882*a^(22/3)*b*c^(2/3)) - ((-1)^(2/3)*Log[3*a - 3*(-1)^
(1/3)*a^(2/3)*c^(1/3)*x + b*x^2])/(354294*(1 + (-1)^(1/3))^2*a^(22/3)*b*c^
(2/3)) + ((-1)^(2/3)*Log[3*a + 3*(-1)^(2/3)*a^(2/3)*c^(1/3)*x + b*x^2])/(1
062882*a^(22/3)*b*c^(2/3)))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2466
```
Int[(u_.)*(Q6_)^(p_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
 x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Sim
p[1/(3^(3*p)*a^(2*p))   Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*
x + b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*
(-1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d,
 0] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coef
f[Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.19

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(b^{3}\_Z^{6}+9b^{2}a\_Z^{4}+27a^{2}c\_Z^{3}+27b\,a^{2}\_Z^{2}+27a^{3}\right)}\frac{\_R^{3}\ln\left(x-\_R\right)}{2\_R^{5}b^{3}+12\_R^{3}a\,b^{2}+27\_R^{2}a^{2}c+18a^{2}b\_R}\right)}{3}$	93
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(b^{3}\_Z^{6}+9b^{2}a\_Z^{4}+27a^{2}c\_Z^{3}+27b\,a^{2}\_Z^{2}+27a^{3}\right)}\frac{\_R^{3}\ln\left(x-\_R\right)}{2\_R^{5}b^{3}+12\_R^{3}a\,b^{2}+27\_R^{2}a^{2}c+18a^{2}b\_R}\right)}{3}$	93

input
```
int(x^3/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x,method=_R
ETURNVERBOSE)
```

output
```
1/3*sum(_R^3/(2*_R^5*b^3+12*_R^3*a*b^2+27*_R^2*a^2*c+18*_R*a^2*b)*ln(x-_R)
,_R=RootOf(_Z^6*b^3+9*_Z^4*a*b^2+27*_Z^3*a^2*c+27*_Z^2*a^2*b+27*a^3))
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^3}{27a^3 + 27a^2bx^2 + 27a^2cx^3 + 9ab^2x^4 + b^3x^6}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^3/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, al
gorithm="fricas")
```

output
```
Exception raised: RuntimeError >> no explicit roots found
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^3}{27a^3 + 27a^2bx^2 + 27a^2cx^3 + 9ab^2x^4 + b^3x^6}\,dx = \text{Timed out}$$

input
```
integrate(x**3/(b**3*x**6+9*a*b**2*x**4+27*a**2*c*x**3+27*a**2*b*x**2+27*a
**3),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^3}{27a^3 + 27a^2bx^2 + 27a^2cx^3 + 9ab^2x^4 + b^3x^6}\,dx = \int \frac{x^3}{b^3x^6 + 9\,ab^2x^4 + 27\,a^2cx^3 + 27\,a^2bx^2 + 27\,a^3}\,dx$$

input
```
integrate(x^3/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, al
gorithm="maxima")
```

output
```
integrate(x^3/(b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2 + 27*a^
3), x)
```

Giac [F]

$$\int \frac{x^3}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx = \int \frac{x^3}{b^3x^6+9\,ab^2x^4+27\,a^2cx^3+27\,a^2bx^2+27\,a^3}\,dx$$

input
```
integrate(x^3/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, al
gorithm="giac")
```

output
```
integrate(x^3/(b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2 + 27*a^
3), x)
```

Mupad [B] (verification not implemented)

Time = 0.88 (sec) , antiderivative size = 1354, normalized size of antiderivative = 2.78

$$\int \frac{x^3}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx = \text{Too large to display}$$

input
```
int(x^3/(27*a^3 + b^3*x^6 + 27*a^2*b*x^2 + 9*a*b^2*x^4 + 27*a^2*c*x^3),x)
```

output
```
symsum(log(4782969*root(10460353203*a^9*b^3*c^6*z^6 - 24794911296*a^8*b^6*
c^4*z^6 - 14348907*a^6*b^2*c^4*z^4 + 314928*a^4*b^3*c^2*z^3 - 531441*a^5*c
^4*z^3 - 19683*a^3*b*c^2*z^2 - 1, z, k)^2*a^9*b^6*c^3 - 729*a^5*b^7*x + 12
9140163*root(10460353203*a^9*b^3*c^6*z^6 - 24794911296*a^8*b^6*c^4*z^6 - 1
4348907*a^6*b^2*c^4*z^4 + 314928*a^4*b^3*c^2*z^3 - 531441*a^5*c^4*z^3 - 19
683*a^3*b*c^2*z^2 - 1, z, k)^3*a^10*b^8*c^3 + 1549681956*root(10460353203*
a^9*b^3*c^6*z^6 - 24794911296*a^8*b^6*c^4*z^6 - 14348907*a^6*b^2*c^4*z^4 +
 314928*a^4*b^3*c^2*z^3 - 531441*a^5*c^4*z^3 - 19683*a^3*b*c^2*z^2 - 1, z,
 k)^4*a^11*b^10*c^3 + 167365651248*root(10460353203*a^9*b^3*c^6*z^6 - 2479
4911296*a^8*b^6*c^4*z^6 - 14348907*a^6*b^2*c^4*z^4 + 314928*a^4*b^3*c^2*z^
3 - 531441*a^5*c^4*z^3 - 19683*a^3*b*c^2*z^2 - 1, z, k)^5*a^12*b^12*c^3 -
94143178827*root(10460353203*a^9*b^3*c^6*z^6 - 24794911296*a^8*b^6*c^4*z^6
 - 14348907*a^6*b^2*c^4*z^4 + 314928*a^4*b^3*c^2*z^3 - 531441*a^5*c^4*z^3
- 19683*a^3*b*c^2*z^2 - 1, z, k)^5*a^13*b^9*c^5 + 98415*root(10460353203*a
^9*b^3*c^6*z^6 - 24794911296*a^8*b^6*c^4*z^6 - 14348907*a^6*b^2*c^4*z^4 +
314928*a^4*b^3*c^2*z^3 - 531441*a^5*c^4*z^3 - 19683*a^3*b*c^2*z^2 - 1, z,
k)*a^7*b^7*c + 4374*root(10460353203*a^9*b^3*c^6*z^6 - 24794911296*a^8*b^6
*c^4*z^6 - 14348907*a^6*b^2*c^4*z^4 + 314928*a^4*b^3*c^2*z^3 - 531441*a^5*
c^4*z^3 - 19683*a^3*b*c^2*z^2 - 1, z, k)*a^6*b^9*x - 2125764*root(10460353
203*a^9*b^3*c^6*z^6 - 24794911296*a^8*b^6*c^4*z^6 - 14348907*a^6*b^2*c^...
```

Reduce [F]

$$\int \frac{x^3}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx \\ = \int \frac{x^3}{b^3x^6+9a\,b^2x^4+27a^2c\,x^3+27a^2b\,x^2+27a^3}dx$$

input
```
int(x^3/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x)
```

output
```
int(x**3/(27*a**3 + 27*a**2*b*x**2 + 27*a**2*c*x**3 + 9*a*b**2*x**4 + b**3
*x**6),x)
```

3.10 $\int \frac{x^2}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 334

$$\int \frac{x^2}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx = \frac{2(-1)^{2/3}\arctan\left(\frac{3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}\right)}{9\sqrt{3}\left(1+\sqrt[3]{-1}\right)^2 a^{11/6}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}c^{2/3}}+\frac{2\arctan\left(\frac{3a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3\sqrt[3]{a}c^{2/3}}}\right)}{27\sqrt{3}a^{11/6}\sqrt{4b-3\sqrt[3]{a}c^{2/3}}c^{2/3}}+\frac{2(-1)^{2/3}\arctan\left(\frac{3(-1)^{2/3}a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\sqrt{a}\sqrt{4b+3\sqrt[3]{-1}\sqrt[3]{a}c^{2/3}}}\right)}{9\sqrt{3}\left(1-\sqrt[3]{-1}\right)\left(1+\sqrt[3]{-1}\right)^2 a^{11/6}\sqrt{4b+3\sqrt[3]{-1}\sqrt[3]{a}c^{2/3}}c^{2/3}}$$

output
```
2/27*(-1)^(2/3)*arctan(1/3*(3*(-1)^(1/3)*a^(2/3)*c^(1/3)-2*b*x)*3^(1/2)/a^
(1/2)/(4*b-3*(-1)^(2/3)*a^(1/3)*c^(2/3))^(1/2))*3^(1/2)/(1+(-1)^(1/3))^2/a
^(11/6)/(4*b-3*(-1)^(2/3)*a^(1/3)*c^(2/3))^(1/2)/c^(2/3)+2/81*arctan(1/3*(
3*a^(2/3)*c^(1/3)+2*b*x)*3^(1/2)/a^(1/2)/(4*b-3*a^(1/3)*c^(2/3))^(1/2))*3^
(1/2)/a^(11/6)/(4*b-3*a^(1/3)*c^(2/3))^(1/2)/c^(2/3)+2/27*(-1)^(2/3)*arcta
n(1/3*(3*(-1)^(2/3)*a^(2/3)*c^(1/3)+2*b*x)*3^(1/2)/a^(1/2)/(4*b+3*(-1)^(1/
3)*a^(1/3)*c^(2/3))^(1/2))*3^(1/2)/(1-(-1)^(1/3))/(1+(-1)^(1/3))^2/a^(11/6
)/(4*b+3*(-1)^(1/3)*a^(1/3)*c^(2/3))^(1/2)/c^(2/3)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.29

$$\int \frac{x^2}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx = \frac{1}{3}\text{RootSum}\left[27a^3+27a^2b\#1^2+27a^2c\#1^3+9ab^2\#1^4 + b^3\#1^6\&,\frac{\log(x-\#1)\#1}{18a^2b+27a^2c\#1+12ab^2\#1^2+2b^3\#1^4}\&\right]$$

input
```
Integrate[x^2/(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^
6),x]
```

output
```
RootSum[27*a^3 + 27*a^2*b*#1^2 + 27*a^2*c*#1^3 + 9*a*b^2*#1^4 + b^3*#1^6 &
 , (Log[x - #1]*#1)/(18*a^2*b + 27*a^2*c*#1 + 12*a*b^2*#1^2 + 2*b^3*#1^4)
& ]/3
```

Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 319, normalized size of antiderivative = 0.96, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {2466, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx$$

$$\Big\downarrow \text{2466}$$

$$19683a^6\int \left(\frac{1}{531441a^{22/3}\left(bx^2+3a^{2/3}\sqrt[3]{c}x+3a\right)c^{2/3}}-\frac{(-1)^{2/3}}{177147\left(1+\sqrt[3]{-1}\right)^2a^{22/3}\left(bx^2-3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}x+3a\right)c\right.$$

$$\Big\downarrow \text{2009}$$

$$19683a^6\left(\frac{2(-1)^{2/3}\arctan\left(\frac{3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}\right)}{177147\sqrt{3}\left(1+\sqrt[3]{-1}\right)^2a^{47/6}c^{2/3}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}+\frac{2\arctan\left(\frac{3a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3\sqrt[3]{a}c^{2/3}}}\right)}{531441\sqrt{3}a^{47/6}c^{2/3}\sqrt{4b-3\sqrt[3]{a}c^{2/3}}}+\frac{2(-1}{531}\right.$$

input
```
Int[x^2/(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^6),x]
```

output
```
19683*a^6*((2*(-1)^(2/3)*ArcTan[(3*(-1)^(1/3)*a^(2/3)*c^(1/3) - 2*b*x)/(Sq
rt[3]*Sqrt[a]*Sqrt[4*b - 3*(-1)^(2/3)*a^(1/3)*c^(2/3)])])/(177147*Sqrt[3]*
(1 + (-1)^(1/3))^2*a^(47/6)*Sqrt[4*b - 3*(-1)^(2/3)*a^(1/3)*c^(2/3)]*c^(2/
3)) + (2*ArcTan[(3*a^(2/3)*c^(1/3) + 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b - 3*
a^(1/3)*c^(2/3)])])/(531441*Sqrt[3]*a^(47/6)*Sqrt[4*b - 3*a^(1/3)*c^(2/3)]
*c^(2/3)) + (2*(-1)^(2/3)*ArcTan[(3*(-1)^(2/3)*a^(2/3)*c^(1/3) + 2*b*x)/(S
qrt[3]*Sqrt[a]*Sqrt[4*b + 3*(-1)^(1/3)*a^(1/3)*c^(2/3)])])/(531441*Sqrt[3]
*a^(47/6)*Sqrt[4*b + 3*(-1)^(1/3)*a^(1/3)*c^(2/3)]*c^(2/3)))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2466
```
Int[(u_.)*(Q6_)^(p_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
 x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Sim
p[1/(3^(3*p)*a^(2*p))   Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*
x + b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*
(-1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d,
 0] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coef
f[Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.28

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(b^3\_Z^6+9b^2a\_Z^4+27a^2c\_Z^3+27b\,a^2\_Z^2+27a^3\right)}\frac{\_R^2\ln\left(x-\_R\right)}{2\_R^5b^3+12\_R^3a\,b^2+27\_R^2a^2c+18a^2b\_R}\right)}{3}$	93
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(b^3\_Z^6+9b^2a\_Z^4+27a^2c\_Z^3+27b\,a^2\_Z^2+27a^3\right)}\frac{\_R^2\ln\left(x-\_R\right)}{2\_R^5b^3+12\_R^3a\,b^2+27\_R^2a^2c+18a^2b\_R}\right)}{3}$	93

input
```
int(x^2/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x,method=_R
ETURNVERBOSE)
```

output
```
1/3*sum(_R^2/(2*_R^5*b^3+12*_R^3*a*b^2+27*_R^2*a^2*c+18*_R*a^2*b)*ln(x-_R)
,_R=RootOf(_Z^6*b^3+9*_Z^4*a*b^2+27*_Z^3*a^2*c+27*_Z^2*a^2*b+27*a^3))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.48 (sec) , antiderivative size = 27094, normalized size of antiderivative = 81.12

$$\int \frac{x^2}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx = \text{Too large to display}$$

input
```
integrate(x^2/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, al
gorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2}{27a^3 + 27a^2bx^2 + 27a^2cx^3 + 9ab^2x^4 + b^3x^6}\,dx = \text{Timed out}$$

input
```
integrate(x**2/(b**3*x**6+9*a*b**2*x**4+27*a**2*c*x**3+27*a**2*b*x**2+27*a
**3),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^2}{27a^3 + 27a^2bx^2 + 27a^2cx^3 + 9ab^2x^4 + b^3x^6}\,dx = \int \frac{x^2}{b^3x^6 + 9\,ab^2x^4 + 27\,a^2cx^3 + 27\,a^2bx^2 + 27\,a^3}\,dx$$

input
```
integrate(x^2/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, al
gorithm="maxima")
```

output
```
integrate(x^2/(b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2 + 27*a^
3), x)
```

Giac [F]

$$\int \frac{x^2}{27a^3 + 27a^2bx^2 + 27a^2cx^3 + 9ab^2x^4 + b^3x^6}\,dx = \int \frac{x^2}{b^3x^6 + 9\,ab^2x^4 + 27\,a^2cx^3 + 27\,a^2bx^2 + 27\,a^3}\,dx$$

input
```
integrate(x^2/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, al
gorithm="giac")
```

output
```
integrate(x^2/(b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2 + 27*a^
3), x)
```

Mupad [B] (verification not implemented)

Time = 23.01 (sec) , antiderivative size = 825, normalized size of antiderivative = 2.47

$$\int \frac{x^2}{27a^3 + 27a^2bx^2 + 27a^2cx^3 + 9ab^2x^4 + b^3x^6}\,dx = \text{Too large to display}$$

input
```
int(x^2/(27*a^3 + b^3*x^6 + 27*a^2*b*x^2 + 9*a*b^2*x^4 + 27*a^2*c*x^3),x)
```

output
```
symsum(log(-27*a^3*b^9*(43046721*root(669462604992*a^11*b^3*c^4*z^6 - 2824
29536481*a^12*c^6*z^6 + 129140163*a^8*c^4*z^4 - 19683*a^4*c^2*z^2 + 1, z,
k)^4*a^8*c^4 - 1062882*root(669462604992*a^11*b^3*c^4*z^6 - 282429536481*a
^12*c^6*z^6 + 129140163*a^8*c^4*z^4 - 19683*a^4*c^2*z^2 + 1, z, k)^3*a^6*c
^3 - 13122*root(669462604992*a^11*b^3*c^4*z^6 - 282429536481*a^12*c^6*z^6
+ 129140163*a^8*c^4*z^4 - 19683*a^4*c^2*z^2 + 1, z, k)^2*a^4*c^2 + 3486784
401*root(669462604992*a^11*b^3*c^4*z^6 - 282429536481*a^12*c^6*z^6 + 12914
0163*a^8*c^4*z^4 - 19683*a^4*c^2*z^2 + 1, z, k)^5*a^10*c^5 + 81*root(66946
2604992*a^11*b^3*c^4*z^6 - 282429536481*a^12*c^6*z^6 + 129140163*a^8*c^4*z
^4 - 19683*a^4*c^2*z^2 + 1, z, k)*a^2*c + 18*root(669462604992*a^11*b^3*c^
4*z^6 - 282429536481*a^12*c^6*z^6 + 129140163*a^8*c^4*z^4 - 19683*a^4*c^2*
z^2 + 1, z, k)*a*b^2*x - 25509168*root(669462604992*a^11*b^3*c^4*z^6 - 282
429536481*a^12*c^6*z^6 + 129140163*a^8*c^4*z^4 - 19683*a^4*c^2*z^2 + 1, z,
 k)^4*a^7*b^3*c^2 - 6198727824*root(669462604992*a^11*b^3*c^4*z^6 - 282429
536481*a^12*c^6*z^6 + 129140163*a^8*c^4*z^4 - 19683*a^4*c^2*z^2 + 1, z, k)
^5*a^9*b^3*c^3 + 5832*root(669462604992*a^11*b^3*c^4*z^6 - 282429536481*a^
12*c^6*z^6 + 129140163*a^8*c^4*z^4 - 19683*a^4*c^2*z^2 + 1, z, k)^2*a^3*b^
2*c*x + 708588*root(669462604992*a^11*b^3*c^4*z^6 - 282429536481*a^12*c^6*
z^6 + 129140163*a^8*c^4*z^4 - 19683*a^4*c^2*z^2 + 1, z, k)^3*a^5*b^2*c^2*x
 + 38263752*root(669462604992*a^11*b^3*c^4*z^6 - 282429536481*a^12*c^6*...
```

Reduce [F]

$$
\int \frac{x^2}{27a^3 + 27a^2bx^2 + 27a^2cx^3 + 9ab^2x^4 + b^3x^6}\,dx
= \int \frac{x^2}{b^3x^6 + 9a\,b^2x^4 + 27a^2c\,x^3 + 27a^2b\,x^2 + 27a^3}dx
$$

input
```
int(x^2/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x)
```

output
```
int(x**2/(27*a**3 + 27*a**2*b*x**2 + 27*a**2*c*x**3 + 9*a*b**2*x**4 + b**3
*x**6),x)
```

3.11 $\int \frac{x}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx$

Optimal result

Integrand size = 44, antiderivative size = 469

$$\begin{aligned}
&\int \frac{x}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx\\
&=-\frac{\arctan\left(\frac{3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}\right)}{9\sqrt{3}\left(1+\sqrt[3]{-1}\right)^2a^{13/6}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}\sqrt[3]{c}}-\frac{\arctan\left(\frac{3a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3\sqrt[3]{a}c^{2/3}}}\right)}{27\sqrt{3}a^{13/6}\sqrt{4b-3\sqrt[3]{a}c^{2/3}}\sqrt[3]{c}}\\
&\quad+\frac{\sqrt[3]{-1}\arctan\left(\frac{3(-1)^{2/3}a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\sqrt{a}\sqrt{4b+3\sqrt[3]{-1}\sqrt[3]{a}c^{2/3}}}\right)}{9\sqrt{3}\left(1-\sqrt[3]{-1}\right)\left(1+\sqrt[3]{-1}\right)^2a^{13/6}\sqrt{4b+3\sqrt[3]{-1}\sqrt[3]{a}c^{2/3}}\sqrt[3]{c}}\\
&\quad-\frac{\log\left(3a+3a^{2/3}\sqrt[3]{c}x+bx^2\right)}{162a^{7/3}c^{2/3}}+\frac{(-1)^{2/3}\log\left(3a-3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}x+bx^2\right)}{54\left(1+\sqrt[3]{-1}\right)^2a^{7/3}c^{2/3}}\\
&\quad-\frac{(-1)^{2/3}\log\left(3a+3(-1)^{2/3}a^{2/3}\sqrt[3]{c}x+bx^2\right)}{162a^{7/3}c^{2/3}}
\end{aligned}$$

output
```
-1/27*arctan(1/3*(3*(-1)^(1/3)*a^(2/3)*c^(1/3)-2*b*x)*3^(1/2)/a^(1/2)/(4*b
-3*(-1)^(2/3)*a^(1/3)*c^(2/3))^(1/2))*3^(1/2)/(1+(-1)^(1/3))^2/a^(13/6)/(4
*b-3*(-1)^(2/3)*a^(1/3)*c^(2/3))^(1/2)/c^(1/3)-1/81*arctan(1/3*(3*a^(2/3)*
c^(1/3)+2*b*x)*3^(1/2)/a^(1/2)/(4*b-3*a^(1/3)*c^(2/3))^(1/2))*3^(1/2)/a^(1
3/6)/(4*b-3*a^(1/3)*c^(2/3))^(1/2)/c^(1/3)+1/27*(-1)^(1/3)*arctan(1/3*(3*(
-1)^(2/3)*a^(2/3)*c^(1/3)+2*b*x)*3^(1/2)/a^(1/2)/(4*b+3*(-1)^(1/3)*a^(1/3)
*c^(2/3))^(1/2))*3^(1/2)/(1-(-1)^(1/3))/(1+(-1)^(1/3))^2/a^(13/6)/(4*b+3*(
-1)^(1/3)*a^(1/3)*c^(2/3))^(1/2)/c^(1/3)-1/162*ln(3*a+3*a^(2/3)*c^(1/3)*x+
b*x^2)/a^(7/3)/c^(2/3)+1/54*(-1)^(2/3)*ln(3*a-3*(-1)^(1/3)*a^(2/3)*c^(1/3)
*x+b*x^2)/(1+(-1)^(1/3))^2/a^(7/3)/c^(2/3)-1/162*(-1)^(2/3)*ln(3*a+3*(-1)^
(2/3)*a^(2/3)*c^(1/3)*x+b*x^2)/a^(7/3)/c^(2/3)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.20

$$\int \frac{x}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx = \frac{1}{3}\text{RootSum}\left[27a^3+27a^2b\#1^2+27a^2c\#1^3+9ab^2\#1^4+b^3\#1^6\&,\frac{\log(x-\#1)}{18a^2b+27a^2c\#1+12ab^2\#1^2+2b^3\#1^4}\&\right]$$

input
```
Integrate[x/(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^6)
,x]
```

output
```
RootSum[27*a^3 + 27*a^2*b*#1^2 + 27*a^2*c*#1^3 + 9*a*b^2*#1^4 + b^3*#1^6 &
 , Log[x - #1]/(18*a^2*b + 27*a^2*c*#1 + 12*a*b^2*#1^2 + 2*b^3*#1^4) & ]/3
```

Rubi [A] (verified)

Time = 1.37 (sec) , antiderivative size = 454, normalized size of antiderivative = 0.97, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {2466, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx$$

$$\Big\downarrow \; 2466$$

$$19683a^6\int \left(-\frac{bx+3a^{2/3}\sqrt[3]{c}}{1594323a^{25/3}c^{2/3}\left(bx^2+3a^{2/3}\sqrt[3]{c}x+3a\right)}+\frac{(-1)^{2/3}bx+3a^{2/3}\sqrt[3]{c}}{531441\left(1+\sqrt[3]{-1}\right)^2a^{25/3}c^{2/3}\left(bx^2-3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}x\right.}\right.$$

$$\Big\downarrow \; 2009$$

$$19683a^6\left(-\frac{\arctan\left(\frac{3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}\right)}{177147\sqrt{3}\left(1+\sqrt[3]{-1}\right)^2a^{49/6}\sqrt[3]{c}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}-\frac{\arctan\left(\frac{3a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3\sqrt[3]{a}c^{2/3}}}\right)}{531441\sqrt{3}a^{49/6}\sqrt[3]{c}\sqrt{4b-3\sqrt[3]{a}c^{2/3}}}+\frac{\sqrt[3]{-}}{531}\right.$$

input
```
Int[x/(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^6),x]
```

output
```
19683*a^6*(-1/177147*ArcTan[(3*(-1)^(1/3)*a^(2/3)*c^(1/3) - 2*b*x)/(Sqrt[3
]*Sqrt[a]*Sqrt[4*b - 3*(-1)^(2/3)*a^(1/3)*c^(2/3)])]/(Sqrt[3]*(1 + (-1)^(1
/3))^2*a^(49/6)*Sqrt[4*b - 3*(-1)^(2/3)*a^(1/3)*c^(2/3)]*c^(1/3)) - ArcTan
[(3*a^(2/3)*c^(1/3) + 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b - 3*a^(1/3)*c^(2/3)
])]/(531441*Sqrt[3]*a^(49/6)*Sqrt[4*b - 3*a^(1/3)*c^(2/3)]*c^(1/3)) + ((-1
)^(1/3)*ArcTan[(3*(-1)^(2/3)*a^(2/3)*c^(1/3) + 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqr
t[4*b + 3*(-1)^(1/3)*a^(1/3)*c^(2/3)])])/(531441*Sqrt[3]*a^(49/6)*Sqrt[4*b
 + 3*(-1)^(1/3)*a^(1/3)*c^(2/3)]*c^(1/3)) - Log[3*a + 3*a^(2/3)*c^(1/3)*x
+ b*x^2]/(3188646*a^(25/3)*c^(2/3)) + ((-1)^(2/3)*Log[3*a - 3*(-1)^(1/3)*a
^(2/3)*c^(1/3)*x + b*x^2])/(1062882*(1 + (-1)^(1/3))^2*a^(25/3)*c^(2/3)) -
 ((-1)^(2/3)*Log[3*a + 3*(-1)^(2/3)*a^(2/3)*c^(1/3)*x + b*x^2])/(3188646*a
^(25/3)*c^(2/3)))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2466
```
Int[(u_.)*(Q6_)^(p_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
 x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Sim
p[1/(3^(3*p)*a^(2*p))   Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*
x + b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*
(-1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d,
 0] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coef
f[Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.19

method	result	size
default	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(b^{3}\_Z^{6}+9b^{2}a\_Z^{4}+27a^{2}c\_Z^{3}+27b\,a^{2}\_Z^{2}+27a^{3}\right)}\frac{\_R\ln\left(x-\_R\right)}{2\_R^{5}b^{3}+12\_R^{3}a\,b^{2}+27\_R^{2}a^{2}c+18a^{2}b\_R}\right)}{3}$	91
risch	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(b^{3}\_Z^{6}+9b^{2}a\_Z^{4}+27a^{2}c\_Z^{3}+27b\,a^{2}\_Z^{2}+27a^{3}\right)}\frac{\_R\ln\left(x-\_R\right)}{2\_R^{5}b^{3}+12\_R^{3}a\,b^{2}+27\_R^{2}a^{2}c+18a^{2}b\_R}\right)}{3}$	91

input
```
int(x/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x,method=_RET
URNVERBOSE)
```

output
```
1/3*sum(_R/(2*_R^5*b^3+12*_R^3*a*b^2+27*_R^2*a^2*c+18*_R*a^2*b)*ln(x-_R),_
R=RootOf(_Z^6*b^3+9*_Z^4*a*b^2+27*_Z^3*a^2*c+27*_Z^2*a^2*b+27*a^3))
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x}{27a^3 + 27a^2bx^2 + 27a^2cx^3 + 9ab^2x^4 + b^3x^6} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, algo
rithm="fricas")
```

output
```
Exception raised: RuntimeError >> no explicit roots found
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x}{27a^3 + 27a^2bx^2 + 27a^2cx^3 + 9ab^2x^4 + b^3x^6} \, dx = \text{Timed out}$$

input
```
integrate(x/(b**3*x**6+9*a*b**2*x**4+27*a**2*c*x**3+27*a**2*b*x**2+27*a**3
),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x}{27a^3 + 27a^2bx^2 + 27a^2cx^3 + 9ab^2x^4 + b^3x^6} \, dx = \int \frac{x}{b^3x^6 + 9\,ab^2x^4 + 27\,a^2cx^3 + 27\,a^2bx^2 + 27\,a^3} \, dx$$

input
```
integrate(x/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, algo
rithm="maxima")
```

output
```
integrate(x/(b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2 + 27*a^3)
, x)
```

Giac [F]

$$\int \frac{x}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx = \int \frac{x}{b^3x^6+9\,ab^2x^4+27\,a^2cx^3+27\,a^2bx^2+27\,a^3}\,dx$$

input
```
integrate(x/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, algo
rithm="giac")
```

output
```
integrate(x/(b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2 + 27*a^3)
, x)
```

Mupad [B] (verification not implemented)

Time = 22.61 (sec) , antiderivative size = 1057, normalized size of antiderivative = 2.25

$$\int \frac{x}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx = \text{Too large to display}$$

input
```
int(x/(27*a^3 + b^3*x^6 + 27*a^2*b*x^2 + 9*a*b^2*x^4 + 27*a^2*c*x^3),x)
```

output
```
symsum(log(b^12*x + 1033121304*root(18075490334784*a^14*b^3*c^4*z^6 - 7625
597484987*a^15*c^6*z^6 + 1162261467*a^10*b*c^4*z^4 + 8503056*a^7*b^3*c^2*z
^3 - 14348907*a^8*c^4*z^3 + 177147*a^5*b^2*c^2*z^2 + b^3, z, k)^4*a^10*b^1
1*c^3 + 167365651248*root(18075490334784*a^14*b^3*c^4*z^6 - 7625597484987*
a^15*c^6*z^6 + 1162261467*a^10*b*c^4*z^4 + 8503056*a^7*b^3*c^2*z^3 - 14348
907*a^8*c^4*z^3 + 177147*a^5*b^2*c^2*z^2 + b^3, z, k)^5*a^12*b^12*c^3 - 94
143178827*root(18075490334784*a^14*b^3*c^4*z^6 - 7625597484987*a^15*c^6*z^
6 + 1162261467*a^10*b*c^4*z^4 + 8503056*a^7*b^3*c^2*z^3 - 14348907*a^8*c^4
*z^3 + 177147*a^5*b^2*c^2*z^2 + b^3, z, k)^5*a^13*b^9*c^5 + 54*root(180754
90334784*a^14*b^3*c^4*z^6 - 7625597484987*a^15*c^6*z^6 + 1162261467*a^10*b
*c^4*z^4 + 8503056*a^7*b^3*c^2*z^3 - 14348907*a^8*c^4*z^3 + 177147*a^5*b^2
*c^2*z^2 + b^3, z, k)*a^2*b^13*x + 177147*root(18075490334784*a^14*b^3*c^4
*z^6 - 7625597484987*a^15*c^6*z^6 + 1162261467*a^10*b*c^4*z^4 + 8503056*a^
7*b^3*c^2*z^3 - 14348907*a^8*c^4*z^3 + 177147*a^5*b^2*c^2*z^2 + b^3, z, k)
^2*a^5*b^11*c^2*x + 17006112*root(18075490334784*a^14*b^3*c^4*z^6 - 762559
7484987*a^15*c^6*z^6 + 1162261467*a^10*b*c^4*z^4 + 8503056*a^7*b^3*c^2*z^3
 - 14348907*a^8*c^4*z^3 + 177147*a^5*b^2*c^2*z^2 + b^3, z, k)^3*a^7*b^12*c
^2*x - 14348907*root(18075490334784*a^14*b^3*c^4*z^6 - 7625597484987*a^15*
c^6*z^6 + 1162261467*a^10*b*c^4*z^4 + 8503056*a^7*b^3*c^2*z^3 - 14348907*a
^8*c^4*z^3 + 177147*a^5*b^2*c^2*z^2 + b^3, z, k)^3*a^8*b^9*c^4*x + 2295...
```

Reduce [F]

$$\int \frac{x}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx = \int \frac{x}{b^3x^6+9a\,b^2x^4+27a^2c\,x^3+27a^2b\,x^2+27a^3}dx$$

input
```
int(x/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x)
```

output
```
int(x/(27*a**3 + 27*a**2*b*x**2 + 27*a**2*c*x**3 + 9*a*b**2*x**4 + b**3*x*
*6),x)
```

3.12 $\int \frac{1}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6} dx$

Optimal result

Integrand size = 42, antiderivative size = 522

$$\begin{aligned}
&\int \frac{1}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx \\
&= -\frac{\sqrt[3]{-1}\left(2\sqrt[3]{-1}b+3\sqrt[3]{a}c^{2/3}\right)\arctan\left(\frac{3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}\right)}{27\sqrt{3}\left(1+\sqrt[3]{-1}\right)^2a^{17/6}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}c^{2/3}} \\
&\quad-\frac{\left(2b-3\sqrt[3]{a}c^{2/3}\right)\arctan\left(\frac{3a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3\sqrt[3]{a}c^{2/3}}}\right)}{81\sqrt{3}a^{17/6}\sqrt{4b-3\sqrt[3]{a}c^{2/3}}c^{2/3}} \\
&\quad-\frac{\left(2(-1)^{2/3}b-3\sqrt[3]{a}c^{2/3}\right)\arctan\left(\frac{3(-1)^{2/3}a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\sqrt{a}\sqrt{4b+3\sqrt[3]{-1}\sqrt[3]{a}c^{2/3}}}\right)}{27\sqrt{3}\left(1-\sqrt[3]{-1}\right)\left(1+\sqrt[3]{-1}\right)^2a^{17/6}\sqrt{4b+3\sqrt[3]{-1}\sqrt[3]{a}c^{2/3}}c^{2/3}} \\
&\quad+\frac{\log\left(3a+3a^{2/3}\sqrt[3]{c}x+bx^2\right)}{162a^{8/3}\sqrt[3]{c}}-\frac{\log\left(3a-3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}x+bx^2\right)}{54\left(1+\sqrt[3]{-1}\right)^2a^{8/3}\sqrt[3]{c}} \\
&\quad-\frac{\sqrt[3]{-1}\log\left(3a+3(-1)^{2/3}a^{2/3}\sqrt[3]{c}x+bx^2\right)}{162a^{8/3}\sqrt[3]{c}}
\end{aligned}$$

output
```
-1/81*(-1)^(1/3)*(2*(-1)^(1/3)*b+3*a^(1/3)*c^(2/3))*arctan(1/3*(3*(-1)^(1/
3)*a^(2/3)*c^(1/3)-2*b*x)*3^(1/2)/a^(1/2)/(4*b-3*(-1)^(2/3)*a^(1/3)*c^(2/3
))^(1/2))*3^(1/2)/(1+(-1)^(1/3))^2/a^(17/6)/(4*b-3*(-1)^(2/3)*a^(1/3)*c^(2
/3))^(1/2)/c^(2/3)-1/243*(2*b-3*a^(1/3)*c^(2/3))*arctan(1/3*(3*a^(2/3)*c^(
1/3)+2*b*x)*3^(1/2)/a^(1/2)/(4*b-3*a^(1/3)*c^(2/3))^(1/2))*3^(1/2)/a^(17/6
)/(4*b-3*a^(1/3)*c^(2/3))^(1/2)/c^(2/3)-1/81*(2*(-1)^(2/3)*b-3*a^(1/3)*c^(
2/3))*arctan(1/3*(3*(-1)^(2/3)*a^(2/3)*c^(1/3)+2*b*x)*3^(1/2)/a^(1/2)/(4*b
+3*(-1)^(1/3)*a^(1/3)*c^(2/3))^(1/2))*3^(1/2)/(1-(-1)^(1/3))/(1+(-1)^(1/3)
)^2/a^(17/6)/(4*b+3*(-1)^(1/3)*a^(1/3)*c^(2/3))^(1/2)/c^(2/3)+1/162*ln(3*a
+3*a^(2/3)*c^(1/3)*x+b*x^2)/a^(8/3)/c^(1/3)-1/54*ln(3*a-3*(-1)^(1/3)*a^(2/
3)*c^(1/3)*x+b*x^2)/(1+(-1)^(1/3))^2/a^(8/3)/c^(1/3)-1/162*(-1)^(1/3)*ln(3
*a+3*(-1)^(2/3)*a^(2/3)*c^(1/3)*x+b*x^2)/a^(8/3)/c^(1/3)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.08 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.19

$$\int \frac{1}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx = \frac{1}{3}\text{RootSum}\left[27a^3+27a^2b\#1^2+27a^2c\#1^3+9ab^2\#1^4 + b^3\#1^6\&,\frac{\log(x-\#1)}{18a^2b\#1+27a^2c\#1^2+12ab^2\#1^3+2b^3\#1^5}\&\right]$$

input
```
Integrate[(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^6)^(
-1),x]
```

output
```
RootSum[27*a^3 + 27*a^2*b*#1^2 + 27*a^2*c*#1^3 + 9*a*b^2*#1^4 + b^3*#1^6 &
 , Log[x - #1]/(18*a^2*b*#1 + 27*a^2*c*#1^2 + 12*a*b^2*#1^3 + 2*b^3*#1^5)
& ]/3
```

Rubi [A] (verified)

Time = 1.67 (sec) , antiderivative size = 507, normalized size of antiderivative = 0.97, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2466, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}dx$$

$$\Big\downarrow \text{2466}$$

$$19683a^6\int \left(-\frac{\sqrt[3]{a}\left(b-3\sqrt[3]{a}c^{2/3}\right)-b\sqrt[3]{c}x}{1594323a^{26/3}c^{2/3}\left(bx^2+3a^{2/3}\sqrt[3]{c}x+3a\right)}+\frac{\sqrt[3]{-1}\sqrt[3]{a}\left(\sqrt[3]{-1}b+3\sqrt[3]{a}c^{2/3}\right)-b\sqrt[3]{c}x}{531441\left(1+\sqrt[3]{-1}\right)^2a^{26/3}c^{2/3}\left(bx^2-3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}x\right.}\right.$$

$$\Big\downarrow \text{2009}$$

$$19683a^6\left(-\frac{\sqrt[3]{-1}\left(3\sqrt[3]{a}c^{2/3}+2\sqrt[3]{-1}b\right)\arctan\left(\frac{3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}\right)}{531441\sqrt{3}\left(1+\sqrt[3]{-1}\right)^2a^{53/6}c^{2/3}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}-\frac{\left(2b-3\sqrt[3]{a}c^{2/3}\right)\arctan\left(\frac{3a^{2/3}\sqrt[3]{c}}{\sqrt{3}\sqrt{a}\sqrt{4b-3}}\right.}{1594323\sqrt{3}a^{53/6}c^{2/3}\sqrt{4b-3\sqrt[3]{}}}\right.$$

input
```
Int[(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^6)^(-1),x]
```

output
```
19683*a^6*(-1/531441*((-1)^(1/3)*(2*(-1)^(1/3)*b + 3*a^(1/3)*c^(2/3))*ArcT
an[(3*(-1)^(1/3)*a^(2/3)*c^(1/3) - 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b - 3*(-
1)^(2/3)*a^(1/3)*c^(2/3)])])/(Sqrt[3]*(1 + (-1)^(1/3))^2*a^(53/6)*Sqrt[4*b
 - 3*(-1)^(2/3)*a^(1/3)*c^(2/3)]*c^(2/3)) - ((2*b - 3*a^(1/3)*c^(2/3))*Arc
Tan[(3*a^(2/3)*c^(1/3) + 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b - 3*a^(1/3)*c^(2
/3)])])/(1594323*Sqrt[3]*a^(53/6)*Sqrt[4*b - 3*a^(1/3)*c^(2/3)]*c^(2/3)) -
 ((2*(-1)^(2/3)*b - 3*a^(1/3)*c^(2/3))*ArcTan[(3*(-1)^(2/3)*a^(2/3)*c^(1/3
) + 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b + 3*(-1)^(1/3)*a^(1/3)*c^(2/3)])])/(1
594323*Sqrt[3]*a^(53/6)*Sqrt[4*b + 3*(-1)^(1/3)*a^(1/3)*c^(2/3)]*c^(2/3))
+ Log[3*a + 3*a^(2/3)*c^(1/3)*x + b*x^2]/(3188646*a^(26/3)*c^(1/3)) - Log[
3*a - 3*(-1)^(1/3)*a^(2/3)*c^(1/3)*x + b*x^2]/(1062882*(1 + (-1)^(1/3))^2*
a^(26/3)*c^(1/3)) - ((-1)^(1/3)*Log[3*a + 3*(-1)^(2/3)*a^(2/3)*c^(1/3)*x +
 b*x^2])/(3188646*a^(26/3)*c^(1/3)))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2466
```
Int[(u_.)*(Q6_)^(p_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
 x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Sim
p[1/(3^(3*p)*a^(2*p))   Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*
x + b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*
(-1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d,
 0] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coef
f[Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.17

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(b^3\_Z^6+9b^2a\_Z^4+27a^2c\_Z^3+27b\,a^2\_Z^2+27a^3\right)}\frac{\ln\left(x-\_R\right)}{2\_R^5b^3+12\_R^3a\,b^2+27\_R^2a^2c+18a^2b\_R}\right)}{3}$	90
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(b^3\_Z^6+9b^2a\_Z^4+27a^2c\_Z^3+27b\,a^2\_Z^2+27a^3\right)}\frac{\ln\left(x-\_R\right)}{2\_R^5b^3+12\_R^3a\,b^2+27\_R^2a^2c+18a^2b\_R}\right)}{3}$	90

input
```
int(1/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x,method=_RET
URNVERBOSE)
```

output
```
1/3*sum(1/(2*_R^5*b^3+12*_R^3*a*b^2+27*_R^2*a^2*c+18*_R*a^2*b)*ln(x-_R),_R
=RootOf(_Z^6*b^3+9*_Z^4*a*b^2+27*_Z^3*a^2*c+27*_Z^2*a^2*b+27*a^3))
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, algo
rithm="fricas")
```

output
```
Exception raised: RuntimeError >> no explicit roots found
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{27a^3 + 27a^2bx^2 + 27a^2cx^3 + 9ab^2x^4 + b^3x^6}\,dx = \text{Timed out}$$

input
```
integrate(1/(b**3*x**6+9*a*b**2*x**4+27*a**2*c*x**3+27*a**2*b*x**2+27*a**3
),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{27a^3 + 27a^2bx^2 + 27a^2cx^3 + 9ab^2x^4 + b^3x^6}\,dx = \int \frac{1}{b^3x^6 + 9\,ab^2x^4 + 27\,a^2cx^3 + 27\,a^2bx^2 + 27\,a^3}\,dx$$

input
```
integrate(1/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, algo
rithm="maxima")
```

output
```
integrate(1/(b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2 + 27*a^3)
, x)
```

Giac [F]

$$\int \frac{1}{27a^3 + 27a^2bx^2 + 27a^2cx^3 + 9ab^2x^4 + b^3x^6}\,dx = \int \frac{1}{b^3x^6 + 9\,ab^2x^4 + 27\,a^2cx^3 + 27\,a^2bx^2 + 27\,a^3}\,dx$$

input
```
integrate(1/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, algo
rithm="giac")
```

output
```
integrate(1/(b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2 + 27*a^3)
, x)
```

Mupad [B] (verification not implemented)

Time = 22.68 (sec) , antiderivative size = 1394, normalized size of antiderivative = 2.67

$$\int \frac{1}{27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6}\,dx = \text{Too large to display}$$

input
```
int(1/(27*a^3 + b^3*x^6 + 27*a^2*b*x^2 + 9*a*b^2*x^4 + 27*a^2*c*x^3),x)
```

output
```
symsum(log(6561*root(488038239039168*a^17*b^3*c^4*z^6 - 205891132094649*a^
18*c^6*z^6 + 10460353203*a^12*b^2*c^4*z^4 - 746143164*a^9*b^3*c^3*z^3 + 38
7420489*a^10*c^5*z^3 + 2657205*a^6*b^4*c^2*z^2 - 2916*a^3*b^5*c*z + b^6, z
, k)^2*a^4*b^12*c^2 - 6*root(488038239039168*a^17*b^3*c^4*z^6 - 2058911320
94649*a^18*c^6*z^6 + 10460353203*a^12*b^2*c^4*z^4 - 746143164*a^9*b^3*c^3*
z^3 + 387420489*a^10*c^5*z^3 + 2657205*a^6*b^4*c^2*z^2 - 2916*a^3*b^5*c*z
+ b^6, z, k)*b^15*x - 4782969*root(488038239039168*a^17*b^3*c^4*z^6 - 2058
91132094649*a^18*c^6*z^6 + 10460353203*a^12*b^2*c^4*z^4 - 746143164*a^9*b^
3*c^3*z^3 + 387420489*a^10*c^5*z^3 + 2657205*a^6*b^4*c^2*z^2 - 2916*a^3*b^
5*c*z + b^6, z, k)^3*a^7*b^11*c^3 - 229582512*root(488038239039168*a^17*b^
3*c^4*z^6 - 205891132094649*a^18*c^6*z^6 + 10460353203*a^12*b^2*c^4*z^4 -
746143164*a^9*b^3*c^3*z^3 + 387420489*a^10*c^5*z^3 + 2657205*a^6*b^4*c^2*z
^2 - 2916*a^3*b^5*c*z + b^6, z, k)^4*a^9*b^13*c^2 - 387420489*root(4880382
39039168*a^17*b^3*c^4*z^6 - 205891132094649*a^18*c^6*z^6 + 10460353203*a^1
2*b^2*c^4*z^4 - 746143164*a^9*b^3*c^3*z^3 + 387420489*a^10*c^5*z^3 + 26572
05*a^6*b^4*c^2*z^2 - 2916*a^3*b^5*c*z + b^6, z, k)^4*a^10*b^10*c^4 + 16736
5651248*root(488038239039168*a^17*b^3*c^4*z^6 - 205891132094649*a^18*c^6*z
^6 + 10460353203*a^12*b^2*c^4*z^4 - 746143164*a^9*b^3*c^3*z^3 + 387420489*
a^10*c^5*z^3 + 2657205*a^6*b^4*c^2*z^2 - 2916*a^3*b^5*c*z + b^6, z, k)^5*a
^12*b^12*c^3 - 94143178827*root(488038239039168*a^17*b^3*c^4*z^6 - 2058...
```

Reduce [F]

$$\int \frac{1}{27a^3 + 27a^2bx^2 + 27a^2cx^3 + 9ab^2x^4 + b^3x^6}\,dx$$
$$= \int \frac{1}{b^3x^6 + 9a\,b^2x^4 + 27a^2c\,x^3 + 27a^2b\,x^2 + 27a^3}dx$$

input
```
int(1/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x)
```

output
```
int(1/(27*a**3 + 27*a**2*b*x**2 + 27*a**2*c*x**3 + 9*a*b**2*x**4 + b**3*x*
*6),x)
```

3.13 $\int \frac{1}{x(27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6)}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 563

$$
\begin{aligned}
&\int \frac{1}{x(27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6)}\,dx \\
&= \frac{\left(b-(-1)^{2/3}\sqrt[3]{a}c^{2/3}\right)\arctan\left(\frac{3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}\right)}{9\sqrt{3}\left(1+\sqrt[3]{-1}\right)^2a^{19/6}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}\sqrt[3]{c}} \\
&+ \frac{\left(b-\sqrt[3]{a}c^{2/3}\right)\arctan\left(\frac{3a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3\sqrt[3]{a}c^{2/3}}}\right)}{27\sqrt{3}a^{19/6}\sqrt{4b-3\sqrt[3]{a}c^{2/3}}\sqrt[3]{c}} \\
&+ \frac{(-1)^{2/3}\left((-1)^{2/3}b-\sqrt[3]{a}c^{2/3}\right)\arctan\left(\frac{3(-1)^{2/3}a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\sqrt{a}\sqrt{4b+3\sqrt[3]{-1}\sqrt[3]{a}c^{2/3}}}\right)}{9\sqrt{3}\left(1-\sqrt[3]{-1}\right)\left(1+\sqrt[3]{-1}\right)^2a^{19/6}\sqrt{4b+3\sqrt[3]{-1}\sqrt[3]{a}c^{2/3}}\sqrt[3]{c}} \\
&+ \frac{\log(x)}{27a^3} - \frac{\left(3\sqrt[3]{a}-\frac{b}{c^{2/3}}\right)\log\left(3a+3a^{2/3}\sqrt[3]{c}x+bx^2\right)}{486a^{10/3}} \\
&- \frac{\left(b+i\sqrt{3}b+6\sqrt[3]{a}c^{2/3}\right)\log\left(3a-3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}x+bx^2\right)}{972a^{10/3}c^{2/3}} \\
&- \frac{\left(3\sqrt[3]{a}-\frac{(-1)^{2/3}b}{c^{2/3}}\right)\log\left(3a+3(-1)^{2/3}a^{2/3}\sqrt[3]{c}x+bx^2\right)}{486a^{10/3}}
\end{aligned}
$$

output
```
1/27*(b-(-1)^(2/3)*a^(1/3)*c^(2/3))*arctan(1/3*(3*(-1)^(1/3)*a^(2/3)*c^(1/
3)-2*b*x)*3^(1/2)/a^(1/2)/(4*b-3*(-1)^(2/3)*a^(1/3)*c^(2/3))^(1/2))*3^(1/2
)/(1+(-1)^(1/3))^2/a^(19/6)/(4*b-3*(-1)^(2/3)*a^(1/3)*c^(2/3))^(1/2)/c^(1/
3)+1/81*(b-a^(1/3)*c^(2/3))*arctan(1/3*(3*a^(2/3)*c^(1/3)+2*b*x)*3^(1/2)/a
^(1/2)/(4*b-3*a^(1/3)*c^(2/3))^(1/2))*3^(1/2)/a^(19/6)/(4*b-3*a^(1/3)*c^(2
/3))^(1/2)/c^(1/3)+1/27*(-1)^(2/3)*((-1)^(2/3)*b-a^(1/3)*c^(2/3))*arctan(1
/3*(3*(-1)^(2/3)*a^(2/3)*c^(1/3)+2*b*x)*3^(1/2)/a^(1/2)/(4*b+3*(-1)^(1/3)*
a^(1/3)*c^(2/3))^(1/2))*3^(1/2)/(1-(-1)^(1/3))/(1+(-1)^(1/3))^2/a^(19/6)/(
4*b+3*(-1)^(1/3)*a^(1/3)*c^(2/3))^(1/2)/c^(1/3)+1/27*ln(x)/a^3-1/486*(3*a^
(1/3)-b/c^(2/3))*ln(3*a+3*a^(2/3)*c^(1/3)*x+b*x^2)/a^(10/3)-1/972*(b+I*3^(
1/2)*b+6*a^(1/3)*c^(2/3))*ln(3*a-3*(-1)^(1/3)*a^(2/3)*c^(1/3)*x+b*x^2)/a^(
10/3)/c^(2/3)-1/486*(3*a^(1/3)-(-1)^(2/3)*b/c^(2/3))*ln(3*a+3*(-1)^(2/3)*a
^(2/3)*c^(1/3)*x+b*x^2)/a^(10/3)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.12 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.28

$$\int \frac{1}{x\left(27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6\right)}\,dx =$$

$$-\frac{-3\log(x)+\text{RootSum}\left[27a^3+27a^2b\#1^2+27a^2c\#1^3+9ab^2\#1^4+b^3\#1^6\&,\frac{27a^2b\log(x-\#1)+27a^2c\log(x-}{18a^2b+27}\right.}{81a^3}$$

input
```
Integrate[1/(x*(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x
^6)),x]
```

output
```
-1/81*(-3*Log[x] + RootSum[27*a^3 + 27*a^2*b*#1^2 + 27*a^2*c*#1^3 + 9*a*b^
2*#1^4 + b^3*#1^6 & , (27*a^2*b*Log[x - #1] + 27*a^2*c*Log[x - #1]*#1 + 9*
a*b^2*Log[x - #1]*#1^2 + b^3*Log[x - #1]*#1^4)/(18*a^2*b + 27*a^2*c*#1 + 1
2*a*b^2*#1^2 + 2*b^3*#1^4) & ])/a^3
```

Rubi [A] (verified)

Time = 2.02 (sec) , antiderivative size = 548, normalized size of antiderivative = 0.97, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {2466, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6)}dx$$

$$\Big\downarrow \; 2466$$

$$19683a^6\int \left(\frac{3a^{2/3}\sqrt[3]{c}\left(2b-3\sqrt[3]{a}c^{2/3}\right)+b\left(b-3\sqrt[3]{a}c^{2/3}\right)x}{4782969a^{28/3}c^{2/3}\left(bx^2+3a^{2/3}\sqrt[3]{c}x+3a\right)}+\frac{1}{531441a^9x}-\frac{3a^{2/3}\sqrt[3]{c}\left(2b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}\right)+\sqrt[3]{-}}{1594323\left(1+\sqrt[3]{-1}\right)^2a^{28/3}c^{2/3}\left(bx^2\right.}\right.$$

$$\Big\downarrow \; 2009$$

$$19683a^6\left(\frac{\left(b-(-1)^{2/3}\sqrt[3]{a}c^{2/3}\right)\arctan\left(\frac{3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}\right)}{177147\sqrt{3}\left(1+\sqrt[3]{-1}\right)^2a^{55/6}\sqrt[3]{c}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}+\frac{\left(b-\sqrt[3]{a}c^{2/3}\right)\arctan\left(\frac{3a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3\sqrt[3]{a}c^{2/3}}}\right)}{531441\sqrt{3}a^{55/6}\sqrt[3]{c}\sqrt{4b-3\sqrt[3]{a}c^{2/3}}}\right.$$

input
```
Int[1/(x*(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^6)),x
]
```

output
```
19683*a^6*(((b - (-1)^(2/3)*a^(1/3)*c^(2/3))*ArcTan[(3*(-1)^(1/3)*a^(2/3)*
c^(1/3) - 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b - 3*(-1)^(2/3)*a^(1/3)*c^(2/3)]
)])/(177147*Sqrt[3]*(1 + (-1)^(1/3))^2*a^(55/6)*Sqrt[4*b - 3*(-1)^(2/3)*a^
(1/3)*c^(2/3)]*c^(1/3)) + ((b - a^(1/3)*c^(2/3))*ArcTan[(3*a^(2/3)*c^(1/3)
 + 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b - 3*a^(1/3)*c^(2/3)])])/(531441*Sqrt[3
]*a^(55/6)*Sqrt[4*b - 3*a^(1/3)*c^(2/3)]*c^(1/3)) + ((-1)^(2/3)*((-1)^(2/3
)*b - a^(1/3)*c^(2/3))*ArcTan[(3*(-1)^(2/3)*a^(2/3)*c^(1/3) + 2*b*x)/(Sqrt
[3]*Sqrt[a]*Sqrt[4*b + 3*(-1)^(1/3)*a^(1/3)*c^(2/3)])])/(531441*Sqrt[3]*a^
(55/6)*Sqrt[4*b + 3*(-1)^(1/3)*a^(1/3)*c^(2/3)]*c^(1/3)) + Log[x]/(531441*
a^9) - ((3*a^(1/3) - b/c^(2/3))*Log[3*a + 3*a^(2/3)*c^(1/3)*x + b*x^2])/(9
565938*a^(28/3)) - ((b + I*Sqrt[3]*b + 6*a^(1/3)*c^(2/3))*Log[3*a - 3*(-1)
^(1/3)*a^(2/3)*c^(1/3)*x + b*x^2])/(19131876*a^(28/3)*c^(2/3)) - ((3*a^(1/
3) - ((-1)^(2/3)*b)/c^(2/3))*Log[3*a + 3*(-1)^(2/3)*a^(2/3)*c^(1/3)*x + b*
x^2])/(9565938*a^(28/3)))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2466
```
Int[(u_.)*(Q6_)^(p_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
 x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Sim
p[1/(3^(3*p)*a^(2*p))   Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*
x + b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*
(-1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d,
 0] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coef
f[Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.24

method	result
default	$\frac{\ln(x)}{27a^3} - \frac{\sum_{\_R=\mathrm{RootOf}\left(b^3\_Z^6+9b^2a\_Z^4+27a^2c\_Z^3+27b\,a^2\_Z^2+27a^3\right)} \frac{\left(\_R^5b^3+9\_R^3a\,b^2+27\_R^2a^2c+27a^2b\_R\right)\ln\left(x-\_R\right)}{2\_R^5b^3+12\_R^3a\,b^2+27\_R^2a^2c+18a^2b\_R}}{81a^3}$
risch	$\left(\sum_{\_R=\mathrm{RootOf}\left(\left(27a^{21}c^6-64a^{20}b^3c^4\right)\_Z^6+\left(243a^{18}c^6-576a^{17}b^3c^4\right)\_Z^5+\left(729a^{15}c^6-1755a^{14}c^4b^3\right)\_Z^4+\left(729a^{12}c^6-1917a^{11}c^4b^3+16a^{10}\right.\right.}\right.$

input
```
int(1/x/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x,method=_R
ETURNVERBOSE)
```

output
```
1/27*ln(x)/a^3-1/81/a^3*sum((_R^5*b^3+9*_R^3*a*b^2+27*_R^2*a^2*c+27*_R*a^2
*b)/(2*_R^5*b^3+12*_R^3*a*b^2+27*_R^2*a^2*c+18*_R*a^2*b)*ln(x-_R),_R=RootO
f(_Z^6*b^3+9*_Z^4*a*b^2+27*_Z^3*a^2*c+27*_Z^2*a^2*b+27*a^3))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x\left(27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6\right)}\,dx = \text{Timed out}$$

input
```
integrate(1/x/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, al
gorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x\left(27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6\right)}\,dx=\text{Timed out}$$

input
```
integrate(1/x/(b**3*x**6+9*a*b**2*x**4+27*a**2*c*x**3+27*a**2*b*x**2+27*a*
*3),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{x\left(27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6\right)}\,dx = \int \frac{1}{(b^3x^6+9\,ab^2x^4+27\,a^2cx^3+27\,a^2bx^2+27\,a^3)x}\,dx$$

input
```
integrate(1/x/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, al
gorithm="maxima")
```

output
```
-1/27*integrate((b^3*x^5 + 9*a*b^2*x^3 + 27*a^2*c*x^2 + 27*a^2*b*x)/(b^3*x
^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2 + 27*a^3), x)/a^3 + 1/27*lo
g(x)/a^3
```

Giac [F]

$$\int \frac{1}{x\left(27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6\right)}\,dx = \int \frac{1}{(b^3x^6+9\,ab^2x^4+27\,a^2cx^3+27\,a^2bx^2+27\,a^3)x}\,dx$$

input
```
integrate(1/x/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x, al
gorithm="giac")
```

output
```
integrate(1/((b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2 + 27*a^3
)*x), x)
```

Mupad [B] (verification not implemented)

Time = 22.20 (sec) , antiderivative size = 4002, normalized size of antiderivative = 7.11

$$\int \frac{1}{x\left(27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6\right)}\,dx = \text{Too large to display}$$

input
```
int(1/(x*(27*a^3 + b^3*x^6 + 27*a^2*b*x^2 + 9*a*b^2*x^4 + 27*a^2*c*x^3)),x
)
```

output
```
log(x)/(27*a^3) + symsum(log(7*root(13177032454057536*a^20*b^3*c^4*z^6 - 5
559060566555523*a^21*c^6*z^6 + 488038239039168*a^17*b^3*c^4*z^5 - 20589113
2094649*a^18*c^6*z^5 + 6119306623755*a^14*b^3*c^4*z^4 - 2541865828329*a^15
*c^6*z^4 + 27506854719*a^11*b^3*c^4*z^3 - 229582512*a^10*b^6*c^2*z^3 - 104
60353203*a^12*c^6*z^3 + 14348907*a^8*b^3*c^4*z^2 + 10097379*a^7*b^6*c^2*z^
2 - 6561*a^4*b^6*c^2*z + b^9, z, k)*b^18*x - 162*root(13177032454057536*a^
20*b^3*c^4*z^6 - 5559060566555523*a^21*c^6*z^6 + 488038239039168*a^17*b^3*
c^4*z^5 - 205891132094649*a^18*c^6*z^5 + 6119306623755*a^14*b^3*c^4*z^4 -
2541865828329*a^15*c^6*z^4 + 27506854719*a^11*b^3*c^4*z^3 - 229582512*a^10
*b^6*c^2*z^3 - 10460353203*a^12*c^6*z^3 + 14348907*a^8*b^3*c^4*z^2 + 10097
379*a^7*b^6*c^2*z^2 - 6561*a^4*b^6*c^2*z + b^9, z, k)^2*a^3*b^18*x + 86093
442*root(13177032454057536*a^20*b^3*c^4*z^6 - 5559060566555523*a^21*c^6*z^
6 + 488038239039168*a^17*b^3*c^4*z^5 - 205891132094649*a^18*c^6*z^5 + 6119
306623755*a^14*b^3*c^4*z^4 - 2541865828329*a^15*c^6*z^4 + 27506854719*a^11
*b^3*c^4*z^3 - 229582512*a^10*b^6*c^2*z^3 - 10460353203*a^12*c^6*z^3 + 143
48907*a^8*b^3*c^4*z^2 + 10097379*a^7*b^6*c^2*z^2 - 6561*a^4*b^6*c^2*z + b^
9, z, k)^3*a^8*b^13*c^3 + 34867844010*root(13177032454057536*a^20*b^3*c^4*
z^6 - 5559060566555523*a^21*c^6*z^6 + 488038239039168*a^17*b^3*c^4*z^5 - 2
05891132094649*a^18*c^6*z^5 + 6119306623755*a^14*b^3*c^4*z^4 - 25418658283
29*a^15*c^6*z^4 + 27506854719*a^11*b^3*c^4*z^3 - 229582512*a^10*b^6*c^2...
```

Reduce [F]

$$\int \frac{1}{x\left(27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6\right)}\,dx$$
$$=\int \frac{1}{b^3x^7+9a\,b^2x^5+27a^2c\,x^4+27a^2b\,x^3+27a^3x}dx$$

input
```
int(1/x/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x)
```

output
```
int(1/(27*a**3*x + 27*a**2*b*x**3 + 27*a**2*c*x**4 + 9*a*b**2*x**5 + b**3*
x**7),x)
```

3.14 $\int \frac{1}{x^2(27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6)}\,dx$

Optimal result

Integrand size = 46, antiderivative size = 645

$$\begin{aligned}\int \frac{1}{x^2(27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6)}\,dx &= -\frac{1}{27a^3x}\\
&+\frac{\left(2(-1)^{2/3}b^2+12\sqrt[3]{-1}\sqrt[3]{a}bc^{2/3}+9a^{2/3}c^{4/3}\right)\arctan\left(\frac{3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}\right)}{81\sqrt{3}\left(1+\sqrt[3]{-1}\right)^2a^{23/6}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}c^{2/3}}\\
&+\frac{\left(2b^2-12\sqrt[3]{a}bc^{2/3}+9a^{2/3}c^{4/3}\right)\arctan\left(\frac{3a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3\sqrt[3]{a}c^{2/3}}}\right)}{243\sqrt{3}a^{23/6}\sqrt{4b-3\sqrt[3]{a}c^{2/3}}c^{2/3}}\\
&+\frac{(-1)^{2/3}\left(2b^2+12\sqrt[3]{-1}\sqrt[3]{a}bc^{2/3}+9(-1)^{2/3}a^{2/3}c^{4/3}\right)\arctan\left(\frac{3(-1)^{2/3}a^{2/3}\sqrt[3]{c}+2bx}{\sqrt{3}\sqrt{a}\sqrt{4b+3\sqrt[3]{-1}\sqrt[3]{a}c^{2/3}}}\right)}{81\sqrt{3}\left(1-\sqrt[3]{-1}\right)\left(1+\sqrt[3]{-1}\right)^2a^{23/6}\sqrt{4b+3\sqrt[3]{-1}\sqrt[3]{a}c^{2/3}}c^{2/3}}\\
&-\frac{\left(2b-3\sqrt[3]{a}c^{2/3}\right)\log\left(3a+3a^{2/3}\sqrt[3]{c}x+bx^2\right)}{486a^{11/3}\sqrt[3]{c}}\\
&+\frac{\left(2b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}\right)\log\left(3a-3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}x+bx^2\right)}{162\left(1+\sqrt[3]{-1}\right)^2a^{11/3}\sqrt[3]{c}}\\
&+\frac{\sqrt[3]{-1}\left(2b+3\sqrt[3]{-1}\sqrt[3]{a}c^{2/3}\right)\log\left(3a+3(-1)^{2/3}a^{2/3}\sqrt[3]{c}x+bx^2\right)}{486a^{11/3}\sqrt[3]{c}}\end{aligned}$$

output
```
-1/27/a^3/x+1/243*(2*(-1)^(2/3)*b^2+12*(-1)^(1/3)*a^(1/3)*b*c^(2/3)+9*a^(2
/3)*c^(4/3))*arctan(1/3*(3*(-1)^(1/3)*a^(2/3)*c^(1/3)-2*b*x)*3^(1/2)/a^(1/
2)/(4*b-3*(-1)^(2/3)*a^(1/3)*c^(2/3))^(1/2))*3^(1/2)/(1+(-1)^(1/3))^2/a^(2
3/6)/(4*b-3*(-1)^(2/3)*a^(1/3)*c^(2/3))^(1/2)/c^(2/3)+1/729*(2*b^2-12*a^(1
/3)*b*c^(2/3)+9*a^(2/3)*c^(4/3))*arctan(1/3*(3*a^(2/3)*c^(1/3)+2*b*x)*3^(1
/2)/a^(1/2)/(4*b-3*a^(1/3)*c^(2/3))^(1/2))*3^(1/2)/a^(23/6)/(4*b-3*a^(1/3)
*c^(2/3))^(1/2)/c^(2/3)+1/243*(-1)^(2/3)*(2*b^2+12*(-1)^(1/3)*a^(1/3)*b*c^
(2/3)+9*(-1)^(2/3)*a^(2/3)*c^(4/3))*arctan(1/3*(3*(-1)^(2/3)*a^(2/3)*c^(1/
3)+2*b*x)*3^(1/2)/a^(1/2)/(4*b+3*(-1)^(1/3)*a^(1/3)*c^(2/3))^(1/2))*3^(1/2
)/(1-(-1)^(1/3))/(1+(-1)^(1/3))^2/a^(23/6)/(4*b+3*(-1)^(1/3)*a^(1/3)*c^(2/
3))^(1/2)/c^(2/3)-1/486*(2*b-3*a^(1/3)*c^(2/3))*ln(3*a+3*a^(2/3)*c^(1/3)*x
+b*x^2)/a^(11/3)/c^(1/3)+1/162*(2*b-3*(-1)^(2/3)*a^(1/3)*c^(2/3))*ln(3*a-3
*(-1)^(1/3)*a^(2/3)*c^(1/3)*x+b*x^2)/(1+(-1)^(1/3))^2/a^(11/3)/c^(1/3)+1/4
86*(-1)^(1/3)*(2*b+3*(-1)^(1/3)*a^(1/3)*c^(2/3))*ln(3*a+3*(-1)^(2/3)*a^(2/
3)*c^(1/3)*x+b*x^2)/a^(11/3)/c^(1/3)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.14 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.25

$$\int \frac{1}{x^2 (27a^3 + 27a^2bx^2 + 27a^2cx^3 + 9ab^2x^4 + b^3x^6)}\,dx =$$
$$-\frac{3 + x\text{RootSum}\left[27a^3 + 27a^2b\#1^2 + 27a^2c\#1^3 + 9ab^2\#1^4 + b^3\#1^6\&, \frac{27a^2b\log(x-\#1)+27a^2c\log(x-\#1)\#1}{18a^2b\#1+27a^2c\#1}\right.}{81a^3x}$$

input
```
Integrate[1/(x^2*(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3
*x^6)),x]
```

output
```
-1/81*(3 + x*RootSum[27*a^3 + 27*a^2*b*#1^2 + 27*a^2*c*#1^3 + 9*a*b^2*#1^4
 + b^3*#1^6 & , (27*a^2*b*Log[x - #1] + 27*a^2*c*Log[x - #1]*#1 + 9*a*b^2*
Log[x - #1]*#1^2 + b^3*Log[x - #1]*#1^4)/(18*a^2*b*#1 + 27*a^2*c*#1^2 + 12
*a*b^2*#1^3 + 2*b^3*#1^5) & ])/(a^3*x)
```

Rubi [A] (verified)

Time = 2.41 (sec) , antiderivative size = 625, normalized size of antiderivative = 0.97, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {2466, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\left(27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6\right)}dx$$

$$\Big\downarrow \text{2466}$$

$$19683a^6\int \left(\frac{\sqrt[3]{a}\left(b^2-9\sqrt[3]{a}c^{2/3}b+9a^{2/3}c^{4/3}\right)-b\left(2b-3\sqrt[3]{a}c^{2/3}\right)\sqrt[3]{c}x}{4782969a^{29/3}c^{2/3}\left(bx^2+3a^{2/3}\sqrt[3]{c}x+3a\right)}-\frac{\sqrt[3]{a}\left((-1)^{2/3}b^2+9\sqrt[3]{-1}\sqrt[3]{a}c^{2/3}b+9a^{2/3}\right.}{1594323\left(1+\sqrt[3]{-1}\right)^2a^{29/3}c^{2/3}}\right.$$

$$\Big\downarrow \text{2009}$$

$$19683a^6\left(\frac{\left(9a^{2/3}c^{4/3}+12\sqrt[3]{-1}\sqrt[3]{a}bc^{2/3}+2(-1)^{2/3}b^2\right)\arctan\left(\frac{3\sqrt[3]{-1}a^{2/3}\sqrt[3]{c}-2bx}{\sqrt{3}\sqrt{a}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}\right)}{1594323\sqrt{3}\left(1+\sqrt[3]{-1}\right)^2a^{59/6}c^{2/3}\sqrt{4b-3(-1)^{2/3}\sqrt[3]{a}c^{2/3}}}+\frac{\left(9a^{2/3}c^{4/3}-12\sqrt[3]{a}b\right.}{478296}\right.$$

input
```
Int[1/(x^2*(27*a^3 + 27*a^2*b*x^2 + 27*a^2*c*x^3 + 9*a*b^2*x^4 + b^3*x^6))
,x]
```

output
```
19683*a^6*(-1/531441*1/(a^9*x) + ((2*(-1)^(2/3)*b^2 + 12*(-1)^(1/3)*a^(1/3
)*b*c^(2/3) + 9*a^(2/3)*c^(4/3))*ArcTan[(3*(-1)^(1/3)*a^(2/3)*c^(1/3) - 2*
b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b - 3*(-1)^(2/3)*a^(1/3)*c^(2/3)])])/(1594323
*Sqrt[3]*(1 + (-1)^(1/3))^2*a^(59/6)*Sqrt[4*b - 3*(-1)^(2/3)*a^(1/3)*c^(2/
3)]*c^(2/3)) + ((2*b^2 - 12*a^(1/3)*b*c^(2/3) + 9*a^(2/3)*c^(4/3))*ArcTan[
(3*a^(2/3)*c^(1/3) + 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b - 3*a^(1/3)*c^(2/3)]
)])/(4782969*Sqrt[3]*a^(59/6)*Sqrt[4*b - 3*a^(1/3)*c^(2/3)]*c^(2/3)) + ((2
*(-1)^(2/3)*b^2 - 12*a^(1/3)*b*c^(2/3) - 9*(-1)^(1/3)*a^(2/3)*c^(4/3))*Arc
Tan[(3*(-1)^(2/3)*a^(2/3)*c^(1/3) + 2*b*x)/(Sqrt[3]*Sqrt[a]*Sqrt[4*b + 3*(
-1)^(1/3)*a^(1/3)*c^(2/3)])])/(4782969*Sqrt[3]*a^(59/6)*Sqrt[4*b + 3*(-1)^
(1/3)*a^(1/3)*c^(2/3)]*c^(2/3)) - ((2*b - 3*a^(1/3)*c^(2/3))*Log[3*a + 3*a
^(2/3)*c^(1/3)*x + b*x^2])/(9565938*a^(29/3)*c^(1/3)) + ((2*b - 3*(-1)^(2/
3)*a^(1/3)*c^(2/3))*Log[3*a - 3*(-1)^(1/3)*a^(2/3)*c^(1/3)*x + b*x^2])/(31
88646*(1 + (-1)^(1/3))^2*a^(29/3)*c^(1/3)) + ((-1)^(1/3)*(2*b + 3*(-1)^(1/
3)*a^(1/3)*c^(2/3))*Log[3*a + 3*(-1)^(2/3)*a^(2/3)*c^(1/3)*x + b*x^2])/(95
65938*a^(29/3)*c^(1/3)))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2466
```
Int[(u_.)*(Q6_)^(p_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
 x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Sim
p[1/(3^(3*p)*a^(2*p))   Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*
x + b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*
(-1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d,
 0] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coef
f[Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.09 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.21

method	result
default	$-\frac{1}{27a^3x}+\frac{\sum_{\_R=\mathrm{RootOf}\left(b^3\_Z^6+9b^2a\_Z^4+27a^2c\_Z^3+27b\,a^2\_Z^2+27a^3\right)}\frac{\left(-\_R^4b^3-9\_R^2a\,b^2-27\_R\,a^2c-27b\,a^2\right)\ln\left(x-\_R\right)}{2\_R^5b^3+12\_R^3a\,b^2+27\_R^2a^2c+18a^2b\_R}}{81a^3}$
risch	$-\frac{1}{27a^3x}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(729a^{24}c^6-1728a^{23}b^3c^4\right)\_Z^6+\left(13122a^{17}b\,c^6-31347a^{16}b^4c^4\right)\_Z^4+\left(-19683c^7a^{14}+52488b^3c^5a^{13}-14472b^6c^3\right.\right.}\right.}{}$

input
```
int(1/x^2/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x,method=
_RETURNVERBOSE)
```

output
```
-1/27/a^3/x+1/81/a^3*sum((-_R^4*b^3-9*_R^2*a*b^2-27*_R*a^2*c-27*a^2*b)/(2*
_R^5*b^3+12*_R^3*a*b^2+27*_R^2*a^2*c+18*_R*a^2*b)*ln(x-_R),_R=RootOf(_Z^6*
b^3+9*_Z^4*a*b^2+27*_Z^3*a^2*c+27*_Z^2*a^2*b+27*a^3))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^2\left(27a^3+27a^2bx^2+27a^2cx^3+9ab^2x^4+b^3x^6\right)}\,dx=\text{Timed out}$$

input
```
integrate(1/x^2/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x,
algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (27a^3 + 27a^2bx^2 + 27a^2cx^3 + 9ab^2x^4 + b^3x^6)} dx = \text{Timed out}$$

input
```
integrate(1/x**2/(b**3*x**6+9*a*b**2*x**4+27*a**2*c*x**3+27*a**2*b*x**2+27
*a**3),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{x^2 (27a^3 + 27a^2bx^2 + 27a^2cx^3 + 9ab^2x^4 + b^3x^6)} dx = \int \frac{1}{(b^3x^6 + 9\,ab^2x^4 + 27\,a^2cx^3 + 27\,a^2bx^2 + 27\,a^3)x^2} dx$$

input
```
integrate(1/x^2/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x,
algorithm="maxima")
```

output
```
-1/27*integrate((b^3*x^4 + 9*a*b^2*x^2 + 27*a^2*c*x + 27*a^2*b)/(b^3*x^6 +
 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2 + 27*a^3), x)/a^3 - 1/27/(a^3*x
)
```

Giac [F]

$$\int \frac{1}{x^2 (27a^3 + 27a^2bx^2 + 27a^2cx^3 + 9ab^2x^4 + b^3x^6)} dx = \int \frac{1}{(b^3x^6 + 9\,ab^2x^4 + 27\,a^2cx^3 + 27\,a^2bx^2 + 27\,a^3)x^2} dx$$

input
```
integrate(1/x^2/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x,
algorithm="giac")
```

output
```
integrate(1/((b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*c*x^3 + 27*a^2*b*x^2 + 27*a^3
)*x^2), x)
```

Mupad [B] (verification not implemented)

Time = 22.51 (sec) , antiderivative size = 2663, normalized size of antiderivative = 4.13

$$\int \frac{1}{x^2 (27a^3 + 27a^2bx^2 + 27a^2cx^3 + 9ab^2x^4 + b^3x^6)} dx = \text{Too large to display}$$

input
```
int(1/(x^2*(27*a^3 + b^3*x^6 + 27*a^2*b*x^2 + 9*a*b^2*x^4 + 27*a^2*c*x^3))
,x)
```

output
```
symsum(log(-282429536481*root(355779876259553472*a^23*b^3*c^4*z^6 - 150094
635296999121*a^24*c^6*z^6 - 45753584909922*a^17*b*c^6*z^4 + 10930023061814
7*a^16*b^4*c^4*z^4 - 753145430616*a^13*b^3*c^5*z^3 + 207657382104*a^12*b^6
*c^3*z^3 + 282429536481*a^14*c^7*z^3 + 258280326*a^9*b^5*c^4*z^2 + 1004423
49*a^8*b^8*c^2*z^2 + 17496*a^4*b^10*c*z + b^12, z, k)*a^23*b^9*(2*b^10*x +
 2541865828329*root(355779876259553472*a^23*b^3*c^4*z^6 - 1500946352969991
21*a^24*c^6*z^6 - 45753584909922*a^17*b*c^6*z^4 + 109300230618147*a^16*b^4
*c^4*z^4 - 753145430616*a^13*b^3*c^5*z^3 + 207657382104*a^12*b^6*c^3*z^3 +
 282429536481*a^14*c^7*z^3 + 258280326*a^9*b^5*c^4*z^2 + 100442349*a^8*b^8
*c^2*z^2 + 17496*a^4*b^10*c*z + b^12, z, k)^4*a^17*c^5 - 45*a*b^8*c + 3874
20489*root(355779876259553472*a^23*b^3*c^4*z^6 - 150094635296999121*a^24*c
^6*z^6 - 45753584909922*a^17*b*c^6*z^4 + 109300230618147*a^16*b^4*c^4*z^4
- 753145430616*a^13*b^3*c^5*z^3 + 207657382104*a^12*b^6*c^3*z^3 + 28242953
6481*a^14*c^7*z^3 + 258280326*a^9*b^5*c^4*z^2 + 100442349*a^8*b^8*c^2*z^2
+ 17496*a^4*b^10*c*z + b^12, z, k)^2*a^10*c^6*x - 401769396*root(355779876
259553472*a^23*b^3*c^4*z^6 - 150094635296999121*a^24*c^6*z^6 - 45753584909
922*a^17*b*c^6*z^4 + 109300230618147*a^16*b^4*c^4*z^4 - 753145430616*a^13*
b^3*c^5*z^3 + 207657382104*a^12*b^6*c^3*z^3 + 282429536481*a^14*c^7*z^3 +
258280326*a^9*b^5*c^4*z^2 + 100442349*a^8*b^8*c^2*z^2 + 17496*a^4*b^10*c*z
 + b^12, z, k)^2*a^9*b^4*c^3 - 2066242608*root(355779876259553472*a^23*...
```

Reduce [F]

$$\int \frac{1}{x^2 \left(27a^3 + 27a^2bx^2 + 27a^2cx^3 + 9ab^2x^4 + b^3x^6\right)}\, dx$$
$$= \frac{-\left(\int \frac{x^4}{b^3x^6+9a\,b^2x^4+27a^2c\,x^3+27a^2b\,x^2+27a^3}dx\right) b^3x - 9\left(\int \frac{x^2}{b^3x^6+9a\,b^2x^4+27a^2c\,x^3+27a^2b\,x^2+27a^3}dx\right) a\,b^2x - 27\left(\int \frac{}{b^3x^6 \cdots}\right.}{27a^3x}$$

input
```
int(1/x^2/(b^3*x^6+9*a*b^2*x^4+27*a^2*c*x^3+27*a^2*b*x^2+27*a^3),x)
```

output
```
( - int(x**4/(27*a**3 + 27*a**2*b*x**2 + 27*a**2*c*x**3 + 9*a*b**2*x**4 +
b**3*x**6),x)*b**3*x - 9*int(x**2/(27*a**3 + 27*a**2*b*x**2 + 27*a**2*c*x*
*3 + 9*a*b**2*x**4 + b**3*x**6),x)*a*b**2*x - 27*int(x/(27*a**3 + 27*a**2*
b*x**2 + 27*a**2*c*x**3 + 9*a*b**2*x**4 + b**3*x**6),x)*a**2*c*x - 27*int(
1/(27*a**3 + 27*a**2*b*x**2 + 27*a**2*c*x**3 + 9*a*b**2*x**4 + b**3*x**6),
x)*a**2*b*x - 1)/(27*a**3*x)
```

3.15 $\int \frac{x^5}{216+108x^2+324x^3+18x^4+x^6}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 395

$$\int \frac{x^5}{216+108x^2+324x^3+18x^4+x^6}\,dx = -\frac{\sqrt[3]{-2}\left(1+\sqrt[3]{-2}3^{2/3}\right)\arctan\left(\frac{3(-2)^{2/3}\sqrt[3]{3}+2x}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{3^{5/6}\sqrt{8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}3^{2/3}}} + \frac{\sqrt[6]{\frac{3}{2}}\left(1-(-3)^{2/3}\sqrt[3]{2}\right)\arctan\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{-3}-\sqrt[3]{2}x\right)}{\sqrt{3\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{\left(1+\sqrt[3]{-1}\right)^2\sqrt{4-3(-3)^{2/3}\sqrt[3]{2}}} - \frac{\left(1-\sqrt[3]{2}3^{2/3}\right)\operatorname{arctanh}\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{3}+\sqrt[3]{2}x\right)}{\sqrt{3\left(-4+3\sqrt[3]{2}3^{2/3}\right)}}\right)}{\sqrt[6]{2}3^{5/6}\sqrt{-4+3\sqrt[3]{2}3^{2/3}}} + \frac{1}{216}\left(36+2^{2/3}\sqrt[3]{3}\left(1+i\sqrt{3}\right)\right)\log\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)+\frac{1}{108}\left(18-(-2)^{2/3}\sqrt[3]{3}\right)\log\left(6+3(-2)^{2/3}\sqrt[3]{3}x+\right.$$

output
```
-1/3*(-2)^(1/3)*(1+(-2)^(1/3)*3^(2/3))*arctan((3*(-2)^(2/3)*3^(1/3)+2*x)/(
24+18*(-2)^(1/3)*3^(2/3))^(1/2))*3^(1/6)/(8+9*I*2^(1/3)*3^(1/6)+3*2^(1/3)*
3^(2/3))^(1/2)+1/2*3^(1/6)*2^(5/6)*(1-(-3)^(2/3)*2^(1/3))*arctan(2^(1/6)*(
3*(-3)^(1/3)-2^(1/3)*x)/(12-9*(-3)^(2/3)*2^(1/3))^(1/2))/(1+(-1)^(1/3))^2/
(4-3*(-3)^(2/3)*2^(1/3))^(1/2)-1/6*(1-2^(1/3)*3^(2/3))*arctanh(2^(1/6)*(3*
3^(1/3)+2^(1/3)*x)/(-12+9*2^(1/3)*3^(2/3))^(1/2))*2^(5/6)*3^(1/6)/(-4+3*2^
(1/3)*3^(2/3))^(1/2)+1/216*(36+2^(2/3)*3^(1/3)*(1+I*3^(1/2)))*ln(6-3*(-3)^
(1/3)*2^(2/3)*x+x^2)+1/108*(18-(-2)^(2/3)*3^(1/3))*ln(6+3*(-2)^(2/3)*3^(1/
3)*x+x^2)+1/108*(18-2^(2/3)*3^(1/3))*ln(6+3*2^(2/3)*3^(1/3)*x+x^2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.15

$$\int \frac{x^5}{216+108x^2+324x^3+18x^4+x^6}\,dx=\frac{1}{6}\text{RootSum}\left[216+108\#1^2+324\#1^3+18\#1^4+\#1^6\&,\frac{\log(x-\#1)\#1^4}{36+162\#1+12\#1^2+\#1^4}\&\right]$$

input
```
Integrate[x^5/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6),x]
```

output
```
RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^4 + #1^6 & , (Log[x - #1]*#1^4)/
(36 + 162*#1 + 12*#1^2 + #1^4) & ]/6
```

Rubi [A] (verified)

Time = 2.26 (sec) , antiderivative size = 402, normalized size of antiderivative = 1.02, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2466, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{x^6+18x^4+324x^3+108x^2+216}\,dx$$

$\big\downarrow$ 2466

$$1259712\int \left(\frac{(-1)^{2/3}\left(\left(1-3(-3)^{2/3}\sqrt[3]{2}\right)x+3\sqrt[3]{-3}2^{2/3}\right)}{3779136\sqrt[3]{2}3^{2/3}\left(1+\sqrt[3]{-1}\right)^2\left(x^2-3\sqrt[3]{-3}2^{2/3}x+6\right)}+\frac{(-1)^{2/3}\left(3(-2)^{2/3}\sqrt[3]{3}-\left(1+3\sqrt[3]{-2}3^{2/3}\right)x\right.}{11337408\sqrt[3]{2}3^{2/3}\left(x^2+3(-2)^{2/3}\sqrt[3]{3}x+6\right)}\right.$$

$\big\downarrow$ 2009

$$1259712\left(\frac{(-1)^{2/3}\left((-1)^{2/3}-\sqrt[3]{2}3^{2/3}\right)\arctan\left(\frac{2x+3(-2)^{2/3}\sqrt[3]{3}}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{1259712\sqrt[6]{2}3^{5/6}\sqrt{4+3\sqrt[3]{-2}3^{2/3}}}-\frac{\left(2(-3)^{2/3}-2^{2/3}\right)\arctan\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{-3}-\right.}{\sqrt{3\left(4-3(-3)^{2/}\right.}}\right.}{419904\,6^{5/6}\left(1+\sqrt[3]{-1}\right)^2\sqrt{4-3(-3)^{2}}}\right.$$

input

```
Int[x^5/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6),x]
```

output

```
1259712*(((-1)^(2/3)*((-1)^(2/3) - 2^(1/3)*3^(2/3))*ArcTan[(3*(-2)^(2/3)*3
^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3))]])/(1259712*2^(1/6)*3^(5/6
)*Sqrt[4 + 3*(-2)^(1/3)*3^(2/3)]) - ((2*(-3)^(2/3) - 2^(2/3))*ArcTan[(2^(1
/6)*(3*(-3)^(1/3) - 2^(1/3)*x))/Sqrt[3*(4 - 3*(-3)^(2/3)*2^(1/3))]])/(4199
04*6^(5/6)*(1 + (-1)^(1/3))^2*Sqrt[4 - 3*(-3)^(2/3)*2^(1/3)]) - ((1 - 2^(1
/3)*3^(2/3))*ArcTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1
/3)*3^(2/3))]])/(1259712*2^(1/6)*3^(5/6)*Sqrt[-4 + 3*2^(1/3)*3^(2/3)]) + (
(36 + 2^(2/3)*3^(1/3) + I*2^(2/3)*3^(5/6))*Log[6 - 3*(-3)^(1/3)*2^(2/3)*x
 + x^2])/272097792 + ((18 - (-2)^(2/3)*3^(1/3))*Log[6 + 3*(-2)^(2/3)*3^(1/3
)*x + x^2])/136048896 + ((18 - 2^(2/3)*3^(1/3))*Log[6 + 3*2^(2/3)*3^(1/3)*
x + x^2])/136048896)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2466
```
Int[(u_.)*(Q6_)^(p_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
 x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Sim
p[1/(3^(3*p)*a^(2*p))   Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*
x + b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*
(-1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d,
 0] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coef
f[Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.05 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.14

method	result	size
default	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)}\frac{\_R^5\ln\left(x-\_R\right)}{\_R^5+12\_R^3+162\_R^2+36\_R}\right)}{6}$	56
risch	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)}\frac{\_R^5\ln\left(x-\_R\right)}{\_R^5+12\_R^3+162\_R^2+36\_R}\right)}{6}$	56

input
```
int(x^5/(x^6+18*x^4+324*x^3+108*x^2+216),x,method=_RETURNVERBOSE)
```

output
```
1/6*sum(_R^5/(_R^5+12*_R^3+162*_R^2+36*_R)*ln(x-_R),_R=RootOf(_Z^6+18*_Z^4
+324*_Z^3+108*_Z^2+216))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^5}{216+108x^2+324x^3+18x^4+x^6}\,dx = \text{Timed out}$$

input
```
integrate(x^5/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.18

$$\int \frac{x^5}{216+108x^2+324x^3+18x^4+x^6}\,dx$$
$$= \text{RootSum}\Big(72662865048t^6 - 72662865048t^5 + 24163559388t^4 - 2646786132t^3 - 6626610t^2 - 4374t - 1$$

input
```
integrate(x**5/(x**6+18*x**4+324*x**3+108*x**2+216),x)
```

output
```
RootSum(72662865048*_t**6 - 72662865048*_t**5 + 24163559388*_t**4 - 264678
6132*_t**3 - 6626610*_t**2 - 4374*_t - 1, Lambda(_t, _t*log(-8923641713104
7376*_t**5/833243797 + 89301949532998128*_t**4/833243797 - 297405602818058
52*_t**3/833243797 + 192466080408420*_t**2/49014341 + 5867255361684*_t/833
243797 + x + 5365044886/2499731391)))
```

Maxima [F]

$$\int \frac{x^5}{216+108x^2+324x^3+18x^4+x^6}\,dx = \int \frac{x^5}{x^6+18\,x^4+324\,x^3+108\,x^2+216}\,dx$$

input
```
integrate(x^5/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="maxima")
```

output
```
integrate(x^5/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

Giac [F]

$$\int \frac{x^5}{216+108x^2+324x^3+18x^4+x^6}\,dx = \int \frac{x^5}{x^6+18\,x^4+324\,x^3+108\,x^2+216}\,dx$$

input
```
integrate(x^5/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="giac")
```

output
```
integrate(x^5/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

Mupad [B] (verification not implemented)

Time = 0.70 (sec) , antiderivative size = 427, normalized size of antiderivative = 1.08

$$\int \frac{x^5}{216+108x^2+324x^3+18x^4+x^6}\,dx$$
$$= \sum_{k=1}^{6} \ln\left(\frac{362797056\left(19236852\,x\,\mathrm{root}(z^6+4374\,z^5+6626610\,z^4+2646786132\,z^3-24163559388\,z^2+7\right.}{}\right.$$
$$\left. -z^5+\frac{421\,z^4}{1266}-\frac{100853\,z^3}{2768742}-\frac{505\,z^2}{5537484}-\frac{z}{16612452}-\frac{1}{72662865048},z,k\right)$$

input
```
int(x^5/(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216),x)
```

output
```
symsum(log((362797056*(19236852*x*root(z^6 + 4374*z^5 + 6626610*z^4 + 2646
786132*z^3 - 24163559388*z^2 + 72662865048*z - 72662865048, z, k) - 191318
76*x - 6482268*x*root(z^6 + 4374*z^5 + 6626610*z^4 + 2646786132*z^3 - 2416
3559388*z^2 + 72662865048*z - 72662865048, z, k)^2 + 742851*x*root(z^6 + 4
374*z^5 + 6626610*z^4 + 2646786132*z^3 - 24163559388*z^2 + 72662865048*z -
 72662865048, z, k)^3 - 4130*x*root(z^6 + 4374*z^5 + 6626610*z^4 + 2646786
132*z^3 - 24163559388*z^2 + 72662865048*z - 72662865048, z, k)^4 + x*root(
z^6 + 4374*z^5 + 6626610*z^4 + 2646786132*z^3 - 24163559388*z^2 + 72662865
048*z - 72662865048, z, k)^5 - 154944576*root(z^6 + 4374*z^5 + 6626610*z^4
 + 2646786132*z^3 - 24163559388*z^2 + 72662865048*z - 72662865048, z, k)^2
 + 17047422*root(z^6 + 4374*z^5 + 6626610*z^4 + 2646786132*z^3 - 241635593
88*z^2 + 72662865048*z - 72662865048, z, k)^3 + 27054*root(z^6 + 4374*z^5
+ 6626610*z^4 + 2646786132*z^3 - 24163559388*z^2 + 72662865048*z - 7266286
5048, z, k)^4 + 9*root(z^6 + 4374*z^5 + 6626610*z^4 + 2646786132*z^3 - 241
63559388*z^2 + 72662865048*z - 72662865048, z, k)^5 + 465542316*root(z^6 +
 4374*z^5 + 6626610*z^4 + 2646786132*z^3 - 24163559388*z^2 + 72662865048*z
 - 72662865048, z, k) - 465542316))/root(z^6 + 4374*z^5 + 6626610*z^4 + 26
46786132*z^3 - 24163559388*z^2 + 72662865048*z - 72662865048, z, k)^5)*roo
t(z^6 - z^5 + (421*z^4)/1266 - (100853*z^3)/2768742 - (505*z^2)/5537484 -
z/16612452 - 1/72662865048, z, k), k, 1, 6)
```

Reduce [F]

$$\int \frac{x^5}{216+108x^2+324x^3+18x^4+x^6}\,dx = \int \frac{x^5}{x^6+18x^4+324x^3+108x^2+216}dx$$

input
```
int(x^5/(x^6+18*x^4+324*x^3+108*x^2+216),x)
```

output
```
int(x**5/(x**6 + 18*x**4 + 324*x**3 + 108*x**2 + 216),x)
```

3.16 $\int \frac{x^4}{216+108x^2+324x^3+18x^4+x^6}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 383

$$\begin{aligned}
&\int \frac{x^4}{216+108x^2+324x^3+18x^4+x^6}\,dx \\
&= \frac{(-1)^{2/3}\left(3(-3)^{2/3}-2^{2/3}\right)\arctan\left(\frac{3\sqrt[3]{-3}2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{9\sqrt[6]{3}\left(1+\sqrt[3]{-1}\right)^2\sqrt{2\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}} \\
&\quad - \frac{\left((-2)^{2/3}-3\ 3^{2/3}\right)\arctan\left(\frac{3(-2)^{2/3}\sqrt[3]{3}+2x}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{27\sqrt[6]{3}\sqrt{8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}3^{2/3}}} \\
&\quad + \frac{\left(2^{2/3}-3\ 3^{2/3}\right)\operatorname{arctanh}\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{3}+\sqrt[3]{2}x\right)}{\sqrt{3\left(-4+3\sqrt[3]{2}3^{2/3}\right)}}\right)}{27\sqrt[6]{3}\sqrt{2\left(-4+3\sqrt[3]{2}3^{2/3}\right)}} + \frac{\log\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)}{6\ 2^{2/3}\sqrt[3]{3}\left(1+\sqrt[3]{-1}\right)^2} \\
&\quad + \frac{\sqrt[3]{-\frac{1}{3}}\log\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)}{18\ 2^{2/3}} - \frac{\log\left(6+3\ 2^{2/3}\sqrt[3]{3}x+x^2\right)}{18\ 2^{2/3}\sqrt[3]{3}}
\end{aligned}$$

output
```
1/27*(-1)^(2/3)*(3*(-3)^(2/3)-2^(2/3))*arctan((3*(-3)^(1/3)*2^(2/3)-2*x)/(
24-18*(-3)^(2/3)*2^(1/3))^(1/2))*3^(5/6)/(1+(-1)^(1/3))^2/(8-6*(-3)^(2/3)*
2^(1/3))^(1/2)-1/81*((-2)^(2/3)-3*3^(2/3))*arctan((3*(-2)^(2/3)*3^(1/3)+2*
x)/(24+18*(-2)^(1/3)*3^(2/3))^(1/2))*3^(5/6)/(8+9*I*2^(1/3)*3^(1/6)+3*2^(1
/3)*3^(2/3))^(1/2)+1/81*(2^(2/3)-3*3^(2/3))*arctanh(2^(1/6)*(3*3^(1/3)+2^(
1/3)*x)/(-12+9*2^(1/3)*3^(2/3))^(1/2))*3^(5/6)/(-8+6*2^(1/3)*3^(2/3))^(1/2
)+1/36*ln(6-3*(-3)^(1/3)*2^(2/3)*x+x^2)*2^(1/3)*3^(2/3)/(1+(-1)^(1/3))^2+1
/108*(-1)^(1/3)*3^(2/3)*ln(6+3*(-2)^(2/3)*3^(1/3)*x+x^2)*2^(1/3)-1/108*ln(
6+3*2^(2/3)*3^(1/3)*x+x^2)*2^(1/3)*3^(2/3)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.16

$$\int \frac{x^4}{216+108x^2+324x^3+18x^4+x^6}\,dx = \frac{1}{6}\text{RootSum}\left[216+108\#1^2+324\#1^3+18\#1^4+\#1^6\&,\frac{\log(x-\#1)\#1^3}{36+162\#1+12\#1^2+\#1^4}\&\right]$$

input
```
Integrate[x^4/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6),x]
```

output
```
RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^4 + #1^6 & , (Log[x - #1]*#1^3)/
(36 + 162*#1 + 12*#1^2 + #1^4) & ]/6
```

Rubi [A] (verified)

Time = 1.78 (sec) , antiderivative size = 379, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2466, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{x^6+18x^4+324x^3+108x^2+216}\,dx$$

$$\Big\downarrow \text{2466}$$

$$1259712\int \left(\frac{(-1)^{2/3}\left(2-\sqrt[3]{-3}2^{2/3}x\right)}{7558272\sqrt[3]{2}3^{2/3}\left(1+\sqrt[3]{-1}\right)^2\left(x^2-3\sqrt[3]{-3}2^{2/3}x+6\right)}-\frac{(-1)^{2/3}\left((-2)^{2/3}\sqrt[3]{3}x+2\right)}{22674816\sqrt[3]{2}3^{2/3}\left(x^2+3(-2)^{2/3}\sqrt[3]{3}x+6\right)}\right.$$

$$\Big\downarrow \text{2009}$$

$$1259712\left(-\frac{\left(2(-1)^{2/3}\sqrt[6]{2}3^{5/6}-9\sqrt{6}\right)\arctan\left(\frac{2x+3(-2)^{2/3}\sqrt[3]{3}}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{204073344\sqrt{4+3\sqrt[3]{-2}3^{2/3}}}+\frac{(-1)^{2/3}\left(3(-3)^{2/3}-2^{2/3}\right)\arctan\left(\frac{\sqrt[6]{2}(3}{\sqrt{3(4}}\right.}{11337408\sqrt[6]{3}\left(1+\sqrt[3]{-1}\right)^2\sqrt{2\left(4-3(\right.}}\right.$$

input
```
Int[x^4/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6),x]
```

output
```
1259712*(-1/204073344*((2*(-1)^(2/3)*2^(1/6)*3^(5/6) - 9*Sqrt[6])*ArcTan[(
3*(-2)^(2/3)*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3))]])/Sqrt[4 +
3*(-2)^(1/3)*3^(2/3)] + ((-1)^(2/3)*(3*(-3)^(2/3) - 2^(2/3))*ArcTan[(2^(1/
6)*(3*(-3)^(1/3) - 2^(1/3)*x))/Sqrt[3*(4 - 3*(-3)^(2/3)*2^(1/3))]])/(11337
408*3^(1/6)*(1 + (-1)^(1/3))^2*Sqrt[2*(4 - 3*(-3)^(2/3)*2^(1/3))]) - ((9 -
 2^(2/3)*3^(1/3))*ArcTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3
*2^(1/3)*3^(2/3))]])/(34012224*Sqrt[6*(-4 + 3*2^(1/3)*3^(2/3))]) + Log[6 -
 3*(-3)^(1/3)*2^(2/3)*x + x^2]/(7558272*2^(2/3)*3^(1/3)*(1 + (-1)^(1/3))^2
) + ((-1/3)^(1/3)*Log[6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2])/(22674816*2^(2/3)
) - Log[6 + 3*2^(2/3)*3^(1/3)*x + x^2]/(22674816*2^(2/3)*3^(1/3)))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2466
```
Int[(u_.)*(Q6_)^(p_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
 x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Sim
p[1/(3^(3*p)*a^(2*p))   Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*
x + b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*
(-1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d,
 0] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coef
f[Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.04 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.15

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6}+18\_Z^{4}+324\_Z^{3}+108\_Z^{2}+216\right)}\frac{\_R^{4}\ln\left(x-\_R\right)}{\_R^{5}+12\_R^{3}+162\_R^{2}+36\_R}\right)}{6}$	56
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6}+18\_Z^{4}+324\_Z^{3}+108\_Z^{2}+216\right)}\frac{\_R^{4}\ln\left(x-\_R\right)}{\_R^{5}+12\_R^{3}+162\_R^{2}+36\_R}\right)}{6}$	56

input
```
int(x^4/(x^6+18*x^4+324*x^3+108*x^2+216),x,method=_RETURNVERBOSE)
```

output
```
1/6*sum(_R^4/(_R^5+12*_R^3+162*_R^2+36*_R)*ln(x-_R),_R=RootOf(_Z^6+18*_Z^4
+324*_Z^3+108*_Z^2+216))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^4}{216 + 108x^2 + 324x^3 + 18x^4 + x^6} \, dx = \text{Timed out}$$

input
```
integrate(x^4/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.17

$$\int \frac{x^4}{216 + 108x^2 + 324x^3 + 18x^4 + x^6} \, dx$$
$$= \text{RootSum}\left(15695178850368t^6 - 2066242608t^4 + 1845163152t^3 - 1180980t^2 - 1944t - 1, \left(t \mapsto t \log\left(\right.\right.\right.$$

input
```
integrate(x**4/(x**6+18*x**4+324*x**3+108*x**2+216),x)
```

output
```
RootSum(15695178850368*_t**6 - 2066242608*_t**4 + 1845163152*_t**3 - 11809
80*_t**2 - 1944*_t - 1, Lambda(_t, _t*log(614714526178551746208*_t**5/5712
1295165 - 1270857362386176*_t**4/57121295165 - 80483053187684376*_t**3/571
21295165 + 72431318325103884*_t**2/57121295165 - 45358602689088*_t/5712129
5165 + x - 44532180783/57121295165)))
```

Maxima [F]

$$\int \frac{x^4}{216+108x^2+324x^3+18x^4+x^6}\,dx = \int \frac{x^4}{x^6+18\,x^4+324\,x^3+108\,x^2+216}\,dx$$

input
```
integrate(x^4/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="maxima")
```

output
```
integrate(x^4/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

Giac [F]

$$\int \frac{x^4}{216+108x^2+324x^3+18x^4+x^6}\,dx = \int \frac{x^4}{x^6+18\,x^4+324\,x^3+108\,x^2+216}\,dx$$

input
```
integrate(x^4/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="giac")
```

output
```
integrate(x^4/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

Mupad [B] (verification not implemented)

Time = 22.52 (sec) , antiderivative size = 390, normalized size of antiderivative = 1.02

$$\int \frac{x^4}{216+108x^2+324x^3+18x^4+x^6}\,dx$$
$$= \sum_{k=1}^{6} \ln\left(-\frac{5038848\left(1377495072\,x+17006112\,x\,\mathrm{root}(z^6+1944\,z^5+1180980\,z^4-1845163152\,z^3+206\ldots\right.}{\ldots}\right.$$
$$\left. -\frac{z^4}{7596}+\frac{217\,z^3}{1845828}-\frac{5\,z^2}{66449808}-\frac{z}{8073651672}-\frac{1}{15695178850368},z,k\right)$$

input
```
int(x^4/(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216),x)
```

output
```
symsum(log(-(5038848*(1377495072*x + 17006112*x*root(z^6 + 1944*z^5 + 1180
980*z^4 - 1845163152*z^3 + 2066242608*z^2 - 15695178850368, z, k) - 104976
*x*root(z^6 + 1944*z^5 + 1180980*z^4 - 1845163152*z^3 + 2066242608*z^2 - 1
5695178850368, z, k)^2 + 158112*x*root(z^6 + 1944*z^5 + 1180980*z^4 - 1845
163152*z^3 + 2066242608*z^2 - 15695178850368, z, k)^3 + 1946*x*root(z^6 +
1944*z^5 + 1180980*z^4 - 1845163152*z^3 + 2066242608*z^2 - 15695178850368,
 z, k)^4 + 3*x*root(z^6 + 1944*z^5 + 1180980*z^4 - 1845163152*z^3 + 206624
2608*z^2 - 15695178850368, z, k)^5 - 4251528*root(z^6 + 1944*z^5 + 1180980
*z^4 - 1845163152*z^3 + 2066242608*z^2 - 15695178850368, z, k)^2 + 3927852
*root(z^6 + 1944*z^5 + 1180980*z^4 - 1845163152*z^3 + 2066242608*z^2 - 156
95178850368, z, k)^3 - 1188*root(z^6 + 1944*z^5 + 1180980*z^4 - 1845163152
*z^3 + 2066242608*z^2 - 15695178850368, z, k)^4 - root(z^6 + 1944*z^5 + 11
80980*z^4 - 1845163152*z^3 + 2066242608*z^2 - 15695178850368, z, k)^5 + 75
58272*root(z^6 + 1944*z^5 + 1180980*z^4 - 1845163152*z^3 + 2066242608*z^2
- 15695178850368, z, k) + 33519046752))/root(z^6 + 1944*z^5 + 1180980*z^4
- 1845163152*z^3 + 2066242608*z^2 - 15695178850368, z, k)^5)*root(z^6 - z^
4/7596 + (217*z^3)/1845828 - (5*z^2)/66449808 - z/8073651672 - 1/156951788
50368, z, k), k, 1, 6)
```

Reduce [F]

$$\int \frac{x^4}{216+108x^2+324x^3+18x^4+x^6}\,dx=\int \frac{x^4}{x^6+18x^4+324x^3+108x^2+216}dx$$

input
```
int(x^4/(x^6+18*x^4+324*x^3+108*x^2+216),x)
```

output
```
int(x**4/(x**6 + 18*x**4 + 324*x**3 + 108*x**2 + 216),x)
```

3.17 $\int \frac{x^3}{216+108x^2+324x^3+18x^4+x^6}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 361

$$\int \frac{x^3}{216+108x^2+324x^3+18x^4+x^6}\,dx = -\frac{\arctan\left(\frac{3\sqrt[3]{-3}2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{6\sqrt[6]{2}3^{5/6}\left(1+\sqrt[3]{-1}\right)^2\sqrt{4-3(-3)^{2/3}\sqrt[3]{2}}} + \frac{\sqrt[3]{-1}\arctan\left(\frac{3(-2)^{2/3}\sqrt[3]{3}+2x}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{9\ 2^{2/3}3^{5/6}\sqrt{8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}3^{2/3}}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{3}+\sqrt[3]{2}x\right)}{\sqrt{3\left(-4+3\sqrt[3]{2}3^{2/3}\right)}}\right)}{18\sqrt[6]{2}3^{5/6}\sqrt{-4+3\sqrt[3]{2}3^{2/3}}} - \frac{(-1)^{2/3}\log\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)}{36\sqrt[3]{2}3^{2/3}\left(1+\sqrt[3]{-1}\right)^2} + \frac{(-1)^{2/3}\log\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)}{108\sqrt[3]{2}3^{2/3}} + \frac{\log\left(6+3\ 2^{2/3}\sqrt[3]{3}x+x^2\right)}{108\sqrt[3]{2}3^{2/3}}$$

output
```
-1/36*arctan((3*(-3)^(1/3)*2^(2/3)-2*x)/(24-18*(-3)^(2/3)*2^(1/3))^(1/2))*
2^(5/6)*3^(1/6)/(1+(-1)^(1/3))^2/(4-3*(-3)^(2/3)*2^(1/3))^(1/2)+1/54*(-1)^
(1/3)*arctan((3*(-2)^(2/3)*3^(1/3)+2*x)/(24+18*(-2)^(1/3)*3^(2/3))^(1/2))*
2^(1/3)*3^(1/6)/(8+9*I*2^(1/3)*3^(1/6)+3*2^(1/3)*3^(2/3))^(1/2)+1/108*arct
anh(2^(1/6)*(3*3^(1/3)+2^(1/3)*x)/(-12+9*2^(1/3)*3^(2/3))^(1/2))*2^(5/6)*3
^(1/6)/(-4+3*2^(1/3)*3^(2/3))^(1/2)-1/216*(-1)^(2/3)*ln(6-3*(-3)^(1/3)*2^(
2/3)*x+x^2)*2^(2/3)*3^(1/3)/(1+(-1)^(1/3))^2+1/648*(-1)^(2/3)*ln(6+3*(-2)^
(2/3)*3^(1/3)*x+x^2)*2^(2/3)*3^(1/3)+1/648*ln(6+3*2^(2/3)*3^(1/3)*x+x^2)*2
^(2/3)*3^(1/3)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.17

$$\int \frac{x^3}{216+108x^2+324x^3+18x^4+x^6}\,dx = \frac{1}{6}\text{RootSum}\left[216+108\#1^2+324\#1^3+18\#1^4+\#1^6\&,\frac{\log(x-\#1)\#1^2}{36+162\#1+12\#1^2+\#1^4}\&\right]$$

input
```
Integrate[x^3/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6),x]
```

output
```
RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^4 + #1^6 & , (Log[x - #1]*#1^2)/
(36 + 162*#1 + 12*#1^2 + #1^4) & ]/6
```

Rubi [A] (verified)

Time = 1.43 (sec) , antiderivative size = 354, normalized size of antiderivative = 0.98, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2466, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{x^6+18x^4+324x^3+108x^2+216}\,dx$$

$\Big\downarrow$ 2466

$$1259712\int \left(-\frac{(-1)^{2/3}x}{22674816\sqrt[3]{2}3^{2/3}\left(1+\sqrt[3]{-1}\right)^2\left(x^2-3\sqrt[3]{-3}2^{2/3}x+6\right)}+\frac{(-1)^{2/3}x}{68024448\sqrt[3]{2}3^{2/3}\left(x^2+3(-2)^{2/3}\sqrt[3]{3}x+6\right.}\right.$$

$\Big\downarrow$ 2009

$$1259712\left(\frac{\sqrt[3]{-1}\arctan\left(\frac{2x+3(-2)^{2/3}\sqrt[3]{3}}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{22674816\sqrt[6]{2}3^{5/6}\sqrt{4+3\sqrt[3]{-2}3^{2/3}}}-\frac{\arctan\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{-3}-\sqrt[3]{2}x\right)}{\sqrt{3\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{7558272\sqrt[6]{2}3^{5/6}\left(1+\sqrt[3]{-1}\right)^2\sqrt{4-3(-3)^{2/3}\sqrt[3]{2}}}+\frac{\text{arctanh}\left(\ldots\right)}{22674816\sqrt[6]{2}3\ldots}\right.$$

input
```
Int[x^3/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6),x]
```

output
```
1259712*(((-1)^(1/3)*ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2
)^(1/3)*3^(2/3))]])/(22674816*2^(1/6)*3^(5/6)*Sqrt[4 + 3*(-2)^(1/3)*3^(2/3
)]) - ArcTan[(2^(1/6)*(3*(-3)^(1/3) - 2^(1/3)*x))/Sqrt[3*(4 - 3*(-3)^(2/3)
*2^(1/3))]]/(7558272*2^(1/6)*3^(5/6)*(1 + (-1)^(1/3))^2*Sqrt[4 - 3*(-3)^(2
/3)*2^(1/3)]) + ArcTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2
^(1/3)*3^(2/3))]]/(22674816*2^(1/6)*3^(5/6)*Sqrt[-4 + 3*2^(1/3)*3^(2/3)])
 - ((-1)^(2/3)*Log[6 - 3*(-3)^(1/3)*2^(2/3)*x + x^2])/(45349632*2^(1/3)*3^(
2/3)*(1 + (-1)^(1/3))^2) + ((-1)^(2/3)*Log[6 + 3*(-2)^(2/3)*3^(1/3)*x + x^
2])/(136048896*2^(1/3)*3^(2/3)) + Log[6 + 3*2^(2/3)*3^(1/3)*x + x^2]/(1360
48896*2^(1/3)*3^(2/3)))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2466
```
Int[(u_.)*(Q6_)^(p_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
 x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Sim
p[1/(3^(3*p)*a^(2*p))   Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*
x + b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*
(-1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d,
 0] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coef
f[Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.04 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.16

method	result	size
default	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\_Z^{6}+18\_Z^{4}+324\_Z^{3}+108\_Z^{2}+216\right)}\frac{\_R^{3}\ln\left(x-\_R\right)}{\_R^{5}+12\_R^{3}+162\_R^{2}+36\_R}\right)}{6}$	56
risch	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\_Z^{6}+18\_Z^{4}+324\_Z^{3}+108\_Z^{2}+216\right)}\frac{\_R^{3}\ln\left(x-\_R\right)}{\_R^{5}+12\_R^{3}+162\_R^{2}+36\_R}\right)}{6}$	56

input
```
int(x^3/(x^6+18*x^4+324*x^3+108*x^2+216),x,method=_RETURNVERBOSE)
```

output
```
1/6*sum(_R^3/(_R^5+12*_R^3+162*_R^2+36*_R)*ln(x-_R),_R=RootOf(_Z^6+18*_Z^4
+324*_Z^3+108*_Z^2+216))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^3}{216 + 108x^2 + 324x^3 + 18x^4 + x^6}\,dx = \text{Timed out}$$

input
```
integrate(x^3/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.17

$$\int \frac{x^3}{216 + 108x^2 + 324x^3 + 18x^4 + x^6}\,dx$$
$$= \text{RootSum}\left(3390158631679488t^6 - 74384733888t^4 - 1332145440t^3 - 1417176t^2 - 1, \left(t \mapsto t\log\left(-\frac{848\cdots}{\cdots}\right.\right.\right.$$

input
```
integrate(x**3/(x**6+18*x**4+324*x**3+108*x**2+216),x)
```

output
```
RootSum(3390158631679488*_t**6 - 74384733888*_t**4 - 1332145440*_t**3 - 14
17176*_t**2 - 1, Lambda(_t, _t*log(-8482372214243328*_t**5/415817 + 221605
5910930560*_t**4/415817 - 2062546612992*_t**3/415817 - 57027208896*_t**2/4
15817 - 416583756*_t/415817 + x - 89938/415817)))
```

Maxima [F]

$$\int \frac{x^3}{216 + 108x^2 + 324x^3 + 18x^4 + x^6}\,dx = \int \frac{x^3}{x^6 + 18\,x^4 + 324\,x^3 + 108\,x^2 + 216}\,dx$$

input
```
integrate(x^3/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="maxima")
```

output
```
integrate(x^3/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

Giac [F]

$$\int \frac{x^3}{216+108x^2+324x^3+18x^4+x^6}\,dx = \int \frac{x^3}{x^6+18\,x^4+324\,x^3+108\,x^2+216}\,dx$$

input
```
integrate(x^3/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="giac")
```

output
```
integrate(x^3/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

Mupad [B] (verification not implemented)

Time = 22.47 (sec) , antiderivative size = 276, normalized size of antiderivative = 0.76

$$\int \frac{x^3}{216+108x^2+324x^3+18x^4+x^6}\,dx$$
$$= \sum_{k=1}^{6} \ln\left(-\frac{23328\left(297538935552\,x - 7992872640\,x\,\mathrm{root}(z^6+1417176\,z^4+1332145440\,z^3+7438473388\right.}{}\right.$$
$$\left. - \frac{z^4}{45576} - \frac{235\,z^3}{598048272} - \frac{z^2}{2392193088} - \frac{1}{3390158631679488}, z, k\right)$$

input
```
int(x^3/(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216),x)
```

output
```
symsum(log(-(23328*(297538935552*x - 7992872640*x*root(z^6 + 1417176*z^4 +
 1332145440*z^3 + 74384733888*z^2 - 3390158631679488, z, k) + 52488*x*root
(z^6 + 1417176*z^4 + 1332145440*z^3 + 74384733888*z^2 - 3390158631679488,
z, k)^3 + 2904*x*root(z^6 + 1417176*z^4 + 1332145440*z^3 + 74384733888*z^2
 - 3390158631679488, z, k)^4 + x*root(z^6 + 1417176*z^4 + 1332145440*z^3 +
 74384733888*z^2 - 3390158631679488, z, k)^5 - 153055008*root(z^6 + 141717
6*z^4 + 1332145440*z^3 + 74384733888*z^2 - 3390158631679488, z, k)^2 - 276
4368*root(z^6 + 1417176*z^4 + 1332145440*z^3 + 74384733888*z^2 - 339015863
1679488, z, k)^3 - 1620*root(z^6 + 1417176*z^4 + 1332145440*z^3 + 74384733
888*z^2 - 3390158631679488, z, k)^4 - 3673320192*root(z^6 + 1417176*z^4 +
1332145440*z^3 + 74384733888*z^2 - 3390158631679488, z, k) + 7240114098432
))/root(z^6 + 1417176*z^4 + 1332145440*z^3 + 74384733888*z^2 - 33901586316
79488, z, k)^5)*root(z^6 - z^4/45576 - (235*z^3)/598048272 - z^2/239219308
8 - 1/3390158631679488, z, k), k, 1, 6)
```

Reduce [F]

$$\int \frac{x^3}{216+108x^2+324x^3+18x^4+x^6}\,dx=\int \frac{x^3}{x^6+18x^4+324x^3+108x^2+216}dx$$

input
```
int(x^3/(x^6+18*x^4+324*x^3+108*x^2+216),x)
```

output
```
int(x**3/(x**6 + 18*x**4 + 324*x**3 + 108*x**2 + 216),x)
```

3.18 $\int \frac{x^2}{216+108x^2+324x^3+18x^4+x^6}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 248

$$\int \frac{x^2}{216+108x^2+324x^3+18x^4+x^6}\,dx = \frac{(-1)^{2/3}\arctan\left(\frac{3\sqrt[3]{-3}2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{27\ 2^{5/6}\sqrt[6]{3}\left(1+\sqrt[3]{-1}\right)^2\sqrt{4-3(-3)^{2/3}\sqrt[3]{2}}} + \frac{(-1)^{2/3}\arctan\left(\frac{3(-2)^{2/3}\sqrt[3]{3}+2x}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{81\sqrt[3]{2}\sqrt[6]{3}\sqrt{8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}3^{2/3}}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{3}+\sqrt[3]{2}x\right)}{\sqrt{3\left(-4+3\sqrt[3]{2}3^{2/3}\right)}}\right)}{81\ 2^{5/6}\sqrt[6]{3}\sqrt{-4+3\sqrt[3]{2}3^{2/3}}}$$

output
```
1/162*(-1)^(2/3)*arctan((3*(-3)^(1/3)*2^(2/3)-2*x)/(24-18*(-3)^(2/3)*2^(1/
3))^(1/2))*2^(1/6)*3^(5/6)/(1+(-1)^(1/3))^2/(4-3*(-3)^(2/3)*2^(1/3))^(1/2)
+1/486*(-1)^(2/3)*arctan((3*(-2)^(2/3)*3^(1/3)+2*x)/(24+18*(-2)^(1/3)*3^(2
/3))^(1/2))*2^(2/3)*3^(5/6)/(8+9*I*2^(1/3)*3^(1/6)+3*2^(1/3)*3^(2/3))^(1/2
)-1/486*arctanh(2^(1/6)*(3*3^(1/3)+2^(1/3)*x)/(-12+9*2^(1/3)*3^(2/3))^(1/2
))*2^(1/6)*3^(5/6)/(-4+3*2^(1/3)*3^(2/3))^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.24

$$\int \frac{x^2}{216+108x^2+324x^3+18x^4+x^6}\,dx = \frac{1}{6}\mathrm{RootSum}\left[216+108\#1^2+324\#1^3+18\#1^4+\#1^6\&,\frac{\log(x-\#1)\#1}{36+162\#1+12\#1^2+\#1^4}\&\right]$$

input
```
Integrate[x^2/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6),x]
```

output
```
RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^4 + #1^6 & , (Log[x - #1]*#1)/(3
6 + 162*#1 + 12*#1^2 + #1^4) & ]/6
```

Rubi [A] (verified)

Time = 1.07 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.97, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2466, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{x^6+18x^4+324x^3+108x^2+216}dx$$

$$\Big\downarrow \text{2466}$$

$$1259712\int \left(\frac{(-1)^{2/3}}{68024448\sqrt[3]{23}^{2/3}\left(x^2+3(-2)^{2/3}\sqrt[3]{3}x+6\right)}+\frac{1}{68024448\sqrt[3]{23}^{2/3}\left(x^2+3\ 2^{2/3}\sqrt[3]{3}x+6\right)}-\frac{}{22674816}\right.$$

$$\Big\downarrow \text{2009}$$

$$1259712\left(\frac{(-1)^{2/3}\arctan\left(\frac{2x+3(-2)^{2/3}\sqrt[3]{3}}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{102036672\ 2^{5/6}\sqrt[6]{3}\sqrt{4+3\sqrt[3]{-2}3^{2/3}}}+\frac{(-1)^{2/3}\arctan\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{-3}-\sqrt[3]{2}x\right)}{\sqrt{3\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{34012224\ 2^{5/6}\sqrt[6]{3}\left(1+\sqrt[3]{-1}\right)^2\sqrt{4-3(-3)^{2/3}\sqrt[3]{2}}}-\frac{\text{arctan}}{10203667}\right.$$

input
```
Int[x^2/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6),x]
```

output
```
1259712*(((-1)^(2/3)*ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2
)^(1/3)*3^(2/3))]])/(102036672*2^(5/6)*3^(1/6)*Sqrt[4 + 3*(-2)^(1/3)*3^(2/
3)]) + ((-1)^(2/3)*ArcTan[(2^(1/6)*(3*(-3)^(1/3) - 2^(1/3)*x))/Sqrt[3*(4 -
 3*(-3)^(2/3)*2^(1/3))]])/(34012224*2^(5/6)*3^(1/6)*(1 + (-1)^(1/3))^2*Sqr
t[4 - 3*(-3)^(2/3)*2^(1/3)]) - ArcTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/S
qrt[3*(-4 + 3*2^(1/3)*3^(2/3))]]/(102036672*2^(5/6)*3^(1/6)*Sqrt[-4 + 3*2^
(1/3)*3^(2/3)]))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2466
```
Int[(u_.)*(Q6_)^(p_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
 x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Sim
p[1/(3^(3*p)*a^(2*p))   Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*
x + b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*
(-1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d,
 0] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coef
f[Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.04 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.23

method	result	size
default	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\_Z^{6}+18\_Z^{4}+324\_Z^{3}+108\_Z^{2}+216\right)}\frac{\_R^{2}\ln\left(x-\_R\right)}{\_R^{5}+12\_R^{3}+162\_R^{2}+36\_R}\right)}{6}$	56
risch	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\_Z^{6}+18\_Z^{4}+324\_Z^{3}+108\_Z^{2}+216\right)}\frac{\_R^{2}\ln\left(x-\_R\right)}{\_R^{5}+12\_R^{3}+162\_R^{2}+36\_R}\right)}{6}$	56

input
```
int(x^2/(x^6+18*x^4+324*x^3+108*x^2+216),x,method=_RETURNVERBOSE)
```

output
```
1/6*sum(_R^2/(_R^5+12*_R^3+162*_R^2+36*_R)*ln(x-_R),_R=RootOf(_Z^6+18*_Z^4
+324*_Z^3+108*_Z^2+216))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1173 vs. 2(162) = 324.

Time = 0.83 (sec) , antiderivative size = 1173, normalized size of antiderivative = 4.73

$$\int \frac{x^2}{216+108x^2+324x^3+18x^4+x^6}\,dx = \text{Too large to display}$$

input
```
integrate(x^2/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="fricas")
```

output
```
1/324*sqrt(1/633)*sqrt(6*18^(2/3) + 8*18^(1/3) + 81)*log(1/211*sqrt(1/633)
*(3*(6*18^(2/3) + 8*18^(1/3) + 81)^2 - 3741*18^(2/3) - 4988*18^(1/3) - 248
67)*sqrt(6*18^(2/3) + 8*18^(1/3) + 81) - 1/422*(6*18^(2/3) + 8*18^(1/3) +
81)^2 + 2*x + 729/211*18^(2/3) + 972/211*18^(1/3) + 8289/422) - 1/324*sqrt
(1/633)*sqrt(6*18^(2/3) + 8*18^(1/3) + 81)*log(-1/211*sqrt(1/633)*(3*(6*18
^(2/3) + 8*18^(1/3) + 81)^2 - 3741*18^(2/3) - 4988*18^(1/3) - 24867)*sqrt(
6*18^(2/3) + 8*18^(1/3) + 81) - 1/422*(6*18^(2/3) + 8*18^(1/3) + 81)^2 + 2
*x + 729/211*18^(2/3) + 972/211*18^(1/3) + 8289/422) - 1/108*sqrt(-1/1899*
18^(2/3) + 1/1266*sqrt(-1/27*(6*18^(2/3) + 8*18^(1/3) + 81)^2 + 36*18^(2/3
) + 48*18^(1/3) + 371) - 4/5697*18^(1/3) + 3/211)*log(1/211*(6*18^(2/3) +
8*18^(1/3) + 81)^2 + 9/211*sqrt(-1/27*(6*18^(2/3) + 8*18^(1/3) + 81)^2 + 3
6*18^(2/3) + 48*18^(1/3) + 371)*(6*18^(2/3) + 8*18^(1/3) + 81) + 3/211*(6*
(6*18^(2/3) + 8*18^(1/3) + 81)^2 + 9*sqrt(-1/27*(6*18^(2/3) + 8*18^(1/3) +
 81)^2 + 36*18^(2/3) + 48*18^(1/3) + 371)*(36*18^(2/3) + 48*18^(1/3) + 275
) - 7482*18^(2/3) - 9976*18^(1/3) - 49734)*sqrt(-1/1899*18^(2/3) + 1/1266*
sqrt(-1/27*(6*18^(2/3) + 8*18^(1/3) + 81)^2 + 36*18^(2/3) + 48*18^(1/3) +
371) - 4/5697*18^(1/3) + 3/211) + 8*x - 1458/211*18^(2/3) - 1944/211*18^(1
/3) - 8289/211) + 1/108*sqrt(-1/1899*18^(2/3) + 1/1266*sqrt(-1/27*(6*18^(2
/3) + 8*18^(1/3) + 81)^2 + 36*18^(2/3) + 48*18^(1/3) + 371) - 4/5697*18^(1
/3) + 3/211)*log(1/211*(6*18^(2/3) + 8*18^(1/3) + 81)^2 + 9/211*sqrt(-1...
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.19

$$\int \frac{x^2}{216+108x^2+324x^3+18x^4+x^6}\,dx$$
$$=\mathrm{RootSum}\left(732274264442769408t^6-2677850419968t^4+2834352t^2-1,\left(t\mapsto t\log\left(101704758950384\right.\right.\right.$$

input
```
integrate(x**2/(x**6+18*x**4+324*x**3+108*x**2+216),x)
```

output
```
RootSum(732274264442769408*_t**6 - 2677850419968*_t**4 + 2834352*_t**2 - 1
, Lambda(_t, _t*log(10170475895038464*_t**5 - 5231726283456*_t**4 - 318099
32496*_t**3 + 19131876*_t**2 + 19683*_t + x - 27/2)))
```

Maxima [F]

$$\int \frac{x^2}{216+108x^2+324x^3+18x^4+x^6}\,dx = \int \frac{x^2}{x^6+18\,x^4+324\,x^3+108\,x^2+216}\,dx$$

input
```
integrate(x^2/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="maxima")
```

output
```
integrate(x^2/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

Giac [F]

$$\int \frac{x^2}{216+108x^2+324x^3+18x^4+x^6}\,dx = \int \frac{x^2}{x^6+18\,x^4+324\,x^3+108\,x^2+216}\,dx$$

input
```
integrate(x^2/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="giac")
```

output
```
integrate(x^2/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

Mupad [B] (verification not implemented)

Time = 22.66 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{216+108x^2+324x^3+18x^4+x^6}\,dx$$
$$= \sum_{k=1}^{6} \ln\left(-\frac{216\left(32134205039616\,x - 1836660096\,\mathrm{root}(z^6 - 2834352\,z^4 + 2677850419968\,z^2 - 73227426\right.}{} \right.$$
$$\left. - \frac{z^4}{273456} + \frac{z^2}{258356853504} - \frac{1}{732274264442769408}, z, k\right)$$

input
```
int(x^2/(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216),x)
```

output
```
symsum(log(-(216*(32134205039616*x - 1836660096*root(z^6 - 2834352*z^4 + 2
677850419968*z^2 - 732274264442769408, z, k)^2 - 1889568*root(z^6 - 283435
2*z^4 + 2677850419968*z^2 - 732274264442769408, z, k)^3 + 972*root(z^6 - 2
834352*z^4 + 2677850419968*z^2 - 732274264442769408, z, k)^4 + root(z^6 -
2834352*z^4 + 2677850419968*z^2 - 732274264442769408, z, k)^5 + 1322395269
12*x*root(z^6 - 2834352*z^4 + 2677850419968*z^2 - 732274264442769408, z, k
) + 204073344*x*root(z^6 - 2834352*z^4 + 2677850419968*z^2 - 7322742644427
69408, z, k)^2 + 139968*x*root(z^6 - 2834352*z^4 + 2677850419968*z^2 - 732
274264442769408, z, k)^3 + 36*x*root(z^6 - 2834352*z^4 + 2677850419968*z^2
 - 732274264442769408, z, k)^4 + 863230245120*root(z^6 - 2834352*z^4 + 267
7850419968*z^2 - 732274264442769408, z, k) + 781932322630656))/root(z^6 -
2834352*z^4 + 2677850419968*z^2 - 732274264442769408, z, k)^5)*root(z^6 -
z^4/273456 + z^2/258356853504 - 1/732274264442769408, z, k), k, 1, 6)
```

Reduce [F]

$$\int \frac{x^2}{216 + 108x^2 + 324x^3 + 18x^4 + x^6}\,dx = \int \frac{x^2}{x^6 + 18x^4 + 324x^3 + 108x^2 + 216}dx$$

input
```
int(x^2/(x^6+18*x^4+324*x^3+108*x^2+216),x)
```

output
```
int(x**2/(x**6 + 18*x**4 + 324*x**3 + 108*x**2 + 216),x)
```

3.19 $\int \frac{x}{216+108x^2+324x^3+18x^4+x^6}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 361

$$\int \frac{x}{216+108x^2+324x^3+18x^4+x^6}\,dx = -\frac{\arctan\left(\frac{3\sqrt[3]{-3}2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{36\sqrt[6]{2}3^{5/6}\left(1+\sqrt[3]{-1}\right)^2\sqrt{4-3(-3)^{2/3}\sqrt[3]{2}}} + \frac{\sqrt[3]{-1}\arctan\left(\frac{3(-2)^{2/3}\sqrt[3]{3}+2x}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{54\ 2^{2/3}3^{5/6}\sqrt{8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}3^{2/3}}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{3}+\sqrt[3]{2}x\right)}{\sqrt{3\left(-4+3\sqrt[3]{2}3^{2/3}\right)}}\right)}{108\sqrt[6]{2}3^{5/6}\sqrt{-4+3\sqrt[3]{2}3^{2/3}}} + \frac{(-1)^{2/3}\log\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)}{216\sqrt[3]{2}3^{2/3}\left(1+\sqrt[3]{-1}\right)^2} - \frac{(-1)^{2/3}\log\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)}{648\sqrt[3]{2}3^{2/3}} - \frac{\log\left(6+3\ 2^{2/3}\sqrt[3]{3}x+x^2\right)}{648\sqrt[3]{2}3^{2/3}}$$

output
```
-1/216*arctan((3*(-3)^(1/3)*2^(2/3)-2*x)/(24-18*(-3)^(2/3)*2^(1/3))^(1/2))
*2^(5/6)*3^(1/6)/(1+(-1)^(1/3))^2/(4-3*(-3)^(2/3)*2^(1/3))^(1/2)+1/324*(-1
)^(1/3)*arctan((3*(-2)^(2/3)*3^(1/3)+2*x)/(24+18*(-2)^(1/3)*3^(2/3))^(1/2)
)*2^(1/3)*3^(1/6)/(8+9*I*2^(1/3)*3^(1/6)+3*2^(1/3)*3^(2/3))^(1/2)+1/648*ar
ctanh(2^(1/6)*(3*3^(1/3)+2^(1/3)*x)/(-12+9*2^(1/3)*3^(2/3))^(1/2))*2^(5/6)
*3^(1/6)/(-4+3*2^(1/3)*3^(2/3))^(1/2)+1/1296*(-1)^(2/3)*ln(6-3*(-3)^(1/3)*
2^(2/3)*x+x^2)*2^(2/3)*3^(1/3)/(1+(-1)^(1/3))^2-1/3888*(-1)^(2/3)*ln(6+3*(
-2)^(2/3)*3^(1/3)*x+x^2)*2^(2/3)*3^(1/3)-1/3888*ln(6+3*2^(2/3)*3^(1/3)*x+x
^2)*2^(2/3)*3^(1/3)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.16

$$\int \frac{x}{216+108x^2+324x^3+18x^4+x^6}\,dx = \frac{1}{6}\text{RootSum}\left[216+108\#1^2+324\#1^3+18\#1^4+\#1^6\&,\frac{\log(x-\#1)}{36+162\#1+12\#1^2+\#1^4}\&\right]$$

input
```
Integrate[x/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6),x]
```

output
```
RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^4 + #1^6 & , Log[x - #1]/(36 + 1
62*#1 + 12*#1^2 + #1^4) & ]/6
```

Rubi [A] (verified)

Time = 1.50 (sec) , antiderivative size = 354, normalized size of antiderivative = 0.98, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {2466, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{x^6+18x^4+324x^3+108x^2+216}\,dx$$

$\Big\downarrow$ 2466

$$1259712\int \left(-\frac{(-1)^{2/3}\left(3\sqrt[3]{-3}2^{2/3}-x\right)}{136048896\sqrt[3]{2}3^{2/3}\left(1+\sqrt[3]{-1}\right)^2\left(x^2-3\sqrt[3]{-3}2^{2/3}x+6\right)}-\frac{(-1)^{2/3}\left(x+3(-2)^{2/3}\sqrt[3]{3}\right)}{408146688\sqrt[3]{2}3^{2/3}\left(x^2+3(-2)^{2/3}\sqrt[3]{3}x\right.}\right.$$

$\Big\downarrow$ 2009

$$1259712\left(\frac{\sqrt[3]{-1}\arctan\left(\frac{2x+3(-2)^{2/3}\sqrt[3]{3}}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{136048896\sqrt[6]{2}3^{5/6}\sqrt{4+3\sqrt[3]{-2}3^{2/3}}}-\frac{\arctan\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{-3}-\sqrt[3]{2}x\right)}{\sqrt{3\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{45349632\sqrt[6]{2}3^{5/6}\left(1+\sqrt[3]{-1}\right)^2\sqrt{4-3(-3)^{2/3}\sqrt[3]{2}}}+\frac{\text{arctanh}\left(\right.}{136048896\sqrt[6]{}}\right.$$

input
```
Int[x/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6),x]
```

output
```
1259712*(((-1)^(1/3)*ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2
)^(1/3)*3^(2/3))]])/(136048896*2^(1/6)*3^(5/6)*Sqrt[4 + 3*(-2)^(1/3)*3^(2/
3)]) - ArcTan[(2^(1/6)*(3*(-3)^(1/3) - 2^(1/3)*x))/Sqrt[3*(4 - 3*(-3)^(2/3
)*2^(1/3))]]/(45349632*2^(1/6)*3^(5/6)*(1 + (-1)^(1/3))^2*Sqrt[4 - 3*(-3)^
(2/3)*2^(1/3)]) + ArcTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3
*2^(1/3)*3^(2/3))]]/(136048896*2^(1/6)*3^(5/6)*Sqrt[-4 + 3*2^(1/3)*3^(2/3)
]) + ((-1)^(2/3)*Log[6 - 3*(-3)^(1/3)*2^(2/3)*x + x^2])/(272097792*2^(1/3)
*3^(2/3)*(1 + (-1)^(1/3))^2) - ((-1)^(2/3)*Log[6 + 3*(-2)^(2/3)*3^(1/3)*x
+ x^2])/(816293376*2^(1/3)*3^(2/3)) - Log[6 + 3*2^(2/3)*3^(1/3)*x + x^2]/(
816293376*2^(1/3)*3^(2/3)))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2466
```
Int[(u_.)*(Q6_)^(p_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
 x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Sim
p[1/(3^(3*p)*a^(2*p))   Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*
x + b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*
(-1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d,
 0] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coef
f[Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.04 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.15

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6}+18\_Z^{4}+324\_Z^{3}+108\_Z^{2}+216\right)}\frac{\_R\ln\left(x-\_R\right)}{\_R^{5}+12\_R^{3}+162\_R^{2}+36\_R}\right)}{6}$	54
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6}+18\_Z^{4}+324\_Z^{3}+108\_Z^{2}+216\right)}\frac{\_R\ln\left(x-\_R\right)}{\_R^{5}+12\_R^{3}+162\_R^{2}+36\_R}\right)}{6}$	54

input
```
int(x/(x^6+18*x^4+324*x^3+108*x^2+216),x,method=_RETURNVERBOSE)
```

output
```
1/6*sum(_R/(_R^5+12*_R^3+162*_R^2+36*_R)*ln(x-_R),_R=RootOf(_Z^6+18*_Z^4+3
24*_Z^3+108*_Z^2+216))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x}{216 + 108x^2 + 324x^3 + 18x^4 + x^6} \, dx = \text{Timed out}$$

input
```
integrate(x/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.17

$$\int \frac{x}{216 + 108x^2 + 324x^3 + 18x^4 + x^6} \, dx = \text{RootSum}\left(158171241119638192128t^6 - 96402615118848t^4 + 287743415040t^3 - 51018336t^2 - 1, \left(t \mapsto\right.\right.$$

input
```
integrate(x/(x**6+18*x**4+324*x**3+108*x**2+216),x)
```

output
```
RootSum(158171241119638192128*_t**6 - 96402615118848*_t**4 + 287743415040*
_t**3 - 51018336*_t**2 - 1, Lambda(_t, _t*log(654183994457211409612 80*_t**
5/415817 + 2480926457425102848*_t**4/415817 - 39451802929737984*_t**3/4158
17 + 118071997444800*_t**2/415817 - 16745884920*_t/415817 + x - 268790/415
817)))
```

Maxima [F]

$$\int \frac{x}{216 + 108x^2 + 324x^3 + 18x^4 + x^6} \, dx = \int \frac{x}{x^6 + 18\,x^4 + 324\,x^3 + 108\,x^2 + 216} \, dx$$

input
```
integrate(x/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="maxima")
```

output
```
integrate(x/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

Giac [F]

$$\int \frac{x}{216+108x^2+324x^3+18x^4+x^6}\,dx=\int \frac{x}{x^6+18\,x^4+324\,x^3+108\,x^2+216}\,dx$$

input
```
integrate(x/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="giac")
```

output
```
integrate(x/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

Mupad [B] (verification not implemented)

Time = 22.26 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.49

$$\int \frac{x}{216+108x^2+324x^3+18x^4+x^6}\,dx=\sum_{k=1}^{6}\ln\left(x+\mathrm{root}\left(z^6-\frac{z^4}{1640736}+\frac{235\,z^3}{129178426752}-\frac{z^2}{3100282242048}-\frac{1}{158171241119638192128},z,k\right)\left(216\,x+\mathrm{root}\left(z^6-\frac{z^4}{1640736}+\frac{235\,z^3}{129178426752}-\frac{z^2}{3100282242048}-\frac{1}{158171241119638192128},z,k\right)\left(51018336\,x\right.\right.\right.$$
$$-\frac{z^4}{1640736}+\frac{235\,z^3}{129178426752}-\frac{z^2}{3100282242048}-\frac{1}{158171241119638192128},z,k\Big)$$

input
```
int(x/(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216),x)
```

output
```
symsum(log(x + root(z^6 - z^4/1640736 + (235*z^3)/129178426752 - z^2/31002
82242048 - 1/15817124111963819212 8, z, k)*(216*x + root(z^6 - z^4/1640736
+ (235*z^3)/129178426752 - z^2/3100282242048 - 1/158171241119638192128, z,
 k)*(51018336*x - root(z^6 - z^4/1640736 + (235*z^3)/129178426752 - z^2/31
00282242048 - 1/158171241119638192128, z, k)*(277947894528*x - root(z^6 -
z^4/1640736 + (235*z^3)/129178426752 - z^2/3100282242048 - 1/1581712411196
38192128, z, k)*(33192121254912*x - root(z^6 - z^4/1640736 + (235*z^3)/129
178426752 - z^2/3100282242048 - 1/158171241119638192128, z, k)*(6940988288
557056*x + 1688973816882216 96) + 28563737812992)))))*root(z^6 - z^4/164073
6 + (235*z^3)/129178426752 - z^2/3100282242048 - 1/158171241119638192128,
z, k), k, 1, 6)
```

Reduce [F]

$$\int \frac{x}{216+108x^2+324x^3+18x^4+x^6}\,dx=\int \frac{x}{x^6+18x^4+324x^3+108x^2+216}dx$$

input
```
int(x/(x^6+18*x^4+324*x^3+108*x^2+216),x)
```

output
```
int(x/(x**6 + 18*x**4 + 324*x**3 + 108*x**2 + 216),x)
```

3.20 $\int \frac{1}{216+108x^2+324x^3+18x^4+x^6}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 383

$$\int \frac{1}{216+108x^2+324x^3+18x^4+x^6}\,dx = \frac{(-1)^{2/3}\left(3(-3)^{2/3}-2^{2/3}\right)\arctan\left(\frac{3\sqrt[3]{-3}2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{324\sqrt[6]{3}\left(1+\sqrt[3]{-1}\right)^2\sqrt{2\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}} - \frac{\left((-2)^{2/3}-3\ 3^{2/3}\right)\arctan\left(\frac{3(-2)^{2/3}\sqrt[3]{3}+2x}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{972\sqrt[6]{3}\sqrt{8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}3^{2/3}}} + \frac{\left(2^{2/3}-3\ 3^{2/3}\right)\operatorname{arctanh}\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{3}+\sqrt[3]{2}x\right)}{\sqrt{3\left(-4+3\sqrt[3]{2}3^{2/3}\right)}}\right)}{972\sqrt[6]{3}\sqrt{2\left(-4+3\sqrt[3]{2}3^{2/3}\right)}} - \frac{\log\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)}{216\ 2^{2/3}\sqrt[3]{3}\left(1+\sqrt[3]{-1}\right)^2} - \frac{\sqrt[3]{-\frac{1}{3}}\log\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)}{648\ 2^{2/3}} + \frac{\log\left(6+3\ 2^{2/3}\sqrt[3]{3}x+x^2\right)}{648\ 2^{2/3}\sqrt[3]{3}}$$

output
```
1/972*(-1)^(2/3)*(3*(-3)^(2/3)-2^(2/3))*arctan((3*(-3)^(1/3)*2^(2/3)-2*x)/
(24-18*(-3)^(2/3)*2^(1/3))^(1/2))*3^(5/6)/(1+(-1)^(1/3))^2/(8-6*(-3)^(2/3)
*2^(1/3))^(1/2)-1/2916*((-2)^(2/3)-3*3^(2/3))*arctan((3*(-2)^(2/3)*3^(1/3)
+2*x)/(24+18*(-2)^(1/3)*3^(2/3))^(1/2))*3^(5/6)/(8+9*I*2^(1/3)*3^(1/6)+3*2
^(1/3)*3^(2/3))^(1/2)+1/2916*(2^(2/3)-3*3^(2/3))*arctanh(2^(1/6)*(3*3^(1/3
)+2^(1/3)*x)/(-12+9*2^(1/3)*3^(2/3))^(1/2))*3^(5/6)/(-8+6*2^(1/3)*3^(2/3))
^(1/2)-1/1296*ln(6-3*(-3)^(1/3)*2^(2/3)*x+x^2)*2^(1/3)*3^(2/3)/(1+(-1)^(1/
3))^2-1/3888*(-1)^(1/3)*3^(2/3)*ln(6+3*(-2)^(2/3)*3^(1/3)*x+x^2)*2^(1/3)+1
/3888*ln(6+3*2^(2/3)*3^(1/3)*x+x^2)*2^(1/3)*3^(2/3)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.16

$$\int \frac{1}{216+108x^2+324x^3+18x^4+x^6}\,dx = \frac{1}{6}\text{RootSum}\left[216+108\#1^2+324\#1^3+18\#1^4+\#1^6\&,\frac{\log(x-\#1)}{36\#1+162\#1^2+12\#1^3+\#1^5}\&\right]$$

input
```
Integrate[(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^(-1),x]
```

output
```
RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^4 + #1^6 & , Log[x - #1]/(36*#1
+ 162*#1^2 + 12*#1^3 + #1^5) & ]/6
```

Rubi [A] (verified)

Time = 1.81 (sec) , antiderivative size = 370, normalized size of antiderivative = 0.97, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {2466, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6+18x^4+324x^3+108x^2+216}\,dx$$

$$\Big\downarrow \text{2466}$$

$$1259712\int \left(\frac{\left(-\frac{1}{3}\right)^{2/3}\left(\sqrt[3]{-6}x+2^{2/3}\left(1-3(-3)^{2/3}\sqrt[3]{2}\right)\right)}{272097792\left(1+\sqrt[3]{-1}\right)^2\left(x^2-3\sqrt[3]{-3}2^{2/3}x+6\right)}-\frac{\sqrt[3]{-6}x+2^{2/3}\left((-1)^{2/3}-3\sqrt[3]{2}3^{2/3}\right)}{816293376\,3^{2/3}\left(x^2+3(-2)^{2/3}\sqrt[3]{3}x+6\right)}+\frac{}{244\ldots}\right.$$

$$\Big\downarrow \text{2009}$$

$$1259712\left(\frac{\left(9-(-2)^{2/3}\sqrt[3]{3}\right)\arctan\left(\frac{2x+3(-2)^{2/3}\sqrt[3]{3}}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{1224440064\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}+\frac{(-1)^{2/3}\left(3(-3)^{2/3}-2^{2/3}\right)\arctan\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{-3}-\sqrt[3]{2}\ldots\right.}{\sqrt{3\left(4-3(-3)^{2/3}\sqrt[3]{\ldots}\right.}}\right.}{408146688\sqrt[6]{3}\left(1+\sqrt[3]{-1}\right)^2\sqrt{2\left(4-3(-3)^{2/3}\sqrt[3]{2}\ldots\right.}}\right.$$

input
```
Int[(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^(-1),x]
```

output
```
1259712*(((9 - (-2)^(2/3)*3^(1/3))*ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sqr
t[6*(4 + 3*(-2)^(1/3)*3^(2/3))]])/(1224440064*Sqrt[6*(4 + 3*(-2)^(1/3)*3^(
2/3))]) + ((-1)^(2/3)*(3*(-3)^(2/3) - 2^(2/3))*ArcTan[(2^(1/6)*(3*(-3)^(1/
3) - 2^(1/3)*x))/Sqrt[3*(4 - 3*(-3)^(2/3)*2^(1/3))]])/(408146688*3^(1/6)*(
1 + (-1)^(1/3))^2*Sqrt[2*(4 - 3*(-3)^(2/3)*2^(1/3))]) - ((9 - 2^(2/3)*3^(1
/3))*ArcTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(2
/3))]])/(1224440064*Sqrt[6*(-4 + 3*2^(1/3)*3^(2/3))]) - Log[6 - 3*(-3)^(1/
3)*2^(2/3)*x + x^2]/(272097792*2^(2/3)*3^(1/3)*(1 + (-1)^(1/3))^2) - ((-1/
3)^(1/3)*Log[6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2])/(816293376*2^(2/3)) + Log[
6 + 3*2^(2/3)*3^(1/3)*x + x^2]/(816293376*2^(2/3)*3^(1/3)))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2466
```
Int[(u_.)*(Q6_)^(p_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
 x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Sim
p[1/(3^(3*p)*a^(2*p))   Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*
x + b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*
(-1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d,
 0] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coef
f[Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.04 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.14

method	result	size
default	$\frac{\left(\sum_{\_R=\text{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)}\frac{\ln\left(x-\_R\right)}{\_R^5+12\_R^3+162\_R^2+36\_R}\right)}{6}$	53
risch	$\frac{\left(\sum_{\_R=\text{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)}\frac{\ln\left(x-\_R\right)}{\_R^5+12\_R^3+162\_R^2+36\_R}\right)}{6}$	53

input
```
int(1/(x^6+18*x^4+324*x^3+108*x^2+216),x,method=_RETURNVERBOSE)
```

output
```
1/6*sum(1/(_R^5+12*_R^3+162*_R^2+36*_R)*ln(x-_R),_R=RootOf(_Z^6+18*_Z^4+32
4*_Z^3+108*_Z^2+216))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{216 + 108x^2 + 324x^3 + 18x^4 + x^6} \, dx = \text{Timed out}$$

input
```
integrate(1/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.17

$$\int \frac{1}{216 + 108x^2 + 324x^3 + 18x^4 + x^6} \, dx$$
$$= \text{RootSum}\left(341649880818418494996 48t^6 - 3470494144278528t^4 - 86087932019712t^3 - 1530550080t^2 +\right.$$

input
```
integrate(1/(x**6+18*x**4+324*x**3+108*x**2+216),x)
```

output
```
RootSum(34164988081841849499648*_t**6 - 3470494144278528*_t**4 - 860879320
19712*_t**3 - 1530550080*_t**2 + 69984*_t - 1, Lambda(_t, _t*log(185904446
699109611410573787136*_t**5/57121295165 + 6377301253267917382766592*_t**4/
57121295165 - 1890463600238856431155 2*_t**3/57121295165 - 4690805529151817
23968*_t**2/57121295165 - 24358640509989936*_t/57121295165 + x + 152427895
956/57121295165)))
```

Maxima [F]

$$\int \frac{1}{216 + 108x^2 + 324x^3 + 18x^4 + x^6}\,dx = \int \frac{1}{x^6 + 18\,x^4 + 324\,x^3 + 108\,x^2 + 216}\,dx$$

input
```
integrate(1/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="maxima")
```

output
```
integrate(1/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

Giac [F]

$$\int \frac{1}{216 + 108x^2 + 324x^3 + 18x^4 + x^6}\,dx = \int \frac{1}{x^6 + 18\,x^4 + 324\,x^3 + 108\,x^2 + 216}\,dx$$

input
```
integrate(1/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="giac")
```

output
```
integrate(1/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

Mupad [B] (verification not implemented)

Time = 22.55 (sec) , antiderivative size = 306, normalized size of antiderivative = 0.80

$$\int \frac{1}{216+108x^2+324x^3+18x^4+x^6}\,dx = \sum_{k=1}^{6} \ln\left(-\mathrm{root}\left(z^6-\frac{z^4}{9844416}-\frac{217\,z^3}{86118951168}-\frac{5\,z^2}{111610160713728}+\frac{z}{488182842961846272}-\frac{1}{341649880818418494996 48},z,k\right)\,x\,6\right.$$

$$+\mathrm{root}\left(z^6-\frac{z^4}{9844416}-\frac{217\,z^3}{86118951168}-\frac{5\,z^2}{111610160713728}+\frac{z}{488182842961846272}-\frac{1}{34164988081841}\right.$$

$$-\mathrm{root}\left(z^6-\frac{z^4}{9844416}-\frac{217\,z^3}{86118951168}-\frac{5\,z^2}{111610160713728}+\frac{z}{488182842961846272}-\frac{1}{34164988081841}\right.$$

$$-\mathrm{root}\left(z^6-\frac{z^4}{9844416}-\frac{217\,z^3}{86118951168}-\frac{5\,z^2}{111610160713728}+\frac{z}{488182842961846272}-\frac{1}{34164988081841}\right.$$

$$-\mathrm{root}\left(z^6-\frac{z^4}{9844416}-\frac{217\,z^3}{86118951168}-\frac{5\,z^2}{111610160713728}+\frac{z}{488182842961846272}-\frac{1}{34164988081841}\right.$$

$$+944784\,\mathrm{root}\left(z^6-\frac{z^4}{9844416}-\frac{217\,z^3}{86118951168}-\frac{5\,z^2}{111610160713728}+\frac{z}{488182842961846272}-34164988\right.$$

$$-16529940864\,\mathrm{root}\left(z^6-\frac{z^4}{9844416}-\frac{217\,z^3}{86118951168}-\frac{5\,z^2}{111610160713728}+\frac{z}{488182842961846272}-34\right.$$

$$-33192121254912\,\mathrm{root}\left(z^6-\frac{z^4}{9844416}-\frac{217\,z^3}{86118951168}-\frac{5\,z^2}{111610160713728}+\frac{z}{488182842961846272}-\right.$$

$$-168897381688221696\,\mathrm{root}\left(z^6-\frac{z^4}{9844416}-\frac{217\,z^3}{86118951168}-\frac{5\,z^2}{111610160713728}+\frac{z}{4881828429618462}\right.$$

$$\left.-\frac{z^4}{9844416}-\frac{217\,z^3}{86118951168}-\frac{5\,z^2}{111610160713728}+\frac{z}{488182842961846272}-\frac{1}{34164988081841849499648},z,k\right)$$

input
```
int(1/(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216),x)
```

output
```
symsum(log(349920*root(z^6 - z^4/9844416 - (217*z^3)/86118951168 - (5*z^2)
/111610160713728 + z/488182842961846272 - 1/34164988081841849499648, z, k)
^2*x - 6*root(z^6 - z^4/9844416 - (217*z^3)/86118951168 - (5*z^2)/11161016
0713728 + z/488182842961846272 - 1/34164988081841849499648, z, k)*x - 6122
200320*root(z^6 - z^4/9844416 - (217*z^3)/86118951168 - (5*z^2)/1116101607
13728 + z/488182842961846272 - 1/34164988081841849499648, z, k)^3*x - 2582
63796059136*root(z^6 - z^4/9844416 - (217*z^3)/86118951168 - (5*z^2)/11161
0160713728 + z/488182842961846272 - 1/34164988081841849499648, z, k)^4*x -
 6940988288557056*root(z^6 - z^4/9844416 - (217*z^3)/86118951168 - (5*z^2)
/111610160713728 + z/488182842961846272 - 1/34164988081841849499648, z, k)
^5*x + 944784*root(z^6 - z^4/9844416 - (217*z^3)/86118951168 - (5*z^2)/111
610160713728 + z/488182842961846272 - 1/34164988081841849499648, z, k)^2 -
 16529940864*root(z^6 - z^4/9844416 - (217*z^3)/86118951168 - (5*z^2)/1116
10160713728 + z/488182842961846272 - 1/34164988081841849499648, z, k)^3 -
33192121254912*root(z^6 - z^4/9844416 - (217*z^3)/86118951168 - (5*z^2)/11
1610160713728 + z/488182842961846272 - 1/34164988081841849499648, z, k)^4
- 168897381688221696*root(z^6 - z^4/9844416 - (217*z^3)/86118951168 - (5*z
^2)/111610160713728 + z/488182842961846272 - 1/34164988081841849499648, z,
 k)^5)*root(z^6 - z^4/9844416 - (217*z^3)/86118951168 - (5*z^2)/1116101607
13728 + z/488182842961846272 - 1/34164988081841849499648, z, k), k, 1, ...
```

Reduce [F]

$$\int \frac{1}{216 + 108x^2 + 324x^3 + 18x^4 + x^6}\,dx = \int \frac{1}{x^6 + 18x^4 + 324x^3 + 108x^2 + 216}dx$$

input
```
int(1/(x^6+18*x^4+324*x^3+108*x^2+216),x)
```

output
```
int(1/(x**6 + 18*x**4 + 324*x**3 + 108*x**2 + 216),x)
```

3.21 $\int \frac{1}{x(216+108x^2+324x^3+18x^4+x^6)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 415

$$\int \frac{1}{x(216+108x^2+324x^3+18x^4+x^6)}\,dx = \frac{(-1)^{2/3}\left((-2)^{2/3}-2\ 3^{2/3}\right)\arctan\left(\frac{3(-2)^{2/3}\sqrt[3]{3}+2x}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{216\sqrt[3]{2}3^{5/6}\sqrt{8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}3^{2/3}}} - \frac{(-1)^{2/3}\left(\sqrt[3]{-3}+3\sqrt[3]{2}\right)\arctan\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{-3}-\sqrt[3]{2}x\right)}{\sqrt{3\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{216\sqrt[6]{6}\left(1+\sqrt[3]{-1}\right)^2\sqrt{4-3(-3)^{2/3}\sqrt[3]{2}}} - \frac{\left(1-\sqrt[3]{2}3^{2/3}\right)\operatorname{arctanh}\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{3}+\sqrt[3]{2}x\right)}{\sqrt{3\left(-4+3\sqrt[3]{2}3^{2/3}\right)}}\right)}{216\sqrt[6]{2}3^{5/6}\sqrt{-4+3\sqrt[3]{2}3^{2/3}}} + \frac{\log(x)}{216} - \frac{\left(36+2^{2/3}\sqrt[3]{3}(1+i\sqrt{3})\right)\log\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)}{46656} - \frac{\left(18-(-2)^{2/3}\sqrt[3]{3}\right)\log\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)}{23328} - \frac{\left(18-2^{2/3}\sqrt[3]{3}\right)\log\left(6+3\ 2^{2/3}\sqrt[3]{3}x+x^2\right)}{23328}$$

output

```
1/1296*(-1)^(2/3)*((-2)^(2/3)-2*3^(2/3))*arctan((3*(-2)^(2/3)*3^(1/3)+2*x)
/(24+18*(-2)^(1/3)*3^(2/3))^(1/2))*2^(2/3)*3^(1/6)/(8+9*I*2^(1/3)*3^(1/6)+
3*2^(1/3)*3^(2/3))^(1/2)-1/1296*(-1)^(2/3)*((-3)^(1/3)+3*2^(1/3))*arctan(2
^(1/6)*(3*(-3)^(1/3)-2^(1/3)*x)/(12-9*(-3)^(2/3)*2^(1/3))^(1/2))*6^(5/6)/(
1+(-1)^(1/3))^2/(4-3*(-3)^(2/3)*2^(1/3))^(1/2)-1/1296*(1-2^(1/3)*3^(2/3))*
arctanh(2^(1/6)*(3*3^(1/3)+2^(1/3)*x)/(-12+9*2^(1/3)*3^(2/3))^(1/2))*2^(5/
6)*3^(1/6)/(-4+3*2^(1/3)*3^(2/3))^(1/2)+1/216*ln(x)-1/46656*(36+2^(2/3)*3^
(1/3)*(1+I*3^(1/2)))*ln(6-3*(-3)^(1/3)*2^(2/3)*x+x^2)-1/23328*(18-(-2)^(2/
3)*3^(1/3))*ln(6+3*(-2)^(2/3)*3^(1/3)*x+x^2)-1/23328*(18-2^(2/3)*3^(1/3))*
ln(6+3*2^(2/3)*3^(1/3)*x+x^2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.25

$$\int \frac{1}{x\left(216+108x^2+324x^3+18x^4+x^6\right)}\,dx = \frac{\log(x)}{216} - \frac{\operatorname{RootSum}\left[216+108\#1^2+324\#1^3+18\#1^4+\#1^6\&, \frac{108\log(x-\#1)+324\log(x-\#1)\#1+18\log(x-\#1)\#1^2+\text{l}}{36+162\#1+12\#1^2+\#1^4}\right.}{1296}$$

input
```
Integrate[1/(x*(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)),x]
```

output
```
Log[x]/216 - RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^4 + #1^6 & , (108*L
og[x - #1] + 324*Log[x - #1]*#1 + 18*Log[x - #1]*#1^2 + Log[x - #1]*#1^4)/
(36 + 162*#1 + 12*#1^2 + #1^4) & ]/1296
```

Rubi [A] (verified)

Time = 2.12 (sec) , antiderivative size = 401, normalized size of antiderivative = 0.97, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2466, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(x^6+18x^4+324x^3+108x^2+216\right)}\,dx$$

$$\Big\downarrow \; 2466$$

$$1259712\int \left(\frac{(-1)^{2/3}\left(6\left(9+\sqrt[3]{-3}2^{2/3}\right)-\left(1-3(-3)^{2/3}\sqrt[3]{2}\right)x\right)}{816293376\sqrt[3]{2}3^{2/3}\left(1+\sqrt[3]{-1}\right)^2\left(x^2-3\sqrt[3]{-3}2^{2/3}x+6\right)}+\frac{1}{272097792x}-\frac{(-1)^{2/3}\left(6\left(9-(-2)^{2/3}\sqrt[3]{\right.\right.}{2448880128\sqrt[3]{2}3^{2/3}\left(x^2\right.}\right.$$

$$\Big\downarrow \; 2009$$

$$1259712\left(\frac{(-1)^{2/3}\left((-2)^{2/3}-2\ 3^{2/3}\right)\arctan\left(\frac{2x+3(-2)^{2/3}\sqrt[3]{3}}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{272097792\ 6^{5/6}\sqrt{4+3\sqrt[3]{-2}3^{2/3}}}-\frac{(-1)^{2/3}\left(\sqrt[3]{-3}+3\sqrt[3]{2}\right)\arctan\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{-\right.}{\sqrt{3\left(4-3(-\right.}}\right.}{272097792\sqrt[6]{6}\left(1+\sqrt[3]{-1}\right)^2\sqrt{4-3(-3}}\right.$$

input
```
Int[1/(x*(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)),x]
```

output
```
1259712*(((-1)^(2/3)*((-2)^(2/3) - 2*3^(2/3))*ArcTan[(3*(-2)^(2/3)*3^(1/3)
 + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3))]])/(272097792*6^(5/6)*Sqrt[4 + 3
*(-2)^(1/3)*3^(2/3)]) - ((-1)^(2/3)*((-3)^(1/3) + 3*2^(1/3))*ArcTan[(2^(1/
6)*(3*(-3)^(1/3) - 2^(1/3)*x))/Sqrt[3*(4 - 3*(-3)^(2/3)*2^(1/3))]])/(27209
7792*6^(1/6)*(1 + (-1)^(1/3))^2*Sqrt[4 - 3*(-3)^(2/3)*2^(1/3)]) - ((1 - 2^
(1/3)*3^(2/3))*ArcTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^
(1/3)*3^(2/3))]])/(272097792*2^(1/6)*3^(5/6)*Sqrt[-4 + 3*2^(1/3)*3^(2/3)])
 + Log[x]/272097792 - ((36 + 2^(2/3)*3^(1/3) + I*2^(2/3)*3^(5/6))*Log[6 -
3*(-3)^(1/3)*2^(2/3)*x + x^2])/58773123072 - ((18 - (-2)^(2/3)*3^(1/3))*Lo
g[6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2])/29386561536 - ((18 - 2^(2/3)*3^(1/3))
*Log[6 + 3*2^(2/3)*3^(1/3)*x + x^2])/29386561536)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2466
```
Int[(u_.)*(Q6_)^(p_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
 x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Sim
p[1/(3^(3*p)*a^(2*p))   Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*
x + b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*
(-1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d,
 0] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coef
f[Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.05 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.18

method	result
risch	$\frac{\left(\sum_{\_R=\text{RootOf}\left(136728\_Z^6+1230552\_Z^5+3682908\_Z^4+3630708\_Z^3-81810\_Z^2+486\_Z-1\right)} \_R \ln\left(-23672342955240\_R^5-2130562\right.\right.}{1944}$
default	$-\frac{\left(\sum_{\_R=\text{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)} \frac{\left(\_R^5+18\_R^3+324\_R^2+108\_R\right)\ln\left(x-\_R\right)}{\_R^5+12\_R^3+162\_R^2+36\_R}\right)}{1296}+\frac{\ln(x)}{216}$

input
```
int(1/x/(x^6+18*x^4+324*x^3+108*x^2+216),x,method=_RETURNVERBOSE)
```

output
```
1/1944*sum(_R*ln(-23672342955240*_R^5-213056277916248*_R^4-637689647288592
*_R^3-628763677061560*_R^2+14004611129596*_R+2499731391*x-55133083786),_R=
RootOf(136728*_Z^6+1230552*_Z^5+3682908*_Z^4+3630708*_Z^3-81810*_Z^2+486*_
Z-1))+1/216*ln(x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x\,(216+108x^2+324x^3+18x^4+x^6)}\,dx = \text{Timed out}$$

input
```
integrate(1/x/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.20

$$\int \frac{1}{x(216+108x^2+324x^3+18x^4+x^6)}\,dx = \frac{\log(x)}{216} + \text{RootSum}\Big(737963742567783949192396 8t^6 + 34164988081841849499648t^5 + 5259880925068537036 8$$

input
```
integrate(1/x/(x**6+18*x**4+324*x**3+108*x**2+216),x)
```

output
```
log(x)/216 + RootSum(7379637425677839491923968*_t**6 + 3416498808184184949
9648*_t**5 + 5259880925068537036 8*_t**4 + 26673506015311872*_t**3 - 309171
116160*_t**2 + 944784*_t - 1, Lambda(_t, _t*log(814557009966881793678336 21
15119297360560128*_t**6/143425799309052440063 + 977068766770806381087358 25
7564745728*_t**5/143425799309052440063 - 116529526608851264288400971539061
538816*_t**4/143425799309052440063 - 23935979498524220254250144071076659 2*
_t**3/143425799309052440063 - 13667831263813709443988734141824 0*_t**2/1434
25799309052440063 + 15631155690676637957354136 96*_t/143425799309052440063
+ x - 31644463150752361900 44/143425799309052440063)))
```

Maxima [F]

$$\int \frac{1}{x(216+108x^2+324x^3+18x^4+x^6)}\,dx = \int \frac{1}{(x^6+18\,x^4+324\,x^3+108\,x^2+216)x}\,dx$$

input
```
integrate(1/x/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="maxima")
```

output
```
-1/216*integrate((x^5 + 18*x^3 + 324*x^2 + 108*x)/(x^6 + 18*x^4 + 324*x^3
+ 108*x^2 + 216), x) + 1/216*log(x)
```

Giac [F]

$$\int \frac{1}{x(216+108x^2+324x^3+18x^4+x^6)}\,dx = \int \frac{1}{(x^6+18\,x^4+324\,x^3+108\,x^2+216)x}\,dx$$

input
```
integrate(1/x/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="giac")
```

output
```
integrate(1/((x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216)*x), x)
```

Mupad [B] (verification not implemented)

Time = 22.14 (sec) , antiderivative size = 432, normalized size of antiderivative = 1.04

$$\int \frac{1}{x(216+108x^2+324x^3+18x^4+x^6)}\,dx = \text{Too large to display}$$

input
```
int(1/(x*(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216)),x)
```

output
```
log(x)/216 + symsum(log(7*root(z^6 + z^5/216 + (421*z^4)/59066496 + (10085
3*z^3)/27902540178432 - (505*z^2)/12053897357082624 + z/781092548738954035
2 - 1/7379637425677839491923968, z, k)*x - 5670000*root(z^6 + z^5/216 + (4
21*z^4)/59066496 + (100853*z^3)/27902540178432 - (505*z^2)/120538973570826
24 + z/7810925487389540352 - 1/7379637425677839491923968, z, k)^2*x + 1546
875947520*root(z^6 + z^5/216 + (421*z^4)/59066496 + (100853*z^3)/279025401
78432 - (505*z^2)/12053897357082624 + z/7810925487389540352 - 1/7379637425
677839491923968, z, k)^3*x - 106961147905609728*root(z^6 + z^5/216 + (421*
z^4)/59066496 + (100853*z^3)/27902540178432 - (505*z^2)/12053897357082624
+ z/7810925487389540352 - 1/7379637425677839491923968, z, k)^4*x - 1405119
95854134018048*root(z^6 + z^5/216 + (421*z^4)/59066496 + (100853*z^3)/2790
2540178432 - (505*z^2)/12053897357082624 + z/7810925487389540352 - 1/73796
37425677839491923968, z, k)^5*x - 45607290567387619000320*root(z^6 + z^5/2
16 + (421*z^4)/59066496 + (100853*z^3)/27902540178432 - (505*z^2)/12053897
357082624 + z/7810925487389540352 - 1/7379637425677839491923968, z, k)^6*x
 + 839808*root(z^6 + z^5/216 + (421*z^4)/59066496 + (100853*z^3)/279025401
78432 - (505*z^2)/12053897357082624 + z/7810925487389540352 - 1/7379637425
677839491923968, z, k)^2 + 594896472576*root(z^6 + z^5/216 + (421*z^4)/590
66496 + (100853*z^3)/27902540178432 - (505*z^2)/12053897357082624 + z/7810
925487389540352 - 1/7379637425677839491923968, z, k)^3 - 84834301304586...
```

Reduce [F]

$$\int \frac{1}{x\left(216+108x^2+324x^3+18x^4+x^6\right)}\,dx=\int \frac{1}{x^7+18x^5+324x^4+108x^3+216x}dx$$

input
```
int(1/x/(x^6+18*x^4+324*x^3+108*x^2+216),x)
```

output
```
int(1/(x**7 + 18*x**5 + 324*x**4 + 108*x**3 + 216*x),x)
```

3.22 $\int \frac{1}{x^2(216+108x^2+324x^3+18x^4+x^6)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 448

$$\int \frac{1}{x^2\left(216+108x^2+324x^3+18x^4+x^6\right)}\,dx$$
$$= -\frac{1}{216x} - \frac{\left(27\sqrt[3]{-6}-(-2)^{2/3}+12\ 3^{2/3}\right)\arctan\left(\frac{3(-2)^{2/3}\sqrt[3]{3}+2x}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{5832\sqrt[6]{3}\sqrt{8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}3^{2/3}}}$$
$$- \frac{(-1)^{2/3}\left(6(-6)^{2/3}+27\sqrt[3]{-3}-\sqrt[3]{2}\right)\arctan\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{-3}-\sqrt[3]{2}x\right)}{\sqrt{3\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{1944\sqrt[6]{6}\left(1+\sqrt[3]{-1}\right)^2\sqrt{4-3(-3)^{2/3}\sqrt[3]{2}}}$$
$$- \frac{\left(\sqrt[3]{2}+27\sqrt[3]{3}-6\ 6^{2/3}\right)\operatorname{arctanh}\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{3}+\sqrt[3]{2}x\right)}{\sqrt{3\left(-4+3\sqrt[3]{2}3^{2/3}\right)}}\right)}{5832\sqrt[6]{6}\sqrt{-4+3\sqrt[3]{2}3^{2/3}}}$$
$$- \frac{(-1)^{2/3}\left(9+\sqrt[3]{-3}2^{2/3}\right)\log\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)}{1296\sqrt[3]{2}3^{2/3}\left(1+\sqrt[3]{-1}\right)^2}$$
$$+ \frac{\left(3(-6)^{2/3}+2\sqrt[3]{-2}\right)\log\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)}{7776\sqrt[3]{3}}$$
$$- \frac{\left(2^{2/3}-3\ 3^{2/3}\right)\log\left(6+3\ 2^{2/3}\sqrt[3]{3}x+x^2\right)}{3888\sqrt[3]{6}}$$

output

```
-1/216/x-1/17496*(27*(-6)^(1/3)-(-2)^(2/3)+12*3^(2/3))*arctan((3*(-2)^(2/3
)*3^(1/3)+2*x)/(24+18*(-2)^(1/3)*3^(2/3))^(1/2))*3^(5/6)/(8+9*I*2^(1/3)*3^
(1/6)+3*2^(1/3)*3^(2/3))^(1/2)-1/11664*(-1)^(2/3)*(6*(-6)^(2/3)+27*(-3)^(1
/3)-2^(1/3))*arctan(2^(1/6)*(3*(-3)^(1/3)-2^(1/3)*x)/(12-9*(-3)^(2/3)*2^(1
/3))^(1/2))*6^(5/6)/(1+(-1)^(1/3))^2/(4-3*(-3)^(2/3)*2^(1/3))^(1/2)-1/3499
2*(2^(1/3)+27*3^(1/3)-6*6^(2/3))*arctanh(2^(1/6)*(3*3^(1/3)+2^(1/3)*x)/(-1
2+9*2^(1/3)*3^(2/3))^(1/2))*6^(5/6)/(-4+3*2^(1/3)*3^(2/3))^(1/2)-1/7776*(-
1)^(2/3)*(9+(-3)^(1/3)*2^(2/3))*ln(6-3*(-3)^(1/3)*2^(2/3)*x+x^2)*2^(2/3)*3
^(1/3)/(1+(-1)^(1/3))^2+1/23328*(3*(-6)^(2/3)+2*(-2)^(1/3))*ln(6+3*(-2)^(2
/3)*3^(1/3)*x+x^2)*3^(2/3)-1/23328*(2^(2/3)-3*3^(2/3))*ln(6+3*2^(2/3)*3^(1
/3)*x+x^2)*6^(2/3)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.24

$$\int \frac{1}{x^2(216+108x^2+324x^3+18x^4+x^6)}\,dx = -\frac{1}{216x} - \frac{\text{RootSum}\left[216+108\#1^2+324\#1^3+18\#1^4+\#1^6\&, \frac{108\log(x-\#1)+324\log(x-\#1)\#1+18\log(x-\#1)\#1^2+\text{l}}{36\#1+162\#1^2+12\#1^3+\#1^5}\right.}{1296}$$

input
```
Integrate[1/(x^2*(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)),x]
```

output
```
-1/216*1/x - RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^4 + #1^6 & , (108*L
og[x - #1] + 324*Log[x - #1]*#1 + 18*Log[x - #1]*#1^2 + Log[x - #1]*#1^4)/
(36*#1 + 162*#1^2 + 12*#1^3 + #1^5) & ]/1296
```

Rubi [A] (verified)

Time = 2.30 (sec) , antiderivative size = 453, normalized size of antiderivative = 1.01, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2466, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2(x^6+18x^4+324x^3+108x^2+216)}\,dx$$

$$\Big\downarrow \ 2466$$

$$1259712\int \left(-\frac{(-1)^{2/3}\left(\left(9+\sqrt[3]{-3}2^{2/3}\right)x-27\sqrt[3]{-3}2^{2/3}-9(-3)^{2/3}\sqrt[3]{2}+1\right)}{816293376\sqrt[3]{2}3^{2/3}\left(1+\sqrt[3]{-1}\right)^2\left(x^2-3\sqrt[3]{-3}2^{2/3}x+6\right)}+\frac{(-1)^{2/3}\left(\left(9-(-2)^{2/3}\sqrt[3]{3}\right)x+\right.}{2448880128\sqrt[3]{2}3^{2/3}\left(x^2\right.}\right.$$

$$\Big\downarrow \ 2009$$

$$1259712\left(\frac{(-1)^{2/3}\left(2+27(-2)^{2/3}\sqrt[3]{3}+12\sqrt[3]{-2}3^{2/3}\right)\arctan\left(\frac{2x+3(-2)^{2/3}\sqrt[3]{3}}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{7346640384\,2^{5/6}\sqrt[6]{3}\sqrt{4+3\sqrt[3]{-2}3^{2/3}}}-\frac{(-1)^{2/3}\left(6(-6)^{2/3}+27\sqrt[3]{-\right.}{2448880128\sqrt[6]{6}\,(}\right.$$

input

```
Int[1/(x^2*(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)),x]
```

output

```
1259712*(-1/272097792*1/x + ((-1)^(2/3)*(2 + 27*(-2)^(2/3)*3^(1/3) + 12*(-
2)^(1/3)*3^(2/3))*ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(
1/3)*3^(2/3))]])/(7346640384*2^(5/6)*3^(1/6)*Sqrt[4 + 3*(-2)^(1/3)*3^(2/3)
]) - ((-1)^(2/3)*(6*(-6)^(2/3) + 27*(-3)^(1/3) - 2^(1/3))*ArcTan[(2^(1/6)*
(3*(-3)^(1/3) - 2^(1/3)*x))/Sqrt[3*(4 - 3*(-3)^(2/3)*2^(1/3))]])/(24488801
28*6^(1/6)*(1 + (-1)^(1/3))^2*Sqrt[4 - 3*(-3)^(2/3)*2^(1/3)]) - ((2^(1/3)
+ 27*3^(1/3) - 6*6^(2/3))*ArcTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3
*(-4 + 3*2^(1/3)*3^(2/3))]])/(7346640384*6^(1/6)*Sqrt[-4 + 3*2^(1/3)*3^(2/
3)]) - ((-1)^(2/3)*(9 + (-3)^(1/3)*2^(2/3))*Log[6 - 3*(-3)^(1/3)*2^(2/3)*x
 + x^2])/(1632586752*2^(1/3)*3^(2/3)*(1 + (-1)^(1/3))^2) - ((-1)^(2/3)*((-
2)^(2/3) - 3*3^(2/3))*Log[6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2])/(4897760256*6
^(1/3)) - ((2^(2/3) - 3*3^(2/3))*Log[6 + 3*2^(2/3)*3^(1/3)*x + x^2])/(4897
760256*6^(1/3)))
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2466

```
Int[(u_.)*(Q6_)^(p_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
 x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Sim
p[1/(3^(3*p)*a^(2*p))   Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*
x + b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*
(-1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d,
 0] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coef
f[Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.05 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.15

method	result
risch	$-\frac{1}{216x}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(633\_Z^{6}+204849\_Z^{4}-5446980\_Z^{3}-80433\_Z^{2}-72\_Z-1\right)}\_R\ln\left(-462040439801351484393\_R^{5}+13642\right.\right.}{}$
default	$-\frac{1}{216x}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6}+18\_Z^{4}+324\_Z^{3}+108\_Z^{2}+216\right)}\frac{\left(-\_R^{4}-18\_R^{2}-324\_R-108\right)\ln\left(x-\_R\right)}{\_R^{5}+12\_R^{3}+162\_R^{2}+36\_R}\right)}{1296}$

input
```
int(1/x^2/(x^6+18*x^4+324*x^3+108*x^2+216),x,method=_RETURNVERBOSE)
```

output
```
-1/216/x+1/11664*sum(_R*ln(-462040439801351484393*_R^5+1364231865933925308
*_R^4-14952374096957448341761 2*_R^3+397631047190316263673604 2*_R^2+4696745
4543463546461111*_R+2470089956940798359 0*x-25597852658707816584),_R=RootOf
(633*_Z^6+204849*_Z^4-5446980*_Z^3-80433*_Z^2-72*_Z-1))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^2\left(216+108x^2+324x^3+18x^4+x^6\right)}\,dx=\text{Timed out}$$

input
```
integrate(1/x^2/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.16

$$\int \frac{1}{x^2 (216+108x^2+324x^3+18x^4+x^6)} \, dx$$
$$= \text{RootSum}\left(15940016839464133302555770888t^6+37916120264603316387844t^4-864367269958950912 0t^3\right.$$
$$-\frac{1}{216x}$$

input
```
integrate(1/x**2/(x**6+18*x**4+324*x**3+108*x**2+216),x)
```

output
```
RootSum(15940016839464133302555770888*_t**6 + 37916120264603316387844*_t**4
- 8643672699589509120*_t**3 - 10942820851968*_t**2 - 839808*_t - 1, Lambda
(_t, _t*log(-49875532761902496003293561236914468028416*_t**5/1235044978470
3991795 + 12625489872431620388005975200497664*_t**4/12350449784703991795 -
 118637692607573771238550798852644864*_t**3/12350449784703991795 + 2704863
249278321478181937787548l6*_t**2/12350449784703991795 + 273914194897479402
961199352*_t/12350449784703991795 + x - 1279892632935390829 2/1235044978470
3991795))) - 1/(216*x)
```

Maxima [F]

$$\int \frac{1}{x^2 (216+108x^2+324x^3+18x^4+x^6)} \, dx = \int \frac{1}{(x^6+18\,x^4+324\,x^3+108\,x^2+216)x^2} \, dx$$

input
```
integrate(1/x^2/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="maxima")
```

output
```
-1/216/x - 1/216*integrate((x^4 + 18*x^2 + 324*x + 108)/(x^6 + 18*x^4 + 32
4*x^3 + 108*x^2 + 216), x)
```

Giac [F]

$$\int \frac{1}{x^2\left(216+108x^2+324x^3+18x^4+x^6\right)}\,dx=\int \frac{1}{(x^6+18\,x^4+324\,x^3+108\,x^2+216)x^2}\,dx$$

input
```
integrate(1/x^2/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="giac")
```

output
```
integrate(1/((x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216)*x^2), x)
```

Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 340, normalized size of antiderivative = 0.76

$$\int \frac{1}{x^2\left(216+108x^2+324x^3+18x^4+x^6\right)}\,dx=\text{Too large to display}$$

input
```
int(1/(x^2*(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216)),x)
```

output
```
symsum(log((5*root(z^6 + (281*z^4)/118132992 - (50435*z^3)/9300846726144 -
 (331*z^2)/48215589428330496 - z/189805489343565830553 6 - 1/15940016839464
13330255577088, z, k))/8 - (root(z^6 + (281*z^4)/118132992 - (50435*z^3)/9
300846726144 - (331*z^2)/48215589428330496 - z/18980548934356583055 36 - 1/
15940016839464133302555 77088, z, k)*x)/216 - 396252*root(z^6 + (281*z^4)/1
18132992 - (50435*z^3)/9300846726144 - (331*z^2)/48215589428330496 - z/189
805489343565830553 6 - 1/15940016839464133302555 77088, z, k)^2*x - 59822967
0528*root(z^6 + (281*z^4)/118132992 - (50435*z^3)/9300846726144 - (331*z^2
)/48215589428330496 - z/189805489343565830553 6 - 1/15940016839464133302555
77088, z, k)^3*x + 82120746212352*root(z^6 + (281*z^4)/118132992 - (50435*
z^3)/9300846726144 - (331*z^2)/48215589428330496 - z/18980548934356583055 3
6 - 1/15940016839464133302555 77088, z, k)^4*x - 6940988288557056*root(z^6
+ (281*z^4)/118132992 - (50435*z^3)/9300846726144 - (331*z^2)/482155894283
30496 - z/189805489343565830553 6 - 1/15940016839464133302555 77088, z, k)^5
*x + 2344464*root(z^6 + (281*z^4)/118132992 - (50435*z^3)/9300846726144 -
(331*z^2)/48215589428330496 - z/189805489343565830553 6 - 1/159400168394641
3330255577088, z, k)^2 - 210297580992*root(z^6 + (281*z^4)/118132992 - (50
435*z^3)/9300846726144 - (331*z^2)/48215589428330496 - z/18980548934356583
05536 - 1/15940016839464133302555 77088, z, k)^3 - 10535082310656*root(z^6
+ (281*z^4)/118132992 - (50435*z^3)/9300846726144 - (331*z^2)/482155894...
```

Reduce [F]

$$\int \frac{1}{x^2 (216 + 108x^2 + 324x^3 + 18x^4 + x^6)}\, dx$$

$$= \frac{-\left(\int \frac{x^4}{x^6+18x^4+324x^3+108x^2+216}dx\right) x - 18\left(\int \frac{x^2}{x^6+18x^4+324x^3+108x^2+216}dx\right) x - 324\left(\int \frac{x}{x^6+18x^4+324x^3+108x^2+216}d\right.}{216x}$$

input
```
int(1/x^2/(x^6+18*x^4+324*x^3+108*x^2+216),x)
```

output
```
( - int(x**4/(x**6 + 18*x**4 + 324*x**3 + 108*x**2 + 216),x)*x - 18*int(x*
*2/(x**6 + 18*x**4 + 324*x**3 + 108*x**2 + 216),x)*x - 324*int(x/(x**6 + 1
8*x**4 + 324*x**3 + 108*x**2 + 216),x)*x - 108*int(1/(x**6 + 18*x**4 + 324
*x**3 + 108*x**2 + 216),x)*x - 1)/(216*x)
```

3.23 $\int \frac{x^8}{(216+108x^2+324x^3+18x^4+x^6)^2} dx$

Optimal result

Integrand size = 26, antiderivative size = 1063

$$\int \frac{x^8}{(216+108x^2+324x^3+18x^4+x^6)^2} dx = \text{Too large to display}$$

output
```
-1/972*(-1)^(1/3)*3^(2/3)*(54+9*(-3)^(1/3)*2^(2/3)+(2-2^(2/3)*(6*(-6)^(2/3
)+27*(-3)^(1/3)))*x)*2^(1/3)/(1+(-1)^(1/3))^4/(4-3*(-3)^(2/3)*2^(1/3))/(6-
3*(-3)^(1/3)*2^(2/3)*x+x^2)-1/4374*(-1)^(1/3)*3^(2/3)*(54-9*(-2)^(2/3)*3^(
1/3)+(2+27*(-2)^(2/3)*3^(1/3)+12*(-2)^(1/3)*3^(2/3))*x)*2^(1/3)/(8+9*I*2^(
1/3)*3^(1/6)+3*2^(1/3)*3^(2/3))/(6+3*(-2)^(2/3)*3^(1/3)*x+x^2)+1/8748*(54-
9*2^(2/3)*3^(1/3)+(2+2^(2/3)*(27*3^(1/3)-6*6^(2/3)))*x)*2^(1/3)*3^(2/3)/(4
-3*2^(1/3)*3^(2/3))/(6+3*2^(2/3)*3^(1/3)*x+x^2)-1/972*I*((-2)^(2/3)+6*3^(2
/3))*arctan((3*(-2)^(2/3)*3^(1/3)+2*x)/(24+18*(-2)^(1/3)*3^(2/3))^(1/2))*2
^(1/6)*3^(2/3)/(1+(-1)^(1/3))^5/(4+3*(-2)^(1/3)*3^(2/3))^(1/2)-1/972*(-1)^
(1/3)*(2+27*(-2)^(2/3)*3^(1/3)+12*(-2)^(1/3)*3^(2/3))*arctan((3*(-2)^(2/3)
*3^(1/3)+2*x)/(24+18*(-2)^(1/3)*3^(2/3))^(1/2))*2^(5/6)*3^(1/6)/(1-(-1)^(1
/3))^2/(1+(-1)^(1/3))^4/(4+3*(-2)^(1/3)*3^(2/3))^(3/2)-1/486*(-1)^(1/3)*(6
*(-6)^(2/3)+27*(-3)^(1/3)-2^(1/3))*arctan(2^(1/6)*(3*(-3)^(1/3)-2^(1/3)*x)
/(12-9*(-3)^(2/3)*2^(1/3))^(1/2))*2^(1/2)*3^(1/6)/(1+(-1)^(1/3))^4/(4-3*(-
3)^(2/3)*2^(1/3))^(3/2)-1/972*(9*3^(1/6)-I*(2^(2/3)-3*3^(2/3)))*arctan(2^(
1/6)*(3*(-3)^(1/3)-2^(1/3)*x)/(12-9*(-3)^(2/3)*2^(1/3))^(1/2))*2^(1/6)*3^(
2/3)/(1+(-1)^(1/3))^5/(4-3*(-3)^(2/3)*2^(1/3))^(1/2)-1/8748*(1+3*2^(1/3)*3
^(2/3))*arctanh(2^(1/6)*(3*3^(1/3)+2^(1/3)*x)/(-12+9*2^(1/3)*3^(2/3))^(1/2
))*2^(5/6)*3^(1/6)/(-4+3*2^(1/3)*3^(2/3))^(1/2)+1/486*(2^(1/3)+27*3^(1/3)-
6*6^(2/3))*arctanh(2^(1/6)*(3*3^(1/3)+2^(1/3)*x)/(-12+9*2^(1/3)*3^(2/3)...
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.16

$$\int \frac{x^8}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx$$
$$= \frac{-7884+324x-3990x^2-11610x^3-203x^4-9x^5}{34182\,(216+108x^2+324x^3+18x^4+x^6)}$$
$$- \frac{\text{RootSum}\left[216+108\#1^2+324\#1^3+18\#1^4+\#1^6\&, \frac{324\log(x-\#1)-96\log(x-\#1)\#1+324\log(x-\#1)\#1^2+\cdots}{36\#1+162\#1^2+12\#1^3\cdots}\right.}{205092}$$

input
```
Integrate[x^8/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]
```

output
```
(-7884 + 324*x - 3990*x^2 - 11610*x^3 - 203*x^4 - 9*x^5)/(34182*(216 + 108
*x^2 + 324*x^3 + 18*x^4 + x^6)) - RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#
1^4 + #1^6 & , (324*Log[x - #1] - 96*Log[x - #1]*#1 + 324*Log[x - #1]*#1^2
 + 406*Log[x - #1]*#1^3 + 9*Log[x - #1]*#1^4)/(36*#1 + 162*#1^2 + 12*#1^3
+ #1^5) & ]/205092
```

Rubi [A] (verified)

Time = 4.50 (sec) , antiderivative size = 1012, normalized size of antiderivative = 0.95, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2466, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{(x^6+18x^4+324x^3+108x^2+216)^2}dx$$

$$\Big\downarrow \text{2466}$$

$$1586874322944\int \left(-\frac{i(27-x)}{771220920950784\sqrt[3]{2}\sqrt[6]{3}\left(1+\sqrt[3]{-1}\right)^5\left(x^2+3(-2)^{2/3}\sqrt[3]{3}x+6\right)}+\frac{27}{6940988288557056\sqrt[3]{2}3\ldots}\right.$$

$$\Big\downarrow \text{2009}$$

$$1586874322944\left(-\frac{\sqrt[3]{-\frac{1}{3}}\left(\left(2+27(-2)^{2/3}\sqrt[3]{3}+12\sqrt[3]{-2}3^{2/3}\right)x+9\left(6-(-2)^{2/3}\sqrt[3]{3}\right)\right)}{2313662762852352\ 2^{2/3}\left(4+3\sqrt[3]{-2}3^{2/3}\right)\left(x^2+3(-2)^{2/3}\sqrt[3]{3}x+6\right)}-\frac{\sqrt[3]{-1}\left(2+27(-2)^{2/3}\sqrt[3]{\ldots}\right.}{231366276\ldots}\right.$$

input
```
Int[x^8/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]
```

output
```
1586874322944*(-1/257073640316928*((-1/3)^(1/3)*(9*(6 + (-3)^(1/3)*2^(2/3)
) + (2 - 3*2^(2/3)*(2*(-6)^(2/3) + 9*(-3)^(1/3)))*x))/(2^(2/3)*(1 + (-1)^(
1/3))^4*(4 - 3*(-3)^(2/3)*2^(1/3))*(6 - 3*(-3)^(1/3)*2^(2/3)*x + x^2)) - (
(-1/3)^(1/3)*(9*(6 - (-2)^(2/3)*3^(1/3)) + (2 + 27*(-2)^(2/3)*3^(1/3) + 12
*(-2)^(1/3)*3^(2/3))*x))/(2313662762852352*2^(2/3)*(4 + 3*(-2)^(1/3)*3^(2/
3))*(6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2)) + (9*(6 - 2^(2/3)*3^(1/3)) + (2 +
2^(2/3)*(27*3^(1/3) - 6*6^(2/3)))*x)/(2313662762852352*2^(2/3)*3^(1/3)*(4
- 3*2^(1/3)*3^(2/3))*(6 + 3*2^(2/3)*3^(1/3)*x + x^2)) - ((I/25707364031692
8)*((-2)^(2/3) + 6*3^(2/3))*ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sqrt[6*(4
+ 3*(-2)^(1/3)*3^(2/3))]])/(2^(5/6)*3^(1/3)*(1 + (-1)^(1/3))^5*Sqrt[4 + 3*
(-2)^(1/3)*3^(2/3)]) - ((-1)^(1/3)*(2 + 27*(-2)^(2/3)*3^(1/3) + 12*(-2)^(1
/3)*3^(2/3))*ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*
3^(2/3))]])/(2313662762852352*2^(1/6)*3^(5/6)*(4 + 3*(-2)^(1/3)*3^(2/3))^(
3/2)) - ((-1)^(1/3)*(6*(-6)^(2/3) + 27*(-3)^(1/3) - 2^(1/3))*ArcTan[(2^(1/
6)*(3*(-3)^(1/3) - 2^(1/3)*x))/Sqrt[3*(4 - 3*(-3)^(2/3)*2^(1/3))]])/(12853
6820158464*Sqrt[2]*3^(5/6)*(1 + (-1)^(1/3))^4*(4 - 3*(-3)^(2/3)*2^(1/3))^(
3/2)) + ((I*2^(2/3) - 9*3^(1/6) - (3*I)*3^(2/3))*ArcTan[(2^(1/6)*(3*(-3)^(
1/3) - 2^(1/3)*x))/Sqrt[3*(4 - 3*(-3)^(2/3)*2^(1/3))]])/(257073640316928*2
^(5/6)*3^(1/3)*(1 + (-1)^(1/3))^5*Sqrt[4 - 3*(-3)^(2/3)*2^(1/3)]) - ((1 +
3*2^(1/3)*3^(2/3))*ArcTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4...
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2466
```
Int[(u_.)*(Q6_)^(p_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
 x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Sim
p[1/(3^(3*p)*a^(2*p))   Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*
x + b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*
(-1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d,
 0] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coef
f[Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.11

method	result
default	$\frac{-\frac{1}{3798}x^5-\frac{203}{34182}x^4-\frac{215}{633}x^3-\frac{665}{5697}x^2+\frac{2}{211}x-\frac{146}{633}}{x^6+18x^4+324x^3+108x^2+216}+\frac{\left(\sum_{\_R=\mathrm{RootOf}(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216)}\frac{(-9\_R^4-406\_R^3-324}{\_R^5+12\_}\right.}{205092}$
risch	$\frac{-\frac{1}{3798}x^5-\frac{203}{34182}x^4-\frac{215}{633}x^3-\frac{665}{5697}x^2+\frac{2}{211}x-\frac{146}{633}}{x^6+18x^4+324x^3+108x^2+216}+\frac{\left(\sum_{\_R=\mathrm{RootOf}(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216)}\frac{(-9\_R^4-406\_R^3-324}{\_R^5+12\_}\right.}{205092}$

input
```
int(x^8/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x,method=_RETURNVERBOSE)
```

output
```
(-1/3798*x^5-203/34182*x^4-215/633*x^3-665/5697*x^2+2/211*x-146/633)/(x^6+
18*x^4+324*x^3+108*x^2+216)+1/205092*sum((-9*_R^4-406*_R^3-324*_R^2+96*_R-
324)/(_R^5+12*_R^3+162*_R^2+36*_R)*ln(x-_R),_R=RootOf(_Z^6+18*_Z^4+324*_Z^
3+108*_Z^2+216))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^8}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx=\text{Timed out}$$

input
```
integrate(x^8/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.11

$$\int \frac{x^8}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx$$
$$= \operatorname{RootSum}\Big(8525601705296418741512336066457 6t^6 + 501051915333854345687 04t^4 + 488857480512774$$
$$+ \frac{-9x^5-203x^4-11610x^3-3990x^2+324x-7884}{34182x^6+615276x^4+11074968x^3+3691656x^2+7383312}$$

input
```
integrate(x**8/(x**6+18*x**4+324*x**3+108*x**2+216)**2,x)
```

output
```
RootSum(85256017052964187415123360664576*_t**6 + 50105191533385434568704*_
t**4 + 48885748051277486016*_t**3 + 865447782603408*_t**2 + 3220532460*_t
+ 4513, Lambda(_t, _t*log(3549203620408417440411919313548348746659076 4032*
_t**5/356900697070792948475845 - 19474160067218837086826809631017022308224
*_t**4/713801394141585896951 69 + 20779963076545132233894582764903396544*_t
**3/356900697070792948475845 + 2026521915436700497216219801203 7344*_t**2/3
56900697070792948475845 + 27519246894921053204907514537 2*_t/35690069707079
2948475845 + x + 1290285191292177289622012/10707020912123788454275 35))) +
(-9*x**5 - 203*x**4 - 11610*x**3 - 3990*x**2 + 324*x - 7884)/(34182*x**6 +
 615276*x**4 + 11074968*x**3 + 3691656*x**2 + 7383312)
```

Maxima [F]

$$\int \frac{x^8}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx = \int \frac{x^8}{(x^6+18\,x^4+324\,x^3+108\,x^2+216)^2}\,dx$$

input
```
integrate(x^8/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="maxima")
```

output
```
-1/34182*(9*x^5 + 203*x^4 + 11610*x^3 + 3990*x^2 - 324*x + 7884)/(x^6 + 18
*x^4 + 324*x^3 + 108*x^2 + 216) - 1/34182*integrate((9*x^4 + 406*x^3 + 324
*x^2 - 96*x + 324)/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

Giac [F]

$$\int \frac{x^8}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx = \int \frac{x^8}{(x^6+18\,x^4+324\,x^3+108\,x^2+216)^2}\,dx$$

input
```
integrate(x^8/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="giac")
```

output
```
integrate(x^8/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216)^2, x)
```

Mupad [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 388, normalized size of antiderivative = 0.37

$$\int \frac{x^8}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx = \text{Too large to display}$$

input
```
int(x^8/(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216)^2,x)
```

output
```
symsum(log((239491904*root(z^6 + (326*z^4)/554702231619 + (8113597*z^3)/14
14999241634398299 2 + (5171*z^2)/5093997269883833877 12 + (505*z)/1336868643
5083133627113728 + 4513/852560170529641874151233606645 76, z, k)*x)/8763068
43 - (275536*x)/638827688547 - (3848128*root(z^6 + (326*z^4)/554702231619
+ (8113597*z^3)/14149992416343982992 + (5171*z^2)/5093997269883833877 12 +
(505*z)/1336868643508313362711372 8 + 4513/852560170529641874151233606645 76
, z, k))/3606201 - (152363520*root(z^6 + (326*z^4)/554702231619 + (8113597
*z^3)/14149992416343982992 + (5171*z^2)/5093997269883833877 12 + (505*z)/13
3686864350831336271137 28 + 4513/852560170529641874151233606645 76, z, k)^2*
x)/44521 - (698075283456*root(z^6 + (326*z^4)/554702231619 + (8113597*z^3)
/14149992416343982992 + (5171*z^2)/5093997269883833877 12 + (505*z)/1336868
643508313362711372 8 + 4513/852560170529641874151233606645 76, z, k)^3*x)/44
521 + (130789789876224*root(z^6 + (326*z^4)/554702231619 + (8113597*z^3)/1
4149992416343982992 + (5171*z^2)/5093997269883833877 12 + (505*z)/133686864
35083133627113728 + 4513/852560170529641874151233606645 76, z, k)^4*x)/211
- 6940988288557056*root(z^6 + (326*z^4)/554702231619 + (8113597*z^3)/14149
992416343982992 + (5171*z^2)/5093997269883833877 12 + (505*z)/1336868643508
3133627113728 + 4513/852560170529641874151233606645 76, z, k)^5*x - (426422
0928*root(z^6 + (326*z^4)/554702231619 + (8113597*z^3)/1414999241634398299
2 + (5171*z^2)/5093997269883833877 12 + (505*z)/1336868643508313362711 37...
```

Reduce [F]

$$\int \frac{x^8}{(216 + 108x^2 + 324x^3 + 18x^4 + x^6)^2}\,dx = \text{too large to display}$$

input
```
int(x^8/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x)
```

output

```
( - 134946*int(x**10/(x**12 + 36*x**10 + 648*x**9 + 540*x**8 + 11664*x**7
+ 109296*x**6 + 69984*x**5 + 19440*x**4 + 139968*x**3 + 46656*x**2 + 46656
),x)*x**6 - 2429028*int(x**10/(x**12 + 36*x**10 + 648*x**9 + 540*x**8 + 11
664*x**7 + 109296*x**6 + 69984*x**5 + 19440*x**4 + 139968*x**3 + 46656*x**
2 + 46656),x)*x**4 - 43722504*int(x**10/(x**12 + 36*x**10 + 648*x**9 + 540
*x**8 + 11664*x**7 + 109296*x**6 + 69984*x**5 + 19440*x**4 + 139968*x**3 +
 46656*x**2 + 46656),x)*x**3 - 14574168*int(x**10/(x**12 + 36*x**10 + 648*
x**9 + 540*x**8 + 11664*x**7 + 109296*x**6 + 69984*x**5 + 19440*x**4 + 139
968*x**3 + 46656*x**2 + 46656),x)*x**2 - 29148336*int(x**10/(x**12 + 36*x*
*10 + 648*x**9 + 540*x**8 + 11664*x**7 + 109296*x**6 + 69984*x**5 + 19440*
x**4 + 139968*x**3 + 46656*x**2 + 46656),x) + 269892*int(x**9/(x**12 + 36*
x**10 + 648*x**9 + 540*x**8 + 11664*x**7 + 109296*x**6 + 69984*x**5 + 1944
0*x**4 + 139968*x**3 + 46656*x**2 + 46656),x)*x**6 + 4858056*int(x**9/(x**
12 + 36*x**10 + 648*x**9 + 540*x**8 + 11664*x**7 + 109296*x**6 + 69984*x**
5 + 19440*x**4 + 139968*x**3 + 46656*x**2 + 46656),x)*x**4 + 87445008*int(
x**9/(x**12 + 36*x**10 + 648*x**9 + 540*x**8 + 11664*x**7 + 109296*x**6 +
69984*x**5 + 19440*x**4 + 139968*x**3 + 46656*x**2 + 46656),x)*x**3 + 2914
8336*int(x**9/(x**12 + 36*x**10 + 648*x**9 + 540*x**8 + 11664*x**7 + 10929
6*x**6 + 69984*x**5 + 19440*x**4 + 139968*x**3 + 46656*x**2 + 46656),x)*x*
*2 + 58296672*int(x**9/(x**12 + 36*x**10 + 648*x**9 + 540*x**8 + 11664*...
```

3.24 $\int \frac{x^7}{(216+108x^2+324x^3+18x^4+x^6)^2} \, dx$

Optimal result

Integrand size = 26, antiderivative size = 1001

$$\int \frac{x^7}{(216+108x^2+324x^3+18x^4+x^6)^2} \, dx = \text{Too large to display}$$

output
```
-1/1944*(4*(-1)^(1/3)*3^(2/3)+18*6^(1/3)-9*((-2)^(2/3)+2*(-1)^(1/3)*3^(2/3
))*x)*2^(1/3)/(1+(-1)^(1/3))^4/(4-3*(-3)^(2/3)*2^(1/3))/(6-3*(-3)^(1/3)*2^
(2/3)*x+x^2)-1/4374*((-6)^(1/3)*(9*(-2)^(1/3)+2*3^(1/3))-9*(1+(-2)^(1/3)*3
^(2/3))*x)/(8+9*I*2^(1/3)*3^(1/6)+3*2^(1/3)*3^(2/3))/(6+3*(-2)^(2/3)*3^(1/
3)*x+x^2)+1/17496*(4-6*2^(1/3)*3^(2/3)-3*(6-2^(2/3)*3^(1/3))*x)*2^(1/3)*3^
(2/3)/(4-3*2^(1/3)*3^(2/3))/(6+3*2^(2/3)*3^(1/3)*x+x^2)+1/5832*(9*I+3^(1/3
)*(2*I*2^(2/3)-9*3^(1/6)+2*2^(2/3)*3^(1/2)))*arctan((3*(-3)^(1/3)*2^(2/3)-
2*x)/(24-18*(-3)^(2/3)*2^(1/3))^(1/2))/(1+(-1)^(1/3))^5/(8-6*(-3)^(2/3)*2^
(1/3))^(1/2)+1/324*(1+(-2)^(1/3)*3^(2/3))*arctan((3*(-2)^(2/3)*3^(1/3)+2*x
)/(24+18*(-2)^(1/3)*3^(2/3))^(1/2))*6^(1/2)/(1-(-1)^(1/3))^2/(1+(-1)^(1/3)
)^4/(4+3*(-2)^(1/3)*3^(2/3))^(3/2)-1/5832*(9*I-3^(1/3)*(4*I*2^(2/3)+9*3^(1
/6)))*arctan((3*(-2)^(2/3)*3^(1/3)+2*x)/(24+18*(-2)^(1/3)*3^(2/3))^(1/2))/
(1+(-1)^(1/3))^5/(8+6*(-2)^(1/3)*3^(2/3))^(1/2)-1/324*(-1)^(1/3)*((-3)^(1/
3)+3*2^(1/3))*arctan(2^(1/6)*(3*(-3)^(1/3)-2^(1/3)*x)/(12-9*(-3)^(2/3)*2^(
1/3))^(1/2))*2^(1/2)*3^(1/6)/(1+(-1)^(1/3))^4/(4-3*(-3)^(2/3)*2^(1/3))^(3/
2)+1/324*(1-2^(1/3)*3^(2/3))*arctanh(2^(1/6)*(3*3^(1/3)+2^(1/3)*x)/(-12+9*
2^(1/3)*3^(2/3))^(1/2))*6^(1/2)/(1-(-1)^(1/3))^2/(1+(-1)^(1/3))^4/(-4+3*2^
(1/3)*3^(2/3))^(3/2)+1/78732*(2*2^(2/3)+3*3^(2/3))*arctanh(2^(1/6)*(3*3^(1
/3)+2^(1/3)*x)/(-12+9*2^(1/3)*3^(2/3))^(1/2))*3^(5/6)/(-8+6*2^(1/3)*3^(2/3
))^(1/2)+1/3888*I*ln(6-3*(-3)^(1/3)*2^(2/3)*x+x^2)*2^(1/3)*3^(1/6)/(1+(...
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.17

$$\int \frac{x^7}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx = \frac{648-96x+432x^2+908x^3-18x^4+73x^5}{68364\,(216+108x^2+324x^3+18x^4+x^6)} + \frac{\text{RootSum}\Big[216+108\#1^2+324\#1^3+18\#1^4+\#1^6\&,\ \frac{96\log(x-\#1)-216\log(x-\#1)\#1+96\log(x-\#1)\#1^2-36}{36\#1+162\#1^2+12\#1^3+}}{410184}$$

input
```
Integrate[x^7/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]
```

output
```
(648 - 96*x + 432*x^2 + 908*x^3 - 18*x^4 + 73*x^5)/(68364*(216 + 108*x^2 +
 324*x^3 + 18*x^4 + x^6)) + RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^4 +
#1^6 & , (96*Log[x - #1] - 216*Log[x - #1]*#1 + 96*Log[x - #1]*#1^2 - 36*L
og[x - #1]*#1^3 + 73*Log[x - #1]*#1^4)/(36*#1 + 162*#1^2 + 12*#1^3 + #1^5)
 & ]/410184
```

Rubi [A] (verified)

Time = 4.48 (sec) , antiderivative size = 954, normalized size of antiderivative = 0.95, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2466, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{(x^6+18x^4+324x^3+108x^2+216)^2}\,dx$$

$$\Big\downarrow \text{2466}$$

$$1586874322944\int\left(\frac{3i\sqrt[3]{2}\sqrt[6]{3}x-i2^{2/3}3^{5/6}-9i\sqrt{3}-3\ 2^{2/3}\sqrt[3]{3}+27}{9254651051409408\left(1+\sqrt[3]{-1}\right)^5\left(x^2-3\sqrt[3]{-3}2^{2/3}x+6\right)}+\frac{2^{2/3}\left(27-9i\sqrt{3}+2i2^{2/3}3^{5}\right.}{9254651051409408\ 2^{2/3}\left(1+\sqrt[3]{-}\right.}\right.$$

$$\Big\downarrow \text{2009}$$

$$1586874322944\left(\frac{2\left(2-3\sqrt[3]{2}3^{2/3}\right)-3\left(6-2^{2/3}\sqrt[3]{3}\right)x}{4627325525704704\ 2^{2/3}\sqrt[3]{3}\left(4-3\sqrt[3]{2}3^{2/3}\right)\left(x^2+3\ 2^{2/3}\sqrt[3]{3}x+6\right)}-\frac{\left(9i-\sqrt[3]{3}\left(4i2^{2/3}+9\sqrt[6]{3}\right)\right.}{9254651051409408\left(1+\right.}\right.$$

input
```
Int[x^7/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]
```

output
```
1586874322944*(-1/1542441841901568*(2*(2*(-1)^(1/3)*3^(2/3) + 9*6^(1/3)) -
 9*((-2)^(2/3) + 2*(-1)^(1/3)*3^(2/3))*x)/(2^(2/3)*(1 + (-1)^(1/3))^4*(4 -
 3*(-3)^(2/3)*2^(1/3))*(6 - 3*(-3)^(1/3)*2^(2/3)*x + x^2)) - ((-6)^(1/3)*(
9*(-2)^(1/3) + 2*3^(1/3)) - 9*(1 + (-2)^(1/3)*3^(2/3))*x)/(13881976577114l
12*(4 + 3*(-2)^(1/3)*3^(2/3))*(6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2)) + (2*(2
- 3*2^(1/3)*3^(2/3)) - 3*(6 - 2^(2/3)*3^(1/3))*x)/(462732552570470 4*2^(2/3
)*3^(1/3)*(4 - 3*2^(1/3)*3^(2/3))*(6 + 3*2^(2/3)*3^(1/3)*x + x^2)) + ((1 +
 (-2)^(1/3)*3^(2/3))*ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2
)^(1/3)*3^(2/3))]])/(771220920950784*Sqrt[6]*(4 + 3*(-2)^(1/3)*3^(2/3))^(3
/2)) - ((9*I - 3^(1/3)*((4*I)*2^(2/3) + 9*3^(1/6)))*ArcTan[(3*(-2)^(2/3)*3
^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3))]])/(9254651051409408*(1 +
(-1)^(1/3))^5*Sqrt[2*(4 + 3*(-2)^(1/3)*3^(2/3))]) - ((-1)^(1/3)*((-3)^(1/3
) + 3*2^(1/3))*ArcTan[(2^(1/6)*(3*(-3)^(1/3) - 2^(1/3)*x))/Sqrt[3*(4 - 3*(
-3)^(2/3)*2^(1/3))]])/(85691213438976*Sqrt[2]*3^(5/6)*(1 + (-1)^(1/3))^4*(
4 - 3*(-3)^(2/3)*2^(1/3))^(3/2)) + ((9*I + 3^(1/3)*((2*I)*2^(2/3) - 9*3^(1
/6) + 2*2^(2/3)*Sqrt[3]))*ArcTan[(2^(1/6)*(3*(-3)^(1/3) - 2^(1/3)*x))/Sqrt
[3*(4 - 3*(-3)^(2/3)*2^(1/3))]])/(9254651051409408*(1 + (-1)^(1/3))^5*Sqrt
[2*(4 - 3*(-3)^(2/3)*2^(1/3))]) + ((1 - 2^(1/3)*3^(2/3))*ArcTanh[(2^(1/6)*
(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(2/3))]])/(7712209209507
84*Sqrt[6]*(-4 + 3*2^(1/3)*3^(2/3))^(3/2)) + ((2*2^(2/3) + 3*3^(2/3))*A...
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2466
```
Int[(u_.)*(Q6_)^(p_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
 x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Sim
p[1/(3^(3*p)*a^(2*p))   Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*
x + b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*
(-1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d,
 0] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coef
f[Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.12

method	result
default	$\frac{\frac{73}{68364}x^5-\frac{1}{3798}x^4+\frac{227}{17091}x^3+\frac{4}{633}x^2-\frac{8}{5697}x+\frac{2}{211}}{x^6+18x^4+324x^3+108x^2+216}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)}\frac{\left(73\_R^4-36\_R^3+96\_\right.}{\_R^5+12\_R}\right.}{410184}$
risch	$\frac{\frac{73}{68364}x^5-\frac{1}{3798}x^4+\frac{227}{17091}x^3+\frac{4}{633}x^2-\frac{8}{5697}x+\frac{2}{211}}{x^6+18x^4+324x^3+108x^2+216}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)}\frac{\left(73\_R^4-36\_R^3+96\_\right.}{\_R^5+12\_R}\right.}{410184}$

input
```
int(x^7/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x,method=_RETURNVERBOSE)
```

output
```
(73/68364*x^5-1/3798*x^4+227/17091*x^3+4/633*x^2-8/5697*x+2/211)/(x^6+18*x
^4+324*x^3+108*x^2+216)+1/410184*sum((73*_R^4-36*_R^3+96*_R^2-216*_R+96)/(
_R^5+12*_R^3+162*_R^2+36*_R)*ln(x-_R),_R=RootOf(_Z^6+18*_Z^4+324*_Z^3+108*
_Z^2+216))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^7}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx=\text{Timed out}$$

input
```
integrate(x^7/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.11

$$\int \frac{x^7}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx$$
$$=\mathrm{RootSum}\Big(589289589870088463413332668913549312t^6-539640290266075248405737472t^4+9218263$$
$$+\frac{73x^5-18x^4+908x^3+432x^2-96x+648}{68364x^6+1230552x^4+22149936x^3+7383312x^2+14766624}$$

input
```
integrate(x**7/(x**6+18*x**4+324*x**3+108*x**2+216)**2,x)
```

output
```
RootSum(589289589870088463413332668913549312*_t**6 - 539640290266075248405
737472*_t**4 + 92182638168509682392064*_t**3 - 553241442069170496*_t**2 -
3759837842016*_t - 7197829, Lambda(_t, _t*log(4299602763972744771400374330
5160746111018438501025999323136*_t**5/154206009791052044490694380303237521
 - 425847662595081946846897154744422251405157209835847680*_t**4/15420600979
1052044490694380303237521 - 3751244612884958815010836944932375407831734108
2112*_t**3/154206009791052044490694380303237521 + 715203759402167526763889
0715531672481920222144*_t**2/154206009791052044490694380303237521 - 442275
46998835297723830291794974310524032*_t/15420600979105204449069438030323752
1 + x - 174573349036676047734132569583024855/15420600979105204449069438030
3237521))) + (73*x**5 - 18*x**4 + 908*x**3 + 432*x**2 - 96*x + 648)/(68364
*x**6 + 1230552*x**4 + 22149936*x**3 + 7383312*x**2 + 14766624)
```

Maxima [F]

$$\int \frac{x^7}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx=\int \frac{x^7}{(x^6+18\,x^4+324\,x^3+108\,x^2+216)^2}\,dx$$

input
```
integrate(x^7/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="maxima")
```

output
```
1/68364*(73*x^5 - 18*x^4 + 908*x^3 + 432*x^2 - 96*x + 648)/(x^6 + 18*x^4 +
 324*x^3 + 108*x^2 + 216) + 1/68364*integrate((73*x^4 - 36*x^3 + 96*x^2 -
216*x + 96)/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

Giac [F]

$$\int \frac{x^7}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx=\int \frac{x^7}{(x^6+18\,x^4+324\,x^3+108\,x^2+216)^2}\,dx$$

input
```
integrate(x^7/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="giac")
```

output
```
integrate(x^7/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216)^2, x)
```

Mupad [B] (verification not implemented)

Time = 22.18 (sec) , antiderivative size = 387, normalized size of antiderivative = 0.39

$$\int \frac{x^7}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx = \text{Too large to display}$$

input
```
int(x^7/(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216)^2,x)
```

output
```
symsum(log((8336932*root(z^6 - (292589*z^4)/319508485412544 + (11805253*z^
3)/7546662622050124624 - (2479189*z^2)/264072818470777948189 9008 - (19897
87*z)/311864717157619341253309046784 - 7197829/5892895898700884634133326 68
913549312, z, k))/97367427 - (480227*x)/851770251396 - (759164282*root(z^6
 - (292589*z^4)/319508485412544 + (11805253*z^3)/7546662622050124624 - (2
479189*z^2)/2640728184707779481899008 - (1989787*z)/311864717157619341253 3
09046784 - 7197829/58928958987008846341333266891354 9312, z, k)*x)/78867615
87 - (207565888*root(z^6 - (292589*z^4)/319508485412544 + (11805253*z^3)/7
546662622050124262 4 - (2479189*z^2)/2640728184707779481899008 - (1989787*z
)/311864717157619341253309046784 - 7197829/58928958987008846341333266891 35
49312, z, k)^2*x)/400689 - (108430970112*root(z^6 - (292589*z^4)/319508485
412544 + (11805253*z^3)/7546662622050124624 - (2479189*z^2)/2640728184707
779481899008 - (1989787*z)/311864717157619341253309046784 - 7197829/589289
5898700884634133326689135 49312, z, k)^3*x)/44521 - (147138513610752*root(z
^6 - (292589*z^4)/319508485412544 + (11805253*z^3)/7546662622050124624 -
(2479189*z^2)/2640728184707779481899008 - (1989787*z)/31186471715761934125
3309046784 - 7197829/58928958987008846341333266891354 9312, z, k)^4*x)/211
- 6940988288557056*root(z^6 - (292589*z^4)/319508485412544 + (11805253*z^3
)/7546662622050124624 - (2479189*z^2)/2640728184707779481899008 - (198978
7*z)/311864717157619341253309046784 - 7197829/5892895898700884634133326...
```

Reduce [F]

$$\int \frac{x^7}{(216 + 108x^2 + 324x^3 + 18x^4 + x^6)^2} \, dx = \text{too large to display}$$

input
```
int(x^7/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x)
```

output
```
( - 2916*int(x**8/(x**12 + 36*x**10 + 648*x**9 + 540*x**8 + 11664*x**7 + 1
09296*x**6 + 69984*x**5 + 19440*x**4 + 139968*x**3 + 46656*x**2 + 46656),x
)*x**6 - 52488*int(x**8/(x**12 + 36*x**10 + 648*x**9 + 540*x**8 + 11664*x*
*7 + 109296*x**6 + 69984*x**5 + 19440*x**4 + 139968*x**3 + 46656*x**2 + 46
656),x)*x**4 - 944784*int(x**8/(x**12 + 36*x**10 + 648*x**9 + 540*x**8 + 1
1664*x**7 + 109296*x**6 + 69984*x**5 + 19440*x**4 + 139968*x**3 + 46656*x*
*2 + 46656),x)*x**3 - 314928*int(x**8/(x**12 + 36*x**10 + 648*x**9 + 540*x
**8 + 11664*x**7 + 109296*x**6 + 69984*x**5 + 19440*x**4 + 139968*x**3 + 4
6656*x**2 + 46656),x)*x**2 - 629856*int(x**8/(x**12 + 36*x**10 + 648*x**9
+ 540*x**8 + 11664*x**7 + 109296*x**6 + 69984*x**5 + 19440*x**4 + 139968*x
**3 + 46656*x**2 + 46656),x) - 125496*int(x**6/(x**12 + 36*x**10 + 648*x**
9 + 540*x**8 + 11664*x**7 + 109296*x**6 + 69984*x**5 + 19440*x**4 + 139968
*x**3 + 46656*x**2 + 46656),x)*x**6 - 2258928*int(x**6/(x**12 + 36*x**10 +
 648*x**9 + 540*x**8 + 11664*x**7 + 109296*x**6 + 69984*x**5 + 19440*x**4
+ 139968*x**3 + 46656*x**2 + 46656),x)*x**4 - 40660704*int(x**6/(x**12 + 3
6*x**10 + 648*x**9 + 540*x**8 + 11664*x**7 + 109296*x**6 + 69984*x**5 + 19
440*x**4 + 139968*x**3 + 46656*x**2 + 46656),x)*x**3 - 13553568*int(x**6/(
x**12 + 36*x**10 + 648*x**9 + 540*x**8 + 11664*x**7 + 109296*x**6 + 69984*
x**5 + 19440*x**4 + 139968*x**3 + 46656*x**2 + 46656),x)*x**2 - 27107136*i
nt(x**6/(x**12 + 36*x**10 + 648*x**9 + 540*x**8 + 11664*x**7 + 109296*x...
```

3.25 $\int \frac{x^6}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 677

$$\int \frac{x^6}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx$$
$$= \frac{9(-2)^{2/3}+\sqrt[3]{6}\left(9+\sqrt[3]{-3}2^{2/3}\right)x}{2916\ 2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)}$$
$$+\frac{9\ 2^{2/3}+\sqrt[3]{-1}3^{2/3}\left(2+3\sqrt[3]{-2}3^{2/3}\right)x}{13122\ 2^{2/3}\left(8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}3^{2/3}\right)\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)}$$
$$+\frac{3\ 2^{2/3}\sqrt[3]{3}-\left(2-3\sqrt[3]{2}3^{2/3}\right)x}{8748\ 2^{2/3}\sqrt[3]{3}\left(4-3\sqrt[3]{2}3^{2/3}\right)\left(6+3\ 2^{2/3}\sqrt[3]{3}x+x^2\right)}$$
$$+\frac{\sqrt[3]{-1}\left(3(-3)^{2/3}-2^{2/3}\right)\arctan\left(\frac{3\sqrt[3]{-3}2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{486\ 6^{5/6}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)^{3/2}}$$
$$+\frac{\left(3(-3)^{2/3}+\sqrt[3]{-1}2^{2/3}\right)\arctan\left(\frac{3(-2)^{2/3}\sqrt[3]{3}+2x}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{486\ 6^{5/6}\left(1-\sqrt[3]{-1}\right)^2\left(1+\sqrt[3]{-1}\right)^4\left(4+3\sqrt[3]{-2}3^{2/3}\right)^{3/2}}$$
$$-\frac{\left(2^{2/3}-3\ 3^{2/3}\right)\operatorname{arctanh}\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{3}+\sqrt[3]{2}x\right)}{\sqrt{3\left(-4+3\sqrt[3]{2}3^{2/3}\right)}}\right)}{486\ 6^{5/6}\left(1-\sqrt[3]{-1}\right)^2\left(1+\sqrt[3]{-1}\right)^4\left(-4+3\sqrt[3]{2}3^{2/3}\right)^{3/2}}$$
$$+\frac{\sqrt[6]{-\frac{1}{3}}\log\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)}{5832\sqrt[3]{2}\left(1+\sqrt[3]{-1}\right)^5}$$
$$-\frac{i\log\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)}{5832\sqrt[3]{2}\sqrt[6]{3}\left(1+\sqrt[3]{-1}\right)^5}+\frac{\log\left(6+3\ 2^{2/3}\sqrt[3]{3}x+x^2\right)}{52488\sqrt[3]{2}3^{2/3}}$$

output
```
1/5832*(9*(-2)^(2/3)+6^(1/3)*(9+(-3)^(1/3)*2^(2/3))*x)*2^(1/3)/(1+(-1)^(1/
3))^4/(4-3*(-3)^(2/3)*2^(1/3))/(6-3*(-3)^(1/3)*2^(2/3)*x+x^2)+1/26244*(9*2
^(2/3)+(-1)^(1/3)*3^(2/3)*(2+3*(-2)^(1/3)*3^(2/3))*x)*2^(1/3)/(8+9*I*2^(1/
3)*3^(1/6)+3*2^(1/3)*3^(2/3))/(6+3*(-2)^(2/3)*3^(1/3)*x+x^2)+1/52488*(3*2^
(2/3)*3^(1/3)-(2-3*2^(1/3)*3^(2/3))*x)*2^(1/3)*3^(2/3)/(4-3*2^(1/3)*3^(2/3
))/(6+3*2^(2/3)*3^(1/3)*x+x^2)+1/2916*(-1)^(1/3)*(3*(-3)^(2/3)-2^(2/3))*ar
ctan((3*(-3)^(1/3)*2^(2/3)-2*x)/(24-18*(-3)^(2/3)*2^(1/3))^(1/2))*6^(1/6)/
(1+(-1)^(1/3))^4/(4-3*(-3)^(2/3)*2^(1/3))^(3/2)+1/2916*(3*(-3)^(2/3)+(-1)^
(1/3)*2^(2/3))*arctan((3*(-2)^(2/3)*3^(1/3)+2*x)/(24+18*(-2)^(1/3)*3^(2/3)
)^(1/2))*6^(1/6)/(1-(-1)^(1/3))^2/(1+(-1)^(1/3))^4/(4+3*(-2)^(1/3)*3^(2/3)
)^(3/2)-1/2916*(2^(2/3)-3*3^(2/3))*arctanh(2^(1/6)*(3*3^(1/3)+2^(1/3)*x)/(
-12+9*2^(1/3)*3^(2/3))^(1/2))*6^(1/6)/(1-(-1)^(1/3))^2/(1+(-1)^(1/3))^4/(-
4+3*2^(1/3)*3^(2/3))^(3/2)+1/34992*(-1)^(1/6)*3^(5/6)*ln(6-3*(-3)^(1/3)*2^
(2/3)*x+x^2)*2^(2/3)/(1+(-1)^(1/3))^5-1/34992*I*ln(6+3*(-2)^(2/3)*3^(1/3)*
x+x^2)*2^(2/3)*3^(5/6)/(1+(-1)^(1/3))^5+1/314928*ln(6+3*2^(2/3)*3^(1/3)*x+
x^2)*2^(2/3)*3^(1/3)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.25

$$\int \frac{x^6}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx = \frac{-96+108x-64x^2-72x^3+73x^4-3x^5}{68364\,(216+108x^2+324x^3+18x^4+x^6)} - \frac{\text{RootSum}\left[216+108\#1^2+324\#1^3+18\#1^4+\#1^6\&,\frac{108\log(x-\#1)-32\log(x-\#1)\#1+108\log(x-\#1)\#1^2-1}{36\#1+162\#1^2+12\#1^3-}\right.}{410184}$$

input
```
Integrate[x^6/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]
```

output
```
(-96 + 108*x - 64*x^2 - 72*x^3 + 73*x^4 - 3*x^5)/(68364*(216 + 108*x^2 + 3
24*x^3 + 18*x^4 + x^6)) - RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^4 + #1
^6 & , (108*Log[x - #1] - 32*Log[x - #1]*#1 + 108*Log[x - #1]*#1^2 - 146*L
og[x - #1]*#1^3 + 3*Log[x - #1]*#1^4)/(36*#1 + 162*#1^2 + 12*#1^3 + #1^5)
& ]/410184
```

Rubi [A] (warning: unable to verify)

Time = 3.25 (sec) , antiderivative size = 638, normalized size of antiderivative = 0.94, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2466, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{(x^6+18x^4+324x^3+108x^2+216)^2}\,dx$$

$$\Big\downarrow \text{2466}$$

$$1586874322944\int \left(\frac{3^{5/6}\left(1+\sqrt[3]{-1}\right)-i\sqrt[3]{2}x}{4627325525704704\ 2^{2/3}\sqrt[6]{3}\left(1+\sqrt[3]{-1}\right)^5\left(x^2+3(-2)^{2/3}\sqrt[3]{3}x+6\right)}-\frac{3i3^{5/}}{4627325525704704\ 6^{2/}}\right.$$

$$\Big\downarrow \text{2009}$$

$$1586874322944\left(\frac{\left(3(-3)^{2/3}+\sqrt[3]{-1}2^{2/3}\right)\arctan\left(\frac{2x+3(-2)^{2/3}\sqrt[3]{3}}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{6940988288557056\ 6^{5/6}\left(4+3\sqrt[3]{-2}3^{2/3}\right)^{3/2}}-\frac{\left(\sqrt[3]{-1}2^{2/3}+3\ 3^{2/3}\right)\arctan\left(\frac{\sqrt[6]{2}(3}{\sqrt{3(4-}}\right.}{771220920950784\ 6^{5/6}\left(1+\sqrt[3]{-1}\right)^4(4}\right.$$

input
```
Int[x^6/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]
```

output
```
1586874322944*(((-1)^(2/3)*(6*3^(1/3) - (-2)^(1/3)*(2*(-1)^(1/3) + 3*2^(1/
3)*3^(2/3))*x))/(3084883683803136*3^(1/3)*(1 + (-1)^(1/3))^4*(4 - 3*(-3)^(
2/3)*2^(1/3))*(6 - 3*(-3)^(1/3)*2^(2/3)*x + x^2)) + (3*2^(2/3)*3^(1/3) + (
-1)^(1/3)*(2 + 3*(-2)^(1/3)*3^(2/3))*x)/(13881976577114112*2^(2/3)*3^(1/3)
*(4 + 3*(-2)^(1/3)*3^(2/3))*(6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2)) + (6*3^(1/
3) - (2*2^(1/3) - 3*6^(2/3))*x)/(27763953154228224*3^(1/3)*(4 - 3*2^(1/3)*
3^(2/3))*(6 + 3*2^(2/3)*3^(1/3)*x + x^2)) + ((3*(-3)^(2/3) + (-1)^(1/3)*2^
(2/3))*ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3
))]])/(6940988288557056*6^(5/6)*(4 + 3*(-2)^(1/3)*3^(2/3))^(3/2)) - (((-1)
^(1/3)*2^(2/3) + 3*3^(2/3))*ArcTan[(2^(1/6)*(3*(-3)^(1/3) - 2^(1/3)*x))/Sq
rt[3*(4 - 3*(-3)^(2/3)*2^(1/3))]])/(771220920950784*6^(5/6)*(1 + (-1)^(1/3
))^4*(4 - 3*(-3)^(2/3)*2^(1/3))^(3/2)) - ((2^(2/3) - 3*3^(2/3))*ArcTanh[(2
^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(2/3))]])/(694098
8288557056*6^(5/6)*(-4 + 3*2^(1/3)*3^(2/3))^(3/2)) + ((-1/3)^(1/6)*Log[6 -
 3*(-3)^(1/3)*2^(2/3)*x + x^2])/(9254651051409408*2^(1/3)*(1 + (-1)^(1/3))
^5) - ((I/9254651051409408)*Log[6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2])/(2^(1/3
)*3^(1/6)*(1 + (-1)^(1/3))^5) + Log[6 + 3*2^(2/3)*3^(1/3)*x + x^2]/(832918
59462684672*2^(1/3)*3^(2/3)))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2466
```
Int[(u_.)*(Q6_)^(p_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
 x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Sim
p[1/(3^(3*p)*a^(2*p))   Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*
x + b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*
(-1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d,
 0] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coef
f[Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.18

method	result
default	$\frac{-\frac{1}{22788}x^5+\frac{73}{68364}x^4-\frac{2}{1899}x^3-\frac{16}{17091}x^2+\frac{1}{633}x-\frac{8}{5697}}{x^6+18x^4+324x^3+108x^2+216}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)}\frac{\left(-3\_R^4+146\_R^3 \right.}{\_R^5+12}\right.}{410184}$
risch	$\frac{-\frac{1}{22788}x^5+\frac{73}{68364}x^4-\frac{2}{1899}x^3-\frac{16}{17091}x^2+\frac{1}{633}x-\frac{8}{5697}}{x^6+18x^4+324x^3+108x^2+216}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)}\frac{\left(-3\_R^4+146\_R^3 \right.}{\_R^5+12}\right.}{410184}$

input
```
int(x^6/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x,method=_RETURNVERBOSE)
```

output
```
(-1/22788*x^5+73/68364*x^4-2/1899*x^3-16/17091*x^2+1/633*x-8/5697)/(x^6+18
*x^4+324*x^3+108*x^2+216)+1/410184*sum((-3*_R^4+146*_R^3-108*_R^2+32*_R-10
8)/(_R^5+12*_R^3+162*_R^2+36*_R)*ln(x-_R),_R=RootOf(_Z^6+18*_Z^4+324*_Z^3+
108*_Z^2+216))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^6}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx=\text{Timed out}$$

input
```
integrate(x^6/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.17

$$\int \frac{x^6}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx$$
$$=\text{RootSum}\Big(397770473162309712803999551516645785 6t^6-101031431941529596105095168 0t^4-20168$$
$$+\frac{-3x^5+73x^4-72x^3-64x^2+108x-96}{68364x^6+1230552x^4+22149936x^3+7383312x^2+14766624}$$

input
```
integrate(x**6/(x**6+18*x**4+324*x**3+108*x**2+216)**2,x)
```

output
```
RootSum(397770473162309712803999551516645785 6*_t**6 - 10103143194152959610
50951680*_t**4 - 20168224477093957151232*_t**3 - 112582856818899648*_t**2
- 50648453064*_t - 880007, Lambda(_t, _t*log(-273655567090018991570649941 4
1439556098619968804064460 8*_t**5/497978553961399002675733956 95 + 118370084
7019604608530864623076435429280504457011 2*_t**4/497978553961399002675733 95
695 - 10570581900446717266374077482873315047787008*_t**3/497978553961399 00
267573395695 - 15525474115694698723875632187927893 23392*_t**2/497978553961
39900267573395695 - 12542923791159140826909003250295928*_t/497978553961399
00267573395695 + x - 2306653387032032241083434829 6/4979785539613990026757 3
395695))) + (-3*x**5 + 73*x**4 - 72*x**3 - 64*x**2 + 108*x - 96)/(68364*x*
*6 + 1230552*x**4 + 22149936*x**3 + 7383312*x**2 + 14766624)
```

Maxima [F]

$$\int \frac{x^6}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx=\int \frac{x^6}{(x^6+18\,x^4+324\,x^3+108\,x^2+216)^2}\,dx$$

input
```
integrate(x^6/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="maxima")
```

output
```
-1/68364*(3*x^5 - 73*x^4 + 72*x^3 + 64*x^2 - 108*x + 96)/(x^6 + 18*x^4 + 3
24*x^3 + 108*x^2 + 216) - 1/68364*integrate((3*x^4 - 146*x^3 + 108*x^2 - 3
2*x + 108)/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

Giac [F]

$$\int \frac{x^6}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx = \int \frac{x^6}{(x^6+18\,x^4+324\,x^3+108\,x^2+216)^2}\,dx$$

input
```
integrate(x^6/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="giac")
```

output
```
integrate(x^6/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216)^2, x)
```

Mupad [B] (verification not implemented)

Time = 22.29 (sec) , antiderivative size = 388, normalized size of antiderivative = 0.57

$$\int \frac{x^6}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx = \text{Too large to display}$$

input
```
int(x^6/(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216)^2,x)
```

output
```
symsum(log((7028852*root(z^6 - (60865*z^4)/239631364059408 - (15496909*z^3
)/305639836193030032627 2 - (168169*z^2)/5941638415592503834272768 - (3971*
z)/311864717157619341253309046784 - 880007/397770473162309712803999551516
457856, z, k))/2628920529 - (1980083*x)/310470256633842 - (235710556*root(
z^6 - (60865*z^4)/239631364059408 - (15496909*z^3)/305639836193030032627 2
- (168169*z^2)/5941638415592503834272768 - (3971*z)/311864717157619341253
309046784 - 880007/397770473162309712803999551516645785 6, z, k)*x)/70980854
283 - (6628544*root(z^6 - (60865*z^4)/239631364059408 - (15496909*z^3)/305
6398361930300326272 - (168169*z^2)/5941638415592503834272768 - (3971*z)/31
1864717157619341253309046784 - 880007/39777047316230971280399955151664578 5
6, z, k)^2*x)/44521 - (141776759808*root(z^6 - (60865*z^4)/239631364059408
 - (15496909*z^3)/3056398361930300326272 - (168169*z^2)/594163841559250383
4272768 - (3971*z)/311864717157619341253309046784 - 880007/397770473162309
712803999551516645 7856, z, k)^3*x)/44521 + (183701926508544*root(z^6 - (60
865*z^4)/239631364059408 - (15496909*z^3)/3056398361930300326272 - (168169
*z^2)/5941638415592503834272768 - (3971*z)/311864717157619341253309046784
- 880007/397770473162309712803999551516645785 6, z, k)^4*x)/211 - 694098828
8557056*root(z^6 - (60865*z^4)/239631364059408 - (15496909*z^3)/3056398361
930300326272 - (168169*z^2)/5941638415592503834272768 - (3971*z)/311864717
157619341253309046784 - 880007/397770473162309712803999551516645785 6, z...
```

Reduce [F]

$$\int \frac{x^6}{(216 + 108x^2 + 324x^3 + 18x^4 + x^6)^2}\, dx = \text{too large to display}$$

input
```
int(x^6/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x)
```

output

```
( - 3402*int(x**10/(x**12 + 36*x**10 + 648*x**9 + 540*x**8 + 11664*x**7 +
109296*x**6 + 69984*x**5 + 19440*x**4 + 139968*x**3 + 46656*x**2 + 46656),
x)*x**6 - 61236*int(x**10/(x**12 + 36*x**10 + 648*x**9 + 540*x**8 + 11664*
x**7 + 109296*x**6 + 69984*x**5 + 19440*x**4 + 139968*x**3 + 46656*x**2 +
46656),x)*x**4 - 1102248*int(x**10/(x**12 + 36*x**10 + 648*x**9 + 540*x**8
 + 11664*x**7 + 109296*x**6 + 69984*x**5 + 19440*x**4 + 139968*x**3 + 4665
6*x**2 + 46656),x)*x**3 - 367416*int(x**10/(x**12 + 36*x**10 + 648*x**9 +
540*x**8 + 11664*x**7 + 109296*x**6 + 69984*x**5 + 19440*x**4 + 139968*x**
3 + 46656*x**2 + 46656),x)*x**2 - 734832*int(x**10/(x**12 + 36*x**10 + 648
*x**9 + 540*x**8 + 11664*x**7 + 109296*x**6 + 69984*x**5 + 19440*x**4 + 13
9968*x**3 + 46656*x**2 + 46656),x) + 6804*int(x**9/(x**12 + 36*x**10 + 648
*x**9 + 540*x**8 + 11664*x**7 + 109296*x**6 + 69984*x**5 + 19440*x**4 + 13
9968*x**3 + 46656*x**2 + 46656),x)*x**6 + 122472*int(x**9/(x**12 + 36*x**1
0 + 648*x**9 + 540*x**8 + 11664*x**7 + 109296*x**6 + 69984*x**5 + 19440*x*
*4 + 139968*x**3 + 46656*x**2 + 46656),x)*x**4 + 2204496*int(x**9/(x**12 +
 36*x**10 + 648*x**9 + 540*x**8 + 11664*x**7 + 109296*x**6 + 69984*x**5 +
19440*x**4 + 139968*x**3 + 46656*x**2 + 46656),x)*x**3 + 734832*int(x**9/(
x**12 + 36*x**10 + 648*x**9 + 540*x**8 + 11664*x**7 + 109296*x**6 + 69984*
x**5 + 19440*x**4 + 139968*x**3 + 46656*x**2 + 46656),x)*x**2 + 1469664*in
t(x**9/(x**12 + 36*x**10 + 648*x**9 + 540*x**8 + 11664*x**7 + 109296*x*...
```

3.26 $\int \frac{x^5}{(216+108x^2+324x^3+18x^4+x^6)^2} \, dx$

Optimal result

Integrand size = 26, antiderivative size = 682

$$\int \frac{x^5}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx$$
$$= \frac{\sqrt[3]{-\frac{1}{3}}\left(4-\sqrt[3]{-3}2^{2/3}x\right)}{1944\ 2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)}$$
$$+\frac{\sqrt[3]{-\frac{1}{3}}\left(4+(-2)^{2/3}\sqrt[3]{3}x\right)}{8748\ 2^{2/3}\left(8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}3^{2/3}\right)\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)}$$
$$-\frac{4+2^{2/3}\sqrt[3]{3}x}{17496\ 2^{2/3}\sqrt[3]{3}\left(4-3\sqrt[3]{2}3^{2/3}\right)\left(6+3\ 2^{2/3}\sqrt[3]{3}x+x^2\right)}$$
$$-\frac{\arctan\left(\frac{3\sqrt[3]{-3}2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{4374\ 2^{5/6}\sqrt[6]{3}\left(1+\sqrt[3]{-1}\right)^4\sqrt{4-3(-3)^{2/3}\sqrt[3]{2}}}$$
$$+\frac{\arctan\left(\frac{3\sqrt[3]{-3}2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{4374\sqrt{3}\left(8-9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}3^{2/3}\right)^{3/2}}$$
$$-\frac{i\arctan\left(\frac{3(-2)^{2/3}\sqrt[3]{3}+2x}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{1458\ 2^{5/6}3^{2/3}\left(1+\sqrt[3]{-1}\right)^5\sqrt{4+3\sqrt[3]{-2}3^{2/3}}}$$
$$-\frac{\arctan\left(\frac{3(-2)^{2/3}\sqrt[3]{3}+2x}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{4374\sqrt{3}\left(8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}3^{2/3}\right)^{3/2}}$$
$$-\frac{\operatorname{arctanh}\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{3}+\sqrt[3]{2}x\right)}{\sqrt{3\left(-4+3\sqrt[3]{2}3^{2/3}\right)}}\right)}{8748\sqrt{6}\left(-4+3\sqrt[3]{2}3^{2/3}\right)^{3/2}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{3}+\sqrt[3]{2}x\right)}{\sqrt{3\left(-4+3\sqrt[3]{2}3^{2/3}\right)}}\right)}{39366\ 2^{5/6}\sqrt[6]{3}\sqrt{-4+3\sqrt[3]{2}3^{2/3}}}$$

output
```
1/11664*(-1)^(1/3)*3^(2/3)*(4-(-3)^(1/3)*2^(2/3)*x)*2^(1/3)/(1+(-1)^(1/3))
^4/(4-3*(-3)^(2/3)*2^(1/3))/(6-3*(-3)^(1/3)*2^(2/3)*x+x^2)+1/52488*(-1)^(1
/3)*3^(2/3)*(4+(-2)^(2/3)*3^(1/3)*x)*2^(1/3)/(8+9*I*2^(1/3)*3^(1/6)+3*2^(1
/3)*3^(2/3))/(6+3*(-2)^(2/3)*3^(1/3)*x+x^2)-1/104976*(4+2^(2/3)*3^(1/3)*x)
*2^(1/3)*3^(2/3)/(4-3*2^(1/3)*3^(2/3))/(6+3*2^(2/3)*3^(1/3)*x+x^2)-1/26244
*arctan((3*(-3)^(1/3)*2^(2/3)-2*x)/(24-18*(-3)^(2/3)*2^(1/3))^(1/2))*2^(1/
6)*3^(5/6)/(1+(-1)^(1/3))^4/(4-3*(-3)^(2/3)*2^(1/3))^(1/2)+1/13122*arctan(
(3*(-3)^(1/3)*2^(2/3)-2*x)/(24-18*(-3)^(2/3)*2^(1/3))^(1/2))*3^(1/2)/(8-9*
I*2^(1/3)*3^(1/6)+3*2^(1/3)*3^(2/3))^(3/2)-1/8748*I*arctan((3*(-2)^(2/3)*3
^(1/3)+2*x)/(24+18*(-2)^(1/3)*3^(2/3))^(1/2))*2^(1/6)*3^(1/3)/(1+(-1)^(1/3
))^5/(4+3*(-2)^(1/3)*3^(2/3))^(1/2)-1/13122*arctan((3*(-2)^(2/3)*3^(1/3)+2
*x)/(24+18*(-2)^(1/3)*3^(2/3))^(1/2))*3^(1/2)/(8+9*I*2^(1/3)*3^(1/6)+3*2^(
1/3)*3^(2/3))^(3/2)-1/52488*arctanh(2^(1/6)*(3*3^(1/3)+2^(1/3)*x)/(-12+9*2
^(1/3)*3^(2/3))^(1/2))*6^(1/2)/(-4+3*2^(1/3)*3^(2/3))^(3/2)-1/236196*arcta
nh(2^(1/6)*(3*3^(1/3)+2^(1/3)*x)/(-12+9*2^(1/3)*3^(2/3))^(1/2))*2^(1/6)*3^
(5/6)/(-4+3*2^(1/3)*3^(2/3))^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.24

$$\int \frac{x^5}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx = \frac{972-144x+648x^2+729x^3-27x^4+4x^5}{615276\,(216+108x^2+324x^3+18x^4+x^6)} + \frac{\text{RootSum}\left[216+108\#1^2+324\#1^3+18\#1^4+\#1^6\&, \frac{144\log(x-\#1)-324\log(x-\#1)\#1+2043\log(x-\#1)\#1^2}{36\#1+162\#1^2+12\#1^3}\right.}{3691656}$$

input
```
Integrate[x^5/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]
```

output
```
(972 - 144*x + 648*x^2 + 729*x^3 - 27*x^4 + 4*x^5)/(615276*(216 + 108*x^2
+ 324*x^3 + 18*x^4 + x^6)) + RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^4 +
 #1^6 & , (144*Log[x - #1] - 324*Log[x - #1]*#1 + 2043*Log[x - #1]*#1^2 -
54*Log[x - #1]*#1^3 + 4*Log[x - #1]*#1^4)/(36*#1 + 162*#1^2 + 12*#1^3 + #1
^5) & ]/3691656
```

Rubi [A] (warning: unable to verify)

Time = 2.66 (sec) , antiderivative size = 666, normalized size of antiderivative = 0.98, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2466, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(x^6+18x^4+324x^3+108x^2+216)^2}\,dx$$

$$\Big\downarrow \text{2466}$$

$$1586874322944 \int \left(-\frac{\sqrt[3]{-\frac{1}{3}}x}{1542441841901568\ 2^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(x^2-3\sqrt[3]{-3}2^{2/3}x+6\right)^2}-\frac{\sqrt[3]{-}}{13881976577114112\ 2^{2/3}}\right.$$

$$\Big\downarrow \text{2009}$$

$$1586874322944\left(-\frac{i\arctan\left(\frac{2x+3(-2)^{2/3}\sqrt[3]{3}}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{2313662762852352\ 2^{5/6}3^{2/3}\left(1+\sqrt[3]{-1}\right)^5\sqrt{4+3\sqrt[3]{-2}3^{2/3}}}-\frac{\arctan\left(\frac{2x+3(-2)^{2/3}\sqrt[3]{3}}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right.}}\right.}{13881976577114112\sqrt{6}\left(4+3\sqrt[3]{}\right.}\right.$$

input
```
Int[x^5/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]
```

output
```
1586874322944*(((-1/3)^(1/3)*(4 - (-3)^(1/3)*2^(2/3)*x))/(3084883683803136
*2^(2/3)*(1 + (-1)^(1/3))^4*(4 - 3*(-3)^(2/3)*2^(1/3))*(6 - 3*(-3)^(1/3)*2
^(2/3)*x + x^2)) + ((-1/3)^(1/3)*(4 + (-2)^(2/3)*3^(1/3)*x))/(277639531542
28224*2^(2/3)*(4 + 3*(-2)^(1/3)*3^(2/3))*(6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2
)) - (4 + 2^(2/3)*3^(1/3)*x)/(27763953154228224*2^(2/3)*3^(1/3)*(4 - 3*2^(
1/3)*3^(2/3))*(6 + 3*2^(2/3)*3^(1/3)*x + x^2)) - ArcTan[(3*(-2)^(2/3)*3^(1
/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3))]]/(13881976577114112*Sqrt[6]*
(4 + 3*(-2)^(1/3)*3^(2/3))^(3/2)) - ((I/2313662762852352)*ArcTan[(3*(-2)^(
2/3)*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3))]])/(2^(5/6)*3^(2/3)*
(1 + (-1)^(1/3))^5*Sqrt[4 + 3*(-2)^(1/3)*3^(2/3)]) - ArcTan[(2^(1/6)*(3*(-
3)^(1/3) - 2^(1/3)*x))/Sqrt[3*(4 - 3*(-3)^(2/3)*2^(1/3))]]/(69409882885570
56*2^(5/6)*3^(1/6)*(1 + (-1)^(1/3))^4*Sqrt[4 - 3*(-3)^(2/3)*2^(1/3)]) + Ar
cTan[(2^(1/6)*(3*(-3)^(1/3) - 2^(1/3)*x))/Sqrt[3*(4 - 3*(-3)^(2/3)*2^(1/3)
)]]/(6940988288557056*Sqrt[3]*(8 - (9*I)*2^(1/3)*3^(1/6) + 3*2^(1/3)*3^(2/
3))^(3/2)) - ArcTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1
/3)*3^(2/3))]]/(13881976577114112*Sqrt[6]*(-4 + 3*2^(1/3)*3^(2/3))^(3/2))
- ArcTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(2/3)
)]]/(62468894597013504*2^(5/6)*3^(1/6)*Sqrt[-4 + 3*2^(1/3)*3^(2/3)]))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2466
```
Int[(u_.)*(Q6_)^(p_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
 x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Sim
p[1/(3^(3*p)*a^(2*p))   Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*
x + b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*
(-1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d,
 0] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coef
f[Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.05 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.18

method	result
default	$\frac{\frac{1}{153819}x^5-\frac{1}{22788}x^4+\frac{1}{844}x^3+\frac{2}{1899}x^2-\frac{4}{17091}x+\frac{1}{633}}{x^6+18x^4+324x^3+108x^2+216}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)}\frac{\left(4\_R^4-54\_R^3+204\right.}{\_R^5+12\_}\right.}{3691656}$
risch	$\frac{\frac{1}{153819}x^5-\frac{1}{22788}x^4+\frac{1}{844}x^3+\frac{2}{1899}x^2-\frac{4}{17091}x+\frac{1}{633}}{x^6+18x^4+324x^3+108x^2+216}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)}\frac{\left(4\_R^4-54\_R^3+204\right.}{\_R^5+12\_}\right.}{3691656}$

input
```
int(x^5/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x,method=_RETURNVERBOSE)
```

output
```
(1/153819*x^5-1/22788*x^4+1/844*x^3+2/1899*x^2-4/17091*x+1/633)/(x^6+18*x^
4+324*x^3+108*x^2+216)+1/3691656*sum((4*_R^4-54*_R^3+2043*_R^2-324*_R+144)
/(_R^5+12*_R^3+162*_R^2+36*_R)*ln(x-_R),_R=RootOf(_Z^6+18*_Z^4+324*_Z^3+10
8*_Z^2+216))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1340 vs. $2(463) = 926$.

Time = 1.45 (sec) , antiderivative size = 1340, normalized size of antiderivative = 1.96

$$\int \frac{x^5}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^5/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="fricas")
```

output
```
1/66449808*(432*x^5 - 2916*x^4 + 78732*x^3 + sqrt(1/633)*(x^6 + 18*x^4 + 3
24*x^3 + 108*x^2 + 216)*sqrt(5034474*18^(2/3) + 9367856*18^(1/3) + 4468745
7)*log(2/1982119441*sqrt(1/633)*(7238020557*(5034474*18^(2/3) + 9367856*18
^(1/3) + 44687457)^2 - 447902374840040617697967 3*18^(2/3) - 83343065225076
61258645112*18^(1/3) - 268625598114228853471204 77)*sqrt(5034474*18^(2/3) +
 9367856*18^(1/3) + 44687457) - 7383041510/9393931*(5034474*18^(2/3) + 936
7856*18^(1/3) + 44687457)^2 + 247458158879850620*x + 51322554549608034633 5
1330/9393931*18^(2/3) + 95498020363770460407535 20/9393931*18^(1/3) + 27278
92823303394003242583 0/9393931) - sqrt(1/633)*(x^6 + 18*x^4 + 324*x^3 + 108
*x^2 + 216)*sqrt(5034474*18^(2/3) + 9367856*18^(1/3) + 44687457)*log(-2/19
82119441*sqrt(1/633)*(7238020557*(5034474*18^(2/3) + 9367856*18^(1/3) + 44
687457)^2 - 447902374840040617697967 3*18^(2/3) - 833430652250766125864511 2
*18^(1/3) - 268625598114228853471204 77)*sqrt(5034474*18^(2/3) + 9367856*18
^(1/3) + 44687457) - 7383041510/9393931*(5034474*18^(2/3) + 9367856*18^(1/
3) + 44687457)^2 + 247458158879850620*x + 513225545496080346335133 0/939393
1*18^(2/3) + 954980203637704604075352 0/9393931*18^(1/3) + 2727892823303394
0032425830/9393931) - 9*(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216)*sqrt(-103
59/211*18^(2/3) + 1/422*sqrt(-1/19683*(5034474*18^(2/3) + 9367856*18^(1/3)
 + 44687457)^2 + 22860116892*18^(2/3) + 3445478701088/81*18^(1/3) + 273974
962699) - 4683928/51273*18^(1/3) + 183899/211)*log(7383041510/44521*(50...
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.15

$$\int \frac{x^5}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx$$
$$= \text{RootSum}\Big(274938951049788473490124490008305567006 72t^6 - 131871818922695008886298319 2576t^4$$
$$+ \frac{4x^5-27x^4+729x^3+648x^2-144x+972}{615276x^6+11074968x^4+199349424x^3+66449808x^2+132899616}$$

input
```
integrate(x**5/(x**6+18*x**4+324*x**3+108*x**2+216)**2,x)
```

output
```
RootSum(274938951049788473490124490008305567006720*_t**6 - 1318718189226950
088862983192576*_t**4 + 12120917704776776448*_t**2 - 39753025, Lambda(_t,
_t*log(9478422590012887239098320545502099502420459520*_t**5/618645397199626
55 - 243458646817775607639654889480814592*_t**4/9811980923071 - 4168255647
506750043178731077966745600*_t**3/61864539719962655 + 1202687744266432861646
2272*_t**2/9811980923071 + 21614261848885979366842800*_t/61864539719962655 +
 x - 308574300024117/39247923692284))) + (4*x**5 - 27*x**4 + 729*x**3 + 64
8*x**2 - 144*x + 972)/(615276*x**6 + 11074968*x**4 + 199349424*x**3 + 6644
9808*x**2 + 132899616)
```

Maxima [F]

$$\int \frac{x^5}{(216+108x^2+324x^3+18x^4+x^6)^2} \, dx = \int \frac{x^5}{(x^6+18\,x^4+324\,x^3+108\,x^2+216)^2} \, dx$$

input
```
integrate(x^5/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="maxima")
```

output
```
1/615276*(4*x^5 - 27*x^4 + 729*x^3 + 648*x^2 - 144*x + 972)/(x^6 + 18*x^4
+ 324*x^3 + 108*x^2 + 216) + 1/615276*integrate((4*x^4 - 54*x^3 + 2043*x^2
 - 324*x + 144)/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

Giac [F]

$$\int \frac{x^5}{(216+108x^2+324x^3+18x^4+x^6)^2} \, dx = \int \frac{x^5}{(x^6+18\,x^4+324\,x^3+108\,x^2+216)^2} \, dx$$

input
```
integrate(x^5/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="giac")
```

output
```
integrate(x^5/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216)^2, x)
```

Mupad [B] (verification not implemented)

Time = 22.75 (sec) , antiderivative size = 299, normalized size of antiderivative = 0.44

$$\int \frac{x^5}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx = \text{Too large to display}$$

input
```
int(x^5/(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216)^2,x)
```

output
```
symsum(log((6305*x)/4967524106141472 - (4477969*root(z^6 - (183899*z^4)/38
34101824950528 + (6209*z^2)/14083883651774823903461376 - 39753025/27493895
10497884734901244900083055670067 2, z, k))/189282278088 - (16340881*root(z^
6 - (183899*z^4)/3834101824950528 + (6209*z^2)/14083883651774823903461376
- 39753025/274938951049788473490124490008305567006 72, z, k)*x)/51106215083
76 - (43348696*root(z^6 - (183899*z^4)/3834101824950528 + (6209*z^2)/14083
883651774823903461376 - 39753025/274938951049788473490124490008305567006 72
, z, k)^2*x)/10818603 - (65333687616*root(z^6 - (183899*z^4)/3834101824950
528 + (6209*z^2)/14083883651774823903461376 - 39753025/2749389510497884734
90124490008305567006 72, z, k)^3*x)/44521 - (40024496812032*root(z^6 - (183
899*z^4)/3834101824950528 + (6209*z^2)/14083883651774823903461376 - 397530
25/274938951049788473490124490008305567006 72, z, k)^4*x)/211 - 69409882885
57056*root(z^6 - (183899*z^4)/3834101824950528 + (6209*z^2)/14083883651774
823903461376 - 39753025/274938951049788473490124490008305567006 72, z, k)^5
*x + (5943884*root(z^6 - (183899*z^4)/3834101824950528 + (6209*z^2)/140838
83651774823903461376 - 39753025/274938951049788473490124490008305567006 72,
 z, k)^2)/400689 + (224442467136*root(z^6 - (183899*z^4)/3834101824950528
+ (6209*z^2)/14083883651774823903461376 - 39753025/27493895104978847349012
4490008305567006 72, z, k)^3)/44521 - (137087493272064*root(z^6 - (183899*z
^4)/3834101824950528 + (6209*z^2)/14083883651774823903461376 - 39753025...
```

Reduce [F]

$$\int \frac{x^5}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx = \text{too large to display}$$

input
```
int(x^5/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x)
```

output
```
(2391120*int(x**6/(x**12 + 36*x**10 + 648*x**9 + 540*x**8 + 11664*x**7 + 1
09296*x**6 + 69984*x**5 + 19440*x**4 + 139968*x**3 + 46656*x**2 + 46656),x
)*x**6 + 43040160*int(x**6/(x**12 + 36*x**10 + 648*x**9 + 540*x**8 + 11664
*x**7 + 109296*x**6 + 69984*x**5 + 19440*x**4 + 139968*x**3 + 46656*x**2 +
 46656),x)*x**4 + 774722880*int(x**6/(x**12 + 36*x**10 + 648*x**9 + 540*x*
*8 + 11664*x**7 + 109296*x**6 + 69984*x**5 + 19440*x**4 + 139968*x**3 + 46
656*x**2 + 46656),x)*x**3 + 258240960*int(x**6/(x**12 + 36*x**10 + 648*x**
9 + 540*x**8 + 11664*x**7 + 109296*x**6 + 69984*x**5 + 19440*x**4 + 139968
*x**3 + 46656*x**2 + 46656),x)*x**2 + 516481920*int(x**6/(x**12 + 36*x**10
 + 648*x**9 + 540*x**8 + 11664*x**7 + 109296*x**6 + 69984*x**5 + 19440*x**
4 + 139968*x**3 + 46656*x**2 + 46656),x) + 314928*int(x**5/(x**12 + 36*x**
10 + 648*x**9 + 540*x**8 + 11664*x**7 + 109296*x**6 + 69984*x**5 + 19440*x
**4 + 139968*x**3 + 46656*x**2 + 46656),x)*x**6 + 5668704*int(x**5/(x**12
+ 36*x**10 + 648*x**9 + 540*x**8 + 11664*x**7 + 109296*x**6 + 69984*x**5 +
 19440*x**4 + 139968*x**3 + 46656*x**2 + 46656),x)*x**4 + 102036672*int(x*
*5/(x**12 + 36*x**10 + 648*x**9 + 540*x**8 + 11664*x**7 + 109296*x**6 + 69
984*x**5 + 19440*x**4 + 139968*x**3 + 46656*x**2 + 46656),x)*x**3 + 340122
24*int(x**5/(x**12 + 36*x**10 + 648*x**9 + 540*x**8 + 11664*x**7 + 109296*
x**6 + 69984*x**5 + 19440*x**4 + 139968*x**3 + 46656*x**2 + 46656),x)*x**2
 + 68024448*int(x**5/(x**12 + 36*x**10 + 648*x**9 + 540*x**8 + 11664*x*...
```

3.27 $\int \frac{x^4}{(216+108x^2+324x^3+18x^4+x^6)^2} dx$

Optimal result

Integrand size = 26, antiderivative size = 850

$$\int \frac{x^4}{(216 + 108x^2 + 324x^3 + 18x^4 + x^6)^2} dx = \text{Too large to display}$$

output
```
1/34992*(-1)^(1/3)*3^(2/3)*(3*(-3)^(1/3)*2^(2/3)-2*x)*2^(1/3)/(1+(-1)^(1/3
))^4/(4-3*(-3)^(2/3)*2^(1/3))/(6-3*(-3)^(1/3)*2^(2/3)*x+x^2)-1/157464*(-1)
^(1/3)*3^(2/3)*(3*(-2)^(2/3)*3^(1/3)+2*x)*2^(1/3)/(8+9*I*2^(1/3)*3^(1/6)+3
*2^(1/3)*3^(2/3))/(6+3*(-2)^(2/3)*3^(1/3)*x+x^2)-1/52488*(3*3^(1/3)+2^(1/3
)*x)/(9*2^(1/3)-4*3^(1/3))/(6+3*2^(2/3)*3^(1/3)*x+x^2)+1/4374*(-1)^(1/3)*a
rctan((3*(-3)^(1/3)*2^(2/3)-2*x)/(24-18*(-3)^(2/3)*2^(1/3))^(1/2))*2^(1/3)
*3^(1/6)/(1+(-1)^(1/3))^4/(8-9*I*2^(1/3)*3^(1/6)+3*2^(1/3)*3^(2/3))^(3/2)-
1/17496*(-1)^(1/3)*arctan((3*(-2)^(2/3)*3^(1/3)+2*x)/(24+18*(-2)^(1/3)*3^(
2/3))^(1/2))*2^(5/6)*3^(1/6)/(1-(-1)^(1/3))^2/(1+(-1)^(1/3))^4/(4+3*(-2)^(
1/3)*3^(2/3))^(3/2)-1/69984*(3^(1/2)+I)*arctan((3*(-2)^(2/3)*3^(1/3)+2*x)/
(24+18*(-2)^(1/3)*3^(2/3))^(1/2))*2^(5/6)*3^(2/3)/(1+(-1)^(1/3))^5/(4+3*(-
2)^(1/3)*3^(2/3))^(1/2)-1/34992*I*arctan(2^(1/6)*(3*(-3)^(1/3)-2^(1/3)*x)/
(12-9*(-3)^(2/3)*2^(1/3))^(1/2))*2^(5/6)*3^(2/3)/(1+(-1)^(1/3))^5/(4-3*(-3
)^(2/3)*2^(1/3))^(1/2)+1/157464*arctanh(2^(1/6)*(3*3^(1/3)+2^(1/3)*x)/(-12
+9*2^(1/3)*3^(2/3))^(1/2))*2^(5/6)*3^(1/6)/(-4+3*2^(1/3)*3^(2/3))^(3/2)+1/
314928*arctanh(2^(1/6)*(3*3^(1/3)+2^(1/3)*x)/(-12+9*2^(1/3)*3^(2/3))^(1/2)
)*2^(5/6)*3^(1/6)/(-4+3*2^(1/3)*3^(2/3))^(1/2)-1/209952*ln(6-3*(-3)^(1/3)*
2^(2/3)*x+x^2)*2^(2/3)*3^(1/3)/(1+(-1)^(1/3))^4+1/209952*I*ln(6+3*(-2)^(2/
3)*3^(1/3)*x+x^2)*2^(2/3)*3^(5/6)/(1+(-1)^(1/3))^5-1/1889568*ln(6+3*2^(2/3
)*3^(1/3)*x+x^2)*2^(2/3)*3^(1/3)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.20

$$\int \frac{x^4}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx$$
$$= \frac{-288+324x-1458x^2-216x^3+8x^4-9x^5}{1230552\,(216+108x^2+324x^3+18x^4+x^6)}$$
$$-\frac{\text{RootSum}\left[216+108\#1^2+324\#1^3+18\#1^4+\#1^6\&,\frac{324\log(x-\#1)-2628\log(x-\#1)\#1+324\log(x-\#1)\#1^2}{36\#1+162\#1^2+12\#1^3}\right.}{7383312}$$

input
```
Integrate[x^4/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]
```

output
```
(-288 + 324*x - 1458*x^2 - 216*x^3 + 8*x^4 - 9*x^5)/(1230552*(216 + 108*x^
2 + 324*x^3 + 18*x^4 + x^6)) - RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^4
 + #1^6 & , (324*Log[x - #1] - 2628*Log[x - #1]*#1 + 324*Log[x - #1]*#1^2
- 16*Log[x - #1]*#1^3 + 9*Log[x - #1]*#1^4)/(36*#1 + 162*#1^2 + 12*#1^3 +
#1^5) & ]/7383312
```

Rubi [A] (warning: unable to verify)

Time = 3.23 (sec) , antiderivative size = 826, normalized size of antiderivative = 0.97, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2466, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(x^6+18x^4+324x^3+108x^2+216)^2}\,dx$$

$$\Big\downarrow \text{2466}$$

$$1586874322944\int \left(-\frac{x+3\ 2^{2/3}\sqrt[3]{3}}{249875578388054016\sqrt[3]{2}3^{2/3}\left(x^2+3\ 2^{2/3}\sqrt[3]{3}x+6\right)}+\frac{6i3^{5/6}-\left(\sqrt[3]{-2}-\right.}{27763953154228224\ 6^{2/3}\left(1+\sqrt[3]{-1}\right.}\right.$$

$$\Big\downarrow \text{2009}$$

$$1586874322944\left(-\frac{\sqrt[3]{-\frac{1}{3}}\left(2x+3(-2)^{2/3}\sqrt[3]{3}\right)}{83291859462684672\ 2^{2/3}\left(4+3\sqrt[3]{-2}3^{2/3}\right)\left(x^2+3(-2)^{2/3}\sqrt[3]{3}x+6\right)}-\frac{(1-i\sqrt{3})\,\text{arct}}{83291859462684672\sqrt[6]{}}\right.$$

input
```
Int[x^4/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]
```

output
```
1586874322944*(((-1/3)^(1/3)*(3*(-3)^(1/3) - 2^(1/3)*x))/(9254651051409408
*(1 + (-1)^(1/3))^4*(4 - 3*(-3)^(2/3)*2^(1/3))*(6 - 3*(-3)^(1/3)*2^(2/3)*x
 + x^2)) - ((-1/3)^(1/3)*(3*(-2)^(2/3)*3^(1/3) + 2*x))/(8329185946268467 2*
2^(2/3)*(4 + 3*(-2)^(1/3)*3^(2/3))*(6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2)) + (
3*3^(1/3) + 2^(1/3)*x)/(83291859462684672*3^(1/3)*(4 - 3*2^(1/3)*3^(2/3))*
(6 + 3*2^(2/3)*3^(1/3)*x + x^2)) - ((-1)^(1/3)*ArcTan[(3*(-2)^(2/3)*3^(1/3
) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3))]])/(41645929731342336*2^(1/6)*3
^(5/6)*(4 + 3*(-2)^(1/3)*3^(2/3))^(3/2)) - ((1 - I*Sqrt[3])*ArcTan[(3*(-2)
^(2/3)*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3))]])/(83291859462684
672*2^(1/6)*3^(1/3)*(3*I + Sqrt[3])*Sqrt[4 + 3*(-2)^(1/3)*3^(2/3)]) - ((I/
9254651051409408)*ArcTan[(2^(1/6)*(3*(-3)^(1/3) - 2^(1/3)*x))/Sqrt[3*(4 -
3*(-3)^(2/3)*2^(1/3))]])/(2^(1/6)*3^(1/3)*(1 + (-1)^(1/3))^5*Sqrt[4 - 3*(-
3)^(2/3)*2^(1/3)]) + ((-1)^(1/3)*ArcTan[(2^(1/6)*(3*(-3)^(1/3) - 2^(1/3)*x
))/Sqrt[3*(4 - 3*(-3)^(2/3)*2^(1/3))]])/(1156831381426176*2^(2/3)*3^(5/6)*
(1 + (-1)^(1/3))^4*(8 - (9*I)*2^(1/3)*3^(1/6) + 3*2^(1/3)*3^(2/3))^(3/2))
+ ArcTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(2/3)
)]]/(41645929731342336*2^(1/6)*3^(5/6)*(-4 + 3*2^(1/3)*3^(2/3))^(3/2)) + A
rcTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(2/3))]]
/(83291859462684672*2^(1/6)*3^(5/6)*Sqrt[-4 + 3*2^(1/3)*3^(2/3)]) - Log[6
- 3*(-3)^(1/3)*2^(2/3)*x + x^2]/(55527906308456448*2^(1/3)*3^(2/3)*(1 +...
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2466
```
Int[(u_.)*(Q6_)^(p_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
 x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Sim
p[1/(3^(3*p)*a^(2*p))   Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*
x + b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*
(-1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d,
 0] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coef
f[Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.05 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.14

method	result
default	$\frac{-\frac{1}{136728}x^5+\frac{1}{153819}x^4-\frac{1}{5697}x^3-\frac{1}{844}x^2+\frac{1}{3798}x-\frac{4}{17091}}{x^6+18x^4+324x^3+108x^2+216}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)}\frac{\left(-9\_R^4+16\_R^3\right.}{\_R^5}\right.}{7383312}$
risch	$\frac{-\frac{1}{136728}x^5+\frac{1}{153819}x^4-\frac{1}{5697}x^3-\frac{1}{844}x^2+\frac{1}{3798}x-\frac{4}{17091}}{x^6+18x^4+324x^3+108x^2+216}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)}\frac{\left(-9\_R^4+16\_R^3\right.}{\_R^5}\right.}{7383312}$

input
```
int(x^4/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x,method=_RETURNVERBOSE)
```

output
```
(-1/136728*x^5+1/153819*x^4-1/5697*x^3-1/844*x^2+1/3798*x-4/17091)/(x^6+18
*x^4+324*x^3+108*x^2+216)+1/7383312*sum((-9*_R^4+16*_R^3-324*_R^2+2628*_R-
324)/(_R^5+12*_R^3+162*_R^2+36*_R)*ln(x-_R),_R=RootOf(_Z^6+18*_Z^4+324*_Z^
3+108*_Z^2+216))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^4}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx=\text{Timed out}$$

input
```
integrate(x^4/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.13

$$\int \frac{x^4}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx$$
$$= \text{RootSum}\Big(18558379195860721960583403075560625772953 6t^6 - 130936735796222356552203337728 0t$$
$$+ \frac{-9x^5+8x^4-216x^3-1458x^2+324x-288}{1230552x^6+22149936x^4+398698848x^3+132899616x^2+265799232}$$

input
```
integrate(x**4/(x**6+18*x**4+324*x**3+108*x**2+216)**2,x)
```

output
```
RootSum(18558379195860721960583403075560625772953 6*_t**6 - 130936735796222
3565522033377280*_t**4 + 43563364870522947446661 12*_t**3 - 405298284548038
7328*_t**2 + 303890718384*_t - 880007, Lambda(_t, _t*log(3908346265795559 3
47684104470733356597641295275928063469158 4*_t**5/4979785539613990026757339
5695 + 883697934622378553891281760141471110239680446257561 6*_t**4/49797855
396139900267573395695 - 26493058134830853258884424959713469570680506777 6*_
t**3/49797855396139900267573395695 + 88613533354736318520151510982615837 62
50624*_t**2/49797855396139900267573395695 - 68232147957490990651139463585 5
601936*_t/49797855396139900267573395695 + x - 213755607708464862242915195 6
8/49797855396139900267573395695))) + (-9*x**5 + 8*x**4 - 216*x**3 - 1458*x
**2 + 324*x - 288)/(1230552*x**6 + 22149936*x**4 + 398698848*x**3 + 132899
616*x**2 + 265799232)
```

Maxima [F]

$$\int \frac{x^4}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx = \int \frac{x^4}{(x^6+18\,x^4+324\,x^3+108\,x^2+216)^2}\,dx$$

input
```
integrate(x^4/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="maxima")
```

output
```
-1/1230552*(9*x^5 - 8*x^4 + 216*x^3 + 1458*x^2 - 324*x + 288)/(x^6 + 18*x^
4 + 324*x^3 + 108*x^2 + 216) - 1/1230552*integrate((9*x^4 - 16*x^3 + 324*x
^2 - 2628*x + 324)/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

Giac [F]

$$\int \frac{x^4}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx=\int \frac{x^4}{(x^6+18\,x^4+324\,x^3+108\,x^2+216)^2}\,dx$$

input
```
integrate(x^4/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="giac")
```

output
```
integrate(x^4/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216)^2, x)
```

Mupad [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 388, normalized size of antiderivative = 0.46

$$\int \frac{x^4}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx=\text{Too large to display}$$

input
```
int(x^4/(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216)^2,x)
```

output
```
symsum(log((24389*root(z^6 - (60865*z^4)/8626729106138688 + (15496909*z^3)
/660182046176944870474752 - (168169*z^2)/7700363386607884969217507328 + (3
971*z)/2425060040617647997585731147792384 - 880007/18558379195860721960583
4030755606257729536, z, k))/851770251396 + (288041*x)/804738905194918464 -
 (1090723*root(z^6 - (60865*z^4)/8626729106138688 + (15496909*z^3)/6601820
46176944870474752 - (168169*z^2)/7700363386607884969217507328 + (3971*z)/2
425060040617647997585731147792384 - 880007/1855837919586072196058340307556
06257729536, z, k)*x)/22997796787692 + (5850124*root(z^6 - (60865*z^4)/862
6729106138688 + (15496909*z^3)/660182046176944870474752 - (168169*z^2)/770
0363386607884969217507328 + (3971*z)/2425060040617647997585731147792384 -
880007/185583791958607219605834030755606257729536, z, k)^2*x)/3606201 - (6
4554687936*root(z^6 - (60865*z^4)/8626729106138688 + (15496909*z^3)/660182
046176944870474752 - (168169*z^2)/7700363386607884969217507328 + (3971*z)/
2425060040617647997585731147792384 - 880007/185583791958607219605834030755
606257729536, z, k)^3*x)/44521 + (31535589897216*root(z^6 - (60865*z^4)/86
26729106138688 + (15496909*z^3)/660182046176944870474752 - (168169*z^2)/77
00363386607884969217507328 + (3971*z)/2425060040617647997585731147792384 -
 880007/185583791958607219605834030755606257729536, z, k)^4*x)/211 - 69409
88288557056*root(z^6 - (60865*z^4)/8626729106138688 + (15496909*z^3)/66018
2046176944870474752 - (168169*z^2)/7700363386607884969217507328 + (3971...
```

Reduce [F]

$$\int \frac{x^4}{(216 + 108x^2 + 324x^3 + 18x^4 + x^6)^2}\,dx = \text{too large to display}$$

input
```
int(x^4/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x)
```

output

```
(14580*int(x**8/(x**12 + 36*x**10 + 648*x**9 + 540*x**8 + 11664*x**7 + 109
296*x**6 + 69984*x**5 + 19440*x**4 + 139968*x**3 + 46656*x**2 + 46656),x)*
x**6 + 262440*int(x**8/(x**12 + 36*x**10 + 648*x**9 + 540*x**8 + 11664*x**
7 + 109296*x**6 + 69984*x**5 + 19440*x**4 + 139968*x**3 + 46656*x**2 + 466
56),x)*x**4 + 4723920*int(x**8/(x**12 + 36*x**10 + 648*x**9 + 540*x**8 + 1
1664*x**7 + 109296*x**6 + 69984*x**5 + 19440*x**4 + 139968*x**3 + 46656*x*
*2 + 46656),x)*x**3 + 1574640*int(x**8/(x**12 + 36*x**10 + 648*x**9 + 540*
x**8 + 11664*x**7 + 109296*x**6 + 69984*x**5 + 19440*x**4 + 139968*x**3 +
46656*x**2 + 46656),x)*x**2 + 3149280*int(x**8/(x**12 + 36*x**10 + 648*x**
9 + 540*x**8 + 11664*x**7 + 109296*x**6 + 69984*x**5 + 19440*x**4 + 139968
*x**3 + 46656*x**2 + 46656),x) + 184680*int(x**6/(x**12 + 36*x**10 + 648*x
**9 + 540*x**8 + 11664*x**7 + 109296*x**6 + 69984*x**5 + 19440*x**4 + 1399
68*x**3 + 46656*x**2 + 46656),x)*x**6 + 3324240*int(x**6/(x**12 + 36*x**10
 + 648*x**9 + 540*x**8 + 11664*x**7 + 109296*x**6 + 69984*x**5 + 19440*x**
4 + 139968*x**3 + 46656*x**2 + 46656),x)*x**4 + 59836320*int(x**6/(x**12 +
 36*x**10 + 648*x**9 + 540*x**8 + 11664*x**7 + 109296*x**6 + 69984*x**5 +
19440*x**4 + 139968*x**3 + 46656*x**2 + 46656),x)*x**3 + 19945440*int(x**6
/(x**12 + 36*x**10 + 648*x**9 + 540*x**8 + 11664*x**7 + 109296*x**6 + 6998
4*x**5 + 19440*x**4 + 139968*x**3 + 46656*x**2 + 46656),x)*x**2 + 39890880
*int(x**6/(x**12 + 36*x**10 + 648*x**9 + 540*x**8 + 11664*x**7 + 109296...
```

3.28 $\int \frac{x^3}{\left(216+108x^2+324x^3+18x^4+x^6\right)^2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 884

$$\int \frac{x^3}{\left(216+108x^2+324x^3+18x^4+x^6\right)^2}\,dx = \text{Too large to display}$$

output
```
1/157464*((-6)^(1/3)*(2*(-3)^(1/3)+9*2^(1/3))-3*x)/(8-9*I*2^(1/3)*3^(1/6)+
3*2^(1/3)*3^(2/3))/(6-3*(-3)^(1/3)*2^(2/3)*x+x^2)-1/157464*((-6)^(1/3)*(9*
(-2)^(1/3)+2*3^(1/3))+3*x)/(8+9*I*2^(1/3)*3^(1/6)+3*2^(1/3)*3^(2/3))/(6+3*
(-2)^(2/3)*3^(1/3)*x+x^2)-1/104976*(2*2^(1/3)-3*6^(2/3)-3^(1/3)*x)/(9*2^(1
/3)-4*3^(1/3))/(6+3*2^(2/3)*3^(1/3)*x+x^2)+1/78732*arctan((3*(-3)^(1/3)*2^
(2/3)-2*x)/(24-18*(-3)^(2/3)*2^(1/3))^(1/2))*3^(1/2)/(8-9*I*2^(1/3)*3^(1/6
)+3*2^(1/3)*3^(2/3))^(3/2)+1/209952*(9*3^(1/6)+I*(2*2^(2/3)-2*I*2^(2/3)*3^
(1/2)-3*3^(2/3)))*arctan((3*(-3)^(1/3)*2^(2/3)-2*x)/(24-18*(-3)^(2/3)*2^(1
/3))^(1/2))*3^(1/3)/(1+(-1)^(1/3))^5/(8-6*(-3)^(2/3)*2^(1/3))^(1/2)-1/7873
2*arctan((3*(-2)^(2/3)*3^(1/3)+2*x)/(24+18*(-2)^(1/3)*3^(2/3))^(1/2))*3^(1
/2)/(8+9*I*2^(1/3)*3^(1/6)+3*2^(1/3)*3^(2/3))^(3/2)-1/209952*(9*3^(1/6)-I*
(4*2^(2/3)+3*3^(2/3)))*arctan((3*(-2)^(2/3)*3^(1/3)+2*x)/(24+18*(-2)^(1/3)
*3^(2/3))^(1/2))*3^(1/3)/(1+(-1)^(1/3))^5/(8+6*(-2)^(1/3)*3^(2/3))^(1/2)-1
/314928*arctanh(2^(1/6)*(3*3^(1/3)+2^(1/3)*x)/(-12+9*2^(1/3)*3^(2/3))^(1/2
))*6^(1/2)/(-4+3*2^(1/3)*3^(2/3))^(3/2)+1/2834352*(2*2^(2/3)-3*3^(2/3))*ar
ctanh(2^(1/6)*(3*3^(1/3)+2^(1/3)*x)/(-12+9*2^(1/3)*3^(2/3))^(1/2))*3^(5/6)
/(-8+6*2^(1/3)*3^(2/3))^(1/2)-1/139968*I*ln(6-3*(-3)^(1/3)*2^(2/3)*x+x^2)*
2^(1/3)*3^(1/6)/(1+(-1)^(1/3))^5+1/279936*(3^(1/2)+I)*ln(6+3*(-2)^(2/3)*3^
(1/3)*x+x^2)*2^(1/3)*3^(1/6)/(1+(-1)^(1/3))^5+1/3779136*ln(6+3*2^(2/3)*3^(
1/3)*x+x^2)*2^(1/3)*3^(2/3)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.19

$$\int \frac{x^3}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx$$
$$= \frac{972-3942x+648x^2+96x^3-27x^4+4x^5}{3691656\,(216+108x^2+324x^3+18x^4+x^6)}$$
$$+ \frac{\text{RootSum}\left[216+108\#1^2+324\#1^3+18\#1^4+\#1^6\&,\ \frac{1971\log(x-\#1)-162\log(x-\#1)\#1+72\log(x-\#1)\#1^2-}{36\#1+162\#1^2+12\#1^3-}\right.}{11074968}$$

input
```
Integrate[x^3/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]
```

output
```
(972 - 3942*x + 648*x^2 + 96*x^3 - 27*x^4 + 4*x^5)/(3691656*(216 + 108*x^2
 + 324*x^3 + 18*x^4 + x^6)) + RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^4
+ #1^6 & , (1971*Log[x - #1] - 162*Log[x - #1]*#1 + 72*Log[x - #1]*#1^2 -
27*Log[x - #1]*#1^3 + 2*Log[x - #1]*#1^4)/(36*#1 + 162*#1^2 + 12*#1^3 + #1
^5) & ]/11074968
```

Rubi [A] (warning: unable to verify)

Time = 3.93 (sec) , antiderivative size = 855, normalized size of antiderivative = 0.97, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2466, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(x^6+18x^4+324x^3+108x^2+216)^2}\,dx$$

$$\Big\downarrow \text{2466}$$

$$1586874322944 \int \left(-\frac{x+3\ 2^{2/3}\sqrt[3]{3}}{83291859462684672\ 2^{2/3}\sqrt[3]{3}\left(x^2+3\ 2^{2/3}\sqrt[3]{3}x+6\right)^2}-\frac{3i\sqrt[3]{2}\sqrt[6]{3}x+i2^{2/3}3^{5/6}-9i\sqrt{3}}{333167437850738688\left(1+\sqrt[3]{-1}\right)^5}\right.$$

$$\Big\downarrow \text{2009}$$

$$1586874322944\left(-\frac{3x+\sqrt[3]{-6}\left(9\sqrt[3]{-2}+2\sqrt[3]{3}\right)}{499751156776108032\left(4+3\sqrt[3]{-2}3^{2/3}\right)\left(x^2+3(-2)^{2/3}\sqrt[3]{3}x+6\right)}+\frac{\left(9i+\sqrt[3]{3}\left(4i2^{2/3}-9\sqrt[6]{3}\right)\right.}{333167437850738688\,(1}\right.$$

input
```
Int[x^3/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]
```

output
```
1586874322944*(-1/55527906308456448*(2*(2*(-1)^(1/3)*3^(2/3) + 9*6^(1/3))
+ 3*(-2)^(2/3)*x)/(2^(2/3)*(1 + (-1)^(1/3))^4*(4 - 3*(-3)^(2/3)*2^(1/3))*(
6 - 3*(-3)^(1/3)*2^(2/3)*x + x^2)) - ((-6)^(1/3)*(9*(-2)^(1/3) + 2*3^(1/3)
) + 3*x)/(49975115677610803 2*(4 + 3*(-2)^(1/3)*3^(2/3))*(6 + 3*(-2)^(2/3)*
3^(1/3)*x + x^2)) + (2*2^(1/3) - 3*6^(2/3) - 3^(1/3)*x)/(1665837189253693
44*3^(1/3)*(4 - 3*2^(1/3)*3^(2/3))*(6 + 3*2^(2/3)*3^(1/3)*x + x^2)) - ArcTa
n[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3))]]/(832918
59462684672*Sqrt[6]*(4 + 3*(-2)^(1/3)*3^(2/3))^(3/2)) + ((9*I + 3^(1/3)*((
4*I)*2^(2/3) - 9*3^(1/6)))*ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sqrt[6*(4 +
 3*(-2)^(1/3)*3^(2/3))]])/(333167437850738688*(1 + (-1)^(1/3))^5*Sqrt[2*(4
 + 3*(-2)^(1/3)*3^(2/3))]) + ArcTan[(2^(1/6)*(3*(-3)^(1/3) - 2^(1/3)*x))/S
qrt[3*(4 - 3*(-3)^(2/3)*2^(1/3))]]/(41645929731342336*Sqrt[3]*(8 - (9*I)*2
^(1/3)*3^(1/6) + 3*2^(1/3)*3^(2/3))^(3/2)) - ((9*I - 3^(1/3)*((2*I)*2^(2/3
) + 9*3^(1/6) + 2*2^(2/3)*Sqrt[3]))*ArcTan[(2^(1/6)*(3*(-3)^(1/3) - 2^(1/3
)*x))/Sqrt[3*(4 - 3*(-3)^(2/3)*2^(1/3))]])/(333167437850738688*(1 + (-1)^(
1/3))^5*Sqrt[2*(4 - 3*(-3)^(2/3)*2^(1/3))]) - ArcTanh[(2^(1/6)*(3*3^(1/3)
+ 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(2/3))]]/(83291859462684672*Sqrt[6]
*(-4 + 3*2^(1/3)*3^(2/3))^(3/2)) - (Sqrt[-4 + 3*2^(1/3)*3^(2/3)]*ArcTanh[(
2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(2/3))]])/(14992
53470328324096*2^(5/6)*3^(1/6)) - ((I/37018604205637632)*Log[6 - 3*(-3)...
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2466
```
Int[(u_.)*(Q6_)^(p_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
 x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Sim
p[1/(3^(3*p)*a^(2*p))   Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*
x + b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*
(-1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d,
 0] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coef
f[Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.05 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.14

method	result
default	$\frac{\frac{1}{922914}x^5-\frac{1}{136728}x^4+\frac{4}{153819}x^3+\frac{1}{5697}x^2-\frac{73}{68364}x+\frac{1}{3798}}{x^6+18x^4+324x^3+108x^2+216}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)}\frac{\left(2\_R^4-27\_R^3\right.}{\_R^5}\right.}{11074968}$
risch	$\frac{\frac{1}{922914}x^5-\frac{1}{136728}x^4+\frac{4}{153819}x^3+\frac{1}{5697}x^2-\frac{73}{68364}x+\frac{1}{3798}}{x^6+18x^4+324x^3+108x^2+216}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)}\frac{\left(2\_R^4-27\_R^3\right.}{\_R^5}\right.}{11074968}$

input
```
int(x^3/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x,method=_RETURNVERBOSE)
```

output
```
(1/922914*x^5-1/136728*x^4+4/153819*x^3+1/5697*x^2-73/68364*x+1/3798)/(x^6
+18*x^4+324*x^3+108*x^2+216)+1/11074968*sum((2*_R^4-27*_R^3+72*_R^2-162*_R
+1971)/(_R^5+12*_R^3+162*_R^2+36*_R)*ln(x-_R),_R=RootOf(_Z^6+18*_Z^4+324*_
Z^3+108*_Z^2+216))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^3}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx = \text{Timed out}$$

input
```
integrate(x^3/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.13

$$\int \frac{x^3}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx$$
$$= \operatorname{RootSum}\Big(128275517001789310191552482058275045342655283 2t^6 - 9063884657755442444262511497$$
$$+ \frac{4x^5-27x^4+96x^3+648x^2-3942x+972}{3691656x^6+66449808x^4+1196096544x^3+398698848x^2+797397696}$$

input
```
integrate(x**3/(x**6+18*x**4+324*x**3+108*x**2+216)**2,x)
```

output
```
RootSum(1282755170017893101915524820582750453426552832*_t**6 - 90638846577
5544244426251149770752*_t**4 - 4300873166389987741684137984*_t**3 - 717000
908921644962816*_t**2 + 135354162312576*_t - 7197829, Lambda(_t, _t*log(17
25793559281044990140955659789188299560400108333936804136148061388 8*_t**5/1
5420600979105204449069438030323752 1 + 2389607400620985524376358853572652 20
7181956324560587684052992*_t**4/15420600979105204449069438030323752 1 - 122
86072160883283930711715948878260078996992193488388096*_t**3/1542060097910 5
20444906943803032375 21 - 5949055357395917316112549601352790975415655841075
2*_t**2/15420600979105204449069438030323752 1 - 175201496798366911236706 41
977137530048272 00*_t/15420600979105204449069438030323752 1 + x + 7664229887
0722961505585528704088733 2/15420600979105204449069438030323752 1))) + (4*x*
*5 - 27*x**4 + 96*x**3 + 648*x**2 - 3942*x + 972)/(3691656*x**6 + 66449808
*x**4 + 1196096544*x**3 + 398698848*x**2 + 797397696)
```

Maxima [F]

$$\int \frac{x^3}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx = \int \frac{x^3}{(x^6+18\,x^4+324\,x^3+108\,x^2+216)^2}\,dx$$

input
```
integrate(x^3/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="maxima")
```

output
```
1/3691656*(4*x^5 - 27*x^4 + 96*x^3 + 648*x^2 - 3942*x + 972)/(x^6 + 18*x^4
 + 324*x^3 + 108*x^2 + 216) + 1/1845828*integrate((2*x^4 - 27*x^3 + 72*x^2
 - 162*x + 1971)/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

Giac [F]

$$\int \frac{x^3}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx = \int \frac{x^3}{(x^6+18\,x^4+324\,x^3+108\,x^2+216)^2}\,dx$$

input
```
integrate(x^3/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="giac")
```

output
```
integrate(x^3/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216)^2, x)
```

Mupad [B] (verification not implemented)

Time = 23.01 (sec) , antiderivative size = 387, normalized size of antiderivative = 0.44

$$\int \frac{x^3}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx = \text{Too large to display}$$

input
```
int(x^3/(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216)^2,x)
```

output
```
symsum(log((11*x)/603554178896188848 - (14059*root(z^6 - (292589*z^4)/4140
82997094657024 - (11805253*z^3)/3520970912943705975865344 - (2479189*z^2)/
44354093106861417422692842209 28 + (1989787*z)/188572668758428308292266454 0
5233577984 - 7197829/12827551700178931019155248205827504534265528 32, z, k)
)/30663729050256 - (5658601*root(z^6 - (292589*z^4)/414082997094657024 - (
11805253*z^3)/3520970912943705975865344 - (2479189*z^2)/443540931068614174
2269284220928 + (1989787*z)/1885726687584283082922664540523357 7984 - 71978
29/128275517001789310191552482058275045342655 2832, z, k)*x)/66233654748552
96 + (6603523*root(z^6 - (292589*z^4)/414082997094657024 - (11805253*z^3)/
3520970912943705975865344 - (2479189*z^2)/4435409310686141742269284220928
+ (1989787*z)/18857266875842830829226645405233577984 - 7197829/12827551700
178931019155248205827504534 26552832, z, k)^2*x)/584204562 - (1762321104*ro
ot(z^6 - (292589*z^4)/414082997094657024 - (11805253*z^3)/3520970912943705
975865344 - (2479189*z^2)/4435409310686141742269284220928 + (1989787*z)/18
857266875842830829226645405233577984 - 7197829/1282755170017893101915524 82
058275045342655 2832, z, k)^3*x)/44521 - (59633904436992*root(z^6 - (292589
*z^4)/414082997094657024 - (11805253*z^3)/3520970912943705975865344 - (247
9189*z^2)/4435409310686141742269284220928 + (1989787*z)/188572668758428308
29226645405233577984 - 7197829/128275517001789310191552482058275045342655 2
832, z, k)^4*x)/211 - 6940988288557056*root(z^6 - (292589*z^4)/41408299...
```

Reduce [F]

$$\int \frac{x^3}{(216 + 108x^2 + 324x^3 + 18x^4 + x^6)^2} \, dx = \text{too large to display}$$

input
```
int(x^3/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x)
```

output
```
(14580*int(x**8/(x**12 + 36*x**10 + 648*x**9 + 540*x**8 + 11664*x**7 + 109
296*x**6 + 69984*x**5 + 19440*x**4 + 139968*x**3 + 46656*x**2 + 46656),x)*
x**6 + 262440*int(x**8/(x**12 + 36*x**10 + 648*x**9 + 540*x**8 + 11664*x**
7 + 109296*x**6 + 69984*x**5 + 19440*x**4 + 139968*x**3 + 46656*x**2 + 466
56),x)*x**4 + 4723920*int(x**8/(x**12 + 36*x**10 + 648*x**9 + 540*x**8 + 1
1664*x**7 + 109296*x**6 + 69984*x**5 + 19440*x**4 + 139968*x**3 + 46656*x*
*2 + 46656),x)*x**3 + 1574640*int(x**8/(x**12 + 36*x**10 + 648*x**9 + 540*
x**8 + 11664*x**7 + 109296*x**6 + 69984*x**5 + 19440*x**4 + 139968*x**3 +
46656*x**2 + 46656),x)*x**2 + 3149280*int(x**8/(x**12 + 36*x**10 + 648*x**
9 + 540*x**8 + 11664*x**7 + 109296*x**6 + 69984*x**5 + 19440*x**4 + 139968
*x**3 + 46656*x**2 + 46656),x) + 184680*int(x**6/(x**12 + 36*x**10 + 648*x
**9 + 540*x**8 + 11664*x**7 + 109296*x**6 + 69984*x**5 + 19440*x**4 + 1399
68*x**3 + 46656*x**2 + 46656),x)*x**6 + 3324240*int(x**6/(x**12 + 36*x**10
 + 648*x**9 + 540*x**8 + 11664*x**7 + 109296*x**6 + 69984*x**5 + 19440*x**
4 + 139968*x**3 + 46656*x**2 + 46656),x)*x**4 + 59836320*int(x**6/(x**12 +
 36*x**10 + 648*x**9 + 540*x**8 + 11664*x**7 + 109296*x**6 + 69984*x**5 +
19440*x**4 + 139968*x**3 + 46656*x**2 + 46656),x)*x**3 + 19945440*int(x**6
/(x**12 + 36*x**10 + 648*x**9 + 540*x**8 + 11664*x**7 + 109296*x**6 + 6998
4*x**5 + 19440*x**4 + 139968*x**3 + 46656*x**2 + 46656),x)*x**2 + 39890880
*int(x**6/(x**12 + 36*x**10 + 648*x**9 + 540*x**8 + 11664*x**7 + 109296...
```

3.29 $\int \frac{x^2}{(216+108x^2+324x^3+18x^4+x^6)^2} \, dx$

Optimal result

Integrand size = 26, antiderivative size = 986

$$\int \frac{x^2}{(216+108x^2+324x^3+18x^4+x^6)^2} \, dx = \text{Too large to display}$$

output
```
-1/209952*(27*(-2)^(2/3)+54*(-1)^(1/3)*3^(2/3)-6^(1/3)*(9+(-3)^(1/3)*2^(2/
3))*x)*2^(1/3)/(1+(-1)^(1/3))^4/(4-3*(-3)^(2/3)*2^(1/3))/(6-3*(-3)^(1/3)*2
^(2/3)*x+x^2)-1/944784*(27*2^(2/3)*(1+(-2)^(1/3)*3^(2/3))-(-1)^(1/3)*3^(2/
3)*(2+3*(-2)^(1/3)*3^(2/3))*x)*2^(1/3)/(8+9*I*2^(1/3)*3^(1/6)+3*2^(1/3)*3^
(2/3))/(6+3*(-2)^(2/3)*3^(1/3)*x+x^2)+1/1889568*(54-9*2^(2/3)*3^(1/3)-(2-3
*2^(1/3)*3^(2/3))*x)*2^(1/3)*3^(2/3)/(4-3*2^(1/3)*3^(2/3))/(6+3*2^(2/3)*3^
(1/3)*x+x^2)-1/52488*(1+I*3^(1/2)+3*2^(1/3)*3^(2/3))*arctan((3*(-3)^(1/3)*
2^(2/3)-2*x)/(24-18*(-3)^(2/3)*2^(1/3))^(1/2))*2^(1/3)*3^(1/6)/(1+(-1)^(1/
3))^4/(8-9*I*2^(1/3)*3^(1/6)+3*2^(1/3)*3^(2/3))^(3/2)+1/104976*(3*(-3)^(2/
3)+(-1)^(1/3)*2^(2/3))*arctan((3*(-2)^(2/3)*3^(1/3)+2*x)/(24+18*(-2)^(1/3)
*3^(2/3))^(1/2))*6^(1/6)/(1-(-1)^(1/3))^2/(1+(-1)^(1/3))^4/(4+3*(-2)^(1/3)
*3^(2/3))^(3/2)+1/209952*(3^(1/2)+I)*arctan((3*(-2)^(2/3)*3^(1/3)+2*x)/(24
+18*(-2)^(1/3)*3^(2/3))^(1/2))*2^(5/6)*3^(2/3)/(1+(-1)^(1/3))^5/(4+3*(-2)^
(1/3)*3^(2/3))^(1/2)+1/104976*I*arctan(2^(1/6)*(3*(-3)^(1/3)-2^(1/3)*x)/(1
2-9*(-3)^(2/3)*2^(1/3))^(1/2))*2^(5/6)*3^(2/3)/(1+(-1)^(1/3))^5/(4-3*(-3)^
(2/3)*2^(1/3))^(1/2)-1/104976*(2^(2/3)-3*3^(2/3))*arctanh(2^(1/6)*(3*3^(1/
3)+2^(1/3)*x)/(-12+9*2^(1/3)*3^(2/3))^(1/2))*6^(1/6)/(1-(-1)^(1/3))^2/(1+(
-1)^(1/3))^4/(-4+3*2^(1/3)*3^(2/3))^(3/2)-1/944784*arctanh(2^(1/6)*(3*3^(1
/3)+2^(1/3)*x)/(-12+9*2^(1/3)*3^(2/3))^(1/2))*2^(5/6)*3^(1/6)/(-4+3*2^(1/3
)*3^(2/3))^(1/2)+1/2519424*(3^(1/2)+I)*ln(6-3*(-3)^(1/3)*2^(2/3)*x+x^2)...
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.17

$$\int \frac{x^2}{(216 + 108x^2 + 324x^3 + 18x^4 + x^6)^2} \, dx$$
$$= \frac{-7884 + 324x - 2724x^2 - 216x^3 + 8x^4 - 9x^5}{7383312\,(216 + 108x^2 + 324x^3 + 18x^4 + x^6)}$$
$$- \frac{\text{RootSum}\left[216 + 108\#1^2 + 324\#1^3 + 18\#1^4 + \#1^6\&, \frac{324\log(x-\#1)+2436\log(x-\#1)\#1+324\log(x-\#1)\#1^2}{36\#1+162\#1^2+12\#1^3}\right.}{44299872}$$

input
```
Integrate[x^2/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]
```

output
```
(-7884 + 324*x - 2724*x^2 - 216*x^3 + 8*x^4 - 9*x^5)/(7383312*(216 + 108*x
^2 + 324*x^3 + 18*x^4 + x^6)) - RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^
4 + #1^6 & , (324*Log[x - #1] + 2436*Log[x - #1]*#1 + 324*Log[x - #1]*#1^2
 - 16*Log[x - #1]*#1^3 + 9*Log[x - #1]*#1^4)/(36*#1 + 162*#1^2 + 12*#1^3 +
 #1^5) & ]/44299872
```

Rubi [A] (verified)

Time = 4.00 (sec) , antiderivative size = 892, normalized size of antiderivative = 0.90, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2466, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(x^6+18x^4+324x^3+108x^2+216)^2}\,dx$$

$$\Big\downarrow \text{2466}$$

$$1586874322944\int \left(\frac{3x+\sqrt[3]{-6}\left(9\sqrt[3]{-2}+\sqrt[3]{3}\right)}{499751156776108032\left(x^2+3(-2)^{2/3}\sqrt[3]{3}x+6\right)^2}-\frac{i\left(18\sqrt[3]{3}-\sqrt[3]{2}(1-i\sqrt{\ }\right.}{333167437850738688\ 2^{2/3}\sqrt[6]{3}\left(1+\sqrt[3]{-1}\right.}\right.$$

$$\Big\downarrow \text{2009}$$

$$1586874322944\left(-\frac{9\left((-6)^{2/3}+6\sqrt[3]{-3}\right)-\left(2\sqrt[3]{-3}+9\sqrt[3]{2}\right)x}{55527906308456448\ 6^{2/3}\left(1+\sqrt[3]{-1}\right)^4\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)\left(x^2-3\sqrt[3]{-3}2^{2/3}x+6\right)}+\frac{}{55527906}\right.$$

input
```
Int[x^2/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^2,x]
```

output
```
1586874322944*(-1/55527906308456448*(9*((-6)^(2/3) + 6*(-3)^(1/3)) - (2*(-
3)^(1/3) + 9*2^(1/3))*x)/(6^(2/3)*(1 + (-1)^(1/3))^4*(4 - 3*(-3)^(2/3)*2^(
1/3))*(6 - 3*(-3)^(1/3)*2^(2/3)*x + x^2)) - (54*(1 + (-2)^(1/3)*3^(2/3)) +
 (-6)^(2/3)*((-2)^(2/3) - 3*3^(2/3))*x)/(2998506940656648192*(4 + 3*(-2)^(
1/3)*3^(2/3))*(6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2)) + (18*(3*2^(1/3) - 3^(1/
3)) - (2*2^(1/3) - 3*6^(2/3))*x)/(999502313552216064*3^(1/3)*(4 - 3*2^(1/3
)*3^(2/3))*(6 + 3*2^(2/3)*3^(1/3)*x + x^2)) + ((-1)^(1/3)*(2 + 3*(-2)^(1/3
)*3^(2/3))*ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^
(2/3))]])/(499751156776108032*2^(1/6)*3^(5/6)*(4 + 3*(-2)^(1/3)*3^(2/3))^(
3/2)) + ((I + Sqrt[3])*ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(
-2)^(1/3)*3^(2/3))]])/(55527906308456448*2^(1/6)*3^(1/3)*(1 + (-1)^(1/3))^
5*Sqrt[4 + 3*(-2)^(1/3)*3^(2/3)]) + ((I/27763953154228224)*ArcTan[(2^(1/6)
*(3*(-3)^(1/3) - 2^(1/3)*x))/Sqrt[3*(4 - 3*(-3)^(2/3)*2^(1/3))]])/(2^(1/6)
*3^(1/3)*(1 + (-1)^(1/3))^5*Sqrt[4 - 3*(-3)^(2/3)*2^(1/3)]) - ((9 + (-3)^(
1/3)*2^(2/3))*ArcTan[(2^(1/6)*(3*(-3)^(1/3) - 2^(1/3)*x))/Sqrt[3*(4 - 3*(-
3)^(2/3)*2^(1/3))]])/(83291859462684672*2^(5/6)*3^(1/6)*(1 + (-1)^(1/3))^4
*(4 - 3*(-3)^(2/3)*2^(1/3))^(3/2)) - ((2^(2/3) - 3*3^(2/3))*ArcTanh[(2^(1/
6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(2/3))]])/(2498755783
88054016*6^(5/6)*(-4 + 3*2^(1/3)*3^(2/3))^(3/2)) - ArcTanh[(2^(1/6)*(3*3^(
1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(2/3))]]/(24987557838805401...
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2466
```
Int[(u_.)*(Q6_)^(p_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
 x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Sim
p[1/(3^(3*p)*a^(2*p))   Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*
x + b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*
(-1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d,
 0] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coef
f[Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.05 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.12

method	result
default	$\frac{-\frac{1}{820368}x^5+\frac{1}{922914}x^4-\frac{1}{34182}x^3-\frac{227}{615276}x^2+\frac{1}{22788}x-\frac{73}{68364}}{x^6+18x^4+324x^3+108x^2+216}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)} \frac{\left(-9\_R^4+16 \ldots\right.}{- \ldots}\right.}{44299872}$
risch	$\frac{-\frac{1}{820368}x^5+\frac{1}{922914}x^4-\frac{1}{34182}x^3-\frac{227}{615276}x^2+\frac{1}{22788}x-\frac{73}{68364}}{x^6+18x^4+324x^3+108x^2+216}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)} \frac{\left(-9\_R^4+16 \ldots\right.}{- \ldots}\right.}{44299872}$

input
```
int(x^2/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x,method=_RETURNVERBOSE)
```

output
```
(-1/820368*x^5+1/922914*x^4-1/34182*x^3-227/615276*x^2+1/22788*x-73/68364)
/(x^6+18*x^4+324*x^3+108*x^2+216)+1/44299872*sum((-9*_R^4+16*_R^3-324*_R^2
-2436*_R-324)/(_R^5+12*_R^3+162*_R^2+36*_R)*ln(x-_R),_R=RootOf(_Z^6+18*_Z^
4+324*_Z^3+108*_Z^2+216))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^2}{\left(216+108x^2+324x^3+18x^4+x^6\right)^2}\,dx=\text{Timed out}$$

input
```
integrate(x^2/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.11

$$\int \frac{x^2}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx$$
$$= \operatorname{RootSum}\Big(8658597397620778437929792538933565560629231616t^6 + 109068095871770168248838645$$
$$+ \frac{-9x^5+8x^4-216x^3-2724x^2+324x-7884}{7383312x^6+132899616x^4+2392193088x^3+797397696x^2+1594795392}$$

input
```
integrate(x**2/(x**6+18*x**4+324*x**3+108*x**2+216)**2,x)
```

output
```
RootSum(8658597397620778437929792538933565560629231616*_t**6 + 10906809587
1770168248838645612544*_t**4 - 492655707593366915713499136*_t**3 + 4037833
1745144603648*_t**2 - 695635011360*_t + 4513, Lambda(_t, _t*log(1014425315
6180418111316128703985934985188161965363171216588*_t**5/35690069707079294
8475845 - 1497965500823593351127094349719750889670502100500048*_t**4/356900
697070792948475845 + 12224097544582728185058987776867078361723699 2*_t**3/
356900697070792948475845 - 57750555242515957230229019385582614538 24*_t**2/
356900697070792948475845 + 96165242200260265765603930470432*_t/71380139414
158589695169 + x - 170591523411296981205455 84/10707020912123788454275 35)))
 + (-9*x**5 + 8*x**4 - 216*x**3 - 2724*x**2 + 324*x - 7884)/(7383312*x**6
+ 132899616*x**4 + 2392193088*x**3 + 797397696*x**2 + 1594795392)
```

Maxima [F]

$$\int \frac{x^2}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx = \int \frac{x^2}{(x^6+18\,x^4+324\,x^3+108\,x^2+216)^2}\,dx$$

input
```
integrate(x^2/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="maxima")
```

output
```
-1/7383312*(9*x^5 - 8*x^4 + 216*x^3 + 2724*x^2 - 324*x + 7884)/(x^6 + 18*x
^4 + 324*x^3 + 108*x^2 + 216) - 1/7383312*integrate((9*x^4 - 16*x^3 + 324*
x^2 + 2436*x + 324)/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

Giac [F]

$$\int \frac{x^2}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx = \int \frac{x^2}{(x^6+18\,x^4+324\,x^3+108\,x^2+216)^2}\,dx$$

input
```
integrate(x^2/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x, algorithm="giac")
```

output
```
integrate(x^2/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216)^2, x)
```

Mupad [B] (verification not implemented)

Time = 22.58 (sec) , antiderivative size = 388, normalized size of antiderivative = 0.39

$$\int \frac{x^2}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx = \text{Too large to display}$$

input
```
int(x^2/(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216)^2,x)
```

output
```
symsum(log((4897*x)/18772949180387057928192 - (8147*root(z^6 + (163*z^4)/1
2940093659208032 - (8113597*z^3)/142599321974220092022546432 + (5171*z^2)/
1108852327671535435567321055232 - (505*z)/62857556252809436097422151350778
59328 + 4513/8658597397620778437929792538933565560629231616, z, k))/110389
4245809216 - (1197643*root(z^6 + (163*z^4)/12940093659208032 - (8113597*z^
3)/142599321974220092022546432 + (5171*z^2)/110885232767153543556732105523
2 - (505*z)/6285755625280943609742215135077859328 + 4513/86585973976207784
37929792538933565560629231616, z, k)*x)/29805144636848832 + (452809*root(z
^6 + (163*z^4)/12940093659208032 - (8113597*z^3)/1425993219742200920225464
32 + (5171*z^2)/1108852327671535435567321055232 - (505*z)/6285755625280943
609742215135077859328 + 4513/865859739762077843792979253893356556062923161
6, z, k)^2*x)/194734854 - (1241776944*root(z^6 + (163*z^4)/129400936592080
32 - (8113597*z^3)/142599321974220092022546432 + (5171*z^2)/11088523276715
35435567321055232 - (505*z)/6285755625280943609742215135077859328 + 4513/8
658597397620778437929792538933565560629231616, z, k)^3*x)/44521 + (4524079
28832*root(z^6 + (163*z^4)/12940093659208032 - (8113597*z^3)/1425993219742
20092022546432 + (5171*z^2)/1108852327671535435567321055232 - (505*z)/6285
755625280943609742215135077859328 + 4513/865859739762077843792979253893356
5560629231616, z, k)^4*x)/211 - 6940988288557056*root(z^6 + (163*z^4)/1294
0093659208032 - (8113597*z^3)/142599321974220092022546432 + (5171*z^2)/...
```

Reduce [F]

$$\int \frac{x^2}{(216+108x^2+324x^3+18x^4+x^6)^2}\,dx = \text{too large to display}$$

input
```
int(x^2/(x^6+18*x^4+324*x^3+108*x^2+216)^2,x)
```

output
```
(14580*int(x**8/(x**12 + 36*x**10 + 648*x**9 + 540*x**8 + 11664*x**7 + 109
296*x**6 + 69984*x**5 + 19440*x**4 + 139968*x**3 + 46656*x**2 + 46656),x)*
x**6 + 262440*int(x**8/(x**12 + 36*x**10 + 648*x**9 + 540*x**8 + 11664*x**
7 + 109296*x**6 + 69984*x**5 + 19440*x**4 + 139968*x**3 + 46656*x**2 + 466
56),x)*x**4 + 4723920*int(x**8/(x**12 + 36*x**10 + 648*x**9 + 540*x**8 + 1
1664*x**7 + 109296*x**6 + 69984*x**5 + 19440*x**4 + 139968*x**3 + 46656*x*
*2 + 46656),x)*x**3 + 1574640*int(x**8/(x**12 + 36*x**10 + 648*x**9 + 540*
x**8 + 11664*x**7 + 109296*x**6 + 69984*x**5 + 19440*x**4 + 139968*x**3 +
46656*x**2 + 46656),x)*x**2 + 3149280*int(x**8/(x**12 + 36*x**10 + 648*x**
9 + 540*x**8 + 11664*x**7 + 109296*x**6 + 69984*x**5 + 19440*x**4 + 139968
*x**3 + 46656*x**2 + 46656),x) + 184680*int(x**6/(x**12 + 36*x**10 + 648*x
**9 + 540*x**8 + 11664*x**7 + 109296*x**6 + 69984*x**5 + 19440*x**4 + 1399
68*x**3 + 46656*x**2 + 46656),x)*x**6 + 3324240*int(x**6/(x**12 + 36*x**10
 + 648*x**9 + 540*x**8 + 11664*x**7 + 109296*x**6 + 69984*x**5 + 19440*x**
4 + 139968*x**3 + 46656*x**2 + 46656),x)*x**4 + 59836320*int(x**6/(x**12 +
 36*x**10 + 648*x**9 + 540*x**8 + 11664*x**7 + 109296*x**6 + 69984*x**5 +
19440*x**4 + 139968*x**3 + 46656*x**2 + 46656),x)*x**3 + 19945440*int(x**6
/(x**12 + 36*x**10 + 648*x**9 + 540*x**8 + 11664*x**7 + 109296*x**6 + 6998
4*x**5 + 19440*x**4 + 139968*x**3 + 46656*x**2 + 46656),x)*x**2 + 39890880
*int(x**6/(x**12 + 36*x**10 + 648*x**9 + 540*x**8 + 11664*x**7 + 109296...
```

3.30 $\int \frac{-x+x^3}{6+2x}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 24

$$\int \frac{-x+x^3}{6+2x}\,dx = 4x-\frac{3x^2}{4}+\frac{x^3}{6}-12\log(3+x)$$

output
```
4*x-3/4*x^2+1/6*x^3-12*ln(3+x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.29

$$\int \frac{-x+x^3}{6+2x}\,dx = \frac{1}{2}\left(\frac{93}{2}+8x-\frac{3x^2}{2}+\frac{x^3}{3}-24\log(3+x)\right)$$

input
```
Integrate[(-x + x^3)/(6 + 2*x),x]
```

output
```
(93/2 + 8*x - (3*x^2)/2 + x^3/3 - 24*Log[3 + x])/2
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2027, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3-x}{2x+6}dx$$
$$\Big\downarrow \text{ 2027}$$
$$\int \frac{x(x^2-1)}{2x+6}dx$$
$$\Big\downarrow \text{ 522}$$
$$\int \left(\frac{x^2}{2}-\frac{3x}{2}-\frac{12}{x+3}+4\right)dx$$
$$\Big\downarrow \text{ 2009}$$
$$\frac{x^3}{6}-\frac{3x^2}{4}+4x-12\log(x+3)$$

input
```
Int[(-x + x^3)/(6 + 2*x),x]
```

output
```
4*x - (3*x^2)/4 + x^3/6 - 12*Log[3 + x]
```

Defintions of rubi rules used

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.88

method	result	size
default	$4x-\frac{3x^{2}}{4}+\frac{x^{3}}{6}-12\ln\left(3+x\right)$	21
risch	$4x-\frac{3x^{2}}{4}+\frac{x^{3}}{6}-12\ln\left(3+x\right)$	21
parallelrisch	$4x-\frac{3x^{2}}{4}+\frac{x^{3}}{6}-12\ln\left(3+x\right)$	21
norman	$4x-\frac{3x^{2}}{4}+\frac{x^{3}}{6}-12\ln\left(6+2x\right)$	23
meijerg	$\frac{3x\left(\frac{4}{9}x^{2}-2x+12\right)}{8}-12\ln\left(1+\frac{x}{3}\right)-\frac{x}{2}$	26

input
```
int((x^3-x)/(6+2*x),x,method=_RETURNVERBOSE)
```

output
```
4*x-3/4*x^2+1/6*x^3-12*ln(3+x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int\frac{-x+x^{3}}{6+2x}\,dx=\frac{1}{6}\,x^{3}-\frac{3}{4}\,x^{2}+4\,x-12\,\log\left(x+3\right)$$

input
```
integrate((x^3-x)/(6+2*x),x, algorithm="fricas")
```

output
```
1/6*x^3 - 3/4*x^2 + 4*x - 12*log(x + 3)
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{-x+x^3}{6+2x}\,dx = \frac{x^3}{6} - \frac{3x^2}{4} + 4x - 12\log(x+3)$$

input
```
integrate((x**3-x)/(6+2*x),x)
```

output
```
x**3/6 - 3*x**2/4 + 4*x - 12*log(x + 3)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{-x+x^3}{6+2x}\,dx = \frac{1}{6}\,x^3 - \frac{3}{4}\,x^2 + 4\,x - 12\,\log(x+3)$$

input
```
integrate((x^3-x)/(6+2*x),x, algorithm="maxima")
```

output
```
1/6*x^3 - 3/4*x^2 + 4*x - 12*log(x + 3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.88

$$\int \frac{-x+x^3}{6+2x}\,dx = \frac{1}{6}\,x^3 - \frac{3}{4}\,x^2 + 4\,x - 12\,\log(|x+3|)$$

input
```
integrate((x^3-x)/(6+2*x),x, algorithm="giac")
```

output
```
1/6*x^3 - 3/4*x^2 + 4*x - 12*log(abs(x + 3))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{-x+x^3}{6+2x}\,dx = 4\,x - 12\,\ln\left(x+3\right) - \frac{3\,x^2}{4} + \frac{x^3}{6}$$

input
```
int(-(x - x^3)/(2*x + 6),x)
```

output
```
4*x - 12*log(x + 3) - (3*x^2)/4 + x^3/6
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{-x+x^3}{6+2x}\,dx = -12\log(x+3) + \frac{x^3}{6} - \frac{3x^2}{4} + 4x$$

input
```
int((x^3-x)/(6+2*x),x)
```

output
```
( - 144*log(x + 3) + 2*x**3 - 9*x**2 + 48*x)/12
```

3.31 $\int \frac{x+x^3}{-1+x} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 26

$$\int \frac{x+x^3}{-1+x} \, dx = 2x + \frac{x^2}{2} + \frac{x^3}{3} + 2\log(1-x)$$

output
```
2*x+1/2*x^2+1/3*x^3+2*ln(1-x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.96

$$\int \frac{x+x^3}{-1+x} \, dx = \frac{1}{6}\left(-17+12x+3x^2+2x^3+12\log(-1+x)\right)$$

input
```
Integrate[(x + x^3)/(-1 + x),x]
```

output
```
(-17 + 12*x + 3*x^2 + 2*x^3 + 12*Log[-1 + x])/6
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {2027, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3+x}{x-1}\,dx$$

$$\Big\downarrow \; 2027$$

$$\int \frac{x\left(x^2+1\right)}{x-1}dx$$

$$\Big\downarrow \; 522$$

$$\int \left(x^2+x+\frac{2}{x-1}+2\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{x^3}{3}+\frac{x^2}{2}+2x+2\log(1-x)$$

input
```
Int[(x + x^3)/(-1 + x),x]
```

output
```
2*x + x^2/2 + x^3/3 + 2*Log[1 - x]
```

Defintions of rubi rules used

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.81

method	result	size
default	$\frac{x^3}{3}+\frac{x^2}{2}+2x+2\ln(x-1)$	21
norman	$\frac{x^3}{3}+\frac{x^2}{2}+2x+2\ln(x-1)$	21
risch	$\frac{x^3}{3}+\frac{x^2}{2}+2x+2\ln(x-1)$	21
parallelrisch	$\frac{x^3}{3}+\frac{x^2}{2}+2x+2\ln(x-1)$	21
meijerg	$\frac{x(4x^2+6x+12)}{12}+2\ln(1-x)+x$	24

input
```
int((x^3+x)/(x-1),x,method=_RETURNVERBOSE)
```

output
```
1/3*x^3+1/2*x^2+2*x+2*ln(x-1)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.77

$$\int \frac{x+x^3}{-1+x}\,dx = \frac{1}{3}\,x^3 + \frac{1}{2}\,x^2 + 2\,x + 2\,\log\left(x-1\right)$$

input
```
integrate((x^3+x)/(x-1),x, algorithm="fricas")
```

output
```
1/3*x^3 + 1/2*x^2 + 2*x + 2*log(x - 1)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.73

$$\int \frac{x+x^3}{-1+x}\,dx = \frac{x^3}{3}+\frac{x^2}{2}+2x+2\log{(x-1)}$$

input
```
integrate((x**3+x)/(x-1),x)
```

output
```
x**3/3 + x**2/2 + 2*x + 2*log(x - 1)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.77

$$\int \frac{x+x^3}{-1+x}\,dx = \frac{1}{3}\,x^3+\frac{1}{2}\,x^2+2\,x+2\,\log\left(x-1\right)$$

input
```
integrate((x^3+x)/(x-1),x, algorithm="maxima")
```

output
```
1/3*x^3 + 1/2*x^2 + 2*x + 2*log(x - 1)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.81

$$\int \frac{x+x^3}{-1+x}\,dx = \frac{1}{3}\,x^3+\frac{1}{2}\,x^2+2\,x+2\,\log\left(|x-1|\right)$$

input
```
integrate((x^3+x)/(x-1),x, algorithm="giac")
```

output
```
1/3*x^3 + 1/2*x^2 + 2*x + 2*log(abs(x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.77

$$\int \frac{x+x^3}{-1+x}\,dx = 2\,x+2\,\ln\left(x-1\right)+\frac{x^2}{2}+\frac{x^3}{3}$$

input
```
int((x + x^3)/(x - 1),x)
```

output
```
2*x + 2*log(x - 1) + x^2/2 + x^3/3
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.77

$$\int \frac{x+x^3}{-1+x}\,dx = 2\log(x-1)+\frac{x^3}{3}+\frac{x^2}{2}+2x$$

input
```
int((x^3+x)/(x-1),x)
```

output
```
(12*log(x - 1) + 2*x**3 + 3*x**2 + 12*x)/6
```

3.32 $\int \frac{-1+x^3}{-1+x} dx$

Optimal result

Integrand size = 11, antiderivative size = 16

$$\int \frac{-1+x^3}{-1+x} dx = x + \frac{x^2}{2} + \frac{x^3}{3}$$

output
```
x+1/2*x^2+1/3*x^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{-1+x^3}{-1+x} dx = x + \frac{x^2}{2} + \frac{x^3}{3}$$

input
```
Integrate[(-1 + x^3)/(-1 + x),x]
```

output
```
x + x^2/2 + x^3/3
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2019, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3-1}{x-1} \, dx$$

$$\Big\downarrow \text{2019}$$

$$\int \left(x^2+x+1\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^3}{3}+\frac{x^2}{2}+x$$

input
```
Int[(-1 + x^3)/(-1 + x),x]
```

output
```
x + x^2/2 + x^3/3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

method	result	size
default	$x+\frac{1}{2}x^{2}+\frac{1}{3}x^{3}$	13
norman	$x+\frac{1}{2}x^{2}+\frac{1}{3}x^{3}$	13
risch	$x+\frac{1}{2}x^{2}+\frac{1}{3}x^{3}$	13
parallelrisch	$x+\frac{1}{2}x^{2}+\frac{1}{3}x^{3}$	13
parts	$x+\frac{1}{2}x^{2}+\frac{1}{3}x^{3}$	13
gosper	$\frac{x(2x^{2}+3x+6)}{6}$	14
meijerg	$\frac{x(4x^{2}+6x+12)}{12}$	14
orering	$\frac{x(2x^{2}+3x+6)(x^{3}-1)}{6(x^{2}+x+1)(x-1)}$	32

input
```
int((x^3-1)/(x-1),x,method=_RETURNVERBOSE)
```

output
```
x+1/2*x^2+1/3*x^3
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{-1+x^3}{-1+x}\,dx=\frac{1}{3}\,x^{3}+\frac{1}{2}\,x^{2}+x$$

input
```
integrate((x^3-1)/(x-1),x, algorithm="fricas")
```

output
```
1/3*x^3 + 1/2*x^2 + x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

$$\int \frac{-1+x^3}{-1+x}\,dx = \frac{x^3}{3} + \frac{x^2}{2} + x$$

input
```
integrate((x**3-1)/(x-1),x)
```

output
```
x**3/3 + x**2/2 + x
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{-1+x^3}{-1+x}\,dx = \frac{1}{3}\,x^3 + \frac{1}{2}\,x^2 + x$$

input
```
integrate((x^3-1)/(x-1),x, algorithm="maxima")
```

output
```
1/3*x^3 + 1/2*x^2 + x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{-1+x^3}{-1+x}\,dx = \frac{1}{3}\,x^3 + \frac{1}{2}\,x^2 + x$$

input
```
integrate((x^3-1)/(x-1),x, algorithm="giac")
```

output
```
1/3*x^3 + 1/2*x^2 + x
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

$$\int \frac{-1+x^3}{-1+x}\,dx=\frac{x\,\left(2\,x^2+3\,x+6\right)}{6}$$

input
```
int((x^3 - 1)/(x - 1),x)
```

output
```
(x*(3*x + 2*x^2 + 6))/6
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

$$\int \frac{-1+x^3}{-1+x}\,dx=\frac{x(2x^2+3x+6)}{6}$$

input
```
int((x^3-1)/(x-1),x)
```

output
```
(x*(2*x**2 + 3*x + 6))/6
```

3.33 $\int \frac{a^2c+a^2dx+2abcx^2+2abdx^3+b^2cx^4+b^2dx^5}{c+dx}\,dx$

Optimal result

Integrand size = 52, antiderivative size = 25

$$\int \frac{a^2c+a^2dx+2abcx^2+2abdx^3+b^2cx^4+b^2dx^5}{c+dx}\,dx=a^2x+\frac{2}{3}abx^3+\frac{b^2x^5}{5}$$

output
```
a^2*x+2/3*a*b*x^3+1/5*b^2*x^5
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{a^2c+a^2dx+2abcx^2+2abdx^3+b^2cx^4+b^2dx^5}{c+dx}\,dx=a^2x+\frac{2}{3}abx^3+\frac{b^2x^5}{5}$$

input
```
Integrate[(a^2*c + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3 + b^2*c*x^4 + b^2*d
*x^5)/(c + d*x),x]
```

output
```
a^2*x + (2*a*b*x^3)/3 + (b^2*x^5)/5
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {2019, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a^2c+a^2dx+2abcx^2+2abdx^3+b^2cx^4+b^2dx^5}{c+dx}\,dx$$

$$\Big\downarrow \text{2019}$$

$$\int \left(a^2+2abx^2+b^2x^4\right)dx$$

$$\Big\downarrow \text{2009}$$

$$a^2x+\frac{2}{3}abx^3+\frac{b^2x^5}{5}$$

input
```
Int[(a^2*c + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3 + b^2*c*x^4 + b^2*d*x^5)/
(c + d*x),x]
```

output
```
a^2*x + (2*a*b*x^3)/3 + (b^2*x^5)/5
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

method	result	size
default	$x\,a^2+\frac{2}{3}a\,x^3b+\frac{1}{5}b^2x^5$	22
norman	$x\,a^2+\frac{2}{3}a\,x^3b+\frac{1}{5}b^2x^5$	22
risch	$x\,a^2+\frac{2}{3}a\,x^3b+\frac{1}{5}b^2x^5$	22
parallelrisch	$x\,a^2+\frac{2}{3}a\,x^3b+\frac{1}{5}b^2x^5$	22
parts	$x\,a^2+\frac{2}{3}a\,x^3b+\frac{1}{5}b^2x^5$	22
gosper	$\frac{x(3b^2x^4+10ab\,x^2+15a^2)}{15}$	25
orering	$\frac{x(3b^2x^4+10ab\,x^2+15a^2)(b^2d\,x^5+b^2c\,x^4+2abd\,x^3+2abc\,x^2+a^2dx+a^2c)}{15(b\,x^2+a)^2(dx+c)}$	85

input
```
int((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(d*x+c),x,
method=_RETURNVERBOSE)
```

output
```
x*a^2+2/3*a*x^3*b+1/5*b^2*x^5
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{a^2c+a^2dx+2abcx^2+2abdx^3+b^2cx^4+b^2dx^5}{c+dx}\,dx=\frac{1}{5}\,b^2x^5+\frac{2}{3}\,abx^3+a^2x$$

input
```
integrate((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(d*x
+c),x, algorithm="fricas")
```

output
```
1/5*b^2*x^5 + 2/3*a*b*x^3 + a^2*x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \frac{a^2c+a^2dx+2abcx^2+2abdx^3+b^2cx^4+b^2dx^5}{c+dx}\,dx=a^2x+\frac{2abx^3}{3}+\frac{b^2x^5}{5}$$

input
```
integrate((b**2*d*x**5+b**2*c*x**4+2*a*b*d*x**3+2*a*b*c*x**2+a**2*d*x+a**2
*c)/(d*x+c),x)
```

output
```
a**2*x + 2*a*b*x**3/3 + b**2*x**5/5
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{a^2c+a^2dx+2abcx^2+2abdx^3+b^2cx^4+b^2dx^5}{c+dx}\,dx=\frac{1}{5}\,b^2x^5+\frac{2}{3}\,abx^3+a^2x$$

input
```
integrate((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(d*x
+c),x, algorithm="maxima")
```

output
```
1/5*b^2*x^5 + 2/3*a*b*x^3 + a^2*x
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{a^2c+a^2dx+2abcx^2+2abdx^3+b^2cx^4+b^2dx^5}{c+dx}\,dx=\frac{1}{5}\,b^2x^5+\frac{2}{3}\,abx^3+a^2x$$

input
```
integrate((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(d*x
+c),x, algorithm="giac")
```

output
```
1/5*b^2*x^5 + 2/3*a*b*x^3 + a^2*x
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{a^2c + a^2dx + 2abcx^2 + 2abdx^3 + b^2cx^4 + b^2dx^5}{c + dx} \, dx = a^2 \, x + \frac{2 \, a \, b \, x^3}{3} + \frac{b^2 \, x^5}{5}$$

input
```
int((a^2*c + b^2*c*x^4 + b^2*d*x^5 + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3)/
(c + d*x),x)
```

output
```
a^2*x + (b^2*x^5)/5 + (2*a*b*x^3)/3
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{a^2c + a^2dx + 2abcx^2 + 2abdx^3 + b^2cx^4 + b^2dx^5}{c + dx} \, dx = \frac{x(3b^2x^4 + 10ab \, x^2 + 15a^2)}{15}$$

input
```
int((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(d*x+c),x)
```

output
```
(x*(15*a**2 + 10*a*b*x**2 + 3*b**2*x**4))/15
```

3.34 $\int \frac{a^2c+a^2dx+2abcx^2+2abdx^3+b^2cx^4+b^2dx^5}{(c+dx)^2} dx$

Optimal result

Integrand size = 52, antiderivative size = 94

$$\int \frac{a^2c+a^2dx+2abcx^2+2abdx^3+b^2cx^4+b^2dx^5}{(c+dx)^2}\,dx = -\frac{bc(bc^2+2ad^2)\,x}{d^4}+\frac{b(bc^2+2ad^2)\,x^2}{2d^3}-\frac{b^2cx^3}{3d^2}+\frac{b^2x^4}{4d}+\frac{(bc^2+ad^2)^2\log(c+dx)}{d^5}$$

output
```
-b*c*(2*a*d^2+b*c^2)*x/d^4+1/2*b*(2*a*d^2+b*c^2)*x^2/d^3-1/3*b^2*c*x^3/d^2
+1/4*b^2*x^4/d+(a*d^2+b*c^2)^2*ln(d*x+c)/d^5
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.84

$$\int \frac{a^2c+a^2dx+2abcx^2+2abdx^3+b^2cx^4+b^2dx^5}{(c+dx)^2}\,dx = \frac{bdx(12ad^2(-2c+dx)+b(-12c^3+6c^2dx-4cd^2x^2+3d^3x^3))+12(bc^2+ad^2)^2\log(c+dx)}{12d^5}$$

input
```
Integrate[(a^2*c + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3 + b^2*c*x^4 + b^2*d
*x^5)/(c + d*x)^2,x]
```

output
```
(b*d*x*(12*a*d^2*(-2*c + d*x) + b*(-12*c^3 + 6*c^2*d*x - 4*c*d^2*x^2 + 3*d
^3*x^3)) + 12*(b*c^2 + a*d^2)^2*Log[c + d*x])/(12*d^5)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.096$, Rules used = {2019, 1380, 27, 476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a^2c + a^2dx + 2abcx^2 + 2abdx^3 + b^2cx^4 + b^2dx^5}{(c+dx)^2} \, dx$$

$$\Big\downarrow \text{2019}$$

$$\int \frac{a^2 + 2abx^2 + b^2x^4}{c+dx} dx$$

$$\Big\downarrow \text{1380}$$

$$\frac{\int \frac{b^2(bx^2+a)^2}{c+dx} dx}{b^2}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{(a+bx^2)^2}{c+dx} dx$$

$$\Big\downarrow \text{476}$$

$$\int \left(\frac{(ad^2+bc^2)^2}{d^4(c+dx)} - \frac{bc(2ad^2+bc^2)}{d^4} + \frac{bx(2ad^2+bc^2)}{d^3} - \frac{b^2cx^2}{d^2} + \frac{b^2x^3}{d}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{(ad^2+bc^2)^2 \log(c+dx)}{d^5} - \frac{bcx(2ad^2+bc^2)}{d^4} + \frac{bx^2(2ad^2+bc^2)}{2d^3} - \frac{b^2cx^3}{3d^2} + \frac{b^2x^4}{4d}$$

input
```
Int[(a^2*c + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3 + b^2*c*x^4 + b^2*d*x^5)/
(c + d*x)^2,x]
```

output
```
-((b*c*(b*c^2 + 2*a*d^2)*x)/d^4) + (b*(b*c^2 + 2*a*d^2)*x^2)/(2*d^3) - (b^
2*c*x^3)/(3*d^2) + (b^2*x^4)/(4*d) + ((b*c^2 + a*d^2)^2*Log[c + d*x])/d^5
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.02

method	result
default	$-\frac{b\left(-\frac{b\,d^{3}x^{4}}{4}+\frac{bc\,x^{3}d^{2}}{3}-\frac{\left(2a\,d^{2}+b\,c^{2}\right)x^{2}d}{2}+xc\left(2a\,d^{2}+b\,c^{2}\right)\right)}{d^{4}}+\frac{\left(a^{2}d^{4}+2ab\,c^{2}d^{2}+b^{2}c^{4}\right)\ln\left(dx+c\right)}{d^{5}}$
risch	$\frac{b^{2}x^{4}}{4d}-\frac{b^{2}c\,x^{3}}{3d^{2}}+\frac{ba\,x^{2}}{d}+\frac{b^{2}c^{2}x^{2}}{2d^{3}}-\frac{2bacx}{d^{2}}-\frac{b^{2}c^{3}x}{d^{4}}+\frac{\ln\left(dx+c\right)a^{2}}{d}+\frac{2\ln\left(dx+c\right)ab\,c^{2}}{d^{3}}+\frac{\ln\left(dx+c\right)b^{2}c^{4}}{d^{5}}$
parallelrisch	$\frac{3x^{4}b^{2}d^{4}-4b^{2}c\,x^{3}d^{3}+12x^{2}ab\,d^{4}+6x^{2}b^{2}c^{2}d^{2}+12\ln\left(dx+c\right)a^{2}d^{4}+24\ln\left(dx+c\right)ab\,c^{2}d^{2}+12\ln\left(dx+c\right)b^{2}c^{4}-24xabc\,d^{3}-12x\,b^{2}c^{3}d}{12d^{5}}$
norman	$\frac{\frac{c\left(2ab\,c^{2}d^{2}+b^{2}c^{4}\right)}{d^{5}}+\frac{b^{2}x^{5}}{4}+\frac{b\left(6a\,d^{2}+b\,c^{2}\right)x^{3}}{6d^{2}}-\frac{c\,b^{2}x^{4}}{12d}-\frac{bc\left(2a\,d^{2}+b\,c^{2}\right)x^{2}}{2d^{3}}}{dx+c}+\frac{\left(a^{2}d^{4}+2ab\,c^{2}d^{2}+b^{2}c^{4}\right)\ln\left(dx+c\right)}{d^{5}}$

input
```
int((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(d*x+c)^2,
x,method=_RETURNVERBOSE)
```

output
```
-b/d^4*(-1/4*b*d^3*x^4+1/3*b*c*x^3*d^2-1/2*(2*a*d^2+b*c^2)*x^2*d+x*c*(2*a*
d^2+b*c^2))+(a^2*d^4+2*a*b*c^2*d^2+b^2*c^4)/d^5*ln(d*x+c)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.12

$$\int \frac{a^2c+a^2dx+2abcx^2+2abdx^3+b^2cx^4+b^2dx^5}{(c+dx)^2}\,dx$$
$$=\frac{3\,b^2d^4x^4-4\,b^2cd^3x^3+6\,(b^2c^2d^2+2\,abd^4)x^2-12\,(b^2c^3d+2\,abcd^3)x+12\,(b^2c^4+2\,abc^2d^2+a^2d^4)\log\left(dx+c\right)}{12\,d^5}$$

input
```
integrate((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(d*x
+c)^2,x, algorithm="fricas")
```

output
```
1/12*(3*b^2*d^4*x^4 - 4*b^2*c*d^3*x^3 + 6*(b^2*c^2*d^2 + 2*a*b*d^4)*x^2 -
12*(b^2*c^3*d + 2*a*b*c*d^3)*x + 12*(b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)*lo
g(d*x + c))/d^5
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.94

$$\int \frac{a^2c+a^2dx+2abcx^2+2abdx^3+b^2cx^4+b^2dx^5}{(c+dx)^2}\,dx = -\frac{b^2cx^3}{3d^2}+\frac{b^2x^4}{4d}+x^2\left(\frac{ab}{d}+\frac{b^2c^2}{2d^3}\right)+x\left(-\frac{2abc}{d^2}-\frac{b^2c^3}{d^4}\right)+\frac{\left(ad^2+bc^2\right)^2\log\left(c+dx\right)}{d^5}$$

input
```
integrate((b**2*d*x**5+b**2*c*x**4+2*a*b*d*x**3+2*a*b*c*x**2+a**2*d*x+a**2
*c)/(d*x+c)**2,x)
```

output
```
-b**2*c*x**3/(3*d**2) + b**2*x**4/(4*d) + x**2*(a*b/d + b**2*c**2/(2*d**3)
) + x*(-2*a*b*c/d**2 - b**2*c**3/d**4) + (a*d**2 + b*c**2)**2*log(c + d*x)
/d**5
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.12

$$\int \frac{a^2c+a^2dx+2abcx^2+2abdx^3+b^2cx^4+b^2dx^5}{(c+dx)^2}\,dx = \frac{3\,b^2d^3x^4-4\,b^2cd^2x^3+6\,(b^2c^2d+2\,abd^3)x^2-12\,(b^2c^3+2\,abcd^2)x}{12\,d^4} + \frac{(b^2c^4+2\,abc^2d^2+a^2d^4)\log\left(dx+c\right)}{d^5}$$

input
```
integrate((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(d*x
+c)^2,x, algorithm="maxima")
```

output
```
1/12*(3*b^2*d^3*x^4 - 4*b^2*c*d^2*x^3 + 6*(b^2*c^2*d + 2*a*b*d^3)*x^2 - 12
*(b^2*c^3 + 2*a*b*c*d^2)*x)/d^4 + (b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)*log(
d*x + c)/d^5
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 365 vs. $2(88) = 176$.

Time = 0.13 (sec) , antiderivative size = 365, normalized size of antiderivative = 3.88

$$\int \frac{a^2c + a^2dx + 2abcx^2 + 2abdx^3 + b^2cx^4 + b^2dx^5}{(c+dx)^2}\,dx =$$

$$-\frac{1}{12}b^2d\left(\frac{(dx+c)^4\left(\frac{20\,c}{dx+c} - \frac{60\,c^2}{(dx+c)^2} + \frac{120\,c^3}{(dx+c)^3} - 3\right)}{d^6} + \frac{60\,c^4\log\left(\frac{|dx+c|}{(dx+c)^2|d|}\right)}{d^6} - \frac{12\,c^5}{(dx+c)d^6}\right)$$
$$-\frac{1}{3}b^2c\left(\frac{(dx+c)^3\left(\frac{6\,c}{dx+c} - \frac{18\,c^2}{(dx+c)^2} - 1\right)}{d^5} - \frac{12\,c^3\log\left(\frac{|dx+c|}{(dx+c)^2|d|}\right)}{d^5} + \frac{3\,c^4}{(dx+c)d^5}\right)$$
$$-abd\left(\frac{(dx+c)^2\left(\frac{6\,c}{dx+c} - 1\right)}{d^4} + \frac{6\,c^2\log\left(\frac{|dx+c|}{(dx+c)^2|d|}\right)}{d^4} - \frac{2\,c^3}{(dx+c)d^4}\right)$$
$$+2\,abc\left(\frac{2\,c\log\left(\frac{|dx+c|}{(dx+c)^2|d|}\right)}{d^3} + \frac{dx+c}{d^3} - \frac{c^2}{(dx+c)d^3}\right)$$
$$-a^2\left(\frac{\log\left(\frac{|dx+c|}{(dx+c)^2|d|}\right)}{d} - \frac{c}{(dx+c)d}\right) - \frac{a^2c}{(dx+c)d}$$

input
```
integrate((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(d*x
+c)^2,x, algorithm="giac")
```

output
```
-1/12*b^2*d*((d*x + c)^4*(20*c/(d*x + c) - 60*c^2/(d*x + c)^2 + 120*c^3/(d
*x + c)^3 - 3)/d^6 + 60*c^4*log(abs(d*x + c)/((d*x + c)^2*abs(d)))/d^6 - 1
2*c^5/((d*x + c)*d^6)) - 1/3*b^2*c*((d*x + c)^3*(6*c/(d*x + c) - 18*c^2/(d
*x + c)^2 - 1)/d^5 - 12*c^3*log(abs(d*x + c)/((d*x + c)^2*abs(d)))/d^5 + 3
*c^4/((d*x + c)*d^5)) - a*b*d*((d*x + c)^2*(6*c/(d*x + c) - 1)/d^4 + 6*c^2
*log(abs(d*x + c)/((d*x + c)^2*abs(d)))/d^4 - 2*c^3/((d*x + c)*d^4)) + 2*a
*b*c*(2*c*log(abs(d*x + c)/((d*x + c)^2*abs(d)))/d^3 + (d*x + c)/d^3 - c^2
/((d*x + c)*d^3)) - a^2*(log(abs(d*x + c)/((d*x + c)^2*abs(d)))/d - c/((d*
x + c)*d)) - a^2*c/((d*x + c)*d)
```

Mupad [B] (verification not implemented)

Time = 22.51 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.13

$$\int \frac{a^2c+a^2dx+2abcx^2+2abdx^3+b^2cx^4+b^2dx^5}{(c+dx)^2}\,dx$$
$$= x^2\left(\frac{b^2\,c^2}{2\,d^3}+\frac{a\,b}{d}\right)+\frac{\ln\left(c+d\,x\right)\,\left(a^2\,d^4+2\,a\,b\,c^2\,d^2+b^2\,c^4\right)}{d^5}$$
$$+\frac{b^2\,x^4}{4\,d}-\frac{b^2\,c\,x^3}{3\,d^2}-\frac{c\,x\,\left(\frac{b^2\,c^2}{d^3}+\frac{2\,a\,b}{d}\right)}{d}$$

input
```
int((a^2*c + b^2*c*x^4 + b^2*d*x^5 + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3)/
(c + d*x)^2,x)
```

output
```
x^2*((b^2*c^2)/(2*d^3) + (a*b)/d) + (log(c + d*x)*(a^2*d^4 + b^2*c^4 + 2*a
*b*c^2*d^2))/d^5 + (b^2*x^4)/(4*d) - (b^2*c*x^3)/(3*d^2) - (c*x*((b^2*c^2)
/d^3 + (2*a*b)/d))/d
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.23

$$\int \frac{a^2c+a^2dx+2abcx^2+2abdx^3+b^2cx^4+b^2dx^5}{(c+dx)^2}\,dx$$
$$= \frac{12\log(dx+c)\,a^2d^4+24\log(dx+c)\,ab\,c^2d^2+12\log(dx+c)\,b^2c^4-24abc\,d^3x+12ab\,d^4x^2-12b^2c^3dx+}{12d^5}$$

input
```
int((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(d*x+c)^2,
x)
```

output
```
(12*log(c + d*x)*a**2*d**4 + 24*log(c + d*x)*a*b*c**2*d**2 + 12*log(c + d*
x)*b**2*c**4 - 24*a*b*c*d**3*x + 12*a*b*d**4*x**2 - 12*b**2*c**3*d*x + 6*b
**2*c**2*d**2*x**2 - 4*b**2*c*d**3*x**3 + 3*b**2*d**4*x**4)/(12*d**5)
```

3.35 $\int \frac{-1+x^3}{1+x+x^2} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 11

$$\int \frac{-1+x^3}{1+x+x^2} \, dx = \frac{1}{2}(1-x)^2$$

output
```
1/2*(1-x)^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{-1+x^3}{1+x+x^2} \, dx = -x+\frac{x^2}{2}$$

input
```
Integrate[(-1 + x^3)/(1 + x + x^2),x]
```

output
```
-x + x^2/2
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2019, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3-1}{x^2+x+1}dx$$

$$\Big\downarrow \text{ 2019}$$

$$\int (x-1)dx$$

$$\Big\downarrow \text{ 17}$$

$$\frac{1}{2}(1-x)^2$$

input
```
Int[(-1 + x^3)/(1 + x + x^2),x]
```

output
```
(1 - x)^2/2
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

method	result	size
gosper	$\frac{x(x-2)}{2}$	7
default	$\frac{1}{2}x^2 - x$	10
norman	$\frac{1}{2}x^2 - x$	10
risch	$\frac{1}{2}x^2 - x$	10
parallelrisch	$\frac{1}{2}x^2 - x$	10
parts	$\frac{1}{2}x^2 - x$	10
orering	$\frac{x(x-2)(x^3-1)}{2(x-1)(x^2+x+1)}$	25

input
```
int((x^3-1)/(x^2+x+1),x,method=_RETURNVERBOSE)
```

output
```
1/2*x*(x-2)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{-1+x^3}{1+x+x^2}\,dx = \frac{1}{2}\,x^2 - x$$

input
```
integrate((x^3-1)/(x^2+x+1),x, algorithm="fricas")
```

output
```
1/2*x^2 - x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.45

$$\int \frac{-1+x^3}{1+x+x^2}\,dx = \frac{x^2}{2} - x$$

input
```
integrate((x**3-1)/(x**2+x+1),x)
```

output
```
x**2/2 - x
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{-1+x^3}{1+x+x^2}\,dx = \frac{1}{2}\,x^2 - x$$

input
```
integrate((x^3-1)/(x^2+x+1),x, algorithm="maxima")
```

output
```
1/2*x^2 - x
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{-1+x^3}{1+x+x^2}\,dx = \frac{1}{2}\,x^2 - x$$

input
```
integrate((x^3-1)/(x^2+x+1),x, algorithm="giac")
```

output
```
1/2*x^2 - x
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.55

$$\int \frac{-1+x^3}{1+x+x^2}\,dx=\frac{x\,(x-2)}{2}$$

input
```
int((x^3 - 1)/(x + x^2 + 1),x)
```

output
```
(x*(x - 2))/2
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.55

$$\int \frac{-1+x^3}{1+x+x^2}\,dx=\frac{x(x-2)}{2}$$

input
```
int((x^3-1)/(x^2+x+1),x)
```

output
```
(x*(x - 2))/2
```

3.36 $\int \frac{a^2c+a^2dx+2abcx^2+2abdx^3+b^2cx^4+b^2dx^5}{a+bx^2}\,dx$

Optimal result

Integrand size = 54, antiderivative size = 31

$$\int \frac{a^2c+a^2dx+2abcx^2+2abdx^3+b^2cx^4+b^2dx^5}{a+bx^2}\,dx = acx+\frac{1}{3}bcx^3+\frac{d(a+bx^2)^2}{4b}$$

output
```
a*c*x+1/3*b*c*x^3+1/4*d*(b*x^2+a)^2/b
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03

$$\int \frac{a^2c+a^2dx+2abcx^2+2abdx^3+b^2cx^4+b^2dx^5}{a+bx^2}\,dx = acx+\frac{1}{2}adx^2+\frac{1}{3}bcx^3+\frac{1}{4}bdx^4$$

input
```
Integrate[(a^2*c + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3 + b^2*c*x^4 + b^2*d
*x^5)/(a + b*x^2),x]
```

output
```
a*c*x + (a*d*x^2)/2 + (b*c*x^3)/3 + (b*d*x^4)/4
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.037$, Rules used = {2019, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a^2c + a^2dx + 2abcx^2 + 2abdx^3 + b^2cx^4 + b^2dx^5}{a + bx^2} \, dx$$

$$\Big\downarrow \text{2019}$$

$$\int \left(ac + adx + bcx^2 + bdx^3\right) dx$$

$$\Big\downarrow \text{2009}$$

$$acx + \frac{1}{2}adx^2 + \frac{1}{3}bcx^3 + \frac{1}{4}bdx^4$$

input
```
Int[(a^2*c + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3 + b^2*c*x^4 + b^2*d*x^5)/
(a + b*x^2),x]
```

output
```
a*c*x + (a*d*x^2)/2 + (b*c*x^3)/3 + (b*d*x^4)/4
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

method	result	size
default	$\frac{1}{4}bd\,x^4+\frac{1}{3}bc\,x^3+\frac{1}{2}ad\,x^2+xac$	27
norman	$\frac{1}{4}bd\,x^4+\frac{1}{3}bc\,x^3+\frac{1}{2}ad\,x^2+xac$	27
risch	$\frac{1}{4}bd\,x^4+\frac{1}{3}bc\,x^3+\frac{1}{2}ad\,x^2+xac$	27
parallelrisch	$\frac{1}{4}bd\,x^4+\frac{1}{3}bc\,x^3+\frac{1}{2}ad\,x^2+xac$	27
parts	$\frac{1}{4}bd\,x^4+\frac{1}{3}bc\,x^3+\frac{1}{2}ad\,x^2+xac$	27
gosper	$\frac{x(3bd\,x^3+4bc\,x^2+6adx+12ac)}{12}$	28
orering	$\frac{x(3bd\,x^3+4bc\,x^2+6adx+12ac)(b^2d\,x^5+b^2c\,x^4+2abd\,x^3+2abc\,x^2+a^2dx+a^2c)}{12(dx+c)(b\,x^2+a)^2}$	88

input
```
int((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(b*x^2+a),
x,method=_RETURNVERBOSE)
```

output
```
1/4*b*d*x^4+1/3*b*c*x^3+1/2*a*d*x^2+x*a*c
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \frac{a^2c+a^2dx+2abcx^2+2abdx^3+b^2cx^4+b^2dx^5}{a+bx^2}\,dx=\frac{1}{4}\,bdx^4+\frac{1}{3}\,bcx^3+\frac{1}{2}\,adx^2+acx$$

input
```
integrate((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(b*x
^2+a),x, algorithm="fricas")
```

output
```
1/4*b*d*x^4 + 1/3*b*c*x^3 + 1/2*a*d*x^2 + a*c*x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int \frac{a^2c+a^2dx+2abcx^2+2abdx^3+b^2cx^4+b^2dx^5}{a+bx^2}\,dx = acx+\frac{adx^2}{2}+\frac{bcx^3}{3}+\frac{bdx^4}{4}$$

input
```
integrate((b**2*d*x**5+b**2*c*x**4+2*a*b*d*x**3+2*a*b*c*x**2+a**2*d*x+a**2
*c)/(b*x**2+a),x)
```

output
```
a*c*x + a*d*x**2/2 + b*c*x**3/3 + b*d*x**4/4
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \frac{a^2c+a^2dx+2abcx^2+2abdx^3+b^2cx^4+b^2dx^5}{a+bx^2}\,dx = \frac{1}{4}\,bdx^4+\frac{1}{3}\,bcx^3+\frac{1}{2}\,adx^2+acx$$

input
```
integrate((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(b*x
^2+a),x, algorithm="maxima")
```

output
```
1/4*b*d*x^4 + 1/3*b*c*x^3 + 1/2*a*d*x^2 + a*c*x
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \frac{a^2c+a^2dx+2abcx^2+2abdx^3+b^2cx^4+b^2dx^5}{a+bx^2}\,dx = \frac{1}{4}\,bdx^4+\frac{1}{3}\,bcx^3+\frac{1}{2}\,adx^2+acx$$

input
```
integrate((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(b*x
^2+a),x, algorithm="giac")
```

output
```
1/4*b*d*x^4 + 1/3*b*c*x^3 + 1/2*a*d*x^2 + a*c*x
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \frac{a^2c + a^2dx + 2abcx^2 + 2abdx^3 + b^2cx^4 + b^2dx^5}{a + bx^2}\,dx = \frac{b\,d\,x^4}{4} + \frac{b\,c\,x^3}{3} + \frac{a\,d\,x^2}{2} + a\,c\,x$$

input
```
int((a^2*c + b^2*c*x^4 + b^2*d*x^5 + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3)/
(a + b*x^2),x)
```

output
```
a*c*x + (a*d*x^2)/2 + (b*c*x^3)/3 + (b*d*x^4)/4
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int \frac{a^2c + a^2dx + 2abcx^2 + 2abdx^3 + b^2cx^4 + b^2dx^5}{a + bx^2}\,dx = \frac{x(3bd\,x^3 + 4bc\,x^2 + 6adx + 12ac)}{12}$$

input
```
int((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(b*x^2+a),
x)
```

output
```
(x*(12*a*c + 6*a*d*x + 4*b*c*x**2 + 3*b*d*x**3))/12
```

3.37 $\int \frac{a^2c+a^2dx+2abcx^2+2abdx^3+b^2cx^4+b^2dx^5}{(a+bx^2)^2}\,dx$

Optimal result

Integrand size = 54, antiderivative size = 14

$$\int \frac{a^2c+a^2dx+2abcx^2+2abdx^3+b^2cx^4+b^2dx^5}{(a+bx^2)^2}\,dx=\frac{(c+dx)^2}{2d}$$

output
```
1/2*(d*x+c)^2/d
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{a^2c+a^2dx+2abcx^2+2abdx^3+b^2cx^4+b^2dx^5}{(a+bx^2)^2}\,dx=cx+\frac{dx^2}{2}$$

input
```
Integrate[(a^2*c + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3 + b^2*c*x^4 + b^2*d
*x^5)/(a + b*x^2)^2,x]
```

output
```
c*x + (d*x^2)/2
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {2019, 2019, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a^2c+a^2dx+2abcx^2+2abdx^3+b^2cx^4+b^2dx^5}{(a+bx^2)^2}dx$$

$$\Big\downarrow \text{2019}$$

$$\int \frac{ac+adx+bcx^2+bdx^3}{a+bx^2}dx$$

$$\Big\downarrow \text{2019}$$

$$\int (c+dx)dx$$

$$\Big\downarrow \text{17}$$

$$\frac{(c+dx)^2}{2d}$$

input
```
Int[(a^2*c + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3 + b^2*c*x^4 + b^2*d*x^5)/
(a + b*x^2)^2,x]
```

output
```
(c + d*x)^2/(2*d)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

method	result	size
gosper	$\frac{x(dx+2c)}{2}$	11
default	$\frac{1}{2}d\,x^2+cx$	11
risch	$\frac{1}{2}d\,x^2+cx$	11
parallelrisch	$\frac{1}{2}d\,x^2+cx$	11
parts	$\frac{1}{2}d\,x^2+cx$	11
norman	$\frac{bc\,x^3+xac-\frac{a^2d}{2b}+\frac{bd\,x^4}{2}}{b\,x^2+a}$	38
orering	$\frac{x(dx+2c)\left(b^2d\,x^5+b^2c\,x^4+2abd\,x^3+2abc\,x^2+a^2dx+a^2c\right)}{2(dx+c)\left(b\,x^2+a\right)^2}$	71

input
```
int((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(b*x^2+a)^
2,x,method=_RETURNVERBOSE)
```

output
```
1/2*x*(d*x+2*c)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{a^2c+a^2dx+2abcx^2+2abdx^3+b^2cx^4+b^2dx^5}{\left(a+bx^2\right)^2}\,dx=\frac{1}{2}\,dx^2+cx$$

input
```
integrate((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(b*x
^2+a)^2,x, algorithm="fricas")
```

output
```
1/2*d*x^2 + c*x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.57

$$\int \frac{a^2c + a^2dx + 2abcx^2 + 2abdx^3 + b^2cx^4 + b^2dx^5}{(a + bx^2)^2}\,dx = cx + \frac{dx^2}{2}$$

input
```
integrate((b**2*d*x**5+b**2*c*x**4+2*a*b*d*x**3+2*a*b*c*x**2+a**2*d*x+a**2
*c)/(b*x**2+a)**2,x)
```

output
```
c*x + d*x**2/2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{a^2c + a^2dx + 2abcx^2 + 2abdx^3 + b^2cx^4 + b^2dx^5}{(a + bx^2)^2}\,dx = \frac{1}{2}\,dx^2 + cx$$

input
```
integrate((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(b*x
^2+a)^2,x, algorithm="maxima")
```

output
```
1/2*d*x^2 + c*x
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{a^2c + a^2dx + 2abcx^2 + 2abdx^3 + b^2cx^4 + b^2dx^5}{(a + bx^2)^2}\,dx = \frac{1}{2}\,dx^2 + cx$$

input
```
integrate((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(b*x
^2+a)^2,x, algorithm="giac")
```

output
```
1/2*d*x^2 + c*x
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{a^2c + a^2dx + 2abcx^2 + 2abdx^3 + b^2cx^4 + b^2dx^5}{(a + bx^2)^2} dx = \frac{d\,x^2}{2} + c\,x$$

input
```
int((a^2*c + b^2*c*x^4 + b^2*d*x^5 + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3)/
(a + b*x^2)^2,x)
```

output
```
c*x + (d*x^2)/2
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{a^2c + a^2dx + 2abcx^2 + 2abdx^3 + b^2cx^4 + b^2dx^5}{(a + bx^2)^2} dx = \frac{x(dx + 2c)}{2}$$

input
```
int((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(b*x^2+a)^
2,x)
```

output
```
(x*(2*c + d*x))/2
```

3.38 $\int \frac{a^2c+a^2dx+2abcx^2+2abdx^3+b^2cx^4+b^2dx^5}{(a+bx^2)^3} dx$

Optimal result

Integrand size = 54, antiderivative size = 42

$$\int \frac{a^2c+a^2dx+2abcx^2+2abdx^3+b^2cx^4+b^2dx^5}{(a+bx^2)^3} dx = \frac{c\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}} + \frac{d\log(a+bx^2)}{2b}$$

output
```
c*arctan(b^(1/2)*x/a^(1/2))/a^(1/2)/b^(1/2)+1/2*d*ln(b*x^2+a)/b
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int \frac{a^2c+a^2dx+2abcx^2+2abdx^3+b^2cx^4+b^2dx^5}{(a+bx^2)^3} dx = \frac{c\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}} + \frac{d\log(a+bx^2)}{2b}$$

input
```
Integrate[(a^2*c + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3 + b^2*c*x^4 + b^2*d
*x^5)/(a + b*x^2)^3,x]
```

output
```
(c*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*Sqrt[b]) + (d*Log[a + b*x^2])/(2*
b)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.093, Rules used = {2019, 2019, 452, 218, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a^2c+a^2dx+2abcx^2+2abdx^3+b^2cx^4+b^2dx^5}{(a+bx^2)^3}dx$$

$$\Big\downarrow \text{2019}$$

$$\int \frac{ac+adx+bcx^2+bdx^3}{(a+bx^2)^2}dx$$

$$\Big\downarrow \text{2019}$$

$$\int \frac{c+dx}{a+bx^2}dx$$

$$\Big\downarrow \text{452}$$

$$c\int \frac{1}{bx^2+a}dx+d\int \frac{x}{bx^2+a}dx$$

$$\Big\downarrow \text{218}$$

$$d\int \frac{x}{bx^2+a}dx+\frac{c\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

$$\Big\downarrow \text{240}$$

$$\frac{c\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}+\frac{d\log\left(a+bx^2\right)}{2b}$$

input
```
Int[(a^2*c + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3 + b^2*c*x^4 + b^2*d*x^5)/
(a + b*x^2)^3,x]
```

output
```
(c*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*Sqrt[b]) + (d*Log[a + b*x^2])/(2*
b)
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 452
```
Int[((c_) + (d_.)*(x_))/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[c   Int[1/
(a + b*x^2), x], x] + Simp[d   Int[x/(a + b*x^2), x], x] /; FreeQ[{a, b, c,
 d}, x] && NeQ[b*c^2 + a*d^2, 0]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.76

method	result	size
default	$\frac{d\ln\left(b\,x^{2}+a\right)}{2b}+\frac{c\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}$	32
risch	$\frac{\ln\left(-\sqrt{-ab}\,x+a\right)c\sqrt{-ab}}{2ab}+\frac{\ln\left(-\sqrt{-ab}\,x+a\right)d}{2b}-\frac{\ln\left(\sqrt{-ab}\,x+a\right)c\sqrt{-ab}}{2ab}+\frac{\ln\left(\sqrt{-ab}\,x+a\right)d}{2b}$	90

input
```
int((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(b*x^2+a)^
3,x,method=_RETURNVERBOSE)
```

output
```
1/2*d*ln(b*x^2+a)/b+c/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 98, normalized size of antiderivative = 2.33

$$\int \frac{a^2c + a^2dx + 2abcx^2 + 2abdx^3 + b^2cx^4 + b^2dx^5}{(a + bx^2)^3} \, dx$$
$$= \left[\frac{ad \log (bx^2 + a) - \sqrt{-ab}c \log \left(\frac{bx^2 - 2\sqrt{-ab}x - a}{bx^2 + a} \right)}{2\,ab}, \frac{ad \log (bx^2 + a) + 2\sqrt{ab}c \arctan \left(\frac{\sqrt{ab}x}{a} \right)}{2\,ab} \right]$$

input
```
integrate((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(b*x
^2+a)^3,x, algorithm="fricas")
```

output
```
[1/2*(a*d*log(b*x^2 + a) - sqrt(-a*b)*c*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(
b*x^2 + a)))/(a*b), 1/2*(a*d*log(b*x^2 + a) + 2*sqrt(a*b)*c*arctan(sqrt(a*
b)*x/a))/(a*b)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 124 vs. 2(37) = 74.

Time = 0.17 (sec) , antiderivative size = 124, normalized size of antiderivative = 2.95

$$\int \frac{a^2c + a^2dx + 2abcx^2 + 2abdx^3 + b^2cx^4 + b^2dx^5}{(a + bx^2)^3} \, dx$$
$$= \left(\frac{d}{2b} - \frac{c\sqrt{-ab^3}}{2ab^2} \right) \log \left(x + \frac{2ab\left(\frac{d}{2b} - \frac{c\sqrt{-ab^3}}{2ab^2} \right) - ad}{bc} \right)$$
$$+ \left(\frac{d}{2b} + \frac{c\sqrt{-ab^3}}{2ab^2} \right) \log \left(x + \frac{2ab\left(\frac{d}{2b} + \frac{c\sqrt{-ab^3}}{2ab^2} \right) - ad}{bc} \right)$$

input
```
integrate((b**2*d*x**5+b**2*c*x**4+2*a*b*d*x**3+2*a*b*c*x**2+a**2*d*x+a**2
*c)/(b*x**2+a)**3,x)
```

output
```
(d/(2*b) - c*sqrt(-a*b**3)/(2*a*b**2))*log(x + (2*a*b*(d/(2*b) - c*sqrt(-a
*b**3)/(2*a*b**2)) - a*d)/(b*c)) + (d/(2*b) + c*sqrt(-a*b**3)/(2*a*b**2))*
log(x + (2*a*b*(d/(2*b) + c*sqrt(-a*b**3)/(2*a*b**2)) - a*d)/(b*c))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.74

$$\int \frac{a^2c + a^2dx + 2abcx^2 + 2abdx^3 + b^2cx^4 + b^2dx^5}{(a + bx^2)^3}\,dx = \frac{c\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}} + \frac{d\log\left(bx^2 + a\right)}{2\,b}$$

input
```
integrate((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(b*x
^2+a)^3,x, algorithm="maxima")
```

output
```
c*arctan(b*x/sqrt(a*b))/sqrt(a*b) + 1/2*d*log(b*x^2 + a)/b
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.74

$$\int \frac{a^2c + a^2dx + 2abcx^2 + 2abdx^3 + b^2cx^4 + b^2dx^5}{(a + bx^2)^3}\,dx = \frac{c\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}} + \frac{d\log\left(bx^2 + a\right)}{2\,b}$$

input
```
integrate((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(b*x
^2+a)^3,x, algorithm="giac")
```

output
```
c*arctan(b*x/sqrt(a*b))/sqrt(a*b) + 1/2*d*log(b*x^2 + a)/b
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.76

$$\int \frac{a^2c+a^2dx+2abcx^2+2abdx^3+b^2cx^4+b^2dx^5}{(a+bx^2)^3}\,dx = \frac{d\,\ln\left(b\,x^2+a\right)}{2\,b}+\frac{c\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}$$

input
```
int((a^2*c + b^2*c*x^4 + b^2*d*x^5 + a^2*d*x + 2*a*b*c*x^2 + 2*a*b*d*x^3)/
(a + b*x^2)^3,x)
```

output
```
(d*log(a + b*x^2))/(2*b) + (c*atan((b^(1/2)*x)/a^(1/2)))/(a^(1/2)*b^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.93

$$\int \frac{a^2c+a^2dx+2abcx^2+2abdx^3+b^2cx^4+b^2dx^5}{(a+bx^2)^3}\,dx = \frac{2\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)c+\log(b\,x^2+a)\,ad}{2ab}$$

input
```
int((b^2*d*x^5+b^2*c*x^4+2*a*b*d*x^3+2*a*b*c*x^2+a^2*d*x+a^2*c)/(b*x^2+a)^
3,x)
```

output
```
(2*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*c + log(a + b*x**2)*a*d)/
(2*a*b)
```

3.39 $\int \frac{1-x}{1+x^3} dx$

Optimal result

Integrand size = 13, antiderivative size = 15

$$\int \frac{1-x}{1+x^3} dx = \log(1+x) - \frac{1}{3}\log\left(1+x^3\right)$$

output
```
ln(1+x)-1/3*ln(x^3+1)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.47

$$\int \frac{1-x}{1+x^3} dx = \frac{2}{3}\log(1+x) - \frac{1}{3}\log\left(1-x+x^2\right)$$

input
```
Integrate[(1 - x)/(1 + x^3),x]
```

output
```
(2*Log[1 + x])/3 - Log[1 - x + x^2]/3
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.47, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2399, 16, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-x}{x^3+1}dx$$

$$\Big\downarrow \text{2399}$$

$$\frac{1}{3}\int \frac{1-2x}{x^2-x+1}dx+\frac{2}{3}\int \frac{1}{x+1}dx$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{3}\int \frac{1-2x}{x^2-x+1}dx+\frac{2}{3}\log(x+1)$$

$$\Big\downarrow \text{1103}$$

$$\frac{2}{3}\log(x+1)-\frac{1}{3}\log\left(x^2-x+1\right)$$

input
```
Int[(1 - x)/(1 + x^3),x]
```

output
```
(2*Log[1 + x])/3 - Log[1 - x + x^2]/3
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 2399
```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Simp[(-r)*((B*r - A*s)/(3*a
*s))   Int[1/(r + s*x), x], x] + Simp[r/(3*a*s)   Int[(r*(B*r + 2*A*s) + s*
(B*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] &
& NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.27

method	result
default	$-\frac{\ln\left(x^{2}-x+1\right)}{3}+\frac{2\ln\left(x+1\right)}{3}$
norman	$-\frac{\ln\left(x^{2}-x+1\right)}{3}+\frac{2\ln\left(x+1\right)}{3}$
risch	$-\frac{\ln\left(x^{2}-x+1\right)}{3}+\frac{2\ln\left(x+1\right)}{3}$
parallelrisch	$-\frac{\ln\left(x^{2}-x+1\right)}{3}+\frac{2\ln\left(x+1\right)}{3}$
meijerg	$\frac{x\ln\left(1+\left(x^{3}\right)^{\frac{1}{3}}\right)}{3\left(x^{3}\right)^{\frac{1}{3}}}-\frac{x\ln\left(1-\left(x^{3}\right)^{\frac{1}{3}}+\left(x^{3}\right)^{\frac{2}{3}}\right)}{6\left(x^{3}\right)^{\frac{1}{3}}}+\frac{x\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(x^{3}\right)^{\frac{1}{3}}}{2-\left(x^{3}\right)^{\frac{1}{3}}}\right)}{3\left(x^{3}\right)^{\frac{1}{3}}}+\frac{x^{2}\ln\left(1+\left(x^{3}\right)^{\frac{1}{3}}\right)}{3\left(x^{3}\right)^{\frac{2}{3}}}-\frac{x^{2}\ln\left(1-\left(x^{3}\right)^{\frac{1}{3}}+(\right.}{6\left(x^{3}\right)^{\frac{2}{3}}}$

input
```
int((1-x)/(x^3+1),x,method=_RETURNVERBOSE)
```

output
```
-1/3*ln(x^2-x+1)+2/3*ln(x+1)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.20

$$\int \frac{1-x}{1+x^3}\,dx=-\frac{1}{3}\,\log\left(x^{2}-x+1\right)+\frac{2}{3}\,\log\left(x+1\right)$$

input
```
integrate((1-x)/(x^3+1),x, algorithm="fricas")
```

output
```
-1/3*log(x^2 - x + 1) + 2/3*log(x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{1-x}{1+x^3}\,dx = \frac{2\log{(x+1)}}{3} - \frac{\log{(x^2-x+1)}}{3}$$

input
```
integrate((1-x)/(x**3+1),x)
```

output
```
2*log(x + 1)/3 - log(x**2 - x + 1)/3
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.20

$$\int \frac{1-x}{1+x^3}\,dx = -\frac{1}{3}\log\left(x^2-x+1\right) + \frac{2}{3}\log\left(x+1\right)$$

input
```
integrate((1-x)/(x^3+1),x, algorithm="maxima")
```

output
```
-1/3*log(x^2 - x + 1) + 2/3*log(x + 1)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.27

$$\int \frac{1-x}{1+x^3}\,dx = -\frac{1}{3}\log\left(x^2-x+1\right) + \frac{2}{3}\log\left(|x+1|\right)$$

input
```
integrate((1-x)/(x^3+1),x, algorithm="giac")
```

output
```
-1/3*log(x^2 - x + 1) + 2/3*log(abs(x + 1))
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.20

$$\int \frac{1-x}{1+x^3}\,dx=\frac{2\,\ln\left(x+1\right)}{3}-\frac{\ln\left(x^2-x+1\right)}{3}$$

input
```
int(-(x - 1)/(x^3 + 1),x)
```

output
```
(2*log(x + 1))/3 - log(x^2 - x + 1)/3
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.20

$$\int \frac{1-x}{1+x^3}\,dx=-\frac{\log(x^2-x+1)}{3}+\frac{2\log(x+1)}{3}$$

input
```
int((1-x)/(x^3+1),x)
```

output
```
( - log(x**2 - x + 1) + 2*log(x + 1))/3
```

3.40 $\int \frac{1+x+4x^2}{x+4x^3} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 11

$$\int \frac{1+x+4x^2}{x+4x^3} \, dx = \frac{1}{2}\arctan(2x)+\log(x)$$

output
```
1/2*arctan(2*x)+ln(x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{1+x+4x^2}{x+4x^3} \, dx = \frac{1}{2}\arctan(2x)+\log(x)$$

input
```
Integrate[(1 + x + 4*x^2)/(x + 4*x^3),x]
```

output
```
ArcTan[2*x]/2 + Log[x]
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2026, 2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{4x^2+x+1}{4x^3+x} \, dx$$

$$\Big\downarrow \text{ 2026}$$

$$\int \frac{4x^2+x+1}{x(4x^2+1)}dx$$

$$\Big\downarrow \text{ 2333}$$

$$\int \left(\frac{1}{4x^2+1}+\frac{1}{x}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\arctan(2x)+\log(x)$$

input
```
Int[(1 + x + 4*x^2)/(x + 4*x^3),x]
```

output
```
ArcTan[2*x]/2 + Log[x]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2026
```
Int[(Fx_.)*(Px_)^(p_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Int[x^(p
*r)*ExpandToSum[Px/x^r, x]^p*Fx, x] /; IGtQ[r, 0]] /; PolyQ[Px, x] && Integ
erQ[p] &&  !MonomialQ[Px, x] && (ILtQ[p, 0] ||  !PolyQ[u, x])
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

method	result	size
default	$\frac{\arctan(2x)}{2}+\ln\left(x\right)$	10
risch	$\frac{\arctan(2x)}{2}+\ln\left(x\right)$	10
meijerg	$\frac{\arctan(2x)}{2}+\ln\left(x\right)+\ln\left(2\right)$	12
parallelrisch	$\ln\left(x\right)-\frac{i\ln\left(x-\frac{i}{2}\right)}{4}+\frac{i\ln\left(x+\frac{i}{2}\right)}{4}$	20

input
```
int((4*x^2+x+1)/(4*x^3+x),x,method=_RETURNVERBOSE)
```

output
```
1/2*arctan(2*x)+ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{1+x+4x^2}{x+4x^3}\,dx=\frac{1}{2}\,\arctan\left(2\,x\right)+\log\left(x\right)$$

input
```
integrate((4*x^2+x+1)/(4*x^3+x),x, algorithm="fricas")
```

output
```
1/2*arctan(2*x) + log(x)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int \frac{1+x+4x^2}{x+4x^3}\,dx=\log{(x)}+\frac{\operatorname{atan}{(2x)}}{2}$$

input
```
integrate((4*x**2+x+1)/(4*x**3+x),x)
```

output
```
log(x) + atan(2*x)/2
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{1+x+4x^2}{x+4x^3}\,dx=\frac{1}{2}\arctan\left(2\,x\right)+\log\left(x\right)$$

input
```
integrate((4*x^2+x+1)/(4*x^3+x),x, algorithm="maxima")
```

output
```
1/2*arctan(2*x) + log(x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

$$\int \frac{1+x+4x^2}{x+4x^3}\,dx=\frac{1}{2}\arctan\left(2\,x\right)+\log\left(|x|\right)$$

input
```
integrate((4*x^2+x+1)/(4*x^3+x),x, algorithm="giac")
```

output
```
1/2*arctan(2*x) + log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.55

$$\int \frac{1+x+4x^2}{x+4x^3}\,dx = \ln(x) - \frac{\operatorname{atan}\left(\frac{17}{32\left(\frac{x}{16}-\frac{1}{8}\right)}+4\right)}{2}$$

input
```
int((x + 4*x^2 + 1)/(x + 4*x^3),x)
```

output
```
log(x) - atan(17/(32*(x/16 - 1/8)) + 4)/2
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{1+x+4x^2}{x+4x^3}\,dx = \frac{atan(2x)}{2} + \log(x)$$

input
```
int((4*x^2+x+1)/(4*x^3+x),x)
```

output
```
(atan(2*x) + 2*log(x))/2
```

3.41 $\int \frac{1-x+3x^2}{-x^2+x^3} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 12

$$\int \frac{1-x+3x^2}{-x^2+x^3} \, dx = \frac{1}{x}+3\log(1-x)$$

output
```
1/x+3*ln(1-x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1-x+3x^2}{-x^2+x^3} \, dx = \frac{1}{x}+3\log(1-x)$$

input
```
Integrate[(1 - x + 3*x^2)/(-x^2 + x^3),x]
```

output
```
x^(-1) + 3*Log[1 - x]
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {2026, 1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{3x^2-x+1}{x^3-x^2} \, dx$$

$$\Big\downarrow \text{2026}$$

$$\int \frac{3x^2-x+1}{(x-1)x^2}dx$$

$$\Big\downarrow \text{1195}$$

$$\int \left(\frac{3}{x-1}-\frac{1}{x^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{x}+3\log(1-x)$$

input
```
Int[(1 - x + 3*x^2)/(-x^2 + x^3),x]
```

output
```
x^(-1) + 3*Log[1 - x]
```

Defintions of rubi rules used

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2026
```
Int[(Fx_.)*(Px_)^(p_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Int[x^(p
*r)*ExpandToSum[Px/x^r, x]^p*Fx, x] /; IGtQ[r, 0]] /; PolyQ[Px, x] && Integ
erQ[p] &&  !MonomialQ[Px, x] && (ILtQ[p, 0] ||  !PolyQ[u, x])
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

method	result	size
default	$3\ln\left(x-1\right)+\frac{1}{x}$	11
norman	$3\ln\left(x-1\right)+\frac{1}{x}$	11
risch	$3\ln\left(x-1\right)+\frac{1}{x}$	11
meijerg	$\frac{1}{x}+3\ln\left(1-x\right)$	13
parallelrisch	$\frac{3\ln(x-1)x+1}{x}$	14

input
```
int((3*x^2-x+1)/(x^3-x^2),x,method=_RETURNVERBOSE)
```

output
```
3*ln(x-1)+1/x
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

$$\int \frac{1-x+3x^2}{-x^2+x^3}\,dx=\frac{3\,x\log\left(x-1\right)+1}{x}$$

input
```
integrate((3*x^2-x+1)/(x^3-x^2),x, algorithm="fricas")
```

output
```
(3*x*log(x - 1) + 1)/x
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{1-x+3x^2}{-x^2+x^3}\,dx = 3\log\left(x-1\right)+\frac{1}{x}$$

input
```
integrate((3*x**2-x+1)/(x**3-x**2),x)
```

output
```
3*log(x - 1) + 1/x
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1-x+3x^2}{-x^2+x^3}\,dx = \frac{1}{x}+3\,\log\left(x-1\right)$$

input
```
integrate((3*x^2-x+1)/(x^3-x^2),x, algorithm="maxima")
```

output
```
1/x + 3*log(x - 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

$$\int \frac{1-x+3x^2}{-x^2+x^3}\,dx = \frac{1}{x}+3\,\log\left(|x-1|\right)$$

input
```
integrate((3*x^2-x+1)/(x^3-x^2),x, algorithm="giac")
```

output
```
1/x + 3*log(abs(x - 1))
```

Mupad [B] (verification not implemented)

Time = 22.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1-x+3x^2}{-x^2+x^3}\,dx=3\,\ln\left(x-1\right)+\frac{1}{x}$$

input
```
int(-(3*x^2 - x + 1)/(x^2 - x^3),x)
```

output
```
3*log(x - 1) + 1/x
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

$$\int \frac{1-x+3x^2}{-x^2+x^3}\,dx=\frac{3\log(x-1)\,x+1}{x}$$

input
```
int((3*x^2-x+1)/(x^3-x^2),x)
```

output
```
(3*log(x - 1)*x + 1)/x
```

3.42 $\int \frac{4+3x+x^2}{x+x^2} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 12

$$\int \frac{4+3x+x^2}{x+x^2} \, dx = x+4\log(x)-2\log(1+x)$$

output
```
x+4*ln(x)-2*ln(1+x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{4+3x+x^2}{x+x^2} \, dx = x+4\log(x)-2\log(1+x)$$

input
```
Integrate[(4 + 3*x + x^2)/(x + x^2),x]
```

output
```
x + 4*Log[x] - 2*Log[1 + x]
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {2026, 1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2+3x+4}{x^2+x}\,dx$$

$$\Big\downarrow \text{2026}$$

$$\int \frac{x^2+3x+4}{x(x+1)}dx$$

$$\Big\downarrow \text{1195}$$

$$\int \left(-\frac{2}{x+1}+\frac{4}{x}+1\right)dx$$

$$\Big\downarrow \text{2009}$$

$$x+4\log(x)-2\log(x+1)$$

input
```
Int[(4 + 3*x + x^2)/(x + x^2),x]
```

output
```
x + 4*Log[x] - 2*Log[1 + x]
```

Defintions of rubi rules used

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2026
```
Int[(Fx_.)*(Px_)^(p_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Int[x^(p
*r)*ExpandToSum[Px/x^r, x]^p*Fx, x] /; IGtQ[r, 0]] /; PolyQ[Px, x] && Integ
erQ[p] &&  !MonomialQ[Px, x] && (ILtQ[p, 0] ||  !PolyQ[u, x])
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

method	result	size
default	$x+4\ln\left(x\right)-2\ln\left(x+1\right)$	13
norman	$x+4\ln\left(x\right)-2\ln\left(x+1\right)$	13
meijerg	$x+4\ln\left(x\right)-2\ln\left(x+1\right)$	13
risch	$x+4\ln\left(x\right)-2\ln\left(x+1\right)$	13
parallelrisch	$x+4\ln\left(x\right)-2\ln\left(x+1\right)$	13

input
```
int((x^2+3*x+4)/(x^2+x),x,method=_RETURNVERBOSE)
```

output
```
x+4*ln(x)-2*ln(x+1)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{4+3x+x^2}{x+x^2}\,dx = x-2\,\log\left(x+1\right)+4\,\log\left(x\right)$$

input
```
integrate((x^2+3*x+4)/(x^2+x),x, algorithm="fricas")
```

output
```
x - 2*log(x + 1) + 4*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{4+3x+x^2}{x+x^2}\,dx = x + 4\log(x) - 2\log(x+1)$$

input
```
integrate((x**2+3*x+4)/(x**2+x),x)
```

output
```
x + 4*log(x) - 2*log(x + 1)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{4+3x+x^2}{x+x^2}\,dx = x - 2\,\log(x+1) + 4\,\log(x)$$

input
```
integrate((x^2+3*x+4)/(x^2+x),x, algorithm="maxima")
```

output
```
x - 2*log(x + 1) + 4*log(x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{4+3x+x^2}{x+x^2}\,dx = x - 2\,\log(|x+1|) + 4\,\log(|x|)$$

input
```
integrate((x^2+3*x+4)/(x^2+x),x, algorithm="giac")
```

output
```
x - 2*log(abs(x + 1)) + 4*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 22.40 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{4+3x+x^2}{x+x^2} \, dx = x - 2 \ln(x+1) + 4 \ln(x)$$

input
```
int((3*x + x^2 + 4)/(x + x^2),x)
```

output
```
x - 2*log(x + 1) + 4*log(x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{4+3x+x^2}{x+x^2} \, dx = -2\log(x+1) + 4\log(x) + x$$

input
```
int((x^2+3*x+4)/(x^2+x),x)
```

output
```
 - 2*log(x + 1) + 4*log(x) + x
```

3.43 $\int \frac{4+x+3x^2}{x+x^3} dx$

Optimal result

Integrand size = 16, antiderivative size = 17

$$\int \frac{4+x+3x^2}{x+x^3} dx = \arctan(x) + 4\log(x) - \frac{1}{2}\log\left(1+x^2\right)$$

output
```
arctan(x)+4*ln(x)-1/2*ln(x^2+1)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{4+x+3x^2}{x+x^3} dx = \arctan(x) + 4\log(x) - \frac{1}{2}\log\left(1+x^2\right)$$

input
```
Integrate[(4 + x + 3*x^2)/(x + x^3),x]
```

output
```
ArcTan[x] + 4*Log[x] - Log[1 + x^2]/2
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {2026, 2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{3x^2+x+4}{x^3+x}\,dx$$
$$\Big\downarrow \text{ 2026}$$
$$\int \frac{3x^2+x+4}{x\,(x^2+1)}dx$$
$$\Big\downarrow \text{ 2333}$$
$$\int \left(\frac{1-x}{x^2+1}+\frac{4}{x}\right)dx$$
$$\Big\downarrow \text{ 2009}$$
$$\arctan(x)-\frac{1}{2}\log\left(x^2+1\right)+4\log(x)$$

input
```
Int[(4 + x + 3*x^2)/(x + x^3),x]
```

output
```
ArcTan[x] + 4*Log[x] - Log[1 + x^2]/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2026
```
Int[(Fx_.)*(Px_)^(p_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Int[x^(p
*r)*ExpandToSum[Px/x^r, x]^p*Fx, x] /; IGtQ[r, 0]] /; PolyQ[Px, x] && Integ
erQ[p] && !MonomialQ[Px, x] && (ILtQ[p, 0] || !PolyQ[u, x])
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

method	result	size
default	$\arctan\left(x\right)+4\ln\left(x\right)-\frac{\ln\left(x^{2}+1\right)}{2}$	16
meijerg	$\arctan\left(x\right)+4\ln\left(x\right)-\frac{\ln\left(x^{2}+1\right)}{2}$	16
risch	$\arctan\left(x\right)+4\ln\left(x\right)-\frac{\ln\left(x^{2}+1\right)}{2}$	16
parallelrisch	$4\ln\left(x\right)-\frac{\ln(x-i)}{2}-\frac{i\ln(x-i)}{2}-\frac{\ln(x+i)}{2}+\frac{i\ln(x+i)}{2}$	36

input
```
int((3*x^2+x+4)/(x^3+x),x,method=_RETURNVERBOSE)
```

output
```
arctan(x)+4*ln(x)-1/2*ln(x^2+1)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{4+x+3x^2}{x+x^3}\,dx=\arctan\left(x\right)-\frac{1}{2}\,\log\left(x^{2}+1\right)+4\,\log\left(x\right)$$

input
```
integrate((3*x^2+x+4)/(x^3+x),x, algorithm="fricas")
```

output
```
arctan(x) - 1/2*log(x^2 + 1) + 4*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{4+x+3x^2}{x+x^3}\,dx = 4\log{(x)} - \frac{\log{(x^2+1)}}{2} + \operatorname{atan}{(x)}$$

input
```
integrate((3*x**2+x+4)/(x**3+x),x)
```

output
```
4*log(x) - log(x**2 + 1)/2 + atan(x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{4+x+3x^2}{x+x^3}\,dx = \arctan\left(x\right) - \frac{1}{2}\,\log\left(x^2+1\right) + 4\,\log\left(x\right)$$

input
```
integrate((3*x^2+x+4)/(x^3+x),x, algorithm="maxima")
```

output
```
arctan(x) - 1/2*log(x^2 + 1) + 4*log(x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

$$\int \frac{4+x+3x^2}{x+x^3}\,dx = \arctan\left(x\right) - \frac{1}{2}\,\log\left(x^2+1\right) + 4\,\log\left(|x|\right)$$

input
```
integrate((3*x^2+x+4)/(x^3+x),x, algorithm="giac")
```

output
```
arctan(x) - 1/2*log(x^2 + 1) + 4*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.35

$$\int \frac{4+x+3x^2}{x+x^3}\,dx = 4\,\ln(x) + \ln(x-\mathrm{i})\,\left(-\frac{1}{2}-\frac{1}{2}\mathrm{i}\right) + \ln(x+1\mathrm{i})\,\left(-\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)$$

input
```
int((x + 3*x^2 + 4)/(x + x^3),x)
```

output
```
4*log(x) - log(x + 1i)*(1/2 - 1i/2) - log(x - 1i)*(1/2 + 1i/2)
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{4+x+3x^2}{x+x^3}\,dx = atan(x) - \frac{\log(x^2+1)}{2} + 4\log(x)$$

input
```
int((3*x^2+x+4)/(x^3+x),x)
```

output
```
(2*atan(x) - log(x**2 + 1) + 8*log(x))/2
```

3.44 $\int \frac{1+x^3}{-x+x^3} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 12

$$\int \frac{1+x^3}{-x+x^3} \, dx = x+\log(1-x)-\log(x)$$

output
```
x+ln(1-x)-ln(x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1+x^3}{-x+x^3} \, dx = x+\log(1-x)-\log(x)$$

input
```
Integrate[(1 + x^3)/(-x + x^3),x]
```

output
```
x + Log[1 - x] - Log[x]
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2026, 2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3+1}{x^3-x}dx$$

$$\Big\downarrow \text{ 2026}$$

$$\int \frac{x^3+1}{x(x^2-1)}dx$$

$$\Big\downarrow \text{ 2333}$$

$$\int \left(-\frac{1}{x}+\frac{1}{x-1}+1\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$x+\log(1-x)-\log(x)$$

input
```
Int[(1 + x^3)/(-x + x^3),x]
```

output
```
x + Log[1 - x] - Log[x]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2026
```
Int[(Fx_.)*(Px_)^(p_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Int[x^(p
*r)*ExpandToSum[Px/x^r, x]^p*Fx, x] /; IGtQ[r, 0]] /; PolyQ[Px, x] && Integ
erQ[p] && !MonomialQ[Px, x] && (ILtQ[p, 0] || !PolyQ[u, x])
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

method	result	size
default	$x+\ln(x-1)-\ln(x)$	11
norman	$x+\ln(x-1)-\ln(x)$	11
risch	$x+\ln(x-1)-\ln(x)$	11
parallelrisch	$x+\ln(x-1)-\ln(x)$	11
meijerg	$-\ln(x)-\frac{i\pi}{2}+\frac{\ln(-x^2+1)}{2}-\frac{i(2ix-2i\,\mathrm{arctanh}(x))}{2}$	33

input
```
int((x^3+1)/(x^3-x),x,method=_RETURNVERBOSE)
```

output
```
x+ln(x-1)-ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1+x^3}{-x+x^3}\,dx = x+\log(x-1)-\log(x)$$

input
```
integrate((x^3+1)/(x^3-x),x, algorithm="fricas")
```

output
```
x + log(x - 1) - log(x)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{1+x^3}{-x+x^3}\,dx = x - \log(x) + \log(x-1)$$

input
```
integrate((x**3+1)/(x**3-x),x)
```

output
```
x - log(x) + log(x - 1)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1+x^3}{-x+x^3}\,dx = x + \log(x-1) - \log(x)$$

input
```
integrate((x^3+1)/(x^3-x),x, algorithm="maxima")
```

output
```
x + log(x - 1) - log(x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1+x^3}{-x+x^3}\,dx = x + \log(|x-1|) - \log(|x|)$$

input
```
integrate((x^3+1)/(x^3-x),x, algorithm="giac")
```

output
```
x + log(abs(x - 1)) - log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1+x^3}{-x+x^3}\,dx = x - 2\,\operatorname{atanh}(2\,x-1)$$

input
```
int(-(x^3 + 1)/(x - x^3),x)
```

output
```
x - 2*atanh(2*x - 1)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1+x^3}{-x+x^3}\,dx = \log(x-1) - \log(x) + x$$

input
```
int((x^3+1)/(x^3-x),x)
```

output
```
log(x - 1) - log(x) + x
```

3.45 $\int \frac{1+x^3}{-x^2+x^3} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 17

$$\int \frac{1+x^3}{-x^2+x^3} \, dx = \frac{1}{x}+x+2\log(1-x)-\log(x)$$

output
```
1/x+x+2*ln(1-x)-ln(x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{1+x^3}{-x^2+x^3} \, dx = \frac{1}{x}+x+2\log(1-x)-\log(x)$$

input
```
Integrate[(1 + x^3)/(-x^2 + x^3),x]
```

output
```
x^(-1) + x + 2*Log[1 - x] - Log[x]
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {2026, 2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3+1}{x^3-x^2} \, dx$$

$$\Big\downarrow \text{ 2026}$$

$$\int \frac{x^3+1}{(x-1)x^2}dx$$

$$\Big\downarrow \text{ 2123}$$

$$\int \left(-\frac{1}{x^2}-\frac{1}{x}+\frac{2}{x-1}+1\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$x+\frac{1}{x}+2\log(1-x)-\log(x)$$

input
```
Int[(1 + x^3)/(-x^2 + x^3),x]
```

output
```
x^(-1) + x + 2*Log[1 - x] - Log[x]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2026
```
Int[(Fx_.)*(Px_)^(p_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Int[x^(p
*r)*ExpandToSum[Px/x^r, x]^p*Fx, x] /; IGtQ[r, 0]] /; PolyQ[Px, x] && Integ
erQ[p] &&  !MonomialQ[Px, x] && (ILtQ[p, 0] ||  !PolyQ[u, x])
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

method	result	size
default	$x+2\ln\left(x-1\right)+\frac{1}{x}-\ln\left(x\right)$	16
risch	$x+2\ln\left(x-1\right)+\frac{1}{x}-\ln\left(x\right)$	16
norman	$\frac{x^{2}+1}{x}-\ln\left(x\right)+2\ln\left(x-1\right)$	21
meijerg	$\frac{1}{x}-\ln\left(x\right)-i\pi+2\ln\left(1-x\right)+x$	22
parallelrisch	$-\frac{\ln(x)x-2\ln(x-1)x-x^{2}-1}{x}$	24

input
```
int((x^3+1)/(x^3-x^2),x,method=_RETURNVERBOSE)
```

output
```
x+2*ln(x-1)+1/x-ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.24

$$\int\frac{1+x^{3}}{-x^{2}+x^{3}}\,dx=\frac{x^{2}+2\,x\log\left(x-1\right)-x\log\left(x\right)+1}{x}$$

input
```
integrate((x^3+1)/(x^3-x^2),x, algorithm="fricas")
```

output
```
(x^2 + 2*x*log(x - 1) - x*log(x) + 1)/x
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \frac{1+x^3}{-x^2+x^3}\,dx = x - \log(x) + 2\log(x-1) + \frac{1}{x}$$

input
```
integrate((x**3+1)/(x**3-x**2),x)
```

output
```
x - log(x) + 2*log(x - 1) + 1/x
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{1+x^3}{-x^2+x^3}\,dx = x + \frac{1}{x} + 2\,\log(x-1) - \log(x)$$

input
```
integrate((x^3+1)/(x^3-x^2),x, algorithm="maxima")
```

output
```
x + 1/x + 2*log(x - 1) - log(x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{1+x^3}{-x^2+x^3}\,dx = x + \frac{1}{x} + 2\,\log(|x-1|) - \log(|x|)$$

input
```
integrate((x^3+1)/(x^3-x^2),x, algorithm="giac")
```

output
```
x + 1/x + 2*log(abs(x - 1)) - log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{1+x^3}{-x^2+x^3}\,dx = x+2\,\ln\left(x-1\right)-\ln\left(x\right)+\frac{1}{x}$$

input
```
int(-(x^3 + 1)/(x^2 - x^3),x)
```

output
```
x + 2*log(x - 1) - log(x) + 1/x
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.24

$$\int \frac{1+x^3}{-x^2+x^3}\,dx = \frac{2\log(x-1)\,x-\log(x)\,x+x^2+1}{x}$$

input
```
int((x^3+1)/(x^3-x^2),x)
```

output
```
(2*log(x - 1)*x - log(x)*x + x**2 + 1)/x
```

3.46 $\int \frac{-1+x^5}{-x+x^3} dx$

Optimal result

Integrand size = 15, antiderivative size = 17

$$\int \frac{-1+x^5}{-x+x^3} dx = x + \frac{x^3}{3} + \log(x) - \log(1+x)$$

output
```
x+1/3*x^3+ln(x)-ln(1+x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{-1+x^5}{-x+x^3} dx = x + \frac{x^3}{3} + \log(x) - \log(1+x)$$

input
```
Integrate[(-1 + x^5)/(-x + x^3),x]
```

output
```
x + x^3/3 + Log[x] - Log[1 + x]
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2026, 2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5-1}{x^3-x}dx$$

$$\Big\downarrow \text{2026}$$

$$\int \frac{x^5-1}{x(x^2-1)}dx$$

$$\Big\downarrow \text{2333}$$

$$\int \left(x^2+\frac{1}{-x-1}+\frac{1}{x}+1\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^3}{3}+x+\log(x)-\log(x+1)$$

input
```
Int[(-1 + x^5)/(-x + x^3),x]
```

output
```
x + x^3/3 + Log[x] - Log[1 + x]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2026
```
Int[(Fx_.)*(Px_)^(p_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Int[x^(p
*r)*ExpandToSum[Px/x^r, x]^p*Fx, x] /; IGtQ[r, 0]] /; PolyQ[Px, x] && Integ
erQ[p] &&  !MonomialQ[Px, x] && (ILtQ[p, 0] ||  !PolyQ[u, x])
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

method	result	size
default	$x+\frac{x^{3}}{3}+\ln\left(x\right)-\ln\left(x+1\right)$	16
norman	$x+\frac{x^{3}}{3}+\ln\left(x\right)-\ln\left(x+1\right)$	16
risch	$x+\frac{x^{3}}{3}+\ln\left(x\right)-\ln\left(x+1\right)$	16
parallelrisch	$x+\frac{x^{3}}{3}+\ln\left(x\right)-\ln\left(x+1\right)$	16
meijerg	$\ln\left(x\right)+\frac{i\pi}{2}-\frac{\ln\left(-x^{2}+1\right)}{2}+\frac{i\left(-\frac{2ix\left(5x^{2}+15\right)}{15}+2i\,\mathrm{arctanh}\left(x\right)\right)}{2}$	38

input
```
int((x^5-1)/(x^3-x),x,method=_RETURNVERBOSE)
```

output
```
x+1/3*x^3+ln(x)-ln(x+1)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{-1+x^5}{-x+x^3}\,dx=\frac{1}{3}\,x^{3}+x-\log\left(x+1\right)+\log\left(x\right)$$

input
```
integrate((x^5-1)/(x^3-x),x, algorithm="fricas")
```

output
```
1/3*x^3 + x - log(x + 1) + log(x)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \frac{-1+x^5}{-x+x^3}\,dx = \frac{x^3}{3} + x + \log(x) - \log(x+1)$$

input
```
integrate((x**5-1)/(x**3-x),x)
```

output
```
x**3/3 + x + log(x) - log(x + 1)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{-1+x^5}{-x+x^3}\,dx = \frac{1}{3}\,x^3 + x - \log(x+1) + \log(x)$$

input
```
integrate((x^5-1)/(x^3-x),x, algorithm="maxima")
```

output
```
1/3*x^3 + x - log(x + 1) + log(x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{-1+x^5}{-x+x^3}\,dx = \frac{1}{3}\,x^3 + x - \log(|x+1|) + \log(|x|)$$

input
```
integrate((x^5-1)/(x^3-x),x, algorithm="giac")
```

output
```
1/3*x^3 + x - log(abs(x + 1)) + log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{-1+x^5}{-x+x^3}\,dx = x - 2\,\operatorname{atanh}(2\,x+1) + \frac{x^3}{3}$$

input
```
int(-(x^5 - 1)/(x - x^3),x)
```

output
```
x - 2*atanh(2*x + 1) + x^3/3
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{-1+x^5}{-x+x^3}\,dx = -\log(x+1) + \log(x) + \frac{x^3}{3} + x$$

input
```
int((x^5-1)/(x^3-x),x)
```

output
```
( - 3*log(x + 1) + 3*log(x) + x**3 + 3*x)/3
```

3.47 $\int \frac{1+x^4}{x^3+x^5} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 18

$$\int \frac{1+x^4}{x^3+x^5} \, dx = -\frac{1}{2x^2} - \log(x) + \log\left(1+x^2\right)$$

output
```
-1/2/x^2-ln(x)+ln(x^2+1)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1+x^4}{x^3+x^5} \, dx = -\frac{1}{2x^2} - \log(x) + \log\left(1+x^2\right)$$

input
```
Integrate[(1 + x^4)/(x^3 + x^5),x]
```

output
```
-1/2*1/x^2 - Log[x] + Log[1 + x^2]
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.33, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {2026, 1579, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4+1}{x^5+x^3}dx$$

$$\Big\downarrow \text{ 2026}$$

$$\int \frac{x^4+1}{x^3(x^2+1)}dx$$

$$\Big\downarrow \text{ 1579}$$

$$\frac{1}{2}\int \frac{x^4+1}{x^4(x^2+1)}dx^2$$

$$\Big\downarrow \text{ 522}$$

$$\frac{1}{2}\int \left(-\frac{1}{x^2}+\frac{1}{x^4}+\frac{2}{x^2+1}\right)dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(-\frac{1}{x^2}-\log(x^2)+2\log(x^2+1)\right)$$

input
```
Int[(1 + x^4)/(x^3 + x^5),x]
```

output
```
(-x^(-2) - Log[x^2] + 2*Log[1 + x^2])/2
```

Defintions of rubi rules used

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 1579
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2026
```
Int[(Fx_.)*(Px_)^(p_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Int[x^(p
*r)*ExpandToSum[Px/x^r, x]^p*Fx, x] /; IGtQ[r, 0]] /; PolyQ[Px, x] && Integ
erQ[p] &&  !MonomialQ[Px, x] && (ILtQ[p, 0] ||  !PolyQ[u, x])
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

method	result	size
default	$-\frac{1}{2x^2}-\ln\left(x\right)+\ln\left(x^2+1\right)$	17
norman	$-\frac{1}{2x^2}-\ln\left(x\right)+\ln\left(x^2+1\right)$	17
meijerg	$-\frac{1}{2x^2}-\ln\left(x\right)+\ln\left(x^2+1\right)$	17
risch	$-\frac{1}{2x^2}-\ln\left(x\right)+\ln\left(x^2+1\right)$	17
parallelrisch	$-\frac{2\ln(x)x^2-2\ln(x^2+1)x^2+1}{2x^2}$	26

input
```
int((x^4+1)/(x^5+x^3),x,method=_RETURNVERBOSE)
```

output
```
-1/2/x^2-ln(x)+ln(x^2+1)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.39

$$\int \frac{1+x^4}{x^3+x^5}\,dx = \frac{2\,x^2\log\left(x^2+1\right)-2\,x^2\log\left(x\right)-1}{2\,x^2}$$

input
```
integrate((x^4+1)/(x^5+x^3),x, algorithm="fricas")
```

output
```
1/2*(2*x^2*log(x^2 + 1) - 2*x^2*log(x) - 1)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{1+x^4}{x^3+x^5}\,dx = -\log\left(x\right)+\log\left(x^2+1\right)-\frac{1}{2x^2}$$

input
```
integrate((x**4+1)/(x**5+x**3),x)
```

output
```
-log(x) + log(x**2 + 1) - 1/(2*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{1+x^4}{x^3+x^5}\,dx = -\frac{1}{2\,x^2}+\log\left(x^2+1\right)-\log\left(x\right)$$

input
```
integrate((x^4+1)/(x^5+x^3),x, algorithm="maxima")
```

output
```
-1/2/x^2 + log(x^2 + 1) - log(x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.28

$$\int \frac{1+x^4}{x^3+x^5}\,dx = \frac{x^2-1}{2\,x^2} + \log\left(x^2+1\right) - \frac{1}{2}\log\left(x^2\right)$$

input
```
integrate((x^4+1)/(x^5+x^3),x, algorithm="giac")
```

output
```
1/2*(x^2 - 1)/x^2 + log(x^2 + 1) - 1/2*log(x^2)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{1+x^4}{x^3+x^5}\,dx = \ln\left(x^2+1\right) - \ln\left(x\right) - \frac{1}{2\,x^2}$$

input
```
int((x^4 + 1)/(x^3 + x^5),x)
```

output
```
log(x^2 + 1) - log(x) - 1/(2*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.39

$$\int \frac{1+x^4}{x^3+x^5}\,dx = \frac{2\log(x^2+1)\,x^2 - 2\log(x)\,x^2 - 1}{2x^2}$$

input
```
int((x^4+1)/(x^5+x^3),x)
```

output
```
(2*log(x**2 + 1)*x**2 - 2*log(x)*x**2 - 1)/(2*x**2)
```

3.48 $\int \frac{-1+x^2}{-2x+x^3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 19

$$\int \frac{-1+x^2}{-2x+x^3}\,dx=\frac{\log(x)}{2}+\frac{1}{4}\log\left(2-x^2\right)$$

output
```
1/2*ln(x)+1/4*ln(-x^2+2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{-1+x^2}{-2x+x^3}\,dx=\frac{\log(x)}{2}+\frac{1}{4}\log\left(2-x^2\right)$$

input
```
Integrate[(-1 + x^2)/(-2*x + x^3),x]
```

output
```
Log[x]/2 + Log[2 - x^2]/4
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.32, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {2026, 354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2-1}{x^3-2x}dx$$

$$\Big\downarrow \text{2026}$$

$$\int \frac{x^2-1}{x(x^2-2)}dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{1-x^2}{x^2(2-x^2)}dx^2$$

$$\Big\downarrow \text{86}$$

$$\frac{1}{2}\int \left(\frac{1}{2x^2}+\frac{1}{2(x^2-2)}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{\log(x^2)}{2}+\frac{1}{2}\log(2-x^2)\right)$$

input
```
Int[(-1 + x^2)/(-2*x + x^3),x]
```

output
```
(Log[x^2]/2 + Log[2 - x^2]/2)/2
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2026
```
Int[(Fx_.)*(Px_)^(p_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Int[x^(p
*r)*ExpandToSum[Px/x^r, x]^p*Fx, x] /; IGtQ[r, 0]] /; PolyQ[Px, x] && Integ
erQ[p] &&  !MonomialQ[Px, x] && (ILtQ[p, 0] ||  !PolyQ[u, x])
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

method	result	size
default	$\frac{\ln\left(x^{2}-2\right)}{4}+\frac{\ln\left(x\right)}{2}$	14
norman	$\frac{\ln\left(x^{2}-2\right)}{4}+\frac{\ln\left(x\right)}{2}$	14
risch	$\frac{\ln\left(x^{2}-2\right)}{4}+\frac{\ln\left(x\right)}{2}$	14
parallelrisch	$\frac{\ln\left(x^{2}-2\right)}{4}+\frac{\ln\left(x\right)}{2}$	14
meijerg	$\frac{\ln\left(x\right)}{2}-\frac{\ln\left(2\right)}{4}+\frac{i \pi}{4}+\frac{\ln\left(1-\frac{x^{2}}{2}\right)}{4}$	24

input
```
int((x^2-1)/(x^3-2*x),x,method=_RETURNVERBOSE)
```

output
```
1/4*ln(x^2-2)+1/2*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \frac{-1+x^2}{-2x+x^3}\,dx = \frac{1}{4}\log\left(x^2-2\right)+\frac{1}{2}\log\left(x\right)$$

input
```
integrate((x^2-1)/(x^3-2*x),x, algorithm="fricas")
```

output
```
1/4*log(x^2 - 2) + 1/2*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.63

$$\int \frac{-1+x^2}{-2x+x^3}\,dx = \frac{\log\left(x\right)}{2}+\frac{\log\left(x^2-2\right)}{4}$$

input
```
integrate((x**2-1)/(x**3-2*x),x)
```

output
```
log(x)/2 + log(x**2 - 2)/4
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \frac{-1+x^2}{-2x+x^3}\,dx = \frac{1}{4}\log\left(x^2-2\right)+\frac{1}{2}\log\left(x\right)$$

input
```
integrate((x^2-1)/(x^3-2*x),x, algorithm="maxima")
```

output
```
1/4*log(x^2 - 2) + 1/2*log(x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int \frac{-1+x^2}{-2x+x^3}\,dx = \frac{1}{4}\log\left(x^2\right) + \frac{1}{4}\log\left(\left|x^2-2\right|\right)$$

input
```
integrate((x^2-1)/(x^3-2*x),x, algorithm="giac")
```

output
```
1/4*log(x^2) + 1/4*log(abs(x^2 - 2))
```

Mupad [B] (verification not implemented)

Time = 22.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \frac{-1+x^2}{-2x+x^3}\,dx = \frac{\ln\left(x^2-2\right)}{4} + \frac{\ln\left(x\right)}{2}$$

input
```
int(-(x^2 - 1)/(2*x - x^3),x)
```

output
```
log(x^2 - 2)/4 + log(x)/2
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{-1+x^2}{-2x+x^3}\,dx = \frac{\log\left(-\sqrt{2}+x\right)}{4} + \frac{\log\left(\sqrt{2}+x\right)}{4} + \frac{\log(x)}{2}$$

input
```
int((x^2-1)/(x^3-2*x),x)
```

output
```
(log( - sqrt(2) + x) + log(sqrt(2) + x) + 2*log(x))/4
```

3.49 $\int \frac{1+x^2}{3x+x^3} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 12

$$\int \frac{1+x^2}{3x+x^3} \, dx = \frac{1}{3} \log \left(3x+x^3\right)$$

output
```
1/3*ln(x^3+3*x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.42

$$\int \frac{1+x^2}{3x+x^3} \, dx = \frac{\log(x)}{3} + \frac{1}{3} \log \left(3+x^2\right)$$

input
```
Integrate[(1 + x^2)/(3*x + x^3),x]
```

output
```
Log[x]/3 + Log[3 + x^2]/3
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2020}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2+1}{x^3+3x} \, dx$$

$$\Big\downarrow \text{ 2020}$$

$$\frac{1}{3} \log \left(x^3+3x\right)$$

input
```
Int[(1 + x^2)/(3*x + x^3),x]
```

output
```
Log[3*x + x^3]/3
```

Defintions of rubi rules used

rule 2020
```
Int[(Pp_)/(Qq_), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Si
mp[Coeff[Pp, x, p]*(Log[RemoveContent[Qq, x]]/(q*Coeff[Qq, x, q])), x] /; E
qQ[p, q - 1] && EqQ[Pp, Simplify[(Coeff[Pp, x, p]/(q*Coeff[Qq, x, q]))*D[Qq
, x]]]] /; PolyQ[Pp, x] && PolyQ[Qq, x]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

method	result	size
default	$\frac{\ln(x(x^2+3))}{3}$	11
risch	$\frac{\ln(x^3+3x)}{3}$	11
norman	$\frac{\ln(x)}{3}+\frac{\ln(x^2+3)}{3}$	14
parallelrisch	$\frac{\ln(x)}{3}+\frac{\ln(x^2+3)}{3}$	14
meijerg	$\frac{\ln(x)}{3}-\frac{\ln(3)}{6}+\frac{\ln\left(1+\frac{x^2}{3}\right)}{3}$	20

input
```
int((x^2+1)/(x^3+3*x),x,method=_RETURNVERBOSE)
```

output
```
1/3*ln(x*(x^2+3))
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1+x^2}{3x+x^3}\,dx=\frac{1}{3}\,\log\left(x^3+3\,x\right)$$

input
```
integrate((x^2+1)/(x^3+3*x),x, algorithm="fricas")
```

output
```
1/3*log(x^3 + 3*x)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{1+x^2}{3x+x^3}\,dx=\frac{\log\left(x^3+3x\right)}{3}$$

input
```
integrate((x**2+1)/(x**3+3*x),x)
```

output
```
log(x**3 + 3*x)/3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1+x^2}{3x+x^3}\,dx = \frac{1}{3}\,\log\left(x^3+3\,x\right)$$

input
```
integrate((x^2+1)/(x^3+3*x),x, algorithm="maxima")
```

output
```
1/3*log(x^3 + 3*x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

$$\int \frac{1+x^2}{3x+x^3}\,dx = \frac{1}{3}\,\log\left(3\left|\frac{1}{3}\,x^3+x\right|\right)$$

input
```
integrate((x^2+1)/(x^3+3*x),x, algorithm="giac")
```

output
```
1/3*log(3*abs(1/3*x^3 + x))
```

Mupad [B] (verification not implemented)

Time = 22.22 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1+x^2}{3x+x^3}\,dx = \frac{\ln\left(x^3+3\,x\right)}{3}$$

input
```
int((x^2 + 1)/(3*x + x^3),x)
```

output
```
log(3*x + x^3)/3
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

$$\int \frac{1+x^2}{3x+x^3}\,dx=\frac{\log(x^2+3)}{3}+\frac{\log(x)}{3}$$

input
```
int((x^2+1)/(x^3+3*x),x)
```

output
```
(log(x**2 + 3) + log(x))/3
```

3.50 $\int \frac{a+3bx^2}{ax+bx^3} dx$

Optimal result

Integrand size = 20, antiderivative size = 10

$$\int \frac{a+3bx^2}{ax+bx^3} dx = \log\left(ax+bx^3\right)$$

output
```
ln(b*x^3+a*x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

$$\int \frac{a+3bx^2}{ax+bx^3} dx = \log(x)+\log\left(a+bx^2\right)$$

input
```
Integrate[(a + 3*b*x^2)/(a*x + b*x^3),x]
```

output
```
Log[x] + Log[a + b*x^2]
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = $\{2020\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+3bx^2}{ax+bx^3}\,dx$$

$$\Big\downarrow \text{2020}$$

$$\log\left(ax+bx^3\right)$$

input
```
Int[(a + 3*b*x^2)/(a*x + b*x^3),x]
```

output
```
Log[a*x + b*x^3]
```

Defintions of rubi rules used

rule 2020
```
Int[(Pp_)/(Qq_), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Si
mp[Coeff[Pp, x, p]*(Log[RemoveContent[Qq, x]]/(q*Coeff[Qq, x, q])), x] /; E
qQ[p, q - 1] && EqQ[Pp, Simplify[(Coeff[Pp, x, p]/(q*Coeff[Qq, x, q]))*D[Qq
, x]]]] /; PolyQ[Pp, x] && PolyQ[Qq, x]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

method	result	size
derivativedivides	$\ln(b\,x^{3}+xa)$	11
default	$\ln(x(b\,x^{2}+a))$	11
risch	$\ln(b\,x^{3}+xa)$	11
norman	$\ln(x)+\ln(b\,x^{2}+a)$	12
parallelrisch	$\ln(x)+\ln(b\,x^{2}+a)$	12

input
```
int((3*b*x^2+a)/(b*x^3+a*x),x,method=_RETURNVERBOSE)
```

output
```
ln(b*x^3+a*x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{a+3bx^2}{ax+bx^3}\,dx=\log\left(bx^3+ax\right)$$

input
```
integrate((3*b*x^2+a)/(b*x^3+a*x),x, algorithm="fricas")
```

output
```
log(b*x^3 + a*x)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{a+3bx^2}{ax+bx^3}\,dx=\log\left(ax+bx^3\right)$$

input
```
integrate((3*b*x**2+a)/(b*x**3+a*x),x)
```

output
```
log(a*x + b*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{a+3bx^2}{ax+bx^3}\,dx=\log\left(bx^3+ax\right)$$

input
```
integrate((3*b*x^2+a)/(b*x^3+a*x),x, algorithm="maxima")
```

output
```
log(b*x^3 + a*x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

$$\int \frac{a+3bx^2}{ax+bx^3}\,dx=\log\left(\left|bx^3+ax\right|\right)$$

input
```
integrate((3*b*x^2+a)/(b*x^3+a*x),x, algorithm="giac")
```

output
```
log(abs(b*x^3 + a*x))
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{a+3bx^2}{ax+bx^3}\,dx=\ln\left(b\,x^3+a\,x\right)$$

input
```
int((a + 3*b*x^2)/(a*x + b*x^3),x)
```

output
```
log(a*x + b*x^3)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

$$\int \frac{a+3bx^2}{ax+bx^3}\,dx = \log\left(b\,x^2+a\right)+\log(x)$$

input
```
int((3*b*x^2+a)/(b*x^3+a*x),x)
```

output
```
log(a + b*x**2) + log(x)
```

3.51 $\int \frac{-2+4x}{-x+x^3} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 17

$$\int \frac{-2+4x}{-x+x^3} \, dx = \log(1-x) + 2\log(x) - 3\log(1+x)$$

output
```
ln(1-x)+2*ln(x)-3*ln(1+x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{-2+4x}{-x+x^3} \, dx = \log(1-x) + 2\log(x) - 3\log(1+x)$$

input
```
Integrate[(-2 + 4*x)/(-x + x^3),x]
```

output
```
Log[1 - x] + 2*Log[x] - 3*Log[1 + x]
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2026, 523, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{4x-2}{x^3-x}dx$$

$$\Big\downarrow \text{2026}$$

$$\int \frac{4x-2}{x(x^2-1)}dx$$

$$\Big\downarrow \text{523}$$

$$\int \left(\frac{2}{x}-\frac{3}{x+1}+\frac{1}{x-1}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\log(1-x)+2\log(x)-3\log(x+1)$$

input
```
Int[(-2 + 4*x)/(-x + x^3),x]
```

output
```
Log[1 - x] + 2*Log[x] - 3*Log[1 + x]
```

Defintions of rubi rules used

rule 523
```
Int[((x_)^(m_.)*((c_) + (d_.)*(x_)))/((a_) + (b_.)*(x_)^2), x_Symbol] :> In
t[ExpandIntegrand[x^m*((c + d*x)/(a + b*x^2)), x], x] /; FreeQ[{a, b, c, d}
, x] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2026
```
Int[(Fx_.)*(Px_)^(p_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Int[x^(p
*r)*ExpandToSum[Px/x^r, x]^p*Fx, x] /; IGtQ[r, 0]] /; PolyQ[Px, x] && Integ
erQ[p] &&  !MonomialQ[Px, x] && (ILtQ[p, 0] ||  !PolyQ[u, x])
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

method	result	size
default	$\ln(x-1)-3\ln(x+1)+2\ln(x)$	16
norman	$\ln(x-1)-3\ln(x+1)+2\ln(x)$	16
risch	$\ln(x-1)-3\ln(x+1)+2\ln(x)$	16
parallelrisch	$\ln(x-1)-3\ln(x+1)+2\ln(x)$	16
meijerg	$2\ln(x)+i\pi-\ln(-x^2+1)-4\,\mathrm{arctanh}(x)$	24

input
```
int((-2+4*x)/(x^3-x),x,method=_RETURNVERBOSE)
```

output
```
ln(x-1)-3*ln(x+1)+2*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{-2+4x}{-x+x^3}\,dx = -3\,\log(x+1)+\log(x-1)+2\,\log(x)$$

input
```
integrate((-2+4*x)/(x^3-x),x, algorithm="fricas")
```

output
```
-3*log(x + 1) + log(x - 1) + 2*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{-2+4x}{-x+x^3}\,dx = 2\log(x) + \log(x-1) - 3\log(x+1)$$

input
```
integrate((-2+4*x)/(x**3-x),x)
```

output
```
2*log(x) + log(x - 1) - 3*log(x + 1)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{-2+4x}{-x+x^3}\,dx = -3\,\log(x+1) + \log(x-1) + 2\,\log(x)$$

input
```
integrate((-2+4*x)/(x^3-x),x, algorithm="maxima")
```

output
```
-3*log(x + 1) + log(x - 1) + 2*log(x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{-2+4x}{-x+x^3}\,dx = -3\,\log(|x+1|) + \log(|x-1|) + 2\,\log(|x|)$$

input
```
integrate((-2+4*x)/(x^3-x),x, algorithm="giac")
```

output
```
-3*log(abs(x + 1)) + log(abs(x - 1)) + 2*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{-2+4x}{-x+x^3}\,dx = \ln(x-1) - 3\ \ln(x+1) + 2\ \ln(x)$$

input
```
int(-(4*x - 2)/(x - x^3),x)
```

output
```
log(x - 1) - 3*log(x + 1) + 2*log(x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{-2+4x}{-x+x^3}\,dx = \log(x-1) - 3\log(x+1) + 2\log(x)$$

input
```
int((-2+4*x)/(x^3-x),x)
```

output
```
log(x - 1) - 3*log(x + 1) + 2*log(x)
```

3.52 $\int \frac{4+x}{4x+x^3} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 23

$$\int \frac{4+x}{4x+x^3} \, dx = \frac{1}{2}\arctan\left(\frac{x}{2}\right)+\log(x)-\frac{1}{2}\log\left(4+x^2\right)$$

output
```
1/2*arctan(1/2*x)+ln(x)-1/2*ln(x^2+4)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{4+x}{4x+x^3} \, dx = \frac{1}{2}\arctan\left(\frac{x}{2}\right)+\log(x)-\frac{1}{2}\log\left(4+x^2\right)$$

input
```
Integrate[(4 + x)/(4*x + x^3),x]
```

output
```
ArcTan[x/2]/2 + Log[x] - Log[4 + x^2]/2
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2026, 523, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x+4}{x^3+4x}dx$$

$$\Big\downarrow \text{2026}$$

$$\int \frac{x+4}{x\left(x^2+4\right)}dx$$

$$\Big\downarrow \text{523}$$

$$\int \left(\frac{1-x}{x^2+4}+\frac{1}{x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\arctan\left(\frac{x}{2}\right)-\frac{1}{2}\log\left(x^2+4\right)+\log(x)$$

input
```
Int[(4 + x)/(4*x + x^3),x]
```

output
```
ArcTan[x/2]/2 + Log[x] - Log[4 + x^2]/2
```

Defintions of rubi rules used

rule 523
```
Int[((x_)^(m_.)*((c_) + (d_.)*(x_)))/((a_) + (b_.)*(x_)^2), x_Symbol] :> In
t[ExpandIntegrand[x^m*((c + d*x)/(a + b*x^2)), x], x] /; FreeQ[{a, b, c, d}
, x] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2026
```
Int[(Fx_.)*(Px_)^(p_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Int[x^(p
*r)*ExpandToSum[Px/x^r, x]^p*Fx, x] /; IGtQ[r, 0]] /; PolyQ[Px, x] && Integ
erQ[p] &&  !MonomialQ[Px, x] && (ILtQ[p, 0] ||  !PolyQ[u, x])
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

method	result	size
default	$\frac{\arctan\left(\frac{x}{2}\right)}{2}+\ln\left(x\right)-\frac{\ln\left(x^{2}+4\right)}{2}$	18
risch	$\frac{\arctan\left(\frac{x}{2}\right)}{2}+\ln\left(x\right)-\frac{\ln\left(x^{2}+4\right)}{2}$	18
meijerg	$\ln\left(x\right)-\ln\left(2\right)-\frac{\ln\left(1+\frac{x^{2}}{4}\right)}{2}+\frac{\arctan\left(\frac{x}{2}\right)}{2}$	24
parallelrisch	$\ln\left(x\right)-\frac{\ln\left(x-2i\right)}{2}-\frac{i\ln\left(x-2i\right)}{4}-\frac{\ln\left(x+2i\right)}{2}+\frac{i\ln\left(x+2i\right)}{4}$	34

input
```
int((4+x)/(x^3+4*x),x,method=_RETURNVERBOSE)
```

output
```
1/2*arctan(1/2*x)+ln(x)-1/2*ln(x^2+4)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int\frac{4+x}{4x+x^{3}}\,dx=\frac{1}{2}\arctan\left(\frac{1}{2}\,x\right)-\frac{1}{2}\log\left(x^{2}+4\right)+\log\left(x\right)$$

input
```
integrate((4+x)/(x^3+4*x),x, algorithm="fricas")
```

output
```
1/2*arctan(1/2*x) - 1/2*log(x^2 + 4) + log(x)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \frac{4+x}{4x+x^3}\,dx = \log(x) - \frac{\log(x^2+4)}{2} + \frac{\operatorname{atan}\left(\frac{x}{2}\right)}{2}$$

input
```
integrate((4+x)/(x**3+4*x),x)
```

output
```
log(x) - log(x**2 + 4)/2 + atan(x/2)/2
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \frac{4+x}{4x+x^3}\,dx = \frac{1}{2}\arctan\left(\frac{1}{2}x\right) - \frac{1}{2}\log\left(x^2+4\right) + \log(x)$$

input
```
integrate((4+x)/(x^3+4*x),x, algorithm="maxima")
```

output
```
1/2*arctan(1/2*x) - 1/2*log(x^2 + 4) + log(x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

$$\int \frac{4+x}{4x+x^3}\,dx = \frac{1}{2}\arctan\left(\frac{1}{2}x\right) - \frac{1}{2}\log\left(x^2+4\right) + \log(|x|)$$

input
```
integrate((4+x)/(x^3+4*x),x, algorithm="giac")
```

output
```
1/2*arctan(1/2*x) - 1/2*log(x^2 + 4) + log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 22.49 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{4+x}{4x+x^3}\,dx = \ln(x) + \ln(x-2\mathrm{i})\left(-\frac{1}{2}-\frac{1}{4}\mathrm{i}\right) + \ln(x+2\mathrm{i})\left(-\frac{1}{2}+\frac{1}{4}\mathrm{i}\right)$$

input
```
int((x + 4)/(4*x + x^3),x)
```

output
```
log(x) - log(x + 2i)*(1/2 - 1i/4) - log(x - 2i)*(1/2 + 1i/4)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \frac{4+x}{4x+x^3}\,dx = \frac{atan\left(\frac{x}{2}\right)}{2} - \frac{\log(x^2+4)}{2} + \log(x)$$

input
```
int((4+x)/(x^3+4*x),x)
```

output
```
(atan(x/2) - log(x**2 + 4) + 2*log(x))/2
```

3.53 $\int \frac{4x^2+x^3}{x+x^3} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 14

$$\int \frac{4x^2+x^3}{x+x^3} \, dx = x-\arctan(x)+2\log\left(1+x^2\right)$$

output
```
x-arctan(x)+2*ln(x^2+1)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{4x^2+x^3}{x+x^3} \, dx = x-\arctan(x)+2\log\left(1+x^2\right)$$

input
```
Integrate[(4*x^2 + x^3)/(x + x^3),x]
```

output
```
x - ArcTan[x] + 2*Log[1 + x^2]
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {2026, 9, 523, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3+4x^2}{x^3+x}dx$$

$$\Big\downarrow \text{ 2026}$$

$$\int \frac{x^3+4x^2}{x\left(x^2+1\right)}dx$$

$$\Big\downarrow \text{ 9}$$

$$\int \frac{x(x+4)}{x^2+1}dx$$

$$\Big\downarrow \text{ 523}$$

$$\int \left(1-\frac{1-4x}{x^2+1}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\arctan(x)+2\log\left(x^2+1\right)+x$$

input
```
Int[(4*x^2 + x^3)/(x + x^3),x]
```

output
```
x - ArcTan[x] + 2*Log[1 + x^2]
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 523
```
Int[((x_)^(m_.)*((c_) + (d_.)*(x_)))/((a_) + (b_.)*(x_)^2), x_Symbol] :> In
t[ExpandIntegrand[x^m*((c + d*x)/(a + b*x^2)), x], x] /; FreeQ[{a, b, c, d}
, x] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2026
```
Int[(Fx_.)*(Px_)^(p_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Int[x^(p
*r)*ExpandToSum[Px/x^r, x]^p*Fx, x] /; IGtQ[r, 0]] /; PolyQ[Px, x] && Integ
erQ[p] &&  !MonomialQ[Px, x] && (ILtQ[p, 0] ||  !PolyQ[u, x])
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

method	result	size
default	$x - \arctan(x) + 2\ln(x^2+1)$	15
meijerg	$x - \arctan(x) + 2\ln(x^2+1)$	15
risch	$x - \arctan(x) + 2\ln(x^2+1)$	15
parallelrisch	$x + 2\ln(x-i) + \frac{i\ln(x-i)}{2} + 2\ln(x+i) - \frac{i\ln(x+i)}{2}$	33

input
```
int((x^3+4*x^2)/(x^3+x),x,method=_RETURNVERBOSE)
```

output
```
x-arctan(x)+2*ln(x^2+1)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{4x^2+x^3}{x+x^3}\,dx = x-\arctan(x)+2\log\left(x^2+1\right)$$

input
```
integrate((x^3+4*x^2)/(x^3+x),x, algorithm="fricas")
```

output
```
x - arctan(x) + 2*log(x^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{4x^2+x^3}{x+x^3}\,dx = x+2\log\left(x^2+1\right)-\operatorname{atan}(x)$$

input
```
integrate((x**3+4*x**2)/(x**3+x),x)
```

output
```
x + 2*log(x**2 + 1) - atan(x)
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{4x^2+x^3}{x+x^3}\,dx = x-\arctan(x)+2\log\left(x^2+1\right)$$

input
```
integrate((x^3+4*x^2)/(x^3+x),x, algorithm="maxima")
```

output
```
x - arctan(x) + 2*log(x^2 + 1)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{4x^2 + x^3}{x + x^3} \, dx = x - \arctan(x) + 2 \log(x^2 + 1)$$

input
```
integrate((x^3+4*x^2)/(x^3+x),x, algorithm="giac")
```

output
```
x - arctan(x) + 2*log(x^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{4x^2 + x^3}{x + x^3} \, dx = x + 2 \ln(x^2 + 1) - \operatorname{atan}(x)$$

input
```
int((4*x^2 + x^3)/(x + x^3),x)
```

output
```
x + 2*log(x^2 + 1) - atan(x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{4x^2 + x^3}{x + x^3} \, dx = -atan(x) + 2 \log(x^2 + 1) + x$$

input
```
int((x^3+4*x^2)/(x^3+x),x)
```

output
```
- atan(x) + 2*log(x**2 + 1) + x
```

3.54 $\int \frac{ax^2+bx^3}{cx^2+dx^3} dx$

Optimal result

Integrand size = 25, antiderivative size = 26

$$\int \frac{ax^2+bx^3}{cx^2+dx^3} dx = \frac{bx}{d} - \frac{(bc-ad)\log(c+dx)}{d^2}$$

output
```
b*x/d-(-a*d+b*c)*ln(d*x+c)/d^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.96

$$\int \frac{ax^2+bx^3}{cx^2+dx^3} dx = \frac{bx}{d} + \frac{(-bc+ad)\log(c+dx)}{d^2}$$

input
```
Integrate[(a*x^2 + b*x^3)/(c*x^2 + d*x^3),x]
```

output
```
(b*x)/d + ((-(b*c) + a*d)*Log[c + d*x])/d^2
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2026, 9, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ax^2+bx^3}{cx^2+dx^3}dx$$
$$\Big\downarrow \text{ 2026}$$
$$\int \frac{ax^2+bx^3}{x^2(c+dx)}dx$$
$$\Big\downarrow \text{ 9}$$
$$\int \frac{a+bx}{c+dx}dx$$
$$\Big\downarrow \text{ 49}$$
$$\int \left(\frac{ad-bc}{d(c+dx)}+\frac{b}{d}\right)dx$$
$$\Big\downarrow \text{ 2009}$$
$$\frac{bx}{d}-\frac{(bc-ad)\log(c+dx)}{d^2}$$

input
```
Int[(a*x^2 + b*x^3)/(c*x^2 + d*x^3),x]
```

output
```
(b*x)/d - ((b*c - a*d)*Log[c + d*x])/d^2
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2026
```
Int[(Fx_.)*(Px_)^(p_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Int[x^(p
*r)*ExpandToSum[Px/x^r, x]^p*Fx, x] /; IGtQ[r, 0]] /; PolyQ[Px, x] && Integ
erQ[p] &&  !MonomialQ[Px, x] && (ILtQ[p, 0] ||  !PolyQ[u, x])
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

method	result	size
default	$\frac{bx}{d}+\frac{(ad-bc)\ln(dx+c)}{d^2}$	26
norman	$\frac{bx}{d}+\frac{(ad-bc)\ln(dx+c)}{d^2}$	26
parallelrisch	$\frac{\ln(dx+c)ad-\ln(dx+c)bc+bdx}{d^2}$	29
risch	$\frac{bx}{d}+\frac{\ln(dx+c)a}{d}-\frac{\ln(dx+c)bc}{d^2}$	32

input
```
int((b*x^3+a*x^2)/(d*x^3+c*x^2),x,method=_RETURNVERBOSE)
```

output
```
b/d*x+(a*d-b*c)/d^2*ln(d*x+c)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.96

$$\int \frac{ax^2 + bx^3}{cx^2 + dx^3}\,dx = \frac{bdx - (bc - ad)\log(dx + c)}{d^2}$$

input
```
integrate((b*x^3+a*x^2)/(d*x^3+c*x^2),x, algorithm="fricas")
```

output
```
(b*d*x - (b*c - a*d)*log(d*x + c))/d^2
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.77

$$\int \frac{ax^2 + bx^3}{cx^2 + dx^3}\,dx = \frac{bx}{d} + \frac{(ad - bc)\log(c + dx)}{d^2}$$

input
```
integrate((b*x**3+a*x**2)/(d*x**3+c*x**2),x)
```

output
```
b*x/d + (a*d - b*c)*log(c + d*x)/d**2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{ax^2 + bx^3}{cx^2 + dx^3}\,dx = \frac{bx}{d} - \frac{(bc - ad)\log(dx + c)}{d^2}$$

input
```
integrate((b*x^3+a*x^2)/(d*x^3+c*x^2),x, algorithm="maxima")
```

output
```
b*x/d - (b*c - a*d)*log(d*x + c)/d^2
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.04

$$\int \frac{ax^2+bx^3}{cx^2+dx^3}\,dx=\frac{bx}{d}-\frac{(bc-ad)\log\left(|dx+c|\right)}{d^2}$$

input
```
integrate((b*x^3+a*x^2)/(d*x^3+c*x^2),x, algorithm="giac")
```

output
```
b*x/d - (b*c - a*d)*log(abs(d*x + c))/d^2
```

Mupad [B] (verification not implemented)

Time = 22.39 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.96

$$\int \frac{ax^2+bx^3}{cx^2+dx^3}\,dx=\frac{\ln\left(c+d\,x\right)\,\left(a\,d-b\,c\right)}{d^2}+\frac{b\,x}{d}$$

input
```
int((a*x^2 + b*x^3)/(c*x^2 + d*x^3),x)
```

output
```
(log(c + d*x)*(a*d - b*c))/d^2 + (b*x)/d
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{ax^2+bx^3}{cx^2+dx^3}\,dx=\frac{\log(dx+c)\,ad-\log(dx+c)\,bc+bdx}{d^2}$$

input
```
int((b*x^3+a*x^2)/(d*x^3+c*x^2),x)
```

output
```
(log(c + d*x)*a*d - log(c + d*x)*b*c + b*d*x)/d**2
```

3.55 $\int \frac{x+x^2}{-2x-x^2+x^3} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 6

$$\int \frac{x+x^2}{-2x-x^2+x^3} \, dx = \log(2-x)$$

output
```
ln(2-x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 4, normalized size of antiderivative = 0.67

$$\int \frac{x+x^2}{-2x-x^2+x^3} \, dx = \log(-2+x)$$

input
```
Integrate[(x + x^2)/(-2*x - x^2 + x^3),x]
```

output
```
Log[-2 + x]
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2019, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2+x}{x^3-x^2-2x}\,dx$$

$$\Big\downarrow \text{2019}$$

$$\int \frac{1}{x-2}dx$$

$$\Big\downarrow \text{16}$$

$$\log(2-x)$$

input
```
Int[(x + x^2)/(-2*x - x^2 + x^3),x]
```

output
```
Log[2 - x]
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.83

method	result	size
default	$\ln(x-2)$	5
norman	$\ln(x-2)$	5
risch	$\ln(x-2)$	5
parallelrisch	$\ln(x-2)$	5

input
```
int((x^2+x)/(x^3-x^2-2*x),x,method=_RETURNVERBOSE)
```

output
```
ln(x-2)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 4, normalized size of antiderivative = 0.67

$$\int \frac{x+x^2}{-2x-x^2+x^3}\,dx = \log(x-2)$$

input
```
integrate((x^2+x)/(x^3-x^2-2*x),x, algorithm="fricas")
```

output
```
log(x - 2)
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.50

$$\int \frac{x+x^2}{-2x-x^2+x^3}\,dx = \log(x-2)$$

input
```
integrate((x**2+x)/(x**3-x**2-2*x),x)
```

output
```
log(x - 2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 4, normalized size of antiderivative = 0.67

$$\int \frac{x+x^2}{-2x-x^2+x^3}\,dx = \log{(x-2)}$$

input
```
integrate((x^2+x)/(x^3-x^2-2*x),x, algorithm="maxima")
```

output
```
log(x - 2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.83

$$\int \frac{x+x^2}{-2x-x^2+x^3}\,dx = \log{(|x-2|)}$$

input
```
integrate((x^2+x)/(x^3-x^2-2*x),x, algorithm="giac")
```

output
```
log(abs(x - 2))
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 4, normalized size of antiderivative = 0.67

$$\int \frac{x+x^2}{-2x-x^2+x^3}\,dx = \ln{(x-2)}$$

input
```
int(-(x + x^2)/(2*x + x^2 - x^3),x)
```

output
```
log(x - 2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 4, normalized size of antiderivative = 0.67

$$\int \frac{x+x^2}{-2x-x^2+x^3}\,dx=\log(x-2)$$

input

```
int((x^2+x)/(x^3-x^2-2*x),x)
```

output

```
log(x - 2)
```

3.56 $\int \frac{1+x^2}{x+2x^2+x^3} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 10

$$\int \frac{1+x^2}{x+2x^2+x^3} \, dx = \frac{2}{1+x} + \log(x)$$

output
```
2/(1+x)+ln(x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1+x^2}{x+2x^2+x^3} \, dx = \frac{2}{1+x} + \log(x)$$

input
```
Integrate[(1 + x^2)/(x + 2*x^2 + x^3),x]
```

output
```
2/(1 + x) + Log[x]
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2026, 1332, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2+1}{x^3+2x^2+x}dx$$
$$\Big\downarrow \text{ 2026}$$
$$\int \frac{x^2+1}{x(x^2+2x+1)}dx$$
$$\Big\downarrow \text{ 1332}$$
$$\int \frac{x^2+1}{x(x+1)^2}dx$$
$$\Big\downarrow \text{ 522}$$
$$\int \left(\frac{1}{x}-\frac{2}{(x+1)^2}\right)dx$$
$$\Big\downarrow \text{ 2009}$$
$$\frac{2}{x+1}+\log(x)$$

input
```
Int[(1 + x^2)/(x + 2*x^2 + x^3),x]
```

output
```
2/(1 + x) + Log[x]
```

Defintions of rubi rules used

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 1332
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.)*((d
_.) + (f_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[1/c^p   Int[(g + h*x)^m*(b/2 +
 c*x)^(2*p)*(d + f*x^2)^q, x], x] /; FreeQ[{a, b, c, d, f, g, h, m, q}, x]
&& EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2026
```
Int[(Fx_.)*(Px_)^(p_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Int[x^(p
*r)*ExpandToSum[Px/x^r, x]^p*Fx, x] /; IGtQ[r, 0]] /; PolyQ[Px, x] && Integ
erQ[p] &&  !MonomialQ[Px, x] && (ILtQ[p, 0] ||  !PolyQ[u, x])
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

method	result	size
default	$\frac{2}{x+1}+\ln\left(x\right)$	11
norman	$\frac{2}{x+1}+\ln\left(x\right)$	11
risch	$\frac{2}{x+1}+\ln\left(x\right)$	11
parallelrisch	$\frac{\ln\left(x\right)x+2+\ln\left(x\right)}{x+1}$	15

input
```
int((x^2+1)/(x^3+2*x^2+x),x,method=_RETURNVERBOSE)
```

output
```
2/(x+1)+ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.40

$$\int \frac{1+x^2}{x+2x^2+x^3}\,dx = \frac{(x+1)\log(x)+2}{x+1}$$

input
```
integrate((x^2+1)/(x^3+2*x^2+x),x, algorithm="fricas")
```

output
```
((x + 1)*log(x) + 2)/(x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

$$\int \frac{1+x^2}{x+2x^2+x^3}\,dx = \log(x) + \frac{2}{x+1}$$

input
```
integrate((x**2+1)/(x**3+2*x**2+x),x)
```

output
```
log(x) + 2/(x + 1)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1+x^2}{x+2x^2+x^3}\,dx = \frac{2}{x+1} + \log(x)$$

input
```
integrate((x^2+1)/(x^3+2*x^2+x),x, algorithm="maxima")
```

output
```
2/(x + 1) + log(x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

$$\int \frac{1+x^2}{x+2x^2+x^3}\,dx = \frac{2}{x+1} + \log(|x|)$$

input
```
integrate((x^2+1)/(x^3+2*x^2+x),x, algorithm="giac")
```

output
```
2/(x + 1) + log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1+x^2}{x+2x^2+x^3}\,dx = \ln(x) + \frac{2}{x+1}$$

input
```
int((x^2 + 1)/(x + 2*x^2 + x^3),x)
```

output
```
log(x) + 2/(x + 1)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.60

$$\int \frac{1+x^2}{x+2x^2+x^3}\,dx = \frac{\log(x)\,x + \log(x) - 2x}{x+1}$$

input
```
int((x^2+1)/(x^3+2*x^2+x),x)
```

output
```
(log(x)*x + log(x) - 2*x)/(x + 1)
```

3.57 $\int \frac{1+x^5}{-10x-3x^2+x^3} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 42

$$\int \frac{1+x^5}{-10x-3x^2+x^3} \, dx = 19x+\frac{3x^2}{2}+\frac{x^3}{3}+\frac{3126}{35}\log(5-x)-\frac{\log(x)}{10}-\frac{31}{14}\log(2+x)$$

output
```
19*x+3/2*x^2+1/3*x^3+3126/35*ln(5-x)-1/10*ln(x)-31/14*ln(2+x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int \frac{1+x^5}{-10x-3x^2+x^3} \, dx = 19x+\frac{3x^2}{2}+\frac{x^3}{3}+\frac{3126}{35}\log(5-x)-\frac{\log(x)}{10}-\frac{31}{14}\log(2+x)$$

input
```
Integrate[(1 + x^5)/(-10*x - 3*x^2 + x^3),x]
```

output
```
19*x + (3*x^2)/2 + x^3/3 + (3126*Log[5 - x])/35 - Log[x]/10 - (31*Log[2 +
x])/14
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {2026, 2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5+1}{x^3-3x^2-10x}dx$$
$$\Big\downarrow \text{2026}$$
$$\int \frac{x^5+1}{x\left(x^2-3x-10\right)}dx$$
$$\Big\downarrow \text{2159}$$
$$\int \left(x^2+3x+\frac{3126}{35(x-5)}-\frac{31}{14(x+2)}-\frac{1}{10x}+19\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{x^3}{3}+\frac{3x^2}{2}+19x+\frac{3126}{35}\log(5-x)-\frac{\log(x)}{10}-\frac{31}{14}\log(x+2)$$

input
```
Int[(1 + x^5)/(-10*x - 3*x^2 + x^3),x]
```

output
```
19*x + (3*x^2)/2 + x^3/3 + (3126*Log[5 - x])/35 - Log[x]/10 - (31*Log[2 +
x])/14
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2026
```
Int[(Fx_.)*(Px_)^(p_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Int[x^(p
*r)*ExpandToSum[Px/x^r, x]^p*Fx, x] /; IGtQ[r, 0]] /; PolyQ[Px, x] && Integ
erQ[p] && !MonomialQ[Px, x] && (ILtQ[p, 0] || !PolyQ[u, x])
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.74

method	result	size
default	$\frac{x^3}{3}+\frac{3x^2}{2}+19x-\frac{31\ln(2+x)}{14}+\frac{3126\ln(x-5)}{35}-\frac{\ln(x)}{10}$	31
norman	$\frac{x^3}{3}+\frac{3x^2}{2}+19x-\frac{31\ln(2+x)}{14}+\frac{3126\ln(x-5)}{35}-\frac{\ln(x)}{10}$	31
risch	$\frac{x^3}{3}+\frac{3x^2}{2}+19x-\frac{31\ln(2+x)}{14}+\frac{3126\ln(x-5)}{35}-\frac{\ln(x)}{10}$	31
parallelrisch	$\frac{x^3}{3}+\frac{3x^2}{2}+19x-\frac{31\ln(2+x)}{14}+\frac{3126\ln(x-5)}{35}-\frac{\ln(x)}{10}$	31

input
```
int((x^5+1)/(x^3-3*x^2-10*x),x,method=_RETURNVERBOSE)
```

output
```
1/3*x^3+3/2*x^2+19*x-31/14*ln(2+x)+3126/35*ln(x-5)-1/10*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.71

$$\int \frac{1+x^5}{-10x-3x^2+x^3}\,dx=\frac{1}{3}\,x^3+\frac{3}{2}\,x^2+19\,x-\frac{31}{14}\,\log\left(x+2\right)+\frac{3126}{35}\,\log\left(x-5\right)-\frac{1}{10}\,\log\left(x\right)$$

input
```
integrate((x^5+1)/(x^3-3*x^2-10*x),x, algorithm="fricas")
```

output
```
1/3*x^3 + 3/2*x^2 + 19*x - 31/14*log(x + 2) + 3126/35*log(x - 5) - 1/10*lo
g(x)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.86

$$\int \frac{1+x^5}{-10x-3x^2+x^3}\,dx = \frac{x^3}{3}+\frac{3x^2}{2}+19x-\frac{\log(x)}{10}+\frac{3126\log(x-5)}{35}-\frac{31\log(x+2)}{14}$$

input
```
integrate((x**5+1)/(x**3-3*x**2-10*x),x)
```

output
```
x**3/3 + 3*x**2/2 + 19*x - log(x)/10 + 3126*log(x - 5)/35 - 31*log(x + 2)/
14
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.71

$$\int \frac{1+x^5}{-10x-3x^2+x^3}\,dx = \frac{1}{3}x^3+\frac{3}{2}x^2+19x-\frac{31}{14}\log(x+2)+\frac{3126}{35}\log(x-5)-\frac{1}{10}\log(x)$$

input
```
integrate((x^5+1)/(x^3-3*x^2-10*x),x, algorithm="maxima")
```

output
```
1/3*x^3 + 3/2*x^2 + 19*x - 31/14*log(x + 2) + 3126/35*log(x - 5) - 1/10*lo
g(x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.79

$$\int \frac{1+x^5}{-10x-3x^2+x^3}\,dx = \frac{1}{3}x^3+\frac{3}{2}x^2+19x-\frac{31}{14}\log(|x+2|)+\frac{3126}{35}\log(|x-5|)-\frac{1}{10}\log(|x|)$$

input
```
integrate((x^5+1)/(x^3-3*x^2-10*x),x, algorithm="giac")
```

output
```
1/3*x^3 + 3/2*x^2 + 19*x - 31/14*log(abs(x + 2)) + 3126/35*log(abs(x - 5))
 - 1/10*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 22.38 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.71

$$\int \frac{1+x^5}{-10x-3x^2+x^3}\,dx = 19\,x - \frac{31\,\ln(x+2)}{14} + \frac{3126\,\ln(x-5)}{35} - \frac{\ln(x)}{10} + \frac{3\,x^2}{2} + \frac{x^3}{3}$$

input
```
int(-(x^5 + 1)/(10*x + 3*x^2 - x^3),x)
```

output
```
19*x - (31*log(x + 2))/14 + (3126*log(x - 5))/35 - log(x)/10 + (3*x^2)/2 +
 x^3/3
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.71

$$\int \frac{1+x^5}{-10x-3x^2+x^3}\,dx = \frac{3126\log(x-5)}{35} - \frac{31\log(x+2)}{14} - \frac{\log(x)}{10} + \frac{x^3}{3} + \frac{3x^2}{2} + 19x$$

input
```
int((x^5+1)/(x^3-3*x^2-10*x),x)
```

output
```
(18756*log(x - 5) - 465*log(x + 2) - 21*log(x) + 70*x**3 + 315*x**2 + 3990
*x)/210
```

3.58 $\int \frac{-x+2x^3}{1-x^2+x^4} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 15

$$\int \frac{-x+2x^3}{1-x^2+x^4} \, dx = \frac{1}{2} \log \left(1-x^2+x^4\right)$$

output
```
1/2*ln(x^4-x^2+1)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{-x+2x^3}{1-x^2+x^4} \, dx = \frac{1}{2} \log \left(1-x^2+x^4\right)$$

input
```
Integrate[(-x + 2*x^3)/(1 - x^2 + x^4),x]
```

output
```
Log[1 - x^2 + x^4]/2
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {2020}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2x^3-x}{x^4-x^2+1}\,dx$$
$$\Big\downarrow \text{2020}$$
$$\frac{1}{2}\log\left(x^4-x^2+1\right)$$

input
```
Int[(-x + 2*x^3)/(1 - x^2 + x^4),x]
```

output
```
Log[1 - x^2 + x^4]/2
```

Defintions of rubi rules used

rule 2020
```
Int[(Pp_)/(Qq_), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Si
mp[Coeff[Pp, x, p]*(Log[RemoveContent[Qq, x]]/(q*Coeff[Qq, x, q])), x] /; E
qQ[p, q - 1] && EqQ[Pp, Simplify[(Coeff[Pp, x, p]/(q*Coeff[Qq, x, q]))*D[Qq
, x]]]] /; PolyQ[Pp, x] && PolyQ[Qq, x]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
default	$\frac{\ln(x^4-x^2+1)}{2}$	14
norman	$\frac{\ln(x^4-x^2+1)}{2}$	14
risch	$\frac{\ln(x^4-x^2+1)}{2}$	14
parallelrisch	$\frac{\ln(x^4-x^2+1)}{2}$	14

input
```
int((2*x^3-x)/(x^4-x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/2*ln(x^4-x^2+1)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{-x+2x^3}{1-x^2+x^4}\,dx = \frac{1}{2}\log\left(x^4-x^2+1\right)$$

input
```
integrate((2*x^3-x)/(x^4-x^2+1),x, algorithm="fricas")
```

output
```
1/2*log(x^4 - x^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.67

$$\int \frac{-x+2x^3}{1-x^2+x^4}\,dx = \frac{\log\left(x^4-x^2+1\right)}{2}$$

input
```
integrate((2*x**3-x)/(x**4-x**2+1),x)
```

output
```
log(x**4 - x**2 + 1)/2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{-x+2x^3}{1-x^2+x^4}\,dx = \frac{1}{2}\log\left(x^4-x^2+1\right)$$

input
```
integrate((2*x^3-x)/(x^4-x^2+1),x, algorithm="maxima")
```

output
```
1/2*log(x^4 - x^2 + 1)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{-x+2x^3}{1-x^2+x^4}\,dx = \frac{1}{2}\log\left(x^4-x^2+1\right)$$

input
```
integrate((2*x^3-x)/(x^4-x^2+1),x, algorithm="giac")
```

output
```
1/2*log(x^4 - x^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{-x+2x^3}{1-x^2+x^4}\,dx = \frac{\ln\left(x^4-x^2+1\right)}{2}$$

input
```
int(-(x - 2*x^3)/(x^4 - x^2 + 1),x)
```

output
```
log(x^4 - x^2 + 1)/2
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.73

$$\int \frac{-x+2x^3}{1-x^2+x^4}\,dx=\frac{\log\left(-\sqrt{3}\,x+x^2+1\right)}{2}+\frac{\log\left(\sqrt{3}\,x+x^2+1\right)}{2}$$

input
```
int((2*x^3-x)/(x^4-x^2+1),x)
```

output
```
(log( - sqrt(3)*x + x**2 + 1) + log(sqrt(3)*x + x**2 + 1))/2
```

3.59 $\int \frac{x+2x^3}{(x^2+x^4)^3} dx$

Optimal result

Integrand size = 17, antiderivative size = 13

$$\int \frac{x+2x^3}{(x^2+x^4)^3} dx = -\frac{1}{4(x^2+x^4)^2}$$

output
```
-1/4/(x^4+x^2)^2
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

$$\int \frac{x+2x^3}{(x^2+x^4)^3} dx = -\frac{1}{4x^4(1+x^2)^2}$$

input
```
Integrate[(x + 2*x^3)/(x^2 + x^4)^3,x]
```

output
```
-1/4*1/(x^4*(1 + x^2)^2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {2021}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2x^3+x}{(x^4+x^2)^3} \, dx$$
$$\Big\downarrow \text{2021}$$
$$-\frac{1}{4(x^4+x^2)^2}$$

input
```
Int[(x + 2*x^3)/(x^2 + x^4)^3,x]
```

output
```
-1/4*1/(x^2 + x^4)^2
```

Defintions of rubi rules used

rule 2021
```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

method	result	size
gosper	$-\frac{1}{4x^4(x^2+1)^2}$	13
norman	$-\frac{1}{4x^4(x^2+1)^2}$	13
risch	$-\frac{1}{4x^4(x^2+1)^2}$	13
parallelrisch	$-\frac{1}{4x^4(x^2+1)^2}$	13
default	$-\frac{1}{4(x^2+1)^2}-\frac{1}{2(x^2+1)}-\frac{1}{4x^4}+\frac{1}{2x^2}$	30
orering	$-\frac{x(x^2+1)(2x^3+x)}{4(2x^2+1)(x^4+x^2)^3}$	34
meijerg	$\frac{1}{2x^2}-\frac{3}{4}+\frac{x^2(5x^2+6)}{2(x^2+1)^2}-\frac{1}{4x^4}-\frac{x^2(7x^2+8)}{4(x^2+1)^2}$	51

input
```
int((2*x^3+x)/(x^4+x^2)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/4/x^4/(x^2+1)^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.23

$$\int \frac{x+2x^3}{(x^2+x^4)^3}\,dx = -\frac{1}{4\,(x^8+2\,x^6+x^4)}$$

input
```
integrate((2*x^3+x)/(x^4+x^2)^3,x, algorithm="fricas")
```

output
```
-1/4/(x^8 + 2*x^6 + x^4)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.31

$$\int \frac{x+2x^3}{(x^2+x^4)^3}\,dx = -\frac{1}{4x^8+8x^6+4x^4}$$

input
```
integrate((2*x**3+x)/(x**4+x**2)**3,x)
```

output
```
-1/(4*x**8 + 8*x**6 + 4*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{x+2x^3}{(x^2+x^4)^3}\,dx = -\frac{1}{4\,(x^4+x^2)^2}$$

input
```
integrate((2*x^3+x)/(x^4+x^2)^3,x, algorithm="maxima")
```

output
```
-1/4/(x^4 + x^2)^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{x+2x^3}{(x^2+x^4)^3}\,dx = -\frac{1}{4\,(x^4+x^2)^2}$$

input
```
integrate((2*x^3+x)/(x^4+x^2)^3,x, algorithm="giac")
```

output
```
-1/4/(x^4 + x^2)^2
```

Mupad [B] (verification not implemented)

Time = 22.47 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.54

$$\int \frac{x+2x^3}{(x^2+x^4)^3}\,dx = -\frac{1}{4\,x^8+8\,x^6+4\,x^4}$$

input
```
int((x + 2*x^3)/(x^2 + x^4)^3,x)
```

output
```
-1/(4*x^4 + 8*x^6 + 4*x^8)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.31

$$\int \frac{x+2x^3}{(x^2+x^4)^3}\,dx = -\frac{1}{4x^4\,(x^4+2x^2+1)}$$

input
```
int((2*x^3+x)/(x^4+x^2)^3,x)
```

output
```
( - 1)/(4*x**4*(x**4 + 2*x**2 + 1))
```

3.60 $\int \frac{-x+2x^3+4x^5}{(3+2x^2+x^4)^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 45

$$\int \frac{-x+2x^3+4x^5}{(3+2x^2+x^4)^2}\,dx=\frac{5-7x^2}{8(3+2x^2+x^4)}+\frac{9\arctan\left(\frac{1+x^2}{\sqrt{2}}\right)}{8\sqrt{2}}$$

output
```
(-7*x^2+5)/(8*x^4+16*x^2+24)+9/16*arctan(1/2*(x^2+1)*2^(1/2))*2^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00

$$\int \frac{-x+2x^3+4x^5}{(3+2x^2+x^4)^2}\,dx=\frac{5-7x^2}{8(3+2x^2+x^4)}+\frac{9\arctan\left(\frac{1+x^2}{\sqrt{2}}\right)}{8\sqrt{2}}$$

input
```
Integrate[(-x + 2*x^3 + 4*x^5)/(3 + 2*x^2 + x^4)^2,x]
```

output
```
(5 - 7*x^2)/(8*(3 + 2*x^2 + x^4)) + (9*ArcTan[(1 + x^2)/Sqrt[2]])/(8*Sqrt[
2])
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.20, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2028, 2194, 25, 2191, 27, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{4x^5+2x^3-x}{\left(x^4+2x^2+3\right)^2} \, dx$$

$$\Big\downarrow \text{ 2028}$$

$$\int \frac{x\left(4x^4+2x^2-1\right)}{\left(x^4+2x^2+3\right)^2}dx$$

$$\Big\downarrow \text{ 2194}$$

$$\frac{1}{2}\int -\frac{-4x^4-2x^2+1}{\left(x^4+2x^2+3\right)^2}dx^2$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{1}{2}\int \frac{-4x^4-2x^2+1}{\left(x^4+2x^2+3\right)^2}dx^2$$

$$\Big\downarrow \text{ 2191}$$

$$\frac{1}{2}\left(\frac{5-7x^2}{4\left(x^4+2x^2+3\right)}-\frac{1}{8}\int -\frac{18}{x^4+2x^2+3}dx^2\right)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{2}\left(\frac{9}{4}\int \frac{1}{x^4+2x^2+3}dx^2+\frac{5-7x^2}{4\left(x^4+2x^2+3\right)}\right)$$

$$\Big\downarrow \text{ 1083}$$

$$\frac{1}{2}\left(\frac{5-7x^2}{4\left(x^4+2x^2+3\right)}-\frac{9}{2}\int \frac{1}{-x^4-8}d\left(2x^2+2\right)\right)$$

$$\Big\downarrow \text{ 217}$$

$$\frac{1}{2}\left(\frac{9\arctan\left(\frac{2x^2+2}{2\sqrt{2}}\right)}{4\sqrt{2}}+\frac{5-7x^2}{4\left(x^4+2x^2+3\right)}\right)$$

input
```
Int[(-x + 2*x^3 + 4*x^5)/(3 + 2*x^2 + x^4)^2,x]
```

output
```
((5 - 7*x^2)/(4*(3 + 2*x^2 + x^4)) + (9*ArcTan[(2 + 2*x^2)/(2*Sqrt[2])])/(
4*Sqrt[2]))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 2028
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.) + (c_.)*(x_)^(t_.))^(p_.),
x_Symbol] :> Int[x^(p*r)*(a + b*x^(s - r) + c*x^(t - r))^p*Fx, x] /; FreeQ[
{a, b, c, r, s, t}, x] && IntegerQ[p] && PosQ[s - r] && PosQ[t - r] &&  !(E
qQ[p, 1] && EqQ[u, 1])
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 2194
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Simp[1/2   Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)
^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ
[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.84

method	result	size
risch	$\frac{-\frac{7x^2}{8}+\frac{5}{8}}{x^4+2x^2+3}+\frac{9\arctan\left(\frac{\left(x^2+1\right)\sqrt{2}}{2}\right)\sqrt{2}}{16}$	38
default	$\frac{-\frac{7x^2}{4}+\frac{5}{4}}{2x^4+4x^2+6}+\frac{9\sqrt{2}\,\arctan\left(\frac{\left(2x^2+2\right)\sqrt{2}}{4}\right)}{16}$	41

input
```
int((4*x^5+2*x^3-x)/(x^4+2*x^2+3)^2,x,method=_RETURNVERBOSE)
```

output
```
(-7/8*x^2+5/8)/(x^4+2*x^2+3)+9/16*arctan(1/2*(x^2+1)*2^(1/2))*2^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.04

$$\int \frac{-x+2x^3+4x^5}{(3+2x^2+x^4)^2}\,dx = \frac{9\sqrt{2}(x^4+2x^2+3)\arctan\left(\frac{1}{2}\sqrt{2}(x^2+1)\right)-14x^2+10}{16(x^4+2x^2+3)}$$

input
```
integrate((4*x^5+2*x^3-x)/(x^4+2*x^2+3)^2,x, algorithm="fricas")
```

output
```
1/16*(9*sqrt(2)*(x^4 + 2*x^2 + 3)*arctan(1/2*sqrt(2)*(x^2 + 1)) - 14*x^2 +
 10)/(x^4 + 2*x^2 + 3)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.98

$$\int \frac{-x+2x^3+4x^5}{(3+2x^2+x^4)^2}\,dx = \frac{5-7x^2}{8x^4+16x^2+24} + \frac{9\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x^2}{2}+\frac{\sqrt{2}}{2}\right)}{16}$$

input
```
integrate((4*x**5+2*x**3-x)/(x**4+2*x**2+3)**2,x)
```

output
```
(5 - 7*x**2)/(8*x**4 + 16*x**2 + 24) + 9*sqrt(2)*atan(sqrt(2)*x**2/2 + sqr
t(2)/2)/16
```

Maxima [F]

$$\int \frac{-x+2x^3+4x^5}{(3+2x^2+x^4)^2}\,dx = \int \frac{4x^5+2x^3-x}{(x^4+2x^2+3)^2}\,dx$$

input
```
integrate((4*x^5+2*x^3-x)/(x^4+2*x^2+3)^2,x, algorithm="maxima")
```

output
```
-1/8*(7*x^2 - 5)/(x^4 + 2*x^2 + 3) + 9/4*integrate(x/(x^4 + 2*x^2 + 3), x)
```

Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.84

$$\int \frac{-x+2x^3+4x^5}{(3+2x^2+x^4)^2}\,dx = \frac{9}{16}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}(x^2+1)\right) - \frac{7\,x^2-5}{8\,(x^4+2\,x^2+3)}$$

input
```
integrate((4*x^5+2*x^3-x)/(x^4+2*x^2+3)^2,x, algorithm="giac")
```

output
```
9/16*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1)) - 1/8*(7*x^2 - 5)/(x^4 + 2*x^2
+ 3)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.93

$$\int \frac{-x+2x^3+4x^5}{(3+2x^2+x^4)^2}\,dx = \frac{9\,\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x^2}{2}+\frac{\sqrt{2}}{2}\right)}{16} - \frac{\frac{7\,x^2}{8}-\frac{5}{8}}{x^4+2\,x^2+3}$$

input
```
int((2*x^3 - x + 4*x^5)/(2*x^2 + x^4 + 3)^2,x)
```

output
```
(9*2^(1/2)*atan(2^(1/2)/2 + (2^(1/2)*x^2)/2))/16 - ((7*x^2)/8 - 5/8)/(2*x^
2 + x^4 + 3)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 256, normalized size of antiderivative = 5.69

$$\int \frac{-x+2x^3+4x^5}{(3+2x^2+x^4)^2}\,dx$$
$$= \frac{-9\sqrt{\sqrt{3}+1}\,\sqrt{\sqrt{3}-1}\,atan\left(\frac{\sqrt{\sqrt{3}-1}\,\sqrt{2}-2x}{\sqrt{\sqrt{3}+1}\,\sqrt{2}}\right)x^4 - 18\sqrt{\sqrt{3}+1}\,\sqrt{\sqrt{3}-1}\,atan\left(\frac{\sqrt{\sqrt{3}-1}\,\sqrt{2}-2x}{\sqrt{\sqrt{3}+1}\,\sqrt{2}}\right)x^2 - 27\sqrt{\sqrt{}}}{}$$

input
```
int((4*x^5+2*x^3-x)/(x^4+2*x^2+3)^2,x)
```

output
```
( - 9*sqrt(sqrt(3) + 1)*sqrt(sqrt(3) - 1)*atan((sqrt(sqrt(3) - 1)*sqrt(2)
- 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2)))*x**4 - 18*sqrt(sqrt(3) + 1)*sqrt(sqrt(
3) - 1)*atan((sqrt(sqrt(3) - 1)*sqrt(2) - 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2))
)*x**2 - 27*sqrt(sqrt(3) + 1)*sqrt(sqrt(3) - 1)*atan((sqrt(sqrt(3) - 1)*sq
rt(2) - 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2))) - 9*sqrt(sqrt(3) + 1)*sqrt(sqrt(
3) - 1)*atan((sqrt(sqrt(3) - 1)*sqrt(2) + 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2))
)*x**4 - 18*sqrt(sqrt(3) + 1)*sqrt(sqrt(3) - 1)*atan((sqrt(sqrt(3) - 1)*sq
rt(2) + 2*x)/(sqrt(sqrt(3) + 1)*sqrt(2)))*x**2 - 27*sqrt(sqrt(3) + 1)*sqrt
(sqrt(3) - 1)*atan((sqrt(sqrt(3) - 1)*sqrt(2) + 2*x)/(sqrt(sqrt(3) + 1)*sq
rt(2))) + 7*x**4 + 31)/(16*(x**4 + 2*x**2 + 3))
```

3.61 $\int \frac{x+x^5}{(1+2x^2+2x^4)^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 59

$$\int \frac{x+x^5}{(1+2x^2+2x^4)^3}\,dx = \frac{3+4x^2}{16(1+2x^2+2x^4)^2} + \frac{1+2x^2}{2(1+2x^2+2x^4)} + \arctan\left(1+2x^2\right)$$

output
```
1/16*(4*x^2+3)/(2*x^4+2*x^2+1)^2+(2*x^2+1)/(4*x^4+4*x^2+2)+arctan(2*x^2+1)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.75

$$\int \frac{x+x^5}{(1+2x^2+2x^4)^3}\,dx = \frac{11+36x^2+48x^4+32x^6}{16(1+2x^2+2x^4)^2} + \arctan\left(1+2x^2\right)$$

input
```
Integrate[(x + x^5)/(1 + 2*x^2 + 2*x^4)^3,x]
```

output
```
(11 + 36*x^2 + 48*x^4 + 32*x^6)/(16*(1 + 2*x^2 + 2*x^4)^2) + ArcTan[1 + 2*
x^2]
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.12, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {2027, 2194, 2191, 27, 1086, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5+x}{(2x^4+2x^2+1)^3}dx$$

$$\Big\downarrow \text{ 2027}$$

$$\int \frac{x(x^4+1)}{(2x^4+2x^2+1)^3}dx$$

$$\Big\downarrow \text{ 2194}$$

$$\frac{1}{2}\int \frac{x^4+1}{(2x^4+2x^2+1)^3}dx^2$$

$$\Big\downarrow \text{ 2191}$$

$$\frac{1}{2}\left(\frac{1}{8}\int \frac{16}{(2x^4+2x^2+1)^2}dx^2+\frac{4x^2+3}{8(2x^4+2x^2+1)^2}\right)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{2}\left(2\int \frac{1}{(2x^4+2x^2+1)^2}dx^2+\frac{4x^2+3}{8(2x^4+2x^2+1)^2}\right)$$

$$\Big\downarrow \text{ 1086}$$

$$\frac{1}{2}\left(2\left(\int \frac{1}{2x^4+2x^2+1}dx^2+\frac{2x^2+1}{2(2x^4+2x^2+1)}\right)+\frac{4x^2+3}{8(2x^4+2x^2+1)^2}\right)$$

$$\Big\downarrow \text{ 1082}$$

$$\frac{1}{2}\left(2\left(\frac{2x^2+1}{2(2x^4+2x^2+1)}-\int \frac{1}{-x^4-1}d(2x^2+1)\right)+\frac{4x^2+3}{8(2x^4+2x^2+1)^2}\right)$$

$$\Big\downarrow \text{ 217}$$

$$\frac{1}{2}\left(2\left(\arctan(2x^2+1)+\frac{2x^2+1}{2(2x^4+2x^2+1)}\right)+\frac{4x^2+3}{8(2x^4+2x^2+1)^2}\right)$$

input
```
Int[(x + x^5)/(1 + 2*x^2 + 2*x^4)^3,x]
```

output
```
((3 + 4*x^2)/(8*(1 + 2*x^2 + 2*x^4)^2) + 2*((1 + 2*x^2)/(2*(1 + 2*x^2 + 2*
x^4)) + ArcTan[1 + 2*x^2]))/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1086
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && ILtQ[p, -1]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 2194
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Simp[1/2   Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)
^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ
[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.69

method	result
default	$\frac{2x^6+3x^4+\frac{9}{4}x^2+\frac{11}{16}}{(2x^4+2x^2+1)^2}+\arctan\left(2x^2+1\right)$
risch	$\frac{2x^6+3x^4+\frac{9}{4}x^2+\frac{11}{16}}{(2x^4+2x^2+1)^2}+\arctan\left(2x^2+1\right)$
parallelrisch	$-\frac{-64i\ln\left(x^2+\frac{1}{2}+\frac{i}{2}\right)x^6-8i\ln\left(x^2+\frac{1}{2}+\frac{i}{2}\right)-5-32i\ln\left(x^2+\frac{1}{2}+\frac{i}{2}\right)x^2+8i\ln\left(x^2+\frac{1}{2}-\frac{i}{2}\right)+24x^8+32i\ln\left(x^2+\frac{1}{2}-\frac{i}{2}\right)x^2-64i\ln(x^2}{16(2x^4+2x^2+}$

input
```
int((x^5+x)/(2*x^4+2*x^2+1)^3,x,method=_RETURNVERBOSE)
```

output
```
2*(x^6+3/2*x^4+9/8*x^2+11/32)/(2*x^4+2*x^2+1)^2+arctan(2*x^2+1)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.27

$$\int \frac{x+x^5}{(1+2x^2+2x^4)^3}\,dx = \frac{32\,x^6+48\,x^4+36\,x^2+16\,(4\,x^8+8\,x^6+8\,x^4+4\,x^2+1)\arctan(2\,x^2+1)+11}{16\,(4\,x^8+8\,x^6+8\,x^4+4\,x^2+1)}$$

input
```
integrate((x^5+x)/(2*x^4+2*x^2+1)^3,x, algorithm="fricas")
```

output
```
1/16*(32*x^6 + 48*x^4 + 36*x^2 + 16*(4*x^8 + 8*x^6 + 8*x^4 + 4*x^2 + 1)*ar
ctan(2*x^2 + 1) + 11)/(4*x^8 + 8*x^6 + 8*x^4 + 4*x^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.78

$$\int \frac{x+x^5}{(1+2x^2+2x^4)^3}\,dx = \frac{32x^6+48x^4+36x^2+11}{64x^8+128x^6+128x^4+64x^2+16} + \operatorname{atan}\left(2x^2+1\right)$$

input
```
integrate((x**5+x)/(2*x**4+2*x**2+1)**3,x)
```

output
```
(32*x**6 + 48*x**4 + 36*x**2 + 11)/(64*x**8 + 128*x**6 + 128*x**4 + 64*x**
2 + 16) + atan(2*x**2 + 1)
```

Maxima [F]

$$\int \frac{x+x^5}{(1+2x^2+2x^4)^3}\,dx = \int \frac{x^5+x}{(2\,x^4+2\,x^2+1)^3}\,dx$$

input
```
integrate((x^5+x)/(2*x^4+2*x^2+1)^3,x, algorithm="maxima")
```

output
```
1/16*(32*x^6 + 48*x^4 + 36*x^2 + 11)/(4*x^8 + 8*x^6 + 8*x^4 + 4*x^2 + 1) +
 2*integrate(x/(2*x^4 + 2*x^2 + 1), x)
```

Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.71

$$\int \frac{x+x^5}{(1+2x^2+2x^4)^3}\,dx = \frac{32\,x^6+48\,x^4+36\,x^2+11}{16\,(2\,x^4+2\,x^2+1)^2}+\arctan\left(2\,x^2+1\right)$$

input
```
integrate((x^5+x)/(2*x^4+2*x^2+1)^3,x, algorithm="giac")
```

output
```
1/16*(32*x^6 + 48*x^4 + 36*x^2 + 11)/(2*x^4 + 2*x^2 + 1)^2 + arctan(2*x^2
+ 1)
```

Mupad [B] (verification not implemented)

Time = 22.08 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.80

$$\int \frac{x+x^5}{(1+2x^2+2x^4)^3}\,dx = \mathrm{atan}\left(2\,x^2+1\right)+\frac{\frac{x^6}{2}+\frac{3\,x^4}{4}+\frac{9\,x^2}{16}+\frac{11}{64}}{x^8+2\,x^6+2\,x^4+x^2+\frac{1}{4}}$$

input
```
int((x + x^5)/(2*x^2 + 2*x^4 + 1)^3,x)
```

output
```
atan(2*x^2 + 1) + ((9*x^2)/16 + (3*x^4)/4 + x^6/2 + 11/64)/(x^2 + 2*x^4 +
2*x^6 + x^8 + 1/4)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 366, normalized size of antiderivative = 6.20

$$\int \frac{x+x^5}{(1+2x^2+2x^4)^3}\,dx$$
$$= \frac{-64\sqrt{\sqrt{2}+1}\,\sqrt{\sqrt{2}-1}\,atan\left(\frac{\sqrt{\sqrt{2}-1}-2x}{\sqrt{\sqrt{2}+1}}\right)x^8-128\sqrt{\sqrt{2}+1}\,\sqrt{\sqrt{2}-1}\,atan\left(\frac{\sqrt{\sqrt{2}-1}-2x}{\sqrt{\sqrt{2}+1}}\right)x^6-128\sqrt{\sqrt{2}}}{}$$

input
```
int((x^5+x)/(2*x^4+2*x^2+1)^3,x)
```

output
```
( - 64*sqrt(sqrt(2) + 1)*sqrt(sqrt(2) - 1)*atan((sqrt(sqrt(2) - 1) - 2*x)/
sqrt(sqrt(2) + 1))*x**8 - 128*sqrt(sqrt(2) + 1)*sqrt(sqrt(2) - 1)*atan((sq
rt(sqrt(2) - 1) - 2*x)/sqrt(sqrt(2) + 1))*x**6 - 128*sqrt(sqrt(2) + 1)*sqr
t(sqrt(2) - 1)*atan((sqrt(sqrt(2) - 1) - 2*x)/sqrt(sqrt(2) + 1))*x**4 - 64
*sqrt(sqrt(2) + 1)*sqrt(sqrt(2) - 1)*atan((sqrt(sqrt(2) - 1) - 2*x)/sqrt(s
qrt(2) + 1))*x**2 - 16*sqrt(sqrt(2) + 1)*sqrt(sqrt(2) - 1)*atan((sqrt(sqrt
(2) - 1) - 2*x)/sqrt(sqrt(2) + 1)) - 64*sqrt(sqrt(2) + 1)*sqrt(sqrt(2) - 1
)*atan((sqrt(sqrt(2) - 1) + 2*x)/sqrt(sqrt(2) + 1))*x**8 - 128*sqrt(sqrt(2
) + 1)*sqrt(sqrt(2) - 1)*atan((sqrt(sqrt(2) - 1) + 2*x)/sqrt(sqrt(2) + 1))
*x**6 - 128*sqrt(sqrt(2) + 1)*sqrt(sqrt(2) - 1)*atan((sqrt(sqrt(2) - 1) +
2*x)/sqrt(sqrt(2) + 1))*x**4 - 64*sqrt(sqrt(2) + 1)*sqrt(sqrt(2) - 1)*atan
((sqrt(sqrt(2) - 1) + 2*x)/sqrt(sqrt(2) + 1))*x**2 - 16*sqrt(sqrt(2) + 1)*
sqrt(sqrt(2) - 1)*atan((sqrt(sqrt(2) - 1) + 2*x)/sqrt(sqrt(2) + 1)) - 16*x
**8 + 16*x**4 + 20*x**2 + 7)/(16*(4*x**8 + 8*x**6 + 8*x**4 + 4*x**2 + 1))
```

3.62 $\int \frac{-84-576x-400x^2+2560x^3}{9+24x-12x^2+80x^3+320x^4} dx$

Optimal result

Integrand size = 38, antiderivative size = 78

$$\int \frac{-84-576x-400x^2+2560x^3}{9+24x-12x^2+80x^3+320x^4} dx = 2\sqrt{11}\arctan\left(\frac{7-40x}{5\sqrt{11}}\right) - 2\sqrt{11}\arctan\left(\frac{57+30x-40x^2+800x^3}{6\sqrt{11}}\right) + 2\log\left(9+24x-12x^2+80x^3+320x^4\right)$$

output
```
2*11^(1/2)*arctan(1/55*(7-40*x)*11^(1/2))-2*11^(1/2)*arctan(1/66*(800*x^3-
40*x^2+30*x+57)*11^(1/2))+2*ln(320*x^4+80*x^3-12*x^2+24*x+9)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.27

$$\int \frac{-84-576x-400x^2+2560x^3}{9+24x-12x^2+80x^3+320x^4} dx = \frac{1}{2}\text{RootSum}\Big[9+24\#1-12\#1^2+80\#1^3 +320\#1^4\&, \frac{-21\log(x-\#1)-144\log(x-\#1)\#1-100\log(x-\#1)\#1^2+640\log(x-\#1)\#1^3}{3-3\#1+30\#1^2+160\#1^3}\&\Big]$$

input
```
Integrate[(-84 - 576*x - 400*x^2 + 2560*x^3)/(9 + 24*x - 12*x^2 + 80*x^3 +
 320*x^4),x]
```

output
```
RootSum[9 + 24*#1 - 12*#1^2 + 80*#1^3 + 320*#1^4 & , (-21*Log[x - #1] - 14
4*Log[x - #1]*#1 - 100*Log[x - #1]*#1^2 + 640*Log[x - #1]*#1^3)/(3 - 3*#1
+ 30*#1^2 + 160*#1^3) & ]/2
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.09, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.079$, Rules used = {2525, 27, 2502}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2560x^3-400x^2-576x-84}{320x^4+80x^3-12x^2+24x+9}dx$$

$$\Big\downarrow \; 2525$$

$$\frac{\int -\frac{56320(20x^2+12x+3)}{320x^4+80x^3-12x^2+24x+9}dx}{1280}+2\log\left(320x^4+80x^3-12x^2+24x+9\right)$$

$$\Big\downarrow \; 27$$

$$2\log\left(320x^4+80x^3-12x^2+24x+9\right)-44\int \frac{20x^2+12x+3}{320x^4+80x^3-12x^2+24x+9}dx$$

$$\Big\downarrow \; 2502$$

$$2\log\left(320x^4+80x^3-12x^2+24x+9\right)-44\left(\frac{\arctan\left(\frac{800x^3-40x^2+30x+57}{6\sqrt{11}}\right)}{2\sqrt{11}}-\frac{\arctan\left(\frac{7-40x}{5\sqrt{11}}\right)}{2\sqrt{11}}\right)$$

input
```
Int[(-84 - 576*x - 400*x^2 + 2560*x^3)/(9 + 24*x - 12*x^2 + 80*x^3 + 320*x
^4),x]
```

output
```
-44*(-1/2*ArcTan[(7 - 40*x)/(5*Sqrt[11])]/Sqrt[11] + ArcTan[(57 + 30*x - 4
0*x^2 + 800*x^3)/(6*Sqrt[11])]/(2*Sqrt[11])) + 2*Log[9 + 24*x - 12*x^2 + 8
0*x^3 + 320*x^4]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2502
```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2 +
 (d_.)*(x_)^3 + (e_.)*(x_)^4), x_Symbol] :> With[{q = Rt[(-C)*(2*e*(B*d - 4
*A*e) + C*(d^2 - 4*c*e)), 2]}, Simp[2*(C^2/q)*ArcTan[(C*d - B*e + 2*C*e*x)/
q], x] - Simp[2*(C^2/q)*ArcTan[C*((4*B*c*C - 3*B^2*d - 4*A*C*d + 12*A*B*e +
 4*C*(2*c*C - B*d + 2*A*e)*x + 4*C*(2*C*d - B*e)*x^2 + 8*C^2*e*x^3)/(q*(B^2
 - 4*A*C))], x]] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B^2*d + 2*C*
(b*C + A*d) - 2*B*(c*C + 2*A*e), 0] && EqQ[2*B^2*c*C - 8*a*C^3 - B^3*d - 4*
A*B*C*d + 4*A*(B^2 + 2*A*C)*e, 0] && NegQ[C*(2*e*(B*d - 4*A*e) + C*(d^2 - 4
*c*e))]
```

rule 2525
```
Int[(Pm_)/(Qn_), x_Symbol] :> With[{m = Expon[Pm, x], n = Expon[Qn, x]}, Si
mp[Coeff[Pm, x, m]*(Log[Qn]/(n*Coeff[Qn, x, n])), x] + Simp[1/(n*Coeff[Qn,
x, n])   Int[ExpandToSum[n*Coeff[Qn, x, n]*Pm - Coeff[Pm, x, m]*D[Qn, x], x
]/Qn, x], x] /; EqQ[m, n - 1]] /; PolyQ[Pm, x] && PolyQ[Qn, x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.06 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.90

method	result
default	$4\left(\frac{1}{2}+\frac{i\sqrt{11}}{4}\right)\ln\left(80x^2+\left(-10i\sqrt{11}+10\right)x-3i\sqrt{11}-9\right)+4\left(\frac{1}{2}-\frac{i\sqrt{11}}{4}\right)\ln\left(80x^2+\left(10i\sqrt{11}+\right.\right.$
risch	$2\ln\left(6400x^4+1600x^3-240x^2+480x+180\right)-2\arctan\left(-\frac{20\sqrt{11}\,x^2}{33}+\frac{5x\sqrt{11}}{11}+\frac{19\sqrt{11}}{22}+\frac{400\sqrt{11}\,x}{33}\right.$

input
```
int((2560*x^3-400*x^2-576*x-84)/(320*x^4+80*x^3-12*x^2+24*x+9),x,method=_R
ETURNVERBOSE)
```

output
```
4*(1/2+1/4*I*11^(1/2))*ln(80*x^2+(-10*I*11^(1/2)+10)*x-3*I*11^(1/2)-9)+4*(
1/2-1/4*I*11^(1/2))*ln(80*x^2+(10*I*11^(1/2)+10)*x+3*I*11^(1/2)-9)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.85

$$\int \frac{-84-576x-400x^2+2560x^3}{9+24x-12x^2+80x^3+320x^4}\,dx = -2\sqrt{11}\arctan\left(\frac{1}{66}\sqrt{11}(800\,x^3-40\,x^2+30\,x+57)\right) - 2\sqrt{11}\arctan\left(\frac{1}{55}\sqrt{11}(40\,x-7)\right)+2\,\log\left(320\,x^4+80\,x^3-12\,x^2+24\,x+9\right)$$

input
```
integrate((2560*x^3-400*x^2-576*x-84)/(320*x^4+80*x^3-12*x^2+24*x+9),x, al
gorithm="fricas")
```

output
```
-2*sqrt(11)*arctan(1/66*sqrt(11)*(800*x^3 - 40*x^2 + 30*x + 57)) - 2*sqrt(
11)*arctan(1/55*sqrt(11)*(40*x - 7)) + 2*log(320*x^4 + 80*x^3 - 12*x^2 + 2
4*x + 9)
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.28

$$\int \frac{-84-576x-400x^2+2560x^3}{9+24x-12x^2+80x^3+320x^4}\,dx = \sqrt{11}\left(-2\,\mathrm{atan}\left(\frac{8\sqrt{11}x}{11}-\frac{7\sqrt{11}}{55}\right) - 2\,\mathrm{atan}\left(\frac{400\sqrt{11}x^3}{33}-\frac{20\sqrt{11}x^2}{33}+\frac{5\sqrt{11}x}{11}+\frac{19\sqrt{11}}{22}\right)\right) + 2\log\left(x^4+\frac{x^3}{4}-\frac{3x^2}{80}+\frac{3x}{40}+\frac{9}{320}\right)$$

input
```
integrate((2560*x**3-400*x**2-576*x-84)/(320*x**4+80*x**3-12*x**2+24*x+9),
x)
```

output
```
sqrt(11)*(-2*atan(8*sqrt(11)*x/11 - 7*sqrt(11)/55) - 2*atan(400*sqrt(11)*x
**3/33 - 20*sqrt(11)*x**2/33 + 5*sqrt(11)*x/11 + 19*sqrt(11)/22)) + 2*log(
x**4 + x**3/4 - 3*x**2/80 + 3*x/40 + 9/320)
```

Maxima [F]

$$\int \frac{-84-576x-400x^2+2560x^3}{9+24x-12x^2+80x^3+320x^4}\,dx=\int \frac{4\,(640\,x^3-100\,x^2-144\,x-21)}{320\,x^4+80\,x^3-12\,x^2+24\,x+9}\,dx$$

input
```
integrate((2560*x^3-400*x^2-576*x-84)/(320*x^4+80*x^3-12*x^2+24*x+9),x, al
gorithm="maxima")
```

output
```
4*integrate((640*x^3 - 100*x^2 - 144*x - 21)/(320*x^4 + 80*x^3 - 12*x^2 +
24*x + 9), x)
```

Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.82

$$\int \frac{-84-576x-400x^2+2560x^3}{9+24x-12x^2+80x^3+320x^4}\,dx = -2\,\sqrt{11}\left(\arctan\left(\frac{1}{66}\,\sqrt{11}(800\,x^3-40\,x^2+30\,x+57)\right)-\arctan\left(-\frac{1}{55}\,\sqrt{11}(40\,x-7)\right)\right) + 2\,\log\left(320\,x^4+80\,x^3-12\,x^2+24\,x+9\right)$$

input
```
integrate((2560*x^3-400*x^2-576*x-84)/(320*x^4+80*x^3-12*x^2+24*x+9),x, al
gorithm="giac")
```

output
```
-2*sqrt(11)*(arctan(1/66*sqrt(11)*(800*x^3 - 40*x^2 + 30*x + 57)) - arctan
(-1/55*sqrt(11)*(40*x - 7))) + 2*log(320*x^4 + 80*x^3 - 12*x^2 + 24*x + 9)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.97

$$\int \frac{-84-576x-400x^2+2560x^3}{9+24x-12x^2+80x^3+320x^4}\,dx = 2\,\ln\left(320\,x^4+80\,x^3-12\,x^2+24\,x+9\right) - 2\,\sqrt{11}\,\mathrm{atan}\left(\frac{8\,\sqrt{11}\,x}{11}-\frac{7\,\sqrt{11}}{55}\right) - 2\,\sqrt{11}\,\mathrm{atan}\left(\frac{400\,\sqrt{11}\,x^3}{33}-\frac{20\,\sqrt{11}\,x^2}{33}+\frac{5\,\sqrt{11}\,x}{11}+\frac{19\,\sqrt{11}}{22}\right)$$

input
```
int(-(576*x + 400*x^2 - 2560*x^3 + 84)/(24*x - 12*x^2 + 80*x^3 + 320*x^4 +
 9),x)
```

output
```
2*log(24*x - 12*x^2 + 80*x^3 + 320*x^4 + 9) - 2*11^(1/2)*atan((8*11^(1/2)*
x)/11 - (7*11^(1/2))/55) - 2*11^(1/2)*atan((5*11^(1/2)*x)/11 + (19*11^(1/2
))/22 - (20*11^(1/2)*x^2)/33 + (400*11^(1/2)*x^3)/33)
```

Reduce [F]

$$\int \frac{-84-576x-400x^2+2560x^3}{9+24x-12x^2+80x^3+320x^4}\,dx = -880\left(\int \frac{x^2}{320x^4+80x^3-12x^2+24x+9}dx\right) - 528\left(\int \frac{x}{320x^4+80x^3-12x^2+24x+9}dx\right) - 132\left(\int \frac{1}{320x^4+80x^3-12x^2+24x+9}dx\right) + 2\log\left(320x^4+80x^3-12x^2+24x+9\right)$$

input
```
int((2560*x^3-400*x^2-576*x-84)/(320*x^4+80*x^3-12*x^2+24*x+9),x)
```

output
```
2*( - 440*int(x**2/(320*x**4 + 80*x**3 - 12*x**2 + 24*x + 9),x) - 264*int(
x/(320*x**4 + 80*x**3 - 12*x**2 + 24*x + 9),x) - 66*int(1/(320*x**4 + 80*x
**3 - 12*x**2 + 24*x + 9),x) + log(320*x**4 + 80*x**3 - 12*x**2 + 24*x + 9
))
```

3.63 $\int -\frac{15-36x+5x^2+12x^3-34x^4+140x^5+15x^6+8x^7-30x^9}{(3+x+x^4)^4}\,dx$

Optimal result

Integrand size = 50, antiderivative size = 60

$$\int -\frac{15-36x+5x^2+12x^3-34x^4+140x^5+15x^6+8x^7-30x^9}{(3+x+x^4)^4}\,dx = \frac{2}{(3+x+x^4)^3}-\frac{3x}{(3+x+x^4)^3}+\frac{5x^2}{(3+x+x^4)^3}+\frac{x^4}{(3+x+x^4)^3}-\frac{5x^6}{(3+x+x^4)^3}$$

output
```
2/(x^4+x+3)^3-3*x/(x^4+x+3)^3+5*x^2/(x^4+x+3)^3+x^4/(x^4+x+3)^3-5*x^6/(x^4
+x+3)^3
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.45

$$\int -\frac{15-36x+5x^2+12x^3-34x^4+140x^5+15x^6+8x^7-30x^9}{(3+x+x^4)^4}\,dx = \frac{2-3x+5x^2+x^4-5x^6}{(3+x+x^4)^3}$$

input
```
Integrate[-((15 - 36*x + 5*x^2 + 12*x^3 - 34*x^4 + 140*x^5 + 15*x^6 + 8*x^
7 - 30*x^9)/(3 + x + x^4)^4),x]
```

output
```
(2 - 3*x + 5*x^2 + x^4 - 5*x^6)/(3 + x + x^4)^3
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.07, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {25, 2527, 27, 2527, 27, 2527, 27, 2527, 27, 2021}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int -\frac{-30x^9+8x^7+15x^6+140x^5-34x^4+12x^3+5x^2-36x+15}{\left(x^4+x+3\right)^4}dx$$

$$\Big\downarrow \text{ 25}$$

$$-\int \frac{-30x^9+8x^7+15x^6+140x^5-34x^4+12x^3+5x^2-36x+15}{\left(x^4+x+3\right)^4}dx$$

$$\Big\downarrow \text{ 2527}$$

$$\frac{1}{6}\int -\frac{6\left(8x^7+50x^5-34x^4+12x^3+5x^2-36x+15\right)}{\left(x^4+x+3\right)^4}dx-\frac{5x^6}{\left(x^4+x+3\right)^3}$$

$$\Big\downarrow \text{ 27}$$

$$-\int \frac{8x^7+50x^5-34x^4+12x^3+5x^2-36x+15}{\left(x^4+x+3\right)^4}dx-\frac{5x^6}{\left(x^4+x+3\right)^3}$$

$$\Big\downarrow \text{ 2527}$$

$$\frac{1}{8}\int -\frac{8\left(50x^5-33x^4+24x^3+5x^2-36x+15\right)}{\left(x^4+x+3\right)^4}dx+\frac{x^4}{\left(x^4+x+3\right)^3}-\frac{5x^6}{\left(x^4+x+3\right)^3}$$

$$\Big\downarrow \text{ 27}$$

$$-\int \frac{50x^5-33x^4+24x^3+5x^2-36x+15}{\left(x^4+x+3\right)^4}dx+\frac{x^4}{\left(x^4+x+3\right)^3}-\frac{5x^6}{\left(x^4+x+3\right)^3}$$

$$\Big\downarrow \text{ 2527}$$

$$\frac{1}{10}\int -\frac{30\left(-11x^4+8x^3-2x+5\right)}{\left(x^4+x+3\right)^4}dx+\frac{x^4}{\left(x^4+x+3\right)^3}-\frac{5x^6}{\left(x^4+x+3\right)^3}+\frac{5x^2}{\left(x^4+x+3\right)^3}$$

$$\Big\downarrow \text{ 27}$$

$$-3\int \frac{-11x^4+8x^3-2x+5}{(x^4+x+3)^4}dx+\frac{x^4}{(x^4+x+3)^3}-\frac{5x^6}{(x^4+x+3)^3}+\frac{5x^2}{(x^4+x+3)^3}$$

$$\Big\downarrow \text{2527}$$

$$-3\left(\frac{x}{(x^4+x+3)^3}-\frac{1}{11}\int -\frac{22(4x^3+1)}{(x^4+x+3)^4}dx\right)+\frac{x^4}{(x^4+x+3)^3}-\frac{5x^6}{(x^4+x+3)^3}+\frac{5x^2}{(x^4+x+3)^3}$$

$$\Big\downarrow \text{27}$$

$$-3\left(2\int \frac{4x^3+1}{(x^4+x+3)^4}dx+\frac{x}{(x^4+x+3)^3}\right)+\frac{x^4}{(x^4+x+3)^3}-\frac{5x^6}{(x^4+x+3)^3}+\frac{5x^2}{(x^4+x+3)^3}$$

$$\Big\downarrow \text{2021}$$

$$\frac{x^4}{(x^4+x+3)^3}-3\left(\frac{x}{(x^4+x+3)^3}-\frac{2}{3(x^4+x+3)^3}\right)-\frac{5x^6}{(x^4+x+3)^3}+\frac{5x^2}{(x^4+x+3)^3}$$

input
```
Int[-((15 - 36*x + 5*x^2 + 12*x^3 - 34*x^4 + 140*x^5 + 15*x^6 + 8*x^7 - 30
*x^9)/(3 + x + x^4)^4),x]
```

output
```
(5*x^2)/(3 + x + x^4)^3 + x^4/(3 + x + x^4)^3 - (5*x^6)/(3 + x + x^4)^3 -
3*(-2/(3*(3 + x + x^4)^3) + x/(3 + x + x^4)^3)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2021
```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

rule 2527
```
Int[(Pm_)*(Qn_)^(p_.), x_Symbol] :> With[{m = Expon[Pm, x], n = Expon[Qn, x
]}, Simp[Coeff[Pm, x, m]*x^(m - n + 1)*(Qn^(p + 1)/((m + n*p + 1)*Coeff[Qn,
 x, n])), x] + Simp[1/((m + n*p + 1)*Coeff[Qn, x, n])   Int[ExpandToSum[(m
+ n*p + 1)*Coeff[Qn, x, n]*Pm - Coeff[Pm, x, m]*x^(m - n)*((m - n + 1)*Qn +
 (p + 1)*x*D[Qn, x]), x]*Qn^p, x], x] /; LtQ[1, n, m + 1] && m + n*p + 1 <
0] /; FreeQ[p, x] && PolyQ[Pm, x] && PolyQ[Qn, x] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.47

method	result	size
default	$\frac{-5x^6+x^4+5x^2-3x+2}{(x^4+x+3)^3}$	28
norman	$\frac{-5x^6+x^4+5x^2-3x+2}{(x^4+x+3)^3}$	28
gosper	$-\frac{5x^6-x^4-5x^2+3x-2}{(x^4+x+3)^3}$	31
risch	$-\frac{5x^6-x^4-5x^2+3x-2}{(x^4+x+3)^3}$	31
parallelrisch	$-\frac{5x^6-x^4-5x^2+3x-2}{(x^4+x+3)^3}$	31
orering	$\frac{(5x^6-x^4-5x^2+3x-2)(-30x^9+8x^7+15x^6+140x^5-34x^4+12x^3+5x^2-36x+15)}{(x^4+x+3)^3(30x^9-8x^7-15x^6-140x^5+34x^4-12x^3-5x^2+36x-15)}$	112

input
```
int(-(-30*x^9+8*x^7+15*x^6+140*x^5-34*x^4+12*x^3+5*x^2-36*x+15)/(x^4+x+3)^
4,x,method=_RETURNVERBOSE)
```

output
```
(-5*x^6+x^4+5*x^2-3*x+2)/(x^4+x+3)^3
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.08

$$\int -\frac{15-36x+5x^2+12x^3-34x^4+140x^5+15x^6+8x^7-30x^9}{(3+x+x^4)^4}\,dx$$
$$=-\frac{5\,x^6-x^4-5\,x^2+3\,x-2}{x^{12}+3\,x^9+9\,x^8+3\,x^6+18\,x^5+27\,x^4+x^3+9\,x^2+27\,x+27}$$

input
```
integrate(-(-30*x^9+8*x^7+15*x^6+140*x^5-34*x^4+12*x^3+5*x^2-36*x+15)/(x^4
+x+3)^4,x, algorithm="fricas")
```

output
```
-(5*x^6 - x^4 - 5*x^2 + 3*x - 2)/(x^12 + 3*x^9 + 9*x^8 + 3*x^6 + 18*x^5 +
27*x^4 + x^3 + 9*x^2 + 27*x + 27)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00

$$\int -\frac{15-36x+5x^2+12x^3-34x^4+140x^5+15x^6+8x^7-30x^9}{(3+x+x^4)^4}\,dx = \frac{-5x^6+x^4+5x^2-3x+2}{x^{12}+3x^9+9x^8+3x^6+18x^5+27x^4+x^3+9x^2+27x+27}$$

input
```
integrate(-(-30*x**9+8*x**7+15*x**6+140*x**5-34*x**4+12*x**3+5*x**2-36*x+1
5)/(x**4+x+3)**4,x)
```

output
```
(-5*x**6 + x**4 + 5*x**2 - 3*x + 2)/(x**12 + 3*x**9 + 9*x**8 + 3*x**6 + 18
*x**5 + 27*x**4 + x**3 + 9*x**2 + 27*x + 27)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.08

$$\int -\frac{15-36x+5x^2+12x^3-34x^4+140x^5+15x^6+8x^7-30x^9}{(3+x+x^4)^4}\,dx = -\frac{5\,x^6-x^4-5\,x^2+3\,x-2}{x^{12}+3\,x^9+9\,x^8+3\,x^6+18\,x^5+27\,x^4+x^3+9\,x^2+27\,x+27}$$

input
```
integrate(-(-30*x^9+8*x^7+15*x^6+140*x^5-34*x^4+12*x^3+5*x^2-36*x+15)/(x^4
+x+3)^4,x, algorithm="maxima")
```

output
```
-(5*x^6 - x^4 - 5*x^2 + 3*x - 2)/(x^12 + 3*x^9 + 9*x^8 + 3*x^6 + 18*x^5 +
27*x^4 + x^3 + 9*x^2 + 27*x + 27)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.50

$$\int -\frac{15-36x+5x^2+12x^3-34x^4+140x^5+15x^6+8x^7-30x^9}{(3+x+x^4)^4}\,dx = -\frac{5\,x^6-x^4-5\,x^2+3\,x-2}{(x^4+x+3)^3}$$

input
```
integrate(-(-30*x^9+8*x^7+15*x^6+140*x^5-34*x^4+12*x^3+5*x^2-36*x+15)/(x^4
+x+3)^4,x, algorithm="giac")
```

output
```
-(5*x^6 - x^4 - 5*x^2 + 3*x - 2)/(x^4 + x + 3)^3
```

Mupad [B] (verification not implemented)

Time = 22.50 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.45

$$\int -\frac{15-36x+5x^2+12x^3-34x^4+140x^5+15x^6+8x^7-30x^9}{(3+x+x^4)^4}\,dx = \frac{-5\,x^6+x^4+5\,x^2-3\,x+2}{(x^4+x+3)^3}$$

input
```
int(-(5*x^2 - 36*x + 12*x^3 - 34*x^4 + 140*x^5 + 15*x^6 + 8*x^7 - 30*x^9 +
 15)/(x + x^4 + 3)^4,x)
```

output
```
(5*x^2 - 3*x + x^4 - 5*x^6 + 2)/(x + x^4 + 3)^3
```

Reduce [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.03

$$\int -\frac{15-36x+5x^2+12x^3-34x^4+140x^5+15x^6+8x^7-30x^9}{(3+x+x^4)^4}\,dx = \frac{-5x^6+x^4+5x^2-3x+2}{x^{12}+3x^9+9x^8+3x^6+18x^5+27x^4+x^3+9x^2+27x+27}$$

input
```
int(-(-30*x^9+8*x^7+15*x^6+140*x^5-34*x^4+12*x^3+5*x^2-36*x+15)/(x^4+x+3)^
4,x)
```

output
```
( - 5*x**6 + x**4 + 5*x**2 - 3*x + 2)/(x**12 + 3*x**9 + 9*x**8 + 3*x**6 +
18*x**5 + 27*x**4 + x**3 + 9*x**2 + 27*x + 27)
```

3.64 $\int \left(\frac{3(-47+228x+120x^2+19x^3)}{(3+x+x^4)^4}+\frac{42-320x-75x^2-8x^3}{(3+x+x^4)^3}+\frac{30x}{(3+x+x^4)^2}\right)$

Optimal result

Integrand size = 61, antiderivative size = 27

$$\int \left(\frac{3(-47+228x+120x^2+19x^3)}{(3+x+x^4)^4}+\frac{42-320x-75x^2-8x^3}{(3+x+x^4)^3}+\frac{30x}{(3+x+x^4)^2}\right)\,dx = \frac{2-3x+5x^2+x^4-5x^6}{(3+x+x^4)^3}$$

output
```
(-5*x^6+x^4+5*x^2-3*x+2)/(x^4+x+3)^3
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \left(\frac{3(-47+228x+120x^2+19x^3)}{(3+x+x^4)^4}+\frac{42-320x-75x^2-8x^3}{(3+x+x^4)^3}+\frac{30x}{(3+x+x^4)^2}\right)\,dx = \frac{2-3x+5x^2+x^4-5x^6}{(3+x+x^4)^3}$$

input
```
Integrate[(3*(-47 + 228*x + 120*x^2 + 19*x^3))/(3 + x + x^4)^4 + (42 - 320
*x - 75*x^2 - 8*x^3)/(3 + x + x^4)^3 + (30*x)/(3 + x + x^4)^2,x]
```

output
```
(2 - 3*x + 5*x^2 + x^4 - 5*x^6)/(3 + x + x^4)^3
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{30x}{(x^4+x+3)^2}+\frac{-8x^3-75x^2-320x+42}{(x^4+x+3)^3}+\frac{3(19x^3+120x^2+228x-47)}{(x^4+x+3)^4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{621}{4}\int \frac{1}{(x^4+x+3)^4}dx+684\int \frac{x}{(x^4+x+3)^4}dx+44\int \frac{1}{(x^4+x+3)^3}dx-320\int \frac{x}{(x^4+x+3)^3}dx+30\int \frac{x}{(x^4+x+3)^2}dx+360\int \frac{x^2}{(x^4+x+3)^4}dx-75\int \frac{x^2}{(x^4+x+3)^3}dx+\frac{1}{(x^4+x+3)^2}-\frac{19}{4(x^4+x+3)^3}$$

input
```
Int[(3*(-47 + 228*x + 120*x^2 + 19*x^3))/(3 + x + x^4)^4 + (42 - 320*x - 7
5*x^2 - 8*x^3)/(3 + x + x^4)^3 + (30*x)/(3 + x + x^4)^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.04

method	result
norman	$\frac{-5x^6+x^4+5x^2-3x+2}{(x^4+x+3)^3}$
parallelrisch	$\frac{-5x^6+x^4+5x^2-3x+2}{(x^4+x+3)^3}$
gosper	$-\frac{5x^6-x^4-5x^2+3x-2}{(x^4+x+3)^3}$
orering	$-\frac{(5x^6-x^4-5x^2+3x-2)(x^4+x+3)\left(\frac{57x^3+360x^2+684x-141}{(x^4+x+3)^4}+\frac{-8x^3-75x^2-320x+42}{(x^4+x+3)^3}+\frac{30x}{(x^4+x+3)^2}\right)}{30x^9-8x^7-15x^6-140x^5+34x^4-12x^3-5x^2+36x-15}$
default	$\frac{\frac{377432}{195075}x^7-\frac{1404328}{195075}x^6+\frac{234517}{195075}x^5+\frac{660506}{195075}x^4-\frac{208792}{195075}x^3-\frac{13339729}{390150}x^2+\frac{89881}{13005}x+\frac{121303}{21675}}{(x^4+x+3)^2}+\left(\sum_{\_R=\text{RootOf}(\_Z^4+\_Z+3)}(3774$
risch	$\frac{\frac{377432}{195075}x^7-\frac{1404328}{195075}x^6+\frac{234517}{195075}x^5+\frac{660506}{195075}x^4-\frac{208792}{195075}x^3-\frac{13339729}{390150}x^2+\frac{89881}{13005}x+\frac{121303}{21675}}{(x^4+x+3)^2}+\left(\sum_{\_R=\text{RootOf}(\_Z^4+\_Z+3)}(3774$

input
```
int(3*(19*x^3+120*x^2+228*x-47)/(x^4+x+3)^4+(-8*x^3-75*x^2-320*x+42)/(x^4+
x+3)^3+30*x/(x^4+x+3)^2,x,method=_RETURNVERBOSE)
```

output
```
(-5*x^6+x^4+5*x^2-3*x+2)/(x^4+x+3)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(30) = 60$.

Time = 0.06 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.41

$$\int \left(\frac{3(-47+228x+120x^2+19x^3)}{(3+x+x^4)^4}+\frac{42-320x-75x^2-8x^3}{(3+x+x^4)^3}+\frac{30x}{(3+x+x^4)^2}\right)dx$$
$$=-\frac{5\,x^6-x^4-5\,x^2+3\,x-2}{x^{12}+3\,x^9+9\,x^8+3\,x^6+18\,x^5+27\,x^4+x^3+9\,x^2+27\,x+27}$$

input
```
integrate(3*(19*x^3+120*x^2+228*x-47)/(x^4+x+3)^4+(-8*x^3-75*x^2-320*x+42)
/(x^4+x+3)^3+30*x/(x^4+x+3)^2,x, algorithm="fricas")
```

output
```
-(5*x^6 - x^4 - 5*x^2 + 3*x - 2)/(x^12 + 3*x^9 + 9*x^8 + 3*x^6 + 18*x^5 +
27*x^4 + x^3 + 9*x^2 + 27*x + 27)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(26) = 52$.

Time = 0.14 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.22

$$\int \left(\frac{3(-47+228x+120x^2+19x^3)}{(3+x+x^4)^4}+\frac{42-320x-75x^2-8x^3}{(3+x+x^4)^3}+\frac{30x}{(3+x+x^4)^2}\right)dx$$
$$=\frac{-5x^6+x^4+5x^2-3x+2}{x^{12}+3x^9+9x^8+3x^6+18x^5+27x^4+x^3+9x^2+27x+27}$$

input
```
integrate(3*(19*x**3+120*x**2+228*x-47)/(x**4+x+3)**4+(-8*x**3-75*x**2-320
*x+42)/(x**4+x+3)**3+30*x/(x**4+x+3)**2,x)
```

output
```
(-5*x**6 + x**4 + 5*x**2 - 3*x + 2)/(x**12 + 3*x**9 + 9*x**8 + 3*x**6 + 18
*x**5 + 27*x**4 + x**3 + 9*x**2 + 27*x + 27)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(30) = 60$.

Time = 0.04 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.41

$$\int \left(\frac{3(-47+228x+120x^2+19x^3)}{(3+x+x^4)^4}+\frac{42-320x-75x^2-8x^3}{(3+x+x^4)^3}+\frac{30x}{(3+x+x^4)^2}\right)dx$$
$$= -\frac{5\,x^6-x^4-5\,x^2+3\,x-2}{x^{12}+3\,x^9+9\,x^8+3\,x^6+18\,x^5+27\,x^4+x^3+9\,x^2+27\,x+27}$$

input
```
integrate(3*(19*x^3+120*x^2+228*x-47)/(x^4+x+3)^4+(-8*x^3-75*x^2-320*x+42)
/(x^4+x+3)^3+30*x/(x^4+x+3)^2,x, algorithm="maxima")
```

output
```
-(5*x^6 - x^4 - 5*x^2 + 3*x - 2)/(x^12 + 3*x^9 + 9*x^8 + 3*x^6 + 18*x^5 +
27*x^4 + x^3 + 9*x^2 + 27*x + 27)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 197 vs. $2(30) = 60$.

Time = 0.12 (sec) , antiderivative size = 197, normalized size of antiderivative = 7.30

$$\int \left(\frac{3(-47+228x+120x^2+19x^3)}{(3+x+x^4)^4}+\frac{42-320x-75x^2-8x^3}{(3+x+x^4)^3}+\frac{30x}{(3+x+x^4)^2}\right)dx$$
$$= \frac{1}{195075}\,x\left(\frac{377432\,x^2-2808656\,x+703551}{x^4+x+3}-\frac{255032\,x^2-1829456\,x+680601}{x^4+x+3}-\frac{7650\,(16\,x^2-128\,x}{x^4+x+3}\right.$$
$$-\frac{2\,(16\,x^3-64\,x^2+x+12)}{51\,(x^4+x+3)}$$
$$+\frac{754864\,x^7-2808656\,x^6+469034\,x^5+1321012\,x^4-417584\,x^3-13339729\,x^2+2696430\,x+218345}{390150\,(x^4+x+3)^2}$$
$$-\frac{510064\,x^{11}-1829456\,x^{10}+453734\,x^9+1402676\,x^8-472048\,x^7-13501313\,x^6+4720744\,x^5+3747}{390150\,(x^4+x+3)^3}$$

input
```
integrate(3*(19*x^3+120*x^2+228*x-47)/(x^4+x+3)^4+(-8*x^3-75*x^2-320*x+42)
/(x^4+x+3)^3+30*x/(x^4+x+3)^2,x, algorithm="giac")
```

output
```
1/195075*x*((377432*x^2 - 2808656*x + 703551)/(x^4 + x + 3) - (255032*x^2
- 1829456*x + 680601)/(x^4 + x + 3) - 7650*(16*x^2 - 128*x + 3)/(x^4 + x +
 3)) - 2/51*(16*x^3 - 64*x^2 + x + 12)/(x^4 + x + 3) + 1/390150*(754864*x^
7 - 2808656*x^6 + 469034*x^5 + 1321012*x^4 - 417584*x^3 - 13339729*x^2 + 2
696430*x + 2183454)/(x^4 + x + 3)^2 - 1/390150*(510064*x^11 - 1829456*x^10
 + 453734*x^9 + 1402676*x^8 - 472048*x^7 - 13501313*x^6 + 4720744*x^5 + 37
47556*x^4 - 10935781*x^3 - 30736107*x^2 + 10203894*x + 4117662)/(x^4 + x +
 3)^3
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \left(\frac{3(-47+228x+120x^2+19x^3)}{(3+x+x^4)^4}+\frac{42-320x-75x^2-8x^3}{(3+x+x^4)^3}+\frac{30x}{(3+x+x^4)^2}\right)\,dx = \frac{-5\,x^6+x^4+5\,x^2-3\,x+2}{{\left(x^4+x+3\right)}^3}$$

input
```
int((684*x + 360*x^2 + 57*x^3 - 141)/(x + x^4 + 3)^4 - (320*x + 75*x^2 + 8
*x^3 - 42)/(x + x^4 + 3)^3 + (30*x)/(x + x^4 + 3)^2,x)
```

output
```
(5*x^2 - 3*x + x^4 - 5*x^6 + 2)/(x + x^4 + 3)^3
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 62, normalized size of antiderivative = 2.30

$$\int \left(\frac{3(-47+228x+120x^2+19x^3)}{(3+x+x^4)^4}+\frac{42-320x-75x^2-8x^3}{(3+x+x^4)^3}+\frac{30x}{(3+x+x^4)^2}\right)\,dx = \frac{-5x^6+x^4+5x^2-3x+2}{x^{12}+3x^9+9x^8+3x^6+18x^5+27x^4+x^3+9x^2+27x+27}$$

input
```
int(3*(19*x^3+120*x^2+228*x-47)/(x^4+x+3)^4+(-8*x^3-75*x^2-320*x+42)/(x^4+
x+3)^3+30*x/(x^4+x+3)^2,x)
```

output
```
( - 5*x**6 + x**4 + 5*x**2 - 3*x + 2)/(x**12 + 3*x**9 + 9*x**8 + 3*x**6 +
18*x**5 + 27*x**4 + x**3 + 9*x**2 + 27*x + 27)
```

3.65 $\int \left(\frac{-3+10x+4x^3-30x^5}{(3+x+x^4)^3}-\frac{3(1+4x^3)(2-3x+5x^2+x^4-5x^6)}{(3+x+x^4)^4}\right) dx$

Optimal result

Integrand size = 60, antiderivative size = 27

$$\int \left(\frac{-3+10x+4x^3-30x^5}{(3+x+x^4)^3}-\frac{3(1+4x^3)(2-3x+5x^2+x^4-5x^6)}{(3+x+x^4)^4}\right) dx = \frac{2-3x+5x^2+x^4-5x^6}{(3+x+x^4)^3}$$

output
```
(-5*x^6+x^4+5*x^2-3*x+2)/(x^4+x+3)^3
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \left(\frac{-3+10x+4x^3-30x^5}{(3+x+x^4)^3}-\frac{3(1+4x^3)(2-3x+5x^2+x^4-5x^6)}{(3+x+x^4)^4}\right) dx = \frac{2-3x+5x^2+x^4-5x^6}{(3+x+x^4)^3}$$

input
```
Integrate[(-3 + 10*x + 4*x^3 - 30*x^5)/(3 + x + x^4)^3 - (3*(1 + 4*x^3)*(2
 - 3*x + 5*x^2 + x^4 - 5*x^6))/(3 + x + x^4)^4,x]
```

output
```
(2 - 3*x + 5*x^2 + x^4 - 5*x^6)/(3 + x + x^4)^3
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{-30x^5+4x^3+10x-3}{\left(x^4+x+3\right)^3}-\frac{3\left(4x^3+1\right)\left(-5x^6+x^4+5x^2-3x+2\right)}{\left(x^4+x+3\right)^4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{144}{11}\int \frac{1}{\left(x^4+x+3\right)^4}dx+\frac{828}{11}\int \frac{x}{\left(x^4+x+3\right)^4}dx-4\int \frac{1}{\left(x^4+x+3\right)^3}dx-20\int \frac{x}{\left(x^4+x+3\right)^3}dx+18\int \frac{x^2}{\left(x^4+x+3\right)^4}dx+\frac{3x^4}{2\left(x^4+x+3\right)^3}-\frac{63x}{22\left(x^4+x+3\right)^3}-\frac{1}{2\left(x^4+x+3\right)^2}+\frac{7}{2\left(x^4+x+3\right)^3}-\frac{10x^6}{\left(x^4+x+3\right)^3}-\frac{5x^3}{\left(x^4+x+3\right)^3}+\frac{5x^2}{\left(x^4+x+3\right)^2}-\frac{12x^2}{\left(x^4+x+3\right)^3}$$

input
```
Int[(-3 + 10*x + 4*x^3 - 30*x^5)/(3 + x + x^4)^3 - (3*(1 + 4*x^3)*(2 - 3*x
 + 5*x^2 + x^4 - 5*x^6))/(3 + x + x^4)^4,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.04

method	result
norman	$\frac{-5x^{6}+x^{4}+5x^{2}-3x+2}{\left(x^{4}+x+3\right)^{3}}$
parallelrisch	$\frac{-5x^{6}+x^{4}+5x^{2}-3x+2}{\left(x^{4}+x+3\right)^{3}}$
gosper	$-\frac{5x^{6}-x^{4}-5x^{2}+3x-2}{\left(x^{4}+x+3\right)^{3}}$
risch	$-\frac{5x^{6}-x^{4}-5x^{2}+3x-2}{\left(x^{4}+x+3\right)^{3}}$
default	$-\frac{-\frac{34568}{195075}x^{7}+\frac{73672}{195075}x^{6}+\frac{15392}{195075}x^{5}-\frac{60494}{195075}x^{4}-\frac{68792}{195075}x^{3}-\frac{583927}{195075}x^{2}+\frac{3356}{13005}x-\frac{2069}{43350}}{\left(x^{4}+x+3\right)^{2}}+\frac{-\frac{34568}{195075}x^{11}+\frac{73672}{195075}x^{10}+\frac{15392}{195075}\cdots}{\cdots}$
orering	$-\frac{\left(5x^{6}-x^{4}-5x^{2}+3x-2\right)\left(x^{4}+x+3\right)\left(\frac{-30x^{5}+4x^{3}+10x-3}{\left(x^{4}+x+3\right)^{3}}-\frac{3\left(4x^{3}+1\right)\left(-5x^{6}+x^{4}+5x^{2}-3x+2\right)}{\left(x^{4}+x+3\right)^{4}}\right)}{30x^{9}-8x^{7}-15x^{6}-140x^{5}+34x^{4}-12x^{3}-5x^{2}+36x-15}$

input
```
int((-30*x^5+4*x^3+10*x-3)/(x^4+x+3)^3-3*(4*x^3+1)*(-5*x^6+x^4+5*x^2-3*x+2
)/(x^4+x+3)^4,x,method=_RETURNVERBOSE)
```

output
```
(-5*x^6+x^4+5*x^2-3*x+2)/(x^4+x+3)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 65 vs. 2(30) = 60.

Time = 0.06 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.41

$$\int \left(\frac{-3+10x+4x^3-30x^5}{(3+x+x^4)^3}-\frac{3(1+4x^3)\left(2-3x+5x^2+x^4-5x^6\right)}{(3+x+x^4)^4}\right)\,dx$$
$$=-\frac{5\,x^{6}-x^{4}-5\,x^{2}+3\,x-2}{x^{12}+3\,x^{9}+9\,x^{8}+3\,x^{6}+18\,x^{5}+27\,x^{4}+x^{3}+9\,x^{2}+27\,x+27}$$

input
```
integrate((-30*x^5+4*x^3+10*x-3)/(x^4+x+3)^3-3*(4*x^3+1)*(-5*x^6+x^4+5*x^2
-3*x+2)/(x^4+x+3)^4,x, algorithm="fricas")
```

output
```
-(5*x^6 - x^4 - 5*x^2 + 3*x - 2)/(x^12 + 3*x^9 + 9*x^8 + 3*x^6 + 18*x^5 +
27*x^4 + x^3 + 9*x^2 + 27*x + 27)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(26) = 52$.

Time = 0.11 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.22

$$\int \left(\frac{-3+10x+4x^3-30x^5}{(3+x+x^4)^3}-\frac{3(1+4x^3)\,(2-3x+5x^2+x^4-5x^6)}{(3+x+x^4)^4}\right)dx = \frac{-5x^6+x^4+5x^2-3x+2}{x^{12}+3x^9+9x^8+3x^6+18x^5+27x^4+x^3+9x^2+27x+27}$$

input
```
integrate((-30*x**5+4*x**3+10*x-3)/(x**4+x+3)**3-3*(4*x**3+1)*(-5*x**6+x**
4+5*x**2-3*x+2)/(x**4+x+3)**4,x)
```

output
```
(-5*x**6 + x**4 + 5*x**2 - 3*x + 2)/(x**12 + 3*x**9 + 9*x**8 + 3*x**6 + 18
*x**5 + 27*x**4 + x**3 + 9*x**2 + 27*x + 27)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(30) = 60$.

Time = 0.04 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.41

$$\int \left(\frac{-3+10x+4x^3-30x^5}{(3+x+x^4)^3}-\frac{3(1+4x^3)\,(2-3x+5x^2+x^4-5x^6)}{(3+x+x^4)^4}\right)dx = -\frac{5\,x^6-x^4-5\,x^2+3\,x-2}{x^{12}+3\,x^9+9\,x^8+3\,x^6+18\,x^5+27\,x^4+x^3+9\,x^2+27\,x+27}$$

input
```
integrate((-30*x^5+4*x^3+10*x-3)/(x^4+x+3)^3-3*(4*x^3+1)*(-5*x^6+x^4+5*x^2
-3*x+2)/(x^4+x+3)^4,x, algorithm="maxima")
```

output
```
-(5*x^6 - x^4 - 5*x^2 + 3*x - 2)/(x^12 + 3*x^9 + 9*x^8 + 3*x^6 + 18*x^5 +
27*x^4 + x^3 + 9*x^2 + 27*x + 27)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 111 vs. $2(30) = 60$.

Time = 0.12 (sec) , antiderivative size = 111, normalized size of antiderivative = 4.11

$$\int \left(\frac{-3+10x+4x^3-30x^5}{(3+x+x^4)^3}-\frac{3(1+4x^3)\left(2-3x+5x^2+x^4-5x^6\right)}{(3+x+x^4)^4}\right)dx$$
$$=\frac{69136\,x^7-147344\,x^6-30784\,x^5+120988\,x^4+137584\,x^3+1167854\,x^2-100680\,x+18621}{390150\,(x^4+x+3)^2}$$
$$-\frac{69136\,x^{11}-147344\,x^{10}-30784\,x^9+190124\,x^8+197648\,x^7+2645788\,x^6-72044\,x^5+129019\,x^4+}{390150\,(x^4+x+3)^3}$$

input
```
integrate((-30*x^5+4*x^3+10*x-3)/(x^4+x+3)^3-3*(4*x^3+1)*(-5*x^6+x^4+5*x^2
-3*x+2)/(x^4+x+3)^4,x, algorithm="giac")
```

output
```
1/390150*(69136*x^7 - 147344*x^6 - 30784*x^5 + 120988*x^4 + 137584*x^3 + 1
167854*x^2 - 100680*x + 18621)/(x^4 + x + 3)^2 - 1/390150*(69136*x^11 - 14
7344*x^10 - 30784*x^9 + 190124*x^8 + 197648*x^7 + 2645788*x^6 - 72044*x^5
+ 129019*x^4 + 1580606*x^3 + 1452132*x^2 + 887031*x - 724437)/(x^4 + x + 3
)^3
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \left(\frac{-3+10x+4x^3-30x^5}{(3+x+x^4)^3}-\frac{3(1+4x^3)\left(2-3x+5x^2+x^4-5x^6\right)}{(3+x+x^4)^4}\right)dx$$
$$=\frac{-5\,x^6+x^4+5\,x^2-3\,x+2}{(x^4+x+3)^3}$$

input
```
int((10*x + 4*x^3 - 30*x^5 - 3)/(x + x^4 + 3)^3 - (3*(4*x^3 + 1)*(5*x^2 -
3*x + x^4 - 5*x^6 + 2))/(x + x^4 + 3)^4,x)
```

output
```
(5*x^2 - 3*x + x^4 - 5*x^6 + 2)/(x + x^4 + 3)^3
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 62, normalized size of antiderivative = 2.30

$$\int \left(\frac{-3+10x+4x^3-30x^5}{(3+x+x^4)^3}-\frac{3(1+4x^3)\,(2-3x+5x^2+x^4-5x^6)}{(3+x+x^4)^4}\right)dx$$
$$=\frac{-5x^6+x^4+5x^2-3x+2}{x^{12}+3x^9+9x^8+3x^6+18x^5+27x^4+x^3+9x^2+27x+27}$$

input
```
int((-30*x^5+4*x^3+10*x-3)/(x^4+x+3)^3-3*(4*x^3+1)*(-5*x^6+x^4+5*x^2-3*x+2
)/(x^4+x+3)^4,x)
```

output
```
( - 5*x**6 + x**4 + 5*x**2 - 3*x + 2)/(x**12 + 3*x**9 + 9*x**8 + 3*x**6 +
18*x**5 + 27*x**4 + x**3 + 9*x**2 + 27*x + 27)
```

3.66 $\int \frac{-1+4x^5}{(1+x+x^5)^2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 11

$$\int \frac{-1+4x^5}{(1+x+x^5)^2}\,dx=-\frac{x}{1+x+x^5}$$

output
```
-x/(x^5+x+1)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{-1+4x^5}{(1+x+x^5)^2}\,dx=-\frac{x}{1+x+x^5}$$

input
```
Integrate[(-1 + 4*x^5)/(1 + x + x^5)^2,x]
```

output
```
-(x/(1 + x + x^5))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {2021}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{4x^5-1}{(x^5+x+1)^2}\,dx$$

$$\Big\downarrow \; 2021$$

$$-\frac{x}{x^5+x+1}$$

input
```
Int[(-1 + 4*x^5)/(1 + x + x^5)^2,x]
```

output
```
-(x/(1 + x + x^5))
```

Defintions of rubi rules used

rule 2021
```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

method	result	size
gosper	$-\frac{x}{x^5+x+1}$	12
norman	$-\frac{x}{x^5+x+1}$	12
risch	$-\frac{x}{x^5+x+1}$	12
parallelrisch	$-\frac{x}{x^5+x+1}$	12
orering	$-\frac{(x^2+x+1)(x^3-x^2+1)x}{(x^5+x+1)^2}$	28
default	$\frac{-3x-1}{7x^2+7x+7}-\frac{-3x^2+5x-1}{7(x^3-x^2+1)}$	41

input
```
int((4*x^5-1)/(x^5+x+1)^2,x,method=_RETURNVERBOSE)
```

output
```
-x/(x^5+x+1)
```

Fricas [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{-1+4x^5}{(1+x+x^5)^2}\,dx=-\frac{x}{x^5+x+1}$$

input
```
integrate((4*x^5-1)/(x^5+x+1)^2,x, algorithm="fricas")
```

output
```
-x/(x^5 + x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int \frac{-1+4x^5}{(1+x+x^5)^2}\,dx=-\frac{x}{x^5+x+1}$$

input
```
integrate((4*x**5-1)/(x**5+x+1)**2,x)
```

output
```
-x/(x**5 + x + 1)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{-1+4x^5}{(1+x+x^5)^2}\,dx=-\frac{x}{x^5+x+1}$$

input
```
integrate((4*x^5-1)/(x^5+x+1)^2,x, algorithm="maxima")
```

output
```
-x/(x^5 + x + 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{-1+4x^5}{(1+x+x^5)^2}\,dx=-\frac{x}{x^5+x+1}$$

input
```
integrate((4*x^5-1)/(x^5+x+1)^2,x, algorithm="giac")
```

output
```
-x/(x^5 + x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{-1+4x^5}{(1+x+x^5)^2}\,dx=-\frac{x}{x^5+x+1}$$

input
```
int((4*x^5 - 1)/(x + x^5 + 1)^2,x)
```

output
```
-x/(x + x^5 + 1)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.27

$$\int \frac{-1+4x^5}{(1+x+x^5)^2}\,dx = \frac{x^5+1}{x^5+x+1}$$

input
```
int((4*x^5-1)/(x^5+x+1)^2,x)
```

output
```
(x**5 + 1)/(x**5 + x + 1)
```

3.67 $\int \frac{1+x^3+x^6}{x+x^5}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 91

$$\int \frac{1+x^3+x^6}{x+x^5}\,dx=\frac{x^2}{2}-\frac{\arctan(x^2)}{2}-\frac{\arctan\left(1-\sqrt{2}x\right)}{2\sqrt{2}}+\frac{\arctan\left(1+\sqrt{2}x\right)}{2\sqrt{2}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{2}x}{1+x^2}\right)}{2\sqrt{2}}+\log(x)-\frac{1}{4}\log\left(1+x^4\right)$$

output
```
1/2*x^2-1/2*arctan(x^2)+1/4*arctan(-1+x*2^(1/2))*2^(1/2)+1/4*arctan(1+x*2^
(1/2))*2^(1/2)-1/4*arctanh(2^(1/2)*x/(x^2+1))*2^(1/2)+ln(x)-1/4*ln(x^4+1)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.11

$$\int \frac{1+x^3+x^6}{x+x^5}\,dx=\frac{1}{8}\left(4x^2-2\left(-2+\sqrt{2}\right)\arctan\left(1-\sqrt{2}x\right)+2\left(2+\sqrt{2}\right)\arctan\left(1+\sqrt{2}x\right)+8\log(x)+\sqrt{2}\log\left(1-\sqrt{2}x+x^2\right)-\sqrt{2}\log\left(1+\sqrt{2}x+x^2\right)-2\log\left(1+x^4\right)\right)$$

input
```
Integrate[(1 + x^3 + x^6)/(x + x^5),x]
```

output
```
(4*x^2 - 2*(-2 + Sqrt[2])*ArcTan[1 - Sqrt[2]*x] + 2*(2 + Sqrt[2])*ArcTan[1
 + Sqrt[2]*x] + 8*Log[x] + Sqrt[2]*Log[1 - Sqrt[2]*x + x^2] - Sqrt[2]*Log[
1 + Sqrt[2]*x + x^2] - 2*Log[1 + x^4])/8
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.23, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {2026, 2372, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6+x^3+1}{x^5+x}\,dx$$

$$\Big\downarrow \text{ 2026}$$

$$\int \frac{x^6+x^3+1}{x(x^4+1)}dx$$

$$\Big\downarrow \text{ 2372}$$

$$\int \left(\frac{x^6+1}{(x^4+1)x}+\frac{x^2}{x^4+1}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{\arctan(x^2)}{2}-\frac{\arctan(1-\sqrt{2}x)}{2\sqrt{2}}+\frac{\arctan(\sqrt{2}x+1)}{2\sqrt{2}}-\frac{1}{4}\log(x^4+1)+\frac{x^2}{2}+\frac{\log(x^2-\sqrt{2}x+1)}{4\sqrt{2}}-\frac{\log(x^2+\sqrt{2}x+1)}{4\sqrt{2}}+\log(x)$$

input
```
Int[(1 + x^3 + x^6)/(x + x^5),x]
```

output
```
x^2/2 - ArcTan[x^2]/2 - ArcTan[1 - Sqrt[2]*x]/(2*Sqrt[2]) + ArcTan[1 + Sqr
t[2]*x]/(2*Sqrt[2]) + Log[x] + Log[1 - Sqrt[2]*x + x^2]/(4*Sqrt[2]) - Log[
1 + Sqrt[2]*x + x^2]/(4*Sqrt[2]) - Log[1 + x^4]/4
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2026
```
Int[(Fx_.)*(Px_)^(p_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Int[x^(p
*r)*ExpandToSum[Px/x^r, x]^p*Fx, x] /; IGtQ[r, 0]] /; PolyQ[Px, x] && Integ
erQ[p] &&  !MonomialQ[Px, x] && (ILtQ[p, 0] ||  !PolyQ[u, x])
```

rule 2372
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.18 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.59

method	result
risch	$\frac{x^2}{2}+\ln(x)+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^4+4\_Z^3+8\_Z^2+4\_Z+1\right)}\_R\ln\left(-\_R^3-5\_R^2-10\_R+3x-5\right)\right)}{4}$
default	$\frac{x^2}{2}-\frac{\arctan(x^2)}{2}+\frac{\sqrt{2}\left(\ln\left(\frac{x^2-x\sqrt{2}+1}{x^2+x\sqrt{2}+1}\right)+2\arctan\left(1+x\sqrt{2}\right)+2\arctan\left(-1+x\sqrt{2}\right)\right)}{8}-\frac{\ln(x^4+1)}{4}+\ln(x)$
meijerg	$\frac{x^2}{2}-\frac{\arctan(x^2)}{2}+\frac{x^3\sqrt{2}\ln\left(1-\sqrt{2}\,(x^4)^{\frac{1}{4}}+\sqrt{x^4}\right)}{8(x^4)^{\frac{3}{4}}}+\frac{x^3\sqrt{2}\arctan\left(\frac{\sqrt{2}\,(x^4)^{\frac{1}{4}}}{2-\sqrt{2}\,(x^4)^{\frac{1}{4}}}\right)}{4(x^4)^{\frac{3}{4}}}-\frac{x^3\sqrt{2}\ln\left(1+\sqrt{2}\,(x^4)^{\frac{1}{4}}+\sqrt{x^4}\right)}{8(x^4)^{\frac{3}{4}}}+$

input
```
int((x^6+x^3+1)/(x^5+x),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2+ln(x)+1/4*sum(_R*ln(-_R^3-5*_R^2-10*_R+3*x-5),_R=RootOf(_Z^4+4*_Z^
3+8*_Z^2+4*_Z+1))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.82

$$\int \frac{1+x^3+x^6}{x+x^5}\,dx = \frac{1}{2}x^2+\frac{1}{4}\left(\sqrt{2}+2\right)\arctan\left(\sqrt{2}x+1\right) +\frac{1}{4}\left(\sqrt{2}-2\right)\arctan\left(\sqrt{2}x-1\right)-\frac{1}{8}\left(\sqrt{2}+2\right)\log\left(x^2+\sqrt{2}x+1\right) +\frac{1}{8}\left(\sqrt{2}-2\right)\log\left(x^2-\sqrt{2}x+1\right)+\log(x)$$

input
```
integrate((x^6+x^3+1)/(x^5+x),x, algorithm="fricas")
```

output
```
1/2*x^2 + 1/4*(sqrt(2) + 2)*arctan(sqrt(2)*x + 1) + 1/4*(sqrt(2) - 2)*arct
an(sqrt(2)*x - 1) - 1/8*(sqrt(2) + 2)*log(x^2 + sqrt(2)*x + 1) + 1/8*(sqrt
(2) - 2)*log(x^2 - sqrt(2)*x + 1) + log(x)
```

Sympy [A] (verification not implemented)

Time = 0.67 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.67

$$\int \frac{1+x^3+x^6}{x+x^5}\,dx = \frac{x^2}{2}+\log(x) +\text{RootSum}\left(256t^4+256t^3+128t^2+16t+1,\left(t\mapsto t\log\left(\frac{1792t^4}{73}+\frac{704t^3}{219}-\frac{3152t^2}{219}-\frac{2584t}{219}+x-\frac{34}{21}\right.\right.\right.$$

input
```
integrate((x**6+x**3+1)/(x**5+x),x)
```

output
```
x**2/2 + log(x) + RootSum(256*_t**4 + 256*_t**3 + 128*_t**2 + 16*_t + 1, L
ambda(_t, _t*log(1792*_t**4/73 + 704*_t**3/219 - 3152*_t**2/219 - 2584*_t/
219 + x - 344/219)))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.09

$$\int \frac{1+x^3+x^6}{x+x^5}\,dx = \frac{1}{4}\sqrt{2}\left(\sqrt{2}+1\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right) - \frac{1}{4}\sqrt{2}\left(\sqrt{2}-1\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right) - \frac{1}{8}\sqrt{2}\left(\sqrt{2}+1\right)\log\left(x^2+\sqrt{2}x+1\right) - \frac{1}{8}\sqrt{2}\left(\sqrt{2}-1\right)\log\left(x^2-\sqrt{2}x+1\right) + \frac{1}{2}x^2 + \log\left(x\right)$$

input
```
integrate((x^6+x^3+1)/(x^5+x),x, algorithm="maxima")
```

output
```
1/4*sqrt(2)*(sqrt(2) + 1)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) - 1/4*sqrt(2
)*(sqrt(2) - 1)*arctan(1/2*sqrt(2)*(2*x - sqrt(2))) - 1/8*sqrt(2)*(sqrt(2)
 + 1)*log(x^2 + sqrt(2)*x + 1) - 1/8*sqrt(2)*(sqrt(2) - 1)*log(x^2 - sqrt(
2)*x + 1) + 1/2*x^2 + log(x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.01

$$\int \frac{1+x^3+x^6}{x+x^5}\,dx = \frac{1}{2}x^2 + \frac{1}{4}\left(\sqrt{2}+2\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right) + \frac{1}{4}\left(\sqrt{2}-2\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right) - \frac{1}{8}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right) + \frac{1}{8}\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right) - \frac{1}{4}\log\left(x^4+1\right) + \log\left(|x|\right)$$

input
```
integrate((x^6+x^3+1)/(x^5+x),x, algorithm="giac")
```

output
```
1/2*x^2 + 1/4*(sqrt(2) + 2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/4*(sqr
t(2) - 2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2))) - 1/8*sqrt(2)*log(x^2 + sqrt
(2)*x + 1) + 1/8*sqrt(2)*log(x^2 - sqrt(2)*x + 1) - 1/4*log(x^4 + 1) + log
(abs(x))
```

Mupad [B] (verification not implemented)

Time = 22.49 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.87

$$\int \frac{1+x^3+x^6}{x+x^5}\,dx = \ln(x) + \left(\sum_{k=1}^{4} \ln\left(\mathrm{root}\left(z^4+z^3+\frac{z^2}{2}+\frac{z}{16}+\frac{1}{256},z,k\right)\left(8\,\mathrm{root}\left(z^4+z^3+\frac{z^2}{2}+\frac{z}{16}+\frac{1}{256},z,k\right)+x+\mathrm{root}\left(z^4+z^3 + z^3+\frac{z^2}{2}+\frac{z}{16}+\frac{1}{256},z,k\right)\right) + \frac{x^2}{2}$$

input
```
int((x^3 + x^6 + 1)/(x + x^5),x)
```

output
```
log(x) + symsum(log(root(z^4 + z^3 + z^2/2 + z/16 + 1/256, z, k)*(8*root(z
^4 + z^3 + z^2/2 + z/16 + 1/256, z, k) + x + 96*root(z^4 + z^3 + z^2/2 + z
/16 + 1/256, z, k)*x + 240*root(z^4 + z^3 + z^2/2 + z/16 + 1/256, z, k)^2*
x + 320*root(z^4 + z^3 + z^2/2 + z/16 + 1/256, z, k)^3*x - 16*root(z^4 + z
^3 + z^2/2 + z/16 + 1/256, z, k)^2 + 8))*root(z^4 + z^3 + z^2/2 + z/16 + 1
/256, z, k), k, 1, 4) + x^2/2
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.34

$$\int \frac{1+x^3+x^6}{x+x^5}\,dx = -\frac{\sqrt{2}\,atan\left(\frac{\sqrt{2}-2x}{\sqrt{2}}\right)}{4}+\frac{atan\left(\frac{\sqrt{2}-2x}{\sqrt{2}}\right)}{2}+\frac{\sqrt{2}\,atan\left(\frac{\sqrt{2}+2x}{\sqrt{2}}\right)}{4}+\frac{atan\left(\frac{\sqrt{2}+2x}{\sqrt{2}}\right)}{2}+\frac{\sqrt{2}\log\left(-\sqrt{2}\,x+x^2+1\right)}{8}-\frac{\sqrt{2}\log\left(\sqrt{2}\,x+x^2+1\right)}{8}-\frac{\log\left(-\sqrt{2}\,x+x^2+1\right)}{4}-\frac{\log\left(\sqrt{2}\,x+x^2+1\right)}{4}+\log(x)+\frac{x^2}{2}$$

input
```
int((x^6+x^3+1)/(x^5+x),x)
```

output
```
( - 2*sqrt(2)*atan((sqrt(2) - 2*x)/sqrt(2)) + 4*atan((sqrt(2) - 2*x)/sqrt(
2)) + 2*sqrt(2)*atan((sqrt(2) + 2*x)/sqrt(2)) + 4*atan((sqrt(2) + 2*x)/sqr
t(2)) + sqrt(2)*log( - sqrt(2)*x + x**2 + 1) - sqrt(2)*log(sqrt(2)*x + x**
2 + 1) - 2*log( - sqrt(2)*x + x**2 + 1) - 2*log(sqrt(2)*x + x**2 + 1) + 8*
log(x) + 4*x**2)/8
```

3.68 $\int \frac{1}{\left(-1+7x^2-7x^4+x^6\right)^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 124

$$\int \frac{1}{\left(-1+7x^2-7x^4+x^6\right)^2}\,dx=\frac{21x}{128\left(1-x^2\right)}+\frac{x\left(41-7x^2\right)}{64\left(1-x^2\right)\left(1-6x^2+x^4\right)}+\frac{5\text{arctanh}(x)}{32}+\frac{\text{arctanh}\left(\sqrt{3-2\sqrt{2}}x\right)}{256\sqrt{2\left(17+12\sqrt{2}\right)}}+\frac{\text{arctanh}\left(\sqrt{3+2\sqrt{2}}x\right)}{256\sqrt{2\left(17-12\sqrt{2}\right)}}$$

output
```
21*x/(-128*x^2+128)+1/64*x*(-7*x^2+41)/(-x^2+1)/(x^4-6*x^2+1)+5/32*arctanh
(x)+1/256*arctanh((2^(1/2)-1)*x)/(3*2^(1/2)+4)+1/256*arctanh((1+2^(1/2))*x
)/(3*2^(1/2)-4)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.06

$$\int \frac{1}{\left(-1+7x^2-7x^4+x^6\right)^2}\,dx=\frac{-\frac{8x\left(103-140x^2+21x^4\right)}{-1+7x^2-7x^4+x^6}-80\log(1-x)-\left(4+3\sqrt{2}\right)\log\left(-1+\sqrt{2}-x\right)+\left(4-3\sqrt{2}\right)\log\left(1+\sqrt{2}-x\right)+8}{1024}$$

input
```
Integrate[(-1 + 7*x^2 - 7*x^4 + x^6)^(-2),x]
```

output
```
((-8*x*(103 - 140*x^2 + 21*x^4))/(-1 + 7*x^2 - 7*x^4 + x^6) - 80*Log[1 - x
] - (4 + 3*Sqrt[2])*Log[-1 + Sqrt[2] - x] + (4 - 3*Sqrt[2])*Log[1 + Sqrt[2
] - x] + 80*Log[1 + x] + (4 + 3*Sqrt[2])*Log[-1 + Sqrt[2] + x] + (-4 + 3*S
qrt[2])*Log[1 + Sqrt[2] + x])/1024
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.98, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2460, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(x^6-7x^4+7x^2-1\right)^2}dx$$

$$\Big\downarrow \text{ 2460}$$

$$\int \left(\frac{29-12x}{64\left(x^2-2x-1\right)^2}-\frac{5}{32\left(x^2-1\right)}+\frac{x+6}{128\left(x^2-2x-1\right)}+\frac{6-x}{128\left(x^2+2x-1\right)}+\frac{12x+29}{64\left(x^2+2x-1\right)^2}+\frac{1}{64(x-1)}\right.$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{5\text{arctanh}(x)}{32}-\frac{41-17x}{256\left(-x^2+2x+1\right)}+\frac{17x+41}{256\left(-x^2-2x+1\right)}+\frac{1}{64(1-x)}-\frac{1}{64(x+1)}+\frac{1}{512}\left(2-7\sqrt{2}\right)\log\left(-x-\sqrt{2}+1\right)+\frac{17\log\left(-x-\sqrt{2}+1\right)}{512\sqrt{2}}+\frac{1}{512}\left(2+7\sqrt{2}\right)\log\left(-x+\sqrt{2}+1\right)-\frac{17\log\left(-x+\sqrt{2}+1\right)}{512\sqrt{2}}-\frac{1}{512}\left(2-7\sqrt{2}\right)\log\left(x-\sqrt{2}+1\right)-\frac{17\log\left(x-\sqrt{2}+1\right)}{512\sqrt{2}}-\frac{1}{512}\left(2+7\sqrt{2}\right)\log\left(x+\sqrt{2}+1\right)+\frac{17\log\left(x+\sqrt{2}+1\right)}{512\sqrt{2}}$$

input
```
Int[(-1 + 7*x^2 - 7*x^4 + x^6)^(-2),x]
```

output
```
1/(64*(1 - x)) - 1/(64*(1 + x)) + (41 + 17*x)/(256*(1 - 2*x - x^2)) - (41
- 17*x)/(256*(1 + 2*x - x^2)) + (5*ArcTanh[x])/32 + (17*Log[1 - Sqrt[2] -
x])/(512*Sqrt[2]) + ((2 - 7*Sqrt[2])*Log[1 - Sqrt[2] - x])/512 - (17*Log[1
 + Sqrt[2] - x])/(512*Sqrt[2]) + ((2 + 7*Sqrt[2])*Log[1 + Sqrt[2] - x])/51
2 - (17*Log[1 - Sqrt[2] + x])/(512*Sqrt[2]) - ((2 - 7*Sqrt[2])*Log[1 - Sqr
t[2] + x])/512 + (17*Log[1 + Sqrt[2] + x])/(512*Sqrt[2]) - ((2 + 7*Sqrt[2]
)*Log[1 + Sqrt[2] + x])/512
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2460
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px /. x -> Sqrt[x]]},
Int[ExpandIntegrand[u*(Qx /. x -> x^2)^p, x], x] /;  !SumQ[NonfreeFactors[Q
x, x]]] /; PolyQ[Px, x^2] && GtQ[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&
 !TrinomialQ[Px, x] && ILtQ[p, 0] && RationalFunctionQ[u, x]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.94

method	result
default	$-\frac{1}{64(x-1)}-\frac{5\ln(x-1)}{64}-\frac{\frac{17x}{2}+\frac{41}{2}}{128(x^2+2x-1)}-\frac{\ln(x^2+2x-1)}{256}+\frac{3\sqrt{2}\,\operatorname{arctanh}\left(\frac{(2x+2)\sqrt{2}}{4}\right)}{512}-\frac{1}{64(x+1)}+\frac{5\ln(x+1)}{64}+\frac{}{128}$
risch	$\frac{-\frac{21}{128}x^5+\frac{35}{32}x^3-\frac{103}{128}x}{x^6-7x^4+7x^2-1}+\frac{\ln\left(x-1+\sqrt{2}\right)}{256}+\frac{3\ln\left(x-1+\sqrt{2}\right)\sqrt{2}}{1024}+\frac{\ln\left(x-1-\sqrt{2}\right)}{256}-\frac{3\ln\left(x-1-\sqrt{2}\right)\sqrt{2}}{1024}+\frac{3\ln\left(1+\sqrt{2}+x\right)\sqrt{2}}{1024}$

input
```
int(1/(x^6-7*x^4+7*x^2-1)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/64/(x-1)-5/64*ln(x-1)-1/128*(17/2*x+41/2)/(x^2+2*x-1)-1/256*ln(x^2+2*x-
1)+3/512*2^(1/2)*arctanh(1/4*(2*x+2)*2^(1/2))-1/64/(x+1)+5/64*ln(x+1)+1/12
8*(-17/2*x+41/2)/(x^2-2*x-1)+1/256*ln(x^2-2*x-1)+3/512*2^(1/2)*arctanh(1/4
*(2*x-2)*2^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 223 vs. $2(82) = 164$.

Time = 0.07 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.80

$$\int \frac{1}{(-1+7x^2-7x^4+x^6)^2}\,dx = -\frac{168\,x^5 - 1120\,x^3 - 3\,\sqrt{2}(x^6 - 7\,x^4 + 7\,x^2 - 1)\log\left(\frac{x^2+2\,\sqrt{2}(x+1)+2\,x+3}{x^2+2\,x-1}\right) - 3\,\sqrt{2}(x^6 - 7\,x^4 + 7\,x^2 - 1)\,\mathrm{lo}}{}$$

input
```
integrate(1/(x^6-7*x^4+7*x^2-1)^2,x, algorithm="fricas")
```

output
```
-1/1024*(168*x^5 - 1120*x^3 - 3*sqrt(2)*(x^6 - 7*x^4 + 7*x^2 - 1)*log((x^2
 + 2*sqrt(2)*(x + 1) + 2*x + 3)/(x^2 + 2*x - 1)) - 3*sqrt(2)*(x^6 - 7*x^4
+ 7*x^2 - 1)*log((x^2 + 2*sqrt(2)*(x - 1) - 2*x + 3)/(x^2 - 2*x - 1)) + 4*
(x^6 - 7*x^4 + 7*x^2 - 1)*log(x^2 + 2*x - 1) - 4*(x^6 - 7*x^4 + 7*x^2 - 1)
*log(x^2 - 2*x - 1) - 80*(x^6 - 7*x^4 + 7*x^2 - 1)*log(x + 1) + 80*(x^6 -
7*x^4 + 7*x^2 - 1)*log(x - 1) + 824*x)/(x^6 - 7*x^4 + 7*x^2 - 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 296 vs. $2(78) = 156$.

Time = 0.90 (sec) , antiderivative size = 296, normalized size of antiderivative = 2.39

$$\int \frac{1}{(-1+7x^2-7x^4+x^6)^2}\,dx$$
$$= \frac{-21x^5+140x^3-103x}{128x^6-896x^4+896x^2-128} - \frac{5\log(x-1)}{64} + \frac{5\log(x+1)}{64} + \left(-\frac{1}{256} + \frac{3\sqrt{2}}{1024}\right)\log\left(x - \frac{8071264001}{202624020} - \frac{471550901878784\left(-\frac{1}{256}+\frac{3\sqrt{2}}{1024}\right)^3}{2979765} + \frac{1299552375287054336\left(-\frac{1}{256}+\right.}{50656005}\right.$$
$$+ \left(-\frac{3\sqrt{2}}{1024} - \frac{1}{256}\right)\log\left(x - \frac{8071264001\sqrt{2}}{270165360} - \frac{8071264001}{202624020} + \frac{1299552375287054336\left(-\frac{3\sqrt{2}}{1024}-\frac{1}{256}\right)^5}{50656005} - \frac{47155090}{}\right.$$
$$+ \left(\frac{1}{256} - \frac{3\sqrt{2}}{1024}\right)\log\left(x - \frac{8071264001\sqrt{2}}{270165360} + \frac{1299552375287054336\left(\frac{1}{256}-\frac{3\sqrt{2}}{1024}\right)^5}{50656005} - \frac{471550901878784\left(\frac{1}{256}-\right.}{2979765}\right.$$
$$+ \left(\frac{1}{256} + \frac{3\sqrt{2}}{1024}\right)\log\left(x - \frac{471550901878784\left(\frac{1}{256}+\frac{3\sqrt{2}}{1024}\right)^3}{2979765} + \frac{1299552375287054336\left(\frac{1}{256}+\frac{3\sqrt{2}}{1024}\right)^5}{50656005} + \frac{80712640}{2026240}\right.$$

input
```
integrate(1/(x**6-7*x**4+7*x**2-1)**2,x)
```

output
```
(-21*x**5 + 140*x**3 - 103*x)/(128*x**6 - 896*x**4 + 896*x**2 - 128) - 5*l
og(x - 1)/64 + 5*log(x + 1)/64 + (-1/256 + 3*sqrt(2)/1024)*log(x - 8071264
001/202624020 - 471550901878784*(-1/256 + 3*sqrt(2)/1024)**3/2979765 + 129
9552375287054336*(-1/256 + 3*sqrt(2)/1024)**5/50656005 + 8071264001*sqrt(2
)/270165360) + (-3*sqrt(2)/1024 - 1/256)*log(x - 8071264001*sqrt(2)/270165
360 - 8071264001/202624020 + 1299552375287054336*(-3*sqrt(2)/1024 - 1/256)
**5/50656005 - 471550901878784*(-3*sqrt(2)/1024 - 1/256)**3/2979765) + (1/
256 - 3*sqrt(2)/1024)*log(x - 8071264001*sqrt(2)/270165360 + 1299552375287
054336*(1/256 - 3*sqrt(2)/1024)**5/50656005 - 471550901878784*(1/256 - 3*s
qrt(2)/1024)**3/2979765 + 8071264001/202624020) + (1/256 + 3*sqrt(2)/1024)
*log(x - 471550901878784*(1/256 + 3*sqrt(2)/1024)**3/2979765 + 12995523752
87054336*(1/256 + 3*sqrt(2)/1024)**5/50656005 + 8071264001/202624020 + 807
1264001*sqrt(2)/270165360)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.92

$$\int \frac{1}{(-1+7x^2-7x^4+x^6)^2}\,dx = -\frac{3}{1024}\sqrt{2}\log\left(\frac{x-\sqrt{2}+1}{x+\sqrt{2}+1}\right) - \frac{3}{1024}\sqrt{2}\log\left(\frac{x-\sqrt{2}-1}{x+\sqrt{2}-1}\right) - \frac{21\,x^5-140\,x^3+103\,x}{128\,(x^6-7\,x^4+7\,x^2-1)} - \frac{1}{256}\,\log\left(x^2+2\,x-1\right) + \frac{1}{256}\,\log\left(x^2-2\,x-1\right) + \frac{5}{64}\,\log\left(x+1\right) - \frac{5}{64}\,\log\left(x-1\right)$$

input
```
integrate(1/(x^6-7*x^4+7*x^2-1)^2,x, algorithm="maxima")
```

output
```
-3/1024*sqrt(2)*log((x - sqrt(2) + 1)/(x + sqrt(2) + 1)) - 3/1024*sqrt(2)*
log((x - sqrt(2) - 1)/(x + sqrt(2) - 1)) - 1/128*(21*x^5 - 140*x^3 + 103*x
)/(x^6 - 7*x^4 + 7*x^2 - 1) - 1/256*log(x^2 + 2*x - 1) + 1/256*log(x^2 - 2
*x - 1) + 5/64*log(x + 1) - 5/64*log(x - 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.08

$$\int \frac{1}{\left(-1+7x^2-7x^4+x^6\right)^2}\,dx = -\frac{3}{1024}\,\sqrt{2}\log\left(\frac{\left|2\,x-2\,\sqrt{2}+2\right|}{\left|2\,x+2\,\sqrt{2}+2\right|}\right) - \frac{3}{1024}\,\sqrt{2}\log\left(\frac{\left|2\,x-2\,\sqrt{2}-2\right|}{\left|2\,x+2\,\sqrt{2}-2\right|}\right) - \frac{21\,x^5-140\,x^3+103\,x}{128\,(x^6-7\,x^4+7\,x^2-1)} - \frac{1}{256}\,\log\left(\left|x^2+2\,x-1\right|\right) + \frac{1}{256}\,\log\left(\left|x^2-2\,x-1\right|\right) + \frac{5}{64}\,\log\left(\left|x+1\right|\right) - \frac{5}{64}\,\log\left(\left|x-1\right|\right)$$

input
```
integrate(1/(x^6-7*x^4+7*x^2-1)^2,x, algorithm="giac")
```

output
```
-3/1024*sqrt(2)*log(abs(2*x - 2*sqrt(2) + 2)/abs(2*x + 2*sqrt(2) + 2)) - 3
/1024*sqrt(2)*log(abs(2*x - 2*sqrt(2) - 2)/abs(2*x + 2*sqrt(2) - 2)) - 1/1
28*(21*x^5 - 140*x^3 + 103*x)/(x^6 - 7*x^4 + 7*x^2 - 1) - 1/256*log(abs(x^
2 + 2*x - 1)) + 1/256*log(abs(x^2 - 2*x - 1)) + 5/64*log(abs(x + 1)) - 5/6
4*log(abs(x - 1))
```

Mupad [B] (verification not implemented)

Time = 21.87 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.02

$$\int \frac{1}{(-1+7x^2-7x^4+x^6)^2}\,dx = -\frac{\mathrm{atan}(x\,1\mathrm{i})\,5\mathrm{i}}{32} - \frac{\frac{21\,x^5}{128}-\frac{35\,x^3}{32}+\frac{103\,x}{128}}{x^6-7\,x^4+7\,x^2-1} - \mathrm{atan}\left(\frac{x\,940311\mathrm{i}}{134217728\,\left(\frac{275445\,\sqrt{2}}{134217728}-\frac{389421}{134217728}\right)} - \frac{\sqrt{2}\,x\,332433\mathrm{i}}{67108864\,\left(\frac{275445\,\sqrt{2}}{134217728}-\frac{389421}{134217728}\right)}\right)\left(\frac{\sqrt{2}\,3\mathrm{i}}{512}-\frac{1}{128}\mathrm{i}\right) - \mathrm{atan}\left(\frac{x\,940311\mathrm{i}}{134217728\,\left(\frac{275445\,\sqrt{2}}{134217728}+\frac{389421}{134217728}\right)} + \frac{\sqrt{2}\,x\,332433\mathrm{i}}{67108864\,\left(\frac{275445\,\sqrt{2}}{134217728}+\frac{389421}{134217728}\right)}\right)\left(\frac{\sqrt{2}\,3\mathrm{i}}{512}+\frac{1}{128}\mathrm{i}\right)$$

input
```
int(1/(7*x^2 - 7*x^4 + x^6 - 1)^2,x)
```

output
```
- (atan(x*1i)*5i)/32 - ((103*x)/128 - (35*x^3)/32 + (21*x^5)/128)/(7*x^2 -
 7*x^4 + x^6 - 1) - atan((x*940311i)/(134217728*((275445*2^(1/2))/13421772
8 - 389421/134217728)) - (2^(1/2)*x*332433i)/(67108864*((275445*2^(1/2))/1
34217728 - 389421/134217728)))*((2^(1/2)*3i)/512 - 1i/128) - atan((x*94031
1i)/(134217728*((275445*2^(1/2))/134217728 + 389421/134217728)) + (2^(1/2)
*x*332433i)/(67108864*((275445*2^(1/2))/134217728 + 389421/134217728)))*((
2^(1/2)*3i)/512 + 1i/128)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 492, normalized size of antiderivative = 3.97

$$\int \frac{1}{(-1+7x^2-7x^4+x^6)^2}\,dx = \text{Too large to display}$$

input
```
int(1/(x^6-7*x^4+7*x^2-1)^2,x)
```

output
```
( - 3*sqrt(2)*log( - sqrt(2) + x - 1)*x**6 + 21*sqrt(2)*log( - sqrt(2) + x
 - 1)*x**4 - 21*sqrt(2)*log( - sqrt(2) + x - 1)*x**2 + 3*sqrt(2)*log( - sq
rt(2) + x - 1) - 3*sqrt(2)*log( - sqrt(2) + x + 1)*x**6 + 21*sqrt(2)*log(
- sqrt(2) + x + 1)*x**4 - 21*sqrt(2)*log( - sqrt(2) + x + 1)*x**2 + 3*sqrt
(2)*log( - sqrt(2) + x + 1) + 3*sqrt(2)*log(sqrt(2) + x - 1)*x**6 - 21*sqr
t(2)*log(sqrt(2) + x - 1)*x**4 + 21*sqrt(2)*log(sqrt(2) + x - 1)*x**2 - 3*
sqrt(2)*log(sqrt(2) + x - 1) + 3*sqrt(2)*log(sqrt(2) + x + 1)*x**6 - 21*sq
rt(2)*log(sqrt(2) + x + 1)*x**4 + 21*sqrt(2)*log(sqrt(2) + x + 1)*x**2 - 3
*sqrt(2)*log(sqrt(2) + x + 1) + 4*log( - sqrt(2) + x - 1)*x**6 - 28*log( -
 sqrt(2) + x - 1)*x**4 + 28*log( - sqrt(2) + x - 1)*x**2 - 4*log( - sqrt(2
) + x - 1) - 4*log( - sqrt(2) + x + 1)*x**6 + 28*log( - sqrt(2) + x + 1)*x
**4 - 28*log( - sqrt(2) + x + 1)*x**2 + 4*log( - sqrt(2) + x + 1) + 4*log(
sqrt(2) + x - 1)*x**6 - 28*log(sqrt(2) + x - 1)*x**4 + 28*log(sqrt(2) + x
- 1)*x**2 - 4*log(sqrt(2) + x - 1) - 4*log(sqrt(2) + x + 1)*x**6 + 28*log(
sqrt(2) + x + 1)*x**4 - 28*log(sqrt(2) + x + 1)*x**2 + 4*log(sqrt(2) + x +
 1) - 80*log(x - 1)*x**6 + 560*log(x - 1)*x**4 - 560*log(x - 1)*x**2 + 80*
log(x - 1) + 80*log(x + 1)*x**6 - 560*log(x + 1)*x**4 + 560*log(x + 1)*x**
2 - 80*log(x + 1) - 168*x**5 + 1120*x**3 - 824*x)/(1024*(x**6 - 7*x**4 + 7
*x**2 - 1))
```

3.69 $\int \frac{1+x^2}{(1-7x^2+7x^4-x^6)^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 111

$$\int \frac{1+x^2}{(1-7x^2+7x^4-x^6)^2}\,dx = \frac{x}{16(1-x^2)}+\frac{x(29-5x^2)}{32(1-6x^2+x^4)}+\frac{\operatorname{arctanh}(x)}{4}+\frac{\operatorname{arctanh}\left(\sqrt{3-2\sqrt{2}}x\right)}{64\sqrt{17+12\sqrt{2}}}-\frac{\operatorname{arctanh}\left(\sqrt{3+2\sqrt{2}}x\right)}{64\sqrt{17-12\sqrt{2}}}$$

output

```
x/(-16*x^2+16)+x*(-5*x^2+29)/(32*x^4-192*x^2+32)+1/4*arctanh(x)+1/64*arcta
nh((2^(1/2)-1)*x)/(3+2*2^(1/2))-1/64*arctanh((1+2^(1/2))*x)/(3-2*2^(1/2))
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.19

$$\int \frac{1+x^2}{(1-7x^2+7x^4-x^6)^2}\,dx = \frac{1}{128}\left(-\frac{4x(31-46x^2+7x^4)}{-1+7x^2-7x^4+x^6} - 16\log(1-x) + \left(3+2\sqrt{2}\right)\log\left(-1+\sqrt{2}-x\right) + \left(-3+2\sqrt{2}\right)\log\left(1+\sqrt{2}-x\right) + 16\log(1+x) - \left(3+2\sqrt{2}\right)\log\left(-1+\sqrt{2}+x\right) + \left(3-2\sqrt{2}\right)\log\left(1+\sqrt{2}+x\right)\right)$$

input
```
Integrate[(1 + x^2)/(1 - 7*x^2 + 7*x^4 - x^6)^2,x]
```

output
```
((-4*x*(31 - 46*x^2 + 7*x^4))/(-1 + 7*x^2 - 7*x^4 + x^6) - 16*Log[1 - x] +
 (3 + 2*Sqrt[2])*Log[-1 + Sqrt[2] - x] + (-3 + 2*Sqrt[2])*Log[1 + Sqrt[2]
- x] + 16*Log[1 + x] - (3 + 2*Sqrt[2])*Log[-1 + Sqrt[2] + x] + (3 - 2*Sqrt
[2])*Log[1 + Sqrt[2] + x])/128
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 245 vs. $2(111) = 222$.

Time = 0.72 (sec) , antiderivative size = 245, normalized size of antiderivative = 2.21, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2460, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2+1}{(-x^6+7x^4-7x^2+1)^2}\,dx$$

$$\Big\downarrow \; 2460$$

$$\int \left(\frac{17-7x}{32\left(x^2-2x-1\right)^2}-\frac{1}{4\left(x^2-1\right)}-\frac{3(x-4)}{64\left(x^2-2x-1\right)}+\frac{3(x+4)}{64\left(x^2+2x-1\right)}+\frac{7x+17}{32\left(x^2+2x-1\right)^2}+\frac{1}{32(x-1)^2}+\right.$$

$$\Big\downarrow \; 2009$$

$$\frac{\operatorname{arctanh}(x)}{4}-\frac{12-5x}{64\left(-x^2+2x+1\right)}+\frac{5x+12}{64\left(-x^2-2x+1\right)}+\frac{1}{32(1-x)}-\frac{1}{32(x+1)}-\frac{3}{256}\left(2+3\sqrt{2}\right)\log\left(-x-\sqrt{2}+1\right)+\frac{5\log\left(-x-\sqrt{2}+1\right)}{128\sqrt{2}}-\frac{3}{256}\left(2-3\sqrt{2}\right)\log\left(-x+\sqrt{2}+1\right)-\frac{5\log\left(-x+\sqrt{2}+1\right)}{128\sqrt{2}}+\frac{3}{256}\left(2+3\sqrt{2}\right)\log\left(x-\sqrt{2}+1\right)-\frac{5\log\left(x-\sqrt{2}+1\right)}{128\sqrt{2}}+\frac{3}{256}\left(2-3\sqrt{2}\right)\log\left(x+\sqrt{2}+1\right)+\frac{5\log\left(x+\sqrt{2}+1\right)}{128\sqrt{2}}$$

input
```
Int[(1 + x^2)/(1 - 7*x^2 + 7*x^4 - x^6)^2,x]
```

output
```
1/(32*(1 - x)) - 1/(32*(1 + x)) + (12 + 5*x)/(64*(1 - 2*x - x^2)) - (12 -
5*x)/(64*(1 + 2*x - x^2)) + ArcTanh[x]/4 + (5*Log[1 - Sqrt[2] - x])/(128*S
qrt[2]) - (3*(2 + 3*Sqrt[2])*Log[1 - Sqrt[2] - x])/256 - (5*Log[1 + Sqrt[2
] - x])/(128*Sqrt[2]) - (3*(2 - 3*Sqrt[2])*Log[1 + Sqrt[2] - x])/256 - (5*
Log[1 - Sqrt[2] + x])/(128*Sqrt[2]) + (3*(2 + 3*Sqrt[2])*Log[1 - Sqrt[2] +
 x])/256 + (5*Log[1 + Sqrt[2] + x])/(128*Sqrt[2]) + (3*(2 - 3*Sqrt[2])*Log
[1 + Sqrt[2] + x])/256
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2460
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px /. x -> Sqrt[x]]},
Int[ExpandIntegrand[u*(Qx /. x -> x^2)^p, x], x] /;  !SumQ[NonfreeFactors[Q
x, x]]] /; PolyQ[Px, x^2] && GtQ[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&
 !TrinomialQ[Px, x] && ILtQ[p, 0] && RationalFunctionQ[u, x]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.05

method	result
default	$-\frac{1}{32(x-1)}-\frac{\ln(x-1)}{8}+\frac{-5x-12}{64x^2+128x-64}+\frac{3\ln(x^2+2x-1)}{128}-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{(2x+2)\sqrt{2}}{4}\right)}{32}-\frac{1}{32(x+1)}+\frac{\ln(x+1)}{8}-\frac{5x}{64(x^2}$
risch	$\frac{-\frac{7}{32}x^5+\frac{23}{16}x^3-\frac{31}{32}x}{x^6-7x^4+7x^2-1}+\frac{3\ln\left(1-\sqrt{2}+x\right)}{128}+\frac{\ln\left(1-\sqrt{2}+x\right)\sqrt{2}}{64}+\frac{3\ln\left(1+\sqrt{2}+x\right)}{128}-\frac{\ln\left(1+\sqrt{2}+x\right)\sqrt{2}}{64}+\frac{\ln(x+1)}{8}-\frac{3\ln\left(2x\right.}{1}$

input

```
int((x^2+1)/(-x^6+7*x^4-7*x^2+1)^2,x,method=_RETURNVERBOSE)
```

output

```
-1/32/(x-1)-1/8*ln(x-1)+1/64*(-5*x-12)/(x^2+2*x-1)+3/128*ln(x^2+2*x-1)-1/3
2*2^(1/2)*arctanh(1/4*(2*x+2)*2^(1/2))-1/32/(x+1)+1/8*ln(x+1)-1/64*(5*x-12
)/(x^2-2*x-1)-3/128*ln(x^2-2*x-1)-1/32*2^(1/2)*arctanh(1/4*(2*x-2)*2^(1/2)
)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 223 vs. 2(75) = 150.

Time = 0.08 (sec) , antiderivative size = 223, normalized size of antiderivative = 2.01

$$\int \frac{1+x^2}{(1-7x^2+7x^4-x^6)^2}\,dx =$$
$$-\frac{28\,x^5-184\,x^3-2\,\sqrt{2}(x^6-7\,x^4+7\,x^2-1)\log\left(\frac{x^2-2\,\sqrt{2}(x+1)+2\,x+3}{x^2+2\,x-1}\right)-2\,\sqrt{2}(x^6-7\,x^4+7\,x^2-1)\log}{}$$

input

```
integrate((x^2+1)/(-x^6+7*x^4-7*x^2+1)^2,x, algorithm="fricas")
```

output

```
-1/128*(28*x^5 - 184*x^3 - 2*sqrt(2)*(x^6 - 7*x^4 + 7*x^2 - 1)*log((x^2 -
2*sqrt(2)*(x + 1) + 2*x + 3)/(x^2 + 2*x - 1)) - 2*sqrt(2)*(x^6 - 7*x^4 + 7
*x^2 - 1)*log((x^2 - 2*sqrt(2)*(x - 1) - 2*x + 3)/(x^2 - 2*x - 1)) - 3*(x^
6 - 7*x^4 + 7*x^2 - 1)*log(x^2 + 2*x - 1) + 3*(x^6 - 7*x^4 + 7*x^2 - 1)*lo
g(x^2 - 2*x - 1) - 16*(x^6 - 7*x^4 + 7*x^2 - 1)*log(x + 1) + 16*(x^6 - 7*x
^4 + 7*x^2 - 1)*log(x - 1) + 124*x)/(x^6 - 7*x^4 + 7*x^2 - 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 272 vs. $2(70) = 140$.

Time = 0.93 (sec) , antiderivative size = 272, normalized size of antiderivative = 2.45

$$\int \frac{1+x^2}{(1-7x^2+7x^4-x^6)^2}\,dx$$

$$= \frac{-7x^5+46x^3-31x}{32x^6-224x^4+224x^2-32} - \frac{\log(x-1)}{8} + \frac{\log(x+1)}{8} + \left(-\frac{3}{128}\right.$$

$$\left.-\frac{\sqrt{2}}{64}\right)\log\left(x - \frac{38423555}{909328} - \frac{38423555\sqrt{2}}{1363992} + \frac{9549859782656\left(-\frac{3}{128}-\frac{\sqrt{2}}{64}\right)^5}{170499} - \frac{56267374592\left(-\frac{3}{128}-\right.}{56833}\right.$$

$$+\left(-\frac{3}{128}\right.$$

$$\left.+\frac{\sqrt{2}}{64}\right)\log\left(x - \frac{38423555}{909328} + \frac{9549859782656\left(-\frac{3}{128}+\frac{\sqrt{2}}{64}\right)^5}{170499} - \frac{56267374592\left(-\frac{3}{128}+\frac{\sqrt{2}}{64}\right)^3}{56833} + \frac{3842355}{13639}\right.$$

$$+\left(\frac{3}{128}\right.$$

$$\left.-\frac{\sqrt{2}}{64}\right)\log\left(x - \frac{38423555\sqrt{2}}{1363992} - \frac{56267374592\left(\frac{3}{128}-\frac{\sqrt{2}}{64}\right)^3}{56833} + \frac{9549859782656\left(\frac{3}{128}-\frac{\sqrt{2}}{64}\right)^5}{170499} + \frac{3842355}{909328}\right.$$

$$+\left(\frac{\sqrt{2}}{64}\right.$$

$$\left.+\frac{3}{128}\right)\log\left(x - \frac{56267374592\left(\frac{\sqrt{2}}{64}+\frac{3}{128}\right)^3}{56833} + \frac{9549859782656\left(\frac{\sqrt{2}}{64}+\frac{3}{128}\right)^5}{170499} + \frac{38423555\sqrt{2}}{1363992} + \frac{3842355}{909328}\right.$$

input
```
integrate((x**2+1)/(-x**6+7*x**4-7*x**2+1)**2,x)
```

output
```
(-7*x**5 + 46*x**3 - 31*x)/(32*x**6 - 224*x**4 + 224*x**2 - 32) - log(x -
1)/8 + log(x + 1)/8 + (-3/128 - sqrt(2)/64)*log(x - 38423555/909328 - 3842
3555*sqrt(2)/1363992 + 9549859782656*(-3/128 - sqrt(2)/64)**5/170499 - 562
67374592*(-3/128 - sqrt(2)/64)**3/56833) + (-3/128 + sqrt(2)/64)*log(x - 3
8423555/909328 + 9549859782656*(-3/128 + sqrt(2)/64)**5/170499 - 562673745
92*(-3/128 + sqrt(2)/64)**3/56833 + 38423555*sqrt(2)/1363992) + (3/128 - s
qrt(2)/64)*log(x - 38423555*sqrt(2)/1363992 - 56267374592*(3/128 - sqrt(2)
/64)**3/56833 + 9549859782656*(3/128 - sqrt(2)/64)**5/170499 + 38423555/90
9328) + (sqrt(2)/64 + 3/128)*log(x - 56267374592*(sqrt(2)/64 + 3/128)**3/5
6833 + 9549859782656*(sqrt(2)/64 + 3/128)**5/170499 + 38423555*sqrt(2)/136
3992 + 38423555/909328)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.03

$$\int \frac{1+x^2}{\left(1-7x^2+7x^4-x^6\right)^2}\,dx = \frac{1}{64}\,\sqrt{2}\log\left(\frac{x-\sqrt{2}+1}{x+\sqrt{2}+1}\right)+\frac{1}{64}\,\sqrt{2}\log\left(\frac{x-\sqrt{2}-1}{x+\sqrt{2}-1}\right) - \frac{7\,x^5-46\,x^3+31\,x}{32\,(x^6-7\,x^4+7\,x^2-1)}+\frac{3}{128}\,\log\left(x^2+2\,x-1\right) - \frac{3}{128}\,\log\left(x^2-2\,x-1\right)+\frac{1}{8}\,\log\left(x+1\right)-\frac{1}{8}\,\log\left(x-1\right)$$

input
```
integrate((x^2+1)/(-x^6+7*x^4-7*x^2+1)^2,x, algorithm="maxima")
```

output
```
1/64*sqrt(2)*log((x - sqrt(2) + 1)/(x + sqrt(2) + 1)) + 1/64*sqrt(2)*log((
x - sqrt(2) - 1)/(x + sqrt(2) - 1)) - 1/32*(7*x^5 - 46*x^3 + 31*x)/(x^6 -
7*x^4 + 7*x^2 - 1) + 3/128*log(x^2 + 2*x - 1) - 3/128*log(x^2 - 2*x - 1) +
 1/8*log(x + 1) - 1/8*log(x - 1)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.21

$$\int \frac{1+x^2}{\left(1-7x^2+7x^4-x^6\right)^2}\,dx = \frac{1}{64}\sqrt{2}\log\left(\frac{|2x-2\sqrt{2}+2|}{|2x+2\sqrt{2}+2|}\right) + \frac{1}{64}\sqrt{2}\log\left(\frac{|2x-2\sqrt{2}-2|}{|2x+2\sqrt{2}-2|}\right) - \frac{7x^5-46x^3+31x}{32\left(x^6-7x^4+7x^2-1\right)} + \frac{3}{128}\log\left(|x^2+2x-1|\right) - \frac{3}{128}\log\left(|x^2-2x-1|\right) + \frac{1}{8}\log\left(|x+1|\right) - \frac{1}{8}\log\left(|x-1|\right)$$

input
```
integrate((x^2+1)/(-x^6+7*x^4-7*x^2+1)^2,x, algorithm="giac")
```

output
```
1/64*sqrt(2)*log(abs(2*x - 2*sqrt(2) + 2)/abs(2*x + 2*sqrt(2) + 2)) + 1/64
*sqrt(2)*log(abs(2*x - 2*sqrt(2) - 2)/abs(2*x + 2*sqrt(2) - 2)) - 1/32*(7*
x^5 - 46*x^3 + 31*x)/(x^6 - 7*x^4 + 7*x^2 - 1) + 3/128*log(abs(x^2 + 2*x -
 1)) - 3/128*log(abs(x^2 - 2*x - 1)) + 1/8*log(abs(x + 1)) - 1/8*log(abs(x
 - 1))
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.12

$$\int \frac{1+x^2}{(1-7x^2+7x^4-x^6)^2}\,dx = -\frac{\mathrm{atan}(x\,1\mathrm{i})\,1\mathrm{i}}{4} - \frac{\frac{7\,x^5}{32}-\frac{23\,x^3}{16}+\frac{31\,x}{32}}{x^6-7\,x^4+7\,x^2-1} + \mathrm{atan}\left(\frac{x\,23313\mathrm{i}}{8192\,\left(\frac{27309\,\sqrt{2}}{32768}-\frac{19317}{16384}\right)} - \frac{\sqrt{2}\,x\,65943\mathrm{i}}{32768\,\left(\frac{27309\,\sqrt{2}}{32768}-\frac{19317}{16384}\right)}\right)\left(\frac{\sqrt{2}\,1\mathrm{i}}{32}-\frac{3}{64}\mathrm{i}\right) + \mathrm{atan}\left(\frac{x\,23313\mathrm{i}}{8192\,\left(\frac{27309\,\sqrt{2}}{32768}+\frac{19317}{16384}\right)} + \frac{\sqrt{2}\,x\,65943\mathrm{i}}{32768\,\left(\frac{27309\,\sqrt{2}}{32768}+\frac{19317}{16384}\right)}\right)\left(\frac{\sqrt{2}\,1\mathrm{i}}{32}+\frac{3}{64}\mathrm{i}\right)$$

input
```
int((x^2 + 1)/(7*x^2 - 7*x^4 + x^6 - 1)^2,x)
```

output
```
atan((x*23313i)/(8192*((27309*2^(1/2))/32768 - 19317/16384)) - (2^(1/2)*x*
65943i)/(32768*((27309*2^(1/2))/32768 - 19317/16384)))*((2^(1/2)*1i)/32 -
3i/64) - ((31*x)/32 - (23*x^3)/16 + (7*x^5)/32)/(7*x^2 - 7*x^4 + x^6 - 1)
- (atan(x*1i)*1i)/4 + atan((x*23313i)/(8192*((27309*2^(1/2))/32768 + 19317
/16384)) + (2^(1/2)*x*65943i)/(32768*((27309*2^(1/2))/32768 + 19317/16384)
))*((2^(1/2)*1i)/32 + 3i/64)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 492, normalized size of antiderivative = 4.43

$$\int \frac{1+x^2}{(1-7x^2+7x^4-x^6)^2}\,dx = \text{Too large to display}$$

input
```
int((x^2+1)/(-x^6+7*x^4-7*x^2+1)^2,x)
```

output
```
(2*sqrt(2)*log( - sqrt(2) + x - 1)*x**6 - 14*sqrt(2)*log( - sqrt(2) + x -
1)*x**4 + 14*sqrt(2)*log( - sqrt(2) + x - 1)*x**2 - 2*sqrt(2)*log( - sqrt(
2) + x - 1) + 2*sqrt(2)*log( - sqrt(2) + x + 1)*x**6 - 14*sqrt(2)*log( - s
qrt(2) + x + 1)*x**4 + 14*sqrt(2)*log( - sqrt(2) + x + 1)*x**2 - 2*sqrt(2)
*log( - sqrt(2) + x + 1) - 2*sqrt(2)*log(sqrt(2) + x - 1)*x**6 + 14*sqrt(2
)*log(sqrt(2) + x - 1)*x**4 - 14*sqrt(2)*log(sqrt(2) + x - 1)*x**2 + 2*sqr
t(2)*log(sqrt(2) + x - 1) - 2*sqrt(2)*log(sqrt(2) + x + 1)*x**6 + 14*sqrt(
2)*log(sqrt(2) + x + 1)*x**4 - 14*sqrt(2)*log(sqrt(2) + x + 1)*x**2 + 2*sq
rt(2)*log(sqrt(2) + x + 1) - 3*log( - sqrt(2) + x - 1)*x**6 + 21*log( - sq
rt(2) + x - 1)*x**4 - 21*log( - sqrt(2) + x - 1)*x**2 + 3*log( - sqrt(2) +
 x - 1) + 3*log( - sqrt(2) + x + 1)*x**6 - 21*log( - sqrt(2) + x + 1)*x**4
 + 21*log( - sqrt(2) + x + 1)*x**2 - 3*log( - sqrt(2) + x + 1) - 3*log(sqr
t(2) + x - 1)*x**6 + 21*log(sqrt(2) + x - 1)*x**4 - 21*log(sqrt(2) + x - 1
)*x**2 + 3*log(sqrt(2) + x - 1) + 3*log(sqrt(2) + x + 1)*x**6 - 21*log(sqr
t(2) + x + 1)*x**4 + 21*log(sqrt(2) + x + 1)*x**2 - 3*log(sqrt(2) + x + 1)
 - 16*log(x - 1)*x**6 + 112*log(x - 1)*x**4 - 112*log(x - 1)*x**2 + 16*log
(x - 1) + 16*log(x + 1)*x**6 - 112*log(x + 1)*x**4 + 112*log(x + 1)*x**2 -
 16*log(x + 1) - 28*x**5 + 184*x**3 - 124*x)/(128*(x**6 - 7*x**4 + 7*x**2
- 1))
```

3.70 $\int \frac{(3-2\sqrt{2}+x^2)^2(-3+2\sqrt{2}+x^2)}{577-408\sqrt{2}-8(-41+29\sqrt{2})x^2-2(-39+28\sqrt{2})x^4-8(-1+\sqrt{2})x^6+x^8}$

Optimal result

Integrand size = 81, antiderivative size = 22

$$\int \frac{(3-2\sqrt{2}+x^2)^2(-3+2\sqrt{2}+x^2)}{577-408\sqrt{2}-8(-41+29\sqrt{2})x^2-2(-39+28\sqrt{2})x^4-8(-1+\sqrt{2})x^6+x^8}\,dx = -\frac{1}{2}\operatorname{arctanh}\left(\frac{2x}{3-2\sqrt{2}+x^2}\right)$$

output
```
-1/2*arctanh(2*x/(3-2*2^(1/2)+x^2))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 139 vs. 2(22) = 44.

Time = 0.20 (sec) , antiderivative size = 139, normalized size of antiderivative = 6.32

$$\int \frac{(3-2\sqrt{2}+x^2)^2(-3+2\sqrt{2}+x^2)}{577-408\sqrt{2}-8(-41+29\sqrt{2})x^2-2(-39+28\sqrt{2})x^4-8(-1+\sqrt{2})x^6+x^8}\,dx = -\frac{(3-2\sqrt{2}+x^2)^2(-17+12\sqrt{2}+(-2+4\sqrt{2})x^2-x^4)(\log(-3+2\sqrt{2}-2x-x^2)-\log(-3+2\sqrt{2}}{4(-577+408\sqrt{2}+8(-41+29\sqrt{2})x^2+(-78+56\sqrt{2})x^4+8(-1+\sqrt{2})x^6-x^8)}$$

input
```
Integrate[((3 - 2*Sqrt[2] + x^2)^2*(-3 + 2*Sqrt[2] + x^2))/(577 - 408*Sqrt
[2] - 8*(-41 + 29*Sqrt[2])*x^2 - 2*(-39 + 28*Sqrt[2])*x^4 - 8*(-1 + Sqrt[2
])*x^6 + x^8),x]
```

output
```
-1/4*((3 - 2*Sqrt[2] + x^2)^2*(-17 + 12*Sqrt[2] + (-2 + 4*Sqrt[2])*x^2 - x
^4)*(Log[-3 + 2*Sqrt[2] - 2*x - x^2] - Log[-3 + 2*Sqrt[2] + 2*x - x^2]))/(
-577 + 408*Sqrt[2] + 8*(-41 + 29*Sqrt[2])*x^2 + (-78 + 56*Sqrt[2])*x^4 + 8
*(-1 + Sqrt[2])*x^6 - x^8)
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 103 vs. $2(22) = 44$.

Time = 0.79 (sec) , antiderivative size = 103, normalized size of antiderivative = 4.68, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {2019, 2019, 1475, 1081, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x^2-2\sqrt{2}+3\right)^2\left(x^2+2\sqrt{2}-3\right)}{x^8-8\left(\sqrt{2}-1\right)x^6-2\left(28\sqrt{2}-39\right)x^4-8\left(29\sqrt{2}-41\right)x^2-408\sqrt{2}+577}dx$$

$$\Big\downarrow \text{2019}$$

$$\int \frac{\left(x^2-2\sqrt{2}+3\right)\left(x^2+2\sqrt{2}-3\right)}{x^6+\left(5-6\sqrt{2}\right)x^4+\left(39-28\sqrt{2}\right)x^2-70\sqrt{2}+99}dx$$

$$\Big\downarrow \text{2019}$$

$$\int \frac{x^2+2\sqrt{2}-3}{x^4+\left(2-4\sqrt{2}\right)x^2-12\sqrt{2}+17}dx$$

$$\Big\downarrow \text{1475}$$

$$\frac{1}{2}\int \frac{1}{x^2-2\sqrt{2\left(-1+\sqrt{2}\right)}x+2\sqrt{2}-3}dx+\frac{1}{2}\int \frac{1}{x^2+2\sqrt{2\left(-1+\sqrt{2}\right)}x+2\sqrt{2}-3}dx$$

$$\Big\downarrow \text{1081}$$

$$\frac{1}{2}\int\left(-\frac{1}{2\left(x-\sqrt{2\left(-1+\sqrt{2}\right)}+1\right)}-\frac{1}{2\left(-x+\sqrt{2\left(-1+\sqrt{2}\right)}+1\right)}\right)dx+$$
$$\frac{1}{2}\int\left(-\frac{1}{2\left(x+\sqrt{2\left(-1+\sqrt{2}\right)}+1\right)}-\frac{1}{2\left(-x-\sqrt{2\left(-1+\sqrt{2}\right)}+1\right)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{1}{2}\log\left(-x+\sqrt{2\left(\sqrt{2}-1\right)}+1\right)-\frac{1}{2}\log\left(x-\sqrt{2\left(\sqrt{2}-1\right)}+1\right)\right)+$$
$$\frac{1}{2}\left(\frac{1}{2}\log\left(-x-\sqrt{2\left(\sqrt{2}-1\right)}+1\right)-\frac{1}{2}\log\left(x+\sqrt{2\left(\sqrt{2}-1\right)}+1\right)\right)$$

input
```
Int[((3 - 2*Sqrt[2] + x^2)^2*(-3 + 2*Sqrt[2] + x^2))/(577 - 408*Sqrt[2] -
8*(-41 + 29*Sqrt[2])*x^2 - 2*(-39 + 28*Sqrt[2])*x^4 - 8*(-1 + Sqrt[2])*x^6
 + x^8),x]
```

output
```
(Log[1 + Sqrt[2*(-1 + Sqrt[2])] - x]/2 - Log[1 - Sqrt[2*(-1 + Sqrt[2])] +
x]/2)/2 + (Log[1 - Sqrt[2*(-1 + Sqrt[2])] - x]/2 - Log[1 + Sqrt[2*(-1 + Sq
rt[2])] + x]/2)/2
```

Defintions of rubi rules used

rule 1081
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[c   Int[ExpandIntegrand[1/((b/2 - q/2 + c*x)*(b/2 + q/2
+ c*x)), x], x], x]] /; FreeQ[{a, b, c}, x] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 1475
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^
2, x], x], x] + Simp[e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] &&
 (GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2]
, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.55

method	result	size
default	$\frac{\ln\left(x^{2}-2\sqrt{2}-2x+3\right)}{4}-\frac{\ln\left(x^{2}-2\sqrt{2}+2x+3\right)}{4}$	34
risch	$\frac{\ln\left(x^{2}-2\sqrt{2}-2x+3\right)}{4}-\frac{\ln\left(x^{2}-2\sqrt{2}+2x+3\right)}{4}$	34
parallelrisch	$\frac{\ln\left(x^{2}-2\sqrt{2}-2x+3\right)}{4}-\frac{\ln\left(x^{2}-2\sqrt{2}+2x+3\right)}{4}$	34

input
```
int((3-2*2^(1/2)+x^2)^2*(-3+2*2^(1/2)+x^2)/(577-408*2^(1/2)-8*(-41+29*2^(1
/2))*x^2-2*(-39+28*2^(1/2))*x^4-8*(2^(1/2)-1)*x^6+x^8),x,method=_RETURNVER
BOSE)
```

output
```
1/4*ln(x^2-2*2^(1/2)-2*x+3)-1/4*ln(x^2-2*2^(1/2)+2*x+3)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.50

$$\int \frac{\left(3-2\sqrt{2}+x^2\right)^2\left(-3+2\sqrt{2}+x^2\right)}{577-408\sqrt{2}-8\left(-41+29\sqrt{2}\right)x^2-2\left(-39+28\sqrt{2}\right)x^4-8\left(-1+\sqrt{2}\right)x^6+x^8}\,dx$$
$$=-\frac{1}{4}\log\left(x^2+2\,x-2\,\sqrt{2}+3\right)+\frac{1}{4}\log\left(x^2-2\,x-2\,\sqrt{2}+3\right)$$

input
```
integrate((3-2*2^(1/2)+x^2)^2*(-3+2*2^(1/2)+x^2)/(577-408*2^(1/2)-8*(-41+2
9*2^(1/2))*x^2-2*(-39+28*2^(1/2))*x^4-8*(2^(1/2)-1)*x^6+x^8),x, algorithm=
"fricas")
```

output
```
-1/4*log(x^2 + 2*x - 2*sqrt(2) + 3) + 1/4*log(x^2 - 2*x - 2*sqrt(2) + 3)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{(3-2\sqrt{2}+x^2)^2(-3+2\sqrt{2}+x^2)}{577-408\sqrt{2}-8(-41+29\sqrt{2})x^2-2(-39+28\sqrt{2})x^4-8(-1+\sqrt{2})x^6+x^8}\,dx$$
$$= \text{Exception raised: PolynomialError}$$

input
```
integrate((3-2*2**(1/2)+x**2)**2*(-3+2*2**(1/2)+x**2)/(577-408*2**(1/2)-8*
(-41+29*2**(1/2))*x**2-2*(-39+28*2**(1/2))*x**4-8*(2**(1/2)-1)*x**6+x**8),
x)
```

output
```
Exception raised: PolynomialError >> 1/(-4893319121142556020618924174780472
2498117708482611714912381696*_t**4 + 346009913306969839800447635927970293
0522480193213103784309 76*sqrt(2)*_t**4 - 159769239484575670917838951113184
628965915778476
```

Maxima [F]

$$\int \frac{(3-2\sqrt{2}+x^2)^2(-3+2\sqrt{2}+x^2)}{577-408\sqrt{2}-8(-41+29\sqrt{2})x^2-2(-39+28\sqrt{2})x^4-8(-1+\sqrt{2})x^6+x^8}\,dx$$
$$= \int \frac{(x^2+2\sqrt{2}-3)(x^2-2\sqrt{2}+3)^2}{x^8-8x^6(\sqrt{2}-1)-2x^4(28\sqrt{2}-39)-8x^2(29\sqrt{2}-41)-408\sqrt{2}+577}\,dx$$

input
```
integrate((3-2*2^(1/2)+x^2)^2*(-3+2*2^(1/2)+x^2)/(577-408*2^(1/2)-8*(-41+2
9*2^(1/2))*x^2-2*(-39+28*2^(1/2))*x^4-8*(2^(1/2)-1)*x^6+x^8),x, algorithm=
"maxima")
```

output
```
integrate((x^2 + 2*sqrt(2) - 3)*(x^2 - 2*sqrt(2) + 3)^2/(x^8 - 8*x^6*(sqrt
(2) - 1) - 2*x^4*(28*sqrt(2) - 39) - 8*x^2*(29*sqrt(2) - 41) - 408*sqrt(2)
 + 577), x)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.59

$$\int \frac{(3-2\sqrt{2}+x^2)^2\,(-3+2\sqrt{2}+x^2)}{577-408\sqrt{2}-8\,(-41+29\sqrt{2})\,x^2-2\,(-39+28\sqrt{2})\,x^4-8\,(-1+\sqrt{2})\,x^6+x^8}\,dx$$
$$=-\frac{1}{4}\log\left(\left|x^2+2\,x-2\,\sqrt{2}+3\right|\right)+\frac{1}{4}\log\left(\left|x^2-2\,x-2\,\sqrt{2}+3\right|\right)$$

input
```
integrate((3-2*2^(1/2)+x^2)^2*(-3+2*2^(1/2)+x^2)/(577-408*2^(1/2)-8*(-41+2
9*2^(1/2))*x^2-2*(-39+28*2^(1/2))*x^4-8*(2^(1/2)-1)*x^6+x^8),x, algorithm=
"giac")
```

output
```
-1/4*log(abs(x^2 + 2*x - 2*sqrt(2) + 3)) + 1/4*log(abs(x^2 - 2*x - 2*sqrt(
2) + 3))
```

Mupad [B] (verification not implemented)

Time = 23.28 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.55

$$\int \frac{(3-2\sqrt{2}+x^2)^2\,(-3+2\sqrt{2}+x^2)}{577-408\sqrt{2}-8\,(-41+29\sqrt{2})\,x^2-2\,(-39+28\sqrt{2})\,x^4-8\,(-1+\sqrt{2})\,x^6+x^8}\,dx$$
$$=-\frac{\mathrm{atanh}\left(\frac{x\,\left(16\,\sqrt{2}-16\right)}{2\,\left(20\,\sqrt{2}+4\,\sqrt{2}\,x^2-4\,x^2-28\right)}\right)}{2}$$

input
```
int(-((x^2 - 2*2^(1/2) + 3)^2*(2*2^(1/2) + x^2 - 3))/(2*x^4*(28*2^(1/2) -
39) + 8*x^2*(29*2^(1/2) - 41) + 408*2^(1/2) - x^8 + 8*x^6*(2^(1/2) - 1) -
577),x)
```

output
```
-atanh((x*(16*2^(1/2) - 16))/(2*(20*2^(1/2) + 4*2^(1/2)*x^2 - 4*x^2 - 28))
)/2
```

Reduce [F]

$$\int \frac{(3-2\sqrt{2}+x^2)^2(-3+2\sqrt{2}+x^2)}{577-408\sqrt{2}-8(-41+29\sqrt{2})x^2-2(-39+28\sqrt{2})x^4-8(-1+\sqrt{2})x^6+x^8}\,dx$$
$$= 6\sqrt{2}\left(\int \frac{x^4}{x^8+4x^6+6x^4-124x^2+1}dx\right) + 4\sqrt{2}\left(\int \frac{x^2}{x^8+4x^6+6x^4-124x^2+1}dx\right) - 2\sqrt{2}\left(\int \frac{1}{x^8+4x^6+6x^4-124x^2+1}dx\right) + \int \frac{x^6}{x^8+4x^6+6x^4-124x^2+1}dx - \left(\int \frac{x^4}{x^8+4x^6+6x^4-124x^2+1}dx\right) + 27\left(\int \frac{x^2}{x^8+4x^6+6x^4-124x^2+1}dx\right) - 3\left(\int \frac{1}{x^8+4x^6+6x^4-124x^2+1}dx\right)$$

input
```
int((3-2*2^(1/2)+x^2)^2*(-3+2*2^(1/2)+x^2)/(577-408*2^(1/2)-8*(-41+29*2^(1
/2))*x^2-2*(-39+28*2^(1/2))*x^4-8*(2^(1/2)-1)*x^6+x^8),x)
```

output
```
6*sqrt(2)*int(x**4/(x**8 + 4*x**6 + 6*x**4 - 124*x**2 + 1),x) + 4*sqrt(2)*
int(x**2/(x**8 + 4*x**6 + 6*x**4 - 124*x**2 + 1),x) - 2*sqrt(2)*int(1/(x**
8 + 4*x**6 + 6*x**4 - 124*x**2 + 1),x) + int(x**6/(x**8 + 4*x**6 + 6*x**4
- 124*x**2 + 1),x) - int(x**4/(x**8 + 4*x**6 + 6*x**4 - 124*x**2 + 1),x) +
 27*int(x**2/(x**8 + 4*x**6 + 6*x**4 - 124*x**2 + 1),x) - 3*int(1/(x**8 +
4*x**6 + 6*x**4 - 124*x**2 + 1),x)
```

3.71 $\int (a+bx) \left(1+\left(ax+\frac{bx^2}{2}\right)^4\right) \, dx$

Optimal result

Integrand size = 22, antiderivative size = 28

$$\int (a+bx) \left(1+\left(ax+\frac{bx^2}{2}\right)^4\right) \, dx = ax+\frac{bx^2}{2}+\frac{1}{160}\left(2ax+bx^2\right)^5$$

output
```
a*x+1/2*b*x^2+1/160*(b*x^2+2*a*x)^5
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 80 vs. 2(28) = 56.

Time = 0.01 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.86

$$\int (a+bx) \left(1+\left(ax+\frac{bx^2}{2}\right)^4\right) \, dx = ax+\frac{bx^2}{2}+\frac{a^5x^5}{5}+\frac{1}{2}a^4bx^6+\frac{1}{2}a^3b^2x^7 +\frac{1}{4}a^2b^3x^8+\frac{1}{16}ab^4x^9+\frac{b^5x^{10}}{160}$$

input
```
Integrate[(a + b*x)*(1 + (a*x + (b*x^2)/2)^4),x]
```

output
```
a*x + (b*x^2)/2 + (a^5*x^5)/5 + (a^4*b*x^6)/2 + (a^3*b^2*x^7)/2 + (a^2*b^3
*x^8)/4 + (a*b^4*x^9)/16 + (b^5*x^10)/160
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {2024, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)\left(\left(ax+\frac{bx^2}{2}\right)^4+1\right)dx$$

$$\Big\downarrow \text{2024}$$

$$\int \left(\left(ax+\frac{bx^2}{2}\right)^4+1\right)d\left(ax+\frac{bx^2}{2}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{5}\left(ax+\frac{bx^2}{2}\right)^5+ax+\frac{bx^2}{2}$$

input
```
Int[(a + b*x)*(1 + (a*x + (b*x^2)/2)^4),x]
```

output
```
a*x + (b*x^2)/2 + (a*x + (b*x^2)/2)^5/5
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2024
```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Simp[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q])   Subst[
Int[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D
[Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &
& PolyQ[Qr, x]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

method	result	size
default	$\frac{\left(xa+\frac{1}{2}b\,x^{2}\right)^{5}}{5}+xa+\frac{b\,x^{2}}{2}$	25
gosper	$\frac{x\left(b^{5}x^{9}+10a\,b^{4}x^{8}+40a^{2}b^{3}x^{7}+80a^{3}b^{2}x^{6}+80a^{4}b\,x^{5}+32a^{5}x^{4}+80bx+160a\right)}{160}$	67
norman	$xa+\frac{1}{5}a^{5}x^{5}+\frac{1}{2}b\,x^{2}+\frac{1}{160}b^{5}x^{10}+\frac{1}{16}a\,b^{4}x^{9}+\frac{1}{4}a^{2}b^{3}x^{8}+\frac{1}{2}a^{3}b^{2}x^{7}+\frac{1}{2}a^{4}b\,x^{6}$	67
risch	$xa+\frac{1}{5}a^{5}x^{5}+\frac{1}{2}b\,x^{2}+\frac{1}{160}b^{5}x^{10}+\frac{1}{16}a\,b^{4}x^{9}+\frac{1}{4}a^{2}b^{3}x^{8}+\frac{1}{2}a^{3}b^{2}x^{7}+\frac{1}{2}a^{4}b\,x^{6}$	67
parallelrisch	$xa+\frac{1}{5}a^{5}x^{5}+\frac{1}{2}b\,x^{2}+\frac{1}{160}b^{5}x^{10}+\frac{1}{16}a\,b^{4}x^{9}+\frac{1}{4}a^{2}b^{3}x^{8}+\frac{1}{2}a^{3}b^{2}x^{7}+\frac{1}{2}a^{4}b\,x^{6}$	67
orering	$\frac{x\left(b^{5}x^{9}+10a\,b^{4}x^{8}+40a^{2}b^{3}x^{7}+80a^{3}b^{2}x^{6}+80a^{4}b\,x^{5}+32a^{5}x^{4}+80bx+160a\right)\left(1+\left(xa+\frac{1}{2}b\,x^{2}\right)^{4}\right)}{10b^{4}x^{8}+80x^{7}a\,b^{3}+240b^{2}x^{6}a^{2}+320a^{3}b\,x^{5}+160x^{4}a^{4}+160}$	129

input
```
int((b*x+a)*(1+(x*a+1/2*b*x^2)^4),x,method=_RETURNVERBOSE)
```

output
```
1/5*(x*a+1/2*b*x^2)^5+x*a+1/2*b*x^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(24) = 48$.

Time = 0.06 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.36

$$\int (a+bx)\left(1+\left(ax+\frac{bx^2}{2}\right)^4\right)\,dx = \frac{1}{160}\,b^5x^{10}+\frac{1}{16}\,ab^4x^9+\frac{1}{4}\,a^2b^3x^8+\frac{1}{2}\,a^3b^2x^7+\frac{1}{2}\,a^4bx^6+\frac{1}{5}\,a^5x^5+\frac{1}{2}\,bx^2+ax$$

input
```
integrate((b*x+a)*(1+(a*x+1/2*b*x^2)^4),x, algorithm="fricas")
```

output
```
1/160*b^5*x^10 + 1/16*a*b^4*x^9 + 1/4*a^2*b^3*x^8 + 1/2*a^3*b^2*x^7 + 1/2*
a^4*b*x^6 + 1/5*a^5*x^5 + 1/2*b*x^2 + a*x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(22) = 44$.

Time = 0.04 (sec) , antiderivative size = 70, normalized size of antiderivative = 2.50

$$\int (a+bx)\left(1+\left(ax+\frac{bx^2}{2}\right)^4\right)\,dx = \frac{a^5x^5}{5}+\frac{a^4bx^6}{2}+\frac{a^3b^2x^7}{2}+\frac{a^2b^3x^8}{4}+\frac{ab^4x^9}{16}+ax+\frac{b^5x^{10}}{160}+\frac{bx^2}{2}$$

input
```
integrate((b*x+a)*(1+(a*x+1/2*b*x**2)**4),x)
```

output
```
a**5*x**5/5 + a**4*b*x**6/2 + a**3*b**2*x**7/2 + a**2*b**3*x**8/4 + a*b**4
*x**9/16 + a*x + b**5*x**10/160 + b*x**2/2
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(24) = 48$.

Time = 0.03 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.36

$$\int (a+bx)\left(1+\left(ax+\frac{bx^2}{2}\right)^4\right)\,dx=\frac{1}{160}\,b^5x^{10}+\frac{1}{16}\,ab^4x^9+\frac{1}{4}\,a^2b^3x^8+\frac{1}{2}\,a^3b^2x^7+\frac{1}{2}\,a^4bx^6+\frac{1}{5}\,a^5x^5+\frac{1}{2}\,bx^2+ax$$

input
```
integrate((b*x+a)*(1+(a*x+1/2*b*x^2)^4),x, algorithm="maxima")
```

output
```
1/160*b^5*x^10 + 1/16*a*b^4*x^9 + 1/4*a^2*b^3*x^8 + 1/2*a^3*b^2*x^7 + 1/2*
a^4*b*x^6 + 1/5*a^5*x^5 + 1/2*b*x^2 + a*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int (a+bx)\left(1+\left(ax+\frac{bx^2}{2}\right)^4\right)\,dx=\frac{1}{160}\,\left(bx^2+2\,ax\right)^5+\frac{1}{2}\,bx^2+ax$$

input
```
integrate((b*x+a)*(1+(a*x+1/2*b*x^2)^4),x, algorithm="giac")
```

output
```
1/160*(b*x^2 + 2*a*x)^5 + 1/2*b*x^2 + a*x
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.36

$$\int (a+bx)\left(1+\left(ax+\frac{bx^2}{2}\right)^4\right)\,dx=\frac{a^5\,x^5}{5}+\frac{a^4\,b\,x^6}{2}+\frac{a^3\,b^2\,x^7}{2}+\frac{a^2\,b^3\,x^8}{4}+\frac{a\,b^4\,x^9}{16}+a\,x+\frac{b^5\,x^{10}}{160}+\frac{b\,x^2}{2}$$

input
```
int(((a*x + (b*x^2)/2)^4 + 1)*(a + b*x),x)
```

output
```
a*x + (b*x^2)/2 + (a^5*x^5)/5 + (b^5*x^10)/160 + (a^4*b*x^6)/2 + (a*b^4*x^
9)/16 + (a^3*b^2*x^7)/2 + (a^2*b^3*x^8)/4
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.36

$$\int (a+bx)\left(1+\left(ax+\frac{bx^2}{2}\right)^4\right)\,dx$$
$$=\frac{x(b^5x^9+10a\,b^4x^8+40a^2b^3x^7+80a^3b^2x^6+80a^4b\,x^5+32a^5x^4+80bx+160a)}{160}$$

input
```
int((b*x+a)*(1+(a*x+1/2*b*x^2)^4),x)
```

output
```
(x*(32*a**5*x**4 + 80*a**4*b*x**5 + 80*a**3*b**2*x**6 + 40*a**2*b**3*x**7
+ 10*a*b**4*x**8 + 160*a + b**5*x**9 + 80*b*x))/160
```

3.72 $\int (a+bx)\left(1+\left(c+ax+\frac{bx^2}{2}\right)^4\right)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 31

$$\int (a+bx)\left(1+\left(c+ax+\frac{bx^2}{2}\right)^4\right)\,dx = ax+\frac{bx^2}{2}+\frac{1}{160}\left(2c+2ax+bx^2\right)^5$$

output
```
a*x+1/2*b*x^2+1/160*(b*x^2+2*a*x+2*c)^5
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 108 vs. 2(31) = 62.

Time = 0.05 (sec) , antiderivative size = 108, normalized size of antiderivative = 3.48

$$\begin{aligned}\int (a+bx)\left(1+\left(c+ax+\frac{bx^2}{2}\right)^4\right)\,dx &= \frac{1}{160}x(2a+bx)\left(80+80c^4+16a^4x^4+32a^3bx^5\right.\\ &\quad+24a^2b^2x^6+8ab^3x^7+b^4x^8+80c^3x(2a+bx)\\ &\quad\left.+40c^2x^2(2a+bx)^2+10cx^3(2a+bx)^3\right)\end{aligned}$$

input
```
Integrate[(a + b*x)*(1 + (c + a*x + (b*x^2)/2)^4),x]
```

output
```
(x*(2*a + b*x)*(80 + 80*c^4 + 16*a^4*x^4 + 32*a^3*b*x^5 + 24*a^2*b^2*x^6 +
 8*a*b^3*x^7 + b^4*x^8 + 80*c^3*x*(2*a + b*x) + 40*c^2*x^2*(2*a + b*x)^2 +
 10*c*x^3*(2*a + b*x)^3))/160
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2024, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)\left(\left(ax+\frac{bx^2}{2}+c\right)^4+1\right)dx$$

$$\Big\downarrow \text{2024}$$

$$\int \left(\left(ax+\frac{bx^2}{2}+c\right)^4+1\right)d\left(ax+\frac{bx^2}{2}+c\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{5}\left(ax+\frac{bx^2}{2}+c\right)^5+ax+\frac{bx^2}{2}+c$$

input
```
Int[(a + b*x)*(1 + (c + a*x + (b*x^2)/2)^4),x]
```

output
```
c + a*x + (b*x^2)/2 + (c + a*x + (b*x^2)/2)^5/5
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2024
```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Simp[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q])   Subst[
Int[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D
[Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &
& PolyQ[Qr, x]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

method	result
default	$\frac{\left(c+xa+\frac{1}{2}b\,x^{2}\right)^{5}}{5}+c+xa+\frac{b\,x^{2}}{2}$
norman	$\left(\frac{1}{4}a^{2}b^{3}+\frac{1}{16}b^{4}c\right)x^{8}+\left(\frac{1}{2}a^{3}b^{2}+\frac{1}{2}a\,b^{3}c\right)x^{7}+\left(\frac{1}{5}a^{5}+2a^{3}bc+\frac{3}{2}a\,b^{2}c^{2}\right)x^{5}+\left(2a^{2}c^{3}+\frac{1}{2}b\,c^{4}+\right.$
gosper	$\frac{x\left(b^{5}x^{9}+10a\,b^{4}x^{8}+40a^{2}b^{3}x^{7}+10b^{4}c\,x^{7}+80a^{3}b^{2}x^{6}+80a\,b^{3}c\,x^{6}+80a^{4}b\,x^{5}+240a^{2}b^{2}c\,x^{5}+40b^{3}c^{2}x^{5}+32a^{5}x^{4}+320x^{4}a^{3}bc+240\right.}{160}$
risch	$\frac{1}{160}b^{5}x^{10}+\frac{1}{16}a\,b^{4}x^{9}+\frac{1}{4}a^{2}b^{3}x^{8}+\frac{1}{16}x^{8}b^{4}c+\frac{1}{2}a^{3}b^{2}x^{7}+\frac{1}{2}x^{7}a\,b^{3}c+\frac{1}{2}a^{4}b\,x^{6}+\frac{3}{2}x^{6}c\,a^{2}b^{2}+\frac{1}{4}x$
parallelrisch	$\frac{1}{160}b^{5}x^{10}+\frac{1}{16}a\,b^{4}x^{9}+\frac{1}{4}a^{2}b^{3}x^{8}+\frac{1}{16}x^{8}b^{4}c+\frac{1}{2}a^{3}b^{2}x^{7}+\frac{1}{2}x^{7}a\,b^{3}c+\frac{1}{2}a^{4}b\,x^{6}+\frac{3}{2}x^{6}c\,a^{2}b^{2}+\frac{1}{4}x$
orering	$\frac{x\left(b^{5}x^{9}+10a\,b^{4}x^{8}+40a^{2}b^{3}x^{7}+10b^{4}c\,x^{7}+80a^{3}b^{2}x^{6}+80a\,b^{3}c\,x^{6}+80a^{4}b\,x^{5}+240a^{2}b^{2}c\,x^{5}+40b^{3}c^{2}x^{5}+32a^{5}x^{4}+320x^{4}a^{3}bc+240\right.}{10b^{4}x^{8}+80x^{7}a\,b^{3}+240b^{2}x^{6}a^{2}+80c\,b^{3}x^{6}+320a^{3}b\,x^{5}+480c\,b^{2}a\,x^{5}+160x^{4}a^{4}+960}$

input
```
int((b*x+a)*(1+(c+x*a+1/2*b*x^2)^4),x,method=_RETURNVERBOSE)
```

output
```
1/5*(c+x*a+1/2*b*x^2)^5+c+x*a+1/2*b*x^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 187 vs. $2(27) = 54$.

Time = 0.06 (sec) , antiderivative size = 187, normalized size of antiderivative = 6.03

$$\int (a+bx)\left(1+\left(c+ax+\frac{bx^2}{2}\right)^4\right)\,dx$$
$$= \frac{1}{160}b^5x^{10}+\frac{1}{16}ab^4x^9+\frac{1}{16}\left(4\,a^2b^3+b^4c\right)x^8 + \frac{1}{2}\left(a^3b^2+ab^3c\right)x^7+\frac{1}{4}\left(2\,a^4b+6\,a^2b^2c+b^3c^2\right)x^6 + \frac{1}{10}\left(2\,a^5+20\,a^3bc+15\,ab^2c^2\right)x^5+\frac{1}{2}\left(2\,a^4c+6\,a^2bc^2+b^2c^3\right)x^4 + 2\left(a^3c^2+abc^3\right)x^3+\frac{1}{2}\left(4\,a^2c^3+bc^4+b\right)x^2+\left(ac^4+a\right)x$$

input

```
integrate((b*x+a)*(1+(c+a*x+1/2*b*x^2)^4),x, algorithm="fricas")
```

output

```
1/160*b^5*x^10 + 1/16*a*b^4*x^9 + 1/16*(4*a^2*b^3 + b^4*c)*x^8 + 1/2*(a^3*
b^2 + a*b^3*c)*x^7 + 1/4*(2*a^4*b + 6*a^2*b^2*c + b^3*c^2)*x^6 + 1/10*(2*a
^5 + 20*a^3*b*c + 15*a*b^2*c^2)*x^5 + 1/2*(2*a^4*c + 6*a^2*b*c^2 + b^2*c^3
)*x^4 + 2*(a^3*c^2 + a*b*c^3)*x^3 + 1/2*(4*a^2*c^3 + b*c^4 + b)*x^2 + (a*c
^4 + a)*x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 194 vs. $2(26) = 52$.

Time = 0.05 (sec) , antiderivative size = 194, normalized size of antiderivative = 6.26

$$\int (a+bx)\left(1+\left(c+ax+\frac{bx^2}{2}\right)^4\right)dx = \frac{ab^4x^9}{16}+\frac{b^5x^{10}}{160}+x^8\left(\frac{a^2b^3}{4}+\frac{b^4c}{16}\right) + x^7\left(\frac{a^3b^2}{2}+\frac{ab^3c}{2}\right) + x^6\left(\frac{a^4b}{2}+\frac{3a^2b^2c}{2}+\frac{b^3c^2}{4}\right) + x^5\left(\frac{a^5}{5}+2a^3bc+\frac{3ab^2c^2}{2}\right) + x^4\left(a^4c+3a^2bc^2+\frac{b^2c^3}{2}\right) + x^3\cdot\left(2a^3c^2+2abc^3\right)+x^2 \cdot\left(2a^2c^3+\frac{bc^4}{2}+\frac{b}{2}\right)+x\left(ac^4+a\right)$$

input
```
integrate((b*x+a)*(1+(c+a*x+1/2*b*x**2)**4),x)
```

output
```
a*b**4*x**9/16 + b**5*x**10/160 + x**8*(a**2*b**3/4 + b**4*c/16) + x**7*(a
**3*b**2/2 + a*b**3*c/2) + x**6*(a**4*b/2 + 3*a**2*b**2*c/2 + b**3*c**2/4)
 + x**5*(a**5/5 + 2*a**3*b*c + 3*a*b**2*c**2/2) + x**4*(a**4*c + 3*a**2*b*
c**2 + b**2*c**3/2) + x**3*(2*a**3*c**2 + 2*a*b*c**3) + x**2*(2*a**2*c**3
+ b*c**4/2 + b/2) + x*(a*c**4 + a)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 187 vs. $2(27) = 54$.

Time = 0.03 (sec) , antiderivative size = 187, normalized size of antiderivative = 6.03

$$\int (a+bx)\left(1+\left(c+ax+\frac{bx^2}{2}\right)^4\right)dx = \frac{1}{160}b^5x^{10}+\frac{1}{16}ab^4x^9+\frac{1}{16}\left(4a^2b^3+b^4c\right)x^8 + \frac{1}{2}\left(a^3b^2+ab^3c\right)x^7+\frac{1}{4}\left(2a^4b+6a^2b^2c+b^3c^2\right)x^6 + \frac{1}{10}\left(2a^5+20a^3bc+15ab^2c^2\right)x^5+\frac{1}{2}\left(2a^4c+6a^2bc^2+b^2c^3\right)x^4 + 2\left(a^3c^2+abc^3\right)x^3+\frac{1}{2}\left(4a^2c^3+bc^4+b\right)x^2+\left(ac^4+a\right)x$$

input
```
integrate((b*x+a)*(1+(c+a*x+1/2*b*x^2)^4),x, algorithm="maxima")
```

output
```
1/160*b^5*x^10 + 1/16*a*b^4*x^9 + 1/16*(4*a^2*b^3 + b^4*c)*x^8 + 1/2*(a^3*
b^2 + a*b^3*c)*x^7 + 1/4*(2*a^4*b + 6*a^2*b^2*c + b^3*c^2)*x^6 + 1/10*(2*a
^5 + 20*a^3*b*c + 15*a*b^2*c^2)*x^5 + 1/2*(2*a^4*c + 6*a^2*b*c^2 + b^2*c^3
)*x^4 + 2*(a^3*c^2 + a*b*c^3)*x^3 + 1/2*(4*a^2*c^3 + b*c^4 + b)*x^2 + (a*c
^4 + a)*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. 2(27) = 54.

Time = 0.11 (sec) , antiderivative size = 88, normalized size of antiderivative = 2.84

$$\int (a+bx)\left(1+\left(c+ax+\frac{bx^2}{2}\right)^4\right)dx = \frac{1}{160}\left(bx^2+2ax\right)^5+\frac{1}{16}\left(bx^2+2ax\right)^4c + \frac{1}{4}\left(bx^2+2ax\right)^3c^2+\frac{1}{2}\left(bx^2+2ax\right)^2c^3 + \frac{1}{2}\left(bx^2+2ax\right)c^4+\frac{1}{2}bx^2+ax$$

input
```
integrate((b*x+a)*(1+(c+a*x+1/2*b*x^2)^4),x, algorithm="giac")
```

output
```
1/160*(b*x^2 + 2*a*x)^5 + 1/16*(b*x^2 + 2*a*x)^4*c + 1/4*(b*x^2 + 2*a*x)^3
*c^2 + 1/2*(b*x^2 + 2*a*x)^2*c^3 + 1/2*(b*x^2 + 2*a*x)*c^4 + 1/2*b*x^2 + a
*x
```

Mupad [B] (verification not implemented)

Time = 21.81 (sec) , antiderivative size = 180, normalized size of antiderivative = 5.81

$$\int (a+bx)\left(1+\left(c+ax+\frac{bx^2}{2}\right)^4\right)\,dx = x^6\left(\frac{a^4\,b}{2}+\frac{3\,a^2\,b^2\,c}{2}+\frac{b^3\,c^2}{4}\right) + x^4\left(a^4\,c+3\,a^2\,b\,c^2+\frac{b^2\,c^3}{2}\right) + x^2\left(2\,a^2\,c^3+\frac{b\,c^4}{2}+\frac{b}{2}\right) + x^5\left(\frac{a^5}{5}+2\,a^3\,b\,c+\frac{3\,a\,b^2\,c^2}{2}\right)+\frac{b^5\,x^{10}}{160} + x^8\left(\frac{a^2\,b^3}{4}+\frac{c\,b^4}{16}\right)+\frac{a\,b^4\,x^9}{16}+a\,x\,\left(c^4+1\right) + \frac{a\,b^2\,x^7\,\left(a^2+b\,c\right)}{2}+2\,a\,c^2\,x^3\,\left(a^2+b\,c\right)$$

input
```
int(((c + a*x + (b*x^2)/2)^4 + 1)*(a + b*x),x)
```

output
```
x^6*((a^4*b)/2 + (b^3*c^2)/4 + (3*a^2*b^2*c)/2) + x^4*(a^4*c + (b^2*c^3)/2
 + 3*a^2*b*c^2) + x^2*(b/2 + (b*c^4)/2 + 2*a^2*c^3) + x^5*(a^5/5 + (3*a*b^
2*c^2)/2 + 2*a^3*b*c) + (b^5*x^10)/160 + x^8*((b^4*c)/16 + (a^2*b^3)/4) +
(a*b^4*x^9)/16 + a*x*(c^4 + 1) + (a*b^2*x^7*(b*c + a^2))/2 + 2*a*c^2*x^3*(
b*c + a^2)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 205, normalized size of antiderivative = 6.61

$$\int (a+bx)\left(1+\left(c+ax+\frac{bx^2}{2}\right)^4\right)\,dx$$
$$= \frac{x(b^5x^9+10a\,b^4x^8+40a^2b^3x^7+10b^4c\,x^7+80a^3b^2x^6+80a\,b^3c\,x^6+80a^4b\,x^5+240a^2b^2c\,x^5+40b^3c^2x^5+$$

input
```
int((b*x+a)*(1+(c+a*x+1/2*b*x^2)^4),x)
```

output
```
(x*(32*a**5*x**4 + 80*a**4*b*x**5 + 160*a**4*c*x**3 + 80*a**3*b**2*x**6 +
320*a**3*b*c*x**4 + 320*a**3*c**2*x**2 + 40*a**2*b**3*x**7 + 240*a**2*b**2
*c*x**5 + 480*a**2*b*c**2*x**3 + 320*a**2*c**3*x + 10*a*b**4*x**8 + 80*a*b
**3*c*x**6 + 240*a*b**2*c**2*x**4 + 320*a*b*c**3*x**2 + 160*a*c**4 + 160*a
 + b**5*x**9 + 10*b**4*c*x**7 + 40*b**3*c**2*x**5 + 80*b**2*c**3*x**3 + 80
*b*c**4*x + 80*b*x))/160
```

3.73 $\int (a+bx)\left(1+\left(ax+\frac{bx^2}{2}\right)^n\right)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 34

$$\int (a+bx)\left(1+\left(ax+\frac{bx^2}{2}\right)^n\right)\,dx=ax+\frac{bx^2}{2}+\frac{\left(ax+\frac{bx^2}{2}\right)^{1+n}}{1+n}$$

output
```
a*x+1/2*b*x^2+(a*x+1/2*b*x^2)^(1+n)/(1+n)
```

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int (a+bx)\left(1+\left(ax+\frac{bx^2}{2}\right)^n\right)\,dx=\frac{x(2a+bx)\left(1+n+\left(ax+\frac{bx^2}{2}\right)^n\right)}{2(1+n)}$$

input
```
Integrate[(a + b*x)*(1 + (a*x + (b*x^2)/2)^n),x]
```

output
```
(x*(2*a + b*x)*(1 + n + (a*x + (b*x^2)/2)^n))/(2*(1 + n))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2024, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)\left(\left(ax+\frac{bx^2}{2}\right)^n+1\right)dx$$
$$\Big\downarrow \text{2024}$$
$$\int \left(\left(ax+\frac{bx^2}{2}\right)^n+1\right)d\left(ax+\frac{bx^2}{2}\right)$$
$$\Big\downarrow \text{2009}$$
$$\frac{\left(ax+\frac{bx^2}{2}\right)^{n+1}}{n+1}+ax+\frac{bx^2}{2}$$

input
```
Int[(a + b*x)*(1 + (a*x + (b*x^2)/2)^n),x]
```

output
```
a*x + (b*x^2)/2 + (a*x + (b*x^2)/2)^(1 + n)/(1 + n)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2024
```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Simp[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q])   Subst[
Int[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D
[Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &
& PolyQ[Qr, x]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

method	result
derivativedivides	$xa+\frac{b\,x^{2}}{2}+\frac{\left(xa+\frac{1}{2}b\,x^{2}\right)^{1+n}}{1+n}$
default	$xa+\frac{b\,x^{2}}{2}+\frac{\left(xa+\frac{1}{2}b\,x^{2}\right)^{1+n}}{1+n}$
risch	$xa+\frac{b\,x^{2}}{2}+\frac{x(bx+2a)\left(\frac{1}{2}\right)^{n}\left(x(bx+2a)\right)^{n}}{2+2n}$
norman	$xa+\frac{ax\,{\mathrm e}^{n\ln\left(xa+\frac{1}{2}b\,x^{2}\right)}}{1+n}+\frac{b\,x^{2}}{2}+\frac{b\,x^{2}{\mathrm e}^{n\ln\left(xa+\frac{1}{2}b\,x^{2}\right)}}{2+2n}$
parallelrisch	$\frac{x^{2}\left(\frac{x(bx+2a)}{2}\right)^{n}b^{2}+x^{2}b^{2}n+b^{2}x^{2}+2x\left(\frac{x(bx+2a)}{2}\right)^{n}ab+2abnx+2abx-4a^{2}n-4a^{2}}{2b(1+n)}$
orering	$\frac{x(bx+2a)\left(2x^{2}b^{2}n+4abnx+3b^{2}x^{2}+2a^{2}n+6abx+2a^{2}\right)\left(1+\left(xa+\frac{1}{2}b\,x^{2}\right)^{n}\right)}{4(1+n)(bx+a)^{2}}-\frac{x^{2}(bx+2a)^{2}\left(b\left(1+\left(xa+\frac{1}{2}b\,x^{2}\right)^{n}\right)+\frac{(bx+\cdots}{\cdots}\right.}{4(1+n)(bx+a)^{2}}$

input
```
int((b*x+a)*(1+(x*a+1/2*b*x^2)^n),x,method=_RETURNVERBOSE)
```

output
```
x*a+1/2*b*x^2+(x*a+1/2*b*x^2)^(1+n)/(1+n)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.41

$$\int (a+bx)\left(1+\left(ax+\frac{bx^2}{2}\right)^n\right)\,dx = \frac{(bn+b)x^2+(bx^2+2\,ax)\left(\frac{1}{2}\,bx^2+ax\right)^n+2\,(an+a)x}{2\,(n+1)}$$

input
```
integrate((b*x+a)*(1+(a*x+1/2*b*x^2)^n),x, algorithm="fricas")
```

output
```
1/2*((b*n + b)*x^2 + (b*x^2 + 2*a*x)*(1/2*b*x^2 + a*x)^n + 2*(a*n + a)*x)/
(n + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 228 vs. $2(26) = 52$.

Time = 23.53 (sec) , antiderivative size = 228, normalized size of antiderivative = 6.71

$$\int (a+bx)\left(1+\left(ax+\frac{bx^2}{2}\right)^n\right)dx = \begin{cases} a\left(x+\frac{\log(x)}{a}\right) & \text{for } b=0 \wedge n= \\ a\left(\frac{nx}{n+1}+\frac{x(ax)^n}{n+1}+\frac{x}{n+1}\right) & \text{for } b=0 \\ ax+\frac{bx^2}{2}+\log(x)+\log\left(\frac{2a}{b}+x\right) & \text{for } n=-1 \\ \frac{2\cdot 2^n abnx}{2\cdot 2^n bn+2\cdot 2^n b}+\frac{2\cdot 2^n abx}{2\cdot 2^n bn+2\cdot 2^n b}+\frac{2^n b^2nx^2}{2\cdot 2^n bn+2\cdot 2^n b}+\frac{2^n b^2x^2}{2\cdot 2^n bn+2\cdot 2^n b}+\frac{2abx(2ax+bx^2)^n}{2\cdot 2^n bn+2\cdot 2^n b}+\frac{b^2x^2(2ax+bx^2)^n}{2\cdot 2^n bn+2\cdot 2^n b} & \text{otherwise} \end{cases}$$

input
```
integrate((b*x+a)*(1+(a*x+1/2*b*x**2)**n),x)
```

output
```
Piecewise((a*(x + log(x)/a), Eq(b, 0) & Eq(n, -1)), (a*(n*x/(n + 1) + x*(a
*x)**n/(n + 1) + x/(n + 1)), Eq(b, 0)), (a*x + b*x**2/2 + log(x) + log(2*a
/b + x), Eq(n, -1)), (2*2**n*a*b*n*x/(2*2**n*b*n + 2*2**n*b) + 2*2**n*a*b*
x/(2*2**n*b*n + 2*2**n*b) + 2**n*b**2*n*x**2/(2*2**n*b*n + 2*2**n*b) + 2**
n*b**2*x**2/(2*2**n*b*n + 2*2**n*b) + 2*a*b*x*(2*a*x + b*x**2)**n/(2*2**n*
b*n + 2*2**n*b) + b**2*x**2*(2*a*x + b*x**2)**n/(2*2**n*b*n + 2*2**n*b), T
rue))
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.53

$$\int (a+bx)\left(1+\left(ax+\frac{bx^2}{2}\right)^n\right)dx = \frac{1}{2}bx^2+ax+\frac{(bx^2+2\,ax)e^{(n\log(bx+2\,a)+n\log(x))}}{2^{n+1}n+2^{n+1}}$$

input
```
integrate((b*x+a)*(1+(a*x+1/2*b*x^2)^n),x, algorithm="maxima")
```

output
```
1/2*b*x^2 + a*x + (b*x^2 + 2*a*x)*e^(n*log(b*x + 2*a) + n*log(x))/(2^(n +
1)*n + 2^(n + 1))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int (a+bx)\left(1+\left(ax+\frac{bx^2}{2}\right)^n\right)\,dx=\frac{1}{2}\,bx^2+ax+\frac{\left(\frac{1}{2}\,bx^2+ax\right)^{n+1}}{n+1}$$

input
```
integrate((b*x+a)*(1+(a*x+1/2*b*x^2)^n),x, algorithm="giac")
```

output
```
1/2*b*x^2 + a*x + (1/2*b*x^2 + a*x)^(n + 1)/(n + 1)
```

Mupad [B] (verification not implemented)

Time = 21.89 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

$$\int (a+bx)\left(1+\left(ax+\frac{bx^2}{2}\right)^n\right)\,dx=\frac{x\,(2\,a+b\,x)\,\left(n+\left(\frac{b\,x^2}{2}+a\,x\right)^n+1\right)}{2\,(n+1)}$$

input
```
int(((a*x + (b*x^2)/2)^n + 1)*(a + b*x),x)
```

output
```
(x*(2*a + b*x)*(n + (a*x + (b*x^2)/2)^n + 1))/(2*(n + 1))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 70, normalized size of antiderivative = 2.06

$$\int (a+bx)\left(1+\left(ax+\frac{bx^2}{2}\right)^n\right)\,dx$$
$$=\frac{x\left(2(b\,x^2+2ax)^n\,a+(b\,x^2+2ax)^n\,bx+2\,2^n an+2\,2^n a+2^n bnx+2^n bx\right)}{2\,2^n\,(n+1)}$$

input
```
int((b*x+a)*(1+(a*x+1/2*b*x^2)^n),x)
```

output
```
(x*(2*(2*a*x + b*x**2)**n*a + (2*a*x + b*x**2)**n*b*x + 2*2**n*a*n + 2*2**
n*a + 2**n*b*n*x + 2**n*b*x))/(2*2**n*(n + 1))
```

3.74 $\int (a+bx)\left(1+\left(c+ax+\frac{bx^2}{2}\right)^n\right)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 35

$$\int (a+bx)\left(1+\left(c+ax+\frac{bx^2}{2}\right)^n\right)\,dx = ax+\frac{bx^2}{2}+\frac{\left(c+ax+\frac{bx^2}{2}\right)^{1+n}}{1+n}$$

output
```
a*x+1/2*b*x^2+(c+a*x+1/2*b*x^2)^(1+n)/(1+n)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 73 vs. 2(35) = 70.

Time = 0.35 (sec) , antiderivative size = 73, normalized size of antiderivative = 2.09

$$\int (a+bx)\left(1+\left(c+ax+\frac{bx^2}{2}\right)^n\right)\,dx = \frac{2c\left(c+ax+\frac{bx^2}{2}\right)^n+2ax\left(1+n+\left(c+ax+\frac{bx^2}{2}\right)^n\right)+bx^2\left(1+n+\left(c+ax+\frac{bx^2}{2}\right)^n\right)}{2(1+n)}$$

input
```
Integrate[(a + b*x)*(1 + (c + a*x + (b*x^2)/2)^n),x]
```

output
```
(2*c*(c + a*x + (b*x^2)/2)^n + 2*a*x*(1 + n + (c + a*x + (b*x^2)/2)^n) + b
*x^2*(1 + n + (c + a*x + (b*x^2)/2)^n))/(2*(1 + n))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2024, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)\left(\left(ax+\frac{bx^2}{2}+c\right)^n+1\right)dx$$

$$\Big\downarrow \text{2024}$$

$$\int \left(\left(ax+\frac{bx^2}{2}+c\right)^n+1\right)d\left(ax+\frac{bx^2}{2}+c\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(ax+\frac{bx^2}{2}+c\right)^{n+1}}{n+1}+ax+\frac{bx^2}{2}+c$$

input
```
Int[(a + b*x)*(1 + (c + a*x + (b*x^2)/2)^n),x]
```

output
```
c + a*x + (b*x^2)/2 + (c + a*x + (b*x^2)/2)^(1 + n)/(1 + n)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2024
```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Simp[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q])   Subst[
Int[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D
[Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &
& PolyQ[Qr, x]
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94

method	result
derivativedivides	$c+xa+\frac{b\,x^{2}}{2}+\frac{\left(c+xa+\frac{1}{2}b\,x^{2}\right)^{1+n}}{1+n}$
default	$c+xa+\frac{b\,x^{2}}{2}+\frac{\left(c+xa+\frac{1}{2}b\,x^{2}\right)^{1+n}}{1+n}$
risch	$xa+\frac{b\,x^{2}}{2}+\frac{\left(b\,x^{2}+2xa+2c\right)\left(\frac{1}{2}\right)^{n}\left(b\,x^{2}+2xa+2c\right)^{n}}{2+2n}$
norman	$xa+\frac{c\,{\mathrm e}^{n\ln\left(c+xa+\frac{1}{2}b\,x^{2}\right)}}{1+n}+\frac{ax\,{\mathrm e}^{n\ln\left(c+xa+\frac{1}{2}b\,x^{2}\right)}}{1+n}+\frac{b\,x^{2}}{2}+\frac{b\,x^{2}{\mathrm e}^{n\ln\left(c+xa+\frac{1}{2}b\,x^{2}\right)}}{2+2n}$
parallelrisch	$\frac{(c+xa+\frac{1}{2}b\,x^{2})^{n}b^{2}x^{2}+x^{2}b^{2}n+b^{2}x^{2}+2(c+xa+\frac{1}{2}b\,x^{2})^{n}abx+2abnx+2abx+2(c+xa+\frac{1}{2}b\,x^{2})^{n}bc-4a^{2}n-2bcn-4a^{2}-2b}{2b(1+n)}$
orering	$\frac{(2b^{3}n\,x^{4}+8a\,b^{2}n\,x^{3}+3b^{3}x^{4}+10a^{2}bn\,x^{2}+12a\,b^{2}x^{3}+4a^{3}nx+14b\,a^{2}x^{2}+6b^{2}c\,x^{2}+4a^{3}x+12abcx+4a^{2}c)(1+(c+xa+\frac{1}{2}b\,x}{4(bx+a)^{2}(1+n)}$

input
```
int((b*x+a)*(1+(c+x*a+1/2*b*x^2)^n),x,method=_RETURNVERBOSE)
```

output
```
c+x*a+1/2*b*x^2+(c+x*a+1/2*b*x^2)^(1+n)/(1+n)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.49

$$\int (a+bx)\left(1+\left(c+ax+\frac{bx^2}{2}\right)^n\right)\,dx$$
$$=\frac{(bn+b)x^2+(bx^2+2\,ax+2\,c)\left(\frac{1}{2}\,bx^2+ax+c\right)^n+2\,(an+a)x}{2\,(n+1)}$$

input
```
integrate((b*x+a)*(1+(c+a*x+1/2*b*x^2)^n),x, algorithm="fricas")
```

output
```
1/2*((b*n + b)*x^2 + (b*x^2 + 2*a*x + 2*c)*(1/2*b*x^2 + a*x + c)^n + 2*(a*
n + a)*x)/(n + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 328 vs. $2(27) = 54$.

Time = 136.09 (sec) , antiderivative size = 328, normalized size of antiderivative = 9.37

$$\int (a+bx)\left(1+\left(c+ax+\frac{bx^2}{2}\right)^n\right)dx$$
$$=\begin{cases} a\left(x+\frac{\log\left(x+\frac{c}{a}\right)}{a}\right) \\ a\left(\frac{anx}{an+a}+\frac{ax(ax+c)^n}{an+a}+\frac{ax}{an+a}+\frac{c(ax+c)^n}{an+a}\right) \\ ax+\frac{bx^2}{2}+\log\left(\frac{a}{b}+x-\frac{\sqrt{a^2-2bc}}{b}\right)+\log\left(\frac{a}{b}+x+\frac{\sqrt{a^2-2bc}}{b}\right) \\ \frac{2\cdot 2^n abnx}{2\cdot 2^n bn+2\cdot 2^n b}+\frac{2\cdot 2^n abx}{2\cdot 2^n bn+2\cdot 2^n b}+\frac{2^n b^2nx^2}{2\cdot 2^n bn+2\cdot 2^n b}+\frac{2^n b^2x^2}{2\cdot 2^n bn+2\cdot 2^n b}+\frac{2abx(2ax+bx^2+2c)^n}{2\cdot 2^n bn+2\cdot 2^n b}+\frac{b^2x^2(2ax+bx^2+2c)^n}{2\cdot 2^n bn+2\cdot 2^n b}+\frac{2bc(2ax+bx^2}{2\cdot 2^n bn+2} \end{cases}$$

input
```
integrate((b*x+a)*(1+(c+a*x+1/2*b*x**2)**n),x)
```

output
```
Piecewise((a*(x + log(x + c/a)/a), Eq(b, 0) & Eq(n, -1)), (a*(a*n*x/(a*n +
 a) + a*x*(a*x + c)**n/(a*n + a) + a*x/(a*n + a) + c*(a*x + c)**n/(a*n + a
)), Eq(b, 0)), (a*x + b*x**2/2 + log(a/b + x - sqrt(a**2 - 2*b*c)/b) + log
(a/b + x + sqrt(a**2 - 2*b*c)/b), Eq(n, -1)), (2*2**n*a*b*n*x/(2*2**n*b*n
+ 2*2**n*b) + 2*2**n*a*b*x/(2*2**n*b*n + 2*2**n*b) + 2**n*b**2*n*x**2/(2*2
**n*b*n + 2*2**n*b) + 2**n*b**2*x**2/(2*2**n*b*n + 2*2**n*b) + 2*a*b*x*(2*
a*x + b*x**2 + 2*c)**n/(2*2**n*b*n + 2*2**n*b) + b**2*x**2*(2*a*x + b*x**2
 + 2*c)**n/(2*2**n*b*n + 2*2**n*b) + 2*b*c*(2*a*x + b*x**2 + 2*c)**n/(2*2*
*n*b*n + 2*2**n*b), True))
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.54

$$\int (a+bx)\left(1+\left(c+ax+\frac{bx^2}{2}\right)^n\right)\,dx=\frac{1}{2}\,bx^2+ax+\frac{(bx^2+2\,ax+2\,c)(bx^2+2\,ax+2\,c)^n}{2^{n+1}n+2^{n+1}}$$

input
```
integrate((b*x+a)*(1+(c+a*x+1/2*b*x^2)^n),x, algorithm="maxima")
```

output
```
1/2*b*x^2 + a*x + (b*x^2 + 2*a*x + 2*c)*(b*x^2 + 2*a*x + 2*c)^n/(2^(n + 1)
*n + 2^(n + 1))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

$$\int (a+bx)\left(1+\left(c+ax+\frac{bx^2}{2}\right)^n\right)\,dx=\frac{1}{2}\,bx^2+ax+c+\frac{\left(\frac{1}{2}\,bx^2+ax+c\right)^{n+1}}{n+1}$$

input
```
integrate((b*x+a)*(1+(c+a*x+1/2*b*x^2)^n),x, algorithm="giac")
```

output
```
1/2*b*x^2 + a*x + c + (1/2*b*x^2 + a*x + c)^(n + 1)/(n + 1)
```

Mupad [B] (verification not implemented)

Time = 21.88 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.66

$$\int (a+bx)\left(1+\left(c+ax+\frac{bx^2}{2}\right)^n\right)\,dx=a\,x+\left(\frac{b\,x^2}{2}+a\,x+c\right)^n\left(\frac{2\,c}{2\,n+2}+\frac{b\,x^2}{2\,n+2}+\frac{2\,a\,x}{2\,n+2}\right)+\frac{b\,x^2}{2}$$

input
```
int(((c + a*x + (b*x^2)/2)^n + 1)*(a + b*x),x)
```

output
```
a*x + (c + a*x + (b*x^2)/2)^n*((2*c)/(2*n + 2) + (b*x^2)/(2*n + 2) + (2*a*
x)/(2*n + 2)) + (b*x^2)/2
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 102, normalized size of antiderivative = 2.91

$$\int (a+bx)\left(1+\left(c+ax+\frac{bx^2}{2}\right)^n\right)\,dx$$
$$=\frac{2(b\,x^2+2ax+2c)^n\,ax+(b\,x^2+2ax+2c)^n\,b\,x^2+2(b\,x^2+2ax+2c)^n\,c+2\,2^nanx+2\,2^nax+2^nbn\,x^2+}{2\,2^n\,(n+1)}$$

input
```
int((b*x+a)*(1+(c+a*x+1/2*b*x^2)^n),x)
```

output
```
(2*(2*a*x + b*x**2 + 2*c)**n*a*x + (2*a*x + b*x**2 + 2*c)**n*b*x**2 + 2*(2
*a*x + b*x**2 + 2*c)**n*c + 2*2**n*a*n*x + 2*2**n*a*x + 2**n*b*n*x**2 + 2*
*n*b*x**2)/(2*2**n*(n + 1))
```

3.75 $\int (a+cx^2)\left(1+\left(ax+\frac{cx^3}{3}\right)^5\right)\,dx$

Optimal result

Integrand size = 24, antiderivative size = 28

$$\int (a+cx^2)\left(1+\left(ax+\frac{cx^3}{3}\right)^5\right)dx=ax+\frac{cx^3}{3}+\frac{(3ax+cx^3)^6}{4374}$$

output
```
a*x+1/3*c*x^3+1/4374*(c*x^3+3*a*x)^6
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 93 vs. 2(28) = 56.

Time = 0.01 (sec) , antiderivative size = 93, normalized size of antiderivative = 3.32

$$\int (a+cx^2)\left(1+\left(ax+\frac{cx^3}{3}\right)^5\right)dx=ax+\frac{cx^3}{3}+\frac{a^6x^6}{6}+\frac{1}{3}a^5cx^8+\frac{5}{18}a^4c^2x^{10}+\frac{10}{81}a^3c^3x^{12}+\frac{5}{162}a^2c^4x^{14}+\frac{1}{243}ac^5x^{16}+\frac{c^6x^{18}}{4374}$$

input
```
Integrate[(a + c*x^2)*(1 + (a*x + (c*x^3)/3)^5),x]
```

output
```
a*x + (c*x^3)/3 + (a^6*x^6)/6 + (a^5*c*x^8)/3 + (5*a^4*c^2*x^10)/18 + (10*
a^3*c^3*x^12)/81 + (5*a^2*c^4*x^14)/162 + (a*c^5*x^16)/243 + (c^6*x^18)/43
74
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2024, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+cx^2)\left(\left(ax+\frac{cx^3}{3}\right)^5+1\right)dx$$

$$\Big\downarrow \text{2024}$$

$$\int \left(\left(ax+\frac{cx^3}{3}\right)^5+1\right)d\left(ax+\frac{cx^3}{3}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{6}\left(ax+\frac{cx^3}{3}\right)^6+ax+\frac{cx^3}{3}$$

input
```
Int[(a + c*x^2)*(1 + (a*x + (c*x^3)/3)^5),x]
```

output
```
a*x + (c*x^3)/3 + (a*x + (c*x^3)/3)^6/6
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2024
```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Simp[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q])   Subst[
Int[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D
[Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &
& PolyQ[Qr, x]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

method	result
default	$\frac{\left(xa+\frac{1}{3}c\,x^{3}\right)^{6}}{6}+xa+\frac{c\,x^{3}}{3}$
norman	$xa+\frac{1}{6}a^{6}x^{6}+\frac{1}{3}c\,x^{3}+\frac{1}{4374}c^{6}x^{18}+\frac{1}{243}a\,c^{5}x^{16}+\frac{5}{162}a^{2}c^{4}x^{14}+\frac{10}{81}a^{3}c^{3}x^{12}+\frac{5}{18}a^{4}c^{2}x^{10}+\frac{1}{3}a^{5}c$
risch	$xa+\frac{1}{6}a^{6}x^{6}+\frac{1}{3}c\,x^{3}+\frac{1}{4374}c^{6}x^{18}+\frac{1}{243}a\,c^{5}x^{16}+\frac{5}{162}a^{2}c^{4}x^{14}+\frac{10}{81}a^{3}c^{3}x^{12}+\frac{5}{18}a^{4}c^{2}x^{10}+\frac{1}{3}a^{5}c$
parallelrisch	$xa+\frac{1}{6}a^{6}x^{6}+\frac{1}{3}c\,x^{3}+\frac{1}{4374}c^{6}x^{18}+\frac{1}{243}a\,c^{5}x^{16}+\frac{5}{162}a^{2}c^{4}x^{14}+\frac{10}{81}a^{3}c^{3}x^{12}+\frac{5}{18}a^{4}c^{2}x^{10}+\frac{1}{3}a^{5}c$
gosper	$\frac{x\left(c^{6}x^{17}+18a\,c^{5}x^{15}+135a^{2}c^{4}x^{13}+540a^{3}c^{3}x^{11}+1215a^{4}c^{2}x^{9}+1458a^{5}c\,x^{7}+729a^{6}x^{5}+1458c\,x^{2}+4374a\right)}{4374}$
orering	$\frac{x\left(c^{6}x^{17}+18a\,c^{5}x^{15}+135a^{2}c^{4}x^{13}+540a^{3}c^{3}x^{11}+1215a^{4}c^{2}x^{9}+1458a^{5}c\,x^{7}+729a^{6}x^{5}+1458c\,x^{2}+4374a\right)\left(1+(xa\right.}{18(c^{4}x^{12}+12x^{10}a\,c^{3}+54x^{8}a^{2}c^{2}-3c^{3}x^{9}+108x^{6}a^{3}c-27x^{7}a\,c^{2}+81x^{4}a^{4}-81x^{5}a^{2}c+9c^{2}x^{6}-81a^{3}x^{3}+54x^{4}ac+81a^{2}x^{2}-27c\,x^{3}}$

input
```
int((c*x^2+a)*(1+(x*a+1/3*c*x^3)^5),x,method=_RETURNVERBOSE)
```

output
```
1/6*(x*a+1/3*c*x^3)^6+x*a+1/3*c*x^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 77 vs. $2(24) = 48$.

Time = 0.06 (sec) , antiderivative size = 77, normalized size of antiderivative = 2.75

$$\int (a+cx^2)\left(1+\left(ax+\frac{cx^3}{3}\right)^5\right)dx = \frac{1}{4374}c^6x^{18}+\frac{1}{243}ac^5x^{16}+\frac{5}{162}a^2c^4x^{14}+\frac{10}{81}a^3c^3x^{12}+\frac{5}{18}a^4c^2x^{10}+\frac{1}{3}a^5cx^8+\frac{1}{6}a^6x^6+\frac{1}{3}cx^3+ax$$

input
```
integrate((c*x^2+a)*(1+(a*x+1/3*c*x^3)^5),x, algorithm="fricas")
```

output
```
1/4374*c^6*x^18 + 1/243*a*c^5*x^16 + 5/162*a^2*c^4*x^14 + 10/81*a^3*c^3*x^
12 + 5/18*a^4*c^2*x^10 + 1/3*a^5*c*x^8 + 1/6*a^6*x^6 + 1/3*c*x^3 + a*x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(22) = 44$.

Time = 0.04 (sec) , antiderivative size = 87, normalized size of antiderivative = 3.11

$$\int (a+cx^2)\left(1+\left(ax+\frac{cx^3}{3}\right)^5\right)dx = \frac{a^6x^6}{6}+\frac{a^5cx^8}{3}+\frac{5a^4c^2x^{10}}{18}+\frac{10a^3c^3x^{12}}{81}+\frac{5a^2c^4x^{14}}{162}+\frac{ac^5x^{16}}{243}+ax+\frac{c^6x^{18}}{4374}+\frac{cx^3}{3}$$

input
```
integrate((c*x**2+a)*(1+(a*x+1/3*c*x**3)**5),x)
```

output
```
a**6*x**6/6 + a**5*c*x**8/3 + 5*a**4*c**2*x**10/18 + 10*a**3*c**3*x**12/81
 + 5*a**2*c**4*x**14/162 + a*c**5*x**16/243 + a*x + c**6*x**18/4374 + c*x*
*3/3
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 77 vs. $2(24) = 48$.

Time = 0.03 (sec) , antiderivative size = 77, normalized size of antiderivative = 2.75

$$\int (a+cx^2)\left(1+\left(ax+\frac{cx^3}{3}\right)^5\right)\,dx=\frac{1}{4374}\,c^6x^{18}+\frac{1}{243}\,ac^5x^{16}+\frac{5}{162}\,a^2c^4x^{14}+\frac{10}{81}\,a^3c^3x^{12}+\frac{5}{18}\,a^4c^2x^{10}+\frac{1}{3}\,a^5cx^8+\frac{1}{6}\,a^6x^6+\frac{1}{3}\,cx^3+ax$$

input
```
integrate((c*x^2+a)*(1+(a*x+1/3*c*x^3)^5),x, algorithm="maxima")
```

output
```
1/4374*c^6*x^18 + 1/243*a*c^5*x^16 + 5/162*a^2*c^4*x^14 + 10/81*a^3*c^3*x^
12 + 5/18*a^4*c^2*x^10 + 1/3*a^5*c*x^8 + 1/6*a^6*x^6 + 1/3*c*x^3 + a*x
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int (a+cx^2)\left(1+\left(ax+\frac{cx^3}{3}\right)^5\right)\,dx=\frac{1}{4374}\,(cx^3+3\,ax)^6+\frac{1}{3}\,cx^3+ax$$

input
```
integrate((c*x^2+a)*(1+(a*x+1/3*c*x^3)^5),x, algorithm="giac")
```

output
```
1/4374*(c*x^3 + 3*a*x)^6 + 1/3*c*x^3 + a*x
```

Mupad [B] (verification not implemented)

Time = 21.81 (sec) , antiderivative size = 77, normalized size of antiderivative = 2.75

$$\int (a+cx^2)\left(1+\left(ax+\frac{cx^3}{3}\right)^5\right)\,dx=\frac{a^6\,x^6}{6}+\frac{a^5\,c\,x^8}{3}+\frac{5\,a^4\,c^2\,x^{10}}{18}+\frac{10\,a^3\,c^3\,x^{12}}{81}+\frac{5\,a^2\,c^4\,x^{14}}{162}+\frac{a\,c^5\,x^{16}}{243}+a\,x+\frac{c^6\,x^{18}}{4374}+\frac{c\,x^3}{3}$$

input
```
int((a + c*x^2)*((a*x + (c*x^3)/3)^5 + 1),x)
```

output
```
a*x + (c*x^3)/3 + (a^6*x^6)/6 + (c^6*x^18)/4374 + (a^5*c*x^8)/3 + (a*c^5*x
^16)/243 + (5*a^4*c^2*x^10)/18 + (10*a^3*c^3*x^12)/81 + (5*a^2*c^4*x^14)/1
62
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 79, normalized size of antiderivative = 2.82

$$\int (a+cx^2)\left(1+\left(ax+\frac{cx^3}{3}\right)^5\right)dx$$
$$=\frac{x(c^6x^{17}+18a\,c^5x^{15}+135a^2c^4x^{13}+540a^3c^3x^{11}+1215a^4c^2x^9+1458a^5c\,x^7+729a^6x^5+1458c\,x^2+437}{4374}$$

input
```
int((c*x^2+a)*(1+(a*x+1/3*c*x^3)^5),x)
```

output
```
(x*(729*a**6*x**5 + 1458*a**5*c*x**7 + 1215*a**4*c**2*x**9 + 540*a**3*c**3
*x**11 + 135*a**2*c**4*x**13 + 18*a*c**5*x**15 + 4374*a + c**6*x**17 + 145
8*c*x**2))/4374
```

3.76 $\int (a+cx^2) \left(1+\left(d+ax+\frac{cx^3}{3}\right)^5\right) \, dx$

Optimal result

Integrand size = 25, antiderivative size = 31

$$\int (a+cx^2) \left(1+\left(d+ax+\frac{cx^3}{3}\right)^5\right) \, dx = ax+\frac{cx^3}{3}+\frac{(3d+3ax+cx^3)^6}{4374}$$

output
```
a*x+1/3*c*x^3+1/4374*(c*x^3+3*a*x+3*d)^6
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 140 vs. 2(31) = 62.

Time = 0.06 (sec) , antiderivative size = 140, normalized size of antiderivative = 4.52

$$\int (a+cx^2) \left(1+\left(d+ax+\frac{cx^3}{3}\right)^5\right) \, dx$$
$$= \frac{x(3a+cx^2)\left(1458+1458d^5+243a^5x^5+405a^4cx^7+270a^3c^2x^9+90a^2c^3x^{11}+15ac^4x^{13}+c^5x^{15}+1215\right.}{4374}$$

input
```
Integrate[(a + c*x^2)*(1 + (d + a*x + (c*x^3)/3)^5),x]
```

output
```
(x*(3*a + c*x^2)*(1458 + 1458*d^5 + 243*a^5*x^5 + 405*a^4*c*x^7 + 270*a^3*
c^2*x^9 + 90*a^2*c^3*x^11 + 15*a*c^4*x^13 + c^5*x^15 + 1215*d^4*(3*a*x + c
*x^3) + 540*d^3*(3*a*x + c*x^3)^2 + 135*d^2*(3*a*x + c*x^3)^3 + 18*d*(3*a*
x + c*x^3)^4))/4374
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2024, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+cx^2)\left(\left(ax+\frac{cx^3}{3}+d\right)^5+1\right)dx$$

$$\Big\downarrow \text{ 2024}$$

$$\int \left(\left(ax+\frac{cx^3}{3}+d\right)^5+1\right)d\left(ax+\frac{cx^3}{3}+d\right)$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{6}\left(ax+\frac{cx^3}{3}+d\right)^6+ax+\frac{cx^3}{3}+d$$

input
```
Int[(a + c*x^2)*(1 + (d + a*x + (c*x^3)/3)^5),x]
```

output
```
d + a*x + (c*x^3)/3 + (d + a*x + (c*x^3)/3)^6/6
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2024
```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Simp[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q])   Subst[
Int[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D
[Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &
& PolyQ[Qr, x]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

method	result
default	$\frac{\left(d+xa+\frac{1}{3}c\,x^{3}\right)^{6}}{6}+d+xa+\frac{c\,x^{3}}{3}$
norman	$\left(\frac{10}{81}a^{3}c^{3}+\frac{5}{162}c^{4}d^{2}\right)x^{12}+\left(\frac{5}{18}a^{4}c^{2}+\frac{10}{27}a\,c^{3}d^{2}\right)x^{10}+\left(\frac{5}{2}a^{4}d^{2}+\frac{5}{3}d^{4}ca\right)x^{4}+\left(\frac{1}{3}a^{5}c+\frac{5}{3}a^{2}c^{2}d^{2}\right)$
risch	$\frac{5}{18}x^{6}c^{2}d^{4}+\frac{10}{3}x^{3}a^{3}d^{3}+\frac{1}{3}x^{3}c\,d^{5}+\frac{1}{3}c\,x^{3}+\frac{5}{81}a\,c^{4}d\,x^{13}+\frac{10}{27}a^{2}c^{3}d\,x^{11}+\frac{10}{27}x^{10}a\,c^{3}d^{2}+\frac{10}{9}x^{9}a^{3}$
parallelrisch	$\frac{5}{18}x^{6}c^{2}d^{4}+\frac{10}{3}x^{3}a^{3}d^{3}+\frac{1}{3}x^{3}c\,d^{5}+\frac{1}{3}c\,x^{3}+\frac{5}{81}a\,c^{4}d\,x^{13}+\frac{10}{27}a^{2}c^{3}d\,x^{11}+\frac{10}{27}x^{10}a\,c^{3}d^{2}+\frac{10}{9}x^{9}a^{3}$
gosper	$x\left(c^{6}x^{17}+18a\,c^{5}x^{15}+18c^{5}d\,x^{14}+135a^{2}c^{4}x^{13}+270a\,c^{4}d\,x^{12}+540a^{3}c^{3}x^{11}+135x^{11}c^{4}d^{2}+1620a^{2}c^{3}d\,x^{10}+1215a^{4}c^{2}x^{9}+1620x\right.$
orering	$\frac{x\left(c^{6}x^{17}+18a\,c^{5}x^{15}+18c^{5}d\,x^{14}+135a^{2}c^{4}x^{13}+270a\,c^{4}d\,x^{12}+540a^{3}c^{3}x^{11}+135x^{11}c^{4}d^{2}+1620a^{2}c^{3}d\,x^{10}+1215a^{4}c^{2}x^{9}+1620x\right.}{18(c\,x^{3}+3xa+3d+3)(c^{4}x^{12}+12x^{10}a\,c^{3}+12d\,c^{3}x^{9}+54x^{8}a^{2}c^{2}-3c^{3}x^{9}+108d\,x^{7}a\,c^{2}+108x^{6}a^{3}c-27x^{7}}$

input
```
int((c*x^2+a)*(1+(d+x*a+1/3*c*x^3)^5),x,method=_RETURNVERBOSE)
```

output
```
1/6*(d+x*a+1/3*c*x^3)^6+d+x*a+1/3*c*x^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 280 vs. $2(27) = 54$.

Time = 0.07 (sec) , antiderivative size = 280, normalized size of antiderivative = 9.03

$$\int (a+cx^2)\left(1+\left(d+ax+\frac{cx^3}{3}\right)^5\right)dx$$
$$= \frac{1}{4374}c^6x^{18} + \frac{1}{243}ac^5x^{16} + \frac{1}{243}c^5dx^{15} + \frac{5}{162}a^2c^4x^{14} + \frac{5}{81}ac^4dx^{13} + \frac{10}{27}a^2c^3dx^{11} + \frac{5}{162}(4a^3c^3+c^4d^2)x^{12} + \frac{5}{54}(3a^4c^2+4ac^3d^2)x^{10} + \frac{10}{81}(9a^3c^2d+c^3d^3)x^9 + \frac{1}{3}(a^5c+5a^2c^2d^2)x^8 + \frac{5}{2}a^2d^4x^2 + \frac{5}{9}(3a^4cd+2ac^2d^3)x^7 + \frac{1}{18}(3a^6+60a^3cd^2+5c^2d^4)x^6 + \frac{1}{3}(3a^5d+10a^2cd^3)x^5 + \frac{5}{6}(3a^4d^2+2acd^4)x^4 + \frac{1}{3}(10a^3d^3+cd^5+c)x^3 + (ad^5+a)x$$

input

```
integrate((c*x^2+a)*(1+(d+a*x+1/3*c*x^3)^5),x, algorithm="fricas")
```

output

```
1/4374*c^6*x^18 + 1/243*a*c^5*x^16 + 1/243*c^5*d*x^15 + 5/162*a^2*c^4*x^14
 + 5/81*a*c^4*d*x^13 + 10/27*a^2*c^3*d*x^11 + 5/162*(4*a^3*c^3 + c^4*d^2)*
x^12 + 5/54*(3*a^4*c^2 + 4*a*c^3*d^2)*x^10 + 10/81*(9*a^3*c^2*d + c^3*d^3)
*x^9 + 1/3*(a^5*c + 5*a^2*c^2*d^2)*x^8 + 5/2*a^2*d^4*x^2 + 5/9*(3*a^4*c*d
+ 2*a*c^2*d^3)*x^7 + 1/18*(3*a^6 + 60*a^3*c*d^2 + 5*c^2*d^4)*x^6 + 1/3*(3*
a^5*d + 10*a^2*c*d^3)*x^5 + 5/6*(3*a^4*d^2 + 2*a*c*d^4)*x^4 + 1/3*(10*a^3*
d^3 + c*d^5 + c)*x^3 + (a*d^5 + a)*x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 314 vs. $2(26) = 52$.

Time = 0.07 (sec) , antiderivative size = 314, normalized size of antiderivative = 10.13

$$\int (a+cx^2)\left(1+\left(d+ax+\frac{cx^3}{3}\right)^5\right)dx$$
$$=\frac{5a^2c^4x^{14}}{162}+\frac{10a^2c^3dx^{11}}{27}+\frac{5a^2d^4x^2}{2}+\frac{ac^5x^{16}}{243}+\frac{5ac^4dx^{13}}{81}+\frac{c^6x^{18}}{4374}+\frac{c^5dx^{15}}{243}+x^{12}\cdot\left(\frac{10a^3c^3}{81}+\frac{5c^4d^2}{162}\right)+x^{10}\cdot\left(\frac{5a^4c^2}{18}+\frac{10ac^3d^2}{27}\right)+x^9\cdot\left(\frac{10a^3c^2d}{9}+\frac{10c^3d^3}{81}\right)+x^8\left(\frac{a^5c}{3}+\frac{5a^2c^2d^2}{3}\right)+x^7\cdot\left(\frac{5a^4cd}{3}+\frac{10ac^2d^3}{9}\right)+x^6\left(\frac{a^6}{6}+\frac{10a^3cd^2}{3}+\frac{5c^2d^4}{18}\right)+x^5\left(a^5d+\frac{10a^2cd^3}{3}\right)+x^4\cdot\left(\frac{5a^4d^2}{2}+\frac{5acd^4}{3}\right)+x^3\cdot\left(\frac{10a^3d^3}{3}+\frac{cd^5}{3}+\frac{c}{3}\right)+x\left(ad^5+a\right)$$

input
```
integrate((c*x**2+a)*(1+(d+a*x+1/3*c*x**3)**5),x)
```

output
```
5*a**2*c**4*x**14/162 + 10*a**2*c**3*d*x**11/27 + 5*a**2*d**4*x**2/2 + a*c
**5*x**16/243 + 5*a*c**4*d*x**13/81 + c**6*x**18/4374 + c**5*d*x**15/243 +
 x**12*(10*a**3*c**3/81 + 5*c**4*d**2/162) + x**10*(5*a**4*c**2/18 + 10*a*
c**3*d**2/27) + x**9*(10*a**3*c**2*d/9 + 10*c**3*d**3/81) + x**8*(a**5*c/3
 + 5*a**2*c**2*d**2/3) + x**7*(5*a**4*c*d/3 + 10*a*c**2*d**3/9) + x**6*(a*
*6/6 + 10*a**3*c*d**2/3 + 5*c**2*d**4/18) + x**5*(a**5*d + 10*a**2*c*d**3/
3) + x**4*(5*a**4*d**2/2 + 5*a*c*d**4/3) + x**3*(10*a**3*d**3/3 + c*d**5/3
 + c/3) + x*(a*d**5 + a)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 280 vs. $2(27) = 54$.

Time = 0.03 (sec) , antiderivative size = 280, normalized size of antiderivative = 9.03

$$\int (a+cx^2)\left(1+\left(d+ax+\frac{cx^3}{3}\right)^5\right)dx$$
$$= \frac{1}{4374}c^6x^{18}+\frac{1}{243}ac^5x^{16}+\frac{1}{243}c^5dx^{15}+\frac{5}{162}a^2c^4x^{14}+\frac{5}{81}ac^4dx^{13}+\frac{10}{27}a^2c^3dx^{11}+\frac{5}{162}\left(4a^3c^3+c^4d^2\right)x^{12}+\frac{5}{54}\left(3a^4c^2+4ac^3d^2\right)x^{10}+\frac{10}{81}\left(9a^3c^2d+c^3d^3\right)x^9+\frac{1}{3}\left(a^5c+5a^2c^2d^2\right)x^8+\frac{5}{2}a^2d^4x^2+\frac{5}{9}\left(3a^4cd+2ac^2d^3\right)x^7+\frac{1}{18}\left(3a^6+60a^3cd^2+5c^2d^4\right)x^6+\frac{1}{3}\left(3a^5d+10a^2cd^3\right)x^5+\frac{5}{6}\left(3a^4d^2+2acd^4\right)x^4+\frac{1}{3}\left(10a^3d^3+cd^5+c\right)x^3+\left(ad^5+a\right)x$$

input
```
integrate((c*x^2+a)*(1+(d+a*x+1/3*c*x^3)^5),x, algorithm="maxima")
```

output
```
1/4374*c^6*x^18 + 1/243*a*c^5*x^16 + 1/243*c^5*d*x^15 + 5/162*a^2*c^4*x^14
 + 5/81*a*c^4*d*x^13 + 10/27*a^2*c^3*d*x^11 + 5/162*(4*a^3*c^3 + c^4*d^2)*
x^12 + 5/54*(3*a^4*c^2 + 4*a*c^3*d^2)*x^10 + 10/81*(9*a^3*c^2*d + c^3*d^3)
*x^9 + 1/3*(a^5*c + 5*a^2*c^2*d^2)*x^8 + 5/2*a^2*d^4*x^2 + 5/9*(3*a^4*c*d
+ 2*a*c^2*d^3)*x^7 + 1/18*(3*a^6 + 60*a^3*c*d^2 + 5*c^2*d^4)*x^6 + 1/3*(3*
a^5*d + 10*a^2*c*d^3)*x^5 + 5/6*(3*a^4*d^2 + 2*a*c*d^4)*x^4 + 1/3*(10*a^3*
d^3 + c*d^5 + c)*x^3 + (a*d^5 + a)*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 105 vs. $2(27) = 54$.

Time = 0.12 (sec) , antiderivative size = 105, normalized size of antiderivative = 3.39

$$\int (a+cx^2)\left(1+\left(d+ax+\frac{cx^3}{3}\right)^5\right)dx = \frac{1}{4374}(cx^3+3ax)^6+\frac{1}{243}(cx^3+3ax)^5d + \frac{5}{162}(cx^3+3ax)^4d^2 + \frac{10}{81}(cx^3+3ax)^3d^3+\frac{5}{18}(cx^3+3ax)^2d^4 + \frac{1}{3}(cx^3+3ax)d^5+\frac{1}{3}cx^3+ax$$

input
```
integrate((c*x^2+a)*(1+(d+a*x+1/3*c*x^3)^5),x, algorithm="giac")
```

output
```
1/4374*(c*x^3 + 3*a*x)^6 + 1/243*(c*x^3 + 3*a*x)^5*d + 5/162*(c*x^3 + 3*a*
x)^4*d^2 + 10/81*(c*x^3 + 3*a*x)^3*d^3 + 5/18*(c*x^3 + 3*a*x)^2*d^4 + 1/3*
(c*x^3 + 3*a*x)*d^5 + 1/3*c*x^3 + a*x
```

Mupad [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 266, normalized size of antiderivative = 8.58

$$\int (a+cx^2)\left(1+\left(d+ax+\frac{cx^3}{3}\right)^5\right)\,dx = x^5\left(a^5\,d+\frac{10\,c\,a^2\,d^3}{3}\right)+x^4\left(\frac{5\,a^4\,d^2}{2}+\frac{5\,c\,a\,d^4}{3}\right) + x^3\left(\frac{10\,a^3\,d^3}{3}+\frac{c\,d^5}{3}+\frac{c}{3}\right) + x^6\left(\frac{a^6}{6}+\frac{10\,a^3\,c\,d^2}{3}+\frac{5\,c^2\,d^4}{18}\right) + \frac{c^6\,x^{18}}{4374}+\frac{a\,c^5\,x^{16}}{243}+a\,x\,(d^5+1) + \frac{c^5\,d\,x^{15}}{243}+\frac{5\,a^2\,c^4\,x^{14}}{162} + \frac{5\,a^2\,d^4\,x^2}{2}+\frac{5\,c^3\,x^{12}\,(4\,a^3+c\,d^2)}{162} + \frac{a^2\,c\,x^8\,(a^3+5\,c\,d^2)}{3}+\frac{10\,a^2\,c^3\,d\,x^{11}}{27} + \frac{5\,a\,c^2\,x^{10}\,(3\,a^3+4\,c\,d^2)}{54} + \frac{10\,c^2\,d\,x^9\,(9\,a^3+c\,d^2)}{81}+\frac{5\,a\,c^4\,d\,x^{13}}{81} + \frac{5\,a\,c\,d\,x^7\,(3\,a^3+2\,c\,d^2)}{9}$$

input

```
int(((d + a*x + (c*x^3)/3)^5 + 1)*(a + c*x^2),x)
```

output

```
x^5*(a^5*d + (10*a^2*c*d^3)/3) + x^4*((5*a^4*d^2)/2 + (5*a*c*d^4)/3) + x^3
*(c/3 + (c*d^5)/3 + (10*a^3*d^3)/3) + x^6*(a^6/6 + (5*c^2*d^4)/18 + (10*a^
3*c*d^2)/3) + (c^6*x^18)/4374 + (a*c^5*x^16)/243 + a*x*(d^5 + 1) + (c^5*d*
x^15)/243 + (5*a^2*c^4*x^14)/162 + (5*a^2*d^4*x^2)/2 + (5*c^3*x^12*(c*d^2
+ 4*a^3))/162 + (a^2*c*x^8*(5*c*d^2 + a^3))/3 + (10*a^2*c^3*d*x^11)/27 + (
5*a*c^2*x^10*(4*c*d^2 + 3*a^3))/54 + (10*c^2*d*x^9*(c*d^2 + 9*a^3))/81 + (
5*a*c^4*d*x^13)/81 + (5*a*c*d*x^7*(2*c*d^2 + 3*a^3))/9
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 292, normalized size of antiderivative = 9.42

$$\int (a+cx^2)\left(1+\left(d+ax+\frac{cx^3}{3}\right)^5\right)dx$$
$$= \frac{x(c^6x^{17}+18a\,c^5x^{15}+18c^5d\,x^{14}+135a^2c^4x^{13}+270a\,c^4d\,x^{12}+540a^3c^3x^{11}+135c^4d^2x^{11}+1620a^2c^3d\,x^{10}}{}$$

input
```
int((c*x^2+a)*(1+(d+a*x+1/3*c*x^3)^5),x)
```

output
```
(x*(729*a**6*x**5 + 1458*a**5*c*x**7 + 4374*a**5*d*x**4 + 1215*a**4*c**2*x
**9 + 7290*a**4*c*d*x**6 + 10935*a**4*d**2*x**3 + 540*a**3*c**3*x**11 + 48
60*a**3*c**2*d*x**8 + 14580*a**3*c*d**2*x**5 + 14580*a**3*d**3*x**2 + 135*
a**2*c**4*x**13 + 1620*a**2*c**3*d*x**10 + 7290*a**2*c**2*d**2*x**7 + 1458
0*a**2*c*d**3*x**4 + 10935*a**2*d**4*x + 18*a*c**5*x**15 + 270*a*c**4*d*x*
*12 + 1620*a*c**3*d**2*x**9 + 4860*a*c**2*d**3*x**6 + 7290*a*c*d**4*x**3 +
 4374*a*d**5 + 4374*a + c**6*x**17 + 18*c**5*d*x**14 + 135*c**4*d**2*x**11
 + 540*c**3*d**3*x**8 + 1215*c**2*d**4*x**5 + 1458*c*d**5*x**2 + 1458*c*x*
*2))/4374
```

3.77 $\int (bx+cx^2)\left(1+\left(\frac{bx^2}{2}+\frac{cx^3}{3}\right)^5\right)\,dx$

Optimal result

Integrand size = 31, antiderivative size = 34

$$\int (bx+cx^2)\left(1+\left(\frac{bx^2}{2}+\frac{cx^3}{3}\right)^5\right)\,dx=\frac{bx^2}{2}+\frac{cx^3}{3}+\frac{x^{12}(3b+2cx)^6}{279936}$$

output
```
1/2*b*x^2+1/3*c*x^3+1/279936*x^12*(2*c*x+3*b)^6
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 98 vs. 2(34) = 68.

Time = 0.01 (sec) , antiderivative size = 98, normalized size of antiderivative = 2.88

$$\int (bx+cx^2)\left(1+\left(\frac{bx^2}{2}+\frac{cx^3}{3}\right)^5\right)\,dx=\frac{bx^2}{2}+\frac{cx^3}{3}+\frac{b^6x^{12}}{384}+\frac{1}{96}b^5cx^{13}+\frac{5}{288}b^4c^2x^{14}+\frac{5}{324}b^3c^3x^{15}+\frac{5}{648}b^2c^4x^{16}+\frac{1}{486}bc^5x^{17}+\frac{c^6x^{18}}{4374}$$

input
```
Integrate[(b*x + c*x^2)*(1 + ((b*x^2)/2 + (c*x^3)/3)^5),x]
```

output
```
(b*x^2)/2 + (c*x^3)/3 + (b^6*x^12)/384 + (b^5*c*x^13)/96 + (5*b^4*c^2*x^14
)/288 + (5*b^3*c^3*x^15)/324 + (5*b^2*c^4*x^16)/648 + (b*c^5*x^17)/486 + (
c^6*x^18)/4374
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.18, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2024, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(bx+cx^2\right)\left(\left(\frac{bx^2}{2}+\frac{cx^3}{3}\right)^5+1\right)dx$$

$$\Big\downarrow \text{2024}$$

$$\int \left(\left(\frac{bx^2}{2}+\frac{cx^3}{3}\right)^5+1\right)d\left(\frac{bx^2}{2}+\frac{cx^3}{3}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{6}\left(\frac{bx^2}{2}+\frac{cx^3}{3}\right)^6+\frac{bx^2}{2}+\frac{cx^3}{3}$$

input
```
Int[(b*x + c*x^2)*(1 + ((b*x^2)/2 + (c*x^3)/3)^5),x]
```

output
```
(b*x^2)/2 + (c*x^3)/3 + ((b*x^2)/2 + (c*x^3)/3)^6/6
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2024
```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Simp[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q])   Subst[
Int[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D
[Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &
& PolyQ[Qr, x]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

method	result
default	$\frac{\left(\frac{1}{2}b\,x^{2}+\frac{1}{3}c\,x^{3}\right)^{6}}{6}+\frac{b\,x^{2}}{2}+\frac{c\,x^{3}}{3}$
gosper	$\frac{x^{2}\left(64c^{6}x^{16}+576b\,c^{5}x^{15}+2160b^{2}c^{4}x^{14}+4320b^{3}c^{3}x^{13}+4860b^{4}c^{2}x^{12}+2916b^{5}c\,x^{11}+729b^{6}x^{10}+93312cx+139968b\right)}{279936}$
norman	$\frac{1}{2}b\,x^{2}+\frac{1}{384}b^{6}x^{12}+\frac{1}{3}c\,x^{3}+\frac{1}{4374}c^{6}x^{18}+\frac{1}{486}b\,c^{5}x^{17}+\frac{5}{648}b^{2}c^{4}x^{16}+\frac{5}{324}b^{3}c^{3}x^{15}+\frac{5}{288}b^{4}c^{2}x^{14}+$
risch	$\frac{1}{2}b\,x^{2}+\frac{1}{384}b^{6}x^{12}+\frac{1}{3}c\,x^{3}+\frac{1}{4374}c^{6}x^{18}+\frac{1}{486}b\,c^{5}x^{17}+\frac{5}{648}b^{2}c^{4}x^{16}+\frac{5}{324}b^{3}c^{3}x^{15}+\frac{5}{288}b^{4}c^{2}x^{14}+$
parallelrisch	$\frac{1}{2}b\,x^{2}+\frac{1}{384}b^{6}x^{12}+\frac{1}{3}c\,x^{3}+\frac{1}{4374}c^{6}x^{18}+\frac{1}{486}b\,c^{5}x^{17}+\frac{5}{648}b^{2}c^{4}x^{16}+\frac{5}{324}b^{3}c^{3}x^{15}+\frac{5}{288}b^{4}c^{2}x^{14}+$
orering	$\frac{x\left(64c^{6}x^{16}+576b\,c^{5}x^{15}+2160b^{2}c^{4}x^{14}+4320b^{3}c^{3}x^{13}+4860b^{4}c^{2}x^{12}+2916b^{5}c\,x^{11}+729b^{6}x^{10}+93312cx+1399\right.}{36(cx+b)(2c\,x^{3}+3b\,x^{2}+6)(16c^{4}x^{12}+96b\,x^{11}c^{3}+216b^{2}x^{10}c^{2}+216b^{3}x^{9}c+81b^{4}x^{8}-48c^{3}x^{9}-216b\,x^{8}c^{2}-324b^{2}x^{7}c-162b^{3}x^{6}+1}$

input
```
int((c*x^2+b*x)*(1+(1/2*b*x^2+1/3*c*x^3)^5),x,method=_RETURNVERBOSE)
```

output
```
1/6*(1/2*b*x^2+1/3*c*x^3)^6+1/2*b*x^2+1/3*c*x^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 80 vs. $2(28) = 56$.

Time = 0.07 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.35

$$\int (bx + cx^2)\left(1 + \left(\frac{bx^2}{2} + \frac{cx^3}{3}\right)^5\right) dx = \frac{1}{4374}c^6x^{18} + \frac{1}{486}bc^5x^{17} + \frac{5}{648}b^2c^4x^{16} + \frac{5}{324}b^3c^3x^{15} + \frac{5}{288}b^4c^2x^{14} + \frac{1}{96}b^5cx^{13} + \frac{1}{384}b^6x^{12} + \frac{1}{3}cx^3 + \frac{1}{2}bx^2$$

input
```
integrate((c*x^2+b*x)*(1+(1/2*b*x^2+1/3*c*x^3)^5),x, algorithm="fricas")
```

output
```
1/4374*c^6*x^18 + 1/486*b*c^5*x^17 + 5/648*b^2*c^4*x^16 + 5/324*b^3*c^3*x^
15 + 5/288*b^4*c^2*x^14 + 1/96*b^5*c*x^13 + 1/384*b^6*x^12 + 1/3*c*x^3 + 1
/2*b*x^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(27) = 54$.

Time = 0.05 (sec) , antiderivative size = 90, normalized size of antiderivative = 2.65

$$\int (bx + cx^2)\left(1 + \left(\frac{bx^2}{2} + \frac{cx^3}{3}\right)^5\right) dx = \frac{b^6x^{12}}{384} + \frac{b^5cx^{13}}{96} + \frac{5b^4c^2x^{14}}{288} + \frac{5b^3c^3x^{15}}{324} + \frac{5b^2c^4x^{16}}{648} + \frac{bc^5x^{17}}{486} + \frac{bx^2}{2} + \frac{c^6x^{18}}{4374} + \frac{cx^3}{3}$$

input
```
integrate((c*x**2+b*x)*(1+(1/2*b*x**2+1/3*c*x**3)**5),x)
```

output
```
b**6*x**12/384 + b**5*c*x**13/96 + 5*b**4*c**2*x**14/288 + 5*b**3*c**3*x**
15/324 + 5*b**2*c**4*x**16/648 + b*c**5*x**17/486 + b*x**2/2 + c**6*x**18/
4374 + c*x**3/3
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 80 vs. 2(28) = 56.

Time = 0.03 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.35

$$\int (bx+cx^2)\left(1+\left(\frac{bx^2}{2}+\frac{cx^3}{3}\right)^5\right)dx = \frac{1}{4374}c^6x^{18}+\frac{1}{486}bc^5x^{17}+\frac{5}{648}b^2c^4x^{16}+\frac{5}{324}b^3c^3x^{15}+\frac{5}{288}b^4c^2x^{14}+\frac{1}{96}b^5cx^{13}+\frac{1}{384}b^6x^{12}+\frac{1}{3}cx^3+\frac{1}{2}bx^2$$

input
```
integrate((c*x^2+b*x)*(1+(1/2*b*x^2+1/3*c*x^3)^5),x, algorithm="maxima")
```

output
```
1/4374*c^6*x^18 + 1/486*b*c^5*x^17 + 5/648*b^2*c^4*x^16 + 5/324*b^3*c^3*x^
15 + 5/288*b^4*c^2*x^14 + 1/96*b^5*c*x^13 + 1/384*b^6*x^12 + 1/3*c*x^3 + 1
/2*b*x^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int (bx+cx^2)\left(1+\left(\frac{bx^2}{2}+\frac{cx^3}{3}\right)^5\right)dx = \frac{1}{279936}\left(2\,cx^3+3\,bx^2\right)^6+\frac{1}{3}cx^3+\frac{1}{2}bx^2$$

input
```
integrate((c*x^2+b*x)*(1+(1/2*b*x^2+1/3*c*x^3)^5),x, algorithm="giac")
```

output
```
1/279936*(2*c*x^3 + 3*b*x^2)^6 + 1/3*c*x^3 + 1/2*b*x^2
```

Mupad [B] (verification not implemented)

Time = 21.80 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.35

$$\int (bx+cx^2)\left(1+\left(\frac{bx^2}{2}+\frac{cx^3}{3}\right)^5\right)\,dx=\frac{b^6\,x^{12}}{384}+\frac{b^5\,c\,x^{13}}{96}+\frac{5\,b^4\,c^2\,x^{14}}{288}+\frac{5\,b^3\,c^3\,x^{15}}{324}+\frac{5\,b^2\,c^4\,x^{16}}{648}+\frac{b\,c^5\,x^{17}}{486}+\frac{b\,x^2}{2}+\frac{c^6\,x^{18}}{4374}+\frac{c\,x^3}{3}$$

input
```
int((b*x + c*x^2)*(((b*x^2)/2 + (c*x^3)/3)^5 + 1),x)
```

output
```
(b*x^2)/2 + (c*x^3)/3 + (b^6*x^12)/384 + (c^6*x^18)/4374 + (b^5*c*x^13)/96
 + (b*c^5*x^17)/486 + (5*b^4*c^2*x^14)/288 + (5*b^3*c^3*x^15)/324 + (5*b^2
*c^4*x^16)/648
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.35

$$\int (bx+cx^2)\left(1+\left(\frac{bx^2}{2}+\frac{cx^3}{3}\right)^5\right)\,dx$$
$$=\frac{x^2(64c^6x^{16}+576b\,c^5x^{15}+2160b^2c^4x^{14}+4320b^3c^3x^{13}+4860b^4c^2x^{12}+2916b^5c\,x^{11}+729b^6x^{10}+93312c}{279936}$$

input
```
int((c*x^2+b*x)*(1+(1/2*b*x^2+1/3*c*x^3)^5),x)
```

output
```
(x**2*(729*b**6*x**10 + 2916*b**5*c*x**11 + 4860*b**4*c**2*x**12 + 4320*b*
*3*c**3*x**13 + 2160*b**2*c**4*x**14 + 576*b*c**5*x**15 + 139968*b + 64*c*
*6*x**16 + 93312*c*x))/279936
```

3.78 $\int (bx+cx^2)\left(1+\left(d+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^5\right)\,dx$

Optimal result

Integrand size = 32, antiderivative size = 39

$$\int (bx+cx^2)\left(1+\left(d+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^5\right)\,dx=\frac{bx^2}{2}+\frac{cx^3}{3}+\frac{(6d+3bx^2+2cx^3)^6}{279936}$$

output
```
1/2*b*x^2+1/3*c*x^3+1/279936*(2*c*x^3+3*b*x^2+6*d)^6
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 146 vs. 2(39) = 78.

Time = 0.07 (sec) , antiderivative size = 146, normalized size of antiderivative = 3.74

$$\int (bx+cx^2)\left(1+\left(d+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^5\right)\,dx$$
$$=\frac{x^2(3b+2cx)\left(46656+46656d^5+243b^5x^{10}+810b^4cx^{11}+1080b^3c^2x^{12}+720b^2c^3x^{13}+240bc^4x^{14}+32c^5x\right.}{279936}$$

input
```
Integrate[(b*x + c*x^2)*(1 + (d + (b*x^2)/2 + (c*x^3)/3)^5),x]
```

output
```
(x^2*(3*b + 2*c*x)*(46656 + 46656*d^5 + 243*b^5*x^10 + 810*b^4*c*x^11 + 10
80*b^3*c^2*x^12 + 720*b^2*c^3*x^13 + 240*b*c^4*x^14 + 32*c^5*x^15 + 19440*
d^4*x^2*(3*b + 2*c*x) + 4320*d^3*x^4*(3*b + 2*c*x)^2 + 540*d^2*x^6*(3*b +
2*c*x)^3 + 36*d*x^8*(3*b + 2*c*x)^4))/279936
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {2024, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (bx + cx^2)\left(\left(\frac{bx^2}{2} + \frac{cx^3}{3} + d\right)^5 + 1\right) dx$$

$$\Big\downarrow \text{2024}$$

$$\int \left(\left(\frac{bx^2}{2} + \frac{cx^3}{3} + d\right)^5 + 1\right) d\left(\frac{bx^2}{2} + \frac{cx^3}{3} + d\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{6}\left(\frac{bx^2}{2} + \frac{cx^3}{3} + d\right)^6 + \frac{bx^2}{2} + \frac{cx^3}{3} + d$$

input
```
Int[(b*x + c*x^2)*(1 + (d + (b*x^2)/2 + (c*x^3)/3)^5),x]
```

output
```
d + (b*x^2)/2 + (c*x^3)/3 + (d + (b*x^2)/2 + (c*x^3)/3)^6/6
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2024
```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Simp[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q])   Subst[
Int[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D
[Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &
& PolyQ[Qr, x]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.85

method	result
default	$\frac{\left(d+\frac{1}{2}b\,x^{2}+\frac{1}{3}c\,x^{3}\right)^{6}}{6}+d+\frac{b\,x^{2}}{2}+\frac{c\,x^{3}}{3}$
norman	$\left(\frac{1}{2}b\,d^{5}+\frac{1}{2}b\right)x^{2}+\left(\frac{5}{324}b^{3}c^{3}+\frac{1}{243}c^{5}d\right)x^{15}+\left(\frac{5}{12}b^{3}d^{3}+\frac{5}{18}c^{2}d^{4}\right)x^{6}+\left(\frac{5}{288}b^{4}c^{2}+\frac{5}{162}b\,c^{4}d\right)x^{14}$
gosper	$\frac{x^{2}\left(64c^{6}x^{16}+576b\,c^{5}x^{15}+2160b^{2}c^{4}x^{14}+4320b^{3}c^{3}x^{13}+1152c^{5}x^{13}d+4860b^{4}c^{2}x^{12}+8640b\,c^{4}d\,x^{12}+2916b^{5}c\,x^{11}+25920b^{2}c^{3}d\,x\right.}{}$
risch	$\frac{5}{18}x^{6}c^{2}d^{4}+\frac{1}{3}x^{3}c\,d^{5}+\frac{1}{32}x^{10}b^{5}d+\frac{1}{3}c\,x^{3}+\frac{5}{162}x^{14}b\,c^{4}d+\frac{5}{54}x^{13}d\,b^{2}c^{3}+\frac{1}{2}b\,x^{2}+\frac{5}{36}x^{12}b^{3}c^{2}d+$
parallelrisch	$\frac{5}{18}x^{6}c^{2}d^{4}+\frac{1}{3}x^{3}c\,d^{5}+\frac{1}{32}x^{10}b^{5}d+\frac{1}{3}c\,x^{3}+\frac{5}{162}x^{14}b\,c^{4}d+\frac{5}{54}x^{13}d\,b^{2}c^{3}+\frac{1}{2}b\,x^{2}+\frac{5}{36}x^{12}b^{3}c^{2}d+$
orering	$\frac{x\left(64c^{6}x^{16}+576b\,c^{5}x^{15}+2160b^{2}c^{4}x^{14}+4320b^{3}c^{3}x^{13}+1152c^{5}x^{13}d+4860b^{4}c^{2}x^{12}+8640b\,c^{4}d\,x^{12}+2916b^{5}c\,x^{11}+25920b^{2}c^{3}d\,x\right.}{36(cx+b)\left(2c\,x^{3}+3b\,x^{2}+6d+6\right)\left(16c^{4}x^{12}+96b\,x^{11}c^{3}+216b^{2}x^{10}c^{2}+216b^{3}x^{9}c+192d\,c^{3}x^{9}+81b^{4}x^{8}+864db\right.}$

input
```
int((c*x^2+b*x)*(1+(d+1/2*b*x^2+1/3*c*x^3)^5),x,method=_RETURNVERBOSE)
```

output
```
1/6*(d+1/2*b*x^2+1/3*c*x^3)^6+d+1/2*b*x^2+1/3*c*x^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 289 vs. $2(33) = 66$.

Time = 0.07 (sec) , antiderivative size = 289, normalized size of antiderivative = 7.41

$$\int (bx + cx^2)\left(1 + \left(d + \frac{bx^2}{2} + \frac{cx^3}{3}\right)^5\right) dx$$
$$= \frac{1}{4374} c^6x^{18} + \frac{1}{486} bc^5x^{17} + \frac{5}{648} b^2c^4x^{16} + \frac{1}{972} (15 b^3c^3 + 4 c^5d)x^{15} + \frac{5}{2592} (9 b^4c^2 + 16 bc^4d)x^{14} + \frac{1}{864} (9 b^5c + 80 b^2c^3d)x^{13} + \frac{5}{6} b^2cd^3x^7 + \frac{1}{10368} (27 b^6 + 1440 b^3c^2d + 320 c^4d^2)x^{12} + \frac{5}{432} (9 b^4cd + 16 bc^3d^2)x^{11} + \frac{5}{6} bcd^4x^5 + \frac{1}{96} (3 b^5d + 40 b^2c^2d^2)x^{10} + \frac{5}{8} b^2d^4x^4 + \frac{5}{324} (27 b^3cd^2 + 8 c^3d^3)x^9 + \frac{5}{288} (9 b^4d^2 + 32 bc^2d^3)x^8 + \frac{5}{36} (3 b^3d^3 + 2 c^2d^4)x^6 + \frac{1}{3} (cd^5 + c)x^3 + \frac{1}{2} (bd^5 + b)x^2$$

input
```
integrate((c*x^2+b*x)*(1+(d+1/2*b*x^2+1/3*c*x^3)^5),x, algorithm="fricas")
```

output
```
1/4374*c^6*x^18 + 1/486*b*c^5*x^17 + 5/648*b^2*c^4*x^16 + 1/972*(15*b^3*c^
3 + 4*c^5*d)*x^15 + 5/2592*(9*b^4*c^2 + 16*b*c^4*d)*x^14 + 1/864*(9*b^5*c
+ 80*b^2*c^3*d)*x^13 + 5/6*b^2*c*d^3*x^7 + 1/10368*(27*b^6 + 1440*b^3*c^2*
d + 320*c^4*d^2)*x^12 + 5/432*(9*b^4*c*d + 16*b*c^3*d^2)*x^11 + 5/6*b*c*d^
4*x^5 + 1/96*(3*b^5*d + 40*b^2*c^2*d^2)*x^10 + 5/8*b^2*d^4*x^4 + 5/324*(27
*b^3*c*d^2 + 8*c^3*d^3)*x^9 + 5/288*(9*b^4*d^2 + 32*b*c^2*d^3)*x^8 + 5/36*
(3*b^3*d^3 + 2*c^2*d^4)*x^6 + 1/3*(c*d^5 + c)*x^3 + 1/2*(b*d^5 + b)*x^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 321 vs. $2(32) = 64$.

Time = 0.08 (sec) , antiderivative size = 321, normalized size of antiderivative = 8.23

$$\int (bx+cx^2)\left(1+\left(d+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^5\right)dx$$
$$=\frac{5b^2c^4x^{16}}{648}+\frac{5b^2cd^3x^7}{6}+\frac{5b^2d^4x^4}{8}+\frac{bc^5x^{17}}{486}+\frac{5bcd^4x^5}{6}+\frac{c^6x^{18}}{4374}+x^{15}\cdot\left(\frac{5b^3c^3}{324}+\frac{c^5d}{243}\right)+x^{14}\cdot\left(\frac{5b^4c^2}{288}+\frac{5bc^4d}{162}\right)+x^{13}\left(\frac{b^5c}{96}+\frac{5b^2c^3d}{54}\right)+x^{12}\left(\frac{b^6}{384}+\frac{5b^3c^2d}{36}+\frac{5c^4d^2}{162}\right)+x^{11}\cdot\left(\frac{5b^4cd}{48}+\frac{5bc^3d^2}{27}\right)+x^{10}\left(\frac{b^5d}{32}+\frac{5b^2c^2d^2}{12}\right)+x^9\cdot\left(\frac{5b^3cd^2}{12}+\frac{10c^3d^3}{81}\right)+x^8\cdot\left(\frac{5b^4d^2}{32}+\frac{5bc^2d^3}{9}\right)+x^6\cdot\left(\frac{5b^3d^3}{12}+\frac{5c^2d^4}{18}\right)+x^3\left(\frac{cd^5}{3}+\frac{c}{3}\right)+x^2\left(\frac{bd^5}{2}+\frac{b}{2}\right)$$

input
```
integrate((c*x**2+b*x)*(1+(d+1/2*b*x**2+1/3*c*x**3)**5),x)
```

output
```
5*b**2*c**4*x**16/648 + 5*b**2*c*d**3*x**7/6 + 5*b**2*d**4*x**4/8 + b*c**5
*x**17/486 + 5*b*c*d**4*x**5/6 + c**6*x**18/4374 + x**15*(5*b**3*c**3/324
+ c**5*d/243) + x**14*(5*b**4*c**2/288 + 5*b*c**4*d/162) + x**13*(b**5*c/9
6 + 5*b**2*c**3*d/54) + x**12*(b**6/384 + 5*b**3*c**2*d/36 + 5*c**4*d**2/1
62) + x**11*(5*b**4*c*d/48 + 5*b*c**3*d**2/27) + x**10*(b**5*d/32 + 5*b**2
*c**2*d**2/12) + x**9*(5*b**3*c*d**2/12 + 10*c**3*d**3/81) + x**8*(5*b**4*
d**2/32 + 5*b*c**2*d**3/9) + x**6*(5*b**3*d**3/12 + 5*c**2*d**4/18) + x**3
*(c*d**5/3 + c/3) + x**2*(b*d**5/2 + b/2)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 289 vs. $2(33) = 66$.

Time = 0.03 (sec) , antiderivative size = 289, normalized size of antiderivative = 7.41

$$\int (bx + cx^2)\left(1 + \left(d + \frac{bx^2}{2} + \frac{cx^3}{3}\right)^5\right) dx$$
$$= \frac{1}{4374} c^6 x^{18} + \frac{1}{486} bc^5 x^{17} + \frac{5}{648} b^2 c^4 x^{16} + \frac{1}{972} (15 b^3 c^3 + 4 c^5 d) x^{15} + \frac{5}{2592} (9 b^4 c^2 + 16 bc^4 d) x^{14} + \frac{1}{864} (9 b^5 c + 80 b^2 c^3 d) x^{13} + \frac{5}{6} b^2 c d^3 x^7 + \frac{1}{10368} (27 b^6 + 1440 b^3 c^2 d + 320 c^4 d^2) x^{12} + \frac{5}{432} (9 b^4 cd + 16 bc^3 d^2) x^{11} + \frac{5}{6} bcd^4 x^5 + \frac{1}{96} (3 b^5 d + 40 b^2 c^2 d^2) x^{10} + \frac{5}{8} b^2 d^4 x^4 + \frac{5}{324} (27 b^3 cd^2 + 8 c^3 d^3) x^9 + \frac{5}{288} (9 b^4 d^2 + 32 bc^2 d^3) x^8 + \frac{5}{36} (3 b^3 d^3 + 2 c^2 d^4) x^6 + \frac{1}{3} (cd^5 + c) x^3 + \frac{1}{2} (bd^5 + b) x^2$$

input
```
integrate((c*x^2+b*x)*(1+(d+1/2*b*x^2+1/3*c*x^3)^5),x, algorithm="maxima")
```

output
```
1/4374*c^6*x^18 + 1/486*b*c^5*x^17 + 5/648*b^2*c^4*x^16 + 1/972*(15*b^3*c^
3 + 4*c^5*d)*x^15 + 5/2592*(9*b^4*c^2 + 16*b*c^4*d)*x^14 + 1/864*(9*b^5*c
+ 80*b^2*c^3*d)*x^13 + 5/6*b^2*c*d^3*x^7 + 1/10368*(27*b^6 + 1440*b^3*c^2*
d + 320*c^4*d^2)*x^12 + 5/432*(9*b^4*c*d + 16*b*c^3*d^2)*x^11 + 5/6*b*c*d^
4*x^5 + 1/96*(3*b^5*d + 40*b^2*c^2*d^2)*x^10 + 5/8*b^2*d^4*x^4 + 5/324*(27
*b^3*c*d^2 + 8*c^3*d^3)*x^9 + 5/288*(9*b^4*d^2 + 32*b*c^2*d^3)*x^8 + 5/36*
(3*b^3*d^3 + 2*c^2*d^4)*x^6 + 1/3*(c*d^5 + c)*x^3 + 1/2*(b*d^5 + b)*x^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 126 vs. $2(33) = 66$.

Time = 0.13 (sec) , antiderivative size = 126, normalized size of antiderivative = 3.23

$$\int (bx + cx^2)\left(1 + \left(d + \frac{bx^2}{2} + \frac{cx^3}{3}\right)^5\right) dx$$
$$= \frac{1}{279936} (2 cx^3 + 3 bx^2)^6 + \frac{1}{7776} (2 cx^3 + 3 bx^2)^5 d + \frac{5}{2592} (2 cx^3 + 3 bx^2)^4 d^2 + \frac{5}{324} (2 cx^3 + 3 bx^2)^3 d^3 + \frac{5}{72} (2 cx^3 + 3 bx^2)^2 d^4 + \frac{1}{6} (2 cx^3 + 3 bx^2) d^5 + \frac{1}{3} cx^3 + \frac{1}{2} bx^2$$

input
```
integrate((c*x^2+b*x)*(1+(d+1/2*b*x^2+1/3*c*x^3)^5),x, algorithm="giac")
```

output
```
1/279936*(2*c*x^3 + 3*b*x^2)^6 + 1/7776*(2*c*x^3 + 3*b*x^2)^5*d + 5/2592*(
2*c*x^3 + 3*b*x^2)^4*d^2 + 5/324*(2*c*x^3 + 3*b*x^2)^3*d^3 + 5/72*(2*c*x^3
 + 3*b*x^2)^2*d^4 + 1/6*(2*c*x^3 + 3*b*x^2)*d^5 + 1/3*c*x^3 + 1/2*b*x^2
```

Mupad [B] (verification not implemented)

Time = 22.37 (sec) , antiderivative size = 273, normalized size of antiderivative = 7.00

$$\int (bx+cx^2)\left(1+\left(d+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^5\right)\,dx$$
$$= x^{13}\left(\frac{b^5\,c}{96}+\frac{5\,d\,b^2\,c^3}{54}\right)+x^{14}\left(\frac{5\,b^4\,c^2}{288}+\frac{5\,d\,b\,c^4}{162}\right)+x^{12}\left(\frac{b^6}{384}+\frac{5\,b^3\,c^2\,d}{36}+\frac{5\,c^4\,d^2}{162}\right)$$
$$+\frac{c^6\,x^{18}}{4374}+x^{15}\left(\frac{5\,b^3\,c^3}{324}+\frac{d\,c^5}{243}\right)+\frac{5\,d^3\,x^6\,(3\,b^3+2\,d\,c^2)}{36}+\frac{b\,c^5\,x^{17}}{486}$$
$$+\frac{5\,b^2\,c^4\,x^{16}}{648}+\frac{b\,x^2\,(d^5+1)}{2}+\frac{5\,b^2\,d^4\,x^4}{8}+\frac{c\,x^3\,(d^5+1)}{3}$$
$$+\frac{5\,b^2\,c\,d^3\,x^7}{6}+\frac{5\,b\,d^2\,x^8\,(9\,b^3+32\,d\,c^2)}{288}+\frac{b^2\,d\,x^{10}\,(3\,b^3+40\,d\,c^2)}{96}$$
$$+\frac{5\,c\,d^2\,x^9\,(27\,b^3+8\,d\,c^2)}{324}+\frac{5\,b\,c\,d^4\,x^5}{6}+\frac{5\,b\,c\,d\,x^{11}\,(9\,b^3+16\,d\,c^2)}{432}$$

input
```
int((b*x + c*x^2)*((d + (b*x^2)/2 + (c*x^3)/3)^5 + 1),x)
```

output
```
x^13*((b^5*c)/96 + (5*b^2*c^3*d)/54) + x^14*((5*b^4*c^2)/288 + (5*b*c^4*d)
/162) + x^12*(b^6/384 + (5*c^4*d^2)/162 + (5*b^3*c^2*d)/36) + (c^6*x^18)/4
374 + x^15*((c^5*d)/243 + (5*b^3*c^3)/324) + (5*d^3*x^6*(2*c^2*d + 3*b^3))
/36 + (b*c^5*x^17)/486 + (5*b^2*c^4*x^16)/648 + (b*x^2*(d^5 + 1))/2 + (5*b
^2*d^4*x^4)/8 + (c*x^3*(d^5 + 1))/3 + (5*b^2*c*d^3*x^7)/6 + (5*b*d^2*x^8*(
32*c^2*d + 9*b^3))/288 + (b^2*d*x^10*(40*c^2*d + 3*b^3))/96 + (5*c*d^2*x^9
*(8*c^2*d + 27*b^3))/324 + (5*b*c*d^4*x^5)/6 + (5*b*c*d*x^11*(16*c^2*d + 9
*b^3))/432
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 293, normalized size of antiderivative = 7.51

$$\int \left(bx+cx^2\right)\left(1+\left(d+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^5\right)\,dx$$
$$=\frac{x^2(64c^6x^{16}+576b\,c^5x^{15}+2160b^2c^4x^{14}+4320b^3c^3x^{13}+1152c^5d\,x^{13}+4860b^4c^2x^{12}+8640b\,c^4d\,x^{12}+29\cdots}{\cdots}$$

input
```
int((c*x^2+b*x)*(1+(d+1/2*b*x^2+1/3*c*x^3)^5),x)
```

output
```
(x**2*(729*b**6*x**10 + 2916*b**5*c*x**11 + 8748*b**5*d*x**8 + 4860*b**4*c
**2*x**12 + 29160*b**4*c*d*x**9 + 43740*b**4*d**2*x**6 + 4320*b**3*c**3*x*
*13 + 38880*b**3*c**2*d*x**10 + 116640*b**3*c*d**2*x**7 + 116640*b**3*d**3
*x**4 + 2160*b**2*c**4*x**14 + 25920*b**2*c**3*d*x**11 + 116640*b**2*c**2*
d**2*x**8 + 233280*b**2*c*d**3*x**5 + 174960*b**2*d**4*x**2 + 576*b*c**5*x
**15 + 8640*b*c**4*d*x**12 + 51840*b*c**3*d**2*x**9 + 155520*b*c**2*d**3*x
**6 + 233280*b*c*d**4*x**3 + 139968*b*d**5 + 139968*b + 64*c**6*x**16 + 11
52*c**5*d*x**13 + 8640*c**4*d**2*x**10 + 34560*c**3*d**3*x**7 + 77760*c**2
*d**4*x**4 + 93312*c*d**5*x + 93312*c*x))/279936
```

3.79 $\int (a+bx+cx^2)\left(1+\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^5\right)\,dx$

Optimal result

Integrand size = 35, antiderivative size = 43

$$\int (a+bx+cx^2)\left(1+\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^5\right)\,dx = ax+\frac{bx^2}{2}+\frac{cx^3}{3}+\frac{(6ax+3bx^2+2cx^3)^6}{279936}$$

output
```
a*x+1/2*b*x^2+1/3*c*x^3+1/279936*(2*c*x^3+3*b*x^2+6*a*x)^6
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 244 vs. 2(43) = 86.

Time = 0.08 (sec) , antiderivative size = 244, normalized size of antiderivative = 5.67

$$\int (a+bx+cx^2)\left(1+\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^5\right)dx$$
$$=\frac{a^6x^6}{6}+\frac{1}{6}a^5x^7(3b+2cx)+\frac{5}{72}a^4x^8(3b+2cx)^2+\frac{5}{324}a^3x^9(3b+2cx)^3+\frac{5a^2x^{10}(3b+2cx)^4}{2592}$$
$$+a\left(x+\frac{b^5x^{11}}{32}+\frac{5}{48}b^4cx^{12}+\frac{5}{36}b^3c^2x^{13}+\frac{5}{54}b^2c^3x^{14}+\frac{5}{162}bc^4x^{15}+\frac{c^5x^{16}}{243}\right)$$
$$+\frac{x^2(729b^6x^{10}+2916b^5cx^{11}+4860b^4c^2x^{12}+4320b^3c^3x^{13}+2160b^2c^4x^{14}+576b(243+c^5x^{15})+64cx(14}{279936}$$

input
```
Integrate[(a + b*x + c*x^2)*(1 + (a*x + (b*x^2)/2 + (c*x^3)/3)^5),x]
```

output
```
(a^6*x^6)/6 + (a^5*x^7*(3*b + 2*c*x))/6 + (5*a^4*x^8*(3*b + 2*c*x)^2)/72 +
 (5*a^3*x^9*(3*b + 2*c*x)^3)/324 + (5*a^2*x^10*(3*b + 2*c*x)^4)/2592 + a*(
x + (b^5*x^11)/32 + (5*b^4*c*x^12)/48 + (5*b^3*c^2*x^13)/36 + (5*b^2*c^3*x
^14)/54 + (5*b*c^4*x^15)/162 + (c^5*x^16)/243) + (x^2*(729*b^6*x^10 + 2916
*b^5*c*x^11 + 4860*b^4*c^2*x^12 + 4320*b^3*c^3*x^13 + 2160*b^2*c^4*x^14 +
576*b*(243 + c^5*x^15) + 64*c*x*(1458 + c^5*x^15)))/279936
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.07, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {2024, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx+cx^2)\left(\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^5+1\right)dx$$

$$\Big\downarrow \; 2024$$

$$\int \left(\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^5+1\right)d\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{6}\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^6+ax+\frac{bx^2}{2}+\frac{cx^3}{3}$$

input
```
Int[(a + b*x + c*x^2)*(1 + (a*x + (b*x^2)/2 + (c*x^3)/3)^5),x]
```

output
```
a*x + (b*x^2)/2 + (c*x^3)/3 + (a*x + (b*x^2)/2 + (c*x^3)/3)^6/6
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2024
```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Simp[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q])   Subst[
Int[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D
[Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &
& PolyQ[Qr, x]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

method	result
default	$\frac{\left(\frac{1}{3}c\,x^3+\frac{1}{2}b\,x^2+xa\right)^6}{6}+\frac{c\,x^3}{3}+\frac{b\,x^2}{2}+xa$
norman	$xa+\left(\frac{1}{243}a\,c^5+\frac{5}{648}b^2c^4\right)x^{16}+\left(\frac{1}{3}a^5c+\frac{5}{8}a^4b^2\right)x^8+\left(\frac{5}{162}ab\,c^4+\frac{5}{324}b^3c^3\right)x^{15}+\Big(\frac{5}{6}a^4bc+\frac{5}{12}$
risch	$\frac{1}{32}x^{11}b^5a+\frac{5}{8}x^8a^4b^2+\frac{1}{3}c\,x^3+\frac{1}{2}b\,x^2+\frac{5}{18}a^4c^2x^{10}+\frac{1}{4374}c^6x^{18}+xa+\frac{1}{243}a\,c^5x^{16}+\frac{1}{384}b^6x^{12}$
parallelrisch	$\frac{1}{32}x^{11}b^5a+\frac{5}{8}x^8a^4b^2+\frac{1}{3}c\,x^3+\frac{1}{2}b\,x^2+\frac{5}{18}a^4c^2x^{10}+\frac{1}{4374}c^6x^{18}+xa+\frac{1}{243}a\,c^5x^{16}+\frac{1}{384}b^6x^{12}$
gosper	$x\big(64c^6x^{17}+576b\,c^5x^{16}+1152a\,c^5x^{15}+2160b^2c^4x^{15}+8640ab\,c^4x^{14}+4320b^3c^3x^{14}+8640a^2c^4x^{13}+25920a\,b^2c^3x^{13}+4860b^4c^2$
orering	$\frac{x\big(64c^6x^{17}+576b\,c^5x^{16}+1152a\,c^5x^{15}+2160b^2c^4x^{15}+8640ab\,c^4x^{14}+4320b^3c^3x^{14}+8640a^2c^4x^{13}+25920a\,b^2c^3x^{13}+4860b^4c^2}{36(16c^4x^{12}+96b\,x^{11}c^3+192x^{10}a\,c^3+216b^2x^{10}c^2+864c^2x^9ab+216b^3x^9c+864x^8a^2c^2+129}$

input
```
int((c*x^2+b*x+a)*(1+(1/3*c*x^3+1/2*b*x^2+x*a)^5),x,method=_RETURNVERBOSE)
```

output
```
1/6*(1/3*c*x^3+1/2*b*x^2+x*a)^6+1/3*c*x^3+1/2*b*x^2+x*a
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 289 vs. $2(37) = 74$.

Time = 0.09 (sec) , antiderivative size = 289, normalized size of antiderivative = 6.72

$$\begin{aligned}
&\int (a+bx+cx^2)\left(1+\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^5\right)dx\\
&=\frac{1}{4374}c^6x^{18}+\frac{1}{486}bc^5x^{17}+\frac{1}{1944}\left(15\,b^2c^4+8\,ac^5\right)x^{16}+\frac{5}{324}\left(b^3c^3+2\,abc^4\right)x^{15}\\
&+\frac{5}{2592}\left(9\,b^4c^2+48\,ab^2c^3+16\,a^2c^4\right)x^{14}+\frac{1}{864}\left(9\,b^5c+120\,ab^3c^2+160\,a^2bc^3\right)x^{13}\\
&+\frac{1}{2}a^5bx^7+\frac{1}{10368}\left(27\,b^6+1080\,ab^4c+4320\,a^2b^2c^2+1280\,a^3c^3\right)x^{12}+\frac{1}{6}a^6x^6\\
&+\frac{1}{288}\left(9\,ab^5+120\,a^2b^3c+160\,a^3bc^2\right)x^{11}+\frac{5}{288}\left(9\,a^2b^4+48\,a^3b^2c+16\,a^4c^2\right)x^{10}\\
&+\frac{5}{12}\left(a^3b^3+2\,a^4bc\right)x^9+\frac{1}{24}\left(15\,a^4b^2+8\,a^5c\right)x^8+\frac{1}{3}cx^3+\frac{1}{2}bx^2+ax
\end{aligned}$$

input
```
integrate((c*x^2+b*x+a)*(1+(a*x+1/2*b*x^2+1/3*c*x^3)^5),x, algorithm="fric
as")
```

output
```
1/4374*c^6*x^18 + 1/486*b*c^5*x^17 + 1/1944*(15*b^2*c^4 + 8*a*c^5)*x^16 +
5/324*(b^3*c^3 + 2*a*b*c^4)*x^15 + 5/2592*(9*b^4*c^2 + 48*a*b^2*c^3 + 16*a
^2*c^4)*x^14 + 1/864*(9*b^5*c + 120*a*b^3*c^2 + 160*a^2*b*c^3)*x^13 + 1/2*
a^5*b*x^7 + 1/10368*(27*b^6 + 1080*a*b^4*c + 4320*a^2*b^2*c^2 + 1280*a^3*c
^3)*x^12 + 1/6*a^6*x^6 + 1/288*(9*a*b^5 + 120*a^2*b^3*c + 160*a^3*b*c^2)*x
^11 + 5/288*(9*a^2*b^4 + 48*a^3*b^2*c + 16*a^4*c^2)*x^10 + 5/12*(a^3*b^3 +
 2*a^4*b*c)*x^9 + 1/24*(15*a^4*b^2 + 8*a^5*c)*x^8 + 1/3*c*x^3 + 1/2*b*x^2
+ a*x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 323 vs. $2(37) = 74$.

Time = 0.10 (sec) , antiderivative size = 323, normalized size of antiderivative = 7.51

$$\int (a + bx + cx^2)\left(1 + \left(ax + \frac{bx^2}{2} + \frac{cx^3}{3}\right)^5\right) dx$$
$$= \frac{a^6x^6}{6} + \frac{a^5bx^7}{2} + ax + \frac{bc^5x^{17}}{486} + \frac{bx^2}{2} + \frac{c^6x^{18}}{4374} + \frac{cx^3}{3} + x^{16}\left(\frac{ac^5}{243} + \frac{5b^2c^4}{648}\right) + x^{15} \cdot \left(\frac{5abc^4}{162} + \frac{5b^3c^3}{324}\right) + x^{14} \cdot \left(\frac{5a^2c^4}{162} + \frac{5ab^2c^3}{54} + \frac{5b^4c^2}{288}\right) + x^{13} \cdot \left(\frac{5a^2bc^3}{27} + \frac{5ab^3c^2}{36} + \frac{b^5c}{96}\right) + x^{12} \cdot \left(\frac{10a^3c^3}{81} + \frac{5a^2b^2c^2}{12} + \frac{5ab^4c}{48} + \frac{b^6}{384}\right) + x^{11} \cdot \left(\frac{5a^3bc^2}{9} + \frac{5a^2b^3c}{12} + \frac{ab^5}{32}\right) + x^{10} \cdot \left(\frac{5a^4c^2}{18} + \frac{5a^3b^2c}{6} + \frac{5a^2b^4}{32}\right) + x^9 \cdot \left(\frac{5a^4bc}{6} + \frac{5a^3b^3}{12}\right) + x^8\left(\frac{a^5c}{3} + \frac{5a^4b^2}{8}\right)$$

input
```
integrate((c*x**2+b*x+a)*(1+(a*x+1/2*b*x**2+1/3*c*x**3)**5),x)
```

output
```
a**6*x**6/6 + a**5*b*x**7/2 + a*x + b*c**5*x**17/486 + b*x**2/2 + c**6*x**
18/4374 + c*x**3/3 + x**16*(a*c**5/243 + 5*b**2*c**4/648) + x**15*(5*a*b*c
**4/162 + 5*b**3*c**3/324) + x**14*(5*a**2*c**4/162 + 5*a*b**2*c**3/54 + 5
*b**4*c**2/288) + x**13*(5*a**2*b*c**3/27 + 5*a*b**3*c**2/36 + b**5*c/96)
+ x**12*(10*a**3*c**3/81 + 5*a**2*b**2*c**2/12 + 5*a*b**4*c/48 + b**6/384)
 + x**11*(5*a**3*b*c**2/9 + 5*a**2*b**3*c/12 + a*b**5/32) + x**10*(5*a**4*
c**2/18 + 5*a**3*b**2*c/6 + 5*a**2*b**4/32) + x**9*(5*a**4*b*c/6 + 5*a**3*
b**3/12) + x**8*(a**5*c/3 + 5*a**4*b**2/8)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 289 vs. $2(37) = 74$.

Time = 0.03 (sec) , antiderivative size = 289, normalized size of antiderivative = 6.72

$$\int (a+bx+cx^2)\left(1+\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^5\right)dx$$
$$=\frac{1}{4374}c^6x^{18}+\frac{1}{486}bc^5x^{17}+\frac{1}{1944}(15\,b^2c^4+8\,ac^5)x^{16}+\frac{5}{324}(b^3c^3+2\,abc^4)x^{15}+\frac{5}{2592}(9\,b^4c^2+48\,ab^2c^3+16\,a^2c^4)x^{14}+\frac{1}{864}(9\,b^5c+120\,ab^3c^2+160\,a^2bc^3)x^{13}+\frac{1}{2}a^5bx^7+\frac{1}{10368}(27\,b^6+1080\,ab^4c+4320\,a^2b^2c^2+1280\,a^3c^3)x^{12}+\frac{1}{6}a^6x^6+\frac{1}{288}(9\,ab^5+120\,a^2b^3c+160\,a^3bc^2)x^{11}+\frac{5}{288}(9\,a^2b^4+48\,a^3b^2c+16\,a^4c^2)x^{10}+\frac{5}{12}(a^3b^3+2\,a^4bc)x^9+\frac{1}{24}(15\,a^4b^2+8\,a^5c)x^8+\frac{1}{3}cx^3+\frac{1}{2}bx^2+ax$$

input
```
integrate((c*x^2+b*x+a)*(1+(a*x+1/2*b*x^2+1/3*c*x^3)^5),x, algorithm="maxi
ma")
```

output
```
1/4374*c^6*x^18 + 1/486*b*c^5*x^17 + 1/1944*(15*b^2*c^4 + 8*a*c^5)*x^16 +
5/324*(b^3*c^3 + 2*a*b*c^4)*x^15 + 5/2592*(9*b^4*c^2 + 48*a*b^2*c^3 + 16*a
^2*c^4)*x^14 + 1/864*(9*b^5*c + 120*a*b^3*c^2 + 160*a^2*b*c^3)*x^13 + 1/2*
a^5*b*x^7 + 1/10368*(27*b^6 + 1080*a*b^4*c + 4320*a^2*b^2*c^2 + 1280*a^3*c
^3)*x^12 + 1/6*a^6*x^6 + 1/288*(9*a*b^5 + 120*a^2*b^3*c + 160*a^3*b*c^2)*x
^11 + 5/288*(9*a^2*b^4 + 48*a^3*b^2*c + 16*a^4*c^2)*x^10 + 5/12*(a^3*b^3 +
 2*a^4*b*c)*x^9 + 1/24*(15*a^4*b^2 + 8*a^5*c)*x^8 + 1/3*c*x^3 + 1/2*b*x^2
+ a*x
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int (a+bx+cx^2)\left(1+\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^5\right)dx=\frac{1}{279936}(2\,cx^3+3\,bx^2+6\,ax)^6+\frac{1}{3}cx^3+\frac{1}{2}bx^2+ax$$

input
```
integrate((c*x^2+b*x+a)*(1+(a*x+1/2*b*x^2+1/3*c*x^3)^5),x, algorithm="giac
")
```

output
```
1/279936*(2*c*x^3 + 3*b*x^2 + 6*a*x)^6 + 1/3*c*x^3 + 1/2*b*x^2 + a*x
```

Mupad [B] (verification not implemented)

Time = 21.96 (sec) , antiderivative size = 270, normalized size of antiderivative = 6.28

$$\int (a+bx+cx^2)\left(1+\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^5\right)\,dx$$
$$= x^{12}\left(\frac{10\,a^3\,c^3}{81}+\frac{5\,a^2\,b^2\,c^2}{12}+\frac{5\,a\,b^4\,c}{48}+\frac{b^6}{384}\right)+a\,x+\frac{b\,x^2}{2}+\frac{c\,x^3}{3}+\frac{a^6\,x^6}{6}+\frac{c^6\,x^{18}}{4374}+\frac{5\,a^2\,x^{10}\,(16\,a^2\,c^2+48\,a\,b^2\,c+9\,b^4)}{288}+\frac{5\,c^2\,x^{14}\,(16\,a^2\,c^2+48\,a\,b^2\,c+9\,b^4)}{2592}+\frac{a^5\,b\,x^7}{2}+\frac{b\,c^5\,x^{17}}{486}+\frac{a^4\,x^8\,(15\,b^2+8\,a\,c)}{24}+\frac{c^4\,x^{16}\,(15\,b^2+8\,a\,c)}{1944}+\frac{a\,b\,x^{11}\,(160\,a^2\,c^2+120\,a\,b^2\,c+9\,b^4)}{288}+\frac{b\,c\,x^{13}\,(160\,a^2\,c^2+120\,a\,b^2\,c+9\,b^4)}{864}+\frac{5\,a^3\,b\,x^9\,(b^2+2\,a\,c)}{12}+\frac{5\,b\,c^3\,x^{15}\,(b^2+2\,a\,c)}{324}$$

input
```
int(((a*x + (b*x^2)/2 + (c*x^3)/3)^5 + 1)*(a + b*x + c*x^2),x)
```

output
```
x^12*(b^6/384 + (10*a^3*c^3)/81 + (5*a^2*b^2*c^2)/12 + (5*a*b^4*c)/48) + a
*x + (b*x^2)/2 + (c*x^3)/3 + (a^6*x^6)/6 + (c^6*x^18)/4374 + (5*a^2*x^10*(
9*b^4 + 16*a^2*c^2 + 48*a*b^2*c))/288 + (5*c^2*x^14*(9*b^4 + 16*a^2*c^2 +
48*a*b^2*c))/2592 + (a^5*b*x^7)/2 + (b*c^5*x^17)/486 + (a^4*x^8*(8*a*c + 1
5*b^2))/24 + (c^4*x^16*(8*a*c + 15*b^2))/1944 + (a*b*x^11*(9*b^4 + 160*a^2
*c^2 + 120*a*b^2*c))/288 + (b*c*x^13*(9*b^4 + 160*a^2*c^2 + 120*a*b^2*c))/
864 + (5*a^3*b*x^9*(2*a*c + b^2))/12 + (5*b*c^3*x^15*(2*a*c + b^2))/324
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 310, normalized size of antiderivative = 7.21

$$\int (a+bx+cx^2)\left(1+\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^5\right)dx$$
$$= \frac{x(64c^6x^{17}+576b\,c^5x^{16}+1152a\,c^5x^{15}+2160b^2c^4x^{15}+8640ab\,c^4x^{14}+4320b^3c^3x^{14}+8640a^2c^4x^{13}+259}{}$$

input
```
int((c*x^2+b*x+a)*(1+(a*x+1/2*b*x^2+1/3*c*x^3)^5),x)
```

output
```
(x*(46656*a**6*x**5 + 139968*a**5*b*x**6 + 93312*a**5*c*x**7 + 174960*a**4
*b**2*x**7 + 233280*a**4*b*c*x**8 + 77760*a**4*c**2*x**9 + 116640*a**3*b**
3*x**8 + 233280*a**3*b**2*c*x**9 + 155520*a**3*b*c**2*x**10 + 34560*a**3*c
**3*x**11 + 43740*a**2*b**4*x**9 + 116640*a**2*b**3*c*x**10 + 116640*a**2*
b**2*c**2*x**11 + 51840*a**2*b*c**3*x**12 + 8640*a**2*c**4*x**13 + 8748*a*
b**5*x**10 + 29160*a*b**4*c*x**11 + 38880*a*b**3*c**2*x**12 + 25920*a*b**2
*c**3*x**13 + 8640*a*b*c**4*x**14 + 1152*a*c**5*x**15 + 279936*a + 729*b**
6*x**11 + 2916*b**5*c*x**12 + 4860*b**4*c**2*x**13 + 4320*b**3*c**3*x**14
+ 2160*b**2*c**4*x**15 + 576*b*c**5*x**16 + 139968*b*x + 64*c**6*x**17 + 9
3312*c*x**2))/279936
```

3.80 $\int (a+bx+cx^2)\left(1+\left(d+ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^5\right)\,dx$

Optimal result

Integrand size = 36, antiderivative size = 46

$$\int (a+bx+cx^2)\left(1+\left(d+ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^5\right)\,dx = ax+\frac{bx^2}{2}+\frac{cx^3}{3}+\frac{(6d+6ax+3bx^2+2cx^3)^6}{279936}$$

output
```
a*x+1/2*b*x^2+1/3*c*x^3+1/279936*(2*c*x^3+3*b*x^2+6*a*x+6*d)^6
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 248 vs. 2(46) = 92.

Time = 0.13 (sec) , antiderivative size = 248, normalized size of antiderivative = 5.39

$$\int (a+bx+cx^2)\left(1+\left(d+ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^5\right)\,dx = \frac{x(6a+x(3b+2cx))\,(46656+46656d^5+7776a^5x^5+243b^5x^{10}+810b^4cx^{11}+1080b^3c^2x^{12}+720b^2c^3x^{13}}{}$$

input
```
Integrate[(a + b*x + c*x^2)*(1 + (d + a*x + (b*x^2)/2 + (c*x^3)/3)^5),x]
```

output
```
(x*(6*a + x*(3*b + 2*c*x))*(46656 + 46656*d^5 + 7776*a^5*x^5 + 243*b^5*x^1
0 + 810*b^4*c*x^11 + 1080*b^3*c^2*x^12 + 720*b^2*c^3*x^13 + 240*b*c^4*x^14
 + 32*c^5*x^15 + 6480*a^4*x^6*(3*b + 2*c*x) + 2160*a^3*x^7*(3*b + 2*c*x)^2
 + 360*a^2*x^8*(3*b + 2*c*x)^3 + 30*a*x^9*(3*b + 2*c*x)^4 + 19440*d^4*x*(6
*a + x*(3*b + 2*c*x)) + 4320*d^3*x^2*(6*a + x*(3*b + 2*c*x))^2 + 540*d^2*x
^3*(6*a + x*(3*b + 2*c*x))^3 + 36*d*x^4*(6*a + x*(3*b + 2*c*x))^4))/279936
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.04, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {2024, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx+cx^2\right)\left(\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}+d\right)^5+1\right)dx$$

$$\Big\downarrow \text{2024}$$

$$\int \left(\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}+d\right)^5+1\right)d\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}+d\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{6}\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}+d\right)^6+ax+\frac{bx^2}{2}+\frac{cx^3}{3}+d$$

input
```
Int[(a + b*x + c*x^2)*(1 + (d + a*x + (b*x^2)/2 + (c*x^3)/3)^5),x]
```

output
```
d + a*x + (b*x^2)/2 + (c*x^3)/3 + (d + a*x + (b*x^2)/2 + (c*x^3)/3)^6/6
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2024
```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Simp[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q])   Subst[
Int[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D
[Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &
& PolyQ[Qr, x]
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.85

method	result
default	$\frac{\left(d+xa+\frac{1}{2}b\,x^{2}+\frac{1}{3}c\,x^{3}\right)^{6}}{6}+d+xa+\frac{b\,x^{2}}{2}+\frac{c\,x^{3}}{3}$
norman	$\left(\frac{1}{243}a\,c^{5}+\frac{5}{648}b^{2}c^{4}\right)x^{16}+\left(\frac{5}{2}a^{2}d^{4}+\frac{1}{2}b\,d^{5}+\frac{1}{2}b\right)x^{2}+\left(\frac{5}{162}ab\,c^{4}+\frac{5}{324}b^{3}c^{3}+\frac{1}{243}c^{5}d\right)x^{15}+\left(\frac{5}{16}\right.$
gosper	$\frac{x\left(64c^{6}x^{17}+576b\,c^{5}x^{16}+1152a\,c^{5}x^{15}+2160b^{2}c^{4}x^{15}+8640ab\,c^{4}x^{14}+4320b^{3}c^{3}x^{14}+1152c^{5}d\,x^{14}+8640a^{2}c^{4}x^{13}+25920a\,b^{2}c^{3}x\right.}{}$
risch	$\frac{1}{32}x^{11}b^{5}a+\frac{5}{18}x^{6}c^{2}d^{4}+\frac{10}{3}x^{3}a^{3}d^{3}+\frac{1}{3}x^{3}c\,d^{5}+\frac{5}{8}x^{8}a^{4}b^{2}+\frac{10}{27}x^{12}ab\,c^{3}d+\frac{5}{6}x^{11}d\,c^{2}b^{2}a+\frac{5}{3}x^{10}a$
parallelrisch	$\frac{1}{32}x^{11}b^{5}a+\frac{5}{18}x^{6}c^{2}d^{4}+\frac{10}{3}x^{3}a^{3}d^{3}+\frac{1}{3}x^{3}c\,d^{5}+\frac{5}{8}x^{8}a^{4}b^{2}+\frac{10}{27}x^{12}ab\,c^{3}d+\frac{5}{6}x^{11}d\,c^{2}b^{2}a+\frac{5}{3}x^{10}a$
orering	Expression too large to display

input
```
int((c*x^2+b*x+a)*(1+(d+x*a+1/2*b*x^2+1/3*c*x^3)^5),x,method=_RETURNVERBOS
E)
```

output
```
1/6*(d+x*a+1/2*b*x^2+1/3*c*x^3)^6+d+x*a+1/2*b*x^2+1/3*c*x^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 773 vs. $2(40) = 80$.

Time = 0.08 (sec) , antiderivative size = 773, normalized size of antiderivative = 16.80

$$\int (a+bx+cx^2)\left(1+\left(d+ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^5\right)dx = \text{Too large to display}$$

input
```
integrate((c*x^2+b*x+a)*(1+(d+a*x+1/2*b*x^2+1/3*c*x^3)^5),x, algorithm="fr
icas")
```

output
```
1/4374*c^6*x^18 + 1/486*b*c^5*x^17 + 1/1944*(15*b^2*c^4 + 8*a*c^5)*x^16 +
1/972*(15*b^3*c^3 + 30*a*b*c^4 + 4*c^5*d)*x^15 + 5/2592*(9*b^4*c^2 + 48*a*
b^2*c^3 + 16*a^2*c^4 + 16*b*c^4*d)*x^14 + 1/2592*(27*b^5*c + 360*a*b^3*c^2
 + 480*a^2*b*c^3 + 80*(3*b^2*c^3 + 2*a*c^4)*d)*x^13 + 1/10368*(27*b^6 + 10
80*a*b^4*c + 4320*a^2*b^2*c^2 + 1280*a^3*c^3 + 320*c^4*d^2 + 480*(3*b^3*c^
2 + 8*a*b*c^3)*d)*x^12 + 1/864*(27*a*b^5 + 360*a^2*b^3*c + 480*a^3*b*c^2 +
 160*b*c^3*d^2 + 10*(9*b^4*c + 72*a*b^2*c^2 + 32*a^2*c^3)*d)*x^11 + 1/864*
(135*a^2*b^4 + 720*a^3*b^2*c + 240*a^4*c^2 + 40*(9*b^2*c^2 + 8*a*c^3)*d^2
+ 9*(3*b^5 + 80*a*b^3*c + 160*a^2*b*c^2)*d)*x^10 + 5/1296*(108*a^3*b^3 + 2
16*a^4*b*c + 32*c^3*d^3 + 108*(b^3*c + 4*a*b*c^2)*d^2 + 9*(9*a*b^4 + 72*a^
2*b^2*c + 32*a^3*c^2)*d)*x^9 + 1/288*(180*a^4*b^2 + 96*a^5*c + 160*b*c^2*d
^3 + 15*(3*b^4 + 48*a*b^2*c + 32*a^2*c^2)*d^2 + 120*(3*a^2*b^3 + 8*a^3*b*c
)*d)*x^8 + 1/36*(18*a^5*b + 10*(3*b^2*c + 4*a*c^2)*d^3 + 45*(a*b^3 + 4*a^2
*b*c)*d^2 + 30*(3*a^3*b^2 + 2*a^4*c)*d)*x^7 + 1/36*(6*a^6 + 90*a^4*b*d + 1
0*c^2*d^4 + 15*(b^3 + 8*a*b*c)*d^3 + 15*(9*a^2*b^2 + 8*a^3*c)*d^2)*x^6 + 1
/6*(6*a^5*d + 30*a^3*b*d^2 + 5*b*c*d^4 + 5*(3*a*b^2 + 4*a^2*c)*d^3)*x^5 +
5/24*(12*a^4*d^2 + 24*a^2*b*d^3 + (3*b^2 + 8*a*c)*d^4)*x^4 + 1/6*(20*a^3*d
^3 + 15*a*b*d^4 + 2*c*d^5 + 2*c)*x^3 + 1/2*(5*a^2*d^4 + b*d^5 + b)*x^2 + (
a*d^5 + a)*x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 930 vs. $2(41) = 82$.

Time = 0.13 (sec) , antiderivative size = 930, normalized size of antiderivative = 20.22

$$\int (a + bx + cx^2)\left(1 + \left(d + ax + \frac{bx^2}{2} + \frac{cx^3}{3}\right)^5\right) dx = \text{Too large to display}$$

input
```
integrate((c*x**2+b*x+a)*(1+(d+a*x+1/2*b*x**2+1/3*c*x**3)**5),x)
```

output
```
b*c**5*x**17/486 + c**6*x**18/4374 + x**16*(a*c**5/243 + 5*b**2*c**4/648)
+ x**15*(5*a*b*c**4/162 + 5*b**3*c**3/324 + c**5*d/243) + x**14*(5*a**2*c*
*4/162 + 5*a*b**2*c**3/54 + 5*b**4*c**2/288 + 5*b*c**4*d/162) + x**13*(5*a
**2*b*c**3/27 + 5*a*b**3*c**2/36 + 5*a*c**4*d/81 + b**5*c/96 + 5*b**2*c**3
*d/54) + x**12*(10*a**3*c**3/81 + 5*a**2*b**2*c**2/12 + 5*a*b**4*c/48 + 10
*a*b*c**3*d/27 + b**6/384 + 5*b**3*c**2*d/36 + 5*c**4*d**2/162) + x**11*(5
*a**3*b*c**2/9 + 5*a**2*b**3*c/12 + 10*a**2*c**3*d/27 + a*b**5/32 + 5*a*b*
*2*c**2*d/6 + 5*b**4*c*d/48 + 5*b*c**3*d**2/27) + x**10*(5*a**4*c**2/18 +
5*a**3*b**2*c/6 + 5*a**2*b**4/32 + 5*a**2*b*c**2*d/3 + 5*a*b**3*c*d/6 + 10
*a*c**3*d**2/27 + b**5*d/32 + 5*b**2*c**2*d**2/12) + x**9*(5*a**4*b*c/6 +
5*a**3*b**3/12 + 10*a**3*c**2*d/9 + 5*a**2*b**2*c*d/2 + 5*a*b**4*d/16 + 5*
a*b*c**2*d**2/3 + 5*b**3*c*d**2/12 + 10*c**3*d**3/81) + x**8*(a**5*c/3 + 5
*a**4*b**2/8 + 10*a**3*b*c*d/3 + 5*a**2*b**3*d/4 + 5*a**2*c**2*d**2/3 + 5*
a*b**2*c*d**2/2 + 5*b**4*d**2/32 + 5*b*c**2*d**3/9) + x**7*(a**5*b/2 + 5*a
**4*c*d/3 + 5*a**3*b**2*d/2 + 5*a**2*b*c*d**2 + 5*a*b**3*d**2/4 + 10*a*c**
2*d**3/9 + 5*b**2*c*d**3/6) + x**6*(a**6/6 + 5*a**4*b*d/2 + 10*a**3*c*d**2
/3 + 15*a**2*b**2*d**2/4 + 10*a*b*c*d**3/3 + 5*b**3*d**3/12 + 5*c**2*d**4/
18) + x**5*(a**5*d + 5*a**3*b*d**2 + 10*a**2*c*d**3/3 + 5*a*b**2*d**3/2 +
5*b*c*d**4/6) + x**4*(5*a**4*d**2/2 + 5*a**2*b*d**3 + 5*a*c*d**4/3 + 5*b**
2*d**4/8) + x**3*(10*a**3*d**3/3 + 5*a*b*d**4/2 + c*d**5/3 + c/3) + x**...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 773 vs. $2(40) = 80$.

Time = 0.04 (sec) , antiderivative size = 773, normalized size of antiderivative = 16.80

$$\int (a+bx+cx^2)\left(1+\left(d+ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^5\right)\,dx = \text{Too large to display}$$

input
```
integrate((c*x^2+b*x+a)*(1+(d+a*x+1/2*b*x^2+1/3*c*x^3)^5),x, algorithm="ma
xima")
```

output
```
1/4374*c^6*x^18 + 1/486*b*c^5*x^17 + 1/1944*(15*b^2*c^4 + 8*a*c^5)*x^16 +
1/972*(15*b^3*c^3 + 30*a*b*c^4 + 4*c^5*d)*x^15 + 5/2592*(9*b^4*c^2 + 48*a*
b^2*c^3 + 16*a^2*c^4 + 16*b*c^4*d)*x^14 + 1/2592*(27*b^5*c + 360*a*b^3*c^2
 + 480*a^2*b*c^3 + 80*(3*b^2*c^3 + 2*a*c^4)*d)*x^13 + 1/10368*(27*b^6 + 10
80*a*b^4*c + 4320*a^2*b^2*c^2 + 1280*a^3*c^3 + 320*c^4*d^2 + 480*(3*b^3*c^
2 + 8*a*b*c^3)*d)*x^12 + 1/864*(27*a*b^5 + 360*a^2*b^3*c + 480*a^3*b*c^2 +
 160*b*c^3*d^2 + 10*(9*b^4*c + 72*a*b^2*c^2 + 32*a^2*c^3)*d)*x^11 + 1/864*
(135*a^2*b^4 + 720*a^3*b^2*c + 240*a^4*c^2 + 40*(9*b^2*c^2 + 8*a*c^3)*d^2
+ 9*(3*b^5 + 80*a*b^3*c + 160*a^2*b*c^2)*d)*x^10 + 5/1296*(108*a^3*b^3 + 2
16*a^4*b*c + 32*c^3*d^3 + 108*(b^3*c + 4*a*b*c^2)*d^2 + 9*(9*a*b^4 + 72*a^
2*b^2*c + 32*a^3*c^2)*d)*x^9 + 1/288*(180*a^4*b^2 + 96*a^5*c + 160*b*c^2*d
^3 + 15*(3*b^4 + 48*a*b^2*c + 32*a^2*c^2)*d^2 + 120*(3*a^2*b^3 + 8*a^3*b*c
)*d)*x^8 + 1/36*(18*a^5*b + 10*(3*b^2*c + 4*a*c^2)*d^3 + 45*(a*b^3 + 4*a^2
*b*c)*d^2 + 30*(3*a^3*b^2 + 2*a^4*c)*d)*x^7 + 1/36*(6*a^6 + 90*a^4*b*d + 1
0*c^2*d^4 + 15*(b^3 + 8*a*b*c)*d^3 + 15*(9*a^2*b^2 + 8*a^3*c)*d^2)*x^6 + 1
/6*(6*a^5*d + 30*a^3*b*d^2 + 5*b*c*d^4 + 5*(3*a*b^2 + 4*a^2*c)*d^3)*x^5 +
5/24*(12*a^4*d^2 + 24*a^2*b*d^3 + (3*b^2 + 8*a*c)*d^4)*x^4 + 1/6*(20*a^3*d
^3 + 15*a*b*d^4 + 2*c*d^5 + 2*c)*x^3 + 1/2*(5*a^2*d^4 + b*d^5 + b)*x^2 + (
a*d^5 + a)*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(40) = 80$.

Time = 0.12 (sec) , antiderivative size = 153, normalized size of antiderivative = 3.33

$$\int (a+bx+cx^2)\left(1+\left(d+ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^5\right)dx$$
$$=\frac{1}{279936}\left(2\,cx^3+3\,bx^2+6\,ax\right)^6+\frac{1}{7776}\left(2\,cx^3+3\,bx^2+6\,ax\right)^5d+\frac{5}{2592}\left(2\,cx^3+3\,bx^2+6\,ax\right)^4d^2+\frac{5}{324}\left(2\,cx^3+3\,bx^2+6\,ax\right)^3d^3+\frac{5}{72}\left(2\,cx^3+3\,bx^2+6\,ax\right)^2d^4+\frac{1}{6}\left(2\,cx^3+3\,bx^2+6\,ax\right)d^5+\frac{1}{3}\,cx^3+\frac{1}{2}\,bx^2+ax$$

input
```
integrate((c*x^2+b*x+a)*(1+(d+a*x+1/2*b*x^2+1/3*c*x^3)^5),x, algorithm="gi
ac")
```

output
```
1/279936*(2*c*x^3 + 3*b*x^2 + 6*a*x)^6 + 1/7776*(2*c*x^3 + 3*b*x^2 + 6*a*x
)^5*d + 5/2592*(2*c*x^3 + 3*b*x^2 + 6*a*x)^4*d^2 + 5/324*(2*c*x^3 + 3*b*x^
2 + 6*a*x)^3*d^3 + 5/72*(2*c*x^3 + 3*b*x^2 + 6*a*x)^2*d^4 + 1/6*(2*c*x^3 +
 3*b*x^2 + 6*a*x)*d^5 + 1/3*c*x^3 + 1/2*b*x^2 + a*x
```

Mupad [B] (verification not implemented)

Time = 22.29 (sec) , antiderivative size = 753, normalized size of antiderivative = 16.37

$$\int (a+bx+cx^2)\left(1+\left(d+ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^5\right)dx = \text{Too large to display}$$

input
```
int(((d + a*x + (b*x^2)/2 + (c*x^3)/3)^5 + 1)*(a + b*x + c*x^2),x)
```

output
```
x^10*((b^5*d)/32 + (5*a^2*b^4)/32 + (5*a^4*c^2)/18 + (5*a^3*b^2*c)/6 + (10
*a*c^3*d^2)/27 + (5*b^2*c^2*d^2)/12 + (5*a*b^3*c*d)/6 + (5*a^2*b*c^2*d)/3)
 + x^8*((a^5*c)/3 + (5*a^4*b^2)/8 + (5*b^4*d^2)/32 + (5*a^2*b^3*d)/4 + (5*
b*c^2*d^3)/9 + (5*a^2*c^2*d^2)/3 + (10*a^3*b*c*d)/3 + (5*a*b^2*c*d^2)/2) +
 x^9*((5*a^3*b^3)/12 + (10*c^3*d^3)/81 + (10*a^3*c^2*d)/9 + (5*b^3*c*d^2)/
12 + (5*a^4*b*c)/6 + (5*a*b^4*d)/16 + (5*a*b*c^2*d^2)/3 + (5*a^2*b^2*c*d)/
2) + x^14*((5*a^2*c^4)/162 + (5*b^4*c^2)/288 + (5*a*b^2*c^3)/54 + (5*b*c^4
*d)/162) + x^12*(b^6/384 + (10*a^3*c^3)/81 + (5*c^4*d^2)/162 + (5*b^3*c^2*
d)/36 + (5*a^2*b^2*c^2)/12 + (5*a*b^4*c)/48 + (10*a*b*c^3*d)/27) + x^6*(a^
6/6 + (5*b^3*d^3)/12 + (5*c^2*d^4)/18 + (10*a^3*c*d^2)/3 + (15*a^2*b^2*d^2
)/4 + (5*a^4*b*d)/2 + (10*a*b*c*d^3)/3) + x^3*(c/3 + (c*d^5)/3 + (10*a^3*d
^3)/3 + (5*a*b*d^4)/2) + x^11*((a*b^5)/32 + (5*a^2*b^3*c)/12 + (5*a^3*b*c^
2)/9 + (10*a^2*c^3*d)/27 + (5*b*c^3*d^2)/27 + (5*b^4*c*d)/48 + (5*a*b^2*c^
2*d)/6) + x^7*((a^5*b)/2 + (5*a*b^3*d^2)/4 + (5*a^3*b^2*d)/2 + (10*a*c^2*d
^3)/9 + (5*b^2*c*d^3)/6 + (5*a^4*c*d)/3 + 5*a^2*b*c*d^2) + x^2*(b/2 + (b*d
^5)/2 + (5*a^2*d^4)/2) + x^13*((b^5*c)/96 + (5*a*b^3*c^2)/36 + (5*a^2*b*c^
3)/27 + (5*b^2*c^3*d)/54 + (5*a*c^4*d)/81) + x^5*(a^5*d + (5*a*b^2*d^3)/2
+ 5*a^3*b*d^2 + (10*a^2*c*d^3)/3 + (5*b*c*d^4)/6) + (c^6*x^18)/4374 + (5*d
^2*x^4*(12*a^4 + 3*b^2*d^2 + 24*a^2*b*d + 8*a*c*d^2))/24 + a*x*(d^5 + 1) +
 (b*c^5*x^17)/486 + (c^3*x^15*(4*c^2*d + 15*b^3 + 30*a*b*c))/972 + (c^4...
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 926, normalized size of antiderivative = 20.13

$$\int (a+bx+cx^2)\left(1+\left(d+ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^5\right)dx = \text{Too large to display}$$

input
```
int((c*x^2+b*x+a)*(1+(d+a*x+1/2*b*x^2+1/3*c*x^3)^5),x)
```

output

```
(x*(46656*a**6*x**5 + 139968*a**5*b*x**6 + 93312*a**5*c*x**7 + 279936*a**5
*d*x**4 + 174960*a**4*b**2*x**7 + 233280*a**4*b*c*x**8 + 699840*a**4*b*d*x
**5 + 77760*a**4*c**2*x**9 + 466560*a**4*c*d*x**6 + 699840*a**4*d**2*x**3
+ 116640*a**3*b**3*x**8 + 233280*a**3*b**2*c*x**9 + 699840*a**3*b**2*d*x**
6 + 155520*a**3*b*c**2*x**10 + 933120*a**3*b*c*d*x**7 + 1399680*a**3*b*d**
2*x**4 + 34560*a**3*c**3*x**11 + 311040*a**3*c**2*d*x**8 + 933120*a**3*c*d
**2*x**5 + 933120*a**3*d**3*x**2 + 43740*a**2*b**4*x**9 + 116640*a**2*b**3
*c*x**10 + 349920*a**2*b**3*d*x**7 + 116640*a**2*b**2*c**2*x**11 + 699840*
a**2*b**2*c*d*x**8 + 1049760*a**2*b**2*d**2*x**5 + 51840*a**2*b*c**3*x**12
 + 466560*a**2*b*c**2*d*x**9 + 1399680*a**2*b*c*d**2*x**6 + 1399680*a**2*b
*d**3*x**3 + 8640*a**2*c**4*x**13 + 103680*a**2*c**3*d*x**10 + 466560*a**2
*c**2*d**2*x**7 + 933120*a**2*c*d**3*x**4 + 699840*a**2*d**4*x + 8748*a*b*
*5*x**10 + 29160*a*b**4*c*x**11 + 87480*a*b**4*d*x**8 + 38880*a*b**3*c**2*
x**12 + 233280*a*b**3*c*d*x**9 + 349920*a*b**3*d**2*x**6 + 25920*a*b**2*c*
*3*x**13 + 233280*a*b**2*c**2*d*x**10 + 699840*a*b**2*c*d**2*x**7 + 699840
*a*b**2*d**3*x**4 + 8640*a*b*c**4*x**14 + 103680*a*b*c**3*d*x**11 + 466560
*a*b*c**2*d**2*x**8 + 933120*a*b*c*d**3*x**5 + 699840*a*b*d**4*x**2 + 1152
*a*c**5*x**15 + 17280*a*c**4*d*x**12 + 103680*a*c**3*d**2*x**9 + 311040*a*
c**2*d**3*x**6 + 466560*a*c*d**4*x**3 + 279936*a*d**5 + 279936*a + 729*b**
6*x**11 + 2916*b**5*c*x**12 + 8748*b**5*d*x**9 + 4860*b**4*c**2*x**13 +...
```

3.81 $\int (a+cx^2)\left(1+\left(ax+\frac{cx^3}{3}\right)^n\right)\,dx$

Optimal result

Integrand size = 24, antiderivative size = 34

$$\int (a+cx^2)\left(1+\left(ax+\frac{cx^3}{3}\right)^n\right)\,dx = ax+\frac{cx^3}{3}+\frac{\left(ax+\frac{cx^3}{3}\right)^{1+n}}{1+n}$$

output
```
a*x+1/3*c*x^3+(a*x+1/3*c*x^3)^(1+n)/(1+n)
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int (a+cx^2)\left(1+\left(ax+\frac{cx^3}{3}\right)^n\right)\,dx = \frac{x(3a+cx^2)\left(1+n+\left(ax+\frac{cx^3}{3}\right)^n\right)}{3(1+n)}$$

input
```
Integrate[(a + c*x^2)*(1 + (a*x + (c*x^3)/3)^n),x]
```

output
```
(x*(3*a + c*x^2)*(1 + n + (a*x + (c*x^3)/3)^n))/(3*(1 + n))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2024, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+cx^2)\left(\left(ax+\frac{cx^3}{3}\right)^n+1\right)dx$$

$$\Big\downarrow \text{2024}$$

$$\int \left(\left(ax+\frac{cx^3}{3}\right)^n+1\right)d\left(ax+\frac{cx^3}{3}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(ax+\frac{cx^3}{3}\right)^{n+1}}{n+1}+ax+\frac{cx^3}{3}$$

input
```
Int[(a + c*x^2)*(1 + (a*x + (c*x^3)/3)^n),x]
```

output
```
a*x + (c*x^3)/3 + (a*x + (c*x^3)/3)^(1 + n)/(1 + n)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2024
```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Simp[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q])   Subst[
Int[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D
[Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &
& PolyQ[Qr, x]
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

method	result
derivativedivides	$xa+\frac{c\,x^{3}}{3}+\frac{\left(xa+\frac{1}{3}c\,x^{3}\right)^{1+n}}{1+n}$
default	$xa+\frac{c\,x^{3}}{3}+\frac{\left(xa+\frac{1}{3}c\,x^{3}\right)^{1+n}}{1+n}$
risch	$xa+\frac{c\,x^{3}}{3}+\frac{x\left(c\,x^{2}+3a\right)\left(\frac{1}{3}\right)^{n}\left(x\left(c\,x^{2}+3a\right)\right)^{n}}{3+3n}$
norman	$xa+\frac{ax\,\mathrm{e}^{n\ln\left(xa+\frac{1}{3}c\,x^{3}\right)}}{1+n}+\frac{c\,x^{3}}{3}+\frac{c\,x^{3}\mathrm{e}^{n\ln\left(xa+\frac{1}{3}c\,x^{3}\right)}}{3+3n}$
parallelrisch	$\frac{x^{3}\left(\frac{x\left(c\,x^{2}+3a\right)}{3}\right)^{n}c^{2}+x^{3}c^{2}n+c^{2}x^{3}+3x\left(\frac{x\left(c\,x^{2}+3a\right)}{3}\right)^{n}ac+3xacn+3xac}{3c(1+n)}$
orering	$\frac{x\left(c\,x^{2}+3a\right)\left(3c^{2}n\,x^{4}+5c^{2}x^{4}+6acn\,x^{2}+12x^{2}ac+3a^{2}n+3a^{2}\right)\left(1+\left(xa+\frac{1}{3}c\,x^{3}\right)^{n}\right)}{9(1+n)\left(c\,x^{2}+a\right)^{2}}-\frac{\left(c\,x^{2}+3a\right)^{2}x^{2}\left(2cx\left(1+\left(xa+\frac{1}{3}c\,x^{3}\right.\right.\right.}{9(1+n)(c}$

input
```
int((c*x^2+a)*(1+(x*a+1/3*c*x^3)^n),x,method=_RETURNVERBOSE)
```

output
```
x*a+1/3*c*x^3+(x*a+1/3*c*x^3)^(1+n)/(1+n)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.41

$$\int\left(a+cx^{2}\right)\left(1+\left(ax+\frac{cx^{3}}{3}\right)^{n}\right)dx$$
$$=\frac{(cn+c)x^{3}+\left(cx^{3}+3\,ax\right)\left(\frac{1}{3}\,cx^{3}+ax\right)^{n}+3\,(an+a)x}{3\,(n+1)}$$

input
```
integrate((c*x^2+a)*(1+(a*x+1/3*c*x^3)^n),x, algorithm="fricas")
```

output
```
1/3*((c*n + c)*x^3 + (c*x^3 + 3*a*x)*(1/3*c*x^3 + a*x)^n + 3*(a*n + a)*x)/
(n + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 190 vs. $2(26) = 52$.

Time = 49.77 (sec) , antiderivative size = 190, normalized size of antiderivative = 5.59

$$\int (a+cx^2)\left(1+\left(ax+\frac{cx^3}{3}\right)^n\right)dx = \begin{cases} \frac{3\cdot 3^n anx}{3\cdot 3^n n+3\cdot 3^n}+\frac{3\cdot 3^n ax}{3\cdot 3^n n+3\cdot 3^n}+\frac{3^n cnx^3}{3\cdot 3^n n+3\cdot 3^n}+\frac{3^n cx^3}{3\cdot 3^n n+3\cdot 3^n}+\frac{3ax(3ax+cx^3)^n}{3\cdot 3^n n+3\cdot 3^n}+\frac{cx^3(3ax+cx^3)^n}{3\cdot 3^n n+3\cdot 3^n} & \text{for } n\neq -1 \\ ax+\frac{cx^3}{3}+\log(x)+\log\left(x-\sqrt{3}\sqrt{-\frac{a}{c}}\right)+\log\left(x+\sqrt{3}\sqrt{-\frac{a}{c}}\right) & \text{otherwise} \end{cases}$$

input
```
integrate((c*x**2+a)*(1+(a*x+1/3*c*x**3)**n),x)
```

output
```
Piecewise((3*3**n*a*n*x/(3*3**n*n + 3*3**n) + 3*3**n*a*x/(3*3**n*n + 3*3**
n) + 3**n*c*n*x**3/(3*3**n*n + 3*3**n) + 3**n*c*x**3/(3*3**n*n + 3*3**n) +
 3*a*x*(3*a*x + c*x**3)**n/(3*3**n*n + 3*3**n) + c*x**3*(3*a*x + c*x**3)**
n/(3*3**n*n + 3*3**n), Ne(n, -1)), (a*x + c*x**3/3 + log(x) + log(x - sqrt
(3)*sqrt(-a/c)) + log(x + sqrt(3)*sqrt(-a/c)), True))
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.59

$$\int (a+cx^2)\left(1+\left(ax+\frac{cx^3}{3}\right)^n\right)dx = \frac{1}{3}cx^3+ax+\frac{(cx^3+3ax)e^{(n\log(cx^2+3a)+n\log(x))}}{3^{n+1}n+3^{n+1}}$$

input
```
integrate((c*x^2+a)*(1+(a*x+1/3*c*x^3)^n),x, algorithm="maxima")
```

output
```
1/3*c*x^3 + a*x + (c*x^3 + 3*a*x)*e^(n*log(c*x^2 + 3*a) + n*log(x))/(3^(n
+ 1)*n + 3^(n + 1))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int (a+cx^2)\left(1+\left(ax+\frac{cx^3}{3}\right)^n\right)\,dx=\frac{1}{3}\,cx^3+ax+\frac{\left(\frac{1}{3}\,cx^3+ax\right)^{n+1}}{n+1}$$

input
```
integrate((c*x^2+a)*(1+(a*x+1/3*c*x^3)^n),x, algorithm="giac")
```

output
```
1/3*c*x^3 + a*x + (1/3*c*x^3 + a*x)^(n + 1)/(n + 1)
```

Mupad [B] (verification not implemented)

Time = 21.85 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.97

$$\int (a+cx^2)\left(1+\left(ax+\frac{cx^3}{3}\right)^n\right)\,dx=\frac{x\,(c\,x^2+3\,a)\,\left(n+\left(\frac{c\,x^3}{3}+a\,x\right)^n+1\right)}{3\,(n+1)}$$

input
```
int((a + c*x^2)*((a*x + (c*x^3)/3)^n + 1),x)
```

output
```
(x*(3*a + c*x^2)*(n + (a*x + (c*x^3)/3)^n + 1))/(3*(n + 1))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.24

$$\int (a+cx^2)\left(1+\left(ax+\frac{cx^3}{3}\right)^n\right)\,dx$$
$$=\frac{x\left(3(c\,x^3+3ax)^n\,a+(c\,x^3+3ax)^n\,c\,x^2+3\,3^n an+3\,3^n a+3^n cn\,x^2+3^n c\,x^2\right)}{3\,3^n\,(n+1)}$$

input
```
int((c*x^2+a)*(1+(a*x+1/3*c*x^3)^n),x)
```

output
```
(x*(3*(3*a*x + c*x**3)**n*a + (3*a*x + c*x**3)**n*c*x**2 + 3*3**n*a*n + 3*
3**n*a + 3**n*c*n*x**2 + 3**n*c*x**2))/(3*3**n*(n + 1))
```

3.82 $\int (bx+cx^2)\left(1+\left(\frac{bx^2}{2}+\frac{cx^3}{3}\right)^n\right)\,dx$

Optimal result

Integrand size = 31, antiderivative size = 44

$$\int (bx+cx^2)\left(1+\left(\frac{bx^2}{2}+\frac{cx^3}{3}\right)^n\right)\,dx=\frac{bx^2}{2}+\frac{cx^3}{3}+\frac{\left(\frac{bx^2}{2}+\frac{cx^3}{3}\right)^{1+n}}{1+n}$$

output
```
1/2*b*x^2+1/3*c*x^3+(1/2*b*x^2+1/3*c*x^3)^(1+n)/(1+n)
```

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.95

$$\int (bx+cx^2)\left(1+\left(\frac{bx^2}{2}+\frac{cx^3}{3}\right)^n\right)\,dx=\frac{x^2(3b+2cx)\left(1+n+\left(\frac{bx^2}{2}+\frac{cx^3}{3}\right)^n\right)}{6(1+n)}$$

input
```
Integrate[(b*x + c*x^2)*(1 + ((b*x^2)/2 + (c*x^3)/3)^n),x]
```

output
```
(x^2*(3*b + 2*c*x)*(1 + n + ((b*x^2)/2 + (c*x^3)/3)^n))/(6*(1 + n))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2024, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (bx+cx^2)\left(\left(\frac{bx^2}{2}+\frac{cx^3}{3}\right)^n+1\right)dx$$

$$\Big\downarrow \ 2024$$

$$\int \left(\left(\frac{bx^2}{2}+\frac{cx^3}{3}\right)^n+1\right)d\left(\frac{bx^2}{2}+\frac{cx^3}{3}\right)$$

$$\Big\downarrow \ 2009$$

$$\frac{\left(\frac{bx^2}{2}+\frac{cx^3}{3}\right)^{n+1}}{n+1}+\frac{bx^2}{2}+\frac{cx^3}{3}$$

input
```
Int[(b*x + c*x^2)*(1 + ((b*x^2)/2 + (c*x^3)/3)^n),x]
```

output
```
(b*x^2)/2 + (c*x^3)/3 + ((b*x^2)/2 + (c*x^3)/3)^(1 + n)/(1 + n)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2024
```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Simp[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q])   Subst[
Int[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D
[Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &
& PolyQ[Qr, x]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.84

method	result
derivativedivides	$\frac{b\,x^{2}}{2}+\frac{c\,x^{3}}{3}+\frac{\left(\frac{1}{2}b\,x^{2}+\frac{1}{3}c\,x^{3}\right)^{1+n}}{1+n}$
default	$\frac{b\,x^{2}}{2}+\frac{c\,x^{3}}{3}+\frac{\left(\frac{1}{2}b\,x^{2}+\frac{1}{3}c\,x^{3}\right)^{1+n}}{1+n}$
risch	$\frac{b\,x^{2}}{2}+\frac{c\,x^{3}}{3}+\frac{x^{2}(2cx+3b)\left(\frac{1}{3}\right)^{n}\left(\frac{1}{2}\right)^{n}\left(x^{2}(2cx+3b)\right)^{n}}{6n+6}$
norman	$\frac{b\,x^{2}}{2}+\frac{c\,x^{3}}{3}+\frac{b\,x^{2}\mathrm{e}^{n\ln\left(\frac{1}{2}b\,x^{2}+\frac{1}{3}c\,x^{3}\right)}}{2+2n}+\frac{c\,x^{3}\mathrm{e}^{n\ln\left(\frac{1}{2}b\,x^{2}+\frac{1}{3}c\,x^{3}\right)}}{3+3n}$
parallelrisch	$\frac{2x^{3}\left(\frac{x^{2}(2cx+3b)}{6}\right)^{n}c^{2}+2x^{3}c^{2}n+2c^{2}x^{3}+3x^{2}\left(\frac{x^{2}(2cx+3b)}{6}\right)^{n}bc+3bcn\,x^{2}+3bc\,x^{2}}{6c(1+n)}$
orering	$\frac{(2cx+3b)x\left(6c^{2}n\,x^{2}+12bcnx+10c^{2}x^{2}+6b^{2}n+20bcx+9b^{2}\right)\left(c\,x^{2}+bx\right)\left(1+\left(\frac{1}{2}b\,x^{2}+\frac{1}{3}c\,x^{3}\right)^{n}\right)}{36(1+n)(cx+b)^{3}}-\frac{x^{2}(2cx+3b)^{2}\Big((2cx+b$

input
```
int((c*x^2+b*x)*(1+(1/2*b*x^2+1/3*c*x^3)^n),x,method=_RETURNVERBOSE)
```

output
```
1/2*b*x^2+1/3*c*x^3+(1/2*b*x^2+1/3*c*x^3)^(1+n)/(1+n)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.30

$$\int (bx+cx^2)\left(1+\left(\frac{bx^2}{2}+\frac{cx^3}{3}\right)^n\right)\,dx$$
$$=\frac{2\,(cn+c)x^3+3\,(bn+b)x^2+(2\,cx^3+3\,bx^2)\left(\frac{1}{3}\,cx^3+\frac{1}{2}\,bx^2\right)^n}{6\,(n+1)}$$

input
```
integrate((c*x^2+b*x)*(1+(1/2*b*x^2+1/3*c*x^3)^n),x, algorithm="fricas")
```

output
```
1/6*(2*(c*n + c)*x^3 + 3*(b*n + b)*x^2 + (2*c*x^3 + 3*b*x^2)*(1/3*c*x^3 +
1/2*b*x^2)^n)/(n + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 189 vs. $2(32) = 64$.

Time = 92.27 (sec) , antiderivative size = 189, normalized size of antiderivative = 4.30

$$\int (bx+cx^2)\left(1+\left(\frac{bx^2}{2}+\frac{cx^3}{3}\right)^n\right)dx = \begin{cases} \frac{3\cdot 6^n bnx^2}{6\cdot 6^n n+6\cdot 6^n}+\frac{3\cdot 6^n bx^2}{6\cdot 6^n n+6\cdot 6^n}+\frac{2\cdot 6^n cnx^3}{6\cdot 6^n n+6\cdot 6^n}+\frac{2\cdot 6^n cx^3}{6\cdot 6^n n+6\cdot 6^n}+\frac{3bx^2(3bx^2+2cx^3)^n}{6\cdot 6^n n+6\cdot 6^n}+\frac{2cx^3(3bx^2+2cx^3)^n}{6\cdot 6^n n+6\cdot 6^n} & \text{for } n\neq -1 \\ \frac{bx^2}{2}+\frac{cx^3}{3}+2\log(x)+\log\left(\frac{3b}{2c}+x\right) & \text{otherwise} \end{cases}$$

input
```
integrate((c*x**2+b*x)*(1+(1/2*b*x**2+1/3*c*x**3)**n),x)
```

output
```
Piecewise((3*6**n*b*n*x**2/(6*6**n*n + 6*6**n) + 3*6**n*b*x**2/(6*6**n*n +
 6*6**n) + 2*6**n*c*n*x**3/(6*6**n*n + 6*6**n) + 2*6**n*c*x**3/(6*6**n*n +
 6*6**n) + 3*b*x**2*(3*b*x**2 + 2*c*x**3)**n/(6*6**n*n + 6*6**n) + 2*c*x**
3*(3*b*x**2 + 2*c*x**3)**n/(6*6**n*n + 6*6**n), Ne(n, -1)), (b*x**2/2 + c*
x**3/3 + 2*log(x) + log(3*b/(2*c) + x), True))
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.61

$$\int (bx+cx^2)\left(1+\left(\frac{bx^2}{2}+\frac{cx^3}{3}\right)^n\right)dx = \frac{1}{3}cx^3+\frac{1}{2}bx^2 + \frac{(2cx^3+3bx^2)e^{(n\log(2cx+3b)+2n\log(x))}}{3^{n+1}2^{n+1}n+3^{n+1}2^{n+1}}$$

input
```
integrate((c*x^2+b*x)*(1+(1/2*b*x^2+1/3*c*x^3)^n),x, algorithm="maxima")
```

output
```
1/3*c*x^3 + 1/2*b*x^2 + (2*c*x^3 + 3*b*x^2)*e^(n*log(2*c*x + 3*b) + 2*n*lo
g(x))/(3^(n + 1)*2^(n + 1)*n + 3^(n + 1)*2^(n + 1))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.82

$$\int (bx+cx^2)\left(1+\left(\frac{bx^2}{2}+\frac{cx^3}{3}\right)^n\right)\,dx=\frac{1}{3}\,cx^3+\frac{1}{2}\,bx^2+\frac{\left(\frac{1}{3}\,cx^3+\frac{1}{2}\,bx^2\right)^{n+1}}{n+1}$$

input
```
integrate((c*x^2+b*x)*(1+(1/2*b*x^2+1/3*c*x^3)^n),x, algorithm="giac")
```

output
```
1/3*c*x^3 + 1/2*b*x^2 + (1/3*c*x^3 + 1/2*b*x^2)^(n + 1)/(n + 1)
```

Mupad [B] (verification not implemented)

Time = 21.93 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.84

$$\int (bx+cx^2)\left(1+\left(\frac{bx^2}{2}+\frac{cx^3}{3}\right)^n\right)\,dx=\frac{x^2\,(3\,b+2\,c\,x)\,\left(n+\left(\frac{c\,x^3}{3}+\frac{b\,x^2}{2}\right)^n+1\right)}{6\,(n+1)}$$

input
```
int((b*x + c*x^2)*(((b*x^2)/2 + (c*x^3)/3)^n + 1),x)
```

output
```
(x^2*(3*b + 2*c*x)*(n + ((b*x^2)/2 + (c*x^3)/3)^n + 1))/(6*(n + 1))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.84

$$\int (bx+cx^2)\left(1+\left(\frac{bx^2}{2}+\frac{cx^3}{3}\right)^n\right)\,dx$$
$$=\frac{x^2\left(3(2c\,x^3+3b\,x^2)^n\,b+2(2c\,x^3+3b\,x^2)^n\,cx+3\,6^nbn+3\,6^nb+2\,6^ncnx+2\,6^ncx\right)}{6\,6^n\,(n+1)}$$

input
```
int((c*x^2+b*x)*(1+(1/2*b*x^2+1/3*c*x^3)^n),x)
```

output
```
(x**2*(3*(3*b*x**2 + 2*c*x**3)**n*b + 2*(3*b*x**2 + 2*c*x**3)**n*c*x + 3*6
**n*b*n + 3*6**n*b + 2*6**n*c*n*x + 2*6**n*c*x))/(6*6**n*(n + 1))
```

3.83 $\int (a+bx+cx^2) \left(1+\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^n\right) dx$

Optimal result

Integrand size = 35, antiderivative size = 50

$$\int (a+bx+cx^2) \left(1+\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^n\right) dx = ax+\frac{bx^2}{2}+\frac{cx^3}{3}+\frac{\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^{1+n}}{1+n}$$

output
```
a*x+1/2*b*x^2+1/3*c*x^3+(a*x+1/2*b*x^2+1/3*c*x^3)^(1+n)/(1+n)
```

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.98

$$\int (a+bx+cx^2) \left(1+\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^n\right) dx = \frac{x(6a+x(3b+2cx))\left(1+n+\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^n\right)}{6(1+n)}$$

input
```
Integrate[(a + b*x + c*x^2)*(1 + (a*x + (b*x^2)/2 + (c*x^3)/3)^n),x]
```

output
```
(x*(6*a + x*(3*b + 2*c*x))*(1 + n + (a*x + (b*x^2)/2 + (c*x^3)/3)^n))/(6*(
1 + n))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {2024, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx+cx^2\right)\left(\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^n+1\right)dx$$

$$\Big\downarrow \text{2024}$$

$$\int \left(\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^n+1\right)d\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^{n+1}}{n+1}+ax+\frac{bx^2}{2}+\frac{cx^3}{3}$$

input
```
Int[(a + b*x + c*x^2)*(1 + (a*x + (b*x^2)/2 + (c*x^3)/3)^n),x]
```

output
```
a*x + (b*x^2)/2 + (c*x^3)/3 + (a*x + (b*x^2)/2 + (c*x^3)/3)^(1 + n)/(1 + n
)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2024
```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Simp[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q])   Subst[
Int[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D
[Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &
& PolyQ[Qr, x]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.86

method	result
derivativedivides	$xa+\frac{b\,x^{2}}{2}+\frac{c\,x^{3}}{3}+\frac{\left(\frac{1}{3}c\,x^{3}+\frac{1}{2}b\,x^{2}+xa\right)^{1+n}}{1+n}$
default	$xa+\frac{b\,x^{2}}{2}+\frac{c\,x^{3}}{3}+\frac{\left(\frac{1}{3}c\,x^{3}+\frac{1}{2}b\,x^{2}+xa\right)^{1+n}}{1+n}$
risch	$\frac{c\,x^{3}}{3}+\frac{b\,x^{2}}{2}+xa+\frac{x\left(2c\,x^{2}+3bx+6a\right)\left(\frac{1}{3}\right)^{n}\left(\frac{1}{2}\right)^{n}\left(x\left(2c\,x^{2}+3bx+6a\right)\right)^{n}}{6n+6}$
norman	$xa+\frac{ax\,{\mathrm e}^{n\ln\left(\frac{1}{3}c\,x^{3}+\frac{1}{2}b\,x^{2}+xa\right)}}{1+n}+\frac{b\,x^{2}}{2}+\frac{c\,x^{3}}{3}+\frac{b\,x^{2}{\mathrm e}^{n\ln\left(\frac{1}{3}c\,x^{3}+\frac{1}{2}b\,x^{2}+xa\right)}}{2+2n}+\frac{c\,x^{3}{\mathrm e}^{n\ln\left(\frac{1}{3}c\,x^{3}+\frac{1}{2}b\,x^{2}+xa\right)}}{3+3n}$
parallelrisch	$\frac{2x^{3}\left(\frac{x\left(2c\,x^{2}+3bx+6a\right)}{6}\right)^{n}c^{2}+2x^{3}c^{2}n+2c^{2}x^{3}+3x^{2}\left(\frac{x\left(2c\,x^{2}+3bx+6a\right)}{6}\right)^{n}bc+3bcn\,x^{2}+3bc\,x^{2}+6x\left(\frac{x\left(2c\,x^{2}+3bx+6a\right)}{6}\right)}{6c(1+n)}$
orering	$\frac{x\left(2c\,x^{2}+3bx+6a\right)\left(6c^{2}n\,x^{4}+12bcn\,x^{3}+10c^{2}x^{4}+12acn\,x^{2}+6x^{2}b^{2}n+20bc\,x^{3}+12abnx+24x^{2}ac+9b^{2}x^{2}+6a^{2}n+18abx+\right.}{36(1+n)(c\,x^{2}+bx+a)^{2}}$

input
```
int((c*x^2+b*x+a)*(1+(1/3*c*x^3+1/2*b*x^2+x*a)^n),x,method=_RETURNVERBOSE)
```

output
```
x*a+1/2*b*x^2+1/3*c*x^3+(1/3*c*x^3+1/2*b*x^2+x*a)^(1+n)/(1+n)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.44

$$\int \left(a+bx+cx^2\right)\left(1+\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^n\right)\,dx = \frac{2\,(cn+c)x^3+3\,(bn+b)x^2+\left(2\,cx^3+3\,bx^2+6\,ax\right)\left(\frac{1}{3}\,cx^3+\frac{1}{2}\,bx^2+ax\right)^n+6\,(an+a)x}{6\,(n+1)}$$

input
```
integrate((c*x^2+b*x+a)*(1+(a*x+1/2*b*x^2+1/3*c*x^3)^n),x, algorithm="fric
as")
```

output
```
1/6*(2*(c*n + c)*x^3 + 3*(b*n + b)*x^2 + (2*c*x^3 + 3*b*x^2 + 6*a*x)*(1/3*
c*x^3 + 1/2*b*x^2 + a*x)^n + 6*(a*n + a)*x)/(n + 1)
```

Sympy [F(-1)]

Timed out.

$$\int \left(a+bx+cx^2\right)\left(1+\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^n\right)\,dx = \text{Timed out}$$

input
```
integrate((c*x**2+b*x+a)*(1+(a*x+1/2*b*x**2+1/3*c*x**3)**n),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.66

$$\int \left(a+bx+cx^2\right)\left(1+\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^n\right)\,dx = \frac{1}{3}\,cx^3+\frac{1}{2}\,bx^2+ax+\frac{\left(2\,cx^3+3\,bx^2+6\,ax\right)e^{\left(n\log\left(2\,cx^2+3\,bx+6\,a\right)+n\log\left(x\right)\right)}}{3^{n+1}2^{n+1}n+3^{n+1}2^{n+1}}$$

input
```
integrate((c*x^2+b*x+a)*(1+(a*x+1/2*b*x^2+1/3*c*x^3)^n),x, algorithm="maxi
ma")
```

output
```
1/3*c*x^3 + 1/2*b*x^2 + a*x + (2*c*x^3 + 3*b*x^2 + 6*a*x)*e^(n*log(2*c*x^2
 + 3*b*x + 6*a) + n*log(x))/(3^(n + 1)*2^(n + 1)*n + 3^(n + 1)*2^(n + 1))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.84

$$\int (a+bx+cx^2)\left(1+\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^n\right)\,dx = \frac{1}{3}\,cx^3+\frac{1}{2}\,bx^2+ax+\frac{\left(\frac{1}{3}\,cx^3+\frac{1}{2}\,bx^2+ax\right)^{n+1}}{n+1}$$

input
```
integrate((c*x^2+b*x+a)*(1+(a*x+1/2*b*x^2+1/3*c*x^3)^n),x, algorithm="giac
")
```

output
```
1/3*c*x^3 + 1/2*b*x^2 + a*x + (1/3*c*x^3 + 1/2*b*x^2 + a*x)^(n + 1)/(n + 1
)
```

Mupad [B] (verification not implemented)

Time = 21.84 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.46

$$\int (a+bx+cx^2)\left(1+\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^n\right)\,dx = a\,x+\left(\frac{3\,b\,x^2}{6\,n+6}+\frac{2\,c\,x^3}{6\,n+6}+\frac{6\,a\,x}{6\,n+6}\right)\left(\frac{c\,x^3}{3}+\frac{b\,x^2}{2}+a\,x\right)^n+\frac{b\,x^2}{2}+\frac{c\,x^3}{3}$$

input
```
int(((a*x + (b*x^2)/2 + (c*x^3)/3)^n + 1)*(a + b*x + c*x^2),x)
```

output
```
a*x + ((3*b*x^2)/(6*n + 6) + (2*c*x^3)/(6*n + 6) + (6*a*x)/(6*n + 6))*(a*x
 + (b*x^2)/2 + (c*x^3)/3)^n + (b*x^2)/2 + (c*x^3)/3
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 131, normalized size of antiderivative = 2.62

$$\int \left(a+bx+cx^2\right)\left(1+\left(ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right)^n\right)dx$$
$$=\frac{x\big(6(2c\,x^3+3b\,x^2+6ax)^n\,a+3(2c\,x^3+3b\,x^2+6ax)^n\,bx+2(2c\,x^3+3b\,x^2+6ax)^n\,c\,x^2+6\,6^n an+6\,6^n}{6\,6^n\,(n+1)}$$

input
```
int((c*x^2+b*x+a)*(1+(a*x+1/2*b*x^2+1/3*c*x^3)^n),x)
```

output
```
(x*(6*(6*a*x + 3*b*x**2 + 2*c*x**3)**n*a + 3*(6*a*x + 3*b*x**2 + 2*c*x**3)
**n*b*x + 2*(6*a*x + 3*b*x**2 + 2*c*x**3)**n*c*x**2 + 6*6**n*a*n + 6*6**n*
a + 3*6**n*b*n*x + 3*6**n*b*x + 2*6**n*c*n*x**2 + 2*6**n*c*x**2))/(6*6**n*
(n + 1))
```

3.84 $\int \left(-4+4x+x^2\right)\left(5-12x+6x^2+x^3\right)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 19

$$\int \left(-4+4x+x^2\right)\left(5-12x+6x^2+x^3\right)\,dx=\frac{1}{6}\left(5-12x+6x^2+x^3\right)^2$$

output
```
1/6*(x^3+6*x^2-12*x+5)^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.74

$$\int \left(-4+4x+x^2\right)\left(5-12x+6x^2+x^3\right)\,dx=-20x+34x^2-\frac{67x^3}{3}+2x^4+2x^5+\frac{x^6}{6}$$

input
```
Integrate[(-4 + 4*x + x^2)*(5 - 12*x + 6*x^2 + x^3),x]
```

output
```
-20*x + 34*x^2 - (67*x^3)/3 + 2*x^4 + 2*x^5 + x^6/6
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {2021}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(x^2+4x-4\right)\left(x^3+6x^2-12x+5\right)\,dx$$

$$\Big\downarrow \text{ 2021}$$

$$\frac{1}{6}\left(x^3+6x^2-12x+5\right)^2$$

input
```
Int[(-4 + 4*x + x^2)*(5 - 12*x + 6*x^2 + x^3),x]
```

output
```
(5 - 12*x + 6*x^2 + x^3)^2/6
```

Defintions of rubi rules used

rule 2021
```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

method	result	size
default	$\frac{(x^3+6x^2-12x+5)^2}{6}$	18
gosper	$\frac{x(x^5+12x^4+12x^3-134x^2+204x-120)}{6}$	27
norman	$\frac{1}{6}x^6+2x^5+2x^4-\frac{67}{3}x^3+34x^2-20x$	30
parallelrisch	$\frac{1}{6}x^6+2x^5+2x^4-\frac{67}{3}x^3+34x^2-20x$	30
risch	$\frac{1}{6}x^6+2x^5+2x^4-\frac{67}{3}x^3+34x^2-20x+\frac{25}{6}$	31
orering	$\frac{x(x^5+12x^4+12x^3-134x^2+204x-120)(x^3+6x^2-12x+5)}{6(x-1)(x^2+7x-5)}$	55

input
```
int((x^2+4*x-4)*(x^3+6*x^2-12*x+5),x,method=_RETURNVERBOSE)
```

output
```
1/6*(x^3+6*x^2-12*x+5)^2
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.53

$$\int (-4+4x+x^2)(5-12x+6x^2+x^3)\,dx=\frac{1}{6}\,x^6+2\,x^5+2\,x^4-\frac{67}{3}\,x^3+34\,x^2-20\,x$$

input
```
integrate((x^2+4*x-4)*(x^3+6*x^2-12*x+5),x, algorithm="fricas")
```

output
```
1/6*x^6 + 2*x^5 + 2*x^4 - 67/3*x^3 + 34*x^2 - 20*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.53

$$\int (-4+4x+x^2)(5-12x+6x^2+x^3)\,dx=\frac{x^6}{6}+2x^5+2x^4-\frac{67x^3}{3}+34x^2-20x$$

input
```
integrate((x**2+4*x-4)*(x**3+6*x**2-12*x+5),x)
```

output
```
x**6/6 + 2*x**5 + 2*x**4 - 67*x**3/3 + 34*x**2 - 20*x
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \left(-4+4x+x^2\right)\left(5-12x+6x^2+x^3\right)\,dx=\frac{1}{6}\left(x^3+6\,x^2-12\,x+5\right)^2$$

input
```
integrate((x^2+4*x-4)*(x^3+6*x^2-12*x+5),x, algorithm="maxima")
```

output
```
1/6*(x^3 + 6*x^2 - 12*x + 5)^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.58

$$\int \left(-4+4x+x^2\right)\left(5-12x+6x^2+x^3\right)\,dx=\frac{5}{3}\,x^3+\frac{1}{6}\left(x^3+6\,x^2-12\,x\right)^2+10\,x^2-20\,x$$

input
```
integrate((x^2+4*x-4)*(x^3+6*x^2-12*x+5),x, algorithm="giac")
```

output
```
5/3*x^3 + 1/6*(x^3 + 6*x^2 - 12*x)^2 + 10*x^2 - 20*x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.53

$$\int \left(-4+4x+x^2\right)\left(5-12x+6x^2+x^3\right)\,dx=\frac{x^6}{6}+2\,x^5+2\,x^4-\frac{67\,x^3}{3}+34\,x^2-20\,x$$

input
```
int((4*x + x^2 - 4)*(6*x^2 - 12*x + x^3 + 5),x)
```

output
```
34*x^2 - 20*x - (67*x^3)/3 + 2*x^4 + 2*x^5 + x^6/6
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.37

$$\int (-4+4x+x^2)\left(5-12x+6x^2+x^3\right)\,dx=\frac{x(x^5+12x^4+12x^3-134x^2+204x-120)}{6}$$

input
```
int((x^2+4*x-4)*(x^3+6*x^2-12*x+5),x)
```

output
```
(x*(x**5 + 12*x**4 + 12*x**3 - 134*x**2 + 204*x - 120))/6
```

3.85 $\int (2x+x^3)(1+4x^2+x^4)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 16

$$\int (2x+x^3)(1+4x^2+x^4)\,dx=\frac{1}{8}(1+4x^2+x^4)^2$$

output
```
1/8*(x^4+4*x^2+1)^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.31

$$\int (2x+x^3)(1+4x^2+x^4)\,dx=x^2+\frac{9x^4}{4}+x^6+\frac{x^8}{8}$$

input
```
Integrate[(2*x + x^3)*(1 + 4*x^2 + x^4),x]
```

output
```
x^2 + (9*x^4)/4 + x^6 + x^8/8
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {2021}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (x^3+2x)\left(x^4+4x^2+1\right)\,dx$$

$$\Big\downarrow \; 2021$$

$$\frac{1}{8}\left(x^4+4x^2+1\right)^2$$

input
```
Int[(2*x + x^3)*(1 + 4*x^2 + x^4),x]
```

output
```
(1 + 4*x^2 + x^4)^2/8
```

Defintions of rubi rules used

rule 2021
```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result	size
default	$\frac{(x^4+4x^2+1)^2}{8}$	15
norman	$\frac{1}{8}x^8+x^6+\frac{9}{4}x^4+x^2$	18
parallelrisch	$\frac{1}{8}x^8+x^6+\frac{9}{4}x^4+x^2$	18
risch	$\frac{1}{8}x^8+x^6+\frac{9}{4}x^4+x^2+\frac{1}{8}$	19
gosper	$\frac{x^2(x^6+8x^4+18x^2+8)}{8}$	21
orering	$\frac{x(x^6+8x^4+18x^2+8)(x^3+2x)}{8x^2+16}$	33

input
```
int((x^3+2*x)*(x^4+4*x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/8*(x^4+4*x^2+1)^2
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int (2x+x^3)(1+4x^2+x^4)\,dx = \frac{1}{8}x^8+x^6+\frac{9}{4}x^4+x^2$$

input
```
integrate((x^3+2*x)*(x^4+4*x^2+1),x, algorithm="fricas")
```

output
```
1/8*x^8 + x^6 + 9/4*x^4 + x^2
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int (2x+x^3)(1+4x^2+x^4)\,dx = \frac{x^8}{8}+x^6+\frac{9x^4}{4}+x^2$$

input
```
integrate((x**3+2*x)*(x**4+4*x**2+1),x)
```

output
```
x**8/8 + x**6 + 9*x**4/4 + x**2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int (2x + x^3) (1 + 4x^2 + x^4) \, dx = \frac{1}{8} (x^4 + 4 x^2 + 1)^2$$

input
```
integrate((x^3+2*x)*(x^4+4*x^2+1),x, algorithm="maxima")
```

output
```
1/8*(x^4 + 4*x^2 + 1)^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.38

$$\int (2x + x^3) (1 + 4x^2 + x^4) \, dx = \frac{1}{4} x^4 + \frac{1}{8} (x^4 + 4 x^2)^2 + x^2$$

input
```
integrate((x^3+2*x)*(x^4+4*x^2+1),x, algorithm="giac")
```

output
```
1/4*x^4 + 1/8*(x^4 + 4*x^2)^2 + x^2
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int (2x + x^3) (1 + 4x^2 + x^4) \, dx = \frac{x^8}{8} + x^6 + \frac{9 x^4}{4} + x^2$$

input
```
int((2*x + x^3)*(4*x^2 + x^4 + 1),x)
```

output
```
x^2 + (9*x^4)/4 + x^6 + x^8/8
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int (2x + x^3)(1 + 4x^2 + x^4)\, dx = \frac{x^2(x^6 + 8x^4 + 18x^2 + 8)}{8}$$

input
```
int((x^3+2*x)*(x^4+4*x^2+1),x)
```

output
```
(x**2*(x**6 + 8*x**4 + 18*x**2 + 8))/8
```

3.86 $\int (1+2x)\left(x+x^2\right)^3\left(-18+7\left(x+x^2\right)^3\right)^2 \, dx$

Optimal result

Integrand size = 26, antiderivative size = 33

$$\int (1+2x)\left(x+x^2\right)^3\left(-18+7\left(x+x^2\right)^3\right)^2 \, dx = 81x^4(1+x)^4-36x^7(1+x)^7+\frac{49}{10}x^{10}(1+x)^{10}$$

output
```
81*x^4*(1+x)^4-36*x^7*(1+x)^7+49/10*x^10*(1+x)^10
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 96 vs. 2(33) = 66.

Time = 0.01 (sec) , antiderivative size = 96, normalized size of antiderivative = 2.91

$$\int (1+2x)\left(x+x^2\right)^3\left(-18+7\left(x+x^2\right)^3\right)^2 \, dx = 81x^4+324x^5+486x^6+288x^7-171x^8-756x^9-\frac{12551x^{10}}{10}-1211x^{11}-\frac{1071x^{12}}{2}+336x^{13}+993x^{14}+\frac{6174x^{15}}{5}+1029x^{16}+588x^{17}+\frac{441x^{18}}{2}+49x^{19}+\frac{49x^{20}}{10}$$

input
```
Integrate[(1 + 2*x)*(x + x^2)^3*(-18 + 7*(x + x^2)^3)^2,x]
```

output
```
81*x^4 + 324*x^5 + 486*x^6 + 288*x^7 - 171*x^8 - 756*x^9 - (12551*x^10)/10
 - 1211*x^11 - (1071*x^12)/2 + 336*x^13 + 993*x^14 + (6174*x^15)/5 + 1029*
x^16 + 588*x^17 + (441*x^18)/2 + 49*x^19 + (49*x^20)/10
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 96 vs. $2(33) = 66$.

Time = 0.75 (sec) , antiderivative size = 96, normalized size of antiderivative = 2.91, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {2027, 2115, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (2x+1)\left(x^2+x\right)^3\left(7\left(x^2+x\right)^3-18\right)^2dx$$

$$\Big\downarrow \text{2027}$$

$$\int x^3(x+1)^3(2x+1)\left(7\left(x^2+x\right)^3-18\right)^2dx$$

$$\Big\downarrow \text{2115}$$

$$\int \left(98x^{19}+931x^{18}+3969x^{17}+9996x^{16}+16464x^{15}+18522x^{14}+13902x^{13}+4368x^{12}-6426x^{11}-13321x^{10}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{49x^{20}}{10}+49x^{19}+\frac{441x^{18}}{2}+588x^{17}+1029x^{16}+\frac{6174x^{15}}{5}+993x^{14}+336x^{13}-\frac{1071x^{12}}{2}-1211x^{11}-\frac{12551x^{10}}{10}-756x^9-171x^8+288x^7+486x^6+324x^5+81x^4$$

input
```
Int[(1 + 2*x)*(x + x^2)^3*(-18 + 7*(x + x^2)^3)^2,x]
```

output
```
81*x^4 + 324*x^5 + 486*x^6 + 288*x^7 - 171*x^8 - 756*x^9 - (12551*x^10)/10
 - 1211*x^11 - (1071*x^12)/2 + 336*x^13 + 993*x^14 + (6174*x^15)/5 + 1029*
x^16 + 588*x^17 + (441*x^18)/2 + 49*x^19 + (49*x^20)/10
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

rule 2115
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^
n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px,
 x] && IntegersQ[m, n]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

method	result
default	$\frac{49(x^2+x)^{10}}{10}-36(x^2+x)^7+81(x^2+x)^4$
gosper	$\frac{(x+1)^3(49x^{13}+343x^{12}+1029x^{11}+1715x^{10}+1715x^9+1029x^8-17x^7-1391x^6-2160x^5-1440x^4-360x^3+810x+810)x^4}{10}$
norman	$81x^4+324x^5+486x^6+288x^7-171x^8-756x^9-\frac{12551}{10}x^{10}-1211x^{11}-\frac{1071}{2}x^{12}+336x^{13}$
risch	$81x^4+324x^5+486x^6+288x^7-171x^8-756x^9-\frac{12551}{10}x^{10}-1211x^{11}-\frac{1071}{2}x^{12}+336x^{13}$
parallelrisch	$81x^4+324x^5+486x^6+288x^7-171x^8-756x^9-\frac{12551}{10}x^{10}-1211x^{11}-\frac{1071}{2}x^{12}+336x^{13}$
orering	$\frac{(49x^{13}+343x^{12}+1029x^{11}+1715x^{10}+1715x^9+1029x^8-17x^7-1391x^6-2160x^5-1440x^4-360x^3+810x+810)x(x^2+x)^3(-1}{10(7x^6+21x^5+21x^4+7x^3-18)^2}$

input
```
int((2*x+1)*(x^2+x)^3*(-18+7*(x^2+x)^3)^2,x,method=_RETURNVERBOSE)
```

output
```
49/10*(x^2+x)^10-36*(x^2+x)^7+81*(x^2+x)^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 86 vs. $2(31) = 62$.

Time = 0.06 (sec) , antiderivative size = 86, normalized size of antiderivative = 2.61

$$\int (1+2x)\left(x+x^2\right)^3\left(-18+7\left(x+x^2\right)^3\right)^2\,dx = \frac{49}{10}\,x^{20}+49\,x^{19}+\frac{441}{2}\,x^{18}+588\,x^{17}+1029\,x^{16}+\frac{6174}{5}\,x^{15}+993\,x^{14}+336\,x^{13} - \frac{1071}{2}\,x^{12}-1211\,x^{11}-\frac{12551}{10}\,x^{10}-756\,x^{9}-171\,x^{8}+288\,x^{7}+486\,x^{6}+324\,x^{5}+81\,x^{4}$$

input
```
integrate((1+2*x)*(x^2+x)^3*(-18+7*(x^2+x)^3)^2,x, algorithm="fricas")
```

output
```
49/10*x^20 + 49*x^19 + 441/2*x^18 + 588*x^17 + 1029*x^16 + 6174/5*x^15 + 9
93*x^14 + 336*x^13 - 1071/2*x^12 - 1211*x^11 - 12551/10*x^10 - 756*x^9 - 1
71*x^8 + 288*x^7 + 486*x^6 + 324*x^5 + 81*x^4
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 94 vs. $2(31) = 62$.

Time = 0.03 (sec) , antiderivative size = 94, normalized size of antiderivative = 2.85

$$\int (1+2x)\left(x+x^2\right)^3\left(-18+7\left(x+x^2\right)^3\right)^2\,dx = \frac{49x^{20}}{10}+49x^{19}+\frac{441x^{18}}{2}+588x^{17}+1029x^{16}+\frac{6174x^{15}}{5}+993x^{14}+336x^{13} - \frac{1071x^{12}}{2}-1211x^{11}-\frac{12551x^{10}}{10}-756x^{9}-171x^{8}+288x^{7}+486x^{6}+324x^{5}+81x^{4}$$

input
```
integrate((1+2*x)*(x**2+x)**3*(-18+7*(x**2+x)**3)**2,x)
```

output
```
49*x**20/10 + 49*x**19 + 441*x**18/2 + 588*x**17 + 1029*x**16 + 6174*x**15
/5 + 993*x**14 + 336*x**13 - 1071*x**12/2 - 1211*x**11 - 12551*x**10/10 -
756*x**9 - 171*x**8 + 288*x**7 + 486*x**6 + 324*x**5 + 81*x**4
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 86 vs. $2(31) = 62$.

Time = 0.03 (sec) , antiderivative size = 86, normalized size of antiderivative = 2.61

$$\int (1+2x)\left(x+x^2\right)^3\left(-18+7\left(x+x^2\right)^3\right)^2\,dx = \frac{49}{10}\,x^{20}+49\,x^{19}+\frac{441}{2}\,x^{18}+588\,x^{17}+1029\,x^{16}+\frac{6174}{5}\,x^{15}+993\,x^{14}+336\,x^{13}-\frac{1071}{2}\,x^{12}-1211\,x^{11}-\frac{12551}{10}\,x^{10}-756\,x^9-171\,x^8+288\,x^7+486\,x^6+324\,x^5+81\,x^4$$

input
```
integrate((1+2*x)*(x^2+x)^3*(-18+7*(x^2+x)^3)^2,x, algorithm="maxima")
```

output
```
49/10*x^20 + 49*x^19 + 441/2*x^18 + 588*x^17 + 1029*x^16 + 6174/5*x^15 + 9
93*x^14 + 336*x^13 - 1071/2*x^12 - 1211*x^11 - 12551/10*x^10 - 756*x^9 - 1
71*x^8 + 288*x^7 + 486*x^6 + 324*x^5 + 81*x^4
```

Giac [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85

$$\int (1+2x)\left(x+x^2\right)^3\left(-18+7\left(x+x^2\right)^3\right)^2\,dx = \frac{49}{10}\left(x^2+x\right)^{10}-36\left(x^2+x\right)^7+81\left(x^2+x\right)^4$$

input
```
integrate((1+2*x)*(x^2+x)^3*(-18+7*(x^2+x)^3)^2,x, algorithm="giac")
```

output
```
49/10*(x^2 + x)^10 - 36*(x^2 + x)^7 + 81*(x^2 + x)^4
```

Mupad [B] (verification not implemented)

Time = 22.08 (sec) , antiderivative size = 86, normalized size of antiderivative = 2.61

$$\int (1+2x)\left(x+x^2\right)^3\left(-18+7\left(x+x^2\right)^3\right)^2\,dx$$
$$=\frac{49\,x^{20}}{10}+49\,x^{19}+\frac{441\,x^{18}}{2}+588\,x^{17}+1029\,x^{16}+\frac{6174\,x^{15}}{5}+993\,x^{14}+336\,x^{13}-\frac{1071\,x^{12}}{2}-1211\,x^{11}-\frac{12551\,x^{10}}{10}-756\,x^9-171\,x^8+288\,x^7+486\,x^6+324\,x^5+81\,x^4$$

input
```
int((2*x + 1)*(x + x^2)^3*(7*(x + x^2)^3 - 18)^2,x)
```

output
```
81*x^4 + 324*x^5 + 486*x^6 + 288*x^7 - 171*x^8 - 756*x^9 - (12551*x^10)/10
 - 1211*x^11 - (1071*x^12)/2 + 336*x^13 + 993*x^14 + (6174*x^15)/5 + 1029*
x^16 + 588*x^17 + (441*x^18)/2 + 49*x^19 + (49*x^20)/10
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 85, normalized size of antiderivative = 2.58

$$\int (1+2x)\left(x+x^2\right)^3\left(-18+7\left(x+x^2\right)^3\right)^2\,dx$$
$$=\frac{x^4(49x^{16}+490x^{15}+2205x^{14}+5880x^{13}+10290x^{12}+12348x^{11}+9930x^{10}+3360x^9-5355x^8-12110}{10}$$

input
```
int((1+2*x)*(x^2+x)^3*(-18+7*(x^2+x)^3)^2,x)
```

output
```
(x**4*(49*x**16 + 490*x**15 + 2205*x**14 + 5880*x**13 + 10290*x**12 + 1234
8*x**11 + 9930*x**10 + 3360*x**9 - 5355*x**8 - 12110*x**7 - 12551*x**6 - 7
560*x**5 - 1710*x**4 + 2880*x**3 + 4860*x**2 + 3240*x + 810))/10
```

3.87 $\int x^3(1+x)^3(1+2x)\left(-18+7x^3(1+x)^3\right)^2\,dx$

Optimal result

Integrand size = 28, antiderivative size = 33

$$\int x^3(1+x)^3(1+2x)\left(-18+7x^3(1+x)^3\right)^2\,dx = 81x^4(1+x)^4-36x^7(1+x)^7+\frac{49}{10}x^{10}(1+x)^{10}$$

output
```
81*x^4*(1+x)^4-36*x^7*(1+x)^7+49/10*x^10*(1+x)^10
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 96 vs. 2(33) = 66.

Time = 0.01 (sec) , antiderivative size = 96, normalized size of antiderivative = 2.91

$$\int x^3(1+x)^3(1+2x)\left(-18+7x^3(1+x)^3\right)^2\,dx = 81x^4+324x^5+486x^6+288x^7-171x^8-756x^9-\frac{12551x^{10}}{10}-1211x^{11}-\frac{1071x^{12}}{2}+336x^{13}+993x^{14}+\frac{6174x^{15}}{5}+1029x^{16}+588x^{17}+\frac{441x^{18}}{2}+49x^{19}+\frac{49x^{20}}{10}$$

input
```
Integrate[x^3*(1 + x)^3*(1 + 2*x)*(-18 + 7*x^3*(1 + x)^3)^2,x]
```

output
```
81*x^4 + 324*x^5 + 486*x^6 + 288*x^7 - 171*x^8 - 756*x^9 - (12551*x^10)/10
 - 1211*x^11 - (1071*x^12)/2 + 336*x^13 + 993*x^14 + (6174*x^15)/5 + 1029*
x^16 + 588*x^17 + (441*x^18)/2 + 49*x^19 + (49*x^20)/10
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 96 vs. $2(33) = 66$.

Time = 0.68 (sec) , antiderivative size = 96, normalized size of antiderivative = 2.91, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2115, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3(x+1)^3(2x+1)\left(7x^3(x+1)^3-18\right)^2dx$$

$$\Big\downarrow \text{2115}$$

$$\int \left(98x^{19}+931x^{18}+3969x^{17}+9996x^{16}+16464x^{15}+18522x^{14}+13902x^{13}+4368x^{12}-6426x^{11}-13321x^{10}-\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{49x^{20}}{10}+49x^{19}+\frac{441x^{18}}{2}+588x^{17}+1029x^{16}+\frac{6174x^{15}}{5}+993x^{14}+336x^{13}-\frac{1071x^{12}}{2}-1211x^{11}-\frac{12551x^{10}}{10}-756x^9-171x^8+288x^7+486x^6+324x^5+81x^4$$

input
```
Int[x^3*(1 + x)^3*(1 + 2*x)*(-18 + 7*x^3*(1 + x)^3)^2,x]
```

output

```
81*x^4 + 324*x^5 + 486*x^6 + 288*x^7 - 171*x^8 - 756*x^9 - (12551*x^10)/10
 - 1211*x^11 - (1071*x^12)/2 + 336*x^13 + 993*x^14 + (6174*x^15)/5 + 1029*
x^16 + 588*x^17 + (441*x^18)/2 + 49*x^19 + (49*x^20)/10
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2115
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^
n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px,
 x] && IntegersQ[m, n]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 85 vs. $2(31) = 62$.

Time = 0.19 (sec) , antiderivative size = 86, normalized size of antiderivative = 2.61

method	result
gosper	$\frac{x^4(49x^{16}+490x^{15}+2205x^{14}+5880x^{13}+10290x^{12}+12348x^{11}+9930x^{10}+3360x^9-5355x^8-12110x^7-12551x^6-7560x^5-171}{10}$
default	$81x^4+324x^5+486x^6+288x^7-171x^8-756x^9-\frac{12551}{10}x^{10}-1211x^{11}-\frac{1071}{2}x^{12}+336x^{13}$
norman	$81x^4+324x^5+486x^6+288x^7-171x^8-756x^9-\frac{12551}{10}x^{10}-1211x^{11}-\frac{1071}{2}x^{12}+336x^{13}$
risch	$81x^4+324x^5+486x^6+288x^7-171x^8-756x^9-\frac{12551}{10}x^{10}-1211x^{11}-\frac{1071}{2}x^{12}+336x^{13}$
parallelrisch	$81x^4+324x^5+486x^6+288x^7-171x^8-756x^9-\frac{12551}{10}x^{10}-1211x^{11}-\frac{1071}{2}x^{12}+336x^{13}$
orering	$\frac{(49x^{13}+343x^{12}+1029x^{11}+1715x^{10}+1715x^9+1029x^8-17x^7-1391x^6-2160x^5-1440x^4-360x^3+810x+810)x^4(x+1)^3(-18}{10(7x^6+21x^5+21x^4+7x^3-18)^2}$

input
```
int(x^3*(x+1)^3*(2*x+1)*(-18+7*x^3*(x+1)^3)^2,x,method=_RETURNVERBOSE)
```

output
```
1/10*x^4*(49*x^16+490*x^15+2205*x^14+5880*x^13+10290*x^12+12348*x^11+9930*
x^10+3360*x^9-5355*x^8-12110*x^7-12551*x^6-7560*x^5-1710*x^4+2880*x^3+4860
*x^2+3240*x+810)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 86 vs. $2(31) = 62$.

Time = 0.06 (sec) , antiderivative size = 86, normalized size of antiderivative = 2.61

$$\int x^3(1+x)^3(1+2x)\left(-18+7x^3(1+x)^3\right)^2\,dx = \frac{49}{10}\,x^{20}+49\,x^{19}+\frac{441}{2}\,x^{18}+588\,x^{17}+1029\,x^{16}+\frac{6174}{5}\,x^{15}+993\,x^{14}+336\,x^{13} - \frac{1071}{2}\,x^{12}-1211\,x^{11}-\frac{12551}{10}\,x^{10}-756\,x^{9}-171\,x^{8}+288\,x^{7}+486\,x^{6}+324\,x^{5}+81\,x^{4}$$

input
```
integrate(x^3*(1+x)^3*(1+2*x)*(-18+7*x^3*(1+x)^3)^2,x, algorithm="fricas")
```

output
```
49/10*x^20 + 49*x^19 + 441/2*x^18 + 588*x^17 + 1029*x^16 + 6174/5*x^15 + 9
93*x^14 + 336*x^13 - 1071/2*x^12 - 1211*x^11 - 12551/10*x^10 - 756*x^9 - 1
71*x^8 + 288*x^7 + 486*x^6 + 324*x^5 + 81*x^4
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 94 vs. $2(31) = 62$.

Time = 0.04 (sec) , antiderivative size = 94, normalized size of antiderivative = 2.85

$$\int x^3(1+x)^3(1+2x)\left(-18+7x^3(1+x)^3\right)^2\,dx = \frac{49x^{20}}{10}+49x^{19}+\frac{441x^{18}}{2}+588x^{17}+1029x^{16}+\frac{6174x^{15}}{5}+993x^{14}+336x^{13} - \frac{1071x^{12}}{2}-1211x^{11}-\frac{12551x^{10}}{10}-756x^{9}-171x^{8}+288x^{7}+486x^{6}+324x^{5}+81x^{4}$$

input
```
integrate(x**3*(1+x)**3*(1+2*x)*(-18+7*x**3*(1+x)**3)**2,x)
```

output
```
49*x**20/10 + 49*x**19 + 441*x**18/2 + 588*x**17 + 1029*x**16 + 6174*x**15
/5 + 993*x**14 + 336*x**13 - 1071*x**12/2 - 1211*x**11 - 12551*x**10/10 -
756*x**9 - 171*x**8 + 288*x**7 + 486*x**6 + 324*x**5 + 81*x**4
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 86 vs. $2(31) = 62$.

Time = 0.03 (sec) , antiderivative size = 86, normalized size of antiderivative = 2.61

$$\int x^3(1+x)^3(1+2x)\left(-18+7x^3(1+x)^3\right)^2\,dx$$
$$= \frac{49}{10}\,x^{20} + 49\,x^{19} + \frac{441}{2}\,x^{18} + 588\,x^{17} + 1029\,x^{16} + \frac{6174}{5}\,x^{15} + 993\,x^{14} + 336\,x^{13} - \frac{1071}{2}\,x^{12} - 1211\,x^{11} - \frac{12551}{10}\,x^{10} - 756\,x^9 - 171\,x^8 + 288\,x^7 + 486\,x^6 + 324\,x^5 + 81\,x^4$$

input
```
integrate(x^3*(1+x)^3*(1+2*x)*(-18+7*x^3*(1+x)^3)^2,x, algorithm="maxima")
```

output
```
49/10*x^20 + 49*x^19 + 441/2*x^18 + 588*x^17 + 1029*x^16 + 6174/5*x^15 + 9
93*x^14 + 336*x^13 - 1071/2*x^12 - 1211*x^11 - 12551/10*x^10 - 756*x^9 - 1
71*x^8 + 288*x^7 + 486*x^6 + 324*x^5 + 81*x^4
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85

$$\int x^3(1+x)^3(1+2x)\left(-18+7x^3(1+x)^3\right)^2\,dx = \frac{49}{10}\left(x^2+x\right)^{10} - 36\left(x^2+x\right)^7 + 81\left(x^2+x\right)^4$$

input
```
integrate(x^3*(1+x)^3*(1+2*x)*(-18+7*x^3*(1+x)^3)^2,x, algorithm="giac")
```

output
```
49/10*(x^2 + x)^10 - 36*(x^2 + x)^7 + 81*(x^2 + x)^4
```

Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 86, normalized size of antiderivative = 2.61

$$\int x^3(1+x)^3(1+2x)\left(-18+7x^3(1+x)^3\right)^2\,dx = \frac{49\,x^{20}}{10}+49\,x^{19}+\frac{441\,x^{18}}{2}+588\,x^{17}+1029\,x^{16}+\frac{6174\,x^{15}}{5}+993\,x^{14}+336\,x^{13} -\frac{1071\,x^{12}}{2}-1211\,x^{11}-\frac{12551\,x^{10}}{10}-756\,x^9-171\,x^8+288\,x^7+486\,x^6+324\,x^5+81\,x^4$$

input
```
int(x^3*(2*x + 1)*(7*x^3*(x + 1)^3 - 18)^2*(x + 1)^3,x)
```

output
```
81*x^4 + 324*x^5 + 486*x^6 + 288*x^7 - 171*x^8 - 756*x^9 - (12551*x^10)/10
 - 1211*x^11 - (1071*x^12)/2 + 336*x^13 + 993*x^14 + (6174*x^15)/5 + 1029*
x^16 + 588*x^17 + (441*x^18)/2 + 49*x^19 + (49*x^20)/10
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 85, normalized size of antiderivative = 2.58

$$\int x^3(1+x)^3(1+2x)\left(-18+7x^3(1+x)^3\right)^2\,dx = \frac{x^4(49x^{16}+490x^{15}+2205x^{14}+5880x^{13}+10290x^{12}+12348x^{11}+9930x^{10}+3360x^9-5355x^8-12110}{10}$$

input
```
int(x^3*(1+x)^3*(1+2*x)*(-18+7*x^3*(1+x)^3)^2,x)
```

output
```
(x**4*(49*x**16 + 490*x**15 + 2205*x**14 + 5880*x**13 + 10290*x**12 + 1234
8*x**11 + 9930*x**10 + 3360*x**9 - 5355*x**8 - 12110*x**7 - 12551*x**6 - 7
560*x**5 - 1710*x**4 + 2880*x**3 + 4860*x**2 + 3240*x + 810))/10
```

3.88 $\int \frac{2-x^2}{(1-6x+x^3)^5} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 14

$$\int \frac{2-x^2}{(1-6x+x^3)^5} \, dx = \frac{1}{12\,(1-6x+x^3)^4}$$

output
```
1/12/(x^3-6*x+1)^4
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{2-x^2}{(1-6x+x^3)^5} \, dx = \frac{1}{12\,(1-6x+x^3)^4}$$

input
```
Integrate[(2 - x^2)/(1 - 6*x + x^3)^5,x]
```

output
```
1/(12*(1 - 6*x + x^3)^4)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {2021}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2-x^2}{(x^3-6x+1)^5}\,dx$$
$$\Big\downarrow \ 2021$$
$$\frac{1}{12\,(x^3-6x+1)^4}$$

input
```
Int[(2 - x^2)/(1 - 6*x + x^3)^5,x]
```

output
```
1/(12*(1 - 6*x + x^3)^4)
```

Defintions of rubi rules used

rule 2021
```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result	size
gosper	$\frac{1}{12(x^3-6x+1)^4}$	13
default	$\frac{1}{12(x^3-6x+1)^4}$	13
norman	$\frac{1}{12(x^3-6x+1)^4}$	13
risch	$\frac{1}{12(x^3-6x+1)^4}$	13
parallelrisch	$\frac{1}{12(x^3-6x+1)^4}$	13
orering	$-\frac{-x^2+2}{12(x^3-6x+1)^4(x^2-2)}$	27

input
```
int((-x^2+2)/(x^3-6*x+1)^5,x,method=_RETURNVERBOSE)
```

output
```
1/12/(x^3-6*x+1)^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. $2(12) = 24$.

Time = 0.06 (sec) , antiderivative size = 57, normalized size of antiderivative = 4.07

$$\int \frac{2-x^2}{(1-6x+x^3)^5}\,dx$$
$$= \frac{1}{12\,(x^{12}-24\,x^{10}+4\,x^9+216\,x^8-72\,x^7-858\,x^6+432\,x^5+1224\,x^4-860\,x^3+216\,x^2-24\,x+1)}$$

input
```
integrate((-x^2+2)/(x^3-6*x+1)^5,x, algorithm="fricas")
```

output
```
1/12/(x^12 - 24*x^10 + 4*x^9 + 216*x^8 - 72*x^7 - 858*x^6 + 432*x^5 + 1224
*x^4 - 860*x^3 + 216*x^2 - 24*x + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. $2(12) = 24$.

Time = 0.11 (sec) , antiderivative size = 56, normalized size of antiderivative = 4.00

$$\int \frac{2-x^2}{(1-6x+x^3)^5}\,dx$$
$$= \frac{1}{12x^{12}-288x^{10}+48x^9+2592x^8-864x^7-10296x^6+5184x^5+14688x^4-10320x^3+2592x^2-288x \cdots}$$

input
```
integrate((-x**2+2)/(x**3-6*x+1)**5,x)
```

output
```
1/(12*x**12 - 288*x**10 + 48*x**9 + 2592*x**8 - 864*x**7 - 10296*x**6 + 51
84*x**5 + 14688*x**4 - 10320*x**3 + 2592*x**2 - 288*x + 12)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{2-x^2}{(1-6x+x^3)^5}\,dx = \frac{1}{12\,(x^3-6\,x+1)^4}$$

input
```
integrate((-x^2+2)/(x^3-6*x+1)^5,x, algorithm="maxima")
```

output
```
1/12/(x^3 - 6*x + 1)^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{2-x^2}{(1-6x+x^3)^5}\,dx = \frac{1}{12\,(x^3-6\,x+1)^4}$$

input
```
integrate((-x^2+2)/(x^3-6*x+1)^5,x, algorithm="giac")
```

output
```
1/12/(x^3 - 6*x + 1)^4
```

Mupad [B] (verification not implemented)

Time = 21.84 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{2-x^2}{\left(1-6x+x^3\right)^5} \, dx = \frac{1}{12 \left(x^3-6 x+1\right)^4}$$

input
```
int(-(x^2 - 2)/(x^3 - 6*x + 1)^5,x)
```

output
```
1/(12*(x^3 - 6*x + 1)^4)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 57, normalized size of antiderivative = 4.07

$$\int \frac{2-x^2}{\left(1-6x+x^3\right)^5} \, dx$$
$$= \frac{1}{12x^{12}-288x^{10}+48x^9+2592x^8-864x^7-10296x^6+5184x^5+14688x^4-10320x^3+2592x^2-288x}$$

input
```
int((-x^2+2)/(x^3-6*x+1)^5,x)
```

output
```
1/(12*(x**12 - 24*x**10 + 4*x**9 + 216*x**8 - 72*x**7 - 858*x**6 + 432*x**
5 + 1224*x**4 - 860*x**3 + 216*x**2 - 24*x + 1))
```

3.89 $\int \frac{2x+x^2}{4+3x^2+x^3} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 15

$$\int \frac{2x+x^2}{4+3x^2+x^3} \, dx = \frac{1}{3} \log \left(4+3x^2+x^3\right)$$

output
```
1/3*ln(x^3+3*x^2+4)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{2x+x^2}{4+3x^2+x^3} \, dx = \frac{1}{3} \log \left(4+3x^2+x^3\right)$$

input
```
Integrate[(2*x + x^2)/(4 + 3*x^2 + x^3),x]
```

output
```
Log[4 + 3*x^2 + x^3]/3
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {2020}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2+2x}{x^3+3x^2+4} \, dx$$
$$\Big\downarrow \text{2020}$$
$$\frac{1}{3}\log\left(x^3+3x^2+4\right)$$

input
```
Int[(2*x + x^2)/(4 + 3*x^2 + x^3),x]
```

output
```
Log[4 + 3*x^2 + x^3]/3
```

Defintions of rubi rules used

rule 2020
```
Int[(Pp_)/(Qq_), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Si
mp[Coeff[Pp, x, p]*(Log[RemoveContent[Qq, x]]/(q*Coeff[Qq, x, q])), x] /; E
qQ[p, q - 1] && EqQ[Pp, Simplify[(Coeff[Pp, x, p]/(q*Coeff[Qq, x, q]))*D[Qq
, x]]]] /; PolyQ[Pp, x] && PolyQ[Qq, x]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
default	$\frac{\ln(x^3+3x^2+4)}{3}$	14
norman	$\frac{\ln(x^3+3x^2+4)}{3}$	14
risch	$\frac{\ln(x^3+3x^2+4)}{3}$	14
parallelrisch	$\frac{\ln(x^3+3x^2+4)}{3}$	14

input
```
int((x^2+2*x)/(x^3+3*x^2+4),x,method=_RETURNVERBOSE)
```

output
```
1/3*ln(x^3+3*x^2+4)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{2x+x^2}{4+3x^2+x^3}\,dx = \frac{1}{3}\,\log\left(x^3+3\,x^2+4\right)$$

input
```
integrate((x^2+2*x)/(x^3+3*x^2+4),x, algorithm="fricas")
```

output
```
1/3*log(x^3 + 3*x^2 + 4)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

$$\int \frac{2x+x^2}{4+3x^2+x^3}\,dx = \frac{\log\left(x^3+3x^2+4\right)}{3}$$

input
```
integrate((x**2+2*x)/(x**3+3*x**2+4),x)
```

output
```
log(x**3 + 3*x**2 + 4)/3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{2x + x^2}{4 + 3x^2 + x^3} \, dx = \frac{1}{3} \log \left(x^3 + 3\,x^2 + 4\right)$$

input
```
integrate((x^2+2*x)/(x^3+3*x^2+4),x, algorithm="maxima")
```

output
```
1/3*log(x^3 + 3*x^2 + 4)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

$$\int \frac{2x + x^2}{4 + 3x^2 + x^3} \, dx = \frac{1}{3} \log \left(\left|x^3 + 3\,x^2 + 4\right|\right)$$

input
```
integrate((x^2+2*x)/(x^3+3*x^2+4),x, algorithm="giac")
```

output
```
1/3*log(abs(x^3 + 3*x^2 + 4))
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{2x + x^2}{4 + 3x^2 + x^3} \, dx = \frac{\ln \left(x^3 + 3\,x^2 + 4\right)}{3}$$

input
```
int((2*x + x^2)/(3*x^2 + x^3 + 4),x)
```

output
```
log(3*x^2 + x^3 + 4)/3
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{2x + x^2}{4 + 3x^2 + x^3} \, dx = \frac{\log(x^3 + 3x^2 + 4)}{3}$$

input
```
int((x^2+2*x)/(x^3+3*x^2+4),x)
```

output
```
log(x**3 + 3*x**2 + 4)/3
```

3.90 $\int \frac{1+x+x^3}{4x+2x^2+x^4} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 17

$$\int \frac{1+x+x^3}{4x+2x^2+x^4} \, dx = \frac{1}{4} \log \left(4x+2x^2+x^4\right)$$

output
```
1/4*ln(x^4+2*x^2+4*x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.18

$$\int \frac{1+x+x^3}{4x+2x^2+x^4} \, dx = \frac{\log(x)}{4} + \frac{1}{4} \log \left(4+2x+x^3\right)$$

input
```
Integrate[(1 + x + x^3)/(4*x + 2*x^2 + x^4),x]
```

output
```
Log[x]/4 + Log[4 + 2*x + x^3]/4
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {2020}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3+x+1}{x^4+2x^2+4x}\,dx$$

$$\Big\downarrow \; 2020$$

$$\frac{1}{4}\log\left(x^4+2x^2+4x\right)$$

input
```
Int[(1 + x + x^3)/(4*x + 2*x^2 + x^4),x]
```

output
```
Log[4*x + 2*x^2 + x^4]/4
```

Defintions of rubi rules used

rule 2020
```
Int[(Pp_)/(Qq_), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Si
mp[Coeff[Pp, x, p]*(Log[RemoveContent[Qq, x]]/(q*Coeff[Qq, x, q])), x] /; E
qQ[p, q - 1] && EqQ[Pp, Simplify[(Coeff[Pp, x, p]/(q*Coeff[Qq, x, q]))*D[Qq
, x]]]] /; PolyQ[Pp, x] && PolyQ[Qq, x]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
default	$\frac{\ln(x(x^3+2x+4))}{4}$	14
risch	$\frac{\ln(x^4+2x^2+4x)}{4}$	16
norman	$\frac{\ln(x)}{4}+\frac{\ln(x^3+2x+4)}{4}$	17
parallelrisch	$\frac{\ln(x)}{4}+\frac{\ln(x^3+2x+4)}{4}$	17

input
```
int((x^3+x+1)/(x^4+2*x^2+4*x),x,method=_RETURNVERBOSE)
```

output
```
1/4*ln(x*(x^3+2*x+4))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{1+x+x^3}{4x+2x^2+x^4}\,dx = \frac{1}{4}\log\left(x^4+2\,x^2+4\,x\right)$$

input
```
integrate((x^3+x+1)/(x^4+2*x^2+4*x),x, algorithm="fricas")
```

output
```
1/4*log(x^4 + 2*x^2 + 4*x)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \frac{1+x+x^3}{4x+2x^2+x^4}\,dx = \frac{\log\left(x^4+2x^2+4x\right)}{4}$$

input
```
integrate((x**3+x+1)/(x**4+2*x**2+4*x),x)
```

output
```
log(x**4 + 2*x**2 + 4*x)/4
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{1+x+x^3}{4x+2x^2+x^4}\,dx=\frac{1}{4}\log\left(x^4+2\,x^2+4\,x\right)$$

input
```
integrate((x^3+x+1)/(x^4+2*x^2+4*x),x, algorithm="maxima")
```

output
```
1/4*log(x^4 + 2*x^2 + 4*x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{1+x+x^3}{4x+2x^2+x^4}\,dx=\frac{1}{4}\log\left(4\left|\frac{1}{4}\,x^4+\frac{1}{2}\,x^2+x\right|\right)$$

input
```
integrate((x^3+x+1)/(x^4+2*x^2+4*x),x, algorithm="giac")
```

output
```
1/4*log(4*abs(1/4*x^4 + 1/2*x^2 + x))
```

Mupad [B] (verification not implemented)

Time = 21.98 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{1+x+x^3}{4x+2x^2+x^4}\,dx=\frac{\ln\left(x\,\left(x^3+2\,x+4\right)\right)}{4}$$

input
```
int((x + x^3 + 1)/(4*x + 2*x^2 + x^4),x)
```

output
```
log(x*(2*x + x^3 + 4))/4
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

$$\int \frac{1+x+x^3}{4x+2x^2+x^4}\,dx = \frac{\log(x^3+2x+4)}{4} + \frac{\log(x)}{4}$$

input
```
int((x^3+x+1)/(x^4+2*x^2+4*x),x)
```

output
```
(log(x**3 + 2*x + 4) + log(x))/4
```

3.91 $\int \frac{-1+x}{1-x+x^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 32

$$\int \frac{-1+x}{1-x+x^2}\,dx = \frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{1}{2}\log\left(1-x+x^2\right)$$

output
```
1/3*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)+1/2*ln(x^2-x+1)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.03

$$\int \frac{-1+x}{1-x+x^2}\,dx = -\frac{\arctan\left(\frac{-1+2x}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{1}{2}\log\left(1-x+x^2\right)$$

input
```
Integrate[(-1 + x)/(1 - x + x^2),x]
```

output
```
-(ArcTan[(-1 + 2*x)/Sqrt[3]]/Sqrt[3]) + Log[1 - x + x^2]/2
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x-1}{x^2-x+1}dx$$

$$\Big\downarrow \text{ 1142}$$

$$\frac{1}{2}\int -\frac{1-2x}{x^2-x+1}dx-\frac{1}{2}\int \frac{1}{x^2-x+1}dx$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{1}{2}\int \frac{1}{x^2-x+1}dx-\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx$$

$$\Big\downarrow \text{ 1083}$$

$$\int \frac{1}{-(2x-1)^2-3}d(2x-1)-\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx$$

$$\Big\downarrow \text{ 217}$$

$$-\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx-\frac{\arctan\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}$$

$$\Big\downarrow \text{ 1103}$$

$$\frac{1}{2}\log\left(x^2-x+1\right)-\frac{\arctan\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}$$

input
```
Int[(-1 + x)/(1 - x + x^2),x]
```

output
```
-(ArcTan[(-1 + 2*x)/Sqrt[3]]/Sqrt[3]) + Log[1 - x + x^2]/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.91

method	result	size
default	$\frac{\ln(x^2-x+1)}{2}-\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{3}$	29
risch	$-\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{3}+\frac{\ln(4x^2-4x+4)}{2}$	31

input
```
int((x-1)/(x^2-x+1),x,method=_RETURNVERBOSE)
```

output
```
1/2*ln(x^2-x+1)-1/3*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

$$\int \frac{-1+x}{1-x+x^2}\,dx = -\frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right)+\frac{1}{2}\,\log\left(x^2-x+1\right)$$

input
```
integrate((x-1)/(x^2-x+1),x, algorithm="fricas")
```

output
```
-1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/2*log(x^2 - x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{-1+x}{1-x+x^2}\,dx = \frac{\log\left(x^2-x+1\right)}{2}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{3}$$

input
```
integrate((x-1)/(x**2-x+1),x)
```

output
```
log(x**2 - x + 1)/2 - sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/3)/3
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

$$\int \frac{-1+x}{1-x+x^2}\,dx = -\frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right)+\frac{1}{2}\,\log\left(x^2-x+1\right)$$

input
```
integrate((x-1)/(x^2-x+1),x, algorithm="maxima")
```

output
```
-1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/2*log(x^2 - x + 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

$$\int \frac{-1+x}{1-x+x^2}\,dx = -\frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right)+\frac{1}{2}\,\log\left(x^2-x+1\right)$$

input
```
integrate((x-1)/(x^2-x+1),x, algorithm="giac")
```

output
```
-1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/2*log(x^2 - x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94

$$\int \frac{-1+x}{1-x+x^2}\,dx = \frac{\ln\left(x^2-x+1\right)}{2}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\,\sqrt{3}\,x}{3}-\frac{\sqrt{3}}{3}\right)}{3}$$

input
```
int((x - 1)/(x^2 - x + 1),x)
```

output
```
log(x^2 - x + 1)/2 - (3^(1/2)*atan((2*3^(1/2)*x)/3 - 3^(1/2)/3))/3
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.84

$$\int \frac{-1+x}{1-x+x^2}\,dx = -\frac{\sqrt{3}\,atan\left(\frac{2x-1}{\sqrt{3}}\right)}{3}+\frac{\log(x^2-x+1)}{2}$$

input
```
int((x-1)/(x^2-x+1),x)
```

output
```
( - 2*sqrt(3)*atan((2*x - 1)/sqrt(3)) + 3*log(x**2 - x + 1))/6
```

3.92 $\int \frac{-1+x^2}{1+x^3} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 32

$$\int \frac{-1+x^2}{1+x^3} \, dx = \frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{1}{2}\log\left(1-x+x^2\right)$$

output
```
1/3*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)+1/2*ln(x^2-x+1)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.03

$$\int \frac{-1+x^2}{1+x^3} \, dx = -\frac{\arctan\left(\frac{-1+2x}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{1}{2}\log\left(1-x+x^2\right)$$

input
```
Integrate[(-1 + x^2)/(1 + x^3),x]
```

output
```
-(ArcTan[(-1 + 2*x)/Sqrt[3]]/Sqrt[3]) + Log[1 - x + x^2]/2
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.03, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {2411, 25, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2-1}{x^3+1}\,dx$$

$$\Big\downarrow \text{ 2411}$$

$$\int -\frac{1-x}{x^2-x+1}dx$$

$$\Big\downarrow \text{ 25}$$

$$-\int \frac{1-x}{x^2-x+1}dx$$

$$\Big\downarrow \text{ 1142}$$

$$\frac{1}{2}\int -\frac{1-2x}{x^2-x+1}dx-\frac{1}{2}\int \frac{1}{x^2-x+1}dx$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{1}{2}\int \frac{1}{x^2-x+1}dx-\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx$$

$$\Big\downarrow \text{ 1083}$$

$$\int \frac{1}{-(2x-1)^2-3}d(2x-1)-\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx$$

$$\Big\downarrow \text{ 217}$$

$$-\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx-\frac{\arctan\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}$$

$$\Big\downarrow \text{ 1103}$$

$$\frac{1}{2}\log\left(x^2-x+1\right)-\frac{\arctan\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}$$

input
```
Int[(-1 + x^2)/(1 + x^3),x]
```

output
```
-(ArcTan[(-1 + 2*x)/Sqrt[3]]/Sqrt[3]) + Log[1 - x + x^2]/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2411
```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = (a/b)^(1/3)}, Simp[q^2/a
   Int[(A + C*q*x)/(q^2 - q*x + x^2), x], x]] /; EqQ[A - B*(a/b)^(1/3) + C*
(a/b)^(2/3), 0]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.91

method	result	size
default	$\frac{\ln\left(x^{2}-x+1\right)}{2}-\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{3}$	29
risch	$-\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{3}+\frac{\ln\left(4x^{2}-4x+4\right)}{2}$	31
meijerg	$\frac{\ln\left(x^{3}+1\right)}{3}-\frac{x\ln\left(1+\left(x^{3}\right)^{\frac{1}{3}}\right)}{3\left(x^{3}\right)^{\frac{1}{3}}}+\frac{x\ln\left(1-\left(x^{3}\right)^{\frac{1}{3}}+\left(x^{3}\right)^{\frac{2}{3}}\right)}{6\left(x^{3}\right)^{\frac{1}{3}}}-\frac{x\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(x^{3}\right)^{\frac{1}{3}}}{2-\left(x^{3}\right)^{\frac{1}{3}}}\right)}{3\left(x^{3}\right)^{\frac{1}{3}}}$	82

input
```
int((x^2-1)/(x^3+1),x,method=_RETURNVERBOSE)
```

output
```
1/2*ln(x^2-x+1)-1/3*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

$$\int \frac{-1+x^2}{1+x^3}\,dx=-\frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right)+\frac{1}{2}\,\log\left(x^{2}-x+1\right)$$

input
```
integrate((x^2-1)/(x^3+1),x, algorithm="fricas")
```

output
```
-1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/2*log(x^2 - x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{-1+x^2}{1+x^3}\,dx = \frac{\log\left(x^2-x+1\right)}{2} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{3}$$

input
```
integrate((x**2-1)/(x**3+1),x)
```

output
```
log(x**2 - x + 1)/2 - sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/3)/3
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

$$\int \frac{-1+x^2}{1+x^3}\,dx = -\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + \frac{1}{2}\log\left(x^2-x+1\right)$$

input
```
integrate((x^2-1)/(x^3+1),x, algorithm="maxima")
```

output
```
-1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/2*log(x^2 - x + 1)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

$$\int \frac{-1+x^2}{1+x^3}\,dx = -\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + \frac{1}{2}\log\left(x^2-x+1\right)$$

input
```
integrate((x^2-1)/(x^3+1),x, algorithm="giac")
```

output
```
-1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/2*log(x^2 - x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94

$$\int \frac{-1+x^2}{1+x^3}\,dx = \frac{\ln\left(x^2-x+1\right)}{2} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}\,x}{3}-\frac{\sqrt{3}}{3}\right)}{3}$$

input
```
int((x^2 - 1)/(x^3 + 1),x)
```

output
```
log(x^2 - x + 1)/2 - (3^(1/2)*atan((2*3^(1/2)*x)/3 - 3^(1/2)/3))/3
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.84

$$\int \frac{-1+x^2}{1+x^3}\,dx = -\frac{\sqrt{3}\,atan\left(\frac{2x-1}{\sqrt{3}}\right)}{3} + \frac{\log(x^2-x+1)}{2}$$

input
```
int((x^2-1)/(x^3+1),x)
```

output
```
( - 2*sqrt(3)*atan((2*x - 1)/sqrt(3)) + 3*log(x**2 - x + 1))/6
```

3.93 $\int \frac{-4+3x}{4-2x+x^2} dx$

Optimal result

Integrand size = 16, antiderivative size = 32

$$\int \frac{-4+3x}{4-2x+x^2} dx = \frac{\arctan\left(\frac{1-x}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{3}{2}\log\left(4-2x+x^2\right)$$

output
```
1/3*arctan(1/3*(1-x)*3^(1/2))*3^(1/2)+3/2*ln(x^2-2*x+4)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.97

$$\int \frac{-4+3x}{4-2x+x^2} dx = -\frac{\arctan\left(\frac{-1+x}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{3}{2}\log\left(4-2x+x^2\right)$$

input
```
Integrate[(-4 + 3*x)/(4 - 2*x + x^2),x]
```

output
```
-(ArcTan[(-1 + x)/Sqrt[3]]/Sqrt[3]) + (3*Log[4 - 2*x + x^2])/2
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.12, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {1142, 27, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{3x-4}{x^2-2x+4}dx$$

$$\Big\downarrow \text{1142}$$

$$\frac{3}{2}\int -\frac{2(1-x)}{x^2-2x+4}dx-\int \frac{1}{x^2-2x+4}dx$$

$$\Big\downarrow \text{27}$$

$$-\int \frac{1}{x^2-2x+4}dx-3\int \frac{1-x}{x^2-2x+4}dx$$

$$\Big\downarrow \text{1083}$$

$$2\int \frac{1}{-(2x-2)^2-12}d(2x-2)-3\int \frac{1-x}{x^2-2x+4}dx$$

$$\Big\downarrow \text{217}$$

$$-3\int \frac{1-x}{x^2-2x+4}dx-\frac{\arctan\left(\frac{2x-2}{2\sqrt{3}}\right)}{\sqrt{3}}$$

$$\Big\downarrow \text{1103}$$

$$\frac{3}{2}\log\left(x^2-2x+4\right)-\frac{\arctan\left(\frac{2x-2}{2\sqrt{3}}\right)}{\sqrt{3}}$$

input
```
Int[(-4 + 3*x)/(4 - 2*x + x^2),x]
```

output
```
-(ArcTan[(-2 + 2*x)/(2*Sqrt[3])]/Sqrt[3]) + (3*Log[4 - 2*x + x^2])/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.84

method	result	size
risch	$\frac{3\ln\left(x^2-2x+4\right)}{2}-\frac{\sqrt{3}\,\arctan\left(\frac{(x-1)\sqrt{3}}{3}\right)}{3}$	27
default	$\frac{3\ln\left(x^2-2x+4\right)}{2}-\frac{\sqrt{3}\,\arctan\left(\frac{(2x-2)\sqrt{3}}{6}\right)}{3}$	29

input
```
int((3*x-4)/(x^2-2*x+4),x,method=_RETURNVERBOSE)
```

output
```
3/2*ln(x^2-2*x+4)-1/3*3^(1/2)*arctan(1/3*(x-1)*3^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int \frac{-4+3x}{4-2x+x^2}\,dx = -\frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(x-1)\right) + \frac{3}{2}\,\log\left(x^2-2\,x+4\right)$$

input
```
integrate((-4+3*x)/(x^2-2*x+4),x, algorithm="fricas")
```

output
```
-1/3*sqrt(3)*arctan(1/3*sqrt(3)*(x - 1)) + 3/2*log(x^2 - 2*x + 4)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.12

$$\int \frac{-4+3x}{4-2x+x^2}\,dx = \frac{3\log\left(x^2-2x+4\right)}{2} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{3}$$

input
```
integrate((-4+3*x)/(x**2-2*x+4),x)
```

output
```
3*log(x**2 - 2*x + 4)/2 - sqrt(3)*atan(sqrt(3)*x/3 - sqrt(3)/3)/3
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int \frac{-4+3x}{4-2x+x^2}\,dx = -\frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(x-1)\right) + \frac{3}{2}\,\log\left(x^2-2\,x+4\right)$$

input
```
integrate((-4+3*x)/(x^2-2*x+4),x, algorithm="maxima")
```

output
```
-1/3*sqrt(3)*arctan(1/3*sqrt(3)*(x - 1)) + 3/2*log(x^2 - 2*x + 4)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int \frac{-4+3x}{4-2x+x^2}\,dx = -\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(x-1)\right)+\frac{3}{2}\log\left(x^2-2x+4\right)$$

input
```
integrate((-4+3*x)/(x^2-2*x+4),x, algorithm="giac")
```

output
```
-1/3*sqrt(3)*arctan(1/3*sqrt(3)*(x - 1)) + 3/2*log(x^2 - 2*x + 4)
```

Mupad [B] (verification not implemented)

Time = 21.76 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94

$$\int \frac{-4+3x}{4-2x+x^2}\,dx = \frac{3\ln\left(x^2-2x+4\right)}{2}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}\,x}{3}-\frac{\sqrt{3}}{3}\right)}{3}$$

input
```
int((3*x - 4)/(x^2 - 2*x + 4),x)
```

output
```
(3*log(x^2 - 2*x + 4))/2 - (3^(1/2)*atan((3^(1/2)*x)/3 - 3^(1/2)/3))/3
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.78

$$\int \frac{-4+3x}{4-2x+x^2}\,dx = -\frac{\sqrt{3}\,atan\left(\frac{x-1}{\sqrt{3}}\right)}{3}+\frac{3\log(x^2-2x+4)}{2}$$

input
```
int((-4+3*x)/(x^2-2*x+4),x)
```

output
```
( - 2*sqrt(3)*atan((x - 1)/sqrt(3)) + 9*log(x**2 - 2*x + 4))/6
```

3.94 $\int \frac{-8+2x+3x^2}{8+x^3} dx$

Optimal result

Integrand size = 18, antiderivative size = 32

$$\int \frac{-8+2x+3x^2}{8+x^3} dx = \frac{\arctan\left(\frac{1-x}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{3}{2}\log\left(4-2x+x^2\right)$$

output
```
1/3*arctan(1/3*(1-x)*3^(1/2))*3^(1/2)+3/2*ln(x^2-2*x+4)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.97

$$\int \frac{-8+2x+3x^2}{8+x^3} dx = -\frac{\arctan\left(\frac{-1+x}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{3}{2}\log\left(4-2x+x^2\right)$$

input
```
Integrate[(-8 + 2*x + 3*x^2)/(8 + x^3),x]
```

output
```
-(ArcTan[(-1 + x)/Sqrt[3]]/Sqrt[3]) + (3*Log[4 - 2*x + x^2])/2
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.12, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {2411, 27, 1142, 27, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{3x^2+2x-8}{x^3+8}\,dx$$

$$\Big\downarrow \text{2411}$$

$$\frac{1}{2}\int -\frac{2(4-3x)}{x^2-2x+4}dx$$

$$\Big\downarrow \text{27}$$

$$-\int \frac{4-3x}{x^2-2x+4}dx$$

$$\Big\downarrow \text{1142}$$

$$\frac{3}{2}\int -\frac{2(1-x)}{x^2-2x+4}dx-\int \frac{1}{x^2-2x+4}dx$$

$$\Big\downarrow \text{27}$$

$$-\int \frac{1}{x^2-2x+4}dx-3\int \frac{1-x}{x^2-2x+4}dx$$

$$\Big\downarrow \text{1083}$$

$$2\int \frac{1}{-(2x-2)^2-12}d(2x-2)-3\int \frac{1-x}{x^2-2x+4}dx$$

$$\Big\downarrow \text{217}$$

$$-3\int \frac{1-x}{x^2-2x+4}dx-\frac{\arctan\left(\frac{2x-2}{2\sqrt{3}}\right)}{\sqrt{3}}$$

$$\Big\downarrow \text{1103}$$

$$\frac{3}{2}\log\left(x^2-2x+4\right)-\frac{\arctan\left(\frac{2x-2}{2\sqrt{3}}\right)}{\sqrt{3}}$$

input
```
Int[(-8 + 2*x + 3*x^2)/(8 + x^3),x]
```

output
```
-(ArcTan[(-2 + 2*x)/(2*Sqrt[3])]/Sqrt[3]) + (3*Log[4 - 2*x + x^2])/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2411
```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = (a/b)^(1/3)}, Simp[q^2/a
   Int[(A + C*q*x)/(q^2 - q*x + x^2), x], x]] /; EqQ[A - B*(a/b)^(1/3) + C*
(a/b)^(2/3), 0]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.84

method	result
risch	$\frac{3\ln\left(x^{2}-2x+4\right)}{2}-\frac{\sqrt{3}\,\arctan\left(\frac{(x-1)\sqrt{3}}{3}\right)}{3}$
default	$\frac{3\ln\left(x^{2}-2x+4\right)}{2}-\frac{\sqrt{3}\,\arctan\left(\frac{(2x-2)\sqrt{3}}{6}\right)}{3}$
meijerg	$-\frac{2x\ln\left(1+\frac{\left(x^{3}\right)^{\frac{1}{3}}}{2}\right)}{3\left(x^{3}\right)^{\frac{1}{3}}}+\frac{x\ln\left(1-\frac{\left(x^{3}\right)^{\frac{1}{3}}}{2}+\frac{\left(x^{3}\right)^{\frac{2}{3}}}{4}\right)}{3\left(x^{3}\right)^{\frac{1}{3}}}-\frac{2x\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(x^{3}\right)^{\frac{1}{3}}}{4-\left(x^{3}\right)^{\frac{1}{3}}}\right)}{3\left(x^{3}\right)^{\frac{1}{3}}}+\ln\left(1+\frac{x^{3}}{8}\right)-\frac{x^{2}\ln\left(1+\frac{\left(x^{3}\right)^{\frac{1}{3}}}{2}\right)}{3\left(x^{3}\right)^{\frac{2}{3}}}$

input
```
int((3*x^2+2*x-8)/(x^3+8),x,method=_RETURNVERBOSE)
```

output
```
3/2*ln(x^2-2*x+4)-1/3*3^(1/2)*arctan(1/3*(x-1)*3^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int \frac{-8+2x+3x^2}{8+x^3}\,dx=-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(x-1)\right)+\frac{3}{2}\log\left(x^2-2x+4\right)$$

input
```
integrate((3*x^2+2*x-8)/(x^3+8),x, algorithm="fricas")
```

output
```
-1/3*sqrt(3)*arctan(1/3*sqrt(3)*(x - 1)) + 3/2*log(x^2 - 2*x + 4)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.12

$$\int \frac{-8+2x+3x^2}{8+x^3}\,dx = \frac{3\log\left(x^2-2x+4\right)}{2} - \frac{\sqrt{3}\operatorname{atan}\left(\frac{\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{3}$$

input
```
integrate((3*x**2+2*x-8)/(x**3+8),x)
```

output
```
3*log(x**2 - 2*x + 4)/2 - sqrt(3)*atan(sqrt(3)*x/3 - sqrt(3)/3)/3
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int \frac{-8+2x+3x^2}{8+x^3}\,dx = -\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(x-1)\right) + \frac{3}{2}\log\left(x^2-2x+4\right)$$

input
```
integrate((3*x^2+2*x-8)/(x^3+8),x, algorithm="maxima")
```

output
```
-1/3*sqrt(3)*arctan(1/3*sqrt(3)*(x - 1)) + 3/2*log(x^2 - 2*x + 4)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int \frac{-8+2x+3x^2}{8+x^3}\,dx = -\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(x-1)\right) + \frac{3}{2}\log\left(x^2-2x+4\right)$$

input
```
integrate((3*x^2+2*x-8)/(x^3+8),x, algorithm="giac")
```

output
```
-1/3*sqrt(3)*arctan(1/3*sqrt(3)*(x - 1)) + 3/2*log(x^2 - 2*x + 4)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94

$$\int \frac{-8+2x+3x^2}{8+x^3}\,dx = \frac{3\,\ln\left(x^2-2\,x+4\right)}{2}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}\,x}{3}-\frac{\sqrt{3}}{3}\right)}{3}$$

input
```
int((2*x + 3*x^2 - 8)/(x^3 + 8),x)
```

output
```
(3*log(x^2 - 2*x + 4))/2 - (3^(1/2)*atan((3^(1/2)*x)/3 - 3^(1/2)/3))/3
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.78

$$\int \frac{-8+2x+3x^2}{8+x^3}\,dx = -\frac{\sqrt{3}\,atan\left(\frac{x-1}{\sqrt{3}}\right)}{3}+\frac{3\log(x^2-2x+4)}{2}$$

input
```
int((3*x^2+2*x-8)/(x^3+8),x)
```

output
```
( - 2*sqrt(3)*atan((x - 1)/sqrt(3)) + 9*log(x**2 - 2*x + 4))/6
```

3.95 $\int \frac{2+x}{-1+2x+x^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 45

$$\int \frac{2+x}{-1+2x+x^2}\,dx=\frac{1}{4}\left(2+\sqrt{2}\right)\log\left(1-\sqrt{2}+x\right)+\frac{1}{4}\left(2-\sqrt{2}\right)\log\left(1+\sqrt{2}+x\right)$$

output
```
1/4*(2+2^(1/2))*ln(1-2^(1/2)+x)+1/4*(2-2^(1/2))*ln(1+2^(1/2)+x)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.93

$$\int \frac{2+x}{-1+2x+x^2}\,dx=\frac{1}{4}\left(\left(2+\sqrt{2}\right)\log\left(-1+\sqrt{2}-x\right)-\left(-2+\sqrt{2}\right)\log\left(1+\sqrt{2}+x\right)\right)$$

input
```
Integrate[(2 + x)/(-1 + 2*x + x^2),x]
```

output
```
((2 + Sqrt[2])*Log[-1 + Sqrt[2] - x] - (-2 + Sqrt[2])*Log[1 + Sqrt[2] + x]
)/4
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x+2}{x^2+2x-1} \, dx$$

$$\Big\downarrow \text{ 1141}$$

$$\int \left(\frac{2-\sqrt{2}}{4 \left(x+\sqrt{2}+1\right)}+\frac{2+\sqrt{2}}{4 \left(x-\sqrt{2}+1\right)}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{4} \left(2+\sqrt{2}\right) \log \left(x-\sqrt{2}+1\right)+\frac{1}{4} \left(2-\sqrt{2}\right) \log \left(x+\sqrt{2}+1\right)$$

input
```
Int[(2 + x)/(-1 + 2*x + x^2),x]
```

output
```
((2 + Sqrt[2])*Log[1 - Sqrt[2] + x])/4 + ((2 - Sqrt[2])*Log[1 + Sqrt[2] +
x])/4
```

Defintions of rubi rules used

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.64

method	result	size
default	$\frac{\ln(x^2+2x-1)}{2}-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{(2x+2)\sqrt{2}}{4}\right)}{2}$	29
risch	$\frac{\ln\left(1-\sqrt{2}+x\right)}{2}+\frac{\ln\left(1-\sqrt{2}+x\right)\sqrt{2}}{4}+\frac{\ln\left(1+\sqrt{2}+x\right)}{2}-\frac{\ln\left(1+\sqrt{2}+x\right)\sqrt{2}}{4}$	48

input
```
int((2+x)/(x^2+2*x-1),x,method=_RETURNVERBOSE)
```

output
```
1/2*ln(x^2+2*x-1)-1/2*2^(1/2)*arctanh(1/4*(2*x+2)*2^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00

$$\int \frac{2+x}{-1+2x+x^2}\,dx = \frac{1}{4}\sqrt{2}\log\left(\frac{x^2-2\sqrt{2}(x+1)+2x+3}{x^2+2x-1}\right)+\frac{1}{2}\log\left(x^2+2x-1\right)$$

input
```
integrate((2+x)/(x^2+2*x-1),x, algorithm="fricas")
```

output
```
1/4*sqrt(2)*log((x^2 - 2*sqrt(2)*(x + 1) + 2*x + 3)/(x^2 + 2*x - 1)) + 1/2
*log(x^2 + 2*x - 1)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.87

$$\int \frac{2+x}{-1+2x+x^2}\,dx = \left(\frac{1}{2}-\frac{\sqrt{2}}{4}\right)\log\left(x+1+\sqrt{2}\right)+\left(\frac{\sqrt{2}}{4}+\frac{1}{2}\right)\log\left(x-\sqrt{2}+1\right)$$

input
```
integrate((2+x)/(x**2+2*x-1),x)
```

output
```
(1/2 - sqrt(2)/4)*log(x + 1 + sqrt(2)) + (sqrt(2)/4 + 1/2)*log(x - sqrt(2)
 + 1)
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.78

$$\int \frac{2+x}{-1+2x+x^2}\,dx=\frac{1}{4}\sqrt{2}\log\left(\frac{x-\sqrt{2}+1}{x+\sqrt{2}+1}\right)+\frac{1}{2}\log\left(x^2+2\,x-1\right)$$

input
```
integrate((2+x)/(x^2+2*x-1),x, algorithm="maxima")
```

output
```
1/4*sqrt(2)*log((x - sqrt(2) + 1)/(x + sqrt(2) + 1)) + 1/2*log(x^2 + 2*x -
 1)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.98

$$\int \frac{2+x}{-1+2x+x^2}\,dx=\frac{1}{4}\sqrt{2}\log\left(\frac{|2\,x-2\,\sqrt{2}+2|}{|2\,x+2\,\sqrt{2}+2|}\right)+\frac{1}{2}\log\left(|x^2+2\,x-1|\right)$$

input
```
integrate((2+x)/(x^2+2*x-1),x, algorithm="giac")
```

output
```
1/4*sqrt(2)*log(abs(2*x - 2*sqrt(2) + 2)/abs(2*x + 2*sqrt(2) + 2)) + 1/2*l
og(abs(x^2 + 2*x - 1))
```

Mupad [B] (verification not implemented)

Time = 21.87 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.76

$$\int \frac{2+x}{-1+2x+x^2}\,dx = \ln\left(x-\sqrt{2}+1\right)\left(\frac{\sqrt{2}}{4}+\frac{1}{2}\right)-\ln\left(x+\sqrt{2}+1\right)\left(\frac{\sqrt{2}}{4}-\frac{1}{2}\right)$$

input
```
int((x + 2)/(2*x + x^2 - 1),x)
```

output
```
log(x - 2^(1/2) + 1)*(2^(1/2)/4 + 1/2) - log(x + 2^(1/2) + 1)*(2^(1/2)/4 -
 1/2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.91

$$\int \frac{2+x}{-1+2x+x^2}\,dx = \frac{\sqrt{2}\log\left(-\sqrt{2}+x+1\right)}{4}-\frac{\sqrt{2}\log\left(\sqrt{2}+x+1\right)}{4}+\frac{\log\left(-\sqrt{2}+x+1\right)}{2}+\frac{\log\left(\sqrt{2}+x+1\right)}{2}$$

input
```
int((2+x)/(x^2+2*x-1),x)
```

output
```
(sqrt(2)*log( - sqrt(2) + x + 1) - sqrt(2)*log(sqrt(2) + x + 1) + 2*log( -
 sqrt(2) + x + 1) + 2*log(sqrt(2) + x + 1))/4
```

3.96 $\int \frac{-4+x^2}{2-5x+x^3} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 45

$$\int \frac{-4+x^2}{2-5x+x^3} \, dx = \frac{1}{4}\left(2+\sqrt{2}\right) \log \left(1-\sqrt{2}+x\right) + \frac{1}{4}\left(2-\sqrt{2}\right) \log \left(1+\sqrt{2}+x\right)$$

output
```
1/4*(2+2^(1/2))*ln(1-2^(1/2)+x)+1/4*(2-2^(1/2))*ln(1+2^(1/2)+x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.93

$$\int \frac{-4+x^2}{2-5x+x^3} \, dx = \frac{1}{4}\left(\left(2+\sqrt{2}\right) \log \left(-1+\sqrt{2}-x\right) - \left(-2+\sqrt{2}\right) \log \left(1+\sqrt{2}+x\right)\right)$$

input
```
Integrate[(-4 + x^2)/(2 - 5*x + x^3),x]
```

output
```
((2 + Sqrt[2])*Log[-1 + Sqrt[2] - x] - (-2 + Sqrt[2])*Log[1 + Sqrt[2] + x]
)/4
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2457, 1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2-4}{x^3-5x+2}dx$$

$$\Big\downarrow \text{ 2457}$$

$$\int \frac{x+2}{x^2+2x-1}dx$$

$$\Big\downarrow \text{ 1141}$$

$$\int \left(\frac{2-\sqrt{2}}{4\left(x+\sqrt{2}+1\right)}+\frac{2+\sqrt{2}}{4\left(x-\sqrt{2}+1\right)}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{4}\left(2+\sqrt{2}\right)\log\left(x-\sqrt{2}+1\right)+\frac{1}{4}\left(2-\sqrt{2}\right)\log\left(x+\sqrt{2}+1\right)$$

input
```
Int[(-4 + x^2)/(2 - 5*x + x^3),x]
```

output
```
((2 + Sqrt[2])*Log[1 - Sqrt[2] + x])/4 + ((2 - Sqrt[2])*Log[1 + Sqrt[2] + 
x])/4
```

Defintions of rubi rules used

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p, 
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2457
```
Int[(u_.)*(Px_)*(Qx_)^(q_), x_Symbol] :> Module[{Rx = PolyGCD[Px, Qx, x]},
Int[u*Rx^(q + 1)*PolynomialQuotient[Px, Rx, x]*PolynomialQuotient[Qx, Rx, x
]^q, x] /; NeQ[Rx, 1]] /; ILtQ[q, 0] && PolyQ[Px, x] && PolyQ[Qx, x]
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.64

method	result	size
default	$\frac{\ln(x^2+2x-1)}{2}-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{(2x+2)\sqrt{2}}{4}\right)}{2}$	29
risch	$\frac{\ln\left(1-\sqrt{2}+x\right)}{2}+\frac{\ln\left(1-\sqrt{2}+x\right)\sqrt{2}}{4}+\frac{\ln\left(1+\sqrt{2}+x\right)}{2}-\frac{\ln\left(1+\sqrt{2}+x\right)\sqrt{2}}{4}$	48

input
```
int((x^2-4)/(x^3-5*x+2),x,method=_RETURNVERBOSE)
```

output
```
1/2*ln(x^2+2*x-1)-1/2*2^(1/2)*arctanh(1/4*(2*x+2)*2^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00

$$\int \frac{-4+x^2}{2-5x+x^3}\,dx=\frac{1}{4}\sqrt{2}\log\left(\frac{x^2-2\sqrt{2}(x+1)+2x+3}{x^2+2x-1}\right)+\frac{1}{2}\log\left(x^2+2x-1\right)$$

input
```
integrate((x^2-4)/(x^3-5*x+2),x, algorithm="fricas")
```

output
```
1/4*sqrt(2)*log((x^2 - 2*sqrt(2)*(x + 1) + 2*x + 3)/(x^2 + 2*x - 1)) + 1/2
*log(x^2 + 2*x - 1)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.87

$$\int \frac{-4+x^2}{2-5x+x^3}\,dx = \left(\frac{1}{2}-\frac{\sqrt{2}}{4}\right)\log\left(x+1+\sqrt{2}\right)+\left(\frac{\sqrt{2}}{4}+\frac{1}{2}\right)\log\left(x-\sqrt{2}+1\right)$$

input
```
integrate((x**2-4)/(x**3-5*x+2),x)
```

output
```
(1/2 - sqrt(2)/4)*log(x + 1 + sqrt(2)) + (sqrt(2)/4 + 1/2)*log(x - sqrt(2)
 + 1)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.78

$$\int \frac{-4+x^2}{2-5x+x^3}\,dx = \frac{1}{4}\sqrt{2}\log\left(\frac{x-\sqrt{2}+1}{x+\sqrt{2}+1}\right)+\frac{1}{2}\log\left(x^2+2x-1\right)$$

input
```
integrate((x^2-4)/(x^3-5*x+2),x, algorithm="maxima")
```

output
```
1/4*sqrt(2)*log((x - sqrt(2) + 1)/(x + sqrt(2) + 1)) + 1/2*log(x^2 + 2*x -
 1)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.98

$$\int \frac{-4+x^2}{2-5x+x^3}\,dx = \frac{1}{4}\sqrt{2}\log\left(\frac{\left|2x-2\sqrt{2}+2\right|}{\left|2x+2\sqrt{2}+2\right|}\right)+\frac{1}{2}\log\left(\left|x^2+2x-1\right|\right)$$

input
```
integrate((x^2-4)/(x^3-5*x+2),x, algorithm="giac")
```

output
```
1/4*sqrt(2)*log(abs(2*x - 2*sqrt(2) + 2)/abs(2*x + 2*sqrt(2) + 2)) + 1/2*l
og(abs(x^2 + 2*x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.76

$$\int \frac{-4+x^2}{2-5x+x^3}\,dx = \ln\left(x-\sqrt{2}+1\right)\left(\frac{\sqrt{2}}{4}+\frac{1}{2}\right)-\ln\left(x+\sqrt{2}+1\right)\left(\frac{\sqrt{2}}{4}-\frac{1}{2}\right)$$

input
```
int((x^2 - 4)/(x^3 - 5*x + 2),x)
```

output
```
log(x - 2^(1/2) + 1)*(2^(1/2)/4 + 1/2) - log(x + 2^(1/2) + 1)*(2^(1/2)/4 -
 1/2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.91

$$\int \frac{-4+x^2}{2-5x+x^3}\,dx = \frac{\sqrt{2}\log\left(-\sqrt{2}+x+1\right)}{4}-\frac{\sqrt{2}\log\left(\sqrt{2}+x+1\right)}{4}+\frac{\log\left(-\sqrt{2}+x+1\right)}{2}+\frac{\log\left(\sqrt{2}+x+1\right)}{2}$$

input
```
int((x^2-4)/(x^3-5*x+2),x)
```

output
```
(sqrt(2)*log( - sqrt(2) + x + 1) - sqrt(2)*log(sqrt(2) + x + 1) + 2*log( -
 sqrt(2) + x + 1) + 2*log(sqrt(2) + x + 1))/4
```

3.97 $\int \frac{-3+2\sqrt{2}+x^2}{17-12\sqrt{2}+\left(2-4\sqrt{2}\right)x^2+x^4}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 22

$$\int \frac{-3+2\sqrt{2}+x^2}{17-12\sqrt{2}+\left(2-4\sqrt{2}\right)x^2+x^4}\,dx=-\frac{1}{2}\operatorname{arctanh}\left(\frac{2x}{3-2\sqrt{2}+x^2}\right)$$

output
```
-1/2*arctanh(2*x/(3-2*2^(1/2)+x^2))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 45 vs. 2(22) = 44.

Time = 0.09 (sec) , antiderivative size = 45, normalized size of antiderivative = 2.05

$$\int \frac{-3+2\sqrt{2}+x^2}{17-12\sqrt{2}+\left(2-4\sqrt{2}\right)x^2+x^4}\,dx=-\frac{1}{4}\log\left(-3+2\sqrt{2}-2x-x^2\right)+\frac{1}{4}\log\left(-3+2\sqrt{2}+2x-x^2\right)$$

input
```
Integrate[(-3 + 2*Sqrt[2] + x^2)/(17 - 12*Sqrt[2] + (2 - 4*Sqrt[2])*x^2 +
x^4),x]
```

output
```
-1/4*Log[-3 + 2*Sqrt[2] - 2*x - x^2] + Log[-3 + 2*Sqrt[2] + 2*x - x^2]/4
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 103 vs. $2(22) = 44$.

Time = 0.44 (sec) , antiderivative size = 103, normalized size of antiderivative = 4.68, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.075$, Rules used = {1475, 1081, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2+2\sqrt{2}-3}{x^4+\left(2-4\sqrt{2}\right)x^2-12\sqrt{2}+17}dx$$

$$\Big\downarrow \text{1475}$$

$$\frac{1}{2}\int \frac{1}{x^2-2\sqrt{2\left(-1+\sqrt{2}\right)}x+2\sqrt{2}-3}dx+\frac{1}{2}\int \frac{1}{x^2+2\sqrt{2\left(-1+\sqrt{2}\right)}x+2\sqrt{2}-3}dx$$

$$\Big\downarrow \text{1081}$$

$$\frac{1}{2}\int \left(-\frac{1}{2\left(x-\sqrt{2\left(-1+\sqrt{2}\right)}+1\right)}-\frac{1}{2\left(-x+\sqrt{2\left(-1+\sqrt{2}\right)}+1\right)}\right)dx+\frac{1}{2}\int \left(-\frac{1}{2\left(x+\sqrt{2\left(-1+\sqrt{2}\right)}+1\right)}-\frac{1}{2\left(-x-\sqrt{2\left(-1+\sqrt{2}\right)}+1\right)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{1}{2}\log\left(-x+\sqrt{2\left(\sqrt{2}-1\right)}+1\right)-\frac{1}{2}\log\left(x-\sqrt{2\left(\sqrt{2}-1\right)}+1\right)\right)+\frac{1}{2}\left(\frac{1}{2}\log\left(-x-\sqrt{2\left(\sqrt{2}-1\right)}+1\right)-\frac{1}{2}\log\left(x+\sqrt{2\left(\sqrt{2}-1\right)}+1\right)\right)$$

input
```
Int[(-3 + 2*Sqrt[2] + x^2)/(17 - 12*Sqrt[2] + (2 - 4*Sqrt[2])*x^2 + x^4),x
]
```

output
```
(Log[1 + Sqrt[2*(-1 + Sqrt[2])] - x]/2 - Log[1 - Sqrt[2*(-1 + Sqrt[2])] +
x]/2)/2 + (Log[1 - Sqrt[2*(-1 + Sqrt[2])] - x]/2 - Log[1 + Sqrt[2*(-1 + Sq
rt[2])] + x]/2)/2
```

Defintions of rubi rules used

rule 1081
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[c   Int[ExpandIntegrand[1/((b/2 - q/2 + c*x)*(b/2 + q/2
+ c*x)), x], x], x]] /; FreeQ[{a, b, c}, x] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 1475
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^
2, x], x], x] + Simp[e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] &&
 (GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2]
, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.55

method	result	size
default	$\frac{\ln\left(x^2-2\sqrt{2}-2x+3\right)}{4}-\frac{\ln\left(x^2-2\sqrt{2}+2x+3\right)}{4}$	34
risch	$\frac{\ln\left(x^2-2\sqrt{2}-2x+3\right)}{4}-\frac{\ln\left(x^2-2\sqrt{2}+2x+3\right)}{4}$	34
parallelrisch	$\frac{\ln\left(x^2-2\sqrt{2}-2x+3\right)}{4}-\frac{\ln\left(x^2-2\sqrt{2}+2x+3\right)}{4}$	34

input
```
int((-3+2*2^(1/2)+x^2)/(17-12*2^(1/2)+(2-4*2^(1/2))*x^2+x^4),x,method=_RET
URNVERBOSE)
```

output
```
1/4*ln(x^2-2*2^(1/2)-2*x+3)-1/4*ln(x^2-2*2^(1/2)+2*x+3)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.50

$$\int \frac{-3+2\sqrt{2}+x^2}{17-12\sqrt{2}+\left(2-4\sqrt{2}\right)x^2+x^4}\,dx = -\frac{1}{4}\log\left(x^2+2\,x-2\,\sqrt{2}+3\right) + \frac{1}{4}\log\left(x^2-2\,x-2\,\sqrt{2}+3\right)$$

input
```
integrate((-3+2*2^(1/2)+x^2)/(17-12*2^(1/2)+(2-4*2^(1/2))*x^2+x^4),x, algo
rithm="fricas")
```

output
```
-1/4*log(x^2 + 2*x - 2*sqrt(2) + 3) + 1/4*log(x^2 - 2*x - 2*sqrt(2) + 3)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{-3+2\sqrt{2}+x^2}{17-12\sqrt{2}+\left(2-4\sqrt{2}\right)x^2+x^4}\,dx = \text{Exception raised: PolynomialError}$$

input
```
integrate((-3+2*2**(1/2)+x**2)/(17-12*2**(1/2)+(2-4*2**(1/2))*x***2+x***4),x
)
```

output
```
Exception raised: PolynomialError >> 1/(-2304*_t**4 + 1024*sqrt(2)*_t**4 -
 32*_t**2 + 64*sqrt(2)*_t**2 - 1) contains an element of the set of genera
tors.
```

Maxima [F]

$$\int \frac{-3+2\sqrt{2}+x^2}{17-12\sqrt{2}+\left(2-4\sqrt{2}\right)x^2+x^4}\,dx = \int \frac{x^2+2\sqrt{2}-3}{x^4-2\,x^2\left(2\sqrt{2}-1\right)-12\sqrt{2}+17}\,dx$$

input
```
integrate((-3+2*2^(1/2)+x^2)/(17-12*2^(1/2)+(2-4*2^(1/2))*x^2+x^4),x, algo
rithm="maxima")
```

output
```
integrate((x^2 + 2*sqrt(2) - 3)/(x^4 - 2*x^2*(2*sqrt(2) - 1) - 12*sqrt(2)
+ 17), x)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.59

$$\int \frac{-3+2\sqrt{2}+x^2}{17-12\sqrt{2}+\left(2-4\sqrt{2}\right)x^2+x^4}\,dx = -\frac{1}{4}\log\left(\left|x^2+2\,x-2\sqrt{2}+3\right|\right) + \frac{1}{4}\log\left(\left|x^2-2\,x-2\sqrt{2}+3\right|\right)$$

input
```
integrate((-3+2*2^(1/2)+x^2)/(17-12*2^(1/2)+(2-4*2^(1/2))*x^2+x^4),x, algo
rithm="giac")
```

output
```
-1/4*log(abs(x^2 + 2*x - 2*sqrt(2) + 3)) + 1/4*log(abs(x^2 - 2*x - 2*sqrt(
2) + 3))
```

Mupad [B] (verification not implemented)

Time = 22.71 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.55

$$\int \frac{-3+2\sqrt{2}+x^2}{17-12\sqrt{2}+\left(2-4\sqrt{2}\right)x^2+x^4}\,dx = -\frac{\operatorname{atanh}\left(\frac{x\left(16\sqrt{2}-16\right)}{2\left(20\sqrt{2}+4\sqrt{2}\,x^2-4\,x^2-28\right)}\right)}{2}$$

input
```
int(-(2*2^(1/2) + x^2 - 3)/(x^2*(4*2^(1/2) - 2) + 12*2^(1/2) - x^4 - 17),x
)
```

output
```
-atanh((x*(16*2^(1/2) - 16))/(2*(20*2^(1/2) + 4*2^(1/2)*x^2 - 4*x^2 - 28))
)/2
```

Reduce [F]

$$\int \frac{-3+2\sqrt{2}+x^2}{17-12\sqrt{2}+\left(2-4\sqrt{2}\right)x^2+x^4}\,dx = 6\sqrt{2}\left(\int \frac{x^4}{x^8+4x^6+6x^4-124x^2+1}dx\right) + 4\sqrt{2}\left(\int \frac{x^2}{x^8+4x^6+6x^4-124x^2+1}dx\right) - 2\sqrt{2}\left(\int \frac{1}{x^8+4x^6+6x^4-124x^2+1}dx\right) + \int \frac{x^6}{x^8+4x^6+6x^4-124x^2+1}dx - \left(\int \frac{x^4}{x^8+4x^6+6x^4-124x^2+1}dx\right) + 27\left(\int \frac{x^2}{x^8+4x^6+6x^4-124x^2+1}dx\right) - 3\left(\int \frac{1}{x^8+4x^6+6x^4-124x^2+1}dx\right)$$

input
```
int((-3+2*2^(1/2)+x^2)/(17-12*2^(1/2)+(2-4*2^(1/2))*x^2+x^4),x)
```

output
```
6*sqrt(2)*int(x**4/(x**8 + 4*x**6 + 6*x**4 - 124*x**2 + 1),x) + 4*sqrt(2)*
int(x**2/(x**8 + 4*x**6 + 6*x**4 - 124*x**2 + 1),x) - 2*sqrt(2)*int(1/(x**
8 + 4*x**6 + 6*x**4 - 124*x**2 + 1),x) + int(x**6/(x**8 + 4*x**6 + 6*x**4
- 124*x**2 + 1),x) - int(x**4/(x**8 + 4*x**6 + 6*x**4 - 124*x**2 + 1),x) +
 27*int(x**2/(x**8 + 4*x**6 + 6*x**4 - 124*x**2 + 1),x) - 3*int(1/(x**8 +
4*x**6 + 6*x**4 - 124*x**2 + 1),x)
```

3.98 $\int \frac{(-3+2\sqrt{2})^2-x^4}{-99+70\sqrt{2}+(-39+28\sqrt{2})x^2+(-5+6\sqrt{2})x^4-x^6}\,dx$

Optimal result

Integrand size = 60, antiderivative size = 22

$$\int \frac{\left(-3+2\sqrt{2}\right)^2-x^4}{-99+70\sqrt{2}+\left(-39+28\sqrt{2}\right)x^2+\left(-5+6\sqrt{2}\right)x^4-x^6}\,dx = -\frac{1}{2}\text{arctanh}\left(\frac{2x}{3-2\sqrt{2}+x^2}\right)$$

output
```
-1/2*arctanh(2*x/(3-2*2^(1/2)+x^2))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 45 vs. 2(22) = 44.

Time = 0.11 (sec) , antiderivative size = 45, normalized size of antiderivative = 2.05

$$\int \frac{\left(-3+2\sqrt{2}\right)^2-x^4}{-99+70\sqrt{2}+\left(-39+28\sqrt{2}\right)x^2+\left(-5+6\sqrt{2}\right)x^4-x^6}\,dx = -\frac{1}{4}\log\left(-3+2\sqrt{2}-2x-x^2\right)+\frac{1}{4}\log\left(-3+2\sqrt{2}+2x-x^2\right)$$

input
```
Integrate[((-3 + 2*Sqrt[2])^2 - x^4)/(-99 + 70*Sqrt[2] + (-39 + 28*Sqrt[2]
)*x^2 + (-5 + 6*Sqrt[2])*x^4 - x^6),x]
```

output
```
-1/4*Log[-3 + 2*Sqrt[2] - 2*x - x^2] + Log[-3 + 2*Sqrt[2] + 2*x - x^2]/4
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(2\sqrt{2}-3\right)^2-x^4}{-x^6+\left(6\sqrt{2}-5\right)x^4+\left(28\sqrt{2}-39\right)x^2+70\sqrt{2}-99}dx$$

$$\Big\downarrow \text{7292}$$

$$\int \frac{x^4-\left(2\sqrt{2}-3\right)^2}{x^6-\left(6\sqrt{2}-5\right)x^4-\left(28\sqrt{2}-39\right)x^2+99\left(1-\frac{70\sqrt{2}}{99}\right)}dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(\frac{x^4}{x^6+5\left(1-\frac{6\sqrt{2}}{5}\right)x^4+39\left(1-\frac{28\sqrt{2}}{39}\right)x^2+99\left(1-\frac{70\sqrt{2}}{99}\right)}+\frac{12\sqrt{2}-17}{x^6+5\left(1-\frac{6\sqrt{2}}{5}\right)x^4+39\left(1-\frac{28\sqrt{2}}{39}\right)x^2+99}\right.$$

$$\Big\downarrow \text{2009}$$

$$\int \frac{x^4}{x^6+5\left(1-\frac{6\sqrt{2}}{5}\right)x^4+39\left(1-\frac{28\sqrt{2}}{39}\right)x^2+99\left(1-\frac{70\sqrt{2}}{99}\right)}dx-\left(17-12\sqrt{2}\right)\int \frac{1}{x^6+5\left(1-\frac{6\sqrt{2}}{5}\right)x^4+39\left(1-\frac{28\sqrt{2}}{39}\right)x^2+99\left(1-\frac{70\sqrt{2}}{99}\right)}dx$$

input
```
Int[((-3 + 2*Sqrt[2])^2 - x^4)/(-99 + 70*Sqrt[2] + (-39 + 28*Sqrt[2])*x^2
+ (-5 + 6*Sqrt[2])*x^4 - x^6),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7292
```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.55

method	result	size
default	$\frac{\ln\left(x^2-2\sqrt{2}-2x+3\right)}{4}-\frac{\ln\left(x^2-2\sqrt{2}+2x+3\right)}{4}$	34
risch	$\frac{\ln\left(x^2-2\sqrt{2}-2x+3\right)}{4}-\frac{\ln\left(x^2-2\sqrt{2}+2x+3\right)}{4}$	34
parallelrisch	$\frac{\ln\left(x^2-2\sqrt{2}-2x+3\right)}{4}-\frac{\ln\left(x^2-2\sqrt{2}+2x+3\right)}{4}$	34

input
```
int(((-3+2*2^(1/2))^2-x^4)/(-99+70*2^(1/2)+(-39+28*2^(1/2))*x^2+(-5+6*2^(1
/2))*x^4-x^6),x,method=_RETURNVERBOSE)
```

output
```
1/4*ln(x^2-2*2^(1/2)-2*x+3)-1/4*ln(x^2-2*2^(1/2)+2*x+3)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.50

$$\int \frac{\left(-3+2\sqrt{2}\right)^2-x^4}{-99+70\sqrt{2}+\left(-39+28\sqrt{2}\right)x^2+\left(-5+6\sqrt{2}\right)x^4-x^6}\,dx$$
$$=-\frac{1}{4}\log\left(x^2+2\,x-2\,\sqrt{2}+3\right)+\frac{1}{4}\log\left(x^2-2\,x-2\,\sqrt{2}+3\right)$$

input
```
integrate(((-3+2*2^(1/2))^2-x^4)/(-99+70*2^(1/2)+(-39+28*2^(1/2))*x^2+(-5+
6*2^(1/2))*x^4-x^6),x, algorithm="fricas")
```

output
```
-1/4*log(x^2 + 2*x - 2*sqrt(2) + 3) + 1/4*log(x^2 - 2*x - 2*sqrt(2) + 3)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{\left(-3+2\sqrt{2}\right)^2-x^4}{-99+70\sqrt{2}+\left(-39+28\sqrt{2}\right)x^2+\left(-5+6\sqrt{2}\right)x^4-x^6}\,dx$$
$$=\text{Exception raised: PolynomialError}$$

input
```
integrate(((-3+2*2**(1/2))**2-x**4)/(-99+70*2**(1/2)+(-39+28*2**(1/2))*x**
2+(-5+6*2**(1/2))*x**4-x**6),x)
```

output
```
Exception raised: PolynomialError >> 1/(-160817378869521623700434126076296
32108508413135104*_t**4 + 113715059131284938253449200806460980121432594954
24*sqrt(2)*_t**4 - 52507653152788951663100478062419214178686830083 2*_t**2
+ 3712851760852
```

Maxima [F]

$$\int \frac{\left(-3+2\sqrt{2}\right)^2-x^4}{-99+70\sqrt{2}+\left(-39+28\sqrt{2}\right)x^2+\left(-5+6\sqrt{2}\right)x^4-x^6}\,dx = \int \frac{x^4-\left(2\sqrt{2}-3\right)^2}{x^6-x^4\left(6\sqrt{2}-5\right)-x^2\left(28\sqrt{2}-39\right)-70\sqrt{2}+99}\,dx$$

input
```
integrate(((-3+2*2^(1/2))^2-x^4)/(-99+70*2^(1/2)+(-39+28*2^(1/2))*x^2+(-5+
6*2^(1/2))*x^4-x^6),x, algorithm="maxima")
```

output
```
integrate((x^4 - (2*sqrt(2) - 3)^2)/(x^6 - x^4*(6*sqrt(2) - 5) - x^2*(28*s
qrt(2) - 39) - 70*sqrt(2) + 99), x)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.59

$$\int \frac{\left(-3+2\sqrt{2}\right)^2-x^4}{-99+70\sqrt{2}+\left(-39+28\sqrt{2}\right)x^2+\left(-5+6\sqrt{2}\right)x^4-x^6}\,dx = -\frac{1}{4}\log\left(\left|x^2+2x-2\sqrt{2}+3\right|\right)+\frac{1}{4}\log\left(\left|x^2-2x-2\sqrt{2}+3\right|\right)$$

input
```
integrate(((-3+2*2^(1/2))^2-x^4)/(-99+70*2^(1/2)+(-39+28*2^(1/2))*x^2+(-5+
6*2^(1/2))*x^4-x^6),x, algorithm="giac")
```

output
```
-1/4*log(abs(x^2 + 2*x - 2*sqrt(2) + 3)) + 1/4*log(abs(x^2 - 2*x - 2*sqrt(
2) + 3))
```

Mupad [B] (verification not implemented)

Time = 22.75 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.55

$$\int \frac{\left(-3+2\sqrt{2}\right)^2-x^4}{-99+70\sqrt{2}+\left(-39+28\sqrt{2}\right)x^2+\left(-5+6\sqrt{2}\right)x^4-x^6}\,dx = -\frac{\operatorname{atanh}\left(\frac{x\left(16\sqrt{2}-16\right)}{2\left(20\sqrt{2}+4\sqrt{2}x^2-4x^2-28\right)}\right)}{2}$$

input
```
int(((2*2^(1/2) - 3)^2 - x^4)/(x^4*(6*2^(1/2) - 5) + x^2*(28*2^(1/2) - 39)
 + 70*2^(1/2) - x^6 - 99),x)
```

output
```
-atanh((x*(16*2^(1/2) - 16))/(2*(20*2^(1/2) + 4*2^(1/2)*x^2 - 4*x^2 - 28))
)/2
```

Reduce [F]

$$\int \frac{\left(-3+2\sqrt{2}\right)^2-x^4}{-99+70\sqrt{2}+\left(-39+28\sqrt{2}\right)x^2+\left(-5+6\sqrt{2}\right)x^4-x^6}\,dx = 6\sqrt{2}\left(\int \frac{x^4}{x^8+4x^6+6x^4-124x^2+1}dx\right) + 4\sqrt{2}\left(\int \frac{x^2}{x^8+4x^6+6x^4-124x^2+1}dx\right) - 2\sqrt{2}\left(\int \frac{1}{x^8+4x^6+6x^4-124x^2+1}dx\right) + \int \frac{x^6}{x^8+4x^6+6x^4-124x^2+1}dx - \left(\int \frac{x^4}{x^8+4x^6+6x^4-124x^2+1}dx\right) + 27\left(\int \frac{x^2}{x^8+4x^6+6x^4-124x^2+1}dx\right) - 3\left(\int \frac{1}{x^8+4x^6+6x^4-124x^2+1}dx\right)$$

input
```
int(((-3+2*2^(1/2))^2-x^4)/(-99+70*2^(1/2)+(-39+28*2^(1/2))*x^2+(-5+6*2^(1
/2))*x^4-x^6),x)
```

output
```
6*sqrt(2)*int(x**4/(x**8 + 4*x**6 + 6*x**4 - 124*x**2 + 1),x) + 4*sqrt(2)*
int(x**2/(x**8 + 4*x**6 + 6*x**4 - 124*x**2 + 1),x) - 2*sqrt(2)*int(1/(x**
8 + 4*x**6 + 6*x**4 - 124*x**2 + 1),x) + int(x**6/(x**8 + 4*x**6 + 6*x**4
- 124*x**2 + 1),x) - int(x**4/(x**8 + 4*x**6 + 6*x**4 - 124*x**2 + 1),x) +
 27*int(x**2/(x**8 + 4*x**6 + 6*x**4 - 124*x**2 + 1),x) - 3*int(1/(x**8 +
4*x**6 + 6*x**4 - 124*x**2 + 1),x)
```

3.99 $\int \frac{(-3+2\sqrt{2}-x^2)(-3+2\sqrt{2}+x^2)}{-99+70\sqrt{2}+(-39+28\sqrt{2})x^2+(-5+6\sqrt{2})x^4-x^6}\,dx$

Optimal result

Integrand size = 69, antiderivative size = 22

$$\int \frac{(-3+2\sqrt{2}-x^2)(-3+2\sqrt{2}+x^2)}{-99+70\sqrt{2}+(-39+28\sqrt{2})x^2+(-5+6\sqrt{2})x^4-x^6}\,dx = -\frac{1}{2}\text{arctanh}\left(\frac{2x}{3-2\sqrt{2}+x^2}\right)$$

output
```
-1/2*arctanh(2*x/(3-2*2^(1/2)+x^2))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 45 vs. 2(22) = 44.

Time = 0.07 (sec) , antiderivative size = 45, normalized size of antiderivative = 2.05

$$\int \frac{(-3+2\sqrt{2}-x^2)(-3+2\sqrt{2}+x^2)}{-99+70\sqrt{2}+(-39+28\sqrt{2})x^2+(-5+6\sqrt{2})x^4-x^6}\,dx = -\frac{1}{4}\log\left(-3+2\sqrt{2}-2x-x^2\right)+\frac{1}{4}\log\left(-3+2\sqrt{2}+2x-x^2\right)$$

input
```
Integrate[((-3 + 2*Sqrt[2] - x^2)*(-3 + 2*Sqrt[2] + x^2))/(-99 + 70*Sqrt[2
] + (-39 + 28*Sqrt[2])*x^2 + (-5 + 6*Sqrt[2])*x^4 - x^6),x]
```

output
```
-1/4*Log[-3 + 2*Sqrt[2] - 2*x - x^2] + Log[-3 + 2*Sqrt[2] + 2*x - x^2]/4
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 103 vs. $2(22) = 44$.

Time = 0.52 (sec) , antiderivative size = 103, normalized size of antiderivative = 4.68, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.058$, Rules used = {2019, 1475, 1081, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(-x^2+2\sqrt{2}-3\right)\left(x^2+2\sqrt{2}-3\right)}{-x^6+\left(6\sqrt{2}-5\right)x^4+\left(28\sqrt{2}-39\right)x^2+70\sqrt{2}-99}dx$$

$$\Big\downarrow \text{2019}$$

$$\int \frac{x^2+2\sqrt{2}-3}{x^4+\left(2-4\sqrt{2}\right)x^2-12\sqrt{2}+17}dx$$

$$\Big\downarrow \text{1475}$$

$$\frac{1}{2}\int \frac{1}{x^2-2\sqrt{2\left(-1+\sqrt{2}\right)}x+2\sqrt{2}-3}dx+\frac{1}{2}\int \frac{1}{x^2+2\sqrt{2\left(-1+\sqrt{2}\right)}x+2\sqrt{2}-3}dx$$

$$\Big\downarrow \text{1081}$$

$$\frac{1}{2}\int \left(-\frac{1}{2\left(x-\sqrt{2\left(-1+\sqrt{2}\right)}+1\right)}-\frac{1}{2\left(-x+\sqrt{2\left(-1+\sqrt{2}\right)}+1\right)}\right)dx+\frac{1}{2}\int \left(-\frac{1}{2\left(x+\sqrt{2\left(-1+\sqrt{2}\right)}+1\right)}-\frac{1}{2\left(-x-\sqrt{2\left(-1+\sqrt{2}\right)}+1\right)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{1}{2}\log\left(-x+\sqrt{2\left(\sqrt{2}-1\right)}+1\right)-\frac{1}{2}\log\left(x-\sqrt{2\left(\sqrt{2}-1\right)}+1\right)\right)+\frac{1}{2}\left(\frac{1}{2}\log\left(-x-\sqrt{2\left(\sqrt{2}-1\right)}+1\right)-\frac{1}{2}\log\left(x+\sqrt{2\left(\sqrt{2}-1\right)}+1\right)\right)$$

input
```
Int[((-3 + 2*Sqrt[2] - x^2)*(-3 + 2*Sqrt[2] + x^2))/(-99 + 70*Sqrt[2] + (-
39 + 28*Sqrt[2])*x^2 + (-5 + 6*Sqrt[2])*x^4 - x^6),x]
```

output
```
(Log[1 + Sqrt[2*(-1 + Sqrt[2])] - x]/2 - Log[1 - Sqrt[2*(-1 + Sqrt[2])] +
x]/2)/2 + (Log[1 - Sqrt[2*(-1 + Sqrt[2])] - x]/2 - Log[1 + Sqrt[2*(-1 + Sq
rt[2])] + x]/2)/2
```

Defintions of rubi rules used

rule 1081
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[c   Int[ExpandIntegrand[1/((b/2 - q/2 + c*x)*(b/2 + q/2
+ c*x)), x], x], x]] /; FreeQ[{a, b, c}, x] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 1475
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^
2, x], x], x] + Simp[e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] &&
 (GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2]
, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.55

method	result	size
default	$\frac{\ln\left(x^2-2\sqrt{2}-2x+3\right)}{4}-\frac{\ln\left(x^2-2\sqrt{2}+2x+3\right)}{4}$	34
risch	$\frac{\ln\left(x^2-2\sqrt{2}-2x+3\right)}{4}-\frac{\ln\left(x^2-2\sqrt{2}+2x+3\right)}{4}$	34
parallelrisch	$\frac{\ln\left(x^2-2\sqrt{2}-2x+3\right)}{4}-\frac{\ln\left(x^2-2\sqrt{2}+2x+3\right)}{4}$	34

input
```
int((-3+2*2^(1/2)-x^2)*(-3+2*2^(1/2)+x^2)/(-99+70*2^(1/2)+(-39+28*2^(1/2))
*x^2+(-5+6*2^(1/2))*x^4-x^6),x,method=_RETURNVERBOSE)
```

output
```
1/4*ln(x^2-2*2^(1/2)-2*x+3)-1/4*ln(x^2-2*2^(1/2)+2*x+3)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.50

$$\int \frac{\left(-3+2\sqrt{2}-x^2\right)\left(-3+2\sqrt{2}+x^2\right)}{-99+70\sqrt{2}+\left(-39+28\sqrt{2}\right)x^2+\left(-5+6\sqrt{2}\right)x^4-x^6}\,dx = -\frac{1}{4}\log\left(x^2+2\,x-2\,\sqrt{2}+3\right)+\frac{1}{4}\log\left(x^2-2\,x-2\,\sqrt{2}+3\right)$$

input
```
integrate((-3+2*2^(1/2)-x^2)*(-3+2*2^(1/2)+x^2)/(-99+70*2^(1/2)+(-39+28*2^
(1/2))*x^2+(-5+6*2^(1/2))*x^4-x^6),x, algorithm="fricas")
```

output
```
-1/4*log(x^2 + 2*x - 2*sqrt(2) + 3) + 1/4*log(x^2 - 2*x - 2*sqrt(2) + 3)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{\left(-3+2\sqrt{2}-x^2\right)\left(-3+2\sqrt{2}+x^2\right)}{-99+70\sqrt{2}+\left(-39+28\sqrt{2}\right)x^2+\left(-5+6\sqrt{2}\right)x^4-x^6}\,dx = \text{Exception raised: PolynomialError}$$

input
```
integrate((-3+2*2**(1/2)-x**2)*(-3+2*2**(1/2)+x**2)/(-99+70*2**(1/2)+(-39+
28*2**(1/2))*x**2+(-5+6*2**(1/2))*x**4-x**6),x)
```

output
```
Exception raised: PolynomialError >> 1/(-160817378869521623700434126076296
32108508413135104*_t**4 + 113715059131284938253449200806460980121432594954
24*sqrt(2)*_t**4 - 525076531527889516631004780624192141786868300832*_t**2
+ 3712851760852
```

Maxima [F]

$$\int \frac{\left(-3+2\sqrt{2}-x^2\right)\left(-3+2\sqrt{2}+x^2\right)}{-99+70\sqrt{2}+\left(-39+28\sqrt{2}\right)x^2+\left(-5+6\sqrt{2}\right)x^4-x^6}\,dx = \int \frac{\left(x^2+2\sqrt{2}-3\right)\left(x^2-2\sqrt{2}+3\right)}{x^6-x^4\left(6\sqrt{2}-5\right)-x^2\left(28\sqrt{2}-39\right)-70\sqrt{2}+99}\,dx$$

input
```
integrate((-3+2*2^(1/2)-x^2)*(-3+2*2^(1/2)+x^2)/(-99+70*2^(1/2)+(-39+28*2^
(1/2))*x^2+(-5+6*2^(1/2))*x^4-x^6),x, algorithm="maxima")
```

output
```
integrate((x^2 + 2*sqrt(2) - 3)*(x^2 - 2*sqrt(2) + 3)/(x^6 - x^4*(6*sqrt(2
) - 5) - x^2*(28*sqrt(2) - 39) - 70*sqrt(2) + 99), x)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.59

$$\int \frac{\left(-3+2\sqrt{2}-x^2\right)\left(-3+2\sqrt{2}+x^2\right)}{-99+70\sqrt{2}+\left(-39+28\sqrt{2}\right)x^2+\left(-5+6\sqrt{2}\right)x^4-x^6}\,dx$$
$$= -\frac{1}{4}\log\left(\left|x^2+2x-2\sqrt{2}+3\right|\right)+\frac{1}{4}\log\left(\left|x^2-2x-2\sqrt{2}+3\right|\right)$$

input
```
integrate((-3+2*2^(1/2)-x^2)*(-3+2*2^(1/2)+x^2)/(-99+70*2^(1/2)+(-39+28*2^
(1/2))*x^2+(-5+6*2^(1/2))*x^4-x^6),x, algorithm="giac")
```

output
```
-1/4*log(abs(x^2 + 2*x - 2*sqrt(2) + 3)) + 1/4*log(abs(x^2 - 2*x - 2*sqrt(
2) + 3))
```

Mupad [B] (verification not implemented)

Time = 22.64 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.55

$$\int \frac{\left(-3+2\sqrt{2}-x^2\right)\left(-3+2\sqrt{2}+x^2\right)}{-99+70\sqrt{2}+\left(-39+28\sqrt{2}\right)x^2+\left(-5+6\sqrt{2}\right)x^4-x^6}\,dx$$
$$= -\frac{\operatorname{atanh}\left(\frac{x\left(16\sqrt{2}-16\right)}{2\left(20\sqrt{2}+4\sqrt{2}\,x^2-4x^2-28\right)}\right)}{2}$$

input
```
int(-((x^2 - 2*2^(1/2) + 3)*(2*2^(1/2) + x^2 - 3))/(x^4*(6*2^(1/2) - 5) +
x^2*(28*2^(1/2) - 39) + 70*2^(1/2) - x^6 - 99),x)
```

output
```
-atanh((x*(16*2^(1/2) - 16))/(2*(20*2^(1/2) + 4*2^(1/2)*x^2 - 4*x^2 - 28))
)/2
```

Reduce [F]

$$\int \frac{\left(-3+2\sqrt{2}-x^2\right)\left(-3+2\sqrt{2}+x^2\right)}{-99+70\sqrt{2}+\left(-39+28\sqrt{2}\right)x^2+\left(-5+6\sqrt{2}\right)x^4-x^6}\,dx$$
$$= 6\sqrt{2}\left(\int \frac{x^4}{x^8+4x^6+6x^4-124x^2+1}dx\right)$$
$$+ 4\sqrt{2}\left(\int \frac{x^2}{x^8+4x^6+6x^4-124x^2+1}dx\right)$$
$$- 2\sqrt{2}\left(\int \frac{1}{x^8+4x^6+6x^4-124x^2+1}dx\right)$$
$$+ \int \frac{x^6}{x^8+4x^6+6x^4-124x^2+1}dx - \left(\int \frac{x^4}{x^8+4x^6+6x^4-124x^2+1}dx\right)$$
$$+ 27\left(\int \frac{x^2}{x^8+4x^6+6x^4-124x^2+1}dx\right) - 3\left(\int \frac{1}{x^8+4x^6+6x^4-124x^2+1}dx\right)$$

input
```
int((-3+2*2^(1/2)-x^2)*(-3+2*2^(1/2)+x^2)/(-99+70*2^(1/2)+(-39+28*2^(1/2))
*x^2+(-5+6*2^(1/2))*x^4-x^6),x)
```

output
```
6*sqrt(2)*int(x**4/(x**8 + 4*x**6 + 6*x**4 - 124*x**2 + 1),x) + 4*sqrt(2)*
int(x**2/(x**8 + 4*x**6 + 6*x**4 - 124*x**2 + 1),x) - 2*sqrt(2)*int(1/(x**
8 + 4*x**6 + 6*x**4 - 124*x**2 + 1),x) + int(x**6/(x**8 + 4*x**6 + 6*x**4
- 124*x**2 + 1),x) - int(x**4/(x**8 + 4*x**6 + 6*x**4 - 124*x**2 + 1),x) +
 27*int(x**2/(x**8 + 4*x**6 + 6*x**4 - 124*x**2 + 1),x) - 3*int(1/(x**8 +
4*x**6 + 6*x**4 - 124*x**2 + 1),x)
```

3.100 $\int \frac{\left(-3+2\sqrt{2}\right)^3-\left(-3+2\sqrt{2}\right)^2x^2-\left(-3+2\sqrt{2}\right)x^4+x^6}{577-408\sqrt{2}+\left(328-232\sqrt{2}\right)x^2+\left(78-56\sqrt{2}\right)x^4+\left(8-8\sqrt{2}\right)x^6+x^8}\,dx$

Optimal result

Integrand size = 99, antiderivative size = 22

$$\int \frac{\left(-3+2\sqrt{2}\right)^3-\left(-3+2\sqrt{2}\right)^2x^2-\left(-3+2\sqrt{2}\right)x^4+x^6}{577-408\sqrt{2}+\left(328-232\sqrt{2}\right)x^2+\left(78-56\sqrt{2}\right)x^4+\left(8-8\sqrt{2}\right)x^6+x^8}\,dx = -\frac{1}{2}\text{arctanh}\left(\frac{2x}{3-2\sqrt{2}+x^2}\right)$$

output

```
-1/2*arctanh(2*x/(3-2*2^(1/2)+x^2))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 179 vs. 2(22) = 44.

Time = 0.15 (sec) , antiderivative size = 179, normalized size of antiderivative = 8.14

$$\int \frac{\left(-3+2\sqrt{2}\right)^3-\left(-3+2\sqrt{2}\right)^2x^2-\left(-3+2\sqrt{2}\right)x^4+x^6}{577-408\sqrt{2}+\left(328-232\sqrt{2}\right)x^2+\left(78-56\sqrt{2}\right)x^4+\left(8-8\sqrt{2}\right)x^6+x^8}\,dx = \frac{\left(-17+12\sqrt{2}+\left(-2+4\sqrt{2}\right)x^2-x^4\right)\left(99-70\sqrt{2}+\left(17-12\sqrt{2}\right)x^2+\left(-3+2\sqrt{2}\right)x^4-x^6\right)\left(\log\left(-3\right.\right.}{4\left(-3+2\sqrt{2}+x^2\right)\left(-577+408\sqrt{2}+8\left(-41+29\sqrt{2}\right)x^2+\left(-78+56\sqrt{2}\right)x^4\right.}$$

input
```
Integrate[((-3 + 2*Sqrt[2])^3 - (-3 + 2*Sqrt[2])^2*x^2 - (-3 + 2*Sqrt[2])*
x^4 + x^6)/(577 - 408*Sqrt[2] + (328 - 232*Sqrt[2])*x^2 + (78 - 56*Sqrt[2]
)*x^4 + (8 - 8*Sqrt[2])*x^6 + x^8),x]
```

output
```
((-17 + 12*Sqrt[2] + (-2 + 4*Sqrt[2])*x^2 - x^4)*(99 - 70*Sqrt[2] + (17 -
12*Sqrt[2])*x^2 + (-3 + 2*Sqrt[2])*x^4 - x^6)*(Log[-3 + 2*Sqrt[2] - 2*x -
x^2] - Log[-3 + 2*Sqrt[2] + 2*x - x^2]))/(4*(-3 + 2*Sqrt[2] + x^2)*(-577 +
 408*Sqrt[2] + 8*(-41 + 29*Sqrt[2])*x^2 + (-78 + 56*Sqrt[2])*x^4 + 8*(-1 +
 Sqrt[2])*x^6 - x^8))
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6-\left(2\sqrt{2}-3\right)x^4-\left(2\sqrt{2}-3\right)^2x^2+\left(2\sqrt{2}-3\right)^3}{x^8+\left(8-8\sqrt{2}\right)x^6+\left(78-56\sqrt{2}\right)x^4+\left(328-232\sqrt{2}\right)x^2-408\sqrt{2}+577}dx$$

$$\Big\downarrow \text{ 7292}$$

$$\int \frac{x^6-\left(2\sqrt{2}-3\right)x^4-\left(2\sqrt{2}-3\right)^2x^2+\left(2\sqrt{2}-3\right)^3}{x^8+\left(8-8\sqrt{2}\right)x^6+\left(78-56\sqrt{2}\right)x^4+\left(328-232\sqrt{2}\right)x^2+577\left(1-\frac{408\sqrt{2}}{577}\right)}dx$$

$$\Big\downarrow \text{ 7293}$$

$$\int \left(\frac{x^6}{x^8+8\left(1-\sqrt{2}\right)x^6+78\left(1-\frac{28\sqrt{2}}{39}\right)x^4+328\left(1-\frac{29\sqrt{2}}{41}\right)x^2+577\left(1-\frac{408\sqrt{2}}{577}\right)}+\frac{}{x^8+8\left(1-\sqrt{2}\right)x^6+78}\right.$$

$$\Big\downarrow \text{ 2009}$$

$$-\left(\left(99-70\sqrt{2}\right)\int \frac{1}{x^8+8\left(1-\sqrt{2}\right)x^6+78\left(1-\frac{28\sqrt{2}}{39}\right)x^4+328\left(1-\frac{29\sqrt{2}}{41}\right)x^2+577\left(1-\frac{408\sqrt{2}}{577}\right)}dx\right)-\left(17-12\sqrt{2}\right)\int \frac{x^2}{x^8+8\left(1-\sqrt{2}\right)x^6+78\left(1-\frac{28\sqrt{2}}{39}\right)x^4+328\left(1-\frac{29\sqrt{2}}{41}\right)x^2+577\left(1-\frac{408\sqrt{2}}{577}\right)}dx+\left(3-2\sqrt{2}\right)\int \frac{x^4}{x^8+8\left(1-\sqrt{2}\right)x^6+78\left(1-\frac{28\sqrt{2}}{39}\right)x^4+328\left(1-\frac{29\sqrt{2}}{41}\right)x^2+577\left(1-\frac{408\sqrt{2}}{577}\right)}dx+\int \frac{x^6}{x^8+8\left(1-\sqrt{2}\right)x^6+78\left(1-\frac{28\sqrt{2}}{39}\right)x^4+328\left(1-\frac{29\sqrt{2}}{41}\right)x^2+577\left(1-\frac{408\sqrt{2}}{577}\right)}dx$$

input
```
Int[((-3 + 2*Sqrt[2])^3 - (-3 + 2*Sqrt[2])^2*x^2 - (-3 + 2*Sqrt[2])*x^4 +
x^6)/(577 - 408*Sqrt[2] + (328 - 232*Sqrt[2])*x^2 + (78 - 56*Sqrt[2])*x^4
+ (8 - 8*Sqrt[2])*x^6 + x^8),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7292
```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.55

method	result	size
default	$\frac{\ln\left(x^2-2\sqrt{2}-2x+3\right)}{4}-\frac{\ln\left(x^2-2\sqrt{2}+2x+3\right)}{4}$	34
risch	$\frac{\ln\left(x^2-2\sqrt{2}-2x+3\right)}{4}-\frac{\ln\left(x^2-2\sqrt{2}+2x+3\right)}{4}$	34
parallelrisch	$\frac{\ln\left(x^2-2\sqrt{2}-2x+3\right)}{4}-\frac{\ln\left(x^2-2\sqrt{2}+2x+3\right)}{4}$	34

input
```
int(((-3+2*2^(1/2))^3-(-3+2*2^(1/2))^2*x^2-(-3+2*2^(1/2))*x^4+x^6)/(577-40
8*2^(1/2)+(328-232*2^(1/2))*x^2+(78-56*2^(1/2))*x^4+(8-8*2^(1/2))*x^6+x^8)
,x,method=_RETURNVERBOSE)
```

output
```
1/4*ln(x^2-2*2^(1/2)-2*x+3)-1/4*ln(x^2-2*2^(1/2)+2*x+3)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.50

$$\int \frac{\left(-3+2\sqrt{2}\right)^3-\left(-3+2\sqrt{2}\right)^2x^2-\left(-3+2\sqrt{2}\right)x^4+x^6}{577-408\sqrt{2}+\left(328-232\sqrt{2}\right)x^2+\left(78-56\sqrt{2}\right)x^4+\left(8-8\sqrt{2}\right)x^6+x^8}\,dx$$
$$= -\frac{1}{4}\log\left(x^2+2x-2\sqrt{2}+3\right)+\frac{1}{4}\log\left(x^2-2x-2\sqrt{2}+3\right)$$

input
```
integrate(((-3+2*2^(1/2))^3-(-3+2*2^(1/2))^2*x^2-(-3+2*2^(1/2))*x^4+x^6)/(
577-408*2^(1/2)+(328-232*2^(1/2))*x^2+(78-56*2^(1/2))*x^4+(8-8*2^(1/2))*x^
6+x^8),x, algorithm="fricas")
```

output
```
-1/4*log(x^2 + 2*x - 2*sqrt(2) + 3) + 1/4*log(x^2 - 2*x - 2*sqrt(2) + 3)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{\left(-3+2\sqrt{2}\right)^3-\left(-3+2\sqrt{2}\right)^2x^2-\left(-3+2\sqrt{2}\right)x^4+x^6}{577-408\sqrt{2}+\left(328-232\sqrt{2}\right)x^2+\left(78-56\sqrt{2}\right)x^4+\left(8-8\sqrt{2}\right)x^6+x^8}\,dx$$
$$= \text{Exception raised: PolynomialError}$$

input
```
integrate(((-3+2*2**(1/2))**3-(-3+2*2**(1/2))**2*x**2-(-3+2*2**(1/2))*x**4
+x**6)/(577-408*2**(1/2)+(328-232*2**(1/2))*x**2+(78-56*2**(1/2))*x**4+(8-
8*2**(1/2))*x**6+x**8),x)
```

output
```
Exception raised: PolynomialError >> 1/(-489331912114255602061892417478047
2498117708482611714912381696*_t**4 + 3460099133069698398004476359279702930
052248019321310378430976*sqrt(2)*_t**4 - 159769239484575670917838951113184
628965915778476
```

Maxima [F]

$$\int \frac{\left(-3+2\sqrt{2}\right)^3-\left(-3+2\sqrt{2}\right)^2x^2-\left(-3+2\sqrt{2}\right)x^4+x^6}{577-408\sqrt{2}+\left(328-232\sqrt{2}\right)x^2+\left(78-56\sqrt{2}\right)x^4+\left(8-8\sqrt{2}\right)x^6+x^8}\,dx$$
$$= \int \frac{x^6-x^4\left(2\sqrt{2}-3\right)-x^2\left(2\sqrt{2}-3\right)^2+\left(2\sqrt{2}-3\right)^3}{x^8-8x^6\left(\sqrt{2}-1\right)-2x^4\left(28\sqrt{2}-39\right)-8x^2\left(29\sqrt{2}-41\right)-408\sqrt{2}+577}\,dx$$

input
```
integrate(((-3+2*2^(1/2))^3-(-3+2*2^(1/2))^2*x^2-(-3+2*2^(1/2))*x^4+x^6)/(
577-408*2^(1/2)+(328-232*2^(1/2))*x^2+(78-56*2^(1/2))*x^4+(8-8*2^(1/2))*x^
6+x^8),x, algorithm="maxima")
```

output
```
integrate((x^6 - x^4*(2*sqrt(2) - 3) - x^2*(2*sqrt(2) - 3)^2 + (2*sqrt(2)
- 3)^3)/(x^8 - 8*x^6*(sqrt(2) - 1) - 2*x^4*(28*sqrt(2) - 39) - 8*x^2*(29*s
qrt(2) - 41) - 408*sqrt(2) + 577), x)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.59

$$\int \frac{(-3+2\sqrt{2})^3-(-3+2\sqrt{2})^2 x^2-(-3+2\sqrt{2}) x^4+x^6}{577-408\sqrt{2}+(328-232\sqrt{2}) x^2+(78-56\sqrt{2}) x^4+(8-8\sqrt{2}) x^6+x^8} dx$$
$$= -\frac{1}{4} \log\left(\left|x^2+2x-2\sqrt{2}+3\right|\right)+\frac{1}{4} \log\left(\left|x^2-2x-2\sqrt{2}+3\right|\right)$$

input
```
integrate(((-3+2*2^(1/2))^3-(-3+2*2^(1/2))^2*x^2-(-3+2*2^(1/2))*x^4+x^6)/(
577-408*2^(1/2)+(328-232*2^(1/2))*x^2+(78-56*2^(1/2))*x^4+(8-8*2^(1/2))*x^
6+x^8),x, algorithm="giac")
```

output
```
-1/4*log(abs(x^2 + 2*x - 2*sqrt(2) + 3)) + 1/4*log(abs(x^2 - 2*x - 2*sqrt(
2) + 3))
```

Mupad [B] (verification not implemented)

Time = 22.84 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.55

$$\int \frac{(-3+2\sqrt{2})^3-(-3+2\sqrt{2})^2 x^2-(-3+2\sqrt{2}) x^4+x^6}{577-408\sqrt{2}+(328-232\sqrt{2}) x^2+(78-56\sqrt{2}) x^4+(8-8\sqrt{2}) x^6+x^8} dx$$
$$= -\frac{\operatorname{atanh}\left(\frac{x\left(16\sqrt{2}-16\right)}{2\left(20\sqrt{2}+4\sqrt{2}x^2-4x^2-28\right)}\right)}{2}$$

input
```
int((x^4*(2*2^(1/2) - 3) - (2*2^(1/2) - 3)^3 + x^2*(2*2^(1/2) - 3)^2 - x^6
)/(x^6*(8*2^(1/2) - 8) + x^4*(56*2^(1/2) - 78) + x^2*(232*2^(1/2) - 328) +
 408*2^(1/2) - x^8 - 577),x)
```

output
```
-atanh((x*(16*2^(1/2) - 16))/(2*(20*2^(1/2) + 4*2^(1/2)*x^2 - 4*x^2 - 28))
)/2
```

Reduce [F]

$$\int \frac{\left(-3+2\sqrt{2}\right)^3-\left(-3+2\sqrt{2}\right)^2x^2-\left(-3+2\sqrt{2}\right)x^4+x^6}{577-408\sqrt{2}+\left(328-232\sqrt{2}\right)x^2+\left(78-56\sqrt{2}\right)x^4+\left(8-8\sqrt{2}\right)x^6+x^8}\,dx$$
$$= 6\sqrt{2}\left(\int \frac{x^4}{x^8+4x^6+6x^4-124x^2+1}dx\right) + 4\sqrt{2}\left(\int \frac{x^2}{x^8+4x^6+6x^4-124x^2+1}dx\right) - 2\sqrt{2}\left(\int \frac{1}{x^8+4x^6+6x^4-124x^2+1}dx\right) + \int \frac{x^6}{x^8+4x^6+6x^4-124x^2+1}dx - \left(\int \frac{x^4}{x^8+4x^6+6x^4-124x^2+1}dx\right) + 27\left(\int \frac{x^2}{x^8+4x^6+6x^4-124x^2+1}dx\right) - 3\left(\int \frac{1}{x^8+4x^6+6x^4-124x^2+1}dx\right)$$

input
```
int(((-3+2*2^(1/2))^3-(-3+2*2^(1/2))^2*x^2-(-3+2*2^(1/2))*x^4+x^6)/(577-40
8*2^(1/2)+(328-232*2^(1/2))*x^2+(78-56*2^(1/2))*x^4+(8-8*2^(1/2))*x^6+x^8)
,x)
```

output
```
6*sqrt(2)*int(x**4/(x**8 + 4*x**6 + 6*x**4 - 124*x**2 + 1),x) + 4*sqrt(2)*
int(x**2/(x**8 + 4*x**6 + 6*x**4 - 124*x**2 + 1),x) - 2*sqrt(2)*int(1/(x**
8 + 4*x**6 + 6*x**4 - 124*x**2 + 1),x) + int(x**6/(x**8 + 4*x**6 + 6*x**4
- 124*x**2 + 1),x) - int(x**4/(x**8 + 4*x**6 + 6*x**4 - 124*x**2 + 1),x) +
 27*int(x**2/(x**8 + 4*x**6 + 6*x**4 - 124*x**2 + 1),x) - 3*int(1/(x**8 +
4*x**6 + 6*x**4 - 124*x**2 + 1),x)
```

3.101 $\int \frac{(3-2\sqrt{2}+x^2)^2(-3+2\sqrt{2}+x^2)}{577-408\sqrt{2}+(328-232\sqrt{2})x^2+(78-56\sqrt{2})x^4+(8-8\sqrt{2})x^6+x^8}\,dx$

Optimal result

Integrand size = 80, antiderivative size = 22

$$\int \frac{(3-2\sqrt{2}+x^2)^2(-3+2\sqrt{2}+x^2)}{577-408\sqrt{2}+(328-232\sqrt{2})x^2+(78-56\sqrt{2})x^4+(8-8\sqrt{2})x^6+x^8}\,dx = -\frac{1}{2}\text{arctanh}\left(\frac{2x}{3-2\sqrt{2}+x^2}\right)$$

output
```
-1/2*arctanh(2*x/(3-2*2^(1/2)+x^2))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 139 vs. 2(22) = 44.

Time = 0.09 (sec) , antiderivative size = 139, normalized size of antiderivative = 6.32

$$\int \frac{(3-2\sqrt{2}+x^2)^2(-3+2\sqrt{2}+x^2)}{577-408\sqrt{2}+(328-232\sqrt{2})x^2+(78-56\sqrt{2})x^4+(8-8\sqrt{2})x^6+x^8}\,dx = -\frac{(3-2\sqrt{2}+x^2)^2(-17+12\sqrt{2}+(-2+4\sqrt{2})x^2-x^4)(\log(-3+2\sqrt{2}-2x-x^2)-\log(-3+2\sqrt{2}}{4(-577+408\sqrt{2}+8(-41+29\sqrt{2})x^2+(-78+56\sqrt{2})x^4+8(-1+\sqrt{2})x^6-x^8)}$$

input
```
Integrate[((3 - 2*Sqrt[2] + x^2)^2*(-3 + 2*Sqrt[2] + x^2))/(577 - 408*Sqrt
[2] + (328 - 232*Sqrt[2])*x^2 + (78 - 56*Sqrt[2])*x^4 + (8 - 8*Sqrt[2])*x^
6 + x^8),x]
```

output
```
-1/4*((3 - 2*Sqrt[2] + x^2)^2*(-17 + 12*Sqrt[2] + (-2 + 4*Sqrt[2])*x^2 - x
^4)*(Log[-3 + 2*Sqrt[2] - 2*x - x^2] - Log[-3 + 2*Sqrt[2] + 2*x - x^2]))/(
-577 + 408*Sqrt[2] + 8*(-41 + 29*Sqrt[2])*x^2 + (-78 + 56*Sqrt[2])*x^4 + 8
*(-1 + Sqrt[2])*x^6 - x^8)
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 103 vs. 2(22) = 44.

Time = 0.69 (sec) , antiderivative size = 103, normalized size of antiderivative = 4.68, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {2019, 2019, 1475, 1081, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x^2-2\sqrt{2}+3\right)^2\left(x^2+2\sqrt{2}-3\right)}{x^8+\left(8-8\sqrt{2}\right)x^6+\left(78-56\sqrt{2}\right)x^4+\left(328-232\sqrt{2}\right)x^2-408\sqrt{2}+577}dx$$

$$\Big\downarrow \text{ 2019}$$

$$\int \frac{\left(x^2-2\sqrt{2}+3\right)\left(x^2+2\sqrt{2}-3\right)}{x^6+\left(5-6\sqrt{2}\right)x^4+\left(39-28\sqrt{2}\right)x^2-70\sqrt{2}+99}dx$$

$$\Big\downarrow \text{ 2019}$$

$$\int \frac{x^2+2\sqrt{2}-3}{x^4+\left(2-4\sqrt{2}\right)x^2-12\sqrt{2}+17}dx$$

$$\Big\downarrow \text{ 1475}$$

$$\frac{1}{2}\int \frac{1}{x^2-2\sqrt{2\left(-1+\sqrt{2}\right)}x+2\sqrt{2}-3}dx+\frac{1}{2}\int \frac{1}{x^2+2\sqrt{2\left(-1+\sqrt{2}\right)}x+2\sqrt{2}-3}dx$$

$$\Big\downarrow \text{ 1081}$$

$$\frac{1}{2}\int \left(-\frac{1}{2\left(x-\sqrt{2\left(-1+\sqrt{2}\right)}+1\right)}-\frac{1}{2\left(-x+\sqrt{2\left(-1+\sqrt{2}\right)}+1\right)}\right)dx+$$
$$\frac{1}{2}\int \left(-\frac{1}{2\left(x+\sqrt{2\left(-1+\sqrt{2}\right)}+1\right)}-\frac{1}{2\left(-x-\sqrt{2\left(-1+\sqrt{2}\right)}+1\right)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{1}{2}\log\left(-x+\sqrt{2\left(\sqrt{2}-1\right)}+1\right)-\frac{1}{2}\log\left(x-\sqrt{2\left(\sqrt{2}-1\right)}+1\right)\right)+$$
$$\frac{1}{2}\left(\frac{1}{2}\log\left(-x-\sqrt{2\left(\sqrt{2}-1\right)}+1\right)-\frac{1}{2}\log\left(x+\sqrt{2\left(\sqrt{2}-1\right)}+1\right)\right)$$

input
```
Int[((3 - 2*Sqrt[2] + x^2)^2*(-3 + 2*Sqrt[2] + x^2))/(577 - 408*Sqrt[2] +
(328 - 232*Sqrt[2])*x^2 + (78 - 56*Sqrt[2])*x^4 + (8 - 8*Sqrt[2])*x^6 + x^
8),x]
```

output
```
(Log[1 + Sqrt[2*(-1 + Sqrt[2])] - x]/2 - Log[1 - Sqrt[2*(-1 + Sqrt[2])] +
x]/2)/2 + (Log[1 - Sqrt[2*(-1 + Sqrt[2])] - x]/2 - Log[1 + Sqrt[2*(-1 + Sq
rt[2])] + x]/2)/2
```

Defintions of rubi rules used

rule 1081
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[c   Int[ExpandIntegrand[1/((b/2 - q/2 + c*x)*(b/2 + q/2
+ c*x)), x], x], x]] /; FreeQ[{a, b, c}, x] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 1475
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^
2, x], x], x] + Simp[e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] &&
 (GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2]
, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.55

method	result	size
default	$\frac{\ln\left(x^2-2\sqrt{2}-2x+3\right)}{4}-\frac{\ln\left(x^2-2\sqrt{2}+2x+3\right)}{4}$	34
risch	$\frac{\ln\left(x^2-2\sqrt{2}-2x+3\right)}{4}-\frac{\ln\left(x^2-2\sqrt{2}+2x+3\right)}{4}$	34
parallelrisch	$\frac{\ln\left(x^2-2\sqrt{2}-2x+3\right)}{4}-\frac{\ln\left(x^2-2\sqrt{2}+2x+3\right)}{4}$	34

input
```
int((3-2*2^(1/2)+x^2)^2*(-3+2*2^(1/2)+x^2)/(577-408*2^(1/2)+(328-232*2^(1/
2))*x^2+(78-56*2^(1/2))*x^4+(8-8*2^(1/2))*x^6+x^8),x,method=_RETURNVERBOSE
)
```

output
```
1/4*ln(x^2-2*2^(1/2)-2*x+3)-1/4*ln(x^2-2*2^(1/2)+2*x+3)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.50

$$\int \frac{\left(3-2\sqrt{2}+x^2\right)^2\left(-3+2\sqrt{2}+x^2\right)}{577-408\sqrt{2}+\left(328-232\sqrt{2}\right)x^2+\left(78-56\sqrt{2}\right)x^4+\left(8-8\sqrt{2}\right)x^6+x^8}\,dx$$
$$=-\frac{1}{4}\log\left(x^2+2\,x-2\,\sqrt{2}+3\right)+\frac{1}{4}\log\left(x^2-2\,x-2\,\sqrt{2}+3\right)$$

input
```
integrate((3-2*2^(1/2)+x^2)^2*(-3+2*2^(1/2)+x^2)/(577-408*2^(1/2)+(328-232
*2^(1/2))*x^2+(78-56*2^(1/2))*x^4+(8-8*2^(1/2))*x^6+x^8),x, algorithm="fri
cas")
```

output
```
-1/4*log(x^2 + 2*x - 2*sqrt(2) + 3) + 1/4*log(x^2 - 2*x - 2*sqrt(2) + 3)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{\left(3-2\sqrt{2}+x^2\right)^2 \left(-3+2\sqrt{2}+x^2\right)}{577-408\sqrt{2}+\left(328-232\sqrt{2}\right)x^2+\left(78-56\sqrt{2}\right)x^4+\left(8-8\sqrt{2}\right)x^6+x^8}\,dx = \text{Exception raised: PolynomialError}$$

input
```
integrate((3-2*2**(1/2)+x**2)**2*(-3+2*2**(1/2)+x**2)/(577-408*2**(1/2)+(3
28-232*2**(1/2))*x**2+(78-56*2**(1/2))*x**4+(8-8*2**(1/2))*x**6+x**8),x)
```

output
```
Exception raised: PolynomialError >> 1/(-489331912114255602061892417478047
2498117708482611714912381696*_t**4 + 346009913306969839800447635927970293O
0522480193213103784309 76*sqrt(2)*_t**4 - 15976923948457567091783895111318 4
628965915778476
```

Maxima [F]

$$\int \frac{\left(3-2\sqrt{2}+x^2\right)^2 \left(-3+2\sqrt{2}+x^2\right)}{577-408\sqrt{2}+\left(328-232\sqrt{2}\right)x^2+\left(78-56\sqrt{2}\right)x^4+\left(8-8\sqrt{2}\right)x^6+x^8}\,dx = \int \frac{\left(x^2+2\sqrt{2}-3\right)\left(x^2-2\sqrt{2}+3\right)^2}{x^8-8\,x^6\left(\sqrt{2}-1\right)-2\,x^4\left(28\,\sqrt{2}-39\right)-8\,x^2\left(29\,\sqrt{2}-41\right)-408\,\sqrt{2}+577}\,dx$$

input
```
integrate((3-2*2^(1/2)+x^2)^2*(-3+2*2^(1/2)+x^2)/(577-408*2^(1/2)+(328-232
*2^(1/2))*x^2+(78-56*2^(1/2))*x^4+(8-8*2^(1/2))*x^6+x^8),x, algorithm="max
ima")
```

output
```
integrate((x^2 + 2*sqrt(2) - 3)*(x^2 - 2*sqrt(2) + 3)^2/(x^8 - 8*x^6*(sqrt
(2) - 1) - 2*x^4*(28*sqrt(2) - 39) - 8*x^2*(29*sqrt(2) - 41) - 408*sqrt(2)
 + 577), x)
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.59

$$\int \frac{(3-2\sqrt{2}+x^2)^2(-3+2\sqrt{2}+x^2)}{577-408\sqrt{2}+(328-232\sqrt{2})x^2+(78-56\sqrt{2})x^4+(8-8\sqrt{2})x^6+x^8}\,dx = -\frac{1}{4}\log\left(\left|x^2+2x-2\sqrt{2}+3\right|\right)+\frac{1}{4}\log\left(\left|x^2-2x-2\sqrt{2}+3\right|\right)$$

input
```
integrate((3-2*2^(1/2)+x^2)^2*(-3+2*2^(1/2)+x^2)/(577-408*2^(1/2)+(328-232
*2^(1/2))*x^2+(78-56*2^(1/2))*x^4+(8-8*2^(1/2))*x^6+x^8),x, algorithm="gia
c")
```

output
```
-1/4*log(abs(x^2 + 2*x - 2*sqrt(2) + 3)) + 1/4*log(abs(x^2 - 2*x - 2*sqrt(
2) + 3))
```

Mupad [B] (verification not implemented)

Time = 22.65 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.55

$$\int \frac{(3-2\sqrt{2}+x^2)^2(-3+2\sqrt{2}+x^2)}{577-408\sqrt{2}+(328-232\sqrt{2})x^2+(78-56\sqrt{2})x^4+(8-8\sqrt{2})x^6+x^8}\,dx = -\frac{\operatorname{atanh}\left(\frac{x\left(16\sqrt{2}-16\right)}{2\left(20\sqrt{2}+4\sqrt{2}\,x^2-4x^2-28\right)}\right)}{2}$$

input
```
int(-((x^2 - 2*2^(1/2) + 3)^2*(2*2^(1/2) + x^2 - 3))/(x^6*(8*2^(1/2) - 8)
+ x^4*(56*2^(1/2) - 78) + x^2*(232*2^(1/2) - 328) + 408*2^(1/2) - x^8 - 57
7),x)
```

output
```
-atanh((x*(16*2^(1/2) - 16))/(2*(20*2^(1/2) + 4*2^(1/2)*x^2 - 4*x^2 - 28))
)/2
```

Reduce [F]

$$\int \frac{\left(3-2\sqrt{2}+x^2\right)^2 \left(-3+2\sqrt{2}+x^2\right)}{577-408\sqrt{2}+\left(328-232\sqrt{2}\right)x^2+\left(78-56\sqrt{2}\right)x^4+\left(8-8\sqrt{2}\right)x^6+x^8}\,dx$$
$$= 6\sqrt{2}\left(\int \frac{x^4}{x^8+4x^6+6x^4-124x^2+1}dx\right) + 4\sqrt{2}\left(\int \frac{x^2}{x^8+4x^6+6x^4-124x^2+1}dx\right) - 2\sqrt{2}\left(\int \frac{1}{x^8+4x^6+6x^4-124x^2+1}dx\right) + \int \frac{x^6}{x^8+4x^6+6x^4-124x^2+1}dx - \left(\int \frac{x^4}{x^8+4x^6+6x^4-124x^2+1}dx\right) + 27\left(\int \frac{x^2}{x^8+4x^6+6x^4-124x^2+1}dx\right) - 3\left(\int \frac{1}{x^8+4x^6+6x^4-124x^2+1}dx\right)$$

input
```
int((3-2*2^(1/2)+x^2)^2*(-3+2*2^(1/2)+x^2)/(577-408*2^(1/2)+(328-232*2^(1/
2))*x^2+(78-56*2^(1/2))*x^4+(8-8*2^(1/2))*x^6+x^8),x)
```

output
```
6*sqrt(2)*int(x**4/(x**8 + 4*x**6 + 6*x**4 - 124*x**2 + 1),x) + 4*sqrt(2)*
int(x**2/(x**8 + 4*x**6 + 6*x**4 - 124*x**2 + 1),x) - 2*sqrt(2)*int(1/(x**
8 + 4*x**6 + 6*x**4 - 124*x**2 + 1),x) + int(x**6/(x**8 + 4*x**6 + 6*x**4
- 124*x**2 + 1),x) - int(x**4/(x**8 + 4*x**6 + 6*x**4 - 124*x**2 + 1),x) +
 27*int(x**2/(x**8 + 4*x**6 + 6*x**4 - 124*x**2 + 1),x) - 3*int(1/(x**8 +
4*x**6 + 6*x**4 - 124*x**2 + 1),x)
```

3.102 $\int \left(a+b\sqrt{x}\right)^3 (d+ex)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 98

$$\int \left(a+b\sqrt{x}\right)^3 (d+ex)\,dx = -\frac{a(b^2d+a^2e)\left(a+b\sqrt{x}\right)^4}{2b^4}+\frac{2(b^2d+3a^2e)\left(a+b\sqrt{x}\right)^5}{5b^4} -\frac{ae\left(a+b\sqrt{x}\right)^6}{b^4}+\frac{2e\left(a+b\sqrt{x}\right)^7}{7b^4}$$

output
```
-1/2*a*(a^2*e+b^2*d)*(a+b*x^(1/2))^4/b^4+2/5*(3*a^2*e+b^2*d)*(a+b*x^(1/2))
^5/b^4-a*e*(a+b*x^(1/2))^6/b^4+2/7*e*(a+b*x^(1/2))^7/b^4
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.71

$$\int \left(a+b\sqrt{x}\right)^3 (d+ex)\,dx = \frac{1}{70}x\left(35a^3(2d+ex)+35ab^2x(3d+2ex) +28a^2b\sqrt{x}(5d+3ex)+4b^3x^{3/2}(7d+5ex)\right)$$

input
```
Integrate[(a + b*Sqrt[x])^3*(d + e*x),x]
```

output
```
(x*(35*a^3*(2*d + e*x) + 35*a*b^2*x*(3*d + 2*e*x) + 28*a^2*b*Sqrt[x]*(5*d
+ 3*e*x) + 4*b^3*x^(3/2)*(7*d + 5*e*x)))/70
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = $\{1732, 522, 2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\sqrt{x}\right)^3 (d+ex)\, dx$$

$$\Big\downarrow \text{ 1732}$$

$$2\int \left(a+b\sqrt{x}\right)^3 \sqrt{x}(d+ex)d\sqrt{x}$$

$$\Big\downarrow \text{ 522}$$

$$2\int \left(\frac{e\left(a+b\sqrt{x}\right)^6}{b^3}-\frac{3ae\left(a+b\sqrt{x}\right)^5}{b^3}+\frac{\left(3ea^2+b^2d\right)\left(a+b\sqrt{x}\right)^4}{b^3}+\frac{a\left(-ea^2-b^2d\right)\left(a+b\sqrt{x}\right)^3}{b^3}\right)d\sqrt{x}$$

$$\Big\downarrow \text{ 2009}$$

$$2\left(\frac{\left(a+b\sqrt{x}\right)^5\left(3a^2e+b^2d\right)}{5b^4}-\frac{a\left(a+b\sqrt{x}\right)^4\left(a^2e+b^2d\right)}{4b^4}+\frac{e\left(a+b\sqrt{x}\right)^7}{7b^4}-\frac{ae\left(a+b\sqrt{x}\right)^6}{2b^4}\right)$$

input
```
Int[(a + b*Sqrt[x])^3*(d + e*x),x]
```

output
```
2*(-1/4*(a*(b^2*d + a^2*e)*(a + b*Sqrt[x])^4)/b^4 + ((b^2*d + 3*a^2*e)*(a
+ b*Sqrt[x])^5)/(5*b^4) - (a*e*(a + b*Sqrt[x])^6)/(2*b^4) + (e*(a + b*Sqrt
[x])^7)/(7*b^4))
```

Defintions of rubi rules used

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 1732
```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Simp[g   Subst[Int[x^(g - 1)*(d + e*x^(g*
n))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q}
, x] && EqQ[n2, 2*n] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.72

method	result
default	$b^3\left(\frac{2e\,x^{\frac{7}{2}}}{7}+\frac{2d\,x^{\frac{5}{2}}}{5}\right)+3b^2a\left(\frac{1}{3}e\,x^3+\frac{1}{2}d\,x^2\right)+3b\,a^2\left(\frac{2e\,x^{\frac{5}{2}}}{5}+\frac{2d\,x^{\frac{3}{2}}}{3}\right)+a^3\left(\frac{1}{2}e\,x^2+dx\right)$
derivativedivides	$\frac{2b^3e\,x^{\frac{7}{2}}}{7}+a\,b^2e\,x^3+\frac{2(3b\,a^2e+b^3d)x^{\frac{5}{2}}}{5}+\frac{(a^3e+3a\,b^2d)x^2}{2}+2a^2bd\,x^{\frac{3}{2}}+a^3dx$
trager	$\frac{a(2b^2e\,x^2+a^2ex+3b^2dx+2b^2ex+2a^2d+a^2e+3d\,b^2+2b^2e)(x-1)}{2}+\frac{2b\,x^{\frac{3}{2}}(5b^2e\,x^2+21a^2ex+7b^2dx+35a^2d)}{35}$
orering	$-\frac{(-110b^6e^2x^5+243a^2b^4e^2x^4-279b^6de\,x^4-147a^4b^2e^2x^3+630a^2b^4de\,x^3-147b^6d^2x^3-105a^4b^2de\,x^2+315a^2b^4d^2x^2+}{210b^2(-b^2x+a^2)^2(ex+d)}$

input
```
int((a+b*x^(1/2))^3*(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
b^3*(2/7*e*x^(7/2)+2/5*d*x^(5/2))+3*b^2*a*(1/3*e*x^3+1/2*d*x^2)+3*b*a^2*(2
/5*e*x^(5/2)+2/3*d*x^(3/2))+a^3*(1/2*e*x^2+d*x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.77

$$\int \left(a+b\sqrt{x}\right)^3 (d+ex)\,dx = ab^2ex^3 + a^3dx + \frac{1}{2}\left(3\,ab^2d + a^3e\right)x^2 + \frac{2}{35}\left(5\,b^3ex^3 + 35\,a^2bdx + 7\,\left(b^3d + 3\,a^2be\right)x^2\right)\sqrt{x}$$

input
```
integrate((a+b*x^(1/2))^3*(e*x+d),x, algorithm="fricas")
```

output
```
a*b^2*e*x^3 + a^3*d*x + 1/2*(3*a*b^2*d + a^3*e)*x^2 + 2/35*(5*b^3*e*x^3 +
35*a^2*b*d*x + 7*(b^3*d + 3*a^2*b*e)*x^2)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.85

$$\int \left(a+b\sqrt{x}\right)^3 (d+ex)\,dx = a^3dx + 2a^2bdx^{\frac{3}{2}} + ab^2ex^3 + \frac{2b^3ex^{\frac{7}{2}}}{7} + \frac{2x^{\frac{5}{2}}\cdot(3a^2be + b^3d)}{5} + \frac{x^2(a^3e + 3ab^2d)}{2}$$

input
```
integrate((a+b*x**(1/2))**3*(e*x+d),x)
```

output
```
a**3*d*x + 2*a**2*b*d*x**(3/2) + a*b**2*e*x**3 + 2*b**3*e*x**(7/2)/7 + 2*x
**(5/2)*(3*a**2*b*e + b**3*d)/5 + x**2*(a**3*e + 3*a*b**2*d)/2
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.72

$$\int \left(a+b\sqrt{x}\right)^3 (d+ex)\,dx = \frac{2}{7}b^3ex^{\frac{7}{2}} + ab^2ex^3 + 2\,a^2bdx^{\frac{3}{2}} + a^3dx + \frac{2}{5}\left(b^3d+3\,a^2be\right)x^{\frac{5}{2}} + \frac{1}{2}\left(3\,ab^2d+a^3e\right)x^2$$

input
```
integrate((a+b*x^(1/2))^3*(e*x+d),x, algorithm="maxima")
```

output
```
2/7*b^3*e*x^(7/2) + a*b^2*e*x^3 + 2*a^2*b*d*x^(3/2) + a^3*d*x + 2/5*(b^3*d
 + 3*a^2*b*e)*x^(5/2) + 1/2*(3*a*b^2*d + a^3*e)*x^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.74

$$\int \left(a+b\sqrt{x}\right)^3 (d+ex)\,dx = \frac{2}{7}b^3ex^{\frac{7}{2}} + ab^2ex^3 + \frac{2}{5}b^3dx^{\frac{5}{2}} + \frac{6}{5}a^2bex^{\frac{5}{2}} + \frac{3}{2}ab^2dx^2 + \frac{1}{2}a^3ex^2 + 2\,a^2bdx^{\frac{3}{2}} + a^3dx$$

input
```
integrate((a+b*x^(1/2))^3*(e*x+d),x, algorithm="giac")
```

output
```
2/7*b^3*e*x^(7/2) + a*b^2*e*x^3 + 2/5*b^3*d*x^(5/2) + 6/5*a^2*b*e*x^(5/2)
+ 3/2*a*b^2*d*x^2 + 1/2*a^3*e*x^2 + 2*a^2*b*d*x^(3/2) + a^3*d*x
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.72

$$\int \left(a+b\sqrt{x}\right)^3 (d+ex)\,dx = x^2\left(\frac{e\,a^3}{2}+\frac{3\,d\,a\,b^2}{2}\right) + x^{5/2}\left(\frac{6\,e\,a^2\,b}{5}+\frac{2\,d\,b^3}{5}\right) + \frac{2\,b^3\,e\,x^{7/2}}{7} + a^3\,d\,x + 2\,a^2\,b\,d\,x^{3/2} + a\,b^2\,e\,x^3$$

input
```
int((a + b*x^(1/2))^3*(d + e*x),x)
```

output
```
x^2*((a^3*e)/2 + (3*a*b^2*d)/2) + x^(5/2)*((2*b^3*d)/5 + (6*a^2*b*e)/5) +
(2*b^3*e*x^(7/2))/7 + a^3*d*x + 2*a^2*b*d*x^(3/2) + a*b^2*e*x^3
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.76

$$\int \left(a+b\sqrt{x}\right)^3 (d+ex)\,dx$$
$$=\frac{x\left(140\sqrt{x}\,a^2bd+84\sqrt{x}\,a^2bex+28\sqrt{x}\,b^3dx+20\sqrt{x}\,b^3e\,x^2+70a^3d+35a^3ex+105a\,b^2dx+70a\,b^2e\,x^2\right)}{70}$$

input
```
int((a+b*x^(1/2))^3*(e*x+d),x)
```

output
```
(x*(140*sqrt(x)*a**2*b*d + 84*sqrt(x)*a**2*b*e*x + 28*sqrt(x)*b**3*d*x + 2
0*sqrt(x)*b**3*e*x**2 + 70*a**3*d + 35*a**3*e*x + 105*a*b**2*d*x + 70*a*b*
*2*e*x**2))/70
```

3.103 $\int \left(a+b\sqrt{x}\right)^2 (d+ex)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 60

$$\int \left(a+b\sqrt{x}\right)^2 (d+ex)\,dx = a^2dx+\frac{4}{3}abdx^{3/2}+\frac{1}{2}\left(b^2d+a^2e\right)x^2+\frac{4}{5}abex^{5/2}+\frac{1}{3}b^2ex^3$$

output
```
a^2*d*x+4/3*a*b*d*x^(3/2)+1/2*(a^2*e+b^2*d)*x^2+4/5*a*b*e*x^(5/2)+1/3*b^2*
e*x^3
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.82

$$\int \left(a+b\sqrt{x}\right)^2 (d+ex)\,dx = \frac{1}{30}x\left(15a^2(2d+ex)+5b^2x(3d+2ex)+8ab\sqrt{x}(5d+3ex)\right)$$

input
```
Integrate[(a + b*Sqrt[x])^2*(d + e*x),x]
```

output
```
(x*(15*a^2*(2*d + e*x) + 5*b^2*x*(3*d + 2*e*x) + 8*a*b*Sqrt[x]*(5*d + 3*e*
x)))/30
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1732, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\sqrt{x}\right)^2 (d+ex)\,dx$$

$$\Big\downarrow \ 1732$$

$$2\int \left(a+b\sqrt{x}\right)^2 \sqrt{x}(d+ex)d\sqrt{x}$$

$$\Big\downarrow \ 522$$

$$2\int \left(b^2ex^{5/2}+2abex^2+\left(ea^2+b^2d\right)x^{3/2}+2abdx+a^2d\sqrt{x}\right)d\sqrt{x}$$

$$\Big\downarrow \ 2009$$

$$2\left(\frac{1}{4}x^2\left(a^2e+b^2d\right)+\frac{1}{2}a^2dx+\frac{2}{3}abdx^{3/2}+\frac{2}{5}abex^{5/2}+\frac{1}{6}b^2ex^3\right)$$

input
```
Int[(a + b*Sqrt[x])^2*(d + e*x),x]
```

output
```
2*((a^2*d*x)/2 + (2*a*b*d*x^(3/2))/3 + ((b^2*d + a^2*e)*x^2)/4 + (2*a*b*e*
x^(5/2))/5 + (b^2*e*x^3)/6)
```

Defintions of rubi rules used

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 1732
```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Simp[g   Subst[Int[x^(g - 1)*(d + e*x^(g*
n))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q}
, x] && EqQ[n2, 2*n] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.82

method	result
derivativedivides	$a^2dx+\frac{4abd\,x^{\frac{3}{2}}}{3}+\frac{(a^2e+d\,b^2)x^2}{2}+\frac{4abe\,x^{\frac{5}{2}}}{5}+\frac{b^2x^3e}{3}$
default	$b^2\left(\frac{1}{3}e\,x^3+\frac{1}{2}d\,x^2\right)+2ab\left(\frac{2e\,x^{\frac{5}{2}}}{5}+\frac{2d\,x^{\frac{3}{2}}}{3}\right)+a^2\left(\frac{1}{2}e\,x^2+dx\right)$
trager	$\frac{(2b^2e\,x^2+3a^2ex+3b^2dx+2b^2ex+6a^2d+3a^2e+3d\,b^2+2b^2e)(x-1)}{6}+\frac{4ab\,x^{\frac{3}{2}}(3ex+5d)}{15}$
orering	$\frac{(-18b^2e^2x^3+21a^2e^2x^2-49b^2de\,x^2+65a^2dex-25b^2d^2x+30a^2d^2)x(a+b\sqrt{x})^2}{30(-b^2x+a^2)(ex+d)}-\frac{x^2(-2b^2e\,x^2+3a^2ex-5b^2dx+10a^2}{15(ex+d)(}$

input
```
int((a+b*x^(1/2))^2*(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
a^2*d*x+4/3*a*b*d*x^(3/2)+1/2*(a^2*e+b^2*d)*x^2+4/5*a*b*e*x^(5/2)+1/3*b^2*
x^3*e
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.87

$$\int\left(a+b\sqrt{x}\right)^2(d+ex)\,dx=\frac{1}{3}\,b^2ex^3+a^2dx+\frac{1}{2}\,\left(b^2d+a^2e\right)x^2+\frac{4}{15}\,\left(3\,abex^2+5\,abdx\right)\sqrt{x}$$

input
```
integrate((a+b*x^(1/2))^2*(e*x+d),x, algorithm="fricas")
```

output
```
1/3*b^2*e*x^3 + a^2*d*x + 1/2*(b^2*d + a^2*e)*x^2 + 4/15*(3*a*b*e*x^2 + 5*
a*b*d*x)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.05

$$\int \left(a+b\sqrt{x}\right)^2 (d+ex)\,dx = a^2dx + \frac{a^2ex^2}{2} + \frac{4abdx^{\frac{3}{2}}}{3} + \frac{4abex^{\frac{5}{2}}}{5} + \frac{b^2dx^2}{2} + \frac{b^2ex^3}{3}$$

input
```
integrate((a+b*x**(1/2))**2*(e*x+d),x)
```

output
```
a**2*d*x + a**2*e*x**2/2 + 4*a*b*d*x**(3/2)/3 + 4*a*b*e*x**(5/2)/5 + b**2*
d*x**2/2 + b**2*e*x**3/3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.80

$$\int \left(a+b\sqrt{x}\right)^2 (d+ex)\,dx = \frac{1}{3}\,b^2ex^3 + \frac{4}{5}\,abex^{\frac{5}{2}} + \frac{4}{3}\,abdx^{\frac{3}{2}} + a^2dx + \frac{1}{2}\,(b^2d + a^2e)x^2$$

input
```
integrate((a+b*x^(1/2))^2*(e*x+d),x, algorithm="maxima")
```

output
```
1/3*b^2*e*x^3 + 4/5*a*b*e*x^(5/2) + 4/3*a*b*d*x^(3/2) + a^2*d*x + 1/2*(b^2
*d + a^2*e)*x^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.83

$$\int \left(a+b\sqrt{x}\right)^2 (d+ex)\,dx = \frac{1}{3}\,b^2ex^3 + \frac{4}{5}\,abex^{\frac{5}{2}} + \frac{1}{2}\,b^2dx^2 + \frac{1}{2}\,a^2ex^2 + \frac{4}{3}\,abdx^{\frac{3}{2}} + a^2dx$$

input
```
integrate((a+b*x^(1/2))^2*(e*x+d),x, algorithm="giac")
```

output
```
1/3*b^2*e*x^3 + 4/5*a*b*e*x^(5/2) + 1/2*b^2*d*x^2 + 1/2*a^2*e*x^2 + 4/3*a*
b*d*x^(3/2) + a^2*d*x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.82

$$\int \left(a+b\sqrt{x}\right)^2 (d+ex)\,dx = x^2\left(\frac{e\,a^2}{2}+\frac{d\,b^2}{2}\right)+\frac{b^2\,e\,x^3}{3}+a^2\,d\,x+\frac{4\,a\,b\,d\,x^{3/2}}{3}+\frac{4\,a\,b\,e\,x^{5/2}}{5}$$

input
```
int((a + b*x^(1/2))^2*(d + e*x),x)
```

output
```
x^2*((a^2*e)/2 + (b^2*d)/2) + (b^2*e*x^3)/3 + a^2*d*x + (4*a*b*d*x^(3/2))/
3 + (4*a*b*e*x^(5/2))/5
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.80

$$\int \left(a+b\sqrt{x}\right)^2 (d+ex)\,dx = \frac{x\left(40\sqrt{x}\,abd+24\sqrt{x}\,abex+30a^2d+15a^2ex+15b^2dx+10b^2e\,x^2\right)}{30}$$

input
```
int((a+b*x^(1/2))^2*(e*x+d),x)
```

output
```
(x*(40*sqrt(x)*a*b*d + 24*sqrt(x)*a*b*e*x + 30*a**2*d + 15*a**2*e*x + 15*b
**2*d*x + 10*b**2*e*x**2))/30
```

3.104 $\int \left(a+b\sqrt{x}\right)(d+ex)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 38

$$\int \left(a+b\sqrt{x}\right)(d+ex)\,dx=\frac{2}{3}bdx^{3/2}+\frac{2}{5}bex^{5/2}+\frac{a(d+ex)^2}{2e}$$

output
```
2/3*b*d*x^(3/2)+2/5*b*e*x^(5/2)+1/2*a*(e*x+d)^2/e
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.84

$$\int \left(a+b\sqrt{x}\right)(d+ex)\,dx=\frac{1}{2}ax(2d+ex)+\frac{2}{15}bx^{3/2}(5d+3ex)$$

input
```
Integrate[(a + b*Sqrt[x])*(d + e*x),x]
```

output
```
(a*x*(2*d + e*x))/2 + (2*b*x^(3/2)*(5*d + 3*e*x))/15
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1732, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\sqrt{x}\right)(d+ex)\,dx$$

$$\Big\downarrow \ 1732$$

$$2\int \left(a+b\sqrt{x}\right)\sqrt{x}(d+ex)d\sqrt{x}$$

$$\Big\downarrow \ 522$$

$$2\int \left(bex^2+aex^{3/2}+bdx+ad\sqrt{x}\right)d\sqrt{x}$$

$$\Big\downarrow \ 2009$$

$$2\left(\frac{adx}{2}+\frac{1}{4}aex^2+\frac{1}{3}bdx^{3/2}+\frac{1}{5}bex^{5/2}\right)$$

input
```
Int[(a + b*Sqrt[x])*(d + e*x),x]
```

output
```
2*((a*d*x)/2 + (b*d*x^(3/2))/3 + (a*e*x^2)/4 + (b*e*x^(5/2))/5)
```

Defintions of rubi rules used

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 1732
```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Simp[g   Subst[Int[x^(g - 1)*(d + e*x^(g*
n))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q}
, x] && EqQ[n2, 2*n] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.71

method	result	size
derivativedivides	$\frac{2be\,x^{\frac{5}{2}}}{5}+\frac{ae\,x^{2}}{2}+\frac{2bd\,x^{\frac{3}{2}}}{3}+adx$	27
default	$b\left(\frac{2e\,x^{\frac{5}{2}}}{5}+\frac{2d\,x^{\frac{3}{2}}}{3}\right)+a\left(\frac{1}{2}e\,x^{2}+dx\right)$	29
trager	$\frac{(x-1)(ex+2d+e)a}{2}+\frac{2b\,x^{\frac{3}{2}}(3ex+5d)}{15}$	30
orering	$\frac{x(21e^{2}x^{2}+65dex+30d^{2})(a+b\sqrt{x})}{30ex+30d}-\frac{x^{2}(3ex+10d)\left(\frac{b(ex+d)}{2\sqrt{x}}+(a+b\sqrt{x})e\right)}{15(ex+d)}$	79

input
```
int((a+b*x^(1/2))*(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
2/5*b*e*x^(5/2)+1/2*a*e*x^2+2/3*b*d*x^(3/2)+a*d*x
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.79

$$\int \left(a+b\sqrt{x}\right)(d+ex)\,dx=\frac{1}{2}\,aex^{2}+adx+\frac{2}{15}\,\left(3\,bex^{2}+5\,bdx\right)\sqrt{x}$$

input
```
integrate((a+b*x^(1/2))*(e*x+d),x, algorithm="fricas")
```

output
```
1/2*a*e*x^2 + a*d*x + 2/15*(3*b*e*x^2 + 5*b*d*x)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.95

$$\int \left(a+b\sqrt{x}\right)(d+ex)\,dx = adx + \frac{aex^2}{2} + \frac{2bdx^{\frac{3}{2}}}{3} + \frac{2bex^{\frac{5}{2}}}{5}$$

input
```
integrate((a+b*x**(1/2))*(e*x+d),x)
```

output
```
a*d*x + a*e*x**2/2 + 2*b*d*x**(3/2)/3 + 2*b*e*x**(5/2)/5
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.68

$$\int \left(a+b\sqrt{x}\right)(d+ex)\,dx = \frac{2}{5}bex^{\frac{5}{2}} + \frac{1}{2}aex^2 + \frac{2}{3}bdx^{\frac{3}{2}} + adx$$

input
```
integrate((a+b*x^(1/2))*(e*x+d),x, algorithm="maxima")
```

output
```
2/5*b*e*x^(5/2) + 1/2*a*e*x^2 + 2/3*b*d*x^(3/2) + a*d*x
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.68

$$\int \left(a+b\sqrt{x}\right)(d+ex)\,dx = \frac{2}{5}bex^{\frac{5}{2}} + \frac{1}{2}aex^2 + \frac{2}{3}bdx^{\frac{3}{2}} + adx$$

input
```
integrate((a+b*x^(1/2))*(e*x+d),x, algorithm="giac")
```

output
```
2/5*b*e*x^(5/2) + 1/2*a*e*x^2 + 2/3*b*d*x^(3/2) + a*d*x
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.68

$$\int \left(a+b\sqrt{x}\right)(d+ex)\,dx = a\,d\,x+\frac{a\,e\,x^2}{2}+\frac{2\,b\,d\,x^{3/2}}{3}+\frac{2\,b\,e\,x^{5/2}}{5}$$

input
```
int((a + b*x^(1/2))*(d + e*x),x)
```

output
```
a*d*x + (a*e*x^2)/2 + (2*b*d*x^(3/2))/3 + (2*b*e*x^(5/2))/5
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.68

$$\int \left(a+b\sqrt{x}\right)(d+ex)\,dx = \frac{x\left(20\sqrt{x}\,bd+12\sqrt{x}\,bex+30ad+15aex\right)}{30}$$

input
```
int((a+b*x^(1/2))*(e*x+d),x)
```

output
```
(x*(20*sqrt(x)*b*d + 12*sqrt(x)*b*e*x + 30*a*d + 15*a*e*x))/30
```

3.105 $\int \left(a+b\sqrt{x}+cx\right)^3 (d+ex)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 166

$$\begin{aligned}\int \left(a+b\sqrt{x}+cx\right)^3 (d+ex)\,dx &= a^3dx+2a^2bdx^{3/2}+\frac{1}{2}a\left(3b^2d+a(3cd+ae)\right)x^2\\&\quad+\frac{2}{5}b\left(b^2d+3a(2cd+ae)\right)x^{5/2}\\&\quad+\left(b^2+ac\right)(cd+ae)x^3+\frac{2}{7}b\left(3c^2d+b^2e+6ace\right)x^{7/2}\\&\quad+\frac{1}{4}c\left(c^2d+3b^2e+3ace\right)x^4+\frac{2}{3}bc^2ex^{9/2}+\frac{1}{5}c^3ex^5\end{aligned}$$

output
```
a^3*d*x+2*a^2*b*d*x^(3/2)+1/2*a*(3*b^2*d+a*(a*e+3*c*d))*x^2+2/5*b*(b^2*d+3
*a*(a*e+2*c*d))*x^(5/2)+(a*c+b^2)*(a*e+c*d)*x^3+2/7*b*(6*a*c*e+b^2*e+3*c^2
*d)*x^(7/2)+1/4*c*(3*a*c*e+3*b^2*e+c^2*d)*x^4+2/3*b*c^2*e*x^(9/2)+1/5*c^3*
e*x^5
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.03

$$\int \left(a+b\sqrt{x}+cx\right)^3(d+ex)\,dx$$
$$=\frac{1}{420}x\left(210a^3(2d+ex)+105b^2cx^2(4d+3ex)+21c^3x^3(5d+4ex)+24b^3x^{3/2}(7d+5ex)\right.$$
$$+40bc^2x^{5/2}(9d+7ex)+42a^2\left(5cx(3d+2ex)+4b\sqrt{x}(5d+3ex)\right)+3ax\left(70b^2(3d+2ex)+35c^2x(4d+3ex)+4\right.$$

input
```
Integrate[(a + b*Sqrt[x] + c*x)^3*(d + e*x),x]
```

output
```
(x*(210*a^3*(2*d + e*x) + 105*b^2*c*x^2*(4*d + 3*e*x) + 21*c^3*x^3*(5*d +
4*e*x) + 24*b^3*x^(3/2)*(7*d + 5*e*x) + 40*b*c^2*x^(5/2)*(9*d + 7*e*x) + 4
2*a^2*(5*c*x*(3*d + 2*e*x) + 4*b*Sqrt[x]*(5*d + 3*e*x)) + 3*a*x*(70*b^2*(3
*d + 2*e*x) + 35*c^2*x*(4*d + 3*e*x) + 48*b*c*Sqrt[x]*(7*d + 5*e*x))))/420
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.14, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2308, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)\left(a+b\sqrt{x}+cx\right)^3\,dx$$
$$\Big\downarrow \ 2308$$
$$\int \left(d\left(a+b\sqrt{x}+cx\right)^3+ex\left(a+b\sqrt{x}+cx\right)^3\right)dx$$
$$\Big\downarrow \ 2009$$
$$a^3dx+\frac{1}{2}a^3ex^2+2a^2bdx^{3/2}+\frac{6}{5}a^2bex^{5/2}+\frac{2}{5}bdx^{5/2}\left(6ac+b^2\right)+cdx^3\left(ac+b^2\right)+\frac{3}{2}adx^2\left(ac+b^2\right)+\frac{2}{7}bex^{7/2}\left(6ac+b^2\right)+\frac{3}{4}cex^4\left(ac+b^2\right)+aex^3\left(ac+b^2\right)+\frac{6}{7}bc^2dx^{7/2}+\frac{2}{3}bc^2ex^{9/2}+\frac{1}{4}c^3dx^4+\frac{1}{5}c^3ex^5$$

input
```
Int[(a + b*Sqrt[x] + c*x)^3*(d + e*x),x]
```

output
```
a^3*d*x + 2*a^2*b*d*x^(3/2) + (3*a*(b^2 + a*c)*d*x^2)/2 + (a^3*e*x^2)/2 +
(2*b*(b^2 + 6*a*c)*d*x^(5/2))/5 + (6*a^2*b*e*x^(5/2))/5 + c*(b^2 + a*c)*d*
x^3 + a*(b^2 + a*c)*e*x^3 + (6*b*c^2*d*x^(7/2))/7 + (2*b*(b^2 + 6*a*c)*e*x
^(7/2))/7 + (c^3*d*x^4)/4 + (3*c*(b^2 + a*c)*e*x^4)/4 + (2*b*c^2*e*x^(9/2)
)/3 + (c^3*e*x^5)/5
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2308
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_.), x_Symbol] :>
 Int[ExpandIntegrand[Pq*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c
, n}, x] && EqQ[n2, 2*n] && PolyQ[Pq, x] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 1.16 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.06

method	result
default	$b^3\left(\frac{2e\,x^{\frac{7}{2}}}{7}+\frac{2d\,x^{\frac{5}{2}}}{5}\right)+3b^2\left(\frac{ce\,x^4}{4}+\frac{(ae+cd)x^3}{3}+\frac{ad\,x^2}{2}\right)+3b\left(\frac{2c^2e\,x^{\frac{9}{2}}}{9}+\frac{2(2ace+c^2d)x^{\frac{7}{2}}}{7}+\frac{2(a^2e+\right.$
derivativedivides	$\frac{c^3e\,x^5}{5}+\frac{2b\,c^2e\,x^{\frac{9}{2}}}{3}+\frac{((a\,c^2+2c\,b^2+c(2ac+b^2))e+c^3d)x^4}{4}+\frac{2((4abc+b(2ac+b^2))e+3b\,c^2d)x^{\frac{7}{2}}}{7}+\frac{((a(2ac+b^2)}{}$
trager	$\frac{(4e\,c^3x^4+15a\,c^2e\,x^3+15b^2ce\,x^3+5c^3d\,x^3+4c^3e\,x^3+20a^2ce\,x^2+20a\,b^2e\,x^2+20a\,c^2d\,x^2+15a\,c^2e\,x^2+20b^2cd\,x^2+15b^2c}{}$
orering	Expression too large to display

input
```
int((a+b*x^(1/2)+c*x)^3*(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
b^3*(2/7*e*x^(7/2)+2/5*d*x^(5/2))+3*b^2*(1/4*c*e*x^4+1/3*(a*e+c*d)*x^3+1/2
*a*d*x^2)+3*b*(2/9*c^2*e*x^(9/2)+2/7*(2*a*c*e+c^2*d)*x^(7/2)+2/5*(a^2*e+2*
a*c*d)*x^(5/2)+2/3*a^2*d*x^(3/2))+1/5*c^3*e*x^5+1/4*(3*a*c^2*e+c^3*d)*x^4+
1/3*(3*a^2*c*e+3*a*c^2*d)*x^3+1/2*(a^3*e+3*a^2*c*d)*x^2+a^3*d*x
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.02

$$\int \left(a+b\sqrt{x}+cx\right)^3(d+ex)\,dx = \frac{1}{5}c^3ex^5 + a^3dx + \frac{1}{4}\left(c^3d + 3\left(b^2c + ac^2\right)e\right)x^4 + \left(\left(b^2c + ac^2\right)d + \left(ab^2 + a^2c\right)e\right)x^3 + \frac{1}{2}\left(a^3e + 3\left(ab^2 + a^2c\right)d\right)x^2 + \frac{2}{105}\left(35\,bc^2ex^4 + 105\,a^2bdx + 15\left(3\,bc^2d + \left(b^3 + 6\,abc\right)e\right)x^3 + 21\left(3\,a^2be + \left(b^3 + 6\,abc\right)d\right)x^2\right)\sqrt{x}$$

input
```
integrate((a+b*x^(1/2)+c*x)^3*(e*x+d),x, algorithm="fricas")
```

output
```
1/5*c^3*e*x^5 + a^3*d*x + 1/4*(c^3*d + 3*(b^2*c + a*c^2)*e)*x^4 + ((b^2*c
+ a*c^2)*d + (a*b^2 + a^2*c)*e)*x^3 + 1/2*(a^3*e + 3*(a*b^2 + a^2*c)*d)*x^
2 + 2/105*(35*b*c^2*e*x^4 + 105*a^2*b*d*x + 15*(3*b*c^2*d + (b^3 + 6*a*b*c
)*e)*x^3 + 21*(3*a^2*b*e + (b^3 + 6*a*b*c)*d)*x^2)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.49

$$\int \left(a+b\sqrt{x}+cx\right)^3(d+ex)\,dx = a^3dx + \frac{a^3ex^2}{2} + 2a^2bdx^{\frac{3}{2}} + \frac{6a^2bex^{\frac{5}{2}}}{5} + \frac{3a^2cdx^2}{2} + a^2cex^3 + \frac{3ab^2dx^2}{2} + ab^2ex^3 + \frac{12abcdx^{\frac{5}{2}}}{5} + \frac{12abcex^{\frac{7}{2}}}{7} + ac^2dx^3 + \frac{3ac^2ex^4}{4} + \frac{2b^3dx^{\frac{5}{2}}}{5} + \frac{2b^3ex^{\frac{7}{2}}}{7} + b^2cdx^3 + \frac{3b^2cex^4}{4} + \frac{6bc^2dx^{\frac{7}{2}}}{7} + \frac{2bc^2ex^{\frac{9}{2}}}{3} + \frac{c^3dx^4}{4} + \frac{c^3ex^5}{5}$$

input
```
integrate((a+b*x**(1/2)+c*x)**3*(e*x+d),x)
```

output
```
a**3*d*x + a**3*e*x**2/2 + 2*a**2*b*d*x**(3/2) + 6*a**2*b*e*x**(5/2)/5 + 3
*a**2*c*d*x**2/2 + a**2*c*e*x**3 + 3*a*b**2*d*x**2/2 + a*b**2*e*x**3 + 12*
a*b*c*d*x**(5/2)/5 + 12*a*b*c*e*x**(7/2)/7 + a*c**2*d*x**3 + 3*a*c**2*e*x*
*4/4 + 2*b**3*d*x**(5/2)/5 + 2*b**3*e*x**(7/2)/7 + b**2*c*d*x**3 + 3*b**2*
c*e*x**4/4 + 6*b*c**2*d*x**(7/2)/7 + 2*b*c**2*e*x**(9/2)/3 + c**3*d*x**4/4
 + c**3*e*x**5/5
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.99

$$\int \left(a+b\sqrt{x}+cx\right)^3(d+ex)\,dx = \frac{1}{5}\,c^3ex^5+\frac{2}{3}\,bc^2ex^{\frac{9}{2}}+2\,a^2bdx^{\frac{3}{2}}+a^3dx+\frac{1}{4}\left(c^3d+3\left(b^2c+ac^2\right)e\right)x^4+\frac{2}{7}\left(3\,bc^2d+\left(b^3+6\,abc\right)e\right)x^{\frac{7}{2}}+\left(\left(b^2c+ac^2\right)d+\left(ab^2+a^2c\right)e\right)x^3+\frac{2}{5}\left(3\,a^2be+\left(b^3+6\,abc\right)d\right)x^{\frac{5}{2}}+\frac{1}{2}\left(a^3e+3\left(ab^2+a^2c\right)d\right)x^2$$

input
```
integrate((a+b*x^(1/2)+c*x)^3*(e*x+d),x, algorithm="maxima")
```

output
```
1/5*c^3*e*x^5 + 2/3*b*c^2*e*x^(9/2) + 2*a^2*b*d*x^(3/2) + a^3*d*x + 1/4*(c
^3*d + 3*(b^2*c + a*c^2)*e)*x^4 + 2/7*(3*b*c^2*d + (b^3 + 6*a*b*c)*e)*x^(7
/2) + ((b^2*c + a*c^2)*d + (a*b^2 + a^2*c)*e)*x^3 + 2/5*(3*a^2*b*e + (b^3
+ 6*a*b*c)*d)*x^(5/2) + 1/2*(a^3*e + 3*(a*b^2 + a^2*c)*d)*x^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.12

$$\int \left(a+b\sqrt{x}+cx\right)^3(d+ex)\,dx = \frac{1}{5}\,c^3ex^5 + \frac{2}{3}\,bc^2ex^{\frac{9}{2}} + \frac{1}{4}\,c^3dx^4 + \frac{3}{4}\,b^2cex^4 + \frac{3}{4}\,ac^2ex^4 + \frac{6}{7}\,bc^2dx^{\frac{7}{2}} + \frac{2}{7}\,b^3ex^{\frac{7}{2}} + \frac{12}{7}\,abcex^{\frac{7}{2}} + b^2cdx^3 + ac^2dx^3 + ab^2ex^3 + a^2cex^3 + \frac{2}{5}\,b^3dx^{\frac{5}{2}} + \frac{12}{5}\,abcdx^{\frac{5}{2}} + \frac{6}{5}\,a^2bex^{\frac{5}{2}} + \frac{3}{2}\,ab^2dx^2 + \frac{3}{2}\,a^2cdx^2 + \frac{1}{2}\,a^3ex^2 + 2\,a^2bdx^{\frac{3}{2}} + a^3dx$$

input
```
integrate((a+b*x^(1/2)+c*x)^3*(e*x+d),x, algorithm="giac")
```

output
```
1/5*c^3*e*x^5 + 2/3*b*c^2*e*x^(9/2) + 1/4*c^3*d*x^4 + 3/4*b^2*c*e*x^4 + 3/
4*a*c^2*e*x^4 + 6/7*b*c^2*d*x^(7/2) + 2/7*b^3*e*x^(7/2) + 12/7*a*b*c*e*x^(
7/2) + b^2*c*d*x^3 + a*c^2*d*x^3 + a*b^2*e*x^3 + a^2*c*e*x^3 + 2/5*b^3*d*x
^(5/2) + 12/5*a*b*c*d*x^(5/2) + 6/5*a^2*b*e*x^(5/2) + 3/2*a*b^2*d*x^2 + 3/
2*a^2*c*d*x^2 + 1/2*a^3*e*x^2 + 2*a^2*b*d*x^(3/2) + a^3*d*x
```

Mupad [B] (verification not implemented)

Time = 22.06 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.90

$$\int \left(a+b\sqrt{x}+cx\right)^3(d+ex)\,dx = x^2\left(\frac{e\,a^3}{2}+\frac{3\,c\,d\,a^2}{2}+\frac{3\,d\,a\,b^2}{2}\right) + x^4\left(\frac{3\,e\,b^2\,c}{4}+\frac{d\,c^3}{4}+\frac{3\,a\,e\,c^2}{4}\right) + x^3\left(b^2+a\,c\right)\left(a\,e+c\,d\right) + \frac{c^3\,e\,x^5}{5} + \frac{2\,b\,x^{5/2}\left(3\,e\,a^2+6\,c\,d\,a+d\,b^2\right)}{5} + \frac{2\,b\,x^{7/2}\left(e\,b^2+3\,d\,c^2+6\,a\,e\,c\right)}{7} + a^3\,d\,x + 2\,a^2\,b\,d\,x^{3/2} + \frac{2\,b\,c^2\,e\,x^{9/2}}{3}$$

input
```
int((d + e*x)*(a + c*x + b*x^(1/2))^3,x)
```

output
```
x^2*((a^3*e)/2 + (3*a*b^2*d)/2 + (3*a^2*c*d)/2) + x^4*((c^3*d)/4 + (3*a*c^
2*e)/4 + (3*b^2*c*e)/4) + x^3*(a*c + b^2)*(a*e + c*d) + (c^3*e*x^5)/5 + (2
*b*x^(5/2)*(3*a^2*e + b^2*d + 6*a*c*d))/5 + (2*b*x^(7/2)*(b^2*e + 3*c^2*d
+ 6*a*c*e))/7 + a^3*d*x + 2*a^2*b*d*x^(3/2) + (2*b*c^2*e*x^(9/2))/3
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.17

$$\int \left(a+b\sqrt{x}+cx\right)^3(d+ex)\,dx = \frac{x\left(840\sqrt{x}\,a^2bd+504\sqrt{x}\,a^2bex+1008\sqrt{x}\,abcdx+720\sqrt{x}\,abce\,x^2+168\sqrt{x}\,b^3dx+120\sqrt{x}\,b^3e\,x^2+360\sqrt{}\right.}{}$$

input
```
int((a+b*x^(1/2)+c*x)^3*(e*x+d),x)
```

output
```
(x*(840*sqrt(x)*a**2*b*d + 504*sqrt(x)*a**2*b*e*x + 1008*sqrt(x)*a*b*c*d*x
 + 720*sqrt(x)*a*b*c*e*x**2 + 168*sqrt(x)*b**3*d*x + 120*sqrt(x)*b**3*e*x*
*2 + 360*sqrt(x)*b*c**2*d*x**2 + 280*sqrt(x)*b*c**2*e*x**3 + 420*a**3*d +
210*a**3*e*x + 630*a**2*c*d*x + 420*a**2*c*e*x**2 + 630*a*b**2*d*x + 420*a
*b**2*e*x**2 + 420*a*c**2*d*x**2 + 315*a*c**2*e*x**3 + 420*b**2*c*d*x**2 +
 315*b**2*c*e*x**3 + 105*c**3*d*x**3 + 84*c**3*e*x**4))/420
```

3.106 $\int \left(a+b\sqrt{x}+cx\right)^2 (d+ex)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 105

$$\int \left(a+b\sqrt{x}+cx\right)^2 (d+ex)\,dx = a^2dx+\frac{4}{3}abdx^{3/2} +\frac{1}{2}\left(b^2d+a(2cd+ae)\right)x^2+\frac{4}{5}b(cd+ae)x^{5/2} +\frac{1}{3}\left(c^2d+b^2e+2ace\right)x^3+\frac{4}{7}bcex^{7/2}+\frac{1}{4}c^2ex^4$$

output
```
a^2*d*x+4/3*a*b*d*x^(3/2)+1/2*(b^2*d+a*(a*e+2*c*d))*x^2+4/5*b*(a*e+c*d)*x^
(5/2)+1/3*(2*a*c*e+b^2*e+c^2*d)*x^3+4/7*b*c*e*x^(7/2)+1/4*c^2*e*x^4
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.92

$$\int \left(a+b\sqrt{x}+cx\right)^2 (d+ex)\,dx = \frac{1}{420}x\left(210a^2(2d+ex) +28a\left(5cx(3d+2ex)+4b\sqrt{x}(5d+3ex)\right) +x\left(70b^2(3d+2ex)+35c^2x(4d+3ex) +48bc\sqrt{x}(7d+5ex)\right)\right)$$

input
```
Integrate[(a + b*Sqrt[x] + c*x)^2*(d + e*x),x]
```

output
```
(x*(210*a^2*(2*d + e*x) + 28*a*(5*c*x*(3*d + 2*e*x) + 4*b*Sqrt[x]*(5*d + 3
*e*x)) + x*(70*b^2*(3*d + 2*e*x) + 35*c^2*x*(4*d + 3*e*x) + 48*b*c*Sqrt[x]
*(7*d + 5*e*x))))/420
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.14, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2308, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)\left(a+b\sqrt{x}+cx\right)^2 dx$$

$$\Big\downarrow \text{2308}$$

$$\int \left(d\left(a+b\sqrt{x}+cx\right)^2+ex\left(a+b\sqrt{x}+cx\right)^2\right)dx$$

$$\Big\downarrow \text{2009}$$

$$a^2dx+\frac{1}{2}a^2ex^2+\frac{1}{2}dx^2\left(2ac+b^2\right)+\frac{1}{3}ex^3\left(2ac+b^2\right)+\frac{4}{3}abdx^{3/2}+\frac{4}{5}abex^{5/2}+\frac{4}{5}bcdx^{5/2}+\frac{4}{7}bcex^{7/2}+\frac{1}{3}c^2dx^3+\frac{1}{4}c^2ex^4$$

input
```
Int[(a + b*Sqrt[x] + c*x)^2*(d + e*x),x]
```

output
```
a^2*d*x + (4*a*b*d*x^(3/2))/3 + ((b^2 + 2*a*c)*d*x^2)/2 + (a^2*e*x^2)/2 +
(4*b*c*d*x^(5/2))/5 + (4*a*b*e*x^(5/2))/5 + (c^2*d*x^3)/3 + ((b^2 + 2*a*c)
*e*x^3)/3 + (4*b*c*e*x^(7/2))/7 + (c^2*e*x^4)/4
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2308
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_.), x_Symbol] :>
 Int[ExpandIntegrand[Pq*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c
, n}, x] && EqQ[n2, 2*n] && PolyQ[Pq, x] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.87

method	result
derivativedivides	$\frac{c^2 e\,x^4}{4}+\frac{4bce\,x^{\frac{7}{2}}}{7}+\frac{((2ac+b^2)e+c^2d)x^3}{3}+\frac{2(2abe+2cbd)x^{\frac{5}{2}}}{5}+\frac{(a^2e+(2ac+b^2)d)x^2}{2}+\frac{4abd\,x^{\frac{3}{2}}}{3}+a^2dx$
default	$b^2\left(\frac{1}{3}e\,x^3+\frac{1}{2}d\,x^2\right)+2b\left(\frac{2ce\,x^{\frac{7}{2}}}{7}+\frac{2(ae+cd)x^{\frac{5}{2}}}{5}+\frac{2ad\,x^{\frac{3}{2}}}{3}\right)+\frac{c^2e\,x^4}{4}+\frac{(2ace+c^2d)x^3}{3}+\frac{(a^2e+2ac\cdots}{2}$
trager	$\frac{(3c^2e\,x^3+8ace\,x^2+4b^2e\,x^2+4c^2d\,x^2+3c^2e\,x^2+6a^2ex+12acdx+8acex+6b^2dx+4b^2ex+4c^2xd+3c^2ex+12a^2d+6a^2e+\cdots}{12}$
orering	$-\frac{(195c^4e^2x^6+253a\,c^3e^2x^5-220b^2c^2e^2x^5+473c^4de\,x^5-306a^2c^2e^2x^4+252a\,b^2c\,e^2x^4+711a\,c^3de\,x^4-558b^2c^2de\,x^4+\cdots}{4\cdots}$

input
```
int((a+b*x^(1/2)+c*x)^2*(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
1/4*c^2*e*x^4+4/7*b*c*e*x^(7/2)+1/3*((2*a*c+b^2)*e+c^2*d)*x^3+2/5*(2*a*b*e
+2*b*c*d)*x^(5/2)+1/2*(a^2*e+(2*a*c+b^2)*d)*x^2+4/3*a*b*d*x^(3/2)+a^2*d*x
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.88

$$\int \left(a+b\sqrt{x}+cx\right)^2(d+ex)\,dx=\frac{1}{4}\,c^2ex^4+a^2dx+\frac{1}{3}\left(c^2d+\left(b^2+2\,ac\right)e\right)x^3+\frac{1}{2}\left(a^2e+\left(b^2+2\,ac\right)d\right)x^2+\frac{4}{105}\left(15\,bcex^3+35\,abdx+21\,(bcd+abe)x^2\right)\sqrt{x}$$

input
```
integrate((a+b*x^(1/2)+c*x)^2*(e*x+d),x, algorithm="fricas")
```

output
```
1/4*c^2*e*x^4 + a^2*d*x + 1/3*(c^2*d + (b^2 + 2*a*c)*e)*x^3 + 1/2*(a^2*e +
 (b^2 + 2*a*c)*d)*x^2 + 4/105*(15*b*c*e*x^3 + 35*a*b*d*x + 21*(b*c*d + a*b
*e)*x^2)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.25

$$\int \left(a+b\sqrt{x}+cx\right)^2(d+ex)\,dx = a^2dx+\frac{a^2ex^2}{2}+\frac{4abdx^{\frac{3}{2}}}{3}+\frac{4abex^{\frac{5}{2}}}{5}+acdx^2+\frac{2acex^3}{3}+\frac{b^2dx^2}{2}+\frac{b^2ex^3}{3}+\frac{4bcdx^{\frac{5}{2}}}{5}+\frac{4bcex^{\frac{7}{2}}}{7}+\frac{c^2dx^3}{3}+\frac{c^2ex^4}{4}$$

input
```
integrate((a+b*x**(1/2)+c*x)**2*(e*x+d),x)
```

output
```
a**2*d*x + a**2*e*x**2/2 + 4*a*b*d*x**(3/2)/3 + 4*a*b*e*x**(5/2)/5 + a*c*d
*x**2 + 2*a*c*e*x**3/3 + b**2*d*x**2/2 + b**2*e*x**3/3 + 4*b*c*d*x**(5/2)/
5 + 4*b*c*e*x**(7/2)/7 + c**2*d*x**3/3 + c**2*e*x**4/4
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.84

$$\int \left(a+b\sqrt{x}+cx\right)^2(d+ex)\,dx = \frac{1}{4}c^2ex^4+\frac{4}{7}bcex^{\frac{7}{2}}+\frac{4}{3}abdx^{\frac{3}{2}}+a^2dx+\frac{1}{3}\left(c^2d+\left(b^2+2\,ac\right)e\right)x^3+\frac{4}{5}(bcd+abe)x^{\frac{5}{2}}+\frac{1}{2}\left(a^2e+\left(b^2+2\,ac\right)d\right)x^2$$

input
```
integrate((a+b*x^(1/2)+c*x)^2*(e*x+d),x, algorithm="maxima")
```

output
```
1/4*c^2*e*x^4 + 4/7*b*c*e*x^(7/2) + 4/3*a*b*d*x^(3/2) + a^2*d*x + 1/3*(c^2
*d + (b^2 + 2*a*c)*e)*x^3 + 4/5*(b*c*d + a*b*e)*x^(5/2) + 1/2*(a^2*e + (b^
2 + 2*a*c)*d)*x^2
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.94

$$\int \left(a+b\sqrt{x}+cx\right)^2(d+ex)\,dx=\frac{1}{4}\,c^2ex^4+\frac{4}{7}\,bcex^{\frac{7}{2}}+\frac{1}{3}\,c^2dx^3+\frac{1}{3}\,b^2ex^3+\frac{2}{3}\,acex^3+\frac{4}{5}\,bcdx^{\frac{5}{2}}+\frac{4}{5}\,abex^{\frac{5}{2}}+\frac{1}{2}\,b^2dx^2+acdx^2+\frac{1}{2}\,a^2ex^2+\frac{4}{3}\,abdx^{\frac{3}{2}}+a^2dx$$

input
```
integrate((a+b*x^(1/2)+c*x)^2*(e*x+d),x, algorithm="giac")
```

output
```
1/4*c^2*e*x^4 + 4/7*b*c*e*x^(7/2) + 1/3*c^2*d*x^3 + 1/3*b^2*e*x^3 + 2/3*a*
c*e*x^3 + 4/5*b*c*d*x^(5/2) + 4/5*a*b*e*x^(5/2) + 1/2*b^2*d*x^2 + a*c*d*x^
2 + 1/2*a^2*e*x^2 + 4/3*a*b*d*x^(3/2) + a^2*d*x
```

Mupad [B] (verification not implemented)

Time = 21.67 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.84

$$\int \left(a+b\sqrt{x}+cx\right)^2(d+ex)\,dx=x^2\left(\frac{e\,a^2}{2}+c\,d\,a+\frac{d\,b^2}{2}\right)+x^3\left(\frac{e\,b^2}{3}+\frac{d\,c^2}{3}+\frac{2\,a\,e\,c}{3}\right)+\frac{4\,b\,x^{5/2}\,(a\,e+c\,d)}{5}+\frac{c^2\,e\,x^4}{4}+a^2\,d\,x+\frac{4\,a\,b\,d\,x^{3/2}}{3}+\frac{4\,b\,c\,e\,x^{7/2}}{7}$$

input
```
int((d + e*x)*(a + c*x + b*x^(1/2))^2,x)
```

output
```
x^2*((a^2*e)/2 + (b^2*d)/2 + a*c*d) + x^3*((b^2*e)/3 + (c^2*d)/3 + (2*a*c*
e)/3) + (4*b*x^(5/2)*(a*e + c*d))/5 + (c^2*e*x^4)/4 + a^2*d*x + (4*a*b*d*x
^(3/2))/3 + (4*b*c*e*x^(7/2))/7
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.93

$$\int \left(a+b\sqrt{x}+cx\right)^2(d+ex)\,dx$$
$$=\frac{x\big(560\sqrt{x}\,abd+336\sqrt{x}\,abex+336\sqrt{x}\,bcdx+240\sqrt{x}\,bce\,x^2+420a^2d+210a^2ex+420acdx+280ace\,x^2}{420}$$

input
```
int((a+b*x^(1/2)+c*x)^2*(e*x+d),x)
```

output
```
(x*(560*sqrt(x)*a*b*d + 336*sqrt(x)*a*b*e*x + 336*sqrt(x)*b*c*d*x + 240*sq
rt(x)*b*c*e*x**2 + 420*a**2*d + 210*a**2*e*x + 420*a*c*d*x + 280*a*c*e*x**
2 + 210*b**2*d*x + 140*b**2*e*x**2 + 140*c**2*d*x**2 + 105*c**2*e*x**3))/4
20
```

3.107 $\int \left(a+b\sqrt{x}+cx\right)(d+ex)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 50

$$\int \left(a+b\sqrt{x}+cx\right)(d+ex)\,dx=adx+\frac{2}{3}bdx^{3/2}+\frac{1}{2}(cd+ae)x^2+\frac{2}{5}bex^{5/2}+\frac{1}{3}cex^3$$

output
```
a*d*x+2/3*b*d*x^(3/2)+1/2*(a*e+c*d)*x^2+2/5*b*e*x^(5/2)+1/3*c*e*x^3
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.90

$$\int \left(a+b\sqrt{x}+cx\right)(d+ex)\,dx=\frac{1}{30}x\left(30ad+20bd\sqrt{x}+15cdx+15aex+12bex^{3/2}+10cex^2\right)$$

input
```
Integrate[(a + b*Sqrt[x] + c*x)*(d + e*x),x]
```

output
```
(x*(30*a*d + 20*b*d*Sqrt[x] + 15*c*d*x + 15*a*e*x + 12*b*e*x^(3/2) + 10*c*
e*x^2))/30
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.12, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2308, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)\left(a+b\sqrt{x}+cx\right)\,dx$$

$$\Big\downarrow \; 2308$$

$$\int \left(a(d+ex)+b\sqrt{x}(d+ex)+cx(d+ex)\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{a(d+ex)^2}{2e}+\frac{2}{3}bdx^{3/2}+\frac{2}{5}bex^{5/2}+\frac{1}{2}cdx^2+\frac{1}{3}cex^3$$

input
```
Int[(a + b*Sqrt[x] + c*x)*(d + e*x),x]
```

output
```
(2*b*d*x^(3/2))/3 + (c*d*x^2)/2 + (2*b*e*x^(5/2))/5 + (c*e*x^3)/3 + (a*(d
+ e*x)^2)/(2*e)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2308
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_.), x_Symbol] :>
 Int[ExpandIntegrand[Pq*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c
, n}, x] && EqQ[n2, 2*n] && PolyQ[Pq, x] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.78

method	result
derivativedivides	$a d x +\frac{2 b d \,x^{\frac{3}{2}}}{3}+\frac{\left(a e +c d \right) x^{2}}{2}+\frac{2 b e \,x^{\frac{5}{2}}}{5}+\frac{c e \,x^{3}}{3}$
default	$b \left(\frac{2 e \,x^{\frac{5}{2}}}{5}+\frac{2 d \,x^{\frac{3}{2}}}{3}\right)+\frac{c e \,x^{3}}{3}+\frac{\left(a e +c d \right) x^{2}}{2}+a d x$
trager	$\frac{\left(2 c e \,x^{2}+3 a e x +3 x c d +2 c e x +6 a d +3 a e +3 c d +2 c e \right) \left(x -1\right)}{6}+\frac{2 b \,x^{\frac{3}{2}} \left(3 e x +5 d \right)}{15}$
orering	$\frac{\left(-18 c \,e^{2} x^{3}+21 a \,e^{2} x^{2}-49 c d e \,x^{2}+65 a d e x -25 c \,d^{2} x +30 a \,d^{2}\right) x \left(a +b \sqrt{x}+c x \right)}{30 \left(-c x +a \right) \left(e x +d \right)}-\frac{x^{2} \left(-2 c e \,x^{2}+3 a e x -5 x c d +10 a d \right) \left(\left(\right.\right.}{15 \left(e x +d \right) \left(-\right.}$

input
```
int((a+b*x^(1/2)+c*x)*(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
a*d*x+2/3*b*d*x^(3/2)+1/2*(a*e+c*d)*x^2+2/5*b*e*x^(5/2)+1/3*c*e*x^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.84

$$\int \left(a+b\sqrt{x}+cx\right)(d+ex)\,dx=\frac{1}{3}\,cex^{3}+adx+\frac{1}{2}\,(cd+ae)x^{2}+\frac{2}{15}\,\left(3\,bex^{2}+5\,bdx\right)\sqrt{x}$$

input
```
integrate((a+b*x^(1/2)+c*x)*(e*x+d),x, algorithm="fricas")
```

output
```
1/3*c*e*x^3 + a*d*x + 1/2*(c*d + a*e)*x^2 + 2/15*(3*b*e*x^2 + 5*b*d*x)*sqr
t(x)
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.06

$$\int \left(a+b\sqrt{x}+cx\right)(d+ex)\,dx = adx + \frac{aex^2}{2} + \frac{2bdx^{\frac{3}{2}}}{3} + \frac{2bex^{\frac{5}{2}}}{5} + \frac{cdx^2}{2} + \frac{cex^3}{3}$$

input
```
integrate((a+b*x**(1/2)+c*x)*(e*x+d),x)
```

output
```
a*d*x + a*e*x**2/2 + 2*b*d*x**(3/2)/3 + 2*b*e*x**(5/2)/5 + c*d*x**2/2 + c*
e*x**3/3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.76

$$\int \left(a+b\sqrt{x}+cx\right)(d+ex)\,dx = \frac{1}{3}\,cex^3 + \frac{2}{5}\,bex^{\frac{5}{2}} + \frac{2}{3}\,bdx^{\frac{3}{2}} + adx + \frac{1}{2}\,(cd+ae)x^2$$

input
```
integrate((a+b*x^(1/2)+c*x)*(e*x+d),x, algorithm="maxima")
```

output
```
1/3*c*e*x^3 + 2/5*b*e*x^(5/2) + 2/3*b*d*x^(3/2) + a*d*x + 1/2*(c*d + a*e)*
x^2
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.80

$$\int \left(a+b\sqrt{x}+cx\right)(d+ex)\,dx = \frac{1}{3}\,cex^3 + \frac{2}{5}\,bex^{\frac{5}{2}} + \frac{1}{2}\,cdx^2 + \frac{1}{2}\,aex^2 + \frac{2}{3}\,bdx^{\frac{3}{2}} + adx$$

input
```
integrate((a+b*x^(1/2)+c*x)*(e*x+d),x, algorithm="giac")
```

output
```
1/3*c*e*x^3 + 2/5*b*e*x^(5/2) + 1/2*c*d*x^2 + 1/2*a*e*x^2 + 2/3*b*d*x^(3/2
) + a*d*x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.78

$$\int \left(a+b\sqrt{x}+cx\right)(d+ex)\,dx=x^2\left(\frac{a\,e}{2}+\frac{c\,d}{2}\right)+a\,d\,x+\frac{2\,b\,d\,x^{3/2}}{3}+\frac{2\,b\,e\,x^{5/2}}{5}+\frac{c\,e\,x^3}{3}$$

input
```
int((d + e*x)*(a + c*x + b*x^(1/2)),x)
```

output
```
x^2*((a*e)/2 + (c*d)/2) + a*d*x + (2*b*d*x^(3/2))/3 + (2*b*e*x^(5/2))/5 +
(c*e*x^3)/3
```

Reduce [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.76

$$\int \left(a+b\sqrt{x}+cx\right)(d+ex)\,dx$$
$$=\frac{x\left(20\sqrt{x}\,bd+12\sqrt{x}\,bex+30ad+15aex+15cdx+10ce\,x^2\right)}{30}$$

input
```
int((a+b*x^(1/2)+c*x)*(e*x+d),x)
```

output
```
(x*(20*sqrt(x)*b*d + 12*sqrt(x)*b*e*x + 30*a*d + 15*a*e*x + 15*c*d*x + 10*
c*e*x**2))/30
```

3.108 $\int \left(a+b\sqrt{x}\right)^3 \left(d+e\sqrt{x}+fx\right) \, dx$

Optimal result

Integrand size = 24, antiderivative size = 116

$$\int \left(a+b\sqrt{x}\right)^3 \left(d+e\sqrt{x}+fx\right) \, dx = -\frac{a(b^2d-abe+a^2f)\left(a+b\sqrt{x}\right)^4}{2b^4} + \frac{2(b^2d-2abe+3a^2f)\left(a+b\sqrt{x}\right)^5}{5b^4} + \frac{(be-3af)\left(a+b\sqrt{x}\right)^6}{3b^4} + \frac{2f\left(a+b\sqrt{x}\right)^7}{7b^4}$$

output
```
-1/2*a*(a^2*f-a*b*e+b^2*d)*(a+b*x^(1/2))^4/b^4+2/5*(3*a^2*f-2*a*b*e+b^2*d)
*(a+b*x^(1/2))^5/b^4+1/3*(-3*a*f+b*e)*(a+b*x^(1/2))^6/b^4+2/7*f*(a+b*x^(1/
2))^7/b^4
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.89

$$\int \left(a+b\sqrt{x}\right)^3 \left(d+e\sqrt{x}+fx\right) \, dx = \frac{1}{210}x\left(35a^3\left(6d+4e\sqrt{x}+3fx\right) + 21ab^2x\left(15d+12e\sqrt{x}+10fx\right) + 21a^2b\sqrt{x}\left(20d+15e\sqrt{x}+12fx\right) + 2b^3x^{3/2}\left(42d+35e\sqrt{x}+30fx\right)\right)$$

input
```
Integrate[(a + b*Sqrt[x])^3*(d + e*Sqrt[x] + f*x),x]
```

output
```
(x*(35*a^3*(6*d + 4*e*Sqrt[x] + 3*f*x) + 21*a*b^2*x*(15*d + 12*e*Sqrt[x] +
 10*f*x) + 21*a^2*b*Sqrt[x]*(20*d + 15*e*Sqrt[x] + 12*f*x) + 2*b^3*x^(3/2)
*(42*d + 35*e*Sqrt[x] + 30*f*x)))/210
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1731, 1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\sqrt{x}\right)^3 \left(d+e\sqrt{x}+fx\right) \, dx$$

$$\Big\downarrow \text{1731}$$

$$2\int \left(a+b\sqrt{x}\right)^3 \sqrt{x}\left(d+fx+e\sqrt{x}\right)d\sqrt{x}$$

$$\Big\downarrow \text{1195}$$

$$2\int \left(\frac{f\left(a+b\sqrt{x}\right)^6}{b^3}+\frac{(be-3af)\left(a+b\sqrt{x}\right)^5}{b^3}+\frac{\left(3fa^2-2bea+b^2d\right)\left(a+b\sqrt{x}\right)^4}{b^3}-\frac{a\left(fa^2-bea+b^2d\right)\left(a+b\sqrt{}\right.}{b^3}\right.$$

$$\Big\downarrow \text{2009}$$

$$2\left(\frac{\left(a+b\sqrt{x}\right)^5\left(3a^2f-2abe+b^2d\right)}{5b^4}-\frac{a\left(a+b\sqrt{x}\right)^4\left(a^2f-abe+b^2d\right)}{4b^4}+\frac{\left(a+b\sqrt{x}\right)^6(be-3af)}{6b^4}+\frac{f\left(a+b\sqrt{x}\right)}{7b^4}\right.$$

input
```
Int[(a + b*Sqrt[x])^3*(d + e*Sqrt[x] + f*x),x]
```

output
```
2*(-1/4*(a*(b^2*d - a*b*e + a^2*f)*(a + b*Sqrt[x])^4)/b^4 + ((b^2*d - 2*a*
b*e + 3*a^2*f)*(a + b*Sqrt[x])^5)/(5*b^4) + ((b*e - 3*a*f)*(a + b*Sqrt[x])
^6)/(6*b^4) + (f*(a + b*Sqrt[x])^7)/(7*b^4))
```

Defintions of rubi rules used

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 1731
```
Int[((a_.) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (e_.)*(x_)^
(n_))^(q_.), x_Symbol] :> With[{g = Denominator[n]}, Simp[g   Subst[Int[x^(
g - 1)*(d + e*x^(g*n))^q*(a + b*x^(g*n) + c*x^(2*g*n))^p, x], x, x^(1/g)],
x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && EqQ[n2, 2*n] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.89

method	result
derivativedivides	$\frac{2b^3 f x^{\frac{7}{2}}}{7}+\frac{(3b^2af+b^3e)x^3}{3}+\frac{2(3ba^2f+3ab^2e+b^3d)x^{\frac{5}{2}}}{5}+\frac{(a^3f+3ba^2e+3ab^2d)x^2}{2}+\frac{2(a^3e+3a^2bd)x^{\frac{3}{2}}}{3}+a$
default	$\frac{b^3 e x^3}{3}+b^2\left(\frac{2bf x^{\frac{7}{2}}}{7}+\frac{2(3ae+bd)x^{\frac{5}{2}}}{5}\right)+b^2af x^3+\frac{(3ba^2e+3ab^2d)x^2}{2}+a^2\left(\frac{6bf x^{\frac{5}{2}}}{5}+\frac{2(ae+3bd)x^{\frac{3}{2}}}{3}\right.$
trager	$\frac{(6b^2af x^2+2b^3x^2e+3a^3fx+9a^2bex+9ab^2dx+6b^2afx+2b^3ex+6a^3d+3a^3f+9ba^2e+9ab^2d+6b^2af+2b^3e)(x-1)}{6}+$
orering	Expression too large to display

input
```
int((a+b*x^(1/2))^3*(d+e*x^(1/2)+f*x),x,method=_RETURNVERBOSE)
```

output
```
2/7*b^3*f*x^(7/2)+1/3*(3*a*b^2*f+b^3*e)*x^3+2/5*(3*a^2*b*f+3*a*b^2*e+b^3*d
)*x^(5/2)+1/2*(a^3*f+3*a^2*b*e+3*a*b^2*d)*x^2+2/3*(a^3*e+3*a^2*b*d)*x^(3/2
)+a^3*d*x
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.91

$$\int \left(a+b\sqrt{x}\right)^3\left(d+e\sqrt{x}+fx\right)\,dx = a^3dx+\frac{1}{3}\left(b^3e+3\,ab^2f\right)x^3+\frac{1}{2}\left(3\,ab^2d+3\,a^2be+a^3f\right)x^2 + \frac{2}{105}\left(15\,b^3fx^3+21\left(b^3d+3\,ab^2e+3\,a^2bf\right)x^2+35\left(3\,a^2bd+a^3e\right)x\right)\sqrt{x}$$

input
```
integrate((a+b*x^(1/2))^3*(d+e*x^(1/2)+f*x),x, algorithm="fricas")
```

output
```
a^3*d*x + 1/3*(b^3*e + 3*a*b^2*f)*x^3 + 1/2*(3*a*b^2*d + 3*a^2*b*e + a^3*f
)*x^2 + 2/105*(15*b^3*f*x^3 + 21*(b^3*d + 3*a*b^2*e + 3*a^2*b*f)*x^2 + 35*
(3*a^2*b*d + a^3*e)*x)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.01

$$\int \left(a+b\sqrt{x}\right)^3\left(d+e\sqrt{x}+fx\right)\,dx = a^3dx+\frac{2b^3fx^{\frac{7}{2}}}{7}+\frac{2x^{\frac{5}{2}}\cdot\left(3a^2bf+3ab^2e+b^3d\right)}{5} + \frac{2x^{\frac{3}{2}}\left(a^3e+3a^2bd\right)}{3}+\frac{x^3\cdot\left(3ab^2f+b^3e\right)}{3} + \frac{x^2\left(a^3f+3a^2be+3ab^2d\right)}{2}$$

input
```
integrate((a+b*x**(1/2))**3*(d+e*x**(1/2)+f*x),x)
```

output
```
a**3*d*x + 2*b**3*f*x**(7/2)/7 + 2*x**(5/2)*(3*a**2*b*f + 3*a*b**2*e + b**
3*d)/5 + 2*x**(3/2)*(a**3*e + 3*a**2*b*d)/3 + x**3*(3*a*b**2*f + b**3*e)/3
 + x**2*(a**3*f + 3*a**2*b*e + 3*a*b**2*d)/2
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.88

$$\int \left(a+b\sqrt{x}\right)^3\left(d+e\sqrt{x}+fx\right)\,dx = \frac{2}{7}\,b^3fx^{\frac{7}{2}} + a^3dx + \frac{1}{3}\left(b^3e+3\,ab^2f\right)x^3 + \frac{2}{5}\left(b^3d+3\,ab^2e+3\,a^2bf\right)x^{\frac{5}{2}} + \frac{1}{2}\left(3\,ab^2d+3\,a^2be+a^3f\right)x^2 + \frac{2}{3}\left(3\,a^2bd+a^3e\right)x^{\frac{3}{2}}$$

input
```
integrate((a+b*x^(1/2))^3*(d+e*x^(1/2)+f*x),x, algorithm="maxima")
```

output
```
2/7*b^3*f*x^(7/2) + a^3*d*x + 1/3*(b^3*e + 3*a*b^2*f)*x^3 + 2/5*(b^3*d + 3
*a*b^2*e + 3*a^2*b*f)*x^(5/2) + 1/2*(3*a*b^2*d + 3*a^2*b*e + a^3*f)*x^2 +
2/3*(3*a^2*b*d + a^3*e)*x^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.96

$$\int \left(a+b\sqrt{x}\right)^3\left(d+e\sqrt{x}+fx\right)\,dx = \frac{2}{7}\,b^3fx^{\frac{7}{2}} + \frac{1}{3}\,b^3ex^3 + ab^2fx^3 + \frac{2}{5}\,b^3dx^{\frac{5}{2}} + \frac{6}{5}\,ab^2ex^{\frac{5}{2}} + \frac{6}{5}\,a^2bfx^{\frac{5}{2}} + \frac{3}{2}\,ab^2dx^2 + \frac{3}{2}\,a^2bex^2 + \frac{1}{2}\,a^3fx^2 + 2\,a^2bdx^{\frac{3}{2}} + \frac{2}{3}\,a^3ex^{\frac{3}{2}} + a^3dx$$

input
```
integrate((a+b*x^(1/2))^3*(d+e*x^(1/2)+f*x),x, algorithm="giac")
```

output
```
2/7*b^3*f*x^(7/2) + 1/3*b^3*e*x^3 + a*b^2*f*x^3 + 2/5*b^3*d*x^(5/2) + 6/5*
a*b^2*e*x^(5/2) + 6/5*a^2*b*f*x^(5/2) + 3/2*a*b^2*d*x^2 + 3/2*a^2*b*e*x^2
+ 1/2*a^3*f*x^2 + 2*a^2*b*d*x^(3/2) + 2/3*a^3*e*x^(3/2) + a^3*d*x
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.87

$$\int \left(a+b\sqrt{x}\right)^3\left(d+e\sqrt{x}+fx\right)\,dx = x^{3/2}\left(\frac{2\,e\,a^3}{3}+2\,b\,d\,a^2\right)+x^3\left(\frac{e\,b^3}{3}+a\,f\,b^2\right)+x^2\left(\frac{f\,a^3}{2}+\frac{3\,e\,a^2\,b}{2}+\frac{3\,d\,a\,b^2}{2}\right)+x^{5/2}\left(\frac{6\,f\,a^2\,b}{5}+\frac{6\,e\,a\,b^2}{5}+\frac{2\,d\,b^3}{5}\right)+\frac{2\,b^3\,f\,x^{7/2}}{7}+a^3\,d\,x$$

input
```
int((a + b*x^(1/2))^3*(d + f*x + e*x^(1/2)),x)
```

output
```
x^(3/2)*((2*a^3*e)/3 + 2*a^2*b*d) + x^3*((b^3*e)/3 + a*b^2*f) + x^2*((a^3*
f)/2 + (3*a*b^2*d)/2 + (3*a^2*b*e)/2) + x^(5/2)*((2*b^3*d)/5 + (6*a*b^2*e)
/5 + (6*a^2*b*f)/5) + (2*b^3*f*x^(7/2))/7 + a^3*d*x
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.94

$$\int \left(a+b\sqrt{x}\right)^3\left(d+e\sqrt{x}+fx\right)\,dx = \frac{x\left(140\sqrt{x}\,a^3e+420\sqrt{x}\,a^2bd+252\sqrt{x}\,a^2bfx+252\sqrt{x}\,a\,b^2ex+84\sqrt{x}\,b^3dx+60\sqrt{x}\,b^3f\,x^2+210a^3d+10\right.}{210}$$

input
```
int((a+b*x^(1/2))^3*(d+e*x^(1/2)+f*x),x)
```

output
```
(x*(140*sqrt(x)*a**3*e + 420*sqrt(x)*a**2*b*d + 252*sqrt(x)*a**2*b*f*x + 2
52*sqrt(x)*a*b**2*e*x + 84*sqrt(x)*b**3*d*x + 60*sqrt(x)*b**3*f*x**2 + 210
*a**3*d + 105*a**3*f*x + 315*a**2*b*e*x + 315*a*b**2*d*x + 210*a*b**2*f*x*
*2 + 70*b**3*e*x**2))/210
```

3.109 $\int \left(a+b\sqrt{x}\right)^2 \left(d+e\sqrt{x}+fx\right) dx$

Optimal result

Integrand size = 24, antiderivative size = 77

$$\int \left(a+b\sqrt{x}\right)^2 \left(d+e\sqrt{x}+fx\right) dx = a^2dx+\frac{2}{3}a(2bd+ae)x^{3/2}+\frac{1}{2}\left(b^2d+2abe+a^2f\right)x^2 + \frac{2}{5}b(be+2af)x^{5/2}+\frac{1}{3}b^2fx^3$$

output
```
a^2*d*x+2/3*a*(a*e+2*b*d)*x^(3/2)+1/2*(a^2*f+2*a*b*e+b^2*d)*x^2+2/5*b*(2*a
*f+b*e)*x^(5/2)+1/3*b^2*f*x^3
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.95

$$\int \left(a+b\sqrt{x}\right)^2 \left(d+e\sqrt{x}+fx\right) dx = \frac{1}{30}x\left(5a^2\left(6d+4e\sqrt{x}+3fx\right) + b^2x\left(15d+12e\sqrt{x}+10fx\right) + 2ab\sqrt{x}\left(20d+15e\sqrt{x}+12fx\right)\right)$$

input
```
Integrate[(a + b*Sqrt[x])^2*(d + e*Sqrt[x] + f*x),x]
```

output
```
(x*(5*a^2*(6*d + 4*e*Sqrt[x] + 3*f*x) + b^2*x*(15*d + 12*e*Sqrt[x] + 10*f*
x) + 2*a*b*Sqrt[x]*(20*d + 15*e*Sqrt[x] + 12*f*x)))/30
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1731, 1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\sqrt{x}\right)^2 \left(d+e\sqrt{x}+fx\right) dx$$

$$\Big\downarrow \text{1731}$$

$$2\int \left(a+b\sqrt{x}\right)^2 \sqrt{x}\left(d+fx+e\sqrt{x}\right) d\sqrt{x}$$

$$\Big\downarrow \text{1195}$$

$$2\int \left(b^2fx^{5/2}+b(be+2af)x^2+\left(fa^2+2bea+b^2d\right)x^{3/2}+a(2bd+ae)x+a^2d\sqrt{x}\right) d\sqrt{x}$$

$$\Big\downarrow \text{2009}$$

$$2\left(\frac{1}{4}x^2\left(a^2f+2abe+b^2d\right)+\frac{1}{2}a^2dx+\frac{1}{3}ax^{3/2}(ae+2bd)+\frac{1}{5}bx^{5/2}(2af+be)+\frac{1}{6}b^2fx^3\right)$$

input
```
Int[(a + b*Sqrt[x])^2*(d + e*Sqrt[x] + f*x),x]
```

output
```
2*((a^2*d*x)/2 + (a*(2*b*d + a*e)*x^(3/2))/3 + ((b^2*d + 2*a*b*e + a^2*f)*
x^2)/4 + (b*(b*e + 2*a*f)*x^(5/2))/5 + (b^2*f*x^3)/6)
```

Defintions of rubi rules used

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 1731
```
Int[((a_.) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (e_.)*(x_)^
(n_))^(q_.), x_Symbol] :> With[{g = Denominator[n]}, Simp[g   Subst[Int[x^(
g - 1)*(d + e*x^(g*n))^q*(a + b*x^(g*n) + c*x^(2*g*n))^p, x], x, x^(1/g)],
x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && EqQ[n2, 2*n] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.91

method	result
derivativedivides	$\frac{b^2x^3f}{3}+\frac{2(2abf+b^2e)x^{\frac{5}{2}}}{5}+\frac{(a^2f+2abe+d\,b^2)x^2}{2}+\frac{2(a^2e+2abd)x^{\frac{3}{2}}}{3}+a^2dx$
default	$\frac{2x^{\frac{5}{2}}b^2e}{5}+\frac{b^2x^3f}{3}+\frac{(2abe+d\,b^2)x^2}{2}+a\left(\frac{4bf\,x^{\frac{5}{2}}}{5}+\frac{2(ae+2bd)x^{\frac{3}{2}}}{3}\right)+a^2\left(\frac{1}{2}f\,x^2+dx\right)$
trager	$\frac{(2b^2f\,x^2+3a^2fx+6abex+3b^2dx+2b^2fx+6a^2d+3a^2f+6abe+3d\,b^2+2b^2f)(x-1)}{6}+\frac{2x^{\frac{3}{2}}(6bafx+3b^2ex+5a^2e+10ab}{15}$
orering	$-\frac{(72x^4b^4f^3a^2+72x^4b^5f^2ae+18b^6e^2f\,x^4-84x^3b^2f^3a^4-112x^3b^3f^2a^3e+196x^3b^4f^2a^2d-119x^3b^4f\,a^2e^2+56x^3b^5fad}{30(-2a\,b^3f^2x^3-b^4ef\,x^3+2a^3b\,f}$

input
```
int((a+b*x^(1/2))^2*(d+e*x^(1/2)+f*x),x,method=_RETURNVERBOSE)
```

output
```
1/3*b^2*x^3*f+2/5*(2*a*b*f+b^2*e)*x^(5/2)+1/2*(a^2*f+2*a*b*e+b^2*d)*x^2+2/
3*(a^2*e+2*a*b*d)*x^(3/2)+a^2*d*x
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.95

$$\int \left(a+b\sqrt{x}\right)^2\left(d+e\sqrt{x}+fx\right)\,dx=\frac{1}{3}\,b^2fx^3+a^2dx+\frac{1}{2}\left(b^2d+2\,abe+a^2f\right)x^2+\frac{2}{15}\left(3\left(b^2e+2\,abf\right)x^2+5\left(2\,abd+a^2e\right)x\right)\sqrt{x}$$

input
```
integrate((a+b*x^(1/2))^2*(d+e*x^(1/2)+f*x),x, algorithm="fricas")
```

output
```
1/3*b^2*f*x^3 + a^2*d*x + 1/2*(b^2*d + 2*a*b*e + a^2*f)*x^2 + 2/15*(3*(b^2
*e + 2*a*b*f)*x^2 + 5*(2*a*b*d + a^2*e)*x)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.29

$$\int \left(a+b\sqrt{x}\right)^2\left(d+e\sqrt{x}+fx\right)\,dx=a^2dx+\frac{2a^2ex^{\frac{3}{2}}}{3}+\frac{a^2fx^2}{2}+\frac{4abdx^{\frac{3}{2}}}{3}+abex^2+\frac{4abfx^{\frac{5}{2}}}{5}+\frac{b^2dx^2}{2}+\frac{2b^2ex^{\frac{5}{2}}}{5}+\frac{b^2fx^3}{3}$$

input
```
integrate((a+b*x**(1/2))**2*(d+e*x**(1/2)+f*x),x)
```

output
```
a**2*d*x + 2*a**2*e*x**(3/2)/3 + a**2*f*x**2/2 + 4*a*b*d*x**(3/2)/3 + a*b*
e*x**2 + 4*a*b*f*x**(5/2)/5 + b**2*d*x**2/2 + 2*b**2*e*x**(5/2)/5 + b**2*f
*x**3/3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.90

$$\int \left(a+b\sqrt{x}\right)^2 \left(d+e\sqrt{x}+fx\right)\,dx = \frac{1}{3}\,b^2fx^3+a^2dx+\frac{2}{5}\left(b^2e+2\,abf\right)x^{\frac{5}{2}} + \frac{1}{2}\left(b^2d+2\,abe+a^2f\right)x^2+\frac{2}{3}\left(2\,abd+a^2e\right)x^{\frac{3}{2}}$$

input
```
integrate((a+b*x^(1/2))^2*(d+e*x^(1/2)+f*x),x, algorithm="maxima")
```

output
```
1/3*b^2*f*x^3 + a^2*d*x + 2/5*(b^2*e + 2*a*b*f)*x^(5/2) + 1/2*(b^2*d + 2*a
*b*e + a^2*f)*x^2 + 2/3*(2*a*b*d + a^2*e)*x^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.97

$$\int \left(a+b\sqrt{x}\right)^2 \left(d+e\sqrt{x}+fx\right)\,dx = \frac{1}{3}\,b^2fx^3+\frac{2}{5}\,b^2ex^{\frac{5}{2}}+\frac{4}{5}\,abfx^{\frac{5}{2}}+\frac{1}{2}\,b^2dx^2 + abex^2+\frac{1}{2}\,a^2fx^2+\frac{4}{3}\,abdx^{\frac{3}{2}}+\frac{2}{3}\,a^2ex^{\frac{3}{2}}+a^2dx$$

input
```
integrate((a+b*x^(1/2))^2*(d+e*x^(1/2)+f*x),x, algorithm="giac")
```

output
```
1/3*b^2*f*x^3 + 2/5*b^2*e*x^(5/2) + 4/5*a*b*f*x^(5/2) + 1/2*b^2*d*x^2 + a*
b*e*x^2 + 1/2*a^2*f*x^2 + 4/3*a*b*d*x^(3/2) + 2/3*a^2*e*x^(3/2) + a^2*d*x
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.90

$$\int \left(a+b\sqrt{x}\right)^2 \left(d+e\sqrt{x}+fx\right)\,dx = x^2\left(\frac{f\,a^2}{2}+e\,a\,b+\frac{d\,b^2}{2}\right)+x^{3/2}\left(\frac{2\,e\,a^2}{3}+\frac{4\,b\,d\,a}{3}\right) + x^{5/2}\left(\frac{2\,e\,b^2}{5}+\frac{4\,a\,f\,b}{5}\right)+\frac{b^2\,f\,x^3}{3}+a^2\,d\,x$$

input
```
int((a + b*x^(1/2))^2*(d + f*x + e*x^(1/2)),x)
```

output
```
x^2*((b^2*d)/2 + (a^2*f)/2 + a*b*e) + x^(3/2)*((2*a^2*e)/3 + (4*a*b*d)/3)
+ x^(5/2)*((2*b^2*e)/5 + (4*a*b*f)/5) + (b^2*f*x^3)/3 + a^2*d*x
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.92

$$\int \left(a+b\sqrt{x}\right)^2 \left(d+e\sqrt{x}+fx\right)\,dx$$
$$=\frac{x\left(20\sqrt{x}\,a^2e+40\sqrt{x}\,abd+24\sqrt{x}\,abfx+12\sqrt{x}\,b^2ex+30a^2d+15a^2fx+30abex+15b^2dx+10b^2f\,x^2\right)}{30}$$

input
```
int((a+b*x^(1/2))^2*(d+e*x^(1/2)+f*x),x)
```

output
```
(x*(20*sqrt(x)*a**2*e + 40*sqrt(x)*a*b*d + 24*sqrt(x)*a*b*f*x + 12*sqrt(x)
*b**2*e*x + 30*a**2*d + 15*a**2*f*x + 30*a*b*e*x + 15*b**2*d*x + 10*b**2*f
*x**2))/30
```

3.110 $\int \left(a+b\sqrt{x}\right) \left(d+e\sqrt{x}+fx\right) \, dx$

Optimal result

Integrand size = 22, antiderivative size = 46

$$\int \left(a+b\sqrt{x}\right) \left(d+e\sqrt{x}+fx\right) \, dx = adx+\frac{2}{3}(bd+ae)x^{3/2}+\frac{1}{2}(be+af)x^2+\frac{2}{5}bfx^{5/2}$$

output
```
a*d*x+2/3*(a*e+b*d)*x^(3/2)+1/2*(a*f+b*e)*x^2+2/5*b*f*x^(5/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.02

$$\int \left(a+b\sqrt{x}\right) \left(d+e\sqrt{x}+fx\right) \, dx = \frac{1}{30}x\left(30ad+20bd\sqrt{x}+20ae\sqrt{x}+15bex+15afx+12bfx^{3/2}\right)$$

input
```
Integrate[(a + b*Sqrt[x])*(d + e*Sqrt[x] + f*x),x]
```

output
```
(x*(30*a*d + 20*b*d*Sqrt[x] + 20*a*e*Sqrt[x] + 15*b*e*x + 15*a*f*x + 12*b*
f*x^(3/2)))/30
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1731, 1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\sqrt{x}\right)\left(d+e\sqrt{x}+fx\right)dx$$

$$\Big\downarrow \; 1731$$

$$2\int \left(a+b\sqrt{x}\right)\sqrt{x}\left(d+fx+e\sqrt{x}\right)d\sqrt{x}$$

$$\Big\downarrow \; 1195$$

$$2\int \left(bfx^2+(be+af)x^{3/2}+(bd+ae)x+ad\sqrt{x}\right)d\sqrt{x}$$

$$\Big\downarrow \; 2009$$

$$2\left(\frac{1}{3}x^{3/2}(ae+bd)+\frac{1}{4}x^2(af+be)+\frac{adx}{2}+\frac{1}{5}bfx^{5/2}\right)$$

input
```
Int[(a + b*Sqrt[x])*(d + e*Sqrt[x] + f*x),x]
```

output
```
2*((a*d*x)/2 + ((b*d + a*e)*x^(3/2))/3 + ((b*e + a*f)*x^2)/4 + (b*f*x^(5/2
))/5)
```

Defintions of rubi rules used

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 1731
```
Int[((a_.) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (e_.)*(x_)^
(n_))^(q_.), x_Symbol] :> With[{g = Denominator[n]}, Simp[g   Subst[Int[x^(
g - 1)*(d + e*x^(g*n))^q*(a + b*x^(g*n) + c*x^(2*g*n))^p, x], x, x^(1/g)],
x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && EqQ[n2, 2*n] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

method	result
derivativedivides	$adx+\frac{2(ae+bd)x^{\frac{3}{2}}}{3}+\frac{(af+eb)x^{2}}{2}+\frac{2bf\,x^{\frac{5}{2}}}{5}$
default	$\frac{x^{2}eb}{2}+\frac{2bf\,x^{\frac{5}{2}}}{5}+\frac{2(ae+bd)x^{\frac{3}{2}}}{3}+a\left(\frac{1}{2}f\,x^{2}+dx\right)$
trager	$\frac{(x-1)(afx+xeb+2ad+af+eb)}{2}+\frac{2x^{\frac{3}{2}}(3bfx+5ae+5bd)}{15}$
orering	$-\frac{(-21a\,b^{2}f^{3}x^{3}-21b^{3}x^{3}f^{2}e+25a^{2}be\,f^{2}x^{2}-65a\,b^{2}d\,f^{2}x^{2}+25a\,b^{2}e^{2}f\,x^{2}+25b^{3}def\,x^{2}+10a^{3}d\,e^{2}+20a^{2}b\,d^{2}e+10a\,b^{2}d^{3}}{30bf(ab\,f^{2}x^{2}+b^{2}x^{2}ef-a^{2}efx+2abdfx-ab\,e^{2}x-b^{2}dex+a^{2}de+ab\,d^{2})}$

input
```
int((a+b*x^(1/2))*(d+e*x^(1/2)+f*x),x,method=_RETURNVERBOSE)
```

output
```
a*d*x+2/3*(a*e+b*d)*x^(3/2)+1/2*(a*f+b*e)*x^2+2/5*b*f*x^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.87

$$\int \left(a+b\sqrt{x}\right)\left(d+e\sqrt{x}+fx\right)\,dx=adx+\frac{1}{2}\,(be+af)x^{2}+\frac{2}{15}\,\left(3\,bfx^{2}+5\,(bd+ae)x\right)\sqrt{x}$$

input
```
integrate((a+b*x^(1/2))*(d+e*x^(1/2)+f*x),x, algorithm="fricas")
```

output
```
a*d*x + 1/2*(b*e + a*f)*x^2 + 2/15*(3*b*f*x^2 + 5*(b*d + a*e)*x)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.91

$$\int \left(a+b\sqrt{x}\right)\left(d+e\sqrt{x}+fx\right)\,dx=adx+\frac{2bfx^{\frac{5}{2}}}{5}+\frac{2x^{\frac{3}{2}}(ae+bd)}{3}+\frac{x^2(af+be)}{2}$$

input
```
integrate((a+b*x**(1/2))*(d+e*x**(1/2)+f*x),x)
```

output
```
a*d*x + 2*b*f*x**(5/2)/5 + 2*x**(3/2)*(a*e + b*d)/3 + x**2*(a*f + b*e)/2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \left(a+b\sqrt{x}\right)\left(d+e\sqrt{x}+fx\right)\,dx=\frac{2}{5}\,bfx^{\frac{5}{2}}+adx+\frac{1}{2}\,(be+af)x^2+\frac{2}{3}\,(bd+ae)x^{\frac{3}{2}}$$

input
```
integrate((a+b*x^(1/2))*(d+e*x^(1/2)+f*x),x, algorithm="maxima")
```

output
```
2/5*b*f*x^(5/2) + a*d*x + 1/2*(b*e + a*f)*x^2 + 2/3*(b*d + a*e)*x^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.87

$$\int \left(a+b\sqrt{x}\right)\left(d+e\sqrt{x}+fx\right)\,dx=\frac{2}{5}\,bfx^{\frac{5}{2}}+\frac{1}{2}\,bex^2+\frac{1}{2}\,afx^2+\frac{2}{3}\,bdx^{\frac{3}{2}}+\frac{2}{3}\,aex^{\frac{3}{2}}+adx$$

input
```
integrate((a+b*x^(1/2))*(d+e*x^(1/2)+f*x),x, algorithm="giac")
```

output
```
2/5*b*f*x^(5/2) + 1/2*b*e*x^2 + 1/2*a*f*x^2 + 2/3*b*d*x^(3/2) + 2/3*a*e*x^
(3/2) + a*d*x
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.83

$$\int \left(a+b\sqrt{x}\right)\left(d+e\sqrt{x}+fx\right)\,dx = x^{3/2}\left(\frac{2\,a\,e}{3}+\frac{2\,b\,d}{3}\right)+x^2\left(\frac{a\,f}{2}+\frac{b\,e}{2}\right)+a\,d\,x+\frac{2\,b\,f\,x^{5/2}}{5}$$

input
```
int((a + b*x^(1/2))*(d + f*x + e*x^(1/2)),x)
```

output
```
x^(3/2)*((2*a*e)/3 + (2*b*d)/3) + x^2*((a*f)/2 + (b*e)/2) + a*d*x + (2*b*f
*x^(5/2))/5
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

$$\int \left(a+b\sqrt{x}\right)\left(d+e\sqrt{x}+fx\right)\,dx = \frac{x\left(20\sqrt{x}\,ae+20\sqrt{x}\,bd+12\sqrt{x}\,bfx+30ad+15afx+15bex\right)}{30}$$

input
```
int((a+b*x^(1/2))*(d+e*x^(1/2)+f*x),x)
```

output
```
(x*(20*sqrt(x)*a*e + 20*sqrt(x)*b*d + 12*sqrt(x)*b*f*x + 30*a*d + 15*a*f*x
 + 15*b*e*x))/30
```

3.111 $\int \left(a+b\sqrt{x}+cx\right)^3 \left(d+e\sqrt{x}+fx\right) \, dx$

Optimal result

Integrand size = 27, antiderivative size = 242

$$\begin{aligned}\int \left(a+b\sqrt{x}+cx\right)^3 \left(d+e\sqrt{x}+fx\right) \, dx &= a^3dx+\frac{2}{3}a^2(3bd+ae)x^{3/2}\\ &+\frac{1}{2}a\left(3b^2d+3abe+a(3cd+af)\right)x^2+\frac{2}{5}\left(b^3d+3ab^2e+3a^2ce+3ab(2cd+af)\right)x^{5/2}\\ &+\frac{1}{3}\left(b^3e+6abce+3b^2(cd+af)+3ac(cd+af)\right)x^3\\ &+\frac{2}{7}\left(3b^2ce+3ac^2e+b^3f+3bc(cd+2af)\right)x^{7/2}\\ &+\frac{1}{4}c\left(c^2d+3b^2f+3c(be+af)\right)x^4+\frac{2}{9}c^2(ce+3bf)x^{9/2}+\frac{1}{5}c^3fx^5\end{aligned}$$

output
```
a^3*d*x+2/3*a^2*(a*e+3*b*d)*x^(3/2)+1/2*a*(3*b^2*d+3*a*b*e+a*(a*f+3*c*d))*
x^2+2/5*(b^3*d+3*a*b^2*e+3*a^2*c*e+3*a*b*(a*f+2*c*d))*x^(5/2)+1/3*(b^3*e+6
*a*b*c*e+3*b^2*(a*f+c*d)+3*a*c*(a*f+c*d))*x^3+2/7*(3*b^2*c*e+3*a*c^2*e+b^3
*f+3*b*c*(2*a*f+c*d))*x^(7/2)+1/4*c*(c^2*d+3*b^2*f+3*c*(a*f+b*e))*x^4+2/9*
c^2*(3*b*f+c*e)*x^(9/2)+1/5*c^3*f*x^5
```

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.12

$$\int \left(a+b\sqrt{x}+cx\right)^3 \left(d+e\sqrt{x}+fx\right)\,dx=\frac{1}{6}a^3x\left(6d+4e\sqrt{x}+3fx\right)+\frac{1}{180}c^3x^4\left(45d+40e\sqrt{x}+36fx\right)+\frac{1}{84}bc^2x^{7/2}\left(72d+63e\sqrt{x}+56fx\right)+b^3\left(\frac{2}{5}dx^{5/2}+\frac{ex^3}{3}+\frac{2}{7}fx^{7/2}\right)+b^2c\left(dx^3+\frac{6}{7}ex^{7/2}+\frac{3fx^4}{4}\right)+\frac{1}{10}a^2\left(cx^2\left(15d+12e\sqrt{x}+10fx\right)+bx^{3/2}\left(20d+15e\sqrt{x}+12fx\right)\right)$$

input
```
Integrate[(a + b*Sqrt[x] + c*x)^3*(d + e*Sqrt[x] + f*x),x]
```

output
```
(a^3*x*(6*d + 4*e*Sqrt[x] + 3*f*x))/6 + (c^3*x^4*(45*d + 40*e*Sqrt[x] + 36
*f*x))/180 + (b*c^2*x^(7/2)*(72*d + 63*e*Sqrt[x] + 56*f*x))/84 + b^3*((2*d
*x^(5/2))/5 + (e*x^3)/3 + (2*f*x^(7/2))/7) + b^2*c*(d*x^3 + (6*e*x^(7/2))/
7 + (3*f*x^4)/4) + (a^2*(c*x^2*(15*d + 12*e*Sqrt[x] + 10*f*x) + b*x^(3/2)*
(20*d + 15*e*Sqrt[x] + 12*f*x)))/10 + (a*x^2*(14*b^2*(15*d + 12*e*Sqrt[x]
+ 10*f*x) + 5*c^2*x*(28*d + 24*e*Sqrt[x] + 21*f*x) + 8*b*c*Sqrt[x]*(42*d +
 35*e*Sqrt[x] + 30*f*x)))/140
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\sqrt{x}+cx\right)^3 \left(d+e\sqrt{x}+fx\right)\,dx$$

$$\Big\downarrow \text{ 2329}$$

$$\int \left(a+b\sqrt{x}+cx\right)^3 \left(d+e\sqrt{x}+fx\right)dx$$

input
```
Int[(a + b*Sqrt[x] + c*x)^3*(d + e*Sqrt[x] + f*x),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2329
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_.), x_Symbol] :>
 Unintegrable[Pq*(a + b*x^n + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, n, p}, x]
 && EqQ[n2, 2*n] && (PolyQ[Pq, x] || PolyQ[Pq, x^n])
```

Maple [A] (verified)

Time = 8.34 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.12

method	result
default	$\frac{b^{3} e \,x^{3}}{3}+b^{2} \left(\frac{2 (b f +3 c e) x^{\frac{7}{2}}}{7}+\frac{2 (3 a e +b d) x^{\frac{5}{2}}}{5}\right)+\frac{(3 b^{2} c f +3 b \,c^{2} e) x^{4}}{4}+\frac{(3 b^{2} a f +6 a b c e +3 b^{2} c d) x^{3}}{3}+\frac{(3 b \,a^{2} e +$
derivativedivides	$\frac{c^{3} f \,x^{5}}{5}+\frac{2 \left(3 b \,c^{2} f +e \,c^{3}\right) x^{\frac{9}{2}}}{9}+\frac{\left(\left(a \,c^{2}+2 c \,b^{2}+c \left(2 a c +b^{2}\right)\right) f +3 b \,c^{2} e +c^{3} d \right) x^{4}}{4}+\frac{2 \left(\left(4 a b c +b \left(2 a c +b^{2}\right)\right) f +\left(a \,c^{2}+2 c \,b\right.}{7}$
trager	$\left(12 c^{3} f \,x^{4}+45 a \,c^{2} f \,x^{3}+45 b^{2} c f \,x^{3}+45 x^{3} b \,c^{2} e +15 c^{3} d \,x^{3}+12 c^{3} f \,x^{3}+60 a^{2} c f \,x^{2}+60 b^{2} a f \,x^{2}+120 x^{2} a b c e +60 a \,c^{2} d \,x^{2}+4\right.$
orering	Expression too large to display

input
```
int((a+b*x^(1/2)+c*x)^3*(d+e*x^(1/2)+f*x),x,method=_RETURNVERBOSE)
```

output
```
1/3*b^3*e*x^3+b^2*(2/7*(b*f+3*c*e)*x^(7/2)+2/5*(3*a*e+b*d)*x^(5/2))+1/4*(3
*b^2*c*f+3*b*c^2*e)*x^4+1/3*(3*a*b^2*f+6*a*b*c*e+3*b^2*c*d)*x^3+1/2*(3*a^2
*b*e+3*a*b^2*d)*x^2+2/9*c^2*(3*b*f+c*e)*x^(9/2)+2/7*(2*a*c*(3*b*f+c*e)+c^2
*(a*e+3*b*d))*x^(7/2)+2/5*(a^2*(3*b*f+c*e)+2*a*c*(a*e+3*b*d))*x^(5/2)+2/3*
a^2*(a*e+3*b*d)*x^(3/2)+1/5*c^3*f*x^5+1/4*(3*a*c^2*f+c^3*d)*x^4+1/3*(3*a^2
*c*f+3*a*c^2*d)*x^3+1/2*(a^3*f+3*a^2*c*d)*x^2+a^3*d*x
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.00

$$\int \left(a+b\sqrt{x}+cx\right)^3\left(d+e\sqrt{x}+fx\right)\,dx$$
$$=\frac{1}{5}c^3fx^5+a^3dx+\frac{1}{4}\left(c^3d+3\,bc^2e+3\left(b^2c+ac^2\right)f\right)x^4$$
$$+\frac{1}{3}\left(3\left(b^2c+ac^2\right)d+\left(b^3+6\,abc\right)e+3\left(ab^2+a^2c\right)f\right)x^3$$
$$+\frac{1}{2}\left(3\,a^2be+a^3f+3\left(ab^2+a^2c\right)d\right)x^2$$
$$+\frac{2}{315}\big(35\,(c^3e+3\,bc^2f)x^4+45\,\big(3\,bc^2d+3\,(b^2c+ac^2)e+(b^3+6\,abc)f\big)x^3+63\,\big(3\,a^2bf+(b^3+6\,abc)$$

input
```
integrate((a+b*x^(1/2)+c*x)^3*(d+e*x^(1/2)+f*x),x, algorithm="fricas")
```

output
```
1/5*c^3*f*x^5 + a^3*d*x + 1/4*(c^3*d + 3*b*c^2*e + 3*(b^2*c + a*c^2)*f)*x^
4 + 1/3*(3*(b^2*c + a*c^2)*d + (b^3 + 6*a*b*c)*e + 3*(a*b^2 + a^2*c)*f)*x^
3 + 1/2*(3*a^2*b*e + a^3*f + 3*(a*b^2 + a^2*c)*d)*x^2 + 2/315*(35*(c^3*e +
 3*b*c^2*f)*x^4 + 45*(3*b*c^2*d + 3*(b^2*c + a*c^2)*e + (b^3 + 6*a*b*c)*f)
*x^3 + 63*(3*a^2*b*f + (b^3 + 6*a*b*c)*d + 3*(a*b^2 + a^2*c)*e)*x^2 + 105*
(3*a^2*b*d + a^3*e)*x)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 386, normalized size of antiderivative = 1.60

$$\int \left(a+b\sqrt{x}+cx\right)^3 \left(d+e\sqrt{x}+fx\right) \, dx = a^3dx + \frac{2a^3ex^{\frac{3}{2}}}{3} + \frac{a^3fx^2}{2} + 2a^2bdx^{\frac{3}{2}} + \frac{3a^2bex^2}{2} + \frac{6a^2bfx^{\frac{5}{2}}}{5} + \frac{3a^2cdx^2}{2} + \frac{6a^2cex^{\frac{5}{2}}}{5} + a^2cfx^3 + \frac{3ab^2dx^2}{2} + \frac{6ab^2ex^{\frac{5}{2}}}{5} + ab^2fx^3 + \frac{12abcdx^{\frac{5}{2}}}{5} + 2abcex^3 + \frac{12abcfx^{\frac{7}{2}}}{7} + ac^2dx^3 + \frac{6ac^2ex^{\frac{7}{2}}}{7} + \frac{3ac^2fx^4}{4} + \frac{2b^3dx^{\frac{5}{2}}}{5} + \frac{b^3ex^3}{3} + \frac{2b^3fx^{\frac{7}{2}}}{7} + b^2cdx^3 + \frac{6b^2cex^{\frac{7}{2}}}{7} + \frac{3b^2cfx^4}{4} + \frac{6bc^2dx^{\frac{7}{2}}}{7} + \frac{3bc^2ex^4}{4} + \frac{2bc^2fx^{\frac{9}{2}}}{3} + \frac{c^3dx^4}{4} + \frac{2c^3ex^{\frac{9}{2}}}{9} + \frac{c^3fx^5}{5}$$

input

```
integrate((a+b*x**(1/2)+c*x)**3*(d+e*x**(1/2)+f*x),x)
```

output

```
a**3*d*x + 2*a**3*e*x**(3/2)/3 + a**3*f*x**2/2 + 2*a**2*b*d*x**(3/2) + 3*a
**2*b*e*x**2/2 + 6*a**2*b*f*x**(5/2)/5 + 3*a**2*c*d*x**2/2 + 6*a**2*c*e*x*
*(5/2)/5 + a**2*c*f*x**3 + 3*a*b**2*d*x**2/2 + 6*a*b**2*e*x**(5/2)/5 + a*b
**2*f*x**3 + 12*a*b*c*d*x**(5/2)/5 + 2*a*b*c*e*x**3 + 12*a*b*c*f*x**(7/2)/
7 + a*c**2*d*x**3 + 6*a*c**2*e*x**(7/2)/7 + 3*a*c**2*f*x**4/4 + 2*b**3*d*x
**(5/2)/5 + b**3*e*x**3/3 + 2*b**3*f*x**(7/2)/7 + b**2*c*d*x**3 + 6*b**2*c
*e*x**(7/2)/7 + 3*b**2*c*f*x**4/4 + 6*b*c**2*d*x**(7/2)/7 + 3*b*c**2*e*x**
4/4 + 2*b*c**2*f*x**(9/2)/3 + c**3*d*x**4/4 + 2*c**3*e*x**(9/2)/9 + c**3*f
*x**5/5
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.98

$$\int \left(a+b\sqrt{x}+cx\right)^3 \left(d+e\sqrt{x}+fx\right)\,dx = \frac{1}{5}c^3fx^5+\frac{2}{9}\left(c^3e+3\,bc^2f\right)x^{\frac{9}{2}}+a^3dx+\frac{1}{4}\left(c^3d+3\,bc^2e+3\left(b^2c+ac^2\right)f\right)x^4 + \frac{2}{7}\left(3\,bc^2d+3\left(b^2c+ac^2\right)e+\left(b^3+6\,abc\right)f\right)x^{\frac{7}{2}} + \frac{1}{3}\left(3\left(b^2c+ac^2\right)d+\left(b^3+6\,abc\right)e+3\left(ab^2+a^2c\right)f\right)x^3 + \frac{2}{5}\left(3\,a^2bf+\left(b^3+6\,abc\right)d+3\left(ab^2+a^2c\right)e\right)x^{\frac{5}{2}} + \frac{1}{2}\left(3\,a^2be+a^3f+3\left(ab^2+a^2c\right)d\right)x^2+\frac{2}{3}\left(3\,a^2bd+a^3e\right)x^{\frac{3}{2}}$$

input
```
integrate((a+b*x^(1/2)+c*x)^3*(d+e*x^(1/2)+f*x),x, algorithm="maxima")
```

output
```
1/5*c^3*f*x^5 + 2/9*(c^3*e + 3*b*c^2*f)*x^(9/2) + a^3*d*x + 1/4*(c^3*d + 3
*b*c^2*e + 3*(b^2*c + a*c^2)*f)*x^4 + 2/7*(3*b*c^2*d + 3*(b^2*c + a*c^2)*e
 + (b^3 + 6*a*b*c)*f)*x^(7/2) + 1/3*(3*(b^2*c + a*c^2)*d + (b^3 + 6*a*b*c)
*e + 3*(a*b^2 + a^2*c)*f)*x^3 + 2/5*(3*a^2*b*f + (b^3 + 6*a*b*c)*d + 3*(a*
b^2 + a^2*c)*e)*x^(5/2) + 1/2*(3*a^2*b*e + a^3*f + 3*(a*b^2 + a^2*c)*d)*x^
2 + 2/3*(3*a^2*b*d + a^3*e)*x^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.17

$$\int \left(a+b\sqrt{x}+cx\right)^3 \left(d+e\sqrt{x}+fx\right)\,dx = \frac{1}{5}\,c^3fx^5+\frac{2}{9}\,c^3ex^{\frac{9}{2}}+\frac{2}{3}\,bc^2fx^{\frac{9}{2}}+\frac{1}{4}\,c^3dx^4+\frac{3}{4}\,bc^2ex^4+\frac{3}{4}\,b^2cfx^4+\frac{3}{4}\,ac^2fx^4+\frac{6}{7}\,bc^2dx^{\frac{7}{2}}+\frac{6}{7}\,b^2cex^{\frac{7}{2}}+\frac{6}{7}\,ac^2ex^{\frac{7}{2}}+\frac{2}{7}\,b^3fx^{\frac{7}{2}}+\frac{12}{7}\,abcfx^{\frac{7}{2}}+b^2cdx^3+ac^2dx^3+\frac{1}{3}\,b^3ex^3+2\,abcex^3+ab^2fx^3+a^2cfx^3+\frac{2}{5}\,b^3dx^{\frac{5}{2}}+\frac{12}{5}\,abcdx^{\frac{5}{2}}+\frac{6}{5}\,ab^2ex^{\frac{5}{2}}+\frac{6}{5}\,a^2cex^{\frac{5}{2}}+\frac{6}{5}\,a^2bfx^{\frac{5}{2}}+\frac{3}{2}\,ab^2dx^2+\frac{3}{2}\,a^2cdx^2+\frac{3}{2}\,a^2bex^2+\frac{1}{2}\,a^3fx^2+2\,a^2bdx^{\frac{3}{2}}+\frac{2}{3}\,a^3ex^{\frac{3}{2}}+a^3dx$$

input
```
integrate((a+b*x^(1/2)+c*x)^3*(d+e*x^(1/2)+f*x),x, algorithm="giac")
```

output
```
1/5*c^3*f*x^5 + 2/9*c^3*e*x^(9/2) + 2/3*b*c^2*f*x^(9/2) + 1/4*c^3*d*x^4 +
3/4*b*c^2*e*x^4 + 3/4*b^2*c*f*x^4 + 3/4*a*c^2*f*x^4 + 6/7*b*c^2*d*x^(7/2)
+ 6/7*b^2*c*e*x^(7/2) + 6/7*a*c^2*e*x^(7/2) + 2/7*b^3*f*x^(7/2) + 12/7*a*b
*c*f*x^(7/2) + b^2*c*d*x^3 + a*c^2*d*x^3 + 1/3*b^3*e*x^3 + 2*a*b*c*e*x^3 +
 a*b^2*f*x^3 + a^2*c*f*x^3 + 2/5*b^3*d*x^(5/2) + 12/5*a*b*c*d*x^(5/2) + 6/
5*a*b^2*e*x^(5/2) + 6/5*a^2*c*e*x^(5/2) + 6/5*a^2*b*f*x^(5/2) + 3/2*a*b^2*
d*x^2 + 3/2*a^2*c*d*x^2 + 3/2*a^2*b*e*x^2 + 1/2*a^3*f*x^2 + 2*a^2*b*d*x^(3
/2) + 2/3*a^3*e*x^(3/2) + a^3*d*x
```

Mupad [B] (verification not implemented)

Time = 21.87 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.96

$$\int \left(a+b\sqrt{x}+cx\right)^3\left(d+e\sqrt{x}+fx\right)\,dx = x^{3/2}\left(\frac{2\,e\,a^3}{3}+2\,b\,d\,a^2\right)$$
$$+x^{9/2}\left(\frac{2\,e\,c^3}{9}+\frac{2\,b\,f\,c^2}{3}\right)+x^3\left(f\,a^2\,c+f\,a\,b^2+2\,e\,a\,b\,c+d\,a\,c^2+\frac{e\,b^3}{3}+d\,b^2\,c\right)$$
$$+x^{5/2}\left(\frac{6\,f\,a^2\,b}{5}+\frac{6\,c\,e\,a^2}{5}+\frac{6\,e\,a\,b^2}{5}+\frac{12\,c\,d\,a\,b}{5}+\frac{2\,d\,b^3}{5}\right)+x^{7/2}\left(\frac{2\,f\,b^3}{7}+\frac{6\,e\,b^2\,c}{7}+\frac{6\,d\,b\,c^2}{7}+\frac{12\,a\,f\,b\,c}{7}+\frac{6\,a}{}\right.$$

input
```
int((a + c*x + b*x^(1/2))^3*(d + f*x + e*x^(1/2)),x)
```

output
```
x^(3/2)*((2*a^3*e)/3 + 2*a^2*b*d) + x^(9/2)*((2*c^3*e)/9 + (2*b*c^2*f)/3)
+ x^3*((b^3*e)/3 + a*c^2*d + a*b^2*f + b^2*c*d + a^2*c*f + 2*a*b*c*e) + x^
(5/2)*((2*b^3*d)/5 + (6*a*b^2*e)/5 + (6*a^2*b*f)/5 + (6*a^2*c*e)/5 + (12*a
*b*c*d)/5) + x^(7/2)*((2*b^3*f)/7 + (6*a*c^2*e)/7 + (6*b*c^2*d)/7 + (6*b^2
*c*e)/7 + (12*a*b*c*f)/7) + x^2*((a^3*f)/2 + (3*a*b^2*d)/2 + (3*a^2*b*e)/2
 + (3*a^2*c*d)/2) + x^4*((c^3*d)/4 + (3*a*c^2*f)/4 + (3*b*c^2*e)/4 + (3*b^
2*c*f)/4) + (c^3*f*x^5)/5 + a^3*d*x
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.21

$$\int \left(a+b\sqrt{x}+cx\right)^3\left(d+e\sqrt{x}+fx\right)\,dx$$
$$=\frac{x\left(3024\sqrt{x}\,abcdx+420b^3e\,x^2+1080\sqrt{x}\,b\,c^2d\,x^2+315c^3d\,x^3+1890a^2cdx+1260a\,c^2d\,x^2+1260b^2cd\,x^2\right.}{}$$

input
```
int((a+b*x^(1/2)+c*x)^3*(d+e*x^(1/2)+f*x),x)
```

output

```
(x*(840*sqrt(x)*a**3*e + 2520*sqrt(x)*a**2*b*d + 1512*sqrt(x)*a**2*b*f*x +
 1512*sqrt(x)*a**2*c*e*x + 1512*sqrt(x)*a*b**2*e*x + 3024*sqrt(x)*a*b*c*d*
x + 2160*sqrt(x)*a*b*c*f*x**2 + 1080*sqrt(x)*a*c**2*e*x**2 + 504*sqrt(x)*b
**3*d*x + 360*sqrt(x)*b**3*f*x**2 + 1080*sqrt(x)*b**2*c*e*x**2 + 1080*sqrt
(x)*b*c**2*d*x**2 + 840*sqrt(x)*b*c**2*f*x**3 + 280*sqrt(x)*c**3*e*x**3 +
1260*a**3*d + 630*a**3*f*x + 1890*a**2*b*e*x + 1890*a**2*c*d*x + 1260*a**2
*c*f*x**2 + 1890*a*b**2*d*x + 1260*a*b**2*f*x**2 + 2520*a*b*c*e*x**2 + 126
0*a*c**2*d*x**2 + 945*a*c**2*f*x**3 + 420*b**3*e*x**2 + 1260*b**2*c*d*x**2
 + 945*b**2*c*f*x**3 + 945*b*c**2*e*x**3 + 315*c**3*d*x**3 + 252*c**3*f*x*
*4))/1260
```

3.112 $\int \left(a+b\sqrt{x}+cx\right)^2 \left(d+e\sqrt{x}+fx\right)\,dx$

Optimal result

Integrand size = 27, antiderivative size = 140

$$\begin{aligned}\int \left(a+b\sqrt{x}+cx\right)^2 \left(d+e\sqrt{x}+fx\right)\,dx &= a^2dx+\frac{2}{3}a(2bd+ae)x^{3/2}\\ &\quad+\frac{1}{2}\left(b^2d+2abe+a(2cd+af)\right)x^2\\ &\quad+\frac{2}{5}\left(b^2e+2ace+2b(cd+af)\right)x^{5/2}\\ &\quad+\frac{1}{3}\left(c^2d+b^2f+2c(be+af)\right)x^3\\ &\quad+\frac{2}{7}c(ce+2bf)x^{7/2}+\frac{1}{4}c^2fx^4\end{aligned}$$

output

```
a^2*d*x+2/3*a*(a*e+2*b*d)*x^(3/2)+1/2*(b^2*d+2*a*b*e+a*(a*f+2*c*d))*x^2+2/
5*(b^2*e+2*a*c*e+2*b*(a*f+c*d))*x^(5/2)+1/3*(c^2*d+b^2*f+2*c*(a*f+b*e))*x^
3+2/7*c*(2*b*f+c*e)*x^(7/2)+1/4*c^2*f*x^4
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.13

$$\int \left(a+b\sqrt{x}+cx\right)^2\left(d+e\sqrt{x}+fx\right)\,dx=\frac{1}{6}a^2x\left(6d+4e\sqrt{x}+3fx\right)+\frac{1}{420}x^2\left(14b^2\left(15d+12e\sqrt{x}+10fx\right)+5c^2x\left(28d+24e\sqrt{x}+21fx\right)+8bc\sqrt{x}\left(42d+35e\sqrt{x}+30fx\right)\right)+a\left(cdx^2+\frac{4}{5}cex^{5/2}+\frac{2}{3}cfx^3+b\left(\frac{4}{3}dx^{3/2}+ex^2+\frac{4}{5}fx^{5/2}\right)\right)$$

input
```
Integrate[(a + b*Sqrt[x] + c*x)^2*(d + e*Sqrt[x] + f*x),x]
```

output
```
(a^2*x*(6*d + 4*e*Sqrt[x] + 3*f*x))/6 + (x^2*(14*b^2*(15*d + 12*e*Sqrt[x]
+ 10*f*x) + 5*c^2*x*(28*d + 24*e*Sqrt[x] + 21*f*x) + 8*b*c*Sqrt[x]*(42*d +
 35*e*Sqrt[x] + 30*f*x)))/420 + a*(c*d*x^2 + (4*c*e*x^(5/2))/5 + (2*c*f*x^
3)/3 + b*((4*d*x^(3/2))/3 + e*x^2 + (4*f*x^(5/2))/5))
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\sqrt{x}+cx\right)^2\left(d+e\sqrt{x}+fx\right)\,dx$$

$$\Big\downarrow \text{ 2329}$$

$$\int \left(a+b\sqrt{x}+cx\right)^2\left(d+e\sqrt{x}+fx\right)dx$$

input
```
Int[(a + b*Sqrt[x] + c*x)^2*(d + e*Sqrt[x] + f*x),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2329
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_.), x_Symbol] :>
 Unintegrable[Pq*(a + b*x^n + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, n, p}, x]
 && EqQ[n2, 2*n] && (PolyQ[Pq, x] || PolyQ[Pq, x^n])
```

Maple [A] (verified)

Time = 2.35 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.91

method	result
derivativedivides	$\frac{c^2 f x^4}{4}+\frac{2(2bcf+c^2e)x^{\frac{7}{2}}}{7}+\frac{((2ac+b^2)f+2bce+c^2d)x^3}{3}+\frac{2(2abf+(2ac+b^2)e+2cbd)x^{\frac{5}{2}}}{5}+\frac{(a^2f+2abe+(2ac+\cdots}{2}$
default	$\frac{2x^{\frac{5}{2}}b^2e}{5}+\frac{(b^2f+2bce)x^3}{3}+\frac{(2abe+db^2)x^2}{2}+\frac{2c(2bf+ce)x^{\frac{7}{2}}}{7}+\frac{2(a(2bf+ce)+c(ae+2bd))x^{\frac{5}{2}}}{5}+\frac{2a(ae+2bd)x\cdots}{3}$
trager	$\frac{(3c^2fx^3+8acfx^2+4b^2fx^2+8x^2bce+4c^2dx^2+3c^2fx^2+6a^2fx+12abex+12acdx+8acfx+6b^2dx+4b^2fx+8bcex+4c^2\cdots}{12}$
orering	Expression too large to display

input
```
int((a+b*x^(1/2)+c*x)^2*(d+e*x^(1/2)+f*x),x,method=_RETURNVERBOSE)
```

output
```
1/4*c^2*f*x^4+2/7*(2*b*c*f+c^2*e)*x^(7/2)+1/3*((2*a*c+b^2)*f+2*b*c*e+c^2*d
)*x^3+2/5*(2*a*b*f+(2*a*c+b^2)*e+2*c*b*d)*x^(5/2)+1/2*(a^2*f+2*a*b*e+(2*a*
c+b^2)*d)*x^2+2/3*(a^2*e+2*a*b*d)*x^(3/2)+a^2*d*x
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.93

$$\int \left(a+b\sqrt{x}+cx\right)^2\left(d+e\sqrt{x}+fx\right)\,dx$$
$$=\frac{1}{4}c^2fx^4+a^2dx+\frac{1}{3}\left(c^2d+2\,bce+\left(b^2+2\,ac\right)f\right)x^3+\frac{1}{2}\left(2\,abe+a^2f+\left(b^2+2\,ac\right)d\right)x^2$$
$$+\frac{2}{105}\left(15\left(c^2e+2\,bcf\right)x^3+21\left(2\,bcd+2\,abf+\left(b^2+2\,ac\right)e\right)x^2+35\left(2\,abd+a^2e\right)x\right)\sqrt{x}$$

input
```
integrate((a+b*x^(1/2)+c*x)^2*(d+e*x^(1/2)+f*x),x, algorithm="fricas")
```

output
```
1/4*c^2*f*x^4 + a^2*d*x + 1/3*(c^2*d + 2*b*c*e + (b^2 + 2*a*c)*f)*x^3 + 1/
2*(2*a*b*e + a^2*f + (b^2 + 2*a*c)*d)*x^2 + 2/105*(15*(c^2*e + 2*b*c*f)*x^
3 + 21*(2*b*c*d + 2*a*b*f + (b^2 + 2*a*c)*e)*x^2 + 35*(2*a*b*d + a^2*e)*x)
*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.47

$$\int \left(a+b\sqrt{x}+cx\right)^2 \left(d+e\sqrt{x}+fx\right)\,dx = a^2dx + \frac{2a^2ex^{\frac{3}{2}}}{3} + \frac{a^2fx^2}{2} + \frac{4abdx^{\frac{3}{2}}}{3} + abex^2 + \frac{4abfx^{\frac{5}{2}}}{5} + acdx^2 + \frac{4acex^{\frac{5}{2}}}{5} + \frac{2acfx^3}{3} + \frac{b^2dx^2}{2} + \frac{2b^2ex^{\frac{5}{2}}}{5} + \frac{b^2fx^3}{3} + \frac{4bcdx^{\frac{5}{2}}}{5} + \frac{2bcex^3}{3} + \frac{4bcfx^{\frac{7}{2}}}{7} + \frac{c^2dx^3}{3} + \frac{2c^2ex^{\frac{7}{2}}}{7} + \frac{c^2fx^4}{4}$$

input
```
integrate((a+b*x**(1/2)+c*x)**2*(d+e*x**(1/2)+f*x),x)
```

output
```
a**2*d*x + 2*a**2*e*x**(3/2)/3 + a**2*f*x**2/2 + 4*a*b*d*x**(3/2)/3 + a*b*
e*x**2 + 4*a*b*f*x**(5/2)/5 + a*c*d*x**2 + 4*a*c*e*x**(5/2)/5 + 2*a*c*f*x*
*3/3 + b**2*d*x**2/2 + 2*b**2*e*x**(5/2)/5 + b**2*f*x**3/3 + 4*b*c*d*x**(5
/2)/5 + 2*b*c*e*x**3/3 + 4*b*c*f*x**(7/2)/7 + c**2*d*x**3/3 + 2*c**2*e*x**
(7/2)/7 + c**2*f*x**4/4
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.90

$$\int \left(a+b\sqrt{x}+cx\right)^2\left(d+e\sqrt{x}+fx\right)\,dx = \frac{1}{4}\,c^2fx^4+\frac{2}{7}\,\left(c^2e+2\,bcf\right)x^{\frac{7}{2}}+a^2dx+\frac{1}{3}\,\left(c^2d+2\,bce+\left(b^2+2\,ac\right)f\right)x^3+\frac{2}{5}\,\left(2\,bcd+2\,abf+\left(b^2+2\,ac\right)e\right)x^{\frac{5}{2}}+\frac{1}{2}\,\left(2\,abe+a^2f+\left(b^2+2\,ac\right)d\right)x^2+\frac{2}{3}\,\left(2\,abd+a^2e\right)x^{\frac{3}{2}}$$

input
```
integrate((a+b*x^(1/2)+c*x)^2*(d+e*x^(1/2)+f*x),x, algorithm="maxima")
```

output
```
1/4*c^2*f*x^4 + 2/7*(c^2*e + 2*b*c*f)*x^(7/2) + a^2*d*x + 1/3*(c^2*d + 2*b
*c*e + (b^2 + 2*a*c)*f)*x^3 + 2/5*(2*b*c*d + 2*a*b*f + (b^2 + 2*a*c)*e)*x^
(5/2) + 1/2*(2*a*b*e + a^2*f + (b^2 + 2*a*c)*d)*x^2 + 2/3*(2*a*b*d + a^2*e
)*x^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.06

$$\int \left(a+b\sqrt{x}+cx\right)^2\left(d+e\sqrt{x}+fx\right)\,dx = \frac{1}{4}\,c^2fx^4+\frac{2}{7}\,c^2ex^{\frac{7}{2}}+\frac{4}{7}\,bcfx^{\frac{7}{2}}+\frac{1}{3}\,c^2dx^3+\frac{2}{3}\,bcex^3+\frac{1}{3}\,b^2fx^3+\frac{2}{3}\,acfx^3+\frac{4}{5}\,bcdx^{\frac{5}{2}}+\frac{2}{5}\,b^2ex^{\frac{5}{2}}+\frac{4}{5}\,acex^{\frac{5}{2}}+\frac{4}{5}\,abfx^{\frac{5}{2}}+\frac{1}{2}\,b^2dx^2+acdx^2+abex^2+\frac{1}{2}\,a^2fx^2+\frac{4}{3}\,abdx^{\frac{3}{2}}+\frac{2}{3}\,a^2ex^{\frac{3}{2}}+a^2dx$$

input
```
integrate((a+b*x^(1/2)+c*x)^2*(d+e*x^(1/2)+f*x),x, algorithm="giac")
```

output
```
1/4*c^2*f*x^4 + 2/7*c^2*e*x^(7/2) + 4/7*b*c*f*x^(7/2) + 1/3*c^2*d*x^3 + 2/
3*b*c*e*x^3 + 1/3*b^2*f*x^3 + 2/3*a*c*f*x^3 + 4/5*b*c*d*x^(5/2) + 2/5*b^2*
e*x^(5/2) + 4/5*a*c*e*x^(5/2) + 4/5*a*b*f*x^(5/2) + 1/2*b^2*d*x^2 + a*c*d*
x^2 + a*b*e*x^2 + 1/2*a^2*f*x^2 + 4/3*a*b*d*x^(3/2) + 2/3*a^2*e*x^(3/2) +
a^2*d*x
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.90

$$\int \left(a+b\sqrt{x}+cx\right)^2\left(d+e\sqrt{x}+fx\right)\,dx$$
$$= x^{3/2}\left(\frac{2\,e\,a^2}{3}+\frac{4\,b\,d\,a}{3}\right)+x^{7/2}\left(\frac{2\,e\,c^2}{7}+\frac{4\,b\,f\,c}{7}\right)+x^2\left(\frac{f\,a^2}{2}+e\,a\,b+c\,d\,a+\frac{d\,b^2}{2}\right) +x^{5/2}\left(\frac{2\,b^2\,e}{5}+\frac{4\,a\,b\,f}{5}+\frac{4\,a\,c\,e}{5}+\frac{4\,b\,c\,d}{5}\right)+x^3\left(\frac{f\,b^2}{3}+\frac{2\,e\,b\,c}{3}+\frac{d\,c^2}{3}+\frac{2\,a\,f\,c}{3}\right)+\frac{c^2\,f\,x^4}{4}+a^2\,d\,x$$

input
```
int((a + c*x + b*x^(1/2))^2*(d + f*x + e*x^(1/2)),x)
```

output
```
x^(3/2)*((2*a^2*e)/3 + (4*a*b*d)/3) + x^(7/2)*((2*c^2*e)/7 + (4*b*c*f)/7)
+ x^2*((b^2*d)/2 + (a^2*f)/2 + a*b*e + a*c*d) + x^(5/2)*((2*b^2*e)/5 + (4*
a*b*f)/5 + (4*a*c*e)/5 + (4*b*c*d)/5) + x^3*((c^2*d)/3 + (b^2*f)/3 + (2*a*
c*f)/3 + (2*b*c*e)/3) + (c^2*f*x^4)/4 + a^2*d*x
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.06

$$\int \left(a+b\sqrt{x}+cx\right)^2\left(d+e\sqrt{x}+fx\right)\,dx$$
$$= \frac{x\left(280\sqrt{x}\,a^2e+560\sqrt{x}\,abd+336\sqrt{x}\,abfx+336\sqrt{x}\,acex+168\sqrt{x}\,b^2ex+336\sqrt{x}\,bcdx+240\sqrt{x}\,bcf\,x^2\right.}{}$$

input
```
int((a+b*x^(1/2)+c*x)^2*(d+e*x^(1/2)+f*x),x)
```

output
```
(x*(280*sqrt(x)*a**2*e + 560*sqrt(x)*a*b*d + 336*sqrt(x)*a*b*f*x + 336*sqr
t(x)*a*c*e*x + 168*sqrt(x)*b**2*e*x + 336*sqrt(x)*b*c*d*x + 240*sqrt(x)*b*
c*f*x**2 + 120*sqrt(x)*c**2*e*x**2 + 420*a**2*d + 210*a**2*f*x + 420*a*b*e
*x + 420*a*c*d*x + 280*a*c*f*x**2 + 210*b**2*d*x + 140*b**2*f*x**2 + 280*b
*c*e*x**2 + 140*c**2*d*x**2 + 105*c**2*f*x**3))/420
```

3.113 $\int \left(a+b\sqrt{x}+cx\right)\left(d+e\sqrt{x}+fx\right)\,dx$

Optimal result

Integrand size = 25, antiderivative size = 63

$$\int \left(a+b\sqrt{x}+cx\right)\left(d+e\sqrt{x}+fx\right)\,dx = adx+\frac{2}{3}(bd+ae)x^{3/2} +\frac{1}{2}(cd+be+af)x^2+\frac{2}{5}(ce+bf)x^{5/2}+\frac{1}{3}cfx^3$$

output
```
a*d*x+2/3*(a*e+b*d)*x^(3/2)+1/2*(a*f+b*e+c*d)*x^2+2/5*(b*f+c*e)*x^(5/2)+1/
3*c*f*x^3
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.08

$$\int \left(a+b\sqrt{x}+cx\right)\left(d+e\sqrt{x}+fx\right)\,dx = \frac{1}{30}x\left(30ad+20bd\sqrt{x}+20ae\sqrt{x}+15cdx+15bex +15afx+12cex^{3/2}+12bfx^{3/2}+10cfx^2\right)$$

input
```
Integrate[(a + b*Sqrt[x] + c*x)*(d + e*Sqrt[x] + f*x),x]
```

output
```
(x*(30*a*d + 20*b*d*Sqrt[x] + 20*a*e*Sqrt[x] + 15*c*d*x + 15*b*e*x + 15*a*
f*x + 12*c*e*x^(3/2) + 12*b*f*x^(3/2) + 10*c*f*x^2))/30
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\sqrt{x}+cx\right)\left(d+e\sqrt{x}+fx\right)\,dx$$

$$\Big\downarrow \; 2329$$

$$\int \left(a+b\sqrt{x}+cx\right)\left(d+e\sqrt{x}+fx\right)dx$$

input
```
Int[(a + b*Sqrt[x] + c*x)*(d + e*Sqrt[x] + f*x),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2329
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_.), x_Symbol] :>
 Unintegrable[Pq*(a + b*x^n + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, n, p}, x]
 && EqQ[n2, 2*n] && (PolyQ[Pq, x] || PolyQ[Pq, x^n])
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.83

method	result
derivativedivides	$adx+\frac{2(ae+bd)x^{\frac{3}{2}}}{3}+\frac{(af+eb+cd)x^{2}}{2}+\frac{2(bf+ce)x^{\frac{5}{2}}}{5}+\frac{cf\,x^{3}}{3}$
default	$\frac{x^{2}eb}{2}+\frac{2(bf+ce)x^{\frac{5}{2}}}{5}+\frac{2(ae+bd)x^{\frac{3}{2}}}{3}+\frac{cf\,x^{3}}{3}+\frac{(af+cd)x^{2}}{2}+adx$
trager	$\frac{(2cf\,x^{2}+3afx+3xeb+3xcd+2cfx+6ad+3af+3eb+3cd+2cf)(x-1)}{6}+\frac{2x^{\frac{3}{2}}(3bfx+3cex+5ae+5bd)}{15}$
orering	$-\frac{(18b^{2}cf^{3}x^{4}+36x^{4}c^{2}f^{2}be+18c^{3}e^{2}fx^{4}-21ab^{2}f^{3}x^{3}+28x^{3}cf^{2}abe+49ac^{2}e^{2}fx^{3}-21b^{3}x^{3}f^{2}e+49b^{2}cdf^{2}x^{3}-42x^{3}c\ldots}{30\ldots}$

input
```
int((a+b*x^(1/2)+c*x)*(d+e*x^(1/2)+f*x),x,method=_RETURNVERBOSE)
```

output
```
a*d*x+2/3*(a*e+b*d)*x^(3/2)+1/2*(a*f+b*e+c*d)*x^2+2/5*(b*f+c*e)*x^(5/2)+1/
3*c*f*x^3
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.87

$$\int \left(a+b\sqrt{x}+cx\right)\left(d+e\sqrt{x}+fx\right)\,dx=\frac{1}{3}\,cfx^3+adx+\frac{1}{2}\,(cd+be+af)x^2+\frac{2}{15}\,\left(3\,(ce+bf)x^2+5\,(bd+ae)x\right)\sqrt{x}$$

input
```
integrate((a+b*x^(1/2)+c*x)*(d+e*x^(1/2)+f*x),x, algorithm="fricas")
```

output
```
1/3*c*f*x^3 + a*d*x + 1/2*(c*d + b*e + a*f)*x^2 + 2/15*(3*(c*e + b*f)*x^2
+ 5*(b*d + a*e)*x)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.35

$$\int \left(a+b\sqrt{x}+cx\right)\left(d+e\sqrt{x}+fx\right)\,dx=adx+\frac{2aex^{\frac{3}{2}}}{3}+\frac{afx^2}{2}+\frac{2bdx^{\frac{3}{2}}}{3}+\frac{bex^2}{2}+\frac{2bfx^{\frac{5}{2}}}{5}+\frac{cdx^2}{2}+\frac{2cex^{\frac{5}{2}}}{5}+\frac{cfx^3}{3}$$

input
```
integrate((a+b*x**(1/2)+c*x)*(d+e*x**(1/2)+f*x),x)
```

output
```
a*d*x + 2*a*e*x**(3/2)/3 + a*f*x**2/2 + 2*b*d*x**(3/2)/3 + b*e*x**2/2 + 2*
b*f*x**(5/2)/5 + c*d*x**2/2 + 2*c*e*x**(5/2)/5 + c*f*x**3/3
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

$$\int \left(a+b\sqrt{x}+cx\right)\left(d+e\sqrt{x}+fx\right)\,dx=\frac{1}{3}\,cfx^3+\frac{2}{5}\,(ce+bf)x^{\frac{5}{2}}+adx+\frac{1}{2}\,(cd+be+af)x^2+\frac{2}{3}\,(bd+ae)x^{\frac{3}{2}}$$

input
```
integrate((a+b*x^(1/2)+c*x)*(d+e*x^(1/2)+f*x),x, algorithm="maxima")
```

output
```
1/3*c*f*x^3 + 2/5*(c*e + b*f)*x^(5/2) + a*d*x + 1/2*(c*d + b*e + a*f)*x^2
+ 2/3*(b*d + a*e)*x^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.97

$$\int \left(a+b\sqrt{x}+cx\right)\left(d+e\sqrt{x}+fx\right)\,dx=\frac{1}{3}\,cfx^3+\frac{2}{5}\,cex^{\frac{5}{2}}+\frac{2}{5}\,bfx^{\frac{5}{2}}+\frac{1}{2}\,cdx^2+\frac{1}{2}\,bex^2+\frac{1}{2}\,afx^2+\frac{2}{3}\,bdx^{\frac{3}{2}}+\frac{2}{3}\,aex^{\frac{3}{2}}+adx$$

input
```
integrate((a+b*x^(1/2)+c*x)*(d+e*x^(1/2)+f*x),x, algorithm="giac")
```

output
```
1/3*c*f*x^3 + 2/5*c*e*x^(5/2) + 2/5*b*f*x^(5/2) + 1/2*c*d*x^2 + 1/2*b*e*x^
2 + 1/2*a*f*x^2 + 2/3*b*d*x^(3/2) + 2/3*a*e*x^(3/2) + a*d*x
```

Mupad [B] (verification not implemented)

Time = 21.71 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.87

$$\int \left(a+b\sqrt{x}+cx\right)\left(d+e\sqrt{x}+fx\right)\,dx=x^2\left(\frac{a\,f}{2}+\frac{b\,e}{2}+\frac{c\,d}{2}\right)+x^{3/2}\left(\frac{2\,a\,e}{3}+\frac{2\,b\,d}{3}\right)+x^{5/2}\left(\frac{2\,b\,f}{5}+\frac{2\,c\,e}{5}\right)+a\,d\,x+\frac{c\,f\,x^3}{3}$$

input
```
int((a + c*x + b*x^(1/2))*(d + f*x + e*x^(1/2)),x)
```

output
```
x^2*((a*f)/2 + (b*e)/2 + (c*d)/2) + x^(3/2)*((2*a*e)/3 + (2*b*d)/3) + x^(5
/2)*((2*b*f)/5 + (2*c*e)/5) + a*d*x + (c*f*x^3)/3
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.89

$$\int \left(a+b\sqrt{x}+cx\right)\left(d+e\sqrt{x}+fx\right)\,dx$$
$$=\frac{x\left(20\sqrt{x}\,ae+20\sqrt{x}\,bd+12\sqrt{x}\,bfx+12\sqrt{x}\,cex+30ad+15afx+15bex+15cdx+10cf\,x^2\right)}{30}$$

input
```
int((a+b*x^(1/2)+c*x)*(d+e*x^(1/2)+f*x),x)
```

output
```
(x*(20*sqrt(x)*a*e + 20*sqrt(x)*b*d + 12*sqrt(x)*b*f*x + 12*sqrt(x)*c*e*x
+ 30*a*d + 15*a*f*x + 15*b*e*x + 15*c*d*x + 10*c*f*x**2))/30
```

3.114 $\int (a+bx)^4(c+dx)^3\,dx$

Optimal result

Integrand size = 15, antiderivative size = 92

$$\int (a+bx)^4(c+dx)^3\,dx = \frac{(bc-ad)^3(a+bx)^5}{5b^4} + \frac{d(bc-ad)^2(a+bx)^6}{2b^4} + \frac{3d^2(bc-ad)(a+bx)^7}{7b^4} + \frac{d^3(a+bx)^8}{8b^4}$$

output
```
1/5*(-a*d+b*c)^3*(b*x+a)^5/b^4+1/2*d*(-a*d+b*c)^2*(b*x+a)^6/b^4+3/7*d^2*(-
a*d+b*c)*(b*x+a)^7/b^4+1/8*d^3*(b*x+a)^8/b^4
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 217 vs. $2(92) = 184$.

Time = 0.04 (sec) , antiderivative size = 217, normalized size of antiderivative = 2.36

$$\int (a+bx)^4(c+dx)^3\,dx = a^4c^3x+\frac{1}{2}a^3c^2(4bc+3ad)x^2+a^2c\left(2b^2c^2+4abcd+a^2d^2\right)x^3 +\frac{1}{4}a\left(4b^3c^3+18ab^2c^2d+12a^2bcd^2+a^3d^3\right)x^4 +\frac{1}{5}b\left(b^3c^3+12ab^2c^2d+18a^2bcd^2+4a^3d^3\right)x^5 +\frac{1}{2}b^2d\left(b^2c^2+4abcd+2a^2d^2\right)x^6 +\frac{1}{7}b^3d^2(3bc+4ad)x^7+\frac{1}{8}b^4d^3x^8$$

input
```
Integrate[(a + b*x)^4*(c + d*x)^3,x]
```

output
```
a^4*c^3*x + (a^3*c^2*(4*b*c + 3*a*d)*x^2)/2 + a^2*c*(2*b^2*c^2 + 4*a*b*c*d
 + a^2*d^2)*x^3 + (a*(4*b^3*c^3 + 18*a*b^2*c^2*d + 12*a^2*b*c*d^2 + a^3*d^
3)*x^4)/4 + (b*(b^3*c^3 + 12*a*b^2*c^2*d + 18*a^2*b*c*d^2 + 4*a^3*d^3)*x^5
)/5 + (b^2*d*(b^2*c^2 + 4*a*b*c*d + 2*a^2*d^2)*x^6)/2 + (b^3*d^2*(3*b*c +
4*a*d)*x^7)/7 + (b^4*d^3*x^8)/8
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)^4(c+dx)^3\,dx$$

$$\Big\downarrow \text{ 49}$$

$$\int \left(\frac{3d^2(a+bx)^6(bc-ad)}{b^3}+\frac{3d(a+bx)^5(bc-ad)^2}{b^3}+\frac{(a+bx)^4(bc-ad)^3}{b^3}+\frac{d^3(a+bx)^7}{b^3}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{3d^2(a+bx)^7(bc-ad)}{7b^4}+\frac{d(a+bx)^6(bc-ad)^2}{2b^4}+\frac{(a+bx)^5(bc-ad)^3}{5b^4}+\frac{d^3(a+bx)^8}{8b^4}$$

input
```
Int[(a + b*x)^4*(c + d*x)^3,x]
```

output
```
((b*c - a*d)^3*(a + b*x)^5)/(5*b^4) + (d*(b*c - a*d)^2*(a + b*x)^6)/(2*b^4
) + (3*d^2*(b*c - a*d)*(a + b*x)^7)/(7*b^4) + (d^3*(a + b*x)^8)/(8*b^4)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 221 vs. $2(84) = 168$.

Time = 0.14 (sec) , antiderivative size = 222, normalized size of antiderivative = 2.41

method	result
norman	$\frac{b^4d^3x^8}{8}+\left(\frac{4}{7}b^3a\,d^3+\frac{3}{7}b^4c\,d^2\right)x^7+\left(a^2b^2d^3+2a\,b^3c\,d^2+\frac{1}{2}b^4c^2d\right)x^6+\left(\frac{4}{5}a^3b\,d^3+\frac{18}{5}a^2b^2c\,d^2\right.$
default	$\frac{b^4d^3x^8}{8}+\frac{(4b^3a\,d^3+3b^4c\,d^2)x^7}{7}+\frac{(6a^2b^2d^3+12a\,b^3c\,d^2+3b^4c^2d)x^6}{6}+\frac{(4a^3b\,d^3+18a^2b^2c\,d^2+12b^3a\,c^2d+b^4c^3)x^5}{5}+$
gosper	$\frac{1}{8}b^4d^3x^8+\frac{4}{7}x^7b^3a\,d^3+\frac{3}{7}x^7b^4c\,d^2+x^6a^2b^2d^3+2x^6a\,b^3c\,d^2+\frac{1}{2}x^6b^4c^2d+\frac{4}{5}x^5a^3b\,d^3+\frac{18}{5}x^5a$
risch	$\frac{1}{8}b^4d^3x^8+\frac{4}{7}x^7b^3a\,d^3+\frac{3}{7}x^7b^4c\,d^2+x^6a^2b^2d^3+2x^6a\,b^3c\,d^2+\frac{1}{2}x^6b^4c^2d+\frac{4}{5}x^5a^3b\,d^3+\frac{18}{5}x^5a$
parallelrisch	$\frac{1}{8}b^4d^3x^8+\frac{4}{7}x^7b^3a\,d^3+\frac{3}{7}x^7b^4c\,d^2+x^6a^2b^2d^3+2x^6a\,b^3c\,d^2+\frac{1}{2}x^6b^4c^2d+\frac{4}{5}x^5a^3b\,d^3+\frac{18}{5}x^5a$
orering	$x\left(35b^4d^3x^7+160a\,b^3d^3x^6+120b^4c\,d^2x^6+280a^2b^2d^3x^5+560a\,b^3c\,d^2x^5+140b^4c^2d\,x^5+224a^3b\,d^3x^4+1008a^2b^2c\,d^2x^4+672a\,b^3\right.$

input
```
int((b*x+a)^4*(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/8*b^4*d^3*x^8+(4/7*b^3*a*d^3+3/7*b^4*c*d^2)*x^7+(a^2*b^2*d^3+2*a*b^3*c*d
^2+1/2*b^4*c^2*d)*x^6+(4/5*a^3*b*d^3+18/5*a^2*b^2*c*d^2+12/5*b^3*a*c^2*d+1
/5*b^4*c^3)*x^5+(1/4*a^4*d^3+3*a^3*b*c*d^2+9/2*a^2*b^2*c^2*d+a*b^3*c^3)*x^
4+(a^4*c*d^2+4*a^3*b*c^2*d+2*a^2*b^2*c^3)*x^3+(3/2*a^4*c^2*d+2*a^3*b*c^3)*
x^2+a^4*c^3*x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 225 vs. $2(84) = 168$.

Time = 0.08 (sec) , antiderivative size = 225, normalized size of antiderivative = 2.45

$$\begin{aligned}\int (a+bx)^4(c+dx)^3\,dx &= \frac{1}{8}\,b^4d^3x^8 + a^4c^3x + \frac{1}{7}\left(3\,b^4cd^2 + 4\,ab^3d^3\right)x^7 \\ &+ \frac{1}{2}\left(b^4c^2d + 4\,ab^3cd^2 + 2\,a^2b^2d^3\right)x^6 \\ &+ \frac{1}{5}\left(b^4c^3 + 12\,ab^3c^2d + 18\,a^2b^2cd^2 + 4\,a^3bd^3\right)x^5 \\ &+ \frac{1}{4}\left(4\,ab^3c^3 + 18\,a^2b^2c^2d + 12\,a^3bcd^2 + a^4d^3\right)x^4 \\ &+ \left(2\,a^2b^2c^3 + 4\,a^3bc^2d + a^4cd^2\right)x^3 + \frac{1}{2}\left(4\,a^3bc^3 + 3\,a^4c^2d\right)x^2\end{aligned}$$

input
```
integrate((b*x+a)^4*(d*x+c)^3,x, algorithm="fricas")
```

output
```
1/8*b^4*d^3*x^8 + a^4*c^3*x + 1/7*(3*b^4*c*d^2 + 4*a*b^3*d^3)*x^7 + 1/2*(b
^4*c^2*d + 4*a*b^3*c*d^2 + 2*a^2*b^2*d^3)*x^6 + 1/5*(b^4*c^3 + 12*a*b^3*c^
2*d + 18*a^2*b^2*c*d^2 + 4*a^3*b*d^3)*x^5 + 1/4*(4*a*b^3*c^3 + 18*a^2*b^2*
c^2*d + 12*a^3*b*c*d^2 + a^4*d^3)*x^4 + (2*a^2*b^2*c^3 + 4*a^3*b*c^2*d + a
^4*c*d^2)*x^3 + 1/2*(4*a^3*b*c^3 + 3*a^4*c^2*d)*x^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 243 vs. $2(80) = 160$.

Time = 0.05 (sec) , antiderivative size = 243, normalized size of antiderivative = 2.64

$$\int (a+bx)^4(c+dx)^3\,dx = a^4c^3x + \frac{b^4d^3x^8}{8} + x^7\cdot\left(\frac{4ab^3d^3}{7} + \frac{3b^4cd^2}{7}\right) + x^6\left(a^2b^2d^3 + 2ab^3cd^2 + \frac{b^4c^2d}{2}\right) + x^5\cdot\left(\frac{4a^3bd^3}{5} + \frac{18a^2b^2cd^2}{5} + \frac{12ab^3c^2d}{5} + \frac{b^4c^3}{5}\right) + x^4\left(\frac{a^4d^3}{4} + 3a^3bcd^2 + \frac{9a^2b^2c^2d}{2} + ab^3c^3\right) + x^3\left(a^4cd^2 + 4a^3bc^2d + 2a^2b^2c^3\right) + x^2\cdot\left(\frac{3a^4c^2d}{2} + 2a^3bc^3\right)$$

input
```
integrate((b*x+a)**4*(d*x+c)**3,x)
```

output
```
a**4*c**3*x + b**4*d**3*x**8/8 + x**7*(4*a*b**3*d**3/7 + 3*b**4*c*d**2/7)
+ x**6*(a**2*b**2*d**3 + 2*a*b**3*c*d**2 + b**4*c**2*d/2) + x**5*(4*a**3*b
*d**3/5 + 18*a**2*b**2*c*d**2/5 + 12*a*b**3*c**2*d/5 + b**4*c**3/5) + x**4
*(a**4*d**3/4 + 3*a**3*b*c*d**2 + 9*a**2*b**2*c**2*d/2 + a*b**3*c**3) + x*
*3*(a**4*c*d**2 + 4*a**3*b*c**2*d + 2*a**2*b**2*c**3) + x**2*(3*a**4*c**2*
d/2 + 2*a**3*b*c**3)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 225 vs. 2(84) = 168.

Time = 0.03 (sec) , antiderivative size = 225, normalized size of antiderivative = 2.45

$$\int (a+bx)^4(c+dx)^3\,dx = \frac{1}{8}\,b^4d^3x^8 + a^4c^3x + \frac{1}{7}\left(3\,b^4cd^2 + 4\,ab^3d^3\right)x^7 + \frac{1}{2}\left(b^4c^2d + 4\,ab^3cd^2 + 2\,a^2b^2d^3\right)x^6 + \frac{1}{5}\left(b^4c^3 + 12\,ab^3c^2d + 18\,a^2b^2cd^2 + 4\,a^3bd^3\right)x^5 + \frac{1}{4}\left(4\,ab^3c^3 + 18\,a^2b^2c^2d + 12\,a^3bcd^2 + a^4d^3\right)x^4 + \left(2\,a^2b^2c^3 + 4\,a^3bc^2d + a^4cd^2\right)x^3 + \frac{1}{2}\left(4\,a^3bc^3 + 3\,a^4c^2d\right)x^2$$

input
```
integrate((b*x+a)^4*(d*x+c)^3,x, algorithm="maxima")
```

output
```
1/8*b^4*d^3*x^8 + a^4*c^3*x + 1/7*(3*b^4*c*d^2 + 4*a*b^3*d^3)*x^7 + 1/2*(b
^4*c^2*d + 4*a*b^3*c*d^2 + 2*a^2*b^2*d^3)*x^6 + 1/5*(b^4*c^3 + 12*a*b^3*c^
2*d + 18*a^2*b^2*c*d^2 + 4*a^3*b*d^3)*x^5 + 1/4*(4*a*b^3*c^3 + 18*a^2*b^2*
c^2*d + 12*a^3*b*c*d^2 + a^4*d^3)*x^4 + (2*a^2*b^2*c^3 + 4*a^3*b*c^2*d + a
^4*c*d^2)*x^3 + 1/2*(4*a^3*b*c^3 + 3*a^4*c^2*d)*x^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 245 vs. 2(84) = 168.

Time = 0.11 (sec) , antiderivative size = 245, normalized size of antiderivative = 2.66

$$\int (a+bx)^4(c+dx)^3\,dx = \frac{1}{8}\,b^4d^3x^8 + \frac{3}{7}\,b^4cd^2x^7 + \frac{4}{7}\,ab^3d^3x^7 + \frac{1}{2}\,b^4c^2dx^6 + 2\,ab^3cd^2x^6 + a^2b^2d^3x^6 + \frac{1}{5}\,b^4c^3x^5 + \frac{12}{5}\,ab^3c^2dx^5 + \frac{18}{5}\,a^2b^2cd^2x^5 + \frac{4}{5}\,a^3bd^3x^5 + ab^3c^3x^4 + \frac{9}{2}\,a^2b^2c^2dx^4 + 3\,a^3bcd^2x^4 + \frac{1}{4}\,a^4d^3x^4 + 2\,a^2b^2c^3x^3 + 4\,a^3bc^2dx^3 + a^4cd^2x^3 + 2\,a^3bc^3x^2 + \frac{3}{2}\,a^4c^2dx^2 + a^4c^3x$$

input
```
integrate((b*x+a)^4*(d*x+c)^3,x, algorithm="giac")
```

output
```
1/8*b^4*d^3*x^8 + 3/7*b^4*c*d^2*x^7 + 4/7*a*b^3*d^3*x^7 + 1/2*b^4*c^2*d*x^
6 + 2*a*b^3*c*d^2*x^6 + a^2*b^2*d^3*x^6 + 1/5*b^4*c^3*x^5 + 12/5*a*b^3*c^2
*d*x^5 + 18/5*a^2*b^2*c*d^2*x^5 + 4/5*a^3*b*d^3*x^5 + a*b^3*c^3*x^4 + 9/2*
a^2*b^2*c^2*d*x^4 + 3*a^3*b*c*d^2*x^4 + 1/4*a^4*d^3*x^4 + 2*a^2*b^2*c^3*x^
3 + 4*a^3*b*c^2*d*x^3 + a^4*c*d^2*x^3 + 2*a^3*b*c^3*x^2 + 3/2*a^4*c^2*d*x^
2 + a^4*c^3*x
```

Mupad [B] (verification not implemented)

Time = 21.78 (sec) , antiderivative size = 208, normalized size of antiderivative = 2.26

$$\int (a+bx)^4(c+dx)^3\,dx = x^4\left(\frac{a^4\,d^3}{4}+3\,a^3\,b\,c\,d^2+\frac{9\,a^2\,b^2\,c^2\,d}{2}+a\,b^3\,c^3\right) + x^5\left(\frac{4\,a^3\,b\,d^3}{5}+\frac{18\,a^2\,b^2\,c\,d^2}{5}+\frac{12\,a\,b^3\,c^2\,d}{5}+\frac{b^4\,c^3}{5}\right) + a^4\,c^3\,x+\frac{b^4\,d^3\,x^8}{8}+\frac{a^3\,c^2\,x^2\,(3\,a\,d+4\,b\,c)}{2} + \frac{b^3\,d^2\,x^7\,(4\,a\,d+3\,b\,c)}{7}+a^2\,c\,x^3\,\left(a^2\,d^2+4\,a\,b\,c\,d+2\,b^2\,c^2\right) + \frac{b^2\,d\,x^6\,\left(2\,a^2\,d^2+4\,a\,b\,c\,d+b^2\,c^2\right)}{2}$$

input
```
int((a + b*x)^4*(c + d*x)^3,x)
```

output
```
x^4*((a^4*d^3)/4 + a*b^3*c^3 + (9*a^2*b^2*c^2*d)/2 + 3*a^3*b*c*d^2) + x^5*
((b^4*c^3)/5 + (4*a^3*b*d^3)/5 + (18*a^2*b^2*c*d^2)/5 + (12*a*b^3*c^2*d)/5
) + a^4*c^3*x + (b^4*d^3*x^8)/8 + (a^3*c^2*x^2*(3*a*d + 4*b*c))/2 + (b^3*d
^2*x^7*(4*a*d + 3*b*c))/7 + a^2*c*x^3*(a^2*d^2 + 2*b^2*c^2 + 4*a*b*c*d) +
(b^2*d*x^6*(2*a^2*d^2 + b^2*c^2 + 4*a*b*c*d))/2
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 247, normalized size of antiderivative = 2.68

$$\int (a+bx)^4(c+dx)^3\,dx$$
$$= \frac{x(35b^4d^3x^7 + 160a\,b^3d^3x^6 + 120b^4c\,d^2x^6 + 280a^2b^2d^3x^5 + 560a\,b^3c\,d^2x^5 + 140b^4c^2d\,x^5 + 224a^3b\,d^3x^4 + 1}{}$$

input
```
int((b*x+a)^4*(d*x+c)^3,x)
```

output
```
(x*(280*a**4*c**3 + 420*a**4*c**2*d*x + 280*a**4*c*d**2*x**2 + 70*a**4*d**
3*x**3 + 560*a**3*b*c**3*x + 1120*a**3*b*c**2*d*x**2 + 840*a**3*b*c*d**2*x
**3 + 224*a**3*b*d**3*x**4 + 560*a**2*b**2*c**3*x**2 + 1260*a**2*b**2*c**2
*d*x**3 + 1008*a**2*b**2*c*d**2*x**4 + 280*a**2*b**2*d**3*x**5 + 280*a*b**
3*c**3*x**3 + 672*a*b**3*c**2*d*x**4 + 560*a*b**3*c*d**2*x**5 + 160*a*b**3
*d**3*x**6 + 56*b**4*c**3*x**4 + 140*b**4*c**2*d*x**5 + 120*b**4*c*d**2*x*
*6 + 35*b**4*d**3*x**7))/280
```

3.115 $\int (a+bx)^4 \left(c^3+3c^2dx+3cd^2x^2+d^3x^3\right) dx$

Optimal result

Integrand size = 35, antiderivative size = 92

$$\int (a+bx)^4 \left(c^3+3c^2dx+3cd^2x^2+d^3x^3\right) dx = \frac{(bc-ad)^3(a+bx)^5}{5b^4}+\frac{d(bc-ad)^2(a+bx)^6}{2b^4}+\frac{3d^2(bc-ad)(a+bx)^7}{7b^4}+\frac{d^3(a+bx)^8}{8b^4}$$

output
```
1/5*(-a*d+b*c)^3*(b*x+a)^5/b^4+1/2*d*(-a*d+b*c)^2*(b*x+a)^6/b^4+3/7*d^2*(-
a*d+b*c)*(b*x+a)^7/b^4+1/8*d^3*(b*x+a)^8/b^4
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 217 vs. 2(92) = 184.

Time = 0.01 (sec) , antiderivative size = 217, normalized size of antiderivative = 2.36

$$\begin{aligned}\int (a+bx)^4 \left(c^3+3c^2dx+3cd^2x^2+d^3x^3\right) dx &= a^4c^3x+\frac{1}{2}a^3c^2(4bc+3ad)x^2+a^2c\left(2b^2c^2+4abcd+a^2d^2\right)x^3\\&+\frac{1}{4}a\left(4b^3c^3+18ab^2c^2d+12a^2bcd^2+a^3d^3\right)x^4\\&+\frac{1}{5}b\left(b^3c^3+12ab^2c^2d+18a^2bcd^2+4a^3d^3\right)x^5\\&+\frac{1}{2}b^2d\left(b^2c^2+4abcd+2a^2d^2\right)x^6+\frac{1}{7}b^3d^2(3bc+4ad)x^7+\frac{1}{8}b^4d^3x^8\end{aligned}$$

input
```
Integrate[(a + b*x)^4*(c^3 + 3*c^2*d*x + 3*c*d^2*x^2 + d^3*x^3),x]
```

output
```
a^4*c^3*x + (a^3*c^2*(4*b*c + 3*a*d)*x^2)/2 + a^2*c*(2*b^2*c^2 + 4*a*b*c*d
 + a^2*d^2)*x^3 + (a*(4*b^3*c^3 + 18*a*b^2*c^2*d + 12*a^2*b*c*d^2 + a^3*d^
3)*x^4)/4 + (b*(b^3*c^3 + 12*a*b^2*c^2*d + 18*a^2*b*c*d^2 + 4*a^3*d^3)*x^5
)/5 + (b^2*d*(b^2*c^2 + 4*a*b*c*d + 2*a^2*d^2)*x^6)/2 + (b^3*d^2*(3*b*c +
4*a*d)*x^7)/7 + (b^4*d^3*x^8)/8
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {2006, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)^4 \left(c^3+3c^2dx+3cd^2x^2+d^3x^3\right) \, dx$$

$$\Big\downarrow \text{2006}$$

$$\int (a+bx)^4(c+dx)^3dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(\frac{3d^2(a+bx)^6(bc-ad)}{b^3}+\frac{3d(a+bx)^5(bc-ad)^2}{b^3}+\frac{(a+bx)^4(bc-ad)^3}{b^3}+\frac{d^3(a+bx)^7}{b^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{3d^2(a+bx)^7(bc-ad)}{7b^4}+\frac{d(a+bx)^6(bc-ad)^2}{2b^4}+\frac{(a+bx)^5(bc-ad)^3}{5b^4}+\frac{d^3(a+bx)^8}{8b^4}$$

input
```
Int[(a + b*x)^4*(c^3 + 3*c^2*d*x + 3*c*d^2*x^2 + d^3*x^3),x]
```

output
```
((b*c - a*d)^3*(a + b*x)^5)/(5*b^4) + (d*(b*c - a*d)^2*(a + b*x)^6)/(2*b^4
) + (3*d^2*(b*c - a*d)*(a + b*x)^7)/(7*b^4) + (d^3*(a + b*x)^8)/(8*b^4)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2006
```
Int[(u_.)*(Px_), x_Symbol] :> With[{a = Rt[Coeff[Px, x, 0], Expon[Px, x]],
b = Rt[Coeff[Px, x, Expon[Px, x]], Expon[Px, x]]}, Int[u*(a + b*x)^Expon[Px
, x], x] /; EqQ[Px, (a + b*x)^Expon[Px, x]]] /; PolyQ[Px, x] && GtQ[Expon[P
x, x], 1] && NeQ[Coeff[Px, x, 0], 0] &&  !MatchQ[Px, (a_.)*(v_)^Expon[Px, x
] /; FreeQ[a, x] && LinearQ[v, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 221 vs. $2(84) = 168$.

Time = 0.14 (sec) , antiderivative size = 222, normalized size of antiderivative = 2.41

method	result
norman	$\frac{b^4 d^3 x^8}{8} + \left(\frac{4}{7} b^3 a\, d^3 + \frac{3}{7} b^4 c\, d^2\right) x^7 + \left(a^2 b^2 d^3 + 2 a\, b^3 c\, d^2 + \frac{1}{2} b^4 c^2 d\right) x^6 + \left(\frac{4}{5} a^3 b\, d^3 + \frac{18}{5} a^2 b^2 c\, d^2\right.$
default	$\frac{b^4 d^3 x^8}{8} + \frac{(4 b^3 a\, d^3 + 3 b^4 c\, d^2) x^7}{7} + \frac{(6 a^2 b^2 d^3 + 12 a\, b^3 c\, d^2 + 3 b^4 c^2 d) x^6}{6} + \frac{(4 a^3 b\, d^3 + 18 a^2 b^2 c\, d^2 + 12 b^3 a\, c^2 d + b^4 c^3) x^5}{5} + \frac{(a}{}$
risch	$\frac{1}{8} b^4 d^3 x^8 + \frac{4}{7} x^7 b^3 a\, d^3 + \frac{3}{7} x^7 b^4 c\, d^2 + x^6 a^2 b^2 d^3 + 2 x^6 a\, b^3 c\, d^2 + \frac{1}{2} x^6 b^4 c^2 d + \frac{4}{5} x^5 a^3 b\, d^3 + \frac{18}{5} x^5 a$
parallelrisch	$\frac{1}{8} b^4 d^3 x^8 + \frac{4}{7} x^7 b^3 a\, d^3 + \frac{3}{7} x^7 b^4 c\, d^2 + x^6 a^2 b^2 d^3 + 2 x^6 a\, b^3 c\, d^2 + \frac{1}{2} x^6 b^4 c^2 d + \frac{4}{5} x^5 a^3 b\, d^3 + \frac{18}{5} x^5 a$
gosper	$\frac{x (35 b^4 d^3 x^7 + 160 a\, b^3 d^3 x^6 + 120 b^4 c\, d^2 x^6 + 280 a^2 b^2 d^3 x^5 + 560 a\, b^3 c\, d^2 x^5 + 140 b^4 c^2 d\, x^5 + 224 a^3 b\, d^3 x^4 + 1008 a^2 b^2 c\, d^2 x^4 + 672 a\, b^3}{}$
orering	$\frac{x (35 b^4 d^3 x^7 + 160 a\, b^3 d^3 x^6 + 120 b^4 c\, d^2 x^6 + 280 a^2 b^2 d^3 x^5 + 560 a\, b^3 c\, d^2 x^5 + 140 b^4 c^2 d\, x^5 + 224 a^3 b\, d^3 x^4 + 1008 a^2 b^2 c\, d^2 x^4 + 672 a\, b^3}{}$

input
```
int((b*x+a)^4*(d^3*x^3+3*c*d^2*x^2+3*c^2*d*x+c^3),x,method=_RETURNVERBOSE)
```

output
```
1/8*b^4*d^3*x^8+(4/7*b^3*a*d^3+3/7*b^4*c*d^2)*x^7+(a^2*b^2*d^3+2*a*b^3*c*d
^2+1/2*b^4*c^2*d)*x^6+(4/5*a^3*b*d^3+18/5*a^2*b^2*c*d^2+12/5*b^3*a*c^2*d+1
/5*b^4*c^3)*x^5+(1/4*a^4*d^3+3*a^3*b*c*d^2+9/2*a^2*b^2*c^2*d+a*b^3*c^3)*x^
4+(a^4*c*d^2+4*a^3*b*c^2*d+2*a^2*b^2*c^3)*x^3+(3/2*a^4*c^2*d+2*a^3*b*c^3)*
x^2+a^4*c^3*x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 225 vs. $2(84) = 168$.

Time = 0.10 (sec) , antiderivative size = 225, normalized size of antiderivative = 2.45

$$\int (a+bx)^4 \left(c^3+3c^2dx+3cd^2x^2+d^3x^3\right) dx = \frac{1}{8}b^4d^3x^8+a^4c^3x+\frac{1}{7}\left(3b^4cd^2+4ab^3d^3\right)x^7+\frac{1}{2}\left(b^4c^2d+4ab^3cd^2+2a^2b^2d^3\right)x^6 + \frac{1}{5}\left(b^4c^3+12ab^3c^2d+18a^2b^2cd^2+4a^3bd^3\right)x^5 + \frac{1}{4}\left(4ab^3c^3+18a^2b^2c^2d+12a^3bcd^2+a^4d^3\right)x^4 + \left(2a^2b^2c^3+4a^3bc^2d+a^4cd^2\right)x^3+\frac{1}{2}\left(4a^3bc^3+3a^4c^2d\right)x^2$$

input
```
integrate((b*x+a)^4*(d^3*x^3+3*c*d^2*x^2+3*c^2*d*x+c^3),x, algorithm="fric
as")
```

output
```
1/8*b^4*d^3*x^8 + a^4*c^3*x + 1/7*(3*b^4*c*d^2 + 4*a*b^3*d^3)*x^7 + 1/2*(b
^4*c^2*d + 4*a*b^3*c*d^2 + 2*a^2*b^2*d^3)*x^6 + 1/5*(b^4*c^3 + 12*a*b^3*c^
2*d + 18*a^2*b^2*c*d^2 + 4*a^3*b*d^3)*x^5 + 1/4*(4*a*b^3*c^3 + 18*a^2*b^2*
c^2*d + 12*a^3*b*c*d^2 + a^4*d^3)*x^4 + (2*a^2*b^2*c^3 + 4*a^3*b*c^2*d + a
^4*c*d^2)*x^3 + 1/2*(4*a^3*b*c^3 + 3*a^4*c^2*d)*x^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 243 vs. $2(80) = 160$.

Time = 0.05 (sec) , antiderivative size = 243, normalized size of antiderivative = 2.64

$$\int (a+bx)^4 \left(c^3+3c^2dx+3cd^2x^2+d^3x^3\right)\,dx$$
$$= a^4c^3x + \frac{b^4d^3x^8}{8} + x^7\cdot\left(\frac{4ab^3d^3}{7}+\frac{3b^4cd^2}{7}\right) + x^6\left(a^2b^2d^3+2ab^3cd^2+\frac{b^4c^2d}{2}\right) + x^5\cdot\left(\frac{4a^3bd^3}{5}+\frac{18a^2b^2cd^2}{5}+\frac{12ab^3c^2d}{5}+\frac{b^4c^3}{5}\right) + x^4\left(\frac{a^4d^3}{4}+3a^3bcd^2+\frac{9a^2b^2c^2d}{2}+ab^3c^3\right) + x^3\left(a^4cd^2+4a^3bc^2d+2a^2b^2c^3\right) + x^2\cdot\left(\frac{3a^4c^2d}{2}+2a^3bc^3\right)$$

input
```
integrate((b*x+a)**4*(d**3*x**3+3*c*d**2*x**2+3*c**2*d*x+c**3),x)
```

output
```
a**4*c**3*x + b**4*d**3*x**8/8 + x**7*(4*a*b**3*d**3/7 + 3*b**4*c*d**2/7)
+ x**6*(a**2*b**2*d**3 + 2*a*b**3*c*d**2 + b**4*c**2*d/2) + x**5*(4*a**3*b
*d**3/5 + 18*a**2*b**2*c*d**2/5 + 12*a*b**3*c**2*d/5 + b**4*c**3/5) + x**4
*(a**4*d**3/4 + 3*a**3*b*c*d**2 + 9*a**2*b**2*c**2*d/2 + a*b**3*c**3) + x*
*3*(a**4*c*d**2 + 4*a**3*b*c**2*d + 2*a**2*b**2*c**3) + x**2*(3*a**4*c**2*
d/2 + 2*a**3*b*c**3)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 225 vs. $2(84) = 168$.

Time = 0.03 (sec) , antiderivative size = 225, normalized size of antiderivative = 2.45

$$\int (a+bx)^4 \left(c^3+3c^2dx+3cd^2x^2+d^3x^3\right)\,dx$$
$$= \frac{1}{8}\,b^4d^3x^8 + a^4c^3x + \frac{1}{7}\left(3\,b^4cd^2+4\,ab^3d^3\right)x^7 + \frac{1}{2}\left(b^4c^2d+4\,ab^3cd^2+2\,a^2b^2d^3\right)x^6 + \frac{1}{5}\left(b^4c^3+12\,ab^3c^2d+18\,a^2b^2cd^2+4\,a^3bd^3\right)x^5 + \frac{1}{4}\left(4\,ab^3c^3+18\,a^2b^2c^2d+12\,a^3bcd^2+a^4d^3\right)x^4 + \left(2\,a^2b^2c^3+4\,a^3bc^2d+a^4cd^2\right)x^3 + \frac{1}{2}\left(4\,a^3bc^3+3\,a^4c^2d\right)x^2$$

input
```
integrate((b*x+a)^4*(d^3*x^3+3*c*d^2*x^2+3*c^2*d*x+c^3),x, algorithm="maxi
ma")
```

output
```
1/8*b^4*d^3*x^8 + a^4*c^3*x + 1/7*(3*b^4*c*d^2 + 4*a*b^3*d^3)*x^7 + 1/2*(b
^4*c^2*d + 4*a*b^3*c*d^2 + 2*a^2*b^2*d^3)*x^6 + 1/5*(b^4*c^3 + 12*a*b^3*c^
2*d + 18*a^2*b^2*c*d^2 + 4*a^3*b*d^3)*x^5 + 1/4*(4*a*b^3*c^3 + 18*a^2*b^2*
c^2*d + 12*a^3*b*c*d^2 + a^4*d^3)*x^4 + (2*a^2*b^2*c^3 + 4*a^3*b*c^2*d + a
^4*c*d^2)*x^3 + 1/2*(4*a^3*b*c^3 + 3*a^4*c^2*d)*x^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 245 vs. $2(84) = 168$.

Time = 0.13 (sec) , antiderivative size = 245, normalized size of antiderivative = 2.66

$$\int (a+bx)^4 \left(c^3+3c^2dx+3cd^2x^2+d^3x^3\right) dx$$
$$= \frac{1}{8}b^4d^3x^8 + \frac{3}{7}b^4cd^2x^7 + \frac{4}{7}ab^3d^3x^7 + \frac{1}{2}b^4c^2dx^6 + 2ab^3cd^2x^6 + a^2b^2d^3x^6 + \frac{1}{5}b^4c^3x^5 + \frac{12}{5}ab^3c^2dx^5 + \frac{18}{5}a^2b^2cd^2x^5 + \frac{4}{5}a^3bd^3x^5 + ab^3c^3x^4 + \frac{9}{2}a^2b^2c^2dx^4 + 3a^3bcd^2x^4 + \frac{1}{4}a^4d^3x^4 + 2a^2b^2c^3x^3 + 4a^3bc^2dx^3 + a^4cd^2x^3 + 2a^3bc^3x^2 + \frac{3}{2}a^4c^2dx^2 + a^4c^3x$$

input
```
integrate((b*x+a)^4*(d^3*x^3+3*c*d^2*x^2+3*c^2*d*x+c^3),x, algorithm="giac
")
```

output
```
1/8*b^4*d^3*x^8 + 3/7*b^4*c*d^2*x^7 + 4/7*a*b^3*d^3*x^7 + 1/2*b^4*c^2*d*x^
6 + 2*a*b^3*c*d^2*x^6 + a^2*b^2*d^3*x^6 + 1/5*b^4*c^3*x^5 + 12/5*a*b^3*c^2
*d*x^5 + 18/5*a^2*b^2*c*d^2*x^5 + 4/5*a^3*b*d^3*x^5 + a*b^3*c^3*x^4 + 9/2*
a^2*b^2*c^2*d*x^4 + 3*a^3*b*c*d^2*x^4 + 1/4*a^4*d^3*x^4 + 2*a^2*b^2*c^3*x^
3 + 4*a^3*b*c^2*d*x^3 + a^4*c*d^2*x^3 + 2*a^3*b*c^3*x^2 + 3/2*a^4*c^2*d*x^
2 + a^4*c^3*x
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 208, normalized size of antiderivative = 2.26

$$\int (a+bx)^4 \left(c^3+3c^2dx+3cd^2x^2+d^3x^3\right) \, dx$$
$$= x^4 \left(\frac{a^4 d^3}{4} + 3 a^3 b c d^2 + \frac{9 a^2 b^2 c^2 d}{2} + a b^3 c^3\right) + x^5 \left(\frac{4 a^3 b d^3}{5} + \frac{18 a^2 b^2 c d^2}{5} + \frac{12 a b^3 c^2 d}{5} + \frac{b^4 c^3}{5}\right) + a^4 c^3 x + \frac{b^4 d^3 x^8}{8} + \frac{a^3 c^2 x^2 (3 a d + 4 b c)}{2} + \frac{b^3 d^2 x^7 (4 a d + 3 b c)}{7} + a^2 c x^3 \left(a^2 d^2 + 4 a b c d + 2 b^2 c^2\right) + \frac{b^2 d x^6 (2 a^2 d^2 + 4 a b c d + b^2 c^2)}{2}$$

input
```
int((a + b*x)^4*(c^3 + d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x),x)
```

output
```
x^4*((a^4*d^3)/4 + a*b^3*c^3 + (9*a^2*b^2*c^2*d)/2 + 3*a^3*b*c*d^2) + x^5*
((b^4*c^3)/5 + (4*a^3*b*d^3)/5 + (18*a^2*b^2*c*d^2)/5 + (12*a*b^3*c^2*d)/5
) + a^4*c^3*x + (b^4*d^3*x^8)/8 + (a^3*c^2*x^2*(3*a*d + 4*b*c))/2 + (b^3*d
^2*x^7*(4*a*d + 3*b*c))/7 + a^2*c*x^3*(a^2*d^2 + 2*b^2*c^2 + 4*a*b*c*d) +
(b^2*d*x^6*(2*a^2*d^2 + b^2*c^2 + 4*a*b*c*d))/2
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 247, normalized size of antiderivative = 2.68

$$\int (a+bx)^4 \left(c^3+3c^2dx+3cd^2x^2+d^3x^3\right) \, dx$$
$$= \frac{x(35b^4d^3x^7 + 160a\,b^3d^3x^6 + 120b^4c\,d^2x^6 + 280a^2b^2d^3x^5 + 560a\,b^3c\,d^2x^5 + 140b^4c^2d\,x^5 + 224a^3b\,d^3x^4 + 1}{}$$

input
```
int((b*x+a)^4*(d^3*x^3+3*c*d^2*x^2+3*c^2*d*x+c^3),x)
```

output

```
(x*(280*a**4*c**3 + 420*a**4*c**2*d*x + 280*a**4*c*d**2*x**2 + 70*a**4*d**
3*x**3 + 560*a**3*b*c**3*x + 1120*a**3*b*c**2*d*x**2 + 840*a**3*b*c*d**2*x
**3 + 224*a**3*b*d**3*x**4 + 560*a**2*b**2*c**3*x**2 + 1260*a**2*b**2*c**2
*d*x**3 + 1008*a**2*b**2*c*d**2*x**4 + 280*a**2*b**2*d**3*x**5 + 280*a*b**
3*c**3*x**3 + 672*a*b**3*c**2*d*x**4 + 560*a*b**3*c*d**2*x**5 + 160*a*b**3
*d**3*x**6 + 56*b**4*c**3*x**4 + 140*b**4*c**2*d*x**5 + 120*b**4*c*d**2*x*
*6 + 35*b**4*d**3*x**7))/280
```

3.116 $\int (c+dx)^3 \left(a^4+4a^3bx+6a^2b^2x^2+4ab^3x^3+b^4x^4\right) d$

Optimal result

Integrand size = 46, antiderivative size = 92

$$\int (c+dx)^3 \left(a^4+4a^3bx+6a^2b^2x^2+4ab^3x^3+b^4x^4\right) dx = \frac{(bc-ad)^3(a+bx)^5}{5b^4}+\frac{d(bc-ad)^2(a+bx)^6}{2b^4}+\frac{3d^2(bc-ad)(a+bx)^7}{7b^4}+\frac{d^3(a+bx)^8}{8b^4}$$

output
```
1/5*(-a*d+b*c)^3*(b*x+a)^5/b^4+1/2*d*(-a*d+b*c)^2*(b*x+a)^6/b^4+3/7*d^2*(-
a*d+b*c)*(b*x+a)^7/b^4+1/8*d^3*(b*x+a)^8/b^4
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 217 vs. 2(92) = 184.

Time = 0.01 (sec) , antiderivative size = 217, normalized size of antiderivative = 2.36

$$\begin{aligned}\int (c+dx)^3 \left(a^4+4a^3bx+6a^2b^2x^2+4ab^3x^3+b^4x^4\right) dx &= a^4c^3x+\frac{1}{2}a^3c^2(4bc+3ad)x^2+a^2c\left(2b^2c^2+4abcd+a^2d^2\right)x^3 \\ &+\frac{1}{4}a\left(4b^3c^3+18ab^2c^2d+12a^2bcd^2+a^3d^3\right)x^4 \\ &+\frac{1}{5}b\left(b^3c^3+12ab^2c^2d+18a^2bcd^2+4a^3d^3\right)x^5 \\ &+\frac{1}{2}b^2d\left(b^2c^2+4abcd+2a^2d^2\right)x^6+\frac{1}{7}b^3d^2(3bc+4ad)x^7+\frac{1}{8}b^4d^3x^8\end{aligned}$$

input
```
Integrate[(c + d*x)^3*(a^4 + 4*a^3*b*x + 6*a^2*b^2*x^2 + 4*a*b^3*x^3 + b^4
*x^4),x]
```

output
```
a^4*c^3*x + (a^3*c^2*(4*b*c + 3*a*d)*x^2)/2 + a^2*c*(2*b^2*c^2 + 4*a*b*c*d
 + a^2*d^2)*x^3 + (a*(4*b^3*c^3 + 18*a*b^2*c^2*d + 12*a^2*b*c*d^2 + a^3*d^
3)*x^4)/4 + (b*(b^3*c^3 + 12*a*b^2*c^2*d + 18*a^2*b*c*d^2 + 4*a^3*d^3)*x^5
)/5 + (b^2*d*(b^2*c^2 + 4*a*b*c*d + 2*a^2*d^2)*x^6)/2 + (b^3*d^2*(3*b*c +
4*a*d)*x^7)/7 + (b^4*d^3*x^8)/8
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {2006, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a^4+4a^3bx+6a^2b^2x^2+4ab^3x^3+b^4x^4\right)(c+dx)^3\,dx$$

$$\Big\downarrow \text{2006}$$

$$\int (a+bx)^4(c+dx)^3dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(\frac{3d^2(a+bx)^6(bc-ad)}{b^3}+\frac{3d(a+bx)^5(bc-ad)^2}{b^3}+\frac{(a+bx)^4(bc-ad)^3}{b^3}+\frac{d^3(a+bx)^7}{b^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{3d^2(a+bx)^7(bc-ad)}{7b^4}+\frac{d(a+bx)^6(bc-ad)^2}{2b^4}+\frac{(a+bx)^5(bc-ad)^3}{5b^4}+\frac{d^3(a+bx)^8}{8b^4}$$

input
```
Int[(c + d*x)^3*(a^4 + 4*a^3*b*x + 6*a^2*b^2*x^2 + 4*a*b^3*x^3 + b^4*x^4),
x]
```

output
```
((b*c - a*d)^3*(a + b*x)^5)/(5*b^4) + (d*(b*c - a*d)^2*(a + b*x)^6)/(2*b^4
) + (3*d^2*(b*c - a*d)*(a + b*x)^7)/(7*b^4) + (d^3*(a + b*x)^8)/(8*b^4)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2006
```
Int[(u_.)*(Px_), x_Symbol] :> With[{a = Rt[Coeff[Px, x, 0], Expon[Px, x]],
b = Rt[Coeff[Px, x, Expon[Px, x]], Expon[Px, x]]}, Int[u*(a + b*x)^Expon[Px
, x], x] /; EqQ[Px, (a + b*x)^Expon[Px, x]]] /; PolyQ[Px, x] && GtQ[Expon[P
x, x], 1] && NeQ[Coeff[Px, x, 0], 0] &&  !MatchQ[Px, (a_.)*(v_)^Expon[Px, x
] /; FreeQ[a, x] && LinearQ[v, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 221 vs. $2(84) = 168$.

Time = 0.13 (sec) , antiderivative size = 222, normalized size of antiderivative = 2.41

method	result
norman	$\frac{b^4d^3x^8}{8}+\left(\frac{4}{7}b^3a\,d^3+\frac{3}{7}b^4c\,d^2\right)x^7+\left(a^2b^2d^3+2a\,b^3c\,d^2+\frac{1}{2}b^4c^2d\right)x^6+\left(\frac{4}{5}a^3b\,d^3+\frac{18}{5}a^2b^2c\,d^2 \ldots$
default	$\frac{b^4d^3x^8}{8}+\frac{(4b^3a\,d^3+3b^4c\,d^2)x^7}{7}+\frac{(6a^2b^2d^3+12a\,b^3c\,d^2+3b^4c^2d)x^6}{6}+\frac{(4a^3b\,d^3+18a^2b^2c\,d^2+12b^3a\,c^2d+b^4c^3)x^5}{5}+ \ldots$
risch	$\frac{1}{8}b^4d^3x^8+\frac{4}{7}x^7b^3a\,d^3+\frac{3}{7}x^7b^4c\,d^2+x^6a^2b^2d^3+2x^6a\,b^3c\,d^2+\frac{1}{2}x^6b^4c^2d+\frac{4}{5}x^5a^3b\,d^3+\frac{18}{5}x^5a \ldots$
parallelrisch	$\frac{1}{8}b^4d^3x^8+\frac{4}{7}x^7b^3a\,d^3+\frac{3}{7}x^7b^4c\,d^2+x^6a^2b^2d^3+2x^6a\,b^3c\,d^2+\frac{1}{2}x^6b^4c^2d+\frac{4}{5}x^5a^3b\,d^3+\frac{18}{5}x^5a \ldots$
gosper	$x(35b^4d^3x^7+160a\,b^3d^3x^6+120b^4c\,d^2x^6+280a^2b^2d^3x^5+560a\,b^3c\,d^2x^5+140b^4c^2d\,x^5+224a^3b\,d^3x^4+1008a^2b^2c\,d^2x^4+672a\,b^3 \ldots$
orering	$x(35b^4d^3x^7+160a\,b^3d^3x^6+120b^4c\,d^2x^6+280a^2b^2d^3x^5+560a\,b^3c\,d^2x^5+140b^4c^2d\,x^5+224a^3b\,d^3x^4+1008a^2b^2c\,d^2x^4+672a\,b^3 \ldots$

input
```
int((d*x+c)^3*(b^4*x^4+4*a*b^3*x^3+6*a^2*b^2*x^2+4*a^3*b*x+a^4),x,method=_
RETURNVERBOSE)
```

output
```
1/8*b^4*d^3*x^8+(4/7*b^3*a*d^3+3/7*b^4*c*d^2)*x^7+(a^2*b^2*d^3+2*a*b^3*c*d
^2+1/2*b^4*c^2*d)*x^6+(4/5*a^3*b*d^3+18/5*a^2*b^2*c*d^2+12/5*b^3*a*c^2*d+1
/5*b^4*c^3)*x^5+(1/4*a^4*d^3+3*a^3*b*c*d^2+9/2*a^2*b^2*c^2*d+a*b^3*c^3)*x^
4+(a^4*c*d^2+4*a^3*b*c^2*d+2*a^2*b^2*c^3)*x^3+(3/2*a^4*c^2*d+2*a^3*b*c^3)*
x^2+a^4*c^3*x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 225 vs. $2(84) = 168$.

Time = 0.11 (sec) , antiderivative size = 225, normalized size of antiderivative = 2.45

$$\begin{aligned}
&\int (c+dx)^3 \left(a^4+4a^3bx+6a^2b^2x^2+4ab^3x^3+b^4x^4\right)\,dx \\
&= \frac{1}{8}\,b^4d^3x^8 + a^4c^3x + \frac{1}{7}\left(3\,b^4cd^2+4\,ab^3d^3\right)x^7 + \frac{1}{2}\left(b^4c^2d+4\,ab^3cd^2+2\,a^2b^2d^3\right)x^6 \\
&\quad + \frac{1}{5}\left(b^4c^3+12\,ab^3c^2d+18\,a^2b^2cd^2+4\,a^3bd^3\right)x^5 \\
&\quad + \frac{1}{4}\left(4\,ab^3c^3+18\,a^2b^2c^2d+12\,a^3bcd^2+a^4d^3\right)x^4 \\
&\quad + \left(2\,a^2b^2c^3+4\,a^3bc^2d+a^4cd^2\right)x^3 + \frac{1}{2}\left(4\,a^3bc^3+3\,a^4c^2d\right)x^2
\end{aligned}$$

input
```
integrate((d*x+c)^3*(b^4*x^4+4*a*b^3*x^3+6*a^2*b^2*x^2+4*a^3*b*x+a^4),x, a
lgorithm="fricas")
```

output
```
1/8*b^4*d^3*x^8 + a^4*c^3*x + 1/7*(3*b^4*c*d^2 + 4*a*b^3*d^3)*x^7 + 1/2*(b
^4*c^2*d + 4*a*b^3*c*d^2 + 2*a^2*b^2*d^3)*x^6 + 1/5*(b^4*c^3 + 12*a*b^3*c^
2*d + 18*a^2*b^2*c*d^2 + 4*a^3*b*d^3)*x^5 + 1/4*(4*a*b^3*c^3 + 18*a^2*b^2*
c^2*d + 12*a^3*b*c*d^2 + a^4*d^3)*x^4 + (2*a^2*b^2*c^3 + 4*a^3*b*c^2*d + a
^4*c*d^2)*x^3 + 1/2*(4*a^3*b*c^3 + 3*a^4*c^2*d)*x^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 243 vs. $2(80) = 160$.

Time = 0.04 (sec) , antiderivative size = 243, normalized size of antiderivative = 2.64

$$\int (c+dx)^3 \left(a^4+4a^3bx+6a^2b^2x^2+4ab^3x^3+b^4x^4\right)\,dx = a^4c^3x + \frac{b^4d^3x^8}{8} + x^7\cdot\left(\frac{4ab^3d^3}{7}+\frac{3b^4cd^2}{7}\right) + x^6\left(a^2b^2d^3+2ab^3cd^2+\frac{b^4c^2d}{2}\right) + x^5\cdot\left(\frac{4a^3bd^3}{5}+\frac{18a^2b^2cd^2}{5}+\frac{12ab^3c^2d}{5}+\frac{b^4c^3}{5}\right) + x^4\left(\frac{a^4d^3}{4}+3a^3bcd^2+\frac{9a^2b^2c^2d}{2}+ab^3c^3\right) + x^3\left(a^4cd^2+4a^3bc^2d+2a^2b^2c^3\right) + x^2\cdot\left(\frac{3a^4c^2d}{2}+2a^3bc^3\right)$$

input
```
integrate((d*x+c)**3*(b**4*x**4+4*a*b**3*x**3+6*a**2*b**2*x**2+4*a**3*b*x+
a**4),x)
```

output
```
a**4*c**3*x + b**4*d**3*x**8/8 + x**7*(4*a*b**3*d**3/7 + 3*b**4*c*d**2/7)
+ x**6*(a**2*b**2*d**3 + 2*a*b**3*c*d**2 + b**4*c**2*d/2) + x**5*(4*a**3*b
*d**3/5 + 18*a**2*b**2*c*d**2/5 + 12*a*b**3*c**2*d/5 + b**4*c**3/5) + x**4
*(a**4*d**3/4 + 3*a**3*b*c*d**2 + 9*a**2*b**2*c**2*d/2 + a*b**3*c**3) + x*
*3*(a**4*c*d**2 + 4*a**3*b*c**2*d + 2*a**2*b**2*c**3) + x**2*(3*a**4*c**2*
d/2 + 2*a**3*b*c**3)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 225 vs. $2(84) = 168$.

Time = 0.03 (sec) , antiderivative size = 225, normalized size of antiderivative = 2.45

$$\int (c+dx)^3 \left(a^4+4a^3bx+6a^2b^2x^2+4ab^3x^3+b^4x^4\right)\,dx = \frac{1}{8}\,b^4d^3x^8 + a^4c^3x + \frac{1}{7}\left(3\,b^4cd^2+4\,ab^3d^3\right)x^7 + \frac{1}{2}\left(b^4c^2d+4\,ab^3cd^2+2\,a^2b^2d^3\right)x^6 + \frac{1}{5}\left(b^4c^3+12\,ab^3c^2d+18\,a^2b^2cd^2+4\,a^3bd^3\right)x^5 + \frac{1}{4}\left(4\,ab^3c^3+18\,a^2b^2c^2d+12\,a^3bcd^2+a^4d^3\right)x^4 + \left(2\,a^2b^2c^3+4\,a^3bc^2d+a^4cd^2\right)x^3 + \frac{1}{2}\left(4\,a^3bc^3+3\,a^4c^2d\right)x^2$$

input
```
integrate((d*x+c)^3*(b^4*x^4+4*a*b^3*x^3+6*a^2*b^2*x^2+4*a^3*b*x+a^4),x, a
lgorithm="maxima")
```

output
```
1/8*b^4*d^3*x^8 + a^4*c^3*x + 1/7*(3*b^4*c*d^2 + 4*a*b^3*d^3)*x^7 + 1/2*(b
^4*c^2*d + 4*a*b^3*c*d^2 + 2*a^2*b^2*d^3)*x^6 + 1/5*(b^4*c^3 + 12*a*b^3*c^
2*d + 18*a^2*b^2*c*d^2 + 4*a^3*b*d^3)*x^5 + 1/4*(4*a*b^3*c^3 + 18*a^2*b^2*
c^2*d + 12*a^3*b*c*d^2 + a^4*d^3)*x^4 + (2*a^2*b^2*c^3 + 4*a^3*b*c^2*d + a
^4*c*d^2)*x^3 + 1/2*(4*a^3*b*c^3 + 3*a^4*c^2*d)*x^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 245 vs. $2(84) = 168$.

Time = 0.13 (sec) , antiderivative size = 245, normalized size of antiderivative = 2.66

$$\begin{aligned}\int (c+dx)^3 \left(a^4+4a^3bx+6a^2b^2x^2+4ab^3x^3+b^4x^4\right) dx &= \frac{1}{8}b^4d^3x^8+\frac{3}{7}b^4cd^2x^7+\frac{4}{7}ab^3d^3x^7+\frac{1}{2}b^4c^2dx^6+2\,ab^3cd^2x^6+a^2b^2d^3x^6+\frac{1}{5}b^4c^3x^5 \\ &+\frac{12}{5}ab^3c^2dx^5+\frac{18}{5}a^2b^2cd^2x^5+\frac{4}{5}a^3bd^3x^5+ab^3c^3x^4+\frac{9}{2}a^2b^2c^2dx^4+3\,a^3bcd^2x^4 \\ &+\frac{1}{4}a^4d^3x^4+2\,a^2b^2c^3x^3+4\,a^3bc^2dx^3+a^4cd^2x^3+2\,a^3bc^3x^2+\frac{3}{2}a^4c^2dx^2+a^4c^3x\end{aligned}$$

input
```
integrate((d*x+c)^3*(b^4*x^4+4*a*b^3*x^3+6*a^2*b^2*x^2+4*a^3*b*x+a^4),x, a
lgorithm="giac")
```

output
```
1/8*b^4*d^3*x^8 + 3/7*b^4*c*d^2*x^7 + 4/7*a*b^3*d^3*x^7 + 1/2*b^4*c^2*d*x^
6 + 2*a*b^3*c*d^2*x^6 + a^2*b^2*d^3*x^6 + 1/5*b^4*c^3*x^5 + 12/5*a*b^3*c^2
*d*x^5 + 18/5*a^2*b^2*c*d^2*x^5 + 4/5*a^3*b*d^3*x^5 + a*b^3*c^3*x^4 + 9/2*
a^2*b^2*c^2*d*x^4 + 3*a^3*b*c*d^2*x^4 + 1/4*a^4*d^3*x^4 + 2*a^2*b^2*c^3*x^
3 + 4*a^3*b*c^2*d*x^3 + a^4*c*d^2*x^3 + 2*a^3*b*c^3*x^2 + 3/2*a^4*c^2*d*x^
2 + a^4*c^3*x
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 208, normalized size of antiderivative = 2.26

$$\int (c+dx)^3\left(a^4+4a^3bx+6a^2b^2x^2+4ab^3x^3+b^4x^4\right)\,dx$$
$$=x^4\left(\frac{a^4\,d^3}{4}+3\,a^3\,b\,c\,d^2+\frac{9\,a^2\,b^2\,c^2\,d}{2}+a\,b^3\,c^3\right)$$
$$+x^5\left(\frac{4\,a^3\,b\,d^3}{5}+\frac{18\,a^2\,b^2\,c\,d^2}{5}+\frac{12\,a\,b^3\,c^2\,d}{5}+\frac{b^4\,c^3}{5}\right)+a^4\,c^3\,x$$
$$+\frac{b^4\,d^3\,x^8}{8}+\frac{a^3\,c^2\,x^2\,(3\,a\,d+4\,b\,c)}{2}+\frac{b^3\,d^2\,x^7\,(4\,a\,d+3\,b\,c)}{7}$$
$$+a^2\,c\,x^3\left(a^2\,d^2+4\,a\,b\,c\,d+2\,b^2\,c^2\right)+\frac{b^2\,d\,x^6\,(2\,a^2\,d^2+4\,a\,b\,c\,d+b^2\,c^2)}{2}$$

input
```
int((c + d*x)^3*(a^4 + b^4*x^4 + 4*a*b^3*x^3 + 6*a^2*b^2*x^2 + 4*a^3*b*x),
x)
```

output
```
x^4*((a^4*d^3)/4 + a*b^3*c^3 + (9*a^2*b^2*c^2*d)/2 + 3*a^3*b*c*d^2) + x^5*
((b^4*c^3)/5 + (4*a^3*b*d^3)/5 + (18*a^2*b^2*c*d^2)/5 + (12*a*b^3*c^2*d)/5
) + a^4*c^3*x + (b^4*d^3*x^8)/8 + (a^3*c^2*x^2*(3*a*d + 4*b*c))/2 + (b^3*d
^2*x^7*(4*a*d + 3*b*c))/7 + a^2*c*x^3*(a^2*d^2 + 2*b^2*c^2 + 4*a*b*c*d) +
(b^2*d*x^6*(2*a^2*d^2 + b^2*c^2 + 4*a*b*c*d))/2
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 247, normalized size of antiderivative = 2.68

$$\int (c+dx)^3\left(a^4+4a^3bx+6a^2b^2x^2+4ab^3x^3+b^4x^4\right)\,dx$$
$$=\frac{x(35b^4d^3x^7+160a\,b^3d^3x^6+120b^4c\,d^2x^6+280a^2b^2d^3x^5+560a\,b^3c\,d^2x^5+140b^4c^2d\,x^5+224a^3b\,d^3x^4+1}{}$$

input
```
int((d*x+c)^3*(b^4*x^4+4*a*b^3*x^3+6*a^2*b^2*x^2+4*a^3*b*x+a^4),x)
```

output

```
(x*(280*a**4*c**3 + 420*a**4*c**2*d*x + 280*a**4*c*d**2*x**2 + 70*a**4*d**
3*x**3 + 560*a**3*b*c**3*x + 1120*a**3*b*c**2*d*x**2 + 840*a**3*b*c*d**2*x
**3 + 224*a**3*b*d**3*x**4 + 560*a**2*b**2*c**3*x**2 + 1260*a**2*b**2*c**2
*d*x**3 + 1008*a**2*b**2*c*d**2*x**4 + 280*a**2*b**2*d**3*x**5 + 280*a*b**
3*c**3*x**3 + 672*a*b**3*c**2*d*x**4 + 560*a*b**3*c*d**2*x**5 + 160*a*b**3
*d**3*x**6 + 56*b**4*c**3*x**4 + 140*b**4*c**2*d*x**5 + 120*b**4*c*d**2*x*
*6 + 35*b**4*d**3*x**7))/280
```

3.117 $\int (c^3+3c^2dx+3cd^2x^2+d^3x^3)(a^4+4a^3bx+6a^2b^2$

Optimal result

Integrand size = 66, antiderivative size = 92

$$\int (c^3+3c^2dx+3cd^2x^2+d^3x^3)(a^4+4a^3bx+6a^2b^2x^2+4ab^3x^3+b^4x^4)\,dx = \frac{(bc-ad)^3(a+bx)^5}{5b^4}+\frac{d(bc-ad)^2(a+bx)^6}{2b^4}+\frac{3d^2(bc-ad)(a+bx)^7}{7b^4}+\frac{d^3(a+bx)^8}{8b^4}$$

output
```
1/5*(-a*d+b*c)^3*(b*x+a)^5/b^4+1/2*d*(-a*d+b*c)^2*(b*x+a)^6/b^4+3/7*d^2*(-
a*d+b*c)*(b*x+a)^7/b^4+1/8*d^3*(b*x+a)^8/b^4
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 217 vs. 2(92) = 184.

Time = 0.01 (sec) , antiderivative size = 217, normalized size of antiderivative = 2.36

$$\begin{aligned}\int (c^3+3c^2dx+3cd^2x^2+d^3x^3)(a^4+4a^3bx+6a^2b^2x^2+4ab^3x^3+b^4x^4)\,dx &= a^4c^3x+\frac{1}{2}a^3c^2(4bc+3ad)x^2+a^2c(2b^2c^2+4abcd+a^2d^2)x^3\\ &+\frac{1}{4}a(4b^3c^3+18ab^2c^2d+12a^2bcd^2+a^3d^3)x^4\\ &+\frac{1}{5}b(b^3c^3+12ab^2c^2d+18a^2bcd^2+4a^3d^3)x^5\\ &+\frac{1}{2}b^2d(b^2c^2+4abcd+2a^2d^2)x^6+\frac{1}{7}b^3d^2(3bc+4ad)x^7+\frac{1}{8}b^4d^3x^8\end{aligned}$$

input
```
Integrate[(c^3 + 3*c^2*d*x + 3*c*d^2*x^2 + d^3*x^3)*(a^4 + 4*a^3*b*x + 6*a
^2*b^2*x^2 + 4*a*b^3*x^3 + b^4*x^4),x]
```

output
```
a^4*c^3*x + (a^3*c^2*(4*b*c + 3*a*d)*x^2)/2 + a^2*c*(2*b^2*c^2 + 4*a*b*c*d
 + a^2*d^2)*x^3 + (a*(4*b^3*c^3 + 18*a*b^2*c^2*d + 12*a^2*b*c*d^2 + a^3*d^
3)*x^4)/4 + (b*(b^3*c^3 + 12*a*b^2*c^2*d + 18*a^2*b*c*d^2 + 4*a^3*d^3)*x^5
)/5 + (b^2*d*(b^2*c^2 + 4*a*b*c*d + 2*a^2*d^2)*x^6)/2 + (b^3*d^2*(3*b*c +
4*a*d)*x^7)/7 + (b^4*d^3*x^8)/8
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.061$, Rules used = {2006, 2006, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a^4+4a^3bx+6a^2b^2x^2+4ab^3x^3+b^4x^4\right)\left(c^3+3c^2dx+3cd^2x^2+d^3x^3\right)dx$$

$$\Big\downarrow \text{2006}$$

$$\int (a+bx)^4\left(c^3+3c^2dx+3cd^2x^2+d^3x^3\right)dx$$

$$\Big\downarrow \text{2006}$$

$$\int (a+bx)^4(c+dx)^3dx$$

$$\Big\downarrow \text{49}$$

$$\int \left(\frac{3d^2(a+bx)^6(bc-ad)}{b^3}+\frac{3d(a+bx)^5(bc-ad)^2}{b^3}+\frac{(a+bx)^4(bc-ad)^3}{b^3}+\frac{d^3(a+bx)^7}{b^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{3d^2(a+bx)^7(bc-ad)}{7b^4}+\frac{d(a+bx)^6(bc-ad)^2}{2b^4}+\frac{(a+bx)^5(bc-ad)^3}{5b^4}+\frac{d^3(a+bx)^8}{8b^4}$$

input
```
Int[(c^3 + 3*c^2*d*x + 3*c*d^2*x^2 + d^3*x^3)*(a^4 + 4*a^3*b*x + 6*a^2*b^2
*x^2 + 4*a*b^3*x^3 + b^4*x^4),x]
```

output
```
((b*c - a*d)^3*(a + b*x)^5)/(5*b^4) + (d*(b*c - a*d)^2*(a + b*x)^6)/(2*b^4
) + (3*d^2*(b*c - a*d)*(a + b*x)^7)/(7*b^4) + (d^3*(a + b*x)^8)/(8*b^4)
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2006
```
Int[(u_.)*(Px_), x_Symbol] :> With[{a = Rt[Coeff[Px, x, 0], Expon[Px, x]],
b = Rt[Coeff[Px, x, Expon[Px, x]], Expon[Px, x]]}, Int[u*(a + b*x)^Expon[Px
, x], x] /; EqQ[Px, (a + b*x)^Expon[Px, x]]] /; PolyQ[Px, x] && GtQ[Expon[P
x, x], 1] && NeQ[Coeff[Px, x, 0], 0] &&  !MatchQ[Px, (a_.)*(v_)^Expon[Px, x
] /; FreeQ[a, x] && LinearQ[v, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 221 vs. $2(84) = 168$.

Time = 0.09 (sec) , antiderivative size = 222, normalized size of antiderivative = 2.41

method	result
norman	$\frac{b^4d^3x^8}{8}+\left(\frac{4}{7}b^3a\,d^3+\frac{3}{7}b^4c\,d^2\right)x^7+\left(a^2b^2d^3+2a\,b^3c\,d^2+\frac{1}{2}b^4c^2d\right)x^6+\left(\frac{4}{5}a^3b\,d^3+\frac{18}{5}a^2b^2c\,d^2\right.$
default	$\frac{b^4d^3x^8}{8}+\frac{(4b^3a\,d^3+3b^4c\,d^2)x^7}{7}+\frac{(6a^2b^2d^3+12a\,b^3c\,d^2+3b^4c^2d)x^6}{6}+\frac{(4a^3b\,d^3+18a^2b^2c\,d^2+12b^3a\,c^2d+b^4c^3)x^5}{5}+$
risch	$\frac{1}{8}b^4d^3x^8+\frac{4}{7}x^7b^3a\,d^3+\frac{3}{7}x^7b^4c\,d^2+x^6a^2b^2d^3+2x^6a\,b^3c\,d^2+\frac{1}{2}x^6b^4c^2d+\frac{4}{5}x^5a^3b\,d^3+\frac{18}{5}x^5a$
parallelrisch	$\frac{1}{8}b^4d^3x^8+\frac{4}{7}x^7b^3a\,d^3+\frac{3}{7}x^7b^4c\,d^2+x^6a^2b^2d^3+2x^6a\,b^3c\,d^2+\frac{1}{2}x^6b^4c^2d+\frac{4}{5}x^5a^3b\,d^3+\frac{18}{5}x^5a$
gosper	$x(35b^4d^3x^7+160a\,b^3d^3x^6+120b^4c\,d^2x^6+280a^2b^2d^3x^5+560a\,b^3c\,d^2x^5+140b^4c^2d\,x^5+224a^3b\,d^3x^4+1008a^2b^2c\,d^2x^4+672a\,b^3$
orering	$x(35b^4d^3x^7+160a\,b^3d^3x^6+120b^4c\,d^2x^6+280a^2b^2d^3x^5+560a\,b^3c\,d^2x^5+140b^4c^2d\,x^5+224a^3b\,d^3x^4+1008a^2b^2c\,d^2x^4+672a\,b^3$

input
```
int((d^3*x^3+3*c*d^2*x^2+3*c^2*d*x+c^3)*(b^4*x^4+4*a*b^3*x^3+6*a^2*b^2*x^2
+4*a^3*b*x+a^4),x,method=_RETURNVERBOSE)
```

output
```
1/8*b^4*d^3*x^8+(4/7*b^3*a*d^3+3/7*b^4*c*d^2)*x^7+(a^2*b^2*d^3+2*a*b^3*c*d
^2+1/2*b^4*c^2*d)*x^6+(4/5*a^3*b*d^3+18/5*a^2*b^2*c*d^2+12/5*b^3*a*c^2*d+1
/5*b^4*c^3)*x^5+(1/4*a^4*d^3+3*a^3*b*c*d^2+9/2*a^2*b^2*c^2*d+a*b^3*c^3)*x^
4+(a^4*c*d^2+4*a^3*b*c^2*d+2*a^2*b^2*c^3)*x^3+(3/2*a^4*c^2*d+2*a^3*b*c^3)*
x^2+a^4*c^3*x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 225 vs. $2(84) = 168$.

Time = 0.07 (sec) , antiderivative size = 225, normalized size of antiderivative = 2.45

$$\begin{aligned}&\int\left(c^3+3c^2dx+3cd^2x^2+d^3x^3\right)\left(a^4+4a^3bx+6a^2b^2x^2+4ab^3x^3+b^4x^4\right)\,dx\\&=\frac{1}{8}\,b^4d^3x^8+a^4c^3x+\frac{1}{7}\left(3\,b^4cd^2+4\,ab^3d^3\right)x^7+\frac{1}{2}\left(b^4c^2d+4\,ab^3cd^2+2\,a^2b^2d^3\right)x^6\\&\quad+\frac{1}{5}\left(b^4c^3+12\,ab^3c^2d+18\,a^2b^2cd^2+4\,a^3bd^3\right)x^5\\&\quad+\frac{1}{4}\left(4\,ab^3c^3+18\,a^2b^2c^2d+12\,a^3bcd^2+a^4d^3\right)x^4\\&\quad+\left(2\,a^2b^2c^3+4\,a^3bc^2d+a^4cd^2\right)x^3+\frac{1}{2}\left(4\,a^3bc^3+3\,a^4c^2d\right)x^2\end{aligned}$$

input
```
integrate((d^3*x^3+3*c*d^2*x^2+3*c^2*d*x+c^3)*(b^4*x^4+4*a*b^3*x^3+6*a^2*b
^2*x^2+4*a^3*b*x+a^4),x, algorithm="fricas")
```

output
```
1/8*b^4*d^3*x^8 + a^4*c^3*x + 1/7*(3*b^4*c*d^2 + 4*a*b^3*d^3)*x^7 + 1/2*(b
^4*c^2*d + 4*a*b^3*c*d^2 + 2*a^2*b^2*d^3)*x^6 + 1/5*(b^4*c^3 + 12*a*b^3*c^
2*d + 18*a^2*b^2*c*d^2 + 4*a^3*b*d^3)*x^5 + 1/4*(4*a*b^3*c^3 + 18*a^2*b^2*
c^2*d + 12*a^3*b*c*d^2 + a^4*d^3)*x^4 + (2*a^2*b^2*c^3 + 4*a^3*b*c^2*d + a
^4*c*d^2)*x^3 + 1/2*(4*a^3*b*c^3 + 3*a^4*c^2*d)*x^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 243 vs. $2(80) = 160$.

Time = 0.04 (sec) , antiderivative size = 243, normalized size of antiderivative = 2.64

$$\int (c^3 + 3c^2dx + 3cd^2x^2 + d^3x^3)(a^4 + 4a^3bx + 6a^2b^2x^2 + 4ab^3x^3 + b^4x^4)\,dx$$
$$= a^4c^3x + \frac{b^4d^3x^8}{8} + x^7 \cdot \left(\frac{4ab^3d^3}{7} + \frac{3b^4cd^2}{7}\right) + x^6\left(a^2b^2d^3 + 2ab^3cd^2 + \frac{b^4c^2d}{2}\right) + x^5 \cdot \left(\frac{4a^3bd^3}{5} + \frac{18a^2b^2cd^2}{5} + \frac{12ab^3c^2d}{5} + \frac{b^4c^3}{5}\right) + x^4\left(\frac{a^4d^3}{4} + 3a^3bcd^2 + \frac{9a^2b^2c^2d}{2} + ab^3c^3\right) + x^3(a^4cd^2 + 4a^3bc^2d + 2a^2b^2c^3) + x^2 \cdot \left(\frac{3a^4c^2d}{2} + 2a^3bc^3\right)$$

input
```
integrate((d**3*x**3+3*c*d**2*x**2+3*c**2*d*x+c**3)*(b**4*x**4+4*a*b**3*x*
*3+6*a**2*b**2*x**2+4*a**3*b*x+a**4),x)
```

output
```
a**4*c**3*x + b**4*d**3*x**8/8 + x**7*(4*a*b**3*d**3/7 + 3*b**4*c*d**2/7)
+ x**6*(a**2*b**2*d**3 + 2*a*b**3*c*d**2 + b**4*c**2*d/2) + x**5*(4*a**3*b
*d**3/5 + 18*a**2*b**2*c*d**2/5 + 12*a*b**3*c**2*d/5 + b**4*c**3/5) + x**4
*(a**4*d**3/4 + 3*a**3*b*c*d**2 + 9*a**2*b**2*c**2*d/2 + a*b**3*c**3) + x*
*3*(a**4*c*d**2 + 4*a**3*b*c**2*d + 2*a**2*b**2*c**3) + x**2*(3*a**4*c**2*
d/2 + 2*a**3*b*c**3)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 225 vs. $2(84) = 168$.

Time = 0.04 (sec) , antiderivative size = 225, normalized size of antiderivative = 2.45

$$\int (c^3 + 3c^2dx + 3cd^2x^2 + d^3x^3) (a^4 + 4a^3bx + 6a^2b^2x^2 + 4ab^3x^3 + b^4x^4) dx$$
$$= \frac{1}{8} b^4d^3x^8 + a^4c^3x + \frac{1}{7} (3 b^4cd^2 + 4 ab^3d^3)x^7 + \frac{1}{2} (b^4c^2d + 4 ab^3cd^2 + 2 a^2b^2d^3)x^6 + \frac{1}{5} (b^4c^3 + 12 ab^3c^2d + 18 a^2b^2cd^2 + 4 a^3bd^3)x^5 + \frac{1}{4} (4 ab^3c^3 + 18 a^2b^2c^2d + 12 a^3bcd^2 + a^4d^3)x^4 + (2 a^2b^2c^3 + 4 a^3bc^2d + a^4cd^2)x^3 + \frac{1}{2} (4 a^3bc^3 + 3 a^4c^2d)x^2$$

input
```
integrate((d^3*x^3+3*c*d^2*x^2+3*c^2*d*x+c^3)*(b^4*x^4+4*a*b^3*x^3+6*a^2*b
^2*x^2+4*a^3*b*x+a^4),x, algorithm="maxima")
```

output
```
1/8*b^4*d^3*x^8 + a^4*c^3*x + 1/7*(3*b^4*c*d^2 + 4*a*b^3*d^3)*x^7 + 1/2*(b
^4*c^2*d + 4*a*b^3*c*d^2 + 2*a^2*b^2*d^3)*x^6 + 1/5*(b^4*c^3 + 12*a*b^3*c^
2*d + 18*a^2*b^2*c*d^2 + 4*a^3*b*d^3)*x^5 + 1/4*(4*a*b^3*c^3 + 18*a^2*b^2*
c^2*d + 12*a^3*b*c*d^2 + a^4*d^3)*x^4 + (2*a^2*b^2*c^3 + 4*a^3*b*c^2*d + a
^4*c*d^2)*x^3 + 1/2*(4*a^3*b*c^3 + 3*a^4*c^2*d)*x^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 245 vs. $2(84) = 168$.

Time = 0.14 (sec) , antiderivative size = 245, normalized size of antiderivative = 2.66

$$\int (c^3 + 3c^2dx + 3cd^2x^2 + d^3x^3) (a^4 + 4a^3bx + 6a^2b^2x^2 + 4ab^3x^3 + b^4x^4) dx$$
$$= \frac{1}{8} b^4d^3x^8 + \frac{3}{7} b^4cd^2x^7 + \frac{4}{7} ab^3d^3x^7 + \frac{1}{2} b^4c^2dx^6 + 2 ab^3cd^2x^6 + a^2b^2d^3x^6 + \frac{1}{5} b^4c^3x^5 + \frac{12}{5} ab^3c^2dx^5 + \frac{18}{5} a^2b^2cd^2x^5 + \frac{4}{5} a^3bd^3x^5 + ab^3c^3x^4 + \frac{9}{2} a^2b^2c^2dx^4 + 3 a^3bcd^2x^4 + \frac{1}{4} a^4d^3x^4 + 2 a^2b^2c^3x^3 + 4 a^3bc^2dx^3 + a^4cd^2x^3 + 2 a^3bc^3x^2 + \frac{3}{2} a^4c^2dx^2 + a^4c^3x$$

input
```
integrate((d^3*x^3+3*c*d^2*x^2+3*c^2*d*x+c^3)*(b^4*x^4+4*a*b^3*x^3+6*a^2*b
^2*x^2+4*a^3*b*x+a^4),x, algorithm="giac")
```

output
```
1/8*b^4*d^3*x^8 + 3/7*b^4*c*d^2*x^7 + 4/7*a*b^3*d^3*x^7 + 1/2*b^4*c^2*d*x^
6 + 2*a*b^3*c*d^2*x^6 + a^2*b^2*d^3*x^6 + 1/5*b^4*c^3*x^5 + 12/5*a*b^3*c^2
*d*x^5 + 18/5*a^2*b^2*c*d^2*x^5 + 4/5*a^3*b*d^3*x^5 + a*b^3*c^3*x^4 + 9/2*
a^2*b^2*c^2*d*x^4 + 3*a^3*b*c*d^2*x^4 + 1/4*a^4*d^3*x^4 + 2*a^2*b^2*c^3*x^
3 + 4*a^3*b*c^2*d*x^3 + a^4*c*d^2*x^3 + 2*a^3*b*c^3*x^2 + 3/2*a^4*c^2*d*x^
2 + a^4*c^3*x
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 208, normalized size of antiderivative = 2.26

$$\int \left(c^3+3c^2dx+3cd^2x^2+d^3x^3\right)\left(a^4+4a^3bx+6a^2b^2x^2+4ab^3x^3+b^4x^4\right)\,dx$$
$$= x^4\left(\frac{a^4\,d^3}{4}+3\,a^3\,b\,c\,d^2+\frac{9\,a^2\,b^2\,c^2\,d}{2}+a\,b^3\,c^3\right) + x^5\left(\frac{4\,a^3\,b\,d^3}{5}+\frac{18\,a^2\,b^2\,c\,d^2}{5}+\frac{12\,a\,b^3\,c^2\,d}{5}+\frac{b^4\,c^3}{5}\right)+a^4\,c^3\,x + \frac{b^4\,d^3\,x^8}{8}+\frac{a^3\,c^2\,x^2\,(3\,a\,d+4\,b\,c)}{2}+\frac{b^3\,d^2\,x^7\,(4\,a\,d+3\,b\,c)}{7} + a^2\,c\,x^3\left(a^2\,d^2+4\,a\,b\,c\,d+2\,b^2\,c^2\right)+\frac{b^2\,d\,x^6\,(2\,a^2\,d^2+4\,a\,b\,c\,d+b^2\,c^2)}{2}$$

input
```
int((c^3 + d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x)*(a^4 + b^4*x^4 + 4*a*b^3*x^3
 + 6*a^2*b^2*x^2 + 4*a^3*b*x),x)
```

output
```
x^4*((a^4*d^3)/4 + a*b^3*c^3 + (9*a^2*b^2*c^2*d)/2 + 3*a^3*b*c*d^2) + x^5*
((b^4*c^3)/5 + (4*a^3*b*d^3)/5 + (18*a^2*b^2*c*d^2)/5 + (12*a*b^3*c^2*d)/5
) + a^4*c^3*x + (b^4*d^3*x^8)/8 + (a^3*c^2*x^2*(3*a*d + 4*b*c))/2 + (b^3*d
^2*x^7*(4*a*d + 3*b*c))/7 + a^2*c*x^3*(a^2*d^2 + 2*b^2*c^2 + 4*a*b*c*d) +
(b^2*d*x^6*(2*a^2*d^2 + b^2*c^2 + 4*a*b*c*d))/2
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 247, normalized size of antiderivative = 2.68

$$\int \left(c^3+3c^2dx+3cd^2x^2+d^3x^3\right)\left(a^4+4a^3bx+6a^2b^2x^2+4ab^3x^3+b^4x^4\right)\,dx$$
$$=\frac{x(35b^4d^3x^7+160a\,b^3d^3x^6+120b^4c\,d^2x^6+280a^2b^2d^3x^5+560a\,b^3c\,d^2x^5+140b^4c^2d\,x^5+224a^3b\,d^3x^4+1}{}$$

input
```
int((d^3*x^3+3*c*d^2*x^2+3*c^2*d*x+c^3)*(b^4*x^4+4*a*b^3*x^3+6*a^2*b^2*x^2
+4*a^3*b*x+a^4),x)
```

output
```
(x*(280*a**4*c**3 + 420*a**4*c**2*d*x + 280*a**4*c*d**2*x**2 + 70*a**4*d**
3*x**3 + 560*a**3*b*c**3*x + 1120*a**3*b*c**2*d*x**2 + 840*a**3*b*c*d**2*x
**3 + 224*a**3*b*d**3*x**4 + 560*a**2*b**2*c**3*x**2 + 1260*a**2*b**2*c**2
*d*x**3 + 1008*a**2*b**2*c*d**2*x**4 + 280*a**2*b**2*d**3*x**5 + 280*a*b**
3*c**3*x**3 + 672*a*b**3*c**2*d*x**4 + 560*a*b**3*c*d**2*x**5 + 160*a*b**3
*d**3*x**6 + 56*b**4*c**3*x**4 + 140*b**4*c**2*d*x**5 + 120*b**4*c*d**2*x*
*6 + 35*b**4*d**3*x**7))/280
```

3.118 $\int \frac{1}{\left(2^{2/3}+x\right)\sqrt{1+x^3}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 145

$$\int \frac{1}{\left(2^{2/3}+x\right)\sqrt{1+x^3}}\,dx=\frac{2\arctan\left(\frac{\sqrt{3}\left(1+\sqrt[3]{2}x\right)}{\sqrt{1+x^3}}\right)}{3\sqrt{3}}+\frac{2\sqrt[3]{2}\sqrt{2+\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

output
```
2/9*arctan(3^(1/2)*(1+2^(1/3)*x)/(x^3+1)^(1/2))*3^(1/2)+2/9*2^(1/3)*(1/2*6
^(1/2)+1/2*2^(1/2))*(1+x)*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*EllipticF((1+x
-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*3^(3/4)/((1+x)/(1+x+3^(1/2))^2)^(1/
2)/(x^3+1)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 20.28 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.02

$$\int \frac{1}{\left(2^{2/3}+x\right)\sqrt{1+x^3}}\,dx = \frac{4i\sqrt{2}\sqrt{\frac{i(1+x)}{3i+\sqrt{3}}}\sqrt{1-x+x^2}\,\text{EllipticPi}\left(\frac{2\sqrt{3}}{i+2i2^{2/3}+\sqrt{3}},\arcsin\left(\frac{\sqrt{i+\sqrt{3}-2ix}}{\sqrt{2}\sqrt[4]{3}}\right),\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)}{\left(1+2\,2^{2/3}-i\sqrt{3}\right)\sqrt{1+x^3}}$$

input
```
Integrate[1/((2^(2/3) + x)*Sqrt[1 + x^3]),x]
```

output
```
((4*I)*Sqrt[2]*Sqrt[(I*(1 + x))/(3*I + Sqrt[3])]*Sqrt[1 - x + x^2]*Ellipti
cPi[(2*Sqrt[3])/(I + (2*I)*2^(2/3) + Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] - (
2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])])/((1 + 2*2^(2/3)
- I*Sqrt[3])*Sqrt[1 + x^3])
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {2559, 759, 2562, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(x+2^{2/3}\right)\sqrt{x^3+1}}dx$$

$$\Big\downarrow \ 2559$$

$$\frac{1}{3}\sqrt[3]{2}\int \frac{1}{\sqrt{x^3+1}}dx+\frac{\int \frac{2^{2/3}-2x}{\left(x+2^{2/3}\right)\sqrt{x^3+1}}dx}{3\ 2^{2/3}}$$

$$\Big\downarrow \ 759$$

$$\frac{\int \frac{2^{2/3}-2x}{\left(x+2^{2/3}\right)\sqrt{x^3+1}}dx}{3\ 2^{2/3}}+\frac{2\sqrt[3]{2}\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

$$\Big\downarrow \text{2562}$$

$$\frac{2}{3}\int \frac{1}{\frac{3\left(\sqrt[3]{2}x+1\right)^2}{x^3+1}+1}d\frac{\sqrt[3]{2}x+1}{\sqrt{x^3+1}}+\frac{2\sqrt[3]{2}\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

$$\Big\downarrow \text{216}$$

$$\frac{2\sqrt[3]{2}\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}+\frac{2\arctan\left(\frac{\sqrt{3}\left(\sqrt[3]{2}x+1\right)}{\sqrt{x^3+1}}\right)}{3\sqrt{3}}$$

input
```
Int[1/((2^(2/3) + x)*Sqrt[1 + x^3]),x]
```

output
```
(2*ArcTan[(Sqrt[3]*(1 + 2^(1/3)*x))/Sqrt[1 + x^3]])/(3*Sqrt[3]) + (2*2^(1/
3)*Sqrt[2 + Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*Ellip
ticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3*3^(1
/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 2559
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Simp[2/
(3*c)   Int[1/Sqrt[a + b*x^3], x], x] + Simp[1/(3*c)   Int[(c - 2*d*x)/((c
+ d*x)*Sqrt[a + b*x^3]), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 - 4*
a*d^3, 0]
```

rule 2562
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[2*(e/d)   Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))
/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0]
&& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Maple [A] (verified)

Time = 1.67 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.96

method	result	size
default	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\operatorname{EllipticPi}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{2^{\frac{2}{3}}-1},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^{3}+1}\left(2^{\frac{2}{3}}-1\right)}$	139
elliptic	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\operatorname{EllipticPi}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{2^{\frac{2}{3}}-1},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^{3}+1}\left(2^{\frac{2}{3}}-1\right)}$	139

input
```
int(1/(2^(2/3)+x)/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1
/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2
)))^(1/2)/(x^3+1)^(1/2)/(2^(2/3)-1)*EllipticPi(((x+1)/(3/2-1/2*I*3^(1/2)))
^(1/2),(-3/2+1/2*I*3^(1/2))/(2^(2/3)-1),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*
3^(1/2)))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.51

$$\int \frac{1}{(2^{2/3}+x)\sqrt{1+x^3}}\,dx = -\frac{1}{9}\sqrt{3}\arctan\left(-\frac{\sqrt{3}\left(5\,x^3-2^{\frac{2}{3}}(x^5+x^2)+2^{\frac{1}{3}}(7\,x^4+4\,x)+2\right)\sqrt{x^3+1}}{6\,(2\,x^6+3\,x^3+1)}\right) + \frac{2}{3}\cdot 2^{\frac{1}{3}}\text{weierstrassPInverse}(0,-4,x)$$

input
```
integrate(1/(2^(2/3)+x)/(x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/9*sqrt(3)*arctan(-1/6*sqrt(3)*(5*x^3 - 2^(2/3)*(x^5 + x^2) + 2^(1/3)*(7
*x^4 + 4*x) + 2)*sqrt(x^3 + 1)/(2*x^6 + 3*x^3 + 1)) + 2/3*2^(1/3)*weierstr
assPInverse(0, -4, x)
```

Sympy [F]

$$\int \frac{1}{(2^{2/3}+x)\sqrt{1+x^3}}\,dx = \int \frac{1}{\sqrt{(x+1)\,(x^2-x+1)}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

input
```
integrate(1/(2**(2/3)+x)/(x**3+1)**(1/2),x)
```

output
```
Integral(1/(sqrt((x + 1)*(x**2 - x + 1))*(x + 2**(2/3))), x)
```

Maxima [F]

$$\int \frac{1}{(2^{2/3}+x)\sqrt{1+x^3}}\,dx = \int \frac{1}{\sqrt{x^3+1}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

input
```
integrate(1/(2^(2/3)+x)/(x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(x^3 + 1)*(x + 2^(2/3))), x)
```

Giac [F]

$$\int \frac{1}{(2^{2/3}+x)\sqrt{1+x^3}}\,dx = \int \frac{1}{\sqrt{x^3+1}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

input
```
integrate(1/(2^(2/3)+x)/(x^3+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(x^3 + 1)*(x + 2^(2/3))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(2^{2/3}+x)\sqrt{1+x^3}}\,dx = \int \frac{1}{\sqrt{x^3+1}\,(x+2^{2/3})}\,dx$$

input
```
int(1/((x^3 + 1)^(1/2)*(x + 2^(2/3))),x)
```

output
```
int(1/((x^3 + 1)^(1/2)*(x + 2^(2/3))), x)
```

Reduce [F]

$$\int \frac{1}{(2^{2/3}+x)\sqrt{1+x^3}}\,dx = \int \frac{1}{\sqrt{x^3+1}\,2^{\frac{2}{3}}+\sqrt{x^3+1}\,x}dx$$

input
```
int(1/(2^(2/3)+x)/(x^3+1)^(1/2),x)
```

output
```
int(1/(sqrt(x**3 + 1)*2**(2/3) + sqrt(x**3 + 1)*x),x)
```

3.119 $\int \frac{1}{\left(2^{2/3}-x\right)\sqrt{1-x^3}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 160

$$\int \frac{1}{\left(2^{2/3}-x\right)\sqrt{1-x^3}}\,dx = -\frac{2\arctan\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{1-x^3}}\right)}{3\sqrt{3}} - \frac{2\sqrt[3]{2}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

output
```
-2/9*arctan(3^(1/2)*(1-2^(1/3)*x)/(-x^3+1)^(1/2))*3^(1/2)-2/9*2^(1/3)*(1/2
*6^(1/2)+1/2*2^(1/2))*(1-x)*((x^2+x+1)/(1+3^(1/2)-x)^2)^(1/2)*EllipticF((1
-3^(1/2)-x)/(1+3^(1/2)-x),I*3^(1/2)+2*I)*3^(3/4)/((1-x)/(1+3^(1/2)-x)^2)^(
1/2)/(-x^3+1)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.18 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.92

$$\int \frac{1}{(2^{2/3}-x)\sqrt{1-x^3}}\,dx = -\frac{4i\sqrt{2}\sqrt{-\frac{i(-1+x)}{3i+\sqrt{3}}}\sqrt{1+x+x^2}\,\text{EllipticPi}\left(\frac{2\sqrt{3}}{i+2i2^{2/3}+\sqrt{3}},\arcsin\left(\frac{\sqrt{i+\sqrt{3}+2ix}}{\sqrt{2}\sqrt[4]{3}}\right),\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)}{\left(1+2\,2^{2/3}-i\sqrt{3}\right)\sqrt{1-x^3}}$$

input
```
Integrate[1/((2^(2/3) - x)*Sqrt[1 - x^3]),x]
```

output
```
((-4*I)*Sqrt[2]*Sqrt[((-I)*(-1 + x))/(3*I + Sqrt[3])]*Sqrt[1 + x + x^2]*El
lipticPi[(2*Sqrt[3])/(I + (2*I)*2^(2/3) + Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3
] + (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])])/((1 + 2*2^(
2/3) - I*Sqrt[3])*Sqrt[1 - x^3])
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {2559, 27, 759, 2562, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(2^{2/3}-x)\sqrt{1-x^3}}\,dx$$

$$\Big\downarrow \text{ 2559}$$

$$\frac{1}{3}\sqrt[3]{2}\int \frac{1}{\sqrt{1-x^3}}dx+\frac{\int \frac{2^{2/3}\left(\sqrt[3]{2}x+1\right)}{(2^{2/3}-x)\sqrt{1-x^3}}dx}{3\,2^{2/3}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{3}\sqrt[3]{2}\int \frac{1}{\sqrt{1-x^3}}dx+\frac{1}{3}\int \frac{\sqrt[3]{2}x+1}{\left(2^{2/3}-x\right)\sqrt{1-x^3}}dx$$

$$\Big\downarrow \textbf{759}$$

$$\frac{1}{3}\int \frac{\sqrt[3]{2}x+1}{\left(2^{2/3}-x\right)\sqrt{1-x^3}}dx-\frac{2\sqrt[3]{2}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

$$\Big\downarrow \textbf{2562}$$

$$-\frac{2}{3}\int \frac{1}{\frac{3\left(1-\sqrt[3]{2}x\right)^2}{1-x^3}+1}d\frac{1-\sqrt[3]{2}x}{\sqrt{1-x^3}}-\frac{2\sqrt[3]{2}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

$$\Big\downarrow \textbf{216}$$

$$-\frac{2\sqrt[3]{2}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}-\frac{2\arctan\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{1-x^3}}\right)}{3\sqrt{3}}$$

input
```
Int[1/((2^(2/3) - x)*Sqrt[1 - x^3]),x]
```

output
```
(-2*ArcTan[(Sqrt[3]*(1 - 2^(1/3)*x))/Sqrt[1 - x^3]])/(3*Sqrt[3]) - (2*2^(1
/3)*Sqrt[2 + Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*Elli
pticF[ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(3*3^(
1/4)*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 2559
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Simp[2/
(3*c)   Int[1/Sqrt[a + b*x^3], x], x] + Simp[1/(3*c)   Int[(c - 2*d*x)/((c
+ d*x)*Sqrt[a + b*x^3]), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 - 4*
a*d^3, 0]
```

rule 2562
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[2*(e/d)   Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))
/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0]
&& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Maple [A] (verified)

Time = 1.64 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.89

method	result	si
default	$\frac{2i\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\text{EllipticPi}\left(\frac{\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\frac{i\sqrt{3}}{-\frac{1}{2}+\frac{i\sqrt{3}}{2}-2^{\frac{2}{3}}},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^{3}+1}\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}-2^{\frac{2}{3}}\right)}$	14
elliptic	$\frac{2i\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\text{EllipticPi}\left(\frac{\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\frac{i\sqrt{3}}{-\frac{1}{2}+\frac{i\sqrt{3}}{2}-2^{\frac{2}{3}}},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^{3}+1}\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}-2^{\frac{2}{3}}\right)}$	14

input
```
int(1/(2^(2/3)-x)/(-x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x-1)/(-3/2+1/2*I*3
^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)/(-1
/2+1/2*I*3^(1/2)-2^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*
3^(1/2))^(1/2),I*3^(1/2)/(-1/2+1/2*I*3^(1/2)-2^(2/3)),(I*3^(1/2)/(-3/2+1/2
*I*3^(1/2)))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.49

$$\int \frac{1}{\left(2^{2/3}-x\right)\sqrt{1-x^3}}\,dx = -\frac{1}{9}\sqrt{3}\arctan\left(-\frac{\sqrt{3}\left(5x^3-2^{\frac{2}{3}}\left(x^5-x^2\right)-2^{\frac{1}{3}}\left(7x^4-4x\right)-2\right)\sqrt{-x^3+1}}{6\left(2x^6-3x^3+1\right)}\right) - \frac{2}{3}i\cdot 2^{\frac{1}{3}}\,\text{weierstrassPInverse}(0,4,x)$$

input
```
integrate(1/(2^(2/3)-x)/(-x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/9*sqrt(3)*arctan(-1/6*sqrt(3)*(5*x^3 - 2^(2/3)*(x^5 - x^2) - 2^(1/3)*(7
*x^4 - 4*x) - 2)*sqrt(-x^3 + 1)/(2*x^6 - 3*x^3 + 1)) - 2/3*I*2^(1/3)*weier
strassPInverse(0, 4, x)
```

Sympy [F]

$$\int \frac{1}{(2^{2/3}-x)\sqrt{1-x^3}}\,dx = -\int \frac{1}{x\sqrt{1-x^3}-2^{\frac{2}{3}}\sqrt{1-x^3}}\,dx$$

input
```
integrate(1/(2**(2/3)-x)/(-x**3+1)**(1/2),x)
```

output
```
-Integral(1/(x*sqrt(1 - x**3) - 2**(2/3)*sqrt(1 - x**3)), x)
```

Maxima [F]

$$\int \frac{1}{(2^{2/3}-x)\sqrt{1-x^3}}\,dx = \int -\frac{1}{\sqrt{-x^3+1}\left(x-2^{\frac{2}{3}}\right)}\,dx$$

input
```
integrate(1/(2^(2/3)-x)/(-x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
-integrate(1/(sqrt(-x^3 + 1)*(x - 2^(2/3))), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(2^{2/3}-x)\sqrt{1-x^3}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(2^(2/3)-x)/(-x^3+1)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[2]%%%} / %%%{%%{[2,0]:[1,0,0,-2]%%},[2]%%%} Error: Bad
Argument
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(2^{2/3}-x\right)\sqrt{1-x^3}}\,dx=-\int \frac{1}{\sqrt{1-x^3}\,\left(x-2^{2/3}\right)}\,dx$$

input
```
int(-1/((1 - x^3)^(1/2)*(x - 2^(2/3))),x)
```

output
```
-int(1/((1 - x^3)^(1/2)*(x - 2^(2/3))), x)
```

Reduce [F]

$$\int \frac{1}{\left(2^{2/3}-x\right)\sqrt{1-x^3}}\,dx=\int \frac{1}{\sqrt{-x^3+1}\,2^{\frac{2}{3}}-\sqrt{-x^3+1}\,x}dx$$

input
```
int(1/(2^(2/3)-x)/(-x^3+1)^(1/2),x)
```

output
```
int(1/(sqrt( - x**3 + 1)*2**(2/3) - sqrt( - x**3 + 1)*x),x)
```

3.120 $\int \frac{1}{\left(2^{2/3}-x\right)\sqrt{-1+x^3}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 163

$$\int \frac{1}{(2^{2/3}-x)\sqrt{-1+x^3}}\,dx = -\frac{2\operatorname{arctanh}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{-1+x^3}}\right)}{3\sqrt{3}} - \frac{2\sqrt[3]{2}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

output
```
-2/9*arctanh(3^(1/2)*(1-2^(1/3)*x)/(x^3-1)^(1/2))*3^(1/2)-2/9*2^(1/3)*(1/2
*6^(1/2)-1/2*2^(1/2))*(1-x)*((x^2+x+1)/(1-3^(1/2)-x)^2)^(1/2)*EllipticF((1
+3^(1/2)-x)/(1-3^(1/2)-x),2*I-I*3^(1/2))*3^(3/4)/(-(1-x)/(1-3^(1/2)-x)^2)^
(1/2)/(x^3-1)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 20.31 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.90

$$\int \frac{1}{(2^{2/3}-x)\sqrt{-1+x^3}}\,dx = -\frac{4i\sqrt{2}\sqrt{-\frac{i(-1+x)}{3i+\sqrt{3}}}\sqrt{1+x+x^2}\,\text{EllipticPi}\left(\frac{2\sqrt{3}}{i+2i2^{2/3}+\sqrt{3}},\arcsin\left(\frac{\sqrt{i+\sqrt{3}+2ix}}{\sqrt{2}\sqrt[4]{3}}\right),\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)}{\left(1+2\,2^{2/3}-i\sqrt{3}\right)\sqrt{-1+x^3}}$$

input
```
Integrate[1/((2^(2/3) - x)*Sqrt[-1 + x^3]),x]
```

output
```
((-4*I)*Sqrt[2]*Sqrt[((-I)*(-1 + x))/(3*I + Sqrt[3])]*Sqrt[1 + x + x^2]*El
lipticPi[(2*Sqrt[3])/(I + (2*I)*2^(2/3) + Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3
] + (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])]])/((1 + 2*2^(
2/3) - I*Sqrt[3])*Sqrt[-1 + x^3])
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2559, 27, 760, 2562, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(2^{2/3}-x)\sqrt{x^3-1}}dx$$

$$\Big\downarrow \text{ 2559}$$

$$\frac{1}{3}\sqrt[3]{2}\int \frac{1}{\sqrt{x^3-1}}dx+\frac{\int \frac{2^{2/3}\left(\sqrt[3]{2}x+1\right)}{(2^{2/3}-x)\sqrt{x^3-1}}dx}{3\ 2^{2/3}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{3}\sqrt[3]{2}\int \frac{1}{\sqrt{x^3-1}}dx+\frac{1}{3}\int \frac{\sqrt[3]{2}x+1}{\left(2^{2/3}-x\right)\sqrt{x^3-1}}dx$$

$$\Big\downarrow \text{760}$$

$$\frac{1}{3}\int \frac{\sqrt[3]{2}x+1}{\left(2^{2/3}-x\right)\sqrt{x^3-1}}dx-\frac{2\sqrt[3]{2}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

$$\Big\downarrow \text{2562}$$

$$-\frac{2}{3}\int \frac{1}{1-\frac{3\left(1-\sqrt[3]{2}x\right)^2}{x^3-1}}d\frac{1-\sqrt[3]{2}x}{\sqrt{x^3-1}}-\frac{2\sqrt[3]{2}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

$$\Big\downarrow \text{219}$$

$$-\frac{2\sqrt[3]{2}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}-\frac{2\text{arctanh}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{x^3-1}}\right)}{3\sqrt{3}}$$

input
```
Int[1/((2^(2/3) - x)*Sqrt[-1 + x^3]),x]
```

output
```
(-2*ArcTanh[(Sqrt[3]*(1 - 2^(1/3)*x))/Sqrt[-1 + x^3]])/(3*Sqrt[3]) - (2*2^
(1/3)*Sqrt[2 - Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*El
lipticF[ArcSin[(1 + Sqrt[3] - x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(3*3
^(1/4)*Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 + x^3])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 2559
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Simp[2/
(3*c)   Int[1/Sqrt[a + b*x^3], x], x] + Simp[1/(3*c)   Int[(c - 2*d*x)/((c
+ d*x)*Sqrt[a + b*x^3]), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 - 4*
a*d^3, 0]
```

rule 2562
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[2*(e/d)   Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))
/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0]
&& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Maple [A] (verified)

Time = 1.64 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.88

method	result	size
default	$-\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticPi}\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-2^{\frac{2}{3}}+1},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^{3}-1}\left(-2^{\frac{2}{3}}+1\right)}$	143
elliptic	$-\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticPi}\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-2^{\frac{2}{3}}+1},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^{3}-1}\left(-2^{\frac{2}{3}}+1\right)}$	143

input
```
int(1/(2^(2/3)-x)/(x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3
^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/
2)))^(1/2)/(x^3-1)^(1/2)/(-2^(2/3)+1)*EllipticPi(((x-1)/(-3/2-1/2*I*3^(1/2
)))^(1/2),(3/2+1/2*I*3^(1/2))/(-2^(2/3)+1),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I
*3^(1/2)))^(1/2))
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\left(2^{2/3}-x\right)\sqrt{-1+x^3}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(2^(2/3)-x)/(x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   catd
ef: division by zero
```

Sympy [F]

$$\int \frac{1}{(2^{2/3}-x)\sqrt{-1+x^3}}\,dx = -\int \frac{1}{x\sqrt{x^3-1}-2^{\frac{2}{3}}\sqrt{x^3-1}}\,dx$$

input
```
integrate(1/(2**(2/3)-x)/(x**3-1)**(1/2),x)
```

output
```
-Integral(1/(x*sqrt(x**3 - 1) - 2**(2/3)*sqrt(x**3 - 1)), x)
```

Maxima [F]

$$\int \frac{1}{(2^{2/3}-x)\sqrt{-1+x^3}}\,dx = \int -\frac{1}{\sqrt{x^3-1}\left(x-2^{\frac{2}{3}}\right)}\,dx$$

input
```
integrate(1/(2^(2/3)-x)/(x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
-integrate(1/(sqrt(x^3 - 1)*(x - 2^(2/3))), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(2^{2/3}-x)\sqrt{-1+x^3}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(2^(2/3)-x)/(x^3-1)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:Unable to divide, perhaps due to rounding error%%%{1,[2
]%%%} / %%%{%%{[2,0]:[1,0,0,-2]%%},[2]%%%} Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(2^{2/3}-x)\sqrt{-1+x^3}}\,dx = -\int \frac{1}{\sqrt{x^3-1}\,(x-2^{2/3})}\,dx$$

input
```
int(-1/((x^3 - 1)^(1/2)*(x - 2^(2/3))),x)
```

output
```
-int(1/((x^3 - 1)^(1/2)*(x - 2^(2/3))), x)
```

Reduce [F]

$$\int \frac{1}{(2^{2/3}-x)\sqrt{-1+x^3}}\,dx = \int \frac{1}{\sqrt{x^3-1}\,2^{\frac{2}{3}}-\sqrt{x^3-1}\,x}dx$$

input
```
int(1/(2^(2/3)-x)/(x^3-1)^(1/2),x)
```

output
```
int(1/(sqrt(x**3 - 1)*2**(2/3) - sqrt(x**3 - 1)*x),x)
```

3.121 $\int \frac{1}{\left(2^{2/3}+x\right)\sqrt{-1-x^3}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 156

$$\int \frac{1}{\left(2^{2/3}+x\right)\sqrt{-1-x^3}}\,dx = \frac{2\operatorname{arctanh}\left(\frac{\sqrt{3}\left(1+\sqrt[3]{2}x\right)}{\sqrt{-1-x^3}}\right)}{3\sqrt{3}} + \frac{2\sqrt[3]{2}\sqrt{2-\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

output
```
2/9*arctanh(3^(1/2)*(1+2^(1/3)*x)/(-x^3-1)^(1/2))*3^(1/2)+2/9*2^(1/3)*(1/2
*6^(1/2)-1/2*2^(1/2))*(1+x)*((x^2-x+1)/(1+x-3^(1/2))^2)^(1/2)*EllipticF((1
+x+3^(1/2))/(1+x-3^(1/2)),2*I-I*3^(1/2))*3^(3/4)/(-(1+x)/(1+x-3^(1/2))^2)^
(1/2)/(-x^3-1)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.14 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.96

$$\int \frac{1}{\left(2^{2/3}+x\right)\sqrt{-1-x^3}}\,dx = \frac{4i\sqrt{2}\sqrt{\frac{i(1+x)}{3i+\sqrt{3}}}\sqrt{1-x+x^2}\,\text{EllipticPi}\left(\frac{2\sqrt{3}}{i+2i2^{2/3}+\sqrt{3}},\arcsin\left(\frac{\sqrt{i+\sqrt{3}-2ix}}{\sqrt{2}\sqrt[4]{3}}\right),\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)}{\left(1+2\,2^{2/3}-i\sqrt{3}\right)\sqrt{-1-x^3}}$$

input
```
Integrate[1/((2^(2/3) + x)*Sqrt[-1 - x^3]),x]
```

output
```
((4*I)*Sqrt[2]*Sqrt[(I*(1 + x))/(3*I + Sqrt[3])]*Sqrt[1 - x + x^2]*Ellipti
cPi[(2*Sqrt[3])/(I + (2*I)*2^(2/3) + Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] - (
2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])])/((1 + 2*2^(2/3)
- I*Sqrt[3])*Sqrt[-1 - x^3])
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2559, 760, 2562, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(x+2^{2/3}\right)\sqrt{-x^3-1}}dx$$

$$\Big\downarrow \text{2559}$$

$$\frac{1}{3}\sqrt[3]{2}\int \frac{1}{\sqrt{-x^3-1}}dx+\frac{\int \frac{2^{2/3}-2x}{\left(x+2^{2/3}\right)\sqrt{-x^3-1}}dx}{3\;2^{2/3}}$$

$$\Big\downarrow \text{760}$$

$$\frac{\int \frac{2^{2/3}-2x}{\left(x+2^{2/3}\right)\sqrt{-x^3-1}}dx}{3\ 2^{2/3}}+\frac{2\sqrt[3]{2}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

$$\Big\downarrow \ 2562$$

$$\frac{2}{3}\int \frac{1}{1-\frac{3\left(\sqrt[3]{2}x+1\right)^2}{-x^3-1}}d\frac{\sqrt[3]{2}x+1}{\sqrt{-x^3-1}}+\frac{2\sqrt[3]{2}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

$$\Big\downarrow \ 219$$

$$\frac{2\sqrt[3]{2}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}+\frac{2\text{arctanh}\left(\frac{\sqrt{3}\left(\sqrt[3]{2}x+1\right)}{\sqrt{-x^3-1}}\right)}{3\sqrt{3}}$$

input
```
Int[1/((2^(2/3) + x)*Sqrt[-1 - x^3]),x]
```

output
```
(2*ArcTanh[(Sqrt[3]*(1 + 2^(1/3)*x))/Sqrt[-1 - x^3]])/(3*Sqrt[3]) + (2*2^(
1/3)*Sqrt[2 - Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*Ell
ipticF[ArcSin[(1 + Sqrt[3] + x)/(1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(3*3^
(1/4)*Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^2)]*Sqrt[-1 - x^3])
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 2559
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Simp[2/
(3*c)   Int[1/Sqrt[a + b*x^3], x], x] + Simp[1/(3*c)   Int[(c - 2*d*x)/((c
+ d*x)*Sqrt[a + b*x^3]), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 - 4*
a*d^3, 0]
```

rule 2562
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[2*(e/d)   Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))
/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0]
&& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Maple [A] (verified)

Time = 1.58 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.89

method	result	size
default	$-\frac{2 i \sqrt{3}\, \sqrt{i \left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\, \sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\, \sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\, \operatorname{EllipticPi}\left(\frac{\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3}, \frac{i\sqrt{3}}{\frac{1}{2}+\frac{i\sqrt{3}}{2}+2^{\frac{2}{3}}}, \sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3-1}\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}+2^{\frac{2}{3}}\right)}$	139
elliptic	$-\frac{2 i \sqrt{3}\, \sqrt{i \left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\, \sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\, \sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\, \operatorname{EllipticPi}\left(\frac{\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3}, \frac{i\sqrt{3}}{\frac{1}{2}+\frac{i\sqrt{3}}{2}+2^{\frac{2}{3}}}, \sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3-1}\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}+2^{\frac{2}{3}}\right)}$	139

input
```
int(1/(2^(2/3)+x)/(-x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x+1)/(3/2+1/2*I*3
^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)/(1/
2+1/2*I*3^(1/2)+2^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3
^(1/2))^(1/2),I*3^(1/2)/(1/2+1/2*I*3^(1/2)+2^(2/3)),(I*3^(1/2)/(3/2+1/2*I*
3^(1/2)))^(1/2))
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(2^{2/3}+x)\sqrt{-1-x^3}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(2^(2/3)+x)/(-x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   catd
ef: division by zero
```

Sympy [F]

$$\int \frac{1}{(2^{2/3}+x)\sqrt{-1-x^3}}\,dx = \int \frac{1}{\sqrt{-(x+1)(x^2-x+1)}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

input
```
integrate(1/(2**(2/3)+x)/(-x**3-1)**(1/2),x)
```

output
```
Integral(1/(sqrt(-(x + 1)*(x**2 - x + 1))*(x + 2**(2/3))), x)
```

Maxima [F]

$$\int \frac{1}{\left(2^{2/3}+x\right)\sqrt{-1-x^3}}\,dx = \int \frac{1}{\sqrt{-x^3-1}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

input
```
integrate(1/(2^(2/3)+x)/(-x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(-x^3 - 1)*(x + 2^(2/3))), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\left(2^{2/3}+x\right)\sqrt{-1-x^3}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(2^(2/3)+x)/(-x^3-1)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:Unable to divide, perhaps due to rounding error%%%{1,[1
]%%%} / %%%{%%{[1,0,0]:[1,0,0,-2]%%},[1]%%%} Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(2^{2/3}+x\right)\sqrt{-1-x^3}}\,dx = \int \frac{1}{\sqrt{-x^3-1}\,\left(x+2^{2/3}\right)}\,dx$$

input
```
int(1/((- x^3 - 1)^(1/2)*(x + 2^(2/3))),x)
```

output
```
int(1/((- x^3 - 1)^(1/2)*(x + 2^(2/3))), x)
```

Reduce [F]

$$\int \frac{1}{(2^{2/3}+x)\sqrt{-1-x^3}}\,dx = -\left(\int \frac{1}{\sqrt{x^3+1}\,2^{\frac{2}{3}}+\sqrt{x^3+1}\,x}dx\right)i$$

input
```
int(1/(2^(2/3)+x)/(-x^3-1)^(1/2),x)
```

output
```
- int(1/(sqrt(x**3 + 1)*2**(2/3) + sqrt(x**3 + 1)*x),x)*i
```

3.122 $\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 280

$$\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=\frac{2\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{a+bx^3}}\right)}{3\sqrt{3}\sqrt{a}\sqrt[3]{b}}+\frac{2\sqrt[3]{2}\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{a}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
2/9*arctan(3^(1/2)*a^(1/6)*(a^(1/3)+2^(1/3)*b^(1/3)*x)/(b*x^3+a)^(1/2))*3^
(1/2)/a^(1/2)/b^(1/3)+2/9*2^(1/3)*(1/2*6^(1/2)+1/2*2^(1/2))*(a^(1/3)+b^(1/
3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3
)*x)^2)^(1/2)*EllipticF(((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/
3)+b^(1/3)*x),I*3^(1/2)+2*I)*3^(3/4)/a^(1/3)/b^(1/3)/(a^(1/3)*(a^(1/3)+b^(
1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.31 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.59

$$\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx = -\frac{2i\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{1-\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+\frac{b^{2/3}x^2}{a^{2/3}}}\,\text{EllipticPi}\left(\frac{i\sqrt{3}}{\sqrt[3]{-1}+2^{2/3}},\arcsin\left(\sqrt{\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right),\sqrt[3]{-1}\right)}{\left(\sqrt[3]{-1}+2^{2/3}\right)\sqrt[3]{b}\sqrt{a+bx^3}}$$

input
```
Integrate[1/((2^(2/3)*a^(1/3) + b^(1/3)*x)*Sqrt[a + b*x^3]),x]
```

output
```
((-2*I)*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[1 - (b
^(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(I*Sqrt[3])/((-1)^(1
/3) + 2^(2/3)), ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1
/3))*a^(1/3))]], (-1)^(1/3)])/(((-1)^(1/3) + 2^(2/3))*b^(1/3)*Sqrt[a + b*x
^3])
```

Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {2559, 759, 2562, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx$$

$$\Big\downarrow \text{2559}$$

$$\frac{\sqrt[3]{2}\int \frac{1}{\sqrt{bx^3+a}}dx}{3\sqrt[3]{a}}+\frac{\int \frac{2^{2/3}\sqrt[3]{a}-2\sqrt[3]{b}x}{\left(\sqrt[3]{b}x+2^{2/3}\sqrt[3]{a}\right)\sqrt{bx^3+a}}dx}{3\ 2^{2/3}\sqrt[3]{a}}$$

$$\Big\downarrow \text{759}$$

$$\frac{\int \frac{2^{2/3}\sqrt[3]{a}-2\sqrt[3]{b}x}{\left(\sqrt[3]{b}x+2^{2/3}\sqrt[3]{a}\right)\sqrt{bx^3+a}}dx}{3\ 2^{2/3}\sqrt[3]{a}}+\frac{2\sqrt[3]{2}\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{a}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{2562}$$

$$\frac{2\int \frac{1}{\frac{3\sqrt[3]{a}\left(\sqrt[3]{2}\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{bx^3+a}+1}d\frac{\sqrt[3]{2}\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt{bx^3+a}}}{3\sqrt[3]{b}}+\frac{2\sqrt[3]{2}\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{a}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{216}$$

$$\frac{2\sqrt[3]{2}\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{a}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{2\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{a+bx^3}}\right)}{3\sqrt{3}\sqrt{a}\sqrt[3]{b}}$$

input

```
Int[1/((2^(2/3)*a^(1/3) + b^(1/3)*x)*Sqrt[a + b*x^3]),x]
```

output
```
(2*ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) + 2^(1/3)*b^(1/3)*x))/Sqrt[a + b*x^3]]
)/(3*Sqrt[3]*Sqrt[a]*b^(1/3)) + (2*2^(1/3)*Sqrt[2 + Sqrt[3]]*(a^(1/3) + b^
(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a
^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x
)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*a^(1/3
)*b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^
(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 2559
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Simp[2/
(3*c)   Int[1/Sqrt[a + b*x^3], x], x] + Simp[1/(3*c)   Int[(c - 2*d*x)/((c
+ d*x)*Sqrt[a + b*x^3]), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 - 4*
a*d^3, 0]
```

rule 2562
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[2*(e/d)   Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))
/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0]
&& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Maple [F]

$$\int \frac{1}{\left(2^{\frac{2}{3}} a^{\frac{1}{3}}+b^{\frac{1}{3}} x \right) \sqrt{b\,x^{3}+a}}d x$$

input
```
int(1/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/2),x)
```

output
```
int(1/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=\text{Timed out}$$

input
```
integrate(1/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/2),x, algorithm="fric
as")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=\int \frac{1}{\sqrt{a+bx^3}\cdot\left(2^{\frac{2}{3}}\sqrt[3]{a}+\sqrt[3]{b}x\right)}\,dx$$

input
```
integrate(1/(2**(2/3)*a**(1/3)+b**(1/3)*x)/(b*x**3+a)**(1/2),x)
```

output
```
Integral(1/(sqrt(a + b*x**3)*(2**(2/3)*a**(1/3) + b**(1/3)*x)), x)
```

Maxima [F]

$$\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=\int \frac{1}{\sqrt{bx^3+a}\left(b^{\frac{1}{3}}x+2^{\frac{2}{3}}a^{\frac{1}{3}}\right)}\,dx$$

input
```
integrate(1/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/2),x, algorithm="maxi
ma")
```

output
```
integrate(1/(sqrt(b*x^3 + a)*(b^(1/3)*x + 2^(2/3)*a^(1/3))), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=\text{Timed out}$$

input
```
integrate(1/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/2),x, algorithm="giac
")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=\int \frac{1}{\sqrt{b\,x^3+a}\,\left(2^{2/3}\,a^{1/3}+b^{1/3}\,x\right)}\,dx$$

input
```
int(1/((a + b*x^3)^(1/2)*(2^(2/3)*a^(1/3) + b^(1/3)*x)),x)
```

output
```
int(1/((a + b*x^3)^(1/2)*(2^(2/3)*a^(1/3) + b^(1/3)*x)), x)
```

Reduce [F]

$$\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=\int \frac{1}{a^{\frac{1}{3}}\sqrt{b\,x^3+a}\,2^{\frac{2}{3}}+b^{\frac{1}{3}}\sqrt{b\,x^3+a}\,x}dx$$

input
```
int(1/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/2),x)
```

output
```
int(1/(a**(1/3)*sqrt(a + b*x**3)*2**(2/3) + b**(1/3)*sqrt(a + b*x**3)*x),x
)
```

3.123 $\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 288

$$\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx=-\frac{2\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{a-bx^3}}\right)}{3\sqrt{3}\sqrt{a}\sqrt[3]{b}}-\frac{2\sqrt[3]{2}\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{a}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$

output
```
-2/9*arctan(3^(1/2)*a^(1/6)*(a^(1/3)-2^(1/3)*b^(1/3)*x)/(-b*x^3+a)^(1/2))*
3^(1/2)/a^(1/2)/b^(1/3)-2/9*2^(1/3)*(1/2*6^(1/2)+1/2*2^(1/2))*(a^(1/3)-b^(
1/3)*x)*((a^(2/3)+a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1+3^(1/2))*a^(1/3)-b^(1
/3)*x)^2)^(1/2)*EllipticF(((1-3^(1/2))*a^(1/3)-b^(1/3)*x)/((1+3^(1/2))*a^(
1/3)-b^(1/3)*x),I*3^(1/2)+2*I)*3^(3/4)/a^(1/3)/b^(1/3)/(a^(1/3)*(a^(1/3)-b
^(1/3)*x)/((1+3^(1/2))*a^(1/3)-b^(1/3)*x)^2)^(1/2)/(-b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.29 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.58

$$\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx = \frac{2i\sqrt{\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+\frac{b^{2/3}x^2}{a^{2/3}}}\,\text{EllipticPi}\left(\frac{i\sqrt{3}}{\sqrt[3]{-1}+2^{2/3}},\arcsin\left(\right.\right.}{\left(\sqrt[3]{-1}+2^{2/3}\right)\sqrt[3]{b}\sqrt{a-bx^3}}$$

input
```
Integrate[1/((2^(2/3)*a^(1/3) - b^(1/3)*x)*Sqrt[a - b*x^3]),x]
```

output
```
((2*I)*Sqrt[(a^(1/3) - b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[1 + (b^
(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(I*Sqrt[3])/((-1)^(1/
3) + 2^(2/3)), ArcSin[Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/
3))*a^(1/3))]], (-1)^(1/3)])/(((-1)^(1/3) + 2^(2/3))*b^(1/3)*Sqrt[a - b*x^
3])
```

Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2559, 27, 759, 2562, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}dx$$

$$\Big\downarrow \text{2559}$$

$$\frac{\sqrt[3]{2}\int \frac{1}{\sqrt{a-bx^3}}dx}{3\sqrt[3]{a}}+\frac{\int \frac{2^{2/3}\left(\sqrt[3]{2}\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}dx}{3\ 2^{2/3}\sqrt[3]{a}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt[3]{2}\int \frac{1}{\sqrt{a-bx^3}}dx}{3\sqrt[3]{a}}+\frac{\int \frac{\sqrt[3]{2}\sqrt[3]{b}x+\sqrt[3]{a}}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}dx}{3\sqrt[3]{a}}$$

$$\Big\downarrow \text{759}$$

$$\frac{\int \frac{\sqrt[3]{2}\sqrt[3]{b}x+\sqrt[3]{a}}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}dx}{3\sqrt[3]{a}}-\frac{2\sqrt[3]{2}\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{a}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$

$$\Big\downarrow \text{2562}$$

$$-\frac{2\int \frac{1}{\frac{3\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x\right)^2}{a-bx^3}+1}d\frac{\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a}\sqrt{a-bx^3}}}{3\sqrt[3]{b}}-\frac{2\sqrt[3]{2}\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{a}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$

$$\Big\downarrow \text{216}$$

$$-\frac{2\sqrt[3]{2}\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{a}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}-\frac{2\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{a-bx^3}}\right)}{3\sqrt{3}\sqrt{a}\sqrt[3]{b}}$$

input
```
Int[1/((2^(2/3)*a^(1/3) - b^(1/3)*x)*Sqrt[a - b*x^3]),x]
```

output
```
(-2*ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(1/3)*b^(1/3)*x))/Sqrt[a - b*x^3]
])/(3*Sqrt[3]*Sqrt[a]*b^(1/3)) - (2*2^(1/3)*Sqrt[2 + Sqrt[3]]*(a^(1/3) - b
^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*
a^(1/3) - b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*
x)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*a^(1/
3)*b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) - b
^(1/3)*x)^2]*Sqrt[a - b*x^3])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 2559
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Simp[2/
(3*c)   Int[1/Sqrt[a + b*x^3], x], x] + Simp[1/(3*c)   Int[(c - 2*d*x)/((c
+ d*x)*Sqrt[a + b*x^3]), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 - 4*
a*d^3, 0]
```

rule 2562
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[2*(e/d)   Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))
/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0]
&& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Maple [F]

$$\int \frac{1}{\left(2^{\frac{2}{3}} a^{\frac{1}{3}}-b^{\frac{1}{3}} x\right) \sqrt{-b\,x^{3}+a}}dx$$

input
```
int(1/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(-b*x^3+a)^(1/2),x)
```

output
```
int(1/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(-b*x^3+a)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx=\text{Timed out}$$

input
```
integrate(1/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(-b*x^3+a)^(1/2),x, algorithm="fri
cas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx = -\int \frac{1}{-2^{\frac{2}{3}}\sqrt[3]{a}\sqrt{a-bx^3}+\sqrt[3]{b}x\sqrt{a-bx^3}}\,dx$$

input
```
integrate(1/(2**(2/3)*a**(1/3)-b**(1/3)*x)/(-b*x**3+a)**(1/2),x)
```

output
```
-Integral(1/(-2**(2/3)*a**(1/3)*sqrt(a - b*x**3) + b**(1/3)*x*sqrt(a - b*x
**3)), x)
```

Maxima [F]

$$\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx = \int -\frac{1}{\sqrt{-bx^3+a}\left(b^{\frac{1}{3}}x-2^{\frac{2}{3}}a^{\frac{1}{3}}\right)}\,dx$$

input
```
integrate(1/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(-b*x^3+a)^(1/2),x, algorithm="max
ima")
```

output
```
-integrate(1/(sqrt(-b*x^3 + a)*(b^(1/3)*x - 2^(2/3)*a^(1/3))), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx = \text{Timed out}$$

input
```
integrate(1/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(-b*x^3+a)^(1/2),x, algorithm="gia
c")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx=\int \frac{1}{\sqrt{a-b\,x^3}\,\left(2^{2/3}\,a^{1/3}-b^{1/3}\,x\right)}\,dx$$

input
```
int(1/((a - b*x^3)^(1/2)*(2^(2/3)*a^(1/3) - b^(1/3)*x)),x)
```

output
```
int(1/((a - b*x^3)^(1/2)*(2^(2/3)*a^(1/3) - b^(1/3)*x)), x)
```

Reduce [F]

$$\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx=\int \frac{1}{a^{\frac{1}{3}}\sqrt{-b\,x^3+a}\,2^{\frac{2}{3}}-b^{\frac{1}{3}}\sqrt{-b\,x^3+a}\,x}dx$$

input
```
int(1/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(-b*x^3+a)^(1/2),x)
```

output
```
int(1/(a**(1/3)*sqrt(a - b*x**3)*2**(2/3) - b**(1/3)*sqrt(a - b*x**3)*x),x
)
```

3.124 $\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 297

$$\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx = -\frac{2\operatorname{arctanh}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{-a+bx^3}}\right)}{3\sqrt{3}\sqrt{a}\sqrt[3]{b}} - \frac{2\sqrt[3]{2}\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{a}\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{-a+bx^3}}$$

output

```
-2/9*arctanh(3^(1/2)*a^(1/6)*(a^(1/3)-2^(1/3)*b^(1/3)*x)/(b*x^3-a)^(1/2))*
3^(1/2)/a^(1/2)/b^(1/3)-2/9*2^(1/3)*(1/2*6^(1/2)-1/2*2^(1/2))*(a^(1/3)-b^(
1/3)*x)*((a^(2/3)+a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1-3^(1/2))*a^(1/3)-b^(1
/3)*x)^2)^(1/2)*EllipticF(((1+3^(1/2))*a^(1/3)-b^(1/3)*x)/((1-3^(1/2))*a^(
1/3)-b^(1/3)*x),2*I-I*3^(1/2))*3^(3/4)/a^(1/3)/b^(1/3)/(-a^(1/3)*(a^(1/3)-
b^(1/3)*x)/((1-3^(1/2))*a^(1/3)-b^(1/3)*x)^2)^(1/2)/(b*x^3-a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.27 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.56

$$\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx = \frac{2i\sqrt{\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+\frac{b^{2/3}x^2}{a^{2/3}}}\,\text{EllipticPi}\left(\frac{i\sqrt{3}}{\sqrt[3]{-1}+2^{2/3}},\text{arcsin}\right.}{\left(\sqrt[3]{-1}+2^{2/3}\right)\sqrt[3]{b}\sqrt{-a+bx^3}}$$

input
```
Integrate[1/((2^(2/3)*a^(1/3) - b^(1/3)*x)*Sqrt[-a + b*x^3]),x]
```

output
```
((2*I)*Sqrt[(a^(1/3) - b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[1 + (b^
(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(I*Sqrt[3])/((-1)^(1/
3) + 2^(2/3)), ArcSin[Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/
3))*a^(1/3))]], (-1)^(1/3)])/(((-1)^(1/3) + 2^(2/3))*b^(1/3)*Sqrt[-a + b*x
^3])
```

Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.139$, Rules used = {2559, 27, 760, 2562, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{bx^3-a}}dx$$

$$\Big\downarrow \text{ 2559}$$

$$\frac{\sqrt[3]{2}\int \frac{1}{\sqrt{bx^3-a}}dx}{3\sqrt[3]{a}}+\frac{\int \frac{2^{2/3}\left(\sqrt[3]{2}\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{bx^3-a}}dx}{3\ 2^{2/3}\sqrt[3]{a}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt[3]{2}\int \frac{1}{\sqrt{bx^3-a}}dx}{3\sqrt[3]{a}}+\frac{\int \frac{\sqrt[3]{2}\sqrt[3]{b}x+\sqrt[3]{a}}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{bx^3-a}}dx}{3\sqrt[3]{a}}$$

$$\Big\downarrow \; 760$$

$$\frac{\int \frac{\sqrt[3]{2}\sqrt[3]{b}x+\sqrt[3]{a}}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{bx^3-a}}dx}{3\sqrt[3]{a}}-\frac{2\sqrt[3]{2}\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{a}\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}$$

$$\Big\downarrow \; 2562$$

$$-\frac{2\int \frac{1}{1-\frac{3\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x\right)^2}{bx^3-a}}d\frac{\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a}\sqrt{bx^3-a}}}{3\sqrt[3]{b}}-\frac{2\sqrt[3]{2}\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{a}\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}$$

$$\Big\downarrow \; 219$$

$$-\frac{2\sqrt[3]{2}\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{a}\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}-\frac{2\operatorname{arctanh}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{bx^3-a}}\right)}{3\sqrt{3}\sqrt{a}\sqrt[3]{b}}$$

input
```
Int[1/((2^(2/3)*a^(1/3) - b^(1/3)*x)*Sqrt[-a + b*x^3]),x]
```

output
```
(-2*ArcTanh[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(1/3)*b^(1/3)*x))/Sqrt[-a + b*x^
3]])/(3*Sqrt[3]*Sqrt[a]*b^(1/3)) - (2*2^(1/3)*Sqrt[2 - Sqrt[3]]*(a^(1/3) -
 b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3]
)*a^(1/3) - b^(1/3)*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - b^(1/3
)*x)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 + 4*Sqrt[3]])/(3*3^(1/4)*a^(
1/3)*b^(1/3)*Sqrt[-((a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3)
 - b^(1/3)*x)^2)]*Sqrt[-a + b*x^3])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 2559
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Simp[2/
(3*c)   Int[1/Sqrt[a + b*x^3], x], x] + Simp[1/(3*c)   Int[(c - 2*d*x)/((c
+ d*x)*Sqrt[a + b*x^3]), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 - 4*
a*d^3, 0]
```

rule 2562
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[2*(e/d)   Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))
/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0]
&& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Maple [F]

$$\int \frac{1}{\left(2^{\frac{2}{3}} a^{\frac{1}{3}}-b^{\frac{1}{3}} x\right) \sqrt{b\,x^{3}-a}}dx$$

input
```
int(1/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(b*x^3-a)^(1/2),x)
```

output
```
int(1/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(b*x^3-a)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx=\text{Timed out}$$

input
```
integrate(1/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(b*x^3-a)^(1/2),x, algorithm="fric
as")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx=-\int \frac{1}{-2^{\frac{2}{3}}\sqrt[3]{a}\sqrt{-a+bx^3}+\sqrt[3]{b}x\sqrt{-a+bx^3}}\,dx$$

input
```
integrate(1/(2**(2/3)*a**(1/3)-b**(1/3)*x)/(b*x**3-a)**(1/2),x)
```

output
```
-Integral(1/(-2**(2/3)*a**(1/3)*sqrt(-a + b*x**3) + b**(1/3)*x*sqrt(-a + b
*x**3)), x)
```

Maxima [F]

$$\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx=\int -\frac{1}{\sqrt{bx^3-a}\left(b^{\frac{1}{3}}x-2^{\frac{2}{3}}a^{\frac{1}{3}}\right)}\,dx$$

input
```
integrate(1/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(b*x^3-a)^(1/2),x, algorithm="maxi
ma")
```

output
```
-integrate(1/(sqrt(b*x^3 - a)*(b^(1/3)*x - 2^(2/3)*a^(1/3))), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx=\text{Timed out}$$

input
```
integrate(1/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(b*x^3-a)^(1/2),x, algorithm="giac
")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx=\int \frac{1}{\sqrt{b\,x^3-a}\,\left(2^{2/3}\,a^{1/3}-b^{1/3}\,x\right)}\,dx$$

input
```
int(1/((b*x^3 - a)^(1/2)*(2^(2/3)*a^(1/3) - b^(1/3)*x)),x)
```

output
```
int(1/((b*x^3 - a)^(1/2)*(2^(2/3)*a^(1/3) - b^(1/3)*x)), x)
```

Reduce [F]

$$\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx=\int \frac{1}{a^{\frac{1}{3}}\sqrt{b\,x^3-a}\,2^{\frac{2}{3}}-b^{\frac{1}{3}}\sqrt{b\,x^3-a}\,x}dx$$

input
```
int(1/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(b*x^3-a)^(1/2),x)
```

output
```
int(1/(a**(1/3)*sqrt( - a + b*x**3)*2**(2/3) - b**(1/3)*sqrt( - a + b*x**3
)*x),x)
```

3.125 $\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 293

$$\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx = \frac{2\operatorname{arctanh}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{-a-bx^3}}\right)}{3\sqrt{3}\sqrt{a}\sqrt[3]{b}} + \frac{2\sqrt[3]{2}\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{a}\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$

output
```
2/9*arctanh(3^(1/2)*a^(1/6)*(a^(1/3)+2^(1/3)*b^(1/3)*x)/(-b*x^3-a)^(1/2))*
3^(1/2)/a^(1/2)/b^(1/3)+2/9*2^(1/3)*(1/2*6^(1/2)-1/2*2^(1/2))*(a^(1/3)+b^(
1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1-3^(1/2))*a^(1/3)+b^(1
/3)*x)^2)^(1/2)*EllipticF(((1+3^(1/2))*a^(1/3)+b^(1/3)*x)/((1-3^(1/2))*a^(
1/3)+b^(1/3)*x),2*I-I*3^(1/2))*3^(3/4)/a^(1/3)/b^(1/3)/(-a^(1/3)*(a^(1/3)+
b^(1/3)*x)/((1-3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)/(-b*x^3-a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.28 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.57

$$\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx =$$
$$-\frac{2i\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{1-\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+\frac{b^{2/3}x^2}{a^{2/3}}}\,\text{EllipticPi}\left(\frac{i\sqrt{3}}{\sqrt[3]{-1}+2^{2/3}},\arcsin\left(\sqrt{\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right),\sqrt[3]{-1}\right)}{\left(\sqrt[3]{-1}+2^{2/3}\right)\sqrt[3]{b}\sqrt{-a-bx^3}}$$

input
```
Integrate[1/((2^(2/3)*a^(1/3) + b^(1/3)*x)*Sqrt[-a - b*x^3]),x]
```

output
```
((-2*I)*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[1 - (b
^(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(I*Sqrt[3])/((-1)^(1
/3) + 2^(2/3)), ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1
/3))*a^(1/3))]], (-1)^(1/3)])/(((-1)^(1/3) + 2^(2/3))*b^(1/3)*Sqrt[-a - b*
x^3])
```

Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2559, 760, 2562, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}dx$$

$$\Big\downarrow \text{2559}$$

$$\frac{\sqrt[3]{2}\int \frac{1}{\sqrt{-bx^3-a}}dx}{3\sqrt[3]{a}}+\frac{\int \frac{2^{2/3}\sqrt[3]{a}-2\sqrt[3]{b}x}{\left(\sqrt[3]{b}x+2^{2/3}\sqrt[3]{a}\right)\sqrt{-bx^3-a}}dx}{3\ 2^{2/3}\sqrt[3]{a}}$$

$$\Big\downarrow \text{760}$$

$$\frac{\int \frac{2^{2/3}\sqrt[3]{a}-2\sqrt[3]{b}x}{\left(\sqrt[3]{b}x+2^{2/3}\sqrt[3]{a}\right)\sqrt{-bx^3-a}}dx}{3\ 2^{2/3}\sqrt[3]{a}}+\frac{2\sqrt[3]{2}\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{a}\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$

$$\Big\downarrow \text{2562}$$

$$\frac{2\int \frac{1}{1-\frac{3\sqrt[3]{a}\left(\sqrt[3]{2}\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{-bx^3-a}}d\frac{\sqrt[3]{2}\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt{-bx^3-a}}}{3\sqrt[3]{b}}+\frac{2\sqrt[3]{2}\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{a}\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$

$$\Big\downarrow \text{219}$$

$$\frac{2\sqrt[3]{2}\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{a}\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}+\frac{2\text{arctanh}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{-a-bx^3}}\right)}{3\sqrt{3}\sqrt{a}\sqrt[3]{b}}$$

input
```
Int[1/((2^(2/3)*a^(1/3) + b^(1/3)*x)*Sqrt[-a - b*x^3]),x]
```

output
```
(2*ArcTanh[(Sqrt[3]*a^(1/6)*(a^(1/3) + 2^(1/3)*b^(1/3)*x))/Sqrt[-a - b*x^3
]])/(3*Sqrt[3]*Sqrt[a]*b^(1/3)) + (2*2^(1/3)*Sqrt[2 - Sqrt[3]]*(a^(1/3) +
b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])
*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) + b^(1/3)
*x)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 + 4*Sqrt[3]])/(3*3^(1/4)*a^(1
/3)*b^(1/3)*Sqrt[-((a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3)
+ b^(1/3)*x)^2)]*Sqrt[-a - b*x^3])
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 2559
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Simp[2/
(3*c)   Int[1/Sqrt[a + b*x^3], x], x] + Simp[1/(3*c)   Int[(c - 2*d*x)/((c
+ d*x)*Sqrt[a + b*x^3]), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 - 4*
a*d^3, 0]
```

rule 2562
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[2*(e/d)   Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))
/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0]
&& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Maple [F]

$$\int \frac{1}{\left(2^{\frac{2}{3}} a^{\frac{1}{3}}+b^{\frac{1}{3}} x\right) \sqrt{-b\,x^{3}-a}}dx$$

input
```
int(1/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(-b*x^3-a)^(1/2),x)
```

output
```
int(1/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(-b*x^3-a)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx = \text{Timed out}$$

input
```
integrate(1/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(-b*x^3-a)^(1/2),x, algorithm="fri
cas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx = \int \frac{1}{\sqrt{-a-bx^3}\cdot\left(2^{\frac{2}{3}}\sqrt[3]{a}+\sqrt[3]{b}x\right)}\,dx$$

input
```
integrate(1/(2**(2/3)*a**(1/3)+b**(1/3)*x)/(-b*x**3-a)**(1/2),x)
```

output
```
Integral(1/(sqrt(-a - b*x**3)*(2**(2/3)*a**(1/3) + b**(1/3)*x)), x)
```

Maxima [F]

$$\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx=\int \frac{1}{\sqrt{-bx^3-a}\left(b^{\frac{1}{3}}x+2^{\frac{2}{3}}a^{\frac{1}{3}}\right)}\,dx$$

input
```
integrate(1/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(-b*x^3-a)^(1/2),x, algorithm="max
ima")
```

output
```
integrate(1/(sqrt(-b*x^3 - a)*(b^(1/3)*x + 2^(2/3)*a^(1/3))), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx=\text{Timed out}$$

input
```
integrate(1/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(-b*x^3-a)^(1/2),x, algorithm="gia
c")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx=\int \frac{1}{\sqrt{-b\,x^3-a}\,\left(2^{2/3}\,a^{1/3}+b^{1/3}\,x\right)}\,dx$$

input
```
int(1/((- a - b*x^3)^(1/2)*(2^(2/3)*a^(1/3) + b^(1/3)*x)),x)
```

output
```
int(1/((- a - b*x^3)^(1/2)*(2^(2/3)*a^(1/3) + b^(1/3)*x)), x)
```

Reduce [F]

$$\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx = -\left(\int \frac{1}{a^{\frac{1}{3}}\sqrt{b\,x^3+a}\,2^{\frac{2}{3}}+b^{\frac{1}{3}}\sqrt{b\,x^3+a}\,x}dx\right)i$$

input
```
int(1/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(-b*x^3-a)^(1/2),x)
```

output
```
 - int(1/(a**(1/3)*sqrt(a + b*x**3)*2**(2/3) + b**(1/3)*sqrt(a + b*x**3)*x
),x)*i
```

3.126 $\int \frac{1}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 249

$$\int \frac{1}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx = \frac{2\arctan\left(\frac{\sqrt{3}\sqrt{c}(c+2dx)}{\sqrt{c^3+4d^3x^3}}\right)}{3\sqrt{3}c^{3/2}d} + \frac{2\sqrt[3]{2}\sqrt{2+\sqrt{3}}\left(c+2^{2/3}dx\right)\sqrt{\frac{c^2-2^{2/3}cdx+2\sqrt[3]{2}d^2x^2}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)c+2^{2/3}dx}{\left(1+\sqrt{3}\right)c+2^{2/3}dx}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}cd\sqrt{\frac{c\left(c+2^{2/3}dx\right)}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}\sqrt{c^3+4d^3x^3}}$$

output
```
2/9*arctan(3^(1/2)*c^(1/2)*(2*d*x+c)/(4*d^3*x^3+c^3)^(1/2))*3^(1/2)/c^(3/2
)/d+2/9*2^(1/3)*(1/2*6^(1/2)+1/2*2^(1/2))*(c+2^(2/3)*d*x)*((c^2-2^(2/3)*c*
d*x+2*2^(1/3)*d^2*x^2)/((1+3^(1/2))*c+2^(2/3)*d*x)^2)^(1/2)*EllipticF(((1-
3^(1/2))*c+2^(2/3)*d*x)/((1+3^(1/2))*c+2^(2/3)*d*x),I*3^(1/2)+2*I)*3^(3/4)
/c/d/(c*(c+2^(2/3)*d*x)/((1+3^(1/2))*c+2^(2/3)*d*x)^2)^(1/2)/(4*d^3*x^3+c^
3)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.32 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.68

$$\int \frac{1}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx = -\frac{i2^{5/6}\sqrt{\frac{\sqrt[3]{2}c+2dx}{\left(1+\sqrt[3]{-1}\right)c}}\sqrt{2^{2/3}-\frac{2\sqrt[3]{2}dx}{c}+\frac{4d^2x^2}{c^2}}\,\text{EllipticPi}\left(\frac{i\sqrt[3]{2}\sqrt{3}}{2+\sqrt[3]{-2}},\arcsin\left(\frac{\sqrt{\frac{\sqrt[3]{2}c+2(-1)^{2/3}dx}{\left(1+\sqrt[3]{-1}\right)c}}}{\sqrt[6]{2}}\right),\sqrt[3]{-1}\right)}{\left(2+\sqrt[3]{-2}\right)d\sqrt{c^3+4d^3x^3}}$$

input
```
Integrate[1/((c + d*x)*Sqrt[c^3 + 4*d^3*x^3]),x]
```

output
```
((-I)*2^(5/6)*Sqrt[(2^(1/3)*c + 2*d*x)/((1 + (-1)^(1/3))*c)]*Sqrt[2^(2/3)
- (2*2^(1/3)*d*x)/c + (4*d^2*x^2)/c^2]*EllipticPi[(I*2^(1/3)*Sqrt[3])/(2 +
 (-2)^(1/3)), ArcSin[Sqrt[(2^(1/3)*c + 2*(-1)^(2/3)*d*x)/((1 + (-1)^(1/3))
*c)]/2^(1/6)], (-1)^(1/3)])/((2 + (-2)^(1/3))*d*Sqrt[c^3 + 4*d^3*x^3])
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2559, 759, 2562, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx$$

$$\Big\downarrow \text{2559}$$

$$\frac{2\int \frac{1}{\sqrt{c^3+4d^3x^3}}dx}{3c}+\frac{\int \frac{c-2dx}{(c+dx)\sqrt{c^3+4d^3x^3}}dx}{3c}$$

$$\Big\downarrow \text{759}$$

$$\frac{\int \frac{c-2dx}{(c+dx)\sqrt{c^3+4d^3x^3}}dx}{3c}+\frac{2\sqrt[3]{2}\sqrt{2+\sqrt{3}}(c+2^{2/3}dx)\sqrt{\frac{c^2-2^{2/3}cdx+2\sqrt[3]{2}d^2x^2}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)c+2^{2/3}dx}{\left(1+\sqrt{3}\right)c+2^{2/3}dx}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}cd\sqrt{\frac{c(c+2^{2/3}dx)}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}\sqrt{c^3+4d^3x^3}}$$

$$\Big\downarrow \; 2562$$

$$\frac{2\int \frac{1}{\frac{3c(c+2dx)^2}{c^3+4d^3x^3}+1}d\frac{c+2dx}{c\sqrt{c^3+4d^3x^3}}}{3d}+\frac{2\sqrt[3]{2}\sqrt{2+\sqrt{3}}(c+2^{2/3}dx)\sqrt{\frac{c^2-2^{2/3}cdx+2\sqrt[3]{2}d^2x^2}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)c+2^{2/3}dx}{\left(1+\sqrt{3}\right)c+2^{2/3}dx}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}cd\sqrt{\frac{c(c+2^{2/3}dx)}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}\sqrt{c^3+4d^3x^3}}$$

$$\Big\downarrow \; 216$$

$$\frac{2\sqrt[3]{2}\sqrt{2+\sqrt{3}}(c+2^{2/3}dx)\sqrt{\frac{c^2-2^{2/3}cdx+2\sqrt[3]{2}d^2x^2}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)c+2^{2/3}dx}{\left(1+\sqrt{3}\right)c+2^{2/3}dx}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}cd\sqrt{\frac{c(c+2^{2/3}dx)}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}\sqrt{c^3+4d^3x^3}}+\frac{2\arctan\left(\frac{\sqrt{3}\sqrt{c}(c+2dx)}{\sqrt{c^3+4d^3x^3}}\right)}{3\sqrt{3}c^{3/2}d}$$

input
```
Int[1/((c + d*x)*Sqrt[c^3 + 4*d^3*x^3]),x]
```

output
```
(2*ArcTan[(Sqrt[3]*Sqrt[c]*(c + 2*d*x))/Sqrt[c^3 + 4*d^3*x^3]])/(3*Sqrt[3]
*c^(3/2)*d) + (2*2^(1/3)*Sqrt[2 + Sqrt[3]]*(c + 2^(2/3)*d*x)*Sqrt[(c^2 - 2
^(2/3)*c*d*x + 2*2^(1/3)*d^2*x^2)/((1 + Sqrt[3])*c + 2^(2/3)*d*x)^2]*Ellip
ticF[ArcSin[((1 - Sqrt[3])*c + 2^(2/3)*d*x)/((1 + Sqrt[3])*c + 2^(2/3)*d*x
)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*c*d*Sqrt[(c*(c + 2^(2/3)*d*x))/((1 + Sqrt[
3])*c + 2^(2/3)*d*x)^2]*Sqrt[c^3 + 4*d^3*x^3])
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 2559
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Simp[2/
(3*c)   Int[1/Sqrt[a + b*x^3], x], x] + Simp[1/(3*c)   Int[(c - 2*d*x)/((c
+ d*x)*Sqrt[a + b*x^3]), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 - 4*
a*d^3, 0]
```

rule 2562
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[2*(e/d)   Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))
/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0]
&& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 494 vs. $2(200) = 400$.

Time = 0.51 (sec) , antiderivative size = 495, normalized size of antiderivative = 1.99

method	result
default	$\frac{2\left(\frac{\left(\frac{2^{\frac{1}{3}}}{4}-\frac{i\sqrt{3}\,2^{\frac{1}{3}}}{4}\right)c}{d}-\frac{\left(\frac{2^{\frac{1}{3}}}{4}+\frac{i\sqrt{3}\,2^{\frac{1}{3}}}{4}\right)c}{d}\right)\sqrt{\frac{x-\frac{\left(\frac{2^{\frac{1}{3}}}{4}+\frac{i\sqrt{3}\,2^{\frac{1}{3}}}{4}\right)c}{d}}{\frac{\left(\frac{2^{\frac{1}{3}}}{4}-\frac{i\sqrt{3}\,2^{\frac{1}{3}}}{4}\right)c}{d}-\frac{\left(\frac{2^{\frac{1}{3}}}{4}+\frac{i\sqrt{3}\,2^{\frac{1}{3}}}{4}\right)c}{d}}}\sqrt{\frac{x+\frac{2^{\frac{1}{3}}c}{2d}}{\frac{\left(\frac{2^{\frac{1}{3}}}{4}+\frac{i\sqrt{3}\,2^{\frac{1}{3}}}{4}\right)c}{d}+\frac{2^{\frac{1}{3}}c}{2d}}}\sqrt{\frac{x-\frac{\left(\frac{2^{\frac{1}{3}}}{4}-i\right.}{}}{\frac{\left(\frac{2^{\frac{1}{3}}}{4}+\frac{i\sqrt{3}\,2^{\frac{1}{3}}}{4}\right)c}{d}-}}}{d\sqrt{4d^3x^3-}}$
elliptic	$\frac{2\left(\frac{\left(\frac{2^{\frac{1}{3}}}{4}-\frac{i\sqrt{3}\,2^{\frac{1}{3}}}{4}\right)c}{d}-\frac{\left(\frac{2^{\frac{1}{3}}}{4}+\frac{i\sqrt{3}\,2^{\frac{1}{3}}}{4}\right)c}{d}\right)\sqrt{\frac{x-\frac{\left(\frac{2^{\frac{1}{3}}}{4}+\frac{i\sqrt{3}\,2^{\frac{1}{3}}}{4}\right)c}{d}}{\frac{\left(\frac{2^{\frac{1}{3}}}{4}-\frac{i\sqrt{3}\,2^{\frac{1}{3}}}{4}\right)c}{d}-\frac{\left(\frac{2^{\frac{1}{3}}}{4}+\frac{i\sqrt{3}\,2^{\frac{1}{3}}}{4}\right)c}{d}}}\sqrt{\frac{x+\frac{2^{\frac{1}{3}}c}{2d}}{\frac{\left(\frac{2^{\frac{1}{3}}}{4}+\frac{i\sqrt{3}\,2^{\frac{1}{3}}}{4}\right)c}{d}+\frac{2^{\frac{1}{3}}c}{2d}}}\sqrt{\frac{x-\frac{\left(\frac{2^{\frac{1}{3}}}{4}-i\right.}{}}{\frac{\left(\frac{2^{\frac{1}{3}}}{4}+\frac{i\sqrt{3}\,2^{\frac{1}{3}}}{4}\right)c}{d}-}}}{d\sqrt{4d^3x^3-}}$

input

```
int(1/(d*x+c)/(4*d^3*x^3+c^3)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
2/d*((1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^
(1/3))*c/d)*((x-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^(1/3)-1/4
*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d))^(1/2)*((
x+1/2*2^(1/3)*c/d)/((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d+1/2*2^(1/3)*c/
d))^(1/2)*((x-(1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^(1/3)+1/4*I
*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d))^(1/2)/(4*d
^3*x^3+c^3)^(1/2)/((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d+c/d)*EllipticPi
(((x-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^(1/3)-1/4*I*3^(1/2)*
2^(1/3))*c/d-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d))^(1/2),((1/4*2^(1/3)
+1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4
*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d+c/d),(((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1
/3))*c/d-(1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^(1/3)+1/4*I*3^(1
/2)*2^(1/3))*c/d+1/2*2^(1/3)*c/d))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.40

$$\int \frac{1}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx$$
$$= \left[-\frac{\sqrt{3}\sqrt{-c}d^2\log\left(\frac{2\,d^6x^6-36\,cd^5x^5-18\,c^2d^4x^4+28\,c^3d^3x^3+18\,c^4d^2x^2-c^6+\sqrt{3}(4\,d^4x^4-10\,cd^3x^3-18\,c^2d^2x^2-8\,c^3dx-c^4)\sqrt{4\,d^3x^3+c^3}}{d^6x^6+6\,cd^5x^5+15\,c^2d^4x^4+20\,c^3d^3x^3+15\,c^4d^2x^2+6\,c^5dx+c^6}\right.}{18\,c^2d^3}\right.$$
$$\left.-\frac{\sqrt{3}\sqrt{c}d^2\arctan\left(\frac{\sqrt{3}\sqrt{4\,d^3x^3+c^3}\,(2\,d^3x^3-6\,cd^2x^2-6\,c^2dx-c^3)\sqrt{c}}{3\,(8\,cd^4x^4+4\,c^2d^3x^3+2\,c^4dx+c^5)}\right)-6\,c\sqrt{d^3}\mathrm{weierstrassPInverse}\left(0,-\frac{c^3}{d^3},x\right)}{9\,c^2d^3}\right]$$

input
```
integrate(1/(d*x+c)/(4*d^3*x^3+c^3)^(1/2),x, algorithm="fricas")
```

output
```
[-1/18*(sqrt(3)*sqrt(-c)*d^2*log((2*d^6*x^6 - 36*c*d^5*x^5 - 18*c^2*d^4*x^
4 + 28*c^3*d^3*x^3 + 18*c^4*d^2*x^2 - c^6 + sqrt(3)*(4*d^4*x^4 - 10*c*d^3*
x^3 - 18*c^2*d^2*x^2 - 8*c^3*d*x - c^4)*sqrt(4*d^3*x^3 + c^3)*sqrt(-c))/(d
^6*x^6 + 6*c*d^5*x^5 + 15*c^2*d^4*x^4 + 20*c^3*d^3*x^3 + 15*c^4*d^2*x^2 +
6*c^5*d*x + c^6)) - 12*c*sqrt(d^3)*weierstrassPInverse(0, -c^3/d^3, x))/(c
^2*d^3), -1/9*(sqrt(3)*sqrt(c)*d^2*arctan(1/3*sqrt(3)*sqrt(4*d^3*x^3 + c^3
)*(2*d^3*x^3 - 6*c*d^2*x^2 - 6*c^2*d*x - c^3)*sqrt(c)/(8*c*d^4*x^4 + 4*c^2
*d^3*x^3 + 2*c^4*d*x + c^5)) - 6*c*sqrt(d^3)*weierstrassPInverse(0, -c^3/d
^3, x))/(c^2*d^3)]
```

Sympy [F]

$$\int \frac{1}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx = \int \frac{1}{(c+dx)\,\sqrt{c^3+4d^3x^3}}\,dx$$

input
```
integrate(1/(d*x+c)/(4*d**3*x**3+c**3)**(1/2),x)
```

output
```
Integral(1/((c + d*x)*sqrt(c**3 + 4*d**3*x**3)), x)
```

Maxima [F]

$$\int \frac{1}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx = \int \frac{1}{\sqrt{4\,d^3x^3+c^3}(dx+c)}\,dx$$

input
```
integrate(1/(d*x+c)/(4*d^3*x^3+c^3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(4*d^3*x^3 + c^3)*(d*x + c)), x)
```

Giac [F]

$$\int \frac{1}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx = \int \frac{1}{\sqrt{4\,d^3x^3+c^3}(dx+c)}\,dx$$

input
```
integrate(1/(d*x+c)/(4*d^3*x^3+c^3)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(4*d^3*x^3 + c^3)*(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx = \int \frac{1}{\sqrt{c^3+4\,d^3\,x^3}\,(c+d\,x)}\,dx$$

input
```
int(1/((c^3 + 4*d^3*x^3)^(1/2)*(c + d*x)),x)
```

output
```
int(1/((c^3 + 4*d^3*x^3)^(1/2)*(c + d*x)), x)
```

Reduce [F]

$$\int \frac{1}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx = \int \frac{\sqrt{4d^3x^3+c^3}}{4d^4x^4+4c\,d^3x^3+c^3dx+c^4}dx$$

input

```
int(1/(d*x+c)/(4*d^3*x^3+c^3)^(1/2),x)
```

output

```
int(sqrt(c**3 + 4*d**3*x**3)/(c**4 + c**3*d*x + 4*c*d**3*x**3 + 4*d**4*x**
4),x)
```

3.127 $\int \frac{1}{(1+\sqrt{3}+x)\sqrt{1+x^3}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 146

$$\int \frac{1}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = \frac{\arctan\left(\frac{\sqrt{3+2\sqrt{3}}(1+x)}{\sqrt{1+x^3}}\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}} + \frac{\sqrt{2+\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{3^{3/4}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

output
```
arctan((3+2*3^(1/2))^(1/2)*(1+x)/(x^3+1)^(1/2))/(9+6*3^(1/2))^(1/2)+1/3*(1
/2*6^(1/2)+1/2*2^(1/2))*(1+x)*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*EllipticF(
(1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*3^(1/4)/((1+x)/(1+x+3^(1/2))^2)
^(1/2)/(x^3+1)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 20.32 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.93

$$\int \frac{1}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = -\frac{4\sqrt{2}\sqrt{\frac{i(1+x)}{3i+\sqrt{3}}}\sqrt{1-x+x^2}\,\text{EllipticPi}\left(\frac{2\sqrt{3}}{3i+(1+2i)\sqrt{3}},\arcsin\left(\frac{\sqrt{i+\sqrt{3}-2ix}}{\sqrt{2}\sqrt[4]{3}}\right),\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)}{\left(3i+(1+2i)\sqrt{3}\right)\sqrt{1+x^3}}$$

input
```
Integrate[1/((1 + Sqrt[3] + x)*Sqrt[1 + x^3]),x]
```

output
```
(-4*Sqrt[2]*Sqrt[(I*(1 + x))/(3*I + Sqrt[3])]*Sqrt[1 - x + x^2]*EllipticPi
[(2*Sqrt[3])/(3*I + (1 + 2*I)*Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] - (2*I)*x]
/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])])/((3*I + (1 + 2*I)*Sqrt[
3])*Sqrt[1 + x^3])
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2560, 27, 759, 2565, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(x+\sqrt{3}+1\right)\sqrt{x^3+1}}dx$$

$$\Big\downarrow \text{2560}$$

$$\frac{\int \frac{1}{\sqrt{x^3+1}}dx}{2\sqrt{3}}-\frac{\int \frac{6\left(x-\sqrt{3}+1\right)}{\left(x+\sqrt{3}+1\right)\sqrt{x^3+1}}dx}{12\sqrt{3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{1}{\sqrt{x^3+1}}dx}{2\sqrt{3}}-\frac{\int \frac{x-\sqrt{3}+1}{\left(x+\sqrt{3}+1\right)\sqrt{x^3+1}}dx}{2\sqrt{3}}$$

$$\Big\downarrow \text{759}$$

$$\frac{\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{3^{3/4}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{\int \frac{x-\sqrt{3}+1}{\left(x+\sqrt{3}+1\right)\sqrt{x^3+1}}dx}{2\sqrt{3}}$$

$$\Big\downarrow \text{2565}$$

$$\frac{\int \frac{1}{\frac{\left(3+2\sqrt{3}\right)(x+1)^2}{x^3+1}+1}d\frac{x+1}{\sqrt{x^3+1}}}{\sqrt{3}}+\frac{\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{3^{3/4}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

$$\Big\downarrow \text{216}$$

$$\frac{\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{3^{3/4}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}+\frac{\arctan\left(\frac{\sqrt{3+2\sqrt{3}}(x+1)}{\sqrt{x^3+1}}\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}}$$

input
```
Int[1/((1 + Sqrt[3] + x)*Sqrt[1 + x^3]),x]
```

output
```
ArcTan[(Sqrt[3 + 2*Sqrt[3]]*(1 + x))/Sqrt[1 + x^3]]/Sqrt[3*(3 + 2*Sqrt[3])
] + (Sqrt[2 + Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*Ell
ipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3^(3
/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 2560
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Simp[-6
*a*(d^3/(c*(b*c^3 - 28*a*d^3)))   Int[1/Sqrt[a + b*x^3], x], x] + Simp[1/(c
*(b*c^3 - 28*a*d^3))   Int[Simp[c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x, x]/((c +
d*x)*Sqrt[a + b*x^3]), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2*c^6 - 20
*a*b*c^3*d^3 - 8*a^2*d^6, 0]
```

rule 2565
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Simp[(1 + k)*(e/d)   S
ubst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x
^3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c
^6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^
3), 0]
```

Maple [A] (verified)

Time = 1.08 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.90

method	result	size
default	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{3}\,\mathrm{EllipticPi}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\frac{\left(-\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}{3},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{x^{3}+1}}$	132
elliptic	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{3}\,\mathrm{EllipticPi}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\frac{\left(-\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}{3},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{x^{3}+1}}$	132

input
```
int(1/(1+3^(1/2)+x)/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^
(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1
/2)))^(1/2)/(x^3+1)^(1/2)*3^(1/2)*EllipticPi(((x+1)/(3/2-1/2*I*3^(1/2)))^(
1/2),1/3*(-3/2+1/2*I*3^(1/2))*3^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^
(1/2)))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.39

$$\int \frac{1}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^{3}}}\,dx = -\frac{1}{6}\sqrt{2\sqrt{3}-3}\arctan\left(\frac{\left(\sqrt{3}(x^{2}-4x-2)-6x-6\right)\sqrt{2\sqrt{3}-3}}{6\sqrt{x^{3}+1}}\right) + \frac{1}{3}\sqrt{3}\mathrm{weierstrassPInverse}(0,-4,x)$$

input
```
integrate(1/(1+3^(1/2)+x)/(x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/6*sqrt(2*sqrt(3) - 3)*arctan(1/6*(sqrt(3)*(x^2 - 4*x - 2) - 6*x - 6)*sq
rt(2*sqrt(3) - 3)/sqrt(x^3 + 1)) + 1/3*sqrt(3)*weierstrassPInverse(0, -4,
x)
```

Sympy [F]

$$\int \frac{1}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = \int \frac{1}{\sqrt{(x+1)\left(x^2-x+1\right)}\left(x+1+\sqrt{3}\right)}\,dx$$

input
```
integrate(1/(1+3**(1/2)+x)/(x**3+1)**(1/2),x)
```

output
```
Integral(1/(sqrt((x + 1)*(x**2 - x + 1))*(x + 1 + sqrt(3))), x)
```

Maxima [F]

$$\int \frac{1}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = \int \frac{1}{\sqrt{x^3+1}\left(x+\sqrt{3}+1\right)}\,dx$$

input
```
integrate(1/(1+3^(1/2)+x)/(x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(x^3 + 1)*(x + sqrt(3) + 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(1+3^(1/2)+x)/(x^3+1)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:Unable to divide, perhaps due to rounding error%%%{1,[2
]%%%} / %%%{%%{[2,4]:[1,0,-3]%%},[2]%%%} Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = \text{Hanged}$$

input
```
int(1/((x^3 + 1)^(1/2)*(x + 3^(1/2) + 1)),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{1}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = -\sqrt{3}\left(\int \frac{\sqrt{x^3+1}}{x^5+2x^4-2x^3+x^2+2x-2}dx\right) + \int \frac{\sqrt{x^3+1}}{x^5+2x^4-2x^3+x^2+2x-2}dx + \int \frac{\sqrt{x^3+1}\,x}{x^5+2x^4-2x^3+x^2+2x-2}dx$$

input
```
int(1/(1+3^(1/2)+x)/(x^3+1)^(1/2),x)
```

output
```
- sqrt(3)*int(sqrt(x**3 + 1)/(x**5 + 2*x**4 - 2*x**3 + x**2 + 2*x - 2),x)
 + int(sqrt(x**3 + 1)/(x**5 + 2*x**4 - 2*x**3 + x**2 + 2*x - 2),x) + int((
sqrt(x**3 + 1)*x)/(x**5 + 2*x**4 - 2*x**3 + x**2 + 2*x - 2),x)
```

3.128 $\int \frac{1}{(1+\sqrt{3}-x)\sqrt{1-x^3}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 164

$$\int \frac{1}{\left(1+\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx = -\frac{\arctan\left(\frac{\sqrt{3+2\sqrt{3}}(1-x)}{\sqrt{1-x^3}}\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}} - \frac{\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right),-7-4\sqrt{3}\right)}{3^{3/4}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

output
```
-arctan((3+2*3^(1/2))^(1/2)*(1-x)/(-x^3+1)^(1/2))/(9+6*3^(1/2))^(1/2)-1/3*
(1/2*6^(1/2)+1/2*2^(1/2))*(1-x)*((x^2+x+1)/(1+3^(1/2)-x)^2)^(1/2)*Elliptic
F((1-3^(1/2)-x)/(1+3^(1/2)-x),I*3^(1/2)+2*I)*3^(1/4)/((1-x)/(1+3^(1/2)-x)^
2)^(1/2)/(-x^3+1)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.17 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.83

$$\int \frac{1}{\left(1+\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx = \frac{4\sqrt{2}\sqrt{-\frac{i(-1+x)}{3i+\sqrt{3}}}\sqrt{1+x+x^2}\,\text{EllipticPi}\left(\frac{2\sqrt{3}}{3i+(1+2i)\sqrt{3}},\arcsin\left(\frac{\sqrt{i+\sqrt{3}+2ix}}{\sqrt{2}\sqrt[4]{3}}\right),\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)}{\left(3i+(1+2i)\sqrt{3}\right)\sqrt{1-x^3}}$$

input
```
Integrate[1/((1 + Sqrt[3] - x)*Sqrt[1 - x^3]),x]
```

output
```
(4*Sqrt[2]*Sqrt[((-I)*(-1 + x))/(3*I + Sqrt[3])]*Sqrt[1 + x + x^2]*Ellipti
cPi[(2*Sqrt[3])/(3*I + (1 + 2*I)*Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] + (2*I)
*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])])/((3*I + (1 + 2*I)*Sq
rt[3])*Sqrt[1 - x^3])
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {2560, 27, 759, 2565, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(-x+\sqrt{3}+1\right)\sqrt{1-x^3}}dx$$

$$\Big\downarrow \text{ 2560}$$

$$\frac{\int \frac{1}{\sqrt{1-x^3}}dx}{2\sqrt{3}}+\frac{\int -\frac{6\left(-x-\sqrt{3}+1\right)}{\left(-x+\sqrt{3}+1\right)\sqrt{1-x^3}}dx}{12\sqrt{3}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{1}{\sqrt{1-x^3}}dx}{2\sqrt{3}} - \frac{\int \frac{-x-\sqrt{3}+1}{\left(-x+\sqrt{3}+1\right)\sqrt{1-x^3}}dx}{2\sqrt{3}}$$

$$\Big\downarrow \text{759}$$

$$-\frac{\int \frac{-x-\sqrt{3}+1}{\left(-x+\sqrt{3}+1\right)\sqrt{1-x^3}}dx}{2\sqrt{3}} - \frac{\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{3^{3/4}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

$$\Big\downarrow \text{2565}$$

$$-\frac{\int \frac{1}{\frac{\left(3+2\sqrt{3}\right)(1-x)^2}{1-x^3}+1}d\frac{1-x}{\sqrt{1-x^3}}}{\sqrt{3}} - \frac{\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{3^{3/4}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

$$\Big\downarrow \text{216}$$

$$-\frac{\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{3^{3/4}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}} - \frac{\arctan\left(\frac{\sqrt{3+2\sqrt{3}}(1-x)}{\sqrt{1-x^3}}\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}}$$

input
```
Int[1/((1 + Sqrt[3] - x)*Sqrt[1 - x^3]),x]
```

output
```
-(ArcTan[(Sqrt[3 + 2*Sqrt[3]]*(1 - x))/Sqrt[1 - x^3]]/Sqrt[3*(3 + 2*Sqrt[3
]])]) - (Sqrt[2 + Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*
EllipticF[ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(3
^(3/4)*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 2560
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Simp[-6
*a*(d^3/(c*(b*c^3 - 28*a*d^3)))   Int[1/Sqrt[a + b*x^3], x], x] + Simp[1/(c
*(b*c^3 - 28*a*d^3))   Int[Simp[c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x, x]/((c +
d*x)*Sqrt[a + b*x^3]), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2*c^6 - 20
*a*b*c^3*d^3 - 8*a^2*d^6, 0]
```

rule 2565
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Simp[(1 + k)*(e/d)   S
ubst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x
^3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c
^6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^
3), 0]
```

Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.87

method	result	siz
default	$\frac{2 i \sqrt{3}\, \sqrt{i \left(x+\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, \sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\, \sqrt{-i \left(x+\frac{1}{2}+\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, \operatorname{EllipticPi}\left(\frac{\sqrt{3}\, \sqrt{i \left(x+\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}}{3}, \frac{i \sqrt{3}}{-\frac{3}{2}+\frac{i \sqrt{3}}{2}-\sqrt{3}}, \sqrt{\frac{i \sqrt{3}}{-\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\right)}{3 \sqrt{-x^{3}+1}\, \left(-\frac{3}{2}+\frac{i \sqrt{3}}{2}-\sqrt{3}\right)}$	14
elliptic	$\frac{2 i \sqrt{3}\, \sqrt{i \left(x+\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, \sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\, \sqrt{-i \left(x+\frac{1}{2}+\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, \operatorname{EllipticPi}\left(\frac{\sqrt{3}\, \sqrt{i \left(x+\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}}{3}, \frac{i \sqrt{3}}{-\frac{3}{2}+\frac{i \sqrt{3}}{2}-\sqrt{3}}, \sqrt{\frac{i \sqrt{3}}{-\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\right)}{3 \sqrt{-x^{3}+1}\, \left(-\frac{3}{2}+\frac{i \sqrt{3}}{2}-\sqrt{3}\right)}$	14

input
```
int(1/(1+3^(1/2)-x)/(-x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x-1)/(-3/2+1/2*I*3
^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)/(-3
/2+1/2*I*3^(1/2)-3^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*
3^(1/2))^(1/2),I*3^(1/2)/(-3/2+1/2*I*3^(1/2)-3^(1/2)),(I*3^(1/2)/(-3/2+1/2
*I*3^(1/2)))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.40

$$\int \frac{1}{\left(1+\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx = -\frac{1}{6}\,\sqrt{2\,\sqrt{3}-3}\arctan\left(\frac{\sqrt{-x^3+1}\left(\sqrt{3}(x^2+4\,x-2)+6\,x-6\right)\sqrt{2\,\sqrt{3}-3}}{6\,(x^3-1)}\right) - \frac{1}{3}i\,\sqrt{3}\text{weierstrassPInverse}(0,4,x)$$

input
```
integrate(1/(1+3^(1/2)-x)/(-x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/6*sqrt(2*sqrt(3) - 3)*arctan(1/6*sqrt(-x^3 + 1)*(sqrt(3)*(x^2 + 4*x - 2
) + 6*x - 6)*sqrt(2*sqrt(3) - 3)/(x^3 - 1)) - 1/3*I*sqrt(3)*weierstrassPIn
verse(0, 4, x)
```

Sympy [F]

$$\int \frac{1}{\left(1+\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx = -\int \frac{1}{x\sqrt{1-x^3}-\sqrt{3}\sqrt{1-x^3}-\sqrt{1-x^3}}\,dx$$

input
```
integrate(1/(1+3**(1/2)-x)/(-x**3+1)**(1/2),x)
```

output
```
-Integral(1/(x*sqrt(1 - x**3) - sqrt(3)*sqrt(1 - x**3) - sqrt(1 - x**3)),
x)
```

Maxima [F]

$$\int \frac{1}{\left(1+\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx = \int -\frac{1}{\sqrt{-x^3+1}\left(x-\sqrt{3}-1\right)}\,dx$$

input
```
integrate(1/(1+3^(1/2)-x)/(-x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
-integrate(1/(sqrt(-x^3 + 1)*(x - sqrt(3) - 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\left(1+\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(1+3^(1/2)-x)/(-x^3+1)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:Unable to divide, perhaps due to rounding error%%%{1,[2
]%%%} / %%%{%%{[2,4]:[1,0,-3]%%},[2]%%%} Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(1+\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx = \text{Hanged}$$

input
```
int(1/((1 - x^3)^(1/2)*(3^(1/2) - x + 1)),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{1}{\left(1+\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx = \sqrt{3}\left(\int \frac{\sqrt{-x^3+1}}{x^5-2x^4-2x^3-x^2+2x+2}dx\right) - \left(\int \frac{\sqrt{-x^3+1}}{x^5-2x^4-2x^3-x^2+2x+2}dx\right) + \int \frac{\sqrt{-x^3+1}\,x}{x^5-2x^4-2x^3-x^2+2x+2}dx$$

input
```
int(1/(1+3^(1/2)-x)/(-x^3+1)^(1/2),x)
```

output
```
sqrt(3)*int(sqrt( - x**3 + 1)/(x**5 - 2*x**4 - 2*x**3 - x**2 + 2*x + 2),x)
 - int(sqrt( - x**3 + 1)/(x**5 - 2*x**4 - 2*x**3 - x**2 + 2*x + 2),x) + in
t((sqrt( - x**3 + 1)*x)/(x**5 - 2*x**4 - 2*x**3 - x**2 + 2*x + 2),x)
```

3.129 $\int \frac{1}{\left(1+\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 167

$$\int \frac{1}{\left(1+\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{3+2\sqrt{3}}(1-x)}{\sqrt{-1+x^3}}\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}} - \frac{\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right),-7+4\sqrt{3}\right)}{3^{3/4}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

output
```
-arctanh((3+2*3^(1/2))^(1/2)*(1-x)/(x^3-1)^(1/2))/(9+6*3^(1/2))^(1/2)-1/3*
(1/2*6^(1/2)-1/2*2^(1/2))*(1-x)*((x^2+x+1)/(1-3^(1/2)-x)^2)^(1/2)*Elliptic
F((1+3^(1/2)-x)/(1-3^(1/2)-x),2*I-I*3^(1/2))*3^(1/4)/(-(1-x)/(1-3^(1/2)-x)
^2)^(1/2)/(x^3-1)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 20.36 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.80

$$\int \frac{1}{\left(1+\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx = \frac{4\sqrt{2}\sqrt{-\frac{i(-1+x)}{3i+\sqrt{3}}}\sqrt{1+x+x^2}\,\text{EllipticPi}\left(\frac{2\sqrt{3}}{3i+(1+2i)\sqrt{3}},\arcsin\left(\frac{\sqrt{i+\sqrt{3}+2ix}}{\sqrt{2}\sqrt[4]{3}}\right),\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)}{\left(3i+(1+2i)\sqrt{3}\right)\sqrt{-1+x^3}}$$

input
```
Integrate[1/((1 + Sqrt[3] - x)*Sqrt[-1 + x^3]),x]
```

output
```
(4*Sqrt[2]*Sqrt[((-I)*(-1 + x))/(3*I + Sqrt[3])]*Sqrt[1 + x + x^2]*Ellipti
cPi[(2*Sqrt[3])/(3*I + (1 + 2*I)*Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] + (2*I)
*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])]])/((3*I + (1 + 2*I)*Sq
rt[3])*Sqrt[-1 + x^3])
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {2560, 27, 760, 2565, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(-x+\sqrt{3}+1\right)\sqrt{x^3-1}}dx$$

$$\Big\downarrow \text{ 2560}$$

$$\frac{\int \frac{1}{\sqrt{x^3-1}}dx}{2\sqrt{3}}-\frac{\int \frac{6\left(-x-\sqrt{3}+1\right)}{\left(-x+\sqrt{3}+1\right)\sqrt{x^3-1}}dx}{12\sqrt{3}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{1}{\sqrt{x^3-1}}dx}{2\sqrt{3}}-\frac{\int \frac{-x-\sqrt{3}+1}{\left(-x+\sqrt{3}+1\right)\sqrt{x^3-1}}dx}{2\sqrt{3}}$$

$$\Big\downarrow \text{760}$$

$$-\frac{\int \frac{-x-\sqrt{3}+1}{\left(-x+\sqrt{3}+1\right)\sqrt{x^3-1}}dx}{2\sqrt{3}}-\frac{\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{3^{3/4}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

$$\Big\downarrow \text{2565}$$

$$-\frac{\int \frac{1}{1-\frac{\left(3+2\sqrt{3}\right)(1-x)^2}{x^3-1}}d\frac{1-x}{\sqrt{x^3-1}}}{\sqrt{3}}-\frac{\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{3^{3/4}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{3^{3/4}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{3+2\sqrt{3}}(1-x)}{\sqrt{x^3-1}}\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}}$$

input
```
Int[1/((1 + Sqrt[3] - x)*Sqrt[-1 + x^3]),x]
```

output
```
-(ArcTanh[(Sqrt[3 + 2*Sqrt[3]]*(1 - x))/Sqrt[-1 + x^3]]/Sqrt[3*(3 + 2*Sqrt
[3])]) - (Sqrt[2 - Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2
]*EllipticF[ArcSin[(1 + Sqrt[3] - x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/
(3^(3/4)*Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 + x^3])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 2560
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Simp[-6
*a*(d^3/(c*(b*c^3 - 28*a*d^3)))   Int[1/Sqrt[a + b*x^3], x], x] + Simp[1/(c
*(b*c^3 - 28*a*d^3))   Int[Simp[c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x, x]/((c +
d*x)*Sqrt[a + b*x^3]), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2*c^6 - 20
*a*b*c^3*d^3 - 8*a^2*d^6, 0]
```

rule 2565
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Simp[(1 + k)*(e/d)   S
ubst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x
^3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c
^6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^
3), 0]
```

Maple [A] (verified)

Time = 1.03 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.79

method	result	size
default	$\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{3}\,\mathrm{EllipticPi}\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},-\frac{\left(\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}{3},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{x^{3}-1}}$	132
elliptic	$\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{3}\,\mathrm{EllipticPi}\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},-\frac{\left(\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}{3},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{x^{3}-1}}$	132

input
```
int(1/(1+3^(1/2)-x)/(x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*
3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1
/2)))^(1/2)/(x^3-1)^(1/2)*3^(1/2)*EllipticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))^
(1/2),-1/3*(3/2+1/2*I*3^(1/2))*3^(1/2),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(
1/2)))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.26

$$\int \frac{1}{\left(1+\sqrt{3}-x\right)\sqrt{-1+x^{3}}}\,dx$$
$$=\frac{1}{12}\sqrt{2\sqrt{3}-3}\log\left(\frac{x^{8}+16\,x^{7}+112\,x^{6}+16\,x^{5}+112\,x^{4}-224\,x^{3}+64\,x^{2}+4\left(2\,x^{6}+18\,x^{5}+42\,x^{4}+8\right.}{}\right.$$
$$+\frac{1}{3}\sqrt{3}\text{weierstrassPInverse}(0,4,x)$$

input
```
integrate(1/(1+3^(1/2)-x)/(x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
1/12*sqrt(2*sqrt(3) - 3)*log((x^8 + 16*x^7 + 112*x^6 + 16*x^5 + 112*x^4 -
224*x^3 + 64*x^2 + 4*(2*x^6 + 18*x^5 + 42*x^4 + 8*x^3 + sqrt(3)*(x^6 + 12*
x^5 + 18*x^4 + 16*x^3 - 12*x^2 - 8) - 24*x + 8)*sqrt(x^3 - 1)*sqrt(2*sqrt(
3) - 3) + 16*sqrt(3)*(x^7 + 2*x^6 + 6*x^5 - 5*x^4 + 2*x^3 - 6*x^2 + 4*x -
4) - 128*x + 112)/(x^8 - 8*x^7 + 16*x^6 + 16*x^5 - 56*x^4 - 32*x^3 + 64*x^
2 + 64*x + 16)) + 1/3*sqrt(3)*weierstrassPInverse(0, 4, x)
```

Sympy [F]

$$\int \frac{1}{\left(1+\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx = -\int \frac{1}{x\sqrt{x^3-1}-\sqrt{3}\sqrt{x^3-1}-\sqrt{x^3-1}}\,dx$$

input
```
integrate(1/(1+3**(1/2)-x)/(x**3-1)**(1/2),x)
```

output
```
-Integral(1/(x*sqrt(x**3 - 1) - sqrt(3)*sqrt(x**3 - 1) - sqrt(x**3 - 1)),
x)
```

Maxima [F]

$$\int \frac{1}{\left(1+\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx = \int -\frac{1}{\sqrt{x^3-1}\left(x-\sqrt{3}-1\right)}\,dx$$

input
```
integrate(1/(1+3^(1/2)-x)/(x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
-integrate(1/(sqrt(x^3 - 1)*(x - sqrt(3) - 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\left(1+\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(1+3^(1/2)-x)/(x^3-1)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[2]%%%} / %%%{%%{[2,4]:[1,0,-3]%%},[2]%%%} Error: Bad Ar
gument Va
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(1+\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx = \text{Hanged}$$

input
```
int(1/((x^3 - 1)^(1/2)*(3^(1/2) - x + 1)),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\begin{aligned}\int \frac{1}{\left(1+\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx = & -\sqrt{3}\left(\int \frac{\sqrt{x^3-1}}{x^5-2x^4-2x^3-x^2+2x+2}dx\right) \\ & +\int \frac{\sqrt{x^3-1}}{x^5-2x^4-2x^3-x^2+2x+2}dx \\ & -\left(\int \frac{\sqrt{x^3-1}\,x}{x^5-2x^4-2x^3-x^2+2x+2}dx\right)\end{aligned}$$

input
```
int(1/(1+3^(1/2)-x)/(x^3-1)^(1/2),x)
```

output
```
 - sqrt(3)*int(sqrt(x**3 - 1)/(x**5 - 2*x**4 - 2*x**3 - x**2 + 2*x + 2),x)
 + int(sqrt(x**3 - 1)/(x**5 - 2*x**4 - 2*x**3 - x**2 + 2*x + 2),x) - int((
sqrt(x**3 - 1)*x)/(x**5 - 2*x**4 - 2*x**3 - x**2 + 2*x + 2),x)
```

3.130 $\int \frac{1}{(1+\sqrt{3}+x)\sqrt{-1-x^3}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 157

$$\int \frac{1}{\left(1+\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{3+2\sqrt{3}}(1+x)}{\sqrt{-1-x^3}}\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}} + \frac{\sqrt{2-\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right),-7+4\sqrt{3}\right)}{3^{3/4}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

output

```
arctanh((3+2*3^(1/2))^(1/2)*(1+x)/(-x^3-1)^(1/2))/(9+6*3^(1/2))^(1/2)+1/3*
(1/2*6^(1/2)-1/2*2^(1/2))*(1+x)*((x^2-x+1)/(1+x-3^(1/2))^2)^(1/2)*Elliptic
F((1+x+3^(1/2))/(1+x-3^(1/2)),2*I-I*3^(1/2))*3^(1/4)/(-(1+x)/(1+x-3^(1/2))
^2)^(1/2)/(-x^3-1)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.12 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.88

$$\int \frac{1}{(1+\sqrt{3}+x)\sqrt{-1-x^3}}\,dx = -\frac{4\sqrt{2}\sqrt{\frac{i(1+x)}{3i+\sqrt{3}}}\sqrt{1-x+x^2}\,\text{EllipticPi}\left(\frac{2\sqrt{3}}{3i+(1+2i)\sqrt{3}},\arcsin\left(\frac{\sqrt{i+\sqrt{3}-2ix}}{\sqrt{2}\sqrt[4]{3}}\right),\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)}{\left(3i+(1+2i)\sqrt{3}\right)\sqrt{-1-x^3}}$$

input
```
Integrate[1/((1 + Sqrt[3] + x)*Sqrt[-1 - x^3]),x]
```

output
```
(-4*Sqrt[2]*Sqrt[(I*(1 + x))/(3*I + Sqrt[3])]*Sqrt[1 - x + x^2]*EllipticPi
[(2*Sqrt[3])/(3*I + (1 + 2*I)*Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] - (2*I)*x]
/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])])/((3*I + (1 + 2*I)*Sqrt[
3])*Sqrt[-1 - x^3])
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {2560, 27, 760, 2565, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(x+\sqrt{3}+1\right)\sqrt{-x^3-1}}dx$$

$$\Big\downarrow \text{ 2560}$$

$$\frac{\int \frac{1}{\sqrt{-x^3-1}}dx}{2\sqrt{3}}+\frac{\int -\frac{6\left(x-\sqrt{3}+1\right)}{\left(x+\sqrt{3}+1\right)\sqrt{-x^3-1}}dx}{12\sqrt{3}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{1}{\sqrt{-x^3-1}}dx}{2\sqrt{3}}-\frac{\int \frac{x-\sqrt{3}+1}{\left(x+\sqrt{3}+1\right)\sqrt{-x^3-1}}dx}{2\sqrt{3}}$$

$$\Big\downarrow \; 760$$

$$\frac{\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{3^{3/4}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}-\frac{\int \frac{x-\sqrt{3}+1}{\left(x+\sqrt{3}+1\right)\sqrt{-x^3-1}}dx}{2\sqrt{3}}$$

$$\Big\downarrow \; 2565$$

$$\frac{\int \frac{1}{1-\frac{\left(3+2\sqrt{3}\right)(x+1)^2}{-x^3-1}}d\frac{x+1}{\sqrt{-x^3-1}}}{\sqrt{3}}+\frac{\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{3^{3/4}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

$$\Big\downarrow \; 219$$

$$\frac{\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{3^{3/4}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}+\frac{\text{arctanh}\left(\frac{\sqrt{3+2\sqrt{3}}(x+1)}{\sqrt{-x^3-1}}\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}}$$

input
```
Int[1/((1 + Sqrt[3] + x)*Sqrt[-1 - x^3]),x]
```

output
```
ArcTanh[(Sqrt[3 + 2*Sqrt[3]]*(1 + x))/Sqrt[-1 - x^3]]/Sqrt[3*(3 + 2*Sqrt[3
])] + (Sqrt[2 - Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*E
llipticF[ArcSin[(1 + Sqrt[3] + x)/(1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(3^
(3/4)*Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^2)]*Sqrt[-1 - x^3])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 2560
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Simp[-6
*a*(d^3/(c*(b*c^3 - 28*a*d^3)))   Int[1/Sqrt[a + b*x^3], x], x] + Simp[1/(c
*(b*c^3 - 28*a*d^3))   Int[Simp[c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x, x]/((c +
d*x)*Sqrt[a + b*x^3]), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2*c^6 - 20
*a*b*c^3*d^3 - 8*a^2*d^6, 0]
```

rule 2565
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Simp[(1 + k)*(e/d)   S
ubst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x
^3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c
^6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^
3), 0]
```

Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.89

method	result	size
default	$-\frac{2 i \sqrt{3}\, \sqrt{i \left(x-\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, \sqrt{\frac{x+1}{\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\, \sqrt{-i \left(x-\frac{1}{2}+\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, \operatorname{EllipticPi}\left(\frac{\sqrt{3}\, \sqrt{i \left(x-\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}}{3}, \frac{i \sqrt{3}}{\frac{3}{2}+\frac{i \sqrt{3}}{2}+\sqrt{3}}, \sqrt{\frac{i \sqrt{3}}{\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\right)}{3 \sqrt{-x^{3}-1}\, \left(\frac{3}{2}+\frac{i \sqrt{3}}{2}+\sqrt{3}\right)}$	139
elliptic	$-\frac{2 i \sqrt{3}\, \sqrt{i \left(x-\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, \sqrt{\frac{x+1}{\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\, \sqrt{-i \left(x-\frac{1}{2}+\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, \operatorname{EllipticPi}\left(\frac{\sqrt{3}\, \sqrt{i \left(x-\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}}{3}, \frac{i \sqrt{3}}{\frac{3}{2}+\frac{i \sqrt{3}}{2}+\sqrt{3}}, \sqrt{\frac{i \sqrt{3}}{\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\right)}{3 \sqrt{-x^{3}-1}\, \left(\frac{3}{2}+\frac{i \sqrt{3}}{2}+\sqrt{3}\right)}$	139

input
```
int(1/(1+3^(1/2)+x)/(-x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x+1)/(3/2+1/2*I*3
^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)/(3/
2+1/2*I*3^(1/2)+3^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3
^(1/2))^(1/2),I*3^(1/2)/(3/2+1/2*I*3^(1/2)+3^(1/2)),(I*3^(1/2)/(3/2+1/2*I*
3^(1/2)))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.36

$$\int \frac{1}{\left(1+\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx$$
$$= \frac{1}{12}\sqrt{2\sqrt{3}-3}\log\left(\frac{x^8-16x^7+112x^6-16x^5+112x^4+224x^3+64x^2-4\left(2x^6-18x^5+42x^4-8\right.}{}\right.$$
$$-\frac{1}{3}i\sqrt{3}\text{weierstrassPInverse}(0,-4,x)$$

input
```
integrate(1/(1+3^(1/2)+x)/(-x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
1/12*sqrt(2*sqrt(3) - 3)*log((x^8 - 16*x^7 + 112*x^6 - 16*x^5 + 112*x^4 +
224*x^3 + 64*x^2 - 4*(2*x^6 - 18*x^5 + 42*x^4 - 8*x^3 + sqrt(3)*(x^6 - 12*
x^5 + 18*x^4 - 16*x^3 - 12*x^2 - 8) + 24*x + 8)*sqrt(-x^3 - 1)*sqrt(2*sqrt
(3) - 3) - 16*sqrt(3)*(x^7 - 2*x^6 + 6*x^5 + 5*x^4 + 2*x^3 + 6*x^2 + 4*x +
 4) + 128*x + 112)/(x^8 + 8*x^7 + 16*x^6 - 16*x^5 - 56*x^4 + 32*x^3 + 64*x
^2 - 64*x + 16)) - 1/3*I*sqrt(3)*weierstrassPInverse(0, -4, x)
```

Sympy [F]

$$\int \frac{1}{\left(1+\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx = \int \frac{1}{\sqrt{-\left(x+1\right)\left(x^2-x+1\right)}\left(x+1+\sqrt{3}\right)}\,dx$$

input
```
integrate(1/(1+3**(1/2)+x)/(-x**3-1)**(1/2),x)
```

output
```
Integral(1/(sqrt(-(x + 1)*(x**2 - x + 1))*(x + 1 + sqrt(3))), x)
```

Maxima [F]

$$\int \frac{1}{\left(1+\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx = \int \frac{1}{\sqrt{-x^3-1}\left(x+\sqrt{3}+1\right)}\,dx$$

input
```
integrate(1/(1+3^(1/2)+x)/(-x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(-x^3 - 1)*(x + sqrt(3) + 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\left(1+\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(1+3^(1/2)+x)/(-x^3-1)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[2]%%%} / %%%{%%{[2,4]:[1,0,-3]%%},[2]%%%} Error: Bad Ar
gument Va
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(1+\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx = \text{Hanged}$$

input
```
int(1/((- x^3 - 1)^(1/2)*(x + 3^(1/2) + 1)),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{1}{\left(1+\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx = i\left(\sqrt{3}\left(\int \frac{\sqrt{x^3+1}}{x^5+2x^4-2x^3+x^2+2x-2}dx\right) - \left(\int \frac{\sqrt{x^3+1}}{x^5+2x^4-2x^3+x^2+2x-2}dx\right) - \left(\int \frac{\sqrt{x^3+1}\,x}{x^5+2x^4-2x^3+x^2+2x-2}dx\right)\right)$$

input
```
int(1/(1+3^(1/2)+x)/(-x^3-1)^(1/2),x)
```

output
```
i*(sqrt(3)*int(sqrt(x**3 + 1)/(x**5 + 2*x**4 - 2*x**3 + x**2 + 2*x - 2),x)
 - int(sqrt(x**3 + 1)/(x**5 + 2*x**4 - 2*x**3 + x**2 + 2*x - 2),x) - int((
sqrt(x**3 + 1)*x)/(x**5 + 2*x**4 - 2*x**3 + x**2 + 2*x - 2),x))
```

3.131 $\int \frac{1}{(3+x)\sqrt{1+x^3}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 329

$$\int \frac{1}{(3+x)\sqrt{1+x^3}}\,dx = \frac{(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\arctan\left(\frac{\sqrt{\frac{13}{2}}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}}{\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}}\right)}{\sqrt{26}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} + \frac{2\sqrt{26+15\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} - \frac{4\sqrt[4]{3}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticPi}\left(97-56\sqrt{3},\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{\sqrt{2-\sqrt{3}}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

output
```
1/26*(1+x)*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*arctan(1/2*26^(1/2)*((1+x)/(1
+x+3^(1/2))^2)^(1/2)/((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2))*26^(1/2)/((1+x)/(1
+x+3^(1/2))^2)^(1/2)/(x^3+1)^(1/2)+2/3*(3/2*6^(1/2)+5/2*2^(1/2))*(1+x)*((x
^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(
1/2)+2*I)*3^(3/4)/((1+x)/(1+x+3^(1/2))^2)^(1/2)/(x^3+1)^(1/2)-4*3^(1/4)*(1
+x)*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*EllipticPi((1+x-3^(1/2))/(1+x+3^(1/2
)),97-56*3^(1/2),I*3^(1/2)+2*I)/(1/2*6^(1/2)-1/2*2^(1/2))/((1+x)/(1+x+3^(1
/2))^2)^(1/2)/(x^3+1)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 20.13 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.39

$$\int \frac{1}{(3+x)\sqrt{1+x^3}}\,dx = -\frac{4\sqrt{2}\sqrt{\frac{i(1+x)}{3i+\sqrt{3}}}\sqrt{1-x+x^2}\,\text{EllipticPi}\left(\frac{2\sqrt{3}}{7i+\sqrt{3}},\arcsin\left(\frac{\sqrt{i+\sqrt{3}-2ix}}{\sqrt{2}\sqrt[4]{3}}\right),\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)}{\left(7i+\sqrt{3}\right)\sqrt{1+x^3}}$$

input
```
Integrate[1/((3 + x)*Sqrt[1 + x^3]),x]
```

output
```
(-4*Sqrt[2]*Sqrt[(I*(1 + x))/(3*I + Sqrt[3])]*Sqrt[1 - x + x^2]*EllipticPi
[(2*Sqrt[3])/(7*I + Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*
3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])])/((7*I + Sqrt[3])*Sqrt[1 + x^3])
```

Rubi [A] (warning: unable to verify)

Time = 1.51 (sec) , antiderivative size = 321, normalized size of antiderivative = 0.98, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {2561, 759, 2567, 25, 2538, 412, 435, 104, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x+3)\sqrt{x^3+1}}dx$$

$$\Big\downarrow \text{ 2561}$$

$$\frac{\int \frac{1}{\sqrt{x^3+1}}dx}{2-\sqrt{3}}-\frac{\int \frac{x+\sqrt{3}+1}{(x+3)\sqrt{x^3+1}}dx}{2-\sqrt{3}}$$

$$\Big\downarrow \text{ 759}$$

$$\frac{2\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(2-\sqrt{3}\right)\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{\int \frac{x+\sqrt{3}+1}{(x+3)\sqrt{x^3+1}}dx}{2-\sqrt{3}}$$

$$\Big\downarrow \text{ 2567}$$

$$\frac{2\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(2-\sqrt{3}\right)\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{4\sqrt[4]{3}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\int -\frac{1}{\sqrt{1-\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\left(-\frac{\left(2-\sqrt{3}\right)\left(x-\sqrt{3}+1\right)}{x+\sqrt{3}+1}+\sqrt{3}+2\right)}d\left(-\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)}{\sqrt{2-\sqrt{3}}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{4\sqrt[4]{3}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\int \frac{1}{\sqrt{1-\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\left(-\frac{\left(2-\sqrt{3}\right)\left(x-\sqrt{3}+1\right)}{x+\sqrt{3}+1}+\sqrt{3}+2\right)}d\left(-\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)}{\sqrt{2-\sqrt{3}}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}+\frac{2\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(2-\sqrt{3}\right)\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

$$\Big\downarrow \text{ 2538}$$

$$\frac{2\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(2-\sqrt{3}\right)\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-$$

$$\frac{4\sqrt[4]{3}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(\left(2-\sqrt{3}\right)\int -\frac{x-\sqrt{3}+1}{\left(x+\sqrt{3}+1\right)\sqrt{1-\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\left(-\frac{\left(7-4\sqrt{3}\right)\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}+4\sqrt{3}+7\right)}d\right.}{\sqrt{2-\sqrt{3}}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right.}}}$$

$\downarrow$ 412

$$\frac{2\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(2-\sqrt{3}\right)\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-$$

$$\frac{4\sqrt[4]{3}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(\left(2-\sqrt{3}\right)\int -\frac{x-\sqrt{3}+1}{\left(x+\sqrt{3}+1\right)\sqrt{1-\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\left(-\frac{\left(7-4\sqrt{3}\right)\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}+4\sqrt{3}+7\right)}d\right.}{\sqrt{2-\sqrt{3}}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x}}$$

$\downarrow$ 435

$$\frac{2\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(2-\sqrt{3}\right)\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-$$

$$\frac{4\sqrt[4]{3}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(\frac{1}{2}\left(2-\sqrt{3}\right)\int \frac{1}{\sqrt{\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\sqrt{\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}+1}\left(\frac{\left(7-4\sqrt{3}\right)\left(x-\sqrt{3}+1\right)}{x+\sqrt{3}+1}+4\sqrt{3}+7\right)}d\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}+\sqrt{}\right.}{\sqrt{2-\sqrt{3}}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

$\downarrow$ 104

$$\frac{2\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(2-\sqrt{3}\right)\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-$$

$$\frac{4\sqrt[4]{3}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(\left(2-\sqrt{3}\right)\int\frac{1}{-\frac{52\left(2-\sqrt{3}\right)\sqrt{\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}+1}}{\sqrt{\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}}-8\sqrt{3}}d\frac{\sqrt{\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}+1}}{\sqrt{\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}}+\sqrt{7-4\sqrt{3}}\left(2+\sqrt{3}\right)\text{Ellip}}{\sqrt{2-\sqrt{3}}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

$$\Big\downarrow \; 217$$

$$\frac{2\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(2-\sqrt{3}\right)\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-$$

$$\frac{4\sqrt[4]{3}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(\sqrt{7-4\sqrt{3}}\left(2+\sqrt{3}\right)\text{EllipticPi}\left(97-56\sqrt{3},\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)+\frac{\sqrt{\frac{1}{26}\left(2-\sqrt{\right.}\right.}}{}\right.}{\sqrt{2-\sqrt{3}}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

input
```
Int[1/((3 + x)*Sqrt[1 + x^3]),x]
```

output
```
(2*Sqrt[2 + Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*Ellip
ticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3^(1/4
)*(2 - Sqrt[3])*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3]) - (4*3^(1
/4)*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*((Sqrt[(2 - Sqrt[3])/2
6]*ArcTan[(Sqrt[(13*(2 - Sqrt[3]))/2]*(1 - Sqrt[3] + x))/(3^(1/4)*(1 + Sqr
t[3] + x))])/(4*3^(1/4)) + Sqrt[7 - 4*Sqrt[3]]*(2 + Sqrt[3])*EllipticPi[97
 - 56*Sqrt[3], ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]
]))/(Sqrt[2 - Sqrt[3]]*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 435
```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^(q_.)*((
e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)
*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^2], x] /; FreeQ[{a, b, c, d,
 e, f, p, q, r}, x] && IntegerQ[(m - 1)/2]
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 2538
```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[a   Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
 f*x^2]), x], x] - Simp[b   Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
 f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

rule 2561
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> With[{q
 = Rt[b/a, 3]}, Simp[-q/((1 + Sqrt[3])*d - c*q)   Int[1/Sqrt[a + b*x^3], x]
, x] + Simp[d/((1 + Sqrt[3])*d - c*q)   Int[(1 + Sqrt[3] + q*x)/((c + d*x)*
Sqrt[a + b*x^3]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2*c^6 - 20*a*b
*c^3*d^3 - 8*a^2*d^6, 0]
```

rule 2567
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{q = Simplify[(1 + Sqrt[3])*(f/e)]}, Simp[4*3^(1/4)*Sqrt[2
- Sqrt[3]]*f*(1 + q*x)*(Sqrt[(1 - q*x + q^2*x^2)/(1 + Sqrt[3] + q*x)^2]/(q*
Sqrt[a + b*x^3]*Sqrt[(1 + q*x)/(1 + Sqrt[3] + q*x)^2]))   Subst[Int[1/(((1
- Sqrt[3])*d - c*q + ((1 + Sqrt[3])*d - c*q)*x)*Sqrt[1 - x^2]*Sqrt[7 - 4*Sq
rt[3] + x^2]), x], x, (-1 + Sqrt[3] - q*x)/(1 + Sqrt[3] + q*x)], x]] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*e^3 - 2*(5 + 3*Sqrt
[3])*a*f^3, 0] && NeQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.37

method	result	size
default	$\frac{\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\operatorname{EllipticPi}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},-\frac{3}{4}+\frac{i\sqrt{3}}{4},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^{3}+1}}$	123
elliptic	$\frac{\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\operatorname{EllipticPi}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},-\frac{3}{4}+\frac{i\sqrt{3}}{4},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^{3}+1}}$	123

input
```
int(1/(3+x)/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(3/2-1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2
))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2))
)^(1/2)/(x^3+1)^(1/2)*EllipticPi(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),-3/4+1/
4*I*3^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))
```

Fricas [F]

$$\int \frac{1}{(3+x)\sqrt{1+x^3}}\,dx = \int \frac{1}{\sqrt{x^3+1}(x+3)}\,dx$$

input
```
integrate(1/(3+x)/(x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(x^3 + 1)/(x^4 + 3*x^3 + x + 3), x)
```

Sympy [F]

$$\int \frac{1}{(3+x)\sqrt{1+x^3}}\,dx = \int \frac{1}{\sqrt{(x+1)\,(x^2-x+1)}\,(x+3)}\,dx$$

input
```
integrate(1/(3+x)/(x**3+1)**(1/2),x)
```

output
```
Integral(1/(sqrt((x + 1)*(x**2 - x + 1))*(x + 3)), x)
```

Maxima [F]

$$\int \frac{1}{(3+x)\sqrt{1+x^3}}\,dx = \int \frac{1}{\sqrt{x^3+1}(x+3)}\,dx$$

input
```
integrate(1/(3+x)/(x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(x^3 + 1)*(x + 3)), x)
```

Giac [F]

$$\int \frac{1}{(3+x)\sqrt{1+x^3}}\,dx = \int \frac{1}{\sqrt{x^3+1}(x+3)}\,dx$$

input
```
integrate(1/(3+x)/(x^3+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(x^3 + 1)*(x + 3)), x)
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.50

$$\int \frac{1}{(3+x)\sqrt{1+x^3}}\,dx = \frac{(3+\sqrt{3}\,1\mathrm{i})\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\Pi\left(-\frac{3}{4}-\frac{\sqrt{3}\,1\mathrm{i}}{4};\operatorname{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{2\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

input
```
int(1/((x^3 + 1)^(1/2)*(x + 3)),x)
```

output
```
((3^(1/2)*1i + 3)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2
)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(
1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi(- (3^(1/2)*1i)/4 - 3/4, asin(((x + 1)/
((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 -
3/2)))/(2*(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - (
(3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2))
```

Reduce [F]

$$\int \frac{1}{(3+x)\sqrt{1+x^3}}\,dx = \int \frac{1}{(x+3)\sqrt{x^3+1}}dx$$

input
```
int(1/(3+x)/(x^3+1)^(1/2),x)
```

output
```
int(1/(3+x)/(x^3+1)^(1/2),x)
```

3.132 $\int \frac{1}{(3+x)\sqrt{1-x^3}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 380

$$\int \frac{1}{(3+x)\sqrt{1-x^3}}\,dx = -\frac{(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\operatorname{arctanh}\left(\frac{\sqrt{7}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}}{2\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}}\right)}{2\sqrt{7}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$
$$-\frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(4+\sqrt{3}\right)\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$
$$-\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\operatorname{EllipticPi}\left(\frac{1}{169}\left(553+304\sqrt{3}\right),\arcsin\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right),-7-4\sqrt{3}\right)}{13\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

output
```
-1/14*(1-x)*((x^2+x+1)/(1+3^(1/2)-x)^2)^(1/2)*arctanh(1/2*7^(1/2)*((1-x)/(
1+3^(1/2)-x)^2)^(1/2)/((x^2+x+1)/(1+3^(1/2)-x)^2)^(1/2))*7^(1/2)/((1-x)/(1
+3^(1/2)-x)^2)^(1/2)/(-x^3+1)^(1/2)-2/3*(1/2*6^(1/2)+1/2*2^(1/2))*(1-x)*((
x^2+x+1)/(1+3^(1/2)-x)^2)^(1/2)*EllipticF((1-3^(1/2)-x)/(1+3^(1/2)-x),I*3^
(1/2)+2*I)*3^(3/4)/(4+3^(1/2))/((1-x)/(1+3^(1/2)-x)^2)^(1/2)/(-x^3+1)^(1/2
)-4/13*3^(1/4)*(1/2*6^(1/2)+1/2*2^(1/2))*(1-x)*((x^2+x+1)/(1+3^(1/2)-x)^2)
^(1/2)*EllipticPi((1-3^(1/2)-x)/(1+3^(1/2)-x),553/169+304/169*3^(1/2),I*3^
(1/2)+2*I)/((1-x)/(1+3^(1/2)-x)^2)^(1/2)/(-x^3+1)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 20.13 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.34

$$\int \frac{1}{(3+x)\sqrt{1-x^3}}\,dx = -\frac{4\sqrt{2}\sqrt{\frac{i(-1+x)}{-3i+\sqrt{3}}}\sqrt{1+x+x^2}\,\text{EllipticPi}\left(\frac{2\sqrt{3}}{5i+\sqrt{3}},\arcsin\left(\frac{\sqrt{-i+\sqrt{3}-2ix}}{\sqrt{2}\sqrt[4]{3}}\right),\frac{2\sqrt{3}}{-3i+\sqrt{3}}\right)}{\left(5i+\sqrt{3}\right)\sqrt{1-x^3}}$$

input
```
Integrate[1/((3 + x)*Sqrt[1 - x^3]),x]
```

output
```
(-4*Sqrt[2]*Sqrt[(I*(-1 + x))/(-3*I + Sqrt[3])]*Sqrt[1 + x + x^2]*Elliptic
Pi[(2*Sqrt[3])/(5*I + Sqrt[3]), ArcSin[Sqrt[-I + Sqrt[3] - (2*I)*x]/(Sqrt[
2]*3^(1/4))], (2*Sqrt[3])/(-3*I + Sqrt[3])])/((5*I + Sqrt[3])*Sqrt[1 - x^3
])
```

Rubi [A] (warning: unable to verify)

Time = 1.55 (sec) , antiderivative size = 371, normalized size of antiderivative = 0.98, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$, Rules used = {2561, 759, 2567, 2538, 412, 435, 104, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x+3)\sqrt{1-x^3}}dx$$

$$\Big\downarrow \; 2561$$

$$\frac{\int \frac{1}{\sqrt{1-x^3}}dx}{4+\sqrt{3}}+\frac{\int \frac{-x+\sqrt{3}+1}{(x+3)\sqrt{1-x^3}}dx}{4+\sqrt{3}}$$

$$\Big\downarrow \; 759$$

$$\frac{\int \frac{-x+\sqrt{3}+1}{(x+3)\sqrt{1-x^3}}dx}{4+\sqrt{3}}-\frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(4+\sqrt{3}\right)\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

$$\Big\downarrow \; 2567$$

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\int \frac{1}{\sqrt{1-\frac{\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{\frac{\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\left(-\frac{\left(4+\sqrt{3}\right)\left(-x-\sqrt{3}+1\right)}{-x+\sqrt{3}+1}-\sqrt{3}+4\right)}d\left(-\frac{-x-\sqrt{3}+}{-x+\sqrt{3}+}\right.}{\left(4+\sqrt{3}\right)\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

$$\frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(4+\sqrt{3}\right)\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

$$\Big\downarrow \; 2538$$

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\left(\left(4-\sqrt{3}\right)\int \frac{1}{\sqrt{1-\frac{\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{\frac{\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\left(-\frac{\left(4+\sqrt{3}\right)^2\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}-8\sqrt{3}+1\right.}\right.}{\left(4+\sqrt{3}\right)}$$

$$\frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(4+\sqrt{3}\right)\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

$$\Big\downarrow \; 412$$

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\left(-(4+\sqrt{3})\int -\frac{-x-\sqrt{3}+1}{\sqrt{1-\frac{\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{\frac{\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\left(-\frac{\left(4+\sqrt{3}\right)^2\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}-8\sqrt{\right.}}\right.}{(4+\sqrt{}}$$

$$\frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(4+\sqrt{3}\right)\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

$\downarrow$ 435

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\left(-\frac{1}{2}(4+\sqrt{3})\int \frac{1}{\sqrt{\frac{\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\sqrt{\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}+1}\left(\frac{\left(4+\sqrt{3}\right)^2\left(-x-\sqrt{3}+1\right)}{-x+\sqrt{3}+1}-8\sqrt{3}+19\right)}\right.}{\left(4+\sqrt{3}\right)\sqrt{\frac{}{(-x}}}$$

$$\frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(4+\sqrt{3}\right)\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

$\downarrow$ 104

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\left(-(4+\sqrt{3})\int \frac{1}{16\sqrt{3}-\frac{28\left(2-\sqrt{3}\right)\sqrt{\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}+1}}{\sqrt{\frac{\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}}}d\frac{\sqrt{\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}+1}}{\sqrt{\frac{\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}}-\frac{1}{169}(4-\sqrt{\right.}}{\left(4+\sqrt{3}\right)\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-}}$$

$$\frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(4+\sqrt{3}\right)\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

$\downarrow$ 219

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\left(\frac{\left(4+\sqrt{3}\right)\operatorname{arctanh}\left(\frac{\sqrt{7\left(2-\sqrt{3}\right)}\left(-x-\sqrt{3}+1\right)}{2\sqrt[4]{3}\left(-x+\sqrt{3}+1\right)}\right)}{8\sqrt[4]{3}\sqrt{7\left(2-\sqrt{3}\right)}}-\frac{1}{169}\left(4-\sqrt{3}\right)\sqrt{7519+4340\sqrt{3}}\,\text{Elli}\right.}{\left(4+\sqrt{3}\right)\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

$$\frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(4+\sqrt{3}\right)\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

input
```
Int[1/((3 + x)*Sqrt[1 - x^3]),x]
```

output
```
(-2*Sqrt[2 + Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*Elli
pticF[ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(3^(1/
4)*(4 + Sqrt[3])*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3]) + (4*3^(
1/4)*Sqrt[2 - Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*(((
4 + Sqrt[3])*ArcTanh[(Sqrt[7*(2 - Sqrt[3])]*(1 - Sqrt[3] - x))/(2*3^(1/4)*
(1 + Sqrt[3] - x))])/(8*3^(1/4)*Sqrt[7*(2 - Sqrt[3])]) - ((4 - Sqrt[3])*Sq
rt[7519 + 4340*Sqrt[3]]*EllipticPi[(553 + 304*Sqrt[3])/169, ArcSin[(1 - Sq
rt[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/169))/((4 + Sqrt[3])*Sqrt[
(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3])
```

Defintions of rubi rules used

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 435
```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^(q_.)*((
e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)
*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^2], x] /; FreeQ[{a, b, c, d,
 e, f, p, q, r}, x] && IntegerQ[(m - 1)/2]
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 2538
```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[a   Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
 f*x^2]), x], x] - Simp[b   Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
 f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

rule 2561
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> With[{q
 = Rt[b/a, 3]}, Simp[-q/((1 + Sqrt[3])*d - c*q)   Int[1/Sqrt[a + b*x^3], x]
, x] + Simp[d/((1 + Sqrt[3])*d - c*q)   Int[(1 + Sqrt[3] + q*x)/((c + d*x)*
Sqrt[a + b*x^3]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2*c^6 - 20*a*b
*c^3*d^3 - 8*a^2*d^6, 0]
```

rule 2567
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{q = Simplify[(1 + Sqrt[3])*(f/e)]}, Simp[4*3^(1/4)*Sqrt[2
- Sqrt[3]]*f*(1 + q*x)*(Sqrt[(1 - q*x + q^2*x^2)/(1 + Sqrt[3] + q*x)^2]/(q*
Sqrt[a + b*x^3]*Sqrt[(1 + q*x)/(1 + Sqrt[3] + q*x)^2]))   Subst[Int[1/(((1
- Sqrt[3])*d - c*q + ((1 + Sqrt[3])*d - c*q)*x)*Sqrt[1 - x^2]*Sqrt[7 - 4*Sq
rt[3] + x^2]), x], x, (-1 + Sqrt[3] - q*x)/(1 + Sqrt[3] + q*x)], x]] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*e^3 - 2*(5 + 3*Sqrt
[3])*a*f^3, 0] && NeQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.35

method	result	size
default	$-\frac{2i\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\operatorname{EllipticPi}\left(\frac{\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\frac{i\sqrt{3}}{\frac{5}{2}+\frac{i\sqrt{3}}{2}},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^{3}+1}\,\left(\frac{5}{2}+\frac{i\sqrt{3}}{2}\right)}$	133
elliptic	$-\frac{2i\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\operatorname{EllipticPi}\left(\frac{\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\frac{i\sqrt{3}}{\frac{5}{2}+\frac{i\sqrt{3}}{2}},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^{3}+1}\,\left(\frac{5}{2}+\frac{i\sqrt{3}}{2}\right)}$	133

input
```
int(1/(3+x)/(-x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x-1)/(-3/2+1/2*I*
3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)/(5
/2+1/2*I*3^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))
^(1/2),I*3^(1/2)/(5/2+1/2*I*3^(1/2)),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2
))
```

Fricas [F]

$$\int \frac{1}{(3+x)\sqrt{1-x^3}}\,dx = \int \frac{1}{\sqrt{-x^3+1}(x+3)}\,dx$$

input
```
integrate(1/(3+x)/(-x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-x^3 + 1)/(x^4 + 3*x^3 - x - 3), x)
```

Sympy [F]

$$\int \frac{1}{(3+x)\sqrt{1-x^3}}\,dx = \int \frac{1}{\sqrt{-(x-1)(x^2+x+1)}(x+3)}\,dx$$

input
```
integrate(1/(3+x)/(-x**3+1)**(1/2),x)
```

output
```
Integral(1/(sqrt(-(x - 1)*(x**2 + x + 1))*(x + 3)), x)
```

Maxima [F]

$$\int \frac{1}{(3+x)\sqrt{1-x^3}}\,dx = \int \frac{1}{\sqrt{-x^3+1}(x+3)}\,dx$$

input
```
integrate(1/(3+x)/(-x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(-x^3 + 1)*(x + 3)), x)
```

Giac [F]

$$\int \frac{1}{(3+x)\sqrt{1-x^3}}\,dx = \int \frac{1}{\sqrt{-x^3+1}(x+3)}\,dx$$

input
```
integrate(1/(3+x)/(-x^3+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(-x^3 + 1)*(x + 3)), x)
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.47

$$\int \frac{1}{(3+x)\sqrt{1-x^3}}\,dx = -\frac{\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{x^3-1}\,\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\Pi\left(\frac{3}{8}+\frac{\sqrt{3}\,1\mathrm{i}}{8};\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{2\sqrt{1-x^3}\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

input
```
int(1/((1 - x^3)^(1/2)*(x + 3)),x)
```

output
```
-(((3^(1/2)*1i)/2 + 3/2)*(x^3 - 1)^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^
(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/
2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi((3^(1/2)*1i)/
8 + 3/8, asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 +
 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(2*(1 - x^3)^(1/2)*(((3^(1/2)*1i)/2 - 1/2)*
((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2)
+ 1) + x^3)^(1/2))
```

Reduce [F]

$$\int \frac{1}{(3+x)\sqrt{1-x^3}}\,dx = -\left(\int \frac{\sqrt{-x^3+1}}{x^4+3x^3-x-3}dx\right)$$

input
```
int(1/(3+x)/(-x^3+1)^(1/2),x)
```

output
```
 - int(sqrt( - x**3 + 1)/(x**4 + 3*x**3 - x - 3),x)
```

3.133 $\int \frac{1}{(3+x)\sqrt{-1+x^3}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 374

$$\int \frac{1}{(3+x)\sqrt{-1+x^3}}\,dx = -\frac{(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\operatorname{arctanh}\left(\frac{\sqrt{7}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}}{2\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}}\right)}{2\sqrt{7}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}} - \frac{2\sqrt{62-35\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right),-7+4\sqrt{3}\right)}{13\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}} - \frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\operatorname{EllipticPi}\left(\frac{1}{169}\left(553+304\sqrt{3}\right),\arcsin\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right),-7-4\sqrt{3}\right)}{13\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

output
```
-1/14*(1-x)*((x^2+x+1)/(1+3^(1/2)-x)^2)^(1/2)*arctanh(1/2*7^(1/2)*((1-x)/(
1+3^(1/2)-x)^2)^(1/2)/((x^2+x+1)/(1+3^(1/2)-x)^2)^(1/2))*7^(1/2)/((1-x)/(1
+3^(1/2)-x)^2)^(1/2)/(x^3-1)^(1/2)-2/39*(5/2*6^(1/2)-7/2*2^(1/2))*(1-x)*((
x^2+x+1)/(1-3^(1/2)-x)^2)^(1/2)*EllipticF((1+3^(1/2)-x)/(1-3^(1/2)-x),2*I-
I*3^(1/2))*3^(3/4)/(-(1-x)/(1-3^(1/2)-x)^2)^(1/2)/(x^3-1)^(1/2)-4/13*3^(1/
4)*(1/2*6^(1/2)+1/2*2^(1/2))*(1-x)*((x^2+x+1)/(1+3^(1/2)-x)^2)^(1/2)*Ellip
ticPi((1-3^(1/2)-x)/(1+3^(1/2)-x),553/169+304/169*3^(1/2),I*3^(1/2)+2*I)/(
(1-x)/(1+3^(1/2)-x)^2)^(1/2)/(x^3-1)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 20.12 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.34

$$\int \frac{1}{(3+x)\sqrt{-1+x^3}}\,dx = -\frac{4\sqrt{2}\sqrt{\frac{i(-1+x)}{-3i+\sqrt{3}}}\sqrt{1+x+x^2}\,\text{EllipticPi}\left(\frac{2\sqrt{3}}{5i+\sqrt{3}},\arcsin\left(\frac{\sqrt{-i+\sqrt{3}-2ix}}{\sqrt{2}\sqrt[4]{3}}\right),\frac{2\sqrt{3}}{-3i+\sqrt{3}}\right)}{\left(5i+\sqrt{3}\right)\sqrt{-1+x^3}}$$

input
```
Integrate[1/((3 + x)*Sqrt[-1 + x^3]),x]
```

output
```
(-4*Sqrt[2]*Sqrt[(I*(-1 + x))/(-3*I + Sqrt[3])]*Sqrt[1 + x + x^2]*Elliptic
Pi[(2*Sqrt[3])/(5*I + Sqrt[3]), ArcSin[Sqrt[-I + Sqrt[3] - (2*I)*x]/(Sqrt[
2]*3^(1/4))], (2*Sqrt[3])/(-3*I + Sqrt[3])])/((5*I + Sqrt[3])*Sqrt[-1 + x^
3])
```

Rubi [A] (warning: unable to verify)

Time = 1.55 (sec) , antiderivative size = 374, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {2561, 760, 2567, 2538, 412, 435, 104, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x+3)\sqrt{x^3-1}} \, dx$$

$$\Big\downarrow \ 2561$$

$$\frac{\int \frac{1}{\sqrt{x^3-1}} dx}{4+\sqrt{3}} + \frac{\int \frac{-x+\sqrt{3}+1}{(x+3)\sqrt{x^3-1}} dx}{4+\sqrt{3}}$$

$$\Big\downarrow \ 760$$

$$\frac{\int \frac{-x+\sqrt{3}+1}{(x+3)\sqrt{x^3-1}} dx}{4+\sqrt{3}} - \frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}} \operatorname{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right), -7+4\sqrt{3}\right)}{\sqrt[4]{3}\left(4+\sqrt{3}\right)\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

$$\Big\downarrow \ 2567$$

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}} \int \frac{1}{\sqrt{1-\frac{\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{\frac{\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\left(-\frac{\left(4+\sqrt{3}\right)\left(-x-\sqrt{3}+1\right)}{-x+\sqrt{3}+1}-\sqrt{3}+4\right)} d\left(-\frac{-x-\sqrt{3}+}{-x+\sqrt{3}+}\right.}{\left(4+\sqrt{3}\right)\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

$$\frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}} \operatorname{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right), -7+4\sqrt{3}\right)}{\sqrt[4]{3}\left(4+\sqrt{3}\right)\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

$$\Big\downarrow \ 2538$$

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\left(\left(4-\sqrt{3}\right)\int \frac{1}{\sqrt{1-\frac{\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{\frac{\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\left(-\frac{\left(4+\sqrt{3}\right)^2\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}-8\sqrt{3}+1\right.}\right.}{\left(4+\sqrt{3}\right)}$$

$$\frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}} \operatorname{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right), -7+4\sqrt{3}\right)}{\sqrt[4]{3}\left(4+\sqrt{3}\right)\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

$$\Big\downarrow \ 412$$

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\left(-\left(4+\sqrt{3}\right)\int -\frac{-x-\sqrt{3}+1}{\sqrt{1-\frac{\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{\frac{\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\left(-\frac{\left(4+\sqrt{3}\right)^2\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}-8\sqrt{}\right.}\right.}{\left(4+\sqrt{}\right.}$$

$$\frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\left(4+\sqrt{3}\right)\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

$\downarrow$ 435

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\left(-\frac{1}{2}\left(4+\sqrt{3}\right)\int \frac{1}{\sqrt{\frac{\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\sqrt{\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}+1}\left(\frac{\left(4+\sqrt{3}\right)^2\left(-x-\sqrt{3}+1\right)}{-x+\sqrt{3}+1}-8\sqrt{3}+19\right)}\right.}{\left(4+\sqrt{3}\right)\sqrt{\frac{}{(-x}}}$$

$$\frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\left(4+\sqrt{3}\right)\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

$\downarrow$ 104

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\left(-\left(4+\sqrt{3}\right)\int \frac{1}{16\sqrt{3}-\frac{28\left(2-\sqrt{3}\right)\sqrt{\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}+1}}{\sqrt{\frac{\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}}}d\frac{\sqrt{\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}+1}}{\sqrt{\frac{\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}}-\frac{1}{169}\left(4-\sqrt{}\right.\right.}{\left(4+\sqrt{3}\right)\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{x^3}}$$

$$\frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\left(4+\sqrt{3}\right)\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

$\downarrow$ 219

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\left(\frac{\left(4+\sqrt{3}\right)\operatorname{arctanh}\left(\frac{\sqrt{7\left(2-\sqrt{3}\right)}\left(-x-\sqrt{3}+1\right)}{2\sqrt[4]{3}\left(-x+\sqrt{3}+1\right)}\right)}{8\sqrt[4]{3}\sqrt{7\left(2-\sqrt{3}\right)}}-\frac{1}{169}\left(4-\sqrt{3}\right)\sqrt{7519+4340\sqrt{3}}\,\text{Elli}\right.}{\left(4+\sqrt{3}\right)\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

$$\frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\left(4+\sqrt{3}\right)\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

input
```
Int[1/((3 + x)*Sqrt[-1 + x^3]),x]
```

output
```
(-2*Sqrt[2 - Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*Elli
pticF[ArcSin[(1 + Sqrt[3] - x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(3^(1/
4)*(4 + Sqrt[3])*Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 + x^3]) + (4
*3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]
*(((4 + Sqrt[3])*ArcTanh[(Sqrt[7*(2 - Sqrt[3])]*(1 - Sqrt[3] - x))/(2*3^(1
/4)*(1 + Sqrt[3] - x))])/(8*3^(1/4)*Sqrt[7*(2 - Sqrt[3])]) - ((4 - Sqrt[3]
)*Sqrt[7519 + 4340*Sqrt[3]]*EllipticPi[(553 + 304*Sqrt[3])/169, ArcSin[(1
- Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/169))/((4 + Sqrt[3])*S
qrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[-1 + x^3])
```

Defintions of rubi rules used

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 435
```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^(q_.)*((
e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)
*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^2], x] /; FreeQ[{a, b, c, d,
 e, f, p, q, r}, x] && IntegerQ[(m - 1)/2]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 2538
```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[a   Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
 f*x^2]), x], x] - Simp[b   Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
 f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

rule 2561
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> With[{q
 = Rt[b/a, 3]}, Simp[-q/((1 + Sqrt[3])*d - c*q)   Int[1/Sqrt[a + b*x^3], x]
, x] + Simp[d/((1 + Sqrt[3])*d - c*q)   Int[(1 + Sqrt[3] + q*x)/((c + d*x)*
Sqrt[a + b*x^3]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2*c^6 - 20*a*b
*c^3*d^3 - 8*a^2*d^6, 0]
```

rule 2567
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{q = Simplify[(1 + Sqrt[3])*(f/e)]}, Simp[4*3^(1/4)*Sqrt[2
- Sqrt[3]]*f*(1 + q*x)*(Sqrt[(1 - q*x + q^2*x^2)/(1 + Sqrt[3] + q*x)^2]/(q*
Sqrt[a + b*x^3]*Sqrt[(1 + q*x)/(1 + Sqrt[3] + q*x)^2]))   Subst[Int[1/(((1
- Sqrt[3])*d - c*q + ((1 + Sqrt[3])*d - c*q)*x)*Sqrt[1 - x^2]*Sqrt[7 - 4*Sq
rt[3] + x^2]), x], x, (-1 + Sqrt[3] - q*x)/(1 + Sqrt[3] + q*x)], x]] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*e^3 - 2*(5 + 3*Sqrt
[3])*a*f^3, 0] && NeQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.33

method	result	size
default	$\frac{\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticPi}\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\frac{3}{8}+\frac{i\sqrt{3}}{8},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{2\sqrt{x^{3}-1}}$	124
elliptic	$\frac{\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticPi}\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\frac{3}{8}+\frac{i\sqrt{3}}{8},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{2\sqrt{x^{3}-1}}$	124

input
```
int(1/(3+x)/(x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*
3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1
/2)))^(1/2)/(x^3-1)^(1/2)*EllipticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),3/
8+1/8*I*3^(1/2),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))
```

Fricas [F]

$$\int \frac{1}{(3+x)\sqrt{-1+x^3}}\,dx = \int \frac{1}{\sqrt{x^3-1}\,(x+3)}\,dx$$

input
```
integrate(1/(3+x)/(x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(x^3 - 1)/(x^4 + 3*x^3 - x - 3), x)
```

Sympy [F]

$$\int \frac{1}{(3+x)\sqrt{-1+x^3}}\,dx = \int \frac{1}{\sqrt{(x-1)\left(x^2+x+1\right)}\,(x+3)}\,dx$$

input
```
integrate(1/(3+x)/(x**3-1)**(1/2),x)
```

output
```
Integral(1/(sqrt((x - 1)*(x**2 + x + 1))*(x + 3)), x)
```

Maxima [F]

$$\int \frac{1}{(3+x)\sqrt{-1+x^3}}\,dx = \int \frac{1}{\sqrt{x^3-1}(x+3)}\,dx$$

input
```
integrate(1/(3+x)/(x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(x^3 - 1)*(x + 3)), x)
```

Giac [F]

$$\int \frac{1}{(3+x)\sqrt{-1+x^3}}\,dx = \int \frac{1}{\sqrt{x^3-1}(x+3)}\,dx$$

input
```
integrate(1/(3+x)/(x^3-1)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(x^3 - 1)*(x + 3)), x)
```

Mupad [B] (verification not implemented)

Time = 21.74 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.44

$$\int \frac{1}{(3+x)\sqrt{-1+x^3}}\,dx = -\frac{\left(3+\sqrt{3}\,1\mathrm{i}\right)\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\Pi\left(\frac{3}{8}+\frac{\sqrt{3}\,1\mathrm{i}}{8};\operatorname{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{4\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

input
```
int(1/((x^3 - 1)^(1/2)*(x + 3)),x)
```

output
```
-((3^(1/2)*1i + 3)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1
/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((
3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi((3^(1/2)*1i)/8 + 3/8, asin((-(x - 1
)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2
- 3/2)))/(4*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*
1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2))
```

Reduce [F]

$$\int \frac{1}{(3+x)\sqrt{-1+x^3}}\,dx = \int \frac{\sqrt{x^3-1}}{x^4+3x^3-x-3}dx$$

input
```
int(1/(3+x)/(x^3-1)^(1/2),x)
```

output
```
int(sqrt(x**3 - 1)/(x**4 + 3*x**3 - x - 3),x)
```

3.134 $\int \frac{1}{(3+x)\sqrt{-1-x^3}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 340

$$\int \frac{1}{(3+x)\sqrt{-1-x^3}}\,dx = \frac{(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\arctan\left(\frac{\sqrt{\frac{13}{2}}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}}{\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}}\right)}{\sqrt{26}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}} + \frac{2(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}} - \frac{4\sqrt[4]{3}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\operatorname{EllipticPi}\left(97-56\sqrt{3},\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{\sqrt{2-\sqrt{3}}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

output
```
1/26*(1+x)*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*arctan(1/2*26^(1/2)*((1+x)/(1
+x+3^(1/2))^2)^(1/2)/((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2))*26^(1/2)/((1+x)/(1
+x+3^(1/2))^2)^(1/2)/(-x^3-1)^(1/2)+2/3*(1+x)*((x^2-x+1)/(1+x-3^(1/2))^2)^
(1/2)*EllipticF((1+x+3^(1/2))/(1+x-3^(1/2)),2*I-I*3^(1/2))*3^(3/4)/(1/2*6^
(1/2)-1/2*2^(1/2))/(-(1+x)/(1+x-3^(1/2))^2)^(1/2)/(-x^3-1)^(1/2)-4*3^(1/4)
*(1+x)*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*EllipticPi((1+x-3^(1/2))/(1+x+3^(
1/2)),97-56*3^(1/2),I*3^(1/2)+2*I)/(1/2*6^(1/2)-1/2*2^(1/2))/((1+x)/(1+x+3
^(1/2))^2)^(1/2)/(-x^3-1)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 20.12 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.38

$$\int \frac{1}{(3+x)\sqrt{-1-x^3}}\,dx = -\frac{4\sqrt{2}\sqrt{\frac{i(1+x)}{3i+\sqrt{3}}}\sqrt{1-x+x^2}\,\text{EllipticPi}\left(\frac{2\sqrt{3}}{7i+\sqrt{3}},\arcsin\left(\frac{\sqrt{i+\sqrt{3}-2ix}}{\sqrt{2}\sqrt[4]{3}}\right),\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)}{\left(7i+\sqrt{3}\right)\sqrt{-1-x^3}}$$

input
```
Integrate[1/((3 + x)*Sqrt[-1 - x^3]),x]
```

output
```
(-4*Sqrt[2]*Sqrt[(I*(1 + x))/(3*I + Sqrt[3])]*Sqrt[1 - x + x^2]*EllipticPi
[(2*Sqrt[3])/(7*I + Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*
3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])])/((7*I + Sqrt[3])*Sqrt[-1 - x^3])
```

Rubi [A] (warning: unable to verify)

Time = 1.54 (sec) , antiderivative size = 321, normalized size of antiderivative = 0.94, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.529$, Rules used = {2561, 760, 2567, 25, 2538, 412, 435, 104, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x+3)\sqrt{-x^3-1}}\,dx$$

$$\Big\downarrow \; 2561$$

$$\frac{\int \frac{1}{\sqrt{-x^3-1}}dx}{2-\sqrt{3}}-\frac{\int \frac{x+\sqrt{3}+1}{(x+3)\sqrt{-x^3-1}}dx}{2-\sqrt{3}}$$

$$\Big\downarrow \; 760$$

$$\frac{2(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}-\frac{\int \frac{x+\sqrt{3}+1}{(x+3)\sqrt{-x^3-1}}dx}{2-\sqrt{3}}$$

$$\Big\downarrow \; 2567$$

$$\frac{2(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}-$$
$$\frac{4\sqrt[4]{3}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\int -\frac{1}{\sqrt{1-\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\left(-\frac{\left(2-\sqrt{3}\right)\left(x-\sqrt{3}+1\right)}{x+\sqrt{3}+1}+\sqrt{3}+2\right)}d\left(-\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)}{\sqrt{2-\sqrt{3}}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

$$\Big\downarrow \; 25$$

$$\frac{4\sqrt[4]{3}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\int \frac{1}{\sqrt{1-\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\left(-\frac{\left(2-\sqrt{3}\right)\left(x-\sqrt{3}+1\right)}{x+\sqrt{3}+1}+\sqrt{3}+2\right)}d\left(-\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)}{\sqrt{2-\sqrt{3}}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}+$$
$$\frac{2(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

$$\Big\downarrow \; 2538$$

$$\frac{2(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}-$$

$$\frac{4\sqrt[4]{3}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(\left(2-\sqrt{3}\right)\int -\frac{x-\sqrt{3}+1}{\left(x+\sqrt{3}+1\right)\sqrt{1-\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\left(-\frac{\left(7-4\sqrt{3}\right)\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}+4\sqrt{3}+7\right)}d\left(\right.\right.}{\sqrt{2-\sqrt{3}}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)}}}$$

$\downarrow$ 412

$$\frac{2(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}-$$

$$\frac{4\sqrt[4]{3}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(\left(2-\sqrt{3}\right)\int -\frac{x-\sqrt{3}+1}{\left(x+\sqrt{3}+1\right)\sqrt{1-\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\left(-\frac{\left(7-4\sqrt{3}\right)\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}+4\sqrt{3}+7\right)}d\left(\right.\right.}{\sqrt{2-\sqrt{3}}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{-x}}$$

$\downarrow$ 435

$$\frac{2(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}-$$

$$\frac{4\sqrt[4]{3}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(\frac{1}{2}\left(2-\sqrt{3}\right)\int \frac{1}{\sqrt{\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\sqrt{\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}+1}\left(\frac{\left(7-4\sqrt{3}\right)\left(x-\sqrt{3}+1\right)}{x+\sqrt{3}+1}+4\sqrt{3}+7\right)}d\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}+\sqrt{\right.}{\sqrt{2-\sqrt{3}}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

$\downarrow$ 104

$$\frac{2(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}-$$

$$\frac{4\sqrt[4]{3}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(\left(2-\sqrt{3}\right)\int\frac{1}{-\frac{52\left(2-\sqrt{3}\right)\sqrt{\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}+1}}{\sqrt{\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}}-8\sqrt{3}}d\frac{\sqrt{\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}+1}}{\sqrt{\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}}+\sqrt{7-4\sqrt{3}}\left(2+\sqrt{3}\right)\text{Ellip}\right.}{\sqrt{2-\sqrt{3}}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

$$\Big\downarrow \ 217$$

$$\frac{2(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}-$$

$$\frac{4\sqrt[4]{3}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(\sqrt{7-4\sqrt{3}}\left(2+\sqrt{3}\right)\text{EllipticPi}\left(97-56\sqrt{3},\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)+\frac{\sqrt{\frac{1}{26}\left(2-\right.}}{}\right.}{\sqrt{2-\sqrt{3}}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

input
```
Int[1/((3 + x)*Sqrt[-1 - x^3]),x]
```

output
```
(2*(1 + x)*Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 + S
qrt[3] + x)/(1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[2 - Sqrt[3]
]*Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^2)]*Sqrt[-1 - x^3]) - (4*3^(1/4)*(1 + x
)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*((Sqrt[(2 - Sqrt[3])/26]*ArcTan[
(Sqrt[(13*(2 - Sqrt[3]))/2]*(1 - Sqrt[3] + x))/(3^(1/4)*(1 + Sqrt[3] + x))
])/(4*3^(1/4)) + Sqrt[7 - 4*Sqrt[3]]*(2 + Sqrt[3])*EllipticPi[97 - 56*Sqrt
[3], ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]]))/(Sqrt[
2 - Sqrt[3]]*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[-1 - x^3])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 435
```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^(q_.)*((
e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)
*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^2], x] /; FreeQ[{a, b, c, d,
 e, f, p, q, r}, x] && IntegerQ[(m - 1)/2]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 2538
```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[a   Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
 f*x^2]), x], x] - Simp[b   Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
 f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

rule 2561
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> With[{q
 = Rt[b/a, 3]}, Simp[-q/((1 + Sqrt[3])*d - c*q)   Int[1/Sqrt[a + b*x^3], x]
, x] + Simp[d/((1 + Sqrt[3])*d - c*q)   Int[(1 + Sqrt[3] + q*x)/((c + d*x)*
Sqrt[a + b*x^3]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2*c^6 - 20*a*b
*c^3*d^3 - 8*a^2*d^6, 0]
```

rule 2567
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{q = Simplify[(1 + Sqrt[3])*(f/e)]}, Simp[4*3^(1/4)*Sqrt[2
- Sqrt[3]]*f*(1 + q*x)*(Sqrt[(1 - q*x + q^2*x^2)/(1 + Sqrt[3] + q*x)^2]/(q*
Sqrt[a + b*x^3]*Sqrt[(1 + q*x)/(1 + Sqrt[3] + q*x)^2]))   Subst[Int[1/(((1
- Sqrt[3])*d - c*q + ((1 + Sqrt[3])*d - c*q)*x)*Sqrt[1 - x^2]*Sqrt[7 - 4*Sq
rt[3] + x^2]), x], x, (-1 + Sqrt[3] - q*x)/(1 + Sqrt[3] + q*x)], x]] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*e^3 - 2*(5 + 3*Sqrt
[3])*a*f^3, 0] && NeQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.39

method	result	size
default	$-\frac{2 i \sqrt{3}\, \sqrt{i \left(x-\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, \sqrt{\frac{x+1}{\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\, \sqrt{-i \left(x-\frac{1}{2}+\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, \operatorname{EllipticPi}\left(\frac{\sqrt{3}\, \sqrt{i \left(x-\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}}{3}, \frac{i \sqrt{3}}{\frac{7}{2}+\frac{i \sqrt{3}}{2}}, \sqrt{\frac{i \sqrt{3}}{\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\right)}{3 \sqrt{-x^{3}-1}\, \left(\frac{7}{2}+\frac{i \sqrt{3}}{2}\right)}$	133
elliptic	$-\frac{2 i \sqrt{3}\, \sqrt{i \left(x-\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, \sqrt{\frac{x+1}{\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\, \sqrt{-i \left(x-\frac{1}{2}+\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, \operatorname{EllipticPi}\left(\frac{\sqrt{3}\, \sqrt{i \left(x-\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}}{3}, \frac{i \sqrt{3}}{\frac{7}{2}+\frac{i \sqrt{3}}{2}}, \sqrt{\frac{i \sqrt{3}}{\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\right)}{3 \sqrt{-x^{3}-1}\, \left(\frac{7}{2}+\frac{i \sqrt{3}}{2}\right)}$	133

input
```
int(1/(3+x)/(-x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x+1)/(3/2+1/2*I*3
^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)/(7/
2+1/2*I*3^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^
(1/2),I*3^(1/2)/(7/2+1/2*I*3^(1/2)),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))
```

Fricas [F]

$$\int \frac{1}{(3+x)\sqrt{-1-x^3}}\,dx = \int \frac{1}{\sqrt{-x^3-1}(x+3)}\,dx$$

input
```
integrate(1/(3+x)/(-x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-x^3 - 1)/(x^4 + 3*x^3 + x + 3), x)
```

Sympy [F]

$$\int \frac{1}{(3+x)\sqrt{-1-x^3}}\,dx = \int \frac{1}{\sqrt{-(x+1)(x^2-x+1)}\,(x+3)}\,dx$$

input
```
integrate(1/(3+x)/(-x**3-1)**(1/2),x)
```

output
```
Integral(1/(sqrt(-(x + 1)*(x**2 - x + 1))*(x + 3)), x)
```

Maxima [F]

$$\int \frac{1}{(3+x)\sqrt{-1-x^3}}\,dx = \int \frac{1}{\sqrt{-x^3-1}(x+3)}\,dx$$

input
```
integrate(1/(3+x)/(-x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(-x^3 - 1)*(x + 3)), x)
```

Giac [F]

$$\int \frac{1}{(3+x)\sqrt{-1-x^3}}\,dx = \int \frac{1}{\sqrt{-x^3-1}(x+3)}\,dx$$

input
```
integrate(1/(3+x)/(-x^3-1)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(-x^3 - 1)*(x + 3)), x)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.53

$$\int \frac{1}{(3+x)\sqrt{-1-x^3}}\,dx = \frac{\left(\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}\right)\sqrt{x^3+1}\,\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1i}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\Pi\left(-\frac{3}{4}-\frac{\sqrt{3}\,1i}{4};\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)}{\sqrt{-x^3-1}\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}}$$

input
```
int(1/((- x^3 - 1)^(1/2)*(x + 3)),x)
```

output
```
(((3^(1/2)*1i)/2 + 3/2)*(x^3 + 1)^(1/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1
/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*
1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi(- (3^(1/2)*1i)/4
 - 3/4, asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3
/2)/((3^(1/2)*1i)/2 - 3/2)))/((- x^3 - 1)^(1/2)*(x^3 - x*(((3^(1/2)*1i)/2
- 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/
2 + 1/2))^(1/2))
```

Reduce [F]

$$\int \frac{1}{(3+x)\sqrt{-1-x^3}}\,dx = \int \frac{1}{(x+3)\sqrt{-x^3-1}}dx$$

input
```
int(1/(3+x)/(-x^3-1)^(1/2),x)
```

output
```
int(1/(3+x)/(-x^3-1)^(1/2),x)
```

3.135 $\int \frac{1}{(c+dx)\sqrt[3]{c^3-d^3x^3}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 137

$$\int \frac{1}{(c+dx)\sqrt[3]{c^3-d^3x^3}}\,dx = -\frac{\sqrt{3}\arctan\left(\frac{1+\frac{\sqrt[3]{2}(c-dx)}{\sqrt[3]{c^3-d^3x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}cd} - \frac{\log\left((c-dx)(c+dx)^2\right)}{4\sqrt[3]{2}cd} + \frac{3\log\left(-d(c-dx)+2^{2/3}d\sqrt[3]{c^3-d^3x^3}\right)}{4\sqrt[3]{2}cd}$$

output
```
-1/4*3^(1/2)*arctan(1/3*(1+2^(1/3)*(-d*x+c)/(-d^3*x^3+c^3)^(1/3))*3^(1/2))
*2^(2/3)/c/d-1/8*ln((-d*x+c)*(d*x+c)^2)*2^(2/3)/c/d+3/8*ln(-d*(-d*x+c)+2^(
2/3)*d*(-d^3*x^3+c^3)^(1/3))*2^(2/3)/c/d
```

Mathematica [A] (verified)

Time = 1.85 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.57

$$\int \frac{1}{(c+dx)\sqrt[3]{c^3-d^3x^3}}\,dx$$
$$= \frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{c^3-d^3x^3}}{\sqrt[3]{2}c-\sqrt[3]{2}dx+\sqrt[3]{c^3-d^3x^3}}\right)+2\log\left(\sqrt{c}\sqrt{d}\left(\sqrt[3]{2}c-\sqrt[3]{2}dx-2\sqrt[3]{c^3-d^3x^3}\right)\right)-\log\left(cd\left(2^{2/3}\right.\right.}{4\sqrt[3]{2}cd}$$

input
```
Integrate[1/((c + d*x)*(c^3 - d^3*x^3)^(1/3)),x]
```

output
```
(2*Sqrt[3]*ArcTan[(Sqrt[3]*(c^3 - d^3*x^3)^(1/3))/(2^(1/3)*c - 2^(1/3)*d*x
 + (c^3 - d^3*x^3)^(1/3))] + 2*Log[Sqrt[c]*Sqrt[d]*(2^(1/3)*c - 2^(1/3)*d*
x - 2*(c^3 - d^3*x^3)^(1/3))] - Log[c*d*(2^(2/3)*c^2 - 2*2^(2/3)*c*d*x + 2
^(2/3)*d^2*x^2 + 2*2^(1/3)*(c - d*x)*(c^3 - d^3*x^3)^(1/3) + 4*(c^3 - d^3*
x^3)^(2/3))])/(4*2^(1/3)*c*d)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {2574}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c+dx)\sqrt[3]{c^3-d^3x^3}}\,dx$$
$$\Big\downarrow \; 2574$$
$$-\frac{\sqrt{3}\arctan\left(\frac{\frac{\sqrt[3]{2}(c-dx)}{\sqrt[3]{c^3-d^3x^3}}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}cd}+\frac{3\log\left(2^{2/3}d\sqrt[3]{c^3-d^3x^3}-d(c-dx)\right)}{4\sqrt[3]{2}cd}-\frac{\log\left((c-dx)(c+dx)^2\right)}{4\sqrt[3]{2}cd}$$

input
```
Int[1/((c + d*x)*(c^3 - d^3*x^3)^(1/3)),x]
```

output
```
-1/2*(Sqrt[3]*ArcTan[(1 + (2^(1/3)*(c - d*x))/(c^3 - d^3*x^3)^(1/3))/Sqrt[
3]])/(2^(1/3)*c*d) - Log[(c - d*x)*(c + d*x)^2]/(4*2^(1/3)*c*d) + (3*Log[-
(d*(c - d*x)) + 2^(2/3)*d*(c^3 - d^3*x^3)^(1/3)])/(4*2^(1/3)*c*d)
```

Defintions of rubi rules used

rule 2574
```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] :> Simp[
Sqrt[3]*(ArcTan[(1 - 2^(1/3)*Rt[b, 3]*((c - d*x)/(d*(a + b*x^3)^(1/3))))/Sq
rt[3]]/(2^(4/3)*Rt[b, 3]*c)), x] + (Simp[Log[(c + d*x)^2*(c - d*x)]/(2^(7/3
)*Rt[b, 3]*c), x] - Simp[(3*Log[Rt[b, 3]*(c - d*x) + 2^(2/3)*d*(a + b*x^3)^
(1/3)])/(2^(7/3)*Rt[b, 3]*c), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 +
a*d^3, 0]
```

Maple [F]

$$\int \frac{1}{(dx + c) \left(-d^3 x^3 + c^3\right)^{\frac{1}{3}}} dx$$

input
```
int(1/(d*x+c)/(-d^3*x^3+c^3)^(1/3),x)
```

output
```
int(1/(d*x+c)/(-d^3*x^3+c^3)^(1/3),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(c + dx)\sqrt[3]{c^3 - d^3 x^3}} dx = \text{Timed out}$$

input
```
integrate(1/(d*x+c)/(-d^3*x^3+c^3)^(1/3),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(c+dx)\sqrt[3]{c^3-d^3x^3}}\,dx = \int \frac{1}{\sqrt[3]{-(-c+dx)\,(c^2+cdx+d^2x^2)}\,(c+dx)}\,dx$$

input
```
integrate(1/(d*x+c)/(-d**3*x**3+c**3)**(1/3),x)
```

output
```
Integral(1/((-(-c + d*x)*(c**2 + c*d*x + d**2*x**2))**(1/3)*(c + d*x)), x)
```

Maxima [F]

$$\int \frac{1}{(c+dx)\sqrt[3]{c^3-d^3x^3}}\,dx = \int \frac{1}{(-d^3x^3+c^3)^{\frac{1}{3}}(dx+c)}\,dx$$

input
```
integrate(1/(d*x+c)/(-d^3*x^3+c^3)^(1/3),x, algorithm="maxima")
```

output
```
integrate(1/((-d^3*x^3 + c^3)^(1/3)*(d*x + c)), x)
```

Giac [F]

$$\int \frac{1}{(c+dx)\sqrt[3]{c^3-d^3x^3}}\,dx = \int \frac{1}{(-d^3x^3+c^3)^{\frac{1}{3}}(dx+c)}\,dx$$

input
```
integrate(1/(d*x+c)/(-d^3*x^3+c^3)^(1/3),x, algorithm="giac")
```

output
```
integrate(1/((-d^3*x^3 + c^3)^(1/3)*(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)\sqrt[3]{c^3-d^3x^3}}\,dx = \int \frac{1}{\left(c^3-d^3\,x^3\right)^{1/3}\,\left(c+d\,x\right)}\,dx$$

input
```
int(1/((c^3 - d^3*x^3)^(1/3)*(c + d*x)),x)
```

output
```
int(1/((c^3 - d^3*x^3)^(1/3)*(c + d*x)), x)
```

Reduce [F]

$$\int \frac{1}{(c+dx)\sqrt[3]{c^3-d^3x^3}}\,dx = \int \frac{1}{\left(-d^3x^3+c^3\right)^{\frac{1}{3}}c+\left(-d^3x^3+c^3\right)^{\frac{1}{3}}dx}dx$$

input
```
int(1/(d*x+c)/(-d^3*x^3+c^3)^(1/3),x)
```

output
```
int(1/((c**3 - d**3*x**3)**(1/3)*c + (c**3 - d**3*x**3)**(1/3)*d*x),x)
```

3.136 $\int \frac{1}{(c+dx)\sqrt[3]{-c^3+d^3x^3}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 139

$$\int \frac{1}{(c+dx)\sqrt[3]{-c^3+d^3x^3}}\,dx=\frac{\sqrt{3}\arctan\left(\frac{1-\frac{\sqrt[3]{2}(c-dx)}{\sqrt[3]{-c^3+d^3x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}cd}+\frac{\log\left((c-dx)(c+dx)^2\right)}{4\sqrt[3]{2}cd}-\frac{3\log\left(d(c-dx)+2^{2/3}d\sqrt[3]{-c^3+d^3x^3}\right)}{4\sqrt[3]{2}cd}$$

output

```
1/4*3^(1/2)*arctan(1/3*(1-2^(1/3)*(-d*x+c)/(d^3*x^3-c^3)^(1/3))*3^(1/2))*2
^(2/3)/c/d+1/8*ln((-d*x+c)*(d*x+c)^2)*2^(2/3)/c/d-3/8*ln(d*(-d*x+c)+2^(2/3
)*d*(d^3*x^3-c^3)^(1/3))*2^(2/3)/c/d
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.91 (sec) , antiderivative size = 311, normalized size of antiderivative = 2.24

$$\int \frac{1}{(c+dx)\sqrt[3]{-c^3+d^3x^3}}\,dx$$
$$= \frac{\sqrt[3]{-\frac{1}{2}}\left(2i\sqrt{3}\text{arctanh}\left(\frac{\sqrt[3]{2}(3+i\sqrt{3})c+\sqrt[3]{2}(-3-i\sqrt{3})dx+2i\sqrt{3}\sqrt[3]{-c^3+d^3x^3}}{6\sqrt[3]{-c^3+d^3x^3}}\right)+2\log\left(\sqrt{c}\sqrt{d}\left(-c+i\sqrt{3}c+dx\right.\right.\right.}{}$$

input
```
Integrate[1/((c + d*x)*(-c^3 + d^3*x^3)^(1/3)),x]
```

output
```
((-1/2)^(1/3)*((2*I)*Sqrt[3]*ArcTanh[(2^(1/3)*(3 + I*Sqrt[3])*c + 2^(1/3)*
(-3 - I*Sqrt[3])*d*x + (2*I)*Sqrt[3]*(-c^3 + d^3*x^3)^(1/3))/(6*(-c^3 + d^
3*x^3)^(1/3))] + 2*Log[Sqrt[c]*Sqrt[d]*(-c + I*Sqrt[3]*c + d*x - I*Sqrt[3]
*d*x + 2*2^(2/3)*(-c^3 + d^3*x^3)^(1/3))] - Log[-(c*d*((1 + I*Sqrt[3])*c^2
 + (1 + I*Sqrt[3])*d^2*x^2 - 2*(-2)^(2/3)*d*x*(-c^3 + d^3*x^3)^(1/3) - 4*2
^(1/3)*(-c^3 + d^3*x^3)^(2/3) + 2*c*((-1 - I*Sqrt[3])*d*x + (-2)^(2/3)*(-c
^3 + d^3*x^3)^(1/3))))]))/(4*c*d)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {2574}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c+dx)\sqrt[3]{d^3x^3-c^3}}\,dx$$
$$\Big\downarrow \; 2574$$

$$\frac{\sqrt{3}\arctan\left(\frac{1-\frac{\sqrt[3]{2}(c-dx)}{\sqrt[3]{d^3x^3-c^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}cd}-\frac{3\log\left(2^{2/3}d\sqrt[3]{d^3x^3-c^3}+d(c-dx)\right)}{4\sqrt[3]{2}cd}+\frac{\log\left((c-dx)(c+dx)^2\right)}{4\sqrt[3]{2}cd}$$

input

```
Int[1/((c + d*x)*(-c^3 + d^3*x^3)^(1/3)),x]
```

output

```
(Sqrt[3]*ArcTan[(1 - (2^(1/3)*(c - d*x))/(-c^3 + d^3*x^3)^(1/3))/Sqrt[3]])
/(2*2^(1/3)*c*d) + Log[(c - d*x)*(c + d*x)^2]/(4*2^(1/3)*c*d) - (3*Log[d*(
c - d*x) + 2^(2/3)*d*(-c^3 + d^3*x^3)^(1/3)])/(4*2^(1/3)*c*d)
```

Defintions of rubi rules used

rule 2574

```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] :> Simp[
Sqrt[3]*(ArcTan[(1 - 2^(1/3)*Rt[b, 3]*((c - d*x)/(d*(a + b*x^3)^(1/3))))/Sq
rt[3]]/(2^(4/3)*Rt[b, 3]*c)), x] + (Simp[Log[(c + d*x)^2*(c - d*x)]/(2^(7/3
)*Rt[b, 3]*c), x] - Simp[(3*Log[Rt[b, 3]*(c - d*x) + 2^(2/3)*d*(a + b*x^3)^
(1/3)])/(2^(7/3)*Rt[b, 3]*c), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 +
a*d^3, 0]
```

Maple [F]

$$\int \frac{1}{(dx+c)\left(d^3x^3-c^3\right)^{\frac{1}{3}}}dx$$

input

```
int(1/(d*x+c)/(d^3*x^3-c^3)^(1/3),x)
```

output

```
int(1/(d*x+c)/(d^3*x^3-c^3)^(1/3),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)\sqrt[3]{-c^3+d^3x^3}}\,dx = \text{Timed out}$$

input
```
integrate(1/(d*x+c)/(d^3*x^3-c^3)^(1/3),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(c+dx)\sqrt[3]{-c^3+d^3x^3}}\,dx = \int \frac{1}{\sqrt[3]{(-c+dx)\,(c^2+cdx+d^2x^2)}\,(c+dx)}\,dx$$

input
```
integrate(1/(d*x+c)/(d**3*x**3-c**3)**(1/3),x)
```

output
```
Integral(1/(((-c + d*x)*(c**2 + c*d*x + d**2*x**2))**(1/3)*(c + d*x)), x)
```

Maxima [F]

$$\int \frac{1}{(c+dx)\sqrt[3]{-c^3+d^3x^3}}\,dx = \int \frac{1}{(d^3x^3-c^3)^{\frac{1}{3}}(dx+c)}\,dx$$

input
```
integrate(1/(d*x+c)/(d^3*x^3-c^3)^(1/3),x, algorithm="maxima")
```

output
```
integrate(1/((d^3*x^3 - c^3)^(1/3)*(d*x + c)), x)
```

Giac [F]

$$\int \frac{1}{(c+dx)\sqrt[3]{-c^3+d^3x^3}}\,dx = \int \frac{1}{(d^3x^3-c^3)^{\frac{1}{3}}(dx+c)}\,dx$$

input
```
integrate(1/(d*x+c)/(d^3*x^3-c^3)^(1/3),x, algorithm="giac")
```

output
```
integrate(1/((d^3*x^3 - c^3)^(1/3)*(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)\sqrt[3]{-c^3+d^3x^3}}\,dx = \int \frac{1}{(d^3\,x^3-c^3)^{1/3}\,(c+d\,x)}\,dx$$

input
```
int(1/((d^3*x^3 - c^3)^(1/3)*(c + d*x)),x)
```

output
```
int(1/((d^3*x^3 - c^3)^(1/3)*(c + d*x)), x)
```

Reduce [F]

$$\int \frac{1}{(c+dx)\sqrt[3]{-c^3+d^3x^3}}\,dx = \int \frac{1}{(d^3x^3-c^3)^{\frac{1}{3}}\,c+(d^3x^3-c^3)^{\frac{1}{3}}\,dx}dx$$

input
```
int(1/(d*x+c)/(d^3*x^3-c^3)^(1/3),x)
```

output
```
int(1/(( - c**3 + d**3*x**3)**(1/3)*c + ( - c**3 + d**3*x**3)**(1/3)*d*x),
x)
```

3.137 $\int \frac{1}{(c+dx)\sqrt[3]{bc^3-bd^3x^3}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 168

$$\int \frac{1}{(c+dx)\sqrt[3]{bc^3-bd^3x^3}}\,dx = -\frac{\sqrt{3}\arctan\left(\frac{1+\frac{\sqrt[3]{2}\sqrt[3]{b}(c-dx)}{\sqrt[3]{bc^3-bd^3x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt[3]{b}cd} - \frac{\log\left((c-dx)(c+dx)^2\right)}{4\sqrt[3]{2}\sqrt[3]{b}cd} + \frac{3\log\left(-\sqrt[3]{b}d(c-dx)+2^{2/3}d\sqrt[3]{bc^3-bd^3x^3}\right)}{4\sqrt[3]{2}\sqrt[3]{b}cd}$$

output

```
-1/4*3^(1/2)*arctan(1/3*(1+2^(1/3)*b^(1/3)*(-d*x+c)/(-b*d^3*x^3+b*c^3)^(1/
3))*3^(1/2))*2^(2/3)/b^(1/3)/c/d-1/8*ln((-d*x+c)*(d*x+c)^2)*2^(2/3)/b^(1/3
)/c/d+3/8*ln(-b^(1/3)*d*(-d*x+c)+2^(2/3)*d*(-b*d^3*x^3+b*c^3)^(1/3))*2^(2/
3)/b^(1/3)/c/d
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.48

$$\int \frac{1}{(c+dx)\sqrt[3]{bc^3-bd^3x^3}}\,dx$$
$$= \frac{\sqrt[3]{c^3-d^3x^3}\left(2\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{c^3-d^3x^3}}{\sqrt[3]{2}c-\sqrt[3]{2}dx+\sqrt[3]{c^3-d^3x^3}}\right)+2\log\left(\sqrt{c}\sqrt{d}\left(\sqrt[3]{2}c-\sqrt[3]{2}dx-2\sqrt[3]{c^3-d^3x^3}\right)\right)-\right.}{4\sqrt[3]{2}cd\sqrt[3]{b\left(c^3-d^3\right.}}$$

input
```
Integrate[1/((c + d*x)*(b*c^3 - b*d^3*x^3)^(1/3)),x]
```

output
```
((c^3 - d^3*x^3)^(1/3)*(2*Sqrt[3]*ArcTan[(Sqrt[3]*(c^3 - d^3*x^3)^(1/3))/(
2^(1/3)*c - 2^(1/3)*d*x + (c^3 - d^3*x^3)^(1/3))] + 2*Log[Sqrt[c]*Sqrt[d]*
(2^(1/3)*c - 2^(1/3)*d*x - 2*(c^3 - d^3*x^3)^(1/3))] - Log[c*d*(2^(2/3)*c^
2 - 2*2^(2/3)*c*d*x + 2^(2/3)*d^2*x^2 + 2*2^(1/3)*(c - d*x)*(c^3 - d^3*x^3
)^(1/3) + 4*(c^3 - d^3*x^3)^(2/3))]))/(4*2^(1/3)*c*d*(b*(c^3 - d^3*x^3))^(
1/3))
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.037$, Rules used = $\{2574\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c+dx)\sqrt[3]{bc^3-bd^3x^3}}\,dx$$
$$\Big\downarrow \; 2574$$

$$-\frac{\sqrt{3}\arctan\left(\frac{\frac{\sqrt[3]{2}\sqrt[3]{b}(c-dx)}{\sqrt[3]{bc^3-bd^3x^3}}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt[3]{b}cd}+\frac{3\log\left(2^{2/3}d\sqrt[3]{bc^3-bd^3x^3}-\sqrt[3]{b}d(c-dx)\right)}{4\sqrt[3]{2}\sqrt[3]{b}cd}-\frac{\log\left((c-dx)(c+dx)^2\right)}{4\sqrt[3]{2}\sqrt[3]{b}cd}$$

input
```
Int[1/((c + d*x)*(b*c^3 - b*d^3*x^3)^(1/3)),x]
```

output
```
-1/2*(Sqrt[3]*ArcTan[(1 + (2^(1/3)*b^(1/3)*(c - d*x))/(b*c^3 - b*d^3*x^3)^
(1/3))/Sqrt[3]])/(2^(1/3)*b^(1/3)*c*d) - Log[(c - d*x)*(c + d*x)^2]/(4*2^(
1/3)*b^(1/3)*c*d) + (3*Log[-(b^(1/3)*d*(c - d*x)) + 2^(2/3)*d*(b*c^3 - b*d
^3*x^3)^(1/3)])/(4*2^(1/3)*b^(1/3)*c*d)
```

Defintions of rubi rules used

rule 2574
```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] :> Simp[
Sqrt[3]*(ArcTan[(1 - 2^(1/3)*Rt[b, 3]*((c - d*x)/(d*(a + b*x^3)^(1/3))))/Sq
rt[3]]/(2^(4/3)*Rt[b, 3]*c)), x] + (Simp[Log[(c + d*x)^2*(c - d*x)]/(2^(7/3
)*Rt[b, 3]*c), x] - Simp[(3*Log[Rt[b, 3]*(c - d*x) + 2^(2/3)*d*(a + b*x^3)^
(1/3)])/(2^(7/3)*Rt[b, 3]*c), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 +
a*d^3, 0]
```

Maple [F]

$$\int \frac{1}{(dx+c)\left(-b\,d^3x^3+b\,c^3\right)^{\frac{1}{3}}}dx$$

input
```
int(1/(d*x+c)/(-b*d^3*x^3+b*c^3)^(1/3),x)
```

output
```
int(1/(d*x+c)/(-b*d^3*x^3+b*c^3)^(1/3),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)\sqrt[3]{bc^3-bd^3x^3}}\,dx = \text{Timed out}$$

input
```
integrate(1/(d*x+c)/(-b*d^3*x^3+b*c^3)^(1/3),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(c+dx)\sqrt[3]{bc^3-bd^3x^3}}\,dx = \int \frac{1}{\sqrt[3]{-b\left(-c+dx\right)\left(c^2+cdx+d^2x^2\right)}\left(c+dx\right)}\,dx$$

input
```
integrate(1/(d*x+c)/(-b*d**3*x**3+b*c**3)**(1/3),x)
```

output
```
Integral(1/((-b*(-c + d*x)*(c**2 + c*d*x + d**2*x**2))**(1/3)*(c + d*x)),
x)
```

Maxima [F]

$$\int \frac{1}{(c+dx)\sqrt[3]{bc^3-bd^3x^3}}\,dx = \int \frac{1}{\left(-bd^3x^3+bc^3\right)^{\frac{1}{3}}(dx+c)}\,dx$$

input
```
integrate(1/(d*x+c)/(-b*d^3*x^3+b*c^3)^(1/3),x, algorithm="maxima")
```

output
```
integrate(1/((-b*d^3*x^3 + b*c^3)^(1/3)*(d*x + c)), x)
```

Giac [F]

$$\int \frac{1}{(c+dx)\sqrt[3]{bc^3-bd^3x^3}}\,dx=\int \frac{1}{(-bd^3x^3+bc^3)^{\frac{1}{3}}(dx+c)}\,dx$$

input
```
integrate(1/(d*x+c)/(-b*d^3*x^3+b*c^3)^(1/3),x, algorithm="giac")
```

output
```
integrate(1/((-b*d^3*x^3 + b*c^3)^(1/3)*(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)\sqrt[3]{bc^3-bd^3x^3}}\,dx=\int \frac{1}{(b\,c^3-b\,d^3\,x^3)^{1/3}\,(c+d\,x)}\,dx$$

input
```
int(1/((b*c^3 - b*d^3*x^3)^(1/3)*(c + d*x)),x)
```

output
```
int(1/((b*c^3 - b*d^3*x^3)^(1/3)*(c + d*x)), x)
```

Reduce [F]

$$\int \frac{1}{(c+dx)\sqrt[3]{bc^3-bd^3x^3}}\,dx=\frac{\int \frac{1}{(-d^3x^3+c^3)^{\frac{1}{3}}c+(-d^3x^3+c^3)^{\frac{1}{3}}dx}dx}{b^{\frac{1}{3}}}$$

input
```
int(1/(d*x+c)/(-b*d^3*x^3+b*c^3)^(1/3),x)
```

output
```
int(1/((c**3 - d**3*x**3)**(1/3)*c + (c**3 - d**3*x**3)**(1/3)*d*x),x)/b**
(1/3)
```

3.138 $\int \frac{1}{(1+x)\sqrt[3]{1-x^3}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 97

$$\int \frac{1}{(1+x)\sqrt[3]{1-x^3}}\,dx = -\frac{\sqrt{3}\arctan\left(\frac{1+\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}} - \frac{\log\left((1-x)(1+x)^2\right)}{4\sqrt[3]{2}} + \frac{3\log\left(-1+x+2^{2/3}\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}$$

output
```
-1/4*3^(1/2)*arctan(1/3*(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)-
1/8*ln((1-x)*(1+x)^2)*2^(2/3)+3/8*ln(-1+x+2^(2/3)*(-x^3+1)^(1/3))*2^(2/3)
```

Mathematica [A] (verified)

Time = 1.30 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.53

$$\int \frac{1}{(1+x)\sqrt[3]{1-x^3}}\,dx = \frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{1-x^3}}{\sqrt[3]{2}-\sqrt[3]{2}x+\sqrt[3]{1-x^3}}\right) + 2\log\left(-\sqrt[3]{2}+\sqrt[3]{2}x+2\sqrt[3]{1-x^3}\right) - \log\left(2^{2/3} - 2\ 2^{2/3}x + 2^{2/3}x^2 -\right.}{4\sqrt[3]{2}}$$

input
```
Integrate[1/((1 + x)*(1 - x^3)^(1/3)),x]
```

output
```
(2*Sqrt[3]*ArcTan[(Sqrt[3]*(1 - x^3)^(1/3))/(2^(1/3) - 2^(1/3)*x + (1 - x^
3)^(1/3))] + 2*Log[-2^(1/3) + 2^(1/3)*x + 2*(1 - x^3)^(1/3)] - Log[2^(2/3)
 - 2*2^(2/3)*x + 2^(2/3)*x^2 - 2*(-1 + x)*(2 - 2*x^3)^(1/3) + 4*(1 - x^3)^
(2/3)])/(4*2^(1/3))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = $\{2574\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x+1)\sqrt[3]{1-x^3}}dx$$

$$\Big\downarrow \text{ 2574}$$

$$-\frac{\sqrt{3}\arctan\left(\frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}}+\frac{3\log\left(2^{2/3}\sqrt[3]{1-x^3}+x-1\right)}{4\sqrt[3]{2}}-\frac{\log\left((1-x)(x+1)^2\right)}{4\sqrt[3]{2}}$$

input
```
Int[1/((1 + x)*(1 - x^3)^(1/3)),x]
```

output
```
-1/2*(Sqrt[3]*ArcTan[(1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]])/2^(
1/3) - Log[(1 - x)*(1 + x)^2]/(4*2^(1/3)) + (3*Log[-1 + x + 2^(2/3)*(1 - x
^3)^(1/3)])/(4*2^(1/3))
```

Defintions of rubi rules used

rule 2574
```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] :> Simp[
Sqrt[3]*(ArcTan[(1 - 2^(1/3)*Rt[b, 3]*((c - d*x)/(d*(a + b*x^3)^(1/3))))/Sq
rt[3]]/(2^(4/3)*Rt[b, 3]*c)), x] + (Simp[Log[(c + d*x)^2*(c - d*x)]/(2^(7/3
)*Rt[b, 3]*c), x] - Simp[(3*Log[Rt[b, 3]*(c - d*x) + 2^(2/3)*d*(a + b*x^3)^
(1/3)])/(2^(7/3)*Rt[b, 3]*c), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 +
a*d^3, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 6.29 (sec) , antiderivative size = 1142, normalized size of antiderivative = 11.77

method	result	size
trager	Expression too large to display	1142

input
```
int(1/(x+1)/(-x^3+1)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
-1/4*ln((6*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*RootOf(_Z^3
-4)^3*x+8*(-x^3+1)^(2/3)*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^
2)*RootOf(_Z^3-4)^2+20*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)
^2*RootOf(_Z^3-4)^2*x-13*(-x^3+1)^(1/3)*RootOf(_Z^3-4)^2*x-18*(-x^3+1)^(1/
3)*RootOf(_Z^3-4)*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*x+13
*(-x^3+1)^(1/3)*RootOf(_Z^3-4)^2+18*(-x^3+1)^(1/3)*RootOf(_Z^3-4)*RootOf(R
ootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)-21*RootOf(_Z^3-4)*x^2-70*RootO
f(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*x^2-6*RootOf(_Z^3-4)*x-36*(
-x^3+1)^(2/3)-20*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*x-21*
RootOf(_Z^3-4)-70*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2))/(x+
1)^2)*RootOf(_Z^3-4)-1/2*ln((6*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)
+4*_Z^2)*RootOf(_Z^3-4)^3*x+8*(-x^3+1)^(2/3)*RootOf(RootOf(_Z^3-4)^2+2*_Z*
RootOf(_Z^3-4)+4*_Z^2)*RootOf(_Z^3-4)^2+20*RootOf(RootOf(_Z^3-4)^2+2*_Z*Ro
otOf(_Z^3-4)+4*_Z^2)^2*RootOf(_Z^3-4)^2*x-13*(-x^3+1)^(1/3)*RootOf(_Z^3-4)
^2*x-18*(-x^3+1)^(1/3)*RootOf(_Z^3-4)*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(
_Z^3-4)+4*_Z^2)*x+13*(-x^3+1)^(1/3)*RootOf(_Z^3-4)^2+18*(-x^3+1)^(1/3)*Roo
tOf(_Z^3-4)*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)-21*RootOf(
_Z^3-4)*x^2-70*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_Z^3-4)+4*_Z^2)*x^2-6*R
ootOf(_Z^3-4)*x-36*(-x^3+1)^(2/3)-20*RootOf(RootOf(_Z^3-4)^2+2*_Z*RootOf(_
Z^3-4)+4*_Z^2)*x-21*RootOf(_Z^3-4)-70*RootOf(RootOf(_Z^3-4)^2+2*_Z*Root...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 298 vs. $2(71) = 142$.

Time = 2.13 (sec) , antiderivative size = 298, normalized size of antiderivative = 3.07

$$\int \frac{1}{(1+x)\sqrt[3]{1-x^3}}\,dx = \frac{1}{6}$$

$$\cdot 2^{\frac{1}{6}}\sqrt{\frac{3}{2}}\arctan\left(\frac{2^{\frac{1}{6}}\sqrt{\frac{3}{2}}\Big(8\cdot 2^{\frac{2}{3}}(x^4+2x^3+2x^2+2x+1)(-x^3+1)^{\frac{2}{3}}+2^{\frac{1}{3}}(13x^6+2x^5+19x^4-4x^3-}{3\,(3x^6-18x^5-3x^4-28x^3-}\right.$$

$$-\frac{1}{24}$$

$$\cdot 2^{\frac{2}{3}}\log\left(\frac{4\cdot 2^{\frac{2}{3}}(-x^3+1)^{\frac{2}{3}}(x^2+1)+2^{\frac{1}{3}}(5x^4+6x^2+5)-2\,(3x^3-x^2+x-3)(-x^3+1)^{\frac{1}{3}}}{x^4+4x^3+6x^2+4x+1}\right)$$

$$+\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(\frac{2^{\frac{2}{3}}(x^2+2x+1)-2\cdot 2^{\frac{1}{3}}(-x^3+1)^{\frac{1}{3}}(x-1)-4\,(-x^3+1)^{\frac{2}{3}}}{x^2+2x+1}\right)$$

input
```
integrate(1/(1+x)/(-x^3+1)^(1/3),x, algorithm="fricas")
```

output
```
1/6*2^(1/6)*sqrt(3/2)*arctan(1/3*2^(1/6)*sqrt(3/2)*(8*2^(2/3)*(x^4 + 2*x^3
 + 2*x^2 + 2*x + 1)*(-x^3 + 1)^(2/3) + 2^(1/3)*(13*x^6 + 2*x^5 + 19*x^4 -
4*x^3 + 19*x^2 + 2*x + 13) - 4*(5*x^5 - 5*x^4 + 6*x^3 - 6*x^2 + 5*x - 5)*(
-x^3 + 1)^(1/3))/(3*x^6 - 18*x^5 - 3*x^4 - 28*x^3 - 3*x^2 - 18*x + 3)) - 1
/24*2^(2/3)*log((4*2^(2/3)*(-x^3 + 1)^(2/3)*(x^2 + 1) + 2^(1/3)*(5*x^4 + 6
*x^2 + 5) - 2*(3*x^3 - x^2 + x - 3)*(-x^3 + 1)^(1/3))/(x^4 + 4*x^3 + 6*x^2
 + 4*x + 1)) + 1/12*2^(2/3)*log((2^(2/3)*(x^2 + 2*x + 1) - 2*2^(1/3)*(-x^3
 + 1)^(1/3)*(x - 1) - 4*(-x^3 + 1)^(2/3))/(x^2 + 2*x + 1))
```

Sympy [F]

$$\int \frac{1}{(1+x)\sqrt[3]{1-x^3}}\,dx = \int \frac{1}{\sqrt[3]{-(x-1)(x^2+x+1)}\,(x+1)}\,dx$$

input
```
integrate(1/(1+x)/(-x**3+1)**(1/3),x)
```

output
```
Integral(1/((-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)), x)
```

Maxima [F]

$$\int \frac{1}{(1+x)\sqrt[3]{1-x^3}}\,dx = \int \frac{1}{(-x^3+1)^{\frac{1}{3}}(x+1)}\,dx$$

input
```
integrate(1/(1+x)/(-x^3+1)^(1/3),x, algorithm="maxima")
```

output
```
integrate(1/((-x^3 + 1)^(1/3)*(x + 1)), x)
```

Giac [F]

$$\int \frac{1}{(1+x)\sqrt[3]{1-x^3}}\,dx = \int \frac{1}{(-x^3+1)^{\frac{1}{3}}(x+1)}\,dx$$

input
```
integrate(1/(1+x)/(-x^3+1)^(1/3),x, algorithm="giac")
```

output
```
integrate(1/((-x^3 + 1)^(1/3)*(x + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(1+x)\sqrt[3]{1-x^3}}\,dx = \int \frac{1}{(1-x^3)^{1/3}\,(x+1)}\,dx$$

input
```
int(1/((1 - x^3)^(1/3)*(x + 1)),x)
```

output
```
int(1/((1 - x^3)^(1/3)*(x + 1)), x)
```

Reduce [F]

$$\int \frac{1}{(1+x)\sqrt[3]{1-x^3}}\,dx = \int \frac{1}{(-x^3+1)^{\frac{1}{3}}\,x+(-x^3+1)^{\frac{1}{3}}}dx$$

input
```
int(1/(1+x)/(-x^3+1)^(1/3),x)
```

output
```
int(1/(( - x**3 + 1)**(1/3)*x + ( - x**3 + 1)**(1/3)),x)
```

3.139 $\int \frac{1}{(c+dx)\sqrt[3]{2c^3+d^3x^3}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 186

$$\int \frac{1}{(c+dx)\sqrt[3]{2c^3+d^3x^3}}\,dx = \frac{\arctan\left(\frac{1+\frac{2dx}{\sqrt[3]{2c^3+d^3x^3}}}{\sqrt{3}}\right)}{2\sqrt{3}cd} - \frac{\sqrt{3}\arctan\left(\frac{1+\frac{2(2c+dx)}{\sqrt[3]{2c^3+d^3x^3}}}{\sqrt{3}}\right)}{2cd} - \frac{\log(c+dx)}{2cd} - \frac{\log\left(-dx+\sqrt[3]{2c^3+d^3x^3}\right)}{4cd} + \frac{3\log\left(d(2c+dx)-d\sqrt[3]{2c^3+d^3x^3}\right)}{4cd}$$

output
```
1/6*arctan(1/3*(1+2*d*x/(d^3*x^3+2*c^3)^(1/3))*3^(1/2))*3^(1/2)/c/d-1/2*3^
(1/2)*arctan(1/3*(1+2*(d*x+2*c)/(d^3*x^3+2*c^3)^(1/3))*3^(1/2))/c/d-1/2*ln
(d*x+c)/c/d-1/4*ln(-d*x+(d^3*x^3+2*c^3)^(1/3))/c/d+3/4*ln(d*(d*x+2*c)-d*(d
^3*x^3+2*c^3)^(1/3))/c/d
```

Mathematica [F]

$$\int \frac{1}{(c+dx)\sqrt[3]{2c^3+d^3x^3}}\,dx=\int \frac{1}{(c+dx)\sqrt[3]{2c^3+d^3x^3}}\,dx$$

input
```
Integrate[1/((c + d*x)*(2*c^3 + d^3*x^3)^(1/3)),x]
```

output
```
Integrate[1/((c + d*x)*(2*c^3 + d^3*x^3)^(1/3)), x]
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.97, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2575, 769, 2576}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c+dx)\sqrt[3]{2c^3+d^3x^3}}dx$$

$$\Big\downarrow \text{2575}$$

$$\frac{\int \frac{1}{\sqrt[3]{2c^3+d^3x^3}}dx}{2c}+\frac{\int \frac{c-dx}{(c+dx)\sqrt[3]{2c^3+d^3x^3}}dx}{2c}$$

$$\Big\downarrow \text{769}$$

$$\frac{\int \frac{c-dx}{(c+dx)\sqrt[3]{2c^3+d^3x^3}}dx}{2c}+\frac{\frac{\arctan\left(\frac{\frac{2dx}{\sqrt[3]{2c^3+d^3x^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}d}-\frac{\log\left(\sqrt[3]{2c^3+d^3x^3}-dx\right)}{2d}}{2c}$$

$$\Big\downarrow \text{2576}$$

$$\frac{\frac{\arctan\left(\frac{\frac{2dx}{\sqrt[3]{2c^3+d^3x^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}d}-\frac{\log\left(\sqrt[3]{2c^3+d^3x^3}-dx\right)}{2d}}{2c}+\frac{-\frac{\sqrt{3}\arctan\left(\frac{\frac{2(2c+dx)}{\sqrt[3]{2c^3+d^3x^3}}+1}{\sqrt{3}}\right)}{d}+\frac{3\log\left(d(2c+dx)-d\sqrt[3]{2c^3+d^3x^3}\right)}{2d}-\frac{\log(c+dx)}{d}}{2c}$$

input
```
Int[1/((c + d*x)*(2*c^3 + d^3*x^3)^(1/3)),x]
```

output
```
(ArcTan[(1 + (2*d*x)/(2*c^3 + d^3*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*d) - Log[-
(d*x) + (2*c^3 + d^3*x^3)^(1/3)]/(2*d))/(2*c) + (-((Sqrt[3]*ArcTan[(1 + (2
*(2*c + d*x))/(2*c^3 + d^3*x^3)^(1/3))/Sqrt[3]])/d) - Log[c + d*x]/d + (3*
Log[d*(2*c + d*x) - d*(2*c^3 + d^3*x^3)^(1/3)])/(2*d))/(2*c)
```

Defintions of rubi rules used

rule 769
```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

rule 2575
```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(a + b*x^3)^(1/3), x], x] + Simp[1/(2*c)   Int[(c - d*x)/((
c + d*x)*(a + b*x^3)^(1/3)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[2*b*c^
3 - a*d^3, 0]
```

rule 2576
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)),
x_Symbol] :> Simp[Sqrt[3]*f*(ArcTan[(1 + 2*Rt[b, 3]*((2*c + d*x)/(d*(a + b*
x^3)^(1/3))))/Sqrt[3]]/(Rt[b, 3]*d)), x] + (Simp[(f*Log[c + d*x])/(Rt[b, 3]
*d), x] - Simp[(3*f*Log[Rt[b, 3]*(2*c + d*x) - d*(a + b*x^3)^(1/3)])/(2*Rt[
b, 3]*d), x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[d*e + c*f, 0] && EqQ[2
*b*c^3 - a*d^3, 0]
```

Maple [F]

$$\int \frac{1}{(dx+c)\left(d^3x^3+2c^3\right)^{\frac{1}{3}}}dx$$

input
```
int(1/(d*x+c)/(d^3*x^3+2*c^3)^(1/3),x)
```

output
```
int(1/(d*x+c)/(d^3*x^3+2*c^3)^(1/3),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)\sqrt[3]{2c^3+d^3x^3}}\,dx = \text{Timed out}$$

input
```
integrate(1/(d*x+c)/(d^3*x^3+2*c^3)^(1/3),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(c+dx)\sqrt[3]{2c^3+d^3x^3}}\,dx = \int \frac{1}{(c+dx)\,\sqrt[3]{2c^3+d^3x^3}}\,dx$$

input
```
integrate(1/(d*x+c)/(d**3*x**3+2*c**3)**(1/3),x)
```

output
```
Integral(1/((c + d*x)*(2*c**3 + d**3*x**3)**(1/3)), x)
```

Maxima [F]

$$\int \frac{1}{(c+dx)\sqrt[3]{2c^3+d^3x^3}}\,dx = \int \frac{1}{(d^3x^3+2\,c^3)^{\frac{1}{3}}(dx+c)}\,dx$$

input
```
integrate(1/(d*x+c)/(d^3*x^3+2*c^3)^(1/3),x, algorithm="maxima")
```

output
```
integrate(1/((d^3*x^3 + 2*c^3)^(1/3)*(d*x + c)), x)
```

Giac [F]

$$\int \frac{1}{(c+dx)\sqrt[3]{2c^3+d^3x^3}}\,dx = \int \frac{1}{(d^3x^3+2\,c^3)^{\frac{1}{3}}(dx+c)}\,dx$$

input
```
integrate(1/(d*x+c)/(d^3*x^3+2*c^3)^(1/3),x, algorithm="giac")
```

output
```
integrate(1/((d^3*x^3 + 2*c^3)^(1/3)*(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)\sqrt[3]{2c^3+d^3x^3}}\,dx = \int \frac{1}{\left(2\,c^3+d^3\,x^3\right)^{1/3}\,\left(c+d\,x\right)}\,dx$$

input
```
int(1/((2*c^3 + d^3*x^3)^(1/3)*(c + d*x)),x)
```

output
```
int(1/((2*c^3 + d^3*x^3)^(1/3)*(c + d*x)), x)
```

Reduce [F]

$$\int \frac{1}{(c+dx)\sqrt[3]{2c^3+d^3x^3}}\,dx = \int \frac{1}{(d^3x^3+2c^3)^{\frac{1}{3}}\,c+(d^3x^3+2c^3)^{\frac{1}{3}}\,dx}dx$$

input
```
int(1/(d*x+c)/(d^3*x^3+2*c^3)^(1/3),x)
```

output
```
int(1/((2*c**3 + d**3*x**3)**(1/3)*c + (2*c**3 + d**3*x**3)**(1/3)*d*x),x)
```

3.140 $\int \frac{1}{(c+dx)\left(2c^3+d^3x^3\right)^{2/3}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 187

$$\int \frac{1}{(c+dx)\left(2c^3+d^3x^3\right)^{2/3}}\,dx = -\frac{\arctan\left(\frac{1+\frac{2dx}{\sqrt[3]{2c^3+d^3x^3}}}{\sqrt{3}}\right)}{2\sqrt{3}c^2d} + \frac{\sqrt{3}\arctan\left(\frac{1+\frac{2(2c+dx)}{\sqrt[3]{2c^3+d^3x^3}}}{\sqrt{3}}\right)}{2c^2d} - \frac{\log(c+dx)}{2c^2d} - \frac{\log\left(dx-\sqrt[3]{2c^3+d^3x^3}\right)}{4c^2d} + \frac{3\log\left(d(2c+dx)-d\sqrt[3]{2c^3+d^3x^3}\right)}{4c^2d}$$

output
```
-1/6*arctan(1/3*(1+2*d*x/(d^3*x^3+2*c^3)^(1/3))*3^(1/2))*3^(1/2)/c^2/d+1/2
*3^(1/2)*arctan(1/3*(1+2*(d*x+2*c)/(d^3*x^3+2*c^3)^(1/3))*3^(1/2))/c^2/d-1
/2*ln(d*x+c)/c^2/d-1/4*ln(d*x-(d^3*x^3+2*c^3)^(1/3))/c^2/d+3/4*ln(d*(d*x+2
*c)-d*(d^3*x^3+2*c^3)^(1/3))/c^2/d
```

Mathematica [F]

$$\int \frac{1}{(c+dx)\left(2c^3+d^3x^3\right)^{2/3}}\,dx = \int \frac{1}{(c+dx)\left(2c^3+d^3x^3\right)^{2/3}}\,dx$$

input
```
Integrate[1/((c + d*x)*(2*c^3 + d^3*x^3)^(2/3)),x]
```

output
```
Integrate[1/((c + d*x)*(2*c^3 + d^3*x^3)^(2/3)), x]
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {2579}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c+dx)\left(2c^3+d^3x^3\right)^{2/3}}\,dx$$

$$\Big\downarrow \text{2579}$$

$$-\frac{\arctan\left(\frac{\frac{2dx}{\sqrt[3]{2c^3+d^3x^3}}+1}{\sqrt{3}}\right)}{2\sqrt{3}c^2d}+\frac{\sqrt{3}\arctan\left(\frac{\frac{2(2c+dx)}{\sqrt[3]{2c^3+d^3x^3}}+1}{\sqrt{3}}\right)}{2c^2d}-\frac{\log(c+dx)}{2c^2d}-\frac{\log\left(dx-\sqrt[3]{2c^3+d^3x^3}\right)}{4c^2d}+\frac{3\log\left(d(2c+dx)-d\sqrt[3]{2c^3+d^3x^3}\right)}{4c^2d}$$

input
```
Int[1/((c + d*x)*(2*c^3 + d^3*x^3)^(2/3)),x]
```

output
```
-1/2*ArcTan[(1 + (2*d*x)/(2*c^3 + d^3*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*c^2*d)
 + (Sqrt[3]*ArcTan[(1 + (2*(2*c + d*x))/(2*c^3 + d^3*x^3)^(1/3))/Sqrt[3]])
/(2*c^2*d) - Log[c + d*x]/(2*c^2*d) - Log[d*x - (2*c^3 + d^3*x^3)^(1/3)]/(
4*c^2*d) + (3*Log[d*(2*c + d*x) - d*(2*c^3 + d^3*x^3)^(1/3)])/(4*c^2*d)
```

Defintions of rubi rules used

rule 2579
```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(2/3)), x_Symbol] :> With[
{q = Rt[b, 3]}, Simp[(-d)*(ArcTan[(1 + 2*q*(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/
(2*Sqrt[3]*q^2*c^2)), x] + (Simp[Sqrt[3]*d*(ArcTan[(1 + 2*q*((2*c + d*x)/(d
*(a + b*x^3)^(1/3))))/Sqrt[3]]/(2*q^2*c^2)), x] - Simp[d*(Log[c + d*x]/(2*q
^2*c^2)), x] - Simp[d*(Log[q*x - (a + b*x^3)^(1/3)]/(4*q^2*c^2)), x] + Simp
[3*d*(Log[q*(2*c + d*x) - d*(a + b*x^3)^(1/3)]/(4*q^2*c^2)), x])] /; FreeQ[
{a, b, c, d}, x] && EqQ[2*b*c^3 - a*d^3, 0]
```

Maple [F]

$$\int \frac{1}{(dx+c)\,(d^3x^3+2c^3)^{\frac{2}{3}}}dx$$

input
```
int(1/(d*x+c)/(d^3*x^3+2*c^3)^(2/3),x)
```

output
```
int(1/(d*x+c)/(d^3*x^3+2*c^3)^(2/3),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)\,(2c^3+d^3x^3)^{2/3}}\,dx = \text{Timed out}$$

input
```
integrate(1/(d*x+c)/(d^3*x^3+2*c^3)^(2/3),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(c+dx)\left(2c^3+d^3x^3\right)^{2/3}}\,dx = \int \frac{1}{(c+dx)\left(2c^3+d^3x^3\right)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(d*x+c)/(d**3*x**3+2*c**3)**(2/3),x)
```

output
```
Integral(1/((c + d*x)*(2*c**3 + d**3*x**3)**(2/3)), x)
```

Maxima [F]

$$\int \frac{1}{(c+dx)\left(2c^3+d^3x^3\right)^{2/3}}\,dx = \int \frac{1}{\left(d^3x^3+2\,c^3\right)^{\frac{2}{3}}(dx+c)}\,dx$$

input
```
integrate(1/(d*x+c)/(d^3*x^3+2*c^3)^(2/3),x, algorithm="maxima")
```

output
```
integrate(1/((d^3*x^3 + 2*c^3)^(2/3)*(d*x + c)), x)
```

Giac [F]

$$\int \frac{1}{(c+dx)\left(2c^3+d^3x^3\right)^{2/3}}\,dx = \int \frac{1}{\left(d^3x^3+2\,c^3\right)^{\frac{2}{3}}(dx+c)}\,dx$$

input
```
integrate(1/(d*x+c)/(d^3*x^3+2*c^3)^(2/3),x, algorithm="giac")
```

output
```
integrate(1/((d^3*x^3 + 2*c^3)^(2/3)*(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)\left(2c^3+d^3x^3\right)^{2/3}}\,dx=\int \frac{1}{{\left(2\,c^3+d^3\,x^3\right)}^{2/3}\,\left(c+d\,x\right)} \,d x$$

input
```
int(1/((2*c^3 + d^3*x^3)^(2/3)*(c + d*x)),x)
```

output
```
int(1/((2*c^3 + d^3*x^3)^(2/3)*(c + d*x)), x)
```

Reduce [F]

$$\int \frac{1}{(c+dx)\left(2c^3+d^3x^3\right)^{2/3}}\,dx=\int \frac{1}{(d^{3} x^{3}+2 c^{3})^{\frac{2}{3}} c +(d^{3} x^{3}+2 c^{3})^{\frac{2}{3}} d x}d x$$

input
```
int(1/(d*x+c)/(d^3*x^3+2*c^3)^(2/3),x)
```

output
```
int(1/((2*c**3 + d**3*x**3)**(2/3)*c + (2*c**3 + d**3*x**3)**(2/3)*d*x),x)
```

3.141 $\int \frac{1}{\left(1+\sqrt[3]{2}x\right)\left(1+x^3\right)^{2/3}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 147

$$\int \frac{1}{\left(1+\sqrt[3]{2}x\right)\left(1+x^3\right)^{2/3}}\,dx = -\frac{\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{1+x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}} + \frac{\sqrt{3}\arctan\left(\frac{1+\frac{2\left(2^{2/3}+x\right)}{\sqrt[3]{1+x^3}}}{\sqrt{3}}\right)}{2^{2/3}} - \frac{\log\left(1+\sqrt[3]{2}x\right)}{2^{2/3}} - \frac{\log\left(x-\sqrt[3]{1+x^3}\right)}{2\,2^{2/3}} + \frac{3\log\left(2+\sqrt[3]{2}x-\sqrt[3]{2}\sqrt[3]{1+x^3}\right)}{2\,2^{2/3}}$$

output
```
-1/6*arctan(1/3*(1+2*x/(x^3+1)^(1/3))*3^(1/2))*2^(1/3)*3^(1/2)+1/2*3^(1/2)
*arctan(1/3*(1+2*(2^(2/3)+x)/(x^3+1)^(1/3))*3^(1/2))*2^(1/3)-1/2*ln(1+2^(1
/3)*x)*2^(1/3)-1/4*ln(x-(x^3+1)^(1/3))*2^(1/3)+3/4*ln(2+2^(1/3)*x-2^(1/3)*
(x^3+1)^(1/3))*2^(1/3)
```

Mathematica [F]

$$\int \frac{1}{\left(1+\sqrt[3]{2}x\right)(1+x^3)^{2/3}}\,dx = \int \frac{1}{\left(1+\sqrt[3]{2}x\right)(1+x^3)^{2/3}}\,dx$$

input
```
Integrate[1/((1 + 2^(1/3)*x)*(1 + x^3)^(2/3)),x]
```

output
```
Integrate[1/((1 + 2^(1/3)*x)*(1 + x^3)^(2/3)), x]
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {2579}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(\sqrt[3]{2}x+1\right)(x^3+1)^{2/3}}\,dx$$

$$\Big\downarrow \text{ 2579}$$

$$-\frac{\arctan\left(\frac{\frac{2x}{\sqrt[3]{x^3+1}}+1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}+\frac{\sqrt{3}\arctan\left(\frac{\frac{2\left(x+2^{2/3}\right)}{\sqrt[3]{x^3+1}}+1}{\sqrt{3}}\right)}{2^{2/3}}-\frac{\log\left(x-\sqrt[3]{x^3+1}\right)}{2\ 2^{2/3}}+\frac{3\log\left(-\sqrt[3]{2}\sqrt[3]{x^3+1}+\sqrt[3]{2}x+2\right)}{2\ 2^{2/3}}-\frac{\log\left(\sqrt[3]{2}x+1\right)}{2^{2/3}}$$

input
```
Int[1/((1 + 2^(1/3)*x)*(1 + x^3)^(2/3)),x]
```

output
```
-(ArcTan[(1 + (2*x)/(1 + x^3)^(1/3))/Sqrt[3]]/(2^(2/3)*Sqrt[3])) + (Sqrt[3
]*ArcTan[(1 + (2*(2^(2/3) + x))/(1 + x^3)^(1/3))/Sqrt[3]])/2^(2/3) - Log[1
 + 2^(1/3)*x]/2^(2/3) - Log[x - (1 + x^3)^(1/3)]/(2*2^(2/3)) + (3*Log[2 +
2^(1/3)*x - 2^(1/3)*(1 + x^3)^(1/3)])/(2*2^(2/3))
```

Defintions of rubi rules used

rule 2579
```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(2/3)), x_Symbol] :> With[
{q = Rt[b, 3]}, Simp[(-d)*(ArcTan[(1 + 2*q*(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/
(2*Sqrt[3]*q^2*c^2)), x] + (Simp[Sqrt[3]*d*(ArcTan[(1 + 2*q*((2*c + d*x)/(d
*(a + b*x^3)^(1/3))))/Sqrt[3]]/(2*q^2*c^2)), x] - Simp[d*(Log[c + d*x]/(2*q
^2*c^2)), x] - Simp[d*(Log[q*x - (a + b*x^3)^(1/3)]/(4*q^2*c^2)), x] + Simp
[3*d*(Log[q*(2*c + d*x) - d*(a + b*x^3)^(1/3)]/(4*q^2*c^2)), x])] /; FreeQ[
{a, b, c, d}, x] && EqQ[2*b*c^3 - a*d^3, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 33.92 (sec) , antiderivative size = 3063, normalized size of antiderivative = 20.84

method	result	size
trager	Expression too large to display	3063

input
```
int(1/(1+2^(1/3)*x)/(x^3+1)^(2/3),x,method=_RETURNVERBOSE)
```

output
```
-1/6*ln(-(-15559137585059152+5665414413224496*x^5*2^(1/3)*RootOf(2^(2/3)+2
^(1/3)*_Z+_Z^2)+3532767618003008*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*2^(1/3)
*x^3+1876782797064098*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*2^(1/3)*x^6+321734
1937824168*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*2^(2/3)*x^4+2081809489180344*
RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*2^(2/3)*x+2613886855325640*RootOf(2^(2/3
)+2^(1/3)*_Z+_Z^2)^2*(x^3+1)^(2/3)*x^3+2884227944870616*RootOf(2^(2/3)+2^(
1/3)*_Z+_Z^2)*(x^3+1)^(2/3)*x^2-8123294120973864*RootOf(2^(2/3)+2^(1/3)*_Z
+_Z^2)*(x^3+1)^(1/3)*x^3-11138422684341672*2^(1/3)*(x^3+1)^(2/3)*x^2+39190
74648194292*2^(2/3)*(x^3+1)^(2/3)*x^3-1315592369000448*(x^3+1)^(1/3)*2^(2/
3)*x^4-6964190009986188*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*(x^3+1)^(1/3)*x^
4+3288980922501120*(x^3+1)^(1/3)*2^(1/3)*x^3-7115580883942020*RootOf(2^(2/
3)+2^(1/3)*_Z+_Z^2)*2^(1/3)*(x^3+1)^(2/3)-20062783627256832*(x^3+1)^(1/3)*
2^(2/3)*x+2161300347926748*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*(x^3+1)^(1/3)
*x-4279877387294208*x^5*2^(2/3)+14712078518823840*x^3-936223178470608*x^6-
1604954020235328*2^(2/3)*x^2-23004340956706368*2^(1/3)*x-1249812750550425б
*2^(1/3)*(x^3+1)^(1/3)+2321435374812274*2^(2/3)*RootOf(2^(2/3)+2^(1/3)*_Z+
_Z^2)*x^6+10197714008127436*2^(2/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*x^3+79
59206999356368*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(1/3)*x^2-912549035791293
6*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*(x^3+1)^(1/3)+3712807561447224*(x^3+1)^(
2/3)*x^4-2960082830251008*(x^3+1)^(1/3)*x^5-32590199706036744*(x^3+1)^(...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 712 vs. $2(112) = 224$.

Time = 3.42 (sec) , antiderivative size = 712, normalized size of antiderivative = 4.84

$$\int \frac{1}{\left(1+\sqrt[3]{2}x\right)\left(1+x^3\right)^{2/3}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(1+2^(1/3)*x)/(x^3+1)^(2/3),x, algorithm="fricas")
```

output
```
1/6*sqrt(3)*2^(1/3)*arctan(-1/3*(13910019318573948542*sqrt(3)*(44297109310
930172741433829405399636654451725916403400759596345420183*x^16 + 469911753
8775772972666874933612662742982197517261565117487967882l0304*x^13 - 168603
21903643326044064702132534629564524232524637546054758296040942 4*x^10 - 197
88063011823765739382929542277926273733302833978765826115583328934 40*x^7 -
144009089168717758142291876308930196860258103687221308438991237030l872*x^4
 - 2^(2/3)*(522710774531251076129959239776547583493948769228855528192 09999
866413*x^15 + 590674547854548577293285820788340778493299281255213360593997
994805172*x^12 + 306314261222931431619887382966630423064822217690279625339
1978577817900*x^9 + 7331049558697577809008352571597039403457968857066730 27
7786114959327080*x^6 + 772324480675629044375977054678087297173944475017351
9635544186114816064*x^3 + 291168089878390092195634857418355141558919044601
5106452608070501424800) + 6*2^(1/3)*(1260135599621632209331474867914912054
3302140685677058235520929344665*x^14 - 555869063001966513924627194919212 67
8478207988900198502271159380897 18*x^11 - 450398920105320599307639536027883
986131793624729303407436233610788504*x^8 - 721888705880948261432517052 6703
941062383389438443735539065108798665 84*x^5 - 338668158068684373436309273 06
784946440569136075137850744247292177454 4*x^2) - 623676430454539792290217 01
2355944404253806601409762924332407805196 80*x)*(x^3 + 1)^(2/3) - 1391001931
8573948542*sqrt(3)*(2024415138676272858287317644091664227603691384672 19...
```

Sympy [F]

$$\int \frac{1}{\left(1+\sqrt[3]{2}x\right)\left(1+x^3\right)^{2/3}}\,dx=\int \frac{1}{\left((x+1)\left(x^2-x+1\right)\right)^{\frac{2}{3}}\cdot\left(\sqrt[3]{2}x+1\right)}\,dx$$

input
```
integrate(1/(1+2**(1/3)*x)/(x**3+1)**(2/3),x)
```

output
```
Integral(1/(((x + 1)*(x**2 - x + 1))**(2/3)*(2**(1/3)*x + 1)), x)
```

Maxima [F]

$$\int \frac{1}{\left(1+\sqrt[3]{2}x\right)(1+x^3)^{2/3}}\,dx=\int \frac{1}{(x^3+1)^{\frac{2}{3}}\left(2^{\frac{1}{3}}x+1\right)}\,dx$$

input
```
integrate(1/(1+2^(1/3)*x)/(x^3+1)^(2/3),x, algorithm="maxima")
```

output
```
integrate(1/((x^3 + 1)^(2/3)*(2^(1/3)*x + 1)), x)
```

Giac [F]

$$\int \frac{1}{\left(1+\sqrt[3]{2}x\right)(1+x^3)^{2/3}}\,dx=\int \frac{1}{(x^3+1)^{\frac{2}{3}}\left(2^{\frac{1}{3}}x+1\right)}\,dx$$

input
```
integrate(1/(1+2^(1/3)*x)/(x^3+1)^(2/3),x, algorithm="giac")
```

output
```
integrate(1/((x^3 + 1)^(2/3)*(2^(1/3)*x + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(1+\sqrt[3]{2}x\right)(1+x^3)^{2/3}}\,dx=\int \frac{1}{(x^3+1)^{2/3}\,(2^{1/3}\,x+1)}\,dx$$

input
```
int(1/((x^3 + 1)^(2/3)*(2^(1/3)*x + 1)),x)
```

output
```
int(1/((x^3 + 1)^(2/3)*(2^(1/3)*x + 1)), x)
```

Reduce [F]

$$\int \frac{1}{\left(1+\sqrt[3]{2}x\right)\left(1+x^3\right)^{2/3}}\,dx = \int \frac{1}{\left(x^3+1\right)^{\frac{2}{3}} 2^{\frac{1}{3}}x + \left(x^3+1\right)^{\frac{2}{3}}}dx$$

input
```
int(1/(1+2^(1/3)*x)/(x^3+1)^(2/3),x)
```

output
```
int(1/((x**3 + 1)**(2/3)*2**(1/3)*x + (x**3 + 1)**(2/3)),x)
```

3.142 $\int \frac{1}{\left(1-\sqrt[3]{2}x\right)\left(1-x^3\right)^{2/3}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 159

$$\int \frac{1}{\left(1-\sqrt[3]{2}x\right)\left(1-x^3\right)^{2/3}}\,dx = -\frac{\sqrt{3}\arctan\left(\frac{1+\frac{2\,2^{2/3}-2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}} + \frac{\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}} + \frac{\log\left(1-\sqrt[3]{2}x\right)}{2^{2/3}} + \frac{\log\left(-x-\sqrt[3]{1-x^3}\right)}{2\,2^{2/3}} - \frac{3\log\left(-2+\sqrt[3]{2}x+\sqrt[3]{2}\sqrt[3]{1-x^3}\right)}{2\,2^{2/3}}$$

output
```
-1/2*3^(1/2)*arctan(1/3*(1+(2*2^(2/3)-2*x)/(-x^3+1)^(1/3))*3^(1/2))*2^(1/3
)+1/6*arctan(1/3*(1-2*x/(-x^3+1)^(1/3))*3^(1/2))*2^(1/3)*3^(1/2)+1/2*ln(1-
2^(1/3)*x)*2^(1/3)+1/4*ln(-x-(-x^3+1)^(1/3))*2^(1/3)-3/4*ln(-2+2^(1/3)*x+2
^(1/3)*(-x^3+1)^(1/3))*2^(1/3)
```

Mathematica [F]

$$\int \frac{1}{\left(1-\sqrt[3]{2}x\right)(1-x^3)^{2/3}}\,dx = \int \frac{1}{\left(1-\sqrt[3]{2}x\right)(1-x^3)^{2/3}}\,dx$$

input
```
Integrate[1/((1 - 2^(1/3)*x)*(1 - x^3)^(2/3)),x]
```

output
```
Integrate[1/((1 - 2^(1/3)*x)*(1 - x^3)^(2/3)), x]
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {2579}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(1-\sqrt[3]{2}x\right)(1-x^3)^{2/3}}\,dx$$

$$\Big\downarrow \text{2579}$$

$$-\frac{\sqrt{3}\arctan\left(\frac{\frac{2\,2^{2/3}-2x}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)}{2^{2/3}}+\frac{\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}+\frac{\log\left(-\sqrt[3]{1-x^3}-x\right)}{2\,2^{2/3}}-\frac{3\log\left(\sqrt[3]{2}\sqrt[3]{1-x^3}+\sqrt[3]{2}x-2\right)}{2\,2^{2/3}}+\frac{\log\left(1-\sqrt[3]{2}x\right)}{2^{2/3}}$$

input
```
Int[1/((1 - 2^(1/3)*x)*(1 - x^3)^(2/3)),x]
```

output
```
-((Sqrt[3]*ArcTan[(1 + (2*2^(2/3) - 2*x)/(1 - x^3)^(1/3))/Sqrt[3]])/2^(2/3
)) + ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(2/3)*Sqrt[3]) + Log[1
 - 2^(1/3)*x]/2^(2/3) + Log[-x - (1 - x^3)^(1/3)]/(2*2^(2/3)) - (3*Log[-2
+ 2^(1/3)*x + 2^(1/3)*(1 - x^3)^(1/3)])/(2*2^(2/3))
```

Defintions of rubi rules used

rule 2579
```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(2/3)), x_Symbol] :> With[
{q = Rt[b, 3]}, Simp[(-d)*(ArcTan[(1 + 2*q*(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/
(2*Sqrt[3]*q^2*c^2)), x] + (Simp[Sqrt[3]*d*(ArcTan[(1 + 2*q*((2*c + d*x)/(d
*(a + b*x^3)^(1/3))))/Sqrt[3]]/(2*q^2*c^2)), x] - Simp[d*(Log[c + d*x]/(2*q
^2*c^2)), x] - Simp[d*(Log[q*x - (a + b*x^3)^(1/3)]/(4*q^2*c^2)), x] + Simp
[3*d*(Log[q*(2*c + d*x) - d*(a + b*x^3)^(1/3)]/(4*q^2*c^2)), x])] /; FreeQ[
{a, b, c, d}, x] && EqQ[2*b*c^3 - a*d^3, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 32.60 (sec) , antiderivative size = 3248, normalized size of antiderivative = 20.43

method	result	size
trager	Expression too large to display	3248

input
```
int(1/(1-2^(1/3)*x)/(-x^3+1)^(2/3),x,method=_RETURNVERBOSE)
```

output
```
1/6*ln((-3712807561447224*(-x^3+1)^(2/3)*x^4+5665414413224496*x^5*2^(1/3)*
RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)+3532767618003008*RootOf(2^(2/3)+2^(1/3)*_Z
+_Z^2)^2*2^(1/3)*x^3-1876782797064098*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*2^
(1/3)*x^6-3217341937824168*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*2^(2/3)*x^4+2
081809489180344*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*2^(2/3)*x-42798773872942
08*x^5*2^(2/3)+14712078518823840*x^3+936223178470608*x^6+1604954020235328*
2^(2/3)*x^2-23004340956706368*2^(1/3)*x+12498127505504256*2^(1/3)*(-x^3+1)
^(1/3)-2321435374812274*2^(2/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*x^6+101977
14008127436*2^(2/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*x^3-7959206999356368*R
ootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(1/3)*x^2-32590199706036744*(-x^3+1)^(2/3
)*x+12827025597754368*x^2*(-x^3+1)^(1/3)-3775614346581480*RootOf(2^(2/3)+2
^(1/3)*_Z+_Z^2)^2*2^(2/3)*(-x^3+1)^(2/3)*x^2-3842311729647552*RootOf(2^(2/
3)+2^(1/3)*_Z+_Z^2)^2*2^(2/3)*(-x^3+1)^(1/3)*x^3+10498622607665136*RootOf(
2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(1/3)*(-x^3+1)^(1/3)*x^4-7759251414704196*RootO
f(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(2/3)*(-x^3+1)^(2/3)*x+2613886855325640*RootO
f(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*2^(1/3)*(-x^3+1)^(2/3)*x+3531674097632562*Roo
tOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(2/3)*(-x^3+1)^(1/3)*x^2-840505690860402*Ro
otOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*2^(1/3)*(-x^3+1)^(1/3)*x^2-116887306390302
84*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(1/3)*(-x^3+1)^(1/3)*x+16049540202353
28*2^(1/3)*x^4-2960082830251008*(-x^3+1)^(1/3)*x^5+10107087250606332*(-...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 720 vs. $2(124) = 248$.

Time = 3.10 (sec) , antiderivative size = 720, normalized size of antiderivative = 4.53

$$\int \frac{1}{\left(1-\sqrt[3]{2}x\right)\left(1-x^3\right)^{2/3}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(1-2^(1/3)*x)/(-x^3+1)^(2/3),x, algorithm="fricas")
```

output

```
1/6*sqrt(3)*2^(1/3)*arctan(1/3*(13910019318573948542*sqrt(3)*(442971093109
3017274143382940539963665445172591640340075959634542018 3*x^16 - 4699117538
7757729726668749336126627429821975172615651174879678821030 4*x^13 - 1686032
190364332604406470213253462956452423252463754605475829604094 24*x^10 + 1978
806301182376573938292954227792627373330283397876582611558332893440*x^7 - 1
44009089168717758142291876308930196860258103687221308438991237030187 2*x^4
+ 2^(2/3)*(52271077453125107612995923977654758349394876922885552819209999 8
66413*x^15 - 5906745478545485772932858207883407784932992812552133605939979
94805172*x^12 + 306314261222931431619887382966630423064822217690279625339 1
978577817900*x^9 - 7331049558697577809008352571597039403457968857066730277
786114959327080*x^6 + 7723244806756290443759770546780872971739444750173519
635544186114816064*x^3 - 2911680898783900921956348574183551415589190446015
106452608070501424800) + 6*2^(1/3)*(12601355996216322093314748679149120543
302140685677058235520929344665*x^14 + 5558690630019665139246271949192126 78
4782079889001985022711593808971 8*x^11 - 4503989201053205993076395360278839
8613179362472930340743623361078850 4*x^8 + 7218887058809482614325170526703 9
410623833894384437355390651087986658 4*x^5 - 3386681580686843734363092730 67
84946440569136075137850744247292177454 4*x^2) + 623676430454539792290217012
3559444042538066014097629243324078051968 0*x)*(-x^3 + 1)^(2/3) + 1391001931
8573948542*sqrt(3)*(2024415138676272858287317644091664227603691384672 19...
```

Sympy [F]

$$\int \frac{1}{\left(1-\sqrt[3]{2}x\right)\left(1-x^3\right)^{2/3}}\,dx=-\int \frac{1}{\sqrt[3]{2}x\left(1-x^3\right)^{\frac{2}{3}}-\left(1-x^3\right)^{\frac{2}{3}}}\,dx$$

input

```
integrate(1/(1-2**(1/3)*x)/(-x**3+1)**(2/3),x)
```

output

```
-Integral(1/(2**(1/3)*x*(1 - x**3)**(2/3) - (1 - x**3)**(2/3)), x)
```

Maxima [F]

$$\int \frac{1}{\left(1-\sqrt[3]{2}x\right)(1-x^3)^{2/3}}\,dx = \int -\frac{1}{(-x^3+1)^{\frac{2}{3}}\left(2^{\frac{1}{3}}x-1\right)}\,dx$$

input
```
integrate(1/(1-2^(1/3)*x)/(-x^3+1)^(2/3),x, algorithm="maxima")
```

output
```
-integrate(1/((-x^3 + 1)^(2/3)*(2^(1/3)*x - 1)), x)
```

Giac [F]

$$\int \frac{1}{\left(1-\sqrt[3]{2}x\right)(1-x^3)^{2/3}}\,dx = \int -\frac{1}{(-x^3+1)^{\frac{2}{3}}\left(2^{\frac{1}{3}}x-1\right)}\,dx$$

input
```
integrate(1/(1-2^(1/3)*x)/(-x^3+1)^(2/3),x, algorithm="giac")
```

output
```
integrate(-1/((-x^3 + 1)^(2/3)*(2^(1/3)*x - 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(1-\sqrt[3]{2}x\right)(1-x^3)^{2/3}}\,dx = -\int \frac{1}{(1-x^3)^{2/3}\,(2^{1/3}\,x-1)}\,dx$$

input
```
int(-1/((1 - x^3)^(2/3)*(2^(1/3)*x - 1)),x)
```

output
```
-int(1/((1 - x^3)^(2/3)*(2^(1/3)*x - 1)), x)
```

Reduce [F]

$$\int \frac{1}{\left(1-\sqrt[3]{2}x\right)(1-x^3)^{2/3}}\,dx = -\left(\int \frac{1}{(-x^3+1)^{\frac{2}{3}}\,2^{\frac{1}{3}}x-(-x^3+1)^{\frac{2}{3}}}dx\right)$$

input
```
int(1/(1-2^(1/3)*x)/(-x^3+1)^(2/3),x)
```

output
```
- int(1/(( - x**3 + 1)**(2/3)*2**(1/3)*x - ( - x**3 + 1)**(2/3)),x)
```

3.143 $\int (c+dx)^4 \sqrt[3]{a+bx^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 304

$$\begin{aligned}
&\int (c+dx)^4 \sqrt[3]{a+bx^3}\,dx \\
&= \frac{3ac^2d^2\sqrt[3]{a+bx^3}}{2b}+\frac{2acd^3x\sqrt[3]{a+bx^3}}{5b}+\frac{ad^4x^2\sqrt[3]{a+bx^3}}{18b} \\
&\quad+\frac{1}{30}\sqrt[3]{a+bx^3}\left(15c^4x+40c^3dx^2+45c^2d^2x^3+24cd^3x^4+5d^4x^5\right) \\
&\quad-\frac{ad(12bc^3-ad^3)\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{9\sqrt{3}b^{5/3}} \\
&\quad+\frac{ac(5bc^3-4ad^3)x\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{10b(a+bx^3)^{2/3}} \\
&\quad-\frac{ad(12bc^3-ad^3)\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{18b^{5/3}}
\end{aligned}$$

output
```
3/2*a*c^2*d^2*(b*x^3+a)^(1/3)/b+2/5*a*c*d^3*x*(b*x^3+a)^(1/3)/b+1/18*a*d^4
*x^2*(b*x^3+a)^(1/3)/b+1/30*(b*x^3+a)^(1/3)*(5*d^4*x^5+24*c*d^3*x^4+45*c^2
*d^2*x^3+40*c^3*d*x^2+15*c^4*x)-1/27*a*d*(-a*d^3+12*b*c^3)*arctan(1/3*(1+2
*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))*3^(1/2)/b^(5/3)+1/10*a*c*(-4*a*d^3+5*
b*c^3)*x*(1+b*x^3/a)^(2/3)*hypergeom([1/3, 2/3],[4/3],-b*x^3/a)/b/(b*x^3+a
)^(2/3)-1/18*a*d*(-a*d^3+12*b*c^3)*ln(b^(1/3)*x-(b*x^3+a)^(1/3))/b^(5/3)
```

Mathematica [A] (verified)

Time = 9.13 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.54

$$\int (c+dx)^4 \sqrt[3]{a+bx^3}\,dx$$
$$= \frac{\sqrt[3]{a+bx^3}\left(6bc^4x \text{ Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{3},\frac{4}{3},-\frac{bx^3}{a}\right)+d(12bc^3-ad^3)\,x^2 \text{ Hypergeometric2F1}\left(-\frac{1}{3},\frac{2}{3}, \right.\right.}{6b\sqrt[3]{1+}}$$

input
```
Integrate[(c + d*x)^4*(a + b*x^3)^(1/3),x]
```

output
```
((a + b*x^3)^(1/3)*(6*b*c^4*x*Hypergeometric2F1[-1/3, 1/3, 4/3, -((b*x^3)/
a)] + d*(12*b*c^3 - a*d^3)*x^2*Hypergeometric2F1[-1/3, 2/3, 5/3, -((b*x^3)
/a)] + d^2*((9*c^2 + d^2*x^2)*(a + b*x^3)*(1 + (b*x^3)/a)^(1/3) + 6*b*c*d*
x^4*Hypergeometric2F1[-1/3, 4/3, 7/3, -((b*x^3)/a)])))/(6*b*(1 + (b*x^3)/a
)^(1/3))
```

Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 373, normalized size of antiderivative = 1.23, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {2392, 27, 2432, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[3]{a+bx^3}(c+dx)^4\,dx$$

$$\Big\downarrow \; 2392$$

$$a\int \frac{15c^4+40dxc^3+45d^2x^2c^2+24d^3x^3c+5d^4x^4}{30\left(bx^3+a\right)^{2/3}}dx+\frac{1}{30}\sqrt[3]{a+bx^3}\left(15c^4x+40c^3dx^2+45c^2d^2x^3+24cd^3x^4+5d^4x^5\right)$$

$$\Big\downarrow \; 27$$

$$\frac{1}{30}a\int \frac{15c^4+40dxc^3+45d^2x^2c^2+24d^3x^3c+5d^4x^4}{\left(bx^3+a\right)^{2/3}}dx+\frac{1}{30}\sqrt[3]{a+bx^3}\left(15c^4x+40c^3dx^2+45c^2d^2x^3+24cd^3x^4+5d^4x^5\right)$$

$$\Big\downarrow \; 2432$$

$$\frac{1}{30}a\int \left(\frac{15c^4}{\left(bx^3+a\right)^{2/3}}+\frac{40dxc^3}{\left(bx^3+a\right)^{2/3}}+\frac{45d^2x^2c^2}{\left(bx^3+a\right)^{2/3}}+\frac{24d^3x^3c}{\left(bx^3+a\right)^{2/3}}+\frac{5d^4x^4}{\left(bx^3+a\right)^{2/3}}\right)dx+\frac{1}{30}\sqrt[3]{a+bx^3}\left(15c^4x+40c^3dx^2+45c^2d^2x^3+24cd^3x^4+5d^4x^5\right)$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{30}a\left(-\frac{40c^3d\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}}+\frac{10ad^4\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}b^{5/3}}-\frac{20c^3d\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{b^{2/3}}+\frac{5a\right.$$

$$\frac{1}{30}\sqrt[3]{a+bx^3}\left(15c^4x+40c^3dx^2+45c^2d^2x^3+24cd^3x^4+5d^4x^5\right)$$

input

```
Int[(c + d*x)^4*(a + b*x^3)^(1/3),x]
```

output
```
((a + b*x^3)^(1/3)*(15*c^4*x + 40*c^3*d*x^2 + 45*c^2*d^2*x^3 + 24*c*d^3*x^
4 + 5*d^4*x^5))/30 + (a*((45*c^2*d^2*(a + b*x^3)^(1/3))/b + (5*d^4*x^2*(a
+ b*x^3)^(1/3))/(3*b) - (40*c^3*d*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1
/3))/Sqrt[3]])/(Sqrt[3]*b^(2/3)) + (10*a*d^4*ArcTan[(1 + (2*b^(1/3)*x)/(a
+ b*x^3)^(1/3))/Sqrt[3]])/(3*Sqrt[3]*b^(5/3)) + (15*c^4*x*(1 + (b*x^3)/a)^
(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)])/(a + b*x^3)^(2/3) +
(6*c*d^3*x^4*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[2/3, 4/3, 7/3, -((b*x
^3)/a)])/(a + b*x^3)^(2/3) - (20*c^3*d*Log[b^(1/3)*x - (a + b*x^3)^(1/3)])
/b^(2/3) + (5*a*d^4*Log[b^(1/3)*x - (a + b*x^3)^(1/3)])/(3*b^(5/3))))/30
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2392
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{q = Expon[Pq
, x], i}, Simp[(a + b*x^n)^p*Sum[Coeff[Pq, x, i]*(x^(i + 1)/(n*p + i + 1)),
 {i, 0, q}], x] + Simp[a*n*p   Int[(a + b*x^n)^(p - 1)*Sum[Coeff[Pq, x, i]*
(x^i/(n*p + i + 1)), {i, 0, q}], x], x]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x
] && IGtQ[(n - 1)/2, 0] && GtQ[p, 0]
```

rule 2432
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

Maple [F]

$$\int (dx+c)^4 \left(b\,x^3+a\right)^{\frac{1}{3}}\,dx$$

input
```
int((d*x+c)^4*(b*x^3+a)^(1/3),x)
```

output
```
int((d*x+c)^4*(b*x^3+a)^(1/3),x)
```

Fricas [F(-1)]

Timed out.

$$\int (c+dx)^4 \sqrt[3]{a+bx^3}\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)^4*(b*x^3+a)^(1/3),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [A] (verification not implemented)

Time = 3.37 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.70

$$\int (c+dx)^4\sqrt[3]{a+bx^3}\,dx = \frac{\sqrt[3]{a}c^4x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{4\sqrt[3]{a}c^3dx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)} + \frac{4\sqrt[3]{a}cd^3x^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{\sqrt[3]{a}d^4x^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{8}{3}\right)} + 6c^2d^2\left(\begin{cases}\frac{\sqrt[3]{a}x^3}{3} & \text{for } b=0\\ \frac{(a+bx^3)^{\frac{4}{3}}}{4b} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((d*x+c)**4*(b*x**3+a)**(1/3),x)
```

output
```
a**(1/3)*c**4*x*gamma(1/3)*hyper((-1/3, 1/3), (4/3,), b*x**3*exp_polar(I*p
i)/a)/(3*gamma(4/3)) + 4*a**(1/3)*c**3*d*x**2*gamma(2/3)*hyper((-1/3, 2/3)
, (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(5/3)) + 4*a**(1/3)*c*d**3*x**
4*gamma(4/3)*hyper((-1/3, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma
(7/3)) + a**(1/3)*d**4*x**5*gamma(5/3)*hyper((-1/3, 5/3), (8/3,), b*x**3*e
xp_polar(I*pi)/a)/(3*gamma(8/3)) + 6*c**2*d**2*Piecewise((a**(1/3)*x**3/3,
 Eq(b, 0)), ((a + b*x**3)**(4/3)/(4*b), True))
```

Maxima [F]

$$\int (c+dx)^4\sqrt[3]{a+bx^3}\,dx = \int \left(bx^3+a\right)^{\frac{1}{3}}(dx+c)^4\,dx$$

input
```
integrate((d*x+c)^4*(b*x^3+a)^(1/3),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(1/3)*(d*x + c)^4, x)
```

Giac [F]

$$\int (c+dx)^4\sqrt[3]{a+bx^3}\,dx = \int \left(bx^3+a\right)^{\frac{1}{3}}(dx+c)^4\,dx$$

input
```
integrate((d*x+c)^4*(b*x^3+a)^(1/3),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(1/3)*(d*x + c)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^4\sqrt[3]{a+bx^3}\,dx = \int \left(b\,x^3+a\right)^{1/3}\,(c+d\,x)^4\,dx$$

input
```
int((a + b*x^3)^(1/3)*(c + d*x)^4,x)
```

output
```
int((a + b*x^3)^(1/3)*(c + d*x)^4, x)
```

Reduce [F]

$$\int (c+dx)^4 \sqrt[3]{a+bx^3}\,dx$$
$$= \frac{135(b\,x^3+a)^{\frac{1}{3}}\,a\,c^2d^2+36(b\,x^3+a)^{\frac{1}{3}}\,ac\,d^3x+5(b\,x^3+a)^{\frac{1}{3}}\,a\,d^4x^2+45(b\,x^3+a)^{\frac{1}{3}}\,b\,c^4x+120(b\,x^3+a)^{\frac{1}{3}}}{}$$

input
```
int((d*x+c)^4*(b*x^3+a)^(1/3),x)
```

output
```
(135*(a + b*x**3)**(1/3)*a*c**2*d**2 + 36*(a + b*x**3)**(1/3)*a*c*d**3*x +
 5*(a + b*x**3)**(1/3)*a*d**4*x**2 + 45*(a + b*x**3)**(1/3)*b*c**4*x + 120
*(a + b*x**3)**(1/3)*b*c**3*d*x**2 + 135*(a + b*x**3)**(1/3)*b*c**2*d**2*x
**3 + 72*(a + b*x**3)**(1/3)*b*c*d**3*x**4 + 15*(a + b*x**3)**(1/3)*b*d**4
*x**5 - 36*int((a + b*x**3)**(1/3)/(a + b*x**3),x)*a**2*c*d**3 + 45*int((a
 + b*x**3)**(1/3)/(a + b*x**3),x)*a*b*c**4 - 10*int(((a + b*x**3)**(1/3)*x
)/(a + b*x**3),x)*a**2*d**4 + 120*int(((a + b*x**3)**(1/3)*x)/(a + b*x**3)
,x)*a*b*c**3*d)/(90*b)
```

3.144 $\int (c+dx)^3 \sqrt[3]{a+bx^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 242

$$\int (c+dx)^3 \sqrt[3]{a+bx^3}\,dx = \frac{3acd^2\sqrt[3]{a+bx^3}}{4b}+\frac{ad^3x\sqrt[3]{a+bx^3}}{10b} + \frac{1}{20}\sqrt[3]{a+bx^3}\left(10c^3x+20c^2dx^2+15cd^2x^3+4d^3x^4\right) - \frac{ac^2d\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}} + \frac{a(5bc^3-ad^3)\,x\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{10b\,(a+bx^3)^{2/3}} - \frac{ac^2d\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2b^{2/3}}$$

output
```
3/4*a*c*d^2*(b*x^3+a)^(1/3)/b+1/10*a*d^3*x*(b*x^3+a)^(1/3)/b+1/20*(b*x^3+a
)^(1/3)*(4*d^3*x^4+15*c*d^2*x^3+20*c^2*d*x^2+10*c^3*x)-1/3*a*c^2*d*arctan(
1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))*3^(1/2)/b^(2/3)+1/10*a*(-a*d^
3+5*b*c^3)*x*(1+b*x^3/a)^(2/3)*hypergeom([1/3, 2/3],[4/3],-b*x^3/a)/b/(b*x
^3+a)^(2/3)-1/2*a*c^2*d*ln(b^(1/3)*x-(b*x^3+a)^(1/3))/b^(2/3)
```

Mathematica [A] (verified)

Time = 8.59 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.59

$$\int (c+dx)^3\sqrt[3]{a+bx^3}\,dx$$
$$=\frac{\sqrt[3]{a+bx^3}\left(4bc^3x\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{3},\frac{4}{3},-\frac{bx^3}{a}\right)+d\left(6bc^2x^2\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right.\right.\right.}{4b\sqrt[3]{1+\frac{bx^3}{a}}}$$

input
```
Integrate[(c + d*x)^3*(a + b*x^3)^(1/3),x]
```

output
```
((a + b*x^3)^(1/3)*(4*b*c^3*x*Hypergeometric2F1[-1/3, 1/3, 4/3, -((b*x^3)/
a)] + d*(6*b*c^2*x^2*Hypergeometric2F1[-1/3, 2/3, 5/3, -((b*x^3)/a)] + d*(
3*c*(a + b*x^3)*(1 + (b*x^3)/a)^(1/3) + b*d*x^4*Hypergeometric2F1[-1/3, 4/
3, 7/3, -((b*x^3)/a)]))))/(4*b*(1 + (b*x^3)/a)^(1/3))
```

Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.97, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {2392, 27, 2427, 27, 2425, 793, 2432, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[3]{a+bx^3}(c+dx)^3\,dx$$

$$\Big\downarrow \text{2392}$$

$$a\int \frac{10c^3+20dxc^2+15d^2x^2c+4d^3x^3}{20\left(bx^3+a\right)^{2/3}}dx+\frac{1}{20}\sqrt[3]{a+bx^3}\left(10c^3x+20c^2dx^2+15cd^2x^3+4d^3x^4\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{20}a\int \frac{10c^3+20dxc^2+15d^2x^2c+4d^3x^3}{(bx^3+a)^{2/3}}dx+$$
$$\frac{1}{20}\sqrt[3]{a+bx^3}\left(10c^3x+20c^2dx^2+15cd^2x^3+4d^3x^4\right)$$

$$\Big\downarrow \text{ 2427}$$

$$\frac{1}{20}a\left(\frac{\int \frac{2\left(20bdxc^2+15bd^2x^2c+2\left(5bc^3-ad^3\right)\right)}{(bx^3+a)^{2/3}}dx}{2b}+\frac{2d^3x\sqrt[3]{a+bx^3}}{b}\right)+$$
$$\frac{1}{20}\sqrt[3]{a+bx^3}\left(10c^3x+20c^2dx^2+15cd^2x^3+4d^3x^4\right)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{20}a\left(\frac{\int \frac{20bdxc^2+15bd^2x^2c+2\left(5bc^3-ad^3\right)}{(bx^3+a)^{2/3}}dx}{b}+\frac{2d^3x\sqrt[3]{a+bx^3}}{b}\right)+$$
$$\frac{1}{20}\sqrt[3]{a+bx^3}\left(10c^3x+20c^2dx^2+15cd^2x^3+4d^3x^4\right)$$

$$\Big\downarrow \text{ 2425}$$

$$\frac{1}{20}a\left(\frac{\int \frac{20bdxc^2+2\left(5bc^3-ad^3\right)}{(bx^3+a)^{2/3}}dx+15bcd^2\int \frac{x^2}{(bx^3+a)^{2/3}}dx}{b}+\frac{2d^3x\sqrt[3]{a+bx^3}}{b}\right)+$$
$$\frac{1}{20}\sqrt[3]{a+bx^3}\left(10c^3x+20c^2dx^2+15cd^2x^3+4d^3x^4\right)$$

$$\Big\downarrow \text{ 793}$$

$$\frac{1}{20}a\left(\frac{\int \frac{20bdxc^2+2\left(5bc^3-ad^3\right)}{(bx^3+a)^{2/3}}dx+15cd^2\sqrt[3]{a+bx^3}}{b}+\frac{2d^3x\sqrt[3]{a+bx^3}}{b}\right)+$$
$$\frac{1}{20}\sqrt[3]{a+bx^3}\left(10c^3x+20c^2dx^2+15cd^2x^3+4d^3x^4\right)$$

$$\Big\downarrow \text{ 2432}$$

$$\frac{1}{20}a\left(\frac{\int \left(\frac{20bdxc^2}{(bx^3+a)^{2/3}}+\frac{2\left(5bc^3-ad^3\right)}{(bx^3+a)^{2/3}}\right)dx+15cd^2\sqrt[3]{a+bx^3}}{b}+\frac{2d^3x\sqrt[3]{a+bx^3}}{b}\right)+$$
$$\frac{1}{20}\sqrt[3]{a+bx^3}\left(10c^3x+20c^2dx^2+15cd^2x^3+4d^3x^4\right)$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{20}a\left(\frac{-\frac{20\sqrt[3]{b}c^2d\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{2x\left(\frac{bx^3}{a}+1\right)^{2/3}(5bc^3-ad^3)\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{(a+bx^3)^{2/3}}-10\sqrt[3]{b}c^2d\log\left(\sqrt[3]{b}x\right.}{b}\right.$$

$$\frac{1}{20}\sqrt[3]{a+bx^3}(10c^3x+20c^2dx^2+15cd^2x^3+4d^3x^4)$$

input
```
Int[(c + d*x)^3*(a + b*x^3)^(1/3),x]
```

output
```
((a + b*x^3)^(1/3)*(10*c^3*x + 20*c^2*d*x^2 + 15*c*d^2*x^3 + 4*d^3*x^4))/2
0 + (a*((2*d^3*x*(a + b*x^3)^(1/3))/b + (15*c*d^2*(a + b*x^3)^(1/3) - (20*
b^(1/3)*c^2*d*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/Sqrt[
3] + (2*(5*b*c^3 - a*d^3)*x*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 2
/3, 4/3, -((b*x^3)/a)])/(a + b*x^3)^(2/3) - 10*b^(1/3)*c^2*d*Log[b^(1/3)*x
 - (a + b*x^3)^(1/3)])/b))/20
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2392
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{q = Expon[Pq
, x], i}, Simp[(a + b*x^n)^p*Sum[Coeff[Pq, x, i]*(x^(i + 1)/(n*p + i + 1)),
 {i, 0, q}], x] + Simp[a*n*p   Int[(a + b*x^n)^(p - 1)*Sum[Coeff[Pq, x, i]*
(x^i/(n*p + i + 1)), {i, 0, q}], x], x]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x
] && IGtQ[(n - 1)/2, 0] && GtQ[p, 0]
```

rule 2425
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Pq, x, n -
 1]   Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq,
x, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[P
q, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

rule 2427
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{q = Expon[Pq, x
]}, With[{Pqq = Coeff[Pq, x, q]}, Simp[Pqq*x^(q - n + 1)*((a + b*x^n)^(p +
1)/(b*(q + n*p + 1))), x] + Simp[1/(b*(q + n*p + 1))   Int[ExpandToSum[b*(q
 + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(q - n + 1)*x^(q - n), x]*(a + b*x^n)^p,
 x], x]] /; NeQ[q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ
[p + (q + 1)/(2*n)])] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

rule 2432
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

Maple [F]

$$\int (dx+c)^3 \left(b\,x^3+a\right)^{\frac{1}{3}} dx$$

input
```
int((d*x+c)^3*(b*x^3+a)^(1/3),x)
```

output
```
int((d*x+c)^3*(b*x^3+a)^(1/3),x)
```

Fricas [F]

$$\int (c+dx)^3 \sqrt[3]{a+bx^3}\,dx = \int \left(bx^3+a\right)^{\frac{1}{3}} (dx+c)^3\,dx$$

input
```
integrate((d*x+c)^3*(b*x^3+a)^(1/3),x, algorithm="fricas")
```

output
```
integral((d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)*(b*x^3 + a)^(1/3), x)
```

Sympy [A] (verification not implemented)

Time = 2.10 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.66

$$\int (c+dx)^3 \sqrt[3]{a+bx^3}\,dx = \frac{\sqrt[3]{a}c^3x\Gamma\left(\frac{1}{3}\right)\,{}_2F_1\left(\begin{matrix} -\frac{1}{3}, \frac{1}{3} \\ \frac{4}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{\sqrt[3]{a}c^2dx^2\Gamma\left(\frac{2}{3}\right)\,{}_2F_1\left(\begin{matrix} -\frac{1}{3}, \frac{2}{3} \\ \frac{5}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{\Gamma\left(\frac{5}{3}\right)} + \frac{\sqrt[3]{a}d^3x^4\Gamma\left(\frac{4}{3}\right)\,{}_2F_1\left(\begin{matrix} -\frac{1}{3}, \frac{4}{3} \\ \frac{7}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)} + 3cd^2\left(\begin{cases} \frac{\sqrt[3]{a}x^3}{3} & \text{for } b=0 \\ \frac{\left(a+bx^3\right)^{\frac{4}{3}}}{4b} & \text{otherwise} \end{cases}\right)$$

input
```
integrate((d*x+c)**3*(b*x**3+a)**(1/3),x)
```

output
```
a**(1/3)*c**3*x*gamma(1/3)*hyper((-1/3, 1/3), (4/3,), b*x**3*exp_polar(I*p
i)/a)/(3*gamma(4/3)) + a**(1/3)*c**2*d*x**2*gamma(2/3)*hyper((-1/3, 2/3),
(5/3,), b*x**3*exp_polar(I*pi)/a)/gamma(5/3) + a**(1/3)*d**3*x**4*gamma(4/
3)*hyper((-1/3, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + 3
*c*d**2*Piecewise((a**(1/3)*x**3/3, Eq(b, 0)), ((a + b*x**3)**(4/3)/(4*b),
 True))
```

Maxima [F]

$$\int (c+dx)^3 \sqrt[3]{a+bx^3}\, dx = \int \left(bx^3+a\right)^{\frac{1}{3}} (dx+c)^3\, dx$$

input
```
integrate((d*x+c)^3*(b*x^3+a)^(1/3),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(1/3)*(d*x + c)^3, x)
```

Giac [F]

$$\int (c+dx)^3 \sqrt[3]{a+bx^3}\, dx = \int \left(bx^3+a\right)^{\frac{1}{3}} (dx+c)^3\, dx$$

input
```
integrate((d*x+c)^3*(b*x^3+a)^(1/3),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(1/3)*(d*x + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^3 \sqrt[3]{a+bx^3}\, dx = \int \left(b\,x^3+a\right)^{1/3}\,\left(c+d\,x\right)^3\,dx$$

input
```
int((a + b*x^3)^(1/3)*(c + d*x)^3,x)
```

output
```
int((a + b*x^3)^(1/3)*(c + d*x)^3, x)
```

Reduce [F]

$$\int (c+dx)^3 \sqrt[3]{a+bx^3}\, dx$$
$$= \frac{15(b\,x^3+a)^{\frac{1}{3}}\,ac\,d^2+2(b\,x^3+a)^{\frac{1}{3}}\,a\,d^3x+10(b\,x^3+a)^{\frac{1}{3}}\,b\,c^3x+20(b\,x^3+a)^{\frac{1}{3}}\,b\,c^2d\,x^2+15(b\,x^3+a)^{\frac{1}{3}}\,bc}{2}$$

input
```
int((d*x+c)^3*(b*x^3+a)^(1/3),x)
```

output
```
(15*(a + b*x**3)**(1/3)*a*c*d**2 + 2*(a + b*x**3)**(1/3)*a*d**3*x + 10*(a
+ b*x**3)**(1/3)*b*c**3*x + 20*(a + b*x**3)**(1/3)*b*c**2*d*x**2 + 15*(a +
 b*x**3)**(1/3)*b*c*d**2*x**3 + 4*(a + b*x**3)**(1/3)*b*d**3*x**4 - 2*int(
(a + b*x**3)**(1/3)/(a + b*x**3),x)*a**2*d**3 + 10*int((a + b*x**3)**(1/3)
/(a + b*x**3),x)*a*b*c**3 + 20*int(((a + b*x**3)**(1/3)*x)/(a + b*x**3),x)
*a*b*c**2*d)/(20*b)
```

3.145 $\int (c+dx)^2 \sqrt[3]{a+bx^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 183

$$\int (c+dx)^2 \sqrt[3]{a+bx^3}\,dx = \frac{1}{6}\left(3c^2x+4cdx^2\right)\sqrt[3]{a+bx^3}+\frac{d^2\left(a+bx^3\right)^{4/3}}{4b} - \frac{2acd\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}b^{2/3}} + \frac{ac^2x\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2\left(a+bx^3\right)^{2/3}} - \frac{acd\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{3b^{2/3}}$$

output

```
1/6*(4*c*d*x^2+3*c^2*x)*(b*x^3+a)^(1/3)+1/4*d^2*(b*x^3+a)^(4/3)/b-2/9*a*c*
d*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))*3^(1/2)/b^(2/3)+1/2*
a*c^2*x*(1+b*x^3/a)^(2/3)*hypergeom([1/3, 2/3],[4/3],-b*x^3/a)/(b*x^3+a)^(
2/3)-1/3*a*c*d*ln(b^(1/3)*x-(b*x^3+a)^(1/3))/b^(2/3)
```

Mathematica [A] (verified)

Time = 8.37 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.61

$$\int (c+dx)^2\sqrt[3]{a+bx^3}\,dx$$
$$=\frac{\sqrt[3]{a+bx^3}\left(4bc^2x\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{3},\frac{4}{3},-\frac{bx^3}{a}\right)+d\left(d(a+bx^3)\sqrt[3]{1+\frac{bx^3}{a}}+4bcx^2\,\text{Hypergeome}\right.\right.}{4b\sqrt[3]{1+\frac{bx^3}{a}}}$$

input
```
Integrate[(c + d*x)^2*(a + b*x^3)^(1/3),x]
```

output
```
((a + b*x^3)^(1/3)*(4*b*c^2*x*Hypergeometric2F1[-1/3, 1/3, 4/3, -((b*x^3)/
a)] + d*(d*(a + b*x^3)*(1 + (b*x^3)/a)^(1/3) + 4*b*c*x^2*Hypergeometric2F1
[-1/3, 2/3, 5/3, -((b*x^3)/a)])))/(4*b*(1 + (b*x^3)/a)^(1/3))
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {2392, 27, 2425, 793, 2432, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[3]{a+bx^3}(c+dx)^2\,dx$$
$$\Big\downarrow \text{2392}$$
$$a\int \frac{6c^2+8dxc+3d^2x^2}{12\left(bx^3+a\right)^{2/3}}dx+\frac{1}{12}\sqrt[3]{a+bx^3}\left(6c^2x+8cdx^2+3d^2x^3\right)$$
$$\Big\downarrow \text{27}$$
$$\frac{1}{12}a\int \frac{6c^2+8dxc+3d^2x^2}{\left(bx^3+a\right)^{2/3}}dx+\frac{1}{12}\sqrt[3]{a+bx^3}\left(6c^2x+8cdx^2+3d^2x^3\right)$$
$$\Big\downarrow \text{2425}$$

$$\frac{1}{12}a\left(\int \frac{6c^2+8dxc}{(bx^3+a)^{2/3}}dx+3d^2\int \frac{x^2}{(bx^3+a)^{2/3}}dx\right)+\frac{1}{12}\sqrt[3]{a+bx^3}\left(6c^2x+8cdx^2+3d^2x^3\right)$$

$\Big\downarrow$ 793

$$\frac{1}{12}a\left(\int \frac{6c^2+8dxc}{(bx^3+a)^{2/3}}dx+\frac{3d^2\sqrt[3]{a+bx^3}}{b}\right)+\frac{1}{12}\sqrt[3]{a+bx^3}\left(6c^2x+8cdx^2+3d^2x^3\right)$$

$\Big\downarrow$ 2432

$$\frac{1}{12}a\left(\int \left(\frac{6c^2}{(bx^3+a)^{2/3}}+\frac{8dxc}{(bx^3+a)^{2/3}}\right)dx+\frac{3d^2\sqrt[3]{a+bx^3}}{b}\right)+\frac{1}{12}\sqrt[3]{a+bx^3}\left(6c^2x+8cdx^2+3d^2x^3\right)$$

$\Big\downarrow$ 2009

$$\frac{1}{12}a\left(-\frac{8cd\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}}-\frac{4cd\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{b^{2/3}}+\frac{6c^2x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3}\right.}{(a+bx^3)^{2/3}}\right.$$
$$\frac{1}{12}\sqrt[3]{a+bx^3}\left(6c^2x+8cdx^2+3d^2x^3\right)$$

input
```
Int[(c + d*x)^2*(a + b*x^3)^(1/3),x]
```

output
```
((a + b*x^3)^(1/3)*(6*c^2*x + 8*c*d*x^2 + 3*d^2*x^3))/12 + (a*((3*d^2*(a +
 b*x^3)^(1/3))/b - (8*c*d*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqr
t[3]])/(Sqrt[3]*b^(2/3)) + (6*c^2*x*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F
1[1/3, 2/3, 4/3, -((b*x^3)/a)])/(a + b*x^3)^(2/3) - (4*c*d*Log[b^(1/3)*x -
 (a + b*x^3)^(1/3)])/b^(2/3)))/12
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2392
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{q = Expon[Pq
, x], i}, Simp[(a + b*x^n)^p*Sum[Coeff[Pq, x, i]*(x^(i + 1)/(n*p + i + 1)),
 {i, 0, q}], x] + Simp[a*n*p   Int[(a + b*x^n)^(p - 1)*Sum[Coeff[Pq, x, i]*
(x^i/(n*p + i + 1)), {i, 0, q}], x], x]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x
] && IGtQ[(n - 1)/2, 0] && GtQ[p, 0]
```

rule 2425
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Pq, x, n -
 1]   Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq,
x, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[P
q, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

rule 2432
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

Maple [F]

$$\int (dx+c)^2 \left(b\,x^3+a\right)^{\frac{1}{3}} dx$$

input
```
int((d*x+c)^2*(b*x^3+a)^(1/3),x)
```

output
```
int((d*x+c)^2*(b*x^3+a)^(1/3),x)
```

Fricas [F]

$$\int (c+dx)^2\sqrt[3]{a+bx^3}\,dx=\int \left(bx^3+a\right)^{\frac{1}{3}}(dx+c)^2\,dx$$

input
```
integrate((d*x+c)^2*(b*x^3+a)^(1/3),x, algorithm="fricas")
```

output
```
integral((d^2*x^2 + 2*c*d*x + c^2)*(b*x^3 + a)^(1/3), x)
```

Sympy [A] (verification not implemented)

Time = 1.66 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.62

$$\int (c+dx)^2\sqrt[3]{a+bx^3}\,dx=\frac{\sqrt[3]{a}c^2x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}+\frac{2\sqrt[3]{a}cdx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)}+d^2\left(\begin{cases}\frac{\sqrt[3]{a}x^3}{3} & \text{for } b=0\\ \frac{(a+bx^3)^{\frac{4}{3}}}{4b} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((d*x+c)**2*(b*x**3+a)**(1/3),x)
```

output
```
a**(1/3)*c**2*x*gamma(1/3)*hyper((-1/3, 1/3), (4/3,), b*x**3*exp_polar(I*p
i)/a)/(3*gamma(4/3)) + 2*a**(1/3)*c*d*x**2*gamma(2/3)*hyper((-1/3, 2/3), (
5/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(5/3)) + d**2*Piecewise((a**(1/3)
*x**3/3, Eq(b, 0)), ((a + b*x**3)**(4/3)/(4*b), True))
```

Maxima [F]

$$\int (c+dx)^2\sqrt[3]{a+bx^3}\,dx = \int \left(bx^3+a\right)^{\frac{1}{3}}(dx+c)^2\,dx$$

input
```
integrate((d*x+c)^2*(b*x^3+a)^(1/3),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(1/3)*(d*x + c)^2, x)
```

Giac [F]

$$\int (c+dx)^2\sqrt[3]{a+bx^3}\,dx = \int \left(bx^3+a\right)^{\frac{1}{3}}(dx+c)^2\,dx$$

input
```
integrate((d*x+c)^2*(b*x^3+a)^(1/3),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(1/3)*(d*x + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^2\sqrt[3]{a+bx^3}\,dx = \int \left(b\,x^3+a\right)^{1/3}\,(c+d\,x)^2\,dx$$

input
```
int((a + b*x^3)^(1/3)*(c + d*x)^2,x)
```

output
```
int((a + b*x^3)^(1/3)*(c + d*x)^2, x)
```

Reduce [F]

$$\int (c+dx)^2\sqrt[3]{a+bx^3}\,dx$$
$$= \frac{3(b\,x^3+a)^{\frac{1}{3}}\,a\,d^2+6(b\,x^3+a)^{\frac{1}{3}}\,b\,c^2x+8(b\,x^3+a)^{\frac{1}{3}}\,bcd\,x^2+3(b\,x^3+a)^{\frac{1}{3}}\,b\,d^2x^3+6\left(\int \frac{1}{(b\,x^3+a)^{\frac{2}{3}}}dx\right)ab\,c}{12b}$$

input
```
int((d*x+c)^2*(b*x^3+a)^(1/3),x)
```

output
```
(3*(a + b*x**3)**(1/3)*a*d**2 + 6*(a + b*x**3)**(1/3)*b*c**2*x + 8*(a + b*
x**3)**(1/3)*b*c*d*x**2 + 3*(a + b*x**3)**(1/3)*b*d**2*x**3 + 6*int((a + b
*x**3)**(1/3)/(a + b*x**3),x)*a*b*c**2 + 8*int(((a + b*x**3)**(1/3)*x)/(a
+ b*x**3),x)*a*b*c*d)/(12*b)
```

3.146 $\int (c+dx)\sqrt[3]{a+bx^3}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 155

$$\int (c+dx)\sqrt[3]{a+bx^3}\,dx = \frac{1}{6}\left(3cx+2dx^2\right)\sqrt[3]{a+bx^3} - \frac{ad\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}b^{2/3}} + \frac{acx\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2\left(a+bx^3\right)^{2/3}} - \frac{ad\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{6b^{2/3}}$$

output
```
1/6*(2*d*x^2+3*c*x)*(b*x^3+a)^(1/3)-1/9*a*d*arctan(1/3*(1+2*b^(1/3)*x/(b*x
^3+a)^(1/3))*3^(1/2))*3^(1/2)/b^(2/3)+1/2*a*c*x*(1+b*x^3/a)^(2/3)*hypergeo
m([1/3, 2/3],[4/3],-b*x^3/a)/(b*x^3+a)^(2/3)-1/6*a*d*ln(b^(1/3)*x-(b*x^3+a
)^(1/3))/b^(2/3)
```

Mathematica [A] (verified)

Time = 7.67 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.48

$$\int (c+dx)\sqrt[3]{a+bx^3}\,dx = \frac{x\sqrt[3]{a+bx^3}\left(2c\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{3},\frac{4}{3},-\frac{bx^3}{a}\right)+dx\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)\right)}{2\sqrt[3]{1+\frac{bx^3}{a}}}$$

input
```
Integrate[(c + d*x)*(a + b*x^3)^(1/3),x]
```

output
```
(x*(a + b*x^3)^(1/3)*(2*c*Hypergeometric2F1[-1/3, 1/3, 4/3, -((b*x^3)/a)]
+ d*x*Hypergeometric2F1[-1/3, 2/3, 5/3, -((b*x^3)/a)]))/(2*(1 + (b*x^3)/a)
^(1/3))
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {2392, 27, 2432, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[3]{a+bx^3}(c+dx)\,dx$$

$$\Big\downarrow \text{2392}$$

$$a\int \frac{3c+2dx}{6\left(bx^3+a\right)^{2/3}}dx+\frac{1}{6}\sqrt[3]{a+bx^3}\left(3cx+2dx^2\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{6}a\int \frac{3c+2dx}{\left(bx^3+a\right)^{2/3}}dx+\frac{1}{6}\sqrt[3]{a+bx^3}\left(3cx+2dx^2\right)$$

$$\Big\downarrow \text{2432}$$

$$\frac{1}{6}a\int\left(\frac{3c}{(bx^3+a)^{2/3}}+\frac{2dx}{(bx^3+a)^{2/3}}\right)dx+\frac{1}{6}\sqrt[3]{a+bx^3}(3cx+2dx^2)$$

$\Big\downarrow$ 2009

$$\frac{1}{6}a\left(-\frac{2d\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}}-\frac{d\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{b^{2/3}}+\frac{3cx\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},\right.}{(a+bx^3)^{2/3}}\right.$$
$$\frac{1}{6}\sqrt[3]{a+bx^3}(3cx+2dx^2)$$

input
```
Int[(c + d*x)*(a + b*x^3)^(1/3),x]
```

output
```
((3*c*x + 2*d*x^2)*(a + b*x^3)^(1/3))/6 + (a*((-2*d*ArcTan[(1 + (2*b^(1/3)
*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*b^(2/3)) + (3*c*x*(1 + (b*x^3)/a
)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)])/(a + b*x^3)^(2/3)
- (d*Log[b^(1/3)*x - (a + b*x^3)^(1/3)])/b^(2/3)))/6
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2392
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{q = Expon[Pq
, x], i}, Simp[(a + b*x^n)^p*Sum[Coeff[Pq, x, i]*(x^(i + 1)/(n*p + i + 1)),
 {i, 0, q}], x] + Simp[a*n*p   Int[(a + b*x^n)^(p - 1)*Sum[Coeff[Pq, x, i]*
(x^i/(n*p + i + 1)), {i, 0, q}], x], x]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x
] && IGtQ[(n - 1)/2, 0] && GtQ[p, 0]
```

rule 2432
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

Maple [F]

$$\int (dx+c)\left(b\,x^{3}+a\right)^{\frac{1}{3}}dx$$

input
```
int((d*x+c)*(b*x^3+a)^(1/3),x)
```

output
```
int((d*x+c)*(b*x^3+a)^(1/3),x)
```

Fricas [F]

$$\int (c+dx)\sqrt[3]{a+bx^3}\,dx=\int \left(bx^{3}+a\right)^{\frac{1}{3}}(dx+c)\,dx$$

input
```
integrate((d*x+c)*(b*x^3+a)^(1/3),x, algorithm="fricas")
```

output
```
integral((b*x^3 + a)^(1/3)*(d*x + c), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.51 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.53

$$\int (c+dx)\sqrt[3]{a+bx^3}\,dx$$
$$=\frac{\sqrt[3]{a}cx\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}+\frac{\sqrt[3]{a}dx^{2}\Gamma\left(\frac{2}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{3},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)}$$

input
```
integrate((d*x+c)*(b*x**3+a)**(1/3),x)
```

output
```
a**(1/3)*c*x*gamma(1/3)*hyper((-1/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/
a)/(3*gamma(4/3)) + a**(1/3)*d*x**2*gamma(2/3)*hyper((-1/3, 2/3), (5/3,),
b*x**3*exp_polar(I*pi)/a)/(3*gamma(5/3))
```

Maxima [F]

$$\int (c+dx)\sqrt[3]{a+bx^3}\,dx = \int \left(bx^3+a\right)^{\frac{1}{3}}(dx+c)\,dx$$

input
```
integrate((d*x+c)*(b*x^3+a)^(1/3),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(1/3)*(d*x + c), x)
```

Giac [F]

$$\int (c+dx)\sqrt[3]{a+bx^3}\,dx = \int \left(bx^3+a\right)^{\frac{1}{3}}(dx+c)\,dx$$

input
```
integrate((d*x+c)*(b*x^3+a)^(1/3),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(1/3)*(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)\sqrt[3]{a+bx^3}\,dx=\int \left(b\,x^3+a\right)^{1/3}\,\left(c+d\,x\right)\,dx$$

input
```
int((a + b*x^3)^(1/3)*(c + d*x),x)
```

output
```
int((a + b*x^3)^(1/3)*(c + d*x), x)
```

Reduce [F]

$$\int (c+dx)\sqrt[3]{a+bx^3}\,dx=\frac{(b\,x^3+a)^{\frac{1}{3}}\,cx}{2}+\frac{(b\,x^3+a)^{\frac{1}{3}}\,d\,x^2}{3}+\frac{\left(\int \frac{1}{(b\,x^3+a)^{\frac{2}{3}}}dx\right)ac}{2}+\frac{\left(\int \frac{x}{(b\,x^3+a)^{\frac{2}{3}}}dx\right)ad}{3}$$

input
```
int((d*x+c)*(b*x^3+a)^(1/3),x)
```

output
```
(3*(a + b*x**3)**(1/3)*c*x + 2*(a + b*x**3)**(1/3)*d*x**2 + 3*int((a + b*x
**3)**(1/3)/(a + b*x**3),x)*a*c + 2*int(((a + b*x**3)**(1/3)*x)/(a + b*x**
3),x)*a*d)/6
```

3.147 $\int \frac{\sqrt[3]{a+bx^3}}{c+dx} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 435

$$\int \frac{\sqrt[3]{a+bx^3}}{c+dx}\,dx = \frac{\sqrt[3]{a+bx^3}}{d}+\frac{x\sqrt[3]{a+bx^3}\,\text{AppellF1}\left(\frac{1}{3},-\frac{1}{3},1,\frac{4}{3},-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)}{c\sqrt[3]{1+\frac{bx^3}{a}}} + \frac{\sqrt[3]{b}c\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}d^2} - \frac{\sqrt[3]{bc^3-ad^3}\arctan\left(\frac{1+\frac{2\sqrt[3]{bc^3-ad^3}x}{c\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}d^2} + \frac{\sqrt[3]{bc^3-ad^3}\arctan\left(\frac{1-\frac{2d\sqrt[3]{a+bx^3}}{\sqrt[3]{bc^3-ad^3}}}{\sqrt{3}}\right)}{\sqrt{3}d^2} + \frac{\sqrt[3]{bc^3-ad^3}\log\left(c^3+d^3x^3\right)}{3d^2}+\frac{\sqrt[3]{b}c\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2d^2} - \frac{\sqrt[3]{bc^3-ad^3}\log\left(\frac{\sqrt[3]{bc^3-ad^3}x}{c}-\sqrt[3]{a+bx^3}\right)}{2d^2} - \frac{\sqrt[3]{bc^3-ad^3}\log\left(\sqrt[3]{bc^3-ad^3}+d\sqrt[3]{a+bx^3}\right)}{2d^2}$$

output
```
(b*x^3+a)^(1/3)/d+x*(b*x^3+a)^(1/3)*AppellF1(1/3,-1/3,1,4/3,-b*x^3/a,-d^3*
x^3/c^3)/c/(1+b*x^3/a)^(1/3)+1/3*b^(1/3)*c*arctan(1/3*(1+2*b^(1/3)*x/(b*x^
3+a)^(1/3))*3^(1/2))*3^(1/2)/d^2-1/3*(-a*d^3+b*c^3)^(1/3)*arctan(1/3*(1+2*
(-a*d^3+b*c^3)^(1/3)*x/c/(b*x^3+a)^(1/3))*3^(1/2))*3^(1/2)/d^2+1/3*(-a*d^3
+b*c^3)^(1/3)*arctan(1/3*(1-2*d*(b*x^3+a)^(1/3)/(-a*d^3+b*c^3)^(1/3))*3^(1
/2))*3^(1/2)/d^2+1/3*(-a*d^3+b*c^3)^(1/3)*ln(d^3*x^3+c^3)/d^2+1/2*b^(1/3)*
c*ln(b^(1/3)*x-(b*x^3+a)^(1/3))/d^2-1/2*(-a*d^3+b*c^3)^(1/3)*ln((-a*d^3+b*
c^3)^(1/3)*x/c-(b*x^3+a)^(1/3))/d^2-1/2*(-a*d^3+b*c^3)^(1/3)*ln((-a*d^3+b*
c^3)^(1/3)+d*(b*x^3+a)^(1/3))/d^2
```

Mathematica [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{c+dx}\,dx = \int \frac{\sqrt[3]{a+bx^3}}{c+dx}\,dx$$

input
```
Integrate[(a + b*x^3)^(1/3)/(c + d*x),x]
```

output
```
Integrate[(a + b*x^3)^(1/3)/(c + d*x), x]
```

Rubi [A] (verified)

Time = 1.20 (sec) , antiderivative size = 435, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2581, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{a+bx^3}}{c+dx}\,dx$$

$$\Big\downarrow \text{2581}$$

$$\int \left(-\frac{cdx\sqrt[3]{a+bx^3}}{c^3+d^3x^3}+\frac{d^2x^2\sqrt[3]{a+bx^3}}{c^3+d^3x^3}+\frac{c^2\sqrt[3]{a+bx^3}}{c^3+d^3x^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{x\sqrt[3]{a+bx^3}\,\text{AppellF1}\left(\frac{1}{3},-\frac{1}{3},1,\frac{4}{3},-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)}{c\sqrt[3]{\frac{bx^3}{a}+1}}-\frac{\sqrt[3]{bc^3-ad^3}\arctan\left(\frac{\frac{2x\sqrt[3]{bc^3-ad^3}}{c\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}d^2}+\frac{\sqrt[3]{bc^3-ad^3}\arctan\left(\frac{1-\frac{2d\sqrt[3]{a+bx^3}}{\sqrt[3]{bc^3-ad^3}}}{\sqrt{3}}\right)}{\sqrt{3}d^2}+\frac{\sqrt[3]{b}c\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}d^2}+\frac{\sqrt[3]{bc^3-ad^3}\log\left(c^3+d^3x^3\right)}{3d^2}-\frac{\sqrt[3]{bc^3-ad^3}\log\left(\frac{x\sqrt[3]{bc^3-ad^3}}{c}-\sqrt[3]{a+bx^3}\right)}{2d^2}-\frac{\sqrt[3]{bc^3-ad^3}\log\left(\sqrt[3]{bc^3-ad^3}+d\sqrt[3]{a+bx^3}\right)}{2d^2}+\frac{\sqrt[3]{b}c\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2d^2}+\frac{\sqrt[3]{a+bx^3}}{d}$$

input

```
Int[(a + b*x^3)^(1/3)/(c + d*x),x]
```

output

```
(a + b*x^3)^(1/3)/d + (x*(a + b*x^3)^(1/3)*AppellF1[1/3, -1/3, 1, 4/3, -((
b*x^3)/a), -((d^3*x^3)/c^3)])/(c*(1 + (b*x^3)/a)^(1/3)) + (b^(1/3)*c*ArcTa
n[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*d^2) - ((b*c^3
- a*d^3)^(1/3)*ArcTan[(1 + (2*(b*c^3 - a*d^3)^(1/3)*x)/(c*(a + b*x^3)^(1/3
)))/Sqrt[3]])/(Sqrt[3]*d^2) + ((b*c^3 - a*d^3)^(1/3)*ArcTan[(1 - (2*d*(a +
 b*x^3)^(1/3))/(b*c^3 - a*d^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*d^2) + ((b*c^3 -
a*d^3)^(1/3)*Log[c^3 + d^3*x^3])/(3*d^2) + (b^(1/3)*c*Log[b^(1/3)*x - (a +
 b*x^3)^(1/3)])/(2*d^2) - ((b*c^3 - a*d^3)^(1/3)*Log[((b*c^3 - a*d^3)^(1/3
)*x)/c - (a + b*x^3)^(1/3)])/(2*d^2) - ((b*c^3 - a*d^3)^(1/3)*Log[(b*c^3 -
 a*d^3)^(1/3) + d*(a + b*x^3)^(1/3)])/(2*d^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2581
```
Int[(Px_.)*((c_) + (d_.)*(x_))^(q_)*((a_) + (b_.)*(x_)^3)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(c^3 + d^3*x^3)^q*(a + b*x^3)^p, Px/(c^2 - c*d*x + d
^2*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p}, x] && PolyQ[Px, x] && ILtQ[q, 0
] && RationalQ[p] && EqQ[Denominator[p], 3]
```

Maple [F]

$$\int \frac{(b\,x^{3}+a)^{\frac{1}{3}}}{dx+c}dx$$

input
```
int((b*x^3+a)^(1/3)/(d*x+c),x)
```

output
```
int((b*x^3+a)^(1/3)/(d*x+c),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^3}}{c+dx}\,dx=\text{Timed out}$$

input
```
integrate((b*x^3+a)^(1/3)/(d*x+c),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{c+dx}\,dx = \int \frac{\sqrt[3]{a+bx^3}}{c+dx}\,dx$$

input
```
integrate((b*x**3+a)**(1/3)/(d*x+c),x)
```

output
```
Integral((a + b*x**3)**(1/3)/(c + d*x), x)
```

Maxima [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{c+dx}\,dx = \int \frac{(bx^3+a)^{\frac{1}{3}}}{dx+c}\,dx$$

input
```
integrate((b*x^3+a)^(1/3)/(d*x+c),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(1/3)/(d*x + c), x)
```

Giac [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{c+dx}\,dx = \int \frac{(bx^3+a)^{\frac{1}{3}}}{dx+c}\,dx$$

input
```
integrate((b*x^3+a)^(1/3)/(d*x+c),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(1/3)/(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^3}}{c+dx}\,dx=\int \frac{{\left(b\,x^3+a\right)}^{1/3}}{c+d\,x} \,d x$$

input
```
int((a + b*x^3)^(1/3)/(c + d*x),x)
```

output
```
int((a + b*x^3)^(1/3)/(c + d*x), x)
```

Reduce [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{c+dx}\,dx=\frac{(b\,x^{3}+a)^{\frac{1}{3}}+\left(\int \frac{(b\,x^{3}+a)^{\frac{1}{3}}}{bd\,x^{4}+bc\,x^{3}+adx+ac}d x \right) a d -\left(\int \frac{(b\,x^{3}+a)^{\frac{1}{3}} x^{2}}{bd\,x^{4}+bc\,x^{3}+adx+ac}d x \right) b c}{d}$$

input
```
int((b*x^3+a)^(1/3)/(d*x+c),x)
```

output
```
((a + b*x**3)**(1/3) + int((a + b*x**3)**(1/3)/(a*c + a*d*x + b*c*x**3 + b
*d*x**4),x)*a*d - int(((a + b*x**3)**(1/3)*x**2)/(a*c + a*d*x + b*c*x**3 +
 b*d*x**4),x)*b*c)/d
```

3.148 $\int \frac{\sqrt[3]{a+bx^3}}{(c+dx)^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 818

$$\int \frac{\sqrt[3]{a+bx^3}}{(c+dx)^2}\,dx = -\frac{c^2\sqrt[3]{a+bx^3}}{d\left(c^3+d^3x^3\right)} - \frac{dx^2\sqrt[3]{a+bx^3}}{c^3+d^3x^3} + \frac{x\sqrt[3]{a+bx^3}\,\text{AppellF1}\left(\frac{1}{3},-\frac{1}{3},2,\frac{4}{3},-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)}{c^2\sqrt[3]{1+\frac{bx^3}{a}}} - \frac{d^3x^4\sqrt[3]{a+bx^3}\,\text{AppellF1}\left(\frac{4}{3},-\frac{1}{3},2,\frac{7}{3},-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)}{2c^5\sqrt[3]{1+\frac{bx^3}{a}}} - \frac{\sqrt[3]{b}\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}d^2} + \frac{2ad\arctan\left(\frac{1+\frac{2\sqrt[3]{bc^3-ad^3}x}{c\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}c\left(bc^3-ad^3\right)^{2/3}} + \frac{\left(3bc^3-2ad^3\right)\arctan\left(\frac{1+\frac{2\sqrt[3]{bc^3-ad^3}x}{c\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}cd^2\left(bc^3-ad^3\right)^{2/3}} - \frac{bc^2\arctan\left(\frac{1-\frac{2d\sqrt[3]{a+bx^3}}{\sqrt[3]{bc^3-ad^3}}}{\sqrt{3}}\right)}{\sqrt{3}d^2\left(bc^3-ad^3\right)^{2/3}} - \frac{bc^2\log\left(c^3+d^3x^3\right)}{6d^2\left(bc^3-ad^3\right)^{2/3}} - \frac{ad\log\left(c^3+d^3x^3\right)}{9c\left(bc^3-ad^3\right)^{2/3}} - \frac{\left(3bc^3-2ad^3\right)\log\left(c^3+d^3x^3\right)}{18cd^2\left(bc^3-ad^3\right)^{2/3}} - \frac{\sqrt[3]{b}\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2d^2} + \frac{ad\log\left(\frac{\sqrt[3]{bc^3-ad^3}x}{c}-\sqrt[3]{a+bx^3}\right)}{3c\left(bc^3-ad^3\right)^{2/3}} + \frac{\left(3bc^3-2ad^3\right)\log\left(\frac{\sqrt[3]{bc^3-ad^3}x}{c}-\sqrt[3]{a+bx^3}\right)}{6cd^2\left(bc^3-ad^3\right)^{2/3}} + \frac{bc^2\log\left(\sqrt[3]{bc^3-ad^3}+d\sqrt[3]{a+bx^3}\right)}{2d^2\left(bc^3-ad^3\right)^{2/3}}$$

output
```
-c^2*(b*x^3+a)^(1/3)/d/(d^3*x^3+c^3)-d*x^2*(b*x^3+a)^(1/3)/(d^3*x^3+c^3)+x
*(b*x^3+a)^(1/3)*AppellF1(1/3,-1/3,2,4/3,-b*x^3/a,-d^3*x^3/c^3)/c^2/(1+b*x
^3/a)^(1/3)-1/2*d^3*x^4*(b*x^3+a)^(1/3)*AppellF1(4/3,-1/3,2,7/3,-b*x^3/a,-
d^3*x^3/c^3)/c^5/(1+b*x^3/a)^(1/3)-1/3*b^(1/3)*arctan(1/3*(1+2*b^(1/3)*x/(
b*x^3+a)^(1/3))*3^(1/2))*3^(1/2)/d^2+2/9*a*d*arctan(1/3*(1+2*(-a*d^3+b*c^3
)^(1/3)*x/c/(b*x^3+a)^(1/3))*3^(1/2))*3^(1/2)/c/(-a*d^3+b*c^3)^(2/3)+1/9*(
-2*a*d^3+3*b*c^3)*arctan(1/3*(1+2*(-a*d^3+b*c^3)^(1/3)*x/c/(b*x^3+a)^(1/3)
)*3^(1/2))*3^(1/2)/c/d^2/(-a*d^3+b*c^3)^(2/3)-1/3*b*c^2*arctan(1/3*(1-2*d*
(b*x^3+a)^(1/3)/(-a*d^3+b*c^3)^(1/3))*3^(1/2))*3^(1/2)/d^2/(-a*d^3+b*c^3)^
(2/3)-1/6*b*c^2*ln(d^3*x^3+c^3)/d^2/(-a*d^3+b*c^3)^(2/3)-1/9*a*d*ln(d^3*x^
3+c^3)/c/(-a*d^3+b*c^3)^(2/3)-1/18*(-2*a*d^3+3*b*c^3)*ln(d^3*x^3+c^3)/c/d^
2/(-a*d^3+b*c^3)^(2/3)-1/2*b^(1/3)*ln(b^(1/3)*x-(b*x^3+a)^(1/3))/d^2+1/3*a
*d*ln((-a*d^3+b*c^3)^(1/3)*x/c-(b*x^3+a)^(1/3))/c/(-a*d^3+b*c^3)^(2/3)+1/6
*(-2*a*d^3+3*b*c^3)*ln((-a*d^3+b*c^3)^(1/3)*x/c-(b*x^3+a)^(1/3))/c/d^2/(-a
*d^3+b*c^3)^(2/3)+1/2*b*c^2*ln((-a*d^3+b*c^3)^(1/3)+d*(b*x^3+a)^(1/3))/d^2
/(-a*d^3+b*c^3)^(2/3)
```

Mathematica [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{(c+dx)^2}\,dx = \int \frac{\sqrt[3]{a+bx^3}}{(c+dx)^2}\,dx$$

input
```
Integrate[(a + b*x^3)^(1/3)/(c + d*x)^2,x]
```

output
```
Integrate[(a + b*x^3)^(1/3)/(c + d*x)^2, x]
```

Rubi [A] (verified)

Time = 1.80 (sec) , antiderivative size = 818, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2581, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{a+bx^3}}{(c+dx)^2}\,dx$$

$$\Big\downarrow \text{ 2581}$$

$$\int \left(-\frac{2c^3dx\sqrt[3]{a+bx^3}}{(c^3+d^3x^3)^2}-\frac{2cd^3x^3\sqrt[3]{a+bx^3}}{(c^3+d^3x^3)^2}+\frac{d^4x^4\sqrt[3]{a+bx^3}}{(c^3+d^3x^3)^2}+\frac{c^4\sqrt[3]{a+bx^3}}{(c^3+d^3x^3)^2}+\frac{3c^2d^2x^2\sqrt[3]{a+bx^3}}{(c^3+d^3x^3)^2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{d^3\sqrt[3]{bx^3+a}\,\text{AppellF1}\left(\frac{4}{3},-\frac{1}{3},2,\frac{7}{3},-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)x^4}{2c^5\sqrt[3]{\frac{bx^3}{a}+1}}-\frac{d\sqrt[3]{bx^3+a}x^2}{c^3+d^3x^3}+\frac{\sqrt[3]{bx^3+a}\,\text{AppellF1}\left(\frac{1}{3},-\frac{1}{3},2,\frac{4}{3},-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)x}{c^2\sqrt[3]{\frac{bx^3}{a}+1}}-\frac{\sqrt[3]{b}\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{bx^3+a}}+1}{\sqrt{3}}\right)}{\sqrt{3}d^2}+\frac{2ad\arctan\left(\frac{\frac{2\sqrt[3]{bc^3-ad^3}x}{c\sqrt[3]{bx^3+a}}+1}{\sqrt{3}}\right)}{3\sqrt{3}c\left(bc^3-ad^3\right)^{2/3}}+\frac{\left(3bc^3-2ad^3\right)\arctan\left(\frac{\frac{2\sqrt[3]{bc^3-ad^3}x}{c\sqrt[3]{bx^3+a}}+1}{\sqrt{3}}\right)}{3\sqrt{3}cd^2\left(bc^3-ad^3\right)^{2/3}}-\frac{bc^2\arctan\left(\frac{1-\frac{2d\sqrt[3]{bx^3+a}}{\sqrt[3]{bc^3-ad^3}}}{\sqrt{3}}\right)}{\sqrt{3}d^2\left(bc^3-ad^3\right)^{2/3}}-\frac{ad\log\left(c^3+d^3x^3\right)}{9c\left(bc^3-ad^3\right)^{2/3}}-\frac{\left(3bc^3-2ad^3\right)\log\left(c^3+d^3x^3\right)}{18cd^2\left(bc^3-ad^3\right)^{2/3}}-\frac{bc^2\log\left(c^3+d^3x^3\right)}{6d^2\left(bc^3-ad^3\right)^{2/3}}-\frac{\sqrt[3]{b}\log\left(\sqrt[3]{b}x-\sqrt[3]{bx^3+a}\right)}{2d^2}+\frac{ad\log\left(\frac{\sqrt[3]{bc^3-ad^3}x}{c}-\sqrt[3]{bx^3+a}\right)}{3c\left(bc^3-ad^3\right)^{2/3}}+\frac{\left(3bc^3-2ad^3\right)\log\left(\frac{\sqrt[3]{bc^3-ad^3}x}{c}-\sqrt[3]{bx^3+a}\right)}{6cd^2\left(bc^3-ad^3\right)^{2/3}}+\frac{bc^2\log\left(\sqrt[3]{bx^3+a}d+\sqrt[3]{bc^3-ad^3}\right)}{2d^2\left(bc^3-ad^3\right)^{2/3}}-\frac{c^2\sqrt[3]{bx^3+a}}{d\left(c^3+d^3x^3\right)}$$

input
```
Int[(a + b*x^3)^(1/3)/(c + d*x)^2,x]
```

output
```
-((c^2*(a + b*x^3)^(1/3))/(d*(c^3 + d^3*x^3))) - (d*x^2*(a + b*x^3)^(1/3))
/(c^3 + d^3*x^3) + (x*(a + b*x^3)^(1/3)*AppellF1[1/3, -1/3, 2, 4/3, -((b*x
^3)/a), -((d^3*x^3)/c^3)])/(c^2*(1 + (b*x^3)/a)^(1/3)) - (d^3*x^4*(a + b*x
^3)^(1/3)*AppellF1[4/3, -1/3, 2, 7/3, -((b*x^3)/a), -((d^3*x^3)/c^3)])/(2*
c^5*(1 + (b*x^3)/a)^(1/3)) - (b^(1/3)*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3
)^(1/3))/Sqrt[3]])/(Sqrt[3]*d^2) + (2*a*d*ArcTan[(1 + (2*(b*c^3 - a*d^3)^(
1/3)*x)/(c*(a + b*x^3)^(1/3)))/Sqrt[3]])/(3*Sqrt[3]*c*(b*c^3 - a*d^3)^(2/3
)) + ((3*b*c^3 - 2*a*d^3)*ArcTan[(1 + (2*(b*c^3 - a*d^3)^(1/3)*x)/(c*(a +
b*x^3)^(1/3)))/Sqrt[3]])/(3*Sqrt[3]*c*d^2*(b*c^3 - a*d^3)^(2/3)) - (b*c^2*
ArcTan[(1 - (2*d*(a + b*x^3)^(1/3))/(b*c^3 - a*d^3)^(1/3))/Sqrt[3]])/(Sqrt
[3]*d^2*(b*c^3 - a*d^3)^(2/3)) - (b*c^2*Log[c^3 + d^3*x^3])/(6*d^2*(b*c^3
- a*d^3)^(2/3)) - (a*d*Log[c^3 + d^3*x^3])/(9*c*(b*c^3 - a*d^3)^(2/3)) - (
(3*b*c^3 - 2*a*d^3)*Log[c^3 + d^3*x^3])/(18*c*d^2*(b*c^3 - a*d^3)^(2/3)) -
 (b^(1/3)*Log[b^(1/3)*x - (a + b*x^3)^(1/3)])/(2*d^2) + (a*d*Log[((b*c^3 -
 a*d^3)^(1/3)*x)/c - (a + b*x^3)^(1/3)])/(3*c*(b*c^3 - a*d^3)^(2/3)) + ((3
*b*c^3 - 2*a*d^3)*Log[((b*c^3 - a*d^3)^(1/3)*x)/c - (a + b*x^3)^(1/3)])/(6
*c*d^2*(b*c^3 - a*d^3)^(2/3)) + (b*c^2*Log[(b*c^3 - a*d^3)^(1/3) + d*(a +
b*x^3)^(1/3)])/(2*d^2*(b*c^3 - a*d^3)^(2/3))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2581
```
Int[(Px_.)*((c_) + (d_.)*(x_))^(q_)*((a_) + (b_.)*(x_)^3)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(c^3 + d^3*x^3)^q*(a + b*x^3)^p, Px/(c^2 - c*d*x + d
^2*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p}, x] && PolyQ[Px, x] && ILtQ[q, 0
] && RationalQ[p] && EqQ[Denominator[p], 3]
```

Maple [F]

$$\int \frac{(b\,x^3+a)^{\frac{1}{3}}}{(dx+c)^2}dx$$

input
```
int((b*x^3+a)^(1/3)/(d*x+c)^2,x)
```

output
```
int((b*x^3+a)^(1/3)/(d*x+c)^2,x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^3}}{(c+dx)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x^3+a)^(1/3)/(d*x+c)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{(c+dx)^2}\,dx = \int \frac{\sqrt[3]{a+bx^3}}{(c+dx)^2}\,dx$$

input
```
integrate((b*x**3+a)**(1/3)/(d*x+c)**2,x)
```

output
```
Integral((a + b*x**3)**(1/3)/(c + d*x)**2, x)
```

Maxima [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{(c+dx)^2}\,dx = \int \frac{(bx^3+a)^{\frac{1}{3}}}{(dx+c)^2}\,dx$$

input
```
integrate((b*x^3+a)^(1/3)/(d*x+c)^2,x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(1/3)/(d*x + c)^2, x)
```

Giac [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{(c+dx)^2}\,dx=\int \frac{(bx^3+a)^{\frac{1}{3}}}{(dx+c)^2}\,dx$$

input
```
integrate((b*x^3+a)^(1/3)/(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(1/3)/(d*x + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^3}}{(c+dx)^2}\,dx=\int \frac{(b\,x^3+a)^{1/3}}{(c+d\,x)^2}\,dx$$

input
```
int((a + b*x^3)^(1/3)/(c + d*x)^2,x)
```

output
```
int((a + b*x^3)^(1/3)/(c + d*x)^2, x)
```

Reduce [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{(c+dx)^2}\,dx=\int \frac{(b\,x^3+a)^{\frac{1}{3}}}{d^2x^2+2cdx+c^2}dx$$

input
```
int((b*x^3+a)^(1/3)/(d*x+c)^2,x)
```

output
```
int((a + b*x**3)**(1/3)/(c**2 + 2*c*d*x + d**2*x**2),x)
```

3.149 $\int \frac{(c+dx)^4}{\sqrt[3]{a+bx^3}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 238

$$\int \frac{(c+dx)^4}{\sqrt[3]{a+bx^3}}\,dx = \frac{3c^2d^2(a+bx^3)^{2/3}}{b}+\frac{4cd^3x(a+bx^3)^{2/3}}{3b}+\frac{d^4x^2(a+bx^3)^{2/3}}{4b} + \frac{c(3bc^3-4ad^3)\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}b^{4/3}} + \frac{d(8bc^3-ad^3)\,x^2\sqrt[3]{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{4b\sqrt[3]{a+bx^3}} - \frac{c(3bc^3-4ad^3)\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{6b^{4/3}}$$

output

```
3*c^2*d^2*(b*x^3+a)^(2/3)/b+4/3*c*d^3*x*(b*x^3+a)^(2/3)/b+1/4*d^4*x^2*(b*x
^3+a)^(2/3)/b+1/9*c*(-4*a*d^3+3*b*c^3)*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)
^(1/3))*3^(1/2))*3^(1/2)/b^(4/3)+1/4*d*(-a*d^3+8*b*c^3)*x^2*(1+b*x^3/a)^(1
/3)*hypergeom([1/3, 2/3],[5/3],-b*x^3/a)/b/(b*x^3+a)^(1/3)-1/6*c*(-4*a*d^3
+3*b*c^3)*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/b^(4/3)
```

Mathematica [A] (verified)

Time = 10.53 (sec) , antiderivative size = 392, normalized size of antiderivative = 1.65

$$\int \frac{(c+dx)^4}{\sqrt[3]{a+bx^3}}\,dx$$

$$= \frac{180b^{4/3}c^3dx^2\sqrt[3]{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)+18b^{4/3}d^4x^5\sqrt[3]{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}}{}$$

input
```
Integrate[(c + d*x)^4/(a + b*x^3)^(1/3),x]
```

output
```
(180*b^(4/3)*c^3*d*x^2*(1 + (b*x^3)/a)^(1/3)*Hypergeometric2F1[1/3, 2/3, 5
/3, -((b*x^3)/a)] + 18*b^(4/3)*d^4*x^5*(1 + (b*x^3)/a)^(1/3)*Hypergeometri
c2F1[1/3, 5/3, 8/3, -((b*x^3)/a)] + 5*c*(54*a*b^(1/3)*c*d^2 + 24*a*b^(1/3)
*d^3*x + 54*b^(4/3)*c*d^2*x^3 + 24*b^(4/3)*d^3*x^4 + 2*Sqrt[3]*(3*b*c^3 -
4*a*d^3)*(a + b*x^3)^(1/3)*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sq
rt[3]] + 2*(-3*b*c^3 + 4*a*d^3)*(a + b*x^3)^(1/3)*Log[1 - (b^(1/3)*x)/(a +
 b*x^3)^(1/3)] + 3*b*c^3*(a + b*x^3)^(1/3)*Log[1 + (b^(2/3)*x^2)/(a + b*x^
3)^(2/3) + (b^(1/3)*x)/(a + b*x^3)^(1/3)] - 4*a*d^3*(a + b*x^3)^(1/3)*Log[
1 + (b^(2/3)*x^2)/(a + b*x^3)^(2/3) + (b^(1/3)*x)/(a + b*x^3)^(1/3)]))/(90
*b^(4/3)*(a + b*x^3)^(1/3))
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.30, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2432, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^4}{\sqrt[3]{a+bx^3}}\,dx$$

$$\Big\downarrow \text{ 2432}$$

$$\int \left(\frac{c^4}{\sqrt[3]{a+bx^3}}+\frac{4c^3dx}{\sqrt[3]{a+bx^3}}+\frac{6c^2d^2x^2}{\sqrt[3]{a+bx^3}}+\frac{4cd^3x^3}{\sqrt[3]{a+bx^3}}+\frac{d^4x^4}{\sqrt[3]{a+bx^3}}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{4acd^3\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}b^{4/3}}+\frac{c^4\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}+\frac{2acd^3\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{3b^{4/3}}-\frac{c^4\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}}+\frac{2c^3dx^2\sqrt[3]{\frac{bx^3}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{\sqrt[3]{a+bx^3}}+\frac{3c^2d^2\left(a+bx^3\right)^{2/3}}{b}+\frac{4cd^3x\left(a+bx^3\right)^{2/3}}{3b}+\frac{d^4x^5\sqrt[3]{\frac{bx^3}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{5}{3},\frac{8}{3},-\frac{bx^3}{a}\right)}{5\sqrt[3]{a+bx^3}}$$

input
```
Int[(c + d*x)^4/(a + b*x^3)^(1/3),x]
```

output
```
(3*c^2*d^2*(a + b*x^3)^(2/3))/b + (4*c*d^3*x*(a + b*x^3)^(2/3))/(3*b) + (c
^4*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*b^(1/3)
) - (4*a*c*d^3*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(3*S
qrt[3]*b^(4/3)) + (2*c^3*d*x^2*(1 + (b*x^3)/a)^(1/3)*Hypergeometric2F1[1/3
, 2/3, 5/3, -((b*x^3)/a)])/(a + b*x^3)^(1/3) + (d^4*x^5*(1 + (b*x^3)/a)^(1
/3)*Hypergeometric2F1[1/3, 5/3, 8/3, -((b*x^3)/a)])/(5*(a + b*x^3)^(1/3))
 - (c^4*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/(2*b^(1/3)) + (2*a*c*d^3*Log
[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/(3*b^(4/3))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2432
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

Maple [F]

$$\int \frac{(dx+c)^4}{(b\,x^3+a)^{\frac{1}{3}}}dx$$

input
```
int((d*x+c)^4/(b*x^3+a)^(1/3),x)
```

output
```
int((d*x+c)^4/(b*x^3+a)^(1/3),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(c+dx)^4}{\sqrt[3]{a+bx^3}}\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)^4/(b*x^3+a)^(1/3),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [A] (verification not implemented)

Time = 2.73 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.87

$$\int \frac{(c+dx)^4}{\sqrt[3]{a+bx^3}}\,dx = 6c^2d^2\left(\begin{cases} \frac{x^3}{3\sqrt[3]{a}} & \text{for } b=0 \\ \frac{(a+bx^3)^{\frac{2}{3}}}{2b} & \text{otherwise} \end{cases}\right) + \frac{c^4x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix} \frac{1}{3},\frac{1}{3} \\ \frac{4}{3} \end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\Gamma\left(\frac{4}{3}\right)} + \frac{4c^3dx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix} \frac{1}{3},\frac{2}{3} \\ \frac{5}{3} \end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\Gamma\left(\frac{5}{3}\right)} + \frac{4cd^3x^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix} \frac{1}{3},\frac{4}{3} \\ \frac{7}{3} \end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\Gamma\left(\frac{7}{3}\right)} + \frac{d^4x^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix} \frac{1}{3},\frac{5}{3} \\ \frac{8}{3} \end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\Gamma\left(\frac{8}{3}\right)}$$

input
```
integrate((d*x+c)**4/(b*x**3+a)**(1/3),x)
```

output
```
6*c**2*d**2*Piecewise((x**3/(3*a**(1/3)), Eq(b, 0)), ((a + b*x**3)**(2/3)/
(2*b), True)) + c**4*x*gamma(1/3)*hyper((1/3, 1/3), (4/3,), b*x**3*exp_pol
ar(I*pi)/a)/(3*a**(1/3)*gamma(4/3)) + 4*c**3*d*x**2*gamma(2/3)*hyper((1/3,
 2/3), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(1/3)*gamma(5/3)) + 4*c*d**
3*x**4*gamma(4/3)*hyper((1/3, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*a
**(1/3)*gamma(7/3)) + d**4*x**5*gamma(5/3)*hyper((1/3, 5/3), (8/3,), b*x**
3*exp_polar(I*pi)/a)/(3*a**(1/3)*gamma(8/3))
```

Maxima [F]

$$\int \frac{(c+dx)^4}{\sqrt[3]{a+bx^3}}\,dx = \int \frac{(dx+c)^4}{(bx^3+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate((d*x+c)^4/(b*x^3+a)^(1/3),x, algorithm="maxima")
```

output
```
-1/6*(2*sqrt(3)*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/
3))/b^(1/3) - log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3
)/x^2)/b^(1/3) + 2*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(1/3))*c^4 + inte
grate((d^4*x^4 + 4*c*d^3*x^3 + 6*c^2*d^2*x^2 + 4*c^3*d*x)/(b*x^3 + a)^(1/3
), x)
```

Giac [F]

$$\int \frac{(c+dx)^4}{\sqrt[3]{a+bx^3}}\,dx = \int \frac{(dx+c)^4}{(bx^3+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate((d*x+c)^4/(b*x^3+a)^(1/3),x, algorithm="giac")
```

output
```
integrate((d*x + c)^4/(b*x^3 + a)^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^4}{\sqrt[3]{a+bx^3}}\,dx = \int \frac{(c+d\,x)^4}{(b\,x^3+a)^{1/3}}\,dx$$

input
```
int((c + d*x)^4/(a + b*x^3)^(1/3),x)
```

output
```
int((c + d*x)^4/(a + b*x^3)^(1/3), x)
```

Reduce [F]

$$\int \frac{(c+dx)^4}{\sqrt[3]{a+bx^3}}\,dx = \left(\int \frac{x^4}{(b\,x^3+a)^{\frac{1}{3}}}dx\right) d^4 + 4\left(\int \frac{x^3}{(b\,x^3+a)^{\frac{1}{3}}}dx\right) c\,d^3 + 6\left(\int \frac{x^2}{(b\,x^3+a)^{\frac{1}{3}}}dx\right) c^2d^2 + 4\left(\int \frac{x}{(b\,x^3+a)^{\frac{1}{3}}}dx\right) c^3d + \left(\int \frac{1}{(b\,x^3+a)^{\frac{1}{3}}}dx\right) c^4$$

input
```
int((d*x+c)^4/(b*x^3+a)^(1/3),x)
```

output
```
int(x**4/(a + b*x**3)**(1/3),x)*d**4 + 4*int(x**3/(a + b*x**3)**(1/3),x)*c
*d**3 + 6*int(x**2/(a + b*x**3)**(1/3),x)*c**2*d**2 + 4*int(x/(a + b*x**3)
**(1/3),x)*c**3*d + int(1/(a + b*x**3)**(1/3),x)*c**4
```

3.150 $\int \frac{(c+dx)^3}{\sqrt[3]{a+bx^3}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 198

$$\int \frac{(c+dx)^3}{\sqrt[3]{a+bx^3}}\,dx = \frac{3cd^2(a+bx^3)^{2/3}}{2b}+\frac{d^3x(a+bx^3)^{2/3}}{3b}+\frac{(3bc^3-ad^3)\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}b^{4/3}}+\frac{3c^2dx^2\sqrt[3]{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{2\sqrt[3]{a+bx^3}}-\frac{(3bc^3-ad^3)\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{6b^{4/3}}$$

output
```
3/2*c*d^2*(b*x^3+a)^(2/3)/b+1/3*d^3*x*(b*x^3+a)^(2/3)/b+1/9*(-a*d^3+3*b*c^
3)*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))*3^(1/2)/b^(4/3)+3/2
*c^2*d*x^2*(1+b*x^3/a)^(1/3)*hypergeom([1/3, 2/3],[5/3],-b*x^3/a)/(b*x^3+a
)^(1/3)-1/6*(-a*d^3+3*b*c^3)*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/b^(4/3)
```

Mathematica [A] (verified)

Time = 10.40 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.45

$$\int \frac{(c+dx)^3}{\sqrt[3]{a+bx^3}}\,dx = \frac{1}{18}\left(\frac{27c^2dx^2\sqrt[3]{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{\sqrt[3]{a+bx^3}}\right.$$
$$+\frac{27\sqrt[3]{b}cd^2(a+bx^3)^{2/3}+6\sqrt[3]{b}d^3x(a+bx^3)^{2/3}+2\sqrt{3}(3bc^3-ad^3)\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)+(-6bc^3+2\cdots}{\cdots}$$

input
```
Integrate[(c + d*x)^3/(a + b*x^3)^(1/3),x]
```

output
```
((27*c^2*d*x^2*(1 + (b*x^3)/a)^(1/3)*Hypergeometric2F1[1/3, 2/3, 5/3, -((b
*x^3)/a)])/(a + b*x^3)^(1/3) + (27*b^(1/3)*c*d^2*(a + b*x^3)^(2/3) + 6*b^(
1/3)*d^3*x*(a + b*x^3)^(2/3) + 2*Sqrt[3]*(3*b*c^3 - a*d^3)*ArcTan[(1 + (2*
b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]] + (-6*b*c^3 + 2*a*d^3)*Log[1 - (b^(
1/3)*x)/(a + b*x^3)^(1/3)] + 3*b*c^3*Log[1 + (b^(2/3)*x^2)/(a + b*x^3)^(2/
3) + (b^(1/3)*x)/(a + b*x^3)^(1/3)] - a*d^3*Log[1 + (b^(2/3)*x^2)/(a + b*x
^3)^(2/3) + (b^(1/3)*x)/(a + b*x^3)^(1/3)])/b^(4/3))/18
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.29, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2432, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^3}{\sqrt[3]{a+bx^3}}\,dx$$

$$\Big\downarrow \text{2432}$$

$$\int \left(\frac{c^3}{\sqrt[3]{a+bx^3}}+\frac{3c^2dx}{\sqrt[3]{a+bx^3}}+\frac{3cd^2x^2}{\sqrt[3]{a+bx^3}}+\frac{d^3x^3}{\sqrt[3]{a+bx^3}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{ad^3\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}b^{4/3}}+\frac{c^3\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}+\frac{ad^3\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{6b^{4/3}}-\frac{c^3\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}}+\frac{3c^2dx^2\sqrt[3]{\frac{bx^3}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{2\sqrt[3]{a+bx^3}}+\frac{3cd^2\left(a+bx^3\right)^{2/3}}{2b}+\frac{d^3x\left(a+bx^3\right)^{2/3}}{3b}$$

input
```
Int[(c + d*x)^3/(a + b*x^3)^(1/3),x]
```

output
```
(3*c*d^2*(a + b*x^3)^(2/3))/(2*b) + (d^3*x*(a + b*x^3)^(2/3))/(3*b) + (c^3
*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*b^(1/3))
- (a*d^3*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(3*Sqrt[3]
*b^(4/3)) + (3*c^2*d*x^2*(1 + (b*x^3)/a)^(1/3)*Hypergeometric2F1[1/3, 2/3,
 5/3, -((b*x^3)/a)])/(2*(a + b*x^3)^(1/3)) - (c^3*Log[-(b^(1/3)*x) + (a +
b*x^3)^(1/3)])/(2*b^(1/3)) + (a*d^3*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])
/(6*b^(4/3))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2432
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

Maple [F]

$$\int \frac{(dx+c)^3}{(b\,x^3+a)^{\frac{1}{3}}}dx$$

input
```
int((d*x+c)^3/(b*x^3+a)^(1/3),x)
```

output
```
int((d*x+c)^3/(b*x^3+a)^(1/3),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(c+dx)^3}{\sqrt[3]{a+bx^3}}\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)^3/(b*x^3+a)^(1/3),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [A] (verification not implemented)

Time = 2.27 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.78

$$\int \frac{(c+dx)^3}{\sqrt[3]{a+bx^3}}\,dx = 3cd^2\left(\begin{cases}\frac{x^3}{3\sqrt[3]{a}} & \text{for } b=0\\ \frac{(a+bx^3)^{\frac{2}{3}}}{2b} & \text{otherwise}\end{cases}\right) + \frac{c^3x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\Gamma\left(\frac{4}{3}\right)} + \frac{c^2dx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{\sqrt[3]{a}\Gamma\left(\frac{5}{3}\right)} + \frac{d^3x^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\Gamma\left(\frac{7}{3}\right)}$$

input
```
integrate((d*x+c)**3/(b*x**3+a)**(1/3),x)
```

output
```
3*c*d**2*Piecewise((x**3/(3*a**(1/3)), Eq(b, 0)), ((a + b*x**3)**(2/3)/(2*
b), True)) + c**3*x*gamma(1/3)*hyper((1/3, 1/3), (4/3,), b*x**3*exp_polar(
I*pi)/a)/(3*a**(1/3)*gamma(4/3)) + c**2*d*x**2*gamma(2/3)*hyper((1/3, 2/3)
, (5/3,), b*x**3*exp_polar(I*pi)/a)/(a**(1/3)*gamma(5/3)) + d**3*x**4*gamm
a(4/3)*hyper((1/3, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(1/3)*gam
ma(7/3))
```

Maxima [F]

$$\int \frac{(c+dx)^3}{\sqrt[3]{a+bx^3}}\,dx = \int \frac{(dx+c)^3}{(bx^3+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate((d*x+c)^3/(b*x^3+a)^(1/3),x, algorithm="maxima")
```

output
```
-1/6*(2*sqrt(3)*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/
3))/b^(1/3) - log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3
)/x^2)/b^(1/3) + 2*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(1/3))*c^3 + inte
grate((d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x)/(b*x^3 + a)^(1/3), x)
```

Giac [F]

$$\int \frac{(c+dx)^3}{\sqrt[3]{a+bx^3}}\,dx = \int \frac{(dx+c)^3}{(bx^3+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate((d*x+c)^3/(b*x^3+a)^(1/3),x, algorithm="giac")
```

output
```
integrate((d*x + c)^3/(b*x^3 + a)^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^3}{\sqrt[3]{a+bx^3}}\,dx = \int \frac{(c+d\,x)^3}{(b\,x^3+a)^{1/3}}\,dx$$

input
```
int((c + d*x)^3/(a + b*x^3)^(1/3),x)
```

output
```
int((c + d*x)^3/(a + b*x^3)^(1/3), x)
```

Reduce [F]

$$\int \frac{(c+dx)^3}{\sqrt[3]{a+bx^3}}\,dx = \left(\int \frac{x^3}{(b\,x^3+a)^{\frac{1}{3}}}dx\right) d^3 + 3\left(\int \frac{x^2}{(b\,x^3+a)^{\frac{1}{3}}}dx\right) c\,d^2 + 3\left(\int \frac{x}{(b\,x^3+a)^{\frac{1}{3}}}dx\right) c^2 d + \left(\int \frac{1}{(b\,x^3+a)^{\frac{1}{3}}}dx\right) c^3$$

input
```
int((d*x+c)^3/(b*x^3+a)^(1/3),x)
```

output
```
int(x**3/(a + b*x**3)**(1/3),x)*d**3 + 3*int(x**2/(a + b*x**3)**(1/3),x)*c
*d**2 + 3*int(x/(a + b*x**3)**(1/3),x)*c**2*d + int(1/(a + b*x**3)**(1/3),
x)*c**3
```

3.151 $\int \frac{(c+dx)^2}{\sqrt[3]{a+bx^3}} dx$

Optimal result

Integrand size = 19, antiderivative size = 147

$$\int \frac{(c+dx)^2}{\sqrt[3]{a+bx^3}} dx = \frac{d^2(a+bx^3)^{2/3}}{2b} + \frac{c^2 \arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}} + \frac{cdx^2\sqrt[3]{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{\sqrt[3]{a+bx^3}} - \frac{c^2\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{b}}$$

output

```
1/2*d^2*(b*x^3+a)^(2/3)/b+1/3*c^2*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3
))*3^(1/2))*3^(1/2)/b^(1/3)+c*d*x^2*(1+b*x^3/a)^(1/3)*hypergeom([1/3, 2/3]
,[5/3],-b*x^3/a)/(b*x^3+a)^(1/3)-1/2*c^2*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/b^
(1/3)
```

Mathematica [A] (verified)

Time = 10.22 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.37

$$\int \frac{(c+dx)^2}{\sqrt[3]{a+bx^3}}\,dx = \frac{d^2(a+bx^3)^{2/3}}{2b} + \frac{c^2\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}} + \frac{cdx^2\sqrt[3]{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{\sqrt[3]{a+bx^3}} - \frac{c^2\log\left(1-\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{b}} + \frac{c^2\log\left(1+\frac{b^{2/3}x^2}{(a+bx^3)^{2/3}}+\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{6\sqrt[3]{b}}$$

input
```
Integrate[(c + d*x)^2/(a + b*x^3)^(1/3),x]
```

output
```
(d^2*(a + b*x^3)^(2/3))/(2*b) + (c^2*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)
^(1/3))/Sqrt[3]])/(Sqrt[3]*b^(1/3)) + (c*d*x^2*(1 + (b*x^3)/a)^(1/3)*Hyper
geometric2F1[1/3, 2/3, 5/3, -((b*x^3)/a)])/(a + b*x^3)^(1/3) - (c^2*Log[1
- (b^(1/3)*x)/(a + b*x^3)^(1/3)])/(3*b^(1/3)) + (c^2*Log[1 + (b^(2/3)*x^2)
/(a + b*x^3)^(2/3) + (b^(1/3)*x)/(a + b*x^3)^(1/3)])/(6*b^(1/3))
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {2425, 793, 2432, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^2}{\sqrt[3]{a+bx^3}}dx$$

$$\Big\downarrow \text{ 2425}$$

$$\int \frac{c^2+2dxc}{\sqrt[3]{bx^3+a}}dx+d^2\int \frac{x^2}{\sqrt[3]{bx^3+a}}dx$$

$$\Big\downarrow \text{ 793}$$

$$\int \frac{c^2+2dxc}{\sqrt[3]{bx^3+a}}dx+\frac{d^2\left(a+bx^3\right)^{2/3}}{2b}$$

$$\Big\downarrow \text{ 2432}$$

$$\int \left(\frac{c^2}{\sqrt[3]{bx^3+a}}+\frac{2dxc}{\sqrt[3]{bx^3+a}}\right)dx+\frac{d^2\left(a+bx^3\right)^{2/3}}{2b}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{c^2\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}-\frac{c^2\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}}+\frac{cdx^2\sqrt[3]{\frac{bx^3}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{\sqrt[3]{a+bx^3}}+\frac{d^2\left(a+bx^3\right)^{2/3}}{2b}$$

input
```
Int[(c + d*x)^2/(a + b*x^3)^(1/3),x]
```

output
```
(d^2*(a + b*x^3)^(2/3))/(2*b) + (c^2*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)
^(1/3))/Sqrt[3]])/(Sqrt[3]*b^(1/3)) + (c*d*x^2*(1 + (b*x^3)/a)^(1/3)*Hyper
geometric2F1[1/3, 2/3, 5/3, -((b*x^3)/a)])/(a + b*x^3)^(1/3) - (c^2*Log[-(
b^(1/3)*x) + (a + b*x^3)^(1/3)])/(2*b^(1/3))
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2425
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Pq, x, n -
 1]   Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq,
x, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[P
q, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

rule 2432
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

Maple [F]

$$\int \frac{(dx+c)^2}{(b\,x^3+a)^{\frac{1}{3}}}dx$$

input
```
int((d*x+c)^2/(b*x^3+a)^(1/3),x)
```

output
```
int((d*x+c)^2/(b*x^3+a)^(1/3),x)
```

Fricas [F]

$$\int \frac{(c+dx)^2}{\sqrt[3]{a+bx^3}}\,dx = \int \frac{(dx+c)^2}{(bx^3+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate((d*x+c)^2/(b*x^3+a)^(1/3),x, algorithm="fricas")
```

output
```
integral((d^2*x^2 + 2*c*d*x + c^2)/(b*x^3 + a)^(1/3), x)
```

Sympy [A] (verification not implemented)

Time = 1.54 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.75

$$\int \frac{(c+dx)^2}{\sqrt[3]{a+bx^3}}\,dx = d^2\left(\begin{cases}\frac{x^3}{3\sqrt[3]{a}} & \text{for } b=0\\ \frac{(a+bx^3)^{\frac{2}{3}}}{2b} & \text{otherwise}\end{cases}\right)+\frac{c^2x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\Gamma\left(\frac{4}{3}\right)} + \frac{2cdx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\Gamma\left(\frac{5}{3}\right)}$$

input
```
integrate((d*x+c)**2/(b*x**3+a)**(1/3),x)
```

output
```
d**2*Piecewise((x**3/(3*a**(1/3)), Eq(b, 0)), ((a + b*x**3)**(2/3)/(2*b),
True)) + c**2*x*gamma(1/3)*hyper((1/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi
)/a)/(3*a**(1/3)*gamma(4/3)) + 2*c*d*x**2*gamma(2/3)*hyper((1/3, 2/3), (5/
3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(1/3)*gamma(5/3))
```

Maxima [F]

$$\int \frac{(c+dx)^2}{\sqrt[3]{a+bx^3}}\,dx = \int \frac{(dx+c)^2}{(bx^3+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate((d*x+c)^2/(b*x^3+a)^(1/3),x, algorithm="maxima")
```

output
```
-1/6*(2*sqrt(3)*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/
3))/b^(1/3) - log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3
)/x^2)/b^(1/3) + 2*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(1/3))*c^2 + inte
grate((d^2*x^2 + 2*c*d*x)/(b*x^3 + a)^(1/3), x)
```

Giac [F]

$$\int \frac{(c+dx)^2}{\sqrt[3]{a+bx^3}}\,dx = \int \frac{(dx+c)^2}{(bx^3+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate((d*x+c)^2/(b*x^3+a)^(1/3),x, algorithm="giac")
```

output
```
integrate((d*x + c)^2/(b*x^3 + a)^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2}{\sqrt[3]{a+bx^3}}\,dx = \int \frac{(c+d\,x)^2}{(b\,x^3+a)^{1/3}}\,dx$$

input
```
int((c + d*x)^2/(a + b*x^3)^(1/3),x)
```

output
```
int((c + d*x)^2/(a + b*x^3)^(1/3), x)
```

Reduce [F]

$$\int \frac{(c+dx)^2}{\sqrt[3]{a+bx^3}}\,dx = \left(\int \frac{x^2}{(b\,x^3+a)^{\frac{1}{3}}}dx\right) d^2 + 2\left(\int \frac{x}{(b\,x^3+a)^{\frac{1}{3}}}dx\right) cd + \left(\int \frac{1}{(b\,x^3+a)^{\frac{1}{3}}}dx\right) c^2$$

input
```
int((d*x+c)^2/(b*x^3+a)^(1/3),x)
```

output
```
int(x**2/(a + b*x**3)**(1/3),x)*d**2 + 2*int(x/(a + b*x**3)**(1/3),x)*c*d
+ int(1/(a + b*x**3)**(1/3),x)*c**2
```

3.152 $\int \frac{c+dx}{\sqrt[3]{a+bx^3}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 124

$$\int \frac{c+dx}{\sqrt[3]{a+bx^3}}\,dx = \frac{c\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}} + \frac{dx^2\sqrt[3]{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{2\sqrt[3]{a+bx^3}} - \frac{c\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{b}}$$

output
```
1/3*c*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))*3^(1/2)/b^(1/3)+
1/2*d*x^2*(1+b*x^3/a)^(1/3)*hypergeom([1/3, 2/3],[5/3],-b*x^3/a)/(b*x^3+a)
^(1/3)-1/2*c*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/b^(1/3)
```

Mathematica [A] (verified)

Time = 10.17 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.31

$$\int \frac{c+dx}{\sqrt[3]{a+bx^3}}\,dx = \frac{1}{6}\left(\frac{3dx^2\sqrt[3]{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{\sqrt[3]{a+bx^3}} + \frac{c\left(2\sqrt{3}\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)-2\log\left(1-\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)+\log\left(1+\frac{b^{2/3}x^2}{(a+bx^3)^{2/3}}+\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)\right)}{\sqrt[3]{b}}\right)$$

input
```
Integrate[(c + d*x)/(a + b*x^3)^(1/3),x]
```

output
```
((3*d*x^2*(1 + (b*x^3)/a)^(1/3)*Hypergeometric2F1[1/3, 2/3, 5/3, -((b*x^3)
/a)])/(a + b*x^3)^(1/3) + (c*(2*Sqrt[3]*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x
^3)^(1/3))/Sqrt[3]] - 2*Log[1 - (b^(1/3)*x)/(a + b*x^3)^(1/3)] + Log[1 + (
b^(2/3)*x^2)/(a + b*x^3)^(2/3) + (b^(1/3)*x)/(a + b*x^3)^(1/3)]))/b^(1/3))
/6
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2432, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{\sqrt[3]{a+bx^3}}\,dx$$

$$\Big\downarrow \text{2432}$$

$$\int \left(\frac{c}{\sqrt[3]{a+bx^3}}+\frac{dx}{\sqrt[3]{a+bx^3}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{c\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}-\frac{c\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}}+\frac{dx^2\sqrt[3]{\frac{bx^3}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{2\sqrt[3]{a+bx^3}}$$

input
```
Int[(c + d*x)/(a + b*x^3)^(1/3),x]
```

output
```
(c*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*b^(1/3)
) + (d*x^2*(1 + (b*x^3)/a)^(1/3)*Hypergeometric2F1[1/3, 2/3, 5/3, -((b*x^3
)/a)])/(2*(a + b*x^3)^(1/3)) - (c*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/(
2*b^(1/3))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2432
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

Maple [F]

$$\int \frac{dx+c}{\left(b\,x^{3}+a\right)^{\frac{1}{3}}}d x$$

input
```
int((d*x+c)/(b*x^3+a)^(1/3),x)
```

output
```
int((d*x+c)/(b*x^3+a)^(1/3),x)
```

Fricas [F]

$$\int \frac{c+dx}{\sqrt[3]{a+bx^3}}\,dx = \int \frac{dx+c}{\left(bx^{3}+a\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate((d*x+c)/(b*x^3+a)^(1/3),x, algorithm="fricas")
```

output
```
integral((d*x + c)/(b*x^3 + a)^(1/3), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.13 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.63

$$\int \frac{c+dx}{\sqrt[3]{a+bx^3}}\,dx = \frac{cx\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\right|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\Gamma\left(\frac{4}{3}\right)} + \frac{dx^{2}\Gamma\left(\frac{2}{3}\right){}_{2}F_{1}\left(\left.\begin{matrix}\frac{1}{3},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\right|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\Gamma\left(\frac{5}{3}\right)}$$

input
```
integrate((d*x+c)/(b*x**3+a)**(1/3),x)
```

output
```
c*x*gamma(1/3)*hyper((1/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(
1/3)*gamma(4/3)) + d*x**2*gamma(2/3)*hyper((1/3, 2/3), (5/3,), b*x**3*exp_
polar(I*pi)/a)/(3*a**(1/3)*gamma(5/3))
```

Maxima [F]

$$\int \frac{c+dx}{\sqrt[3]{a+bx^3}}\,dx = \int \frac{dx+c}{(bx^3+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate((d*x+c)/(b*x^3+a)^(1/3),x, algorithm="maxima")
```

output
```
-1/6*(2*sqrt(3)*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/
3))/b^(1/3) - log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3
)/x^2)/b^(1/3) + 2*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(1/3))*c + d*inte
grate(x/(b*x^3 + a)^(1/3), x)
```

Giac [F]

$$\int \frac{c+dx}{\sqrt[3]{a+bx^3}}\,dx = \int \frac{dx+c}{(bx^3+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate((d*x+c)/(b*x^3+a)^(1/3),x, algorithm="giac")
```

output
```
integrate((d*x + c)/(b*x^3 + a)^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx}{\sqrt[3]{a+bx^3}}\,dx = \int \frac{c+d\,x}{(b\,x^3+a)^{1/3}}\,dx$$

input
```
int((c + d*x)/(a + b*x^3)^(1/3),x)
```

output
```
int((c + d*x)/(a + b*x^3)^(1/3), x)
```

Reduce [F]

$$\int \frac{c+dx}{\sqrt[3]{a+bx^3}}\,dx = \left(\int \frac{x}{(b\,x^3+a)^{\frac{1}{3}}}dx\right)d + \left(\int \frac{1}{(b\,x^3+a)^{\frac{1}{3}}}dx\right)c$$

input
```
int((d*x+c)/(b*x^3+a)^(1/3),x)
```

output
```
int(x/(a + b*x**3)**(1/3),x)*d + int(1/(a + b*x**3)**(1/3),x)*c
```

3.153 $\int \frac{1}{(c+dx)\sqrt[3]{a+bx^3}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 333

$$\int \frac{1}{(c+dx)\sqrt[3]{a+bx^3}}\,dx = -\frac{dx^2\sqrt[3]{1+\frac{bx^3}{a}}\operatorname{AppellF1}\left(\frac{2}{3},\frac{1}{3},1,\frac{5}{3},-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)}{2c^2\sqrt[3]{a+bx^3}} + \frac{\arctan\left(\frac{1+\frac{2\sqrt[3]{bc^3-ad^3}x}{c\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{bc^3-ad^3}} - \frac{\arctan\left(\frac{1-\frac{2d\sqrt[3]{a+bx^3}}{\sqrt[3]{bc^3-ad^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{bc^3-ad^3}} + \frac{\log\left(c^3+d^3x^3\right)}{3\sqrt[3]{bc^3-ad^3}} - \frac{\log\left(\frac{\sqrt[3]{bc^3-ad^3}x}{c}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{bc^3-ad^3}} - \frac{\log\left(\sqrt[3]{bc^3-ad^3}+d\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{bc^3-ad^3}}$$

output
```
-1/2*d*x^2*(1+b*x^3/a)^(1/3)*AppellF1(2/3,1/3,1,5/3,-b*x^3/a,-d^3*x^3/c^3)
/c^2/(b*x^3+a)^(1/3)+1/3*arctan(1/3*(1+2*(-a*d^3+b*c^3)^(1/3)*x/c/(b*x^3+a
)^(1/3))*3^(1/2))*3^(1/2)/(-a*d^3+b*c^3)^(1/3)-1/3*arctan(1/3*(1-2*d*(b*x^
3+a)^(1/3)/(-a*d^3+b*c^3)^(1/3))*3^(1/2))*3^(1/2)/(-a*d^3+b*c^3)^(1/3)+1/3
*ln(d^3*x^3+c^3)/(-a*d^3+b*c^3)^(1/3)-1/2*ln((-a*d^3+b*c^3)^(1/3)*x/c-(b*x
^3+a)^(1/3))/(-a*d^3+b*c^3)^(1/3)-1/2*ln((-a*d^3+b*c^3)^(1/3)+d*(b*x^3+a)^
(1/3))/(-a*d^3+b*c^3)^(1/3)
```

Mathematica [F]

$$\int \frac{1}{(c+dx)\sqrt[3]{a+bx^3}}\,dx=\int \frac{1}{(c+dx)\sqrt[3]{a+bx^3}}\,dx$$

input
```
Integrate[1/((c + d*x)*(a + b*x^3)^(1/3)),x]
```

output
```
Integrate[1/((c + d*x)*(a + b*x^3)^(1/3)), x]
```

Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2581, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[3]{a+bx^3}(c+dx)}\,dx$$

$$\Big\downarrow \text{ 2581}$$

$$\int \left(-\frac{cdx}{\sqrt[3]{a+bx^3}\,(c^3+d^3x^3)}+\frac{d^2x^2}{\sqrt[3]{a+bx^3}\,(c^3+d^3x^3)}+\frac{c^2}{\sqrt[3]{a+bx^3}\,(c^3+d^3x^3)}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{dx^2\sqrt[3]{\frac{bx^3}{a}+1}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{3},1,\frac{5}{3},-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)}{2c^2\sqrt[3]{a+bx^3}}+\frac{\arctan\left(\frac{\frac{2x\sqrt[3]{bc^3-ad^3}}{c\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{bc^3-ad^3}}-\frac{\arctan\left(\frac{1-\frac{2d\sqrt[3]{a+bx^3}}{\sqrt[3]{bc^3-ad^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{bc^3-ad^3}}+\frac{\log\left(c^3+d^3x^3\right)}{3\sqrt[3]{bc^3-ad^3}}-\frac{\log\left(\frac{x\sqrt[3]{bc^3-ad^3}}{c}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{bc^3-ad^3}}-\frac{\log\left(\sqrt[3]{bc^3-ad^3}+d\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{bc^3-ad^3}}$$

input
```
Int[1/((c + d*x)*(a + b*x^3)^(1/3)),x]
```

output
```
-1/2*(d*x^2*(1 + (b*x^3)/a)^(1/3)*AppellF1[2/3, 1/3, 1, 5/3, -((b*x^3)/a),
 -((d^3*x^3)/c^3)])/(c^2*(a + b*x^3)^(1/3)) + ArcTan[(1 + (2*(b*c^3 - a*d^
3)^(1/3)*x)/(c*(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*(b*c^3 - a*d^3)^(1/3)
) - ArcTan[(1 - (2*d*(a + b*x^3)^(1/3))/(b*c^3 - a*d^3)^(1/3))/Sqrt[3]]/(S
qrt[3]*(b*c^3 - a*d^3)^(1/3)) + Log[c^3 + d^3*x^3]/(3*(b*c^3 - a*d^3)^(1/3
)) - Log[((b*c^3 - a*d^3)^(1/3)*x)/c - (a + b*x^3)^(1/3)]/(2*(b*c^3 - a*d^
3)^(1/3)) - Log[(b*c^3 - a*d^3)^(1/3) + d*(a + b*x^3)^(1/3)]/(2*(b*c^3 - a
*d^3)^(1/3))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2581
```
Int[(Px_.)*((c_) + (d_.)*(x_))^(q_)*((a_) + (b_.)*(x_)^3)^(p_.), x_Symbol]
 :> Int[ExpandIntegrand[(c^3 + d^3*x^3)^q*(a + b*x^3)^p, Px/(c^2 - c*d*x + d
^2*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p}, x] && PolyQ[Px, x] && ILtQ[q, 0
] && RationalQ[p] && EqQ[Denominator[p], 3]
```

Maple [F]

$$\int \frac{1}{(dx+c)\,(b\,x^{3}+a)^{\frac{1}{3}}}dx$$

input
```
int(1/(d*x+c)/(b*x^3+a)^(1/3),x)
```

output
```
int(1/(d*x+c)/(b*x^3+a)^(1/3),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)\sqrt[3]{a+bx^3}}\,dx = \text{Timed out}$$

input
```
integrate(1/(d*x+c)/(b*x^3+a)^(1/3),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(c+dx)\sqrt[3]{a+bx^3}}\,dx = \int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx)}\,dx$$

input
```
integrate(1/(d*x+c)/(b*x**3+a)**(1/3),x)
```

output
```
Integral(1/((a + b*x**3)**(1/3)*(c + d*x)), x)
```

Maxima [F]

$$\int \frac{1}{(c+dx)\sqrt[3]{a+bx^3}}\,dx = \int \frac{1}{(bx^3+a)^{\frac{1}{3}}(dx+c)}\,dx$$

input
```
integrate(1/(d*x+c)/(b*x^3+a)^(1/3),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^(1/3)*(d*x + c)), x)
```

Giac [F]

$$\int \frac{1}{(c+dx)\sqrt[3]{a+bx^3}}\,dx = \int \frac{1}{(bx^3+a)^{\frac{1}{3}}(dx+c)}\,dx$$

input
```
integrate(1/(d*x+c)/(b*x^3+a)^(1/3),x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a)^(1/3)*(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)\sqrt[3]{a+bx^3}}\,dx = \int \frac{1}{(b\,x^3+a)^{1/3}\,(c+d\,x)}\,dx$$

input
```
int(1/((a + b*x^3)^(1/3)*(c + d*x)),x)
```

output
```
int(1/((a + b*x^3)^(1/3)*(c + d*x)), x)
```

Reduce [F]

$$\int \frac{1}{(c+dx)\sqrt[3]{a+bx^3}}\,dx = \int \frac{1}{(b\,x^3+a)^{\frac{1}{3}}\,c+(b\,x^3+a)^{\frac{1}{3}}\,dx}dx$$

input
```
int(1/(d*x+c)/(b*x^3+a)^(1/3),x)
```

output
```
int(1/((a + b*x**3)**(1/3)*c + (a + b*x**3)**(1/3)*d*x),x)
```

3.154 $\int \frac{1}{(c+dx)^2 \sqrt[3]{a+bx^3}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 761

$$\int \frac{1}{(c+dx)^2\sqrt[3]{a+bx^3}}\,dx = \frac{c^2d^2(a+bx^3)^{2/3}}{(bc^3-ad^3)\,(c^3+d^3x^3)} - \frac{cd^3x(a+bx^3)^{2/3}}{(bc^3-ad^3)\,(c^3+d^3x^3)} - \frac{dx^2\sqrt[3]{1+\frac{bx^3}{a}}\,\mathrm{AppellF1}\left(\frac{2}{3},\frac{1}{3},2,\frac{5}{3},-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)}{c^3\sqrt[3]{a+bx^3}} + \frac{d^4x^5\sqrt[3]{1+\frac{bx^3}{a}}\,\mathrm{AppellF1}\left(\frac{5}{3},\frac{1}{3},2,\frac{8}{3},-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)}{5c^6\sqrt[3]{a+bx^3}} + \frac{2ad^3\arctan\left(\frac{1+\frac{2\sqrt[3]{bc^3-ad^3}x}{c\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}c\,(bc^3-ad^3)^{4/3}} + \frac{(3bc^3-2ad^3)\arctan\left(\frac{1+\frac{2\sqrt[3]{bc^3-ad^3}x}{c\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}c\,(bc^3-ad^3)^{4/3}} - \frac{bc^2\arctan\left(\frac{1-\frac{2d\sqrt[3]{a+bx^3}}{\sqrt[3]{bc^3-ad^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,(bc^3-ad^3)^{4/3}} + \frac{bc^2\log(c^3+d^3x^3)}{6\,(bc^3-ad^3)^{4/3}} + \frac{ad^3\log(c^3+d^3x^3)}{9c\,(bc^3-ad^3)^{4/3}} + \frac{(3bc^3-2ad^3)\log(c^3+d^3x^3)}{18c\,(bc^3-ad^3)^{4/3}} - \frac{ad^3\log\left(\frac{\sqrt[3]{bc^3-ad^3}x}{c}-\sqrt[3]{a+bx^3}\right)}{3c\,(bc^3-ad^3)^{4/3}} - \frac{(3bc^3-2ad^3)\log\left(\frac{\sqrt[3]{bc^3-ad^3}x}{c}-\sqrt[3]{a+bx^3}\right)}{6c\,(bc^3-ad^3)^{4/3}} - \frac{bc^2\log\left(\sqrt[3]{bc^3-ad^3}+d\sqrt[3]{a+bx^3}\right)}{2\,(bc^3-ad^3)^{4/3}}$$

output
```
c^2*d^2*(b*x^3+a)^(2/3)/(-a*d^3+b*c^3)/(d^3*x^3+c^3)-c*d^3*x*(b*x^3+a)^(2/
3)/(-a*d^3+b*c^3)/(d^3*x^3+c^3)-d*x^2*(1+b*x^3/a)^(1/3)*AppellF1(2/3,1/3,2
,5/3,-b*x^3/a,-d^3*x^3/c^3)/c^3/(b*x^3+a)^(1/3)+1/5*d^4*x^5*(1+b*x^3/a)^(1
/3)*AppellF1(5/3,1/3,2,8/3,-b*x^3/a,-d^3*x^3/c^3)/c^6/(b*x^3+a)^(1/3)+2/9*
a*d^3*arctan(1/3*(1+2*(-a*d^3+b*c^3)^(1/3)*x/c/(b*x^3+a)^(1/3))*3^(1/2))*3
^(1/2)/c/(-a*d^3+b*c^3)^(4/3)+1/9*(-2*a*d^3+3*b*c^3)*arctan(1/3*(1+2*(-a*d
^3+b*c^3)^(1/3)*x/c/(b*x^3+a)^(1/3))*3^(1/2))*3^(1/2)/c/(-a*d^3+b*c^3)^(4/
3)-1/3*b*c^2*arctan(1/3*(1-2*d*(b*x^3+a)^(1/3)/(-a*d^3+b*c^3)^(1/3))*3^(1/
2))*3^(1/2)/(-a*d^3+b*c^3)^(4/3)+1/6*b*c^2*ln(d^3*x^3+c^3)/(-a*d^3+b*c^3)^
(4/3)+1/9*a*d^3*ln(d^3*x^3+c^3)/c/(-a*d^3+b*c^3)^(4/3)+1/18*(-2*a*d^3+3*b*
c^3)*ln(d^3*x^3+c^3)/c/(-a*d^3+b*c^3)^(4/3)-1/3*a*d^3*ln((-a*d^3+b*c^3)^(1
/3)*x/c-(b*x^3+a)^(1/3))/c/(-a*d^3+b*c^3)^(4/3)-1/6*(-2*a*d^3+3*b*c^3)*ln(
(-a*d^3+b*c^3)^(1/3)*x/c-(b*x^3+a)^(1/3))/c/(-a*d^3+b*c^3)^(4/3)-1/2*b*c^2
*ln((-a*d^3+b*c^3)^(1/3)+d*(b*x^3+a)^(1/3))/(-a*d^3+b*c^3)^(4/3)
```

Mathematica [F]

$$\int \frac{1}{(c+dx)^2\sqrt[3]{a+bx^3}}\,dx = \int \frac{1}{(c+dx)^2\sqrt[3]{a+bx^3}}\,dx$$

input
```
Integrate[1/((c + d*x)^2*(a + b*x^3)^(1/3)),x]
```

output
```
Integrate[1/((c + d*x)^2*(a + b*x^3)^(1/3)), x]
```

Rubi [A] (verified)

Time = 1.72 (sec) , antiderivative size = 761, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2581, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[3]{a+bx^3}(c+dx)^2}\,dx$$

$$\Big\downarrow \text{2581}$$

$$\int \left(-\frac{2c^3dx}{\sqrt[3]{a+bx^3}\,(c^3+d^3x^3)^2}-\frac{2cd^3x^3}{\sqrt[3]{a+bx^3}\,(c^3+d^3x^3)^2}+\frac{d^4x^4}{\sqrt[3]{a+bx^3}\,(c^3+d^3x^3)^2}+\frac{c^4}{\sqrt[3]{a+bx^3}\,(c^3+d^3x^3)^2}+\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{dx^2\sqrt[3]{\frac{bx^3}{a}+1}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{3},2,\frac{5}{3},-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)}{c^3\sqrt[3]{a+bx^3}}+$$

$$\frac{d^4x^5\sqrt[3]{\frac{bx^3}{a}+1}\,\text{AppellF1}\left(\frac{5}{3},\frac{1}{3},2,\frac{8}{3},-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)}{5c^6\sqrt[3]{a+bx^3}}+\frac{2ad^3\arctan\left(\frac{\frac{2x\sqrt[3]{bc^3-ad^3}}{c\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}c\,(bc^3-ad^3)^{4/3}}+$$

$$\frac{(3bc^3-2ad^3)\arctan\left(\frac{\frac{2x\sqrt[3]{bc^3-ad^3}}{c\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}c\,(bc^3-ad^3)^{4/3}}-\frac{bc^2\arctan\left(\frac{1-\frac{2d\sqrt[3]{a+bx^3}}{\sqrt[3]{bc^3-ad^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,(bc^3-ad^3)^{4/3}}-$$

$$\frac{cd^3x\left(a+bx^3\right)^{2/3}}{(c^3+d^3x^3)\,(bc^3-ad^3)}+\frac{ad^3\log\left(c^3+d^3x^3\right)}{9c\,(bc^3-ad^3)^{4/3}}+\frac{(3bc^3-2ad^3)\log\left(c^3+d^3x^3\right)}{18c\,(bc^3-ad^3)^{4/3}}-$$

$$\frac{ad^3\log\left(\frac{x\sqrt[3]{bc^3-ad^3}}{c}-\sqrt[3]{a+bx^3}\right)}{3c\,(bc^3-ad^3)^{4/3}}-\frac{(3bc^3-2ad^3)\log\left(\frac{x\sqrt[3]{bc^3-ad^3}}{c}-\sqrt[3]{a+bx^3}\right)}{6c\,(bc^3-ad^3)^{4/3}}+$$

$$\frac{bc^2\log\left(c^3+d^3x^3\right)}{6\,(bc^3-ad^3)^{4/3}}-\frac{bc^2\log\left(\sqrt[3]{bc^3-ad^3}+d\sqrt[3]{a+bx^3}\right)}{2\,(bc^3-ad^3)^{4/3}}+\frac{c^2d^2\left(a+bx^3\right)^{2/3}}{(c^3+d^3x^3)\,(bc^3-ad^3)}$$

input
```
Int[1/((c + d*x)^2*(a + b*x^3)^(1/3)),x]
```

output
```
(c^2*d^2*(a + b*x^3)^(2/3))/((b*c^3 - a*d^3)*(c^3 + d^3*x^3)) - (c*d^3*x*(
a + b*x^3)^(2/3))/((b*c^3 - a*d^3)*(c^3 + d^3*x^3)) - (d*x^2*(1 + (b*x^3)/
a)^(1/3)*AppellF1[2/3, 1/3, 2, 5/3, -((b*x^3)/a), -((d^3*x^3)/c^3)])/(c^3*
(a + b*x^3)^(1/3)) + (d^4*x^5*(1 + (b*x^3)/a)^(1/3)*AppellF1[5/3, 1/3, 2,
8/3, -((b*x^3)/a), -((d^3*x^3)/c^3)])/(5*c^6*(a + b*x^3)^(1/3)) + (2*a*d^3
*ArcTan[(1 + (2*(b*c^3 - a*d^3)^(1/3)*x)/(c*(a + b*x^3)^(1/3)))/Sqrt[3]])/
(3*Sqrt[3]*c*(b*c^3 - a*d^3)^(4/3)) + ((3*b*c^3 - 2*a*d^3)*ArcTan[(1 + (2*
(b*c^3 - a*d^3)^(1/3)*x)/(c*(a + b*x^3)^(1/3)))/Sqrt[3]])/(3*Sqrt[3]*c*(b*
c^3 - a*d^3)^(4/3)) - (b*c^2*ArcTan[(1 - (2*d*(a + b*x^3)^(1/3))/(b*c^3 -
a*d^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*(b*c^3 - a*d^3)^(4/3)) + (b*c^2*Log[c^3 +
 d^3*x^3])/(6*(b*c^3 - a*d^3)^(4/3)) + (a*d^3*Log[c^3 + d^3*x^3])/(9*c*(b*
c^3 - a*d^3)^(4/3)) + ((3*b*c^3 - 2*a*d^3)*Log[c^3 + d^3*x^3])/(18*c*(b*c^
3 - a*d^3)^(4/3)) - (a*d^3*Log[((b*c^3 - a*d^3)^(1/3)*x)/c - (a + b*x^3)^(
1/3)])/(3*c*(b*c^3 - a*d^3)^(4/3)) - ((3*b*c^3 - 2*a*d^3)*Log[((b*c^3 - a*
d^3)^(1/3)*x)/c - (a + b*x^3)^(1/3)])/(6*c*(b*c^3 - a*d^3)^(4/3)) - (b*c^2
*Log[(b*c^3 - a*d^3)^(1/3) + d*(a + b*x^3)^(1/3)])/(2*(b*c^3 - a*d^3)^(4/3
))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2581
```
Int[(Px_.)*((c_) + (d_.)*(x_))^(q_)*((a_) + (b_.)*(x_)^3)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(c^3 + d^3*x^3)^q*(a + b*x^3)^p, Px/(c^2 - c*d*x + d
^2*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p}, x] && PolyQ[Px, x] && ILtQ[q, 0
] && RationalQ[p] && EqQ[Denominator[p], 3]
```

Maple [F]

$$\int \frac{1}{(dx+c)^2 (b\,x^3+a)^{\frac{1}{3}}}dx$$

input
```
int(1/(d*x+c)^2/(b*x^3+a)^(1/3),x)
```

output
```
int(1/(d*x+c)^2/(b*x^3+a)^(1/3),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)^2 \sqrt[3]{a+bx^3}}\,dx = \text{Timed out}$$

input
```
integrate(1/(d*x+c)^2/(b*x^3+a)^(1/3),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(c+dx)^2 \sqrt[3]{a+bx^3}}\,dx = \int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx)^2}\,dx$$

input
```
integrate(1/(d*x+c)**2/(b*x**3+a)**(1/3),x)
```

output
```
Integral(1/((a + b*x**3)**(1/3)*(c + d*x)**2), x)
```

Maxima [F]

$$\int \frac{1}{(c+dx)^2 \sqrt[3]{a+bx^3}}\,dx = \int \frac{1}{(bx^3+a)^{\frac{1}{3}}(dx+c)^2}\,dx$$

input
```
integrate(1/(d*x+c)^2/(b*x^3+a)^(1/3),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^(1/3)*(d*x + c)^2), x)
```

Giac [F]

$$\int \frac{1}{(c+dx)^2\sqrt[3]{a+bx^3}}\,dx = \int \frac{1}{(bx^3+a)^{\frac{1}{3}}(dx+c)^2}\,dx$$

input
```
integrate(1/(d*x+c)^2/(b*x^3+a)^(1/3),x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a)^(1/3)*(d*x + c)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)^2\sqrt[3]{a+bx^3}}\,dx = \int \frac{1}{(b\,x^3+a)^{1/3}\,(c+d\,x)^2}\,dx$$

input
```
int(1/((a + b*x^3)^(1/3)*(c + d*x)^2),x)
```

output
```
int(1/((a + b*x^3)^(1/3)*(c + d*x)^2), x)
```

Reduce [F]

$$\int \frac{1}{(c+dx)^2\sqrt[3]{a+bx^3}}\,dx = \int \frac{1}{(b\,x^3+a)^{\frac{1}{3}}\,c^2+2\,(b\,x^3+a)^{\frac{1}{3}}\,cdx+(b\,x^3+a)^{\frac{1}{3}}\,d^2x^2}dx$$

input
```
int(1/(d*x+c)^2/(b*x^3+a)^(1/3),x)
```

output
```
int(1/((a + b*x**3)**(1/3)*c**2 + 2*(a + b*x**3)**(1/3)*c*d*x + (a + b*x**
3)**(1/3)*d**2*x**2),x)
```

3.155 $\int \frac{(c+dx)^4}{(a+bx^3)^{2/3}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 231

$$\int \frac{(c+dx)^4}{(a+bx^3)^{2/3}}\,dx = \frac{6c^2d^2\sqrt[3]{a+bx^3}}{b}+\frac{2cd^3x\sqrt[3]{a+bx^3}}{b}+\frac{d^4x^2\sqrt[3]{a+bx^3}}{3b}-\frac{2d(6bc^3-ad^3)\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}b^{5/3}}+\frac{c(bc^3-2ad^3)\,x\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{b\,(a+bx^3)^{2/3}}-\frac{d(6bc^3-ad^3)\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{3b^{5/3}}$$

output

```
6*c^2*d^2*(b*x^3+a)^(1/3)/b+2*c*d^3*x*(b*x^3+a)^(1/3)/b+1/3*d^4*x^2*(b*x^3
+a)^(1/3)/b-2/9*d*(-a*d^3+6*b*c^3)*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/
3))*3^(1/2))*3^(1/2)/b^(5/3)+c*(-2*a*d^3+b*c^3)*x*(1+b*x^3/a)^(2/3)*hyperg
eom([1/3, 2/3],[4/3],-b*x^3/a)/b/(b*x^3+a)^(2/3)-1/3*d*(-a*d^3+6*b*c^3)*ln
(b^(1/3)*x-(b*x^3+a)^(1/3))/b^(5/3)
```

Mathematica [A] (verified)

Time = 10.16 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.72

$$\int \frac{(c+dx)^4}{(a+bx^3)^{2/3}}\,dx = \frac{3bc^4x\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)+d\Big((6bc^3-ad^3)\,x^2\,\text{Hypergeo}$$

input
```
Integrate[(c + d*x)^4/(a + b*x^3)^(2/3),x]
```

output
```
(3*b*c^4*x*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3
)/a)] + d*((6*b*c^3 - a*d^3)*x^2*Hypergeometric2F1[2/3, 1, 5/3, (b*x^3)/(a
 + b*x^3)] + d*((18*c^2 + d^2*x^2)*(a + b*x^3) + 3*b*c*d*x^4*(1 + (b*x^3)/
a)^(2/3)*Hypergeometric2F1[2/3, 4/3, 7/3, -((b*x^3)/a)])))/(3*b*(a + b*x^3
)^(2/3))
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.32, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2432, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^4}{(a+bx^3)^{2/3}}\,dx$$

$$\Big\downarrow \ 2432$$

$$\int \left(\frac{c^4}{(a+bx^3)^{2/3}}+\frac{4c^3dx}{(a+bx^3)^{2/3}}+\frac{6c^2d^2x^2}{(a+bx^3)^{2/3}}+\frac{4cd^3x^3}{(a+bx^3)^{2/3}}+\frac{d^4x^4}{(a+bx^3)^{2/3}}\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{4c^3d\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}}+\frac{2ad^4\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}b^{5/3}}-\frac{2c^3d\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{b^{2/3}}+\frac{ad^4\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{3b^{5/3}}+\frac{c^4x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{(a+bx^3)^{2/3}}+\frac{6c^2d^2\sqrt[3]{a+bx^3}}{b}+\frac{cd^3x^4\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{4}{3},\frac{7}{3},-\frac{bx^3}{a}\right)}{(a+bx^3)^{2/3}}+\frac{d^4x^2\sqrt[3]{a+bx^3}}{3b}$$

input
```
Int[(c + d*x)^4/(a + b*x^3)^(2/3),x]
```

output
```
(6*c^2*d^2*(a + b*x^3)^(1/3))/b + (d^4*x^2*(a + b*x^3)^(1/3))/(3*b) - (4*c
^3*d*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*b^(2/
3)) + (2*a*d^4*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(3*S
qrt[3]*b^(5/3)) + (c^4*x*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3,
 4/3, -((b*x^3)/a)])/(a + b*x^3)^(2/3) + (c*d^3*x^4*(1 + (b*x^3)/a)^(2/3)*
Hypergeometric2F1[2/3, 4/3, 7/3, -((b*x^3)/a)])/(a + b*x^3)^(2/3) - (2*c^3
*d*Log[b^(1/3)*x - (a + b*x^3)^(1/3)])/b^(2/3) + (a*d^4*Log[b^(1/3)*x - (a
 + b*x^3)^(1/3)])/(3*b^(5/3))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2432
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

Maple [F]

$$\int \frac{(dx+c)^4}{(b\,x^3+a)^{\frac{2}{3}}}dx$$

input
```
int((d*x+c)^4/(b*x^3+a)^(2/3),x)
```

output
```
int((d*x+c)^4/(b*x^3+a)^(2/3),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(c+dx)^4}{(a+bx^3)^{2/3}}\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)^4/(b*x^3+a)^(2/3),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [A] (verification not implemented)

Time = 2.76 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.88

$$\int \frac{(c+dx)^4}{(a+bx^3)^{2/3}}\,dx = 6c^2d^2\left(\begin{cases}\frac{x^3}{3a^{\frac{2}{3}}} & \text{for } b=0\\ \frac{\sqrt[3]{a+bx^3}}{b} & \text{otherwise}\end{cases}\right) + \frac{c^4x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{2}{3}\\ \frac{4}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{4}{3}\right)} + \frac{4c^3dx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{2}{3},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{5}{3}\right)} + \frac{4cd^3x^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{2}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{7}{3}\right)} + \frac{d^4x^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{2}{3},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{8}{3}\right)}$$

input
```
integrate((d*x+c)**4/(b*x**3+a)**(2/3),x)
```

output
```
6*c**2*d**2*Piecewise((x**3/(3*a**(2/3)), Eq(b, 0)), ((a + b*x**3)**(1/3)/
b, True)) + c**4*x*gamma(1/3)*hyper((1/3, 2/3), (4/3,), b*x**3*exp_polar(I
*pi)/a)/(3*a**(2/3)*gamma(4/3)) + 4*c**3*d*x**2*gamma(2/3)*hyper((2/3, 2/3
), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(2/3)*gamma(5/3)) + 4*c*d**3*x*
*4*gamma(4/3)*hyper((2/3, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(2
/3)*gamma(7/3)) + d**4*x**5*gamma(5/3)*hyper((2/3, 5/3), (8/3,), b*x**3*ex
p_polar(I*pi)/a)/(3*a**(2/3)*gamma(8/3))
```

Maxima [F]

$$\int \frac{(c+dx)^4}{(a+bx^3)^{2/3}}\,dx = \int \frac{(dx+c)^4}{(bx^3+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate((d*x+c)^4/(b*x^3+a)^(2/3),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^4/(b*x^3 + a)^(2/3), x)
```

Giac [F]

$$\int \frac{(c+dx)^4}{(a+bx^3)^{2/3}}\,dx = \int \frac{(dx+c)^4}{(bx^3+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate((d*x+c)^4/(b*x^3+a)^(2/3),x, algorithm="giac")
```

output
```
integrate((d*x + c)^4/(b*x^3 + a)^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^4}{(a+bx^3)^{2/3}}\,dx=\int \frac{{\left(c+d\,x\right)}^4}{{\left(b\,x^3+a\right)}^{2/3}} \,d x$$

input
```
int((c + d*x)^4/(a + b*x^3)^(2/3),x)
```

output
```
int((c + d*x)^4/(a + b*x^3)^(2/3), x)
```

Reduce [F]

$$\int \frac{(c+dx)^4}{(a+bx^3)^{2/3}}\,dx=\frac{6(b\,x^3+a)^{\frac{1}{3}}\,c^2 d^2+\left(\int \frac{x^4}{(b\,x^3+a)^{\frac{2}{3}}}dx\right) b\,d^4+4\left(\int \frac{x^3}{(b\,x^3+a)^{\frac{2}{3}}}dx\right) bc\,d^3+4\left(\int \frac{x}{(b\,x^3+a)^{\frac{2}{3}}}dx\right)}{b}$$

input
```
int((d*x+c)^4/(b*x^3+a)^(2/3),x)
```

output
```
(6*(a + b*x**3)**(1/3)*c**2*d**2 + int(x**4/(a + b*x**3)**(2/3),x)*b*d**4
+ 4*int(x**3/(a + b*x**3)**(2/3),x)*b*c*d**3 + 4*int(x/(a + b*x**3)**(2/3)
,x)*b*c**3*d + int(1/(a + b*x**3)**(2/3),x)*b*c**4)/b
```

3.156 $\int \frac{(c+dx)^3}{(a+bx^3)^{2/3}} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 187

$$\int \frac{(c+dx)^3}{(a+bx^3)^{2/3}} \, dx = \frac{3cd^2\sqrt[3]{a+bx^3}}{b} + \frac{d^3x\sqrt[3]{a+bx^3}}{2b} - \frac{\sqrt{3}c^2d\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{b^{2/3}} + \frac{(2bc^3-ad^3)\,x\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2b\,(a+bx^3)^{2/3}} - \frac{3c^2d\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2b^{2/3}}$$

output
```
3*c*d^2*(b*x^3+a)^(1/3)/b+1/2*d^3*x*(b*x^3+a)^(1/3)/b-3^(1/2)*c^2*d*arctan
(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^(2/3)+1/2*(-a*d^3+2*b*c^3)
*x*(1+b*x^3/a)^(2/3)*hypergeom([1/3, 2/3],[4/3],-b*x^3/a)/b/(b*x^3+a)^(2/3
)-3/2*c^2*d*ln(b^(1/3)*x-(b*x^3+a)^(1/3))/b^(2/3)
```

Mathematica [A] (verified)

Time = 10.13 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.78

$$\int \frac{(c+dx)^3}{(a+bx^3)^{2/3}}\,dx = \frac{4bc^3x\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)+d\left(6bc^2x^2\,\text{Hypergeometric2I}\right.}{4l}$$

input
```
Integrate[(c + d*x)^3/(a + b*x^3)^(2/3),x]
```

output
```
(4*b*c^3*x*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3
)/a)] + d*(6*b*c^2*x^2*Hypergeometric2F1[2/3, 1, 5/3, (b*x^3)/(a + b*x^3)]
 + d*(12*c*(a + b*x^3) + b*d*x^4*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[2
/3, 4/3, 7/3, -((b*x^3)/a)])))/(4*b*(a + b*x^3)^(2/3))
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {2427, 2425, 793, 2432, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^3}{(a+bx^3)^{2/3}}dx$$

$$\Big\downarrow \text{2427}$$

$$\frac{\int \frac{2bc^3+6bdxc^2+6bd^2x^2c-ad^3}{(bx^3+a)^{2/3}}dx}{2b}+\frac{d^3x\sqrt[3]{a+bx^3}}{2b}$$

$$\Big\downarrow \text{2425}$$

$$\frac{\int \frac{2bc^3+6bdxc^2-ad^3}{(bx^3+a)^{2/3}}dx+6bcd^2\int \frac{x^2}{(bx^3+a)^{2/3}}dx}{2b}+\frac{d^3x\sqrt[3]{a+bx^3}}{2b}$$

$$\Big\downarrow \text{793}$$

$$\frac{\int \frac{2bc^3+6bdxc^2-ad^3}{(bx^3+a)^{2/3}}dx+6cd^2\sqrt[3]{a+bx^3}}{2b}+\frac{d^3x\sqrt[3]{a+bx^3}}{2b}$$

$\Big\downarrow$ 2432

$$\frac{\int \left(\frac{2b\left(1-\frac{ad^3}{2bc^3}\right)c^3}{(bx^3+a)^{2/3}}+\frac{6bdxc^2}{(bx^3+a)^{2/3}}\right)dx+6cd^2\sqrt[3]{a+bx^3}}{2b}+\frac{d^3x\sqrt[3]{a+bx^3}}{2b}$$

$\Big\downarrow$ 2009

$$\frac{-2\sqrt{3}\sqrt[3]{b}c^2d\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)+\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}(2bc^3-ad^3)\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{(a+bx^3)^{2/3}}-3\sqrt[3]{b}c^2d\log\left(\sqrt[3]{b}x-\right.}{2b}$$

$$\frac{d^3x\sqrt[3]{a+bx^3}}{2b}$$

input
```
Int[(c + d*x)^3/(a + b*x^3)^(2/3),x]
```

output
```
(d^3*x*(a + b*x^3)^(1/3))/(2*b) + (6*c*d^2*(a + b*x^3)^(1/3) - 2*Sqrt[3]*b
^(1/3)*c^2*d*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]] + ((2*b
*c^3 - a*d^3)*x*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((
b*x^3)/a)])/(a + b*x^3)^(2/3) - 3*b^(1/3)*c^2*d*Log[b^(1/3)*x - (a + b*x^3
)^(1/3)])/(2*b)
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2425
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Pq, x, n -
 1]   Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq,
x, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[P
q, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

rule 2427
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{q = Expon[Pq, x
]}, With[{Pqq = Coeff[Pq, x, q]}, Simp[Pqq*x^(q - n + 1)*((a + b*x^n)^(p +
1)/(b*(q + n*p + 1))), x] + Simp[1/(b*(q + n*p + 1))   Int[ExpandToSum[b*(q
 + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(q - n + 1)*x^(q - n), x]*(a + b*x^n)^p,
 x], x]] /; NeQ[q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ
[p + (q + 1)/(2*n)])] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

rule 2432
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

Maple [F]

$$\int \frac{(dx+c)^3}{(b\,x^3+a)^{\frac{2}{3}}}dx$$

input
```
int((d*x+c)^3/(b*x^3+a)^(2/3),x)
```

output
```
int((d*x+c)^3/(b*x^3+a)^(2/3),x)
```

Fricas [F]

$$\int \frac{(c+dx)^3}{(a+bx^3)^{2/3}}\,dx = \int \frac{(dx+c)^3}{(bx^3+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate((d*x+c)^3/(b*x^3+a)^(2/3),x, algorithm="fricas")
```

output
```
integral((d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)/(b*x^3 + a)^(2/3), x)
```

Sympy [A] (verification not implemented)

Time = 2.07 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.82

$$\int \frac{(c+dx)^3}{(a+bx^3)^{2/3}}\,dx = 3cd^2\left(\begin{cases}\frac{x^3}{3a^{\frac{2}{3}}} & \text{for } b=0\\ \frac{\sqrt[3]{a+bx^3}}{b} & \text{otherwise}\end{cases}\right)+\frac{c^3x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{2}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{4}{3}\right)}+\frac{c^2dx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{a^{\frac{2}{3}}\Gamma\left(\frac{5}{3}\right)}+\frac{d^3x^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{7}{3}\right)}$$

input
```
integrate((d*x+c)**3/(b*x**3+a)**(2/3),x)
```

output
```
3*c*d**2*Piecewise((x**3/(3*a**(2/3)), Eq(b, 0)), ((a + b*x**3)**(1/3)/b,
True)) + c**3*x*gamma(1/3)*hyper((1/3, 2/3), (4/3,), b*x**3*exp_polar(I*pi
)/a)/(3*a**(2/3)*gamma(4/3)) + c**2*d*x**2*gamma(2/3)*hyper((2/3, 2/3), (5
/3,), b*x**3*exp_polar(I*pi)/a)/(a**(2/3)*gamma(5/3)) + d**3*x**4*gamma(4/
3)*hyper((2/3, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(2/3)*gamma(7
/3))
```

Maxima [F]

$$\int \frac{(c+dx)^3}{(a+bx^3)^{2/3}}\,dx = \int \frac{(dx+c)^3}{(bx^3+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate((d*x+c)^3/(b*x^3+a)^(2/3),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^3/(b*x^3 + a)^(2/3), x)
```

Giac [F]

$$\int \frac{(c+dx)^3}{(a+bx^3)^{2/3}}\,dx = \int \frac{(dx+c)^3}{(bx^3+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate((d*x+c)^3/(b*x^3+a)^(2/3),x, algorithm="giac")
```

output
```
integrate((d*x + c)^3/(b*x^3 + a)^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^3}{(a+bx^3)^{2/3}}\,dx = \int \frac{(c+d\,x)^3}{(b\,x^3+a)^{2/3}}\,dx$$

input
```
int((c + d*x)^3/(a + b*x^3)^(2/3),x)
```

output
```
int((c + d*x)^3/(a + b*x^3)^(2/3), x)
```

Reduce [F]

$$\int \frac{(c+dx)^3}{(a+bx^3)^{2/3}}\,dx = \frac{3(b\,x^3+a)^{\frac{1}{3}}\,c\,d^2 + \left(\int \frac{x^3}{(b\,x^3+a)^{\frac{2}{3}}}dx\right) b\,d^3 + 3\left(\int \frac{x}{(b\,x^3+a)^{\frac{2}{3}}}dx\right) b\,c^2 d + \left(\int \frac{1}{(b\,x^3+a)^{\frac{2}{3}}}dx\right) b}{b}$$

input
```
int((d*x+c)^3/(b*x^3+a)^(2/3),x)
```

output
```
(3*(a + b*x**3)**(1/3)*c*d**2 + int(x**3/(a + b*x**3)**(2/3),x)*b*d**3 + 3
*int(x/(a + b*x**3)**(2/3),x)*b*c**2*d + int(1/(a + b*x**3)**(2/3),x)*b*c*
*3)/b
```

3.157 $\int \frac{(c+dx)^2}{(a+bx^3)^{2/3}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 141

$$\int \frac{(c+dx)^2}{(a+bx^3)^{2/3}}\,dx = \frac{d^2\sqrt[3]{a+bx^3}}{b}-\frac{2cd\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}}+\frac{c^2x\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{(a+bx^3)^{2/3}}-\frac{cd\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{b^{2/3}}$$

output

```
d^2*(b*x^3+a)^(1/3)/b-2/3*c*d*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3
^(1/2))*3^(1/2)/b^(2/3)+c^2*x*(1+b*x^3/a)^(2/3)*hypergeom([1/3, 2/3],[4/3]
,-b*x^3/a)/(b*x^3+a)^(2/3)-c*d*ln(b^(1/3)*x-(b*x^3+a)^(1/3))/b^(2/3)
```

Mathematica [A] (verified)

Time = 10.06 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.67

$$\int \frac{(c+dx)^2}{(a+bx^3)^{2/3}}\,dx = \frac{bc^2x\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)+d\Big(d(a+bx^3)+bcx^2\,\text{Hyperge}}{b\,(a+bx^3)^{2/3}}$$

input
```
Integrate[(c + d*x)^2/(a + b*x^3)^(2/3),x]
```

output
```
(b*c^2*x*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)/
a)] + d*(d*(a + b*x^3) + b*c*x^2*Hypergeometric2F1[2/3, 1, 5/3, (b*x^3)/(a
 + b*x^3)]))/(b*(a + b*x^3)^(2/3))
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {2425, 793, 2432, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^2}{(a+bx^3)^{2/3}}dx$$

$$\Big\downarrow \text{2425}$$

$$\int \frac{c^2+2dxc}{(bx^3+a)^{2/3}}dx+d^2\int \frac{x^2}{(bx^3+a)^{2/3}}dx$$

$$\Big\downarrow \text{793}$$

$$\int \frac{c^2+2dxc}{(bx^3+a)^{2/3}}dx+\frac{d^2\sqrt[3]{a+bx^3}}{b}$$

$$\Big\downarrow \text{2432}$$

$$\int \left(\frac{c^2}{(bx^3+a)^{2/3}}+\frac{2dxc}{(bx^3+a)^{2/3}}\right)dx+\frac{d^2\sqrt[3]{a+bx^3}}{b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2cd\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}}-\frac{cd\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{b^{2/3}}+\frac{c^2x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{(a+bx^3)^{2/3}}+\frac{d^2\sqrt[3]{a+bx^3}}{b}$$

input
```
Int[(c + d*x)^2/(a + b*x^3)^(2/3),x]
```

output
```
(d^2*(a + b*x^3)^(1/3))/b - (2*c*d*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(
1/3))/Sqrt[3]])/(Sqrt[3]*b^(2/3)) + (c^2*x*(1 + (b*x^3)/a)^(2/3)*Hypergeom
etric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)])/(a + b*x^3)^(2/3) - (c*d*Log[b^(1/3
)*x - (a + b*x^3)^(1/3)])/b^(2/3)
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2425
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Pq, x, n -
 1]   Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq,
x, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[P
q, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

rule 2432
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

Maple [F]

$$\int \frac{(dx+c)^2}{(b\,x^3+a)^{\frac{2}{3}}}dx$$

input
```
int((d*x+c)^2/(b*x^3+a)^(2/3),x)
```

output
```
int((d*x+c)^2/(b*x^3+a)^(2/3),x)
```

Fricas [F]

$$\int \frac{(c+dx)^2}{(a+bx^3)^{2/3}}\,dx = \int \frac{(dx+c)^2}{(bx^3+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate((d*x+c)^2/(b*x^3+a)^(2/3),x, algorithm="fricas")
```

output
```
integral((d^2*x^2 + 2*c*d*x + c^2)/(b*x^3 + a)^(2/3), x)
```

Sympy [A] (verification not implemented)

Time = 1.59 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.77

$$\int \frac{(c+dx)^2}{(a+bx^3)^{2/3}}\,dx = d^2\left(\begin{cases} \frac{x^3}{3a^{\frac{2}{3}}} & \text{for } b=0 \\ \frac{\sqrt[3]{a+bx^3}}{b} & \text{otherwise} \end{cases}\right) + \frac{c^2x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{2}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{4}{3}\right)} + \frac{2cdx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{5}{3}\right)}$$

input
```
integrate((d*x+c)**2/(b*x**3+a)**(2/3),x)
```

output
```
d**2*Piecewise((x**3/(3*a**(2/3)), Eq(b, 0)), ((a + b*x**3)**(1/3)/b, True
)) + c**2*x*gamma(1/3)*hyper((1/3, 2/3), (4/3,), b*x**3*exp_polar(I*pi)/a)
/(3*a**(2/3)*gamma(4/3)) + 2*c*d*x**2*gamma(2/3)*hyper((2/3, 2/3), (5/3,),
 b*x**3*exp_polar(I*pi)/a)/(3*a**(2/3)*gamma(5/3))
```

Maxima [F]

$$\int \frac{(c+dx)^2}{(a+bx^3)^{2/3}}\,dx = \int \frac{(dx+c)^2}{(bx^3+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate((d*x+c)^2/(b*x^3+a)^(2/3),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^2/(b*x^3 + a)^(2/3), x)
```

Giac [F]

$$\int \frac{(c+dx)^2}{(a+bx^3)^{2/3}}\,dx = \int \frac{(dx+c)^2}{(bx^3+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate((d*x+c)^2/(b*x^3+a)^(2/3),x, algorithm="giac")
```

output
```
integrate((d*x + c)^2/(b*x^3 + a)^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2}{(a+bx^3)^{2/3}}\,dx = \int \frac{(c+d\,x)^2}{(b\,x^3+a)^{2/3}}\,dx$$

input
```
int((c + d*x)^2/(a + b*x^3)^(2/3),x)
```

output
```
int((c + d*x)^2/(a + b*x^3)^(2/3), x)
```

Reduce [F]

$$\int \frac{(c+dx)^2}{(a+bx^3)^{2/3}}\,dx = \frac{(b\,x^3+a)^{\frac{1}{3}}\,d^2+2\left(\int \frac{x}{(b\,x^3+a)^{\frac{2}{3}}}dx\right)bcd+\left(\int \frac{1}{(b\,x^3+a)^{\frac{2}{3}}}dx\right)b\,c^2}{b}$$

input
```
int((d*x+c)^2/(b*x^3+a)^(2/3),x)
```

output
```
((a + b*x**3)**(1/3)*d**2 + 2*int(x/(a + b*x**3)**(2/3),x)*b*c*d + int(1/(
a + b*x**3)**(2/3),x)*b*c**2)/b
```

3.158 $\int \frac{c+dx}{(a+bx^3)^{2/3}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 121

$$\int \frac{c+dx}{(a+bx^3)^{2/3}}\,dx = -\frac{d\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}} + \frac{cx\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{(a+bx^3)^{2/3}} - \frac{d\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2b^{2/3}}$$

output
```
-1/3*d*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))*3^(1/2)/b^(2/3)
+c*x*(1+b*x^3/a)^(2/3)*hypergeom([1/3, 2/3],[4/3],-b*x^3/a)/(b*x^3+a)^(2/3
)-1/2*d*ln(b^(1/3)*x-(b*x^3+a)^(1/3))/b^(2/3)
```

Mathematica [A] (verified)

Time = 10.04 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.64

$$\int \frac{c+dx}{(a+bx^3)^{2/3}}\,dx = \frac{x\left(2c\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)+dx\,\text{Hypergeometric2F1}\left(\frac{2}{3},1\right.\right.}{2\,(a+bx^3)^{2/3}}$$

input
```
Integrate[(c + d*x)/(a + b*x^3)^(2/3),x]
```

output
```
(x*(2*c*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)/a
)] + d*x*Hypergeometric2F1[2/3, 1, 5/3, (b*x^3)/(a + b*x^3)]))/(2*(a + b*x
^3)^(2/3))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2432, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{(a+bx^3)^{2/3}}\,dx$$

$$\Big\downarrow \ 2432$$

$$\int \left(\frac{c}{(a+bx^3)^{2/3}}+\frac{dx}{(a+bx^3)^{2/3}}\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{d\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}}-\frac{d\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2b^{2/3}}+\frac{cx\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{(a+bx^3)^{2/3}}$$

input
```
Int[(c + d*x)/(a + b*x^3)^(2/3),x]
```

output
```
-((d*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*b^(2/
3))) + (c*x*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^
3)/a)])/(a + b*x^3)^(2/3) - (d*Log[b^(1/3)*x - (a + b*x^3)^(1/3)])/(2*b^(2
/3))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2432
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

Maple [F]

$$\int \frac{dx+c}{(b\,x^3+a)^{\frac{2}{3}}}dx$$

input
```
int((d*x+c)/(b*x^3+a)^(2/3),x)
```

output
```
int((d*x+c)/(b*x^3+a)^(2/3),x)
```

Fricas [F]

$$\int \frac{c+dx}{(a+bx^3)^{2/3}}\,dx = \int \frac{dx+c}{(bx^3+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate((d*x+c)/(b*x^3+a)^(2/3),x, algorithm="fricas")
```

output
```
integral((d*x + c)/(b*x^3 + a)^(2/3), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.14 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.64

$$\int \frac{c+dx}{(a+bx^3)^{2/3}}\,dx = \frac{cx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{2}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{4}{3}\right)} + \frac{dx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{5}{3}\right)}$$

input
```
integrate((d*x+c)/(b*x**3+a)**(2/3),x)
```

output
```
c*x*gamma(1/3)*hyper((1/3, 2/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(
2/3)*gamma(4/3)) + d*x**2*gamma(2/3)*hyper((2/3, 2/3), (5/3,), b*x**3*exp_
polar(I*pi)/a)/(3*a**(2/3)*gamma(5/3))
```

Maxima [F]

$$\int \frac{c+dx}{(a+bx^3)^{2/3}}\,dx = \int \frac{dx+c}{(bx^3+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate((d*x+c)/(b*x^3+a)^(2/3),x, algorithm="maxima")
```

output
```
integrate((d*x + c)/(b*x^3 + a)^(2/3), x)
```

Giac [F]

$$\int \frac{c+dx}{\left(a+bx^3\right)^{2/3}}\,dx=\int \frac{dx+c}{\left(bx^3+a\right)^{\frac{2}{3}}}\,dx$$

input
```
integrate((d*x+c)/(b*x^3+a)^(2/3),x, algorithm="giac")
```

output
```
integrate((d*x + c)/(b*x^3 + a)^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx}{\left(a+bx^3\right)^{2/3}}\,dx=\int \frac{c+d\,x}{\left(b\,x^3+a\right)^{2/3}}\,dx$$

input
```
int((c + d*x)/(a + b*x^3)^(2/3),x)
```

output
```
int((c + d*x)/(a + b*x^3)^(2/3), x)
```

Reduce [F]

$$\int \frac{c+dx}{\left(a+bx^3\right)^{2/3}}\,dx=\left(\int \frac{x}{\left(b\,x^3+a\right)^{\frac{2}{3}}}dx\right)d+\left(\int \frac{1}{\left(b\,x^3+a\right)^{\frac{2}{3}}}dx\right)c$$

input
```
int((d*x+c)/(b*x^3+a)^(2/3),x)
```

output
```
int(x/(a + b*x**3)**(2/3),x)*d + int(1/(a + b*x**3)**(2/3),x)*c
```

3.159 $\int \frac{1}{(c+dx)(a+bx^3)^{2/3}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 332

$$\int \frac{1}{(c+dx)(a+bx^3)^{2/3}}\,dx = \frac{x\left(1+\frac{bx^3}{a}\right)^{2/3}\operatorname{AppellF1}\left(\frac{1}{3},\frac{2}{3},1,\frac{4}{3},-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)}{c\,(a+bx^3)^{2/3}} + \frac{d\arctan\left(\frac{1+\frac{2\sqrt[3]{bc^3-ad^3}\,x}{c\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,(bc^3-ad^3)^{2/3}} - \frac{d\arctan\left(\frac{1-\frac{2d\sqrt[3]{a+bx^3}}{\sqrt[3]{bc^3-ad^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,(bc^3-ad^3)^{2/3}} - \frac{d\log(c^3+d^3x^3)}{3\,(bc^3-ad^3)^{2/3}} + \frac{d\log\left(\frac{\sqrt[3]{bc^3-ad^3}\,x}{c}-\sqrt[3]{a+bx^3}\right)}{2\,(bc^3-ad^3)^{2/3}} + \frac{d\log\left(\sqrt[3]{bc^3-ad^3}+d\sqrt[3]{a+bx^3}\right)}{2\,(bc^3-ad^3)^{2/3}}$$

output
```
x*(1+b*x^3/a)^(2/3)*AppellF1(1/3,2/3,1,4/3,-b*x^3/a,-d^3*x^3/c^3)/c/(b*x^3
+a)^(2/3)+1/3*d*arctan(1/3*(1+2*(-a*d^3+b*c^3)^(1/3)*x/c/(b*x^3+a)^(1/3))*
3^(1/2))*3^(1/2)/(-a*d^3+b*c^3)^(2/3)-1/3*d*arctan(1/3*(1-2*d*(b*x^3+a)^(1
/3)/(-a*d^3+b*c^3)^(1/3))*3^(1/2))*3^(1/2)/(-a*d^3+b*c^3)^(2/3)-1/3*d*ln(d
^3*x^3+c^3)/(-a*d^3+b*c^3)^(2/3)+1/2*d*ln((-a*d^3+b*c^3)^(1/3)*x/c-(b*x^3+
a)^(1/3))/(-a*d^3+b*c^3)^(2/3)+1/2*d*ln((-a*d^3+b*c^3)^(1/3)+d*(b*x^3+a)^(
1/3))/(-a*d^3+b*c^3)^(2/3)
```

Mathematica [F]

$$\int \frac{1}{(c+dx)(a+bx^3)^{2/3}}\,dx = \int \frac{1}{(c+dx)(a+bx^3)^{2/3}}\,dx$$

input
```
Integrate[1/((c + d*x)*(a + b*x^3)^(2/3)),x]
```

output
```
Integrate[1/((c + d*x)*(a + b*x^3)^(2/3)), x]
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2581, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^3)^{2/3}(c+dx)}\,dx$$

$$\Big\downarrow \text{2581}$$

$$\int \left(-\frac{cdx}{(a+bx^3)^{2/3}(c^3+d^3x^3)}+\frac{d^2x^2}{(a+bx^3)^{2/3}(c^3+d^3x^3)}+\frac{c^2}{(a+bx^3)^{2/3}(c^3+d^3x^3)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{AppellF1}\left(\frac{1}{3},\frac{2}{3},1,\frac{4}{3},-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)}{c\,(a+bx^3)^{2/3}}+\frac{d\arctan\left(\frac{\frac{2x\sqrt[3]{bc^3-ad^3}}{c\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,(bc^3-ad^3)^{2/3}}-\frac{d\arctan\left(\frac{1-\frac{2d\sqrt[3]{a+bx^3}}{\sqrt[3]{bc^3-ad^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,(bc^3-ad^3)^{2/3}}-\frac{d\log\left(c^3+d^3x^3\right)}{3\,(bc^3-ad^3)^{2/3}}+\frac{d\log\left(\frac{x\sqrt[3]{bc^3-ad^3}}{c}-\sqrt[3]{a+bx^3}\right)}{2\,(bc^3-ad^3)^{2/3}}+\frac{d\log\left(\sqrt[3]{bc^3-ad^3}+d\sqrt[3]{a+bx^3}\right)}{2\,(bc^3-ad^3)^{2/3}}$$

input
```
Int[1/((c + d*x)*(a + b*x^3)^(2/3)),x]
```

output
```
(x*(1 + (b*x^3)/a)^(2/3)*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d^3*x
^3)/c^3)])/(c*(a + b*x^3)^(2/3)) + (d*ArcTan[(1 + (2*(b*c^3 - a*d^3)^(1/3)
*x)/(c*(a + b*x^3)^(1/3)))/Sqrt[3]])/(Sqrt[3]*(b*c^3 - a*d^3)^(2/3)) - (d*
ArcTan[(1 - (2*d*(a + b*x^3)^(1/3))/(b*c^3 - a*d^3)^(1/3))/Sqrt[3]])/(Sqrt
[3]*(b*c^3 - a*d^3)^(2/3)) - (d*Log[c^3 + d^3*x^3])/(3*(b*c^3 - a*d^3)^(2/
3)) + (d*Log[((b*c^3 - a*d^3)^(1/3)*x)/c - (a + b*x^3)^(1/3)])/(2*(b*c^3 -
 a*d^3)^(2/3)) + (d*Log[(b*c^3 - a*d^3)^(1/3) + d*(a + b*x^3)^(1/3)])/(2*(
b*c^3 - a*d^3)^(2/3))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2581
```
Int[(Px_.)*((c_) + (d_.)*(x_))^(q_)*((a_) + (b_.)*(x_)^3)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(c^3 + d^3*x^3)^q*(a + b*x^3)^p, Px/(c^2 - c*d*x + d
^2*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p}, x] && PolyQ[Px, x] && ILtQ[q, 0
] && RationalQ[p] && EqQ[Denominator[p], 3]
```

Maple [F]

$$\int \frac{1}{(dx+c)\,(b\,x^3+a)^{\frac{2}{3}}}dx$$

input
```
int(1/(d*x+c)/(b*x^3+a)^(2/3),x)
```

output
```
int(1/(d*x+c)/(b*x^3+a)^(2/3),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)\,(a+bx^3)^{2/3}}\,dx = \text{Timed out}$$

input
```
integrate(1/(d*x+c)/(b*x^3+a)^(2/3),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(c+dx)\,(a+bx^3)^{2/3}}\,dx = \int \frac{1}{(a+bx^3)^{\frac{2}{3}}\,(c+dx)}\,dx$$

input
```
integrate(1/(d*x+c)/(b*x**3+a)**(2/3),x)
```

output
```
Integral(1/((a + b*x**3)**(2/3)*(c + d*x)), x)
```

Maxima [F]

$$\int \frac{1}{(c+dx)\left(a+bx^3\right)^{2/3}}\,dx = \int \frac{1}{(bx^3+a)^{\frac{2}{3}}(dx+c)}\,dx$$

input
```
integrate(1/(d*x+c)/(b*x^3+a)^(2/3),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^(2/3)*(d*x + c)), x)
```

Giac [F]

$$\int \frac{1}{(c+dx)\left(a+bx^3\right)^{2/3}}\,dx = \int \frac{1}{(bx^3+a)^{\frac{2}{3}}(dx+c)}\,dx$$

input
```
integrate(1/(d*x+c)/(b*x^3+a)^(2/3),x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a)^(2/3)*(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)\left(a+bx^3\right)^{2/3}}\,dx = \int \frac{1}{\left(b\,x^3+a\right)^{2/3}\,\left(c+d\,x\right)}\,dx$$

input
```
int(1/((a + b*x^3)^(2/3)*(c + d*x)),x)
```

output
```
int(1/((a + b*x^3)^(2/3)*(c + d*x)), x)
```

Reduce [F]

$$\int \frac{1}{(c+dx)\left(a+bx^3\right)^{2/3}}\,dx = \int \frac{1}{\left(b\,x^3+a\right)^{\frac{2}{3}} c+\left(b\,x^3+a\right)^{\frac{2}{3}} d x}dx$$

input
```
int(1/(d*x+c)/(b*x^3+a)^(2/3),x)
```

output
```
int(1/((a + b*x**3)**(2/3)*c + (a + b*x**3)**(2/3)*d*x),x)
```

3.160 $\int \frac{1}{(c+dx)^2 (a+bx^3)^{2/3}} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 760

$$\int \frac{1}{(c+dx)^2 (a+bx^3)^{2/3}}\,dx = \frac{c^2 d^2 \sqrt[3]{a+bx^3}}{(bc^3-ad^3)(c^3+d^3x^3)} + \frac{d^4 x^2 \sqrt[3]{a+bx^3}}{(bc^3-ad^3)(c^3+d^3x^3)} + \frac{x\left(1+\frac{bx^3}{a}\right)^{2/3} \operatorname{AppellF1}\left(\frac{1}{3},\frac{2}{3},2,\frac{4}{3},-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)}{c^2 (a+bx^3)^{2/3}} - \frac{d^3 x^4 \left(1+\frac{bx^3}{a}\right)^{2/3} \operatorname{AppellF1}\left(\frac{4}{3},\frac{2}{3},2,\frac{7}{3},-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)}{2c^5 (a+bx^3)^{2/3}} + \frac{2ad^4 \arctan\left(\frac{1+\frac{2\sqrt[3]{bc^3-ad^3}\,x}{c\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}c\,(bc^3-ad^3)^{5/3}} + \frac{2d(3bc^3-ad^3)\arctan\left(\frac{1+\frac{2\sqrt[3]{bc^3-ad^3}\,x}{c\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}c\,(bc^3-ad^3)^{5/3}} - \frac{2bc^2 d \arctan\left(\frac{1-\frac{2d\sqrt[3]{a+bx^3}}{\sqrt[3]{bc^3-ad^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,(bc^3-ad^3)^{5/3}} - \frac{bc^2 d\log(c^3+d^3x^3)}{3(bc^3-ad^3)^{5/3}} - \frac{ad^4\log(c^3+d^3x^3)}{9c\,(bc^3-ad^3)^{5/3}} - \frac{d(3bc^3-ad^3)\log(c^3+d^3x^3)}{9c\,(bc^3-ad^3)^{5/3}} + \frac{ad^4\log\left(\frac{\sqrt[3]{bc^3-ad^3}\,x}{c}-\sqrt[3]{a+bx^3}\right)}{3c\,(bc^3-ad^3)^{5/3}} + \frac{d(3bc^3-ad^3)\log\left(\frac{\sqrt[3]{bc^3-ad^3}\,x}{c}-\sqrt[3]{a+bx^3}\right)}{3c\,(bc^3-ad^3)^{5/3}} + \frac{bc^2 d\log\left(\sqrt[3]{bc^3-ad^3}+d\sqrt[3]{a+bx^3}\right)}{(bc^3-ad^3)^{5/3}}$$

output
```
c^2*d^2*(b*x^3+a)^(1/3)/(-a*d^3+b*c^3)/(d^3*x^3+c^3)+d^4*x^2*(b*x^3+a)^(1/
3)/(-a*d^3+b*c^3)/(d^3*x^3+c^3)+x*(1+b*x^3/a)^(2/3)*AppellF1(1/3,2/3,2,4/3
,-b*x^3/a,-d^3*x^3/c^3)/c^2/(b*x^3+a)^(2/3)-1/2*d^3*x^4*(1+b*x^3/a)^(2/3)*
AppellF1(4/3,2/3,2,7/3,-b*x^3/a,-d^3*x^3/c^3)/c^5/(b*x^3+a)^(2/3)+2/9*a*d^
4*arctan(1/3*(1+2*(-a*d^3+b*c^3)^(1/3)*x/c/(b*x^3+a)^(1/3))*3^(1/2))*3^(1/
2)/c/(-a*d^3+b*c^3)^(5/3)+2/9*d*(-a*d^3+3*b*c^3)*arctan(1/3*(1+2*(-a*d^3+b
*c^3)^(1/3)*x/c/(b*x^3+a)^(1/3))*3^(1/2))*3^(1/2)/c/(-a*d^3+b*c^3)^(5/3)-2
/3*b*c^2*d*arctan(1/3*(1-2*d*(b*x^3+a)^(1/3)/(-a*d^3+b*c^3)^(1/3))*3^(1/2)
)*3^(1/2)/(-a*d^3+b*c^3)^(5/3)-1/3*b*c^2*d*ln(d^3*x^3+c^3)/(-a*d^3+b*c^3)^
(5/3)-1/9*a*d^4*ln(d^3*x^3+c^3)/c/(-a*d^3+b*c^3)^(5/3)-1/9*d*(-a*d^3+3*b*c
^3)*ln(d^3*x^3+c^3)/c/(-a*d^3+b*c^3)^(5/3)+1/3*a*d^4*ln((-a*d^3+b*c^3)^(1/
3)*x/c-(b*x^3+a)^(1/3))/c/(-a*d^3+b*c^3)^(5/3)+1/3*d*(-a*d^3+3*b*c^3)*ln((
-a*d^3+b*c^3)^(1/3)*x/c-(b*x^3+a)^(1/3))/c/(-a*d^3+b*c^3)^(5/3)+b*c^2*d*ln
((-a*d^3+b*c^3)^(1/3)+d*(b*x^3+a)^(1/3))/(-a*d^3+b*c^3)^(5/3)
```

Mathematica [F]

$$\int \frac{1}{(c+dx)^2 (a+bx^3)^{2/3}}\,dx=\int \frac{1}{(c+dx)^2 (a+bx^3)^{2/3}}\,dx$$

input
```
Integrate[1/((c + d*x)^2*(a + b*x^3)^(2/3)),x]
```

output
```
Integrate[1/((c + d*x)^2*(a + b*x^3)^(2/3)), x]
```

Rubi [A] (verified)

Time = 1.77 (sec) , antiderivative size = 760, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2581, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+bx^3\right)^{2/3}(c+dx)^2}dx$$

$$\Big\downarrow \text{2581}$$

$$\int \left(-\frac{2c^3dx}{\left(a+bx^3\right)^{2/3}\left(c^3+d^3x^3\right)^2}-\frac{2cd^3x^3}{\left(a+bx^3\right)^{2/3}\left(c^3+d^3x^3\right)^2}+\frac{d^4x^4}{\left(a+bx^3\right)^{2/3}\left(c^3+d^3x^3\right)^2}+\frac{c^4}{\left(a+bx^3\right)^{2/3}\left(c^3+d^3\right.}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{d^3x^4\left(\frac{bx^3}{a}+1\right)^{2/3}\operatorname{AppellF1}\left(\frac{4}{3},\frac{2}{3},2,\frac{7}{3},-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)}{2c^5\left(a+bx^3\right)^{2/3}}+\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\operatorname{AppellF1}\left(\frac{1}{3},\frac{2}{3},2,\frac{4}{3},-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)}{c^2\left(a+bx^3\right)^{2/3}}+\frac{2d\left(3bc^3-ad^3\right)\arctan\left(\frac{\frac{2x\sqrt[3]{bc^3-ad^3}}{c\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}c\left(bc^3-ad^3\right)^{5/3}}+\frac{2ad^4\arctan\left(\frac{\frac{2x\sqrt[3]{bc^3-ad^3}}{c\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}c\left(bc^3-ad^3\right)^{5/3}}-\frac{2bc^2d\arctan\left(\frac{1-\frac{2d\sqrt[3]{a+bx^3}}{\sqrt[3]{bc^3-ad^3}}}{\sqrt{3}}\right)}{\sqrt{3}\left(bc^3-ad^3\right)^{5/3}}-\frac{d\left(3bc^3-ad^3\right)\log\left(c^3+d^3x^3\right)}{9c\left(bc^3-ad^3\right)^{5/3}}+\frac{d\left(3bc^3-ad^3\right)\log\left(\frac{x\sqrt[3]{bc^3-ad^3}}{c}-\sqrt[3]{a+bx^3}\right)}{3c\left(bc^3-ad^3\right)^{5/3}}-\frac{ad^4\log\left(c^3+d^3x^3\right)}{9c\left(bc^3-ad^3\right)^{5/3}}+\frac{ad^4\log\left(\frac{x\sqrt[3]{bc^3-ad^3}}{c}-\sqrt[3]{a+bx^3}\right)}{3c\left(bc^3-ad^3\right)^{5/3}}+\frac{d^4x^2\sqrt[3]{a+bx^3}}{\left(c^3+d^3x^3\right)\left(bc^3-ad^3\right)}-\frac{bc^2d\log\left(c^3+d^3x^3\right)}{3\left(bc^3-ad^3\right)^{5/3}}+\frac{bc^2d\log\left(\sqrt[3]{bc^3-ad^3}+d\sqrt[3]{a+bx^3}\right)}{\left(bc^3-ad^3\right)^{5/3}}+\frac{c^2d^2\sqrt[3]{a+bx^3}}{\left(c^3+d^3x^3\right)\left(bc^3-ad^3\right)}$$

input
```
Int[1/((c + d*x)^2*(a + b*x^3)^(2/3)),x]
```

output
```
(c^2*d^2*(a + b*x^3)^(1/3))/((b*c^3 - a*d^3)*(c^3 + d^3*x^3)) + (d^4*x^2*(
a + b*x^3)^(1/3))/((b*c^3 - a*d^3)*(c^3 + d^3*x^3)) + (x*(1 + (b*x^3)/a)^(
2/3)*AppellF1[1/3, 2/3, 2, 4/3, -((b*x^3)/a), -((d^3*x^3)/c^3)])/(c^2*(a +
 b*x^3)^(2/3)) - (d^3*x^4*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 2, 7/3,
 -((b*x^3)/a), -((d^3*x^3)/c^3)])/(2*c^5*(a + b*x^3)^(2/3)) + (2*a*d^4*Arc
Tan[(1 + (2*(b*c^3 - a*d^3)^(1/3)*x)/(c*(a + b*x^3)^(1/3)))/Sqrt[3]])/(3*S
qrt[3]*c*(b*c^3 - a*d^3)^(5/3)) + (2*d*(3*b*c^3 - a*d^3)*ArcTan[(1 + (2*(b
*c^3 - a*d^3)^(1/3)*x)/(c*(a + b*x^3)^(1/3)))/Sqrt[3]])/(3*Sqrt[3]*c*(b*c^
3 - a*d^3)^(5/3)) - (2*b*c^2*d*ArcTan[(1 - (2*d*(a + b*x^3)^(1/3))/(b*c^3
- a*d^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*(b*c^3 - a*d^3)^(5/3)) - (b*c^2*d*Log[c
^3 + d^3*x^3])/(3*(b*c^3 - a*d^3)^(5/3)) - (a*d^4*Log[c^3 + d^3*x^3])/(9*c
*(b*c^3 - a*d^3)^(5/3)) - (d*(3*b*c^3 - a*d^3)*Log[c^3 + d^3*x^3])/(9*c*(b
*c^3 - a*d^3)^(5/3)) + (a*d^4*Log[((b*c^3 - a*d^3)^(1/3)*x)/c - (a + b*x^3
)^(1/3)])/(3*c*(b*c^3 - a*d^3)^(5/3)) + (d*(3*b*c^3 - a*d^3)*Log[((b*c^3 -
 a*d^3)^(1/3)*x)/c - (a + b*x^3)^(1/3)])/(3*c*(b*c^3 - a*d^3)^(5/3)) + (b*
c^2*d*Log[(b*c^3 - a*d^3)^(1/3) + d*(a + b*x^3)^(1/3)])/(b*c^3 - a*d^3)^(5
/3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2581
```
Int[(Px_.)*((c_) + (d_.)*(x_))^(q_)*((a_) + (b_.)*(x_)^3)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(c^3 + d^3*x^3)^q*(a + b*x^3)^p, Px/(c^2 - c*d*x + d
^2*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p}, x] && PolyQ[Px, x] && ILtQ[q, 0
] && RationalQ[p] && EqQ[Denominator[p], 3]
```

Maple [F]

$$\int \frac{1}{(dx+c)^2 (b\,x^3+a)^{\frac{2}{3}}}dx$$

input
```
int(1/(d*x+c)^2/(b*x^3+a)^(2/3),x)
```

output
```
int(1/(d*x+c)^2/(b*x^3+a)^(2/3),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)^2 (a+bx^3)^{2/3}}\,dx = \text{Timed out}$$

input
```
integrate(1/(d*x+c)^2/(b*x^3+a)^(2/3),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(c+dx)^2 (a+bx^3)^{2/3}}\,dx = \int \frac{1}{(a+bx^3)^{\frac{2}{3}} (c+dx)^2}\,dx$$

input
```
integrate(1/(d*x+c)**2/(b*x**3+a)**(2/3),x)
```

output
```
Integral(1/((a + b*x**3)**(2/3)*(c + d*x)**2), x)
```

Maxima [F]

$$\int \frac{1}{(c+dx)^2 (a+bx^3)^{2/3}}\,dx = \int \frac{1}{(bx^3+a)^{\frac{2}{3}} (dx+c)^2}\,dx$$

input
```
integrate(1/(d*x+c)^2/(b*x^3+a)^(2/3),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^(2/3)*(d*x + c)^2), x)
```

Giac [F]

$$\int \frac{1}{(c+dx)^2 (a+bx^3)^{2/3}}\,dx = \int \frac{1}{(bx^3+a)^{\frac{2}{3}}(dx+c)^2}\,dx$$

input
```
integrate(1/(d*x+c)^2/(b*x^3+a)^(2/3),x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a)^(2/3)*(d*x + c)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)^2 (a+bx^3)^{2/3}}\,dx = \int \frac{1}{(b\,x^3+a)^{2/3}\,(c+d\,x)^2}\,dx$$

input
```
int(1/((a + b*x^3)^(2/3)*(c + d*x)^2),x)
```

output
```
int(1/((a + b*x^3)^(2/3)*(c + d*x)^2), x)
```

Reduce [F]

$$\int \frac{1}{(c+dx)^2 (a+bx^3)^{2/3}}\,dx = \int \frac{1}{(b\,x^3+a)^{\frac{2}{3}}\,c^2+2\,(b\,x^3+a)^{\frac{2}{3}}\,cdx+(b\,x^3+a)^{\frac{2}{3}}\,d^2x^2}dx$$

input
```
int(1/(d*x+c)^2/(b*x^3+a)^(2/3),x)
```

output
```
int(1/((a + b*x**3)**(2/3)*c**2 + 2*(a + b*x**3)**(2/3)*c*d*x + (a + b*x**
3)**(2/3)*d**2*x**2),x)
```

3.161 $\int (d+ex)^3 (a+cx^4) \, dx$

Optimal result

Integrand size = 15, antiderivative size = 62

$$\int (d+ex)^3 (a+cx^4) \, dx = \frac{1}{5}cd^3x^5 + \frac{1}{2}cd^2ex^6 + \frac{3}{7}cde^2x^7 + \frac{1}{8}ce^3x^8 + \frac{a(d+ex)^4}{4e}$$

output
```
1/5*c*d^3*x^5+1/2*c*d^2*e*x^6+3/7*c*d*e^2*x^7+1/8*c*e^3*x^8+1/4*a*(e*x+d)^
4/e
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.37

$$\int (d+ex)^3 (a+cx^4) \, dx = ad^3x + \frac{3}{2}ad^2ex^2 + ade^2x^3 + \frac{1}{4}ae^3x^4 + \frac{1}{5}cd^3x^5 + \frac{1}{2}cd^2ex^6 + \frac{3}{7}cde^2x^7 + \frac{1}{8}ce^3x^8$$

input
```
Integrate[(d + e*x)^3*(a + c*x^4),x]
```

output
```
a*d^3*x + (3*a*d^2*e*x^2)/2 + a*d*e^2*x^3 + (a*e^3*x^4)/4 + (c*d^3*x^5)/5
 + (c*d^2*e*x^6)/2 + (3*c*d*e^2*x^7)/7 + (c*e^3*x^8)/8
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.45, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+cx^4)(d+ex)^3\,dx$$

$$\Big\downarrow \text{2389}$$

$$\int \left(\frac{(d+ex)^3(ae^4+cd^4)}{e^4}-\frac{4cd^3(d+ex)^4}{e^4}+\frac{6cd^2(d+ex)^5}{e^4}+\frac{c(d+ex)^7}{e^4}-\frac{4cd(d+ex)^6}{e^4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{(d+ex)^4(ae^4+cd^4)}{4e^5}-\frac{4cd^3(d+ex)^5}{5e^5}+\frac{cd^2(d+ex)^6}{e^5}+\frac{c(d+ex)^8}{8e^5}-\frac{4cd(d+ex)^7}{7e^5}$$

input
```
Int[(d + e*x)^3*(a + c*x^4),x]
```

output
```
((c*d^4 + a*e^4)*(d + e*x)^4)/(4*e^5) - (4*c*d^3*(d + e*x)^5)/(5*e^5) + (c
*d^2*(d + e*x)^6)/e^5 - (4*c*d*(d + e*x)^7)/(7*e^5) + (c*(d + e*x)^8)/(8*e
^5)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2389
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.19

method	result	size
gosper	$\frac{1}{8}c\,e^{3}x^{8}+\frac{3}{7}cd\,e^{2}x^{7}+\frac{1}{2}c\,d^{2}e\,x^{6}+\frac{1}{5}c\,d^{3}x^{5}+\frac{1}{4}a\,e^{3}x^{4}+ad\,e^{2}x^{3}+\frac{3}{2}a\,d^{2}e\,x^{2}+a\,d^{3}x$	74
default	$\frac{1}{8}c\,e^{3}x^{8}+\frac{3}{7}cd\,e^{2}x^{7}+\frac{1}{2}c\,d^{2}e\,x^{6}+\frac{1}{5}c\,d^{3}x^{5}+\frac{1}{4}a\,e^{3}x^{4}+ad\,e^{2}x^{3}+\frac{3}{2}a\,d^{2}e\,x^{2}+a\,d^{3}x$	74
norman	$\frac{1}{8}c\,e^{3}x^{8}+\frac{3}{7}cd\,e^{2}x^{7}+\frac{1}{2}c\,d^{2}e\,x^{6}+\frac{1}{5}c\,d^{3}x^{5}+\frac{1}{4}a\,e^{3}x^{4}+ad\,e^{2}x^{3}+\frac{3}{2}a\,d^{2}e\,x^{2}+a\,d^{3}x$	74
risch	$\frac{1}{8}c\,e^{3}x^{8}+\frac{3}{7}cd\,e^{2}x^{7}+\frac{1}{2}c\,d^{2}e\,x^{6}+\frac{1}{5}c\,d^{3}x^{5}+\frac{1}{4}a\,e^{3}x^{4}+ad\,e^{2}x^{3}+\frac{3}{2}a\,d^{2}e\,x^{2}+a\,d^{3}x$	74
parallelrisch	$\frac{1}{8}c\,e^{3}x^{8}+\frac{3}{7}cd\,e^{2}x^{7}+\frac{1}{2}c\,d^{2}e\,x^{6}+\frac{1}{5}c\,d^{3}x^{5}+\frac{1}{4}a\,e^{3}x^{4}+ad\,e^{2}x^{3}+\frac{3}{2}a\,d^{2}e\,x^{2}+a\,d^{3}x$	74
orering	$\frac{x\left(35e^{3}c\,x^{7}+120cd\,e^{2}x^{6}+140d^{2}ec\,x^{5}+56c\,d^{3}x^{4}+70e^{3}a\,x^{3}+280ad\,e^{2}x^{2}+420d^{2}eax+280d^{3}a\right)}{280}$	76

input
```
int((e*x+d)^3*(c*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/8*c*e^3*x^8+3/7*c*d*e^2*x^7+1/2*c*d^2*e*x^6+1/5*c*d^3*x^5+1/4*a*e^3*x^4+
a*d*e^2*x^3+3/2*a*d^2*e*x^2+a*d^3*x
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.18

$$\int (d+ex)^3\left(a+cx^4\right)\,dx=\frac{1}{8}\,ce^3x^8+\frac{3}{7}\,cde^2x^7+\frac{1}{2}\,cd^2ex^6+\frac{1}{5}\,cd^3x^5+\frac{1}{4}\,ae^3x^4+ade^2x^3+\frac{3}{2}\,ad^2ex^2+ad^3x$$

input
```
integrate((e*x+d)^3*(c*x^4+a),x, algorithm="fricas")
```

output
```
1/8*c*e^3*x^8 + 3/7*c*d*e^2*x^7 + 1/2*c*d^2*e*x^6 + 1/5*c*d^3*x^5 + 1/4*a*
e^3*x^4 + a*d*e^2*x^3 + 3/2*a*d^2*e*x^2 + a*d^3*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.37

$$\int (d+ex)^3 \left(a+cx^4\right) dx = ad^3x + \frac{3ad^2ex^2}{2} + ade^2x^3 + \frac{ae^3x^4}{4} + \frac{cd^3x^5}{5} + \frac{cd^2ex^6}{2} + \frac{3cde^2x^7}{7} + \frac{ce^3x^8}{8}$$

input
```
integrate((e*x+d)**3*(c*x**4+a),x)
```

output
```
a*d**3*x + 3*a*d**2*e*x**2/2 + a*d*e**2*x**3 + a*e**3*x**4/4 + c*d**3*x**5
/5 + c*d**2*e*x**6/2 + 3*c*d*e**2*x**7/7 + c*e**3*x**8/8
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.18

$$\int (d+ex)^3 \left(a+cx^4\right) dx = \frac{1}{8}ce^3x^8 + \frac{3}{7}cde^2x^7 + \frac{1}{2}cd^2ex^6 + \frac{1}{5}cd^3x^5 + \frac{1}{4}ae^3x^4 + ade^2x^3 + \frac{3}{2}ad^2ex^2 + ad^3x$$

input
```
integrate((e*x+d)^3*(c*x^4+a),x, algorithm="maxima")
```

output
```
1/8*c*e^3*x^8 + 3/7*c*d*e^2*x^7 + 1/2*c*d^2*e*x^6 + 1/5*c*d^3*x^5 + 1/4*a*
e^3*x^4 + a*d*e^2*x^3 + 3/2*a*d^2*e*x^2 + a*d^3*x
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.18

$$\int (d+ex)^3\left(a+cx^4\right)\,dx=\frac{1}{8}ce^3x^8+\frac{3}{7}cde^2x^7+\frac{1}{2}cd^2ex^6+\frac{1}{5}cd^3x^5+\frac{1}{4}ae^3x^4+ade^2x^3+\frac{3}{2}ad^2ex^2+ad^3x$$

input
```
integrate((e*x+d)^3*(c*x^4+a),x, algorithm="giac")
```

output
```
1/8*c*e^3*x^8 + 3/7*c*d*e^2*x^7 + 1/2*c*d^2*e*x^6 + 1/5*c*d^3*x^5 + 1/4*a*
e^3*x^4 + a*d*e^2*x^3 + 3/2*a*d^2*e*x^2 + a*d^3*x
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.18

$$\int (d+ex)^3\left(a+cx^4\right)\,dx=\frac{c\,d^3\,x^5}{5}+a\,d^3\,x+\frac{c\,d^2\,e\,x^6}{2}+\frac{3\,a\,d^2\,e\,x^2}{2}+\frac{3\,c\,d\,e^2\,x^7}{7}+a\,d\,e^2\,x^3+\frac{c\,e^3\,x^8}{8}+\frac{a\,e^3\,x^4}{4}$$

input
```
int((a + c*x^4)*(d + e*x)^3,x)
```

output
```
(a*e^3*x^4)/4 + (c*d^3*x^5)/5 + (c*e^3*x^8)/8 + a*d^3*x + (3*a*d^2*e*x^2)/
2 + a*d*e^2*x^3 + (c*d^2*e*x^6)/2 + (3*c*d*e^2*x^7)/7
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.21

$$\int (d+ex)^3 \left(a+cx^4\right)\,dx$$
$$=\frac{x(35c\,e^3x^7+120cd\,e^2x^6+140c\,d^2e\,x^5+56c\,d^3x^4+70a\,e^3x^3+280ad\,e^2x^2+420a\,d^2ex+280a\,d^3)}{280}$$

input
```
int((e*x+d)^3*(c*x^4+a),x)
```

output
```
(x*(280*a*d**3 + 420*a*d**2*e*x + 280*a*d*e**2*x**2 + 70*a*e**3*x**3 + 56*
c*d**3*x**4 + 140*c*d**2*e*x**5 + 120*c*d*e**2*x**6 + 35*c*e**3*x**7))/280
```

3.162 $\int (d+ex)^2 \left(a+cx^4\right) \, dx$

Optimal result

Integrand size = 15, antiderivative size = 48

$$\int (d+ex)^2 \left(a+cx^4\right) \, dx = \frac{1}{5}cd^2x^5 + \frac{1}{3}cdex^6 + \frac{1}{7}ce^2x^7 + \frac{a(d+ex)^3}{3e}$$

output
```
1/5*c*d^2*x^5+1/3*c*d*e*x^6+1/7*c*e^2*x^7+1/3*a*(e*x+d)^3/e
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.19

$$\int (d+ex)^2 \left(a+cx^4\right) \, dx = ad^2x + adex^2 + \frac{1}{3}ae^2x^3 + \frac{1}{5}cd^2x^5 + \frac{1}{3}cdex^6 + \frac{1}{7}ce^2x^7$$

input
```
Integrate[(d + e*x)^2*(a + c*x^4),x]
```

output
```
a*d^2*x + a*d*e*x^2 + (a*e^2*x^3)/3 + (c*d^2*x^5)/5 + (c*d*e*x^6)/3 + (c*e
^2*x^7)/7
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.19, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+cx^4\right) (d+ex)^2 \, dx$$

$$\Big\downarrow \text{ 2389}$$

$$\int \left(ad^2+2adex+ae^2x^2+cd^2x^4+2cdex^5+ce^2x^6\right) \, dx$$

$$\Big\downarrow \text{ 2009}$$

$$ad^2x+adex^2+\frac{1}{3}ae^2x^3+\frac{1}{5}cd^2x^5+\frac{1}{3}cdex^6+\frac{1}{7}ce^2x^7$$

input
```
Int[(d + e*x)^2*(a + c*x^4),x]
```

output
```
a*d^2*x + a*d*e*x^2 + (a*e^2*x^3)/3 + (c*d^2*x^5)/5 + (c*d*e*x^6)/3 + (c*e
^2*x^7)/7
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2389
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.04

method	result	size
gosper	$\frac{1}{7}c\,e^{2}x^{7}+\frac{1}{3}cde\,x^{6}+\frac{1}{5}c\,d^{2}x^{5}+\frac{1}{3}a\,e^{2}x^{3}+ade\,x^{2}+a\,d^{2}x$	50
default	$\frac{1}{7}c\,e^{2}x^{7}+\frac{1}{3}cde\,x^{6}+\frac{1}{5}c\,d^{2}x^{5}+\frac{1}{3}a\,e^{2}x^{3}+ade\,x^{2}+a\,d^{2}x$	50
norman	$\frac{1}{7}c\,e^{2}x^{7}+\frac{1}{3}cde\,x^{6}+\frac{1}{5}c\,d^{2}x^{5}+\frac{1}{3}a\,e^{2}x^{3}+ade\,x^{2}+a\,d^{2}x$	50
risch	$\frac{1}{7}c\,e^{2}x^{7}+\frac{1}{3}cde\,x^{6}+\frac{1}{5}c\,d^{2}x^{5}+\frac{1}{3}a\,e^{2}x^{3}+ade\,x^{2}+a\,d^{2}x$	50
parallelrisch	$\frac{1}{7}c\,e^{2}x^{7}+\frac{1}{3}cde\,x^{6}+\frac{1}{5}c\,d^{2}x^{5}+\frac{1}{3}a\,e^{2}x^{3}+ade\,x^{2}+a\,d^{2}x$	50
orering	$\frac{x\left(15c\,e^{2}x^{6}+35cde\,x^{5}+21c\,d^{2}x^{4}+35a\,e^{2}x^{2}+105adex+105a\,d^{2}\right)}{105}$	52

input
```
int((e*x+d)^2*(c*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/7*c*e^2*x^7+1/3*c*d*e*x^6+1/5*c*d^2*x^5+1/3*a*e^2*x^3+a*d*e*x^2+a*d^2*x
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.02

$$\int (d+ex)^2\left(a+cx^4\right)\,dx=\frac{1}{7}\,ce^2x^7+\frac{1}{3}\,cdex^6+\frac{1}{5}\,cd^2x^5+\frac{1}{3}\,ae^2x^3+adex^2+ad^2x$$

input
```
integrate((e*x+d)^2*(c*x^4+a),x, algorithm="fricas")
```

output
```
1/7*c*e^2*x^7 + 1/3*c*d*e*x^6 + 1/5*c*d^2*x^5 + 1/3*a*e^2*x^3 + a*d*e*x^2
+ a*d^2*x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.12

$$\int (d+ex)^2 \left(a+cx^4\right) \, dx = ad^2x + adex^2 + \frac{ae^2x^3}{3} + \frac{cd^2x^5}{5} + \frac{cdex^6}{3} + \frac{ce^2x^7}{7}$$

input
```
integrate((e*x+d)**2*(c*x**4+a),x)
```

output
```
a*d**2*x + a*d*e*x**2 + a*e**2*x**3/3 + c*d**2*x**5/5 + c*d*e*x**6/3 + c*e
**2*x**7/7
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.02

$$\int (d+ex)^2 \left(a+cx^4\right) \, dx = \frac{1}{7}\,ce^2x^7 + \frac{1}{3}\,cdex^6 + \frac{1}{5}\,cd^2x^5 + \frac{1}{3}\,ae^2x^3 + adex^2 + ad^2x$$

input
```
integrate((e*x+d)^2*(c*x^4+a),x, algorithm="maxima")
```

output
```
1/7*c*e^2*x^7 + 1/3*c*d*e*x^6 + 1/5*c*d^2*x^5 + 1/3*a*e^2*x^3 + a*d*e*x^2
+ a*d^2*x
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.02

$$\int (d+ex)^2 \left(a+cx^4\right) \, dx = \frac{1}{7}\,ce^2x^7 + \frac{1}{3}\,cdex^6 + \frac{1}{5}\,cd^2x^5 + \frac{1}{3}\,ae^2x^3 + adex^2 + ad^2x$$

input
```
integrate((e*x+d)^2*(c*x^4+a),x, algorithm="giac")
```

output
```
1/7*c*e^2*x^7 + 1/3*c*d*e*x^6 + 1/5*c*d^2*x^5 + 1/3*a*e^2*x^3 + a*d*e*x^2
+ a*d^2*x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.02

$$\int (d+ex)^2\left(a+cx^4\right)\,dx=\frac{c\,d^2\,x^5}{5}+a\,d^2\,x+\frac{c\,d\,e\,x^6}{3}+a\,d\,e\,x^2+\frac{c\,e^2\,x^7}{7}+\frac{a\,e^2\,x^3}{3}$$

input
```
int((a + c*x^4)*(d + e*x)^2,x)
```

output
```
(a*e^2*x^3)/3 + (c*d^2*x^5)/5 + (c*e^2*x^7)/7 + a*d^2*x + a*d*e*x^2 + (c*d
*e*x^6)/3
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.06

$$\int (d+ex)^2\left(a+cx^4\right)\,dx=\frac{x(15c\,e^2x^6+35cde\,x^5+21c\,d^2x^4+35a\,e^2x^2+105adex+105a\,d^2)}{105}$$

input
```
int((e*x+d)^2*(c*x^4+a),x)
```

output
```
(x*(105*a*d**2 + 105*a*d*e*x + 35*a*e**2*x**2 + 21*c*d**2*x**4 + 35*c*d*e*
x**5 + 15*c*e**2*x**6))/105
```

3.163 $\int (d+ex)(a+cx^4)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 34

$$\int (d+ex)(a+cx^4)\,dx=\frac{1}{5}cdx^5+\frac{1}{6}cex^6+\frac{a(d+ex)^2}{2e}$$

output
```
1/5*c*d*x^5+1/6*c*e*x^6+1/2*a*(e*x+d)^2/e
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.94

$$\int (d+ex)(a+cx^4)\,dx=adx+\frac{1}{2}aex^2+\frac{1}{5}cdx^5+\frac{1}{6}cex^6$$

input
```
Integrate[(d + e*x)*(a + c*x^4),x]
```

output
```
a*d*x + (a*e*x^2)/2 + (c*d*x^5)/5 + (c*e*x^6)/6
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.94, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+cx^4)(d+ex)\,dx$$

$$\Big\downarrow \text{ 2389}$$

$$\int (ad+aex+cdx^4+cex^5)\,dx$$

$$\Big\downarrow \text{ 2009}$$

$$adx+\frac{1}{2}aex^2+\frac{1}{5}cdx^5+\frac{1}{6}cex^6$$

input
```
Int[(d + e*x)*(a + c*x^4),x]
```

output
```
a*d*x + (a*e*x^2)/2 + (c*d*x^5)/5 + (c*e*x^6)/6
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2389
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.79

method	result	size
gosper	$\frac{1}{6}ce\,x^6+\frac{1}{5}cd\,x^5+\frac{1}{2}ae\,x^2+adx$	27
default	$\frac{1}{6}ce\,x^6+\frac{1}{5}cd\,x^5+\frac{1}{2}ae\,x^2+adx$	27
norman	$\frac{1}{6}ce\,x^6+\frac{1}{5}cd\,x^5+\frac{1}{2}ae\,x^2+adx$	27
risch	$\frac{1}{6}ce\,x^6+\frac{1}{5}cd\,x^5+\frac{1}{2}ae\,x^2+adx$	27
parallelrisch	$\frac{1}{6}ce\,x^6+\frac{1}{5}cd\,x^5+\frac{1}{2}ae\,x^2+adx$	27
orering	$\frac{x(5ce\,x^5+6cd\,x^4+15aex+30ad)}{30}$	28

input
```
int((e*x+d)*(c*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/6*c*e*x^6+1/5*c*d*x^5+1/2*a*e*x^2+a*d*x
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

$$\int (d+ex)\left(a+cx^4\right)\,dx=\frac{1}{6}\,cex^6+\frac{1}{5}\,cdx^5+\frac{1}{2}\,aex^2+adx$$

input
```
integrate((e*x+d)*(c*x^4+a),x, algorithm="fricas")
```

output
```
1/6*c*e*x^6 + 1/5*c*d*x^5 + 1/2*a*e*x^2 + a*d*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.85

$$\int (d+ex)\left(a+cx^4\right)\,dx=adx+\frac{aex^2}{2}+\frac{cdx^5}{5}+\frac{cex^6}{6}$$

input
```
integrate((e*x+d)*(c*x**4+a),x)
```

output
```
a*d*x + a*e*x**2/2 + c*d*x**5/5 + c*e*x**6/6
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

$$\int (d+ex)\left(a+cx^4\right)\,dx=\frac{1}{6}\,cex^6+\frac{1}{5}\,cdx^5+\frac{1}{2}\,aex^2+adx$$

input
```
integrate((e*x+d)*(c*x^4+a),x, algorithm="maxima")
```

output
```
1/6*c*e*x^6 + 1/5*c*d*x^5 + 1/2*a*e*x^2 + a*d*x
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

$$\int (d+ex)\left(a+cx^4\right)\,dx=\frac{1}{6}\,cex^6+\frac{1}{5}\,cdx^5+\frac{1}{2}\,aex^2+adx$$

input
```
integrate((e*x+d)*(c*x^4+a),x, algorithm="giac")
```

output
```
1/6*c*e*x^6 + 1/5*c*d*x^5 + 1/2*a*e*x^2 + a*d*x
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

$$\int (d+ex)\left(a+cx^4\right)\,dx=\frac{c\,e\,x^6}{6}+\frac{c\,d\,x^5}{5}+\frac{a\,e\,x^2}{2}+a\,d\,x$$

input
```
int((a + c*x^4)*(d + e*x),x)
```

output
```
a*d*x + (a*e*x^2)/2 + (c*d*x^5)/5 + (c*e*x^6)/6
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.79

$$\int (d+ex)\left(a+cx^4\right)\,dx=\frac{x(5ce\,x^5+6cd\,x^4+15aex+30ad)}{30}$$

input
```
int((e*x+d)*(c*x^4+a),x)
```

output
```
(x*(30*a*d + 15*a*e*x + 6*c*d*x**4 + 5*c*e*x**5))/30
```

3.164 $\int (a+cx^4)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 12

$$\int (a+cx^4)\,dx=ax+\frac{cx^5}{5}$$

output
```
a*x+1/5*c*x^5
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int (a+cx^4)\,dx=ax+\frac{cx^5}{5}$$

input
```
Integrate[a + c*x^4,x]
```

output
```
a*x + (c*x^5)/5
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+cx^4)\,dx$$

$$\Big\downarrow \text{2009}$$

$$ax+\frac{cx^5}{5}$$

input
```
Int[a + c*x^4,x]
```

output
```
a*x + (c*x^5)/5
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

method	result	size
gosper	$xa+\frac{1}{5}c\,x^{5}$	11
default	$xa+\frac{1}{5}c\,x^{5}$	11
norman	$xa+\frac{1}{5}c\,x^{5}$	11
risch	$xa+\frac{1}{5}c\,x^{5}$	11
parallelrisch	$xa+\frac{1}{5}c\,x^{5}$	11
parts	$xa+\frac{1}{5}c\,x^{5}$	11
orering	$\frac{x(c\,x^{4}+5a)}{5}$	13

input
```
int(c*x^4+a,x,method=_RETURNVERBOSE)
```

output
```
x*a+1/5*c*x^5
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \left(a+cx^4\right)\,dx=\frac{1}{5}\,cx^5+ax$$

input
```
integrate(c*x^4+a,x, algorithm="fricas")
```

output
```
1/5*c*x^5 + a*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int (a + cx^4) \, dx = ax + \frac{cx^5}{5}$$

input
```
integrate(c*x**4+a,x)
```

output
```
a*x + c*x**5/5
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int (a + cx^4) \, dx = \frac{1}{5} cx^5 + ax$$

input
```
integrate(c*x^4+a,x, algorithm="maxima")
```

output
```
1/5*c*x^5 + a*x
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int (a + cx^4) \, dx = \frac{1}{5} cx^5 + ax$$

input
```
integrate(c*x^4+a,x, algorithm="giac")
```

output
```
1/5*c*x^5 + a*x
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int (a + cx^4) \, dx = \frac{c\,x^5}{5} + a\,x$$

input
```
int(a + c*x^4,x)
```

output
```
a*x + (c*x^5)/5
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int (a + cx^4) \, dx = \frac{x(c\,x^4 + 5a)}{5}$$

input
```
int(c*x^4+a,x)
```

output
```
(x*(5*a + c*x**4))/5
```

3.165 $\int \frac{a+cx^4}{d+ex}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 69

$$\int \frac{a+cx^4}{d+ex}\,dx=-\frac{cd^3x}{e^4}+\frac{cd^2x^2}{2e^3}-\frac{cdx^3}{3e^2}+\frac{cx^4}{4e}+\frac{(cd^4+ae^4)\log(d+ex)}{e^5}$$

output
```
-c*d^3*x/e^4+1/2*c*d^2*x^2/e^3-1/3*c*d*x^3/e^2+1/4*c*x^4/e+(a*e^4+c*d^4)*l
n(e*x+d)/e^5
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.88

$$\int \frac{a+cx^4}{d+ex}\,dx=\frac{cex(-12d^3+6d^2ex-4de^2x^2+3e^3x^3)+12(cd^4+ae^4)\log(d+ex)}{12e^5}$$

input
```
Integrate[(a + c*x^4)/(d + e*x),x]
```

output
```
(c*e*x*(-12*d^3 + 6*d^2*e*x - 4*d*e^2*x^2 + 3*e^3*x^3) + 12*(c*d^4 + a*e^4
)*Log[d + e*x])/(12*e^5)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+cx^4}{d+ex}\,dx$$
$$\Big\downarrow \text{ 2389}$$
$$\int \left(\frac{ae^4+cd^4}{e^4(d+ex)}-\frac{cd^3}{e^4}+\frac{cd^2x}{e^3}-\frac{cdx^2}{e^2}+\frac{cx^3}{e}\right)dx$$
$$\Big\downarrow \text{ 2009}$$
$$\frac{(ae^4+cd^4)\log(d+ex)}{e^5}-\frac{cd^3x}{e^4}+\frac{cd^2x^2}{2e^3}-\frac{cdx^3}{3e^2}+\frac{cx^4}{4e}$$

input
```
Int[(a + c*x^4)/(d + e*x),x]
```

output
```
-((c*d^3*x)/e^4) + (c*d^2*x^2)/(2*e^3) - (c*d*x^3)/(3*e^2) + (c*x^4)/(4*e)
 + ((c*d^4 + a*e^4)*Log[d + e*x])/e^5
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2389
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.88

method	result	size
default	$-\frac{c\left(-\frac{1}{4}x^{4}e^{3}+\frac{1}{3}d\,e^{2}x^{3}-\frac{1}{2}e\,x^{2}d^{2}+d^{3}x\right)}{e^{4}}+\frac{\left(e^{4}a+c\,d^{4}\right)\ln\left(ex+d\right)}{e^{5}}$	61
norman	$-\frac{c\,d^{3}x}{e^{4}}+\frac{c\,d^{2}x^{2}}{2e^{3}}-\frac{cd\,x^{3}}{3e^{2}}+\frac{c\,x^{4}}{4e}+\frac{\left(e^{4}a+c\,d^{4}\right)\ln\left(ex+d\right)}{e^{5}}$	64
risch	$\frac{c\,x^{4}}{4e}-\frac{cd\,x^{3}}{3e^{2}}+\frac{c\,d^{2}x^{2}}{2e^{3}}-\frac{c\,d^{3}x}{e^{4}}+\frac{\ln\left(ex+d\right)a}{e}+\frac{\ln\left(ex+d\right)c\,d^{4}}{e^{5}}$	68
parallelrisch	$\frac{3x^{4}c\,e^{4}-4cd\,x^{3}e^{3}+6x^{2}c\,d^{2}e^{2}+12\ln\left(ex+d\right)a\,e^{4}+12\ln\left(ex+d\right)c\,d^{4}-12c\,d^{3}xe}{12e^{5}}$	70

input
```
int((c*x^4+a)/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
-c/e^4*(-1/4*x^4*e^3+1/3*d*e^2*x^3-1/2*e*x^2*d^2+d^3*x)+(a*e^4+c*d^4)*ln(e
*x+d)/e^5
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.93

$$\int \frac{a+cx^4}{d+ex}\,dx=\frac{3\,ce^4x^4-4\,cde^3x^3+6\,cd^2e^2x^2-12\,cd^3ex+12\,(cd^4+ae^4)\log(ex+d)}{12\,e^5}$$

input
```
integrate((c*x^4+a)/(e*x+d),x, algorithm="fricas")
```

output
```
1/12*(3*c*e^4*x^4 - 4*c*d*e^3*x^3 + 6*c*d^2*e^2*x^2 - 12*c*d^3*e*x + 12*(c
*d^4 + a*e^4)*log(e*x + d))/e^5
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.91

$$\int \frac{a+cx^4}{d+ex}\,dx = -\frac{cd^3x}{e^4}+\frac{cd^2x^2}{2e^3}-\frac{cdx^3}{3e^2}+\frac{cx^4}{4e}+\frac{(ae^4+cd^4)\log(d+ex)}{e^5}$$

input
```
integrate((c*x**4+a)/(e*x+d),x)
```

output
```
-c*d**3*x/e**4 + c*d**2*x**2/(2*e**3) - c*d*x**3/(3*e**2) + c*x**4/(4*e) +
 (a*e**4 + c*d**4)*log(d + e*x)/e**5
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.93

$$\int \frac{a+cx^4}{d+ex}\,dx = \frac{3\,ce^3x^4-4\,cde^2x^3+6\,cd^2ex^2-12\,cd^3x}{12\,e^4}+\frac{(cd^4+ae^4)\log(ex+d)}{e^5}$$

input
```
integrate((c*x^4+a)/(e*x+d),x, algorithm="maxima")
```

output
```
1/12*(3*c*e^3*x^4 - 4*c*d*e^2*x^3 + 6*c*d^2*e*x^2 - 12*c*d^3*x)/e^4 + (c*d
^4 + a*e^4)*log(e*x + d)/e^5
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.94

$$\int \frac{a+cx^4}{d+ex}\,dx = \frac{3\,ce^3x^4-4\,cde^2x^3+6\,cd^2ex^2-12\,cd^3x}{12\,e^4}+\frac{(cd^4+ae^4)\log(|ex+d|)}{e^5}$$

input
```
integrate((c*x^4+a)/(e*x+d),x, algorithm="giac")
```

output
```
1/12*(3*c*e^3*x^4 - 4*c*d*e^2*x^3 + 6*c*d^2*e*x^2 - 12*c*d^3*x)/e^4 + (c*d
^4 + a*e^4)*log(abs(e*x + d))/e^5
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.91

$$\int \frac{a+cx^4}{d+ex}\,dx=\frac{c\,x^4}{4\,e}+\frac{\ln\left(d+e\,x\right)\,\left(c\,d^4+a\,e^4\right)}{e^5}+\frac{c\,d^2\,x^2}{2\,e^3}-\frac{c\,d\,x^3}{3\,e^2}-\frac{c\,d^3\,x}{e^4}$$

input
```
int((a + c*x^4)/(d + e*x),x)
```

output
```
(c*x^4)/(4*e) + (log(d + e*x)*(a*e^4 + c*d^4))/e^5 + (c*d^2*x^2)/(2*e^3) -
 (c*d*x^3)/(3*e^2) - (c*d^3*x)/e^4
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00

$$\int \frac{a+cx^4}{d+ex}\,dx = \frac{12\log(ex+d)\,a\,e^4+12\log(ex+d)\,c\,d^4-12c\,d^3ex+6c\,d^2e^2x^2-4cd\,e^3x^3+3c\,e^4x^4}{12e^5}$$

input
```
int((c*x^4+a)/(e*x+d),x)
```

output
```
(12*log(d + e*x)*a*e**4 + 12*log(d + e*x)*c*d**4 - 12*c*d**3*e*x + 6*c*d**
2*e**2*x**2 - 4*c*d*e**3*x**3 + 3*c*e**4*x**4)/(12*e**5)
```

3.166 $\int \frac{a+cx^4}{(d+ex)^2} dx$

Optimal result

Integrand size = 15, antiderivative size = 70

$$\int \frac{a+cx^4}{(d+ex)^2} dx = \frac{3cd^2x}{e^4} - \frac{cdx^2}{e^3} + \frac{cx^3}{3e^2} - \frac{cd^4+ae^4}{e^5(d+ex)} - \frac{4cd^3\log(d+ex)}{e^5}$$

output
```
3*c*d^2*x/e^4-c*d*x^2/e^3+1/3*c*x^3/e^2-(a*e^4+c*d^4)/e^5/(e*x+d)-4*c*d^3*
ln(e*x+d)/e^5
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.94

$$\int \frac{a+cx^4}{(d+ex)^2} dx = \frac{9cd^2ex - 3cde^2x^2 + ce^3x^3 - \frac{3(cd^4+ae^4)}{d+ex} - 12cd^3\log(d+ex)}{3e^5}$$

input
```
Integrate[(a + c*x^4)/(d + e*x)^2,x]
```

output
```
(9*c*d^2*e*x - 3*c*d*e^2*x^2 + c*e^3*x^3 - (3*(c*d^4 + a*e^4))/(d + e*x) -
 12*c*d^3*Log[d + e*x])/(3*e^5)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+cx^4}{(d+ex)^2}\,dx$$

$$\Big\downarrow \text{2389}$$

$$\int \left(\frac{ae^4+cd^4}{e^4(d+ex)^2}-\frac{4cd^3}{e^4(d+ex)}+\frac{3cd^2}{e^4}-\frac{2cdx}{e^3}+\frac{cx^2}{e^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{ae^4+cd^4}{e^5(d+ex)}-\frac{4cd^3\log(d+ex)}{e^5}+\frac{3cd^2x}{e^4}-\frac{cdx^2}{e^3}+\frac{cx^3}{3e^2}$$

input
```
Int[(a + c*x^4)/(d + e*x)^2,x]
```

output
```
(3*c*d^2*x)/e^4 - (c*d*x^2)/e^3 + (c*x^3)/(3*e^2) - (c*d^4 + a*e^4)/(e^5*(
d + e*x)) - (4*c*d^3*Log[d + e*x])/e^5
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2389
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.96

method	result	size
default	$\frac{c\left(\frac{1}{3}e^{2}x^{3}-de\,x^{2}+3d^{2}x\right)}{e^{4}}-\frac{e^{4}a+c\,d^{4}}{e^{5}\left(ex+d\right)}-\frac{4c\,d^{3}\ln\left(ex+d\right)}{e^{5}}$	67
norman	$\frac{-\frac{e^{4}a+4c\,d^{4}}{e^{5}}+\frac{c\,x^{4}}{3e}-\frac{2cd\,x^{3}}{3e^{2}}+\frac{2c\,d^{2}x^{2}}{e^{3}}}{ex+d}-\frac{4c\,d^{3}\ln\left(ex+d\right)}{e^{5}}$	74
risch	$\frac{c\,x^{3}}{3e^{2}}-\frac{cd\,x^{2}}{e^{3}}+\frac{3c\,d^{2}x}{e^{4}}-\frac{a}{e\left(ex+d\right)}-\frac{c\,d^{4}}{e^{5}\left(ex+d\right)}-\frac{4c\,d^{3}\ln\left(ex+d\right)}{e^{5}}$	75
parallelrisch	$-\frac{-x^{4}c\,e^{4}+2cd\,x^{3}e^{3}+12\ln\left(ex+d\right)xc\,d^{3}e-6x^{2}c\,d^{2}e^{2}+12\ln\left(ex+d\right)c\,d^{4}+3e^{4}a+12c\,d^{4}}{3e^{5}\left(ex+d\right)}$	83

input
```
int((c*x^4+a)/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
c/e^4*(1/3*e^2*x^3-d*e*x^2+3*d^2*x)-(a*e^4+c*d^4)/e^5/(e*x+d)-4*c*d^3*ln(e
*x+d)/e^5
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.24

$$\int\frac{a+cx^4}{(d+ex)^2}\,dx=\frac{ce^4x^4-2\,cde^3x^3+6\,cd^2e^2x^2+9\,cd^3ex-3\,cd^4-3\,ae^4-12\,(cd^3ex+cd^4)\log\left(ex+d\right)}{3\,(e^6x+de^5)}$$

input
```
integrate((c*x^4+a)/(e*x+d)^2,x, algorithm="fricas")
```

output
```
1/3*(c*e^4*x^4 - 2*c*d*e^3*x^3 + 6*c*d^2*e^2*x^2 + 9*c*d^3*e*x - 3*c*d^4 -
 3*a*e^4 - 12*(c*d^3*e*x + c*d^4)*log(e*x + d))/(e^6*x + d*e^5)
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00

$$\int \frac{a+cx^4}{(d+ex)^2}\,dx = -\frac{4cd^3\log{(d+ex)}}{e^5} + \frac{3cd^2x}{e^4} - \frac{cdx^2}{e^3} + \frac{cx^3}{3e^2} + \frac{-ae^4-cd^4}{de^5+e^6x}$$

input
```
integrate((c*x**4+a)/(e*x+d)**2,x)
```

output
```
-4*c*d**3*log(d + e*x)/e**5 + 3*c*d**2*x/e**4 - c*d*x**2/e**3 + c*x**3/(3*
e**2) + (-a*e**4 - c*d**4)/(d*e**5 + e**6*x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.01

$$\int \frac{a+cx^4}{(d+ex)^2}\,dx = -\frac{cd^4+ae^4}{e^6x+de^5} - \frac{4\,cd^3\log{(ex+d)}}{e^5} + \frac{ce^2x^3-3\,cdex^2+9\,cd^2x}{3\,e^4}$$

input
```
integrate((c*x^4+a)/(e*x+d)^2,x, algorithm="maxima")
```

output
```
-(c*d^4 + a*e^4)/(e^6*x + d*e^5) - 4*c*d^3*log(e*x + d)/e^5 + 1/3*(c*e^2*x
^3 - 3*c*d*e*x^2 + 9*c*d^2*x)/e^4
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.36

$$\int \frac{a+cx^4}{(d+ex)^2}\,dx = -\frac{1}{3}\,c\left(\frac{(ex+d)^3\left(\frac{6\,d}{ex+d}-\frac{18\,d^2}{(ex+d)^2}-1\right)}{e^5} - \frac{12\,d^3\log\left(\frac{|ex+d|}{(ex+d)^2|e|}\right)}{e^5} + \frac{3\,d^4}{(ex+d)e^5}\right) - \frac{a}{(ex+d)e}$$

input
```
integrate((c*x^4+a)/(e*x+d)^2,x, algorithm="giac")
```

output
```
-1/3*c*((e*x + d)^3*(6*d/(e*x + d) - 18*d^2/(e*x + d)^2 - 1)/e^5 - 12*d^3*
log(abs(e*x + d)/((e*x + d)^2*abs(e)))/e^5 + 3*d^4/((e*x + d)*e^5)) - a/((
e*x + d)*e)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.06

$$\int \frac{a+cx^4}{(d+ex)^2}\,dx = \frac{c\,x^3}{3\,e^2} - \frac{c\,d^4+a\,e^4}{e\,\left(x\,e^5+d\,e^4\right)} - \frac{4\,c\,d^3\,\ln\left(d+e\,x\right)}{e^5} - \frac{c\,d\,x^2}{e^3} + \frac{3\,c\,d^2\,x}{e^4}$$

input
```
int((a + c*x^4)/(d + e*x)^2,x)
```

output
```
(c*x^3)/(3*e^2) - (a*e^4 + c*d^4)/(e*(d*e^4 + e^5*x)) - (4*c*d^3*log(d + e
*x))/e^5 - (c*d*x^2)/e^3 + (3*c*d^2*x)/e^4
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.29

$$\int \frac{a+cx^4}{(d+ex)^2}\,dx = \frac{-12\log(ex+d)\,c\,d^5-12\log(ex+d)\,c\,d^4ex+3a\,e^5x+12c\,d^4ex+6c\,d^3e^2x^2-2c\,d^2e^3x^3+cd\,e^4x^4}{3d\,e^5\,(ex+d)}$$

input
```
int((c*x^4+a)/(e*x+d)^2,x)
```

output
```
( - 12*log(d + e*x)*c*d**5 - 12*log(d + e*x)*c*d**4*e*x + 3*a*e**5*x + 12*
c*d**4*e*x + 6*c*d**3*e**2*x**2 - 2*c*d**2*e**3*x**3 + c*d*e**4*x**4)/(3*d
*e**5*(d + e*x))
```

3.167 $\int (d+ex)^3 \left(a+cx^4\right)^2 \, dx$

Optimal result

Integrand size = 17, antiderivative size = 119

$$\int (d+ex)^3 \left(a+cx^4\right)^2 \, dx = \frac{2}{5}acd^3x^5 + acd^2ex^6 + \frac{6}{7}acde^2x^7 + \frac{1}{4}ace^3x^8 + \frac{1}{9}c^2d^3x^9 + \frac{3}{10}c^2d^2ex^{10} + \frac{3}{11}c^2de^2x^{11} + \frac{1}{12}c^2e^3x^{12} + \frac{a^2(d+ex)^4}{4e}$$

output
```
2/5*a*c*d^3*x^5+a*c*d^2*e*x^6+6/7*a*c*d*e^2*x^7+1/4*a*c*e^3*x^8+1/9*c^2*d^
3*x^9+3/10*c^2*d^2*e*x^10+3/11*c^2*d*e^2*x^11+1/12*c^2*e^3*x^12+1/4*a^2*(e
*x+d)^4/e
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.24

$$\int (d+ex)^3 \left(a+cx^4\right)^2 \, dx = a^2d^3x + \frac{3}{2}a^2d^2ex^2 + a^2de^2x^3 + \frac{1}{4}a^2e^3x^4 + \frac{2}{5}acd^3x^5 + acd^2ex^6 + \frac{6}{7}acde^2x^7 + \frac{1}{4}ace^3x^8 + \frac{1}{9}c^2d^3x^9 + \frac{3}{10}c^2d^2ex^{10} + \frac{3}{11}c^2de^2x^{11} + \frac{1}{12}c^2e^3x^{12}$$

input
```
Integrate[(d + e*x)^3*(a + c*x^4)^2,x]
```

output
```
a^2*d^3*x + (3*a^2*d^2*e*x^2)/2 + a^2*d*e^2*x^3 + (a^2*e^3*x^4)/4 + (2*a*c
*d^3*x^5)/5 + a*c*d^2*e*x^6 + (6*a*c*d*e^2*x^7)/7 + (a*c*e^3*x^8)/4 + (c^2
*d^3*x^9)/9 + (3*c^2*d^2*e*x^10)/10 + (3*c^2*d*e^2*x^11)/11 + (c^2*e^3*x^1
2)/12
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {2017, 2389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+cx^4)^2 (d+ex)^3 \, dx$$

$$\Big\downarrow \text{2017}$$

$$\int (cx^4+a)^2 \left((d+ex)^3 - e^3x^3\right) dx + \frac{e^3(a+cx^4)^3}{12c}$$

$$\Big\downarrow \text{2389}$$

$$\int (3c^2de^2x^{10} + 3c^2d^2ex^9 + c^2d^3x^8 + 6acde^2x^6 + 6acd^2ex^5 + 2acd^3x^4 + 3a^2de^2x^2 + 3a^2d^2ex + a^2d^3) \, dx + \frac{e^3(a+cx^4)^3}{12c}$$

$$\Big\downarrow \text{2009}$$

$$a^2d^3x + \frac{3}{2}a^2d^2ex^2 + a^2de^2x^3 + \frac{2}{5}acd^3x^5 + acd^2ex^6 + \frac{6}{7}acde^2x^7 + \frac{e^3(a+cx^4)^3}{12c} + \frac{1}{9}c^2d^3x^9 + \frac{3}{10}c^2d^2ex^{10} + \frac{3}{11}c^2de^2x^{11}$$

input
```
Int[(d + e*x)^3*(a + c*x^4)^2,x]
```

output
```
a^2*d^3*x + (3*a^2*d^2*e*x^2)/2 + a^2*d*e^2*x^3 + (2*a*c*d^3*x^5)/5 + a*c*
d^2*e*x^6 + (6*a*c*d*e^2*x^7)/7 + (c^2*d^3*x^9)/9 + (3*c^2*d^2*e*x^10)/10
+ (3*c^2*d*e^2*x^11)/11 + (e^3*(a + c*x^4)^3)/(12*c)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2017
```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

rule 2389
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.10

method	result
gosper	$\frac{1}{12}c^2e^3x^{12}+\frac{3}{11}c^2d\,e^2x^{11}+\frac{3}{10}c^2d^2e\,x^{10}+\frac{1}{9}c^2d^3x^9+\frac{1}{4}ac\,e^3x^8+\frac{6}{7}acd\,e^2x^7+ac\,d^2e\,x^6+\frac{2}{5}ac$
default	$\frac{1}{12}c^2e^3x^{12}+\frac{3}{11}c^2d\,e^2x^{11}+\frac{3}{10}c^2d^2e\,x^{10}+\frac{1}{9}c^2d^3x^9+\frac{1}{4}ac\,e^3x^8+\frac{6}{7}acd\,e^2x^7+ac\,d^2e\,x^6+\frac{2}{5}ac$
norman	$\frac{1}{12}c^2e^3x^{12}+\frac{3}{11}c^2d\,e^2x^{11}+\frac{3}{10}c^2d^2e\,x^{10}+\frac{1}{9}c^2d^3x^9+\frac{1}{4}ac\,e^3x^8+\frac{6}{7}acd\,e^2x^7+ac\,d^2e\,x^6+\frac{2}{5}ac$
risch	$\frac{1}{12}c^2e^3x^{12}+\frac{3}{11}c^2d\,e^2x^{11}+\frac{3}{10}c^2d^2e\,x^{10}+\frac{1}{9}c^2d^3x^9+\frac{1}{4}ac\,e^3x^8+\frac{6}{7}acd\,e^2x^7+ac\,d^2e\,x^6+\frac{2}{5}ac$
parallelrisch	$\frac{1}{12}c^2e^3x^{12}+\frac{3}{11}c^2d\,e^2x^{11}+\frac{3}{10}c^2d^2e\,x^{10}+\frac{1}{9}c^2d^3x^9+\frac{1}{4}ac\,e^3x^8+\frac{6}{7}acd\,e^2x^7+ac\,d^2e\,x^6+\frac{2}{5}ac$
orering	$\frac{x(1155e^3c^2x^{11}+3780c^2d\,e^2x^{10}+4158d^2e\,c^2x^9+1540c^2d^3x^8+3465ac\,e^3x^7+11880acd\,e^2x^6+13860ac\,d^2e\,x^5+5544ac\,d^3x^4+3}{13860}$

input
```
int((e*x+d)^3*(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/12*c^2*e^3*x^12+3/11*c^2*d*e^2*x^11+3/10*c^2*d^2*e*x^10+1/9*c^2*d^3*x^9+
1/4*a*c*e^3*x^8+6/7*a*c*d*e^2*x^7+a*c*d^2*e*x^6+2/5*a*c*d^3*x^5+1/4*a^2*e^
3*x^4+a^2*d*e^2*x^3+3/2*a^2*d^2*e*x^2+a^2*d^3*x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.09

$$\int (d+ex)^3 \left(a+cx^4\right)^2 \, dx = \frac{1}{12} c^2e^3x^{12} + \frac{3}{11} c^2de^2x^{11} + \frac{3}{10} c^2d^2ex^{10} + \frac{1}{9} c^2d^3x^9 + \frac{1}{4} ace^3x^8 + \frac{6}{7} acde^2x^7 + acd^2ex^6 + \frac{2}{5} acd^3x^5 + \frac{1}{4} a^2e^3x^4 + a^2de^2x^3 + \frac{3}{2} a^2d^2ex^2 + a^2d^3x$$

input
```
integrate((e*x+d)^3*(c*x^4+a)^2,x, algorithm="fricas")
```

output
```
1/12*c^2*e^3*x^12 + 3/11*c^2*d*e^2*x^11 + 3/10*c^2*d^2*e*x^10 + 1/9*c^2*d^
3*x^9 + 1/4*a*c*e^3*x^8 + 6/7*a*c*d*e^2*x^7 + a*c*d^2*e*x^6 + 2/5*a*c*d^3*
x^5 + 1/4*a^2*e^3*x^4 + a^2*d*e^2*x^3 + 3/2*a^2*d^2*e*x^2 + a^2*d^3*x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.29

$$\int (d+ex)^3 \left(a+cx^4\right)^2 \, dx = a^2d^3x + \frac{3a^2d^2ex^2}{2} + a^2de^2x^3 + \frac{a^2e^3x^4}{4} + \frac{2acd^3x^5}{5} + acd^2ex^6 + \frac{6acde^2x^7}{7} + \frac{ace^3x^8}{4} + \frac{c^2d^3x^9}{9} + \frac{3c^2d^2ex^{10}}{10} + \frac{3c^2de^2x^{11}}{11} + \frac{c^2e^3x^{12}}{12}$$

input
```
integrate((e*x+d)**3*(c*x**4+a)**2,x)
```

output
```
a**2*d**3*x + 3*a**2*d**2*e*x**2/2 + a**2*d*e**2*x**3 + a**2*e**3*x**4/4 +
 2*a*c*d**3*x**5/5 + a*c*d**2*e*x**6 + 6*a*c*d*e**2*x**7/7 + a*c*e**3*x**8
/4 + c**2*d**3*x**9/9 + 3*c**2*d**2*e*x**10/10 + 3*c**2*d*e**2*x**11/11 +
c**2*e**3*x**12/12
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.09

$$\int (d+ex)^3 \left(a+cx^4\right)^2 \, dx = \frac{1}{12} c^2 e^3 x^{12} + \frac{3}{11} c^2 d e^2 x^{11} + \frac{3}{10} c^2 d^2 e x^{10} + \frac{1}{9} c^2 d^3 x^9 + \frac{1}{4} a c e^3 x^8 + \frac{6}{7} a c d e^2 x^7 + a c d^2 e x^6 + \frac{2}{5} a c d^3 x^5 + \frac{1}{4} a^2 e^3 x^4 + a^2 d e^2 x^3 + \frac{3}{2} a^2 d^2 e x^2 + a^2 d^3 x$$

input
```
integrate((e*x+d)^3*(c*x^4+a)^2,x, algorithm="maxima")
```

output
```
1/12*c^2*e^3*x^12 + 3/11*c^2*d*e^2*x^11 + 3/10*c^2*d^2*e*x^10 + 1/9*c^2*d^
3*x^9 + 1/4*a*c*e^3*x^8 + 6/7*a*c*d*e^2*x^7 + a*c*d^2*e*x^6 + 2/5*a*c*d^3*
x^5 + 1/4*a^2*e^3*x^4 + a^2*d*e^2*x^3 + 3/2*a^2*d^2*e*x^2 + a^2*d^3*x
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.09

$$\int (d+ex)^3 \left(a+cx^4\right)^2 \, dx = \frac{1}{12} c^2 e^3 x^{12} + \frac{3}{11} c^2 d e^2 x^{11} + \frac{3}{10} c^2 d^2 e x^{10} + \frac{1}{9} c^2 d^3 x^9 + \frac{1}{4} a c e^3 x^8 + \frac{6}{7} a c d e^2 x^7 + a c d^2 e x^6 + \frac{2}{5} a c d^3 x^5 + \frac{1}{4} a^2 e^3 x^4 + a^2 d e^2 x^3 + \frac{3}{2} a^2 d^2 e x^2 + a^2 d^3 x$$

input
```
integrate((e*x+d)^3*(c*x^4+a)^2,x, algorithm="giac")
```

output
```
1/12*c^2*e^3*x^12 + 3/11*c^2*d*e^2*x^11 + 3/10*c^2*d^2*e*x^10 + 1/9*c^2*d^
3*x^9 + 1/4*a*c*e^3*x^8 + 6/7*a*c*d*e^2*x^7 + a*c*d^2*e*x^6 + 2/5*a*c*d^3*
x^5 + 1/4*a^2*e^3*x^4 + a^2*d*e^2*x^3 + 3/2*a^2*d^2*e*x^2 + a^2*d^3*x
```

Mupad [B] (verification not implemented)

Time = 21.50 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.09

$$\int (d+ex)^3 \left(a+cx^4\right)^2 \, dx = a^2 \, d^3 \, x + \frac{3 \, a^2 \, d^2 \, e \, x^2}{2} + a^2 \, d \, e^2 \, x^3 + \frac{a^2 \, e^3 \, x^4}{4} + \frac{2 \, a \, c \, d^3 \, x^5}{5} + a \, c \, d^2 \, e \, x^6 + \frac{6 \, a \, c \, d \, e^2 \, x^7}{7} + \frac{a \, c \, e^3 \, x^8}{4} + \frac{c^2 \, d^3 \, x^9}{9} + \frac{3 \, c^2 \, d^2 \, e \, x^{10}}{10} + \frac{3 \, c^2 \, d \, e^2 \, x^{11}}{11} + \frac{c^2 \, e^3 \, x^{12}}{12}$$

input
```
int((a + c*x^4)^2*(d + e*x)^3,x)
```

output
```
a^2*d^3*x + (a^2*e^3*x^4)/4 + (c^2*d^3*x^9)/9 + (c^2*e^3*x^12)/12 + (3*a^2
*d^2*e*x^2)/2 + a^2*d*e^2*x^3 + (3*c^2*d^2*e*x^10)/10 + (3*c^2*d*e^2*x^11)
/11 + (2*a*c*d^3*x^5)/5 + (a*c*e^3*x^8)/4 + a*c*d^2*e*x^6 + (6*a*c*d*e^2*x
^7)/7
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.12

$$\int (d+ex)^3 \left(a+cx^4\right)^2 \, dx = \frac{x(1155c^2e^3x^{11} + 3780c^2d\,e^2x^{10} + 4158c^2d^2e\,x^9 + 1540c^2d^3x^8 + 3465ac\,e^3x^7 + 11880acd\,e^2x^6 + 13860ac}{13860}$$

input
```
int((e*x+d)^3*(c*x^4+a)^2,x)
```

output
```
(x*(13860*a**2*d**3 + 20790*a**2*d**2*e*x + 13860*a**2*d*e**2*x**2 + 3465*
a**2*e**3*x**3 + 5544*a*c*d**3*x**4 + 13860*a*c*d**2*e*x**5 + 11880*a*c*d*
e**2*x**6 + 3465*a*c*e**3*x**7 + 1540*c**2*d**3*x**8 + 4158*c**2*d**2*e*x*
*9 + 3780*c**2*d*e**2*x**10 + 1155*c**2*e**3*x**11))/13860
```

3.168 $\int (d+ex)^2 \left(a+cx^4\right)^2 \, dx$

Optimal result

Integrand size = 17, antiderivative size = 91

$$\int (d+ex)^2 \left(a+cx^4\right)^2 \, dx = \frac{2}{5}acd^2x^5+\frac{2}{3}acdex^6+\frac{2}{7}ace^2x^7+\frac{1}{9}c^2d^2x^9+\frac{1}{5}c^2dex^{10}+\frac{1}{11}c^2e^2x^{11}+\frac{a^2(d+ex)^3}{3e}$$

output
```
2/5*a*c*d^2*x^5+2/3*a*c*d*e*x^6+2/7*a*c*e^2*x^7+1/9*c^2*d^2*x^9+1/5*c^2*d*
e*x^10+1/11*c^2*e^2*x^11+1/3*a^2*(e*x+d)^3/e
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.14

$$\int (d+ex)^2 \left(a+cx^4\right)^2 \, dx = a^2d^2x+a^2dex^2+\frac{1}{3}a^2e^2x^3+\frac{2}{5}acd^2x^5+\frac{2}{3}acdex^6+\frac{2}{7}ace^2x^7+\frac{1}{9}c^2d^2x^9+\frac{1}{5}c^2dex^{10}+\frac{1}{11}c^2e^2x^{11}$$

input
```
Integrate[(d + e*x)^2*(a + c*x^4)^2,x]
```

output
```
a^2*d^2*x + a^2*d*e*x^2 + (a^2*e^2*x^3)/3 + (2*a*c*d^2*x^5)/5 + (2*a*c*d*e
*x^6)/3 + (2*a*c*e^2*x^7)/7 + (c^2*d^2*x^9)/9 + (c^2*d*e*x^10)/5 + (c^2*e^
2*x^11)/11
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.14, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+cx^4)^2 (d+ex)^2 \, dx$$

$$\Big\downarrow \text{ 2389}$$

$$\int \left(a^2d^2+2a^2dex+a^2e^2x^2+2acd^2x^4+4acdex^5+2ace^2x^6+c^2d^2x^8+2c^2dex^9+c^2e^2x^{10}\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$a^2d^2x+a^2dex^2+\frac{1}{3}a^2e^2x^3+\frac{2}{5}acd^2x^5+\frac{2}{3}acdex^6+\frac{2}{7}ace^2x^7+\frac{1}{9}c^2d^2x^9+\frac{1}{5}c^2dex^{10}+\frac{1}{11}c^2e^2x^{11}$$

input
```
Int[(d + e*x)^2*(a + c*x^4)^2,x]
```

output

```
a^2*d^2*x + a^2*d*e*x^2 + (a^2*e^2*x^3)/3 + (2*a*c*d^2*x^5)/5 + (2*a*c*d*e
*x^6)/3 + (2*a*c*e^2*x^7)/7 + (c^2*d^2*x^9)/9 + (c^2*d*e*x^10)/5 + (c^2*e^
2*x^11)/11
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2389
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00

method	result
gosper	$\frac{1}{11}c^2e^2x^{11}+\frac{1}{5}c^2de\,x^{10}+\frac{1}{9}c^2d^2x^9+\frac{2}{7}ac\,e^2x^7+\frac{2}{3}acde\,x^6+\frac{2}{5}ac\,d^2x^5+\frac{1}{3}x^3a^2e^2+a^2de\,x^2+$
default	$\frac{1}{11}c^2e^2x^{11}+\frac{1}{5}c^2de\,x^{10}+\frac{1}{9}c^2d^2x^9+\frac{2}{7}ac\,e^2x^7+\frac{2}{3}acde\,x^6+\frac{2}{5}ac\,d^2x^5+\frac{1}{3}x^3a^2e^2+a^2de\,x^2+$
norman	$\frac{1}{11}c^2e^2x^{11}+\frac{1}{5}c^2de\,x^{10}+\frac{1}{9}c^2d^2x^9+\frac{2}{7}ac\,e^2x^7+\frac{2}{3}acde\,x^6+\frac{2}{5}ac\,d^2x^5+\frac{1}{3}x^3a^2e^2+a^2de\,x^2+$
risch	$\frac{1}{11}c^2e^2x^{11}+\frac{1}{5}c^2de\,x^{10}+\frac{1}{9}c^2d^2x^9+\frac{2}{7}ac\,e^2x^7+\frac{2}{3}acde\,x^6+\frac{2}{5}ac\,d^2x^5+\frac{1}{3}x^3a^2e^2+a^2de\,x^2+$
parallelrisch	$\frac{1}{11}c^2e^2x^{11}+\frac{1}{5}c^2de\,x^{10}+\frac{1}{9}c^2d^2x^9+\frac{2}{7}ac\,e^2x^7+\frac{2}{3}acde\,x^6+\frac{2}{5}ac\,d^2x^5+\frac{1}{3}x^3a^2e^2+a^2de\,x^2+$
orering	$\frac{x\left(315c^2e^2x^{10}+693c^2de\,x^9+385c^2d^2x^8+990ac\,e^2x^6+2310acde\,x^5+1386d^2x^4ac+1155a^2e^2x^2+3465a^2dex+3465a^2d^2\right)}{3465}$

input
```
int((e*x+d)^2*(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/11*c^2*e^2*x^11+1/5*c^2*d*e*x^10+1/9*c^2*d^2*x^9+2/7*a*c*e^2*x^7+2/3*a*c
*d*e*x^6+2/5*a*c*d^2*x^5+1/3*x^3*a^2*e^2+a^2*d*e*x^2+x*a^2*d^2
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.99

$$\int (d+ex)^2\left(a+cx^4\right)^2\,dx=\frac{1}{11}\,c^2e^2x^{11}+\frac{1}{5}\,c^2dex^{10}+\frac{1}{9}\,c^2d^2x^9+\frac{2}{7}\,ace^2x^7+\frac{2}{3}\,acdex^6+\frac{2}{5}\,acd^2x^5+\frac{1}{3}\,a^2e^2x^3+a^2dex^2+a^2d^2x$$

input
```
integrate((e*x+d)^2*(c*x^4+a)^2,x, algorithm="fricas")
```

output
```
1/11*c^2*e^2*x^11 + 1/5*c^2*d*e*x^10 + 1/9*c^2*d^2*x^9 + 2/7*a*c*e^2*x^7 +
 2/3*a*c*d*e*x^6 + 2/5*a*c*d^2*x^5 + 1/3*a^2*e^2*x^3 + a^2*d*e*x^2 + a^2*d
^2*x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.15

$$\int (d+ex)^2 \left(a+cx^4\right)^2 \, dx = a^2d^2x + a^2dex^2 + \frac{a^2e^2x^3}{3} + \frac{2acd^2x^5}{5} + \frac{2acdex^6}{3} + \frac{2ace^2x^7}{7} + \frac{c^2d^2x^9}{9} + \frac{c^2dex^{10}}{5} + \frac{c^2e^2x^{11}}{11}$$

input
```
integrate((e*x+d)**2*(c*x**4+a)**2,x)
```

output
```
a**2*d**2*x + a**2*d*e*x**2 + a**2*e**2*x**3/3 + 2*a*c*d**2*x**5/5 + 2*a*c
*d*e*x**6/3 + 2*a*c*e**2*x**7/7 + c**2*d**2*x**9/9 + c**2*d*e*x**10/5 + c*
*2*e**2*x**11/11
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.99

$$\int (d+ex)^2 \left(a+cx^4\right)^2 \, dx = \frac{1}{11}\,c^2e^2x^{11} + \frac{1}{5}\,c^2dex^{10} + \frac{1}{9}\,c^2d^2x^9 + \frac{2}{7}\,ace^2x^7 + \frac{2}{3}\,acdex^6 + \frac{2}{5}\,acd^2x^5 + \frac{1}{3}\,a^2e^2x^3 + a^2dex^2 + a^2d^2x$$

input
```
integrate((e*x+d)^2*(c*x^4+a)^2,x, algorithm="maxima")
```

output
```
1/11*c^2*e^2*x^11 + 1/5*c^2*d*e*x^10 + 1/9*c^2*d^2*x^9 + 2/7*a*c*e^2*x^7 +
 2/3*a*c*d*e*x^6 + 2/5*a*c*d^2*x^5 + 1/3*a^2*e^2*x^3 + a^2*d*e*x^2 + a^2*d
^2*x
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.99

$$\int (d+ex)^2 \left(a+cx^4\right)^2 \, dx = \frac{1}{11}c^2e^2x^{11} + \frac{1}{5}c^2dex^{10} + \frac{1}{9}c^2d^2x^9 + \frac{2}{7}ace^2x^7 + \frac{2}{3}acdex^6 + \frac{2}{5}acd^2x^5 + \frac{1}{3}a^2e^2x^3 + a^2dex^2 + a^2d^2x$$

input
```
integrate((e*x+d)^2*(c*x^4+a)^2,x, algorithm="giac")
```

output
```
1/11*c^2*e^2*x^11 + 1/5*c^2*d*e*x^10 + 1/9*c^2*d^2*x^9 + 2/7*a*c*e^2*x^7 +
 2/3*a*c*d*e*x^6 + 2/5*a*c*d^2*x^5 + 1/3*a^2*e^2*x^3 + a^2*d*e*x^2 + a^2*d
^2*x
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.99

$$\int (d+ex)^2 \left(a+cx^4\right)^2 \, dx = a^2 d^2 x + a^2 d e x^2 + \frac{a^2 e^2 x^3}{3} + \frac{2 a c d^2 x^5}{5} + \frac{2 a c d e x^6}{3} + \frac{2 a c e^2 x^7}{7} + \frac{c^2 d^2 x^9}{9} + \frac{c^2 d e x^{10}}{5} + \frac{c^2 e^2 x^{11}}{11}$$

input
```
int((a + c*x^4)^2*(d + e*x)^2,x)
```

output
```
a^2*d^2*x + (a^2*e^2*x^3)/3 + (c^2*d^2*x^9)/9 + (c^2*e^2*x^11)/11 + (2*a*c
*d^2*x^5)/5 + (2*a*c*e^2*x^7)/7 + a^2*d*e*x^2 + (c^2*d*e*x^10)/5 + (2*a*c*
d*e*x^6)/3
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.01

$$\int (d+ex)^2 \left(a+cx^4\right)^2 dx$$
$$= \frac{x(315c^2e^2x^{10} + 693c^2de\,x^9 + 385c^2d^2x^8 + 990ac\,e^2x^6 + 2310acde\,x^5 + 1386ac\,d^2x^4 + 1155a^2e^2x^2 + 3465}{3465}$$

input
```
int((e*x+d)^2*(c*x^4+a)^2,x)
```

output
```
(x*(3465*a**2*d**2 + 3465*a**2*d*e*x + 1155*a**2*e**2*x**2 + 1386*a*c*d**2
*x**4 + 2310*a*c*d*e*x**5 + 990*a*c*e**2*x**6 + 385*c**2*d**2*x**8 + 693*c
**2*d*e*x**9 + 315*c**2*e**2*x**10))/3465
```

3.169 $\int (d+ex) \left(a+cx^4\right)^2 \, dx$

Optimal result

Integrand size = 15, antiderivative size = 60

$$\int (d+ex) \left(a+cx^4\right)^2 \, dx = \frac{2}{5}acdx^5+\frac{1}{3}acex^6+\frac{1}{9}c^2dx^9+\frac{1}{10}c^2ex^{10}+\frac{a^2(d+ex)^2}{2e}$$

output
```
2/5*a*c*d*x^5+1/3*a*c*e*x^6+1/9*c^2*d*x^9+1/10*c^2*e*x^10+1/2*a^2*(e*x+d)^
2/e
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00

$$\int (d+ex) \left(a+cx^4\right)^2 \, dx = a^2dx+\frac{1}{2}a^2ex^2+\frac{2}{5}acdx^5+\frac{1}{3}acex^6+\frac{1}{9}c^2dx^9+\frac{1}{10}c^2ex^{10}$$

input
```
Integrate[(d + e*x)*(a + c*x^4)^2,x]
```

output
```
a^2*d*x + (a^2*e*x^2)/2 + (2*a*c*d*x^5)/5 + (a*c*e*x^6)/3 + (c^2*d*x^9)/9
+ (c^2*e*x^10)/10
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+cx^4\right)^2 (d+ex) \, dx$$
$$\Big\downarrow \text{2389}$$
$$\int \left(a^2d+a^2ex+2acdx^4+2acex^5+c^2dx^8+c^2ex^9\right)dx$$
$$\Big\downarrow \text{2009}$$
$$a^2dx+\frac{1}{2}a^2ex^2+\frac{2}{5}acdx^5+\frac{1}{3}acex^6+\frac{1}{9}c^2dx^9+\frac{1}{10}c^2ex^{10}$$

input
```
Int[(d + e*x)*(a + c*x^4)^2,x]
```

output
```
a^2*d*x + (a^2*e*x^2)/2 + (2*a*c*d*x^5)/5 + (a*c*e*x^6)/3 + (c^2*d*x^9)/9
+ (c^2*e*x^10)/10
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2389
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.85

method	result	size
gosper	$\frac{1}{10}c^2e\,x^{10}+\frac{1}{9}c^2d\,x^9+\frac{1}{3}x^6ace+\frac{2}{5}acd\,x^5+\frac{1}{2}a^2e\,x^2+a^2dx$	51
default	$\frac{1}{10}c^2e\,x^{10}+\frac{1}{9}c^2d\,x^9+\frac{1}{3}x^6ace+\frac{2}{5}acd\,x^5+\frac{1}{2}a^2e\,x^2+a^2dx$	51
norman	$\frac{1}{10}c^2e\,x^{10}+\frac{1}{9}c^2d\,x^9+\frac{1}{3}x^6ace+\frac{2}{5}acd\,x^5+\frac{1}{2}a^2e\,x^2+a^2dx$	51
risch	$\frac{1}{10}c^2e\,x^{10}+\frac{1}{9}c^2d\,x^9+\frac{1}{3}x^6ace+\frac{2}{5}acd\,x^5+\frac{1}{2}a^2e\,x^2+a^2dx$	51
parallelrisch	$\frac{1}{10}c^2e\,x^{10}+\frac{1}{9}c^2d\,x^9+\frac{1}{3}x^6ace+\frac{2}{5}acd\,x^5+\frac{1}{2}a^2e\,x^2+a^2dx$	51
orering	$\frac{x\left(9c^2e\,x^9+10c^2d\,x^8+30ace\,x^5+36d\,x^4ac+45a^2ex+90a^2d\right)}{90}$	52

input
```
int((e*x+d)*(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/10*c^2*e*x^10+1/9*c^2*d*x^9+1/3*x^6*a*c*e+2/5*a*c*d*x^5+1/2*a^2*e*x^2+a^
2*d*x
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.83

$$\int (d+ex)\left(a+cx^4\right)^2\,dx=\frac{1}{10}\,c^2ex^{10}+\frac{1}{9}\,c^2dx^9+\frac{1}{3}\,acex^6+\frac{2}{5}\,acdx^5+\frac{1}{2}\,a^2ex^2+a^2dx$$

input
```
integrate((e*x+d)*(c*x^4+a)^2,x, algorithm="fricas")
```

output
```
1/10*c^2*e*x^10 + 1/9*c^2*d*x^9 + 1/3*a*c*e*x^6 + 2/5*a*c*d*x^5 + 1/2*a^2*
e*x^2 + a^2*d*x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.97

$$\int (d+ex)\left(a+cx^4\right)^2\,dx=a^2dx+\frac{a^2ex^2}{2}+\frac{2acdx^5}{5}+\frac{acex^6}{3}+\frac{c^2dx^9}{9}+\frac{c^2ex^{10}}{10}$$

input
```
integrate((e*x+d)*(c*x**4+a)**2,x)
```

output
```
a**2*d*x + a**2*e*x**2/2 + 2*a*c*d*x**5/5 + a*c*e*x**6/3 + c**2*d*x**9/9 +
 c**2*e*x**10/10
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.83

$$\int (d+ex)\left(a+cx^4\right)^2\,dx=\frac{1}{10}\,c^2ex^{10}+\frac{1}{9}\,c^2dx^9+\frac{1}{3}\,acex^6+\frac{2}{5}\,acdx^5+\frac{1}{2}\,a^2ex^2+a^2dx$$

input
```
integrate((e*x+d)*(c*x^4+a)^2,x, algorithm="maxima")
```

output
```
1/10*c^2*e*x^10 + 1/9*c^2*d*x^9 + 1/3*a*c*e*x^6 + 2/5*a*c*d*x^5 + 1/2*a^2*
e*x^2 + a^2*d*x
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.83

$$\int (d+ex)\left(a+cx^4\right)^2\,dx=\frac{1}{10}\,c^2ex^{10}+\frac{1}{9}\,c^2dx^9+\frac{1}{3}\,acex^6+\frac{2}{5}\,acdx^5+\frac{1}{2}\,a^2ex^2+a^2dx$$

input
```
integrate((e*x+d)*(c*x^4+a)^2,x, algorithm="giac")
```

output
```
1/10*c^2*e*x^10 + 1/9*c^2*d*x^9 + 1/3*a*c*e*x^6 + 2/5*a*c*d*x^5 + 1/2*a^2*
e*x^2 + a^2*d*x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.83

$$\int (d+ex)\left(a+cx^4\right)^2\,dx=\frac{e\,a^2\,x^2}{2}+d\,a^2\,x+\frac{e\,a\,c\,x^6}{3}+\frac{2\,d\,a\,c\,x^5}{5}+\frac{e\,c^2\,x^{10}}{10}+\frac{d\,c^2\,x^9}{9}$$

input
```
int((a + c*x^4)^2*(d + e*x),x)
```

output
```
(a^2*e*x^2)/2 + (c^2*d*x^9)/9 + (c^2*e*x^10)/10 + a^2*d*x + (2*a*c*d*x^5)/
5 + (a*c*e*x^6)/3
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.85

$$\int (d+ex)\left(a+cx^4\right)^2\,dx=\frac{x(9c^2e\,x^9+10c^2d\,x^8+30ace\,x^5+36acd\,x^4+45a^2ex+90a^2d)}{90}$$

input
```
int((e*x+d)*(c*x^4+a)^2,x)
```

output
```
(x*(90*a**2*d + 45*a**2*e*x + 36*a*c*d*x**4 + 30*a*c*e*x**5 + 10*c**2*d*x*
*8 + 9*c**2*e*x**9))/90
```

3.170 $\int (a+cx^4)^2 \, dx$

Optimal result

Integrand size = 9, antiderivative size = 25

$$\int (a+cx^4)^2 \, dx = a^2x + \frac{2}{5}acx^5 + \frac{c^2x^9}{9}$$

output
```
a^2*x+2/5*a*c*x^5+1/9*c^2*x^9
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (a+cx^4)^2 \, dx = a^2x + \frac{2}{5}acx^5 + \frac{c^2x^9}{9}$$

input
```
Integrate[(a + c*x^4)^2,x]
```

output
```
a^2*x + (2*a*c*x^5)/5 + (c^2*x^9)/9
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {747, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a + cx^4)^2 \, dx$$

$$\Big\downarrow \text{ 747}$$

$$\int (a^2 + 2acx^4 + c^2x^8) \, dx$$

$$\Big\downarrow \text{ 2009}$$

$$a^2x + \frac{2}{5}acx^5 + \frac{c^2x^9}{9}$$

input
```
Int[(a + c*x^4)^2,x]
```

output
```
a^2*x + (2*a*c*x^5)/5 + (c^2*x^9)/9
```

Defintions of rubi rules used

rule 747
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

method	result	size
gosper	$x\,a^{2}+\frac{2}{5}x^{5}ac+\frac{1}{9}c^{2}x^{9}$	22
default	$x\,a^{2}+\frac{2}{5}x^{5}ac+\frac{1}{9}c^{2}x^{9}$	22
norman	$x\,a^{2}+\frac{2}{5}x^{5}ac+\frac{1}{9}c^{2}x^{9}$	22
risch	$x\,a^{2}+\frac{2}{5}x^{5}ac+\frac{1}{9}c^{2}x^{9}$	22
parallelrisch	$x\,a^{2}+\frac{2}{5}x^{5}ac+\frac{1}{9}c^{2}x^{9}$	22
orering	$\frac{x\left(5x^{8}c^{2}+18x^{4}ac+45a^{2}\right)}{45}$	25

input
```
int((c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
x*a^2+2/5*x^5*a*c+1/9*c^2*x^9
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \left(a+cx^{4}\right)^{2}\,dx=\frac{1}{9}\,c^{2}x^{9}+\frac{2}{5}\,acx^{5}+a^{2}x$$

input
```
integrate((c*x^4+a)^2,x, algorithm="fricas")
```

output
```
1/9*c^2*x^9 + 2/5*a*c*x^5 + a^2*x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \left(a+cx^4\right)^2\,dx=a^2x+\frac{2acx^5}{5}+\frac{c^2x^9}{9}$$

input
```
integrate((c*x**4+a)**2,x)
```

output
```
a**2*x + 2*a*c*x**5/5 + c**2*x**9/9
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \left(a+cx^4\right)^2\,dx=\frac{1}{9}\,c^2x^9+\frac{2}{5}\,acx^5+a^2x$$

input
```
integrate((c*x^4+a)^2,x, algorithm="maxima")
```

output
```
1/9*c^2*x^9 + 2/5*a*c*x^5 + a^2*x
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \left(a+cx^4\right)^2\,dx=\frac{1}{9}\,c^2x^9+\frac{2}{5}\,acx^5+a^2x$$

input
```
integrate((c*x^4+a)^2,x, algorithm="giac")
```

output
```
1/9*c^2*x^9 + 2/5*a*c*x^5 + a^2*x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \left(a+cx^4\right)^2\,dx=a^2\,x+\frac{2\,a\,c\,x^5}{5}+\frac{c^2\,x^9}{9}$$

input
```
int((a + c*x^4)^2,x)
```

output
```
a^2*x + (c^2*x^9)/9 + (2*a*c*x^5)/5
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \left(a+cx^4\right)^2\,dx=\frac{x(5c^2x^8+18ac\,x^4+45a^2)}{45}$$

input
```
int((c*x^4+a)^2,x)
```

output
```
(x*(45*a**2 + 18*a*c*x**4 + 5*c**2*x**8))/45
```

3.171 $\int \frac{(a+cx^4)^2}{d+ex} dx$

Optimal result

Integrand size = 17, antiderivative size = 178

$$\int \frac{(a+cx^4)^2}{d+ex}\,dx = -\frac{cd^3(cd^4+2ae^4)\,x}{e^8}+\frac{cd^2(cd^4+2ae^4)\,x^2}{2e^7} -\frac{cd(cd^4+2ae^4)\,x^3}{3e^6}+\frac{c(cd^4+2ae^4)\,x^4}{4e^5}-\frac{c^2d^3x^5}{5e^4} +\frac{c^2d^2x^6}{6e^3}-\frac{c^2dx^7}{7e^2}+\frac{c^2x^8}{8e}+\frac{(cd^4+ae^4)^2\log(d+ex)}{e^9}$$

output
```
-c*d^3*(2*a*e^4+c*d^4)*x/e^8+1/2*c*d^2*(2*a*e^4+c*d^4)*x^2/e^7-1/3*c*d*(2*
a*e^4+c*d^4)*x^3/e^6+1/4*c*(2*a*e^4+c*d^4)*x^4/e^5-1/5*c^2*d^3*x^5/e^4+1/6
*c^2*d^2*x^6/e^3-1/7*c^2*d*x^7/e^2+1/8*c^2*x^8/e+(a*e^4+c*d^4)^2*ln(e*x+d)
/e^9
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.82

$$\int \frac{(a+cx^4)^2}{d+ex}\,dx$$
$$= \frac{cx(140ae^4(-12d^3+6d^2ex-4de^2x^2+3e^3x^3)+c(-840d^7+420d^6ex-280d^5e^2x^2+210d^4e^3x^3-168d^3 \cdots}{840e^8}$$
$$+\frac{(cd^4+ae^4)^2\log(d+ex)}{e^9}$$

input
```
Integrate[(a + c*x^4)^2/(d + e*x),x]
```

output
```
(c*x*(140*a*e^4*(-12*d^3 + 6*d^2*e*x - 4*d*e^2*x^2 + 3*e^3*x^3) + c*(-840*
d^7 + 420*d^6*e*x - 280*d^5*e^2*x^2 + 210*d^4*e^3*x^3 - 168*d^3*e^4*x^4 +
140*d^2*e^5*x^5 - 120*d*e^6*x^6 + 105*e^7*x^7)))/(840*e^8) + ((c*d^4 + a*e
^4)^2*Log[d + e*x])/e^9
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^4)^2}{d+ex}\,dx$$
$$\Big\downarrow \text{2389}$$
$$\int \left(\frac{(ae^4+cd^4)^2}{e^8(d+ex)}-\frac{cdx^2(2ae^4+cd^4)}{e^6}+\frac{cx^3(2ae^4+cd^4)}{e^5}-\frac{cd^3(2ae^4+cd^4)}{e^8}+\frac{cd^2x(2ae^4+cd^4)}{e^7}-\frac{c^2d^3x^4}{e^4}+ \cdots\right.$$
$$\Big\downarrow \text{2009}$$

$$\frac{(ae^4+cd^4)^2\log(d+ex)}{e^9}-\frac{cdx^3(2ae^4+cd^4)}{3e^6}+\frac{cx^4(2ae^4+cd^4)}{4e^5}-\frac{cd^3x(2ae^4+cd^4)}{e^8}+\frac{cd^2x^2(2ae^4+cd^4)}{2e^7}-\frac{c^2d^3x^5}{5e^4}+\frac{c^2d^2x^6}{6e^3}-\frac{c^2dx^7}{7e^2}+\frac{c^2x^8}{8e}$$

input
```
Int[(a + c*x^4)^2/(d + e*x),x]
```

output
```
-((c*d^3*(c*d^4 + 2*a*e^4)*x)/e^8) + (c*d^2*(c*d^4 + 2*a*e^4)*x^2)/(2*e^7)
 - (c*d*(c*d^4 + 2*a*e^4)*x^3)/(3*e^6) + (c*(c*d^4 + 2*a*e^4)*x^4)/(4*e^5)
 - (c^2*d^3*x^5)/(5*e^4) + (c^2*d^2*x^6)/(6*e^3) - (c^2*d*x^7)/(7*e^2) + (
c^2*x^8)/(8*e) + ((c*d^4 + a*e^4)^2*Log[d + e*x])/e^9
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2389
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.93

method	result
default	$-\frac{c\left(-\frac{c e^7 x^8}{8}+\frac{c d e^6 x^7}{7}-\frac{c d^2 x^6 e^5}{6}+\frac{c d^3 x^5 e^4}{5}-\frac{(2e^4 a+c d^4) x^4 e^3}{4}+\frac{d(2e^4 a+c d^4) x^3 e^2}{3}-\frac{d^2(2e^4 a+c d^4) x^2 e}{2}+x d^3(2e^4 a+c d^4)\right)}{e^8} \cdots$
norman	$\frac{c^2x^8}{8e}+\frac{c(2e^4a+c d^4)x^4}{4e^5}-\frac{c^2 d x^7}{7e^2}+\frac{c^2d^2x^6}{6e^3}-\frac{c^2d^3x^5}{5e^4}-\frac{cd(2e^4a+c d^4)x^3}{3e^6}+\frac{c d^2(2e^4a+c d^4)x^2}{2e^7}-\frac{c d^3(2e^4a+c d}{e^8} \cdots$
risch	$\frac{c^2x^8}{8e}-\frac{c^2 d x^7}{7e^2}+\frac{c^2d^2x^6}{6e^3}-\frac{c^2d^3x^5}{5e^4}+\frac{ca x^4}{2e}+\frac{c^2d^4x^4}{4e^5}-\frac{2cad x^3}{3e^2}-\frac{c^2d^5x^3}{3e^6}+\frac{ca d^2x^2}{e^3}+\frac{c^2d^6x^2}{2e^7}-\frac{2ca d^3x}{e^4} \cdots$
parallelrisch	$\frac{105x^8c^2e^8-120c^2d x^7e^7+140c^2d^2x^6e^6-168c^2d^3x^5e^5+420x^4ac e^8+210x^4c^2d^4e^4-560x^3acd e^7-280x^3c^2d^5e^3+840x^2ac d^2e \cdots}{840e^9}$

input
```
int((c*x^4+a)^2/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
-c/e^8*(-1/8*c*e^7*x^8+1/7*c*d*e^6*x^7-1/6*c*d^2*x^6*e^5+1/5*c*d^3*x^5*e^4
-1/4*(2*a*e^4+c*d^4)*x^4*e^3+1/3*d*(2*a*e^4+c*d^4)*x^3*e^2-1/2*d^2*(2*a*e^
4+c*d^4)*x^2*e+x*d^3*(2*a*e^4+c*d^4))+(a^2*e^8+2*a*c*d^4*e^4+c^2*d^8)/e^9*
ln(e*x+d)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.04

$$\int \frac{(a+cx^4)^2}{d+ex}\,dx$$
$$= \frac{105\,c^2e^8x^8 - 120\,c^2de^7x^7 + 140\,c^2d^2e^6x^6 - 168\,c^2d^3e^5x^5 + 210\,(c^2d^4e^4 + 2\,ace^8)x^4 - 280\,(c^2d^5e^3 + 2\,acd}{840}$$

input
```
integrate((c*x^4+a)^2/(e*x+d),x, algorithm="fricas")
```

output
```
1/840*(105*c^2*e^8*x^8 - 120*c^2*d*e^7*x^7 + 140*c^2*d^2*e^6*x^6 - 168*c^2
*d^3*e^5*x^5 + 210*(c^2*d^4*e^4 + 2*a*c*e^8)*x^4 - 280*(c^2*d^5*e^3 + 2*a*
c*d*e^7)*x^3 + 420*(c^2*d^6*e^2 + 2*a*c*d^2*e^6)*x^2 - 840*(c^2*d^7*e + 2*
a*c*d^3*e^5)*x + 840*(c^2*d^8 + 2*a*c*d^4*e^4 + a^2*e^8)*log(e*x + d))/e^9
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.99

$$\begin{aligned}\int \frac{(a+cx^4)^2}{d+ex}\,dx = &-\frac{c^2d^3x^5}{5e^4} + \frac{c^2d^2x^6}{6e^3} - \frac{c^2dx^7}{7e^2} + \frac{c^2x^8}{8e} + x^4\left(\frac{ac}{2e} + \frac{c^2d^4}{4e^5}\right)\\ &+ x^3\left(-\frac{2acd}{3e^2} - \frac{c^2d^5}{3e^6}\right) + x^2\left(\frac{acd^2}{e^3} + \frac{c^2d^6}{2e^7}\right)\\ &+ x\left(-\frac{2acd^3}{e^4} - \frac{c^2d^7}{e^8}\right) + \frac{(ae^4+cd^4)^2\log(d+ex)}{e^9}\end{aligned}$$

input
```
integrate((c*x**4+a)**2/(e*x+d),x)
```

output
```
-c**2*d**3*x**5/(5*e**4) + c**2*d**2*x**6/(6*e**3) - c**2*d*x**7/(7*e**2)
+ c**2*x**8/(8*e) + x**4*(a*c/(2*e) + c**2*d**4/(4*e**5)) + x**3*(-2*a*c*d
/(3*e**2) - c**2*d**5/(3*e**6)) + x**2*(a*c*d**2/e**3 + c**2*d**6/(2*e**7)
) + x*(-2*a*c*d**3/e**4 - c**2*d**7/e**8) + (a*e**4 + c*d**4)**2*log(d + e
*x)/e**9
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.04

$$\int \frac{(a+cx^4)^2}{d+ex}\,dx = \frac{105\,c^2e^7x^8 - 120\,c^2de^6x^7 + 140\,c^2d^2e^5x^6 - 168\,c^2d^3e^4x^5 + 210\,(c^2d^4e^3 + 2\,ace^7)x^4 - 280\,(c^2d^5e^2 + 2\,acd}{840\,e^8} + \frac{(c^2d^8 + 2\,acd^4e^4 + a^2e^8)\log{(ex+d)}}{e^9}$$

input
```
integrate((c*x^4+a)^2/(e*x+d),x, algorithm="maxima")
```

output
```
1/840*(105*c^2*e^7*x^8 - 120*c^2*d*e^6*x^7 + 140*c^2*d^2*e^5*x^6 - 168*c^2
*d^3*e^4*x^5 + 210*(c^2*d^4*e^3 + 2*a*c*e^7)*x^4 - 280*(c^2*d^5*e^2 + 2*a*
c*d*e^6)*x^3 + 420*(c^2*d^6*e + 2*a*c*d^2*e^5)*x^2 - 840*(c^2*d^7 + 2*a*c*
d^3*e^4)*x)/e^8 + (c^2*d^8 + 2*a*c*d^4*e^4 + a^2*e^8)*log(e*x + d)/e^9
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.06

$$\int \frac{(a+cx^4)^2}{d+ex}\,dx = \frac{105\,c^2e^7x^8 - 120\,c^2de^6x^7 + 140\,c^2d^2e^5x^6 - 168\,c^2d^3e^4x^5 + 210\,c^2d^4e^3x^4 + 420\,ace^7x^4 - 280\,c^2d^5e^2x^3 - 5}{840\,e^8} + \frac{(c^2d^8 + 2\,acd^4e^4 + a^2e^8)\log{(|ex+d|)}}{e^9}$$

input
```
integrate((c*x^4+a)^2/(e*x+d),x, algorithm="giac")
```

output
```
1/840*(105*c^2*e^7*x^8 - 120*c^2*d*e^6*x^7 + 140*c^2*d^2*e^5*x^6 - 168*c^2
*d^3*e^4*x^5 + 210*c^2*d^4*e^3*x^4 + 420*a*c*e^7*x^4 - 280*c^2*d^5*e^2*x^3
 - 560*a*c*d*e^6*x^3 + 420*c^2*d^6*e*x^2 + 840*a*c*d^2*e^5*x^2 - 840*c^2*d
^7*x - 1680*a*c*d^3*e^4*x)/e^8 + (c^2*d^8 + 2*a*c*d^4*e^4 + a^2*e^8)*log(a
bs(e*x + d))/e^9
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.08

$$\int \frac{(a+cx^4)^2}{d+ex}\,dx = x^4\left(\frac{c^2\,d^4}{4\,e^5}+\frac{a\,c}{2\,e}\right)+\frac{\ln\left(d+e\,x\right)\,\left(a^2\,e^8+2\,a\,c\,d^4\,e^4+c^2\,d^8\right)}{e^9} + \frac{c^2\,x^8}{8\,e}-\frac{c^2\,d\,x^7}{7\,e^2}-\frac{d\,x^3\,\left(\frac{c^2\,d^4}{e^5}+\frac{2\,a\,c}{e}\right)}{3\,e}-\frac{d^3\,x\,\left(\frac{c^2\,d^4}{e^5}+\frac{2\,a\,c}{e}\right)}{e^3} + \frac{c^2\,d^2\,x^6}{6\,e^3}-\frac{c^2\,d^3\,x^5}{5\,e^4}+\frac{d^2\,x^2\,\left(\frac{c^2\,d^4}{e^5}+\frac{2\,a\,c}{e}\right)}{2\,e^2}$$

input
```
int((a + c*x^4)^2/(d + e*x),x)
```

output
```
x^4*((c^2*d^4)/(4*e^5) + (a*c)/(2*e)) + (log(d + e*x)*(a^2*e^8 + c^2*d^8 +
 2*a*c*d^4*e^4))/e^9 + (c^2*x^8)/(8*e) - (c^2*d*x^7)/(7*e^2) - (d*x^3*((c^
2*d^4)/e^5 + (2*a*c)/e))/(3*e) - (d^3*x*((c^2*d^4)/e^5 + (2*a*c)/e))/e^3 +
 (c^2*d^2*x^6)/(6*e^3) - (c^2*d^3*x^5)/(5*e^4) + (d^2*x^2*((c^2*d^4)/e^5 +
 (2*a*c)/e))/(2*e^2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.11

$$\int \frac{(a+cx^4)^2}{d+ex}\,dx = \frac{840\log(ex+d)\,a^2e^8+1680\log(ex+d)\,ac\,d^4e^4+840\log(ex+d)\,c^2d^8-1680ac\,d^3e^5x+840ac\,d^2e^6x^2-$$

input
```
int((c*x^4+a)^2/(e*x+d),x)
```

output
```
(840*log(d + e*x)*a**2*e**8 + 1680*log(d + e*x)*a*c*d**4*e**4 + 840*log(d
+ e*x)*c**2*d**8 - 1680*a*c*d**3*e**5*x + 840*a*c*d**2*e**6*x**2 - 560*a*c
*d*e**7*x**3 + 420*a*c*e**8*x**4 - 840*c**2*d**7*e*x + 420*c**2*d**6*e**2*
x**2 - 280*c**2*d**5*e**3*x**3 + 210*c**2*d**4*e**4*x**4 - 168*c**2*d**3*e
**5*x**5 + 140*c**2*d**2*e**6*x**6 - 120*c**2*d*e**7*x**7 + 105*c**2*e**8*
x**8)/(840*e**9)
```

3.172 $\int \frac{\left(a+cx^4\right)^2}{(d+ex)^2} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 178

$$\int \frac{\left(a+cx^4\right)^2}{(d+ex)^2} \, dx = \frac{cd^2(7cd^4+6ae^4)\,x}{e^8} - \frac{cd(3cd^4+2ae^4)\,x^2}{e^7} + \frac{c(5cd^4+2ae^4)\,x^3}{3e^6} - \frac{c^2d^3x^4}{e^5} + \frac{3c^2d^2x^5}{5e^4} - \frac{c^2dx^6}{3e^3} + \frac{c^2x^7}{7e^2} - \frac{\left(cd^4+ae^4\right)^2}{e^9(d+ex)} - \frac{8cd^3(cd^4+ae^4)\log(d+ex)}{e^9}$$

output
```
c*d^2*(6*a*e^4+7*c*d^4)*x/e^8-c*d*(2*a*e^4+3*c*d^4)*x^2/e^7+1/3*c*(2*a*e^4
+5*c*d^4)*x^3/e^6-c^2*d^3*x^4/e^5+3/5*c^2*d^2*x^5/e^4-1/3*c^2*d*x^6/e^3+1/
7*c^2*x^7/e^2-(a*e^4+c*d^4)^2/e^9/(e*x+d)-8*c*d^3*(a*e^4+c*d^4)*ln(e*x+d)/
e^9
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.00

$$\int \frac{(a+cx^4)^2}{(d+ex)^2}\,dx = \frac{cd^2(7cd^4+6ae^4)\,x}{e^8} - \frac{cd(3cd^4+2ae^4)\,x^2}{e^7} + \frac{c(5cd^4+2ae^4)\,x^3}{3e^6} - \frac{c^2d^3x^4}{e^5} + \frac{3c^2d^2x^5}{5e^4} - \frac{c^2dx^6}{3e^3} + \frac{c^2x^7}{7e^2} - \frac{(cd^4+ae^4)^2}{e^9(d+ex)} - \frac{8cd^3(cd^4+ae^4)\log(d+ex)}{e^9}$$

input
```
Integrate[(a + c*x^4)^2/(d + e*x)^2,x]
```

output
```
(c*d^2*(7*c*d^4 + 6*a*e^4)*x)/e^8 - (c*d*(3*c*d^4 + 2*a*e^4)*x^2)/e^7 + (c
*(5*c*d^4 + 2*a*e^4)*x^3)/(3*e^6) - (c^2*d^3*x^4)/e^5 + (3*c^2*d^2*x^5)/(5
*e^4) - (c^2*d*x^6)/(3*e^3) + (c^2*x^7)/(7*e^2) - (c*d^4 + a*e^4)^2/(e^9*(
d + e*x)) - (8*c*d^3*(c*d^4 + a*e^4)*Log[d + e*x])/e^9
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^4)^2}{(d+ex)^2}\,dx$$

$$\Big\downarrow \text{2389}$$

$$\int \left(\frac{(ae^4+cd^4)^2}{e^8(d+ex)^2} - \frac{2cdx(2ae^4+3cd^4)}{e^7} + \frac{cx^2(2ae^4+5cd^4)}{e^6} - \frac{8cd^3(ae^4+cd^4)}{e^8(d+ex)} + \frac{cd^2(6ae^4+7cd^4)}{e^8} - \frac{4c^2d^3x^3}{e^5}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\left(ae^4+cd^4\right)^2}{e^9(d+ex)}-\frac{cdx^2\left(2ae^4+3cd^4\right)}{e^7}+\frac{cx^3\left(2ae^4+5cd^4\right)}{3e^6}-\frac{8cd^3\left(ae^4+cd^4\right)\log(d+ex)}{e^9}+\frac{cd^2x\left(6ae^4+7cd^4\right)}{e^8}-\frac{c^2d^3x^4}{e^5}+\frac{3c^2d^2x^5}{5e^4}-\frac{c^2dx^6}{3e^3}+\frac{c^2x^7}{7e^2}$$

input
```
Int[(a + c*x^4)^2/(d + e*x)^2,x]
```

output
```
(c*d^2*(7*c*d^4 + 6*a*e^4)*x)/e^8 - (c*d*(3*c*d^4 + 2*a*e^4)*x^2)/e^7 + (c
*(5*c*d^4 + 2*a*e^4)*x^3)/(3*e^6) - (c^2*d^3*x^4)/e^5 + (3*c^2*d^2*x^5)/(5
*e^4) - (c^2*d*x^6)/(3*e^3) + (c^2*x^7)/(7*e^2) - (c*d^4 + a*e^4)^2/(e^9*(
d + e*x)) - (8*c*d^3*(c*d^4 + a*e^4)*Log[d + e*x])/e^9
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2389
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.97

method	result
default	$\frac{c\left(\frac{1}{7}cx^7e^6-\frac{1}{3}cdx^6e^5+\frac{3}{5}cd^2x^5e^4-cd^3x^4e^3+\frac{2}{3}ae^6x^3+\frac{5}{3}cd^4e^2x^3-2ade^5x^2-3cd^5ex^2+6xad^2e^4+7xcd^6\right)}{e^8}-\frac{a^2e^8+2acd^4\cdots}{e^9(ex+\cdots}$
norman	$\frac{-\frac{a^2e^8+8acd^4e^4+8d^8c^2}{e^9}+\frac{c^2x^8}{7e}+\frac{2c\left(e^4a+cd^4\right)x^4}{3e^5}-\frac{4c^2dx^7}{21e^2}+\frac{4c^2d^2x^6}{15e^3}-\frac{2c^2d^3x^5}{5e^4}-\frac{4dc\left(e^4a+cd^4\right)x^3}{3e^6}+\frac{4d^2c\left(e^4a+cd^4\right)x^2}{e^7}}{ex+d}-\frac{8cd\cdots}{}$
risch	$\frac{c^2x^7}{7e^2}-\frac{c^2dx^6}{3e^3}+\frac{3c^2d^2x^5}{5e^4}-\frac{c^2d^3x^4}{e^5}+\frac{2cax^3}{3e^2}+\frac{5c^2d^4x^3}{3e^6}-\frac{2cadx^2}{e^3}-\frac{3c^2d^5x^2}{e^7}+\frac{6cxad^2}{e^4}+\frac{7c^2xd^6}{e^8}-\frac{a^2\cdots}{e(ex\cdots}$
parallelrisch	$-\frac{-15x^8c^2e^8+20c^2dx^7e^7-28c^2d^2x^6e^6+42c^2d^3x^5e^5-70x^4ace^8-70x^4c^2d^4e^4+140x^3acde^7+140x^3c^2d^5e^3+840\ln(ex+d)x\cdots}{\cdots}$

input
```
int((c*x^4+a)^2/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
c/e^8*(1/7*c*x^7*e^6-1/3*c*d*x^6*e^5+3/5*c*d^2*x^5*e^4-c*d^3*x^4*e^3+2/3*a
*e^6*x^3+5/3*c*d^4*e^2*x^3-2*a*d*e^5*x^2-3*c*d^5*e*x^2+6*x*a*d^2*e^4+7*x*c
*d^6)-(a^2*e^8+2*a*c*d^4*e^4+c^2*d^8)/e^9/(e*x+d)-8*c*d^3*(a*e^4+c*d^4)*ln
(e*x+d)/e^9
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.30

$$\int \frac{(a+cx^4)^2}{(d+ex)^2}\,dx$$
$$= \frac{15\,c^2e^8x^8 - 20\,c^2de^7x^7 + 28\,c^2d^2e^6x^6 - 42\,c^2d^3e^5x^5 - 105\,c^2d^8 - 210\,acd^4e^4 - 105\,a^2e^8 + 70\,(c^2d^4e^4 + ac}{}$$

input
```
integrate((c*x^4+a)^2/(e*x+d)^2,x, algorithm="fricas")
```

output
```
1/105*(15*c^2*e^8*x^8 - 20*c^2*d*e^7*x^7 + 28*c^2*d^2*e^6*x^6 - 42*c^2*d^3
*e^5*x^5 - 105*c^2*d^8 - 210*a*c*d^4*e^4 - 105*a^2*e^8 + 70*(c^2*d^4*e^4 +
 a*c*e^8)*x^4 - 140*(c^2*d^5*e^3 + a*c*d*e^7)*x^3 + 420*(c^2*d^6*e^2 + a*c
*d^2*e^6)*x^2 + 105*(7*c^2*d^7*e + 6*a*c*d^3*e^5)*x - 840*(c^2*d^8 + a*c*d
^4*e^4 + (c^2*d^7*e + a*c*d^3*e^5)*x)*log(e*x + d))/(e^10*x + d*e^9)
```

Sympy [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.12

$$\int \frac{(a+cx^4)^2}{(d+ex)^2}\,dx = -\frac{c^2d^3x^4}{e^5} + \frac{3c^2d^2x^5}{5e^4} - \frac{c^2dx^6}{3e^3} + \frac{c^2x^7}{7e^2} - \frac{8cd^3(ae^4+cd^4)\log(d+ex)}{e^9} + x^3\cdot\left(\frac{2ac}{3e^2} + \frac{5c^2d^4}{3e^6}\right) + x^2\left(-\frac{2acd}{e^3} - \frac{3c^2d^5}{e^7}\right) + x\left(\frac{6acd^2}{e^4} + \frac{7c^2d^6}{e^8}\right) + \frac{-a^2e^8 - 2acd^4e^4 - c^2d^8}{de^9 + e^{10}x}$$

input
```
integrate((c*x**4+a)**2/(e*x+d)**2,x)
```

output
```
-c**2*d**3*x**4/e**5 + 3*c**2*d**2*x**5/(5*e**4) - c**2*d*x**6/(3*e**3) +
c**2*x**7/(7*e**2) - 8*c*d**3*(a*e**4 + c*d**4)*log(d + e*x)/e**9 + x**3*(
2*a*c/(3*e**2) + 5*c**2*d**4/(3*e**6)) + x**2*(-2*a*c*d/e**3 - 3*c**2*d**5
/e**7) + x*(6*a*c*d**2/e**4 + 7*c**2*d**6/e**8) + (-a**2*e**8 - 2*a*c*d**4
*e**4 - c**2*d**8)/(d*e**9 + e**10*x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.10

$$\int \frac{(a+cx^4)^2}{(d+ex)^2}\,dx = -\frac{c^2d^8+2\,acd^4e^4+a^2e^8}{e^{10}x+de^9} + \frac{15\,c^2e^6x^7-35\,c^2de^5x^6+63\,c^2d^2e^4x^5-105\,c^2d^3e^3x^4+35\,(5\,c^2d^4e^2+2\,ace^6)x^3-105\,(3\,c^2d^5e+2\,acde}{105\,e^8} - \frac{8\,(c^2d^7+acd^3e^4)\log\left(ex+d\right)}{e^9}$$

input
```
integrate((c*x^4+a)^2/(e*x+d)^2,x, algorithm="maxima")
```

output
```
-(c^2*d^8 + 2*a*c*d^4*e^4 + a^2*e^8)/(e^10*x + d*e^9) + 1/105*(15*c^2*e^6*
x^7 - 35*c^2*d*e^5*x^6 + 63*c^2*d^2*e^4*x^5 - 105*c^2*d^3*e^3*x^4 + 35*(5*
c^2*d^4*e^2 + 2*a*c*e^6)*x^3 - 105*(3*c^2*d^5*e + 2*a*c*d*e^5)*x^2 + 105*(
7*c^2*d^6 + 6*a*c*d^2*e^4)*x)/e^8 - 8*(c^2*d^7 + a*c*d^3*e^4)*log(e*x + d)
/e^9
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.42

$$\int \frac{(a+cx^4)^2}{(d+ex)^2}\,dx = \frac{\left(15\,c^2-\frac{140\,c^2d}{ex+d}+\frac{588\,c^2d^2}{(ex+d)^2}-\frac{1470\,c^2d^3}{(ex+d)^3}+\frac{70\,(35\,c^2d^4e^4+ace^8)}{(ex+d)^4e^4}-\frac{420\,(7\,c^2d^5e^5+acde^9)}{(ex+d)^5e^5}+\frac{420\,(7\,c^2d^6e^6+3\,acd^2e^{10})}{(ex+d)^6e^6}\right)(ex+d}{105\,e^9} + \frac{8\,(c^2d^7+acd^3e^4)\log\left(\frac{|ex+d|}{(ex+d)^2|e|}\right)}{e^9} - \frac{\frac{c^2d^8e^7}{ex+d}+\frac{2\,acd^4e^{11}}{ex+d}+\frac{a^2e^{15}}{ex+d}}{e^{16}}$$

input
```
integrate((c*x^4+a)^2/(e*x+d)^2,x, algorithm="giac")
```

output
```
1/105*(15*c^2 - 140*c^2*d/(e*x + d) + 588*c^2*d^2/(e*x + d)^2 - 1470*c^2*d
^3/(e*x + d)^3 + 70*(35*c^2*d^4*e^4 + a*c*e^8)/((e*x + d)^4*e^4) - 420*(7*
c^2*d^5*e^5 + a*c*d*e^9)/((e*x + d)^5*e^5) + 420*(7*c^2*d^6*e^6 + 3*a*c*d^
2*e^10)/((e*x + d)^6*e^6))*(e*x + d)^7/e^9 + 8*(c^2*d^7 + a*c*d^3*e^4)*log
(abs(e*x + d)/((e*x + d)^2*abs(e)))/e^9 - (c^2*d^8*e^7/(e*x + d) + 2*a*c*d
^4*e^11/(e*x + d) + a^2*e^15/(e*x + d))/e^16
```

Mupad [B] (verification not implemented)

Time = 21.46 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.47

$$\int \frac{(a+cx^4)^2}{(d+ex)^2}\,dx = x\left(\frac{2\,d\,\left(\frac{2\,d\,\left(\frac{5\,c^2\,d^4}{e^6}+\frac{2\,a\,c}{e^2}\right)}{e}-\frac{4\,c^2\,d^5}{e^7}\right)}{e}-\frac{d^2\,\left(\frac{5\,c^2\,d^4}{e^6}+\frac{2\,a\,c}{e^2}\right)}{e^2}\right) + x^3\left(\frac{5\,c^2\,d^4}{3\,e^6}+\frac{2\,a\,c}{3\,e^2}\right)-x^2\left(\frac{d\,\left(\frac{5\,c^2\,d^4}{e^6}+\frac{2\,a\,c}{e^2}\right)}{e}-\frac{2\,c^2\,d^5}{e^7}\right) - \frac{\ln\left(d+e\,x\right)\,\left(8\,c^2\,d^7+8\,a\,c\,d^3\,e^4\right)}{e^9}+\frac{c^2\,x^7}{7\,e^2} - \frac{a^2\,e^8+2\,a\,c\,d^4\,e^4+c^2\,d^8}{e\,\left(x\,e^9+d\,e^8\right)}-\frac{c^2\,d\,x^6}{3\,e^3}+\frac{3\,c^2\,d^2\,x^5}{5\,e^4}-\frac{c^2\,d^3\,x^4}{e^5}$$

input
```
int((a + c*x^4)^2/(d + e*x)^2,x)
```

output
```
x*((2*d*((2*d*((5*c^2*d^4)/e^6 + (2*a*c)/e^2))/e - (4*c^2*d^5)/e^7))/e - (
d^2*((5*c^2*d^4)/e^6 + (2*a*c)/e^2))/e^2) + x^3*((5*c^2*d^4)/(3*e^6) + (2*
a*c)/(3*e^2)) - x^2*((d*((5*c^2*d^4)/e^6 + (2*a*c)/e^2))/e - (2*c^2*d^5)/e
^7) - (log(d + e*x)*(8*c^2*d^7 + 8*a*c*d^3*e^4))/e^9 + (c^2*x^7)/(7*e^2) -
 (a^2*e^8 + c^2*d^8 + 2*a*c*d^4*e^4)/(e*(d*e^8 + e^9*x)) - (c^2*d*x^6)/(3*
e^3) + (3*c^2*d^2*x^5)/(5*e^4) - (c^2*d^3*x^4)/e^5
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.36

$$\int \frac{(a+cx^4)^2}{(d+ex)^2}\,dx$$
$$= \frac{-840\log(ex+d)\,ac\,d^5e^4 - 840\log(ex+d)\,ac\,d^4e^5x - 840\log(ex+d)\,c^2d^9 - 840\log(ex+d)\,c^2d^8ex + 1}{}$$

input
```
int((c*x^4+a)^2/(e*x+d)^2,x)
```

output
```
( - 840*log(d + e*x)*a*c*d**5*e**4 - 840*log(d + e*x)*a*c*d**4*e**5*x - 84
0*log(d + e*x)*c**2*d**9 - 840*log(d + e*x)*c**2*d**8*e*x + 105*a**2*e**9*
x + 840*a*c*d**4*e**5*x + 420*a*c*d**3*e**6*x**2 - 140*a*c*d**2*e**7*x**3
+ 70*a*c*d*e**8*x**4 + 840*c**2*d**8*e*x + 420*c**2*d**7*e**2*x**2 - 140*c
**2*d**6*e**3*x**3 + 70*c**2*d**5*e**4*x**4 - 42*c**2*d**4*e**5*x**5 + 28*
c**2*d**3*e**6*x**6 - 20*c**2*d**2*e**7*x**7 + 15*c**2*d*e**8*x**8)/(105*d
*e**9*(d + e*x))
```

3.173 $\int \frac{(d+ex)^3}{a+cx^4} dx$

Optimal result

Integrand size = 17, antiderivative size = 248

$$\int \frac{(d+ex)^3}{a+cx^4} dx = \frac{3d^2e\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{c}} - \frac{d\left(\sqrt{c}d^2+3\sqrt{a}e^2\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}c^{3/4}} + \frac{d\left(\sqrt{c}d^2+3\sqrt{a}e^2\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}c^{3/4}} + \frac{d\left(\sqrt{c}d^2-3\sqrt{a}e^2\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x}{\sqrt{a}+\sqrt{c}x^2}\right)}{2\sqrt{2}a^{3/4}c^{3/4}} + \frac{e^3\log\left(a+cx^4\right)}{4c}$$

output
```
3/2*d^2*e*arctan(c^(1/2)*x^2/a^(1/2))/a^(1/2)/c^(1/2)+1/4*d*(c^(1/2)*d^2+3
*a^(1/2)*e^2)*arctan(-1+2^(1/2)*c^(1/4)*x/a^(1/4))*2^(1/2)/a^(3/4)/c^(3/4)
+1/4*d*(c^(1/2)*d^2+3*a^(1/2)*e^2)*arctan(1+2^(1/2)*c^(1/4)*x/a^(1/4))*2^(
1/2)/a^(3/4)/c^(3/4)+1/4*d*(c^(1/2)*d^2-3*a^(1/2)*e^2)*arctanh(2^(1/2)*a^(
1/4)*c^(1/4)*x/(a^(1/2)+c^(1/2)*x^2))*2^(1/2)/a^(3/4)/c^(3/4)+1/4*e^3*ln(c
*x^4+a)/c
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.30

$$\int \frac{(d+ex)^3}{a+cx^4}\,dx = \frac{-2\sqrt[4]{a}\sqrt[4]{c}d\left(\sqrt{2}\sqrt{c}d^2+6\sqrt[4]{a}\sqrt[4]{c}de+3\sqrt{2}\sqrt{a}e^2\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)+2\sqrt[4]{a}\sqrt[4]{c}d\left(\sqrt{2}\sqrt{c}d^2-6\sqrt[4]{a}\sqrt[4]{c}de\right.}{}$$

input
```
Integrate[(d + e*x)^3/(a + c*x^4),x]
```

output
```
(-2*a^(1/4)*c^(1/4)*d*(Sqrt[2]*Sqrt[c]*d^2 + 6*a^(1/4)*c^(1/4)*d*e + 3*Sqr
t[2]*Sqrt[a]*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 2*a^(1/4)*c^(1
/4)*d*(Sqrt[2]*Sqrt[c]*d^2 - 6*a^(1/4)*c^(1/4)*d*e + 3*Sqrt[2]*Sqrt[a]*e^2
)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)] - Sqrt[2]*c^(1/4)*(a^(1/4)*Sqrt[
c]*d^3 - 3*a^(3/4)*d*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c
]*x^2] + Sqrt[2]*c^(1/4)*(a^(1/4)*Sqrt[c]*d^3 - 3*a^(3/4)*d*e^2)*Log[Sqrt[
a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] + 2*a*e^3*Log[a + c*x^4])/(8
*a*c)
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.29, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2415, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^3}{a+cx^4}\,dx$$

$$\Big\downarrow \ 2415$$

$$\int \left(\frac{d^3+3de^2x^2}{a+cx^4}+\frac{x\left(3d^2e+e^3x^2\right)}{a+cx^4}\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{d\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\left(3\sqrt{a}e^2+\sqrt{c}d^2\right)}{2\sqrt{2}a^{3/4}c^{3/4}}+\frac{d\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)\left(3\sqrt{a}e^2+\sqrt{c}d^2\right)}{2\sqrt{2}a^{3/4}c^{3/4}}-\frac{d\left(\sqrt{c}d^2-3\sqrt{a}e^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}c^{3/4}}+\frac{d\left(\sqrt{c}d^2-3\sqrt{a}e^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}c^{3/4}}+\frac{3d^2e\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{c}}+\frac{e^3\log\left(a+cx^4\right)}{4c}$$

input
```
Int[(d + e*x)^3/(a + c*x^4),x]
```

output
```
(3*d^2*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*Sqrt[a]*Sqrt[c]) - (d*(Sqrt[c]*
d^2 + 3*Sqrt[a]*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a
^(3/4)*c^(3/4)) + (d*(Sqrt[c]*d^2 + 3*Sqrt[a]*e^2)*ArcTan[1 + (Sqrt[2]*c^(
1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*c^(3/4)) - (d*(Sqrt[c]*d^2 - 3*Sqrt[a
]*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*
a^(3/4)*c^(3/4)) + (d*(Sqrt[c]*d^2 - 3*Sqrt[a]*e^2)*Log[Sqrt[a] + Sqrt[2]*
a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(3/4)) + (e^3*Log[a
 + c*x^4])/(4*c)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2415
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.17 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.22

method	result
risch	$\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+a\right)}\frac{\left(\_R^3e^3+3\_R^2de^2+3\_Rd^2e+d^3\right)\ln\left(x-\_R\right)}{\_R^3}}{4c}$
default	$\frac{d^3\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{8a}+\frac{3d^2e\arctan\left(\sqrt{\frac{c}{a}}\,x^2\right)}{2\sqrt{ac}}+\frac{3de^2\sqrt{2}\,(}{}$

input

```
int((e*x+d)^3/(c*x^4+a),x,method=_RETURNVERBOSE)
```

output

```
1/4/c*sum((_R^3*e^3+3*_R^2*d*e^2+3*_R*d^2*e+d^3)/_R^3*ln(x-_R),_R=RootOf(_
Z^4*c+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 15.66 (sec) , antiderivative size = 141845, normalized size of antiderivative = 571.96

$$\int \frac{(d+ex)^3}{a+cx^4}\,dx = \text{Too large to display}$$

input

```
integrate((e*x+d)^3/(c*x^4+a),x, algorithm="fricas")
```

output

```
Too large to include
```

Sympy [A] (verification not implemented)

Time = 3.87 (sec) , antiderivative size = 384, normalized size of antiderivative = 1.55

$$\int \frac{(d+ex)^3}{a+cx^4}\,dx$$
$$= \text{RootSum}\left(256t^4a^3c^4 - 256t^3a^3c^3e^3 + t^2\cdot\left(96a^3c^2e^6 + 480a^2c^3d^4e^2\right) + t\left(-16a^3ce^9 + 192a^2c^2d^4e^5 - 48a\right.\right.$$

input
```
integrate((e*x+d)**3/(c*x**4+a),x)
```

output
```
RootSum(256*_t**4*a**3*c**4 - 256*_t**3*a**3*c**3*e**3 + _t**2*(96*a**3*c*
*2*e**6 + 480*a**2*c**3*d**4*e**2) + _t*(-16*a**3*c*e**9 + 192*a**2*c**2*d
**4*e**5 - 48*a*c**3*d**8*e) + a**3*e**12 + 3*a**2*c*d**4*e**8 + 3*a*c**2*
d**8*e**4 + c**3*d**12, Lambda(_t, _t*log(x + (1728*_t**3*a**4*c**3*e**6 +
 960*_t**3*a**3*c**4*d**4*e**2 - 1296*_t**2*a**4*c**2*e**9 - 2016*_t**2*a*
*3*c**3*d**4*e**5 + 48*_t**2*a**2*c**4*d**8*e + 324*_t*a**4*c*e**12 + 4716
*_t*a**3*c**2*d**4*e**8 + 1452*_t*a**2*c**3*d**8*e**4 + 4*_t*a*c**4*d**12
- 27*a**4*e**15 + 1119*a**3*c*d**4*e**11 - 609*a**2*c**2*d**8*e**7 - 91*a*
c**3*d**12*e**3)/(729*a**3*c*d**3*e**12 - 1053*a**2*c**2*d**7*e**8 - 117*a
*c**3*d**11*e**4 + c**4*d**15))))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.25

$$\begin{aligned}\int \frac{(d+ex)^3}{a+cx^4}\,dx &= \frac{\sqrt{2}\left(\sqrt{2}a^{\frac{3}{4}}c^{\frac{1}{4}}e^3 + cd^3 - 3\sqrt{a}\sqrt{c}de^2\right)\log\left(\sqrt{c}x^2 + \sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x + \sqrt{a}\right)}{8\,a^{\frac{3}{4}}c^{\frac{5}{4}}} \\
&+ \frac{\sqrt{2}\left(\sqrt{2}a^{\frac{3}{4}}c^{\frac{1}{4}}e^3 - cd^3 + 3\sqrt{a}\sqrt{c}de^2\right)\log\left(\sqrt{c}x^2 - \sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x + \sqrt{a}\right)}{8\,a^{\frac{3}{4}}c^{\frac{5}{4}}} \\
&+ \frac{\left(\sqrt{2}a^{\frac{1}{4}}c^{\frac{5}{4}}d^3 + 3\sqrt{2}a^{\frac{3}{4}}c^{\frac{3}{4}}de^2 - 6\sqrt{a}cd^2e\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{4\,a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{c}}c^{\frac{5}{4}}} \\
&+ \frac{\left(\sqrt{2}a^{\frac{1}{4}}c^{\frac{5}{4}}d^3 + 3\sqrt{2}a^{\frac{3}{4}}c^{\frac{3}{4}}de^2 + 6\sqrt{a}cd^2e\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{4\,a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{c}}c^{\frac{5}{4}}}\end{aligned}$$

input
```
integrate((e*x+d)^3/(c*x^4+a),x, algorithm="maxima")
```

output
```
1/8*sqrt(2)*(sqrt(2)*a^(3/4)*c^(1/4)*e^3 + c*d^3 - 3*sqrt(a)*sqrt(c)*d*e^2
)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(5/4))
 + 1/8*sqrt(2)*(sqrt(2)*a^(3/4)*c^(1/4)*e^3 - c*d^3 + 3*sqrt(a)*sqrt(c)*d*
e^2)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(5/
4)) + 1/4*(sqrt(2)*a^(1/4)*c^(5/4)*d^3 + 3*sqrt(2)*a^(3/4)*c^(3/4)*d*e^2 -
 6*sqrt(a)*c*d^2*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1
/4))/sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(5/4)) + 1/4*
(sqrt(2)*a^(1/4)*c^(5/4)*d^3 + 3*sqrt(2)*a^(3/4)*c^(3/4)*d*e^2 + 6*sqrt(a)
*c*d^2*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(
sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(5/4))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.27

$$\int \frac{(d+ex)^3}{a+cx^4}\,dx = \frac{e^3\log\left(|cx^4+a|\right)}{4\,c} + \frac{\sqrt{2}\left(3\,\sqrt{2}\sqrt{ac}c^2d^2e+\left(ac^3\right)^{\frac{1}{4}}c^2d^3+3\left(ac^3\right)^{\frac{3}{4}}de^2\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4\,ac^3} + \frac{\sqrt{2}\left(3\,\sqrt{2}\sqrt{ac}c^2d^2e+\left(ac^3\right)^{\frac{1}{4}}c^2d^3+3\left(ac^3\right)^{\frac{3}{4}}de^2\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4\,ac^3} + \frac{\sqrt{2}\left(\left(ac^3\right)^{\frac{1}{4}}c^2d^3-3\left(ac^3\right)^{\frac{3}{4}}de^2\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{8\,ac^3} - \frac{\sqrt{2}\left(\left(ac^3\right)^{\frac{1}{4}}c^2d^3-3\left(ac^3\right)^{\frac{3}{4}}de^2\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{8\,ac^3}$$

input
```
integrate((e*x+d)^3/(c*x^4+a),x, algorithm="giac")
```

output
```
1/4*e^3*log(abs(c*x^4 + a))/c + 1/4*sqrt(2)*(3*sqrt(2)*sqrt(a*c)*c^2*d^2*e
 + (a*c^3)^(1/4)*c^2*d^3 + 3*(a*c^3)^(3/4)*d*e^2)*arctan(1/2*sqrt(2)*(2*x
+ sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a*c^3) + 1/4*sqrt(2)*(3*sqrt(2)*sqrt(
a*c)*c^2*d^2*e + (a*c^3)^(1/4)*c^2*d^3 + 3*(a*c^3)^(3/4)*d*e^2)*arctan(1/2
*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a*c^3) + 1/8*sqrt(2)*((
a*c^3)^(1/4)*c^2*d^3 - 3*(a*c^3)^(3/4)*d*e^2)*log(x^2 + sqrt(2)*x*(a/c)^(1
/4) + sqrt(a/c))/(a*c^3) - 1/8*sqrt(2)*((a*c^3)^(1/4)*c^2*d^3 - 3*(a*c^3)^
(3/4)*d*e^2)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a*c^3)
```

Mupad [B] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 894, normalized size of antiderivative = 3.60

$$\int \frac{(d+ex)^3}{a+cx^4}\,dx = \sum_{k=1}^{4} \ln\Big(-c\,d^2\,\Big(-3\,c\,d^5\,e^2+5\,a\,d\,e^6+3\,a\,e^7\,x$$

$$+\mathrm{root}\big(256\,a^3\,c^4\,z^4-256\,a^3\,c^3\,e^3\,z^3+480\,a^2\,c^3\,d^4\,e^2\,z^2+96\,a^3\,c^2\,e^6\,z^2+192\,a^2\,c^2\,d^4\,e^5\,z-48\,a\,c^3\,d^8\,e\,z$$

$$+\mathrm{root}\big(256\,a^3\,c^4\,z^4-256\,a^3\,c^3\,e^3\,z^3+480\,a^2\,c^3\,d^4\,e^2\,z^2+96\,a^3\,c^2\,e^6\,z^2+192\,a^2\,c^2\,d^4\,e^5\,z-48\,a\,c^3\,d^8\,e\,z-16$$

$$-5\,c\,d^4\,e^3\,x$$

$$-\mathrm{root}\big(256\,a^3\,c^4\,z^4-256\,a^3\,c^3\,e^3\,z^3+480\,a^2\,c^3\,d^4\,e^2\,z^2+96\,a^3\,c^2\,e^6\,z^2+192\,a^2\,c^2\,d^4\,e^5\,z-48\,a\,c^3\,d^8\,e\,z$$

$$+\mathrm{root}\big(256\,a^3\,c^4\,z^4-256\,a^3\,c^3\,e^3\,z^3+480\,a^2\,c^3\,d^4\,e^2\,z^2+96\,a^3\,c^2\,e^6\,z^2+192\,a^2\,c^2\,d^4\,e^5\,z-48\,a\,c^3\,d^8\,e\,z-16$$

$$-\mathrm{root}\big(256\,a^3\,c^4\,z^4-256\,a^3\,c^3\,e^3\,z^3+480\,a^2\,c^3\,d^4\,e^2\,z^2+96\,a^3\,c^2\,e^6\,z^2+192\,a^2\,c^2\,d^4\,e^5\,z-48\,a\,c^3\,d^8\,e\,z-16$$

$$-256\,a^3\,c^3\,e^3\,z^3+480\,a^2\,c^3\,d^4\,e^2\,z^2+96\,a^3\,c^2\,e^6\,z^2+192\,a^2\,c^2\,d^4\,e^5\,z-48\,a\,c^3\,d^8\,e\,z$$

$$-16\,a^3\,c\,e^9\,z+3\,a^2\,c\,d^4\,e^8+3\,a\,c^2\,d^8\,e^4+c^3\,d^{12}+a^3\,e^{12},z,k\big)$$

input
```
int((d + e*x)^3/(a + c*x^4),x)
```

output
```
symsum(log(-2*c*d^2*(5*a*d*e^6 - 3*c*d^5*e^2 + 3*a*e^7*x + 8*root(256*a^3*
c^4*z^4 - 256*a^3*c^3*e^3*z^3 + 480*a^2*c^3*d^4*e^2*z^2 + 96*a^3*c^2*e^6*z
^2 + 192*a^2*c^2*d^4*e^5*z - 48*a*c^3*d^8*e*z - 16*a^3*c*e^9*z + 3*a^2*c*d
^4*e^8 + 3*a*c^2*d^8*e^4 + c^3*d^12 + a^3*e^12, z, k)^2*a*c^2*d + 2*root(2
56*a^3*c^4*z^4 - 256*a^3*c^3*e^3*z^3 + 480*a^2*c^3*d^4*e^2*z^2 + 96*a^3*c^
2*e^6*z^2 + 192*a^2*c^2*d^4*e^5*z - 48*a*c^3*d^8*e*z - 16*a^3*c*e^9*z + 3*
a^2*c*d^4*e^8 + 3*a*c^2*d^8*e^4 + c^3*d^12 + a^3*e^12, z, k)*c^2*d^4*x - 5
*c*d^4*e^3*x - 24*root(256*a^3*c^4*z^4 - 256*a^3*c^3*e^3*z^3 + 480*a^2*c^3
*d^4*e^2*z^2 + 96*a^3*c^2*e^6*z^2 + 192*a^2*c^2*d^4*e^5*z - 48*a*c^3*d^8*e
*z - 16*a^3*c*e^9*z + 3*a^2*c*d^4*e^8 + 3*a*c^2*d^8*e^4 + c^3*d^12 + a^3*e
^12, z, k)^2*a*c^2*e*x + 32*root(256*a^3*c^4*z^4 - 256*a^3*c^3*e^3*z^3 + 4
80*a^2*c^3*d^4*e^2*z^2 + 96*a^3*c^2*e^6*z^2 + 192*a^2*c^2*d^4*e^5*z - 48*a
*c^3*d^8*e*z - 16*a^3*c*e^9*z + 3*a^2*c*d^4*e^8 + 3*a*c^2*d^8*e^4 + c^3*d^
12 + a^3*e^12, z, k)*a*c*d*e^3 - 6*root(256*a^3*c^4*z^4 - 256*a^3*c^3*e^3*
z^3 + 480*a^2*c^3*d^4*e^2*z^2 + 96*a^3*c^2*e^6*z^2 + 192*a^2*c^2*d^4*e^5*z
 - 48*a*c^3*d^8*e*z - 16*a^3*c*e^9*z + 3*a^2*c*d^4*e^8 + 3*a*c^2*d^8*e^4 +
 c^3*d^12 + a^3*e^12, z, k)*a*c*e^4*x))*root(256*a^3*c^4*z^4 - 256*a^3*c^3
*e^3*z^3 + 480*a^2*c^3*d^4*e^2*z^2 + 96*a^3*c^2*e^6*z^2 + 192*a^2*c^2*d^4*
e^5*z - 48*a*c^3*d^8*e*z - 16*a^3*c*e^9*z + 3*a^2*c*d^4*e^8 + 3*a*c^2*d^8*
e^4 + c^3*d^12 + a^3*e^12, z, k), k, 1, 4)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 433, normalized size of antiderivative = 1.75

$$\int \frac{(d+ex)^3}{a+cx^4}\,dx = \frac{-6c^{\frac{1}{4}}a^{\frac{3}{4}}\sqrt{2}\,atan\left(\frac{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{c}\,x}{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)d\,e^2-2c^{\frac{3}{4}}a^{\frac{1}{4}}\sqrt{2}\,atan\left(\frac{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{c}\,x}{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)d^3-12\sqrt{c}\,\sqrt{a}\,atan\left(\frac{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{c}\,x}{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right.}{}$$

input
```
int((e*x+d)^3/(c*x^4+a),x)
```

output

```
( - 6*c**(1/4)*a**(3/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c
)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*d*e**2 - 2*c**(3/4)*a**(1/4)*sqrt(2)*ata
n((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*d
**3 - 12*sqrt(c)*sqrt(a)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x)/(c
**(1/4)*a**(1/4)*sqrt(2)))*d**2*e + 6*c**(1/4)*a**(3/4)*sqrt(2)*atan((c**(
1/4)*a**(1/4)*sqrt(2) + 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*d*e**2 +
 2*c**(3/4)*a**(1/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(c)*x
)/(c**(1/4)*a**(1/4)*sqrt(2)))*d**3 - 12*sqrt(c)*sqrt(a)*atan((c**(1/4)*a*
*(1/4)*sqrt(2) + 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*d**2*e + 3*c**(
1/4)*a**(3/4)*sqrt(2)*log( - c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(
c)*x**2)*d*e**2 - 3*c**(1/4)*a**(3/4)*sqrt(2)*log(c**(1/4)*a**(1/4)*sqrt(2
)*x + sqrt(a) + sqrt(c)*x**2)*d*e**2 - c**(3/4)*a**(1/4)*sqrt(2)*log( - c*
*(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c)*x**2)*d**3 + c**(3/4)*a**(1/
4)*sqrt(2)*log(c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c)*x**2)*d**3
+ 2*log( - c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c)*x**2)*a*e**3 +
2*log(c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c)*x**2)*a*e**3)/(8*a*c
)
```

3.174 $\int \frac{(d+ex)^2}{a+cx^4} dx$

Optimal result

Integrand size = 17, antiderivative size = 220

$$\int \frac{(d+ex)^2}{a+cx^4} dx = \frac{de\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{c}} - \frac{\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}c^{3/4}} + \frac{\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}c^{3/4}} + \frac{\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x}{\sqrt{a}+\sqrt{c}x^2}\right)}{2\sqrt{2}a^{3/4}c^{3/4}}$$

output
```
d*e*arctan(c^(1/2)*x^2/a^(1/2))/a^(1/2)/c^(1/2)+1/4*(c^(1/2)*d^2+a^(1/2)*e
^2)*arctan(-1+2^(1/2)*c^(1/4)*x/a^(1/4))*2^(1/2)/a^(3/4)/c^(3/4)+1/4*(c^(1
/2)*d^2+a^(1/2)*e^2)*arctan(1+2^(1/2)*c^(1/4)*x/a^(1/4))*2^(1/2)/a^(3/4)/c
^(3/4)+1/4*(c^(1/2)*d^2-a^(1/2)*e^2)*arctanh(2^(1/2)*a^(1/4)*c^(1/4)*x/(a^
(1/2)+c^(1/2)*x^2))*2^(1/2)/a^(3/4)/c^(3/4)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.10

$$\int \frac{(d+ex)^2}{a+cx^4}\,dx$$
$$= \frac{-2\left(\sqrt{2}\sqrt{c}d^2+4\sqrt[4]{a}\sqrt[4]{c}de+\sqrt{2}\sqrt{a}e^2\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)+2\left(\sqrt{2}\sqrt{c}d^2-4\sqrt[4]{a}\sqrt[4]{c}de+\sqrt{2}\sqrt{a}e^2\right)\mathrm{arct}}{8a^{3}}$$

input
```
Integrate[(d + e*x)^2/(a + c*x^4),x]
```

output
```
(-2*(Sqrt[2]*Sqrt[c]*d^2 + 4*a^(1/4)*c^(1/4)*d*e + Sqrt[2]*Sqrt[a]*e^2)*Ar
cTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 2*(Sqrt[2]*Sqrt[c]*d^2 - 4*a^(1/4)
*c^(1/4)*d*e + Sqrt[2]*Sqrt[a]*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)
] - Sqrt[2]*(Sqrt[c]*d^2 - Sqrt[a]*e^2)*(Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(
1/4)*x + Sqrt[c]*x^2] - Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*
x^2]))/(8*a^(3/4)*c^(3/4))
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.32, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2415, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^2}{a+cx^4}\,dx$$
$$\Big\downarrow \; 2415$$
$$\int \left(\frac{d^2+e^2x^2}{a+cx^4}+\frac{2dex}{a+cx^4}\right)dx$$
$$\Big\downarrow \; 2009$$

$$-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\left(\sqrt{a}e^2+\sqrt{c}d^2\right)}{2\sqrt{2}a^{3/4}c^{3/4}}+\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)\left(\sqrt{a}e^2+\sqrt{c}d^2\right)}{2\sqrt{2}a^{3/4}c^{3/4}}-\frac{\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}c^{3/4}}+\frac{\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}c^{3/4}}+\frac{de\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{c}}$$

input
```
Int[(d + e*x)^2/(a + c*x^4),x]
```

output
```
(d*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(Sqrt[a]*Sqrt[c]) - ((Sqrt[c]*d^2 + Sq
rt[a]*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*c^(
3/4)) + ((Sqrt[c]*d^2 + Sqrt[a]*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4
)])/(2*Sqrt[2]*a^(3/4)*c^(3/4)) - ((Sqrt[c]*d^2 - Sqrt[a]*e^2)*Log[Sqrt[a]
 - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(3/4)) +
 ((Sqrt[c]*d^2 - Sqrt[a]*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sq
rt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(3/4))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2415
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.26 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.20

method	result
risch	$\frac{\sum_{\_R=\mathrm{RootOf}\left(c\,\_Z^{4}+a\right)}\frac{\left(\_R^{2}e^{2}+2\,\_R de+d^{2}\right)\ln\left(x-\_R\right)}{\_R^{3}}}{4c}$
default	$\frac{d^{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^{2}+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^{2}-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{8a}+\frac{de\arctan\left(\sqrt{\frac{c}{a}}\,x^{2}\right)}{\sqrt{ac}}+\frac{e^{2}\sqrt{2}\left(\ln\left(\right.\right.}{}$

input
```
int((e*x+d)^2/(c*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/4/c*sum((_R^2*e^2+2*_R*d*e+d^2)/_R^3*ln(x-_R),_R=RootOf(_Z^4*c+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.04 (sec) , antiderivative size = 86139, normalized size of antiderivative = 391.54

$$\int \frac{(d+ex)^2}{a+cx^4}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^2/(c*x^4+a),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [A] (verification not implemented)

Time = 2.44 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.26

$$\int \frac{(d+ex)^2}{a+cx^4}\,dx$$
$$=\operatorname{RootSum}\left(256t^4a^3c^3+192t^2a^2c^2d^2e^2+t\left(32a^2cde^5-32ac^2d^5e\right)+a^2e^8+2acd^4e^4+c^2d^8,\left(t\mapsto t\log\left(\right.\right.\right.$$

input
```
integrate((e*x+d)**2/(c*x**4+a),x)
```

output
```
RootSum(256*_t**4*a**3*c**3 + 192*_t**2*a**2*c**2*d**2*e**2 + _t*(32*a**2*
c*d*e**5 - 32*a*c**2*d**5*e) + a**2*e**8 + 2*a*c*d**4*e**4 + c**2*d**8, La
mbda(_t, _t*log(x + (64*_t**3*a**4*c**2*e**6 + 448*_t**3*a**3*c**3*d**4*e*
*2 - 160*_t**2*a**3*c**2*d**3*e**5 + 32*_t**2*a**2*c**3*d**7*e + 60*_t*a**
3*c*d**2*e**8 + 256*_t*a**2*c**2*d**6*e**4 + 4*_t*a*c**3*d**10 + 6*a**3*d*
e**11 - 24*a**2*c*d**5*e**7 - 30*a*c**2*d**9*e**3)/(a**3*e**12 - 33*a**2*c
*d**4*e**8 - 33*a*c**2*d**8*e**4 + c**3*d**12))))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.25

$$\int \frac{(d+ex)^2}{a+cx^4}\,dx = \frac{\sqrt{2}\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\log\left(\sqrt{c}x^2+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{8\,a^{\frac{3}{4}}c^{\frac{3}{4}}} - \frac{\sqrt{2}\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\log\left(\sqrt{c}x^2-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{8\,a^{\frac{3}{4}}c^{\frac{3}{4}}} + \frac{\left(\sqrt{2}a^{\frac{1}{4}}c^{\frac{3}{4}}d^2+\sqrt{2}a^{\frac{3}{4}}c^{\frac{1}{4}}e^2-4\sqrt{a}\sqrt{c}de\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{4\,a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{c}}c^{\frac{3}{4}}} + \frac{\left(\sqrt{2}a^{\frac{1}{4}}c^{\frac{3}{4}}d^2+\sqrt{2}a^{\frac{3}{4}}c^{\frac{1}{4}}e^2+4\sqrt{a}\sqrt{c}de\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{4\,a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{c}}c^{\frac{3}{4}}}$$

input
```
integrate((e*x+d)^2/(c*x^4+a),x, algorithm="maxima")
```

output
```
1/8*sqrt(2)*(sqrt(c)*d^2 - sqrt(a)*e^2)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*
c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)) - 1/8*sqrt(2)*(sqrt(c)*d^2 - sqrt(a
)*e^2)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(
3/4)) + 1/4*(sqrt(2)*a^(1/4)*c^(3/4)*d^2 + sqrt(2)*a^(3/4)*c^(1/4)*e^2 - 4
*sqrt(a)*sqrt(c)*d*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^
(1/4))/sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(3/4)) + 1/
4*(sqrt(2)*a^(1/4)*c^(3/4)*d^2 + sqrt(2)*a^(3/4)*c^(1/4)*e^2 + 4*sqrt(a)*s
qrt(c)*d*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqr
t(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(3/4))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.30

$$\int \frac{(d+ex)^2}{a+cx^4}\,dx = \frac{\sqrt{2}\left(2\sqrt{2}\sqrt{ac}c^2de+(ac^3)^{\frac{1}{4}}c^2d^2+(ac^3)^{\frac{3}{4}}e^2\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4ac^3} + \frac{\sqrt{2}\left(2\sqrt{2}\sqrt{ac}c^2de+(ac^3)^{\frac{1}{4}}c^2d^2+(ac^3)^{\frac{3}{4}}e^2\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4ac^3} + \frac{\sqrt{2}\left((ac^3)^{\frac{1}{4}}c^2d^2-(ac^3)^{\frac{3}{4}}e^2\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{8ac^3} - \frac{\sqrt{2}\left((ac^3)^{\frac{1}{4}}c^2d^2-(ac^3)^{\frac{3}{4}}e^2\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{8ac^3}$$

input
```
integrate((e*x+d)^2/(c*x^4+a),x, algorithm="giac")
```

output
```
1/4*sqrt(2)*(2*sqrt(2)*sqrt(a*c)*c^2*d*e + (a*c^3)^(1/4)*c^2*d^2 + (a*c^3)
^(3/4)*e^2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a
*c^3) + 1/4*sqrt(2)*(2*sqrt(2)*sqrt(a*c)*c^2*d*e + (a*c^3)^(1/4)*c^2*d^2 +
 (a*c^3)^(3/4)*e^2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(
1/4))/(a*c^3) + 1/8*sqrt(2)*((a*c^3)^(1/4)*c^2*d^2 - (a*c^3)^(3/4)*e^2)*lo
g(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a*c^3) - 1/8*sqrt(2)*((a*c^3)^
(1/4)*c^2*d^2 - (a*c^3)^(3/4)*e^2)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(
a/c))/(a*c^3)
```

Mupad [B] (verification not implemented)

Time = 21.72 (sec) , antiderivative size = 556, normalized size of antiderivative = 2.53

$$\int \frac{(d+ex)^2}{a+cx^4}\,dx$$

$$= \sum_{k=1}^{4} \ln\Big(3\,c^2\,d^4\,e^2 - a\,c\,e^6 + 4\,c^2\,d^3\,e^3\,x - \mathrm{root}\big(256\,a^3\,c^3\,z^4 + 192\,a^2\,c^2\,d^2\,e^2\,z^2 + 32\,a^2\,c\,d\,e^5\,z - 32\,a\,c^2\,d^5\,e\,z + 2\,a\,c\,d^4\,e^4 + c^2\,d^8 + a^2\,e^8, z, k\big)\;c^3\,d^4\,x\,4$$

$$-\mathrm{root}\big(256\,a^3\,c^3\,z^4 + 192\,a^2\,c^2\,d^2\,e^2\,z^2 + 32\,a^2\,c\,d\,e^5\,z - 32\,a\,c^2\,d^5\,e\,z + 2\,a\,c\,d^4\,e^4 + c^2\,d^8 + a^2\,e^8, z, k\big)^2\,a$$

$$+\mathrm{root}\big(256\,a^3\,c^3\,z^4 + 192\,a^2\,c^2\,d^2\,e^2\,z^2 + 32\,a^2\,c\,d\,e^5\,z - 32\,a\,c^2\,d^5\,e\,z + 2\,a\,c\,d^4\,e^4 + c^2\,d^8 + a^2\,e^8, z, k\big)\;a\,c^2\,e^4\,x\,4 - \mathrm{root}\big(256\,a^3\,c^3\,z^4 + 192\,a^2\,c^2\,d^2\,e^2\,z^2 + 32\,a^2\,c\,d\,e^5\,z - 32\,a\,c^2\,d^5\,e\,z + 2\,a\,c\,d^4\,e^4 + c^2\,d^8 + a^2\,e^8, z, k\big)\;a\,c^2\,d\,e^3\,16$$

$$+\mathrm{root}\big(256\,a^3\,c^3\,z^4 + 192\,a^2\,c^2\,d^2\,e^2\,z^2 + 32\,a^2\,c\,d\,e^5\,z - 32\,a\,c^2\,d^5\,e\,z + 2\,a\,c\,d^4\,e^4 + c^2\,d^8 + a^2\,e^8, z, k\big)^2\,a$$

$$+ 192\,a^2\,c^2\,d^2\,e^2\,z^2 + 32\,a^2\,c\,d\,e^5\,z - 32\,a\,c^2\,d^5\,e\,z + 2\,a\,c\,d^4\,e^4 + c^2\,d^8 + a^2\,e^8, z, k\big)$$

input
```
int((d + e*x)^2/(a + c*x^4),x)
```

output
```
symsum(log(3*c^2*d^4*e^2 - a*c*e^6 + 4*c^2*d^3*e^3*x - 4*root(256*a^3*c^3*
z^4 + 192*a^2*c^2*d^2*e^2*z^2 + 32*a^2*c*d*e^5*z - 32*a*c^2*d^5*e*z + 2*a*
c*d^4*e^4 + c^2*d^8 + a^2*e^8, z, k)*c^3*d^4*x - 16*root(256*a^3*c^3*z^4 +
 192*a^2*c^2*d^2*e^2*z^2 + 32*a^2*c*d*e^5*z - 32*a*c^2*d^5*e*z + 2*a*c*d^4
*e^4 + c^2*d^8 + a^2*e^8, z, k)^2*a*c^3*d^2 + 4*root(256*a^3*c^3*z^4 + 192
*a^2*c^2*d^2*e^2*z^2 + 32*a^2*c*d*e^5*z - 32*a*c^2*d^5*e*z + 2*a*c*d^4*e^4
 + c^2*d^8 + a^2*e^8, z, k)*a*c^2*e^4*x - 16*root(256*a^3*c^3*z^4 + 192*a^
2*c^2*d^2*e^2*z^2 + 32*a^2*c*d*e^5*z - 32*a*c^2*d^5*e*z + 2*a*c*d^4*e^4 +
c^2*d^8 + a^2*e^8, z, k)*a*c^2*d*e^3 + 32*root(256*a^3*c^3*z^4 + 192*a^2*c
^2*d^2*e^2*z^2 + 32*a^2*c*d*e^5*z - 32*a*c^2*d^5*e*z + 2*a*c*d^4*e^4 + c^2
*d^8 + a^2*e^8, z, k)^2*a*c^3*d*e*x)*root(256*a^3*c^3*z^4 + 192*a^2*c^2*d^
2*e^2*z^2 + 32*a^2*c*d*e^5*z - 32*a*c^2*d^5*e*z + 2*a*c*d^4*e^4 + c^2*d^8
+ a^2*e^8, z, k), k, 1, 4)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.69

$$\int \frac{(d+ex)^2}{a+cx^4}\,dx = \frac{-2c^{\frac{1}{4}}a^{\frac{3}{4}}\sqrt{2}\,atan\left(\frac{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{c}\,x}{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)e^2-2c^{\frac{3}{4}}a^{\frac{1}{4}}\sqrt{2}\,atan\left(\frac{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{c}\,x}{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)d^2-8\sqrt{c}\,\sqrt{a}\,atan\left(\frac{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{c}\,x}{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)\cdots}{}$$

input
```
int((e*x+d)^2/(c*x^4+a),x)
```

output
```
( - 2*c**(1/4)*a**(3/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c
)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*e**2 - 2*c**(3/4)*a**(1/4)*sqrt(2)*atan(
(c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*d**
2 - 8*sqrt(c)*sqrt(a)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x)/(c**(
1/4)*a**(1/4)*sqrt(2)))*d*e + 2*c**(1/4)*a**(3/4)*sqrt(2)*atan((c**(1/4)*a
**(1/4)*sqrt(2) + 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*e**2 + 2*c**(3
/4)*a**(1/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(c)*x)/(c**(1
/4)*a**(1/4)*sqrt(2)))*d**2 - 8*sqrt(c)*sqrt(a)*atan((c**(1/4)*a**(1/4)*sq
rt(2) + 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*d*e + c**(1/4)*a**(3/4)*
sqrt(2)*log( - c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c)*x**2)*e**2
- c**(1/4)*a**(3/4)*sqrt(2)*log(c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sq
rt(c)*x**2)*e**2 - c**(3/4)*a**(1/4)*sqrt(2)*log( - c**(1/4)*a**(1/4)*sqrt
(2)*x + sqrt(a) + sqrt(c)*x**2)*d**2 + c**(3/4)*a**(1/4)*sqrt(2)*log(c**(1
/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c)*x**2)*d**2)/(8*a*c)
```

3.175 $\int \frac{d+ex}{a+cx^4}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 167

$$\int \frac{d+ex}{a+cx^4}\,dx=\frac{e\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{c}}-\frac{d\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{c}}+\frac{d\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{c}}+\frac{d\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x}{\sqrt{a}+\sqrt{c}x^2}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{c}}$$

output
```
1/2*e*arctan(c^(1/2)*x^2/a^(1/2))/a^(1/2)/c^(1/2)+1/4*d*arctan(-1+2^(1/2)*
c^(1/4)*x/a^(1/4))*2^(1/2)/a^(3/4)/c^(1/4)+1/4*d*arctan(1+2^(1/2)*c^(1/4)*
x/a^(1/4))*2^(1/2)/a^(3/4)/c^(1/4)+1/4*d*arctanh(2^(1/2)*a^(1/4)*c^(1/4)*x
/(a^(1/2)+c^(1/2)*x^2))*2^(1/2)/a^(3/4)/c^(1/4)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.10

$$\int \frac{d+ex}{a+cx^4}\,dx=\frac{-2\left(\sqrt{2}\sqrt[4]{c}d+2\sqrt[4]{a}e\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)+2\left(\sqrt{2}\sqrt[4]{c}d-2\sqrt[4]{a}e\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)+\sqrt{2}\sqrt[4]{c}d(-\log}{8a^{3/4}\sqrt{c}}$$

input
```
Integrate[(d + e*x)/(a + c*x^4),x]
```

output
```
(-2*(Sqrt[2]*c^(1/4)*d + 2*a^(1/4)*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/
4)] + 2*(Sqrt[2]*c^(1/4)*d - 2*a^(1/4)*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a
^(1/4)] + Sqrt[2]*c^(1/4)*d*(-Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sq
rt[c]*x^2] + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]))/(8*a
^(3/4)*Sqrt[c])
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.31, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2415, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex}{a+cx^4}\,dx$$
$$\Big\downarrow \ 2415$$
$$\int \left(\frac{d}{a+cx^4}+\frac{ex}{a+cx^4}\right)dx$$
$$\Big\downarrow \ 2009$$
$$-\frac{d\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{c}}+\frac{d\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{c}}-\frac{d\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{c}}+\frac{d\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{c}}+\frac{e\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{c}}$$

input
```
Int[(d + e*x)/(a + c*x^4),x]
```

output
```
(e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*Sqrt[a]*Sqrt[c]) - (d*ArcTan[1 - (Sqr
t[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*c^(1/4)) + (d*ArcTan[1 + (Sqr
t[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*c^(1/4)) - (d*Log[Sqrt[a] - S
qrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(1/4)) + (d*
Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)
*c^(1/4))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2415
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.16 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.19

method	result	size
risch	$\frac{\sum_{\_R=\operatorname{RootOf}\left(c\_Z^4+a\right)}\frac{\left(\_Re+d\right)\ln\left(x-\_R\right)}{\_R^3}}{4c}$	32
default	$\frac{d\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{8a}+\frac{e\arctan\left(\sqrt{\frac{c}{a}}\,x^2\right)}{2\sqrt{ac}}$	124

input
```
int((e*x+d)/(c*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/4/c*sum((_R*e+d)/_R^3*ln(x-_R),_R=RootOf(_Z^4*c+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.79 (sec) , antiderivative size = 41851, normalized size of antiderivative = 250.60

$$\int \frac{d+ex}{a+cx^4}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)/(c*x^4+a),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.74

$$\int \frac{d+ex}{a+cx^4}\,dx$$
$$= \text{RootSum}\left(256t^4a^3c^2 + 32t^2a^2ce^2 - 16tacd^2e + ae^4 + cd^4, \left(t \mapsto t\log\left(x + \frac{-128t^3a^3ce^2 - 16t^2a^2cd^2e -}{4ade^4}\right.\right.\right.$$

input
```
integrate((e*x+d)/(c*x**4+a),x)
```

output
```
RootSum(256*_t**4*a**3*c**2 + 32*_t**2*a**2*c*e**2 - 16*_t*a*c*d**2*e + a*
e**4 + c*d**4, Lambda(_t, _t*log(x + (-128*_t**3*a**3*c*e**2 - 16*_t**2*a*
*2*c*d**2*e - 8*_t*a**2*e**4 - 4*_t*a*c*d**4 + 5*a*d**2*e**3)/(4*a*d*e**4
- c*d**5))))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.24

$$\int \frac{d+ex}{a+cx^4}\,dx = \frac{\sqrt{2}d\log\left(\sqrt{c}x^2+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{8\,a^{\frac{3}{4}}c^{\frac{1}{4}}} - \frac{\sqrt{2}d\log\left(\sqrt{c}x^2-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{8\,a^{\frac{3}{4}}c^{\frac{1}{4}}} + \frac{\left(\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}d-2\sqrt{a}e\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{4\,a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{c}}c^{\frac{1}{4}}} + \frac{\left(\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}d+2\sqrt{a}e\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{4\,a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{c}}c^{\frac{1}{4}}}$$

input
```
integrate((e*x+d)/(c*x^4+a),x, algorithm="maxima")
```

output
```
1/8*sqrt(2)*d*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3
/4)*c^(1/4)) - 1/8*sqrt(2)*d*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x +
 sqrt(a))/(a^(3/4)*c^(1/4)) + 1/4*(sqrt(2)*a^(1/4)*c^(1/4)*d - 2*sqrt(a)*e
)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*
sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(1/4)) + 1/4*(sqrt(2)*a^(1/4)*c
^(1/4)*d + 2*sqrt(a)*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*
c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(1/4))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.28

$$\int \frac{d+ex}{a+cx^4}\,dx = \frac{\sqrt{2}(ac^3)^{\frac{1}{4}}\,d\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{8\,ac} - \frac{\sqrt{2}(ac^3)^{\frac{1}{4}}\,d\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{8\,ac} - \frac{\sqrt{2}\left(\sqrt{2}\sqrt{ac}ce-(ac^3)^{\frac{1}{4}}\,cd\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4\,ac^2} - \frac{\sqrt{2}\left(\sqrt{2}\sqrt{ac}ce-(ac^3)^{\frac{1}{4}}\,cd\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4\,ac^2}$$

input
```
integrate((e*x+d)/(c*x^4+a),x, algorithm="giac")
```

output
```
1/8*sqrt(2)*(a*c^3)^(1/4)*d*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(
a*c) - 1/8*sqrt(2)*(a*c^3)^(1/4)*d*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(
a/c))/(a*c) - 1/4*sqrt(2)*(sqrt(2)*sqrt(a*c)*c*e - (a*c^3)^(1/4)*c*d)*arct
an(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a*c^2) - 1/4*sqrt
(2)*(sqrt(2)*sqrt(a*c)*c*e - (a*c^3)^(1/4)*c*d)*arctan(1/2*sqrt(2)*(2*x -
sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a*c^2)
```

Mupad [B] (verification not implemented)

Time = 21.46 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.96

$$\int \frac{d+ex}{a+cx^4}\,dx = \begin{cases} -\frac{2\,d+3\,e\,x}{6\,c\,x^3} & \text{if} \\ \frac{\mathrm{atan}\left(\frac{\sqrt{2}\,c^{1/4}\,x}{a^{1/4}}-1\right)\left(2\,a^{1/4}\,e+\sqrt{2}\,c^{1/4}\,d\right)}{4\,a^{3/4}\,\sqrt{c}} - \frac{\mathrm{atan}\left(\frac{\sqrt{2}\,c^{1/4}\,x}{a^{1/4}}+1\right)\left(4\,a^{1/4}\,e-2\,\sqrt{2}\,c^{1/4}\,d\right)}{8\,a^{3/4}\,\sqrt{c}} + \frac{\sqrt{2}\,d\,\ln\left(\frac{\sqrt{a}+\sqrt{c}\,x^2+\sqrt{2}\,a^{1/4}\,c^{1/4}\,x}{\sqrt{a}+\sqrt{c}\,x^2-\sqrt{2}\,a^{1/4}\,c^{1/4}\,x}\right)}{8\,a^{3/4}\,c^{1/4}} & \text{if} \end{cases}$$

input
```
int((d + e*x)/(a + c*x^4),x)
```

output
```
piecewise(a == 0, -(2*d + 3*e*x)/(6*c*x^3), a ~= 0, (atan((2^(1/2)*c^(1/4)
*x)/a^(1/4) - 1)*(2*a^(1/4)*e + 2^(1/2)*c^(1/4)*d))/(4*a^(3/4)*c^(1/2)) -
(atan((2^(1/2)*c^(1/4)*x)/a^(1/4) + 1)*(4*a^(1/4)*e - 2*2^(1/2)*c^(1/4)*d)
)/(8*a^(3/4)*c^(1/2)) + (2^(1/2)*d*log((a^(1/2) + c^(1/2)*x^2 + 2^(1/2)*a^
(1/4)*c^(1/4)*x)/(a^(1/2) + c^(1/2)*x^2 - 2^(1/2)*a^(1/4)*c^(1/4)*x)))/(8*
a^(3/4)*c^(1/4)))
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.28

$$\int \frac{d+ex}{a+cx^4}\,dx$$
$$= \frac{\sqrt{c}\left(-2c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}\,atan\left(\frac{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{c}\,x}{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)d-4\sqrt{a}\,atan\left(\frac{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{c}\,x}{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)e+2c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}\,atan\left(\frac{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}+2\sqrt{c}\,x}{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)\right.}{}$$

input
```
int((e*x+d)/(c*x^4+a),x)
```

output
```
(sqrt(c)*( - 2*c**(1/4)*a**(1/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) -
 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*d - 4*sqrt(a)*atan((c**(1/4)*a*
*(1/4)*sqrt(2) - 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*e + 2*c**(1/4)*
a**(1/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(c)*x)/(c**(1/4)*
a**(1/4)*sqrt(2)))*d - 4*sqrt(a)*atan((c**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(
c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*e - c**(1/4)*a**(1/4)*sqrt(2)*log( - c*
*(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c)*x**2)*d + c**(1/4)*a**(1/4)*
sqrt(2)*log(c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c)*x**2)*d))/(8*a
*c)
```

3.176 $\int \frac{1}{a+cx^4}\,dx$

Optimal result

Integrand size = 9, antiderivative size = 134

$$\int \frac{1}{a+cx^4}\,dx = -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{c}}+\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{c}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x}{\sqrt{a}+\sqrt{c}x^2}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{c}}$$

output
```
1/4*arctan(-1+2^(1/2)*c^(1/4)*x/a^(1/4))*2^(1/2)/a^(3/4)/c^(1/4)+1/4*arcta
n(1+2^(1/2)*c^(1/4)*x/a^(1/4))*2^(1/2)/a^(3/4)/c^(1/4)+1/4*arctanh(2^(1/2)
*a^(1/4)*c^(1/4)*x/(a^(1/2)+c^(1/2)*x^2))*2^(1/2)/a^(3/4)/c^(1/4)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.00

$$\int \frac{1}{a+cx^4}\,dx = \frac{-2\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)+2\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)-\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)+\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{\right.}}{4\sqrt{2}a^{3/4}\sqrt[4]{c}}$$

input
```
Integrate[(a + c*x^4)^(-1),x]
```

output
```
(-2*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*c^(1/4
)*x)/a^(1/4)] - Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] + L
og[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*
c^(1/4))
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.49, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.889$, Rules used = {755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+cx^4}dx$$

$$\Big\downarrow \text{755}$$

$$\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{a}}+\frac{\int \frac{\sqrt{c}x^2+\sqrt{a}}{cx^4+a}dx}{2\sqrt{a}}$$

$$\Big\downarrow \text{1476}$$

$$\frac{\frac{\int \frac{1}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}+\frac{\int \frac{1}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}}{2\sqrt{a}}+\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{a}}$$

$$\Big\downarrow \text{1082}$$

$$\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{a}}+\frac{\frac{\int \frac{1}{-\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)^2-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\int \frac{1}{-\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)^2-1}d\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}$$

$$\Big\downarrow \text{217}$$

$$\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}$$

$$\Big\downarrow \text{1479}$$

$$\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{\sqrt[4]{c}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}\right)}{\sqrt[4]{c}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{\sqrt[4]{c}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}\right)}{\sqrt[4]{c}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt{c}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt[4]{a}\sqrt{c}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}$$

$$\Big\downarrow \text{1103}$$

$$\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}$$

input

```
Int[(a + c*x^4)^(-1),x]
```

output

```
(-(ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(Sqrt[2]*a^(1/4)*c^(1/4))) + Ar
cTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(Sqrt[2]*a^(1/4)*c^(1/4)))/(2*Sqrt[a
]) + (-1/2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]/(Sqrt[2]
*a^(1/4)*c^(1/4)) + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]
/(2*Sqrt[2]*a^(1/4)*c^(1/4)))/(2*Sqrt[a])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.18 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.20

method	result	size
risch	$\frac{\sum_{\_R=\mathrm{RootOf}\left(c\,\_Z^4+a\right)}\frac{\ln\left(x-\_R\right)}{\_R^3}}{4c}$	27
default	$\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{8a}$	102

input
```
int(1/(c*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/4/c*sum(1/_R^3*ln(x-_R),_R=RootOf(_Z^4*c+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.84

$$\int \frac{1}{a+cx^4}\,dx = \frac{1}{4}\left(-\frac{1}{a^3c}\right)^{\frac{1}{4}}\log\left(a\left(-\frac{1}{a^3c}\right)^{\frac{1}{4}}+x\right) + \frac{1}{4}i\left(-\frac{1}{a^3c}\right)^{\frac{1}{4}}\log\left(i\,a\left(-\frac{1}{a^3c}\right)^{\frac{1}{4}}+x\right) - \frac{1}{4}i\left(-\frac{1}{a^3c}\right)^{\frac{1}{4}}\log\left(-i\,a\left(-\frac{1}{a^3c}\right)^{\frac{1}{4}}+x\right) - \frac{1}{4}\left(-\frac{1}{a^3c}\right)^{\frac{1}{4}}\log\left(-a\left(-\frac{1}{a^3c}\right)^{\frac{1}{4}}+x\right)$$

input
```
integrate(1/(c*x^4+a),x, algorithm="fricas")
```

output
```
1/4*(-1/(a^3*c))^(1/4)*log(a*(-1/(a^3*c))^(1/4) + x) + 1/4*I*(-1/(a^3*c))^
(1/4)*log(I*a*(-1/(a^3*c))^(1/4) + x) - 1/4*I*(-1/(a^3*c))^(1/4)*log(-I*a*
(-1/(a^3*c))^(1/4) + x) - 1/4*(-1/(a^3*c))^(1/4)*log(-a*(-1/(a^3*c))^(1/4)
 + x)
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.15

$$\int \frac{1}{a+cx^4}\,dx = \operatorname{RootSum}\left(256t^4a^3c+1,\left(t\mapsto t\log\left(4ta+x\right)\right)\right)$$

input
```
integrate(1/(c*x**4+a),x)
```

output
```
RootSum(256*_t**4*a**3*c + 1, Lambda(_t, _t*log(4*_t*a + x)))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.26

$$\int \frac{1}{a+cx^4}\,dx = \frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{4\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}} + \frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{4\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}} + \frac{\sqrt{2}\log\left(\sqrt{c}x^2+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{8a^{\frac{3}{4}}c^{\frac{1}{4}}} - \frac{\sqrt{2}\log\left(\sqrt{c}x^2-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{8a^{\frac{3}{4}}c^{\frac{1}{4}}}$$

input
```
integrate(1/(c*x^4+a),x, algorithm="maxima")
```

output
```
1/4*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/sqr
t(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))) + 1/4*sqrt(2)*arctan(1
/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/
(sqrt(a)*sqrt(sqrt(a)*sqrt(c))) + 1/8*sqrt(2)*log(sqrt(c)*x^2 + sqrt(2)*a^
(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(1/4)) - 1/8*sqrt(2)*log(sqrt(c)*x^2
 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(1/4))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 179 vs. 2(87) = 174.

Time = 0.11 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.34

$$\int \frac{1}{a+cx^4}\,dx = \frac{\sqrt{2}(ac^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4ac} + \frac{\sqrt{2}(ac^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4ac} + \frac{\sqrt{2}(ac^3)^{\frac{1}{4}}\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{8ac} - \frac{\sqrt{2}(ac^3)^{\frac{1}{4}}\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{8ac}$$

input
```
integrate(1/(c*x^4+a),x, algorithm="giac")
```

output
```
1/4*sqrt(2)*(a*c^3)^(1/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(
a/c)^(1/4))/(a*c) + 1/4*sqrt(2)*(a*c^3)^(1/4)*arctan(1/2*sqrt(2)*(2*x - sq
rt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a*c) + 1/8*sqrt(2)*(a*c^3)^(1/4)*log(x^2
+ sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a*c) - 1/8*sqrt(2)*(a*c^3)^(1/4)*log
(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a*c)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.25

$$\int \frac{1}{a+cx^4}\,dx=-\frac{\operatorname{atan}\left(\frac{c^{1/4}\,x}{(-a)^{1/4}}\right)+\operatorname{atanh}\left(\frac{c^{1/4}\,x}{(-a)^{1/4}}\right)}{2\,(-a)^{3/4}\,c^{1/4}}$$

input
```
int(1/(a + c*x^4),x)
```

output
```
-(atan((c^(1/4)*x)/(-a)^(1/4)) + atanh((c^(1/4)*x)/(-a)^(1/4)))/(2*(-a)^(3
/4)*c^(1/4))
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.84

$$\int \frac{1}{a+cx^4}\,dx =\frac{\sqrt{2}\left(-2atan\left(\frac{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{c}\,x}{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)+2atan\left(\frac{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}+2\sqrt{c}\,x}{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)-\log\left(-c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)+\log\left(c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{\right.\right)}{8c^{\frac{1}{4}}a^{\frac{3}{4}}}$$

input
```
int(1/(c*x^4+a),x)
```

output
```
(c**(3/4)*a**(1/4)*sqrt(2)*( - 2*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(
c)*x)/(c**(1/4)*a**(1/4)*sqrt(2))) + 2*atan((c**(1/4)*a**(1/4)*sqrt(2) + 2
*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2))) - log( - c**(1/4)*a**(1/4)*sqrt(2
)*x + sqrt(a) + sqrt(c)*x**2) + log(c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a)
+ sqrt(c)*x**2)))/(8*a*c)
```

3.177 $\int \frac{1}{(d+ex)(a+cx^4)}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 331

$$\begin{aligned}
\int \frac{1}{(d+ex)\left(a+cx^4\right)}\,dx = &-\frac{\sqrt{c}d^2e\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\left(cd^4+ae^4\right)} \\
&-\frac{\sqrt[4]{c}d\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\left(cd^4+ae^4\right)} \\
&+\frac{\sqrt[4]{c}d\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\left(cd^4+ae^4\right)} \\
&+\frac{\sqrt[4]{c}d\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x}{\sqrt{a}+\sqrt{c}x^2}\right)}{2\sqrt{2}a^{3/4}\left(cd^4+ae^4\right)} \\
&+\frac{e^3\log(d+ex)}{cd^4+ae^4}-\frac{e^3\log\left(a+cx^4\right)}{4\left(cd^4+ae^4\right)}
\end{aligned}$$

output
```
-1/2*c^(1/2)*d^2*e*arctan(c^(1/2)*x^2/a^(1/2))/a^(1/2)/(a*e^4+c*d^4)+1/4*c
^(1/4)*d*(c^(1/2)*d^2+a^(1/2)*e^2)*arctan(-1+2^(1/2)*c^(1/4)*x/a^(1/4))*2^
(1/2)/a^(3/4)/(a*e^4+c*d^4)+1/4*c^(1/4)*d*(c^(1/2)*d^2+a^(1/2)*e^2)*arctan
(1+2^(1/2)*c^(1/4)*x/a^(1/4))*2^(1/2)/a^(3/4)/(a*e^4+c*d^4)+1/4*c^(1/4)*d*
(c^(1/2)*d^2-a^(1/2)*e^2)*arctanh(2^(1/2)*a^(1/4)*c^(1/4)*x/(a^(1/2)+c^(1/
2)*x^2))*2^(1/2)/a^(3/4)/(a*e^4+c*d^4)+e^3*ln(e*x+d)/(a*e^4+c*d^4)-e^3*ln(
c*x^4+a)/(4*a*e^4+4*c*d^4)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 404, normalized size of antiderivative = 1.22

$$\int \frac{1}{(d+ex)(a+cx^4)}\,dx = \frac{-2\sqrt[4]{c}d\left(\sqrt{2}\sqrt{c}d^2 - 2\sqrt[4]{a}\sqrt[4]{c}de + \sqrt{2}\sqrt{a}e^2\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right) + 2\sqrt[4]{c}d\left(\sqrt{2}\sqrt{c}d^2 + 2\sqrt[4]{a}\sqrt[4]{c}de + \sqrt{2}\sqrt{c}\right.}{}$$

input
```
Integrate[1/((d + e*x)*(a + c*x^4)),x]
```

output
```
(-2*c^(1/4)*d*(Sqrt[2]*Sqrt[c]*d^2 - 2*a^(1/4)*c^(1/4)*d*e + Sqrt[2]*Sqrt[
a]*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 2*c^(1/4)*d*(Sqrt[2]*Sqr
t[c]*d^2 + 2*a^(1/4)*c^(1/4)*d*e + Sqrt[2]*Sqrt[a]*e^2)*ArcTan[1 + (Sqrt[2
]*c^(1/4)*x)/a^(1/4)] + 8*a^(3/4)*e^3*Log[d + e*x] - Sqrt[2]*c^(3/4)*d^3*L
og[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] + Sqrt[2]*Sqrt[a]*c^
(1/4)*d*e^2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] + Sqrt[
2]*c^(3/4)*d^3*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] - Sq
rt[2]*Sqrt[a]*c^(1/4)*d*e^2*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt
[c]*x^2] - 2*a^(3/4)*e^3*Log[a + c*x^4])/(8*a^(3/4)*(c*d^4 + a*e^4))
```

Rubi [A] (verified)

Time = 1.08 (sec) , antiderivative size = 416, normalized size of antiderivative = 1.26, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+cx^4)(d+ex)}dx$$

$$\Big\downarrow \text{ 7276}$$

$$\int \left(\frac{e^4}{(d+ex)(ae^4+cd^4)}+\frac{c(d^3-d^2ex+de^2x^2-e^3x^3)}{(a+cx^4)(ae^4+cd^4)}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{\sqrt[4]{c}d\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)(\sqrt{a}e^2+\sqrt{c}d^2)}{2\sqrt{2}a^{3/4}(ae^4+cd^4)}+\frac{\sqrt[4]{c}d\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)(\sqrt{a}e^2+\sqrt{c}d^2)}{2\sqrt{2}a^{3/4}(ae^4+cd^4)}-\frac{\sqrt[4]{c}d(\sqrt{c}d^2-\sqrt{a}e^2)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}(ae^4+cd^4)}+\frac{\sqrt[4]{c}d(\sqrt{c}d^2-\sqrt{a}e^2)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}(ae^4+cd^4)}-\frac{\sqrt{c}d^2e\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{a}(ae^4+cd^4)}-\frac{e^3\log(a+cx^4)}{4(ae^4+cd^4)}+\frac{e^3\log(d+ex)}{ae^4+cd^4}$$

input
```
Int[1/((d + e*x)*(a + c*x^4)),x]
```

output
```
-1/2*(Sqrt[c]*d^2*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(Sqrt[a]*(c*d^4 + a*e^4
)) - (c^(1/4)*d*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)
/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)) + (c^(1/4)*d*(Sqrt[c]*d^2 +
 Sqrt[a]*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*
(c*d^4 + a*e^4)) + (e^3*Log[d + e*x])/(c*d^4 + a*e^4) - (c^(1/4)*d*(Sqrt[c
]*d^2 - Sqrt[a]*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2
])/(4*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)) + (c^(1/4)*d*(Sqrt[c]*d^2 - Sqrt[a]
*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a
^(3/4)*(c*d^4 + a*e^4)) - (e^3*Log[a + c*x^4])/(4*(c*d^4 + a*e^4))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.30 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.64

method	result
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(1+\left(a^4e^4+a^3c\,d^4\right)\_Z^4+4a^3e^3\_Z^3+6a^2e^2\_Z^2+4ae\_Z\right)}\_R\ln\left(\left(\left(5a^3e^6-3a^2d^4e^2c\right)\_R^3+\left(15a^2e^5-3a\,d^4ec\right)\_R^2\right.\right.\right.}{4}$
default	$\frac{c\left(\frac{d^3\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{8a}-\frac{d^2e\arctan\left(\sqrt{\frac{c}{a}}\,x^2\right)}{2\sqrt{ac}}+\frac{d\,e^2\sqrt{2}\left(\ln\left(\frac{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x}{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x}\right.\right.}{}\right.}{e^4a+c\,d^4}$

input
```
int(1/(e*x+d)/(c*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/4*sum(_R*ln(((5*a^3*e^6-3*a^2*c*d^4*e^2)*_R^3+(15*a^2*e^5-3*a*c*d^4*e)*_
R^2+(15*a*e^4-c*d^4)*_R+5*e^3)*x+(6*a^3*d*e^5-2*a^2*c*d^5*e)*_R^3+(13*a^2*
d*e^4-a*c*d^5)*_R^2+8*a*d*e^3*_R+d*e^2),_R=RootOf(1+(a^4*e^4+a^3*c*d^4)*_Z
^4+4*a^3*e^3*_Z^3+6*a^2*e^2*_Z^2+4*a*e*_Z))+e^3*ln(e*x+d)/(a*e^4+c*d^4)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 93.92 (sec) , antiderivative size = 352864, normalized size of antiderivative = 1066.05

$$\int \frac{1}{(d+ex)(a+cx^4)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*x+d)/(c*x^4+a),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)(a+cx^4)}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x+d)/(c*x**4+a),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.04

$$\int \frac{1}{(d+ex)(a+cx^4)}\,dx = \frac{e^3\log(ex+d)}{cd^4+ae^4} - \frac{c\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{3}{4}}c^{\frac{1}{4}}e^3-cd^3+\sqrt{a}\sqrt{c}de^2\right)\log\left(\sqrt{c}x^2+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{5}{4}}} + \frac{\sqrt{2}\left(\sqrt{2}a^{\frac{3}{4}}c^{\frac{1}{4}}e^3+cd^3-\sqrt{a}\sqrt{c}de^2\right)\log\left(\sqrt{c}x^2-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{5}{4}}} - \right.}{8}$$

input
```
integrate(1/(e*x+d)/(c*x^4+a),x, algorithm="maxima")
```

output
```
e^3*log(e*x + d)/(c*d^4 + a*e^4) - 1/8*c*(sqrt(2)*(sqrt(2)*a^(3/4)*c^(1/4)
*e^3 - c*d^3 + sqrt(a)*sqrt(c)*d*e^2)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c^
(1/4)*x + sqrt(a))/(a^(3/4)*c^(5/4)) + sqrt(2)*(sqrt(2)*a^(3/4)*c^(1/4)*e^
3 + c*d^3 - sqrt(a)*sqrt(c)*d*e^2)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/
4)*x + sqrt(a))/(a^(3/4)*c^(5/4)) - 2*(sqrt(2)*a^(1/4)*c^(5/4)*d^3 + sqrt(
2)*a^(3/4)*c^(3/4)*d*e^2 + 2*sqrt(a)*c*d^2*e)*arctan(1/2*sqrt(2)*(2*sqrt(c
)*x + sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a
)*sqrt(c))*c^(5/4)) - 2*(sqrt(2)*a^(1/4)*c^(5/4)*d^3 + sqrt(2)*a^(3/4)*c^(
3/4)*d*e^2 - 2*sqrt(a)*c*d^2*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*
a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(
5/4)))/(c*d^4 + a*e^4)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 377, normalized size of antiderivative = 1.14

$$\begin{aligned}\int \frac{1}{(d+ex)(a+cx^4)}\,dx &= \frac{e^4\log(|ex+d|)}{cd^4e+ae^5} - \frac{e^3\log(|cx^4+a|)}{4(cd^4+ae^4)} \\ &\quad + \frac{(ac^3)^{\frac{1}{4}}cd\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}ac^2d^2+\sqrt{2}\sqrt{ac}ace^2-2(ac^3)^{\frac{1}{4}}acde\right)} \\ &\quad + \frac{(ac^3)^{\frac{1}{4}}cd\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}ac^2d^2+\sqrt{2}\sqrt{ac}ace^2+2(ac^3)^{\frac{1}{4}}acde\right)} \\ &\quad + \frac{\left((ac^3)^{\frac{1}{4}}c^2d^3-(ac^3)^{\frac{3}{4}}de^2\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{4\left(\sqrt{2}ac^3d^4+\sqrt{2}a^2c^2e^4\right)} \\ &\quad - \frac{\left((ac^3)^{\frac{1}{4}}c^2d^3-(ac^3)^{\frac{3}{4}}de^2\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{4\left(\sqrt{2}ac^3d^4+\sqrt{2}a^2c^2e^4\right)}\end{aligned}$$

input
```
integrate(1/(e*x+d)/(c*x^4+a),x, algorithm="giac")
```

output
```
e^4*log(abs(e*x + d))/(c*d^4*e + a*e^5) - 1/4*e^3*log(abs(c*x^4 + a))/(c*d
^4 + a*e^4) + 1/2*(a*c^3)^(1/4)*c*d*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c
)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a*c^2*d^2 + sqrt(2)*sqrt(a*c)*a*c*e^2 - 2*(
a*c^3)^(1/4)*a*c*d*e) + 1/2*(a*c^3)^(1/4)*c*d*arctan(1/2*sqrt(2)*(2*x - sq
rt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a*c^2*d^2 + sqrt(2)*sqrt(a*c)*a*c
*e^2 + 2*(a*c^3)^(1/4)*a*c*d*e) + 1/4*((a*c^3)^(1/4)*c^2*d^3 - (a*c^3)^(3/
4)*d*e^2)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a*c^3*d^4
+ sqrt(2)*a^2*c^2*e^4) - 1/4*((a*c^3)^(1/4)*c^2*d^3 - (a*c^3)^(3/4)*d*e^2)
*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a*c^3*d^4 + sqrt(2)
*a^2*c^2*e^4)
```

Mupad [B] (verification not implemented)

Time = 22.05 (sec) , antiderivative size = 874, normalized size of antiderivative = 2.64

$$\int \frac{1}{(d+ex)(a+cx^4)}\,dx$$
$$= \left(\sum_{k=1}^{4} \ln\left(\mathrm{root}\left(256\,a^3\,c\,d^4\,z^4+256\,a^4\,e^4\,z^4+256\,a^3\,e^3\,z^3+96\,a^2\,e^2\,z^2+16\,a\,e\,z+1,z,k\right)\,c^4\,e\,\left(d\,e^2+5\,e^3\,x+\right.\right.\right.$$
$$\left.\left. +256\,a^4\,e^4\,z^4+256\,a^3\,e^3\,z^3+96\,a^2\,e^2\,z^2+16\,a\,e\,z+1,z,k\right)\right)+\frac{e^3\,\ln\left(d+e\,x\right)}{c\,d^4+a\,e^4}$$

input
```
int(1/((a + c*x^4)*(d + e*x)),x)
```

output
```
symsum(log(root(256*a^3*c*d^4*z^4 + 256*a^4*e^4*z^4 + 256*a^3*e^3*z^3 + 96
*a^2*e^2*z^2 + 16*a*e*z + 1, z, k)*c^4*e*(d*e^2 + 5*e^3*x + 240*root(256*a
^3*c*d^4*z^4 + 256*a^4*e^4*z^4 + 256*a^3*e^3*z^3 + 96*a^2*e^2*z^2 + 16*a*e
*z + 1, z, k)^2*a^2*e^5*x + 320*root(256*a^3*c*d^4*z^4 + 256*a^4*e^4*z^4 +
 256*a^3*e^3*z^3 + 96*a^2*e^2*z^2 + 16*a*e*z + 1, z, k)^3*a^3*e^6*x + 32*r
oot(256*a^3*c*d^4*z^4 + 256*a^4*e^4*z^4 + 256*a^3*e^3*z^3 + 96*a^2*e^2*z^2
 + 16*a*e*z + 1, z, k)*a*d*e^3 + 60*root(256*a^3*c*d^4*z^4 + 256*a^4*e^4*z
^4 + 256*a^3*e^3*z^3 + 96*a^2*e^2*z^2 + 16*a*e*z + 1, z, k)*a*e^4*x - 4*ro
ot(256*a^3*c*d^4*z^4 + 256*a^4*e^4*z^4 + 256*a^3*e^3*z^3 + 96*a^2*e^2*z^2
+ 16*a*e*z + 1, z, k)*c*d^4*x - 16*root(256*a^3*c*d^4*z^4 + 256*a^4*e^4*z^
4 + 256*a^3*e^3*z^3 + 96*a^2*e^2*z^2 + 16*a*e*z + 1, z, k)^2*a*c*d^5 + 208
*root(256*a^3*c*d^4*z^4 + 256*a^4*e^4*z^4 + 256*a^3*e^3*z^3 + 96*a^2*e^2*z
^2 + 16*a*e*z + 1, z, k)^2*a^2*d*e^4 + 384*root(256*a^3*c*d^4*z^4 + 256*a^
4*e^4*z^4 + 256*a^3*e^3*z^3 + 96*a^2*e^2*z^2 + 16*a*e*z + 1, z, k)^3*a^3*d
*e^5 - 128*root(256*a^3*c*d^4*z^4 + 256*a^4*e^4*z^4 + 256*a^3*e^3*z^3 + 96
*a^2*e^2*z^2 + 16*a*e*z + 1, z, k)^3*a^2*c*d^5*e - 192*root(256*a^3*c*d^4*
z^4 + 256*a^4*e^4*z^4 + 256*a^3*e^3*z^3 + 96*a^2*e^2*z^2 + 16*a*e*z + 1, z
, k)^3*a^2*c*d^4*e^2*x - 48*root(256*a^3*c*d^4*z^4 + 256*a^4*e^4*z^4 + 256
*a^3*e^3*z^3 + 96*a^2*e^2*z^2 + 16*a*e*z + 1, z, k)^2*a*c*d^4*e*x))*root(2
56*a^3*c*d^4*z^4 + 256*a^4*e^4*z^4 + 256*a^3*e^3*z^3 + 96*a^2*e^2*z^2 +...
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 454, normalized size of antiderivative = 1.37

$$\int \frac{1}{(d+ex)(a+cx^4)}\,dx$$
$$= \frac{-2c^{\frac{1}{4}}a^{\frac{3}{4}}\sqrt{2}\,atan\left(\frac{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{c}\,x}{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)d\,e^2 - 2c^{\frac{3}{4}}a^{\frac{1}{4}}\sqrt{2}\,atan\left(\frac{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{c}\,x}{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)d^3 + 4\sqrt{c}\,\sqrt{a}\,atan\left(\frac{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{c}\,x}{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)}{}$$

input
```
int(1/(e*x+d)/(c*x^4+a),x)
```

output
```
( - 2*c**(1/4)*a**(3/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c
)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*d*e**2 - 2*c**(3/4)*a**(1/4)*sqrt(2)*ata
n((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*d
**3 + 4*sqrt(c)*sqrt(a)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x)/(c*
*(1/4)*a**(1/4)*sqrt(2)))*d**2*e + 2*c**(1/4)*a**(3/4)*sqrt(2)*atan((c**(1
/4)*a**(1/4)*sqrt(2) + 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*d*e**2 +
2*c**(3/4)*a**(1/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(c)*x)
/(c**(1/4)*a**(1/4)*sqrt(2)))*d**3 + 4*sqrt(c)*sqrt(a)*atan((c**(1/4)*a**(
1/4)*sqrt(2) + 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*d**2*e + c**(1/4)
*a**(3/4)*sqrt(2)*log( - c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c)*x
**2)*d*e**2 - c**(1/4)*a**(3/4)*sqrt(2)*log(c**(1/4)*a**(1/4)*sqrt(2)*x +
sqrt(a) + sqrt(c)*x**2)*d*e**2 - c**(3/4)*a**(1/4)*sqrt(2)*log( - c**(1/4)
*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c)*x**2)*d**3 + c**(3/4)*a**(1/4)*sqr
t(2)*log(c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c)*x**2)*d**3 - 2*lo
g( - c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c)*x**2)*a*e**3 - 2*log(
c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c)*x**2)*a*e**3 + 8*log(d + e
*x)*a*e**3)/(8*a*(a*e**4 + c*d**4))
```

3.178 $\int \frac{1}{(d+ex)^2(a+cx^4)}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 443

$$\begin{aligned}\int \frac{1}{(d+ex)^2\,(a+cx^4)}\,dx &= -\frac{e^3}{(cd^4+ae^4)\,(d+ex)}-\frac{\sqrt{c}de(cd^4-ae^4)\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{\sqrt{a}\,(cd^4+ae^4)^2}\\
&\quad-\frac{\sqrt[4]{c}\big(\sqrt{c}d^2(cd^4-3ae^4)+\sqrt{a}e^2(3cd^4-ae^4)\big)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\,(cd^4+ae^4)^2}\\
&\quad+\frac{\sqrt[4]{c}\big(\sqrt{c}d^2(cd^4-3ae^4)+\sqrt{a}e^2(3cd^4-ae^4)\big)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\,(cd^4+ae^4)^2}\\
&\quad+\frac{\sqrt[4]{c}\big(\sqrt{c}d^2(cd^4-3ae^4)-\sqrt{a}e^2(3cd^4-ae^4)\big)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x}{\sqrt{a}+\sqrt{c}x^2}\right)}{2\sqrt{2}a^{3/4}\,(cd^4+ae^4)^2}\\
&\quad+\frac{4cd^3e^3\log(d+ex)}{(cd^4+ae^4)^2}-\frac{cd^3e^3\log(a+cx^4)}{(cd^4+ae^4)^2}\end{aligned}$$

output
```
-e^3/(a*e^4+c*d^4)/(e*x+d)-c^(1/2)*d*e*(-a*e^4+c*d^4)*arctan(c^(1/2)*x^2/a
^(1/2))/a^(1/2)/(a*e^4+c*d^4)^2+1/4*c^(1/4)*(c^(1/2)*d^2*(-3*a*e^4+c*d^4)+
a^(1/2)*e^2*(-a*e^4+3*c*d^4))*arctan(-1+2^(1/2)*c^(1/4)*x/a^(1/4))*2^(1/2)
/a^(3/4)/(a*e^4+c*d^4)^2+1/4*c^(1/4)*(c^(1/2)*d^2*(-3*a*e^4+c*d^4)+a^(1/2)
*e^2*(-a*e^4+3*c*d^4))*arctan(1+2^(1/2)*c^(1/4)*x/a^(1/4))*2^(1/2)/a^(3/4)
/(a*e^4+c*d^4)^2+1/4*c^(1/4)*(c^(1/2)*d^2*(-3*a*e^4+c*d^4)-a^(1/2)*e^2*(-a
*e^4+3*c*d^4))*arctanh(2^(1/2)*a^(1/4)*c^(1/4)*x/(a^(1/2)+c^(1/2)*x^2))*2^
(1/2)/a^(3/4)/(a*e^4+c*d^4)^2+4*c*d^3*e^3*ln(e*x+d)/(a*e^4+c*d^4)^2-c*d^3*
e^3*ln(c*x^4+a)/(a*e^4+c*d^4)^2
```

Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 524, normalized size of antiderivative = 1.18

$$\int \frac{1}{(d+ex)^2\,(a+cx^4)}\,dx = \frac{-\frac{8e^3(cd^4+ae^4)}{d+ex} + \frac{2\sqrt[4]{c}\left(-\sqrt{c}d^2+\sqrt{a}e^2\right)\left(\sqrt{2}cd^4-4\sqrt[4]{a}c^{3/4}d^3e+4\sqrt{2}\sqrt{a}\sqrt{c}d^2e^2-4a^{3/4}\sqrt[4]{c}de^3+\sqrt{2}ae^4\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{a^{3/4}} + 2\sqrt[4]{\ }}{}$$

input
```
Integrate[1/((d + e*x)^2*(a + c*x^4)),x]
```

output
```
((-8*e^3*(c*d^4 + a*e^4))/(d + e*x) + (2*c^(1/4)*(-(Sqrt[c]*d^2) + Sqrt[a]
*e^2)*(Sqrt[2]*c*d^4 - 4*a^(1/4)*c^(3/4)*d^3*e + 4*Sqrt[2]*Sqrt[a]*Sqrt[c]
*d^2*e^2 - 4*a^(3/4)*c^(1/4)*d*e^3 + Sqrt[2]*a*e^4)*ArcTan[1 - (Sqrt[2]*c^
(1/4)*x)/a^(1/4)])/a^(3/4) + (2*c^(1/4)*(Sqrt[c]*d^2 - Sqrt[a]*e^2)*(Sqrt[
2]*c*d^4 + 4*a^(1/4)*c^(3/4)*d^3*e + 4*Sqrt[2]*Sqrt[a]*Sqrt[c]*d^2*e^2 + 4
*a^(3/4)*c^(1/4)*d*e^3 + Sqrt[2]*a*e^4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(
1/4)])/a^(3/4) + 32*c*d^3*e^3*Log[d + e*x] - (Sqrt[2]*c^(1/4)*(c^(3/2)*d^6
 - 3*Sqrt[a]*c*d^4*e^2 - 3*a*Sqrt[c]*d^2*e^4 + a^(3/2)*e^6)*Log[Sqrt[a] -
Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/a^(3/4) + (Sqrt[2]*c^(1/4)*(c^(3
/2)*d^6 - 3*Sqrt[a]*c*d^4*e^2 - 3*a*Sqrt[c]*d^2*e^4 + a^(3/2)*e^6)*Log[Sqr
t[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/a^(3/4) - 8*c*d^3*e^3*Log
[a + c*x^4])/(8*(c*d^4 + a*e^4)^2)
```

Rubi [A] (verified)

Time = 1.58 (sec) , antiderivative size = 552, normalized size of antiderivative = 1.25, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+cx^4)(d+ex)^2}\,dx$$

$$\Big\downarrow \text{ 7276}$$

$$\int \left(\frac{e^4}{(d+ex)^2(ae^4+cd^4)}+\frac{4cd^3e^4}{(d+ex)(ae^4+cd^4)^2}+\frac{c\left(-2dex(cd^4-ae^4)+e^2x^2(3cd^4-ae^4)+d^2(cd^4-3ae^4)\right.}{(a+cx^4)(ae^4+cd^4)^2}\right.$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{\sqrt[4]{c}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\left(\sqrt{a}e^2(3cd^4-ae^4)+\sqrt{c}d^2(cd^4-3ae^4)\right)}{2\sqrt{2}a^{3/4}(ae^4+cd^4)^2}+\frac{\sqrt[4]{c}\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)\left(\sqrt{a}e^2(3cd^4-ae^4)+\sqrt{c}d^2(cd^4-3ae^4)\right)}{2\sqrt{2}a^{3/4}(ae^4+cd^4)^2}-\frac{\sqrt[4]{c}\left(\sqrt{c}d^2(cd^4-3ae^4)-\sqrt{a}e^2(3cd^4-ae^4)\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}(ae^4+cd^4)^2}+\frac{\sqrt[4]{c}\left(\sqrt{c}d^2(cd^4-3ae^4)-\sqrt{a}e^2(3cd^4-ae^4)\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}(ae^4+cd^4)^2}-\frac{\sqrt{c}de\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)(cd^4-ae^4)}{\sqrt{a}(ae^4+cd^4)^2}-\frac{e^3}{(d+ex)(ae^4+cd^4)}-\frac{cd^3e^3\log(a+cx^4)}{(ae^4+cd^4)^2}+\frac{4cd^3e^3\log(d+ex)}{(ae^4+cd^4)^2}$$

input
```
Int[1/((d + e*x)^2*(a + c*x^4)),x]
```

output
```
-(e^3/((c*d^4 + a*e^4)*(d + e*x))) - (Sqrt[c]*d*e*(c*d^4 - a*e^4)*ArcTan[(
Sqrt[c]*x^2)/Sqrt[a]])/(Sqrt[a]*(c*d^4 + a*e^4)^2) - (c^(1/4)*(Sqrt[c]*d^2
*(c*d^4 - 3*a*e^4) + Sqrt[a]*e^2*(3*c*d^4 - a*e^4))*ArcTan[1 - (Sqrt[2]*c^
(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^2) + (c^(1/4)*(Sqrt[
c]*d^2*(c*d^4 - 3*a*e^4) + Sqrt[a]*e^2*(3*c*d^4 - a*e^4))*ArcTan[1 + (Sqrt
[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^2) + (4*c*d^3*
e^3*Log[d + e*x])/(c*d^4 + a*e^4)^2 - (c^(1/4)*(Sqrt[c]*d^2*(c*d^4 - 3*a*e
^4) - Sqrt[a]*e^2*(3*c*d^4 - a*e^4))*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)
*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^2) + (c^(1/4)*(Sqrt[
c]*d^2*(c*d^4 - 3*a*e^4) - Sqrt[a]*e^2*(3*c*d^4 - a*e^4))*Log[Sqrt[a] + Sq
rt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)
^2) - (c*d^3*e^3*Log[a + c*x^4])/(c*d^4 + a*e^4)^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 354, normalized size of antiderivative = 0.80

method	result
default	$-\frac{c\left(\frac{\left(3a\,d^{2}e^{4}-c\,d^{6}\right)\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^{2}+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^{2}-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{8a}+\frac{\left(-2ad\,e^{5}+2c\,d^{5}e\right)\arctan\left(\sqrt{\frac{c}{a}}\right.}{2\sqrt{ac}}\right.}{\left(e^{4}a+c\,d^{4}\right)^{2}}$
risch	$-\frac{e^{3}}{\left(e^{4}a+c\,d^{4}\right)\left(ex+d\right)}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(a^{5}e^{8}+2c\,d^{4}a^{4}e^{4}+d^{8}c^{2}a^{3}\right)\_Z^{4}+16a^{3}c\,d^{3}e^{3}\_Z^{3}+20a^{2}c\,d^{2}e^{2}\_Z^{2}+8acde\_Z+c\right)}\_R\ln\left(\left(\left(5a^{5}\right.\right.\right.\right.}{}$

input
```
int(1/(e*x+d)^2/(c*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
-c/(a*e^4+c*d^4)^2*(1/8*(3*a*d^2*e^4-c*d^6)*(a/c)^(1/4)/a*2^(1/2)*(ln((x^2
+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)
))+2*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/c)^(1/4)*x-1))+1/
2*(-2*a*d*e^5+2*c*d^5*e)/(a*c)^(1/2)*arctan((c/a)^(1/2)*x^2)+1/8*(a*e^6-3*
c*d^4*e^2)/c/(a/c)^(1/4)*2^(1/2)*(ln((x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2
))/(x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+2*arctan(2^(1/2)/(a/c)^(1/4)*x
+1)+2*arctan(2^(1/2)/(a/c)^(1/4)*x-1))+d^3*e^3*ln(c*x^4+a))-e^3/(a*e^4+c*d
^4)/(e*x+d)+4*c*d^3*e^3*ln(e*x+d)/(a*e^4+c*d^4)^2
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^2 (a+cx^4)} dx = \text{Timed out}$$

input
```
integrate(1/(e*x+d)^2/(c*x^4+a),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^2 (a+cx^4)} dx = \text{Timed out}$$

input
```
integrate(1/(e*x+d)**2/(c*x**4+a),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 561, normalized size of antiderivative = 1.27

$$\int \frac{1}{(d+ex)^2(a+cx^4)}\,dx = \frac{4\,cd^3e^3\log(ex+d)}{c^2d^8+2\,acd^4e^4+a^2e^8} - \frac{e^3}{cd^5+ade^4+(cd^4e+ae^5)x} - \frac{c\left(\frac{\sqrt{2}\left(4\,\sqrt{2}a^{\frac{3}{4}}c^{\frac{5}{4}}d^3e^3-c^2d^6+3\,\sqrt{a}c^{\frac{3}{2}}d^4e^2+3\,acd^2e^4-a^{\frac{3}{2}}\sqrt{c}e^6\right)\log\left(\sqrt{c}x^2+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{5}{4}}} + \frac{\sqrt{2}\left(4\,\sqrt{2}a^{\frac{3}{4}}c^{\frac{5}{4}}d^3e^3+c^2d^6-3\,\sqrt{a}c^{\frac{3}{2}}d^4e\right.}{}\right.}{}$$

input
```
integrate(1/(e*x+d)^2/(c*x^4+a),x, algorithm="maxima")
```

output
```
4*c*d^3*e^3*log(e*x + d)/(c^2*d^8 + 2*a*c*d^4*e^4 + a^2*e^8) - e^3/(c*d^5
 + a*d*e^4 + (c*d^4*e + a*e^5)*x) - 1/8*c*(sqrt(2)*(4*sqrt(2)*a^(3/4)*c^(5/
4)*d^3*e^3 - c^2*d^6 + 3*sqrt(a)*c^(3/2)*d^4*e^2 + 3*a*c*d^2*e^4 - a^(3/2)
*sqrt(c)*e^6)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3
/4)*c^(5/4)) + sqrt(2)*(4*sqrt(2)*a^(3/4)*c^(5/4)*d^3*e^3 + c^2*d^6 - 3*sq
rt(a)*c^(3/2)*d^4*e^2 - 3*a*c*d^2*e^4 + a^(3/2)*sqrt(c)*e^6)*log(sqrt(c)*x
^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(5/4)) - 2*(sqrt(2)*a
^(1/4)*c^(9/4)*d^6 + 3*sqrt(2)*a^(3/4)*c^(7/4)*d^4*e^2 - 3*sqrt(2)*a^(5/4)
*c^(5/4)*d^2*e^4 - sqrt(2)*a^(7/4)*c^(3/4)*e^6 + 4*sqrt(a)*c^2*d^5*e - 4*a
^(3/2)*c*d*e^5)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))
/sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(5/4)) - 2*(sqrt(
2)*a^(1/4)*c^(9/4)*d^6 + 3*sqrt(2)*a^(3/4)*c^(7/4)*d^4*e^2 - 3*sqrt(2)*a^(
5/4)*c^(5/4)*d^2*e^4 - sqrt(2)*a^(7/4)*c^(3/4)*e^6 - 4*sqrt(a)*c^2*d^5*e +
 4*a^(3/2)*c*d*e^5)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1
/4))/sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(5/4)))/(c^2*
d^8 + 2*a*c*d^4*e^4 + a^2*e^8)
```

Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 668, normalized size of antiderivative = 1.51

$$\int \frac{1}{(d+ex)^2 (a+cx^4)} dx = \text{Too large to display}$$

input
```
integrate(1/(e*x+d)^2/(c*x^4+a),x, algorithm="giac")
```

output
```
4*c*d^3*e^4*log(abs(e*x + d))/(c^2*d^8*e + 2*a*c*d^4*e^5 + a^2*e^9) - c*d^
3*e^3*log(abs(c*x^4 + a))/(c^2*d^8 + 2*a*c*d^4*e^4 + a^2*e^8) + 1/2*((a*c^
3)^(1/4)*c^2*d^2 - (a*c^3)^(3/4)*e^2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a
/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a*c^3*d^4 + sqrt(2)*a^2*c^2*e^4 + 4*sqrt(
2)*sqrt(a*c)*a*c^2*d^2*e^2 - 4*(a*c^3)^(1/4)*a*c^2*d^3*e - 4*(a*c^3)^(3/4)
*a*d*e^3) + 1/2*((a*c^3)^(1/4)*c^2*d^2 - (a*c^3)^(3/4)*e^2)*arctan(1/2*sqr
t(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a*c^3*d^4 + sqrt(2)
*a^2*c^2*e^4 + 4*sqrt(2)*sqrt(a*c)*a*c^2*d^2*e^2 + 4*(a*c^3)^(1/4)*a*c^2*d
^3*e + 4*(a*c^3)^(3/4)*a*d*e^3) + 1/8*(sqrt(2)*(a*c^3)^(1/4)*c^3*d^6 - 3*s
qrt(2)*(a*c^3)^(1/4)*a*c^2*d^2*e^4 - 3*sqrt(2)*(a*c^3)^(3/4)*c*d^4*e^2 + s
qrt(2)*(a*c^3)^(3/4)*a*e^6)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(
a*c^4*d^8 + 2*a^2*c^3*d^4*e^4 + a^3*c^2*e^8) - 1/8*(sqrt(2)*(a*c^3)^(1/4)*
c^3*d^6 - 3*sqrt(2)*(a*c^3)^(1/4)*a*c^2*d^2*e^4 - 3*sqrt(2)*(a*c^3)^(3/4)*
c*d^4*e^2 + sqrt(2)*(a*c^3)^(3/4)*a*e^6)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) +
 sqrt(a/c))/(a*c^4*d^8 + 2*a^2*c^3*d^4*e^4 + a^3*c^2*e^8) - (c*d^4*e^3 + a
*e^7)/((c*d^4 + a*e^4)^2*(e*x + d))
```

Mupad [B] (verification not implemented)

Time = 22.42 (sec) , antiderivative size = 2436, normalized size of antiderivative = 5.50

$$\int \frac{1}{(d+ex)^2 (a+cx^4)} dx = \text{Too large to display}$$

input
```
int(1/((a + c*x^4)*(d + e*x)^2),x)
```

output
```
symsum(log((c^5*d*e^6 + c^5*e^7*x + 16*root(512*a^4*c*d^4*e^4*z^4 + 256*a^
3*c^2*d^8*z^4 + 256*a^5*e^8*z^4 + 1024*a^3*c*d^3*e^3*z^3 + 320*a^2*c*d^2*e
^2*z^2 + 32*a*c*d*e*z + c, z, k)^3*a^4*c^4*e^13 + 256*root(512*a^4*c*d^4*e
^4*z^4 + 256*a^3*c^2*d^8*z^4 + 256*a^5*e^8*z^4 + 1024*a^3*c*d^3*e^3*z^3 +
320*a^2*c*d^2*e^2*z^2 + 32*a*c*d*e*z + c, z, k)^2*a^2*c^5*d^3*e^8 + 496*ro
ot(512*a^4*c*d^4*e^4*z^4 + 256*a^3*c^2*d^8*z^4 + 256*a^5*e^8*z^4 + 1024*a^
3*c*d^3*e^3*z^3 + 320*a^2*c*d^2*e^2*z^2 + 32*a*c*d*e*z + c, z, k)^3*a^2*c^
6*d^8*e^5 + 528*root(512*a^4*c*d^4*e^4*z^4 + 256*a^3*c^2*d^8*z^4 + 256*a^5
*e^8*z^4 + 1024*a^3*c*d^3*e^3*z^3 + 320*a^2*c*d^2*e^2*z^2 + 32*a*c*d*e*z +
 c, z, k)^3*a^3*c^5*d^4*e^9 - 128*root(512*a^4*c*d^4*e^4*z^4 + 256*a^3*c^2
*d^8*z^4 + 256*a^5*e^8*z^4 + 1024*a^3*c*d^3*e^3*z^3 + 320*a^2*c*d^2*e^2*z^
2 + 32*a*c*d*e*z + c, z, k)^4*a^2*c^7*d^13*e^2 + 128*root(512*a^4*c*d^4*e^
4*z^4 + 256*a^3*c^2*d^8*z^4 + 256*a^5*e^8*z^4 + 1024*a^3*c*d^3*e^3*z^3 + 3
20*a^2*c*d^2*e^2*z^2 + 32*a*c*d*e*z + c, z, k)^4*a^3*c^6*d^9*e^6 + 640*roo
t(512*a^4*c*d^4*e^4*z^4 + 256*a^3*c^2*d^8*z^4 + 256*a^5*e^8*z^4 + 1024*a^3
*c*d^3*e^3*z^3 + 320*a^2*c*d^2*e^2*z^2 + 32*a*c*d*e*z + c, z, k)^4*a^4*c^5
*d^5*e^10 + 32*root(512*a^4*c*d^4*e^4*z^4 + 256*a^3*c^2*d^8*z^4 + 256*a^5*
e^8*z^4 + 1024*a^3*c*d^3*e^3*z^3 + 320*a^2*c*d^2*e^2*z^2 + 32*a*c*d*e*z +
c, z, k)*a*c^5*d^2*e^7 - 16*root(512*a^4*c*d^4*e^4*z^4 + 256*a^3*c^2*d^8*z
^4 + 256*a^5*e^8*z^4 + 1024*a^3*c*d^3*e^3*z^3 + 320*a^2*c*d^2*e^2*z^2 +...
```

Reduce [F]

$$\int \frac{1}{(d+ex)^2\,(a+cx^4)}\,dx=\int \frac{1}{(ex+d)^2\,(c\,x^4+a)}dx$$

input
```
int(1/(e*x+d)^2/(c*x^4+a),x)
```

output
```
int(1/(e*x+d)^2/(c*x^4+a),x)
```

3.179 $\int \frac{(d+ex)^3}{(a+cx^4)^2} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 284

$$\begin{aligned}\int \frac{(d+ex)^3}{(a+cx^4)^2}\,dx &= -\frac{e^3}{4c(a+cx^4)}+\frac{x(d^3+3d^2ex+3de^2x^2)}{4a(a+cx^4)}+\frac{3d^2e\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{c}}\\
&\quad-\frac{3d\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}c^{3/4}}\\
&\quad+\frac{3d\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}c^{3/4}}\\
&\quad+\frac{3d\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x}{\sqrt{a}+\sqrt{c}x^2}\right)}{8\sqrt{2}a^{7/4}c^{3/4}}\end{aligned}$$

output

```
-1/4*e^3/c/(c*x^4+a)+1/4*x*(3*d*e^2*x^2+3*d^2*e*x+d^3)/a/(c*x^4+a)+3/4*d^2
*e*arctan(c^(1/2)*x^2/a^(1/2))/a^(3/2)/c^(1/2)+3/16*d*(c^(1/2)*d^2+a^(1/2)
*e^2)*arctan(-1+2^(1/2)*c^(1/4)*x/a^(1/4))*2^(1/2)/a^(7/4)/c^(3/4)+3/16*d*
(c^(1/2)*d^2+a^(1/2)*e^2)*arctan(1+2^(1/2)*c^(1/4)*x/a^(1/4))*2^(1/2)/a^(7
/4)/c^(3/4)+3/16*d*(c^(1/2)*d^2-a^(1/2)*e^2)*arctanh(2^(1/2)*a^(1/4)*c^(1/
4)*x/(a^(1/2)+c^(1/2)*x^2))*2^(1/2)/a^(7/4)/c^(3/4)
```

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.22

$$\int \frac{(d+ex)^3}{(a+cx^4)^2}\,dx = \frac{-\frac{8a(ae^3-cdx(d^2+3dex+3e^2x^2))}{a+cx^4} - 6\sqrt[4]{a}\sqrt[4]{c}d\left(\sqrt{2}\sqrt{c}d^2+4\sqrt[4]{a}\sqrt[4]{c}de+\sqrt{2}\sqrt{a}e^2\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)+6\sqrt[4]{a}\sqrt[4]{}}{}$$

input
```
Integrate[(d + e*x)^3/(a + c*x^4)^2,x]
```

output
```
((-8*a*(a*e^3 - c*d*x*(d^2 + 3*d*e*x + 3*e^2*x^2)))/(a + c*x^4) - 6*a^(1/4
)*c^(1/4)*d*(Sqrt[2]*Sqrt[c]*d^2 + 4*a^(1/4)*c^(1/4)*d*e + Sqrt[2]*Sqrt[a]
*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 6*a^(1/4)*c^(1/4)*d*(Sqrt[
2]*Sqrt[c]*d^2 - 4*a^(1/4)*c^(1/4)*d*e + Sqrt[2]*Sqrt[a]*e^2)*ArcTan[1 + (
Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 3*Sqrt[2]*c^(1/4)*(-(a^(1/4)*Sqrt[c]*d^3) +
a^(3/4)*d*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] + 3*
Sqrt[2]*c^(1/4)*(a^(1/4)*Sqrt[c]*d^3 - a^(3/4)*d*e^2)*Log[Sqrt[a] + Sqrt[2
]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(32*a^2*c)
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.25, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {2393, 27, 2006, 2415, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^3}{(a+cx^4)^2}\,dx$$

$$\Big\downarrow \; 2393$$

$$-\frac{\int -\frac{3(d^3+2exd^2+e^2x^2d)}{cx^4+a}dx}{4a} - \frac{ae^3-cx\left(d^3+3d^2ex+3de^2x^2\right)}{4ac\left(a+cx^4\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{3\int \frac{d^3+2exd^2+e^2x^2d}{cx^4+a}dx}{4a} - \frac{ae^3 - cx\left(d^3+3d^2ex+3de^2x^2\right)}{4ac\left(a+cx^4\right)}$$

$\Big\downarrow$ 2006

$$\frac{3\int \frac{\left(d^{3/2}+ex\sqrt{d}\right)^2}{cx^4+a}dx}{4a} - \frac{ae^3 - cx\left(d^3+3d^2ex+3de^2x^2\right)}{4ac\left(a+cx^4\right)}$$

$\Big\downarrow$ 2415

$$\frac{3\int \left(\frac{2exd^2}{cx^4+a}+\frac{d^3+e^2x^2d}{cx^4+a}\right)dx}{4a} - \frac{ae^3 - cx\left(d^3+3d^2ex+3de^2x^2\right)}{4ac\left(a+cx^4\right)}$$

$\Big\downarrow$ 2009

$$\frac{3\left(-\frac{d\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\left(\sqrt{a}e^2+\sqrt{c}d^2\right)}{2\sqrt{2}a^{3/4}c^{3/4}}+\frac{d\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)\left(\sqrt{a}e^2+\sqrt{c}d^2\right)}{2\sqrt{2}a^{3/4}c^{3/4}}-\frac{d\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}c^{3/4}}-\right.}{4a}$$

$$\frac{ae^3 - cx\left(d^3+3d^2ex+3de^2x^2\right)}{4ac\left(a+cx^4\right)}$$

input

```
Int[(d + e*x)^3/(a + c*x^4)^2,x]
```

output

```
-1/4*(a*e^3 - c*x*(d^3 + 3*d^2*e*x + 3*d*e^2*x^2))/(a*c*(a + c*x^4)) + (3*
((d^2*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(Sqrt[a]*Sqrt[c]) - (d*(Sqrt[c]*d^2
 + Sqrt[a]*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4
)*c^(3/4)) + (d*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)
/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*c^(3/4)) - (d*(Sqrt[c]*d^2 - Sqrt[a]*e^2)*Lo
g[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c
^(3/4)) + (d*(Sqrt[c]*d^2 - Sqrt[a]*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(
1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(3/4))))/(4*a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2006
```
Int[(u_.)*(Px_), x_Symbol] :> With[{a = Rt[Coeff[Px, x, 0], Expon[Px, x]],
b = Rt[Coeff[Px, x, Expon[Px, x]], Expon[Px, x]]}, Int[u*(a + b*x)^Expon[Px
, x], x] /; EqQ[Px, (a + b*x)^Expon[Px, x]]] /; PolyQ[Px, x] && GtQ[Expon[P
x, x], 1] && NeQ[Coeff[Px, x, 0], 0] &&  !MatchQ[Px, (a_.)*(v_)^Expon[Px, x
] /; FreeQ[a, x] && LinearQ[v, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2393
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   In
t[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(
p + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n
, 0] && LtQ[p, -1]
```

rule 2415
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.20 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.35

method	result
risch	$\frac{\frac{3d\,e^{2}x^{3}}{4a}+\frac{3d^{2}e\,x^{2}}{4a}+\frac{d^{3}x}{4a}-\frac{e^{3}}{4c}}{c\,x^{4}+a}+\frac{3d\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^{4}+a\right)}\frac{\left(e^{2}\_R^{2}+2de\_R+d^{2}\right)\ln\left(x-\_R\right)}{\_R^{3}}\right)}{16ac}$
default	$d^{3}\left(\frac{x}{4a(c\,x^{4}+a)}+\frac{3\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^{2}+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^{2}-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{32a^{2}}\right)+3d^{2}e\left(\frac{}{4a(c}\right.$

input
```
int((e*x+d)^3/(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(3/4*d*e^2/a*x^3+3/4*d^2*e/a*x^2+1/4*d^3/a*x-1/4*e^3/c)/(c*x^4+a)+3/16*d/a
/c*sum((_R^2*e^2+2*_R*d*e+d^2)/_R^3*ln(x-_R),_R=RootOf(_Z^4*c+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 15.25 (sec) , antiderivative size = 91191, normalized size of antiderivative = 321.10

$$\int \frac{(d+ex)^3}{(a+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^3/(c*x^4+a)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [A] (verification not implemented)

Time = 9.11 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.23

$$\int \frac{(d+ex)^3}{(a+cx^4)^2}\,dx$$
$$= \mathrm{RootSum}\Big(65536t^4a^7c^3 + 27648t^2a^4c^2d^4e^2 + t\big(3456a^3cd^4e^5 - 3456a^2c^2d^8e\big) + 81a^2d^4e^8 + 162acd^8e^4 + 8$$
$$+ \frac{-ae^3 + cd^3x + 3cd^2ex^2 + 3cde^2x^3}{4a^2c + 4ac^2x^4}$$

input
```
integrate((e*x+d)**3/(c*x**4+a)**2,x)
```

output
```
RootSum(65536*_t**4*a**7*c**3 + 27648*_t**2*a**4*c**2*d**4*e**2 + _t*(3456
*a**3*c*d**4*e**5 - 3456*a**2*c**2*d**8*e) + 81*a**2*d**4*e**8 + 162*a*c*d
**8*e**4 + 81*c**2*d**12, Lambda(_t, _t*log(x + (4096*_t**3*a**7*c**2*e**6
 + 28672*_t**3*a**6*c**3*d**4*e**2 - 7680*_t**2*a**5*c**2*d**4*e**5 + 1536
*_t**2*a**4*c**3*d**8*e + 2160*_t*a**4*c*d**4*e**8 + 9216*_t*a**3*c**2*d**
8*e**4 + 144*_t*a**2*c**3*d**12 + 162*a**3*d**4*e**11 - 648*a**2*c*d**8*e*
*7 - 810*a*c**2*d**12*e**3)/(27*a**3*d**3*e**12 - 891*a**2*c*d**7*e**8 - 8
91*a*c**2*d**11*e**4 + 27*c**3*d**15))) + (-a*e**3 + c*d**3*x + 3*c*d**2*
e*x**2 + 3*c*d*e**2*x**3)/(4*a**2*c + 4*a*c**2*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.17

$$\int \frac{(d+ex)^3}{(a+cx^4)^2}\,dx$$
$$= \frac{3\,d\left(\frac{\sqrt{2}(\sqrt{c}d^2-\sqrt{a}e^2)\log\left(\sqrt{c}x^2+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{3}{4}}} - \frac{\sqrt{2}(\sqrt{c}d^2-\sqrt{a}e^2)\log\left(\sqrt{c}x^2-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{3}{4}}} + \frac{2\left(\sqrt{2}a^{\frac{1}{4}}c^{\frac{3}{4}}d^2+\sqrt{2}a^{\frac{3}{4}}c^{\frac{1}{4}}e^2-4\sqrt{}\right.}{a^{\frac{3}{4}}}\right.}{32\,a}$$
$$+ \frac{3\,cde^2x^3 + 3\,cd^2ex^2 + cd^3x - ae^3}{4\,(ac^2x^4 + a^2c)}$$

input
```
integrate((e*x+d)^3/(c*x^4+a)^2,x, algorithm="maxima")
```

output
```
3/32*d*(sqrt(2)*(sqrt(c)*d^2 - sqrt(a)*e^2)*log(sqrt(c)*x^2 + sqrt(2)*a^(1
/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)) - sqrt(2)*(sqrt(c)*d^2 - sqrt(a
)*e^2)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(
3/4)) + 2*(sqrt(2)*a^(1/4)*c^(3/4)*d^2 + sqrt(2)*a^(3/4)*c^(1/4)*e^2 - 4*s
qrt(a)*sqrt(c)*d*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1
/4))/sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(3/4)) + 2*(s
qrt(2)*a^(1/4)*c^(3/4)*d^2 + sqrt(2)*a^(3/4)*c^(1/4)*e^2 + 4*sqrt(a)*sqrt(
c)*d*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sq
rt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(3/4)))/a + 1/4*(3*c*d*e^
2*x^3 + 3*c*d^2*e*x^2 + c*d^3*x - a*e^3)/(a*c^2*x^4 + a^2*c)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.21

$$\begin{aligned}\int \frac{(d+ex)^3}{(a+cx^4)^2}\,dx &= \frac{3\,cde^2x^3+3\,cd^2ex^2+cd^3x-ae^3}{4\,(cx^4+a)ac} \\ &+ \frac{3\,\sqrt{2}\Big(2\,\sqrt{2}\sqrt{ac}c^2d^2e+(ac^3)^{\frac{1}{4}}\,c^2d^3+(ac^3)^{\frac{3}{4}}\,de^2\Big)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16\,a^2c^3} \\ &+ \frac{3\,\sqrt{2}\Big(2\,\sqrt{2}\sqrt{ac}c^2d^2e+(ac^3)^{\frac{1}{4}}\,c^2d^3+(ac^3)^{\frac{3}{4}}\,de^2\Big)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16\,a^2c^3} \\ &+ \frac{3\,\sqrt{2}\Big((ac^3)^{\frac{1}{4}}\,c^2d^3-(ac^3)^{\frac{3}{4}}\,de^2\Big)\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{32\,a^2c^3} \\ &- \frac{3\,\sqrt{2}\Big((ac^3)^{\frac{1}{4}}\,c^2d^3-(ac^3)^{\frac{3}{4}}\,de^2\Big)\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{32\,a^2c^3}\end{aligned}$$

input
```
integrate((e*x+d)^3/(c*x^4+a)^2,x, algorithm="giac")
```

output
```
1/4*(3*c*d*e^2*x^3 + 3*c*d^2*e*x^2 + c*d^3*x - a*e^3)/((c*x^4 + a)*a*c) +
3/16*sqrt(2)*(2*sqrt(2)*sqrt(a*c)*c^2*d^2*e + (a*c^3)^(1/4)*c^2*d^3 + (a*c
^3)^(3/4)*d*e^2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4
))/(a^2*c^3) + 3/16*sqrt(2)*(2*sqrt(2)*sqrt(a*c)*c^2*d^2*e + (a*c^3)^(1/4)
*c^2*d^3 + (a*c^3)^(3/4)*d*e^2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1
/4))/(a/c)^(1/4))/(a^2*c^3) + 3/32*sqrt(2)*((a*c^3)^(1/4)*c^2*d^3 - (a*c^3
)^(3/4)*d*e^2)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^2*c^3) - 3/
32*sqrt(2)*((a*c^3)^(1/4)*c^2*d^3 - (a*c^3)^(3/4)*d*e^2)*log(x^2 - sqrt(2)
*x*(a/c)^(1/4) + sqrt(a/c))/(a^2*c^3)
```

Mupad [B] (verification not implemented)

Time = 22.35 (sec) , antiderivative size = 670, normalized size of antiderivative = 2.36

$$\int \frac{(d+ex)^3}{(a+cx^4)^2}\,dx$$

$$= \left(\sum_{k=1}^{4} \ln\left(\frac{c\,d^2\,\left(27\,c\,d^5\,e^2 - 9\,a\,d\,e^6 + 36\,c\,d^4\,e^3\,x - \mathrm{root}(65536\,a^7\,c^3\,z^4 + 27648\,a^4\,c^2\,d^4\,e^2\,z^2 + 3456\,a^3\,c\,d \right.}{}\right.\right.$$

$$+ 27648\,a^4\,c^2\,d^4\,e^2\,z^2 + 3456\,a^3\,c\,d^4\,e^5\,z - 3456\,a^2\,c^2\,d^8\,e\,z + 162\,a\,c\,d^8\,e^4 + 81\,a^2\,d^4\,e^8$$

$$\left.\left. + 81\,c^2\,d^{12}, z, k)\right)\right) + \frac{\frac{d^3\,x}{4\,a} - \frac{e^3}{4\,c} + \frac{3\,d^2\,e\,x^2}{4\,a} + \frac{3\,d\,e^2\,x^3}{4\,a}}{c\,x^4 + a}$$

input
```
int((d + e*x)^3/(a + c*x^4)^2,x)
```

output
```
symsum(log((3*c*d^2*(27*c*d^5*e^2 - 9*a*d*e^6 + 36*c*d^4*e^3*x - 256*root(
65536*a^7*c^3*z^4 + 27648*a^4*c^2*d^4*e^2*z^2 + 3456*a^3*c*d^4*e^5*z - 345
6*a^2*c^2*d^8*e*z + 162*a*c*d^8*e^4 + 81*a^2*d^4*e^8 + 81*c^2*d^12, z, k)^
2*a^3*c^2*d - 48*root(65536*a^7*c^3*z^4 + 27648*a^4*c^2*d^4*e^2*z^2 + 3456
*a^3*c*d^4*e^5*z - 3456*a^2*c^2*d^8*e*z + 162*a*c*d^8*e^4 + 81*a^2*d^4*e^8
 + 81*c^2*d^12, z, k)*a*c^2*d^4*x + 48*root(65536*a^7*c^3*z^4 + 27648*a^4*
c^2*d^4*e^2*z^2 + 3456*a^3*c*d^4*e^5*z - 3456*a^2*c^2*d^8*e*z + 162*a*c*d^
8*e^4 + 81*a^2*d^4*e^8 + 81*c^2*d^12, z, k)*a^2*c*e^4*x + 512*root(65536*a
^7*c^3*z^4 + 27648*a^4*c^2*d^4*e^2*z^2 + 3456*a^3*c*d^4*e^5*z - 3456*a^2*c
^2*d^8*e*z + 162*a*c*d^8*e^4 + 81*a^2*d^4*e^8 + 81*c^2*d^12, z, k)^2*a^3*c
^2*e*x - 192*root(65536*a^7*c^3*z^4 + 27648*a^4*c^2*d^4*e^2*z^2 + 3456*a^3
*c*d^4*e^5*z - 3456*a^2*c^2*d^8*e*z + 162*a*c*d^8*e^4 + 81*a^2*d^4*e^8 + 8
1*c^2*d^12, z, k)*a^2*c*d*e^3))/(64*a^3))*root(65536*a^7*c^3*z^4 + 27648*a
^4*c^2*d^4*e^2*z^2 + 3456*a^3*c*d^4*e^5*z - 3456*a^2*c^2*d^8*e*z + 162*a*c
*d^8*e^4 + 81*a^2*d^4*e^8 + 81*c^2*d^12, z, k), k, 1, 4) + ((d^3*x)/(4*a)
- e^3/(4*c) + (3*d^2*e*x^2)/(4*a) + (3*d*e^2*x^3)/(4*a))/(a + c*x^4)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 836, normalized size of antiderivative = 2.94

$$\int \frac{(d+ex)^3}{(a+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
int((e*x+d)^3/(c*x^4+a)^2,x)
```

output
```
( - 6*c**(1/4)*a**(3/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c
)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a*d*e**2 - 6*c**(1/4)*a**(3/4)*sqrt(2)*a
tan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))
*c*d*e**2*x**4 - 6*c**(3/4)*a**(1/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(
2) - 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a*d**3 - 6*c**(3/4)*a**(1/4
)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x)/(c**(1/4)*a**(1/4
)*sqrt(2)))*c*d**3*x**4 - 24*sqrt(c)*sqrt(a)*atan((c**(1/4)*a**(1/4)*sqrt(
2) - 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a*d**2*e - 24*sqrt(c)*sqrt(
a)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(
2)))*c*d**2*e*x**4 + 6*c**(1/4)*a**(3/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*s
qrt(2) + 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a*d*e**2 + 6*c**(1/4)*a
**(3/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(c)*x)/(c**(1/4)*a
**(1/4)*sqrt(2)))*c*d*e**2*x**4 + 6*c**(3/4)*a**(1/4)*sqrt(2)*atan((c**(1/
4)*a**(1/4)*sqrt(2) + 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a*d**3 + 6
*c**(3/4)*a**(1/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(c)*x)/
(c**(1/4)*a**(1/4)*sqrt(2)))*c*d**3*x**4 - 24*sqrt(c)*sqrt(a)*atan((c**(1/
4)*a**(1/4)*sqrt(2) + 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a*d**2*e -
 24*sqrt(c)*sqrt(a)*atan((c**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(c)*x)/(c**(1/
4)*a**(1/4)*sqrt(2)))*c*d**2*e*x**4 + 3*c**(1/4)*a**(3/4)*sqrt(2)*log( - c
**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c)*x**2)*a*d*e**2 + 3*c**(1...
```

3.180 $\int \frac{(d+ex)^2}{(a+cx^4)^2} dx$

Optimal result

Integrand size = 17, antiderivative size = 250

$$\int \frac{(d+ex)^2}{(a+cx^4)^2} dx = \frac{x(d+ex)^2}{4a(a+cx^4)} + \frac{de \arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{c}} - \frac{(3\sqrt{c}d^2+\sqrt{a}e^2)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}c^{3/4}} + \frac{(3\sqrt{c}d^2+\sqrt{a}e^2)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}c^{3/4}} + \frac{(3\sqrt{c}d^2-\sqrt{a}e^2)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x}{\sqrt{a}+\sqrt{c}x^2}\right)}{8\sqrt{2}a^{7/4}c^{3/4}}$$

output
```
1/4*x*(e*x+d)^2/a/(c*x^4+a)+1/2*d*e*arctan(c^(1/2)*x^2/a^(1/2))/a^(3/2)/c^
(1/2)+1/16*(3*c^(1/2)*d^2+a^(1/2)*e^2)*arctan(-1+2^(1/2)*c^(1/4)*x/a^(1/4)
)*2^(1/2)/a^(7/4)/c^(3/4)+1/16*(3*c^(1/2)*d^2+a^(1/2)*e^2)*arctan(1+2^(1/2
)*c^(1/4)*x/a^(1/4))*2^(1/2)/a^(7/4)/c^(3/4)+1/16*(3*c^(1/2)*d^2-a^(1/2)*e
^2)*arctanh(2^(1/2)*a^(1/4)*c^(1/4)*x/(a^(1/2)+c^(1/2)*x^2))*2^(1/2)/a^(7/
4)/c^(3/4)
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.28

$$\int \frac{(d+ex)^2}{(a+cx^4)^2}\,dx$$
$$= \frac{\frac{8ax(d+ex)^2}{a+cx^4} - \frac{2\sqrt[4]{a}\left(3\sqrt{2}\sqrt{c}d^2+8\sqrt[4]{a}\sqrt[4]{c}de+\sqrt{2}\sqrt{a}e^2\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{c^{3/4}} + \frac{2\sqrt[4]{a}\left(3\sqrt{2}\sqrt{c}d^2-8\sqrt[4]{a}\sqrt[4]{c}de+\sqrt{2}\sqrt{a}e^2\right)\arctan\left(1\right.}{c^{3/4}}}{32}$$

input
```
Integrate[(d + e*x)^2/(a + c*x^4)^2,x]
```

output
```
((8*a*x*(d + e*x)^2)/(a + c*x^4) - (2*a^(1/4)*(3*Sqrt[2]*Sqrt[c]*d^2 + 8*a
^(1/4)*c^(1/4)*d*e + Sqrt[2]*Sqrt[a]*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a
^(1/4)])/c^(3/4) + (2*a^(1/4)*(3*Sqrt[2]*Sqrt[c]*d^2 - 8*a^(1/4)*c^(1/4)*d
*e + Sqrt[2]*Sqrt[a]*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/c^(3/4)
 + (Sqrt[2]*(-3*a^(1/4)*Sqrt[c]*d^2 + a^(3/4)*e^2)*Log[Sqrt[a] - Sqrt[2]*a
^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^(3/4) + (Sqrt[2]*(3*a^(1/4)*Sqrt[c]*d^2
 - a^(3/4)*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^
(3/4))/(32*a^2)
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.31, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {2394, 25, 2415, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^2}{(a+cx^4)^2}\,dx$$
$$\Big\downarrow \; 2394$$
$$\frac{x(d+ex)^2}{4a\left(a+cx^4\right)} - \frac{\int -\frac{3d^2+4exd+e^2x^2}{cx^4+a}dx}{4a}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{3d^2+4exd+e^2x^2}{cx^4+a}dx}{4a}+\frac{x(d+ex)^2}{4a\left(a+cx^4\right)}$$

$$\Big\downarrow \; 2415$$

$$\frac{\int \left(\frac{4dex}{cx^4+a}+\frac{3d^2+e^2x^2}{cx^4+a}\right)dx}{4a}+\frac{x(d+ex)^2}{4a\left(a+cx^4\right)}$$

$$\Big\downarrow \; 2009$$

$$\frac{-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\left(\sqrt{a}e^2+3\sqrt{c}d^2\right)}{2\sqrt{2}a^{3/4}c^{3/4}}+\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)\left(\sqrt{a}e^2+3\sqrt{c}d^2\right)}{2\sqrt{2}a^{3/4}c^{3/4}}-\frac{\left(3\sqrt{c}d^2-\sqrt{a}e^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}c^{3/4}}+\frac{(3\cdots}{\cdots}}{4a}$$

$$\frac{x(d+ex)^2}{4a\left(a+cx^4\right)}$$

input
```
Int[(d + e*x)^2/(a + c*x^4)^2,x]
```

output
```
(x*(d + e*x)^2)/(4*a*(a + c*x^4)) + ((2*d*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])
/(Sqrt[a]*Sqrt[c]) - ((3*Sqrt[c]*d^2 + Sqrt[a]*e^2)*ArcTan[1 - (Sqrt[2]*c^
(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*c^(3/4)) + ((3*Sqrt[c]*d^2 + Sqrt[a]
*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*c^(3/4))
 - ((3*Sqrt[c]*d^2 - Sqrt[a]*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x
+ Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(3/4)) + ((3*Sqrt[c]*d^2 - Sqrt[a]*e^
2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3
/4)*c^(3/4)))/(4*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2394
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   Int[ExpandToSum[n
*(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x
] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

rule 2415
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.17 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.36

method	result
risch	$\frac{\frac{e^2x^3}{4a}+\frac{de\,x^2}{2a}+\frac{d^2x}{4a}}{c\,x^4+a}+\frac{\sum_{\_R=\operatorname{RootOf}\left(c\_Z^4+a\right)}\frac{\left(e^2\_R^2+4de\_R+3d^2\right)\ln\left(x-\_R\right)}{\_R^3}}{16ca}$
default	$d^2\left(\frac{x}{4a(c\,x^4+a)}+\frac{3\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{32a^2}\right)+2de\left(\frac{x}{4a(c\,x\right.$

input
```
int((e*x+d)^2/(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(1/4*e^2/a*x^3+1/2*d*e/a*x^2+1/4*d^2/a*x)/(c*x^4+a)+1/16/c/a*sum((_R^2*e^2
+4*_R*d*e+3*d^2)/_R^3*ln(x-_R),_R=RootOf(_Z^4*c+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.85 (sec) , antiderivative size = 90963, normalized size of antiderivative = 363.85

$$\int \frac{(d+ex)^2}{(a+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^2/(c*x^4+a)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [A] (verification not implemented)

Time = 2.53 (sec) , antiderivative size = 318, normalized size of antiderivative = 1.27

$$\int \frac{(d+ex)^2}{(a+cx^4)^2}\,dx$$
$$= \text{RootSum}\Big(65536t^4a^7c^3 + 11264t^2a^4c^2d^2e^2 + t\left(256a^3cde^5 - 2304a^2c^2d^5e\right) + a^2e^8 + 82acd^4e^4 + 81c^2d^8,$$
$$+ \frac{d^2x + 2dex^2 + e^2x^3}{4a^2 + 4acx^4}$$

input
```
integrate((e*x+d)**2/(c*x**4+a)**2,x)
```

output
```
RootSum(65536*_t**4*a**7*c**3 + 11264*_t**2*a**4*c**2*d**2*e**2 + _t*(256*
a**3*c*d*e**5 - 2304*a**2*c**2*d**5*e) + a**2*e**8 + 82*a*c*d**4*e**4 + 81
*c**2*d**8, Lambda(_t, _t*log(x + (4096*_t**3*a**7*c**2*e**6 + 356352*_t**
3*a**6*c**3*d**4*e**2 - 23552*_t**2*a**5*c**2*d**3*e**5 + 27648*_t**2*a**4
*c**3*d**7*e + 912*_t*a**4*c*d**2*e**8 + 43584*_t*a**3*c**2*d**6*e**4 + 38
88*_t*a**2*c**3*d**10 + 12*a**3*d*e**11 - 1088*a**2*c*d**5*e**7 - 7020*a*c
**2*d**9*e**3)/(a**3*e**12 - 649*a**2*c*d**4*e**8 - 5841*a*c**2*d**8*e**4
+ 729*c**3*d**12))) + (d**2*x + 2*d*e*x**2 + e**2*x**3)/(4*a**2 + 4*a*c*x
**4)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 318, normalized size of antiderivative = 1.27

$$\int \frac{(d+ex)^2}{(a+cx^4)^2}\,dx = \frac{e^2x^3+2\,dex^2+d^2x}{4\,(acx^4+a^2)}$$

$$+\frac{\frac{\sqrt{2}(3\,\sqrt{c}d^2-\sqrt{a}e^2)\log\left(\sqrt{c}x^2+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{3}{4}}}-\frac{\sqrt{2}(3\,\sqrt{c}d^2-\sqrt{a}e^2)\log\left(\sqrt{c}x^2-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{3}{4}}}+\frac{2\left(3\,\sqrt{2}a^{\frac{1}{4}}c^{\frac{3}{4}}d^2+\sqrt{2}a^{\frac{3}{4}}c^{\frac{1}{4}}e^2-8\right.}{a^{\frac{3}{4}}}}{32\,a}$$

input
```
integrate((e*x+d)^2/(c*x^4+a)^2,x, algorithm="maxima")
```

output
```
1/4*(e^2*x^3 + 2*d*e*x^2 + d^2*x)/(a*c*x^4 + a^2) + 1/32*(sqrt(2)*(3*sqrt(
c)*d^2 - sqrt(a)*e^2)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a
))/(a^(3/4)*c^(3/4)) - sqrt(2)*(3*sqrt(c)*d^2 - sqrt(a)*e^2)*log(sqrt(c)*x
^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)) + 2*(3*sqrt(2)
*a^(1/4)*c^(3/4)*d^2 + sqrt(2)*a^(3/4)*c^(1/4)*e^2 - 8*sqrt(a)*sqrt(c)*d*e
)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*
sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(3/4)) + 2*(3*sqrt(2)*a^(1/4)*c
^(3/4)*d^2 + sqrt(2)*a^(3/4)*c^(1/4)*e^2 + 8*sqrt(a)*sqrt(c)*d*e)*arctan(1
/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/
(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(3/4)))/a
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.30

$$\int \frac{(d+ex)^2}{(a+cx^4)^2}\,dx$$
$$= \frac{e^2x^3+2\,dex^2+d^2x}{4\,(cx^4+a)a}$$
$$+ \frac{\sqrt{2}\Big(4\sqrt{2}\sqrt{ac}c^2de+3\,(ac^3)^{\frac{1}{4}}\,c^2d^2+(ac^3)^{\frac{3}{4}}\,e^2\Big)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16\,a^2c^3}$$
$$+ \frac{\sqrt{2}\Big(4\sqrt{2}\sqrt{ac}c^2de+3\,(ac^3)^{\frac{1}{4}}\,c^2d^2+(ac^3)^{\frac{3}{4}}\,e^2\Big)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16\,a^2c^3}$$
$$+ \frac{\sqrt{2}\Big(3\,(ac^3)^{\frac{1}{4}}\,c^2d^2-(ac^3)^{\frac{3}{4}}\,e^2\Big)\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{32\,a^2c^3}$$
$$- \frac{\sqrt{2}\Big(3\,(ac^3)^{\frac{1}{4}}\,c^2d^2-(ac^3)^{\frac{3}{4}}\,e^2\Big)\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{32\,a^2c^3}$$

input
```
integrate((e*x+d)^2/(c*x^4+a)^2,x, algorithm="giac")
```

output
```
1/4*(e^2*x^3 + 2*d*e*x^2 + d^2*x)/((c*x^4 + a)*a) + 1/16*sqrt(2)*(4*sqrt(2
)*sqrt(a*c)*c^2*d*e + 3*(a*c^3)^(1/4)*c^2*d^2 + (a*c^3)^(3/4)*e^2)*arctan(
1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^2*c^3) + 1/16*sqrt
(2)*(4*sqrt(2)*sqrt(a*c)*c^2*d*e + 3*(a*c^3)^(1/4)*c^2*d^2 + (a*c^3)^(3/4)
*e^2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^2*c^3
) + 1/32*sqrt(2)*(3*(a*c^3)^(1/4)*c^2*d^2 - (a*c^3)^(3/4)*e^2)*log(x^2 + s
qrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^2*c^3) - 1/32*sqrt(2)*(3*(a*c^3)^(1/4
)*c^2*d^2 - (a*c^3)^(3/4)*e^2)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c)
)/(a^2*c^3)
```

Mupad [B] (verification not implemented)

Time = 21.75 (sec) , antiderivative size = 391, normalized size of antiderivative = 1.56

$$\int \frac{(d+ex)^2}{(a+cx^4)^2}\,dx = \frac{\frac{d^2\,x}{4\,a}+\frac{e^2\,x^3}{4\,a}+\frac{d\,e\,x^2}{2\,a}}{c\,x^4+a}+\left(\sum_{k=1}^{4}\ln\left(\frac{39\,c^2\,d^4\,e^2-a\,c\,e^6}{64\,a^3}\right.\right.$$
$$-\mathrm{root}\left(65536\,a^7\,c^3\,z^4+11264\,a^4\,c^2\,d^2\,e^2\,z^2-2304\,a^2\,c^2\,d^5\,e\,z+256\,a^3\,c\,d\,e^5\,z+82\,a\,c\,d^4\,e^4+81\,c^2\,d^8+a^2\,e^8,\right.$$
$$\left.+\frac{5\,c^2\,d^3\,e^3\,x}{8\,a^3}\right)\mathrm{root}\left(65536\,a^7\,c^3\,z^4+11264\,a^4\,c^2\,d^2\,e^2\,z^2-2304\,a^2\,c^2\,d^5\,e\,z\right.$$
$$\left.\left.+256\,a^3\,c\,d\,e^5\,z+82\,a\,c\,d^4\,e^4+81\,c^2\,d^8+a^2\,e^8,z,k\right)\right)$$

input
```
int((d + e*x)^2/(a + c*x^4)^2,x)
```

output
```
((d^2*x)/(4*a) + (e^2*x^3)/(4*a) + (d*e*x^2)/(2*a))/(a + c*x^4) + symsum(l
og((39*c^2*d^4*e^2 - a*c*e^6)/(64*a^3) - root(65536*a^7*c^3*z^4 + 11264*a^
4*c^2*d^2*e^2*z^2 - 2304*a^2*c^2*d^5*e*z + 256*a^3*c*d*e^5*z + 82*a*c*d^4*
e^4 + 81*c^2*d^8 + a^2*e^8, z, k)*(root(65536*a^7*c^3*z^4 + 11264*a^4*c^2*
d^2*e^2*z^2 - 2304*a^2*c^2*d^5*e*z + 256*a^3*c*d*e^5*z + 82*a*c*d^4*e^4 +
81*c^2*d^8 + a^2*e^8, z, k)*(12*c^3*d^2 - 16*c^3*d*e*x) + (x*(18*a*c^3*d^4
 - 2*a^2*c^2*e^4))/(8*a^3) + (2*c^2*d*e^3)/a) + (5*c^2*d^3*e^3*x)/(8*a^3))
*root(65536*a^7*c^3*z^4 + 11264*a^4*c^2*d^2*e^2*z^2 - 2304*a^2*c^2*d^5*e*z
 + 256*a^3*c*d*e^5*z + 82*a*c*d^4*e^4 + 81*c^2*d^8 + a^2*e^8, z, k), k, 1,
 4)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 805, normalized size of antiderivative = 3.22

$$\int \frac{(d+ex)^2}{(a+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
int((e*x+d)^2/(c*x^4+a)^2,x)
```

output

```
( - 2*c**(1/4)*a**(3/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c
)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a*e**2 - 2*c**(1/4)*a**(3/4)*sqrt(2)*ata
n((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*c
*e**2*x**4 - 6*c**(3/4)*a**(1/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) -
 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a*d**2 - 6*c**(3/4)*a**(1/4)*sq
rt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sq
rt(2)))*c*d**2*x**4 - 16*sqrt(c)*sqrt(a)*atan((c**(1/4)*a**(1/4)*sqrt(2) -
 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a*d*e - 16*sqrt(c)*sqrt(a)*atan
((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*c*
d*e*x**4 + 2*c**(1/4)*a**(3/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) + 2
*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a*e**2 + 2*c**(1/4)*a**(3/4)*sqrt
(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt
(2)))*c*e**2*x**4 + 6*c**(3/4)*a**(1/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sq
rt(2) + 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a*d**2 + 6*c**(3/4)*a**(
1/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(c)*x)/(c**(1/4)*a**(
1/4)*sqrt(2)))*c*d**2*x**4 - 16*sqrt(c)*sqrt(a)*atan((c**(1/4)*a**(1/4)*sq
rt(2) + 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a*d*e - 16*sqrt(c)*sqrt(
a)*atan((c**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(
2)))*c*d*e*x**4 + c**(1/4)*a**(3/4)*sqrt(2)*log( - c**(1/4)*a**(1/4)*sqrt(
2)*x + sqrt(a) + sqrt(c)*x**2)*a*e**2 + c**(1/4)*a**(3/4)*sqrt(2)*log( ...
```

3.181 $\int \frac{d+ex}{(a+cx^4)^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 189

$$\int \frac{d+ex}{(a+cx^4)^2}\,dx = \frac{x(d+ex)}{4a(a+cx^4)} + \frac{e\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{c}} - \frac{3d\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{c}} + \frac{3d\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{c}} + \frac{3d\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x}{\sqrt{a}+\sqrt{c}x^2}\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{c}}$$

output

```
1/4*x*(e*x+d)/a/(c*x^4+a)+1/4*e*arctan(c^(1/2)*x^2/a^(1/2))/a^(3/2)/c^(1/2
)+3/16*d*arctan(-1+2^(1/2)*c^(1/4)*x/a^(1/4))*2^(1/2)/a^(7/4)/c^(1/4)+3/16
*d*arctan(1+2^(1/2)*c^(1/4)*x/a^(1/4))*2^(1/2)/a^(7/4)/c^(1/4)+3/16*d*arct
anh(2^(1/2)*a^(1/4)*c^(1/4)*x/(a^(1/2)+c^(1/2)*x^2))*2^(1/2)/a^(7/4)/c^(1/
4)
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.19

$$\int \frac{d+ex}{\left(a+cx^4\right)^2}\,dx$$
$$= \frac{\frac{8a^{3/4}x(d+ex)}{a+cx^4} - \frac{2\left(3\sqrt{2}\sqrt[4]{c}d+4\sqrt[4]{a}e\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{c}} + \frac{2\left(3\sqrt{2}\sqrt[4]{c}d-4\sqrt[4]{a}e\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{c}} - \frac{3\sqrt{2}d\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{\ldots}\right.}{\sqrt[4]{c}\ldots}}{32a^{7/4}}$$

input
```
Integrate[(d + e*x)/(a + c*x^4)^2,x]
```

output
```
((8*a^(3/4)*x*(d + e*x))/(a + c*x^4) - (2*(3*Sqrt[2]*c^(1/4)*d + 4*a^(1/4)
*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/Sqrt[c] + (2*(3*Sqrt[2]*c^(1/
4)*d - 4*a^(1/4)*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/Sqrt[c] - (3*
Sqrt[2]*d*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^(1/4)
+ (3*Sqrt[2]*d*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^(
1/4))/(32*a^(7/4))
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.30, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {2394, 25, 2415, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex}{\left(a+cx^4\right)^2}\,dx$$
$$\Big\downarrow \text{2394}$$
$$\frac{x(d+ex)}{4a\left(a+cx^4\right)} - \frac{\int -\frac{3d+2ex}{cx^4+a}dx}{4a}$$
$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{3d+2ex}{cx^4+a}dx}{4a}+\frac{x(d+ex)}{4a\left(a+cx^4\right)}$$

$\Big\downarrow$ 2415

$$\frac{\int \left(\frac{3d}{cx^4+a}+\frac{2ex}{cx^4+a}\right)dx}{4a}+\frac{x(d+ex)}{4a\left(a+cx^4\right)}$$

$\Big\downarrow$ 2009

$$\frac{-\frac{3d\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{c}}+\frac{3d\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{c}}-\frac{3d\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{c}}+\frac{3d\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{c}}+\frac{e\text{ arc}}{}}{4a}$$
$$\frac{x(d+ex)}{4a\left(a+cx^4\right)}$$

input
```
Int[(d + e*x)/(a + c*x^4)^2,x]
```

output
```
(x*(d + e*x))/(4*a*(a + c*x^4)) + ((e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(Sqrt
[a]*Sqrt[c]) - (3*d*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^
(3/4)*c^(1/4)) + (3*d*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*
a^(3/4)*c^(1/4)) - (3*d*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*
x^2])/(4*Sqrt[2]*a^(3/4)*c^(1/4)) + (3*d*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(
1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(1/4)))/(4*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2394
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   Int[ExpandToSum[n
*(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x
] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

rule 2415
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.25 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.35

method	result
risch	$\frac{\frac{e\,x^2}{4a}+\frac{dx}{4a}}{c\,x^4+a}+\frac{\sum_{\_R=\operatorname{RootOf}\left(c\_Z^4+a\right)}\frac{\left(2e\_R+3d\right)\ln\left(x-\_R\right)}{\_R^3}}{16ca}$
default	$d\left(\frac{x}{4a(c\,x^4+a)}+\frac{3\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{32a^2}\right)+e\left(\frac{x^2}{4a(c\,x^4+a}\right.$

input
```
int((e*x+d)/(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(1/4*e/a*x^2+1/4*d/a*x)/(c*x^4+a)+1/16/c/a*sum((2*_R*e+3*d)/_R^3*ln(x-_R),
_R=RootOf(_Z^4*c+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.16 (sec) , antiderivative size = 43065, normalized size of antiderivative = 227.86

$$\int \frac{d+ex}{(a+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)/(c*x^4+a)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [A] (verification not implemented)

Time = 0.90 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.82

$$\int \frac{d+ex}{(a+cx^4)^2}\,dx$$
$$=\operatorname{RootSum}\left(65536t^4a^7c^2+2048t^2a^4ce^2-1152ta^2cd^2e+16ae^4+81cd^4,\left(t\mapsto t\log\left(x+\frac{-32768t^3a^6ce^2}{}\right.\right.\right.$$
$$+\frac{dx+ex^2}{4a^2+4acx^4}$$

input
```
integrate((e*x+d)/(c*x**4+a)**2,x)
```

output
```
RootSum(65536*_t**4*a**7*c**2 + 2048*_t**2*a**4*c*e**2 - 1152*_t*a**2*c*d*
*2*e + 16*a*e**4 + 81*c*d**4, Lambda(_t, _t*log(x + (-32768*_t**3*a**6*c*e
**2 - 4608*_t**2*a**4*c*d**2*e - 512*_t*a**3*e**4 - 1296*_t*a**2*c*d**4 +
360*a*d**2*e**3)/(192*a*d*e**4 - 243*c*d**5)))) + (d*x + e*x**2)/(4*a**2 +
 4*a*c*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.26

$$\int \frac{d+ex}{(a+cx^4)^2}\,dx=\frac{ex^2+dx}{4\,(acx^4+a^2)}$$
$$+\frac{\frac{3\sqrt{2}d\log\left(\sqrt{c}x^2+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{1}{4}}}-\frac{3\sqrt{2}d\log\left(\sqrt{c}x^2-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{1}{4}}}+\frac{2\left(3\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}d-4\sqrt{a}e\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{c}}c^{\frac{1}{4}}}+}{32\,a}$$

input
```
integrate((e*x+d)/(c*x^4+a)^2,x, algorithm="maxima")
```

output
```
1/4*(e*x^2 + d*x)/(a*c*x^4 + a^2) + 1/32*(3*sqrt(2)*d*log(sqrt(c)*x^2 + sq
rt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(1/4)) - 3*sqrt(2)*d*log(sqr
t(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(1/4)) + 2*(3*s
qrt(2)*a^(1/4)*c^(1/4)*d - 4*sqrt(a)*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x +
sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt
(c))*c^(1/4)) + 2*(3*sqrt(2)*a^(1/4)*c^(1/4)*d + 4*sqrt(a)*e)*arctan(1/2*s
qrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(a^(
3/4)*sqrt(sqrt(a)*sqrt(c))*c^(1/4)))/a
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.26

$$\int \frac{d+ex}{(a+cx^4)^2}\,dx = \frac{3\sqrt{2}(ac^3)^{\frac{1}{4}}\,d\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{32\,a^2c} - \frac{3\sqrt{2}(ac^3)^{\frac{1}{4}}\,d\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{32\,a^2c} + \frac{ex^2+dx}{4\,(cx^4+a)a} + \frac{\sqrt{2}\left(2\sqrt{2}\sqrt{ac}ce+3\,(ac^3)^{\frac{1}{4}}\,cd\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16\,a^2c^2} + \frac{\sqrt{2}\left(2\sqrt{2}\sqrt{ac}ce+3\,(ac^3)^{\frac{1}{4}}\,cd\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16\,a^2c^2}$$

input
```
integrate((e*x+d)/(c*x^4+a)^2,x, algorithm="giac")
```

output
```
3/32*sqrt(2)*(a*c^3)^(1/4)*d*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/
(a^2*c) - 3/32*sqrt(2)*(a*c^3)^(1/4)*d*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + s
qrt(a/c))/(a^2*c) + 1/4*(e*x^2 + d*x)/((c*x^4 + a)*a) + 1/16*sqrt(2)*(2*sq
rt(2)*sqrt(a*c)*c*e + 3*(a*c^3)^(1/4)*c*d)*arctan(1/2*sqrt(2)*(2*x + sqrt(
2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^2*c^2) + 1/16*sqrt(2)*(2*sqrt(2)*sqrt(a*c)
*c*e + 3*(a*c^3)^(1/4)*c*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))
/(a/c)^(1/4))/(a^2*c^2)
```

Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.49

$$\int \frac{d+ex}{(a+cx^4)^2}\,dx$$
$$= \left(\sum_{k=1}^{4} \ln\left(\frac{c^2\left(3\,d\,e^2+2\,e^3\,x-\mathrm{root}(65536\,a^7\,c^2\,z^4+2048\,a^4\,c\,e^2\,z^2-1152\,a^2\,c\,d^2\,e\,z+81\,c\,d^4+16\,a\,e^4,\right.}{}\right.\right.$$
$$\left.\left.+2048\,a^4\,c\,e^2\,z^2-1152\,a^2\,c\,d^2\,e\,z+81\,c\,d^4+16\,a\,e^4,z,k\right)\right)+\frac{\frac{e\,x^2}{4\,a}+\frac{d\,x}{4\,a}}{c\,x^4+a}$$

input
```
int((d + e*x)/(a + c*x^4)^2,x)
```

output
```
symsum(log((c^2*(3*d*e^2 + 2*e^3*x - 192*root(65536*a^7*c^2*z^4 + 2048*a^4
*c*e^2*z^2 - 1152*a^2*c*d^2*e*z + 81*c*d^4 + 16*a*e^4, z, k)^2*a^3*c*d + 1
28*root(65536*a^7*c^2*z^4 + 2048*a^4*c*e^2*z^2 - 1152*a^2*c*d^2*e*z + 81*c
*d^4 + 16*a*e^4, z, k)^2*a^3*c*e*x - 36*root(65536*a^7*c^2*z^4 + 2048*a^4*
c*e^2*z^2 - 1152*a^2*c*d^2*e*z + 81*c*d^4 + 16*a*e^4, z, k)*a*c*d^2*x))/(1
6*a^3))*root(65536*a^7*c^2*z^4 + 2048*a^4*c*e^2*z^2 - 1152*a^2*c*d^2*e*z +
 81*c*d^4 + 16*a*e^4, z, k), k, 1, 4) + ((e*x^2)/(4*a) + (d*x)/(4*a))/(a +
 c*x^4)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 468, normalized size of antiderivative = 2.48

$$\int \frac{d+ex}{(a+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
int((e*x+d)/(c*x^4+a)^2,x)
```

output

```
( - 6*c**(3/4)*a**(1/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c
)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a*d - 6*c**(3/4)*a**(1/4)*sqrt(2)*atan((
c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*c*d*
x**4 - 8*sqrt(c)*sqrt(a)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x)/(c
**(1/4)*a**(1/4)*sqrt(2)))*a*e - 8*sqrt(c)*sqrt(a)*atan((c**(1/4)*a**(1/4)
*sqrt(2) - 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*c*e*x**4 + 6*c**(3/4)
*a**(1/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(c)*x)/(c**(1/4)
*a**(1/4)*sqrt(2)))*a*d + 6*c**(3/4)*a**(1/4)*sqrt(2)*atan((c**(1/4)*a**(1
/4)*sqrt(2) + 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*c*d*x**4 - 8*sqrt(
c)*sqrt(a)*atan((c**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(c)*x)/(c**(1/4)*a**(1/
4)*sqrt(2)))*a*e - 8*sqrt(c)*sqrt(a)*atan((c**(1/4)*a**(1/4)*sqrt(2) + 2*s
qrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*c*e*x**4 - 3*c**(3/4)*a**(1/4)*sqrt
(2)*log( - c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c)*x**2)*a*d - 3*c
**(3/4)*a**(1/4)*sqrt(2)*log( - c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sq
rt(c)*x**2)*c*d*x**4 + 3*c**(3/4)*a**(1/4)*sqrt(2)*log(c**(1/4)*a**(1/4)*s
qrt(2)*x + sqrt(a) + sqrt(c)*x**2)*a*d + 3*c**(3/4)*a**(1/4)*sqrt(2)*log(c
**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c)*x**2)*c*d*x**4 + 8*a*c*d*x
+ 8*a*c*e*x**2)/(32*a**2*c*(a + c*x**4))
```

3.182 $\int \frac{1}{\left(a+cx^4\right)^2}\,dx$

Optimal result

Integrand size = 9, antiderivative size = 151

$$\int \frac{1}{\left(a+cx^4\right)^2}\,dx = \frac{x}{4a\left(a+cx^4\right)} - \frac{3\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{c}} + \frac{3\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{c}} + \frac{3\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x}{\sqrt{a}+\sqrt{c}x^2}\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{c}}$$

output
```
1/4*x/a/(c*x^4+a)+3/16*arctan(-1+2^(1/2)*c^(1/4)*x/a^(1/4))*2^(1/2)/a^(7/4
)/c^(1/4)+3/16*arctan(1+2^(1/2)*c^(1/4)*x/a^(1/4))*2^(1/2)/a^(7/4)/c^(1/4)
+3/16*arctanh(2^(1/2)*a^(1/4)*c^(1/4)*x/(a^(1/2)+c^(1/2)*x^2))*2^(1/2)/a^(
7/4)/c^(1/4)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.21

$$\int \frac{1}{(a+cx^4)^2}\,dx = \frac{\frac{8a^{3/4}x}{a+cx^4} - \frac{6\sqrt{2}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt[4]{c}} + \frac{6\sqrt{2}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt[4]{c}} - \frac{3\sqrt{2}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{\sqrt[4]{c}} + \frac{3\sqrt{2}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{\sqrt[4]{c}}}{32a^{7/4}}$$

input
```
Integrate[(a + c*x^4)^(-2),x]
```

output
```
((8*a^(3/4)*x)/(a + c*x^4) - (6*Sqrt[2]*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(
1/4)])/c^(1/4) + (6*Sqrt[2]*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/c^(1/
4) - (3*Sqrt[2]*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^
(1/4) + (3*Sqrt[2]*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])
/c^(1/4))/(32*a^(7/4))
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.49, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {749, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+cx^4)^2}\,dx$$

$$\Big\downarrow \text{ 749}$$

$$\frac{3\int \frac{1}{cx^4+a}dx}{4a} + \frac{x}{4a(a+cx^4)}$$

$$\Big\downarrow \text{ 755}$$

$$\frac{3\left(\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{a}}+\frac{\int \frac{\sqrt{c}x^2+\sqrt{a}}{cx^4+a}dx}{2\sqrt{a}}\right)}{4a}+\frac{x}{4a\left(a+cx^4\right)}$$

$$\Big\downarrow \text{1476}$$

$$\frac{3\left(\frac{\frac{\int \frac{1}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}+\frac{\int \frac{1}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}}{2\sqrt{a}}+\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{a}}\right)}{4a}+\frac{x}{4a\left(a+cx^4\right)}$$

$$\Big\downarrow \text{1082}$$

$$\frac{3\left(\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{a}}+\frac{\frac{\int \frac{1}{-\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)^2-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\int \frac{1}{-\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)^2-1}d\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}\right)}{4a}+\frac{x}{4a\left(a+cx^4\right)}$$

$$\Big\downarrow \text{217}$$

$$\frac{3\left(\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}\right)}{4a}+\frac{x}{4a\left(a+cx^4\right)}$$

$$\Big\downarrow \text{1479}$$

$$\frac{3\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{\sqrt[4]{c}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}\right)}{\sqrt[4]{c}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}\right)}{4a}+\frac{x}{4a\left(a+cx^4\right)}$$

$\Big\downarrow$ 25

$$\frac{3\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{\sqrt[4]{c}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}\right)}{\sqrt[4]{c}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}\right)}{4a}+\frac{x}{4a\left(a+cx^4\right)}$$

$\Big\downarrow$ 27

$$\frac{3\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt{c}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt[4]{a}\sqrt{c}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}\right)}{4a}+\frac{x}{4a\left(a+cx^4\right)}$$

$\Big\downarrow$ 1103

$$\frac{3\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}\right)}{4a}+\frac{x}{4a\left(a+cx^4\right)}$$

input
```
Int[(a + c*x^4)^(-2),x]
```

output
```
x/(4*a*(a + c*x^4)) + (3*((-(ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(Sqrt
[2]*a^(1/4)*c^(1/4))) + ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(Sqrt[2]*a
^(1/4)*c^(1/4)))/(2*Sqrt[a]) + (-1/2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)
*x + Sqrt[c]*x^2]/(Sqrt[2]*a^(1/4)*c^(1/4)) + Log[Sqrt[a] + Sqrt[2]*a^(1/4
)*c^(1/4)*x + Sqrt[c]*x^2]/(2*Sqrt[2]*a^(1/4)*c^(1/4)))/(2*Sqrt[a])))/(4*a
)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 749
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Simp[(n*(p + 1) + 1)/(a*n*(p + 1))   Int[(a + b*x^
n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Inte
gerQ[2*p] || Denominator[p + 1/n] < Denominator[p])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.16 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.30

method	result	size
risch	$\frac{x}{4a(c\,x^4+a)}+\frac{3\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+a\right)}\frac{\ln\left(x-\_R\right)}{\_R^3}\right)}{16ca}$	46
default	$\frac{x}{4a(c\,x^4+a)}+\frac{3\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{32a^2}$	118

input
```
int(1/(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*x/a/(c*x^4+a)+3/16/c/a*sum(1/_R^3*ln(x-_R),_R=RootOf(_Z^4*c+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.21

$$\int \frac{1}{(a+cx^4)^2}\,dx$$
$$= \frac{3\left(acx^4+a^2\right)\left(-\frac{1}{a^7c}\right)^{\frac{1}{4}}\log\left(a^2\left(-\frac{1}{a^7c}\right)^{\frac{1}{4}}+x\right)-3\left(-i\,acx^4-i\,a^2\right)\left(-\frac{1}{a^7c}\right)^{\frac{1}{4}}\log\left(i\,a^2\left(-\frac{1}{a^7c}\right)^{\frac{1}{4}}+x\right)-3\left(i\,ac\right.}{16\left(acx^4+\right.}$$

input
```
integrate(1/(c*x^4+a)^2,x, algorithm="fricas")
```

output
```
1/16*(3*(a*c*x^4 + a^2)*(-1/(a^7*c))^(1/4)*log(a^2*(-1/(a^7*c))^(1/4) + x)
 - 3*(-I*a*c*x^4 - I*a^2)*(-1/(a^7*c))^(1/4)*log(I*a^2*(-1/(a^7*c))^(1/4)
+ x) - 3*(I*a*c*x^4 + I*a^2)*(-1/(a^7*c))^(1/4)*log(-I*a^2*(-1/(a^7*c))^(1
/4) + x) - 3*(a*c*x^4 + a^2)*(-1/(a^7*c))^(1/4)*log(-a^2*(-1/(a^7*c))^(1/4
) + x) + 4*x)/(a*c*x^4 + a^2)
```

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.26

$$\int \frac{1}{(a+cx^4)^2}\,dx = \frac{x}{4a^2+4acx^4} + \operatorname{RootSum}\left(65536t^4a^7c+81, \left(t \mapsto t\log\left(\frac{16ta^2}{3}+x\right)\right)\right)$$

input
```
integrate(1/(c*x**4+a)**2,x)
```

output
```
x/(4*a**2 + 4*a*c*x**4) + RootSum(65536*_t**4*a**7*c + 81, Lambda(_t, _t*l
og(16*_t*a**2/3 + x)))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.25

$$\int \frac{1}{(a+cx^4)^2}\,dx = \frac{x}{4(acx^4+a^2)} + \frac{3\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}} + \frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}} + \frac{\sqrt{2}\log\left(\sqrt{c}x^2+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{1}{4}}} - \frac{\sqrt{2}\log\left(\sqrt{c}x^2 \cdots\right.}{\cdots}\right.}{32a}$$

input
```
integrate(1/(c*x^4+a)^2,x, algorithm="maxima")
```

output
```
1/4*x/(a*c*x^4 + a^2) + 3/32*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x +
sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt
(c))) + 2*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4
))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))) + sqrt(2)*log(sq
rt(c)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(1/4)) - sqrt(
2)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(1/4)
))/a
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.28

$$\int \frac{1}{(a+cx^4)^2}\,dx = \frac{x}{4\,(cx^4+a)a} + \frac{3\,\sqrt{2}(ac^3)^{\frac{1}{4}} \arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\,\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16\,a^2c} + \frac{3\,\sqrt{2}(ac^3)^{\frac{1}{4}} \arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\,\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16\,a^2c} + \frac{3\,\sqrt{2}(ac^3)^{\frac{1}{4}} \log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{32\,a^2c} - \frac{3\,\sqrt{2}(ac^3)^{\frac{1}{4}} \log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{32\,a^2c}$$

input
```
integrate(1/(c*x^4+a)^2,x, algorithm="giac")
```

output
```
1/4*x/((c*x^4 + a)*a) + 3/16*sqrt(2)*(a*c^3)^(1/4)*arctan(1/2*sqrt(2)*(2*x
 + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^2*c) + 3/16*sqrt(2)*(a*c^3)^(1/4)*
arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^2*c) + 3/32
*sqrt(2)*(a*c^3)^(1/4)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^2*c
) - 3/32*sqrt(2)*(a*c^3)^(1/4)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c)
)/(a^2*c)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.38

$$\int \frac{1}{(a+cx^4)^2}\,dx = \frac{x}{4\,a\,\left(c\,x^4+a\right)} + \frac{3\,\mathrm{atan}\left(\frac{c^{1/4}\,x}{(-a)^{1/4}}\right)}{8\,(-a)^{7/4}\,c^{1/4}} + \frac{3\,\mathrm{atanh}\left(\frac{c^{1/4}\,x}{(-a)^{1/4}}\right)}{8\,(-a)^{7/4}\,c^{1/4}}$$

input
```
int(1/(a + c*x^4)^2,x)
```

output
```
x/(4*a*(a + c*x^4)) + (3*atan((c^(1/4)*x)/(-a)^(1/4)))/(8*(-a)^(7/4)*c^(1/
4)) + (3*atanh((c^(1/4)*x)/(-a)^(1/4)))/(8*(-a)^(7/4)*c^(1/4))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 305, normalized size of antiderivative = 2.02

$$\int \frac{1}{(a+cx^4)^2}\,dx$$
$$= \frac{-6c^{\frac{3}{4}}a^{\frac{5}{4}}\sqrt{2}\,atan\left(\frac{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{c}\,x}{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)-6c^{\frac{7}{4}}a^{\frac{1}{4}}\sqrt{2}\,atan\left(\frac{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{c}\,x}{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)x^4+6c^{\frac{3}{4}}a^{\frac{5}{4}}\sqrt{2}\,atan\left(\frac{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}+2\sqrt{c}\,x}{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)+}{}$$

input
```
int(1/(c*x^4+a)^2,x)
```

output
```
( - 6*c**(3/4)*a**(1/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c
)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a - 6*c**(3/4)*a**(1/4)*sqrt(2)*atan((c*
*(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*c*x**4
 + 6*c**(3/4)*a**(1/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(c)
*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a + 6*c**(3/4)*a**(1/4)*sqrt(2)*atan((c**
(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*c*x**4
- 3*c**(3/4)*a**(1/4)*sqrt(2)*log( - c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a)
 + sqrt(c)*x**2)*a - 3*c**(3/4)*a**(1/4)*sqrt(2)*log( - c**(1/4)*a**(1/4)*
sqrt(2)*x + sqrt(a) + sqrt(c)*x**2)*c*x**4 + 3*c**(3/4)*a**(1/4)*sqrt(2)*l
og(c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c)*x**2)*a + 3*c**(3/4)*a*
*(1/4)*sqrt(2)*log(c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c)*x**2)*c
*x**4 + 8*a*c*x)/(32*a**2*c*(a + c*x**4))
```

3.183 $\int \frac{1}{(d+ex)(a+cx^4)^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 703

$$\int \frac{1}{(d+ex)\,(a+cx^4)^2}\,dx = \frac{e^3}{4\,(cd^4+ae^4)\,(a+cx^4)} + \frac{cx(d^3-d^2ex+de^2x^2)}{4a\,(cd^4+ae^4)\,(a+cx^4)} - \frac{\sqrt{c}d^2e^5\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\,(cd^4+ae^4)^2} - \frac{\sqrt{c}d^2e\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4a^{3/2}\,(cd^4+ae^4)} - \frac{\sqrt[4]{c}de^4(\sqrt{c}d^2+\sqrt{a}e^2)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\,(cd^4+ae^4)^2} - \frac{\sqrt[4]{c}d(3\sqrt{c}d^2+\sqrt{a}e^2)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\,(cd^4+ae^4)} + \frac{\sqrt[4]{c}de^4(\sqrt{c}d^2+\sqrt{a}e^2)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\,(cd^4+ae^4)^2} + \frac{\sqrt[4]{c}d(3\sqrt{c}d^2+\sqrt{a}e^2)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\,(cd^4+ae^4)} + \frac{\sqrt[4]{c}de^4(\sqrt{c}d^2-\sqrt{a}e^2)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x}{\sqrt{a}+\sqrt{c}x^2}\right)}{2\sqrt{2}a^{3/4}\,(cd^4+ae^4)^2} + \frac{\sqrt[4]{c}d(3\sqrt{c}d^2-\sqrt{a}e^2)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x}{\sqrt{a}+\sqrt{c}x^2}\right)}{8\sqrt{2}a^{7/4}\,(cd^4+ae^4)} + \frac{e^7\log(d+ex)}{(cd^4+ae^4)^2} - \frac{e^7\log(a+cx^4)}{4\,(cd^4+ae^4)^2}$$

output
```
1/4*e^3/(a*e^4+c*d^4)/(c*x^4+a)+1/4*c*x*(d*e^2*x^2-d^2*e*x+d^3)/a/(a*e^4+c
*d^4)/(c*x^4+a)-1/2*c^(1/2)*d^2*e^5*arctan(c^(1/2)*x^2/a^(1/2))/a^(1/2)/(a
*e^4+c*d^4)^2-1/4*c^(1/2)*d^2*e*arctan(c^(1/2)*x^2/a^(1/2))/a^(3/2)/(a*e^4
+c*d^4)+1/4*c^(1/4)*d*e^4*(c^(1/2)*d^2+a^(1/2)*e^2)*arctan(-1+2^(1/2)*c^(1
/4)*x/a^(1/4))*2^(1/2)/a^(3/4)/(a*e^4+c*d^4)^2+1/16*c^(1/4)*d*(3*c^(1/2)*d
^2+a^(1/2)*e^2)*arctan(-1+2^(1/2)*c^(1/4)*x/a^(1/4))*2^(1/2)/a^(7/4)/(a*e^
4+c*d^4)+1/4*c^(1/4)*d*e^4*(c^(1/2)*d^2+a^(1/2)*e^2)*arctan(1+2^(1/2)*c^(1
/4)*x/a^(1/4))*2^(1/2)/a^(3/4)/(a*e^4+c*d^4)^2+1/16*c^(1/4)*d*(3*c^(1/2)*d
^2+a^(1/2)*e^2)*arctan(1+2^(1/2)*c^(1/4)*x/a^(1/4))*2^(1/2)/a^(7/4)/(a*e^4
+c*d^4)+1/4*c^(1/4)*d*e^4*(c^(1/2)*d^2-a^(1/2)*e^2)*arctanh(2^(1/2)*a^(1/4
)*c^(1/4)*x/(a^(1/2)+c^(1/2)*x^2))*2^(1/2)/a^(3/4)/(a*e^4+c*d^4)^2+1/16*c^
(1/4)*d*(3*c^(1/2)*d^2-a^(1/2)*e^2)*arctanh(2^(1/2)*a^(1/4)*c^(1/4)*x/(a^(
1/2)+c^(1/2)*x^2))*2^(1/2)/a^(7/4)/(a*e^4+c*d^4)+e^7*ln(e*x+d)/(a*e^4+c*d^
4)^2-1/4*e^7*ln(c*x^4+a)/(a*e^4+c*d^4)^2
```

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 558, normalized size of antiderivative = 0.79

$$\int \frac{1}{(d+ex)(a+cx^4)^2}\,dx = \frac{\frac{8(cd^4+ae^4)(ae^3+cdx(d^2-dex+e^2x^2))}{a(a+cx^4)} - \frac{2\sqrt[4]{c}d\left(3\sqrt{2}c^{3/2}d^6-4\sqrt[4]{a}c^{5/4}d^5e+\sqrt{2}\sqrt{a}cd^4e^2+7\sqrt{2}a\sqrt{c}d^2e^4-12a^{5/4}\sqrt[4]{c}de^5+5\sqrt{2}a^{3/2}e^6\right)\text{arc}}{a^{7/4}}}{}$$

input
```
Integrate[1/((d + e*x)*(a + c*x^4)^2),x]
```

output
```
((8*(c*d^4 + a*e^4)*(a*e^3 + c*d*x*(d^2 - d*e*x + e^2*x^2)))/(a*(a + c*x^4
)) - (2*c^(1/4)*d*(3*Sqrt[2]*c^(3/2)*d^6 - 4*a^(1/4)*c^(5/4)*d^5*e + Sqrt[
2]*Sqrt[a]*c*d^4*e^2 + 7*Sqrt[2]*a*Sqrt[c]*d^2*e^4 - 12*a^(5/4)*c^(1/4)*d*
e^5 + 5*Sqrt[2]*a^(3/2)*e^6)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/a^(7
/4) + (2*c^(1/4)*d*(3*Sqrt[2]*c^(3/2)*d^6 + 4*a^(1/4)*c^(5/4)*d^5*e + Sqrt
[2]*Sqrt[a]*c*d^4*e^2 + 7*Sqrt[2]*a*Sqrt[c]*d^2*e^4 + 12*a^(5/4)*c^(1/4)*d
*e^5 + 5*Sqrt[2]*a^(3/2)*e^6)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/a^(
7/4) + 32*e^7*Log[d + e*x] + (Sqrt[2]*c^(1/4)*(-3*c^(3/2)*d^7 + Sqrt[a]*c*
d^5*e^2 - 7*a*Sqrt[c]*d^3*e^4 + 5*a^(3/2)*d*e^6)*Log[Sqrt[a] - Sqrt[2]*a^(
1/4)*c^(1/4)*x + Sqrt[c]*x^2])/a^(7/4) + (Sqrt[2]*c^(1/4)*(3*c^(3/2)*d^7 -
 Sqrt[a]*c*d^5*e^2 + 7*a*Sqrt[c]*d^3*e^4 - 5*a^(3/2)*d*e^6)*Log[Sqrt[a] +
Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/a^(7/4) - 8*e^7*Log[a + c*x^4])/
(32*(c*d^4 + a*e^4)^2)
```

Rubi [A] (verified)

Time = 1.92 (sec) , antiderivative size = 855, normalized size of antiderivative = 1.22, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+cx^4\right)^2(d+ex)}\,dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(\frac{e^8}{(d+ex)\left(ae^4+cd^4\right)^2}-\frac{ce^4\left(-d^3+d^2ex-de^2x^2+e^3x^3\right)}{\left(a+cx^4\right)\left(ae^4+cd^4\right)^2}+\frac{c\left(d^3-d^2ex+de^2x^2-e^3x^3\right)}{\left(a+cx^4\right)^2\left(ae^4+cd^4\right)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\log(d+ex)e^7}{(cd^4+ae^4)^2}-\frac{\log(cx^4+a)\,e^7}{4(cd^4+ae^4)^2}-\frac{\sqrt{c}d^2\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)e^5}{2\sqrt{a}\,(cd^4+ae^4)^2}-\frac{\sqrt[4]{c}d(\sqrt{c}d^2+\sqrt{a}e^2)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)e^4}{2\sqrt{2}a^{3/4}\,(cd^4+ae^4)^2}+\frac{\sqrt[4]{c}d(\sqrt{c}d^2+\sqrt{a}e^2)\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)e^4}{2\sqrt{2}a^{3/4}\,(cd^4+ae^4)^2}-\frac{\sqrt[4]{c}d(\sqrt{c}d^2-\sqrt{a}e^2)\log\left(\sqrt{c}x^2-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}\right)e^4}{4\sqrt{2}a^{3/4}\,(cd^4+ae^4)^2}+\frac{\sqrt[4]{c}d(\sqrt{c}d^2-\sqrt{a}e^2)\log\left(\sqrt{c}x^2+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}\right)e^4}{4\sqrt{2}a^{3/4}\,(cd^4+ae^4)^2}-\frac{\sqrt{c}d^2\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)e}{4a^{3/2}\,(cd^4+ae^4)}+\frac{ae^3+cx(d^3-exd^2+e^2x^2d)}{4a\,(cd^4+ae^4)\,(cx^4+a)}-\frac{\sqrt[4]{c}d(3\sqrt{c}d^2+\sqrt{a}e^2)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\,(cd^4+ae^4)}+\frac{\sqrt[4]{c}d(3\sqrt{c}d^2+\sqrt{a}e^2)\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{8\sqrt{2}a^{7/4}\,(cd^4+ae^4)}-\frac{\sqrt[4]{c}d(3\sqrt{c}d^2-\sqrt{a}e^2)\log\left(\sqrt{c}x^2-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}\right)}{16\sqrt{2}a^{7/4}\,(cd^4+ae^4)}+\frac{\sqrt[4]{c}d(3\sqrt{c}d^2-\sqrt{a}e^2)\log\left(\sqrt{c}x^2+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}\right)}{16\sqrt{2}a^{7/4}\,(cd^4+ae^4)}$$

input
```
Int[1/((d + e*x)*(a + c*x^4)^2),x]
```

output
```
(a*e^3 + c*x*(d^3 - d^2*e*x + d*e^2*x^2))/(4*a*(c*d^4 + a*e^4)*(a + c*x^4)
) - (Sqrt[c]*d^2*e^5*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*Sqrt[a]*(c*d^4 + a*
e^4)^2) - (Sqrt[c]*d^2*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(4*a^(3/2)*(c*d^4
+ a*e^4)) - (c^(1/4)*d*e^4*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*ArcTan[1 - (Sqrt[2]
*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^2) - (c^(1/4)*d*(
3*Sqrt[c]*d^2 + Sqrt[a]*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*S
qrt[2]*a^(7/4)*(c*d^4 + a*e^4)) + (c^(1/4)*d*e^4*(Sqrt[c]*d^2 + Sqrt[a]*e^
2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^4 + a*
e^4)^2) + (c^(1/4)*d*(3*Sqrt[c]*d^2 + Sqrt[a]*e^2)*ArcTan[1 + (Sqrt[2]*c^(
1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(c*d^4 + a*e^4)) + (e^7*Log[d + e*x])
/(c*d^4 + a*e^4)^2 - (c^(1/4)*d*e^4*(Sqrt[c]*d^2 - Sqrt[a]*e^2)*Log[Sqrt[a
] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^4 +
a*e^4)^2) - (c^(1/4)*d*(3*Sqrt[c]*d^2 - Sqrt[a]*e^2)*Log[Sqrt[a] - Sqrt[2]
*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*(c*d^4 + a*e^4)) +
(c^(1/4)*d*e^4*(Sqrt[c]*d^2 - Sqrt[a]*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c
^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^2) + (c^(1/4)*
d*(3*Sqrt[c]*d^2 - Sqrt[a]*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x +
Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*(c*d^4 + a*e^4)) - (e^7*Log[a + c*x^4])/
(4*(c*d^4 + a*e^4)^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 430, normalized size of antiderivative = 0.61

method	result
default	$\frac{c\left(\frac{\frac{d\,e^{2}\left(e^{4}a+c\,d^{4}\right)x^{3}}{4a}-\frac{d^{2}e\left(e^{4}a+c\,d^{4}\right)x^{2}}{4a}+\frac{d^{3}\left(e^{4}a+c\,d^{4}\right)x}{4a}+\frac{e^{3}\left(e^{4}a+c\,d^{4}\right)}{4c}}{c\,x^{4}+a}+\frac{\frac{\left(7a\,d^{3}e^{4}+3c\,d^{7}\right)\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^{2}+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^{2}-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\,\mathrm{arct}\right.}{8a}}{}\right.}{}$
risch	$\frac{\frac{cd\,e^{2}x^{3}}{4a\left(e^{4}a+c\,d^{4}\right)}-\frac{d^{2}ce\,x^{2}}{4a\left(e^{4}a+c\,d^{4}\right)}+\frac{d^{3}cx}{4a\left(e^{4}a+c\,d^{4}\right)}+\frac{e^{3}}{4e^{4}a+4c\,d^{4}}}{c\,x^{4}+a}+\frac{e^{7}\ln(ex+d)}{a^{2}e^{8}+2ac\,d^{4}e^{4}+d^{8}c^{2}}+\frac{\left(\_R=\mathrm{RootOf}\left(\left(a^{9}e^{8}+2a^{8}c\,d^{4}e^{4}+a^{7}c^{2}d^{8}\right)\_\right.\right.}{}$

input
```
int(1/(e*x+d)/(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
c/(a*e^4+c*d^4)^2*((1/4*d*e^2*(a*e^4+c*d^4)/a*x^3-1/4*d^2*e*(a*e^4+c*d^4)/
a*x^2+1/4*d^3*(a*e^4+c*d^4)/a*x+1/4*e^3*(a*e^4+c*d^4)/c)/(c*x^4+a)+1/4/a*(
1/8*(7*a*d^3*e^4+3*c*d^7)*(a/c)^(1/4)/a*2^(1/2)*(ln((x^2+(a/c)^(1/4)*x*2^(
1/2)+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+2*arctan(2^(1/2
)/(a/c)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/c)^(1/4)*x-1))+1/2*(-6*a*d^2*e^5-2*
c*d^6*e)/(a*c)^(1/2)*arctan((c/a)^(1/2)*x^2)+1/8*(5*a*d*e^6+c*d^5*e^2)/c/(
a/c)^(1/4)*2^(1/2)*(ln((x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2+(a/c)^
(1/4)*x*2^(1/2)+(a/c)^(1/2)))+2*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+2*arctan(2
^(1/2)/(a/c)^(1/4)*x-1))-a*e^7/c*ln(c*x^4+a)))+e^7*ln(e*x+d)/(a*e^4+c*d^4)
^2
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)\left(a+cx^4\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x+d)/(c*x^4+a)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)(a+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x+d)/(c*x**4+a)**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 601, normalized size of antiderivative = 0.85

$$\int \frac{1}{(d+ex)(a+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*x+d)/(c*x^4+a)^2,x, algorithm="maxima")
```

output
```
e^7*log(e*x + d)/(c^2*d^8 + 2*a*c*d^4*e^4 + a^2*e^8) - 1/32*c*(sqrt(2)*(4*
sqrt(2)*a^(7/4)*c^(1/4)*e^7 - 3*c^2*d^7 + sqrt(a)*c^(3/2)*d^5*e^2 - 7*a*c*
d^3*e^4 + 5*a^(3/2)*sqrt(c)*d*e^6)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c^(1/
4)*x + sqrt(a))/(a^(3/4)*c^(5/4)) + sqrt(2)*(4*sqrt(2)*a^(7/4)*c^(1/4)*e^7
 + 3*c^2*d^7 - sqrt(a)*c^(3/2)*d^5*e^2 + 7*a*c*d^3*e^4 - 5*a^(3/2)*sqrt(c)
*d*e^6)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^
(5/4)) - 2*(3*sqrt(2)*a^(1/4)*c^(9/4)*d^7 + sqrt(2)*a^(3/4)*c^(7/4)*d^5*e^
2 + 7*sqrt(2)*a^(5/4)*c^(5/4)*d^3*e^4 + 5*sqrt(2)*a^(7/4)*c^(3/4)*d*e^6 +
4*sqrt(a)*c^2*d^6*e + 12*a^(3/2)*c*d^2*e^5)*arctan(1/2*sqrt(2)*(2*sqrt(c)*
x + sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*
sqrt(c))*c^(5/4)) - 2*(3*sqrt(2)*a^(1/4)*c^(9/4)*d^7 + sqrt(2)*a^(3/4)*c^(
7/4)*d^5*e^2 + 7*sqrt(2)*a^(5/4)*c^(5/4)*d^3*e^4 + 5*sqrt(2)*a^(7/4)*c^(3/
4)*d*e^6 - 4*sqrt(a)*c^2*d^6*e - 12*a^(3/2)*c*d^2*e^5)*arctan(1/2*sqrt(2)*
(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sq
rt(sqrt(a)*sqrt(c))*c^(5/4)))/(a*c^2*d^8 + 2*a^2*c*d^4*e^4 + a^3*e^8) + 1/
4*(c*d*e^2*x^3 - c*d^2*e*x^2 + c*d^3*x + a*e^3)/(a^2*c*d^4 + a^3*e^4 + (a*
c^2*d^4 + a^2*c*e^4)*x^4)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 795, normalized size of antiderivative = 1.13

$$\int \frac{1}{(d+ex)(a+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*x+d)/(c*x^4+a)^2,x, algorithm="giac")
```

output
```
e^8*log(abs(e*x + d))/(c^2*d^8*e + 2*a*c*d^4*e^5 + a^2*e^9) - 1/4*e^7*log(
abs(c*x^4 + a))/(c^2*d^8 + 2*a*c*d^4*e^4 + a^2*e^8) + 1/8*(4*sqrt(2)*sqrt(
a*c)*c^2*d^2*e + 3*(a*c^3)^(1/4)*c^2*d^3 + 5*(a*c^3)^(3/4)*d*e^2)*arctan(1
/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a^2*c^3*d^4 +
 sqrt(2)*a^3*c^2*e^4 + 4*sqrt(2)*sqrt(a*c)*a^2*c^2*d^2*e^2 - 4*(a*c^3)^(1/
4)*a^2*c^2*d^3*e - 4*(a*c^3)^(3/4)*a^2*d*e^3) + 1/8*(4*sqrt(2)*sqrt(a*c)*c
^2*d^2*e + 3*(a*c^3)^(1/4)*c^2*d^3 + 5*(a*c^3)^(3/4)*d*e^2)*arctan(1/2*sqr
t(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a^2*c^3*d^4 + sqrt(
2)*a^3*c^2*e^4 + 4*sqrt(2)*sqrt(a*c)*a^2*c^2*d^2*e^2 + 4*(a*c^3)^(1/4)*a^2
*c^2*d^3*e + 4*(a*c^3)^(3/4)*a^2*d*e^3) + 1/32*(3*sqrt(2)*(a*c^3)^(1/4)*c^
3*d^7 + 7*sqrt(2)*(a*c^3)^(1/4)*a*c^2*d^3*e^4 - sqrt(2)*(a*c^3)^(3/4)*c*d^
5*e^2 - 5*sqrt(2)*(a*c^3)^(3/4)*a*d*e^6)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) +
 sqrt(a/c))/(a^2*c^4*d^8 + 2*a^3*c^3*d^4*e^4 + a^4*c^2*e^8) - 1/32*(3*sqrt
(2)*(a*c^3)^(1/4)*c^3*d^7 + 7*sqrt(2)*(a*c^3)^(1/4)*a*c^2*d^3*e^4 - sqrt(2
)*(a*c^3)^(3/4)*c*d^5*e^2 - 5*sqrt(2)*(a*c^3)^(3/4)*a*d*e^6)*log(x^2 - sqr
t(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^2*c^4*d^8 + 2*a^3*c^3*d^4*e^4 + a^4*c^2
*e^8) + 1/4*(a*c*d^4*e^3 + a^2*e^7 + (c^2*d^5*e^2 + a*c*d*e^6)*x^3 - (c^2*
d^6*e + a*c*d^2*e^5)*x^2 + (c^2*d^7 + a*c*d^3*e^4)*x)/((c*d^4 + a*e^4)^2*(
c*x^4 + a)*a)
```

Mupad [B] (verification not implemented)

Time = 22.28 (sec) , antiderivative size = 1591, normalized size of antiderivative = 2.26

$$\int \frac{1}{(d+ex)\left(a+cx^4\right)^2}\,dx = \text{Too large to display}$$

input
```
int(1/((a + c*x^4)^2*(d + e*x)),x)
```

output
```
e^3/(4*(a^2*e^4 + c^2*d^4*x^4 + a*c*d^4 + a*c*e^4*x^4)) + symsum(log((81*c
^5*d^5*e^6 + 64*a*c^4*d*e^10)/(256*(a^6*e^8 + a^4*c^2*d^8 + 2*a^5*c*d^4*e^
4)) + root(131072*a^8*c*d^4*e^4*z^4 + 65536*a^7*c^2*d^8*z^4 + 65536*a^9*e^
8*z^4 + 65536*a^7*e^7*z^3 + 5120*a^4*c*d^4*e^2*z^2 + 24576*a^5*e^6*z^2 + 1
152*a^2*c*d^4*e*z + 4096*a^3*e^5*z + 81*c*d^4 + 256*a*e^4, z, k)*(root(131
072*a^8*c*d^4*e^4*z^4 + 65536*a^7*c^2*d^8*z^4 + 65536*a^9*e^8*z^4 + 65536*
a^7*e^7*z^3 + 5120*a^4*c*d^4*e^2*z^2 + 24576*a^5*e^6*z^2 + 1152*a^2*c*d^4*
e*z + 4096*a^3*e^5*z + 81*c*d^4 + 256*a*e^4, z, k)*(root(131072*a^8*c*d^4*
e^4*z^4 + 65536*a^7*c^2*d^8*z^4 + 65536*a^9*e^8*z^4 + 65536*a^7*e^7*z^3 +
5120*a^4*c*d^4*e^2*z^2 + 24576*a^5*e^6*z^2 + 1152*a^2*c*d^4*e*z + 4096*a^3
*e^5*z + 81*c*d^4 + 256*a*e^4, z, k)*(root(131072*a^8*c*d^4*e^4*z^4 + 6553
6*a^7*c^2*d^8*z^4 + 65536*a^9*e^8*z^4 + 65536*a^7*e^7*z^3 + 5120*a^4*c*d^4
*e^2*z^2 + 24576*a^5*e^6*z^2 + 1152*a^2*c*d^4*e*z + 4096*a^3*e^5*z + 81*c*
d^4 + 256*a*e^4, z, k)*((98304*a^9*c^4*d*e^14 - 32768*a^6*c^7*d^13*e^2 + 3
2768*a^7*c^6*d^9*e^6 + 163840*a^8*c^5*d^5*e^10)/(256*(a^6*e^8 + a^4*c^2*d^
8 + 2*a^5*c*d^4*e^4)) + (x*(81920*a^9*c^4*e^15 - 49152*a^6*c^7*d^12*e^3 -
16384*a^7*c^6*d^8*e^7 + 114688*a^8*c^5*d^4*e^11))/(256*(a^6*e^8 + a^4*c^2*
d^8 + 2*a^5*c*d^4*e^4))) + (52224*a^7*c^4*d*e^13 - 3072*a^4*c^7*d^13*e + 1
3312*a^5*c^6*d^9*e^5 + 68608*a^6*c^5*d^5*e^9)/(256*(a^6*e^8 + a^4*c^2*d^8
+ 2*a^5*c*d^4*e^4)) + (x*(61440*a^7*c^4*e^14 - 8192*a^4*c^7*d^12*e^2 - ...
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 1946, normalized size of antiderivative = 2.77

$$\int \frac{1}{(d+ex)\left(a+cx^4\right)^2}\,dx = \text{Too large to display}$$

input
```
int(1/(e*x+d)/(c*x^4+a)^2,x)
```

output

```
( - 10*c**(1/4)*a**(3/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(
c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a**2*d*e**6 - 2*c**(1/4)*a**(3/4)*sqrt(
2)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(
2)))*a*c*d**5*e**2 - 10*c**(1/4)*a**(3/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*
sqrt(2) - 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a*c*d*e**6*x**4 - 2*c*
*(1/4)*a**(3/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x)/(c*
*(1/4)*a**(1/4)*sqrt(2)))*c**2*d**5*e**2*x**4 - 14*c**(3/4)*a**(1/4)*sqrt(
2)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(
2)))*a**2*d**3*e**4 - 6*c**(3/4)*a**(1/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*
sqrt(2) - 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a*c*d**7 - 14*c**(3/4)
*a**(1/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x)/(c**(1/4)
*a**(1/4)*sqrt(2)))*a*c*d**3*e**4*x**4 - 6*c**(3/4)*a**(1/4)*sqrt(2)*atan(
(c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*c**
2*d**7*x**4 + 24*sqrt(c)*sqrt(a)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(
c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a**2*d**2*e**5 + 8*sqrt(c)*sqrt(a)*atan
((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a*
c*d**6*e + 24*sqrt(c)*sqrt(a)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*
x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a*c*d**2*e**5*x**4 + 8*sqrt(c)*sqrt(a)*ata
n((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*c
**2*d**6*e*x**4 + 10*c**(1/4)*a**(3/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*...
```

3.184 $\int \frac{1}{(d+ex)^2(a+cx^4)^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 937

$$\int \frac{1}{(d+ex)^2\,(a+cx^4)^2}\,dx = \text{Too large to display}$$

output
```
-e^7/(a*e^4+c*d^4)^2/(e*x+d)+c*d^3*e^3/(a*e^4+c*d^4)^2/(c*x^4+a)+1/4*c*x*(
d^2*(-3*a*e^4+c*d^4)-2*d*e*(-a*e^4+c*d^4)*x+e^2*(-a*e^4+3*c*d^4)*x^2)/a/(a
*e^4+c*d^4)^2/(c*x^4+a)-c^(1/2)*d*e^5*(-a*e^4+3*c*d^4)*arctan(c^(1/2)*x^2/
a^(1/2))/a^(1/2)/(a*e^4+c*d^4)^3-1/2*c^(1/2)*d*e*(-a*e^4+c*d^4)*arctan(c^(
1/2)*x^2/a^(1/2))/a^(3/2)/(a*e^4+c*d^4)^2+1/16*c^(1/4)*(3*c^(1/2)*d^2*(-3*
a*e^4+c*d^4)+a^(1/2)*e^2*(-a*e^4+3*c*d^4))*arctan(-1+2^(1/2)*c^(1/4)*x/a^(
1/4))*2^(1/2)/a^(7/4)/(a*e^4+c*d^4)^2+1/4*c^(1/4)*e^4*(c^(1/2)*d^2*(-3*a*e
^4+5*c*d^4)+a^(1/2)*e^2*(-a*e^4+7*c*d^4))*arctan(-1+2^(1/2)*c^(1/4)*x/a^(1
/4))*2^(1/2)/a^(3/4)/(a*e^4+c*d^4)^3+1/16*c^(1/4)*(3*c^(1/2)*d^2*(-3*a*e^4
+c*d^4)+a^(1/2)*e^2*(-a*e^4+3*c*d^4))*arctan(1+2^(1/2)*c^(1/4)*x/a^(1/4))*
2^(1/2)/a^(7/4)/(a*e^4+c*d^4)^2+1/4*c^(1/4)*e^4*(c^(1/2)*d^2*(-3*a*e^4+5*c
*d^4)+a^(1/2)*e^2*(-a*e^4+7*c*d^4))*arctan(1+2^(1/2)*c^(1/4)*x/a^(1/4))*2^
(1/2)/a^(3/4)/(a*e^4+c*d^4)^3+1/16*c^(1/4)*(3*c^(1/2)*d^2*(-3*a*e^4+c*d^4)
-a^(1/2)*e^2*(-a*e^4+3*c*d^4))*arctanh(2^(1/2)*a^(1/4)*c^(1/4)*x/(a^(1/2)+
c^(1/2)*x^2))*2^(1/2)/a^(7/4)/(a*e^4+c*d^4)^2+1/4*c^(1/4)*e^4*(c^(1/2)*d^2
*(-3*a*e^4+5*c*d^4)-a^(1/2)*e^2*(-a*e^4+7*c*d^4))*arctanh(2^(1/2)*a^(1/4)*
c^(1/4)*x/(a^(1/2)+c^(1/2)*x^2))*2^(1/2)/a^(3/4)/(a*e^4+c*d^4)^3+8*c*d^3*e
^7*ln(e*x+d)/(a*e^4+c*d^4)^3-2*c*d^3*e^7*ln(c*x^4+a)/(a*e^4+c*d^4)^3
```

Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 807, normalized size of antiderivative = 0.86

$$\int \frac{1}{(d+ex)^2 (a+cx^4)^2} \, dx$$

$$= \frac{-\frac{32e^7(cd^4+ae^4)}{d+ex} + \frac{8c(cd^4+ae^4)(cd^4x(d^2-2dex+3e^2x^2)+ae^3(4d^3-3d^2ex+2de^2x^2-e^3x^3))}{a(a+cx^4)} + \frac{2\sqrt[4]{c}\left(-3\sqrt{2}c^{5/2}d^{10}+8\sqrt[4]{a}c^{9/4}d^9e-3\sqrt{}\right.}{}}{}$$

input
```
Integrate[1/((d + e*x)^2*(a + c*x^4)^2),x]
```

output
```
((-32*e^7*(c*d^4 + a*e^4))/(d + e*x) + (8*c*(c*d^4 + a*e^4)*(c*d^4*x*(d^2
- 2*d*e*x + 3*e^2*x^2) + a*e^3*(4*d^3 - 3*d^2*e*x + 2*d*e^2*x^2 - e^3*x^3)
))/(a*(a + c*x^4)) + (2*c^(1/4)*(-3*Sqrt[2]*c^(5/2)*d^10 + 8*a^(1/4)*c^(9/
4)*d^9*e - 3*Sqrt[2]*Sqrt[a]*c^2*d^8*e^2 - 14*Sqrt[2]*a*c^(3/2)*d^6*e^4 +
48*a^(5/4)*c^(5/4)*d^5*e^5 - 30*Sqrt[2]*a^(3/2)*c*d^4*e^6 + 21*Sqrt[2]*a^2
*Sqrt[c]*d^2*e^8 - 24*a^(9/4)*c^(1/4)*d*e^9 + 5*Sqrt[2]*a^(5/2)*e^10)*ArcT
an[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/a^(7/4) + (2*c^(1/4)*(3*Sqrt[2]*c^(5/
2)*d^10 + 8*a^(1/4)*c^(9/4)*d^9*e + 3*Sqrt[2]*Sqrt[a]*c^2*d^8*e^2 + 14*Sqr
t[2]*a*c^(3/2)*d^6*e^4 + 48*a^(5/4)*c^(5/4)*d^5*e^5 + 30*Sqrt[2]*a^(3/2)*c
*d^4*e^6 - 21*Sqrt[2]*a^2*Sqrt[c]*d^2*e^8 - 24*a^(9/4)*c^(1/4)*d*e^9 - 5*S
qrt[2]*a^(5/2)*e^10)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/a^(7/4) + 25
6*c*d^3*e^7*Log[d + e*x] - (Sqrt[2]*c^(1/4)*(3*c^(5/2)*d^10 - 3*Sqrt[a]*c^
2*d^8*e^2 + 14*a*c^(3/2)*d^6*e^4 - 30*a^(3/2)*c*d^4*e^6 - 21*a^2*Sqrt[c]*d
^2*e^8 + 5*a^(5/2)*e^10)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]
*x^2])/a^(7/4) + (Sqrt[2]*c^(1/4)*(3*c^(5/2)*d^10 - 3*Sqrt[a]*c^2*d^8*e^2
+ 14*a*c^(3/2)*d^6*e^4 - 30*a^(3/2)*c*d^4*e^6 - 21*a^2*Sqrt[c]*d^2*e^8 + 5
*a^(5/2)*e^10)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/a^(
7/4) - 64*c*d^3*e^7*Log[a + c*x^4])/(32*(c*d^4 + a*e^4)^3)
```

Rubi [A] (verified)

Time = 3.11 (sec) , antiderivative size = 1141, normalized size of antiderivative = 1.22, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+cx^4)^2 (d+ex)^2} \, dx$$

$$\Big\downarrow \; 7293$$

$$\int \left(\frac{e^8}{(d+ex)^2 (ae^4+cd^4)^2} + \frac{8cd^3e^8}{(d+ex)(ae^4+cd^4)^3} + \frac{ce^4\left(-2dex\left(3cd^4-ae^4\right)+e^2x^2\left(7cd^4-ae^4\right)+d^2\left(5cd^4-\right.\right.}{(a+cx^4)(ae^4+cd^4)^3}\right.$$

$$\Big\downarrow \; 2009$$

$$\frac{8cd^3\log(d+ex)e^7}{(cd^4+ae^4)^3}-\frac{2cd^3\log\left(cx^4+a\right)e^7}{(cd^4+ae^4)^3}-\frac{e^7}{(cd^4+ae^4)^2\,(d+ex)}-\frac{\sqrt{c}d\left(3cd^4-ae^4\right)\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)e^5}{\sqrt{a}\,(cd^4+ae^4)^3}-\frac{\sqrt[4]{c}\left(\sqrt{c}\left(5cd^4-3ae^4\right)d^2+\sqrt{a}e^2\left(7cd^4-ae^4\right)\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)e^4}{2\sqrt{2}a^{3/4}\,(cd^4+ae^4)^3}+\frac{\sqrt[4]{c}\left(\sqrt{c}\left(5cd^4-3ae^4\right)d^2+\sqrt{a}e^2\left(7cd^4-ae^4\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)e^4}{2\sqrt{2}a^{3/4}\,(cd^4+ae^4)^3}-\frac{\sqrt[4]{c}\left(\sqrt{c}d^2\left(5cd^4-3ae^4\right)-\sqrt{a}e^2\left(7cd^4-ae^4\right)\right)\log\left(\sqrt{c}x^2-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}\right)e^4}{4\sqrt{2}a^{3/4}\,(cd^4+ae^4)^3}+\frac{\sqrt[4]{c}\left(\sqrt{c}d^2\left(5cd^4-3ae^4\right)-\sqrt{a}e^2\left(7cd^4-ae^4\right)\right)\log\left(\sqrt{c}x^2+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}\right)e^4}{4\sqrt{2}a^{3/4}\,(cd^4+ae^4)^3}-\frac{\sqrt{c}d\left(cd^4-ae^4\right)\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)e}{2a^{3/2}\,(cd^4+ae^4)^2}+\frac{c\left(4ad^3e^3+x\left(\left(cd^4-3ae^4\right)d^2-2e\left(cd^4-ae^4\right)xd+e^2\left(3cd^4-ae^4\right)x^2\right)\right)}{4a\,(cd^4+ae^4)^2\,(cx^4+a)}-\frac{\sqrt[4]{c}\left(3\sqrt{c}\left(cd^4-3ae^4\right)d^2+\sqrt{a}e^2\left(3cd^4-ae^4\right)\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\,(cd^4+ae^4)^2}+\frac{\sqrt[4]{c}\left(3\sqrt{c}\left(cd^4-3ae^4\right)d^2+\sqrt{a}e^2\left(3cd^4-ae^4\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{8\sqrt{2}a^{7/4}\,(cd^4+ae^4)^2}-\frac{\sqrt[4]{c}\left(3\sqrt{c}d^2\left(cd^4-3ae^4\right)-\sqrt{a}e^2\left(3cd^4-ae^4\right)\right)\log\left(\sqrt{c}x^2-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}\right)}{16\sqrt{2}a^{7/4}\,(cd^4+ae^4)^2}+\frac{\sqrt[4]{c}\left(3\sqrt{c}d^2\left(cd^4-3ae^4\right)-\sqrt{a}e^2\left(3cd^4-ae^4\right)\right)\log\left(\sqrt{c}x^2+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}\right)}{16\sqrt{2}a^{7/4}\,(cd^4+ae^4)^2}$$

input
```
Int[1/((d + e*x)^2*(a + c*x^4)^2),x]
```

output
```
-(e^7/((c*d^4 + a*e^4)^2*(d + e*x))) + (c*(4*a*d^3*e^3 + x*(d^2*(c*d^4 - 3
*a*e^4) - 2*d*e*(c*d^4 - a*e^4)*x + e^2*(3*c*d^4 - a*e^4)*x^2)))/(4*a*(c*d
^4 + a*e^4)^2*(a + c*x^4)) - (Sqrt[c]*d*e^5*(3*c*d^4 - a*e^4)*ArcTan[(Sqrt
[c]*x^2)/Sqrt[a]])/(Sqrt[a]*(c*d^4 + a*e^4)^3) - (Sqrt[c]*d*e*(c*d^4 - a*e
^4)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*a^(3/2)*(c*d^4 + a*e^4)^2) - (c^(1/4
)*(3*Sqrt[c]*d^2*(c*d^4 - 3*a*e^4) + Sqrt[a]*e^2*(3*c*d^4 - a*e^4))*ArcTan
[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(c*d^4 + a*e^4)^2) -
 (c^(1/4)*e^4*(Sqrt[c]*d^2*(5*c*d^4 - 3*a*e^4) + Sqrt[a]*e^2*(7*c*d^4 - a*
e^4))*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^4 +
 a*e^4)^3) + (c^(1/4)*(3*Sqrt[c]*d^2*(c*d^4 - 3*a*e^4) + Sqrt[a]*e^2*(3*c*
d^4 - a*e^4))*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*
(c*d^4 + a*e^4)^2) + (c^(1/4)*e^4*(Sqrt[c]*d^2*(5*c*d^4 - 3*a*e^4) + Sqrt[
a]*e^2*(7*c*d^4 - a*e^4))*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt
[2]*a^(3/4)*(c*d^4 + a*e^4)^3) + (8*c*d^3*e^7*Log[d + e*x])/(c*d^4 + a*e^4
)^3 - (c^(1/4)*(3*Sqrt[c]*d^2*(c*d^4 - 3*a*e^4) - Sqrt[a]*e^2*(3*c*d^4 - a
*e^4))*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]
*a^(7/4)*(c*d^4 + a*e^4)^2) - (c^(1/4)*e^4*(Sqrt[c]*d^2*(5*c*d^4 - 3*a*e^4
) - Sqrt[a]*e^2*(7*c*d^4 - a*e^4))*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x
 + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^3) + (c^(1/4)*(3*Sqrt[
c]*d^2*(c*d^4 - 3*a*e^4) - Sqrt[a]*e^2*(3*c*d^4 - a*e^4))*Log[Sqrt[a] +...
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 534, normalized size of antiderivative = 0.57

method	result
default	$-\frac{c\left(\frac{\frac{e^2\left(a^2e^8-2ac\,d^4e^4-3d^8c^2\right)x^3}{4a}-\frac{de\left(a^2e^8-d^8c^2\right)x^2}{2a}+\frac{d^2\left(3a^2e^8+2ac\,d^4e^4-d^8c^2\right)x}{4a}-d^3e^3\left(e^4a+c\,d^4\right)}{c\,x^4+a}+\frac{\left(21a^2e^8d^2-14ac\,d^6e^4-3c^2d^{10}\right)\cdots}{\cdots}\right.}{\cdots}$
risch	$\frac{-\frac{c\,e^3\left(5e^4a-3c\,d^4\right)x^4}{4a\left(e^4a+c\,d^4\right)^2}+\frac{cd\,e^2x^3}{4a\left(e^4a+c\,d^4\right)}-\frac{d^2ce\,x^2}{4a\left(e^4a+c\,d^4\right)}+\frac{d^3cx}{4a\left(e^4a+c\,d^4\right)}-\frac{e^3\left(e^4a-c\,d^4\right)}{\left(e^4a+c\,d^4\right)^2}}{(ex+d)(c\,x^4+a)}+\frac{\left(\_R=\mathrm{RootOf}\left(\left(a^{10}e^{12}+3c\,d^4a^9e^8+3d^8c^2a^8e\cdots\right.\right.\right.}{\cdots}$

input
```
int(1/(e*x+d)^2/(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-c/(a*e^4+c*d^4)^3*((1/4*e^2*(a^2*e^8-2*a*c*d^4*e^4-3*c^2*d^8)/a*x^3-1/2*d
*e*(a^2*e^8-c^2*d^8)/a*x^2+1/4*d^2*(3*a^2*e^8+2*a*c*d^4*e^4-c^2*d^8)/a*x-d
^3*e^3*(a*e^4+c*d^4))/(c*x^4+a)+1/4/a*(1/8*(21*a^2*d^2*e^8-14*a*c*d^6*e^4-
3*c^2*d^10)*(a/c)^(1/4)/a*2^(1/2)*(ln((x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/
2))/(x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+2*arctan(2^(1/2)/(a/c)^(1/4)*
x+1)+2*arctan(2^(1/2)/(a/c)^(1/4)*x-1))+1/2*(-12*a^2*d*e^9+24*a*c*d^5*e^5+
4*c^2*d^9*e)/(a*c)^(1/2)*arctan((c/a)^(1/2)*x^2)+1/8*(5*a^2*e^10-30*a*c*d^
4*e^6-3*c^2*d^8*e^2)/c/(a/c)^(1/4)*2^(1/2)*(ln((x^2-(a/c)^(1/4)*x*2^(1/2)+
(a/c)^(1/2))/(x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+2*arctan(2^(1/2)/(a/
c)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/c)^(1/4)*x-1))+8*a*d^3*e^7*ln(c*x^4+a)))
-e^7/(a*e^4+c*d^4)^2/(e*x+d)+8*c*d^3*e^7*ln(e*x+d)/(a*e^4+c*d^4)^3
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^2\left(a+cx^4\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x+d)^2/(c*x^4+a)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^2 (a+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x+d)**2/(c*x**4+a)**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 961, normalized size of antiderivative = 1.03

$$\int \frac{1}{(d+ex)^2 (a+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*x+d)^2/(c*x^4+a)^2,x, algorithm="maxima")
```

output
```
8*c*d^3*e^7*log(e*x + d)/(c^3*d^12 + 3*a*c^2*d^8*e^4 + 3*a^2*c*d^4*e^8 + a
^3*e^12) - 1/32*c*(sqrt(2)*(32*sqrt(2)*a^(7/4)*c^(5/4)*d^3*e^7 - 3*c^3*d^1
0 + 3*sqrt(a)*c^(5/2)*d^8*e^2 - 14*a*c^2*d^6*e^4 + 30*a^(3/2)*c^(3/2)*d^4*
e^6 + 21*a^2*c*d^2*e^8 - 5*a^(5/2)*sqrt(c)*e^10)*log(sqrt(c)*x^2 + sqrt(2)
*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(5/4)) + sqrt(2)*(32*sqrt(2)*a^(7
/4)*c^(5/4)*d^3*e^7 + 3*c^3*d^10 - 3*sqrt(a)*c^(5/2)*d^8*e^2 + 14*a*c^2*d^
6*e^4 - 30*a^(3/2)*c^(3/2)*d^4*e^6 - 21*a^2*c*d^2*e^8 + 5*a^(5/2)*sqrt(c)*
e^10)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(5
/4)) - 2*(3*sqrt(2)*a^(1/4)*c^(13/4)*d^10 + 3*sqrt(2)*a^(3/4)*c^(11/4)*d^8
*e^2 + 14*sqrt(2)*a^(5/4)*c^(9/4)*d^6*e^4 + 30*sqrt(2)*a^(7/4)*c^(7/4)*d^4
*e^6 - 21*sqrt(2)*a^(9/4)*c^(5/4)*d^2*e^8 - 5*sqrt(2)*a^(11/4)*c^(3/4)*e^1
0 + 8*sqrt(a)*c^3*d^9*e + 48*a^(3/2)*c^2*d^5*e^5 - 24*a^(5/2)*c*d*e^9)*arc
tan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(
c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(5/4)) - 2*(3*sqrt(2)*a^(1/4)*c^(13/
4)*d^10 + 3*sqrt(2)*a^(3/4)*c^(11/4)*d^8*e^2 + 14*sqrt(2)*a^(5/4)*c^(9/4)*
d^6*e^4 + 30*sqrt(2)*a^(7/4)*c^(7/4)*d^4*e^6 - 21*sqrt(2)*a^(9/4)*c^(5/4)*
d^2*e^8 - 5*sqrt(2)*a^(11/4)*c^(3/4)*e^10 - 8*sqrt(a)*c^3*d^9*e - 48*a^(3/
2)*c^2*d^5*e^5 + 24*a^(5/2)*c*d*e^9)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqr
t(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c)
)*c^(5/4)))/(a*c^3*d^12 + 3*a^2*c^2*d^8*e^4 + 3*a^3*c*d^4*e^8 + a^4*e^1...
```

Giac [A] (verification not implemented)

Time = 4.34 (sec) , antiderivative size = 1145, normalized size of antiderivative = 1.22

$$\int \frac{1}{(d+ex)^2 (a+cx^4)^2} \, dx = \text{Too large to display}$$

input
```
integrate(1/(e*x+d)^2/(c*x^4+a)^2,x, algorithm="giac")
```

output
```
8*c*d^3*e^8*log(abs(e*x + d))/(c^3*d^12*e + 3*a*c^2*d^8*e^5 + 3*a^2*c*d^4*
e^9 + a^3*e^13) - 2*c*d^3*e^7*log(abs(c*x^4 + a))/(c^3*d^12 + 3*a*c^2*d^8*
e^4 + 3*a^2*c*d^4*e^8 + a^3*e^12) + 1/8*(3*sqrt(2)*a*c^2*d*e^3 + 5*sqrt(2)
*sqrt(a*c)*c^2*d^3*e + 3*(a*c^3)^(1/4)*c^2*d^4 - 5*(a*c^3)^(1/4)*a*c*e^4 +
 6*(a*c^3)^(3/4)*d^2*e^2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(
a/c)^(1/4))/(sqrt(2)*a^2*c^3*d^6 + 9*sqrt(2)*a^3*c^2*d^2*e^4 + 9*sqrt(2)*s
qrt(a*c)*a^2*c^2*d^4*e^2 + sqrt(2)*sqrt(a*c)*a^3*c*e^6 - 6*(a*c^3)^(1/4)*a
^2*c^2*d^5*e - 6*(a*c^3)^(1/4)*a^3*c*d*e^5 - 16*(a*c^3)^(3/4)*a^2*d^3*e^3)
 - 1/8*(3*sqrt(2)*a*c^2*d*e^3 - 5*sqrt(2)*sqrt(a*c)*c^2*d^3*e - 3*(a*c^3)^
(1/4)*c^2*d^4 + 5*(a*c^3)^(1/4)*a*c*e^4 - 6*(a*c^3)^(3/4)*d^2*e^2)*arctan(
1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a^2*c^3*d^6
+ 9*sqrt(2)*a^3*c^2*d^2*e^4 + 9*sqrt(2)*sqrt(a*c)*a^2*c^2*d^4*e^2 + sqrt(2
)*sqrt(a*c)*a^3*c*e^6 + 6*(a*c^3)^(1/4)*a^2*c^2*d^5*e + 6*(a*c^3)^(1/4)*a^
3*c*d*e^5 + 16*(a*c^3)^(3/4)*a^2*d^3*e^3) + 1/32*(3*sqrt(2)*(a*c^3)^(1/4)*
c^4*d^10 + 14*sqrt(2)*(a*c^3)^(1/4)*a*c^3*d^6*e^4 - 21*sqrt(2)*(a*c^3)^(1/
4)*a^2*c^2*d^2*e^8 - 3*sqrt(2)*(a*c^3)^(3/4)*c^2*d^8*e^2 - 30*sqrt(2)*(a*c
^3)^(3/4)*a*c*d^4*e^6 + 5*sqrt(2)*(a*c^3)^(3/4)*a^2*e^10)*log(x^2 + sqrt(2
)*x*(a/c)^(1/4) + sqrt(a/c))/(a^2*c^5*d^12 + 3*a^3*c^4*d^8*e^4 + 3*a^4*c^3
*d^4*e^8 + a^5*c^2*e^12) - 1/32*(3*sqrt(2)*(a*c^3)^(1/4)*c^4*d^10 + 14*sqr
t(2)*(a*c^3)^(1/4)*a*c^3*d^6*e^4 - 21*sqrt(2)*(a*c^3)^(1/4)*a^2*c^2*d^2...
```

Mupad [B] (verification not implemented)

Time = 23.38 (sec) , antiderivative size = 2246, normalized size of antiderivative = 2.40

$$\int \frac{1}{(d+ex)^2 (a+cx^4)^2} dx = \text{Too large to display}$$

input
```
int(1/((a + c*x^4)^2*(d + e*x)^2),x)
```

output
```
symsum(log(root(196608*a^9*c*d^4*e^8*z^4 + 196608*a^8*c^2*d^8*e^4*z^4 + 65
536*a^7*c^3*d^12*z^4 + 65536*a^10*e^12*z^4 + 524288*a^7*c*d^3*e^7*z^3 + 18
1248*a^5*c*d^2*e^6*z^2 + 17408*a^4*c^2*d^6*e^2*z^2 + 2304*a^2*c^2*d^5*e*z
+ 19200*a^3*c*d*e^5*z + 625*a*c*e^4 + 81*c^2*d^4, z, k)*((120*a*c^8*d^14*e
^3 + 2664*a^2*c^7*d^10*e^7 - 10904*a^3*c^6*d^6*e^11 + 19320*a^4*c^5*d^2*e^
15)/(256*(a^8*e^16 + a^4*c^4*d^16 + 4*a^7*c*d^4*e^12 + 4*a^5*c^3*d^12*e^4
+ 6*a^6*c^2*d^8*e^8)) + root(196608*a^9*c*d^4*e^8*z^4 + 196608*a^8*c^2*d^8
*e^4*z^4 + 65536*a^7*c^3*d^12*z^4 + 65536*a^10*e^12*z^4 + 524288*a^7*c*d^3
*e^7*z^3 + 181248*a^5*c*d^2*e^6*z^2 + 17408*a^4*c^2*d^6*e^2*z^2 + 2304*a^2
*c^2*d^5*e*z + 19200*a^3*c*d*e^5*z + 625*a*c*e^4 + 81*c^2*d^4, z, k)*((409
6*a^3*c^8*d^15*e^4 + 54272*a^4*c^7*d^11*e^8 - 2048*a^5*c^6*d^7*e^12 + 1443
84*a^6*c^5*d^3*e^16)/(256*(a^8*e^16 + a^4*c^4*d^16 + 4*a^7*c*d^4*e^12 + 4*
a^5*c^3*d^12*e^4 + 6*a^6*c^2*d^8*e^8)) + root(196608*a^9*c*d^4*e^8*z^4 + 1
96608*a^8*c^2*d^8*e^4*z^4 + 65536*a^7*c^3*d^12*z^4 + 65536*a^10*e^12*z^4 +
 524288*a^7*c*d^3*e^7*z^3 + 181248*a^5*c*d^2*e^6*z^2 + 17408*a^4*c^2*d^6*e
^2*z^2 + 2304*a^2*c^2*d^5*e*z + 19200*a^3*c*d*e^5*z + 625*a*c*e^4 + 81*c^2
*d^4, z, k)*(root(196608*a^9*c*d^4*e^8*z^4 + 196608*a^8*c^2*d^8*e^4*z^4 +
65536*a^7*c^3*d^12*z^4 + 65536*a^10*e^12*z^4 + 524288*a^7*c*d^3*e^7*z^3 +
181248*a^5*c*d^2*e^6*z^2 + 17408*a^4*c^2*d^6*e^2*z^2 + 2304*a^2*c^2*d^5*e*
z + 19200*a^3*c*d*e^5*z + 625*a*c*e^4 + 81*c^2*d^4, z, k)*((98304*a^11*...
```

Reduce [B] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 5777, normalized size of antiderivative = 6.17

$$\int \frac{1}{(d+ex)^2 (a+cx^4)^2} \, dx = \text{Too large to display}$$

input
```
int(1/(e*x+d)^2/(c*x^4+a)^2,x)
```

output

```
(10*c**(1/4)*a**(3/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*
x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a**3*d**2*e**10 + 10*c**(1/4)*a**(3/4)*sqr
t(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqr
t(2)))*a**3*d*e**11*x - 60*c**(1/4)*a**(3/4)*sqrt(2)*atan((c**(1/4)*a**(1/
4)*sqrt(2) - 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a**2*c*d**6*e**6 -
60*c**(1/4)*a**(3/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x
)/(c**(1/4)*a**(1/4)*sqrt(2)))*a**2*c*d**5*e**7*x + 10*c**(1/4)*a**(3/4)*s
qrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*s
qrt(2)))*a**2*c*d**2*e**10*x**4 + 10*c**(1/4)*a**(3/4)*sqrt(2)*atan((c**(1
/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a**2*c*d*
e**11*x**5 - 6*c**(1/4)*a**(3/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) -
 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a*c**2*d**10*e**2 - 6*c**(1/4)*
a**(3/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x)/(c**(1/4)*
a**(1/4)*sqrt(2)))*a*c**2*d**9*e**3*x - 60*c**(1/4)*a**(3/4)*sqrt(2)*atan(
(c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a*c
**2*d**6*e**6*x**4 - 60*c**(1/4)*a**(3/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*
sqrt(2) - 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a*c**2*d**5*e**7*x**5
- 6*c**(1/4)*a**(3/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*
x)/(c**(1/4)*a**(1/4)*sqrt(2)))*c**3*d**10*e**2*x**4 - 6*c**(1/4)*a**(3/4)
*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x)/(c**(1/4)*a**(1...
```

3.185 $\int \frac{(d+ex)^3}{(a+cx^4)^3}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 328

$$\begin{aligned}\int \frac{(d+ex)^3}{(a+cx^4)^3}\,dx = & -\frac{e^3}{8c\,(a+cx^4)^2}+\frac{x(d^3+3d^2ex+3de^2x^2)}{8a\,(a+cx^4)^2} \\ & +\frac{x(7d^3+18d^2ex+15de^2x^2)}{32a^2\,(a+cx^4)}+\frac{9d^2e\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{c}} \\ & -\frac{3d\left(7\sqrt{c}d^2+5\sqrt{a}e^2\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}c^{3/4}} \\ & +\frac{3d\left(7\sqrt{c}d^2+5\sqrt{a}e^2\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}c^{3/4}} \\ & +\frac{3d\left(7\sqrt{c}d^2-5\sqrt{a}e^2\right)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x}{\sqrt{a}+\sqrt{c}x^2}\right)}{64\sqrt{2}a^{11/4}c^{3/4}}\end{aligned}$$

output
```
-1/8*e^3/c/(c*x^4+a)^2+1/8*x*(3*d*e^2*x^2+3*d^2*e*x+d^3)/a/(c*x^4+a)^2+1/3
2*x*(15*d*e^2*x^2+18*d^2*e*x+7*d^3)/a^2/(c*x^4+a)+9/16*d^2*e*arctan(c^(1/2
)*x^2/a^(1/2))/a^(5/2)/c^(1/2)+3/128*d*(7*c^(1/2)*d^2+5*a^(1/2)*e^2)*arcta
n(-1+2^(1/2)*c^(1/4)*x/a^(1/4))*2^(1/2)/a^(11/4)/c^(3/4)+3/128*d*(7*c^(1/2
)*d^2+5*a^(1/2)*e^2)*arctan(1+2^(1/2)*c^(1/4)*x/a^(1/4))*2^(1/2)/a^(11/4)/
c^(3/4)+3/128*d*(7*c^(1/2)*d^2-5*a^(1/2)*e^2)*arctanh(2^(1/2)*a^(1/4)*c^(1
/4)*x/(a^(1/2)+c^(1/2)*x^2))*2^(1/2)/a^(11/4)/c^(3/4)
```

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 388, normalized size of antiderivative = 1.18

$$\int \frac{(d+ex)^3}{(a+cx^4)^3}\,dx$$
$$= \frac{\frac{8adx(7d^2+18dex+15e^2x^2)}{a+cx^4} - \frac{32a^2(ae^3-cdx(d^2+3dex+3e^2x^2))}{c(a+cx^4)^2} - \frac{6\sqrt[4]{a}d\left(7\sqrt{2}\sqrt{c}d^2+24\sqrt[4]{a}\sqrt[4]{c}de+5\sqrt{2}\sqrt{a}e^2\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{c^{3/4}}}{}$$

input
```
Integrate[(d + e*x)^3/(a + c*x^4)^3,x]
```

output
```
((8*a*d*x*(7*d^2 + 18*d*e*x + 15*e^2*x^2))/(a + c*x^4) - (32*a^2*(a*e^3 -
c*d*x*(d^2 + 3*d*e*x + 3*e^2*x^2)))/(c*(a + c*x^4)^2) - (6*a^(1/4)*d*(7*Sq
rt[2]*Sqrt[c]*d^2 + 24*a^(1/4)*c^(1/4)*d*e + 5*Sqrt[2]*Sqrt[a]*e^2)*ArcTan
[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/c^(3/4) + (6*a^(1/4)*d*(7*Sqrt[2]*Sqrt[
c]*d^2 - 24*a^(1/4)*c^(1/4)*d*e + 5*Sqrt[2]*Sqrt[a]*e^2)*ArcTan[1 + (Sqrt[
2]*c^(1/4)*x)/a^(1/4)])/c^(3/4) + (3*Sqrt[2]*(-7*a^(1/4)*Sqrt[c]*d^3 + 5*a
^(3/4)*d*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^(3
/4) + (3*Sqrt[2]*(7*a^(1/4)*Sqrt[c]*d^3 - 5*a^(3/4)*d*e^2)*Log[Sqrt[a] + S
qrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^(3/4))/(256*a^3)
```

Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 408, normalized size of antiderivative = 1.24, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {2393, 25, 2394, 27, 2415, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^3}{(a+cx^4)^3}dx$$

$$\Big\downarrow \text{2393}$$

$$-\frac{\int -\frac{7d^3+18exd^2+15e^2x^2d}{(cx^4+a)^2}dx}{8a}-\frac{ae^3-cx(d^3+3d^2ex+3de^2x^2)}{8ac(a+cx^4)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{7d^3+18exd^2+15e^2x^2d}{(cx^4+a)^2}dx}{8a}-\frac{ae^3-cx(d^3+3d^2ex+3de^2x^2)}{8ac(a+cx^4)^2}$$

$$\Big\downarrow \text{2394}$$

$$\frac{\frac{x(7d^3+18d^2ex+15de^2x^2)}{4a(a+cx^4)}-\frac{\int -\frac{3(7d^3+12exd^2+5e^2x^2d)}{cx^4+a}dx}{4a}}{8a}-\frac{ae^3-cx(d^3+3d^2ex+3de^2x^2)}{8ac(a+cx^4)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{3\int \frac{7d^3+12exd^2+5e^2x^2d}{cx^4+a}dx}{4a}+\frac{x(7d^3+18d^2ex+15de^2x^2)}{4a(a+cx^4)}}{8a}-\frac{ae^3-cx(d^3+3d^2ex+3de^2x^2)}{8ac(a+cx^4)^2}$$

$$\Big\downarrow \text{2415}$$

$$\frac{\frac{3\int \left(\frac{12exd^2}{cx^4+a}+\frac{7d^3+5e^2x^2d}{cx^4+a}\right)dx}{4a}+\frac{x(7d^3+18d^2ex+15de^2x^2)}{4a(a+cx^4)}}{8a}-\frac{ae^3-cx(d^3+3d^2ex+3de^2x^2)}{8ac(a+cx^4)^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{3\left(-\frac{d\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\left(5\sqrt{a}e^2+7\sqrt{c}d^2\right)}{2\sqrt{2}a^{3/4}c^{3/4}}+\frac{d\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)\left(5\sqrt{a}e^2+7\sqrt{c}d^2\right)}{2\sqrt{2}a^{3/4}c^{3/4}}-\frac{d\left(7\sqrt{c}d^2-5\sqrt{a}e^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}c^{3/4}}+\frac{d\left(7\sqrt{c}d^2-\right.}{}\right.}{4a}$$

$$8a$$

$$\frac{ae^3-cx\left(d^3+3d^2ex+3de^2x^2\right)}{8ac\left(a+cx^4\right)^2}$$

input
```
Int[(d + e*x)^3/(a + c*x^4)^3,x]
```

output
```
-1/8*(a*e^3 - c*x*(d^3 + 3*d^2*e*x + 3*d*e^2*x^2))/(a*c*(a + c*x^4)^2) + (
(x*(7*d^3 + 18*d^2*e*x + 15*d*e^2*x^2))/(4*a*(a + c*x^4)) + (3*((6*d^2*e*A
rcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(Sqrt[a]*Sqrt[c]) - (d*(7*Sqrt[c]*d^2 + 5*Sq
rt[a]*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*c^(
3/4)) + (d*(7*Sqrt[c]*d^2 + 5*Sqrt[a]*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/
a^(1/4)])/(2*Sqrt[2]*a^(3/4)*c^(3/4)) - (d*(7*Sqrt[c]*d^2 - 5*Sqrt[a]*e^2)
*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4
)*c^(3/4)) + (d*(7*Sqrt[c]*d^2 - 5*Sqrt[a]*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1
/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(3/4))))/(4*a))/(8*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2393
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   In
t[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(
p + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n
, 0] && LtQ[p, -1]
```

rule 2394
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   Int[ExpandToSum[n
*(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x
] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

rule 2415
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.18 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.43

method	result
risch	$\frac{\frac{15cd\,e^2x^7}{32a^2}+\frac{9d^2ec\,x^6}{16a^2}+\frac{7c\,d^3x^5}{32a^2}+\frac{27d\,e^2x^3}{32a}+\frac{15d^2e\,x^2}{16a}+\frac{11d^3x}{32a}-\frac{e^3}{8c}}{(c\,x^4+a)^2}+\frac{3d\left(\sum_{\_R=\text{RootOf}\left(c\_Z^4+a\right)}\frac{\left(5e^2\_R^2+12de\_R+7d^2\right)\ln\left(x-\_\right.}{\_R^3}\right.}{128a^2c}$
default	$d^3\left(\frac{x}{8a(c\,x^4+a)^2}+\frac{\frac{7x}{32a\left(c\,x^4+a\right)}+\frac{21\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{256a^2}}{a}\right)+3d^{\cdot}$

input
```
int((e*x+d)^3/(c*x^4+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(15/32*c*d*e^2/a^2*x^7+9/16*d^2*e*c/a^2*x^6+7/32*c*d^3/a^2*x^5+27/32*d*e^2
/a*x^3+15/16*d^2*e/a*x^2+11/32*d^3/a*x-1/8*e^3/c)/(c*x^4+a)^2+3/128/a^2*d/
c*sum((5*_R^2*e^2+12*_R*d*e+7*d^2)/_R^3*ln(x-_R),_R=RootOf(_Z^4*c+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 15.07 (sec) , antiderivative size = 95566, normalized size of antiderivative = 291.36

$$\int \frac{(d+ex)^3}{(a+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^3/(c*x^4+a)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [A] (verification not implemented)

Time = 6.25 (sec) , antiderivative size = 413, normalized size of antiderivative = 1.26

$$\int \frac{(d+ex)^3}{(a+cx^4)^3}\,dx$$
$$= \text{RootSum}\Big(268435456t^4a^{11}c^3 + 63111168t^2a^6c^2d^4e^2 + t\big(4147200a^4cd^4e^5 - 8128512a^3c^2d^8e\big) + 50625a^2c$$
$$+ \frac{-4a^2e^3 + 11acd^3x + 30acd^2ex^2 + 27acde^2x^3 + 7c^2d^3x^5 + 18c^2d^2ex^6 + 15c^2de^2x^7}{32a^4c + 64a^3c^2x^4 + 32a^2c^3x^8}$$

input
```
integrate((e*x+d)**3/(c*x**4+a)**3,x)
```

output
```
RootSum(268435456*_t**4*a**11*c**3 + 63111168*_t**2*a**6*c**2*d**4*e**2 +
_t*(4147200*a**4*c*d**4*e**5 - 8128512*a**3*c**2*d**8*e) + 50625*a**2*d**4
*e**8 + 245106*a*c*d**8*e**4 + 194481*c**2*d**12, Lambda(_t, _t*log(x + (2
62144000*_t**3*a**10*c**2*e**6 + 3714056192*_t**3*a**9*c**3*d**4*e**2 - 53
9688960*_t**2*a**7*c**2*d**4*e**5 + 202309632*_t**2*a**6*c**3*d**8*e + 773
28000*_t*a**5*c*d**4*e**8 + 660699648*_t*a**4*c**2*d**8*e**4 + 19361664*_t
*a**3*c**3*d**12 + 3037500*a**3*d**4*e**11 - 26360640*a**2*c*d**8*e**7 - 6
0566940*a*c**2*d**12*e**3)/(421875*a**3*d**3*e**12 - 29598075*a**2*c*d**7*
e**8 - 58012227*a*c**2*d**11*e**4 + 3176523*c**3*d**15))))) + (-4*a**2*e**3
 + 11*a*c*d**3*x + 30*a*c*d**2*e*x**2 + 27*a*c*d*e**2*x**3 + 7*c**2*d**3*x
**5 + 18*c**2*d**2*e*x**6 + 15*c**2*d*e**2*x**7)/(32*a**4*c + 64*a**3*c**2
*x**4 + 32*a**2*c**3*x**8)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 392, normalized size of antiderivative = 1.20

$$\int \frac{(d+ex)^3}{(a+cx^4)^3}\,dx$$
$$= \frac{15\,c^2de^2x^7+18\,c^2d^2ex^6+7\,c^2d^3x^5+27\,acde^2x^3+30\,acd^2ex^2+11\,acd^3x-4\,a^2e^3}{32\,(a^2c^3x^8+2\,a^3c^2x^4+a^4c)}$$
$$+ \frac{3\,d\left(\frac{\sqrt{2}(7\,\sqrt{c}d^2-5\,\sqrt{a}e^2)\log\left(\sqrt{c}x^2+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{3}{4}}} - \frac{\sqrt{2}(7\,\sqrt{c}d^2-5\,\sqrt{a}e^2)\log\left(\sqrt{c}x^2-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{3}{4}}} + \frac{2\left(7\,\sqrt{2}a^{\frac{1}{4}}c^{\frac{3}{4}}d^2+5\,\sqrt{2}c\right.}{}\right.}{256}$$

input
```
integrate((e*x+d)^3/(c*x^4+a)^3,x, algorithm="maxima")
```

output
```
1/32*(15*c^2*d*e^2*x^7 + 18*c^2*d^2*e*x^6 + 7*c^2*d^3*x^5 + 27*a*c*d*e^2*x
^3 + 30*a*c*d^2*e*x^2 + 11*a*c*d^3*x - 4*a^2*e^3)/(a^2*c^3*x^8 + 2*a^3*c^2
*x^4 + a^4*c) + 3/256*d*(sqrt(2)*(7*sqrt(c)*d^2 - 5*sqrt(a)*e^2)*log(sqrt(
c)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)) - sqrt(2)*
(7*sqrt(c)*d^2 - 5*sqrt(a)*e^2)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*
x + sqrt(a))/(a^(3/4)*c^(3/4)) + 2*(7*sqrt(2)*a^(1/4)*c^(3/4)*d^2 + 5*sqrt
(2)*a^(3/4)*c^(1/4)*e^2 - 24*sqrt(a)*sqrt(c)*d*e)*arctan(1/2*sqrt(2)*(2*sq
rt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sq
rt(a)*sqrt(c))*c^(3/4)) + 2*(7*sqrt(2)*a^(1/4)*c^(3/4)*d^2 + 5*sqrt(2)*a^(
3/4)*c^(1/4)*e^2 + 24*sqrt(a)*sqrt(c)*d*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x
 - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*s
qrt(c))*c^(3/4)))/a^2
```

Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 392, normalized size of antiderivative = 1.20

$$\int \frac{(d+ex)^3}{(a+cx^4)^3}\,dx = \frac{3\sqrt{2}\left(12\sqrt{2}\sqrt{ac}c^2d^2e+7\left(ac^3\right)^{\frac{1}{4}}c^2d^3+5\left(ac^3\right)^{\frac{3}{4}}de^2\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{128a^3c^3} + \frac{3\sqrt{2}\left(12\sqrt{2}\sqrt{ac}c^2d^2e+7\left(ac^3\right)^{\frac{1}{4}}c^2d^3+5\left(ac^3\right)^{\frac{3}{4}}de^2\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{128a^3c^3} + \frac{3\sqrt{2}\left(7\left(ac^3\right)^{\frac{1}{4}}c^2d^3-5\left(ac^3\right)^{\frac{3}{4}}de^2\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{256a^3c^3} - \frac{3\sqrt{2}\left(7\left(ac^3\right)^{\frac{1}{4}}c^2d^3-5\left(ac^3\right)^{\frac{3}{4}}de^2\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{256a^3c^3} + \frac{15c^2de^2x^7+18c^2d^2ex^6+7c^2d^3x^5+27acde^2x^3+30acd^2ex^2+11acd^3x-4a^2e^3}{32\left(cx^4+a\right)^2a^2c}$$

input
```
integrate((e*x+d)^3/(c*x^4+a)^3,x, algorithm="giac")
```

output
```
3/128*sqrt(2)*(12*sqrt(2)*sqrt(a*c)*c^2*d^2*e + 7*(a*c^3)^(1/4)*c^2*d^3 +
5*(a*c^3)^(3/4)*d*e^2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c
)^(1/4))/(a^3*c^3) + 3/128*sqrt(2)*(12*sqrt(2)*sqrt(a*c)*c^2*d^2*e + 7*(a*
c^3)^(1/4)*c^2*d^3 + 5*(a*c^3)^(3/4)*d*e^2)*arctan(1/2*sqrt(2)*(2*x - sqrt
(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^3*c^3) + 3/256*sqrt(2)*(7*(a*c^3)^(1/4)*c
^2*d^3 - 5*(a*c^3)^(3/4)*d*e^2)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c
))/(a^3*c^3) - 3/256*sqrt(2)*(7*(a*c^3)^(1/4)*c^2*d^3 - 5*(a*c^3)^(3/4)*d*
e^2)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^3*c^3) + 1/32*(15*c^2
*d*e^2*x^7 + 18*c^2*d^2*e*x^6 + 7*c^2*d^3*x^5 + 27*a*c*d*e^2*x^3 + 30*a*c*
d^2*e*x^2 + 11*a*c*d^3*x - 4*a^2*e^3)/((c*x^4 + a)^2*a^2*c)
```

Mupad [B] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 721, normalized size of antiderivative = 2.20

$$\int \frac{(d+ex)^3}{(a+cx^4)^3}\,dx = \frac{\frac{11\,d^3\,x}{32\,a}-\frac{e^3}{8\,c}+\frac{7\,c\,d^3\,x^5}{32\,a^2}+\frac{15\,d^2\,e\,x^2}{16\,a}+\frac{27\,d\,e^2\,x^3}{32\,a}+\frac{9\,c\,d^2\,e\,x^6}{16\,a^2}+\frac{15\,c\,d\,e^2\,x^7}{32\,a^2}}{a^2+2\,a\,c\,x^4+c^2\,x^8}$$
$$+\left(\sum_{k=1}^{4}\ln\left(\frac{c\,d^2\,\Big(6867\,c\,d^5\,e^2-1125\,a\,d\,e^6+7992\,c\,d^4\,e^3\,x-\mathrm{root}\big(268435456\,a^{11}\,c^3\,z^4+63111168\,a^6\,c^2\,\cdots}{}\right.\right.$$
$$+63111168\,a^6\,c^2\,d^4\,e^2\,z^2-8128512\,a^3\,c^2\,d^8\,e\,z+4147200\,a^4\,c\,d^4\,e^5\,z$$
$$\left.+245106\,a\,c\,d^8\,e^4+50625\,a^2\,d^4\,e^8+194481\,c^2\,d^{12},z,k\big)\right)$$

input
```
int((d + e*x)^3/(a + c*x^4)^3,x)
```

output
```
((11*d^3*x)/(32*a) - e^3/(8*c) + (7*c*d^3*x^5)/(32*a^2) + (15*d^2*e*x^2)/(
16*a) + (27*d*e^2*x^3)/(32*a) + (9*c*d^2*e*x^6)/(16*a^2) + (15*c*d*e^2*x^7
)/(32*a^2))/(a^2 + c^2*x^8 + 2*a*c*x^4) + symsum(log((3*c*d^2*(6867*c*d^5*
e^2 - 1125*a*d*e^6 + 7992*c*d^4*e^3*x - 114688*root(268435456*a^11*c^3*z^4
 + 63111168*a^6*c^2*d^4*e^2*z^2 - 8128512*a^3*c^2*d^8*e*z + 4147200*a^4*c*
d^4*e^5*z + 245106*a*c*d^8*e^4 + 50625*a^2*d^4*e^8 + 194481*c^2*d^12, z, k
)^2*a^5*c^2*d + 9600*root(268435456*a^11*c^3*z^4 + 63111168*a^6*c^2*d^4*e^
2*z^2 - 8128512*a^3*c^2*d^8*e*z + 4147200*a^4*c*d^4*e^5*z + 245106*a*c*d^8
*e^4 + 50625*a^2*d^4*e^8 + 194481*c^2*d^12, z, k)*a^3*c*e^4*x - 18816*root
(268435456*a^11*c^3*z^4 + 63111168*a^6*c^2*d^4*e^2*z^2 - 8128512*a^3*c^2*d
^8*e*z + 4147200*a^4*c*d^4*e^5*z + 245106*a*c*d^8*e^4 + 50625*a^2*d^4*e^8
+ 194481*c^2*d^12, z, k)*a^2*c^2*d^4*x + 196608*root(268435456*a^11*c^3*z^
4 + 63111168*a^6*c^2*d^4*e^2*z^2 - 8128512*a^3*c^2*d^8*e*z + 4147200*a^4*c
*d^4*e^5*z + 245106*a*c*d^8*e^4 + 50625*a^2*d^4*e^8 + 194481*c^2*d^12, z,
k)^2*a^5*c^2*e*x - 46080*root(268435456*a^11*c^3*z^4 + 63111168*a^6*c^2*d^
4*e^2*z^2 - 8128512*a^3*c^2*d^8*e*z + 4147200*a^4*c*d^4*e^5*z + 245106*a*c
*d^8*e^4 + 50625*a^2*d^4*e^8 + 194481*c^2*d^12, z, k)*a^3*c*d*e^3))/(32768
*a^6))*root(268435456*a^11*c^3*z^4 + 63111168*a^6*c^2*d^4*e^2*z^2 - 812851
2*a^3*c^2*d^8*e*z + 4147200*a^4*c*d^4*e^5*z + 245106*a*c*d^8*e^4 + 50625*a
^2*d^4*e^8 + 194481*c^2*d^12, z, k), k, 1, 4)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 1303, normalized size of antiderivative = 3.97

$$\int \frac{(d+ex)^3}{(a+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
int((e*x+d)^3/(c*x^4+a)^3,x)
```

output
```
( - 30*c**(1/4)*a**(3/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(
c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a**2*d*e**2 - 60*c**(1/4)*a**(3/4)*sqrt
(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt
(2)))*a*c*d*e**2*x**4 - 30*c**(1/4)*a**(3/4)*sqrt(2)*atan((c**(1/4)*a**(1/
4)*sqrt(2) - 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*c**2*d*e**2*x**8 -
42*c**(3/4)*a**(1/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x
)/(c**(1/4)*a**(1/4)*sqrt(2)))*a**2*d**3 - 84*c**(3/4)*a**(1/4)*sqrt(2)*at
an((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*
a*c*d**3*x**4 - 42*c**(3/4)*a**(1/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(
2) - 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*c**2*d**3*x**8 - 144*sqrt(c
)*sqrt(a)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x)/(c**(1/4)*a**(1/4
)*sqrt(2)))*a**2*d**2*e - 288*sqrt(c)*sqrt(a)*atan((c**(1/4)*a**(1/4)*sqrt
(2) - 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a*c*d**2*e*x**4 - 144*sqrt
(c)*sqrt(a)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x)/(c**(1/4)*a**(1
/4)*sqrt(2)))*c**2*d**2*e*x**8 + 30*c**(1/4)*a**(3/4)*sqrt(2)*atan((c**(1/
4)*a**(1/4)*sqrt(2) + 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a**2*d*e**
2 + 60*c**(1/4)*a**(3/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(
c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a*c*d*e**2*x**4 + 30*c**(1/4)*a**(3/4)*
sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*
sqrt(2)))*c**2*d*e**2*x**8 + 42*c**(3/4)*a**(1/4)*sqrt(2)*atan((c**(1/4...
```

3.186 $\int \frac{(d+ex)^2}{(a+cx^4)^3} dx$

Optimal result

Integrand size = 17, antiderivative size = 288

$$\int \frac{(d+ex)^2}{(a+cx^4)^3} dx = \frac{x(d+ex)^2}{8a(a+cx^4)^2} + \frac{x(7d^2+12dex+5e^2x^2)}{32a^2(a+cx^4)} + \frac{3de\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{c}} - \frac{(21\sqrt{c}d^2+5\sqrt{a}e^2)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}c^{3/4}} + \frac{(21\sqrt{c}d^2+5\sqrt{a}e^2)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}c^{3/4}} + \frac{(21\sqrt{c}d^2-5\sqrt{a}e^2)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x}{\sqrt{a}+\sqrt{c}x^2}\right)}{64\sqrt{2}a^{11/4}c^{3/4}}$$

output
```
1/8*x*(e*x+d)^2/a/(c*x^4+a)^2+1/32*x*(5*e^2*x^2+12*d*e*x+7*d^2)/a^2/(c*x^4
+a)+3/8*d*e*arctan(c^(1/2)*x^2/a^(1/2))/a^(5/2)/c^(1/2)+1/128*(21*c^(1/2)*
d^2+5*a^(1/2)*e^2)*arctan(-1+2^(1/2)*c^(1/4)*x/a^(1/4))*2^(1/2)/a^(11/4)/c
^(3/4)+1/128*(21*c^(1/2)*d^2+5*a^(1/2)*e^2)*arctan(1+2^(1/2)*c^(1/4)*x/a^(
1/4))*2^(1/2)/a^(11/4)/c^(3/4)+1/128*(21*c^(1/2)*d^2-5*a^(1/2)*e^2)*arctan
h(2^(1/2)*a^(1/4)*c^(1/4)*x/(a^(1/2)+c^(1/2)*x^2))*2^(1/2)/a^(11/4)/c^(3/4
)
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 358, normalized size of antiderivative = 1.24

$$\int \frac{(d+ex)^2}{(a+cx^4)^3}\,dx$$
$$= \frac{\frac{32a^2x(d+ex)^2}{(a+cx^4)^2} + \frac{8ax(7d^2+12dex+5e^2x^2)}{a+cx^4} - \frac{2\sqrt[4]{a}\left(21\sqrt{2}\sqrt{c}d^2+48\sqrt[4]{a}\sqrt[4]{c}de+5\sqrt{2}\sqrt{a}e^2\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{c^{3/4}} + \frac{2\sqrt[4]{a}\left(21\sqrt{2}\sqrt{c}d^2 \cdots\right.}{\cdots}}{\cdots}$$

input
```
Integrate[(d + e*x)^2/(a + c*x^4)^3,x]
```

output
```
((32*a^2*x*(d + e*x)^2)/(a + c*x^4)^2 + (8*a*x*(7*d^2 + 12*d*e*x + 5*e^2*x
^2))/(a + c*x^4) - (2*a^(1/4)*(21*Sqrt[2]*Sqrt[c]*d^2 + 48*a^(1/4)*c^(1/4)
*d*e + 5*Sqrt[2]*Sqrt[a]*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/c^(
3/4) + (2*a^(1/4)*(21*Sqrt[2]*Sqrt[c]*d^2 - 48*a^(1/4)*c^(1/4)*d*e + 5*Sqr
t[2]*Sqrt[a]*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/c^(3/4) + (Sqrt
[2]*(-21*a^(1/4)*Sqrt[c]*d^2 + 5*a^(3/4)*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4
)*c^(1/4)*x + Sqrt[c]*x^2])/c^(3/4) + (Sqrt[2]*(21*a^(1/4)*Sqrt[c]*d^2 - 5
*a^(3/4)*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^(3
/4))/(256*a^3)
```

Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 374, normalized size of antiderivative = 1.30, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {2394, 25, 2394, 25, 2415, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^2}{(a+cx^4)^3}\,dx$$
$$\Big\downarrow \text{ 2394}$$
$$\frac{x(d+ex)^2}{8a(a+cx^4)^2} - \frac{\int -\frac{7d^2+12exd+5e^2x^2}{(cx^4+a)^2}dx}{8a}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{7d^2+12exd+5e^2x^2}{(cx^4+a)^2}dx}{8a}+\frac{x(d+ex)^2}{8a\left(a+cx^4\right)^2}$$

$$\Big\downarrow \; 2394$$

$$\frac{\frac{x(7d^2+12dex+5e^2x^2)}{4a(a+cx^4)}-\frac{\int -\frac{21d^2+24exd+5e^2x^2}{cx^4+a}dx}{4a}}{8a}+\frac{x(d+ex)^2}{8a\left(a+cx^4\right)^2}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{21d^2+24exd+5e^2x^2}{cx^4+a}dx}{4a}+\frac{x(7d^2+12dex+5e^2x^2)}{4a(a+cx^4)}}{8a}+\frac{x(d+ex)^2}{8a\left(a+cx^4\right)^2}$$

$$\Big\downarrow \; 2415$$

$$\frac{\frac{\int \left(\frac{24dex}{cx^4+a}+\frac{21d^2+5e^2x^2}{cx^4+a}\right)dx}{4a}+\frac{x(7d^2+12dex+5e^2x^2)}{4a(a+cx^4)}}{8a}+\frac{x(d+ex)^2}{8a\left(a+cx^4\right)^2}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\left(5\sqrt{a}e^2+21\sqrt{c}d^2\right)}{2\sqrt{2}a^{3/4}c^{3/4}}+\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)\left(5\sqrt{a}e^2+21\sqrt{c}d^2\right)}{2\sqrt{2}a^{3/4}c^{3/4}}-\frac{\left(21\sqrt{c}d^2-5\sqrt{a}e^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}c^{3/4}}+\frac{\left(21\sqrt{c}d^2-5\sqrt{c}\right.}{}}{4a}}{8a}$$

$$\frac{x(d+ex)^2}{8a\left(a+cx^4\right)^2}$$

input
```
Int[(d + e*x)^2/(a + c*x^4)^3,x]
```

output
```
(x*(d + e*x)^2)/(8*a*(a + c*x^4)^2) + ((x*(7*d^2 + 12*d*e*x + 5*e^2*x^2))/
(4*a*(a + c*x^4)) + ((12*d*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(Sqrt[a]*Sqrt[
c]) - ((21*Sqrt[c]*d^2 + 5*Sqrt[a]*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(
1/4)])/(2*Sqrt[2]*a^(3/4)*c^(3/4)) + ((21*Sqrt[c]*d^2 + 5*Sqrt[a]*e^2)*Arc
Tan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*c^(3/4)) - ((21*S
qrt[c]*d^2 - 5*Sqrt[a]*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt
[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(3/4)) + ((21*Sqrt[c]*d^2 - 5*Sqrt[a]*e^2)*
Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)
*c^(3/4)))/(4*a))/(8*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2394
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   Int[ExpandToSum[n
*(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x
] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

rule 2415
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.27 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.44

method	result
risch	$\frac{\frac{5c\,e^2x^7}{32a^2}+\frac{3cde\,x^6}{8a^2}+\frac{7c\,d^2x^5}{32a^2}+\frac{9e^2x^3}{32a}+\frac{5de\,x^2}{8a}+\frac{11d^2x}{32a}}{(c\,x^4+a)^2}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+a\right)}\frac{\left(5e^2\_R^2+24de\_R+21d^2\right)\ln\left(x-\_R\right)}{\_R^3}}{128a^2c}$
default	$d^2\left(\frac{x}{8a(c\,x^4+a)^2}+\frac{\frac{7x}{32a\left(c\,x^4+a\right)}+\frac{21\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{256a^2}}{a}\right)+2de$

input
```
int((e*x+d)^2/(c*x^4+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(5/32*c*e^2/a^2*x^7+3/8*c*d*e/a^2*x^6+7/32*c*d^2/a^2*x^5+9/32*e^2/a*x^3+5/
8*d*e/a*x^2+11/32*d^2/a*x)/(c*x^4+a)^2+1/128/a^2/c*sum((5*_R^2*e^2+24*_R*d
*e+21*d^2)/_R^3*ln(x-_R),_R=RootOf(_Z^4*c+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 8.11 (sec) , antiderivative size = 91420, normalized size of antiderivative = 317.43

$$\int \frac{(d+ex)^2}{(a+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^2/(c*x^4+a)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [A] (verification not implemented)

Time = 3.72 (sec) , antiderivative size = 374, normalized size of antiderivative = 1.30

$$\int \frac{(d+ex)^2}{(a+cx^4)^3}\,dx$$
$$= \text{RootSum}\left(268435456t^4a^{11}c^3 + 25755648t^2a^6c^2d^2e^2 + t(307200a^4cde^5 - 5419008a^3c^2d^5e) + 625a^2e^8 + \right.$$
$$+ \frac{11ad^2x + 20adex^2 + 9ae^2x^3 + 7cd^2x^5 + 12cdex^6 + 5ce^2x^7}{32a^4 + 64a^3cx^4 + 32a^2c^2x^8}$$

input
```
integrate((e*x+d)**2/(c*x**4+a)**3,x)
```

output
```
RootSum(268435456*_t**4*a**11*c**3 + 25755648*_t**2*a**6*c**2*d**2*e**2 +
_t*(307200*a**4*c*d*e**5 - 5419008*a**3*c**2*d**5*e) + 625*a**2*e**8 + 111
906*a*c*d**4*e**4 + 194481*c**2*d**8, Lambda(_t, _t*log(x + (262144000*_t*
*3*a**10*c**2*e**6 + 46110081024*_t**3*a**9*c**3*d**4*e**2 - 1645608960*_t
**2*a**7*c**2*d**3*e**5 + 3641573376*_t**2*a**6*c**3*d**7*e + 32688000*_t*
a**5*c*d**2*e**8 + 3128219136*_t*a**4*c**2*d**6*e**4 + 522764928*_t*a**3*c
**3*d**10 + 225000*a**3*d*e**11 - 43338240*a**2*c*d**5*e**7 - 523431720*a*
c**2*d**9*e**3)/(15625*a**3*e**12 - 21357225*a**2*c*d**4*e**8 - 376741449*
a*c**2*d**8*e**4 + 85766121*c**3*d**12))) + (11*a*d**2*x + 20*a*d*e*x**2
+ 9*a*e**2*x**3 + 7*c*d**2*x**5 + 12*c*d*e*x**6 + 5*c*e**2*x**7)/(32*a**4
+ 64*a**3*c*x**4 + 32*a**2*c**2*x**8)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 364, normalized size of antiderivative = 1.26

$$\int \frac{(d+ex)^2}{(a+cx^4)^3}\,dx = \frac{5\,ce^2x^7 + 12\,cdex^6 + 7\,cd^2x^5 + 9\,ae^2x^3 + 20\,adex^2 + 11\,ad^2x}{32\,(a^2c^2x^8 + 2\,a^3cx^4 + a^4)}$$
$$+ \frac{\frac{\sqrt{2}(21\,\sqrt{c}d^2 - 5\,\sqrt{a}e^2)\log\left(\sqrt{c}x^2 + \sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x + \sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{3}{4}}} - \frac{\sqrt{2}(21\,\sqrt{c}d^2 - 5\,\sqrt{a}e^2)\log\left(\sqrt{c}x^2 - \sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x + \sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{3}{4}}} + \frac{2\left(21\,\sqrt{2}a^{\frac{1}{4}}c^{\frac{3}{4}}d^2 + 5\,\sqrt{2}a^{\frac{3}{4}}c\right.}{}}{256\,a}$$

input
```
integrate((e*x+d)^2/(c*x^4+a)^3,x, algorithm="maxima")
```

output
```
1/32*(5*c*e^2*x^7 + 12*c*d*e*x^6 + 7*c*d^2*x^5 + 9*a*e^2*x^3 + 20*a*d*e*x^
2 + 11*a*d^2*x)/(a^2*c^2*x^8 + 2*a^3*c*x^4 + a^4) + 1/256*(sqrt(2)*(21*sqr
t(c)*d^2 - 5*sqrt(a)*e^2)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*x + sq
rt(a))/(a^(3/4)*c^(3/4)) - sqrt(2)*(21*sqrt(c)*d^2 - 5*sqrt(a)*e^2)*log(sq
rt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)) + 2*(21
*sqrt(2)*a^(1/4)*c^(3/4)*d^2 + 5*sqrt(2)*a^(3/4)*c^(1/4)*e^2 - 48*sqrt(a)*
sqrt(c)*d*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/sq
rt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(3/4)) + 2*(21*sqrt(
2)*a^(1/4)*c^(3/4)*d^2 + 5*sqrt(2)*a^(3/4)*c^(1/4)*e^2 + 48*sqrt(a)*sqrt(c
)*d*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqr
t(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(3/4)))/a^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 358, normalized size of antiderivative = 1.24

$$
\begin{aligned}
\int \frac{(d+ex)^2}{(a+cx^4)^3}\,dx
&= \frac{5\,ce^2x^7+12\,cdex^6+7\,cd^2x^5+9\,ae^2x^3+20\,adex^2+11\,ad^2x}{32\,(cx^4+a)^2a^2} \\
&\quad + \frac{\sqrt{2}\left(24\,\sqrt{2}\sqrt{ac}c^2de+21\,(ac^3)^{\frac14}\,c^2d^2+5\,(ac^3)^{\frac34}\,e^2\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac14}\right)}{2\,\left(\frac{a}{c}\right)^{\frac14}}\right)}{128\,a^3c^3} \\
&\quad + \frac{\sqrt{2}\left(24\,\sqrt{2}\sqrt{ac}c^2de+21\,(ac^3)^{\frac14}\,c^2d^2+5\,(ac^3)^{\frac34}\,e^2\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac14}\right)}{2\,\left(\frac{a}{c}\right)^{\frac14}}\right)}{128\,a^3c^3} \\
&\quad + \frac{\sqrt{2}\left(21\,(ac^3)^{\frac14}\,c^2d^2-5\,(ac^3)^{\frac34}\,e^2\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac14}+\sqrt{\frac{a}{c}}\right)}{256\,a^3c^3} \\
&\quad - \frac{\sqrt{2}\left(21\,(ac^3)^{\frac14}\,c^2d^2-5\,(ac^3)^{\frac34}\,e^2\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac14}+\sqrt{\frac{a}{c}}\right)}{256\,a^3c^3}
\end{aligned}
$$

input
```
integrate((e*x+d)^2/(c*x^4+a)^3,x, algorithm="giac")
```

output
```
1/32*(5*c*e^2*x^7 + 12*c*d*e*x^6 + 7*c*d^2*x^5 + 9*a*e^2*x^3 + 20*a*d*e*x^
2 + 11*a*d^2*x)/((c*x^4 + a)^2*a^2) + 1/128*sqrt(2)*(24*sqrt(2)*sqrt(a*c)*
c^2*d*e + 21*(a*c^3)^(1/4)*c^2*d^2 + 5*(a*c^3)^(3/4)*e^2)*arctan(1/2*sqrt(
2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^3*c^3) + 1/128*sqrt(2)*(24*
sqrt(2)*sqrt(a*c)*c^2*d*e + 21*(a*c^3)^(1/4)*c^2*d^2 + 5*(a*c^3)^(3/4)*e^2
)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^3*c^3) +
1/256*sqrt(2)*(21*(a*c^3)^(1/4)*c^2*d^2 - 5*(a*c^3)^(3/4)*e^2)*log(x^2 + s
qrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^3*c^3) - 1/256*sqrt(2)*(21*(a*c^3)^(1
/4)*c^2*d^2 - 5*(a*c^3)^(3/4)*e^2)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(
a/c))/(a^3*c^3)
```

Mupad [B] (verification not implemented)

Time = 21.86 (sec) , antiderivative size = 676, normalized size of antiderivative = 2.35

$$\int \frac{(d+ex)^2}{(a+cx^4)^3}\,dx = \frac{\frac{11\,d^2\,x}{32\,a}+\frac{9\,e^2\,x^3}{32\,a}+\frac{7\,c\,d^2\,x^5}{32\,a^2}+\frac{5\,c\,e^2\,x^7}{32\,a^2}+\frac{5\,d\,e\,x^2}{8\,a}+\frac{3\,c\,d\,e\,x^6}{8\,a^2}}{a^2+2\,a\,c\,x^4+c^2\,x^8}$$
$$+\left(\sum_{k=1}^{4}\ln\left(-\frac{c\left(125\,a\,e^6-9891\,c\,d^4\,e^2+\mathrm{root}(268435456\,a^{11}\,c^3\,z^4+25755648\,a^6\,c^2\,d^2\,e^2\,z^2-5419008\,a\right.}{}\right.\right.$$
$$+25755648\,a^6\,c^2\,d^2\,e^2\,z^2-5419008\,a^3\,c^2\,d^5\,e\,z+307200\,a^4\,c\,d\,e^5\,z+111906\,a\,c\,d^4\,e^4$$
$$\left.\left.+194481\,c^2\,d^8+625\,a^2\,e^8,z,k\right)\right)$$

input
```
int((d + e*x)^2/(a + c*x^4)^3,x)
```

output
```
((11*d^2*x)/(32*a) + (9*e^2*x^3)/(32*a) + (7*c*d^2*x^5)/(32*a^2) + (5*c*e^
2*x^7)/(32*a^2) + (5*d*e*x^2)/(8*a) + (3*c*d*e*x^6)/(8*a^2))/(a^2 + c^2*x^
8 + 2*a*c*x^4) + symsum(log(-(c*(125*a*e^6 - 9891*c*d^4*e^2 + 344064*root(
268435456*a^11*c^3*z^4 + 25755648*a^6*c^2*d^2*e^2*z^2 - 5419008*a^3*c^2*d^
5*e*z + 307200*a^4*c*d*e^5*z + 111906*a*c*d^4*e^4 + 194481*c^2*d^8 + 625*a
^2*e^8, z, k)^2*a^5*c^2*d^2 - 8784*c*d^3*e^3*x - 3200*root(268435456*a^11*
c^3*z^4 + 25755648*a^6*c^2*d^2*e^2*z^2 - 5419008*a^3*c^2*d^5*e*z + 307200*
a^4*c*d*e^5*z + 111906*a*c*d^4*e^4 + 194481*c^2*d^8 + 625*a^2*e^8, z, k)*a
^3*c*e^4*x + 56448*root(268435456*a^11*c^3*z^4 + 25755648*a^6*c^2*d^2*e^2*
z^2 - 5419008*a^3*c^2*d^5*e*z + 307200*a^4*c*d*e^5*z + 111906*a*c*d^4*e^4
+ 194481*c^2*d^8 + 625*a^2*e^8, z, k)*a^2*c^2*d^4*x + 30720*root(268435456
*a^11*c^3*z^4 + 25755648*a^6*c^2*d^2*e^2*z^2 - 5419008*a^3*c^2*d^5*e*z + 3
07200*a^4*c*d*e^5*z + 111906*a*c*d^4*e^4 + 194481*c^2*d^8 + 625*a^2*e^8, z
, k)*a^3*c*d*e^3 - 393216*root(268435456*a^11*c^3*z^4 + 25755648*a^6*c^2*d
^2*e^2*z^2 - 5419008*a^3*c^2*d^5*e*z + 307200*a^4*c*d*e^5*z + 111906*a*c*d
^4*e^4 + 194481*c^2*d^8 + 625*a^2*e^8, z, k)^2*a^5*c^2*d*e*x))/(32768*a^6)
)*root(268435456*a^11*c^3*z^4 + 25755648*a^6*c^2*d^2*e^2*z^2 - 5419008*a^3
*c^2*d^5*e*z + 307200*a^4*c*d*e^5*z + 111906*a*c*d^4*e^4 + 194481*c^2*d^8
+ 625*a^2*e^8, z, k), k, 1, 4)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 1265, normalized size of antiderivative = 4.39

$$\int \frac{(d+ex)^2}{(a+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
int((e*x+d)^2/(c*x^4+a)^3,x)
```

output

```
( - 10*c**(1/4)*a**(3/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(
c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a**2*e**2 - 20*c**(1/4)*a**(3/4)*sqrt(2
)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2
)))*a*c*e**2*x**4 - 10*c**(1/4)*a**(3/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*s
qrt(2) - 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*c**2*e**2*x**8 - 42*c**
(3/4)*a**(1/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x)/(c**
(1/4)*a**(1/4)*sqrt(2)))*a**2*d**2 - 84*c**(3/4)*a**(1/4)*sqrt(2)*atan((c*
*(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a*c*d*
*2*x**4 - 42*c**(3/4)*a**(1/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2
*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*c**2*d**2*x**8 - 96*sqrt(c)*sqrt(
a)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(
2)))*a**2*d*e - 192*sqrt(c)*sqrt(a)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sq
rt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a*c*d*e*x**4 - 96*sqrt(c)*sqrt(a)*at
an((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*
c**2*d*e*x**8 + 10*c**(1/4)*a**(3/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(
2) + 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a**2*e**2 + 20*c**(1/4)*a**
(3/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(c)*x)/(c**(1/4)*a**
(1/4)*sqrt(2)))*a*c*e**2*x**4 + 10*c**(1/4)*a**(3/4)*sqrt(2)*atan((c**(1/4
)*a**(1/4)*sqrt(2) + 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*c**2*e**2*x
**8 + 42*c**(3/4)*a**(1/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) + 2*...
```

3.187 $\int \frac{d+ex}{(a+cx^4)^3} dx$

Optimal result

Integrand size = 15, antiderivative size = 214

$$\begin{aligned}\int \frac{d+ex}{(a+cx^4)^3} dx &= \frac{x(d+ex)}{8a(a+cx^4)^2} + \frac{x(7d+6ex)}{32a^2(a+cx^4)} \\ &+ \frac{3e\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{c}} - \frac{21d\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}\sqrt[4]{c}} \\ &+ \frac{21d\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}\sqrt[4]{c}} + \frac{21d\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x}{\sqrt{a}+\sqrt{c}x^2}\right)}{64\sqrt{2}a^{11/4}\sqrt[4]{c}}\end{aligned}$$

output
```
1/8*x*(e*x+d)/a/(c*x^4+a)^2+1/32*x*(6*e*x+7*d)/a^2/(c*x^4+a)+3/16*e*arctan
(c^(1/2)*x^2/a^(1/2))/a^(5/2)/c^(1/2)+21/128*d*arctan(-1+2^(1/2)*c^(1/4)*x
/a^(1/4))*2^(1/2)/a^(11/4)/c^(1/4)+21/128*d*arctan(1+2^(1/2)*c^(1/4)*x/a^(
1/4))*2^(1/2)/a^(11/4)/c^(1/4)+21/128*d*arctanh(2^(1/2)*a^(1/4)*c^(1/4)*x/
(a^(1/2)+c^(1/2)*x^2))*2^(1/2)/a^(11/4)/c^(1/4)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.16

$$\int \frac{d+ex}{(a+cx^4)^3}\,dx$$

$$= \frac{\frac{32a^{7/4}x(d+ex)}{(a+cx^4)^2} + \frac{8a^{3/4}x(7d+6ex)}{a+cx^4} - \frac{6\left(7\sqrt{2}\sqrt[4]{c}d+8\sqrt[4]{a}e\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{c}} + \frac{6\left(7\sqrt{2}\sqrt[4]{c}d-8\sqrt[4]{a}e\right)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{c}} - \cdots}{256a^{11/4}}$$

input
```
Integrate[(d + e*x)/(a + c*x^4)^3,x]
```

output
```
((32*a^(7/4)*x*(d + e*x))/(a + c*x^4)^2 + (8*a^(3/4)*x*(7*d + 6*e*x))/(a +
 c*x^4) - (6*(7*Sqrt[2]*c^(1/4)*d + 8*a^(1/4)*e)*ArcTan[1 - (Sqrt[2]*c^(1/
4)*x)/a^(1/4)])/Sqrt[c] + (6*(7*Sqrt[2]*c^(1/4)*d - 8*a^(1/4)*e)*ArcTan[1
+ (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/Sqrt[c] - (21*Sqrt[2]*d*Log[Sqrt[a] - Sqrt
[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^(1/4) + (21*Sqrt[2]*d*Log[Sqrt[a]
+ Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^(1/4))/(256*a^(11/4))
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.31, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {2394, 25, 2394, 27, 2415, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex}{(a+cx^4)^3}\,dx$$

$$\Big\downarrow \text{2394}$$

$$\frac{x(d+ex)}{8a(a+cx^4)^2} - \frac{\int -\frac{7d+6ex}{(cx^4+a)^2}dx}{8a}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{7d+6ex}{(cx^4+a)^2}dx}{8a}+\frac{x(d+ex)}{8a\left(a+cx^4\right)^2}$$

$\Big\downarrow$ 2394

$$\frac{\frac{x(7d+6ex)}{4a(a+cx^4)}-\frac{\int -\frac{3(7d+4ex)}{cx^4+a}dx}{4a}}{8a}+\frac{x(d+ex)}{8a\left(a+cx^4\right)^2}$$

$\Big\downarrow$ 27

$$\frac{\frac{3\int \frac{7d+4ex}{cx^4+a}dx}{4a}+\frac{x(7d+6ex)}{4a(a+cx^4)}}{8a}+\frac{x(d+ex)}{8a\left(a+cx^4\right)^2}$$

$\Big\downarrow$ 2415

$$\frac{\frac{3\int \left(\frac{7d}{cx^4+a}+\frac{4ex}{cx^4+a}\right)dx}{4a}+\frac{x(7d+6ex)}{4a(a+cx^4)}}{8a}+\frac{x(d+ex)}{8a\left(a+cx^4\right)^2}$$

$\Big\downarrow$ 2009

$$\frac{\frac{3\left(-\frac{7d\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{c}}+\frac{7d\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{c}}-\frac{7d\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{c}}+\frac{7d\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt[4]{c}}+\frac{2e\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{c}}\right)}{4a}}{8a}$$

$$\frac{x(d+ex)}{8a\left(a+cx^4\right)^2}$$

input
```
Int[(d + e*x)/(a + c*x^4)^3,x]
```

output
```
(x*(d + e*x))/(8*a*(a + c*x^4)^2) + ((x*(7*d + 6*e*x))/(4*a*(a + c*x^4)) +
 (3*((2*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(Sqrt[a]*Sqrt[c]) - (7*d*ArcTan[1
 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*c^(1/4)) + (7*d*ArcTan
[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*c^(1/4)) - (7*d*Log[
Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(
1/4)) + (7*d*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sq
rt[2]*a^(3/4)*c^(1/4))))/(4*a))/(8*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2394
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   Int[ExpandToSum[n
*(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x
] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

rule 2415
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[x^ii*((Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2))/(a + b*x^n)), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.25 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.40

method	result
risch	$\frac{\frac{3ce\,x^{6}}{16a^{2}}+\frac{7cd\,x^{5}}{32a^{2}}+\frac{5e\,x^{2}}{16a}+\frac{11dx}{32a}}{\left(c\,x^{4}+a\right)^{2}}+\frac{3\left(\sum_{\_R=\operatorname{RootOf}\left(c\_Z^{4}+a\right)}\frac{\left(4e\_R+7d\right)\ln\left(x-\_R\right)}{\_R^{3}}\right)}{128a^{2}c}$
default	$d\left(\frac{x}{8a(c\,x^{4}+a)^{2}}+\frac{\frac{7x}{32a\left(c\,x^{4}+a\right)}+\frac{21\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^{2}+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^{2}-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{256a^{2}}}{a}\right)+e\Bigg($

input
```
int((e*x+d)/(c*x^4+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(3/16*c*e/a^2*x^6+7/32*c*d/a^2*x^5+5/16*e/a*x^2+11/32*d/a*x)/(c*x^4+a)^2+3
/128/a^2/c*sum((4*_R*e+7*d)/_R^3*ln(x-_R),_R=RootOf(_Z^4*c+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.67 (sec) , antiderivative size = 43180, normalized size of antiderivative = 201.78

$$\int \frac{d+ex}{(a+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)/(c*x^4+a)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [A] (verification not implemented)

Time = 0.98 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.90

$$\int \frac{d+ex}{(a+cx^4)^3}\,dx$$
$$= \operatorname{RootSum}\left(268435456t^4a^{11}c^2 + 4718592t^2a^6ce^2 - 2709504ta^3cd^2e + 20736ae^4 + 194481cd^4, \left(t \mapsto t\log\right.\right.$$
$$+ \frac{11adx + 10aex^2 + 7cdx^5 + 6cex^6}{32a^4 + 64a^3cx^4 + 32a^2c^2x^8}$$

input
```
integrate((e*x+d)/(c*x**4+a)**3,x)
```

output
```
RootSum(268435456*_t**4*a**11*c**2 + 4718592*_t**2*a**6*c*e**2 - 2709504*_
t*a**3*c*d**2*e + 20736*a*e**4 + 194481*c*d**4, Lambda(_t, _t*log(x + (-67
108864*_t**3*a**9*c*e**2 - 9633792*_t**2*a**6*c*d**2*e - 589824*_t*a**4*e*
*4 - 2765952*_t*a**3*c*d**4 + 423360*a*d**2*e**3)/(193536*a*d*e**4 - 45378
9*c*d**5)))) + (11*a*d*x + 10*a*e*x**2 + 7*c*d*x**5 + 6*c*e*x**6)/(32*a**4
 + 64*a**3*c*x**4 + 32*a**2*c**2*x**8)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.26

$$\int \frac{d+ex}{(a+cx^4)^3}\,dx = \frac{6\,cex^6+7\,cdx^5+10\,aex^2+11\,adx}{32\,(a^2c^2x^8+2\,a^3cx^4+a^4)} + \frac{3\left(\frac{7\,\sqrt{2}d\log\left(\sqrt{c}x^2+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{1}{4}}} - \frac{7\,\sqrt{2}d\log\left(\sqrt{c}x^2-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{1}{4}}} + \frac{2\left(7\,\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}d-8\,\sqrt{a}e\right)\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{c}x+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{a}\sqrt{c}}}\right)}{a^{\frac{3}{4}}\sqrt{\sqrt{a}\sqrt{c}}c^{\frac{1}{4}}}\right.}{256\,a^2}$$

input
```
integrate((e*x+d)/(c*x^4+a)^3,x, algorithm="maxima")
```

output
```
1/32*(6*c*e*x^6 + 7*c*d*x^5 + 10*a*e*x^2 + 11*a*d*x)/(a^2*c^2*x^8 + 2*a^3*
c*x^4 + a^4) + 3/256*(7*sqrt(2)*d*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c^(1/4
)*x + sqrt(a))/(a^(3/4)*c^(1/4)) - 7*sqrt(2)*d*log(sqrt(c)*x^2 - sqrt(2)*a
^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(1/4)) + 2*(7*sqrt(2)*a^(1/4)*c^(1/
4)*d - 8*sqrt(a)*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1
/4))/sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(1/4)) + 2*(7
*sqrt(2)*a^(1/4)*c^(1/4)*d + 8*sqrt(a)*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x
- sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sq
rt(c))*c^(1/4)))/a^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.20

$$\int \frac{d+ex}{(a+cx^4)^3}\,dx = \frac{21\sqrt{2}(ac^3)^{\frac{1}{4}}\,d\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{256\,a^3c} - \frac{21\sqrt{2}(ac^3)^{\frac{1}{4}}\,d\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{256\,a^3c} + \frac{3\sqrt{2}\left(4\sqrt{2}\sqrt{ac}ce+7\,(ac^3)^{\frac{1}{4}}\,cd\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{128\,a^3c^2} + \frac{3\sqrt{2}\left(4\sqrt{2}\sqrt{ac}ce+7\,(ac^3)^{\frac{1}{4}}\,cd\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{128\,a^3c^2} + \frac{6\,cex^6+7\,cdx^5+10\,aex^2+11\,adx}{32\,(cx^4+a)^2a^2}$$

input
```
integrate((e*x+d)/(c*x^4+a)^3,x, algorithm="giac")
```

output
```
21/256*sqrt(2)*(a*c^3)^(1/4)*d*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c)
)/(a^3*c) - 21/256*sqrt(2)*(a*c^3)^(1/4)*d*log(x^2 - sqrt(2)*x*(a/c)^(1/4)
 + sqrt(a/c))/(a^3*c) + 3/128*sqrt(2)*(4*sqrt(2)*sqrt(a*c)*c*e + 7*(a*c^3)
^(1/4)*c*d)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a
^3*c^2) + 3/128*sqrt(2)*(4*sqrt(2)*sqrt(a*c)*c*e + 7*(a*c^3)^(1/4)*c*d)*ar
ctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^3*c^2) + 1/32
*(6*c*e*x^6 + 7*c*d*x^5 + 10*a*e*x^2 + 11*a*d*x)/((c*x^4 + a)^2*a^2)
```

Mupad [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.47

$$\int \frac{d+ex}{(a+cx^4)^3}\,dx = \frac{\frac{5\,e\,x^2}{16\,a}+\frac{11\,d\,x}{32\,a}+\frac{7\,c\,d\,x^5}{32\,a^2}+\frac{3\,c\,e\,x^6}{16\,a^2}}{a^2+2\,a\,c\,x^4+c^2\,x^8} + \left(\sum_{k=1}^{4}\ln\left(\frac{c^2\left(63\,d\,e^2+36\,e^3\,x-\mathrm{root}(268435456\,a^{11}\,c^2\,z^4+4718592\,a^6\,c\,e^2\,z^2-2709504\,a^3\,c\,d^2\,e\,z+\right.}{}\right.\right.$$

$$\left.\left. + 4718592\,a^6\,c\,e^2\,z^2-2709504\,a^3\,c\,d^2\,e\,z+194481\,c\,d^4+20736\,a\,e^4,z,k)\right)$$

input
```
int((d + e*x)/(a + c*x^4)^3,x)
```

output
```
((5*e*x^2)/(16*a) + (11*d*x)/(32*a) + (7*c*d*x^5)/(32*a^2) + (3*c*e*x^6)/(
16*a^2))/(a^2 + c^2*x^8 + 2*a*c*x^4) + symsum(log((3*c^2*(63*d*e^2 + 36*e^
3*x - 7168*root(268435456*a^11*c^2*z^4 + 4718592*a^6*c*e^2*z^2 - 2709504*a
^3*c*d^2*e*z + 194481*c*d^4 + 20736*a*e^4, z, k)^2*a^5*c*d - 1176*root(268
435456*a^11*c^2*z^4 + 4718592*a^6*c*e^2*z^2 - 2709504*a^3*c*d^2*e*z + 1944
81*c*d^4 + 20736*a*e^4, z, k)*a^2*c*d^2*x + 4096*root(268435456*a^11*c^2*z
^4 + 4718592*a^6*c*e^2*z^2 - 2709504*a^3*c*d^2*e*z + 194481*c*d^4 + 20736*
a*e^4, z, k)^2*a^5*c*e*x))/(2048*a^6))*root(268435456*a^11*c^2*z^4 + 47185
92*a^6*c*e^2*z^2 - 2709504*a^3*c*d^2*e*z + 194481*c*d^4 + 20736*a*e^4, z,
k), k, 1, 4)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 740, normalized size of antiderivative = 3.46

$$\int \frac{d+ex}{(a+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
int((e*x+d)/(c*x^4+a)^3,x)
```

output
```
( - 42*c**(3/4)*a**(1/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(
c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a**2*d - 84*c**(3/4)*a**(1/4)*sqrt(2)*a
tan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))
*a*c*d*x**4 - 42*c**(3/4)*a**(1/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2)
 - 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*c**2*d*x**8 - 48*sqrt(c)*sqrt
(a)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt
(2)))*a**2*e - 96*sqrt(c)*sqrt(a)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt
(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a*c*e*x**4 - 48*sqrt(c)*sqrt(a)*atan((
c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*c**2
*e*x**8 + 42*c**(3/4)*a**(1/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) + 2
*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a**2*d + 84*c**(3/4)*a**(1/4)*sqr
t(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqr
t(2)))*a*c*d*x**4 + 42*c**(3/4)*a**(1/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*s
qrt(2) + 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*c**2*d*x**8 - 48*sqrt(c
)*sqrt(a)*atan((c**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(c)*x)/(c**(1/4)*a**(1/4
)*sqrt(2)))*a**2*e - 96*sqrt(c)*sqrt(a)*atan((c**(1/4)*a**(1/4)*sqrt(2) +
2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a*c*e*x**4 - 48*sqrt(c)*sqrt(a)*
atan((c**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2))
)*c**2*e*x**8 - 21*c**(3/4)*a**(1/4)*sqrt(2)*log( - c**(1/4)*a**(1/4)*sqrt
(2)*x + sqrt(a) + sqrt(c)*x**2)*a**2*d - 42*c**(3/4)*a**(1/4)*sqrt(2)*l...
```

3.188 $\int \frac{1}{(a+cx^4)^3} dx$

Optimal result

Integrand size = 9, antiderivative size = 168

$$\int \frac{1}{(a+cx^4)^3} dx = \frac{x}{8a(a+cx^4)^2} + \frac{7x}{32a^2(a+cx^4)} - \frac{21\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}\sqrt[4]{c}} + \frac{21\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}\sqrt[4]{c}} + \frac{21\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x}{\sqrt{a}+\sqrt{c}x^2}\right)}{64\sqrt{2}a^{11/4}\sqrt[4]{c}}$$

output
```
1/8*x/a/(c*x^4+a)^2+7/32*x/a^2/(c*x^4+a)+21/128*arctan(-1+2^(1/2)*c^(1/4)*
x/a^(1/4))*2^(1/2)/a^(11/4)/c^(1/4)+21/128*arctan(1+2^(1/2)*c^(1/4)*x/a^(1
/4))*2^(1/2)/a^(11/4)/c^(1/4)+21/128*arctanh(2^(1/2)*a^(1/4)*c^(1/4)*x/(a^
(1/2)+c^(1/2)*x^2))*2^(1/2)/a^(11/4)/c^(1/4)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.19

$$\int \frac{1}{(a+cx^4)^3}\,dx = \frac{\frac{32a^{7/4}x}{(a+cx^4)^2}+\frac{56a^{3/4}x}{a+cx^4}-\frac{42\sqrt{2}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt[4]{c}}+\frac{42\sqrt{2}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt[4]{c}}-\frac{21\sqrt{2}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{\sqrt[4]{c}}+\frac{21\sqrt{2}}{}}{256a^{11/4}}$$

input
```
Integrate[(a + c*x^4)^(-3),x]
```

output
```
((32*a^(7/4)*x)/(a + c*x^4)^2 + (56*a^(3/4)*x)/(a + c*x^4) - (42*Sqrt[2]*A
rcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/c^(1/4) + (42*Sqrt[2]*ArcTan[1 + (
Sqrt[2]*c^(1/4)*x)/a^(1/4)])/c^(1/4) - (21*Sqrt[2]*Log[Sqrt[a] - Sqrt[2]*a
^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^(1/4) + (21*Sqrt[2]*Log[Sqrt[a] + Sqrt[
2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^(1/4))/(256*a^(11/4))
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.49, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.111, Rules used = {749, 749, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+cx^4)^3}\,dx$$

$$\Big\downarrow \; 749$$

$$\frac{7\int \frac{1}{(cx^4+a)^2}dx}{8a}+\frac{x}{8a\,(a+cx^4)^2}$$

$$\Big\downarrow \; 749$$

$$\frac{7\left(\frac{3\int \frac{1}{cx^4+a}dx}{4a}+\frac{x}{4a(a+cx^4)}\right)}{8a}+\frac{x}{8a\left(a+cx^4\right)^2}$$

$$\Big\downarrow \text{755}$$

$$\frac{7\left(\frac{3\left(\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{a}}+\frac{\int \frac{\sqrt{c}x^2+\sqrt{a}}{cx^4+a}dx}{2\sqrt{a}}\right)}{4a}+\frac{x}{4a(a+cx^4)}\right)}{8a}+\frac{x}{8a\left(a+cx^4\right)^2}$$

$$\Big\downarrow \text{1476}$$

$$\frac{7\left(\frac{3\left(\frac{\frac{\int \frac{1}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}+\frac{\int \frac{1}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}}{2\sqrt{a}}+\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{a}}\right)}{4a}+\frac{x}{4a(a+cx^4)}\right)}{8a}+\frac{x}{8a\left(a+cx^4\right)^2}$$

$$\Big\downarrow \text{1082}$$

$$\frac{7\left(\frac{3\left(\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{a}}+\frac{\frac{\int \frac{1}{-\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)^2-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\int \frac{1}{-\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)^2-1}d\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}\right)}{4a}+\frac{x}{4a(a+cx^4)}\right)}{8a}+\frac{x}{8a\left(a+cx^4\right)^2}$$

$\downarrow$ 217

$$\frac{7\left(\frac{3\left(\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}\right)}{4a}+\frac{x}{4a(a+cx^4)}\right)}{8a}+\frac{x}{8a\left(a+cx^4\right)^2}$$

$\downarrow$ 1479

$$\frac{7\left(\frac{3\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{\sqrt[4]{c}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}\right)}{\sqrt[4]{c}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}\right)}{4a}+\frac{x}{4a(a+cx^4)}\right)}{8a}+\frac{x}{8a\left(a+cx^4\right)^2}$$

$$\Big\downarrow \ 25$$

$$\frac{7\left(\frac{3\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{\sqrt[4]{c}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}\right)}{\sqrt[4]{c}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}\right)}{4a}+\frac{x}{4a(a+cx^4)}\right)}{8a}+\frac{x}{8a\left(a+cx^4\right)^2}$$

$$\Big\downarrow \ 27$$

$$\frac{7\left(\frac{3\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt{c}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt[4]{a}\sqrt{c}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}\right)}{4a}+\frac{x}{4a(a+cx^4)}\right)}{8a}+\frac{x}{8a\left(a+cx^4\right)^2}$$

$$\Big\downarrow \; 1103$$

$$\frac{7\left(\frac{3\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}\right)}{4a}+\frac{x}{4a(a+cx^4)}\right)}{8a}+\frac{x}{8a\left(a+cx^4\right)^2}$$

input
```
Int[(a + c*x^4)^(-3),x]
```

output
```
x/(8*a*(a + c*x^4)^2) + (7*(x/(4*a*(a + c*x^4)) + (3*((-(ArcTan[1 - (Sqrt[
2]*c^(1/4)*x)/a^(1/4)]/(Sqrt[2]*a^(1/4)*c^(1/4))) + ArcTan[1 + (Sqrt[2]*c^
(1/4)*x)/a^(1/4)]/(Sqrt[2]*a^(1/4)*c^(1/4)))/(2*Sqrt[a]) + (-1/2*Log[Sqrt[
a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]/(Sqrt[2]*a^(1/4)*c^(1/4)) +
Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]/(2*Sqrt[2]*a^(1/4)*
c^(1/4)))/(2*Sqrt[a])))/(4*a)))/(8*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 749
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Simp[(n*(p + 1) + 1)/(a*n*(p + 1))   Int[(a + b*x^
n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Inte
gerQ[2*p] || Denominator[p + 1/n] < Denominator[p])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.16 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.34

method	result	size
risch	$\frac{\frac{7c\,x^{5}}{32a^{2}}+\frac{11x}{32a}}{\left(c\,x^{4}+a\right)^{2}}+\frac{21\left(\sum_{\_R=\operatorname{RootOf}\left(c\,\_Z^{4}+a\right)}\frac{\ln\left(x-\_R\right)}{\_R^{3}}\right)}{128a^{2}c}$	57
default	$\frac{x}{8a\left(c\,x^{4}+a\right)^{2}}+\frac{\frac{7x}{32a\left(c\,x^{4}+a\right)}+\frac{21\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^{2}+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^{2}-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{256a^{2}}}{a}$	139

input
```
int(1/(c*x^4+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(7/32*c/a^2*x^5+11/32/a*x)/(c*x^4+a)^2+21/128/a^2/c*sum(1/_R^3*ln(x-_R),_R
=RootOf(_Z^4*c+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.52

$$\int \frac{1}{(a+cx^4)^3}\,dx = \frac{28\,cx^5 + 21\,(a^2c^2x^8 + 2\,a^3cx^4 + a^4)\left(-\frac{1}{a^{11}c}\right)^{\frac{1}{4}}\log\left(a^3\left(-\frac{1}{a^{11}c}\right)^{\frac{1}{4}} + x\right) - 21\,(-i\,a^2c^2x^8 - 2i\,a^3cx^4 - i\,a^4)(-$$

input
```
integrate(1/(c*x^4+a)^3,x, algorithm="fricas")
```

output
```
1/128*(28*c*x^5 + 21*(a^2*c^2*x^8 + 2*a^3*c*x^4 + a^4)*(-1/(a^11*c))^(1/4)
*log(a^3*(-1/(a^11*c))^(1/4) + x) - 21*(-I*a^2*c^2*x^8 - 2*I*a^3*c*x^4 - I
*a^4)*(-1/(a^11*c))^(1/4)*log(I*a^3*(-1/(a^11*c))^(1/4) + x) - 21*(I*a^2*c
^2*x^8 + 2*I*a^3*c*x^4 + I*a^4)*(-1/(a^11*c))^(1/4)*log(-I*a^3*(-1/(a^11*c
))^(1/4) + x) - 21*(a^2*c^2*x^8 + 2*a^3*c*x^4 + a^4)*(-1/(a^11*c))^(1/4)*l
og(-a^3*(-1/(a^11*c))^(1/4) + x) + 44*a*x)/(a^2*c^2*x^8 + 2*a^3*c*x^4 + a^
4)
```

Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.38

$$\int \frac{1}{(a+cx^4)^3}\,dx = \frac{11ax + 7cx^5}{32a^4 + 64a^3cx^4 + 32a^2c^2x^8} + \operatorname{RootSum}\left(268435456t^4a^{11}c + 194481, \left(t \mapsto t\log\left(\frac{128ta^3}{21} + x\right)\right)\right)$$

input
```
integrate(1/(c*x**4+a)**3,x)
```

output
```
(11*a*x + 7*c*x**5)/(32*a**4 + 64*a**3*c*x**4 + 32*a**2*c**2*x**8) + RootS
um(268435456*_t**4*a**11*c + 194481, Lambda(_t, _t*log(128*_t*a**3/21 + x)
))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.26

$$\int \frac{1}{(a+cx^4)^3}\,dx = \frac{7\,cx^5+11\,ax}{32\,(a^2c^2x^8+2\,a^3cx^4+a^4)} + \frac{21\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}}+\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}}+\frac{\sqrt{2}\log\left(\sqrt{c}x^2+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{1}{4}}}-\frac{\sqrt{2}\log\left(\sqrt{c}x\right.}{}\right.}{256\,a^2}$$

input
```
integrate(1/(c*x^4+a)^3,x, algorithm="maxima")
```

output
```
1/32*(7*c*x^5 + 11*a*x)/(a^2*c^2*x^8 + 2*a^3*c*x^4 + a^4) + 21/256*(2*sqrt
(2)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a
)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))) + 2*sqrt(2)*arctan(1/2*sqrt(2)
*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*s
qrt(sqrt(a)*sqrt(c))) + sqrt(2)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*
x + sqrt(a))/(a^(3/4)*c^(1/4)) - sqrt(2)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)
*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(1/4)))/a^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.21

$$\begin{aligned}\int \frac{1}{(a+cx^4)^3}\,dx &= \frac{21\sqrt{2}(ac^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{128\,a^3c} \\ &+ \frac{21\sqrt{2}(ac^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{128\,a^3c} \\ &+ \frac{21\sqrt{2}(ac^3)^{\frac{1}{4}}\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{256\,a^3c} \\ &- \frac{21\sqrt{2}(ac^3)^{\frac{1}{4}}\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{256\,a^3c} + \frac{7\,cx^5+11\,ax}{32\,(cx^4+a)^2a^2}\end{aligned}$$

input
```
integrate(1/(c*x^4+a)^3,x, algorithm="giac")
```

output
```
21/128*sqrt(2)*(a*c^3)^(1/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4)
)/(a/c)^(1/4))/(a^3*c) + 21/128*sqrt(2)*(a*c^3)^(1/4)*arctan(1/2*sqrt(2)*(
2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^3*c) + 21/256*sqrt(2)*(a*c^3)^(
1/4)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^3*c) - 21/256*sqrt(2)
*(a*c^3)^(1/4)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^3*c) + 1/32
*(7*c*x^5 + 11*a*x)/((c*x^4 + a)^2*a^2)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.48

$$\int \frac{1}{(a+cx^4)^3}\,dx=\frac{\frac{11\,x}{32\,a}+\frac{7\,c\,x^5}{32\,a^2}}{a^2+2\,a\,c\,x^4+c^2\,x^8}-\frac{21\,\mathrm{atan}\left(\frac{c^{1/4}\,x}{(-a)^{1/4}}\right)}{64\,(-a)^{11/4}\,c^{1/4}}-\frac{21\,\mathrm{atanh}\left(\frac{c^{1/4}\,x}{(-a)^{1/4}}\right)}{64\,(-a)^{11/4}\,c^{1/4}}$$

input
```
int(1/(a + c*x^4)^3,x)
```

output
```
((11*x)/(32*a) + (7*c*x^5)/(32*a^2))/(a^2 + c^2*x^8 + 2*a*c*x^4) - (21*ata
n((c^(1/4)*x)/(-a)^(1/4)))/(64*(-a)^(11/4)*c^(1/4)) - (21*atanh((c^(1/4)*x
)/(-a)^(1/4)))/(64*(-a)^(11/4)*c^(1/4))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 474, normalized size of antiderivative = 2.82

$$\int \frac{1}{(a+cx^4)^3}\,dx=\text{Too large to display}$$

input
```
int(1/(c*x^4+a)^3,x)
```

output
```
( - 42*c**(3/4)*a**(1/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(
c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a**2 - 84*c**(3/4)*a**(1/4)*sqrt(2)*ata
n((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a
*c*x**4 - 42*c**(3/4)*a**(1/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2
*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*c**2*x**8 + 42*c**(3/4)*a**(1/4)*
sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*
sqrt(2)))*a**2 + 84*c**(3/4)*a**(1/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt
(2) + 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a*c*x**4 + 42*c**(3/4)*a**
(1/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(c)*x)/(c**(1/4)*a**
(1/4)*sqrt(2)))*c**2*x**8 - 21*c**(3/4)*a**(1/4)*sqrt(2)*log( - c**(1/4)*a
**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c)*x**2)*a**2 - 42*c**(3/4)*a**(1/4)*sq
rt(2)*log( - c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c)*x**2)*a*c*x**
4 - 21*c**(3/4)*a**(1/4)*sqrt(2)*log( - c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt
(a) + sqrt(c)*x**2)*c**2*x**8 + 21*c**(3/4)*a**(1/4)*sqrt(2)*log(c**(1/4)*
a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c)*x**2)*a**2 + 42*c**(3/4)*a**(1/4)*s
qrt(2)*log(c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c)*x**2)*a*c*x**4
+ 21*c**(3/4)*a**(1/4)*sqrt(2)*log(c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) +
 sqrt(c)*x**2)*c**2*x**8 + 88*a**2*c*x + 56*a*c**2*x**5)/(256*a**3*c*(a**2
 + 2*a*c*x**4 + c**2*x**8))
```

3.189 $\int \frac{1}{(d+ex)(a+cx^4)^3} dx$

Optimal result

Integrand size = 17, antiderivative size = 1133

$$\int \frac{1}{(d+ex)(a+cx^4)^3} dx = \text{Too large to display}$$

output
```
1/8*e^3/(a*e^4+c*d^4)/(c*x^4+a)^2+1/8*c*x*(d*e^2*x^2-d^2*e*x+d^3)/a/(a*e^4
+c*d^4)/(c*x^4+a)^2+1/4*e^7/(a*e^4+c*d^4)^2/(c*x^4+a)+1/4*c*e^4*x*(d*e^2*x
^2-d^2*e*x+d^3)/a/(a*e^4+c*d^4)^2/(c*x^4+a)+1/32*c*x*(5*d*e^2*x^2-6*d^2*e*
x+7*d^3)/a^2/(a*e^4+c*d^4)/(c*x^4+a)-1/2*c^(1/2)*d^2*e^9*arctan(c^(1/2)*x^
2/a^(1/2))/a^(1/2)/(a*e^4+c*d^4)^3-1/4*c^(1/2)*d^2*e^5*arctan(c^(1/2)*x^2/
a^(1/2))/a^(3/2)/(a*e^4+c*d^4)^2-3/16*c^(1/2)*d^2*e*arctan(c^(1/2)*x^2/a^(
1/2))/a^(5/2)/(a*e^4+c*d^4)+1/4*c^(1/4)*d*e^8*(c^(1/2)*d^2+a^(1/2)*e^2)*ar
ctan(-1+2^(1/2)*c^(1/4)*x/a^(1/4))*2^(1/2)/a^(3/4)/(a*e^4+c*d^4)^3+1/16*c^
(1/4)*d*e^4*(3*c^(1/2)*d^2+a^(1/2)*e^2)*arctan(-1+2^(1/2)*c^(1/4)*x/a^(1/4
))*2^(1/2)/a^(7/4)/(a*e^4+c*d^4)^2+1/128*c^(1/4)*d*(21*c^(1/2)*d^2+5*a^(1/
2)*e^2)*arctan(-1+2^(1/2)*c^(1/4)*x/a^(1/4))*2^(1/2)/a^(11/4)/(a*e^4+c*d^4
)+1/4*c^(1/4)*d*e^8*(c^(1/2)*d^2+a^(1/2)*e^2)*arctan(1+2^(1/2)*c^(1/4)*x/a
^(1/4))*2^(1/2)/a^(3/4)/(a*e^4+c*d^4)^3+1/16*c^(1/4)*d*e^4*(3*c^(1/2)*d^2+
a^(1/2)*e^2)*arctan(1+2^(1/2)*c^(1/4)*x/a^(1/4))*2^(1/2)/a^(7/4)/(a*e^4+c*
d^4)^2+1/128*c^(1/4)*d*(21*c^(1/2)*d^2+5*a^(1/2)*e^2)*arctan(1+2^(1/2)*c^(
1/4)*x/a^(1/4))*2^(1/2)/a^(11/4)/(a*e^4+c*d^4)+1/4*c^(1/4)*d*e^8*(c^(1/2)*
d^2-a^(1/2)*e^2)*arctanh(2^(1/2)*a^(1/4)*c^(1/4)*x/(a^(1/2)+c^(1/2)*x^2))*
2^(1/2)/a^(3/4)/(a*e^4+c*d^4)^3+1/16*c^(1/4)*d*e^4*(3*c^(1/2)*d^2-a^(1/2)*
e^2)*arctanh(2^(1/2)*a^(1/4)*c^(1/4)*x/(a^(1/2)+c^(1/2)*x^2))*2^(1/2)/a^(7
/4)/(a*e^4+c*d^4)^2+1/128*c^(1/4)*d*(21*c^(1/2)*d^2-5*a^(1/2)*e^2)*arct...
```

Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 835, normalized size of antiderivative = 0.74

$$\int \frac{1}{(d+ex)\left(a+cx^4\right)^3}\,dx$$

$$= \frac{\frac{32(cd^4+ae^4)^2(ae^3+cdx(d^2-dex+e^2x^2))}{a(a+cx^4)^2}+\frac{8(cd^4+ae^4)(8a^2e^7+c^2d^5x(7d^2-6dex+5e^2x^2)+acde^4x(15d^2-14dex+13e^2x^2))}{a^2(a+cx^4)}-\frac{2\sqrt[4]{c}d\Big(21}{}}{}$$

input
```
Integrate[1/((d + e*x)*(a + c*x^4)^3),x]
```

output
```
((32*(c*d^4 + a*e^4)^2*(a*e^3 + c*d*x*(d^2 - d*e*x + e^2*x^2)))/(a*(a + c*
x^4)^2) + (8*(c*d^4 + a*e^4)*(8*a^2*e^7 + c^2*d^5*x*(7*d^2 - 6*d*e*x + 5*e
^2*x^2) + a*c*d*e^4*x*(15*d^2 - 14*d*e*x + 13*e^2*x^2)))/(a^2*(a + c*x^4))
 - (2*c^(1/4)*d*(21*Sqrt[2]*c^(5/2)*d^10 - 24*a^(1/4)*c^(9/4)*d^9*e + 5*Sq
rt[2]*Sqrt[a]*c^2*d^8*e^2 + 66*Sqrt[2]*a*c^(3/2)*d^6*e^4 - 80*a^(5/4)*c^(5
/4)*d^5*e^5 + 18*Sqrt[2]*a^(3/2)*c*d^4*e^6 + 77*Sqrt[2]*a^2*Sqrt[c]*d^2*e^
8 - 120*a^(9/4)*c^(1/4)*d*e^9 + 45*Sqrt[2]*a^(5/2)*e^10)*ArcTan[1 - (Sqrt[
2]*c^(1/4)*x)/a^(1/4)])/a^(11/4) + (2*c^(1/4)*d*(21*Sqrt[2]*c^(5/2)*d^10 +
 24*a^(1/4)*c^(9/4)*d^9*e + 5*Sqrt[2]*Sqrt[a]*c^2*d^8*e^2 + 66*Sqrt[2]*a*c
^(3/2)*d^6*e^4 + 80*a^(5/4)*c^(5/4)*d^5*e^5 + 18*Sqrt[2]*a^(3/2)*c*d^4*e^6
 + 77*Sqrt[2]*a^2*Sqrt[c]*d^2*e^8 + 120*a^(9/4)*c^(1/4)*d*e^9 + 45*Sqrt[2]
*a^(5/2)*e^10)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/a^(11/4) + 256*e^1
1*Log[d + e*x] + (Sqrt[2]*c^(1/4)*(-21*c^(5/2)*d^11 + 5*Sqrt[a]*c^2*d^9*e^
2 - 66*a*c^(3/2)*d^7*e^4 + 18*a^(3/2)*c*d^5*e^6 - 77*a^2*Sqrt[c]*d^3*e^8 +
 45*a^(5/2)*d*e^10)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]
)/a^(11/4) + (Sqrt[2]*c^(1/4)*(21*c^(5/2)*d^11 - 5*Sqrt[a]*c^2*d^9*e^2 + 6
6*a*c^(3/2)*d^7*e^4 - 18*a^(3/2)*c*d^5*e^6 + 77*a^2*Sqrt[c]*d^3*e^8 - 45*a
^(5/2)*d*e^10)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/a^(
11/4) - 64*e^11*Log[a + c*x^4])/(256*(c*d^4 + a*e^4)^3)
```

Rubi [A] (verified)

Time = 2.95 (sec) , antiderivative size = 1352, normalized size of antiderivative = 1.19, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+cx^4)^3 (d+ex)} \, dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(\frac{e^{12}}{(d+ex)(ae^4+cd^4)^3} - \frac{ce^4\left(-d^3+d^2ex-de^2x^2+e^3x^3\right)}{(a+cx^4)^2 (ae^4+cd^4)^2} + \frac{c\left(d^3-d^2ex+de^2x^2-e^3x^3\right)}{(a+cx^4)^3 (ae^4+cd^4)} - \frac{ce^8\left(-d^3+d^2e\right.}{(a+cx^4)}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{\log(d+ex)e^{11}}{(cd^4+ae^4)^3}-\frac{\log\left(cx^4+a\right)e^{11}}{4\left(cd^4+ae^4\right)^3}-\frac{\sqrt{c}d^2\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)e^9}{2\sqrt{a}\left(cd^4+ae^4\right)^3}-\frac{\sqrt[4]{c}d(\sqrt{c}d^2+\sqrt{a}e^2)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)e^8}{2\sqrt{2}a^{3/4}\left(cd^4+ae^4\right)^3}+\frac{\sqrt[4]{c}d(\sqrt{c}d^2+\sqrt{a}e^2)\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)e^8}{2\sqrt{2}a^{3/4}\left(cd^4+ae^4\right)^3}-\frac{\sqrt[4]{c}d(\sqrt{c}d^2-\sqrt{a}e^2)\log\left(\sqrt{c}x^2-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}\right)e^8}{4\sqrt{2}a^{3/4}\left(cd^4+ae^4\right)^3}+\frac{\sqrt[4]{c}d(\sqrt{c}d^2-\sqrt{a}e^2)\log\left(\sqrt{c}x^2+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}\right)e^8}{4\sqrt{2}a^{3/4}\left(cd^4+ae^4\right)^3}-\frac{\sqrt{c}d^2\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)e^5}{4a^{3/2}\left(cd^4+ae^4\right)^2}+\frac{\left(ae^3+cx\left(d^3-exd^2+e^2x^2d\right)\right)e^4}{4a\left(cd^4+ae^4\right)^2\left(cx^4+a\right)}-\frac{\sqrt[4]{c}d(3\sqrt{c}d^2+\sqrt{a}e^2)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)e^4}{8\sqrt{2}a^{7/4}\left(cd^4+ae^4\right)^2}+\frac{\sqrt[4]{c}d(3\sqrt{c}d^2+\sqrt{a}e^2)\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)e^4}{8\sqrt{2}a^{7/4}\left(cd^4+ae^4\right)^2}-\frac{\sqrt[4]{c}d(3\sqrt{c}d^2-\sqrt{a}e^2)\log\left(\sqrt{c}x^2-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}\right)e^4}{16\sqrt{2}a^{7/4}\left(cd^4+ae^4\right)^2}+\frac{\sqrt[4]{c}d(3\sqrt{c}d^2-\sqrt{a}e^2)\log\left(\sqrt{c}x^2+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}\right)e^4}{16\sqrt{2}a^{7/4}\left(cd^4+ae^4\right)^2}-\frac{3\sqrt{c}d^2\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)e}{16a^{5/2}\left(cd^4+ae^4\right)}+\frac{ae^3+cx\left(d^3-exd^2+e^2x^2d\right)}{8a\left(cd^4+ae^4\right)\left(cx^4+a\right)^2}-\frac{\sqrt[4]{c}d(21\sqrt{c}d^2+5\sqrt{a}e^2)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}\left(cd^4+ae^4\right)}+\frac{\sqrt[4]{c}d(21\sqrt{c}d^2+5\sqrt{a}e^2)\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{64\sqrt{2}a^{11/4}\left(cd^4+ae^4\right)}-\frac{\sqrt[4]{c}d(21\sqrt{c}d^2-5\sqrt{a}e^2)\log\left(\sqrt{c}x^2-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}\right)}{128\sqrt{2}a^{11/4}\left(cd^4+ae^4\right)}+\frac{\sqrt[4]{c}d(21\sqrt{c}d^2-5\sqrt{a}e^2)\log\left(\sqrt{c}x^2+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}\right)}{128\sqrt{2}a^{11/4}\left(cd^4+ae^4\right)}+\frac{cx\left(7d^3-6exd^2+5e^2x^2d\right)}{32a^2\left(cd^4+ae^4\right)\left(cx^4+a\right)}$$

input
```
Int[1/((d + e*x)*(a + c*x^4)^3),x]
```

output
```
(c*x*(7*d^3 - 6*d^2*e*x + 5*d*e^2*x^2))/(32*a^2*(c*d^4 + a*e^4)*(a + c*x^4
)) + (a*e^3 + c*x*(d^3 - d^2*e*x + d*e^2*x^2))/(8*a*(c*d^4 + a*e^4)*(a + c
*x^4)^2) + (e^4*(a*e^3 + c*x*(d^3 - d^2*e*x + d*e^2*x^2)))/(4*a*(c*d^4 + a
*e^4)^2*(a + c*x^4)) - (Sqrt[c]*d^2*e^9*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*
Sqrt[a]*(c*d^4 + a*e^4)^3) - (Sqrt[c]*d^2*e^5*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]
])/(4*a^(3/2)*(c*d^4 + a*e^4)^2) - (3*Sqrt[c]*d^2*e*ArcTan[(Sqrt[c]*x^2)/S
qrt[a]])/(16*a^(5/2)*(c*d^4 + a*e^4)) - (c^(1/4)*d*e^8*(Sqrt[c]*d^2 + Sqrt
[a]*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^
4 + a*e^4)^3) - (c^(1/4)*d*e^4*(3*Sqrt[c]*d^2 + Sqrt[a]*e^2)*ArcTan[1 - (S
qrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(c*d^4 + a*e^4)^2) - (c^(1/
4)*d*(21*Sqrt[c]*d^2 + 5*Sqrt[a]*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/
4)])/(64*Sqrt[2]*a^(11/4)*(c*d^4 + a*e^4)) + (c^(1/4)*d*e^8*(Sqrt[c]*d^2 +
 Sqrt[a]*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*
(c*d^4 + a*e^4)^3) + (c^(1/4)*d*e^4*(3*Sqrt[c]*d^2 + Sqrt[a]*e^2)*ArcTan[1
 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(c*d^4 + a*e^4)^2) + (
c^(1/4)*d*(21*Sqrt[c]*d^2 + 5*Sqrt[a]*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/
a^(1/4)])/(64*Sqrt[2]*a^(11/4)*(c*d^4 + a*e^4)) + (e^11*Log[d + e*x])/(c*d
^4 + a*e^4)^3 - (c^(1/4)*d*e^8*(Sqrt[c]*d^2 - Sqrt[a]*e^2)*Log[Sqrt[a] - S
qrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4
)^3) - (c^(1/4)*d*e^4*(3*Sqrt[c]*d^2 - Sqrt[a]*e^2)*Log[Sqrt[a] - Sqrt[...
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 677, normalized size of antiderivative = 0.60

method	result
default	$c\left(\frac{\frac{c d\,e^{2}\left(13a^{2}e^{8}+18ac\,d^{4}e^{4}+5d^{8}c^{2}\right)x^{7}}{32a^{2}}-\frac{d^{2}ec\left(7a^{2}e^{8}+10ac\,d^{4}e^{4}+3d^{8}c^{2}\right)x^{6}}{16a^{2}}+\frac{c\,d^{3}\left(15a^{2}e^{8}+22ac\,d^{4}e^{4}+7d^{8}c^{2}\right)x^{5}}{32a^{2}}+\left(\frac{1}{4}a\,e^{11}+\frac{1}{4}d^{4}e^{7}c\right)x^{4}+\frac{d\,e}{}}{\left(c\,x^{4}+a\right.}\right.$
risch	Expression too large to display

input

```
int(1/(e*x+d)/(c*x^4+a)^3,x,method=_RETURNVERBOSE)
```

output

```
c/(a*e^4+c*d^4)^3*((1/32*c*d*e^2*(13*a^2*e^8+18*a*c*d^4*e^4+5*c^2*d^8)/a^2
*x^7-1/16*d^2*e*c*(7*a^2*e^8+10*a*c*d^4*e^4+3*c^2*d^8)/a^2*x^6+1/32*c*d^3*
(15*a^2*e^8+22*a*c*d^4*e^4+7*c^2*d^8)/a^2*x^5+(1/4*a*e^11+1/4*d^4*e^7*c)*x
^4+1/32*d*e^2*(17*a^2*e^8+26*a*c*d^4*e^4+9*c^2*d^8)/a*x^3-1/16*d^2*e*(9*a^
2*e^8+14*a*c*d^4*e^4+5*c^2*d^8)/a*x^2+1/32*d^3*(19*a^2*e^8+30*a*c*d^4*e^4+
11*c^2*d^8)/a*x+1/8*e^3*(3*a^2*e^8+4*a*c*d^4*e^4+c^2*d^8)/c)/(c*x^4+a)^2+1
/32/a^2*(1/8*(77*a^2*d^3*e^8+66*a*c*d^7*e^4+21*c^2*d^11)*(a/c)^(1/4)/a*2^(
1/2)*(ln((x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*x*2^(1/2
)+(a/c)^(1/2)))+2*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/c)^(
1/4)*x-1))+1/2*(-60*a^2*d^2*e^9-40*a*c*d^6*e^5-12*c^2*d^10*e)/(a*c)^(1/2)*
arctan((c/a)^(1/2)*x^2)+1/8*(45*a^2*d*e^10+18*a*c*d^5*e^6+5*c^2*d^9*e^2)/c
/(a/c)^(1/4)*2^(1/2)*(ln((x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2+(a/c
)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+2*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+2*arctan
(2^(1/2)/(a/c)^(1/4)*x-1))-8*a^2*e^11/c*ln(c*x^4+a)))+e^11*ln(e*x+d)/(a*e^
4+c*d^4)^3
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)\,(a+cx^4)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x+d)/(c*x^4+a)^3,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)\,(a+cx^4)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x+d)/(c*x**4+a)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 1015, normalized size of antiderivative = 0.90

$$\int \frac{1}{(d+ex)\,(a+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*x+d)/(c*x^4+a)^3,x, algorithm="maxima")
```

output
```
e^11*log(e*x + d)/(c^3*d^12 + 3*a*c^2*d^8*e^4 + 3*a^2*c*d^4*e^8 + a^3*e^12
) - 1/256*c*(sqrt(2)*(32*sqrt(2)*a^(11/4)*c^(1/4)*e^11 - 21*c^3*d^11 + 5*s
qrt(a)*c^(5/2)*d^9*e^2 - 66*a*c^2*d^7*e^4 + 18*a^(3/2)*c^(3/2)*d^5*e^6 - 7
7*a^2*c*d^3*e^8 + 45*a^(5/2)*sqrt(c)*d*e^10)*log(sqrt(c)*x^2 + sqrt(2)*a^(
1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(5/4)) + sqrt(2)*(32*sqrt(2)*a^(11/4)
*c^(1/4)*e^11 + 21*c^3*d^11 - 5*sqrt(a)*c^(5/2)*d^9*e^2 + 66*a*c^2*d^7*e^4
 - 18*a^(3/2)*c^(3/2)*d^5*e^6 + 77*a^2*c*d^3*e^8 - 45*a^(5/2)*sqrt(c)*d*e^
10)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(5/4
)) - 2*(21*sqrt(2)*a^(1/4)*c^(13/4)*d^11 + 5*sqrt(2)*a^(3/4)*c^(11/4)*d^9*
e^2 + 66*sqrt(2)*a^(5/4)*c^(9/4)*d^7*e^4 + 18*sqrt(2)*a^(7/4)*c^(7/4)*d^5*
e^6 + 77*sqrt(2)*a^(9/4)*c^(5/4)*d^3*e^8 + 45*sqrt(2)*a^(11/4)*c^(3/4)*d*e
^10 + 24*sqrt(a)*c^3*d^10*e + 80*a^(3/2)*c^2*d^6*e^5 + 120*a^(5/2)*c*d^2*e
^9)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a
)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(5/4)) - 2*(21*sqrt(2)*a^(1/4
)*c^(13/4)*d^11 + 5*sqrt(2)*a^(3/4)*c^(11/4)*d^9*e^2 + 66*sqrt(2)*a^(5/4)*
c^(9/4)*d^7*e^4 + 18*sqrt(2)*a^(7/4)*c^(7/4)*d^5*e^6 + 77*sqrt(2)*a^(9/4)*
c^(5/4)*d^3*e^8 + 45*sqrt(2)*a^(11/4)*c^(3/4)*d*e^10 - 24*sqrt(a)*c^3*d^10
*e - 80*a^(3/2)*c^2*d^6*e^5 - 120*a^(5/2)*c*d^2*e^9)*arctan(1/2*sqrt(2)*(2
*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt
(sqrt(a)*sqrt(c))*c^(5/4)))/(a^2*c^3*d^12 + 3*a^3*c^2*d^8*e^4 + 3*a^4*c...
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 1311, normalized size of antiderivative = 1.16

$$\int \frac{1}{(d+ex)\,(a+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*x+d)/(c*x^4+a)^3,x, algorithm="giac")
```

output
```
e^12*log(abs(e*x + d))/(c^3*d^12*e + 3*a*c^2*d^8*e^5 + 3*a^2*c*d^4*e^9 + a
^3*e^13) - 1/4*e^11*log(abs(c*x^4 + a))/(c^3*d^12 + 3*a*c^2*d^8*e^4 + 3*a^
2*c*d^4*e^8 + a^3*e^12) - 1/64*(75*sqrt(2)*a*c^2*d^2*e^3 - 51*sqrt(2)*sqrt
(a*c)*c^2*d^4*e - 21*(a*c^3)^(1/4)*c^2*d^5 - 45*(a*c^3)^(1/4)*a*c*d*e^4 -
122*(a*c^3)^(3/4)*d^3*e^2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/
(a/c)^(1/4))/(sqrt(2)*a^3*c^3*d^6 + 9*sqrt(2)*a^4*c^2*d^2*e^4 + 9*sqrt(2)*
sqrt(a*c)*a^3*c^2*d^4*e^2 + sqrt(2)*sqrt(a*c)*a^4*c*e^6 - 6*(a*c^3)^(1/4)*
a^3*c^2*d^5*e - 6*(a*c^3)^(1/4)*a^4*c*d*e^5 - 16*(a*c^3)^(3/4)*a^3*d^3*e^3
) + 1/64*(75*sqrt(2)*a*c^2*d^2*e^3 + 51*sqrt(2)*sqrt(a*c)*c^2*d^4*e + 21*(
a*c^3)^(1/4)*c^2*d^5 + 45*(a*c^3)^(1/4)*a*c*d*e^4 + 122*(a*c^3)^(3/4)*d^3*
e^2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*
a^3*c^3*d^6 + 9*sqrt(2)*a^4*c^2*d^2*e^4 + 9*sqrt(2)*sqrt(a*c)*a^3*c^2*d^4*
e^2 + sqrt(2)*sqrt(a*c)*a^4*c*e^6 + 6*(a*c^3)^(1/4)*a^3*c^2*d^5*e + 6*(a*c
^3)^(1/4)*a^4*c*d*e^5 + 16*(a*c^3)^(3/4)*a^3*d^3*e^3) + 1/256*(21*sqrt(2)*
(a*c^3)^(1/4)*c^4*d^11 + 66*sqrt(2)*(a*c^3)^(1/4)*a*c^3*d^7*e^4 + 77*sqrt(
2)*(a*c^3)^(1/4)*a^2*c^2*d^3*e^8 - 5*sqrt(2)*(a*c^3)^(3/4)*c^2*d^9*e^2 - 1
8*sqrt(2)*(a*c^3)^(3/4)*a*c*d^5*e^6 - 45*sqrt(2)*(a*c^3)^(3/4)*a^2*d*e^10)
*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^3*c^5*d^12 + 3*a^4*c^4*d^
8*e^4 + 3*a^5*c^3*d^4*e^8 + a^6*c^2*e^12) - 1/256*(21*sqrt(2)*(a*c^3)^(1/4
)*c^4*d^11 + 66*sqrt(2)*(a*c^3)^(1/4)*a*c^3*d^7*e^4 + 77*sqrt(2)*(a*c^3...
```

Mupad [B] (verification not implemented)

Time = 23.61 (sec) , antiderivative size = 2720, normalized size of antiderivative = 2.40

$$\int \frac{1}{(d+ex)\left(a+cx^4\right)^3}\,dx = \text{Too large to display}$$

input
```
int(1/((a + c*x^4)^3*(d + e*x)),x)
```

output
```
symsum(log((194481*c^7*d^13*e^6 + 871362*a*c^6*d^9*e^10 + 425984*a^3*c^4*d
*e^18 + 1148881*a^2*c^5*d^5*e^14)/(1048576*(a^12*e^16 + a^8*c^4*d^16 + 4*a
^11*c*d^4*e^12 + 4*a^9*c^3*d^12*e^4 + 6*a^10*c^2*d^8*e^8)) + root(80530636
8*a^12*c^2*d^8*e^4*z^4 + 805306368*a^13*c*d^4*e^8*z^4 + 268435456*a^11*c^3
*d^12*z^4 + 268435456*a^14*e^12*z^4 + 268435456*a^11*e^11*z^3 + 43057152*a
^7*c*d^4*e^6*z^2 + 11599872*a^6*c^2*d^8*e^2*z^2 + 100663296*a^8*e^10*z^2 +
 9652224*a^4*c*d^4*e^5*z + 2709504*a^3*c^2*d^8*e*z + 16777216*a^5*e^9*z +
676881*a*c*d^4*e^4 + 194481*c^2*d^8 + 1048576*a^2*e^8, z, k)*(root(8053063
68*a^12*c^2*d^8*e^4*z^4 + 805306368*a^13*c*d^4*e^8*z^4 + 268435456*a^11*c^
3*d^12*z^4 + 268435456*a^14*e^12*z^4 + 268435456*a^11*e^11*z^3 + 43057152*
a^7*c*d^4*e^6*z^2 + 11599872*a^6*c^2*d^8*e^2*z^2 + 100663296*a^8*e^10*z^2
+ 9652224*a^4*c*d^4*e^5*z + 2709504*a^3*c^2*d^8*e*z + 16777216*a^5*e^9*z +
 676881*a*c*d^4*e^4 + 194481*c^2*d^8 + 1048576*a^2*e^8, z, k)*(root(805306
368*a^12*c^2*d^8*e^4*z^4 + 805306368*a^13*c*d^4*e^8*z^4 + 268435456*a^11*c
^3*d^12*z^4 + 268435456*a^14*e^12*z^4 + 268435456*a^11*e^11*z^3 + 43057152
*a^7*c*d^4*e^6*z^2 + 11599872*a^6*c^2*d^8*e^2*z^2 + 100663296*a^8*e^10*z^2
 + 9652224*a^4*c*d^4*e^5*z + 2709504*a^3*c^2*d^8*e*z + 16777216*a^5*e^9*z
+ 676881*a*c*d^4*e^4 + 194481*c^2*d^8 + 1048576*a^2*e^8, z, k)*(root(80530
6368*a^12*c^2*d^8*e^4*z^4 + 805306368*a^13*c*d^4*e^8*z^4 + 268435456*a^11*
c^3*d^12*z^4 + 268435456*a^14*e^12*z^4 + 268435456*a^11*e^11*z^3 + 4305...
```

Reduce [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 4499, normalized size of antiderivative = 3.97

$$\int \frac{1}{(d+ex)(a+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
int(1/(e*x+d)/(c*x^4+a)^3,x)
```

output

```
( - 90*c**(1/4)*a**(3/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(
c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a**4*d*e**10 - 36*c**(1/4)*a**(3/4)*sqr
t(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqr
t(2)))*a**3*c*d**5*e**6 - 180*c**(1/4)*a**(3/4)*sqrt(2)*atan((c**(1/4)*a**
(1/4)*sqrt(2) - 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a**3*c*d*e**10*x
**4 - 10*c**(1/4)*a**(3/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqr
t(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a**2*c**2*d**9*e**2 - 72*c**(1/4)*a**
(3/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x)/(c**(1/4)*a**
(1/4)*sqrt(2)))*a**2*c**2*d**5*e**6*x**4 - 90*c**(1/4)*a**(3/4)*sqrt(2)*at
an((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*
a**2*c**2*d*e**10*x**8 - 20*c**(1/4)*a**(3/4)*sqrt(2)*atan((c**(1/4)*a**(1
/4)*sqrt(2) - 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a*c**3*d**9*e**2*x
**4 - 36*c**(1/4)*a**(3/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqr
t(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a*c**3*d**5*e**6*x**8 - 10*c**(1/4)*a
**(3/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x)/(c**(1/4)*a
**(1/4)*sqrt(2)))*c**4*d**9*e**2*x**8 - 154*c**(3/4)*a**(1/4)*sqrt(2)*atan
((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a*
*4*d**3*e**8 - 132*c**(3/4)*a**(1/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(
2) - 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a**3*c*d**7*e**4 - 308*c**(
3/4)*a**(1/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x)/(c...
```

3.190 $\int \frac{1}{(d+ex)^2 (a+cx^4)^3} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 1533

$$\int \frac{1}{(d+ex)^2 (a+cx^4)^3} \, dx = \text{Too large to display}$$

output
```
-e^11/(a*e^4+c*d^4)^3/(e*x+d)+1/2*c*d^3*e^3/(a*e^4+c*d^4)^2/(c*x^4+a)^2+1/
8*c*x*(d^2*(-3*a*e^4+c*d^4)-2*d*e*(-a*e^4+c*d^4)*x+e^2*(-a*e^4+3*c*d^4)*x^
2)/a/(a*e^4+c*d^4)^2/(c*x^4+a)^2+2*c*d^3*e^7/(a*e^4+c*d^4)^3/(c*x^4+a)+1/3
2*c*x*(7*d^2*(-3*a*e^4+c*d^4)-12*d*e*(-a*e^4+c*d^4)*x+5*e^2*(-a*e^4+3*c*d^
4)*x^2)/a^2/(a*e^4+c*d^4)^2/(c*x^4+a)+1/4*c*e^4*x*(d^2*(-3*a*e^4+5*c*d^4)-
2*d*e*(-a*e^4+3*c*d^4)*x+e^2*(-a*e^4+7*c*d^4)*x^2)/a/(a*e^4+c*d^4)^3/(c*x^
4+a)-c^(1/2)*d*e^9*(-a*e^4+5*c*d^4)*arctan(c^(1/2)*x^2/a^(1/2))/a^(1/2)/(a
*e^4+c*d^4)^4-1/2*c^(1/2)*d*e^5*(-a*e^4+3*c*d^4)*arctan(c^(1/2)*x^2/a^(1/2
))/a^(3/2)/(a*e^4+c*d^4)^3-3/8*c^(1/2)*d*e*(-a*e^4+c*d^4)*arctan(c^(1/2)*x
^2/a^(1/2))/a^(5/2)/(a*e^4+c*d^4)^2+1/128*c^(1/4)*(21*c^(1/2)*d^2*(-3*a*e^
4+c*d^4)+5*a^(1/2)*e^2*(-a*e^4+3*c*d^4))*arctan(-1+2^(1/2)*c^(1/4)*x/a^(1/
4))*2^(1/2)/a^(11/4)/(a*e^4+c*d^4)^2+1/16*c^(1/4)*e^4*(3*c^(1/2)*d^2*(-3*a
*e^4+5*c*d^4)+a^(1/2)*e^2*(-a*e^4+7*c*d^4))*arctan(-1+2^(1/2)*c^(1/4)*x/a^
(1/4))*2^(1/2)/a^(7/4)/(a*e^4+c*d^4)^3+1/4*c^(1/4)*e^8*(3*c^(1/2)*d^2*(-a*
e^4+3*c*d^4)+a^(1/2)*e^2*(-a*e^4+11*c*d^4))*arctan(-1+2^(1/2)*c^(1/4)*x/a^
(1/4))*2^(1/2)/a^(3/4)/(a*e^4+c*d^4)^4+1/128*c^(1/4)*(21*c^(1/2)*d^2*(-3*a
*e^4+c*d^4)+5*a^(1/2)*e^2*(-a*e^4+3*c*d^4))*arctan(1+2^(1/2)*c^(1/4)*x/a^(
1/4))*2^(1/2)/a^(11/4)/(a*e^4+c*d^4)^2+1/16*c^(1/4)*e^4*(3*c^(1/2)*d^2*(-3
*a*e^4+5*c*d^4)+a^(1/2)*e^2*(-a*e^4+7*c*d^4))*arctan(1+2^(1/2)*c^(1/4)*x/a
^(1/4))*2^(1/2)/a^(7/4)/(a*e^4+c*d^4)^3+1/4*c^(1/4)*e^8*(3*c^(1/2)*d^2*...
```

Mathematica [A] (verified)

Time = 1.33 (sec) , antiderivative size = 1115, normalized size of antiderivative = 0.73

$$\int \frac{1}{(d+ex)^2 (a+cx^4)^3} \, dx = \text{Too large to display}$$

input
```
Integrate[1/((d + e*x)^2*(a + c*x^4)^3),x]
```

output

```
((-256*e^11*(c*d^4 + a*e^4))/(d + e*x) + (8*c*(c*d^4 + a*e^4)*(c^2*d^8*x*(
7*d^2 - 12*d*e*x + 15*e^2*x^2) + 2*a*c*d^4*e^4*x*(13*d^2 - 24*d*e*x + 33*e
^2*x^2) + a^2*e^7*(64*d^3 - 45*d^2*e*x + 28*d*e^2*x^2 - 13*e^3*x^3)))/(a^2
*(a + c*x^4)) + (32*c*(c*d^4 + a*e^4)^2*(c*d^4*x*(d^2 - 2*d*e*x + 3*e^2*x^
2) + a*e^3*(4*d^3 - 3*d^2*e*x + 2*d*e^2*x^2 - e^3*x^3)))/(a*(a + c*x^4)^2)
 - (6*c^(1/4)*(7*Sqrt[2]*c^(7/2)*d^14 - 16*a^(1/4)*c^(13/4)*d^13*e + 5*Sqr
t[2]*Sqrt[a]*c^3*d^12*e^2 + 33*Sqrt[2]*a*c^(5/2)*d^10*e^4 - 80*a^(5/4)*c^(
9/4)*d^9*e^5 + 27*Sqrt[2]*a^(3/2)*c^2*d^8*e^6 + 77*Sqrt[2]*a^2*c^(3/2)*d^6
*e^8 - 240*a^(9/4)*c^(5/4)*d^5*e^9 + 135*Sqrt[2]*a^(5/2)*c*d^4*e^10 - 77*S
qrt[2]*a^3*Sqrt[c]*d^2*e^12 + 80*a^(13/4)*c^(1/4)*d*e^13 - 15*Sqrt[2]*a^(7
/2)*e^14)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/a^(11/4) + (6*c^(1/4)*(
7*Sqrt[2]*c^(7/2)*d^14 + 16*a^(1/4)*c^(13/4)*d^13*e + 5*Sqrt[2]*Sqrt[a]*c^
3*d^12*e^2 + 33*Sqrt[2]*a*c^(5/2)*d^10*e^4 + 80*a^(5/4)*c^(9/4)*d^9*e^5 +
27*Sqrt[2]*a^(3/2)*c^2*d^8*e^6 + 77*Sqrt[2]*a^2*c^(3/2)*d^6*e^8 + 240*a^(9
/4)*c^(5/4)*d^5*e^9 + 135*Sqrt[2]*a^(5/2)*c*d^4*e^10 - 77*Sqrt[2]*a^3*Sqrt
[c]*d^2*e^12 - 80*a^(13/4)*c^(1/4)*d*e^13 - 15*Sqrt[2]*a^(7/2)*e^14)*ArcTa
n[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/a^(11/4) + 3072*c*d^3*e^11*Log[d + e*x
] - (3*Sqrt[2]*c^(1/4)*(7*c^(7/2)*d^14 - 5*Sqrt[a]*c^3*d^12*e^2 + 33*a*c^(
5/2)*d^10*e^4 - 27*a^(3/2)*c^2*d^8*e^6 + 77*a^2*c^(3/2)*d^6*e^8 - 135*a^(5
/2)*c*d^4*e^10 - 77*a^3*Sqrt[c]*d^2*e^12 + 15*a^(7/2)*e^14)*Log[Sqrt[a]...
```

Rubi [A] (verified)

Time = 4.90 (sec) , antiderivative size = 1830, normalized size of antiderivative = 1.19, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+cx^4)^3 (d+ex)^2} \, dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(\frac{e^{12}}{(d+ex)^2 (ae^4+cd^4)^3}+\frac{12cd^3e^{12}}{(d+ex) (ae^4+cd^4)^4}+\frac{ce^4 \left(-2dex \left(3cd^4-ae^4\right)+e^2x^2 \left(7cd^4-ae^4\right)+d^2 \left(5cd^4-\right.\right.}{(a+cx^4)^2 (ae^4+cd^4)^3}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{12cd^3\log(d+ex)e^{11}}{(cd^4+ae^4)^4}-\frac{3cd^3\log\left(cx^4+a\right)e^{11}}{(cd^4+ae^4)^4}-\frac{e^{11}}{(cd^4+ae^4)^3\,(d+ex)}-\frac{\sqrt{c}d\left(5cd^4-ae^4\right)\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)e^9}{\sqrt{a}\,(cd^4+ae^4)^4}-\frac{\sqrt[4]{c}\left(3\sqrt{c}\left(3cd^4-ae^4\right)d^2+\sqrt{a}e^2\left(11cd^4-ae^4\right)\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)e^8}{2\sqrt{2}a^{3/4}\,(cd^4+ae^4)^4}+\frac{\sqrt[4]{c}\left(3\sqrt{c}\left(3cd^4-ae^4\right)d^2+\sqrt{a}e^2\left(11cd^4-ae^4\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)e^8}{2\sqrt{2}a^{3/4}\,(cd^4+ae^4)^4}-\frac{\sqrt[4]{c}\left(9c^{3/2}d^6-11\sqrt{a}ce^2d^4-3a\sqrt{c}e^4d^2+a^{3/2}e^6\right)\log\left(\sqrt{c}x^2-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}\right)e^8}{4\sqrt{2}a^{3/4}\,(cd^4+ae^4)^4}+\frac{\sqrt[4]{c}\left(9c^{3/2}d^6-11\sqrt{a}ce^2d^4-3a\sqrt{c}e^4d^2+a^{3/2}e^6\right)\log\left(\sqrt{c}x^2+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}\right)e^8}{4\sqrt{2}a^{3/4}\,(cd^4+ae^4)^4}-\frac{\sqrt{c}d\left(3cd^4-ae^4\right)\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)e^5}{2a^{3/2}\,(cd^4+ae^4)^3}+\frac{c\left(8ad^3e^3+x\left(\left(5cd^4-3ae^4\right)d^2-2e\left(3cd^4-ae^4\right)xd+e^2\left(7cd^4-ae^4\right)x^2\right)\right)e^4}{4a\,(cd^4+ae^4)^3\,(cx^4+a)}-\frac{\sqrt[4]{c}\left(3\sqrt{c}\left(5cd^4-3ae^4\right)d^2+\sqrt{a}e^2\left(7cd^4-ae^4\right)\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)e^4}{8\sqrt{2}a^{7/4}\,(cd^4+ae^4)^3}+\frac{\sqrt[4]{c}\left(3\sqrt{c}\left(5cd^4-3ae^4\right)d^2+\sqrt{a}e^2\left(7cd^4-ae^4\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)e^4}{8\sqrt{2}a^{7/4}\,(cd^4+ae^4)^3}-\frac{\sqrt[4]{c}\left(3\sqrt{c}d^2\left(5cd^4-3ae^4\right)-\sqrt{a}e^2\left(7cd^4-ae^4\right)\right)\log\left(\sqrt{c}x^2-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}\right)e^4}{16\sqrt{2}a^{7/4}\,(cd^4+ae^4)^3}+\frac{\sqrt[4]{c}\left(3\sqrt{c}d^2\left(5cd^4-3ae^4\right)-\sqrt{a}e^2\left(7cd^4-ae^4\right)\right)\log\left(\sqrt{c}x^2+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}\right)e^4}{16\sqrt{2}a^{7/4}\,(cd^4+ae^4)^3}-\frac{3\sqrt{c}d\left(cd^4-ae^4\right)\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)e}{8a^{5/2}\,(cd^4+ae^4)^2}+\frac{c\left(4ad^3e^3+x\left(\left(cd^4-3ae^4\right)d^2-2e\left(cd^4-ae^4\right)xd+e^2\left(3cd^4-ae^4\right)x^2\right)\right)}{8a\,(cd^4+ae^4)^2\,(cx^4+a)^2}-\frac{\sqrt[4]{c}\left(21\sqrt{c}\left(cd^4-3ae^4\right)d^2+5\sqrt{a}e^2\left(3cd^4-ae^4\right)\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}\,(cd^4+ae^4)^2}+\frac{\sqrt[4]{c}\left(21\sqrt{c}\left(cd^4-3ae^4\right)d^2+5\sqrt{a}e^2\left(3cd^4-ae^4\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{64\sqrt{2}a^{11/4}\,(cd^4+ae^4)^2}-\frac{\sqrt[4]{c}\left(21\sqrt{c}d^2\left(cd^4-3ae^4\right)-5\sqrt{a}e^2\left(3cd^4-ae^4\right)\right)\log\left(\sqrt{c}x^2-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}\right)}{128\sqrt{2}a^{11/4}\,(cd^4+ae^4)^2}+\frac{\sqrt[4]{c}\left(21\sqrt{c}d^2\left(cd^4-3ae^4\right)-5\sqrt{a}e^2\left(3cd^4-ae^4\right)\right)\log\left(\sqrt{c}x^2+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}\right)}{128\sqrt{2}a^{11/4}\,(cd^4+ae^4)^2}+\frac{cx\left(7\left(cd^4-3ae^4\right)d^2-12e\left(cd^4-ae^4\right)xd+5e^2\left(3cd^4-ae^4\right)x^2\right)}{32a^2\,(cd^4+ae^4)^2\,(cx^4+a)}$$

input
```
Int[1/((d + e*x)^2*(a + c*x^4)^3),x]
```

output
```
-(e^11/((c*d^4 + a*e^4)^3*(d + e*x))) + (c*x*(7*d^2*(c*d^4 - 3*a*e^4) - 12
*d*e*(c*d^4 - a*e^4)*x + 5*e^2*(3*c*d^4 - a*e^4)*x^2))/(32*a^2*(c*d^4 + a*
e^4)^2*(a + c*x^4)) + (c*(4*a*d^3*e^3 + x*(d^2*(c*d^4 - 3*a*e^4) - 2*d*e*(
c*d^4 - a*e^4)*x + e^2*(3*c*d^4 - a*e^4)*x^2)))/(8*a*(c*d^4 + a*e^4)^2*(a
+ c*x^4)^2) + (c*e^4*(8*a*d^3*e^3 + x*(d^2*(5*c*d^4 - 3*a*e^4) - 2*d*e*(3*
c*d^4 - a*e^4)*x + e^2*(7*c*d^4 - a*e^4)*x^2)))/(4*a*(c*d^4 + a*e^4)^3*(a
+ c*x^4)) - (Sqrt[c]*d*e^9*(5*c*d^4 - a*e^4)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]]
)/(Sqrt[a]*(c*d^4 + a*e^4)^4) - (Sqrt[c]*d*e^5*(3*c*d^4 - a*e^4)*ArcTan[(S
qrt[c]*x^2)/Sqrt[a]])/(2*a^(3/2)*(c*d^4 + a*e^4)^3) - (3*Sqrt[c]*d*e*(c*d^
4 - a*e^4)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(8*a^(5/2)*(c*d^4 + a*e^4)^2) -
(c^(1/4)*(21*Sqrt[c]*d^2*(c*d^4 - 3*a*e^4) + 5*Sqrt[a]*e^2*(3*c*d^4 - a*e^
4))*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*(c*d^4 +
 a*e^4)^2) - (c^(1/4)*e^4*(3*Sqrt[c]*d^2*(5*c*d^4 - 3*a*e^4) + Sqrt[a]*e^2
*(7*c*d^4 - a*e^4))*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^
(7/4)*(c*d^4 + a*e^4)^3) - (c^(1/4)*e^8*(3*Sqrt[c]*d^2*(3*c*d^4 - a*e^4) +
 Sqrt[a]*e^2*(11*c*d^4 - a*e^4))*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/
(2*Sqrt[2]*a^(3/4)*(c*d^4 + a*e^4)^4) + (c^(1/4)*(21*Sqrt[c]*d^2*(c*d^4 -
3*a*e^4) + 5*Sqrt[a]*e^2*(3*c*d^4 - a*e^4))*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)
/a^(1/4)])/(64*Sqrt[2]*a^(11/4)*(c*d^4 + a*e^4)^2) + (c^(1/4)*e^4*(3*Sqrt[
c]*d^2*(5*c*d^4 - 3*a*e^4) + Sqrt[a]*e^2*(7*c*d^4 - a*e^4))*ArcTan[1 + ...
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 829, normalized size of antiderivative = 0.54

method	result
default	$-\frac{c\left(\frac{c e^2\left(13a^3e^{12}-53a^2c\,d^4e^8-81a\,c^2d^8e^4-15c^3d^{12}\right)x^7}{32a^2}-\frac{cde\left(7a^3e^{12}-5a^2c\,d^4e^8-15a\,c^2d^8e^4-3c^3d^{12}\right)x^6}{8a^2}+\frac{c\,d^2\left(45a^3e^{12}+19a^2c\,d^4e^8-33\right.}{32a^2}\right.}{}$
risch	Expression too large to display

input
```
int(1/(e*x+d)^2/(c*x^4+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-c/(a*e^4+c*d^4)^4*((1/32*c*e^2*(13*a^3*e^12-53*a^2*c*d^4*e^8-81*a*c^2*d^8
*e^4-15*c^3*d^12)/a^2*x^7-1/8*c*d*e*(7*a^3*e^12-5*a^2*c*d^4*e^8-15*a*c^2*d
^8*e^4-3*c^3*d^12)/a^2*x^6+1/32*c*d^2*(45*a^3*e^12+19*a^2*c*d^4*e^8-33*a*c
^2*d^8*e^4-7*c^3*d^12)/a^2*x^5+(-2*a*c*d^3*e^11-2*c^2*d^7*e^7)*x^4+1/32*e^
2*(17*a^3*e^12-57*a^2*c*d^4*e^8-101*a*c^2*d^8*e^4-27*c^3*d^12)/a*x^3-1/8*d
*e*(9*a^3*e^12-3*a^2*c*d^4*e^8-17*a*c^2*d^8*e^4-5*c^3*d^12)/a*x^2+1/32*d^2
*(57*a^3*e^12+39*a^2*c*d^4*e^8-29*a*c^2*d^8*e^4-11*c^3*d^12)/a*x-5/2*a^2*d
^3*e^11-3*a*d^7*e^7*c-1/2*d^11*e^3*c^2)/(c*x^4+a)^2+3/32/a^2*(1/8*(77*a^3*
d^2*e^12-77*a^2*c*d^6*e^8-33*a*c^2*d^10*e^4-7*c^3*d^14)*(a/c)^(1/4)/a*2^(1
/2)*(ln((x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*x*2^(1/2)
+(a/c)^(1/2)))+2*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/c)^(1
/4)*x-1))+1/2*(-40*a^3*d*e^13+120*a^2*c*d^5*e^9+40*a*c^2*d^9*e^5+8*c^3*d^1
3*e)/(a*c)^(1/2)*arctan((c/a)^(1/2)*x^2)+1/8*(15*a^3*e^14-135*a^2*c*d^4*e^
10-27*a*c^2*d^8*e^6-5*c^3*d^12*e^2)/c/(a/c)^(1/4)*2^(1/2)*(ln((x^2-(a/c)^(
1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+2*arc
tan(2^(1/2)/(a/c)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/c)^(1/4)*x-1))+32*a^2*d^3
*e^11*ln(c*x^4+a)))-e^11/(a*e^4+c*d^4)^3/(e*x+d)+12*c*d^3*e^11*ln(e*x+d)/(
a*e^4+c*d^4)^4
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^2 (a+cx^4)^3} \, dx = \text{Timed out}$$

input
```
integrate(1/(e*x+d)^2/(c*x^4+a)^3,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^2 (a+cx^4)^3} \, dx = \text{Timed out}$$

input
```
integrate(1/(e*x+d)**2/(c*x**4+a)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 1564, normalized size of antiderivative = 1.02

$$\int \frac{1}{(d+ex)^2 (a+cx^4)^3} \, dx = \text{Too large to display}$$

input
```
integrate(1/(e*x+d)^2/(c*x^4+a)^3,x, algorithm="maxima")
```

output
```
12*c*d^3*e^11*log(e*x + d)/(c^4*d^16 + 4*a*c^3*d^12*e^4 + 6*a^2*c^2*d^8*e^
8 + 4*a^3*c*d^4*e^12 + a^4*e^16) - 3/256*c*(sqrt(2)*(128*sqrt(2)*a^(11/4)*
c^(5/4)*d^3*e^11 - 7*c^4*d^14 + 5*sqrt(a)*c^(7/2)*d^12*e^2 - 33*a*c^3*d^10
*e^4 + 27*a^(3/2)*c^(5/2)*d^8*e^6 - 77*a^2*c^2*d^6*e^8 + 135*a^(5/2)*c^(3/
2)*d^4*e^10 + 77*a^3*c*d^2*e^12 - 15*a^(7/2)*sqrt(c)*e^14)*log(sqrt(c)*x^2
 + sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(5/4)) + sqrt(2)*(128*s
qrt(2)*a^(11/4)*c^(5/4)*d^3*e^11 + 7*c^4*d^14 - 5*sqrt(a)*c^(7/2)*d^12*e^2
 + 33*a*c^3*d^10*e^4 - 27*a^(3/2)*c^(5/2)*d^8*e^6 + 77*a^2*c^2*d^6*e^8 - 1
35*a^(5/2)*c^(3/2)*d^4*e^10 - 77*a^3*c*d^2*e^12 + 15*a^(7/2)*sqrt(c)*e^14)
*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(5/4))
- 2*(7*sqrt(2)*a^(1/4)*c^(17/4)*d^14 + 5*sqrt(2)*a^(3/4)*c^(15/4)*d^12*e^2
 + 33*sqrt(2)*a^(5/4)*c^(13/4)*d^10*e^4 + 27*sqrt(2)*a^(7/4)*c^(11/4)*d^8*
e^6 + 77*sqrt(2)*a^(9/4)*c^(9/4)*d^6*e^8 + 135*sqrt(2)*a^(11/4)*c^(7/4)*d^
4*e^10 - 77*sqrt(2)*a^(13/4)*c^(5/4)*d^2*e^12 - 15*sqrt(2)*a^(15/4)*c^(3/4
)*e^14 + 16*sqrt(a)*c^4*d^13*e + 80*a^(3/2)*c^3*d^9*e^5 + 240*a^(5/2)*c^2*
d^5*e^9 - 80*a^(7/2)*c*d*e^13)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a
^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(a^(3/4)*sqrt(sqrt(a)*sqrt(c))*c^(5
/4)) - 2*(7*sqrt(2)*a^(1/4)*c^(17/4)*d^14 + 5*sqrt(2)*a^(3/4)*c^(15/4)*d^1
2*e^2 + 33*sqrt(2)*a^(5/4)*c^(13/4)*d^10*e^4 + 27*sqrt(2)*a^(7/4)*c^(11/4)
*d^8*e^6 + 77*sqrt(2)*a^(9/4)*c^(9/4)*d^6*e^8 + 135*sqrt(2)*a^(11/4)*c^...
```

Giac [A] (verification not implemented)

Time = 28.69 (sec) , antiderivative size = 1809, normalized size of antiderivative = 1.18

$$\int \frac{1}{(d+ex)^2 (a+cx^4)^3} dx = \text{Too large to display}$$

input
```
integrate(1/(e*x+d)^2/(c*x^4+a)^3,x, algorithm="giac")
```

output
```
12*c*d^3*e^12*log(abs(e*x + d))/(c^4*d^16*e + 4*a*c^3*d^12*e^5 + 6*a^2*c^2
*d^8*e^9 + 4*a^3*c*d^4*e^13 + a^4*e^17) - 3*c*d^3*e^11*log(abs(c*x^4 + a))
/(c^4*d^16 + 4*a*c^3*d^12*e^4 + 6*a^2*c^2*d^8*e^8 + 4*a^3*c*d^4*e^12 + a^4
*e^16) - 3/64*(32*sqrt(2)*a*c^3*d^3*e^3 - 20*sqrt(2)*sqrt(a*c)*c^3*d^5*e -
 20*sqrt(2)*sqrt(a*c)*a*c^2*d*e^5 - 7*(a*c^3)^(1/4)*c^3*d^6 - 3*(a*c^3)^(1
/4)*a*c^2*d^2*e^4 - 53*(a*c^3)^(3/4)*c*d^4*e^2 + 15*(a*c^3)^(3/4)*a*e^6)*a
rctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a^3*c^
4*d^8 + 34*sqrt(2)*a^4*c^3*d^4*e^4 + sqrt(2)*a^5*c^2*e^8 + 16*sqrt(2)*sqrt
(a*c)*a^3*c^3*d^6*e^2 + 16*sqrt(2)*sqrt(a*c)*a^4*c^2*d^2*e^6 - 8*(a*c^3)^(
1/4)*a^3*c^3*d^7*e - 40*(a*c^3)^(1/4)*a^4*c^2*d^3*e^5 - 40*(a*c^3)^(3/4)*a
^3*c*d^5*e^3 - 8*(a*c^3)^(3/4)*a^4*d*e^7) + 3/64*(32*sqrt(2)*a*c^3*d^3*e^3
 + 20*sqrt(2)*sqrt(a*c)*c^3*d^5*e + 20*sqrt(2)*sqrt(a*c)*a*c^2*d*e^5 + 7*(
a*c^3)^(1/4)*c^3*d^6 + 3*(a*c^3)^(1/4)*a*c^2*d^2*e^4 + 53*(a*c^3)^(3/4)*c*
d^4*e^2 - 15*(a*c^3)^(3/4)*a*e^6)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^
(1/4))/(a/c)^(1/4))/(sqrt(2)*a^3*c^4*d^8 + 34*sqrt(2)*a^4*c^3*d^4*e^4 + sq
rt(2)*a^5*c^2*e^8 + 16*sqrt(2)*sqrt(a*c)*a^3*c^3*d^6*e^2 + 16*sqrt(2)*sqrt
(a*c)*a^4*c^2*d^2*e^6 + 8*(a*c^3)^(1/4)*a^3*c^3*d^7*e + 40*(a*c^3)^(1/4)*a
^4*c^2*d^3*e^5 + 40*(a*c^3)^(3/4)*a^3*c*d^5*e^3 + 8*(a*c^3)^(3/4)*a^4*d*e^
7) + 3/256*(7*sqrt(2)*(a*c^3)^(1/4)*c^5*d^14 + 33*sqrt(2)*(a*c^3)^(1/4)*a*
c^4*d^10*e^4 + 77*sqrt(2)*(a*c^3)^(1/4)*a^2*c^3*d^6*e^8 - 77*sqrt(2)*(a...
```

Mupad [B] (verification not implemented)

Time = 25.24 (sec) , antiderivative size = 3572, normalized size of antiderivative = 2.33

$$\int \frac{1}{(d+ex)^2 (a+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
int(1/((a + c*x^4)^3*(d + e*x)^2),x)
```

output
```
symsum(log((194481*c^9*d^17*e^6 + 1527012*a*c^8*d^13*e^10 + 4100625*a^4*c^
5*d*e^22 + 1926342*a^2*c^7*d^9*e^14 - 3102300*a^3*c^6*d^5*e^18)/(1048576*(
a^14*e^24 + a^8*c^6*d^24 + 6*a^13*c*d^4*e^20 + 6*a^9*c^5*d^20*e^4 + 15*a^1
0*c^4*d^16*e^8 + 20*a^11*c^3*d^12*e^12 + 15*a^12*c^2*d^8*e^16)) + root(161
0612736*a^13*c^2*d^8*e^8*z^4 + 1073741824*a^12*c^3*d^12*e^4*z^4 + 10737418
24*a^14*c*d^4*e^12*z^4 + 268435456*a^11*c^4*d^16*z^4 + 268435456*a^15*e^16
*z^4 + 3221225472*a^11*c*d^3*e^11*z^3 + 239468544*a^7*c^2*d^6*e^6*z^2 + 39
518208*a^6*c^3*d^10*e^2*z^2 + 1153105920*a^8*c*d^2*e^10*z^2 + 32071680*a^4
*c^2*d^5*e^5*z + 5419008*a^3*c^3*d^9*e*z + 124416000*a^5*c*d*e^9*z + 11380
50*a*c^2*d^4*e^4 + 4100625*a^2*c*e^8 + 194481*c^3*d^8, z, k)*(root(1610612
736*a^13*c^2*d^8*e^8*z^4 + 1073741824*a^12*c^3*d^12*e^4*z^4 + 1073741824*a
^14*c*d^4*e^12*z^4 + 268435456*a^11*c^4*d^16*z^4 + 268435456*a^15*e^16*z^4
 + 3221225472*a^11*c*d^3*e^11*z^3 + 239468544*a^7*c^2*d^6*e^6*z^2 + 395182
08*a^6*c^3*d^10*e^2*z^2 + 1153105920*a^8*c*d^2*e^10*z^2 + 32071680*a^4*c^2
*d^5*e^5*z + 5419008*a^3*c^3*d^9*e*z + 124416000*a^5*c*d*e^9*z + 1138050*a
*c^2*d^4*e^4 + 4100625*a^2*c*e^8 + 194481*c^3*d^8, z, k)*(root(1610612736*
a^13*c^2*d^8*e^8*z^4 + 1073741824*a^12*c^3*d^12*e^4*z^4 + 1073741824*a^14*
c*d^4*e^12*z^4 + 268435456*a^11*c^4*d^16*z^4 + 268435456*a^15*e^16*z^4 + 3
221225472*a^11*c*d^3*e^11*z^3 + 239468544*a^7*c^2*d^6*e^6*z^2 + 39518208*a
^6*c^3*d^10*e^2*z^2 + 1153105920*a^8*c*d^2*e^10*z^2 + 32071680*a^4*c^2*...
```

Reduce [B] (verification not implemented)

Time = 2.50 (sec) , antiderivative size = 11727, normalized size of antiderivative = 7.65

$$\int \frac{1}{(d+ex)^2 (a+cx^4)^3} \, dx = \text{Too large to display}$$

input
```
int(1/(e*x+d)^2/(c*x^4+a)^3,x)
```

output

```
(90*c**(1/4)*a**(3/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*
x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a**5*d**2*e**14 + 90*c**(1/4)*a**(3/4)*sqr
t(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqr
t(2)))*a**5*d*e**15*x - 810*c**(1/4)*a**(3/4)*sqrt(2)*atan((c**(1/4)*a**(1
/4)*sqrt(2) - 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a**4*c*d**6*e**10
- 810*c**(1/4)*a**(3/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c
)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a**4*c*d**5*e**11*x + 180*c**(1/4)*a**(3
/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x)/(c**(1/4)*a**(1
/4)*sqrt(2)))*a**4*c*d**2*e**14*x**4 + 180*c**(1/4)*a**(3/4)*sqrt(2)*atan(
(c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a**
4*c*d*e**15*x**5 - 162*c**(1/4)*a**(3/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*s
qrt(2) - 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a**3*c**2*d**10*e**6 -
162*c**(1/4)*a**(3/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*
x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a**3*c**2*d**9*e**7*x - 1620*c**(1/4)*a**(
3/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x)/(c**(1/4)*a**(
1/4)*sqrt(2)))*a**3*c**2*d**6*e**10*x**4 - 1620*c**(1/4)*a**(3/4)*sqrt(2)*
atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2))
)*a**3*c**2*d**5*e**11*x**5 + 90*c**(1/4)*a**(3/4)*sqrt(2)*atan((c**(1/4)*
a**(1/4)*sqrt(2) - 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a**3*c**2*d**
2*e**14*x**8 + 90*c**(1/4)*a**(3/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqr...
```

3.191 $\int (d+ex)^3\sqrt{a+cx^4}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 355

$$\int (d+ex)^3\sqrt{a+cx^4}\,dx = \frac{3}{4}d^2ex^2\sqrt{a+cx^4}+\frac{6ade^2x\sqrt{a+cx^4}}{5\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)} + \frac{1}{15}dx\left(5d^2+9e^2x^2\right)\sqrt{a+cx^4}+\frac{e^3(a+cx^4)^{3/2}}{6c}+\frac{3ad^2e\operatorname{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{4\sqrt{c}} - \frac{6a^{5/4}de^2\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5c^{3/4}\sqrt{a+cx^4}} + \frac{a^{3/4}d\left(5\sqrt{c}d^2+9\sqrt{a}e^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{15c^{3/4}\sqrt{a+cx^4}}$$

output
```
3/4*d^2*e*x^2*(c*x^4+a)^(1/2)+6/5*a*d*e^2*x*(c*x^4+a)^(1/2)/c^(1/2)/(a^(1/
2)+c^(1/2)*x^2)+1/15*d*x*(9*e^2*x^2+5*d^2)*(c*x^4+a)^(1/2)+1/6*e^3*(c*x^4+
a)^(3/2)/c+3/4*a*d^2*e*arctanh(c^(1/2)*x^2/(c*x^4+a)^(1/2))/c^(1/2)-6/5*a^
(5/4)*d*e^2*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2
)*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))/c^(3/4)/(c*x^4+a
)^(1/2)+1/15*a^(3/4)*d*(5*c^(1/2)*d^2+9*a^(1/2)*e^2)*(a^(1/2)+c^(1/2)*x^2)
*((c*x^4+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(c^(1/4
)*x/a^(1/4)),1/2*2^(1/2))/c^(3/4)/(c*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.18 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.52

$$\int (d+ex)^3\sqrt{a+cx^4}\,dx$$
$$=\frac{\sqrt{a+cx^4}\left(2ae^3\sqrt{1+\frac{cx^4}{a}}+9cd^2ex^2\sqrt{1+\frac{cx^4}{a}}+2ce^3x^4\sqrt{1+\frac{cx^4}{a}}+9\sqrt{a}\sqrt{c}d^2e\operatorname{arcsinh}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)+12cd^3x\,\mathrm{I}\right.}{12c\sqrt{1+\frac{cx^4}{a}}}$$

input
```
Integrate[(d + e*x)^3*Sqrt[a + c*x^4],x]
```

output
```
(Sqrt[a + c*x^4]*(2*a*e^3*Sqrt[1 + (c*x^4)/a] + 9*c*d^2*e*x^2*Sqrt[1 + (c*
x^4)/a] + 2*c*e^3*x^4*Sqrt[1 + (c*x^4)/a] + 9*Sqrt[a]*Sqrt[c]*d^2*e*ArcSin
h[(Sqrt[c]*x^2)/Sqrt[a]] + 12*c*d^3*x*Hypergeometric2F1[-1/2, 1/4, 5/4, -(
(c*x^4)/a)] + 12*c*d*e^2*x^3*Hypergeometric2F1[-1/2, 3/4, 7/4, -((c*x^4)/a
)]))/(12*c*Sqrt[1 + (c*x^4)/a])
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2424, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+cx^4}(d+ex)^3\,dx$$

$$\Big\downarrow \; 2424$$

$$\int \left(\sqrt{a+cx^4}\left(d^3+3de^2x^2\right)+x\sqrt{a+cx^4}\left(3d^2e+e^3x^2\right)\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{a^{3/4}d\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(9\sqrt{a}e^2+5\sqrt{c}d^2\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{15c^{3/4}\sqrt{a+cx^4}} - \frac{6a^{5/4}de^2\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5c^{3/4}\sqrt{a+cx^4}} + \frac{3ad^2e\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{4\sqrt{c}} + \frac{1}{15}dx\sqrt{a+cx^4}\left(5d^2+9e^2x^2\right)+\frac{3}{4}d^2ex^2\sqrt{a+cx^4}+\frac{6ade^2x\sqrt{a+cx^4}}{5\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)}+\frac{e^3\left(a+cx^4\right)^{3/2}}{6c}$$

input

```
Int[(d + e*x)^3*Sqrt[a + c*x^4],x]
```

output

```
(3*d^2*e*x^2*Sqrt[a + c*x^4])/4 + (6*a*d*e^2*x*Sqrt[a + c*x^4])/(5*Sqrt[c]
*(Sqrt[a] + Sqrt[c]*x^2)) + (d*x*(5*d^2 + 9*e^2*x^2)*Sqrt[a + c*x^4])/15 +
 (e^3*(a + c*x^4)^(3/2))/(6*c) + (3*a*d^2*e*ArcTanh[(Sqrt[c]*x^2)/Sqrt[a +
 c*x^4]])/(4*Sqrt[c]) - (6*a^(5/4)*d*e^2*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a +
 c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)]
, 1/2])/(5*c^(3/4)*Sqrt[a + c*x^4]) + (a^(3/4)*d*(5*Sqrt[c]*d^2 + 9*Sqrt[a
]*e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]
*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(15*c^(3/4)*Sqrt[a + c*x^4
])
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2424

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.26 (sec) , antiderivative size = 261, normalized size of antiderivative = 0.74

method	result
risch	$\frac{(10x^4e^3c+36d\,e^2x^3c+45c\,d^2e\,x^2+20c\,d^3x+10e^3a)\sqrt{c\,x^4+a}}{60c}+\frac{ad\left(\frac{20d^2\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\sqrt{c\,x^4+a}}+\frac{36ie^2\sqrt{a}\sqrt{1-\cdots}}{\cdots}\right)}{\cdots}$
default	$d^3\left(\frac{x\sqrt{c\,x^4+a}}{3}+\frac{2a\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{3\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\sqrt{c\,x^4+a}}\right)+\frac{e^3(c\,x^4+a)^{\frac{3}{2}}}{6c}+3d\,e^2\left(\frac{x^3\sqrt{c\,x^4+a}}{5}+\frac{2ia^{\frac{3}{2}}\sqrt{1\cdots}}{\cdots}\right.$
elliptic	$\frac{e^3x^4\sqrt{c\,x^4+a}}{6}+\frac{3d\,e^2x^3\sqrt{c\,x^4+a}}{5}+\frac{3d^2e\,x^2\sqrt{c\,x^4+a}}{4}+\frac{d^3x\sqrt{c\,x^4+a}}{3}+\frac{e^3a\sqrt{c\,x^4+a}}{6c}+\frac{2d^3a\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\mathrm{Ellip}\cdots}{3\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\sqrt{c\,x^4+a}\cdots}$

input
```
int((e*x+d)^3*(c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/60*(10*c*e^3*x^4+36*c*d*e^2*x^3+45*c*d^2*e*x^2+20*c*d^3*x+10*a*e^3)*(c*x
^4+a)^(1/2)/c+1/30*a*d*(20*d^2/(I*c^(1/2)/a^(1/2))^(1/2)*(1-I*c^(1/2)*x^2/
a^(1/2))^(1/2)*(1+I*c^(1/2)*x^2/a^(1/2))^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x
*(I*c^(1/2)/a^(1/2))^(1/2),I)+36*I*e^2*a^(1/2)/(I*c^(1/2)/a^(1/2))^(1/2)*(
1-I*c^(1/2)*x^2/a^(1/2))^(1/2)*(1+I*c^(1/2)*x^2/a^(1/2))^(1/2)/(c*x^4+a)^(
1/2)/c^(1/2)*(EllipticF(x*(I*c^(1/2)/a^(1/2))^(1/2),I)-EllipticE(x*(I*c^(1
/2)/a^(1/2))^(1/2),I))+45/2*d*e*ln(c^(1/2)*x^2+(c*x^4+a)^(1/2))/c^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.53

$$\int (d+ex)^3\sqrt{a+cx^4}\,dx$$
$$=\frac{144\,a\sqrt{c}de^2x\left(-\frac{a}{c}\right)^{\frac{3}{4}}E\left(\arcsin\left(\frac{\left(-\frac{a}{c}\right)^{\frac{1}{4}}}{x}\right)\mid -1\right)+45\,a\sqrt{c}d^2ex\log\left(-2\,cx^4-2\,\sqrt{cx^4+a}\sqrt{c}x^2-a\right)+16\,(5\cdots}{\cdots}$$

input
```
integrate((e*x+d)^3*(c*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
1/120*(144*a*sqrt(c)*d*e^2*x*(-a/c)^(3/4)*elliptic_e(arcsin((-a/c)^(1/4)/x
), -1) + 45*a*sqrt(c)*d^2*e*x*log(-2*c*x^4 - 2*sqrt(c*x^4 + a)*sqrt(c)*x^2
 - a) + 16*(5*c*d^3 - 9*a*d*e^2)*sqrt(c)*x*(-a/c)^(3/4)*elliptic_f(arcsin(
(-a/c)^(1/4)/x), -1) + 2*(10*c*e^3*x^5 + 36*c*d*e^2*x^4 + 45*c*d^2*e*x^3 +
 20*c*d^3*x^2 + 10*a*e^3*x + 72*a*d*e^2)*sqrt(c*x^4 + a))/(c*x)
```

Sympy [A] (verification not implemented)

Time = 2.37 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.49

$$\int (d+ex)^3\sqrt{a+cx^4}\,dx = \frac{\sqrt{a}d^3x\Gamma\left(\frac{1}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)} + \frac{3\sqrt{a}d^2ex^2\sqrt{1+\frac{cx^4}{a}}}{4} + \frac{3\sqrt{a}de^2x^3\Gamma\left(\frac{3}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{7}{4}\right)} + \frac{3ad^2e\,\mathrm{asinh}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4\sqrt{c}} + e^3\left(\begin{cases}\frac{\sqrt{a}x^4}{4} & \text{for } c=0\\ \frac{(a+cx^4)^{\frac{3}{2}}}{6c} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((e*x+d)**3*(c*x**4+a)**(1/2),x)
```

output
```
sqrt(a)*d**3*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), c*x**4*exp_polar(I*pi
)/a)/(4*gamma(5/4)) + 3*sqrt(a)*d**2*e*x**2*sqrt(1 + c*x**4/a)/4 + 3*sqrt(
a)*d*e**2*x**3*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), c*x**4*exp_polar(I*pi
)/a)/(4*gamma(7/4)) + 3*a*d**2*e*asinh(sqrt(c)*x**2/sqrt(a))/(4*sqrt(c)) +
 e**3*Piecewise((sqrt(a)*x**4/4, Eq(c, 0)), ((a + c*x**4)**(3/2)/(6*c), Tr
ue))
```

Maxima [F]

$$\int (d+ex)^3\sqrt{a+cx^4}\,dx=\int \sqrt{cx^4+a}(ex+d)^3\,dx$$

input
```
integrate((e*x+d)^3*(c*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^4 + a)*(e*x + d)^3, x)
```

Giac [F]

$$\int (d+ex)^3\sqrt{a+cx^4}\,dx=\int \sqrt{cx^4+a}(ex+d)^3\,dx$$

input
```
integrate((e*x+d)^3*(c*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^4 + a)*(e*x + d)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)^3\sqrt{a+cx^4}\,dx=\int \sqrt{c\,x^4+a}\,(d+e\,x)^3\,dx$$

input
```
int((a + c*x^4)^(1/2)*(d + e*x)^3,x)
```

output
```
int((a + c*x^4)^(1/2)*(d + e*x)^3, x)
```

Reduce [F]

$$\int (d+ex)^3\sqrt{a+cx^4}\,dx$$
$$= \frac{20\sqrt{c\,x^4+a}\,a\,e^3+40\sqrt{c\,x^4+a}\,c\,d^3x+90\sqrt{c\,x^4+a}\,c\,d^2e\,x^2+72\sqrt{c\,x^4+a}\,cd\,e^2x^3+20\sqrt{c\,x^4+a}\,c\,e^3x^4}{}$$

input
```
int((e*x+d)^3*(c*x^4+a)^(1/2),x)
```

output
```
(20*sqrt(a + c*x**4)*a*e**3 + 40*sqrt(a + c*x**4)*c*d**3*x + 90*sqrt(a + c
*x**4)*c*d**2*e*x**2 + 72*sqrt(a + c*x**4)*c*d*e**2*x**3 + 20*sqrt(a + c*x
**4)*c*e**3*x**4 - 45*sqrt(c)*log(sqrt(a + c*x**4) - sqrt(c)*x**2)*a*d**2*
e + 45*sqrt(c)*log(sqrt(a + c*x**4) + sqrt(c)*x**2)*a*d**2*e + 80*int(sqrt
(a + c*x**4)/(a + c*x**4),x)*a*c*d**3 + 144*int((sqrt(a + c*x**4)*x**2)/(a
 + c*x**4),x)*a*c*d*e**2)/(120*c)
```

3.192 $\int (d+ex)^2\sqrt{a+cx^4}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 326

$$\begin{aligned}\int (d+ex)^2\sqrt{a+cx^4}\,dx &= \frac{1}{2}dex^2\sqrt{a+cx^4}+\frac{2ae^2x\sqrt{a+cx^4}}{5\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)}\\ &\quad+\frac{1}{15}x\left(5d^2+3e^2x^2\right)\sqrt{a+cx^4}+\frac{ade\,\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{2\sqrt{c}}\\ &\quad-\frac{2a^{5/4}e^2\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5c^{3/4}\sqrt{a+cx^4}}\\ &\quad+\frac{a^{3/4}\left(5\sqrt{c}d^2+3\sqrt{a}e^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{15c^{3/4}\sqrt{a+cx^4}}\end{aligned}$$

output
```
1/2*d*e*x^2*(c*x^4+a)^(1/2)+2/5*a*e^2*x*(c*x^4+a)^(1/2)/c^(1/2)/(a^(1/2)+c
^(1/2)*x^2)+1/15*x*(3*e^2*x^2+5*d^2)*(c*x^4+a)^(1/2)+1/2*a*d*e*arctanh(c^(
1/2)*x^2/(c*x^4+a)^(1/2))/c^(1/2)-2/5*a^(5/4)*e^2*(a^(1/2)+c^(1/2)*x^2)*((
c*x^4+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*EllipticE(sin(2*arctan(c^(1/4)*x/a
^(1/4))),1/2*2^(1/2))/c^(3/4)/(c*x^4+a)^(1/2)+1/15*a^(3/4)*(5*c^(1/2)*d^2+
3*a^(1/2)*e^2)*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(
1/2)*InverseJacobiAM(2*arctan(c^(1/4)*x/a^(1/4)),1/2*2^(1/2))/c^(3/4)/(c*x
^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.16 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.45

$$\int (d+ex)^2\sqrt{a+cx^4}\,dx = \frac{\sqrt{a+cx^4}\left(6\sqrt{c}d^2x\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4},\frac{5}{4},-\frac{cx^4}{a}\right)+e\left(3d\left(\sqrt{c}x^2\sqrt{1+\frac{cx^4}{a}}+\sqrt{a}\text{arcsinh}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)\right)\right.\right.}{6\sqrt{c}\sqrt{1+\frac{cx^4}{a}}}$$

input
```
Integrate[(d + e*x)^2*Sqrt[a + c*x^4],x]
```

output
```
(Sqrt[a + c*x^4]*(6*Sqrt[c]*d^2*x*Hypergeometric2F1[-1/2, 1/4, 5/4, -((c*x
^4)/a)] + e*(3*d*(Sqrt[c]*x^2*Sqrt[1 + (c*x^4)/a] + Sqrt[a]*ArcSinh[(Sqrt[
c]*x^2)/Sqrt[a]]) + 2*Sqrt[c]*e*x^3*Hypergeometric2F1[-1/2, 3/4, 7/4, -((c
*x^4)/a)])))/(6*Sqrt[c]*Sqrt[1 + (c*x^4)/a])
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2424, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+cx^4}(d+ex)^2\,dx$$

$\Big\downarrow$ 2424

$$\int \left(\sqrt{a+cx^4}\left(d^2+e^2x^2\right)+2dex\sqrt{a+cx^4}\right)dx$$

$\Big\downarrow$ 2009

$$\frac{a^{3/4}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(3\sqrt{a}e^2+5\sqrt{c}d^2\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{15c^{3/4}\sqrt{a+cx^4}}-\frac{2a^{5/4}e^2\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5c^{3/4}\sqrt{a+cx^4}}+\frac{ade\,\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{2\sqrt{c}}+\frac{1}{15}x\sqrt{a+cx^4}\left(5d^2+3e^2x^2\right)+\frac{1}{2}dex^2\sqrt{a+cx^4}+\frac{2ae^2x\sqrt{a+cx^4}}{5\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)}$$

input
```
Int[(d + e*x)^2*Sqrt[a + c*x^4],x]
```

output
```
(d*e*x^2*Sqrt[a + c*x^4])/2 + (2*a*e^2*x*Sqrt[a + c*x^4])/(5*Sqrt[c]*(Sqrt
[a] + Sqrt[c]*x^2)) + (x*(5*d^2 + 3*e^2*x^2)*Sqrt[a + c*x^4])/15 + (a*d*e*
ArcTanh[(Sqrt[c]*x^2)/Sqrt[a + c*x^4]])/(2*Sqrt[c]) - (2*a^(5/4)*e^2*(Sqrt
[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2
*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(5*c^(3/4)*Sqrt[a + c*x^4]) + (a^(3/4)
*(5*Sqrt[c]*d^2 + 3*Sqrt[a]*e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/
(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/
(15*c^(3/4)*Sqrt[a + c*x^4])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2424
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.06 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.72

method	result
risch	$\frac{x(6e^2x^2+15dex+10d^2)\sqrt{cx^4+a}}{30}+\frac{a\left(\frac{10d^2\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\sqrt{cx^4+a}}+\frac{6ie^2\sqrt{a}\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\left(\mathrm{EllipticF}(\right.}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\sqrt{cx^4}}\right.}{15}$
default	$d^2\left(\frac{x\sqrt{cx^4+a}}{3}+\frac{2a\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{3\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\sqrt{cx^4+a}}\right)+e^2\left(\frac{x^3\sqrt{cx^4+a}}{5}+\frac{2ia^{\frac{3}{2}}\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}}{5}\right.$
elliptic	$\frac{e^2x^3\sqrt{cx^4+a}}{5}+\frac{de\,x^2\sqrt{cx^4+a}}{2}+\frac{d^2x\sqrt{cx^4+a}}{3}+\frac{2a\,d^2\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{3\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\sqrt{cx^4+a}}+\frac{ade\ln\left(2\sqrt{c}\,x^2+2\sqrt{}\right.}{2\sqrt{c}}$

input
```
int((e*x+d)^2*(c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/30*x*(6*e^2*x^2+15*d*e*x+10*d^2)*(c*x^4+a)^(1/2)+1/15*a*(10*d^2/(I*c^(1/
2)/a^(1/2))^(1/2)*(1-I*c^(1/2)*x^2/a^(1/2))^(1/2)*(1+I*c^(1/2)*x^2/a^(1/2)
)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I*c^(1/2)/a^(1/2))^(1/2),I)+6*I*e^2*a
^(1/2)/(I*c^(1/2)/a^(1/2))^(1/2)*(1-I*c^(1/2)*x^2/a^(1/2))^(1/2)*(1+I*c^(1
/2)*x^2/a^(1/2))^(1/2)/(c*x^4+a)^(1/2)/c^(1/2)*(EllipticF(x*(I*c^(1/2)/a^(
1/2))^(1/2),I)-EllipticE(x*(I*c^(1/2)/a^(1/2))^(1/2),I))+15/2*d*e*ln(c^(1/
2)*x^2+(c*x^4+a)^(1/2))/c^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.51

$$\int (d+ex)^2\sqrt{a+cx^4}\,dx$$
$$=\frac{24\,a\sqrt{c}e^2x\left(-\frac{a}{c}\right)^{\frac{3}{4}}E\left(\arcsin\left(\frac{\left(-\frac{a}{c}\right)^{\frac{1}{4}}}{x}\right)\mid -1\right)+15\,a\sqrt{c}dex\log\left(-2\,cx^4-2\,\sqrt{cx^4+a}\sqrt{c}x^2-a\right)+8\left(5\,cd^2\right.}{60\,cx}$$

input
```
integrate((e*x+d)^2*(c*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
1/60*(24*a*sqrt(c)*e^2*x*(-a/c)^(3/4)*elliptic_e(arcsin((-a/c)^(1/4)/x), -
1) + 15*a*sqrt(c)*d*e*x*log(-2*c*x^4 - 2*sqrt(c*x^4 + a)*sqrt(c)*x^2 - a)
+ 8*(5*c*d^2 - 3*a*e^2)*sqrt(c)*x*(-a/c)^(3/4)*elliptic_f(arcsin((-a/c)^(1
/4)/x), -1) + 2*(6*c*e^2*x^4 + 15*c*d*e*x^3 + 10*c*d^2*x^2 + 12*a*e^2)*sqr
t(c*x^4 + a))/(c*x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.41 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.42

$$\int (d+ex)^2\sqrt{a+cx^4}\,dx = \frac{\sqrt{a}d^2x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)} + \frac{\sqrt{a}dex^2\sqrt{1+\frac{cx^4}{a}}}{2} + \frac{\sqrt{a}e^2x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{7}{4}\right)} + \frac{ade\operatorname{asinh}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{c}}$$

input
```
integrate((e*x+d)**2*(c*x**4+a)**(1/2),x)
```

output
```
sqrt(a)*d**2*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), c*x**4*exp_polar(I*pi
)/a)/(4*gamma(5/4)) + sqrt(a)*d*e*x**2*sqrt(1 + c*x**4/a)/2 + sqrt(a)*e**2
*x**3*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), c*x**4*exp_polar(I*pi)/a)/(4*g
amma(7/4)) + a*d*e*asinh(sqrt(c)*x**2/sqrt(a))/(2*sqrt(c))
```

Maxima [F]

$$\int (d+ex)^2\sqrt{a+cx^4}\,dx = \int \sqrt{cx^4+a}(ex+d)^2\,dx$$

input
```
integrate((e*x+d)^2*(c*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^4 + a)*(e*x + d)^2, x)
```

Giac [F]

$$\int (d+ex)^2\sqrt{a+cx^4}\,dx=\int \sqrt{cx^4+a}(ex+d)^2\,dx$$

input
```
integrate((e*x+d)^2*(c*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^4 + a)*(e*x + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)^2\sqrt{a+cx^4}\,dx=\int \sqrt{c\,x^4+a}\,(d+e\,x)^2\,dx$$

input
```
int((a + c*x^4)^(1/2)*(d + e*x)^2,x)
```

output
```
int((a + c*x^4)^(1/2)*(d + e*x)^2, x)
```

Reduce [F]

$$\int (d+ex)^2\sqrt{a+cx^4}\,dx = \frac{20\sqrt{c\,x^4+a}\,c\,d^2x+30\sqrt{c\,x^4+a}\,cde\,x^2+12\sqrt{c\,x^4+a}\,c\,e^2x^3-15\sqrt{c}\log\left(\sqrt{c\,x^4+a}-\sqrt{c}\,x^2\right)ade+15\sqrt{}}{60c}$$

input
```
int((e*x+d)^2*(c*x^4+a)^(1/2),x)
```

output
```
(20*sqrt(a + c*x**4)*c*d**2*x + 30*sqrt(a + c*x**4)*c*d*e*x**2 + 12*sqrt(a
 + c*x**4)*c*e**2*x**3 - 15*sqrt(c)*log(sqrt(a + c*x**4) - sqrt(c)*x**2)*a
*d*e + 15*sqrt(c)*log(sqrt(a + c*x**4) + sqrt(c)*x**2)*a*d*e + 40*int(sqrt
(a + c*x**4)/(a + c*x**4),x)*a*c*d**2 + 24*int((sqrt(a + c*x**4)*x**2)/(a
+ c*x**4),x)*a*c*e**2)/(60*c)
```

3.193 $\int (d+ex)\sqrt{a+cx^4}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 158

$$\int (d+ex)\sqrt{a+cx^4}\,dx = \frac{1}{3}dx\sqrt{a+cx^4}+\frac{1}{4}ex^2\sqrt{a+cx^4}+\frac{ae\operatorname{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{4\sqrt{c}} + \frac{a^{3/4}d\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{3\sqrt[4]{c}\sqrt{a+cx^4}}$$

output
```
1/3*d*x*(c*x^4+a)^(1/2)+1/4*e*x^2*(c*x^4+a)^(1/2)+1/4*a*e*arctanh(c^(1/2)*
x^2/(c*x^4+a)^(1/2))/c^(1/2)+1/3*a^(3/4)*d*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+a
)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(c^(1/4)*x/a^(1/4
)),1/2*2^(1/2))/c^(1/4)/(c*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 8.47 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.69

$$\int (d+ex)\sqrt{a+cx^4}\,dx = \frac{\sqrt{a+cx^4}\left(\sqrt{c}ex^2\sqrt{1+\frac{cx^4}{a}}+\sqrt{a}e\operatorname{arcsinh}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)+4\sqrt{c}dx\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4},\frac{5}{4},-\frac{cx^4}{a}\right)\right)}{4\sqrt{c}\sqrt{1+\frac{cx^4}{a}}}$$

input
```
Integrate[(d + e*x)*Sqrt[a + c*x^4],x]
```

output
```
(Sqrt[a + c*x^4]*(Sqrt[c]*e*x^2*Sqrt[1 + (c*x^4)/a] + Sqrt[a]*e*ArcSinh[(S
qrt[c]*x^2)/Sqrt[a]] + 4*Sqrt[c]*d*x*Hypergeometric2F1[-1/2, 1/4, 5/4, -((
c*x^4)/a)]))/(4*Sqrt[c]*Sqrt[1 + (c*x^4)/a])
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2424, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+cx^4}(d+ex)\,dx$$

$$\Big\downarrow \text{2424}$$

$$\int \left(d\sqrt{a+cx^4}+ex\sqrt{a+cx^4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^{3/4}d\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{3\sqrt[4]{c}\sqrt{a+cx^4}}+\frac{ae\,\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{4\sqrt{c}}+\frac{1}{3}dx\sqrt{a+cx^4}+\frac{1}{4}ex^2\sqrt{a+cx^4}$$

input
```
Int[(d + e*x)*Sqrt[a + c*x^4],x]
```

output
```
(d*x*Sqrt[a + c*x^4])/3 + (e*x^2*Sqrt[a + c*x^4])/4 + (a*e*ArcTanh[(Sqrt[c
]*x^2)/Sqrt[a + c*x^4]])/(4*Sqrt[c]) + (a^(3/4)*d*(Sqrt[a] + Sqrt[c]*x^2)*
Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)
/a^(1/4)], 1/2])/(3*c^(1/4)*Sqrt[a + c*x^4])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2424
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.63 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.75

method	result	size
risch	$\frac{x(3ex+4d)\sqrt{c\,x^{4}+a}}{12}+\frac{2ad\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{3\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}}+\frac{ae\ln\left(\sqrt{c}\,x^{2}+\sqrt{c\,x^{4}+a}\right)}{4\sqrt{c}}$	119
default	$d\left(\frac{x\sqrt{c\,x^{4}+a}}{3}+\frac{2a\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{3\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}}\right)+e\left(\frac{x^{2}\sqrt{c\,x^{4}+a}}{4}+\frac{a\ln\left(\sqrt{c}\,x^{2}+\sqrt{c\,x^{4}+a}\right)}{4\sqrt{c}}\right)$	129
elliptic	$\frac{e\,x^{2}\sqrt{c\,x^{4}+a}}{4}+\frac{dx\sqrt{c\,x^{4}+a}}{3}+\frac{2ad\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{3\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}}+\frac{ae\ln\left(2\sqrt{c}\,x^{2}+2\sqrt{c\,x^{4}+a}\right)}{4\sqrt{c}}$	130

input
```
int((e*x+d)*(c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/12*x*(3*e*x+4*d)*(c*x^4+a)^(1/2)+2/3*a*d/(I*c^(1/2)/a^(1/2))^(1/2)*(1-I*
c^(1/2)*x^2/a^(1/2))^(1/2)*(1+I*c^(1/2)*x^2/a^(1/2))^(1/2)/(c*x^4+a)^(1/2)
*EllipticF(x*(I*c^(1/2)/a^(1/2))^(1/2),I)+1/4*a*e*ln(c^(1/2)*x^2+(c*x^4+a)
^(1/2))/c^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.59

$$\int (d+ex)\sqrt{a+cx^4}\,dx$$
$$=\frac{16\,c^{\frac{3}{2}}d\left(-\frac{a}{c}\right)^{\frac{3}{4}}F\left(\arcsin\left(\frac{\left(-\frac{a}{c}\right)^{\frac{1}{4}}}{x}\right)\,|\,-1\right)+3\,a\sqrt{c}e\log\left(-2\,cx^{4}-2\,\sqrt{cx^{4}+a}\sqrt{c}x^{2}-a\right)+2\,\sqrt{cx^{4}+a}(3\,cex}{24\,c}$$

input
```
integrate((e*x+d)*(c*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
1/24*(16*c^(3/2)*d*(-a/c)^(3/4)*elliptic_f(arcsin((-a/c)^(1/4)/x), -1) + 3
*a*sqrt(c)*e*log(-2*c*x^4 - 2*sqrt(c*x^4 + a)*sqrt(c)*x^2 - a) + 2*sqrt(c*
x^4 + a)*(3*c*e*x^2 + 4*c*d*x))/c
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.00 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.56

$$\int (d+ex)\sqrt{a+cx^4}\,dx = \frac{\sqrt{a}dx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)} + \frac{\sqrt{a}ex^2\sqrt{1+\frac{cx^4}{a}}}{4} + \frac{ae\operatorname{asinh}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4\sqrt{c}}$$

input
```
integrate((e*x+d)*(c*x**4+a)**(1/2),x)
```

output
```
sqrt(a)*d*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), c*x**4*exp_polar(I*pi)/a
)/(4*gamma(5/4)) + sqrt(a)*e*x**2*sqrt(1 + c*x**4/a)/4 + a*e*asinh(sqrt(c)
*x**2/sqrt(a))/(4*sqrt(c))
```

Maxima [F]

$$\int (d+ex)\sqrt{a+cx^4}\,dx = \int \sqrt{cx^4+a}(ex+d)\,dx$$

input
```
integrate((e*x+d)*(c*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^4 + a)*(e*x + d), x)
```

Giac [F]

$$\int (d+ex)\sqrt{a+cx^4}\,dx=\int \sqrt{cx^4+a}(ex+d)\,dx$$

input
```
integrate((e*x+d)*(c*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^4 + a)*(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)\sqrt{a+cx^4}\,dx=\int \sqrt{c\,x^4+a}\,(d+e\,x)\,dx$$

input
```
int((a + c*x^4)^(1/2)*(d + e*x),x)
```

output
```
int((a + c*x^4)^(1/2)*(d + e*x), x)
```

Reduce [F]

$$\int (d+ex)\sqrt{a+cx^4}\,dx$$
$$=\frac{8\sqrt{c\,x^4+a}\,cdx+6\sqrt{c\,x^4+a}\,ce\,x^2-3\sqrt{c}\log\left(\sqrt{c\,x^4+a}-\sqrt{c}\,x^2\right)ae+3\sqrt{c}\log\left(\sqrt{c\,x^4+a}+\sqrt{c}\,x^2\right)ae-}{24c}$$

input
```
int((e*x+d)*(c*x^4+a)^(1/2),x)
```

output
```
(8*sqrt(a + c*x**4)*c*d*x + 6*sqrt(a + c*x**4)*c*e*x**2 - 3*sqrt(c)*log(sq
rt(a + c*x**4) - sqrt(c)*x**2)*a*e + 3*sqrt(c)*log(sqrt(a + c*x**4) + sqrt
(c)*x**2)*a*e + 16*int(sqrt(a + c*x**4)/(a + c*x**4),x)*a*c*d)/(24*c)
```

3.194 $\int \sqrt{a+cx^4}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 105

$$\int \sqrt{a+cx^4}\,dx=\frac{1}{3}x\sqrt{a+cx^4}+\frac{a^{3/4}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{3\sqrt[4]{c}\sqrt{a+cx^4}}$$

output
```
1/3*x*(c*x^4+a)^(1/2)+1/3*a^(3/4)*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+a)/(a^(1/2
)+c^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(c^(1/4)*x/a^(1/4)),1/2*2^
(1/2))/c^(1/4)/(c*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.19 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.85

$$\int \sqrt{a+cx^4}\,dx=\frac{x(a+cx^4)-\frac{2ia\sqrt{1+\frac{cx^4}{a}}\operatorname{EllipticF}\left(i\operatorname{arcsinh}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}x\right),-1\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}}}{3\sqrt{a+cx^4}}$$

input
```
Integrate[Sqrt[a + c*x^4],x]
```

output
```
(x*(a + c*x^4) - ((2*I)*a*Sqrt[1 + (c*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[(I*
Sqrt[c])/Sqrt[a]]*x], -1])/Sqrt[(I*Sqrt[c])/Sqrt[a]])/(3*Sqrt[a + c*x^4])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {748, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+cx^4}\,dx$$

$$\Big\downarrow \text{748}$$

$$\frac{2}{3}a\int \frac{1}{\sqrt{cx^4+a}}dx+\frac{1}{3}x\sqrt{a+cx^4}$$

$$\Big\downarrow \text{761}$$

$$\frac{a^{3/4}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{3\sqrt[4]{c}\sqrt{a+cx^4}}+\frac{1}{3}x\sqrt{a+cx^4}$$

input
```
Int[Sqrt[a + c*x^4],x]
```

output
```
(x*Sqrt[a + c*x^4])/3 + (a^(3/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/
(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/
(3*c^(1/4)*Sqrt[a + c*x^4])
```

Defintions of rubi rules used

rule 748
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Simp[a*n*(p/(n*p + 1))   Int[(a + b*x^n)^(p - 1), x], x] /; Fre
eQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || LtQ[Denominat
or[p + 1/n], Denominator[p]])
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.36 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.81

method	result	size
default	$\frac{x\sqrt{c\,x^4+a}}{3}+\frac{2a\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{3\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}$	85
risch	$\frac{x\sqrt{c\,x^4+a}}{3}+\frac{2a\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{3\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}$	85
elliptic	$\frac{x\sqrt{c\,x^4+a}}{3}+\frac{2a\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{3\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}$	85

input
```
int((c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*x*(c*x^4+a)^(1/2)+2/3*a/(I*c^(1/2)/a^(1/2))^(1/2)*(1-I*c^(1/2)*x^2/a^(
1/2))^(1/2)*(1+I*c^(1/2)*x^2/a^(1/2))^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I
*c^(1/2)/a^(1/2))^(1/2),I)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.39

$$\int \sqrt{a+cx^4}\,dx = \frac{2}{3}\sqrt{c}\left(-\frac{a}{c}\right)^{\frac{3}{4}}F(\arcsin\left(\frac{\left(-\frac{a}{c}\right)^{\frac{1}{4}}}{x}\right)|-1)+\frac{1}{3}\sqrt{cx^4+a}\,x$$

input
```
integrate((c*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
2/3*sqrt(c)*(-a/c)^(3/4)*elliptic_f(arcsin((-a/c)^(1/4)/x), -1) + 1/3*sqrt
(c*x^4 + a)*x
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.55 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.35

$$\int \sqrt{a+cx^4}\,dx = \frac{\sqrt{a}x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((c*x**4+a)**(1/2),x)
```

output
```
sqrt(a)*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), c*x**4*exp_polar(I*pi)/a)/
(4*gamma(5/4))
```

Maxima [F]

$$\int \sqrt{a+cx^4}\,dx = \int \sqrt{cx^4+a}\,dx$$

input
```
integrate((c*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^4 + a), x)
```

Giac [F]

$$\int \sqrt{a+cx^4}\,dx = \int \sqrt{cx^4+a}\,dx$$

input
```
integrate((c*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^4 + a), x)
```

Mupad [B] (verification not implemented)

Time = 21.43 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.35

$$\int \sqrt{a+cx^4}\,dx = \frac{x\sqrt{c\,x^4+a}\,{}_2\mathrm{F}_1\left(-\frac{1}{2},\frac{1}{4};\ \frac{5}{4};\ -\frac{c\,x^4}{a}\right)}{\sqrt{\frac{c\,x^4}{a}+1}}$$

input
```
int((a + c*x^4)^(1/2),x)
```

output
```
(x*(a + c*x^4)^(1/2)*hypergeom([-1/2, 1/4], 5/4, -(c*x^4)/a))/((c*x^4)/a +
 1)^(1/2)
```

Reduce [F]

$$\int \sqrt{a+cx^4}\,dx = \frac{\sqrt{c\,x^4+a}\,x}{3} + \frac{2\left(\int \frac{\sqrt{c\,x^4+a}}{c\,x^4+a}dx\right)a}{3}$$

input
```
int((c*x^4+a)^(1/2),x)
```

output
```
(sqrt(a + c*x**4)*x + 2*int(sqrt(a + c*x**4)/(a + c*x**4),x)*a)/3
```

3.195 $\int \frac{\sqrt{a+cx^4}}{d+ex} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 605

$$\begin{aligned}\int \frac{\sqrt{a+cx^4}}{d+ex} \, dx &= \frac{\sqrt{a+cx^4}}{2e} - \frac{\sqrt{c}dx\sqrt{a+cx^4}}{e^2\left(\sqrt{a}+\sqrt{c}x^2\right)} + \frac{\sqrt{cd^4+ae^4}\text{arctanh}\left(\frac{\sqrt{cd^4+ae^4}x}{de\sqrt{a+cx^4}}\right)}{2e^3} \\ &\quad + \frac{\sqrt{c}d^2\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{2e^3} - \frac{\sqrt{cd^4+ae^4}\text{arctanh}\left(\frac{ae^2+cd^2x^2}{\sqrt{cd^4+ae^4}\sqrt{a+cx^4}}\right)}{2e^3} \\ &\quad + \frac{\sqrt[4]{a}\sqrt[4]{c}d\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{e^2\sqrt{a+cx^4}} \\ &\quad - \frac{\sqrt[4]{a}c^{3/4}d^3\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{e^2\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\sqrt{a+cx^4}} \\ &\quad - \frac{\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\left(cd^4+ae^4\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticPi}\left(\frac{\left(\sqrt{c}d^2+\sqrt{a}e^2\right)^2}{4\sqrt{a}\sqrt{c}d^2e^2},2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4\sqrt[4]{a}\sqrt[4]{c}de^4\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\sqrt{a+cx^4}}\end{aligned}$$

output
```
1/2*(c*x^4+a)^(1/2)/e-c^(1/2)*d*x*(c*x^4+a)^(1/2)/e^2/(a^(1/2)+c^(1/2)*x^2
)+1/2*(a*e^4+c*d^4)^(1/2)*arctanh((a*e^4+c*d^4)^(1/2)*x/d/e/(c*x^4+a)^(1/2
))/e^3+1/2*c^(1/2)*d^2*arctanh(c^(1/2)*x^2/(c*x^4+a)^(1/2))/e^3-1/2*(a*e^4
+c*d^4)^(1/2)*arctanh((c*d^2*x^2+a*e^2)/(a*e^4+c*d^4)^(1/2)/(c*x^4+a)^(1/2
))/e^3+a^(1/4)*c^(1/4)*d*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+a)/(a^(1/2)+c^(1/2)
*x^2)^2)^(1/2)*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))/e^2
/(c*x^4+a)^(1/2)-a^(1/4)*c^(3/4)*d^3*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+a)/(a^(
1/2)+c^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(c^(1/4)*x/a^(1/4)),1/2
*2^(1/2))/e^2/(c^(1/2)*d^2+a^(1/2)*e^2)/(c*x^4+a)^(1/2)-1/4*(c^(1/2)*d^2-a
^(1/2)*e^2)*(a*e^4+c*d^4)*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+a)/(a^(1/2)+c^(1/2
)*x^2)^2)^(1/2)*EllipticPi(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/4*(c^(1/2)*d
^2+a^(1/2)*e^2)^2/a^(1/2)/c^(1/2)/d^2/e^2,1/2*2^(1/2))/a^(1/4)/c^(1/4)/d/e
^4/(c^(1/2)*d^2+a^(1/2)*e^2)/(c*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.41 (sec) , antiderivative size = 421, normalized size of antiderivative = 0.70

$$\int \frac{\sqrt{a+cx^4}}{d+ex}\,dx$$

$$= \frac{-2\sqrt{a}c^{3/4}d^2e^2\sqrt{1+\frac{cx^4}{a}}E\left(i\operatorname{arcsinh}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}x\right)\middle|-1\right)+2c^{3/4}d^2\left(i\sqrt{c}d^2+\sqrt{a}e^2\right)\sqrt{1+\frac{cx^4}{a}}\operatorname{EllipticF}\left(i\operatorname{arcs}\right.}{}$$

input
```
Integrate[Sqrt[a + c*x^4]/(d + e*x),x]
```

output
```
(-2*Sqrt[a]*c^(3/4)*d^2*e^2*Sqrt[1 + (c*x^4)/a]*EllipticE[I*ArcSinh[Sqrt[(
I*Sqrt[c])/Sqrt[a]]*x], -1] + 2*c^(3/4)*d^2*(I*Sqrt[c]*d^2 + Sqrt[a]*e^2)*
Sqrt[1 + (c*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1]
+ Sqrt[(I*Sqrt[c])/Sqrt[a]]*(-2*(-1)^(1/4)*a^(1/4)*(c*d^4 + a*e^4)*Sqrt[1
+ (c*x^4)/a]*EllipticPi[(I*Sqrt[a]*e^2)/(Sqrt[c]*d^2), ArcSin[((-1)^(3/4)*
c^(1/4)*x)/a^(1/4)], -1] + c^(1/4)*d*e*(e^2*(a + c*x^4) - 2*Sqrt[-(c*d^4)
- a*e^4]*Sqrt[a + c*x^4]*ArcTan[(Sqrt[c]*(d^2 - e^2*x^2) + e^2*Sqrt[a + c*
x^4])/Sqrt[-(c*d^4) - a*e^4]] - Sqrt[c]*d^2*Sqrt[a + c*x^4]*Log[-(Sqrt[c]*
x^2) + Sqrt[a + c*x^4]])))/(2*Sqrt[(I*Sqrt[c])/Sqrt[a]]*c^(1/4)*d*e^4*Sqrt
[a + c*x^4])
```

Rubi [A] (verified)

Time = 2.02 (sec) , antiderivative size = 660, normalized size of antiderivative = 1.09, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.842$, Rules used = {2267, 1524, 27, 1512, 27, 761, 1510, 1577, 493, 25, 719, 224, 219, 488, 219, 2223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+cx^4}}{d+ex}dx$$

$$\Big\downarrow \text{2267}$$

$$d\int \frac{\sqrt{cx^4+a}}{d^2-e^2x^2}dx-e\int \frac{x\sqrt{cx^4+a}}{d^2-e^2x^2}dx$$

$$\Big\downarrow \text{1524}$$

$$d\left(\frac{\left(ae^4+cd^4\right)\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a}(d^2-e^2x^2)\sqrt{cx^4+a}}dx}{e^2\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)}-\frac{\int \frac{\sqrt{c}\left(\sqrt{c}d^2-\sqrt{a}e^2+\sqrt{c}\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)x^2\right)}{\sqrt{cx^4+a}}dx}{e^2\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)}\right)-e\int \frac{x\sqrt{cx^4+a}}{d^2-e^2x^2}dx$$

$$\Big\downarrow \text{27}$$

$$d\left(\frac{\left(ae^4+cd^4\right)\int \frac{\sqrt{c}x^2+\sqrt{a}}{\left(d^2-e^2x^2\right)\sqrt{cx^4+a}}dx}{\sqrt{a}e^2\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)}-\frac{\sqrt{c}\int \frac{\sqrt{c}d^2-\sqrt{a}e^2+\sqrt{c}\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)x^2}{\sqrt{cx^4+a}}dx}{e^2\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)}\right)-e\int \frac{x\sqrt{cx^4+a}}{d^2-e^2x^2}dx$$

$\big\downarrow$ 1512

$$d\left(\frac{\left(ae^4+cd^4\right)\int \frac{\sqrt{c}x^2+\sqrt{a}}{\left(d^2-e^2x^2\right)\sqrt{cx^4+a}}dx}{\sqrt{a}e^2\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)}-\frac{\sqrt{c}\left(2\sqrt{c}d^2\int \frac{1}{\sqrt{cx^4+a}}dx-\left(\sqrt{a}e^2+\sqrt{c}d^2\right)\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{a}\sqrt{cx^4+a}}dx\right)}{e^2\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)}\right)-e\int \frac{x\sqrt{cx^4+a}}{d^2-e^2x^2}dx$$

$\big\downarrow$ 27

$$d\left(\frac{\left(ae^4+cd^4\right)\int \frac{\sqrt{c}x^2+\sqrt{a}}{\left(d^2-e^2x^2\right)\sqrt{cx^4+a}}dx}{\sqrt{a}e^2\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)}-\frac{\sqrt{c}\left(2\sqrt{c}d^2\int \frac{1}{\sqrt{cx^4+a}}dx-\frac{\left(\sqrt{a}e^2+\sqrt{c}d^2\right)\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+a}}dx}{\sqrt{a}}\right)}{e^2\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)}\right)-e\int \frac{x\sqrt{cx^4+a}}{d^2-e^2x^2}dx$$

$\big\downarrow$ 761

$$d\left(\frac{\left(ae^4+cd^4\right)\int \frac{\sqrt{c}x^2+\sqrt{a}}{\left(d^2-e^2x^2\right)\sqrt{cx^4+a}}dx}{\sqrt{a}e^2\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)}-\frac{\sqrt{c}\left(\frac{\sqrt[4]{c}d^2\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{\sqrt[4]{a}\sqrt{a+cx^4}}-\frac{\left(\sqrt{a}e^2+\sqrt{c}d^2\right)\cdots}{\cdots}\right.}{e^2\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)}\right.$$

$$e\int \frac{x\sqrt{cx^4+a}}{d^2-e^2x^2}dx$$

$\big\downarrow$ 1510

$$d\left(\frac{\left(ae^4+cd^4\right)\int \frac{\sqrt{c}x^2+\sqrt{a}}{(d^2-e^2x^2)\sqrt{cx^4+a}}dx}{\sqrt{a}e^2\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)}-\frac{\sqrt{c}\left(\frac{\sqrt[4]{c}d^2\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{\sqrt[4]{a}\sqrt{a+cx^4}}-\frac{\left(\sqrt{a}e^2+\sqrt{c}d^2\right)}{}\right.}{e^2\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)}\right.$$

$$e\int \frac{x\sqrt{cx^4+a}}{d^2-e^2x^2}dx$$

$$\Big\downarrow \text{1577}$$

$$d\left(\frac{\left(ae^4+cd^4\right)\int \frac{\sqrt{c}x^2+\sqrt{a}}{(d^2-e^2x^2)\sqrt{cx^4+a}}dx}{\sqrt{a}e^2\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)}-\frac{\sqrt{c}\left(\frac{\sqrt[4]{c}d^2\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{\sqrt[4]{a}\sqrt{a+cx^4}}-\frac{\left(\sqrt{a}e^2+\sqrt{c}d^2\right)}{}\right.}{e^2\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)}\right.$$

$$\frac{1}{2}e\int \frac{\sqrt{cx^4+a}}{d^2-e^2x^2}dx^2$$

$$\Big\downarrow \text{493}$$

$$d\left(\frac{(ae^4+cd^4)\int \frac{\sqrt{c}x^2+\sqrt{a}}{(d^2-e^2x^2)\sqrt{cx^4+a}}dx}{\sqrt{a}e^2\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)}-\frac{\sqrt{c}\left(\frac{\sqrt[4]{c}d^2(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{\sqrt[4]{a}\sqrt{a+cx^4}}-\frac{(\sqrt{a}e^2+\sqrt{c}d^2)}{}\right.}{e^2\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)}\right.$$

$$\frac{1}{2}e\left(-\frac{\int -\frac{ae^2+cd^2x^2}{(d^2-e^2x^2)\sqrt{cx^4+a}}dx^2}{e^2}-\frac{\sqrt{a+cx^4}}{e^2}\right)$$

$\downarrow$ 25

$$d\left(\frac{(ae^4+cd^4)\int \frac{\sqrt{c}x^2+\sqrt{a}}{(d^2-e^2x^2)\sqrt{cx^4+a}}dx}{\sqrt{a}e^2\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)}-\frac{\sqrt{c}\left(\frac{\sqrt[4]{c}d^2(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{\sqrt[4]{a}\sqrt{a+cx^4}}-\frac{(\sqrt{a}e^2+\sqrt{c}d^2)}{}\right.}{e^2\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)}\right.$$

$$\frac{1}{2}e\left(\frac{\int \frac{ae^2+cd^2x^2}{(d^2-e^2x^2)\sqrt{cx^4+a}}dx^2}{e^2}-\frac{\sqrt{a+cx^4}}{e^2}\right)$$

$\downarrow$ 719

$$d\left(\frac{\left(ae^4+cd^4\right)\int\frac{\sqrt{c}x^2+\sqrt{a}}{(d^2-e^2x^2)\sqrt{cx^4+a}}dx}{\sqrt{a}e^2\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)}-\frac{\sqrt{c}\left(\frac{\sqrt[4]{c}d^2(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{\sqrt[4]{a}\sqrt{a+cx^4}}-\frac{(\sqrt{a}e^2+\sqrt{c}d^2)}{}\right.}{e^2\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)}\right.$$

$$\frac{1}{2}e\left(\frac{\frac{\left(ae^4+cd^4\right)\int\frac{1}{\left(d^2-e^2x^2\right)\sqrt{cx^4+a}}dx^2}{e^2}-\frac{cd^2\int\frac{1}{\sqrt{cx^4+a}}dx^2}{e^2}}{e^2}-\frac{\sqrt{a+cx^4}}{e^2}\right)$$

$$\Big\downarrow \text{224}$$

$$d\left(\frac{\left(ae^4+cd^4\right)\int\frac{\sqrt{c}x^2+\sqrt{a}}{(d^2-e^2x^2)\sqrt{cx^4+a}}dx}{\sqrt{a}e^2\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)}-\frac{\sqrt{c}\left(\frac{\sqrt[4]{c}d^2(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{\sqrt[4]{a}\sqrt{a+cx^4}}-\frac{(\sqrt{a}e^2+\sqrt{c}d^2)}{}\right.}{e^2\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)}\right.$$

$$\frac{1}{2}e\left(\frac{\frac{\left(ae^4+cd^4\right)\int\frac{1}{\left(d^2-e^2x^2\right)\sqrt{cx^4+a}}dx^2}{e^2}-\frac{cd^2\int\frac{1}{1-cx^4}d\frac{x^2}{\sqrt{cx^4+a}}}{e^2}}{e^2}-\frac{\sqrt{a+cx^4}}{e^2}\right)$$

$$\Big\downarrow \text{219}$$

$$d\left(\frac{\left(ae^4+cd^4\right)\int \frac{\sqrt{c}x^2+\sqrt{a}}{\left(d^2-e^2x^2\right)\sqrt{cx^4+a}}dx}{\sqrt{a}e^2\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)}-\frac{\sqrt{c}\left(\frac{\sqrt[4]{c}d^2\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{\sqrt[4]{a}\sqrt{a+cx^4}}-\frac{\left(\sqrt{a}e^2+\sqrt{c}d^2\right)}{}\right.}{e^2\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)}\right.$$

$$\frac{1}{2}e\left(\frac{\frac{\left(ae^4+cd^4\right)\int \frac{1}{\left(d^2-e^2x^2\right)\sqrt{cx^4+a}}dx^2}{e^2}-\frac{\sqrt{c}d^2\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{e^2}}{e^2}-\frac{\sqrt{a+cx^4}}{e^2}\right)$$

$\downarrow$ 488

$$d\left(\frac{\left(ae^4+cd^4\right)\int \frac{\sqrt{c}x^2+\sqrt{a}}{\left(d^2-e^2x^2\right)\sqrt{cx^4+a}}dx}{\sqrt{a}e^2\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)}-\frac{\sqrt{c}\left(\frac{\sqrt[4]{c}d^2\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{\sqrt[4]{a}\sqrt{a+cx^4}}-\frac{\left(\sqrt{a}e^2+\sqrt{c}d^2\right)}{}\right.}{e^2\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)}\right.$$

$$\frac{1}{2}e\left(\frac{-\frac{\left(ae^4+cd^4\right)\int \frac{1}{cd^4+ae^4-x^4}d\frac{-ae^2-cd^2x^2}{\sqrt{cx^4+a}}}{e^2}-\frac{\sqrt{c}d^2\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{e^2}}{e^2}-\frac{\sqrt{a+cx^4}}{e^2}\right)$$

$\downarrow$ 219

$$d\left(\frac{\left(ae^4+cd^4\right)\int \frac{\sqrt{c}x^2+\sqrt{a}}{\left(d^2-e^2x^2\right)\sqrt{cx^4+a}}dx}{\sqrt{a}e^2\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)}-\frac{\sqrt{c}\left(\frac{\sqrt[4]{c}d^2\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{\sqrt[4]{a}\sqrt{a+cx^4}}-\frac{\left(\sqrt{a}e^2+\sqrt{c}d^2\right)\cdots}{\cdots}\right.}{e^2\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)}\right.$$

$$\frac{1}{2}e\left(\frac{-\frac{\sqrt{c}d^2\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{e^2}-\frac{\sqrt{ae^4+cd^4}\text{arctanh}\left(\frac{-ae^2-cd^2x^2}{\sqrt{a+cx^4}\sqrt{ae^4+cd^4}}\right)}{e^2}}{e^2}-\frac{\sqrt{a+cx^4}}{e^2}\right)$$

$$\Big\downarrow \; 2223$$

$$d\left(\frac{\left(ae^4+cd^4\right)\left(\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{\sqrt{a}}{d^2}-\frac{\sqrt{c}}{e^2}\right)\text{EllipticPi}\left(\frac{\left(\sqrt{c}d^2+\sqrt{a}e^2\right)^2}{4\sqrt{a}\sqrt{c}d^2e^2},2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+cx^4}}+\frac{\left(\sqrt{a}e^2+\sqrt{c}d^2\right)\text{arctanh}\left(\cdots\right)}{2de\sqrt{ae^4+cd^4}}\cdots\right.}{\sqrt{a}e^2\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)}\right.$$

$$\frac{1}{2}e\left(\frac{-\frac{\sqrt{c}d^2\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{e^2}-\frac{\sqrt{ae^4+cd^4}\text{arctanh}\left(\frac{-ae^2-cd^2x^2}{\sqrt{a+cx^4}\sqrt{ae^4+cd^4}}\right)}{e^2}}{e^2}-\frac{\sqrt{a+cx^4}}{e^2}\right)$$

input
```
Int[Sqrt[a + c*x^4]/(d + e*x),x]
```

output
```
-1/2*(e*(-(Sqrt[a + c*x^4]/e^2) + (-((Sqrt[c]*d^2*ArcTanh[(Sqrt[c]*x^2)/Sq
rt[a + c*x^4]])/e^2) - (Sqrt[c*d^4 + a*e^4]*ArcTanh[(-(a*e^2) - c*d^2*x^2)
/(Sqrt[c*d^4 + a*e^4]*Sqrt[a + c*x^4])])/e^2)/e^2)) + d*(-((Sqrt[c]*(-(((S
qrt[c]*d^2 + Sqrt[a]*e^2)*(-((x*Sqrt[a + c*x^4])/(Sqrt[a] + Sqrt[c]*x^2))
+ (a^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2
)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(c^(1/4)*Sqrt[a + c*x^
4])))/Sqrt[a]) + (c^(1/4)*d^2*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sq
rt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(a^
(1/4)*Sqrt[a + c*x^4])))/(e^2*((Sqrt[c]*d^2)/Sqrt[a] + e^2))) + ((c*d^4 +
a*e^4)*(((Sqrt[c]*d^2 + Sqrt[a]*e^2)*ArcTanh[(Sqrt[c*d^4 + a*e^4]*x)/(d*e*
Sqrt[a + c*x^4])])/(2*d*e*Sqrt[c*d^4 + a*e^4]) + ((Sqrt[a]/d^2 - Sqrt[c]/e
^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*El
lipticPi[(Sqrt[c]*d^2 + Sqrt[a]*e^2)^2/(4*Sqrt[a]*Sqrt[c]*d^2*e^2), 2*ArcT
an[(c^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(1/4)*c^(1/4)*Sqrt[a + c*x^4])))/(Sqrt
[a]*e^2*((Sqrt[c]*d^2)/Sqrt[a] + e^2)))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 493
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*((a + b*x^2)^p/(d*(n + 2*p + 1))), x] + Simp[2*(p/(d*(n +
 2*p + 1)))   Int[(c + d*x)^n*(a + b*x^2)^(p - 1)*(a*d - b*c*x), x], x] /;
FreeQ[{a, b, c, d, n}, x] && GtQ[p, 0] && NeQ[n + 2*p + 1, 0] && ( !Rationa
lQ[n] || LtQ[n, 1]) &&  !ILtQ[n + 2*p, 0] && IntQuadraticQ[a, 0, b, c, d, n
, p, x]
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1512
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + c*x^4], x], x] - Simp[e/q
 Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c
, d, e}, x] && PosQ[c/a]
```

rule 1524
```
Int[Sqrt[(a_) + (c_.)*(x_)^4]/((d_) + (e_.)*(x_)^2), x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Simp[(c*d^2 + a*e^2)/(e*(e - d*q))   Int[(1 + q*x^2)/((d + e*
x^2)*Sqrt[a + c*x^4]), x], x] - Simp[1/(e*(e - d*q))   Int[(c*d + a*e*q - (
c*e - a*d*q^3)*x^2)/Sqrt[a + c*x^4], x], x]] /; FreeQ[{a, c, d, e}, x] && N
eQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

rule 1577
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, c, d, e, p, q}, x]
```

rule 2223
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTanh[Rt[(-c)*
(d/e) - a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[(-c)*(d/e) - a*(e/d), 2]
)), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^
2)]/(4*d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*Arc
Tan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[c*(d/e) + a*(e
/d)]
```

rule 2267
```
Int[((a_) + (c_.)*(x_)^4)^(p_.)/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[d
Int[(a + c*x^4)^p/(d^2 - e^2*x^2), x], x] - Simp[e   Int[x*((a + c*x^4)^p/(
d^2 - e^2*x^2)), x], x] /; FreeQ[{a, c, d, e}, x] && IntegerQ[p + 1/2]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.53 (sec) , antiderivative size = 404, normalized size of antiderivative = 0.67

method	result
default	$\frac{\sqrt{c x^4+a}}{2e} - \frac{c d^3 \sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}, i\right)}{e^4\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\sqrt{c x^4+a}} + \frac{d^2\sqrt{c}\,\ln\left(2\sqrt{c}\,x^2+2\sqrt{c x^4+a}\right)}{2e^3} - \frac{i\sqrt{c}\,d\sqrt{a}\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\sqrt{1+\cdots}}{\cdots}$
elliptic	$\frac{\sqrt{c x^4+a}}{2e} - \frac{c d^3 \sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}, i\right)}{e^4\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\sqrt{c x^4+a}} + \frac{d^2\sqrt{c}\,\ln\left(2\sqrt{c}\,x^2+2\sqrt{c x^4+a}\right)}{2e^3} - \frac{i\sqrt{c}\,d\sqrt{a}\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\sqrt{1+\cdots}}{\cdots}$
risch	$\frac{\sqrt{c x^4+a}}{2e} - \frac{-\frac{\left(e^4 a+c d^4\right)\left(-\frac{\operatorname{arctanh}\left(\frac{\frac{2c x^2 d^2}{e^2}+2a}{2\sqrt{a+\frac{c d^4}{e^4}}\sqrt{c x^4+a}}\right)}{2\sqrt{a+\frac{c d^4}{e^4}}}+\frac{e\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticPi}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}, -\frac{i\sqrt{a}\,e^2}{\sqrt{c}\,d^2}, \frac{\sqrt{-\frac{i\sqrt{c}}{\sqrt{a}}}}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}}\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,d\sqrt{c x^4+a}}\right)}{e^4} + \frac{c d\left(\cdots\right.}{\cdots}}{\cdots}$

input
```
int((c*x^4+a)^(1/2)/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
1/2*(c*x^4+a)^(1/2)/e-c*d^3/e^4/(I*c^(1/2)/a^(1/2))^(1/2)*(1-I*c^(1/2)*x^2
/a^(1/2))^(1/2)*(1+I*c^(1/2)*x^2/a^(1/2))^(1/2)/(c*x^4+a)^(1/2)*EllipticF(
x*(I*c^(1/2)/a^(1/2))^(1/2),I)+1/2*d^2/e^3*c^(1/2)*ln(2*c^(1/2)*x^2+2*(c*x
^4+a)^(1/2))-I*c^(1/2)*d/e^2*a^(1/2)/(I*c^(1/2)/a^(1/2))^(1/2)*(1-I*c^(1/2
)*x^2/a^(1/2))^(1/2)*(1+I*c^(1/2)*x^2/a^(1/2))^(1/2)/(c*x^4+a)^(1/2)*(Elli
pticF(x*(I*c^(1/2)/a^(1/2))^(1/2),I)-EllipticE(x*(I*c^(1/2)/a^(1/2))^(1/2)
,I))+(a*e^4+c*d^4)/e^5*(-1/2/(a+c*d^4/e^4)^(1/2)*arctanh(1/2*(2*c*x^2*d^2/
e^2+2*a)/(a+c*d^4/e^4)^(1/2)/(c*x^4+a)^(1/2))+1/(I*c^(1/2)/a^(1/2))^(1/2)/
d*e*(1-I*c^(1/2)*x^2/a^(1/2))^(1/2)*(1+I*c^(1/2)*x^2/a^(1/2))^(1/2)/(c*x^4
+a)^(1/2)*EllipticPi(x*(I*c^(1/2)/a^(1/2))^(1/2),-I/c^(1/2)*a^(1/2)/d^2*e^
2,(-I/a^(1/2)*c^(1/2))^(1/2)/(I*c^(1/2)/a^(1/2))^(1/2)))
```

Fricas [F]

$$\int \frac{\sqrt{a+cx^4}}{d+ex}\,dx = \int \frac{\sqrt{cx^4+a}}{ex+d}\,dx$$

input
```
integrate((c*x^4+a)^(1/2)/(e*x+d),x, algorithm="fricas")
```

output
```
integral(sqrt(c*x^4 + a)/(e*x + d), x)
```

Sympy [F]

$$\int \frac{\sqrt{a+cx^4}}{d+ex}\,dx = \int \frac{\sqrt{a+cx^4}}{d+ex}\,dx$$

input
```
integrate((c*x**4+a)**(1/2)/(e*x+d),x)
```

output
```
Integral(sqrt(a + c*x**4)/(d + e*x), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+cx^4}}{d+ex}\,dx = \int \frac{\sqrt{cx^4+a}}{ex+d}\,dx$$

input
```
integrate((c*x^4+a)^(1/2)/(e*x+d),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^4 + a)/(e*x + d), x)
```

Giac [F]

$$\int \frac{\sqrt{a+cx^4}}{d+ex}\,dx = \int \frac{\sqrt{cx^4+a}}{ex+d}\,dx$$

input
```
integrate((c*x^4+a)^(1/2)/(e*x+d),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^4 + a)/(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+cx^4}}{d+ex}\,dx = \int \frac{\sqrt{c\,x^4+a}}{d+e\,x}\,dx$$

input
```
int((a + c*x^4)^(1/2)/(d + e*x),x)
```

output
```
int((a + c*x^4)^(1/2)/(d + e*x), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+cx^4}}{d+ex}\,dx = \int \frac{\sqrt{c\,x^4+a}}{ex+d}dx$$

input
```
int((c*x^4+a)^(1/2)/(e*x+d),x)
```

output
```
int((c*x^4+a)^(1/2)/(e*x+d),x)
```

3.196 $\int \frac{\sqrt{a+cx^4}}{(d+ex)^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 648

$$\begin{aligned}
\int \frac{\sqrt{a+cx^4}}{(d+ex)^2}\,dx &= \frac{2\sqrt{c}x\sqrt{a+cx^4}}{e^2\left(\sqrt{a}+\sqrt{c}x^2\right)} - \frac{d\sqrt{a+cx^4}}{e\left(d^2-e^2x^2\right)} + \frac{x\sqrt{a+cx^4}}{d^2-e^2x^2} \\
&- \frac{cd^3\operatorname{arctanh}\left(\frac{\sqrt{cd^4+ae^4}x}{de\sqrt{a+cx^4}}\right)}{e^3\sqrt{cd^4+ae^4}} - \frac{\sqrt{c}d\operatorname{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{e^3} + \frac{cd^3\operatorname{arctanh}\left(\frac{ae^2+cd^2x^2}{\sqrt{cd^4+ae^4}\sqrt{a+cx^4}}\right)}{e^3\sqrt{cd^4+ae^4}} \\
&- \frac{2\sqrt[4]{a}\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{e^2\sqrt{a+cx^4}} \\
&+ \frac{\sqrt[4]{a}\sqrt[4]{c}\left(2\sqrt{c}d^2+\sqrt{a}e^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{e^2\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\sqrt{a+cx^4}} \\
&+ \frac{c^{3/4}d^2\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticPi}\left(\frac{\left(\sqrt{c}d^2+\sqrt{a}e^2\right)^2}{4\sqrt{a}\sqrt{c}d^2e^2},2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}e^4\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\sqrt{a+cx^4}}
\end{aligned}$$

output
```
2*c^(1/2)*x*(c*x^4+a)^(1/2)/e^2/(a^(1/2)+c^(1/2)*x^2)-d*(c*x^4+a)^(1/2)/e/
(-e^2*x^2+d^2)+x*(c*x^4+a)^(1/2)/(-e^2*x^2+d^2)-c*d^3*arctanh((a*e^4+c*d^4
)^(1/2)*x/d/e/(c*x^4+a)^(1/2))/e^3/(a*e^4+c*d^4)^(1/2)-c^(1/2)*d*arctanh(c
^(1/2)*x^2/(c*x^4+a)^(1/2))/e^3+c*d^3*arctanh((c*d^2*x^2+a*e^2)/(a*e^4+c*d
^4)^(1/2)/(c*x^4+a)^(1/2))/e^3/(a*e^4+c*d^4)^(1/2)-2*a^(1/4)*c^(1/4)*(a^(1
/2)+c^(1/2)*x^2)*((c*x^4+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*EllipticE(sin(2
*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))/e^2/(c*x^4+a)^(1/2)+a^(1/4)*c^(1/
4)*(2*c^(1/2)*d^2+a^(1/2)*e^2)*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+a)/(a^(1/2)+c
^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(c^(1/4)*x/a^(1/4)),1/2*2^(1/
2))/e^2/(c^(1/2)*d^2+a^(1/2)*e^2)/(c*x^4+a)^(1/2)+1/2*c^(3/4)*d^2*(c^(1/2)
*d^2-a^(1/2)*e^2)*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+a)/(a^(1/2)+c^(1/2)*x^2)^2
)^(1/2)*EllipticPi(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/4*(c^(1/2)*d^2+a^(1/
2)*e^2)^2/a^(1/2)/c^(1/2)/d^2/e^2,1/2*2^(1/2))/a^(1/4)/e^4/(c^(1/2)*d^2+a^
(1/2)*e^2)/(c*x^4+a)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 12.73 (sec) , antiderivative size = 394, normalized size of antiderivative = 0.61

$$\int \frac{\sqrt{a+cx^4}}{(d+ex)^2} \, dx$$

$$= \frac{-\frac{e^3(a+cx^4)}{d+ex} - \frac{2cd^3e\sqrt{a+cx^4}\arctan\left(\frac{\sqrt{c}\left(d^2-e^2x^2\right)+e^2\sqrt{a+cx^4}}{\sqrt{-cd^4-ae^4}}\right)}{\sqrt{-cd^4-ae^4}} - 2ia\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}e^2\sqrt{1+\frac{cx^4}{a}}E\left(i\text{arcsinh}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}x\right)\middle|-1\right) -}{}$$

input
```
Integrate[Sqrt[a + c*x^4]/(d + e*x)^2,x]
```

output
```
(-((e^3*(a + c*x^4))/(d + e*x)) - (2*c*d^3*e*Sqrt[a + c*x^4]*ArcTan[(Sqrt[
c]*(d^2 - e^2*x^2) + e^2*Sqrt[a + c*x^4])/Sqrt[-(c*d^4) - a*e^4]])/Sqrt[-(
c*d^4) - a*e^4] - (2*I)*a*Sqrt[(I*Sqrt[c])/Sqrt[a]]*e^2*Sqrt[1 + (c*x^4)/a
]*EllipticE[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1] - (2*Sqrt[c]*(I*Sq
rt[c]*d^2 + Sqrt[a]*e^2)*Sqrt[1 + (c*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[(I*S
qrt[c])/Sqrt[a]]*x], -1])/Sqrt[(I*Sqrt[c])/Sqrt[a]] + 2*(-1)^(1/4)*a^(1/4)
*c^(3/4)*d^2*Sqrt[1 + (c*x^4)/a]*EllipticPi[(I*Sqrt[a]*e^2)/(Sqrt[c]*d^2),
 ArcSin[((-1)^(3/4)*c^(1/4)*x)/a^(1/4)], -1] + Sqrt[c]*d*e*Sqrt[a + c*x^4]
*Log[-(Sqrt[c]*x^2) + Sqrt[a + c*x^4]])/(e^4*Sqrt[a + c*x^4])
```

Rubi [A] (verified)

Time = 4.37 (sec) , antiderivative size = 975, normalized size of antiderivative = 1.50, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.632$, Rules used = {2584, 2255, 27, 1577, 492, 605, 224, 219, 488, 219, 2259, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+cx^4}}{(d+ex)^2}dx$$

$$\Big\downarrow \ 2584$$

$$\int \frac{\sqrt{a+cx^4}\left(d^2-2dex+e^2x^2\right)}{\left(d^2-e^2x^2\right)^2}dx$$

$$\Big\downarrow \ 2255$$

$$\int -\frac{2dex\sqrt{cx^4+a}}{\left(d^2-e^2x^2\right)^2}dx+\int \frac{\left(d^2+e^2x^2\right)\sqrt{cx^4+a}}{\left(d^2-e^2x^2\right)^2}dx$$

$$\Big\downarrow \ 27$$

$$\int \frac{\left(d^2+e^2x^2\right)\sqrt{cx^4+a}}{\left(d^2-e^2x^2\right)^2}dx-2de\int \frac{x\sqrt{cx^4+a}}{\left(d^2-e^2x^2\right)^2}dx$$

$$\Big\downarrow \ 1577$$

$$\int \frac{\left(d^2+e^2x^2\right)\sqrt{cx^4+a}}{\left(d^2-e^2x^2\right)^2}dx-de\int \frac{\sqrt{cx^4+a}}{\left(d^2-e^2x^2\right)^2}dx^2$$

$$\Big\downarrow \ 492$$

$$\int \frac{(d^2+e^2x^2)\sqrt{cx^4+a}}{(d^2-e^2x^2)^2}dx - de\left(\frac{\sqrt{a+cx^4}}{e^2(d^2-e^2x^2)} - \frac{c\int \frac{x^2}{(d^2-e^2x^2)\sqrt{cx^4+a}}dx^2}{e^2}\right)$$

$\downarrow$ 605

$$\int \frac{(d^2+e^2x^2)\sqrt{cx^4+a}}{(d^2-e^2x^2)^2}dx - de\left(\frac{\sqrt{a+cx^4}}{e^2(d^2-e^2x^2)} - \frac{c\left(\frac{d^2\int \frac{1}{(d^2-e^2x^2)\sqrt{cx^4+a}}dx^2}{e^2} - \frac{\int \frac{1}{\sqrt{cx^4+a}}dx^2}{e^2}\right)}{e^2}\right)$$

$\downarrow$ 224

$$\int \frac{(d^2+e^2x^2)\sqrt{cx^4+a}}{(d^2-e^2x^2)^2}dx - de\left(\frac{\sqrt{a+cx^4}}{e^2(d^2-e^2x^2)} - \frac{c\left(\frac{d^2\int \frac{1}{(d^2-e^2x^2)\sqrt{cx^4+a}}dx^2}{e^2} - \frac{\int \frac{1}{1-cx^4}d\frac{x^2}{\sqrt{cx^4+a}}}{e^2}\right)}{e^2}\right)$$

$\downarrow$ 219

$$\int \frac{(d^2+e^2x^2)\sqrt{cx^4+a}}{(d^2-e^2x^2)^2}dx - de\left(\frac{\sqrt{a+cx^4}}{e^2(d^2-e^2x^2)} - \frac{c\left(\frac{d^2\int \frac{1}{(d^2-e^2x^2)\sqrt{cx^4+a}}dx^2}{e^2} - \frac{\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{\sqrt{c}e^2}\right)}{e^2}\right)$$

$\downarrow$ 488

$$\int \frac{(d^2+e^2x^2)\sqrt{cx^4+a}}{(d^2-e^2x^2)^2}dx - de\left(\frac{\sqrt{a+cx^4}}{e^2(d^2-e^2x^2)} - \frac{c\left(-\frac{d^2\int \frac{1}{cd^4+ae^4-x^4}d\frac{-ae^2-cd^2x^2}{\sqrt{cx^4+a}}}{e^2} - \frac{\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{\sqrt{c}e^2}\right)}{e^2}\right)$$

$\downarrow$ 219

$$\int \frac{\left(d^2+e^2x^2\right)\sqrt{cx^4+a}}{\left(d^2-e^2x^2\right)^2}dx - de\left(\frac{\sqrt{a+cx^4}}{e^2\left(d^2-e^2x^2\right)} - \frac{c\left(-\frac{d^2\text{arctanh}\left(\frac{-ae^2-cd^2x^2}{\sqrt{a+cx^4}\sqrt{ae^4+cd^4}}\right)}{e^2\sqrt{ae^4+cd^4}} - \frac{\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{\sqrt{c}e^2}\right)}{e^2}\right)$$

$$\Big\downarrow \text{ 2259}$$

$$\int \left(-\frac{4cd^4}{e^4\left(d^2-e^2x^2\right)\sqrt{cx^4+a}} + \frac{3cd^2}{e^4\sqrt{cx^4+a}} + \frac{cx^2}{e^2\sqrt{cx^4+a}} + \frac{cd^4+ae^4}{2e^4(ex-d)^2\sqrt{cx^4+a}} + \frac{cd^4+ae^4}{2e^4(d+ex)^2\sqrt{cx^4+a}}\right.$$
$$de\left(\frac{\sqrt{a+cx^4}}{e^2\left(d^2-e^2x^2\right)} - \frac{c\left(-\frac{d^2\text{arctanh}\left(\frac{-ae^2-cd^2x^2}{\sqrt{a+cx^4}\sqrt{ae^4+cd^4}}\right)}{e^2\sqrt{ae^4+cd^4}} - \frac{\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{\sqrt{c}e^2}\right)}{e^2}\right)$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{2c^{5/4}\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{\frac{cx^4+a}{\left(\sqrt{c}x^2+\sqrt{a}\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)d^4}{\sqrt[4]{a}e^4\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\sqrt{cx^4+a}}-\frac{c\operatorname{arctanh}\left(\frac{\sqrt{cd^4+ae^4}x}{de\sqrt{cx^4+a}}\right)d^3}{e^3\sqrt{cd^4+ae^4}}+\frac{3c^{3/4}\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{\frac{cx^4+a}{\left(\sqrt{c}x^2+\sqrt{a}\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)d^2}{2\sqrt[4]{a}e^4\sqrt{cx^4+a}}+\frac{c^{3/4}\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{\frac{cx^4+a}{\left(\sqrt{c}x^2+\sqrt{a}\right)^2}}\operatorname{EllipticPi}\left(\frac{\left(\sqrt{c}d^2+\sqrt{a}e^2\right)^2}{4\sqrt{a}\sqrt{c}d^2e^2},2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)d^2}{2\sqrt[4]{a}e^4\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\sqrt{cx^4+a}}-e\left(\frac{\sqrt{cx^4+a}}{e^2\left(d^2-e^2x^2\right)}-\frac{c\left(-\frac{\operatorname{arctanh}\left(\frac{-ae^2-cd^2x^2}{\sqrt{cd^4+ae^4}\sqrt{cx^4+a}}\right)d^2}{e^2\sqrt{cd^4+ae^4}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{cx^4+a}}\right)}{\sqrt{c}e^2}\right)}{e^2}\right)d-\frac{2\sqrt[4]{a}\sqrt[4]{c}\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{\frac{cx^4+a}{\left(\sqrt{c}x^2+\sqrt{a}\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{e^2\sqrt{cx^4+a}}+\frac{\sqrt[4]{c}\left(cd^4+ae^4\right)\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{\frac{cx^4+a}{\left(\sqrt{c}x^2+\sqrt{a}\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}e^4\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\sqrt{cx^4+a}}+\frac{\sqrt[4]{a}\sqrt[4]{c}\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{\frac{cx^4+a}{\left(\sqrt{c}x^2+\sqrt{a}\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2e^2\sqrt{cx^4+a}}+\frac{\sqrt{cx^4+a}}{2e(d-ex)}-\frac{\sqrt{cx^4+a}}{2e(d+ex)}+\frac{2\sqrt{c}x\sqrt{cx^4+a}}{e^2\left(\sqrt{c}x^2+\sqrt{a}\right)}$$

input
```
Int[Sqrt[a + c*x^4]/(d + e*x)^2,x]
```

output
```
Sqrt[a + c*x^4]/(2*e*(d - e*x)) - Sqrt[a + c*x^4]/(2*e*(d + e*x)) + (2*Sqr
t[c]*x*Sqrt[a + c*x^4])/(e^2*(Sqrt[a] + Sqrt[c]*x^2)) - (c*d^3*ArcTanh[(Sq
rt[c*d^4 + a*e^4]*x)/(d*e*Sqrt[a + c*x^4])])/(e^3*Sqrt[c*d^4 + a*e^4]) - d
*e*(Sqrt[a + c*x^4]/(e^2*(d^2 - e^2*x^2)) - (c*(-(ArcTanh[(Sqrt[c]*x^2)/Sq
rt[a + c*x^4]]/(Sqrt[c]*e^2)) - (d^2*ArcTanh[(-(a*e^2) - c*d^2*x^2)/(Sqrt[
c*d^4 + a*e^4]*Sqrt[a + c*x^4])])/(e^2*Sqrt[c*d^4 + a*e^4])))/e^2) - (2*a^
(1/4)*c^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*
x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(e^2*Sqrt[a + c*x^4
]) + (3*c^(3/4)*d^2*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sq
rt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*e^
4*Sqrt[a + c*x^4]) + (a^(1/4)*c^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*
x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1
/2])/(2*e^2*Sqrt[a + c*x^4]) - (2*c^(5/4)*d^4*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt
[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(
1/4)], 1/2])/(a^(1/4)*e^4*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*Sqrt[a + c*x^4]) + (
c^(1/4)*(c*d^4 + a*e^4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a]
+ Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4
)*e^4*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*Sqrt[a + c*x^4]) + (c^(3/4)*d^2*(Sqrt[c]
*d^2 - Sqrt[a]*e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sq
rt[c]*x^2)^2]*EllipticPi[(Sqrt[c]*d^2 + Sqrt[a]*e^2)^2/(4*Sqrt[a]*Sqrt[...
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 492
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*((a + b*x^2)^p/(d*(n + 1))), x] - Simp[2*b*(p/(d*(n + 1))
)   Int[x*(c + d*x)^(n + 1)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c,
d, n}, x] && GtQ[p, 0] && (IntegerQ[p] || LtQ[n, -1]) && NeQ[n, -1] &&  !IL
tQ[n + 2*p + 1, 0] && IntQuadraticQ[a, 0, b, c, d, n, p, x]
```

rule 605
```
Int[((x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)), x_Symbol]
:> Simp[1/d   Int[x^(m - 1)*(a + b*x^2)^p, x], x] - Simp[c/d   Int[x^(m - 1
)*((a + b*x^2)^p/(c + d*x)), x], x] /; FreeQ[{a, b, c, d, p}, x] && IGtQ[m,
 0] && LtQ[-1, p, 0]
```

rule 1577
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, c, d, e, p, q}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2255
```
Int[(Pr_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Module[{r = Expon[Pr, x], k}, Int[Sum[Coeff[Pr, x, 2*k]*x^(2*k), {k, 0,
 r/2}]*(d + e*x^2)^q*(a + c*x^4)^p, x] + Int[x*Sum[Coeff[Pr, x, 2*k + 1]*x^
(2*k), {k, 0, (r - 1)/2}]*(d + e*x^2)^q*(a + c*x^4)^p, x]] /; FreeQ[{a, c,
d, e, p, q}, x] && PolyQ[Pr, x] &&  !PolyQ[Pr, x^2]
```

rule 2259
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Int[ExpandIntegrand[1/Sqrt[a + c*x^4], Px*(d + e*x^2)^q*(a + c*x^4)^(p
+ 1/2), x], x] /; FreeQ[{a, c, d, e}, x] && PolyQ[Px, x] && IntegerQ[p + 1/
2] && IntegerQ[q]
```

rule 2584
```
Int[((c_) + (d_.)*(x_)^(n_.))^(q_)*((a_) + (b_.)*(x_)^(nn_.))^(p_), x_Symbo
l] :> Int[ExpandToSum[(c - d*x^n)^(-q), x]*((a + b*x^nn)^p/(c^2 - d^2*x^(2*
n))^(-q)), x] /; FreeQ[{a, b, c, d, n, nn, p}, x] &&  !IntegerQ[p] && ILtQ[
q, 0] && IGtQ[Log[2, nn/n], 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.51 (sec) , antiderivative size = 402, normalized size of antiderivative = 0.62

method	result
default	$-\frac{\sqrt{c\,x^4+a}}{e(ex+d)}+\frac{2c\,d^2\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{e^4\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}-\frac{\sqrt{c}\,d\ln\left(2\sqrt{c}\,x^2+2\sqrt{c\,x^4+a}\right)}{e^3}+\frac{2i\sqrt{c}\,\sqrt{a}\,\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{}}{}$
elliptic	$-\frac{\sqrt{c\,x^4+a}}{e(ex+d)}+\frac{2c\,d^2\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{e^4\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}-\frac{\sqrt{c}\,d\ln\left(2\sqrt{c}\,x^2+2\sqrt{c\,x^4+a}\right)}{e^3}+\frac{2i\sqrt{c}\,\sqrt{a}\,\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{}}{}$

input
```
int((c*x^4+a)^(1/2)/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/e*(c*x^4+a)^(1/2)/(e*x+d)+2*c*d^2/e^4/(I*c^(1/2)/a^(1/2))^(1/2)*(1-I*c^
(1/2)*x^2/a^(1/2))^(1/2)*(1+I*c^(1/2)*x^2/a^(1/2))^(1/2)/(c*x^4+a)^(1/2)*E
llipticF(x*(I*c^(1/2)/a^(1/2))^(1/2),I)-c^(1/2)*d/e^3*ln(2*c^(1/2)*x^2+2*(
c*x^4+a)^(1/2))+2*I*c^(1/2)/e^2*a^(1/2)/(I*c^(1/2)/a^(1/2))^(1/2)*(1-I*c^(
1/2)*x^2/a^(1/2))^(1/2)*(1+I*c^(1/2)*x^2/a^(1/2))^(1/2)/(c*x^4+a)^(1/2)*(E
llipticF(x*(I*c^(1/2)/a^(1/2))^(1/2),I)-EllipticE(x*(I*c^(1/2)/a^(1/2))^(1
/2),I))-2*c*d^3/e^5*(-1/2/(a+c*d^4/e^4)^(1/2)*arctanh(1/2*(2*c*x^2*d^2/e^2
+2*a)/(a+c*d^4/e^4)^(1/2)/(c*x^4+a)^(1/2))+1/(I*c^(1/2)/a^(1/2))^(1/2)/d*e
*(1-I*c^(1/2)*x^2/a^(1/2))^(1/2)*(1+I*c^(1/2)*x^2/a^(1/2))^(1/2)/(c*x^4+a)
^(1/2)*EllipticPi(x*(I*c^(1/2)/a^(1/2))^(1/2),-I/c^(1/2)*a^(1/2)/d^2*e^2,(
-I/a^(1/2)*c^(1/2))^(1/2)/(I*c^(1/2)/a^(1/2))^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+cx^4}}{(d+ex)^2}\,dx = \text{Timed out}$$

input
```
integrate((c*x^4+a)^(1/2)/(e*x+d)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+cx^4}}{(d+ex)^2}\,dx = \int \frac{\sqrt{a+cx^4}}{(d+ex)^2}\,dx$$

input
```
integrate((c*x**4+a)**(1/2)/(e*x+d)**2,x)
```

output
```
Integral(sqrt(a + c*x**4)/(d + e*x)**2, x)
```

Maxima [F]

$$\int \frac{\sqrt{a+cx^4}}{(d+ex)^2}\,dx = \int \frac{\sqrt{cx^4+a}}{(ex+d)^2}\,dx$$

input
```
integrate((c*x^4+a)^(1/2)/(e*x+d)^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^4 + a)/(e*x + d)^2, x)
```

Giac [F]

$$\int \frac{\sqrt{a+cx^4}}{(d+ex)^2}\,dx = \int \frac{\sqrt{cx^4+a}}{(ex+d)^2}\,dx$$

input
```
integrate((c*x^4+a)^(1/2)/(e*x+d)^2,x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^4 + a)/(e*x + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+cx^4}}{(d+ex)^2}\,dx = \int \frac{\sqrt{c\,x^4+a}}{(d+e\,x)^2}\,dx$$

input
```
int((a + c*x^4)^(1/2)/(d + e*x)^2,x)
```

output
```
int((a + c*x^4)^(1/2)/(d + e*x)^2, x)
```

Reduce [F]

$$\int \frac{\sqrt{a+cx^4}}{(d+ex)^2}\,dx = \int \frac{\sqrt{c\,x^4+a}}{(ex+d)^2}dx$$

input
```
int((c*x^4+a)^(1/2)/(e*x+d)^2,x)
```

output
```
int((c*x^4+a)^(1/2)/(e*x+d)^2,x)
```

3.197 $\int \frac{\sqrt{a+cx^4}}{(d+ex)^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 811

$$\int \frac{\sqrt{a+cx^4}}{(d+ex)^3}\,dx = -\frac{c^{3/2}d^3x\sqrt{a+cx^4}}{e^2\left(cd^4+ae^4\right)\left(\sqrt{a}+\sqrt{c}x^2\right)}+\frac{dx\sqrt{a+cx^4}}{\left(d^2-e^2x^2\right)^2}$$
$$-\frac{cd^3x\sqrt{a+cx^4}}{\left(cd^4+ae^4\right)\left(d^2-e^2x^2\right)}+\frac{\left(d^2\left(cd^4-ae^4\right)-e^2\left(3cd^4+ae^4\right)x^2\right)\sqrt{a+cx^4}}{2e\left(cd^4+ae^4\right)\left(d^2-e^2x^2\right)^2}$$
$$+\frac{cd^2\left(cd^4+3ae^4\right)\operatorname{arctanh}\left(\frac{\sqrt{cd^4+ae^4}x}{de\sqrt{a+cx^4}}\right)}{2e^3\left(cd^4+ae^4\right)^{3/2}}+\frac{\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{2e^3}$$
$$-\frac{cd^2\left(cd^4+3ae^4\right)\operatorname{arctanh}\left(\frac{ae^2+cd^2x^2}{\sqrt{cd^4+ae^4}\sqrt{a+cx^4}}\right)}{2e^3\left(cd^4+ae^4\right)^{3/2}}$$
$$+\frac{\sqrt[4]{a}c^{5/4}d^3\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{e^2\left(cd^4+ae^4\right)\sqrt{a+cx^4}}$$
$$-\frac{\sqrt[4]{a}c^{3/4}d\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{e^2\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\sqrt{a+cx^4}}$$
$$-\frac{c^{3/4}d\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\left(cd^4+3ae^4\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticPi}\left(\frac{\left(\sqrt{c}d^2+\sqrt{a}e^2\right)^2}{4\sqrt{a}\sqrt{c}d^2e^2},2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right.\right.}{4\sqrt[4]{a}e^4\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\left(cd^4+ae^4\right)\sqrt{a+cx^4}}$$

output
```
-c^(3/2)*d^3*x*(c*x^4+a)^(1/2)/e^2/(a*e^4+c*d^4)/(a^(1/2)+c^(1/2)*x^2)+d*x
*(c*x^4+a)^(1/2)/(-e^2*x^2+d^2)^2-c*d^3*x*(c*x^4+a)^(1/2)/(a*e^4+c*d^4)/(-
e^2*x^2+d^2)+1/2*(d^2*(-a*e^4+c*d^4)-e^2*(a*e^4+3*c*d^4)*x^2)*(c*x^4+a)^(1
/2)/e/(a*e^4+c*d^4)/(-e^2*x^2+d^2)^2+1/2*c*d^2*(3*a*e^4+c*d^4)*arctanh((a*
e^4+c*d^4)^(1/2)*x/d/e/(c*x^4+a)^(1/2))/e^3/(a*e^4+c*d^4)^(3/2)+1/2*c^(1/2
)*arctanh(c^(1/2)*x^2/(c*x^4+a)^(1/2))/e^3-1/2*c*d^2*(3*a*e^4+c*d^4)*arcta
nh((c*d^2*x^2+a*e^2)/(a*e^4+c*d^4)^(1/2)/(c*x^4+a)^(1/2))/e^3/(a*e^4+c*d^4
)^(3/2)+a^(1/4)*c^(5/4)*d^3*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+a)/(a^(1/2)+c^(1
/2)*x^2)^2)^(1/2)*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))/
e^2/(a*e^4+c*d^4)/(c*x^4+a)^(1/2)-a^(1/4)*c^(3/4)*d*(a^(1/2)+c^(1/2)*x^2)*
((c*x^4+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(c^(1/4)
*x/a^(1/4)),1/2*2^(1/2))/e^2/(c^(1/2)*d^2+a^(1/2)*e^2)/(c*x^4+a)^(1/2)-1/4
*c^(3/4)*d*(c^(1/2)*d^2-a^(1/2)*e^2)*(3*a*e^4+c*d^4)*(a^(1/2)+c^(1/2)*x^2)
*((c*x^4+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*EllipticPi(sin(2*arctan(c^(1/4)
*x/a^(1/4))),1/4*(c^(1/2)*d^2+a^(1/2)*e^2)^2/a^(1/2)/c^(1/2)/d^2/e^2,1/2*2
^(1/2))/a^(1/4)/e^4/(c^(1/2)*d^2+a^(1/2)*e^2)/(a*e^4+c*d^4)/(c*x^4+a)^(1/2
)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 12.01 (sec) , antiderivative size = 989, normalized size of antiderivative = 1.22

$$\int \frac{\sqrt{a+cx^4}}{(d+ex)^3} dx = \text{Too large to display}$$

input
```
Integrate[Sqrt[a + c*x^4]/(d + e*x)^3,x]
```

output
```
(-(e^3*(c*d^4 + a*e^4)^2*(a + c*x^4)) + 2*c*d^3*e^3*(c*d^4 + a*e^4)*(d + e
*x)*(a + c*x^4) - 2*c^2*d^6*e*Sqrt[-(c*d^4) - a*e^4]*(d + e*x)^2*Sqrt[a +
c*x^4]*ArcTan[(Sqrt[c]*(d^2 - e^2*x^2) + e^2*Sqrt[a + c*x^4])/Sqrt[-(c*d^4
) - a*e^4]] - 6*a*c*d^2*e^5*Sqrt[-(c*d^4) - a*e^4]*(d + e*x)^2*Sqrt[a + c*
x^4]*ArcTan[(Sqrt[c]*(d^2 - e^2*x^2) + e^2*Sqrt[a + c*x^4])/Sqrt[-(c*d^4)
- a*e^4]] + (2*I)*a*Sqrt[(I*Sqrt[c])/Sqrt[a]]*c*d^3*e^2*(c*d^4 + a*e^4)*(d
 + e*x)^2*Sqrt[1 + (c*x^4)/a]*EllipticE[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]
]*x], -1] + ((2*I)*c^2*d^5*(c*d^4 + a*e^4)*(d + e*x)^2*Sqrt[1 + (c*x^4)/a]
*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1])/Sqrt[(I*Sqrt[c])/S
qrt[a]] - (2*I)*a*Sqrt[(I*Sqrt[c])/Sqrt[a]]*c*d^3*e^2*(c*d^4 + a*e^4)*(d +
 e*x)^2*Sqrt[1 + (c*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*
x], -1] + ((4*I)*a*c*d*e^4*(c*d^4 + a*e^4)*(d + e*x)^2*Sqrt[1 + (c*x^4)/a]
*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1])/Sqrt[(I*Sqrt[c])/S
qrt[a]] - 2*(-1)^(1/4)*a^(1/4)*c^(7/4)*d^5*(c*d^4 + a*e^4)*(d + e*x)^2*Sqr
t[1 + (c*x^4)/a]*EllipticPi[(I*Sqrt[a]*e^2)/(Sqrt[c]*d^2), ArcSin[((-1)^(3
/4)*c^(1/4)*x)/a^(1/4)], -1] - 6*(-1)^(1/4)*a^(5/4)*c^(3/4)*d*e^4*(c*d^4 +
 a*e^4)*(d + e*x)^2*Sqrt[1 + (c*x^4)/a]*EllipticPi[(I*Sqrt[a]*e^2)/(Sqrt[c
]*d^2), ArcSin[((-1)^(3/4)*c^(1/4)*x)/a^(1/4)], -1] - c^(5/2)*d^8*e*(d + e
*x)^2*Sqrt[a + c*x^4]*Log[-(Sqrt[c]*x^2) + Sqrt[a + c*x^4]] - 2*a*c^(3/2)*
d^4*e^5*(d + e*x)^2*Sqrt[a + c*x^4]*Log[-(Sqrt[c]*x^2) + Sqrt[a + c*x^4...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+cx^4}}{(d+ex)^3}\,dx$$

$$\Big\downarrow \text{ 2584}$$

$$\int \frac{\sqrt{a+cx^4}\left(d^3-3d^2ex+3de^2x^2-e^3x^3\right)}{\left(d^2-e^2x^2\right)^3}dx$$

$$\Big\downarrow \text{ 2006}$$

$$\int \frac{\sqrt{a+cx^4}(d-ex)^3}{\left(d^2-e^2x^2\right)^3}dx$$

$$\Big\downarrow \text{ 2003}$$

$$\int \frac{\sqrt{a+cx^4}}{(d+ex)^3}dx$$

$$\Big\downarrow \text{ 2584}$$

$$\int \frac{\sqrt{a+cx^4}\left(d^3-3d^2ex+3de^2x^2-e^3x^3\right)}{\left(d^2-e^2x^2\right)^3}dx$$

$$\Big\downarrow \text{ 2006}$$

$$\int \frac{\sqrt{a+cx^4}(d-ex)^3}{\left(d^2-e^2x^2\right)^3}dx$$

$$\Big\downarrow \text{ 2003}$$

$$\int \frac{\sqrt{a+cx^4}}{(d+ex)^3}dx$$

$$\Big\downarrow \text{ 2584}$$

$$\int \frac{\sqrt{a+cx^4}\left(d^3-3d^2ex+3de^2x^2-e^3x^3\right)}{\left(d^2-e^2x^2\right)^3}dx$$

$$\Big\downarrow \text{ 2006}$$

$$\int \frac{\sqrt{a+cx^4}(d-ex)^3}{\left(d^2-e^2x^2\right)^3}dx$$

$$\Big\downarrow \text{ 2003}$$

$$\int \frac{\sqrt{a+cx^4}}{(d+ex)^3}dx$$

$$\Big\downarrow \text{ 2584}$$

$$\int \frac{\sqrt{a+cx^4}\left(d^3-3d^2ex+3de^2x^2-e^3x^3\right)}{\left(d^2-e^2x^2\right)^3}dx$$

$$\Big\downarrow \text{ 2006}$$

$$\int \frac{\sqrt{a+cx^4}(d-ex)^3}{\left(d^2-e^2x^2\right)^3}dx$$

$$\Big\downarrow \text{ 2003}$$

$$\int \frac{\sqrt{a+cx^4}}{(d+ex)^3}dx$$

$$\Big\downarrow \text{ 2584}$$

$$\int \frac{\sqrt{a+cx^4}\left(d^3-3d^2ex+3de^2x^2-e^3x^3\right)}{\left(d^2-e^2x^2\right)^3}dx$$

$$\Big\downarrow \text{ 2006}$$

$$\int \frac{\sqrt{a+cx^4}(d-ex)^3}{\left(d^2-e^2x^2\right)^3}dx$$

$$\Big\downarrow \text{ 2003}$$

$$\int \frac{\sqrt{a+cx^4}}{(d+ex)^3}dx$$

$$\Big\downarrow \text{ 2584}$$

$$\int \frac{\sqrt{a+cx^4}\left(d^3-3d^2ex+3de^2x^2-e^3x^3\right)}{\left(d^2-e^2x^2\right)^3}dx$$

$$\Big\downarrow \text{ 2006}$$

$$\int \frac{\sqrt{a+cx^4}(d-ex)^3}{\left(d^2-e^2x^2\right)^3}dx$$

$$\Big\downarrow \text{ 2003}$$

$$\int \frac{\sqrt{a+cx^4}}{(d+ex)^3}dx$$

$$\Big\downarrow \text{ 2584}$$

$$\int \frac{\sqrt{a+cx^4}\left(d^3-3d^2ex+3de^2x^2-e^3x^3\right)}{\left(d^2-e^2x^2\right)^3}dx$$

$$\Big\downarrow \text{ 2006}$$

$$\int \frac{\sqrt{a+cx^4}(d-ex)^3}{\left(d^2-e^2x^2\right)^3}dx$$

$$\Big\downarrow \text{ 2003}$$

$$\int \frac{\sqrt{a+cx^4}}{(d+ex)^3}dx$$

$$\Big\downarrow \text{ 2584}$$

$$\int \frac{\sqrt{a+cx^4}\left(d^3-3d^2ex+3de^2x^2-e^3x^3\right)}{\left(d^2-e^2x^2\right)^3}dx$$

$$\Big\downarrow \text{ 2006}$$

$$\int \frac{\sqrt{a+cx^4}(d-ex)^3}{(d^2-e^2x^2)^3}dx$$

$$\Big\downarrow \text{2003}$$

$$\int \frac{\sqrt{a+cx^4}}{(d+ex)^3}dx$$

$$\Big\downarrow \text{2584}$$

$$\int \frac{\sqrt{a+cx^4}(d^3-3d^2ex+3de^2x^2-e^3x^3)}{(d^2-e^2x^2)^3}dx$$

$$\Big\downarrow \text{2006}$$

$$\int \frac{\sqrt{a+cx^4}(d-ex)^3}{(d^2-e^2x^2)^3}dx$$

$$\Big\downarrow \text{2003}$$

$$\int \frac{\sqrt{a+cx^4}}{(d+ex)^3}dx$$

$$\Big\downarrow \text{2584}$$

$$\int \frac{\sqrt{a+cx^4}(d^3-3d^2ex+3de^2x^2-e^3x^3)}{(d^2-e^2x^2)^3}dx$$

$$\Big\downarrow \text{2006}$$

$$\int \frac{\sqrt{a+cx^4}(d-ex)^3}{(d^2-e^2x^2)^3}dx$$

$$\Big\downarrow \text{2003}$$

$$\int \frac{\sqrt{a+cx^4}}{(d+ex)^3}dx$$

input
```
Int[Sqrt[a + c*x^4]/(d + e*x)^3,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2003
```
Int[(u_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :
> Int[u*(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}
, x] && EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&
 !IntegerQ[n]))
```

rule 2006
```
Int[(u_.)*(Px_), x_Symbol] :> With[{a = Rt[Coeff[Px, x, 0], Expon[Px, x]],
b = Rt[Coeff[Px, x, Expon[Px, x]], Expon[Px, x]]}, Int[u*(a + b*x)^Expon[Px
, x], x] /; EqQ[Px, (a + b*x)^Expon[Px, x]]] /; PolyQ[Px, x] && GtQ[Expon[P
x, x], 1] && NeQ[Coeff[Px, x, 0], 0] &&  !MatchQ[Px, (a_.)*(v_)^Expon[Px, x
] /; FreeQ[a, x] && LinearQ[v, x]]
```

rule 2584
```
Int[((c_) + (d_.)*(x_)^(n_.))^(q_)*((a_) + (b_.)*(x_)^(nn_.))^(p_), x_Symbo
l] :> Int[ExpandToSum[(c - d*x^n)^(-q), x]*((a + b*x^nn)^p/(c^2 - d^2*x^(2*
n))^(-q)), x] /; FreeQ[{a, b, c, d, n, nn, p}, x] &&  !IntegerQ[p] && ILtQ[
q, 0] && IGtQ[Log[2, nn/n], 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.65 (sec) , antiderivative size = 509, normalized size of antiderivative = 0.63

method	result
default	$-\frac{\sqrt{c\,x^{4}+a}}{2e\left(ex+d\right)^{2}}+\frac{c\,d^{3}\sqrt{c\,x^{4}+a}}{\left(e^{4}a+c\,d^{4}\right)e\left(ex+d\right)}+\frac{\left(-\frac{3cd}{e^{4}}+\frac{cd\left(e^{4}a+2c\,d^{4}\right)}{e^{4}\left(e^{4}a+c\,d^{4}\right)}\right)\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}}+\frac{\sqrt{c}\,\ln\left(2\sqrt{c}\right.}{}$
elliptic	$-\frac{\sqrt{c\,x^{4}+a}}{2e\left(ex+d\right)^{2}}+\frac{c\,d^{3}\sqrt{c\,x^{4}+a}}{\left(e^{4}a+c\,d^{4}\right)e\left(ex+d\right)}+\frac{\left(-\frac{3cd}{e^{4}}+\frac{cd\left(e^{4}a+2c\,d^{4}\right)}{e^{4}\left(e^{4}a+c\,d^{4}\right)}\right)\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}}+\frac{\sqrt{c}\,\ln\left(2\sqrt{c}\right.}{}$

input
```
int((c*x^4+a)^(1/2)/(e*x+d)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2/e*(c*x^4+a)^(1/2)/(e*x+d)^2+c*d^3/(a*e^4+c*d^4)/e*(c*x^4+a)^(1/2)/(e*
x+d)+(-3*c*d/e^4+c*d*(a*e^4+2*c*d^4)/e^4/(a*e^4+c*d^4))/(I*c^(1/2)/a^(1/2)
)^(1/2)*(1-I*c^(1/2)*x^2/a^(1/2))^(1/2)*(1+I*c^(1/2)*x^2/a^(1/2))^(1/2)/(c
*x^4+a)^(1/2)*EllipticF(x*(I*c^(1/2)/a^(1/2))^(1/2),I)+1/2*c^(1/2)/e^3*ln(
2*c^(1/2)*x^2+2*(c*x^4+a)^(1/2))-I*d^3/e^2*c^(3/2)/(a*e^4+c*d^4)*a^(1/2)/(
I*c^(1/2)/a^(1/2))^(1/2)*(1-I*c^(1/2)*x^2/a^(1/2))^(1/2)*(1+I*c^(1/2)*x^2/
a^(1/2))^(1/2)/(c*x^4+a)^(1/2)*(EllipticF(x*(I*c^(1/2)/a^(1/2))^(1/2),I)-E
llipticE(x*(I*c^(1/2)/a^(1/2))^(1/2),I))+c*d^2*(3*a*e^4+c*d^4)/(a*e^4+c*d^
4)/e^5*(-1/2/(a+c*d^4/e^4)^(1/2)*arctanh(1/2*(2*c*x^2*d^2/e^2+2*a)/(a+c*d^
4/e^4)^(1/2)/(c*x^4+a)^(1/2))+1/(I*c^(1/2)/a^(1/2))^(1/2)/d*e*(1-I*c^(1/2)
*x^2/a^(1/2))^(1/2)*(1+I*c^(1/2)*x^2/a^(1/2))^(1/2)/(c*x^4+a)^(1/2)*Ellipt
icPi(x*(I*c^(1/2)/a^(1/2))^(1/2),-I/c^(1/2)*a^(1/2)/d^2*e^2,(-I/a^(1/2)*c^
(1/2))^(1/2)/(I*c^(1/2)/a^(1/2))^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+cx^4}}{(d+ex)^3}\,dx = \text{Timed out}$$

input
```
integrate((c*x^4+a)^(1/2)/(e*x+d)^3,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+cx^4}}{(d+ex)^3}\,dx = \int \frac{\sqrt{a+cx^4}}{(d+ex)^3}\,dx$$

input
```
integrate((c*x**4+a)**(1/2)/(e*x+d)**3,x)
```

output
```
Integral(sqrt(a + c*x**4)/(d + e*x)**3, x)
```

Maxima [F]

$$\int \frac{\sqrt{a+cx^4}}{(d+ex)^3}\,dx = \int \frac{\sqrt{cx^4+a}}{(ex+d)^3}\,dx$$

input
```
integrate((c*x^4+a)^(1/2)/(e*x+d)^3,x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^4 + a)/(e*x + d)^3, x)
```

Giac [F]

$$\int \frac{\sqrt{a+cx^4}}{(d+ex)^3}\,dx = \int \frac{\sqrt{cx^4+a}}{(ex+d)^3}\,dx$$

input
```
integrate((c*x^4+a)^(1/2)/(e*x+d)^3,x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^4 + a)/(e*x + d)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+cx^4}}{(d+ex)^3}\,dx = \int \frac{\sqrt{c\,x^4+a}}{(d+e\,x)^3}\,dx$$

input
```
int((a + c*x^4)^(1/2)/(d + e*x)^3,x)
```

output
```
int((a + c*x^4)^(1/2)/(d + e*x)^3, x)
```

Reduce [F]

$$\int \frac{\sqrt{a+cx^4}}{(d+ex)^3}\,dx = \int \frac{\sqrt{c\,x^4+a}}{(ex+d)^3}dx$$

input
```
int((c*x^4+a)^(1/2)/(e*x+d)^3,x)
```

output
```
int((c*x^4+a)^(1/2)/(e*x+d)^3,x)
```

3.198 $\int (d+ex)^3 \left(a+cx^4\right)^{3/2} dx$

Optimal result

Integrand size = 19, antiderivative size = 414

$$\begin{aligned}\int (d+ex)^3 \left(a+cx^4\right)^{3/2} dx &= \frac{9}{16}ad^2ex^2\sqrt{a+cx^4}+\frac{4a^2de^2x\sqrt{a+cx^4}}{5\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)}\\
&+\frac{2}{35}adx\left(5d^2+7e^2x^2\right)\sqrt{a+cx^4}+\frac{3}{8}d^2ex^2\left(a+cx^4\right)^{3/2}\\
&+\frac{1}{21}dx\left(3d^2+7e^2x^2\right)\left(a+cx^4\right)^{3/2}+\frac{e^3\left(a+cx^4\right)^{5/2}}{10c}+\frac{9a^2d^2e\operatorname{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{16\sqrt{c}}\\
&-\frac{4a^{9/4}de^2\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5c^{3/4}\sqrt{a+cx^4}}\\
&+\frac{2a^{7/4}d\left(5\sqrt{c}d^2+7\sqrt{a}e^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{35c^{3/4}\sqrt{a+cx^4}}\end{aligned}$$

output
```
9/16*a*d^2*e*x^2*(c*x^4+a)^(1/2)+4/5*a^2*d*e^2*x*(c*x^4+a)^(1/2)/c^(1/2)/(
a^(1/2)+c^(1/2)*x^2)+2/35*a*d*x*(7*e^2*x^2+5*d^2)*(c*x^4+a)^(1/2)+3/8*d^2*
e*x^2*(c*x^4+a)^(3/2)+1/21*d*x*(7*e^2*x^2+3*d^2)*(c*x^4+a)^(3/2)+1/10*e^3*
(c*x^4+a)^(5/2)/c+9/16*a^2*d^2*e*arctanh(c^(1/2)*x^2/(c*x^4+a)^(1/2))/c^(1
/2)-4/5*a^(9/4)*d*e^2*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+a)/(a^(1/2)+c^(1/2)*x^
2)^2)^(1/2)*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))/c^(3/4
)/(c*x^4+a)^(1/2)+2/35*a^(7/4)*d*(5*c^(1/2)*d^2+7*a^(1/2)*e^2)*(a^(1/2)+c^
(1/2)*x^2)*((c*x^4+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arc
tan(c^(1/4)*x/a^(1/4)),1/2*2^(1/2))/c^(3/4)/(c*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.60 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.45

$$\int (d+ex)^3 (a+cx^4)^{3/2}\,dx = \frac{1}{80}\sqrt{a+cx^4}\left(\frac{8e^3(a+cx^4)^2}{c} + 15d^2e\left(5ax^2+2cx^6+\frac{3a^{5/2}\sqrt{1+\frac{cx^4}{a}}\operatorname{arcsinh}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{\sqrt{c}\,(a+cx^4)}\right) + \frac{80ad^3x\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{4},\frac{5}{4},-\frac{cx^4}{a}\right)}{\sqrt{1+\frac{cx^4}{a}}} + \frac{80}{}\right.$$

input
```
Integrate[(d + e*x)^3*(a + c*x^4)^(3/2),x]
```

output
```
(Sqrt[a + c*x^4]*((8*e^3*(a + c*x^4)^2)/c + 15*d^2*e*(5*a*x^2 + 2*c*x^6 +
(3*a^(5/2)*Sqrt[1 + (c*x^4)/a]*ArcSinh[(Sqrt[c]*x^2)/Sqrt[a]])/(Sqrt[c]*(a
 + c*x^4))) + (80*a*d^3*x*Hypergeometric2F1[-3/2, 1/4, 5/4, -((c*x^4)/a)])
/Sqrt[1 + (c*x^4)/a] + (80*a*d*e^2*x^3*Hypergeometric2F1[-3/2, 3/4, 7/4, -
((c*x^4)/a)])/Sqrt[1 + (c*x^4)/a]))/80
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 414, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2424, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+cx^4\right)^{3/2} (d+ex)^3 \, dx$$

$$\Big\downarrow \text{2424}$$

$$\int \left(\left(a+cx^4\right)^{3/2} \left(d^3+3de^2x^2\right)+x\left(a+cx^4\right)^{3/2} \left(3d^2e+e^3x^2\right)\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2a^{7/4}d\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(7\sqrt{a}e^2+5\sqrt{c}d^2\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{35c^{3/4}\sqrt{a+cx^4}}-\frac{4a^{9/4}de^2\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5c^{3/4}\sqrt{a+cx^4}}+\frac{9a^2d^2e\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{16\sqrt{c}}+\frac{4a^2de^2x\sqrt{a+cx^4}}{5\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)}+\frac{1}{21}dx\left(a+cx^4\right)^{3/2}\left(3d^2+7e^2x^2\right)+\frac{2}{35}adx\sqrt{a+cx^4}\left(5d^2+7e^2x^2\right)+\frac{3}{8}d^2ex^2\left(a+cx^4\right)^{3/2}+\frac{9}{16}ad^2ex^2\sqrt{a+cx^4}+\frac{e^3\left(a+cx^4\right)^{5/2}}{10c}$$

input
```
Int[(d + e*x)^3*(a + c*x^4)^(3/2),x]
```

output
```
(9*a*d^2*e*x^2*Sqrt[a + c*x^4])/16 + (4*a^2*d*e^2*x*Sqrt[a + c*x^4])/(5*Sq
rt[c]*(Sqrt[a] + Sqrt[c]*x^2)) + (2*a*d*x*(5*d^2 + 7*e^2*x^2)*Sqrt[a + c*x
^4])/35 + (3*d^2*e*x^2*(a + c*x^4)^(3/2))/8 + (d*x*(3*d^2 + 7*e^2*x^2)*(a
+ c*x^4)^(3/2))/21 + (e^3*(a + c*x^4)^(5/2))/(10*c) + (9*a^2*d^2*e*ArcTanh
[(Sqrt[c]*x^2)/Sqrt[a + c*x^4]])/(16*Sqrt[c]) - (4*a^(9/4)*d*e^2*(Sqrt[a]
+ Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*Arc
Tan[(c^(1/4)*x)/a^(1/4)], 1/2])/(5*c^(3/4)*Sqrt[a + c*x^4]) + (2*a^(7/4)*d
*(5*Sqrt[c]*d^2 + 7*Sqrt[a]*e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/
(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/
(35*c^(3/4)*Sqrt[a + c*x^4])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2424
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.28 (sec) , antiderivative size = 315, normalized size of antiderivative = 0.76

method	result
risch	$\frac{\left(168x^{8}e^{3}c^{2}+560d\,e^{2}x^{7}c^{2}+630d^{2}e\,x^{6}c^{2}+240d^{3}x^{5}c^{2}+336x^{4}a\,e^{3}c+1232a\,x^{3}c\,d\,e^{2}+1575a\,d^{2}e\,x^{2}c+720a\,d^{3}x c+168a^{2}e^{3}\right)\sqrt{c\,x^{4}+a}}{1680c}+$
default	$d^{3}\left(\frac{c\,x^{5}\sqrt{c\,x^{4}+a}}{7}+\frac{3a x\sqrt{c\,x^{4}+a}}{7}+\frac{4a^{2}\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{7\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}}\right)+\frac{e^{3}\left(c\,x^{4}+a\right)^{\frac{5}{2}}}{10c}+3d\,e^{2}\left(\frac{c\,x^{7}}{}\right.$
elliptic	$\frac{e^{3}c\,x^{8}\sqrt{c\,x^{4}+a}}{10}+\frac{c d\,e^{2}x^{7}\sqrt{c\,x^{4}+a}}{3}+\frac{3d^{2}e c\,x^{6}\sqrt{c\,x^{4}+a}}{8}+\frac{c\,d^{3}x^{5}\sqrt{c\,x^{4}+a}}{7}+\frac{e^{3}a\,x^{4}\sqrt{c\,x^{4}+a}}{5}+\frac{11a d\,e^{2}x^{3}\sqrt{c\,x^{4}+a}}{15}+\frac{15a}{}$

input
```
int((e*x+d)^3*(c*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/1680*(168*c^2*e^3*x^8+560*c^2*d*e^2*x^7+630*c^2*d^2*e*x^6+240*c^2*d^3*x^
5+336*a*c*e^3*x^4+1232*a*c*d*e^2*x^3+1575*a*c*d^2*e*x^2+720*a*c*d^3*x+168*
a^2*e^3)/c*(c*x^4+a)^(1/2)+1/280*a^2*d*(160*d^2/(I*c^(1/2)/a^(1/2))^(1/2)*
(1-I*c^(1/2)*x^2/a^(1/2))^(1/2)*(1+I*c^(1/2)*x^2/a^(1/2))^(1/2)/(c*x^4+a)^
(1/2)*EllipticF(x*(I*c^(1/2)/a^(1/2))^(1/2),I)+224*I*e^2*a^(1/2)/(I*c^(1/2
)/a^(1/2))^(1/2)*(1-I*c^(1/2)*x^2/a^(1/2))^(1/2)*(1+I*c^(1/2)*x^2/a^(1/2))
^(1/2)/(c*x^4+a)^(1/2)/c^(1/2)*(EllipticF(x*(I*c^(1/2)/a^(1/2))^(1/2),I)-E
llipticE(x*(I*c^(1/2)/a^(1/2))^(1/2),I))+315/2*d*e*ln(c^(1/2)*x^2+(c*x^4+a
)^(1/2))/c^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 250, normalized size of antiderivative = 0.60

$$\int (d+ex)^3 \left(a+cx^4\right)^{3/2} dx = \frac{2688\, a^2\sqrt{c}de^2x\left(-\frac{a}{c}\right)^{\frac{3}{4}} E(\arcsin\left(\frac{\left(-\frac{a}{c}\right)^{\frac{1}{4}}}{x}\right) |-1) + 945\, a^2\sqrt{c}d^2ex\log\left(-2\,cx^4-2\,\sqrt{cx^4+a}\sqrt{}\right.}{}$$

input
```
integrate((e*x+d)^3*(c*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
1/3360*(2688*a^2*sqrt(c)*d*e^2*x*(-a/c)^(3/4)*elliptic_e(arcsin((-a/c)^(1/
4)/x), -1) + 945*a^2*sqrt(c)*d^2*e*x*log(-2*c*x^4 - 2*sqrt(c*x^4 + a)*sqrt
(c)*x^2 - a) + 384*(5*a*c*d^3 - 7*a^2*d*e^2)*sqrt(c)*x*(-a/c)^(3/4)*ellipt
ic_f(arcsin((-a/c)^(1/4)/x), -1) + 2*(168*c^2*e^3*x^9 + 560*c^2*d*e^2*x^8
+ 630*c^2*d^2*e*x^7 + 240*c^2*d^3*x^6 + 336*a*c*e^3*x^5 + 1232*a*c*d*e^2*x
^4 + 1575*a*c*d^2*e*x^3 + 720*a*c*d^3*x^2 + 168*a^2*e^3*x + 1344*a^2*d*e^2
)*sqrt(c*x^4 + a))/(c*x)
```

Sympy [A] (verification not implemented)

Time = 5.78 (sec) , antiderivative size = 434, normalized size of antiderivative = 1.05

$$\int (d+ex)^3 \left(a+cx^4\right)^{3/2} dx = \frac{a^{\frac{3}{2}} d^3 x \Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{1}{4} \\ \frac{5}{4} \end{matrix}\middle| \frac{cx^4 e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)} + \frac{3a^{\frac{3}{2}} d^2 e x^2 \sqrt{1+\frac{cx^4}{a}}}{4} + \frac{3a^{\frac{3}{2}} d^2 e x^2}{16\sqrt{1+\frac{cx^4}{a}}} + \frac{3a^{\frac{3}{2}} d e^2 x^3 \Gamma\left(\frac{3}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{3}{4} \\ \frac{7}{4} \end{matrix}\middle| \frac{cx^4 e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{7}{4}\right)} + \frac{\sqrt{a} c d^3 x^5 \Gamma\left(\frac{5}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{5}{4} \\ \frac{9}{4} \end{matrix}\middle| \frac{cx^4 e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{9}{4}\right)} + \frac{9\sqrt{a} c d^2 e x^6}{16\sqrt{1+\frac{cx^4}{a}}} + \frac{3\sqrt{a} c d e^2 x^7 \Gamma\left(\frac{7}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{7}{4} \\ \frac{11}{4} \end{matrix}\middle| \frac{cx^4 e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{11}{4}\right)} + \frac{9a^2 d^2 e \operatorname{asinh}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{16\sqrt{c}} + ae^3 \left(\begin{cases} \frac{\sqrt{a}x^4}{4} & \text{for } c = 0 \\ \frac{(a+cx^4)^{\frac{3}{2}}}{6c} & \text{otherwise} \end{cases}\right) + ce^3 \left(\begin{cases} -\frac{a^2\sqrt{a+cx^4}}{15c^2} + \frac{ax^4\sqrt{a+cx^4}}{30c} + \frac{x^8\sqrt{a+cx^4}}{10} & \text{for } c \neq 0 \\ \frac{\sqrt{a}x^8}{8} & \text{otherwise} \end{cases}\right) + \frac{3c^2 d^2 e x^{10}}{8\sqrt{a}\sqrt{1+\frac{cx^4}{a}}}$$

input
```
integrate((e*x+d)**3*(c*x**4+a)**(3/2),x)
```

output
```
a**(3/2)*d**3*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), c*x**4*exp_polar(I*p
i)/a)/(4*gamma(5/4)) + 3*a**(3/2)*d**2*e*x**2*sqrt(1 + c*x**4/a)/4 + 3*a**
(3/2)*d**2*e*x**2/(16*sqrt(1 + c*x**4/a)) + 3*a**(3/2)*d*e**2*x**3*gamma(3
/4)*hyper((-1/2, 3/4), (7/4,), c*x**4*exp_polar(I*pi)/a)/(4*gamma(7/4)) +
sqrt(a)*c*d**3*x**5*gamma(5/4)*hyper((-1/2, 5/4), (9/4,), c*x**4*exp_polar
(I*pi)/a)/(4*gamma(9/4)) + 9*sqrt(a)*c*d**2*e*x**6/(16*sqrt(1 + c*x**4/a))
 + 3*sqrt(a)*c*d*e**2*x**7*gamma(7/4)*hyper((-1/2, 7/4), (11/4,), c*x**4*e
xp_polar(I*pi)/a)/(4*gamma(11/4)) + 9*a**2*d**2*e*asinh(sqrt(c)*x**2/sqrt(
a))/(16*sqrt(c)) + a*e**3*Piecewise((sqrt(a)*x**4/4, Eq(c, 0)), ((a + c*x*
*4)**(3/2)/(6*c), True)) + c*e**3*Piecewise((-a**2*sqrt(a + c*x**4)/(15*c*
*2) + a*x**4*sqrt(a + c*x**4)/(30*c) + x**8*sqrt(a + c*x**4)/10, Ne(c, 0))
, (sqrt(a)*x**8/8, True)) + 3*c**2*d**2*e*x**10/(8*sqrt(a)*sqrt(1 + c*x**4
/a))
```

Maxima [F]

$$\int (d+ex)^3 \left(a+cx^4\right)^{3/2} \, dx = \int \left(cx^4+a\right)^{\frac{3}{2}} (ex+d)^3 \, dx$$

input
```
integrate((e*x+d)^3*(c*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*x^4 + a)^(3/2)*(e*x + d)^3, x)
```

Giac [F]

$$\int (d+ex)^3 \left(a+cx^4\right)^{3/2} \, dx = \int \left(cx^4+a\right)^{\frac{3}{2}} (ex+d)^3 \, dx$$

input
```
integrate((e*x+d)^3*(c*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((c*x^4 + a)^(3/2)*(e*x + d)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)^3 \left(a+cx^4\right)^{3/2} \, dx = \int \left(c\,x^4+a\right)^{3/2}\,{\left(d+e\,x\right)}^3 \,d x$$

input
```
int((a + c*x^4)^(3/2)*(d + e*x)^3,x)
```

output
```
int((a + c*x^4)^(3/2)*(d + e*x)^3, x)
```

Reduce [F]

$$\int (d+ex)^3 \left(a+cx^4\right)^{3/2} \, dx = \frac{336\sqrt{c\,x^4+a}\,a^2 e^3+1440\sqrt{c\,x^4+a}\,ac\,d^3x+3150\sqrt{c\,x^4+a}\,ac\,d^2e\,x^2+2464\sqrt{c\,x^4+a}\,acd\,e}{}$$

input
```
int((e*x+d)^3*(c*x^4+a)^(3/2),x)
```

output
```
(336*sqrt(a + c*x**4)*a**2*e**3 + 1440*sqrt(a + c*x**4)*a*c*d**3*x + 3150*
sqrt(a + c*x**4)*a*c*d**2*e*x**2 + 2464*sqrt(a + c*x**4)*a*c*d*e**2*x**3 +
 672*sqrt(a + c*x**4)*a*c*e**3*x**4 + 480*sqrt(a + c*x**4)*c**2*d**3*x**5
+ 1260*sqrt(a + c*x**4)*c**2*d**2*e*x**6 + 1120*sqrt(a + c*x**4)*c**2*d*e*
*2*x**7 + 336*sqrt(a + c*x**4)*c**2*e**3*x**8 - 945*sqrt(c)*log(sqrt(a + c
*x**4) - sqrt(c)*x**2)*a**2*d**2*e + 945*sqrt(c)*log(sqrt(a + c*x**4) + sq
rt(c)*x**2)*a**2*d**2*e + 1920*int(sqrt(a + c*x**4)/(a + c*x**4),x)*a**2*c
*d**3 + 2688*int((sqrt(a + c*x**4)*x**2)/(a + c*x**4),x)*a**2*c*d*e**2)/(3
360*c)
```

3.199 $\int (d+ex)^2 \left(a+cx^4\right)^{3/2} dx$

Optimal result

Integrand size = 19, antiderivative size = 382

$$\int (d+ex)^2 \left(a+cx^4\right)^{3/2} dx = \frac{3}{8}adex^2\sqrt{a+cx^4} + \frac{4a^2e^2x\sqrt{a+cx^4}}{15\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)} + \frac{2}{105}ax\left(15d^2+7e^2x^2\right)\sqrt{a+cx^4} + \frac{1}{4}dex^2\left(a+cx^4\right)^{3/2} + \frac{1}{63}x\left(9d^2+7e^2x^2\right)\left(a+cx^4\right)^{3/2} + \frac{3a^2de\operatorname{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{8\sqrt{c}} - \frac{4a^{9/4}e^2\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15c^{3/4}\sqrt{a+cx^4}} + \frac{2a^{7/4}\left(15\sqrt{c}d^2+7\sqrt{a}e^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{105c^{3/4}\sqrt{a+cx^4}}$$

output
```
3/8*a*d*e*x^2*(c*x^4+a)^(1/2)+4/15*a^2*e^2*x*(c*x^4+a)^(1/2)/c^(1/2)/(a^(1
/2)+c^(1/2)*x^2)+2/105*a*x*(7*e^2*x^2+15*d^2)*(c*x^4+a)^(1/2)+1/4*d*e*x^2*
(c*x^4+a)^(3/2)+1/63*x*(7*e^2*x^2+9*d^2)*(c*x^4+a)^(3/2)+3/8*a^2*d*e*arcta
nh(c^(1/2)*x^2/(c*x^4+a)^(1/2))/c^(1/2)-4/15*a^(9/4)*e^2*(a^(1/2)+c^(1/2)*
x^2)*((c*x^4+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*EllipticE(sin(2*arctan(c^(1
/4)*x/a^(1/4))),1/2*2^(1/2))/c^(3/4)/(c*x^4+a)^(1/2)+2/105*a^(7/4)*(15*c^(
1/2)*d^2+7*a^(1/2)*e^2)*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+a)/(a^(1/2)+c^(1/2)*
x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(c^(1/4)*x/a^(1/4)),1/2*2^(1/2))/c^(
3/4)/(c*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.21 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.41

$$\int (d+ex)^2 \left(a+cx^4\right)^{3/2}\,dx = \frac{\sqrt{a+cx^4}\left(24a\sqrt{c}d^2x\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{4},\frac{5}{4},-\frac{cx^4}{a}\right)+e\left(3\sqrt{c}dx^2(5a+2cx^4)\sqrt{1+\cdots}\right.\right.}{24\sqrt{c}\sqrt{1+\cdots}}$$

input
```
Integrate[(d + e*x)^2*(a + c*x^4)^(3/2),x]
```

output
```
(Sqrt[a + c*x^4]*(24*a*Sqrt[c]*d^2*x*Hypergeometric2F1[-3/2, 1/4, 5/4, -((
c*x^4)/a)] + e*(3*Sqrt[c]*d*x^2*(5*a + 2*c*x^4)*Sqrt[1 + (c*x^4)/a] + 9*a^
(3/2)*d*ArcSinh[(Sqrt[c]*x^2)/Sqrt[a]] + 8*a*Sqrt[c]*e*x^3*Hypergeometric2
F1[-3/2, 3/4, 7/4, -((c*x^4)/a)])))/(24*Sqrt[c]*Sqrt[1 + (c*x^4)/a])
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 382, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2424, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+cx^4\right)^{3/2}(d+ex)^2\,dx$$

$$\Big\downarrow \text{2424}$$

$$\int \left(\left(a+cx^4\right)^{3/2}\left(d^2+e^2x^2\right)+2dex\left(a+cx^4\right)^{3/2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2a^{7/4}(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}(7\sqrt{a}e^2+15\sqrt{c}d^2)\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{105c^{3/4}\sqrt{a+cx^4}} - \frac{4a^{9/4}e^2(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15c^{3/4}\sqrt{a+cx^4}} + \frac{3a^2de\,\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{8\sqrt{c}} + \frac{4a^2e^2x\sqrt{a+cx^4}}{15\sqrt{c}(\sqrt{a}+\sqrt{c}x^2)} + \frac{2}{105}ax\sqrt{a+cx^4}(15d^2+7e^2x^2) + \frac{1}{63}x(a+cx^4)^{3/2}(9d^2+7e^2x^2) + \frac{3}{8}adex^2\sqrt{a+cx^4} + \frac{1}{4}dex^2(a+cx^4)^{3/2}$$

input
```
Int[(d + e*x)^2*(a + c*x^4)^(3/2),x]
```

output
```
(3*a*d*e*x^2*Sqrt[a + c*x^4])/8 + (4*a^2*e^2*x*Sqrt[a + c*x^4])/(15*Sqrt[c
]*(Sqrt[a] + Sqrt[c]*x^2)) + (2*a*x*(15*d^2 + 7*e^2*x^2)*Sqrt[a + c*x^4])/
105 + (d*e*x^2*(a + c*x^4)^(3/2))/4 + (x*(9*d^2 + 7*e^2*x^2)*(a + c*x^4)^(
3/2))/63 + (3*a^2*d*e*ArcTanh[(Sqrt[c]*x^2)/Sqrt[a + c*x^4]])/(8*Sqrt[c])
- (4*a^(9/4)*e^2*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[
c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(15*c^(3/4)*Sqrt
[a + c*x^4]) + (2*a^(7/4)*(15*Sqrt[c]*d^2 + 7*Sqrt[a]*e^2)*(Sqrt[a] + Sqrt
[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c
^(1/4)*x)/a^(1/4)], 1/2])/(105*c^(3/4)*Sqrt[a + c*x^4])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2424
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.14 (sec) , antiderivative size = 265, normalized size of antiderivative = 0.69

method	result
risch	$\frac{x(280c\,e^2x^6+630cde\,x^5+360c\,d^2x^4+616a\,e^2x^2+1575adex+1080a\,d^2)\sqrt{c\,x^4+a}}{2520} + \frac{a^2\left(\frac{240d^2\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\text{EllipticF}\left(x\sqrt{\cdots}\right.}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\sqrt{c\,x^4+a}}\right.}{}$
default	$d^2\left(\frac{c\,x^5\sqrt{c\,x^4+a}}{7}+\frac{3ax\sqrt{c\,x^4+a}}{7}+\frac{4a^2\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\text{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{7\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\sqrt{c\,x^4+a}}\right)+e^2\left(\frac{c\,x^7\sqrt{c\,x^4+a}}{9}+\frac{11a\,x^3\sqrt{\cdots}}{4\cdots}\right.$
elliptic	$\frac{c\,e^2x^7\sqrt{c\,x^4+a}}{9}+\frac{cde\,x^6\sqrt{c\,x^4+a}}{4}+\frac{c\,d^2x^5\sqrt{c\,x^4+a}}{7}+\frac{11a\,e^2x^3\sqrt{c\,x^4+a}}{45}+\frac{5ade\,x^2\sqrt{c\,x^4+a}}{8}+\frac{3a\,d^2x\sqrt{c\,x^4+a}}{7}+\frac{4a^2d^2\cdots}{\cdots}$

input
```
int((e*x+d)^2*(c*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2520*x*(280*c*e^2*x^6+630*c*d*e*x^5+360*c*d^2*x^4+616*a*e^2*x^2+1575*a*d
*e*x+1080*a*d^2)*(c*x^4+a)^(1/2)+1/420*a^2*(240*d^2/(I*c^(1/2)/a^(1/2))^(1
/2)*(1-I*c^(1/2)*x^2/a^(1/2))^(1/2)*(1+I*c^(1/2)*x^2/a^(1/2))^(1/2)/(c*x^4
+a)^(1/2)*EllipticF(x*(I*c^(1/2)/a^(1/2))^(1/2),I)+112*I*e^2*a^(1/2)/(I*c^
(1/2)/a^(1/2))^(1/2)*(1-I*c^(1/2)*x^2/a^(1/2))^(1/2)*(1+I*c^(1/2)*x^2/a^(1
/2))^(1/2)/(c*x^4+a)^(1/2)/c^(1/2)*(EllipticF(x*(I*c^(1/2)/a^(1/2))^(1/2),
I)-EllipticE(x*(I*c^(1/2)/a^(1/2))^(1/2),I))+315/2*d*e*ln(c^(1/2)*x^2+(c*x
^4+a)^(1/2))/c^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.55

$$\int (d+ex)^2\,(a+cx^4)^{3/2}\,dx = \frac{1344\,a^2\sqrt{c}e^2x\left(-\frac{a}{c}\right)^{\frac{3}{4}}E(\arcsin\left(\frac{\left(-\frac{a}{c}\right)^{\frac{1}{4}}}{x}\right)\,|-1)+945\,a^2\sqrt{c}dex\log\left(-2\,cx^4-2\,\sqrt{cx^4+a}\sqrt{c}x\cdots\right.}{\cdots}$$

input
```
integrate((e*x+d)^2*(c*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
1/5040*(1344*a^2*sqrt(c)*e^2*x*(-a/c)^(3/4)*elliptic_e(arcsin((-a/c)^(1/4)
/x), -1) + 945*a^2*sqrt(c)*d*e*x*log(-2*c*x^4 - 2*sqrt(c*x^4 + a)*sqrt(c)*
x^2 - a) + 192*(15*a*c*d^2 - 7*a^2*e^2)*sqrt(c)*x*(-a/c)^(3/4)*elliptic_f(
arcsin((-a/c)^(1/4)/x), -1) + 2*(280*c^2*e^2*x^8 + 630*c^2*d*e*x^7 + 360*c
^2*d^2*x^6 + 616*a*c*e^2*x^4 + 1575*a*c*d*e*x^3 + 1080*a*c*d^2*x^2 + 672*a
^2*e^2)*sqrt(c*x^4 + a))/(c*x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 5.13 (sec) , antiderivative size = 316, normalized size of antiderivative = 0.83

$$\int (d+ex)^2 \left(a+cx^4\right)^{3/2} dx = \frac{a^{\frac{3}{2}} d^2 x \Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{1}{4} \\ \frac{5}{4} \end{matrix} \middle| \frac{cx^4 e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)} + \frac{a^{\frac{3}{2}} dex^2 \sqrt{1+\frac{cx^4}{a}}}{2} + \frac{a^{\frac{3}{2}} dex^2}{8\sqrt{1+\frac{cx^4}{a}}} + \frac{a^{\frac{3}{2}} e^2 x^3 \Gamma\left(\frac{3}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{3}{4} \\ \frac{7}{4} \end{matrix} \middle| \frac{cx^4 e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{7}{4}\right)} + \frac{\sqrt{a} c d^2 x^5 \Gamma\left(\frac{5}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{5}{4} \\ \frac{9}{4} \end{matrix} \middle| \frac{cx^4 e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{9}{4}\right)} + \frac{3\sqrt{a} cdex^6}{8\sqrt{1+\frac{cx^4}{a}}} + \frac{\sqrt{a} ce^2 x^7 \Gamma\left(\frac{7}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{7}{4} \\ \frac{11}{4} \end{matrix} \middle| \frac{cx^4 e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{11}{4}\right)} + \frac{3a^2 de \operatorname{asinh}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{8\sqrt{c}} + \frac{c^2 dex^{10}}{4\sqrt{a}\sqrt{1+\frac{cx^4}{a}}}$$

input
```
integrate((e*x+d)**2*(c*x**4+a)**(3/2),x)
```

output
```
a**(3/2)*d**2*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), c*x**4*exp_polar(I*p
i)/a)/(4*gamma(5/4)) + a**(3/2)*d*e*x**2*sqrt(1 + c*x**4/a)/2 + a**(3/2)*d
*e*x**2/(8*sqrt(1 + c*x**4/a)) + a**(3/2)*e**2*x**3*gamma(3/4)*hyper((-1/2
, 3/4), (7/4,), c*x**4*exp_polar(I*pi)/a)/(4*gamma(7/4)) + sqrt(a)*c*d**2*
x**5*gamma(5/4)*hyper((-1/2, 5/4), (9/4,), c*x**4*exp_polar(I*pi)/a)/(4*ga
mma(9/4)) + 3*sqrt(a)*c*d*e*x**6/(8*sqrt(1 + c*x**4/a)) + sqrt(a)*c*e**2*x
**7*gamma(7/4)*hyper((-1/2, 7/4), (11/4,), c*x**4*exp_polar(I*pi)/a)/(4*ga
mma(11/4)) + 3*a**2*d*e*asinh(sqrt(c)*x**2/sqrt(a))/(8*sqrt(c)) + c**2*d*e
*x**10/(4*sqrt(a)*sqrt(1 + c*x**4/a))
```

Maxima [F]

$$\int (d+ex)^2 \left(a+cx^4\right)^{3/2} \, dx = \int \left(cx^4+a\right)^{\frac{3}{2}} (ex+d)^2 \, dx$$

input
```
integrate((e*x+d)^2*(c*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*x^4 + a)^(3/2)*(e*x + d)^2, x)
```

Giac [F]

$$\int (d+ex)^2 \left(a+cx^4\right)^{3/2} \, dx = \int \left(cx^4+a\right)^{\frac{3}{2}} (ex+d)^2 \, dx$$

input
```
integrate((e*x+d)^2*(c*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((c*x^4 + a)^(3/2)*(e*x + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)^2 \left(a+cx^4\right)^{3/2} \, dx = \int \left(c\,x^4+a\right)^{3/2} \left(d+e\,x\right)^2 \, dx$$

input
```
int((a + c*x^4)^(3/2)*(d + e*x)^2,x)
```

output
```
int((a + c*x^4)^(3/2)*(d + e*x)^2, x)
```

Reduce [F]

$$\int (d+ex)^2 \left(a+cx^4\right)^{3/2} \, dx = \frac{2160\sqrt{c\,x^4+a}\,ac\,d^2x+3150\sqrt{c\,x^4+a}\,acde\,x^2+1232\sqrt{c\,x^4+a}\,ac\,e^2x^3+720\sqrt{c\,x^4+a}\,c^2d}{}$$

input
```
int((e*x+d)^2*(c*x^4+a)^(3/2),x)
```

output
```
(2160*sqrt(a + c*x**4)*a*c*d**2*x + 3150*sqrt(a + c*x**4)*a*c*d*e*x**2 + 1
232*sqrt(a + c*x**4)*a*c*e**2*x**3 + 720*sqrt(a + c*x**4)*c**2*d**2*x**5 +
 1260*sqrt(a + c*x**4)*c**2*d*e*x**6 + 560*sqrt(a + c*x**4)*c**2*e**2*x**7
 - 945*sqrt(c)*log(sqrt(a + c*x**4) - sqrt(c)*x**2)*a**2*d*e + 945*sqrt(c)
*log(sqrt(a + c*x**4) + sqrt(c)*x**2)*a**2*d*e + 2880*int(sqrt(a + c*x**4)
/(a + c*x**4),x)*a**2*c*d**2 + 1344*int((sqrt(a + c*x**4)*x**2)/(a + c*x**
4),x)*a**2*c*e**2)/(5040*c)
```

3.200 $\int (d+ex)\left(a+cx^4\right)^{3/2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 198

$$\begin{aligned}\int (d+ex)\left(a+cx^4\right)^{3/2}\,dx &= \frac{2}{7}adx\sqrt{a+cx^4}+\frac{3}{16}aex^2\sqrt{a+cx^4}\\&+\frac{1}{7}dx\left(a+cx^4\right)^{3/2}+\frac{1}{8}ex^2\left(a+cx^4\right)^{3/2}+\frac{3a^2e\,\mathrm{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{16\sqrt{c}}\\&+\frac{2a^{7/4}d\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\mathrm{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{7\sqrt[4]{c}\sqrt{a+cx^4}}\end{aligned}$$

output

```
2/7*a*d*x*(c*x^4+a)^(1/2)+3/16*a*e*x^2*(c*x^4+a)^(1/2)+1/7*d*x*(c*x^4+a)^(
3/2)+1/8*e*x^2*(c*x^4+a)^(3/2)+3/16*a^2*e*arctanh(c^(1/2)*x^2/(c*x^4+a)^(1
/2))/c^(1/2)+2/7*a^(7/4)*d*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+a)/(a^(1/2)+c^(1/
2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(c^(1/4)*x/a^(1/4)),1/2*2^(1/2))/
c^(1/4)/(c*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.18 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.61

$$\int (d+ex)\left(a+cx^4\right)^{3/2}\,dx = \frac{\sqrt{a+cx^4}\left(\sqrt{c}ex^2(5a+2cx^4)\sqrt{1+\frac{cx^4}{a}}+3a^{3/2}e\operatorname{arcsinh}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)+16a\sqrt{c}dx\,\text{Hypergeometric}$$

$$16\sqrt{c}\sqrt{1+\frac{cx^4}{a}}$$

input
```
Integrate[(d + e*x)*(a + c*x^4)^(3/2),x]
```

output
```
(Sqrt[a + c*x^4]*(Sqrt[c]*e*x^2*(5*a + 2*c*x^4)*Sqrt[1 + (c*x^4)/a] + 3*a^
(3/2)*e*ArcSinh[(Sqrt[c]*x^2)/Sqrt[a]] + 16*a*Sqrt[c]*d*x*Hypergeometric2F
1[-3/2, 1/4, 5/4, -((c*x^4)/a)]))/(16*Sqrt[c]*Sqrt[1 + (c*x^4)/a])
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2424, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+cx^4\right)^{3/2}(d+ex)\,dx$$

$$\Big\downarrow \text{ 2424}$$

$$\int \left(d\left(a+cx^4\right)^{3/2}+ex\left(a+cx^4\right)^{3/2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2a^{7/4}d\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{7\sqrt[4]{c}\sqrt{a+cx^4}}+\frac{3a^2e\,\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{16\sqrt{c}}+\frac{2}{7}adx\sqrt{a+cx^4}+\frac{1}{7}dx\left(a+cx^4\right)^{3/2}+\frac{3}{16}aex^2\sqrt{a+cx^4}+\frac{1}{8}ex^2\left(a+cx^4\right)^{3/2}$$

input
```
Int[(d + e*x)*(a + c*x^4)^(3/2),x]
```

output
```
(2*a*d*x*Sqrt[a + c*x^4])/7 + (3*a*e*x^2*Sqrt[a + c*x^4])/16 + (d*x*(a + c
*x^4)^(3/2))/7 + (e*x^2*(a + c*x^4)^(3/2))/8 + (3*a^2*e*ArcTanh[(Sqrt[c]*x
^2)/Sqrt[a + c*x^4]])/(16*Sqrt[c]) + (2*a^(7/4)*d*(Sqrt[a] + Sqrt[c]*x^2)*
Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)
/a^(1/4)], 1/2])/(7*c^(1/4)*Sqrt[a + c*x^4])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2424
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.65 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.70

method	result
risch	$\frac{x(14ce\,x^5+16cd\,x^4+35aex+48ad)\sqrt{c\,x^4+a}}{112}+\frac{4a^2d\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{7\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}+\frac{3a^2e\ln\left(\sqrt{c}\,x^2+\sqrt{c\,x^4+a}\right)}{16\sqrt{c}}$
default	$d\left(\frac{c\,x^5\sqrt{c\,x^4+a}}{7}+\frac{3ax\sqrt{c\,x^4+a}}{7}+\frac{4a^2\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{7\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}\right)+e\left(\frac{3a^2\ln\left(\sqrt{c}\,x^2+\sqrt{c\,x^4+a}\right)}{16\sqrt{c}}+\right.$
elliptic	$\frac{ce\,x^6\sqrt{c\,x^4+a}}{8}+\frac{cd\,x^5\sqrt{c\,x^4+a}}{7}+\frac{5ae\,x^2\sqrt{c\,x^4+a}}{16}+\frac{3adx\sqrt{c\,x^4+a}}{7}+\frac{4a^2d\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{7\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}+$

input
```
int((e*x+d)*(c*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/112*x*(14*c*e*x^5+16*c*d*x^4+35*a*e*x+48*a*d)*(c*x^4+a)^(1/2)+4/7*a^2*d/
(I*c^(1/2)/a^(1/2))^(1/2)*(1-I*c^(1/2)*x^2/a^(1/2))^(1/2)*(1+I*c^(1/2)*x^2
/a^(1/2))^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I*c^(1/2)/a^(1/2))^(1/2),I)+3
/16*a^2*e*ln(c^(1/2)*x^2+(c*x^4+a)^(1/2))/c^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.59

$$\int (d+ex)\left(a+cx^4\right)^{3/2}\,dx=\frac{128\,ac^{\frac{3}{2}}d\left(-\frac{a}{c}\right)^{\frac{3}{4}}F(\arcsin\left(\frac{\left(-\frac{a}{c}\right)^{\frac{1}{4}}}{x}\right)\,|-1)+21\,a^2\sqrt{c}e\log\left(-2\,cx^4-2\,\sqrt{cx^4+a}\sqrt{c}x^2-a\right)+}{224\,c}$$

input
```
integrate((e*x+d)*(c*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
1/224*(128*a*c^(3/2)*d*(-a/c)^(3/4)*elliptic_f(arcsin((-a/c)^(1/4)/x), -1)
 + 21*a^2*sqrt(c)*e*log(-2*c*x^4 - 2*sqrt(c*x^4 + a)*sqrt(c)*x^2 - a) + 2*
(14*c^2*e*x^6 + 16*c^2*d*x^5 + 35*a*c*e*x^2 + 48*a*c*d*x)*sqrt(c*x^4 + a))
/c
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.24 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.08

$$\int (d+ex)\left(a+cx^4\right)^{3/2}\,dx = \frac{a^{\frac{3}{2}}dx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)} + \frac{a^{\frac{3}{2}}ex^2\sqrt{1+\frac{cx^4}{a}}}{4} + \frac{a^{\frac{3}{2}}ex^2}{16\sqrt{1+\frac{cx^4}{a}}} + \frac{\sqrt{a}cdx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{9}{4}\right)} + \frac{3\sqrt{a}cex^6}{16\sqrt{1+\frac{cx^4}{a}}} + \frac{3a^2e\operatorname{asinh}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{16\sqrt{c}} + \frac{c^2ex^{10}}{8\sqrt{a}\sqrt{1+\frac{cx^4}{a}}}$$

input
```
integrate((e*x+d)*(c*x**4+a)**(3/2),x)
```

output
```
a**(3/2)*d*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), c*x**4*exp_polar(I*pi)/
a)/(4*gamma(5/4)) + a**(3/2)*e*x**2*sqrt(1 + c*x**4/a)/4 + a**(3/2)*e*x**2
/(16*sqrt(1 + c*x**4/a)) + sqrt(a)*c*d*x**5*gamma(5/4)*hyper((-1/2, 5/4),
(9/4,), c*x**4*exp_polar(I*pi)/a)/(4*gamma(9/4)) + 3*sqrt(a)*c*e*x**6/(16*
sqrt(1 + c*x**4/a)) + 3*a**2*e*asinh(sqrt(c)*x**2/sqrt(a))/(16*sqrt(c)) +
c**2*e*x**10/(8*sqrt(a)*sqrt(1 + c*x**4/a))
```

Maxima [F]

$$\int (d+ex)\left(a+cx^4\right)^{3/2}\,dx=\int \left(cx^4+a\right)^{\frac{3}{2}}(ex+d)\,dx$$

input
```
integrate((e*x+d)*(c*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*x^4 + a)^(3/2)*(e*x + d), x)
```

Giac [F]

$$\int (d+ex)\left(a+cx^4\right)^{3/2}\,dx=\int \left(cx^4+a\right)^{\frac{3}{2}}(ex+d)\,dx$$

input
```
integrate((e*x+d)*(c*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((c*x^4 + a)^(3/2)*(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)\left(a+cx^4\right)^{3/2}\,dx=\int \left(c\,x^4+a\right)^{3/2}\,(d+e\,x)\,dx$$

input
```
int((a + c*x^4)^(3/2)*(d + e*x),x)
```

output
```
int((a + c*x^4)^(3/2)*(d + e*x), x)
```

Reduce [F]

$$\int (d+ex) \left(a+cx^4\right)^{3/2} dx = \frac{96\sqrt{c\,x^4+a}\,acdx+70\sqrt{c\,x^4+a}\,ace\,x^2+32\sqrt{c\,x^4+a}\,c^2d\,x^5+28\sqrt{c\,x^4+a}\,c^2e\,x^6-21\sqrt{c}\,\mathrm{l}}{224}$$

input
```
int((e*x+d)*(c*x^4+a)^(3/2),x)
```

output
```
(96*sqrt(a + c*x**4)*a*c*d*x + 70*sqrt(a + c*x**4)*a*c*e*x**2 + 32*sqrt(a
+ c*x**4)*c**2*d*x**5 + 28*sqrt(a + c*x**4)*c**2*e*x**6 - 21*sqrt(c)*log(s
qrt(a + c*x**4) - sqrt(c)*x**2)*a**2*e + 21*sqrt(c)*log(sqrt(a + c*x**4) +
 sqrt(c)*x**2)*a**2*e + 128*int(sqrt(a + c*x**4)/(a + c*x**4),x)*a**2*c*d)
/(224*c)
```

3.201 $\int (a+cx^4)^{3/2}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 122

$$\int (a+cx^4)^{3/2}\,dx = \frac{2}{7}ax\sqrt{a+cx^4} + \frac{1}{7}x(a+cx^4)^{3/2} + \frac{2a^{7/4}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{7\sqrt[4]{c}\sqrt{a+cx^4}}$$

output
```
2/7*a*x*(c*x^4+a)^(1/2)+1/7*x*(c*x^4+a)^(3/2)+2/7*a^(7/4)*(a^(1/2)+c^(1/2)
*x^2)*((c*x^4+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(c
^(1/4)*x/a^(1/4)),1/2*2^(1/2))/c^(1/4)/(c*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.70 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.39

$$\int (a+cx^4)^{3/2}\,dx = \frac{ax\sqrt{a+cx^4}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{4},\frac{5}{4},-\frac{cx^4}{a}\right)}{\sqrt{1+\frac{cx^4}{a}}}$$

input
```
Integrate[(a + c*x^4)^(3/2),x]
```

output
```
(a*x*Sqrt[a + c*x^4]*Hypergeometric2F1[-3/2, 1/4, 5/4, -((c*x^4)/a)])/Sqrt
[1 + (c*x^4)/a]
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.04, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {748, 748, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+cx^4\right)^{3/2}\,dx$$

$$\Big\downarrow \; 748$$

$$\frac{6}{7}a\int \sqrt{cx^4+a}dx+\frac{1}{7}x\left(a+cx^4\right)^{3/2}$$

$$\Big\downarrow \; 748$$

$$\frac{6}{7}a\left(\frac{2}{3}a\int \frac{1}{\sqrt{cx^4+a}}dx+\frac{1}{3}x\sqrt{a+cx^4}\right)+\frac{1}{7}x\left(a+cx^4\right)^{3/2}$$

$$\Big\downarrow \; 761$$

$$\frac{6}{7}a\left(\frac{a^{3/4}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{3\sqrt[4]{c}\sqrt{a+cx^4}}+\frac{1}{3}x\sqrt{a+cx^4}\right)+\frac{1}{7}x\left(a+cx^4\right)^{3/2}$$

input
```
Int[(a + c*x^4)^(3/2),x]
```

output
```
(x*(a + c*x^4)^(3/2))/7 + (6*a*((x*Sqrt[a + c*x^4])/3 + (a^(3/4)*(Sqrt[a]
+ Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*Arc
Tan[(c^(1/4)*x)/a^(1/4)], 1/2])/(3*c^(1/4)*Sqrt[a + c*x^4])))/7
```

Defintions of rubi rules used

rule 748
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Simp[a*n*(p/(n*p + 1))   Int[(a + b*x^n)^(p - 1), x], x] /; Fre
eQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || LtQ[Denominat
or[p + 1/n], Denominator[p]])
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.36 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.79

method	result	size
risch	$\frac{x\left(c\,x^{4}+3a\right)\sqrt{c\,x^{4}+a}}{7}+\frac{4a^{2}\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{7\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}}$	96
default	$\frac{c\,x^{5}\sqrt{c\,x^{4}+a}}{7}+\frac{3ax\sqrt{c\,x^{4}+a}}{7}+\frac{4a^{2}\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{7\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}}$	103
elliptic	$\frac{c\,x^{5}\sqrt{c\,x^{4}+a}}{7}+\frac{3ax\sqrt{c\,x^{4}+a}}{7}+\frac{4a^{2}\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{7\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}}$	103

input
```
int((c*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/7*x*(c*x^4+3*a)*(c*x^4+a)^(1/2)+4/7*a^2/(I*c^(1/2)/a^(1/2))^(1/2)*(1-I*c
^(1/2)*x^2/a^(1/2))^(1/2)*(1+I*c^(1/2)*x^2/a^(1/2))^(1/2)/(c*x^4+a)^(1/2)*
EllipticF(x*(I*c^(1/2)/a^(1/2))^(1/2),I)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.42

$$\int \left(a+cx^4\right)^{3/2} dx = \frac{4}{7} a\sqrt{c}\left(-\frac{a}{c}\right)^{\frac{3}{4}} F(\arcsin\left(\frac{\left(-\frac{a}{c}\right)^{\frac{1}{4}}}{x}\right) | -1) + \frac{1}{7} \left(cx^5 + 3\,ax\right)\sqrt{cx^4 + a}$$

input
```
integrate((c*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
4/7*a*sqrt(c)*(-a/c)^(3/4)*elliptic_f(arcsin((-a/c)^(1/4)/x), -1) + 1/7*(c
*x^5 + 3*a*x)*sqrt(c*x^4 + a)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.56 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.30

$$\int \left(a + cx^4\right)^{3/2} dx = \frac{a^{\frac{3}{2}} x\Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{3}{2}, \frac{1}{4} \\ \frac{5}{4} \end{matrix}\middle| \frac{cx^4 e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((c*x**4+a)**(3/2),x)
```

output
```
a**(3/2)*x*gamma(1/4)*hyper((-3/2, 1/4), (5/4,), c*x**4*exp_polar(I*pi)/a)
/(4*gamma(5/4))
```

Maxima [F]

$$\int \left(a+cx^4\right)^{3/2}\,dx=\int \left(cx^4+a\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((c*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*x^4 + a)^(3/2), x)
```

Giac [F]

$$\int \left(a+cx^4\right)^{3/2}\,dx=\int \left(cx^4+a\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((c*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((c*x^4 + a)^(3/2), x)
```

Mupad [B] (verification not implemented)

Time = 21.44 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.30

$$\int \left(a+cx^4\right)^{3/2}\,dx=\frac{x\left(c\,x^4+a\right)^{3/2}{}_2\mathrm{F}_1\left(-\frac{3}{2},\frac{1}{4};\ \frac{5}{4};\ -\frac{c\,x^4}{a}\right)}{\left(\frac{c\,x^4}{a}+1\right)^{3/2}}$$

input
```
int((a + c*x^4)^(3/2),x)
```

output
```
(x*(a + c*x^4)^(3/2)*hypergeom([-3/2, 1/4], 5/4, -(c*x^4)/a))/((c*x^4)/a +
 1)^(3/2)
```

Reduce [F]

$$\int \left(a+cx^4\right)^{3/2}\,dx=\frac{3\sqrt{c\,x^4+a}\,ax}{7}+\frac{\sqrt{c\,x^4+a}\,c\,x^5}{7}+\frac{4\left(\int\frac{\sqrt{c\,x^4+a}}{c\,x^4+a}dx\right)a^2}{7}$$

input
```
int((c*x^4+a)^(3/2),x)
```

output
```
(3*sqrt(a + c*x**4)*a*x + sqrt(a + c*x**4)*c*x**5 + 4*int(sqrt(a + c*x**4)
/(a + c*x**4),x)*a**2)/7
```

3.202 $\int \frac{(a+cx^4)^{3/2}}{d+ex} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 800

$$\int \frac{(a+cx^4)^{3/2}}{d+ex} \, dx = \frac{(cd^4+ae^4)\sqrt{a+cx^4}}{2e^5} - \frac{cd^3x\sqrt{a+cx^4}}{3e^4}$$
$$+ \frac{cd^2x^2\sqrt{a+cx^4}}{4e^3} - \frac{cdx^3\sqrt{a+cx^4}}{5e^2} - \frac{\sqrt{c}d(5cd^4+7ae^4)\,x\sqrt{a+cx^4}}{5e^6\left(\sqrt{a}+\sqrt{c}x^2\right)}$$
$$+ \frac{(a+cx^4)^{3/2}}{6e} + \frac{(cd^4+ae^4)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{cd^4+ae^4}x}{de\sqrt{a+cx^4}}\right)}{2e^7}$$
$$+ \frac{\sqrt{c}d^2(2cd^4+3ae^4)\operatorname{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{4e^7} - \frac{(cd^4+ae^4)^{3/2}\operatorname{arctanh}\left(\frac{ae^2+cd^2x^2}{\sqrt{cd^4+ae^4}\sqrt{a+cx^4}}\right)}{2e^7}$$
$$+ \frac{\sqrt[4]{a}\sqrt[4]{c}d(5cd^4+7ae^4)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5e^6\sqrt{a+cx^4}}$$
$$- \frac{\sqrt[4]{a}\sqrt[4]{c}d\left(15c^{3/2}d^6+5\sqrt{a}cd^4e^2+23a\sqrt{c}d^2e^4+3a^{3/2}e^6\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\right.\right.}{15e^6\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\sqrt{a+cx^4}}$$
$$- \frac{\left(\sqrt{c}d^2-\sqrt{a}e^2\right)(cd^4+ae^4)^2\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticPi}\left(\frac{\left(\sqrt{c}d^2+\sqrt{a}e^2\right)^2}{4\sqrt{a}\sqrt{c}d^2e^2},2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4\sqrt[4]{a}\sqrt[4]{c}de^8\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\sqrt{a+cx^4}}$$

output
```
1/2*(a*e^4+c*d^4)*(c*x^4+a)^(1/2)/e^5-1/3*c*d^3*x*(c*x^4+a)^(1/2)/e^4+1/4*
c*d^2*x^2*(c*x^4+a)^(1/2)/e^3-1/5*c*d*x^3*(c*x^4+a)^(1/2)/e^2-1/5*c^(1/2)*
d*(7*a*e^4+5*c*d^4)*x*(c*x^4+a)^(1/2)/e^6/(a^(1/2)+c^(1/2)*x^2)+1/6*(c*x^4
+a)^(3/2)/e+1/2*(a*e^4+c*d^4)^(3/2)*arctanh((a*e^4+c*d^4)^(1/2)*x/d/e/(c*x
^4+a)^(1/2))/e^7+1/4*c^(1/2)*d^2*(3*a*e^4+2*c*d^4)*arctanh(c^(1/2)*x^2/(c*
x^4+a)^(1/2))/e^7-1/2*(a*e^4+c*d^4)^(3/2)*arctanh((c*d^2*x^2+a*e^2)/(a*e^4
+c*d^4)^(1/2)/(c*x^4+a)^(1/2))/e^7+1/5*a^(1/4)*c^(1/4)*d*(7*a*e^4+5*c*d^4)
*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*EllipticE
(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))/e^6/(c*x^4+a)^(1/2)-1/15*a^
(1/4)*c^(1/4)*d*(15*c^(3/2)*d^6+5*a^(1/2)*c*d^4*e^2+23*a*c^(1/2)*d^2*e^4+3
*a^(3/2)*e^6)*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1
/2)*InverseJacobiAM(2*arctan(c^(1/4)*x/a^(1/4)),1/2*2^(1/2))/e^6/(c^(1/2)*
d^2+a^(1/2)*e^2)/(c*x^4+a)^(1/2)-1/4*(c^(1/2)*d^2-a^(1/2)*e^2)*(a*e^4+c*d^
4)^2*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*Ellip
ticPi(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/4*(c^(1/2)*d^2+a^(1/2)*e^2)^2/a^(
1/2)/c^(1/2)/d^2/e^2,1/2*2^(1/2))/a^(1/4)/c^(1/4)/d/e^8/(c^(1/2)*d^2+a^(1/
2)*e^2)/(c*x^4+a)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 12.56 (sec) , antiderivative size = 530, normalized size of antiderivative = 0.66

$$\int \frac{(a+cx^4)^{3/2}}{d+ex}\,dx = \frac{-12\sqrt{a}c^{3/4}d^2e^2(5cd^4+7ae^4)\sqrt{1+\frac{cx^4}{a}}E\left(i\operatorname{arcsinh}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}x\right)\middle|-1\right)+4c^{3/4}d^2\left(15ic^{3/2}\right.}{}$$

input
```
Integrate[(a + c*x^4)^(3/2)/(d + e*x),x]
```

output
```
(-12*Sqrt[a]*c^(3/4)*d^2*e^2*(5*c*d^4 + 7*a*e^4)*Sqrt[1 + (c*x^4)/a]*Ellip
ticE[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1] + 4*c^(3/4)*d^2*((15*I)*c
^(3/2)*d^6 + 15*Sqrt[a]*c*d^4*e^2 + (25*I)*a*Sqrt[c]*d^2*e^4 + 21*a^(3/2)*
e^6)*Sqrt[1 + (c*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x],
 -1] + Sqrt[(I*Sqrt[c])/Sqrt[a]]*(-60*(-1)^(1/4)*a^(1/4)*(c*d^4 + a*e^4)^2
*Sqrt[1 + (c*x^4)/a]*EllipticPi[(I*Sqrt[a]*e^2)/(Sqrt[c]*d^2), ArcSin[((-1
)^(3/4)*c^(1/4)*x)/a^(1/4)], -1] + c^(1/4)*d*e*(e^2*(a + c*x^4)*(40*a*e^4
+ c*(30*d^4 - 20*d^3*e*x + 15*d^2*e^2*x^2 - 12*d*e^3*x^3 + 10*e^4*x^4)) +
60*(-(c*d^4) - a*e^4)^(3/2)*Sqrt[a + c*x^4]*ArcTan[(Sqrt[c]*(d^2 - e^2*x^2
) + e^2*Sqrt[a + c*x^4])/Sqrt[-(c*d^4) - a*e^4]] - 15*Sqrt[c]*d^2*(2*c*d^4
 + 3*a*e^4)*Sqrt[a + c*x^4]*Log[-(Sqrt[c]*x^2) + Sqrt[a + c*x^4]])))/(60*S
qrt[(I*Sqrt[c])/Sqrt[a]]*c^(1/4)*d*e^8*Sqrt[a + c*x^4])
```

Rubi [A] (verified)

Time = 3.58 (sec) , antiderivative size = 876, normalized size of antiderivative = 1.10, number of steps used = 22, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.105, Rules used = {2267, 1531, 27, 1577, 493, 25, 682, 27, 719, 224, 219, 488, 219, 2223, 2427, 2427, 27, 1512, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+cx^4\right)^{3/2}}{d+ex}dx$$

$$\Big\downarrow \text{2267}$$

$$d\int \frac{\left(cx^4+a\right)^{3/2}}{d^2-e^2x^2}dx-e\int \frac{x\left(cx^4+a\right)^{3/2}}{d^2-e^2x^2}dx$$

$$\Big\downarrow \text{1531}$$

$$d\left(\frac{\left(ae^4+cd^4\right)^2\int \frac{\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\left(\sqrt{c}x^2+\sqrt{a}\right)}{\left(d^2-e^2x^2\right)\sqrt{cx^4+a}}dx}{e^6\left(cd^4-ae^4\right)}-\frac{\int \frac{c^2e^4\left(cd^4-ae^4\right)x^6+c^2d^2e^2\left(cd^4-ae^4\right)x^4+c\left(cd^4-ae^4\right)\left(cd^4+2ae^4\right)x^2+\sqrt{a}\sqrt{c}\left(\sqrt{c}d^2\right.}{\sqrt{cx^4+a}}}{e^6\left(cd^4-ae^4\right)}\right.$$

$$e\int \frac{x\left(cx^4+a\right)^{3/2}}{d^2-e^2x^2}dx$$

$$\Big\downarrow \text{27}$$

$$d\left(\frac{\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\left(ae^4+cd^4\right)^2\int\frac{\sqrt{c}x^2+\sqrt{a}}{(d^2-e^2x^2)\sqrt{cx^4+a}}dx}{e^6\left(cd^4-ae^4\right)}-\frac{\int\frac{c^2e^4(cd^4-ae^4)x^6+c^2d^2e^2(cd^4-ae^4)x^4+c(cd^4-ae^4)(cd^4+2ae^4)x^2+}{\sqrt{cx^4+a}}}{e^6\left(cd^4-ae^4\right)}\right.$$

$$e\int\frac{x\left(cx^4+a\right)^{3/2}}{d^2-e^2x^2}dx$$

$\downarrow$ 1577

$$d\left(\frac{\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\left(ae^4+cd^4\right)^2\int\frac{\sqrt{c}x^2+\sqrt{a}}{(d^2-e^2x^2)\sqrt{cx^4+a}}dx}{e^6\left(cd^4-ae^4\right)}-\frac{\int\frac{c^2e^4(cd^4-ae^4)x^6+c^2d^2e^2(cd^4-ae^4)x^4+c(cd^4-ae^4)(cd^4+2ae^4)x^2+}{\sqrt{cx^4+a}}}{e^6\left(cd^4-ae^4\right)}\right.$$

$$\frac{1}{2}e\int\frac{\left(cx^4+a\right)^{3/2}}{d^2-e^2x^2}dx^2$$

$\downarrow$ 493

$$d\left(\frac{\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\left(ae^4+cd^4\right)^2\int\frac{\sqrt{c}x^2+\sqrt{a}}{(d^2-e^2x^2)\sqrt{cx^4+a}}dx}{e^6\left(cd^4-ae^4\right)}-\frac{\int\frac{c^2e^4(cd^4-ae^4)x^6+c^2d^2e^2(cd^4-ae^4)x^4+c(cd^4-ae^4)(cd^4+2ae^4)x^2+}{\sqrt{cx^4+a}}}{e^6\left(cd^4-ae^4\right)}\right.$$

$$\frac{1}{2}e\left(-\frac{\int-\frac{(ae^2+cd^2x^2)\sqrt{cx^4+a}}{d^2-e^2x^2}dx^2}{e^2}-\frac{\left(a+cx^4\right)^{3/2}}{3e^2}\right)$$

$\downarrow$ 25

$$d\left(\frac{\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\left(ae^4+cd^4\right)^2\int\frac{\sqrt{c}x^2+\sqrt{a}}{(d^2-e^2x^2)\sqrt{cx^4+a}}dx}{e^6\left(cd^4-ae^4\right)}-\frac{\int\frac{c^2e^4(cd^4-ae^4)x^6+c^2d^2e^2(cd^4-ae^4)x^4+c(cd^4-ae^4)(cd^4+2ae^4)x^2+}{\sqrt{cx^4+a}}}{e^6\left(cd^4-ae^4\right)}\right.$$

$$\frac{1}{2}e\left(\frac{\int\frac{(ae^2+cd^2x^2)\sqrt{cx^4+a}}{d^2-e^2x^2}dx^2}{e^2}-\frac{\left(a+cx^4\right)^{3/2}}{3e^2}\right)$$

$\downarrow$ 682

$$d\left(\frac{\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\left(ae^4+cd^4\right)^2\int\frac{\sqrt{c}x^2+\sqrt{a}}{(d^2-e^2x^2)\sqrt{cx^4+a}}dx}{e^6\left(cd^4-ae^4\right)}-\frac{\int\frac{c^2e^4(cd^4-ae^4)x^6+c^2d^2e^2(cd^4-ae^4)x^4+c(cd^4-ae^4)(cd^4+2ae^4)x^2+}{\sqrt{cx^4+a}}}{e^6\left(cd^4-ae^4\right)}\right.$$

$$\frac{1}{2}e\left(\frac{\frac{\int\frac{c\left(a\left(cd^4+2ae^4\right)e^2+cd^2\left(2cd^4+3ae^4\right)x^2\right)}{\left(d^2-e^2x^2\right)\sqrt{cx^4+a}}dx^2}{2ce^4}-\frac{\sqrt{a+cx^4}\left(2\left(ae^4+cd^4\right)+cd^2e^2x^2\right)}{2e^4}}{e^2}-\frac{\left(a+cx^4\right)^{3/2}}{3e^2}\right)$$

$\downarrow$ 27

$$d\left(\frac{\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\left(ae^4+cd^4\right)^2\int\frac{\sqrt{c}x^2+\sqrt{a}}{(d^2-e^2x^2)\sqrt{cx^4+a}}dx}{e^6\left(cd^4-ae^4\right)}-\frac{\int\frac{c^2e^4(cd^4-ae^4)x^6+c^2d^2e^2(cd^4-ae^4)x^4+c(cd^4-ae^4)(cd^4+2ae^4)x^2+}{\sqrt{cx^4+a}}}{e^6\left(cd^4-ae^4\right)}\right.$$

$$\frac{1}{2}e\left(\frac{\frac{\int\frac{a\left(cd^4+2ae^4\right)e^2+cd^2\left(2cd^4+3ae^4\right)x^2}{\left(d^2-e^2x^2\right)\sqrt{cx^4+a}}dx^2}{2e^4}-\frac{\sqrt{a+cx^4}(2(ae^4+cd^4)+cd^2e^2x^2)}{2e^4}}{e^2}-\frac{\left(a+cx^4\right)^{3/2}}{3e^2}\right)$$

$$\Big\downarrow \text{719}$$

$$d\left(\frac{\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\left(ae^4+cd^4\right)^2\int\frac{\sqrt{c}x^2+\sqrt{a}}{(d^2-e^2x^2)\sqrt{cx^4+a}}dx}{e^6\left(cd^4-ae^4\right)}-\frac{\int\frac{c^2e^4(cd^4-ae^4)x^6+c^2d^2e^2(cd^4-ae^4)x^4+c(cd^4-ae^4)(cd^4+2ae^4)x^2+}{\sqrt{cx^4+a}}}{e^6\left(cd^4-ae^4\right)}\right.$$

$$\frac{1}{2}e\left(\frac{\frac{\frac{2\left(ae^4+cd^4\right)^2\int\frac{1}{\left(d^2-e^2x^2\right)\sqrt{cx^4+a}}dx^2}{e^2}-\frac{cd^2\left(3ae^4+2cd^4\right)\int\frac{1}{\sqrt{cx^4+a}}dx^2}{e^2}}{2e^4}-\frac{\sqrt{a+cx^4}(2(ae^4+cd^4)+cd^2e^2x^2)}{2e^4}}{e^2}-\frac{\left(a+cx^4\right)^{3/2}}{3e^2}\right)$$

$$\Big\downarrow \text{224}$$

$$d\left(\frac{\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\left(ae^4+cd^4\right)^2\int\frac{\sqrt{c}x^2+\sqrt{a}}{(d^2-e^2x^2)\sqrt{cx^4+a}}dx}{e^6\left(cd^4-ae^4\right)}-\frac{\int\frac{c^2e^4(cd^4-ae^4)x^6+c^2d^2e^2(cd^4-ae^4)x^4+c(cd^4-ae^4)(cd^4+2ae^4)x^2+}{\sqrt{cx^4+a}}}{e^6\left(cd^4-ae^4\right)}\right.$$

$$\frac{1}{2}e\left(\frac{\frac{\frac{2\left(ae^4+cd^4\right)^2\int\frac{1}{\left(d^2-e^2x^2\right)\sqrt{cx^4+a}}dx^2}{e^2}-\frac{cd^2\left(3ae^4+2cd^4\right)\int\frac{1}{1-cx^4}d\frac{x^2}{\sqrt{cx^4+a}}}{e^2}}{2e^4}-\frac{\sqrt{a+cx^4}(2(ae^4+cd^4)+cd^2e^2x^2)}{2e^4}}{e^2}-\frac{\left(a+cx^4\right)^{3/2}}{3e^2}\right)$$

$$\Big\downarrow \text{219}$$

$$d\left(\frac{\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\left(ae^4+cd^4\right)^2\int\frac{\sqrt{c}x^2+\sqrt{a}}{(d^2-e^2x^2)\sqrt{cx^4+a}}dx}{e^6\left(cd^4-ae^4\right)}-\frac{\int\frac{c^2e^4(cd^4-ae^4)x^6+c^2d^2e^2(cd^4-ae^4)x^4+c(cd^4-ae^4)(cd^4+2ae^4)x^2+}{\sqrt{cx^4+a}}}{e^6\left(cd^4-ae^4\right)}\right.$$

$$\frac{1}{2}e\left(\frac{\frac{\frac{2\left(ae^4+cd^4\right)^2\int\frac{1}{\left(d^2-e^2x^2\right)\sqrt{cx^4+a}}dx^2}{e^2}-\frac{\sqrt{c}d^2\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)\left(3ae^4+2cd^4\right)}{e^2}}{2e^4}-\frac{\sqrt{a+cx^4}(2(ae^4+cd^4)+cd^2e^2x^2)}{2e^4}}{e^2}-\frac{\left(a+cx^4\right)^{3/2}}{3e^2}\right.$$

$$\Big\downarrow \text{488}$$

$$d\left(\frac{\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\left(ae^4+cd^4\right)^2\int \frac{\sqrt{c}x^2+\sqrt{a}}{\left(d^2-e^2x^2\right)\sqrt{cx^4+a}}dx}{e^6\left(cd^4-ae^4\right)}-\frac{\int \frac{c^2e^4\left(cd^4-ae^4\right)x^6+c^2d^2e^2\left(cd^4-ae^4\right)x^4+c\left(cd^4-ae^4\right)\left(cd^4+2ae^4\right)x^2+}{\sqrt{cx^4+a}}}{e^6\left(cd^4-ae^4\right)}\right.$$

$$\frac{1}{2}e\left(\frac{\frac{-\frac{2\left(ae^4+cd^4\right)^2\int \frac{1}{cd^4+ae^4-x^4}d\frac{-ae^2-cd^2x^2}{\sqrt{cx^4+a}}}{e^2}-\frac{\sqrt{c}d^2\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)\left(3ae^4+2cd^4\right)}{e^2}}{2e^4}-\frac{\sqrt{a+cx^4}\left(2\left(ae^4+cd^4\right)+cd^2e^2x^2\right)}{2e^4}}{e^2}-\frac{\left(a+cx^4\right)}{3e^2}\right.$$

$$\Big\downarrow \ 219$$

$$d\left(\frac{\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\left(ae^4+cd^4\right)^2\int \frac{\sqrt{c}x^2+\sqrt{a}}{\left(d^2-e^2x^2\right)\sqrt{cx^4+a}}dx}{e^6\left(cd^4-ae^4\right)}-\frac{\int \frac{c^2e^4\left(cd^4-ae^4\right)x^6+c^2d^2e^2\left(cd^4-ae^4\right)x^4+c\left(cd^4-ae^4\right)\left(cd^4+2ae^4\right)x^2+}{\sqrt{cx^4+a}}}{e^6\left(cd^4-ae^4\right)}\right.$$

$$\frac{1}{2}e\left(\frac{\frac{-\frac{\sqrt{c}d^2\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)\left(3ae^4+2cd^4\right)}{e^2}-\frac{2\left(ae^4+cd^4\right)^{3/2}\text{arctanh}\left(\frac{-ae^2-cd^2x^2}{\sqrt{a+cx^4}\sqrt{ae^4+cd^4}}\right)}{e^2}}{2e^4}-\frac{\sqrt{a+cx^4}\left(2\left(ae^4+cd^4\right)+cd^2e^2x^2\right)}{2e^4}}{e^2}-\frac{\left(a+\right.}{3}\right.$$

$$\Big\downarrow \ 2223$$

$$d\left(\frac{\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\left(ae^4+cd^4\right)^2\left(\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{\sqrt{a}}{d^2}-\frac{\sqrt{c}}{e^2}\right)\text{EllipticPi}\left(\frac{\left(\sqrt{c}d^2+\sqrt{a}e^2\right)^2}{4\sqrt{a}\sqrt{c}d^2e^2},2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+cx^4}}+\frac{\left(\sqrt{a}e\right.}{}\right.}{e^6\left(cd^4-ae^4\right)}\right.$$

$$\frac{1}{2}e\left(\frac{\frac{-\frac{\sqrt{c}d^2\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)\left(3ae^4+2cd^4\right)}{e^2}-\frac{2\left(ae^4+cd^4\right)^{3/2}\text{arctanh}\left(\frac{-ae^2-cd^2x^2}{\sqrt{a+cx^4}\sqrt{ae^4+cd^4}}\right)}{e^2}}{2e^4}-\frac{\sqrt{a+cx^4}\left(2\left(ae^4+cd^4\right)+cd^2e^2x^2\right)}{2e^4}}{e^2}-\frac{\left(a+\right.}{3}\right.$$

$$\Big\downarrow \ 2427$$

$$d\left(\frac{\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\left(ae^4+cd^4\right)^2\left(\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{\sqrt{a}}{d^2}-\frac{\sqrt{c}}{e^2}\right)\text{EllipticPi}\left(\frac{\left(\sqrt{c}d^2+\sqrt{a}e^2\right)^2}{4\sqrt{a}\sqrt{c}d^2e^2},2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+cx^4}}+\frac{\left(\sqrt{a}e\cdots\right.}{\cdots}\right.}{e^6\left(cd^4-ae^4\right)}\cdots\right.$$

$$\frac{1}{2}e\left(\frac{-\frac{\frac{\sqrt{c}d^2\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)\left(3ae^4+2cd^4\right)}{e^2}-\frac{2\left(ae^4+cd^4\right)^{3/2}\text{arctanh}\left(\frac{-ae^2-cd^2x^2}{\sqrt{a+cx^4}\sqrt{ae^4+cd^4}}\right)}{e^2}}{2e^4}-\frac{\sqrt{a+cx^4}\left(2\left(ae^4+cd^4\right)+cd^2e^2x^2\right)}{2e^4}}{e^2}-\frac{(a+\cdots}{3\cdots}\right.$$

$$\Big\downarrow \text{ 2427}$$

$$d\left(\frac{\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\left(ae^4+cd^4\right)^2\left(\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{\sqrt{a}}{d^2}-\frac{\sqrt{c}}{e^2}\right)\text{EllipticPi}\left(\frac{\left(\sqrt{c}d^2+\sqrt{a}e^2\right)^2}{4\sqrt{a}\sqrt{c}d^2e^2},2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+cx^4}}+\frac{\left(\sqrt{a}e\cdots\right.}{\cdots}\right.}{e^6\left(cd^4-ae^4\right)}\cdots\right.$$

$$\frac{1}{2}e\left(\frac{-\frac{\frac{\sqrt{c}d^2\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)\left(3ae^4+2cd^4\right)}{e^2}-\frac{2\left(ae^4+cd^4\right)^{3/2}\text{arctanh}\left(\frac{-ae^2-cd^2x^2}{\sqrt{a+cx^4}\sqrt{ae^4+cd^4}}\right)}{e^2}}{2e^4}-\frac{\sqrt{a+cx^4}\left(2\left(ae^4+cd^4\right)+cd^2e^2x^2\right)}{2e^4}}{e^2}-\frac{(a+\cdots}{3\cdots}\right.$$

$$\Big\downarrow \text{ 27}$$

$$d\left(\frac{\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\left(ae^4+cd^4\right)^2\left(\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{\sqrt{a}}{d^2}-\frac{\sqrt{c}}{e^2}\right)\text{EllipticPi}\left(\frac{\left(\sqrt{c}d^2+\sqrt{a}e^2\right)^2}{4\sqrt{a}\sqrt{c}d^2e^2},2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+cx^4}}+\frac{\left(\sqrt{a}e\cdots\right.}{\cdots}\right.}{e^6\left(cd^4-ae^4\right)}\cdots\right.$$

$$\frac{1}{2}e\left(\frac{-\frac{\frac{\sqrt{c}d^2\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)\left(3ae^4+2cd^4\right)}{e^2}-\frac{2\left(ae^4+cd^4\right)^{3/2}\text{arctanh}\left(\frac{-ae^2-cd^2x^2}{\sqrt{a+cx^4}\sqrt{ae^4+cd^4}}\right)}{e^2}}{2e^4}-\frac{\sqrt{a+cx^4}\left(2\left(ae^4+cd^4\right)+cd^2e^2x^2\right)}{2e^4}}{e^2}-\frac{(a+\cdots}{3\cdots}\cdots\right.$$

$\downarrow$ 1512

$$d\left(\frac{\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\left(ae^4+cd^4\right)^2\left(\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{\sqrt{a}}{d^2}-\frac{\sqrt{c}}{e^2}\right)\text{EllipticPi}\left(\frac{\left(\sqrt{c}d^2+\sqrt{a}e^2\right)^2}{4\sqrt{a}\sqrt{c}d^2e^2},2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+cx^4}}+\frac{\left(\sqrt{a}e\cdots\right.}{\cdots}\right.}{e^6\left(cd^4-ae^4\right)}\cdots\right.$$

$$\frac{1}{2}e\left(\frac{-\frac{\frac{\sqrt{c}d^2\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)\left(3ae^4+2cd^4\right)}{e^2}-\frac{2\left(ae^4+cd^4\right)^{3/2}\text{arctanh}\left(\frac{-ae^2-cd^2x^2}{\sqrt{a+cx^4}\sqrt{ae^4+cd^4}}\right)}{e^2}}{2e^4}-\frac{\sqrt{a+cx^4}\left(2\left(ae^4+cd^4\right)+cd^2e^2x^2\right)}{2e^4}}{e^2}-\frac{(a+\cdots}{3\cdots}\cdots\right.$$

$\downarrow$ 27

$$d\left(\frac{\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\left(ae^4+cd^4\right)^2\left(\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{\sqrt{a}}{d^2}-\frac{\sqrt{c}}{e^2}\right)\text{EllipticPi}\left(\frac{\left(\sqrt{c}d^2+\sqrt{a}e^2\right)^2}{4\sqrt{a}\sqrt{c}d^2e^2},2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+cx^4}}+\frac{\left(\sqrt{a}e\right.}{}\right.}{e^6\left(cd^4-ae^4\right)}\right.$$

$$\frac{1}{2}e\left(\frac{\frac{-\frac{\sqrt{c}d^2\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)\left(3ae^4+2cd^4\right)}{e^2}-\frac{2\left(ae^4+cd^4\right)^{3/2}\text{arctanh}\left(\frac{-ae^2-cd^2x^2}{\sqrt{a+cx^4}\sqrt{ae^4+cd^4}}\right)}{e^2}}{2e^4}-\frac{\sqrt{a+cx^4}\left(2\left(ae^4+cd^4\right)+cd^2e^2x^2\right)}{2e^4}}{e^2}-\frac{(a+}{3}\right.$$

$\downarrow$ 761

$$d\left(\frac{\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\left(ae^4+cd^4\right)^2\left(\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{\sqrt{a}}{d^2}-\frac{\sqrt{c}}{e^2}\right)\text{EllipticPi}\left(\frac{\left(\sqrt{c}d^2+\sqrt{a}e^2\right)^2}{4\sqrt{a}\sqrt{c}d^2e^2},2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+cx^4}}+\frac{\left(\sqrt{a}e\right.}{}\right.}{e^6\left(cd^4-ae^4\right)}\right.$$

$$\frac{1}{2}e\left(\frac{\frac{-\frac{\sqrt{c}d^2\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)\left(3ae^4+2cd^4\right)}{e^2}-\frac{2\left(ae^4+cd^4\right)^{3/2}\text{arctanh}\left(\frac{-ae^2-cd^2x^2}{\sqrt{a+cx^4}\sqrt{ae^4+cd^4}}\right)}{e^2}}{2e^4}-\frac{\sqrt{a+cx^4}\left(2\left(ae^4+cd^4\right)+cd^2e^2x^2\right)}{2e^4}}{e^2}-\frac{(a+}{3}\right.$$

$\downarrow$ 1510

$$d\left(\frac{\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\left(cd^4+ae^4\right)^2\left(\frac{\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\text{arctanh}\left(\frac{\sqrt{cd^4+ae^4}x}{de\sqrt{cx^4+a}}\right)}{2de\sqrt{cd^4+ae^4}}+\frac{\left(\frac{\sqrt{a}}{d^2}-\frac{\sqrt{c}}{e^2}\right)\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{\frac{cx^4+a}{\left(\sqrt{c}x^2+\sqrt{a}\right)^2}}\,\text{EllipticPi}\left(\frac{(\sqrt{\cdots}}{4\cdots}\right.}{4\sqrt[4]{a}\sqrt[4]{c}\sqrt{cx^4+a}\cdots}\right.}{e^6\left(cd^4-ae^4\right)}\right.$$

$$\frac{1}{2}e\left(\frac{\frac{-\frac{\sqrt{c}\left(2cd^4+3ae^4\right)\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{cx^4+a}}\right)d^2}{e^2}-\frac{2\left(cd^4+ae^4\right)^{3/2}\text{arctanh}\left(\frac{-ae^2-cd^2x^2}{\sqrt{cd^4+ae^4}\sqrt{cx^4+a}}\right)}{e^2}}{2e^4}-\frac{\left(cd^2e^2x^2+2\left(cd^4+ae^4\right)\right)\sqrt{cx^4+a}}{2e^4}}{e^2}-\frac{\left(cx^4\cdots\right.}{3\cdots}\right.$$

input

```
Int[(a + c*x^4)^(3/2)/(d + e*x),x]
```

output

```
-1/2*(e*(-1/3*(a + c*x^4)^(3/2)/e^2 + (-1/2*((2*(c*d^4 + a*e^4) + c*d^2*e^
2*x^2)*Sqrt[a + c*x^4])/e^4 + (-((Sqrt[c]*d^2*(2*c*d^4 + 3*a*e^4)*ArcTanh[
(Sqrt[c]*x^2)/Sqrt[a + c*x^4]])/e^2) - (2*(c*d^4 + a*e^4)^(3/2)*ArcTanh[(-
(a*e^2) - c*d^2*x^2)/(Sqrt[c*d^4 + a*e^4]*Sqrt[a + c*x^4])])/e^2)/(2*e^4))
/e^2)) + d*(-((((c*e^4*(c*d^4 - a*e^4)*x^3*Sqrt[a + c*x^4])/5 + ((5*c^2*d^2
*e^2*(c*d^4 - a*e^4)*x*Sqrt[a + c*x^4])/3 + (c^(3/2)*(-3*(c*d^4 - a*e^4)*(
5*c*d^4 + 7*a*e^4)*(-((x*Sqrt[a + c*x^4])/(Sqrt[a] + Sqrt[c]*x^2)) + (a^(1
/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*El
lipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(c^(1/4)*Sqrt[a + c*x^4])) +
(a^(1/4)*(15*c^2*d^8 - 10*Sqrt[a]*c^(3/2)*d^6*e^2 + 18*a*c*d^4*e^4 - 20*a^
(3/2)*Sqrt[c]*d^2*e^6 - 3*a^2*e^8)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4
)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2]
)/(c^(1/4)*Sqrt[a + c*x^4])))/3)/(5*c))/(e^6*(c*d^4 - a*e^4))) + ((Sqrt[c]
*d^2 - Sqrt[a]*e^2)*(c*d^4 + a*e^4)^2*(((Sqrt[c]*d^2 + Sqrt[a]*e^2)*ArcTan
h[(Sqrt[c*d^4 + a*e^4]*x)/(d*e*Sqrt[a + c*x^4])])/(2*d*e*Sqrt[c*d^4 + a*e^
4]) + ((Sqrt[a]/d^2 - Sqrt[c]/e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4
)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticPi[(Sqrt[c]*d^2 + Sqrt[a]*e^2)^2/(4*S
qrt[a]*Sqrt[c]*d^2*e^2), 2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(1/4)*c
^(1/4)*Sqrt[a + c*x^4]))))/(e^6*(c*d^4 - a*e^4)))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 493
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*((a + b*x^2)^p/(d*(n + 2*p + 1))), x] + Simp[2*(p/(d*(n +
 2*p + 1)))   Int[(c + d*x)^n*(a + b*x^2)^(p - 1)*(a*d - b*c*x), x], x] /;
FreeQ[{a, b, c, d, n}, x] && GtQ[p, 0] && NeQ[n + 2*p + 1, 0] && ( !Rationa
lQ[n] || LtQ[n, 1]) &&  !ILtQ[n + 2*p, 0] && IntQuadraticQ[a, 0, b, c, d, n
, p, x]
```

rule 682
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Simp[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)))   Int[(d + e*x)
^m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*
d*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x
], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !
RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1512
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + c*x^4], x], x] - Simp[e/q
 Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c
, d, e}, x] && PosQ[c/a]
```

rule 1531
```
Int[((a_) + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symbol] :> Simp[-(c
*d^2 + a*e^2)^(p + 1/2)/(e^(2*p)*(c*d^2 - a*e^2))   Int[(a*d*Rt[c/a, 2] + a
*e + (c*d + a*e*Rt[c/a, 2])*x^2)/((d + e*x^2)*Sqrt[a + c*x^4]), x], x] + Si
mp[1/(e^(2*p)*(c*d^2 - a*e^2))   Int[(1/Sqrt[a + c*x^4])*ExpandToSum[(e^(2*
p)*(c*d^2 - a*e^2)*(a + c*x^4)^(p + 1/2) + (c*d^2 + a*e^2)^(p + 1/2)*(a*d*R
t[c/a, 2] + a*e + (c*d + a*e*Rt[c/a, 2])*x^2))/(d + e*x^2), x], x], x] /; F
reeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p - 1/2, 0] && NeQ[c
*d^2 - a*e^2, 0] && PosQ[c/a]
```

rule 1577
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, c, d, e, p, q}, x]
```

rule 2223
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTanh[Rt[(-c)*
(d/e) - a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[(-c)*(d/e) - a*(e/d), 2]
)), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^
2)]/(4*d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*Arc
Tan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[c*(d/e) + a*(e
/d)]
```

rule 2267
```
Int[((a_) + (c_.)*(x_)^4)^(p_.)/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[d
Int[(a + c*x^4)^p/(d^2 - e^2*x^2), x], x] - Simp[e   Int[x*((a + c*x^4)^p/(
d^2 - e^2*x^2)), x], x] /; FreeQ[{a, c, d, e}, x] && IntegerQ[p + 1/2]
```

rule 2427
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{q = Expon[Pq, x
]}, With[{Pqq = Coeff[Pq, x, q]}, Simp[Pqq*x^(q - n + 1)*((a + b*x^n)^(p +
1)/(b*(q + n*p + 1))), x] + Simp[1/(b*(q + n*p + 1))   Int[ExpandToSum[b*(q
 + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(q - n + 1)*x^(q - n), x]*(a + b*x^n)^p,
 x], x]] /; NeQ[q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ
[p + (q + 1)/(2*n)])] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.19 (sec) , antiderivative size = 576, normalized size of antiderivative = 0.72

method	result
risch	$\frac{(10x^4ce^4-12cdx^3e^3+15x^2cd^2e^2-20cd^3xe+40e^4a+30cd^4)\sqrt{cx^4+a}}{60e^5} - \frac{-\frac{30(a^2e^8+2acd^4e^4+d^8c^2)\left(-\frac{\operatorname{arctanh}\left(\frac{\frac{2cx^2d^2}{e^2}+2a}{2\sqrt{a+\frac{cd^4}{e^4}}\sqrt{cx^4+\cdots}}\right.}{2\sqrt{a+\frac{cd^4}{e^4}}}\right.}{\cdots}}{\cdots}$
default	$\frac{cx^4\sqrt{cx^4+a}}{6e} - \frac{cdx^3\sqrt{cx^4+a}}{5e^2} + \frac{cd^2x^2\sqrt{cx^4+a}}{4e^3} - \frac{cd^3x\sqrt{cx^4+a}}{3e^4} + \frac{\left(\frac{c(2e^4a+cd^4)}{e^5} - \frac{2ca}{3e}\right)\sqrt{cx^4+a}}{2c} + \frac{\left(-\frac{cd^3(2e^4a+cd^4)}{e^8}\cdots\right.}{\cdots}$
elliptic	$\frac{cx^4\sqrt{cx^4+a}}{6e} - \frac{cdx^3\sqrt{cx^4+a}}{5e^2} + \frac{cd^2x^2\sqrt{cx^4+a}}{4e^3} - \frac{cd^3x\sqrt{cx^4+a}}{3e^4} + \frac{\left(\frac{c(2e^4a+cd^4)}{e^5} - \frac{2ca}{3e}\right)\sqrt{cx^4+a}}{2c} + \frac{\left(-\frac{cd^3(2e^4a+cd^4)}{e^8}\cdots\right.}{\cdots}$

input
```
int((c*x^4+a)^(3/2)/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
1/60*(10*c*e^4*x^4-12*c*d*e^3*x^3+15*c*d^2*e^2*x^2-20*c*d^3*e*x+40*a*e^4+3
0*c*d^4)*(c*x^4+a)^(1/2)/e^5-1/30/e^5*(-30*(a^2*e^8+2*a*c*d^4*e^4+c^2*d^8)
/e^4*(-1/2/(a+c*d^4/e^4)^(1/2)*arctanh(1/2*(2*c*x^2*d^2/e^2+2*a)/(a+c*d^4/
e^4)^(1/2)/(c*x^4+a)^(1/2))+1/(I*c^(1/2)/a^(1/2))^(1/2)/d*e*(1-I*c^(1/2)*x
^2/a^(1/2))^(1/2)*(1+I*c^(1/2)*x^2/a^(1/2))^(1/2)/(c*x^4+a)^(1/2)*Elliptic
Pi(x*(I*c^(1/2)/a^(1/2))^(1/2),-I/c^(1/2)*a^(1/2)/d^2*e^2,(-I/a^(1/2)*c^(1
/2))^(1/2)/(I*c^(1/2)/a^(1/2))^(1/2)))+c*d/e^3*(6*I*e^2*(7*a*e^4+5*c*d^4)*
a^(1/2)/(I*c^(1/2)/a^(1/2))^(1/2)*(1-I*c^(1/2)*x^2/a^(1/2))^(1/2)*(1+I*c^(
1/2)*x^2/a^(1/2))^(1/2)/(c*x^4+a)^(1/2)/c^(1/2)*(EllipticF(x*(I*c^(1/2)/a^
(1/2))^(1/2),I)-EllipticE(x*(I*c^(1/2)/a^(1/2))^(1/2),I))-15/2*d*e*(3*a*e^
4+2*c*d^4)*ln(c^(1/2)*x^2+(c*x^4+a)^(1/2))/c^(1/2)+30*c*d^6/(I*c^(1/2)/a^(
1/2))^(1/2)*(1-I*c^(1/2)*x^2/a^(1/2))^(1/2)*(1+I*c^(1/2)*x^2/a^(1/2))^(1/2
)/(c*x^4+a)^(1/2)*EllipticF(x*(I*c^(1/2)/a^(1/2))^(1/2),I)+50*a*d^2*e^4/(I
*c^(1/2)/a^(1/2))^(1/2)*(1-I*c^(1/2)*x^2/a^(1/2))^(1/2)*(1+I*c^(1/2)*x^2/a
^(1/2))^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I*c^(1/2)/a^(1/2))^(1/2),I)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+cx^4)^{3/2}}{d+ex}\,dx = \text{Timed out}$$

input
```
integrate((c*x^4+a)^(3/2)/(e*x+d),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(a+cx^4)^{3/2}}{d+ex}\,dx = \int \frac{(a+cx^4)^{\frac{3}{2}}}{d+ex}\,dx$$

input
```
integrate((c*x**4+a)**(3/2)/(e*x+d),x)
```

output
```
Integral((a + c*x**4)**(3/2)/(d + e*x), x)
```

Maxima [F]

$$\int \frac{(a+cx^4)^{3/2}}{d+ex}\,dx = \int \frac{(cx^4+a)^{\frac{3}{2}}}{ex+d}\,dx$$

input
```
integrate((c*x^4+a)^(3/2)/(e*x+d),x, algorithm="maxima")
```

output
```
integrate((c*x^4 + a)^(3/2)/(e*x + d), x)
```

Giac [F]

$$\int \frac{(a+cx^4)^{3/2}}{d+ex}\,dx = \int \frac{(cx^4+a)^{\frac{3}{2}}}{ex+d}\,dx$$

input
```
integrate((c*x^4+a)^(3/2)/(e*x+d),x, algorithm="giac")
```

output
```
integrate((c*x^4 + a)^(3/2)/(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+cx^4)^{3/2}}{d+ex}\,dx = \int \frac{(c\,x^4+a)^{3/2}}{d+e\,x}\,dx$$

input
```
int((a + c*x^4)^(3/2)/(d + e*x),x)
```

output
```
int((a + c*x^4)^(3/2)/(d + e*x), x)
```

Reduce [F]

$$\int \frac{(a+cx^4)^{3/2}}{d+ex}\,dx = \int \frac{(c\,x^4+a)^{\frac{3}{2}}}{ex+d}dx$$

input
```
int((c*x^4+a)^(3/2)/(e*x+d),x)
```

output
```
int((c*x^4+a)^(3/2)/(e*x+d),x)
```

3.203 $\int \frac{(a+cx^4)^{3/2}}{(d+ex)^2} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 831

$$\int \frac{(a+cx^4)^{3/2}}{(d+ex)^2} \, dx = -\frac{3cd^3\sqrt{a+cx^4}}{e^5} + \frac{cd^2x\sqrt{a+cx^4}}{e^4} - \frac{3cdx^2\sqrt{a+cx^4}}{2e^3} + \frac{cx^3\sqrt{a+cx^4}}{5e^2} + \frac{6\sqrt{c}(5cd^4+2ae^4)\,x\sqrt{a+cx^4}}{5e^6\left(\sqrt{a}+\sqrt{c}x^2\right)} + \frac{\left(a+\frac{cd^4}{e^4}\right)x\sqrt{a+cx^4}}{d^2-e^2x^2} - \frac{d(a+cx^4)^{3/2}}{e\,(d^2-e^2x^2)} - \frac{3cd^3\sqrt{cd^4+ae^4}\text{arctanh}\left(\frac{\sqrt{cd^4+ae^4}x}{de\sqrt{a+cx^4}}\right)}{e^7} - \frac{3\sqrt{c}d(2cd^4+ae^4)\,\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{2e^7} + \frac{3cd^3\sqrt{cd^4+ae^4}\text{arctanh}\left(\frac{ae^2+cd^2x^2}{\sqrt{cd^4+ae^4}\sqrt{a+cx^4}}\right)}{e^7} - \frac{6\sqrt[4]{a}\sqrt[4]{c}(5cd^4+2ae^4)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5e^6\sqrt{a+cx^4}} + \frac{2\sqrt[4]{a}\sqrt[4]{c}\left(15c^{3/2}d^6+5\sqrt{a}cd^4e^2+8a\sqrt{c}d^2e^4+3a^{3/2}e^6\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right.\right.}{5e^6\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\sqrt{a+cx^4}} + \frac{3c^{3/4}d^2\left(\sqrt{c}d^2-\sqrt{a}e^2\right)(cd^4+ae^4)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\,\text{EllipticPi}\left(\frac{(\sqrt{c}d^2+\sqrt{a}e^2)^2}{4\sqrt{a}\sqrt{c}d^2e^2}, 2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\right.}{2\sqrt[4]{a}e^8\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\sqrt{a+cx^4}}$$

output
```
-3*c*d^3*(c*x^4+a)^(1/2)/e^5+c*d^2*x*(c*x^4+a)^(1/2)/e^4-3/2*c*d*x^2*(c*x^
4+a)^(1/2)/e^3+1/5*c*x^3*(c*x^4+a)^(1/2)/e^2+6/5*c^(1/2)*(2*a*e^4+5*c*d^4)
*x*(c*x^4+a)^(1/2)/e^6/(a^(1/2)+c^(1/2)*x^2)+(a+c*d^4/e^4)*x*(c*x^4+a)^(1/
2)/(-e^2*x^2+d^2)-d*(c*x^4+a)^(3/2)/e/(-e^2*x^2+d^2)-3*c*d^3*(a*e^4+c*d^4)
^(1/2)*arctanh((a*e^4+c*d^4)^(1/2)*x/d/e/(c*x^4+a)^(1/2))/e^7-3/2*c^(1/2)*
d*(a*e^4+2*c*d^4)*arctanh(c^(1/2)*x^2/(c*x^4+a)^(1/2))/e^7+3*c*d^3*(a*e^4+
c*d^4)^(1/2)*arctanh((c*d^2*x^2+a*e^2)/(a*e^4+c*d^4)^(1/2)/(c*x^4+a)^(1/2)
)/e^7-6/5*a^(1/4)*c^(1/4)*(2*a*e^4+5*c*d^4)*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+
a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4)
)),1/2*2^(1/2))/e^6/(c*x^4+a)^(1/2)+2/5*a^(1/4)*c^(1/4)*(15*c^(3/2)*d^6+5*
a^(1/2)*c*d^4*e^2+8*a*c^(1/2)*d^2*e^4+3*a^(3/2)*e^6)*(a^(1/2)+c^(1/2)*x^2)
*((c*x^4+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(c^(1/4
)*x/a^(1/4)),1/2*2^(1/2))/e^6/(c^(1/2)*d^2+a^(1/2)*e^2)/(c*x^4+a)^(1/2)+3/
2*c^(3/4)*d^2*(c^(1/2)*d^2-a^(1/2)*e^2)*(a*e^4+c*d^4)*(a^(1/2)+c^(1/2)*x^2
)*((c*x^4+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*EllipticPi(sin(2*arctan(c^(1/4
)*x/a^(1/4))),1/4*(c^(1/2)*d^2+a^(1/2)*e^2)^2/a^(1/2)/c^(1/2)/d^2/e^2,1/2*
2^(1/2))/a^(1/4)/e^8/(c^(1/2)*d^2+a^(1/2)*e^2)/(c*x^4+a)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 12.93 (sec) , antiderivative size = 549, normalized size of antiderivative = 0.66

$$\int \frac{(a+cx^4)^{3/2}}{(d+ex)^2}\,dx = \frac{12\sqrt{a}\sqrt{c}e^2(5cd^4+2ae^4)(d+ex)\sqrt{1+\frac{cx^4}{a}}E\left(i\operatorname{arcsinh}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}x\right)\middle|-1\right)-4\sqrt{c}(15ic^{3/}}{}$$

input
```
Integrate[(a + c*x^4)^(3/2)/(d + e*x)^2,x]
```

output
```
(12*Sqrt[a]*Sqrt[c]*e^2*(5*c*d^4 + 2*a*e^4)*(d + e*x)*Sqrt[1 + (c*x^4)/a]*
EllipticE[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1] - 4*Sqrt[c]*((15*I)*
c^(3/2)*d^6 + 15*Sqrt[a]*c*d^4*e^2 + (10*I)*a*Sqrt[c]*d^2*e^4 + 6*a^(3/2)*
e^6)*(d + e*x)*Sqrt[1 + (c*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sq
rt[a]]*x], -1] + Sqrt[(I*Sqrt[c])/Sqrt[a]]*(-(e^3*(a + c*x^4)*(10*a*e^4 +
c*(30*d^4 + 10*d^3*e*x - 5*d^2*e^2*x^2 + 3*d*e^3*x^3 - 2*e^4*x^4))) + 60*c
*d^3*e*Sqrt[-(c*d^4) - a*e^4]*(d + e*x)*Sqrt[a + c*x^4]*ArcTan[(Sqrt[c]*(d
^2 - e^2*x^2) + e^2*Sqrt[a + c*x^4])/Sqrt[-(c*d^4) - a*e^4]] + 60*(-1)^(1/
4)*a^(1/4)*c^(3/4)*d^2*(c*d^4 + a*e^4)*(d + e*x)*Sqrt[1 + (c*x^4)/a]*Ellip
ticPi[(I*Sqrt[a]*e^2)/(Sqrt[c]*d^2), ArcSin[((-1)^(3/4)*c^(1/4)*x)/a^(1/4)
], -1] + 15*Sqrt[c]*d*e*(2*c*d^4 + a*e^4)*(d + e*x)*Sqrt[a + c*x^4]*Log[-(
Sqrt[c]*x^2) + Sqrt[a + c*x^4]]))/(10*Sqrt[(I*Sqrt[c])/Sqrt[a]]*e^8*(d + e
*x)*Sqrt[a + c*x^4])
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1816 vs. 2(831) = 1662.

Time = 5.37 (sec) , antiderivative size = 1816, normalized size of antiderivative = 2.19, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.684$, Rules used = {2584, 2255, 27, 1577, 492, 591, 719, 224, 219, 488, 219, 2259, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^4)^{3/2}}{(d+ex)^2}dx$$

$$\Big\downarrow \text{2584}$$

$$\int \frac{(a+cx^4)^{3/2}(d^2-2dex+e^2x^2)}{(d^2-e^2x^2)^2}dx$$

$$\Big\downarrow \text{2255}$$

$$\int -\frac{2dex(cx^4+a)^{3/2}}{(d^2-e^2x^2)^2}dx+\int \frac{(d^2+e^2x^2)(cx^4+a)^{3/2}}{(d^2-e^2x^2)^2}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{(d^2+e^2x^2)(cx^4+a)^{3/2}}{(d^2-e^2x^2)^2}dx-2de\int \frac{x(cx^4+a)^{3/2}}{(d^2-e^2x^2)^2}dx$$

$$\Big\downarrow \; 1577$$

$$\int \frac{\left(d^2+e^2x^2\right)\left(cx^4+a\right)^{3/2}}{\left(d^2-e^2x^2\right)^2}dx-de\int \frac{\left(cx^4+a\right)^{3/2}}{\left(d^2-e^2x^2\right)^2}dx^2$$

$$\Big\downarrow \; 492$$

$$\int \frac{\left(d^2+e^2x^2\right)\left(cx^4+a\right)^{3/2}}{\left(d^2-e^2x^2\right)^2}dx-de\left(\frac{\left(a+cx^4\right)^{3/2}}{e^2\left(d^2-e^2x^2\right)}-\frac{3c\int \frac{x^2\sqrt{cx^4+a}}{d^2-e^2x^2}dx^2}{e^2}\right)$$

$$\Big\downarrow \; 591$$

$$\int \frac{\left(d^2+e^2x^2\right)\left(cx^4+a\right)^{3/2}}{\left(d^2-e^2x^2\right)^2}dx-de\left(\frac{\left(a+cx^4\right)^{3/2}}{e^2\left(d^2-e^2x^2\right)}-\frac{3c\left(\frac{\int \frac{ad^2e^2+\left(2cd^4+ae^4\right)x^2}{\left(d^2-e^2x^2\right)\sqrt{cx^4+a}}dx^2}{2e^4}-\frac{\sqrt{a+cx^4}\left(2d^2+e^2x^2\right)}{2e^4}\right)}{e^2}\right)$$

$$\Big\downarrow \; 719$$

$$\int \frac{\left(d^2+e^2x^2\right)\left(cx^4+a\right)^{3/2}}{\left(d^2-e^2x^2\right)^2}dx-de\left(\frac{\left(a+cx^4\right)^{3/2}}{e^2\left(d^2-e^2x^2\right)}-\frac{3c\left(\frac{\frac{2d^2\left(ae^4+cd^4\right)\int \frac{1}{\left(d^2-e^2x^2\right)\sqrt{cx^4+a}}dx^2}{e^2}-\frac{\left(ae^4+2cd^4\right)\int \frac{1}{\sqrt{cx^4+a}}dx^2}{e^2}}{2e^4}-\frac{\sqrt{a+cx^4}\left(2d^2+e^2x^2\right)}{2e^4}\right)}{e^2}\right)$$

$$\Big\downarrow \; 224$$

$$\int \frac{\left(d^2+e^2x^2\right)\left(cx^4+a\right)^{3/2}}{\left(d^2-e^2x^2\right)^2}dx-de\left(\frac{\left(a+cx^4\right)^{3/2}}{e^2\left(d^2-e^2x^2\right)}-\frac{3c\left(\frac{\frac{2d^2\left(ae^4+cd^4\right)\int \frac{1}{\left(d^2-e^2x^2\right)\sqrt{cx^4+a}}dx^2}{e^2}-\frac{\left(ae^4+2cd^4\right)\int \frac{1}{1-cx^4}d\frac{x^2}{\sqrt{cx^4+a}}}{e^2}}{2e^4}-\frac{\sqrt{a+cx^4}\left(2d^2+e^2x^2\right)}{2e^4}\right)}{e^2}\right)$$

$\Big\downarrow$ 219

$$\int \frac{(d^2+e^2x^2)(cx^4+a)^{3/2}}{(d^2-e^2x^2)^2}dx-de\left(\frac{(a+cx^4)^{3/2}}{e^2(d^2-e^2x^2)}-\frac{3c\left(\frac{\frac{2d^2(ae^4+cd^4)\int \frac{1}{(d^2-e^2x^2)\sqrt{cx^4+a}}dx^2}{e^2}-\frac{\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)(ae^4+2cd^4)}{\sqrt{c}e^2}}{2e^4}-\frac{\sqrt{a+cx^4}(2d^2+e^2x^2)}{2e^4}\right)}{e^2}\right)$$

$\Big\downarrow$ 488

$$\int \frac{(d^2+e^2x^2)(cx^4+a)^{3/2}}{(d^2-e^2x^2)^2}dx-de\left(\frac{(a+cx^4)^{3/2}}{e^2(d^2-e^2x^2)}-\frac{3c\left(\frac{-\frac{2d^2(ae^4+cd^4)\int \frac{1}{cd^4+ae^4-x^4}d\frac{-ae^2-cd^2x^2}{\sqrt{cx^4+a}}}{e^2}-\frac{\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)(ae^4+2cd^4)}{\sqrt{c}e^2}}{2e^4}-\frac{\sqrt{a+cx^4}(2d^2+e^2x^2)}{2e^4}\right)}{e^2}\right)$$

$\Big\downarrow$ 219

$$\int \frac{(d^2+e^2x^2)(cx^4+a)^{3/2}}{(d^2-e^2x^2)^2}dx-de\left(\frac{(a+cx^4)^{3/2}}{e^2(d^2-e^2x^2)}-\frac{3c\left(\frac{-\frac{\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)(ae^4+2cd^4)}{\sqrt{c}e^2}-\frac{2d^2\sqrt{ae^4+cd^4}\,\text{arctanh}\left(\frac{-ae^2-cd^2x^2}{\sqrt{a+cx^4}\sqrt{ae^4+cd^4}}\right)}{e^2}}{2e^4}-\frac{\sqrt{a+cx^4}(2d^2+e^2x^2)}{2e^4}\right)}{e^2}\right)$$

$\Big\downarrow$ 2259

$$\int \left(\frac{c^2x^6}{e^2\sqrt{cx^4+a}} + \frac{3c^2d^2x^4}{e^4\sqrt{cx^4+a}} + \frac{c\left(5cd^4+2ae^4\right)x^2}{e^6\sqrt{cx^4+a}} + \frac{cd^2\left(7cd^4+6ae^4\right)}{e^8\sqrt{cx^4+a}} + \frac{8cd^4\left(cd^4+ae^4\right)}{e^8\left(e^2x^2-d^2\right)\sqrt{cx^4+a}} + \frac{(cd}{2e^8(ex-}\right.$$

$$de\left(\frac{\left(a+cx^4\right)^{3/2}}{e^2\left(d^2-e^2x^2\right)} - \frac{3c\left(\frac{-\frac{\operatorname{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)\left(ae^4+2cd^4\right)}{\sqrt{c}e^2} - \frac{2d^2\sqrt{ae^4+cd^4}\operatorname{arctanh}\left(\frac{-ae^2-cd^2x^2}{\sqrt{a+cx^4}\sqrt{ae^4+cd^4}}\right)}{e^2}}{2e^4} - \frac{\sqrt{a+cx^4}\left(2d^2+e^2x^2\right)}{2e^4} \right)}{e^2} \right.$$

$$\Big\downarrow \; 2009$$

$$-\frac{4c^{5/4}\left(cd^4+ae^4\right)\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{\frac{cx^4+a}{\left(\sqrt{c}x^2+\sqrt{a}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)d^4}{\sqrt[4]{a}e^8\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\sqrt{cx^4+a}}-\frac{3c\sqrt{cd^4+ae^4}\,\text{arctanh}\left(\frac{\sqrt{cd^4+ae^4}x}{de\sqrt{cx^4+a}}\right)d^3}{e^7}+$$

$$\frac{c^{3/4}\left(7cd^4+6ae^4\right)\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{\frac{cx^4+a}{\left(\sqrt{c}x^2+\sqrt{a}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)d^2}{2\sqrt[4]{a}e^8\sqrt{cx^4+a}}-$$

$$\frac{a^{3/4}c^{3/4}\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{\frac{cx^4+a}{\left(\sqrt{c}x^2+\sqrt{a}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)d^2}{2e^4\sqrt{cx^4+a}}+$$

$$\frac{2c^{3/4}\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\left(cd^4+ae^4\right)\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{\frac{cx^4+a}{\left(\sqrt{c}x^2+\sqrt{a}\right)^2}}\,\text{EllipticPi}\left(\frac{\sqrt{a}\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)^2}{4\sqrt{c}d^2e^2},2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)d}{\sqrt[4]{a}e^8\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\sqrt{cx^4+a}}$$

$$\frac{c^{3/4}\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\left(cd^4+ae^4\right)\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{\frac{cx^4+a}{\left(\sqrt{c}x^2+\sqrt{a}\right)^2}}\,\text{EllipticPi}\left(\frac{\left(\sqrt{c}d^2+\sqrt{a}e^2\right)^2}{4\sqrt{a}\sqrt{c}d^2e^2},2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)d^2}{2\sqrt[4]{a}e^8\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\sqrt{cx^4+a}}+\frac{cx\sqrt{cx^4+a}d^2}{e^4}-$$

$$e\left(\frac{\left(cx^4+a\right)^{3/2}}{e^2\left(d^2-e^2x^2\right)}-\frac{3c\left(-\frac{-\frac{2\sqrt{cd^4+ae^4}\,\text{arctanh}\left(\frac{-ae^2-cd^2x^2}{\sqrt{cd^4+ae^4}\sqrt{cx^4+a}}\right)d^2}{e^2}-\frac{\left(2cd^4+ae^4\right)\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{cx^4+a}}\right)}{\sqrt{c}e^2}}{2e^4}-\frac{\left(2d^2+e^2x^2\right)\sqrt{cx^4+a}}{2e^4}\right)}{e^2}\right.$$

$$\frac{\sqrt[4]{a}\sqrt[4]{c}\left(cd^4+ae^4\right)\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{\frac{cx^4+a}{\left(\sqrt{c}x^2+\sqrt{a}\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{e^6\sqrt{cx^4+a}}-$$

$$\frac{\sqrt[4]{a}\sqrt[4]{c}\left(5cd^4+2ae^4\right)\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{\frac{cx^4+a}{\left(\sqrt{c}x^2+\sqrt{a}\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{e^6\sqrt{cx^4+a}}+$$

$$\frac{3a^{5/4}\sqrt[4]{c}\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{\frac{cx^4+a}{\left(\sqrt{c}x^2+\sqrt{a}\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5e^2\sqrt{cx^4+a}}+$$

$$\frac{\sqrt[4]{c}\left(cd^4+ae^4\right)^2\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{\frac{cx^4+a}{\left(\sqrt{c}x^2+\sqrt{a}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}e^8\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\sqrt{cx^4+a}}+$$

$$\frac{\sqrt[4]{a}\sqrt[4]{c}\left(5cd^4+2ae^4\right)\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{\frac{cx^4+a}{\left(\sqrt{c}x^2+\sqrt{a}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2e^6\sqrt{cx^4+a}}-$$

$$\frac{3a^{5/4}\sqrt[4]{c}\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{\frac{cx^4+a}{\left(\sqrt{c}x^2+\sqrt{a}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{10e^2\sqrt{cx^4+a}}+\frac{cx^3\sqrt{cx^4+a}}{5e^2}+$$

$$\frac{\left(cd^4+ae^4\right)\sqrt{cx^4+a}}{2e^5(d-ex)}-\frac{\left(cd^4+ae^4\right)\sqrt{cx^4+a}}{2e^5(d+ex)}+\frac{\sqrt{c}\left(cd^4+ae^4\right)x\sqrt{cx^4+a}}{e^6\left(\sqrt{c}x^2+\sqrt{a}\right)}+$$

input
```
Int[(a + c*x^4)^(3/2)/(d + e*x)^2,x]
```

output
```
(c*d^2*x*Sqrt[a + c*x^4])/e^4 + (c*x^3*Sqrt[a + c*x^4])/(5*e^2) + ((c*d^4
+ a*e^4)*Sqrt[a + c*x^4])/(2*e^5*(d - e*x)) - ((c*d^4 + a*e^4)*Sqrt[a + c*
x^4])/(2*e^5*(d + e*x)) - (3*a*Sqrt[c]*x*Sqrt[a + c*x^4])/(5*e^2*(Sqrt[a]
+ Sqrt[c]*x^2)) + (Sqrt[c]*(c*d^4 + a*e^4)*x*Sqrt[a + c*x^4])/(e^6*(Sqrt[a
] + Sqrt[c]*x^2)) + (Sqrt[c]*(5*c*d^4 + 2*a*e^4)*x*Sqrt[a + c*x^4])/(e^6*(
Sqrt[a] + Sqrt[c]*x^2)) - (3*c*d^3*Sqrt[c*d^4 + a*e^4]*ArcTanh[(Sqrt[c*d^4
 + a*e^4]*x)/(d*e*Sqrt[a + c*x^4])])/e^7 - d*e*((a + c*x^4)^(3/2)/(e^2*(d^
2 - e^2*x^2)) - (3*c*(-1/2*((2*d^2 + e^2*x^2)*Sqrt[a + c*x^4])/e^4 + (-(((
2*c*d^4 + a*e^4)*ArcTanh[(Sqrt[c]*x^2)/Sqrt[a + c*x^4]])/(Sqrt[c]*e^2)) -
(2*d^2*Sqrt[c*d^4 + a*e^4]*ArcTanh[(-(a*e^2) - c*d^2*x^2)/(Sqrt[c*d^4 + a*
e^4]*Sqrt[a + c*x^4])])/e^2)/(2*e^4)))/e^2) + (3*a^(5/4)*c^(1/4)*(Sqrt[a]
+ Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*Arc
Tan[(c^(1/4)*x)/a^(1/4)], 1/2])/(5*e^2*Sqrt[a + c*x^4]) - (a^(1/4)*c^(1/4)
*(c*d^4 + a*e^4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[
c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(e^6*Sqrt[a + c*
x^4]) - (a^(1/4)*c^(1/4)*(5*c*d^4 + 2*a*e^4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[
(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1
/4)], 1/2])/(e^6*Sqrt[a + c*x^4]) - (a^(3/4)*c^(3/4)*d^2*(Sqrt[a] + Sqrt[c
]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(
1/4)*x)/a^(1/4)], 1/2])/(2*e^4*Sqrt[a + c*x^4]) - (3*a^(5/4)*c^(1/4)*(S...
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 492
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*((a + b*x^2)^p/(d*(n + 1))), x] - Simp[2*b*(p/(d*(n + 1))
)   Int[x*(c + d*x)^(n + 1)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c,
d, n}, x] && GtQ[p, 0] && (IntegerQ[p] || LtQ[n, -1]) && NeQ[n, -1] &&  !IL
tQ[n + 2*p + 1, 0] && IntQuadraticQ[a, 0, b, c, d, n, p, x]
```

rule 591
```
Int[(x_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-(c + d*x)^(n + 1))*(a + b*x^2)^p*((c*(2*p + 1) - d*(n + 2*p + 1)*x)/
(d^2*(n + 2*p + 1)*(n + 2*p + 2))), x] + Simp[2*(p/(d^2*(n + 2*p + 1)*(n +
2*p + 2)))   Int[(c + d*x)^n*(a + b*x^2)^(p - 1)*Simp[a*c*d*n + (b*c^2*(2*p
 + 1) + a*d^2*(n + 2*p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d, n}, x] &&
GtQ[p, 0] && LeQ[-1, n, 0] &&  !ILtQ[n + 2*p, 0]
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1577
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, c, d, e, p, q}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2255
```
Int[(Pr_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Module[{r = Expon[Pr, x], k}, Int[Sum[Coeff[Pr, x, 2*k]*x^(2*k), {k, 0,
 r/2}]*(d + e*x^2)^q*(a + c*x^4)^p, x] + Int[x*Sum[Coeff[Pr, x, 2*k + 1]*x^
(2*k), {k, 0, (r - 1)/2}]*(d + e*x^2)^q*(a + c*x^4)^p, x]] /; FreeQ[{a, c,
d, e, p, q}, x] && PolyQ[Pr, x] &&  !PolyQ[Pr, x^2]
```

rule 2259
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Int[ExpandIntegrand[1/Sqrt[a + c*x^4], Px*(d + e*x^2)^q*(a + c*x^4)^(p
+ 1/2), x], x] /; FreeQ[{a, c, d, e}, x] && PolyQ[Px, x] && IntegerQ[p + 1/
2] && IntegerQ[q]
```

rule 2584
```
Int[((c_) + (d_.)*(x_)^(n_.))^(q_)*((a_) + (b_.)*(x_)^(nn_.))^(p_), x_Symbo
l] :> Int[ExpandToSum[(c - d*x^n)^(-q), x]*((a + b*x^nn)^p/(c^2 - d^2*x^(2*
n))^(-q)), x] /; FreeQ[{a, b, c, d, n, nn, p}, x] &&  !IntegerQ[p] && ILtQ[
q, 0] && IGtQ[Log[2, nn/n], 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.48 (sec) , antiderivative size = 604, normalized size of antiderivative = 0.73

method	result
default	$-\frac{(e^4a+c\,d^4)\sqrt{c\,x^4+a}}{e^5(ex+d)}+\frac{c\,x^3\sqrt{c\,x^4+a}}{5e^2}-\frac{cd\,x^2\sqrt{c\,x^4+a}}{2e^3}+\frac{c\,d^2x\sqrt{c\,x^4+a}}{e^4}-\frac{2c\,d^3\sqrt{c\,x^4+a}}{e^5}+\frac{\left(\frac{c\,d^2\left(6e^4a+7c\,d^4\right)}{e^8}-\frac{d^2c\left(e^4a\right.}{e^8}\right.}{}$
elliptic	$-\frac{(e^4a+c\,d^4)\sqrt{c\,x^4+a}}{e^5(ex+d)}+\frac{c\,x^3\sqrt{c\,x^4+a}}{5e^2}-\frac{cd\,x^2\sqrt{c\,x^4+a}}{2e^3}+\frac{c\,d^2x\sqrt{c\,x^4+a}}{e^4}-\frac{2c\,d^3\sqrt{c\,x^4+a}}{e^5}+\frac{\left(\frac{c\,d^2\left(6e^4a+7c\,d^4\right)}{e^8}-\frac{d^2c\left(e^4a\right.}{e^8}\right.}{}$
risch	Expression too large to display

input
```
int((c*x^4+a)^(3/2)/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
-(a*e^4+c*d^4)/e^5*(c*x^4+a)^(1/2)/(e*x+d)+1/5*c*x^3*(c*x^4+a)^(1/2)/e^2-1
/2*c*d*x^2*(c*x^4+a)^(1/2)/e^3+c*d^2*x*(c*x^4+a)^(1/2)/e^4-2*c*d^3*(c*x^4+
a)^(1/2)/e^5+(c*d^2*(6*a*e^4+7*c*d^4)/e^8-d^2*c*(a*e^4+c*d^4)/e^8-c*d^2/e^
4*a)/(I*c^(1/2)/a^(1/2))^(1/2)*(1-I*c^(1/2)*x^2/a^(1/2))^(1/2)*(1+I*c^(1/2
)*x^2/a^(1/2))^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I*c^(1/2)/a^(1/2))^(1/2)
,I)+1/2*(-2*c*d/e^7*(2*a*e^4+3*c*d^4)+c*d/e^3*a)*ln(2*c^(1/2)*x^2+2*(c*x^4
+a)^(1/2))/c^(1/2)+I*(c/e^6*(2*a*e^4+5*c*d^4)+c*(a*e^4+c*d^4)/e^6-3/5*c/e^
2*a)*a^(1/2)/(I*c^(1/2)/a^(1/2))^(1/2)*(1-I*c^(1/2)*x^2/a^(1/2))^(1/2)*(1+
I*c^(1/2)*x^2/a^(1/2))^(1/2)/(c*x^4+a)^(1/2)/c^(1/2)*(EllipticF(x*(I*c^(1/
2)/a^(1/2))^(1/2),I)-EllipticE(x*(I*c^(1/2)/a^(1/2))^(1/2),I))-6*c*d^3/e^9
*(a*e^4+c*d^4)*(-1/2/(a+c*d^4/e^4)^(1/2)*arctanh(1/2*(2*c*x^2*d^2/e^2+2*a)
/(a+c*d^4/e^4)^(1/2)/(c*x^4+a)^(1/2))+1/(I*c^(1/2)/a^(1/2))^(1/2)/d*e*(1-I
*c^(1/2)*x^2/a^(1/2))^(1/2)*(1+I*c^(1/2)*x^2/a^(1/2))^(1/2)/(c*x^4+a)^(1/2
)*EllipticPi(x*(I*c^(1/2)/a^(1/2))^(1/2),-I/c^(1/2)*a^(1/2)/d^2*e^2,(-I/a^
(1/2)*c^(1/2))^(1/2)/(I*c^(1/2)/a^(1/2))^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+cx^4)^{3/2}}{(d+ex)^2}\,dx = \text{Timed out}$$

input
```
integrate((c*x^4+a)^(3/2)/(e*x+d)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(a+cx^4)^{3/2}}{(d+ex)^2}\,dx = \int \frac{(a+cx^4)^{\frac{3}{2}}}{(d+ex)^2}\,dx$$

input
```
integrate((c*x**4+a)**(3/2)/(e*x+d)**2,x)
```

output
```
Integral((a + c*x**4)**(3/2)/(d + e*x)**2, x)
```

Maxima [F]

$$\int \frac{(a+cx^4)^{3/2}}{(d+ex)^2}\,dx = \int \frac{(cx^4+a)^{\frac{3}{2}}}{(ex+d)^2}\,dx$$

input
```
integrate((c*x^4+a)^(3/2)/(e*x+d)^2,x, algorithm="maxima")
```

output
```
integrate((c*x^4 + a)^(3/2)/(e*x + d)^2, x)
```

Giac [F]

$$\int \frac{(a+cx^4)^{3/2}}{(d+ex)^2}\,dx = \int \frac{(cx^4+a)^{\frac{3}{2}}}{(ex+d)^2}\,dx$$

input
```
integrate((c*x^4+a)^(3/2)/(e*x+d)^2,x, algorithm="giac")
```

output
```
integrate((c*x^4 + a)^(3/2)/(e*x + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+cx^4)^{3/2}}{(d+ex)^2}\,dx = \int \frac{(c\,x^4+a)^{3/2}}{(d+e\,x)^2}\,dx$$

input
```
int((a + c*x^4)^(3/2)/(d + e*x)^2,x)
```

output
```
int((a + c*x^4)^(3/2)/(d + e*x)^2, x)
```

Reduce [F]

$$\int \frac{(a+cx^4)^{3/2}}{(d+ex)^2}\,dx = \int \frac{(c\,x^4+a)^{\frac{3}{2}}}{(ex+d)^2}dx$$

input
```
int((c*x^4+a)^(3/2)/(e*x+d)^2,x)
```

output
```
int((c*x^4+a)^(3/2)/(e*x+d)^2,x)
```

3.204 $\int \frac{(d+ex)^3}{\sqrt{a+cx^4}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 295

$$\int \frac{(d+ex)^3}{\sqrt{a+cx^4}}\,dx = \frac{e^3\sqrt{a+cx^4}}{2c}+\frac{3de^2x\sqrt{a+cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)}+\frac{3d^2e\operatorname{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{2\sqrt{c}} - \frac{3\sqrt[4]{a}de^2\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{c^{3/4}\sqrt{a+cx^4}} + \frac{d\left(\sqrt{c}d^2+3\sqrt{a}e^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}c^{3/4}\sqrt{a+cx^4}}$$

output

```
1/2*e^3*(c*x^4+a)^(1/2)/c+3*d*e^2*x*(c*x^4+a)^(1/2)/c^(1/2)/(a^(1/2)+c^(1/
2)*x^2)+3/2*d^2*e*arctanh(c^(1/2)*x^2/(c*x^4+a)^(1/2))/c^(1/2)-3*a^(1/4)*d
*e^2*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*Ellip
ticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))/c^(3/4)/(c*x^4+a)^(1/2)
+1/2*d*(c^(1/2)*d^2+3*a^(1/2)*e^2)*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+a)/(a^(1/
2)+c^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(c^(1/4)*x/a^(1/4)),1/2*2
^(1/2))/a^(1/4)/c^(3/4)/(c*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.18 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.53

$$\int \frac{(d+ex)^3}{\sqrt{a+cx^4}}\,dx = \frac{e^3\sqrt{a+cx^4}}{2c}+\frac{3d^2e\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{2\sqrt{c}} + \frac{d^3x\sqrt{1+\frac{cx^4}{a}}\ \text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{cx^4}{a}\right)}{\sqrt{a+cx^4}} + \frac{de^2x^3\sqrt{1+\frac{cx^4}{a}}\ \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-\frac{cx^4}{a}\right)}{\sqrt{a+cx^4}}$$

input
```
Integrate[(d + e*x)^3/Sqrt[a + c*x^4],x]
```

output
```
(e^3*Sqrt[a + c*x^4])/(2*c) + (3*d^2*e*ArcTanh[(Sqrt[c]*x^2)/Sqrt[a + c*x^
4]])/(2*Sqrt[c]) + (d^3*x*Sqrt[1 + (c*x^4)/a]*Hypergeometric2F1[1/4, 1/2,
5/4, -((c*x^4)/a)])/Sqrt[a + c*x^4] + (d*e^2*x^3*Sqrt[1 + (c*x^4)/a]*Hyper
geometric2F1[1/2, 3/4, 7/4, -((c*x^4)/a)])/Sqrt[a + c*x^4]
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2424, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^3}{\sqrt{a+cx^4}}dx$$

$$\Big\downarrow \ 2424$$

$$\int \left(\frac{d^3+3de^2x^2}{\sqrt{a+cx^4}}+\frac{x(3d^2e+e^3x^2)}{\sqrt{a+cx^4}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{d(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}(3\sqrt{a}e^2+\sqrt{c}d^2)\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}c^{3/4}\sqrt{a+cx^4}}-\frac{3\sqrt[4]{a}de^2(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{c^{3/4}\sqrt{a+cx^4}}+\frac{3d^2e\,\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{2\sqrt{c}}+\frac{3de^2x\sqrt{a+cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)}+\frac{e^3\sqrt{a+cx^4}}{2c}$$

input
```
Int[(d + e*x)^3/Sqrt[a + c*x^4],x]
```

output
```
(e^3*Sqrt[a + c*x^4])/(2*c) + (3*d*e^2*x*Sqrt[a + c*x^4])/(Sqrt[c]*(Sqrt[a
] + Sqrt[c]*x^2)) + (3*d^2*e*ArcTanh[(Sqrt[c]*x^2)/Sqrt[a + c*x^4]])/(2*Sq
rt[c]) - (3*a^(1/4)*d*e^2*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a
] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(c^(3/4
)*Sqrt[a + c*x^4]) + (d*(Sqrt[c]*d^2 + 3*Sqrt[a]*e^2)*(Sqrt[a] + Sqrt[c]*x
^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4
)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*c^(3/4)*Sqrt[a + c*x^4])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2424
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && !PolyQ[Pq, x^(n/2)]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.18 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.74

method	result
default	$\frac{d^{3}\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}}+\frac{e^{3}\sqrt{c\,x^{4}+a}}{2c}+\frac{3id\,e^{2}\sqrt{a}\,\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\left(\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)-\right.}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}\,\sqrt{c}}$
risch	$\frac{e^{3}\sqrt{c\,x^{4}+a}}{2c}+d\left(\frac{d^{2}\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}}+\frac{3ie^{2}\sqrt{a}\,\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\left(\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)\right.}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}\,\sqrt{c}}\right.$
elliptic	$\frac{e^{3}\sqrt{c\,x^{4}+a}}{2c}+\frac{d^{3}\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}}+\frac{3d^{2}e\ln\left(2\sqrt{c}\,x^{2}+2\sqrt{c\,x^{4}+a}\right)}{2\sqrt{c}}+\frac{3id\,e^{2}\sqrt{a}\,\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+}}{}$

input
```
int((e*x+d)^3/(c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
d^3/(I*c^(1/2)/a^(1/2))^(1/2)*(1-I*c^(1/2)*x^2/a^(1/2))^(1/2)*(1+I*c^(1/2)
*x^2/a^(1/2))^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I*c^(1/2)/a^(1/2))^(1/2),
I)+1/2*e^3*(c*x^4+a)^(1/2)/c+3*I*d*e^2*a^(1/2)/(I*c^(1/2)/a^(1/2))^(1/2)*(
1-I*c^(1/2)*x^2/a^(1/2))^(1/2)*(1+I*c^(1/2)*x^2/a^(1/2))^(1/2)/(c*x^4+a)^(
1/2)/c^(1/2)*(EllipticF(x*(I*c^(1/2)/a^(1/2))^(1/2),I)-EllipticE(x*(I*c^(1
/2)/a^(1/2))^(1/2),I))+3/2*d^2*e*ln(c^(1/2)*x^2+(c*x^4+a)^(1/2))/c^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.52

$$\int \frac{(d+ex)^3}{\sqrt{a+cx^4}}\,dx$$
$$=\frac{12\,a\sqrt{c}de^{2}x\left(-\frac{a}{c}\right)^{\frac{3}{4}}E(\arcsin\left(\frac{\left(-\frac{a}{c}\right)^{\frac{1}{4}}}{x}\right)\,|-1)+3\,a\sqrt{c}d^{2}ex\log\left(-2\,cx^{4}-2\,\sqrt{cx^{4}+a}\sqrt{c}x^{2}-a\right)+4\,(cd^{3}-}{4\,acx}$$

input
```
integrate((e*x+d)^3/(c*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
1/4*(12*a*sqrt(c)*d*e^2*x*(-a/c)^(3/4)*elliptic_e(arcsin((-a/c)^(1/4)/x),
-1) + 3*a*sqrt(c)*d^2*e*x*log(-2*c*x^4 - 2*sqrt(c*x^4 + a)*sqrt(c)*x^2 - a
) + 4*(c*d^3 - 3*a*d*e^2)*sqrt(c)*x*(-a/c)^(3/4)*elliptic_f(arcsin((-a/c)^
(1/4)/x), -1) + 2*(a*e^3*x + 6*a*d*e^2)*sqrt(c*x^4 + a))/(a*c*x)
```

Sympy [A] (verification not implemented)

Time = 2.07 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.48

$$\int \frac{(d+ex)^3}{\sqrt{a+cx^4}}\,dx = e^3\left(\begin{cases}\frac{x^4}{4\sqrt{a}} & \text{for } c=0\\ \frac{\sqrt{a+cx^4}}{2c} & \text{otherwise}\end{cases}\right) + \frac{3d^2e\operatorname{asinh}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{c}} + \frac{d^3x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)} + \frac{3de^2x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate((e*x+d)**3/(c*x**4+a)**(1/2),x)
```

output
```
e**3*Piecewise((x**4/(4*sqrt(a)), Eq(c, 0)), (sqrt(a + c*x**4)/(2*c), True
)) + 3*d**2*e*asinh(sqrt(c)*x**2/sqrt(a))/(2*sqrt(c)) + d**3*x*gamma(1/4)*
hyper((1/4, 1/2), (5/4,), c*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(5/4))
 + 3*d*e**2*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), c*x**4*exp_polar(I*p
i)/a)/(4*sqrt(a)*gamma(7/4))
```

Maxima [F]

$$\int \frac{(d+ex)^3}{\sqrt{a+cx^4}}\,dx = \int \frac{(ex+d)^3}{\sqrt{cx^4+a}}\,dx$$

input
```
integrate((e*x+d)^3/(c*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((e*x + d)^3/sqrt(c*x^4 + a), x)
```

Giac [F]

$$\int \frac{(d+ex)^3}{\sqrt{a+cx^4}}\,dx = \int \frac{(ex+d)^3}{\sqrt{cx^4+a}}\,dx$$

input
```
integrate((e*x+d)^3/(c*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((e*x + d)^3/sqrt(c*x^4 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^3}{\sqrt{a+cx^4}}\,dx = \int \frac{(d+e\,x)^3}{\sqrt{c\,x^4+a}}\,dx$$

input
```
int((d + e*x)^3/(a + c*x^4)^(1/2),x)
```

output
```
int((d + e*x)^3/(a + c*x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{(d+ex)^3}{\sqrt{a+cx^4}}\,dx$$
$$= \frac{-3\sqrt{c}\,\sqrt{c\,x^4+a}\log\left(\sqrt{c\,x^4+a}-\sqrt{c}\,x^2\right)d^2e+3\sqrt{c}\,\sqrt{c\,x^4+a}\log\left(\sqrt{c\,x^4+a}+\sqrt{c}\,x^2\right)d^2e+2\sqrt{c}\,\sqrt{c\,x^4}}{}$$

input
```
int((e*x+d)^3/(c*x^4+a)^(1/2),x)
```

output

```
( - 3*sqrt(c)*sqrt(a + c*x**4)*log(sqrt(a + c*x**4) - sqrt(c)*x**2)*d**2*e
 + 3*sqrt(c)*sqrt(a + c*x**4)*log(sqrt(a + c*x**4) + sqrt(c)*x**2)*d**2*e
+ 2*sqrt(c)*sqrt(a + c*x**4)*e**3*x**2 + 4*sqrt(a + c*x**4)*int(sqrt(a + c
*x**4)/(a + c*x**4),x)*c*d**3 + 12*sqrt(a + c*x**4)*int((sqrt(a + c*x**4)*
x**2)/(a + c*x**4),x)*c*d*e**2 + 4*sqrt(c)*int(sqrt(a + c*x**4)/(a + c*x**
4),x)*c*d**3*x**2 + 12*sqrt(c)*int((sqrt(a + c*x**4)*x**2)/(a + c*x**4),x)
*c*d*e**2*x**2 - 3*log(sqrt(a + c*x**4) - sqrt(c)*x**2)*c*d**2*e*x**2 + 3*
log(sqrt(a + c*x**4) + sqrt(c)*x**2)*c*d**2*e*x**2 + 2*a*e**3 + 2*c*e**3*x
**4)/(4*c*(sqrt(a + c*x**4) + sqrt(c)*x**2))
```

3.205 $\int \frac{(d+ex)^2}{\sqrt{a+cx^4}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 264

$$\int \frac{(d+ex)^2}{\sqrt{a+cx^4}}\,dx = \frac{e^2x\sqrt{a+cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)}+\frac{de\,\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{\sqrt{c}} - \frac{\sqrt[4]{a}e^2\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{c^{3/4}\sqrt{a+cx^4}} + \frac{\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}c^{3/4}\sqrt{a+cx^4}}$$

output
```
e^2*x*(c*x^4+a)^(1/2)/c^(1/2)/(a^(1/2)+c^(1/2)*x^2)+d*e*arctanh(c^(1/2)*x^
2/(c*x^4+a)^(1/2))/c^(1/2)-a^(1/4)*e^2*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+a)/(a
^(1/2)+c^(1/2)*x^2)^2)^(1/2)*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/
2*2^(1/2))/c^(3/4)/(c*x^4+a)^(1/2)+1/2*(c^(1/2)*d^2+a^(1/2)*e^2)*(a^(1/2)+
c^(1/2)*x^2)*((c*x^4+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*a
rctan(c^(1/4)*x/a^(1/4)),1/2*2^(1/2))/a^(1/4)/c^(3/4)/(c*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.18 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.50

$$\int \frac{(d+ex)^2}{\sqrt{a+cx^4}}\,dx = \frac{d e\operatorname{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{\sqrt{c}} + \frac{d^2x\sqrt{1+\frac{cx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{cx^4}{a}\right)}{\sqrt{a+cx^4}} + \frac{e^2x^3\sqrt{1+\frac{cx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-\frac{cx^4}{a}\right)}{3\sqrt{a+cx^4}}$$

input
```
Integrate[(d + e*x)^2/Sqrt[a + c*x^4],x]
```

output
```
(d*e*ArcTanh[(Sqrt[c]*x^2)/Sqrt[a + c*x^4]])/Sqrt[c] + (d^2*x*Sqrt[1 + (c*
x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((c*x^4)/a)])/Sqrt[a + c*x^4] +
(e^2*x^3*Sqrt[1 + (c*x^4)/a]*Hypergeometric2F1[1/2, 3/4, 7/4, -((c*x^4)/a)
])/(3*Sqrt[a + c*x^4])
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2424, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^2}{\sqrt{a+cx^4}}\,dx$$

$$\Big\downarrow \text{ 2424}$$

$$\int \left(\frac{d^2+e^2x^2}{\sqrt{a+cx^4}}+\frac{2dex}{\sqrt{a+cx^4}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2c^{3/4}\sqrt{a+cx^4}}-\frac{\sqrt[4]{a}e^2\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{c^{3/4}\sqrt{a+cx^4}}+\frac{de\,\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{\sqrt{c}}+\frac{e^2x\sqrt{a+cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)}$$

input
```
Int[(d + e*x)^2/Sqrt[a + c*x^4],x]
```

output
```
(e^2*x*Sqrt[a + c*x^4])/(Sqrt[c]*(Sqrt[a] + Sqrt[c]*x^2)) + (d*e*ArcTanh[(
Sqrt[c]*x^2)/Sqrt[a + c*x^4]])/Sqrt[c] - (a^(1/4)*e^2*(Sqrt[a] + Sqrt[c]*x
^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4
)*x)/a^(1/4)], 1/2])/(c^(3/4)*Sqrt[a + c*x^4]) + (a^(1/4)*((Sqrt[c]*d^2)/S
qrt[a] + e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*
x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*c^(3/4)*Sqrt[a +
 c*x^4])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2424
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.74 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.75

method	result
default	$\frac{d^2\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}+\frac{ie^2\sqrt{a}\,\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\left(\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)-\operatorname{EllipticE}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},\right.\right.}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}\,\sqrt{c}}$
elliptic	$\frac{d^2\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}+\frac{de\ln\left(2\sqrt{c}\,x^2+2\sqrt{c\,x^4+a}\right)}{\sqrt{c}}+\frac{ie^2\sqrt{a}\,\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\left(\operatorname{EllipticF}\left(x\right.\right.}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+}}$

input
```
int((e*x+d)^2/(c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
d^2/(I*c^(1/2)/a^(1/2))^(1/2)*(1-I*c^(1/2)*x^2/a^(1/2))^(1/2)*(1+I*c^(1/2)
*x^2/a^(1/2))^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I*c^(1/2)/a^(1/2))^(1/2),
I)+I*e^2*a^(1/2)/(I*c^(1/2)/a^(1/2))^(1/2)*(1-I*c^(1/2)*x^2/a^(1/2))^(1/2)
*(1+I*c^(1/2)*x^2/a^(1/2))^(1/2)/(c*x^4+a)^(1/2)/c^(1/2)*(EllipticF(x*(I*c
^(1/2)/a^(1/2))^(1/2),I)-EllipticE(x*(I*c^(1/2)/a^(1/2))^(1/2),I))+d*e*ln(
c^(1/2)*x^2+(c*x^4+a)^(1/2))/c^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.52

$$\int \frac{(d+ex)^2}{\sqrt{a+cx^4}}\,dx$$
$$=\frac{2\,a\sqrt{c}e^2x\left(-\frac{a}{c}\right)^{\frac{3}{4}}E(\arcsin\left(\frac{\left(-\frac{a}{c}\right)^{\frac{1}{4}}}{x}\right)\,|\,-1)+a\sqrt{c}dex\log\left(-2\,cx^4-2\,\sqrt{cx^4+a}\sqrt{c}x^2-a\right)+2\,(cd^2-ae^2)}{2\,acx}$$

input
```
integrate((e*x+d)^2/(c*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
1/2*(2*a*sqrt(c)*e^2*x*(-a/c)^(3/4)*elliptic_e(arcsin((-a/c)^(1/4)/x), -1)
 + a*sqrt(c)*d*e*x*log(-2*c*x^4 - 2*sqrt(c*x^4 + a)*sqrt(c)*x^2 - a) + 2*(
c*d^2 - a*e^2)*sqrt(c)*x*(-a/c)^(3/4)*elliptic_f(arcsin((-a/c)^(1/4)/x), -
1) + 2*sqrt(c*x^4 + a)*a*e^2)/(a*c*x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.99 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.40

$$\int \frac{(d+ex)^2}{\sqrt{a+cx^4}}\,dx = \frac{de\,\mathrm{asinh}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{\sqrt{c}} + \frac{d^2x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)} + \frac{e^2x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate((e*x+d)**2/(c*x**4+a)**(1/2),x)
```

output
```
d*e*asinh(sqrt(c)*x**2/sqrt(a))/sqrt(c) + d**2*x*gamma(1/4)*hyper((1/4, 1/
2), (5/4,), c*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(5/4)) + e**2*x**3*g
amma(3/4)*hyper((1/2, 3/4), (7/4,), c*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*g
amma(7/4))
```

Maxima [F]

$$\int \frac{(d+ex)^2}{\sqrt{a+cx^4}}\,dx = \int \frac{(ex+d)^2}{\sqrt{cx^4+a}}\,dx$$

input
```
integrate((e*x+d)^2/(c*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((e*x + d)^2/sqrt(c*x^4 + a), x)
```

Giac [F]

$$\int \frac{(d+ex)^2}{\sqrt{a+cx^4}}\,dx = \int \frac{(ex+d)^2}{\sqrt{cx^4+a}}\,dx$$

input
```
integrate((e*x+d)^2/(c*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((e*x + d)^2/sqrt(c*x^4 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^2}{\sqrt{a+cx^4}}\,dx = \int \frac{(d+e\,x)^2}{\sqrt{c\,x^4+a}}\,dx$$

input
```
int((d + e*x)^2/(a + c*x^4)^(1/2),x)
```

output
```
int((d + e*x)^2/(a + c*x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{(d+ex)^2}{\sqrt{a+cx^4}}\,dx = \frac{-\sqrt{c}\log\left(\sqrt{c\,x^4+a}-\sqrt{c}\,x^2\right)de+\sqrt{c}\log\left(\sqrt{c\,x^4+a}+\sqrt{c}\,x^2\right)de+2\left(\int\frac{\sqrt{c\,x^4+a}}{c\,x^4+a}dx\right)c\,d^2+2\left(\int\frac{\sqrt{c\,x^4+a}\,x^2}{c\,x^4+a}\right.}{2c}$$

input
```
int((e*x+d)^2/(c*x^4+a)^(1/2),x)
```

output
```
( - sqrt(c)*log(sqrt(a + c*x**4) - sqrt(c)*x**2)*d*e + sqrt(c)*log(sqrt(a
+ c*x**4) + sqrt(c)*x**2)*d*e + 2*int(sqrt(a + c*x**4)/(a + c*x**4),x)*c*d
**2 + 2*int((sqrt(a + c*x**4)*x**2)/(a + c*x**4),x)*c*e**2)/(2*c)
```

3.206 $\int \frac{d+ex}{\sqrt{a+cx^4}} dx$

Optimal result

Integrand size = 17, antiderivative size = 121

$$\int \frac{d+ex}{\sqrt{a+cx^4}} dx = \frac{e\operatorname{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{2\sqrt{c}} + \frac{d(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+cx^4}}$$

output
```
1/2*e*arctanh(c^(1/2)*x^2/(c*x^4+a)^(1/2))/c^(1/2)+1/2*d*(a^(1/2)+c^(1/2)*
x^2)*((c*x^4+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(c^
(1/4)*x/a^(1/4)),1/2*2^(1/2))/a^(1/4)/c^(1/4)/(c*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.65

$$\int \frac{d+ex}{\sqrt{a+cx^4}} dx = \frac{e\operatorname{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{2\sqrt{c}} + \frac{dx\sqrt{1+\frac{cx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{cx^4}{a}\right)}{\sqrt{a+cx^4}}$$

input
```
Integrate[(d + e*x)/Sqrt[a + c*x^4],x]
```

output
```
(e*ArcTanh[(Sqrt[c]*x^2)/Sqrt[a + c*x^4]])/(2*Sqrt[c]) + (d*x*Sqrt[1 + (c*
x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((c*x^4)/a)])/Sqrt[a + c*x^4]
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2424, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex}{\sqrt{a+cx^4}}\,dx$$

$$\Big\downarrow \text{2424}$$

$$\int \left(\frac{d}{\sqrt{a+cx^4}}+\frac{ex}{\sqrt{a+cx^4}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{d\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+cx^4}}+\frac{e\,\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{2\sqrt{c}}$$

input
```
Int[(d + e*x)/Sqrt[a + c*x^4],x]
```

output
```
(e*ArcTanh[(Sqrt[c]*x^2)/Sqrt[a + c*x^4]])/(2*Sqrt[c]) + (d*(Sqrt[a] + Sqr
t[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(
c^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*c^(1/4)*Sqrt[a + c*x^4])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2424
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.48 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.79

method	result	size
default	$\frac{d\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\;\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}+\frac{e\ln\left(\sqrt{c}\,x^2+\sqrt{c\,x^4+a}\right)}{2\sqrt{c}}$	96
elliptic	$\frac{d\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\;\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}+\frac{e\ln\left(2\sqrt{c}\,x^2+2\sqrt{c\,x^4+a}\right)}{2\sqrt{c}}$	99

input
```
int((e*x+d)/(c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
d/(I*c^(1/2)/a^(1/2))^(1/2)*(1-I*c^(1/2)*x^2/a^(1/2))^(1/2)*(1+I*c^(1/2)*x
^2/a^(1/2))^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I*c^(1/2)/a^(1/2))^(1/2),I)
+1/2*e*ln(c^(1/2)*x^2+(c*x^4+a)^(1/2))/c^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.60

$$\int \frac{d+ex}{\sqrt{a+cx^4}}\,dx = \frac{4\,c^{\frac{3}{2}}d\left(-\frac{a}{c}\right)^{\frac{3}{4}}F\left(\arcsin\left(\frac{\left(-\frac{a}{c}\right)^{\frac{1}{4}}}{x}\right)\middle|\,-1\right)+a\sqrt{c}e\log\left(-2\,cx^4-2\,\sqrt{cx^4+a}\sqrt{c}x^2-a\right)}{4\,ac}$$

input
```
integrate((e*x+d)/(c*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
1/4*(4*c^(3/2)*d*(-a/c)^(3/4)*elliptic_f(arcsin((-a/c)^(1/4)/x), -1) + a*s
qrt(c)*e*log(-2*c*x^4 - 2*sqrt(c*x^4 + a)*sqrt(c)*x^2 - a))/(a*c)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.29 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.50

$$\int \frac{d+ex}{\sqrt{a+cx^4}}\,dx = \frac{e\,\mathrm{asinh}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{c}} + \frac{dx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\,\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((e*x+d)/(c*x**4+a)**(1/2),x)
```

output
```
e*asinh(sqrt(c)*x**2/sqrt(a))/(2*sqrt(c)) + d*x*gamma(1/4)*hyper((1/4, 1/2
), (5/4,), c*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(5/4))
```

Maxima [F]

$$\int \frac{d+ex}{\sqrt{a+cx^4}}\,dx = \int \frac{ex+d}{\sqrt{cx^4+a}}\,dx$$

input
```
integrate((e*x+d)/(c*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((e*x + d)/sqrt(c*x^4 + a), x)
```

Giac [F]

$$\int \frac{d+ex}{\sqrt{a+cx^4}}\,dx = \int \frac{ex+d}{\sqrt{cx^4+a}}\,dx$$

input
```
integrate((e*x+d)/(c*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((e*x + d)/sqrt(c*x^4 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex}{\sqrt{a+cx^4}}\,dx = \int \frac{d+e\,x}{\sqrt{c\,x^4+a}}\,dx$$

input
```
int((d + e*x)/(a + c*x^4)^(1/2),x)
```

output
```
int((d + e*x)/(a + c*x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{d+ex}{\sqrt{a+cx^4}}\,dx$$
$$= \frac{-\sqrt{c}\log\left(\sqrt{c\,x^4+a}-\sqrt{c}\,x^2\right)e+\sqrt{c}\log\left(\sqrt{c\,x^4+a}+\sqrt{c}\,x^2\right)e+4\left(\int \frac{\sqrt{c\,x^4+a}}{c\,x^4+a}dx\right)cd}{4c}$$

input
```
int((e*x+d)/(c*x^4+a)^(1/2),x)
```

output
```
( - sqrt(c)*log(sqrt(a + c*x**4) - sqrt(c)*x**2)*e + sqrt(c)*log(sqrt(a +
c*x**4) + sqrt(c)*x**2)*e + 4*int(sqrt(a + c*x**4)/(a + c*x**4),x)*c*d)/(4
*c)
```

3.207 $\int \frac{1}{\sqrt{a+cx^4}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 88

$$\int \frac{1}{\sqrt{a+cx^4}}\,dx=\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+cx^4}}$$

output
```
1/2*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*Invers
eJacobiAM(2*arctan(c^(1/4)*x/a^(1/4)),1/2*2^(1/2))/a^(1/4)/c^(1/4)/(c*x^4+
a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.03 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.84

$$\int \frac{1}{\sqrt{a+cx^4}}\,dx=-\frac{i\sqrt{1+\frac{cx^4}{a}}\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}x\right),-1\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\sqrt{a+cx^4}}$$

input
```
Integrate[1/Sqrt[a + c*x^4],x]
```

output
```
((-I)*Sqrt[1 + (c*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x]
, -1])/(Sqrt[(I*Sqrt[c])/Sqrt[a]]*Sqrt[a + c*x^4])
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+cx^4}}\,dx$$

$$\Big\downarrow \; 761$$

$$\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+cx^4}}$$

input
```
Int[1/Sqrt[a + c*x^4],x]
```

output
```
((Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Ellip
ticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*c^(1/4)*Sqrt[a + c*x^
4])
```

Defintions of rubi rules used

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}}$	70
elliptic	$\frac{\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}}$	70

input
```
int(1/(c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/(I*c^(1/2)/a^(1/2))^(1/2)*(1-I*c^(1/2)*x^2/a^(1/2))^(1/2)*(1+I*c^(1/2)*x
^2/a^(1/2))^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I*c^(1/2)/a^(1/2))^(1/2),I)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.33

$$\int \frac{1}{\sqrt{a+cx^4}}\,dx = -\frac{\sqrt{a}\left(-\frac{c}{a}\right)^{\frac{3}{4}}F\left(\arcsin\left(x\left(-\frac{c}{a}\right)^{\frac{1}{4}}\right)\middle|-1\right)}{c}$$

input
```
integrate(1/(c*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
-sqrt(a)*(-c/a)^(3/4)*elliptic_f(arcsin(x*(-c/a)^(1/4)), -1)/c
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.50 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.41

$$\int \frac{1}{\sqrt{a+cx^4}}\,dx = \frac{x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate(1/(c*x**4+a)**(1/2),x)
```

output
```
x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), c*x**4*exp_polar(I*pi)/a)/(4*sqrt(a
)*gamma(5/4))
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+cx^4}}\,dx = \int \frac{1}{\sqrt{cx^4+a}}\,dx$$

input
```
integrate(1/(c*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(c*x^4 + a), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a+cx^4}}\,dx = \int \frac{1}{\sqrt{cx^4+a}}\,dx$$

input
```
integrate(1/(c*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(c*x^4 + a), x)
```

Mupad [B] (verification not implemented)

Time = 21.25 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.42

$$\int \frac{1}{\sqrt{a+cx^4}}\,dx=\frac{x\,\sqrt{\frac{c\,x^4}{a}+1}\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{1}{2};\ \frac{5}{4};\ -\frac{c\,x^4}{a}\right)}{\sqrt{c\,x^4+a}}$$

input
```
int(1/(a + c*x^4)^(1/2),x)
```

output
```
(x*((c*x^4)/a + 1)^(1/2)*hypergeom([1/4, 1/2], 5/4, -(c*x^4)/a))/(a + c*x^
4)^(1/2)
```

Reduce [F]

$$\int \frac{1}{\sqrt{a+cx^4}}\,dx=\int \frac{\sqrt{c\,x^4+a}}{c\,x^4+a}dx$$

input
```
int(1/(c*x^4+a)^(1/2),x)
```

output
```
int(sqrt(a + c*x**4)/(a + c*x**4),x)
```

3.208 $\int \frac{1}{(d+ex)\sqrt{a+cx^4}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 401

$$\int \frac{1}{(d+ex)\sqrt{a+cx^4}}\,dx = \frac{e\,\text{arctanh}\left(\frac{\sqrt{cd^4+ae^4}\,x}{de\sqrt{a+cx^4}}\right)}{2\sqrt{cd^4+ae^4}} - \frac{e\,\text{arctanh}\left(\frac{ae^2+cd^2x^2}{\sqrt{cd^4+ae^4}\sqrt{a+cx^4}}\right)}{2\sqrt{cd^4+ae^4}} + \frac{\sqrt[4]{c}d\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\sqrt{a+cx^4}} - \frac{\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticPi}\left(\frac{\left(\sqrt{c}d^2+\sqrt{a}e^2\right)^2}{4\sqrt{a}\sqrt{c}d^2e^2},2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4\sqrt[4]{a}\sqrt[4]{c}d\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\sqrt{a+cx^4}}$$

output
```
1/2*e*arctanh((a*e^4+c*d^4)^(1/2)*x/d/e/(c*x^4+a)^(1/2))/(a*e^4+c*d^4)^(1/
2)-1/2*e*arctanh((c*d^2*x^2+a*e^2)/(a*e^4+c*d^4)^(1/2)/(c*x^4+a)^(1/2))/(a
*e^4+c*d^4)^(1/2)+1/2*c^(1/4)*d*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+a)/(a^(1/2)+
c^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(c^(1/4)*x/a^(1/4)),1/2*2^(1
/2))/a^(1/4)/(c^(1/2)*d^2+a^(1/2)*e^2)/(c*x^4+a)^(1/2)-1/4*(c^(1/2)*d^2-a^
(1/2)*e^2)*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)
*EllipticPi(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/4*(c^(1/2)*d^2+a^(1/2)*e^2)
^2/a^(1/2)/c^(1/2)/d^2/e^2,1/2*2^(1/2))/a^(1/4)/c^(1/4)/d/(c^(1/2)*d^2+a^(
1/2)*e^2)/(c*x^4+a)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.38 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.50

$$\int \frac{1}{(d+ex)\sqrt{a+cx^4}}\,dx = \frac{\sqrt{1+\frac{cx^4}{a}}\left(-2\sqrt[4]{-1}\sqrt[4]{a}\sqrt{1+\frac{cd^4}{ae^4}}e\,\text{EllipticPi}\left(\frac{i\sqrt{a}e^2}{\sqrt{c}d^2},\arcsin\left(\frac{(-1)^{3/4}\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)+\sqrt[4]{c}d\log\left(\frac{-d}{cd^2x^2+ae^2\left(1+\right.}\right.\right.}{2\sqrt[4]{c}d\sqrt{1+\frac{cd^4}{ae^4}}e\sqrt{a+cx^4}}$$

input
```
Integrate[1/((d + e*x)*Sqrt[a + c*x^4]),x]
```

output
```
(Sqrt[1 + (c*x^4)/a]*(-2*(-1)^(1/4)*a^(1/4)*Sqrt[1 + (c*d^4)/(a*e^4)]*e*El
lipticPi[(I*Sqrt[a]*e^2)/(Sqrt[c]*d^2), ArcSin[((-1)^(3/4)*c^(1/4)*x)/a^(1
/4)], -1] + c^(1/4)*d*Log[(-d^2 + e^2*x^2)/(c*d^2*x^2 + a*e^2*(1 + Sqrt[1
+ (c*d^4)/(a*e^4)]*Sqrt[1 + (c*x^4)/a]))]))/(2*c^(1/4)*d*Sqrt[1 + (c*d^4)/
(a*e^4)]*e*Sqrt[a + c*x^4])
```

Rubi [A] (verified)

Time = 1.10 (sec) , antiderivative size = 431, normalized size of antiderivative = 1.07, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {2263, 1541, 27, 761, 1577, 488, 219, 2223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+cx^4}(d+ex)}\,dx$$

$$\Big\downarrow \text{ 2263}$$

$$d\int \frac{1}{(d^2-e^2x^2)\sqrt{cx^4+a}}dx - e\int \frac{x}{(d^2-e^2x^2)\sqrt{cx^4+a}}dx$$

$$\Big\downarrow \text{ 1541}$$

$$d\left(\frac{\sqrt{c}\int \frac{1}{\sqrt{cx^4+a}}dx}{\sqrt{a}e^2+\sqrt{c}d^2}+\frac{\sqrt{a}e^2\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a}(d^2-e^2x^2)\sqrt{cx^4+a}}dx}{\sqrt{a}e^2+\sqrt{c}d^2}\right)-e\int \frac{x}{(d^2-e^2x^2)\sqrt{cx^4+a}}dx$$

$$\Big\downarrow \ 27$$

$$d\left(\frac{\sqrt{c}\int \frac{1}{\sqrt{cx^4+a}}dx}{\sqrt{a}e^2+\sqrt{c}d^2}+\frac{e^2\int \frac{\sqrt{c}x^2+\sqrt{a}}{(d^2-e^2x^2)\sqrt{cx^4+a}}dx}{\sqrt{a}e^2+\sqrt{c}d^2}\right)-e\int \frac{x}{(d^2-e^2x^2)\sqrt{cx^4+a}}dx$$

$$\Big\downarrow \ 761$$

$$d\left(\frac{e^2\int \frac{\sqrt{c}x^2+\sqrt{a}}{(d^2-e^2x^2)\sqrt{cx^4+a}}dx}{\sqrt{a}e^2+\sqrt{c}d^2}+\frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt{a+cx^4}\left(\sqrt{a}e^2+\sqrt{c}d^2\right)}\right)- e\int \frac{x}{(d^2-e^2x^2)\sqrt{cx^4+a}}dx$$

$$\Big\downarrow \ 1577$$

$$d\left(\frac{e^2\int \frac{\sqrt{c}x^2+\sqrt{a}}{(d^2-e^2x^2)\sqrt{cx^4+a}}dx}{\sqrt{a}e^2+\sqrt{c}d^2}+\frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt{a+cx^4}\left(\sqrt{a}e^2+\sqrt{c}d^2\right)}\right)- \frac{1}{2}e\int \frac{1}{(d^2-e^2x^2)\sqrt{cx^4+a}}dx^2$$

$$\Big\downarrow \ 488$$

$$d\left(\frac{e^2\int \frac{\sqrt{c}x^2+\sqrt{a}}{(d^2-e^2x^2)\sqrt{cx^4+a}}dx}{\sqrt{a}e^2+\sqrt{c}d^2}+\frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt{a+cx^4}\left(\sqrt{a}e^2+\sqrt{c}d^2\right)}\right)+ \frac{1}{2}e\int \frac{1}{cd^4+ae^4-x^4}d\frac{-ae^2-cd^2x^2}{\sqrt{cx^4+a}}$$

$$\Big\downarrow \ 219$$

$$d\left(\frac{e^2\int \frac{\sqrt{c}x^2+\sqrt{a}}{(d^2-e^2x^2)\sqrt{cx^4+a}}dx}{\sqrt{a}e^2+\sqrt{c}d^2}+\frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt{a+cx^4}\left(\sqrt{a}e^2+\sqrt{c}d^2\right)}\right)+\frac{e\text{arctanh}\left(\frac{-ae^2-cd^2x^2}{\sqrt{a+cx^4}\sqrt{ae^4+cd^4}}\right)}{2\sqrt{ae^4+cd^4}}$$

$\downarrow$ 2223

$$d\left(\frac{e^2\left(\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{\sqrt{a}}{d^2}-\frac{\sqrt{c}}{e^2}\right)\text{EllipticPi}\left(\frac{\left(\sqrt{c}d^2+\sqrt{a}e^2\right)^2}{4\sqrt{a}\sqrt{c}d^2e^2},2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+cx^4}}+\frac{\left(\sqrt{a}e^2+\sqrt{c}d^2\right)\text{arctanh}\left(\frac{x\sqrt{ae^4+cd^4}}{de\sqrt{a+cx^4}}\right)}{2de\sqrt{ae^4+cd^4}}\right.}{\sqrt{a}e^2+\sqrt{c}d^2}\right.$$

$$\frac{e\text{arctanh}\left(\frac{-ae^2-cd^2x^2}{\sqrt{a+cx^4}\sqrt{ae^4+cd^4}}\right)}{2\sqrt{ae^4+cd^4}}$$

input
```
Int[1/((d + e*x)*Sqrt[a + c*x^4]),x]
```

output
```
(e*ArcTanh[(-(a*e^2) - c*d^2*x^2)/(Sqrt[c*d^4 + a*e^4]*Sqrt[a + c*x^4])])/
(2*Sqrt[c*d^4 + a*e^4]) + d*((c^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*
x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1
/2])/(2*a^(1/4)*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*Sqrt[a + c*x^4]) + (e^2*(((Sqr
t[c]*d^2 + Sqrt[a]*e^2)*ArcTanh[(Sqrt[c*d^4 + a*e^4]*x)/(d*e*Sqrt[a + c*x^
4])])/(2*d*e*Sqrt[c*d^4 + a*e^4]) + ((Sqrt[a]/d^2 - Sqrt[c]/e^2)*(Sqrt[a]
+ Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticPi[(Sqr
t[c]*d^2 + Sqrt[a]*e^2)^2/(4*Sqrt[a]*Sqrt[c]*d^2*e^2), 2*ArcTan[(c^(1/4)*x
)/a^(1/4)], 1/2])/(4*a^(1/4)*c^(1/4)*Sqrt[a + c*x^4])))/(Sqrt[c]*d^2 + Sqr
t[a]*e^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1541
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Simp[(c*d + a*e*q)/(c*d^2 - a*e^2)   Int[1/Sqrt[a + c*x^4
], x], x] - Simp[(a*e*(e + d*q))/(c*d^2 - a*e^2)   Int[(1 + q*x^2)/((d + e*
x^2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

rule 1577
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, c, d, e, p, q}, x]
```

rule 2223
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTanh[Rt[(-c)*
(d/e) - a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[(-c)*(d/e) - a*(e/d), 2]
)), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^
2)]/(4*d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*Arc
Tan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[c*(d/e) + a*(e
/d)]
```

rule 2263
```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Simp[d
  Int[1/((d^2 - e^2*x^2)*Sqrt[a + c*x^4]), x], x] - Simp[e   Int[x/((d^2 -
e^2*x^2)*Sqrt[a + c*x^4]), x], x] /; FreeQ[{a, c, d, e}, x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.41 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.42

method	result	size
default	$\frac{-\frac{\operatorname{arctanh}\left(\frac{\frac{2c x^2 d^2}{e^2}+2a}{2\sqrt{a+\frac{c d^4}{e^4}}\sqrt{c x^4+a}}\right)}{2\sqrt{a+\frac{c d^4}{e^4}}}+\frac{e\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\operatorname{EllipticPi}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},-\frac{i\sqrt{a}\,e^2}{\sqrt{c}\,d^2},\frac{\sqrt{-\frac{i\sqrt{c}}{\sqrt{a}}}}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}}\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,d\sqrt{c x^4+a}}}{e}$	169
elliptic	$\frac{-\frac{\operatorname{arctanh}\left(\frac{\frac{2c x^2 d^2}{e^2}+2a}{2\sqrt{a+\frac{c d^4}{e^4}}\sqrt{c x^4+a}}\right)}{2\sqrt{a+\frac{c d^4}{e^4}}}+\frac{e\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\operatorname{EllipticPi}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},-\frac{i\sqrt{a}\,e^2}{\sqrt{c}\,d^2},\frac{\sqrt{-\frac{i\sqrt{c}}{\sqrt{a}}}}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}}\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,d\sqrt{c x^4+a}}}{e}$	169

input
```
int(1/(e*x+d)/(c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/e*(-1/2/(a+c*d^4/e^4)^(1/2)*arctanh(1/2*(2*c*x^2*d^2/e^2+2*a)/(a+c*d^4/e
^4)^(1/2)/(c*x^4+a)^(1/2))+1/(I*c^(1/2)/a^(1/2))^(1/2)/d*e*(1-I*c^(1/2)*x^
2/a^(1/2))^(1/2)*(1+I*c^(1/2)*x^2/a^(1/2))^(1/2)/(c*x^4+a)^(1/2)*EllipticP
i(x*(I*c^(1/2)/a^(1/2))^(1/2),-I/c^(1/2)*a^(1/2)/d^2*e^2,(-I/a^(1/2)*c^(1/
2))^(1/2)/(I*c^(1/2)/a^(1/2))^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)\sqrt{a+cx^4}}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x+d)/(c*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(d+ex)\sqrt{a+cx^4}}\,dx = \int \frac{1}{\sqrt{a+cx^4}\,(d+ex)}\,dx$$

input
```
integrate(1/(e*x+d)/(c*x**4+a)**(1/2),x)
```

output
```
Integral(1/(sqrt(a + c*x**4)*(d + e*x)), x)
```

Maxima [F]

$$\int \frac{1}{(d+ex)\sqrt{a+cx^4}}\,dx = \int \frac{1}{\sqrt{cx^4+a}(ex+d)}\,dx$$

input
```
integrate(1/(e*x+d)/(c*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(c*x^4 + a)*(e*x + d)), x)
```

Giac [F]

$$\int \frac{1}{(d+ex)\sqrt{a+cx^4}}\,dx = \int \frac{1}{\sqrt{cx^4+a}(ex+d)}\,dx$$

input
```
integrate(1/(e*x+d)/(c*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(c*x^4 + a)*(e*x + d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)\sqrt{a+cx^4}}\,dx = \int \frac{1}{\sqrt{c\,x^4+a}\,(d+e\,x)}\,dx$$

input
```
int(1/((a + c*x^4)^(1/2)*(d + e*x)),x)
```

output
```
int(1/((a + c*x^4)^(1/2)*(d + e*x)), x)
```

Reduce [F]

$$\int \frac{1}{(d+ex)\sqrt{a+cx^4}}\,dx = \int \frac{\sqrt{c\,x^4+a}}{ce\,x^5+cd\,x^4+aex+ad}dx$$

input
```
int(1/(e*x+d)/(c*x^4+a)^(1/2),x)
```

output
```
int(sqrt(a + c*x**4)/(a*d + a*e*x + c*d*x**4 + c*e*x**5),x)
```

3.209 $\int \frac{1}{(d+ex)^2\sqrt{a+cx^4}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 605

$$\begin{aligned}\int \frac{1}{(d+ex)^2\sqrt{a+cx^4}}\,dx &= -\frac{e^3\sqrt{a+cx^4}}{(cd^4+ae^4)\,(d+ex)}+\frac{\sqrt{c}e^2x\sqrt{a+cx^4}}{(cd^4+ae^4)\left(\sqrt{a}+\sqrt{c}x^2\right)} \\
&\quad+\frac{cd^3\operatorname{earctanh}\left(\frac{\sqrt{cd^4+ae^4}x}{de\sqrt{a+cx^4}}\right)}{(cd^4+ae^4)^{3/2}}-\frac{cd^3\operatorname{earctanh}\left(\frac{ae^2+cd^2x^2}{\sqrt{cd^4+ae^4}\sqrt{a+cx^4}}\right)}{(cd^4+ae^4)^{3/2}} \\
&\quad-\frac{\sqrt[4]{a}\sqrt[4]{c}e^2\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{(cd^4+ae^4)\sqrt{a+cx^4}} \\
&\quad+\frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\sqrt{a+cx^4}} \\
&\quad-\frac{c^{3/4}d^2\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticPi}\left(\frac{\left(\sqrt{c}d^2+\sqrt{a}e^2\right)^2}{4\sqrt{a}\sqrt{c}d^2e^2},2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\left(\sqrt{c}d^2+\sqrt{a}e^2\right)(cd^4+ae^4)\sqrt{a+cx^4}}\end{aligned}$$

output
```
-e^3*(c*x^4+a)^(1/2)/(a*e^4+c*d^4)/(e*x+d)+c^(1/2)*e^2*x*(c*x^4+a)^(1/2)/(
a*e^4+c*d^4)/(a^(1/2)+c^(1/2)*x^2)+c*d^3*e*arctanh((a*e^4+c*d^4)^(1/2)*x/d
/e/(c*x^4+a)^(1/2))/(a*e^4+c*d^4)^(3/2)-c*d^3*e*arctanh((c*d^2*x^2+a*e^2)/
(a*e^4+c*d^4)^(1/2)/(c*x^4+a)^(1/2))/(a*e^4+c*d^4)^(3/2)-a^(1/4)*c^(1/4)*e
^2*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*Ellipti
cE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))/(a*e^4+c*d^4)/(c*x^4+a)^(
1/2)+1/2*c^(1/4)*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+a)/(a^(1/2)+c^(1/2)*x^2)^2)
^(1/2)*InverseJacobiAM(2*arctan(c^(1/4)*x/a^(1/4)),1/2*2^(1/2))/a^(1/4)/(c
^(1/2)*d^2+a^(1/2)*e^2)/(c*x^4+a)^(1/2)-1/2*c^(3/4)*d^2*(c^(1/2)*d^2-a^(1/
2)*e^2)*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*El
lipticPi(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/4*(c^(1/2)*d^2+a^(1/2)*e^2)^2/
a^(1/2)/c^(1/2)/d^2/e^2,1/2*2^(1/2))/a^(1/4)/(c^(1/2)*d^2+a^(1/2)*e^2)/(a*
e^4+c*d^4)/(c*x^4+a)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 11.57 (sec) , antiderivative size = 448, normalized size of antiderivative = 0.74

$$\int \frac{1}{(d+ex)^2\sqrt{a+cx^4}}\,dx =$$

$$-\frac{\sqrt{a}\sqrt{c}e^2\sqrt{-cd^4-ae^4}(d+ex)\sqrt{1+\frac{cx^4}{a}}E\left(i\text{arcsinh}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}x\right)\middle|-1\right)+i\sqrt{c}\left(\sqrt{c}d^2+i\sqrt{a}e^2\right)\sqrt{-cd^4-}}{}$$

input
```
Integrate[1/((d + e*x)^2*Sqrt[a + c*x^4]),x]
```

output
```
-((Sqrt[a]*Sqrt[c]*e^2*Sqrt[-(c*d^4) - a*e^4]*(d + e*x)*Sqrt[1 + (c*x^4)/a
]*EllipticE[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1] + I*Sqrt[c]*(Sqrt[
c]*d^2 + I*Sqrt[a]*e^2)*Sqrt[-(c*d^4) - a*e^4]*(d + e*x)*Sqrt[1 + (c*x^4)/
a]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1] - Sqrt[(I*Sqrt[c]
)/Sqrt[a]]*(e^3*Sqrt[-(c*d^4) - a*e^4]*(a + c*x^4) - 2*c*d^3*e*(d + e*x)*S
qrt[a + c*x^4]*ArcTan[(Sqrt[c]*(d^2 - e^2*x^2) + e^2*Sqrt[a + c*x^4])/Sqrt
[-(c*d^4) - a*e^4]] + 2*(-1)^(1/4)*a^(1/4)*c^(3/4)*d^2*Sqrt[-(c*d^4) - a*e
^4]*(d + e*x)*Sqrt[1 + (c*x^4)/a]*EllipticPi[(I*Sqrt[a]*e^2)/(Sqrt[c]*d^2)
, ArcSin[((-1)^(3/4)*c^(1/4)*x)/a^(1/4)], -1]))/(Sqrt[(I*Sqrt[c])/Sqrt[a]]
*(-(c*d^4) - a*e^4)^(3/2)*(d + e*x)*Sqrt[a + c*x^4]))
```

Rubi [A] (verified)

Time = 2.15 (sec) , antiderivative size = 634, normalized size of antiderivative = 1.05, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.789$, Rules used = {2265, 25, 2280, 27, 1577, 488, 219, 2233, 25, 27, 1510, 2227, 27, 761, 2223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+cx^4}(d+ex)^2}dx$$

$$\Big\downarrow \text{2265}$$

$$-\frac{c\int -\frac{d^3-exd^2+e^2x^2d+e^3x^3}{(d+ex)\sqrt{cx^4+a}}dx}{ae^4+cd^4}-\frac{e^3\sqrt{a+cx^4}}{(d+ex)\left(ae^4+cd^4\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{c\int \frac{d^3-exd^2+e^2x^2d+e^3x^3}{(d+ex)\sqrt{cx^4+a}}dx}{ae^4+cd^4}-\frac{e^3\sqrt{a+cx^4}}{(d+ex)\left(ae^4+cd^4\right)}$$

$$\Big\downarrow \text{2280}$$

$$\frac{c\left(\int \frac{d^4+2e^2x^2d^2-e^4x^4}{(d^2-e^2x^2)\sqrt{cx^4+a}}dx+\int -\frac{2d^3ex}{(d^2-e^2x^2)\sqrt{cx^4+a}}dx\right)}{ae^4+cd^4}-\frac{e^3\sqrt{a+cx^4}}{(d+ex)\left(ae^4+cd^4\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{c\left(\int \frac{d^4+2e^2x^2d^2-e^4x^4}{(d^2-e^2x^2)\sqrt{cx^4+a}}dx-2d^3e\int \frac{x}{(d^2-e^2x^2)\sqrt{cx^4+a}}dx\right)}{ae^4+cd^4}-\frac{e^3\sqrt{a+cx^4}}{(d+ex)\left(ae^4+cd^4\right)}$$

$$\Big\downarrow \text{1577}$$

$$\frac{c\left(\int \frac{d^4+2e^2x^2d^2-e^4x^4}{(d^2-e^2x^2)\sqrt{cx^4+a}}dx-d^3e\int \frac{1}{(d^2-e^2x^2)\sqrt{cx^4+a}}dx^2\right)}{ae^4+cd^4}-\frac{e^3\sqrt{a+cx^4}}{(d+ex)\left(ae^4+cd^4\right)}$$

$$\Big\downarrow \text{488}$$

$$\frac{c\left(\int \frac{d^4+2e^2x^2d^2-e^4x^4}{(d^2-e^2x^2)\sqrt{cx^4+a}}dx+d^3e\int \frac{1}{cd^4+ae^4-x^4}d\frac{-ae^2-cd^2x^2}{\sqrt{cx^4+a}}\right)}{ae^4+cd^4}-\frac{e^3\sqrt{a+cx^4}}{(d+ex)\left(ae^4+cd^4\right)}$$

$$\Big\downarrow \text{219}$$

$$\frac{c\left(\int \frac{d^4+2e^2x^2d^2-e^4x^4}{(d^2-e^2x^2)\sqrt{cx^4+a}}dx+\frac{d^3e\operatorname{arctanh}\left(\frac{-ae^2-cd^2x^2}{\sqrt{a+cx^4}\sqrt{ae^4+cd^4}}\right)}{\sqrt{ae^4+cd^4}}\right)}{ae^4+cd^4}-\frac{e^3\sqrt{a+cx^4}}{(d+ex)\left(ae^4+cd^4\right)}$$

$$\Big\downarrow \text{ 2233}$$

$$\frac{c\left(-\frac{\int -\frac{\sqrt{c}e^2\left(\left(\sqrt{c}d^2+\sqrt{a}e^2\right)d^2+e^2\left(\sqrt{c}d^2-\sqrt{a}e^2\right)x^2\right)}{\left(d^2-e^2x^2\right)\sqrt{cx^4+a}}dx}{ce^2}-\frac{\sqrt{a}e^2\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{a}\sqrt{cx^4+a}}dx}{\sqrt{c}}+\frac{d^3e\operatorname{arctanh}\left(\frac{-ae^2-cd^2x^2}{\sqrt{a+cx^4}\sqrt{ae^4+cd^4}}\right)}{\sqrt{ae^4+cd^4}}\right)}{ae^4+cd^4}-\frac{e^3\sqrt{a+cx^4}}{(d+ex)\left(ae^4+cd^4\right)}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{c\left(\frac{\int \frac{\sqrt{c}e^2\left(\left(\sqrt{c}d^2+\sqrt{a}e^2\right)d^2+e^2\left(\sqrt{c}d^2-\sqrt{a}e^2\right)x^2\right)}{\left(d^2-e^2x^2\right)\sqrt{cx^4+a}}dx}{ce^2}-\frac{\sqrt{a}e^2\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{a}\sqrt{cx^4+a}}dx}{\sqrt{c}}+\frac{d^3e\operatorname{arctanh}\left(\frac{-ae^2-cd^2x^2}{\sqrt{a+cx^4}\sqrt{ae^4+cd^4}}\right)}{\sqrt{ae^4+cd^4}}\right)}{ae^4+cd^4}-\frac{e^3\sqrt{a+cx^4}}{(d+ex)\left(ae^4+cd^4\right)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{c\left(\frac{\int \frac{\left(\sqrt{c}d^2+\sqrt{a}e^2\right)d^2+e^2\left(\sqrt{c}d^2-\sqrt{a}e^2\right)x^2}{\left(d^2-e^2x^2\right)\sqrt{cx^4+a}}dx}{\sqrt{c}}-\frac{e^2\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+a}}dx}{\sqrt{c}}+\frac{d^3e\operatorname{arctanh}\left(\frac{-ae^2-cd^2x^2}{\sqrt{a+cx^4}\sqrt{ae^4+cd^4}}\right)}{\sqrt{ae^4+cd^4}}\right)}{ae^4+cd^4}-\frac{e^3\sqrt{a+cx^4}}{(d+ex)\left(ae^4+cd^4\right)}$$

$$\Big\downarrow \text{ 1510}$$

$$\frac{c\left(\frac{\int \frac{\left(\sqrt{c}d^2+\sqrt{a}e^2\right)d^2+e^2\left(\sqrt{c}d^2-\sqrt{a}e^2\right)x^2}{\left(d^2-e^2x^2\right)\sqrt{cx^4+a}}dx}{\sqrt{c}}-\frac{e^2\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt[4]{c}\sqrt{a+cx^4}}-\frac{x\sqrt{a+cx^4}}{\sqrt{a}+\sqrt{c}x^2}\right)}{\sqrt{c}}+\frac{d^3e\operatorname{arctanh}\left(\cdots\right)}{\sqrt{ae\cdots}}\right)}{ae^4+cd^4}-\frac{e^3\sqrt{a+cx^4}}{(d+ex)\left(ae^4+cd^4\right)}$$

$$\Big\downarrow \; 2227$$

$$\frac{c\left(\frac{\frac{2\sqrt{a}\sqrt{c}d^4e^2\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a}\left(d^2-e^2x^2\right)\sqrt{cx^4+a}}dx}{\sqrt{a}e^2+\sqrt{c}d^2}+\frac{\left(ae^4+cd^4\right)\int \frac{1}{\sqrt{cx^4+a}}dx}{\sqrt{a}e^2+\sqrt{c}d^2}}{\sqrt{c}}-\frac{e^2\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt[4]{c}\sqrt{a+cx^4}}-\frac{x\sqrt{a+cx^4}}{\sqrt{a}+\sqrt{c}x^2}\right)}{\sqrt{c}}\right.}{ae^4+cd^4}$$

$$\frac{e^3\sqrt{a+cx^4}}{(d+ex)\left(ae^4+cd^4\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{c\left(\frac{\frac{2\sqrt{c}d^4e^2\int \frac{\sqrt{c}x^2+\sqrt{a}}{\left(d^2-e^2x^2\right)\sqrt{cx^4+a}}dx}{\sqrt{a}e^2+\sqrt{c}d^2}+\frac{\left(ae^4+cd^4\right)\int \frac{1}{\sqrt{cx^4+a}}dx}{\sqrt{a}e^2+\sqrt{c}d^2}}{\sqrt{c}}-\frac{e^2\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt[4]{c}\sqrt{a+cx^4}}-\frac{x\sqrt{a+cx^4}}{\sqrt{a}+\sqrt{c}x^2}\right)}{\sqrt{c}}+\right.}{ae^4+cd^4}$$

$$\frac{e^3\sqrt{a+cx^4}}{(d+ex)\left(ae^4+cd^4\right)}$$

$$\Big\downarrow \; 761$$

$$\frac{c\left(\frac{\frac{2\sqrt{c}d^4e^2\int \frac{\sqrt{c}x^2+\sqrt{a}}{\left(d^2-e^2x^2\right)\sqrt{cx^4+a}}dx}{\sqrt{a}e^2+\sqrt{c}d^2}+\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(ae^4+cd^4\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+cx^4}\left(\sqrt{a}e^2+\sqrt{c}d^2\right)}}{\sqrt{c}}-\frac{e^2\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\cdots}{\sqrt[4]{c}\sqrt{a\cdots}}\right.}{}\right.}{ae^4+cd^4}$$

$$\frac{e^3\sqrt{a+cx^4}}{(d+ex)\left(ae^4+cd^4\right)}$$

$$\Big\downarrow \; 2223$$

$$c\left(\frac{\frac{2\sqrt{c}d^4e^2\left(\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{\sqrt{a}}{d^2}-\frac{\sqrt{c}}{e^2}\right)\text{EllipticPi}\left(\frac{\left(\sqrt{c}d^2+\sqrt{a}e^2\right)^2}{4\sqrt{a}\sqrt{c}d^2e^2},2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+cx^4}}+\frac{\left(\sqrt{a}e^2+\sqrt{c}d^2\right)\text{arctanh}\left(\frac{x\sqrt{ae^4+cd^4}}{de\sqrt{a+cx^4}}\right)}{2de\sqrt{ae^4+cd^4}}\right)}{\sqrt{a}e^2+\sqrt{c}d^2}}{\sqrt{c}}+\right.$$

$$\frac{e^3\sqrt{a+cx^4}}{(d+ex)\left(ae^4+cd^4\right)}$$

input
```
Int[1/((d + e*x)^2*Sqrt[a + c*x^4]),x]
```

output
```
-((e^3*Sqrt[a + c*x^4])/((c*d^4 + a*e^4)*(d + e*x))) + (c*((d^3*e*ArcTanh[
(-(a*e^2) - c*d^2*x^2)/(Sqrt[c*d^4 + a*e^4]*Sqrt[a + c*x^4])])/Sqrt[c*d^4
 + a*e^4] - (e^2*(-((x*Sqrt[a + c*x^4])/(Sqrt[a] + Sqrt[c]*x^2)) + (a^(1/4)
*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Ellip
ticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(c^(1/4)*Sqrt[a + c*x^4])))/Sqrt
[c] + (((c*d^4 + a*e^4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a]
 + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4
)*c^(1/4)*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*Sqrt[a + c*x^4]) + (2*Sqrt[c]*d^4*e^
2*(((Sqrt[c]*d^2 + Sqrt[a]*e^2)*ArcTanh[(Sqrt[c*d^4 + a*e^4]*x)/(d*e*Sqrt[
a + c*x^4])])/(2*d*e*Sqrt[c*d^4 + a*e^4]) + ((Sqrt[a]/d^2 - Sqrt[c]/e^2)*(
Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Ellipti
cPi[(Sqrt[c]*d^2 + Sqrt[a]*e^2)^2/(4*Sqrt[a]*Sqrt[c]*d^2*e^2), 2*ArcTan[(c
^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(1/4)*c^(1/4)*Sqrt[a + c*x^4])))/(Sqrt[c]*d
^2 + Sqrt[a]*e^2))/Sqrt[c]))/(c*d^4 + a*e^4)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1577
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, c, d, e, p, q}, x]
```

rule 2223
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTanh[Rt[(-c)*
(d/e) - a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[(-c)*(d/e) - a*(e/d), 2]
)), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^
2)]/(4*d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*Arc
Tan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[c*(d/e) + a*(e
/d)]
```

rule 2227
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[c/a, 2]}, Simp[(A*(c*d + a*e*q) - a*B*(e + d*q)
)/(c*d^2 - a*e^2)   Int[1/Sqrt[a + c*x^4], x], x] + Simp[a*(B*d - A*e)*((e
+ d*q)/(c*d^2 - a*e^2))   Int[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + c*x^4]), x]
, x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
 && NeQ[c*A^2 - a*B^2, 0]
```

rule 2233
```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :>
With[{q = Rt[c/a, 2], A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff
[P4x, x, 4]}, Simp[-C/(e*q)   Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] + Sim
p[1/(c*e)   Int[(A*c*e + a*C*d*q + (B*c*e - C*(c*d - a*e*q))*x^2)/((d + e*x
^2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[P4x, x^2,
2] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

rule 2265
```
Int[((d_) + (e_.)*(x_))^(q_)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[e
^3*(d + e*x)^(q + 1)*(Sqrt[a + c*x^4]/((q + 1)*(c*d^4 + a*e^4))), x] + Simp
[c/((q + 1)*(c*d^4 + a*e^4))   Int[((d + e*x)^(q + 1)/Sqrt[a + c*x^4])*Simp
[d^3*(q + 1) - d^2*e*(q + 1)*x + d*e^2*(q + 1)*x^2 - e^3*(q + 3)*x^3, x], x
], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^4 + a*e^4, 0] && ILtQ[q, -1]
```

rule 2280
```
Int[(Px_)/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Wit
h[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1], C = Coeff[Px, x, 2], D = Coeff
[Px, x, 3]}, Int[(x*(B*d - A*e + (d*D - C*e)*x^2))/((d^2 - e^2*x^2)*Sqrt[a
+ c*x^4]), x] + Int[(A*d + (C*d - B*e)*x^2 - D*e*x^4)/((d^2 - e^2*x^2)*Sqrt
[a + c*x^4]), x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Px, x] && LeQ[Expon[Px
, x], 3] && NeQ[c*d^4 + a*e^4, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.48 (sec) , antiderivative size = 421, normalized size of antiderivative = 0.70

method	result
default	$-\frac{e^3\sqrt{c\,x^4+a}}{(e^4a+c\,d^4)(ex+d)}-\frac{d^2c\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{(e^4a+c\,d^4)\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}+\frac{i\sqrt{c}\,e^2\sqrt{a}\,\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\Big(\mathrm{EllipticF}\Big(x}{(e^4a+c\,d^4)\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c}}$
elliptic	$-\frac{e^3\sqrt{c\,x^4+a}}{(e^4a+c\,d^4)(ex+d)}-\frac{d^2c\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{(e^4a+c\,d^4)\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}+\frac{i\sqrt{c}\,e^2\sqrt{a}\,\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\Big(\mathrm{EllipticF}\Big(x}{(e^4a+c\,d^4)\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c}}$

input
```
int(1/(e*x+d)^2/(c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-e^3*(c*x^4+a)^(1/2)/(a*e^4+c*d^4)/(e*x+d)-d^2*c/(a*e^4+c*d^4)/(I*c^(1/2)/
a^(1/2))^(1/2)*(1-I*c^(1/2)*x^2/a^(1/2))^(1/2)*(1+I*c^(1/2)*x^2/a^(1/2))^(
1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I*c^(1/2)/a^(1/2))^(1/2),I)+I*c^(1/2)*e^
2/(a*e^4+c*d^4)*a^(1/2)/(I*c^(1/2)/a^(1/2))^(1/2)*(1-I*c^(1/2)*x^2/a^(1/2)
)^(1/2)*(1+I*c^(1/2)*x^2/a^(1/2))^(1/2)/(c*x^4+a)^(1/2)*(EllipticF(x*(I*c^
(1/2)/a^(1/2))^(1/2),I)-EllipticE(x*(I*c^(1/2)/a^(1/2))^(1/2),I))+2/(a*e^4
+c*d^4)*c*d^3/e*(-1/2/(a+c*d^4/e^4)^(1/2)*arctanh(1/2*(2*c*x^2*d^2/e^2+2*a
)/(a+c*d^4/e^4)^(1/2)/(c*x^4+a)^(1/2))+1/(I*c^(1/2)/a^(1/2))^(1/2)/d*e*(1-
I*c^(1/2)*x^2/a^(1/2))^(1/2)*(1+I*c^(1/2)*x^2/a^(1/2))^(1/2)/(c*x^4+a)^(1/
2)*EllipticPi(x*(I*c^(1/2)/a^(1/2))^(1/2),-I/c^(1/2)*a^(1/2)/d^2*e^2,(-I/a
^(1/2)*c^(1/2))^(1/2)/(I*c^(1/2)/a^(1/2))^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^2\sqrt{a+cx^4}}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x+d)^2/(c*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(d+ex)^2\sqrt{a+cx^4}}\,dx = \int \frac{1}{\sqrt{a+cx^4}\,(d+ex)^2}\,dx$$

input
```
integrate(1/(e*x+d)**2/(c*x**4+a)**(1/2),x)
```

output
```
Integral(1/(sqrt(a + c*x**4)*(d + e*x)**2), x)
```

Maxima [F]

$$\int \frac{1}{(d+ex)^2\sqrt{a+cx^4}}\,dx = \int \frac{1}{\sqrt{cx^4+a}(ex+d)^2}\,dx$$

input
```
integrate(1/(e*x+d)^2/(c*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(c*x^4 + a)*(e*x + d)^2), x)
```

Giac [F]

$$\int \frac{1}{(d+ex)^2\sqrt{a+cx^4}}\,dx = \int \frac{1}{\sqrt{cx^4+a}(ex+d)^2}\,dx$$

input
```
integrate(1/(e*x+d)^2/(c*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(c*x^4 + a)*(e*x + d)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^2\sqrt{a+cx^4}}\,dx = \int \frac{1}{\sqrt{c\,x^4+a}\,(d+e\,x)^2}\,dx$$

input
```
int(1/((a + c*x^4)^(1/2)*(d + e*x)^2),x)
```

output
```
int(1/((a + c*x^4)^(1/2)*(d + e*x)^2), x)
```

Reduce [F]

$$\int \frac{1}{(d+ex)^2\sqrt{a+cx^4}}\,dx = \int \frac{1}{(ex+d)^2\sqrt{c\,x^4+a}}dx$$

input
```
int(1/(e*x+d)^2/(c*x^4+a)^(1/2),x)
```

output
```
int(1/(e*x+d)^2/(c*x^4+a)^(1/2),x)
```

3.210 $\int \frac{1}{(d+ex)^3\sqrt{a+cx^4}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 655

$$\begin{aligned}
&\int \frac{1}{(d+ex)^3\sqrt{a+cx^4}}\,dx \\
&= -\frac{e^3\sqrt{a+cx^4}}{2(cd^4+ae^4)(d+ex)^2} - \frac{3cd^3e^3\sqrt{a+cx^4}}{(cd^4+ae^4)^2(d+ex)} + \frac{3c^{3/2}d^3e^2x\sqrt{a+cx^4}}{(cd^4+ae^4)^2\left(\sqrt{a}+\sqrt{c}x^2\right)} \\
&\quad + \frac{3cd^2e(cd^4-ae^4)\,\mathrm{arctanh}\left(\frac{\sqrt{cd^4+ae^4}\,x}{de\sqrt{a+cx^4}}\right)}{2(cd^4+ae^4)^{5/2}} - \frac{3cd^2e(cd^4-ae^4)\,\mathrm{arctanh}\left(\frac{ae^2+cd^2x^2}{\sqrt{cd^4+ae^4}\sqrt{a+cx^4}}\right)}{2(cd^4+ae^4)^{5/2}} \\
&\quad - \frac{3\sqrt[4]{a}c^{5/4}d^3e^2\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{(cd^4+ae^4)^2\sqrt{a+cx^4}} \\
&\quad + \frac{c^{3/4}d\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\mathrm{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}(cd^4+ae^4)\sqrt{a+cx^4}} \\
&\quad - \frac{3c^{3/4}d\left(\sqrt{c}d^2-\sqrt{a}e^2\right)^2\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\mathrm{EllipticPi}\left(\frac{\left(\sqrt{c}d^2+\sqrt{a}e^2\right)^2}{4\sqrt{a}\sqrt{c}d^2e^2},2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4\sqrt[4]{a}(cd^4+ae^4)^2\sqrt{a+cx^4}}
\end{aligned}$$

output
```
-1/2*e^3*(c*x^4+a)^(1/2)/(a*e^4+c*d^4)/(e*x+d)^2-3*c*d^3*e^3*(c*x^4+a)^(1/
2)/(a*e^4+c*d^4)^2/(e*x+d)+3*c^(3/2)*d^3*e^2*x*(c*x^4+a)^(1/2)/(a*e^4+c*d^
4)^2/(a^(1/2)+c^(1/2)*x^2)+3/2*c*d^2*e*(-a*e^4+c*d^4)*arctanh((a*e^4+c*d^4
)^(1/2)*x/d/e/(c*x^4+a)^(1/2))/(a*e^4+c*d^4)^(5/2)-3/2*c*d^2*e*(-a*e^4+c*d
^4)*arctanh((c*d^2*x^2+a*e^2)/(a*e^4+c*d^4)^(1/2)/(c*x^4+a)^(1/2))/(a*e^4+
c*d^4)^(5/2)-3*a^(1/4)*c^(5/4)*d^3*e^2*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+a)/(a
^(1/2)+c^(1/2)*x^2)^2)^(1/2)*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/
2*2^(1/2))/(a*e^4+c*d^4)^2/(c*x^4+a)^(1/2)+1/2*c^(3/4)*d*(a^(1/2)+c^(1/2)*
x^2)*((c*x^4+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(c^
(1/4)*x/a^(1/4)),1/2*2^(1/2))/a^(1/4)/(a*e^4+c*d^4)/(c*x^4+a)^(1/2)-3/4*c^
(3/4)*d*(c^(1/2)*d^2-a^(1/2)*e^2)^2*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+a)/(a^(1
/2)+c^(1/2)*x^2)^2)^(1/2)*EllipticPi(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/4*
(c^(1/2)*d^2+a^(1/2)*e^2)^2/a^(1/2)/c^(1/2)/d^2/e^2,1/2*2^(1/2))/a^(1/4)/(
a*e^4+c*d^4)^2/(c*x^4+a)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 12.35 (sec) , antiderivative size = 614, normalized size of antiderivative = 0.94

$$\int \frac{1}{(d+ex)^3\sqrt{a+cx^4}}\,dx =$$

$$-\frac{\frac{cd^4e^3\sqrt{a+cx^4}}{(d+ex)^2}+\frac{ae^7\sqrt{a+cx^4}}{(d+ex)^2}+\frac{6cd^3e^3\sqrt{a+cx^4}}{d+ex}-\frac{6c^2d^6e\arctan\left(\frac{\sqrt{c}\left(d^2-e^2x^2\right)+e^2\sqrt{a+cx^4}}{\sqrt{-cd^4-ae^4}}\right)}{\sqrt{-cd^4-ae^4}}+\frac{6acd^2e^5\arctan\left(\frac{\sqrt{c}\left(d^2-e^2x^2\right)+e^2\sqrt{}}{\sqrt{-cd^4-ae^4}}\right.}{\sqrt{-cd^4-ae^4}}}{}$$

input
```
Integrate[1/((d + e*x)^3*Sqrt[a + c*x^4]),x]
```

output
```
-1/2*((c*d^4*e^3*Sqrt[a + c*x^4])/(d + e*x)^2 + (a*e^7*Sqrt[a + c*x^4])/(d
 + e*x)^2 + (6*c*d^3*e^3*Sqrt[a + c*x^4])/(d + e*x) - (6*c^2*d^6*e*ArcTan[
(Sqrt[c]*(d^2 - e^2*x^2) + e^2*Sqrt[a + c*x^4])/Sqrt[-(c*d^4) - a*e^4]])/S
qrt[-(c*d^4) - a*e^4] + (6*a*c*d^2*e^5*ArcTan[(Sqrt[c]*(d^2 - e^2*x^2) + e
^2*Sqrt[a + c*x^4])/Sqrt[-(c*d^4) - a*e^4]])/Sqrt[-(c*d^4) - a*e^4] + ((6*
I)*a*Sqrt[(I*Sqrt[c])/Sqrt[a]]*c*d^3*e^2*Sqrt[1 + (c*x^4)/a]*EllipticE[I*A
rcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1])/Sqrt[a + c*x^4] + ((2*I)*c*d*(-2
*c*d^4 - (3*I)*Sqrt[a]*Sqrt[c]*d^2*e^2 + a*e^4)*Sqrt[1 + (c*x^4)/a]*Ellipt
icF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1])/(Sqrt[(I*Sqrt[c])/Sqrt[a]
]*Sqrt[a + c*x^4]) + (6*(-1)^(1/4)*a^(1/4)*c^(7/4)*d^5*Sqrt[1 + (c*x^4)/a]
*EllipticPi[(I*Sqrt[a]*e^2)/(Sqrt[c]*d^2), ArcSin[((-1)^(3/4)*c^(1/4)*x)/a
^(1/4)], -1])/Sqrt[a + c*x^4] - (6*(-1)^(1/4)*a^(5/4)*c^(3/4)*d*e^4*Sqrt[1
 + (c*x^4)/a]*EllipticPi[(I*Sqrt[a]*e^2)/(Sqrt[c]*d^2), ArcSin[((-1)^(3/4)
*c^(1/4)*x)/a^(1/4)], -1])/Sqrt[a + c*x^4])/(c*d^4 + a*e^4)^2
```

Rubi [A] (verified)

Time = 2.78 (sec) , antiderivative size = 677, normalized size of antiderivative = 1.03, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.895$, Rules used = {2265, 27, 2277, 25, 2280, 27, 1577, 488, 219, 2233, 25, 27, 1510, 2227, 27, 761, 2223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+cx^4}(d+ex)^3}dx$$

$$\Big\downarrow \text{2265}$$

$$-\frac{c\int -\frac{2(d^3-exd^2+e^2x^2d)}{(d+ex)^2\sqrt{cx^4+a}}dx}{2(ae^4+cd^4)}-\frac{e^3\sqrt{a+cx^4}}{2(d+ex)^2(ae^4+cd^4)}$$

$$\Big\downarrow \text{27}$$

$$\frac{c\int \frac{d^3-exd^2+e^2x^2d}{(d+ex)^2\sqrt{cx^4+a}}dx}{ae^4+cd^4}-\frac{e^3\sqrt{a+cx^4}}{2(d+ex)^2(ae^4+cd^4)}$$

$$\Big\downarrow \text{2277}$$

$$\frac{c\left(-\frac{\int -\frac{3ce^2x^2d^4+3ce^3x^3d^3+\left(cd^4-2ae^4\right)d^2-e\left(2cd^4-ae^4\right)xd}{(d+ex)\sqrt{cx^4+a}}dx}{ae^4+cd^4}-\frac{3d^3e^3\sqrt{a+cx^4}}{(d+ex)(ae^4+cd^4)}\right)}{ae^4+cd^4}-\frac{e^3\sqrt{a+cx^4}}{2(d+ex)^2\left(ae^4+cd^4\right)}$$

$\Big\downarrow$ 25

$$\frac{c\left(\frac{\int \frac{3ce^2x^2d^4+3ce^3x^3d^3+\left(cd^4-2ae^4\right)d^2-e\left(2cd^4-ae^4\right)xd}{(d+ex)\sqrt{cx^4+a}}dx}{ae^4+cd^4}-\frac{3d^3e^3\sqrt{a+cx^4}}{(d+ex)(ae^4+cd^4)}\right)}{ae^4+cd^4}-\frac{e^3\sqrt{a+cx^4}}{2(d+ex)^2\left(ae^4+cd^4\right)}$$

$\Big\downarrow$ 2280

$$\frac{c\left(\frac{\int \frac{\left(-e\left(cd^4-2ae^4\right)d^2-e\left(2cd^4-ae^4\right)d^2\right)x}{\left(d^2-e^2x^2\right)\sqrt{cx^4+a}}dx+\int \frac{-3cd^3e^4x^4+\left(3ce^2d^5+e^2\left(2cd^4-ae^4\right)d\right)x^2+d^3\left(cd^4-2ae^4\right)}{\left(d^2-e^2x^2\right)\sqrt{cx^4+a}}dx}{ae^4+cd^4}-\frac{3d^3e^3\sqrt{a+cx^4}}{(d+ex)(ae^4+cd^4)}\right)}{ae^4+cd^4}-\frac{e^3\sqrt{a+cx^4}}{2(d+ex)^2\left(ae^4+cd^4\right)}$$

$\Big\downarrow$ 27

$$\frac{c\left(\frac{\int \frac{-3cd^3e^4x^4+\left(3ce^2d^5+e^2\left(2cd^4-ae^4\right)d\right)x^2+d^3\left(cd^4-2ae^4\right)}{\left(d^2-e^2x^2\right)\sqrt{cx^4+a}}dx-3d^2e\left(cd^4-ae^4\right)\int \frac{x}{\left(d^2-e^2x^2\right)\sqrt{cx^4+a}}dx}{ae^4+cd^4}-\frac{3d^3e^3\sqrt{a+cx^4}}{(d+ex)(ae^4+cd^4)}\right)}{ae^4+cd^4}-\frac{e^3\sqrt{a+cx^4}}{2(d+ex)^2\left(ae^4+cd^4\right)}$$

$\Big\downarrow$ 1577

$$\frac{c\left(\frac{\int \frac{-3cd^3e^4x^4+\left(3ce^2d^5+e^2\left(2cd^4-ae^4\right)d\right)x^2+d^3\left(cd^4-2ae^4\right)}{\left(d^2-e^2x^2\right)\sqrt{cx^4+a}}dx-\frac{3}{2}d^2e\left(cd^4-ae^4\right)\int \frac{1}{\left(d^2-e^2x^2\right)\sqrt{cx^4+a}}dx^2}{ae^4+cd^4}-\frac{3d^3e^3\sqrt{a+cx^4}}{(d+ex)(ae^4+cd^4)}\right)}{ae^4+cd^4}-\frac{e^3\sqrt{a+cx^4}}{2(d+ex)^2\left(ae^4+cd^4\right)}$$

$\Big\downarrow$ 488

$$\frac{c\left(\frac{\frac{3}{2}d^2e\left(cd^4-ae^4\right)\int \frac{1}{cd^4+ae^4-x^4}d\frac{-ae^2-cd^2x^2}{\sqrt{cx^4+a}}+\int \frac{-3cd^3e^4x^4+\left(3ce^2d^5+e^2\left(2cd^4-ae^4\right)d\right)x^2+d^3\left(cd^4-2ae^4\right)}{\left(d^2-e^2x^2\right)\sqrt{cx^4+a}}dx}{ae^4+cd^4}-\frac{3d^3e^3\sqrt{a+cx^4}}{(d+ex)(ae^4+cd^4)}\right)}{ae^4+cd^4}-\frac{e^3\sqrt{a+cx^4}}{2(d+ex)^2\left(ae^4+cd^4\right)}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{c\left(\frac{\int \frac{-3cd^3e^4x^4+\left(3ce^2d^5+e^2\left(2cd^4-ae^4\right)d\right)x^2+d^3\left(cd^4-2ae^4\right)}{\left(d^2-e^2x^2\right)\sqrt{cx^4+a}}dx+\frac{3d^2e\left(cd^4-ae^4\right)\text{arctanh}\left(\frac{-ae^2-cd^2x^2}{\sqrt{a+cx^4}\sqrt{ae^4+cd^4}}\right)}{2\sqrt{ae^4+cd^4}}}{ae^4+cd^4}-\frac{3d^3e^3\sqrt{a+cx^4}}{(d+ex)(ae^4+cd^4)}\right)}{ae^4+cd^4}-\frac{e^3\sqrt{a+cx^4}}{2(d+ex)^2\left(ae^4+cd^4\right)}$$

$$\Big\downarrow \text{ 2233}$$

$$\frac{c\left(\frac{-3\sqrt{a}\sqrt{c}d^3e^2\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{a}\sqrt{cx^4+a}}dx-\frac{\int -\frac{cde^2\left(\left(cd^4+3\sqrt{a}\sqrt{c}e^2d^2-2ae^4\right)d^2+e^2\left(2cd^4-3\sqrt{a}\sqrt{c}e^2d^2-ae^4\right)x^2\right)}{\left(d^2-e^2x^2\right)\sqrt{cx^4+a}}dx}{ce^2}+\frac{3d^2e\left(cd^4-ae^4\right)\text{arctanh}\left(\frac{-ae^2-cd^2x^2}{\sqrt{a+cx^4}\sqrt{ae^4+cd^4}}\right)}{2\sqrt{ae^4+cd^4}}}{ae^4+cd^4}-\frac{3d^3e^3\sqrt{a+cx^4}}{(d+ex)(ae^4+cd^4)}\right)}{ae^4+cd^4}-\frac{e^3\sqrt{a+cx^4}}{2(d+ex)^2\left(ae^4+cd^4\right)}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{c\left(\frac{-3\sqrt{a}\sqrt{c}d^3e^2\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{a}\sqrt{cx^4+a}}dx+\frac{\int \frac{cde^2\left(\left(cd^4+3\sqrt{a}\sqrt{c}e^2d^2-2ae^4\right)d^2+e^2\left(2cd^4-3\sqrt{a}\sqrt{c}e^2d^2-ae^4\right)x^2\right)}{\left(d^2-e^2x^2\right)\sqrt{cx^4+a}}dx}{ce^2}+\frac{3d^2e\left(cd^4-ae^4\right)\text{arctanh}\left(\frac{-ae^2-cd^2x^2}{\sqrt{a+cx^4}\sqrt{ae^4+cd^4}}\right)}{2\sqrt{ae^4+cd^4}}}{ae^4+cd^4}-\frac{3d^3e^3\sqrt{a+cx^4}}{(d+ex)(ae^4+cd^4)}\right)}{ae^4+cd^4}-\frac{e^3\sqrt{a+cx^4}}{2(d+ex)^2\left(ae^4+cd^4\right)}$$

$$\Big\downarrow \text{ 27}$$

$$c\left(\frac{-3\sqrt{c}d^3e^2\int\frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+a}}dx+d\int\frac{\left(cd^4+3\sqrt{a}\sqrt{c}e^2d^2-2ae^4\right)d^2+e^2\left(2cd^4-3\sqrt{a}\sqrt{c}e^2d^2-ae^4\right)x^2}{\left(d^2-e^2x^2\right)\sqrt{cx^4+a}}dx+\frac{3d^2e\left(cd^4-ae^4\right)\text{arctanh}\left(\frac{-ae^2-cd^2x^2}{\sqrt{a+cx^4}\sqrt{ae^4+cd^4}}\right)}{2\sqrt{ae^4+cd^4}}}{ae^4+cd^4}\right.$$

$$\overline{\qquad ae^4+cd^4 \qquad}$$

$$\frac{e^3\sqrt{a+cx^4}}{2(d+ex)^2\left(ae^4+cd^4\right)}$$

$\downarrow$ 1510

$$c\left(\frac{d\int\frac{\left(cd^4+3\sqrt{a}\sqrt{c}e^2d^2-2ae^4\right)d^2+e^2\left(2cd^4-3\sqrt{a}\sqrt{c}e^2d^2-ae^4\right)x^2}{\left(d^2-e^2x^2\right)\sqrt{cx^4+a}}dx-3\sqrt{c}d^3e^2\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt[4]{c}\sqrt{a+cx^4}}-\frac{x\sqrt{a+c}}{\sqrt{a}+\sqrt{c}}\right.}{ae^4+cd^4}\right.$$

$$\overline{\qquad ae^4+cd^4 \qquad}$$

$$\frac{e^3\sqrt{a+cx^4}}{2(d+ex)^2\left(ae^4+cd^4\right)}$$

$\downarrow$ 2227

$$c\left(\frac{d\left(\left(ae^4+cd^4\right)\int\frac{1}{\sqrt{cx^4+a}}dx+3\sqrt{a}d^2e^2\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\int\frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a}\left(d^2-e^2x^2\right)\sqrt{cx^4+a}}dx\right)-3\sqrt{c}d^3e^2\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}}{\sqrt[4]{}}\right.\right.}{\sqrt[4]{c}\sqrt{a+cx^4}}\right.}{ae^4+cd^4}\right.$$

$$\overline{\qquad ae^4+cd^4 \qquad}$$

$$\frac{e^3\sqrt{a+cx^4}}{2(d+ex)^2\left(ae^4+cd^4\right)}$$

$\downarrow$ 27

$$\frac{c\left(\frac{d\left(\left(ae^4+cd^4\right)\int \frac{1}{\sqrt{cx^4+a}}dx+3d^2e^2\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\int \frac{\sqrt{c}x^2+\sqrt{a}}{\left(d^2-e^2x^2\right)\sqrt{cx^4+a}}dx\right)-3\sqrt{c}d^3e^2\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\right.}{\sqrt[4]{c}\sqrt{a+cx^4}}\right.}{ae^4+cd^4}\right.}{ae^4+cd^4}$$

$$\frac{e^3\sqrt{a+cx^4}}{2(d+ex)^2\left(ae^4+cd^4\right)}$$

$\downarrow$ 761

$$\frac{c\left(\frac{d\left(3d^2e^2\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\int \frac{\sqrt{c}x^2+\sqrt{a}}{\left(d^2-e^2x^2\right)\sqrt{cx^4+a}}dx+\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(ae^4+cd^4\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+cx^4}}\right)-3\sqrt{c}d^3e^2\left(\frac{\sqrt[4]{a}(\sqrt{}}{}\right.}{ae^4+cd^4}\right.}{ae^4+cd}$$

$$\frac{e^3\sqrt{a+cx^4}}{2(d+ex)^2\left(ae^4+cd^4\right)}$$

$\downarrow$ 2223

$$\frac{c\left(d\left(3d^2e^2\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\left(\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{\sqrt{a}}{d^2}-\frac{\sqrt{c}}{e^2}\right)\text{EllipticPi}\left(\frac{\left(\sqrt{c}d^2+\sqrt{a}e^2\right)^2}{4\sqrt{a}\sqrt{c}d^2e^2},2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+cx^4}}+\frac{\left(\sqrt{a}e^2+\sqrt{c}d^2\right)\text{arctanh}\left(\right.}{2de\sqrt{ae^4+cd^4}}\right.\right.\right.}{}$$

$$\frac{e^3\sqrt{a+cx^4}}{2(d+ex)^2\left(ae^4+cd^4\right)}$$

input
```
Int[1/((d + e*x)^3*Sqrt[a + c*x^4]),x]
```

output
```
-1/2*(e^3*Sqrt[a + c*x^4])/((c*d^4 + a*e^4)*(d + e*x)^2) + (c*((-3*d^3*e^3
*Sqrt[a + c*x^4])/((c*d^4 + a*e^4)*(d + e*x)) + ((3*d^2*e*(c*d^4 - a*e^4)*
ArcTanh[(-(a*e^2) - c*d^2*x^2)/(Sqrt[c*d^4 + a*e^4]*Sqrt[a + c*x^4])])/(2*
Sqrt[c*d^4 + a*e^4]) - 3*Sqrt[c]*d^3*e^2*(-((x*Sqrt[a + c*x^4])/(Sqrt[a] +
 Sqrt[c]*x^2)) + (a^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a
] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(c^(1/4
)*Sqrt[a + c*x^4])) + d*(((c*d^4 + a*e^4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a
+ c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)
], 1/2])/(2*a^(1/4)*c^(1/4)*Sqrt[a + c*x^4]) + 3*d^2*e^2*(Sqrt[c]*d^2 - Sq
rt[a]*e^2)*(((Sqrt[c]*d^2 + Sqrt[a]*e^2)*ArcTanh[(Sqrt[c*d^4 + a*e^4]*x)/(
d*e*Sqrt[a + c*x^4])])/(2*d*e*Sqrt[c*d^4 + a*e^4]) + ((Sqrt[a]/d^2 - Sqrt[
c]/e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2
]*EllipticPi[(Sqrt[c]*d^2 + Sqrt[a]*e^2)^2/(4*Sqrt[a]*Sqrt[c]*d^2*e^2), 2*
ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(1/4)*c^(1/4)*Sqrt[a + c*x^4]))))/
(c*d^4 + a*e^4)))/(c*d^4 + a*e^4)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1577
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, c, d, e, p, q}, x]
```

rule 2223
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTanh[Rt[(-c)*
(d/e) - a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[(-c)*(d/e) - a*(e/d), 2]
)), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^
2)]/(4*d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*Arc
Tan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[c*(d/e) + a*(e
/d)]
```

rule 2227
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[c/a, 2]}, Simp[(A*(c*d + a*e*q) - a*B*(e + d*q)
)/(c*d^2 - a*e^2)   Int[1/Sqrt[a + c*x^4], x], x] + Simp[a*(B*d - A*e)*((e
+ d*q)/(c*d^2 - a*e^2))   Int[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + c*x^4]), x]
, x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
 && NeQ[c*A^2 - a*B^2, 0]
```

rule 2233
```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :>
With[{q = Rt[c/a, 2], A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff
[P4x, x, 4]}, Simp[-C/(e*q)   Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] + Sim
p[1/(c*e)   Int[(A*c*e + a*C*d*q + (B*c*e - C*(c*d - a*e*q))*x^2)/((d + e*x
^2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[P4x, x^2,
2] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

rule 2265
```
Int[((d_) + (e_.)*(x_))^(q_)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[e
^3*(d + e*x)^(q + 1)*(Sqrt[a + c*x^4]/((q + 1)*(c*d^4 + a*e^4))), x] + Simp
[c/((q + 1)*(c*d^4 + a*e^4))   Int[((d + e*x)^(q + 1)/Sqrt[a + c*x^4])*Simp
[d^3*(q + 1) - d^2*e*(q + 1)*x + d*e^2*(q + 1)*x^2 - e^3*(q + 3)*x^3, x], x
], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^4 + a*e^4, 0] && ILtQ[q, -1]
```

rule 2277
```
Int[((Px_)*((d_) + (e_.)*(x_))^(q_))/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :
> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1], C = Coeff[Px, x, 2], D =
Coeff[Px, x, 3]}, Simp[(-(d^3*D - C*d^2*e + B*d*e^2 - A*e^3))*(d + e*x)^(q
+ 1)*(Sqrt[a + c*x^4]/((q + 1)*(c*d^4 + a*e^4))), x] + Simp[1/((q + 1)*(c*d
^4 + a*e^4))   Int[((d + e*x)^(q + 1)/Sqrt[a + c*x^4])*Simp[(q + 1)*(a*e*(d
^2*D - C*d*e + B*e^2) + A*d*(c*d^2)) - (e*(q + 1)*(A*c*d^2 + a*e*(d*D - C*e
)) - B*d*(c*d^2*(q + 1)))*x + (q + 1)*(D*e*(a*e^2) + c*d*(C*d^2 - e*(B*d -
A*e)))*x^2 + c*(q + 3)*(d^3*D - C*d^2*e + B*d*e^2 - A*e^3)*x^3, x], x], x]]
 /; FreeQ[{a, c, d, e}, x] && PolyQ[Px, x] && LeQ[Expon[Px, x], 3] && NeQ[c
*d^4 + a*e^4, 0] && LtQ[q, -1]
```

rule 2280
```
Int[(Px_)/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Wit
h[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1], C = Coeff[Px, x, 2], D = Coeff
[Px, x, 3]}, Int[(x*(B*d - A*e + (d*D - C*e)*x^2))/((d^2 - e^2*x^2)*Sqrt[a
+ c*x^4]), x] + Int[(A*d + (C*d - B*e)*x^2 - D*e*x^4)/((d^2 - e^2*x^2)*Sqrt
[a + c*x^4]), x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Px, x] && LeQ[Expon[Px
, x], 3] && NeQ[c*d^4 + a*e^4, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.64 (sec) , antiderivative size = 483, normalized size of antiderivative = 0.74

method	result
default	$-\frac{e^3\sqrt{c\,x^4+a}}{2(e^4a+c\,d^4)(ex+d)^2}-\frac{3c\,d^3e^3\sqrt{c\,x^4+a}}{(e^4a+c\,d^4)^2(ex+d)}+\frac{cd(e^4a-2c\,d^4)\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{(e^4a+c\,d^4)^2\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\sqrt{c\,x^4+a}}+\frac{3id^3e^2c^{\frac{3}{2}}\sqrt{}}{}$
elliptic	$-\frac{e^3\sqrt{c\,x^4+a}}{2(e^4a+c\,d^4)(ex+d)^2}-\frac{3c\,d^3e^3\sqrt{c\,x^4+a}}{(e^4a+c\,d^4)^2(ex+d)}+\frac{cd(e^4a-2c\,d^4)\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{(e^4a+c\,d^4)^2\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\sqrt{c\,x^4+a}}+\frac{3id^3e^2c^{\frac{3}{2}}\sqrt{}}{}$

input
```
int(1/(e*x+d)^3/(c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*e^3*(c*x^4+a)^(1/2)/(a*e^4+c*d^4)/(e*x+d)^2-3*c*d^3*e^3*(c*x^4+a)^(1/
2)/(a*e^4+c*d^4)^2/(e*x+d)+c*d*(a*e^4-2*c*d^4)/(a*e^4+c*d^4)^2/(I*c^(1/2)/
a^(1/2))^(1/2)*(1-I*c^(1/2)*x^2/a^(1/2))^(1/2)*(1+I*c^(1/2)*x^2/a^(1/2))^(
1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I*c^(1/2)/a^(1/2))^(1/2),I)+3*I*d^3*e^2*
c^(3/2)/(a*e^4+c*d^4)^2*a^(1/2)/(I*c^(1/2)/a^(1/2))^(1/2)*(1-I*c^(1/2)*x^2
/a^(1/2))^(1/2)*(1+I*c^(1/2)*x^2/a^(1/2))^(1/2)/(c*x^4+a)^(1/2)*(EllipticF
(x*(I*c^(1/2)/a^(1/2))^(1/2),I)-EllipticE(x*(I*c^(1/2)/a^(1/2))^(1/2),I))-
3*c*d^2*(a*e^4-c*d^4)/(a*e^4+c*d^4)^2/e*(-1/2/(a+c*d^4/e^4)^(1/2)*arctanh(
1/2*(2*c*x^2*d^2/e^2+2*a)/(a+c*d^4/e^4)^(1/2)/(c*x^4+a)^(1/2))+1/(I*c^(1/2
)/a^(1/2))^(1/2)/d*e*(1-I*c^(1/2)*x^2/a^(1/2))^(1/2)*(1+I*c^(1/2)*x^2/a^(1
/2))^(1/2)/(c*x^4+a)^(1/2)*EllipticPi(x*(I*c^(1/2)/a^(1/2))^(1/2),-I/c^(1/
2)*a^(1/2)/d^2*e^2,(-I/a^(1/2)*c^(1/2))^(1/2)/(I*c^(1/2)/a^(1/2))^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^3\sqrt{a+cx^4}}\,dx=\text{Timed out}$$

input
```
integrate(1/(e*x+d)^3/(c*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(d+ex)^3\sqrt{a+cx^4}}\,dx = \int \frac{1}{\sqrt{a+cx^4}\,(d+ex)^3}\,dx$$

input
```
integrate(1/(e*x+d)**3/(c*x**4+a)**(1/2),x)
```

output
```
Integral(1/(sqrt(a + c*x**4)*(d + e*x)**3), x)
```

Maxima [F]

$$\int \frac{1}{(d+ex)^3\sqrt{a+cx^4}}\,dx = \int \frac{1}{\sqrt{cx^4+a}(ex+d)^3}\,dx$$

input
```
integrate(1/(e*x+d)^3/(c*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(c*x^4 + a)*(e*x + d)^3), x)
```

Giac [F]

$$\int \frac{1}{(d+ex)^3\sqrt{a+cx^4}}\,dx = \int \frac{1}{\sqrt{cx^4+a}(ex+d)^3}\,dx$$

input
```
integrate(1/(e*x+d)^3/(c*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(c*x^4 + a)*(e*x + d)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^3\sqrt{a+cx^4}}\,dx = \int \frac{1}{\sqrt{c\,x^4+a}\,(d+e\,x)^3}\,dx$$

input
```
int(1/((a + c*x^4)^(1/2)*(d + e*x)^3),x)
```

output
```
int(1/((a + c*x^4)^(1/2)*(d + e*x)^3), x)
```

Reduce [F]

$$\int \frac{1}{(d+ex)^3\sqrt{a+cx^4}}\,dx = \int \frac{1}{(ex+d)^3\,\sqrt{c\,x^4+a}}dx$$

input
```
int(1/(e*x+d)^3/(c*x^4+a)^(1/2),x)
```

output
```
int(1/(e*x+d)^3/(c*x^4+a)^(1/2),x)
```

3.211 $\int \frac{(d+ex)^3}{(a+cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 307

$$\int \frac{(d+ex)^3}{(a+cx^4)^{3/2}}\,dx = -\frac{e^3}{2c\sqrt{a+cx^4}} + \frac{x(d^3+3d^2ex+3de^2x^2)}{2a\sqrt{a+cx^4}} - \frac{3de^2x\sqrt{a+cx^4}}{2a\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)} + \frac{3de^2\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{3/4}c^{3/4}\sqrt{a+cx^4}} + \frac{d\left(\sqrt{c}d^2-3\sqrt{a}e^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4a^{5/4}c^{3/4}\sqrt{a+cx^4}}$$

output
```
-1/2*e^3/c/(c*x^4+a)^(1/2)+1/2*x*(3*d*e^2*x^2+3*d^2*e*x+d^3)/a/(c*x^4+a)^(
1/2)-3/2*d*e^2*x*(c*x^4+a)^(1/2)/a/c^(1/2)/(a^(1/2)+c^(1/2)*x^2)+3/2*d*e^2
*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*EllipticE
(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))/a^(3/4)/c^(3/4)/(c*x^4+a)^(
1/2)+1/4*d*(c^(1/2)*d^2-3*a^(1/2)*e^2)*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+a)/(a
^(1/2)+c^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(c^(1/4)*x/a^(1/4)),1
/2*2^(1/2))/a^(5/4)/c^(3/4)/(c*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.11 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.41

$$\int \frac{(d+ex)^3}{(a+cx^4)^{3/2}}\,dx = \frac{-ae^3+cd^3x+3cd^2ex^2+cd^3x\sqrt{1+\frac{cx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{cx^4}{a}\right)+2cde^2}{2ac\sqrt{a+cx^4}}$$

input
```
Integrate[(d + e*x)^3/(a + c*x^4)^(3/2),x]
```

output
```
(-(a*e^3) + c*d^3*x + 3*c*d^2*e*x^2 + c*d^3*x*Sqrt[1 + (c*x^4)/a]*Hypergeo
metric2F1[1/4, 1/2, 5/4, -((c*x^4)/a)] + 2*c*d*e^2*x^3*Sqrt[1 + (c*x^4)/a]
*Hypergeometric2F1[3/4, 3/2, 7/4, -((c*x^4)/a)])/(2*a*c*Sqrt[a + c*x^4])
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 295, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {2393, 25, 27, 1512, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^3}{(a+cx^4)^{3/2}}\,dx$$

$$\Big\downarrow \text{2393}$$

$$-\frac{\int -\frac{d(d^2-3e^2x^2)}{\sqrt{cx^4+a}}dx}{2a}-\frac{ae^3-cx\left(d^3+3d^2ex+3de^2x^2\right)}{2ac\sqrt{a+cx^4}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{d(d^2-3e^2x^2)}{\sqrt{cx^4+a}}dx}{2a}-\frac{ae^3-cx\left(d^3+3d^2ex+3de^2x^2\right)}{2ac\sqrt{a+cx^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{d\int \frac{d^2-3e^2x^2}{\sqrt{cx^4+a}}dx}{2a}-\frac{ae^3-cx\left(d^3+3d^2ex+3de^2x^2\right)}{2ac\sqrt{a+cx^4}}$$

$\big\downarrow$ 1512

$$\frac{d\left(\left(d^2-\frac{3\sqrt{a}e^2}{\sqrt{c}}\right)\int \frac{1}{\sqrt{cx^4+a}}dx+\frac{3\sqrt{a}e^2\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{a}\sqrt{cx^4+a}}dx}{\sqrt{c}}\right)}{2a}-\frac{ae^3-cx\left(d^3+3d^2ex+3de^2x^2\right)}{2ac\sqrt{a+cx^4}}$$

$\big\downarrow$ 27

$$\frac{d\left(\left(d^2-\frac{3\sqrt{a}e^2}{\sqrt{c}}\right)\int \frac{1}{\sqrt{cx^4+a}}dx+\frac{3e^2\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+a}}dx}{\sqrt{c}}\right)}{2a}-\frac{ae^3-cx\left(d^3+3d^2ex+3de^2x^2\right)}{2ac\sqrt{a+cx^4}}$$

$\big\downarrow$ 761

$$\frac{d\left(\frac{3e^2\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+a}}dx}{\sqrt{c}}+\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(d^2-\frac{3\sqrt{a}e^2}{\sqrt{c}}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+cx^4}}\right)}{2a}-\frac{ae^3-cx\left(d^3+3d^2ex+3de^2x^2\right)}{2ac\sqrt{a+cx^4}}$$

$\big\downarrow$ 1510

$$\frac{d\left(\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(d^2-\frac{3\sqrt{a}e^2}{\sqrt{c}}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+cx^4}}+\frac{3e^2\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right.}{\sqrt[4]{c}\sqrt{a+cx^4}}\right.}{\sqrt{c}}\right.}{2a}$$
$$\frac{ae^3-cx\left(d^3+3d^2ex+3de^2x^2\right)}{2ac\sqrt{a+cx^4}}$$

input
```
Int[(d + e*x)^3/(a + c*x^4)^(3/2),x]
```

output
```
-1/2*(a*e^3 - c*x*(d^3 + 3*d^2*e*x + 3*d*e^2*x^2))/(a*c*Sqrt[a + c*x^4]) +
 (d*((3*e^2*(-((x*Sqrt[a + c*x^4])/(Sqrt[a] + Sqrt[c]*x^2)) + (a^(1/4)*(Sq
rt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE
[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(c^(1/4)*Sqrt[a + c*x^4])))/Sqrt[c]
+ ((d^2 - (3*Sqrt[a]*e^2)/Sqrt[c])*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4
)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2]
)/(2*a^(1/4)*c^(1/4)*Sqrt[a + c*x^4])))/(2*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1512
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + c*x^4], x], x] - Simp[e/q
 Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c
, d, e}, x] && PosQ[c/a]
```

rule 2393
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   In
t[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(
p + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n
, 0] && LtQ[p, -1]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.83 (sec) , antiderivative size = 245, normalized size of antiderivative = 0.80

method	result
elliptic	$-\frac{2c\left(-\frac{3d e^2 x^3}{4ac}-\frac{3d^2 e x^2}{4ca}-\frac{d^3 x}{4ac}+\frac{e^3}{4c^2}\right)}{\sqrt{c\left(\frac{a}{c}+x^4\right)}}+\frac{d^3\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{2a\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}-\frac{3id\,e^2\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,(\mathrm{E}}{2\sqrt{a}\,}$
default	$d^3\left(\frac{x}{2a\sqrt{c\left(\frac{a}{c}+x^4\right)}}+\frac{\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{2a\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}\right)-\frac{e^3}{2c\sqrt{c\,x^4+a}}+3d\,e^2\left(\frac{x^3}{2a\sqrt{c\left(\frac{a}{c}+x^4\right)}}-\frac{i\sqrt{1-}}{}\right.$

input
```
int((e*x+d)^3/(c*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*c*(-3/4*d*e^2/a/c*x^3-3/4*d^2*e/c/a*x^2-1/4*d^3/a/c*x+1/4*e^3/c^2)/(c*(
a/c+x^4))^(1/2)+1/2*d^3/a/(I*c^(1/2)/a^(1/2))^(1/2)*(1-I*c^(1/2)*x^2/a^(1/
2))^(1/2)*(1+I*c^(1/2)*x^2/a^(1/2))^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I*c
^(1/2)/a^(1/2))^(1/2),I)-3/2*I*d*e^2/a^(1/2)/(I*c^(1/2)/a^(1/2))^(1/2)*(1-
I*c^(1/2)*x^2/a^(1/2))^(1/2)*(1+I*c^(1/2)*x^2/a^(1/2))^(1/2)/(c*x^4+a)^(1/
2)/c^(1/2)*(EllipticF(x*(I*c^(1/2)/a^(1/2))^(1/2),I)-EllipticE(x*(I*c^(1/2
)/a^(1/2))^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.52

$$\int \frac{(d+ex)^3}{(a+cx^4)^{3/2}}\,dx = \frac{3\,(cde^2x^4+ade^2)\sqrt{a}\left(-\frac{c}{a}\right)^{\frac{3}{4}} E\left(\arcsin\left(x\left(-\frac{c}{a}\right)^{\frac{1}{4}}\right)|-1\right) - \left((cd^3+3\,cde^2)x^4+ad^3+3\right.}{2\,(ac^2}$$

input
```
integrate((e*x+d)^3/(c*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
1/2*(3*(c*d*e^2*x^4 + a*d*e^2)*sqrt(a)*(-c/a)^(3/4)*elliptic_e(arcsin(x*(-
c/a)^(1/4)), -1) - ((c*d^3 + 3*c*d*e^2)*x^4 + a*d^3 + 3*a*d*e^2)*sqrt(a)*(
-c/a)^(3/4)*elliptic_f(arcsin(x*(-c/a)^(1/4)), -1) + (3*c*d*e^2*x^3 + 3*c*
d^2*e*x^2 + c*d^3*x - a*e^3)*sqrt(c*x^4 + a))/(a*c^2*x^4 + a^2*c)
```

Sympy [F]

$$\int \frac{(d+ex)^3}{(a+cx^4)^{3/2}}\,dx = \int \frac{(d+ex)^3}{(a+cx^4)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x+d)**3/(c*x**4+a)**(3/2),x)
```

output
```
Integral((d + e*x)**3/(a + c*x**4)**(3/2), x)
```

Maxima [F]

$$\int \frac{(d+ex)^3}{(a+cx^4)^{3/2}}\,dx = \int \frac{(ex+d)^3}{(cx^4+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x+d)^3/(c*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((e*x + d)^3/(c*x^4 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{(d+ex)^3}{(a+cx^4)^{3/2}}\,dx = \int \frac{(ex+d)^3}{(cx^4+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x+d)^3/(c*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((e*x + d)^3/(c*x^4 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^3}{(a+cx^4)^{3/2}}\,dx = \int \frac{(d+e\,x)^3}{(c\,x^4+a)^{3/2}}\,dx$$

input
```
int((d + e*x)^3/(a + c*x^4)^(3/2),x)
```

output
```
int((d + e*x)^3/(a + c*x^4)^(3/2), x)
```

Reduce [F]

$$\int \frac{(d+ex)^3}{(a+cx^4)^{3/2}}\,dx = \frac{-\sqrt{c}\,\sqrt{c\,x^4+a}\,a^2e^3+9\sqrt{c}\,\sqrt{c\,x^4+a}\,ac\,d^2e\,x^2-2\sqrt{c}\,\sqrt{c\,x^4+a}\,ac\,e^3x^4+12\sqrt{c}\,\sqrt{c\,x^4}}{}$$

input
```
int((e*x+d)^3/(c*x^4+a)^(3/2),x)
```

output
```
( - sqrt(c)*sqrt(a + c*x**4)*a**2*e**3 + 9*sqrt(c)*sqrt(a + c*x**4)*a*c*d*
*2*e*x**2 - 2*sqrt(c)*sqrt(a + c*x**4)*a*c*e**3*x**4 + 12*sqrt(c)*sqrt(a +
 c*x**4)*c**2*d**2*e*x**6 + 4*sqrt(a + c*x**4)*int(sqrt(a + c*x**4)/(a**2
+ 2*a*c*x**4 + c**2*x**8),x)*a**2*c**2*d**3*x**2 + 4*sqrt(a + c*x**4)*int(
sqrt(a + c*x**4)/(a**2 + 2*a*c*x**4 + c**2*x**8),x)*a*c**3*d**3*x**6 + 12*
sqrt(a + c*x**4)*int((sqrt(a + c*x**4)*x**2)/(a**2 + 2*a*c*x**4 + c**2*x**
8),x)*a**2*c**2*d*e**2*x**2 + 12*sqrt(a + c*x**4)*int((sqrt(a + c*x**4)*x*
*2)/(a**2 + 2*a*c*x**4 + c**2*x**8),x)*a*c**3*d*e**2*x**6 + 2*sqrt(c)*int(
sqrt(a + c*x**4)/(a**2 + 2*a*c*x**4 + c**2*x**8),x)*a**3*c*d**3 + 6*sqrt(c
)*int(sqrt(a + c*x**4)/(a**2 + 2*a*c*x**4 + c**2*x**8),x)*a**2*c**2*d**3*x
**4 + 4*sqrt(c)*int(sqrt(a + c*x**4)/(a**2 + 2*a*c*x**4 + c**2*x**8),x)*a*
c**3*d**3*x**8 + 6*sqrt(c)*int((sqrt(a + c*x**4)*x**2)/(a**2 + 2*a*c*x**4
+ c**2*x**8),x)*a**3*c*d*e**2 + 18*sqrt(c)*int((sqrt(a + c*x**4)*x**2)/(a*
*2 + 2*a*c*x**4 + c**2*x**8),x)*a**2*c**2*d*e**2*x**4 + 12*sqrt(c)*int((sq
rt(a + c*x**4)*x**2)/(a**2 + 2*a*c*x**4 + c**2*x**8),x)*a*c**3*d*e**2*x**8
 + 3*a**2*c*d**2*e - 2*a**2*c*e**3*x**2 + 15*a*c**2*d**2*e*x**4 - 2*a*c**2
*e**3*x**6 + 12*c**3*d**2*e*x**8)/(2*a*c*(2*sqrt(a + c*x**4)*a*c*x**2 + 2*
sqrt(a + c*x**4)*c**2*x**6 + sqrt(c)*a**2 + 3*sqrt(c)*a*c*x**4 + 2*sqrt(c)
*c**2*x**8))
```

3.212 $\int \frac{(d+ex)^2}{(a+cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 270

$$\int \frac{(d+ex)^2}{(a+cx^4)^{3/2}}\,dx = \frac{x(d+ex)^2}{2a\sqrt{a+cx^4}} - \frac{e^2x\sqrt{a+cx^4}}{2a\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)} + \frac{e^2\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{3/4}c^{3/4}\sqrt{a+cx^4}} + \frac{\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4a^{5/4}c^{3/4}\sqrt{a+cx^4}}$$

output
```
1/2*x*(e*x+d)^2/a/(c*x^4+a)^(1/2)-1/2*e^2*x*(c*x^4+a)^(1/2)/a/c^(1/2)/(a^(
1/2)+c^(1/2)*x^2)+1/2*e^2*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+a)/(a^(1/2)+c^(1/2
)*x^2)^2)^(1/2)*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))/a^
(3/4)/c^(3/4)/(c*x^4+a)^(1/2)+1/4*(c^(1/2)*d^2-a^(1/2)*e^2)*(a^(1/2)+c^(1/
2)*x^2)*((c*x^4+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan
(c^(1/4)*x/a^(1/4)),1/2*2^(1/2))/a^(5/4)/c^(3/4)/(c*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.10 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.40

$$\int \frac{(d+ex)^2}{(a+cx^4)^{3/2}}\,dx = \frac{x\left(3d(d+2ex)+3d^2\sqrt{1+\frac{cx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{cx^4}{a}\right)+2e^2x^2\sqrt{1+\frac{cx^4}{a}}\right.}{6a\sqrt{a+cx^4}}$$

input
```
Integrate[(d + e*x)^2/(a + c*x^4)^(3/2),x]
```

output
```
(x*(3*d*(d + 2*e*x) + 3*d^2*Sqrt[1 + (c*x^4)/a]*Hypergeometric2F1[1/4, 1/2
, 5/4, -((c*x^4)/a)] + 2*e^2*x^2*Sqrt[1 + (c*x^4)/a]*Hypergeometric2F1[3/4
, 3/2, 7/4, -((c*x^4)/a)]))/(6*a*Sqrt[a + c*x^4])
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 268, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {2394, 25, 1512, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^2}{(a+cx^4)^{3/2}}\,dx$$

$$\Big\downarrow \; 2394$$

$$\frac{x(d+ex)^2}{2a\sqrt{a+cx^4}} - \frac{\int -\frac{d^2-e^2x^2}{\sqrt{cx^4+a}}dx}{2a}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{d^2-e^2x^2}{\sqrt{cx^4+a}}dx}{2a} + \frac{x(d+ex)^2}{2a\sqrt{a+cx^4}}$$

$$\Big\downarrow \; 1512$$

$$\frac{\left(d^2-\frac{\sqrt{a}e^2}{\sqrt{c}}\right)\int \frac{1}{\sqrt{cx^4+a}}dx+\frac{\sqrt{a}e^2\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{a}\sqrt{cx^4+a}}dx}{\sqrt{c}}}{2a}+\frac{x(d+ex)^2}{2a\sqrt{a+cx^4}}$$

$$\Big\downarrow \; 27$$

$$\frac{\left(d^2-\frac{\sqrt{a}e^2}{\sqrt{c}}\right)\int \frac{1}{\sqrt{cx^4+a}}dx+\frac{e^2\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+a}}dx}{\sqrt{c}}}{2a}+\frac{x(d+ex)^2}{2a\sqrt{a+cx^4}}$$

$$\Big\downarrow \; 761$$

$$\frac{\frac{e^2\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+a}}dx}{\sqrt{c}}+\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(d^2-\frac{\sqrt{a}e^2}{\sqrt{c}}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+cx^4}}}{2a}+\frac{x(d+ex)^2}{2a\sqrt{a+cx^4}}$$

$$\Big\downarrow \; 1510$$

$$\frac{\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(d^2-\frac{\sqrt{a}e^2}{\sqrt{c}}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+cx^4}}+\frac{e^2\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt[4]{c}\sqrt{a+cx^4}}-\frac{x\cdots}{\sqrt{\cdots}}\right.}{\sqrt{c}}}{2a}+$$

$$\frac{x(d+ex)^2}{2a\sqrt{a+cx^4}}$$

input

```
Int[(d + e*x)^2/(a + c*x^4)^(3/2),x]
```

output

```
(x*(d + e*x)^2)/(2*a*Sqrt[a + c*x^4]) + ((e^2*(-((x*Sqrt[a + c*x^4])/(Sqrt
[a] + Sqrt[c]*x^2)) + (a^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(S
qrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(c
^(1/4)*Sqrt[a + c*x^4])))/Sqrt[c] + ((d^2 - (Sqrt[a]*e^2)/Sqrt[c])*(Sqrt[a
] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*A
rcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*c^(1/4)*Sqrt[a + c*x^4]))/(2*
a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1512
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + c*x^4], x], x] - Simp[e/q
 Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c
, d, e}, x] && PosQ[c/a]
```

rule 2394
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   Int[ExpandToSum[n
*(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x
] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.77 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.86

method	result
elliptic	$-\frac{2c\left(-\frac{e^{2}x^{3}}{4ac}-\frac{de\,x^{2}}{2ca}-\frac{d^{2}x}{4ac}\right)}{\sqrt{c\left(\frac{a}{c}+x^{4}\right)}}+\frac{d^{2}\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{2a\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}}-\frac{ie^{2}\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\left(\operatorname{EllipticF}\left(x\sqrt{\right.\right.}{2\sqrt{a}\,\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}}}$
default	$d^{2}\left(\frac{x}{2a\sqrt{c\left(\frac{a}{c}+x^{4}\right)}}+\frac{\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{2a\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}}\right)+e^{2}\left(\frac{x^{3}}{2a\sqrt{c\left(\frac{a}{c}+x^{4}\right)}}-\frac{i\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\left(\right.}{2}\right.$

input
```
int((e*x+d)^2/(c*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*c*(-1/4*e^2/a/c*x^3-1/2*d*e/c/a*x^2-1/4*d^2/a/c*x)/(c*(a/c+x^4))^(1/2)+
1/2*d^2/a/(I*c^(1/2)/a^(1/2))^(1/2)*(1-I*c^(1/2)*x^2/a^(1/2))^(1/2)*(1+I*c
^(1/2)*x^2/a^(1/2))^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I*c^(1/2)/a^(1/2))^
(1/2),I)-1/2*I*e^2/a^(1/2)/(I*c^(1/2)/a^(1/2))^(1/2)*(1-I*c^(1/2)*x^2/a^(1
/2))^(1/2)*(1+I*c^(1/2)*x^2/a^(1/2))^(1/2)/(c*x^4+a)^(1/2)/c^(1/2)*(Ellipt
icF(x*(I*c^(1/2)/a^(1/2))^(1/2),I)-EllipticE(x*(I*c^(1/2)/a^(1/2))^(1/2),I
))
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.53

$$\int \frac{(d+ex)^2}{(a+cx^4)^{3/2}}\,dx=\frac{(ce^2x^4+ae^2)\sqrt{a}\left(-\frac{c}{a}\right)^{\frac{3}{4}}E\left(\arcsin\left(x\left(-\frac{c}{a}\right)^{\frac{1}{4}}\right)\mid-1\right)-((cd^2+ce^2)x^4+ad^2+ae^2)\sqrt{a}(}{2\left(ac^2x^4+a^2c\right)}$$

input
```
integrate((e*x+d)^2/(c*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
1/2*((c*e^2*x^4 + a*e^2)*sqrt(a)*(-c/a)^(3/4)*elliptic_e(arcsin(x*(-c/a)^(
1/4)), -1) - ((c*d^2 + c*e^2)*x^4 + a*d^2 + a*e^2)*sqrt(a)*(-c/a)^(3/4)*el
liptic_f(arcsin(x*(-c/a)^(1/4)), -1) + (c*e^2*x^3 + 2*c*d*e*x^2 + c*d^2*x)
*sqrt(c*x^4 + a))/(a*c^2*x^4 + a^2*c)
```

Sympy [F]

$$\int \frac{(d+ex)^2}{(a+cx^4)^{3/2}}\,dx = \int \frac{(d+ex)^2}{(a+cx^4)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x+d)**2/(c*x**4+a)**(3/2),x)
```

output
```
Integral((d + e*x)**2/(a + c*x**4)**(3/2), x)
```

Maxima [F]

$$\int \frac{(d+ex)^2}{(a+cx^4)^{3/2}}\,dx = \int \frac{(ex+d)^2}{(cx^4+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x+d)^2/(c*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((e*x + d)^2/(c*x^4 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{(d+ex)^2}{(a+cx^4)^{3/2}}\,dx = \int \frac{(ex+d)^2}{(cx^4+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x+d)^2/(c*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((e*x + d)^2/(c*x^4 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^2}{(a+cx^4)^{3/2}}\,dx = \int \frac{(d+e\,x)^2}{(c\,x^4+a)^{3/2}}\,dx$$

input
```
int((d + e*x)^2/(a + c*x^4)^(3/2),x)
```

output
```
int((d + e*x)^2/(a + c*x^4)^(3/2), x)
```

Reduce [F]

$$\int \frac{(d+ex)^2}{(a+cx^4)^{3/2}}\,dx = \frac{2\sqrt{c}\,\sqrt{c\,x^4+a}\,de\,x^2+\sqrt{c\,x^4+a}\left(\int \frac{\sqrt{c\,x^4+a}}{c^2x^8+2ac\,x^4+a^2}dx\right)ac\,d^2x^2+\sqrt{c\,x^4+a}\left(\int \frac{\sqrt{c\,x^4+}}{c^2x^8+2ac}\right.}{}$$

input
```
int((e*x+d)^2/(c*x^4+a)^(3/2),x)
```

output

```
(2*sqrt(c)*sqrt(a + c*x**4)*d*e*x**2 + sqrt(a + c*x**4)*int(sqrt(a + c*x**
4)/(a**2 + 2*a*c*x**4 + c**2*x**8),x)*a*c*d**2*x**2 + sqrt(a + c*x**4)*int
((sqrt(a + c*x**4)*x**2)/(a**2 + 2*a*c*x**4 + c**2*x**8),x)*a*c*e**2*x**2
+ sqrt(c)*int(sqrt(a + c*x**4)/(a**2 + 2*a*c*x**4 + c**2*x**8),x)*a**2*d**
2 + sqrt(c)*int(sqrt(a + c*x**4)/(a**2 + 2*a*c*x**4 + c**2*x**8),x)*a*c*d*
*2*x**4 + sqrt(c)*int((sqrt(a + c*x**4)*x**2)/(a**2 + 2*a*c*x**4 + c**2*x*
*8),x)*a**2*e**2 + sqrt(c)*int((sqrt(a + c*x**4)*x**2)/(a**2 + 2*a*c*x**4
+ c**2*x**8),x)*a*c*e**2*x**4 + a*d*e + 2*c*d*e*x**4)/(a*(sqrt(a + c*x**4)
*c*x**2 + sqrt(c)*a + sqrt(c)*c*x**4))
```

3.213 $\int \frac{d+ex}{(a+cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 114

$$\int \frac{d+ex}{(a+cx^4)^{3/2}}\,dx = \frac{x(d+ex)}{2a\sqrt{a+cx^4}} + \frac{d\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4a^{5/4}\sqrt[4]{c}\sqrt{a+cx^4}}$$

output
```
1/2*x*(e*x+d)/a/(c*x^4+a)^(1/2)+1/4*d*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+a)/(a^
(1/2)+c^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(c^(1/4)*x/a^(1/4)),1/
2*2^(1/2))/a^(5/4)/c^(1/4)/(c*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.52

$$\int \frac{d+ex}{(a+cx^4)^{3/2}}\,dx = \frac{x\left(d+ex+d\sqrt{1+\frac{cx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{cx^4}{a}\right)\right)}{2a\sqrt{a+cx^4}}$$

input
```
Integrate[(d + e*x)/(a + c*x^4)^(3/2),x]
```

output
```
(x*(d + e*x + d*Sqrt[1 + (c*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((c*
x^4)/a)]))/(2*a*Sqrt[a + c*x^4])
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {2394, 25, 27, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex}{(a+cx^4)^{3/2}}\,dx$$

$$\Big\downarrow \text{2394}$$

$$\frac{x(d+ex)}{2a\sqrt{a+cx^4}}-\frac{\int -\frac{d}{\sqrt{cx^4+a}}dx}{2a}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{d}{\sqrt{cx^4+a}}dx}{2a}+\frac{x(d+ex)}{2a\sqrt{a+cx^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{d\int \frac{1}{\sqrt{cx^4+a}}dx}{2a}+\frac{x(d+ex)}{2a\sqrt{a+cx^4}}$$

$$\Big\downarrow \text{761}$$

$$\frac{d(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4a^{5/4}\sqrt[4]{c}\sqrt{a+cx^4}}+\frac{x(d+ex)}{2a\sqrt{a+cx^4}}$$

input
```
Int[(d + e*x)/(a + c*x^4)^(3/2),x]
```

output
```
(x*(d + e*x))/(2*a*Sqrt[a + c*x^4]) + (d*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a +
 c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)]
, 1/2])/(4*a^(5/4)*c^(1/4)*Sqrt[a + c*x^4])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 2394
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   Int[ExpandToSum[n
*(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x
] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.48 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.01

method	result	size
default	$d\left(\frac{x}{2a\sqrt{c\left(\frac{a}{c}+x^{4}\right)}}+\frac{\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}, i\right)}{2a\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}}\right)+\frac{e\,x^{2}}{2\sqrt{c\,x^{4}+a}\,a}$	115
elliptic	$-\frac{2c\left(-\frac{e\,x^{2}}{4ca}-\frac{dx}{4ac}\right)}{\sqrt{c\left(\frac{a}{c}+x^{4}\right)}}+\frac{d\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}, i\right)}{2a\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}}$	115

input
```
int((e*x+d)/(c*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
d*(1/2/a*x/(c*(a/c+x^4))^(1/2)+1/2/a/(I*c^(1/2)/a^(1/2))^(1/2)*(1-I*c^(1/2
)*x^2/a^(1/2))^(1/2)*(1+I*c^(1/2)*x^2/a^(1/2))^(1/2)/(c*x^4+a)^(1/2)*Ellip
ticF(x*(I*c^(1/2)/a^(1/2))^(1/2),I))+1/2*e/(c*x^4+a)^(1/2)/a*x^2
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.67

$$\int \frac{d+ex}{(a+cx^4)^{3/2}}\,dx = -\frac{(cdx^4+ad)\sqrt{a}\left(-\frac{c}{a}\right)^{\frac{3}{4}} F\left(\arcsin\left(x\left(-\frac{c}{a}\right)^{\frac{1}{4}}\right) \mid -1\right) - \sqrt{cx^4+a}(cex^2+cdx)}{2\,(ac^2x^4+a^2c)}$$

input
```
integrate((e*x+d)/(c*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
-1/2*((c*d*x^4 + a*d)*sqrt(a)*(-c/a)^(3/4)*elliptic_f(arcsin(x*(-c/a)^(1/4
)), -1) - sqrt(c*x^4 + a)*(c*e*x^2 + c*d*x))/(a*c^2*x^4 + a^2*c)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.52 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.54

$$\int \frac{d+ex}{(a+cx^4)^{3/2}}\,dx = \frac{dx\Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\begin{matrix} \frac{1}{4}, \frac{3}{2} \\ \frac{5}{4} \end{matrix}\middle| \frac{cx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{5}{4}\right)} + \frac{ex^2}{2a^{\frac{3}{2}}\sqrt{1+\frac{cx^4}{a}}}$$

input
```
integrate((e*x+d)/(c*x**4+a)**(3/2),x)
```

output
```
d*x*gamma(1/4)*hyper((1/4, 3/2), (5/4,), c*x**4*exp_polar(I*pi)/a)/(4*a**(
3/2)*gamma(5/4)) + e*x**2/(2*a**(3/2)*sqrt(1 + c*x**4/a))
```

Maxima [F]

$$\int \frac{d+ex}{(a+cx^4)^{3/2}}\,dx=\int \frac{ex+d}{(cx^4+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x+d)/(c*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((e*x + d)/(c*x^4 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{d+ex}{(a+cx^4)^{3/2}}\,dx=\int \frac{ex+d}{(cx^4+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x+d)/(c*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((e*x + d)/(c*x^4 + a)^(3/2), x)
```

Mupad [B] (verification not implemented)

Time = 21.61 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.50

$$\int \frac{d+ex}{(a+cx^4)^{3/2}}\,dx=\frac{e\,x^2}{2\,a\,\sqrt{c\,x^4+a}}+\frac{d\,x\,{\left(\frac{c\,x^4}{a}+1\right)}^{3/2}\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{3}{2};\ \frac{5}{4};\ -\frac{c\,x^4}{a}\right)}{{\left(c\,x^4+a\right)}^{3/2}}$$

input
```
int((d + e*x)/(a + c*x^4)^(3/2),x)
```

output
```
(e*x^2)/(2*a*(a + c*x^4)^(1/2)) + (d*x*((c*x^4)/a + 1)^(3/2)*hypergeom([1/
4, 3/2], 5/4, -(c*x^4)/a))/(a + c*x^4)^(3/2)
```

Reduce [F]

$$\int \frac{d+ex}{(a+cx^4)^{3/2}}\,dx = \frac{2\sqrt{c}\,\sqrt{c\,x^4+a}\,e\,x^2+2\sqrt{c\,x^4+a}\left(\int \frac{\sqrt{c\,x^4+a}}{c^2x^8+2ac\,x^4+a^2}dx\right)acd\,x^2+2\sqrt{c}\left(\int \frac{\sqrt{c\,x^4+a}}{c^2x^8+2ac\,x^4+a^2}d\right.}{2a\left(\sqrt{c\,x^4+a}\,c\,x^2+\sqrt{c}\,a+\sqrt{c}\,c\right.}$$

input
```
int((e*x+d)/(c*x^4+a)^(3/2),x)
```

output
```
(2*sqrt(c)*sqrt(a + c*x**4)*e*x**2 + 2*sqrt(a + c*x**4)*int(sqrt(a + c*x**
4)/(a**2 + 2*a*c*x**4 + c**2*x**8),x)*a*c*d*x**2 + 2*sqrt(c)*int(sqrt(a +
c*x**4)/(a**2 + 2*a*c*x**4 + c**2*x**8),x)*a**2*d + 2*sqrt(c)*int(sqrt(a +
 c*x**4)/(a**2 + 2*a*c*x**4 + c**2*x**8),x)*a*c*d*x**4 + a*e + 2*c*e*x**4)
/(2*a*(sqrt(a + c*x**4)*c*x**2 + sqrt(c)*a + sqrt(c)*c*x**4))
```

3.214 $\int \frac{1}{(a+cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 108

$$\int \frac{1}{(a+cx^4)^{3/2}}\,dx = \frac{x}{2a\sqrt{a+cx^4}} + \frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4a^{5/4}\sqrt[4]{c}\sqrt{a+cx^4}}$$

output
```
1/2*x/a/(c*x^4+a)^(1/2)+1/4*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+a)/(a^(1/2)+c^(1
/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(c^(1/4)*x/a^(1/4)),1/2*2^(1/2))
/a^(5/4)/c^(1/4)/(c*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.55 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.51

$$\int \frac{1}{(a+cx^4)^{3/2}}\,dx = \frac{x+x\sqrt{1+\frac{cx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{cx^4}{a}\right)}{2a\sqrt{a+cx^4}}$$

input
```
Integrate[(a + c*x^4)^(-3/2),x]
```

output
```
(x + x*Sqrt[1 + (c*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((c*x^4)/a)])
/(2*a*Sqrt[a + c*x^4])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {749, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+cx^4)^{3/2}}\,dx$$

$$\Big\downarrow \text{749}$$

$$\frac{\int \frac{1}{\sqrt{cx^4+a}}dx}{2a}+\frac{x}{2a\sqrt{a+cx^4}}$$

$$\Big\downarrow \text{761}$$

$$\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4a^{5/4}\sqrt[4]{c}\sqrt{a+cx^4}}+\frac{x}{2a\sqrt{a+cx^4}}$$

input
```
Int[(a + c*x^4)^(-3/2),x]
```

output
```
x/(2*a*Sqrt[a + c*x^4]) + ((Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[
a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(
5/4)*c^(1/4)*Sqrt[a + c*x^4])
```

Defintions of rubi rules used

rule 749
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Simp[(n*(p + 1) + 1)/(a*n*(p + 1))   Int[(a + b*x^
n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Inte
gerQ[2*p] || Denominator[p + 1/n] < Denominator[p])
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.87

method	result	size
default	$\frac{x}{2a\sqrt{c\left(\frac{a}{c}+x^4\right)}}+\frac{\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{2a\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}$	94
elliptic	$\frac{x}{2a\sqrt{c\left(\frac{a}{c}+x^4\right)}}+\frac{\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{2a\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}$	94

input
```
int(1/(c*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/a*x/(c*(a/c+x^4))^(1/2)+1/2/a/(I*c^(1/2)/a^(1/2))^(1/2)*(1-I*c^(1/2)*x
^2/a^(1/2))^(1/2)*(1+I*c^(1/2)*x^2/a^(1/2))^(1/2)/(c*x^4+a)^(1/2)*Elliptic
F(x*(I*c^(1/2)/a^(1/2))^(1/2),I)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.59

$$\int \frac{1}{(a+cx^4)^{3/2}}\,dx = -\frac{(cx^4+a)\sqrt{a}\left(-\frac{c}{a}\right)^{\frac{3}{4}}F\left(\arcsin\left(x\left(-\frac{c}{a}\right)^{\frac{1}{4}}\right)|-1\right) - \sqrt{cx^4+a}\,cx}{2\left(ac^2x^4+a^2c\right)}$$

input
```
integrate(1/(c*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
-1/2*((c*x^4 + a)*sqrt(a)*(-c/a)^(3/4)*elliptic_f(arcsin(x*(-c/a)^(1/4)),
-1) - sqrt(c*x^4 + a)*c*x)/(a*c^2*x^4 + a^2*c)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.56 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.33

$$\int \frac{1}{(a+cx^4)^{3/2}}\,dx = \frac{x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{3}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate(1/(c*x**4+a)**(3/2),x)
```

output
```
x*gamma(1/4)*hyper((1/4, 3/2), (5/4,), c*x**4*exp_polar(I*pi)/a)/(4*a**(3/
2)*gamma(5/4))
```

Maxima [F]

$$\int \frac{1}{(a+cx^4)^{3/2}}\,dx = \int \frac{1}{(cx^4+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(c*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*x^4 + a)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(a+cx^4)^{3/2}}\,dx = \int \frac{1}{(cx^4+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(c*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((c*x^4 + a)^(-3/2), x)
```

Mupad [B] (verification not implemented)

Time = 21.47 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.34

$$\int \frac{1}{(a+cx^4)^{3/2}}\,dx = \frac{x\left(\frac{c\,x^4}{a}+1\right)^{3/2}\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{3}{2};\ \frac{5}{4};\ -\frac{c\,x^4}{a}\right)}{(c\,x^4+a)^{3/2}}$$

input
```
int(1/(a + c*x^4)^(3/2),x)
```

output
```
(x*((c*x^4)/a + 1)^(3/2)*hypergeom([1/4, 3/2], 5/4, -(c*x^4)/a))/(a + c*x^
4)^(3/2)
```

Reduce [F]

$$\int \frac{1}{(a+cx^4)^{3/2}}\,dx = \int \frac{\sqrt{c\,x^4+a}}{c^2x^8+2ac\,x^4+a^2}dx$$

input
```
int(1/(c*x^4+a)^(3/2),x)
```

output
```
int(sqrt(a + c*x**4)/(a**2 + 2*a*c*x**4 + c**2*x**8),x)
```

3.215 $\int \frac{1}{(d+ex)(a+cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 676

$$\begin{aligned}\int \frac{1}{(d+ex)(a+cx^4)^{3/2}}\,dx &= \frac{e(ae^2-cd^2x^2)}{2a(cd^4+ae^4)\sqrt{a+cx^4}}+\frac{cdx(d^2+e^2x^2)}{2a(cd^4+ae^4)\sqrt{a+cx^4}}\\
&-\frac{\sqrt{c}de^2x\sqrt{a+cx^4}}{2a(cd^4+ae^4)\left(\sqrt{a}+\sqrt{c}x^2\right)}+\frac{e^5\operatorname{arctanh}\left(\frac{\sqrt{cd^4+ae^4}x}{de\sqrt{a+cx^4}}\right)}{2(cd^4+ae^4)^{3/2}}-\frac{e^5\operatorname{arctanh}\left(\frac{ae^2+cd^2x^2}{\sqrt{cd^4+ae^4}\sqrt{a+cx^4}}\right)}{2(cd^4+ae^4)^{3/2}}\\
&+\frac{\sqrt[4]{c}de^2\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{3/4}(cd^4+ae^4)\sqrt{a+cx^4}}\\
&+\frac{\sqrt[4]{c}d\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4a^{5/4}\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\sqrt{a+cx^4}}\\
&-\frac{e^4\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticPi}\left(\frac{\left(\sqrt{c}d^2+\sqrt{a}e^2\right)^2}{4\sqrt{a}\sqrt{c}d^2e^2},2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4\sqrt[4]{a}\sqrt[4]{c}d\left(\sqrt{c}d^2+\sqrt{a}e^2\right)(cd^4+ae^4)\sqrt{a+cx^4}}\end{aligned}$$

output
```
1/2*e*(-c*d^2*x^2+a*e^2)/a/(a*e^4+c*d^4)/(c*x^4+a)^(1/2)+1/2*c*d*x*(e^2*x^
2+d^2)/a/(a*e^4+c*d^4)/(c*x^4+a)^(1/2)-1/2*c^(1/2)*d*e^2*x*(c*x^4+a)^(1/2)
/a/(a*e^4+c*d^4)/(a^(1/2)+c^(1/2)*x^2)+1/2*e^5*arctanh((a*e^4+c*d^4)^(1/2)
*x/d/e/(c*x^4+a)^(1/2))/(a*e^4+c*d^4)^(3/2)-1/2*e^5*arctanh((c*d^2*x^2+a*e
^2)/(a*e^4+c*d^4)^(1/2)/(c*x^4+a)^(1/2))/(a*e^4+c*d^4)^(3/2)+1/2*c^(1/4)*d
*e^2*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*Ellip
ticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))/a^(3/4)/(a*e^4+c*d^4)/(
c*x^4+a)^(1/2)+1/4*c^(1/4)*d*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+a)/(a^(1/2)+c^(
1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(c^(1/4)*x/a^(1/4)),1/2*2^(1/2)
)/a^(5/4)/(c^(1/2)*d^2+a^(1/2)*e^2)/(c*x^4+a)^(1/2)-1/4*e^4*(c^(1/2)*d^2-a
^(1/2)*e^2)*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2
)*EllipticPi(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/4*(c^(1/2)*d^2+a^(1/2)*e^2
)^2/a^(1/2)/c^(1/2)/d^2/e^2,1/2*2^(1/2))/a^(1/4)/c^(1/4)/d/(c^(1/2)*d^2+a^
(1/2)*e^2)/(a*e^4+c*d^4)/(c*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.24 (sec) , antiderivative size = 455, normalized size of antiderivative = 0.67

$$\int \frac{1}{(d+ex)\left(a+cx^4\right)^{3/2}}\,dx =$$

$$-\frac{-\sqrt{a}c^{3/4}d^2e^2\sqrt{-cd^4-ae^4}\sqrt{1+\frac{cx^4}{a}}E\left(i\operatorname{arcsinh}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}x\right)\middle|-1\right)+c^{3/4}d^2\left(-i\sqrt{c}d^2+\sqrt{a}e^2\right)\sqrt{-cd^4-ae^4}}{}$$

input
```
Integrate[1/((d + e*x)*(a + c*x^4)^(3/2)),x]
```

output
```
-1/2*(-(Sqrt[a]*c^(3/4)*d^2*e^2*Sqrt[-(c*d^4) - a*e^4]*Sqrt[1 + (c*x^4)/a]
*EllipticE[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1]) + c^(3/4)*d^2*((-I
)*Sqrt[c]*d^2 + Sqrt[a]*e^2)*Sqrt[-(c*d^4) - a*e^4]*Sqrt[1 + (c*x^4)/a]*El
lipticF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1] + Sqrt[(I*Sqrt[c])/Sqr
t[a]]*(c^(1/4)*d*(Sqrt[-(c*d^4) - a*e^4]*(a*e^3 + c*d*x*(d^2 - d*e*x + e^2
*x^2)) + 2*a*e^5*Sqrt[a + c*x^4]*ArcTan[(Sqrt[c]*(d^2 - e^2*x^2) + e^2*Sqr
t[a + c*x^4])/Sqrt[-(c*d^4) - a*e^4]]) - 2*(-1)^(1/4)*a^(5/4)*e^4*Sqrt[-(c
*d^4) - a*e^4]*Sqrt[1 + (c*x^4)/a]*EllipticPi[(I*Sqrt[a]*e^2)/(Sqrt[c]*d^2
), ArcSin[((-1)^(3/4)*c^(1/4)*x)/a^(1/4)], -1]))/(a*Sqrt[(I*Sqrt[c])/Sqrt[
a]]*c^(1/4)*d*(-(c*d^4) - a*e^4)^(3/2)*Sqrt[a + c*x^4])
```

Rubi [A] (verified)

Time = 2.40 (sec) , antiderivative size = 727, normalized size of antiderivative = 1.08, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.895$, Rules used = {2267, 1548, 27, 1577, 496, 25, 27, 488, 219, 2223, 2397, 25, 27, 1512, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+cx^4)^{3/2}(d+ex)}dx$$

$$\Big\downarrow \text{2267}$$

$$d\int \frac{1}{(d^2-e^2x^2)(cx^4+a)^{3/2}}dx-e\int \frac{x}{(d^2-e^2x^2)(cx^4+a)^{3/2}}dx$$

$$\Big\downarrow \text{1548}$$

$$d\left(\frac{\int \frac{\frac{c^{3/2}e^4x^4}{\sqrt{a}}+ce^2\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)x^2+\sqrt{c}\left(\frac{cd^4}{\sqrt{a}}+\sqrt{c}e^2d^2+\sqrt{a}e^4\right)}{(cx^4+a)^{3/2}}dx}{\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)(ae^4+cd^4)}+\frac{e^6\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a}(d^2-e^2x^2)\sqrt{cx^4+a}}dx}{\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)(ae^4+cd^4)}\right)-e\int \frac{x}{(d^2-e^2x^2)(cx^4+a)^{3/2}}dx$$

$$\Big\downarrow \text{27}$$

$$d\left(\frac{\int \frac{\frac{c^{3/2}e^4x^4}{\sqrt{a}}+ce^2\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)x^2+\sqrt{c}\left(\frac{cd^4}{\sqrt{a}}+\sqrt{c}e^2d^2+\sqrt{a}e^4\right)}{(cx^4+a)^{3/2}}dx}{\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)(ae^4+cd^4)}+\frac{e^6\int \frac{\sqrt{c}x^2+\sqrt{a}}{(d^2-e^2x^2)\sqrt{cx^4+a}}dx}{\sqrt{a}\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)(ae^4+cd^4)}\right)-e\int \frac{x}{(d^2-e^2x^2)(cx^4+a)^{3/2}}dx$$

$\Big\downarrow$ 1577

$$d\left(\frac{\int \frac{\frac{c^{3/2}e^4x^4}{\sqrt{a}}+ce^2\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)x^2+\sqrt{c}\left(\frac{cd^4}{\sqrt{a}}+\sqrt{c}e^2d^2+\sqrt{a}e^4\right)}{(cx^4+a)^{3/2}}dx}{\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)(ae^4+cd^4)}+\frac{e^6\int \frac{\sqrt{c}x^2+\sqrt{a}}{(d^2-e^2x^2)\sqrt{cx^4+a}}dx}{\sqrt{a}\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)(ae^4+cd^4)}\right)-\frac{1}{2}e\int \frac{1}{(d^2-e^2x^2)(cx^4+a)^{3/2}}dx^2$$

$\Big\downarrow$ 496

$$d\left(\frac{\int \frac{\frac{c^{3/2}e^4x^4}{\sqrt{a}}+ce^2\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)x^2+\sqrt{c}\left(\frac{cd^4}{\sqrt{a}}+\sqrt{c}e^2d^2+\sqrt{a}e^4\right)}{(cx^4+a)^{3/2}}dx}{\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)(ae^4+cd^4)}+\frac{e^6\int \frac{\sqrt{c}x^2+\sqrt{a}}{(d^2-e^2x^2)\sqrt{cx^4+a}}dx}{\sqrt{a}\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)(ae^4+cd^4)}\right)-\frac{1}{2}e\left(-\frac{\int -\frac{ae^4}{(d^2-e^2x^2)\sqrt{cx^4+a}}dx^2}{a(ae^4+cd^4)}-\frac{ae^2-cd^2x^2}{a\sqrt{a+cx^4}(ae^4+cd^4)}\right)$$

$\Big\downarrow$ 25

$$d\left(\frac{\int \frac{\frac{c^{3/2}e^4x^4}{\sqrt{a}}+ce^2\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)x^2+\sqrt{c}\left(\frac{cd^4}{\sqrt{a}}+\sqrt{c}e^2d^2+\sqrt{a}e^4\right)}{(cx^4+a)^{3/2}}dx}{\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)(ae^4+cd^4)}+\frac{e^6\int \frac{\sqrt{c}x^2+\sqrt{a}}{(d^2-e^2x^2)\sqrt{cx^4+a}}dx}{\sqrt{a}\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)(ae^4+cd^4)}\right)-\frac{1}{2}e\left(\frac{\int \frac{ae^4}{(d^2-e^2x^2)\sqrt{cx^4+a}}dx^2}{a(ae^4+cd^4)}-\frac{ae^2-cd^2x^2}{a\sqrt{a+cx^4}(ae^4+cd^4)}\right)$$

$\Big\downarrow$ 27

$$d\left(\frac{\int \frac{\frac{c^{3/2}e^4x^4}{\sqrt{a}}+ce^2\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)x^2+\sqrt{c}\left(\frac{cd^4}{\sqrt{a}}+\sqrt{c}e^2d^2+\sqrt{a}e^4\right)}{(cx^4+a)^{3/2}}dx}{\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)(ae^4+cd^4)}+\frac{e^6\int \frac{\sqrt{c}x^2+\sqrt{a}}{(d^2-e^2x^2)\sqrt{cx^4+a}}dx}{\sqrt{a}\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)(ae^4+cd^4)}\right)-\frac{1}{2}e\left(\frac{e^4\int \frac{1}{(d^2-e^2x^2)\sqrt{cx^4+a}}dx^2}{ae^4+cd^4}-\frac{ae^2-cd^2x^2}{a\sqrt{a+cx^4}(ae^4+cd^4)}\right)$$

$$\Big\downarrow \text{488}$$

$$d\left(\frac{\int \frac{\frac{c^{3/2}e^4x^4}{\sqrt{a}}+ce^2\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)x^2+\sqrt{c}\left(\frac{cd^4}{\sqrt{a}}+\sqrt{c}e^2d^2+\sqrt{a}e^4\right)}{(cx^4+a)^{3/2}}dx}{\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)(ae^4+cd^4)}+\frac{e^6\int \frac{\sqrt{c}x^2+\sqrt{a}}{(d^2-e^2x^2)\sqrt{cx^4+a}}dx}{\sqrt{a}\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)(ae^4+cd^4)}\right)-\frac{1}{2}e\left(-\frac{e^4\int \frac{1}{cd^4+ae^4-x^4}d\frac{-ae^2-cd^2x^2}{\sqrt{cx^4+a}}}{ae^4+cd^4}-\frac{ae^2-cd^2x^2}{a\sqrt{a+cx^4}(ae^4+cd^4)}\right)$$

$$\Big\downarrow \text{219}$$

$$d\left(\frac{\int \frac{\frac{c^{3/2}e^4x^4}{\sqrt{a}}+ce^2\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)x^2+\sqrt{c}\left(\frac{cd^4}{\sqrt{a}}+\sqrt{c}e^2d^2+\sqrt{a}e^4\right)}{(cx^4+a)^{3/2}}dx}{\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)(ae^4+cd^4)}+\frac{e^6\int \frac{\sqrt{c}x^2+\sqrt{a}}{(d^2-e^2x^2)\sqrt{cx^4+a}}dx}{\sqrt{a}\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)(ae^4+cd^4)}\right)-\frac{1}{2}e\left(-\frac{e^4\text{arctanh}\left(\frac{-ae^2-cd^2x^2}{\sqrt{a+cx^4}\sqrt{ae^4+cd^4}}\right)}{(ae^4+cd^4)^{3/2}}-\frac{ae^2-cd^2x^2}{a\sqrt{a+cx^4}(ae^4+cd^4)}\right)$$

$$\Big\downarrow \text{2223}$$

$$d\left(\frac{\int \frac{\frac{c^{3/2}e^4x^4}{\sqrt{a}}+ce^2\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)x^2+\sqrt{c}\left(\frac{cd^4}{\sqrt{a}}+\sqrt{c}e^2d^2+\sqrt{a}e^4\right)}{(cx^4+a)^{3/2}}dx}{\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)(ae^4+cd^4)}+\frac{e^6\left(\frac{(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\left(\frac{\sqrt{a}}{d^2}-\frac{\sqrt{c}}{e^2}\right)\text{EllipticPi}\left(\frac{(\sqrt{c}d^2+\sqrt{a}e^2)}{4\sqrt{a}\sqrt{c}d^2e^2}\right.}{4\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+cx^4}}\right.}{\sqrt{a}\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e\right.}\right.$$

$$-\frac{1}{2}e\left(-\frac{e^4\text{arctanh}\left(\frac{-ae^2-cd^2x^2}{\sqrt{a+cx^4}\sqrt{ae^4+cd^4}}\right)}{(ae^4+cd^4)^{3/2}}-\frac{ae^2-cd^2x^2}{a\sqrt{a+cx^4}(ae^4+cd^4)}\right)$$

$$\Big\downarrow \text{2397}$$

$$d\left(\frac{\frac{cx(d^2+e^2x^2)\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)}{2a\sqrt{a+cx^4}}-\frac{\int -\frac{c^{3/2}\left(cd^4+\sqrt{a}\sqrt{c}e^2d^2+2ae^4-\sqrt{a}\sqrt{c}e^2\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)x^2\right)}{\sqrt{a}\sqrt{cx^4+a}}dx}{2ac}}{\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)(ae^4+cd^4)}+\frac{e^6\left(\frac{(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{\sqrt{a}}{d^2}-\frac{\sqrt{c}}{e^2}\right)}{4\sqrt[4]{}}\right.}{}\right.$$

$$\frac{1}{2}e\left(-\frac{e^4\text{arctanh}\left(\frac{-ae^2-cd^2x^2}{\sqrt{a+cx^4}\sqrt{ae^4+cd^4}}\right)}{(ae^4+cd^4)^{3/2}}-\frac{ae^2-cd^2x^2}{a\sqrt{a+cx^4}\,(ae^4+cd^4)}\right)$$

$$\Big\downarrow \; 25$$

$$d\left(\frac{\frac{\int \frac{c^{3/2}\left(cd^4+\sqrt{a}\sqrt{c}e^2d^2+2ae^4-\sqrt{c}e^2\left(\sqrt{c}d^2+\sqrt{a}e^2\right)x^2\right)}{\sqrt{a}\sqrt{cx^4+a}}dx}{2ac}+\frac{cx(d^2+e^2x^2)\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)}{2a\sqrt{a+cx^4}}}{\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)(ae^4+cd^4)}+\frac{e^6\left(\frac{(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{\sqrt{a}}{d^2}-\frac{\sqrt{c}}{e^2}\right)\text{Ellip}}{4\sqrt[4]{a}\sqrt[4]{}}\right.}{}\right.$$

$$\frac{1}{2}e\left(-\frac{e^4\text{arctanh}\left(\frac{-ae^2-cd^2x^2}{\sqrt{a+cx^4}\sqrt{ae^4+cd^4}}\right)}{(ae^4+cd^4)^{3/2}}-\frac{ae^2-cd^2x^2}{a\sqrt{a+cx^4}\,(ae^4+cd^4)}\right)$$

$$\Big\downarrow \; 27$$

$$d\left(\frac{\frac{\sqrt{c}\int \frac{cd^4+\sqrt{a}\sqrt{c}e^2d^2+2ae^4-\sqrt{c}e^2\left(\sqrt{c}d^2+\sqrt{a}e^2\right)x^2}{\sqrt{cx^4+a}}dx}{2a^{3/2}}+\frac{cx(d^2+e^2x^2)\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)}{2a\sqrt{a+cx^4}}}{\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)(ae^4+cd^4)}+\frac{e^6\left(\frac{(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{\sqrt{a}}{d^2}-\frac{\sqrt{c}}{e^2}\right)\text{Elliptic}}{4\sqrt[4]{a}\sqrt[4]{c}\sqrt{}}\right.}{}\right.$$

$$\frac{1}{2}e\left(-\frac{e^4\text{arctanh}\left(\frac{-ae^2-cd^2x^2}{\sqrt{a+cx^4}\sqrt{ae^4+cd^4}}\right)}{(ae^4+cd^4)^{3/2}}-\frac{ae^2-cd^2x^2}{a\sqrt{a+cx^4}\,(ae^4+cd^4)}\right)$$

$$\Big\downarrow \; 1512$$

$$d\left(\frac{\frac{\sqrt{c}\left(\left(ae^4+cd^4\right)\int\frac{1}{\sqrt{cx^4+a}}dx+\sqrt{a}e^2\left(\sqrt{a}e^2+\sqrt{c}d^2\right)\int\frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{a}\sqrt{cx^4+a}}dx\right)}{2a^{3/2}}+\frac{cx\left(d^2+e^2x^2\right)\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)}{2a\sqrt{a+cx^4}}}{\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)\left(ae^4+cd^4\right)}+\frac{e^6\left(\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\cdots\right.}{\cdots}\right.}{\cdots}\right.$$

$$\frac{1}{2}e\left(-\frac{e^4\text{arctanh}\left(\frac{-ae^2-cd^2x^2}{\sqrt{a+cx^4}\sqrt{ae^4+cd^4}}\right)}{\left(ae^4+cd^4\right)^{3/2}}-\frac{ae^2-cd^2x^2}{a\sqrt{a+cx^4}\left(ae^4+cd^4\right)}\right)$$

$\downarrow$ 27

$$d\left(\frac{\frac{\sqrt{c}\left(\left(ae^4+cd^4\right)\int\frac{1}{\sqrt{cx^4+a}}dx+e^2\left(\sqrt{a}e^2+\sqrt{c}d^2\right)\int\frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+a}}dx\right)}{2a^{3/2}}+\frac{cx\left(d^2+e^2x^2\right)\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)}{2a\sqrt{a+cx^4}}}{\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)\left(ae^4+cd^4\right)}+\frac{e^6\left(\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{\sqrt{a}}{d^2}\cdots\right.}{\cdots}\right.}{\cdots}\right.$$

$$\frac{1}{2}e\left(-\frac{e^4\text{arctanh}\left(\frac{-ae^2-cd^2x^2}{\sqrt{a+cx^4}\sqrt{ae^4+cd^4}}\right)}{\left(ae^4+cd^4\right)^{3/2}}-\frac{ae^2-cd^2x^2}{a\sqrt{a+cx^4}\left(ae^4+cd^4\right)}\right)$$

$\downarrow$ 761

$$d\left(\frac{\frac{\sqrt{c}\left(e^2\left(\sqrt{a}e^2+\sqrt{c}d^2\right)\int\frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+a}}dx+\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(ae^4+cd^4\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+cx^4}}\right)}{2a^{3/2}}+\frac{cx\left(d^2+e^2x^2\right)\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)}{2a\sqrt{a+cx^4}}}{\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)\left(ae^4+cd^4\right)}\cdots\right.$$

$$\frac{1}{2}e\left(-\frac{e^4\text{arctanh}\left(\frac{-ae^2-cd^2x^2}{\sqrt{a+cx^4}\sqrt{ae^4+cd^4}}\right)}{\left(ae^4+cd^4\right)^{3/2}}-\frac{ae^2-cd^2x^2}{a\sqrt{a+cx^4}\left(ae^4+cd^4\right)}\right)$$

$\downarrow$ 1510

$$d\left(\frac{\frac{\sqrt{c}\left(\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(ae^4+cd^4\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+cx^4}}+e^2\left(\sqrt{a}e^2+\sqrt{c}d^2\right)\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\cdots\right.\right.}{\sqrt[4]{c}\sqrt{a+cx^4}}\right.\right.}{2a^{3/2}}}{\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)\left(ae^4+cd^4\right)}\right.$$

$$\frac{1}{2}e\left(-\frac{e^4\text{arctanh}\left(\frac{-ae^2-cd^2x^2}{\sqrt{a+cx^4}\sqrt{ae^4+cd^4}}\right)}{\left(ae^4+cd^4\right)^{3/2}}-\frac{ae^2-cd^2x^2}{a\sqrt{a+cx^4}\left(ae^4+cd^4\right)}\right)$$

input

```
Int[1/((d + e*x)*(a + c*x^4)^(3/2)),x]
```

output

```
-1/2*(e*(-((a*e^2 - c*d^2*x^2)/(a*(c*d^4 + a*e^4)*Sqrt[a + c*x^4])) - (e^4
*ArcTanh[(-(a*e^2) - c*d^2*x^2)/(Sqrt[c*d^4 + a*e^4]*Sqrt[a + c*x^4])])/(c
*d^4 + a*e^4)^(3/2))) + d*(((c*((Sqrt[c]*d^2)/Sqrt[a] + e^2)*x*(d^2 + e^2*
x^2))/(2*a*Sqrt[a + c*x^4]) + (Sqrt[c]*(e^2*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*(-
((x*Sqrt[a + c*x^4])/(Sqrt[a] + Sqrt[c]*x^2)) + (a^(1/4)*(Sqrt[a] + Sqrt[c
]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(
1/4)*x)/a^(1/4)], 1/2])/(c^(1/4)*Sqrt[a + c*x^4])) + ((c*d^4 + a*e^4)*(Sqr
t[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[
2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*c^(1/4)*Sqrt[a + c*x^4])))
/(2*a^(3/2)))/(((Sqrt[c]*d^2)/Sqrt[a] + e^2)*(c*d^4 + a*e^4)) + (e^6*(((Sq
rt[c]*d^2 + Sqrt[a]*e^2)*ArcTanh[(Sqrt[c*d^4 + a*e^4]*x)/(d*e*Sqrt[a + c*x
^4])])/(2*d*e*Sqrt[c*d^4 + a*e^4]) + ((Sqrt[a]/d^2 - Sqrt[c]/e^2)*(Sqrt[a]
 + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticPi[(Sq
rt[c]*d^2 + Sqrt[a]*e^2)^2/(4*Sqrt[a]*Sqrt[c]*d^2*e^2), 2*ArcTan[(c^(1/4)*
x)/a^(1/4)], 1/2])/(4*a^(1/4)*c^(1/4)*Sqrt[a + c*x^4])))/(Sqrt[a]*((Sqrt[c
]*d^2)/Sqrt[a] + e^2)*(c*d^4 + a*e^4)))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 496
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(a*d + b*c*x))*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1)*(b*c^2
 + a*d^2))), x] + Simp[1/(2*a*(p + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^n*(a
 + b*x^2)^(p + 1)*Simp[b*c^2*(2*p + 3) + a*d^2*(n + 2*p + 3) + b*c*d*(n + 2
*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[p, -1] && IntQuad
raticQ[a, 0, b, c, d, n, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1512
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + c*x^4], x], x] - Simp[e/q
 Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c
, d, e}, x] && PosQ[c/a]
```

rule 1548
```
Int[((a_) + (c_.)*(x_)^4)^(p_)/((d_.) + (e_.)*(x_)^2), x_Symbol] :> Simp[-(
c*d^2 + a*e^2)^(p + 1/2)/(e^(2*p)*(Rt[c/a, 2]*d - e))   Int[(1 + Rt[c/a, 2]
*x^2)/((d + e*x^2)*Sqrt[a + c*x^4]), x], x] + Simp[(c*d^2 + a*e^2)^(p + 1/2
)/(Rt[c/a, 2]*d - e)   Int[(a + c*x^4)^p*ExpandToSum[((Rt[c/a, 2]*d - e)*(c
*d^2 + a*e^2)^(-p - 1/2) + ((1 + Rt[c/a, 2]*x^2)*(a + c*x^4)^(-p - 1/2))/e^
(2*p))/(d + e*x^2), x], x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e
^2, 0] && ILtQ[p + 1/2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

rule 1577
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, c, d, e, p, q}, x]
```

rule 2223
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTanh[Rt[(-c)*
(d/e) - a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[(-c)*(d/e) - a*(e/d), 2]
)), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^
2)]/(4*d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*Arc
Tan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[c*(d/e) + a*(e
/d)]
```

rule 2267
```
Int[((a_) + (c_.)*(x_)^4)^(p_.)/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[d
Int[(a + c*x^4)^p/(d^2 - e^2*x^2), x], x] - Simp[e   Int[x*((a + c*x^4)^p/(
d^2 - e^2*x^2)), x], x] /; FreeQ[{a, c, d, e}, x] && IntegerQ[p + 1/2]
```

rule 2397
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, S
imp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]
 + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))   Int[(a + b*x^n)^(p + 1)*
ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x]] /; GeQ[q,
n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.43 (sec) , antiderivative size = 496, normalized size of antiderivative = 0.73

method	result
default	$-\frac{2c\left(-\frac{d e^2 x^3}{4a\left(e^4 a+c d^4\right)}+\frac{d^2 e x^2}{4a\left(e^4 a+c d^4\right)}-\frac{d^3 x}{4a\left(e^4 a+c d^4\right)}-\frac{e^3}{4\left(e^4 a+c d^4\right)c}\right)}{\sqrt{c\left(\frac{a}{c}+x^4\right)}}+\frac{d^3 c\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{2a\left(e^4 a+c d^4\right)\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c x^4+a}}-$
elliptic	$-\frac{2c\left(-\frac{d e^2 x^3}{4a\left(e^4 a+c d^4\right)}+\frac{d^2 e x^2}{4a\left(e^4 a+c d^4\right)}-\frac{d^3 x}{4a\left(e^4 a+c d^4\right)}-\frac{e^3}{4\left(e^4 a+c d^4\right)c}\right)}{\sqrt{c\left(\frac{a}{c}+x^4\right)}}+\frac{d^3 c\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{2a\left(e^4 a+c d^4\right)\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c x^4+a}}-$

input
```
int(1/(e*x+d)/(c*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*c*(-1/4*d*e^2/a/(a*e^4+c*d^4)*x^3+1/4*d^2*e/a/(a*e^4+c*d^4)*x^2-1/4*d^3
/a/(a*e^4+c*d^4)*x-1/4*e^3/(a*e^4+c*d^4)/c)/(c*(a/c+x^4))^(1/2)+1/2*d^3*c/
a/(a*e^4+c*d^4)/(I*c^(1/2)/a^(1/2))^(1/2)*(1-I*c^(1/2)*x^2/a^(1/2))^(1/2)*
(1+I*c^(1/2)*x^2/a^(1/2))^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I*c^(1/2)/a^(
1/2))^(1/2),I)-1/2*I*c^(1/2)*d*e^2/a^(1/2)/(a*e^4+c*d^4)/(I*c^(1/2)/a^(1/2
))^(1/2)*(1-I*c^(1/2)*x^2/a^(1/2))^(1/2)*(1+I*c^(1/2)*x^2/a^(1/2))^(1/2)/(
c*x^4+a)^(1/2)*(EllipticF(x*(I*c^(1/2)/a^(1/2))^(1/2),I)-EllipticE(x*(I*c^
(1/2)/a^(1/2))^(1/2),I))+e^3/(a*e^4+c*d^4)*(-1/2/(a+c*d^4/e^4)^(1/2)*arcta
nh(1/2*(2*c*x^2*d^2/e^2+2*a)/(a+c*d^4/e^4)^(1/2)/(c*x^4+a)^(1/2))+1/(I*c^(
1/2)/a^(1/2))^(1/2)/d*e*(1-I*c^(1/2)*x^2/a^(1/2))^(1/2)*(1+I*c^(1/2)*x^2/a
^(1/2))^(1/2)/(c*x^4+a)^(1/2)*EllipticPi(x*(I*c^(1/2)/a^(1/2))^(1/2),-I/c^
(1/2)*a^(1/2)/d^2*e^2,(-I/a^(1/2)*c^(1/2))^(1/2)/(I*c^(1/2)/a^(1/2))^(1/2)
))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)\left(a+cx^4\right)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x+d)/(c*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(d+ex)\left(a+cx^4\right)^{3/2}}\,dx = \int \frac{1}{\left(a+cx^4\right)^{\frac{3}{2}}(d+ex)}\,dx$$

input
```
integrate(1/(e*x+d)/(c*x**4+a)**(3/2),x)
```

output
```
Integral(1/((a + c*x**4)**(3/2)*(d + e*x)), x)
```

Maxima [F]

$$\int \frac{1}{(d+ex)(a+cx^4)^{3/2}}\,dx = \int \frac{1}{(cx^4+a)^{\frac{3}{2}}(ex+d)}\,dx$$

input
```
integrate(1/(e*x+d)/(c*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((c*x^4 + a)^(3/2)*(e*x + d)), x)
```

Giac [F]

$$\int \frac{1}{(d+ex)(a+cx^4)^{3/2}}\,dx = \int \frac{1}{(cx^4+a)^{\frac{3}{2}}(ex+d)}\,dx$$

input
```
integrate(1/(e*x+d)/(c*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((c*x^4 + a)^(3/2)*(e*x + d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)(a+cx^4)^{3/2}}\,dx = \int \frac{1}{(c\,x^4+a)^{3/2}\,(d+e\,x)}\,dx$$

input
```
int(1/((a + c*x^4)^(3/2)*(d + e*x)),x)
```

output
```
int(1/((a + c*x^4)^(3/2)*(d + e*x)), x)
```

Reduce [F]

$$\int \frac{1}{(d+ex)\,(a+cx^4)^{3/2}}\,dx = \int \frac{\sqrt{c\,x^4+a}}{c^2e\,x^9+c^2d\,x^8+2ace\,x^5+2acd\,x^4+a^2ex+a^2d}dx$$

input
```
int(1/(e*x+d)/(c*x^4+a)^(3/2),x)
```

output
```
int(sqrt(a + c*x**4)/(a**2*d + a**2*e*x + 2*a*c*d*x**4 + 2*a*c*e*x**5 + c*
*2*d*x**8 + c**2*e*x**9),x)
```

3.216 $\int \frac{1}{(d+ex)^2(a+cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 882

$$
\begin{aligned}
\int \frac{1}{(d+ex)^2(a+cx^4)^{3/2}}\,dx &= \frac{e^4x}{(cd^4+ae^4)(d^2-e^2x^2)\sqrt{a+cx^4}} \\
&+ \frac{de(ae^2-cd^2x^2)}{a(cd^4+ae^4)(d^2-e^2x^2)\sqrt{a+cx^4}} + \frac{cx(d^2(cd^4-5ae^4)+3e^2(cd^4-ae^4)x^2)}{2a(cd^4+ae^4)^2\sqrt{a+cx^4}} \\
&- \frac{3\sqrt{c}e^2(cd^4-ae^4)x\sqrt{a+cx^4}}{2a(cd^4+ae^4)^2(\sqrt{a}+\sqrt{c}x^2)} + \frac{de^3(cd^4-2ae^4)\sqrt{a+cx^4}}{a(cd^4+ae^4)^2(d^2-e^2x^2)} \\
&+ \frac{3cd^3e^5\,\mathrm{arctanh}\left(\frac{\sqrt{cd^4+ae^4}\,x}{de\sqrt{a+cx^4}}\right)}{(cd^4+ae^4)^{5/2}} - \frac{3cd^3e^5\,\mathrm{arctanh}\left(\frac{ae^2+cd^2x^2}{\sqrt{cd^4+ae^4}\sqrt{a+cx^4}}\right)}{(cd^4+ae^4)^{5/2}} \\
&+ \frac{3\sqrt[4]{c}e^2(cd^4-ae^4)(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\,E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{3/4}(cd^4+ae^4)^2\sqrt{a+cx^4}} \\
&+ \frac{\sqrt[4]{c}(cd^4-2\sqrt{a}\sqrt{c}d^2e^2+3ae^4)(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\,\mathrm{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4a^{5/4}(\sqrt{c}d^2+\sqrt{a}e^2)(cd^4+ae^4)\sqrt{a+cx^4}} \\
&- \frac{3c^{3/4}d^2e^4(\sqrt{c}d^2-\sqrt{a}e^2)(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\,\mathrm{EllipticPi}\left(\frac{(\sqrt{c}d^2+\sqrt{a}e^2)^2}{4\sqrt{a}\sqrt{c}d^2e^2},2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}(\sqrt{c}d^2+\sqrt{a}e^2)(cd^4+ae^4)^2\sqrt{a+cx^4}}
\end{aligned}
$$

output
```
e^4*x/(a*e^4+c*d^4)/(-e^2*x^2+d^2)/(c*x^4+a)^(1/2)+d*e*(-c*d^2*x^2+a*e^2)/
a/(a*e^4+c*d^4)/(-e^2*x^2+d^2)/(c*x^4+a)^(1/2)+1/2*c*x*(d^2*(-5*a*e^4+c*d^
4)+3*e^2*(-a*e^4+c*d^4)*x^2)/a/(a*e^4+c*d^4)^2/(c*x^4+a)^(1/2)-3/2*c^(1/2)
*e^2*(-a*e^4+c*d^4)*x*(c*x^4+a)^(1/2)/a/(a*e^4+c*d^4)^2/(a^(1/2)+c^(1/2)*x
^2)+d*e^3*(-2*a*e^4+c*d^4)*(c*x^4+a)^(1/2)/a/(a*e^4+c*d^4)^2/(-e^2*x^2+d^2
)+3*c*d^3*e^5*arctanh((a*e^4+c*d^4)^(1/2)*x/d/e/(c*x^4+a)^(1/2))/(a*e^4+c*
d^4)^(5/2)-3*c*d^3*e^5*arctanh((c*d^2*x^2+a*e^2)/(a*e^4+c*d^4)^(1/2)/(c*x^
4+a)^(1/2))/(a*e^4+c*d^4)^(5/2)+3/2*c^(1/4)*e^2*(-a*e^4+c*d^4)*(a^(1/2)+c^
(1/2)*x^2)*((c*x^4+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*EllipticE(sin(2*arcta
n(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))/a^(3/4)/(a*e^4+c*d^4)^2/(c*x^4+a)^(1/2)
+1/4*c^(1/4)*(c*d^4-2*a^(1/2)*c^(1/2)*d^2*e^2+3*a*e^4)*(a^(1/2)+c^(1/2)*x^
2)*((c*x^4+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(c^(1
/4)*x/a^(1/4)),1/2*2^(1/2))/a^(5/4)/(c^(1/2)*d^2+a^(1/2)*e^2)/(a*e^4+c*d^4
)/(c*x^4+a)^(1/2)-3/2*c^(3/4)*d^2*e^4*(c^(1/2)*d^2-a^(1/2)*e^2)*(a^(1/2)+c
^(1/2)*x^2)*((c*x^4+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*EllipticPi(sin(2*arc
tan(c^(1/4)*x/a^(1/4))),1/4*(c^(1/2)*d^2+a^(1/2)*e^2)^2/a^(1/2)/c^(1/2)/d^
2/e^2,1/2*2^(1/2))/a^(1/4)/(c^(1/2)*d^2+a^(1/2)*e^2)/(a*e^4+c*d^4)^2/(c*x^
4+a)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 13.19 (sec) , antiderivative size = 578, normalized size of antiderivative = 0.66

$$\int \frac{1}{(d+ex)^2\,(a+cx^4)^{3/2}}\,dx = \frac{3\sqrt{a}\sqrt{c}e^2\sqrt{-cd^4-ae^4}(-cd^4+ae^4)\,(d+ex)\sqrt{1+\frac{cx^4}{a}}E\Big(i\text{arcsinh}\Big(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}x\Big)}{}$$

input
```
Integrate[1/((d + e*x)^2*(a + c*x^4)^(3/2)),x]
```

output
```
(3*Sqrt[a]*Sqrt[c]*e^2*Sqrt[-(c*d^4) - a*e^4]*(-(c*d^4) + a*e^4)*(d + e*x)
*Sqrt[1 + (c*x^4)/a]*EllipticE[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1]
 + Sqrt[c]*Sqrt[-(c*d^4) - a*e^4]*((-I)*c^(3/2)*d^6 + 3*Sqrt[a]*c*d^4*e^2
+ (5*I)*a*Sqrt[c]*d^2*e^4 - 3*a^(3/2)*e^6)*(d + e*x)*Sqrt[1 + (c*x^4)/a]*E
llipticF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1] + Sqrt[(I*Sqrt[c])/Sq
rt[a]]*(Sqrt[-(c*d^4) - a*e^4]*(-2*a^2*e^7 + c^2*d^4*x*(d^3 - d^2*e*x + d*
e^2*x^2 + 3*e^3*x^3) + a*c*e^3*(4*d^4 + d^3*e*x - d^2*e^2*x^2 + d*e^3*x^3
- 3*e^4*x^4)) + 12*a*c*d^3*e^5*(d + e*x)*Sqrt[a + c*x^4]*ArcTan[(Sqrt[c]*(
d^2 - e^2*x^2) + e^2*Sqrt[a + c*x^4])/Sqrt[-(c*d^4) - a*e^4]] - 12*(-1)^(1
/4)*a^(5/4)*c^(3/4)*d^2*e^4*Sqrt[-(c*d^4) - a*e^4]*(d + e*x)*Sqrt[1 + (c*x
^4)/a]*EllipticPi[(I*Sqrt[a]*e^2)/(Sqrt[c]*d^2), ArcSin[((-1)^(3/4)*c^(1/4
)*x)/a^(1/4)], -1]))/(2*a*Sqrt[(I*Sqrt[c])/Sqrt[a]]*(-(c*d^4) - a*e^4)^(5/
2)*(d + e*x)*Sqrt[a + c*x^4])
```

Rubi [A] (verified)

Time = 5.06 (sec) , antiderivative size = 1336, normalized size of antiderivative = 1.51, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.632$, Rules used = {2584, 2255, 27, 1577, 496, 25, 27, 679, 488, 219, 2259, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+cx^4)^{3/2}(d+ex)^2}dx$$

$$\Big\downarrow \text{2584}$$

$$\int \frac{d^2-2dex+e^2x^2}{(a+cx^4)^{3/2}(d^2-e^2x^2)^2}dx$$

$$\Big\downarrow \text{2255}$$

$$\int -\frac{2dex}{(d^2-e^2x^2)^2(cx^4+a)^{3/2}}dx+\int \frac{d^2+e^2x^2}{(d^2-e^2x^2)^2(cx^4+a)^{3/2}}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{d^2+e^2x^2}{(d^2-e^2x^2)^2(cx^4+a)^{3/2}}dx-2de\int \frac{x}{(d^2-e^2x^2)^2(cx^4+a)^{3/2}}dx$$

$$\Big\downarrow \text{1577}$$

$$\int \frac{d^2+e^2x^2}{(d^2-e^2x^2)^2 (cx^4+a)^{3/2}}dx - de\int \frac{1}{(d^2-e^2x^2)^2 (cx^4+a)^{3/2}}dx^2$$

$\Big\downarrow$ 496

$$\int \frac{d^2+e^2x^2}{(d^2-e^2x^2)^2 (cx^4+a)^{3/2}}dx - de\left(-\frac{\int -\frac{e^2(2ae^2-cd^2x^2)}{(d^2-e^2x^2)^2\sqrt{cx^4+a}}dx^2}{a(ae^4+cd^4)} - \frac{ae^2-cd^2x^2}{a\sqrt{a+cx^4}(d^2-e^2x^2)(ae^4+cd^4)}\right)$$

$\Big\downarrow$ 25

$$\int \frac{d^2+e^2x^2}{(d^2-e^2x^2)^2 (cx^4+a)^{3/2}}dx - de\left(\frac{\int \frac{e^2(2ae^2-cd^2x^2)}{(d^2-e^2x^2)^2\sqrt{cx^4+a}}dx^2}{a(ae^4+cd^4)} - \frac{ae^2-cd^2x^2}{a\sqrt{a+cx^4}(d^2-e^2x^2)(ae^4+cd^4)}\right)$$

$\Big\downarrow$ 27

$$\int \frac{d^2+e^2x^2}{(d^2-e^2x^2)^2 (cx^4+a)^{3/2}}dx - de\left(\frac{e^2\int \frac{2ae^2-cd^2x^2}{(d^2-e^2x^2)^2\sqrt{cx^4+a}}dx^2}{a(ae^4+cd^4)} - \frac{ae^2-cd^2x^2}{a\sqrt{a+cx^4}(d^2-e^2x^2)(ae^4+cd^4)}\right)$$

$\Big\downarrow$ 679

$$\int \frac{d^2+e^2x^2}{(d^2-e^2x^2)^2 (cx^4+a)^{3/2}}dx - de\left(\frac{e^2\left(\frac{3acd^2e^2\int \frac{1}{(d^2-e^2x^2)\sqrt{cx^4+a}}dx^2}{ae^4+cd^4} - \frac{\sqrt{a+cx^4}(cd^4-2ae^4)}{(d^2-e^2x^2)(ae^4+cd^4)}\right)}{a(ae^4+cd^4)} - \frac{ae^2-cd^2x^2}{a\sqrt{a+cx^4}(d^2-e^2x^2)(ae^4+cd^4)}\right)$$

$\Big\downarrow$ 488

$$\int \frac{d^2+e^2x^2}{(d^2-e^2x^2)^2 (cx^4+a)^{3/2}}dx - de\left(\frac{e^2\left(-\frac{3acd^2e^2\int \frac{1}{cd^4+ae^4-x^4}d\frac{-ae^2-cd^2x^2}{\sqrt{cx^4+a}}}{ae^4+cd^4} - \frac{\sqrt{a+cx^4}(cd^4-2ae^4)}{(d^2-e^2x^2)(ae^4+cd^4)}\right)}{a(ae^4+cd^4)} - \frac{ae^2-cd^2x^2}{a\sqrt{a+cx^4}(d^2-e^2x^2)(ae^4+cd^4)}\right)$$

$$\Big\downarrow \ 219$$

$$\int \frac{d^2+e^2x^2}{\left(d^2-e^2x^2\right)^2 \left(cx^4+a\right)^{3/2}}dx-de\left(\frac{e^2\left(-\frac{3acd^2e^2\text{arctanh}\left(\frac{-ae^2-cd^2x^2}{\sqrt{a+cx^4}\sqrt{ae^4+cd^4}}\right)}{\left(ae^4+cd^4\right)^{3/2}}-\frac{\sqrt{a+cx^4}\left(cd^4-2ae^4\right)}{\left(d^2-e^2x^2\right)\left(ae^4+cd^4\right)}\right)}{a\left(ae^4+cd^4\right)}-\frac{ae^2-cd^2x^2}{a\sqrt{a+cx^4}\left(d^2-e^2x^2\right)\left(ae^4+cd^4\right)}\right)$$

$$\Big\downarrow \ 2259$$

$$\int \left(\frac{4cd^4e^4}{\left(cd^4+ae^4\right)^2\left(d^2-e^2x^2\right)\sqrt{cx^4+a}}+\frac{e^4}{2\left(cd^4+ae^4\right)(d-ex)^2\sqrt{cx^4+a}}+\frac{e^4}{2\left(cd^4+ae^4\right)(d+ex)^2\sqrt{cx^4+a}}+\right.$$
$$de\left(\frac{e^2\left(-\frac{3acd^2e^2\text{arctanh}\left(\frac{-ae^2-cd^2x^2}{\sqrt{a+cx^4}\sqrt{ae^4+cd^4}}\right)}{\left(ae^4+cd^4\right)^{3/2}}-\frac{\sqrt{a+cx^4}\left(cd^4-2ae^4\right)}{\left(d^2-e^2x^2\right)\left(ae^4+cd^4\right)}\right)}{a\left(ae^4+cd^4\right)}-\frac{ae^2-cd^2x^2}{a\sqrt{a+cx^4}\left(d^2-e^2x^2\right)\left(ae^4+cd^4\right)}\right)$$

$$\Big\downarrow \ 2009$$

$$\frac{\sqrt{cx^4+a}e^7}{2(cd^4+ae^4)^2(d-ex)} - \frac{\sqrt{cx^4+a}e^7}{2(cd^4+ae^4)^2(d+ex)} - \frac{\sqrt[4]{a}\sqrt[4]{c}(\sqrt{c}x^2+\sqrt{a})\sqrt{\frac{cx^4+a}{(\sqrt{c}x^2+\sqrt{a})^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)e^6}{(cd^4+ae^4)^2\sqrt{cx^4+a}} + \frac{\sqrt{c}x\sqrt{cx^4+a}e^6}{(cd^4+ae^4)^2(\sqrt{c}x^2+\sqrt{a})} + \frac{3cd^3\operatorname{arctanh}\left(\frac{\sqrt{cd^4+ae^4}x}{de\sqrt{cx^4+a}}\right)e^5}{(cd^4+ae^4)^{5/2}} + \frac{\sqrt[4]{c}(\sqrt{c}x^2+\sqrt{a})\sqrt{\frac{cx^4+a}{(\sqrt{c}x^2+\sqrt{a})^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)e^4}{2\sqrt[4]{a}(\sqrt{c}d^2+\sqrt{a}e^2)(cd^4+ae^4)\sqrt{cx^4+a}} + \frac{2c^{5/4}d^4(\sqrt{c}x^2+\sqrt{a})\sqrt{\frac{cx^4+a}{(\sqrt{c}x^2+\sqrt{a})^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)e^4}{\sqrt[4]{a}(\sqrt{c}d^2+\sqrt{a}e^2)(cd^4+ae^4)^2\sqrt{cx^4+a}} - \frac{3c^{3/4}d^2(\sqrt{c}d^2-\sqrt{a}e^2)(\sqrt{c}x^2+\sqrt{a})\sqrt{\frac{cx^4+a}{(\sqrt{c}x^2+\sqrt{a})^2}}\operatorname{EllipticPi}\left(\frac{(\sqrt{c}d^2+\sqrt{a}e^2)^2}{4\sqrt{a}\sqrt{c}d^2e^2},2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)e^4}{2\sqrt[4]{a}(\sqrt{c}d^2+\sqrt{a}e^2)(cd^4+ae^4)^2\sqrt{cx^4+a}} + \frac{\sqrt[4]{c}(3cd^4-ae^4)(\sqrt{c}x^2+\sqrt{a})\sqrt{\frac{cx^4+a}{(\sqrt{c}x^2+\sqrt{a})^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)e^2}{2a^{3/4}(cd^4+ae^4)^2\sqrt{cx^4+a}} - \frac{\sqrt{c}(3cd^4-ae^4)x\sqrt{cx^4+a}e^2}{2a(cd^4+ae^4)^2(\sqrt{c}x^2+\sqrt{a})} - d\left(\frac{e^2\left(-\frac{3acd^2\operatorname{arctanh}\left(\frac{-ae^2-cd^2x^2}{\sqrt{cd^4+ae^4}\sqrt{cx^4+a}}\right)e^2}{(cd^4+ae^4)^{3/2}} - \frac{(cd^4-2ae^4)\sqrt{cx^4+a}}{(cd^4+ae^4)(d^2-e^2x^2)}\right)}{a(cd^4+ae^4)} - \frac{ae^2-cd^2x^2}{a(cd^4+ae^4)(d^2-e^2x^2)\sqrt{cx^4+a}}\right)e - \frac{\sqrt[4]{c}\left(-ae^6+3cd^4e^2-\frac{\sqrt{c}d^2(cd^4-3ae^4)}{\sqrt{a}}\right)(\sqrt{c}x^2+\sqrt{a})\sqrt{\frac{cx^4+a}{(\sqrt{c}x^2+\sqrt{a})^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4a^{3/4}(cd^4+ae^4)^2\sqrt{cx^4+a}} + \frac{cx((cd^4-3ae^4)d^2+e^2(3cd^4-ae^4)x^2)}{2a(cd^4+ae^4)^2\sqrt{cx^4+a}}$$

input
```
Int[1/((d + e*x)^2*(a + c*x^4)^(3/2)),x]
```

output
```
(c*x*(d^2*(c*d^4 - 3*a*e^4) + e^2*(3*c*d^4 - a*e^4)*x^2))/(2*a*(c*d^4 + a*
e^4)^2*Sqrt[a + c*x^4]) + (e^7*Sqrt[a + c*x^4])/(2*(c*d^4 + a*e^4)^2*(d -
e*x)) - (e^7*Sqrt[a + c*x^4])/(2*(c*d^4 + a*e^4)^2*(d + e*x)) + (Sqrt[c]*e
^6*x*Sqrt[a + c*x^4])/((c*d^4 + a*e^4)^2*(Sqrt[a] + Sqrt[c]*x^2)) - (Sqrt[
c]*e^2*(3*c*d^4 - a*e^4)*x*Sqrt[a + c*x^4])/(2*a*(c*d^4 + a*e^4)^2*(Sqrt[a
] + Sqrt[c]*x^2)) + (3*c*d^3*e^5*ArcTanh[(Sqrt[c*d^4 + a*e^4]*x)/(d*e*Sqrt
[a + c*x^4])])/(c*d^4 + a*e^4)^(5/2) - d*e*(-((a*e^2 - c*d^2*x^2)/(a*(c*d^
4 + a*e^4)*(d^2 - e^2*x^2)*Sqrt[a + c*x^4])) + (e^2*(-(((c*d^4 - 2*a*e^4)*
Sqrt[a + c*x^4])/((c*d^4 + a*e^4)*(d^2 - e^2*x^2))) - (3*a*c*d^2*e^2*ArcTa
nh[(-(a*e^2) - c*d^2*x^2)/(Sqrt[c*d^4 + a*e^4]*Sqrt[a + c*x^4])])/(c*d^4 +
 a*e^4)^(3/2)))/(a*(c*d^4 + a*e^4))) - (a^(1/4)*c^(1/4)*e^6*(Sqrt[a] + Sqr
t[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(
c^(1/4)*x)/a^(1/4)], 1/2])/((c*d^4 + a*e^4)^2*Sqrt[a + c*x^4]) + (c^(1/4)*
e^2*(3*c*d^4 - a*e^4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] +
Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(3/4)*
(c*d^4 + a*e^4)^2*Sqrt[a + c*x^4]) + (2*c^(5/4)*d^4*e^4*(Sqrt[a] + Sqrt[c]
*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1
/4)*x)/a^(1/4)], 1/2])/(a^(1/4)*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*(c*d^4 + a*e^4
)^2*Sqrt[a + c*x^4]) + (c^(1/4)*e^4*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^
4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], ...
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 496
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(a*d + b*c*x))*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1)*(b*c^2
 + a*d^2))), x] + Simp[1/(2*a*(p + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^n*(a
 + b*x^2)^(p + 1)*Simp[b*c^2*(2*p + 3) + a*d^2*(n + 2*p + 3) + b*c*d*(n + 2
*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[p, -1] && IntQuad
raticQ[a, 0, b, c, d, n, p, x]
```

rule 679
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[(c*d*f + a*e*g)/(c*d^2 + a*e^2)
 Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
 p}, x] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 1577
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, c, d, e, p, q}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2255
```
Int[(Pr_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Module[{r = Expon[Pr, x], k}, Int[Sum[Coeff[Pr, x, 2*k]*x^(2*k), {k, 0,
 r/2}]*(d + e*x^2)^q*(a + c*x^4)^p, x] + Int[x*Sum[Coeff[Pr, x, 2*k + 1]*x^
(2*k), {k, 0, (r - 1)/2}]*(d + e*x^2)^q*(a + c*x^4)^p, x]] /; FreeQ[{a, c,
d, e, p, q}, x] && PolyQ[Pr, x] &&  !PolyQ[Pr, x^2]
```

rule 2259
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Int[ExpandIntegrand[1/Sqrt[a + c*x^4], Px*(d + e*x^2)^q*(a + c*x^4)^(p
+ 1/2), x], x] /; FreeQ[{a, c, d, e}, x] && PolyQ[Px, x] && IntegerQ[p + 1/
2] && IntegerQ[q]
```

rule 2584
```
Int[((c_) + (d_.)*(x_)^(n_.))^(q_)*((a_) + (b_.)*(x_)^(nn_.))^(p_), x_Symbo
l] :> Int[ExpandToSum[(c - d*x^n)^(-q), x]*((a + b*x^nn)^p/(c^2 - d^2*x^(2*
n))^(-q)), x] /; FreeQ[{a, b, c, d, n, nn, p}, x] &&  !IntegerQ[p] && ILtQ[
q, 0] && IGtQ[Log[2, nn/n], 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.52 (sec) , antiderivative size = 642, normalized size of antiderivative = 0.73

method	result
default	$-\frac{e^{7}\sqrt{c\,x^{4}+a}}{\left(e^{4}a+c\,d^{4}\right)^{2}\left(ex+d\right)}-\frac{2c\left(\frac{e^{2}\left(e^{4}a-3c\,d^{4}\right)x^{3}}{4a\left(e^{4}a+c\,d^{4}\right)^{2}}-\frac{de\left(e^{4}a-c\,d^{4}\right)x^{2}}{2a\left(e^{4}a+c\,d^{4}\right)^{2}}+\frac{d^{2}\left(3e^{4}a-c\,d^{4}\right)x}{4a\left(e^{4}a+c\,d^{4}\right)^{2}}-\frac{d^{3}e^{3}}{\left(e^{4}a+c\,d^{4}\right)^{2}}\right)}{\sqrt{c\left(\frac{a}{c}+x^{4}\right)}}+\frac{\left(-\frac{c\,d^{2}e^{4}}{\left(e^{4}a+c\,d^{4}\right)^{2}}-\frac{c\,d^{2}\left(3e^{4}\right.}{2a\left(e^{4}a+\right.}\right.}{}$
elliptic	$-\frac{e^{7}\sqrt{c\,x^{4}+a}}{\left(e^{4}a+c\,d^{4}\right)^{2}\left(ex+d\right)}-\frac{2c\left(\frac{e^{2}\left(e^{4}a-3c\,d^{4}\right)x^{3}}{4a\left(e^{4}a+c\,d^{4}\right)^{2}}-\frac{de\left(e^{4}a-c\,d^{4}\right)x^{2}}{2a\left(e^{4}a+c\,d^{4}\right)^{2}}+\frac{d^{2}\left(3e^{4}a-c\,d^{4}\right)x}{4a\left(e^{4}a+c\,d^{4}\right)^{2}}-\frac{d^{3}e^{3}}{\left(e^{4}a+c\,d^{4}\right)^{2}}\right)}{\sqrt{c\left(\frac{a}{c}+x^{4}\right)}}+\frac{\left(-\frac{c\,d^{2}e^{4}}{\left(e^{4}a+c\,d^{4}\right)^{2}}-\frac{c\,d^{2}\left(3e^{4}\right.}{2a\left(e^{4}a+\right.}\right.}{}$

input
```
int(1/(e*x+d)^2/(c*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-e^7/(a*e^4+c*d^4)^2*(c*x^4+a)^(1/2)/(e*x+d)-2*c*(1/4*e^2*(a*e^4-3*c*d^4)/
a/(a*e^4+c*d^4)^2*x^3-1/2*d*e*(a*e^4-c*d^4)/a/(a*e^4+c*d^4)^2*x^2+1/4*d^2*
(3*a*e^4-c*d^4)/a/(a*e^4+c*d^4)^2*x-d^3*e^3/(a*e^4+c*d^4)^2)/(c*(a/c+x^4))
^(1/2)+(-c*d^2*e^4/(a*e^4+c*d^4)^2-1/2*c*d^2*(3*a*e^4-c*d^4)/a/(a*e^4+c*d^
4)^2)/(I*c^(1/2)/a^(1/2))^(1/2)*(1-I*c^(1/2)*x^2/a^(1/2))^(1/2)*(1+I*c^(1/
2)*x^2/a^(1/2))^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I*c^(1/2)/a^(1/2))^(1/2
),I)+I*(e^6*c/(a*e^4+c*d^4)^2+1/2*c*e^2*(a*e^4-3*c*d^4)/a/(a*e^4+c*d^4)^2)
*a^(1/2)/(I*c^(1/2)/a^(1/2))^(1/2)*(1-I*c^(1/2)*x^2/a^(1/2))^(1/2)*(1+I*c^
(1/2)*x^2/a^(1/2))^(1/2)/(c*x^4+a)^(1/2)/c^(1/2)*(EllipticF(x*(I*c^(1/2)/a
^(1/2))^(1/2),I)-EllipticE(x*(I*c^(1/2)/a^(1/2))^(1/2),I))+6*c*d^3*e^3/(a*
e^4+c*d^4)^2*(-1/2/(a+c*d^4/e^4)^(1/2)*arctanh(1/2*(2*c*x^2*d^2/e^2+2*a)/(
a+c*d^4/e^4)^(1/2)/(c*x^4+a)^(1/2))+1/(I*c^(1/2)/a^(1/2))^(1/2)/d*e*(1-I*c
^(1/2)*x^2/a^(1/2))^(1/2)*(1+I*c^(1/2)*x^2/a^(1/2))^(1/2)/(c*x^4+a)^(1/2)*
EllipticPi(x*(I*c^(1/2)/a^(1/2))^(1/2),-I/c^(1/2)*a^(1/2)/d^2*e^2,(-I/a^(1
/2)*c^(1/2))^(1/2)/(I*c^(1/2)/a^(1/2))^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^2 (a+cx^4)^{3/2}} dx = \text{Timed out}$$

input
```
integrate(1/(e*x+d)^2/(c*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(d+ex)^2 (a+cx^4)^{3/2}} dx = \int \frac{1}{(a+cx^4)^{\frac{3}{2}} (d+ex)^2} dx$$

input
```
integrate(1/(e*x+d)**2/(c*x**4+a)**(3/2),x)
```

output
```
Integral(1/((a + c*x**4)**(3/2)*(d + e*x)**2), x)
```

Maxima [F]

$$\int \frac{1}{(d+ex)^2 (a+cx^4)^{3/2}} dx = \int \frac{1}{(cx^4+a)^{\frac{3}{2}} (ex+d)^2} dx$$

input
```
integrate(1/(e*x+d)^2/(c*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((c*x^4 + a)^(3/2)*(e*x + d)^2), x)
```

Giac [F]

$$\int \frac{1}{(d+ex)^2 (a+cx^4)^{3/2}} \, dx = \int \frac{1}{(cx^4+a)^{\frac{3}{2}} (ex+d)^2} \, dx$$

input
```
integrate(1/(e*x+d)^2/(c*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((c*x^4 + a)^(3/2)*(e*x + d)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^2 (a+cx^4)^{3/2}} \, dx = \int \frac{1}{(c\,x^4+a)^{3/2}\,(d+e\,x)^2} \, dx$$

input
```
int(1/((a + c*x^4)^(3/2)*(d + e*x)^2),x)
```

output
```
int(1/((a + c*x^4)^(3/2)*(d + e*x)^2), x)
```

Reduce [F]

$$\int \frac{1}{(d+ex)^2 (a+cx^4)^{3/2}} \, dx = \int \frac{\sqrt{c\,x^4+a}}{c^2e^2x^{10}+2c^2de\,x^9+c^2d^2x^8+2ac\,e^2x^6+4acde\,x^5+2ac\,d^2x^4+a^2e^2x^2+}$$

input
```
int(1/(e*x+d)^2/(c*x^4+a)^(3/2),x)
```

output
```
int(sqrt(a + c*x**4)/(a**2*d**2 + 2*a**2*d*e*x + a**2*e**2*x**2 + 2*a*c*d*
*2*x**4 + 4*a*c*d*e*x**5 + 2*a*c*e**2*x**6 + c**2*d**2*x**8 + 2*c**2*d*e*x
**9 + c**2*e**2*x**10),x)
```

3.217 $\int (c+dx)^3 \left(a+bx^4\right)^p \, dx$

Optimal result

Integrand size = 17, antiderivative size = 177

$$\int (c+dx)^3 \left(a+bx^4\right)^p \, dx = \frac{d^3\left(a+bx^4\right)^{1+p}}{4b(1+p)} + c^3x\left(a+bx^4\right)^p \left(1+\frac{bx^4}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{4},-p,\frac{5}{4},-\frac{bx^4}{a}\right) + \frac{3}{2}c^2dx^2\left(a+bx^4\right)^p \left(1+\frac{bx^4}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^4}{a}\right) + cd^2x^3\left(a+bx^4\right)^p \left(1+\frac{bx^4}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{3}{4},-p,\frac{7}{4},-\frac{bx^4}{a}\right)$$

output
```
1/4*d^3*(b*x^4+a)^(p+1)/b/(p+1)+c^3*x*(b*x^4+a)^p*hypergeom([1/4, -p],[5/4
],-b*x^4/a)/((1+b*x^4/a)^p)+3/2*c^2*d*x^2*(b*x^4+a)^p*hypergeom([1/2, -p],
[3/2],-b*x^4/a)/((1+b*x^4/a)^p)+c*d^2*x^3*(b*x^4+a)^p*hypergeom([3/4, -p],
[7/4],-b*x^4/a)/((1+b*x^4/a)^p)
```

Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.86

$$\int (c+dx)^3 \left(a+bx^4\right)^p \, dx$$
$$= \frac{\left(a+bx^4\right)^p \left(1+\frac{bx^4}{a}\right)^{-p} \left(4bc^3(1+p)x \operatorname{Hypergeometric2F1}\left(\frac{1}{4},-p,\frac{5}{4},-\frac{bx^4}{a}\right)+d\Big(6bc^2(1+p)x^2 \operatorname{Hyperge}\right.}{}$$

input
```
Integrate[(c + d*x)^3*(a + b*x^4)^p,x]
```

output
```
((a + b*x^4)^p*(4*b*c^3*(1 + p)*x*Hypergeometric2F1[1/4, -p, 5/4, -((b*x^4
)/a)] + d*(6*b*c^2*(1 + p)*x^2*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^4)/a
)] + d*(d*(a + b*x^4)*(1 + (b*x^4)/a)^p + 4*b*c*(1 + p)*x^3*Hypergeometric
2F1[3/4, -p, 7/4, -((b*x^4)/a)])))/(4*b*(1 + p)*(1 + (b*x^4)/a)^p)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2424, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3 \left(a+bx^4\right)^p \, dx$$
$$\Big\downarrow \text{ 2424}$$
$$\int \left(\left(c^3+3cd^2x^2\right)\left(a+bx^4\right)^p+x\left(3c^2d+d^3x^2\right)\left(a+bx^4\right)^p\right)dx$$
$$\Big\downarrow \text{ 2009}$$

$$c^3x(a+bx^4)^p\left(\frac{bx^4}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{4},-p,\frac{5}{4},-\frac{bx^4}{a}\right)+\frac{3}{2}c^2dx^2(a+bx^4)^p\left(\frac{bx^4}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^4}{a}\right)+cd^2x^3(a+bx^4)^p\left(\frac{bx^4}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{3}{4},-p,\frac{7}{4},-\frac{bx^4}{a}\right)+\frac{d^3(a+bx^4)^{p+1}}{4b(p+1)}$$

input
```
Int[(c + d*x)^3*(a + b*x^4)^p,x]
```

output
```
(d^3*(a + b*x^4)^(1 + p))/(4*b*(1 + p)) + (c^3*x*(a + b*x^4)^p*Hypergeomet
ric2F1[1/4, -p, 5/4, -((b*x^4)/a)])/(1 + (b*x^4)/a)^p + (3*c^2*d*x^2*(a +
b*x^4)^p*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^4)/a)])/(2*(1 + (b*x^4)/a)
^p) + (c*d^2*x^3*(a + b*x^4)^p*Hypergeometric2F1[3/4, -p, 7/4, -((b*x^4)/a
)])/(1 + (b*x^4)/a)^p
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2424
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Maple [F]

$$\int (dx+c)^3 \left(b\,x^4+a\right)^p dx$$

input
```
int((d*x+c)^3*(b*x^4+a)^p,x)
```

output
```
int((d*x+c)^3*(b*x^4+a)^p,x)
```

Fricas [F]

$$\int (c+dx)^3 \left(a+bx^4\right)^p \, dx = \int (dx+c)^3 (bx^4+a)^p \, dx$$

input
```
integrate((d*x+c)^3*(b*x^4+a)^p,x, algorithm="fricas")
```

output
```
integral((d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)*(b*x^4 + a)^p, x)
```

Sympy [A] (verification not implemented)

Time = 24.64 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.88

$$\int (c+dx)^3 \left(a+bx^4\right)^p \, dx = \frac{a^p c^3 x \Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\begin{matrix} \frac{1}{4}, -p \\ \frac{5}{4} \end{matrix}\middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)} + \frac{3a^p c^2 d x^2 {}_2F_1\left(\begin{matrix} \frac{1}{2}, -p \\ \frac{3}{2} \end{matrix}\middle| \frac{bx^4 e^{i\pi}}{a}\right)}{2} + \frac{3a^p c d^2 x^3 \Gamma\left(\frac{3}{4}\right) {}_2F_1\left(\begin{matrix} \frac{3}{4}, -p \\ \frac{7}{4} \end{matrix}\middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{7}{4}\right)} + d^3 \left(\begin{cases} \frac{a^p x^4}{4} & \text{for}\ b = 0 \\ \frac{\begin{cases} \frac{(a+bx^4)^{p+1}}{p+1} & \text{for}\ p \neq -1 \\ \log\left(a+bx^4\right) & \text{otherwise} \end{cases}}{4b} & \text{otherwise} \end{cases}\right)$$

input
```
integrate((d*x+c)**3*(b*x**4+a)**p,x)
```

output
```
a**p*c**3*x*gamma(1/4)*hyper((1/4, -p), (5/4,), b*x**4*exp_polar(I*pi)/a)/
(4*gamma(5/4)) + 3*a**p*c**2*d*x**2*hyper((1/2, -p), (3/2,), b*x**4*exp_po
lar(I*pi)/a)/2 + 3*a**p*c*d**2*x**3*gamma(3/4)*hyper((3/4, -p), (7/4,), b*
x**4*exp_polar(I*pi)/a)/(4*gamma(7/4)) + d**3*Piecewise((a**p*x**4/4, Eq(b
, 0)), (Piecewise(((a + b*x**4)**(p + 1)/(p + 1), Ne(p, -1)), (log(a + b*x
**4), True))/(4*b), True))
```

Maxima [F]

$$\int (c+dx)^3 \left(a+bx^4\right)^p \, dx = \int (dx+c)^3 (bx^4+a)^p \, dx$$

input
```
integrate((d*x+c)^3*(b*x^4+a)^p,x, algorithm="maxima")
```

output
```
integrate((d*x + c)^3*(b*x^4 + a)^p, x)
```

Giac [F]

$$\int (c+dx)^3 \left(a+bx^4\right)^p \, dx = \int (dx+c)^3 (bx^4+a)^p \, dx$$

input
```
integrate((d*x+c)^3*(b*x^4+a)^p,x, algorithm="giac")
```

output
```
integrate((d*x + c)^3*(b*x^4 + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^3 \left(a+bx^4\right)^p \, dx = \int \left(b\,x^4+a\right)^p \left(c+d\,x\right)^3 dx$$

input
```
int((a + b*x^4)^p*(c + d*x)^3,x)
```

output
```
int((a + b*x^4)^p*(c + d*x)^3, x)
```

Reduce [F]

$$\int (c+dx)^3 \left(a+bx^4\right)^p \,dx = \text{too large to display}$$

input
```
int((d*x+c)^3*(b*x^4+a)^p,x)
```

output
```
(32*(a + b*x**4)**p*a*d**3*p**3 + 48*(a + b*x**4)**p*a*d**3*p**2 + 22*(a +
 b*x**4)**p*a*d**3*p + 3*(a + b*x**4)**p*a*d**3 + 32*(a + b*x**4)**p*b*c**
3*p**3*x + 72*(a + b*x**4)**p*b*c**3*p**2*x + 52*(a + b*x**4)**p*b*c**3*p*
x + 12*(a + b*x**4)**p*b*c**3*x + 96*(a + b*x**4)**p*b*c**2*d*p**3*x**2 +
192*(a + b*x**4)**p*b*c**2*d*p**2*x**2 + 114*(a + b*x**4)**p*b*c**2*d*p*x*
*2 + 18*(a + b*x**4)**p*b*c**2*d*x**2 + 96*(a + b*x**4)**p*b*c*d**2*p**3*x
**3 + 168*(a + b*x**4)**p*b*c*d**2*p**2*x**3 + 84*(a + b*x**4)**p*b*c*d**2
*p*x**3 + 12*(a + b*x**4)**p*b*c*d**2*x**3 + 32*(a + b*x**4)**p*b*d**3*p**
3*x**4 + 48*(a + b*x**4)**p*b*d**3*p**2*x**4 + 22*(a + b*x**4)**p*b*d**3*p
*x**4 + 3*(a + b*x**4)**p*b*d**3*x**4 + 4096*int((a + b*x**4)**p/(32*a*p**
3 + 48*a*p**2 + 22*a*p + 3*a + 32*b*p**3*x**4 + 48*b*p**2*x**4 + 22*b*p*x*
*4 + 3*b*x**4),x)*a*b*c**3*p**7 + 15360*int((a + b*x**4)**p/(32*a*p**3 + 4
8*a*p**2 + 22*a*p + 3*a + 32*b*p**3*x**4 + 48*b*p**2*x**4 + 22*b*p*x**4 +
3*b*x**4),x)*a*b*c**3*p**6 + 23296*int((a + b*x**4)**p/(32*a*p**3 + 48*a*p
**2 + 22*a*p + 3*a + 32*b*p**3*x**4 + 48*b*p**2*x**4 + 22*b*p*x**4 + 3*b*x
**4),x)*a*b*c**3*p**5 + 18240*int((a + b*x**4)**p/(32*a*p**3 + 48*a*p**2 +
 22*a*p + 3*a + 32*b*p**3*x**4 + 48*b*p**2*x**4 + 22*b*p*x**4 + 3*b*x**4),
x)*a*b*c**3*p**4 + 7744*int((a + b*x**4)**p/(32*a*p**3 + 48*a*p**2 + 22*a*
p + 3*a + 32*b*p**3*x**4 + 48*b*p**2*x**4 + 22*b*p*x**4 + 3*b*x**4),x)*a*b
*c**3*p**3 + 1680*int((a + b*x**4)**p/(32*a*p**3 + 48*a*p**2 + 22*a*p +...
```

3.218 $\int (c+dx)^2 (a+bx^4)^p \, dx$

Optimal result

Integrand size = 17, antiderivative size = 148

$$\int (c+dx)^2 (a+bx^4)^p \, dx = c^2x(a+bx^4)^p \left(1+\frac{bx^4}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{4}, -p, \frac{5}{4}, -\frac{bx^4}{a}\right) + cdx^2(a+bx^4)^p \left(1+\frac{bx^4}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{2}, -p, \frac{3}{2}, -\frac{bx^4}{a}\right) + \frac{1}{3}d^2x^3(a+bx^4)^p \left(1+\frac{bx^4}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{3}{4}, -p, \frac{7}{4}, -\frac{bx^4}{a}\right)$$

output

```
c^2*x*(b*x^4+a)^p*hypergeom([1/4, -p],[5/4],-b*x^4/a)/((1+b*x^4/a)^p)+c*d*
x^2*(b*x^4+a)^p*hypergeom([1/2, -p],[3/2],-b*x^4/a)/((1+b*x^4/a)^p)+1/3*d^
2*x^3*(b*x^4+a)^p*hypergeom([3/4, -p],[7/4],-b*x^4/a)/((1+b*x^4/a)^p)
```

Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.68

$$\int (c+dx)^2 \left(a+bx^4\right)^p \, dx = \frac{1}{3}x\left(a+bx^4\right)^p \left(1+\frac{bx^4}{a}\right)^{-p} \left(3c^2 \text{Hypergeometric2F1}\left(\frac{1}{4},-p,\frac{5}{4},-\frac{bx^4}{a}\right)+dx\left(3c\,\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^4}{a}\right)+dx\,\text{Hypergeometric2F1}\left(\frac{3}{4},-p,\frac{7}{4},-\frac{bx^4}{a}\right)\right)\right)$$

input
```
Integrate[(c + d*x)^2*(a + b*x^4)^p,x]
```

output
```
(x*(a + b*x^4)^p*(3*c^2*Hypergeometric2F1[1/4, -p, 5/4, -((b*x^4)/a)] + d*
x*(3*c*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^4)/a)] + d*x*Hypergeometric2
F1[3/4, -p, 7/4, -((b*x^4)/a)])))/(3*(1 + (b*x^4)/a)^p)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2424, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2 \left(a+bx^4\right)^p \, dx$$

$$\Big\downarrow \; 2424$$

$$\int \left(\left(c^2+d^2x^2\right)\left(a+bx^4\right)^p+2cdx\left(a+bx^4\right)^p\right)dx$$

$$\Big\downarrow \; 2009$$

$$c^2x(a+bx^4)^p\left(\frac{bx^4}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{4},-p,\frac{5}{4},-\frac{bx^4}{a}\right)+ cdx^2(a+bx^4)^p\left(\frac{bx^4}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^4}{a}\right)+ \frac{1}{3}d^2x^3(a+bx^4)^p\left(\frac{bx^4}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{3}{4},-p,\frac{7}{4},-\frac{bx^4}{a}\right)$$

input
```
Int[(c + d*x)^2*(a + b*x^4)^p,x]
```

output
```
(c^2*x*(a + b*x^4)^p*Hypergeometric2F1[1/4, -p, 5/4, -((b*x^4)/a)])/(1 + (
b*x^4)/a)^p + (c*d*x^2*(a + b*x^4)^p*Hypergeometric2F1[1/2, -p, 3/2, -((b*
x^4)/a)])/(1 + (b*x^4)/a)^p + (d^2*x^3*(a + b*x^4)^p*Hypergeometric2F1[3/4
, -p, 7/4, -((b*x^4)/a)])/(3*(1 + (b*x^4)/a)^p)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2424
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Maple [F]

$$\int (dx+c)^2\left(b\,x^4+a\right)^p dx$$

input
```
int((d*x+c)^2*(b*x^4+a)^p,x)
```

output
```
int((d*x+c)^2*(b*x^4+a)^p,x)
```

Fricas [F]

$$\int (c+dx)^2 \left(a+bx^4\right)^p \, dx = \int (dx+c)^2 (bx^4+a)^p \, dx$$

input
```
integrate((d*x+c)^2*(b*x^4+a)^p,x, algorithm="fricas")
```

output
```
integral((d^2*x^2 + 2*c*d*x + c^2)*(b*x^4 + a)^p, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 22.77 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.72

$$\int (c+dx)^2 \left(a+bx^4\right)^p \, dx = \frac{a^p c^2 x \Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\begin{matrix} \frac{1}{4}, -p \\ \frac{5}{4} \end{matrix} \middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)} + a^p c d x^2 {}_2F_1\left(\begin{matrix} \frac{1}{2}, -p \\ \frac{3}{2} \end{matrix} \middle| \frac{bx^4 e^{i\pi}}{a}\right) + \frac{a^p d^2 x^3 \Gamma\left(\frac{3}{4}\right) {}_2F_1\left(\begin{matrix} \frac{3}{4}, -p \\ \frac{7}{4} \end{matrix} \middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate((d*x+c)**2*(b*x**4+a)**p,x)
```

output
```
a**p*c**2*x*gamma(1/4)*hyper((1/4, -p), (5/4,), b*x**4*exp_polar(I*pi)/a)/
(4*gamma(5/4)) + a**p*c*d*x**2*hyper((1/2, -p), (3/2,), b*x**4*exp_polar(I
*pi)/a) + a**p*d**2*x**3*gamma(3/4)*hyper((3/4, -p), (7/4,), b*x**4*exp_po
lar(I*pi)/a)/(4*gamma(7/4))
```

Maxima [F]

$$\int (c+dx)^2 \left(a+bx^4\right)^p \, dx = \int (dx+c)^2 (bx^4+a)^p \, dx$$

input
```
integrate((d*x+c)^2*(b*x^4+a)^p,x, algorithm="maxima")
```

output
```
integrate((d*x + c)^2*(b*x^4 + a)^p, x)
```

Giac [F]

$$\int (c+dx)^2 \left(a+bx^4\right)^p \, dx = \int (dx+c)^2 (bx^4+a)^p \, dx$$

input
```
integrate((d*x+c)^2*(b*x^4+a)^p,x, algorithm="giac")
```

output
```
integrate((d*x + c)^2*(b*x^4 + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^2 \left(a+bx^4\right)^p \, dx = \int \left(b\,x^4+a\right)^p \, (c+d\,x)^2 \, dx$$

input
```
int((a + b*x^4)^p*(c + d*x)^2,x)
```

output
```
int((a + b*x^4)^p*(c + d*x)^2, x)
```

Reduce [F]

$$\int (c+dx)^2 \left(a+bx^4\right)^p \, dx = \text{Too large to display}$$

input
```
int((d*x+c)^2*(b*x^4+a)^p,x)
```

output
```
(8*(a + b*x**4)**p*c**2*p**2*x + 10*(a + b*x**4)**p*c**2*p*x + 3*(a + b*x*
*4)**p*c**2*x + 16*(a + b*x**4)**p*c*d*p**2*x**2 + 16*(a + b*x**4)**p*c*d*
p*x**2 + 3*(a + b*x**4)**p*c*d*x**2 + 8*(a + b*x**4)**p*d**2*p**2*x**3 + 6
*(a + b*x**4)**p*d**2*p*x**3 + (a + b*x**4)**p*d**2*x**3 + 1024*int((a + b
*x**4)**p/(32*a*p**3 + 48*a*p**2 + 22*a*p + 3*a + 32*b*p**3*x**4 + 48*b*p*
*2*x**4 + 22*b*p*x**4 + 3*b*x**4),x)*a*c**2*p**6 + 2816*int((a + b*x**4)**
p/(32*a*p**3 + 48*a*p**2 + 22*a*p + 3*a + 32*b*p**3*x**4 + 48*b*p**2*x**4
 + 22*b*p*x**4 + 3*b*x**4),x)*a*c**2*p**5 + 3008*int((a + b*x**4)**p/(32*a*
p**3 + 48*a*p**2 + 22*a*p + 3*a + 32*b*p**3*x**4 + 48*b*p**2*x**4 + 22*b*p
*x**4 + 3*b*x**4),x)*a*c**2*p**4 + 1552*int((a + b*x**4)**p/(32*a*p**3 + 4
8*a*p**2 + 22*a*p + 3*a + 32*b*p**3*x**4 + 48*b*p**2*x**4 + 22*b*p*x**4 +
3*b*x**4),x)*a*c**2*p**3 + 384*int((a + b*x**4)**p/(32*a*p**3 + 48*a*p**2
 + 22*a*p + 3*a + 32*b*p**3*x**4 + 48*b*p**2*x**4 + 22*b*p*x**4 + 3*b*x**4)
,x)*a*c**2*p**2 + 36*int((a + b*x**4)**p/(32*a*p**3 + 48*a*p**2 + 22*a*p +
 3*a + 32*b*p**3*x**4 + 48*b*p**2*x**4 + 22*b*p*x**4 + 3*b*x**4),x)*a*c**2
*p + 1024*int(((a + b*x**4)**p*x**2)/(32*a*p**3 + 48*a*p**2 + 22*a*p + 3*a
 + 32*b*p**3*x**4 + 48*b*p**2*x**4 + 22*b*p*x**4 + 3*b*x**4),x)*a*d**2*p**
6 + 2304*int(((a + b*x**4)**p*x**2)/(32*a*p**3 + 48*a*p**2 + 22*a*p + 3*a
 + 32*b*p**3*x**4 + 48*b*p**2*x**4 + 22*b*p*x**4 + 3*b*x**4),x)*a*d**2*p**5
 + 1984*int(((a + b*x**4)**p*x**2)/(32*a*p**3 + 48*a*p**2 + 22*a*p + 3*...
```

3.219 $\int (c+dx)\left(a+bx^4\right)^p\,dx$

Optimal result

Integrand size = 15, antiderivative size = 96

$$\int (c+dx)\left(a+bx^4\right)^p\,dx=cx\left(a+bx^4\right)^p\left(1+\frac{bx^4}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{4},-p,\frac{5}{4},-\frac{bx^4}{a}\right)+\frac{1}{2}dx^2\left(a+bx^4\right)^p\left(1+\frac{bx^4}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^4}{a}\right)$$

output
```
c*x*(b*x^4+a)^p*hypergeom([1/4, -p],[5/4],-b*x^4/a)/((1+b*x^4/a)^p)+1/2*d*
x^2*(b*x^4+a)^p*hypergeom([1/2, -p],[3/2],-b*x^4/a)/((1+b*x^4/a)^p)
```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.76

$$\int (c+dx)\left(a+bx^4\right)^p\,dx=\frac{1}{2}x\left(a+bx^4\right)^p\left(1+\frac{bx^4}{a}\right)^{-p}\left(2c\,\text{Hypergeometric2F1}\left(\frac{1}{4},-p,\frac{5}{4},-\frac{bx^4}{a}\right)+dx\,\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^4}{a}\right)\right)$$

input
```
Integrate[(c + d*x)*(a + b*x^4)^p,x]
```

output
```
(x*(a + b*x^4)^p*(2*c*Hypergeometric2F1[1/4, -p, 5/4, -((b*x^4)/a)] + d*x*
Hypergeometric2F1[1/2, -p, 3/2, -((b*x^4)/a)]))/(2*(1 + (b*x^4)/a)^p)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2424, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx) \left(a+bx^4\right)^p \, dx$$

$$\Big\downarrow \text{2424}$$

$$\int \left(c\left(a+bx^4\right)^p + dx\left(a+bx^4\right)^p\right) dx$$

$$\Big\downarrow \text{2009}$$

$$cx\left(a+bx^4\right)^p \left(\frac{bx^4}{a}+1\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{4},-p,\frac{5}{4},-\frac{bx^4}{a}\right) + \frac{1}{2}dx^2\left(a+bx^4\right)^p \left(\frac{bx^4}{a}+1\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^4}{a}\right)$$

input
```
Int[(c + d*x)*(a + b*x^4)^p,x]
```

output
```
(c*x*(a + b*x^4)^p*Hypergeometric2F1[1/4, -p, 5/4, -((b*x^4)/a)])/(1 + (b*
x^4)/a)^p + (d*x^2*(a + b*x^4)^p*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^4)
/a)])/(2*(1 + (b*x^4)/a)^p)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2424
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && !PolyQ[Pq, x^(n/2)]
```

Maple [F]

$$\int (dx+c)\left(b\,x^{4}+a\right)^{p}dx$$

input
```
int((d*x+c)*(b*x^4+a)^p,x)
```

output
```
int((d*x+c)*(b*x^4+a)^p,x)
```

Fricas [F]

$$\int (c+dx)\left(a+bx^4\right)^p\,dx=\int (dx+c)\left(bx^4+a\right)^p\,dx$$

input
```
integrate((d*x+c)*(b*x^4+a)^p,x, algorithm="fricas")
```

output
```
integral((d*x + c)*(b*x^4 + a)^p, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 14.26 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.68

$$\int (c+dx)\left(a+bx^4\right)^p\,dx = \frac{a^p c x \Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\begin{matrix}\frac{1}{4}, -p \\ \frac{5}{4}\end{matrix}\middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)} + \frac{a^p d x^2 {}_2F_1\left(\begin{matrix}\frac{1}{2}, -p \\ \frac{3}{2}\end{matrix}\middle| \frac{bx^4 e^{i\pi}}{a}\right)}{2}$$

input
```
integrate((d*x+c)*(b*x**4+a)**p,x)
```

output
```
a**p*c*x*gamma(1/4)*hyper((1/4, -p), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*
gamma(5/4)) + a**p*d*x**2*hyper((1/2, -p), (3/2,), b*x**4*exp_polar(I*pi)/
a)/2
```

Maxima [F]

$$\int (c+dx)\left(a+bx^4\right)^p\,dx = \int (dx+c)\left(bx^4+a\right)^p\,dx$$

input
```
integrate((d*x+c)*(b*x^4+a)^p,x, algorithm="maxima")
```

output
```
integrate((d*x + c)*(b*x^4 + a)^p, x)
```

Giac [F]

$$\int (c+dx)\left(a+bx^4\right)^p\,dx = \int (dx+c)\left(bx^4+a\right)^p\,dx$$

input
```
integrate((d*x+c)*(b*x^4+a)^p,x, algorithm="giac")
```

output
```
integrate((d*x + c)*(b*x^4 + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx) \left(a+bx^4\right)^p \, dx = \int \left(b\,x^4+a\right)^p\,(c+d\,x)\, dx$$

input
```
int((a + b*x^4)^p*(c + d*x),x)
```

output
```
int((a + b*x^4)^p*(c + d*x), x)
```

Reduce [F]

$$\int (c+dx) \left(a+bx^4\right)^p \, dx = \frac{4(b\,x^4+a)^p\,cpx+2(b\,x^4+a)^p\,cx+4(b\,x^4+a)^p\,dp\,x^2+(b\,x^4+a)^p\,d\,x^2+128\Big(\int \frac{(b\,x^4+a)^p}{8b\,p^2x^4+6bp\,x^4+b\,x^4+8a\,p^2+}$$

input
```
int((d*x+c)*(b*x^4+a)^p,x)
```

output
```
(4*(a + b*x**4)**p*c*p*x + 2*(a + b*x**4)**p*c*x + 4*(a + b*x**4)**p*d*p*x
**2 + (a + b*x**4)**p*d*x**2 + 128*int((a + b*x**4)**p/(8*a*p**2 + 6*a*p +
 a + 8*b*p**2*x**4 + 6*b*p*x**4 + b*x**4),x)*a*c*p**4 + 160*int((a + b*x**
4)**p/(8*a*p**2 + 6*a*p + a + 8*b*p**2*x**4 + 6*b*p*x**4 + b*x**4),x)*a*c*
p**3 + 64*int((a + b*x**4)**p/(8*a*p**2 + 6*a*p + a + 8*b*p**2*x**4 + 6*b*
p*x**4 + b*x**4),x)*a*c*p**2 + 8*int((a + b*x**4)**p/(8*a*p**2 + 6*a*p + a
 + 8*b*p**2*x**4 + 6*b*p*x**4 + b*x**4),x)*a*c*p + 128*int(((a + b*x**4)**
p*x)/(8*a*p**2 + 6*a*p + a + 8*b*p**2*x**4 + 6*b*p*x**4 + b*x**4),x)*a*d*p
**4 + 128*int(((a + b*x**4)**p*x)/(8*a*p**2 + 6*a*p + a + 8*b*p**2*x**4 +
6*b*p*x**4 + b*x**4),x)*a*d*p**3 + 40*int(((a + b*x**4)**p*x)/(8*a*p**2 +
6*a*p + a + 8*b*p**2*x**4 + 6*b*p*x**4 + b*x**4),x)*a*d*p**2 + 4*int(((a +
 b*x**4)**p*x)/(8*a*p**2 + 6*a*p + a + 8*b*p**2*x**4 + 6*b*p*x**4 + b*x**4
),x)*a*d*p)/(2*(8*p**2 + 6*p + 1))
```

3.220 $\int (a+bx^4)^p \, dx$

Optimal result

Integrand size = 9, antiderivative size = 44

$$\int (a+bx^4)^p \, dx = x(a+bx^4)^p \left(1+\frac{bx^4}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{4},-p,\frac{5}{4},-\frac{bx^4}{a}\right)$$

output
```
x*(b*x^4+a)^p*hypergeom([1/4, -p],[5/4],-b*x^4/a)/((1+b*x^4/a)^p)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int (a+bx^4)^p \, dx = x(a+bx^4)^p \left(1+\frac{bx^4}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{4},-p,\frac{5}{4},-\frac{bx^4}{a}\right)$$

input
```
Integrate[(a + b*x^4)^p,x]
```

output
```
(x*(a + b*x^4)^p*Hypergeometric2F1[1/4, -p, 5/4, -((b*x^4)/a)])/(1 + (b*x^
4)/a)^p
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {779, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^4\right)^p\,dx$$

$$\Big\downarrow \text{779}$$

$$\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}\int \left(\frac{bx^4}{a}+1\right)^p dx$$

$$\Big\downarrow \text{778}$$

$$x\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{4},-p,\frac{5}{4},-\frac{bx^4}{a}\right)$$

input
```
Int[(a + b*x^4)^p,x]
```

output
```
(x*(a + b*x^4)^p*Hypergeometric2F1[1/4, -p, 5/4, -((b*x^4)/a)])/(1 + (b*x^
4)/a)^p
```

Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Si
mplify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \left(b\,x^{4}+a\right)^{p}dx$$

input
```
int((b*x^4+a)^p,x)
```

output
```
int((b*x^4+a)^p,x)
```

Fricas [F]

$$\int \left(a+bx^4\right)^p\,dx=\int \left(bx^{4}+a\right)^{p}dx$$

input
```
integrate((b*x^4+a)^p,x, algorithm="fricas")
```

output
```
integral((b*x^4 + a)^p, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.61 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.77

$$\int \left(a+bx^4\right)^p\,dx=\frac{a^{p}x\Gamma\left(\frac{1}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{4},-p\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^{4}e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((b*x**4+a)**p,x)
```

output
```
a**p*x*gamma(1/4)*hyper((1/4, -p), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*ga
mma(5/4))
```

Maxima [F]

$$\int (a+bx^4)^p \, dx = \int (bx^4+a)^p \, dx$$

input
```
integrate((b*x^4+a)^p,x, algorithm="maxima")
```

output
```
integrate((b*x^4 + a)^p, x)
```

Giac [F]

$$\int (a+bx^4)^p \, dx = \int (bx^4+a)^p \, dx$$

input
```
integrate((b*x^4+a)^p,x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^p, x)
```

Mupad [B] (verification not implemented)

Time = 21.55 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.93

$$\int (a+bx^4)^p \, dx = \frac{x\,(b\,x^4+a)^p\,{}_2\mathrm{F}_1\left(\frac{1}{4},-p;\ \frac{5}{4};\ -\frac{b\,x^4}{a}\right)}{\left(\frac{b\,x^4}{a}+1\right)^p}$$

input
```
int((a + b*x^4)^p,x)
```

output
```
(x*(a + b*x^4)^p*hypergeom([1/4, -p], 5/4, -(b*x^4)/a))/((b*x^4)/a + 1)^p
```

Reduce [F]

$$\int \left(a+bx^4\right)^p \, dx$$
$$= \frac{\left(b\,x^4+a\right)^p x+16\left(\int \frac{\left(b\,x^4+a\right)^p}{4bp\,x^4+b\,x^4+4ap+a}dx\right)a\,p^2+4\left(\int \frac{\left(b\,x^4+a\right)^p}{4bp\,x^4+b\,x^4+4ap+a}dx\right)ap}{4p+1}$$

input

```
int((b*x^4+a)^p,x)
```

output

```
((a + b*x**4)**p*x + 16*int((a + b*x**4)**p/(4*a*p + a + 4*b*p*x**4 + b*x*
*4),x)*a*p**2 + 4*int((a + b*x**4)**p/(4*a*p + a + 4*b*p*x**4 + b*x**4),x)
*a*p)/(4*p + 1)
```

3.221 $\int \frac{(a+bx^4)^p}{c+dx}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 257

$$\begin{aligned}\int \frac{(a+bx^4)^p}{c+dx}\,dx &= \frac{x(a+bx^4)^p\left(1+\frac{bx^4}{a}\right)^{-p}\text{AppellF1}\left(\frac{1}{4},-p,1,\frac{5}{4},-\frac{bx^4}{a},\frac{d^4x^4}{c^4}\right)}{c}\\
&\quad-\frac{dx^2(a+bx^4)^p\left(1+\frac{bx^4}{a}\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-p,1,\frac{3}{2},-\frac{bx^4}{a},\frac{d^4x^4}{c^4}\right)}{2c^2}\\
&\quad+\frac{d^2x^3(a+bx^4)^p\left(1+\frac{bx^4}{a}\right)^{-p}\text{AppellF1}\left(\frac{3}{4},-p,1,\frac{7}{4},-\frac{bx^4}{a},\frac{d^4x^4}{c^4}\right)}{3c^3}\\
&\quad-\frac{d^3(a+bx^4)^{1+p}\,\text{Hypergeometric2F1}\left(1,1+p,2+p,\frac{d^4(a+bx^4)}{bc^4+ad^4}\right)}{4\,(bc^4+ad^4)\,(1+p)}\end{aligned}$$

output
```
x*(b*x^4+a)^p*AppellF1(1/4,1,-p,5/4,d^4*x^4/c^4,-b*x^4/a)/c/((1+b*x^4/a)^p
)-1/2*d*x^2*(b*x^4+a)^p*AppellF1(1/2,1,-p,3/2,d^4*x^4/c^4,-b*x^4/a)/c^2/((
1+b*x^4/a)^p)+1/3*d^2*x^3*(b*x^4+a)^p*AppellF1(3/4,1,-p,7/4,d^4*x^4/c^4,-b
*x^4/a)/c^3/((1+b*x^4/a)^p)-1/4*d^3*(b*x^4+a)^(p+1)*hypergeom([1, p+1],[2+
p],d^4*(b*x^4+a)/(a*d^4+b*c^4))/(a*d^4+b*c^4)/(p+1)
```

Mathematica [F]

$$\int \frac{(a+bx^4)^p}{c+dx}\,dx = \int \frac{(a+bx^4)^p}{c+dx}\,dx$$

input
```
Integrate[(a + b*x^4)^p/(c + d*x),x]
```

output
```
Integrate[(a + b*x^4)^p/(c + d*x), x]
```

Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.02, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.588, Rules used = {2584, 1791, 1569, 1577, 504, 334, 333, 353, 78, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^4)^p}{c+dx}dx$$

$$\Big\downarrow \text{2584}$$

$$\int \frac{(c-dx)(a+bx^4)^p}{c^2-d^2x^2}dx$$

$$\Big\downarrow \text{1791}$$

$$c\int \frac{(bx^4+a)^p}{c^2-d^2x^2}dx - d\int \frac{x(bx^4+a)^p}{c^2-d^2x^2}dx$$

$$\Big\downarrow \text{1569}$$

$$c\int \left(\frac{c^2(bx^4+a)^p}{c^4-d^4x^4} - \frac{d^2x^2(bx^4+a)^p}{d^4x^4-c^4}\right)dx - d\int \frac{x(bx^4+a)^p}{c^2-d^2x^2}dx$$

$$\Big\downarrow \text{1577}$$

$$c\int \left(\frac{c^2(bx^4+a)^p}{c^4-d^4x^4} - \frac{d^2x^2(bx^4+a)^p}{d^4x^4-c^4}\right)dx - \frac{1}{2}d\int \frac{(bx^4+a)^p}{c^2-d^2x^2}dx^2$$

$$\Big\downarrow \text{504}$$

$$c\int \left(\frac{c^2 \left(bx^4+a\right)^p}{c^4-d^4x^4}-\frac{d^2x^2 \left(bx^4+a\right)^p}{d^4x^4-c^4}\right)dx-\frac{1}{2}d\left(d^2\int \frac{x^2 \left(bx^4+a\right)^p}{c^4-d^4x^4}dx^2+c^2\int \frac{\left(bx^4+a\right)^p}{c^4-d^4x^4}dx^2\right)$$

$$\Big\downarrow \text{334}$$

$$c\int \left(\frac{c^2 \left(bx^4+a\right)^p}{c^4-d^4x^4}-\frac{d^2x^2 \left(bx^4+a\right)^p}{d^4x^4-c^4}\right)dx-\frac{1}{2}d\left(d^2\int \frac{x^2 \left(bx^4+a\right)^p}{c^4-d^4x^4}dx^2+c^2\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}\int \frac{\left(\frac{bx^4}{a}+1\right)^p}{c^4-d^4x^4}dx^2\right)$$

$$\Big\downarrow \text{333}$$

$$c\int \left(\frac{c^2 \left(bx^4+a\right)^p}{c^4-d^4x^4}-\frac{d^2x^2 \left(bx^4+a\right)^p}{d^4x^4-c^4}\right)dx-\frac{1}{2}d\left(d^2\int \frac{x^2 \left(bx^4+a\right)^p}{c^4-d^4x^4}dx^2+\frac{x^2\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-p,1,\frac{3}{2},-\frac{bx^4}{a},\frac{d^4x^4}{c^4}\right)}{c^2}\right)$$

$$\Big\downarrow \text{353}$$

$$c\int \left(\frac{c^2 \left(bx^4+a\right)^p}{c^4-d^4x^4}-\frac{d^2x^2 \left(bx^4+a\right)^p}{d^4x^4-c^4}\right)dx-\frac{1}{2}d\left(\frac{1}{2}d^2\int \frac{\left(bx^4+a\right)^p}{c^4-d^4x^4}dx^4+\frac{x^2\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-p,1,\frac{3}{2},-\frac{bx^4}{a},\frac{d^4x^4}{c^4}\right)}{c^2}\right)$$

$$\Big\downarrow \text{78}$$

$$c\int \left(\frac{c^2 \left(bx^4+a\right)^p}{c^4-d^4x^4}-\frac{d^2x^2 \left(bx^4+a\right)^p}{d^4x^4-c^4}\right)dx-\frac{1}{2}d\left(\frac{x^2\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-p,1,\frac{3}{2},-\frac{bx^4}{a},\frac{d^4x^4}{c^4}\right)}{c^2}+\frac{d^2\left(a+bx^4\right)^{p+1}\text{Hypergeometric2F1}\left(1,p+\right.}{2(p+1)\left(ad^4+bc^4\right)}\right.$$

$$\Big\downarrow \text{2009}$$

$$c\left(\frac{d^2x^3(a+bx^4)^p\left(\frac{bx^4}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{3}{4},-p,1,\frac{7}{4},-\frac{bx^4}{a},\frac{d^4x^4}{c^4}\right)}{3c^4}+\frac{x(a+bx^4)^p\left(\frac{bx^4}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{4},-p,\right.}{c^2}\right.$$

$$\frac{1}{2}d\left(\frac{x^2(a+bx^4)^p\left(\frac{bx^4}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-p,1,\frac{3}{2},-\frac{bx^4}{a},\frac{d^4x^4}{c^4}\right)}{c^2}+\frac{d^2(a+bx^4)^{p+1}\text{Hypergeometric2F1}\left(1,p+\right.}{2(p+1)(ad^4+bc^4)}\right.$$

input
```
Int[(a + b*x^4)^p/(c + d*x),x]
```

output
```
c*((x*(a + b*x^4)^p*AppellF1[1/4, -p, 1, 5/4, -((b*x^4)/a), (d^4*x^4)/c^4]
)/(c^2*(1 + (b*x^4)/a)^p) + (d^2*x^3*(a + b*x^4)^p*AppellF1[3/4, -p, 1, 7/
4, -((b*x^4)/a), (d^4*x^4)/c^4])/(3*c^4*(1 + (b*x^4)/a)^p)) - (d*((x^2*(a
+ b*x^4)^p*AppellF1[1/2, -p, 1, 3/2, -((b*x^4)/a), (d^4*x^4)/c^4])/(c^2*(1
 + (b*x^4)/a)^p) + (d^2*(a + b*x^4)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2
+ p, (d^4*(a + b*x^4))/(b*c^4 + a*d^4)])/(2*(b*c^4 + a*d^4)*(1 + p))))/2
```

Defintions of rubi rules used

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 334
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[
(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p, q}, x] &&
NeQ[b*c - a*d, 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 504
```
Int[((a_) + (b_.)*(x_)^2)^(p_)/((c_) + (d_.)*(x_)), x_Symbol] :> Simp[c   I
nt[(a + b*x^2)^p/(c^2 - d^2*x^2), x], x] - Simp[d   Int[x*((a + b*x^2)^p/(c
^2 - d^2*x^2)), x], x] /; FreeQ[{a, b, c, d, p}, x]
```

rule 1569
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int
[ExpandIntegrand[(a + c*x^4)^p, (d/(d^2 - e^2*x^4) - e*(x^2/(d^2 - e^2*x^4)
))^(-q), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !
IntegerQ[p] && ILtQ[q, 0]
```

rule 1577
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, c, d, e, p, q}, x]
```

rule 1791
```
Int[((A_) + (B_.)*(x_)^(m_.))*((d_) + (e_.)*(x_)^(n_))^(q_.)*((a_) + (c_.)*
(x_)^(n2_))^(p_.), x_Symbol] :> Simp[A   Int[(d + e*x^n)^q*(a + c*x^(2*n))^
p, x], x] + Simp[B   Int[x^m*(d + e*x^n)^q*(a + c*x^(2*n))^p, x], x] /; Fre
eQ[{a, c, d, e, A, B, m, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[m - n + 1, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2584
```
Int[((c_) + (d_.)*(x_)^(n_.))^(q_)*((a_) + (b_.)*(x_)^(nn_.))^(p_), x_Symbo
l] :> Int[ExpandToSum[(c - d*x^n)^(-q), x]*((a + b*x^nn)^p/(c^2 - d^2*x^(2*
n))^(-q)), x] /; FreeQ[{a, b, c, d, n, nn, p}, x] &&  !IntegerQ[p] && ILtQ[
q, 0] && IGtQ[Log[2, nn/n], 0]
```

Maple [F]

$$\int \frac{(b\,x^4+a)^p}{dx+c}dx$$

input
```
int((b*x^4+a)^p/(d*x+c),x)
```

output
```
int((b*x^4+a)^p/(d*x+c),x)
```

Fricas [F]

$$\int \frac{(a+bx^4)^p}{c+dx}\,dx = \int \frac{(bx^4+a)^p}{dx+c}\,dx$$

input
```
integrate((b*x^4+a)^p/(d*x+c),x, algorithm="fricas")
```

output
```
integral((b*x^4 + a)^p/(d*x + c), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^4)^p}{c+dx}\,dx = \text{Timed out}$$

input
```
integrate((b*x**4+a)**p/(d*x+c),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^4)^p}{c+dx}\,dx = \int \frac{(bx^4+a)^p}{dx+c}\,dx$$

input
```
integrate((b*x^4+a)^p/(d*x+c),x, algorithm="maxima")
```

output
```
integrate((b*x^4 + a)^p/(d*x + c), x)
```

Giac [F]

$$\int \frac{(a+bx^4)^p}{c+dx}\,dx = \int \frac{(bx^4+a)^p}{dx+c}\,dx$$

input
```
integrate((b*x^4+a)^p/(d*x+c),x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^p/(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^4)^p}{c+dx}\,dx = \int \frac{(b\,x^4+a)^p}{c+d\,x}\,dx$$

input
```
int((a + b*x^4)^p/(c + d*x),x)
```

output
```
int((a + b*x^4)^p/(c + d*x), x)
```

Reduce [F]

$$\int \frac{(a+bx^4)^p}{c+dx}\,dx = \frac{(b\,x^4+a)^p + 4\left(\int \frac{(b\,x^4+a)^p}{bd\,x^5+bc\,x^4+adx+ac}dx\right)adp - 4\left(\int \frac{(b\,x^4+a)^p x^3}{bd\,x^5+bc\,x^4+adx+ac}dx\right)bcp}{4dp}$$

input
```
int((b*x^4+a)^p/(d*x+c),x)
```

output
```
((a + b*x**4)**p + 4*int((a + b*x**4)**p/(a*c + a*d*x + b*c*x**4 + b*d*x**
5),x)*a*d*p - 4*int(((a + b*x**4)**p*x**3)/(a*c + a*d*x + b*c*x**4 + b*d*x
**5),x)*b*c*p)/(4*d*p)
```

3.222 $\int \frac{(a+bx^4)^p}{(c+dx)^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 452

$$\begin{aligned}\int \frac{(a+bx^4)^p}{(c+dx)^2}\,dx &= \frac{x(a+bx^4)^p\left(1+\frac{bx^4}{a}\right)^{-p}\text{AppellF1}\left(\frac{1}{4},-p,2,\frac{5}{4},-\frac{bx^4}{a},\frac{d^4x^4}{c^4}\right)}{c^2}\\
&\quad-\frac{dx^2(a+bx^4)^p\left(1+\frac{bx^4}{a}\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-p,2,\frac{3}{2},-\frac{bx^4}{a},\frac{d^4x^4}{c^4}\right)}{c^3}\\
&\quad+\frac{d^2x^3(a+bx^4)^p\left(1+\frac{bx^4}{a}\right)^{-p}\text{AppellF1}\left(\frac{3}{4},-p,2,\frac{7}{4},-\frac{bx^4}{a},\frac{d^4x^4}{c^4}\right)}{c^4}\\
&\quad+\frac{3d^4x^5(a+bx^4)^p\left(1+\frac{bx^4}{a}\right)^{-p}\text{AppellF1}\left(\frac{5}{4},-p,2,\frac{9}{4},-\frac{bx^4}{a},\frac{d^4x^4}{c^4}\right)}{5c^6}\\
&\quad-\frac{d^5x^6(a+bx^4)^p\left(1+\frac{bx^4}{a}\right)^{-p}\text{AppellF1}\left(\frac{3}{2},-p,2,\frac{5}{2},-\frac{bx^4}{a},\frac{d^4x^4}{c^4}\right)}{3c^7}\\
&\quad+\frac{d^6x^7(a+bx^4)^p\left(1+\frac{bx^4}{a}\right)^{-p}\text{AppellF1}\left(\frac{7}{4},-p,2,\frac{11}{4},-\frac{bx^4}{a},\frac{d^4x^4}{c^4}\right)}{7c^8}\\
&\quad-\frac{bc^3d^3(a+bx^4)^{1+p}\,\text{Hypergeometric2F1}\left(2,1+p,2+p,\frac{d^4(a+bx^4)}{bc^4+ad^4}\right)}{(bc^4+ad^4)^2(1+p)}\end{aligned}$$

output
```
x*(b*x^4+a)^p*AppellF1(1/4,2,-p,5/4,d^4*x^4/c^4,-b*x^4/a)/c^2/((1+b*x^4/a)
^p)-d*x^2*(b*x^4+a)^p*AppellF1(1/2,2,-p,3/2,d^4*x^4/c^4,-b*x^4/a)/c^3/((1+
b*x^4/a)^p)+d^2*x^3*(b*x^4+a)^p*AppellF1(3/4,2,-p,7/4,d^4*x^4/c^4,-b*x^4/a
)/c^4/((1+b*x^4/a)^p)+3/5*d^4*x^5*(b*x^4+a)^p*AppellF1(5/4,2,-p,9/4,d^4*x^
4/c^4,-b*x^4/a)/c^6/((1+b*x^4/a)^p)-1/3*d^5*x^6*(b*x^4+a)^p*AppellF1(3/2,2
,-p,5/2,d^4*x^4/c^4,-b*x^4/a)/c^7/((1+b*x^4/a)^p)+1/7*d^6*x^7*(b*x^4+a)^p*
AppellF1(7/4,2,-p,11/4,d^4*x^4/c^4,-b*x^4/a)/c^8/((1+b*x^4/a)^p)-b*c^3*d^3
*(b*x^4+a)^(p+1)*hypergeom([2, p+1],[2+p],d^4*(b*x^4+a)/(a*d^4+b*c^4))/(a*
d^4+b*c^4)^2/(p+1)
```

Mathematica [F]

$$\int \frac{(a+bx^4)^p}{(c+dx)^2}\,dx=\int \frac{(a+bx^4)^p}{(c+dx)^2}\,dx$$

input
```
Integrate[(a + b*x^4)^p/(c + d*x)^2,x]
```

output
```
Integrate[(a + b*x^4)^p/(c + d*x)^2, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^4)^p}{(c+dx)^2}\,dx$$

$$\Big\downarrow \text{ 2584}$$

$$\int \frac{(c^2-2cdx+d^2x^2)\,(a+bx^4)^p}{(c^2-d^2x^2)^2}dx$$

$$\Big\downarrow \text{ 2255}$$

$$\int -\frac{2cdx\,(bx^4+a)^p}{(c^2-d^2x^2)^2}dx+\int \frac{(c^2+d^2x^2)\,(bx^4+a)^p}{(c^2-d^2x^2)^2}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{(c^2+d^2x^2)(bx^4+a)^p}{(c^2-d^2x^2)^2}dx - 2cd\int \frac{x(bx^4+a)^p}{(c^2-d^2x^2)^2}dx$$

$$\Big\downarrow \text{1577}$$

$$\int \frac{(c^2+d^2x^2)(bx^4+a)^p}{(c^2-d^2x^2)^2}dx - cd\int \frac{(bx^4+a)^p}{(c^2-d^2x^2)^2}dx^2$$

$$\Big\downarrow \text{505}$$

$$\int \frac{(c^2+d^2x^2)(bx^4+a)^p}{(c^2-d^2x^2)^2}dx - cd\int \left(\frac{c^4(bx^4+a)^p}{(c^4-d^4x^4)^2} + \frac{2c^2d^2x^2(bx^4+a)^p}{(c^4-d^4x^4)^2} + \frac{d^4x^4(bx^4+a)^p}{(d^4x^4-c^4)^2}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\int \frac{(c^2+d^2x^2)(bx^4+a)^p}{(c^2-d^2x^2)^2}dx - cd\left(\frac{x^2(a+bx^4)^p\left(\frac{bx^4}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-p,2,\frac{3}{2},-\frac{bx^4}{a},\frac{d^4x^4}{c^4}\right)}{c^4} + \frac{d^4x^6(a+bx^4)^p\left(\frac{bx^4}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{3}{2},-\right.}{3c^8}\right.$$

$$\Big\downarrow \text{2261}$$

$$\int \frac{(c^2+d^2x^2)(bx^4+a)^p}{(c^2-d^2x^2)^2}dx - cd\left(\frac{x^2(a+bx^4)^p\left(\frac{bx^4}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-p,2,\frac{3}{2},-\frac{bx^4}{a},\frac{d^4x^4}{c^4}\right)}{c^4} + \frac{d^4x^6(a+bx^4)^p\left(\frac{bx^4}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{3}{2},-\right.}{3c^8}\right.$$

input
```
Int[(a + b*x^4)^p/(c + d*x)^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 505
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x^2)^p, (c/(c^2 - d^2*x^2) - d*(x/(c^2 - d^2*x^2)))^(
-n), x], x] /; FreeQ[{a, b, c, d, p}, x] && ILtQ[n, -1] && PosQ[a/b]
```

rule 1577
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, c, d, e, p, q}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2255
```
Int[(Pr_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Module[{r = Expon[Pr, x], k}, Int[Sum[Coeff[Pr, x, 2*k]*x^(2*k), {k, 0,
 r/2}]*(d + e*x^2)^q*(a + c*x^4)^p, x] + Int[x*Sum[Coeff[Pr, x, 2*k + 1]*x^
(2*k), {k, 0, (r - 1)/2}]*(d + e*x^2)^q*(a + c*x^4)^p, x]] /; FreeQ[{a, c,
d, e, p, q}, x] && PolyQ[Pr, x] &&  !PolyQ[Pr, x^2]
```

rule 2261
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Unintegrable[Px*(d + e*x^2)^q*(a + c*x^4)^p, x] /; FreeQ[{a, c, d, e,
p, q}, x] && PolyQ[Px, x]
```

rule 2584
```
Int[((c_) + (d_.)*(x_)^(n_.))^(q_)*((a_) + (b_.)*(x_)^(nn_.))^(p_), x_Symbo
l] :> Int[ExpandToSum[(c - d*x^n)^(-q), x]*((a + b*x^nn)^p/(c^2 - d^2*x^(2*
n))^(-q)), x] /; FreeQ[{a, b, c, d, n, nn, p}, x] &&  !IntegerQ[p] && ILtQ[
q, 0] && IGtQ[Log[2, nn/n], 0]
```

Maple [F]

$$\int \frac{(b\,x^4+a)^p}{(dx+c)^2}dx$$

input
```
int((b*x^4+a)^p/(d*x+c)^2,x)
```

output
```
int((b*x^4+a)^p/(d*x+c)^2,x)
```

Fricas [F]

$$\int \frac{(a+bx^4)^p}{(c+dx)^2}\,dx = \int \frac{(bx^4+a)^p}{(dx+c)^2}\,dx$$

input
```
integrate((b*x^4+a)^p/(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral((b*x^4 + a)^p/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^4)^p}{(c+dx)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x**4+a)**p/(d*x+c)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^4)^p}{(c+dx)^2}\,dx = \int \frac{(bx^4+a)^p}{(dx+c)^2}\,dx$$

input
```
integrate((b*x^4+a)^p/(d*x+c)^2,x, algorithm="maxima")
```

output
```
integrate((b*x^4 + a)^p/(d*x + c)^2, x)
```

Giac [F]

$$\int \frac{(a+bx^4)^p}{(c+dx)^2}\,dx = \int \frac{(bx^4+a)^p}{(dx+c)^2}\,dx$$

input
```
integrate((b*x^4+a)^p/(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^p/(d*x + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^4)^p}{(c+dx)^2}\,dx = \int \frac{(b\,x^4+a)^p}{(c+d\,x)^2}\,dx$$

input
```
int((a + b*x^4)^p/(c + d*x)^2,x)
```

output
```
int((a + b*x^4)^p/(c + d*x)^2, x)
```

Reduce [F]

$$\int \frac{(a+bx^4)^p}{(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
int((b*x^4+a)^p/(d*x+c)^2,x)
```

output
```
((a + b*x**4)**p + 16*int((a + b*x**4)**p/(4*a*c**2*p - a*c**2 + 8*a*c*d*p
*x - 2*a*c*d*x + 4*a*d**2*p*x**2 - a*d**2*x**2 + 4*b*c**2*p*x**4 - b*c**2*
x**4 + 8*b*c*d*p*x**5 - 2*b*c*d*x**5 + 4*b*d**2*p*x**6 - b*d**2*x**6),x)*a
*c*d*p**2 - 4*int((a + b*x**4)**p/(4*a*c**2*p - a*c**2 + 8*a*c*d*p*x - 2*a
*c*d*x + 4*a*d**2*p*x**2 - a*d**2*x**2 + 4*b*c**2*p*x**4 - b*c**2*x**4 + 8
*b*c*d*p*x**5 - 2*b*c*d*x**5 + 4*b*d**2*p*x**6 - b*d**2*x**6),x)*a*c*d*p +
 16*int((a + b*x**4)**p/(4*a*c**2*p - a*c**2 + 8*a*c*d*p*x - 2*a*c*d*x + 4
*a*d**2*p*x**2 - a*d**2*x**2 + 4*b*c**2*p*x**4 - b*c**2*x**4 + 8*b*c*d*p*x
**5 - 2*b*c*d*x**5 + 4*b*d**2*p*x**6 - b*d**2*x**6),x)*a*d**2*p**2*x - 4*i
nt((a + b*x**4)**p/(4*a*c**2*p - a*c**2 + 8*a*c*d*p*x - 2*a*c*d*x + 4*a*d*
*2*p*x**2 - a*d**2*x**2 + 4*b*c**2*p*x**4 - b*c**2*x**4 + 8*b*c*d*p*x**5 -
 2*b*c*d*x**5 + 4*b*d**2*p*x**6 - b*d**2*x**6),x)*a*d**2*p*x - 16*int(((a
+ b*x**4)**p*x**3)/(4*a*c**2*p - a*c**2 + 8*a*c*d*p*x - 2*a*c*d*x + 4*a*d*
*2*p*x**2 - a*d**2*x**2 + 4*b*c**2*p*x**4 - b*c**2*x**4 + 8*b*c*d*p*x**5 -
 2*b*c*d*x**5 + 4*b*d**2*p*x**6 - b*d**2*x**6),x)*b*c**2*p**2 + 4*int(((a
+ b*x**4)**p*x**3)/(4*a*c**2*p - a*c**2 + 8*a*c*d*p*x - 2*a*c*d*x + 4*a*d*
*2*p*x**2 - a*d**2*x**2 + 4*b*c**2*p*x**4 - b*c**2*x**4 + 8*b*c*d*p*x**5 -
 2*b*c*d*x**5 + 4*b*d**2*p*x**6 - b*d**2*x**6),x)*b*c**2*p - 16*int(((a +
b*x**4)**p*x**3)/(4*a*c**2*p - a*c**2 + 8*a*c*d*p*x - 2*a*c*d*x + 4*a*d**2
*p*x**2 - a*d**2*x**2 + 4*b*c**2*p*x**4 - b*c**2*x**4 + 8*b*c*d*p*x**5 ...
```

3.223 $\int (d+ex)^3 \left(a+bx^2+cx^4\right) \, dx$

Optimal result

Integrand size = 20, antiderivative size = 102

$$\int (d+ex)^3 \left(a+bx^2+cx^4\right) \, dx = \frac{1}{3}bd^3x^3 + \frac{3}{4}bd^2ex^4 + \frac{1}{5}d\left(cd^2+3be^2\right)x^5 + \frac{1}{6}e\left(3cd^2+be^2\right)x^6 + \frac{3}{7}cde^2x^7 + \frac{1}{8}ce^3x^8 + \frac{a(d+ex)^4}{4e}$$

output
```
1/3*b*d^3*x^3+3/4*b*d^2*e*x^4+1/5*d*(3*b*e^2+c*d^2)*x^5+1/6*e*(b*e^2+3*c*d
^2)*x^6+3/7*c*d*e^2*x^7+1/8*c*e^3*x^8+1/4*a*(e*x+d)^4/e
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.20

$$\int (d+ex)^3 \left(a+bx^2+cx^4\right) \, dx = ad^3x + \frac{3}{2}ad^2ex^2 + \frac{1}{3}d\left(bd^2+3ae^2\right)x^3 + \frac{1}{4}e\left(3bd^2+ae^2\right)x^4 + \frac{1}{5}d\left(cd^2+3be^2\right)x^5 + \frac{1}{6}e\left(3cd^2+be^2\right)x^6 + \frac{3}{7}cde^2x^7 + \frac{1}{8}ce^3x^8$$

input
```
Integrate[(d + e*x)^3*(a + b*x^2 + c*x^4),x]
```

output
```
a*d^3*x + (3*a*d^2*e*x^2)/2 + (d*(b*d^2 + 3*a*e^2)*x^3)/3 + (e*(3*b*d^2 +
a*e^2)*x^4)/4 + (d*(c*d^2 + 3*b*e^2)*x^5)/5 + (e*(3*c*d^2 + b*e^2)*x^6)/6
+ (3*c*d*e^2*x^7)/7 + (c*e^3*x^8)/8
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.16, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^3 \left(a+bx^2+cx^4\right) \, dx$$

$$\Big\downarrow \text{2200}$$

$$\int \left(\frac{(d+ex)^3 \left(ae^4+bd^2e^2+cd^4\right)}{e^4}-\frac{2(d+ex)^4 \left(bde^2+2cd^3\right)}{e^4}+\frac{(d+ex)^5 \left(be^2+6cd^2\right)}{e^4}+\frac{c(d+ex)^7}{e^4}-\frac{4cd(d\cdots}{e\cdots}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{(d+ex)^4 \left(ae^4+bd^2e^2+cd^4\right)}{4e^5}+\frac{(d+ex)^6 \left(be^2+6cd^2\right)}{6e^5}-\frac{2d(d+ex)^5 \left(be^2+2cd^2\right)}{5e^5}+\frac{c(d+ex)^8}{8e^5}-\frac{4cd(d+ex)^7}{7e^5}$$

input
```
Int[(d + e*x)^3*(a + b*x^2 + c*x^4),x]
```

output
```
((c*d^4 + b*d^2*e^2 + a*e^4)*(d + e*x)^4)/(4*e^5) - (2*d*(2*c*d^2 + b*e^2)
*(d + e*x)^5)/(5*e^5) + ((6*c*d^2 + b*e^2)*(d + e*x)^6)/(6*e^5) - (4*c*d*(
d + e*x)^7)/(7*e^5) + (c*(d + e*x)^8)/(8*e^5)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2200
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[Expa
ndIntegrand[Px*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c}, x] && Poly
Q[Px, x] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.06

method	result
norman	$\frac{c\,e^3x^8}{8}+\frac{3cd\,e^2x^7}{7}+\left(\frac{1}{6}b\,e^3+\frac{1}{2}d^2ec\right)x^6+\left(\frac{3}{5}bd\,e^2+\frac{1}{5}c\,d^3\right)x^5+\left(\frac{1}{4}e^3a+\frac{3}{4}b\,d^2e\right)x^4+\left(ad\,e^2+\right.$
default	$\frac{c\,e^3x^8}{8}+\frac{3cd\,e^2x^7}{7}+\frac{(b\,e^3+3d^2ec)x^6}{6}+\frac{(3bd\,e^2+c\,d^3)x^5}{5}+\frac{(e^3a+3b\,d^2e)x^4}{4}+\frac{(3ad\,e^2+b\,d^3)x^3}{3}+\frac{3a\,d^2e\,x^2}{2}+a\,c$
gosper	$\frac{1}{8}c\,e^3x^8+\frac{3}{7}cd\,e^2x^7+\frac{1}{6}b\,e^3x^6+\frac{1}{2}c\,d^2e\,x^6+\frac{3}{5}x^5bd\,e^2+\frac{1}{5}c\,d^3x^5+\frac{1}{4}a\,e^3x^4+\frac{3}{4}b\,d^2e\,x^4+ad\,e$
risch	$\frac{1}{8}c\,e^3x^8+\frac{3}{7}cd\,e^2x^7+\frac{1}{6}b\,e^3x^6+\frac{1}{2}c\,d^2e\,x^6+\frac{3}{5}x^5bd\,e^2+\frac{1}{5}c\,d^3x^5+\frac{1}{4}a\,e^3x^4+\frac{3}{4}b\,d^2e\,x^4+ad\,e$
parallelrisch	$\frac{1}{8}c\,e^3x^8+\frac{3}{7}cd\,e^2x^7+\frac{1}{6}b\,e^3x^6+\frac{1}{2}c\,d^2e\,x^6+\frac{3}{5}x^5bd\,e^2+\frac{1}{5}c\,d^3x^5+\frac{1}{4}a\,e^3x^4+\frac{3}{4}b\,d^2e\,x^4+ad\,e$
orering	$\frac{x(105e^3c\,x^7+360cd\,e^2x^6+140b\,e^3x^5+420d^2ec\,x^5+504bd\,e^2x^4+168c\,d^3x^4+210e^3a\,x^3+630b\,d^2e\,x^3+840ad\,e^2x^2+280b\,d^3x^2-}{840}$

input
```
int((e*x+d)^3*(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/8*c*e^3*x^8+3/7*c*d*e^2*x^7+(1/6*b*e^3+1/2*d^2*e*c)*x^6+(3/5*b*d*e^2+1/5
*c*d^3)*x^5+(1/4*e^3*a+3/4*b*d^2*e)*x^4+(a*d*e^2+1/3*b*d^3)*x^3+3/2*a*d^2*
e*x^2+a*d^3*x
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.06

$$\int (d+ex)^3 \left(a+bx^2+cx^4\right) dx = \frac{1}{8} ce^3x^8 + \frac{3}{7} cde^2x^7 + \frac{1}{6} \left(3 cd^2e + be^3\right)x^6 + \frac{3}{2} ad^2ex^2 + \frac{1}{5} \left(cd^3 + 3 bde^2\right)x^5 + ad^3x + \frac{1}{4} \left(3 bd^2e + ae^3\right)x^4 + \frac{1}{3} \left(bd^3 + 3 ade^2\right)x^3$$

input
```
integrate((e*x+d)^3*(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
1/8*c*e^3*x^8 + 3/7*c*d*e^2*x^7 + 1/6*(3*c*d^2*e + b*e^3)*x^6 + 3/2*a*d^2*
e*x^2 + 1/5*(c*d^3 + 3*b*d*e^2)*x^5 + a*d^3*x + 1/4*(3*b*d^2*e + a*e^3)*x^
4 + 1/3*(b*d^3 + 3*a*d*e^2)*x^3
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.17

$$\int (d+ex)^3 \left(a+bx^2+cx^4\right) dx = ad^3x + \frac{3ad^2ex^2}{2} + \frac{3cde^2x^7}{7} + \frac{ce^3x^8}{8} + x^6 \left(\frac{be^3}{6} + \frac{cd^2e}{2}\right) + x^5 \cdot \left(\frac{3bde^2}{5} + \frac{cd^3}{5}\right) + x^4 \left(\frac{ae^3}{4} + \frac{3bd^2e}{4}\right) + x^3 \left(ade^2 + \frac{bd^3}{3}\right)$$

input
```
integrate((e*x+d)**3*(c*x**4+b*x**2+a),x)
```

output
```
a*d**3*x + 3*a*d**2*e*x**2/2 + 3*c*d*e**2*x**7/7 + c*e**3*x**8/8 + x**6*(b
*e**3/6 + c*d**2*e/2) + x**5*(3*b*d*e**2/5 + c*d**3/5) + x**4*(a*e**3/4 +
3*b*d**2*e/4) + x**3*(a*d*e**2 + b*d**3/3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.06

$$\int (d+ex)^3\left(a+bx^2+cx^4\right)\,dx = \frac{1}{8}\,ce^3x^8 + \frac{3}{7}\,cde^2x^7 + \frac{1}{6}\left(3\,cd^2e + be^3\right)x^6 + \frac{3}{2}\,ad^2ex^2 + \frac{1}{5}\left(cd^3 + 3\,bde^2\right)x^5 + ad^3x + \frac{1}{4}\left(3\,bd^2e + ae^3\right)x^4 + \frac{1}{3}\left(bd^3 + 3\,ade^2\right)x^3$$

input
```
integrate((e*x+d)^3*(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
1/8*c*e^3*x^8 + 3/7*c*d*e^2*x^7 + 1/6*(3*c*d^2*e + b*e^3)*x^6 + 3/2*a*d^2*
e*x^2 + 1/5*(c*d^3 + 3*b*d*e^2)*x^5 + a*d^3*x + 1/4*(3*b*d^2*e + a*e^3)*x^
4 + 1/3*(b*d^3 + 3*a*d*e^2)*x^3
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.09

$$\int (d+ex)^3\left(a+bx^2+cx^4\right)\,dx = \frac{1}{8}\,ce^3x^8 + \frac{3}{7}\,cde^2x^7 + \frac{1}{2}\,cd^2ex^6 + \frac{1}{6}\,be^3x^6 + \frac{1}{5}\,cd^3x^5 + \frac{3}{5}\,bde^2x^5 + \frac{3}{4}\,bd^2ex^4 + \frac{1}{4}\,ae^3x^4 + \frac{1}{3}\,bd^3x^3 + ade^2x^3 + \frac{3}{2}\,ad^2ex^2 + ad^3x$$

input
```
integrate((e*x+d)^3*(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
1/8*c*e^3*x^8 + 3/7*c*d*e^2*x^7 + 1/2*c*d^2*e*x^6 + 1/6*b*e^3*x^6 + 1/5*c*
d^3*x^5 + 3/5*b*d*e^2*x^5 + 3/4*b*d^2*e*x^4 + 1/4*a*e^3*x^4 + 1/3*b*d^3*x^
3 + a*d*e^2*x^3 + 3/2*a*d^2*e*x^2 + a*d^3*x
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.05

$$\int (d+ex)^3 \left(a+bx^2+cx^4\right) \, dx = x^3 \left(\frac{b\,d^3}{3}+a\,d\,e^2\right)+x^4 \left(\frac{3\,b\,d^2\,e}{4}+\frac{a\,e^3}{4}\right) + x^5 \left(\frac{c\,d^3}{5}+\frac{3\,b\,d\,e^2}{5}\right)+x^6 \left(\frac{c\,d^2\,e}{2}+\frac{b\,e^3}{6}\right) + \frac{c\,e^3\,x^8}{8}+a\,d^3\,x+\frac{3\,a\,d^2\,e\,x^2}{2}+\frac{3\,c\,d\,e^2\,x^7}{7}$$

input
```
int((d + e*x)^3*(a + b*x^2 + c*x^4),x)
```

output
```
x^3*((b*d^3)/3 + a*d*e^2) + x^4*((a*e^3)/4 + (3*b*d^2*e)/4) + x^5*((c*d^3)
/5 + (3*b*d*e^2)/5) + x^6*((b*e^3)/6 + (c*d^2*e)/2) + (c*e^3*x^8)/8 + a*d^
3*x + (3*a*d^2*e*x^2)/2 + (3*c*d*e^2*x^7)/7
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.11

$$\int (d+ex)^3 \left(a+bx^2+cx^4\right) \, dx = \frac{x(105c\,e^3x^7+360cd\,e^2x^6+140b\,e^3x^5+420c\,d^2e\,x^5+504bd\,e^2x^4+168c\,d^3x^4+210a\,e^3x^3+630b\,d^2e\,x^3}{840}$$

input
```
int((e*x+d)^3*(c*x^4+b*x^2+a),x)
```

output
```
(x*(840*a*d**3 + 1260*a*d**2*e*x + 840*a*d*e**2*x**2 + 210*a*e**3*x**3 + 2
80*b*d**3*x**2 + 630*b*d**2*e*x**3 + 504*b*d*e**2*x**4 + 140*b*e**3*x**5 +
 168*c*d**3*x**4 + 420*c*d**2*e*x**5 + 360*c*d*e**2*x**6 + 105*c*e**3*x**7
))/840
```

3.224 $\int (d+ex)^2 \left(a+bx^2+cx^4\right) \, dx$

Optimal result

Integrand size = 20, antiderivative size = 76

$$\int (d+ex)^2 \left(a+bx^2+cx^4\right) \, dx = \frac{1}{3}bd^2x^3+\frac{1}{2}bdex^4+\frac{1}{5}\left(cd^2+be^2\right)x^5+\frac{1}{3}cdex^6+\frac{1}{7}ce^2x^7+\frac{a(d+ex)^3}{3e}$$

output
```
1/3*b*d^2*x^3+1/2*b*d*e*x^4+1/5*(b*e^2+c*d^2)*x^5+1/3*c*d*e*x^6+1/7*c*e^2*
x^7+1/3*a*(e*x+d)^3/e
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.07

$$\int (d+ex)^2 \left(a+bx^2+cx^4\right) \, dx = ad^2x+adex^2+\frac{1}{3}\left(bd^2+ae^2\right)x^3+\frac{1}{2}bdex^4+\frac{1}{5}\left(cd^2+be^2\right)x^5+\frac{1}{3}cdex^6+\frac{1}{7}ce^2x^7$$

input
```
Integrate[(d + e*x)^2*(a + b*x^2 + c*x^4),x]
```

output
```
a*d^2*x + a*d*e*x^2 + ((b*d^2 + a*e^2)*x^3)/3 + (b*d*e*x^4)/2 + ((c*d^2 +
b*e^2)*x^5)/5 + (c*d*e*x^6)/3 + (c*e^2*x^7)/7
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.07, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^2 \left(a+bx^2+cx^4\right) \, dx$$

$$\Big\downarrow \text{2200}$$

$$\int \left(x^2\left(ae^2+bd^2\right)+ad^2+2adex+x^4\left(be^2+cd^2\right)+2bdex^3+2cdex^5+ce^2x^6\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}x^3\left(ae^2+bd^2\right)+ad^2x+adex^2+\frac{1}{5}x^5\left(be^2+cd^2\right)+\frac{1}{2}bdex^4+\frac{1}{3}cdex^6+\frac{1}{7}ce^2x^7$$

input
```
Int[(d + e*x)^2*(a + b*x^2 + c*x^4),x]
```

output

```
a*d^2*x + a*d*e*x^2 + ((b*d^2 + a*e^2)*x^3)/3 + (b*d*e*x^4)/2 + ((c*d^2 +
b*e^2)*x^5)/5 + (c*d*e*x^6)/3 + (c*e^2*x^7)/7
```


Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2200
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[Expa
ndIntegrand[Px*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c}, x] && Poly
Q[Px, x] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.95

method	result	size
default	$\frac{c\,e^{2} x^{7}}{7}+\frac{cde\,x^{6}}{3}+\frac{\left(b\,e^{2}+c\,d^{2}\right) x^{5}}{5}+\frac{bde\,x^{4}}{2}+\frac{\left(a\,e^{2}+b\,d^{2}\right) x^{3}}{3}+ade\,x^{2}+a\,d^{2} x$	72
norman	$\frac{c\,e^{2} x^{7}}{7}+\frac{cde\,x^{6}}{3}+\left(\frac{b\,e^{2}}{5}+\frac{c\,d^{2}}{5}\right) x^{5}+\frac{bde\,x^{4}}{2}+\left(\frac{a\,e^{2}}{3}+\frac{b\,d^{2}}{3}\right) x^{3}+ade\,x^{2}+a\,d^{2} x$	74
gosper	$\frac{1}{7}c\,e^{2} x^{7}+\frac{1}{3}cde\,x^{6}+\frac{1}{5}x^{5} b\,e^{2}+\frac{1}{5}c\,d^{2} x^{5}+\frac{1}{2}bde\,x^{4}+\frac{1}{3}a\,e^{2} x^{3}+\frac{1}{3}b\,d^{2} x^{3}+ade\,x^{2}+a\,d^{2} x$	76
risch	$\frac{1}{7}c\,e^{2} x^{7}+\frac{1}{3}cde\,x^{6}+\frac{1}{5}x^{5} b\,e^{2}+\frac{1}{5}c\,d^{2} x^{5}+\frac{1}{2}bde\,x^{4}+\frac{1}{3}a\,e^{2} x^{3}+\frac{1}{3}b\,d^{2} x^{3}+ade\,x^{2}+a\,d^{2} x$	76
parallelrisch	$\frac{1}{7}c\,e^{2} x^{7}+\frac{1}{3}cde\,x^{6}+\frac{1}{5}x^{5} b\,e^{2}+\frac{1}{5}c\,d^{2} x^{5}+\frac{1}{2}bde\,x^{4}+\frac{1}{3}a\,e^{2} x^{3}+\frac{1}{3}b\,d^{2} x^{3}+ade\,x^{2}+a\,d^{2} x$	76
orering	$\frac{x\left(30c\,e^{2} x^{6}+70cde\,x^{5}+42b\,e^{2} x^{4}+42c\,d^{2} x^{4}+105bde\,x^{3}+70a\,e^{2} x^{2}+70b\,d^{2} x^{2}+210adex+210a\,d^{2}\right)}{210}$	78

input
```
int((e*x+d)^2*(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/7*c*e^2*x^7+1/3*c*d*e*x^6+1/5*(b*e^2+c*d^2)*x^5+1/2*b*d*e*x^4+1/3*(a*e^2
+b*d^2)*x^3+a*d*e*x^2+a*d^2*x
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.93

$$\int (d+ex)^2\left(a+bx^2+cx^4\right)\,dx=\frac{1}{7}\,ce^2x^7+\frac{1}{3}\,cdex^6+\frac{1}{2}\,bdex^4+\frac{1}{5}\,(cd^2+be^2)x^5+adex^2+ad^2x+\frac{1}{3}\,(bd^2+ae^2)x^3$$

input
```
integrate((e*x+d)^2*(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
1/7*c*e^2*x^7 + 1/3*c*d*e*x^6 + 1/2*b*d*e*x^4 + 1/5*(c*d^2 + b*e^2)*x^5 +
a*d*e*x^2 + a*d^2*x + 1/3*(b*d^2 + a*e^2)*x^3
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.03

$$\int (d+ex)^2 \left(a+bx^2+cx^4\right) dx = ad^2x + adex^2 + \frac{bdex^4}{2} + \frac{cdex^6}{3} + \frac{ce^2x^7}{7} + x^5\left(\frac{be^2}{5} + \frac{cd^2}{5}\right) + x^3\left(\frac{ae^2}{3} + \frac{bd^2}{3}\right)$$

input
```
integrate((e*x+d)**2*(c*x**4+b*x**2+a),x)
```

output
```
a*d**2*x + a*d*e*x**2 + b*d*e*x**4/2 + c*d*e*x**6/3 + c*e**2*x**7/7 + x**5
*(b*e**2/5 + c*d**2/5) + x**3*(a*e**2/3 + b*d**2/3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.93

$$\int (d+ex)^2 \left(a+bx^2+cx^4\right) dx = \frac{1}{7}ce^2x^7 + \frac{1}{3}cdex^6 + \frac{1}{2}bdex^4 + \frac{1}{5}\left(cd^2+be^2\right)x^5 + adex^2 + ad^2x + \frac{1}{3}\left(bd^2+ae^2\right)x^3$$

input
```
integrate((e*x+d)^2*(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
1/7*c*e^2*x^7 + 1/3*c*d*e*x^6 + 1/2*b*d*e*x^4 + 1/5*(c*d^2 + b*e^2)*x^5 +
a*d*e*x^2 + a*d^2*x + 1/3*(b*d^2 + a*e^2)*x^3
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.99

$$\int (d+ex)^2\left(a+bx^2+cx^4\right)\,dx = \frac{1}{7}\,ce^2x^7+\frac{1}{3}\,cdex^6+\frac{1}{5}\,cd^2x^5+\frac{1}{5}\,be^2x^5+\frac{1}{2}\,bdex^4+\frac{1}{3}\,bd^2x^3+\frac{1}{3}\,ae^2x^3+adex^2+ad^2x$$

input
```
integrate((e*x+d)^2*(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
1/7*c*e^2*x^7 + 1/3*c*d*e*x^6 + 1/5*c*d^2*x^5 + 1/5*b*e^2*x^5 + 1/2*b*d*e*
x^4 + 1/3*b*d^2*x^3 + 1/3*a*e^2*x^3 + a*d*e*x^2 + a*d^2*x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.96

$$\int (d+ex)^2\left(a+bx^2+cx^4\right)\,dx = x^3\left(\frac{b\,d^2}{3}+\frac{a\,e^2}{3}\right)+x^5\left(\frac{c\,d^2}{5}+\frac{b\,e^2}{5}\right)+\frac{c\,e^2\,x^7}{7}+a\,d^2\,x+a\,d\,e\,x^2+\frac{b\,d\,e\,x^4}{2}+\frac{c\,d\,e\,x^6}{3}$$

input
```
int((d + e*x)^2*(a + b*x^2 + c*x^4),x)
```

output
```
x^3*((a*e^2)/3 + (b*d^2)/3) + x^5*((b*e^2)/5 + (c*d^2)/5) + (c*e^2*x^7)/7
+ a*d^2*x + a*d*e*x^2 + (b*d*e*x^4)/2 + (c*d*e*x^6)/3
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.01

$$\int (d+ex)^2 \left(a+bx^2+cx^4\right) \, dx = \frac{x(30c\,e^2x^6+70cde\,x^5+42b\,e^2x^4+42c\,d^2x^4+105bde\,x^3+70a\,e^2x^2+70b\,d^2x^2+210adex+210a\,d^2)}{210}$$

input

```
int((e*x+d)^2*(c*x^4+b*x^2+a),x)
```

output

```
(x*(210*a*d**2 + 210*a*d*e*x + 70*a*e**2*x**2 + 70*b*d**2*x**2 + 105*b*d*e
*x**3 + 42*b*e**2*x**4 + 42*c*d**2*x**4 + 70*c*d*e*x**5 + 30*c*e**2*x**6))
/210
```

3.225 $\int (d+ex) \left(a+bx^2+cx^4\right) \, dx$

Optimal result

Integrand size = 18, antiderivative size = 52

$$\int (d+ex) \left(a+bx^2+cx^4\right) \, dx = \frac{1}{3}bdx^3 + \frac{1}{4}bex^4 + \frac{1}{5}cdx^5 + \frac{1}{6}cex^6 + \frac{a(d+ex)^2}{2e}$$

output
```
1/3*b*d*x^3+1/4*b*e*x^4+1/5*c*d*x^5+1/6*c*e*x^6+1/2*a*(e*x+d)^2/e
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.96

$$\int (d+ex) \left(a+bx^2+cx^4\right) \, dx = adx + \frac{1}{2}aex^2 + \frac{1}{3}bdx^3 + \frac{1}{4}bex^4 + \frac{1}{5}cdx^5 + \frac{1}{6}cex^6$$

input
```
Integrate[(d + e*x)*(a + b*x^2 + c*x^4),x]
```

output
```
a*d*x + (a*e*x^2)/2 + (b*d*x^3)/3 + (b*e*x^4)/4 + (c*d*x^5)/5 + (c*e*x^6)/
6
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.96, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)\left(a+bx^2+cx^4\right)\,dx$$
$$\Big\downarrow \text{2200}$$
$$\int \left(ad+aex+bdx^2+bex^3+cdx^4+cex^5\right)dx$$
$$\Big\downarrow \text{2009}$$
$$adx+\frac{1}{2}aex^2+\frac{1}{3}bdx^3+\frac{1}{4}bex^4+\frac{1}{5}cdx^5+\frac{1}{6}cex^6$$

input
```
Int[(d + e*x)*(a + b*x^2 + c*x^4),x]
```

output
```
a*d*x + (a*e*x^2)/2 + (b*d*x^3)/3 + (b*e*x^4)/4 + (c*d*x^5)/5 + (c*e*x^6)/
6
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2200
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[Expa
ndIntegrand[Px*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c}, x] && Poly
Q[Px, x] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.79

method	result	size
gosper	$\frac{1}{6}ce\,x^{6}+\frac{1}{5}cd\,x^{5}+\frac{1}{4}be\,x^{4}+\frac{1}{3}bd\,x^{3}+\frac{1}{2}ae\,x^{2}+adx$	41
default	$\frac{1}{6}ce\,x^{6}+\frac{1}{5}cd\,x^{5}+\frac{1}{4}be\,x^{4}+\frac{1}{3}bd\,x^{3}+\frac{1}{2}ae\,x^{2}+adx$	41
norman	$\frac{1}{6}ce\,x^{6}+\frac{1}{5}cd\,x^{5}+\frac{1}{4}be\,x^{4}+\frac{1}{3}bd\,x^{3}+\frac{1}{2}ae\,x^{2}+adx$	41
risch	$\frac{1}{6}ce\,x^{6}+\frac{1}{5}cd\,x^{5}+\frac{1}{4}be\,x^{4}+\frac{1}{3}bd\,x^{3}+\frac{1}{2}ae\,x^{2}+adx$	41
parallelrisch	$\frac{1}{6}ce\,x^{6}+\frac{1}{5}cd\,x^{5}+\frac{1}{4}be\,x^{4}+\frac{1}{3}bd\,x^{3}+\frac{1}{2}ae\,x^{2}+adx$	41
orering	$\frac{x\left(10ce\,x^{5}+12cd\,x^{4}+15be\,x^{3}+20bd\,x^{2}+30aex+60ad\right)}{60}$	42

input
```
int((e*x+d)*(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/6*c*e*x^6+1/5*c*d*x^5+1/4*b*e*x^4+1/3*b*d*x^3+1/2*a*e*x^2+a*d*x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.77

$$\int (d+ex)\left(a+bx^2+cx^4\right)\,dx=\frac{1}{6}\,cex^6+\frac{1}{5}\,cdx^5+\frac{1}{4}\,bex^4+\frac{1}{3}\,bdx^3+\frac{1}{2}\,aex^2+adx$$

input
```
integrate((e*x+d)*(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
1/6*c*e*x^6 + 1/5*c*d*x^5 + 1/4*b*e*x^4 + 1/3*b*d*x^3 + 1/2*a*e*x^2 + a*d*
x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.88

$$\int (d+ex)\left(a+bx^2+cx^4\right)\,dx=adx+\frac{aex^2}{2}+\frac{bdx^3}{3}+\frac{bex^4}{4}+\frac{cdx^5}{5}+\frac{cex^6}{6}$$

input
```
integrate((e*x+d)*(c*x**4+b*x**2+a),x)
```

output
```
a*d*x + a*e*x**2/2 + b*d*x**3/3 + b*e*x**4/4 + c*d*x**5/5 + c*e*x**6/6
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.77

$$\int (d+ex)\left(a+bx^2+cx^4\right)\,dx=\frac{1}{6}\,cex^6+\frac{1}{5}\,cdx^5+\frac{1}{4}\,bex^4+\frac{1}{3}\,bdx^3+\frac{1}{2}\,aex^2+adx$$

input
```
integrate((e*x+d)*(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
1/6*c*e*x^6 + 1/5*c*d*x^5 + 1/4*b*e*x^4 + 1/3*b*d*x^3 + 1/2*a*e*x^2 + a*d*
x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.77

$$\int (d+ex)\left(a+bx^2+cx^4\right)\,dx=\frac{1}{6}\,cex^6+\frac{1}{5}\,cdx^5+\frac{1}{4}\,bex^4+\frac{1}{3}\,bdx^3+\frac{1}{2}\,aex^2+adx$$

input
```
integrate((e*x+d)*(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
1/6*c*e*x^6 + 1/5*c*d*x^5 + 1/4*b*e*x^4 + 1/3*b*d*x^3 + 1/2*a*e*x^2 + a*d*
x
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.77

$$\int (d+ex)\left(a+bx^2+cx^4\right)\,dx=\frac{c\,e\,x^6}{6}+\frac{c\,d\,x^5}{5}+\frac{b\,e\,x^4}{4}+\frac{b\,d\,x^3}{3}+\frac{a\,e\,x^2}{2}+a\,d\,x$$

input
```
int((d + e*x)*(a + b*x^2 + c*x^4),x)
```

output
```
a*d*x + (a*e*x^2)/2 + (b*d*x^3)/3 + (b*e*x^4)/4 + (c*d*x^5)/5 + (c*e*x^6)/
6
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.79

$$\int (d+ex)\left(a+bx^2+cx^4\right)\,dx=\frac{x(10ce\,x^5+12cd\,x^4+15be\,x^3+20bd\,x^2+30aex+60ad)}{60}$$

input
```
int((e*x+d)*(c*x^4+b*x^2+a),x)
```

output
```
(x*(60*a*d + 30*a*e*x + 20*b*d*x**2 + 15*b*e*x**3 + 12*c*d*x**4 + 10*c*e*x
**5))/60
```

3.226 $\int (a+bx^2+cx^4)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 20

$$\int (a+bx^2+cx^4)\,dx = ax+\frac{bx^3}{3}+\frac{cx^5}{5}$$

output
```
a*x+1/3*b*x^3+1/5*c*x^5
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int (a+bx^2+cx^4)\,dx = ax+\frac{bx^3}{3}+\frac{cx^5}{5}$$

input
```
Integrate[a + b*x^2 + c*x^4,x]
```

output
```
a*x + (b*x^3)/3 + (c*x^5)/5
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a + bx^2 + cx^4) \, dx$$

$$\Big\downarrow \text{ 2009}$$

$$ax + \frac{bx^3}{3} + \frac{cx^5}{5}$$

input
```
Int[a + b*x^2 + c*x^4,x]
```

output
```
a*x + (b*x^3)/3 + (c*x^5)/5
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

method	result	size
gosper	$xa+\frac{1}{3}b\,x^{3}+\frac{1}{5}c\,x^{5}$	17
default	$xa+\frac{1}{3}b\,x^{3}+\frac{1}{5}c\,x^{5}$	17
norman	$xa+\frac{1}{3}b\,x^{3}+\frac{1}{5}c\,x^{5}$	17
risch	$xa+\frac{1}{3}b\,x^{3}+\frac{1}{5}c\,x^{5}$	17
parallelrisch	$xa+\frac{1}{3}b\,x^{3}+\frac{1}{5}c\,x^{5}$	17
parts	$xa+\frac{1}{3}b\,x^{3}+\frac{1}{5}c\,x^{5}$	17
orering	$\frac{x\left(3c\,x^{4}+5b\,x^{2}+15a\right)}{15}$	20

input
```
int(c*x^4+b*x^2+a,x,method=_RETURNVERBOSE)
```

output
```
x*a+1/3*b*x^3+1/5*c*x^5
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \left(a+bx^2+cx^4\right)\,dx=\frac{1}{5}\,cx^{5}+\frac{1}{3}\,bx^{3}+ax$$

input
```
integrate(c*x^4+b*x^2+a,x, algorithm="fricas")
```

output
```
1/5*c*x^5 + 1/3*b*x^3 + a*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.75

$$\int \left(a + bx^2 + cx^4\right) dx = ax + \frac{bx^3}{3} + \frac{cx^5}{5}$$

input
```
integrate(c*x**4+b*x**2+a,x)
```

output
```
a*x + b*x**3/3 + c*x**5/5
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \left(a + bx^2 + cx^4\right) dx = \frac{1}{5}\,cx^5 + \frac{1}{3}\,bx^3 + ax$$

input
```
integrate(c*x^4+b*x^2+a,x, algorithm="maxima")
```

output
```
1/5*c*x^5 + 1/3*b*x^3 + a*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \left(a + bx^2 + cx^4\right) dx = \frac{1}{5}\,cx^5 + \frac{1}{3}\,bx^3 + ax$$

input
```
integrate(c*x^4+b*x^2+a,x, algorithm="giac")
```

output
```
1/5*c*x^5 + 1/3*b*x^3 + a*x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int (a + bx^2 + cx^4) \, dx = \frac{c\,x^5}{5} + \frac{b\,x^3}{3} + a\,x$$

input
```
int(a + b*x^2 + c*x^4,x)
```

output
```
a*x + (b*x^3)/3 + (c*x^5)/5
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int (a + bx^2 + cx^4) \, dx = \frac{x(3c\,x^4 + 5b\,x^2 + 15a)}{15}$$

input
```
int(c*x^4+b*x^2+a,x)
```

output
```
(x*(15*a + 5*b*x**2 + 3*c*x**4))/15
```

3.227 $\int \frac{a+bx^2+cx^4}{d+ex}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 92

$$\int \frac{a+bx^2+cx^4}{d+ex}\,dx=-\frac{d(cd^2+be^2)\,x}{e^4}+\frac{(cd^2+be^2)\,x^2}{2e^3}-\frac{cdx^3}{3e^2}+\frac{cx^4}{4e}+\frac{(cd^4+bd^2e^2+ae^4)\log(d+ex)}{e^5}$$

output
```
-d*(b*e^2+c*d^2)*x/e^4+1/2*(b*e^2+c*d^2)*x^2/e^3-1/3*c*d*x^3/e^2+1/4*c*x^4
/e+(a*e^4+b*d^2*e^2+c*d^4)*ln(e*x+d)/e^5
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.90

$$\int \frac{a+bx^2+cx^4}{d+ex}\,dx=\frac{6be^3x(-2d+ex)+cex(-12d^3+6d^2ex-4de^2x^2+3e^3x^3)+12(cd^4+bd^2e^2+ae^4)\log(d+ex)}{12e^5}$$

input
```
Integrate[(a + b*x^2 + c*x^4)/(d + e*x),x]
```

output
```
(6*b*e^3*x*(-2*d + e*x) + c*e*x*(-12*d^3 + 6*d^2*e*x - 4*d*e^2*x^2 + 3*e^3
*x^3) + 12*(c*d^4 + b*d^2*e^2 + a*e^4)*Log[d + e*x])/(12*e^5)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^2+cx^4}{d+ex}\,dx$$

$$\Big\downarrow \text{2389}$$

$$\int \left(\frac{ae^4+bd^2e^2+cd^4}{e^4(d+ex)}+\frac{d(-be^2-cd^2)}{e^4}+\frac{x(be^2+cd^2)}{e^3}-\frac{cdx^2}{e^2}+\frac{cx^3}{e}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\log(d+ex)\left(ae^4+bd^2e^2+cd^4\right)}{e^5}-\frac{dx\left(be^2+cd^2\right)}{e^4}+\frac{x^2\left(be^2+cd^2\right)}{2e^3}-\frac{cdx^3}{3e^2}+\frac{cx^4}{4e}$$

input
```
Int[(a + b*x^2 + c*x^4)/(d + e*x),x]
```

output
```
-((d*(c*d^2 + b*e^2)*x)/e^4) + ((c*d^2 + b*e^2)*x^2)/(2*e^3) - (c*d*x^3)/(
3*e^2) + (c*x^4)/(4*e) + ((c*d^4 + b*d^2*e^2 + a*e^4)*Log[d + e*x])/e^5
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2389
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.95

method	result	size
default	$-\frac{-\frac{x^4 e^3 c}{4}+\frac{d e^2 x^3 c}{3}-\frac{\left(b e^2+c d^2\right) x^2 e}{2}+x d\left(b e^2+c d^2\right)}{e^4}+\frac{\left(e^4 a+b d^2 e^2+c d^4\right) \ln(e x+d)}{e^5}$	87
norman	$-\frac{d\left(b e^2+c d^2\right) x}{e^4}+\frac{\left(b e^2+c d^2\right) x^2}{2 e^3}-\frac{c d x^3}{3 e^2}+\frac{c x^4}{4 e}+\frac{\left(e^4 a+b d^2 e^2+c d^4\right) \ln(e x+d)}{e^5}$	87
risch	$\frac{c x^4}{4 e}-\frac{c d x^3}{3 e^2}+\frac{b x^2}{2 e}+\frac{c d^2 x^2}{2 e^3}-\frac{b d x}{e^2}-\frac{c d^3 x}{e^4}+\frac{\ln(e x+d) a}{e}+\frac{\ln(e x+d) b d^2}{e^3}+\frac{\ln(e x+d) c d^4}{e^5}$	99
parallelrisch	$\frac{3 x^4 c e^4-4 c d x^3 e^3+6 x^2 b e^4+6 x^2 c d^2 e^2+12 \ln(e x+d) a e^4+12 \ln(e x+d) b d^2 e^2+12 \ln(e x+d) c d^4-12 x b d e^3-12 c d^3 x e}{12 e^5}$	102

input
```
int((c*x^4+b*x^2+a)/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
-1/e^4*(-1/4*x^4*e^3*c+1/3*d*e^2*x^3*c-1/2*(b*e^2+c*d^2)*x^2*e+x*d*(b*e^2+
c*d^2))+(a*e^4+b*d^2*e^2+c*d^4)*ln(e*x+d)/e^5
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.95

$$\int \frac{a+bx^2+cx^4}{d+ex}\,dx = \frac{3\,ce^4x^4-4\,cde^3x^3+6\,(cd^2e^2+be^4)x^2-12\,(cd^3e+bde^3)x+12\,(cd^4+bd^2e^2+ae^4)\log(ex+d)}{12\,e^5}$$

input
```
integrate((c*x^4+b*x^2+a)/(e*x+d),x, algorithm="fricas")
```

output
```
1/12*(3*c*e^4*x^4 - 4*c*d*e^3*x^3 + 6*(c*d^2*e^2 + b*e^4)*x^2 - 12*(c*d^3*
e + b*d*e^3)*x + 12*(c*d^4 + b*d^2*e^2 + a*e^4)*log(e*x + d))/e^5
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.92

$$\int \frac{a+bx^2+cx^4}{d+ex}\,dx = -\frac{cdx^3}{3e^2}+\frac{cx^4}{4e}+x^2\left(\frac{b}{2e}+\frac{cd^2}{2e^3}\right)+x\left(-\frac{bd}{e^2}-\frac{cd^3}{e^4}\right) + \frac{(ae^4+bd^2e^2+cd^4)\log(d+ex)}{e^5}$$

input
```
integrate((c*x**4+b*x**2+a)/(e*x+d),x)
```

output
```
-c*d*x**3/(3*e**2) + c*x**4/(4*e) + x**2*(b/(2*e) + c*d**2/(2*e**3)) + x*(
-b*d/e**2 - c*d**3/e**4) + (a*e**4 + b*d**2*e**2 + c*d**4)*log(d + e*x)/e*
*5
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.95

$$\int \frac{a+bx^2+cx^4}{d+ex}\,dx = \frac{3\,ce^3x^4-4\,cde^2x^3+6\,(cd^2e+be^3)x^2-12\,(cd^3+bde^2)x}{12\,e^4} + \frac{(cd^4+bd^2e^2+ae^4)\log(ex+d)}{e^5}$$

input
```
integrate((c*x^4+b*x^2+a)/(e*x+d),x, algorithm="maxima")
```

output
```
1/12*(3*c*e^3*x^4 - 4*c*d*e^2*x^3 + 6*(c*d^2*e + b*e^3)*x^2 - 12*(c*d^3 +
b*d*e^2)*x)/e^4 + (c*d^4 + b*d^2*e^2 + a*e^4)*log(e*x + d)/e^5
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.98

$$\int \frac{a+bx^2+cx^4}{d+ex}\,dx = \frac{3\,ce^3x^4-4\,cde^2x^3+6\,cd^2ex^2+6\,be^3x^2-12\,cd^3x-12\,bde^2x}{12\,e^4} + \frac{(cd^4+bd^2e^2+ae^4)\log(|ex+d|)}{e^5}$$

input
```
integrate((c*x^4+b*x^2+a)/(e*x+d),x, algorithm="giac")
```

output
```
1/12*(3*c*e^3*x^4 - 4*c*d*e^2*x^3 + 6*c*d^2*e*x^2 + 6*b*e^3*x^2 - 12*c*d^3
*x - 12*b*d*e^2*x)/e^4 + (c*d^4 + b*d^2*e^2 + a*e^4)*log(abs(e*x + d))/e^5
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.98

$$\int \frac{a+bx^2+cx^4}{d+ex}\,dx = x^2\left(\frac{b}{2\,e}+\frac{c\,d^2}{2\,e^3}\right) + \frac{\ln(d+e\,x)\,(c\,d^4+b\,d^2\,e^2+a\,e^4)}{e^5} + \frac{c\,x^4}{4\,e} - \frac{d\,x\left(\frac{b}{e}+\frac{c\,d^2}{e^3}\right)}{e} - \frac{c\,d\,x^3}{3\,e^2}$$

input
```
int((a + b*x^2 + c*x^4)/(d + e*x),x)
```

output
```
x^2*(b/(2*e) + (c*d^2)/(2*e^3)) + (log(d + e*x)*(a*e^4 + c*d^4 + b*d^2*e^2
))/e^5 + (c*x^4)/(4*e) - (d*x*(b/e + (c*d^2)/e^3))/e - (c*d*x^3)/(3*e^2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.10

$$\int \frac{a+bx^2+cx^4}{d+ex}\,dx$$
$$= \frac{12\log(ex+d)\,a\,e^4+12\log(ex+d)\,b\,d^2e^2+12\log(ex+d)\,c\,d^4-12bd\,e^3x+6b\,e^4x^2-12c\,d^3ex+6c\,d^2e}{12e^5}$$

input
```
int((c*x^4+b*x^2+a)/(e*x+d),x)
```

output
```
(12*log(d + e*x)*a*e**4 + 12*log(d + e*x)*b*d**2*e**2 + 12*log(d + e*x)*c*
d**4 - 12*b*d*e**3*x + 6*b*e**4*x**2 - 12*c*d**3*e*x + 6*c*d**2*e**2*x**2
- 4*c*d*e**3*x**3 + 3*c*e**4*x**4)/(12*e**5)
```

3.228 $\int \frac{a+bx^2+cx^4}{(d+ex)^2} dx$

Optimal result

Integrand size = 20, antiderivative size = 94

$$\int \frac{a+bx^2+cx^4}{(d+ex)^2} dx = \frac{(3cd^2+be^2)x}{e^4} - \frac{cdx^2}{e^3} + \frac{cx^3}{3e^2} - \frac{cd^4+bd^2e^2+ae^4}{e^5(d+ex)} - \frac{2d(2cd^2+be^2)\log(d+ex)}{e^5}$$

output
```
(b*e^2+3*c*d^2)*x/e^4-c*d*x^2/e^3+1/3*c*x^3/e^2-(a*e^4+b*d^2*e^2+c*d^4)/e^
5/(e*x+d)-2*d*(b*e^2+2*c*d^2)*ln(e*x+d)/e^5
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.97

$$\int \frac{a+bx^2+cx^4}{(d+ex)^2} dx = \frac{3e(3cd^2+be^2)x - 3cde^2x^2 + ce^3x^3 - \frac{3(cd^4+bd^2e^2+ae^4)}{d+ex} - 6(2cd^3+bde^2)\log(d+ex)}{3e^5}$$

input
```
Integrate[(a + b*x^2 + c*x^4)/(d + e*x)^2,x]
```

output
```
(3*e*(3*c*d^2 + b*e^2)*x - 3*c*d*e^2*x^2 + c*e^3*x^3 - (3*(c*d^4 + b*d^2*e
^2 + a*e^4))/(d + e*x) - 6*(2*c*d^3 + b*d*e^2)*Log[d + e*x])/(3*e^5)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^2+cx^4}{(d+ex)^2} \, dx$$
$$\Big\downarrow \text{ 2389}$$
$$\int \left(\frac{ae^4+bd^2e^2+cd^4}{e^4(d+ex)^2}-\frac{2\left(bde^2+2cd^3\right)}{e^4(d+ex)}+\frac{be^2+3cd^2}{e^4}-\frac{2cdx}{e^3}+\frac{cx^2}{e^2}\right)dx$$
$$\Big\downarrow \text{ 2009}$$
$$-\frac{ae^4+bd^2e^2+cd^4}{e^5(d+ex)}-\frac{2d\left(be^2+2cd^2\right)\log(d+ex)}{e^5}+\frac{x\left(be^2+3cd^2\right)}{e^4}-\frac{cdx^2}{e^3}+\frac{cx^3}{3e^2}$$

input
```
Int[(a + b*x^2 + c*x^4)/(d + e*x)^2,x]
```

output
```
((3*c*d^2 + b*e^2)*x)/e^4 - (c*d*x^2)/e^3 + (c*x^3)/(3*e^2) - (c*d^4 + b*d
^2*e^2 + a*e^4)/(e^5*(d + e*x)) - (2*d*(2*c*d^2 + b*e^2)*Log[d + e*x])/e^5
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2389
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.98

method	result
default	$\frac{\frac{1}{3}c\,e^{2}x^{3}-cde\,x^{2}+b\,e^{2}x+3c\,d^{2}x}{e^{4}}-\frac{e^{4}a+b\,d^{2}e^{2}+c\,d^{4}}{e^{5}(ex+d)}-\frac{2d\left(b\,e^{2}+2c\,d^{2}\right)\ln(ex+d)}{e^{5}}$
norman	$\frac{\frac{\left(b\,e^{2}+2c\,d^{2}\right)x^{2}}{e^{3}}-\frac{e^{4}a+2b\,d^{2}e^{2}+4c\,d^{4}}{e^{5}}+\frac{c\,x^{4}}{3e}-\frac{2cd\,x^{3}}{3e^{2}}}{ex+d}-\frac{2d\left(b\,e^{2}+2c\,d^{2}\right)\ln(ex+d)}{e^{5}}$
risch	$\frac{c\,x^{3}}{3e^{2}}-\frac{cd\,x^{2}}{e^{3}}+\frac{bx}{e^{2}}+\frac{3c\,d^{2}x}{e^{4}}-\frac{a}{e(ex+d)}-\frac{b\,d^{2}}{e^{3}(ex+d)}-\frac{c\,d^{4}}{e^{5}(ex+d)}-\frac{2d\ln(ex+d)b}{e^{3}}-\frac{4c\,d^{3}\ln(ex+d)}{e^{5}}$
parallelrisch	$-\frac{-x^{4}c\,e^{4}+2cd\,x^{3}e^{3}+6\ln(ex+d)xbd\,e^{3}+12\ln(ex+d)xc\,d^{3}e-3x^{2}b\,e^{4}-6x^{2}c\,d^{2}e^{2}+6\ln(ex+d)b\,d^{2}e^{2}+12\ln(ex+d)c\,d^{4}+3e^{4}a}{3e^{5}(ex+d)}$

input
```
int((c*x^4+b*x^2+a)/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
1/e^4*(1/3*c*e^2*x^3-c*d*e*x^2+b*e^2*x+3*c*d^2*x)-(a*e^4+b*d^2*e^2+c*d^4)/
e^5/(e*x+d)-2*d*(b*e^2+2*c*d^2)*ln(e*x+d)/e^5
```

Fricas [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.39

$$\int \frac{a+bx^2+cx^4}{(d+ex)^2}\,dx = \frac{ce^4x^4-2\,cde^3x^3-3\,cd^4-3\,bd^2e^2-3\,ae^4+3\,(2\,cd^2e^2+be^4)x^2+3\,(3\,cd^3e+bde^3)x-6\,(2\,cd^4+bd^2e^2}{3\,(e^6x+de^5)}$$

input
```
integrate((c*x^4+b*x^2+a)/(e*x+d)^2,x, algorithm="fricas")
```

output
```
1/3*(c*e^4*x^4 - 2*c*d*e^3*x^3 - 3*c*d^4 - 3*b*d^2*e^2 - 3*a*e^4 + 3*(2*c*
d^2*e^2 + b*e^4)*x^2 + 3*(3*c*d^3*e + b*d*e^3)*x - 6*(2*c*d^4 + b*d^2*e^2
+ (2*c*d^3*e + b*d*e^3)*x)*log(e*x + d))/(e^6*x + d*e^5)
```

Sympy [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.98

$$\int \frac{a+bx^2+cx^4}{(d+ex)^2}\,dx = -\frac{cdx^2}{e^3}+\frac{cx^3}{3e^2}-\frac{2d(be^2+2cd^2)\log{(d+ex)}}{e^5}+x\left(\frac{b}{e^2}+\frac{3cd^2}{e^4}\right)+\frac{-ae^4-bd^2e^2-cd^4}{de^5+e^6x}$$

input
```
integrate((c*x**4+b*x**2+a)/(e*x+d)**2,x)
```

output
```
-c*d*x**2/e**3 + c*x**3/(3*e**2) - 2*d*(b*e**2 + 2*c*d**2)*log(d + e*x)/e*
*5 + x*(b/e**2 + 3*c*d**2/e**4) + (-a*e**4 - b*d**2*e**2 - c*d**4)/(d*e**5
 + e**6*x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.02

$$\int \frac{a+bx^2+cx^4}{(d+ex)^2}\,dx = -\frac{cd^4+bd^2e^2+ae^4}{e^6x+de^5}+\frac{ce^2x^3-3\,cdex^2+3\,(3\,cd^2+be^2)x}{3\,e^4}-\frac{2\,(2\,cd^3+bde^2)\log{(ex+d)}}{e^5}$$

input
```
integrate((c*x^4+b*x^2+a)/(e*x+d)^2,x, algorithm="maxima")
```

output
```
-(c*d^4 + b*d^2*e^2 + a*e^4)/(e^6*x + d*e^5) + 1/3*(c*e^2*x^3 - 3*c*d*e*x^
2 + 3*(3*c*d^2 + b*e^2)*x)/e^4 - 2*(2*c*d^3 + b*d*e^2)*log(e*x + d)/e^5
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.56

$$\int \frac{a+bx^2+cx^4}{(d+ex)^2}\,dx = -\frac{1}{3}\,c\left(\frac{(ex+d)^3\left(\frac{6\,d}{ex+d}-\frac{18\,d^2}{(ex+d)^2}-1\right)}{e^5}-\frac{12\,d^3\log\left(\frac{|ex+d|}{(ex+d)^2|e|}\right)}{e^5}+\frac{3\,d^4}{(ex+d)e^5}\right) + b\left(\frac{2\,d\log\left(\frac{|ex+d|}{(ex+d)^2|e|}\right)}{e^3}+\frac{ex+d}{e^3}-\frac{d^2}{(ex+d)e^3}\right)-\frac{a}{(ex+d)e}$$

input
```
integrate((c*x^4+b*x^2+a)/(e*x+d)^2,x, algorithm="giac")
```

output
```
-1/3*c*((e*x + d)^3*(6*d/(e*x + d) - 18*d^2/(e*x + d)^2 - 1)/e^5 - 12*d^3*
log(abs(e*x + d)/((e*x + d)^2*abs(e)))/e^5 + 3*d^4/((e*x + d)*e^5)) + b*(2
*d*log(abs(e*x + d)/((e*x + d)^2*abs(e)))/e^3 + (e*x + d)/e^3 - d^2/((e*x
+ d)*e^3)) - a/((e*x + d)*e)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.05

$$\int \frac{a+bx^2+cx^4}{(d+ex)^2}\,dx = x\left(\frac{b}{e^2}+\frac{3\,c\,d^2}{e^4}\right)+\frac{c\,x^3}{3\,e^2}-\frac{\ln\left(d+e\,x\right)\,\left(4\,c\,d^3+2\,b\,d\,e^2\right)}{e^5} - \frac{c\,d^4+b\,d^2\,e^2+a\,e^4}{e\,\left(x\,e^5+d\,e^4\right)}-\frac{c\,d\,x^2}{e^3}$$

input
```
int((a + b*x^2 + c*x^4)/(d + e*x)^2,x)
```

output
```
x*(b/e^2 + (3*c*d^2)/e^4) + (c*x^3)/(3*e^2) - (log(d + e*x)*(4*c*d^3 + 2*b
*d*e^2))/e^5 - (a*e^4 + c*d^4 + b*d^2*e^2)/(e*(d*e^4 + e^5*x)) - (c*d*x^2)
/e^3
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.50

$$\int \frac{a+bx^2+cx^4}{(d+ex)^2}\,dx$$
$$= \frac{-6\log(ex+d)\,b\,d^3e^2-6\log(ex+d)\,b\,d^2e^3x-12\log(ex+d)\,c\,d^5-12\log(ex+d)\,c\,d^4ex+3a\,e^5x+6b}{3d\,e^5\,(ex+d)}$$

input
```
int((c*x^4+b*x^2+a)/(e*x+d)^2,x)
```

output
```
( - 6*log(d + e*x)*b*d**3*e**2 - 6*log(d + e*x)*b*d**2*e**3*x - 12*log(d +
 e*x)*c*d**5 - 12*log(d + e*x)*c*d**4*e*x + 3*a*e**5*x + 6*b*d**2*e**3*x +
 3*b*d*e**4*x**2 + 12*c*d**4*e*x + 6*c*d**3*e**2*x**2 - 2*c*d**2*e**3*x**3
 + c*d*e**4*x**4)/(3*d*e**5*(d + e*x))
```

3.229 $\int (d+ex)^3 \left(a+bx^2+cx^4\right)^2 \, dx$

Optimal result

Integrand size = 22, antiderivative size = 235

$$\begin{aligned}\int (d+ex)^3 \left(a+bx^2+cx^4\right)^2 \, dx = {} & \frac{2}{3}abd^3x^3 + \frac{3}{2}abd^2ex^4 + \frac{1}{5}d\left(b^2d^2+2acd^2+6abe^2\right)x^5 \\ & + \frac{1}{6}e\left(3b^2d^2+6acd^2+2abe^2\right)x^6 \\ & + \frac{1}{7}d\left(2bcd^2+3b^2e^2+6ace^2\right)x^7 \\ & + \frac{1}{8}e\left(6bcd^2+b^2e^2+2ace^2\right)x^8 \\ & + \frac{1}{9}cd\left(cd^2+6be^2\right)x^9 + \frac{1}{10}ce\left(3cd^2+2be^2\right)x^{10} \\ & + \frac{3}{11}c^2de^2x^{11} + \frac{1}{12}c^2e^3x^{12} + \frac{a^2(d+ex)^4}{4e}\end{aligned}$$

output
```
2/3*a*b*d^3*x^3+3/2*a*b*d^2*e*x^4+1/5*d*(6*a*b*e^2+2*a*c*d^2+b^2*d^2)*x^5+
1/6*e*(2*a*b*e^2+6*a*c*d^2+3*b^2*d^2)*x^6+1/7*d*(6*a*c*e^2+3*b^2*e^2+2*b*c
*d^2)*x^7+1/8*e*(2*a*c*e^2+b^2*e^2+6*b*c*d^2)*x^8+1/9*c*d*(6*b*e^2+c*d^2)*
x^9+1/10*c*e*(2*b*e^2+3*c*d^2)*x^10+3/11*c^2*d*e^2*x^11+1/12*c^2*e^3*x^12+
1/4*a^2*(e*x+d)^4/e
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.10

$$\int (d+ex)^3 \left(a+bx^2+cx^4\right)^2 \, dx = a^2d^3x + \frac{3}{2}a^2d^2ex^2 + \frac{1}{3}ad\left(2bd^2+3ae^2\right)x^3 + \frac{1}{4}ae\left(6bd^2+ae^2\right)x^4 + \frac{1}{5}d\left(b^2d^2+2acd^2+6abe^2\right)x^5 + \frac{1}{6}e\left(3b^2d^2+6acd^2+2abe^2\right)x^6 + \frac{1}{7}d\left(2bcd^2+3b^2e^2+6ace^2\right)x^7 + \frac{1}{8}e\left(6bcd^2+b^2e^2+2ace^2\right)x^8 + \frac{1}{9}cd\left(cd^2+6be^2\right)x^9 + \frac{1}{10}ce\left(3cd^2+2be^2\right)x^{10} + \frac{3}{11}c^2de^2x^{11} + \frac{1}{12}c^2e^3x^{12}$$

input
```
Integrate[(d + e*x)^3*(a + b*x^2 + c*x^4)^2,x]
```

output
```
a^2*d^3*x + (3*a^2*d^2*e*x^2)/2 + (a*d*(2*b*d^2 + 3*a*e^2)*x^3)/3 + (a*e*(
6*b*d^2 + a*e^2)*x^4)/4 + (d*(b^2*d^2 + 2*a*c*d^2 + 6*a*b*e^2)*x^5)/5 + (e
*(3*b^2*d^2 + 6*a*c*d^2 + 2*a*b*e^2)*x^6)/6 + (d*(2*b*c*d^2 + 3*b^2*e^2 +
6*a*c*e^2)*x^7)/7 + (e*(6*b*c*d^2 + b^2*e^2 + 2*a*c*e^2)*x^8)/8 + (c*d*(c*
d^2 + 6*b*e^2)*x^9)/9 + (c*e*(3*c*d^2 + 2*b*e^2)*x^10)/10 + (3*c^2*d*e^2*x
^11)/11 + (c^2*e^3*x^12)/12
```

Rubi [A] (verified)

Time = 1.05 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.40, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {2200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^3 \left(a+bx^2+cx^4\right)^2 \, dx$$

$$\Big\downarrow \text{ 2200}$$

$$\int \left(\frac{(d+ex)^7 (2ace^4 + b^2e^4 + 30bcd^2e^2 + 70c^2d^4)}{e^8} - \frac{4d(d+ex)^6 (2ace^4 + b^2e^4 + 10bcd^2e^2 + 14c^2d^4)}{e^8} + \frac{2(d+e\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{(d+ex)^8 (2ace^4 + b^2e^4 + 30bcd^2e^2 + 70c^2d^4)}{8e^9} - \frac{4d(d+ex)^7 (2ace^4 + b^2e^4 + 10bcd^2e^2 + 14c^2d^4)}{7e^9} + \frac{(d+ex)^6 (abe^6 + 6acd^2e^4 + 3b^2d^2e^4 + 15bcd^4e^2 + 14c^2d^6)}{3e^9} - \frac{4d(d+ex)^5 (be^2 + 2cd^2) (ae^4 + bd^2e^2 + cd^4)}{5e^9} + \frac{(d+ex)^4 (ae^4 + bd^2e^2 + cd^4)^2}{4e^9} + \frac{c(d+ex)^{10} (be^2 + 14cd^2)}{5e^9} - \frac{4cd(d+ex)^9 (3be^2 + 14cd^2)}{9e^9} + \frac{c^2(d+ex)^{12}}{12e^9} - \frac{8c^2d(d+ex)^{11}}{11e^9}$$

input
```
Int[(d + e*x)^3*(a + b*x^2 + c*x^4)^2,x]
```

output
```
((c*d^4 + b*d^2*e^2 + a*e^4)^2*(d + e*x)^4)/(4*e^9) - (4*d*(2*c*d^2 + b*e^
2)*(c*d^4 + b*d^2*e^2 + a*e^4)*(d + e*x)^5)/(5*e^9) + ((14*c^2*d^6 + 15*b*
c*d^4*e^2 + 3*b^2*d^2*e^4 + 6*a*c*d^2*e^4 + a*b*e^6)*(d + e*x)^6)/(3*e^9)
- (4*d*(14*c^2*d^4 + 10*b*c*d^2*e^2 + b^2*e^4 + 2*a*c*e^4)*(d + e*x)^7)/(7
*e^9) + ((70*c^2*d^4 + 30*b*c*d^2*e^2 + b^2*e^4 + 2*a*c*e^4)*(d + e*x)^8)/
(8*e^9) - (4*c*d*(14*c*d^2 + 3*b*e^2)*(d + e*x)^9)/(9*e^9) + (c*(14*c*d^2
+ b*e^2)*(d + e*x)^10)/(5*e^9) - (8*c^2*d*(d + e*x)^11)/(11*e^9) + (c^2*(d
 + e*x)^12)/(12*e^9)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2200
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[Expa
ndIntegrand[Px*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c}, x] && Poly
Q[Px, x] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.01

method	result
default	$\frac{c^2e^3x^{12}}{12}+\frac{3c^2d\,e^2x^{11}}{11}+\frac{(2e^3bc+3d^2e\,c^2)x^{10}}{10}+\frac{(6bcd\,e^2+c^2d^3)x^9}{9}+\frac{(6bc\,d^2e+e^3(2ac+b^2))x^8}{8}+\frac{(2bc\,d^3+3d\,e^2(2ac$
norman	$\frac{c^2e^3x^{12}}{12}+\frac{3c^2d\,e^2x^{11}}{11}+\left(\frac{1}{5}e^3bc+\frac{3}{10}d^2e\,c^2\right)x^{10}+\left(\frac{2}{3}bcd\,e^2+\frac{1}{9}c^2d^3\right)x^9+\left(\frac{1}{4}ac\,e^3+\frac{1}{8}b^2e^3+\frac{3}{4}bc$
gosper	$\frac{1}{12}c^2e^3x^{12}+\frac{3}{11}c^2d\,e^2x^{11}+\frac{1}{5}x^{10}e^3bc+\frac{3}{10}c^2d^2e\,x^{10}+\frac{2}{3}x^9bcd\,e^2+\frac{1}{9}c^2d^3x^9+\frac{1}{4}ac\,e^3x^8+\frac{1}{8}x^8b$
risch	$\frac{1}{12}c^2e^3x^{12}+\frac{3}{11}c^2d\,e^2x^{11}+\frac{1}{5}x^{10}e^3bc+\frac{3}{10}c^2d^2e\,x^{10}+\frac{2}{3}x^9bcd\,e^2+\frac{1}{9}c^2d^3x^9+\frac{1}{4}ac\,e^3x^8+\frac{1}{8}x^8b$
parallelrisch	$\frac{1}{12}c^2e^3x^{12}+\frac{3}{11}c^2d\,e^2x^{11}+\frac{1}{5}x^{10}e^3bc+\frac{3}{10}c^2d^2e\,x^{10}+\frac{2}{3}x^9bcd\,e^2+\frac{1}{9}c^2d^3x^9+\frac{1}{4}ac\,e^3x^8+\frac{1}{8}x^8b$
orering	$\frac{x(2310e^3c^2x^{11}+7560c^2d\,e^2x^{10}+5544bc\,e^3x^9+8316c^2d^2e\,x^9+18480bcd\,e^2x^8+3080c^2d^3x^8+6930ac\,e^3x^7+3465b^2e^3x^7+2079}{}$

input
```
int((e*x+d)^3*(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/12*c^2*e^3*x^12+3/11*c^2*d*e^2*x^11+1/10*(2*b*c*e^3+3*c^2*d^2*e)*x^10+1/
9*(6*b*c*d*e^2+c^2*d^3)*x^9+1/8*(6*b*c*d^2*e+e^3*(2*a*c+b^2))*x^8+1/7*(2*b
*c*d^3+3*d*e^2*(2*a*c+b^2))*x^7+1/6*(3*d^2*e*(2*a*c+b^2)+2*a*b*e^3)*x^6+1/
5*(d^3*(2*a*c+b^2)+6*a*b*d*e^2)*x^5+1/4*(a^2*e^3+6*a*b*d^2*e)*x^4+1/3*(3*a
^2*d*e^2+2*a*b*d^3)*x^3+3/2*a^2*d^2*e*x^2+a^2*d^3*x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.00

$$\begin{aligned}\int (d+ex)^3\left(a+bx^2+cx^4\right)^2\,dx=&\ \frac{1}{12}\,c^2e^3x^{12}+\frac{3}{11}\,c^2de^2x^{11}\\&+\frac{1}{10}\left(3\,c^2d^2e+2\,bce^3\right)x^{10}+\frac{1}{9}\left(c^2d^3+6\,bcde^2\right)x^9\\&+\frac{1}{8}\left(6\,bcd^2e+\left(b^2+2\,ac\right)e^3\right)x^8\\&+\frac{1}{7}\left(2\,bcd^3+3\left(b^2+2\,ac\right)de^2\right)x^7\\&+\frac{3}{2}\,a^2d^2ex^2+\frac{1}{6}\left(2\,abe^3+3\left(b^2+2\,ac\right)d^2e\right)x^6\\&+a^2d^3x+\frac{1}{5}\left(6\,abde^2+\left(b^2+2\,ac\right)d^3\right)x^5\\&+\frac{1}{4}\left(6\,abd^2e+a^2e^3\right)x^4+\frac{1}{3}\left(2\,abd^3+3\,a^2de^2\right)x^3\end{aligned}$$

input
```
integrate((e*x+d)^3*(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

output
```
1/12*c^2*e^3*x^12 + 3/11*c^2*d*e^2*x^11 + 1/10*(3*c^2*d^2*e + 2*b*c*e^3)*x
^10 + 1/9*(c^2*d^3 + 6*b*c*d*e^2)*x^9 + 1/8*(6*b*c*d^2*e + (b^2 + 2*a*c)*e
^3)*x^8 + 1/7*(2*b*c*d^3 + 3*(b^2 + 2*a*c)*d*e^2)*x^7 + 3/2*a^2*d^2*e*x^2
+ 1/6*(2*a*b*e^3 + 3*(b^2 + 2*a*c)*d^2*e)*x^6 + a^2*d^3*x + 1/5*(6*a*b*d*e
^2 + (b^2 + 2*a*c)*d^3)*x^5 + 1/4*(6*a*b*d^2*e + a^2*e^3)*x^4 + 1/3*(2*a*b
*d^3 + 3*a^2*d*e^2)*x^3
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.18

$$\int (d+ex)^3\left(a+bx^2+cx^4\right)^2\,dx = a^2d^3x+\frac{3a^2d^2ex^2}{2}+\frac{3c^2de^2x^{11}}{11}+\frac{c^2e^3x^{12}}{12}+x^{10}\left(\frac{bce^3}{5}+\frac{3c^2d^2e}{10}\right)+x^9\cdot\left(\frac{2bcde^2}{3}+\frac{c^2d^3}{9}\right)+x^8\left(\frac{ace^3}{4}+\frac{b^2e^3}{8}+\frac{3bcd^2e}{4}\right)+x^7\cdot\left(\frac{6acde^2}{7}+\frac{3b^2de^2}{7}+\frac{2bcd^3}{7}\right)+x^6\left(\frac{abe^3}{3}+acd^2e+\frac{b^2d^2e}{2}\right)+x^5\cdot\left(\frac{6abde^2}{5}+\frac{2acd^3}{5}+\frac{b^2d^3}{5}\right)+x^4\left(\frac{a^2e^3}{4}+\frac{3abd^2e}{2}\right)+x^3\left(a^2de^2+\frac{2abd^3}{3}\right)$$

input
```
integrate((e*x+d)**3*(c*x**4+b*x**2+a)**2,x)
```

output
```
a**2*d**3*x + 3*a**2*d**2*e*x**2/2 + 3*c**2*d*e**2*x**11/11 + c**2*e**3*x*
*12/12 + x**10*(b*c*e**3/5 + 3*c**2*d**2*e/10) + x**9*(2*b*c*d*e**2/3 + c*
*2*d**3/9) + x**8*(a*c*e**3/4 + b**2*e**3/8 + 3*b*c*d**2*e/4) + x**7*(6*a*
c*d*e**2/7 + 3*b**2*d*e**2/7 + 2*b*c*d**3/7) + x**6*(a*b*e**3/3 + a*c*d**2
*e + b**2*d**2*e/2) + x**5*(6*a*b*d*e**2/5 + 2*a*c*d**3/5 + b**2*d**3/5) +
 x**4*(a**2*e**3/4 + 3*a*b*d**2*e/2) + x**3*(a**2*d*e**2 + 2*a*b*d**3/3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.00

$$\int (d+ex)^3 \left(a+bx^2+cx^4\right)^2 \, dx = \frac{1}{12} c^2 e^3 x^{12} + \frac{3}{11} c^2 d e^2 x^{11} + \frac{1}{10} \left(3 c^2 d^2 e + 2 b c e^3\right) x^{10} + \frac{1}{9} \left(c^2 d^3 + 6 b c d e^2\right) x^9 + \frac{1}{8} \left(6 b c d^2 e + \left(b^2 + 2 a c\right) e^3\right) x^8 + \frac{1}{7} \left(2 b c d^3 + 3 \left(b^2 + 2 a c\right) d e^2\right) x^7 + \frac{3}{2} a^2 d^2 e x^2 + \frac{1}{6} \left(2 a b e^3 + 3 \left(b^2 + 2 a c\right) d^2 e\right) x^6 + a^2 d^3 x + \frac{1}{5} \left(6 a b d e^2 + \left(b^2 + 2 a c\right) d^3\right) x^5 + \frac{1}{4} \left(6 a b d^2 e + a^2 e^3\right) x^4 + \frac{1}{3} \left(2 a b d^3 + 3 a^2 d e^2\right) x^3$$

input
```
integrate((e*x+d)^3*(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/12*c^2*e^3*x^12 + 3/11*c^2*d*e^2*x^11 + 1/10*(3*c^2*d^2*e + 2*b*c*e^3)*x
^10 + 1/9*(c^2*d^3 + 6*b*c*d*e^2)*x^9 + 1/8*(6*b*c*d^2*e + (b^2 + 2*a*c)*e
^3)*x^8 + 1/7*(2*b*c*d^3 + 3*(b^2 + 2*a*c)*d*e^2)*x^7 + 3/2*a^2*d^2*e*x^2
+ 1/6*(2*a*b*e^3 + 3*(b^2 + 2*a*c)*d^2*e)*x^6 + a^2*d^3*x + 1/5*(6*a*b*d*e
^2 + (b^2 + 2*a*c)*d^3)*x^5 + 1/4*(6*a*b*d^2*e + a^2*e^3)*x^4 + 1/3*(2*a*b
*d^3 + 3*a^2*d*e^2)*x^3
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.11

$$\int (d+ex)^3 \left(a+bx^2+cx^4\right)^2 \, dx = \frac{1}{12}c^2e^3x^{12} + \frac{3}{11}c^2de^2x^{11} + \frac{3}{10}c^2d^2ex^{10} + \frac{1}{5}bce^3x^{10} + \frac{1}{9}c^2d^3x^9 + \frac{2}{3}bcde^2x^9 + \frac{3}{4}bcd^2ex^8 + \frac{1}{8}b^2e^3x^8 + \frac{1}{4}ace^3x^8 + \frac{2}{7}bcd^3x^7 + \frac{3}{7}b^2de^2x^7 + \frac{6}{7}acde^2x^7 + \frac{1}{2}b^2d^2ex^6 + acd^2ex^6 + \frac{1}{3}abe^3x^6 + \frac{1}{5}b^2d^3x^5 + \frac{2}{5}acd^3x^5 + \frac{6}{5}abde^2x^5 + \frac{3}{2}abd^2ex^4 + \frac{1}{4}a^2e^3x^4 + \frac{2}{3}abd^3x^3 + a^2de^2x^3 + \frac{3}{2}a^2d^2ex^2 + a^2d^3x$$

input
```
integrate((e*x+d)^3*(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/12*c^2*e^3*x^12 + 3/11*c^2*d*e^2*x^11 + 3/10*c^2*d^2*e*x^10 + 1/5*b*c*e^
3*x^10 + 1/9*c^2*d^3*x^9 + 2/3*b*c*d*e^2*x^9 + 3/4*b*c*d^2*e*x^8 + 1/8*b^2
*e^3*x^8 + 1/4*a*c*e^3*x^8 + 2/7*b*c*d^3*x^7 + 3/7*b^2*d*e^2*x^7 + 6/7*a*c
*d*e^2*x^7 + 1/2*b^2*d^2*e*x^6 + a*c*d^2*e*x^6 + 1/3*a*b*e^3*x^6 + 1/5*b^2
*d^3*x^5 + 2/5*a*c*d^3*x^5 + 6/5*a*b*d*e^2*x^5 + 3/2*a*b*d^2*e*x^4 + 1/4*a
^2*e^3*x^4 + 2/3*a*b*d^3*x^3 + a^2*d*e^2*x^3 + 3/2*a^2*d^2*e*x^2 + a^2*d^3
*x
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.03

$$\int (d+ex)^3 \left(a+bx^2+cx^4\right)^2 \, dx = x^5 \left(\frac{b^2 d^3}{5} + \frac{6\,a\,b\,d\,e^2}{5} + \frac{2\,a\,c\,d^3}{5}\right) + x^8 \left(\frac{b^2 e^3}{8} + \frac{3\,c\,b\,d^2\,e}{4} + \frac{a\,c\,e^3}{4}\right) + x^3 \left(a^2\,d\,e^2 + \frac{2\,b\,a\,d^3}{3}\right) + x^4 \left(\frac{a^2\,e^3}{4} + \frac{3\,b\,a\,d^2\,e}{2}\right) + x^9 \left(\frac{c^2\,d^3}{9} + \frac{2\,b\,c\,d\,e^2}{3}\right) + x^{10} \left(\frac{3\,c^2\,d^2\,e}{10} + \frac{b\,c\,e^3}{5}\right) + a^2\,d^3\,x + \frac{e\,x^6\,(3\,b^2\,d^2 + 2\,a\,b\,e^2 + 6\,a\,c\,d^2)}{6} + \frac{d\,x^7\,(3\,b^2\,e^2 + 2\,c\,b\,d^2 + 6\,a\,c\,e^2)}{7} + \frac{c^2\,e^3\,x^{12}}{12} + \frac{3\,a^2\,d^2\,e\,x^2}{2} + \frac{3\,c^2\,d\,e^2\,x^{11}}{11}$$

input
```
int((d + e*x)^3*(a + b*x^2 + c*x^4)^2,x)
```

output
```
x^5*((b^2*d^3)/5 + (2*a*c*d^3)/5 + (6*a*b*d*e^2)/5) + x^8*((b^2*e^3)/8 + (
a*c*e^3)/4 + (3*b*c*d^2*e)/4) + x^3*(a^2*d*e^2 + (2*a*b*d^3)/3) + x^4*((a^
2*e^3)/4 + (3*a*b*d^2*e)/2) + x^9*((c^2*d^3)/9 + (2*b*c*d*e^2)/3) + x^10*(
(3*c^2*d^2*e)/10 + (b*c*e^3)/5) + a^2*d^3*x + (e*x^6*(3*b^2*d^2 + 2*a*b*e^
2 + 6*a*c*d^2))/6 + (d*x^7*(3*b^2*e^2 + 6*a*c*e^2 + 2*b*c*d^2))/7 + (c^2*e
^3*x^12)/12 + (3*a^2*d^2*e*x^2)/2 + (3*c^2*d*e^2*x^11)/11
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.12

$$\int (d+ex)^3 \left(a+bx^2+cx^4\right)^2 \, dx = \frac{x(2310c^2e^3x^{11} + 7560c^2d\,e^2x^{10} + 5544bc\,e^3x^9 + 8316c^2d^2e\,x^9 + 18480bcd\,e^2x^8 + 3080c^2d^3x^8 + 6930ac\,e^3}{}$$

input
```
int((e*x+d)^3*(c*x^4+b*x^2+a)^2,x)
```

output
```
(x*(27720*a**2*d**3 + 41580*a**2*d**2*e*x + 27720*a**2*d*e**2*x**2 + 6930*
a**2*e**3*x**3 + 18480*a*b*d**3*x**2 + 41580*a*b*d**2*e*x**3 + 33264*a*b*d
*e**2*x**4 + 9240*a*b*e**3*x**5 + 11088*a*c*d**3*x**4 + 27720*a*c*d**2*e*x
**5 + 23760*a*c*d*e**2*x**6 + 6930*a*c*e**3*x**7 + 5544*b**2*d**3*x**4 + 1
3860*b**2*d**2*e*x**5 + 11880*b**2*d*e**2*x**6 + 3465*b**2*e**3*x**7 + 792
0*b*c*d**3*x**6 + 20790*b*c*d**2*e*x**7 + 18480*b*c*d*e**2*x**8 + 5544*b*c
*e**3*x**9 + 3080*c**2*d**3*x**8 + 8316*c**2*d**2*e*x**9 + 7560*c**2*d*e**
2*x**10 + 2310*c**2*e**3*x**11))/27720
```

3.230 $\int (d+ex)^2 \left(a+bx^2+cx^4\right)^2 \, dx$

Optimal result

Integrand size = 22, antiderivative size = 177

$$\begin{aligned}\int (d+ex)^2 \left(a+bx^2+cx^4\right)^2 \, dx = {} & a^2d^2x + a^2dex^2 + \frac{1}{3}a\left(2bd^2+ae^2\right)x^3 + abdex^4 \\ & + \frac{1}{5}\left(b^2d^2+2acd^2+2abe^2\right)x^5 + \frac{1}{3}\left(b^2+2ac\right)dex^6 \\ & + \frac{1}{7}\left(2bcd^2+b^2e^2+2ace^2\right)x^7 + \frac{1}{2}bcdex^8 \\ & + \frac{1}{9}c\left(cd^2+2be^2\right)x^9 + \frac{1}{5}c^2dex^{10} + \frac{1}{11}c^2e^2x^{11}\end{aligned}$$

output
```
a^2*d^2*x+a^2*d*e*x^2+1/3*a*(a*e^2+2*b*d^2)*x^3+a*b*d*e*x^4+1/5*(2*a*b*e^2
+2*a*c*d^2+b^2*d^2)*x^5+1/3*(2*a*c+b^2)*d*e*x^6+1/7*(2*a*c*e^2+b^2*e^2+2*b
*c*d^2)*x^7+1/2*b*c*d*e*x^8+1/9*c*(2*b*e^2+c*d^2)*x^9+1/5*c^2*d*e*x^10+1/1
1*c^2*e^2*x^11
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.00

$$\int (d+ex)^2 \left(a+bx^2+cx^4\right)^2 \, dx = a^2d^2x + a^2dex^2 + \frac{1}{3}a\left(2bd^2+ae^2\right)x^3 + abdex^4 + \frac{1}{5}\left(b^2d^2+2acd^2+2abe^2\right)x^5 + \frac{1}{3}\left(b^2+2ac\right)dex^6 + \frac{1}{7}\left(2bcd^2+b^2e^2+2ace^2\right)x^7 + \frac{1}{2}bcdex^8 + \frac{1}{9}c\left(cd^2+2be^2\right)x^9 + \frac{1}{5}c^2dex^{10} + \frac{1}{11}c^2e^2x^{11}$$

input
```
Integrate[(d + e*x)^2*(a + b*x^2 + c*x^4)^2,x]
```

output
```
a^2*d^2*x + a^2*d*e*x^2 + (a*(2*b*d^2 + a*e^2)*x^3)/3 + a*b*d*e*x^4 + ((b^
2*d^2 + 2*a*c*d^2 + 2*a*b*e^2)*x^5)/5 + ((b^2 + 2*a*c)*d*e*x^6)/3 + ((2*b*
c*d^2 + b^2*e^2 + 2*a*c*e^2)*x^7)/7 + (b*c*d*e*x^8)/2 + (c*(c*d^2 + 2*b*e^
2)*x^9)/9 + (c^2*d*e*x^10)/5 + (c^2*e^2*x^11)/11
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {2200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^2 \left(a+bx^2+cx^4\right)^2 \, dx$$

$$\Big\downarrow \text{2200}$$

$$\int \left(a^2d^2+2a^2dex+x^6\left(2ace^2+b^2e^2+2bcd^2\right)+x^4\left(2abe^2+2acd^2+b^2d^2\right)+2dex^5\left(2ac+b^2\right)+ax^2\left(ae^2+2bd^2\right)\right.$$

$$\Big\downarrow \text{2009}$$

$$a^2d^2x+a^2dex^2+\frac{1}{7}x^7\left(2ace^2+b^2e^2+2bcd^2\right)+\frac{1}{5}x^5\left(2abe^2+2acd^2+b^2d^2\right)+\frac{1}{3}dex^6\left(2ac+b^2\right)+\frac{1}{3}ax^3\left(ae^2+2bd^2\right)+abdex^4+\frac{1}{9}cx^9\left(2be^2+cd^2\right)+\frac{1}{2}bcdex^8+\frac{1}{5}c^2dex^{10}+\frac{1}{11}c^2e^2x^{11}$$

input
```
Int[(d + e*x)^2*(a + b*x^2 + c*x^4)^2,x]
```

output
```
a^2*d^2*x + a^2*d*e*x^2 + (a*(2*b*d^2 + a*e^2)*x^3)/3 + a*b*d*e*x^4 + ((b^
2*d^2 + 2*a*c*d^2 + 2*a*b*e^2)*x^5)/5 + ((b^2 + 2*a*c)*d*e*x^6)/3 + ((2*b*
c*d^2 + b^2*e^2 + 2*a*c*e^2)*x^7)/7 + (b*c*d*e*x^8)/2 + (c*(c*d^2 + 2*b*e^
2)*x^9)/9 + (c^2*d*e*x^10)/5 + (c^2*e^2*x^11)/11
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2200
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[Expa
ndIntegrand[Px*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c}, x] && Poly
Q[Px, x] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.92

method	result
default	$\frac{c^2e^2x^{11}}{11}+\frac{c^2de\,x^{10}}{5}+\frac{\left(2bc\,e^2+c^2d^2\right)x^9}{9}+\frac{bcde\,x^8}{2}+\frac{\left(2bc\,d^2+e^2\left(2ac+b^2\right)\right)x^7}{7}+\frac{\left(2ac+b^2\right)de\,x^6}{3}+\frac{\left(d^2\left(2ac+b^2\right)+2\right.}{5}$
norman	$\frac{c^2e^2x^{11}}{11}+\frac{c^2de\,x^{10}}{5}+\left(\frac{2}{9}bc\,e^2+\frac{1}{9}c^2d^2\right)x^9+\frac{bcde\,x^8}{2}+\left(\frac{2}{7}ac\,e^2+\frac{1}{7}b^2e^2+\frac{2}{7}bc\,d^2\right)x^7+\left(\frac{2}{3}acde+\right.$
gosper	$\frac{1}{11}c^2e^2x^{11}+\frac{1}{5}c^2de\,x^{10}+\frac{2}{9}x^9bc\,e^2+\frac{1}{9}c^2d^2x^9+\frac{1}{2}bcde\,x^8+\frac{2}{7}ac\,e^2x^7+\frac{1}{7}x^7b^2e^2+\frac{2}{7}x^7bc\,d^2+$
risch	$\frac{1}{11}c^2e^2x^{11}+\frac{1}{5}c^2de\,x^{10}+\frac{2}{9}x^9bc\,e^2+\frac{1}{9}c^2d^2x^9+\frac{1}{2}bcde\,x^8+\frac{2}{7}ac\,e^2x^7+\frac{1}{7}x^7b^2e^2+\frac{2}{7}x^7bc\,d^2+$
parallelrisch	$\frac{1}{11}c^2e^2x^{11}+\frac{1}{5}c^2de\,x^{10}+\frac{2}{9}x^9bc\,e^2+\frac{1}{9}c^2d^2x^9+\frac{1}{2}bcde\,x^8+\frac{2}{7}ac\,e^2x^7+\frac{1}{7}x^7b^2e^2+\frac{2}{7}x^7bc\,d^2+$
orering	$\frac{x\left(630c^2e^2x^{10}+1386c^2de\,x^9+1540bc\,e^2x^8+770c^2d^2x^8+3465bcde\,x^7+1980ac\,e^2x^6+990b^2e^2x^6+1980bc\,d^2x^6+4620acde\,x^5+\right.}{6930}$

input
```
int((e*x+d)^2*(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/11*c^2*e^2*x^11+1/5*c^2*d*e*x^10+1/9*(2*b*c*e^2+c^2*d^2)*x^9+1/2*b*c*d*e
*x^8+1/7*(2*b*c*d^2+e^2*(2*a*c+b^2))*x^7+1/3*(2*a*c+b^2)*d*e*x^6+1/5*(d^2*
(2*a*c+b^2)+2*a*b*e^2)*x^5+a*b*d*e*x^4+1/3*(a^2*e^2+2*a*b*d^2)*x^3+a^2*d*e
*x^2+x*a^2*d^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.91

$$\int (d+ex)^2\left(a+bx^2+cx^4\right)^2\,dx = \frac{1}{11}\,c^2e^2x^{11} + \frac{1}{5}\,c^2dex^{10} + \frac{1}{2}\,bcdex^8 + \frac{1}{9}\left(c^2d^2+2\,bce^2\right)x^9 + \frac{1}{3}\left(b^2+2\,ac\right)dex^6 + abdex^4 + \frac{1}{7}\left(2\,bcd^2+\left(b^2+2\,ac\right)e^2\right)x^7 + a^2dex^2 + \frac{1}{5}\left(2\,abe^2+\left(b^2+2\,ac\right)d^2\right)x^5 + a^2d^2x + \frac{1}{3}\left(2\,abd^2+a^2e^2\right)x^3$$

input
```
integrate((e*x+d)^2*(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

output
```
1/11*c^2*e^2*x^11 + 1/5*c^2*d*e*x^10 + 1/2*b*c*d*e*x^8 + 1/9*(c^2*d^2 + 2*
b*c*e^2)*x^9 + 1/3*(b^2 + 2*a*c)*d*e*x^6 + a*b*d*e*x^4 + 1/7*(2*b*c*d^2 +
(b^2 + 2*a*c)*e^2)*x^7 + a^2*d*e*x^2 + 1/5*(2*a*b*e^2 + (b^2 + 2*a*c)*d^2)
*x^5 + a^2*d^2*x + 1/3*(2*a*b*d^2 + a^2*e^2)*x^3
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.10

$$\int (d+ex)^2\left(a+bx^2+cx^4\right)^2\,dx = a^2d^2x + a^2dex^2 + abdex^4 + \frac{bcdex^8}{2} + \frac{c^2dex^{10}}{5} + \frac{c^2e^2x^{11}}{11} + x^9\cdot\left(\frac{2bce^2}{9}+\frac{c^2d^2}{9}\right) + x^7\cdot\left(\frac{2ace^2}{7}+\frac{b^2e^2}{7}+\frac{2bcd^2}{7}\right) + x^6\cdot\left(\frac{2acde}{3}+\frac{b^2de}{3}\right) + x^5\cdot\left(\frac{2abe^2}{5}+\frac{2acd^2}{5}+\frac{b^2d^2}{5}\right) + x^3\left(\frac{a^2e^2}{3}+\frac{2abd^2}{3}\right)$$

input
```
integrate((e*x+d)**2*(c*x**4+b*x**2+a)**2,x)
```

output
```
a**2*d**2*x + a**2*d*e*x**2 + a*b*d*e*x**4 + b*c*d*e*x**8/2 + c**2*d*e*x**
10/5 + c**2*e**2*x**11/11 + x**9*(2*b*c*e**2/9 + c**2*d**2/9) + x**7*(2*a*
c*e**2/7 + b**2*e**2/7 + 2*b*c*d**2/7) + x**6*(2*a*c*d*e/3 + b**2*d*e/3) +
 x**5*(2*a*b*e**2/5 + 2*a*c*d**2/5 + b**2*d**2/5) + x**3*(a**2*e**2/3 + 2*
a*b*d**2/3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.91

$$\int (d+ex)^2 \left(a+bx^2+cx^4\right)^2 dx = \frac{1}{11}c^2e^2x^{11} + \frac{1}{5}c^2dex^{10} + \frac{1}{2}bcdex^8 + \frac{1}{9}\left(c^2d^2+2bce^2\right)x^9 + \frac{1}{3}\left(b^2+2ac\right)dex^6 + abdex^4 + \frac{1}{7}\left(2bcd^2+\left(b^2+2ac\right)e^2\right)x^7 + a^2dex^2 + \frac{1}{5}\left(2abe^2+\left(b^2+2ac\right)d^2\right)x^5 + a^2d^2x + \frac{1}{3}\left(2abd^2+a^2e^2\right)x^3$$

input
```
integrate((e*x+d)^2*(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/11*c^2*e^2*x^11 + 1/5*c^2*d*e*x^10 + 1/2*b*c*d*e*x^8 + 1/9*(c^2*d^2 + 2*
b*c*e^2)*x^9 + 1/3*(b^2 + 2*a*c)*d*e*x^6 + a*b*d*e*x^4 + 1/7*(2*b*c*d^2 +
(b^2 + 2*a*c)*e^2)*x^7 + a^2*d*e*x^2 + 1/5*(2*a*b*e^2 + (b^2 + 2*a*c)*d^2)
*x^5 + a^2*d^2*x + 1/3*(2*a*b*d^2 + a^2*e^2)*x^3
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.01

$$\int (d+ex)^2\left(a+bx^2+cx^4\right)^2\,dx = \frac{1}{11}c^2e^2x^{11}+\frac{1}{5}c^2dex^{10}+\frac{1}{9}c^2d^2x^9+\frac{2}{9}bce^2x^9+\frac{1}{2}bcdex^8+\frac{2}{7}bcd^2x^7+\frac{1}{7}b^2e^2x^7+\frac{2}{7}ace^2x^7+\frac{1}{3}b^2dex^6+\frac{2}{3}acdex^6+\frac{1}{5}b^2d^2x^5+\frac{2}{5}acd^2x^5+\frac{2}{5}abe^2x^5+abdex^4+\frac{2}{3}abd^2x^3+\frac{1}{3}a^2e^2x^3+a^2dex^2+a^2d^2x$$

input
```
integrate((e*x+d)^2*(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/11*c^2*e^2*x^11 + 1/5*c^2*d*e*x^10 + 1/9*c^2*d^2*x^9 + 2/9*b*c*e^2*x^9 +
 1/2*b*c*d*e*x^8 + 2/7*b*c*d^2*x^7 + 1/7*b^2*e^2*x^7 + 2/7*a*c*e^2*x^7 + 1
/3*b^2*d*e*x^6 + 2/3*a*c*d*e*x^6 + 1/5*b^2*d^2*x^5 + 2/5*a*c*d^2*x^5 + 2/5
*a*b*e^2*x^5 + a*b*d*e*x^4 + 2/3*a*b*d^2*x^3 + 1/3*a^2*e^2*x^3 + a^2*d*e*x
^2 + a^2*d^2*x
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.93

$$\int (d+ex)^2\left(a+bx^2+cx^4\right)^2\,dx = x^5\left(\frac{b^2d^2}{5}+\frac{2abe^2}{5}+\frac{2acd^2}{5}\right)+x^7\left(\frac{b^2e^2}{7}+\frac{2cbd^2}{7}+\frac{2ace^2}{7}\right)+x^3\left(\frac{a^2e^2}{3}+\frac{2bad^2}{3}\right)+x^9\left(\frac{c^2d^2}{9}+\frac{2bce^2}{9}\right)+a^2d^2x+\frac{c^2e^2x^{11}}{11}+\frac{dex^6\left(b^2+2ac\right)}{3}+a^2dex^2+\frac{c^2dex^{10}}{5}+abdex^4+\frac{bcdex^8}{2}$$

input
```
int((d + e*x)^2*(a + b*x^2 + c*x^4)^2,x)
```

output
```
x^5*((b^2*d^2)/5 + (2*a*b*e^2)/5 + (2*a*c*d^2)/5) + x^7*((b^2*e^2)/7 + (2*
a*c*e^2)/7 + (2*b*c*d^2)/7) + x^3*((a^2*e^2)/3 + (2*a*b*d^2)/3) + x^9*((c^
2*d^2)/9 + (2*b*c*e^2)/9) + a^2*d^2*x + (c^2*e^2*x^11)/11 + (d*e*x^6*(2*a*
c + b^2))/3 + a^2*d*e*x^2 + (c^2*d*e*x^10)/5 + a*b*d*e*x^4 + (b*c*d*e*x^8)
/2
```

Reduce [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.03

$$\int (d+ex)^2 \left(a+bx^2+cx^4\right)^2 \, dx$$
$$= \frac{x(630c^2e^2x^{10} + 1386c^2de\,x^9 + 1540bc\,e^2x^8 + 770c^2d^2x^8 + 3465bcde\,x^7 + 1980ac\,e^2x^6 + 990b^2e^2x^6 + 198}{}$$

input
```
int((e*x+d)^2*(c*x^4+b*x^2+a)^2,x)
```

output
```
(x*(6930*a**2*d**2 + 6930*a**2*d*e*x + 2310*a**2*e**2*x**2 + 4620*a*b*d**2
*x**2 + 6930*a*b*d*e*x**3 + 2772*a*b*e**2*x**4 + 2772*a*c*d**2*x**4 + 4620
*a*c*d*e*x**5 + 1980*a*c*e**2*x**6 + 1386*b**2*d**2*x**4 + 2310*b**2*d*e*x
**5 + 990*b**2*e**2*x**6 + 1980*b*c*d**2*x**6 + 3465*b*c*d*e*x**7 + 1540*b
*c*e**2*x**8 + 770*c**2*d**2*x**8 + 1386*c**2*d*e*x**9 + 630*c**2*e**2*x**
10))/6930
```

3.231 $\int (d+ex) \left(a+bx^2+cx^4\right)^2 \, dx$

Optimal result

Integrand size = 20, antiderivative size = 112

$$\int (d+ex) \left(a+bx^2+cx^4\right)^2 \, dx = a^2dx+\frac{1}{2}a^2ex^2+\frac{2}{3}abdx^3+\frac{1}{2}abex^4 +\frac{1}{5}\left(b^2+2ac\right)dx^5+\frac{1}{6}\left(b^2+2ac\right)ex^6 +\frac{2}{7}bcdx^7+\frac{1}{4}bcex^8+\frac{1}{9}c^2dx^9+\frac{1}{10}c^2ex^{10}$$

output
```
a^2*d*x+1/2*a^2*e*x^2+2/3*a*b*d*x^3+1/2*a*b*e*x^4+1/5*(2*a*c+b^2)*d*x^5+1/
6*(2*a*c+b^2)*e*x^6+2/7*b*c*d*x^7+1/4*b*c*e*x^8+1/9*c^2*d*x^9+1/10*c^2*e*x
^10
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.87

$$\int (d+ex) \left(a+bx^2+cx^4\right)^2 \, dx = \frac{630a^2x(2d+ex)+42b^2x^5(6d+5ex)+45bcx^7(8d+7ex)+14c^2x^9(10d+9ex)+42a(5bx^3(4d+3ex)+}{1260}$$

input
```
Integrate[(d + e*x)*(a + b*x^2 + c*x^4)^2,x]
```

output
```
(630*a^2*x*(2*d + e*x) + 42*b^2*x^5*(6*d + 5*e*x) + 45*b*c*x^7*(8*d + 7*e*
x) + 14*c^2*x^9*(10*d + 9*e*x) + 42*a*(5*b*x^3*(4*d + 3*e*x) + 2*c*x^5*(6*
d + 5*e*x)))/1260
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)\left(a+bx^2+cx^4\right)^2\,dx$$

$$\Big\downarrow \text{2200}$$

$$\int \left(a^2d+a^2ex+dx^4\left(2ac+b^2\right)+ex^5\left(2ac+b^2\right)+2abdx^2+2abex^3+2bcdx^6+2bcex^7+c^2dx^8+c^2ex^9\right)dx$$

$$\Big\downarrow \text{2009}$$

$$a^2dx+\frac{1}{2}a^2ex^2+\frac{1}{5}dx^5\left(2ac+b^2\right)+\frac{1}{6}ex^6\left(2ac+b^2\right)+\frac{2}{3}abdx^3+\frac{1}{2}abex^4+\frac{2}{7}bcdx^7+\frac{1}{4}bcex^8+\frac{1}{9}c^2dx^9+\frac{1}{10}c^2ex^{10}$$

input
```
Int[(d + e*x)*(a + b*x^2 + c*x^4)^2,x]
```

output
```
a^2*d*x + (a^2*e*x^2)/2 + (2*a*b*d*x^3)/3 + (a*b*e*x^4)/2 + ((b^2 + 2*a*c)
*d*x^5)/5 + ((b^2 + 2*a*c)*e*x^6)/6 + (2*b*c*d*x^7)/7 + (b*c*e*x^8)/4 + (c
^2*d*x^9)/9 + (c^2*e*x^10)/10
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2200
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[Expa
ndIntegrand[Px*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c}, x] && Poly
Q[Px, x] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.85

method	result
default	$a^2dx+\frac{a^2e\,x^2}{2}+\frac{2abd\,x^3}{3}+\frac{abe\,x^4}{2}+\frac{(2ac+b^2)\,d\,x^5}{5}+\frac{(2ac+b^2)\,e\,x^6}{6}+\frac{2bcd\,x^7}{7}+\frac{bce\,x^8}{4}+\frac{c^2d\,x^9}{9}+\frac{c^2e\,x^{10}}{10}$
norman	$\frac{c^2e\,x^{10}}{10}+\frac{c^2d\,x^9}{9}+\frac{bce\,x^8}{4}+\frac{2bcd\,x^7}{7}+\left(\frac{1}{3}ace+\frac{1}{6}b^2e\right)x^6+\left(\frac{2}{5}acd+\frac{1}{5}d\,b^2\right)x^5+\frac{abe\,x^4}{2}+\frac{2abd\,x^3}{3}+$
gosper	$\frac{1}{10}c^2e\,x^{10}+\frac{1}{9}c^2d\,x^9+\frac{1}{4}bce\,x^8+\frac{2}{7}bcd\,x^7+\frac{1}{3}x^6ace+\frac{1}{6}x^6b^2e+\frac{2}{5}acd\,x^5+\frac{1}{5}x^5d\,b^2+\frac{1}{2}abe\,x^4$
risch	$\frac{1}{10}c^2e\,x^{10}+\frac{1}{9}c^2d\,x^9+\frac{1}{4}bce\,x^8+\frac{2}{7}bcd\,x^7+\frac{1}{3}x^6ace+\frac{1}{6}x^6b^2e+\frac{2}{5}acd\,x^5+\frac{1}{5}x^5d\,b^2+\frac{1}{2}abe\,x^4$
parallelrisch	$\frac{1}{10}c^2e\,x^{10}+\frac{1}{9}c^2d\,x^9+\frac{1}{4}bce\,x^8+\frac{2}{7}bcd\,x^7+\frac{1}{3}x^6ace+\frac{1}{6}x^6b^2e+\frac{2}{5}acd\,x^5+\frac{1}{5}x^5d\,b^2+\frac{1}{2}abe\,x^4$
orering	$\frac{x(126c^2e\,x^9+140c^2d\,x^8+315bce\,x^7+360cbd\,x^6+420ace\,x^5+210b^2e\,x^5+504d\,x^4ac+252b^2d\,x^4+630abe\,x^3+840abd\,x^2+630a^2}{1260}$

input
```
int((e*x+d)*(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
a^2*d*x+1/2*a^2*e*x^2+2/3*a*b*d*x^3+1/2*a*b*e*x^4+1/5*(2*a*c+b^2)*d*x^5+1/
6*(2*a*c+b^2)*e*x^6+2/7*b*c*d*x^7+1/4*b*c*e*x^8+1/9*c^2*d*x^9+1/10*c^2*e*x
^10
```

Fricas [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.84

$$\int (d+ex) \left(a+bx^2+cx^4\right)^2 \, dx = \frac{1}{10} c^2ex^{10}+\frac{1}{9} c^2dx^9+\frac{1}{4} bcex^8+\frac{2}{7} bcdx^7+\frac{1}{6} \left(b^2+2 ac\right)ex^6 +\frac{1}{2} abex^4+\frac{1}{5} \left(b^2+2 ac\right)dx^5+\frac{2}{3} abdx^3+\frac{1}{2} a^2ex^2+a^2dx$$

input
```
integrate((e*x+d)*(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

output
```
1/10*c^2*e*x^10 + 1/9*c^2*d*x^9 + 1/4*b*c*e*x^8 + 2/7*b*c*d*x^7 + 1/6*(b^2
 + 2*a*c)*e*x^6 + 1/2*a*b*e*x^4 + 1/5*(b^2 + 2*a*c)*d*x^5 + 2/3*a*b*d*x^3
+ 1/2*a^2*e*x^2 + a^2*d*x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.04

$$\int (d+ex) \left(a+bx^2+cx^4\right)^2 \, dx = a^2dx+\frac{a^2ex^2}{2}+\frac{2abdx^3}{3}+\frac{abex^4}{2}+\frac{2bcdx^7}{7}+\frac{bcex^8}{4}+\frac{c^2dx^9}{9} +\frac{c^2ex^{10}}{10}+x^6\left(\frac{ace}{3}+\frac{b^2e}{6}\right)+x^5\cdot\left(\frac{2acd}{5}+\frac{b^2d}{5}\right)$$

input
```
integrate((e*x+d)*(c*x**4+b*x**2+a)**2,x)
```

output
```
a**2*d*x + a**2*e*x**2/2 + 2*a*b*d*x**3/3 + a*b*e*x**4/2 + 2*b*c*d*x**7/7
+ b*c*e*x**8/4 + c**2*d*x**9/9 + c**2*e*x**10/10 + x**6*(a*c*e/3 + b**2*e/
6) + x**5*(2*a*c*d/5 + b**2*d/5)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.84

$$\int (d+ex)\left(a+bx^2+cx^4\right)^2\,dx=\frac{1}{10}\,c^2ex^{10}+\frac{1}{9}\,c^2dx^9+\frac{1}{4}\,bcex^8+\frac{2}{7}\,bcdx^7+\frac{1}{6}\,(b^2+2\,ac)ex^6+\frac{1}{2}\,abex^4+\frac{1}{5}\,(b^2+2\,ac)dx^5+\frac{2}{3}\,abdx^3+\frac{1}{2}\,a^2ex^2+a^2dx$$

input
```
integrate((e*x+d)*(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/10*c^2*e*x^10 + 1/9*c^2*d*x^9 + 1/4*b*c*e*x^8 + 2/7*b*c*d*x^7 + 1/6*(b^2
 + 2*a*c)*e*x^6 + 1/2*a*b*e*x^4 + 1/5*(b^2 + 2*a*c)*d*x^5 + 2/3*a*b*d*x^3
+ 1/2*a^2*e*x^2 + a^2*d*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.89

$$\int (d+ex)\left(a+bx^2+cx^4\right)^2\,dx=\frac{1}{10}\,c^2ex^{10}+\frac{1}{9}\,c^2dx^9+\frac{1}{4}\,bcex^8+\frac{2}{7}\,bcdx^7+\frac{1}{6}\,b^2ex^6+\frac{1}{3}\,acex^6+\frac{1}{5}\,b^2dx^5+\frac{2}{5}\,acdx^5+\frac{1}{2}\,abex^4+\frac{2}{3}\,abdx^3+\frac{1}{2}\,a^2ex^2+a^2dx$$

input
```
integrate((e*x+d)*(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/10*c^2*e*x^10 + 1/9*c^2*d*x^9 + 1/4*b*c*e*x^8 + 2/7*b*c*d*x^7 + 1/6*b^2*
e*x^6 + 1/3*a*c*e*x^6 + 1/5*b^2*d*x^5 + 2/5*a*c*d*x^5 + 1/2*a*b*e*x^4 + 2/
3*a*b*d*x^3 + 1/2*a^2*e*x^2 + a^2*d*x
```

Mupad [B] (verification not implemented)

Time = 21.41 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.84

$$\int (d+ex)\left(a+bx^2+cx^4\right)^2\,dx=\frac{a^2\,e\,x^2}{2}+\frac{c^2\,d\,x^9}{9}+\frac{c^2\,e\,x^{10}}{10}+\frac{d\,x^5\,(b^2+2\,a\,c)}{5}+\frac{e\,x^6\,(b^2+2\,a\,c)}{6}+a^2\,d\,x+\frac{2\,a\,b\,d\,x^3}{3}+\frac{a\,b\,e\,x^4}{2}+\frac{2\,b\,c\,d\,x^7}{7}+\frac{b\,c\,e\,x^8}{4}$$

input
```
int((d + e*x)*(a + b*x^2 + c*x^4)^2,x)
```

output
```
(a^2*e*x^2)/2 + (c^2*d*x^9)/9 + (c^2*e*x^10)/10 + (d*x^5*(2*a*c + b^2))/5
+ (e*x^6*(2*a*c + b^2))/6 + a^2*d*x + (2*a*b*d*x^3)/3 + (a*b*e*x^4)/2 + (2
*b*c*d*x^7)/7 + (b*c*e*x^8)/4
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.90

$$\int (d+ex)\left(a+bx^2+cx^4\right)^2\,dx=\frac{x(126c^2e\,x^9+140c^2d\,x^8+315bce\,x^7+360bcd\,x^6+420ace\,x^5+210b^2e\,x^5+504acd\,x^4+252b^2d\,x^4+63}{1260}$$

input
```
int((e*x+d)*(c*x^4+b*x^2+a)^2,x)
```

output
```
(x*(1260*a**2*d + 630*a**2*e*x + 840*a*b*d*x**2 + 630*a*b*e*x**3 + 504*a*c
*d*x**4 + 420*a*c*e*x**5 + 252*b**2*d*x**4 + 210*b**2*e*x**5 + 360*b*c*d*x
**6 + 315*b*c*e*x**7 + 140*c**2*d*x**8 + 126*c**2*e*x**9))/1260
```

3.232 $\int (a+bx^2+cx^4)^2 \, dx$

Optimal result

Integrand size = 14, antiderivative size = 49

$$\int (a+bx^2+cx^4)^2 \, dx = a^2x + \frac{2}{3}abx^3 + \frac{1}{5}(b^2+2ac)\,x^5 + \frac{2}{7}bcx^7 + \frac{c^2x^9}{9}$$

output
```
a^2*x+2/3*a*b*x^3+1/5*(2*a*c+b^2)*x^5+2/7*b*c*x^7+1/9*c^2*x^9
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int (a+bx^2+cx^4)^2 \, dx = a^2x + \frac{2}{3}abx^3 + \frac{1}{5}(b^2+2ac)\,x^5 + \frac{2}{7}bcx^7 + \frac{c^2x^9}{9}$$

input
```
Integrate[(a + b*x^2 + c*x^4)^2,x]
```

output
```
a^2*x + (2*a*b*x^3)/3 + ((b^2 + 2*a*c)*x^5)/5 + (2*b*c*x^7)/7 + (c^2*x^9)/
9
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1403, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^2+cx^4)^2\,dx$$

$$\Big\downarrow \text{1403}$$

$$\int \left(a^2+b^2x^4\left(\frac{2ac}{b^2}+1\right)+2abx^2+2bcx^6+c^2x^8\right)dx$$

$$\Big\downarrow \text{2009}$$

$$a^2x+\frac{1}{5}x^5(2ac+b^2)+\frac{2}{3}abx^3+\frac{2}{7}bcx^7+\frac{c^2x^9}{9}$$

input
```
Int[(a + b*x^2 + c*x^4)^2,x]
```

output
```
a^2*x + (2*a*b*x^3)/3 + ((b^2 + 2*a*c)*x^5)/5 + (2*b*c*x^7)/7 + (c^2*x^9)/
9
```

Defintions of rubi rules used

rule 1403
```
Int[((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int[ExpandInte
grand[(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a
*c, 0] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.86

method	result	size
default	$x\,a^{2}+\frac{2 a\,x^{3} b}{3}+\frac{\left(2 a c +b^{2}\right) x^{5}}{5}+\frac{2 b c\,x^{7}}{7}+\frac{c^{2} x^{9}}{9}$	42
norman	$\frac{c^{2} x^{9}}{9}+\frac{2 b c\,x^{7}}{7}+\left(\frac{2 a c}{5}+\frac{b^{2}}{5}\right) x^{5}+\frac{2 a\,x^{3} b}{3}+x\,a^{2}$	43
gosper	$\frac{1}{9} c^{2} x^{9}+\frac{2}{7} b c\,x^{7}+\frac{2}{5} x^{5} a c +\frac{1}{5} b^{2} x^{5}+\frac{2}{3} a\,x^{3} b +x\,a^{2}$	44
risch	$\frac{1}{9} c^{2} x^{9}+\frac{2}{7} b c\,x^{7}+\frac{2}{5} x^{5} a c +\frac{1}{5} b^{2} x^{5}+\frac{2}{3} a\,x^{3} b +x\,a^{2}$	44
parallelrisch	$\frac{1}{9} c^{2} x^{9}+\frac{2}{7} b c\,x^{7}+\frac{2}{5} x^{5} a c +\frac{1}{5} b^{2} x^{5}+\frac{2}{3} a\,x^{3} b +x\,a^{2}$	44
orering	$\frac{x \left(35 x^{8} c^{2}+90 b c\,x^{6}+126 x^{4} a c +63 b^{2} x^{4}+210 a b\,x^{2}+315 a^{2}\right)}{315}$	47

input
```
int((c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
x*a^2+2/3*a*x^3*b+1/5*(2*a*c+b^2)*x^5+2/7*b*c*x^7+1/9*c^2*x^9
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.84

$$\int \left(a+bx^2+cx^4\right)^2\,dx=\frac{1}{9}\,c^2x^9+\frac{2}{7}\,bcx^7+\frac{1}{5}\,\left(b^2+2\,ac\right)x^5+\frac{2}{3}\,abx^3+a^2x$$

input
```
integrate((c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

output
```
1/9*c^2*x^9 + 2/7*b*c*x^7 + 1/5*(b^2 + 2*a*c)*x^5 + 2/3*a*b*x^3 + a^2*x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.98

$$\int \left(a+bx^2+cx^4\right)^2 \, dx = a^2x + \frac{2abx^3}{3} + \frac{2bcx^7}{7} + \frac{c^2x^9}{9} + x^5 \cdot \left(\frac{2ac}{5} + \frac{b^2}{5}\right)$$

input
```
integrate((c*x**4+b*x**2+a)**2,x)
```

output
```
a**2*x + 2*a*b*x**3/3 + 2*b*c*x**7/7 + c**2*x**9/9 + x**5*(2*a*c/5 + b**2/
5)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.92

$$\int \left(a+bx^2+cx^4\right)^2 \, dx = \frac{1}{9}\,c^2x^9 + \frac{2}{7}\,bcx^7 + \frac{1}{5}\,b^2x^5 + a^2x + \frac{2}{15}\,(3\,cx^5 + 5\,bx^3)a$$

input
```
integrate((c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/9*c^2*x^9 + 2/7*b*c*x^7 + 1/5*b^2*x^5 + a^2*x + 2/15*(3*c*x^5 + 5*b*x^3)
*a
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.88

$$\int \left(a+bx^2+cx^4\right)^2 \, dx = \frac{1}{9}\,c^2x^9 + \frac{2}{7}\,bcx^7 + \frac{1}{5}\,b^2x^5 + \frac{2}{5}\,acx^5 + \frac{2}{3}\,abx^3 + a^2x$$

input
```
integrate((c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/9*c^2*x^9 + 2/7*b*c*x^7 + 1/5*b^2*x^5 + 2/5*a*c*x^5 + 2/3*a*b*x^3 + a^2*
x
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.86

$$\int \left(a+bx^2+cx^4\right)^2\,dx = a^2\,x+x^5\left(\frac{b^2}{5}+\frac{2\,a\,c}{5}\right)+\frac{c^2\,x^9}{9}+\frac{2\,a\,b\,x^3}{3}+\frac{2\,b\,c\,x^7}{7}$$

input
```
int((a + b*x^2 + c*x^4)^2,x)
```

output
```
a^2*x + x^5*((2*a*c)/5 + b^2/5) + (c^2*x^9)/9 + (2*a*b*x^3)/3 + (2*b*c*x^7
)/7
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.94

$$\int \left(a+bx^2+cx^4\right)^2\,dx = \frac{x(35c^2x^8+90bc\,x^6+126ac\,x^4+63b^2x^4+210ab\,x^2+315a^2)}{315}$$

input
```
int((c*x^4+b*x^2+a)^2,x)
```

output
```
(x*(315*a**2 + 210*a*b*x**2 + 126*a*c*x**4 + 63*b**2*x**4 + 90*b*c*x**6 +
35*c**2*x**8))/315
```

3.233 $\int \frac{(a+bx^2+cx^4)^2}{d+ex}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 270

$$\begin{aligned}\int \frac{(a+bx^2+cx^4)^2}{d+ex}\,dx = & -\frac{d(cd^2+be^2)\,(cd^4+bd^2e^2+2ae^4)\,x}{e^8} \\ & + \frac{(cd^2+be^2)\,(cd^4+bd^2e^2+2ae^4)\,x^2}{2e^7} \\ & - \frac{d(c^2d^4+b^2e^4+2c(bd^2e^2+ae^4))\,x^3}{3e^6} \\ & + \frac{(c^2d^4+b^2e^4+2c(bd^2e^2+ae^4))\,x^4}{4e^5} \\ & - \frac{cd(cd^2+2be^2)\,x^5}{5e^4} + \frac{c(cd^2+2be^2)\,x^6}{6e^3} - \frac{c^2dx^7}{7e^2} \\ & + \frac{c^2x^8}{8e} + \frac{(cd^4+bd^2e^2+ae^4)^2\log(d+ex)}{e^9}\end{aligned}$$

output
```
-d*(b*e^2+c*d^2)*(2*a*e^4+b*d^2*e^2+c*d^4)*x/e^8+1/2*(b*e^2+c*d^2)*(2*a*e^
4+b*d^2*e^2+c*d^4)*x^2/e^7-1/3*d*(c^2*d^4+b^2*e^4+2*c*(a*e^4+b*d^2*e^2))*x
^3/e^6+1/4*(c^2*d^4+b^2*e^4+2*c*(a*e^4+b*d^2*e^2))*x^4/e^5-1/5*c*d*(2*b*e^
2+c*d^2)*x^5/e^4+1/6*c*(2*b*e^2+c*d^2)*x^6/e^3-1/7*c^2*d*x^7/e^2+1/8*c^2*x
^8/e+(a*e^4+b*d^2*e^2+c*d^4)^2*ln(e*x+d)/e^9
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 268, normalized size of antiderivative = 0.99

$$\int \frac{(a+bx^2+cx^4)^2}{d+ex}\,dx$$
$$= \frac{x(c^2(-840d^7+420d^6ex-280d^5e^2x^2+210d^4e^3x^3-168d^3e^4x^4+140d^2e^5x^5-120de^6x^6+105e^7x^7)+7}{}$$
$$+\frac{(cd^4+bd^2e^2+ae^4)^2\log(d+ex)}{e^9}$$

input
```
Integrate[(a + b*x^2 + c*x^4)^2/(d + e*x),x]
```

output
```
(x*(c^2*(-840*d^7 + 420*d^6*e*x - 280*d^5*e^2*x^2 + 210*d^4*e^3*x^3 - 168*
d^3*e^4*x^4 + 140*d^2*e^5*x^5 - 120*d*e^6*x^6 + 105*e^7*x^7) + 70*b*e^4*(1
2*a*e^2*(-2*d + e*x) + b*(-12*d^3 + 6*d^2*e*x - 4*d*e^2*x^2 + 3*e^3*x^3))
+ 28*c*e^2*(5*a*e^2*(-12*d^3 + 6*d^2*e*x - 4*d*e^2*x^2 + 3*e^3*x^3) + b*(-
60*d^5 + 30*d^4*e*x - 20*d^3*e^2*x^2 + 15*d^2*e^3*x^3 - 12*d*e^4*x^4 + 10*
e^5*x^5))))/(840*e^8) + ((c*d^4 + b*d^2*e^2 + a*e^4)^2*Log[d + e*x])/e^9
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {2389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2+cx^4)^2}{d+ex}\,dx$$
$$\Big\downarrow \text{2389}$$
$$\int \left(\frac{dx^2(-2c(ae^4+bd^2e^2)-b^2e^4-c^2d^4)}{e^6}+\frac{x^3(2c(ae^4+bd^2e^2)+b^2e^4+c^2d^4)}{e^5}+\frac{(ae^4+bd^2e^2+cd^4)^2}{e^8(d+ex)}-\frac{d(be}{}\right.$$
$$\Big\downarrow \text{2009}$$

$$-\frac{dx^3\left(2c\left(ae^4+bd^2e^2\right)+b^2e^4+c^2d^4\right)}{3e^6}+\frac{x^4\left(2c\left(ae^4+bd^2e^2\right)+b^2e^4+c^2d^4\right)}{4e^5}+\frac{\log(d+ex)\left(ae^4+bd^2e^2+cd^4\right)^2}{e^9}-\frac{dx\left(be^2+cd^2\right)\left(2ae^4+bd^2e^2+cd^4\right)}{e^8}+\frac{x^2\left(be^2+cd^2\right)\left(2ae^4+bd^2e^2+cd^4\right)}{2e^7}-\frac{cdx^5\left(2be^2+cd^2\right)}{5e^4}+\frac{cx^6\left(2be^2+cd^2\right)}{6e^3}-\frac{c^2dx^7}{7e^2}+\frac{c^2x^8}{8e}$$

input

```
Int[(a + b*x^2 + c*x^4)^2/(d + e*x),x]
```

output

```
-((d*(c*d^2 + b*e^2)*(c*d^4 + b*d^2*e^2 + 2*a*e^4)*x)/e^8) + ((c*d^2 + b*e
^2)*(c*d^4 + b*d^2*e^2 + 2*a*e^4)*x^2)/(2*e^7) - (d*(c^2*d^4 + b^2*e^4 + 2
*c*(b*d^2*e^2 + a*e^4))*x^3)/(3*e^6) + ((c^2*d^4 + b^2*e^4 + 2*c*(b*d^2*e^
2 + a*e^4))*x^4)/(4*e^5) - (c*d*(c*d^2 + 2*b*e^2)*x^5)/(5*e^4) + (c*(c*d^2
 + 2*b*e^2)*x^6)/(6*e^3) - (c^2*d*x^7)/(7*e^2) + (c^2*x^8)/(8*e) + ((c*d^4
 + b*d^2*e^2 + a*e^4)^2*Log[d + e*x])/e^9
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2389

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.18

method	result
norman	$\frac{c^2x^8}{8e}+\frac{\left(2ab\,e^6+2ac\,d^2e^4+b^2d^2e^4+2bc\,d^4e^2+c^2d^6\right)x^2}{2e^7}+\frac{\left(2ac\,e^4+b^2e^4+2bc\,d^2e^2+c^2d^4\right)x^4}{4e^5}+\frac{c\left(2b\,e^2+c\,d^2\right)x^6}{6e^3}-\frac{c^2d\,x}{7e^2}$
default	$-\frac{-\frac{c^2x^8e^7}{8}+\frac{c^2d\,x^7e^6}{7}+\frac{\left(-e^5\left(b\,e^2+c\,d^2\right)c-e^7cb\right)x^6}{6}+\frac{\left(d\left(b\,e^2+c\,d^2\right)c\,e^4+cd\,e^6b\right)x^5}{5}+\frac{\left(-e^5\left(b\,e^2+c\,d^2\right)b-e^3c\left(2e^4a+b\,d^2e^2+c\,d^4\right)\right)x}{4}}{e}$
risch	$\frac{x^4b^2}{4e}-\frac{b^2d^3x}{e^4}-\frac{c^2d^7x}{e^8}+\frac{\ln(ex+d)b^2d^4}{e^5}+\frac{\ln(ex+d)d^8c^2}{e^9}-\frac{2abdx}{e^2}-\frac{2ac\,d^3x}{e^4}-\frac{2bc\,d^5x}{e^6}+\frac{2\ln(ex+d)ab\,d^2}{e^3}+$
parallelrisch	$280x^6bc\,e^8-280x^3b^2d\,e^7+840x^2ab\,e^8+420x^2b^2d^2e^6-840x\,b^2d^3e^5+840\ln(ex+d)b^2d^4e^4+210x^4b^2e^8-560x^3acd\,e^7+840x^2ac$

input
```
int((c*x^4+b*x^2+a)^2/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
1/8*c^2*x^8/e+1/2/e^7*(2*a*b*e^6+2*a*c*d^2*e^4+b^2*d^2*e^4+2*b*c*d^4*e^2+c
^2*d^6)*x^2+1/4/e^5*(2*a*c*e^4+b^2*e^4+2*b*c*d^2*e^2+c^2*d^4)*x^4+1/6*c*(2
*b*e^2+c*d^2)*x^6/e^3-1/7*c^2*d*x^7/e^2-1/3*d/e^6*(2*a*c*e^4+b^2*e^4+2*b*c
*d^2*e^2+c^2*d^4)*x^3-d*(2*a*b*e^6+2*a*c*d^2*e^4+b^2*d^2*e^4+2*b*c*d^4*e^2
+c^2*d^6)/e^8*x-1/5*c*d*(2*b*e^2+c*d^2)*x^5/e^4+(a^2*e^8+2*a*b*d^2*e^6+2*a
*c*d^4*e^4+b^2*d^4*e^4+2*b*c*d^6*e^2+c^2*d^8)/e^9*ln(e*x+d)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.13

$$\int \frac{(a+bx^2+cx^4)^2}{d+ex}\,dx = \frac{105\,c^2e^8x^8 - 120\,c^2de^7x^7 + 140\,(c^2d^2e^6 + 2\,bce^8)x^6 - 168\,(c^2d^3e^5 + 2\,bcde^7)x^5 + 210\,(c^2d^4e^4 + 2\,bcd^2e^6 \cdots}{}$$

input
```
integrate((c*x^4+b*x^2+a)^2/(e*x+d),x, algorithm="fricas")
```

output
```
1/840*(105*c^2*e^8*x^8 - 120*c^2*d*e^7*x^7 + 140*(c^2*d^2*e^6 + 2*b*c*e^8)
*x^6 - 168*(c^2*d^3*e^5 + 2*b*c*d*e^7)*x^5 + 210*(c^2*d^4*e^4 + 2*b*c*d^2*
e^6 + (b^2 + 2*a*c)*e^8)*x^4 - 280*(c^2*d^5*e^3 + 2*b*c*d^3*e^5 + (b^2 + 2
*a*c)*d*e^7)*x^3 + 420*(c^2*d^6*e^2 + 2*b*c*d^4*e^4 + 2*a*b*e^8 + (b^2 + 2
*a*c)*d^2*e^6)*x^2 - 840*(c^2*d^7*e + 2*b*c*d^5*e^3 + 2*a*b*d*e^7 + (b^2 +
 2*a*c)*d^3*e^5)*x + 840*(c^2*d^8 + 2*b*c*d^6*e^2 + 2*a*b*d^2*e^6 + a^2*e^
8 + (b^2 + 2*a*c)*d^4*e^4)*log(e*x + d))/e^9
```

Sympy [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.14

$$\int \frac{(a+bx^2+cx^4)^2}{d+ex}\,dx = -\frac{c^2dx^7}{7e^2}+\frac{c^2x^8}{8e}+x^6\left(\frac{bc}{3e}+\frac{c^2d^2}{6e^3}\right) +x^5\left(-\frac{2bcd}{5e^2}-\frac{c^2d^3}{5e^4}\right)+x^4\left(\frac{ac}{2e}+\frac{b^2}{4e}+\frac{bcd^2}{2e^3}+\frac{c^2d^4}{4e^5}\right) +x^3\left(-\frac{2acd}{3e^2}-\frac{b^2d}{3e^2}-\frac{2bcd^3}{3e^4}-\frac{c^2d^5}{3e^6}\right) +x^2\left(\frac{ab}{e}+\frac{acd^2}{e^3}+\frac{b^2d^2}{2e^3}+\frac{bcd^4}{e^5}+\frac{c^2d^6}{2e^7}\right) +x\left(-\frac{2abd}{e^2}-\frac{2acd^3}{e^4}-\frac{b^2d^3}{e^4}-\frac{2bcd^5}{e^6}-\frac{c^2d^7}{e^8}\right) +\frac{(ae^4+bd^2e^2+cd^4)^2\log(d+ex)}{e^9}$$

input
```
integrate((c*x**4+b*x**2+a)**2/(e*x+d),x)
```

output
```
-c**2*d*x**7/(7*e**2) + c**2*x**8/(8*e) + x**6*(b*c/(3*e) + c**2*d**2/(6*e
**3)) + x**5*(-2*b*c*d/(5*e**2) - c**2*d**3/(5*e**4)) + x**4*(a*c/(2*e) +
b**2/(4*e) + b*c*d**2/(2*e**3) + c**2*d**4/(4*e**5)) + x**3*(-2*a*c*d/(3*e
**2) - b**2*d/(3*e**2) - 2*b*c*d**3/(3*e**4) - c**2*d**5/(3*e**6)) + x**2*
(a*b/e + a*c*d**2/e**3 + b**2*d**2/(2*e**3) + b*c*d**4/e**5 + c**2*d**6/(2
*e**7)) + x*(-2*a*b*d/e**2 - 2*a*c*d**3/e**4 - b**2*d**3/e**4 - 2*b*c*d**5
/e**6 - c**2*d**7/e**8) + (a*e**4 + b*d**2*e**2 + c*d**4)**2*log(d + e*x)/
e**9
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.13

$$\int \frac{(a+bx^2+cx^4)^2}{d+ex}\,dx = \frac{105\,c^2e^7x^8-120\,c^2de^6x^7+140\,(c^2d^2e^5+2\,bce^7)x^6-168\,(c^2d^3e^4+2\,bcde^6)x^5+210\,(c^2d^4e^3+2\,bcd^2e^5-}{} + \frac{(c^2d^8+2\,bcd^6e^2+2\,abd^2e^6+a^2e^8+(b^2+2\,ac)d^4e^4)\log(ex+d)}{e^9}$$

input
```
integrate((c*x^4+b*x^2+a)^2/(e*x+d),x, algorithm="maxima")
```

output
```
1/840*(105*c^2*e^7*x^8 - 120*c^2*d*e^6*x^7 + 140*(c^2*d^2*e^5 + 2*b*c*e^7)
*x^6 - 168*(c^2*d^3*e^4 + 2*b*c*d*e^6)*x^5 + 210*(c^2*d^4*e^3 + 2*b*c*d^2*
e^5 + (b^2 + 2*a*c)*e^7)*x^4 - 280*(c^2*d^5*e^2 + 2*b*c*d^3*e^4 + (b^2 + 2
*a*c)*d*e^6)*x^3 + 420*(c^2*d^6*e + 2*b*c*d^4*e^3 + 2*a*b*e^7 + (b^2 + 2*a
*c)*d^2*e^5)*x^2 - 840*(c^2*d^7 + 2*b*c*d^5*e^2 + 2*a*b*d*e^6 + (b^2 + 2*a
*c)*d^3*e^4)*x)/e^8 + (c^2*d^8 + 2*b*c*d^6*e^2 + 2*a*b*d^2*e^6 + a^2*e^8 +
 (b^2 + 2*a*c)*d^4*e^4)*log(e*x + d)/e^9
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.32

$$\int \frac{(a+bx^2+cx^4)^2}{d+ex}\,dx$$
$$= \frac{105\,c^2e^7x^8 - 120\,c^2de^6x^7 + 140\,c^2d^2e^5x^6 + 280\,bce^7x^6 - 168\,c^2d^3e^4x^5 - 336\,bcde^6x^5 + 210\,c^2d^4e^3x^4 + 42}{}$$
$$+ \frac{(c^2d^8 + 2\,bcd^6e^2 + b^2d^4e^4 + 2\,acd^4e^4 + 2\,abd^2e^6 + a^2e^8)\log(|ex+d|)}{e^9}$$

input
```
integrate((c*x^4+b*x^2+a)^2/(e*x+d),x, algorithm="giac")
```

output
```
1/840*(105*c^2*e^7*x^8 - 120*c^2*d*e^6*x^7 + 140*c^2*d^2*e^5*x^6 + 280*b*c
*e^7*x^6 - 168*c^2*d^3*e^4*x^5 - 336*b*c*d*e^6*x^5 + 210*c^2*d^4*e^3*x^4 +
 420*b*c*d^2*e^5*x^4 + 210*b^2*e^7*x^4 + 420*a*c*e^7*x^4 - 280*c^2*d^5*e^2
*x^3 - 560*b*c*d^3*e^4*x^3 - 280*b^2*d*e^6*x^3 - 560*a*c*d*e^6*x^3 + 420*c
^2*d^6*e*x^2 + 840*b*c*d^4*e^3*x^2 + 420*b^2*d^2*e^5*x^2 + 840*a*c*d^2*e^5
*x^2 + 840*a*b*e^7*x^2 - 840*c^2*d^7*x - 1680*b*c*d^5*e^2*x - 840*b^2*d^3*
e^4*x - 1680*a*c*d^3*e^4*x - 1680*a*b*d*e^6*x)/e^8 + (c^2*d^8 + 2*b*c*d^6*
e^2 + b^2*d^4*e^4 + 2*a*c*d^4*e^4 + 2*a*b*d^2*e^6 + a^2*e^8)*log(abs(e*x +
 d))/e^9
```

Mupad [B] (verification not implemented)

Time = 21.51 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.29

$$\int \frac{(a+bx^2+cx^4)^2}{d+ex}\,dx$$
$$= x^2\left(\frac{d^2\left(\frac{b^2+2\,a\,c}{e}+\frac{d^2\left(\frac{c^2\,d^2}{e^3}+\frac{2\,b\,c}{e}\right)}{e^2}\right)}{2\,e^2}+\frac{a\,b}{e}\right)+x^6\left(\frac{c^2\,d^2}{6\,e^3}+\frac{b\,c}{3\,e}\right)+x^4\left(\frac{b^2+2\,a\,c}{4\,e}+\frac{d^2\left(\frac{c^2\,d^2}{e^3}+\frac{2\,b\,c}{e}\right)}{4\,e^2}\right)+\frac{\ln(d+e\,x)\,\left(a^2\,e^8+2\,a\,b\,d^2\,e^6+2\,a\,c\,d^4\,e^4+b^2\,d^4\,e^4+2\,b\,c\,d^6\,e^2+c^2\,d^8\right)}{e^9}+\frac{c^2\,x^8}{8\,e}-\frac{d\,x^3\left(\frac{b^2+2\,a\,c}{e}+\frac{d^2\left(\frac{c^2\,d^2}{e^3}+\frac{2\,b\,c}{e}\right)}{e^2}\right)}{3\,e}-\frac{c^2\,d\,x^7}{7\,e^2}-\frac{d\,x^5\left(\frac{c^2\,d^2}{e^3}+\frac{2\,b\,c}{e}\right)}{5\,e}-\frac{d\,x\left(\frac{d^2\left(\frac{b^2+2\,a\,c}{e}+\frac{d^2\left(\frac{c^2\,d^2}{e^3}+\frac{2\,b\,c}{e}\right)}{e^2}\right)}{e^2}+\frac{2\,a\,b}{e}\right)}{e}$$

input
```
int((a + b*x^2 + c*x^4)^2/(d + e*x),x)
```

output
```
x^2*((d^2*((2*a*c + b^2)/e + (d^2*((c^2*d^2)/e^3 + (2*b*c)/e))/e^2))/(2*e^
2) + (a*b)/e) + x^6*((c^2*d^2)/(6*e^3) + (b*c)/(3*e)) + x^4*((2*a*c + b^2)
/(4*e) + (d^2*((c^2*d^2)/e^3 + (2*b*c)/e))/(4*e^2)) + (log(d + e*x)*(a^2*e
^8 + c^2*d^8 + b^2*d^4*e^4 + 2*a*b*d^2*e^6 + 2*a*c*d^4*e^4 + 2*b*c*d^6*e^2
))/e^9 + (c^2*x^8)/(8*e) - (d*x^3*((2*a*c + b^2)/e + (d^2*((c^2*d^2)/e^3 +
 (2*b*c)/e))/e^2))/(3*e) - (c^2*d*x^7)/(7*e^2) - (d*x^5*((c^2*d^2)/e^3 + (
2*b*c)/e))/(5*e) - (d*x*((d^2*((2*a*c + b^2)/e + (d^2*((c^2*d^2)/e^3 + (2*
b*c)/e))/e^2))/e^2 + (2*a*b)/e))/e
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 386, normalized size of antiderivative = 1.43

$$\int \frac{(a+bx^2+cx^4)^2}{d+ex}\,dx$$
$$= \frac{420ac\,e^8x^4-840c^2d^7ex+420c^2d^6e^2x^2-280c^2d^5e^3x^3+210c^2d^4e^4x^4-168c^2d^3e^5x^5+140c^2d^2e^6x^6-12}{}$$

input
```
int((c*x^4+b*x^2+a)^2/(e*x+d),x)
```

output
```
(840*log(d + e*x)*a**2*e**8 + 1680*log(d + e*x)*a*b*d**2*e**6 + 1680*log(d
 + e*x)*a*c*d**4*e**4 + 840*log(d + e*x)*b**2*d**4*e**4 + 1680*log(d + e*x
)*b*c*d**6*e**2 + 840*log(d + e*x)*c**2*d**8 - 1680*a*b*d*e**7*x + 840*a*b
*e**8*x**2 - 1680*a*c*d**3*e**5*x + 840*a*c*d**2*e**6*x**2 - 560*a*c*d*e**
7*x**3 + 420*a*c*e**8*x**4 - 840*b**2*d**3*e**5*x + 420*b**2*d**2*e**6*x**
2 - 280*b**2*d*e**7*x**3 + 210*b**2*e**8*x**4 - 1680*b*c*d**5*e**3*x + 840
*b*c*d**4*e**4*x**2 - 560*b*c*d**3*e**5*x**3 + 420*b*c*d**2*e**6*x**4 - 33
6*b*c*d*e**7*x**5 + 280*b*c*e**8*x**6 - 840*c**2*d**7*e*x + 420*c**2*d**6*
e**2*x**2 - 280*c**2*d**5*e**3*x**3 + 210*c**2*d**4*e**4*x**4 - 168*c**2*d
**3*e**5*x**5 + 140*c**2*d**2*e**6*x**6 - 120*c**2*d*e**7*x**7 + 105*c**2*
e**8*x**8)/(840*e**9)
```

3.234 $\int \frac{(a+bx^2+cx^4)^2}{(d+ex)^2} dx$

Optimal result

Integrand size = 22, antiderivative size = 286

$$\begin{aligned}\int \frac{(a+bx^2+cx^4)^2}{(d+ex)^2} dx &= \frac{(7c^2d^6+10bcd^4e^2+3b^2d^2e^4+6acd^2e^4+2abe^6)\,x}{e^8} \\ &\quad - \frac{d(3c^2d^4+4bcd^2e^2+b^2e^4+2ace^4)\,x^2}{e^7} \\ &\quad + \frac{(5c^2d^4+6bcd^2e^2+b^2e^4+2ace^4)\,x^3}{3e^6} - \frac{cd(cd^2+be^2)\,x^4}{e^5} \\ &\quad + \frac{c(3cd^2+2be^2)\,x^5}{5e^4} - \frac{c^2dx^6}{3e^3} + \frac{c^2x^7}{7e^2} - \frac{(cd^4+bd^2e^2+ae^4)^2}{e^9(d+ex)} \\ &\quad - \frac{4d(2cd^2+be^2)\,(cd^4+bd^2e^2+ae^4)\log(d+ex)}{e^9}\end{aligned}$$

output
```
(2*a*b*e^6+6*a*c*d^2*e^4+3*b^2*d^2*e^4+10*b*c*d^4*e^2+7*c^2*d^6)*x/e^8-d*(
2*a*c*e^4+b^2*e^4+4*b*c*d^2*e^2+3*c^2*d^4)*x^2/e^7+1/3*(2*a*c*e^4+b^2*e^4+
6*b*c*d^2*e^2+5*c^2*d^4)*x^3/e^6-c*d*(b*e^2+c*d^2)*x^4/e^5+1/5*c*(2*b*e^2+
3*c*d^2)*x^5/e^4-1/3*c^2*d*x^6/e^3+1/7*c^2*x^7/e^2-(a*e^4+b*d^2*e^2+c*d^4)
^2/e^9/(e*x+d)-4*d*(b*e^2+2*c*d^2)*(a*e^4+b*d^2*e^2+c*d^4)*ln(e*x+d)/e^9
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2+cx^4)^2}{(d+ex)^2}\,dx = \frac{(7c^2d^6+10bcd^4e^2+3b^2d^2e^4+6acd^2e^4+2abe^6)\,x}{e^8} - \frac{d(3c^2d^4+4bcd^2e^2+b^2e^4+2ace^4)\,x^2}{e^7} + \frac{(5c^2d^4+6bcd^2e^2+b^2e^4+2ace^4)\,x^3}{3e^6} - \frac{cd(cd^2+be^2)\,x^4}{e^5} + \frac{c(3cd^2+2be^2)\,x^5}{5e^4} - \frac{c^2dx^6}{3e^3} + \frac{c^2x^7}{7e^2} - \frac{(cd^4+bd^2e^2+ae^4)^2}{e^9(d+ex)} - \frac{4d(2cd^2+be^2)\,(cd^4+bd^2e^2+ae^4)\log(d+ex)}{e^9}$$

input
```
Integrate[(a + b*x^2 + c*x^4)^2/(d + e*x)^2,x]
```

output
```
((7*c^2*d^6 + 10*b*c*d^4*e^2 + 3*b^2*d^2*e^4 + 6*a*c*d^2*e^4 + 2*a*b*e^6)*
x)/e^8 - (d*(3*c^2*d^4 + 4*b*c*d^2*e^2 + b^2*e^4 + 2*a*c*e^4)*x^2)/e^7 + (
(5*c^2*d^4 + 6*b*c*d^2*e^2 + b^2*e^4 + 2*a*c*e^4)*x^3)/(3*e^6) - (c*d*(c*d
^2 + b*e^2)*x^4)/e^5 + (c*(3*c*d^2 + 2*b*e^2)*x^5)/(5*e^4) - (c^2*d*x^6)/(
3*e^3) + (c^2*x^7)/(7*e^2) - (c*d^4 + b*d^2*e^2 + a*e^4)^2/(e^9*(d + e*x))
 - (4*d*(2*c*d^2 + b*e^2)*(c*d^4 + b*d^2*e^2 + a*e^4)*Log[d + e*x])/e^9
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {2389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2+cx^4)^2}{(d+ex)^2}\,dx$$

$$\Big\downarrow \text{ 2389}$$

$$\int \left(-\frac{2dx(2ace^4+b^2e^4+4bcd^2e^2+3c^2d^4)}{e^7}+\frac{x^2(2ace^4+b^2e^4+6bcd^2e^2+5c^2d^4)}{e^6}+\frac{2abe^6+6acd^2e^4+3b^2d^2e}{e^8}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{dx^2(2ace^4+b^2e^4+4bcd^2e^2+3c^2d^4)}{e^7}+\frac{x^3(2ace^4+b^2e^4+6bcd^2e^2+5c^2d^4)}{3e^6}+\frac{x(2abe^6+6acd^2e^4+3b^2d^2e^4+10bcd^4e^2+7c^2d^6)}{e^8}-\frac{(ae^4+bd^2e^2+cd^4)^2}{e^9(d+ex)}-\frac{4d(be^2+2cd^2)\log(d+ex)(ae^4+bd^2e^2+cd^4)}{e^9}-\frac{cdx^4(be^2+cd^2)}{e^5}+\frac{cx^5(2be^2+3cd^2)}{5e^4}-\frac{c^2dx^6}{3e^3}+\frac{c^2x^7}{7e^2}$$

input
```
Int[(a + b*x^2 + c*x^4)^2/(d + e*x)^2,x]
```

output
```
((7*c^2*d^6 + 10*b*c*d^4*e^2 + 3*b^2*d^2*e^4 + 6*a*c*d^2*e^4 + 2*a*b*e^6)*
x)/e^8 - (d*(3*c^2*d^4 + 4*b*c*d^2*e^2 + b^2*e^4 + 2*a*c*e^4)*x^2)/e^7 + (
(5*c^2*d^4 + 6*b*c*d^2*e^2 + b^2*e^4 + 2*a*c*e^4)*x^3)/(3*e^6) - (c*d*(c*d
^2 + b*e^2)*x^4)/e^5 + (c*(3*c*d^2 + 2*b*e^2)*x^5)/(5*e^4) - (c^2*d*x^6)/(
3*e^3) + (c^2*x^7)/(7*e^2) - (c*d^4 + b*d^2*e^2 + a*e^4)^2/(e^9*(d + e*x))
 - (4*d*(2*c*d^2 + b*e^2)*(c*d^4 + b*d^2*e^2 + a*e^4)*Log[d + e*x])/e^9
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2389
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.17

method	result
norman	$\frac{-\frac{a^2e^8+4ab\,d^2e^6+8ac\,d^4e^4+4b^2d^4e^4+12bc\,d^6e^2+8d^8c^2}{e^9}+\frac{c^2x^8}{7e}+\frac{2\left(ab\,e^6+2ac\,d^2e^4+b^2d^2e^4+3bc\,d^4e^2+2c^2d^6\right)x^2}{e^7}+\frac{\left(2ac\,e^4+b^2e^4+3bc\right.}{3e^5}}{ex+d}$
default	$\frac{\frac{1}{7}c^2x^7e^6-\frac{1}{3}c^2d\,x^6e^5+\frac{2}{5}bc\,e^6x^5+\frac{3}{5}c^2d^2e^4x^5-bcd\,e^5x^4-c^2d^3e^3x^4+\frac{2}{3}ac\,e^6x^3+\frac{1}{3}b^2e^6x^3+2bc\,d^2e^4x^3+\frac{5}{3}c^2d^4e^2x^3-2acd\,e^5x^2}{e^8}$
risch	$-\frac{4d\ln(ex+d)ab}{e^3}-\frac{d^8c^2}{e^9(ex+d)}-\frac{4d^3\ln(ex+d)b^2}{e^5}-\frac{8d^7\ln(ex+d)c^2}{e^9}+\frac{2bc\,x^5}{5e^2}+\frac{2ac\,x^3}{3e^2}+\frac{5c^2d^4x^3}{3e^6}-\frac{b^2d\,x^2}{e^3}-\frac{3c}{}$
parallelrisch	$-\frac{-42x^6bc\,e^8+70x^3b^2d\,e^7-210x^2ab\,e^8-210x^2b^2d^2e^6+420\ln(ex+d)b^2d^4e^4+420\ln(ex+d)x\,b^2d^3e^5+420ab\,d^2e^6+1260bc\,d^6}{}$

input
```
int((c*x^4+b*x^2+a)^2/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
(-(a^2*e^8+4*a*b*d^2*e^6+8*a*c*d^4*e^4+4*b^2*d^4*e^4+12*b*c*d^6*e^2+8*c^2*
d^8)/e^9+1/7*c^2*x^8/e+2/e^7*(a*b*e^6+2*a*c*d^2*e^4+b^2*d^2*e^4+3*b*c*d^4*
e^2+2*c^2*d^6)*x^2+1/3*(2*a*c*e^4+b^2*e^4+3*b*c*d^2*e^2+2*c^2*d^4)/e^5*x^4
+2/15*c*(3*b*e^2+2*c*d^2)/e^3*x^6-4/21*c^2*d*x^7/e^2-2/3*d*(2*a*c*e^4+b^2*
e^4+3*b*c*d^2*e^2+2*c^2*d^4)/e^6*x^3-1/5*d*c*(3*b*e^2+2*c*d^2)/e^4*x^5)/(e
*x+d)-4*d/e^9*(a*b*e^6+2*a*c*d^2*e^4+b^2*d^2*e^4+3*b*c*d^4*e^2+2*c^2*d^6)*
ln(e*x+d)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 409, normalized size of antiderivative = 1.43

$$\int \frac{(a+bx^2+cx^4)^2}{(d+ex)^2}\,dx$$
$$=\frac{15\,c^2e^8x^8-20\,c^2de^7x^7-105\,c^2d^8-210\,bcd^6e^2-210\,abd^2e^6-105\,a^2e^8-105\,(b^2+2\,ac)d^4e^4+14\,(2\,c^2d}{}$$

input
```
integrate((c*x^4+b*x^2+a)^2/(e*x+d)^2,x, algorithm="fricas")
```

output
```
1/105*(15*c^2*e^8*x^8 - 20*c^2*d*e^7*x^7 - 105*c^2*d^8 - 210*b*c*d^6*e^2 -
 210*a*b*d^2*e^6 - 105*a^2*e^8 - 105*(b^2 + 2*a*c)*d^4*e^4 + 14*(2*c^2*d^2
*e^6 + 3*b*c*e^8)*x^6 - 21*(2*c^2*d^3*e^5 + 3*b*c*d*e^7)*x^5 + 35*(2*c^2*d
^4*e^4 + 3*b*c*d^2*e^6 + (b^2 + 2*a*c)*e^8)*x^4 - 70*(2*c^2*d^5*e^3 + 3*b*
c*d^3*e^5 + (b^2 + 2*a*c)*d*e^7)*x^3 + 210*(2*c^2*d^6*e^2 + 3*b*c*d^4*e^4
+ a*b*e^8 + (b^2 + 2*a*c)*d^2*e^6)*x^2 + 105*(7*c^2*d^7*e + 10*b*c*d^5*e^3
 + 2*a*b*d*e^7 + 3*(b^2 + 2*a*c)*d^3*e^5)*x - 420*(2*c^2*d^8 + 3*b*c*d^6*e
^2 + a*b*d^2*e^6 + (b^2 + 2*a*c)*d^4*e^4 + (2*c^2*d^7*e + 3*b*c*d^5*e^3 +
a*b*d*e^7 + (b^2 + 2*a*c)*d^3*e^5)*x)*log(e*x + d))/(e^10*x + d*e^9)
```

Sympy [A] (verification not implemented)

Time = 0.75 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.20

$$\int \frac{(a+bx^2+cx^4)^2}{(d+ex)^2}\,dx = -\frac{c^2dx^6}{3e^3}+\frac{c^2x^7}{7e^2} - \frac{4d(be^2+2cd^2)\left(ae^4+bd^2e^2+cd^4\right)\log(d+ex)}{e^9} + x^5\cdot\left(\frac{2bc}{5e^2}+\frac{3c^2d^2}{5e^4}\right)+x^4\left(-\frac{bcd}{e^3}-\frac{c^2d^3}{e^5}\right) + x^3\cdot\left(\frac{2ac}{3e^2}+\frac{b^2}{3e^2}+\frac{2bcd^2}{e^4}+\frac{5c^2d^4}{3e^6}\right) + x^2\left(-\frac{2acd}{e^3}-\frac{b^2d}{e^3}-\frac{4bcd^3}{e^5}-\frac{3c^2d^5}{e^7}\right) + x\left(\frac{2ab}{e^2}+\frac{6acd^2}{e^4}+\frac{3b^2d^2}{e^4}+\frac{10bcd^4}{e^6}+\frac{7c^2d^6}{e^8}\right) + \frac{-a^2e^8-2abd^2e^6-2acd^4e^4-b^2d^4e^4-2bcd^6e^2-c^2d^8}{de^9+e^{10}x}$$

input
```
integrate((c*x**4+b*x**2+a)**2/(e*x+d)**2,x)
```

output
```
-c**2*d*x**6/(3*e**3) + c**2*x**7/(7*e**2) - 4*d*(b*e**2 + 2*c*d**2)*(a*e*
*4 + b*d**2*e**2 + c*d**4)*log(d + e*x)/e**9 + x**5*(2*b*c/(5*e**2) + 3*c*
*2*d**2/(5*e**4)) + x**4*(-b*c*d/e**3 - c**2*d**3/e**5) + x**3*(2*a*c/(3*e
**2) + b**2/(3*e**2) + 2*b*c*d**2/e**4 + 5*c**2*d**4/(3*e**6)) + x**2*(-2*
a*c*d/e**3 - b**2*d/e**3 - 4*b*c*d**3/e**5 - 3*c**2*d**5/e**7) + x*(2*a*b/
e**2 + 6*a*c*d**2/e**4 + 3*b**2*d**2/e**4 + 10*b*c*d**4/e**6 + 7*c**2*d**6
/e**8) + (-a**2*e**8 - 2*a*b*d**2*e**6 - 2*a*c*d**4*e**4 - b**2*d**4*e**4
- 2*b*c*d**6*e**2 - c**2*d**8)/(d*e**9 + e**10*x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.10

$$\int \frac{(a+bx^2+cx^4)^2}{(d+ex)^2}\,dx = -\frac{c^2d^8+2\,bcd^6e^2+2\,abd^2e^6+a^2e^8+(b^2+2\,ac)d^4e^4}{e^{10}x+de^9}$$
$$+\frac{15\,c^2e^6x^7-35\,c^2de^5x^6+21\,(3\,c^2d^2e^4+2\,bce^6)x^5-105\,(c^2d^3e^3+bcde^5)x^4+35\,(5\,c^2d^4e^2+6\,bcd^2e^4+}{}$$
$$-\frac{4\,(2\,c^2d^7+3\,bcd^5e^2+abde^6+(b^2+2\,ac)d^3e^4)\log{(ex+d)}}{e^9}$$

input
```
integrate((c*x^4+b*x^2+a)^2/(e*x+d)^2,x, algorithm="maxima")
```

output
```
-(c^2*d^8 + 2*b*c*d^6*e^2 + 2*a*b*d^2*e^6 + a^2*e^8 + (b^2 + 2*a*c)*d^4*e^
4)/(e^10*x + d*e^9) + 1/105*(15*c^2*e^6*x^7 - 35*c^2*d*e^5*x^6 + 21*(3*c^2
*d^2*e^4 + 2*b*c*e^6)*x^5 - 105*(c^2*d^3*e^3 + b*c*d*e^5)*x^4 + 35*(5*c^2*
d^4*e^2 + 6*b*c*d^2*e^4 + (b^2 + 2*a*c)*e^6)*x^3 - 105*(3*c^2*d^5*e + 4*b*
c*d^3*e^3 + (b^2 + 2*a*c)*d*e^5)*x^2 + 105*(7*c^2*d^6 + 10*b*c*d^4*e^2 + 2
*a*b*e^6 + 3*(b^2 + 2*a*c)*d^2*e^4)*x)/e^8 - 4*(2*c^2*d^7 + 3*b*c*d^5*e^2
+ a*b*d*e^6 + (b^2 + 2*a*c)*d^3*e^4)*log(e*x + d)/e^9
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 428, normalized size of antiderivative = 1.50

$$\int \frac{(a+bx^2+cx^4)^2}{(d+ex)^2}\,dx$$
$$= \frac{\left(15\,c^2 - \frac{140\,c^2 d}{ex+d} + \frac{42\,(14\,c^2d^2e^2+bce^4)}{(ex+d)^2e^2} - \frac{105\,(14\,c^2d^3e^3+3\,bcde^5)}{(ex+d)^3e^3} + \frac{35\,(70\,c^2d^4e^4+30\,bcd^2e^6+b^2e^8+2\,ace^8)}{(ex+d)^4e^4} - \frac{210\,(14\,c^2d^5e^5+10}{(ex}\right.}{105\,e^9}$$
$$+ \frac{4\,(2\,c^2d^7+3\,bcd^5e^2+b^2d^3e^4+2\,acd^3e^4+abde^6)\log\left(\frac{|ex+d|}{(ex+d)^2|e|}\right)}{e^9}$$
$$- \frac{\frac{c^2d^8e^7}{ex+d} + \frac{2\,bcd^6e^9}{ex+d} + \frac{b^2d^4e^{11}}{ex+d} + \frac{2\,acd^4e^{11}}{ex+d} + \frac{2\,abd^2e^{13}}{ex+d} + \frac{a^2e^{15}}{ex+d}}{e^{16}}$$

input
```
integrate((c*x^4+b*x^2+a)^2/(e*x+d)^2,x, algorithm="giac")
```

output
```
1/105*(15*c^2 - 140*c^2*d/(e*x + d) + 42*(14*c^2*d^2*e^2 + b*c*e^4)/((e*x
+ d)^2*e^2) - 105*(14*c^2*d^3*e^3 + 3*b*c*d*e^5)/((e*x + d)^3*e^3) + 35*(7
0*c^2*d^4*e^4 + 30*b*c*d^2*e^6 + b^2*e^8 + 2*a*c*e^8)/((e*x + d)^4*e^4) -
210*(14*c^2*d^5*e^5 + 10*b*c*d^3*e^7 + b^2*d*e^9 + 2*a*c*d*e^9)/((e*x + d)
^5*e^5) + 210*(14*c^2*d^6*e^6 + 15*b*c*d^4*e^8 + 3*b^2*d^2*e^10 + 6*a*c*d^
2*e^10 + a*b*e^12)/((e*x + d)^6*e^6))*(e*x + d)^7/e^9 + 4*(2*c^2*d^7 + 3*b
*c*d^5*e^2 + b^2*d^3*e^4 + 2*a*c*d^3*e^4 + a*b*d*e^6)*log(abs(e*x + d)/((e
*x + d)^2*abs(e)))/e^9 - (c^2*d^8*e^7/(e*x + d) + 2*b*c*d^6*e^9/(e*x + d)
+ b^2*d^4*e^11/(e*x + d) + 2*a*c*d^4*e^11/(e*x + d) + 2*a*b*d^2*e^13/(e*x
+ d) + a^2*e^15/(e*x + d))/e^16
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 690, normalized size of antiderivative = 2.41

$$\int \frac{(a+bx^2+cx^4)^2}{(d+ex)^2}\,dx$$

$$= x^5\left(\frac{3\,c^2\,d^2}{5\,e^4}+\frac{2\,b\,c}{5\,e^2}\right)-x^2\left(\frac{d\left(\frac{b^2+2\,a\,c}{e^2}+\frac{2\,d\left(\frac{2\,d\left(\frac{3\,c^2\,d^2}{e^4}+\frac{2\,b\,c}{e^2}\right)}{e}-\frac{2\,c^2\,d^3}{e^5}\right)}{e}-\frac{d^2\left(\frac{3\,c^2\,d^2}{e^4}+\frac{2\,b\,c}{e^2}\right)}{e^2}\right)}{e}-\frac{d^2\left(\frac{2\,d\left(\frac{3\,c^2\,d^2}{e^4}+\frac{2\,b\,c}{e^2}\right)}{e}-\frac{2\,c^2\,d^3}{e^5}\right)}{2\,e^2}\right)$$

$$+x\left(\frac{2\,d\left(\frac{2\,d\left(\frac{b^2+2\,a\,c}{e^2}+\frac{2\,d\left(\frac{2\,d\left(\frac{3\,c^2\,d^2}{e^4}+\frac{2\,b\,c}{e^2}\right)}{e}-\frac{2\,c^2\,d^3}{e^5}\right)}{e}-\frac{d^2\left(\frac{3\,c^2\,d^2}{e^4}+\frac{2\,b\,c}{e^2}\right)}{e^2}\right)}{e}-\frac{d^2\left(\frac{2\,d\left(\frac{3\,c^2\,d^2}{e^4}+\frac{2\,b\,c}{e^2}\right)}{e}-\frac{2\,c^2\,d^3}{e^5}\right)}{e^2}\right)}{e}\right.$$

$$\left.-\frac{d^2\left(\frac{b^2+2\,a\,c}{e^2}+\frac{2\,d\left(\frac{2\,d\left(\frac{3\,c^2\,d^2}{e^4}+\frac{2\,b\,c}{e^2}\right)}{e}-\frac{2\,c^2\,d^3}{e^5}\right)}{e}-\frac{d^2\left(\frac{3\,c^2\,d^2}{e^4}+\frac{2\,b\,c}{e^2}\right)}{e^2}\right)}{e^2}+\frac{2\,a\,b}{e^2}\right.$$

input
```
int((a + b*x^2 + c*x^4)^2/(d + e*x)^2,x)
```

output
```
x^5*((3*c^2*d^2)/(5*e^4) + (2*b*c)/(5*e^2)) - x^2*((d*((2*a*c + b^2)/e^2 +
 (2*d*((2*d*((3*c^2*d^2)/e^4 + (2*b*c)/e^2))/e - (2*c^2*d^3)/e^5))/e - (d^
2*((3*c^2*d^2)/e^4 + (2*b*c)/e^2))/e^2))/e - (d^2*((2*d*((3*c^2*d^2)/e^4 +
 (2*b*c)/e^2))/e - (2*c^2*d^3)/e^5))/(2*e^2)) + x*((2*d*((2*d*((2*a*c + b^
2)/e^2 + (2*d*((2*d*((3*c^2*d^2)/e^4 + (2*b*c)/e^2))/e - (2*c^2*d^3)/e^5))
/e - (d^2*((3*c^2*d^2)/e^4 + (2*b*c)/e^2))/e^2))/e - (d^2*((2*d*((3*c^2*d^
2)/e^4 + (2*b*c)/e^2))/e - (2*c^2*d^3)/e^5))/e^2))/e - (d^2*((2*a*c + b^2)
/e^2 + (2*d*((2*d*((3*c^2*d^2)/e^4 + (2*b*c)/e^2))/e - (2*c^2*d^3)/e^5))/e
 - (d^2*((3*c^2*d^2)/e^4 + (2*b*c)/e^2))/e^2))/e^2 + (2*a*b)/e^2) - x^4*((
d*((3*c^2*d^2)/e^4 + (2*b*c)/e^2))/(2*e) - (c^2*d^3)/(2*e^5)) + x^3*((2*a*
c + b^2)/(3*e^2) + (2*d*((2*d*((3*c^2*d^2)/e^4 + (2*b*c)/e^2))/e - (2*c^2*
d^3)/e^5))/(3*e) - (d^2*((3*c^2*d^2)/e^4 + (2*b*c)/e^2))/(3*e^2)) - (log(d
 + e*x)*(8*c^2*d^7 + 4*b^2*d^3*e^4 + 4*a*b*d*e^6 + 8*a*c*d^3*e^4 + 12*b*c*
d^5*e^2))/e^9 + (c^2*x^7)/(7*e^2) - (a^2*e^8 + c^2*d^8 + b^2*d^4*e^4 + 2*a
*b*d^2*e^6 + 2*a*c*d^4*e^4 + 2*b*c*d^6*e^2)/(e*(d*e^8 + e^9*x)) - (c^2*d*x
^6)/(3*e^3)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 491, normalized size of antiderivative = 1.72

$$\int \frac{(a+bx^2+cx^4)^2}{(d+ex)^2}\,dx$$
$$= \frac{-420\log(ex+d)\,b^2d^5e^4+420b^2d^4e^5x+210b^2d^3e^6x^2-70b^2d^2e^7x^3+35b^2d\,e^8x^4+105bc\,d^3e^6x^4-63bc\,d}{}$$

input
```
int((c*x^4+b*x^2+a)^2/(e*x+d)^2,x)
```

output
```
( - 420*log(d + e*x)*a*b*d**3*e**6 - 420*log(d + e*x)*a*b*d**2*e**7*x - 84
0*log(d + e*x)*a*c*d**5*e**4 - 840*log(d + e*x)*a*c*d**4*e**5*x - 420*log(
d + e*x)*b**2*d**5*e**4 - 420*log(d + e*x)*b**2*d**4*e**5*x - 1260*log(d +
 e*x)*b*c*d**7*e**2 - 1260*log(d + e*x)*b*c*d**6*e**3*x - 840*log(d + e*x)
*c**2*d**9 - 840*log(d + e*x)*c**2*d**8*e*x + 105*a**2*e**9*x + 420*a*b*d*
*2*e**7*x + 210*a*b*d*e**8*x**2 + 840*a*c*d**4*e**5*x + 420*a*c*d**3*e**6*
x**2 - 140*a*c*d**2*e**7*x**3 + 70*a*c*d*e**8*x**4 + 420*b**2*d**4*e**5*x
+ 210*b**2*d**3*e**6*x**2 - 70*b**2*d**2*e**7*x**3 + 35*b**2*d*e**8*x**4 +
 1260*b*c*d**6*e**3*x + 630*b*c*d**5*e**4*x**2 - 210*b*c*d**4*e**5*x**3 +
105*b*c*d**3*e**6*x**4 - 63*b*c*d**2*e**7*x**5 + 42*b*c*d*e**8*x**6 + 840*
c**2*d**8*e*x + 420*c**2*d**7*e**2*x**2 - 140*c**2*d**6*e**3*x**3 + 70*c**
2*d**5*e**4*x**4 - 42*c**2*d**4*e**5*x**5 + 28*c**2*d**3*e**6*x**6 - 20*c*
*2*d**2*e**7*x**7 + 15*c**2*d*e**8*x**8)/(105*d*e**9*(d + e*x))
```

3.235 $\int \frac{(d+ex)^3}{a+bx^2+cx^4} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 270

$$\int \frac{(d+ex)^3}{a+bx^2+cx^4} \, dx = \frac{d\left(3e^2+\frac{2cd^2-3be^2}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{d\left(3e^2-\frac{2cd^2-3be^2}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b+\sqrt{b^2-4ac}}} - \frac{e(6cd^2-be^2)\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c\sqrt{b^2-4ac}} + \frac{e^3\log(a+bx^2+cx^4)}{4c}$$

output
```
1/2*d*(3*e^2+(-3*b*e^2+2*c*d^2)/(-4*a*c+b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)
*x/(b-(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1
/2)+1/2*d*(3*e^2-(-3*b*e^2+2*c*d^2)/(-4*a*c+b^2)^(1/2))*arctan(2^(1/2)*c^(
1/2)*x/(b+(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/c^(1/2)/(b+(-4*a*c+b^2)^(1/2)
)^(1/2)-1/2*e*(-b*e^2+6*c*d^2)*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/c/(
-4*a*c+b^2)^(1/2)+1/4*e^3*ln(c*x^4+b*x^2+a)/c
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.12

$$\int \frac{(d+ex)^3}{a+bx^2+cx^4}\,dx = \frac{\frac{2\sqrt{2}\sqrt{c}d\left(2cd^2+3\left(-b+\sqrt{b^2-4ac}\right)e^2\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}}+\frac{2\sqrt{2}\sqrt{c}\left(-2cd^3+3\left(b+\sqrt{b^2-4ac}\right)de^2\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b+\sqrt{b^2-4ac}}}+(6cd^2e}{}$$

input
```
Integrate[(d + e*x)^3/(a + b*x^2 + c*x^4),x]
```

output
```
((2*Sqrt[2]*Sqrt[c]*d*(2*c*d^2 + 3*(-b + Sqrt[b^2 - 4*a*c])*e^2)*ArcTan[(S
qrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/Sqrt[b - Sqrt[b^2 - 4*a*c]
] + (2*Sqrt[2]*Sqrt[c]*(-2*c*d^3 + 3*(b + Sqrt[b^2 - 4*a*c])*d*e^2)*ArcTan
[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/Sqrt[b + Sqrt[b^2 - 4*a
*c]] + (6*c*d^2*e + (-b + Sqrt[b^2 - 4*a*c])*e^3)*Log[-b + Sqrt[b^2 - 4*a*
c] - 2*c*x^2] + (-6*c*d^2*e + (b + Sqrt[b^2 - 4*a*c])*e^3)*Log[b + Sqrt[b^
2 - 4*a*c] + 2*c*x^2])/(4*c*Sqrt[b^2 - 4*a*c])
```

Rubi [A] (verified)

Time = 1.04 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.01, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {2202, 1480, 218, 1576, 27, 1142, 1083, 219, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^3}{a+bx^2+cx^4}\,dx$$

$$\Big\downarrow \text{2202}$$

$$\int \frac{d^3+3e^2x^2d}{cx^4+bx^2+a}dx+\int \frac{x\left(x^2e^3+3d^2e\right)}{cx^4+bx^2+a}dx$$

$$\Big\downarrow \text{1480}$$

$$\frac{1}{2}d\left(\frac{2cd^2-3be^2}{\sqrt{b^2-4ac}}+3e^2\right)\int\frac{1}{cx^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx+\frac{1}{2}d\left(3e^2-\frac{2cd^2-3be^2}{\sqrt{b^2-4ac}}\right)\int\frac{1}{cx^2+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}dx+\int\frac{x\left(x^2e^3+3d^2e\right)}{cx^4+bx^2+a}dx$$

$$\Big\downarrow \text{ 218}$$

$$\int\frac{x\left(x^2e^3+3d^2e\right)}{cx^4+bx^2+a}dx+\frac{d\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{2cd^2-3be^2}{\sqrt{b^2-4ac}}+3e^2\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{d\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(3e^2-\frac{2cd^2-3be^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}$$

$$\Big\downarrow \text{ 1576}$$

$$\frac{1}{2}\int\frac{e\left(3d^2+e^2x^2\right)}{cx^4+bx^2+a}dx^2+\frac{d\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{2cd^2-3be^2}{\sqrt{b^2-4ac}}+3e^2\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{d\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(3e^2-\frac{2cd^2-3be^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{2}e\int\frac{3d^2+e^2x^2}{cx^4+bx^2+a}dx^2+\frac{d\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{2cd^2-3be^2}{\sqrt{b^2-4ac}}+3e^2\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{d\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(3e^2-\frac{2cd^2-3be^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}$$

$$\Big\downarrow \text{ 1142}$$

$$\frac{1}{2}e\left(\frac{\left(6cd^2-be^2\right)\int\frac{1}{cx^4+bx^2+a}dx^2}{2c}+\frac{e^2\int\frac{2cx^2+b}{cx^4+bx^2+a}dx^2}{2c}\right)+\frac{d\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{2cd^2-3be^2}{\sqrt{b^2-4ac}}+3e^2\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{d\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(3e^2-\frac{2cd^2-3be^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}$$

$$\Big\downarrow \text{ 1083}$$

$$\frac{1}{2}e\left(\frac{e^2\int\frac{2cx^2+b}{cx^4+bx^2+a}dx^2}{2c}-\frac{\left(6cd^2-be^2\right)\int\frac{1}{-x^4+b^2-4ac}d\left(2cx^2+b\right)}{c}\right)+\frac{d\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{2cd^2-3be^2}{\sqrt{b^2-4ac}}+3e^2\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{d\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(3e^2-\frac{2cd^2-3be^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}e\left(\frac{e^2\int \frac{2cx^2+b}{cx^4+bx^2+a}dx^2}{2c}-\frac{(6cd^2-be^2)\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}}\right)+\frac{d\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{2cd^2-3be^2}{\sqrt{b^2-4ac}}+3e^2\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{d\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(3e^2-\frac{2cd^2-3be^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}$$

$$\Big\downarrow \text{1103}$$

$$\frac{d\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{2cd^2-3be^2}{\sqrt{b^2-4ac}}+3e^2\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{d\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(3e^2-\frac{2cd^2-3be^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{1}{2}e\left(\frac{e^2\log\left(a+bx^2+cx^4\right)}{2c}-\frac{(6cd^2-be^2)\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}}\right)$$

input
```
Int[(d + e*x)^3/(a + b*x^2 + c*x^4),x]
```

output
```
(d*(3*e^2 + (2*c*d^2 - 3*b*e^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]
*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a
*c]]) + (d*(3*e^2 - (2*c*d^2 - 3*b*e^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]
*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b + Sqrt[b
^2 - 4*a*c]]) + (e*(-(((6*c*d^2 - b*e^2)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 -
4*a*c]])/(c*Sqrt[b^2 - 4*a*c])) + (e^2*Log[a + b*x^2 + c*x^4])/(2*c)))/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1576
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x]
, x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

rule 2202
```
Int[(Pn_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{n
 = Expon[Pn, x], k}, Int[Sum[Coeff[Pn, x, 2*k]*x^(2*k), {k, 0, n/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pn, x, 2*k + 1]*x^(2*k), {k, 0, (n -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pn, x]
 &&  !PolyQ[Pn, x^2]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.15 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.24

method	result
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+\_Z^2b+a\right)}\frac{\left(\_R^3e^3+3\_R^2d\,e^2+3\_R\,d^2e+d^3\right)\ln\left(x-\_R\right)}{2\_R^3c+b\_R}\right)}{2}$
default	$4c\left(-\frac{\sqrt{-4ac+b^2}\left(-\frac{\left(-\sqrt{-4ac+b^2}\,e^3+b\,e^3-6d^2ec\right)\ln\left(-2c\,x^2+\sqrt{-4ac+b^2}-b\right)}{4c}+\frac{\left(-3\sqrt{-4ac+b^2}\,d\,e^2+3bd\,e^2-2c\,d^3\right)\sqrt{2}\,\mathrm{arctanh}\left(\frac{\cdots}{\sqrt{(-b\cdots}}\right.}{2\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\cdots\right.}{4(4ac-b^2)c}\cdots\right.$

input
```
int((e*x+d)^3/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*sum((_R^3*e^3+3*_R^2*d*e^2+3*_R*d^2*e+d^3)/(2*_R^3*c+_R*b)*ln(x-_R),_R
=RootOf(_Z^4*c+_Z^2*b+a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(d+ex)^3}{a+bx^2+cx^4}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)^3/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^3}{a+bx^2+cx^4}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)**3/(c*x**4+b*x**2+a),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(d+ex)^3}{a+bx^2+cx^4}\,dx = \int \frac{(ex+d)^3}{cx^4+bx^2+a}\,dx$$

input
```
integrate((e*x+d)^3/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
integrate((e*x + d)^3/(c*x^4 + b*x^2 + a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3317 vs. $2(228) = 456$.

Time = 0.99 (sec) , antiderivative size = 3317, normalized size of antiderivative = 12.29

$$\int \frac{(d+ex)^3}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^3/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
1/4*e^3*log(abs(c*x^4 + b*x^2 + a))/c + 1/8*(3*(2*b^4*c^2 - 16*a*b^2*c^3 +
 32*a^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^
4 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c +
2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c - 16*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^2 - 8*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^2 + 4*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^3 - 2*(b^2 - 4*a*c)*b^2*c^2 +
 8*(b^2 - 4*a*c)*a*c^3)*c^2*d*e^2 + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*b^4*c^2 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - 2*sqr
t(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^3 - 2*b^4*c^3 + 16*sqrt(2)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^4 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a*b*c^4 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^4 + 16*a*b^2
*c^4 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^5 - 32*a^2*c^5 + 2*(b
^2 - 4*a*c)*b^2*c^3 - 8*(b^2 - 4*a*c)*a*c^4)*d^3*abs(c) + 2*(2*b^3*c^5 - 8
*a*b*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c
^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^4 +
 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^4 - sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^5 - 2*(b^2 - 4*
a*c)*b*c^5)*d^3 - 3*(2*b^4*c^4 - 8*a*b^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c...
```

Mupad [B] (verification not implemented)

Time = 23.06 (sec) , antiderivative size = 9076, normalized size of antiderivative = 33.61

$$\int \frac{(d+ex)^3}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
int((d + e*x)^3/(a + b*x^2 + c*x^4),x)
```

output
```
symsum(log(b^2*d^3*e^6 + 6*c^2*d^7*e^2 - 8*root(128*a^2*b^2*c^3*z^4 - 16*a
*b^4*c^2*z^4 - 256*a^3*c^4*z^4 + 16*a*b^4*c*e^3*z^3 - 128*a^2*b^2*c^2*e^3*
z^3 + 256*a^3*c^3*e^3*z^3 + 240*a^2*b*c^2*d^2*e^4*z^2 + 120*a*b^2*c^2*d^4*
e^2*z^2 - 60*a*b^3*c*d^2*e^4*z^2 + 40*a^2*b^2*c*e^6*z^2 + 16*a*b*c^3*d^6*z
^2 - 480*a^2*c^3*d^4*e^2*z^2 - 96*a^3*c^2*e^6*z^2 - 4*b^3*c^2*d^6*z^2 - 4*
a*b^4*e^6*z^2 - 48*a^2*b*c*d^2*e^7*z + 48*a*b^2*c*d^4*e^5*z - 16*a*b*c^2*d
^6*e^3*z - 192*a^2*c^2*d^4*e^5*z - 12*b^2*c^2*d^8*e*z + 4*b^3*c*d^6*e^3*z
+ 12*a*b^3*d^2*e^7*z + 48*a*c^3*d^8*e*z + 16*a^3*c*e^9*z - 4*a^2*b^2*e^9*z
 - 6*a*b*c*d^6*e^6 - 3*b^2*c*d^8*e^4 - 3*b*c^2*d^10*e^2 - 3*a^2*c*d^4*e^8
- 3*a*c^2*d^8*e^4 - 3*a^2*b*d^2*e^10 - 3*a*b^2*d^4*e^8 - b^3*d^6*e^6 - c^3
*d^12 - a^3*e^12, z, k)^3*b^3*c^2*x + 3*b^2*d^2*e^7*x + 10*c^2*d^6*e^3*x -
 3*a*b*d*e^8 + 4*root(128*a^2*b^2*c^3*z^4 - 16*a*b^4*c^2*z^4 - 256*a^3*c^4
*z^4 + 16*a*b^4*c*e^3*z^3 - 128*a^2*b^2*c^2*e^3*z^3 + 256*a^3*c^3*e^3*z^3
+ 240*a^2*b*c^2*d^2*e^4*z^2 + 120*a*b^2*c^2*d^4*e^2*z^2 - 60*a*b^3*c*d^2*e
^4*z^2 + 40*a^2*b^2*c*e^6*z^2 + 16*a*b*c^3*d^6*z^2 - 480*a^2*c^3*d^4*e^2*z
^2 - 96*a^3*c^2*e^6*z^2 - 4*b^3*c^2*d^6*z^2 - 4*a*b^4*e^6*z^2 - 48*a^2*b*c
*d^2*e^7*z + 48*a*b^2*c*d^4*e^5*z - 16*a*b*c^2*d^6*e^3*z - 192*a^2*c^2*d^4
*e^5*z - 12*b^2*c^2*d^8*e*z + 4*b^3*c*d^6*e^3*z + 12*a*b^3*d^2*e^7*z + 48*
a*c^3*d^8*e*z + 16*a^3*c*e^9*z - 4*a^2*b^2*e^9*z - 6*a*b*c*d^6*e^6 - 3*b^2
*c*d^8*e^4 - 3*b*c^2*d^10*e^2 - 3*a^2*c*d^4*e^8 - 3*a*c^2*d^8*e^4 - 3*a...
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 1120, normalized size of antiderivative = 4.15

$$\int \frac{(d+ex)^3}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
int((e*x+d)^3/(c*x^4+b*x^2+a),x)
```

output

```
(2*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sq
rt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b*e**3 -
12*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sq
rt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*c*d**2*e
- 12*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b)
 - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*c*d*e**2 + 2*sqrt(a)*sqrt(2
*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqr
t(2*sqrt(c)*sqrt(a) + b))*b*c*d**3 + 6*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) + b)
*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) +
 b))*a*b*d*e**2 - 4*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(
c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*c*d**3 + 2*s
qrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c
)*sqrt(a) - b) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b*e**3 - 12*s
qrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c
)*sqrt(a) - b) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*c*d**2*e + 12
*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) + 2
*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*c*d*e**2 - 2*sqrt(a)*sqrt(2*sqr
t(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) + 2*sqrt(c)*x)/sqrt(2*
sqrt(c)*sqrt(a) + b))*b*c*d**3 - 6*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) + b)*ata
n((sqrt(2*sqrt(c)*sqrt(a) - b) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + ...
```

3.236 $\int \frac{(d+ex)^2}{a+bx^2+cx^4} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 224

$$\int \frac{(d+ex)^2}{a+bx^2+cx^4} \, dx = \frac{\left(e^2+\frac{2cd^2-be^2}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(e^2-\frac{2cd^2-be^2}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b+\sqrt{b^2-4ac}}} - \frac{2de\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

output
```
1/2*(e^2+(-b*e^2+2*c*d^2)/(-4*a*c+b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)*x/(b-
(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/
2*(e^2-(-b*e^2+2*c*d^2)/(-4*a*c+b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)*x/(b+(-
4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)-2*d*
e*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.09

$$\int \frac{(d+ex)^2}{a+bx^2+cx^4}\,dx = \frac{\frac{\sqrt{2}\left(2cd^2+\left(-b+\sqrt{b^2-4ac}\right)e^2\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\left(-2cd^2+\left(b+\sqrt{b^2-4ac}\right)e^2\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{c}\sqrt{b+\sqrt{b^2-4ac}}}+2de\log\left(-b+\sqrt{\right.}{2\sqrt{b^2-4ac}}$$

input
```
Integrate[(d + e*x)^2/(a + b*x^2 + c*x^4),x]
```

output
```
((Sqrt[2]*(2*c*d^2 + (-b + Sqrt[b^2 - 4*a*c])*e^2)*ArcTan[(Sqrt[2]*Sqrt[c]
*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) +
(Sqrt[2]*(-2*c*d^2 + (b + Sqrt[b^2 - 4*a*c])*e^2)*ArcTan[(Sqrt[2]*Sqrt[c]*
x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + 2
*d*e*Log[-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2] - 2*d*e*Log[b + Sqrt[b^2 - 4*a*
c] + 2*c*x^2])/(2*Sqrt[b^2 - 4*a*c])
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {2202, 27, 1432, 1083, 219, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^2}{a+bx^2+cx^4}\,dx$$

$$\Big\downarrow \text{ 2202}$$

$$\int \frac{d^2+e^2x^2}{cx^4+bx^2+a}dx+\int \frac{2dex}{cx^4+bx^2+a}dx$$

$$\Big\downarrow \text{ 27}$$

$$\int \frac{d^2+e^2x^2}{cx^4+bx^2+a}dx+2de\int \frac{x}{cx^4+bx^2+a}dx$$

$$\Big\downarrow \text{1432}$$

$$\int \frac{d^2+e^2x^2}{cx^4+bx^2+a}dx+de\int \frac{1}{cx^4+bx^2+a}dx^2$$

$$\Big\downarrow \text{1083}$$

$$\int \frac{d^2+e^2x^2}{cx^4+bx^2+a}dx-2de\int \frac{1}{-x^4+b^2-4ac}d(2cx^2+b)$$

$$\Big\downarrow \text{219}$$

$$\int \frac{d^2+e^2x^2}{cx^4+bx^2+a}dx-\frac{2de\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

$$\Big\downarrow \text{1480}$$

$$\frac{1}{2}\left(\frac{2cd^2-be^2}{\sqrt{b^2-4ac}}+e^2\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx+\frac{1}{2}\left(e^2-\frac{2cd^2-be^2}{\sqrt{b^2-4ac}}\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}dx-\frac{2de\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

$$\Big\downarrow \text{218}$$

$$\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{2cd^2-be^2}{\sqrt{b^2-4ac}}+e^2\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(e^2-\frac{2cd^2-be^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}-\frac{2de\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

input
```
Int[(d + e*x)^2/(a + b*x^2 + c*x^4),x]
```

output
```
((e^2 + (2*c*d^2 - b*e^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sq
rt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]])
+ ((e^2 - (2*c*d^2 - b*e^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/
Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]
) - (2*d*e*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/Sqrt[b^2 - 4*a*c]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1432
```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 2202
```
Int[(Pn_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{n
 = Expon[Pn, x], k}, Int[Sum[Coeff[Pn, x, 2*k]*x^(2*k), {k, 0, n/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pn, x, 2*k + 1]*x^(2*k), {k, 0, (n -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pn, x]
 &&  !PolyQ[Pn, x^2]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.21 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.24

method	result
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+\_Z^2b+a\right)}\frac{\left(\_R^2e^2+2\_Rde+d^2\right)\ln\left(x-\_R\right)}{2\_R^3c+b\_R}\right)}{2}$
default	$4c\left(-\frac{\sqrt{-4ac+b^2}\left(de\ln\left(-2c\,x^2+\sqrt{-4ac+b^2}-b\right)+\frac{\left(-\sqrt{-4ac+b^2}\,e^2+b\,e^2-2c\,d^2\right)\sqrt{2}\,\mathrm{arctanh}\left(\frac{cx\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{4(4ac-b^2)c}-\frac{\sqrt{-4ac}\ldots}{\ldots}\right.$

input
```
int((e*x+d)^2/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*sum((_R^2*e^2+2*_R*d*e+d^2)/(2*_R^3*c+_R*b)*ln(x-_R),_R=RootOf(_Z^4*c+
_Z^2*b+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 83.15 (sec) , antiderivative size = 540080, normalized size of antiderivative = 2411.07

$$\int \frac{(d+ex)^2}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^2/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^2}{a+bx^2+cx^4}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)**2/(c*x**4+b*x**2+a),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(d+ex)^2}{a+bx^2+cx^4}\,dx = \int \frac{(ex+d)^2}{cx^4+bx^2+a}\,dx$$

input
```
integrate((e*x+d)^2/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
integrate((e*x + d)^2/(c*x^4 + b*x^2 + a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1723 vs. $2(186) = 372$.

Time = 0.93 (sec) , antiderivative size = 1723, normalized size of antiderivative = 7.69

$$\int \frac{(d+ex)^2}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^2/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
(b^2*c^2 - 4*a*c^3 - 2*b*c^3 + c^4)*sqrt(b^2 - 4*a*c)*d*e*log(x^2 + 1/2*(b
 - sqrt(b^2 - 4*a*c))/c)/((b^4 - 8*a*b^2*c - 2*b^3*c + 16*a^2*c^2 + 8*a*b*
c^2 + b^2*c^2 - 4*a*c^3)*c^2) + 1/2*(b^5*c - 8*a*b^3*c^2 - 2*b^4*c^2 + 16*
a^2*b*c^3 + 8*a*b^2*c^3 + b^3*c^3 - 4*a*b*c^4 + (b^4*c - 6*a*b^2*c^2 - 2*b
^3*c^2 + 8*a^2*c^3 + 4*a*b*c^3 + b^2*c^3 - 2*a*c^4)*sqrt(b^2 - 4*a*c))*d*e
*log(x^2 + 1/2*(b + sqrt(b^2 - 4*a*c))/c)/((a*b^4 - 8*a^2*b^2*c - 2*a*b^3*
c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c^3)*c^2) + 1/4*((sqrt(2)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a*b^2*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c - 2*b^4*c
 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^2 + 8*sqrt(2)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
b^2*c^2 + 16*a*b^2*c^2 + 2*b^3*c^2 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a*c^3 - 32*a^2*c^3 - 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
b*c^2 + 2*(b^2 - 4*a*c)*b^2*c - 8*(b^2 - 4*a*c)*a*c^2 - 2*(b^2 - 4*a*c)*b*
c^2)*d^2 - 2*(2*a*b^2*c^2 - 8*a^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a*b^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b...
```

Mupad [B] (verification not implemented)

Time = 22.54 (sec) , antiderivative size = 3046, normalized size of antiderivative = 13.60

$$\int \frac{(d+ex)^2}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
int((d + e*x)^2/(a + b*x^2 + c*x^4),x)
```

output
```
symsum(log(3*c^2*d^4*e^2 - a*c*e^6 - 8*root(16*a*b^4*c*z^4 - 128*a^2*b^2*c
^2*z^4 + 256*a^3*c^3*z^4 - 48*a*b^2*c*d^2*e^2*z^2 - 16*a^2*b*c*e^4*z^2 - 1
6*a*b*c^2*d^4*z^2 + 192*a^2*c^2*d^2*e^2*z^2 + 4*b^3*c*d^4*z^2 + 4*a*b^3*e^
4*z^2 + 8*b^2*c*d^5*e*z + 32*a^2*c*d*e^5*z - 32*a*c^2*d^5*e*z - 8*a*b^2*d*
e^5*z + 2*b*c*d^6*e^2 + 2*a*c*d^4*e^4 + 2*a*b*d^2*e^6 + b^2*d^4*e^4 + c^2*
d^8 + a^2*e^8, z, k)^3*b^3*c^2*x + 4*c^2*d^3*e^3*x + 4*root(16*a*b^4*c*z^4
 - 128*a^2*b^2*c^2*z^4 + 256*a^3*c^3*z^4 - 48*a*b^2*c*d^2*e^2*z^2 - 16*a^2
*b*c*e^4*z^2 - 16*a*b*c^2*d^4*z^2 + 192*a^2*c^2*d^2*e^2*z^2 + 4*b^3*c*d^4*
z^2 + 4*a*b^3*e^4*z^2 + 8*b^2*c*d^5*e*z + 32*a^2*c*d*e^5*z - 32*a*c^2*d^5*
e*z - 8*a*b^2*d*e^5*z + 2*b*c*d^6*e^2 + 2*a*c*d^4*e^4 + 2*a*b*d^2*e^6 + b^
2*d^4*e^4 + c^2*d^8 + a^2*e^8, z, k)^2*b^2*c^2*d^2 + b*c*d^2*e^4 - 4*root(
16*a*b^4*c*z^4 - 128*a^2*b^2*c^2*z^4 + 256*a^3*c^3*z^4 - 48*a*b^2*c*d^2*e^
2*z^2 - 16*a^2*b*c*e^4*z^2 - 16*a*b*c^2*d^4*z^2 + 192*a^2*c^2*d^2*e^2*z^2
+ 4*b^3*c*d^4*z^2 + 4*a*b^3*e^4*z^2 + 8*b^2*c*d^5*e*z + 32*a^2*c*d*e^5*z -
 32*a*c^2*d^5*e*z - 8*a*b^2*d*e^5*z + 2*b*c*d^6*e^2 + 2*a*c*d^4*e^4 + 2*a*
b*d^2*e^6 + b^2*d^4*e^4 + c^2*d^8 + a^2*e^8, z, k)*c^3*d^4*x - 16*root(16*
a*b^4*c*z^4 - 128*a^2*b^2*c^2*z^4 + 256*a^3*c^3*z^4 - 48*a*b^2*c*d^2*e^2*z
^2 - 16*a^2*b*c*e^4*z^2 - 16*a*b*c^2*d^4*z^2 + 192*a^2*c^2*d^2*e^2*z^2 + 4
*b^3*c*d^4*z^2 + 4*a*b^3*e^4*z^2 + 8*b^2*c*d^5*e*z + 32*a^2*c*d*e^5*z - 32
*a*c^2*d^5*e*z - 8*a*b^2*d*e^5*z + 2*b*c*d^6*e^2 + 2*a*c*d^4*e^4 + 2*a*...
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 863, normalized size of antiderivative = 3.85

$$\int \frac{(d+ex)^2}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
int((e*x+d)^2/(c*x^4+b*x^2+a),x)
```

output
```
( - 8*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2
*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*c*d*e
- 4*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b)
- 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*c*e**2 + 2*sqrt(a)*sqrt(2*sq
rt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2
*sqrt(c)*sqrt(a) + b))*b*c*d**2 + 2*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) + b)*at
an((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b)
)*a*b*e**2 - 4*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sq
rt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*c*d**2 - 8*sqrt(2
*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqr
t(a) - b) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*c*d*e + 4*sqrt(a)*
sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) + 2*sqrt(c)*
x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*c*e**2 - 2*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a
) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqr
t(a) + b))*b*c*d**2 - 2*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*s
qrt(c)*sqrt(a) - b) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b*e**2 +
 4*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) +
 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*c*d**2 + 2*sqrt(a)*sqrt(2*sqr
t(c)*sqrt(a) - b)*log( - sqrt(2*sqrt(c)*sqrt(a) - b)*x + sqrt(a) + sqrt(c)
*x**2)*a*c*e**2 - sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) - b)*log( - sqrt(2*sqr...
```

3.237 $\int \frac{d+ex}{a+bx^2+cx^4}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 189

$$\int \frac{d+ex}{a+bx^2+cx^4}\,dx = \frac{\sqrt{2}\sqrt{c}d\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{2}\sqrt{c}d\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}} - \frac{e\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

output
```
2^(1/2)*c^(1/2)*d*arctan(2^(1/2)*c^(1/2)*x/(b-(-4*a*c+b^2)^(1/2))^(1/2))/(
-4*a*c+b^2)^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-2^(1/2)*c^(1/2)*d*arctan(2^
(1/2)*c^(1/2)*x/(b+(-4*a*c+b^2)^(1/2))^(1/2))/(-4*a*c+b^2)^(1/2)/(b+(-4*a*
c+b^2)^(1/2))^(1/2)-e*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)
^(1/2)
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.03

$$\int \frac{d+ex}{a+bx^2+cx^4}\,dx$$
$$=\frac{\frac{2\sqrt{2}\sqrt{c}d\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}}-\frac{2\sqrt{2}\sqrt{c}d\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b+\sqrt{b^2-4ac}}}+e\left(\log\left(-b+\sqrt{b^2-4ac}-2cx^2\right)-\log\left(b+\sqrt{b^2-\right.\right.}{2\sqrt{b^2-4ac}}$$

input
```
Integrate[(d + e*x)/(a + b*x^2 + c*x^4),x]
```

output
```
((2*Sqrt[2]*Sqrt[c]*d*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c
]]])/Sqrt[b - Sqrt[b^2 - 4*a*c]] - (2*Sqrt[2]*Sqrt[c]*d*ArcTan[(Sqrt[2]*Sq
rt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/Sqrt[b + Sqrt[b^2 - 4*a*c]] + e*(Lo
g[-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2] - Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2]
))/(2*Sqrt[b^2 - 4*a*c])
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {2202, 27, 1406, 218, 1432, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex}{a+bx^2+cx^4}\,dx$$
$$\Big\downarrow \text{2202}$$
$$\int \frac{d}{cx^4+bx^2+a}dx+\int \frac{ex}{cx^4+bx^2+a}dx$$
$$\Big\downarrow \text{27}$$
$$d\int \frac{1}{cx^4+bx^2+a}dx+e\int \frac{x}{cx^4+bx^2+a}dx$$

$$\Big\downarrow \text{1406}$$

$$d\left(\frac{c\int \frac{1}{cx^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx}{\sqrt{b^2-4ac}}-\frac{c\int \frac{1}{cx^2+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}dx}{\sqrt{b^2-4ac}}\right)+e\int \frac{x}{cx^4+bx^2+a}dx$$

$$\Big\downarrow \text{218}$$

$$e\int \frac{x}{cx^4+bx^2+a}dx+d\left(\frac{\sqrt{2}\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2}\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}\right)$$

$$\Big\downarrow \text{1432}$$

$$\frac{1}{2}e\int \frac{1}{cx^4+bx^2+a}dx^2+d\left(\frac{\sqrt{2}\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2}\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}\right)$$

$$\Big\downarrow \text{1083}$$

$$d\left(\frac{\sqrt{2}\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2}\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}\right)-e\int \frac{1}{-x^4+b^2-4ac}d(2cx^2+b)$$

$$\Big\downarrow \text{219}$$

$$d\left(\frac{\sqrt{2}\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2}\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}\right)-\frac{e\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

input
```
Int[(d + e*x)/(a + b*x^2 + c*x^4),x]
```

output
```
d*((Sqrt[2]*Sqrt[c]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]
])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[2]*Sqrt[c]*ArcT
an[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sq
rt[b + Sqrt[b^2 - 4*a*c]])) - (e*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])
/Sqrt[b^2 - 4*a*c]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1406
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Simp[c/q   Int[1/(b/2 - q/2 + c*x^2), x], x] - Simp[c/q   I
nt[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c
, 0] && PosQ[b^2 - 4*a*c]
```

rule 1432
```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

rule 2202
```
Int[(Pn_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{n
 = Expon[Pn, x], k}, Int[Sum[Coeff[Pn, x, 2*k]*x^(2*k), {k, 0, n/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pn, x, 2*k + 1]*x^(2*k), {k, 0, (n -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pn, x]
 &&  !PolyQ[Pn, x^2]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.23

method	result
risch	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(c\_Z^4+\_Z^2 b+a\right)}\frac{\left(\_Re+d\right)\ln\left(x-\_R\right)}{2\_R^3 c+b\_R}\right)}{2}$
default	$4c\left(-\frac{\sqrt{-4ac+b^2}\left(\frac{e\ln\left(-2c\,x^2+\sqrt{-4ac+b^2}-b\right)}{4c}-\frac{d\sqrt{2}\,\operatorname{arctanh}\left(\frac{cx\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{8ac-2b^2}+\frac{\sqrt{-4ac+b^2}\left(\frac{e\ln\left(2c\,x^2+\sqrt{-4ac+b^2}+b\right)}{4c}\right.}{8ac-}\right.$

input
```
int((e*x+d)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*sum((_R*e+d)/(2*_R^3*c+_R*b)*ln(x-_R),_R=RootOf(_Z^4*c+_Z^2*b+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.89 (sec) , antiderivative size = 398481, normalized size of antiderivative = 2108.37

$$\int \frac{d+ex}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{d+ex}{a+bx^2+cx^4}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)/(c*x**4+b*x**2+a),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{d+ex}{a+bx^2+cx^4}\,dx = \int \frac{ex+d}{cx^4+bx^2+a}\,dx$$

input
```
integrate((e*x+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
integrate((e*x + d)/(c*x^4 + b*x^2 + a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1342 vs. $2(149) = 298$.

Time $= 0.79$ (sec) , antiderivative size $= 1342$, normalized size of antiderivative $= 7.10$

$$\int \frac{d+ex}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
1/4*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4 - 8*sqrt(2)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b^2*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3
*c - 2*b^4*c + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^2 + 8*sqrt
(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*b^2*c^2 + 16*a*b^2*c^2 + 2*b^3*c^2 - 4*sqrt(2)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a*c^3 - 32*a^2*c^3 - 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*b*c^2 + 2*(b^2 - 4*a*c)*b^2*c - 8*(b^2 - 4*a*c)*a*c^2 - 2*(b^2
 - 4*a*c)*b*c^2)*d*arctan(2*sqrt(1/2)*x/sqrt((b + sqrt(b^2 - 4*a*c))/c))/(
(a*b^4 - 8*a^2*b^2*c - 2*a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 -
4*a^2*c^3)*abs(c)) + 1/4*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4 - 8*
sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c - 2*sqrt(2)*sqrt(b*c - sqr
t(b^2 - 4*a*c)*c)*b^3*c + 2*b^4*c + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a^2*c^2 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + sqrt(2)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^2 - 16*a*b^2*c^2 + 2*b^3*c^2 - 4*sq
rt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^3 + 32*a^2*c^3 - 8*a*b*c^3 - sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3 + 4*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c + 2*sqrt(2)*sqrt(b...
```

Mupad [B] (verification not implemented)

Time = 22.00 (sec) , antiderivative size = 1308, normalized size of antiderivative = 6.92

$$\int \frac{d+ex}{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
int((d + e*x)/(a + b*x^2 + c*x^4),x)
```

output
```
symsum(log(c^2*(d*e^2 + e^3*x + 4*root(128*a^2*b^2*c*z^4 - 256*a^3*c^2*z^4
 - 16*a*b^4*z^4 + 16*a*b*c*d^2*z^2 - 32*a^2*c*e^2*z^2 + 8*a*b^2*e^2*z^2 -
4*b^3*d^2*z^2 + 16*a*c*d^2*e*z - 4*b^2*d^2*e*z - b*d^2*e^2 - c*d^4 - a*e^4
, z, k)^2*b^2*d - 8*root(128*a^2*b^2*c*z^4 - 256*a^3*c^2*z^4 - 16*a*b^4*z^
4 + 16*a*b*c*d^2*z^2 - 32*a^2*c*e^2*z^2 + 8*a*b^2*e^2*z^2 - 4*b^3*d^2*z^2
+ 16*a*c*d^2*e*z - 4*b^2*d^2*e*z - b*d^2*e^2 - c*d^4 - a*e^4, z, k)^3*b^3*
x - 16*root(128*a^2*b^2*c*z^4 - 256*a^3*c^2*z^4 - 16*a*b^4*z^4 + 16*a*b*c*
d^2*z^2 - 32*a^2*c*e^2*z^2 + 8*a*b^2*e^2*z^2 - 4*b^3*d^2*z^2 + 16*a*c*d^2*
e*z - 4*b^2*d^2*e*z - b*d^2*e^2 - c*d^4 - a*e^4, z, k)^2*a*c*d + 2*root(12
8*a^2*b^2*c*z^4 - 256*a^3*c^2*z^4 - 16*a*b^4*z^4 + 16*a*b*c*d^2*z^2 - 32*a
^2*c*e^2*z^2 + 8*a*b^2*e^2*z^2 - 4*b^3*d^2*z^2 + 16*a*c*d^2*e*z - 4*b^2*d^
2*e*z - b*d^2*e^2 - c*d^4 - a*e^4, z, k)*b*e^2*x - 4*root(128*a^2*b^2*c*z^
4 - 256*a^3*c^2*z^4 - 16*a*b^4*z^4 + 16*a*b*c*d^2*z^2 - 32*a^2*c*e^2*z^2 +
 8*a*b^2*e^2*z^2 - 4*b^3*d^2*z^2 + 16*a*c*d^2*e*z - 4*b^2*d^2*e*z - b*d^2*
e^2 - c*d^4 - a*e^4, z, k)*c*d^2*x - 4*root(128*a^2*b^2*c*z^4 - 256*a^3*c^
2*z^4 - 16*a*b^4*z^4 + 16*a*b*c*d^2*z^2 - 32*a^2*c*e^2*z^2 + 8*a*b^2*e^2*z
^2 - 4*b^3*d^2*z^2 + 16*a*c*d^2*e*z - 4*b^2*d^2*e*z - b*d^2*e^2 - c*d^4 -
a*e^4, z, k)^2*b^2*e*x + 4*root(128*a^2*b^2*c*z^4 - 256*a^3*c^2*z^4 - 16*a
*b^4*z^4 + 16*a*b*c*d^2*z^2 - 32*a^2*c*e^2*z^2 + 8*a*b^2*e^2*z^2 - 4*b^3*d
^2*z^2 + 16*a*c*d^2*e*z - 4*b^2*d^2*e*z - b*d^2*e^2 - c*d^4 - a*e^4, z,...
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 467, normalized size of antiderivative = 2.47

$$\int \frac{d+ex}{a+bx^2+cx^4}\,dx$$
$$= \frac{-4\sqrt{2\sqrt{c}\,\sqrt{a}+b}\,\sqrt{2\sqrt{c}\,\sqrt{a}-b}\,atan\left(\frac{\sqrt{2\sqrt{c}\,\sqrt{a}-b}-2\sqrt{c}\,x}{\sqrt{2\sqrt{c}\,\sqrt{a}+b}}\right)ae+2\sqrt{a}\,\sqrt{2\sqrt{c}\,\sqrt{a}+b}\,atan\left(\frac{\sqrt{2\sqrt{c}\,\sqrt{a}-b}-2\sqrt{c}\,x}{\sqrt{2\sqrt{c}\,\sqrt{a}+b}}\right)}{}$$

input
```
int((e*x+d)/(c*x^4+b*x^2+a),x)
```

output
```
( - 4*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2
*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*e + 2*
sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*
sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*b*d - 4*sqrt(c)*sqrt(2*sqrt(c)*sqr
t(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*
sqrt(a) + b))*a*d - 4*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) -
 b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a
) + b))*a*e - 2*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*s
qrt(a) - b) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*b*d + 4*sqrt(c)*sq
rt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) + 2*sqrt(c)*x)
/sqrt(2*sqrt(c)*sqrt(a) + b))*a*d - sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) - b)*lo
g( - sqrt(2*sqrt(c)*sqrt(a) - b)*x + sqrt(a) + sqrt(c)*x**2)*b*d + sqrt(a)
*sqrt(2*sqrt(c)*sqrt(a) - b)*log(sqrt(2*sqrt(c)*sqrt(a) - b)*x + sqrt(a) +
 sqrt(c)*x**2)*b*d - 2*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) - b)*log( - sqrt(2*s
qrt(c)*sqrt(a) - b)*x + sqrt(a) + sqrt(c)*x**2)*a*d + 2*sqrt(c)*sqrt(2*sqr
t(c)*sqrt(a) - b)*log(sqrt(2*sqrt(c)*sqrt(a) - b)*x + sqrt(a) + sqrt(c)*x*
*2)*a*d)/(4*a*(4*a*c - b**2))
```

3.238 $\int \frac{1}{a+bx^2+cx^4} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 150

$$\int \frac{1}{a+bx^2+cx^4} \, dx = \frac{\sqrt{2}\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{2}\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}}$$

output
```
2^(1/2)*c^(1/2)*arctan(2^(1/2)*c^(1/2)*x/(b-(-4*a*c+b^2)^(1/2))^(1/2))/(-4
*a*c+b^2)^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-2^(1/2)*c^(1/2)*arctan(2^(1/2
)*c^(1/2)*x/(b+(-4*a*c+b^2)^(1/2))^(1/2))/(-4*a*c+b^2)^(1/2)/(b+(-4*a*c+b^
2)^(1/2))^(1/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.86

$$\int \frac{1}{a+bx^2+cx^4} \, dx = \frac{\sqrt{2}\sqrt{c}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}}$$

input
```
Integrate[(a + b*x^2 + c*x^4)^(-1),x]
```

output
```
(Sqrt[2]*Sqrt[c]*(ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]]/
Sqrt[b - Sqrt[b^2 - 4*a*c]] - ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2
 - 4*a*c]]]/Sqrt[b + Sqrt[b^2 - 4*a*c]]))/Sqrt[b^2 - 4*a*c]
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1406, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+bx^2+cx^4}\,dx$$

$$\Big\downarrow \ 1406$$

$$\frac{c\int \frac{1}{cx^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx}{\sqrt{b^2-4ac}}-\frac{c\int \frac{1}{cx^2+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}dx}{\sqrt{b^2-4ac}}$$

$$\Big\downarrow \ 218$$

$$\frac{\sqrt{2}\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2}\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}$$

input
```
Int[(a + b*x^2 + c*x^4)^(-1),x]
```

output
```
(Sqrt[2]*Sqrt[c]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/
(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[2]*Sqrt[c]*ArcTan[
(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[
b + Sqrt[b^2 - 4*a*c]])
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1406
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Simp[c/q   Int[1/(b/2 - q/2 + c*x^2), x], x] - Simp[c/q   I
nt[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c
, 0] && PosQ[b^2 - 4*a*c]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.05 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.25

method	result	size
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+\_Z^2 b+a\right)}\frac{\ln\left(x-\_R\right)}{2\_R^3 c+b\_R}\right)}{2}$	38
default	$4c\left(-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{cx\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{4\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}-\frac{\sqrt{2}\,\arctan\left(\frac{cx\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{4\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)$	117

input
```
int(1/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*sum(1/(2*_R^3*c+_R*b)*ln(x-_R),_R=RootOf(_Z^4*c+_Z^2*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 613 vs. $2(114) = 228$.

Time = 0.12 (sec) , antiderivative size = 613, normalized size of antiderivative = 4.09

$$\int \frac{1}{a+bx^2+cx^4}\,dx = -\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{b+\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\log\left(2cx+\sqrt{\frac{1}{2}}\left(b^2-4ac-\frac{ab^3-4a^2bc}{\sqrt{a^2b^2-4a^3c}}\right)\sqrt{-\frac{b+\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\right)$$
$$+\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{b+\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\log\left(2cx-\sqrt{\frac{1}{2}}\left(b^2-4ac-\frac{ab^3-4a^2bc}{\sqrt{a^2b^2-4a^3c}}\right)\sqrt{-\frac{b+\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\right)$$
$$-\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{b-\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\log\left(2cx+\sqrt{\frac{1}{2}}\left(b^2-4ac+\frac{ab^3-4a^2bc}{\sqrt{a^2b^2-4a^3c}}\right)\sqrt{-\frac{b-\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\right)$$
$$+\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{b-\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\log\left(2cx-\sqrt{\frac{1}{2}}\left(b^2-4ac+\frac{ab^3-4a^2bc}{\sqrt{a^2b^2-4a^3c}}\right)\sqrt{-\frac{b-\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\right)$$

input
```
integrate(1/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
-1/2*sqrt(1/2)*sqrt(-(b + (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^
2 - 4*a^2*c))*log(2*c*x + sqrt(1/2)*(b^2 - 4*a*c - (a*b^3 - 4*a^2*b*c)/sqr
t(a^2*b^2 - 4*a^3*c))*sqrt(-(b + (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c)
)/(a*b^2 - 4*a^2*c))) + 1/2*sqrt(1/2)*sqrt(-(b + (a*b^2 - 4*a^2*c)/sqrt(a^
2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))*log(2*c*x - sqrt(1/2)*(b^2 - 4*a*c -
(a*b^3 - 4*a^2*b*c)/sqrt(a^2*b^2 - 4*a^3*c))*sqrt(-(b + (a*b^2 - 4*a^2*c)/
sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))) - 1/2*sqrt(1/2)*sqrt(-(b - (a
*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))*log(2*c*x + sq
rt(1/2)*(b^2 - 4*a*c + (a*b^3 - 4*a^2*b*c)/sqrt(a^2*b^2 - 4*a^3*c))*sqrt(-
(b - (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))) + 1/2*
sqrt(1/2)*sqrt(-(b - (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4
*a^2*c))*log(2*c*x - sqrt(1/2)*(b^2 - 4*a*c + (a*b^3 - 4*a^2*b*c)/sqrt(a^2
*b^2 - 4*a^3*c))*sqrt(-(b - (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*
b^2 - 4*a^2*c)))
```

Sympy [A] (verification not implemented)

Time = 0.69 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.58

$$\int \frac{1}{a + bx^2 + cx^4}\,dx$$
$$= \operatorname{RootSum}\left(t^4 \cdot \left(256a^3c^2 - 128a^2b^2c + 16ab^4\right) + t^2\left(-16abc + 4b^3\right) + c, \left(t \mapsto t\log\left(x + \frac{32t^3a^2bc - 8t^3a\right.\right.\right.$$

input
```
integrate(1/(c*x**4+b*x**2+a),x)
```

output
```
RootSum(_t**4*(256*a**3*c**2 - 128*a**2*b**2*c + 16*a*b**4) + _t**2*(-16*a
*b*c + 4*b**3) + c, Lambda(_t, _t*log(x + (32*_t**3*a**2*b*c - 8*_t**3*a*b
**3 + 4*_t*a*c - 2*_t*b**2)/c)))
```

Maxima [F]

$$\int \frac{1}{a+bx^2+cx^4}\,dx=\int \frac{1}{cx^4+bx^2+a}\,dx$$

input
```
integrate(1/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
integrate(1/(c*x^4 + b*x^2 + a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1026 vs. $2(114) = 228$.

Time = 0.35 (sec) , antiderivative size = 1026, normalized size of antiderivative = 6.84

$$\int \frac{1}{a+bx^2+cx^4}\,dx=\text{Too large to display}$$

input
```
integrate(1/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
1/4*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4 - 8*sqrt(2)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b^2*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3
*c - 2*b^4*c + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^2 + 8*sqrt
(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*b^2*c^2 + 16*a*b^2*c^2 + 2*b^3*c^2 - 4*sqrt(2)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a*c^3 - 32*a^2*c^3 - 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*b*c^2 + 2*(b^2 - 4*a*c)*b^2*c - 8*(b^2 - 4*a*c)*a*c^2 - 2*(b^2
 - 4*a*c)*b*c^2)*arctan(2*sqrt(1/2)*x/sqrt((b + sqrt(b^2 - 4*a*c))/c))/((a
*b^4 - 8*a^2*b^2*c - 2*a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*
a^2*c^3)*abs(c)) + 1/4*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4 - 8*sq
rt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c - 2*sqrt(2)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*b^3*c + 2*b^4*c + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*a^2*c^2 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + sqrt(2)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^2 - 16*a*b^2*c^2 - 2*b^3*c^2 - 4*sqrt
(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^3 + 32*a^2*c^3 + 8*a*b*c^3 + sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3 - 4*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c - 2*sqrt(2)*sqrt(b^2...
```

Mupad [B] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 763, normalized size of antiderivative = 5.09

$$\int \frac{1}{a+bx^2+cx^4}\,dx =$$

$$-\mathrm{atan}\left(\frac{b^4\,x\,1\mathrm{i}+b\,x\,\sqrt{-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6}\,1\mathrm{i}+a^2}{4\,a\,b^4\,\sqrt{-\frac{b^3+\sqrt{-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6}-4\,a\,b\,c}{128\,a^3\,c^2-64\,a^2\,b^2\,c+8\,a\,b^4}}+64\,a^3\,c^2\,\sqrt{-\frac{b^3+\sqrt{-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6}-4\,a}{128\,a^3\,c^2-64\,a^2\,b^2\,c+8\,a\,b^4}}}\right.$$

$$-\mathrm{atan}\left(\frac{b^4\,x\,1\mathrm{i}-b\,x\,\sqrt{-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6}\,1\mathrm{i}+a^2\,c^2}{4\,a\,b^4\,\sqrt{\frac{\sqrt{-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6}-b^3+4\,a\,b\,c}{128\,a^3\,c^2-64\,a^2\,b^2\,c+8\,a\,b^4}}+64\,a^3\,c^2\,\sqrt{\frac{\sqrt{-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6}-b^3+4\,a\,b\,c}{128\,a^3\,c^2-64\,a^2\,b^2\,c+8\,a\,b^4}}-}\right.$$

input
```
int(1/(a + b*x^2 + c*x^4),x)
```

output
```
- atan((b^4*x*1i + b*x*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1
/2)*1i + a^2*c^2*x*16i - a*b^2*c*x*8i)/(4*a*b^4*(-(b^3 + (b^6 - 64*a^3*c^3
 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - 4*a*b*c)/(8*a*b^4 + 128*a^3*c^2 -
64*a^2*b^2*c))^(1/2) + 64*a^3*c^2*(-(b^3 + (b^6 - 64*a^3*c^3 + 48*a^2*b^2*
c^2 - 12*a*b^4*c)^(1/2) - 4*a*b*c)/(8*a*b^4 + 128*a^3*c^2 - 64*a^2*b^2*c))
^(1/2) - 32*a^2*b^2*c*(-(b^3 + (b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b
^4*c)^(1/2) - 4*a*b*c)/(8*a*b^4 + 128*a^3*c^2 - 64*a^2*b^2*c))^(1/2)))*(-(
b^3 + (b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - 4*a*b*c)/(8
*a*b^4 + 128*a^3*c^2 - 64*a^2*b^2*c))^(1/2)*2i - atan((b^4*x*1i - b*x*(b^6
 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2)*1i + a^2*c^2*x*16i - a*
b^2*c*x*8i)/(4*a*b^4*(((b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1
/2) - b^3 + 4*a*b*c)/(8*a*b^4 + 128*a^3*c^2 - 64*a^2*b^2*c))^(1/2) + 64*a^
3*c^2*(((b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - b^3 + 4*a
*b*c)/(8*a*b^4 + 128*a^3*c^2 - 64*a^2*b^2*c))^(1/2) - 32*a^2*b^2*c*(((b^6
- 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - b^3 + 4*a*b*c)/(8*a*b^
4 + 128*a^3*c^2 - 64*a^2*b^2*c))^(1/2)))*(((b^6 - 64*a^3*c^3 + 48*a^2*b^2*
c^2 - 12*a*b^4*c)^(1/2) - b^3 + 4*a*b*c)/(8*a*b^4 + 128*a^3*c^2 - 64*a^2*b
^2*c))^(1/2)*2i
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 351, normalized size of antiderivative = 2.34

$$\int \frac{1}{a+bx^2+cx^4}\,dx = \frac{2\sqrt{a}\,\sqrt{2\sqrt{c}\,\sqrt{a}+b}\,atan\left(\frac{\sqrt{2\sqrt{c}\,\sqrt{a}-b}-2\sqrt{c}\,x}{\sqrt{2\sqrt{c}\,\sqrt{a}+b}}\right)b-4\sqrt{c}\,\sqrt{2\sqrt{c}\,\sqrt{a}+b}\,atan\left(\frac{\sqrt{2\sqrt{c}\,\sqrt{a}-b}-2\sqrt{c}\,x}{\sqrt{2\sqrt{c}\,\sqrt{a}+b}}\right)a-2\sqrt{a}}{}$$

input
```
int(1/(c*x^4+b*x^2+a),x)
```

output

```
(2*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) -
 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*b - 4*sqrt(c)*sqrt(2*sqrt(c)*sq
rt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)
*sqrt(a) + b))*a - 2*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt
(c)*sqrt(a) - b) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*b + 4*sqrt(c)
*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) + 2*sqrt(c)
*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a - sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) - b)*l
og( - sqrt(2*sqrt(c)*sqrt(a) - b)*x + sqrt(a) + sqrt(c)*x**2)*b + sqrt(a)*
sqrt(2*sqrt(c)*sqrt(a) - b)*log(sqrt(2*sqrt(c)*sqrt(a) - b)*x + sqrt(a) +
sqrt(c)*x**2)*b - 2*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) - b)*log( - sqrt(2*sqrt
(c)*sqrt(a) - b)*x + sqrt(a) + sqrt(c)*x**2)*a + 2*sqrt(c)*sqrt(2*sqrt(c)*
sqrt(a) - b)*log(sqrt(2*sqrt(c)*sqrt(a) - b)*x + sqrt(a) + sqrt(c)*x**2)*a
)/(4*a*(4*a*c - b**2))
```

3.239 $\int \frac{1}{(d+ex)(a+bx^2+cx^4)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 372

$$\int \frac{1}{(d+ex)(a+bx^2+cx^4)}\,dx = \frac{\sqrt{c}d\left(e^2+\frac{2cd^2+be^2}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}\,(cd^4+bd^2e^2+ae^4)} + \frac{\sqrt{c}d\left(e^2-\frac{2cd^2+be^2}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b+\sqrt{b^2-4ac}}\,(cd^4+bd^2e^2+ae^4)} + \frac{e(2cd^2+be^2)\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2\sqrt{b^2-4ac}\,(cd^4+bd^2e^2+ae^4)} + \frac{e^3\log(d+ex)}{cd^4+bd^2e^2+ae^4} - \frac{e^3\log(a+bx^2+cx^4)}{4(cd^4+bd^2e^2+ae^4)}$$

output
```
1/2*c^(1/2)*d*(e^2+(b*e^2+2*c*d^2)/(-4*a*c+b^2)^(1/2))*arctan(2^(1/2)*c^(1
/2)*x/(b-(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)/(
a*e^4+b*d^2*e^2+c*d^4)+1/2*c^(1/2)*d*(e^2-(b*e^2+2*c*d^2)/(-4*a*c+b^2)^(1/
2))*arctan(2^(1/2)*c^(1/2)*x/(b+(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/(b+(-4*
a*c+b^2)^(1/2))^(1/2)/(a*e^4+b*d^2*e^2+c*d^4)+1/2*e*(b*e^2+2*c*d^2)*arctan
h((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2)/(a*e^4+b*d^2*e^2+c*d^
4)+e^3*ln(e*x+d)/(a*e^4+b*d^2*e^2+c*d^4)-e^3*ln(c*x^4+b*x^2+a)/(4*a*e^4+4*
b*d^2*e^2+4*c*d^4)
```

Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 366, normalized size of antiderivative = 0.98

$$\int \frac{1}{(d+ex)(a+bx^2+cx^4)}\,dx = \frac{\frac{2\sqrt{2}\sqrt{c}d\left(2cd^2+\left(b+\sqrt{b^2-4ac}\right)e^2\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{2\sqrt{2}\sqrt{c}d\left(2cd^2+\left(b-\sqrt{b^2-4ac}\right)e^2\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}} + 4e^3\log(d+}{4\left(cd^4+bd^2e^2+a\right.}$$

input
```
Integrate[1/((d + e*x)*(a + b*x^2 + c*x^4)),x]
```

output
```
((2*Sqrt[2]*Sqrt[c]*d*(2*c*d^2 + (b + Sqrt[b^2 - 4*a*c])*e^2)*ArcTan[(Sqrt
[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - S
qrt[b^2 - 4*a*c]]) - (2*Sqrt[2]*Sqrt[c]*d*(2*c*d^2 + (b - Sqrt[b^2 - 4*a*c
])*e^2)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2
 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + 4*e^3*Log[d + e*x] - ((2*c*d^2*e
+ (b + Sqrt[b^2 - 4*a*c])*e^3)*Log[-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2])/Sqrt
[b^2 - 4*a*c] + ((2*c*d^2*e + (b - Sqrt[b^2 - 4*a*c])*e^3)*Log[b + Sqrt[b^
2 - 4*a*c] + 2*c*x^2])/Sqrt[b^2 - 4*a*c])/(4*(c*d^4 + b*d^2*e^2 + a*e^4))
```

Rubi [A] (verified)

Time = 1.58 (sec) , antiderivative size = 372, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {7279, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex)(a+bx^2+cx^4)}\,dx$$

$$\Big\downarrow \text{7279}$$

$$\int \left(\frac{(d-ex)(be^2+cd^2+ce^2x^2)}{(a+bx^2+cx^4)(ae^4+bd^2e^2+cd^4)} + \frac{e^4}{(d+ex)(ae^4+bd^2e^2+cd^4)}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{\sqrt{c}d\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{be^2+2cd^2}{\sqrt{b^2-4ac}}+e^2\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}\,(ae^4+bd^2e^2+cd^4)}+\frac{\sqrt{c}d\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(e^2-\frac{be^2+2cd^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{\sqrt{b^2-4ac}+b}\,(ae^4+bd^2e^2+cd^4)}+\frac{e(be^2+2cd^2)\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2\sqrt{b^2-4ac}\,(ae^4+bd^2e^2+cd^4)}-\frac{e^3\log\left(a+bx^2+cx^4\right)}{4\left(ae^4+bd^2e^2+cd^4\right)}+\frac{e^3\log(d+ex)}{ae^4+bd^2e^2+cd^4}$$

input
```
Int[1/((d + e*x)*(a + b*x^2 + c*x^4)),x]
```

output
```
(Sqrt[c]*d*(e^2 + (2*c*d^2 + b*e^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqr
t[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b - Sqrt[b^2 - 4*a*c]]
*(c*d^4 + b*d^2*e^2 + a*e^4)) + (Sqrt[c]*d*(e^2 - (2*c*d^2 + b*e^2)/Sqrt[b
^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqr
t[2]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(c*d^4 + b*d^2*e^2 + a*e^4)) + (e*(2*c*d^
2 + b*e^2)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*Sqrt[b^2 - 4*a*c]*
(c*d^4 + b*d^2*e^2 + a*e^4)) + (e^3*Log[d + e*x])/(c*d^4 + b*d^2*e^2 + a*e
^4) - (e^3*Log[a + b*x^2 + c*x^4])/(4*(c*d^4 + b*d^2*e^2 + a*e^4))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7279
```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 307, normalized size of antiderivative = 0.83

method	result
default	$\frac{4c\left(-\frac{\sqrt{-4ac+b^{2}}\left(\sqrt{-4ac+b^{2}}\,e^{2}+b\,e^{2}+2c\,d^{2}\right)\left(-\frac{e\ln\left(-2c\,x^{2}+\sqrt{-4ac+b^{2}}-b\right)}{4c}-\frac{d\sqrt{2}\,\operatorname{arctanh}\left(\frac{cx\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^{2}}\right)c}}\right)}{2\sqrt{\left(-b+\sqrt{-4ac+b^{2}}\right)c}}\right)}{16ac-4b^{2}}-\frac{\sqrt{-4ac+b^{2}}\left(\sqrt{-4\cdots\right.}{\cdots}\right.}{e^{4}a+b\,d^{2}e^{2}+c\,d^{4}}$
risch	$\frac{e^{3}\ln(ex+d)}{e^{4}a+b\,d^{2}e^{2}+c\,d^{4}}+\frac{\left(\_R=\operatorname{RootOf}\left(\left(16a^{4}c^{2}e^{4}-8a^{3}b^{2}c\,e^{4}+16a^{3}b\,c^{2}d^{2}e^{2}+16a^{3}c^{3}d^{4}+b^{4}a^{2}e^{4}-8a^{2}b^{3}c\,d^{2}e^{2}-8a^{2}b^{2}c^{2}d^{4}+a\,b^{5}d^{2}e^{2}+a\,b^{4}c\,d^{4}\cdots\right.\right.\right.}{\cdots}$

input
```
int(1/(e*x+d)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
4/(a*e^4+b*d^2*e^2+c*d^4)*c*(-(-4*a*c+b^2)^(1/2)*((-4*a*c+b^2)^(1/2)*e^2+b
*e^2+2*c*d^2)/(16*a*c-4*b^2)*(-1/4*e/c*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)-1
/2*d*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-
4*a*c+b^2)^(1/2))*c)^(1/2)))-(-4*a*c+b^2)^(1/2)*((-4*a*c+b^2)^(1/2)*e^2-b*
e^2-2*c*d^2)/(16*a*c-4*b^2)*(-1/4*e/c*ln(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)+1/2
*d*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c
+b^2)^(1/2))*c)^(1/2))))+e^3*ln(e*x+d)/(a*e^4+b*d^2*e^2+c*d^4)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)\left(a+bx^2+cx^4\right)}\,dx=\text{Timed out}$$

input
```
integrate(1/(e*x+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)\left(a+bx^2+cx^4\right)}\,dx=\text{Timed out}$$

input
```
integrate(1/(e*x+d)/(c*x**4+b*x**2+a),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{(d+ex)\left(a+bx^2+cx^4\right)}\,dx=\int \frac{1}{(cx^4+bx^2+a)(ex+d)}\,dx$$

input
```
integrate(1/(e*x+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
e^3*log(e*x + d)/(c*d^4 + b*d^2*e^2 + a*e^4) - integrate((c*e^3*x^3 - c*d*
e^2*x^2 - c*d^3 - b*d*e^2 + (c*d^2*e + b*e^3)*x)/(c*x^4 + b*x^2 + a), x)/(
c*d^4 + b*d^2*e^2 + a*e^4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19589 vs. $2(330) = 660$.

Time = 2.97 (sec) , antiderivative size = 19589, normalized size of antiderivative = 52.66

$$\int \frac{1}{(d+ex)\left(a+bx^2+cx^4\right)}\,dx=\text{Too large to display}$$

input
```
integrate(1/(e*x+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
e^4*log(abs(e*x + d))/(c*d^4*e + b*d^2*e^3 + a*e^5) - 1/4*e^3*log(abs(c*x^
4 + b*x^2 + a))/(c*d^4 + b*d^2*e^2 + a*e^4) + 1/8*(2*(2*b^3*c^9 - 8*a*b*c^
10 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^7 + 4
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^8 + 2*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^8 - sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^9 - 2*(b^2 - 4*a*c)*b
*c^9)*d^19 + 9*(2*b^4*c^8 - 8*a*b^2*c^9 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^6 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a*b^2*c^7 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*b^3*c^7 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*b^2*c^8 - 2*(b^2 - 4*a*c)*b^2*c^8)*d^17*e^2 + 8*(4*b^5*c^7 -
 14*a*b^3*c^8 - 8*a^2*b*c^9 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*b^5*c^5 + 7*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a*b^3*c^6 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*b^4*c^6 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a^2*b*c^7 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a*b^2*c^7 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*b^3*c^7 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*
c^8 - 4*(b^2 - 4*a*c)*b^3*c^7 - 2*(b^2 - 4*a*c)*a*b*c^8)*d^15*e^4 + 14*(2*
b^6*c^6 - 4*a*b^4*c^7 - 16*a^2*b^2*c^8 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt...
```

Mupad [B] (verification not implemented)

Time = 22.13 (sec) , antiderivative size = 1881, normalized size of antiderivative = 5.06

$$\int \frac{1}{(d+ex)(a+bx^2+cx^4)}\,dx = \text{Too large to display}$$

input
```
int(1/((d + e*x)*(a + b*x^2 + c*x^4)),x)
```

output
```
symsum(log(root(256*a^3*b*c^2*d^2*e^2*z^4 - 128*a^2*b^3*c*d^2*e^2*z^4 + 16
*a*b^5*d^2*e^2*z^4 - 128*a^3*b^2*c*e^4*z^4 + 16*a*b^4*c*d^4*z^4 - 128*a^2*
b^2*c^2*d^4*z^4 + 256*a^4*c^2*e^4*z^4 + 256*a^3*c^3*d^4*z^4 + 16*a^2*b^4*e
^4*z^4 - 128*a^2*b^2*c*e^3*z^3 + 256*a^3*c^2*e^3*z^3 + 16*a*b^4*e^3*z^3 -
40*a*b^2*c*e^2*z^2 - 16*a*b*c^2*d^2*z^2 + 96*a^2*c^2*e^2*z^2 + 4*b^3*c*d^2
*z^2 + 4*b^4*e^2*z^2 - 4*b^2*c*e*z + 16*a*c^2*e*z + c^2, z, k)*(root(256*a
^3*b*c^2*d^2*e^2*z^4 - 128*a^2*b^3*c*d^2*e^2*z^4 + 16*a*b^5*d^2*e^2*z^4 -
128*a^3*b^2*c*e^4*z^4 + 16*a*b^4*c*d^4*z^4 - 128*a^2*b^2*c^2*d^4*z^4 + 256
*a^4*c^2*e^4*z^4 + 256*a^3*c^3*d^4*z^4 + 16*a^2*b^4*e^4*z^4 - 128*a^2*b^2*
c*e^3*z^3 + 256*a^3*c^2*e^3*z^3 + 16*a*b^4*e^3*z^3 - 40*a*b^2*c*e^2*z^2 -
16*a*b*c^2*d^2*z^2 + 96*a^2*c^2*e^2*z^2 + 4*b^3*c*d^2*z^2 + 4*b^4*e^2*z^2
- 4*b^2*c*e*z + 16*a*c^2*e*z + c^2, z, k)*(root(256*a^3*b*c^2*d^2*e^2*z^4
- 128*a^2*b^3*c*d^2*e^2*z^4 + 16*a*b^5*d^2*e^2*z^4 - 128*a^3*b^2*c*e^4*z^4
 + 16*a*b^4*c*d^4*z^4 - 128*a^2*b^2*c^2*d^4*z^4 + 256*a^4*c^2*e^4*z^4 + 25
6*a^3*c^3*d^4*z^4 + 16*a^2*b^4*e^4*z^4 - 128*a^2*b^2*c*e^3*z^3 + 256*a^3*c
^2*e^3*z^3 + 16*a*b^4*e^3*z^3 - 40*a*b^2*c*e^2*z^2 - 16*a*b*c^2*d^2*z^2 +
96*a^2*c^2*e^2*z^2 + 4*b^3*c*d^2*z^2 + 4*b^4*e^2*z^2 - 4*b^2*c*e*z + 16*a*
c^2*e*z + c^2, z, k)*(x*(240*a^2*c^4*e^6 + 12*b^4*c^2*e^6 - 108*a*b^2*c^3*
e^6 - 48*a*c^5*d^4*e^2 + 12*b^2*c^4*d^4*e^2 + 24*b^3*c^3*d^2*e^4 - 96*a*b*
c^4*d^2*e^4) - root(256*a^3*b*c^2*d^2*e^2*z^4 - 128*a^2*b^3*c*d^2*e^2*z...
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 1383, normalized size of antiderivative = 3.72

$$\int \frac{1}{(d+ex)\,(a+bx^2+cx^4)}\,dx = \text{Too large to display}$$

input
```
int(1/(e*x+d)/(c*x^4+b*x^2+a),x)
```

output
```
(2*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sq
rt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b*e**3 +
4*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqr
t(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*c*d**2*e -
 4*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) -
 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*c*d*e**2 + 2*sqrt(a)*sqrt(2*s
qrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(
2*sqrt(c)*sqrt(a) + b))*b**2*d*e**2 + 2*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b
)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a)
+ b))*b*c*d**3 - 2*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c
)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b*d*e**2 - 4*
sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*
sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*c*d**3 + 2*sqrt(2*sqrt(c)*sqrt(a
) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) + 2*s
qrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b*e**3 + 4*sqrt(2*sqrt(c)*sqrt(a)
 + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) + 2*sq
rt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*c*d**2*e + 4*sqrt(a)*sqrt(2*sqrt(c
)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) + 2*sqrt(c)*x)/sqrt(2*sqr
t(c)*sqrt(a) + b))*a*c*d*e**2 - 2*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan
((sqrt(2*sqrt(c)*sqrt(a) - b) + 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b...
```

3.240 $\int \frac{1}{(d+ex)^2(a+bx^2+cx^4)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 569

$$\int \frac{1}{(d+ex)^2\,(a+bx^2+cx^4)}\,dx = -\frac{e^3}{(cd^4+bd^2e^2+ae^4)\,(d+ex)}$$
$$+\frac{\sqrt{c}\left(2c^2d^6+cd^2e^2\left(bd^2+3\sqrt{b^2-4ac}d^2-6ae^2\right)+\left(b+\sqrt{b^2-4ac}\right)e^4(bd^2-ae^2)\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}\,(cd^4+bd^2e^2+ae^4)^2}$$
$$-\frac{\sqrt{c}\left(2c^2d^6+cd^2e^2\left(bd^2-3\sqrt{b^2-4ac}d^2-6ae^2\right)+\left(b-\sqrt{b^2-4ac}\right)e^4(bd^2-ae^2)\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}\,(cd^4+bd^2e^2+ae^4)^2}$$
$$+\frac{de(2c^2d^4+2bcd^2e^2+b^2e^4-2ace^4)\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\,(cd^4+bd^2e^2+ae^4)^2}$$
$$+\frac{2de^3(2cd^2+be^2)\log(d+ex)}{(cd^4+bd^2e^2+ae^4)^2}-\frac{de^3(2cd^2+be^2)\log\left(a+bx^2+cx^4\right)}{2\,(cd^4+bd^2e^2+ae^4)^2}$$

output
```
-e^3/(a*e^4+b*d^2*e^2+c*d^4)/(e*x+d)+1/2*c^(1/2)*(2*c^2*d^6+c*d^2*e^2*(b*d
^2+3*(-4*a*c+b^2)^(1/2)*d^2-6*a*e^2)+(b+(-4*a*c+b^2)^(1/2))*e^4*(-a*e^2+b*
d^2))*arctan(2^(1/2)*c^(1/2)*x/(b-(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/(-4*a
*c+b^2)^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)/(a*e^4+b*d^2*e^2+c*d^4)^2-1/2*c
^(1/2)*(2*c^2*d^6+c*d^2*e^2*(b*d^2-3*(-4*a*c+b^2)^(1/2)*d^2-6*a*e^2)+(b-(-
4*a*c+b^2)^(1/2))*e^4*(-a*e^2+b*d^2))*arctan(2^(1/2)*c^(1/2)*x/(b+(-4*a*c+
b^2)^(1/2))^(1/2))*2^(1/2)/(-4*a*c+b^2)^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
/(a*e^4+b*d^2*e^2+c*d^4)^2+d*e*(-2*a*c*e^4+b^2*e^4+2*b*c*d^2*e^2+2*c^2*d^4
)*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2)/(a*e^4+b*d^2*
e^2+c*d^4)^2+2*d*e^3*(b*e^2+2*c*d^2)*ln(e*x+d)/(a*e^4+b*d^2*e^2+c*d^4)^2-1
/2*d*e^3*(b*e^2+2*c*d^2)*ln(c*x^4+b*x^2+a)/(a*e^4+b*d^2*e^2+c*d^4)^2
```

Mathematica [A] (verified)

Time = 1.39 (sec) , antiderivative size = 585, normalized size of antiderivative = 1.03

$$\int \frac{1}{(d+ex)^2(a+bx^2+cx^4)}\,dx = \frac{-\frac{2e^3(cd^4+bd^2e^2+ae^4)}{d+ex}+\frac{\sqrt{2}\sqrt{c}\left(2c^2d^6+cd^2e^2\left(bd^2+3\sqrt{b^2-4ac}d^2-6ae^2\right)+\left(b+\sqrt{b^2-4ac}\right)e^4\left(bd^2-ae^2\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\sqrt{}}{}}{}$$

input
```
Integrate[1/((d + e*x)^2*(a + b*x^2 + c*x^4)),x]
```

output
```
((-2*e^3*(c*d^4 + b*d^2*e^2 + a*e^4))/(d + e*x) + (Sqrt[2]*Sqrt[c]*(2*c^2*
d^6 + c*d^2*e^2*(b*d^2 + 3*Sqrt[b^2 - 4*a*c]*d^2 - 6*a*e^2) + (b + Sqrt[b^
2 - 4*a*c])*e^4*(b*d^2 - a*e^2))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[
b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]
*Sqrt[c]*(-2*c^2*d^6 + (-b + Sqrt[b^2 - 4*a*c])*e^4*(b*d^2 - a*e^2) + c*d^
2*e^2*(-(b*d^2) + 3*Sqrt[b^2 - 4*a*c]*d^2 + 6*a*e^2))*ArcTan[(Sqrt[2]*Sqrt
[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2
- 4*a*c]]) + 4*(2*c*d^3*e^3 + b*d*e^5)*Log[d + e*x] - (d*e*(2*c^2*d^4 + b*
(b + Sqrt[b^2 - 4*a*c])*e^4 + 2*c*e^2*(b*d^2 + Sqrt[b^2 - 4*a*c]*d^2 - a*e
^2))*Log[-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2])/Sqrt[b^2 - 4*a*c] + (d*e*(2*c^
2*d^4 + b*(b - Sqrt[b^2 - 4*a*c])*e^4 - 2*c*e^2*(-(b*d^2) + Sqrt[b^2 - 4*a
*c]*d^2 + a*e^2))*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/Sqrt[b^2 - 4*a*c])
/(2*(c*d^4 + b*d^2*e^2 + a*e^4)^2)
```

Rubi [A] (verified)

Time = 3.13 (sec) , antiderivative size = 569, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {7279, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex)^2 (a+bx^2+cx^4)} \, dx$$

$$\Big\downarrow \text{7279}$$

$$\int \left(\frac{-2dex\left(-ace^4+b^2e^4+2bcd^2e^2+c^2d^4\right)+ce^2x^2\left(-ae^4+bd^2e^2+3cd^4\right)-abe^6-3acd^2e^4+b^2d^2e^4+2bcd^4e}{\left(a+bx^2+cx^4\right)\left(ae^4+bd^2e^2+cd^4\right)^2} \right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(cd^2e^2\left(3d^2\sqrt{b^2-4ac}-6ae^2+bd^2\right)+e^4\left(\sqrt{b^2-4ac}+b\right)\left(bd^2-ae^2\right)+2c^2d^6\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}\left(ae^4+bd^2e^2+cd^4\right)^2}-\frac{\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(cd^2e^2\left(-3d^2\sqrt{b^2-4ac}-6ae^2+bd^2\right)+e^4\left(b-\sqrt{b^2-4ac}\right)\left(bd^2-ae^2\right)+2c^2d^6\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}\left(ae^4+bd^2e^2+cd^4\right)^2}+\frac{d\,\text{earctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(-2ace^4+b^2e^4+2bcd^2e^2+2c^2d^4\right)}{\sqrt{b^2-4ac}\left(ae^4+bd^2e^2+cd^4\right)^2}-\frac{de^3\left(be^2+2cd^2\right)\log\left(a+bx^2+cx^4\right)}{2\left(ae^4+bd^2e^2+cd^4\right)^2}-\frac{e^3}{(d+ex)\left(ae^4+bd^2e^2+cd^4\right)}+\frac{2de^3\left(be^2+2cd^2\right)\log(d+ex)}{\left(ae^4+bd^2e^2+cd^4\right)^2}$$

input
```
Int[1/((d + e*x)^2*(a + b*x^2 + c*x^4)),x]
```

output

```
-(e^3/((c*d^4 + b*d^2*e^2 + a*e^4)*(d + e*x))) + (Sqrt[c]*(2*c^2*d^6 + c*d
^2*e^2*(b*d^2 + 3*Sqrt[b^2 - 4*a*c]*d^2 - 6*a*e^2) + (b + Sqrt[b^2 - 4*a*c
])*e^4*(b*d^2 - a*e^2))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a
*c]]])/(Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]*(c*d^4 + b*d
^2*e^2 + a*e^4)^2) - (Sqrt[c]*(2*c^2*d^6 + c*d^2*e^2*(b*d^2 - 3*Sqrt[b^2 -
 4*a*c]*d^2 - 6*a*e^2) + (b - Sqrt[b^2 - 4*a*c])*e^4*(b*d^2 - a*e^2))*ArcT
an[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b^2 - 4
*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(c*d^4 + b*d^2*e^2 + a*e^4)^2) + (d*e*(2
*c^2*d^4 + 2*b*c*d^2*e^2 + b^2*e^4 - 2*a*c*e^4)*ArcTanh[(b + 2*c*x^2)/Sqrt
[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*c]*(c*d^4 + b*d^2*e^2 + a*e^4)^2) + (2*d*e
^3*(2*c*d^2 + b*e^2)*Log[d + e*x])/(c*d^4 + b*d^2*e^2 + a*e^4)^2 - (d*e^3*
(2*c*d^2 + b*e^2)*Log[a + b*x^2 + c*x^4])/(2*(c*d^4 + b*d^2*e^2 + a*e^4)^2
)
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7279

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 637, normalized size of antiderivative = 1.12

method	result
default	$4c\left(\frac{\sqrt{-4ac+b^2}\left(-\frac{\left(-2\sqrt{-4ac+b^2}\,bd\,e^5-4\sqrt{-4ac+b^2}\,c\,d^3e^3+4acd\,e^5-2b^2d\,e^5-4bc\,d^3e^3-4c^2d^5e\right)\ln\left(-2c\,x^2+\sqrt{-4ac+b^2}-b\right)}{4c}+\frac{\left(-a\sqrt{-4ac+\right.}{}\right.}{16ac-4b^2}\right.$
risch	Expression too large to display

input
```
int(1/(e*x+d)^2/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
4/(a*e^4+b*d^2*e^2+c*d^4)^2*c*((-4*a*c+b^2)^(1/2)/(16*a*c-4*b^2)*(-1/4*(-2
*(-4*a*c+b^2)^(1/2)*b*d*e^5-4*(-4*a*c+b^2)^(1/2)*c*d^3*e^3+4*a*c*d*e^5-2*b
^2*d*e^5-4*b*c*d^3*e^3-4*c^2*d^5*e)/c*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)+1/
2*(-a*(-4*a*c+b^2)^(1/2)*e^6+(-4*a*c+b^2)^(1/2)*b*d^2*e^4+3*c*(-4*a*c+b^2)
^(1/2)*d^4*e^2-a*b*e^6-6*a*c*d^2*e^4+b^2*d^2*e^4+b*c*d^4*e^2+2*c^2*d^6)*2^
(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b
^2)^(1/2))*c)^(1/2)))+(-4*a*c+b^2)^(1/2)/(16*a*c-4*b^2)*(1/4*(2*(-4*a*c+b^
2)^(1/2)*b*d*e^5+4*(-4*a*c+b^2)^(1/2)*c*d^3*e^3+4*a*c*d*e^5-2*b^2*d*e^5-4*
b*c*d^3*e^3-4*c^2*d^5*e)/c*ln(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)+1/2*(a*(-4*a*c
+b^2)^(1/2)*e^6-(-4*a*c+b^2)^(1/2)*b*d^2*e^4-3*c*(-4*a*c+b^2)^(1/2)*d^4*e^
2-a*b*e^6-6*a*c*d^2*e^4+b^2*d^2*e^4+b*c*d^4*e^2+2*c^2*d^6)*2^(1/2)/((b+(-4
*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1
/2))))-e^3/(a*e^4+b*d^2*e^2+c*d^4)/(e*x+d)+2*d*e^3*(b*e^2+2*c*d^2)*ln(e*x+
d)/(a*e^4+b*d^2*e^2+c*d^4)^2
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^2(a+bx^2+cx^4)}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x+d)^2/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^2(a+bx^2+cx^4)}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x+d)**2/(c*x**4+b*x**2+a),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{(d+ex)^2 (a+bx^2+cx^4)} dx = \int \frac{1}{(cx^4+bx^2+a)(ex+d)^2} dx$$

input
```
integrate(1/(e*x+d)^2/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

output
```
-e^3/(c*d^5 + b*d^3*e^2 + a*d*e^4 + (c*d^4*e + b*d^2*e^3 + a*e^5)*x) + 2*(
2*c*d^3*e^3 + b*d*e^5)*log(e*x + d)/(c^2*d^8 + 2*b*c*d^6*e^2 + 2*a*b*d^2*e
^6 + a^2*e^8 + (b^2 + 2*a*c)*d^4*e^4) - integrate(-(c^2*d^6 + 2*b*c*d^4*e^
2 - a*b*e^6 + (b^2 - 3*a*c)*d^2*e^4 - 2*(2*c^2*d^3*e^3 + b*c*d*e^5)*x^3 +
(3*c^2*d^4*e^2 + b*c*d^2*e^4 - a*c*e^6)*x^2 - 2*(c^2*d^5*e + 2*b*c*d^3*e^3
 + (b^2 - a*c)*d*e^5)*x)/(c*x^4 + b*x^2 + a), x)/(c^2*d^8 + 2*b*c*d^6*e^2
+ 2*a*b*d^2*e^6 + a^2*e^8 + (b^2 + 2*a*c)*d^4*e^4)
```

Giac [F]

$$\int \frac{1}{(d+ex)^2 (a+bx^2+cx^4)} dx = \int \frac{1}{(cx^4+bx^2+a)(ex+d)^2} dx$$

input
```
integrate(1/(e*x+d)^2/(c*x^4+b*x^2+a),x, algorithm="giac")
```

output
```
sage0*x
```

Mupad [B] (verification not implemented)

Time = 23.99 (sec) , antiderivative size = 4722, normalized size of antiderivative = 8.30

$$\int \frac{1}{(d+ex)^2 (a+bx^2+cx^4)} dx = \text{Too large to display}$$

input
```
int(1/((d + e*x)^2*(a + b*x^2 + c*x^4)),x)
```

output
```
symsum(log(root(512*a^4*b*c^2*d^2*e^6*z^4 + 512*a^3*b*c^3*d^6*e^2*z^4 - 25
6*a^3*b^3*c*d^2*e^6*z^4 - 96*a^2*b^4*c*d^4*e^4*z^4 + 32*a*b^5*c*d^6*e^2*z^
4 - 256*a^2*b^3*c^2*d^6*e^2*z^4 + 16*a*b^6*d^4*e^4*z^4 - 128*a^4*b^2*c*e^8
*z^4 + 16*a*b^4*c^2*d^8*z^4 + 512*a^4*c^3*d^4*e^4*z^4 + 32*a^2*b^5*d^2*e^6
*z^4 - 128*a^2*b^2*c^3*d^8*z^4 + 256*a^5*c^2*e^8*z^4 + 256*a^3*c^4*d^8*z^4
 + 16*a^3*b^4*e^8*z^4 + 512*a^3*b*c^2*d*e^5*z^3 - 256*a^2*b^3*c*d*e^5*z^3
+ 64*a*b^4*c*d^3*e^3*z^3 - 512*a^2*b^2*c^2*d^3*e^3*z^3 + 32*a*b^5*d*e^5*z^
3 + 1024*a^3*c^3*d^3*e^3*z^3 - 112*a*b^2*c^2*d^2*e^2*z^2 + 8*b^4*c*d^2*e^2
*z^2 + 48*a^2*b*c^2*e^4*z^2 - 28*a*b^3*c*e^4*z^2 - 16*a*b*c^3*d^4*z^2 + 32
0*a^2*c^3*d^2*e^2*z^2 + 4*b^3*c^2*d^4*z^2 + 4*b^5*e^4*z^2 + 32*a*c^3*d*e*z
 - 8*b^2*c^2*d*e*z + c^3, z, k)*((32*a*c^5*d^2*e^7 + 16*b*c^5*d^4*e^5)/(a^
2*e^8 + c^2*d^8 + b^2*d^4*e^4 + 2*a*b*d^2*e^6 + 2*a*c*d^4*e^4 + 2*b*c*d^6*
e^2) + root(512*a^4*b*c^2*d^2*e^6*z^4 + 512*a^3*b*c^3*d^6*e^2*z^4 - 256*a^
3*b^3*c*d^2*e^6*z^4 - 96*a^2*b^4*c*d^4*e^4*z^4 + 32*a*b^5*c*d^6*e^2*z^4 -
256*a^2*b^3*c^2*d^6*e^2*z^4 + 16*a*b^6*d^4*e^4*z^4 - 128*a^4*b^2*c*e^8*z^4
 + 16*a*b^4*c^2*d^8*z^4 + 512*a^4*c^3*d^4*e^4*z^4 + 32*a^2*b^5*d^2*e^6*z^4
 - 128*a^2*b^2*c^3*d^8*z^4 + 256*a^5*c^2*e^8*z^4 + 256*a^3*c^4*d^8*z^4 + 1
6*a^3*b^4*e^8*z^4 + 512*a^3*b*c^2*d*e^5*z^3 - 256*a^2*b^3*c*d*e^5*z^3 + 64
*a*b^4*c*d^3*e^3*z^3 - 512*a^2*b^2*c^2*d^3*e^3*z^3 + 32*a*b^5*d*e^5*z^3 +
1024*a^3*c^3*d^3*e^3*z^3 - 112*a*b^2*c^2*d^2*e^2*z^2 + 8*b^4*c*d^2*e^2*...
```

Reduce [F]

$$\int \frac{1}{(d+ex)^2 (a+bx^2+cx^4)} dx = \int \frac{1}{(ex+d)^2 (c\,x^4+b\,x^2+a)}dx$$

input
```
int(1/(e*x+d)^2/(c*x^4+b*x^2+a),x)
```

output
```
int(1/(e*x+d)^2/(c*x^4+b*x^2+a),x)
```

3.241 $\int \frac{(d+ex)^3}{(a+bx^2+cx^4)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 446

$$\begin{aligned}\int \frac{(d+ex)^3}{(a+bx^2+cx^4)^2}\,dx &= \frac{dx((b^2-2ac)d^2-3abe^2+c(bd^2-6ae^2)x^2)}{2a(b^2-4ac)(a+bx^2+cx^4)}-\frac{e(3bd^2-2ae^2+(6cd^2-be^2)x^2)}{2(b^2-4ac)(a+bx^2+cx^4)}\\
&+\frac{\sqrt{c}d\left(b^2d^2+b\left(\sqrt{b^2-4ac}d^2+12ae^2\right)-6a\left(2cd^2+\sqrt{b^2-4ac}e^2\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}\\
&+\frac{\sqrt{c}d\left(bd^2-6ae^2-\frac{b^2d^2-12acd^2+12abe^2}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a(b^2-4ac)\sqrt{b+\sqrt{b^2-4ac}}}\\
&+\frac{e(6cd^2-be^2)\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}\end{aligned}$$

output
```
1/2*d*x*((-2*a*c+b^2)*d^2-3*a*b*e^2+c*(-6*a*e^2+b*d^2)*x^2)/a/(-4*a*c+b^2)
/(c*x^4+b*x^2+a)-1/2*e*(3*b*d^2-2*a*e^2+(-b*e^2+6*c*d^2)*x^2)/(-4*a*c+b^2)
/(c*x^4+b*x^2+a)+1/4*c^(1/2)*d*(b^2*d^2+b*((-4*a*c+b^2)^(1/2)*d^2+12*a*e^2
)-6*a*(2*c*d^2+(-4*a*c+b^2)^(1/2)*e^2))*arctan(2^(1/2)*c^(1/2)*x/(b-(-4*a*
c+b^2)^(1/2))^(1/2))*2^(1/2)/a/(-4*a*c+b^2)^(3/2)/(b-(-4*a*c+b^2)^(1/2))^(
1/2)+1/4*c^(1/2)*d*(b*d^2-6*a*e^2-(12*a*b*e^2-12*a*c*d^2+b^2*d^2)/(-4*a*c+
b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)*x/(b+(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)
/a/(-4*a*c+b^2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)+e*(-b*e^2+6*c*d^2)*arctanh((2
*c*x^2+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(3/2)
```

Mathematica [A] (verified)

Time = 1.54 (sec) , antiderivative size = 477, normalized size of antiderivative = 1.07

$$\int \frac{(d+ex)^3}{\left(a+bx^2+cx^4\right)^2}\,dx = \frac{1}{4}\left(\frac{-4a^2e^3-2bd^3x(b+cx^2)+2abe(3d^2+3dex-e^2x^2)+4acdx(d^2+3dex+3e^2x^2)}{a\left(-b^2+4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\sqrt{2}\sqrt{c}d\left(b^2d^2+b\left(\sqrt{b^2-4ac}d^2+12ae^2\right)-6a\left(2cd^2+\sqrt{b^2-4ac}e^2\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{a\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{2}\sqrt{c}d\left(b^2d^2-12acd^2-b\sqrt{b^2-4ac}d^2+12abe^2+6a\sqrt{b^2-4ac}e^2\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{a\left(b^2-4ac\right)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}} + \frac{2(-6cd^2e+be^3)\log\left(-b+\sqrt{b^2-4ac}-2cx^2\right)}{\left(b^2-4ac\right)^{3/2}} - \frac{2(-6cd^2e+be^3)\log\left(b+\sqrt{b^2-4ac}+2cx^2\right)}{\left(b^2-4ac\right)^{3/2}}\right)$$

input
```
Integrate[(d + e*x)^3/(a + b*x^2 + c*x^4)^2,x]
```

output
```
((-4*a^2*e^3 - 2*b*d^3*x*(b + c*x^2) + 2*a*b*e*(3*d^2 + 3*d*e*x - e^2*x^2)
 + 4*a*c*d*x*(d^2 + 3*d*e*x + 3*e^2*x^2))/(a*(-b^2 + 4*a*c)*(a + b*x^2 + c
*x^4)) + (Sqrt[2]*Sqrt[c]*d*(b^2*d^2 + b*(Sqrt[b^2 - 4*a*c]*d^2 + 12*a*e^2
) - 6*a*(2*c*d^2 + Sqrt[b^2 - 4*a*c]*e^2))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt
[b - Sqrt[b^2 - 4*a*c]]])/(a*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c
]]) - (Sqrt[2]*Sqrt[c]*d*(b^2*d^2 - 12*a*c*d^2 - b*Sqrt[b^2 - 4*a*c]*d^2 +
 12*a*b*e^2 + 6*a*Sqrt[b^2 - 4*a*c]*e^2)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b
 + Sqrt[b^2 - 4*a*c]]])/(a*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]
) + (2*(-6*c*d^2*e + b*e^3)*Log[-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2])/(b^2 -
4*a*c)^(3/2) - (2*(-6*c*d^2*e + b*e^3)*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2
])/(b^2 - 4*a*c)^(3/2))/4
```

Rubi [A] (verified)

Time = 1.37 (sec) , antiderivative size = 420, normalized size of antiderivative = 0.94, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {2202, 1492, 25, 27, 1480, 218, 1576, 27, 1159, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^3}{(a+bx^2+cx^4)^2}dx$$

$$\Big\downarrow \text{ 2202}$$

$$\int \frac{d^3+3e^2x^2d}{(cx^4+bx^2+a)^2}dx+\int \frac{x(x^2e^3+3d^2e)}{(cx^4+bx^2+a)^2}dx$$

$$\Big\downarrow \text{ 1492}$$

$$-\frac{\int -\frac{d(b^2d^2-6acd^2+3abe^2+c(bd^2-6ae^2)x^2)}{cx^4+bx^2+a}dx}{2a(b^2-4ac)}+\int \frac{x(x^2e^3+3d^2e)}{(cx^4+bx^2+a)^2}dx+\frac{dx(d^2(b^2-2ac)+cx^2(bd^2-6ae^2)-3abe^2)}{2a(b^2-4ac)(a+bx^2+cx^4)}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{d(b^2d^2-6acd^2+3abe^2+c(bd^2-6ae^2)x^2)}{cx^4+bx^2+a}dx}{2a(b^2-4ac)}+\int \frac{x(x^2e^3+3d^2e)}{(cx^4+bx^2+a)^2}dx+\frac{dx(d^2(b^2-2ac)+cx^2(bd^2-6ae^2)-3abe^2)}{2a(b^2-4ac)(a+bx^2+cx^4)}$$

$$\Big\downarrow \text{27}$$

$$\frac{d\int \frac{b^2d^2-6acd^2+3abe^2+c(bd^2-6ae^2)x^2}{cx^4+bx^2+a}dx}{2a(b^2-4ac)}+\int \frac{x(x^2e^3+3d^2e)}{(cx^4+bx^2+a)^2}dx+\frac{dx(d^2(b^2-2ac)+cx^2(bd^2-6ae^2)-3abe^2)}{2a(b^2-4ac)(a+bx^2+cx^4)}$$

$$\Big\downarrow \text{1480}$$

$$\frac{d\left(\frac{1}{2}c\left(\frac{12abe^2-12acd^2+b^2d^2}{\sqrt{b^2-4ac}}-6ae^2+bd^2\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx+\frac{1}{2}c\left(-\frac{12abe^2-12acd^2+b^2d^2}{\sqrt{b^2-4ac}}-6ae^2+bd^2\right)\int \frac{}{cx^2+\frac{1}{2}(}\right.}{2a(b^2-4ac)}$$

$$\int \frac{x(x^2e^3+3d^2e)}{(cx^4+bx^2+a)^2}dx+\frac{dx(d^2(b^2-2ac)+cx^2(bd^2-6ae^2)-3abe^2)}{2a(b^2-4ac)(a+bx^2+cx^4)}$$

$$\Big\downarrow \text{218}$$

$$\int \frac{x(x^2e^3+3d^2e)}{(cx^4+bx^2+a)^2}dx+\frac{d\left(\frac{\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{12abe^2-12acd^2+b^2d^2}{\sqrt{b^2-4ac}}-6ae^2+bd^2\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-\frac{12abe^2-12acd^2+b^2d^2}{\sqrt{b^2-4ac}}-6ae^2+bd^2\right)}{\sqrt{2}\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2a(b^2-4ac)}+\frac{dx(d^2(b^2-2ac)+cx^2(bd^2-6ae^2)-3abe^2)}{2a(b^2-4ac)(a+bx^2+cx^4)}$$

$$\Big\downarrow \text{1576}$$

$$\frac{1}{2}\int \frac{e(3d^2+e^2x^2)}{(cx^4+bx^2+a)^2}dx^2+\frac{d\left(\frac{\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{12abe^2-12acd^2+b^2d^2}{\sqrt{b^2-4ac}}-6ae^2+bd^2\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-\frac{12abe^2-12acd^2+b^2d^2}{\sqrt{b^2-4ac}}-6ae^2+bd^2\right)}{\sqrt{2}\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2a(b^2-4ac)}+\frac{dx(d^2(b^2-2ac)+cx^2(bd^2-6ae^2)-3abe^2)}{2a(b^2-4ac)(a+bx^2+cx^4)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}e\int \frac{3d^2+e^2x^2}{\left(cx^4+bx^2+a\right)^2}dx^2+\frac{d\left(\frac{\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{12abe^2-12acd^2+b^2d^2}{\sqrt{b^2-4ac}}-6ae^2+bd^2\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-\frac{12abe^2-12acd^2+b^2d^2}{\sqrt{b^2-4ac}}-6ae^2+bd^2\right)}{\sqrt{2}\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2a\left(b^2-4ac\right)}+\frac{dx\left(d^2\left(b^2-2ac\right)+cx^2\left(bd^2-6ae^2\right)-3abe^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$\Big\downarrow \; 1159$$

$$\frac{1}{2}e\left(-\frac{\left(6cd^2-be^2\right)\int \frac{1}{cx^4+bx^2+a}dx^2}{b^2-4ac}-\frac{-2ae^2+x^2\left(6cd^2-be^2\right)+3bd^2}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\right)+\frac{d\left(\frac{\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{12abe^2-12acd^2+b^2d^2}{\sqrt{b^2-4ac}}-6ae^2+bd^2\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-\frac{12abe^2-12acd^2+b^2d^2}{\sqrt{b^2-4ac}}-6ae^2+bd^2\right)}{\sqrt{2}\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2a\left(b^2-4ac\right)}+\frac{dx\left(d^2\left(b^2-2ac\right)+cx^2\left(bd^2-6ae^2\right)-3abe^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$\Big\downarrow \; 1083$$

$$\frac{1}{2}e\left(\frac{2\left(6cd^2-be^2\right)\int \frac{1}{-x^4+b^2-4ac}d\left(2cx^2+b\right)}{b^2-4ac}-\frac{-2ae^2+x^2\left(6cd^2-be^2\right)+3bd^2}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\right)+\frac{d\left(\frac{\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{12abe^2-12acd^2+b^2d^2}{\sqrt{b^2-4ac}}-6ae^2+bd^2\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-\frac{12abe^2-12acd^2+b^2d^2}{\sqrt{b^2-4ac}}-6ae^2+bd^2\right)}{\sqrt{2}\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2a\left(b^2-4ac\right)}+\frac{dx\left(d^2\left(b^2-2ac\right)+cx^2\left(bd^2-6ae^2\right)-3abe^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$\Big\downarrow \; 219$$

$$\frac{d\left(\frac{\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{12abe^2-12acd^2+b^2d^2}{\sqrt{b^2-4ac}}-6ae^2+bd^2\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-\frac{12abe^2-12acd^2+b^2d^2}{\sqrt{b^2-4ac}}-6ae^2+bd^2\right)}{\sqrt{2}\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2a\left(b^2-4ac\right)}+\frac{1}{2}e\left(\frac{2\left(6cd^2-be^2\right)\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}-\frac{-2ae^2+x^2\left(6cd^2-be^2\right)+3bd^2}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\right)+\frac{dx\left(d^2\left(b^2-2ac\right)+cx^2\left(bd^2-6ae^2\right)-3abe^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

input
```
Int[(d + e*x)^3/(a + b*x^2 + c*x^4)^2,x]
```

output
```
(d*x*((b^2 - 2*a*c)*d^2 - 3*a*b*e^2 + c*(b*d^2 - 6*a*e^2)*x^2))/(2*a*(b^2
- 4*a*c)*(a + b*x^2 + c*x^4)) + (d*((Sqrt[c]*(b*d^2 - 6*a*e^2 + (b^2*d^2 -
 12*a*c*d^2 + 12*a*b*e^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sq
rt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[
c]*(b*d^2 - 6*a*e^2 - (b^2*d^2 - 12*a*c*d^2 + 12*a*b*e^2)/Sqrt[b^2 - 4*a*c
])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[
b + Sqrt[b^2 - 4*a*c]])))/(2*a*(b^2 - 4*a*c)) + (e*(-((3*b*d^2 - 2*a*e^2 +
 (6*c*d^2 - b*e^2)*x^2)/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4))) + (2*(6*c*d^2
 - b*e^2)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)))/
2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1159
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Simp[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &
& LtQ[p, -1] && NeQ[p, -3/2]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1492
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2
 - 4*a*c))   Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p +
7)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 LtQ[p, -1] && IntegerQ[2*p]
```

rule 1576
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x]
, x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

rule 2202
```
Int[(Pn_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{n
 = Expon[Pn, x], k}, Int[Sum[Coeff[Pn, x, 2*k]*x^(2*k), {k, 0, n/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pn, x, 2*k + 1]*x^(2*k), {k, 0, (n -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pn, x]
 &&  !PolyQ[Pn, x^2]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.25 (sec) , antiderivative size = 293, normalized size of antiderivative = 0.66

method	result
risch	$\frac{\frac{cd\left(6a\,e^{2}-b\,d^{2}\right)x^{3}}{2\left(4ac-b^{2}\right)a}-\frac{e\left(b\,e^{2}-6c\,d^{2}\right)x^{2}}{2\left(4ac-b^{2}\right)}+\frac{d\left(3ab\,e^{2}+2ac\,d^{2}-b^{2}d^{2}\right)x}{2\left(4ac-b^{2}\right)a}-\frac{e\left(2a\,e^{2}-3b\,d^{2}\right)}{2\left(4ac-b^{2}\right)}}{c\,x^{4}+b\,x^{2}+a}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^{4}+\_Z^{2}b+a\right)}\frac{\left(\frac{cd\left(6a\,e^{2}-b\,d^{2}\right)\ldots}{\left(4ac-b^{2}\right)\ldots}\right.}{\ldots}\right.}{\ldots}$
default	$16c^{2}\left(-\frac{-\frac{d\left(-4\sqrt{-4ac+b^{2}}\,ac\,d^{2}+\sqrt{-4ac+b^{2}}\,b^{2}d^{2}+24a^{2}c\,e^{2}-6a\,b^{2}e^{2}-4abc\,d^{2}+b^{3}d^{2}\right)x}{16ac}-\frac{e\left(4\sqrt{-4ac+b^{2}}\,ac\,e^{2}-\sqrt{-4ac+b^{2}}\,b^{2}e^{2}-4abc\,e^{2}+\ldots\right.}{16c^{2}}}{x^{2}+\frac{b}{2c}-\frac{\sqrt{-4ac+b^{2}}}{2c}}\ldots\right.$

input
```
int((e*x+d)^3/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(1/2*c*d*(6*a*e^2-b*d^2)/(4*a*c-b^2)/a*x^3-1/2*e*(b*e^2-6*c*d^2)/(4*a*c-b^
2)*x^2+1/2*d*(3*a*b*e^2+2*a*c*d^2-b^2*d^2)/(4*a*c-b^2)/a*x-1/2*e*(2*a*e^2-
3*b*d^2)/(4*a*c-b^2))/(c*x^4+b*x^2+a)+1/4*sum((c*d*(6*a*e^2-b*d^2)/(4*a*c-
b^2)/a*_R^2-2*e*(b*e^2-6*c*d^2)/(4*a*c-b^2)*_R-d*(3*a*b*e^2-6*a*c*d^2+b^2*
d^2)/(4*a*c-b^2)/a)/(2*_R^3*c+_R*b)*ln(x-_R),_R=RootOf(_Z^4*c+_Z^2*b+a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(d+ex)^3}{(a+bx^2+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)^3/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^3}{(a+bx^2+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)**3/(c*x**4+b*x**2+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(d+ex)^3}{(a+bx^2+cx^4)^2}\,dx = \int \frac{(ex+d)^3}{(cx^4+bx^2+a)^2}\,dx$$

input
```
integrate((e*x+d)^3/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

output
```
-1/2*(3*a*b*d^2*e - 2*a^2*e^3 - (b*c*d^3 - 6*a*c*d*e^2)*x^3 + (6*a*c*d^2*e
 - a*b*e^3)*x^2 + (3*a*b*d*e^2 - (b^2 - 2*a*c)*d^3)*x)/((a*b^2*c - 4*a^2*c
^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2) - 1/2*integrate(-(3
*a*b*d*e^2 + (b^2 - 6*a*c)*d^3 + (b*c*d^3 - 6*a*c*d*e^2)*x^2 - 2*(6*a*c*d^
2*e - a*b*e^3)*x)/(c*x^4 + b*x^2 + a), x)/(a*b^2 - 4*a^2*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5647 vs. $2(396) = 792$.

Time = 1.17 (sec) , antiderivative size = 5647, normalized size of antiderivative = 12.66

$$\int \frac{(d+ex)^3}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^3/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/2*(b*c*d^3*x^3 - 6*a*c*d*e^2*x^3 - 6*a*c*d^2*e*x^2 + a*b*e^3*x^2 + b^2*d
^3*x - 2*a*c*d^3*x - 3*a*b*d*e^2*x - 3*a*b*d^2*e + 2*a^2*e^3)/((c*x^4 + b*
x^2 + a)*(a*b^2 - 4*a^2*c)) - 1/16*((2*b^3*c^2 - 8*a*b*c^3 - sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b*c^2)*(a*b^2 - 4*a^2*c)^2*
d^3 - 6*(2*a*b^2*c^2 - 8*a^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a*b*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
*c^2 - 2*(b^2 - 4*a*c)*a*c^2)*(a*b^2 - 4*a^2*c)^2*d*e^2 - 2*(sqrt(2)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6 - 14*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a^2*b^4*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c - 2*a*b^
6*c + 64*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^2 + 20*sqrt(2)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a*b^4*c^2 + 28*a^2*b^4*c^2 - 96*sqrt(2)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a^4*c^3 - 48*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 -
 10*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 128*a^3*b^2*c^3
+ 24*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^4 + 192*a^4*c^4 + 2*...
```

Mupad [B] (verification not implemented)

Time = 22.48 (sec) , antiderivative size = 5441, normalized size of antiderivative = 12.20

$$\int \frac{(d+ex)^3}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
int((d + e*x)^3/(a + b*x^2 + c*x^4)^2,x)
```

output
```
symsum(log(root(1572864*a^8*b^2*c^5*z^4 - 983040*a^7*b^4*c^4*z^4 + 327680*
a^6*b^6*c^3*z^4 - 61440*a^5*b^8*c^2*z^4 + 6144*a^4*b^10*c*z^4 - 1048576*a^
9*c^6*z^4 - 256*a^3*b^12*z^4 + 208896*a^6*b*c^4*d^2*e^4*z^2 + 1728*a^2*b^8
*c*d^4*e^2*z^2 - 1536*a^3*b^7*c*d^2*e^4*z^2 + 294912*a^5*b^2*c^4*d^4*e^2*z
^2 - 147456*a^5*b^3*c^3*d^2*e^4*z^2 - 49152*a^4*b^4*c^3*d^4*e^2*z^2 + 3225
6*a^4*b^5*c^2*d^2*e^4*z^2 - 4608*a^3*b^6*c^2*d^4*e^2*z^2 - 96*a*b^10*d^4*e
^2*z^2 - 1536*a^4*b^6*c*e^6*z^2 + 61440*a^5*b*c^5*d^6*z^2 + 432*a*b^9*c*d^
6*z^2 - 442368*a^6*c^5*d^4*e^2*z^2 - 144*a^2*b^9*d^2*e^4*z^2 - 8192*a^6*b^
2*c^3*e^6*z^2 + 6144*a^5*b^4*c^2*e^6*z^2 - 61440*a^4*b^3*c^4*d^6*z^2 + 240
64*a^3*b^5*c^3*d^6*z^2 - 4608*a^2*b^7*c^2*d^6*z^2 + 128*a^3*b^8*e^6*z^2 -
16*b^11*d^6*z^2 - 2016*a*b^6*c^2*d^8*e*z + 912*a*b^7*c*d^6*e^3*z + 47616*a
^4*b^2*c^3*d^4*e^5*z + 35584*a^3*b^3*c^3*d^6*e^3*z - 14976*a^3*b^4*c^2*d^4
*e^5*z - 9408*a^2*b^5*c^2*d^6*e^3*z - 6912*a^4*b^3*c^2*d^2*e^7*z - 47616*a
^3*b^2*c^4*d^8*e*z - 46080*a^4*b*c^4*d^6*e^3*z + 14976*a^2*b^4*c^3*d^8*e*z
 + 9216*a^5*b*c^3*d^2*e^7*z + 2016*a^2*b^6*c*d^4*e^5*z + 1728*a^3*b^5*c*d^
2*e^7*z - 55296*a^5*c^4*d^4*e^5*z - 144*a^2*b^7*d^2*e^7*z + 55296*a^4*c^5*
d^8*e*z - 96*a*b^8*d^4*e^5*z + 96*b^8*c*d^8*e*z - 16*b^9*d^6*e^3*z - 2592*
a^3*b*c^3*d^6*e^6 - 2592*a^2*b*c^4*d^10*e^2 + 672*a*b^3*c^3*d^10*e^2 + 198
*a*b^4*c^2*d^8*e^4 - 153*a^2*b^4*c*d^4*e^8 - 48*a^3*b^3*c*d^2*e^10 - 138*a
*b^5*c*d^6*e^6 + 624*a^2*b^3*c^2*d^6*e^6 - 576*a^2*b^2*c^3*d^8*e^4 + 36...
```

Reduce [B] (verification not implemented)

Time = 0.67 (sec) , antiderivative size = 5447, normalized size of antiderivative = 12.21

$$\int \frac{(d+ex)^3}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
int((e*x+d)^3/(c*x^4+b*x^2+a)^2,x)
```

output
```
(8*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sq
rt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**3*b**2*e
**3 - 48*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqr
t(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**3*
b*c*d**2*e + 8*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*ata
n((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))
*a**2*b**3*e**3*x**2 - 48*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(
a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sq
rt(a) + b))*a**2*b**2*c*d**2*e*x**2 + 8*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2
*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqr
t(2*sqrt(c)*sqrt(a) + b))*a**2*b**2*c*e**3*x**4 - 48*sqrt(2*sqrt(c)*sqrt(a
) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*s
qrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*b*c**2*d**2*e*x**4 - 24*sqrt(a
)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c
)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**3*b*c*d*e**2 - 6*sqrt(a)*sqrt(2*sqrt(
c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sq
rt(c)*sqrt(a) + b))*a**2*b**3*d*e**2 + 16*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) +
 b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a
) + b))*a**2*b**2*c*d**3 - 24*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sq
rt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a...
```

3.242 $\int \frac{(d+ex)^2}{(a+bx^2+cx^4)^2} dx$

Optimal result

Integrand size = 22, antiderivative size = 410

$$\begin{aligned}
&\int \frac{(d+ex)^2}{(a+bx^2+cx^4)^2} dx \\
&= -\frac{de(b+2cx^2)}{(b^2-4ac)(a+bx^2+cx^4)} + \frac{x((b^2-2ac)d^2-abe^2+c(bd^2-2ae^2)x^2)}{2a(b^2-4ac)(a+bx^2+cx^4)} \\
&+ \frac{\sqrt{c}\left(b^2d^2+b\left(\sqrt{b^2-4ac}d^2+4ae^2\right)-2a\left(6cd^2+\sqrt{b^2-4ac}e^2\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} \\
&+ \frac{\sqrt{c}\left(bd^2-2ae^2-\frac{b^2d^2-12acd^2+4abe^2}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a(b^2-4ac)\sqrt{b+\sqrt{b^2-4ac}}} + \frac{4cde\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}
\end{aligned}$$

output
```
-d*e*(2*c*x^2+b)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+1/2*x*((-2*a*c+b^2)*d^2-a*b*
e^2+c*(-2*a*e^2+b*d^2)*x^2)/a/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+1/4*c^(1/2)*(b^
2*d^2+b*((-4*a*c+b^2)^(1/2)*d^2+4*a*e^2)-2*a*(6*c*d^2+(-4*a*c+b^2)^(1/2)*e
^2))*arctan(2^(1/2)*c^(1/2)*x/(b-(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/a/(-4*
a*c+b^2)^(3/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/4*c^(1/2)*(b*d^2-2*a*e^2-(4*
a*b*e^2-12*a*c*d^2+b^2*d^2)/(-4*a*c+b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)*x/(
b+(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/a/(-4*a*c+b^2)/(b+(-4*a*c+b^2)^(1/2))
^(1/2)+4*c*d*e*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(3/2)
```

Mathematica [A] (verified)

Time = 1.42 (sec) , antiderivative size = 423, normalized size of antiderivative = 1.03

$$\int \frac{(d+ex)^2}{(a+bx^2+cx^4)^2}\,dx = \frac{1}{4}\left(\frac{4acx(d+ex)^2+2abe(2d+ex)-2bd^2x(b+cx^2)}{a(-b^2+4ac)(a+bx^2+cx^4)} + \frac{\sqrt{2}\sqrt{c}\left(b^2d^2+b\left(\sqrt{b^2-4ac}d^2+4ae^2\right)-2a\left(6cd^2+\sqrt{b^2-4ac}e^2\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{a(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{2}\sqrt{c}\left(-b^2d^2+12acd^2+b\sqrt{b^2-4ac}d^2-4abe^2-2a\sqrt{b^2-4ac}e^2\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{a(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}} - \frac{8cde\log\left(-b+\sqrt{b^2-4ac}-2cx^2\right)}{(b^2-4ac)^{3/2}} + \frac{8cde\log\left(b+\sqrt{b^2-4ac}+2cx^2\right)}{(b^2-4ac)^{3/2}}\right)$$

input

```
Integrate[(d + e*x)^2/(a + b*x^2 + c*x^4)^2,x]
```

output

```
((4*a*c*x*(d + e*x)^2 + 2*a*b*e*(2*d + e*x) - 2*b*d^2*x*(b + c*x^2))/(a*(-
b^2 + 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*Sqrt[c]*(b^2*d^2 + b*(Sqrt[b^
2 - 4*a*c]*d^2 + 4*a*e^2) - 2*a*(6*c*d^2 + Sqrt[b^2 - 4*a*c]*e^2))*ArcTan[
(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(a*(b^2 - 4*a*c)^(3/2)*S
qrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(-(b^2*d^2) + 12*a*c*d^2 +
b*Sqrt[b^2 - 4*a*c]*d^2 - 4*a*b*e^2 - 2*a*Sqrt[b^2 - 4*a*c]*e^2)*ArcTan[(S
qrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(a*(b^2 - 4*a*c)^(3/2)*Sqr
t[b + Sqrt[b^2 - 4*a*c]]) - (8*c*d*e*Log[-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2]
)/(b^2 - 4*a*c)^(3/2) + (8*c*d*e*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^
2 - 4*a*c)^(3/2))/4
```

Rubi [A] (verified)

Time = 1.13 (sec) , antiderivative size = 382, normalized size of antiderivative = 0.93, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {2202, 27, 1432, 1086, 1083, 219, 1492, 25, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^2}{(a+bx^2+cx^4)^2}\,dx$$

$$\Big\downarrow \; 2202$$

$$\int \frac{d^2+e^2x^2}{(cx^4+bx^2+a)^2}dx+\int \frac{2dex}{(cx^4+bx^2+a)^2}dx$$

$$\Big\downarrow \; 27$$

$$\int \frac{d^2+e^2x^2}{(cx^4+bx^2+a)^2}dx+2de\int \frac{x}{(cx^4+bx^2+a)^2}dx$$

$$\Big\downarrow \; 1432$$

$$\int \frac{d^2+e^2x^2}{(cx^4+bx^2+a)^2}dx+de\int \frac{1}{(cx^4+bx^2+a)^2}dx^2$$

$$\Big\downarrow \; 1086$$

$$de\left(-\frac{2c\int \frac{1}{cx^4+bx^2+a}dx^2}{b^2-4ac}-\frac{b+2cx^2}{(b^2-4ac)\,(a+bx^2+cx^4)}\right)+\int \frac{d^2+e^2x^2}{(cx^4+bx^2+a)^2}dx$$

$$\Big\downarrow \; 1083$$

$$de\left(\frac{4c\int \frac{1}{-x^4+b^2-4ac}d(2cx^2+b)}{b^2-4ac}-\frac{b+2cx^2}{(b^2-4ac)\,(a+bx^2+cx^4)}\right)+\int \frac{d^2+e^2x^2}{(cx^4+bx^2+a)^2}dx$$

$$\Big\downarrow \; 219$$

$$\int \frac{d^2+e^2x^2}{(cx^4+bx^2+a)^2}dx+de\left(\frac{4c\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}-\frac{b+2cx^2}{(b^2-4ac)\,(a+bx^2+cx^4)}\right)$$

$$\Big\downarrow \; 1492$$

$$-\frac{\int -\frac{b^2d^2-6acd^2+abe^2+c(bd^2-2ae^2)x^2}{cx^4+bx^2+a}dx}{2a\,(b^2-4ac)}+$$
$$de\left(\frac{4c\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}-\frac{b+2cx^2}{(b^2-4ac)\,(a+bx^2+cx^4)}\right)+$$
$$\frac{x\left(d^2\left(b^2-2ac\right)+cx^2\left(bd^2-2ae^2\right)-abe^2\right)}{2a\,(b^2-4ac)\,(a+bx^2+cx^4)}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{b^2d^2-6acd^2+abe^2+c(bd^2-2ae^2)x^2}{cx^4+bx^2+a}dx}{2a(b^2-4ac)}+de\left(\frac{4c\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}-\frac{b+2cx^2}{(b^2-4ac)(a+bx^2+cx^4)}\right)+\frac{x(d^2(b^2-2ac)+cx^2(bd^2-2ae^2)-abe^2)}{2a(b^2-4ac)(a+bx^2+cx^4)}$$

$\Big\downarrow$ 1480

$$\frac{\frac{1}{2}c\left(\frac{4abe^2-12acd^2+b^2d^2}{\sqrt{b^2-4ac}}-2ae^2+bd^2\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx+\frac{1}{2}c\left(-\frac{4abe^2-12acd^2+b^2d^2}{\sqrt{b^2-4ac}}-2ae^2+bd^2\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b+\sqrt{b}\right.}}{2a(b^2-4ac)}$$
$$de\left(\frac{4c\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}-\frac{b+2cx^2}{(b^2-4ac)(a+bx^2+cx^4)}\right)+\frac{x(d^2(b^2-2ac)+cx^2(bd^2-2ae^2)-abe^2)}{2a(b^2-4ac)(a+bx^2+cx^4)}$$

$\Big\downarrow$ 218

$$\frac{\frac{\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{4abe^2-12acd^2+b^2d^2}{\sqrt{b^2-4ac}}-2ae^2+bd^2\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-\frac{4abe^2-12acd^2+b^2d^2}{\sqrt{b^2-4ac}}-2ae^2+bd^2\right)}{\sqrt{2}\sqrt{\sqrt{b^2-4ac}+b}}}{2a(b^2-4ac)}+$$
$$de\left(\frac{4c\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}-\frac{b+2cx^2}{(b^2-4ac)(a+bx^2+cx^4)}\right)+\frac{x(d^2(b^2-2ac)+cx^2(bd^2-2ae^2)-abe^2)}{2a(b^2-4ac)(a+bx^2+cx^4)}$$

input
```
Int[(d + e*x)^2/(a + b*x^2 + c*x^4)^2,x]
```

output
```
(x*((b^2 - 2*a*c)*d^2 - a*b*e^2 + c*(b*d^2 - 2*a*e^2)*x^2))/(2*a*(b^2 - 4*
a*c)*(a + b*x^2 + c*x^4)) + ((Sqrt[c]*(b*d^2 - 2*a*e^2 + (b^2*d^2 - 12*a*c
*d^2 + 4*a*b*e^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - S
qrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[c]*(b*d^
2 - 2*a*e^2 - (b^2*d^2 - 12*a*c*d^2 + 4*a*b*e^2)/Sqrt[b^2 - 4*a*c])*ArcTan
[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b + Sqrt[
b^2 - 4*a*c]]))/(2*a*(b^2 - 4*a*c)) + d*e*(-((b + 2*c*x^2)/((b^2 - 4*a*c)*
(a + b*x^2 + c*x^4))) + (4*c*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^
2 - 4*a*c)^(3/2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1086
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && ILtQ[p, -1]
```

rule 1432
```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1492
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2
 - 4*a*c))   Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p +
7)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 LtQ[p, -1] && IntegerQ[2*p]
```

rule 2202
```
Int[(Pn_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{n
 = Expon[Pn, x], k}, Int[Sum[Coeff[Pn, x, 2*k]*x^(2*k), {k, 0, n/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pn, x, 2*k + 1]*x^(2*k), {k, 0, (n -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pn, x]
 &&  !PolyQ[Pn, x^2]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.22 (sec) , antiderivative size = 255, normalized size of antiderivative = 0.62

method	result
risch	$\frac{\frac{c\left(2a\,e^{2}-b\,d^{2}\right)x^{3}}{2\left(4ac-b^{2}\right)a}+\frac{2cde\,x^{2}}{4ac-b^{2}}+\frac{\left(ab\,e^{2}+2ac\,d^{2}-b^{2}d^{2}\right)x}{2a\left(4ac-b^{2}\right)}+\frac{bde}{4ac-b^{2}}}{c\,x^{4}+b\,x^{2}+a}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^{4}+\_Z^{2}b+a\right)}\frac{\left(\frac{c\left(2a\,e^{2}-b\,d^{2}\right)\_R^{2}}{\left(4ac-b^{2}\right)a}+\frac{8cde\_R}{4ac-b^{2}}-a\right.}{2\_R^{3}c+b}\right.}{4}$
default	$16c^{2}\left(-\frac{-\frac{\left(-4\sqrt{-4ac+b^{2}}\,ac\,d^{2}+\sqrt{-4ac+b^{2}}\,b^{2}d^{2}+8a^{2}c\,e^{2}-2a\,b^{2}e^{2}-4abc\,d^{2}+b^{3}d^{2}\right)x}{16ac}-\frac{de\left(4ac-b^{2}\right)}{4c}}{x^{2}+\frac{b}{2c}-\frac{\sqrt{-4ac+b^{2}}}{2c}}+\frac{4\sqrt{-4ac+b^{2}}\,ade\ln\left(-2c\,x^{2}+\sqrt{-4ac\right.}{4c}\right.$

input
```
int((e*x+d)^2/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(1/2*c*(2*a*e^2-b*d^2)/(4*a*c-b^2)/a*x^3+2*c*d*e/(4*a*c-b^2)*x^2+1/2*(a*b*
e^2+2*a*c*d^2-b^2*d^2)/a/(4*a*c-b^2)*x+b*d*e/(4*a*c-b^2))/(c*x^4+b*x^2+a)+
1/4*sum((c*(2*a*e^2-b*d^2)/(4*a*c-b^2)/a*_R^2+8*c*d*e/(4*a*c-b^2)*_R-(a*b*
e^2-6*a*c*d^2+b^2*d^2)/(4*a*c-b^2)/a)/(2*_R^3*c+_R*b)*ln(x-_R),_R=RootOf(_
Z^4*c+_Z^2*b+a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(d+ex)^2}{(a+bx^2+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)^2/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^2}{(a+bx^2+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)**2/(c*x**4+b*x**2+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(d+ex)^2}{(a+bx^2+cx^4)^2}\,dx = \int \frac{(ex+d)^2}{(cx^4+bx^2+a)^2}\,dx$$

input
```
integrate((e*x+d)^2/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

output
```
-1/2*(4*a*c*d*e*x^2 + 2*a*b*d*e - (b*c*d^2 - 2*a*c*e^2)*x^3 + (a*b*e^2 - (
b^2 - 2*a*c)*d^2)*x)/((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b
^3 - 4*a^2*b*c)*x^2) - 1/2*integrate((8*a*c*d*e*x - a*b*e^2 - (b^2 - 6*a*c
)*d^2 - (b*c*d^2 - 2*a*c*e^2)*x^2)/(c*x^4 + b*x^2 + a), x)/(a*b^2 - 4*a^2*
c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5199 vs. 2(361) = 722.

Time = 1.37 (sec) , antiderivative size = 5199, normalized size of antiderivative = 12.68

$$\int \frac{(d+ex)^2}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^2/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/2*(b*c*d^2*x^3 - 2*a*c*e^2*x^3 - 4*a*c*d*e*x^2 + b^2*d^2*x - 2*a*c*d^2*x
 - a*b*e^2*x - 2*a*b*d*e)/((c*x^4 + b*x^2 + a)*(a*b^2 - 4*a^2*c)) + 1/16*(
(2*b^3*c^2 - 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c
 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^
2 - 4*a*c)*b*c^2)*(a*b^2 - 4*a^2*c)^2*d^2 - 2*(2*a*b^2*c^2 - 8*a^2*c^3 - s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2 + 4*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c + 2*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c - sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^2 - 2*(b^2 - 4*a*c)*a*c^2)*(a*b^2
 - 4*a^2*c)^2*e^2 + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6 - 14*
sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c - 2*sqrt(2)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a*b^5*c - 2*a*b^6*c + 64*sqrt(2)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a^3*b^2*c^2 + 20*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^
3*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 + 28*a^2*b^4*c^2
 - 96*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*c^3 - 48*sqrt(2)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 - 10*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a^2*b^2*c^3 - 128*a^3*b^2*c^3 + 24*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a^3*c^4 + 192*a^4*c^4 + 2*(b^2 - 4*a*c)*a*b^4*c - 20*(b^2 - 4*a*...
```

Mupad [B] (verification not implemented)

Time = 22.38 (sec) , antiderivative size = 4118, normalized size of antiderivative = 10.04

$$\int \frac{(d+ex)^2}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
int((d + e*x)^2/(a + b*x^2 + c*x^4)^2,x)
```

output
```
symsum(log((8*a^3*c^4*e^6 + 5*b^3*c^4*d^6 + 6*a^2*b^2*c^3*e^6 - 312*a^2*c^
5*d^4*e^2 - 3*b^4*c^3*d^4*e^2 - 36*a*b*c^5*d^6 + 82*a*b^2*c^4*d^4*e^2 + 3*
a*b^3*c^3*d^2*e^4 + 4*a^2*b*c^4*d^2*e^4)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3
*b^4*c + 48*a^4*b^2*c^2)) - root(1572864*a^8*b^2*c^5*z^4 - 983040*a^7*b^4*
c^4*z^4 + 327680*a^6*b^6*c^3*z^4 - 61440*a^5*b^8*c^2*z^4 + 6144*a^4*b^10*c
*z^4 - 1048576*a^9*c^6*z^4 - 256*a^3*b^12*z^4 + 576*a^2*b^8*c*d^2*e^2*z^2
+ 122880*a^5*b^2*c^4*d^2*e^2*z^2 - 22528*a^4*b^4*c^3*d^2*e^2*z^2 - 1024*a^
3*b^6*c^2*d^2*e^2*z^2 - 32*a*b^10*d^2*e^2*z^2 + 12288*a^6*b*c^4*e^4*z^2 +
61440*a^5*b*c^5*d^4*z^2 + 432*a*b^9*c*d^4*z^2 - 180224*a^6*c^5*d^2*e^2*z^2
 - 8192*a^5*b^3*c^3*e^4*z^2 + 1536*a^4*b^5*c^2*e^4*z^2 - 61440*a^4*b^3*c^4
*d^4*z^2 + 24064*a^3*b^5*c^3*d^4*z^2 - 4608*a^2*b^7*c^2*d^4*z^2 - 16*a^2*b
^9*e^4*z^2 - 16*b^11*d^4*z^2 - 1344*a*b^6*c^2*d^5*e*z + 128*a*b^7*c*d^3*e^
3*z + 64*a^2*b^6*c*d*e^5*z + 6144*a^3*b^3*c^3*d^3*e^3*z - 1536*a^2*b^5*c^2
*d^3*e^3*z - 31744*a^3*b^2*c^4*d^5*e*z + 9984*a^2*b^4*c^3*d^5*e*z - 8192*a
^4*b*c^4*d^3*e^3*z + 3072*a^4*b^2*c^3*d*e^5*z - 768*a^3*b^4*c^2*d*e^5*z +
36864*a^4*c^5*d^5*e*z - 4096*a^5*c^4*d*e^5*z + 64*b^8*c*d^5*e*z - 1824*a^2
*b*c^4*d^6*e^2 - 544*a^3*b*c^3*d^2*e^6 + 464*a*b^3*c^3*d^6*e^2 + 70*a*b^4*
c^2*d^4*e^4 - 18*a*b^5*c*d^2*e^6 - 192*a^2*b^2*c^3*d^4*e^4 + 80*a^2*b^3*c^
2*d^2*e^6 - 34*b^5*c^2*d^6*e^2 - 1312*a^3*c^4*d^4*e^4 - 24*a^3*b^2*c^2*e^8
 - 9*b^6*c*d^4*e^4 - 9*a^2*b^4*c*e^8 + 360*a*b^2*c^4*d^8 - 25*b^4*c^3*d...
```

Reduce [B] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 4951, normalized size of antiderivative = 12.08

$$\int \frac{(d+ex)^2}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
int((e*x+d)^2/(c*x^4+b*x^2+a)^2,x)
```

output

```
( - 32*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(
2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**3*b*
c*d*e - 32*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((s
qrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**
2*b**2*c*d*e*x**2 - 32*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a)
- b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(
a) + b))*a**2*b*c**2*d*e*x**4 - 8*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan
((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*
a**3*b*c*e**2 - 2*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)
*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*b**3*e**2 +
 16*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b)
- 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*b**2*c*d**2 - 8*sqrt(a)*s
qrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x
)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*b**2*c*e**2*x**2 - 8*sqrt(a)*sqrt(2*sq
rt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2
*sqrt(c)*sqrt(a) + b))*a**2*b*c**2*e**2*x**4 - 2*sqrt(a)*sqrt(2*sqrt(c)*sq
rt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)
*sqrt(a) + b))*a*b**4*d**2 - 2*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((s
qrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b
**4*e**2*x**2 + 16*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqr...
```

3.243 $\int \frac{d+ex}{(a+bx^2+cx^4)^2} dx$

Optimal result

Integrand size = 20, antiderivative size = 330

$$\int \frac{d+ex}{(a+bx^2+cx^4)^2} dx = -\frac{e(b+2cx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} + \frac{dx(b^2-2ac+bcx^2)}{2a(b^2-4ac)(a+bx^2+cx^4)} + \frac{\sqrt{c}\left(b^2-12ac+b\sqrt{b^2-4ac}\right) d \arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{c}\left(b^2-12ac-b\sqrt{b^2-4ac}\right) d \arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}} + \frac{2ce\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}$$

output

```
-1/2*e*(2*c*x^2+b)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+1/2*d*x*(b*c*x^2-2*a*c+b^2
)/a/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+1/4*c^(1/2)*(b^2-12*a*c+b*(-4*a*c+b^2)^(1
/2))*d*arctan(2^(1/2)*c^(1/2)*x/(b-(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/a/(-
4*a*c+b^2)^(3/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/4*c^(1/2)*(b^2-12*a*c-b*(-
4*a*c+b^2)^(1/2))*d*arctan(2^(1/2)*c^(1/2)*x/(b+(-4*a*c+b^2)^(1/2))^(1/2))
*2^(1/2)/a/(-4*a*c+b^2)^(3/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)+2*c*e*arctanh((
2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(3/2)
```

Mathematica [A] (verified)

Time = 0.97 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.03

$$\int \frac{d+ex}{(a+bx^2+cx^4)^2}\,dx = \frac{1}{4}\left(\frac{2abe+4acx(d+ex)-2bdx(b+cx^2)}{a(-b^2+4ac)(a+bx^2+cx^4)} + \frac{\sqrt{2}\sqrt{c}\left(b^2-12ac+b\sqrt{b^2-4ac}\right)d\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{a(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{2}\sqrt{c}\left(-b^2+12ac+b\sqrt{b^2-4ac}\right)d\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{a(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}} - \frac{4ce\log\left(-b+\sqrt{b^2-4ac}-2cx^2\right)}{(b^2-4ac)^{3/2}} + \frac{4ce\log\left(b+\sqrt{b^2-4ac}+2cx^2\right)}{(b^2-4ac)^{3/2}}\right)$$

input

```
Integrate[(d + e*x)/(a + b*x^2 + c*x^4)^2,x]
```

output

```
((2*a*b*e + 4*a*c*x*(d + e*x) - 2*b*d*x*(b + c*x^2))/(a*(-b^2 + 4*a*c)*(a
+ b*x^2 + c*x^4)) + (Sqrt[2]*Sqrt[c]*(b^2 - 12*a*c + b*Sqrt[b^2 - 4*a*c])*
d*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(a*(b^2 - 4*a*c
)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(-b^2 + 12*a*c + b
*Sqrt[b^2 - 4*a*c])*d*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c
]]])/(a*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - (4*c*e*Log[-b +
 Sqrt[b^2 - 4*a*c] - 2*c*x^2])/(b^2 - 4*a*c)^(3/2) + (4*c*e*Log[b + Sqrt[b
^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(3/2))/4
```

Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 316, normalized size of antiderivative = 0.96, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {2202, 27, 1405, 25, 1432, 1086, 1083, 219, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex}{(a+bx^2+cx^4)^2}\,dx$$

$\Big\downarrow$ 2202

$$\int \frac{d}{(cx^4+bx^2+a)^2}dx+\int \frac{ex}{(cx^4+bx^2+a)^2}dx$$

$\Big\downarrow$ 27

$$d\int \frac{1}{(cx^4+bx^2+a)^2}dx+e\int \frac{x}{(cx^4+bx^2+a)^2}dx$$

$\Big\downarrow$ 1405

$$d\left(\frac{x\left(-2ac+b^2+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\int -\frac{b^2+cx^2b-6ac}{cx^4+bx^2+a}dx}{2a\left(b^2-4ac\right)}\right)+e\int \frac{x}{(cx^4+bx^2+a)^2}dx$$

$\Big\downarrow$ 25

$$d\left(\frac{\int \frac{b^2+cx^2b-6ac}{cx^4+bx^2+a}dx}{2a\left(b^2-4ac\right)}+\frac{x\left(-2ac+b^2+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\right)+e\int \frac{x}{(cx^4+bx^2+a)^2}dx$$

$\Big\downarrow$ 1432

$$d\left(\frac{\int \frac{b^2+cx^2b-6ac}{cx^4+bx^2+a}dx}{2a\left(b^2-4ac\right)}+\frac{x\left(-2ac+b^2+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\right)+\frac{1}{2}e\int \frac{1}{(cx^4+bx^2+a)^2}dx^2$$

$\Big\downarrow$ 1086

$$d\left(\frac{\int \frac{b^2+cx^2b-6ac}{cx^4+bx^2+a}dx}{2a\left(b^2-4ac\right)}+\frac{x\left(-2ac+b^2+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\right)+$$
$$\frac{1}{2}e\left(-\frac{2c\int \frac{1}{cx^4+bx^2+a}dx^2}{b^2-4ac}-\frac{b+2cx^2}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\right)$$

$\Big\downarrow$ 1083

$$d\left(\frac{\int \frac{b^2+cx^2b-6ac}{cx^4+bx^2+a}dx}{2a\left(b^2-4ac\right)}+\frac{x\left(-2ac+b^2+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\right)+$$
$$\frac{1}{2}e\left(\frac{4c\int \frac{1}{-x^4+b^2-4ac}d\left(2cx^2+b\right)}{b^2-4ac}-\frac{b+2cx^2}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\right)$$

$\Big\downarrow$ 219

$$d\left(\frac{\int \frac{b^2+cx^2b-6ac}{cx^4+bx^2+a}dx}{2a\left(b^2-4ac\right)}+\frac{x\left(-2ac+b^2+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\right)+\frac{1}{2}e\left(\frac{4c\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}-\frac{b+2cx^2}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\right)$$

$\downarrow$ 1480

$$d\left(\frac{\frac{1}{2}c\left(\frac{b^2-12ac}{\sqrt{b^2-4ac}}+b\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx+\frac{1}{2}c\left(b-\frac{b^2-12ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}dx}{2a\left(b^2-4ac\right)}+\frac{x\left(-2ac+b^2+bcx^2\right.}{2a\left(b^2-4ac\right)\left(a+bx^2+\right.}\right.$$

$$\frac{1}{2}e\left(\frac{4c\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}-\frac{b+2cx^2}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\right)$$

$\downarrow$ 218

$$d\left(\frac{\frac{\sqrt{c}\left(\frac{b^2-12ac}{\sqrt{b^2-4ac}}+b\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(b-\frac{b^2-12ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{\sqrt{b^2-4ac}+b}}}{2a\left(b^2-4ac\right)}+\frac{x\left(-2ac+b^2+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\right)+$$

$$\frac{1}{2}e\left(\frac{4c\,\text{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}-\frac{b+2cx^2}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}\right)$$

input
```
Int[(d + e*x)/(a + b*x^2 + c*x^4)^2,x]
```

output
```
d*((x*(b^2 - 2*a*c + b*c*x^2))/(2*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (
(Sqrt[c]*(b + (b^2 - 12*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)
/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sq
rt[c]*(b - (b^2 - 12*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sq
rt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(2*a*(b
^2 - 4*a*c))) + (e*(-((b + 2*c*x^2)/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4))) +
 (4*c*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1086
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && ILtQ[p, -1]
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1432
```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 2202
```
Int[(Pn_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{n
 = Expon[Pn, x], k}, Int[Sum[Coeff[Pn, x, 2*k]*x^(2*k), {k, 0, n/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pn, x, 2*k + 1]*x^(2*k), {k, 0, (n -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pn, x]
 &&  !PolyQ[Pn, x^2]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.19 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.63

method	result
risch	$\frac{-\frac{x^3 bcd}{2\left(4ac-b^2\right)a}+\frac{x^2 ce}{4ac-b^2}+\frac{d\left(2ac-b^2\right)x}{2a\left(4ac-b^2\right)}+\frac{eb}{8ac-2b^2}}{c x^4+b x^2+a}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+\_Z^2 b+a\right)}\frac{\left(-\frac{\_R^2 bcd}{\left(4ac-b^2\right)a}+\frac{4ce\_R}{4ac-b^2}+\frac{d\left(6ac-b^2\right)}{a\left(4ac-b^2\right)}\right)\ln\left(x-\_\cdots\right.}{2\_R^3 c+b\_R}\cdots}{4}$
default	$16c^2\left(-\frac{\frac{\frac{\left(4\sqrt{-4ac+b^2}\,ac-\sqrt{-4ac+b^2}\,b^2+4abc-b^3\right)dx}{16a\,c^2}-\frac{e\left(4ac-b^2\right)}{8c^2}}{x^2+\frac{b}{2c}-\frac{\sqrt{-4ac+b^2}}{2c}}+\frac{2\sqrt{-4ac+b^2}\,ae\ln\left(-2c\,x^2+\sqrt{-4ac+b^2}-b\right)+\frac{\left(-12\sqrt{-4ac+b^2}\,acd+\cdots\right.}{\cdots}}{8c\cdots}}{4\left(4ac-b^2\right)^2}\cdots\right.$

input
```
int((e*x+d)/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(-1/2/(4*a*c-b^2)/a*x^3*b*c*d+1/(4*a*c-b^2)*x^2*c*e+1/2*d*(2*a*c-b^2)/a/(4
*a*c-b^2)*x+1/2*b*e/(4*a*c-b^2))/(c*x^4+b*x^2+a)+1/4*sum((-1/(4*a*c-b^2)/a
*_R^2*b*c*d+4*c*e/(4*a*c-b^2)*_R+d*(6*a*c-b^2)/a/(4*a*c-b^2))/(2*_R^3*c+_R
*b)*ln(x-_R),_R=RootOf(_Z^4*c+_Z^2*b+a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{d+ex}{(a+bx^2+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{d+ex}{(a+bx^2+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)/(c*x**4+b*x**2+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{d+ex}{\left(a+bx^2+cx^4\right)^2}\,dx=\int \frac{ex+d}{\left(cx^4+bx^2+a\right)^2}\,dx$$

input
```
integrate((e*x+d)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/2*(b*c*d*x^3 - 2*a*c*e*x^2 - a*b*e + (b^2 - 2*a*c)*d*x)/((a*b^2*c - 4*a^
2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2) + 1/2*integrate(
(b*c*d*x^2 - 4*a*c*e*x + (b^2 - 6*a*c)*d)/(c*x^4 + b*x^2 + a), x)/(a*b^2 -
 4*a^2*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3429 vs. $2(278) = 556$.

Time = 1.02 (sec) , antiderivative size = 3429, normalized size of antiderivative = 10.39

$$\int \frac{d+ex}{\left(a+bx^2+cx^4\right)^2}\,dx=\text{Too large to display}$$

input
```
integrate((e*x+d)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/2*(b*c*d*x^3 - 2*a*c*e*x^2 + b^2*d*x - 2*a*c*d*x - a*b*e)/((c*x^4 + b*x^
2 + a)*(a*b^2 - 4*a^2*c)) - 1/16*((2*b^3*c^2 - 8*a*b*c^3 - sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b*c^2)*(a*b^2 - 4*a^2*c)^2*d
- 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6 - 14*sqrt(2)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a*b^5*c - 2*a*b^6*c + 64*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2
*c^2 + 20*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 + sqrt(2)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 + 28*a^2*b^4*c^2 - 96*sqrt(2)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*c^3 - 48*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a^3*b*c^3 - 10*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3
- 128*a^3*b^2*c^3 + 24*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^4 + 1
92*a^4*c^4 + 2*(b^2 - 4*a*c)*a*b^4*c - 20*(b^2 - 4*a*c)*a^2*b^2*c^2 + 48*(
b^2 - 4*a*c)*a^3*c^3)*d*abs(a*b^2 - 4*a^2*c) + (2*a^2*b^7*c^2 - 40*a^3*b^5
*c^3 + 224*a^4*b^3*c^4 - 384*a^5*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^2*b^7 + 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a^2*b^6*c - 112*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c ...
```

Mupad [B] (verification not implemented)

Time = 22.96 (sec) , antiderivative size = 2382, normalized size of antiderivative = 7.22

$$\int \frac{d+ex}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
int((d + e*x)/(a + b*x^2 + c*x^4)^2,x)
```

output
```
((b*e)/(2*(4*a*c - b^2)) + (c*e*x^2)/(4*a*c - b^2) + (d*x*(2*a*c - b^2))/(
2*a*(4*a*c - b^2)) - (b*c*d*x^3)/(2*a*(4*a*c - b^2)))/(a + b*x^2 + c*x^4)
+ symsum(log((5*b^3*c^4*d^3 - 96*a^2*c^5*d*e^2 - 36*a*b*c^5*d^3 + 16*a*b^2
*c^4*d*e^2)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) - r
oot(1572864*a^8*b^2*c^5*z^4 - 983040*a^7*b^4*c^4*z^4 + 327680*a^6*b^6*c^3*
z^4 - 61440*a^5*b^8*c^2*z^4 + 6144*a^4*b^10*c*z^4 - 1048576*a^9*c^6*z^4 -
256*a^3*b^12*z^4 + 61440*a^5*b*c^5*d^2*z^2 + 432*a*b^9*c*d^2*z^2 + 24576*a
^5*b^2*c^4*e^2*z^2 - 6144*a^4*b^4*c^3*e^2*z^2 + 512*a^3*b^6*c^2*e^2*z^2 -
61440*a^4*b^3*c^4*d^2*z^2 + 24064*a^3*b^5*c^3*d^2*z^2 - 4608*a^2*b^7*c^2*d
^2*z^2 - 32768*a^6*c^5*e^2*z^2 - 16*b^11*d^2*z^2 - 672*a*b^6*c^2*d^2*e*z -
 15872*a^3*b^2*c^4*d^2*e*z + 4992*a^2*b^4*c^3*d^2*e*z + 18432*a^4*c^5*d^2*
e*z + 32*b^8*c*d^2*e*z - 960*a^2*b*c^4*d^2*e^2 + 240*a*b^3*c^3*d^2*e^2 - 1
6*b^5*c^2*d^2*e^2 + 360*a*b^2*c^4*d^4 - 256*a^3*c^4*e^4 - 25*b^4*c^3*d^4 -
 1296*a^2*c^5*d^4, z, k)*(root(1572864*a^8*b^2*c^5*z^4 - 983040*a^7*b^4*c^
4*z^4 + 327680*a^6*b^6*c^3*z^4 - 61440*a^5*b^8*c^2*z^4 + 6144*a^4*b^10*c*z
^4 - 1048576*a^9*c^6*z^4 - 256*a^3*b^12*z^4 + 61440*a^5*b*c^5*d^2*z^2 + 43
2*a*b^9*c*d^2*z^2 + 24576*a^5*b^2*c^4*e^2*z^2 - 6144*a^4*b^4*c^3*e^2*z^2 +
 512*a^3*b^6*c^2*e^2*z^2 - 61440*a^4*b^3*c^4*d^2*z^2 + 24064*a^3*b^5*c^3*d
^2*z^2 - 4608*a^2*b^7*c^2*d^2*z^2 - 32768*a^6*c^5*e^2*z^2 - 16*b^11*d^2*z^
2 - 672*a*b^6*c^2*d^2*e*z - 15872*a^3*b^2*c^4*d^2*e*z + 4992*a^2*b^4*c^...
```

Reduce [B] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 2923, normalized size of antiderivative = 8.86

$$\int \frac{d+ex}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
int((e*x+d)/(c*x^4+b*x^2+a)^2,x)
```

output
```
( - 16*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(
2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**3*b*
c*e - 16*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqr
t(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*
b**2*c*e*x**2 - 16*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)
*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) +
 b))*a**2*b*c**2*e*x**4 + 16*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqr
t(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*
b**2*c*d - 2*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt
(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b**4*d + 16*sqrt(a)
*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)
*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b**3*c*d*x**2 + 16*sqrt(a)*sqrt(2*sqrt(
c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sq
rt(c)*sqrt(a) + b))*a*b**2*c**2*d*x**4 - 2*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a)
+ b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(
a) + b))*b**5*d*x**2 - 2*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*
sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*b**4*c*d*
x**4 - 24*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a)
 - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**3*b*c*d + 2*sqrt(c)*s
qrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c...
```

3.244 $\int \frac{1}{(a+bx^2+cx^4)^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 252

$$\int \frac{1}{(a+bx^2+cx^4)^2}\,dx = \frac{x(b^2-2ac+bcx^2)}{2a(b^2-4ac)(a+bx^2+cx^4)} + \frac{\sqrt{c}\left(b^2-12ac+b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{c}\left(b^2-12ac-b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}$$

output

```
1/2*x*(b*c*x^2-2*a*c+b^2)/a/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+1/4*c^(1/2)*(b^2-
12*a*c+b*(-4*a*c+b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)*x/(b-(-4*a*c+b^2)^(1/2
))^(1/2))*2^(1/2)/a/(-4*a*c+b^2)^(3/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/4*c^
(1/2)*(b^2-12*a*c-b*(-4*a*c+b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)*x/(b+(-4*a*
c+b^2)^(1/2))^(1/2))*2^(1/2)/a/(-4*a*c+b^2)^(3/2)/(b+(-4*a*c+b^2)^(1/2))^(
1/2)
```

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.96

$$\int \frac{1}{(a+bx^2+cx^4)^2}\,dx = \frac{\frac{2x(b^2-2ac+bcx^2)}{(b^2-4ac)(a+bx^2+cx^4)}+\frac{\sqrt{2}\sqrt{c}\left(b^2-12ac+b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\sqrt{c}\left(-b^2+12ac+b\sqrt{b^2-4ac}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}}{4a}$$

input
```
Integrate[(a + b*x^2 + c*x^4)^(-2),x]
```

output
```
((2*x*(b^2 - 2*a*c + b*c*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt
[2]*Sqrt[c]*(b^2 - 12*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x
)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4
*a*c]]) + (Sqrt[2]*Sqrt[c]*(-b^2 + 12*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(S
qrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[
b + Sqrt[b^2 - 4*a*c]]))/(4*a)
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 236, normalized size of antiderivative = 0.94, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {1405, 25, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2+cx^4)^2}\,dx$$

$$\Big\downarrow \; 1405$$

$$\frac{x(-2ac+b^2+bcx^2)}{2a(b^2-4ac)(a+bx^2+cx^4)}-\frac{\int -\frac{b^2+cx^2b-6ac}{cx^4+bx^2+a}dx}{2a(b^2-4ac)}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{b^2+cx^2b-6ac}{cx^4+bx^2+a}dx}{2a\left(b^2-4ac\right)}+\frac{x\left(-2ac+b^2+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$\Big\downarrow \text{ 1480}$$

$$\frac{\frac{1}{2}c\left(\frac{b^2-12ac}{\sqrt{b^2-4ac}}+b\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b-\sqrt{b^2-4ac}\right)}dx+\frac{1}{2}c\left(b-\frac{b^2-12ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{cx^2+\frac{1}{2}\left(b+\sqrt{b^2-4ac}\right)}dx}{2a\left(b^2-4ac\right)}+\frac{x\left(-2ac+b^2+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{\frac{\sqrt{c}\left(\frac{b^2-12ac}{\sqrt{b^2-4ac}}+b\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(b-\frac{b^2-12ac}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{\sqrt{b^2-4ac}+b}}}{2a\left(b^2-4ac\right)}+\frac{x\left(-2ac+b^2+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

input
```
Int[(a + b*x^2 + c*x^4)^(-2),x]
```

output
```
(x*(b^2 - 2*a*c + b*c*x^2))/(2*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + ((Sq
rt[c]*(b + (b^2 - 12*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sq
rt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[
c]*(b - (b^2 - 12*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[
b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(2*a*(b^2
- 4*a*c))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.11 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.60

method	result
risch	$\frac{-\frac{b\,x^3 c}{2a\left(4ac-b^2\right)}+\frac{\left(2ac-b^2\right)x}{2a\left(4ac-b^2\right)}}{c\,x^4+b\,x^2+a}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+\_Z^2 b+a\right)}\frac{\left(-\frac{bc\_R^2}{4ac-b^2}+\frac{6ac-b^2}{4ac-b^2}\right)\ln\left(x-\_R\right)}{2\_R^3 c+b\_R}}{4a}$
default	$16c^2\left(-\frac{-\frac{\left(-b\sqrt{-4ac+b^2}+4ac-b^2\right)x}{16a\,c^2\left(x^2+\frac{b}{2c}-\frac{\sqrt{-4ac+b^2}}{2c}\right)}-\frac{\left(b^2-12ac+b\sqrt{-4ac+b^2}\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{cx\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{16ac\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}}{4\sqrt{-4ac+b^2}\,(4ac-b^2)}-\frac{\left(b\sqrt{-4ac+b^2}+4ac-b^2\right)x}{16a\,c^2\left(x^2+\frac{\sqrt{-4ac+b^2}}{2c}+\frac{b}{2c}\right.}\right.$

input
```
int(1/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(-1/2*b/a/(4*a*c-b^2)*x^3*c+1/2*(2*a*c-b^2)/a/(4*a*c-b^2)*x)/(c*x^4+b*x^2+
a)+1/4/a*sum((-b*c/(4*a*c-b^2)*_R^2+(6*a*c-b^2)/(4*a*c-b^2))/(2*_R^3*c+_R*
b)*ln(x-_R),_R=RootOf(_Z^4*c+_Z^2*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2309 vs. $2(206) = 412$.

Time = 0.23 (sec) , antiderivative size = 2309, normalized size of antiderivative = 9.16

$$\int \frac{1}{(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

output
```
1/4*(2*b*c*x^3 + sqrt(1/2)*((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c
+ (a*b^3 - 4*a^2*b*c)*x^2)*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 + (a^3*b
^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*sqrt((b^4 - 18*a*b^2*c +
81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*
b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))*log((5*b^4*c^2 - 81*a*b
^2*c^3 + 324*a^2*c^4)*x + 1/2*sqrt(1/2)*(b^8 - 23*a*b^6*c + 190*a^2*b^4*c^
2 - 672*a^3*b^2*c^3 + 864*a^4*c^4 - (a^3*b^9 - 20*a^4*b^7*c + 144*a^5*b^5*
c^2 - 448*a^6*b^3*c^3 + 512*a^7*b*c^4)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2
)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))*sqrt(-(b^5 - 15
*a*b^3*c + 60*a^2*b*c^2 + (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^
6*c^3)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a
^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a
^6*c^3))) - sqrt(1/2)*((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*
b^3 - 4*a^2*b*c)*x^2)*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 + (a^3*b^6 -
12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*sqrt((b^4 - 18*a*b^2*c + 81*a^
2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 -
 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))*log((5*b^4*c^2 - 81*a*b^2*c^
3 + 324*a^2*c^4)*x - 1/2*sqrt(1/2)*(b^8 - 23*a*b^6*c + 190*a^2*b^4*c^2 - 6
72*a^3*b^2*c^3 + 864*a^4*c^4 - (a^3*b^9 - 20*a^4*b^7*c + 144*a^5*b^5*c^2 -
 448*a^6*b^3*c^3 + 512*a^7*b*c^4)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/...
```

Sympy [A] (verification not implemented)

Time = 119.81 (sec) , antiderivative size = 394, normalized size of antiderivative = 1.56

$$\int \frac{1}{(a+bx^2+cx^4)^2} \, dx = \frac{-bcx^3 + x(2ac-b^2)}{8a^3c - 2a^2b^2 + x^4 \cdot (8a^2c^2 - 2ab^2c) + x^2 \cdot (8a^2bc - 2ab^3)} + \text{RootSum}\left(t^4 \cdot (1048576a^9c^6 - 1572864a^8b^2c^5 + 983040a^7b^4c^4 - 327680a^6b^6c^3 + 61440a^5b^8c^2 - 6144a\right.$$

input
```
integrate(1/(c*x**4+b*x**2+a)**2,x)
```

output
```
(-b*c*x**3 + x*(2*a*c - b**2))/(8*a**3*c - 2*a**2*b**2 + x**4*(8*a**2*c**2
 - 2*a*b**2*c) + x**2*(8*a**2*b*c - 2*a*b**3)) + RootSum(_t**4*(1048576*a*
*9*c**6 - 1572864*a**8*b**2*c**5 + 983040*a**7*b**4*c**4 - 327680*a**6*b**
6*c**3 + 61440*a**5*b**8*c**2 - 6144*a**4*b**10*c + 256*a**3*b**12) + _t**
2*(-61440*a**5*b*c**5 + 61440*a**4*b**3*c**4 - 24064*a**3*b**5*c**3 + 4608
*a**2*b**7*c**2 - 432*a*b**9*c + 16*b**11) + 1296*a**2*c**5 - 360*a*b**2*c
**4 + 25*b**4*c**3, Lambda(_t, _t*log(x + (32768*_t**3*a**7*b*c**4 - 28672
*_t**3*a**6*b**3*c**3 + 9216*_t**3*a**5*b**5*c**2 - 1280*_t**3*a**4*b**7*c
 + 64*_t**3*a**3*b**9 + 1728*_t*a**4*c**4 - 2304*_t*a**3*b**2*c**3 + 740*_
t*a**2*b**4*c**2 - 92*_t*a*b**6*c + 4*_t*b**8)/(324*a**2*c**4 - 81*a*b**2*
c**3 + 5*b**4*c**2))))
```

Maxima [F]

$$\int \frac{1}{(a+bx^2+cx^4)^2} \, dx = \int \frac{1}{(cx^4+bx^2+a)^2} \, dx$$

input
```
integrate(1/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/2*(b*c*x^3 + (b^2 - 2*a*c)*x)/((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a
^3*c + (a*b^3 - 4*a^2*b*c)*x^2) + 1/2*integrate((b*c*x^2 + b^2 - 6*a*c)/(c
*x^4 + b*x^2 + a), x)/(a*b^2 - 4*a^2*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2682 vs. $2(206) = 412$.

Time = 0.40 (sec) , antiderivative size = 2682, normalized size of antiderivative = 10.64

$$\int \frac{1}{(a+bx^2+cx^4)^2} \, dx = \text{Too large to display}$$

input

```
integrate(1/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output

```
1/2*(b*c*x^3 + b^2*x - 2*a*c*x)/((c*x^4 + b*x^2 + a)*(a*b^2 - 4*a^2*c)) -
1/16*(2*a^2*b^7*c^2 - 40*a^3*b^5*c^3 + 224*a^4*b^3*c^4 - 384*a^5*b*c^5 - s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^7 + 20*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c + 2*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c - 112*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^2 - 32*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^2 - sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^2 + 192*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b*c^3 + 96*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^3 + 16*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^3 - 48*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^4 - 2*(b^2 -
 4*a*c)*a^2*b^5*c^2 + 32*(b^2 - 4*a*c)*a^3*b^3*c^3 - 96*(b^2 - 4*a*c)*a^4*
b*c^4 + (2*b^3*c^2 - 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^2
 - 2*(b^2 - 4*a*c)*b*c^2)*(a*b^2 - 4*a^2*c)^2 - 2*(sqrt(2)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a*b^6 - 14*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^
4*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c - 2*a*b^6*c + 6...
```

Mupad [B] (verification not implemented)

Time = 23.58 (sec) , antiderivative size = 6404, normalized size of antiderivative = 25.41

$$\int \frac{1}{\left(a+bx^2+cx^4\right)^2}\,dx = \text{Too large to display}$$

input
```
int(1/(a + b*x^2 + c*x^4)^2,x)
```

output
```
((x*(2*a*c - b^2))/(2*a*(4*a*c - b^2)) - (b*c*x^3)/(2*a*(4*a*c - b^2)))/(a
 + b*x^2 + c*x^4) + atan(((((6144*a^5*c^6 + 16*a*b^8*c^2 - 288*a^2*b^6*c^3
 + 1920*a^3*b^4*c^4 - 5632*a^4*b^2*c^5)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*
b^4*c + 48*a^4*b^2*c^2)) - (x*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 384
0*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a
*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24
*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144
*a^8*b^2*c^5)))^(1/2)*(1024*a^5*b*c^5 - 16*a^2*b^7*c^2 + 192*a^3*b^5*c^3 -
 768*a^4*b^3*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11 + b^2
*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^
5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(3
2*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^
6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2) + (x*(72*a^2*c^5 + b^
4*c^3 - 14*a*b^2*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11 +
 b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^
3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2)
)/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^
6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2)*1i - (((6144*a^5*
c^6 + 16*a*b^8*c^2 - 288*a^2*b^6*c^3 + 1920*a^3*b^4*c^4 - 5632*a^4*b^2*c^5
)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) + (x*(-(b^...
```

Reduce [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 2409, normalized size of antiderivative = 9.56

$$\int \frac{1}{\left(a+bx^2+cx^4\right)^2}\,dx = \text{Too large to display}$$

input
```
int(1/(c*x^4+b*x^2+a)^2,x)
```

output
```
(16*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b)
- 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*b*c - 2*sqrt(a)*sqrt(2*sq
rt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2
*sqrt(c)*sqrt(a) + b))*a*b**3 + 16*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*ata
n((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))
*a*b**2*c*x**2 + 16*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(
c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*b*c**2*x**4
- 2*sqrt(a)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b)
- 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*b**4*x**2 - 2*sqrt(a)*sqrt(2*s
qrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(
2*sqrt(c)*sqrt(a) + b))*b**3*c*x**4 - 24*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) +
b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a)
 + b))*a**3*c + 2*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)
*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*b**2 - 24*s
qrt(c)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*s
qrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**2*b*c*x**2 - 24*sqrt(c)*sqrt(2*s
qrt(c)*sqrt(a) + b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(
2*sqrt(c)*sqrt(a) + b))*a**2*c**2*x**4 + 2*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a)
+ b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(
a) + b))*a*b**3*x**2 + 2*sqrt(c)*sqrt(2*sqrt(c)*sqrt(a) + b)*atan((sqrt...
```

3.245 $\int \frac{1}{(d+ex)\left(a+bx^2+cx^4\right)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 1006

$$
\begin{aligned}
\int \frac{1}{(d+ex)\left(a+bx^2+cx^4\right)^2}\,dx &= \frac{e\left(bcd^2+b^2e^2-2ace^2+c\left(2cd^2+be^2\right)x^2\right)}{2\left(b^2-4ac\right)\left(cd^4+bd^2e^2+ae^4\right)\left(a+bx^2+cx^4\right)} \\
&- \frac{dx\left(abce^2-\left(b^2-2ac\right)\left(cd^2+be^2\right)-c\left(bcd^2+b^2e^2-2ace^2\right)x^2\right)}{2a\left(b^2-4ac\right)\left(cd^4+bd^2e^2+ae^4\right)\left(a+bx^2+cx^4\right)} \\
&+ \frac{\sqrt{c}de^4\left(e^2+\frac{2cd^2+be^2}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}\left(cd^4+bd^2e^2+ae^4\right)^2} \\
&+ \frac{\sqrt{c}d\left(b^3e^2+bc\left(\sqrt{b^2-4ac}d^2-8ae^2\right)+b^2\left(cd^2+\sqrt{b^2-4ac}e^2\right)-2ac\left(6cd^2+\sqrt{b^2-4ac}e^2\right)\right)\arctan\left(\cdots\right.}{2\sqrt{2}a\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}\left(cd^4+bd^2e^2+ae^4\right)} \\
&+ \frac{\sqrt{c}de^4\left(e^2-\frac{2cd^2+be^2}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b+\sqrt{b^2-4ac}}\left(cd^4+bd^2e^2+ae^4\right)^2} \\
&+ \frac{\sqrt{c}d\left(bcd^2+b^2e^2-2ace^2-\frac{b^2cd^2-12ac^2d^2+b^3e^2-8abce^2}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\left(b^2-4ac\right)\sqrt{b+\sqrt{b^2-4ac}}\left(cd^4+bd^2e^2+ae^4\right)} \\
&+ \frac{e^5\left(2cd^2+be^2\right)\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2\sqrt{b^2-4ac}\left(cd^4+bd^2e^2+ae^4\right)^2} - \frac{ce\left(2cd^2+be^2\right)\operatorname{arctanh}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}\left(cd^4+bd^2e^2+ae^4\right)} \\
&+ \frac{e^7\log(d+ex)}{\left(cd^4+bd^2e^2+ae^4\right)^2} - \frac{e^7\log\left(a+bx^2+cx^4\right)}{4\left(cd^4+bd^2e^2+ae^4\right)^2}
\end{aligned}
$$

output
```
1/2*e*(b*c*d^2+b^2*e^2-2*a*c*e^2+c*(b*e^2+2*c*d^2)*x^2)/(-4*a*c+b^2)/(a*e^
4+b*d^2*e^2+c*d^4)/(c*x^4+b*x^2+a)-1/2*d*x*(a*b*c*e^2-(-2*a*c+b^2)*(b*e^2+
c*d^2)-c*(-2*a*c*e^2+b^2*e^2+b*c*d^2)*x^2)/a/(-4*a*c+b^2)/(a*e^4+b*d^2*e^2
+c*d^4)/(c*x^4+b*x^2+a)+1/2*c^(1/2)*d*e^4*(e^2+(b*e^2+2*c*d^2)/(-4*a*c+b^2
)^(1/2))*arctan(2^(1/2)*c^(1/2)*x/(b-(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/(b
-(-4*a*c+b^2)^(1/2))^(1/2)/(a*e^4+b*d^2*e^2+c*d^4)^2+1/4*c^(1/2)*d*(b^3*e^
2+b*c*((-4*a*c+b^2)^(1/2)*d^2-8*a*e^2)+b^2*(c*d^2+(-4*a*c+b^2)^(1/2)*e^2)-
2*a*c*(6*c*d^2+(-4*a*c+b^2)^(1/2)*e^2))*arctan(2^(1/2)*c^(1/2)*x/(b-(-4*a*
c+b^2)^(1/2))^(1/2))*2^(1/2)/a/(-4*a*c+b^2)^(3/2)/(b-(-4*a*c+b^2)^(1/2))^(
1/2)/(a*e^4+b*d^2*e^2+c*d^4)+1/2*c^(1/2)*d*e^4*(e^2-(b*e^2+2*c*d^2)/(-4*a*
c+b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)*x/(b+(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/
2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)/(a*e^4+b*d^2*e^2+c*d^4)^2+1/4*c^(1/2)*d*(b
*c*d^2+b^2*e^2-2*a*c*e^2-(-8*a*b*c*e^2-12*a*c^2*d^2+b^3*e^2+b^2*c*d^2)/(-4
*a*c+b^2)^(1/2))*arctan(2^(1/2)*c^(1/2)*x/(b+(-4*a*c+b^2)^(1/2))^(1/2))*2^
(1/2)/a/(-4*a*c+b^2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)/(a*e^4+b*d^2*e^2+c*d^4)+
1/2*e^5*(b*e^2+2*c*d^2)*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^
2)^(1/2)/(a*e^4+b*d^2*e^2+c*d^4)^2-c*e*(b*e^2+2*c*d^2)*arctanh((2*c*x^2+b)
/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(3/2)/(a*e^4+b*d^2*e^2+c*d^4)+e^7*ln(e*x
+d)/(a*e^4+b*d^2*e^2+c*d^4)^2-1/4*e^7*ln(c*x^4+b*x^2+a)/(a*e^4+b*d^2*e^2+c
*d^4)^2
```

Mathematica [A] (verified)

Time = 5.35 (sec) , antiderivative size = 1031, normalized size of antiderivative = 1.02

$$\int \frac{1}{(d+ex)\left(a+bx^2+cx^4\right)^2}\,dx$$

$$= \frac{-\frac{2(cd^4+bd^2e^2+ae^4)\left(-2a^2ce^3+bd(cd^2+be^2)x(b+cx^2)+a\left(b^2e^3+bce(d^2-3dex+e^2x^2)-2c^2dx(d^2-dex+e^2x^2)\right)\right)}{a(-b^2+4ac)(a+bx^2+cx^4)} - \frac{\sqrt{2}\sqrt{c}d\left(-b^4d^2e^4-b^3\left(2\right.\right.}{}}{}$$

input
```
Integrate[1/((d + e*x)*(a + b*x^2 + c*x^4)^2),x]
```

output
```
((-2*(c*d^4 + b*d^2*e^2 + a*e^4)*(-2*a^2*c*e^3 + b*d*(c*d^2 + b*e^2)*x*(b
+ c*x^2) + a*(b^2*e^3 + b*c*e*(d^2 - 3*d*e*x + e^2*x^2) - 2*c^2*d*x*(d^2 -
 d*e*x + e^2*x^2))))/(a*(-b^2 + 4*a*c)*(a + b*x^2 + c*x^4)) - (Sqrt[2]*Sqr
t[c]*d*(-(b^4*d^2*e^4) - b^3*(2*c*d^4*e^2 + Sqrt[b^2 - 4*a*c]*d^2*e^4 + 3*
a*e^6) + b*c*(-(c*Sqrt[b^2 - 4*a*c]*d^6) + 20*a*c*d^4*e^2 + a*Sqrt[b^2 - 4
*a*c]*d^2*e^4 + 16*a^2*e^6) - b^2*(c^2*d^6 + 2*c*Sqrt[b^2 - 4*a*c]*d^4*e^2
 - 3*a*c*d^2*e^4 + 3*a*Sqrt[b^2 - 4*a*c]*e^6) + 2*a*c*(6*c^2*d^6 + c*Sqrt[
b^2 - 4*a*c]*d^4*e^2 + 14*a*c*d^2*e^4 + 5*a*Sqrt[b^2 - 4*a*c]*e^6))*ArcTan
[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(a*(b^2 - 4*a*c)^(3/2)*
Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*d*(-(b^4*d^2*e^4) + b^3*(-
2*c*d^4*e^2 + Sqrt[b^2 - 4*a*c]*d^2*e^4 - 3*a*e^6) + b*c*(c*Sqrt[b^2 - 4*a
*c]*d^6 + 20*a*c*d^4*e^2 - a*Sqrt[b^2 - 4*a*c]*d^2*e^4 + 16*a^2*e^6) + 2*a
*c*(6*c^2*d^6 - c*Sqrt[b^2 - 4*a*c]*d^4*e^2 + 14*a*c*d^2*e^4 - 5*a*Sqrt[b^
2 - 4*a*c]*e^6) + b^2*(-(c^2*d^6) + 2*c*Sqrt[b^2 - 4*a*c]*d^4*e^2 + 3*a*c*
d^2*e^4 + 3*a*Sqrt[b^2 - 4*a*c]*e^6))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b +
Sqrt[b^2 - 4*a*c]]])/(a*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) +
 4*e^7*Log[d + e*x] + ((4*c^3*d^6*e - b^2*(b + Sqrt[b^2 - 4*a*c])*e^7 + 2*
a*c*(3*b + 2*Sqrt[b^2 - 4*a*c])*e^7 + 6*c^2*(b*d^4*e^3 + 2*a*d^2*e^5))*Log
[-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2])/(b^2 - 4*a*c)^(3/2) - ((4*c^3*d^6*e +
2*a*c*(3*b - 2*Sqrt[b^2 - 4*a*c])*e^7 + b^2*(-b + Sqrt[b^2 - 4*a*c])*e^...
```

Rubi [A] (verified)

Time = 5.06 (sec) , antiderivative size = 1006, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex)\left(a+bx^2+cx^4\right)^2}\,dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(-\frac{e^4(ex-d)\left(be^2+cd^2+ce^2x^2\right)}{\left(a+bx^2+cx^4\right)\left(ae^4+bd^2e^2+cd^4\right)^2}+\frac{(d-ex)\left(be^2+cd^2+ce^2x^2\right)}{\left(a+bx^2+cx^4\right)^2\left(ae^4+bd^2e^2+cd^4\right)}+\frac{e^8}{(d+ex)\left(ae^4+bd^2e^2+c\right.}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{\log(d+ex)e^7}{(cd^4+be^2d^2+ae^4)^2}-\frac{\log\left(cx^4+bx^2+a\right)e^7}{4\left(cd^4+be^2d^2+ae^4\right)^2}+\frac{\left(2cd^2+be^2\right)\operatorname{arctanh}\left(\frac{2cx^2+b}{\sqrt{b^2-4ac}}\right)e^5}{2\sqrt{b^2-4ac}\left(cd^4+be^2d^2+ae^4\right)^2}+$$
$$\frac{\sqrt{c}d\left(e^2+\frac{2cd^2+be^2}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)e^4}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}\left(cd^4+be^2d^2+ae^4\right)^2}+\frac{\sqrt{c}d\left(e^2-\frac{2cd^2+be^2}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)e^4}{\sqrt{2}\sqrt{b+\sqrt{b^2-4ac}}\left(cd^4+be^2d^2+ae^4\right)^2}-$$
$$\frac{c\left(2cd^2+be^2\right)\operatorname{arctanh}\left(\frac{2cx^2+b}{\sqrt{b^2-4ac}}\right)e}{\left(b^2-4ac\right)^{3/2}\left(cd^4+be^2d^2+ae^4\right)}+\frac{\left(bcd^2+b^2e^2-2ace^2+c\left(2cd^2+be^2\right)x^2\right)e}{2\left(b^2-4ac\right)\left(cd^4+be^2d^2+ae^4\right)\left(cx^4+bx^2+a\right)}+$$
$$\frac{\sqrt{c}d\left(e^2b^3+\left(cd^2+\sqrt{b^2-4ac}e^2\right)b^2+c\left(\sqrt{b^2-4ac}d^2-8ae^2\right)b-2ac\left(6cd^2+\sqrt{b^2-4ac}e^2\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b-\sqrt{b^2-}}}\right.}{2\sqrt{2}a\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}\left(cd^4+be^2d^2+ae^4\right)}$$
$$\frac{\sqrt{c}d\left(bcd^2+b^2e^2-2ace^2-\frac{e^2b^3+cd^2b^2-8ace^2b-12ac^2d^2}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\left(b^2-4ac\right)\sqrt{b+\sqrt{b^2-4ac}}\left(cd^4+be^2d^2+ae^4\right)}-$$
$$\frac{dx\left(abce^2-c\left(bcd^2+b^2e^2-2ace^2\right)x^2-\left(b^2-2ac\right)\left(cd^2+be^2\right)\right)}{2a\left(b^2-4ac\right)\left(cd^4+be^2d^2+ae^4\right)\left(cx^4+bx^2+a\right)}$$

input
```
Int[1/((d + e*x)*(a + b*x^2 + c*x^4)^2),x]
```

output
```
(e*(b*c*d^2 + b^2*e^2 - 2*a*c*e^2 + c*(2*c*d^2 + b*e^2)*x^2))/(2*(b^2 - 4*
a*c)*(c*d^4 + b*d^2*e^2 + a*e^4)*(a + b*x^2 + c*x^4)) - (d*x*(a*b*c*e^2 -
(b^2 - 2*a*c)*(c*d^2 + b*e^2) - c*(b*c*d^2 + b^2*e^2 - 2*a*c*e^2)*x^2))/(2
*a*(b^2 - 4*a*c)*(c*d^4 + b*d^2*e^2 + a*e^4)*(a + b*x^2 + c*x^4)) + (Sqrt[
c]*d*e^4*(e^2 + (2*c*d^2 + b*e^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[
c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b - Sqrt[b^2 - 4*a*c]]*(
c*d^4 + b*d^2*e^2 + a*e^4)^2) + (Sqrt[c]*d*(b^3*e^2 + b*c*(Sqrt[b^2 - 4*a*
c]*d^2 - 8*a*e^2) + b^2*(c*d^2 + Sqrt[b^2 - 4*a*c]*e^2) - 2*a*c*(6*c*d^2 +
 Sqrt[b^2 - 4*a*c]*e^2))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*
a*c]]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]*(c*d^
4 + b*d^2*e^2 + a*e^4)) + (Sqrt[c]*d*e^4*(e^2 - (2*c*d^2 + b*e^2)/Sqrt[b^2
 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[
2]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(c*d^4 + b*d^2*e^2 + a*e^4)^2) + (Sqrt[c]*d
*(b*c*d^2 + b^2*e^2 - 2*a*c*e^2 - (b^2*c*d^2 - 12*a*c^2*d^2 + b^3*e^2 - 8*
a*b*c*e^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2
 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(c*d^4
 + b*d^2*e^2 + a*e^4)) + (e^5*(2*c*d^2 + b*e^2)*ArcTanh[(b + 2*c*x^2)/Sqrt
[b^2 - 4*a*c]])/(2*Sqrt[b^2 - 4*a*c]*(c*d^4 + b*d^2*e^2 + a*e^4)^2) - (c*e
*(2*c*d^2 + b*e^2)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/((b^2 - 4*a*c
)^(3/2)*(c*d^4 + b*d^2*e^2 + a*e^4)) + (e^7*Log[d + e*x])/(c*d^4 + b*d^...
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 1568, normalized size of antiderivative = 1.56

method	result	size
default	Expression too large to display	1568
risch	Expression too large to display	7157

input
```
int(1/(e*x+d)/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/(a*e^4+b*d^2*e^2+c*d^4)^2*((1/2*c*d*(2*a^2*c*e^6-a*b^2*e^6+a*b*c*d^2*e^4
+2*a*c^2*d^4*e^2-b^3*d^2*e^4-2*b^2*c*d^4*e^2-b*c^2*d^6)/a/(4*a*c-b^2)*x^3-
1/2*c*e*(a*b*e^6+2*a*c*d^2*e^4+b^2*d^2*e^4+3*b*c*d^4*e^2+2*c^2*d^6)/(4*a*c
-b^2)*x^2+1/2*d*(3*a^2*b*c*e^6+2*a^2*c^2*d^2*e^4-a*b^3*e^6+2*a*b^2*c*d^2*e
^4+5*a*b*c^2*d^4*e^2+2*a*c^3*d^6-b^4*d^2*e^4-2*b^3*c*d^4*e^2-b^2*c^2*d^6)/
a/(4*a*c-b^2)*x+1/2*e*(2*a^2*c*e^6-a*b^2*e^6+a*b*c*d^2*e^4+2*a*c^2*d^4*e^2
-b^3*d^2*e^4-2*b^2*c*d^4*e^2-b*c^2*d^6)/(4*a*c-b^2))/(c*x^4+b*x^2+a)+2/a/(
4*a*c-b^2)*c*(1/(16*a*c-4*b^2)*(-1/4*(-12*(-4*a*c+b^2)^(1/2)*a^2*b*c*e^7-2
4*(-4*a*c+b^2)^(1/2)*a^2*c^2*d^2*e^5+2*(-4*a*c+b^2)^(1/2)*a*b^3*e^7-12*(-4
*a*c+b^2)^(1/2)*a*b*c^2*d^4*e^3-8*(-4*a*c+b^2)^(1/2)*a*c^3*d^6*e+32*a^3*e^
7*c^2-16*a^2*e^7*c*b^2+2*a*e^7*b^4)/c*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)+1/
2*(16*(-4*a*c+b^2)^(1/2)*a^2*b*c*d*e^6+28*(-4*a*c+b^2)^(1/2)*a^2*c^2*d^3*e
^4-3*(-4*a*c+b^2)^(1/2)*a*b^3*d*e^6+3*(-4*a*c+b^2)^(1/2)*a*b^2*c*d^3*e^4+2
0*(-4*a*c+b^2)^(1/2)*a*b*c^2*d^5*e^2+12*(-4*a*c+b^2)^(1/2)*a*c^3*d^7-(-4*a
*c+b^2)^(1/2)*b^4*d^3*e^4-2*(-4*a*c+b^2)^(1/2)*b^3*c*d^5*e^2-(-4*a*c+b^2)^
(1/2)*b^2*c^2*d^7-40*a^3*c^2*d*e^6+22*a^2*b^2*c*d*e^6-4*a^2*b*c^2*d^3*e^4-
8*a^2*c^3*d^5*e^2-3*a*b^4*d*e^6+5*a*b^3*c*d^3*e^4+10*a*b^2*c^2*d^5*e^2+4*a
*b*c^3*d^7-b^5*d^3*e^4-2*b^4*c*d^5*e^2-b^3*c^2*d^7)*2^(1/2)/((-b+(-4*a*c+b
^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))
)+1/(16*a*c-4*b^2)*(1/4*(-12*(-4*a*c+b^2)^(1/2)*a^2*b*c*e^7-24*(-4*a*c+...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)\left(a+bx^2+cx^4\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x+d)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)\left(a+bx^2+cx^4\right)^2}\,dx=\text{Timed out}$$

input
```
integrate(1/(e*x+d)/(c*x**4+b*x**2+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{(d+ex)\left(a+bx^2+cx^4\right)^2}\,dx=\int \frac{1}{(cx^4+bx^2+a)^2(ex+d)}\,dx$$

input
```
integrate(1/(e*x+d)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

output
```
e^7*log(e*x + d)/(c^2*d^8 + 2*b*c*d^6*e^2 + 2*a*b*d^2*e^6 + a^2*e^8 + (b^2
 + 2*a*c)*d^4*e^4) + 1/2*(a*b*c*d^2*e + (a*b^2 - 2*a^2*c)*e^3 + (b*c^2*d^3
 + (b^2*c - 2*a*c^2)*d*e^2)*x^3 + (2*a*c^2*d^2*e + a*b*c*e^3)*x^2 + ((b^2*
c - 2*a*c^2)*d^3 + (b^3 - 3*a*b*c)*d*e^2)*x)/((a^2*b^2*c - 4*a^3*c^2)*d^4
+ (a^2*b^3 - 4*a^3*b*c)*d^2*e^2 + (a^3*b^2 - 4*a^4*c)*e^4 + ((a*b^2*c^2 -
4*a^2*c^3)*d^4 + (a*b^3*c - 4*a^2*b*c^2)*d^2*e^2 + (a^2*b^2*c - 4*a^3*c^2)
*e^4)*x^4 + ((a*b^3*c - 4*a^2*b*c^2)*d^4 + (a*b^4 - 4*a^2*b^2*c)*d^2*e^2 +
 (a^2*b^3 - 4*a^3*b*c)*e^4)*x^2) + 1/2*integrate(-(2*(a*b^2*c - 4*a^2*c^2)
*e^7*x^3 - (b^2*c^2 - 6*a*c^3)*d^7 - (2*b^3*c - 11*a*b*c^2)*d^5*e^2 - (b^4
 - 2*a*b^2*c - 14*a^2*c^2)*d^3*e^4 - (3*a*b^3 - 13*a^2*b*c)*d*e^6 - (b*c^3
*d^7 + 2*(b^2*c^2 - a*c^3)*d^5*e^2 + (b^3*c - a*b*c^2)*d^3*e^4 + (3*a*b^2*
c - 10*a^2*c^2)*d*e^6)*x^2 - 2*(2*a*c^3*d^6*e + 3*a*b*c^2*d^4*e^3 + 6*a^2*
c^2*d^2*e^5 - (a*b^3 - 5*a^2*b*c)*e^7)*x)/(c*x^4 + b*x^2 + a), x)/((a*b^2*
c^2 - 4*a^2*c^3)*d^8 + 2*(a*b^3*c - 4*a^2*b*c^2)*d^6*e^2 + (a*b^4 - 2*a^2*
b^2*c - 8*a^3*c^2)*d^4*e^4 + 2*(a^2*b^3 - 4*a^3*b*c)*d^2*e^6 + (a^3*b^2 -
4*a^4*c)*e^8)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 130635 vs. $2(912) = 1824$.

Time = 15.97 (sec) , antiderivative size = 130635, normalized size of antiderivative = 129.86

$$\int \frac{1}{(d+ex)\left(a+bx^2+cx^4\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*x+d)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output
```
e^8*log(abs(e*x + d))/(c^2*d^8*e + 2*b*c*d^6*e^3 + b^2*d^4*e^5 + 2*a*c*d^4
*e^5 + 2*a*b*d^2*e^7 + a^2*e^9) - 1/4*e^7*log(abs(c*x^4 + b*x^2 + a))/(c^2
*d^8 + 2*b*c*d^6*e^2 + b^2*d^4*e^4 + 2*a*c*d^4*e^4 + 2*a*b*d^2*e^6 + a^2*e
^8) + 1/16*((2*a^4*b^11*c^14 - 56*a^5*b^9*c^15 + 576*a^6*b^7*c^16 - 2816*a
^7*b^5*c^17 + 6656*a^8*b^3*c^18 - 6144*a^9*b*c^19 - sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^11*c^12 + 28*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^9*c^13 + 2*sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^10*c^13 - 288*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^7*c^14 - 48*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^8*c^14 - sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^9*c^14 + 1408*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^5*c^15 + 384*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^6*c^15 + 24*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^7*c^15 - 3328*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^8*b^3*c^16 - 1
280*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^4*c^16
 - 192*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^5*c
^16 + 3072*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^9*b
*c^17 + 1536*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^8
*b^2*c^17 + 640*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*...
```

Mupad [B] (verification not implemented)

Time = 28.71 (sec) , antiderivative size = 27673, normalized size of antiderivative = 27.51

$$\int \frac{1}{(d+ex)(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
int(1/((d + e*x)*(a + b*x^2 + c*x^4)^2),x)
```

output
```
symsum(log(root(12582912*a^11*b*c^7*d^6*e^10*z^5 + 12582912*a^10*b*c^8*d^1
0*e^6*z^5 + 4194304*a^12*b*c^6*d^2*e^14*z^5 + 4194304*a^9*b*c^9*d^14*e^2*z
^5 - 35840*a^6*b^12*c*d^4*e^12*z^5 - 24576*a^7*b^11*c*d^2*e^14*z^5 - 21504
*a^5*b^13*c*d^6*e^10*z^5 - 3072*a^4*b^14*c*d^8*e^8*z^5 + 1024*a^3*b^15*c*d
^10*e^6*z^5 - 14680064*a^10*b^3*c^6*d^6*e^10*z^5 - 14680064*a^9*b^3*c^7*d^
10*e^6*z^5 - 11927552*a^9*b^4*c^6*d^8*e^8*z^5 + 8257536*a^8*b^6*c^5*d^8*e^
8*z^5 - 6291456*a^11*b^3*c^5*d^2*e^14*z^5 - 6291456*a^8*b^3*c^8*d^14*e^2*z
^5 - 5505024*a^10*b^4*c^5*d^4*e^12*z^5 + 5505024*a^9*b^5*c^5*d^6*e^10*z^5
+ 5505024*a^8*b^5*c^6*d^10*e^6*z^5 - 5505024*a^8*b^4*c^7*d^12*e^4*z^5 + 45
87520*a^9*b^6*c^4*d^4*e^12*z^5 + 4587520*a^7*b^6*c^6*d^12*e^4*z^5 + 393216
0*a^10*b^5*c^4*d^2*e^14*z^5 + 3932160*a^7*b^5*c^7*d^14*e^2*z^5 + 3145728*a
^10*b^2*c^7*d^8*e^8*z^5 - 2580480*a^7*b^8*c^4*d^8*e^8*z^5 - 1720320*a^8*b^
8*c^3*d^4*e^12*z^5 - 1720320*a^6*b^8*c^5*d^12*e^4*z^5 - 1310720*a^9*b^7*c^
3*d^2*e^14*z^5 - 1310720*a^6*b^7*c^6*d^14*e^2*z^5 - 573440*a^7*b^9*c^3*d^6
*e^10*z^5 - 573440*a^6*b^9*c^4*d^10*e^6*z^5 + 372736*a^6*b^10*c^3*d^8*e^8*
z^5 + 344064*a^7*b^10*c^2*d^4*e^12*z^5 + 344064*a^5*b^10*c^4*d^12*e^4*z^5
+ 245760*a^8*b^9*c^2*d^2*e^14*z^5 + 245760*a^5*b^9*c^5*d^14*e^2*z^5 + 1720
32*a^6*b^11*c^2*d^6*e^10*z^5 + 172032*a^5*b^11*c^3*d^10*e^6*z^5 - 35840*a^
4*b^12*c^3*d^12*e^4*z^5 - 24576*a^4*b^11*c^4*d^14*e^2*z^5 - 21504*a^4*b^13
*c^2*d^10*e^6*z^5 - 10752*a^5*b^12*c^2*d^8*e^8*z^5 + 1536*a^3*b^14*c^2*...
```

Reduce [B] (verification not implemented)

Time = 4.33 (sec) , antiderivative size = 17358, normalized size of antiderivative = 17.25

$$\int \frac{1}{(d+ex)(a+bx^2+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
int(1/(e*x+d)/(c*x^4+b*x^2+a)^2,x)
```

output
```
(24*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*s
qrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**4*b**2*
c*e**7 + 48*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((
sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a*
*4*b*c**2*d**2*e**5 - 4*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a)
 - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt
(a) + b))*a**3*b**4*e**7 + 24*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*s
qrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c
)*sqrt(a) + b))*a**3*b**3*c*e**7*x**2 + 24*sqrt(2*sqrt(c)*sqrt(a) + b)*sqr
t(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/
sqrt(2*sqrt(c)*sqrt(a) + b))*a**3*b**2*c**2*d**4*e**3 + 48*sqrt(2*sqrt(c)*
sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b)
 - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**3*b**2*c**2*d**2*e**5*x**2
 + 24*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) - b)*atan((sqrt(2
*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a) + b))*a**3*b**
2*c**2*e**7*x**4 + 16*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)*sqrt(a) -
 b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt(c)*sqrt(a
) + b))*a**3*b*c**3*d**6*e + 48*sqrt(2*sqrt(c)*sqrt(a) + b)*sqrt(2*sqrt(c)
*sqrt(a) - b)*atan((sqrt(2*sqrt(c)*sqrt(a) - b) - 2*sqrt(c)*x)/sqrt(2*sqrt
(c)*sqrt(a) + b))*a**3*b*c**3*d**2*e**5*x**4 - 4*sqrt(2*sqrt(c)*sqrt(a)...
```

3.246 $\int \frac{1}{(d+ex)^2(a+bx^2+cx^4)^2} dx$

Optimal result

Integrand size = 22, antiderivative size = 1689

$$\int \frac{1}{(d+ex)^2 (a+bx^2+cx^4)^2} dx = \text{Too large to display}$$

output
```
-e^7/(a*e^4+b*d^2*e^2+c*d^4)^2/(e*x+d)+d*e*(2*b^2*c*d^2*e^2-4*a*c^2*d^2*e^
2+b^3*e^4+b*c*(-3*a*e^4+c*d^4)+c*(-2*a*c*e^4+b^2*e^4+2*b*c*d^2*e^2+2*c^2*d
^4)*x^2)/(-4*a*c+b^2)/(a*e^4+b*d^2*e^2+c*d^4)^2/(c*x^4+b*x^2+a)-1/2*x*(a*b
*c*e^2*(-a*e^4+b*d^2*e^2+3*c*d^4)-(-2*a*c+b^2)*(-a*b*e^6-3*a*c*d^2*e^4+b^2
*d^2*e^4+2*b*c*d^4*e^2+c^2*d^6)-c*(b^3*d^2*e^4+b*c*d^2*(-5*a*e^4+c*d^4)-2*
a*c*e^2*(-a*e^4+3*c*d^4)+b^2*(-a*e^6+2*c*d^4*e^2))*x^2)/a/(-4*a*c+b^2)/(a*
e^4+b*d^2*e^2+c*d^4)^2/(c*x^4+b*x^2+a)+1/2*c^(1/2)*e^4*(10*c^2*d^6+c*d^2*e
^2*(9*b*d^2+7*(-4*a*c+b^2)^(1/2)*d^2-6*a*e^2)+(b+(-4*a*c+b^2)^(1/2))*e^4*(
-a*e^2+3*b*d^2))*arctan(2^(1/2)*c^(1/2)*x/(b-(-4*a*c+b^2)^(1/2))^(1/2))*2^
(1/2)/(-4*a*c+b^2)^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)/(a*e^4+b*d^2*e^2+c*d
^4)^3+1/4*c^(1/2)*(b^4*d^2*e^4+b^3*(2*c*d^4*e^2+(-4*a*c+b^2)^(1/2)*d^2*e^4
-a*e^6)+b*c*(c*(-4*a*c+b^2)^(1/2)*d^6-12*a*c*d^4*e^2-5*a*(-4*a*c+b^2)^(1/2
)*d^2*e^4+8*a^2*e^6)-2*a*c*(6*c^2*d^6+3*c*(-4*a*c+b^2)^(1/2)*d^4*e^2-18*a*
c*d^2*e^4-a*(-4*a*c+b^2)^(1/2)*e^6)+b^2*(c^2*d^6+2*c*(-4*a*c+b^2)^(1/2)*d^
4*e^2-11*a*c*d^2*e^4-a*(-4*a*c+b^2)^(1/2)*e^6))*arctan(2^(1/2)*c^(1/2)*x/(
b-(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)/a/(-4*a*c+b^2)^(3/2)/(b-(-4*a*c+b^2)^
(1/2))^(1/2)/(a*e^4+b*d^2*e^2+c*d^4)^2-1/2*c^(1/2)*e^4*(10*c^2*d^6+c*d^2*e
^2*(9*b*d^2-7*(-4*a*c+b^2)^(1/2)*d^2-6*a*e^2)+(b-(-4*a*c+b^2)^(1/2))*e^4*(
-a*e^2+3*b*d^2))*arctan(2^(1/2)*c^(1/2)*x/(b+(-4*a*c+b^2)^(1/2))^(1/2))*2^
(1/2)/(-4*a*c+b^2)^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)/(a*e^4+b*d^2*e^2+...
```

Mathematica [A] (verified)

Time = 8.08 (sec) , antiderivative size = 2086, normalized size of antiderivative = 1.24

$$\int \frac{1}{(d+ex)^2 (a+bx^2+cx^4)^2} dx = \text{Result too large to show}$$

input
```
Integrate[1/((d + e*x)^2*(a + b*x^2 + c*x^4)^2),x]
```

output
```
-(e^7/((c*d^4 + b*d^2*e^2 + a*e^4)^2*(d + e*x))) + (-2*a*b*c^2*d^5*e - 4*a
*b^2*c*d^3*e^3 + 8*a^2*c^2*d^3*e^3 - 2*a*b^3*d*e^5 + 6*a^2*b*c*d*e^5 - b^2
*c^2*d^6*x + 2*a*c^3*d^6*x - 2*b^3*c*d^4*e^2*x + 7*a*b*c^2*d^4*e^2*x - b^4
*d^2*e^4*x + 6*a*b^2*c*d^2*e^4*x - 6*a^2*c^2*d^2*e^4*x + a*b^3*e^6*x - 3*a
^2*b*c*e^6*x - 4*a*c^3*d^5*e*x^2 - 4*a*b*c^2*d^3*e^3*x^2 - 2*a*b^2*c*d*e^5
*x^2 + 4*a^2*c^2*d*e^5*x^2 - b*c^3*d^6*x^3 - 2*b^2*c^2*d^4*e^2*x^3 + 6*a*c
^3*d^4*e^2*x^3 - b^3*c*d^2*e^4*x^3 + 5*a*b*c^2*d^2*e^4*x^3 + a*b^2*c*e^6*x
^3 - 2*a^2*c^2*e^6*x^3)/(2*a*(-b^2 + 4*a*c)*(c*d^4 + b*d^2*e^2 + a*e^4)^2*
(a + b*x^2 + c*x^4)) + (Sqrt[c]*(b^2*c^3*d^10 - 12*a*c^4*d^10 + b*c^3*Sqrt
[b^2 - 4*a*c]*d^10 + 3*b^3*c^2*d^8*e^2 - 24*a*b*c^3*d^8*e^2 + 3*b^2*c^2*Sq
rt[b^2 - 4*a*c]*d^8*e^2 - 6*a*c^3*Sqrt[b^2 - 4*a*c]*d^8*e^2 + 3*b^4*c*d^6*
e^4 - 2*a*b^2*c^2*d^6*e^4 - 56*a^2*c^3*d^6*e^4 + 3*b^3*c*Sqrt[b^2 - 4*a*c]
*d^6*e^4 - 10*a*b*c^2*Sqrt[b^2 - 4*a*c]*d^6*e^4 + b^5*d^4*e^6 + 8*a*b^3*c*
d^4*e^6 - 40*a^2*b*c^2*d^4*e^6 + b^4*Sqrt[b^2 - 4*a*c]*d^4*e^6 + 10*a*b^2*
c*Sqrt[b^2 - 4*a*c]*d^4*e^6 - 60*a^2*c^2*Sqrt[b^2 - 4*a*c]*d^4*e^6 + 6*a*b
^4*d^2*e^8 - 39*a^2*b^2*c*d^2*e^8 + 84*a^3*c^2*d^2*e^8 + 6*a*b^3*Sqrt[b^2
- 4*a*c]*d^2*e^8 - 27*a^2*b*c*Sqrt[b^2 - 4*a*c]*d^2*e^8 - 3*a^2*b^3*e^10 +
 16*a^3*b*c*e^10 - 3*a^2*b^2*Sqrt[b^2 - 4*a*c]*e^10 + 10*a^3*c*Sqrt[b^2 -
4*a*c]*e^10)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*S
qrt[2]*a*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]*(c*d^4 + b*d^2...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex)^2 (a+bx^2+cx^4)^2} \, dx$$

$$\Big\downarrow \text{ 7293}$$

$$\int \left(\frac{e^4 \left(-2dex \left(-ace^4+2b^2e^4+5bcd^2e^2+3c^2d^4\right)+ce^2x^2 \left(-ae^4+3bd^2e^2+7cd^4\right)-abe^6-3acd^2e^4+3b^2d^2e^4 \right.}{(a+bx^2+cx^4) \left(ae^4+bd^2e^2+cd^4\right)^3}\right.$$

$$\Big\downarrow \text{ 7239}$$

$$\int \frac{1}{(d+ex)^2 (a+bx^2+cx^4)^2} dx$$

$$\Big\downarrow \text{ 7293}$$

$$\int \left(\frac{e^4 \left(-2dex \left(-ace^4 + 2b^2e^4 + 5bcd^2e^2 + 3c^2d^4\right) + ce^2x^2 \left(-ae^4 + 3bd^2e^2 + 7cd^4\right) - abe^6 - 3acd^2e^4 + 3b^2d^2e^4 -}{\left(a + bx^2 + cx^4\right) \left(ae^4 + bd^2e^2 + cd^4\right)^3}\right.$$

$$\Big\downarrow \textbf{7239}$$

$$\int \frac{1}{(d+ex)^2 \left(a+bx^2+cx^4\right)^2} dx$$

$$\Big\downarrow \textbf{7293}$$

$$\int \left(\frac{e^4 \left(-2dex \left(-ace^4 + 2b^2e^4 + 5bcd^2e^2 + 3c^2d^4\right) + ce^2x^2 \left(-ae^4 + 3bd^2e^2 + 7cd^4\right) - abe^6 - 3acd^2e^4 + 3b^2d^2e^4 -}{\left(a + bx^2 + cx^4\right) \left(ae^4 + bd^2e^2 + cd^4\right)^3}\right.$$

$$\Big\downarrow \textbf{7239}$$

$$\int \frac{1}{(d+ex)^2 \left(a+bx^2+cx^4\right)^2} dx$$

$$\Big\downarrow \textbf{7293}$$

$$\int \left(\frac{e^4 \left(-2dex \left(-ace^4 + 2b^2e^4 + 5bcd^2e^2 + 3c^2d^4\right) + ce^2x^2 \left(-ae^4 + 3bd^2e^2 + 7cd^4\right) - abe^6 - 3acd^2e^4 + 3b^2d^2e^4 -}{\left(a + bx^2 + cx^4\right) \left(ae^4 + bd^2e^2 + cd^4\right)^3}\right.$$

$$\Big\downarrow \textbf{7239}$$

$$\int \frac{1}{(d+ex)^2 \left(a+bx^2+cx^4\right)^2} dx$$

$$\Big\downarrow \textbf{7293}$$

$$\int \left(\frac{e^4 \left(-2dex \left(-ace^4 + 2b^2e^4 + 5bcd^2e^2 + 3c^2d^4\right) + ce^2x^2 \left(-ae^4 + 3bd^2e^2 + 7cd^4\right) - abe^6 - 3acd^2e^4 + 3b^2d^2e^4 -}{\left(a + bx^2 + cx^4\right) \left(ae^4 + bd^2e^2 + cd^4\right)^3}\right.$$

$$\Big\downarrow \textbf{7239}$$

$$\int \frac{1}{(d+ex)^2 \left(a+bx^2+cx^4\right)^2} dx$$

$$\Big\downarrow \textbf{7293}$$

$$\int \left(\frac{e^4 \left(-2dex \left(-ace^4 + 2b^2e^4 + 5bcd^2e^2 + 3c^2d^4\right) + ce^2x^2 \left(-ae^4 + 3bd^2e^2 + 7cd^4\right) - abe^6 - 3acd^2e^4 + 3b^2d^2e^4 -}{\left(a + bx^2 + cx^4\right) \left(ae^4 + bd^2e^2 + cd^4\right)^3}\right.$$

$$\Big\downarrow \textbf{7239}$$

$$\int \frac{1}{(d+ex)^2 (a+bx^2+cx^4)^2} dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(\frac{e^4 \left(-2dex \left(-ace^4+2b^2e^4+5bcd^2e^2+3c^2d^4\right)+ce^2x^2 \left(-ae^4+3bd^2e^2+7cd^4\right)-abe^6-3acd^2e^4+3b^2d^2e^4 - \right.}{(a+bx^2+cx^4) \left(ae^4+bd^2e^2+cd^4\right)^3} \right.$$

$$\Big\downarrow \text{7239}$$

$$\int \frac{1}{(d+ex)^2 (a+bx^2+cx^4)^2} dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(\frac{e^4 \left(-2dex \left(-ace^4+2b^2e^4+5bcd^2e^2+3c^2d^4\right)+ce^2x^2 \left(-ae^4+3bd^2e^2+7cd^4\right)-abe^6-3acd^2e^4+3b^2d^2e^4 - \right.}{(a+bx^2+cx^4) \left(ae^4+bd^2e^2+cd^4\right)^3} \right.$$

$$\Big\downarrow \text{7239}$$

$$\int \frac{1}{(d+ex)^2 (a+bx^2+cx^4)^2} dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(\frac{e^4 \left(-2dex \left(-ace^4+2b^2e^4+5bcd^2e^2+3c^2d^4\right)+ce^2x^2 \left(-ae^4+3bd^2e^2+7cd^4\right)-abe^6-3acd^2e^4+3b^2d^2e^4 - \right.}{(a+bx^2+cx^4) \left(ae^4+bd^2e^2+cd^4\right)^3} \right.$$

$$\Big\downarrow \text{7239}$$

$$\int \frac{1}{(d+ex)^2 (a+bx^2+cx^4)^2} dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(\frac{e^4 \left(-2dex \left(-ace^4+2b^2e^4+5bcd^2e^2+3c^2d^4\right)+ce^2x^2 \left(-ae^4+3bd^2e^2+7cd^4\right)-abe^6-3acd^2e^4+3b^2d^2e^4 - \right.}{(a+bx^2+cx^4) \left(ae^4+bd^2e^2+cd^4\right)^3} \right.$$

$$\Big\downarrow \text{7239}$$

$$\int \frac{1}{(d+ex)^2 (a+bx^2+cx^4)^2} dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(\frac{e^4\left(-2dex\left(-ace^4+2b^2e^4+5bcd^2e^2+3c^2d^4\right)+ce^2x^2\left(-ae^4+3bd^2e^2+7cd^4\right)-abe^6-3acd^2e^4+3b^2d^2e^4\right.}{\left(a+bx^2+cx^4\right)\left(ae^4+bd^2e^2+cd^4\right)^3}\right.$$

$$\Big\downarrow \text{ 7239}$$

$$\int \frac{1}{(d+ex)^2\left(a+bx^2+cx^4\right)^2}dx$$

$$\Big\downarrow \text{ 7293}$$

$$\int \left(\frac{e^4\left(-2dex\left(-ace^4+2b^2e^4+5bcd^2e^2+3c^2d^4\right)+ce^2x^2\left(-ae^4+3bd^2e^2+7cd^4\right)-abe^6-3acd^2e^4+3b^2d^2e^4\right.}{\left(a+bx^2+cx^4\right)\left(ae^4+bd^2e^2+cd^4\right)^3}\right.$$

$$\Big\downarrow \text{ 7239}$$

$$\int \frac{1}{(d+ex)^2\left(a+bx^2+cx^4\right)^2}dx$$

$$\Big\downarrow \text{ 7293}$$

$$\int \left(\frac{e^4\left(-2dex\left(-ace^4+2b^2e^4+5bcd^2e^2+3c^2d^4\right)+ce^2x^2\left(-ae^4+3bd^2e^2+7cd^4\right)-abe^6-3acd^2e^4+3b^2d^2e^4\right.}{\left(a+bx^2+cx^4\right)\left(ae^4+bd^2e^2+cd^4\right)^3}\right.$$

$$\Big\downarrow \text{ 7239}$$

$$\int \frac{1}{(d+ex)^2\left(a+bx^2+cx^4\right)^2}dx$$

$$\Big\downarrow \text{ 7293}$$

$$\int \left(\frac{e^4\left(-2dex\left(-ace^4+2b^2e^4+5bcd^2e^2+3c^2d^4\right)+ce^2x^2\left(-ae^4+3bd^2e^2+7cd^4\right)-abe^6-3acd^2e^4+3b^2d^2e^4\right.}{\left(a+bx^2+cx^4\right)\left(ae^4+bd^2e^2+cd^4\right)^3}\right.$$

$$\Big\downarrow \text{ 7239}$$

$$\int \frac{1}{(d+ex)^2\left(a+bx^2+cx^4\right)^2}dx$$

$$\Big\downarrow \text{ 7293}$$

$$\int \left(\frac{e^4\left(-2dex\left(-ace^4+2b^2e^4+5bcd^2e^2+3c^2d^4\right)+ce^2x^2\left(-ae^4+3bd^2e^2+7cd^4\right)-abe^6-3acd^2e^4+3b^2d^2e^4\right.}{\left(a+bx^2+cx^4\right)\left(ae^4+bd^2e^2+cd^4\right)^3}\right.$$

$$\Big\downarrow \text{ 7239}$$

$$\int \frac{1}{(d+ex)^2 (a+bx^2+cx^4)^2} dx$$

input
```
Int[1/((d + e*x)^2*(a + b*x^2 + c*x^4)^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7239
```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 2539, normalized size of antiderivative = 1.50

method	result	size
default	Expression too large to display	2539
risch	Expression too large to display	11714

input
```
int(1/(e*x+d)^2/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output

```
-1/(a*e^4+b*d^2*e^2+c*d^4)^3*((1/2*c*(2*a^3*c*e^10-a^2*b^2*e^10-3*a^2*b*c*
d^2*e^8-4*a^2*c^2*d^4*e^6-4*a*b^2*c*d^4*e^6-10*a*b*c^2*d^6*e^4-6*a*c^3*d^8
*e^2+b^4*d^4*e^6+3*b^3*c*d^6*e^4+3*b^2*c^2*d^8*e^2+b*c^3*d^10)/a/(4*a*c-b^
2)*x^3-c*d*e*(2*a^2*c*e^8-a*b^2*e^8-b^3*d^2*e^6-3*b^2*c*d^4*e^4-4*b*c^2*d^
6*e^2-2*c^3*d^8)/(4*a*c-b^2)*x^2+1/2*(3*a^3*b*c*e^10+6*a^3*c^2*d^2*e^8-a^2
*b^3*e^10-3*a^2*b^2*c*d^2*e^8+2*a^2*b*c^2*d^4*e^6+4*a^2*c^3*d^6*e^4-5*a*b^
3*c*d^4*e^6-12*a*b^2*c^2*d^6*e^4-9*a*b*c^3*d^8*e^2-2*a*c^4*d^10+b^5*d^4*e^
6+3*b^4*c*d^6*e^4+3*b^3*c^2*d^8*e^2+b^2*c^3*d^10)/a/(4*a*c-b^2)*x-d*e*(3*a
^2*b*c*e^8+4*a^2*c^2*d^2*e^6-a*b^3*e^8+a*b^2*c*d^2*e^6+6*a*b*c^2*d^4*e^4+4
*a*c^3*d^6*e^2-b^4*d^2*e^6-3*b^3*c*d^4*e^4-3*b^2*c^2*d^6*e^2-b*c^3*d^8)/(4
*a*c-b^2))/(c*x^4+b*x^2+a)+2/a/(4*a*c-b^2)*c*(1/(16*a*c-4*b^2)*(-1/4*(-48*
(-4*a*c+b^2)^(1/2)*a^3*c^2*d*e^9+48*(-4*a*c+b^2)^(1/2)*a^2*b^2*c*d*e^9+96*
(-4*a*c+b^2)^(1/2)*a^2*b*c^2*d^3*e^7+96*(-4*a*c+b^2)^(1/2)*a^2*c^3*d^5*e^5
-8*(-4*a*c+b^2)^(1/2)*a*b^4*d*e^9-16*(-4*a*c+b^2)^(1/2)*a*b^3*c*d^3*e^7+32
*(-4*a*c+b^2)^(1/2)*a*b*c^3*d^7*e^3+16*(-4*a*c+b^2)^(1/2)*a*c^4*d^9*e-128*
a^3*d*e^9*b*c^2-256*a^3*d^3*e^7*c^3+64*a^2*d*e^9*b^3*c+128*a^2*d^3*e^7*c^2
*b^2-8*a*d*e^9*b^5-16*a*d^3*e^7*b^4*c)/c*ln(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)
+1/2*(-4*a*b*c^4*d^10+22*a^3*b^2*c*e^10+240*a^3*c^3*d^4*e^6+6*a*b^5*d^2*e^
8+24*a^2*c^4*d^8*e^2+3*b^5*c*d^6*e^4+3*b^4*c^2*d^8*e^2-12*(-4*a*c+b^2)^(1/
2)*a*c^4*d^10+(-4*a*c+b^2)^(1/2)*b^5*d^4*e^6+(-4*a*c+b^2)^(1/2)*b^2*c^3...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^2 (a+bx^2+cx^4)^2} dx = \text{Timed out}$$

input

```
integrate(1/(e*x+d)^2/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

output

```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^2 (a+bx^2+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x+d)**2/(c*x**4+b*x**2+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{(d+ex)^2 (a+bx^2+cx^4)^2}\,dx = \int \frac{1}{(cx^4+bx^2+a)^2(ex+d)^2}\,dx$$

input
```
integrate(1/(e*x+d)^2/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

output
```
4*(2*c*d^3*e^7 + b*d*e^9)*log(e*x + d)/(c^3*d^12 + 3*b*c^2*d^10*e^2 + 3*a^
2*b*d^2*e^10 + a^3*e^12 + 3*(b^2*c + a*c^2)*d^8*e^4 + (b^3 + 6*a*b*c)*d^6*
e^6 + 3*(a*b^2 + a^2*c)*d^4*e^8) + 1/2*(2*a*b*c^2*d^6*e + 4*(a*b^2*c - 2*a
^2*c^2)*d^4*e^3 + 2*(a*b^3 - 3*a^2*b*c)*d^2*e^5 - 2*(a^2*b^2 - 4*a^3*c)*e^
7 + (b*c^3*d^6*e + 2*(b^2*c^2 - 3*a*c^3)*d^4*e^3 + (b^3*c - 5*a*b*c^2)*d^2
*e^5 - (3*a*b^2*c - 10*a^2*c^2)*e^7)*x^4 + (b*c^3*d^7 + 2*(b^2*c^2 - a*c^3
)*d^5*e^2 + (b^3*c - a*b*c^2)*d^3*e^4 + (a*b^2*c - 2*a^2*c^2)*d*e^6)*x^3 +
 ((b^2*c^2 + 2*a*c^3)*d^6*e + (2*b^3*c - 3*a*b*c^2)*d^4*e^3 + (b^4 - 4*a*b
^2*c + 2*a^2*c^2)*d^2*e^5 - (3*a*b^3 - 11*a^2*b*c)*e^7)*x^2 + ((b^2*c^2 -
2*a*c^3)*d^7 + (2*b^3*c - 5*a*b*c^2)*d^5*e^2 + (b^4 - 2*a*b^2*c - 2*a^2*c^
2)*d^3*e^4 + (a*b^3 - 3*a^2*b*c)*d*e^6)*x)/((a^2*b^2*c^2 - 4*a^3*c^3)*d^9
+ 2*(a^2*b^3*c - 4*a^3*b*c^2)*d^7*e^2 + (a^2*b^4 - 2*a^3*b^2*c - 8*a^4*c^2
)*d^5*e^4 + 2*(a^3*b^3 - 4*a^4*b*c)*d^3*e^6 + (a^4*b^2 - 4*a^5*c)*d*e^8 +
((a*b^2*c^3 - 4*a^2*c^4)*d^8*e + 2*(a*b^3*c^2 - 4*a^2*b*c^3)*d^6*e^3 + (a*
b^4*c - 2*a^2*b^2*c^2 - 8*a^3*c^3)*d^4*e^5 + 2*(a^2*b^3*c - 4*a^3*b*c^2)*d
^2*e^7 + (a^3*b^2*c - 4*a^4*c^2)*e^9)*x^5 + ((a*b^2*c^3 - 4*a^2*c^4)*d^9 +
 2*(a*b^3*c^2 - 4*a^2*b*c^3)*d^7*e^2 + (a*b^4*c - 2*a^2*b^2*c^2 - 8*a^3*c^
3)*d^5*e^4 + 2*(a^2*b^3*c - 4*a^3*b*c^2)*d^3*e^6 + (a^3*b^2*c - 4*a^4*c^2)
*d*e^8)*x^4 + ((a*b^3*c^2 - 4*a^2*b*c^3)*d^8*e + 2*(a*b^4*c - 4*a^2*b^2*c^
2)*d^6*e^3 + (a*b^5 - 2*a^2*b^3*c - 8*a^3*b*c^2)*d^4*e^5 + 2*(a^2*b^4 -...
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^2 (a+bx^2+cx^4)^2} \, dx = \text{Timed out}$$

input
```
integrate(1/(e*x+d)^2/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 39.81 (sec) , antiderivative size = 45031, normalized size of antiderivative = 26.66

$$\int \frac{1}{(d+ex)^2 (a+bx^2+cx^4)^2} dx = \text{Too large to display}$$

input
```
int(1/((d + e*x)^2*(a + b*x^2 + c*x^4)^2),x)
```

output
```
symsum(log(root(62914560*a^12*b*c^8*d^10*e^14*z^5 + 62914560*a^11*b*c^9*d^
14*e^10*z^5 + 31457280*a^13*b*c^7*d^6*e^18*z^5 + 31457280*a^10*b*c^10*d^18
*e^6*z^5 + 6291456*a^14*b*c^6*d^2*e^22*z^5 + 6291456*a^9*b*c^11*d^22*e^2*z
^5 - 115200*a^7*b^13*c*d^6*e^18*z^5 - 90624*a^8*b^12*c*d^4*e^20*z^5 - 7680
0*a^6*b^14*c*d^8*e^16*z^5 - 36864*a^9*b^11*c*d^2*e^22*z^5 - 21504*a^5*b^15
*c*d^10*e^14*z^5 + 1536*a^4*b^16*c*d^12*e^12*z^5 + 1536*a^3*b^17*c*d^14*e^
10*z^5 - 90439680*a^10*b^4*c^7*d^12*e^12*z^5 - 63897600*a^11*b^4*c^6*d^8*e
^16*z^5 - 63897600*a^9*b^4*c^8*d^16*e^8*z^5 + 62914560*a^11*b^2*c^8*d^12*e
^12*z^5 + 39321600*a^12*b^2*c^7*d^8*e^16*z^5 + 39321600*a^10*b^2*c^9*d^16*
e^8*z^5 + 35782656*a^9*b^6*c^6*d^12*e^12*z^5 - 31457280*a^11*b^3*c^7*d^10*
e^14*z^5 - 31457280*a^10*b^3*c^8*d^14*e^10*z^5 + 30474240*a^10*b^6*c^5*d^8
*e^16*z^5 + 30474240*a^8*b^6*c^7*d^16*e^8*z^5 + 29884416*a^9*b^7*c^5*d^10*
e^14*z^5 + 29884416*a^8*b^7*c^6*d^14*e^10*z^5 - 29097984*a^10*b^5*c^6*d^10
*e^14*z^5 - 29097984*a^9*b^5*c^7*d^14*e^10*z^5 - 26214400*a^12*b^3*c^6*d^6
*e^18*z^5 - 26214400*a^9*b^3*c^9*d^18*e^6*z^5 - 17694720*a^12*b^4*c^5*d^4*
e^20*z^5 - 17694720*a^8*b^4*c^9*d^20*e^4*z^5 + 12779520*a^11*b^6*c^4*d^4*e
^20*z^5 + 12779520*a^7*b^6*c^8*d^20*e^4*z^5 - 10076160*a^8*b^9*c^4*d^10*e^
14*z^5 - 10076160*a^7*b^9*c^5*d^14*e^10*z^5 + 9830400*a^10*b^7*c^4*d^6*e^1
8*z^5 + 9830400*a^7*b^7*c^7*d^18*e^6*z^5 - 9437184*a^13*b^3*c^5*d^2*e^22*z
^5 - 9437184*a^8*b^3*c^10*d^22*e^2*z^5 + 6291456*a^13*b^2*c^6*d^4*e^20*...
```

Reduce [F]

$$\int \frac{1}{(d+ex)^2 (a+bx^2+cx^4)^2} dx = \int \frac{1}{(ex+d)^2 (c\,x^4+b\,x^2+a)^2}dx$$

input
```
int(1/(e*x+d)^2/(c*x^4+b*x^2+a)^2,x)
```

output

```
int(1/(e*x+d)^2/(c*x^4+b*x^2+a)^2,x)
```

3.247 $\int (d+ex)^3\sqrt{a+bx^2+cx^4}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 546

$$\int (d+ex)^3\sqrt{a+bx^2+cx^4}\,dx = \frac{d(5bcd^2-6b^2e^2+18ace^2)\,x\sqrt{a+bx^2+cx^4}}{15c^{3/2}\left(\sqrt{a}+\sqrt{c}x^2\right)}$$
$$+\frac{e(6cd^2-be^2)\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{16c^2}$$
$$+\frac{dx(5cd^2+3be^2+9ce^2x^2)\sqrt{a+bx^2+cx^4}}{15c}+\frac{e^3(a+bx^2+cx^4)^{3/2}}{6c}$$
$$-\frac{(b^2-4ac)\,e(6cd^2-be^2)\operatorname{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{32c^{5/2}}$$
$$-\frac{\sqrt[4]{a}d(5bcd^2-6b^2e^2+18ace^2)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{15c^{7/4}\sqrt{a+bx^2+cx^4}}$$
$$+\frac{\sqrt[4]{a}\left(b+2\sqrt{a}\sqrt{c}\right)d\left(5cd^2-6be^2+9\sqrt{a}\sqrt{c}e^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\right.}{30c^{7/4}\sqrt{a+bx^2+cx^4}}$$

output
```
1/15*d*(18*a*c*e^2-6*b^2*e^2+5*b*c*d^2)*x*(c*x^4+b*x^2+a)^(1/2)/c^(3/2)/(a
^(1/2)+c^(1/2)*x^2)+1/16*e*(-b*e^2+6*c*d^2)*(2*c*x^2+b)*(c*x^4+b*x^2+a)^(1
/2)/c^2+1/15*d*x*(9*c*e^2*x^2+3*b*e^2+5*c*d^2)*(c*x^4+b*x^2+a)^(1/2)/c+1/6
*e^3*(c*x^4+b*x^2+a)^(3/2)/c-1/32*(-4*a*c+b^2)*e*(-b*e^2+6*c*d^2)*arctanh(
1/2*(2*c*x^2+b)/c^(1/2)/(c*x^4+b*x^2+a)^(1/2))/c^(5/2)-1/15*a^(1/4)*d*(18*
a*c*e^2-6*b^2*e^2+5*b*c*d^2)*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^(1/
2)+c^(1/2)*x^2)^2)^(1/2)*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2
-b/a^(1/2)/c^(1/2))^(1/2))/c^(7/4)/(c*x^4+b*x^2+a)^(1/2)+1/30*a^(1/4)*(b+2
*a^(1/2)*c^(1/2))*d*(5*c*d^2-6*b*e^2+9*a^(1/2)*c^(1/2)*e^2)*(a^(1/2)+c^(1/
2)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*
arctan(c^(1/4)*x/a^(1/4)),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))/c^(7/4)/(c*x^4+
b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 14.29 (sec) , antiderivative size = 707, normalized size of antiderivative = 1.29

$$\int (d+ex)^3\sqrt{a+bx^2+cx^4}\,dx$$

$$= \frac{2\sqrt{c}(a+bx^2+cx^4)\left(-15b^2e^3+2bce(45d^2+24dex+5e^2x^2)+4c(10ae^3+cx(20d^3+45d^2ex+36de^2x^2}{}$$

input
```
Integrate[(d + e*x)^3*Sqrt[a + b*x^2 + c*x^4],x]
```

output
```
(2*Sqrt[c]*(a + b*x^2 + c*x^4)*(-15*b^2*e^3 + 2*b*c*e*(45*d^2 + 24*d*e*x +
 5*e^2*x^2) + 4*c*(10*a*e^3 + c*x*(20*d^3 + 45*d^2*e*x + 36*d*e^2*x^2 + 10
*e^3*x^3))) + ((-8*I)*Sqrt[2]*Sqrt[c]*(-b + Sqrt[b^2 - 4*a*c])*d*(-5*b*c*d
^2 + 6*b^2*e^2 - 18*a*c*e^2)*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - S
qrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 -
4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (
b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] + (8*I)*Sqrt[2]*Sqrt[c]*d*
(-6*b^3*e^2 + b*c*(-5*Sqrt[b^2 - 4*a*c]*d^2 + 24*a*e^2) + b^2*(5*c*d^2 + 6
*Sqrt[b^2 - 4*a*c]*e^2) - 2*a*c*(10*c*d^2 + 9*Sqrt[b^2 - 4*a*c]*e^2))*Sqrt
[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt
[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSinh[Sqrt
[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[
b^2 - 4*a*c])] - 15*(b^2 - 4*a*c)*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*e*(-6*c*
d^2 + b*e^2)*Sqrt[a + b*x^2 + c*x^4]*Log[b + 2*c*x^2 - 2*Sqrt[c]*Sqrt[a +
b*x^2 + c*x^4]])/Sqrt[c/(b + Sqrt[b^2 - 4*a*c])])/(480*c^(5/2)*Sqrt[a + b*
x^2 + c*x^4])
```

Rubi [A] (verified)

Time = 1.50 (sec) , antiderivative size = 526, normalized size of antiderivative = 0.96, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.542$, Rules used = {2202, 1490, 27, 1511, 27, 1416, 1509, 1576, 27, 1160, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^3\sqrt{a+bx^2+cx^4}\,dx$$

$$\Big\downarrow \text{2202}$$

$$\int \left(d^3+3e^2x^2d\right)\sqrt{cx^4+bx^2+a}dx+\int x\left(x^2e^3+3d^2e\right)\sqrt{cx^4+bx^2+a}dx$$

$$\Big\downarrow \text{1490}$$

$$\frac{\int \frac{d\left(\left(5bcd^2-6b^2e^2+18ace^2\right)x^2+a\left(10cd^2-3be^2\right)\right)}{\sqrt{cx^4+bx^2+a}}dx}{15c}+\int x\left(x^2e^3+3d^2e\right)\sqrt{cx^4+bx^2+a}dx+\frac{dx\sqrt{a+bx^2+cx^4}\left(3be^2+5cd^2+9ce^2x^2\right)}{15c}$$

$$\Big\downarrow \text{27}$$

$$\frac{d\int \frac{(5bcd^2-6b^2e^2+18ace^2)x^2+a(10cd^2-3be^2)}{\sqrt{cx^4+bx^2+a}}dx}{15c}+\int x\left(x^2e^3+3d^2e\right)\sqrt{cx^4+bx^2+a}dx+\frac{dx\sqrt{a+bx^2+cx^4}\left(3be^2+5cd^2+9ce^2x^2\right)}{15c}$$

$\downarrow$ 1511

$$\frac{d\left(\frac{\sqrt{a}(2\sqrt{a}\sqrt{c}+b)(9\sqrt{a}\sqrt{c}e^2-6be^2+5cd^2)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}-\frac{\sqrt{a}(18ace^2-6b^2e^2+5bcd^2)\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{a}\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}\right)}{15c}+\int x\left(x^2e^3+3d^2e\right)\sqrt{cx^4+bx^2+a}dx+\frac{dx\sqrt{a+bx^2+cx^4}\left(3be^2+5cd^2+9ce^2x^2\right)}{15c}$$

$\downarrow$ 27

$$\frac{d\left(\frac{\sqrt{a}(2\sqrt{a}\sqrt{c}+b)(9\sqrt{a}\sqrt{c}e^2-6be^2+5cd^2)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}-\frac{(18ace^2-6b^2e^2+5bcd^2)\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}\right)}{15c}+\int x\left(x^2e^3+3d^2e\right)\sqrt{cx^4+bx^2+a}dx+\frac{dx\sqrt{a+bx^2+cx^4}\left(3be^2+5cd^2+9ce^2x^2\right)}{15c}$$

$\downarrow$ 1416

$$\frac{d\left(\frac{\sqrt[4]{a}(2\sqrt{a}\sqrt{c}+b)(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}(9\sqrt{a}\sqrt{c}e^2-6be^2+5cd^2)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2c^{3/4}\sqrt{a+bx^2+cx^4}}-\frac{(18ace^2-6b^2e^2+5bc\ldots}{\ldots}\right.}{15c}$$

$$\int x\left(x^2e^3+3d^2e\right)\sqrt{cx^4+bx^2+a}dx+\frac{dx\sqrt{a+bx^2+cx^4}\left(3be^2+5cd^2+9ce^2x^2\right)}{15c}$$

$\downarrow$ 1509

$$\int x\left(x^2e^3+3d^2e\right)\sqrt{cx^4+bx^2+a}dx+$$

$$\frac{d\left(\frac{\sqrt[4]{a}(2\sqrt{a}\sqrt{c}+b)(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}(9\sqrt{a}\sqrt{c}e^2-6be^2+5cd^2)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2c^{3/4}\sqrt{a+bx^2+cx^4}}-\frac{(18ace^2-6b^2e^2+5bc\ldots}{\ldots}\right.}{15c}$$

$$\frac{dx\sqrt{a+bx^2+cx^4}\left(3be^2+5cd^2+9ce^2x^2\right)}{15c}$$

$$\Big\downarrow \text{1576}$$

$$\frac{1}{2}\int e\left(3d^2+e^2x^2\right)\sqrt{cx^4+bx^2+a}dx^2+$$

$$\frac{d\left(\frac{\sqrt[4]{a}\left(2\sqrt{a}\sqrt{c}+b\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(9\sqrt{a}\sqrt{c}e^2-6be^2+5cd^2\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2c^{3/4}\sqrt{a+bx^2+cx^4}}-\frac{\left(18ace^2-6b^2e^2+5bc\right.}{}\right.}{15c}$$

$$\frac{dx\sqrt{a+bx^2+cx^4}\left(3be^2+5cd^2+9ce^2x^2\right)}{15c}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}e\int\left(3d^2+e^2x^2\right)\sqrt{cx^4+bx^2+a}dx^2+$$

$$\frac{d\left(\frac{\sqrt[4]{a}\left(2\sqrt{a}\sqrt{c}+b\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(9\sqrt{a}\sqrt{c}e^2-6be^2+5cd^2\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2c^{3/4}\sqrt{a+bx^2+cx^4}}-\frac{\left(18ace^2-6b^2e^2+5bc\right.}{}\right.}{15c}$$

$$\frac{dx\sqrt{a+bx^2+cx^4}\left(3be^2+5cd^2+9ce^2x^2\right)}{15c}$$

$$\Big\downarrow \text{1160}$$

$$\frac{1}{2}e\left(\frac{\left(6cd^2-be^2\right)\int\sqrt{cx^4+bx^2+a}dx^2}{2c}+\frac{e^2\left(a+bx^2+cx^4\right)^{3/2}}{3c}\right)+$$

$$\frac{d\left(\frac{\sqrt[4]{a}\left(2\sqrt{a}\sqrt{c}+b\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(9\sqrt{a}\sqrt{c}e^2-6be^2+5cd^2\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2c^{3/4}\sqrt{a+bx^2+cx^4}}-\frac{\left(18ace^2-6b^2e^2+5bc\right.}{}\right.}{15c}$$

$$\frac{dx\sqrt{a+bx^2+cx^4}\left(3be^2+5cd^2+9ce^2x^2\right)}{15c}$$

$$\Big\downarrow \text{1087}$$

$$\frac{1}{2}e\left(\frac{(6cd^2-be^2)\left(\frac{(b+2cx^2)\sqrt{a+bx^2+cx^4}}{4c}-\frac{(b^2-4ac)\int\frac{1}{\sqrt{cx^4+bx^2+a}}dx^2}{8c}\right)}{2c}+\frac{e^2(a+bx^2+cx^4)^{3/2}}{3c}\right)+$$

$$d\left(\frac{\sqrt[4]{a}(2\sqrt{a}\sqrt{c}+b)(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}(9\sqrt{a}\sqrt{c}e^2-6be^2+5cd^2)\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2c^{3/4}\sqrt{a+bx^2+cx^4}}-\frac{(18ace^2-6b^2e^2+5bc\ldots}{\ldots}\right.$$

$$\overline{\qquad\qquad 15c}$$

$$\frac{dx\sqrt{a+bx^2+cx^4}(3be^2+5cd^2+9ce^2x^2)}{15c}$$

$$\downarrow \text{1092}$$

$$\frac{1}{2}e\left(\frac{(6cd^2-be^2)\left(\frac{(b+2cx^2)\sqrt{a+bx^2+cx^4}}{4c}-\frac{(b^2-4ac)\int\frac{1}{4c-x^4}d\frac{2cx^2+b}{\sqrt{cx^4+bx^2+a}}}{4c}\right)}{2c}+\frac{e^2(a+bx^2+cx^4)^{3/2}}{3c}\right)+$$

$$d\left(\frac{\sqrt[4]{a}(2\sqrt{a}\sqrt{c}+b)(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}(9\sqrt{a}\sqrt{c}e^2-6be^2+5cd^2)\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2c^{3/4}\sqrt{a+bx^2+cx^4}}-\frac{(18ace^2-6b^2e^2+5bc\ldots}{\ldots}\right.$$

$$\overline{\qquad\qquad 15c}$$

$$\frac{dx\sqrt{a+bx^2+cx^4}(3be^2+5cd^2+9ce^2x^2)}{15c}$$

$$\downarrow \text{219}$$

$$d\left(\frac{\sqrt[4]{a}(2\sqrt{a}\sqrt{c}+b)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(9\sqrt{a}\sqrt{c}e^2-6be^2+5cd^2\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2c^{3/4}\sqrt{a+bx^2+cx^4}}-\frac{\left(18ace^2-6b^2e^2+5bc\right.}{}\right)$$
$$\overline{\qquad 15c \qquad}$$

$$\frac{1}{2}e\left(\frac{\left(6cd^2-be^2\right)\left(\frac{(b+2cx^2)\sqrt{a+bx^2+cx^4}}{4c}-\frac{(b^2-4ac)\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{8c^{3/2}}\right)}{2c}+\frac{e^2\left(a+bx^2+cx^4\right)^{3/2}}{3c}\right)+$$

$$\frac{dx\sqrt{a+bx^2+cx^4}\left(3be^2+5cd^2+9ce^2x^2\right)}{15c}$$

input

```
Int[(d + e*x)^3*Sqrt[a + b*x^2 + c*x^4],x]
```

output

```
(d*x*(5*c*d^2 + 3*b*e^2 + 9*c*e^2*x^2)*Sqrt[a + b*x^2 + c*x^4])/(15*c) + (
e*((e^2*(a + b*x^2 + c*x^4)^(3/2))/(3*c) + ((6*c*d^2 - b*e^2)*(((b + 2*c*x
^2)*Sqrt[a + b*x^2 + c*x^4])/(4*c) - ((b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^2)/
(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(8*c^(3/2))))/(2*c)))/2 + (d*(-(((5*
b*c*d^2 - 6*b^2*e^2 + 18*a*c*e^2)*(-((x*Sqrt[a + b*x^2 + c*x^4])/(Sqrt[a]
+ Sqrt[c]*x^2)) + (a^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4
)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 -
 b/(Sqrt[a]*Sqrt[c]))/4])/(c^(1/4)*Sqrt[a + b*x^2 + c*x^4])))/Sqrt[c]) + (
a^(1/4)*(b + 2*Sqrt[a]*Sqrt[c])*(5*c*d^2 - 6*b*e^2 + 9*Sqrt[a]*Sqrt[c]*e^2
)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)
^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])
/(2*c^(3/4)*Sqrt[a + b*x^2 + c*x^4])))/(15*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1490
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Simp[2*(p/(c*(4*p + 1)*(4*p + 3)))
 Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3)
- b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 1576
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x]
, x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

rule 2202
```
Int[(Pn_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{n
 = Expon[Pn, x], k}, Int[Sum[Coeff[Pn, x, 2*k]*x^(2*k), {k, 0, n/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pn, x, 2*k + 1]*x^(2*k), {k, 0, (n -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pn, x]
 &&  !PolyQ[Pn, x^2]
```

Maple [A] (verified)

Time = 4.05 (sec) , antiderivative size = 687, normalized size of antiderivative = 1.26

method	result
elliptic	$\frac{e^{3} x^{4} \sqrt{c\,x^{4}+b\,x^{2}+a}}{6}+\frac{3 d\,e^{2} x^{3} \sqrt{c\,x^{4}+b\,x^{2}+a}}{5}+\frac{\left(\frac{1}{6} b\,e^{3}+3 d^{2} e c \right) x^{2} \sqrt{c\,x^{4}+b\,x^{2}+a}}{4 c}+\frac{\left(\frac{3}{5} b d\,e^{2}+c\,d^{3}\right) x \sqrt{c\,x^{4}+b\,x^{2}+a}}{3 c}+\frac{\left(\frac{e^{3} a}{3}+\right.}{}$
risch	$\frac{\left(40 x^{4} e^{3} c^{2}+144 c^{2} d\,e^{2} x^{3}+10 b c\,e^{3} x^{2}+180 c^{2} d^{2} e\,x^{2}+48 b d\,e^{2} x c+80 c^{2} d^{3} x+40 a c\,e^{3}-15 b^{2} e^{3}+90 b c\,d^{2} e \right) \sqrt{c\,x^{4}+b\,x^{2}+a}}{240 c^{2}}-\frac{\frac{8 c d \left(18 a c\,e^{2}\right.}{}}{}$
default	$d^{3} \left(\frac{x \sqrt{c\,x^{4}+b\,x^{2}+a}}{3}+\frac{a \sqrt{2}\, \sqrt{4-\frac{2 \left(-b+\sqrt{-4 a c+b^{2}}\right) x^{2}}{a}}\, \sqrt{4+\frac{2 \left(b+\sqrt{-4 a c+b^{2}}\right) x^{2}}{a}}\, \operatorname{EllipticF}\left(\frac{x \sqrt{2}\, \sqrt{\frac{-b+\sqrt{-4 a c+b^{2}}}{a}}}{2}, \frac{\sqrt{-4+\frac{2 b \left(b+\sqrt{}\right.}{}}}{2}\right.}{6 \sqrt{\frac{-b+\sqrt{-4 a c+b^{2}}}{a}}\, \sqrt{c\,x^{4}+b\,x^{2}+a}}\right.$

input

```
int((e*x+d)^3*(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
1/6*e^3*x^4*(c*x^4+b*x^2+a)^(1/2)+3/5*d*e^2*x^3*(c*x^4+b*x^2+a)^(1/2)+1/4*
(1/6*b*e^3+3*d^2*e*c)/c*x^2*(c*x^4+b*x^2+a)^(1/2)+1/3*(3/5*b*d*e^2+c*d^3)/
c*x*(c*x^4+b*x^2+a)^(1/2)+1/2*(1/3*e^3*a+3*b*d^2*e-3/4*b/c*(1/6*b*e^3+3*d^
2*e*c))/c*(c*x^4+b*x^2+a)^(1/2)+1/4*(d^3*a-1/3*a/c*(3/5*b*d*e^2+c*d^3))*2^
(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2
)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*Ell
ipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*
a*c+b^2)^(1/2))/a/c)^(1/2))+1/2*(3*d^2*e*a-1/2*a/c*(1/6*b*e^3+3*d^2*e*c)-1
/2*b/c*(1/3*e^3*a+3*b*d^2*e-3/4*b/c*(1/6*b*e^3+3*d^2*e*c)))*ln((2*c*x^2+b)
/c^(1/2)+2*(c*x^4+b*x^2+a)^(1/2))/c^(1/2)-1/2*(6/5*a*d*e^2+b*d^3-2/3*b/c*(
3/5*b*d*e^2+c*d^3))*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(
-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(
c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+
(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)
)-EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b
+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 625, normalized size of antiderivative = 1.14

$$\int (d+ex)^3\sqrt{a+bx^2+cx^4}\,dx$$

$$= \frac{32\sqrt{\frac{1}{2}}\left((5\,bc^2d^3-6\,(b^2c-3\,ac^2)de^2)x\sqrt{\frac{b^2-4\,ac}{c^2}}-(5\,b^2cd^3-6\,(b^3-3\,abc)de^2)x\right)\sqrt{c}\sqrt{\frac{c\sqrt{\frac{b^2-4\,ac}{c^2}}-b}{c}}E(\mathrm{arcs}}{}$$

input
```
integrate((e*x+d)^3*(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
1/960*(32*sqrt(1/2)*((5*b*c^2*d^3 - 6*(b^2*c - 3*a*c^2)*d*e^2)*x*sqrt((b^2
 - 4*a*c)/c^2) - (5*b^2*c*d^3 - 6*(b^3 - 3*a*b*c)*d*e^2)*x)*sqrt(c)*sqrt((
c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)*elliptic_e(arcsin(sqrt(1/2)*sqrt((c*sqrt
((b^2 - 4*a*c)/c^2) - b)/c)/x), 1/2*(b*c*sqrt((b^2 - 4*a*c)/c^2) + b^2 - 2
*a*c)/(a*c)) - 32*sqrt(1/2)*((5*(b*c^2 - 2*c^3)*d^3 - 3*(2*b^2*c - (6*a +
b)*c^2)*d*e^2)*x*sqrt((b^2 - 4*a*c)/c^2) - (5*(b^2*c + 2*b*c^2)*d^3 - 3*(2
*b^3 - (6*a*b - b^2)*c)*d*e^2)*x)*sqrt(c)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2)
- b)/c)*elliptic_f(arcsin(sqrt(1/2)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c
)/x), 1/2*(b*c*sqrt((b^2 - 4*a*c)/c^2) + b^2 - 2*a*c)/(a*c)) + 15*(6*(b^2*
c - 4*a*c^2)*d^2*e - (b^3 - 4*a*b*c)*e^3)*sqrt(c)*x*log(8*c^2*x^4 + 8*b*c*
x^2 + b^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) + 4*a*c) + 4*(
40*c^3*e^3*x^5 + 144*c^3*d*e^2*x^4 + 80*b*c^2*d^3 - 96*(b^2*c - 3*a*c^2)*d
*e^2 + 10*(18*c^3*d^2*e + b*c^2*e^3)*x^3 + 16*(5*c^3*d^3 + 3*b*c^2*d*e^2)*
x^2 + 5*(18*b*c^2*d^2*e - (3*b^2*c - 8*a*c^2)*e^3)*x)*sqrt(c*x^4 + b*x^2 +
 a))/(c^3*x)
```

Sympy [F]

$$\int (d+ex)^3\sqrt{a+bx^2+cx^4}\,dx = \int (d+ex)^3\sqrt{a+bx^2+cx^4}\,dx$$

input
```
integrate((e*x+d)**3*(c*x**4+b*x**2+a)**(1/2),x)
```

output
```
Integral((d + e*x)**3*sqrt(a + b*x**2 + c*x**4), x)
```

Maxima [F]

$$\int (d+ex)^3\sqrt{a+bx^2+cx^4}\,dx=\int \sqrt{cx^4+bx^2+a}(ex+d)^3\,dx$$

input
```
integrate((e*x+d)^3*(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^4 + b*x^2 + a)*(e*x + d)^3, x)
```

Giac [F]

$$\int (d+ex)^3\sqrt{a+bx^2+cx^4}\,dx=\int \sqrt{cx^4+bx^2+a}(ex+d)^3\,dx$$

input
```
integrate((e*x+d)^3*(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^4 + b*x^2 + a)*(e*x + d)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)^3\sqrt{a+bx^2+cx^4}\,dx=\int (d+e\,x)^3\,\sqrt{c\,x^4+b\,x^2+a}\,dx$$

input
```
int((d + e*x)^3*(a + b*x^2 + c*x^4)^(1/2),x)
```

output
```
int((d + e*x)^3*(a + b*x^2 + c*x^4)^(1/2), x)
```

Reduce [F]

$$\int (d+ex)^3\sqrt{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
int((e*x+d)^3*(c*x^4+b*x^2+a)^(1/2),x)
```

output
```
(80*sqrt(a + b*x**2 + c*x**4)*a*c**2*e**3 - 30*sqrt(a + b*x**2 + c*x**4)*b
**2*c*e**3 + 180*sqrt(a + b*x**2 + c*x**4)*b*c**2*d**2*e + 96*sqrt(a + b*x
**2 + c*x**4)*b*c**2*d*e**2*x + 20*sqrt(a + b*x**2 + c*x**4)*b*c**2*e**3*x
**2 + 160*sqrt(a + b*x**2 + c*x**4)*c**3*d**3*x + 360*sqrt(a + b*x**2 + c*
x**4)*c**3*d**2*e*x**2 + 288*sqrt(a + b*x**2 + c*x**4)*c**3*d*e**2*x**3 +
80*sqrt(a + b*x**2 + c*x**4)*c**3*e**3*x**4 + 60*sqrt(c)*log(sqrt(a + b*x*
*2 + c*x**4) - sqrt(c)*x**2)*a*b*c*e**3 - 360*sqrt(c)*log(sqrt(a + b*x**2
+ c*x**4) - sqrt(c)*x**2)*a*c**2*d**2*e - 15*sqrt(c)*log(sqrt(a + b*x**2 +
 c*x**4) - sqrt(c)*x**2)*b**3*e**3 + 90*sqrt(c)*log(sqrt(a + b*x**2 + c*x*
*4) - sqrt(c)*x**2)*b**2*c*d**2*e - 60*sqrt(c)*log(sqrt(a + b*x**2 + c*x**
4) + sqrt(c)*x**2)*a*b*c*e**3 + 360*sqrt(c)*log(sqrt(a + b*x**2 + c*x**4)
+ sqrt(c)*x**2)*a*c**2*d**2*e + 15*sqrt(c)*log(sqrt(a + b*x**2 + c*x**4) +
 sqrt(c)*x**2)*b**3*e**3 - 90*sqrt(c)*log(sqrt(a + b*x**2 + c*x**4) + sqrt
(c)*x**2)*b**2*c*d**2*e - 96*int(sqrt(a + b*x**2 + c*x**4)/(a**2 + 2*a*b*x
**2 + a*c*x**4 + b**2*x**4 + b*c*x**6),x)*a**2*b*c**2*d*e**2 + 320*int(sqr
t(a + b*x**2 + c*x**4)/(a**2 + 2*a*b*x**2 + a*c*x**4 + b**2*x**4 + b*c*x**
6),x)*a**2*c**3*d**3 + 576*int((sqrt(a + b*x**2 + c*x**4)*x**4)/(a**2 + 2*
a*b*x**2 + a*c*x**4 + b**2*x**4 + b*c*x**6),x)*a*b*c**3*d*e**2 - 192*int((
sqrt(a + b*x**2 + c*x**4)*x**4)/(a**2 + 2*a*b*x**2 + a*c*x**4 + b**2*x**4
+ b*c*x**6),x)*b**3*c**2*d*e**2 + 160*int((sqrt(a + b*x**2 + c*x**4)*x*...
```

3.248 $\int (d+ex)^2\sqrt{a+bx^2+cx^4}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 491

$$\int (d+ex)^2\sqrt{a+bx^2+cx^4}\,dx$$

$$= \frac{(5bcd^2-2b^2e^2+6ace^2)\,x\sqrt{a+bx^2+cx^4}}{15c^{3/2}\left(\sqrt{a}+\sqrt{c}x^2\right)} + \frac{de(b+2cx^2)\sqrt{a+bx^2+cx^4}}{4c}$$

$$+ \frac{x(5cd^2+be^2+3ce^2x^2)\sqrt{a+bx^2+cx^4}}{15c} - \frac{(b^2-4ac)\,de\operatorname{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{8c^{3/2}}$$

$$- \frac{\sqrt[4]{a}(5bcd^2-2b^2e^2+6ace^2)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{15c^{7/4}\sqrt{a+bx^2+cx^4}}$$

$$+ \frac{\sqrt[4]{a}\left(b+2\sqrt{a}\sqrt{c}\right)\left(5cd^2-2be^2+3\sqrt{a}\sqrt{c}e^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\ \ldots\right.}{30c^{7/4}\sqrt{a+bx^2+cx^4}}$$

output
```
1/15*(6*a*c*e^2-2*b^2*e^2+5*b*c*d^2)*x*(c*x^4+b*x^2+a)^(1/2)/c^(3/2)/(a^(1
/2)+c^(1/2)*x^2)+1/4*d*e*(2*c*x^2+b)*(c*x^4+b*x^2+a)^(1/2)/c+1/15*x*(3*c*e
^2*x^2+b*e^2+5*c*d^2)*(c*x^4+b*x^2+a)^(1/2)/c-1/8*(-4*a*c+b^2)*d*e*arctanh
(1/2*(2*c*x^2+b)/c^(1/2)/(c*x^4+b*x^2+a)^(1/2))/c^(3/2)-1/15*a^(1/4)*(6*a*
c*e^2-2*b^2*e^2+5*b*c*d^2)*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2)
+c^(1/2)*x^2)^2)^(1/2)*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b
/a^(1/2)/c^(1/2))^(1/2))/c^(7/4)/(c*x^4+b*x^2+a)^(1/2)+1/30*a^(1/4)*(b+2*a
^(1/2)*c^(1/2))*(5*c*d^2-2*b*e^2+3*a^(1/2)*c^(1/2)*e^2)*(a^(1/2)+c^(1/2)*x
^2)*((c*x^4+b*x^2+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arct
an(c^(1/4)*x/a^(1/4)),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))/c^(7/4)/(c*x^4+b*x^
2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 12.80 (sec) , antiderivative size = 640, normalized size of antiderivative = 1.30

$$\int (d+ex)^2\sqrt{a+bx^2+cx^4}\,dx$$
$$= \frac{-2i\left(-b+\sqrt{b^2-4ac}\right)\left(-5bcd^2+2b^2e^2-6ace^2\right)\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{\frac{2b-2\sqrt{b^2-4ac}+4cx^2}{b-\sqrt{b^2-4ac}}}E\left(i\operatorname{arcsinh}\left(\sqrt{2}\sqrt{\frac{}{b}}\right.\right.}{}$$

input
```
Integrate[(d + e*x)^2*Sqrt[a + b*x^2 + c*x^4],x]
```

output
```
((-2*I)*(-b + Sqrt[b^2 - 4*a*c])*(-5*b*c*d^2 + 2*b^2*e^2 - 6*a*c*e^2)*Sqrt
[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*
Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticE[I*ArcSinh[
Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - S
qrt[b^2 - 4*a*c])] + (2*I)*(-2*b^3*e^2 + b*c*(-5*Sqrt[b^2 - 4*a*c]*d^2 + 8
*a*e^2) + b^2*(5*c*d^2 + 2*Sqrt[b^2 - 4*a*c]*e^2) - 2*a*c*(10*c*d^2 + 3*Sq
rt[b^2 - 4*a*c]*e^2))*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2
 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a
*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b +
 Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] + Sqrt[c]*Sqrt[c/(b + Sqrt[b^
2 - 4*a*c])]*(2*Sqrt[c]*(a + b*x^2 + c*x^4)*(b*e*(15*d + 4*e*x) + 2*c*x*(1
0*d^2 + 15*d*e*x + 6*e^2*x^2)) + 15*(b^2 - 4*a*c)*d*e*Sqrt[a + b*x^2 + c*x
^4]*Log[b + 2*c*x^2 - 2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4]]))/(120*c^2*Sqrt[c
/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] (verified)

Time = 1.10 (sec) , antiderivative size = 474, normalized size of antiderivative = 0.97, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$, Rules used = {2202, 27, 1432, 1087, 1092, 219, 1490, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^2\sqrt{a+bx^2+cx^4}\,dx$$

$$\Big\downarrow \text{2202}$$

$$\int \left(d^2+e^2x^2\right)\sqrt{cx^4+bx^2+a}dx+\int 2dex\sqrt{cx^4+bx^2+a}dx$$

$$\Big\downarrow \text{27}$$

$$\int \left(d^2+e^2x^2\right)\sqrt{cx^4+bx^2+a}dx+2de\int x\sqrt{cx^4+bx^2+a}dx$$

$$\Big\downarrow \text{1432}$$

$$\int \left(d^2+e^2x^2\right)\sqrt{cx^4+bx^2+a}dx+de\int \sqrt{cx^4+bx^2+a}dx^2$$

$$\Big\downarrow \text{1087}$$

$$de\left(\frac{(b+2cx^2)\sqrt{a+bx^2+cx^4}}{4c}-\frac{(b^2-4ac)\int\frac{1}{\sqrt{cx^4+bx^2+a}}dx^2}{8c}\right)+\int(d^2+e^2x^2)\sqrt{cx^4+bx^2+a}dx$$

$$\Big\downarrow \text{1092}$$

$$de\left(\frac{(b+2cx^2)\sqrt{a+bx^2+cx^4}}{4c}-\frac{(b^2-4ac)\int\frac{1}{4c-x^4}d\frac{2cx^2+b}{\sqrt{cx^4+bx^2+a}}}{4c}\right)+\int(d^2+e^2x^2)\sqrt{cx^4+bx^2+a}dx$$

$$\Big\downarrow \text{219}$$

$$\int(d^2+e^2x^2)\sqrt{cx^4+bx^2+a}dx+de\left(\frac{(b+2cx^2)\sqrt{a+bx^2+cx^4}}{4c}-\frac{(b^2-4ac)\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{8c^{3/2}}\right)$$

$$\Big\downarrow \text{1490}$$

$$\frac{\int\frac{(5bcd^2-2b^2e^2+6ace^2)x^2+a(10cd^2-be^2)}{\sqrt{cx^4+bx^2+a}}dx}{15c}+de\left(\frac{(b+2cx^2)\sqrt{a+bx^2+cx^4}}{4c}-\frac{(b^2-4ac)\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{8c^{3/2}}\right)+\frac{x\sqrt{a+bx^2+cx^4}(be^2+5cd^2+3ce^2x^2)}{15c}$$

$$\Big\downarrow \text{1511}$$

$$\frac{\frac{\sqrt{a}(2\sqrt{a}\sqrt{c}+b)(3\sqrt{a}\sqrt{c}e^2-2be^2+5cd^2)\int\frac{1}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}-\frac{\sqrt{a}(6ace^2-2b^2e^2+5bcd^2)\int\frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{a}\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}}{15c}+de\left(\frac{(b+2cx^2)\sqrt{a+bx^2+cx^4}}{4c}-\frac{(b^2-4ac)\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{8c^{3/2}}\right)+\frac{x\sqrt{a+bx^2+cx^4}(be^2+5cd^2+3ce^2x^2)}{15c}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\sqrt{a}(2\sqrt{a}\sqrt{c}+b)\left(3\sqrt{a}\sqrt{c}e^2-2be^2+5cd^2\right)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}-\frac{\left(6ace^2-2b^2e^2+5bcd^2\right)\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}}{15c}+de\left(\frac{\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{4c}-\frac{\left(b^2-4ac\right)\operatorname{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{8c^{3/2}}\right)+\frac{x\sqrt{a+bx^2+cx^4}\left(be^2+5cd^2+3ce^2x^2\right)}{15c}$$

$$\Big\downarrow \text{1416}$$

$$\frac{\frac{\sqrt[4]{a}(2\sqrt{a}\sqrt{c}+b)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(3\sqrt{a}\sqrt{c}e^2-2be^2+5cd^2\right)\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2c^{3/4}\sqrt{a+bx^2+cx^4}}-\frac{\left(6ace^2-2b^2e^2+5bcd^2\right)\int}{\sqrt{c}}}{15c}$$

$$de\left(\frac{\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{4c}-\frac{\left(b^2-4ac\right)\operatorname{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{8c^{3/2}}\right)+\frac{x\sqrt{a+bx^2+cx^4}\left(be^2+5cd^2+3ce^2x^2\right)}{15c}$$

$$\Big\downarrow \text{1509}$$

$$\frac{\frac{\sqrt[4]{a}(2\sqrt{a}\sqrt{c}+b)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(3\sqrt{a}\sqrt{c}e^2-2be^2+5cd^2\right)\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2c^{3/4}\sqrt{a+bx^2+cx^4}}-\frac{\left(6ace^2-2b^2e^2+5bcd^2\right)\Big(}{}}{15c}$$

$$de\left(\frac{\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{4c}-\frac{\left(b^2-4ac\right)\operatorname{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{8c^{3/2}}\right)+\frac{x\sqrt{a+bx^2+cx^4}\left(be^2+5cd^2+3ce^2x^2\right)}{15c}$$

input
```
Int[(d + e*x)^2*Sqrt[a + b*x^2 + c*x^4],x]
```

output
```
(x*(5*c*d^2 + b*e^2 + 3*c*e^2*x^2)*Sqrt[a + b*x^2 + c*x^4])/(15*c) + d*e*(
((b + 2*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(4*c) - ((b^2 - 4*a*c)*ArcTanh[(b
+ 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(8*c^(3/2))) + (-(((5*b*c
*d^2 - 2*b^2*e^2 + 6*a*c*e^2)*(-((x*Sqrt[a + b*x^2 + c*x^4])/(Sqrt[a] + Sq
rt[c]*x^2)) + (a^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(S
qrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(
Sqrt[a]*Sqrt[c]))/4])/(c^(1/4)*Sqrt[a + b*x^2 + c*x^4])))/Sqrt[c]) + (a^(1
/4)*(b + 2*Sqrt[a]*Sqrt[c])*(5*c*d^2 - 2*b*e^2 + 3*Sqrt[a]*Sqrt[c]*e^2)*(S
qrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*
EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*
c^(3/4)*Sqrt[a + b*x^2 + c*x^4]))/(15*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1432
```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

rule 1490
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Simp[2*(p/(c*(4*p + 1)*(4*p + 3)))
 Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3)
- b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 2202
```
Int[(Pn_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{n
 = Expon[Pn, x], k}, Int[Sum[Coeff[Pn, x, 2*k]*x^(2*k), {k, 0, n/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pn, x, 2*k + 1]*x^(2*k), {k, 0, (n -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pn, x]
 &&  !PolyQ[Pn, x^2]
```

Maple [A] (verified)

Time = 3.39 (sec) , antiderivative size = 561, normalized size of antiderivative = 1.14

method	result
elliptic	$\frac{e^2x^3\sqrt{c\,x^4+b\,x^2+a}}{5}+\frac{de\,x^2\sqrt{c\,x^4+b\,x^2+a}}{2}+\frac{\left(\frac{b\,e^2}{5}+c\,d^2\right)x\sqrt{c\,x^4+b\,x^2+a}}{3c}+\frac{bde\sqrt{c\,x^4+b\,x^2+a}}{4c}+\frac{\left(a\,d^2-\frac{a\left(\frac{b\,e^2}{5}+c\,d^2\right)}{3c}\right)\sqrt{\cdots}}{\cdots}$
risch	$\frac{(12c\,e^2x^3+30cde\,x^2+4b\,e^2x+20c\,d^2x+15bde)\sqrt{c\,x^4+b\,x^2+a}}{60c}-\frac{-\frac{15\left(4ac-b^2\right)de\ln\left(\frac{\frac{b}{2}+c\,x^2}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2+a}\right)}{2\sqrt{c}}+\frac{\left(24ac\,e^2-8b^2e^2+20bc\cdots\right.}{\cdots}}{\cdots}$
default	$d^2\left(\frac{x\sqrt{c\,x^4+b\,x^2+a}}{3}+\frac{a\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\mathrm{EllipticF}\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2},\frac{\sqrt{-4+\frac{2b\left(b+\sqrt{\cdots}\right.}{\cdots}}}{2}\cdots\right.}{6\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,\sqrt{c\,x^4+b\,x^2+a}}\right.\cdots$

input

```
int((e*x+d)^2*(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
1/5*e^2*x^3*(c*x^4+b*x^2+a)^(1/2)+1/2*d*e*x^2*(c*x^4+b*x^2+a)^(1/2)+1/3*(1
/5*b*e^2+c*d^2)/c*x*(c*x^4+b*x^2+a)^(1/2)+1/4*b*d*e/c*(c*x^4+b*x^2+a)^(1/2
)+1/4*(a*d^2-1/3*a/c*(1/5*b*e^2+c*d^2))*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a
)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/
2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*
c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))+1/2*
(a*d*e-1/4*b^2/c*d*e)*ln((2*c*x^2+b)/c^(1/2)+2*(c*x^4+b*x^2+a)^(1/2))/c^(1
/2)-1/2*(2/5*a*e^2+b*d^2-2/3*b/c*(1/5*b*e^2+c*d^2))*a*2^(1/2)/((-b+(-4*a*c
+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-
4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2)
)*(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(
b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2
)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 551, normalized size of antiderivative = 1.12

$$\int (d+ex)^2\sqrt{a+bx^2+cx^4}\,dx =$$

$$-\frac{15\,(b^2c-4\,ac^2)\sqrt{c}dex\log\left(8\,c^2x^4+8\,bcx^2+b^2+4\,\sqrt{cx^4+bx^2+a}(2\,cx^2+b)\sqrt{c}+4\,ac\right)-8\,\sqrt{\frac{1}{2}}\Big((5\,b}{}$$

input
```
integrate((e*x+d)^2*(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
-1/240*(15*(b^2*c - 4*a*c^2)*sqrt(c)*d*e*x*log(8*c^2*x^4 + 8*b*c*x^2 + b^2
 + 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) + 4*a*c) - 8*sqrt(1/2)*
((5*b*c^2*d^2 - 2*(b^2*c - 3*a*c^2)*e^2)*x*sqrt((b^2 - 4*a*c)/c^2) - (5*b^
2*c*d^2 - 2*(b^3 - 3*a*b*c)*e^2)*x)*sqrt(c)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2
) - b)/c)*elliptic_e(arcsin(sqrt(1/2)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)
/c)/x), 1/2*(b*c*sqrt((b^2 - 4*a*c)/c^2) + b^2 - 2*a*c)/(a*c)) + 8*sqrt(1/
2)*((5*(b*c^2 - 2*c^3)*d^2 - (2*b^2*c - (6*a + b)*c^2)*e^2)*x*sqrt((b^2 -
4*a*c)/c^2) - (5*(b^2*c + 2*b*c^2)*d^2 - (2*b^3 - (6*a*b - b^2)*c)*e^2)*x)
*sqrt(c)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)*elliptic_f(arcsin(sqrt(1/
2)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)/x), 1/2*(b*c*sqrt((b^2 - 4*a*c)
/c^2) + b^2 - 2*a*c)/(a*c)) - 4*(12*c^3*e^2*x^4 + 30*c^3*d*e*x^3 + 15*b*c^
2*d*e*x + 20*b*c^2*d^2 - 8*(b^2*c - 3*a*c^2)*e^2 + 4*(5*c^3*d^2 + b*c^2*e^
2)*x^2)*sqrt(c*x^4 + b*x^2 + a))/(c^3*x)
```

Sympy [F]

$$\int (d+ex)^2\sqrt{a+bx^2+cx^4}\,dx = \int (d+ex)^2\,\sqrt{a+bx^2+cx^4}\,dx$$

input
```
integrate((e*x+d)**2*(c*x**4+b*x**2+a)**(1/2),x)
```

output
```
Integral((d + e*x)**2*sqrt(a + b*x**2 + c*x**4), x)
```

Maxima [F]

$$\int (d+ex)^2\sqrt{a+bx^2+cx^4}\,dx = \int \sqrt{cx^4+bx^2+a}(ex+d)^2\,dx$$

input
```
integrate((e*x+d)^2*(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^4 + b*x^2 + a)*(e*x + d)^2, x)
```

Giac [F]

$$\int (d+ex)^2\sqrt{a+bx^2+cx^4}\,dx = \int \sqrt{cx^4+bx^2+a}(ex+d)^2\,dx$$

input
```
integrate((e*x+d)^2*(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^4 + b*x^2 + a)*(e*x + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)^2\sqrt{a+bx^2+cx^4}\,dx = \int (d+e\,x)^2\,\sqrt{c\,x^4+b\,x^2+a}\,dx$$

input
```
int((d + e*x)^2*(a + b*x^2 + c*x^4)^(1/2),x)
```

output
```
int((d + e*x)^2*(a + b*x^2 + c*x^4)^(1/2), x)
```

Reduce [F]

$$\int (d+ex)^2\sqrt{a+bx^2+cx^4}\,dx = \text{Too large to display}$$

input
```
int((e*x+d)^2*(c*x^4+b*x^2+a)^(1/2),x)
```

output
```
(30*sqrt(a + b*x**2 + c*x**4)*b*c*d*e + 8*sqrt(a + b*x**2 + c*x**4)*b*c*e*
*2*x + 40*sqrt(a + b*x**2 + c*x**4)*c**2*d**2*x + 60*sqrt(a + b*x**2 + c*x
**4)*c**2*d*e*x**2 + 24*sqrt(a + b*x**2 + c*x**4)*c**2*e**2*x**3 - 60*sqrt
(c)*log(sqrt(a + b*x**2 + c*x**4) - sqrt(c)*x**2)*a*c*d*e + 15*sqrt(c)*log
(sqrt(a + b*x**2 + c*x**4) - sqrt(c)*x**2)*b**2*d*e + 60*sqrt(c)*log(sqrt(
a + b*x**2 + c*x**4) + sqrt(c)*x**2)*a*c*d*e - 15*sqrt(c)*log(sqrt(a + b*x
**2 + c*x**4) + sqrt(c)*x**2)*b**2*d*e - 8*int(sqrt(a + b*x**2 + c*x**4)/(
a**2 + 2*a*b*x**2 + a*c*x**4 + b**2*x**4 + b*c*x**6),x)*a**2*b*c*e**2 + 80
*int(sqrt(a + b*x**2 + c*x**4)/(a**2 + 2*a*b*x**2 + a*c*x**4 + b**2*x**4 +
 b*c*x**6),x)*a**2*c**2*d**2 + 48*int((sqrt(a + b*x**2 + c*x**4)*x**4)/(a*
*2 + 2*a*b*x**2 + a*c*x**4 + b**2*x**4 + b*c*x**6),x)*a*b*c**2*e**2 - 16*i
nt((sqrt(a + b*x**2 + c*x**4)*x**4)/(a**2 + 2*a*b*x**2 + a*c*x**4 + b**2*x
**4 + b*c*x**6),x)*b**3*c*e**2 + 40*int((sqrt(a + b*x**2 + c*x**4)*x**4)/(
a**2 + 2*a*b*x**2 + a*c*x**4 + b**2*x**4 + b*c*x**6),x)*b**2*c**2*d**2 + 4
8*int((sqrt(a + b*x**2 + c*x**4)*x**2)/(a**2 + 2*a*b*x**2 + a*c*x**4 + b**
2*x**4 + b*c*x**6),x)*a**2*c**2*e**2 - 24*int((sqrt(a + b*x**2 + c*x**4)*x
**2)/(a**2 + 2*a*b*x**2 + a*c*x**4 + b**2*x**4 + b*c*x**6),x)*a*b**2*c*e**
2 + 120*int((sqrt(a + b*x**2 + c*x**4)*x**2)/(a**2 + 2*a*b*x**2 + a*c*x**4
 + b**2*x**4 + b*c*x**6),x)*a*b*c**2*d**2 - 120*int((sqrt(a + b*x**2 + c*x
**4)*x)/(a**2 + 2*a*b*x**2 + a*c*x**4 + b**2*x**4 + b*c*x**6),x)*a**2*c...
```

3.249 $\int (d+ex)\sqrt{a+bx^2+cx^4}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 397

$$\int (d+ex)\sqrt{a+bx^2+cx^4}\,dx = \frac{1}{3}dx\sqrt{a+bx^2+cx^4}+\frac{bdx\sqrt{a+bx^2+cx^4}}{3\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)}+\frac{e(b+2cx^2)\sqrt{a+bx^2+cx^4}}{8c}-\frac{(b^2-4ac)\,\operatorname{earctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{16c^{3/2}}-\frac{\sqrt[4]{a}bd\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{3c^{3/4}\sqrt{a+bx^2+cx^4}}+\frac{\sqrt[4]{a}\left(b+2\sqrt{a}\sqrt{c}\right)d\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{6c^{3/4}\sqrt{a+bx^2+cx^4}}$$

output
```
1/3*d*x*(c*x^4+b*x^2+a)^(1/2)+1/3*b*d*x*(c*x^4+b*x^2+a)^(1/2)/c^(1/2)/(a^(
1/2)+c^(1/2)*x^2)+1/8*e*(2*c*x^2+b)*(c*x^4+b*x^2+a)^(1/2)/c-1/16*(-4*a*c+b
^2)*e*arctanh(1/2*(2*c*x^2+b)/c^(1/2)/(c*x^4+b*x^2+a)^(1/2))/c^(3/2)-1/3*a
^(1/4)*b*d*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2)+c^(1/2)*x^2)^2)
^(1/2)*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2)
)^(1/2))/c^(3/4)/(c*x^4+b*x^2+a)^(1/2)+1/6*a^(1/4)*(b+2*a^(1/2)*c^(1/2))*d
*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*Inv
erseJacobiAM(2*arctan(c^(1/4)*x/a^(1/4)),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))/
c^(3/4)/(c*x^4+b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.87 (sec) , antiderivative size = 579, normalized size of antiderivative = 1.46

$$\int (d+ex)\sqrt{a+bx^2+cx^4}\,dx$$

$$= \frac{1}{48}\Bigg(\frac{6be\sqrt{a+bx^2+cx^4}}{c} + 16dx\sqrt{a+bx^2+cx^4} + 12ex^2\sqrt{a+bx^2+cx^4}$$

$$- \frac{16i\sqrt{2}ab\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}d\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}E\left(i\operatorname{arcsinh}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)}{c\sqrt{a+bx^2+cx^4}}$$

$$- \frac{16i\sqrt{2}a\sqrt{b^2-4ac}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}d\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\text{EllipticF}\bigg(i\operatorname{arcsinh}\bigg(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x}{c\sqrt{a+bx^2+cx^4}}$$

$$+ \frac{3b^2e\log\left(b+2cx^2-2\sqrt{c}\sqrt{a+bx^2+cx^4}\right)}{c^{3/2}}$$

$$- \frac{12ae\log\left(b+2cx^2-2\sqrt{c}\sqrt{a+bx^2+cx^4}\right)}{\sqrt{c}}\Bigg)$$

input
```
Integrate[(d + e*x)*Sqrt[a + b*x^2 + c*x^4],x]
```

output
```
((6*b*e*Sqrt[a + b*x^2 + c*x^4])/c + 16*d*x*Sqrt[a + b*x^2 + c*x^4] + 12*e
*x^2*Sqrt[a + b*x^2 + c*x^4] - ((16*I)*Sqrt[2]*a*b*Sqrt[c/(b + Sqrt[b^2 -
4*a*c])]*d*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]
*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*EllipticE
[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a
*c])/(b - Sqrt[b^2 - 4*a*c])])/(c*Sqrt[a + b*x^2 + c*x^4]) - ((16*I)*Sqrt[
2]*a*Sqrt[b^2 - 4*a*c]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*d*Sqrt[(b - Sqrt[b^
2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c]
 + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b
 + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])
])/(c*Sqrt[a + b*x^2 + c*x^4]) + (3*b^2*e*Log[b + 2*c*x^2 - 2*Sqrt[c]*Sqrt
[a + b*x^2 + c*x^4]])/c^(3/2) - (12*a*e*Log[b + 2*c*x^2 - 2*Sqrt[c]*Sqrt[a
 + b*x^2 + c*x^4]])/Sqrt[c])/48
```

Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 403, normalized size of antiderivative = 1.02, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2202, 27, 1404, 1432, 1087, 1092, 219, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)\sqrt{a+bx^2+cx^4}\,dx$$

$$\Big\downarrow \text{2202}$$

$$\int d\sqrt{cx^4+bx^2+a}dx+\int ex\sqrt{cx^4+bx^2+a}dx$$

$$\Big\downarrow \text{27}$$

$$d\int \sqrt{cx^4+bx^2+a}dx+e\int x\sqrt{cx^4+bx^2+a}dx$$

$$\Big\downarrow \text{1404}$$

$$d\left(\frac{1}{3}\int \frac{bx^2+2a}{\sqrt{cx^4+bx^2+a}}dx+\frac{1}{3}x\sqrt{a+bx^2+cx^4}\right)+e\int x\sqrt{cx^4+bx^2+a}dx$$

$$\Big\downarrow \text{1432}$$

$$d\left(\frac{1}{3}\int \frac{bx^2+2a}{\sqrt{cx^4+bx^2+a}}dx+\frac{1}{3}x\sqrt{a+bx^2+cx^4}\right)+\frac{1}{2}e\int \sqrt{cx^4+bx^2+a}dx^2$$

$$\Big\downarrow \text{1087}$$

$$\frac{1}{2}e\left(\frac{(b+2cx^2)\sqrt{a+bx^2+cx^4}}{4c}-\frac{(b^2-4ac)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx^2}{8c}\right)+ d\left(\frac{1}{3}\int \frac{bx^2+2a}{\sqrt{cx^4+bx^2+a}}dx+\frac{1}{3}x\sqrt{a+bx^2+cx^4}\right)$$

$$\Big\downarrow \text{1092}$$

$$\frac{1}{2}e\left(\frac{(b+2cx^2)\sqrt{a+bx^2+cx^4}}{4c}-\frac{(b^2-4ac)\int \frac{1}{4c-x^4}d\frac{2cx^2+b}{\sqrt{cx^4+bx^2+a}}}{4c}\right)+ d\left(\frac{1}{3}\int \frac{bx^2+2a}{\sqrt{cx^4+bx^2+a}}dx+\frac{1}{3}x\sqrt{a+bx^2+cx^4}\right)$$

$$\Big\downarrow \text{219}$$

$$d\left(\frac{1}{3}\int \frac{bx^2+2a}{\sqrt{cx^4+bx^2+a}}dx+\frac{1}{3}x\sqrt{a+bx^2+cx^4}\right)+\frac{1}{2}e\left(\frac{(b+2cx^2)\sqrt{a+bx^2+cx^4}}{4c}-\frac{(b^2-4ac)\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{8c^{3/2}}\right)$$

$\Big\downarrow$ 1511

$$d\left(\frac{1}{3}\left(\sqrt{a}\left(2\sqrt{a}+\frac{b}{\sqrt{c}}\right)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx-\frac{\sqrt{a}b\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{a}\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}\right)+\frac{1}{3}x\sqrt{a+bx^2+cx^4}\right)+\frac{1}{2}e\left(\frac{(b+2cx^2)\sqrt{a+bx^2+cx^4}}{4c}-\frac{(b^2-4ac)\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{8c^{3/2}}\right)$$

$\Big\downarrow$ 27

$$d\left(\frac{1}{3}\left(\sqrt{a}\left(2\sqrt{a}+\frac{b}{\sqrt{c}}\right)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx-\frac{b\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}\right)+\frac{1}{3}x\sqrt{a+bx^2+cx^4}\right)+\frac{1}{2}e\left(\frac{(b+2cx^2)\sqrt{a+bx^2+cx^4}}{4c}-\frac{(b^2-4ac)\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{8c^{3/2}}\right)$$

$\Big\downarrow$ 1416

$$d\left(\frac{1}{3}\left(\frac{\sqrt[4]{a}\left(2\sqrt{a}+\frac{b}{\sqrt{c}}\right)(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}-\frac{b\int \frac{\sqrt{a}-\sqrt{c}x}{\sqrt{cx^4+bx^2}}}{\sqrt{c}}\right.\right.$$
$$\frac{1}{2}e\left(\frac{(b+2cx^2)\sqrt{a+bx^2+cx^4}}{4c}-\frac{(b^2-4ac)\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{8c^{3/2}}\right)$$

$\Big\downarrow$ 1509

$$d\left(\frac{1}{3}\left(\frac{\sqrt[4]{a}\left(2\sqrt{a}+\frac{b}{\sqrt{c}}\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}-\frac{b\left(\frac{\sqrt[4]{a}(\sqrt{a}+\sqrt{}\right.}{}\right.}{}\right.\right.$$

$$\frac{1}{2}e\left(\frac{\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{4c}-\frac{\left(b^2-4ac\right)\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{8c^{3/2}}\right)$$

input
```
Int[(d + e*x)*Sqrt[a + b*x^2 + c*x^4],x]
```

output
```
(e*(((b + 2*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(4*c) - ((b^2 - 4*a*c)*ArcTanh
[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(8*c^(3/2))))/2 + d*(
(x*Sqrt[a + b*x^2 + c*x^4])/3 + (-((b*(-((x*Sqrt[a + b*x^2 + c*x^4])/(Sqrt
[a] + Sqrt[c]*x^2)) + (a^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c
*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)],
(2 - b/(Sqrt[a]*Sqrt[c]))/4])/(c^(1/4)*Sqrt[a + b*x^2 + c*x^4])))/Sqrt[c])
 + (a^(1/4)*(2*Sqrt[a] + b/Sqrt[c])*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^
2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/
4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*c^(1/4)*Sqrt[a + b*x^2 + c*x^4]))/3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1404
```
Int[((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*((a + b
*x^2 + c*x^4)^p/(4*p + 1)), x] + Simp[2*(p/(4*p + 1))   Int[(2*a + b*x^2)*(
a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*
c, 0] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1432
```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 2202
```
Int[(Pn_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{n
 = Expon[Pn, x], k}, Int[Sum[Coeff[Pn, x, 2*k]*x^(2*k), {k, 0, n/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pn, x, 2*k + 1]*x^(2*k), {k, 0, (n -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pn, x]
 &&  !PolyQ[Pn, x^2]
```

Maple [A] (verified)

Time = 1.62 (sec) , antiderivative size = 453, normalized size of antiderivative = 1.14

method	result
risch	$\frac{(6ce\,x^2+8xcd+3eb)\sqrt{c\,x^4+b\,x^2+a}}{24c}+\frac{\frac{3e(4ac-b^2)\ln\left(\frac{\frac{b}{2}+c\,x^2}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2+a}\right)}{2\sqrt{c}}+\frac{4acd\sqrt{2}\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\sqrt{4+\frac{2\left(b+\sqrt{-4ac+\cdots}\right.}{a}}}{\sqrt{\frac{-b+\sqrt{\cdots}}{}}}}{}$
default	$d\left(\frac{x\sqrt{c\,x^4+b\,x^2+a}}{3}+\frac{a\sqrt{2}\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\text{EllipticF}\left(\frac{x\sqrt{2}\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2},\frac{\sqrt{-4+\frac{2b\left(b+\sqrt{-\cdots}\right.}{a}}}{2}\right.}{6\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\sqrt{c\,x^4+b\,x^2+a}}\right.$
elliptic	$\frac{e\,x^2\sqrt{c\,x^4+b\,x^2+a}}{4}+\frac{dx\sqrt{c\,x^4+b\,x^2+a}}{3}+\frac{eb\sqrt{c\,x^4+b\,x^2+a}}{8c}+\frac{ad\sqrt{2}\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\text{Ellipti}\cdots}{6\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\sqrt{c\cdots}}$

input
```
int((e*x+d)*(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/24*(6*c*e*x^2+8*c*d*x+3*b*e)/c*(c*x^4+b*x^2+a)^(1/2)+1/24/c*(3/2*e*(4*a*
c-b^2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))/c^(1/2)+4*a*c*d*2^(
1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)
^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*Elli
pticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a
*c+b^2)^(1/2))/a/c)^(1/2))-4*c*b*d*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(
1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))
/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*
x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1
/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)
,1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 391, normalized size of antiderivative = 0.98

$$\int (d+ex)\sqrt{a+bx^2+cx^4}\,dx =$$

$$-\frac{3\,(b^2-4\,ac)\sqrt{c}ex\log\left(8\,c^2x^4+8\,bcx^2+b^2+4\,\sqrt{cx^4+bx^2+a}(2\,cx^2+b)\sqrt{c}+4\,ac\right)-16\,\sqrt{\frac{1}{2}}\left(bcdx\sqrt{\right.}$$

input
```
integrate((e*x+d)*(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
-1/96*(3*(b^2 - 4*a*c)*sqrt(c)*e*x*log(8*c^2*x^4 + 8*b*c*x^2 + b^2 + 4*sqr
t(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) + 4*a*c) - 16*sqrt(1/2)*(b*c*d*
x*sqrt((b^2 - 4*a*c)/c^2) - b^2*d*x)*sqrt(c)*sqrt((c*sqrt((b^2 - 4*a*c)/c^
2) - b)/c)*elliptic_e(arcsin(sqrt(1/2)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b
)/c)/x), 1/2*(b*c*sqrt((b^2 - 4*a*c)/c^2) + b^2 - 2*a*c)/(a*c)) + 16*sqrt(
1/2)*((b*c - 2*c^2)*d*x*sqrt((b^2 - 4*a*c)/c^2) - (b^2 + 2*b*c)*d*x)*sqrt(
c)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)*elliptic_f(arcsin(sqrt(1/2)*sqr
t((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)/x), 1/2*(b*c*sqrt((b^2 - 4*a*c)/c^2)
+ b^2 - 2*a*c)/(a*c)) - 4*(6*c^2*e*x^3 + 8*c^2*d*x^2 + 3*b*c*e*x + 8*b*c*d
)*sqrt(c*x^4 + b*x^2 + a))/(c^2*x)
```

Sympy [F]

$$\int (d+ex)\sqrt{a+bx^2+cx^4}\,dx = \int (d+ex)\sqrt{a+bx^2+cx^4}\,dx$$

input
```
integrate((e*x+d)*(c*x**4+b*x**2+a)**(1/2),x)
```

output
```
Integral((d + e*x)*sqrt(a + b*x**2 + c*x**4), x)
```

Maxima [F]

$$\int (d+ex)\sqrt{a+bx^2+cx^4}\,dx = \int \sqrt{cx^4+bx^2+a}(ex+d)\,dx$$

input
```
integrate((e*x+d)*(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^4 + b*x^2 + a)*(e*x + d), x)
```

Giac [F]

$$\int (d+ex)\sqrt{a+bx^2+cx^4}\,dx = \int \sqrt{cx^4+bx^2+a}(ex+d)\,dx$$

input
```
integrate((e*x+d)*(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^4 + b*x^2 + a)*(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)\sqrt{a+bx^2+cx^4}\,dx = \int (d+e\,x)\,\sqrt{c\,x^4+b\,x^2+a}\,dx$$

input
```
int((d + e*x)*(a + b*x^2 + c*x^4)^(1/2),x)
```

output
```
int((d + e*x)*(a + b*x^2 + c*x^4)^(1/2), x)
```

Reduce [F]

$$\int (d+ex)\sqrt{a+bx^2+cx^4}\,dx$$
$$= \frac{6\sqrt{c\,x^4+b\,x^2+a}\,bce+16\sqrt{c\,x^4+b\,x^2+a}\,c^2dx+12\sqrt{c\,x^4+b\,x^2+a}\,c^2e\,x^2-12\sqrt{c}\log\left(\sqrt{c\,x^4+b\,x^2+\right.}{}$$

input
```
int((e*x+d)*(c*x^4+b*x^2+a)^(1/2),x)
```

output
```
(6*sqrt(a + b*x**2 + c*x**4)*b*c*e + 16*sqrt(a + b*x**2 + c*x**4)*c**2*d*x
 + 12*sqrt(a + b*x**2 + c*x**4)*c**2*e*x**2 - 12*sqrt(c)*log(sqrt(a + b*x*
*2 + c*x**4) - sqrt(c)*x**2)*a*c*e + 3*sqrt(c)*log(sqrt(a + b*x**2 + c*x**
4) - sqrt(c)*x**2)*b**2*e + 12*sqrt(c)*log(sqrt(a + b*x**2 + c*x**4) + sqr
t(c)*x**2)*a*c*e - 3*sqrt(c)*log(sqrt(a + b*x**2 + c*x**4) + sqrt(c)*x**2)
*b**2*e + 32*int(sqrt(a + b*x**2 + c*x**4)/(a**2 + 2*a*b*x**2 + a*c*x**4 +
 b**2*x**4 + b*c*x**6),x)*a**2*c**2*d + 16*int((sqrt(a + b*x**2 + c*x**4)*
x**4)/(a**2 + 2*a*b*x**2 + a*c*x**4 + b**2*x**4 + b*c*x**6),x)*b**2*c**2*d
 + 48*int((sqrt(a + b*x**2 + c*x**4)*x**2)/(a**2 + 2*a*b*x**2 + a*c*x**4 +
 b**2*x**4 + b*c*x**6),x)*a*b*c**2*d - 24*int((sqrt(a + b*x**2 + c*x**4)*x
)/(a**2 + 2*a*b*x**2 + a*c*x**4 + b**2*x**4 + b*c*x**6),x)*a**2*c**2*e + 6
*int((sqrt(a + b*x**2 + c*x**4)*x)/(a**2 + 2*a*b*x**2 + a*c*x**4 + b**2*x*
*4 + b*c*x**6),x)*a*b**2*c*e)/(48*c**2)
```

3.250 $\int \sqrt{a+bx^2+cx^4}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 309

$$\begin{aligned}\int \sqrt{a+bx^2+cx^4}\,dx &= \frac{1}{3}x\sqrt{a+bx^2+cx^4}+\frac{bx\sqrt{a+bx^2+cx^4}}{3\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)} \\ &\quad -\frac{\sqrt[4]{a}b\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{3c^{3/4}\sqrt{a+bx^2+cx^4}} \\ &\quad +\frac{\sqrt[4]{a}\left(b+2\sqrt{a}\sqrt{c}\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{6c^{3/4}\sqrt{a+bx^2+cx^4}}\end{aligned}$$

output
```
1/3*x*(c*x^4+b*x^2+a)^(1/2)+1/3*b*x*(c*x^4+b*x^2+a)^(1/2)/c^(1/2)/(a^(1/2)
+c^(1/2)*x^2)-1/3*a^(1/4)*b*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2
)+c^(1/2)*x^2)^2)^(1/2)*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-
b/a^(1/2)/c^(1/2))^(1/2))/c^(3/4)/(c*x^4+b*x^2+a)^(1/2)+1/6*a^(1/4)*(b+2*a
^(1/2)*c^(1/2))*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2)+c^(1/2)*x^
2)^2)^(1/2)*InverseJacobiAM(2*arctan(c^(1/4)*x/a^(1/4)),1/2*(2-b/a^(1/2)/c
^(1/2))^(1/2))/c^(3/4)/(c*x^4+b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.85 (sec) , antiderivative size = 445, normalized size of antiderivative = 1.44

$$\int \sqrt{a+bx^2+cx^4}\,dx$$
$$= \frac{4c\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x(a+bx^2+cx^4)+ib\left(-b+\sqrt{b^2-4ac}\right)\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{\frac{2b-2\sqrt{b^2-4ac}+4cx^2}{b-\sqrt{b^2-4ac}}}E\left(i\text{arcsinh}\left(\sqrt{\right.\right.}{}$$

input
```
Integrate[Sqrt[a + b*x^2 + c*x^4],x]
```

output
```
(4*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x*(a + b*x^2 + c*x^4) + I*b*(-b + Sqr
t[b^2 - 4*a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a
*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*
EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[
b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] - I*(-b^2 + 4*a*c + b*Sqrt[b^2 - 4*
a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt
[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[
I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*
c])/(b - Sqrt[b^2 - 4*a*c])])/(12*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[a
 + b*x^2 + c*x^4])
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {1404, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+bx^2+cx^4}\,dx$$
$$\Big\downarrow \ 1404$$

$$\frac{1}{3}\int \frac{bx^2+2a}{\sqrt{cx^4+bx^2+a}}dx+\frac{1}{3}x\sqrt{a+bx^2+cx^4}$$

$$\Big\downarrow \text{1511}$$

$$\frac{1}{3}\left(\sqrt{a}\left(2\sqrt{a}+\frac{b}{\sqrt{c}}\right)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx-\frac{\sqrt{a}b\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{a}\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}\right)+\frac{1}{3}x\sqrt{a+bx^2+cx^4}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(\sqrt{a}\left(2\sqrt{a}+\frac{b}{\sqrt{c}}\right)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx-\frac{b\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}\right)+\frac{1}{3}x\sqrt{a+bx^2+cx^4}$$

$$\Big\downarrow \text{1416}$$

$$\frac{1}{3}\left(\frac{\sqrt[4]{a}\left(2\sqrt{a}+\frac{b}{\sqrt{c}}\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}-\frac{b\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+bx^2+a}}d}{\sqrt{c}}\right.$$

$$\frac{1}{3}x\sqrt{a+bx^2+cx^4}$$

$$\Big\downarrow \text{1509}$$

$$\frac{1}{3}\left(\frac{\sqrt[4]{a}\left(2\sqrt{a}+\frac{b}{\sqrt{c}}\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}-\frac{b\left(\frac{\sqrt[4]{a}(\sqrt{a}+\sqrt{c}x^2}{}\right.}{}\right.$$

$$\frac{1}{3}x\sqrt{a+bx^2+cx^4}$$

input
```
Int[Sqrt[a + b*x^2 + c*x^4],x]
```

output
```
(x*Sqrt[a + b*x^2 + c*x^4])/3 + (-((b*(-((x*Sqrt[a + b*x^2 + c*x^4])/(Sqrt
[a] + Sqrt[c]*x^2)) + (a^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c
*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)],
(2 - b/(Sqrt[a]*Sqrt[c]))/4])/(c^(1/4)*Sqrt[a + b*x^2 + c*x^4])))/Sqrt[c])
 + (a^(1/4)*(2*Sqrt[a] + b/Sqrt[c])*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^
2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/
4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*c^(1/4)*Sqrt[a + b*x^2 + c*x^4]))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1404
```
Int[((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*((a + b
*x^2 + c*x^4)^p/(4*p + 1)), x] + Simp[2*(p/(4*p + 1))   Int[(2*a + b*x^2)*(
a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*
c, 0] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

Maple [A] (verified)

Time = 1.15 (sec) , antiderivative size = 379, normalized size of antiderivative = 1.23

method	result
default	$\frac{x\sqrt{c\,x^4+b\,x^2+a}}{3}+\frac{a\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\mathrm{EllipticF}\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2},\frac{\sqrt{-4+\frac{2b\left(b+\sqrt{-4ac+\cdots\right.}}{ac}}}{2}\cdots\right.}{6\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,\sqrt{c\,x^4+b\,x^2+a}}\cdots$
risch	$\frac{x\sqrt{c\,x^4+b\,x^2+a}}{3}+\frac{a\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\mathrm{EllipticF}\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2},\frac{\sqrt{-4+\frac{2b\left(b+\sqrt{-4ac+\cdots\right.}}{ac}}}{2}\cdots\right.}{6\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,\sqrt{c\,x^4+b\,x^2+a}}\cdots$
elliptic	$\frac{x\sqrt{c\,x^4+b\,x^2+a}}{3}+\frac{a\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\mathrm{EllipticF}\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2},\frac{\sqrt{-4+\frac{2b\left(b+\sqrt{-4ac+\cdots\right.}}{ac}}}{2}\cdots\right.}{6\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,\sqrt{c\,x^4+b\,x^2+a}}\cdots$

input
```
int((c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*x*(c*x^4+b*x^2+a)^(1/2)+1/6*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2
)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*
x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)
^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/6*b*a*2^
(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2
)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+
(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(
1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2
)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c
)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 301, normalized size of antiderivative = 0.97

$$\int \sqrt{a+bx^2+cx^4}\,dx$$
$$= \frac{\sqrt{\frac{1}{2}}\left(bcx\sqrt{\frac{b^2-4\,ac}{c^2}}-b^2x\right)\sqrt{c}\sqrt{\frac{c\sqrt{\frac{b^2-4\,ac}{c^2}}-b}{c}}E\left(\arcsin\left(\frac{\sqrt{\frac{1}{2}}\sqrt{\frac{c\sqrt{\frac{b^2-4\,ac}{c^2}}-b}{c}}}{x}\right)\mid\frac{bc\sqrt{\frac{b^2-4\,ac}{c^2}}+b^2-2\,ac}{2\,ac}\right)-\sqrt{\frac{1}{2}}\left((bc-\right.}{}$$

input
```
integrate((c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
1/6*(sqrt(1/2)*(b*c*x*sqrt((b^2 - 4*a*c)/c^2) - b^2*x)*sqrt(c)*sqrt((c*sqr
t((b^2 - 4*a*c)/c^2) - b)/c)*elliptic_e(arcsin(sqrt(1/2)*sqrt((c*sqrt((b^2
 - 4*a*c)/c^2) - b)/c)/x), 1/2*(b*c*sqrt((b^2 - 4*a*c)/c^2) + b^2 - 2*a*c)
/(a*c)) - sqrt(1/2)*((b*c - 2*c^2)*x*sqrt((b^2 - 4*a*c)/c^2) - (b^2 + 2*b*
c)*x)*sqrt(c)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)*elliptic_f(arcsin(sq
rt(1/2)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)/x), 1/2*(b*c*sqrt((b^2 - 4
*a*c)/c^2) + b^2 - 2*a*c)/(a*c)) + 2*sqrt(c*x^4 + b*x^2 + a)*(c^2*x^2 + b*
c))/(c^2*x)
```

Sympy [F]

$$\int \sqrt{a+bx^2+cx^4}\,dx = \int \sqrt{a+bx^2+cx^4}\,dx$$

input
```
integrate((c*x**4+b*x**2+a)**(1/2),x)
```

output
```
Integral(sqrt(a + b*x**2 + c*x**4), x)
```

Maxima [F]

$$\int \sqrt{a+bx^2+cx^4}\,dx = \int \sqrt{cx^4+bx^2+a}\,dx$$

input
```
integrate((c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^4 + b*x^2 + a), x)
```

Giac [F]

$$\int \sqrt{a+bx^2+cx^4}\,dx = \int \sqrt{cx^4+bx^2+a}\,dx$$

input
```
integrate((c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^4 + b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+bx^2+cx^4}\,dx = \int \sqrt{c\,x^4+b\,x^2+a}\,dx$$

input
```
int((a + b*x^2 + c*x^4)^(1/2),x)
```

output
```
int((a + b*x^2 + c*x^4)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{a+bx^2+cx^4}\,dx = \frac{\sqrt{c\,x^4+b\,x^2+a}\,x}{3} + \frac{2\left(\int \frac{\sqrt{c\,x^4+b\,x^2+a}}{c\,x^4+b\,x^2+a}dx\right)a}{3} + \frac{\left(\int \frac{\sqrt{c\,x^4+b\,x^2+a}\,x^2}{c\,x^4+b\,x^2+a}dx\right)b}{3}$$

input
```
int((c*x^4+b*x^2+a)^(1/2),x)
```

output
```
(sqrt(a + b*x**2 + c*x**4)*x + 2*int(sqrt(a + b*x**2 + c*x**4)/(a + b*x**2
 + c*x**4),x)*a + int((sqrt(a + b*x**2 + c*x**4)*x**2)/(a + b*x**2 + c*x**
4),x)*b)/3
```

3.251 $\int \frac{\sqrt{a+bx^2+cx^4}}{d+ex}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 794

$$
\begin{aligned}
\int \frac{\sqrt{a+bx^2+cx^4}}{d+ex}\,dx &= \frac{\sqrt{a+bx^2+cx^4}}{2e} - \frac{\sqrt{c}dx\sqrt{a+bx^2+cx^4}}{e^2\left(\sqrt{a}+\sqrt{c}x^2\right)} \\
&+ \frac{\sqrt{cd^4+bd^2e^2+ae^4}\operatorname{arctanh}\left(\frac{\sqrt{cd^4+bd^2e^2+ae^4}x}{de\sqrt{a+bx^2+cx^4}}\right)}{2e^3} \\
&+ \frac{(2cd^2+be^2)\operatorname{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{c}e^3} \\
&- \frac{\sqrt{cd^4+bd^2e^2+ae^4}\operatorname{arctanh}\left(\frac{bd^2+2ae^2+(2cd^2+be^2)x^2}{2\sqrt{cd^4+bd^2e^2+ae^4}\sqrt{a+bx^2+cx^4}}\right)}{2e^3} \\
&+ \frac{\sqrt[4]{a}\sqrt[4]{c}d\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{e^2\sqrt{a+bx^2+cx^4}} \\
&- \frac{\sqrt[4]{a}d(2cd^2+be^2)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{c}e^2\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\sqrt{a+bx^2+cx^4}} \\
&- \frac{\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\left(cd^4+bd^2e^2+ae^4\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticPi}\left(\frac{\left(\sqrt{c}d^2+\sqrt{a}e^2\right)^2}{4\sqrt{a}\sqrt{c}d^2e^2},2\arctan\left(\frac{\sqrt[4]{}}{\sqrt[4]{}}\right.\right.}{4\sqrt[4]{a}\sqrt[4]{c}de^4\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\sqrt{a+bx^2+cx^4}}
\end{aligned}
$$

output
```
1/2*(c*x^4+b*x^2+a)^(1/2)/e-c^(1/2)*d*x*(c*x^4+b*x^2+a)^(1/2)/e^2/(a^(1/2)
+c^(1/2)*x^2)+1/2*(a*e^4+b*d^2*e^2+c*d^4)^(1/2)*arctanh((a*e^4+b*d^2*e^2+c
*d^4)^(1/2)*x/d/e/(c*x^4+b*x^2+a)^(1/2))/e^3+1/4*(b*e^2+2*c*d^2)*arctanh(1
/2*(2*c*x^2+b)/c^(1/2)/(c*x^4+b*x^2+a)^(1/2))/c^(1/2)/e^3-1/2*(a*e^4+b*d^2
*e^2+c*d^4)^(1/2)*arctanh(1/2*(b*d^2+2*a*e^2+(b*e^2+2*c*d^2)*x^2)/(a*e^4+b
*d^2*e^2+c*d^4)^(1/2)/(c*x^4+b*x^2+a)^(1/2))/e^3+a^(1/4)*c^(1/4)*d*(a^(1/2
)+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*EllipticE(s
in(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))/e^2/(c*x^
4+b*x^2+a)^(1/2)-1/2*a^(1/4)*d*(b*e^2+2*c*d^2)*(a^(1/2)+c^(1/2)*x^2)*((c*x
^4+b*x^2+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(c^(1/4
)*x/a^(1/4)),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))/c^(1/4)/e^2/(c^(1/2)*d^2+a^(
1/2)*e^2)/(c*x^4+b*x^2+a)^(1/2)-1/4*(c^(1/2)*d^2-a^(1/2)*e^2)*(a*e^4+b*d^2
*e^2+c*d^4)*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2)+c^(1/2)*x^2)^2
)^(1/2)*EllipticPi(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/4*(c^(1/2)*d^2+a^(1/
2)*e^2)^2/a^(1/2)/c^(1/2)/d^2/e^2,1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))/a^(1/4)
/c^(1/4)/d/e^4/(c^(1/2)*d^2+a^(1/2)*e^2)/(c*x^4+b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 20.47 (sec) , antiderivative size = 5994, normalized size of antiderivative = 7.55

$$\int \frac{\sqrt{a+bx^2+cx^4}}{d+ex}\,dx = \text{Result too large to show}$$

input
```
Integrate[Sqrt[a + b*x^2 + c*x^4]/(d + e*x),x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 2.80 (sec) , antiderivative size = 846, normalized size of antiderivative = 1.07, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {2266, 1523, 27, 1511, 27, 1416, 1509, 1576, 1162, 1269, 1092, 219, 1154, 219, 2222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2+cx^4}}{d+ex}dx$$

$$\Big\downarrow \text{ 2266}$$

$$d\int \frac{\sqrt{cx^4+bx^2+a}}{d^2-e^2x^2}dx-e\int \frac{x\sqrt{cx^4+bx^2+a}}{d^2-e^2x^2}dx$$

$$\Big\downarrow \text{ 1523}$$

$$d\left(\frac{(ae^4+bd^2e^2+cd^4)\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a}(d^2-e^2x^2)\sqrt{cx^4+bx^2+a}}dx}{e^2\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)}-\frac{\int \frac{cd^2+be^2-\sqrt{a}\sqrt{c}e^2+c\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)x^2}{\sqrt{cx^4+bx^2+a}}dx}{e^2\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)}\right)-e\int \frac{x\sqrt{cx^4+bx^2+a}}{d^2-e^2x^2}dx$$

$$\Big\downarrow \text{ 27}$$

$$d\left(\frac{(ae^4+bd^2e^2+cd^4)\int \frac{\sqrt{c}x^2+\sqrt{a}}{(d^2-e^2x^2)\sqrt{cx^4+bx^2+a}}dx}{\sqrt{a}e^2\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)}-\frac{\int \frac{cd^2+be^2-\sqrt{a}\sqrt{c}e^2+c\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)x^2}{\sqrt{cx^4+bx^2+a}}dx}{e^2\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)}\right)-e\int \frac{x\sqrt{cx^4+bx^2+a}}{d^2-e^2x^2}dx$$

$$\Big\downarrow \text{ 1511}$$

$$d\left(\frac{(ae^4+bd^2e^2+cd^4)\int \frac{\sqrt{c}x^2+\sqrt{a}}{(d^2-e^2x^2)\sqrt{cx^4+bx^2+a}}dx}{\sqrt{a}e^2\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)}-\frac{(be^2+2cd^2)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx-\sqrt{c}(\sqrt{a}e^2+\sqrt{c}d^2)\int \frac{\sqrt{a}-\sqrt{}}{\sqrt{a}\sqrt{cx^4+}}}{e^2\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)}\right.$$
$$e\int \frac{x\sqrt{cx^4+bx^2+a}}{d^2-e^2x^2}dx$$

$$\Big\downarrow \text{ 27}$$

$$d\left(\frac{\left(ae^4+bd^2e^2+cd^4\right)\int\frac{\sqrt{c}x^2+\sqrt{a}}{(d^2-e^2x^2)\sqrt{cx^4+bx^2+a}}dx}{\sqrt{a}e^2\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)}-\frac{\left(be^2+2cd^2\right)\int\frac{1}{\sqrt{cx^4+bx^2+a}}dx-\frac{\sqrt{c}(\sqrt{a}e^2+\sqrt{c}d^2)\int\frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{a}}}{e^2\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)}\right.$$

$$e\int\frac{x\sqrt{cx^4+bx^2+a}}{d^2-e^2x^2}dx$$

$\downarrow$ 1416

$$d\left(\frac{\left(ae^4+bd^2e^2+cd^4\right)\int\frac{\sqrt{c}x^2+\sqrt{a}}{(d^2-e^2x^2)\sqrt{cx^4+bx^2+a}}dx}{\sqrt{a}e^2\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)}-\frac{\frac{(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}(be^2+2cd^2)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\right.\right.}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}}{e^2\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)}\right.$$

$$e\int\frac{x\sqrt{cx^4+bx^2+a}}{d^2-e^2x^2}dx$$

$\downarrow$ 1509

$$d\left(\frac{\left(ae^4+bd^2e^2+cd^4\right)\int\frac{\sqrt{c}x^2+\sqrt{a}}{(d^2-e^2x^2)\sqrt{cx^4+bx^2+a}}dx}{\sqrt{a}e^2\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)}-\frac{(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}(be^2+2cd^2)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\right.\right.}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}\right.$$

$$e\int\frac{x\sqrt{cx^4+bx^2+a}}{d^2-e^2x^2}dx$$

$\downarrow$ 1576

$$d\left(\frac{\left(ae^4+bd^2e^2+cd^4\right)\int \frac{\sqrt{c}x^2+\sqrt{a}}{(d^2-e^2x^2)\sqrt{cx^4+bx^2+a}}dx}{\sqrt{a}e^2\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)}-\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(be^2+2cd^2\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\right.\right.}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}\right.$$

$$\frac{1}{2}e\int \frac{\sqrt{cx^4+bx^2+a}}{d^2-e^2x^2}dx^2$$

$\downarrow$ 1162

$$d\left(\frac{\left(ae^4+bd^2e^2+cd^4\right)\int \frac{\sqrt{c}x^2+\sqrt{a}}{(d^2-e^2x^2)\sqrt{cx^4+bx^2+a}}dx}{\sqrt{a}e^2\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)}-\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(be^2+2cd^2\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\right.\right.}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}\right.$$

$$\frac{1}{2}e\left(\frac{\int \frac{bd^2+2ae^2+\left(2cd^2+be^2\right)x^2}{(d^2-e^2x^2)\sqrt{cx^4+bx^2+a}}dx^2}{2e^2}-\frac{\sqrt{a+bx^2+cx^4}}{e^2}\right)$$

$\downarrow$ 1269

$$d\left(\frac{\left(ae^4+bd^2e^2+cd^4\right)\int \frac{\sqrt{c}x^2+\sqrt{a}}{(d^2-e^2x^2)\sqrt{cx^4+bx^2+a}}dx}{\sqrt{a}e^2\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)}-\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(be^2+2cd^2\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\right.\right.}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}\right.$$

$$\frac{1}{2}e\left(\frac{\frac{2\left(ae^4+bd^2e^2+cd^4\right)\int \frac{1}{\left(d^2-e^2x^2\right)\sqrt{cx^4+bx^2+a}}dx^2}{e^2}-\left(b+\frac{2cd^2}{e^2}\right)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx^2}{2e^2}-\frac{\sqrt{a+bx^2+cx^4}}{e^2}\right)$$

$$\Big\downarrow \text{1092}$$

$$d\left(\frac{\left(ae^4+bd^2e^2+cd^4\right)\int \frac{\sqrt{c}x^2+\sqrt{a}}{(d^2-e^2x^2)\sqrt{cx^4+bx^2+a}}dx}{\sqrt{a}e^2\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)}-\frac{(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}(be^2+2cd^2)\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{}{\sqrt{}}\right.\right.}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}\right.$$

$$\frac{1}{2}e\left(\frac{\frac{2\left(ae^4+bd^2e^2+cd^4\right)\int \frac{1}{\left(d^2-e^2x^2\right)\sqrt{cx^4+bx^2+a}}dx^2}{e^2}-2\left(b+\frac{2cd^2}{e^2}\right)\int \frac{1}{4c-x^4}d\frac{2cx^2+b}{\sqrt{cx^4+bx^2+a}}}{2e^2}-\frac{\sqrt{a+bx^2+cx^4}}{e^2}\right)$$

$$\Big\downarrow \text{219}$$

$$d\left(\frac{\left(ae^4+bd^2e^2+cd^4\right)\int \frac{\sqrt{c}x^2+\sqrt{a}}{(d^2-e^2x^2)\sqrt{cx^4+bx^2+a}}dx}{\sqrt{a}e^2\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)}-\frac{(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}(be^2+2cd^2)\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{}{\sqrt{}}\right.\right.}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}\right.$$

$$\frac{1}{2}e\left(\frac{\frac{2\left(ae^4+bd^2e^2+cd^4\right)\int \frac{1}{\left(d^2-e^2x^2\right)\sqrt{cx^4+bx^2+a}}dx^2}{e^2}-\frac{\left(b+\frac{2cd^2}{e^2}\right)\operatorname{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{\sqrt{c}}}{2e^2}-\frac{\sqrt{a+bx^2+cx^4}}{e^2}\right)$$

$$\Big\downarrow \text{1154}$$

$$d\left(\frac{\left(ae^4+bd^2e^2+cd^4\right)\int \frac{\sqrt{c}x^2+\sqrt{a}}{(d^2-e^2x^2)\sqrt{cx^4+bx^2+a}}dx}{\sqrt{a}e^2\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)}-\frac{(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}(be^2+2cd^2)\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\right.\right.}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}\right.$$

$$\frac{1}{2}e\left(\frac{-\frac{4\left(ae^4+bd^2e^2+cd^4\right)\int \frac{1}{4\left(cd^4+be^2d^2+ae^4\right)-x^4}d\left(-\frac{bd^2+2ae^2+\left(2cd^2+be^2\right)x^2}{\sqrt{cx^4+bx^2+a}}\right)}{e^2}-\frac{\left(b+\frac{2cd^2}{e^2}\right)\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{\sqrt{c}}}{2e^2}-\frac{\sqrt{a+b}}{e}\right.$$

$\Big\downarrow$ 219

$$d\left(\frac{\left(ae^4+bd^2e^2+cd^4\right)\int \frac{\sqrt{c}x^2+\sqrt{a}}{(d^2-e^2x^2)\sqrt{cx^4+bx^2+a}}dx}{\sqrt{a}e^2\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)}-\frac{(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}(be^2+2cd^2)\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\right.\right.}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}\right.$$

$$\frac{1}{2}e\left(\frac{\frac{2\sqrt{ae^4+bd^2e^2+cd^4}\,\text{arctanh}\left(\frac{2ae^2+x^2\left(be^2+2cd^2\right)+bd^2}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^4+bd^2e^2+cd^4}}\right)}{e^2}-\frac{\left(b+\frac{2cd^2}{e^2}\right)\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{\sqrt{c}}}{2e^2}-\frac{\sqrt{a+bx^2+cx^4}}{e^2}\right)$$

$\Big\downarrow$ 2222

$$d\left(\frac{(cd^4+be^2d^2+ae^4)\left(\frac{(\sqrt{c}d^2+\sqrt{a}e^2)\text{arctanh}\left(\frac{\sqrt{cd^4+be^2d^2+ae^4}x}{de\sqrt{cx^4+bx^2+a}}\right)}{2de\sqrt{cd^4+be^2d^2+ae^4}}+\frac{\left(\frac{\sqrt{a}}{d^2}-\frac{\sqrt{c}}{e^2}\right)(\sqrt{c}x^2+\sqrt{a})\sqrt{\frac{cx^4+bx^2+a}{(\sqrt{c}x^2+\sqrt{a})^2}}\text{EllipticPi}\left(\frac{(\sqrt{c}d^2+\ldots}{4\sqrt{a}\sqrt{\ldots}}\right.}{4\sqrt[4]{a}\sqrt[4]{c}\sqrt{cx^4+bx^2\ldots}}\right.}{\sqrt{a}e^2\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)}\right.$$

$$\frac{1}{2}e\left(\frac{\frac{2\sqrt{cd^4+be^2d^2+ae^4}\text{arctanh}\left(\frac{bd^2+2ae^2+(2cd^2+be^2)x^2}{2\sqrt{cd^4+be^2d^2+ae^4}\sqrt{cx^4+bx^2+a}}\right)}{e^2}-\frac{\left(\frac{2cd^2}{e^2}+b\right)\text{arctanh}\left(\frac{2cx^2+b}{2\sqrt{c}\sqrt{cx^4+bx^2+a}}\right)}{\sqrt{c}}}{2e^2}-\frac{\sqrt{cx^4+bx^2+a}}{e^2}\right)$$

input
```
Int[Sqrt[a + b*x^2 + c*x^4]/(d + e*x),x]
```

output
```
-1/2*(e*(-(Sqrt[a + b*x^2 + c*x^4]/e^2) + (-(((b + (2*c*d^2)/e^2)*ArcTanh[
(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/Sqrt[c]) + (2*Sqrt[c*d
^4 + b*d^2*e^2 + a*e^4]*ArcTanh[(b*d^2 + 2*a*e^2 + (2*c*d^2 + b*e^2)*x^2)/
(2*Sqrt[c*d^4 + b*d^2*e^2 + a*e^4]*Sqrt[a + b*x^2 + c*x^4])])/e^2)/(2*e^2)
)) + d*(-((-((Sqrt[c]*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*(-((x*Sqrt[a + b*x^2 + c
*x^4])/(Sqrt[a] + Sqrt[c]*x^2)) + (a^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a
 + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x
)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(c^(1/4)*Sqrt[a + b*x^2 + c*x^4]
)))/Sqrt[a]) + ((2*c*d^2 + b*e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2
+ c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)
], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*a^(1/4)*c^(1/4)*Sqrt[a + b*x^2 + c*x^4
]))/(e^2*((Sqrt[c]*d^2)/Sqrt[a] + e^2))) + ((c*d^4 + b*d^2*e^2 + a*e^4)*((
(Sqrt[c]*d^2 + Sqrt[a]*e^2)*ArcTanh[(Sqrt[c*d^4 + b*d^2*e^2 + a*e^4]*x)/(d
*e*Sqrt[a + b*x^2 + c*x^4])])/(2*d*e*Sqrt[c*d^4 + b*d^2*e^2 + a*e^4]) + ((
Sqrt[a]/d^2 - Sqrt[c]/e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4
)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticPi[(Sqrt[c]*d^2 + Sqrt[a]*e^2)^2/(4*S
qrt[a]*Sqrt[c]*d^2*e^2), 2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sq
rt[c]))/4])/(4*a^(1/4)*c^(1/4)*Sqrt[a + b*x^2 + c*x^4])))/(Sqrt[a]*e^2*((S
qrt[c]*d^2)/Sqrt[a] + e^2)))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1162
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Simp[p/(e*(m + 2*p + 1))   Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d -
b*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x
] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&
!ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 1523

```
Int[Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(c*d^2 - b*d*e + a*e^2)/(e*(e - d*q))   I
nt[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x] - Simp[1/(e*(e
 - d*q))   Int[(c*d - b*e + a*e*q - (c*e - a*d*q^3)*x^2)/Sqrt[a + b*x^2 + c
*x^4], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c
*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

rule 1576

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x]
, x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

rule 2222

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTanh[Rt[b - c*(d/e) - a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
b - c*(d/e) - a*(e/d), 2])), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a +
 b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*d*e*q*Sqrt[a + b*x^2 + c*x^4]))*Ell
ipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x], 1/2 - b/(4*a*q^2)], x]]
/; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &&
 EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[-b + c*(d/e) + a*(e/d)]
```

rule 2266
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.)/((d_) + (e_.)*(x_)), x_Symbo
l] :> Simp[d   Int[(a + b*x^2 + c*x^4)^p/(d^2 - e^2*x^2), x], x] - Simp[e
 Int[x*((a + b*x^2 + c*x^4)^p/(d^2 - e^2*x^2)), x], x] /; FreeQ[{a, b, c, d
, e}, x] && IntegerQ[p + 1/2]
```

Maple [A] (verified)

Time = 5.06 (sec) , antiderivative size = 755, normalized size of antiderivative = 0.95

method	result
default	$\frac{\sqrt{c\,x^{4}+b\,x^{2}+a}}{2e}-\frac{d\left(b\,e^{2}+c\,d^{2}\right)\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^{2}}\right)x^{2}}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^{2}}\right)x^{2}}{a}}\,\operatorname{EllipticF}\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^{2}}}{a}}}{2},\frac{\sqrt{-4+\frac{2b(}{}}}{}\right.}{4e^{4}\sqrt{\frac{-b+\sqrt{-4ac+b^{2}}}{a}}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}$
elliptic	$\frac{\sqrt{c\,x^{4}+b\,x^{2}+a}}{2e}-\frac{d\left(b\,e^{2}+c\,d^{2}\right)\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^{2}}\right)x^{2}}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^{2}}\right)x^{2}}{a}}\,\operatorname{EllipticF}\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^{2}}}{a}}}{2},\frac{\sqrt{-4+\frac{2b(}{}}}{}\right.}{4e^{4}\sqrt{\frac{-b+\sqrt{-4ac+b^{2}}}{a}}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}$
risch	$\frac{\sqrt{c\,x^{4}+b\,x^{2}+a}}{2e}-\frac{-\frac{e\left(b\,e^{2}+2c\,d^{2}\right)\ln\left(\frac{\frac{b}{2}+c\,x^{2}}{\sqrt{c}}+\sqrt{c\,x^{4}+b\,x^{2}+a}\right)}{2\sqrt{c}}+\frac{c\,d^{3}\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^{2}}\right)x^{2}}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^{2}}\right)x^{2}}{a}}\,\operatorname{EllipticF}\left(\right.}{2\sqrt{\frac{-b+\sqrt{-4ac+b^{2}}}{a}}\,\sqrt{c\,x^{4}+}}}{}$

input
```
int((c*x^4+b*x^2+a)^(1/2)/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
1/2*(c*x^4+b*x^2+a)^(1/2)/e-1/4*d*(b*e^2+c*d^2)/e^4*2^(1/2)/((-b+(-4*a*c+b
^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*
a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)
*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)
^(1/2))+1/2*(1/e^3*(b*e^2+c*d^2)-1/2/e*b)*ln((2*c*x^2+b)/c^(1/2)+2*(c*x^4+
b*x^2+a)^(1/2))/c^(1/2)+1/2*c*d/e^2*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^
(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2)
)/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2
*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(
1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2
),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)))+(a*e^4+b*d^2*e^2+c*d^4)/
e^5*(-1/2/(c*d^4/e^4+b*d^2/e^2+a)^(1/2)*arctanh(1/2*(2*c*x^2*d^2/e^2+b*d^2
/e^2+b*x^2+2*a)/(c*d^4/e^4+b*d^2/e^2+a)^(1/2)/(c*x^4+b*x^2+a)^(1/2))+2^(1/
2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)/d*e*(1-1/2*(-b+(-4*a*c+b^2)^(1/2))/a*
x^2)^(1/2)*(1+1/2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2
)*EllipticPi(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),2/(-b+(-4*a*c
+b^2)^(1/2))*a/d^2*e^2,(-1/2*(b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*2^(1/2)/((-b+
(-4*a*c+b^2)^(1/2))/a)^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2+cx^4}}{d+ex}\,dx = \text{Timed out}$$

input
```
integrate((c*x^4+b*x^2+a)^(1/2)/(e*x+d),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2+cx^4}}{d+ex}\,dx = \int \frac{\sqrt{a+bx^2+cx^4}}{d+ex}\,dx$$

input
```
integrate((c*x**4+b*x**2+a)**(1/2)/(e*x+d),x)
```

output
```
Integral(sqrt(a + b*x**2 + c*x**4)/(d + e*x), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2+cx^4}}{d+ex}\,dx = \int \frac{\sqrt{cx^4+bx^2+a}}{ex+d}\,dx$$

input
```
integrate((c*x^4+b*x^2+a)^(1/2)/(e*x+d),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^4 + b*x^2 + a)/(e*x + d), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2+cx^4}}{d+ex}\,dx = \int \frac{\sqrt{cx^4+bx^2+a}}{ex+d}\,dx$$

input
```
integrate((c*x^4+b*x^2+a)^(1/2)/(e*x+d),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^4 + b*x^2 + a)/(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2+cx^4}}{d+ex}\,dx=\int \frac{\sqrt{c\,x^4+b\,x^2+a}}{d+e\,x}\,dx$$

input
```
int((a + b*x^2 + c*x^4)^(1/2)/(d + e*x),x)
```

output
```
int((a + b*x^2 + c*x^4)^(1/2)/(d + e*x), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2+cx^4}}{d+ex}\,dx=\int \frac{\sqrt{c\,x^4+b\,x^2+a}}{ex+d}dx$$

input
```
int((c*x^4+b*x^2+a)^(1/2)/(e*x+d),x)
```

output
```
int((c*x^4+b*x^2+a)^(1/2)/(e*x+d),x)
```

3.252 $\int \frac{\sqrt{a+bx^2+cx^4}}{(d+ex)^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 855

$$\int \frac{\sqrt{a+bx^2+cx^4}}{(d+ex)^2}\,dx = \frac{2\sqrt{c}x\sqrt{a+bx^2+cx^4}}{e^2\left(\sqrt{a}+\sqrt{c}x^2\right)} - \frac{d\sqrt{a+bx^2+cx^4}}{e\left(d^2-e^2x^2\right)}$$
$$+ \frac{x\sqrt{a+bx^2+cx^4}}{d^2-e^2x^2} - \frac{d(2cd^2+be^2)\,\mathrm{arctanh}\left(\frac{\sqrt{cd^4+bd^2e^2+ae^4}\,x}{de\sqrt{a+bx^2+cx^4}}\right)}{2e^3\sqrt{cd^4+bd^2e^2+ae^4}}$$
$$- \frac{\sqrt{c}d\mathrm{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{e^3} + \frac{d(2cd^2+be^2)\,\mathrm{arctanh}\left(\frac{bd^2+2ae^2+(2cd^2+be^2)x^2}{2\sqrt{cd^4+bd^2e^2+ae^4}\sqrt{a+bx^2+cx^4}}\right)}{2e^3\sqrt{cd^4+bd^2e^2+ae^4}}$$
$$- \frac{2\sqrt[4]{a}\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{e^2\sqrt{a+bx^2+cx^4}}$$
$$+ \frac{\sqrt[4]{a}\left(4cd^2+be^2+2\sqrt{a}\sqrt{c}e^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\mathrm{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{c}e^2\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\sqrt{a+bx^2+cx^4}}$$
$$+ \frac{\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\left(2cd^2+be^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\mathrm{EllipticPi}\left(\frac{\left(\sqrt{c}d^2+\sqrt{a}e^2\right)^2}{4\sqrt{a}\sqrt{c}d^2e^2},2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\right.}{4\sqrt[4]{a}\sqrt[4]{c}e^4\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\sqrt{a+bx^2+cx^4}}$$

output
```
2*c^(1/2)*x*(c*x^4+b*x^2+a)^(1/2)/e^2/(a^(1/2)+c^(1/2)*x^2)-d*(c*x^4+b*x^2
+a)^(1/2)/e/(-e^2*x^2+d^2)+x*(c*x^4+b*x^2+a)^(1/2)/(-e^2*x^2+d^2)-1/2*d*(b
*e^2+2*c*d^2)*arctanh((a*e^4+b*d^2*e^2+c*d^4)^(1/2)*x/d/e/(c*x^4+b*x^2+a)^
(1/2))/e^3/(a*e^4+b*d^2*e^2+c*d^4)^(1/2)-c^(1/2)*d*arctanh(1/2*(2*c*x^2+b)
/c^(1/2)/(c*x^4+b*x^2+a)^(1/2))/e^3+1/2*d*(b*e^2+2*c*d^2)*arctanh(1/2*(b*d
^2+2*a*e^2+(b*e^2+2*c*d^2)*x^2)/(a*e^4+b*d^2*e^2+c*d^4)^(1/2)/(c*x^4+b*x^2
+a)^(1/2))/e^3/(a*e^4+b*d^2*e^2+c*d^4)^(1/2)-2*a^(1/4)*c^(1/4)*(a^(1/2)+c^
(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*EllipticE(sin(2
*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))/e^2/(c*x^4+b*
x^2+a)^(1/2)+1/2*a^(1/4)*(4*c*d^2+b*e^2+2*a^(1/2)*c^(1/2)*e^2)*(a^(1/2)+c^
(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM
(2*arctan(c^(1/4)*x/a^(1/4)),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))/c^(1/4)/e^2/
(c^(1/2)*d^2+a^(1/2)*e^2)/(c*x^4+b*x^2+a)^(1/2)+1/4*(c^(1/2)*d^2-a^(1/2)*e
^2)*(b*e^2+2*c*d^2)*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2)+c^(1/2
)*x^2)^2)^(1/2)*EllipticPi(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/4*(c^(1/2)*d
^2+a^(1/2)*e^2)^2/a^(1/2)/c^(1/2)/d^2/e^2,1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))
/a^(1/4)/c^(1/4)/e^4/(c^(1/2)*d^2+a^(1/2)*e^2)/(c*x^4+b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 19.07 (sec) , antiderivative size = 4221, normalized size of antiderivative = 4.94

$$\int \frac{\sqrt{a+bx^2+cx^4}}{(d+ex)^2}\,dx = \text{Result too large to show}$$

input
```
Integrate[Sqrt[a + b*x^2 + c*x^4]/(d + e*x)^2,x]
```

output
```
-(Sqrt[a + b*x^2 + c*x^4]/(e*(d + e*x))) + ((I*(-b + Sqrt[b^2 - 4*a*c])*Sq
rt[1 - (2*c*x^2)/(-b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 - (2*c*x^2)/(-b + Sqrt[b
^2 - 4*a*c])]*(EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[-(c/(-b - Sqrt[b^2 - 4*a*c
]))]*x], (-b - Sqrt[b^2 - 4*a*c])/(-b + Sqrt[b^2 - 4*a*c])] - EllipticF[I*
ArcSinh[Sqrt[2]*Sqrt[-(c/(-b - Sqrt[b^2 - 4*a*c]))]*x], (-b - Sqrt[b^2 - 4
*a*c])/(-b + Sqrt[b^2 - 4*a*c])]))/(Sqrt[2]*Sqrt[-(c/(-b - Sqrt[b^2 - 4*a*
c]))]*e*Sqrt[a + b*x^2 + c*x^4]) - (I*Sqrt[2]*c*d^2*Sqrt[1 - (2*c*x^2)/(-b
 - Sqrt[b^2 - 4*a*c])]*Sqrt[1 - (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]*Ellipt
icF[I*ArcSinh[Sqrt[2]*Sqrt[-(c/(-b - Sqrt[b^2 - 4*a*c]))]*x], (-b - Sqrt[b
^2 - 4*a*c])/(-b + Sqrt[b^2 - 4*a*c])])/(Sqrt[-(c/(-b - Sqrt[b^2 - 4*a*c])
)]*e^3*Sqrt[a + b*x^2 + c*x^4]) - (I*b*Sqrt[1 - (2*c*x^2)/(-b - Sqrt[b^2 -
 4*a*c])]*Sqrt[1 - (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSinh
[Sqrt[2]*Sqrt[-(c/(-b - Sqrt[b^2 - 4*a*c]))]*x], (-b - Sqrt[b^2 - 4*a*c])/
(-b + Sqrt[b^2 - 4*a*c])])/(Sqrt[2]*Sqrt[-(c/(-b - Sqrt[b^2 - 4*a*c]))]*e*
Sqrt[a + b*x^2 + c*x^4]) - (4*c*(Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/Sqrt[2
] + Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2])*d^3*(-(Sqrt[-(b/c) - Sqrt[
b^2 - 4*a*c]/c]/Sqrt[2]) + x)^2*Sqrt[(Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c]*(-(
Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2]) + x))/((Sqrt[-(b/c) - Sqrt[b^2
 - 4*a*c]/c]/Sqrt[2] + Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2])*(-(Sqrt
[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/Sqrt[2]) + x))]*Sqrt[(Sqrt[(-b - Sqrt[b^...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2+cx^4}}{(d+ex)^2} dx$$

$$\Big\downarrow \text{ 7299}$$

$$\int \frac{\sqrt{a+bx^2+cx^4}}{(d+ex)^2} dx$$

input
```
Int[Sqrt[a + b*x^2 + c*x^4]/(d + e*x)^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 749, normalized size of antiderivative = 0.88

method	result
default	$-\frac{\sqrt{c\,x^4+b\,x^2+a}}{e(ex+d)}+\frac{\left(\frac{b\,e^2+3c\,d^2}{e^4}-\frac{c\,d^2}{e^4}\right)\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\ \mathrm{EllipticF}\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2},\right.}{4\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,\sqrt{c\,x^4+b\,x^2+a}}$
elliptic	$-\frac{\sqrt{c\,x^4+b\,x^2+a}}{e(ex+d)}+\frac{\left(\frac{b\,e^2+3c\,d^2}{e^4}-\frac{c\,d^2}{e^4}\right)\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\ \mathrm{EllipticF}\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2},\right.}{4\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,\sqrt{c\,x^4+b\,x^2+a}}$

input
```
int((c*x^4+b*x^2+a)^(1/2)/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/e*(c*x^4+b*x^2+a)^(1/2)/(e*x+d)+1/4*((b*e^2+3*c*d^2)/e^4-c*d^2/e^4)*2^(
1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)
^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*Elli
pticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a
*c+b^2)^(1/2))/a/c)^(1/2))-c^(1/2)*d/e^3*ln((2*c*x^2+b)/c^(1/2)+2*(c*x^4+b
*x^2+a)^(1/2))-c/e^2*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+
(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/
(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b
+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2
))-EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(
b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)))-d/e^5*(b*e^2+2*c*d^2)*(-1/2/(c*d^4/e^4+
b*d^2/e^2+a)^(1/2)*arctanh(1/2*(2*c*x^2*d^2/e^2+b*d^2/e^2+b*x^2+2*a)/(c*d^
4/e^4+b*d^2/e^2+a)^(1/2)/(c*x^4+b*x^2+a)^(1/2))+2^(1/2)/((-b+(-4*a*c+b^2)^
(1/2))/a)^(1/2)/d*e*(1-1/2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(1+1/2*(b+
(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticPi(1/2*x*2^
(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),2/(-b+(-4*a*c+b^2)^(1/2))*a/d^2*e^
2,(-1/2*(b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a
)^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2+cx^4}}{(d+ex)^2}\,dx = \text{Timed out}$$

input
```
integrate((c*x^4+b*x^2+a)^(1/2)/(e*x+d)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2+cx^4}}{(d+ex)^2}\,dx = \int \frac{\sqrt{a+bx^2+cx^4}}{(d+ex)^2}\,dx$$

input
```
integrate((c*x**4+b*x**2+a)**(1/2)/(e*x+d)**2,x)
```

output
```
Integral(sqrt(a + b*x**2 + c*x**4)/(d + e*x)**2, x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2+cx^4}}{(d+ex)^2}\,dx = \int \frac{\sqrt{cx^4+bx^2+a}}{(ex+d)^2}\,dx$$

input
```
integrate((c*x^4+b*x^2+a)^(1/2)/(e*x+d)^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^4 + b*x^2 + a)/(e*x + d)^2, x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2+cx^4}}{(d+ex)^2}\,dx = \int \frac{\sqrt{cx^4+bx^2+a}}{(ex+d)^2}\,dx$$

input
```
integrate((c*x^4+b*x^2+a)^(1/2)/(e*x+d)^2,x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^4 + b*x^2 + a)/(e*x + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2+cx^4}}{(d+ex)^2}\,dx=\int \frac{\sqrt{c\,x^4+b\,x^2+a}}{{\left(d+e\,x\right)}^2}\,dx$$

input
```
int((a + b*x^2 + c*x^4)^(1/2)/(d + e*x)^2,x)
```

output
```
int((a + b*x^2 + c*x^4)^(1/2)/(d + e*x)^2, x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2+cx^4}}{(d+ex)^2}\,dx=\int \frac{\sqrt{c\,x^4+b\,x^2+a}}{\left(ex+d\right)^2}dx$$

input
```
int((c*x^4+b*x^2+a)^(1/2)/(e*x+d)^2,x)
```

output
```
int((c*x^4+b*x^2+a)^(1/2)/(e*x+d)^2,x)
```

3.253 $\int \frac{\sqrt{a+bx^2+cx^4}}{(d+ex)^3} dx$

Optimal result

Integrand size = 24, antiderivative size = 1140

$$\int \frac{\sqrt{a+bx^2+cx^4}}{(d+ex)^3} dx = \text{Too large to display}$$

output
```
-1/2*c^(1/2)*d*(b*e^2+2*c*d^2)*x*(c*x^4+b*x^2+a)^(1/2)/e^2/(a*e^4+b*d^2*e^
2+c*d^4)/(a^(1/2)+c^(1/2)*x^2)+d*x*(c*x^4+b*x^2+a)^(1/2)/(-e^2*x^2+d^2)^2-
1/2*d*(b*e^2+2*c*d^2)*x*(c*x^4+b*x^2+a)^(1/2)/(a*e^4+b*d^2*e^2+c*d^4)/(-e^
2*x^2+d^2)+1/2*(d^2*(-a*e^4+c*d^4)-e^2*(a*e^4+2*b*d^2*e^2+3*c*d^4)*x^2)*(c
*x^4+b*x^2+a)^(1/2)/e/(a*e^4+b*d^2*e^2+c*d^4)/(-e^2*x^2+d^2)^2+1/4*(a*b*e^
6+6*a*c*d^2*e^4+3*b*c*d^4*e^2+2*c^2*d^6)*arctanh((a*e^4+b*d^2*e^2+c*d^4)^(
1/2)*x/d/e/(c*x^4+b*x^2+a)^(1/2))/e^3/(a*e^4+b*d^2*e^2+c*d^4)^(3/2)+1/2*c^
(1/2)*arctanh(1/2*(2*c*x^2+b)/c^(1/2)/(c*x^4+b*x^2+a)^(1/2))/e^3-1/4*(a*b*
e^6+6*a*c*d^2*e^4+3*b*c*d^4*e^2+2*c^2*d^6)*arctanh(1/2*(b*d^2+2*a*e^2+(b*e
^2+2*c*d^2)*x^2)/(a*e^4+b*d^2*e^2+c*d^4)^(1/2)/(c*x^4+b*x^2+a)^(1/2))/e^3/
(a*e^4+b*d^2*e^2+c*d^4)^(3/2)+1/2*a^(1/4)*c^(1/4)*d*(b*e^2+2*c*d^2)*(a^(1/
2)+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*EllipticE(
sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))/e^2/(a*e
^4+b*d^2*e^2+c*d^4)/(c*x^4+b*x^2+a)^(1/2)-a^(1/4)*c^(3/4)*d*(a^(1/2)+c^(1/
2)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*
arctan(c^(1/4)*x/a^(1/4)),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))/e^2/(c^(1/2)*d^
2+a^(1/2)*e^2)/(c*x^4+b*x^2+a)^(1/2)-1/8*(c^(1/4)*d-a^(1/4)*e)*(c^(1/4)*d+
a^(1/4)*e)*(a*b*e^6+6*a*c*d^2*e^4+3*b*c*d^4*e^2+2*c^2*d^6)*(a^(1/2)+c^(1/2
)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*EllipticPi(sin(2*ar
ctan(c^(1/4)*x/a^(1/4))),1/4*(c^(1/2)*d^2+a^(1/2)*e^2)^2/a^(1/2)/c^(1/2...
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 21.10 (sec) , antiderivative size = 8346, normalized size of antiderivative = 7.32

$$\int \frac{\sqrt{a+bx^2+cx^4}}{(d+ex)^3}\,dx = \text{Result too large to show}$$

input
```
Integrate[Sqrt[a + b*x^2 + c*x^4]/(d + e*x)^3,x]
```

output
```
Result too large to show
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2+cx^4}}{(d+ex)^3} \, dx$$
$$\Big\downarrow \ 7299$$
$$\int \frac{\sqrt{a+bx^2+cx^4}}{(d+ex)^3}dx$$

input
```
Int[Sqrt[a + b*x^2 + c*x^4]/(d + e*x)^3,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [A] (verified)

Time = 1.26 (sec) , antiderivative size = 917, normalized size of antiderivative = 0.80

method	result
default	$-\frac{\sqrt{c\,x^{4}+b\,x^{2}+a}}{2e(ex+d)^{2}}+\frac{d(b\,e^{2}+2c\,d^{2})\sqrt{c\,x^{4}+b\,x^{2}+a}}{2e(e^{4}a+b\,d^{2}e^{2}+c\,d^{4})(ex+d)}+\frac{\left(-\frac{3cd}{e^{4}}+\frac{cd\left(2e^{4}a+3b\,d^{2}e^{2}+4c\,d^{4}\right)}{2e^{4}\left(e^{4}a+b\,d^{2}e^{2}+c\,d^{4}\right)}\right)\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^{2}}\right)x^{2}}{a}}\,\sqrt{4+\frac{2(b+}{}}}{4\sqrt{\frac{-b+\sqrt{-4ac+b}}{a}}}$
elliptic	$-\frac{\sqrt{c\,x^{4}+b\,x^{2}+a}}{2e(ex+d)^{2}}+\frac{d(b\,e^{2}+2c\,d^{2})\sqrt{c\,x^{4}+b\,x^{2}+a}}{2e(e^{4}a+b\,d^{2}e^{2}+c\,d^{4})(ex+d)}+\frac{\left(-\frac{3cd}{e^{4}}+\frac{cd\left(2e^{4}a+3b\,d^{2}e^{2}+4c\,d^{4}\right)}{2e^{4}\left(e^{4}a+b\,d^{2}e^{2}+c\,d^{4}\right)}\right)\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^{2}}\right)x^{2}}{a}}\,\sqrt{4+\frac{2(b+}{}}}{4\sqrt{\frac{-b+\sqrt{-4ac+b}}{a}}}$

input
```
int((c*x^4+b*x^2+a)^(1/2)/(e*x+d)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2/e*(c*x^4+b*x^2+a)^(1/2)/(e*x+d)^2+1/2*d*(b*e^2+2*c*d^2)/e/(a*e^4+b*d^
2*e^2+c*d^4)*(c*x^4+b*x^2+a)^(1/2)/(e*x+d)+1/4*(-3*c*d/e^4+1/2*c*d/e^4*(2*
a*e^4+3*b*d^2*e^2+4*c*d^4)/(a*e^4+b*d^2*e^2+c*d^4))*2^(1/2)/((-b+(-4*a*c+b
^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*
a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)
*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)
^(1/2))+1/2*c^(1/2)/e^3*ln((2*c*x^2+b)/c^(1/2)+2*(c*x^4+b*x^2+a)^(1/2))+1/
4*c*d*(b*e^2+2*c*d^2)/e^2/(a*e^4+b*d^2*e^2+c*d^4)*a*2^(1/2)/((-b+(-4*a*c+b
^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*
a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*
(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+
(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^
(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)))+1/2*(a*b*e
^6+6*a*c*d^2*e^4+3*b*c*d^4*e^2+2*c^2*d^6)/(a*e^4+b*d^2*e^2+c*d^4)/e^5*(-1/
2/(c*d^4/e^4+b*d^2/e^2+a)^(1/2)*arctanh(1/2*(2*c*x^2*d^2/e^2+b*d^2/e^2+b*x
^2+2*a)/(c*d^4/e^4+b*d^2/e^2+a)^(1/2)/(c*x^4+b*x^2+a)^(1/2))+2^(1/2)/((-b+
(-4*a*c+b^2)^(1/2))/a)^(1/2)/d*e*(1-1/2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/
2)*(1+1/2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*Ellipt
icPi(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),2/(-b+(-4*a*c+b^2)^(1
/2))*a/d^2*e^2,(-1/2*(b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*2^(1/2)/((-b+(-4*a...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2+cx^4}}{(d+ex)^3}\,dx = \text{Timed out}$$

input
```
integrate((c*x^4+b*x^2+a)^(1/2)/(e*x+d)^3,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2+cx^4}}{(d+ex)^3}\,dx = \int \frac{\sqrt{a+bx^2+cx^4}}{(d+ex)^3}\,dx$$

input
```
integrate((c*x**4+b*x**2+a)**(1/2)/(e*x+d)**3,x)
```

output
```
Integral(sqrt(a + b*x**2 + c*x**4)/(d + e*x)**3, x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2+cx^4}}{(d+ex)^3}\,dx = \int \frac{\sqrt{cx^4+bx^2+a}}{(ex+d)^3}\,dx$$

input
```
integrate((c*x^4+b*x^2+a)^(1/2)/(e*x+d)^3,x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^4 + b*x^2 + a)/(e*x + d)^3, x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2+cx^4}}{(d+ex)^3}\,dx = \int \frac{\sqrt{cx^4+bx^2+a}}{(ex+d)^3}\,dx$$

input
```
integrate((c*x^4+b*x^2+a)^(1/2)/(e*x+d)^3,x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^4 + b*x^2 + a)/(e*x + d)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2+cx^4}}{(d+ex)^3}\,dx = \int \frac{\sqrt{c\,x^4+b\,x^2+a}}{{\left(d+e\,x\right)}^3}\,dx$$

input
```
int((a + b*x^2 + c*x^4)^(1/2)/(d + e*x)^3,x)
```

output
```
int((a + b*x^2 + c*x^4)^(1/2)/(d + e*x)^3, x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2+cx^4}}{(d+ex)^3}\,dx = \int \frac{\sqrt{c\,x^4+b\,x^2+a}}{\left(ex+d\right)^3}dx$$

input
```
int((c*x^4+b*x^2+a)^(1/2)/(e*x+d)^3,x)
```

output
```
int((c*x^4+b*x^2+a)^(1/2)/(e*x+d)^3,x)
```

3.254 $\int (d+ex)^3 \left(a+bx^2+cx^4\right)^{3/2} dx$

Optimal result

Integrand size = 24, antiderivative size = 799

$$
\begin{aligned}
&\int (d+ex)^3 \left(a+bx^2+cx^4\right)^{3/2} dx = \\
&-\frac{d(6b^3cd^2-48abc^2d^2-8b^4e^2+57ab^2ce^2-84a^2c^2e^2)\,x\sqrt{a+bx^2+cx^4}}{105c^{5/2}\left(\sqrt{a}+\sqrt{c}x^2\right)} \\
&-\frac{3(b^2-4ac)\,e(6cd^2-be^2)\,(b+2cx^2)\sqrt{a+bx^2+cx^4}}{256c^3} \\
&+\frac{dx(3b^2cd^2+30ac^2d^2-4b^3e^2+9abce^2+3c(3bcd^2-4b^2e^2+14ace^2)\,x^2)\sqrt{a+bx^2+cx^4}}{105c^2} \\
&+\frac{e(6cd^2-be^2)\,(b+2cx^2)\left(a+bx^2+cx^4\right)^{3/2}}{32c^2} \\
&+\frac{dx(3(cd^2+be^2)+7ce^2x^2)\left(a+bx^2+cx^4\right)^{3/2}}{21c} \\
&+\frac{e^3(a+bx^2+cx^4)^{5/2}}{10c}+\frac{3(b^2-4ac)^2\,e(6cd^2-be^2)\,\mathrm{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{512c^{7/2}} \\
&+\frac{\sqrt[4]{a}d(6b^3cd^2-48abc^2d^2-8b^4e^2+57ab^2ce^2-84a^2c^2e^2)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\right.}{105c^{11/4}\sqrt{a+bx^2+cx^4}} \\
&-\frac{\sqrt[4]{a}\left(b+2\sqrt{a}\sqrt{c}\right)d\left(6b^2cd^2-9\sqrt{a}bc^{3/2}d^2-30ac^2d^2-8b^3e^2+12\sqrt{a}b^2\sqrt{c}e^2+33abce^2-42a^{3/2}c^{3/2}e^2\right)\left(\right.}{210c^{11/4}\sqrt{a+bx^2+cx^4}}
\end{aligned}
$$

output
```
-1/105*d*(-84*a^2*c^2*e^2+57*a*b^2*c*e^2-48*a*b*c^2*d^2-8*b^4*e^2+6*b^3*c*
d^2)*x*(c*x^4+b*x^2+a)^(1/2)/c^(5/2)/(a^(1/2)+c^(1/2)*x^2)-3/256*(-4*a*c+b
^2)*e*(-b*e^2+6*c*d^2)*(2*c*x^2+b)*(c*x^4+b*x^2+a)^(1/2)/c^3+1/105*d*x*(3*
b^2*c*d^2+30*a*c^2*d^2-4*b^3*e^2+9*a*b*c*e^2+3*c*(14*a*c*e^2-4*b^2*e^2+3*b
*c*d^2)*x^2)*(c*x^4+b*x^2+a)^(1/2)/c^2+1/32*e*(-b*e^2+6*c*d^2)*(2*c*x^2+b)
*(c*x^4+b*x^2+a)^(3/2)/c^2+1/21*d*x*(7*c*e^2*x^2+3*b*e^2+3*c*d^2)*(c*x^4+b
*x^2+a)^(3/2)/c+1/10*e^3*(c*x^4+b*x^2+a)^(5/2)/c+3/512*(-4*a*c+b^2)^2*e*(-
b*e^2+6*c*d^2)*arctanh(1/2*(2*c*x^2+b)/c^(1/2)/(c*x^4+b*x^2+a)^(1/2))/c^(7
/2)+1/105*a^(1/4)*d*(-84*a^2*c^2*e^2+57*a*b^2*c*e^2-48*a*b*c^2*d^2-8*b^4*e
^2+6*b^3*c*d^2)*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2)+c^(1/2)*x^
2)^2)^(1/2)*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^
(1/2))^(1/2))/c^(11/4)/(c*x^4+b*x^2+a)^(1/2)-1/210*a^(1/4)*(b+2*a^(1/2)*c^
(1/2))*d*(6*b^2*c*d^2-9*a^(1/2)*b*c^(3/2)*d^2-30*a*c^2*d^2-8*b^3*e^2+12*a^
(1/2)*b^2*c^(1/2)*e^2+33*a*b*c*e^2-42*a^(3/2)*c^(3/2)*e^2)*(a^(1/2)+c^(1/2
)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*a
rctan(c^(1/4)*x/a^(1/4)),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))/c^(11/4)/(c*x^4+
b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 12.68 (sec) , antiderivative size = 2681, normalized size of antiderivative = 3.36

$$\int (d+ex)^3 \left(a+bx^2+cx^4\right)^{3/2} dx = \text{Result too large to show}$$

input
```
Integrate[(d + e*x)^3*(a + b*x^2 + c*x^4)^(3/2),x]
```

output
```
(-2*Sqrt[c]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*(a + b*x^2 + c*x^4)*(-315*b^4*
e^3 + 2*b^3*c*e*(945*d^2 + 512*d*e*x + 105*e^2*x^2) - 12*b^2*c*(-175*a*e^3
 + c*x*(64*d^3 + 105*d^2*e*x + 64*d*e^2*x^2 + 14*e^3*x^3)) - 8*b*c^2*(3*a*
e*(525*d^2 + 256*d*e*x + 49*e^2*x^2) + 2*c*x^3*(384*d^3 + 945*d^2*e*x + 80
0*d*e^2*x^2 + 231*e^3*x^3)) - 16*c^2*(168*a^2*e^3 + 2*c^2*x^5*(120*d^3 + 3
15*d^2*e*x + 280*d*e^2*x^2 + 84*e^3*x^3) + a*c*x*(720*d^3 + 1575*d^2*e*x +
 1232*d*e^2*x^2 + 336*e^3*x^3))) + (768*I)*Sqrt[2]*b^3*c^(3/2)*(b - Sqrt[b
^2 - 4*a*c])*d^3*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*
a*c])]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*(EllipticE[I*ArcSinh[Sq
rt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqr
t[b^2 - 4*a*c])] - EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*
c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])]) + (6144*I)*Sqrt
[2]*a*b*c^(5/2)*(-b + Sqrt[b^2 - 4*a*c])*d^3*Sqrt[(b + Sqrt[b^2 - 4*a*c] +
 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*
c])]*(EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b +
 Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] - EllipticF[I*ArcSinh[Sqrt[2]
*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2
 - 4*a*c])]) + (7296*I)*Sqrt[2]*a*b^2*c^(3/2)*(b - Sqrt[b^2 - 4*a*c])*d*e^
2*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[1 +
 (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*(EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c...
```

Rubi [A] (verified)

Time = 2.16 (sec) , antiderivative size = 749, normalized size of antiderivative = 0.94, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {2202, 1490, 27, 1490, 25, 1511, 27, 1416, 1509, 1576, 27, 1160, 1087, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^3 \left(a+bx^2+cx^4\right)^{3/2} dx$$

$$\Big\downarrow \text{ 2202}$$

$$\int \left(d^3+3e^2x^2d\right)\left(cx^4+bx^2+a\right)^{3/2}dx+\int x\left(x^2e^3+3d^2e\right)\left(cx^4+bx^2+a\right)^{3/2}dx$$

$$\Big\downarrow \text{ 1490}$$

$$\frac{\int 3d\left(\left(3bcd^2-4b^2e^2+14ace^2\right)x^2+a\left(6cd^2-be^2\right)\right)\sqrt{cx^4+bx^2+a}dx}{21c}+$$
$$\int x\left(x^2e^3+3d^2e\right)\left(cx^4+bx^2+a\right)^{3/2}dx+\frac{dx\left(a+bx^2+cx^4\right)^{3/2}\left(3\left(be^2+cd^2\right)+7ce^2x^2\right)}{21c}$$

$\downarrow$ 27

$$\frac{d\int\left(\left(3bcd^2-4b^2e^2+14ace^2\right)x^2+a\left(6cd^2-be^2\right)\right)\sqrt{cx^4+bx^2+a}dx}{7c}+$$
$$\int x\left(x^2e^3+3d^2e\right)\left(cx^4+bx^2+a\right)^{3/2}dx+\frac{dx\left(a+bx^2+cx^4\right)^{3/2}\left(3\left(be^2+cd^2\right)+7ce^2x^2\right)}{21c}$$

$\downarrow$ 1490

$$\frac{d\left(\frac{\int-\frac{\left(-8e^2b^4+6cd^2b^3+57ace^2b^2-48ac^2d^2b-84a^2c^2e^2\right)x^2+a\left(-4e^2b^3+3cd^2b^2+24ace^2b-60ac^2d^2\right)}{\sqrt{cx^4+bx^2+a}}dx}{15c}+\frac{x\sqrt{a+bx^2+cx^4}\left(3cx^2\left(14ace^2-4b^2e^2+3bc\right.\right.}{15c}\right.}{7c}$$
$$\int x\left(x^2e^3+3d^2e\right)\left(cx^4+bx^2+a\right)^{3/2}dx+\frac{dx\left(a+bx^2+cx^4\right)^{3/2}\left(3\left(be^2+cd^2\right)+7ce^2x^2\right)}{21c}$$

$\downarrow$ 25

$$\frac{d\left(\frac{x\sqrt{a+bx^2+cx^4}\left(3cx^2\left(14ace^2-4b^2e^2+3bcd^2\right)+9abce^2+30ac^2d^2-4b^3e^2+3b^2cd^2\right)}{15c}-\frac{\int\frac{\left(-8e^2b^4+6cd^2b^3+57ace^2b^2-48ac^2d^2b-84a^2c^2e^2\right)x^2+a\left(\right.}{\sqrt{cx^4+bx^2+a}}}{15c}\right.}{7c}$$
$$\int x\left(x^2e^3+3d^2e\right)\left(cx^4+bx^2+a\right)^{3/2}dx+\frac{dx\left(a+bx^2+cx^4\right)^{3/2}\left(3\left(be^2+cd^2\right)+7ce^2x^2\right)}{21c}$$

$\downarrow$ 1511

$$\frac{d\left(\frac{x\sqrt{a+bx^2+cx^4}\left(3cx^2\left(14ace^2-4b^2e^2+3bcd^2\right)+9abce^2+30ac^2d^2-4b^3e^2+3b^2cd^2\right)}{15c}-\frac{\sqrt{a}\left(-84a^2c^2e^2+57ab^2ce^2+\sqrt{a}\sqrt{c}\left(24abce^2-60ac^2d^2-4b^3e^2\right.\right.}{\sqrt{c}}\right.}{7c}$$
$$\int x\left(x^2e^3+3d^2e\right)\left(cx^4+bx^2+a\right)^{3/2}dx+\frac{dx\left(a+bx^2+cx^4\right)^{3/2}\left(3\left(be^2+cd^2\right)+7ce^2x^2\right)}{21c}$$

$\downarrow$ 27

$$\frac{d\left(\frac{x\sqrt{a+bx^2+cx^4}\left(3cx^2\left(14ace^2-4b^2e^2+3bcd^2\right)+9abce^2+30ac^2d^2-4b^3e^2+3b^2cd^2\right)}{15c}-\frac{\sqrt{a}\left(-84a^2c^2e^2+57ab^2ce^2+\sqrt{a}\sqrt{c}\left(24abce^2-60ac^2d^2-4b^3e^2\cdots\right.\right.}{\sqrt{c}\cdots}\cdots\right)}{7c\cdots}$$

$$\int x\left(x^2e^3+3d^2e\right)\left(cx^4+bx^2+a\right)^{3/2}dx+\frac{dx\left(a+bx^2+cx^4\right)^{3/2}\left(3\left(be^2+cd^2\right)+7ce^2x^2\right)}{21c}$$

$\Big\downarrow$ 1416

$$\frac{d\left(\frac{x\sqrt{a+bx^2+cx^4}\left(3cx^2\left(14ace^2-4b^2e^2+3bcd^2\right)+9abce^2+30ac^2d^2-4b^3e^2+3b^2cd^2\right)}{15c}-\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(-84a^2c^2e^2+57ab^2ce^2+\cdots\right.}{\cdots}\cdots\right)}{\cdots}$$

$$\int x\left(x^2e^3+3d^2e\right)\left(cx^4+bx^2+a\right)^{3/2}dx+\frac{dx\left(a+bx^2+cx^4\right)^{3/2}\left(3\left(be^2+cd^2\right)+7ce^2x^2\right)}{21c}$$

$\Big\downarrow$ 1509

$$\int x\left(x^2e^3+3d^2e\right)\left(cx^4+bx^2+a\right)^{3/2}dx+$$

$$\frac{d\left(\frac{x\sqrt{a+bx^2+cx^4}\left(3cx^2\left(14ace^2-4b^2e^2+3bcd^2\right)+9abce^2+30ac^2d^2-4b^3e^2+3b^2cd^2\right)}{15c}-\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(-84a^2c^2e^2+57ab^2ce^2+\cdots\right.}{\cdots}\cdots\right)}{\cdots}$$

$$\frac{dx\left(a+bx^2+cx^4\right)^{3/2}\left(3\left(be^2+cd^2\right)+7ce^2x^2\right)}{21c}$$

$\Big\downarrow$ 1576

$$\frac{1}{2}\int e\left(3d^2+e^2x^2\right)\left(cx^4+bx^2+a\right)^{3/2}dx^2+$$

$$d\left(\frac{x\sqrt{a+bx^2+cx^4}\left(3cx^2\left(14ace^2-4b^2e^2+3bcd^2\right)+9abce^2+30ac^2d^2-4b^3e^2+3b^2cd^2\right)}{15c}-\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(-84a^2c^2e^2+57ab^2ce^2+\cdots\right.}{\cdots}\right.$$

$$\frac{dx\left(a+bx^2+cx^4\right)^{3/2}\left(3\left(be^2+cd^2\right)+7ce^2x^2\right)}{21c}$$

$\downarrow$ 27

$$\frac{1}{2}e\int\left(3d^2+e^2x^2\right)\left(cx^4+bx^2+a\right)^{3/2}dx^2+$$

$$d\left(\frac{x\sqrt{a+bx^2+cx^4}\left(3cx^2\left(14ace^2-4b^2e^2+3bcd^2\right)+9abce^2+30ac^2d^2-4b^3e^2+3b^2cd^2\right)}{15c}-\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(-84a^2c^2e^2+57ab^2ce^2+\cdots\right.}{\cdots}\right.$$

$$\frac{dx\left(a+bx^2+cx^4\right)^{3/2}\left(3\left(be^2+cd^2\right)+7ce^2x^2\right)}{21c}$$

$\downarrow$ 1160

$$\frac{1}{2}e\left(\frac{(6cd^2-be^2)\int (cx^4+bx^2+a)^{3/2}\,dx^2}{2c}+\frac{e^2(a+bx^2+cx^4)^{5/2}}{5c}\right)+$$

$$\frac{d\left(\frac{x\sqrt{a+bx^2+cx^4}(3cx^2(14ace^2-4b^2e^2+3bcd^2)+9abce^2+30ac^2d^2-4b^3e^2+3b^2cd^2)}{15c}-\frac{\sqrt[4]{a}(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}(-84a^2c^2e^2+57ab^2ce^2+\cdots}{\cdots}\right.}{\cdots}$$

$$\frac{dx(a+bx^2+cx^4)^{3/2}(3(be^2+cd^2)+7ce^2x^2)}{21c}$$

$\downarrow$ 1087

$$\frac{1}{2}e\left(\frac{(6cd^2-be^2)\left(\frac{(b+2cx^2)(a+bx^2+cx^4)^{3/2}}{8c}-\frac{3(b^2-4ac)\int \sqrt{cx^4+bx^2+a}dx^2}{16c}\right)}{2c}+\frac{e^2(a+bx^2+cx^4)^{5/2}}{5c}\right)+$$

$$\frac{d\left(\frac{x\sqrt{a+bx^2+cx^4}(3cx^2(14ace^2-4b^2e^2+3bcd^2)+9abce^2+30ac^2d^2-4b^3e^2+3b^2cd^2)}{15c}-\frac{\sqrt[4]{a}(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}(-84a^2c^2e^2+57ab^2ce^2+\cdots}{\cdots}\right.}{\cdots}$$

$$\frac{dx(a+bx^2+cx^4)^{3/2}(3(be^2+cd^2)+7ce^2x^2)}{21c}$$

$\downarrow$ 1087

$$\frac{1}{2}e\left(\frac{(6cd^2-be^2)\left(\frac{(b+2cx^2)(a+bx^2+cx^4)^{3/2}}{8c}-\frac{3(b^2-4ac)\left(\frac{(b+2cx^2)\sqrt{a+bx^2+cx^4}}{4c}-\frac{(b^2-4ac)\int\frac{1}{\sqrt{cx^4+bx^2+a}}dx^2}{8c}\right)}{16c}\right)}{2c}+\frac{e^2(a+bx\right.$$

$$d\left(\frac{x\sqrt{a+bx^2+cx^4}(3cx^2(14ace^2-4b^2e^2+3bcd^2)+9abce^2+30ac^2d^2-4b^3e^2+3b^2cd^2)}{15c}-\frac{\sqrt[4]{a}(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}(-84a^2c^2e^2+57ab^2ce^2+\right.$$

$$\frac{dx(a+bx^2+cx^4)^{3/2}(3(be^2+cd^2)+7ce^2x^2)}{21c}$$

$\downarrow$ 1092

$$\frac{1}{2}e\left(\frac{\left(6cd^2-be^2\right)\left(\frac{\left(b+2cx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\left(\frac{\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{4c}-\frac{\left(b^2-4ac\right)\int\frac{1}{4c-x^4}d\frac{2cx^2+b}{\sqrt{cx^4+bx^2+a}}}{4c}\right)}{16c}\right)}{2c}+\frac{e^2\left(a+\right.}{}\right.$$

$$d\left(\frac{x\sqrt{a+bx^2+cx^4}\left(3cx^2\left(14ace^2-4b^2e^2+3bcd^2\right)+9abce^2+30ac^2d^2-4b^3e^2+3b^2cd^2\right)}{15c}-\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(-84a^2c^2e^2+57ab^2ce^2+\right.}{}\right.$$

$$\frac{dx\left(a+bx^2+cx^4\right)^{3/2}\left(3\left(be^2+cd^2\right)+7ce^2x^2\right)}{21c}$$

$\downarrow$ 219

$$d\left(\frac{x\sqrt{a+bx^2+cx^4}\left(3cx^2\left(14ace^2-4b^2e^2+3bcd^2\right)+9abce^2+30ac^2d^2-4b^3e^2+3b^2cd^2\right)}{15c}-\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(-84a^2c^2e^2+57ab^2ce^2+\right.}{}\right.$$

$$\frac{1}{2}e\left(\frac{\left(6cd^2-be^2\right)\left(\frac{\left(b+2cx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\left(\frac{\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{4c}-\frac{\left(b^2-4ac\right)\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{8c^{3/2}}\right)}{16c}\right)}{2c}+\frac{e}{}\right.$$

$$\frac{dx\left(a+bx^2+cx^4\right)^{3/2}\left(3\left(be^2+cd^2\right)+7ce^2x^2\right)}{21c}$$

input
```
Int[(d + e*x)^3*(a + b*x^2 + c*x^4)^(3/2),x]
```

output
```
(d*x*(3*(c*d^2 + b*e^2) + 7*c*e^2*x^2)*(a + b*x^2 + c*x^4)^(3/2))/(21*c) +
 (e*((e^2*(a + b*x^2 + c*x^4)^(5/2))/(5*c) + ((6*c*d^2 - b*e^2)*(((b + 2*c
*x^2)*(a + b*x^2 + c*x^4)^(3/2))/(8*c) - (3*(b^2 - 4*a*c)*(((b + 2*c*x^2)*
Sqrt[a + b*x^2 + c*x^4])/(4*c) - ((b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^2)/(2*S
qrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(8*c^(3/2))))/(16*c)))/(2*c)))/2 + (d*((
x*(3*b^2*c*d^2 + 30*a*c^2*d^2 - 4*b^3*e^2 + 9*a*b*c*e^2 + 3*c*(3*b*c*d^2 -
 4*b^2*e^2 + 14*a*c*e^2)*x^2)*Sqrt[a + b*x^2 + c*x^4])/(15*c) - (-(((6*b^3
*c*d^2 - 48*a*b*c^2*d^2 - 8*b^4*e^2 + 57*a*b^2*c*e^2 - 84*a^2*c^2*e^2)*(-(
(x*Sqrt[a + b*x^2 + c*x^4])/(Sqrt[a] + Sqrt[c]*x^2)) + (a^(1/4)*(Sqrt[a] +
 Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Elliptic
E[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(c^(1/4)*Sq
rt[a + b*x^2 + c*x^4])))/Sqrt[c]) + (a^(1/4)*(6*b^3*c*d^2 - 48*a*b*c^2*d^2
 - 8*b^4*e^2 + 57*a*b^2*c*e^2 - 84*a^2*c^2*e^2 + Sqrt[a]*Sqrt[c]*(3*b^2*c*
d^2 - 60*a*c^2*d^2 - 4*b^3*e^2 + 24*a*b*c*e^2))*(Sqrt[a] + Sqrt[c]*x^2)*Sq
rt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1
/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*c^(3/4)*Sqrt[a + b*x^2 +
 c*x^4]))/(15*c)))/(7*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1490
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Simp[2*(p/(c*(4*p + 1)*(4*p + 3)))
 Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3)
- b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 1576
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x]
, x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

rule 2202
```
Int[(Pn_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{n
 = Expon[Pn, x], k}, Int[Sum[Coeff[Pn, x, 2*k]*x^(2*k), {k, 0, n/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pn, x, 2*k + 1]*x^(2*k), {k, 0, (n -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pn, x]
 &&  !PolyQ[Pn, x^2]
```

Maple [A] (verified)

Time = 4.58 (sec) , antiderivative size = 1300, normalized size of antiderivative = 1.63

method	result	size
risch	Expression too large to display	1300
default	Expression too large to display	1592
elliptic	Expression too large to display	1671

input
```
int((e*x+d)^3*(c*x^4+b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/26880/c^3*(2688*c^4*e^3*x^8+8960*c^4*d*e^2*x^7+3696*b*c^3*e^3*x^6+10080*
c^4*d^2*e*x^6+12800*b*c^3*d*e^2*x^5+3840*c^4*d^3*x^5+5376*a*c^3*e^3*x^4+16
8*b^2*c^2*e^3*x^4+15120*b*c^3*d^2*e*x^4+19712*a*c^3*d*e^2*x^3+768*b^2*c^2*
d*e^2*x^3+6144*b*c^3*d^3*x^3+1176*a*b*c^2*e^3*x^2+25200*a*c^3*d^2*e*x^2-21
0*b^3*c*e^3*x^2+1260*b^2*c^2*d^2*e*x^2+6144*a*b*c^2*d*e^2*x+11520*a*c^3*d^
3*x-1024*b^3*c*d*e^2*x+768*b^2*c^2*d^3*x+2688*a^2*c^2*e^3-2100*a*b^2*c*e^3
+12600*a*b*c^2*d^2*e+315*b^4*e^3-1890*b^3*c*d^2*e)*(c*x^4+b*x^2+a)^(1/2)-1
/26880/c^3*(128*c*d*(84*a^2*c^2*e^2-57*a*b^2*c*e^2+48*a*b*c^2*d^2+8*b^4*e^
2-6*b^3*c*d^2)*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*
c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4
+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a
*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-Ell
ipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*
a*c+b^2)^(1/2))/a/c)^(1/2)))+315/2*e*(16*a^2*b*c^2*e^2-96*a^2*c^3*d^2-8*a*
b^3*c*e^2+48*a*b^2*c^2*d^2+b^5*e^2-6*b^4*c*d^2)*ln((1/2*b+c*x^2)/c^(1/2)+(
c*x^4+b*x^2+a)^(1/2))/c^(1/2)-3840*a^2*c^3*d^3*2^(1/2)/((-b+(-4*a*c+b^2)^(
1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b
^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b
+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2
))+192*a*b^2*c^2*d^3*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b...
```

Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 981, normalized size of antiderivative = 1.23

$$\int (d+ex)^3 \left(a+bx^2+cx^4\right)^{3/2} dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^3*(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
-1/107520*(512*sqrt(1/2)*((6*(b^3*c^2 - 8*a*b*c^3)*d^3 - (8*b^4*c - 57*a*b
^2*c^2 + 84*a^2*c^3)*d*e^2)*x*sqrt((b^2 - 4*a*c)/c^2) - (6*(b^4*c - 8*a*b^
2*c^2)*d^3 - (8*b^5 - 57*a*b^3*c + 84*a^2*b*c^2)*d*e^2)*x)*sqrt(c)*sqrt((c
*sqrt((b^2 - 4*a*c)/c^2) - b)/c)*elliptic_e(arcsin(sqrt(1/2)*sqrt((c*sqrt(
(b^2 - 4*a*c)/c^2) - b)/c)/x), 1/2*(b*c*sqrt((b^2 - 4*a*c)/c^2) + b^2 - 2*
a*c)/(a*c)) - 512*sqrt(1/2)*((3*(2*b^3*c^2 + 20*a*c^4 - (16*a*b + b^2)*c^3
)*d^3 - (8*b^4*c + 12*(7*a^2 + 2*a*b)*c^3 - (57*a*b^2 + 4*b^3)*c^2)*d*e^2)
*x*sqrt((b^2 - 4*a*c)/c^2) - (3*(2*b^4*c - 20*a*b*c^3 - (16*a*b^2 - b^3)*c
^2)*d^3 - (8*b^5 + 12*(7*a^2*b - 2*a*b^2)*c^2 - (57*a*b^3 - 4*b^4)*c)*d*e^
2)*x)*sqrt(c)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)*elliptic_f(arcsin(sq
rt(1/2)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)/x), 1/2*(b*c*sqrt((b^2 - 4
*a*c)/c^2) + b^2 - 2*a*c)/(a*c)) + 315*(6*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^
3)*d^2*e - (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*e^3)*sqrt(c)*x*log(8*c^2*x^4 +
 8*b*c*x^2 + b^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) + 4*a*c
) - 4*(2688*c^5*e^3*x^9 + 8960*c^5*d*e^2*x^8 + 336*(30*c^5*d^2*e + 11*b*c^
4*e^3)*x^7 + 1280*(3*c^5*d^3 + 10*b*c^4*d*e^2)*x^6 + 168*(90*b*c^4*d^2*e +
 (b^2*c^3 + 32*a*c^4)*e^3)*x^5 + 256*(24*b*c^4*d^3 + (3*b^2*c^3 + 77*a*c^4
)*d*e^2)*x^4 - 1536*(b^3*c^2 - 8*a*b*c^3)*d^3 + 256*(8*b^4*c - 57*a*b^2*c^
2 + 84*a^2*c^3)*d*e^2 + 42*(30*(b^2*c^3 + 20*a*c^4)*d^2*e - (5*b^3*c^2 - 2
8*a*b*c^3)*e^3)*x^3 + 256*(3*(b^2*c^3 + 15*a*c^4)*d^3 - 4*(b^3*c^2 - 6*...
```

Sympy [F]

$$\int (d+ex)^3 \left(a+bx^2+cx^4\right)^{3/2} dx = \int (d+ex)^3 \left(a+bx^2+cx^4\right)^{\frac{3}{2}} dx$$

input
```
integrate((e*x+d)**3*(c*x**4+b*x**2+a)**(3/2),x)
```

output
```
Integral((d + e*x)**3*(a + b*x**2 + c*x**4)**(3/2), x)
```

Maxima [F]

$$\int (d+ex)^3 \left(a+bx^2+cx^4\right)^{3/2} dx = \int \left(cx^4+bx^2+a\right)^{\frac{3}{2}} (ex+d)^3 \, dx$$

input
```
integrate((e*x+d)^3*(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*x^4 + b*x^2 + a)^(3/2)*(e*x + d)^3, x)
```

Giac [F]

$$\int (d+ex)^3 \left(a+bx^2+cx^4\right)^{3/2} dx = \int \left(cx^4+bx^2+a\right)^{\frac{3}{2}} (ex+d)^3 \, dx$$

input
```
integrate((e*x+d)^3*(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((c*x^4 + b*x^2 + a)^(3/2)*(e*x + d)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)^3 \left(a+bx^2+cx^4\right)^{3/2} dx = \int (d+e\,x)^3 \left(c\,x^4+b\,x^2+a\right)^{3/2} dx$$

input
```
int((d + e*x)^3*(a + b*x^2 + c*x^4)^(3/2),x)
```

output
```
int((d + e*x)^3*(a + b*x^2 + c*x^4)^(3/2), x)
```

Reduce [F]

$$\int (d+ex)^3 \left(a+bx^2+cx^4\right)^{3/2} dx = \int (ex+d)^3 \left(c\,x^4+b\,x^2+a\right)^{\frac{3}{2}} dx$$

input
```
int((e*x+d)^3*(c*x^4+b*x^2+a)^(3/2),x)
```

output
```
int((e*x+d)^3*(c*x^4+b*x^2+a)^(3/2),x)
```

3.255 $\int (d+ex)^2 \left(a+bx^2+cx^4\right)^{3/2} dx$

Optimal result

Integrand size = 24, antiderivative size = 733

$$\int (d+ex)^2 \left(a+bx^2+cx^4\right)^{3/2} dx = -\frac{(18b^3cd^2-144abc^2d^2-8b^4e^2+57ab^2ce^2-84a^2c^2e^2)\,x\sqrt{a+bx^2+cx^4}}{315c^{5/2}\left(\sqrt{a}+\sqrt{c}x^2\right)}$$
$$-\frac{3(b^2-4ac)\,de(b+2cx^2)\sqrt{a+bx^2+cx^4}}{64c^2}$$
$$+\frac{x(9b^2cd^2+90ac^2d^2-4b^3e^2+9abce^2+3c(9bcd^2-4b^2e^2+14ace^2)\,x^2)\sqrt{a+bx^2+cx^4}}{315c^2}$$
$$+\frac{de(b+2cx^2)\left(a+bx^2+cx^4\right)^{3/2}}{8c}+\frac{x(3(3cd^2+be^2)+7ce^2x^2)\left(a+bx^2+cx^4\right)^{3/2}}{63c}$$
$$+\frac{3(b^2-4ac)^2\,de\,\mathrm{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{128c^{5/2}}$$
$$+\frac{\sqrt[4]{a}(18b^3cd^2-144abc^2d^2-8b^4e^2+57ab^2ce^2-84a^2c^2e^2)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\right.}{315c^{11/4}\sqrt{a+bx^2+cx^4}}$$
$$-\frac{\sqrt[4]{a}\left(b+2\sqrt{a}\sqrt{c}\right)\left(18b^2cd^2-27\sqrt{a}bc^{3/2}d^2-90ac^2d^2-8b^3e^2+12\sqrt{a}b^2\sqrt{c}e^2+33abce^2-42a^{3/2}c^{3/2}e^2\right)\left(\right.}{630c^{11/4}\sqrt{a+bx^2+cx^4}}$$

output
```
-1/315*(-84*a^2*c^2*e^2+57*a*b^2*c*e^2-144*a*b*c^2*d^2-8*b^4*e^2+18*b^3*c*
d^2)*x*(c*x^4+b*x^2+a)^(1/2)/c^(5/2)/(a^(1/2)+c^(1/2)*x^2)-3/64*(-4*a*c+b^
2)*d*e*(2*c*x^2+b)*(c*x^4+b*x^2+a)^(1/2)/c^2+1/315*x*(9*b^2*c*d^2+90*a*c^2
*d^2-4*b^3*e^2+9*a*b*c*e^2+3*c*(14*a*c*e^2-4*b^2*e^2+9*b*c*d^2)*x^2)*(c*x^
4+b*x^2+a)^(1/2)/c^2+1/8*d*e*(2*c*x^2+b)*(c*x^4+b*x^2+a)^(3/2)/c+1/63*x*(7
*c*e^2*x^2+3*b*e^2+9*c*d^2)*(c*x^4+b*x^2+a)^(3/2)/c+3/128*(-4*a*c+b^2)^2*d
*e*arctanh(1/2*(2*c*x^2+b)/c^(1/2)/(c*x^4+b*x^2+a)^(1/2))/c^(5/2)+1/315*a^
(1/4)*(-84*a^2*c^2*e^2+57*a*b^2*c*e^2-144*a*b*c^2*d^2-8*b^4*e^2+18*b^3*c*d
^2)*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*
EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2)
)/c^(11/4)/(c*x^4+b*x^2+a)^(1/2)-1/630*a^(1/4)*(b+2*a^(1/2)*c^(1/2))*(18*b
^2*c*d^2-27*a^(1/2)*b*c^(3/2)*d^2-90*a*c^2*d^2-8*b^3*e^2+12*a^(1/2)*b^2*c^
(1/2)*e^2+33*a*b*c*e^2-42*a^(3/2)*c^(3/2)*e^2)*(a^(1/2)+c^(1/2)*x^2)*((c*x
^4+b*x^2+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(c^(1/4
)*x/a^(1/4)),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))/c^(11/4)/(c*x^4+b*x^2+a)^(1/
2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 15.10 (sec) , antiderivative size = 838, normalized size of antiderivative = 1.14

$$\int (d+ex)^2 \left(a+bx^2+cx^4\right)^{3/2} dx = \frac{32i\left(-b+\sqrt{b^2-4ac}\right)\left(-18b^3cd^2+144abc^2d^2+8b^4e^2-57ab^2ce^2+84a^2c^2e^2\right)\sqrt{\frac{b+\sqrt{b^2-4ac}+2}{b+\sqrt{b^2-4ac}}}}{}$$

input
```
Integrate[(d + e*x)^2*(a + b*x^2 + c*x^4)^(3/2),x]
```

output
```
((32*I)*(-b + Sqrt[b^2 - 4*a*c])*(-18*b^3*c*d^2 + 144*a*b*c^2*d^2 + 8*b^4*
e^2 - 57*a*b^2*c*e^2 + 84*a^2*c^2*e^2)*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x
^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b
 - Sqrt[b^2 - 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 -
4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] - (32*I)*(-8
*b^5*e^2 + 12*a*b*c^2*(12*Sqrt[b^2 - 4*a*c]*d^2 - 11*a*e^2) + b^3*c*(-18*S
qrt[b^2 - 4*a*c]*d^2 + 65*a*e^2) + 2*b^4*(9*c*d^2 + 4*Sqrt[b^2 - 4*a*c]*e^
2) + 12*a^2*c^2*(30*c*d^2 + 7*Sqrt[b^2 - 4*a*c]*e^2) - 3*a*b^2*c*(54*c*d^2
 + 19*Sqrt[b^2 - 4*a*c]*e^2))*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b +
Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b
^2 - 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*
x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] + Sqrt[c]*Sqrt[c/(b +
 Sqrt[b^2 - 4*a*c])]*(2*Sqrt[c]*(a + b*x^2 + c*x^4)*(-(b^3*e*(945*d + 256*
e*x)) + 6*b^2*c*x*(96*d^2 + 105*d*e*x + 32*e^2*x^2) + 4*b*c*(3*a*e*(525*d
+ 128*e*x) + 2*c*x^3*(576*d^2 + 945*d*e*x + 400*e^2*x^2)) + 8*c^2*x*(10*c*
x^4*(36*d^2 + 63*d*e*x + 28*e^2*x^2) + a*(1080*d^2 + 1575*d*e*x + 616*e^2*
x^2))) - 945*(b^2 - 4*a*c)^2*d*e*Sqrt[a + b*x^2 + c*x^4]*Log[b + 2*c*x^2 -
 2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4]]))/(40320*c^3*Sqrt[c/(b + Sqrt[b^2 - 4*
a*c])]*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] (verified)

Time = 1.61 (sec) , antiderivative size = 702, normalized size of antiderivative = 0.96, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {2202, 27, 1432, 1087, 1087, 1092, 219, 1490, 1490, 25, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^2 \left(a+bx^2+cx^4\right)^{3/2} \, dx$$

$$\Big\downarrow \; 2202$$

$$\int \left(d^2+e^2x^2\right) \left(cx^4+bx^2+a\right)^{3/2}dx+\int 2dex\left(cx^4+bx^2+a\right)^{3/2}dx$$

$$\Big\downarrow \; 27$$

$$\int \left(d^2+e^2x^2\right) \left(cx^4+bx^2+a\right)^{3/2}dx+2de\int x\left(cx^4+bx^2+a\right)^{3/2}dx$$

$$\Big\downarrow \text{1432}$$

$$\int \left(d^2+e^2x^2\right)\left(cx^4+bx^2+a\right)^{3/2}dx+de\int \left(cx^4+bx^2+a\right)^{3/2}dx^2$$

$$\Big\downarrow \text{1087}$$

$$de\left(\frac{\left(b+2cx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\int \sqrt{cx^4+bx^2+a}dx^2}{16c}\right)+\int \left(d^2+e^2x^2\right)\left(cx^4+bx^2+a\right)^{3/2}dx$$

$$\Big\downarrow \text{1087}$$

$$de\left(\frac{\left(b+2cx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\left(\frac{\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{4c}-\frac{\left(b^2-4ac\right)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx^2}{8c}\right)}{16c}\right)+\int \left(d^2+e^2x^2\right)\left(cx^4+bx^2+a\right)^{3/2}dx$$

$$\Big\downarrow \text{1092}$$

$$de\left(\frac{\left(b+2cx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\left(\frac{\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{4c}-\frac{\left(b^2-4ac\right)\int \frac{1}{4c-x^4}d\frac{2cx^2+b}{\sqrt{cx^4+bx^2+a}}}{4c}\right)}{16c}\right)+\int \left(d^2+e^2x^2\right)\left(cx^4+bx^2+a\right)^{3/2}dx$$

$$\Big\downarrow \text{219}$$

$$\int \left(d^2+e^2x^2\right)\left(cx^4+bx^2+a\right)^{3/2}dx+de\left(\frac{\left(b+2cx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\left(\frac{\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{4c}-\frac{\left(b^2-4ac\right)\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{8c^{3/2}}\right)}{16c}\right)$$

$$\Big\downarrow \text{1490}$$

$$\frac{\int \left(\left(9bcd^2-4b^2e^2+14ace^2\right)x^2+a\left(18cd^2-be^2\right)\right)\sqrt{cx^4+bx^2+a}dx}{21c}+de\left(\frac{\left(b+2cx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\left(\frac{\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{4c}-\frac{\left(b^2-4ac\right)\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{8c^{3/2}}\right)}{16c}\right)+\frac{x\left(a+bx^2+cx^4\right)^{3/2}\left(3\left(be^2+3cd^2\right)+7ce^2x^2\right)}{63c}$$

$\Big\downarrow$ 1490

$$\frac{\frac{\int -\frac{\left(-8e^2b^4+18cd^2b^3+57ace^2b^2-144ac^2d^2b-84a^2c^2e^2\right)x^2+a\left(-4e^2b^3+9cd^2b^2+24ace^2b-180ac^2d^2\right)}{\sqrt{cx^4+bx^2+a}}dx}{15c}+\frac{x\sqrt{a+bx^2+cx^4}\left(3cx^2\left(14ace^2-4b^2e^2+9bcd^2\right.\right.}{15c}}{21c}+de\left(\frac{\left(b+2cx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\left(\frac{\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{4c}-\frac{\left(b^2-4ac\right)\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{8c^{3/2}}\right)}{16c}\right)+\frac{x\left(a+bx^2+cx^4\right)^{3/2}\left(3\left(be^2+3cd^2\right)+7ce^2x^2\right)}{63c}$$

$\Big\downarrow$ 25

$$\frac{\frac{x\sqrt{a+bx^2+cx^4}\left(3cx^2\left(14ace^2-4b^2e^2+9bcd^2\right)+9abce^2+90ac^2d^2-4b^3e^2+9b^2cd^2\right)}{15c}-\frac{\int \frac{\left(-8e^2b^4+18cd^2b^3+57ace^2b^2-144ac^2d^2b-84a^2c^2e^2\right)x^2+a\left(-\right.}{\sqrt{cx^4+bx^2+a}}}{15c}}{21c}+de\left(\frac{\left(b+2cx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\left(\frac{\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{4c}-\frac{\left(b^2-4ac\right)\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{8c^{3/2}}\right)}{16c}\right)+\frac{x\left(a+bx^2+cx^4\right)^{3/2}\left(3\left(be^2+3cd^2\right)+7ce^2x^2\right)}{63c}$$

$\Big\downarrow$ 1511

$$\frac{\frac{x\sqrt{a+bx^2+cx^4}\left(3cx^2\left(14ace^2-4b^2e^2+9bcd^2\right)+9abce^2+90ac^2d^2-4b^3e^2+9b^2cd^2\right)}{15c}-\frac{\frac{\sqrt{a}\left(2\sqrt{a}\sqrt{c}+b\right)\left(-42a^{3/2}c^{3/2}e^2+12\sqrt{a}b^2\sqrt{c}e^2-27\sqrt{a}bc^{3/2}d^2+33\cdots\right.}{\sqrt{c}}}{\cdots}}{21c}$$

$$de\left(\frac{\left(b+2cx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\left(\frac{\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{4c}-\frac{\left(b^2-4ac\right)\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{8c^{3/2}}\right)}{16c}\right)+$$

$$\frac{x\left(a+bx^2+cx^4\right)^{3/2}\left(3\left(be^2+3cd^2\right)+7ce^2x^2\right)}{63c}$$

$$\Big\downarrow\ 27$$

$$\frac{\frac{x\sqrt{a+bx^2+cx^4}\left(3cx^2\left(14ace^2-4b^2e^2+9bcd^2\right)+9abce^2+90ac^2d^2-4b^3e^2+9b^2cd^2\right)}{15c}-\frac{\frac{\sqrt{a}\left(2\sqrt{a}\sqrt{c}+b\right)\left(-42a^{3/2}c^{3/2}e^2+12\sqrt{a}b^2\sqrt{c}e^2-27\sqrt{a}bc^{3/2}d^2+33\cdots\right.}{\sqrt{c}}}{\cdots}}{21c}$$

$$de\left(\frac{\left(b+2cx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\left(\frac{\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{4c}-\frac{\left(b^2-4ac\right)\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{8c^{3/2}}\right)}{16c}\right)+$$

$$\frac{x\left(a+bx^2+cx^4\right)^{3/2}\left(3\left(be^2+3cd^2\right)+7ce^2x^2\right)}{63c}$$

$$\Big\downarrow\ 1416$$

$$\frac{\frac{x\sqrt{a+bx^2+cx^4}\left(3cx^2\left(14ace^2-4b^2e^2+9bcd^2\right)+9abce^2+90ac^2d^2-4b^3e^2+9b^2cd^2\right)}{15c}-\frac{\sqrt[4]{a}\left(2\sqrt{a}\sqrt{c}+b\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(-42a^{3/2}c^{3/2}e^2+\cdots\right.}{\cdots}}{\cdots}$$

$$de\left(\frac{\left(b+2cx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\left(\frac{\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{4c}-\frac{\left(b^2-4ac\right)\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{8c^{3/2}}\right)}{16c}\right)+$$

$$\frac{x\left(a+bx^2+cx^4\right)^{3/2}\left(3\left(be^2+3cd^2\right)+7ce^2x^2\right)}{63c}$$

$$\Big\downarrow\ 1509$$

$$\frac{\frac{x\sqrt{a+bx^2+cx^4}\left(3cx^2\left(14ace^2-4b^2e^2+9bcd^2\right)+9abce^2+90ac^2d^2-4b^3e^2+9b^2cd^2\right)}{15c}-\frac{\sqrt[4]{a}\left(2\sqrt{a}\sqrt{c}+b\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(-42a^{3/2}c^{3/2}e^2+\right.}{}}{}$$

$$de\left(\frac{\left(b+2cx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\left(\frac{\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{4c}-\frac{\left(b^2-4ac\right)\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{8c^{3/2}}\right)}{16c}\right)+$$

$$\frac{x\left(a+bx^2+cx^4\right)^{3/2}\left(3\left(be^2+3cd^2\right)+7ce^2x^2\right)}{63c}$$

input
```
Int[(d + e*x)^2*(a + b*x^2 + c*x^4)^(3/2),x]
```

output
```
(x*(3*(3*c*d^2 + b*e^2) + 7*c*e^2*x^2)*(a + b*x^2 + c*x^4)^(3/2))/(63*c) +
 d*e*(((b + 2*c*x^2)*(a + b*x^2 + c*x^4)^(3/2))/(8*c) - (3*(b^2 - 4*a*c)*(
((b + 2*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(4*c) - ((b^2 - 4*a*c)*ArcTanh[(b
+ 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(8*c^(3/2))))/(16*c)) + (
(x*(9*b^2*c*d^2 + 90*a*c^2*d^2 - 4*b^3*e^2 + 9*a*b*c*e^2 + 3*c*(9*b*c*d^2
- 4*b^2*e^2 + 14*a*c*e^2)*x^2)*Sqrt[a + b*x^2 + c*x^4])/(15*c) - (-(((18*b
^3*c*d^2 - 144*a*b*c^2*d^2 - 8*b^4*e^2 + 57*a*b^2*c*e^2 - 84*a^2*c^2*e^2)*
(-((x*Sqrt[a + b*x^2 + c*x^4])/(Sqrt[a] + Sqrt[c]*x^2)) + (a^(1/4)*(Sqrt[a
] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Ellip
ticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(c^(1/4)
*Sqrt[a + b*x^2 + c*x^4])))/Sqrt[c]) + (a^(1/4)*(b + 2*Sqrt[a]*Sqrt[c])*(1
8*b^2*c*d^2 - 27*Sqrt[a]*b*c^(3/2)*d^2 - 90*a*c^2*d^2 - 8*b^3*e^2 + 12*Sqr
t[a]*b^2*Sqrt[c]*e^2 + 33*a*b*c*e^2 - 42*a^(3/2)*c^(3/2)*e^2)*(Sqrt[a] + S
qrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[
2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*c^(3/4)*Sq
rt[a + b*x^2 + c*x^4]))/(15*c))/(21*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1432
```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

rule 1490
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Simp[2*(p/(c*(4*p + 1)*(4*p + 3)))
 Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3)
- b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 2202
```
Int[(Pn_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{n
 = Expon[Pn, x], k}, Int[Sum[Coeff[Pn, x, 2*k]*x^(2*k), {k, 0, n/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pn, x, 2*k + 1]*x^(2*k), {k, 0, (n -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pn, x]
 &&  !PolyQ[Pn, x^2]
```

Maple [A] (verified)

Time = 3.78 (sec) , antiderivative size = 1087, normalized size of antiderivative = 1.48

method	result	size
elliptic	Expression too large to display	1087
risch	Expression too large to display	1116
default	Expression too large to display	1269

input
```
int((e*x+d)^2*(c*x^4+b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/9*c*e^2*x^7*(c*x^4+b*x^2+a)^(1/2)+1/4*c*d*e*x^6*(c*x^4+b*x^2+a)^(1/2)+1/
7*(10/9*b*c*e^2+c^2*d^2)/c*x^5*(c*x^4+b*x^2+a)^(1/2)+3/8*b*d*e*x^4*(c*x^4+
b*x^2+a)^(1/2)+1/5*(11/9*a*c*e^2+b^2*e^2+2*b*c*d^2-6/7*b/c*(10/9*b*c*e^2+c
^2*d^2))/c*x^3*(c*x^4+b*x^2+a)^(1/2)+1/4*(5/2*a*c*d*e+1/8*b^2*d*e)/c*x^2*(
c*x^4+b*x^2+a)^(1/2)+1/3*(2*a*b*e^2+2*a*c*d^2+b^2*d^2-4/5*b/c*(11/9*a*c*e^
2+b^2*e^2+2*b*c*d^2-6/7*b/c*(10/9*b*c*e^2+c^2*d^2))-5/7*a/c*(10/9*b*c*e^2+
c^2*d^2))/c*x*(c*x^4+b*x^2+a)^(1/2)+1/2*(5/2*a*b*d*e-3/4*b/c*(5/2*a*c*d*e+
1/8*b^2*d*e))/c*(c*x^4+b*x^2+a)^(1/2)+1/4*(a^2*d^2-1/3*a/c*(2*a*b*e^2+2*a*
c*d^2+b^2*d^2-4/5*b/c*(11/9*a*c*e^2+b^2*e^2+2*b*c*d^2-6/7*b/c*(10/9*b*c*e^
2+c^2*d^2))-5/7*a/c*(10/9*b*c*e^2+c^2*d^2)))*2^(1/2)/((-b+(-4*a*c+b^2)^(1/
2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2
)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(
-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))
+1/2*(2*a^2*d*e-1/2*a/c*(5/2*a*c*d*e+1/8*b^2*d*e)-1/2*b/c*(5/2*a*b*d*e-3/4
*b/c*(5/2*a*c*d*e+1/8*b^2*d*e)))*ln((2*c*x^2+b)/c^(1/2)+2*(c*x^4+b*x^2+a)^
(1/2))/c^(1/2)-1/2*(a^2*e^2+2*a*b*d^2-3/5*a/c*(11/9*a*c*e^2+b^2*e^2+2*b*c*
d^2-6/7*b/c*(10/9*b*c*e^2+c^2*d^2))-2/3*b/c*(2*a*b*e^2+2*a*c*d^2+b^2*d^2-4
/5*b/c*(11/9*a*c*e^2+b^2*e^2+2*b*c*d^2-6/7*b/c*(10/9*b*c*e^2+c^2*d^2))-5/7
*a/c*(10/9*b*c*e^2+c^2*d^2)))*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*
(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a...
```

Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 834, normalized size of antiderivative = 1.14

$$\int (d+ex)^2 \left(a+bx^2+cx^4\right)^{3/2} dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^2*(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
1/80640*(945*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*sqrt(c)*d*e*x*log(8*c^2*x^
4 + 8*b*c*x^2 + b^2 + 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) + 4*
a*c) - 128*sqrt(1/2)*((18*(b^3*c^2 - 8*a*b*c^3)*d^2 - (8*b^4*c - 57*a*b^2*
c^2 + 84*a^2*c^3)*e^2)*x*sqrt((b^2 - 4*a*c)/c^2) - (18*(b^4*c - 8*a*b^2*c^
2)*d^2 - (8*b^5 - 57*a*b^3*c + 84*a^2*b*c^2)*e^2)*x)*sqrt(c)*sqrt((c*sqrt(
(b^2 - 4*a*c)/c^2) - b)/c)*elliptic_e(arcsin(sqrt(1/2)*sqrt((c*sqrt((b^2 -
 4*a*c)/c^2) - b)/c)/x), 1/2*(b*c*sqrt((b^2 - 4*a*c)/c^2) + b^2 - 2*a*c)/(
a*c)) + 128*sqrt(1/2)*((9*(2*b^3*c^2 + 20*a*c^4 - (16*a*b + b^2)*c^3)*d^2
- (8*b^4*c + 12*(7*a^2 + 2*a*b)*c^3 - (57*a*b^2 + 4*b^3)*c^2)*e^2)*x*sqrt(
(b^2 - 4*a*c)/c^2) - (9*(2*b^4*c - 20*a*b*c^3 - (16*a*b^2 - b^3)*c^2)*d^2
- (8*b^5 + 12*(7*a^2*b - 2*a*b^2)*c^2 - (57*a*b^3 - 4*b^4)*c)*e^2)*x)*sqrt
(c)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)*elliptic_f(arcsin(sqrt(1/2)*sq
rt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)/x), 1/2*(b*c*sqrt((b^2 - 4*a*c)/c^2)
 + b^2 - 2*a*c)/(a*c)) + 4*(2240*c^5*e^2*x^8 + 5040*c^5*d*e*x^7 + 7560*b*c
^4*d*e*x^5 + 320*(9*c^5*d^2 + 10*b*c^4*e^2)*x^6 + 630*(b^2*c^3 + 20*a*c^4)
*d*e*x^3 + 64*(72*b*c^4*d^2 + (3*b^2*c^3 + 77*a*c^4)*e^2)*x^4 - 315*(3*b^3
*c^2 - 20*a*b*c^3)*d*e*x - 1152*(b^3*c^2 - 8*a*b*c^3)*d^2 + 64*(8*b^4*c -
57*a*b^2*c^2 + 84*a^2*c^3)*e^2 + 64*(9*(b^2*c^3 + 15*a*c^4)*d^2 - 4*(b^3*c
^2 - 6*a*b*c^3)*e^2)*x^2)*sqrt(c*x^4 + b*x^2 + a))/(c^4*x)
```

Sympy [F]

$$\int (d+ex)^2 \left(a+bx^2+cx^4\right)^{3/2} \, dx = \int (d+ex)^2 \left(a+bx^2+cx^4\right)^{\frac{3}{2}} \, dx$$

input
```
integrate((e*x+d)**2*(c*x**4+b*x**2+a)**(3/2),x)
```

output
```
Integral((d + e*x)**2*(a + b*x**2 + c*x**4)**(3/2), x)
```

Maxima [F]

$$\int (d+ex)^2 \left(a+bx^2+cx^4\right)^{3/2} dx = \int \left(cx^4+bx^2+a\right)^{\frac{3}{2}}(ex+d)^2 dx$$

input
```
integrate((e*x+d)^2*(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*x^4 + b*x^2 + a)^(3/2)*(e*x + d)^2, x)
```

Giac [F]

$$\int (d+ex)^2 \left(a+bx^2+cx^4\right)^{3/2} dx = \int \left(cx^4+bx^2+a\right)^{\frac{3}{2}}(ex+d)^2 dx$$

input
```
integrate((e*x+d)^2*(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((c*x^4 + b*x^2 + a)^(3/2)*(e*x + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)^2 \left(a+bx^2+cx^4\right)^{3/2} dx = \int (d+e\,x)^2 \left(c\,x^4+b\,x^2+a\right)^{3/2} dx$$

input
```
int((d + e*x)^2*(a + b*x^2 + c*x^4)^(3/2),x)
```

output
```
int((d + e*x)^2*(a + b*x^2 + c*x^4)^(3/2), x)
```

Reduce [F]

$$\int (d+ex)^2 \left(a+bx^2+cx^4\right)^{3/2} dx = \text{too large to display}$$

input
```
int((e*x+d)^2*(c*x^4+b*x^2+a)^(3/2),x)
```

output
```
(12600*sqrt(a + b*x**2 + c*x**4)*a*b*c**2*d*e + 3072*sqrt(a + b*x**2 + c*x
**4)*a*b*c**2*e**2*x + 17280*sqrt(a + b*x**2 + c*x**4)*a*c**3*d**2*x + 252
00*sqrt(a + b*x**2 + c*x**4)*a*c**3*d*e*x**2 + 9856*sqrt(a + b*x**2 + c*x*
*4)*a*c**3*e**2*x**3 - 1890*sqrt(a + b*x**2 + c*x**4)*b**3*c*d*e - 512*sqr
t(a + b*x**2 + c*x**4)*b**3*c*e**2*x + 1152*sqrt(a + b*x**2 + c*x**4)*b**2
*c**2*d**2*x + 1260*sqrt(a + b*x**2 + c*x**4)*b**2*c**2*d*e*x**2 + 384*sqr
t(a + b*x**2 + c*x**4)*b**2*c**2*e**2*x**3 + 9216*sqrt(a + b*x**2 + c*x**4
)*b*c**3*d**2*x**3 + 15120*sqrt(a + b*x**2 + c*x**4)*b*c**3*d*e*x**4 + 640
0*sqrt(a + b*x**2 + c*x**4)*b*c**3*e**2*x**5 + 5760*sqrt(a + b*x**2 + c*x*
*4)*c**4*d**2*x**5 + 10080*sqrt(a + b*x**2 + c*x**4)*c**4*d*e*x**6 + 4480*
sqrt(a + b*x**2 + c*x**4)*c**4*e**2*x**7 - 15120*sqrt(c)*log(sqrt(a + b*x*
*2 + c*x**4) - sqrt(c)*x**2)*a**2*c**2*d*e + 7560*sqrt(c)*log(sqrt(a + b*x
**2 + c*x**4) - sqrt(c)*x**2)*a*b**2*c*d*e - 945*sqrt(c)*log(sqrt(a + b*x*
*2 + c*x**4) - sqrt(c)*x**2)*b**4*d*e + 15120*sqrt(c)*log(sqrt(a + b*x**2
+ c*x**4) + sqrt(c)*x**2)*a**2*c**2*d*e - 7560*sqrt(c)*log(sqrt(a + b*x**2
 + c*x**4) + sqrt(c)*x**2)*a*b**2*c*d*e + 945*sqrt(c)*log(sqrt(a + b*x**2
+ c*x**4) + sqrt(c)*x**2)*b**4*d*e - 3072*int(sqrt(a + b*x**2 + c*x**4)/(a
**2 + 2*a*b*x**2 + a*c*x**4 + b**2*x**4 + b*c*x**6),x)*a**3*b*c**2*e**2 +
23040*int(sqrt(a + b*x**2 + c*x**4)/(a**2 + 2*a*b*x**2 + a*c*x**4 + b**2*x
**4 + b*c*x**6),x)*a**3*c**3*d**2 + 512*int(sqrt(a + b*x**2 + c*x**4)/(...
```

3.256 $\int (d+ex)\left(a+bx^2+cx^4\right)^{3/2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 512

$$\int (d+ex)\left(a+bx^2+cx^4\right)^{3/2}\,dx = -\frac{2b(b^2-8ac)\,dx\sqrt{a+bx^2+cx^4}}{35c^{3/2}\left(\sqrt{a}+\sqrt{c}x^2\right)}$$
$$-\frac{3(b^2-4ac)\,e(b+2cx^2)\sqrt{a+bx^2+cx^4}}{128c^2}$$
$$+\frac{dx(b^2+10ac+3bcx^2)\sqrt{a+bx^2+cx^4}}{35c}+\frac{1}{7}dx\left(a+bx^2+cx^4\right)^{3/2}$$
$$+\frac{e(b+2cx^2)\left(a+bx^2+cx^4\right)^{3/2}}{16c}+\frac{3(b^2-4ac)^2\,e\operatorname{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{256c^{5/2}}$$
$$+\frac{2\sqrt[4]{a}b(b^2-8ac)\,d\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{35c^{7/4}\sqrt{a+bx^2+cx^4}}$$
$$-\frac{\sqrt[4]{a}\left(\sqrt{a}(b^2-20ac)+\frac{2b(b^2-8ac)}{\sqrt{c}}\right)d\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right.\right.}{70c^{5/4}\sqrt{a+bx^2+cx^4}}$$

output
```
-2/35*b*(-8*a*c+b^2)*d*x*(c*x^4+b*x^2+a)^(1/2)/c^(3/2)/(a^(1/2)+c^(1/2)*x^
2)-3/128*(-4*a*c+b^2)*e*(2*c*x^2+b)*(c*x^4+b*x^2+a)^(1/2)/c^2+1/35*d*x*(3*
b*c*x^2+10*a*c+b^2)*(c*x^4+b*x^2+a)^(1/2)/c+1/7*d*x*(c*x^4+b*x^2+a)^(3/2)+
1/16*e*(2*c*x^2+b)*(c*x^4+b*x^2+a)^(3/2)/c+3/256*(-4*a*c+b^2)^2*e*arctanh(
1/2*(2*c*x^2+b)/c^(1/2)/(c*x^4+b*x^2+a)^(1/2))/c^(5/2)+2/35*a^(1/4)*b*(-8*
a*c+b^2)*d*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2)+c^(1/2)*x^2)^2)
^(1/2)*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2)
)^(1/2))/c^(7/4)/(c*x^4+b*x^2+a)^(1/2)-1/70*a^(1/4)*(a^(1/2)*(-20*a*c+b^2)
+2*b*(-8*a*c+b^2)/c^(1/2))*d*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^(1/
2)+c^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(c^(1/4)*x/a^(1/4)),1/2*(
2-b/a^(1/2)/c^(1/2))^(1/2))/c^(5/4)/(c*x^4+b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 12.79 (sec) , antiderivative size = 645, normalized size of antiderivative = 1.26

$$\int (d+ex)\left(a+bx^2+cx^4\right)^{3/2}\,dx=\frac{-128i\sqrt{2}b\sqrt{c}(b^2-8ac)\left(-b+\sqrt{b^2-4ac}\right)d\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}E\left(i\text{arcsinh}\left(\right.\right.}{}$$

input
```
Integrate[(d + e*x)*(a + b*x^2 + c*x^4)^(3/2),x]
```

output
```
((-128*I)*Sqrt[2]*b*Sqrt[c]*(b^2 - 8*a*c)*(-b + Sqrt[b^2 - 4*a*c])*d*Sqrt[
(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[
b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[
2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b
^2 - 4*a*c])] + (128*I)*Sqrt[2]*Sqrt[c]*(-b^4 + 9*a*b^2*c - 20*a^2*c^2 + b
^3*Sqrt[b^2 - 4*a*c] - 8*a*b*c*Sqrt[b^2 - 4*a*c])*d*Sqrt[(b - Sqrt[b^2 - 4
*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*
c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sq
rt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] + S
qrt[c/(b + Sqrt[b^2 - 4*a*c])]*(2*Sqrt[c]*(a + b*x^2 + c*x^4)*(-105*b^3*e
+ 2*b^2*c*x*(64*d + 35*e*x) + 40*c^2*x*(2*c*x^4*(8*d + 7*e*x) + a*(48*d +
35*e*x)) + 4*b*c*(175*a*e + 2*c*x^3*(128*d + 105*e*x))) - 105*(b^2 - 4*a*c
)^2*e*Sqrt[a + b*x^2 + c*x^4]*Log[b + 2*c*x^2 - 2*Sqrt[c]*Sqrt[a + b*x^2 +
 c*x^4]]))/(8960*c^(5/2)*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[a + b*x^2 +
c*x^4])
```

Rubi [A] (verified)

Time = 1.28 (sec) , antiderivative size = 521, normalized size of antiderivative = 1.02, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.636$, Rules used = {2202, 27, 1404, 1432, 1087, 1087, 1092, 219, 1490, 25, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)\left(a+bx^2+cx^4\right)^{3/2}dx$$

$$\Big\downarrow \text{2202}$$

$$\int d\left(cx^4+bx^2+a\right)^{3/2}dx+\int ex\left(cx^4+bx^2+a\right)^{3/2}dx$$

$$\Big\downarrow \text{27}$$

$$d\int \left(cx^4+bx^2+a\right)^{3/2}dx+e\int x\left(cx^4+bx^2+a\right)^{3/2}dx$$

$$\Big\downarrow \text{1404}$$

$$d\left(\frac{3}{7}\int \left(bx^2+2a\right)\sqrt{cx^4+bx^2+a}dx+\frac{1}{7}x\left(a+bx^2+cx^4\right)^{3/2}\right)+e\int x\left(cx^4+bx^2+a\right)^{3/2}dx$$

$$\Big\downarrow \text{1432}$$

$$d\left(\frac{3}{7}\int \left(bx^2+2a\right)\sqrt{cx^4+bx^2+a}dx+\frac{1}{7}x\left(a+bx^2+cx^4\right)^{3/2}\right)+\frac{1}{2}e\int \left(cx^4+bx^2+a\right)^{3/2}dx^2$$

$$\Big\downarrow \text{1087}$$

$$\frac{1}{2}e\left(\frac{\left(b+2cx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\int \sqrt{cx^4+bx^2+a}dx^2}{16c}\right)+ d\left(\frac{3}{7}\int \left(bx^2+2a\right)\sqrt{cx^4+bx^2+a}dx+\frac{1}{7}x\left(a+bx^2+cx^4\right)^{3/2}\right)$$

$$\Big\downarrow \text{1087}$$

$$\frac{1}{2}e\left(\frac{\left(b+2cx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\left(\frac{\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{4c}-\frac{\left(b^2-4ac\right)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx^2}{8c}\right)}{16c}\right)+ d\left(\frac{3}{7}\int \left(bx^2+2a\right)\sqrt{cx^4+bx^2+a}dx+\frac{1}{7}x\left(a+bx^2+cx^4\right)^{3/2}\right)$$

$$\Big\downarrow \text{1092}$$

$$\frac{1}{2}e\left(\frac{\left(b+2cx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\left(\frac{\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{4c}-\frac{\left(b^2-4ac\right)\int \frac{1}{4c-x^4}d\frac{2cx^2+b}{\sqrt{cx^4+bx^2+a}}}{4c}\right)}{16c}\right)+ d\left(\frac{3}{7}\int \left(bx^2+2a\right)\sqrt{cx^4+bx^2+a}dx+\frac{1}{7}x\left(a+bx^2+cx^4\right)^{3/2}\right)$$

$$\Big\downarrow \text{219}$$

$$d\left(\frac{3}{7}\int \left(bx^2+2a\right)\sqrt{cx^4+bx^2+a}dx+\frac{1}{7}x\left(a+bx^2+cx^4\right)^{3/2}\right)+ \frac{1}{2}e\left(\frac{\left(b+2cx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\left(\frac{\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{4c}-\frac{\left(b^2-4ac\right)\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{8c^{3/2}}\right)}{16c}\right)$$

$$\Big\downarrow \text{1490}$$

$$d\left(\frac{3}{7}\left(\frac{\int -\frac{2b(b^2-8ac)x^2+a(b^2-20ac)}{\sqrt{cx^4+bx^2+a}}dx}{15c}+\frac{x\sqrt{a+bx^2+cx^4}\left(10ac+b^2+3bcx^2\right)}{15c}\right)+\frac{1}{7}x\left(a+bx^2+cx^4\right)^{3/2}\right)+\frac{1}{2}e\left(\frac{\left(b+2cx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\left(\frac{(b+2cx^2)\sqrt{a+bx^2+cx^4}}{4c}-\frac{(b^2-4ac)\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{8c^{3/2}}\right)}{16c}\right)$$

$\downarrow$ 25

$$d\left(\frac{3}{7}\left(\frac{x\left(10ac+b^2+3bcx^2\right)\sqrt{a+bx^2+cx^4}}{15c}-\frac{\int \frac{2b(b^2-8ac)x^2+a(b^2-20ac)}{\sqrt{cx^4+bx^2+a}}dx}{15c}\right)+\frac{1}{7}x\left(a+bx^2+cx^4\right)^{3/2}\right)+\frac{1}{2}e\left(\frac{\left(b+2cx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\left(\frac{(b+2cx^2)\sqrt{a+bx^2+cx^4}}{4c}-\frac{(b^2-4ac)\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{8c^{3/2}}\right)}{16c}\right)$$

$\downarrow$ 1511

$$d\left(\frac{3}{7}\left(\frac{x\left(10ac+b^2+3bcx^2\right)\sqrt{a+bx^2+cx^4}}{15c}-\frac{\sqrt{a}\left(\sqrt{a}(b^2-20ac)+\frac{2b(b^2-8ac)}{\sqrt{c}}\right)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx-\frac{2\sqrt{a}b(b^2-8ac)}{}}{15c}\right.\right.$$
$$\frac{1}{2}e\left(\frac{\left(b+2cx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\left(\frac{(b+2cx^2)\sqrt{a+bx^2+cx^4}}{4c}-\frac{(b^2-4ac)\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{8c^{3/2}}\right)}{16c}\right)$$

$\downarrow$ 27

$$d\left(\frac{3}{7}\left(\frac{x\left(10ac+b^2+3bcx^2\right)\sqrt{a+bx^2+cx^4}}{15c}-\frac{\sqrt{a}\left(\sqrt{a}\left(b^2-20ac\right)+\frac{2b\left(b^2-8ac\right)}{\sqrt{c}}\right)\int\frac{1}{\sqrt{cx^4+bx^2+a}}dx-\frac{2b\left(b^2-8ac\right)\int}{\sqrt{}}}{15c}\right.\right.$$

$$\frac{1}{2}e\left(\frac{\left(b+2cx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\left(\frac{\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{4c}-\frac{\left(b^2-4ac\right)\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{8c^{3/2}}\right)}{16c}\right)$$

$\downarrow$ 1416

$$d\left(\frac{3}{7}\left(\frac{x\left(10ac+b^2+3bcx^2\right)\sqrt{a+bx^2+cx^4}}{15c}-\frac{\frac{\sqrt[4]{a}\left(\sqrt{a}\left(b^2-20ac\right)+\frac{2b\left(b^2-8ac\right)}{\sqrt{c}}\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\,\text{arcta}\right.}{2\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}}{15c}\right.\right.$$

$$\frac{1}{2}e\left(\frac{\left(b+2cx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\left(\frac{\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{4c}-\frac{\left(b^2-4ac\right)\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{8c^{3/2}}\right)}{16c}\right)$$

$\downarrow$ 1509

$$d\left(\frac{3}{7}\left(\frac{x\left(10ac+b^2+3bcx^2\right)\sqrt{a+bx^2+cx^4}}{15c}-\frac{\sqrt[4]{a}\left(\sqrt{a}\left(b^2-20ac\right)+\frac{2b\left(b^2-8ac\right)}{\sqrt{c}}\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\,\text{arcta}\right.}{2\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}\right.\right.$$

$$\frac{1}{2}e\left(\frac{\left(b+2cx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\left(\frac{\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{4c}-\frac{\left(b^2-4ac\right)\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{8c^{3/2}}\right)}{16c}\right)$$

input
```
Int[(d + e*x)*(a + b*x^2 + c*x^4)^(3/2),x]
```

output
```
(e*(((b + 2*c*x^2)*(a + b*x^2 + c*x^4)^(3/2))/(8*c) - (3*(b^2 - 4*a*c)*(((
b + 2*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(4*c) - ((b^2 - 4*a*c)*ArcTanh[(b +
2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(8*c^(3/2))))/(16*c)))/2 +
d*((x*(a + b*x^2 + c*x^4)^(3/2))/7 + (3*((x*(b^2 + 10*a*c + 3*b*c*x^2)*Sqr
t[a + b*x^2 + c*x^4])/(15*c) - ((-2*b*(b^2 - 8*a*c)*(-((x*Sqrt[a + b*x^2 +
 c*x^4])/(Sqrt[a] + Sqrt[c]*x^2)) + (a^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[
(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)
*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(c^(1/4)*Sqrt[a + b*x^2 + c*x^
4])))/Sqrt[c] + (a^(1/4)*(Sqrt[a]*(b^2 - 20*a*c) + (2*b*(b^2 - 8*a*c))/Sqr
t[c])*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*
x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))
/4])/(2*c^(1/4)*Sqrt[a + b*x^2 + c*x^4]))/(15*c)))/7)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1404
```
Int[((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*((a + b
*x^2 + c*x^4)^p/(4*p + 1)), x] + Simp[2*(p/(4*p + 1))   Int[(2*a + b*x^2)*(
a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*
c, 0] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1432
```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

rule 1490
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Simp[2*(p/(c*(4*p + 1)*(4*p + 3)))
 Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3)
- b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 2202
```
Int[(Pn_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{n
 = Expon[Pn, x], k}, Int[Sum[Coeff[Pn, x, 2*k]*x^(2*k), {k, 0, n/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pn, x, 2*k + 1]*x^(2*k), {k, 0, (n -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pn, x]
 &&  !PolyQ[Pn, x^2]
```

Maple [A] (verified)

Time = 2.12 (sec) , antiderivative size = 690, normalized size of antiderivative = 1.35

method	result
elliptic	$\frac{ce\,x^6\sqrt{c\,x^4+b\,x^2+a}}{8}+\frac{cd\,x^5\sqrt{c\,x^4+b\,x^2+a}}{7}+\frac{3eb\,x^4\sqrt{c\,x^4+b\,x^2+a}}{16}+\frac{8bd\,x^3\sqrt{c\,x^4+b\,x^2+a}}{35}+\frac{\left(\frac{5}{4}ace+\frac{1}{16}b^2e\right)x^2\sqrt{c\,x^4+b\,x^2+}}{4c}$
risch	$\frac{\left(560e\,x^6c^3+640d\,x^5c^3+840be\,x^4c^2+1024bd\,x^3c^2+1400a\,c^2e\,x^2+70b^2ce\,x^2+1920a\,c^2dx+128b^2cdx+700abce-105b^3e\right)\sqrt{c\,x^4+b\,x^2}}{4480c^2}$
default	$d\left(\frac{c\,x^5\sqrt{c\,x^4+b\,x^2+a}}{7}+\frac{8b\,x^3\sqrt{c\,x^4+b\,x^2+a}}{35}+\frac{\left(\frac{9ac}{7}+\frac{3b^2}{35}\right)x\sqrt{c\,x^4+b\,x^2+a}}{3c}+\frac{\left(a^2-\frac{\left(\frac{9ac}{7}+\frac{3b^2}{35}\right)a}{3c}\right)\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+}\right.}{a}}}{}\right.$

input
```
int((e*x+d)*(c*x^4+b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/8*c*e*x^6*(c*x^4+b*x^2+a)^(1/2)+1/7*c*d*x^5*(c*x^4+b*x^2+a)^(1/2)+3/16*e
*b*x^4*(c*x^4+b*x^2+a)^(1/2)+8/35*b*d*x^3*(c*x^4+b*x^2+a)^(1/2)+1/4*(5/4*a
*c*e+1/16*b^2*e)/c*x^2*(c*x^4+b*x^2+a)^(1/2)+1/3*(9/7*a*c*d+3/35*d*b^2)/c*
x*(c*x^4+b*x^2+a)^(1/2)+1/2*(5/4*a*b*e-3/4*b/c*(5/4*a*c*e+1/16*b^2*e))/c*(
c*x^4+b*x^2+a)^(1/2)+1/4*(a^2*d-1/3*a/c*(9/7*a*c*d+3/35*d*b^2))*2^(1/2)/((
-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*
(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1
/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)
^(1/2))/a/c)^(1/2))+1/2*(a^2*e-1/2*a/c*(5/4*a*c*e+1/16*b^2*e)-1/2*b/c*(5/4
*a*b*e-3/4*b/c*(5/4*a*c*e+1/16*b^2*e)))*ln((2*c*x^2+b)/c^(1/2)+2*(c*x^4+b*
x^2+a)^(1/2))/c^(1/2)-1/2*(46/35*a*b*d-2/3*b/c*(9/7*a*c*d+3/35*d*b^2))*a*2
^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^
2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b
+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^
(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/
2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/
c)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 541, normalized size of antiderivative = 1.06

$$\int (d+ex)\left(a+bx^2+cx^4\right)^{3/2}\,dx = \frac{105\left(b^4-8\,ab^2c+16\,a^2c^2\right)\sqrt{c}ex\log\left(8\,c^2x^4+8\,bcx^2+b^2+4\,\sqrt{cx^4+bx^2+a}(2\,cx^2+b)\sqrt{c}\right.}{}$$

input
```
integrate((e*x+d)*(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
1/17920*(105*(b^4 - 8*a*b^2*c + 16*a^2*c^2)*sqrt(c)*e*x*log(8*c^2*x^4 + 8*
b*c*x^2 + b^2 + 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) + 4*a*c) -
 512*sqrt(1/2)*((b^3*c - 8*a*b*c^2)*d*x*sqrt((b^2 - 4*a*c)/c^2) - (b^4 - 8
*a*b^2*c)*d*x)*sqrt(c)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)*elliptic_e(
arcsin(sqrt(1/2)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)/x), 1/2*(b*c*sqrt
((b^2 - 4*a*c)/c^2) + b^2 - 2*a*c)/(a*c)) + 256*sqrt(1/2)*((2*b^3*c + 20*a
*c^3 - (16*a*b + b^2)*c^2)*d*x*sqrt((b^2 - 4*a*c)/c^2) - (2*b^4 - 20*a*b*c
^2 - (16*a*b^2 - b^3)*c)*d*x)*sqrt(c)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)
/c)*elliptic_f(arcsin(sqrt(1/2)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)/x)
, 1/2*(b*c*sqrt((b^2 - 4*a*c)/c^2) + b^2 - 2*a*c)/(a*c)) + 4*(560*c^4*e*x^
7 + 640*c^4*d*x^6 + 840*b*c^3*e*x^5 + 1024*b*c^3*d*x^4 + 70*(b^2*c^2 + 20*
a*c^3)*e*x^3 + 128*(b^2*c^2 + 15*a*c^3)*d*x^2 - 35*(3*b^3*c - 20*a*b*c^2)*
e*x - 256*(b^3*c - 8*a*b*c^2)*d)*sqrt(c*x^4 + b*x^2 + a))/(c^3*x)
```

Sympy [F]

$$\int (d+ex) \left(a+bx^2+cx^4\right)^{3/2} \, dx = \int (d+ex) \left(a+bx^2+cx^4\right)^{\frac{3}{2}} \, dx$$

input
```
integrate((e*x+d)*(c*x**4+b*x**2+a)**(3/2),x)
```

output
```
Integral((d + e*x)*(a + b*x**2 + c*x**4)**(3/2), x)
```

Maxima [F]

$$\int (d+ex) \left(a+bx^2+cx^4\right)^{3/2} \, dx = \int \left(cx^4+bx^2+a\right)^{\frac{3}{2}}(ex+d) \, dx$$

input
```
integrate((e*x+d)*(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*x^4 + b*x^2 + a)^(3/2)*(e*x + d), x)
```

Giac [F]

$$\int (d+ex)\left(a+bx^2+cx^4\right)^{3/2}\,dx=\int \left(cx^4+bx^2+a\right)^{\frac{3}{2}}(ex+d)\,dx$$

input
```
integrate((e*x+d)*(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((c*x^4 + b*x^2 + a)^(3/2)*(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)\left(a+bx^2+cx^4\right)^{3/2}\,dx=\int (d+e\,x)\,\left(c\,x^4+b\,x^2+a\right)^{3/2}\,dx$$

input
```
int((d + e*x)*(a + b*x^2 + c*x^4)^(3/2),x)
```

output
```
int((d + e*x)*(a + b*x^2 + c*x^4)^(3/2), x)
```

Reduce [F]

$$\int (d+ex)\left(a+bx^2+cx^4\right)^{3/2}\,dx=\text{Too large to display}$$

input
```
int((e*x+d)*(c*x^4+b*x^2+a)^(3/2),x)
```

output

```
(1400*sqrt(a + b*x**2 + c*x**4)*a*b*c**2*e + 3840*sqrt(a + b*x**2 + c*x**4
)*a*c**3*d*x + 2800*sqrt(a + b*x**2 + c*x**4)*a*c**3*e*x**2 - 210*sqrt(a +
 b*x**2 + c*x**4)*b**3*c*e + 256*sqrt(a + b*x**2 + c*x**4)*b**2*c**2*d*x +
 140*sqrt(a + b*x**2 + c*x**4)*b**2*c**2*e*x**2 + 2048*sqrt(a + b*x**2 + c
*x**4)*b*c**3*d*x**3 + 1680*sqrt(a + b*x**2 + c*x**4)*b*c**3*e*x**4 + 1280
*sqrt(a + b*x**2 + c*x**4)*c**4*d*x**5 + 1120*sqrt(a + b*x**2 + c*x**4)*c*
*4*e*x**6 - 1680*sqrt(c)*log(sqrt(a + b*x**2 + c*x**4) - sqrt(c)*x**2)*a**
2*c**2*e + 840*sqrt(c)*log(sqrt(a + b*x**2 + c*x**4) - sqrt(c)*x**2)*a*b**
2*c*e - 105*sqrt(c)*log(sqrt(a + b*x**2 + c*x**4) - sqrt(c)*x**2)*b**4*e +
 1680*sqrt(c)*log(sqrt(a + b*x**2 + c*x**4) + sqrt(c)*x**2)*a**2*c**2*e -
840*sqrt(c)*log(sqrt(a + b*x**2 + c*x**4) + sqrt(c)*x**2)*a*b**2*c*e + 105
*sqrt(c)*log(sqrt(a + b*x**2 + c*x**4) + sqrt(c)*x**2)*b**4*e + 5120*int(s
qrt(a + b*x**2 + c*x**4)/(a**2 + 2*a*b*x**2 + a*c*x**4 + b**2*x**4 + b*c*x
**6),x)*a**3*c**3*d - 256*int(sqrt(a + b*x**2 + c*x**4)/(a**2 + 2*a*b*x**2
 + a*c*x**4 + b**2*x**4 + b*c*x**6),x)*a**2*b**2*c**2*d + 4096*int((sqrt(a
 + b*x**2 + c*x**4)*x**4)/(a**2 + 2*a*b*x**2 + a*c*x**4 + b**2*x**4 + b*c*
x**6),x)*a*b**2*c**3*d - 512*int((sqrt(a + b*x**2 + c*x**4)*x**4)/(a**2 +
2*a*b*x**2 + a*c*x**4 + b**2*x**4 + b*c*x**6),x)*b**4*c**2*d + 9216*int((s
qrt(a + b*x**2 + c*x**4)*x**2)/(a**2 + 2*a*b*x**2 + a*c*x**4 + b**2*x**4 +
 b*c*x**6),x)*a**2*b*c**3*d - 768*int((sqrt(a + b*x**2 + c*x**4)*x**2)/...
```

3.257 $\int (a + bx^2 + cx^4)^{3/2} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 381

$$\int (a+bx^2+cx^4)^{3/2}\,dx = -\frac{2b(b^2-8ac)\,x\sqrt{a+bx^2+cx^4}}{35c^{3/2}\left(\sqrt{a}+\sqrt{c}x^2\right)} + \frac{x(b^2+10ac+3bcx^2)\sqrt{a+bx^2+cx^4}}{35c} + \frac{1}{7}x\left(a+bx^2+cx^4\right)^{3/2} + \frac{2\sqrt[4]{a}b(b^2-8ac)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{35c^{7/4}\sqrt{a+bx^2+cx^4}} - \frac{\sqrt[4]{a}\left(\sqrt{a}(b^2-20ac)+\frac{2b(b^2-8ac)}{\sqrt{c}}\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{70c^{5/4}\sqrt{a+bx^2+cx^4}}$$

output

```
-2/35*b*(-8*a*c+b^2)*x*(c*x^4+b*x^2+a)^(1/2)/c^(3/2)/(a^(1/2)+c^(1/2)*x^2)
+1/35*x*(3*b*c*x^2+10*a*c+b^2)*(c*x^4+b*x^2+a)^(1/2)/c+1/7*x*(c*x^4+b*x^2+
a)^(3/2)+2/35*a^(1/4)*b*(-8*a*c+b^2)*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+b*x^2+a
)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))
),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))/c^(7/4)/(c*x^4+b*x^2+a)^(1/2)-1/70*a^(1
/4)*(a^(1/2)*(-20*a*c+b^2)+2*b*(-8*a*c+b^2)/c^(1/2))*(a^(1/2)+c^(1/2)*x^2)
*((c*x^4+b*x^2+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(
c^(1/4)*x/a^(1/4)),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))/c^(5/4)/(c*x^4+b*x^2+a
)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.76 (sec) , antiderivative size = 533, normalized size of antiderivative = 1.40

$$\int \left(a+bx^2+cx^4\right)^{3/2}\,dx = \frac{2c\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x(15a^2c+a(b^2+23bcx^2+20c^2x^4)+x^2(b^3+9b^2cx^2+13bc^2x^4+5c^3x^6))-ib}{}$$

input
```
Integrate[(a + b*x^2 + c*x^4)^(3/2),x]
```

output
```
(2*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x*(15*a^2*c + a*(b^2 + 23*b*c*x^2 + 2
0*c^2*x^4) + x^2*(b^3 + 9*b^2*c*x^2 + 13*b*c^2*x^4 + 5*c^3*x^6)) - I*b*(b^
2 - 8*a*c)*(-b + Sqrt[b^2 - 4*a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)
/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b -
Sqrt[b^2 - 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a
*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] + I*(-b^4 + 9*a
*b^2*c - 20*a^2*c^2 + b^3*Sqrt[b^2 - 4*a*c] - 8*a*b*c*Sqrt[b^2 - 4*a*c])*S
qrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b -
 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSi
nh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b
- Sqrt[b^2 - 4*a*c])])/(70*c^2*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[a + b*
x^2 + c*x^4])
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 383, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {1404, 1490, 25, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2+cx^4\right)^{3/2} dx$$

$$\Big\downarrow \; 1404$$

$$\frac{3}{7}\int \left(bx^2+2a\right)\sqrt{cx^4+bx^2+a}dx+\frac{1}{7}x\left(a+bx^2+cx^4\right)^{3/2}$$

$$\Big\downarrow \; 1490$$

$$\frac{3}{7}\left(\frac{\int -\frac{2b(b^2-8ac)x^2+a(b^2-20ac)}{\sqrt{cx^4+bx^2+a}}dx}{15c}+\frac{x\sqrt{a+bx^2+cx^4}\left(10ac+b^2+3bcx^2\right)}{15c}\right)+\frac{1}{7}x\left(a+bx^2+cx^4\right)^{3/2}$$

$$\Big\downarrow \; 25$$

$$\frac{3}{7}\left(\frac{x\left(10ac+b^2+3bcx^2\right)\sqrt{a+bx^2+cx^4}}{15c}-\frac{\int \frac{2b(b^2-8ac)x^2+a(b^2-20ac)}{\sqrt{cx^4+bx^2+a}}dx}{15c}\right)+\frac{1}{7}x\left(a+bx^2+cx^4\right)^{3/2}$$

$$\Big\downarrow \; 1511$$

$$\frac{3}{7}\left(\frac{x\left(10ac+b^2+3bcx^2\right)\sqrt{a+bx^2+cx^4}}{15c}-\frac{\sqrt{a}\left(\sqrt{a}\left(b^2-20ac\right)+\frac{2b(b^2-8ac)}{\sqrt{c}}\right)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx-\frac{2\sqrt{a}b(b^2-8ac)\int \cdots}{\sqrt{\cdots}}}{15c}\cdots\right.$$

$$\frac{1}{7}x\left(a+bx^2+cx^4\right)^{3/2}$$

$$\Big\downarrow \; 27$$

$$\frac{3}{7}\left(\frac{x\left(10ac+b^2+3bcx^2\right)\sqrt{a+bx^2+cx^4}}{15c}-\frac{\sqrt{a}\left(\sqrt{a}\left(b^2-20ac\right)+\frac{2b(b^2-8ac)}{\sqrt{c}}\right)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx-\frac{2b(b^2-8ac)\int \frac{\sqrt{c}\cdots}{\sqrt{cx\cdots}}}{\sqrt{c}}}{15c}\cdots\right.$$

$$\frac{1}{7}x\left(a+bx^2+cx^4\right)^{3/2}$$

$$\Big\downarrow \; 1416$$

$$\frac{3}{7}\left(\frac{x\left(10ac+b^2+3bcx^2\right)\sqrt{a+bx^2+cx^4}}{15c}-\frac{\frac{\sqrt[4]{a}\left(\sqrt{a}(b^2-20ac)+\frac{2b\left(b^2-8ac\right)}{\sqrt{c}}\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\cdots\right.\right.}{2\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}}{15c}\right.$$

$$\frac{1}{7}x\left(a+bx^2+cx^4\right)^{3/2}$$

$$\Big\downarrow \text{1509}$$

$$\frac{3}{7}\left(\frac{x\left(10ac+b^2+3bcx^2\right)\sqrt{a+bx^2+cx^4}}{15c}-\frac{\sqrt[4]{a}\left(\sqrt{a}(b^2-20ac)+\frac{2b\left(b^2-8ac\right)}{\sqrt{c}}\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\cdots\right.\right.}{2\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}\right.$$

$$\frac{1}{7}x\left(a+bx^2+cx^4\right)^{3/2}$$

input

```
Int[(a + b*x^2 + c*x^4)^(3/2),x]
```

output

```
(x*(a + b*x^2 + c*x^4)^(3/2))/7 + (3*((x*(b^2 + 10*a*c + 3*b*c*x^2)*Sqrt[a
 + b*x^2 + c*x^4])/(15*c) - ((-2*b*(b^2 - 8*a*c)*(-((x*Sqrt[a + b*x^2 + c*
x^4])/(Sqrt[a] + Sqrt[c]*x^2)) + (a^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a
+ b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)
/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(c^(1/4)*Sqrt[a + b*x^2 + c*x^4])
))/Sqrt[c] + (a^(1/4)*(Sqrt[a]*(b^2 - 20*a*c) + (2*b*(b^2 - 8*a*c))/Sqrt[c
])*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2
)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4]
)/(2*c^(1/4)*Sqrt[a + b*x^2 + c*x^4]))/(15*c)))/7
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1404
```
Int[((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*((a + b
*x^2 + c*x^4)^p/(4*p + 1)), x] + Simp[2*(p/(4*p + 1))   Int[(2*a + b*x^2)*(
a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*
c, 0] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1490
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Simp[2*(p/(c*(4*p + 1)*(4*p + 3)))
 Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3)
- b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

Maple [A] (verified)

Time = 1.65 (sec) , antiderivative size = 471, normalized size of antiderivative = 1.24

method	result
default	$\frac{c\,x^5\sqrt{c\,x^4+b\,x^2+a}}{7}+\frac{8b\,x^3\sqrt{c\,x^4+b\,x^2+a}}{35}+\frac{\left(\frac{9ac}{7}+\frac{3b^2}{35}\right)x\sqrt{c\,x^4+b\,x^2+a}}{3c}+\frac{\left(a^2-\frac{\left(\frac{9ac}{7}+\frac{3b^2}{35}\right)a}{3c}\right)\sqrt{2}\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x}{a}}\cdots}{\cdots}$
elliptic	$\frac{c\,x^5\sqrt{c\,x^4+b\,x^2+a}}{7}+\frac{8b\,x^3\sqrt{c\,x^4+b\,x^2+a}}{35}+\frac{\left(\frac{9ac}{7}+\frac{3b^2}{35}\right)x\sqrt{c\,x^4+b\,x^2+a}}{3c}+\frac{\left(a^2-\frac{\left(\frac{9ac}{7}+\frac{3b^2}{35}\right)a}{3c}\right)\sqrt{2}\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x}{a}}\cdots}{\cdots}$
risch	$\frac{x\left(5c^2x^4+8bc\,x^2+15ac+b^2\right)\sqrt{c\,x^4+b\,x^2+a}}{35c}+\frac{-\frac{b\left(8ac-b^2\right)a\sqrt{2}\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\left(\mathrm{EllipticF}\left(\frac{x\sqrt{2}\sqrt{\cdots}}{\cdots}\cdots\right.\right.}{\sqrt{\frac{-b+\sqrt{-4a\cdots}}{a}}\cdots}}{\cdots}$

input
```
int((c*x^4+b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/7*c*x^5*(c*x^4+b*x^2+a)^(1/2)+8/35*b*x^3*(c*x^4+b*x^2+a)^(1/2)+1/3*(9/7*
a*c+3/35*b^2)/c*x*(c*x^4+b*x^2+a)^(1/2)+1/4*(a^2-1/3*(9/7*a*c+3/35*b^2)/c*
a)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/
a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2
)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b
+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/2*(46/35*a*b-2/3*(9/7*a*c+3/35*b^2)/c*b
)*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))
/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/
2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2)
)/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*
2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2
))/a/c)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 396, normalized size of antiderivative = 1.04

$$\int \left(a+bx^2+cx^4\right)^{3/2}\,dx =$$

$$-\frac{2\sqrt{\frac{1}{2}}\left((b^3c-8\,abc^2)x\sqrt{\frac{b^2-4\,ac}{c^2}}-(b^4-8\,ab^2c)x\right)\sqrt{c}\sqrt{\frac{c\sqrt{\frac{b^2-4\,ac}{c^2}}-b}{c}}E(\arcsin\left(\frac{\sqrt{\frac{1}{2}}\sqrt{\frac{c\sqrt{\frac{b^2-4\,ac}{c^2}}-b}{c}}}{x}\right)\mid\frac{bc\sqrt{\frac{b^2-4\,ac}{c^2}}}{2\,c}}{}$$

input
```
integrate((c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
-1/70*(2*sqrt(1/2)*((b^3*c - 8*a*b*c^2)*x*sqrt((b^2 - 4*a*c)/c^2) - (b^4 -
 8*a*b^2*c)*x)*sqrt(c)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)*elliptic_e(
arcsin(sqrt(1/2)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)/x), 1/2*(b*c*sqrt
((b^2 - 4*a*c)/c^2) + b^2 - 2*a*c)/(a*c)) - sqrt(1/2)*((2*b^3*c + 20*a*c^3
 - (16*a*b + b^2)*c^2)*x*sqrt((b^2 - 4*a*c)/c^2) - (2*b^4 - 20*a*b*c^2 - (
16*a*b^2 - b^3)*c)*x)*sqrt(c)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)*elli
ptic_f(arcsin(sqrt(1/2)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)/x), 1/2*(b
*c*sqrt((b^2 - 4*a*c)/c^2) + b^2 - 2*a*c)/(a*c)) - 2*(5*c^4*x^6 + 8*b*c^3*
x^4 - 2*b^3*c + 16*a*b*c^2 + (b^2*c^2 + 15*a*c^3)*x^2)*sqrt(c*x^4 + b*x^2
+ a))/(c^3*x)
```

Sympy [F]

$$\int \left(a+bx^2+cx^4\right)^{3/2}\,dx=\int \left(a+bx^2+cx^4\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((c*x**4+b*x**2+a)**(3/2),x)
```

output
```
Integral((a + b*x**2 + c*x**4)**(3/2), x)
```

Maxima [F]

$$\int \left(a+bx^2+cx^4\right)^{3/2}\,dx=\int \left(cx^4+bx^2+a\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*x^4 + b*x^2 + a)^(3/2), x)
```

Giac [F]

$$\int \left(a+bx^2+cx^4\right)^{3/2}\,dx=\int \left(cx^4+bx^2+a\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((c*x^4 + b*x^2 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+bx^2+cx^4\right)^{3/2} dx = \int \left(c\,x^4+b\,x^2+a\right)^{3/2} dx$$

input
```
int((a + b*x^2 + c*x^4)^(3/2),x)
```

output
```
int((a + b*x^2 + c*x^4)^(3/2), x)
```

Reduce [F]

$$\int \left(a+bx^2+cx^4\right)^{3/2} dx = \frac{15\sqrt{c\,x^4+b\,x^2+a}\,acx+\sqrt{c\,x^4+b\,x^2+a}\,b^2x+8\sqrt{c\,x^4+b\,x^2+a}\,bc\,x^3+5\sqrt{c\,x^4+b\,x^2+a}$$

input
```
int((c*x^4+b*x^2+a)^(3/2),x)
```

output
```
(15*sqrt(a + b*x**2 + c*x**4)*a*c*x + sqrt(a + b*x**2 + c*x**4)*b**2*x + 8
*sqrt(a + b*x**2 + c*x**4)*b*c*x**3 + 5*sqrt(a + b*x**2 + c*x**4)*c**2*x**
5 + 20*int(sqrt(a + b*x**2 + c*x**4)/(a + b*x**2 + c*x**4),x)*a**2*c - int
(sqrt(a + b*x**2 + c*x**4)/(a + b*x**2 + c*x**4),x)*a*b**2 + 16*int((sqrt(
a + b*x**2 + c*x**4)*x**2)/(a + b*x**2 + c*x**4),x)*a*b*c - 2*int((sqrt(a
 + b*x**2 + c*x**4)*x**2)/(a + b*x**2 + c*x**4),x)*b**3)/(35*c)
```

3.258 $\int \frac{\left(a+bx^2+cx^4\right)^{3/2}}{d+ex} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 1150

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{d+ex} \, dx = \text{Too large to display}$$

output
```
-1/15*d*(6*b*e^2+5*c*d^2)*x*(c*x^4+b*x^2+a)^(1/2)/e^4-1/5*c*d*x^3*(c*x^4+b
*x^2+a)^(1/2)/e^2-1/15*d*(21*a*c*e^4+3*b^2*e^4+20*b*c*d^2*e^2+15*c^2*d^4)*
x*(c*x^4+b*x^2+a)^(1/2)/c^(1/2)/e^6/(a^(1/2)+c^(1/2)*x^2)+1/16*(8*c^2*d^4+
10*b*c*d^2*e^2+b^2*e^4+8*a*c*e^4+2*c*e^2*(b*e^2+2*c*d^2)*x^2)*(c*x^4+b*x^2
+a)^(1/2)/c/e^5+1/6*(c*x^4+b*x^2+a)^(3/2)/e+1/2*(a*e^4+b*d^2*e^2+c*d^4)^(3
/2)*arctanh((a*e^4+b*d^2*e^2+c*d^4)^(1/2)*x/d/e/(c*x^4+b*x^2+a)^(1/2))/e^7
+1/32*(b*e^2+2*c*d^2)*(12*a*c*e^4-b^2*e^4+8*b*c*d^2*e^2+8*c^2*d^4)*arctanh
(1/2*(2*c*x^2+b)/c^(1/2)/(c*x^4+b*x^2+a)^(1/2))/c^(3/2)/e^7-1/2*(a*e^4+b*d
^2*e^2+c*d^4)^(3/2)*arctanh(1/2*(b*d^2+2*a*e^2+(b*e^2+2*c*d^2)*x^2)/(a*e^4
+b*d^2*e^2+c*d^4)^(1/2)/(c*x^4+b*x^2+a)^(1/2))/e^7+1/15*a^(1/4)*d*(21*a*c*
e^4+3*b^2*e^4+20*b*c*d^2*e^2+15*c^2*d^4)*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+b*x
^2+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1
/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))/c^(3/4)/e^6/(c*x^4+b*x^2+a)^(1/2)-1
/30*a^(1/4)*d*(30*c^(5/2)*d^6+10*a^(1/2)*c^2*d^4*e^2+3*a^(1/2)*b^2*e^6+2*a
^(1/2)*c*e^4*(3*a*e^2+7*b*d^2)+6*b*c^(1/2)*e^4*(4*a*e^2+3*b*d^2)+c^(3/2)*(
46*a*d^2*e^4+50*b*d^4*e^2))*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2
)+c^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(c^(1/4)*x/a^(1/4)),1/2*(2
-b/a^(1/2)/c^(1/2))^(1/2))/c^(3/4)/e^6/(c^(1/2)*d^2+a^(1/2)*e^2)/(c*x^4+b*
x^2+a)^(1/2)-1/4*(c^(1/2)*d^2-a^(1/2)*e^2)*(a*e^4+b*d^2*e^2+c*d^4)^2*(a^(1
/2)+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*Ellipt...
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 23.86 (sec) , antiderivative size = 12989, normalized size of antiderivative = 11.29

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{d+ex}\,dx = \text{Result too large to show}$$

input
```
Integrate[(a + b*x^2 + c*x^4)^(3/2)/(d + e*x),x]
```

output
```
Result too large to show
```

Rubi [A] (warning: unable to verify)

Time = 5.91 (sec) , antiderivative size = 1131, normalized size of antiderivative = 0.98, number of steps used = 23, number of rules used = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.917$, Rules used = {2266, 1529, 27, 1576, 1162, 1231, 27, 1269, 1092, 219, 1154, 219, 1786, 27, 414, 2207, 2207, 27, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2+cx^4\right)^{3/2}}{d+ex}\,dx$$

$$\Big\downarrow \text{2266}$$

$$d\int \frac{\left(cx^4+bx^2+a\right)^{3/2}}{d^2-e^2x^2}dx-e\int \frac{x\left(cx^4+bx^2+a\right)^{3/2}}{d^2-e^2x^2}dx$$

$$\Big\downarrow \text{1529}$$

$$d\left(\frac{\left(e^2\left(\sqrt{b^2-4ac}+b\right)+2cd^2\right)\left(ae^4+bd^2e^2+cd^4\right)\int \frac{2cx^2+b-\sqrt{b^2-4ac}}{(d^2-e^2x^2)\sqrt{cx^4+bx^2+a}}dx}{4ce^6}-\frac{\int \frac{2\left(2c^3e^4x^6+2c^2e^2\left(cd^2+2be^2\right)x^4+2c(\right.}{}}{} \right.$$

$$e\int \frac{x\left(cx^4+bx^2+a\right)^{3/2}}{d^2-e^2x^2}dx$$

$$\Big\downarrow \text{27}$$

$$d\left(\frac{\left(e^2\left(\sqrt{b^2-4ac}+b\right)+2cd^2\right)\left(ae^4+bd^2e^2+cd^4\right)\int \frac{2cx^2+b-\sqrt{b^2-4ac}}{(d^2-e^2x^2)\sqrt{cx^4+bx^2+a}}dx}{4ce^6}-\frac{\int \frac{2c^3e^4x^6+2c^2e^2\left(cd^2+2be^2\right)x^4+2c(c^2}{}}{} \right.$$

$$e\int \frac{x\left(cx^4+bx^2+a\right)^{3/2}}{d^2-e^2x^2}dx$$

$$\Big\downarrow \text{1576}$$

$$d\left(\frac{\left(e^2\left(\sqrt{b^2-4ac}+b\right)+2cd^2\right)\left(ae^4+bd^2e^2+cd^4\right)\int \frac{2cx^2+b-\sqrt{b^2-4ac}}{(d^2-e^2x^2)\sqrt{cx^4+bx^2+a}}dx}{4ce^6}-\frac{\int \frac{2c^3e^4x^6+2c^2e^2(cd^2+2be^2)x^4+2c(c^2\cdots}{\cdots}}{\cdots}\right.$$

$$\frac{1}{2}e\int \frac{\left(cx^4+bx^2+a\right)^{3/2}}{d^2-e^2x^2}dx^2$$

$\Big\downarrow$ 1162

$$d\left(\frac{\left(e^2\left(\sqrt{b^2-4ac}+b\right)+2cd^2\right)\left(ae^4+bd^2e^2+cd^4\right)\int \frac{2cx^2+b-\sqrt{b^2-4ac}}{(d^2-e^2x^2)\sqrt{cx^4+bx^2+a}}dx}{4ce^6}-\frac{\int \frac{2c^3e^4x^6+2c^2e^2(cd^2+2be^2)x^4+2c(c^2\cdots}{\cdots}}{\cdots}\right.$$

$$\frac{1}{2}e\left(\frac{\int \frac{(bd^2+2ae^2+(2cd^2+be^2)x^2)\sqrt{cx^4+bx^2+a}}{d^2-e^2x^2}dx^2}{2e^2}-\frac{\left(a+bx^2+cx^4\right)^{3/2}}{3e^2}\right)$$

$\Big\downarrow$ 1231

$$d\left(\frac{\left(e^2\left(\sqrt{b^2-4ac}+b\right)+2cd^2\right)\left(ae^4+bd^2e^2+cd^4\right)\int \frac{2cx^2+b-\sqrt{b^2-4ac}}{(d^2-e^2x^2)\sqrt{cx^4+bx^2+a}}dx}{4ce^6}-\frac{\int \frac{2c^3e^4x^6+2c^2e^2(cd^2+2be^2)x^4+2c(c^2\cdots}{\cdots}}{\cdots}\right.$$

$$\frac{1}{2}e\left(\frac{-\frac{\int -\frac{\left(2cd^2+be^2\right)\left(4bcd^2+b^2e^2+4ace^2\right)d^2+4c\left(2ae^3+bd^2e\right)^2+\left(2cd^2+be^2\right)\left(8c^2d^4+8bce^2d^2-b^2e^4+12ace^4\right)x^2}{2\left(d^2-e^2x^2\right)\sqrt{cx^4+bx^2+a}}dx^2}{4ce^4}-\frac{\sqrt{a+bx^2+cx^4}\left(8ace^4+b^2e^4-\cdots\right.}{\cdots}}{2e^2}\right.$$

$\Big\downarrow$ 27

$$d\left(\frac{\left(e^2\left(\sqrt{b^2-4ac}+b\right)+2cd^2\right)\left(ae^4+bd^2e^2+cd^4\right)\int \frac{2cx^2+b-\sqrt{b^2-4ac}}{(d^2-e^2x^2)\sqrt{cx^4+bx^2+a}}dx}{4ce^6}-\frac{\int \frac{2c^3e^4x^6+2c^2e^2(cd^2+2be^2)x^4+2c(c^2\cdots}{\cdots}}{\cdots}\right.$$

$$\frac{1}{2}e\left(\frac{\frac{\int \frac{\left(2cd^2+be^2\right)\left(4bcd^2+b^2e^2+4ace^2\right)d^2+4c\left(2ae^3+bd^2e\right)^2+\left(2cd^2+be^2\right)\left(8c^2d^4+8bce^2d^2-b^2e^4+12ace^4\right)x^2}{\left(d^2-e^2x^2\right)\sqrt{cx^4+bx^2+a}}dx^2}{8ce^4}-\frac{\sqrt{a+bx^2+cx^4}\left(8ace^4+b^2e^4+2ce\cdots\right.}{4\cdots}}{2e^2}\right.$$

$\Big\downarrow$ 1269

$$d\left(\frac{\left(e^2\left(\sqrt{b^2-4ac}+b\right)+2cd^2\right)\left(ae^4+bd^2e^2+cd^4\right)\int \frac{2cx^2+b-\sqrt{b^2-4ac}}{(d^2-e^2x^2)\sqrt{cx^4+bx^2+a}}dx}{4ce^6}-\int \frac{2c^3e^4x^6+2c^2e^2\left(cd^2+2be^2\right)x^4+2c(c^2 \cdots}{\cdots}\right.$$

$$\frac{1}{2}e\left(\frac{\frac{\frac{16c\left(ae^4+bd^2e^2+cd^4\right)^2\int \frac{1}{\left(d^2-e^2x^2\right)\sqrt{cx^4+bx^2+a}}dx^2}{e^2}-\frac{\left(be^2+2cd^2\right)\left(12ace^4-b^2e^4+8bcd^2e^2+8c^2d^4\right)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx^2}{e^2}}{8ce^4}-\frac{\sqrt{a+bx^2+cx^4}\left(8ace^4 \cdots\right.}{\cdots}}{2e^2}\right.$$

$\downarrow$ 1092

$$d\left(\frac{\left(e^2\left(\sqrt{b^2-4ac}+b\right)+2cd^2\right)\left(ae^4+bd^2e^2+cd^4\right)\int \frac{2cx^2+b-\sqrt{b^2-4ac}}{(d^2-e^2x^2)\sqrt{cx^4+bx^2+a}}dx}{4ce^6}-\int \frac{2c^3e^4x^6+2c^2e^2\left(cd^2+2be^2\right)x^4+2c(c^2 \cdots}{\cdots}\right.$$

$$\frac{1}{2}e\left(\frac{\frac{\frac{16c\left(ae^4+bd^2e^2+cd^4\right)^2\int \frac{1}{\left(d^2-e^2x^2\right)\sqrt{cx^4+bx^2+a}}dx^2}{e^2}-\frac{2\left(be^2+2cd^2\right)\left(12ace^4-b^2e^4+8bcd^2e^2+8c^2d^4\right)\int \frac{1}{4c-x^4}d\frac{2cx^2+b}{\sqrt{cx^4+bx^2+a}}}{e^2}}{8ce^4}-\frac{\sqrt{a+bx^2+cx^4}\left(\cdots\right.}{\cdots}}{2e^2}\right.$$

$\downarrow$ 219

$$d\left(\frac{\left(e^2\left(\sqrt{b^2-4ac}+b\right)+2cd^2\right)\left(ae^4+bd^2e^2+cd^4\right)\int \frac{2cx^2+b-\sqrt{b^2-4ac}}{(d^2-e^2x^2)\sqrt{cx^4+bx^2+a}}dx}{4ce^6}-\int \frac{2c^3e^4x^6+2c^2e^2\left(cd^2+2be^2\right)x^4+2c(c^2 \cdots}{\cdots}\right.$$

$$\frac{1}{2}e\left(\frac{\frac{\frac{16c\left(ae^4+bd^2e^2+cd^4\right)^2\int \frac{1}{\left(d^2-e^2x^2\right)\sqrt{cx^4+bx^2+a}}dx^2}{e^2}-\frac{\left(be^2+2cd^2\right)\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)\left(12ace^4-b^2e^4+8bcd^2e^2+8c^2d^4\right)}{\sqrt{c}e^2}}{8ce^4}-\frac{\sqrt{a+bx^2 \cdots}}{\cdots}}{2e^2}\right.$$

$\downarrow$ 1154

$$d\left(\frac{\left(e^2\left(\sqrt{b^2-4ac}+b\right)+2cd^2\right)\left(ae^4+bd^2e^2+cd^4\right)\int \frac{2cx^2+b-\sqrt{b^2-4ac}}{(d^2-e^2x^2)\sqrt{cx^4+bx^2+a}}dx}{4ce^6}-\int \frac{2c^3e^4x^6+2c^2e^2\left(cd^2+2be^2\right)x^4+2c\left(c^2\cdots\right.}{\cdots}\right.$$

$$\frac{1}{2}e\left(\frac{\frac{-\frac{32c\left(ae^4+bd^2e^2+cd^4\right)^2\int \frac{1}{4\left(cd^4+be^2d^2+ae^4\right)-x^4}d\left(-\frac{bd^2+2ae^2+\left(2cd^2+be^2\right)x^2}{\sqrt{cx^4+bx^2+a}}\right)}{e^2}-\frac{\left(be^2+2cd^2\right)\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)\left(12ace^4-b^2e^4+8\cdots\right.}{\sqrt{c}e^2}}{8ce^4}}{2e^2}\cdots\right.$$

$\downarrow$ 219

$$d\left(\frac{\left(e^2\left(\sqrt{b^2-4ac}+b\right)+2cd^2\right)\left(ae^4+bd^2e^2+cd^4\right)\int \frac{2cx^2+b-\sqrt{b^2-4ac}}{(d^2-e^2x^2)\sqrt{cx^4+bx^2+a}}dx}{4ce^6}-\int \frac{2c^3e^4x^6+2c^2e^2\left(cd^2+2be^2\right)x^4+2c\left(c^2\cdots\right.}{\cdots}\right.$$

$$\frac{1}{2}e\left(\frac{\frac{\frac{16c\left(ae^4+bd^2e^2+cd^4\right)^{3/2}\text{arctanh}\left(\frac{2ae^2+x^2\left(be^2+2cd^2\right)+bd^2}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^4+bd^2e^2+cd^4}}\right)}{e^2}-\frac{\left(be^2+2cd^2\right)\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)\left(12ace^4-b^2e^4+8bcd^2e^2+8c^2\cdots\right.}{\sqrt{c}e^2}}{8ce^4}}{2e^2}\cdots\right.$$

$\downarrow$ 1786

$$d\left(\frac{\sqrt{\frac{2a}{b-\sqrt{b^2-4ac}}+x^2}\sqrt{-\sqrt{b^2-4ac}+b+2cx^2}\left(e^2\left(\sqrt{b^2-4ac}+b\right)+2cd^2\right)\left(ae^4+bd^2e^2+cd^4\right)\int \frac{\sqrt{2}\sqrt{2cx^2+b-\cdots}}{\sqrt{x^2+\frac{2a}{b-\sqrt{b^2-4ac}}}\cdots}}{4\sqrt{2}ce^6\sqrt{a+bx^2+cx^4}}\cdots\right.$$

$$\frac{1}{2}e\left(\frac{\frac{\frac{16c\left(ae^4+bd^2e^2+cd^4\right)^{3/2}\text{arctanh}\left(\frac{2ae^2+x^2\left(be^2+2cd^2\right)+bd^2}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^4+bd^2e^2+cd^4}}\right)}{e^2}-\frac{\left(be^2+2cd^2\right)\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)\left(12ace^4-b^2e^4+8bcd^2e^2+8c^2\cdots\right.}{\sqrt{c}e^2}}{8ce^4}}{2e^2}\cdots\right.$$

$\downarrow$ 27

$$d\left(\frac{\sqrt{\frac{2a}{b-\sqrt{b^2-4ac}}+x^2}\sqrt{-\sqrt{b^2-4ac}+b+2cx^2}\left(e^2\left(\sqrt{b^2-4ac}+b\right)+2cd^2\right)\left(ae^4+bd^2e^2+cd^4\right)\int\frac{\sqrt{2cx^2+b-\sqrt{}}}{\sqrt{x^2+\frac{2a}{b-\sqrt{b^2-4ac}}}}}{4ce^6\sqrt{a+bx^2+cx^4}}\right.$$

$$\frac{1}{2}e\left(\frac{\frac{\frac{16c\left(ae^4+bd^2e^2+cd^4\right)^{3/2}\operatorname{arctanh}\left(\frac{2ae^2+x^2\left(be^2+2cd^2\right)+bd^2}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^4+bd^2e^2+cd^4}}\right)}{e^2}-\frac{\left(be^2+2cd^2\right)\operatorname{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)\left(12ace^4-b^2e^4+8bcd^2e^2+8c^2\right.}{\sqrt{c}e^2}}{8ce^4}}{2e^2}\right.$$

$\downarrow$ 414

$$d\left(\frac{\left(b-\sqrt{b^2-4ac}\right)^{5/2}\left(\frac{2a}{b-\sqrt{b^2-4ac}}+x^2\right)\left(e^2\left(\sqrt{b^2-4ac}+b\right)+2cd^2\right)\left(ae^4+bd^2e^2+cd^4\right)\text{EllipticPi}\left(\frac{\left(b-\sqrt{b^2-}\right.}{2cd}\right.}{8ac^{3/2}d^2e^6\sqrt{\frac{\left(b-\sqrt{b^2-4ac}\right)^2\left(\frac{2a}{b-\sqrt{b^2-4ac}}+x^2\right)}{a\left(-\sqrt{b^2-4ac}+b+2cx^2\right)}}\sqrt{a+bx^2+}}\right.$$

$$\frac{1}{2}e\left(\frac{\frac{\frac{16c\left(ae^4+bd^2e^2+cd^4\right)^{3/2}\operatorname{arctanh}\left(\frac{2ae^2+x^2\left(be^2+2cd^2\right)+bd^2}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^4+bd^2e^2+cd^4}}\right)}{e^2}-\frac{\left(be^2+2cd^2\right)\operatorname{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)\left(12ace^4-b^2e^4+8bcd^2e^2+8c^2\right.}{\sqrt{c}e^2}}{8ce^4}}{2e^2}\right.$$

$\downarrow$ 2207

$$d\left(\frac{\left(b-\sqrt{b^2-4ac}\right)^{5/2}\left(2cd^2+\left(b+\sqrt{b^2-4ac}\right)e^2\right)\left(cd^4+be^2d^2+ae^4\right)\left(x^2+\frac{2a}{b-\sqrt{b^2-4ac}}\right)\text{EllipticPi}\left(\frac{\left(b-\sqrt{b^2-}\right.}{2cd}\right.}{8ac^{3/2}d^2e^6\sqrt{\frac{\left(b-\sqrt{b^2-4ac}\right)^2\left(x^2+\frac{2a}{b-\sqrt{b^2-4ac}}\right)}{a\left(2cx^2+b-\sqrt{b^2-4ac}\right)}}\sqrt{cx^4+bx^2}}\right.$$

$$\frac{1}{2}e\left(\frac{\frac{\frac{16c\left(cd^4+be^2d^2+ae^4\right)^{3/2}\operatorname{arctanh}\left(\frac{bd^2+2ae^2+\left(2cd^2+be^2\right)x^2}{2\sqrt{cd^4+be^2d^2+ae^4}\sqrt{cx^4+bx^2+a}}\right)}{e^2}-\frac{\left(2cd^2+be^2\right)\left(8c^2d^4+8bce^2d^2-b^2e^4+12ace^4\right)\operatorname{arctanh}\left(\frac{2cx^2+b}{2\sqrt{c}\sqrt{cx^4+bx^2+}}\right.}{\sqrt{c}e^2}}{8ce^4}}{2e^2}\right.$$

$\downarrow$ 2207

$$d\left(\frac{\left(b-\sqrt{b^2-4ac}\right)^{5/2}\left(2cd^2+\left(b+\sqrt{b^2-4ac}\right)e^2\right)\left(cd^4+be^2d^2+ae^4\right)\left(x^2+\frac{2a}{b-\sqrt{b^2-4ac}}\right)\text{EllipticPi}\left(\frac{\left(b-\sqrt{b^2-\cdots}\right.}{2ca\cdots}\cdots\right.}{8ac^{3/2}d^2e^6\sqrt{\frac{\left(b-\sqrt{b^2-4ac}\right)^2\left(x^2+\frac{2a}{b-\sqrt{b^2-4ac}}\right)}{a\left(2cx^2+b-\sqrt{b^2-4ac}\right)}}\sqrt{cx^4+bx^2\cdots}}\cdots\right.$$

$$\frac{1}{2}e\left(\frac{\frac{\frac{16c\left(cd^4+be^2d^2+ae^4\right)^{3/2}\text{arctanh}\left(\frac{bd^2+2ae^2+\left(2cd^2+be^2\right)x^2}{2\sqrt{cd^4+be^2d^2+ae^4}\sqrt{cx^4+bx^2+a}}\right)}{e^2}-\frac{\left(2cd^2+be^2\right)\left(8c^2d^4+8bce^2d^2-b^2e^4+12ace^4\right)\text{arctanh}\left(\frac{2cx^2+b}{2\sqrt{c}\sqrt{cx^4+bx^2-\cdots}}\cdots\right.}{\sqrt{c}e^2}\cdots}{8ce^4}\cdots}{2e^2}\cdots\right.$$

$$\Big\downarrow \; 27$$

$$d\left(\frac{\left(b-\sqrt{b^2-4ac}\right)^{5/2}\left(2cd^2+\left(b+\sqrt{b^2-4ac}\right)e^2\right)\left(cd^4+be^2d^2+ae^4\right)\left(x^2+\frac{2a}{b-\sqrt{b^2-4ac}}\right)\text{EllipticPi}\left(\frac{\left(b-\sqrt{b^2-\cdots}\right.}{2ca\cdots}\cdots\right.}{8ac^{3/2}d^2e^6\sqrt{\frac{\left(b-\sqrt{b^2-4ac}\right)^2\left(x^2+\frac{2a}{b-\sqrt{b^2-4ac}}\right)}{a\left(2cx^2+b-\sqrt{b^2-4ac}\right)}}\sqrt{cx^4+bx^2\cdots}}\cdots\right.$$

$$\frac{1}{2}e\left(\frac{\frac{\frac{16c\left(cd^4+be^2d^2+ae^4\right)^{3/2}\text{arctanh}\left(\frac{bd^2+2ae^2+\left(2cd^2+be^2\right)x^2}{2\sqrt{cd^4+be^2d^2+ae^4}\sqrt{cx^4+bx^2+a}}\right)}{e^2}-\frac{\left(2cd^2+be^2\right)\left(8c^2d^4+8bce^2d^2-b^2e^4+12ace^4\right)\text{arctanh}\left(\frac{2cx^2+b}{2\sqrt{c}\sqrt{cx^4+bx^2-\cdots}}\cdots\right.}{\sqrt{c}e^2}\cdots}{8ce^4}\cdots}{2e^2}\cdots\right.$$

$$\Big\downarrow \; 1511$$

$$d\left(\frac{\left(b-\sqrt{b^2-4ac}\right)^{5/2}\left(2cd^2+\left(b+\sqrt{b^2-4ac}\right)e^2\right)\left(cd^4+be^2d^2+ae^4\right)\left(x^2+\frac{2a}{b-\sqrt{b^2-4ac}}\right)\text{EllipticPi}\left(\frac{\left(b-\sqrt{b^2-\cdots}\right.}{2ca\cdots}\cdots\right.}{8ac^{3/2}d^2e^6\sqrt{\frac{\left(b-\sqrt{b^2-4ac}\right)^2\left(x^2+\frac{2a}{b-\sqrt{b^2-4ac}}\right)}{a\left(2cx^2+b-\sqrt{b^2-4ac}\right)}}\sqrt{cx^4+bx^2\cdots}}\cdots\right.$$

$$\frac{1}{2}e\left(\frac{\frac{\frac{16c\left(cd^4+be^2d^2+ae^4\right)^{3/2}\text{arctanh}\left(\frac{bd^2+2ae^2+\left(2cd^2+be^2\right)x^2}{2\sqrt{cd^4+be^2d^2+ae^4}\sqrt{cx^4+bx^2+a}}\right)}{e^2}-\frac{\left(2cd^2+be^2\right)\left(8c^2d^4+8bce^2d^2-b^2e^4+12ace^4\right)\text{arctanh}\left(\frac{2cx^2+b}{2\sqrt{c}\sqrt{cx^4+bx^2-\cdots}}\cdots\right.}{\sqrt{c}e^2}\cdots}{8ce^4}\cdots}{2e^2}\cdots\right.$$

$$\Big\downarrow \; 27$$

$$d\left(\frac{\left(b-\sqrt{b^2-4ac}\right)^{5/2}\left(2cd^2+\left(b+\sqrt{b^2-4ac}\right)e^2\right)\left(cd^4+be^2d^2+ae^4\right)\left(x^2+\frac{2a}{b-\sqrt{b^2-4ac}}\right)\text{EllipticPi}\left(\frac{\left(b-\sqrt{b^2-\right.}{2ca}\right.}{8ac^{3/2}d^2e^6\sqrt{\frac{\left(b-\sqrt{b^2-4ac}\right)^2\left(x^2+\frac{2a}{b-\sqrt{b^2-4ac}}\right)}{a\left(2cx^2+b-\sqrt{b^2-4ac}\right)}}\sqrt{cx^4+bx^2}}\right.$$

$$\frac{1}{2}e\left(\frac{\frac{\frac{16c\left(cd^4+be^2d^2+ae^4\right)^{3/2}\text{arctanh}\left(\frac{bd^2+2ae^2+\left(2cd^2+be^2\right)x^2}{2\sqrt{cd^4+be^2d^2+ae^4}\sqrt{cx^4+bx^2+a}}\right)}{e^2}-\frac{\left(2cd^2+be^2\right)\left(8c^2d^4+8bce^2d^2-b^2e^4+12ace^4\right)\text{arctanh}\left(\frac{2cx^2+b}{2\sqrt{c}\sqrt{cx^4+bx^2-}}\right.}{\sqrt{c}e^2}}{8ce^4}}{2e^2}\right.$$

$\downarrow$ 1416

$$d\left(\frac{\left(b-\sqrt{b^2-4ac}\right)^{5/2}\left(2cd^2+\left(b+\sqrt{b^2-4ac}\right)e^2\right)\left(cd^4+be^2d^2+ae^4\right)\left(x^2+\frac{2a}{b-\sqrt{b^2-4ac}}\right)\text{EllipticPi}\left(\frac{\left(b-\sqrt{b^2-\right.}{2ca}\right.}{8ac^{3/2}d^2e^6\sqrt{\frac{\left(b-\sqrt{b^2-4ac}\right)^2\left(x^2+\frac{2a}{b-\sqrt{b^2-4ac}}\right)}{a\left(2cx^2+b-\sqrt{b^2-4ac}\right)}}\sqrt{cx^4+bx^2}}\right.$$

$$\frac{1}{2}e\left(\frac{\frac{\frac{16c\left(cd^4+be^2d^2+ae^4\right)^{3/2}\text{arctanh}\left(\frac{bd^2+2ae^2+\left(2cd^2+be^2\right)x^2}{2\sqrt{cd^4+be^2d^2+ae^4}\sqrt{cx^4+bx^2+a}}\right)}{e^2}-\frac{\left(2cd^2+be^2\right)\left(8c^2d^4+8bce^2d^2-b^2e^4+12ace^4\right)\text{arctanh}\left(\frac{2cx^2+b}{2\sqrt{c}\sqrt{cx^4+bx^2-}}\right.}{\sqrt{c}e^2}}{8ce^4}}{2e^2}\right.$$

$\downarrow$ 1509

$$d\left(\frac{\left(b-\sqrt{b^2-4ac}\right)^{5/2}\left(2cd^2+\left(b+\sqrt{b^2-4ac}\right)e^2\right)\left(cd^4+be^2d^2+ae^4\right)\left(x^2+\frac{2a}{b-\sqrt{b^2-4ac}}\right)\text{EllipticPi}\left(\frac{\left(b-\sqrt{b^2-\right.}{2ca\right.}}{8ac^{3/2}d^2e^6\sqrt{\frac{\left(b-\sqrt{b^2-4ac}\right)^2\left(x^2+\frac{2a}{b-\sqrt{b^2-4ac}}\right)}{a\left(2cx^2+b-\sqrt{b^2-4ac}\right)}}\sqrt{cx^4+bx^2}}\right.$$

$$\frac{1}{2}e\left(\frac{\frac{\frac{16c\left(cd^4+be^2d^2+ae^4\right)^{3/2}\text{arctanh}\left(\frac{bd^2+2ae^2+\left(2cd^2+be^2\right)x^2}{2\sqrt{cd^4+be^2d^2+ae^4}\sqrt{cx^4+bx^2+a}}\right)}{e^2}-\frac{\left(2cd^2+be^2\right)\left(8c^2d^4+8bce^2d^2-b^2e^4+12ace^4\right)\text{arctanh}\left(\frac{2cx^2+b}{2\sqrt{c}\sqrt{cx^4+bx^2-}}\right.}{\sqrt{c}e^2}}{8ce^4}}{2e^2}\right.$$

input
```
Int[(a + b*x^2 + c*x^4)^(3/2)/(d + e*x),x]
```

output
```
-1/2*(e*(-1/3*(a + b*x^2 + c*x^4)^(3/2)/e^2 + (-1/4*((8*c^2*d^4 + 10*b*c*d
^2*e^2 + b^2*e^4 + 8*a*c*e^4 + 2*c*e^2*(2*c*d^2 + b*e^2)*x^2)*Sqrt[a + b*x
^2 + c*x^4])/(c*e^4) + (-(((2*c*d^2 + b*e^2)*(8*c^2*d^4 + 8*b*c*d^2*e^2 -
b^2*e^4 + 12*a*c*e^4)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*
x^4])])/(Sqrt[c]*e^2)) + (16*c*(c*d^4 + b*d^2*e^2 + a*e^4)^(3/2)*ArcTanh[(
b*d^2 + 2*a*e^2 + (2*c*d^2 + b*e^2)*x^2)/(2*Sqrt[c*d^4 + b*d^2*e^2 + a*e^4
]*Sqrt[a + b*x^2 + c*x^4])])/e^2)/(8*c*e^4))/(2*e^2))) + d*(-1/2*((2*c^2*e
^4*x^3*Sqrt[a + b*x^2 + c*x^4])/5 + ((2*c^2*e^2*(5*c*d^2 + 6*b*e^2)*x*Sqrt
[a + b*x^2 + c*x^4])/3 + (c^2*((-2*(15*c^2*d^4 + 20*b*c*d^2*e^2 + 3*b^2*e^
4 + 21*a*c*e^4)*(-((x*Sqrt[a + b*x^2 + c*x^4])/(Sqrt[a] + Sqrt[c]*x^2)) +
(a^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[
c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c
]))/4])/(c^(1/4)*Sqrt[a + b*x^2 + c*x^4])))/Sqrt[c] + ((15*b*c*d^4 + 15*b^
2*d^2*e^2 + 20*a*c*d^2*e^2 + 33*a*b*e^4 - 15*Sqrt[b^2 - 4*a*c]*(c*d^4 + b*
d^2*e^2 + a*e^4) + (2*Sqrt[a]*(15*c^2*d^4 + 20*b*c*d^2*e^2 + 3*b^2*e^4 + 2
1*a*c*e^4))/Sqrt[c])*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqr
t[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sq
rt[a]*Sqrt[c]))/4])/(2*a^(1/4)*c^(1/4)*Sqrt[a + b*x^2 + c*x^4])))/3)/(5*c)
)/(c*e^6) + ((b - Sqrt[b^2 - 4*a*c])^(5/2)*(2*c*d^2 + (b + Sqrt[b^2 - 4*a*
c])*e^2)*(c*d^4 + b*d^2*e^2 + a*e^4)*((2*a)/(b - Sqrt[b^2 - 4*a*c]) + x...
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 414
```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1162
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Simp[p/(e*(m + 2*p + 1))   Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d -
b*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x
] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&
!ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 1529
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(-(2*c*d - e*(b + q)))*((c*d^2 -
 b*d*e + a*e^2)^(p - 1/2)/(4*c*e^(2*p)))   Int[(b - q + 2*c*x^2)/((d + e*x^
2)*Sqrt[a + b*x^2 + c*x^4]), x], x] + Simp[1/(4*c*e^(2*p))   Int[(1/Sqrt[a
+ b*x^2 + c*x^4])*ExpandToSum[(4*c*e^(2*p)*(a + b*x^2 + c*x^4)^(p + 1/2) +
(2*c*d - e*(b + q))*(c*d^2 - b*d*e + a*e^2)^(p - 1/2)*(b - q + 2*c*x^2))/(d
 + e*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p - 1/2, 0] && PosQ[b^2 - 4*a*c] &
& PosQ[c/a]
```

rule 1576
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x]
, x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

rule 1786
```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((f_) + (g_.)*(x_)^(n_))^(r_.)*((a_) + (
b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[(a + b*x^n + c*x
^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d + (c*x^n)/e)^FracPart[p])
   Int[(d + e*x^n)^(p + q)*(f + g*x^n)^r*(a/d + (c/e)*x^n)^p, x], x] /; Fre
eQ[{a, b, c, d, e, f, g, n, p, q, r}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c
, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p]
```

rule 2207
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{n =
 Expon[Px, x^2], e = Coeff[Px, x^2, Expon[Px, x^2]]}, Simp[e*x^(2*n - 3)*((
a + b*x^2 + c*x^4)^(p + 1)/(c*(2*n + 4*p + 1))), x] + Simp[1/(c*(2*n + 4*p
+ 1))   Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*n + 4*p + 1)*Px - a*e*(2
*n - 3)*x^(2*n - 4) - b*e*(2*n + 2*p - 1)*x^(2*n - 2) - c*e*(2*n + 4*p + 1)
*x^(2*n), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Px, x^2] && Expon[
Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

rule 2266
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.)/((d_) + (e_.)*(x_)), x_Symbo
l] :> Simp[d   Int[(a + b*x^2 + c*x^4)^p/(d^2 - e^2*x^2), x], x] - Simp[e
 Int[x*((a + b*x^2 + c*x^4)^p/(d^2 - e^2*x^2)), x], x] /; FreeQ[{a, b, c, d
, e}, x] && IntegerQ[p + 1/2]
```

Maple [A] (verified)

Time = 4.13 (sec) , antiderivative size = 1017, normalized size of antiderivative = 0.88

method	result	size
risch	Expression too large to display	1017
default	Expression too large to display	1295
elliptic	Expression too large to display	1295

input
```
int((c*x^4+b*x^2+a)^(3/2)/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
1/240/c*(40*c^2*e^4*x^4-48*c^2*d*e^3*x^3+70*b*c*e^4*x^2+60*c^2*d^2*e^2*x^2
-96*b*c*d*e^3*x-80*c^2*d^3*e*x+160*a*c*e^4+15*b^2*e^4+150*b*c*d^2*e^2+120*
c^2*d^4)*(c*x^4+b*x^2+a)^(1/2)/e^5-1/240/e^5/c*(-15/2/e^2*(12*a*b*c*e^6+24
*a*c^2*d^2*e^4-b^3*e^6+6*b^2*c*d^2*e^4+24*b*c^2*d^4*e^2+16*c^3*d^6)*ln((1/
2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))/c^(1/2)+4*d*(24*a*b*e^6+25*a*c*d
^2*e^4+15*b^2*d^2*e^4+30*b*c*d^4*e^2+15*c^2*d^6)*c/e^3*2^(1/2)/((-b+(-4*a*
c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(
-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1
/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a
/c)^(1/2))-240*(a^2*e^8+2*a*b*d^2*e^6+2*a*c*d^4*e^4+b^2*d^4*e^4+2*b*c*d^6*
e^2+c^2*d^8)*c/e^4*(-1/2/(c*d^4/e^4+b*d^2/e^2+a)^(1/2)*arctanh(1/2*(2*c*x^
2*d^2/e^2+b*d^2/e^2+b*x^2+2*a)/(c*d^4/e^4+b*d^2/e^2+a)^(1/2)/(c*x^4+b*x^2+
a)^(1/2))+2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)/d*e*(1-1/2*(-b+(-4*a*c
+b^2)^(1/2))/a*x^2)^(1/2)*(1+1/2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^
4+b*x^2+a)^(1/2)*EllipticPi(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2
),2/(-b+(-4*a*c+b^2)^(1/2))*a/d^2*e^2,(-1/2*(b+(-4*a*c+b^2)^(1/2))/a)^(1/2
)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)))-8*c*d/e*(21*a*c*e^4+3*b^2*e^
4+20*b*c*d^2*e^2+15*c^2*d^4)*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(
4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2
)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{d+ex}\,dx = \text{Timed out}$$

input
```
integrate((c*x^4+b*x^2+a)^(3/2)/(e*x+d),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{d+ex}\,dx = \int \frac{(a+bx^2+cx^4)^{\frac{3}{2}}}{d+ex}\,dx$$

input
```
integrate((c*x**4+b*x**2+a)**(3/2)/(e*x+d),x)
```

output
```
Integral((a + b*x**2 + c*x**4)**(3/2)/(d + e*x), x)
```

Maxima [F]

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{d+ex}\,dx = \int \frac{(cx^4+bx^2+a)^{\frac{3}{2}}}{ex+d}\,dx$$

input
```
integrate((c*x^4+b*x^2+a)^(3/2)/(e*x+d),x, algorithm="maxima")
```

output
```
integrate((c*x^4 + b*x^2 + a)^(3/2)/(e*x + d), x)
```

Giac [F]

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{d+ex}\,dx = \int \frac{(cx^4+bx^2+a)^{\frac{3}{2}}}{ex+d}\,dx$$

input
```
integrate((c*x^4+b*x^2+a)^(3/2)/(e*x+d),x, algorithm="giac")
```

output
```
integrate((c*x^4 + b*x^2 + a)^(3/2)/(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{d+ex}\,dx = \int \frac{(c\,x^4+b\,x^2+a)^{3/2}}{d+e\,x}\,dx$$

input
```
int((a + b*x^2 + c*x^4)^(3/2)/(d + e*x),x)
```

output
```
int((a + b*x^2 + c*x^4)^(3/2)/(d + e*x), x)
```

Reduce [F]

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{d+ex}\,dx = \int \frac{(c\,x^4+b\,x^2+a)^{\frac{3}{2}}}{ex+d}dx$$

input
```
int((c*x^4+b*x^2+a)^(3/2)/(e*x+d),x)
```

output
```
int((c*x^4+b*x^2+a)^(3/2)/(e*x+d),x)
```

3.259 $\int \frac{(a+bx^2+cx^4)^{3/2}}{(d+ex)^2} dx$

Optimal result

Integrand size = 24, antiderivative size = 1195

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{(d+ex)^2}\,dx = \text{Too large to display}$$

output
```
1/5*(2*b*e^2+5*c*d^2)*x*(c*x^4+b*x^2+a)^(1/2)/e^4+1/5*c*x^3*(c*x^4+b*x^2+a
)^(1/2)/e^2+1/5*(12*a*c*e^4+b^2*e^4+25*b*c*d^2*e^2+30*c^2*d^4)*x*(c*x^4+b*
x^2+a)^(1/2)/c^(1/2)/e^6/(a^(1/2)+c^(1/2)*x^2)+(a*e^4+b*d^2*e^2+c*d^4)*x*(
c*x^4+b*x^2+a)^(1/2)/e^4/(-e^2*x^2+d^2)-3/4*d*(2*c*e^2*x^2+3*b*e^2+4*c*d^2
)*(c*x^4+b*x^2+a)^(1/2)/e^5-d*(c*x^4+b*x^2+a)^(3/2)/e/(-e^2*x^2+d^2)-3/2*d
*(b*e^2+2*c*d^2)*(a*e^4+b*d^2*e^2+c*d^4)^(1/2)*arctanh((a*e^4+b*d^2*e^2+c*
d^4)^(1/2)*x/d/e/(c*x^4+b*x^2+a)^(1/2))/e^7-3/8*d*(4*a*c*e^4+b^2*e^4+8*b*c
*d^2*e^2+8*c^2*d^4)*arctanh(1/2*(2*c*x^2+b)/c^(1/2)/(c*x^4+b*x^2+a)^(1/2))
/c^(1/2)/e^7+3/2*d*(b*e^2+2*c*d^2)*(a*e^4+b*d^2*e^2+c*d^4)^(1/2)*arctanh(1
/2*(b*d^2+2*a*e^2+(b*e^2+2*c*d^2)*x^2)/(a*e^4+b*d^2*e^2+c*d^4)^(1/2)/(c*x^
4+b*x^2+a)^(1/2))/e^7-1/5*a^(1/4)*(12*a*c*e^4+b^2*e^4+25*b*c*d^2*e^2+30*c^
2*d^4)*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/
2)*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1
/2))/c^(3/4)/e^6/(c*x^4+b*x^2+a)^(1/2)+1/10*a^(1/4)*(60*c^(5/2)*d^6+20*a^(
1/2)*c^2*d^4*e^2+a^(1/2)*b^2*e^6+8*b*c^(1/2)*e^4*(a*e^2+2*b*d^2)+6*a^(1/2)
*c*e^4*(2*a*e^2+3*b*d^2)+c^(3/2)*(32*a*d^2*e^4+70*b*d^4*e^2))*(a^(1/2)+c^(
1/2)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(
2*arctan(c^(1/4)*x/a^(1/4)),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))/c^(3/4)/e^6/(
c^(1/2)*d^2+a^(1/2)*e^2)/(c*x^4+b*x^2+a)^(1/2)+3/4*(c^(1/2)*d^2-a^(1/2)*e^
2)*(b*e^2+2*c*d^2)*(a*e^4+b*d^2*e^2+c*d^4)*(a^(1/2)+c^(1/2)*x^2)*((c*x^...
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 22.70 (sec) , antiderivative size = 11129, normalized size of antiderivative = 9.31

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{(d+ex)^2}\,dx = \text{Result too large to show}$$

input
```
Integrate[(a + b*x^2 + c*x^4)^(3/2)/(d + e*x)^2,x]
```

output
```
Result too large to show
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{(d+ex)^2}\,dx$$
$$\Big\downarrow \text{7299}$$
$$\int \frac{(a+bx^2+cx^4)^{3/2}}{(d+ex)^2}dx$$

input
```
Int[(a + b*x^2 + c*x^4)^(3/2)/(d + e*x)^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [A] (verified)

Time = 5.63 (sec) , antiderivative size = 1199, normalized size of antiderivative = 1.00

method	result	size
default	Expression too large to display	1199
elliptic	Expression too large to display	1199
risch	Expression too large to display	1746

input
```
int((c*x^4+b*x^2+a)^(3/2)/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
-(a*e^4+b*d^2*e^2+c*d^4)/e^5*(c*x^4+b*x^2+a)^(1/2)/(e*x+d)+1/5*c*x^3*(c*x^
4+b*x^2+a)^(1/2)/e^2-1/2*c*d/e^3*x^2*(c*x^4+b*x^2+a)^(1/2)+1/3*(c/e^4*(2*b
*e^2+3*c*d^2)-4/5*c/e^2*b)/c*x*(c*x^4+b*x^2+a)^(1/2)+1/2*(-4*c*d/e^5*(b*e^
2+c*d^2)+3/2*c*d/e^3*b)/c*(c*x^4+b*x^2+a)^(1/2)+1/4*((2*a*b*e^6+6*a*c*d^2*
e^4+3*b^2*d^2*e^4+10*b*c*d^4*e^2+7*c^2*d^6)/e^8-d^2/e^8*c*(a*e^4+b*d^2*e^2
+c*d^4)-1/3*(c/e^4*(2*b*e^2+3*c*d^2)-4/5*c/e^2*b)/c*a)*2^(1/2)/((-b+(-4*a*
c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(
-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1
/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a
/c)^(1/2))+1/2*(-2*d/e^7*(2*a*c*e^4+b^2*e^4+4*b*c*d^2*e^2+3*c^2*d^4)+c*d/e
^3*a-1/2*(-4*c*d/e^5*(b*e^2+c*d^2)+3/2*c*d/e^3*b)/c*b)*ln((2*c*x^2+b)/c^(1
/2)+2*(c*x^4+b*x^2+a)^(1/2))/c^(1/2)-1/2*(1/e^6*(2*a*c*e^4+b^2*e^4+6*b*c*d
^2*e^2+5*c^2*d^4)+(a*e^4+b*d^2*e^2+c*d^4)/e^6*c-3/5*c/e^2*a-2/3*(c/e^4*(2*
b*e^2+3*c*d^2)-4/5*c/e^2*b)/c*b)*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/
2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a
*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*
2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2
))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1
/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)))-3*d/e^9*(a*b*e^6+2*a*c*d^2*
e^4+b^2*d^2*e^4+3*b*c*d^4*e^2+2*c^2*d^6)*(-1/2/(c*d^4/e^4+b*d^2/e^2+a)^...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{(d+ex)^2}\,dx = \text{Timed out}$$

input
```
integrate((c*x^4+b*x^2+a)^(3/2)/(e*x+d)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{(d+ex)^2}\,dx = \int \frac{(a+bx^2+cx^4)^{\frac{3}{2}}}{(d+ex)^2}\,dx$$

input
```
integrate((c*x**4+b*x**2+a)**(3/2)/(e*x+d)**2,x)
```

output
```
Integral((a + b*x**2 + c*x**4)**(3/2)/(d + e*x)**2, x)
```

Maxima [F]

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{(d+ex)^2}\,dx = \int \frac{(cx^4+bx^2+a)^{\frac{3}{2}}}{(ex+d)^2}\,dx$$

input
```
integrate((c*x^4+b*x^2+a)^(3/2)/(e*x+d)^2,x, algorithm="maxima")
```

output
```
integrate((c*x^4 + b*x^2 + a)^(3/2)/(e*x + d)^2, x)
```

Giac [F]

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{(d+ex)^2}\,dx = \int \frac{(cx^4+bx^2+a)^{\frac{3}{2}}}{(ex+d)^2}\,dx$$

input
```
integrate((c*x^4+b*x^2+a)^(3/2)/(e*x+d)^2,x, algorithm="giac")
```

output
```
integrate((c*x^4 + b*x^2 + a)^(3/2)/(e*x + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{(d+ex)^2}\,dx=\int \frac{{\left(c\,x^4+b\,x^2+a\right)}^{3/2}}{{\left(d+e\,x\right)}^2}\,dx$$

input
```
int((a + b*x^2 + c*x^4)^(3/2)/(d + e*x)^2,x)
```

output
```
int((a + b*x^2 + c*x^4)^(3/2)/(d + e*x)^2, x)
```

Reduce [F]

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{(d+ex)^2}\,dx=\int \frac{\left(c\,x^{4}+b\,x^{2}+a\right)^{\frac{3}{2}}}{\left(ex+d\right)^{2}}dx$$

input
```
int((c*x^4+b*x^2+a)^(3/2)/(e*x+d)^2,x)
```

output
```
int((c*x^4+b*x^2+a)^(3/2)/(e*x+d)^2,x)
```

3.260 $\int \frac{(d+ex)^3}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 380

$$\begin{aligned}\int \frac{(d+ex)^3}{\sqrt{a+bx^2+cx^4}}\,dx &= \frac{e^3\sqrt{a+bx^2+cx^4}}{2c}+\frac{3de^2x\sqrt{a+bx^2+cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)}+\frac{e(6cd^2-be^2)\operatorname{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4c^{3/2}} \\ &\quad -\frac{3\sqrt[4]{a}de^2\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{c^{3/4}\sqrt{a+bx^2+cx^4}} \\ &\quad +\frac{d\left(\sqrt{c}d^2+3\sqrt{a}e^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}c^{3/4}\sqrt{a+bx^2+cx^4}}\end{aligned}$$

output

```
1/2*e^3*(c*x^4+b*x^2+a)^(1/2)/c+3*d*e^2*x*(c*x^4+b*x^2+a)^(1/2)/c^(1/2)/(a
^(1/2)+c^(1/2)*x^2)+1/4*e*(-b*e^2+6*c*d^2)*arctanh(1/2*(2*c*x^2+b)/c^(1/2)
/(c*x^4+b*x^2+a)^(1/2))/c^(3/2)-3*a^(1/4)*d*e^2*(a^(1/2)+c^(1/2)*x^2)*((c*
x^4+b*x^2+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*EllipticE(sin(2*arctan(c^(1/4)
*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))/c^(3/4)/(c*x^4+b*x^2+a)^(1/2
)+1/2*d*(c^(1/2)*d^2+3*a^(1/2)*e^2)*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+b*x^2+a)
/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(c^(1/4)*x/a^(1/4)
),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))/a^(1/4)/c^(3/4)/(c*x^4+b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.48 (sec) , antiderivative size = 541, normalized size of antiderivative = 1.42

$$\int \frac{(d+ex)^3}{\sqrt{a+bx^2+cx^4}}\,dx$$
$$= \frac{3i\sqrt{2}\sqrt{c}\left(-b+\sqrt{b^2-4ac}\right)de^2\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}E\left(i\text{arcsinh}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x\right)|\frac{b+\sqrt{b^2-4a}}{b-\sqrt{b^2-4a}}\right.}{}$$

input
```
Integrate[(d + e*x)^3/Sqrt[a + b*x^2 + c*x^4],x]
```

output
```
((3*I)*Sqrt[2]*Sqrt[c]*(-b + Sqrt[b^2 - 4*a*c])*d*e^2*Sqrt[(b - Sqrt[b^2 -
 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] +
2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b +
Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] -
 I*Sqrt[2]*Sqrt[c]*d*(2*c*d^2 + 3*(-b + Sqrt[b^2 - 4*a*c])*e^2)*Sqrt[(b -
Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 -
 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sq
rt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 -
4*a*c])] + Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*e*(2*Sqrt[c]*e^2*(a + b*x^2 + c
*x^4) + (-6*c*d^2 + b*e^2)*Sqrt[a + b*x^2 + c*x^4]*Log[b + 2*c*x^2 - 2*Sqr
t[c]*Sqrt[a + b*x^2 + c*x^4]]))/(4*c^(3/2)*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]
*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 379, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {2202, 1511, 27, 1416, 1509, 1576, 27, 1160, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^3}{\sqrt{a+bx^2+cx^4}}dx$$

$\Big\downarrow$ 2202

$$\int \frac{d^3+3e^2x^2d}{\sqrt{cx^4+bx^2+a}}dx+\int \frac{x\left(x^2e^3+3d^2e\right)}{\sqrt{cx^4+bx^2+a}}dx$$

$\Big\downarrow$ 1511

$$\int \frac{x\left(x^2e^3+3d^2e\right)}{\sqrt{cx^4+bx^2+a}}dx+d\left(\frac{3\sqrt{a}e^2}{\sqrt{c}}+d^2\right)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx-\frac{3\sqrt{a}de^2\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{a}\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}$$

$\Big\downarrow$ 27

$$\int \frac{x\left(x^2e^3+3d^2e\right)}{\sqrt{cx^4+bx^2+a}}dx+d\left(\frac{3\sqrt{a}e^2}{\sqrt{c}}+d^2\right)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx-\frac{3de^2\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}$$

$\Big\downarrow$ 1416

$$\int \frac{x\left(x^2e^3+3d^2e\right)}{\sqrt{cx^4+bx^2+a}}dx-\frac{3de^2\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}+\frac{d\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{3\sqrt{a}e^2}{\sqrt{c}}+d^2\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}$$

$\Big\downarrow$ 1509

$$\int \frac{x\left(x^2e^3+3d^2e\right)}{\sqrt{cx^4+bx^2+a}}dx+\frac{d\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{3\sqrt{a}e^2}{\sqrt{c}}+d^2\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}-\frac{3de^2\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}-\frac{x\sqrt{a+bx^2+cx^4}}{\sqrt{a}+\sqrt{c}x^2}\right)}{\sqrt{c}}$$

$\Big\downarrow$ 1576

$$\frac{1}{2}\int \frac{e\left(3d^2+e^2x^2\right)}{\sqrt{cx^4+bx^2+a}}dx^2+\frac{d\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{3\sqrt{a}e^2}{\sqrt{c}}+d^2\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}-\frac{3de^2\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}-\frac{x\sqrt{a+bx^2+cx^4}}{\sqrt{a}+\sqrt{c}x^2}\right)}{\sqrt{c}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}e\int \frac{3d^2+e^2x^2}{\sqrt{cx^4+bx^2+a}}dx^2+\frac{d\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{3\sqrt{a}e^2}{\sqrt{c}}+d^2\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}-\frac{3de^2\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}-\frac{x\sqrt{a+bx^2+cx^4}}{\sqrt{a}+\sqrt{c}x^2}\right)}{\sqrt{c}}$$

$$\Big\downarrow \text{1160}$$

$$\frac{1}{2}e\left(\frac{\left(6cd^2-be^2\right)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx^2}{2c}+\frac{e^2\sqrt{a+bx^2+cx^4}}{c}\right)+\frac{d\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{3\sqrt{a}e^2}{\sqrt{c}}+d^2\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}-\frac{3de^2\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}-\frac{x\sqrt{a+bx^2+cx^4}}{\sqrt{a}+\sqrt{c}x^2}\right)}{\sqrt{c}}$$

$$\Big\downarrow \text{1092}$$

$$\frac{1}{2}e\left(\frac{\left(6cd^2-be^2\right)\int \frac{1}{4c-x^4}d\frac{2cx^2+b}{\sqrt{cx^4+bx^2+a}}}{c}+\frac{e^2\sqrt{a+bx^2+cx^4}}{c}\right)+\frac{d\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{3\sqrt{a}e^2}{\sqrt{c}}+d^2\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}-\frac{3de^2\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}-\frac{x\sqrt{a+bx^2+cx^4}}{\sqrt{a}+\sqrt{c}x^2}\right)}{\sqrt{c}}$$

$$\Big\downarrow \text{219}$$

$$\frac{d\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{3\sqrt{a}e^2}{\sqrt{c}}+d^2\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}-\frac{3de^2\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}-\frac{x\sqrt{a+bx^2+cx^4}}{\sqrt{a}+\sqrt{c}x^2}\right)}{\sqrt{c}}+\frac{1}{2}e\left(\frac{\left(6cd^2-be^2\right)\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2c^{3/2}}+\frac{e^2\sqrt{a+bx^2+cx^4}}{c}\right)$$

input
```
Int[(d + e*x)^3/Sqrt[a + b*x^2 + c*x^4],x]
```

output
```
(e*((e^2*Sqrt[a + b*x^2 + c*x^4])/c + ((6*c*d^2 - b*e^2)*ArcTanh[(b + 2*c*
x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(2*c^(3/2))))/2 - (3*d*e^2*(-((
x*Sqrt[a + b*x^2 + c*x^4])/(Sqrt[a] + Sqrt[c]*x^2)) + (a^(1/4)*(Sqrt[a] +
Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE
[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(c^(1/4)*Sqr
t[a + b*x^2 + c*x^4])))/Sqrt[c] + (d*(d^2 + (3*Sqrt[a]*e^2)/Sqrt[c])*(Sqrt
[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Ell
ipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*a^(
1/4)*c^(1/4)*Sqrt[a + b*x^2 + c*x^4])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 1576
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x]
, x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

rule 2202
```
Int[(Pn_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{n
 = Expon[Pn, x], k}, Int[Sum[Coeff[Pn, x, 2*k]*x^(2*k), {k, 0, n/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pn, x, 2*k + 1]*x^(2*k), {k, 0, (n -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pn, x]
 &&  !PolyQ[Pn, x^2]
```

Maple [A] (verified)

Time = 3.06 (sec) , antiderivative size = 440, normalized size of antiderivative = 1.16

method	result
elliptic	$\frac{e^{3} \sqrt{c\,x^{4}+b\,x^{2}+a}}{2 c}+\frac{d^{3} \sqrt{2}\, \sqrt{4-\frac{2 \left(-b+\sqrt{-4 a c+b^{2}}\right) x^{2}}{a}}\, \sqrt{4+\frac{2 \left(b+\sqrt{-4 a c+b^{2}}\right) x^{2}}{a}}\, \operatorname{EllipticF}\left(\frac{x \sqrt{2}\, \sqrt{\frac{-b+\sqrt{-4 a c+b^{2}}}{a}}}{2}, \frac{\sqrt{-4+\frac{2 b \left(b+\sqrt{-4a\right.}}{a c}}}{2}\right.}{4 \sqrt{\frac{-b+\sqrt{-4 a c+b^{2}}}{a}}\, \sqrt{c\,x^{4}+b\,x^{2}+a}}$
risch	$\frac{e^{3} \sqrt{c\,x^{4}+b\,x^{2}+a}}{2 c}-\frac{\frac{e \left(b\,e^{2}-6 c\,d^{2}\right) \ln\left(\frac{\frac{b}{2}+c\,x^{2}}{\sqrt{c}}+\sqrt{c\,x^{4}+b\,x^{2}+a}\right)}{2 \sqrt{c}}-\frac{c\,d^{3} \sqrt{2}\, \sqrt{4-\frac{2 \left(-b+\sqrt{-4 a c+b^{2}}\right) x^{2}}{a}}\, \sqrt{4+\frac{2 \left(b+\sqrt{-4 a c+b^{2}}\right) x^{2}}{a}}\, \operatorname{EllipticF}\left(\right.}{2 \sqrt{\frac{-b+\sqrt{-4 a c+b^{2}}}{a}}\, \sqrt{c\,x^{4}}}}{}$
default	$\frac{d^{3} \sqrt{2}\, \sqrt{4-\frac{2 \left(-b+\sqrt{-4 a c+b^{2}}\right) x^{2}}{a}}\, \sqrt{4+\frac{2 \left(b+\sqrt{-4 a c+b^{2}}\right) x^{2}}{a}}\, \operatorname{EllipticF}\left(\frac{x \sqrt{2}\, \sqrt{\frac{-b+\sqrt{-4 a c+b^{2}}}{a}}}{2}, \frac{\sqrt{-4+\frac{2 b \left(b+\sqrt{-4 a c+b^{2}}\right)}{a c}}}{2}\right)}{4 \sqrt{\frac{-b+\sqrt{-4 a c+b^{2}}}{a}}\, \sqrt{c\,x^{4}+b\,x^{2}+a}}+e^{3} \left(\sqrt{c}\right.$

input
```
int((e*x+d)^3/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*e^3*(c*x^4+b*x^2+a)^(1/2)/c+1/4*d^3*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a
)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/
2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*
c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))+1/2*
(3*d^2*e-1/2*b/c*e^3)*ln((2*c*x^2+b)/c^(1/2)+2*(c*x^4+b*x^2+a)^(1/2))/c^(1
/2)-3/2*d*e^2*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c
+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+
b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*
c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-Elli
pticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a
*c+b^2)^(1/2))/a/c)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 403, normalized size of antiderivative = 1.06

$$\int \frac{(d+ex)^3}{\sqrt{a+bx^2+cx^4}}\,dx = \frac{12\sqrt{\frac{1}{2}}\left(acde^2x\sqrt{\frac{b^2-4\,ac}{c^2}}-abde^2x\right)\sqrt{c}\sqrt{\frac{c\sqrt{\frac{b^2-4\,ac}{c^2}}-b}{c}}E(\arcsin\left(\frac{\sqrt{\frac{1}{2}}\sqrt{\frac{c\sqrt{\frac{b^2-4\,ac}{c^2}}-b}{c}}}{x}\right)\mid\frac{bc\sqrt{\frac{b^2-4\,ac}{c^2}}+b^2-2\,ac}{2\,ac})+4}{}$$

input
```
integrate((e*x+d)^3/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
1/8*(12*sqrt(1/2)*(a*c*d*e^2*x*sqrt((b^2 - 4*a*c)/c^2) - a*b*d*e^2*x)*sqrt
(c)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)*elliptic_e(arcsin(sqrt(1/2)*sq
rt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)/x), 1/2*(b*c*sqrt((b^2 - 4*a*c)/c^2)
 + b^2 - 2*a*c)/(a*c)) + 4*sqrt(1/2)*((c^2*d^3 - 3*a*c*d*e^2)*x*sqrt((b^2
- 4*a*c)/c^2) + (b*c*d^3 + 3*a*b*d*e^2)*x)*sqrt(c)*sqrt((c*sqrt((b^2 - 4*a
*c)/c^2) - b)/c)*elliptic_f(arcsin(sqrt(1/2)*sqrt((c*sqrt((b^2 - 4*a*c)/c^
2) - b)/c)/x), 1/2*(b*c*sqrt((b^2 - 4*a*c)/c^2) + b^2 - 2*a*c)/(a*c)) - (6
*a*c*d^2*e - a*b*e^3)*sqrt(c)*x*log(8*c^2*x^4 + 8*b*c*x^2 + b^2 - 4*sqrt(c
*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) + 4*a*c) + 4*(a*c*e^3*x + 6*a*c*d*
e^2)*sqrt(c*x^4 + b*x^2 + a))/(a*c^2*x)
```

Sympy [F]

$$\int \frac{(d+ex)^3}{\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{(d+ex)^3}{\sqrt{a+bx^2+cx^4}}\,dx$$

input
```
integrate((e*x+d)**3/(c*x**4+b*x**2+a)**(1/2),x)
```

output
```
Integral((d + e*x)**3/sqrt(a + b*x**2 + c*x**4), x)
```

Maxima [F]

$$\int \frac{(d+ex)^3}{\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{(ex+d)^3}{\sqrt{cx^4+bx^2+a}}\,dx$$

input
```
integrate((e*x+d)^3/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((e*x + d)^3/sqrt(c*x^4 + b*x^2 + a), x)
```

Giac [F]

$$\int \frac{(d+ex)^3}{\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{(ex+d)^3}{\sqrt{cx^4+bx^2+a}}\,dx$$

input
```
integrate((e*x+d)^3/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((e*x + d)^3/sqrt(c*x^4 + b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^3}{\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{{\left(d+e\,x\right)}^3}{\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

input
```
int((d + e*x)^3/(a + b*x^2 + c*x^4)^(1/2),x)
```

output
```
int((d + e*x)^3/(a + b*x^2 + c*x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{(d+ex)^3}{\sqrt{a+bx^2+cx^4}}\,dx=\text{too large to display}$$

input
```
int((e*x+d)^3/(c*x^4+b*x^2+a)^(1/2),x)
```

output

```
(4*sqrt(4*a*c - b**2)*sqrt( - 4*a*c + b**2)*atan((4*sqrt(c)*sqrt(a + b*x**
2 + c*x**4)*a*b*c - sqrt(c)*sqrt(a + b*x**2 + c*x**4)*b**3 + 4*a**2*c**2 -
 a*b**2*c + 4*a*b*c**2*x**2 - b**3*c*x**2)/(sqrt(4*a*c - b**2)*sqrt( - 4*a
*c + b**2)*a*c))*a*c*e**3 + 2*sqrt(4*a*c - b**2)*sqrt( - 4*a*c + b**2)*ata
n((4*sqrt(c)*sqrt(a + b*x**2 + c*x**4)*a*b*c - sqrt(c)*sqrt(a + b*x**2 + c
*x**4)*b**3 + 4*a**2*c**2 - a*b**2*c + 4*a*b*c**2*x**2 - b**3*c*x**2)/(sqr
t(4*a*c - b**2)*sqrt( - 4*a*c + b**2)*a*c))*b**2*e**3 + 16*sqrt(c)*int(sqr
t(a + b*x**2 + c*x**4)/(a**2 + 2*a*b*x**2 + a*c*x**4 + b**2*x**4 + b*c*x**
6),x)*a**2*b*c**2*d**3 - 4*sqrt(c)*int(sqrt(a + b*x**2 + c*x**4)/(a**2 + 2
*a*b*x**2 + a*c*x**4 + b**2*x**4 + b*c*x**6),x)*a*b**3*c*d**3 + 16*sqrt(c)
*int((sqrt(a + b*x**2 + c*x**4)*x**5)/(a**2 + 2*a*b*x**2 + a*c*x**4 + b**2
*x**4 + b*c*x**6),x)*a*b**2*c**2*e**3 - 4*sqrt(c)*int((sqrt(a + b*x**2 + c
*x**4)*x**5)/(a**2 + 2*a*b*x**2 + a*c*x**4 + b**2*x**4 + b*c*x**6),x)*b**4
*c*e**3 + 48*sqrt(c)*int((sqrt(a + b*x**2 + c*x**4)*x**4)/(a**2 + 2*a*b*x*
*2 + a*c*x**4 + b**2*x**4 + b*c*x**6),x)*a*b**2*c**2*d*e**2 - 12*sqrt(c)*i
nt((sqrt(a + b*x**2 + c*x**4)*x**4)/(a**2 + 2*a*b*x**2 + a*c*x**4 + b**2*x
**4 + b*c*x**6),x)*b**4*c*d*e**2 + 48*sqrt(c)*int((sqrt(a + b*x**2 + c*x**
4)*x**2)/(a**2 + 2*a*b*x**2 + a*c*x**4 + b**2*x**4 + b*c*x**6),x)*a**2*b*c
**2*d*e**2 - 12*sqrt(c)*int((sqrt(a + b*x**2 + c*x**4)*x**2)/(a**2 + 2*a*b
*x**2 + a*c*x**4 + b**2*x**4 + b*c*x**6),x)*a*b**3*c*d*e**2 + 16*sqrt(c...
```

3.261 $\int \frac{(d+ex)^2}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 334

$$\int \frac{(d+ex)^2}{\sqrt{a+bx^2+cx^4}}\,dx = \frac{e^2x\sqrt{a+bx^2+cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)} + \frac{de\,\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{\sqrt{c}} - \frac{\sqrt[4]{a}e^2\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{c^{3/4}\sqrt{a+bx^2+cx^4}} + \frac{\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}c^{3/4}\sqrt{a+bx^2+cx^4}}$$

output
```
e^2*x*(c*x^4+b*x^2+a)^(1/2)/c^(1/2)/(a^(1/2)+c^(1/2)*x^2)+d*e*arctanh(1/2*
(2*c*x^2+b)/c^(1/2)/(c*x^4+b*x^2+a)^(1/2))/c^(1/2)-a^(1/4)*e^2*(a^(1/2)+c^
(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*EllipticE(sin(2
*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))/c^(3/4)/(c*x^
4+b*x^2+a)^(1/2)+1/2*(c^(1/2)*d^2+a^(1/2)*e^2)*(a^(1/2)+c^(1/2)*x^2)*((c*x
^4+b*x^2+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(c^(1/4
)*x/a^(1/4)),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))/a^(1/4)/c^(3/4)/(c*x^4+b*x^2
+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.20 (sec) , antiderivative size = 490, normalized size of antiderivative = 1.47

$$\int \frac{(d+ex)^2}{\sqrt{a+bx^2+cx^4}}\,dx = \frac{i\left(\left(-b+\sqrt{b^2-4ac}\right)e^2\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{\frac{2b-2\sqrt{b^2-4ac}+4cx^2}{b-\sqrt{b^2-4ac}}}E\left(i\text{arcsinh}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)-\right.}{}$$

input
```
Integrate[(d + e*x)^2/Sqrt[a + b*x^2 + c*x^4],x]
```

output
```
((I/4)*((-b + Sqrt[b^2 - 4*a*c])*e^2*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2
)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b -
 Sqrt[b^2 - 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*
a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] - (2*c*d^2 + (
-b + Sqrt[b^2 - 4*a*c])*e^2)*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + S
qrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^
2 - 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x
], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] + (4*I)*Sqrt[c]*Sqrt[c
/(b + Sqrt[b^2 - 4*a*c])]*d*e*Sqrt[a + b*x^2 + c*x^4]*Log[b + 2*c*x^2 - 2*
Sqrt[c]*Sqrt[a + b*x^2 + c*x^4]]))/(c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt
[a + b*x^2 + c*x^4])
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {2202, 27, 1432, 1092, 219, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^2}{\sqrt{a+bx^2+cx^4}}\,dx$$

$$\Big\downarrow \text{2202}$$

$$\int \frac{d^2+e^2x^2}{\sqrt{cx^4+bx^2+a}}dx+\int \frac{2dex}{\sqrt{cx^4+bx^2+a}}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{d^2+e^2x^2}{\sqrt{cx^4+bx^2+a}}dx+2de\int \frac{x}{\sqrt{cx^4+bx^2+a}}dx$$

$$\Big\downarrow \text{1432}$$

$$\int \frac{d^2+e^2x^2}{\sqrt{cx^4+bx^2+a}}dx+de\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx^2$$

$$\Big\downarrow \text{1092}$$

$$\int \frac{d^2+e^2x^2}{\sqrt{cx^4+bx^2+a}}dx+2de\int \frac{1}{4c-x^4}d\frac{2cx^2+b}{\sqrt{cx^4+bx^2+a}}$$

$$\Big\downarrow \text{219}$$

$$\int \frac{d^2+e^2x^2}{\sqrt{cx^4+bx^2+a}}dx+\frac{de\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{\sqrt{c}}$$

$$\Big\downarrow \text{1511}$$

$$\left(\frac{\sqrt{a}e^2}{\sqrt{c}}+d^2\right)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx-\frac{\sqrt{a}e^2\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{a}\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}+\frac{de\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{\sqrt{c}}$$

$$\Big\downarrow \text{27}$$

$$\left(\frac{\sqrt{a}e^2}{\sqrt{c}}+d^2\right)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx-\frac{e^2\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}+\frac{de\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{\sqrt{c}}$$

$$\Big\downarrow \text{1416}$$

$$-\frac{e^2\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}+\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{\sqrt{a}e^2}{\sqrt{c}}+d^2\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}+\frac{de\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{\sqrt{c}}$$

$$\Big\downarrow \text{1509}$$

$$\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{\sqrt{a}e^2}{\sqrt{c}}+d^2\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}-\frac{e^2\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}-\frac{x\sqrt{a+bx^2+cx^4}}{\sqrt{a}+\sqrt{c}x^2}\right)}{\sqrt{c}}+\frac{de\,\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{\sqrt{c}}$$

input
```
Int[(d + e*x)^2/Sqrt[a + b*x^2 + c*x^4],x]
```

output
```
(d*e*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/Sqrt[c] -
 (e^2*(-((x*Sqrt[a + b*x^2 + c*x^4])/(Sqrt[a] + Sqrt[c]*x^2)) + (a^(1/4)*(
Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]
*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(c
^(1/4)*Sqrt[a + b*x^2 + c*x^4])))/Sqrt[c] + ((d^2 + (Sqrt[a]*e^2)/Sqrt[c])
*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^
2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/
(2*a^(1/4)*c^(1/4)*Sqrt[a + b*x^2 + c*x^4])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1432
```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 2202
```
Int[(Pn_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{n
 = Expon[Pn, x], k}, Int[Sum[Coeff[Pn, x, 2*k]*x^(2*k), {k, 0, n/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pn, x, 2*k + 1]*x^(2*k), {k, 0, (n -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pn, x]
 &&  !PolyQ[Pn, x^2]
```

Maple [A] (verified)

Time = 1.38 (sec) , antiderivative size = 401, normalized size of antiderivative = 1.20

method	result
default	$\frac{d^{2}\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^{2}}\right)x^{2}}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^{2}}\right)x^{2}}{a}}\,\operatorname{EllipticF}\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^{2}}}{a}}}{2},\frac{\sqrt{-4+\frac{2b\left(b+\sqrt{-4ac+b^{2}}\right)}{ac}}}{2}\right)}{4\sqrt{\frac{-b+\sqrt{-4ac+b^{2}}}{a}}\,\sqrt{c\,x^{4}+b\,x^{2}+a}} - \frac{e^{2}a\sqrt{2}\,\sqrt{}}{}$
elliptic	$\frac{d^{2}\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^{2}}\right)x^{2}}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^{2}}\right)x^{2}}{a}}\,\operatorname{EllipticF}\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^{2}}}{a}}}{2},\frac{\sqrt{-4+\frac{2b\left(b+\sqrt{-4ac+b^{2}}\right)}{ac}}}{2}\right)}{4\sqrt{\frac{-b+\sqrt{-4ac+b^{2}}}{a}}\,\sqrt{c\,x^{4}+b\,x^{2}+a}} + \frac{de\ln\left(\frac{2c}{}\right.}{}$

input
```
int((e*x+d)^2/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*d^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1
/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)
^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2
*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/2*e^2*a*2^(1/2)/((-b+(-4*a*c+b^2)^
(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+
b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(Ell
ipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*
a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2
))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)))+d*e*ln((1/2*b+
c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))/c^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.11

$$\int \frac{(d+ex)^2}{\sqrt{a+bx^2+cx^4}}\,dx$$

$$= \frac{ac^{\frac{3}{2}}dex\log\left(8\,c^2x^4+8\,bcx^2+b^2+4\,\sqrt{cx^4+bx^2+a}(2\,cx^2+b)\sqrt{c}+4\,ac\right)+2\,\sqrt{cx^4+bx^2+a}ace^2+\sqrt{}}{}$$

input
```
integrate((e*x+d)^2/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
1/2*(a*c^(3/2)*d*e*x*log(8*c^2*x^4 + 8*b*c*x^2 + b^2 + 4*sqrt(c*x^4 + b*x^
2 + a)*(2*c*x^2 + b)*sqrt(c) + 4*a*c) + 2*sqrt(c*x^4 + b*x^2 + a)*a*c*e^2
+ sqrt(1/2)*(a*c*e^2*x*sqrt((b^2 - 4*a*c)/c^2) - a*b*e^2*x)*sqrt(c)*sqrt((
c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)*elliptic_e(arcsin(sqrt(1/2)*sqrt((c*sqrt
((b^2 - 4*a*c)/c^2) - b)/c)/x), 1/2*(b*c*sqrt((b^2 - 4*a*c)/c^2) + b^2 - 2
*a*c)/(a*c)) + sqrt(1/2)*((c^2*d^2 - a*c*e^2)*x*sqrt((b^2 - 4*a*c)/c^2) +
(b*c*d^2 + a*b*e^2)*x)*sqrt(c)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)*ell
iptic_f(arcsin(sqrt(1/2)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)/x), 1/2*(
b*c*sqrt((b^2 - 4*a*c)/c^2) + b^2 - 2*a*c)/(a*c)))/(a*c^2*x)
```

Sympy [F]

$$\int \frac{(d+ex)^2}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{(d+ex)^2}{\sqrt{a+bx^2+cx^4}}\,dx$$

input
```
integrate((e*x+d)**2/(c*x**4+b*x**2+a)**(1/2),x)
```

output
```
Integral((d + e*x)**2/sqrt(a + b*x**2 + c*x**4), x)
```

Maxima [F]

$$\int \frac{(d+ex)^2}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{(ex+d)^2}{\sqrt{cx^4+bx^2+a}}\,dx$$

input
```
integrate((e*x+d)^2/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((e*x + d)^2/sqrt(c*x^4 + b*x^2 + a), x)
```

Giac [F]

$$\int \frac{(d+ex)^2}{\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{(ex+d)^2}{\sqrt{cx^4+bx^2+a}}\,dx$$

input
```
integrate((e*x+d)^2/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((e*x + d)^2/sqrt(c*x^4 + b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^2}{\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{(d+e\,x)^2}{\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

input
```
int((d + e*x)^2/(a + b*x^2 + c*x^4)^(1/2),x)
```

output
```
int((d + e*x)^2/(a + b*x^2 + c*x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{(d+ex)^2}{\sqrt{a+bx^2+cx^4}}\,dx$$
$$=\frac{-\sqrt{c}\log\left(\sqrt{c\,x^4+b\,x^2+a}-\sqrt{c}\,x^2\right)de+\sqrt{c}\log\left(\sqrt{c\,x^4+b\,x^2+a}+\sqrt{c}\,x^2\right)de+\left(\int \frac{\sqrt{c\,x^4+b\,x^2+a}}{bc\,x^6+ac\,x^4+b^2x^4+2ab\,x^2}\right.}{}$$

input
```
int((e*x+d)^2/(c*x^4+b*x^2+a)^(1/2),x)
```

output

```
( - sqrt(c)*log(sqrt(a + b*x**2 + c*x**4) - sqrt(c)*x**2)*d*e + sqrt(c)*lo
g(sqrt(a + b*x**2 + c*x**4) + sqrt(c)*x**2)*d*e + int(sqrt(a + b*x**2 + c*
x**4)/(a**2 + 2*a*b*x**2 + a*c*x**4 + b**2*x**4 + b*c*x**6),x)*a*c*d**2 +
int((sqrt(a + b*x**2 + c*x**4)*x**4)/(a**2 + 2*a*b*x**2 + a*c*x**4 + b**2*
x**4 + b*c*x**6),x)*b*c*e**2 + int((sqrt(a + b*x**2 + c*x**4)*x**2)/(a**2
+ 2*a*b*x**2 + a*c*x**4 + b**2*x**4 + b*c*x**6),x)*a*c*e**2 + int((sqrt(a
+ b*x**2 + c*x**4)*x**2)/(a**2 + 2*a*b*x**2 + a*c*x**4 + b**2*x**4 + b*c*x
**6),x)*b*c*d**2 - 2*int((sqrt(a + b*x**2 + c*x**4)*x)/(a**2 + 2*a*b*x**2
+ a*c*x**4 + b**2*x**4 + b*c*x**6),x)*a*c*d*e)/c
```

3.262 $\int \frac{d+ex}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 160

$$\int \frac{d+ex}{\sqrt{a+bx^2+cx^4}}\,dx = \frac{e\,\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{c}} + \frac{d\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}$$

output
```
1/2*e*arctanh(1/2*(2*c*x^2+b)/c^(1/2)/(c*x^4+b*x^2+a)^(1/2))/c^(1/2)+1/2*d
*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*Inv
erseJacobiAM(2*arctan(c^(1/4)*x/a^(1/4)),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))/
a^(1/4)/c^(1/4)/(c*x^4+b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.40 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.44

$$\int \frac{d+ex}{\sqrt{a+bx^2+cx^4}}\,dx = -\frac{id\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x\right),\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\sqrt{a+bx^2+cx^4}} - \frac{e\log\left(b+2cx^2-2\sqrt{c}\sqrt{a+bx^2+cx^4}\right)}{2\sqrt{c}}$$

input
```
Integrate[(d + e*x)/Sqrt[a + b*x^2 + c*x^4],x]
```

output
```
((-I)*d*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sq
rt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt
[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*
a*c])])/(Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[a + b*x^2 + c*x^4])
- (e*Log[b + 2*c*x^2 - 2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4]])/(2*Sqrt[c])
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {2202, 27, 1416, 1432, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex}{\sqrt{a+bx^2+cx^4}}dx$$

$$\Big\downarrow \; 2202$$

$$\int \frac{d}{\sqrt{cx^4+bx^2+a}}dx + \int \frac{ex}{\sqrt{cx^4+bx^2+a}}dx$$

$$\Big\downarrow \ 27$$

$$d\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx+e\int \frac{x}{\sqrt{cx^4+bx^2+a}}dx$$

$$\Big\downarrow \ 1416$$

$$e\int \frac{x}{\sqrt{cx^4+bx^2+a}}dx+\frac{d\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}$$

$$\Big\downarrow \ 1432$$

$$\frac{1}{2}e\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx^2+\frac{d\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}$$

$$\Big\downarrow \ 1092$$

$$e\int \frac{1}{4c-x^4}d\frac{2cx^2+b}{\sqrt{cx^4+bx^2+a}}+\frac{d\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}$$

$$\Big\downarrow \ 219$$

$$\frac{d\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}+\frac{e\text{arctanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{c}}$$

input
```
Int[(d + e*x)/Sqrt[a + b*x^2 + c*x^4],x]
```

output
```
(e*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(2*Sqrt[c])
 + (d*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*
x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))
/4])/(2*a^(1/4)*c^(1/4)*Sqrt[a + b*x^2 + c*x^4])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1432
```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

rule 2202
```
Int[(Pn_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{n
 = Expon[Pn, x], k}, Int[Sum[Coeff[Pn, x, 2*k]*x^(2*k), {k, 0, n/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pn, x, 2*k + 1]*x^(2*k), {k, 0, (n -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pn, x]
 &&  !PolyQ[Pn, x^2]
```

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.13

method	result
default	$\frac{d\sqrt{2}\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\text{EllipticF}\left(\frac{x\sqrt{2}\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2},\frac{\sqrt{-4+\frac{2b\left(b+\sqrt{-4ac+b^2}\right)}{ac}}}{2}\right)}{4\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\sqrt{c\,x^4+b\,x^2+a}}+\frac{e\ln\left(\frac{\frac{b}{2}+c}{\sqrt{}}\right.}{}$
elliptic	$\frac{d\sqrt{2}\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\text{EllipticF}\left(\frac{x\sqrt{2}\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2},\frac{\sqrt{-4+\frac{2b\left(b+\sqrt{-4ac+b^2}\right)}{ac}}}{2}\right)}{4\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\sqrt{c\,x^4+b\,x^2+a}}+\frac{e\ln\left(\frac{2c\,x^2}{\sqrt{}}\right.}{}$

input
```
int((e*x+d)/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*d*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2
))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(
1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b
*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))+1/2*e*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+
b*x^2+a)^(1/2))/c^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.18

$$\int \frac{d+ex}{\sqrt{a+bx^2+cx^4}}\,dx$$
$$= \frac{2\sqrt{\frac{1}{2}}\left(cd\sqrt{\frac{b^2-4\,ac}{c^2}}+bd\right)\sqrt{c}\sqrt{\frac{c\sqrt{\frac{b^2-4\,ac}{c^2}}-b}{c}}F(\arcsin\left(\frac{\sqrt{\frac{1}{2}}\sqrt{\frac{c\sqrt{\frac{b^2-4\,ac}{c^2}}-b}{c}}}{x}\right) \mid \frac{bc\sqrt{\frac{b^2-4\,ac}{c^2}}+b^2-2\,ac}{2\,ac})+a\sqrt{c}e\log\left(8\right.}{4\,ac}$$

input
```
integrate((e*x+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
1/4*(2*sqrt(1/2)*(c*d*sqrt((b^2 - 4*a*c)/c^2) + b*d)*sqrt(c)*sqrt((c*sqrt(
(b^2 - 4*a*c)/c^2) - b)/c)*elliptic_f(arcsin(sqrt(1/2)*sqrt((c*sqrt((b^2 -
 4*a*c)/c^2) - b)/c)/x), 1/2*(b*c*sqrt((b^2 - 4*a*c)/c^2) + b^2 - 2*a*c)/(
a*c)) + a*sqrt(c)*e*log(8*c^2*x^4 + 8*b*c*x^2 + b^2 + 4*sqrt(c*x^4 + b*x^2
 + a)*(2*c*x^2 + b)*sqrt(c) + 4*a*c))/(a*c)
```

Sympy [F]

$$\int \frac{d+ex}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{d+ex}{\sqrt{a+bx^2+cx^4}}\,dx$$

input
```
integrate((e*x+d)/(c*x**4+b*x**2+a)**(1/2),x)
```

output
```
Integral((d + e*x)/sqrt(a + b*x**2 + c*x**4), x)
```

Maxima [F]

$$\int \frac{d+ex}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{ex+d}{\sqrt{cx^4+bx^2+a}}\,dx$$

input
```
integrate((e*x+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((e*x + d)/sqrt(c*x^4 + b*x^2 + a), x)
```

Giac [F]

$$\int \frac{d+ex}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{ex+d}{\sqrt{cx^4+bx^2+a}}\,dx$$

input
```
integrate((e*x+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((e*x + d)/sqrt(c*x^4 + b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{d+e\,x}{\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

input
```
int((d + e*x)/(a + b*x^2 + c*x^4)^(1/2),x)
```

output
```
int((d + e*x)/(a + b*x^2 + c*x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{d+ex}{\sqrt{a+bx^2+cx^4}}\,dx$$
$$= \frac{-\sqrt{c}\log\left(\sqrt{c\,x^4+b\,x^2+a}-\sqrt{c}\,x^2\right)e+\sqrt{c}\log\left(\sqrt{c\,x^4+b\,x^2+a}+\sqrt{c}\,x^2\right)e+2\left(\int \frac{\sqrt{c\,x^4+b\,x^2+a}}{bc\,x^6+ac\,x^4+b^2x^4+2ab\,x^2+}\right.}{2c}$$

input
```
int((e*x+d)/(c*x^4+b*x^2+a)^(1/2),x)
```

output
```
( - sqrt(c)*log(sqrt(a + b*x**2 + c*x**4) - sqrt(c)*x**2)*e + sqrt(c)*log(
sqrt(a + b*x**2 + c*x**4) + sqrt(c)*x**2)*e + 2*int(sqrt(a + b*x**2 + c*x*
*4)/(a**2 + 2*a*b*x**2 + a*c*x**4 + b**2*x**4 + b*c*x**6),x)*a*c*d + 2*int
((sqrt(a + b*x**2 + c*x**4)*x**2)/(a**2 + 2*a*b*x**2 + a*c*x**4 + b**2*x**
4 + b*c*x**6),x)*b*c*d - 2*int((sqrt(a + b*x**2 + c*x**4)*x)/(a**2 + 2*a*b
*x**2 + a*c*x**4 + b**2*x**4 + b*c*x**6),x)*a*c*e)/(2*c)
```

3.263 $\int \frac{1}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 114

$$\int \frac{1}{\sqrt{a+bx^2+cx^4}}\,dx = \frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}$$

output
```
1/2*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*
InverseJacobiAM(2*arctan(c^(1/4)*x/a^(1/4)),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2
))/a^(1/4)/c^(1/4)/(c*x^4+b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.63

$$\int \frac{1}{\sqrt{a+bx^2+cx^4}}\,dx = -\frac{i\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x\right),\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\sqrt{a+bx^2+cx^4}}$$

input
```
Integrate[1/Sqrt[a + b*x^2 + c*x^4],x]
```

output
```
((-I)*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt
[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c
/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*
c])])/(Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = $\{1416\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+bx^2+cx^4}}\,dx$$

$$\Big\downarrow \ 1416$$

$$\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}$$

input
```
Int[1/Sqrt[a + b*x^2 + c*x^4],x]
```

output
```
((Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^
2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/
(2*a^(1/4)*c^(1/4)*Sqrt[a + b*x^2 + c*x^4])
```

Defintions of rubi rules used

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.26

method	result	size
default	$\frac{\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^{2}}\right)x^{2}}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^{2}}\right)x^{2}}{a}}\,\mathrm{EllipticF}\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^{2}}}{a}}}{2},\frac{\sqrt{-4+\frac{2b\left(b+\sqrt{-4ac+b^{2}}\right)}{ac}}}{2}\right)}{4\sqrt{\frac{-b+\sqrt{-4ac+b^{2}}}{a}}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}$	144
elliptic	$\frac{\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^{2}}\right)x^{2}}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^{2}}\right)x^{2}}{a}}\,\mathrm{EllipticF}\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^{2}}}{a}}}{2},\frac{\sqrt{-4+\frac{2b\left(b+\sqrt{-4ac+b^{2}}\right)}{ac}}}{2}\right)}{4\sqrt{\frac{-b+\sqrt{-4ac+b^{2}}}{a}}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}$	144

input
```
int(1/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))
/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/
2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(
b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.06

$$\int \frac{1}{\sqrt{a+bx^2+cx^4}}\,dx = -\frac{\sqrt{\frac{1}{2}}\left(a\sqrt{\frac{b^2-4\,ac}{a^2}}+b\right)\sqrt{\frac{a\sqrt{\frac{b^2-4\,ac}{a^2}}-b}{a}}F(\arcsin\left(\sqrt{\frac{1}{2}}x\sqrt{\frac{a\sqrt{\frac{b^2-4\,ac}{a^2}}-b}{a}}\right)\mid\frac{ab\sqrt{\frac{b^2-4\,ac}{a^2}}+b^2-2\,ac}{2\,ac})}{2\,\sqrt{ac}}$$

input
```
integrate(1/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*sqrt(1/2)*(a*sqrt((b^2 - 4*a*c)/a^2) + b)*sqrt((a*sqrt((b^2 - 4*a*c)/
a^2) - b)/a)*elliptic_f(arcsin(sqrt(1/2)*x*sqrt((a*sqrt((b^2 - 4*a*c)/a^2)
 - b)/a)), 1/2*(a*b*sqrt((b^2 - 4*a*c)/a^2) + b^2 - 2*a*c)/(a*c))/(sqrt(a)
*c)
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{1}{\sqrt{a+bx^2+cx^4}}\,dx$$

input
```
integrate(1/(c*x**4+b*x**2+a)**(1/2),x)
```

output
```
Integral(1/sqrt(a + b*x**2 + c*x**4), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{1}{\sqrt{cx^4+bx^2+a}}\,dx$$

input
```
integrate(1/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(c*x^4 + b*x^2 + a), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{1}{\sqrt{cx^4+bx^2+a}}\,dx$$

input
```
integrate(1/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(c*x^4 + b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{1}{\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

input
```
int(1/(a + b*x^2 + c*x^4)^(1/2),x)
```

output
```
int(1/(a + b*x^2 + c*x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{\sqrt{c\,x^4+b\,x^2+a}}{c\,x^4+b\,x^2+a}dx$$

input
```
int(1/(c*x^4+b*x^2+a)^(1/2),x)
```

output
```
int(sqrt(a + b*x**2 + c*x**4)/(a + b*x**2 + c*x**4),x)
```

3.264 $\int \frac{1}{(d+ex)\sqrt{a+bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 512

$$\int \frac{1}{(d+ex)\sqrt{a+bx^2+cx^4}}\,dx = \frac{\operatorname{earctanh}\left(\frac{\sqrt{cd^4+bd^2e^2+ae^4}\,x}{de\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{cd^4+bd^2e^2+ae^4}} - \frac{\operatorname{earctanh}\left(\frac{bd^2+2ae^2+(2cd^2+be^2)x^2}{2\sqrt{cd^4+bd^2e^2+ae^4}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{cd^4+bd^2e^2+ae^4}}$$
$$+ \frac{\sqrt[4]{c}d\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\sqrt{a+bx^2+cx^4}}$$
$$- \frac{\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticPi}\left(\frac{\left(\sqrt{c}d^2+\sqrt{a}e^2\right)^2}{4\sqrt{a}\sqrt{c}d^2e^2},2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{4\sqrt[4]{a}\sqrt[4]{c}d\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\sqrt{a+bx^2+cx^4}}$$

output
```
1/2*e*arctanh((a*e^4+b*d^2*e^2+c*d^4)^(1/2)*x/d/e/(c*x^4+b*x^2+a)^(1/2))/(
a*e^4+b*d^2*e^2+c*d^4)^(1/2)-1/2*e*arctanh(1/2*(b*d^2+2*a*e^2+(b*e^2+2*c*d
^2)*x^2)/(a*e^4+b*d^2*e^2+c*d^4)^(1/2)/(c*x^4+b*x^2+a)^(1/2))/(a*e^4+b*d^2
*e^2+c*d^4)^(1/2)+1/2*c^(1/4)*d*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^
(1/2)+c^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(c^(1/4)*x/a^(1/4)),1/
2*(2-b/a^(1/2)/c^(1/2))^(1/2))/a^(1/4)/(c^(1/2)*d^2+a^(1/2)*e^2)/(c*x^4+b*
x^2+a)^(1/2)-1/4*(c^(1/2)*d^2-a^(1/2)*e^2)*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+b
*x^2+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*EllipticPi(sin(2*arctan(c^(1/4)*x/a
^(1/4))),1/4*(c^(1/2)*d^2+a^(1/2)*e^2)^2/a^(1/2)/c^(1/2)/d^2/e^2,1/2*(2-b/
a^(1/2)/c^(1/2))^(1/2))/a^(1/4)/c^(1/4)/d/(c^(1/2)*d^2+a^(1/2)*e^2)/(c*x^4
+b*x^2+a)^(1/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1725 vs. $2(512) = 1024$.

Time = 17.09 (sec) , antiderivative size = 1725, normalized size of antiderivative = 3.37

$$\int \frac{1}{(d+ex)\sqrt{a+bx^2+cx^4}}\,dx = \text{Too large to display}$$

input
```
Integrate[1/((d + e*x)*Sqrt[a + b*x^2 + c*x^4]),x]
```

output
```
(2*(Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/Sqrt[2] + Sqrt[-(b/c) + Sqrt[b^2 -
4*a*c]/c]/Sqrt[2])*(-(Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/Sqrt[2]) + x)^2*S
qrt[(Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c]*(-(Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c
]/Sqrt[2]) + x))/((Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/Sqrt[2] + Sqrt[-(b/c
) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2])*(-(Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/Sq
rt[2]) + x))]*Sqrt[(Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c]*(Sqrt[-(b/c) + Sqrt[b
^2 - 4*a*c]/c]/Sqrt[2] + x))/((Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/Sqrt[2]
- Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2])*(-(Sqrt[-(b/c) - Sqrt[b^2 -
4*a*c]/c]/Sqrt[2]) + x))]*Sqrt[((Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c] - Sqrt[(
-b + Sqrt[b^2 - 4*a*c])/c])*(Sqrt[2]*Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c] + 2*
x))/((Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c] + Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c])
*(Sqrt[2]*Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c] - 2*x))]*((-d + (Sqrt[-(b/c) -
Sqrt[b^2 - 4*a*c]/c]*e)/Sqrt[2])*EllipticF[ArcSin[Sqrt[((Sqrt[(-b - Sqrt[b
^2 - 4*a*c])/c] - Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c])*(Sqrt[2]*Sqrt[(-b - Sq
rt[b^2 - 4*a*c])/c] + 2*x))/((Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c] + Sqrt[(-b
+ Sqrt[b^2 - 4*a*c])/c])*(Sqrt[2]*Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c] - 2*x))
]], (Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c] + Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c])^
2/(Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c] - Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c])^2]
 - Sqrt[2]*Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c]*e*EllipticPi[((Sqrt[-(b/c) - S
qrt[b^2 - 4*a*c]/c]/Sqrt[2] + Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2...
```

Rubi [A] (verified)

Time = 1.47 (sec) , antiderivative size = 540, normalized size of antiderivative = 1.05, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2262, 1540, 27, 1416, 1576, 1154, 219, 2222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex)\sqrt{a+bx^2+cx^4}}\,dx$$

$$\Big\downarrow \text{ 2262}$$

$$d\int \frac{1}{(d^2-e^2x^2)\sqrt{cx^4+bx^2+a}}dx - e\int \frac{x}{(d^2-e^2x^2)\sqrt{cx^4+bx^2+a}}dx$$

$$\Big\downarrow \text{ 1540}$$

$$d\left(\frac{\sqrt{a}e^2\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a}(d^2-e^2x^2)\sqrt{cx^4+bx^2+a}}dx}{\sqrt{a}e^2+\sqrt{c}d^2}+\frac{\sqrt{c}\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{a}e^2+\sqrt{c}d^2}\right)-e\int \frac{x}{(d^2-e^2x^2)\sqrt{cx^4+bx^2+a}}dx$$

$$\Big\downarrow \text{ 27}$$

$$d\left(\frac{e^2\int \frac{\sqrt{c}x^2+\sqrt{a}}{(d^2-e^2x^2)\sqrt{cx^4+bx^2+a}}dx}{\sqrt{a}e^2+\sqrt{c}d^2}+\frac{\sqrt{c}\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{a}e^2+\sqrt{c}d^2}\right)-e\int \frac{x}{(d^2-e^2x^2)\sqrt{cx^4+bx^2+a}}dx$$

$$\Big\downarrow \text{ 1416}$$

$$d\left(\frac{e^2\int \frac{\sqrt{c}x^2+\sqrt{a}}{(d^2-e^2x^2)\sqrt{cx^4+bx^2+a}}dx}{\sqrt{a}e^2+\sqrt{c}d^2}+\frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\sqrt{a+bx^2+cx^4}\left(\sqrt{a}e^2+\sqrt{c}d^2\right)}\right)-e\int \frac{x}{(d^2-e^2x^2)\sqrt{cx^4+bx^2+a}}dx$$

$$\Big\downarrow \text{ 1576}$$

$$d\left(\frac{e^2\int \frac{\sqrt{c}x^2+\sqrt{a}}{(d^2-e^2x^2)\sqrt{cx^4+bx^2+a}}dx}{\sqrt{a}e^2+\sqrt{c}d^2}+\frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\sqrt{a+bx^2+cx^4}\left(\sqrt{a}e^2+\sqrt{c}d^2\right)}\right)-\frac{1}{2}e\int \frac{1}{(d^2-e^2x^2)\sqrt{cx^4+bx^2+a}}dx^2$$

$$\Big\downarrow \text{ 1154}$$

$$d\left(\frac{e^2\int \frac{\sqrt{c}x^2+\sqrt{a}}{(d^2-e^2x^2)\sqrt{cx^4+bx^2+a}}dx}{\sqrt{a}e^2+\sqrt{c}d^2}+\frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\sqrt{a+bx^2+cx^4}\left(\sqrt{a}e^2+\sqrt{c}d^2\right)}\right)-e\int \frac{1}{4\left(cd^4+be^2d^2+ae^4\right)-x^4}d\left(-\frac{bd^2+2ae^2+\left(2cd^2+be^2\right)x^2}{\sqrt{cx^4+bx^2+a}}\right)$$

$$\Big\downarrow \text{ 219}$$

$$d\left(\frac{e^2\int\frac{\sqrt{c}x^2+\sqrt{a}}{(d^2-e^2x^2)\sqrt{cx^4+bx^2+a}}dx}{\sqrt{a}e^2+\sqrt{c}d^2}+\frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\sqrt{a+bx^2+cx^4}\left(\sqrt{a}e^2+\sqrt{c}d^2\right)}\right)$$

$$\frac{e\operatorname{arctanh}\left(\frac{2ae^2+x^2(be^2+2cd^2)+bd^2}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^4+bd^2e^2+cd^4}}\right)}{2\sqrt{ae^4+bd^2e^2+cd^4}}$$

$\big\downarrow$ 2222

$$d\left(\frac{e^2\left(\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{\sqrt{a}}{d^2}-\frac{\sqrt{c}}{e^2}\right)\operatorname{EllipticPi}\left(\frac{\left(\sqrt{c}d^2+\sqrt{a}e^2\right)^2}{4\sqrt{a}\sqrt{c}d^2e^2},2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{4\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}+\frac{\left(\sqrt{a}e^2+\sqrt{c}d^2\right)\operatorname{arctanh}\left(\frac{x\cdots}{\cdots}\right)}{2de\sqrt{ae^4+bd^2e^2\cdots}}\right.}{\sqrt{a}e^2+\sqrt{c}d^2}\right.$$

$$\frac{e\operatorname{arctanh}\left(\frac{2ae^2+x^2(be^2+2cd^2)+bd^2}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^4+bd^2e^2+cd^4}}\right)}{2\sqrt{ae^4+bd^2e^2+cd^4}}$$

input
```
Int[1/((d + e*x)*Sqrt[a + b*x^2 + c*x^4]),x]
```

output
```
-1/2*(e*ArcTanh[(b*d^2 + 2*a*e^2 + (2*c*d^2 + b*e^2)*x^2)/(2*Sqrt[c*d^4 +
b*d^2*e^2 + a*e^4]*Sqrt[a + b*x^2 + c*x^4])])/Sqrt[c*d^4 + b*d^2*e^2 + a*e
^4] + d*((c^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a
] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[
a]*Sqrt[c]))/4])/(2*a^(1/4)*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*Sqrt[a + b*x^2 + c
*x^4]) + (e^2*(((Sqrt[c]*d^2 + Sqrt[a]*e^2)*ArcTanh[(Sqrt[c*d^4 + b*d^2*e^
2 + a*e^4]*x)/(d*e*Sqrt[a + b*x^2 + c*x^4])])/(2*d*e*Sqrt[c*d^4 + b*d^2*e^
2 + a*e^4]) + ((Sqrt[a]/d^2 - Sqrt[c]/e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a
 + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticPi[(Sqrt[c]*d^2 + Sqr
t[a]*e^2)^2/(4*Sqrt[a]*Sqrt[c]*d^2*e^2), 2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2
 - b/(Sqrt[a]*Sqrt[c]))/4])/(4*a^(1/4)*c^(1/4)*Sqrt[a + b*x^2 + c*x^4])))/
(Sqrt[c]*d^2 + Sqrt[a]*e^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1540
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Simp[(c*d + a*e*q)/(c*d^2 - a*e^2)   Int[1
/Sqrt[a + b*x^2 + c*x^4], x], x] - Simp[(a*e*(e + d*q))/(c*d^2 - a*e^2)   I
nt[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

rule 1576
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x]
, x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

rule 2222
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTanh[Rt[b - c*(d/e) - a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
b - c*(d/e) - a*(e/d), 2])), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a +
 b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*d*e*q*Sqrt[a + b*x^2 + c*x^4]))*Ell
ipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x], 1/2 - b/(4*a*q^2)], x]
/; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &&
 EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[-b + c*(d/e) + a*(e/d)]
```

rule 2262
```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_Sym
bol] :> Simp[d   Int[1/((d^2 - e^2*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x] -
Simp[e   Int[x/((d^2 - e^2*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x] /; FreeQ[{
a, b, c, d, e}, x]
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 281, normalized size of antiderivative = 0.55

method	result
default	$\frac{-\frac{\operatorname{arctanh}\left(\frac{\frac{2c\,x^2 d^2}{e^2}+\frac{b\,d^2}{e^2}+b\,x^2+2a}{2\sqrt{\frac{c\,d^4}{e^4}+\frac{b\,d^2}{e^2}+a}\,\sqrt{c\,x^4+b\,x^2+a}}\right)}{2\sqrt{\frac{c\,d^4}{e^4}+\frac{b\,d^2}{e^2}+a}}+\frac{\sqrt{2}\,e\sqrt{1-\frac{\left(-b+\sqrt{-4ac+b^2}\right)x^2}{2a}}\,\sqrt{1+\frac{\left(b+\sqrt{-4ac+b^2}\right)x^2}{2a}}\,\operatorname{EllipticPi}\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2},\frac{}{(}\right.}{\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,d\sqrt{c\,x^4+b\,x^2+a}}}{e}$
elliptic	$\frac{-\frac{\operatorname{arctanh}\left(\frac{\frac{2c\,x^2 d^2}{e^2}+\frac{b\,d^2}{e^2}+b\,x^2+2a}{2\sqrt{\frac{c\,d^4}{e^4}+\frac{b\,d^2}{e^2}+a}\,\sqrt{c\,x^4+b\,x^2+a}}\right)}{2\sqrt{\frac{c\,d^4}{e^4}+\frac{b\,d^2}{e^2}+a}}+\frac{\sqrt{2}\,e\sqrt{1-\frac{\left(-b+\sqrt{-4ac+b^2}\right)x^2}{2a}}\,\sqrt{1+\frac{\left(b+\sqrt{-4ac+b^2}\right)x^2}{2a}}\,\operatorname{EllipticPi}\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2},\frac{}{(}\right.}{\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,d\sqrt{c\,x^4+b\,x^2+a}}}{e}$

input
```
int(1/(e*x+d)/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/e*(-1/2/(c*d^4/e^4+b*d^2/e^2+a)^(1/2)*arctanh(1/2*(2*c*x^2*d^2/e^2+b*d^2
/e^2+b*x^2+2*a)/(c*d^4/e^4+b*d^2/e^2+a)^(1/2)/(c*x^4+b*x^2+a)^(1/2))+2^(1/
2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)/d*e*(1-1/2*(-b+(-4*a*c+b^2)^(1/2))/a*
x^2)^(1/2)*(1+1/2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2
)*EllipticPi(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),2/(-b+(-4*a*c
+b^2)^(1/2))*a/d^2*e^2,(-1/2*(b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*2^(1/2)/((-b+
(-4*a*c+b^2)^(1/2))/a)^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)\sqrt{a+bx^2+cx^4}}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(d+ex)\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{1}{(d+ex)\,\sqrt{a+bx^2+cx^4}}\,dx$$

input
```
integrate(1/(e*x+d)/(c*x**4+b*x**2+a)**(1/2),x)
```

output
```
Integral(1/((d + e*x)*sqrt(a + b*x**2 + c*x**4)), x)
```

Maxima [F]

$$\int \frac{1}{(d+ex)\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{1}{\sqrt{cx^4+bx^2+a}(ex+d)}\,dx$$

input
```
integrate(1/(e*x+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(c*x^4 + b*x^2 + a)*(e*x + d)), x)
```

Giac [F]

$$\int \frac{1}{(d+ex)\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{1}{\sqrt{cx^4+bx^2+a}(ex+d)}\,dx$$

input
```
integrate(1/(e*x+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(c*x^4 + b*x^2 + a)*(e*x + d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{1}{(d+e\,x)\,\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

input
```
int(1/((d + e*x)*(a + b*x^2 + c*x^4)^(1/2)),x)
```

output
```
int(1/((d + e*x)*(a + b*x^2 + c*x^4)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{(d+ex)\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{1}{(ex+d)\sqrt{c\,x^4+b\,x^2+a}}dx$$

input
```
int(1/(e*x+d)/(c*x^4+b*x^2+a)^(1/2),x)
```

output
```
int(1/(e*x+d)/(c*x^4+b*x^2+a)^(1/2),x)
```

3.265 $\int \frac{1}{(d+ex)^2\sqrt{a+bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 816

$$
\begin{aligned}
\int \frac{1}{(d+ex)^2\sqrt{a+bx^2+cx^4}}\,dx &= -\frac{e^3\sqrt{a+bx^2+cx^4}}{(cd^4+bd^2e^2+ae^4)(d+ex)} \\
&+\frac{\sqrt{c}e^2x\sqrt{a+bx^2+cx^4}}{(cd^4+bd^2e^2+ae^4)\left(\sqrt{a}+\sqrt{c}x^2\right)}+\frac{de(2cd^2+be^2)\operatorname{arctanh}\left(\frac{\sqrt{cd^4+bd^2e^2+ae^4}x}{de\sqrt{a+bx^2+cx^4}}\right)}{2\left(cd^4+bd^2e^2+ae^4\right)^{3/2}} \\
&-\frac{de(2cd^2+be^2)\operatorname{arctanh}\left(\frac{bd^2+2ae^2+(2cd^2+be^2)x^2}{2\sqrt{cd^4+bd^2e^2+ae^4}\sqrt{a+bx^2+cx^4}}\right)}{2\left(cd^4+bd^2e^2+ae^4\right)^{3/2}} \\
&-\frac{\sqrt[4]{a}\sqrt[4]{c}e^2\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{(cd^4+bd^2e^2+ae^4)\sqrt{a+bx^2+cx^4}} \\
&+\frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\sqrt{a+bx^2+cx^4}} \\
&-\frac{\left(\sqrt{c}d^2-\sqrt{a}e^2\right)(2cd^2+be^2)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticPi}\left(\frac{\left(\sqrt{c}d^2+\sqrt{a}e^2\right)^2}{4\sqrt{a}\sqrt{c}d^2e^2},2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\right.}{4\sqrt[4]{a}\sqrt[4]{c}\left(\sqrt{c}d^2+\sqrt{a}e^2\right)(cd^4+bd^2e^2+ae^4)\sqrt{a+bx^2+cx^4}}
\end{aligned}
$$

output
```
-e^3*(c*x^4+b*x^2+a)^(1/2)/(a*e^4+b*d^2*e^2+c*d^4)/(e*x+d)+c^(1/2)*e^2*x*(
c*x^4+b*x^2+a)^(1/2)/(a*e^4+b*d^2*e^2+c*d^4)/(a^(1/2)+c^(1/2)*x^2)+1/2*d*e
*(b*e^2+2*c*d^2)*arctanh((a*e^4+b*d^2*e^2+c*d^4)^(1/2)*x/d/e/(c*x^4+b*x^2+
a)^(1/2))/(a*e^4+b*d^2*e^2+c*d^4)^(3/2)-1/2*d*e*(b*e^2+2*c*d^2)*arctanh(1/
2*(b*d^2+2*a*e^2+(b*e^2+2*c*d^2)*x^2)/(a*e^4+b*d^2*e^2+c*d^4)^(1/2)/(c*x^4
+b*x^2+a)^(1/2))/(a*e^4+b*d^2*e^2+c*d^4)^(3/2)-a^(1/4)*c^(1/4)*e^2*(a^(1/2
)+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*EllipticE(s
in(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))/(a*e^4+b*
d^2*e^2+c*d^4)/(c*x^4+b*x^2+a)^(1/2)+1/2*c^(1/4)*(a^(1/2)+c^(1/2)*x^2)*((c
*x^4+b*x^2+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(c^(1
/4)*x/a^(1/4)),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))/a^(1/4)/(c^(1/2)*d^2+a^(1/
2)*e^2)/(c*x^4+b*x^2+a)^(1/2)-1/4*(c^(1/2)*d^2-a^(1/2)*e^2)*(b*e^2+2*c*d^2
)*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*El
lipticPi(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/4*(c^(1/2)*d^2+a^(1/2)*e^2)^2/
a^(1/2)/c^(1/2)/d^2/e^2,1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))/a^(1/4)/c^(1/4)/(
c^(1/2)*d^2+a^(1/2)*e^2)/(a*e^4+b*d^2*e^2+c*d^4)/(c*x^4+b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 18.15 (sec) , antiderivative size = 4019, normalized size of antiderivative = 4.93

$$\int \frac{1}{(d+ex)^2\sqrt{a+bx^2+cx^4}}\,dx = \text{Result too large to show}$$

input
```
Integrate[1/((d + e*x)^2*Sqrt[a + b*x^2 + c*x^4]),x]
```

output
```
-((e^3*Sqrt[a + b*x^2 + c*x^4])/((c*d^4 + b*d^2*e^2 + a*e^4)*(d + e*x))) +
 (((I/2)*(-b + Sqrt[b^2 - 4*a*c])*e^2*Sqrt[1 - (2*c*x^2)/(-b - Sqrt[b^2 -
4*a*c])]*Sqrt[1 - (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]*(EllipticE[I*ArcSinh
[Sqrt[2]*Sqrt[-(c/(-b - Sqrt[b^2 - 4*a*c]))]*x], (-b - Sqrt[b^2 - 4*a*c])/
(-b + Sqrt[b^2 - 4*a*c])] - EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[-(c/(-b - Sqr
t[b^2 - 4*a*c]))]*x], (-b - Sqrt[b^2 - 4*a*c])/(-b + Sqrt[b^2 - 4*a*c])]))
/(Sqrt[2]*Sqrt[-(c/(-b - Sqrt[b^2 - 4*a*c]))]*Sqrt[a + b*x^2 + c*x^4]) + (
I*c*d^2*Sqrt[1 - (2*c*x^2)/(-b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 - (2*c*x^2)/(-
b + Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[-(c/(-b - Sqrt[b^
2 - 4*a*c]))]*x], (-b - Sqrt[b^2 - 4*a*c])/(-b + Sqrt[b^2 - 4*a*c])])/(Sqr
t[2]*Sqrt[-(c/(-b - Sqrt[b^2 - 4*a*c]))]*Sqrt[a + b*x^2 + c*x^4]) + (4*c*(
Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/Sqrt[2] + Sqrt[-(b/c) + Sqrt[b^2 - 4*a*
c]/c]/Sqrt[2])*d^3*(-(Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/Sqrt[2]) + x)^2*S
qrt[(Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c]*(-(Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c
]/Sqrt[2]) + x))/((Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/Sqrt[2] + Sqrt[-(b/c
) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2])*(-(Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/Sq
rt[2]) + x))]*Sqrt[(Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c]*(Sqrt[-(b/c) + Sqrt[b
^2 - 4*a*c]/c]/Sqrt[2] + x))/((Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/Sqrt[2]
- Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2])*(-(Sqrt[-(b/c) - Sqrt[b^2 -
4*a*c]/c]/Sqrt[2]) + x))]*Sqrt[((Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c] - Sqr...
```

Rubi [A] (verified)

Time = 2.93 (sec) , antiderivative size = 814, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {2264, 25, 2279, 27, 1576, 1154, 219, 2232, 25, 27, 1509, 2226, 27, 1416, 2222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex)^2\sqrt{a+bx^2+cx^4}}\,dx$$

$$\Big\downarrow \text{2264}$$

$$-\frac{\int -\frac{ce^3x^3+cde^2x^2-cd^2ex+d(cd^2+be^2)}{(d+ex)\sqrt{cx^4+bx^2+a}}dx}{ae^4+bd^2e^2+cd^4}-\frac{e^3\sqrt{a+bx^2+cx^4}}{(d+ex)\left(ae^4+bd^2e^2+cd^4\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{ce^3x^3+cde^2x^2-cd^2ex+d(cd^2+be^2)}{(d+ex)\sqrt{cx^4+bx^2+a}}dx}{ae^4+bd^2e^2+cd^4}-\frac{e^3\sqrt{a+bx^2+cx^4}}{(d+ex)\left(ae^4+bd^2e^2+cd^4\right)}$$

$$\Big\downarrow \; 2279$$

$$\frac{\int \frac{-ce^4x^4+2cd^2e^2x^2+d^2(cd^2+be^2)}{(d^2-e^2x^2)\sqrt{cx^4+bx^2+a}}dx+\int \frac{(-ced^3-e(cd^2+be^2)d)x}{(d^2-e^2x^2)\sqrt{cx^4+bx^2+a}}dx}{ae^4+bd^2e^2+cd^4}-\frac{e^3\sqrt{a+bx^2+cx^4}}{(d+ex)\left(ae^4+bd^2e^2+cd^4\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{-ce^4x^4+2cd^2e^2x^2+d^2(cd^2+be^2)}{(d^2-e^2x^2)\sqrt{cx^4+bx^2+a}}dx-de\left(be^2+2cd^2\right)\int \frac{x}{(d^2-e^2x^2)\sqrt{cx^4+bx^2+a}}dx}{ae^4+bd^2e^2+cd^4}-\frac{e^3\sqrt{a+bx^2+cx^4}}{(d+ex)\left(ae^4+bd^2e^2+cd^4\right)}$$

$$\Big\downarrow \; 1576$$

$$\frac{\int \frac{-ce^4x^4+2cd^2e^2x^2+d^2(cd^2+be^2)}{(d^2-e^2x^2)\sqrt{cx^4+bx^2+a}}dx-\frac{1}{2}de\left(be^2+2cd^2\right)\int \frac{1}{(d^2-e^2x^2)\sqrt{cx^4+bx^2+a}}dx^2}{ae^4+bd^2e^2+cd^4}-\frac{e^3\sqrt{a+bx^2+cx^4}}{(d+ex)\left(ae^4+bd^2e^2+cd^4\right)}$$

$$\Big\downarrow \; 1154$$

$$\frac{\int \frac{-ce^4x^4+2cd^2e^2x^2+d^2(cd^2+be^2)}{(d^2-e^2x^2)\sqrt{cx^4+bx^2+a}}dx+de\left(be^2+2cd^2\right)\int \frac{1}{4(cd^4+be^2d^2+ae^4)-x^4}d\left(-\frac{bd^2+2ae^2+(2cd^2+be^2)x^2}{\sqrt{cx^4+bx^2+a}}\right)}{ae^4+bd^2e^2+cd^4}-\frac{e^3\sqrt{a+bx^2+cx^4}}{(d+ex)\left(ae^4+bd^2e^2+cd^4\right)}$$

$$\Big\downarrow \; 219$$

$$\frac{\int \frac{-ce^4x^4+2cd^2e^2x^2+d^2(cd^2+be^2)}{(d^2-e^2x^2)\sqrt{cx^4+bx^2+a}}dx-\frac{de(be^2+2cd^2)\text{arctanh}\left(\frac{2ae^2+x^2\left(be^2+2cd^2\right)+bd^2}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^4+bd^2e^2+cd^4}}\right)}{2\sqrt{ae^4+bd^2e^2+cd^4}}}{ae^4+bd^2e^2+cd^4}-\frac{e^3\sqrt{a+bx^2+cx^4}}{(d+ex)\left(ae^4+bd^2e^2+cd^4\right)}$$

$$\Big\downarrow \; 2232$$

$$\frac{-\frac{\int -\frac{ce^2\left(\left(cd^2+be^2+\sqrt{a}\sqrt{c}e^2\right)d^2+\sqrt{c}e^2\left(\sqrt{c}d^2-\sqrt{a}e^2\right)x^2\right)}{\left(d^2-e^2x^2\right)\sqrt{cx^4+bx^2+a}}dx}{ce^2}-\sqrt{a}\sqrt{c}e^2\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{a}\sqrt{cx^4+bx^2+a}}dx-\frac{de(be^2+2cd^2)\text{arctanh}\left(\frac{2ae^2+x^2\left(be^2+\cdots\right.}{2\sqrt{a+bx^2+cx^4}\sqrt{a\cdots}}\right.}{2\sqrt{ae^4+bd^2e^2+cd^4}}}{ae^4+bd^2e^2+cd^4}-\frac{e^3\sqrt{a+bx^2+cx^4}}{(d+ex)\left(ae^4+bd^2e^2+cd^4\right)}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{\int \frac{ce^2\left(\left(cd^2+be^2+\sqrt{a}\sqrt{c}e^2\right)d^2+\sqrt{c}e^2\left(\sqrt{c}d^2-\sqrt{a}e^2\right)x^2\right)}{\left(d^2-e^2x^2\right)\sqrt{cx^4+bx^2+a}}dx}{ce^2}-\sqrt{a}\sqrt{c}e^2\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{a}\sqrt{cx^4+bx^2+a}}dx-\frac{de\left(be^2+2cd^2\right)\text{arctanh}\left(\frac{2ae^2+x^2\left(be^2+2cd^2\right)\ldots}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^4+\ldots}}\right.}{2\sqrt{ae^4+bd^2e^2+cd^4}}}{ae^4+bd^2e^2+cd^4}$$
$$\frac{e^3\sqrt{a+bx^2+cx^4}}{(d+ex)\left(ae^4+bd^2e^2+cd^4\right)}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{\left(cd^2+be^2+\sqrt{a}\sqrt{c}e^2\right)d^2+\sqrt{c}e^2\left(\sqrt{c}d^2-\sqrt{a}e^2\right)x^2}{\left(d^2-e^2x^2\right)\sqrt{cx^4+bx^2+a}}dx-\sqrt{c}e^2\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+bx^2+a}}dx-\frac{de\left(be^2+2cd^2\right)\text{arctanh}\left(\frac{2ae^2+x^2\left(be^2+2cd^2\right)+bd^2}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^4+bd^2e^2+c\ldots}}\right.}{2\sqrt{ae^4+bd^2e^2+cd^4}}}{ae^4+bd^2e^2+cd^4}$$
$$\frac{e^3\sqrt{a+bx^2+cx^4}}{(d+ex)\left(ae^4+bd^2e^2+cd^4\right)}$$

$$\Big\downarrow \ 1509$$

$$\frac{\int \frac{\left(cd^2+be^2+\sqrt{a}\sqrt{c}e^2\right)d^2+\sqrt{c}e^2\left(\sqrt{c}d^2-\sqrt{a}e^2\right)x^2}{\left(d^2-e^2x^2\right)\sqrt{cx^4+bx^2+a}}dx-\sqrt{c}e^2\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}-\frac{x\ldots}{\ldots}\right.}{ae^4+bd^2e^2+cd^4}$$
$$\frac{e^3\sqrt{a+bx^2+cx^4}}{(d+ex)\left(ae^4+bd^2e^2+cd^4\right)}$$

$$\Big\downarrow \ 2226$$

$$\frac{\frac{\sqrt{a}d^2e^2\left(be^2+2cd^2\right)\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a}\left(d^2-e^2x^2\right)\sqrt{cx^4+bx^2+a}}dx}{\sqrt{a}e^2+\sqrt{c}d^2}+\frac{\sqrt{c}\left(ae^4+bd^2e^2+cd^4\right)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{a}e^2+\sqrt{c}d^2}-\sqrt{c}e^2\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(\ldots\right)}{\sqrt[4]{c}\sqrt{a+\ldots}}\right.}{ae^4+bd^2e^2+cd^4}$$
$$\frac{e^3\sqrt{a+bx^2+cx^4}}{(d+ex)\left(ae^4+bd^2e^2+cd^4\right)}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{d^2e^2(be^2+2cd^2)\int \frac{\sqrt{c}x^2+\sqrt{a}}{(d^2-e^2x^2)\sqrt{cx^4+bx^2+a}}dx}{\sqrt{a}e^2+\sqrt{c}d^2}+\frac{\sqrt{c}(ae^4+bd^2e^2+cd^4)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{a}e^2+\sqrt{c}d^2}-\sqrt{c}e^2\left(\frac{\sqrt[4]{a}(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}E\left(2\,\mathrm{arc}\right.}{\sqrt[4]{c}\sqrt{a+bx^2+c}}\right.}{ae^4+bd^2e^2+cd^4}$$

$$\frac{e^3\sqrt{a+bx^2+cx^4}}{(d+ex)(ae^4+bd^2e^2+cd^4)}$$

$\big\downarrow$ 1416

$$\frac{\frac{d^2e^2(be^2+2cd^2)\int \frac{\sqrt{c}x^2+\sqrt{a}}{(d^2-e^2x^2)\sqrt{cx^4+bx^2+a}}dx}{\sqrt{a}e^2+\sqrt{c}d^2}+\frac{\sqrt[4]{c}(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}(ae^4+bd^2e^2+cd^4)\,\mathrm{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right.}{2\sqrt[4]{a}\sqrt{a+bx^2+cx^4}(\sqrt{a}e^2+\sqrt{c}d^2)}}{}$$

$$\frac{e^3\sqrt{a+bx^2+cx^4}}{(d+ex)(ae^4+bd^2e^2+cd^4)}$$

$\big\downarrow$ 2222

$$\frac{-\sqrt{c}\left(\frac{\sqrt[4]{a}(\sqrt{c}x^2+\sqrt{a})\sqrt{\frac{cx^4+bx^2+a}{(\sqrt{c}x^2+\sqrt{a})^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{\sqrt[4]{c}\sqrt{cx^4+bx^2+a}}-\frac{x\sqrt{cx^4+bx^2+a}}{\sqrt{c}x^2+\sqrt{a}}\right)e^2+\frac{d^2(2cd^2+be^2)\left(\frac{(\sqrt{c}d^2+\sqrt{a}e^2)\mathrm{arct}}{2de\sqrt{}}\right.}{}}{}$$

$$\frac{e^3\sqrt{cx^4+bx^2+a}}{(cd^4+be^2d^2+ae^4)(d+ex)}$$

input
```
Int[1/((d + e*x)^2*Sqrt[a + b*x^2 + c*x^4]),x]
```

output
```
-((e^3*Sqrt[a + b*x^2 + c*x^4])/((c*d^4 + b*d^2*e^2 + a*e^4)*(d + e*x))) +
 (-1/2*(d*e*(2*c*d^2 + b*e^2)*ArcTanh[(b*d^2 + 2*a*e^2 + (2*c*d^2 + b*e^2)
*x^2)/(2*Sqrt[c*d^4 + b*d^2*e^2 + a*e^4]*Sqrt[a + b*x^2 + c*x^4])])/Sqrt[c
*d^4 + b*d^2*e^2 + a*e^4] - Sqrt[c]*e^2*(-((x*Sqrt[a + b*x^2 + c*x^4])/(Sq
rt[a] + Sqrt[c]*x^2)) + (a^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 +
 c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)]
, (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(c^(1/4)*Sqrt[a + b*x^2 + c*x^4])) + (c^(1
/4)*(c*d^4 + b*d^2*e^2 + a*e^4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 +
c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)],
 (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*a^(1/4)*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*Sqrt
[a + b*x^2 + c*x^4]) + (d^2*e^2*(2*c*d^2 + b*e^2)*(((Sqrt[c]*d^2 + Sqrt[a]
*e^2)*ArcTanh[(Sqrt[c*d^4 + b*d^2*e^2 + a*e^4]*x)/(d*e*Sqrt[a + b*x^2 + c*
x^4])])/(2*d*e*Sqrt[c*d^4 + b*d^2*e^2 + a*e^4]) + ((Sqrt[a]/d^2 - Sqrt[c]/
e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x
^2)^2]*EllipticPi[(Sqrt[c]*d^2 + Sqrt[a]*e^2)^2/(4*Sqrt[a]*Sqrt[c]*d^2*e^2
), 2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(4*a^(1/4)
*c^(1/4)*Sqrt[a + b*x^2 + c*x^4])))/(Sqrt[c]*d^2 + Sqrt[a]*e^2))/(c*d^4 +
b*d^2*e^2 + a*e^4)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1576
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x]
, x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

rule 2222
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTanh[Rt[b - c*(d/e) - a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
b - c*(d/e) - a*(e/d), 2])), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a +
 b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*d*e*q*Sqrt[a + b*x^2 + c*x^4]))*Ell
ipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x], 1/2 - b/(4*a*q^2)], x]]
/; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &&
 EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[-b + c*(d/e) + a*(e/d)]
```

rule 2226
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[c/a, 2]}, Simp[(A*(c*d + a*e*q)
- a*B*(e + d*q))/(c*d^2 - a*e^2)   Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] +
Simp[a*(B*d - A*e)*((e + d*q)/(c*d^2 - a*e^2))   Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] &&
NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && NeQ[c*A^2 - a*B^2, 0]
```

rule 2232
```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[c/a, 2], A = Coeff[P4x, x, 0], B = Coeff[P4x, x
, 2], C = Coeff[P4x, x, 4]}, Simp[-C/(e*q)   Int[(1 - q*x^2)/Sqrt[a + b*x^2
 + c*x^4], x], x] + Simp[1/(c*e)   Int[(A*c*e + a*C*d*q + (B*c*e - C*(c*d -
 a*e*q))*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b
, c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
&&  !GtQ[b^2 - 4*a*c, 0]
```

rule 2264
```
Int[((d_) + (e_.)*(x_))^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Sy
mbol] :> Simp[e^3*(d + e*x)^(q + 1)*(Sqrt[a + b*x^2 + c*x^4]/((q + 1)*(c*d^
4 + b*d^2*e^2 + a*e^4))), x] + Simp[1/((q + 1)*(c*d^4 + b*d^2*e^2 + a*e^4))
   Int[((d + e*x)^(q + 1)/Sqrt[a + b*x^2 + c*x^4])*Simp[d*(q + 1)*(c*d^2 +
b*e^2) - e*(c*d^2*(q + 1) + b*e^2*(q + 2))*x + c*d*e^2*(q + 1)*x^2 - c*e^3*
(q + 3)*x^3, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[c*d^4 + b*d^2*e
^2 + a*e^4, 0] && ILtQ[q, -1]
```

rule 2279
```
Int[(Px_)/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x
_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1], C = Coeff[Px, x
, 2], D = Coeff[Px, x, 3]}, Int[(x*(B*d - A*e + (d*D - C*e)*x^2))/((d^2 - e
^2*x^2)*Sqrt[a + b*x^2 + c*x^4]), x] + Int[(A*d + (C*d - B*e)*x^2 - D*e*x^4
)/((d^2 - e^2*x^2)*Sqrt[a + b*x^2 + c*x^4]), x]] /; FreeQ[{a, b, c, d, e},
x] && PolyQ[Px, x] && LeQ[Expon[Px, x], 3] && NeQ[c*d^4 + b*d^2*e^2 + a*e^4
, 0]
```

Maple [A] (verified)

Time = 0.91 (sec) , antiderivative size = 771, normalized size of antiderivative = 0.94

method	result
default	$-\frac{e^{3}\sqrt{c\,x^{4}+b\,x^{2}+a}}{\left(e^{4}a+b\,d^{2}e^{2}+c\,d^{4}\right)\left(ex+d\right)}-\frac{d^{2}c\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^{2}}\right)x^{2}}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^{2}}\right)x^{2}}{a}}\,\mathrm{EllipticF}\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^{2}}}{a}}}{2},\sqrt{-4}\right.}{4\left(e^{4}a+b\,d^{2}e^{2}+c\,d^{4}\right)\sqrt{\frac{-b+\sqrt{-4ac+b^{2}}}{a}}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}$
elliptic	$-\frac{e^{3}\sqrt{c\,x^{4}+b\,x^{2}+a}}{\left(e^{4}a+b\,d^{2}e^{2}+c\,d^{4}\right)\left(ex+d\right)}-\frac{d^{2}c\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^{2}}\right)x^{2}}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^{2}}\right)x^{2}}{a}}\,\mathrm{EllipticF}\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^{2}}}{a}}}{2},\sqrt{-4}\right.}{4\left(e^{4}a+b\,d^{2}e^{2}+c\,d^{4}\right)\sqrt{\frac{-b+\sqrt{-4ac+b^{2}}}{a}}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}$

input
```
int(1/(e*x+d)^2/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-e^3*(c*x^4+b*x^2+a)^(1/2)/(a*e^4+b*d^2*e^2+c*d^4)/(e*x+d)-1/4*d^2*c/(a*e^
4+b*d^2*e^2+c*d^4)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*
a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x
^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2
),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/2*c*e^2/(a*e^4+b*d^2*e^
2+c*d^4)*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)
^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2
+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2
)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE
(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^
2)^(1/2))/a/c)^(1/2)))+d*(b*e^2+2*c*d^2)/(a*e^4+b*d^2*e^2+c*d^4)/e*(-1/2/(
c*d^4/e^4+b*d^2/e^2+a)^(1/2)*arctanh(1/2*(2*c*x^2*d^2/e^2+b*d^2/e^2+b*x^2+
2*a)/(c*d^4/e^4+b*d^2/e^2+a)^(1/2)/(c*x^4+b*x^2+a)^(1/2))+2^(1/2)/((-b+(-4
*a*c+b^2)^(1/2))/a)^(1/2)/d*e*(1-1/2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*
(1+1/2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticP
i(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),2/(-b+(-4*a*c+b^2)^(1/2)
)*a/d^2*e^2,(-1/2*(b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2
)^(1/2))/a)^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^2\sqrt{a+bx^2+cx^4}}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x+d)^2/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(d+ex)^2\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{1}{(d+ex)^2\,\sqrt{a+bx^2+cx^4}}\,dx$$

input
```
integrate(1/(e*x+d)**2/(c*x**4+b*x**2+a)**(1/2),x)
```

output
```
Integral(1/((d + e*x)**2*sqrt(a + b*x**2 + c*x**4)), x)
```

Maxima [F]

$$\int \frac{1}{(d+ex)^2\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{1}{\sqrt{cx^4+bx^2+a}\,(ex+d)^2}\,dx$$

input
```
integrate(1/(e*x+d)^2/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(c*x^4 + b*x^2 + a)*(e*x + d)^2), x)
```

Giac [F]

$$\int \frac{1}{(d+ex)^2\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{1}{\sqrt{cx^4+bx^2+a}(ex+d)^2}\,dx$$

input
```
integrate(1/(e*x+d)^2/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(c*x^4 + b*x^2 + a)*(e*x + d)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^2\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{1}{(d+e\,x)^2\,\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

input
```
int(1/((d + e*x)^2*(a + b*x^2 + c*x^4)^(1/2)),x)
```

output
```
int(1/((d + e*x)^2*(a + b*x^2 + c*x^4)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{(d+ex)^2\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{1}{(ex+d)^2\,\sqrt{c\,x^4+b\,x^2+a}}dx$$

input
```
int(1/(e*x+d)^2/(c*x^4+b*x^2+a)^(1/2),x)
```

output
```
int(1/(e*x+d)^2/(c*x^4+b*x^2+a)^(1/2),x)
```

3.266 $\int \frac{(d+ex)^3}{(a+bx^2+cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 481

$$\begin{aligned}
\int \frac{(d+ex)^3}{(a+bx^2+cx^4)^{3/2}}\,dx &= \frac{dx((b^2-2ac)\,d^2-3abe^2+c(bd^2-6ae^2)\,x^2)}{a\,(b^2-4ac)\,\sqrt{a+bx^2+cx^4}} \\
&- \frac{e(3bd^2-2ae^2+(6cd^2-be^2)\,x^2)}{(b^2-4ac)\,\sqrt{a+bx^2+cx^4}} - \frac{\sqrt{c}d(bd^2-6ae^2)\,x\sqrt{a+bx^2+cx^4}}{a\,(b^2-4ac)\,(\sqrt{a}+\sqrt{c}x^2)} \\
&+ \frac{\sqrt[4]{c}d(bd^2-6ae^2)\,(\sqrt{a}+\sqrt{c}x^2)\,\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{a^{3/4}\,(b^2-4ac)\,\sqrt{a+bx^2+cx^4}} \\
&- \frac{d(\sqrt{c}d^2-3\sqrt{a}e^2)\,(\sqrt{a}+\sqrt{c}x^2)\,\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2a^{3/4}\,(b-2\sqrt{a}\sqrt{c})\,\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}
\end{aligned}$$

output
```
d*x*((-2*a*c+b^2)*d^2-3*a*b*e^2+c*(-6*a*e^2+b*d^2)*x^2)/a/(-4*a*c+b^2)/(c*
x^4+b*x^2+a)^(1/2)-e*(3*b*d^2-2*a*e^2+(-b*e^2+6*c*d^2)*x^2)/(-4*a*c+b^2)/(
c*x^4+b*x^2+a)^(1/2)-c^(1/2)*d*(-6*a*e^2+b*d^2)*x*(c*x^4+b*x^2+a)^(1/2)/a/
(-4*a*c+b^2)/(a^(1/2)+c^(1/2)*x^2)+c^(1/4)*d*(-6*a*e^2+b*d^2)*(a^(1/2)+c^(
1/2)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*EllipticE(sin(2*
arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))/a^(3/4)/(-4*a*
c+b^2)/(c*x^4+b*x^2+a)^(1/2)-1/2*d*(c^(1/2)*d^2-3*a^(1/2)*e^2)*(a^(1/2)+c^
(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM
(2*arctan(c^(1/4)*x/a^(1/4)),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))/a^(3/4)/(b-2
*a^(1/2)*c^(1/2))/c^(1/4)/(c*x^4+b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 12.07 (sec) , antiderivative size = 551, normalized size of antiderivative = 1.15

$$\int \frac{(d+ex)^3}{(a+bx^2+cx^4)^{3/2}}\,dx =$$

$$-\frac{-4\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}(2a^2e^3+bd^3x(b+cx^2)-a(be(3d^2+3dex-e^2x^2)+2cdx(d^2+3dex+3e^2x^2)))+i(-b-\ldots}{\ldots}$$

input
```
Integrate[(d + e*x)^3/(a + b*x^2 + c*x^4)^(3/2),x]
```

output
```
-1/4*(-4*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*(2*a^2*e^3 + b*d^3*x*(b + c*x^2)
- a*(b*e*(3*d^2 + 3*d*e*x - e^2*x^2) + 2*c*d*x*(d^2 + 3*d*e*x + 3*e^2*x^2)
)) + I*(-b + Sqrt[b^2 - 4*a*c])*d*(b*d^2 - 6*a*e^2)*Sqrt[(b + Sqrt[b^2 - 4
*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c]
+ 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b
+ Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])]
 - I*d*(-(b^2*d^2) + 4*a*c*d^2 + b*Sqrt[b^2 - 4*a*c]*d^2 - 6*a*Sqrt[b^2 -
4*a*c]*e^2)*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])
]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Elli
pticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2
- 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/(a*(b^2 - 4*a*c)*Sqrt[c/(b + Sqrt[b^2
- 4*a*c])]*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] (verified)

Time = 1.17 (sec) , antiderivative size = 461, normalized size of antiderivative = 0.96, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {2202, 1492, 27, 1511, 27, 1416, 1509, 1576, 27, 1158}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^3}{(a+bx^2+cx^4)^{3/2}} \, dx$$

$$\Big\downarrow \text{ 2202}$$

$$\int \frac{d^3+3e^2x^2d}{(cx^4+bx^2+a)^{3/2}}dx+\int \frac{x(x^2e^3+3d^2e)}{(cx^4+bx^2+a)^{3/2}}dx$$

$$\Big\downarrow \text{ 1492}$$

$$-\frac{\int \frac{d(c(bd^2-6ae^2)x^2+a(2cd^2-3be^2))}{\sqrt{cx^4+bx^2+a}}dx}{a(b^2-4ac)}+\int \frac{x(x^2e^3+3d^2e)}{(cx^4+bx^2+a)^{3/2}}dx+\frac{dx(d^2(b^2-2ac)+cx^2(bd^2-6ae^2)-3abe^2)}{a(b^2-4ac)\sqrt{a+bx^2+cx^4}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{d\int \frac{c(bd^2-6ae^2)x^2+a(2cd^2-3be^2)}{\sqrt{cx^4+bx^2+a}}dx}{a(b^2-4ac)}+\int \frac{x(x^2e^3+3d^2e)}{(cx^4+bx^2+a)^{3/2}}dx+\frac{dx(d^2(b^2-2ac)+cx^2(bd^2-6ae^2)-3abe^2)}{a(b^2-4ac)\sqrt{a+bx^2+cx^4}}$$

$$\Big\downarrow \text{ 1511}$$

$$-\frac{d\left(\sqrt{a}(2\sqrt{a}\sqrt{c}+b)(\sqrt{c}d^2-3\sqrt{a}e^2)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx-\sqrt{a}\sqrt{c}(bd^2-6ae^2)\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{a}\sqrt{cx^4+bx^2+a}}dx\right)}{a(b^2-4ac)}+\int \frac{x(x^2e^3+3d^2e)}{(cx^4+bx^2+a)^{3/2}}dx+\frac{dx(d^2(b^2-2ac)+cx^2(bd^2-6ae^2)-3abe^2)}{a(b^2-4ac)\sqrt{a+bx^2+cx^4}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{d\left(\sqrt{a}\left(2\sqrt{a}\sqrt{c}+b\right)\left(\sqrt{c}d^2-3\sqrt{a}e^2\right)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx-\sqrt{c}\left(bd^2-6ae^2\right)\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+bx^2+a}}dx\right)}{a\left(b^2-4ac\right)}+\int \frac{x\left(x^2e^3+3d^2e\right)}{\left(cx^4+bx^2+a\right)^{3/2}}dx+\frac{dx\left(d^2\left(b^2-2ac\right)+cx^2\left(bd^2-6ae^2\right)-3abe^2\right)}{a\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}$$

$$\Big\downarrow \text{1416}$$

$$-\frac{d\left(\frac{\sqrt[4]{a}\left(2\sqrt{a}\sqrt{c}+b\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{c}d^2-3\sqrt{a}e^2\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}-\sqrt{c}\left(bd^2-6ae^2\right)\int \frac{\sqrt{\ldots}}{\sqrt{c\ldots}}\right.}{a\left(b^2-4ac\right)}$$

$$\int \frac{x\left(x^2e^3+3d^2e\right)}{\left(cx^4+bx^2+a\right)^{3/2}}dx+\frac{dx\left(d^2\left(b^2-2ac\right)+cx^2\left(bd^2-6ae^2\right)-3abe^2\right)}{a\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}$$

$$\Big\downarrow \text{1509}$$

$$\int \frac{x\left(x^2e^3+3d^2e\right)}{\left(cx^4+bx^2+a\right)^{3/2}}dx-$$

$$\frac{d\left(\frac{\sqrt[4]{a}\left(2\sqrt{a}\sqrt{c}+b\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{c}d^2-3\sqrt{a}e^2\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}-\sqrt{c}\left(bd^2-6ae^2\right)\left(\frac{\sqrt[4]{a}(\ldots}{\ldots}\right.\right.}{a\left(b^2-4ac\right)}$$

$$\frac{dx\left(d^2\left(b^2-2ac\right)+cx^2\left(bd^2-6ae^2\right)-3abe^2\right)}{a\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}$$

$$\Big\downarrow \text{1576}$$

$$\frac{1}{2}\int \frac{e\left(3d^2+e^2x^2\right)}{\left(cx^4+bx^2+a\right)^{3/2}}dx^2-$$

$$\frac{d\left(\frac{\sqrt[4]{a}\left(2\sqrt{a}\sqrt{c}+b\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{c}d^2-3\sqrt{a}e^2\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}-\sqrt{c}\left(bd^2-6ae^2\right)\left(\frac{\sqrt[4]{a}(\ldots}{\ldots}\right.\right.}{a\left(b^2-4ac\right)}$$

$$\frac{dx\left(d^2\left(b^2-2ac\right)+cx^2\left(bd^2-6ae^2\right)-3abe^2\right)}{a\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}e\int \frac{3d^2+e^2x^2}{(cx^4+bx^2+a)^{3/2}}dx^2 -$$

$$\frac{d\left(\frac{\sqrt[4]{a}(2\sqrt{a}\sqrt{c}+b)(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}(\sqrt{c}d^2-3\sqrt{a}e^2)\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}-\sqrt{c}(bd^2-6ae^2)\left(\cdots\right)\right)}{a(b^2-4ac)}$$

$$\frac{dx(d^2(b^2-2ac)+cx^2(bd^2-6ae^2)-3abe^2)}{a(b^2-4ac)\sqrt{a+bx^2+cx^4}}$$

$$\Big\downarrow \; 1158$$

$$-\frac{d\left(\frac{\sqrt[4]{a}(2\sqrt{a}\sqrt{c}+b)(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}(\sqrt{c}d^2-3\sqrt{a}e^2)\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}-\sqrt{c}(bd^2-6ae^2)\left(\cdots\right)\right)}{a(b^2-4ac)}$$

$$\frac{dx(d^2(b^2-2ac)+cx^2(bd^2-6ae^2)-3abe^2)}{a(b^2-4ac)\sqrt{a+bx^2+cx^4}}-\frac{e(-2ae^2+x^2(6cd^2-be^2)+3bd^2)}{(b^2-4ac)\sqrt{a+bx^2+cx^4}}$$

input
```
Int[(d + e*x)^3/(a + b*x^2 + c*x^4)^(3/2),x]
```

output
```
(d*x*((b^2 - 2*a*c)*d^2 - 3*a*b*e^2 + c*(b*d^2 - 6*a*e^2)*x^2))/(a*(b^2 -
4*a*c)*Sqrt[a + b*x^2 + c*x^4]) - (e*(3*b*d^2 - 2*a*e^2 + (6*c*d^2 - b*e^2
)*x^2))/((b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4]) - (d*(-(Sqrt[c]*(b*d^2 - 6
*a*e^2)*(-((x*Sqrt[a + b*x^2 + c*x^4])/(Sqrt[a] + Sqrt[c]*x^2)) + (a^(1/4)
*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^
2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/
(c^(1/4)*Sqrt[a + b*x^2 + c*x^4]))) + (a^(1/4)*(b + 2*Sqrt[a]*Sqrt[c])*(Sq
rt[c]*d^2 - 3*Sqrt[a]*e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4
)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 -
 b/(Sqrt[a]*Sqrt[c]))/4])/(2*c^(1/4)*Sqrt[a + b*x^2 + c*x^4])))/(a*(b^2 -
4*a*c))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1158
```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[-2*((b*d - 2*a*e + (2*c*d - b*e)*x)/((b^2 - 4*a*c)*Sqrt[a + b*x
+ c*x^2])), x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1492
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2
 - 4*a*c))   Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p +
7)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 LtQ[p, -1] && IntegerQ[2*p]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 1576
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x]
, x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

rule 2202
```
Int[(Pn_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{n
 = Expon[Pn, x], k}, Int[Sum[Coeff[Pn, x, 2*k]*x^(2*k), {k, 0, n/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pn, x, 2*k + 1]*x^(2*k), {k, 0, (n -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pn, x]
 &&  !PolyQ[Pn, x^2]
```

Maple [A] (verified)

Time = 1.78 (sec) , antiderivative size = 603, normalized size of antiderivative = 1.25

method	result
elliptic	$-\frac{2c\left(-\frac{d\left(6a\,e^2-b\,d^2\right)x^3}{2a\left(4ac-b^2\right)}+\frac{e\left(b\,e^2-6c\,d^2\right)x^2}{2c\left(4ac-b^2\right)}-\frac{d\left(3ab\,e^2+2ac\,d^2-b^2d^2\right)x}{2ac\left(4ac-b^2\right)}+\frac{e\left(2a\,e^2-3b\,d^2\right)}{2\left(4ac-b^2\right)c}\right)}{\sqrt{\left(x^4+\frac{b\,x^2}{c}+\frac{a}{c}\right)c}}+\frac{\left(\frac{d^3}{a}-\frac{d\left(3ab\,e^2+2ac\,d^2-b^2d^2\right)}{\left(4ac-b^2\right)a}\right)\sqrt{2}\,\sqrt{4-}}{}$
default	Expression too large to display

input
```
int((e*x+d)^3/(c*x^4+b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*c*(-1/2*d*(6*a*e^2-b*d^2)/a/(4*a*c-b^2)*x^3+1/2*e*(b*e^2-6*c*d^2)/c/(4*
a*c-b^2)*x^2-1/2*d*(3*a*b*e^2+2*a*c*d^2-b^2*d^2)/a/c/(4*a*c-b^2)*x+1/2*e*(
2*a*e^2-3*b*d^2)/(4*a*c-b^2)/c)/((x^4+b/c*x^2+a/c)*c)^(1/2)+1/4*(d^3/a-d*(
3*a*b*e^2+2*a*c*d^2-b^2*d^2)/(4*a*c-b^2)/a)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2
))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)
^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-
4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))+
1/2*c*d*(6*a*e^2-b*d^2)/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1
/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/
a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x
*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/
2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),
1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 805, normalized size of antiderivative = 1.67

$$\int \frac{(d+ex)^3}{(a+bx^2+cx^4)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^3/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
-1/2*(sqrt(1/2)*(a*b^2*c*d^3 - 6*a^2*b*c*d*e^2 + (b^2*c^2*d^3 - 6*a*b*c^2*
d*e^2)*x^4 + (b^3*c*d^3 - 6*a*b^2*c*d*e^2)*x^2 - (a^2*b*c*d^3 - 6*a^3*c*d*
e^2 + (a*b*c^2*d^3 - 6*a^2*c^2*d*e^2)*x^4 + (a*b^2*c*d^3 - 6*a^2*b*c*d*e^2
)*x^2)*sqrt((b^2 - 4*a*c)/a^2))*sqrt(a)*sqrt((a*sqrt((b^2 - 4*a*c)/a^2) -
b)/a)*elliptic_e(arcsin(sqrt(1/2)*x*sqrt((a*sqrt((b^2 - 4*a*c)/a^2) - b)/a
)), 1/2*(a*b*sqrt((b^2 - 4*a*c)/a^2) + b^2 - 2*a*c)/(a*c)) - sqrt(1/2)*((2
*a^2*b + a*b^2)*c*d^3 + ((2*a*b + b^2)*c^2*d^3 - 3*(a*b^2*c + 2*a*b*c^2)*d
*e^2)*x^4 - 3*(a^2*b^2 + 2*a^2*b*c)*d*e^2 + ((2*a*b^2 + b^3)*c*d^3 - 3*(a*
b^3 + 2*a*b^2*c)*d*e^2)*x^2 + ((2*a^3 - a^2*b)*c*d^3 + ((2*a^2 - a*b)*c^2*
d^3 - 3*(a^2*b*c - 2*a^2*c^2)*d*e^2)*x^4 - 3*(a^3*b - 2*a^3*c)*d*e^2 + ((2
*a^2*b - a*b^2)*c*d^3 - 3*(a^2*b^2 - 2*a^2*b*c)*d*e^2)*x^2)*sqrt((b^2 - 4*
a*c)/a^2))*sqrt(a)*sqrt((a*sqrt((b^2 - 4*a*c)/a^2) - b)/a)*elliptic_f(arcs
in(sqrt(1/2)*x*sqrt((a*sqrt((b^2 - 4*a*c)/a^2) - b)/a)), 1/2*(a*b*sqrt((b^
2 - 4*a*c)/a^2) + b^2 - 2*a*c)/(a*c)) + 2*(3*a^2*b*c*d^2*e - 2*a^3*c*e^3 -
 (a*b*c^2*d^3 - 6*a^2*c^2*d*e^2)*x^3 + (6*a^2*c^2*d^2*e - a^2*b*c*e^3)*x^2
 + (3*a^2*b*c*d*e^2 - (a*b^2*c - 2*a^2*c^2)*d^3)*x)*sqrt(c*x^4 + b*x^2 + a
))/(a^3*b^2*c - 4*a^4*c^2 + (a^2*b^2*c^2 - 4*a^3*c^3)*x^4 + (a^2*b^3*c - 4
*a^3*b*c^2)*x^2)
```

Sympy [F]

$$\int \frac{(d+ex)^3}{(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{(d+ex)^3}{(a+bx^2+cx^4)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x+d)**3/(c*x**4+b*x**2+a)**(3/2),x)
```

output
```
Integral((d + e*x)**3/(a + b*x**2 + c*x**4)**(3/2), x)
```

Maxima [F]

$$\int \frac{(d+ex)^3}{(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{(ex+d)^3}{(cx^4+bx^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x+d)^3/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((e*x + d)^3/(c*x^4 + b*x^2 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{(d+ex)^3}{(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{(ex+d)^3}{(cx^4+bx^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x+d)^3/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((e*x + d)^3/(c*x^4 + b*x^2 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^3}{(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{(d+e\,x)^3}{(c\,x^4+b\,x^2+a)^{3/2}}\,dx$$

input
```
int((d + e*x)^3/(a + b*x^2 + c*x^4)^(3/2),x)
```

output
```
int((d + e*x)^3/(a + b*x^2 + c*x^4)^(3/2), x)
```

Reduce [F]

$$\int \frac{(d+ex)^3}{(a+bx^2+cx^4)^{3/2}}\,dx = \text{too large to display}$$

input
```
int((e*x+d)^3/(c*x^4+b*x^2+a)^(3/2),x)
```

output
```
(48*sqrt(c)*sqrt(a + b*x**2 + c*x**4)*int(sqrt(a + b*x**2 + c*x**4)/(a**2
+ 2*a*b*x**2 + 2*a*c*x**4 + b**2*x**4 + 2*b*c*x**6 + c**2*x**8),x)*a**3*b*
c**2*d**3 + 96*sqrt(c)*sqrt(a + b*x**2 + c*x**4)*int(sqrt(a + b*x**2 + c*x
**4)/(a**2 + 2*a*b*x**2 + 2*a*c*x**4 + b**2*x**4 + 2*b*c*x**6 + c**2*x**8)
,x)*a**3*c**3*d**3*x**2 - 8*sqrt(c)*sqrt(a + b*x**2 + c*x**4)*int(sqrt(a +
 b*x**2 + c*x**4)/(a**2 + 2*a*b*x**2 + 2*a*c*x**4 + b**2*x**4 + 2*b*c*x**6
 + c**2*x**8),x)*a**2*b**3*c*d**3 + 96*sqrt(c)*sqrt(a + b*x**2 + c*x**4)*i
nt(sqrt(a + b*x**2 + c*x**4)/(a**2 + 2*a*b*x**2 + 2*a*c*x**4 + b**2*x**4 +
 2*b*c*x**6 + c**2*x**8),x)*a**2*b**2*c**2*d**3*x**2 + 336*sqrt(c)*sqrt(a
+ b*x**2 + c*x**4)*int(sqrt(a + b*x**2 + c*x**4)/(a**2 + 2*a*b*x**2 + 2*a*
c*x**4 + b**2*x**4 + 2*b*c*x**6 + c**2*x**8),x)*a**2*b*c**3*d**3*x**4 + 22
4*sqrt(c)*sqrt(a + b*x**2 + c*x**4)*int(sqrt(a + b*x**2 + c*x**4)/(a**2 +
2*a*b*x**2 + 2*a*c*x**4 + b**2*x**4 + 2*b*c*x**6 + c**2*x**8),x)*a**2*c**4
*d**3*x**6 - sqrt(c)*sqrt(a + b*x**2 + c*x**4)*int(sqrt(a + b*x**2 + c*x**
4)/(a**2 + 2*a*b*x**2 + 2*a*c*x**4 + b**2*x**4 + 2*b*c*x**6 + c**2*x**8),x
)*a*b**5*d**3 - 26*sqrt(c)*sqrt(a + b*x**2 + c*x**4)*int(sqrt(a + b*x**2 +
 c*x**4)/(a**2 + 2*a*b*x**2 + 2*a*c*x**4 + b**2*x**4 + 2*b*c*x**6 + c**2*x
**8),x)*a*b**4*c*d**3*x**2 - 8*sqrt(c)*sqrt(a + b*x**2 + c*x**4)*int(sqrt(
a + b*x**2 + c*x**4)/(a**2 + 2*a*b*x**2 + 2*a*c*x**4 + b**2*x**4 + 2*b*c*x
**6 + c**2*x**8),x)*a*b**3*c**2*d**3*x**4 + 208*sqrt(c)*sqrt(a + b*x**2...
```

3.267 $\int \frac{(d+ex)^2}{(a+bx^2+cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 456

$$\begin{aligned}\int \frac{(d+ex)^2}{(a+bx^2+cx^4)^{3/2}}\,dx &= -\frac{2de(b+2cx^2)}{(b^2-4ac)\sqrt{a+bx^2+cx^4}} \\
&+\frac{x((b^2-2ac)\,d^2-abe^2+c(bd^2-2ae^2)\,x^2)}{a\,(b^2-4ac)\sqrt{a+bx^2+cx^4}}-\frac{\sqrt{c}(bd^2-2ae^2)\,x\sqrt{a+bx^2+cx^4}}{a\,(b^2-4ac)\left(\sqrt{a}+\sqrt{c}x^2\right)} \\
&+\frac{\sqrt[4]{c}(bd^2-2ae^2)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{a^{3/4}\,(b^2-4ac)\sqrt{a+bx^2+cx^4}} \\
&-\frac{\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2a^{3/4}\left(b-2\sqrt{a}\sqrt{c}\right)\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}\end{aligned}$$

output
```
-2*d*e*(2*c*x^2+b)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^(1/2)+x*((-2*a*c+b^2)*d^2-
a*b*e^2+c*(-2*a*e^2+b*d^2)*x^2)/a/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^(1/2)-c^(1/
2)*(-2*a*e^2+b*d^2)*x*(c*x^4+b*x^2+a)^(1/2)/a/(-4*a*c+b^2)/(a^(1/2)+c^(1/2
)*x^2)+c^(1/4)*(-2*a*e^2+b*d^2)*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^
(1/2)+c^(1/2)*x^2)^2)^(1/2)*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2
*(2-b/a^(1/2)/c^(1/2))^(1/2))/a^(3/4)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^(1/2)-1
/2*(c^(1/2)*d^2-a^(1/2)*e^2)*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^(1/
2)+c^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(c^(1/4)*x/a^(1/4)),1/2*(
2-b/a^(1/2)/c^(1/2))^(1/2))/a^(3/4)/(b-2*a^(1/2)*c^(1/2))/c^(1/4)/(c*x^4+b
*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.84 (sec) , antiderivative size = 517, normalized size of antiderivative = 1.13

$$\int \frac{(d+ex)^2}{(a+bx^2+cx^4)^{3/2}}\,dx =$$

$$-\frac{4\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\left(2acx(d+ex)^2+abe(2d+ex)-bd^2x(b+cx^2)\right)+i\left(-b+\sqrt{b^2-4ac}\right)\left(bd^2-2ae^2\right)\sqrt{\frac{b+\sqrt{b^2\cdots}}{b+\cdots}}\cdots}{\cdots}$$

input
```
Integrate[(d + e*x)^2/(a + b*x^2 + c*x^4)^(3/2),x]
```

output
```
-1/4*(4*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*(2*a*c*x*(d + e*x)^2 + a*b*e*(2*d
+ e*x) - b*d^2*x*(b + c*x^2)) + I*(-b + Sqrt[b^2 - 4*a*c])*(b*d^2 - 2*a*e^
2)*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2
*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticE[I*A
rcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])
/(b - Sqrt[b^2 - 4*a*c])] - I*(-(b^2*d^2) + 4*a*c*d^2 + b*Sqrt[b^2 - 4*a*c
]*d^2 - 2*a*Sqrt[b^2 - 4*a*c]*e^2)*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/
(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - S
qrt[b^2 - 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*
c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/(a*(b^2 - 4*a*c
)*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 438, normalized size of antiderivative = 0.96, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {2202, 27, 1432, 1088, 1492, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^2}{(a+bx^2+cx^4)^{3/2}}\,dx$$

$$\Big\downarrow \; 2202$$

$$\int \frac{d^2+e^2x^2}{(cx^4+bx^2+a)^{3/2}}dx+\int \frac{2dex}{(cx^4+bx^2+a)^{3/2}}dx$$

$$\Big\downarrow \; 27$$

$$\int \frac{d^2+e^2x^2}{(cx^4+bx^2+a)^{3/2}}dx+2de\int \frac{x}{(cx^4+bx^2+a)^{3/2}}dx$$

$$\Big\downarrow \; 1432$$

$$\int \frac{d^2+e^2x^2}{(cx^4+bx^2+a)^{3/2}}dx+de\int \frac{1}{(cx^4+bx^2+a)^{3/2}}dx^2$$

$$\Big\downarrow \; 1088$$

$$\int \frac{d^2+e^2x^2}{(cx^4+bx^2+a)^{3/2}}dx-\frac{2de\left(b+2cx^2\right)}{(b^2-4ac)\sqrt{a+bx^2+cx^4}}$$

$$\Big\downarrow \; 1492$$

$$-\frac{\int \frac{c\left(bd^2-2ae^2\right)x^2+a\left(2cd^2-be^2\right)}{\sqrt{cx^4+bx^2+a}}dx}{a\left(b^2-4ac\right)}+\frac{x\left(d^2\left(b^2-2ac\right)+cx^2\left(bd^2-2ae^2\right)-abe^2\right)}{a\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}-\frac{2de\left(b+2cx^2\right)}{(b^2-4ac)\sqrt{a+bx^2+cx^4}}$$

$$\Big\downarrow \; 1511$$

$$-\frac{\sqrt{a}\left(2\sqrt{a}\sqrt{c}+b\right)\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx-\sqrt{a}\sqrt{c}\left(bd^2-2ae^2\right)\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{a}\sqrt{cx^4+bx^2+a}}dx}{a\left(b^2-4ac\right)}+\frac{x\left(d^2\left(b^2-2ac\right)+cx^2\left(bd^2-2ae^2\right)-abe^2\right)}{a\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}-\frac{2de\left(b+2cx^2\right)}{(b^2-4ac)\sqrt{a+bx^2+cx^4}}$$

$$\Big\downarrow \; 27$$

$$-\frac{\sqrt{a}(2\sqrt{a}\sqrt{c}+b)\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx-\sqrt{c}(bd^2-2ae^2)\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+bx^2+a}}dx}{a\left(b^2-4ac\right)}+\frac{x\left(d^2\left(b^2-2ac\right)+cx^2\left(bd^2-2ae^2\right)-abe^2\right)}{a\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}-\frac{2de\left(b+2cx^2\right)}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}$$

$$\Big\downarrow \; 1416$$

$$-\frac{\frac{\sqrt[4]{a}(2\sqrt{a}\sqrt{c}+b)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}-\sqrt{c}(bd^2-2ae^2)\int \frac{\sqrt{a}-\sqrt{}}{\sqrt{cx^4+b}}}{a\left(b^2-4ac\right)}$$

$$\frac{x\left(d^2\left(b^2-2ac\right)+cx^2\left(bd^2-2ae^2\right)-abe^2\right)}{a\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}-\frac{2de\left(b+2cx^2\right)}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}$$

$$\Big\downarrow \; 1509$$

$$-\frac{\frac{\sqrt[4]{a}(2\sqrt{a}\sqrt{c}+b)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}-\sqrt{c}(bd^2-2ae^2)\left(\frac{\sqrt[4]{a}(\sqrt{}}{}\right.}{a\left(b^2-4ac\right)}$$

$$\frac{x\left(d^2\left(b^2-2ac\right)+cx^2\left(bd^2-2ae^2\right)-abe^2\right)}{a\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}-\frac{2de\left(b+2cx^2\right)}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}$$

input
```
Int[(d + e*x)^2/(a + b*x^2 + c*x^4)^(3/2),x]
```

output
```
(-2*d*e*(b + 2*c*x^2))/((b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4]) + (x*((b^2
- 2*a*c)*d^2 - a*b*e^2 + c*(b*d^2 - 2*a*e^2)*x^2))/(a*(b^2 - 4*a*c)*Sqrt[a
 + b*x^2 + c*x^4]) - (-(Sqrt[c]*(b*d^2 - 2*a*e^2)*(-((x*Sqrt[a + b*x^2 + c
*x^4])/(Sqrt[a] + Sqrt[c]*x^2)) + (a^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a
 + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x
)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(c^(1/4)*Sqrt[a + b*x^2 + c*x^4]
))) + (a^(1/4)*(b + 2*Sqrt[a]*Sqrt[c])*(Sqrt[c]*d^2 - Sqrt[a]*e^2)*(Sqrt[a
] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Ellip
ticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*c^(1/
4)*Sqrt[a + b*x^2 + c*x^4]))/(a*(b^2 - 4*a*c))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1432
```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

rule 1492
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2
 - 4*a*c))   Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p +
7)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 LtQ[p, -1] && IntegerQ[2*p]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
 - 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 2202
```
Int[(Pn_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{n
 = Expon[Pn, x], k}, Int[Sum[Coeff[Pn, x, 2*k]*x^(2*k), {k, 0, n/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pn, x, 2*k + 1]*x^(2*k), {k, 0, (n -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pn, x]
 &&  !PolyQ[Pn, x^2]
```

Maple [A] (verified)

Time = 1.45 (sec) , antiderivative size = 572, normalized size of antiderivative = 1.25

method	result
elliptic	$-\frac{2c\left(-\frac{\left(2a\,e^{2}-b\,d^{2}\right)x^{3}}{2a\left(4ac-b^{2}\right)}-\frac{2de\,x^{2}}{4ac-b^{2}}-\frac{\left(ab\,e^{2}+2ac\,d^{2}-b^{2}d^{2}\right)x}{2ac\left(4ac-b^{2}\right)}-\frac{bde}{c\left(4ac-b^{2}\right)}\right)}{\sqrt{\left(x^{4}+\frac{b\,x^{2}}{c}+\frac{a}{c}\right)c}}+\frac{\left(\frac{d^{2}}{a}-\frac{ab\,e^{2}+2ac\,d^{2}-b^{2}d^{2}}{a\left(4ac-b^{2}\right)}\right)\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^{2}}\right)x}{a}}}{4}$
default	$d^{2}\left(-\frac{2c\left(\frac{b\,x^{3}}{2a\left(4ac-b^{2}\right)}-\frac{\left(2ac-b^{2}\right)x}{2a\left(4ac-b^{2}\right)c}\right)}{\sqrt{\left(x^{4}+\frac{b\,x^{2}}{c}+\frac{a}{c}\right)c}}+\frac{\left(\frac{1}{a}-\frac{2ac-b^{2}}{a\left(4ac-b^{2}\right)}\right)\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^{2}}\right)x^{2}}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^{2}}\right)x^{2}}{a}}\,\operatorname{EllipticF}\left(\right.}{4\sqrt{\frac{-b+\sqrt{-4ac+b^{2}}}{a}}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}\right.$

input
```
int((e*x+d)^2/(c*x^4+b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*c*(-1/2*(2*a*e^2-b*d^2)/a/(4*a*c-b^2)*x^3-2*d*e/(4*a*c-b^2)*x^2-1/2*(a*
b*e^2+2*a*c*d^2-b^2*d^2)/a/c/(4*a*c-b^2)*x-b*d*e/c/(4*a*c-b^2))/((x^4+b/c*
x^2+a/c)*c)^(1/2)+1/4*(d^2/a-(a*b*e^2+2*a*c*d^2-b^2*d^2)/a/(4*a*c-b^2))*2^
(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2
)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*Ell
ipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*
a*c+b^2)^(1/2))/a/c)^(1/2))+1/2*c*(2*a*e^2-b*d^2)/(4*a*c-b^2)*2^(1/2)/((-b
+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4
+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^
2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(
-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(-4
*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 765, normalized size of antiderivative = 1.68

$$\int \frac{(d+ex)^2}{(a+bx^2+cx^4)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^2/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
-1/2*(sqrt(1/2)*(a*b^2*c*d^2 - 2*a^2*b*c*e^2 + (b^2*c^2*d^2 - 2*a*b*c^2*e^
2)*x^4 + (b^3*c*d^2 - 2*a*b^2*c*e^2)*x^2 - (a^2*b*c*d^2 - 2*a^3*c*e^2 + (a
*b*c^2*d^2 - 2*a^2*c^2*e^2)*x^4 + (a*b^2*c*d^2 - 2*a^2*b*c*e^2)*x^2)*sqrt(
(b^2 - 4*a*c)/a^2))*sqrt(a)*sqrt((a*sqrt((b^2 - 4*a*c)/a^2) - b)/a)*ellipt
ic_e(arcsin(sqrt(1/2)*x*sqrt((a*sqrt((b^2 - 4*a*c)/a^2) - b)/a)), 1/2*(a*b
*sqrt((b^2 - 4*a*c)/a^2) + b^2 - 2*a*c)/(a*c)) - sqrt(1/2)*(((2*a*b + b^2)
*c^2*d^2 - (a*b^2*c + 2*a*b*c^2)*e^2)*x^4 + (2*a^2*b + a*b^2)*c*d^2 - (a^2
*b^2 + 2*a^2*b*c)*e^2 + ((2*a*b^2 + b^3)*c*d^2 - (a*b^3 + 2*a*b^2*c)*e^2)*
x^2 + (((2*a^2 - a*b)*c^2*d^2 - (a^2*b*c - 2*a^2*c^2)*e^2)*x^4 + (2*a^3 -
a^2*b)*c*d^2 - (a^3*b - 2*a^3*c)*e^2 + ((2*a^2*b - a*b^2)*c*d^2 - (a^2*b^2
 - 2*a^2*b*c)*e^2)*x^2)*sqrt((b^2 - 4*a*c)/a^2))*sqrt(a)*sqrt((a*sqrt((b^2
 - 4*a*c)/a^2) - b)/a)*elliptic_f(arcsin(sqrt(1/2)*x*sqrt((a*sqrt((b^2 - 4
*a*c)/a^2) - b)/a)), 1/2*(a*b*sqrt((b^2 - 4*a*c)/a^2) + b^2 - 2*a*c)/(a*c)
) + 2*(4*a^2*c^2*d*e*x^2 + 2*a^2*b*c*d*e - (a*b*c^2*d^2 - 2*a^2*c^2*e^2)*x
^3 + (a^2*b*c*e^2 - (a*b^2*c - 2*a^2*c^2)*d^2)*x)*sqrt(c*x^4 + b*x^2 + a))
/(a^3*b^2*c - 4*a^4*c^2 + (a^2*b^2*c^2 - 4*a^3*c^3)*x^4 + (a^2*b^3*c - 4*a
^3*b*c^2)*x^2)
```

Sympy [F]

$$\int \frac{(d+ex)^2}{(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{(d+ex)^2}{(a+bx^2+cx^4)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x+d)**2/(c*x**4+b*x**2+a)**(3/2),x)
```

output
```
Integral((d + e*x)**2/(a + b*x**2 + c*x**4)**(3/2), x)
```

Maxima [F]

$$\int \frac{(d+ex)^2}{(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{(ex+d)^2}{(cx^4+bx^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x+d)^2/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((e*x + d)^2/(c*x^4 + b*x^2 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{(d+ex)^2}{(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{(ex+d)^2}{(cx^4+bx^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x+d)^2/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((e*x + d)^2/(c*x^4 + b*x^2 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^2}{(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{(d+e\,x)^2}{(c\,x^4+b\,x^2+a)^{3/2}}\,dx$$

input
```
int((d + e*x)^2/(a + b*x^2 + c*x^4)^(3/2),x)
```

output
```
int((d + e*x)^2/(a + b*x^2 + c*x^4)^(3/2), x)
```

Reduce [F]

$$\int \frac{(d+ex)^2}{(a+bx^2+cx^4)^{3/2}}\,dx = \text{too large to display}$$

input
```
int((e*x+d)^2/(c*x^4+b*x^2+a)^(3/2),x)
```

output

```
(8*sqrt(c)*sqrt(a + b*x**2 + c*x**4)*b*d*e + 16*sqrt(c)*sqrt(a + b*x**2 +
c*x**4)*c*d*e*x**2 + 4*sqrt(a + b*x**2 + c*x**4)*int(sqrt(a + b*x**2 + c*x
**4)/(a**2 + 2*a*b*x**2 + 2*a*c*x**4 + b**2*x**4 + 2*b*c*x**6 + c**2*x**8)
,x)*a*b*c*d**2 + 8*sqrt(a + b*x**2 + c*x**4)*int(sqrt(a + b*x**2 + c*x**4)
/(a**2 + 2*a*b*x**2 + 2*a*c*x**4 + b**2*x**4 + 2*b*c*x**6 + c**2*x**8),x)*
a*c**2*d**2*x**2 - sqrt(a + b*x**2 + c*x**4)*int(sqrt(a + b*x**2 + c*x**4)
/(a**2 + 2*a*b*x**2 + 2*a*c*x**4 + b**2*x**4 + 2*b*c*x**6 + c**2*x**8),x)*
b**3*d**2 - 2*sqrt(a + b*x**2 + c*x**4)*int(sqrt(a + b*x**2 + c*x**4)/(a**
2 + 2*a*b*x**2 + 2*a*c*x**4 + b**2*x**4 + 2*b*c*x**6 + c**2*x**8),x)*b**2*
c*d**2*x**2 + 4*sqrt(a + b*x**2 + c*x**4)*int((sqrt(a + b*x**2 + c*x**4)*x
**2)/(a**2 + 2*a*b*x**2 + 2*a*c*x**4 + b**2*x**4 + 2*b*c*x**6 + c**2*x**8)
,x)*a*b*c*e**2 + 8*sqrt(a + b*x**2 + c*x**4)*int((sqrt(a + b*x**2 + c*x**4
)*x**2)/(a**2 + 2*a*b*x**2 + 2*a*c*x**4 + b**2*x**4 + 2*b*c*x**6 + c**2*x*
*8),x)*a*c**2*e**2*x**2 - sqrt(a + b*x**2 + c*x**4)*int((sqrt(a + b*x**2 +
 c*x**4)*x**2)/(a**2 + 2*a*b*x**2 + 2*a*c*x**4 + b**2*x**4 + 2*b*c*x**6 +
c**2*x**8),x)*b**3*e**2 - 2*sqrt(a + b*x**2 + c*x**4)*int((sqrt(a + b*x**2
 + c*x**4)*x**2)/(a**2 + 2*a*b*x**2 + 2*a*c*x**4 + b**2*x**4 + 2*b*c*x**6
+ c**2*x**8),x)*b**2*c*e**2*x**2 + 8*sqrt(c)*int(sqrt(a + b*x**2 + c*x**4)
/(a**2 + 2*a*b*x**2 + 2*a*c*x**4 + b**2*x**4 + 2*b*c*x**6 + c**2*x**8),x)*
a**2*c*d**2 - 2*sqrt(c)*int(sqrt(a + b*x**2 + c*x**4)/(a**2 + 2*a*b*x**...
```

3.268 $\int \frac{d+ex}{(a+bx^2+cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 394

$$\begin{aligned}\int \frac{d+ex}{(a+bx^2+cx^4)^{3/2}}\,dx &= -\frac{e(b+2cx^2)}{(b^2-4ac)\sqrt{a+bx^2+cx^4}} \\ &+ \frac{dx(b^2-2ac+bcx^2)}{a(b^2-4ac)\sqrt{a+bx^2+cx^4}} - \frac{b\sqrt{c}dx\sqrt{a+bx^2+cx^4}}{a(b^2-4ac)\left(\sqrt{a}+\sqrt{c}x^2\right)} \\ &+ \frac{b\sqrt[4]{c}d\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{a^{3/4}(b^2-4ac)\sqrt{a+bx^2+cx^4}} \\ &- \frac{\sqrt[4]{c}d\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2a^{3/4}\left(b-2\sqrt{a}\sqrt{c}\right)\sqrt{a+bx^2+cx^4}}\end{aligned}$$

output

```
-e*(2*c*x^2+b)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^(1/2)+d*x*(b*c*x^2-2*a*c+b^2)/
a/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^(1/2)-b*c^(1/2)*d*x*(c*x^4+b*x^2+a)^(1/2)/a
/(-4*a*c+b^2)/(a^(1/2)+c^(1/2)*x^2)+b*c^(1/4)*d*(a^(1/2)+c^(1/2)*x^2)*((c*
x^4+b*x^2+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*EllipticE(sin(2*arctan(c^(1/4)
*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))/a^(3/4)/(-4*a*c+b^2)/(c*x^4+
b*x^2+a)^(1/2)-1/2*c^(1/4)*d*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^(1/
2)+c^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(c^(1/4)*x/a^(1/4)),1/2*(
2-b/a^(1/2)/c^(1/2))^(1/2))/a^(3/4)/(b-2*a^(1/2)*c^(1/2))/(c*x^4+b*x^2+a)^
(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.17 (sec) , antiderivative size = 470, normalized size of antiderivative = 1.19

$$\int \frac{d+ex}{(a+bx^2+cx^4)^{3/2}}\,dx =$$

$$-\frac{4\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}(abe+2acx(d+ex)-bdx(b+cx^2))+ib\left(-b+\sqrt{b^2-4ac}\right)d\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{\frac{2b-2\sqrt{b^2-4ac}}{b-\sqrt{b^2-4}}}}{}$$

input
```
Integrate[(d + e*x)/(a + b*x^2 + c*x^4)^(3/2),x]
```

output
```
-1/4*(4*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*(a*b*e + 2*a*c*x*(d + e*x) - b*d*x
*(b + c*x^2)) + I*b*(-b + Sqrt[b^2 - 4*a*c])*d*Sqrt[(b + Sqrt[b^2 - 4*a*c]
 + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c
*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqr
t[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] - I*
(-b^2 + 4*a*c + b*Sqrt[b^2 - 4*a*c])*d*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x
^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b
 - Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 -
4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/(a*(b^2 - 4
*a*c)*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 388, normalized size of antiderivative = 0.98, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {2202, 27, 1405, 27, 1432, 1088, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex}{(a+bx^2+cx^4)^{3/2}}dx$$

$$\Big\downarrow \text{ 2202}$$

$$\int \frac{d}{(cx^4+bx^2+a)^{3/2}}dx+\int \frac{ex}{(cx^4+bx^2+a)^{3/2}}dx$$

$$\Big\downarrow \text{27}$$

$$d\int \frac{1}{(cx^4+bx^2+a)^{3/2}}dx+e\int \frac{x}{(cx^4+bx^2+a)^{3/2}}dx$$

$$\Big\downarrow \text{1405}$$

$$d\left(\frac{x(-2ac+b^2+bcx^2)}{a(b^2-4ac)\sqrt{a+bx^2+cx^4}}-\frac{\int \frac{c(bx^2+2a)}{\sqrt{cx^4+bx^2+a}}dx}{a(b^2-4ac)}\right)+e\int \frac{x}{(cx^4+bx^2+a)^{3/2}}dx$$

$$\Big\downarrow \text{27}$$

$$d\left(\frac{x(-2ac+b^2+bcx^2)}{a(b^2-4ac)\sqrt{a+bx^2+cx^4}}-\frac{c\int \frac{bx^2+2a}{\sqrt{cx^4+bx^2+a}}dx}{a(b^2-4ac)}\right)+e\int \frac{x}{(cx^4+bx^2+a)^{3/2}}dx$$

$$\Big\downarrow \text{1432}$$

$$d\left(\frac{x(-2ac+b^2+bcx^2)}{a(b^2-4ac)\sqrt{a+bx^2+cx^4}}-\frac{c\int \frac{bx^2+2a}{\sqrt{cx^4+bx^2+a}}dx}{a(b^2-4ac)}\right)+\frac{1}{2}e\int \frac{1}{(cx^4+bx^2+a)^{3/2}}dx^2$$

$$\Big\downarrow \text{1088}$$

$$d\left(\frac{x(-2ac+b^2+bcx^2)}{a(b^2-4ac)\sqrt{a+bx^2+cx^4}}-\frac{c\int \frac{bx^2+2a}{\sqrt{cx^4+bx^2+a}}dx}{a(b^2-4ac)}\right)-\frac{e(b+2cx^2)}{(b^2-4ac)\sqrt{a+bx^2+cx^4}}$$

$$\Big\downarrow \text{1511}$$

$$d\left(\frac{x(-2ac+b^2+bcx^2)}{a(b^2-4ac)\sqrt{a+bx^2+cx^4}}-\frac{c\left(\sqrt{a}\left(2\sqrt{a}+\frac{b}{\sqrt{c}}\right)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx-\frac{\sqrt{a}b\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{a}\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}\right)}{a(b^2-4ac)}\right)-\frac{e(b+2cx^2)}{(b^2-4ac)\sqrt{a+bx^2+cx^4}}$$

$$\Big\downarrow \text{27}$$

$$d\left(\frac{x\left(-2ac+b^2+bcx^2\right)}{a\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}-\frac{c\left(\sqrt{a}\left(2\sqrt{a}+\frac{b}{\sqrt{c}}\right)\int\frac{1}{\sqrt{cx^4+bx^2+a}}dx-\frac{b\int\frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}\right)}{a\left(b^2-4ac\right)}\right)-\frac{e\left(b+2cx^2\right)}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}$$

$\Big\downarrow$ 1416

$$d\left(\frac{x\left(-2ac+b^2+bcx^2\right)}{a\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}-\frac{c\left(\frac{\sqrt[4]{a}\left(2\sqrt{a}+\frac{b}{\sqrt{c}}\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}-\right.}{a\left(b^2-4ac\right)}\right.$$

$$\frac{e\left(b+2cx^2\right)}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}$$

$\Big\downarrow$ 1509

$$d\left(\frac{x\left(-2ac+b^2+bcx^2\right)}{a\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}-\frac{c\left(\frac{\sqrt[4]{a}\left(2\sqrt{a}+\frac{b}{\sqrt{c}}\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}-\right.}{a\left(b^2-4a\right.}\right.$$

$$\frac{e\left(b+2cx^2\right)}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}$$

input
```
Int[(d + e*x)/(a + b*x^2 + c*x^4)^(3/2),x]
```

output
```
-((e*(b + 2*c*x^2))/((b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4])) + d*((x*(b^2
- 2*a*c + b*c*x^2))/(a*(b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4]) - (c*(-((b*(
-((x*Sqrt[a + b*x^2 + c*x^4])/(Sqrt[a] + Sqrt[c]*x^2)) + (a^(1/4)*(Sqrt[a]
 + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Ellipt
icE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(c^(1/4)*
Sqrt[a + b*x^2 + c*x^4])))/Sqrt[c]) + (a^(1/4)*(2*Sqrt[a] + b/Sqrt[c])*(Sq
rt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*E
llipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*c
^(1/4)*Sqrt[a + b*x^2 + c*x^4])))/(a*(b^2 - 4*a*c)))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1432
```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 2202
```
Int[(Pn_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{n
 = Expon[Pn, x], k}, Int[Sum[Coeff[Pn, x, 2*k]*x^(2*k), {k, 0, n/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pn, x, 2*k + 1]*x^(2*k), {k, 0, (n -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pn, x]
 &&  !PolyQ[Pn, x^2]
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 520, normalized size of antiderivative = 1.32

method	result
default	$d\left(-\frac{2c\left(\frac{b\,x^3}{2a\left(4ac-b^2\right)}-\frac{\left(2ac-b^2\right)x}{2a\left(4ac-b^2\right)c}\right)}{\sqrt{\left(x^4+\frac{b\,x^2}{c}+\frac{a}{c}\right)c}}+\frac{\left(\frac{1}{a}-\frac{2ac-b^2}{a\left(4ac-b^2\right)}\right)\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\operatorname{EllipticF}\left(\frac{x}{\cdots}\right.}{4\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,\sqrt{c\,x^4+b\,x^2+a}}\right.$
elliptic	$-\frac{2c\left(\frac{bd\,x^3}{2a\left(4ac-b^2\right)}-\frac{e\,x^2}{4ac-b^2}-\frac{d\left(2ac-b^2\right)x}{2a\left(4ac-b^2\right)c}-\frac{eb}{2\left(4ac-b^2\right)c}\right)}{\sqrt{\left(x^4+\frac{b\,x^2}{c}+\frac{a}{c}\right)c}}+\frac{\left(\frac{d}{a}-\frac{d\left(2ac-b^2\right)}{a\left(4ac-b^2\right)}\right)\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\cdots}{4\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\cdots}$

input
```
int((e*x+d)/(c*x^4+b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
d*(-2*c*(1/2*b/a/(4*a*c-b^2)*x^3-1/2*(2*a*c-b^2)/a/(4*a*c-b^2)/c*x)/((x^4+
b/c*x^2+a/c)*c)^(1/2)+1/4*(1/a-(2*a*c-b^2)/a/(4*a*c-b^2))*2^(1/2)/((-b+(-4
*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(
b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2
^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2)
)/a/c)^(1/2))-1/2*b/(4*a*c-b^2)*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2
)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*
x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2
^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2)
)/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/
2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))))+e/(c*x^4+b*x^2+a)^(1/2)*(2*
c*x^2+b)/(4*a*c-b^2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 487, normalized size of antiderivative = 1.24

$$\int \frac{d+ex}{(a+bx^2+cx^4)^{3/2}}\,dx =$$

$$-\frac{\sqrt{\frac{1}{2}}\left(b^2cdx^4+b^3dx^2+ab^2d-\left(abcdx^4+ab^2dx^2+a^2bd\right)\sqrt{\frac{b^2-4\,ac}{a^2}}\right)\sqrt{a}\sqrt{\frac{a\sqrt{\frac{b^2-4\,ac}{a^2}}-b}{a}}E\left(\arcsin\left(\sqrt{\frac{1}{2}}x\sqrt{\frac{a\sqrt{}}{}}\right.\right.}{}$$

input
```
integrate((e*x+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
-1/2*(sqrt(1/2)*(b^2*c*d*x^4 + b^3*d*x^2 + a*b^2*d - (a*b*c*d*x^4 + a*b^2*
d*x^2 + a^2*b*d)*sqrt((b^2 - 4*a*c)/a^2))*sqrt(a)*sqrt((a*sqrt((b^2 - 4*a*
c)/a^2) - b)/a)*elliptic_e(arcsin(sqrt(1/2)*x*sqrt((a*sqrt((b^2 - 4*a*c)/a
^2) - b)/a)), 1/2*(a*b*sqrt((b^2 - 4*a*c)/a^2) + b^2 - 2*a*c)/(a*c)) - sqr
t(1/2)*((2*a*b + b^2)*c*d*x^4 + (2*a*b^2 + b^3)*d*x^2 + (2*a^2*b + a*b^2)*
d + ((2*a^2 - a*b)*c*d*x^4 + (2*a^2*b - a*b^2)*d*x^2 + (2*a^3 - a^2*b)*d)*
sqrt((b^2 - 4*a*c)/a^2))*sqrt(a)*sqrt((a*sqrt((b^2 - 4*a*c)/a^2) - b)/a)*e
lliptic_f(arcsin(sqrt(1/2)*x*sqrt((a*sqrt((b^2 - 4*a*c)/a^2) - b)/a)), 1/2
*(a*b*sqrt((b^2 - 4*a*c)/a^2) + b^2 - 2*a*c)/(a*c)) - 2*(a*b*c*d*x^3 - 2*a
^2*c*e*x^2 - a^2*b*e + (a*b^2 - 2*a^2*c)*d*x)*sqrt(c*x^4 + b*x^2 + a))/(a^
3*b^2 - 4*a^4*c + (a^2*b^2*c - 4*a^3*c^2)*x^4 + (a^2*b^3 - 4*a^3*b*c)*x^2)
```

Sympy [F]

$$\int \frac{d+ex}{(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{d+ex}{(a+bx^2+cx^4)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x+d)/(c*x**4+b*x**2+a)**(3/2),x)
```

output
```
Integral((d + e*x)/(a + b*x**2 + c*x**4)**(3/2), x)
```

Maxima [F]

$$\int \frac{d+ex}{(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{ex+d}{(cx^4+bx^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((e*x + d)/(c*x^4 + b*x^2 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{d+ex}{(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{ex+d}{(cx^4+bx^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((e*x + d)/(c*x^4 + b*x^2 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex}{(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{d+e\,x}{(c\,x^4+b\,x^2+a)^{3/2}}\,dx$$

input
```
int((d + e*x)/(a + b*x^2 + c*x^4)^(3/2),x)
```

output
```
int((d + e*x)/(a + b*x^2 + c*x^4)^(3/2), x)
```

Reduce [F]

$$\int \frac{d+ex}{(a+bx^2+cx^4)^{3/2}}\,dx = \frac{\sqrt{c\,x^4+b\,x^2+a}\,be+2\sqrt{c\,x^4+b\,x^2+a}\,ce\,x^2+4\left(\int \frac{\sqrt{c\,x^4+b\,x^2+a}}{c^2x^8+2bc\,x^6+2ac\,x^4+b^2x^4+2ab\,x^2 -}\right.}{}$$

input
```
int((e*x+d)/(c*x^4+b*x^2+a)^(3/2),x)
```

output
```
(sqrt(a + b*x**2 + c*x**4)*b*e + 2*sqrt(a + b*x**2 + c*x**4)*c*e*x**2 + 4*
int(sqrt(a + b*x**2 + c*x**4)/(a**2 + 2*a*b*x**2 + 2*a*c*x**4 + b**2*x**4
+ 2*b*c*x**6 + c**2*x**8),x)*a**2*c*d - int(sqrt(a + b*x**2 + c*x**4)/(a**
2 + 2*a*b*x**2 + 2*a*c*x**4 + b**2*x**4 + 2*b*c*x**6 + c**2*x**8),x)*a*b**
2*d + 4*int(sqrt(a + b*x**2 + c*x**4)/(a**2 + 2*a*b*x**2 + 2*a*c*x**4 + b*
*2*x**4 + 2*b*c*x**6 + c**2*x**8),x)*a*b*c*d*x**2 + 4*int(sqrt(a + b*x**2
+ c*x**4)/(a**2 + 2*a*b*x**2 + 2*a*c*x**4 + b**2*x**4 + 2*b*c*x**6 + c**2*
x**8),x)*a*c**2*d*x**4 - int(sqrt(a + b*x**2 + c*x**4)/(a**2 + 2*a*b*x**2
+ 2*a*c*x**4 + b**2*x**4 + 2*b*c*x**6 + c**2*x**8),x)*b**3*d*x**2 - int(sq
rt(a + b*x**2 + c*x**4)/(a**2 + 2*a*b*x**2 + 2*a*c*x**4 + b**2*x**4 + 2*b*
c*x**6 + c**2*x**8),x)*b**2*c*d*x**4)/(4*a**2*c - a*b**2 + 4*a*b*c*x**2 +
4*a*c**2*x**4 - b**3*x**2 - b**2*c*x**4)
```

3.269 $\int \frac{1}{(a+bx^2+cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 353

$$\begin{aligned}\int \frac{1}{(a+bx^2+cx^4)^{3/2}}\,dx &= \frac{x(b^2-2ac+bcx^2)}{a(b^2-4ac)\sqrt{a+bx^2+cx^4}} \\ &- \frac{b\sqrt{c}x\sqrt{a+bx^2+cx^4}}{a(b^2-4ac)\left(\sqrt{a}+\sqrt{c}x^2\right)} \\ &+ \frac{b\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{a^{3/4}(b^2-4ac)\sqrt{a+bx^2+cx^4}} \\ &- \frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2a^{3/4}\left(b-2\sqrt{a}\sqrt{c}\right)\sqrt{a+bx^2+cx^4}}\end{aligned}$$

output
```
x*(b*c*x^2-2*a*c+b^2)/a/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^(1/2)-b*c^(1/2)*x*(c*
x^4+b*x^2+a)^(1/2)/a/(-4*a*c+b^2)/(a^(1/2)+c^(1/2)*x^2)+b*c^(1/4)*(a^(1/2)
+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*EllipticE(si
n(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))/a^(3/4)/(-
4*a*c+b^2)/(c*x^4+b*x^2+a)^(1/2)-1/2*c^(1/4)*(a^(1/2)+c^(1/2)*x^2)*((c*x^4
+b*x^2+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(c^(1/4)*
x/a^(1/4)),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))/a^(3/4)/(b-2*a^(1/2)*c^(1/2))/
(c*x^4+b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.03 (sec) , antiderivative size = 456, normalized size of antiderivative = 1.29

$$\int \frac{1}{(a+bx^2+cx^4)^{3/2}}\,dx =$$

$$-\frac{-4\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x(b^2-2ac+bcx^2)+ib\left(-b+\sqrt{b^2-4ac}\right)\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{\frac{2b-2\sqrt{b^2-4ac}+4cx^2}{b-\sqrt{b^2-4ac}}}E\Bigg(i\text{arcsinh}}{}$$

input
```
Integrate[(a + b*x^2 + c*x^4)^(-3/2),x]
```

output
```
-1/4*(-4*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x*(b^2 - 2*a*c + b*c*x^2) + I*b*(
-b + Sqrt[b^2 - 4*a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b
^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4
*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b
 + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] - I*(-b^2 + 4*a*c + b*Sqrt[
b^2 - 4*a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c
])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*El
lipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^
2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/(a*(b^2 - 4*a*c)*Sqrt[c/(b + Sqrt[b^
2 - 4*a*c])]*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 348, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1405, 27, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2+cx^4)^{3/2}}\,dx$$

$$\Big\downarrow \text{1405}$$

$$\frac{x\left(-2ac+b^2+bcx^2\right)}{a\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}-\frac{\int \frac{c\left(bx^2+2a\right)}{\sqrt{cx^4+bx^2+a}}dx}{a\left(b^2-4ac\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{x\left(-2ac+b^2+bcx^2\right)}{a\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}-\frac{c\int \frac{bx^2+2a}{\sqrt{cx^4+bx^2+a}}dx}{a\left(b^2-4ac\right)}$$

$$\Big\downarrow \; 1511$$

$$\frac{x\left(-2ac+b^2+bcx^2\right)}{a\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}-\frac{c\left(\sqrt{a}\left(2\sqrt{a}+\frac{b}{\sqrt{c}}\right)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx-\frac{\sqrt{a}b\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{a}\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}\right)}{a\left(b^2-4ac\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{x\left(-2ac+b^2+bcx^2\right)}{a\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}-\frac{c\left(\sqrt{a}\left(2\sqrt{a}+\frac{b}{\sqrt{c}}\right)\int \frac{1}{\sqrt{cx^4+bx^2+a}}dx-\frac{b\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}\right)}{a\left(b^2-4ac\right)}$$

$$\Big\downarrow \; 1416$$

$$\frac{x\left(-2ac+b^2+bcx^2\right)}{a\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}-\frac{c\left(\frac{\sqrt[4]{a}\left(2\sqrt{a}+\frac{b}{\sqrt{c}}\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}-\frac{b\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+bx^2+a}}dx}{\sqrt{c}}\right)}{a\left(b^2-4ac\right)}$$

$$\Big\downarrow \; 1509$$

$$\frac{x\left(-2ac+b^2+bcx^2\right)}{a\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}-\frac{c\left(\frac{\sqrt[4]{a}\left(2\sqrt{a}+\frac{b}{\sqrt{c}}\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}-\frac{b\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\right.}{\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}\right.}{\sqrt{}}\right.}{a\left(b^2-4ac\right)}$$

input
```
Int[(a + b*x^2 + c*x^4)^(-3/2),x]
```

output
```
(x*(b^2 - 2*a*c + b*c*x^2))/(a*(b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4]) - (c
*(-((b*(-((x*Sqrt[a + b*x^2 + c*x^4])/(Sqrt[a] + Sqrt[c]*x^2)) + (a^(1/4)*
(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2
]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(
c^(1/4)*Sqrt[a + b*x^2 + c*x^4])))/Sqrt[c]) + (a^(1/4)*(2*Sqrt[a] + b/Sqrt
[c])*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x
^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/
4])/(2*c^(1/4)*Sqrt[a + b*x^2 + c*x^4])))/(a*(b^2 - 4*a*c))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 481, normalized size of antiderivative = 1.36

method	result
default	$-\frac{2c\left(\frac{b\,x^{3}}{2a\left(4ac-b^{2}\right)}-\frac{\left(2ac-b^{2}\right)x}{2a\left(4ac-b^{2}\right)c}\right)}{\sqrt{\left(x^{4}+\frac{b\,x^{2}}{c}+\frac{a}{c}\right)c}}+\frac{\left(\frac{1}{a}-\frac{2ac-b^{2}}{a\left(4ac-b^{2}\right)}\right)\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^{2}}\right)x^{2}}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^{2}}\right)x^{2}}{a}}\,\mathrm{EllipticF}\left(\frac{x\sqrt{2}\,\sqrt{}}{}\right.}{4\sqrt{\frac{-b+\sqrt{-4ac+b^{2}}}{a}}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}$
elliptic	$-\frac{2c\left(\frac{b\,x^{3}}{2a\left(4ac-b^{2}\right)}-\frac{\left(2ac-b^{2}\right)x}{2a\left(4ac-b^{2}\right)c}\right)}{\sqrt{\left(x^{4}+\frac{b\,x^{2}}{c}+\frac{a}{c}\right)c}}+\frac{\left(\frac{1}{a}-\frac{2ac-b^{2}}{a\left(4ac-b^{2}\right)}\right)\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^{2}}\right)x^{2}}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^{2}}\right)x^{2}}{a}}\,\mathrm{EllipticF}\left(\frac{x\sqrt{2}\,\sqrt{}}{}\right.}{4\sqrt{\frac{-b+\sqrt{-4ac+b^{2}}}{a}}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}$

input
```
int(1/(c*x^4+b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*c*(1/2*b/a/(4*a*c-b^2)*x^3-1/2*(2*a*c-b^2)/a/(4*a*c-b^2)/c*x)/((x^4+b/c
*x^2+a/c)*c)^(1/2)+1/4*(1/a-(2*a*c-b^2)/a/(4*a*c-b^2))*2^(1/2)/((-b+(-4*a*
c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(
-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1
/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a
/c)^(1/2))-1/2*b/(4*a*c-b^2)*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(
4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2
)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1
/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a
/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(
-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 452, normalized size of antiderivative = 1.28

$$\int \frac{1}{(a+bx^2+cx^4)^{3/2}}\,dx =$$

$$-\frac{\sqrt{\frac{1}{2}}\left(b^2cx^4+b^3x^2+ab^2-(abcx^4+ab^2x^2+a^2b)\sqrt{\frac{b^2-4\,ac}{a^2}}\right)\sqrt{a}\sqrt{\frac{a\sqrt{\frac{b^2-4\,ac}{a^2}}-b}{a}}E(\arcsin\left(\sqrt{\frac{1}{2}}x\sqrt{\frac{a\sqrt{\frac{b^2-4\,ac}{a^2}}-}{a}}\right.}{}$$

input
```
integrate(1/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
-1/2*(sqrt(1/2)*(b^2*c*x^4 + b^3*x^2 + a*b^2 - (a*b*c*x^4 + a*b^2*x^2 + a^
2*b)*sqrt((b^2 - 4*a*c)/a^2))*sqrt(a)*sqrt((a*sqrt((b^2 - 4*a*c)/a^2) - b)
/a)*elliptic_e(arcsin(sqrt(1/2)*x*sqrt((a*sqrt((b^2 - 4*a*c)/a^2) - b)/a))
, 1/2*(a*b*sqrt((b^2 - 4*a*c)/a^2) + b^2 - 2*a*c)/(a*c)) - sqrt(1/2)*((2*a
*b + b^2)*c*x^4 + 2*a^2*b + a*b^2 + (2*a*b^2 + b^3)*x^2 + ((2*a^2 - a*b)*c
*x^4 + 2*a^3 - a^2*b + (2*a^2*b - a*b^2)*x^2)*sqrt((b^2 - 4*a*c)/a^2))*sqr
t(a)*sqrt((a*sqrt((b^2 - 4*a*c)/a^2) - b)/a)*elliptic_f(arcsin(sqrt(1/2)*x
*sqrt((a*sqrt((b^2 - 4*a*c)/a^2) - b)/a)), 1/2*(a*b*sqrt((b^2 - 4*a*c)/a^2
) + b^2 - 2*a*c)/(a*c)) - 2*(a*b*c*x^3 + (a*b^2 - 2*a^2*c)*x)*sqrt(c*x^4 +
 b*x^2 + a))/(a^3*b^2 - 4*a^4*c + (a^2*b^2*c - 4*a^3*c^2)*x^4 + (a^2*b^3 -
 4*a^3*b*c)*x^2)
```

Sympy [F]

$$\int \frac{1}{(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{1}{(a+bx^2+cx^4)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(c*x**4+b*x**2+a)**(3/2),x)
```

output
```
Integral((a + b*x**2 + c*x**4)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(a + bx^2 + cx^4)^{3/2}}\,dx = \int \frac{1}{(cx^4 + bx^2 + a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*x^4 + b*x^2 + a)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(a + bx^2 + cx^4)^{3/2}}\,dx = \int \frac{1}{(cx^4 + bx^2 + a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((c*x^4 + b*x^2 + a)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a + bx^2 + cx^4)^{3/2}}\,dx = \int \frac{1}{(c\,x^4 + b\,x^2 + a)^{3/2}}\,dx$$

input
```
int(1/(a + b*x^2 + c*x^4)^(3/2),x)
```

output
```
int(1/(a + b*x^2 + c*x^4)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{\sqrt{c\,x^4+b\,x^2+a}}{c^2x^8+2bc\,x^6+2ac\,x^4+b^2x^4+2ab\,x^2+a^2}dx$$

input
```
int(1/(c*x^4+b*x^2+a)^(3/2),x)
```

output
```
int(sqrt(a + b*x**2 + c*x**4)/(a**2 + 2*a*b*x**2 + 2*a*c*x**4 + b**2*x**4
+ 2*b*c*x**6 + c**2*x**8),x)
```

3.270 $\int \frac{1}{(d+ex)(a+bx^2+cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 1010

$$\int \frac{1}{(d+ex)\,(a+bx^2+cx^4)^{3/2}}\,dx = \text{Too large to display}$$

output
```
e*(b*c*d^2+b^2*e^2-2*a*c*e^2+c*(b*e^2+2*c*d^2)*x^2)/(-4*a*c+b^2)/(a*e^4+b*
d^2*e^2+c*d^4)/(c*x^4+b*x^2+a)^(1/2)+d*x*(c*(-2*a*c+b^2)*d^2+b*(-3*a*c+b^2
)*e^2+c*(-2*a*c*e^2+b^2*e^2+b*c*d^2)*x^2)/a/(-4*a*c+b^2)/(a*e^4+b*d^2*e^2+
c*d^4)/(c*x^4+b*x^2+a)^(1/2)-c^(1/2)*d*(-2*a*c*e^2+b^2*e^2+b*c*d^2)*x*(c*x
^4+b*x^2+a)^(1/2)/a/(-4*a*c+b^2)/(a*e^4+b*d^2*e^2+c*d^4)/(a^(1/2)+c^(1/2)*
x^2)+1/2*e^5*arctanh((a*e^4+b*d^2*e^2+c*d^4)^(1/2)*x/d/e/(c*x^4+b*x^2+a)^(
1/2))/(a*e^4+b*d^2*e^2+c*d^4)^(3/2)-1/2*e^5*arctanh(1/2*(b*d^2+2*a*e^2+(b*
e^2+2*c*d^2)*x^2)/(a*e^4+b*d^2*e^2+c*d^4)^(1/2)/(c*x^4+b*x^2+a)^(1/2))/(a*
e^4+b*d^2*e^2+c*d^4)^(3/2)+c^(1/4)*d*(-2*a*c*e^2+b^2*e^2+b*c*d^2)*(a^(1/2)
+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*EllipticE(si
n(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))/a^(3/4)/(-
4*a*c+b^2)/(a*e^4+b*d^2*e^2+c*d^4)/(c*x^4+b*x^2+a)^(1/2)-1/2*c^(3/4)*d*(a^
(1/2)+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*Inverse
JacobiAM(2*arctan(c^(1/4)*x/a^(1/4)),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))/a^(3
/4)/(b-2*a^(1/2)*c^(1/2))/(c^(1/2)*d^2+a^(1/2)*e^2)/(c*x^4+b*x^2+a)^(1/2)-
1/4*e^4*(c^(1/2)*d^2-a^(1/2)*e^2)*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+b*x^2+a)/(
a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*EllipticPi(sin(2*arctan(c^(1/4)*x/a^(1/4))),
1/4*(c^(1/2)*d^2+a^(1/2)*e^2)^2/a^(1/2)/c^(1/2)/d^2/e^2,1/2*(2-b/a^(1/2)/c
^(1/2))^(1/2))/a^(1/4)/c^(1/4)/d/(c^(1/2)*d^2+a^(1/2)*e^2)/(a*e^4+b*d^2*e^
2+c*d^4)/(c*x^4+b*x^2+a)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 18.36 (sec) , antiderivative size = 4965, normalized size of antiderivative = 4.92

$$\int \frac{1}{(d+ex)\,(a+bx^2+cx^4)^{3/2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[1/((d + e*x)*(a + b*x^2 + c*x^4)^(3/2)),x]
```

output
```
(-(a*b*c*d^2*e) - a*b^2*e^3 + 2*a^2*c*e^3 - b^2*c*d^3*x + 2*a*c^2*d^3*x -
b^3*d*e^2*x + 3*a*b*c*d*e^2*x - 2*a*c^2*d^2*e*x^2 - a*b*c*e^3*x^2 - b*c^2*
d^3*x^3 - b^2*c*d*e^2*x^3 + 2*a*c^2*d*e^2*x^3)/(a*(-b^2 + 4*a*c)*(c*d^4 +
b*d^2*e^2 + a*e^4)*Sqrt[a + b*x^2 + c*x^4]) + (((I/2)*b*c*(-b + Sqrt[b^2 -
 4*a*c])*d^3*Sqrt[1 - (2*c*x^2)/(-b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 - (2*c*x^
2)/(-b + Sqrt[b^2 - 4*a*c])]*(EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[-(c/(-b - S
qrt[b^2 - 4*a*c]))]*x], (-b - Sqrt[b^2 - 4*a*c])/(-b + Sqrt[b^2 - 4*a*c])]
 - EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[-(c/(-b - Sqrt[b^2 - 4*a*c]))]*x], (-b
 - Sqrt[b^2 - 4*a*c])/(-b + Sqrt[b^2 - 4*a*c])]))/(Sqrt[2]*Sqrt[-(c/(-b -
Sqrt[b^2 - 4*a*c]))]*Sqrt[a + b*x^2 + c*x^4]) + ((I/2)*b^2*(-b + Sqrt[b^2
- 4*a*c])*d*e^2*Sqrt[1 - (2*c*x^2)/(-b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 - (2*c
*x^2)/(-b + Sqrt[b^2 - 4*a*c])]*(EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[-(c/(-b
- Sqrt[b^2 - 4*a*c]))]*x], (-b - Sqrt[b^2 - 4*a*c])/(-b + Sqrt[b^2 - 4*a*c
])] - EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[-(c/(-b - Sqrt[b^2 - 4*a*c]))]*x],
(-b - Sqrt[b^2 - 4*a*c])/(-b + Sqrt[b^2 - 4*a*c])]))/(Sqrt[2]*Sqrt[-(c/(-b
 - Sqrt[b^2 - 4*a*c]))]*Sqrt[a + b*x^2 + c*x^4]) - (I*a*c*(-b + Sqrt[b^2 -
 4*a*c])*d*e^2*Sqrt[1 - (2*c*x^2)/(-b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 - (2*c*
x^2)/(-b + Sqrt[b^2 - 4*a*c])]*(EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[-(c/(-b -
 Sqrt[b^2 - 4*a*c]))]*x], (-b - Sqrt[b^2 - 4*a*c])/(-b + Sqrt[b^2 - 4*a*c]
)] - EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[-(c/(-b - Sqrt[b^2 - 4*a*c]))]*x]...
```

Rubi [A] (verified)

Time = 3.57 (sec) , antiderivative size = 1036, normalized size of antiderivative = 1.03, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {2266, 1547, 27, 1576, 1165, 27, 1154, 219, 2206, 27, 1511, 27, 1416, 1509, 2222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex)\left(a+bx^2+cx^4\right)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 2266}$$

$$d\int \frac{1}{\left(d^2-e^2x^2\right)\left(cx^4+bx^2+a\right)^{3/2}}dx-e\int \frac{x}{\left(d^2-e^2x^2\right)\left(cx^4+bx^2+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 1547}$$

$$d\left(\frac{\int \frac{\frac{c^{3/2}x^4e^4}{\sqrt{a}}+be^4+\sqrt{a}\sqrt{c}e^4+cd^2e^2+\frac{\sqrt{c}\left(cd^2+be^2+\sqrt{a}\sqrt{c}e^2\right)x^2e^2}{\sqrt{a}}+\frac{\sqrt{c}d^2\left(cd^2+be^2\right)}{\sqrt{a}}}{(cx^4+bx^2+a)^{3/2}}dx}{\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)(ae^4+bd^2e^2+cd^4)}+\frac{e^6\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a}(d^2-e^2x^2)\sqrt{cx^4+bx^2+a}}dx}{\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)(ae^4+bd^2e^2+cd^4)}\right)-e\int \frac{x}{(d^2-e^2x^2)\left(cx^4+bx^2+a\right)^{3/2}}dx$$

$$\Big\downarrow \; 27$$

$$d\left(\frac{\int \frac{\frac{c^{3/2}x^4e^4}{\sqrt{a}}+be^4+\sqrt{a}\sqrt{c}e^4+cd^2e^2+\frac{\sqrt{c}\left(cd^2+be^2+\sqrt{a}\sqrt{c}e^2\right)x^2e^2}{\sqrt{a}}+\frac{\sqrt{c}d^2\left(cd^2+be^2\right)}{\sqrt{a}}}{(cx^4+bx^2+a)^{3/2}}dx}{\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)(ae^4+bd^2e^2+cd^4)}+\frac{e^6\int \frac{\sqrt{c}x^2+\sqrt{a}}{(d^2-e^2x^2)\sqrt{cx^4+bx^2+a}}dx}{\sqrt{a}\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)(ae^4+bd^2e^2+cd^4)}\right)-e\int \frac{x}{(d^2-e^2x^2)\left(cx^4+bx^2+a\right)^{3/2}}dx$$

$$\Big\downarrow \; 1576$$

$$d\left(\frac{\int \frac{\frac{c^{3/2}x^4e^4}{\sqrt{a}}+be^4+\sqrt{a}\sqrt{c}e^4+cd^2e^2+\frac{\sqrt{c}\left(cd^2+be^2+\sqrt{a}\sqrt{c}e^2\right)x^2e^2}{\sqrt{a}}+\frac{\sqrt{c}d^2\left(cd^2+be^2\right)}{\sqrt{a}}}{(cx^4+bx^2+a)^{3/2}}dx}{\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)(ae^4+bd^2e^2+cd^4)}+\frac{e^6\int \frac{\sqrt{c}x^2+\sqrt{a}}{(d^2-e^2x^2)\sqrt{cx^4+bx^2+a}}dx}{\sqrt{a}\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)(ae^4+bd^2e^2+cd^4)}\right)-\frac{1}{2}e\int \frac{1}{(d^2-e^2x^2)\left(cx^4+bx^2+a\right)^{3/2}}dx^2$$

$$\Big\downarrow \; 1165$$

$$d\left(\frac{\int \frac{\frac{c^{3/2}x^4e^4}{\sqrt{a}}+be^4+\sqrt{a}\sqrt{c}e^4+cd^2e^2+\frac{\sqrt{c}\left(cd^2+be^2+\sqrt{a}\sqrt{c}e^2\right)x^2e^2}{\sqrt{a}}+\frac{\sqrt{c}d^2\left(cd^2+be^2\right)}{\sqrt{a}}}{(cx^4+bx^2+a)^{3/2}}dx}{\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)(ae^4+bd^2e^2+cd^4)}+\frac{e^6\int \frac{\sqrt{c}x^2+\sqrt{a}}{(d^2-e^2x^2)\sqrt{cx^4+bx^2+a}}dx}{\sqrt{a}\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)(ae^4+bd^2e^2+cd^4)}\right)-\frac{1}{2}e\left(-\frac{2\int -\frac{(b^2-4ac)e^4}{2(d^2-e^2x^2)\sqrt{cx^4+bx^2+a}}dx^2}{(b^2-4ac)(ae^4+bd^2e^2+cd^4)}-\frac{2\left(-2ace^2+b^2e^2+cx^2\left(be^2+2cd^2\right)+bcd^2\right)}{(b^2-4ac)\sqrt{a+bx^2+cx^4}\,(ae^4+bd^2e^2+cd^4)}\right)$$

$$\Big\downarrow \; 27$$

$$d\left(\frac{\int \frac{\frac{c^{3/2}x^4e^4}{\sqrt{a}}+be^4+\sqrt{a}\sqrt{c}e^4+cd^2e^2+\frac{\sqrt{c}\left(cd^2+be^2+\sqrt{a}\sqrt{c}e^2\right)x^2e^2}{\sqrt{a}}+\frac{\sqrt{c}d^2\left(cd^2+be^2\right)}{\sqrt{a}}}{\left(cx^4+bx^2+a\right)^{3/2}}dx}{\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)\left(ae^4+bd^2e^2+cd^4\right)}+\frac{e^6\int \frac{\sqrt{c}x^2+\sqrt{a}}{\left(d^2-e^2x^2\right)\sqrt{cx^4+bx^2+a}}dx}{\sqrt{a}\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)\left(ae^4+bd^2e^2+cd^4\right)}\right)-$$
$$\frac{1}{2}e\left(\frac{e^4\int \frac{1}{\left(d^2-e^2x^2\right)\sqrt{cx^4+bx^2+a}}dx^2}{ae^4+bd^2e^2+cd^4}-\frac{2\left(-2ace^2+b^2e^2+cx^2\left(be^2+2cd^2\right)+bcd^2\right)}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}\left(ae^4+bd^2e^2+cd^4\right)}\right)$$

$\Big\downarrow$ 1154

$$d\left(\frac{\int \frac{\frac{c^{3/2}x^4e^4}{\sqrt{a}}+be^4+\sqrt{a}\sqrt{c}e^4+cd^2e^2+\frac{\sqrt{c}\left(cd^2+be^2+\sqrt{a}\sqrt{c}e^2\right)x^2e^2}{\sqrt{a}}+\frac{\sqrt{c}d^2\left(cd^2+be^2\right)}{\sqrt{a}}}{\left(cx^4+bx^2+a\right)^{3/2}}dx}{\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)\left(ae^4+bd^2e^2+cd^4\right)}+\frac{e^6\int \frac{\sqrt{c}x^2+\sqrt{a}}{\left(d^2-e^2x^2\right)\sqrt{cx^4+bx^2+a}}dx}{\sqrt{a}\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)\left(ae^4+bd^2e^2+cd^4\right)}\right)-$$
$$\frac{1}{2}e\left(-\frac{2e^4\int \frac{1}{4\left(cd^4+be^2d^2+ae^4\right)-x^4}d\left(-\frac{bd^2+2ae^2+\left(2cd^2+be^2\right)x^2}{\sqrt{cx^4+bx^2+a}}\right)}{ae^4+bd^2e^2+cd^4}-\frac{2\left(-2ace^2+b^2e^2+cx^2\left(be^2+2cd^2\right)+bcd^2\right)}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}\left(ae^4+bd^2e^2+cd^4\right)}\right)$$

$\Big\downarrow$ 219

$$d\left(\frac{\int \frac{\frac{c^{3/2}x^4e^4}{\sqrt{a}}+be^4+\sqrt{a}\sqrt{c}e^4+cd^2e^2+\frac{\sqrt{c}\left(cd^2+be^2+\sqrt{a}\sqrt{c}e^2\right)x^2e^2}{\sqrt{a}}+\frac{\sqrt{c}d^2\left(cd^2+be^2\right)}{\sqrt{a}}}{\left(cx^4+bx^2+a\right)^{3/2}}dx}{\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)\left(ae^4+bd^2e^2+cd^4\right)}+\frac{e^6\int \frac{\sqrt{c}x^2+\sqrt{a}}{\left(d^2-e^2x^2\right)\sqrt{cx^4+bx^2+a}}dx}{\sqrt{a}\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)\left(ae^4+bd^2e^2+cd^4\right)}\right)-$$
$$\frac{1}{2}e\left(\frac{e^4\text{arctanh}\left(\frac{2ae^2+x^2\left(be^2+2cd^2\right)+bd^2}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^4+bd^2e^2+cd^4}}\right)}{\left(ae^4+bd^2e^2+cd^4\right)^{3/2}}-\frac{2\left(-2ace^2+b^2e^2+cx^2\left(be^2+2cd^2\right)+bcd^2\right)}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}\left(ae^4+bd^2e^2+cd^4\right)}\right)$$

$\Big\downarrow$ 2206

$$d\left(\frac{\frac{x\left(\sqrt{a}e^2+\sqrt{c}d^2\right)\left(cx^2\left(-2ace^2+b^2e^2+bcd^2\right)-3abce^2-2ac^2d^2+b^3e^2+b^2cd^2\right)}{a^{3/2}\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}-\frac{\int \frac{\sqrt{c}\left(\sqrt{c}\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\left(bcd^2+b^2e^2-2ace^2\right)x^2+a\left(2c^2d^4+2\sqrt{a}c^{3/}\cdots\right.\right.}{\sqrt{a}\sqrt{cx^4+bx^2+a}}\cdots}{a\left(b^2-4ac\right)}}{\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)\left(ae^4+bd^2e^2+cd^4\right)}\right.$$
$$\frac{1}{2}e\left(\frac{e^4\text{arctanh}\left(\frac{2ae^2+x^2\left(be^2+2cd^2\right)+bd^2}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^4+bd^2e^2+cd^4}}\right)}{\left(ae^4+bd^2e^2+cd^4\right)^{3/2}}-\frac{2\left(-2ace^2+b^2e^2+cx^2\left(be^2+2cd^2\right)+bcd^2\right)}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}\left(ae^4+bd^2e^2+cd^4\right)}\right)$$

$\Big\downarrow$ 27

$$d\left(\frac{\frac{x(\sqrt{a}e^2+\sqrt{c}d^2)(cx^2(-2ace^2+b^2e^2+bcd^2)-3abce^2-2ac^2d^2+b^3e^2+b^2cd^2)}{a^{3/2}(b^2-4ac)\sqrt{a+bx^2+cx^4}}-\frac{\sqrt{c}\int\frac{\sqrt{c}\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\left(bcd^2+b^2e^2-2ace^2\right)x^2+a\left(2c^2d^4+2\sqrt{a}c^{3/}\cdots\right.}{\sqrt{cx^4+bx^2+a}}\cdots}{a^{3/2}(b^2-4ac)\cdots}}{\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)(ae^4+bd^2e^2+cd^4)}\cdots\right.$$

$$\frac{1}{2}e\left(\frac{e^4\text{arctanh}\left(\frac{2ae^2+x^2(be^2+2cd^2)+bd^2}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^4+bd^2e^2+cd^4}}\right)}{(ae^4+bd^2e^2+cd^4)^{3/2}}-\frac{2\left(-2ace^2+b^2e^2+cx^2\left(be^2+2cd^2\right)+bcd^2\right)}{(b^2-4ac)\sqrt{a+bx^2+cx^4}(ae^4+bd^2e^2+cd^4)}\right)$$

$$\Big\downarrow \text{1511}$$

$$d\left(\frac{\frac{x(\sqrt{a}e^2+\sqrt{c}d^2)(cx^2(-2ace^2+b^2e^2+bcd^2)-3abce^2-2ac^2d^2+b^3e^2+b^2cd^2)}{a^{3/2}(b^2-4ac)\sqrt{a+bx^2+cx^4}}-\frac{\sqrt{c}\left(\sqrt{a}\sqrt{c}(2\sqrt{a}\sqrt{c}+b)(ae^4+bd^2e^2+cd^4)\int\frac{1}{\sqrt{cx^4+bx^2+a}}dx\cdots\right.}{a^{3/2}(\cdots}}{\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)(ae^4+bd^2e^2+cd^4)}\cdots\right.$$

$$\frac{1}{2}e\left(\frac{e^4\text{arctanh}\left(\frac{2ae^2+x^2(be^2+2cd^2)+bd^2}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^4+bd^2e^2+cd^4}}\right)}{(ae^4+bd^2e^2+cd^4)^{3/2}}-\frac{2\left(-2ace^2+b^2e^2+cx^2\left(be^2+2cd^2\right)+bcd^2\right)}{(b^2-4ac)\sqrt{a+bx^2+cx^4}(ae^4+bd^2e^2+cd^4)}\right)$$

$$\Big\downarrow \text{27}$$

$$d\left(\frac{\frac{x(\sqrt{a}e^2+\sqrt{c}d^2)(cx^2(-2ace^2+b^2e^2+bcd^2)-3abce^2-2ac^2d^2+b^3e^2+b^2cd^2)}{a^{3/2}(b^2-4ac)\sqrt{a+bx^2+cx^4}}-\frac{\sqrt{c}\left(\sqrt{a}\sqrt{c}(2\sqrt{a}\sqrt{c}+b)(ae^4+bd^2e^2+cd^4)\int\frac{1}{\sqrt{cx^4+bx^2+a}}dx\cdots\right.}{a^{3/2}(b^2\cdots}}{\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)(ae^4+bd^2e^2+cd^4)}\cdots\right.$$

$$\frac{1}{2}e\left(\frac{e^4\text{arctanh}\left(\frac{2ae^2+x^2(be^2+2cd^2)+bd^2}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^4+bd^2e^2+cd^4}}\right)}{(ae^4+bd^2e^2+cd^4)^{3/2}}-\frac{2\left(-2ace^2+b^2e^2+cx^2\left(be^2+2cd^2\right)+bcd^2\right)}{(b^2-4ac)\sqrt{a+bx^2+cx^4}(ae^4+bd^2e^2+cd^4)}\right)$$

$$\Big\downarrow \text{1416}$$

$$d\left(\frac{\frac{x(\sqrt{a}e^2+\sqrt{c}d^2)(cx^2(-2ace^2+b^2e^2+bcd^2)-3abce^2-2ac^2d^2+b^3e^2+b^2cd^2)}{a^{3/2}(b^2-4ac)\sqrt{a+bx^2+cx^4}}-\frac{\sqrt{c}\left(\frac{\sqrt[4]{a}\sqrt[4]{c}(2\sqrt{a}\sqrt{c}+b)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(ae^4+bd^2\cdots\right.}{2\sqrt{a+bx^2\cdots}}\cdots\right.}{\cdots}}{\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)(ae^4+bd^2e^2+\cdots}\cdots\right.$$

$$\frac{1}{2}e\left(\frac{e^4\text{arctanh}\left(\frac{2ae^2+x^2(be^2+2cd^2)+bd^2}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^4+bd^2e^2+cd^4}}\right)}{(ae^4+bd^2e^2+cd^4)^{3/2}}-\frac{2\left(-2ace^2+b^2e^2+cx^2\left(be^2+2cd^2\right)+bcd^2\right)}{(b^2-4ac)\sqrt{a+bx^2+cx^4}(ae^4+bd^2e^2+cd^4)}\right)$$

$\Big\downarrow$ 1509

$$d\left(\frac{\int \frac{\sqrt{c}x^2+\sqrt{a}}{(d^2-e^2x^2)\sqrt{cx^4+bx^2+a}}dx e^6}{\sqrt{a}\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)(cd^4+be^2d^2+ae^4)}+\frac{(\sqrt{c}d^2+\sqrt{a}e^2)x\left(e^2b^3+cd^2b^2-3ace^2b-2ac^2d^2+c\left(bcd^2+b^2e^2-2ace^2\right)x^2\right)}{a^{3/2}(b^2-4ac)\sqrt{cx^4+bx^2+a}}-\frac{\sqrt{c}\left(\frac{\sqrt[4]{a}(b\ldots}{\ldots}\right.}{\ldots}\right.$$

$$\frac{1}{2}e\left(\frac{e^4\text{arctanh}\left(\frac{bd^2+2ae^2+(2cd^2+be^2)x^2}{2\sqrt{cd^4+be^2d^2+ae^4}\sqrt{cx^4+bx^2+a}}\right)}{(cd^4+be^2d^2+ae^4)^{3/2}}-\frac{2\left(bcd^2+b^2e^2-2ace^2+c\left(2cd^2+be^2\right)x^2\right)}{(b^2-4ac)(cd^4+be^2d^2+ae^4)\sqrt{cx^4+bx^2+a}}\right)$$

$\Big\downarrow$ 2222

$$d\left(\frac{\left(\frac{(\sqrt{c}d^2+\sqrt{a}e^2)\text{arctanh}\left(\frac{\sqrt{cd^4+be^2d^2+ae^4}x}{de\sqrt{cx^4+bx^2+a}}\right)}{2de\sqrt{cd^4+be^2d^2+ae^4}}+\frac{\left(\frac{\sqrt{a}}{d^2}-\frac{\sqrt{c}}{e^2}\right)(\sqrt{c}x^2+\sqrt{a})\sqrt{\frac{cx^4+bx^2+a}{\left(\sqrt{c}x^2+\sqrt{a}\right)^2}}\text{EllipticPi}\left(\frac{\left(\sqrt{c}d^2+\sqrt{a}e^2\right)^2}{4\sqrt{a}\sqrt{c}d^2e^2},2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\ldots\right.}{4\sqrt[4]{a}\sqrt[4]{c}\sqrt{cx^4+bx^2+a}}\ldots\right.}{\sqrt{a}\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)(cd^4+be^2d^2+ae^4)}\ldots\right.$$

$$\frac{1}{2}e\left(\frac{e^4\text{arctanh}\left(\frac{bd^2+2ae^2+(2cd^2+be^2)x^2}{2\sqrt{cd^4+be^2d^2+ae^4}\sqrt{cx^4+bx^2+a}}\right)}{(cd^4+be^2d^2+ae^4)^{3/2}}-\frac{2\left(bcd^2+b^2e^2-2ace^2+c\left(2cd^2+be^2\right)x^2\right)}{(b^2-4ac)(cd^4+be^2d^2+ae^4)\sqrt{cx^4+bx^2+a}}\right)$$

input
```
Int[1/((d + e*x)*(a + b*x^2 + c*x^4)^(3/2)),x]
```

output
```
-1/2*(e*((-2*(b*c*d^2 + b^2*e^2 - 2*a*c*e^2 + c*(2*c*d^2 + b*e^2)*x^2))/((
b^2 - 4*a*c)*(c*d^4 + b*d^2*e^2 + a*e^4)*Sqrt[a + b*x^2 + c*x^4]) + (e^4*A
rcTanh[(b*d^2 + 2*a*e^2 + (2*c*d^2 + b*e^2)*x^2)/(2*Sqrt[c*d^4 + b*d^2*e^2
 + a*e^4]*Sqrt[a + b*x^2 + c*x^4])])/(c*d^4 + b*d^2*e^2 + a*e^4)^(3/2))) +
 d*((((Sqrt[c]*d^2 + Sqrt[a]*e^2)*x*(b^2*c*d^2 - 2*a*c^2*d^2 + b^3*e^2 - 3
*a*b*c*e^2 + c*(b*c*d^2 + b^2*e^2 - 2*a*c*e^2)*x^2))/(a^(3/2)*(b^2 - 4*a*c
)*Sqrt[a + b*x^2 + c*x^4]) - (Sqrt[c]*(-((Sqrt[c]*d^2 + Sqrt[a]*e^2)*(b*c*
d^2 + b^2*e^2 - 2*a*c*e^2)*(-((x*Sqrt[a + b*x^2 + c*x^4])/(Sqrt[a] + Sqrt[
c]*x^2)) + (a^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt
[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqr
t[a]*Sqrt[c]))/4])/(c^(1/4)*Sqrt[a + b*x^2 + c*x^4]))) + (a^(1/4)*(b + 2*S
qrt[a]*Sqrt[c])*c^(1/4)*(c*d^4 + b*d^2*e^2 + a*e^4)*(Sqrt[a] + Sqrt[c]*x^2
)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(
c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*Sqrt[a + b*x^2 + c*x
^4])))/(a^(3/2)*(b^2 - 4*a*c)))/(((Sqrt[c]*d^2)/Sqrt[a] + e^2)*(c*d^4 + b*
d^2*e^2 + a*e^4)) + (e^6*(((Sqrt[c]*d^2 + Sqrt[a]*e^2)*ArcTanh[(Sqrt[c*d^4
 + b*d^2*e^2 + a*e^4]*x)/(d*e*Sqrt[a + b*x^2 + c*x^4])])/(2*d*e*Sqrt[c*d^4
 + b*d^2*e^2 + a*e^4]) + ((Sqrt[a]/d^2 - Sqrt[c]/e^2)*(Sqrt[a] + Sqrt[c]*x
^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticPi[(Sqrt[c
]*d^2 + Sqrt[a]*e^2)^2/(4*Sqrt[a]*Sqrt[c]*d^2*e^2), 2*ArcTan[(c^(1/4)*x...
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1165
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d
+ e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p
+ 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
 b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && LtQ[p, -1]
 && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 1547
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_)/((d_.) + (e_.)*(x_)^2), x_Sym
bol] :> Simp[-(c*d^2 - b*d*e + a*e^2)^(p + 1/2)/(e^(2*p)*(Rt[c/a, 2]*d - e)
)   Int[(1 + Rt[c/a, 2]*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]
+ Simp[(c*d^2 - b*d*e + a*e^2)^(p + 1/2)/(Rt[c/a, 2]*d - e)   Int[(a + b*x^
2 + c*x^4)^p*ExpandToSum[((Rt[c/a, 2]*d - e)*(c*d^2 - b*d*e + a*e^2)^(-p -
1/2) + ((1 + Rt[c/a, 2]*x^2)*(a + b*x^2 + c*x^4)^(-p - 1/2))/e^(2*p))/(d +
e*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && N
eQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[p + 1/2, 0] && NeQ[c*d^2 - a*e^2, 0] &
& PosQ[c/a]
```

rule 1576
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x]
, x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

rule 2206
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c
*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Px,
a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*
p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x
^2] && Expon[Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 2222
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTanh[Rt[b - c*(d/e) - a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
b - c*(d/e) - a*(e/d), 2])), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a +
 b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*d*e*q*Sqrt[a + b*x^2 + c*x^4]))*Ell
ipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x], 1/2 - b/(4*a*q^2)], x]]
/; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &&
 EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[-b + c*(d/e) + a*(e/d)]
```

rule 2266
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.)/((d_) + (e_.)*(x_)), x_Symbo
l] :> Simp[d   Int[(a + b*x^2 + c*x^4)^p/(d^2 - e^2*x^2), x], x] - Simp[e
 Int[x*((a + b*x^2 + c*x^4)^p/(d^2 - e^2*x^2)), x], x] /; FreeQ[{a, b, c, d
, e}, x] && IntegerQ[p + 1/2]
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 1111, normalized size of antiderivative = 1.10

method	result	size
default	Expression too large to display	1111
elliptic	Expression too large to display	1111

input
```
int(1/(e*x+d)/(c*x^4+b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*c*(-1/2*d*(2*a*c*e^2-b^2*e^2-b*c*d^2)/a/(4*a*c-b^2)/(a*e^4+b*d^2*e^2+c*
d^4)*x^3+1/2*e*(b*e^2+2*c*d^2)/(4*a*c-b^2)/(a*e^4+b*d^2*e^2+c*d^4)*x^2-1/2
*d*(3*a*b*c*e^2+2*a*c^2*d^2-b^3*e^2-b^2*c*d^2)/a/(4*a*c-b^2)/(a*e^4+b*d^2*
e^2+c*d^4)/c*x-1/2*e*(2*a*c*e^2-b^2*e^2-b*c*d^2)/(a*e^4+b*d^2*e^2+c*d^4)/(
4*a*c-b^2)/c)/((x^4+b/c*x^2+a/c)*c)^(1/2)+1/4*((b*e^2+c*d^2)*d/a/(a*e^4+b*
d^2*e^2+c*d^4)-d*(3*a*b*c*e^2+2*a*c^2*d^2-b^3*e^2-b^2*c*d^2)/a/(4*a*c-b^2)
/(a*e^4+b*d^2*e^2+c*d^4))*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(
-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/
2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/
a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))+1/2*(2*a*c*e^2-b^2
*e^2-b*c*d^2)*c*d/(4*a*c-b^2)/(a*e^4+b*d^2*e^2+c*d^4)*2^(1/2)/((-b+(-4*a*c
+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-
4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2)
)*(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(
b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2
)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)))+e^3/(a*e
^4+b*d^2*e^2+c*d^4)*(-1/2/(c*d^4/e^4+b*d^2/e^2+a)^(1/2)*arctanh(1/2*(2*c*x
^2*d^2/e^2+b*d^2/e^2+b*x^2+2*a)/(c*d^4/e^4+b*d^2/e^2+a)^(1/2)/(c*x^4+b*x^2
+a)^(1/2))+2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)/d*e*(1-1/2*(-b+(-4*a*
c+b^2)^(1/2))/a*x^2)^(1/2)*(1+1/2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)\left(a+bx^2+cx^4\right)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(d+ex)\left(a+bx^2+cx^4\right)^{3/2}}\,dx = \int \frac{1}{(d+ex)\left(a+bx^2+cx^4\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(e*x+d)/(c*x**4+b*x**2+a)**(3/2),x)
```

output
```
Integral(1/((d + e*x)*(a + b*x**2 + c*x**4)**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{(d+ex)\left(a+bx^2+cx^4\right)^{3/2}}\,dx = \int \frac{1}{\left(cx^4+bx^2+a\right)^{\frac{3}{2}}(ex+d)}\,dx$$

input
```
integrate(1/(e*x+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((c*x^4 + b*x^2 + a)^(3/2)*(e*x + d)), x)
```

Giac [F]

$$\int \frac{1}{(d+ex)\left(a+bx^2+cx^4\right)^{3/2}}\,dx = \int \frac{1}{\left(cx^4+bx^2+a\right)^{\frac{3}{2}}(ex+d)}\,dx$$

input
```
integrate(1/(e*x+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((c*x^4 + b*x^2 + a)^(3/2)*(e*x + d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)\left(a+bx^2+cx^4\right)^{3/2}}\,dx=\int \frac{1}{\left(d+e\,x\right)\,\left(c\,x^4+b\,x^2+a\right)^{3/2}}\,dx$$

input
```
int(1/((d + e*x)*(a + b*x^2 + c*x^4)^(3/2)),x)
```

output
```
int(1/((d + e*x)*(a + b*x^2 + c*x^4)^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{(d+ex)\left(a+bx^2+cx^4\right)^{3/2}}\,dx=\int \frac{1}{(ex+d)\left(c\,x^4+b\,x^2+a\right)^{\frac{3}{2}}}dx$$

input
```
int(1/(e*x+d)/(c*x^4+b*x^2+a)^(3/2),x)
```

output
```
int(1/(e*x+d)/(c*x^4+b*x^2+a)^(3/2),x)
```

3.271 $\int \frac{1}{(d+ex)^2 (a+bx^2+cx^4)^{3/2}} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 1490

$$\int \frac{1}{(d+ex)^2 (a+bx^2+cx^4)^{3/2}} \, dx = \text{Too large to display}$$

output
```
e^4*x/(a*e^4+b*d^2*e^2+c*d^4)/(-e^2*x^2+d^2)/(c*x^4+b*x^2+a)^(1/2)+2*d*e*(
b*c*d^2+b^2*e^2-2*a*c*e^2+c*(b*e^2+2*c*d^2)*x^2)/(-4*a*c+b^2)/(a*e^4+b*d^2
*e^2+c*d^4)/(-e^2*x^2+d^2)/(c*x^4+b*x^2+a)^(1/2)-x*(a*e^2*(-b*e^2+2*c*d^2)
*(b*c*d^2+(-2*a*c+b^2)*e^2)-(c*(-2*a*c+b^2)*d^2+b*(-3*a*c+b^2)*e^2)*(-3*a*
e^4+b*d^2*e^2+c*d^4)-c*((b*c*d^2+(-2*a*c+b^2)*e^2)*(-3*a*e^4+b*d^2*e^2+c*d
^4)-a*e^2*(-b^2*e^4+4*c^2*d^4))*x^2)/a/(-4*a*c+b^2)/(a*e^4+b*d^2*e^2+c*d^4
)^2/(c*x^4+b*x^2+a)^(1/2)-c^(1/2)*(b^3*d^2*e^4+b*c*d^2*(-5*a*e^4+c*d^4)-6*
a*c*e^2*(-a*e^4+c*d^4)+2*b^2*(-a*e^6+c*d^4*e^2))*x*(c*x^4+b*x^2+a)^(1/2)/a
/(-4*a*c+b^2)/(a*e^4+b*d^2*e^2+c*d^4)^2/(a^(1/2)+c^(1/2)*x^2)-d*e^3*(-8*a*
c*e^4+3*b^2*e^4+4*b*c*d^2*e^2+4*c^2*d^4)*(c*x^4+b*x^2+a)^(1/2)/(-4*a*c+b^2
)/(a*e^4+b*d^2*e^2+c*d^4)^2/(-e^2*x^2+d^2)+3/2*d*e^5*(b*e^2+2*c*d^2)*arcta
nh((a*e^4+b*d^2*e^2+c*d^4)^(1/2)*x/d/e/(c*x^4+b*x^2+a)^(1/2))/(a*e^4+b*d^2
*e^2+c*d^4)^(5/2)-3/2*d*e^5*(b*e^2+2*c*d^2)*arctanh(1/2*(b*d^2+2*a*e^2+(b*
e^2+2*c*d^2)*x^2)/(a*e^4+b*d^2*e^2+c*d^4)^(1/2)/(c*x^4+b*x^2+a)^(1/2))/(a*
e^4+b*d^2*e^2+c*d^4)^(5/2)+c^(1/4)*(b^3*d^2*e^4+b*c*d^2*(-5*a*e^4+c*d^4)-6
*a*c*e^2*(-a*e^4+c*d^4)+2*b^2*(-a*e^6+c*d^4*e^2))*(a^(1/2)+c^(1/2)*x^2)*((
c*x^4+b*x^2+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*EllipticE(sin(2*arctan(c^(1/
4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))/a^(3/4)/(-4*a*c+b^2)/(a*e^
4+b*d^2*e^2+c*d^4)^2/(c*x^4+b*x^2+a)^(1/2)-1/2*c^(1/4)*(c^(3/2)*d^4-2*a^(1
/2)*c*d^2*e^2-2*a^(1/2)*b*e^4+c^(1/2)*(3*a*e^4+b*d^2*e^2))*(a^(1/2)+c^(...
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 20.72 (sec) , antiderivative size = 10488, normalized size of antiderivative = 7.04

$$\int \frac{1}{(d+ex)^2 (a+bx^2+cx^4)^{3/2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[1/((d + e*x)^2*(a + b*x^2 + c*x^4)^(3/2)),x]
```

output
```
Result too large to show
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex)^2 (a+bx^2+cx^4)^{3/2}} \, dx$$
$$\Big\downarrow \text{ 7299}$$
$$\int \frac{1}{(d+ex)^2 (a+bx^2+cx^4)^{3/2}} dx$$

input
```
Int[1/((d + e*x)^2*(a + b*x^2 + c*x^4)^(3/2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 1545, normalized size of antiderivative = 1.04

method	result	size
default	Expression too large to display	1545
elliptic	Expression too large to display	1545

input
```
int(1/(e*x+d)^2/(c*x^4+b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-e^7/(a*e^4+b*d^2*e^2+c*d^4)^2*(c*x^4+b*x^2+a)^(1/2)/(e*x+d)-2*c*(1/2*(2*a
^2*c*e^6-a*b^2*e^6-5*a*b*c*d^2*e^4-6*a*c^2*d^4*e^2+b^3*d^2*e^4+2*b^2*c*d^4
*e^2+b*c^2*d^6)/a/(4*a*c-b^2)/(a*e^4+b*d^2*e^2+c*d^4)^2*x^3-d*e*(2*a*c*e^4
-b^2*e^4-2*b*c*d^2*e^2-2*c^2*d^4)/(4*a*c-b^2)/(a*e^4+b*d^2*e^2+c*d^4)^2*x^
2+1/2*(3*a^2*b*c*e^6+6*a^2*c^2*d^2*e^4-a*b^3*e^6-6*a*b^2*c*d^2*e^4-7*a*b*c
^2*d^4*e^2-2*a*c^3*d^6+b^4*d^2*e^4+2*b^3*c*d^4*e^2+b^2*c^2*d^6)/a/(4*a*c-b
^2)/c/(a*e^4+b*d^2*e^2+c*d^4)^2*x-d*e*(3*a*b*c*e^4+4*a*c^2*d^2*e^2-b^3*e^4
-2*b^2*c*d^2*e^2-b*c^2*d^4)/(4*a*c-b^2)/(a*e^4+b*d^2*e^2+c*d^4)^2/c)/((x^4
+b/c*x^2+a/c)*c)^(1/2)+1/4*(-c*d^2*e^4/(a*e^4+b*d^2*e^2+c*d^4)^2-(a*b*e^6+
3*a*c*d^2*e^4-b^2*d^2*e^4-2*b*c*d^4*e^2-c^2*d^6)/a/(a*e^4+b*d^2*e^2+c*d^4)
^2+(3*a^2*b*c*e^6+6*a^2*c^2*d^2*e^4-a*b^3*e^6-6*a*b^2*c*d^2*e^4-7*a*b*c^2*
d^4*e^2-2*a*c^3*d^6+b^4*d^2*e^4+2*b^3*c*d^4*e^2+b^2*c^2*d^6)/a/(4*a*c-b^2)
/(a*e^4+b*d^2*e^2+c*d^4)^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2
*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(
1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2)
)/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/2*(e^6*c/(a*e^
4+b*d^2*e^2+c*d^4)^2+c*(2*a^2*c*e^6-a*b^2*e^6-5*a*b*c*d^2*e^4-6*a*c^2*d^4*
e^2+b^3*d^2*e^4+2*b^2*c*d^4*e^2+b*c^2*d^6)/a/(4*a*c-b^2)/(a*e^4+b*d^2*e^2+
c*d^4)^2)*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2
)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^2 (a+bx^2+cx^4)^{3/2}} \, dx = \text{Timed out}$$

input
```
integrate(1/(e*x+d)^2/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(d+ex)^2\left(a+bx^2+cx^4\right)^{3/2}}\,dx = \int \frac{1}{\left(d+ex\right)^2\left(a+bx^2+cx^4\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(e*x+d)**2/(c*x**4+b*x**2+a)**(3/2),x)
```

output
```
Integral(1/((d + e*x)**2*(a + b*x**2 + c*x**4)**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{(d+ex)^2\left(a+bx^2+cx^4\right)^{3/2}}\,dx = \int \frac{1}{\left(cx^4+bx^2+a\right)^{\frac{3}{2}}\left(ex+d\right)^2}\,dx$$

input
```
integrate(1/(e*x+d)^2/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((c*x^4 + b*x^2 + a)^(3/2)*(e*x + d)^2), x)
```

Giac [F]

$$\int \frac{1}{(d+ex)^2\left(a+bx^2+cx^4\right)^{3/2}}\,dx = \int \frac{1}{\left(cx^4+bx^2+a\right)^{\frac{3}{2}}\left(ex+d\right)^2}\,dx$$

input
```
integrate(1/(e*x+d)^2/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^2 (a+bx^2+cx^4)^{3/2}} \, dx = \int \frac{1}{(d+e\,x)^2 \left(c\,x^4+b\,x^2+a\right)^{3/2}} \, dx$$

input
```
int(1/((d + e*x)^2*(a + b*x^2 + c*x^4)^(3/2)),x)
```

output
```
int(1/((d + e*x)^2*(a + b*x^2 + c*x^4)^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{(d+ex)^2 (a+bx^2+cx^4)^{3/2}} \, dx = \int \frac{1}{(ex+d)^2 \left(c\,x^4+b\,x^2+a\right)^{\frac{3}{2}}} dx$$

input
```
int(1/(e*x+d)^2/(c*x^4+b*x^2+a)^(3/2),x)
```

output
```
int(1/(e*x+d)^2/(c*x^4+b*x^2+a)^(3/2),x)
```

3.272 $\int \frac{\sqrt{2abx^2+b^2x^4}}{c+dx^2} dx$

Optimal result

Integrand size = 29, antiderivative size = 111

$$\int \frac{\sqrt{2abx^2+b^2x^4}}{c+dx^2} dx = \frac{\sqrt{bx^2(2a+bx^2)}}{dx} - \frac{\sqrt{bc-2ad}\sqrt{bx^2(2a+bx^2)}\arctan\left(\frac{\sqrt{d}\sqrt{2a+bx^2}}{\sqrt{bc-2ad}}\right)}{d^{3/2}x\sqrt{2a+bx^2}}$$

output
```
(b*x^2*(b*x^2+2*a))^(1/2)/d/x-(-2*a*d+b*c)^(1/2)*(b*x^2*(b*x^2+2*a))^(1/2)
*arctan(d^(1/2)*(b*x^2+2*a)^(1/2)/(-2*a*d+b*c)^(1/2))/d^(3/2)/x/(b*x^2+2*a
)^(1/2)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{2abx^2+b^2x^4}}{c+dx^2} dx = \frac{bx\sqrt{2a+bx^2}\left(\sqrt{d}\sqrt{2a+bx^2} - \sqrt{bc-2ad}\arctan\left(\frac{\sqrt{d}\sqrt{2a+bx^2}}{\sqrt{bc-2ad}}\right)\right)}{d^{3/2}\sqrt{bx^2(2a+bx^2)}}$$

input
```
Integrate[Sqrt[2*a*b*x^2 + b^2*x^4]/(c + d*x^2),x]
```

output
```
(b*x*Sqrt[2*a + b*x^2]*(Sqrt[d]*Sqrt[2*a + b*x^2] - Sqrt[b*c - 2*a*d]*ArcT
an[(Sqrt[d]*Sqrt[2*a + b*x^2])/Sqrt[b*c - 2*a*d]]))/(d^(3/2)*Sqrt[b*x^2*(2
*a + b*x^2)])
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.15, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {1466, 353, 60, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{2abx^2+b^2x^4}}{c+dx^2} \, dx$$

$$\Big\downarrow \text{ 1466}$$

$$\frac{\sqrt{2abx^2+b^2x^4}\int \frac{x\sqrt{b^2x^2+2ab}}{dx^2+c}dx}{x\sqrt{2ab+b^2x^2}}$$

$$\Big\downarrow \text{ 353}$$

$$\frac{\sqrt{2abx^2+b^2x^4}\int \frac{\sqrt{b^2x^2+2ab}}{dx^2+c}dx^2}{2x\sqrt{2ab+b^2x^2}}$$

$$\Big\downarrow \text{ 60}$$

$$\frac{\sqrt{2abx^2+b^2x^4}\left(\frac{2\sqrt{2ab+b^2x^2}}{d}-\frac{b(bc-2ad)\int \frac{1}{\sqrt{b^2x^2+2ab}(dx^2+c)}dx^2}{d}\right)}{2x\sqrt{2ab+b^2x^2}}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{\sqrt{2abx^2+b^2x^4}\left(\frac{2\sqrt{2ab+b^2x^2}}{d}-\frac{2(bc-2ad)\int \frac{1}{\frac{dx^4}{b^2}+c-\frac{2ad}{b}}d\sqrt{b^2x^2+2ab}}{bd}\right)}{2x\sqrt{2ab+b^2x^2}}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{\sqrt{2abx^2+b^2x^4}\left(\frac{2\sqrt{2ab+b^2x^2}}{d}-\frac{2\sqrt{b}\sqrt{bc-2ad}\arctan\left(\frac{\sqrt{d}\sqrt{2ab+b^2x^2}}{\sqrt{b}\sqrt{bc-2ad}}\right)}{d^{3/2}}\right)}{2x\sqrt{2ab+b^2x^2}}$$

input
```
Int[Sqrt[2*a*b*x^2 + b^2*x^4]/(c + d*x^2),x]
```

output
```
(Sqrt[2*a*b*x^2 + b^2*x^4]*((2*Sqrt[2*a*b + b^2*x^2])/d - (2*Sqrt[b]*Sqrt[
b*c - 2*a*d]*ArcTan[(Sqrt[d]*Sqrt[2*a*b + b^2*x^2])/(Sqrt[b]*Sqrt[b*c - 2*
a*d])])/d^(3/2)))/(2*x*Sqrt[2*a*b + b^2*x^2])
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 1466
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbo
l] :> Simp[(b*x^2 + c*x^4)^FracPart[p]/(x^(2*FracPart[p])*(b + c*x^2)^FracP
art[p])   Int[x^(2*p)*(d + e*x^2)^q*(b + c*x^2)^p, x], x] /; FreeQ[{b, c, d
, e, p, q}, x] &&  !IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 363 vs. $2(95) = 190$.

Time = 0.50 (sec) , antiderivative size = 364, normalized size of antiderivative = 3.28

method	result
default	$\frac{\sqrt{b^2x^4+2abx^2}\left(-2\ln\left(-\frac{2\left(\sqrt{-cd}\,b^2x+\sqrt{b\left(bx^2+2a\right)}\sqrt{\frac{b(2ad-bc)}{d}}\,d+2abd\right)}{-dx+\sqrt{-cd}}\right)abd+\ln\left(-\frac{2\left(\sqrt{-cd}\,b^2x+\sqrt{b\left(bx^2+2a\right)}\sqrt{\frac{b(2ad-}{}}\right.}{-dx+\sqrt{-cd}}\right.\right.}{}$
pseudoelliptic	$\frac{b\left(2\ln\left(\frac{\sqrt{b^2x^2+2ab}\sqrt{\frac{2abd-cb^2}{d}}\,d-\sqrt{-b^2cd}\,bx+2abd}{bdx+\sqrt{-b^2cd}}\right)abd-2\ln\left(\frac{\sqrt{b^2x^2+2ab}\sqrt{\frac{2abd-cb^2}{d}}\,d+\sqrt{-b^2cd}\,bx+2abd}{bdx-\sqrt{-b^2cd}}\right)abd-\ln\left(\right.\right.}{2}$
risch	$\frac{\sqrt{bx^2(bx^2+2a)}}{dx}+\frac{(2ad-bc)\left(-\frac{\ln\left(\frac{\frac{2b(2ad-bc)}{d}+\frac{2b^2\sqrt{-cd}\left(x-\frac{\sqrt{-cd}}{d}\right)}{d}+2\sqrt{\frac{b(2ad-bc)}{d}}\sqrt{b^2\left(x-\frac{\sqrt{-cd}}{d}\right)^2+\frac{2b^2\sqrt{-cd}\left(x-\right.}{d}}}{x-\frac{\sqrt{-cd}}{d}}\right)}{2d\sqrt{\frac{b(2ad-bc)}{d}}}\right.}{}$

input
```
int((b^2*x^4+2*a*b*x^2)^(1/2)/(d*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
1/2*(b^2*x^4+2*a*b*x^2)^(1/2)*(-2*ln(-2*((-c*d)^(1/2)*b^2*x+(b*(b*x^2+2*a)
)^(1/2)*(b*(2*a*d-b*c)/d)^(1/2)*d+2*a*b*d)/(-d*x+(-c*d)^(1/2)))*a*b*d+ln(-
2*((-c*d)^(1/2)*b^2*x+(b*(b*x^2+2*a))^(1/2)*(b*(2*a*d-b*c)/d)^(1/2)*d+2*a*
b*d)/(-d*x+(-c*d)^(1/2)))*b^2*c-2*ln(-2*((-c*d)^(1/2)*b^2*x-(b*(b*x^2+2*a)
)^(1/2)*(b*(2*a*d-b*c)/d)^(1/2)*d-2*a*b*d)/(d*x+(-c*d)^(1/2)))*a*b*d+ln(-2
*((-c*d)^(1/2)*b^2*x-(b*(b*x^2+2*a))^(1/2)*(b*(2*a*d-b*c)/d)^(1/2)*d-2*a*b
*d)/(d*x+(-c*d)^(1/2)))*b^2*c+2*(b*(b*x^2+2*a))^(1/2)*(b*(2*a*d-b*c)/d)^(1
/2)*d)/x/(b*(b*x^2+2*a))^(1/2)/d^2/(b*(2*a*d-b*c)/d)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.99

$$\int \frac{\sqrt{2abx^2+b^2x^4}}{c+dx^2}\,dx = \left[\frac{x\sqrt{-\frac{b^2c-2\,abd}{d}}\log\left(\frac{b^2dx^3-(b^2c-4\,abd)x-2\,\sqrt{b^2x^4+2\,abx^2}d\sqrt{-\frac{b^2c-2\,abd}{d}}}{dx^3+cx}\right)+2\,\sqrt{b^2x^4+2\,abx^2}}{2\,dx}, \frac{x\sqrt{\frac{b^2c-2\,abd}{d}}\arctan\left(\right.}{}\right.$$

input
```
integrate((b^2*x^4+2*a*b*x^2)^(1/2)/(d*x^2+c),x, algorithm="fricas")
```

output
```
[1/2*(x*sqrt(-(b^2*c - 2*a*b*d)/d)*log((b^2*d*x^3 - (b^2*c - 4*a*b*d)*x -
2*sqrt(b^2*x^4 + 2*a*b*x^2)*d*sqrt(-(b^2*c - 2*a*b*d)/d))/(d*x^3 + c*x)) +
 2*sqrt(b^2*x^4 + 2*a*b*x^2))/(d*x), (x*sqrt((b^2*c - 2*a*b*d)/d)*arctan(-
sqrt(b^2*x^4 + 2*a*b*x^2)*d*sqrt((b^2*c - 2*a*b*d)/d)/((b^2*c - 2*a*b*d)*x
)) + sqrt(b^2*x^4 + 2*a*b*x^2))/(d*x)]
```

Sympy [F]

$$\int \frac{\sqrt{2abx^2+b^2x^4}}{c+dx^2}\,dx = \int \frac{\sqrt{bx^2\cdot(2a+bx^2)}}{c+dx^2}\,dx$$

input
```
integrate((b**2*x**4+2*a*b*x**2)**(1/2)/(d*x**2+c),x)
```

output
```
Integral(sqrt(b*x**2*(2*a + b*x**2))/(c + d*x**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{2abx^2+b^2x^4}}{c+dx^2}\,dx = \int \frac{\sqrt{b^2x^4+2\,abx^2}}{dx^2+c}\,dx$$

input
```
integrate((b^2*x^4+2*a*b*x^2)^(1/2)/(d*x^2+c),x, algorithm="maxima")
```

output
```
integrate(sqrt(b^2*x^4 + 2*a*b*x^2)/(d*x^2 + c), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 203 vs. 2(95) = 190.

Time = 0.14 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.83

$$\int \frac{\sqrt{2abx^2+b^2x^4}}{c+dx^2}\,dx$$
$$= -\frac{(b^2c\mathrm{sgn}(x)-2\,abd\mathrm{sgn}(x))\arctan\left(\frac{\sqrt{b^2x^2+2\,ab}d}{\sqrt{b^2cd-2\,abd^2}}\right)}{\sqrt{b^2cd-2\,abd^2}d}+\frac{\sqrt{b^2x^2+2\,ab}\mathrm{sgn}(x)}{d}$$
$$+\frac{\left(b^2c\arctan\left(\frac{\sqrt{2}\sqrt{ab}d}{\sqrt{b^2cd-2\,abd^2}}\right)-2\,abd\arctan\left(\frac{\sqrt{2}\sqrt{ab}d}{\sqrt{b^2cd-2\,abd^2}}\right)-\sqrt{2}\sqrt{b^2cd-2\,abd^2}\sqrt{ab}\right)\mathrm{sgn}(x)}{\sqrt{b^2cd-2\,abd^2}d}$$

input
```
integrate((b^2*x^4+2*a*b*x^2)^(1/2)/(d*x^2+c),x, algorithm="giac")
```

output
```
-(b^2*c*sgn(x) - 2*a*b*d*sgn(x))*arctan(sqrt(b^2*x^2 + 2*a*b)*d/sqrt(b^2*c
*d - 2*a*b*d^2))/(sqrt(b^2*c*d - 2*a*b*d^2)*d) + sqrt(b^2*x^2 + 2*a*b)*sgn
(x)/d + (b^2*c*arctan(sqrt(2)*sqrt(a*b)*d/sqrt(b^2*c*d - 2*a*b*d^2)) - 2*a
*b*d*arctan(sqrt(2)*sqrt(a*b)*d/sqrt(b^2*c*d - 2*a*b*d^2)) - sqrt(2)*sqrt(
b^2*c*d - 2*a*b*d^2)*sqrt(a*b))*sgn(x)/(sqrt(b^2*c*d - 2*a*b*d^2)*d)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{2abx^2+b^2x^4}}{c+dx^2}\,dx=\int \frac{\sqrt{b^2\,x^4+2\,a\,b\,x^2}}{d\,x^2+c}\,dx$$

input
```
int((b^2*x^4 + 2*a*b*x^2)^(1/2)/(c + d*x^2),x)
```

output
```
int((b^2*x^4 + 2*a*b*x^2)^(1/2)/(c + d*x^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.90

$$\int \frac{\sqrt{2abx^2+b^2x^4}}{c+dx^2}\,dx = \frac{\sqrt{b}\left(-\sqrt{d}\,\sqrt{-2ad+bc}\,atan\left(\frac{\sqrt{b}\,\sqrt{b\,x^2+2a}\,dx+2ad+bd\,x^2}{\sqrt{d}\,\sqrt{b\,x^2+2a}\,\sqrt{-2ad+bc}+\sqrt{d}\,\sqrt{b}\,\sqrt{-2ad+bc}\,x}\right)+\sqrt{b\,x^2+2a}\,d\right)}{d^2}$$

input
```
int((b^2*x^4+2*a*b*x^2)^(1/2)/(d*x^2+c),x)
```

output
```
(sqrt(b)*( - sqrt(d)*sqrt( - 2*a*d + b*c)*atan((sqrt(b)*sqrt(2*a + b*x**2)
*d*x + 2*a*d + b*d*x**2)/(sqrt(d)*sqrt(2*a + b*x**2)*sqrt( - 2*a*d + b*c)
+ sqrt(d)*sqrt(b)*sqrt( - 2*a*d + b*c)*x)) + sqrt(2*a + b*x**2)*d))/d**2
```

3.273 $\int \frac{\sqrt{bx^2(2a+bx^2)}}{c+dx^2} \, dx$

Optimal result

Integrand size = 28, antiderivative size = 111

$$\int \frac{\sqrt{bx^2(2a+bx^2)}}{c+dx^2} \, dx = \frac{\sqrt{bx^2(2a+bx^2)}}{dx} - \frac{\sqrt{bc-2ad}\sqrt{bx^2(2a+bx^2)}\arctan\left(\frac{\sqrt{d}\sqrt{2a+bx^2}}{\sqrt{bc-2ad}}\right)}{d^{3/2}x\sqrt{2a+bx^2}}$$

output
```
(b*x^2*(b*x^2+2*a))^(1/2)/d/x-(-2*a*d+b*c)^(1/2)*(b*x^2*(b*x^2+2*a))^(1/2)
*arctan(d^(1/2)*(b*x^2+2*a)^(1/2)/(-2*a*d+b*c)^(1/2))/d^(3/2)/x/(b*x^2+2*a
)^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{bx^2(2a+bx^2)}}{c+dx^2} \, dx = \frac{bx\sqrt{2a+bx^2}\left(\sqrt{d}\sqrt{2a+bx^2} - \sqrt{bc-2ad}\arctan\left(\frac{\sqrt{d}\sqrt{2a+bx^2}}{\sqrt{bc-2ad}}\right)\right)}{d^{3/2}\sqrt{bx^2(2a+bx^2)}}$$

input
```
Integrate[Sqrt[b*x^2*(2*a + b*x^2)]/(c + d*x^2),x]
```

output
```
(b*x*Sqrt[2*a + b*x^2]*(Sqrt[d]*Sqrt[2*a + b*x^2] - Sqrt[b*c - 2*a*d]*ArcT
an[(Sqrt[d]*Sqrt[2*a + b*x^2])/Sqrt[b*c - 2*a*d]]))/(d^(3/2)*Sqrt[b*x^2*(2
*a + b*x^2)])
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.15, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {2048, 1466, 353, 60, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{bx^2 (2a+bx^2)}}{c+dx^2} \, dx$$

$$\Big\downarrow \text{ 2048}$$

$$\int \frac{\sqrt{2abx^2+b^2x^4}}{c+dx^2}dx$$

$$\Big\downarrow \text{ 1466}$$

$$\frac{\sqrt{2abx^2+b^2x^4}\int \frac{x\sqrt{b^2x^2+2ab}}{dx^2+c}dx}{x\sqrt{2ab+b^2x^2}}$$

$$\Big\downarrow \text{ 353}$$

$$\frac{\sqrt{2abx^2+b^2x^4}\int \frac{\sqrt{b^2x^2+2ab}}{dx^2+c}dx^2}{2x\sqrt{2ab+b^2x^2}}$$

$$\Big\downarrow \text{ 60}$$

$$\frac{\sqrt{2abx^2+b^2x^4}\left(\frac{2\sqrt{2ab+b^2x^2}}{d}-\frac{b(bc-2ad)\int \frac{1}{\sqrt{b^2x^2+2ab}\left(dx^2+c\right)}dx^2}{d}\right)}{2x\sqrt{2ab+b^2x^2}}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{\sqrt{2abx^2+b^2x^4}\left(\frac{2\sqrt{2ab+b^2x^2}}{d}-\frac{2(bc-2ad)\int \frac{1}{\frac{dx^4}{b^2}+c-\frac{2ad}{b}}d\sqrt{b^2x^2+2ab}}{bd}\right)}{2x\sqrt{2ab+b^2x^2}}$$

$$\big\downarrow \text{ 218}$$

$$\frac{\sqrt{2abx^2+b^2x^4}\left(\frac{2\sqrt{2ab+b^2x^2}}{d}-\frac{2\sqrt{b}\sqrt{bc-2ad}\arctan\left(\frac{\sqrt{d}\sqrt{2ab+b^2x^2}}{\sqrt{b}\sqrt{bc-2ad}}\right)}{d^{3/2}}\right)}{2x\sqrt{2ab+b^2x^2}}$$

input
```
Int[Sqrt[b*x^2*(2*a + b*x^2)]/(c + d*x^2),x]
```

output
```
(Sqrt[2*a*b*x^2 + b^2*x^4]*((2*Sqrt[2*a*b + b^2*x^2])/d - (2*Sqrt[b]*Sqrt[
b*c - 2*a*d]*ArcTan[(Sqrt[d]*Sqrt[2*a*b + b^2*x^2])/(Sqrt[b]*Sqrt[b*c - 2*
a*d])])/d^(3/2)))/(2*x*Sqrt[2*a*b + b^2*x^2])
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 1466
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbo
l] :> Simp[(b*x^2 + c*x^4)^FracPart[p]/(x^(2*FracPart[p])*(b + c*x^2)^FracP
art[p])   Int[x^(2*p)*(d + e*x^2)^q*(b + c*x^2)^p, x], x] /; FreeQ[{b, c, d
, e, p, q}, x] &&  !IntegerQ[p]
```

rule 2048
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))*((c_) + (d_.)*(x_)^(n_.)))^(p_)
, x_Symbol] :> Int[u*(a*c*e + (b*c + a*d)*e*x^n + b*d*e*x^(2*n))^p, x] /; F
reeQ[{a, b, c, d, e, n, p}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 362 vs. $2(95) = 190$.

Time = 0.31 (sec) , antiderivative size = 363, normalized size of antiderivative = 3.27

method	result
default	$\frac{\sqrt{b x^2(b x^2+2a)}\left(-2\ln\left(-\frac{2\left(\sqrt{-cd}\,b^2x+\sqrt{b\left(b x^2+2a\right)}\sqrt{\frac{b(2ad-bc)}{d}}\,d+2abd\right)}{-dx+\sqrt{-cd}}\right)abd+\ln\left(-\frac{2\left(\sqrt{-cd}\,b^2x+\sqrt{b\left(b x^2+2a\right)}\sqrt{\frac{b(2ad\cdots}{d}}\cdots\right.}{-dx+\sqrt{-cd}}\cdots\right.\right.}{\cdots}$
pseudoelliptic	$\frac{b\left(2\ln\left(\frac{\sqrt{b^2x^2+2ab}\sqrt{\frac{2abd-c\,b^2}{d}}\,d-\sqrt{-b^2cd}\,bx+2abd}{bdx+\sqrt{-b^2cd}}\right)abd-2\ln\left(\frac{\sqrt{b^2x^2+2ab}\sqrt{\frac{2abd-c\,b^2}{d}}\,d+\sqrt{-b^2cd}\,bx+2abd}{bdx-\sqrt{-b^2cd}}\right)abd-\ln\left(\frac{\sqrt{b^2\cdots}}{\cdots}\right.\cdots\right.}{2\cdots}$
risch	$\frac{\sqrt{b x^2(b x^2+2a)}}{dx}+\frac{(2ad-bc)\left(-\frac{\ln\left(\frac{\frac{2b(2ad-bc)}{d}+\frac{2b^2\sqrt{-cd}\left(x-\frac{\sqrt{-cd}}{d}\right)}{d}+2\sqrt{\frac{b(2ad-bc)}{d}}\sqrt{b^2\left(x-\frac{\sqrt{-cd}}{d}\right)^2+\frac{2b^2\sqrt{-cd}\left(x-\frac{\sqrt{\cdots}}{d}\right)}{d}\cdots}}{x-\frac{\sqrt{-cd}}{d}}\right)}{2d\sqrt{\frac{b(2ad-bc)}{d}}}\cdots\right)}{\cdots}$

input
```
int((b*x^2*(b*x^2+2*a))^(1/2)/(d*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
1/2*(b*x^2*(b*x^2+2*a))^(1/2)*(-2*ln(-2*((-c*d)^(1/2)*b^2*x+(b*(b*x^2+2*a)
)^(1/2)*(b*(2*a*d-b*c)/d)^(1/2)*d+2*a*b*d)/(-d*x+(-c*d)^(1/2)))*a*b*d+ln(-
2*((-c*d)^(1/2)*b^2*x+(b*(b*x^2+2*a))^(1/2)*(b*(2*a*d-b*c)/d)^(1/2)*d+2*a*
b*d)/(-d*x+(-c*d)^(1/2)))*b^2*c-2*ln(-2*((-c*d)^(1/2)*b^2*x-(b*(b*x^2+2*a)
)^(1/2)*(b*(2*a*d-b*c)/d)^(1/2)*d-2*a*b*d)/(d*x+(-c*d)^(1/2)))*a*b*d+ln(-2
*((-c*d)^(1/2)*b^2*x-(b*(b*x^2+2*a))^(1/2)*(b*(2*a*d-b*c)/d)^(1/2)*d-2*a*b
*d)/(d*x+(-c*d)^(1/2)))*b^2*c+2*(b*(b*x^2+2*a))^(1/2)*(b*(2*a*d-b*c)/d)^(1
/2)*d)/x/(b*(b*x^2+2*a))^(1/2)/d^2/(b*(2*a*d-b*c)/d)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.99

$$\int \frac{\sqrt{bx^2(2a+bx^2)}}{c+dx^2}\,dx = \left[\frac{x\sqrt{-\frac{b^2c-2\,abd}{d}}\log\left(\frac{b^2dx^3-(b^2c-4\,abd)x-2\,\sqrt{b^2x^4+2\,abx^2}d\sqrt{-\frac{b^2c-2\,abd}{d}}}{dx^3+cx}\right)+2\,\sqrt{b^2x^4+2\,abx^2}}{2\,dx}, \frac{x\sqrt{\frac{b^2c-2\,abd}{d}}\arctan\left(\right.}{}\right.$$

input
```
integrate((b*x^2*(b*x^2+2*a))^(1/2)/(d*x^2+c),x, algorithm="fricas")
```

output
```
[1/2*(x*sqrt(-(b^2*c - 2*a*b*d)/d)*log((b^2*d*x^3 - (b^2*c - 4*a*b*d)*x -
2*sqrt(b^2*x^4 + 2*a*b*x^2)*d*sqrt(-(b^2*c - 2*a*b*d)/d))/(d*x^3 + c*x)) +
 2*sqrt(b^2*x^4 + 2*a*b*x^2))/(d*x), (x*sqrt((b^2*c - 2*a*b*d)/d)*arctan(-
sqrt(b^2*x^4 + 2*a*b*x^2)*d*sqrt((b^2*c - 2*a*b*d)/d)/((b^2*c - 2*a*b*d)*x
)) + sqrt(b^2*x^4 + 2*a*b*x^2))/(d*x)]
```

Sympy [F]

$$\int \frac{\sqrt{bx^2(2a+bx^2)}}{c+dx^2}\,dx = \int \frac{\sqrt{bx^2\cdot(2a+bx^2)}}{c+dx^2}\,dx$$

input
```
integrate((b*x**2*(b*x**2+2*a))**(1/2)/(d*x**2+c),x)
```

output
```
Integral(sqrt(b*x**2*(2*a + b*x**2))/(c + d*x**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{bx^2 (2a + bx^2)}}{c + dx^2} \, dx = \int \frac{\sqrt{(bx^2 + 2\,a)bx^2}}{dx^2 + c} \, dx$$

input
```
integrate((b*x^2*(b*x^2+2*a))^(1/2)/(d*x^2+c),x, algorithm="maxima")
```

output
```
integrate(sqrt((b*x^2 + 2*a)*b*x^2)/(d*x^2 + c), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 203 vs. $2(95) = 190$.

Time = 0.14 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.83

$$\int \frac{\sqrt{bx^2 (2a + bx^2)}}{c + dx^2} \, dx = -\frac{(b^2 c \operatorname{sgn}(x) - 2\,abd \operatorname{sgn}(x)) \arctan\left(\frac{\sqrt{b^2x^2 + 2\,ab}d}{\sqrt{b^2cd - 2\,abd^2}}\right)}{\sqrt{b^2cd - 2\,abd^2}d} + \frac{\sqrt{b^2x^2 + 2\,ab}\operatorname{sgn}(x)}{d} + \frac{\left(b^2 c \arctan\left(\frac{\sqrt{2}\sqrt{ab}d}{\sqrt{b^2cd - 2\,abd^2}}\right) - 2\,abd \arctan\left(\frac{\sqrt{2}\sqrt{ab}d}{\sqrt{b^2cd - 2\,abd^2}}\right) - \sqrt{2}\sqrt{b^2cd - 2\,abd^2}\sqrt{ab}\right)\operatorname{sgn}(x)}{\sqrt{b^2cd - 2\,abd^2}d}$$

input
```
integrate((b*x^2*(b*x^2+2*a))^(1/2)/(d*x^2+c),x, algorithm="giac")
```

output
```
-(b^2*c*sgn(x) - 2*a*b*d*sgn(x))*arctan(sqrt(b^2*x^2 + 2*a*b)*d/sqrt(b^2*c
*d - 2*a*b*d^2))/(sqrt(b^2*c*d - 2*a*b*d^2)*d) + sqrt(b^2*x^2 + 2*a*b)*sgn
(x)/d + (b^2*c*arctan(sqrt(2)*sqrt(a*b)*d/sqrt(b^2*c*d - 2*a*b*d^2)) - 2*a
*b*d*arctan(sqrt(2)*sqrt(a*b)*d/sqrt(b^2*c*d - 2*a*b*d^2)) - sqrt(2)*sqrt(
b^2*c*d - 2*a*b*d^2)*sqrt(a*b))*sgn(x)/(sqrt(b^2*c*d - 2*a*b*d^2)*d)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{bx^2 (2a + bx^2)}}{c + dx^2} \, dx = \int \frac{\sqrt{b\,x^2\,\left(b\,x^2+2\,a\right)}}{d\,x^2+c} \, dx$$

input
```
int((b*x^2*(2*a + b*x^2))^(1/2)/(c + d*x^2),x)
```

output
```
int((b*x^2*(2*a + b*x^2))^(1/2)/(c + d*x^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.90

$$\int \frac{\sqrt{bx^2 (2a + bx^2)}}{c + dx^2} \, dx = \frac{\sqrt{b}\left(-\sqrt{d}\,\sqrt{-2ad+bc}\,atan\left(\frac{\sqrt{b}\,\sqrt{b\,x^2+2a}\,dx+2ad+bd\,x^2}{\sqrt{d}\,\sqrt{b\,x^2+2a}\,\sqrt{-2ad+bc}+\sqrt{d}\,\sqrt{b}\,\sqrt{-2ad+bc}\,x}\right)+\sqrt{b\,x^2+2a}\,d\right)}{d^2}$$

input
```
int((b*x^2*(b*x^2+2*a))^(1/2)/(d*x^2+c),x)
```

output
```
(sqrt(b)*( - sqrt(d)*sqrt( - 2*a*d + b*c)*atan((sqrt(b)*sqrt(2*a + b*x**2)
*d*x + 2*a*d + b*d*x**2)/(sqrt(d)*sqrt(2*a + b*x**2)*sqrt( - 2*a*d + b*c)
+ sqrt(d)*sqrt(b)*sqrt( - 2*a*d + b*c)*x)) + sqrt(2*a + b*x**2)*d))/d**2
```

3.274 $\int \frac{\sqrt{-a^2+(a+bx^2)^2}}{c+dx^2}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 111

$$\int \frac{\sqrt{-a^2+(a+bx^2)^2}}{c+dx^2}\,dx = \frac{\sqrt{bx^2(2a+bx^2)}}{dx} - \frac{\sqrt{bc-2ad}\sqrt{bx^2(2a+bx^2)}\arctan\left(\frac{\sqrt{d}\sqrt{2a+bx^2}}{\sqrt{bc-2ad}}\right)}{d^{3/2}x\sqrt{2a+bx^2}}$$

output
```
(b*x^2*(b*x^2+2*a))^(1/2)/d/x-(-2*a*d+b*c)^(1/2)*(b*x^2*(b*x^2+2*a))^(1/2)
*arctan(d^(1/2)*(b*x^2+2*a)^(1/2)/(-2*a*d+b*c)^(1/2))/d^(3/2)/x/(b*x^2+2*a
)^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{-a^2+(a+bx^2)^2}}{c+dx^2}\,dx = \frac{bx\sqrt{2a+bx^2}\left(\sqrt{d}\sqrt{2a+bx^2}-\sqrt{bc-2ad}\arctan\left(\frac{\sqrt{d}\sqrt{2a+bx^2}}{\sqrt{bc-2ad}}\right)\right)}{d^{3/2}\sqrt{bx^2(2a+bx^2)}}$$

input
```
Integrate[Sqrt[-a^2 + (a + b*x^2)^2]/(c + d*x^2),x]
```

output
```
(b*x*Sqrt[2*a + b*x^2]*(Sqrt[d]*Sqrt[2*a + b*x^2] - Sqrt[b*c - 2*a*d]*ArcT
an[(Sqrt[d]*Sqrt[2*a + b*x^2])/Sqrt[b*c - 2*a*d]]))/(d^(3/2)*Sqrt[b*x^2*(2
*a + b*x^2)])
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.15, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2087, 1466, 353, 60, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\left(a+bx^2\right)^2-a^2}}{c+dx^2}dx$$

$$\Big\downarrow \ 2087$$

$$\int \frac{\sqrt{2abx^2+b^2x^4}}{c+dx^2}dx$$

$$\Big\downarrow \ 1466$$

$$\frac{\sqrt{2abx^2+b^2x^4}\int \frac{x\sqrt{b^2x^2+2ab}}{dx^2+c}dx}{x\sqrt{2ab+b^2x^2}}$$

$$\Big\downarrow \ 353$$

$$\frac{\sqrt{2abx^2+b^2x^4}\int \frac{\sqrt{b^2x^2+2ab}}{dx^2+c}dx^2}{2x\sqrt{2ab+b^2x^2}}$$

$$\Big\downarrow \ 60$$

$$\frac{\sqrt{2abx^2+b^2x^4}\left(\frac{2\sqrt{2ab+b^2x^2}}{d}-\frac{b(bc-2ad)\int \frac{1}{\sqrt{b^2x^2+2ab}\left(dx^2+c\right)}dx^2}{d}\right)}{2x\sqrt{2ab+b^2x^2}}$$

$$\Big\downarrow \ 73$$

$$\frac{\sqrt{2abx^2+b^2x^4}\left(\frac{2\sqrt{2ab+b^2x^2}}{d}-\frac{2(bc-2ad)\int \frac{1}{\frac{dx^4}{b^2}+c-\frac{2ad}{b}}d\sqrt{b^2x^2+2ab}}{bd}\right)}{2x\sqrt{2ab+b^2x^2}}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{\sqrt{2abx^2+b^2x^4}\left(\frac{2\sqrt{2ab+b^2x^2}}{d}-\frac{2\sqrt{b}\sqrt{bc-2ad}\arctan\left(\frac{\sqrt{d}\sqrt{2ab+b^2x^2}}{\sqrt{b}\sqrt{bc-2ad}}\right)}{d^{3/2}}\right)}{2x\sqrt{2ab+b^2x^2}}$$

input
```
Int[Sqrt[-a^2 + (a + b*x^2)^2]/(c + d*x^2),x]
```

output
```
(Sqrt[2*a*b*x^2 + b^2*x^4]*((2*Sqrt[2*a*b + b^2*x^2])/d - (2*Sqrt[b]*Sqrt[
b*c - 2*a*d]*ArcTan[(Sqrt[d]*Sqrt[2*a*b + b^2*x^2])/(Sqrt[b]*Sqrt[b*c - 2*
a*d])])/d^(3/2)))/(2*x*Sqrt[2*a*b + b^2*x^2])
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 1466
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbo
l] :> Simp[(b*x^2 + c*x^4)^FracPart[p]/(x^(2*FracPart[p])*(b + c*x^2)^FracP
art[p])   Int[x^(2*p)*(d + e*x^2)^q*(b + c*x^2)^p, x], x] /; FreeQ[{b, c, d
, e, p, q}, x] &&  !IntegerQ[p]
```

rule 2087
```
Int[(u_)^(q_.)*(v_)^(p_.), x_Symbol] :> Int[ExpandToSum[u, x]^q*ExpandToSum
[v, x]^p, x] /; FreeQ[{p, q}, x] && BinomialQ[u, x] && TrinomialQ[v, x] &&
 !(BinomialMatchQ[u, x] && TrinomialMatchQ[v, x])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 363 vs. $2(95) = 190$.

Time = 0.31 (sec) , antiderivative size = 364, normalized size of antiderivative = 3.28

method	result
default	$\sqrt{b^2x^4+2abx^2}\left(-2\ln\left(-\frac{2\left(\sqrt{-cd}\,b^2x+\sqrt{b\left(bx^2+2a\right)}\sqrt{\frac{b(2ad-bc)}{d}}\,d+2abd\right)}{-dx+\sqrt{-cd}}\right)abd+\ln\left(-\frac{2\left(\sqrt{-cd}\,b^2x+\sqrt{b\left(bx^2+2a\right)}\sqrt{\frac{b(2ad-}{d}}\right.}{-dx+\sqrt{-cd}}\right.\right.$
pseudoelliptic	$b\left(2\ln\left(\frac{\sqrt{b^2x^2+2ab}\sqrt{\frac{2abd-cb^2}{d}}\,d-\sqrt{-b^2cd}\,bx+2abd}{bdx+\sqrt{-b^2cd}}\right)abd-2\ln\left(\frac{\sqrt{b^2x^2+2ab}\sqrt{\frac{2abd-cb^2}{d}}\,d+\sqrt{-b^2cd}\,bx+2abd}{bdx-\sqrt{-b^2cd}}\right)abd-\ln\left(\right.\right.$
risch	$\frac{\sqrt{bx^2(bx^2+2a)}}{dx}+\frac{(2ad-bc)\left(-\frac{\ln\left(\frac{\frac{2b(2ad-bc)}{d}+\frac{2b^2\sqrt{-cd}\left(x-\frac{\sqrt{-cd}}{d}\right)}{d}+2\sqrt{\frac{b(2ad-bc)}{d}}\sqrt{b^2\left(x-\frac{\sqrt{-cd}}{d}\right)^2+\frac{2b^2\sqrt{-cd}\left(x-\frac{\sqrt{-}}{d}\right.}{d}}}{x-\frac{\sqrt{-cd}}{d}}\right)}{2d\sqrt{\frac{b(2ad-bc)}{d}}}\right.}{}$

input
```
int((-a^2+(b*x^2+a)^2)^(1/2)/(d*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
1/2*(b^2*x^4+2*a*b*x^2)^(1/2)*(-2*ln(-2*((-c*d)^(1/2)*b^2*x+(b*(b*x^2+2*a)
)^(1/2)*(b*(2*a*d-b*c)/d)^(1/2)*d+2*a*b*d)/(-d*x+(-c*d)^(1/2)))*a*b*d+ln(-
2*((-c*d)^(1/2)*b^2*x+(b*(b*x^2+2*a))^(1/2)*(b*(2*a*d-b*c)/d)^(1/2)*d+2*a*
b*d)/(-d*x+(-c*d)^(1/2)))*b^2*c-2*ln(-2*((-c*d)^(1/2)*b^2*x-(b*(b*x^2+2*a)
)^(1/2)*(b*(2*a*d-b*c)/d)^(1/2)*d-2*a*b*d)/(d*x+(-c*d)^(1/2)))*a*b*d+ln(-2
*((-c*d)^(1/2)*b^2*x-(b*(b*x^2+2*a))^(1/2)*(b*(2*a*d-b*c)/d)^(1/2)*d-2*a*b
*d)/(d*x+(-c*d)^(1/2)))*b^2*c+2*(b*(b*x^2+2*a))^(1/2)*(b*(2*a*d-b*c)/d)^(1
/2)*d)/x/(b*(b*x^2+2*a))^(1/2)/d^2/(b*(2*a*d-b*c)/d)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.99

$$\int \frac{\sqrt{-a^2+(a+bx^2)^2}}{c+dx^2}\,dx = \left[\frac{x\sqrt{-\frac{b^2c-2\,abd}{d}}\log\left(\frac{b^2dx^3-(b^2c-4\,abd)x-2\,\sqrt{b^2x^4+2\,abx^2}d\sqrt{-\frac{b^2c-2\,abd}{d}}}{dx^3+cx}\right)+2\,\sqrt{b^2x^4+2\,abx^2}}{2\,dx}, \frac{x\sqrt{\frac{b^2c-2\,abd}{d}}\arctan\left(\right.}{}\right.$$

input
```
integrate((-a^2+(b*x^2+a)^2)^(1/2)/(d*x^2+c),x, algorithm="fricas")
```

output
```
[1/2*(x*sqrt(-(b^2*c - 2*a*b*d)/d)*log((b^2*d*x^3 - (b^2*c - 4*a*b*d)*x -
2*sqrt(b^2*x^4 + 2*a*b*x^2)*d*sqrt(-(b^2*c - 2*a*b*d)/d))/(d*x^3 + c*x)) +
 2*sqrt(b^2*x^4 + 2*a*b*x^2))/(d*x), (x*sqrt((b^2*c - 2*a*b*d)/d)*arctan(-
sqrt(b^2*x^4 + 2*a*b*x^2)*d*sqrt((b^2*c - 2*a*b*d)/d)/((b^2*c - 2*a*b*d)*x
)) + sqrt(b^2*x^4 + 2*a*b*x^2))/(d*x)]
```

Sympy [F]

$$\int \frac{\sqrt{-a^2+(a+bx^2)^2}}{c+dx^2}\,dx = \int \frac{\sqrt{bx^2\cdot(2a+bx^2)}}{c+dx^2}\,dx$$

input
```
integrate((-a**2+(b*x**2+a)**2)**(1/2)/(d*x**2+c),x)
```

output
```
Integral(sqrt(b*x**2*(2*a + b*x**2))/(c + d*x**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{-a^2+(a+bx^2)^2}}{c+dx^2}\,dx = \int \frac{\sqrt{(bx^2+a)^2-a^2}}{dx^2+c}\,dx$$

input
```
integrate((-a^2+(b*x^2+a)^2)^(1/2)/(d*x^2+c),x, algorithm="maxima")
```

output
```
integrate(sqrt((b*x^2 + a)^2 - a^2)/(d*x^2 + c), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 203 vs. $2(95) = 190$.

Time = 0.15 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.83

$$\begin{aligned}&\int \frac{\sqrt{-a^2+(a+bx^2)^2}}{c+dx^2}\,dx \\ &= -\frac{(b^2c\,\mathrm{sgn}(x)-2\,abd\,\mathrm{sgn}(x))\arctan\left(\frac{\sqrt{b^2x^2+2\,ab}d}{\sqrt{b^2cd-2\,abd^2}}\right)}{\sqrt{b^2cd-2\,abd^2}d}+\frac{\sqrt{b^2x^2+2\,ab}\,\mathrm{sgn}(x)}{d} \\ &\quad+\frac{\left(b^2c\arctan\left(\frac{\sqrt{2}\sqrt{ab}d}{\sqrt{b^2cd-2\,abd^2}}\right)-2\,abd\arctan\left(\frac{\sqrt{2}\sqrt{ab}d}{\sqrt{b^2cd-2\,abd^2}}\right)-\sqrt{2}\sqrt{b^2cd-2\,abd^2}\sqrt{ab}\right)\mathrm{sgn}(x)}{\sqrt{b^2cd-2\,abd^2}d}\end{aligned}$$

input
```
integrate((-a^2+(b*x^2+a)^2)^(1/2)/(d*x^2+c),x, algorithm="giac")
```

output
```
-(b^2*c*sgn(x) - 2*a*b*d*sgn(x))*arctan(sqrt(b^2*x^2 + 2*a*b)*d/sqrt(b^2*c
*d - 2*a*b*d^2))/(sqrt(b^2*c*d - 2*a*b*d^2)*d) + sqrt(b^2*x^2 + 2*a*b)*sgn
(x)/d + (b^2*c*arctan(sqrt(2)*sqrt(a*b)*d/sqrt(b^2*c*d - 2*a*b*d^2)) - 2*a
*b*d*arctan(sqrt(2)*sqrt(a*b)*d/sqrt(b^2*c*d - 2*a*b*d^2)) - sqrt(2)*sqrt(
b^2*c*d - 2*a*b*d^2)*sqrt(a*b))*sgn(x)/(sqrt(b^2*c*d - 2*a*b*d^2)*d)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{-a^2+(a+bx^2)^2}}{c+dx^2}\,dx=\int \frac{\sqrt{{\left(b\,x^2+a\right)}^2-a^2}}{d\,x^2+c}\,dx$$

input
```
int(((a + b*x^2)^2 - a^2)^(1/2)/(c + d*x^2),x)
```

output
```
int(((a + b*x^2)^2 - a^2)^(1/2)/(c + d*x^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.90

$$\int \frac{\sqrt{-a^2+(a+bx^2)^2}}{c+dx^2}\,dx = \frac{\sqrt{b}\left(-\sqrt{d}\,\sqrt{-2ad+bc}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{b\,x^2+2a}\,dx+2ad+bd\,x^2}{\sqrt{d}\,\sqrt{b\,x^2+2a}\,\sqrt{-2ad+bc}+\sqrt{d}\,\sqrt{b}\,\sqrt{-2ad+bc}\,x}\right)+\sqrt{b\,x^2+2a}\,d\right)}{d^2}$$

input
```
int((-a^2+(b*x^2+a)^2)^(1/2)/(d*x^2+c),x)
```

output
```
(sqrt(b)*( - sqrt(d)*sqrt( - 2*a*d + b*c)*atan((sqrt(b)*sqrt(2*a + b*x**2)
*d*x + 2*a*d + b*d*x**2)/(sqrt(d)*sqrt(2*a + b*x**2)*sqrt( - 2*a*d + b*c)
+ sqrt(d)*sqrt(b)*sqrt( - 2*a*d + b*c)*x)) + sqrt(2*a + b*x**2)*d))/d**2
```

3.275 $\int \frac{\sqrt{acx^2+bcx^4}}{d+ex^2} dx$

Optimal result

Integrand size = 27, antiderivative size = 105

$$\int \frac{\sqrt{acx^2+bcx^4}}{d+ex^2} dx = \frac{\sqrt{acx^2+bcx^4}}{ex} - \frac{\sqrt{bd-ae}\sqrt{acx^2+bcx^4}\arctan\left(\frac{\sqrt{e}\sqrt{a+bx^2}}{\sqrt{bd-ae}}\right)}{e^{3/2}x\sqrt{a+bx^2}}$$

output
```
(b*c*x^4+a*c*x^2)^(1/2)/e/x-(-a*e+b*d)^(1/2)*(b*c*x^4+a*c*x^2)^(1/2)*arcta
n(e^(1/2)*(b*x^2+a)^(1/2)/(-a*e+b*d)^(1/2))/e^(3/2)/x/(b*x^2+a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{acx^2+bcx^4}}{d+ex^2} dx = \frac{cx\sqrt{a+bx^2}\left(\sqrt{e}\sqrt{a+bx^2}-\sqrt{bd-ae}\arctan\left(\frac{\sqrt{e}\sqrt{a+bx^2}}{\sqrt{bd-ae}}\right)\right)}{e^{3/2}\sqrt{cx^2(a+bx^2)}}$$

input
```
Integrate[Sqrt[a*c*x^2 + b*c*x^4]/(d + e*x^2),x]
```

output
```
(c*x*Sqrt[a + b*x^2]*(Sqrt[e]*Sqrt[a + b*x^2] - Sqrt[b*d - a*e]*ArcTan[(Sq
rt[e]*Sqrt[a + b*x^2])/Sqrt[b*d - a*e]]))/(e^(3/2)*Sqrt[c*x^2*(a + b*x^2)]
)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.14, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1466, 353, 60, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{acx^2+bcx^4}}{d+ex^2}dx$$

$$\Big\downarrow \text{ 1466}$$

$$\frac{\sqrt{acx^2+bcx^4}\int \frac{x\sqrt{bcx^2+ac}}{ex^2+d}dx}{x\sqrt{ac+bcx^2}}$$

$$\Big\downarrow \text{ 353}$$

$$\frac{\sqrt{acx^2+bcx^4}\int \frac{\sqrt{bcx^2+ac}}{ex^2+d}dx^2}{2x\sqrt{ac+bcx^2}}$$

$$\Big\downarrow \text{ 60}$$

$$\frac{\sqrt{acx^2+bcx^4}\left(\frac{2\sqrt{ac+bcx^2}}{e}-\frac{c(bd-ae)\int \frac{1}{\sqrt{bcx^2+ac}(ex^2+d)}dx^2}{e}\right)}{2x\sqrt{ac+bcx^2}}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{\sqrt{acx^2+bcx^4}\left(\frac{2\sqrt{ac+bcx^2}}{e}-\frac{2(bd-ae)\int \frac{1}{\frac{ex^4}{bc}+d-\frac{ae}{b}}d\sqrt{bcx^2+ac}}{be}\right)}{2x\sqrt{ac+bcx^2}}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{\sqrt{acx^2+bcx^4}\left(\frac{2\sqrt{ac+bcx^2}}{e}-\frac{2\sqrt{c}\sqrt{bd-ae}\arctan\left(\frac{\sqrt{e}\sqrt{ac+bcx^2}}{\sqrt{c}\sqrt{bd-ae}}\right)}{e^{3/2}}\right)}{2x\sqrt{ac+bcx^2}}$$

input
```
Int[Sqrt[a*c*x^2 + b*c*x^4]/(d + e*x^2),x]
```

output
```
(Sqrt[a*c*x^2 + b*c*x^4]*((2*Sqrt[a*c + b*c*x^2])/e - (2*Sqrt[c]*Sqrt[b*d
- a*e]*ArcTan[(Sqrt[e]*Sqrt[a*c + b*c*x^2])/(Sqrt[c]*Sqrt[b*d - a*e])])/e^
(3/2)))/(2*x*Sqrt[a*c + b*c*x^2])
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 1466
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbo
l] :> Simp[(b*x^2 + c*x^4)^FracPart[p]/(x^(2*FracPart[p])*(b + c*x^2)^FracP
art[p])   Int[x^(2*p)*(d + e*x^2)^q*(b + c*x^2)^p, x], x] /; FreeQ[{b, c, d
, e, p, q}, x] &&  !IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 194 vs. $2(89) = 178$.

Time = 0.52 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.86

method	result
pseudoelliptic	$-\frac{-2\sqrt{-\frac{c(ae-bd)}{d}}\,\sqrt{-de}\,\sqrt{(b\,x^2+a)c}+c(ae-bd)\left(\ln\left(\frac{bcdx+\sqrt{-\frac{c(ae-bd)}{d}}\,\sqrt{\left(b\,x^2+a\right)c}\,d-ac\sqrt{-de}}{\sqrt{-de}\,x+d}\right)-\ln\left(\frac{bcdx+\sqrt{-\frac{c(ae-bd)}{d}}\cdots}{-\sqrt{\cdots}}\right)\cdots\right.}{2\sqrt{-\frac{c(ae-bd)}{d}}\,\sqrt{-de}\,e}$
default	$\frac{\sqrt{bc\,x^4+x^2ac}\left(-\ln\left(-\frac{2\left(\sqrt{-de}\,bcx+\sqrt{\left(b\,x^2+a\right)c}\,\sqrt{\frac{c(ae-bd)}{e}}\,e+ace\right)}{-ex+\sqrt{-de}}\right)ace+\ln\left(-\frac{2\left(\sqrt{-de}\,bcx+\sqrt{\left(b\,x^2+a\right)c}\,\sqrt{\frac{c(ae-bd)}{e}}\,e+ace\cdots\right)}{-ex+\sqrt{-de}}\cdots\right)\cdots\right.}{\cdots}$
risch	$\frac{\sqrt{c\,x^2(b\,x^2+a)}}{ex}+\frac{(ae-bd)\left(-\frac{\ln\left(\frac{\frac{2c(ae-bd)}{e}+\frac{2bc\sqrt{-de}\left(x-\frac{\sqrt{-de}}{e}\right)}{e}+2\sqrt{\frac{c(ae-bd)}{e}}\sqrt{bc\left(x-\frac{\sqrt{-de}}{e}\right)^2+\frac{2bc\sqrt{-de}\left(x-\frac{\sqrt{-de}}{e}\right)}{e}\cdots}}{x-\frac{\sqrt{-de}}{e}}\right)}{2e\sqrt{\frac{c(ae-bd)}{e}}}\cdots\right.}{\cdots}$

input
```
int((b*c*x^4+a*c*x^2)^(1/2)/(e*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-2*(-c/d*(a*e-b*d))^(1/2)*(-d*e)^(1/2)*((b*x^2+a)*c)^(1/2)+c*(a*e-b*
d)*(ln((b*c*d*x+(-c/d*(a*e-b*d))^(1/2)*((b*x^2+a)*c)^(1/2)*d-a*c*(-d*e)^(1
/2))/((-d*e)^(1/2)*x+d))-ln((b*c*d*x+(-c/d*(a*e-b*d))^(1/2)*((b*x^2+a)*c)^
(1/2)*d+a*c*(-d*e)^(1/2))/(-(-d*e)^(1/2)*x+d))))/(-c/d*(a*e-b*d))^(1/2)/(-
d*e)^(1/2)/e
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 313, normalized size of antiderivative = 2.98

$$\int \frac{\sqrt{acx^2+bcx^4}}{d+ex^2}\,dx = \left[\frac{x\sqrt{-\frac{bcd-ace}{e}}\log\left(\frac{b^2ce^2x^5-2\left(3\,b^2cde-4\,abce^2\right)x^3+\left(b^2cd^2-8\,abcde+8\,a^2ce^2\right)x-4\,\sqrt{bcx^4+acx^2}\left(be^2x^2-bde+2\,ae^2\right)\sqrt{-\frac{bcd-ace}{e}}}{e^2x^5+2\,dex^3+d^2x}\right)+}{4\,ex}\right.$$

input
```
integrate((b*c*x^4+a*c*x^2)^(1/2)/(e*x^2+d),x, algorithm="fricas")
```

output
```
[1/4*(x*sqrt(-(b*c*d - a*c*e)/e)*log((b^2*c*e^2*x^5 - 2*(3*b^2*c*d*e - 4*a
*b*c*e^2)*x^3 + (b^2*c*d^2 - 8*a*b*c*d*e + 8*a^2*c*e^2)*x - 4*sqrt(b*c*x^4
 + a*c*x^2)*(b*e^2*x^2 - b*d*e + 2*a*e^2)*sqrt(-(b*c*d - a*c*e)/e))/(e^2*x
^5 + 2*d*e*x^3 + d^2*x)) + 4*sqrt(b*c*x^4 + a*c*x^2))/(e*x), 1/2*(x*sqrt((
b*c*d - a*c*e)/e)*arctan(-1/2*sqrt(b*c*x^4 + a*c*x^2)*(b*e*x^2 - b*d + 2*a
*e)*sqrt((b*c*d - a*c*e)/e)/((b^2*c*d - a*b*c*e)*x^3 + (a*b*c*d - a^2*c*e)
*x)) + 2*sqrt(b*c*x^4 + a*c*x^2))/(e*x)]
```

Sympy [F]

$$\int \frac{\sqrt{acx^2+bcx^4}}{d+ex^2}\,dx = \int \frac{\sqrt{cx^2\left(a+bx^2\right)}}{d+ex^2}\,dx$$

input
```
integrate((b*c*x**4+a*c*x**2)**(1/2)/(e*x**2+d),x)
```

output
```
Integral(sqrt(c*x**2*(a + b*x**2))/(d + e*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{acx^2 + bcx^4}}{d + ex^2} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*c*x^4+a*c*x^2)^(1/2)/(e*x^2+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 182 vs. $2(89) = 178$.

Time = 0.15 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.73

$$\int \frac{\sqrt{acx^2 + bcx^4}}{d + ex^2} \, dx = -\frac{(bcd\text{sgn}(x) - ace\text{sgn}(x)) \arctan\left(\frac{\sqrt{bcx^2+ac}e}{\sqrt{bcde-ace^2}}\right)}{\sqrt{bcde - ace^2}e} + \frac{\sqrt{bcx^2 + ac}\text{sgn}(x)}{e} + \frac{\left(bcd \arctan\left(\frac{\sqrt{ac}e}{\sqrt{bcde-ace^2}}\right) - ace \arctan\left(\frac{\sqrt{ac}e}{\sqrt{bcde-ace^2}}\right) - \sqrt{bcde - ace^2}\sqrt{ac}\right)\text{sgn}(x)}{\sqrt{bcde - ace^2}e}$$

input
```
integrate((b*c*x^4+a*c*x^2)^(1/2)/(e*x^2+d),x, algorithm="giac")
```

output
```
-(b*c*d*sgn(x) - a*c*e*sgn(x))*arctan(sqrt(b*c*x^2 + a*c)*e/sqrt(b*c*d*e -
 a*c*e^2))/(sqrt(b*c*d*e - a*c*e^2)*e) + sqrt(b*c*x^2 + a*c)*sgn(x)/e + (b
*c*d*arctan(sqrt(a*c)*e/sqrt(b*c*d*e - a*c*e^2)) - a*c*e*arctan(sqrt(a*c)*
e/sqrt(b*c*d*e - a*c*e^2)) - sqrt(b*c*d*e - a*c*e^2)*sqrt(a*c))*sgn(x)/(sq
rt(b*c*d*e - a*c*e^2)*e)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{acx^2+bcx^4}}{d+ex^2}\,dx=\int \frac{\sqrt{b\,c\,x^4+a\,c\,x^2}}{e\,x^2+d}\,dx$$

input
```
int((a*c*x^2 + b*c*x^4)^(1/2)/(d + e*x^2),x)
```

output
```
int((a*c*x^2 + b*c*x^4)^(1/2)/(d + e*x^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.89

$$\int \frac{\sqrt{acx^2+bcx^4}}{d+ex^2}\,dx=\frac{\sqrt{c}\left(-\sqrt{e}\,\sqrt{-ae+bd}\,atan\left(\frac{\sqrt{b}\,\sqrt{b\,x^2+a}\,ex+ae+be\,x^2}{\sqrt{e}\,\sqrt{b\,x^2+a}\,\sqrt{-ae+bd}+\sqrt{e}\,\sqrt{b}\,\sqrt{-ae+bd}\,x}\right)+\sqrt{b\,x^2+a}\,e\right)}{e^2}$$

input
```
int((b*c*x^4+a*c*x^2)^(1/2)/(e*x^2+d),x)
```

output
```
(sqrt(c)*( - sqrt(e)*sqrt( - a*e + b*d)*atan((sqrt(b)*sqrt(a + b*x**2)*e*x
 + a*e + b*e*x**2)/(sqrt(e)*sqrt(a + b*x**2)*sqrt( - a*e + b*d) + sqrt(e)*
sqrt(b)*sqrt( - a*e + b*d)*x)) + sqrt(a + b*x**2)*e))/e**2
```

3.276 $\int \frac{\sqrt{cx^2(a+bx^2)}}{d+ex^2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 105

$$\int \frac{\sqrt{cx^2(a+bx^2)}}{d+ex^2}\,dx=\frac{\sqrt{acx^2+bcx^4}}{ex}-\frac{\sqrt{bd-ae}\sqrt{acx^2+bcx^4}\arctan\left(\frac{\sqrt{e}\sqrt{a+bx^2}}{\sqrt{bd-ae}}\right)}{e^{3/2}x\sqrt{a+bx^2}}$$

output
```
(b*c*x^4+a*c*x^2)^(1/2)/e/x-(-a*e+b*d)^(1/2)*(b*c*x^4+a*c*x^2)^(1/2)*arcta
n(e^(1/2)*(b*x^2+a)^(1/2)/(-a*e+b*d)^(1/2))/e^(3/2)/x/(b*x^2+a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{cx^2(a+bx^2)}}{d+ex^2}\,dx=\frac{cx\sqrt{a+bx^2}\left(\sqrt{e}\sqrt{a+bx^2}-\sqrt{bd-ae}\arctan\left(\frac{\sqrt{e}\sqrt{a+bx^2}}{\sqrt{bd-ae}}\right)\right)}{e^{3/2}\sqrt{cx^2(a+bx^2)}}$$

input
```
Integrate[Sqrt[c*x^2*(a + b*x^2)]/(d + e*x^2),x]
```

output
```
(c*x*Sqrt[a + b*x^2]*(Sqrt[e]*Sqrt[a + b*x^2] - Sqrt[b*d - a*e]*ArcTan[(Sq
rt[e]*Sqrt[a + b*x^2])/Sqrt[b*d - a*e]]))/(e^(3/2)*Sqrt[c*x^2*(a + b*x^2)]
)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.14, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2048, 1466, 353, 60, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{cx^2(a+bx^2)}}{d+ex^2}dx$$

$$\Big\downarrow \text{2048}$$

$$\int \frac{\sqrt{acx^2+bcx^4}}{d+ex^2}dx$$

$$\Big\downarrow \text{1466}$$

$$\frac{\sqrt{acx^2+bcx^4}\int \frac{x\sqrt{bcx^2+ac}}{ex^2+d}dx}{x\sqrt{ac+bcx^2}}$$

$$\Big\downarrow \text{353}$$

$$\frac{\sqrt{acx^2+bcx^4}\int \frac{\sqrt{bcx^2+ac}}{ex^2+d}dx^2}{2x\sqrt{ac+bcx^2}}$$

$$\Big\downarrow \text{60}$$

$$\frac{\sqrt{acx^2+bcx^4}\left(\frac{2\sqrt{ac+bcx^2}}{e}-\frac{c(bd-ae)\int \frac{1}{\sqrt{bcx^2+ac}(ex^2+d)}dx^2}{e}\right)}{2x\sqrt{ac+bcx^2}}$$

$$\Big\downarrow \text{73}$$

$$\frac{\sqrt{acx^2+bcx^4}\left(\frac{2\sqrt{ac+bcx^2}}{e}-\frac{2(bd-ae)\int \frac{1}{\frac{ex^4}{bc}+d-\frac{ae}{b}}d\sqrt{bcx^2+ac}}{be}\right)}{2x\sqrt{ac+bcx^2}}$$

$$\Big\downarrow \text{218}$$

$$\frac{\sqrt{acx^2+bcx^4}\left(\frac{2\sqrt{ac+bcx^2}}{e}-\frac{2\sqrt{c}\sqrt{bd-ae}\arctan\left(\frac{\sqrt{e}\sqrt{ac+bcx^2}}{\sqrt{c}\sqrt{bd-ae}}\right)}{e^{3/2}}\right)}{2x\sqrt{ac+bcx^2}}$$

input
```
Int[Sqrt[c*x^2*(a + b*x^2)]/(d + e*x^2),x]
```

output
```
(Sqrt[a*c*x^2 + b*c*x^4]*((2*Sqrt[a*c + b*c*x^2])/e - (2*Sqrt[c]*Sqrt[b*d
- a*e]*ArcTan[(Sqrt[e]*Sqrt[a*c + b*c*x^2])/(Sqrt[c]*Sqrt[b*d - a*e])])/e^
(3/2)))/(2*x*Sqrt[a*c + b*c*x^2])
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 1466
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbo
l] :> Simp[(b*x^2 + c*x^4)^FracPart[p]/(x^(2*FracPart[p])*(b + c*x^2)^FracP
art[p])   Int[x^(2*p)*(d + e*x^2)^q*(b + c*x^2)^p, x], x] /; FreeQ[{b, c, d
, e, p, q}, x] &&  !IntegerQ[p]
```

rule 2048
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))*((c_) + (d_.)*(x_)^(n_.)))^(p_)
, x_Symbol] :> Int[u*(a*c*e + (b*c + a*d)*e*x^n + b*d*e*x^(2*n))^p, x] /; F
reeQ[{a, b, c, d, e, n, p}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 194 vs. $2(89) = 178$.

Time = 0.32 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.86

method	result
pseudoelliptic	$-\frac{-2\sqrt{-\frac{c(ae-bd)}{d}}\,\sqrt{-de}\,\sqrt{(b\,x^2+a)c}+c(ae-bd)\left(\ln\left(\frac{bcdx+\sqrt{-\frac{c(ae-bd)}{d}}\,\sqrt{\left(b\,x^2+a\right)c}\,d-ac\sqrt{-de}}{\sqrt{-de}\,x+d}\right)-\ln\left(\frac{bcdx+\sqrt{-\frac{c(ae-bd)}{d}}\cdots}{-\sqrt{\cdots}}\right.\right.}{2\sqrt{-\frac{c(ae-bd)}{d}}\,\sqrt{-de}\,e}$
default	$\frac{\sqrt{c\,x^2(b\,x^2+a)}\left(-\ln\left(-\frac{2\left(\sqrt{-de}\,bcx+\sqrt{\left(b\,x^2+a\right)c}\,\sqrt{\frac{c(ae-bd)}{e}}\,e+ace\right)}{-ex+\sqrt{-de}}\right)ace+\ln\left(-\frac{2\left(\sqrt{-de}\,bcx+\sqrt{\left(b\,x^2+a\right)c}\,\sqrt{\frac{c(ae-bd)}{e}}\,e+a\cdots\right.}{-ex+\sqrt{-de}}\right.\right.}{\cdots}$
risch	$\frac{\sqrt{c\,x^2(b\,x^2+a)}}{ex}+\frac{(ae-bd)\left(-\frac{\ln\left(\frac{\frac{2c(ae-bd)}{e}+\frac{2bc\sqrt{-de}\left(x-\frac{\sqrt{-de}}{e}\right)}{e}+2\sqrt{\frac{c(ae-bd)}{e}}\sqrt{bc\left(x-\frac{\sqrt{-de}}{e}\right)^2+\frac{2bc\sqrt{-de}\left(x-\frac{\sqrt{-de}}{e}\right)}{e}\cdots}}{x-\frac{\sqrt{-de}}{e}}\right)}{2e\sqrt{\frac{c(ae-bd)}{e}}}\cdots\right.}{\cdots}$

input
```
int((c*x^2*(b*x^2+a))^(1/2)/(e*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-2*(-c/d*(a*e-b*d))^(1/2)*(-d*e)^(1/2)*((b*x^2+a)*c)^(1/2)+c*(a*e-b*
d)*(ln((b*c*d*x+(-c/d*(a*e-b*d))^(1/2)*((b*x^2+a)*c)^(1/2)*d-a*c*(-d*e)^(1
/2))/((-d*e)^(1/2)*x+d))-ln((b*c*d*x+(-c/d*(a*e-b*d))^(1/2)*((b*x^2+a)*c)^
(1/2)*d+a*c*(-d*e)^(1/2))/(-(-d*e)^(1/2)*x+d))))/(-c/d*(a*e-b*d))^(1/2)/(-
d*e)^(1/2)/e
```

Fricas [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 313, normalized size of antiderivative = 2.98

$$\int \frac{\sqrt{cx^2(a+bx^2)}}{d+ex^2}\,dx = \left[\frac{x\sqrt{-\frac{bcd-ace}{e}}\log\left(\frac{b^2ce^2x^5-2\,(3\,b^2cde-4\,abce^2)x^3+(b^2cd^2-8\,abcde+8\,a^2ce^2)x-4\,\sqrt{bcx^4+acx^2}\,(be^2x^2-bde+2\,ae^2)\sqrt{-\frac{bcd-ace}{e}}}{e^2x^5+2\,dex^3+d^2x}\right)+}{4\,ex}\right.$$

input
```
integrate((c*x^2*(b*x^2+a))^(1/2)/(e*x^2+d),x, algorithm="fricas")
```

output
```
[1/4*(x*sqrt(-(b*c*d - a*c*e)/e)*log((b^2*c*e^2*x^5 - 2*(3*b^2*c*d*e - 4*a
*b*c*e^2)*x^3 + (b^2*c*d^2 - 8*a*b*c*d*e + 8*a^2*c*e^2)*x - 4*sqrt(b*c*x^4
 + a*c*x^2)*(b*e^2*x^2 - b*d*e + 2*a*e^2)*sqrt(-(b*c*d - a*c*e)/e))/(e^2*x
^5 + 2*d*e*x^3 + d^2*x)) + 4*sqrt(b*c*x^4 + a*c*x^2))/(e*x), 1/2*(x*sqrt((
b*c*d - a*c*e)/e)*arctan(-1/2*sqrt(b*c*x^4 + a*c*x^2)*(b*e*x^2 - b*d + 2*a
*e)*sqrt((b*c*d - a*c*e)/e)/((b^2*c*d - a*b*c*e)*x^3 + (a*b*c*d - a^2*c*e)
*x)) + 2*sqrt(b*c*x^4 + a*c*x^2))/(e*x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{cx^2(a+bx^2)}}{d+ex^2}\,dx = \text{Timed out}$$

input
```
integrate((c*x**2*(b*x**2+a))**(1/2)/(e*x**2+d),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{cx^2\,(a+bx^2)}}{d+ex^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^2*(b*x^2+a))^(1/2)/(e*x^2+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 182 vs. 2(89) = 178.

Time = 0.13 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.73

$$\begin{aligned}
&\int \frac{\sqrt{cx^2\,(a+bx^2)}}{d+ex^2}\,dx \\
&= -\frac{(bcd\text{sgn}(x)-ace\text{sgn}(x))\arctan\left(\frac{\sqrt{bcx^2+ace}}{\sqrt{bcde-ace^2}}\right)}{\sqrt{bcde-ace^2}\,e}+\frac{\sqrt{bcx^2+ac}\text{sgn}(x)}{e} \\
&\quad+\frac{\left(bcd\arctan\left(\frac{\sqrt{ace}}{\sqrt{bcde-ace^2}}\right)-ace\arctan\left(\frac{\sqrt{ace}}{\sqrt{bcde-ace^2}}\right)-\sqrt{bcde-ace^2}\sqrt{ac}\right)\text{sgn}(x)}{\sqrt{bcde-ace^2}\,e}
\end{aligned}$$

input
```
integrate((c*x^2*(b*x^2+a))^(1/2)/(e*x^2+d),x, algorithm="giac")
```

output
```
-(b*c*d*sgn(x) - a*c*e*sgn(x))*arctan(sqrt(b*c*x^2 + a*c)*e/sqrt(b*c*d*e -
 a*c*e^2))/(sqrt(b*c*d*e - a*c*e^2)*e) + sqrt(b*c*x^2 + a*c)*sgn(x)/e + (b
*c*d*arctan(sqrt(a*c)*e/sqrt(b*c*d*e - a*c*e^2)) - a*c*e*arctan(sqrt(a*c)*
e/sqrt(b*c*d*e - a*c*e^2)) - sqrt(b*c*d*e - a*c*e^2)*sqrt(a*c))*sgn(x)/(sq
rt(b*c*d*e - a*c*e^2)*e)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{cx^2 (a+bx^2)}}{d+ex^2}\,dx = \int \frac{\sqrt{c\,x^2\,\left(b\,x^2+a\right)}}{e\,x^2+d}\,dx$$

input
```
int((c*x^2*(a + b*x^2))^(1/2)/(d + e*x^2),x)
```

output
```
int((c*x^2*(a + b*x^2))^(1/2)/(d + e*x^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.89

$$\int \frac{\sqrt{cx^2 (a+bx^2)}}{d+ex^2}\,dx = \frac{\sqrt{c}\left(-\sqrt{e}\,\sqrt{-ae+bd}\,atan\left(\frac{\sqrt{b}\,\sqrt{b\,x^2+a}\,ex+ae+be\,x^2}{\sqrt{e}\,\sqrt{b\,x^2+a}\,\sqrt{-ae+bd}+\sqrt{e}\,\sqrt{b}\,\sqrt{-ae+bd}\,x}\right)+\sqrt{b\,x^2+a}\,e\right)}{e^2}$$

input
```
int((c*x^2*(b*x^2+a))^(1/2)/(e*x^2+d),x)
```

output
```
(sqrt(c)*( - sqrt(e)*sqrt( - a*e + b*d)*atan((sqrt(b)*sqrt(a + b*x**2)*e*x
 + a*e + b*e*x**2)/(sqrt(e)*sqrt(a + b*x**2)*sqrt( - a*e + b*d) + sqrt(e)*
sqrt(b)*sqrt( - a*e + b*d)*x)) + sqrt(a + b*x**2)*e))/e**2
```

3.277 $\int \frac{\sqrt{x^2(ac+bcx^2)}}{d+ex^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 105

$$\int \frac{\sqrt{x^2\left(ac+bcx^2\right)}}{d+ex^2}\,dx=\frac{\sqrt{acx^2+bcx^4}}{ex}-\frac{\sqrt{bd-ae}\sqrt{acx^2+bcx^4}\arctan\left(\frac{\sqrt{e}\sqrt{a+bx^2}}{\sqrt{bd-ae}}\right)}{e^{3/2}x\sqrt{a+bx^2}}$$

output
```
(b*c*x^4+a*c*x^2)^(1/2)/e/x-(-a*e+b*d)^(1/2)*(b*c*x^4+a*c*x^2)^(1/2)*arcta
n(e^(1/2)*(b*x^2+a)^(1/2)/(-a*e+b*d)^(1/2))/e^(3/2)/x/(b*x^2+a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{x^2\left(ac+bcx^2\right)}}{d+ex^2}\,dx=\frac{cx\sqrt{a+bx^2}\left(\sqrt{e}\sqrt{a+bx^2}-\sqrt{bd-ae}\arctan\left(\frac{\sqrt{e}\sqrt{a+bx^2}}{\sqrt{bd-ae}}\right)\right)}{e^{3/2}\sqrt{cx^2\left(a+bx^2\right)}}$$

input
```
Integrate[Sqrt[x^2*(a*c + b*c*x^2)]/(d + e*x^2),x]
```

output
```
(c*x*Sqrt[a + b*x^2]*(Sqrt[e]*Sqrt[a + b*x^2] - Sqrt[b*d - a*e]*ArcTan[(Sq
rt[e]*Sqrt[a + b*x^2])/Sqrt[b*d - a*e]]))/(e^(3/2)*Sqrt[c*x^2*(a + b*x^2)]
)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.14, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2048, 1466, 353, 60, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x^2 (ac+bcx^2)}}{d+ex^2} \, dx$$

$$\Big\downarrow \text{2048}$$

$$\int \frac{\sqrt{acx^2+bcx^4}}{d+ex^2}dx$$

$$\Big\downarrow \text{1466}$$

$$\frac{\sqrt{acx^2+bcx^4}\int \frac{x\sqrt{bcx^2+ac}}{ex^2+d}dx}{x\sqrt{ac+bcx^2}}$$

$$\Big\downarrow \text{353}$$

$$\frac{\sqrt{acx^2+bcx^4}\int \frac{\sqrt{bcx^2+ac}}{ex^2+d}dx^2}{2x\sqrt{ac+bcx^2}}$$

$$\Big\downarrow \text{60}$$

$$\frac{\sqrt{acx^2+bcx^4}\left(\frac{2\sqrt{ac+bcx^2}}{e}-\frac{c(bd-ae)\int \frac{1}{\sqrt{bcx^2+ac}\left(ex^2+d\right)}dx^2}{e}\right)}{2x\sqrt{ac+bcx^2}}$$

$$\Big\downarrow \text{73}$$

$$\frac{\sqrt{acx^2+bcx^4}\left(\frac{2\sqrt{ac+bcx^2}}{e}-\frac{2(bd-ae)\int \frac{1}{\frac{ex^4}{bc}+d-\frac{ae}{b}}d\sqrt{bcx^2+ac}}{be}\right)}{2x\sqrt{ac+bcx^2}}$$

$$\Big\downarrow \text{218}$$

$$\frac{\sqrt{acx^2+bcx^4}\left(\frac{2\sqrt{ac+bcx^2}}{e}-\frac{2\sqrt{c}\sqrt{bd-ae}\arctan\left(\frac{\sqrt{e}\sqrt{ac+bcx^2}}{\sqrt{c}\sqrt{bd-ae}}\right)}{e^{3/2}}\right)}{2x\sqrt{ac+bcx^2}}$$

input
```
Int[Sqrt[x^2*(a*c + b*c*x^2)]/(d + e*x^2),x]
```

output
```
(Sqrt[a*c*x^2 + b*c*x^4]*((2*Sqrt[a*c + b*c*x^2])/e - (2*Sqrt[c]*Sqrt[b*d
- a*e]*ArcTan[(Sqrt[e]*Sqrt[a*c + b*c*x^2])/(Sqrt[c]*Sqrt[b*d - a*e])])/e^
(3/2)))/(2*x*Sqrt[a*c + b*c*x^2])
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 1466
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbo
l] :> Simp[(b*x^2 + c*x^4)^FracPart[p]/(x^(2*FracPart[p])*(b + c*x^2)^FracP
art[p])   Int[x^(2*p)*(d + e*x^2)^q*(b + c*x^2)^p, x], x] /; FreeQ[{b, c, d
, e, p, q}, x] &&  !IntegerQ[p]
```

rule 2048
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))*((c_) + (d_.)*(x_)^(n_.)))^(p_)
, x_Symbol] :> Int[u*(a*c*e + (b*c + a*d)*e*x^n + b*d*e*x^(2*n))^p, x] /; F
reeQ[{a, b, c, d, e, n, p}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 194 vs. $2(89) = 178$.

Time = 0.32 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.86

method	result
pseudoelliptic	$-\frac{-2\sqrt{-\frac{c(ae-bd)}{d}}\,\sqrt{-de}\,\sqrt{(b\,x^2+a)c}+c(ae-bd)\left(\ln\left(\frac{bcdx+\sqrt{-\frac{c(ae-bd)}{d}}\,\sqrt{\left(b\,x^2+a\right)c}\,d-ac\sqrt{-de}}{\sqrt{-de}\,x+d}\right)-\ln\left(\frac{bcdx+\sqrt{-\frac{c(ae-bd)}{d}}}{-\sqrt{}}\right.\right.}{2\sqrt{-\frac{c(ae-bd)}{d}}\,\sqrt{-de}\,e}$
default	$\frac{\sqrt{c\,x^2(b\,x^2+a)}\left(-\ln\left(-\frac{2\left(\sqrt{-de}\,bcx+\sqrt{\left(b\,x^2+a\right)c}\,\sqrt{\frac{c(ae-bd)}{e}}\,e+ace\right)}{-ex+\sqrt{-de}}\right)ace+\ln\left(-\frac{2\left(\sqrt{-de}\,bcx+\sqrt{\left(b\,x^2+a\right)c}\,\sqrt{\frac{c(ae-bd)}{e}}\,e+a\right.}{-ex+\sqrt{-de}}\right.\right.}{}$
risch	$\frac{\sqrt{c\,x^2(b\,x^2+a)}}{ex}+\frac{(ae-bd)\left(-\frac{\ln\left(\frac{\frac{2c(ae-bd)}{e}+\frac{2bc\sqrt{-de}\left(x-\frac{\sqrt{-de}}{e}\right)}{e}+2\sqrt{\frac{c(ae-bd)}{e}}\sqrt{bc\left(x-\frac{\sqrt{-de}}{e}\right)^2+\frac{2bc\sqrt{-de}\left(x-\frac{\sqrt{-de}}{e}\right)}{e}}}{x-\frac{\sqrt{-de}}{e}}\right.}{2e\sqrt{\frac{c(ae-bd)}{e}}}\right.}{}$

input
```
int((x^2*(b*c*x^2+a*c))^(1/2)/(e*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-2*(-c/d*(a*e-b*d))^(1/2)*(-d*e)^(1/2)*((b*x^2+a)*c)^(1/2)+c*(a*e-b*
d)*(ln((b*c*d*x+(-c/d*(a*e-b*d))^(1/2)*((b*x^2+a)*c)^(1/2)*d-a*c*(-d*e)^(1
/2))/((-d*e)^(1/2)*x+d))-ln((b*c*d*x+(-c/d*(a*e-b*d))^(1/2)*((b*x^2+a)*c)^
(1/2)*d+a*c*(-d*e)^(1/2))/(-(-d*e)^(1/2)*x+d))))/(-c/d*(a*e-b*d))^(1/2)/(-
d*e)^(1/2)/e
```

Fricas [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 313, normalized size of antiderivative = 2.98

$$\int \frac{\sqrt{x^2(ac+bcx^2)}}{d+ex^2}\,dx = \left[\frac{x\sqrt{-\frac{bcd-ace}{e}}\log\left(\frac{b^2ce^2x^5-2\,(3\,b^2cde-4\,abce^2)x^3+(b^2cd^2-8\,abcde+8\,a^2ce^2)x-4\,\sqrt{bcx^4+acx^2}\,(be^2x^2-bde+2\,ae^2)\sqrt{-\frac{bcd-ace}{e}}}{e^2x^5+2\,dex^3+d^2x}\right)+}{4\,ex}\right.$$

input
```
integrate((x^2*(b*c*x^2+a*c))^(1/2)/(e*x^2+d),x, algorithm="fricas")
```

output
```
[1/4*(x*sqrt(-(b*c*d - a*c*e)/e)*log((b^2*c*e^2*x^5 - 2*(3*b^2*c*d*e - 4*a
*b*c*e^2)*x^3 + (b^2*c*d^2 - 8*a*b*c*d*e + 8*a^2*c*e^2)*x - 4*sqrt(b*c*x^4
 + a*c*x^2)*(b*e^2*x^2 - b*d*e + 2*a*e^2)*sqrt(-(b*c*d - a*c*e)/e))/(e^2*x
^5 + 2*d*e*x^3 + d^2*x)) + 4*sqrt(b*c*x^4 + a*c*x^2))/(e*x), 1/2*(x*sqrt((
b*c*d - a*c*e)/e)*arctan(-1/2*sqrt(b*c*x^4 + a*c*x^2)*(b*e*x^2 - b*d + 2*a
*e)*sqrt((b*c*d - a*c*e)/e)/((b^2*c*d - a*b*c*e)*x^3 + (a*b*c*d - a^2*c*e)
*x)) + 2*sqrt(b*c*x^4 + a*c*x^2))/(e*x)]
```

Sympy [F]

$$\int \frac{\sqrt{x^2(ac+bcx^2)}}{d+ex^2}\,dx = \int \frac{\sqrt{cx^2(a+bx^2)}}{d+ex^2}\,dx$$

input
```
integrate((x**2*(b*c*x**2+a*c))**(1/2)/(e*x**2+d),x)
```

output
```
Integral(sqrt(c*x**2*(a + b*x**2))/(d + e*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{x^2 (ac + bcx^2)}}{d + ex^2} dx = \text{Exception raised: ValueError}$$

input
```
integrate((x^2*(b*c*x^2+a*c))^(1/2)/(e*x^2+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 182 vs. 2(89) = 178.

Time = 0.14 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.73

$$\int \frac{\sqrt{x^2 (ac + bcx^2)}}{d + ex^2} dx = -\frac{(bcd\text{sgn}(x) - ace\text{sgn}(x)) \arctan\left(\frac{\sqrt{bcx^2+ace}}{\sqrt{bcde-ace^2}}\right)}{\sqrt{bcde - ace^2}e} + \frac{\sqrt{bcx^2 + ac}\text{sgn}(x)}{e} + \frac{\left(bcd \arctan\left(\frac{\sqrt{ace}}{\sqrt{bcde-ace^2}}\right) - ace \arctan\left(\frac{\sqrt{ace}}{\sqrt{bcde-ace^2}}\right) - \sqrt{bcde - ace^2}\sqrt{ac}\right)\text{sgn}(x)}{\sqrt{bcde - ace^2}e}$$

input
```
integrate((x^2*(b*c*x^2+a*c))^(1/2)/(e*x^2+d),x, algorithm="giac")
```

output
```
-(b*c*d*sgn(x) - a*c*e*sgn(x))*arctan(sqrt(b*c*x^2 + a*c)*e/sqrt(b*c*d*e -
 a*c*e^2))/(sqrt(b*c*d*e - a*c*e^2)*e) + sqrt(b*c*x^2 + a*c)*sgn(x)/e + (b
*c*d*arctan(sqrt(a*c)*e/sqrt(b*c*d*e - a*c*e^2)) - a*c*e*arctan(sqrt(a*c)*
e/sqrt(b*c*d*e - a*c*e^2)) - sqrt(b*c*d*e - a*c*e^2)*sqrt(a*c))*sgn(x)/(sq
rt(b*c*d*e - a*c*e^2)*e)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{x^2(ac+bcx^2)}}{d+ex^2}\,dx = \int \frac{\sqrt{x^2\,\left(b\,c\,x^2+a\,c\right)}}{e\,x^2+d}\,dx$$

input
```
int((x^2*(a*c + b*c*x^2))^(1/2)/(d + e*x^2),x)
```

output
```
int((x^2*(a*c + b*c*x^2))^(1/2)/(d + e*x^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.89

$$\int \frac{\sqrt{x^2(ac+bcx^2)}}{d+ex^2}\,dx = \frac{\sqrt{c}\left(-\sqrt{e}\,\sqrt{-ae+bd}\,atan\left(\frac{\sqrt{b}\,\sqrt{b\,x^2+a}\,ex+ae+be\,x^2}{\sqrt{e}\,\sqrt{b\,x^2+a}\,\sqrt{-ae+bd}+\sqrt{e}\,\sqrt{b}\,\sqrt{-ae+bd}\,x}\right)+\sqrt{b\,x^2+a}\,e\right)}{e^2}$$

input
```
int((x^2*(b*c*x^2+a*c))^(1/2)/(e*x^2+d),x)
```

output
```
(sqrt(c)*( - sqrt(e)*sqrt( - a*e + b*d)*atan((sqrt(b)*sqrt(a + b*x**2)*e*x
 + a*e + b*e*x**2)/(sqrt(e)*sqrt(a + b*x**2)*sqrt( - a*e + b*d) + sqrt(e)*
sqrt(b)*sqrt( - a*e + b*d)*x)) + sqrt(a + b*x**2)*e))/e**2
```

3.278 $\int \frac{\sqrt{cx(ax+bx^3)}}{d+ex^2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 105

$$\int \frac{\sqrt{cx(ax+bx^3)}}{d+ex^2}\,dx=\frac{\sqrt{acx^2+bcx^4}}{ex}-\frac{\sqrt{bd-ae}\sqrt{acx^2+bcx^4}\arctan\left(\frac{\sqrt{e}\sqrt{a+bx^2}}{\sqrt{bd-ae}}\right)}{e^{3/2}x\sqrt{a+bx^2}}$$

output
```
(b*c*x^4+a*c*x^2)^(1/2)/e/x-(-a*e+b*d)^(1/2)*(b*c*x^4+a*c*x^2)^(1/2)*arcta
n(e^(1/2)*(b*x^2+a)^(1/2)/(-a*e+b*d)^(1/2))/e^(3/2)/x/(b*x^2+a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{cx(ax+bx^3)}}{d+ex^2}\,dx=\frac{cx\sqrt{a+bx^2}\left(\sqrt{e}\sqrt{a+bx^2}-\sqrt{bd-ae}\arctan\left(\frac{\sqrt{e}\sqrt{a+bx^2}}{\sqrt{bd-ae}}\right)\right)}{e^{3/2}\sqrt{cx^2(a+bx^2)}}$$

input
```
Integrate[Sqrt[c*x*(a*x + b*x^3)]/(d + e*x^2),x]
```

output
```
(c*x*Sqrt[a + b*x^2]*(Sqrt[e]*Sqrt[a + b*x^2] - Sqrt[b*d - a*e]*ArcTan[(Sq
rt[e]*Sqrt[a + b*x^2])/Sqrt[b*d - a*e]]))/(e^(3/2)*Sqrt[c*x^2*(a + b*x^2)]
)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.14, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2087, 1466, 353, 60, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{cx\left(ax+bx^3\right)}}{d+ex^2}dx$$

$$\Big\downarrow \; 2087$$

$$\int \frac{\sqrt{acx^2+bcx^4}}{d+ex^2}dx$$

$$\Big\downarrow \; 1466$$

$$\frac{\sqrt{acx^2+bcx^4}\int \frac{x\sqrt{bcx^2+ac}}{ex^2+d}dx}{x\sqrt{ac+bcx^2}}$$

$$\Big\downarrow \; 353$$

$$\frac{\sqrt{acx^2+bcx^4}\int \frac{\sqrt{bcx^2+ac}}{ex^2+d}dx^2}{2x\sqrt{ac+bcx^2}}$$

$$\Big\downarrow \; 60$$

$$\frac{\sqrt{acx^2+bcx^4}\left(\frac{2\sqrt{ac+bcx^2}}{e}-\frac{c(bd-ae)\int \frac{1}{\sqrt{bcx^2+ac}\left(ex^2+d\right)}dx^2}{e}\right)}{2x\sqrt{ac+bcx^2}}$$

$$\Big\downarrow \; 73$$

$$\frac{\sqrt{acx^2+bcx^4}\left(\frac{2\sqrt{ac+bcx^2}}{e}-\frac{2(bd-ae)\int \frac{1}{\frac{ex^4}{bc}+d-\frac{ae}{b}}d\sqrt{bcx^2+ac}}{be}\right)}{2x\sqrt{ac+bcx^2}}$$

$$\Big\downarrow \text{218}$$

$$\frac{\sqrt{acx^2+bcx^4}\left(\frac{2\sqrt{ac+bcx^2}}{e}-\frac{2\sqrt{c}\sqrt{bd-ae}\arctan\left(\frac{\sqrt{e}\sqrt{ac+bcx^2}}{\sqrt{c}\sqrt{bd-ae}}\right)}{e^{3/2}}\right)}{2x\sqrt{ac+bcx^2}}$$

input
```
Int[Sqrt[c*x*(a*x + b*x^3)]/(d + e*x^2),x]
```

output
```
(Sqrt[a*c*x^2 + b*c*x^4]*((2*Sqrt[a*c + b*c*x^2])/e - (2*Sqrt[c]*Sqrt[b*d
- a*e]*ArcTan[(Sqrt[e]*Sqrt[a*c + b*c*x^2])/(Sqrt[c]*Sqrt[b*d - a*e])])/e^
(3/2)))/(2*x*Sqrt[a*c + b*c*x^2])
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 1466
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbo
l] :> Simp[(b*x^2 + c*x^4)^FracPart[p]/(x^(2*FracPart[p])*(b + c*x^2)^FracP
art[p])   Int[x^(2*p)*(d + e*x^2)^q*(b + c*x^2)^p, x], x] /; FreeQ[{b, c, d
, e, p, q}, x] &&  !IntegerQ[p]
```

rule 2087
```
Int[(u_)^(q_.)*(v_)^(p_.), x_Symbol] :> Int[ExpandToSum[u, x]^q*ExpandToSum
[v, x]^p, x] /; FreeQ[{p, q}, x] && BinomialQ[u, x] && TrinomialQ[v, x] &&
 !(BinomialMatchQ[u, x] && TrinomialMatchQ[v, x])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 194 vs. $2(89) = 178$.

Time = 0.32 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.86

method	result
pseudoelliptic	$-\frac{-2\sqrt{-\frac{c(ae-bd)}{d}}\sqrt{-de}\sqrt{(bx^2+a)c}+c(ae-bd)\left(\ln\left(\frac{bcdx+\sqrt{-\frac{c(ae-bd)}{d}}\sqrt{(bx^2+a)c}\,d-ac\sqrt{-de}}{\sqrt{-de}\,x+d}\right)-\ln\left(\frac{bcdx+\sqrt{-\frac{c(ae-bd)}{d}}\cdots}{-\sqrt{\cdots}}\right.\cdots}{2\sqrt{-\frac{c(ae-bd)}{d}}\sqrt{-de}\,e}$
default	$\frac{\sqrt{cx^2(bx^2+a)}\left(-\ln\left(-\frac{2\left(\sqrt{-de}\,bcx+\sqrt{(bx^2+a)c}\sqrt{\frac{c(ae-bd)}{e}}\,e+ace\right)}{-ex+\sqrt{-de}}\right)ace+\ln\left(-\frac{2\left(\sqrt{-de}\,bcx+\sqrt{(bx^2+a)c}\sqrt{\frac{c(ae-bd)}{e}}\,e+a\cdots\right.}{-ex+\sqrt{-de}}\right.\cdots}{\cdots}$
risch	$\frac{\sqrt{cx^2(bx^2+a)}}{ex}+\frac{(ae-bd)\left(-\frac{\ln\left(\frac{\frac{2c(ae-bd)}{e}+\frac{2bc\sqrt{-de}\left(x-\frac{\sqrt{-de}}{e}\right)}{e}+2\sqrt{\frac{c(ae-bd)}{e}}\sqrt{bc\left(x-\frac{\sqrt{-de}}{e}\right)^2+\frac{2bc\sqrt{-de}\left(x-\frac{\sqrt{-de}}{e}\right)}{e}\cdots}}{x-\frac{\sqrt{-de}}{e}}\right)}{2e\sqrt{\frac{c(ae-bd)}{e}}}\cdots\right)}{\cdots}$

input
```
int((c*x*(b*x^3+a*x))^(1/2)/(e*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-2*(-c/d*(a*e-b*d))^(1/2)*(-d*e)^(1/2)*((b*x^2+a)*c)^(1/2)+c*(a*e-b*
d)*(ln((b*c*d*x+(-c/d*(a*e-b*d))^(1/2)*((b*x^2+a)*c)^(1/2)*d-a*c*(-d*e)^(1
/2))/((-d*e)^(1/2)*x+d))-ln((b*c*d*x+(-c/d*(a*e-b*d))^(1/2)*((b*x^2+a)*c)^
(1/2)*d+a*c*(-d*e)^(1/2))/(-(-d*e)^(1/2)*x+d))))/(-c/d*(a*e-b*d))^(1/2)/(-
d*e)^(1/2)/e
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 313, normalized size of antiderivative = 2.98

$$\int \frac{\sqrt{cx\,(ax+bx^3)}}{d+ex^2}\,dx$$
$$= \left[\frac{x\sqrt{-\frac{bcd-ace}{e}}\log\left(\frac{b^2ce^2x^5-2\,(3\,b^2cde-4\,abce^2)x^3+(b^2cd^2-8\,abcde+8\,a^2ce^2)x-4\,\sqrt{bcx^4+acx^2}\,(be^2x^2-bde+2\,ae^2)\sqrt{-\frac{bcd-ace}{e}}}{e^2x^5+2\,dex^3+d^2x}\right)+}{4\,ex} \right.$$

input
```
integrate((c*x*(b*x^3+a*x))^(1/2)/(e*x^2+d),x, algorithm="fricas")
```

output
```
[1/4*(x*sqrt(-(b*c*d - a*c*e)/e)*log((b^2*c*e^2*x^5 - 2*(3*b^2*c*d*e - 4*a
*b*c*e^2)*x^3 + (b^2*c*d^2 - 8*a*b*c*d*e + 8*a^2*c*e^2)*x - 4*sqrt(b*c*x^4
 + a*c*x^2)*(b*e^2*x^2 - b*d*e + 2*a*e^2)*sqrt(-(b*c*d - a*c*e)/e))/(e^2*x
^5 + 2*d*e*x^3 + d^2*x)) + 4*sqrt(b*c*x^4 + a*c*x^2))/(e*x), 1/2*(x*sqrt((
b*c*d - a*c*e)/e)*arctan(-1/2*sqrt(b*c*x^4 + a*c*x^2)*(b*e*x^2 - b*d + 2*a
*e)*sqrt((b*c*d - a*c*e)/e)/((b^2*c*d - a*b*c*e)*x^3 + (a*b*c*d - a^2*c*e)
*x)) + 2*sqrt(b*c*x^4 + a*c*x^2))/(e*x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{cx\,(ax+bx^3)}}{d+ex^2}\,dx = \text{Timed out}$$

input
```
integrate((c*x*(b*x**3+a*x))**(1/2)/(e*x**2+d),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{cx\,(ax+bx^3)}}{d+ex^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x*(b*x^3+a*x))^(1/2)/(e*x^2+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 182 vs. 2(89) = 178.

Time = 0.15 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.73

$$\int \frac{\sqrt{cx\,(ax+bx^3)}}{d+ex^2}\,dx = -\frac{(bcd\text{sgn}(x)-ace\text{sgn}(x))\arctan\left(\frac{\sqrt{bcx^2+ace}}{\sqrt{bcde-ace^2}}\right)}{\sqrt{bcde-ace^2}\,e}+\frac{\sqrt{bcx^2+ac}\text{sgn}(x)}{e}+\frac{\left(bcd\arctan\left(\frac{\sqrt{ace}}{\sqrt{bcde-ace^2}}\right)-ace\arctan\left(\frac{\sqrt{ace}}{\sqrt{bcde-ace^2}}\right)-\sqrt{bcde-ace^2}\sqrt{ac}\right)\text{sgn}(x)}{\sqrt{bcde-ace^2}\,e}$$

input
```
integrate((c*x*(b*x^3+a*x))^(1/2)/(e*x^2+d),x, algorithm="giac")
```

output
```
-(b*c*d*sgn(x) - a*c*e*sgn(x))*arctan(sqrt(b*c*x^2 + a*c)*e/sqrt(b*c*d*e -
 a*c*e^2))/(sqrt(b*c*d*e - a*c*e^2)*e) + sqrt(b*c*x^2 + a*c)*sgn(x)/e + (b
*c*d*arctan(sqrt(a*c)*e/sqrt(b*c*d*e - a*c*e^2)) - a*c*e*arctan(sqrt(a*c)*
e/sqrt(b*c*d*e - a*c*e^2)) - sqrt(b*c*d*e - a*c*e^2)*sqrt(a*c))*sgn(x)/(sq
rt(b*c*d*e - a*c*e^2)*e)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{cx\,(ax+bx^3)}}{d+ex^2}\,dx = \int \frac{\sqrt{c\,x\,\left(b\,x^3+a\,x\right)}}{e\,x^2+d}\,dx$$

input
```
int((c*x*(a*x + b*x^3))^(1/2)/(d + e*x^2),x)
```

output
```
int((c*x*(a*x + b*x^3))^(1/2)/(d + e*x^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.89

$$\int \frac{\sqrt{cx\,(ax+bx^3)}}{d+ex^2}\,dx = \frac{\sqrt{c}\left(-\sqrt{e}\,\sqrt{-ae+bd}\,atan\left(\frac{\sqrt{b}\,\sqrt{b\,x^2+a}\,ex+ae+be\,x^2}{\sqrt{e}\,\sqrt{b\,x^2+a}\,\sqrt{-ae+bd}+\sqrt{e}\,\sqrt{b}\,\sqrt{-ae+bd}\,x}\right)+\sqrt{b\,x^2+a}\,e\right)}{e^2}$$

input
```
int((c*x*(b*x^3+a*x))^(1/2)/(e*x^2+d),x)
```

output
```
(sqrt(c)*( - sqrt(e)*sqrt( - a*e + b*d)*atan((sqrt(b)*sqrt(a + b*x**2)*e*x
 + a*e + b*e*x**2)/(sqrt(e)*sqrt(a + b*x**2)*sqrt( - a*e + b*d) + sqrt(e)*
sqrt(b)*sqrt( - a*e + b*d)*x)) + sqrt(a + b*x**2)*e))/e**2
```

3.279 $\int \frac{\sqrt{c(ax^2+bx^4)}}{d+ex^2} dx$

Optimal result

Integrand size = 27, antiderivative size = 105

$$\int \frac{\sqrt{c(ax^2+bx^4)}}{d+ex^2} dx = \frac{\sqrt{acx^2+bcx^4}}{ex} - \frac{\sqrt{bd-ae}\sqrt{acx^2+bcx^4}\arctan\left(\frac{\sqrt{e}\sqrt{a+bx^2}}{\sqrt{bd-ae}}\right)}{e^{3/2}x\sqrt{a+bx^2}}$$

output
```
(b*c*x^4+a*c*x^2)^(1/2)/e/x-(-a*e+b*d)^(1/2)*(b*c*x^4+a*c*x^2)^(1/2)*arcta
n(e^(1/2)*(b*x^2+a)^(1/2)/(-a*e+b*d)^(1/2))/e^(3/2)/x/(b*x^2+a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{c(ax^2+bx^4)}}{d+ex^2} dx = \frac{cx\sqrt{a+bx^2}\left(\sqrt{e}\sqrt{a+bx^2}-\sqrt{bd-ae}\arctan\left(\frac{\sqrt{e}\sqrt{a+bx^2}}{\sqrt{bd-ae}}\right)\right)}{e^{3/2}\sqrt{cx^2(a+bx^2)}}$$

input
```
Integrate[Sqrt[c*(a*x^2 + b*x^4)]/(d + e*x^2),x]
```

output
```
(c*x*Sqrt[a + b*x^2]*(Sqrt[e]*Sqrt[a + b*x^2] - Sqrt[b*d - a*e]*ArcTan[(Sq
rt[e]*Sqrt[a + b*x^2])/Sqrt[b*d - a*e]]))/(e^(3/2)*Sqrt[c*x^2*(a + b*x^2)]
)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.14, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2087, 1466, 353, 60, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c(ax^2+bx^4)}}{d+ex^2}dx$$

$$\Big\downarrow \; 2087$$

$$\int \frac{\sqrt{acx^2+bcx^4}}{d+ex^2}dx$$

$$\Big\downarrow \; 1466$$

$$\frac{\sqrt{acx^2+bcx^4}\int \frac{x\sqrt{bcx^2+ac}}{ex^2+d}dx}{x\sqrt{ac+bcx^2}}$$

$$\Big\downarrow \; 353$$

$$\frac{\sqrt{acx^2+bcx^4}\int \frac{\sqrt{bcx^2+ac}}{ex^2+d}dx^2}{2x\sqrt{ac+bcx^2}}$$

$$\Big\downarrow \; 60$$

$$\frac{\sqrt{acx^2+bcx^4}\left(\frac{2\sqrt{ac+bcx^2}}{e}-\frac{c(bd-ae)\int \frac{1}{\sqrt{bcx^2+ac}(ex^2+d)}dx^2}{e}\right)}{2x\sqrt{ac+bcx^2}}$$

$$\Big\downarrow \; 73$$

$$\frac{\sqrt{acx^2+bcx^4}\left(\frac{2\sqrt{ac+bcx^2}}{e}-\frac{2(bd-ae)\int \frac{1}{\frac{ex^4}{bc}+d-\frac{ae}{b}}d\sqrt{bcx^2+ac}}{be}\right)}{2x\sqrt{ac+bcx^2}}$$

$$\Big\downarrow \text{218}$$

$$\frac{\sqrt{acx^2+bcx^4}\left(\frac{2\sqrt{ac+bcx^2}}{e}-\frac{2\sqrt{c}\sqrt{bd-ae}\arctan\left(\frac{\sqrt{e}\sqrt{ac+bcx^2}}{\sqrt{c}\sqrt{bd-ae}}\right)}{e^{3/2}}\right)}{2x\sqrt{ac+bcx^2}}$$

input
```
Int[Sqrt[c*(a*x^2 + b*x^4)]/(d + e*x^2),x]
```

output
```
(Sqrt[a*c*x^2 + b*c*x^4]*((2*Sqrt[a*c + b*c*x^2])/e - (2*Sqrt[c]*Sqrt[b*d
- a*e]*ArcTan[(Sqrt[e]*Sqrt[a*c + b*c*x^2])/(Sqrt[c]*Sqrt[b*d - a*e])])/e^
(3/2)))/(2*x*Sqrt[a*c + b*c*x^2])
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 1466
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbo
l] :> Simp[(b*x^2 + c*x^4)^FracPart[p]/(x^(2*FracPart[p])*(b + c*x^2)^FracP
art[p])   Int[x^(2*p)*(d + e*x^2)^q*(b + c*x^2)^p, x], x] /; FreeQ[{b, c, d
, e, p, q}, x] &&  !IntegerQ[p]
```

rule 2087
```
Int[(u_)^(q_.)*(v_)^(p_.), x_Symbol] :> Int[ExpandToSum[u, x]^q*ExpandToSum
[v, x]^p, x] /; FreeQ[{p, q}, x] && BinomialQ[u, x] && TrinomialQ[v, x] &&
 !(BinomialMatchQ[u, x] && TrinomialMatchQ[v, x])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 194 vs. $2(89) = 178$.

Time = 0.33 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.86

method	result
pseudoelliptic	$-\frac{-2\sqrt{-\frac{c(ae-bd)}{d}}\sqrt{-de}\sqrt{(bx^2+a)c}+c(ae-bd)\left(\ln\left(\frac{bcdx+\sqrt{-\frac{c(ae-bd)}{d}}\sqrt{(bx^2+a)c}\,d-ac\sqrt{-de}}{\sqrt{-de}\,x+d}\right)-\ln\left(\frac{bcdx+\sqrt{-\frac{c(ae-bd)}{d}}\cdots}{-\sqrt{\cdots}}\right.\cdots\right.}{2\sqrt{-\frac{c(ae-bd)}{d}}\sqrt{-de}\,e}$
default	$\frac{\sqrt{cx^2(bx^2+a)}\left(-\ln\left(-\frac{2\left(\sqrt{-de}\,bcx+\sqrt{(bx^2+a)c}\sqrt{\frac{c(ae-bd)}{e}}\,e+ace\right)}{-ex+\sqrt{-de}}\right)ace+\ln\left(-\frac{2\left(\sqrt{-de}\,bcx+\sqrt{(bx^2+a)c}\sqrt{\frac{c(ae-bd)}{e}}\,e+a\cdots\right.}{-ex+\sqrt{-de}}\right.\cdots\right.}{\cdots}$
risch	$\frac{\sqrt{cx^2(bx^2+a)}}{ex}+\frac{(ae-bd)\left(-\frac{\ln\left(\frac{\frac{2c(ae-bd)}{e}+\frac{2bc\sqrt{-de}\left(x-\frac{\sqrt{-de}}{e}\right)}{e}+2\sqrt{\frac{c(ae-bd)}{e}}\sqrt{bc\left(x-\frac{\sqrt{-de}}{e}\right)^2+\frac{2bc\sqrt{-de}\left(x-\frac{\sqrt{-de}}{e}\right)}{e}\cdots}}{x-\frac{\sqrt{-de}}{e}}\right)}{2e\sqrt{\frac{c(ae-bd)}{e}}}\cdots\right)}{\cdots}$

input
```
int((c*(b*x^4+a*x^2))^(1/2)/(e*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-2*(-c/d*(a*e-b*d))^(1/2)*(-d*e)^(1/2)*((b*x^2+a)*c)^(1/2)+c*(a*e-b*
d)*(ln((b*c*d*x+(-c/d*(a*e-b*d))^(1/2)*((b*x^2+a)*c)^(1/2)*d-a*c*(-d*e)^(1
/2))/((-d*e)^(1/2)*x+d))-ln((b*c*d*x+(-c/d*(a*e-b*d))^(1/2)*((b*x^2+a)*c)^
(1/2)*d+a*c*(-d*e)^(1/2))/(-(-d*e)^(1/2)*x+d))))/(-c/d*(a*e-b*d))^(1/2)/(-
d*e)^(1/2)/e
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 313, normalized size of antiderivative = 2.98

$$\int \frac{\sqrt{c(ax^2+bx^4)}}{d+ex^2}\,dx = \left[\frac{x\sqrt{-\frac{bcd-ace}{e}}\log\left(\frac{b^2ce^2x^5-2\,(3\,b^2cde-4\,abce^2)x^3+(b^2cd^2-8\,abcde+8\,a^2ce^2)x-4\,\sqrt{bcx^4+acx^2}\,(be^2x^2-bde+2\,ae^2)\sqrt{-\frac{bcd-ace}{e}}}{e^2x^5+2\,dex^3+d^2x}\right)+}{4\,ex}\right.$$

input
```
integrate((c*(b*x^4+a*x^2))^(1/2)/(e*x^2+d),x, algorithm="fricas")
```

output
```
[1/4*(x*sqrt(-(b*c*d - a*c*e)/e)*log((b^2*c*e^2*x^5 - 2*(3*b^2*c*d*e - 4*a
*b*c*e^2)*x^3 + (b^2*c*d^2 - 8*a*b*c*d*e + 8*a^2*c*e^2)*x - 4*sqrt(b*c*x^4
 + a*c*x^2)*(b*e^2*x^2 - b*d*e + 2*a*e^2)*sqrt(-(b*c*d - a*c*e)/e))/(e^2*x
^5 + 2*d*e*x^3 + d^2*x)) + 4*sqrt(b*c*x^4 + a*c*x^2))/(e*x), 1/2*(x*sqrt((
b*c*d - a*c*e)/e)*arctan(-1/2*sqrt(b*c*x^4 + a*c*x^2)*(b*e*x^2 - b*d + 2*a
*e)*sqrt((b*c*d - a*c*e)/e)/((b^2*c*d - a*b*c*e)*x^3 + (a*b*c*d - a^2*c*e)
*x)) + 2*sqrt(b*c*x^4 + a*c*x^2))/(e*x)]
```

Sympy [F]

$$\int \frac{\sqrt{c(ax^2+bx^4)}}{d+ex^2}\,dx = \int \frac{\sqrt{cx^2(a+bx^2)}}{d+ex^2}\,dx$$

input
```
integrate((c*(b*x**4+a*x**2))**(1/2)/(e*x**2+d),x)
```

output
```
Integral(sqrt(c*x**2*(a + b*x**2))/(d + e*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c\,(ax^2+bx^4)}}{d+ex^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*(b*x^4+a*x^2))^(1/2)/(e*x^2+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 182 vs. 2(89) = 178.

Time = 0.13 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.73

$$\begin{aligned}\int \frac{\sqrt{c\,(ax^2+bx^4)}}{d+ex^2}\,dx &= -\frac{(bcd\,\mathrm{sgn}(x)-ace\,\mathrm{sgn}(x))\arctan\left(\frac{\sqrt{bcx^2+ac}e}{\sqrt{bcde-ace^2}}\right)}{\sqrt{bcde-ace^2}\,e}+\frac{\sqrt{bcx^2+ac}\,\mathrm{sgn}(x)}{e}\\&\quad+\frac{\left(bcd\arctan\left(\frac{\sqrt{ac}e}{\sqrt{bcde-ace^2}}\right)-ace\arctan\left(\frac{\sqrt{ac}e}{\sqrt{bcde-ace^2}}\right)-\sqrt{bcde-ace^2}\sqrt{ac}\right)\mathrm{sgn}(x)}{\sqrt{bcde-ace^2}\,e}\end{aligned}$$

input
```
integrate((c*(b*x^4+a*x^2))^(1/2)/(e*x^2+d),x, algorithm="giac")
```

output
```
-(b*c*d*sgn(x) - a*c*e*sgn(x))*arctan(sqrt(b*c*x^2 + a*c)*e/sqrt(b*c*d*e -
 a*c*e^2))/(sqrt(b*c*d*e - a*c*e^2)*e) + sqrt(b*c*x^2 + a*c)*sgn(x)/e + (b
*c*d*arctan(sqrt(a*c)*e/sqrt(b*c*d*e - a*c*e^2)) - a*c*e*arctan(sqrt(a*c)*
e/sqrt(b*c*d*e - a*c*e^2)) - sqrt(b*c*d*e - a*c*e^2)*sqrt(a*c))*sgn(x)/(sq
rt(b*c*d*e - a*c*e^2)*e)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c\,(ax^2+bx^4)}}{d+ex^2}\,dx = \int \frac{\sqrt{c\,\left(b\,x^4+a\,x^2\right)}}{e\,x^2+d}\,dx$$

input
```
int((c*(a*x^2 + b*x^4))^(1/2)/(d + e*x^2),x)
```

output
```
int((c*(a*x^2 + b*x^4))^(1/2)/(d + e*x^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.89

$$\int \frac{\sqrt{c\,(ax^2+bx^4)}}{d+ex^2}\,dx = \frac{\sqrt{c}\left(-\sqrt{e}\,\sqrt{-ae+bd}\,atan\left(\frac{\sqrt{b}\,\sqrt{b\,x^2+a}\,ex+ae+be\,x^2}{\sqrt{e}\,\sqrt{b\,x^2+a}\,\sqrt{-ae+bd}+\sqrt{e}\,\sqrt{b}\,\sqrt{-ae+bd}\,x}\right)+\sqrt{b\,x^2+a}\,e\right)}{e^2}$$

input
```
int((c*(b*x^4+a*x^2))^(1/2)/(e*x^2+d),x)
```

output
```
(sqrt(c)*( - sqrt(e)*sqrt( - a*e + b*d)*atan((sqrt(b)*sqrt(a + b*x**2)*e*x
 + a*e + b*e*x**2)/(sqrt(e)*sqrt(a + b*x**2)*sqrt( - a*e + b*d) + sqrt(e)*
sqrt(b)*sqrt( - a*e + b*d)*x)) + sqrt(a + b*x**2)*e))/e**2
```

3.280 $\int \frac{\sqrt{x(acx+bcx^3)}}{d+ex^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 105

$$\int \frac{\sqrt{x(acx+bcx^3)}}{d+ex^2}\,dx=\frac{\sqrt{acx^2+bcx^4}}{ex}-\frac{\sqrt{bd-ae}\sqrt{acx^2+bcx^4}\arctan\left(\frac{\sqrt{e}\sqrt{a+bx^2}}{\sqrt{bd-ae}}\right)}{e^{3/2}x\sqrt{a+bx^2}}$$

output
```
(b*c*x^4+a*c*x^2)^(1/2)/e/x-(-a*e+b*d)^(1/2)*(b*c*x^4+a*c*x^2)^(1/2)*arcta
n(e^(1/2)*(b*x^2+a)^(1/2)/(-a*e+b*d)^(1/2))/e^(3/2)/x/(b*x^2+a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{x(acx+bcx^3)}}{d+ex^2}\,dx=\frac{cx\sqrt{a+bx^2}\left(\sqrt{e}\sqrt{a+bx^2}-\sqrt{bd-ae}\arctan\left(\frac{\sqrt{e}\sqrt{a+bx^2}}{\sqrt{bd-ae}}\right)\right)}{e^{3/2}\sqrt{cx^2(a+bx^2)}}$$

input
```
Integrate[Sqrt[x*(a*c*x + b*c*x^3)]/(d + e*x^2),x]
```

output
```
(c*x*Sqrt[a + b*x^2]*(Sqrt[e]*Sqrt[a + b*x^2] - Sqrt[b*d - a*e]*ArcTan[(Sq
rt[e]*Sqrt[a + b*x^2])/Sqrt[b*d - a*e]]))/(e^(3/2)*Sqrt[c*x^2*(a + b*x^2)]
)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.14, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2087, 1466, 353, 60, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x\left(acx+bcx^3\right)}}{d+ex^2}dx$$

$$\Big\downarrow \ 2087$$

$$\int \frac{\sqrt{acx^2+bcx^4}}{d+ex^2}dx$$

$$\Big\downarrow \ 1466$$

$$\frac{\sqrt{acx^2+bcx^4}\int \frac{x\sqrt{bcx^2+ac}}{ex^2+d}dx}{x\sqrt{ac+bcx^2}}$$

$$\Big\downarrow \ 353$$

$$\frac{\sqrt{acx^2+bcx^4}\int \frac{\sqrt{bcx^2+ac}}{ex^2+d}dx^2}{2x\sqrt{ac+bcx^2}}$$

$$\Big\downarrow \ 60$$

$$\frac{\sqrt{acx^2+bcx^4}\left(\frac{2\sqrt{ac+bcx^2}}{e}-\frac{c(bd-ae)\int \frac{1}{\sqrt{bcx^2+ac}\left(ex^2+d\right)}dx^2}{e}\right)}{2x\sqrt{ac+bcx^2}}$$

$$\Big\downarrow \ 73$$

$$\frac{\sqrt{acx^2+bcx^4}\left(\frac{2\sqrt{ac+bcx^2}}{e}-\frac{2(bd-ae)\int \frac{1}{\frac{ex^4}{bc}+d-\frac{ae}{b}}d\sqrt{bcx^2+ac}}{be}\right)}{2x\sqrt{ac+bcx^2}}$$

$$\Big\downarrow \text{218}$$

$$\frac{\sqrt{acx^2+bcx^4}\left(\frac{2\sqrt{ac+bcx^2}}{e}-\frac{2\sqrt{c}\sqrt{bd-ae}\arctan\left(\frac{\sqrt{e}\sqrt{ac+bcx^2}}{\sqrt{c}\sqrt{bd-ae}}\right)}{e^{3/2}}\right)}{2x\sqrt{ac+bcx^2}}$$

input
```
Int[Sqrt[x*(a*c*x + b*c*x^3)]/(d + e*x^2),x]
```

output
```
(Sqrt[a*c*x^2 + b*c*x^4]*((2*Sqrt[a*c + b*c*x^2])/e - (2*Sqrt[c]*Sqrt[b*d
- a*e]*ArcTan[(Sqrt[e]*Sqrt[a*c + b*c*x^2])/(Sqrt[c]*Sqrt[b*d - a*e])])/e^
(3/2)))/(2*x*Sqrt[a*c + b*c*x^2])
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 1466
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbo
l] :> Simp[(b*x^2 + c*x^4)^FracPart[p]/(x^(2*FracPart[p])*(b + c*x^2)^FracP
art[p])   Int[x^(2*p)*(d + e*x^2)^q*(b + c*x^2)^p, x], x] /; FreeQ[{b, c, d
, e, p, q}, x] &&  !IntegerQ[p]
```

rule 2087
```
Int[(u_)^(q_.)*(v_)^(p_.), x_Symbol] :> Int[ExpandToSum[u, x]^q*ExpandToSum
[v, x]^p, x] /; FreeQ[{p, q}, x] && BinomialQ[u, x] && TrinomialQ[v, x] &&
 !(BinomialMatchQ[u, x] && TrinomialMatchQ[v, x])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 194 vs. $2(89) = 178$.

Time = 0.33 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.86

method	result
pseudoelliptic	$-\frac{-2\sqrt{-\frac{c(ae-bd)}{d}}\,\sqrt{-de}\,\sqrt{(b\,x^2+a)c}+c(ae-bd)\left(\ln\left(\frac{bcdx+\sqrt{-\frac{c(ae-bd)}{d}}\,\sqrt{\left(b\,x^2+a\right)c}\,d-ac\sqrt{-de}}{\sqrt{-de}\,x+d}\right)-\ln\left(\frac{bcdx+\sqrt{-\frac{c(ae-bd)}{d}}}{-\sqrt{}}\right.\right.}{2\sqrt{-\frac{c(ae-bd)}{d}}\,\sqrt{-de}\,e}$
default	$\frac{\sqrt{c\,x^2(b\,x^2+a)}\left(-\ln\left(-\frac{2\left(\sqrt{-de}\,bcx+\sqrt{\left(b\,x^2+a\right)c}\,\sqrt{\frac{c(ae-bd)}{e}}\,e+ace\right)}{-ex+\sqrt{-de}}\right)ace+\ln\left(-\frac{2\left(\sqrt{-de}\,bcx+\sqrt{\left(b\,x^2+a\right)c}\,\sqrt{\frac{c(ae-bd)}{e}}\,e+a\right.}{-ex+\sqrt{-de}}\right.\right.}{}$
risch	$\frac{\sqrt{c\,x^2(b\,x^2+a)}}{ex}+\frac{(ae-bd)\left(-\frac{\ln\left(\frac{\frac{2c(ae-bd)}{e}+\frac{2bc\sqrt{-de}\left(x-\frac{\sqrt{-de}}{e}\right)}{e}+2\sqrt{\frac{c(ae-bd)}{e}}\,\sqrt{bc\left(x-\frac{\sqrt{-de}}{e}\right)^2+\frac{2bc\sqrt{-de}\left(x-\frac{\sqrt{-de}}{e}\right)}{e}}}{x-\frac{\sqrt{-de}}{e}}\right)}{2e\sqrt{\frac{c(ae-bd)}{e}}}\right.}{}$

input
```
int((x*(b*c*x^3+a*c*x))^(1/2)/(e*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-2*(-c/d*(a*e-b*d))^(1/2)*(-d*e)^(1/2)*((b*x^2+a)*c)^(1/2)+c*(a*e-b*
d)*(ln((b*c*d*x+(-c/d*(a*e-b*d))^(1/2)*((b*x^2+a)*c)^(1/2)*d-a*c*(-d*e)^(1
/2))/((-d*e)^(1/2)*x+d))-ln((b*c*d*x+(-c/d*(a*e-b*d))^(1/2)*((b*x^2+a)*c)^
(1/2)*d+a*c*(-d*e)^(1/2))/(-(-d*e)^(1/2)*x+d))))/(-c/d*(a*e-b*d))^(1/2)/(-
d*e)^(1/2)/e
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 313, normalized size of antiderivative = 2.98

$$\int \frac{\sqrt{x\left(acx+bcx^3\right)}}{d+ex^2}\,dx = \left[\frac{x\sqrt{-\frac{bcd-ace}{e}}\log\left(\frac{b^2ce^2x^5-2\left(3\,b^2cde-4\,abce^2\right)x^3+\left(b^2cd^2-8\,abcde+8\,a^2ce^2\right)x-4\,\sqrt{bcx^4+acx^2}\left(be^2x^2-bde+2\,ae^2\right)\sqrt{-\frac{bcd-ace}{e}}}{e^2x^5+2\,dex^3+d^2x}\right)+}{4\,ex}\right.$$

input
```
integrate((x*(b*c*x^3+a*c*x))^(1/2)/(e*x^2+d),x, algorithm="fricas")
```

output
```
[1/4*(x*sqrt(-(b*c*d - a*c*e)/e)*log((b^2*c*e^2*x^5 - 2*(3*b^2*c*d*e - 4*a
*b*c*e^2)*x^3 + (b^2*c*d^2 - 8*a*b*c*d*e + 8*a^2*c*e^2)*x - 4*sqrt(b*c*x^4
 + a*c*x^2)*(b*e^2*x^2 - b*d*e + 2*a*e^2)*sqrt(-(b*c*d - a*c*e)/e))/(e^2*x
^5 + 2*d*e*x^3 + d^2*x)) + 4*sqrt(b*c*x^4 + a*c*x^2))/(e*x), 1/2*(x*sqrt((
b*c*d - a*c*e)/e)*arctan(-1/2*sqrt(b*c*x^4 + a*c*x^2)*(b*e*x^2 - b*d + 2*a
*e)*sqrt((b*c*d - a*c*e)/e)/((b^2*c*d - a*b*c*e)*x^3 + (a*b*c*d - a^2*c*e)
*x)) + 2*sqrt(b*c*x^4 + a*c*x^2))/(e*x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{x\left(acx+bcx^3\right)}}{d+ex^2}\,dx = \text{Timed out}$$

input
```
integrate((x*(b*c*x**3+a*c*x))**(1/2)/(e*x**2+d),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{x\,(acx+bcx^3)}}{d+ex^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((x*(b*c*x^3+a*c*x))^(1/2)/(e*x^2+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 182 vs. 2(89) = 178.

Time = 0.14 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.73

$$\begin{aligned}
&\int \frac{\sqrt{x\,(acx+bcx^3)}}{d+ex^2}\,dx \\
&= -\frac{(bcd\,\mathrm{sgn}(x)-ace\,\mathrm{sgn}(x))\arctan\left(\frac{\sqrt{bcx^2+ac}e}{\sqrt{bcde-ace^2}}\right)}{\sqrt{bcde-ace^2}\,e}+\frac{\sqrt{bcx^2+ac}\,\mathrm{sgn}(x)}{e} \\
&\quad+\frac{\left(bcd\arctan\left(\frac{\sqrt{ac}e}{\sqrt{bcde-ace^2}}\right)-ace\arctan\left(\frac{\sqrt{ac}e}{\sqrt{bcde-ace^2}}\right)-\sqrt{bcde-ace^2}\sqrt{ac}\right)\mathrm{sgn}(x)}{\sqrt{bcde-ace^2}\,e}
\end{aligned}$$

input
```
integrate((x*(b*c*x^3+a*c*x))^(1/2)/(e*x^2+d),x, algorithm="giac")
```

output
```
-(b*c*d*sgn(x) - a*c*e*sgn(x))*arctan(sqrt(b*c*x^2 + a*c)*e/sqrt(b*c*d*e -
 a*c*e^2))/(sqrt(b*c*d*e - a*c*e^2)*e) + sqrt(b*c*x^2 + a*c)*sgn(x)/e + (b
*c*d*arctan(sqrt(a*c)*e/sqrt(b*c*d*e - a*c*e^2)) - a*c*e*arctan(sqrt(a*c)*
e/sqrt(b*c*d*e - a*c*e^2)) - sqrt(b*c*d*e - a*c*e^2)*sqrt(a*c))*sgn(x)/(sq
rt(b*c*d*e - a*c*e^2)*e)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{x\,(acx+bcx^3)}}{d+ex^2}\,dx = \int \frac{\sqrt{x\,\left(b\,c\,x^3+a\,c\,x\right)}}{e\,x^2+d}\,dx$$

input
```
int((x*(a*c*x + b*c*x^3))^(1/2)/(d + e*x^2),x)
```

output
```
int((x*(a*c*x + b*c*x^3))^(1/2)/(d + e*x^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.89

$$\int \frac{\sqrt{x\,(acx+bcx^3)}}{d+ex^2}\,dx = \frac{\sqrt{c}\left(-\sqrt{e}\,\sqrt{-ae+bd}\,atan\left(\frac{\sqrt{b}\,\sqrt{b\,x^2+a}\,ex+ae+be\,x^2}{\sqrt{e}\,\sqrt{b\,x^2+a}\,\sqrt{-ae+bd}+\sqrt{e}\,\sqrt{b}\,\sqrt{-ae+bd}\,x}\right)+\sqrt{b\,x^2+a}\,e\right)}{e^2}$$

input
```
int((x*(b*c*x^3+a*c*x))^(1/2)/(e*x^2+d),x)
```

output
```
(sqrt(c)*( - sqrt(e)*sqrt( - a*e + b*d)*atan((sqrt(b)*sqrt(a + b*x**2)*e*x
 + a*e + b*e*x**2)/(sqrt(e)*sqrt(a + b*x**2)*sqrt( - a*e + b*d) + sqrt(e)*
sqrt(b)*sqrt( - a*e + b*d)*x)) + sqrt(a + b*x**2)*e))/e**2
```

3.281 $\int \frac{1}{(c+dx+ex^2)\sqrt{a+bx^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 1613

$$\int \frac{1}{(c+dx+ex^2)\sqrt{a+bx^4}}\,dx = \text{Too large to display}$$

output
```
-1/2*e^2*arctanh(2^(1/2)*(2*a*e^4+b*(d^4-4*c*d^2*e+2*c^2*e^2+d^3*(-4*c*e+d
^2)^(1/2)-2*c*d*e*(-4*c*e+d^2)^(1/2)))^(1/2)*x/e/(d+(-4*c*e+d^2)^(1/2))/(b
*x^4+a)^(1/2))*2^(1/2)/(-4*c*e+d^2)^(1/2)/(2*a*e^4+b*(d^4-4*c*d^2*e+2*c^2*
e^2+d^3*(-4*c*e+d^2)^(1/2)-2*c*d*e*(-4*c*e+d^2)^(1/2)))^(1/2)+1/2*e^2*arct
anh(1/2*2^(1/2)*(8*a*e^4+2*b*(2*d^4-8*c*d^2*e+4*c^2*e^2-2*d^3*(-4*c*e+d^2)
^(1/2)+4*c*d*e*(-4*c*e+d^2)^(1/2)))^(1/2)*x/e/(d-(-4*c*e+d^2)^(1/2))/(b*x^
4+a)^(1/2))*2^(1/2)/(-4*c*e+d^2)^(1/2)/(2*a*e^4+b*(d^4-4*c*d^2*e+2*c^2*e^2
-d^3*(-4*c*e+d^2)^(1/2)+2*c*d*e*(-4*c*e+d^2)^(1/2)))^(1/2)-1/2*e^2*arctanh
(1/4*(4*a*e^2+b*(d-(-4*c*e+d^2)^(1/2))^2*x^2)*2^(1/2)/(b*d^4-4*b*c*d^2*e+2
*b*c^2*e^2+2*a*e^4-b*d*(-4*c*e+d^2)^(1/2)*(-2*c*e+d^2))^(1/2)/(b*x^4+a)^(1
/2))*2^(1/2)/(-4*c*e+d^2)^(1/2)/(b*d^4-4*b*c*d^2*e+2*b*c^2*e^2+2*a*e^4-b*d
*(-4*c*e+d^2)^(1/2)*(-2*c*e+d^2))^(1/2)+1/2*e^2*arctanh(1/4*(4*a*e^2+b*(d+
(-4*c*e+d^2)^(1/2))^2*x^2)*2^(1/2)/(b*d^4-4*b*c*d^2*e+2*b*c^2*e^2+2*a*e^4+
b*d*(-4*c*e+d^2)^(1/2)*(-2*c*e+d^2))^(1/2)/(b*x^4+a)^(1/2))*2^(1/2)/(-4*c*
e+d^2)^(1/2)/(b*d^4-4*b*c*d^2*e+2*b*c^2*e^2+2*a*e^4+b*d*(-4*c*e+d^2)^(1/2)
*(-2*c*e+d^2))^(1/2)+1/2*b^(1/4)*e*(d-(-4*c*e+d^2)^(1/2))*(a^(1/2)+b^(1/2)
*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(b
^(1/4)*x/a^(1/4)),1/2*2^(1/2))/a^(1/4)/(-4*c*e+d^2)^(1/2)/(2*a^(1/2)*e^2+b
^(1/2)*(d^2-2*c*e-d*(-4*c*e+d^2)^(1/2)))/(b*x^4+a)^(1/2)-1/2*b^(1/4)*e*(d+
(-4*c*e+d^2)^(1/2))*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x...
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 11.69 (sec) , antiderivative size = 455, normalized size of antiderivative = 0.28

$$\int \frac{1}{(c+dx+ex^2)\sqrt{a+bx^4}}\,dx =$$

$$-\frac{i\sqrt{1+\frac{bx^4}{a}}\left(\left(-d^2+\sqrt{d^4-4cd^2e}\right)\text{EllipticPi}\left(\frac{2i\sqrt{a}e^2}{\sqrt{b}\left(d^2-2ce+\sqrt{d^4-4cd^2e}\right)}, i\text{arcsinh}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}x\right),-1\right)+(d^2+ \cdots}{2\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}c\sqrt{d^4-4cd^2e}\sqrt{a+b \cdots}}$$

$$+\sqrt{b}d\text{RootSum}\Big[a^2e^2-2a\sqrt{b}d^2\#1+4a\sqrt{b}ce\#1+4bc^2\#1^2-2ae^2\#1^2+2\sqrt{b}d^2\#1^3$$

$$-4\sqrt{b}ce\#1^3$$

$$+e^2\#1^4\&,\frac{\log\left(-\sqrt{b}x^2+\sqrt{a+bx^4}-\#1\right)\#1}{-a\sqrt{b}d^2+2a\sqrt{b}ce+4bc^2\#1-2ae^2\#1+3\sqrt{b}d^2\#1^2-6\sqrt{b}ce\#1^2+2e^2\#1^3}\&\Big]$$

input
```
Integrate[1/((c + d*x + e*x^2)*Sqrt[a + b*x^4]),x]
```

output
```
((-1/2*I)*Sqrt[1 + (b*x^4)/a]*((-d^2 + Sqrt[d^4 - 4*c*d^2*e])*EllipticPi[(
(2*I)*Sqrt[a]*e^2)/(Sqrt[b]*(d^2 - 2*c*e + Sqrt[d^4 - 4*c*d^2*e])), I*ArcS
inh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*x], -1] + (d^2 + Sqrt[d^4 - 4*c*d^2*e])*Elli
pticPi[((-2*I)*Sqrt[a]*e^2)/(Sqrt[b]*(-d^2 + 2*c*e + Sqrt[d^4 - 4*c*d^2*e]
)), I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*x], -1]))/(Sqrt[(I*Sqrt[b])/Sqrt[a
]]*c*Sqrt[d^4 - 4*c*d^2*e]*Sqrt[a + b*x^4]) + Sqrt[b]*d*RootSum[a^2*e^2 -
2*a*Sqrt[b]*d^2*#1 + 4*a*Sqrt[b]*c*e*#1 + 4*b*c^2*#1^2 - 2*a*e^2*#1^2 + 2*
Sqrt[b]*d^2*#1^3 - 4*Sqrt[b]*c*e*#1^3 + e^2*#1^4 & , (Log[-(Sqrt[b]*x^2) +
 Sqrt[a + b*x^4] - #1]*#1)/(-(a*Sqrt[b]*d^2) + 2*a*Sqrt[b]*c*e + 4*b*c^2*#
1 - 2*a*e^2*#1 + 3*Sqrt[b]*d^2*#1^2 - 6*Sqrt[b]*c*e*#1^2 + 2*e^2*#1^3) & ]
```

Rubi [A] (warning: unable to verify)

Time = 20.67 (sec) , antiderivative size = 1590, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {7279, 7239, 7279, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+bx^4}\,(c+dx+ex^2)}dx$$

$$\Big\downarrow \text{7279}$$

$$\int \left(\frac{2e}{\sqrt{a+bx^4}\sqrt{d^2-4ce}\left(-\sqrt{d^2-4ce}+d+2ex\right)}-\frac{2e}{\sqrt{a+bx^4}\sqrt{d^2-4ce}\left(\sqrt{d^2-4ce}+d+2ex\right)}\right)dx$$

$$\Big\downarrow \text{7239}$$

$$\int \frac{1}{\sqrt{a+bx^4}\,(c+dx+ex^2)}dx$$

$$\Big\downarrow \text{7279}$$

$$\int \left(\frac{2e}{\sqrt{a+bx^4}\sqrt{d^2-4ce}\left(-\sqrt{d^2-4ce}+d+2ex\right)}-\frac{2e}{\sqrt{a+bx^4}\sqrt{d^2-4ce}\left(\sqrt{d^2-4ce}+d+2ex\right)}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{bd^4-4bced^2-b\sqrt{d^2-4ce}(d^2-2ce)d+2ae^4+2bc^2e^2}\,x}{e\left(d-\sqrt{d^2-4ce}\right)\sqrt{bx^4+a}}\right)e^2}{\sqrt{2}\sqrt{d^2-4ce}\sqrt{2ae^4+b\left(d^4-\sqrt{d^2-4ce}\,d^3-4ced^2+2ce\sqrt{d^2-4ce}\,d+2c^2e^2\right)}}-$$

$$\frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{bd^4-4bced^2+b\sqrt{d^2-4ce}(d^2-2ce)d+2ae^4+2bc^2e^2}\,x}{e\left(d+\sqrt{d^2-4ce}\right)\sqrt{bx^4+a}}\right)e^2}{\sqrt{2}\sqrt{d^2-4ce}\sqrt{2ae^4+b\left(d^4+\sqrt{d^2-4ce}\,d^3-4ced^2-2ce\sqrt{d^2-4ce}\,d+2c^2e^2\right)}}-$$

$$\frac{\operatorname{arctanh}\left(\frac{4ae^2+b\left(d-\sqrt{d^2-4ce}\right)^2x^2}{2\sqrt{2}\sqrt{bd^4-4bced^2-b\sqrt{d^2-4ce}(d^2-2ce)d+2ae^4+2bc^2e^2}\sqrt{bx^4+a}}\right)e^2}{\sqrt{2}\sqrt{d^2-4ce}\sqrt{bd^4-4bced^2-b\sqrt{d^2-4ce}\left(d^2-2ce\right)d+2ae^4+2bc^2e^2}}+$$

$$\frac{\operatorname{arctanh}\left(\frac{4ae^2+b\left(d+\sqrt{d^2-4ce}\right)^2x^2}{2\sqrt{2}\sqrt{bd^4-4bced^2+b\sqrt{d^2-4ce}(d^2-2ce)d+2ae^4+2bc^2e^2}\sqrt{bx^4+a}}\right)e^2}{\sqrt{2}\sqrt{d^2-4ce}\sqrt{bd^4-4bced^2+b\sqrt{d^2-4ce}\left(d^2-2ce\right)d+2ae^4+2bc^2e^2}}+$$

$$\frac{\sqrt[4]{b}\left(d-\sqrt{d^2-4ce}\right)\left(\sqrt{b}x^2+\sqrt{a}\right)\sqrt{\frac{bx^4+a}{\left(\sqrt{b}x^2+\sqrt{a}\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)e}{2\sqrt[4]{a}\sqrt{d^2-4ce}\left(2\sqrt{a}e^2+\sqrt{b}\left(d^2-\sqrt{d^2-4ce}\,d-2ce\right)\right)\sqrt{bx^4+a}}-$$

$$\frac{\sqrt[4]{b}\left(d+\sqrt{d^2-4ce}\right)\left(\sqrt{b}x^2+\sqrt{a}\right)\sqrt{\frac{bx^4+a}{\left(\sqrt{b}x^2+\sqrt{a}\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)e}{2\sqrt[4]{a}\sqrt{d^2-4ce}\left(2\sqrt{a}e^2+\sqrt{b}\left(d^2+\sqrt{d^2-4ce}\,d-2ce\right)\right)\sqrt{bx^4+a}}+$$

$$\frac{\left(2\sqrt{a}e^2-\sqrt{b}\left(d^2-\sqrt{d^2-4ce}\,d-2ce\right)\right)\left(\sqrt{b}x^2+\sqrt{a}\right)\sqrt{\frac{bx^4+a}{\left(\sqrt{b}x^2+\sqrt{a}\right)^2}}\operatorname{EllipticPi}\left(\frac{\sqrt{a}\left(4e^2+\frac{\sqrt{b}\left(d-\sqrt{d^2-4ce}\right)^2}{\sqrt{a}}\right)^2}{16\sqrt{b}e^2\left(d-\sqrt{d^2-4ce}\right)^2},2\,\text{ar}\ldots\right.}{2\sqrt[4]{a}\sqrt[4]{b}\sqrt{d^2-4ce}\left(d-\sqrt{d^2-4ce}\right)\left(2\sqrt{a}e^2+\sqrt{b}\left(d^2-\sqrt{d^2-4ce}\,d-2ce\right)\right)\sqrt{bx^4+a}}$$

$$\frac{\left(2\sqrt{a}e^2-\sqrt{b}\left(d^2+\sqrt{d^2-4ce}\,d-2ce\right)\right)\left(\sqrt{b}x^2+\sqrt{a}\right)\sqrt{\frac{bx^4+a}{\left(\sqrt{b}x^2+\sqrt{a}\right)^2}}\operatorname{EllipticPi}\left(\frac{\sqrt{a}\left(4e^2+\frac{\sqrt{b}\left(d+\sqrt{d^2-4ce}\right)^2}{\sqrt{a}}\right)^2}{16\sqrt{b}e^2\left(d+\sqrt{d^2-4ce}\right)^2},2\,\text{ar}\ldots\right.}{2\sqrt[4]{a}\sqrt[4]{b}\sqrt{d^2-4ce}\left(d+\sqrt{d^2-4ce}\right)\left(2\sqrt{a}e^2+\sqrt{b}\left(d^2+\sqrt{d^2-4ce}\,d-2ce\right)\right)\sqrt{bx^4+a}}$$

input
```
Int[1/((c + d*x + e*x^2)*Sqrt[a + b*x^4]),x]
```

output
```
(e^2*ArcTanh[(Sqrt[2]*Sqrt[b*d^4 - 4*b*c*d^2*e + 2*b*c^2*e^2 + 2*a*e^4 - b
*d*Sqrt[d^2 - 4*c*e]*(d^2 - 2*c*e)]*x)/(e*(d - Sqrt[d^2 - 4*c*e])*Sqrt[a +
 b*x^4])])/(Sqrt[2]*Sqrt[d^2 - 4*c*e]*Sqrt[2*a*e^4 + b*(d^4 - 4*c*d^2*e +
2*c^2*e^2 - d^3*Sqrt[d^2 - 4*c*e] + 2*c*d*e*Sqrt[d^2 - 4*c*e])]) - (e^2*Ar
cTanh[(Sqrt[2]*Sqrt[b*d^4 - 4*b*c*d^2*e + 2*b*c^2*e^2 + 2*a*e^4 + b*d*Sqrt
[d^2 - 4*c*e]*(d^2 - 2*c*e)]*x)/(e*(d + Sqrt[d^2 - 4*c*e])*Sqrt[a + b*x^4]
)])/(Sqrt[2]*Sqrt[d^2 - 4*c*e]*Sqrt[2*a*e^4 + b*(d^4 - 4*c*d^2*e + 2*c^2*e
^2 + d^3*Sqrt[d^2 - 4*c*e] - 2*c*d*e*Sqrt[d^2 - 4*c*e])]) - (e^2*ArcTanh[(
4*a*e^2 + b*(d - Sqrt[d^2 - 4*c*e])^2*x^2)/(2*Sqrt[2]*Sqrt[b*d^4 - 4*b*c*d
^2*e + 2*b*c^2*e^2 + 2*a*e^4 - b*d*Sqrt[d^2 - 4*c*e]*(d^2 - 2*c*e)]*Sqrt[a
 + b*x^4])])/(Sqrt[2]*Sqrt[d^2 - 4*c*e]*Sqrt[b*d^4 - 4*b*c*d^2*e + 2*b*c^2
*e^2 + 2*a*e^4 - b*d*Sqrt[d^2 - 4*c*e]*(d^2 - 2*c*e)]) + (e^2*ArcTanh[(4*a
*e^2 + b*(d + Sqrt[d^2 - 4*c*e])^2*x^2)/(2*Sqrt[2]*Sqrt[b*d^4 - 4*b*c*d^2*
e + 2*b*c^2*e^2 + 2*a*e^4 + b*d*Sqrt[d^2 - 4*c*e]*(d^2 - 2*c*e)]*Sqrt[a +
b*x^4])])/(Sqrt[2]*Sqrt[d^2 - 4*c*e]*Sqrt[b*d^4 - 4*b*c*d^2*e + 2*b*c^2*e^
2 + 2*a*e^4 + b*d*Sqrt[d^2 - 4*c*e]*(d^2 - 2*c*e)]) + (b^(1/4)*e*(d - Sqrt
[d^2 - 4*c*e])*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]
*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*Sqrt[d^
2 - 4*c*e]*(2*Sqrt[a]*e^2 + Sqrt[b]*(d^2 - 2*c*e - d*Sqrt[d^2 - 4*c*e]))*S
qrt[a + b*x^4]) - (b^(1/4)*e*(d + Sqrt[d^2 - 4*c*e])*(Sqrt[a] + Sqrt[b]...
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7239
```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

rule 7279
```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.77 (sec) , antiderivative size = 1153, normalized size of antiderivative = 0.71

method	result	size
default	Expression too large to display	1153
elliptic	Expression too large to display	1153

input

```
int(1/(e*x^2+d*x+c)/(b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
-1/2/(-4*c*e+d^2)^(1/2)/(1/2*b/e^4*d^4-1/2*b/e^4*d^3*(-4*c*e+d^2)^(1/2)-2*
b/e^3*d^2*c+b/e^3*c*d*(-4*c*e+d^2)^(1/2)+b/e^2*c^2+a)^(1/2)*arctanh(1/2/(1
/2*b/e^4*d^4-1/2*b/e^4*d^3*(-4*c*e+d^2)^(1/2)-2*b/e^3*d^2*c+b/e^3*c*d*(-4*
c*e+d^2)^(1/2)+b/e^2*c^2+a)^(1/2)/(b*x^4+a)^(1/2)*b*x^2/e^2*d^2-1/2/(1/2*b
/e^4*d^4-1/2*b/e^4*d^3*(-4*c*e+d^2)^(1/2)-2*b/e^3*d^2*c+b/e^3*c*d*(-4*c*e+
d^2)^(1/2)+b/e^2*c^2+a)^(1/2)/(b*x^4+a)^(1/2)*b*x^2/e^2*d*(-4*c*e+d^2)^(1/
2)-1/(1/2*b/e^4*d^4-1/2*b/e^4*d^3*(-4*c*e+d^2)^(1/2)-2*b/e^3*d^2*c+b/e^3*c
*d*(-4*c*e+d^2)^(1/2)+b/e^2*c^2+a)^(1/2)/(b*x^4+a)^(1/2)*b*x^2/e*c+1/(1/2*
b/e^4*d^4-1/2*b/e^4*d^3*(-4*c*e+d^2)^(1/2)-2*b/e^3*d^2*c+b/e^3*c*d*(-4*c*e
+d^2)^(1/2)+b/e^2*c^2+a)^(1/2)/(b*x^4+a)^(1/2)*a)-2/(-4*c*e+d^2)^(1/2)/(I*
b^(1/2)/a^(1/2))^(1/2)*e/(-d+(-4*c*e+d^2)^(1/2))*(1-I*b^(1/2)/a^(1/2)*x^2)
^(1/2)*(1+I*b^(1/2)/a^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticPi(x*(I*b^(
1/2)/a^(1/2))^(1/2),-4*I/b^(1/2)*a^(1/2)*e^2/(-d+(-4*c*e+d^2)^(1/2))^2,(-I
*b^(1/2)/a^(1/2))^(1/2)/(I*b^(1/2)/a^(1/2))^(1/2))+1/2/(-4*c*e+d^2)^(1/2)/
(1/2*b/e^4*d^4+1/2*b/e^4*d^3*(-4*c*e+d^2)^(1/2)-2*b/e^3*d^2*c-b/e^3*c*d*(-
4*c*e+d^2)^(1/2)+b/e^2*c^2+a)^(1/2)*arctanh(1/2/(1/2*b/e^4*d^4+1/2*b/e^4*d
^3*(-4*c*e+d^2)^(1/2)-2*b/e^3*d^2*c-b/e^3*c*d*(-4*c*e+d^2)^(1/2)+b/e^2*c^2
+a)^(1/2)/(b*x^4+a)^(1/2)*b*x^2/e^2*d^2+1/2/(1/2*b/e^4*d^4+1/2*b/e^4*d^3*(
-4*c*e+d^2)^(1/2)-2*b/e^3*d^2*c-b/e^3*c*d*(-4*c*e+d^2)^(1/2)+b/e^2*c^2+a)^
(1/2)/(b*x^4+a)^(1/2)*b*x^2/e^2*d*(-4*c*e+d^2)^(1/2)-1/(1/2*b/e^4*d^4+1...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx+ex^2)\sqrt{a+bx^4}}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(c+dx+ex^2)\sqrt{a+bx^4}}\,dx = \int \frac{1}{\sqrt{a+bx^4}\,(c+dx+ex^2)}\,dx$$

input
```
integrate(1/(e*x**2+d*x+c)/(b*x**4+a)**(1/2),x)
```

output
```
Integral(1/(sqrt(a + b*x**4)*(c + d*x + e*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{(c+dx+ex^2)\sqrt{a+bx^4}}\,dx = \int \frac{1}{\sqrt{bx^4+a}(ex^2+dx+c)}\,dx$$

input
```
integrate(1/(e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*x^4 + a)*(e*x^2 + d*x + c)), x)
```

Giac [F]

$$\int \frac{1}{(c+dx+ex^2)\sqrt{a+bx^4}}\,dx = \int \frac{1}{\sqrt{bx^4+a}(ex^2+dx+c)}\,dx$$

input
```
integrate(1/(e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(b*x^4 + a)*(e*x^2 + d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx+ex^2)\sqrt{a+bx^4}}\,dx = \int \frac{1}{\sqrt{b\,x^4+a}\,\left(e\,x^2+d\,x+c\right)}\,dx$$

input
```
int(1/((a + b*x^4)^(1/2)*(c + d*x + e*x^2)),x)
```

output
```
int(1/((a + b*x^4)^(1/2)*(c + d*x + e*x^2)), x)
```

Reduce [F]

$$\int \frac{1}{(c+dx+ex^2)\sqrt{a+bx^4}}\,dx = \int \frac{\sqrt{b\,x^4+a}}{be\,x^6+bd\,x^5+bc\,x^4+ae\,x^2+adx+ac}dx$$

input
```
int(1/(e*x^2+d*x+c)/(b*x^4+a)^(1/2),x)
```

output
```
int(sqrt(a + b*x**4)/(a*c + a*d*x + a*e*x**2 + b*c*x**4 + b*d*x**5 + b*e*x
**6),x)
```

3.282 $\int \frac{\sqrt{a+bx^2+cx^4}}{ad-cdx^4} dx$

Optimal result

Integrand size = 30, antiderivative size = 145

$$\int \frac{\sqrt{a+bx^2+cx^4}}{ad-cdx^4} dx = -\frac{\sqrt{b-2\sqrt{a}\sqrt{c}}\text{arctanh}\left(\frac{\sqrt{b-2\sqrt{a}\sqrt{c}}x}{\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{a}\sqrt{c}d} + \frac{\sqrt{b+2\sqrt{a}\sqrt{c}}\text{arctanh}\left(\frac{\sqrt{b+2\sqrt{a}\sqrt{c}}x}{\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{a}\sqrt{c}d}$$

output
```
-1/4*(b-2*a^(1/2)*c^(1/2))^(1/2)*arctanh((b-2*a^(1/2)*c^(1/2))^(1/2)*x/(c*
x^4+b*x^2+a)^(1/2))/a^(1/2)/c^(1/2)/d+1/4*(b+2*a^(1/2)*c^(1/2))^(1/2)*arct
anh((b+2*a^(1/2)*c^(1/2))^(1/2)*x/(c*x^4+b*x^2+a)^(1/2))/a^(1/2)/c^(1/2)/d
```

Mathematica [A] (verified)

Time = 0.88 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{a+bx^2+cx^4}}{ad-cdx^4} dx = \frac{-\sqrt{-b-2\sqrt{a}\sqrt{c}}\arctan\left(\frac{\sqrt{-b-2\sqrt{a}\sqrt{c}}x}{\sqrt{a+bx^2+cx^4}}\right) + \sqrt{-b+2\sqrt{a}\sqrt{c}}\arctan\left(\frac{\sqrt{-b+2\sqrt{a}\sqrt{c}}x}{\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{a}\sqrt{c}d}$$

input
```
Integrate[Sqrt[a + b*x^2 + c*x^4]/(a*d - c*d*x^4),x]
```

output
```
(-(Sqrt[-b - 2*Sqrt[a]*Sqrt[c]]*ArcTan[(Sqrt[-b - 2*Sqrt[a]*Sqrt[c]]*x)/Sq
rt[a + b*x^2 + c*x^4]]) + Sqrt[-b + 2*Sqrt[a]*Sqrt[c]]*ArcTan[(Sqrt[-b + 2
*Sqrt[a]*Sqrt[c]]*x)/Sqrt[a + b*x^2 + c*x^4]])/(4*Sqrt[a]*Sqrt[c]*d)
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.32, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2517, 1406, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2+cx^4}}{ad-cdx^4} \, dx$$

$$\Big\downarrow \ 2517$$

$$\frac{\int \frac{1}{\frac{(b^2-4ac)x^4}{(cx^4+bx^2+a)^2}-\frac{2bx^2}{cx^4+bx^2+a}+1}d\frac{x}{\sqrt{cx^4+bx^2+a}}}{d}$$

$$\Big\downarrow \ 1406$$

$$\frac{\frac{(b^2-4ac)\int \frac{1}{\frac{(b^2-4ac)x^2}{cx^4+bx^2+a}-b-2\sqrt{a}\sqrt{c}}d\frac{x}{\sqrt{cx^4+bx^2+a}}}{4\sqrt{a}\sqrt{c}}-\frac{(b^2-4ac)\int \frac{1}{\frac{(b^2-4ac)x^2}{cx^4+bx^2+a}-b+2\sqrt{a}\sqrt{c}}d\frac{x}{\sqrt{cx^4+bx^2+a}}}{4\sqrt{a}\sqrt{c}}}{d}$$

$$\Big\downarrow \ 221$$

$$\frac{\frac{(b^2-4ac)\text{arctanh}\left(\frac{x\sqrt{2\sqrt{a}\sqrt{c}+b}}{\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{a}\sqrt{c}(b-2\sqrt{a}\sqrt{c})\sqrt{2\sqrt{a}\sqrt{c}+b}}-\frac{(b^2-4ac)\text{arctanh}\left(\frac{x\sqrt{b-2\sqrt{a}\sqrt{c}}}{\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{a}\sqrt{c}\sqrt{b-2\sqrt{a}\sqrt{c}}(2\sqrt{a}\sqrt{c}+b)}}{d}$$

input
```
Int[Sqrt[a + b*x^2 + c*x^4]/(a*d - c*d*x^4),x]
```

output
```
(-1/4*((b^2 - 4*a*c)*ArcTanh[(Sqrt[b - 2*Sqrt[a]*Sqrt[c]]*x)/Sqrt[a + b*x^
2 + c*x^4]])/(Sqrt[a]*Sqrt[b - 2*Sqrt[a]*Sqrt[c]]*(b + 2*Sqrt[a]*Sqrt[c])*
Sqrt[c]) + ((b^2 - 4*a*c)*ArcTanh[(Sqrt[b + 2*Sqrt[a]*Sqrt[c]]*x)/Sqrt[a +
 b*x^2 + c*x^4]])/(4*Sqrt[a]*(b - 2*Sqrt[a]*Sqrt[c])*Sqrt[b + 2*Sqrt[a]*Sq
rt[c]]*Sqrt[c]))/d
```

Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1406
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Simp[c/q   Int[1/(b/2 - q/2 + c*x^2), x], x] - Simp[c/q   I
nt[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c
, 0] && PosQ[b^2 - 4*a*c]
```

rule 2517
```
Int[Sqrt[v_]/((d_) + (e_.)*(x_)^4), x_Symbol] :> With[{a = Coeff[v, x, 0],
b = Coeff[v, x, 2], c = Coeff[v, x, 4]}, Simp[a/d   Subst[Int[1/(1 - 2*b*x^
2 + (b^2 - 4*a*c)*x^4), x], x, x/Sqrt[v]], x] /; EqQ[c*d + a*e, 0] && PosQ[
a*c]] /; FreeQ[{d, e}, x] && PolyQ[v, x^2, 2]
```

Maple [A] (verified)

Time = 1.09 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.72

method	result	size
pseudoelliptic	$\frac{\sqrt{-2\sqrt{ac}-b}\,\arctan\left(\frac{\sqrt{c\,x^{4}+b\,x^{2}+a}}{x\sqrt{-2\sqrt{ac}-b}}\right)-\sqrt{2\sqrt{ac}-b}\,\arctan\left(\frac{\sqrt{c\,x^{4}+b\,x^{2}+a}}{x\sqrt{2\sqrt{ac}-b}}\right)}{4\sqrt{ac}\,d}$	105
default	$\frac{\left(-\frac{\left(2\sqrt{ac}-b\right)\arctan\left(\frac{\sqrt{c\,x^{4}+b\,x^{2}+a}\,\sqrt{2}}{x\sqrt{4\sqrt{ac}-2b}}\right)}{2\sqrt{ac}\,\sqrt{4\sqrt{ac}-2b}}-\frac{\left(2\sqrt{ac}+b\right)\arctan\left(\frac{\sqrt{c\,x^{4}+b\,x^{2}+a}\,\sqrt{2}}{x\sqrt{-4\sqrt{ac}-2b}}\right)}{2\sqrt{ac}\,\sqrt{-4\sqrt{ac}-2b}}\right)\sqrt{2}}{2d}$	140
elliptic	$\frac{2\left(-\frac{\left(2\sqrt{ac}-b\right)\arctan\left(\frac{\sqrt{c\,x^{4}+b\,x^{2}+a}\,\sqrt{2}}{x\sqrt{4\sqrt{ac}-2b}}\right)}{8\sqrt{ac}\,\sqrt{4\sqrt{ac}-2b}}-\frac{\left(2\sqrt{ac}+b\right)\arctan\left(\frac{\sqrt{c\,x^{4}+b\,x^{2}+a}\,\sqrt{2}}{x\sqrt{-4\sqrt{ac}-2b}}\right)}{8\sqrt{ac}\,\sqrt{-4\sqrt{ac}-2b}}\right)\sqrt{2}}{d}$	140

input
```
int((c*x^4+b*x^2+a)^(1/2)/(-c*d*x^4+a*d),x,method=_RETURNVERBOSE)
```

output
```
1/4*((-2*(a*c)^(1/2)-b)^(1/2)*arctan(1/x*(c*x^4+b*x^2+a)^(1/2)/(-2*(a*c)^(
1/2)-b)^(1/2))-(2*(a*c)^(1/2)-b)^(1/2)*arctan(1/x*(c*x^4+b*x^2+a)^(1/2)/(2
*(a*c)^(1/2)-b)^(1/2)))/(a*c)^(1/2)/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 603 vs. $2(105) = 210$.

Time = 1.38 (sec) , antiderivative size = 603, normalized size of antiderivative = 4.16

$$\int \frac{\sqrt{a+bx^2+cx^4}}{ad-cdx^4}\,dx$$
$$= \frac{1}{8}\sqrt{\frac{2\,acd^2\sqrt{\frac{1}{acd^4}}+b}{acd^2}}\log\left(\frac{\sqrt{cx^4+bx^2+a}\left(ad^2\sqrt{\frac{1}{acd^4}}+x^2\right)+\left(acd^3x^3\sqrt{\frac{1}{acd^4}}+adx\right)\sqrt{\frac{2\,acd^2\sqrt{\frac{1}{acd^4}}+b}{acd^2}}}{cx^4-a}\right)$$
$$-\frac{1}{8}\sqrt{\frac{2\,acd^2\sqrt{\frac{1}{acd^4}}+b}{acd^2}}\log\left(\frac{\sqrt{cx^4+bx^2+a}\left(ad^2\sqrt{\frac{1}{acd^4}}+x^2\right)-\left(acd^3x^3\sqrt{\frac{1}{acd^4}}+adx\right)\sqrt{\frac{2\,acd^2\sqrt{\frac{1}{acd^4}}+b}{acd^2}}}{cx^4-a}\right)$$
$$+\frac{1}{8}\sqrt{-\frac{2\,acd^2\sqrt{\frac{1}{acd^4}}-b}{acd^2}}\log\left(-\frac{\sqrt{cx^4+bx^2+a}\left(ad^2\sqrt{\frac{1}{acd^4}}-x^2\right)+\left(acd^3x^3\sqrt{\frac{1}{acd^4}}-adx\right)\sqrt{-\frac{2\,acd^2\sqrt{\frac{1}{acd^4}}-b}{acd^2}}}{cx^4-a}\right)$$
$$-\frac{1}{8}\sqrt{-\frac{2\,acd^2\sqrt{\frac{1}{acd^4}}-b}{acd^2}}\log\left(-\frac{\sqrt{cx^4+bx^2+a}\left(ad^2\sqrt{\frac{1}{acd^4}}-x^2\right)-\left(acd^3x^3\sqrt{\frac{1}{acd^4}}-adx\right)\sqrt{-\frac{2\,acd^2\sqrt{\frac{1}{acd^4}}-b}{acd^2}}}{cx^4-a}\right)$$

input
```
integrate((c*x^4+b*x^2+a)^(1/2)/(-c*d*x^4+a*d),x, algorithm="fricas")
```

output
```
1/8*sqrt((2*a*c*d^2*sqrt(1/(a*c*d^4)) + b)/(a*c*d^2))*log((sqrt(c*x^4 + b*
x^2 + a)*(a*d^2*sqrt(1/(a*c*d^4)) + x^2) + (a*c*d^3*x^3*sqrt(1/(a*c*d^4))
+ a*d*x)*sqrt((2*a*c*d^2*sqrt(1/(a*c*d^4)) + b)/(a*c*d^2)))/(c*x^4 - a)) -
 1/8*sqrt((2*a*c*d^2*sqrt(1/(a*c*d^4)) + b)/(a*c*d^2))*log((sqrt(c*x^4 + b
*x^2 + a)*(a*d^2*sqrt(1/(a*c*d^4)) + x^2) - (a*c*d^3*x^3*sqrt(1/(a*c*d^4))
 + a*d*x)*sqrt((2*a*c*d^2*sqrt(1/(a*c*d^4)) + b)/(a*c*d^2)))/(c*x^4 - a))
+ 1/8*sqrt(-(2*a*c*d^2*sqrt(1/(a*c*d^4)) - b)/(a*c*d^2))*log(-(sqrt(c*x^4
+ b*x^2 + a)*(a*d^2*sqrt(1/(a*c*d^4)) - x^2) + (a*c*d^3*x^3*sqrt(1/(a*c*d^
4)) - a*d*x)*sqrt(-(2*a*c*d^2*sqrt(1/(a*c*d^4)) - b)/(a*c*d^2)))/(c*x^4 -
a)) - 1/8*sqrt(-(2*a*c*d^2*sqrt(1/(a*c*d^4)) - b)/(a*c*d^2))*log(-(sqrt(c*
x^4 + b*x^2 + a)*(a*d^2*sqrt(1/(a*c*d^4)) - x^2) - (a*c*d^3*x^3*sqrt(1/(a*
c*d^4)) - a*d*x)*sqrt(-(2*a*c*d^2*sqrt(1/(a*c*d^4)) - b)/(a*c*d^2)))/(c*x^
4 - a))
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2+cx^4}}{ad-cdx^4}\,dx = -\frac{\int \frac{\sqrt{a+bx^2+cx^4}}{-a+cx^4}\,dx}{d}$$

input
```
integrate((c*x**4+b*x**2+a)**(1/2)/(-c*d*x**4+a*d),x)
```

output
```
-Integral(sqrt(a + b*x**2 + c*x**4)/(-a + c*x**4), x)/d
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2+cx^4}}{ad-cdx^4}\,dx = \int -\frac{\sqrt{cx^4+bx^2+a}}{cdx^4-ad}\,dx$$

input
```
integrate((c*x^4+b*x^2+a)^(1/2)/(-c*d*x^4+a*d),x, algorithm="maxima")
```

output
```
-integrate(sqrt(c*x^4 + b*x^2 + a)/(c*d*x^4 - a*d), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2+cx^4}}{ad-cdx^4}\,dx=\int -\frac{\sqrt{cx^4+bx^2+a}}{cdx^4-ad}\,dx$$

input
```
integrate((c*x^4+b*x^2+a)^(1/2)/(-c*d*x^4+a*d),x, algorithm="giac")
```

output
```
integrate(-sqrt(c*x^4 + b*x^2 + a)/(c*d*x^4 - a*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2+cx^4}}{ad-cdx^4}\,dx=\int \frac{\sqrt{c\,x^4+b\,x^2+a}}{a\,d-c\,d\,x^4}\,dx$$

input
```
int((a + b*x^2 + c*x^4)^(1/2)/(a*d - c*d*x^4),x)
```

output
```
int((a + b*x^2 + c*x^4)^(1/2)/(a*d - c*d*x^4), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2+cx^4}}{ad-cdx^4}\,dx=\frac{\int \frac{\sqrt{c\,x^4+b\,x^2+a}}{-c\,x^4+a}dx}{d}$$

input
```
int((c*x^4+b*x^2+a)^(1/2)/(-c*d*x^4+a*d),x)
```

output
```
int(sqrt(a + b*x**2 + c*x**4)/(a - c*x**4),x)/d
```

3.283 $\int \frac{\sqrt{a+bx^2-cx^4}}{ad+cdx^4}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 239

$$\int \frac{\sqrt{a+bx^2-cx^4}}{ad+cdx^4}\,dx = -\frac{\sqrt{b+\sqrt{b^2+4ac}}\arctan\left(\frac{\sqrt{b+\sqrt{b^2+4ac}}\,x\left(b-\sqrt{b^2+4ac}-2cx^2\right)}{2\sqrt{2}\sqrt{a}\sqrt{c}\sqrt{a+bx^2-cx^4}}\right)}{2\sqrt{2}\sqrt{a}\sqrt{c}d} + \frac{\sqrt{-b+\sqrt{b^2+4ac}}\,\text{arctanh}\left(\frac{\sqrt{-b+\sqrt{b^2+4ac}}\,x\left(b+\sqrt{b^2+4ac}-2cx^2\right)}{2\sqrt{2}\sqrt{a}\sqrt{c}\sqrt{a+bx^2-cx^4}}\right)}{2\sqrt{2}\sqrt{a}\sqrt{c}d}$$

output
```
-1/4*(b+(4*a*c+b^2)^(1/2))^(1/2)*arctan(1/4*(b+(4*a*c+b^2)^(1/2))^(1/2)*x*
(b-(4*a*c+b^2)^(1/2)-2*c*x^2)*2^(1/2)/a^(1/2)/c^(1/2)/(-c*x^4+b*x^2+a)^(1/
2))*2^(1/2)/a^(1/2)/c^(1/2)/d+1/4*(-b+(4*a*c+b^2)^(1/2))^(1/2)*arctanh(1/4
*(-b+(4*a*c+b^2)^(1/2))^(1/2)*x*(b+(4*a*c+b^2)^(1/2)-2*c*x^2)*2^(1/2)/a^(1
/2)/c^(1/2)/(-c*x^4+b*x^2+a)^(1/2))*2^(1/2)/a^(1/2)/c^(1/2)/d
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.84 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.63

$$\int \frac{\sqrt{a+bx^2-cx^4}}{ad+cdx^4}\,dx = \frac{i\left(\sqrt{-b-2i\sqrt{a}\sqrt{c}}\arctan\left(\frac{\sqrt{-b-2i\sqrt{a}\sqrt{c}}x}{\sqrt{a+bx^2-cx^4}}\right)-\sqrt{-b+2i\sqrt{a}\sqrt{c}}\arctan\left(\frac{\sqrt{-b+2i\sqrt{a}\sqrt{c}}x}{\sqrt{a+bx^2-cx^4}}\right)\right)}{4\sqrt{a}\sqrt{c}d}$$

input
```
Integrate[Sqrt[a + b*x^2 - c*x^4]/(a*d + c*d*x^4),x]
```

output
```
((I/4)*(Sqrt[-b - (2*I)*Sqrt[a]*Sqrt[c]]*ArcTan[(Sqrt[-b - (2*I)*Sqrt[a]*S
qrt[c]]*x)/Sqrt[a + b*x^2 - c*x^4]] - Sqrt[-b + (2*I)*Sqrt[a]*Sqrt[c]]*Arc
Tan[(Sqrt[-b + (2*I)*Sqrt[a]*Sqrt[c]]*x)/Sqrt[a + b*x^2 - c*x^4]]))/(Sqrt[
a]*Sqrt[c]*d)
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.033$, Rules used = {2518}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2-cx^4}}{ad+cdx^4}\,dx$$

$$\Big\downarrow \text{2518}$$

$$\frac{\sqrt{\sqrt{4ac+b^2}-b}\,\text{arctanh}\left(\frac{x\sqrt{\sqrt{4ac+b^2}-b}\left(\sqrt{4ac+b^2}+b-2cx^2\right)}{2\sqrt{2}\sqrt{a}\sqrt{c}\sqrt{a+bx^2-cx^4}}\right)}{2\sqrt{2}\sqrt{a}\sqrt{c}d}-\frac{\sqrt{\sqrt{4ac+b^2}+b}\arctan\left(\frac{x\sqrt{\sqrt{4ac+b^2}+b}\left(-\sqrt{4ac+b^2}+b-2cx^2\right)}{2\sqrt{2}\sqrt{a}\sqrt{c}\sqrt{a+bx^2-cx^4}}\right)}{2\sqrt{2}\sqrt{a}\sqrt{c}d}$$

input

```
Int[Sqrt[a + b*x^2 - c*x^4]/(a*d + c*d*x^4),x]
```

output

```
-1/2*(Sqrt[b + Sqrt[b^2 + 4*a*c]]*ArcTan[(Sqrt[b + Sqrt[b^2 + 4*a*c]]*x*(b
 - Sqrt[b^2 + 4*a*c] - 2*c*x^2))/(2*Sqrt[2]*Sqrt[a]*Sqrt[c]*Sqrt[a + b*x^2
 - c*x^4])])/(Sqrt[2]*Sqrt[a]*Sqrt[c]*d) + (Sqrt[-b + Sqrt[b^2 + 4*a*c]]*A
rcTanh[(Sqrt[-b + Sqrt[b^2 + 4*a*c]]*x*(b + Sqrt[b^2 + 4*a*c] - 2*c*x^2))/
(2*Sqrt[2]*Sqrt[a]*Sqrt[c]*Sqrt[a + b*x^2 - c*x^4])])/(2*Sqrt[2]*Sqrt[a]*S
qrt[c]*d)
```

Defintions of rubi rules used

rule 2518

```
Int[Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]/((d_) + (e_.)*(x_)^4), x_Symbo
l] :> With[{q = Sqrt[b^2 - 4*a*c]}, Simp[(-a)*(Sqrt[b + q]/(2*Sqrt[2]*Rt[(-
a)*c, 2]*d))*ArcTan[Sqrt[b + q]*x*((b - q + 2*c*x^2)/(2*Sqrt[2]*Rt[(-a)*c,
2]*Sqrt[a + b*x^2 + c*x^4]))], x] + Simp[a*(Sqrt[-b + q]/(2*Sqrt[2]*Rt[(-a)
*c, 2]*d))*ArcTanh[Sqrt[-b + q]*x*((b + q + 2*c*x^2)/(2*Sqrt[2]*Rt[(-a)*c,
2]*Sqrt[a + b*x^2 + c*x^4]))], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d
+ a*e, 0] && NegQ[a*c]
```

Maple [A] (verified)

Time = 1.84 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.47

method	result
pseudoelliptic	$\frac{\sqrt{2\sqrt{4ac+b^2}+2b}\left(b-\sqrt{4ac+b^2}\right)\left(\ln\left(\frac{c\,x^4+\sqrt{-c\,x^4+b\,x^2+a}\,\sqrt{2\sqrt{4ac+b^2}+2b}\,x-\sqrt{4ac+b^2}\,x^2-b\,x^2-a}{x^2}\right)-\ln\left(\frac{-c\,x^4+\sqrt{-c\,x^4+b\,x^2}\cdots}{\cdots}\right.\cdots}{\cdots}$
elliptic	$\frac{\left(\frac{\sqrt{b+\sqrt{4ac+b^2}}\,b\ln\left(\frac{\sqrt{-c\,x^4+b\,x^2+a}\,\sqrt{2}\,\sqrt{b+\sqrt{4ac+b^2}}}{x}-\frac{-c\,x^4+b\,x^2+a}{x^2}-\sqrt{4ac+b^2}\right)}{16dac}-\frac{b^2\arctan\left(\frac{2\sqrt{b+\sqrt{4ac+b^2}}-\frac{2\sqrt{-c\,x^4+b\,x^2}\cdots}{x}}{2\sqrt{-b+\sqrt{4ac+b^2}}}\right.\cdots}{8dac\sqrt{-b+\sqrt{4ac+b^2}}}\cdots\right.}{\cdots}$
default	$\frac{\left(\frac{\sqrt{b+\sqrt{4ac+b^2}}\,b\ln\left(\frac{\sqrt{-c\,x^4+b\,x^2+a}\,\sqrt{2}\,\sqrt{b+\sqrt{4ac+b^2}}}{x}-\frac{-c\,x^4+b\,x^2+a}{x^2}-\sqrt{4ac+b^2}\right)}{16ac}-\frac{\left(b+\sqrt{4ac+b^2}\right)b\arctan\left(\frac{2\sqrt{b+\sqrt{4ac+b^2}}-2\cdots}{2\sqrt{-b+\cdots}}\right.\cdots}{8ac\sqrt{-b+\sqrt{4ac+b^2}}}\cdots\right.}{\cdots}$

input
```
int((-c*x^4+b*x^2+a)^(1/2)/(c*d*x^4+a*d),x,method=_RETURNVERBOSE)
```

output
```
1/32/(2*(4*a*c+b^2)^(1/2)-2*b)^(1/2)*((2*(4*a*c+b^2)^(1/2)+2*b)^(1/2)*(b-(
4*a*c+b^2)^(1/2))*(ln((c*x^4+(-c*x^4+b*x^2+a)^(1/2)*(2*(4*a*c+b^2)^(1/2)+2
*b)^(1/2)*x-(4*a*c+b^2)^(1/2)*x^2-b*x^2-a)/x^2)-ln((-c*x^4+(-c*x^4+b*x^2+a
)^(1/2)*(2*(4*a*c+b^2)^(1/2)+2*b)^(1/2)*x+(4*a*c+b^2)^(1/2)*x^2+b*x^2+a)/x
^2))*(2*(4*a*c+b^2)^(1/2)-2*b)^(1/2)+16*a*c*(arctan(((2*(4*a*c+b^2)^(1/2)+
2*b)^(1/2)*x-2*(-c*x^4+b*x^2+a)^(1/2))/x/(2*(4*a*c+b^2)^(1/2)-2*b)^(1/2))-
arctan(((2*(4*a*c+b^2)^(1/2)+2*b)^(1/2)*x+2*(-c*x^4+b*x^2+a)^(1/2))/x/(2*(
4*a*c+b^2)^(1/2)-2*b)^(1/2))))/a/c/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 669 vs. $2(187) = 374$.

Time = 1.43 (sec) , antiderivative size = 669, normalized size of antiderivative = 2.80

$$\int \frac{\sqrt{a+bx^2-cx^4}}{ad+cdx^4}\,dx =$$

$$-\frac{1}{8}\sqrt{\frac{2\,acd^2\sqrt{-\frac{1}{acd^4}}-b}{acd^2}}\log\left(-\frac{\sqrt{-cx^4+bx^2+a}ad^2\sqrt{-\frac{1}{acd^4}}+\sqrt{-cx^4+bx^2+a}x^2+\left(acd^3x^3\sqrt{-\frac{1}{acd}}\right.}{cx^4+a}\right.$$

$$+\frac{1}{8}\sqrt{\frac{2\,acd^2\sqrt{-\frac{1}{acd^4}}-b}{acd^2}}\log\left(-\frac{\sqrt{-cx^4+bx^2+a}ad^2\sqrt{-\frac{1}{acd^4}}+\sqrt{-cx^4+bx^2+a}x^2-\left(acd^3x^3\sqrt{-\frac{1}{acd}}\right.}{cx^4+a}\right.$$

$$-\frac{1}{8}\sqrt{-\frac{2\,acd^2\sqrt{-\frac{1}{acd^4}}+b}{acd^2}}\log\left(\frac{\sqrt{-cx^4+bx^2+a}ad^2\sqrt{-\frac{1}{acd^4}}-\sqrt{-cx^4+bx^2+a}x^2+\left(acd^3x^3\sqrt{-\frac{1}{acd}}\right.}{cx^4+a}\right.$$

$$+\frac{1}{8}\sqrt{-\frac{2\,acd^2\sqrt{-\frac{1}{acd^4}}+b}{acd^2}}\log\left(\frac{\sqrt{-cx^4+bx^2+a}ad^2\sqrt{-\frac{1}{acd^4}}-\sqrt{-cx^4+bx^2+a}x^2-\left(acd^3x^3\sqrt{-\frac{1}{acd}}\right.}{cx^4+a}\right.$$

input
```
integrate((-c*x^4+b*x^2+a)^(1/2)/(c*d*x^4+a*d),x, algorithm="fricas")
```

output
```
-1/8*sqrt((2*a*c*d^2*sqrt(-1/(a*c*d^4)) - b)/(a*c*d^2))*log(-(sqrt(-c*x^4
+ b*x^2 + a)*a*d^2*sqrt(-1/(a*c*d^4)) + sqrt(-c*x^4 + b*x^2 + a)*x^2 + (a*
c*d^3*x^3*sqrt(-1/(a*c*d^4)) - a*d*x)*sqrt((2*a*c*d^2*sqrt(-1/(a*c*d^4)) -
 b)/(a*c*d^2)))/(c*x^4 + a)) + 1/8*sqrt((2*a*c*d^2*sqrt(-1/(a*c*d^4)) - b)
/(a*c*d^2))*log(-(sqrt(-c*x^4 + b*x^2 + a)*a*d^2*sqrt(-1/(a*c*d^4)) + sqrt
(-c*x^4 + b*x^2 + a)*x^2 - (a*c*d^3*x^3*sqrt(-1/(a*c*d^4)) - a*d*x)*sqrt((
2*a*c*d^2*sqrt(-1/(a*c*d^4)) - b)/(a*c*d^2)))/(c*x^4 + a)) - 1/8*sqrt(-(2*
a*c*d^2*sqrt(-1/(a*c*d^4)) + b)/(a*c*d^2))*log((sqrt(-c*x^4 + b*x^2 + a)*a
*d^2*sqrt(-1/(a*c*d^4)) - sqrt(-c*x^4 + b*x^2 + a)*x^2 + (a*c*d^3*x^3*sqrt
(-1/(a*c*d^4)) + a*d*x)*sqrt(-(2*a*c*d^2*sqrt(-1/(a*c*d^4)) + b)/(a*c*d^2)
))/(c*x^4 + a)) + 1/8*sqrt(-(2*a*c*d^2*sqrt(-1/(a*c*d^4)) + b)/(a*c*d^2))*
log((sqrt(-c*x^4 + b*x^2 + a)*a*d^2*sqrt(-1/(a*c*d^4)) - sqrt(-c*x^4 + b*x
^2 + a)*x^2 - (a*c*d^3*x^3*sqrt(-1/(a*c*d^4)) + a*d*x)*sqrt(-(2*a*c*d^2*sq
rt(-1/(a*c*d^4)) + b)/(a*c*d^2)))/(c*x^4 + a))
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2-cx^4}}{ad+cdx^4}\,dx = \frac{\int \frac{\sqrt{a+bx^2-cx^4}}{a+cx^4}\,dx}{d}$$

input
```
integrate((-c*x**4+b*x**2+a)**(1/2)/(c*d*x**4+a*d),x)
```

output
```
Integral(sqrt(a + b*x**2 - c*x**4)/(a + c*x**4), x)/d
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2-cx^4}}{ad+cdx^4}\,dx = \int \frac{\sqrt{-cx^4+bx^2+a}}{cdx^4+ad}\,dx$$

input
```
integrate((-c*x^4+b*x^2+a)^(1/2)/(c*d*x^4+a*d),x, algorithm="maxima")
```

output
```
integrate(sqrt(-c*x^4 + b*x^2 + a)/(c*d*x^4 + a*d), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2-cx^4}}{ad+cdx^4}\,dx=\int \frac{\sqrt{-cx^4+bx^2+a}}{cdx^4+ad}\,dx$$

input
```
integrate((-c*x^4+b*x^2+a)^(1/2)/(c*d*x^4+a*d),x, algorithm="giac")
```

output
```
integrate(sqrt(-c*x^4 + b*x^2 + a)/(c*d*x^4 + a*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2-cx^4}}{ad+cdx^4}\,dx=\int \frac{\sqrt{-c\,x^4+b\,x^2+a}}{c\,d\,x^4+a\,d}\,dx$$

input
```
int((a + b*x^2 - c*x^4)^(1/2)/(a*d + c*d*x^4),x)
```

output
```
int((a + b*x^2 - c*x^4)^(1/2)/(a*d + c*d*x^4), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2-cx^4}}{ad+cdx^4}\,dx=\frac{\int \frac{\sqrt{-c\,x^4+b\,x^2+a}}{c\,x^4+a}dx}{d}$$

input
```
int((-c*x^4+b*x^2+a)^(1/2)/(c*d*x^4+a*d),x)
```

output
```
int(sqrt(a + b*x**2 - c*x**4)/(a + c*x**4),x)/d
```

3.284 $\int (r+sx)^m \left(a+b(r+sx)^5\right)^p \, dx$

Optimal result

Integrand size = 21, antiderivative size = 80

$$\int (r+sx)^m \left(a+b(r+sx)^5\right)^p \, dx = \frac{(r+sx)^{1+m} \left(a+b(r+sx)^5\right)^p \left(1+\frac{b(r+sx)^5}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1+m}{5},-p,\frac{6+m}{5},-\frac{b(r+sx)^5}{a}\right)}{(1+m)s}$$

output
```
(s*x+r)^(1+m)*(a+b*(s*x+r)^5)^p*hypergeom([-p, 1/5+1/5*m],[6/5+1/5*m],-b*(
s*x+r)^5/a)/(1+m)/s/((1+b*(s*x+r)^5/a)^p)
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.02

$$\int (r+sx)^m \left(a+b(r+sx)^5\right)^p \, dx = \frac{(r+sx)^{1+m} \left(a+b(r+sx)^5\right)^p \left(1+\frac{b(r+sx)^5}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1+m}{5},-p,1+\frac{1+m}{5},-\frac{b(r+sx)^5}{a}\right)}{(1+m)s}$$

input
```
Integrate[(r + s*x)^m*(a + b*(r + s*x)^5)^p,x]
```

output
```
((r + s*x)^(1 + m)*(a + b*(r + s*x)^5)^p*Hypergeometric2F1[(1 + m)/5, -p,
1 + (1 + m)/5, -((b*(r + s*x)^5)/a)])/((1 + m)*s*(1 + (b*(r + s*x)^5)/a)^p
)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {895, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (r+sx)^m \left(a+b(r+sx)^5\right)^p dx$$

$$\Big\downarrow \text{895}$$

$$\frac{\int (r+sx)^m \left(b(r+sx)^5+a\right)^p d(r+sx)}{s}$$

$$\Big\downarrow \text{889}$$

$$\frac{\left(a+b(r+sx)^5\right)^p \left(\frac{b(r+sx)^5}{a}+1\right)^{-p} \int (r+sx)^m \left(\frac{b(r+sx)^5}{a}+1\right)^p d(r+sx)}{s}$$

$$\Big\downarrow \text{888}$$

$$\frac{(r+sx)^{m+1} \left(a+b(r+sx)^5\right)^p \left(\frac{b(r+sx)^5}{a}+1\right)^{-p} \text{Hypergeometric2F1}\left(\frac{m+1}{5},-p,\frac{m+6}{5},-\frac{b(r+sx)^5}{a}\right)}{(m+1)s}$$

input
```
Int[(r + s*x)^m*(a + b*(r + s*x)^5)^p,x]
```

output
```
((r + s*x)^(1 + m)*(a + b*(r + s*x)^5)^p*Hypergeometric2F1[(1 + m)/5, -p,
(6 + m)/5, -((b*(r + s*x)^5)/a)])/((1 + m)*s*(1 + (b*(r + s*x)^5)/a)^p)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 895
```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Simp[u^m/(Coeff
icient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[
{a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

Maple [F]

$$\int (sx+r)^m \left(a+b(sx+r)^5\right)^p dx$$

input
```
int((s*x+r)^m*(a+b*(s*x+r)^5)^p,x)
```

output
```
int((s*x+r)^m*(a+b*(s*x+r)^5)^p,x)
```

Fricas [F]

$$\int (r+sx)^m \left(a+b(r+sx)^5\right)^p \, dx = \int \left((sx+r)^5 b+a\right)^p (sx+r)^m \, dx$$

input
```
integrate((s*x+r)^m*(a+b*(s*x+r)^5)^p,x, algorithm="fricas")
```

output
```
integral((b*s^5*x^5 + 5*b*r*s^4*x^4 + 10*b*r^2*s^3*x^3 + 10*b*r^3*s^2*x^2
+ 5*b*r^4*s*x + b*r^5 + a)^p*(s*x + r)^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int (r+sx)^m \left(a+b(r+sx)^5\right)^p \, dx = \text{Timed out}$$

input
```
integrate((s*x+r)**m*(a+b*(s*x+r)**5)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (r+sx)^m \left(a+b(r+sx)^5\right)^p \, dx = \int \left((sx+r)^5 b+a\right)^p (sx+r)^m \, dx$$

input
```
integrate((s*x+r)^m*(a+b*(s*x+r)^5)^p,x, algorithm="maxima")
```

output
```
integrate(((s*x + r)^5*b + a)^p*(s*x + r)^m, x)
```

Giac [F]

$$\int (r+sx)^m \left(a+b(r+sx)^5\right)^p \, dx = \int \left((sx+r)^5 b+a\right)^p (sx+r)^m \, dx$$

input
```
integrate((s*x+r)^m*(a+b*(s*x+r)^5)^p,x, algorithm="giac")
```

output
```
integrate(((s*x + r)^5*b + a)^p*(s*x + r)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (r+sx)^m \left(a+b(r+sx)^5\right)^p \, dx = \int (r+s\,x)^m \left(a+b\,(r+s\,x)^5\right)^p \, dx$$

input
```
int((r + s*x)^m*(a + b*(r + s*x)^5)^p,x)
```

output
```
int((r + s*x)^m*(a + b*(r + s*x)^5)^p, x)
```

Reduce [F]

$$\int (r+sx)^m \left(a+b(r+sx)^5\right)^p \, dx = \text{too large to display}$$

input
```
int((s*x+r)^m*(a+b*(s*x+r)^5)^p,x)
```

output

```
((r + s*x)**m*(a + b*r**5 + 5*b*r**4*s*x + 10*b*r**3*s**2*x**2 + 10*b*r**2
*s**3*x**3 + 5*b*r*s**4*x**4 + b*s**5*x**5)**p*a*p + (r + s*x)**m*(a + b*r
**5 + 5*b*r**4*s*x + 10*b*r**3*s**2*x**2 + 10*b*r**2*s**3*x**3 + 5*b*r*s**
4*x**4 + b*s**5*x**5)**p*b*m*r**5 + (r + s*x)**m*(a + b*r**5 + 5*b*r**4*s*
x + 10*b*r**3*s**2*x**2 + 10*b*r**2*s**3*x**3 + 5*b*r*s**4*x**4 + b*s**5*x
**5)**p*b*m*r**4*s*x + 5*(r + s*x)**m*(a + b*r**5 + 5*b*r**4*s*x + 10*b*r*
*3*s**2*x**2 + 10*b*r**2*s**3*x**3 + 5*b*r*s**4*x**4 + b*s**5*x**5)**p*b*p
*r**5 + 5*(r + s*x)**m*(a + b*r**5 + 5*b*r**4*s*x + 10*b*r**3*s**2*x**2 +
10*b*r**2*s**3*x**3 + 5*b*r*s**4*x**4 + b*s**5*x**5)**p*b*p*r**4*s*x - int
(((r + s*x)**m*(a + b*r**5 + 5*b*r**4*s*x + 10*b*r**3*s**2*x**2 + 10*b*r**
2*s**3*x**3 + 5*b*r*s**4*x**4 + b*s**5*x**5)**p*x**5)/(a*m**2*r + a*m**2*s
*x + 10*a*m*p*r + 10*a*m*p*s*x + a*m*r + a*m*s*x + 25*a*p**2*r + 25*a*p**2
*s*x + 5*a*p*r + 5*a*p*s*x + b*m**2*r**6 + 6*b*m**2*r**5*s*x + 15*b*m**2*r
**4*s**2*x**2 + 20*b*m**2*r**3*s**3*x**3 + 15*b*m**2*r**2*s**4*x**4 + 6*b*
m**2*r*s**5*x**5 + b*m**2*s**6*x**6 + 10*b*m*p*r**6 + 60*b*m*p*r**5*s*x +
150*b*m*p*r**4*s**2*x**2 + 200*b*m*p*r**3*s**3*x**3 + 150*b*m*p*r**2*s**4*
x**4 + 60*b*m*p*r*s**5*x**5 + 10*b*m*p*s**6*x**6 + b*m*r**6 + 6*b*m*r**5*s
*x + 15*b*m*r**4*s**2*x**2 + 20*b*m*r**3*s**3*x**3 + 15*b*m*r**2*s**4*x**4
 + 6*b*m*r*s**5*x**5 + b*m*s**6*x**6 + 25*b*p**2*r**6 + 150*b*p**2*r**5*s*
x + 375*b*p**2*r**4*s**2*x**2 + 500*b*p**2*r**3*s**3*x**3 + 375*b*p**2*...
```

3.285 $\int (r+sx)^m (a+br^5+5br^4sx+10br^3s^2x^2+10br^2s^3$

Optimal result

Integrand size = 67, antiderivative size = 80

$$\int (r+sx)^m \left(a+br^5+5br^4sx+10br^3s^2x^2+10br^2s^3x^3+5brs^4x^4+bs^5x^5\right)^p\,dx = \frac{(r+sx)^{1+m}\left(a+b(r+sx)^5\right)^p\left(1+\frac{b(r+sx)^5}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1+m}{5},-p,\frac{6+m}{5},-\frac{b(r+sx)^5}{a}\right)}{(1+m)s}$$

output
```
(s*x+r)^(1+m)*(a+b*(s*x+r)^5)^p*hypergeom([-p, 1/5+1/5*m],[6/5+1/5*m],-b*(
s*x+r)^5/a)/(1+m)/s/((1+b*(s*x+r)^5/a)^p)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.02

$$\int (r+sx)^m \left(a+br^5+5br^4sx+10br^3s^2x^2+10br^2s^3x^3+5brs^4x^4+bs^5x^5\right)^p\,dx = \frac{(r+sx)^{1+m}\left(a+b(r+sx)^5\right)^p\left(1+\frac{b(r+sx)^5}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1+m}{5},-p,1+\frac{1+m}{5},-\frac{b(r+sx)^5}{a}\right)}{(1+m)s}$$

input
```
Integrate[(r + s*x)^m*(a + b*r^5 + 5*b*r^4*s*x + 10*b*r^3*s^2*x^2 + 10*b*r
^2*s^3*x^3 + 5*b*r*s^4*x^4 + b*s^5*x^5)^p,x]
```

output
```
((r + s*x)^(1 + m)*(a + b*(r + s*x)^5)^p*Hypergeometric2F1[(1 + m)/5, -p,
1 + (1 + m)/5, -((b*(r + s*x)^5)/a)])/((1 + m)*s*(1 + (b*(r + s*x)^5)/a)^p
)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {2509, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (r+sx)^m \left(a+br^5+5br^4sx+10br^3s^2x^2+10br^2s^3x^3+5brs^4x^4+bs^5x^5\right)^p dx$$

$$\Big\downarrow \text{2509}$$

$$\frac{\int (r+sx)^m \left(b(r+sx)^5+a\right)^p d(r+sx)}{s}$$

$$\Big\downarrow \text{889}$$

$$\frac{\left(a+b(r+sx)^5\right)^p \left(\frac{b(r+sx)^5}{a}+1\right)^{-p} \int (r+sx)^m \left(\frac{b(r+sx)^5}{a}+1\right)^p d(r+sx)}{s}$$

$$\Big\downarrow \text{888}$$

$$\frac{(r+sx)^{m+1} \left(a+b(r+sx)^5\right)^p \left(\frac{b(r+sx)^5}{a}+1\right)^{-p} \text{Hypergeometric2F1}\left(\frac{m+1}{5},-p,\frac{m+6}{5},-\frac{b(r+sx)^5}{a}\right)}{(m+1)s}$$

input
```
Int[(r + s*x)^m*(a + b*r^5 + 5*b*r^4*s*x + 10*b*r^3*s^2*x^2 + 10*b*r^2*s^3
*x^3 + 5*b*r*s^4*x^4 + b*s^5*x^5)^p,x]
```

output
```
((r + s*x)^(1 + m)*(a + b*(r + s*x)^5)^p*Hypergeometric2F1[(1 + m)/5, -p,
(6 + m)/5, -((b*(r + s*x)^5)/a)])/((1 + m)*s*(1 + (b*(r + s*x)^5)/a)^p)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 2509
```
Int[(Pn_)^(p_.)*((g_) + (h_.)*(x_))^(m_.), x_Symbol] :> With[{Px = Pn /. x
-> (x - g)/h}, Simp[1/h   Subst[Int[x^m*ExpandToSum[Px, x]^p, x], x, g + h*
x], x] /; BinomialQ[Px, x]] /; FreeQ[{g, h, m, p}, x] && PolyQ[Pn, x]
```

Maple [F]

$$\int (sx+r)^m \left(b\,s^5x^5+5br\,s^4x^4+10b\,r^2s^3x^3+10b\,r^3s^2x^2+5b\,r^4sx+b\,r^5+a\right)^p dx$$

input
```
int((s*x+r)^m*(b*s^5*x^5+5*b*r*s^4*x^4+10*b*r^2*s^3*x^3+10*b*r^3*s^2*x^2+5
*b*r^4*s*x+b*r^5+a)^p,x)
```

output
```
int((s*x+r)^m*(b*s^5*x^5+5*b*r*s^4*x^4+10*b*r^2*s^3*x^3+10*b*r^3*s^2*x^2+5
*b*r^4*s*x+b*r^5+a)^p,x)
```

Fricas [F]

$$\int (r+sx)^m\left(a+br^5+5br^4sx+10br^3s^2x^2+10br^2s^3x^3+5brs^4x^4+bs^5x^5\right)^p\,dx = \int \left(bs^5x^5+5\,brs^4x^4+10\,br^2s^3x^3+10\,br^3s^2x^2+5\,br^4sx+br^5+a\right)^p(sx+r)^m\,dx$$

input
```
integrate((s*x+r)^m*(b*s^5*x^5+5*b*r*s^4*x^4+10*b*r^2*s^3*x^3+10*b*r^3*s^2
*x^2+5*b*r^4*s*x+b*r^5+a)^p,x, algorithm="fricas")
```

output
```
integral((b*s^5*x^5 + 5*b*r*s^4*x^4 + 10*b*r^2*s^3*x^3 + 10*b*r^3*s^2*x^2
+ 5*b*r^4*s*x + b*r^5 + a)^p*(s*x + r)^m, x)
```

Sympy [F]

$$\int (r+sx)^m\left(a+br^5+5br^4sx+10br^3s^2x^2+10br^2s^3x^3+5brs^4x^4+bs^5x^5\right)^p\,dx = \int (r+sx)^m\left(a+br^5+5br^4sx+10br^3s^2x^2+10br^2s^3x^3+5brs^4x^4+bs^5x^5\right)^p\,dx$$

input
```
integrate((s*x+r)**m*(b*s**5*x**5+5*b*r*s**4*x**4+10*b*r**2*s**3*x**3+10*b
*r**3*s**2*x**2+5*b*r**4*s*x+b*r**5+a)**p,x)
```

output
```
Integral((r + s*x)**m*(a + b*r**5 + 5*b*r**4*s*x + 10*b*r**3*s**2*x**2 + 1
0*b*r**2*s**3*x**3 + 5*b*r*s**4*x**4 + b*s**5*x**5)**p, x)
```

Maxima [F]

$$\int (r+sx)^m \left(a+br^5+5br^4sx+10br^3s^2x^2+10br^2s^3x^3+5brs^4x^4+bs^5x^5\right)^p \, dx = \int \left(bs^5x^5+5\,brs^4x^4+10\,br^2s^3x^3+10\,br^3s^2x^2+5\,br^4sx+br^5+a\right)^p (sx+r)^m \, dx$$

input
```
integrate((s*x+r)^m*(b*s^5*x^5+5*b*r*s^4*x^4+10*b*r^2*s^3*x^3+10*b*r^3*s^2
*x^2+5*b*r^4*s*x+b*r^5+a)^p,x, algorithm="maxima")
```

output
```
integrate((b*s^5*x^5 + 5*b*r*s^4*x^4 + 10*b*r^2*s^3*x^3 + 10*b*r^3*s^2*x^2
 + 5*b*r^4*s*x + b*r^5 + a)^p*(s*x + r)^m, x)
```

Giac [F]

$$\int (r+sx)^m \left(a+br^5+5br^4sx+10br^3s^2x^2+10br^2s^3x^3+5brs^4x^4+bs^5x^5\right)^p \, dx = \int \left(bs^5x^5+5\,brs^4x^4+10\,br^2s^3x^3+10\,br^3s^2x^2+5\,br^4sx+br^5+a\right)^p (sx+r)^m \, dx$$

input
```
integrate((s*x+r)^m*(b*s^5*x^5+5*b*r*s^4*x^4+10*b*r^2*s^3*x^3+10*b*r^3*s^2
*x^2+5*b*r^4*s*x+b*r^5+a)^p,x, algorithm="giac")
```

output
```
integrate((b*s^5*x^5 + 5*b*r*s^4*x^4 + 10*b*r^2*s^3*x^3 + 10*b*r^3*s^2*x^2
 + 5*b*r^4*s*x + b*r^5 + a)^p*(s*x + r)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (r+sx)^m \left(a+br^5+5br^4sx+10br^3s^2x^2+10br^2s^3x^3+5brs^4x^4+bs^5x^5\right)^p\,dx$$
$$=\int (r+s\,x)^m\,\left(b\,r^5+5\,b\,r^4\,s\,x+10\,b\,r^3\,s^2\,x^2+10\,b\,r^2\,s^3\,x^3+5\,b\,r\,s^4\,x^4+b\,s^5\,x^5+a\right)^p\,dx$$

input
```
int((r + s*x)^m*(a + b*r^5 + b*s^5*x^5 + 10*b*r^3*s^2*x^2 + 10*b*r^2*s^3*x
^3 + 5*b*r^4*s*x + 5*b*r*s^4*x^4)^p,x)
```

output
```
int((r + s*x)^m*(a + b*r^5 + b*s^5*x^5 + 10*b*r^3*s^2*x^2 + 10*b*r^2*s^3*x
^3 + 5*b*r^4*s*x + 5*b*r*s^4*x^4)^p, x)
```

Reduce [F]

$$\int (r+sx)^m \left(a+br^5+5br^4sx+10br^3s^2x^2+10br^2s^3x^3+5brs^4x^4+bs^5x^5\right)^p\,dx$$
$$= \text{too large to display}$$

input
```
int((s*x+r)^m*(b*s^5*x^5+5*b*r*s^4*x^4+10*b*r^2*s^3*x^3+10*b*r^3*s^2*x^2+5
*b*r^4*s*x+b*r^5+a)^p,x)
```

output

```
((r + s*x)**m*(a + b*r**5 + 5*b*r**4*s*x + 10*b*r**3*s**2*x**2 + 10*b*r**2
*s**3*x**3 + 5*b*r*s**4*x**4 + b*s**5*x**5)**p*a*p + (r + s*x)**m*(a + b*r
**5 + 5*b*r**4*s*x + 10*b*r**3*s**2*x**2 + 10*b*r**2*s**3*x**3 + 5*b*r*s**
4*x**4 + b*s**5*x**5)**p*b*m*r**5 + (r + s*x)**m*(a + b*r**5 + 5*b*r**4*s*
x + 10*b*r**3*s**2*x**2 + 10*b*r**2*s**3*x**3 + 5*b*r*s**4*x**4 + b*s**5*x
**5)**p*b*m*r**4*s*x + 5*(r + s*x)**m*(a + b*r**5 + 5*b*r**4*s*x + 10*b*r*
*3*s**2*x**2 + 10*b*r**2*s**3*x**3 + 5*b*r*s**4*x**4 + b*s**5*x**5)**p*b*p
*r**5 + 5*(r + s*x)**m*(a + b*r**5 + 5*b*r**4*s*x + 10*b*r**3*s**2*x**2 +
10*b*r**2*s**3*x**3 + 5*b*r*s**4*x**4 + b*s**5*x**5)**p*b*p*r**4*s*x - int
(((r + s*x)**m*(a + b*r**5 + 5*b*r**4*s*x + 10*b*r**3*s**2*x**2 + 10*b*r**
2*s**3*x**3 + 5*b*r*s**4*x**4 + b*s**5*x**5)**p*x**5)/(a*m**2*r + a*m**2*s
*x + 10*a*m*p*r + 10*a*m*p*s*x + a*m*r + a*m*s*x + 25*a*p**2*r + 25*a*p**2
*s*x + 5*a*p*r + 5*a*p*s*x + b*m**2*r**6 + 6*b*m**2*r**5*s*x + 15*b*m**2*r
**4*s**2*x**2 + 20*b*m**2*r**3*s**3*x**3 + 15*b*m**2*r**2*s**4*x**4 + 6*b*
m**2*r*s**5*x**5 + b*m**2*s**6*x**6 + 10*b*m*p*r**6 + 60*b*m*p*r**5*s*x +
150*b*m*p*r**4*s**2*x**2 + 200*b*m*p*r**3*s**3*x**3 + 150*b*m*p*r**2*s**4*
x**4 + 60*b*m*p*r*s**5*x**5 + 10*b*m*p*s**6*x**6 + b*m*r**6 + 6*b*m*r**5*s
*x + 15*b*m*r**4*s**2*x**2 + 20*b*m*r**3*s**3*x**3 + 15*b*m*r**2*s**4*x**4
 + 6*b*m*r*s**5*x**5 + b*m*s**6*x**6 + 25*b*p**2*r**6 + 150*b*p**2*r**5*s*
x + 375*b*p**2*r**4*s**2*x**2 + 500*b*p**2*r**3*s**3*x**3 + 375*b*p**2*...
```

3.286 $\int \frac{(2+3x)^3}{216+108x^2+324x^3+18x^4+x^6}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 437

$$\int \frac{(2+3x)^3}{216+108x^2+324x^3+18x^4+x^6}\,dx$$
$$= \frac{\left(297\sqrt[3]{-6}+158(-2)^{2/3}+12\ 3^{2/3}\right)\arctan\left(\frac{3(-2)^{2/3}\sqrt[3]{3}+2x}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{486\sqrt[6]{3}\sqrt{8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}3^{2/3}}}$$
$$+\frac{(-1)^{2/3}\left(6(-6)^{2/3}+297\sqrt[3]{-3}+158\sqrt[3]{2}\right)\arctan\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{-3}-\sqrt[3]{2}x\right)}{\sqrt{3\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{162\sqrt[6]{6}\left(1+\sqrt[3]{-1}\right)^2\sqrt{4-3(-3)^{2/3}\sqrt[3]{2}}}$$
$$-\frac{\left(158\sqrt[3]{2}-297\sqrt[3]{3}+6\ 6^{2/3}\right)\operatorname{arctanh}\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{3}+\sqrt[3]{2}x\right)}{\sqrt{3\left(-4+3\sqrt[3]{2}3^{2/3}\right)}}\right)}{486\sqrt[6]{6}\sqrt{-4+3\sqrt[3]{2}3^{2/3}}}$$
$$-\frac{\left(21(-3)^{2/3}+2\ 2^{2/3}\right)\log\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)}{108\sqrt[3]{6}\left(1+\sqrt[3]{-1}\right)^2}$$
$$+\frac{\left(21(-6)^{2/3}-4\sqrt[3]{-2}\right)\log\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)}{648\sqrt[3]{3}}$$
$$+\frac{\left(2\ 2^{2/3}+21\ 3^{2/3}\right)\log\left(6+3\ 2^{2/3}\sqrt[3]{3}x+x^2\right)}{324\sqrt[3]{6}}$$

output
```
1/1458*(297*(-6)^(1/3)+158*(-2)^(2/3)+12*3^(2/3))*arctan((3*(-2)^(2/3)*3^(
1/3)+2*x)/(24+18*(-2)^(1/3)*3^(2/3))^(1/2))*3^(5/6)/(8+9*I*2^(1/3)*3^(1/6)
+3*2^(1/3)*3^(2/3))^(1/2)+1/972*(-1)^(2/3)*(6*(-6)^(2/3)+297*(-3)^(1/3)+15
8*2^(1/3))*arctan(2^(1/6)*(3*(-3)^(1/3)-2^(1/3)*x)/(12-9*(-3)^(2/3)*2^(1/3
))^(1/2))*6^(5/6)/(1+(-1)^(1/3))^2/(4-3*(-3)^(2/3)*2^(1/3))^(1/2)-1/2916*(
158*2^(1/3)-297*3^(1/3)+6*6^(2/3))*arctanh(2^(1/6)*(3*3^(1/3)+2^(1/3)*x)/(
-12+9*2^(1/3)*3^(2/3))^(1/2))*6^(5/6)/(-4+3*2^(1/3)*3^(2/3))^(1/2)-1/648*(
21*(-3)^(2/3)+2*2^(2/3))*ln(6-3*(-3)^(1/3)*2^(2/3)*x+x^2)*6^(2/3)/(1+(-1)^
(1/3))^2+1/1944*(21*(-6)^(2/3)-4*(-2)^(1/3))*ln(6+3*(-2)^(2/3)*3^(1/3)*x+x
^2)*3^(2/3)+1/1944*(2*2^(2/3)+21*3^(2/3))*ln(6+3*2^(2/3)*3^(1/3)*x+x^2)*6^
(2/3)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.20

$$\int \frac{(2+3x)^3}{216+108x^2+324x^3+18x^4+x^6}\,dx = \frac{81}{2}\text{RootSum}\Big[125128+64608\#1-39612\#1^2+7292\#1^3+222\#1^4-12\#1^5 +\#1^6\&,\frac{\log(2+3x-\#1)\#1^3}{10768-13204\#1+3646\#1^2+148\#1^3-10\#1^4+\#1^5}\&\Big]$$

input
```
Integrate[(2 + 3*x)^3/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6),x]
```

output
```
(81*RootSum[125128 + 64608*#1 - 39612*#1^2 + 7292*#1^3 + 222*#1^4 - 12*#1^
5 + #1^6 & , (Log[2 + 3*x - #1]*#1^3)/(10768 - 13204*#1 + 3646*#1^2 + 148*
#1^3 - 10*#1^4 + #1^5) & ])/2
```

Rubi [A] (verified)

Time = 2.34 (sec) , antiderivative size = 449, normalized size of antiderivative = 1.03, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2466, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(3x+2)^3}{x^6+18x^4+324x^3+108x^2+216}\,dx$$

$$\Big\downarrow \text{2466}$$

$$1259712\int \left(-\frac{(-1)^{2/3}\left(\left(63-2\sqrt[3]{-3}2^{2/3}\right)x+2\left(79+6(-3)^{2/3}\sqrt[3]{2}+27\sqrt[3]{-3}2^{2/3}\right)\right)}{68024448\sqrt[3]{2}3^{2/3}\left(1+\sqrt[3]{-1}\right)^2\left(x^2-3\sqrt[3]{-3}2^{2/3}x+6\right)}+\frac{(-1)^{2/3}\left(\left(63+2(-2)^{2/3}\right.\right.}{204073344}\right.$$

$$\Big\downarrow \text{2009}$$

$$1259712\left(\frac{(-1)^{2/3}\left(316-297(-2)^{2/3}\sqrt[3]{3}-12\sqrt[3]{-2}3^{2/3}\right)\arctan\left(\frac{2x+3(-2)^{2/3}\sqrt[3]{3}}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{612220032\ 2^{5/6}\sqrt[6]{3}\sqrt{4+3\sqrt[3]{-2}3^{2/3}}}+\frac{(-1)^{2/3}\left(6(-6)^{2/3}+29\right.}{20407334}\right.$$

input
```
Int[(2 + 3*x)^3/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6),x]
```

output
```
1259712*(((-1)^(2/3)*(316 - 297*(-2)^(2/3)*3^(1/3) - 12*(-2)^(1/3)*3^(2/3)
)*ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3))]])
/(612220032*2^(5/6)*3^(1/6)*Sqrt[4 + 3*(-2)^(1/3)*3^(2/3)]) + ((-1)^(2/3)*
(6*(-6)^(2/3) + 297*(-3)^(1/3) + 158*2^(1/3))*ArcTan[(2^(1/6)*(3*(-3)^(1/3
) - 2^(1/3)*x))/Sqrt[3*(4 - 3*(-3)^(2/3)*2^(1/3))]])/(204073344*6^(1/6)*(1
 + (-1)^(1/3))^2*Sqrt[4 - 3*(-3)^(2/3)*2^(1/3)]) - ((158*2^(1/3) - 297*3^(
1/3) + 6*6^(2/3))*ArcTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3
*2^(1/3)*3^(2/3))]])/(612220032*6^(1/6)*Sqrt[-4 + 3*2^(1/3)*3^(2/3)]) - ((
63*(-1)^(2/3)*2^(1/3) + 4*3^(1/3))*Log[6 - 3*(-3)^(1/3)*2^(2/3)*x + x^2])/
(136048896*6^(2/3)*(1 + (-1)^(1/3))^2) + ((-1)^(2/3)*(2*(-2)^(2/3) + 21*3^
(2/3))*Log[6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2])/(408146688*6^(1/3)) + ((2*2^
(2/3) + 21*3^(2/3))*Log[6 + 3*2^(2/3)*3^(1/3)*x + x^2])/(408146688*6^(1/3)
))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2466
```
Int[(u_.)*(Q6_)^(p_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
 x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Sim
p[1/(3^(3*p)*a^(2*p))   Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*
x + b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*
(-1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d,
 0] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coef
f[Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.17 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.16

method	result	size
default	$\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6}+18\_Z^{4}+324\_Z^{3}+108\_Z^{2}+216\right)}\frac{\left(27\_R^{3}+54\_R^{2}+36\_R+8\right)\ln\left(x-\_R\right)}{\_R^{5}+12\_R^{3}+162\_R^{2}+36\_R}}{6}$	68
risch	$\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6}+18\_Z^{4}+324\_Z^{3}+108\_Z^{2}+216\right)}\frac{\left(27\_R^{3}+54\_R^{2}+36\_R+8\right)\ln\left(x-\_R\right)}{\_R^{5}+12\_R^{3}+162\_R^{2}+36\_R}}{6}$	68

input
```
int((2+3*x)^3/(x^6+18*x^4+324*x^3+108*x^2+216),x,method=_RETURNVERBOSE)
```

output
```
1/6*sum((27*_R^3+54*_R^2+36*_R+8)/(_R^5+12*_R^3+162*_R^2+36*_R)*ln(x-_R),_
R=RootOf(_Z^6+18*_Z^4+324*_Z^3+108*_Z^2+216))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(2+3x)^3}{216+108x^2+324x^3+18x^4+x^6}\,dx=\text{Timed out}$$

input
```
integrate((2+3*x)^3/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="fricas"
)
```

output
```
Timed out
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.15

$$\int \frac{(2+3x)^3}{216+108x^2+324x^3+18x^4+x^6}\,dx$$
$$=\operatorname{RootSum}\Big(667284923473473623040t^6-14578786729520035200t^4-3149265778550660160t^3+567077919\ldots$$

input
```
integrate((2+3*x)**3/(x**6+18*x**4+324*x**3+108*x**2+216),x)
```

output
```
RootSum(66728492347347362304*_t**6 - 1457878672952003520*_t**4 - 314926577
855066016*_t**3 + 56707791944836392*_t**2 - 2072881129241808*_t - 38264280
19721, Lambda(_t, _t*log(-2996459005882230993967462856612309614913664000*_
t**5/24149354047666879671477078857336613502051 - 2200726920901623743671570
1957319757221395328*_t**4/24149354047666879671477078857336613502051 + 8111
7847403442138269983662552341850714924480*_t**3/241493540476668796714770788
57336613502051 + 15514735570255880082526639018859911997121600*_t**2/241493
54047666879671477078857336613502051 - 134937888689894711332515799739123991
094092*_t/1857642619051298436267467604410508730927 + x + 21670774544285125
054338536642513993968044/24149354047666879671477078857336613502051))
```

Maxima [F]

$$\int \frac{(2+3x)^3}{216+108x^2+324x^3+18x^4+x^6}\,dx=\int \frac{(3\,x+2)^3}{x^6+18\,x^4+324\,x^3+108\,x^2+216}\,dx$$

input
```
integrate((2+3*x)^3/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="maxima"
)
```

output
```
integrate((3*x + 2)^3/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

Giac [F]

$$\int \frac{(2+3x)^3}{216+108x^2+324x^3+18x^4+x^6}\,dx = \int \frac{(3\,x+2)^3}{x^6+18\,x^4+324\,x^3+108\,x^2+216}\,dx$$

input
```
integrate((2+3*x)^3/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="giac")
```

output
```
integrate((3*x + 2)^3/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

Mupad [B] (verification not implemented)

Time = 22.45 (sec) , antiderivative size = 338, normalized size of antiderivative = 0.77

$$\int \frac{(2+3x)^3}{216+108x^2+324x^3+18x^4+x^6}\,dx = \text{Too large to display}$$

input
```
int((3*x + 2)^3/(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216),x)
```

output
```
symsum(log(23112246375*x - 1060028634492*root(z^6 - (26885*z^4)/1230552 -
(25402517*z^3)/5382434448 + (1482023821*z^2)/1743908761152 - (59238715399*
z)/1906964230319712 - 3826428019721/66728492347347362304, z, k) - 21273515
2480*root(z^6 - (26885*z^4)/1230552 - (25402517*z^3)/5382434448 + (1482023
821*z^2)/1743908761152 - (59238715399*z)/1906964230319712 - 3826428019721/
66728492347347362304, z, k)*x - 317857955976*root(z^6 - (26885*z^4)/123055
2 - (25402517*z^3)/5382434448 + (1482023821*z^2)/1743908761152 - (59238715
399*z)/1906964230319712 - 3826428019721/66728492347347362304, z, k)^2*x +
2097976012992*root(z^6 - (26885*z^4)/1230552 - (25402517*z^3)/5382434448 +
 (1482023821*z^2)/1743908761152 - (59238715399*z)/1906964230319712 - 38264
28019721/66728492347347362304, z, k)^3*x + 27299197336896*root(z^6 - (2688
5*z^4)/1230552 - (25402517*z^3)/5382434448 + (1482023821*z^2)/174390876115
2 - (59238715399*z)/1906964230319712 - 3826428019721/66728492347347362304,
 z, k)^4*x - 72301961339136*root(z^6 - (26885*z^4)/1230552 - (25402517*z^3
)/5382434448 + (1482023821*z^2)/1743908761152 - (59238715399*z)/1906964230
319712 - 3826428019721/66728492347347362304, z, k)^5*x + 9560202531264*roo
t(z^6 - (26885*z^4)/1230552 - (25402517*z^3)/5382434448 + (1482023821*z^2)
/1743908761152 - (59238715399*z)/1906964230319712 - 3826428019721/66728492
347347362304, z, k)^2 + 33332166587232*root(z^6 - (26885*z^4)/1230552 - (2
5402517*z^3)/5382434448 + (1482023821*z^2)/1743908761152 - (59238715399...
```

Reduce [F]

$$\int \frac{(2+3x)^3}{216+108x^2+324x^3+18x^4+x^6}\,dx$$
$$= 27\left(\int \frac{x^3}{x^6+18x^4+324x^3+108x^2+216}dx\right)$$
$$+ 54\left(\int \frac{x^2}{x^6+18x^4+324x^3+108x^2+216}dx\right)$$
$$+ 36\left(\int \frac{x}{x^6+18x^4+324x^3+108x^2+216}dx\right)$$
$$+ 8\left(\int \frac{1}{x^6+18x^4+324x^3+108x^2+216}dx\right)$$

input
```
int((2+3*x)^3/(x^6+18*x^4+324*x^3+108*x^2+216),x)
```

output
```
27*int(x**3/(x**6 + 18*x**4 + 324*x**3 + 108*x**2 + 216),x) + 54*int(x**2/
(x**6 + 18*x**4 + 324*x**3 + 108*x**2 + 216),x) + 36*int(x/(x**6 + 18*x**4
 + 324*x**3 + 108*x**2 + 216),x) + 8*int(1/(x**6 + 18*x**4 + 324*x**3 + 10
8*x**2 + 216),x)
```

3.287 $\int \frac{(2+3x)^2}{216+108x^2+324x^3+18x^4+x^6} \, dx$

Optimal result

Integrand size = 30, antiderivative size = 455

$$\int \frac{(2+3x)^2}{216+108x^2+324x^3+18x^4+x^6}\,dx$$

$$= \frac{\left(18\sqrt[3]{-6}+25(-2)^{2/3}+6\ 3^{2/3}\right)\arctan\left(\frac{3(-2)^{2/3}\sqrt[3]{3}+2x}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{486\sqrt[6]{3}\sqrt{8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}3^{2/3}}}$$
$$+\frac{(-1)^{2/3}\left(25+3(-3)^{2/3}\sqrt[3]{2}+9\sqrt[3]{-3}2^{2/3}\right)\arctan\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{-3}-\sqrt[3]{2}x\right)}{\sqrt{3\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{81\ 2^{5/6}\sqrt[6]{3}\left(1+\sqrt[3]{-1}\right)^2\sqrt{4-3(-3)^{2/3}\sqrt[3]{2}}}$$
$$-\frac{\left(25-9\ 2^{2/3}\sqrt[3]{3}+3\sqrt[3]{2}3^{2/3}\right)\operatorname{arctanh}\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{3}+\sqrt[3]{2}x\right)}{\sqrt{3\left(-4+3\sqrt[3]{2}3^{2/3}\right)}}\right)}{243\ 2^{5/6}\sqrt[6]{3}\sqrt{-4+3\sqrt[3]{2}3^{2/3}}}$$
$$+\frac{\left(-\frac{1}{6}\right)^{2/3}\left(\sqrt[3]{-3}+3\sqrt[3]{2}\right)\log\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)}{54\left(1+\sqrt[3]{-1}\right)^2}$$
$$-\frac{\left((-6)^{2/3}+\sqrt[3]{-2}\right)\log\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)}{324\sqrt[3]{3}}$$
$$+\frac{\left(1-\sqrt[3]{2}3^{2/3}\right)\log\left(6+3\ 2^{2/3}\sqrt[3]{3}x+x^2\right)}{162\ 2^{2/3}\sqrt[3]{3}}$$

output
```
1/1458*(18*(-6)^(1/3)+25*(-2)^(2/3)+6*3^(2/3))*arctan((3*(-2)^(2/3)*3^(1/3
)+2*x)/(24+18*(-2)^(1/3)*3^(2/3))^(1/2))*3^(5/6)/(8+9*I*2^(1/3)*3^(1/6)+3*
2^(1/3)*3^(2/3))^(1/2)+1/486*(-1)^(2/3)*(25+3*(-3)^(2/3)*2^(1/3)+9*(-3)^(1
/3)*2^(2/3))*arctan(2^(1/6)*(3*(-3)^(1/3)-2^(1/3)*x)/(12-9*(-3)^(2/3)*2^(1
/3))^(1/2))*2^(1/6)*3^(5/6)/(1+(-1)^(1/3))^2/(4-3*(-3)^(2/3)*2^(1/3))^(1/2
)-1/1458*(25-9*2^(2/3)*3^(1/3)+3*2^(1/3)*3^(2/3))*arctanh(2^(1/6)*(3*3^(1/
3)+2^(1/3)*x)/(-12+9*2^(1/3)*3^(2/3))^(1/2))*2^(1/6)*3^(5/6)/(-4+3*2^(1/3)
*3^(2/3))^(1/2)+1/324*(-1)^(2/3)*6^(1/3)*((-3)^(1/3)+3*2^(1/3))*ln(6-3*(-3
)^(1/3)*2^(2/3)*x+x^2)/(1+(-1)^(1/3))^2-1/972*((-6)^(2/3)+(-2)^(1/3))*ln(6
+3*(-2)^(2/3)*3^(1/3)*x+x^2)*3^(2/3)+1/972*(1-2^(1/3)*3^(2/3))*ln(6+3*2^(2
/3)*3^(1/3)*x+x^2)*2^(1/3)*3^(2/3)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.19

$$\int \frac{(2+3x)^2}{216+108x^2+324x^3+18x^4+x^6}\,dx = \frac{81}{2}\text{RootSum}\left[125128+64608\#1-39612\#1^2+7292\#1^3+222\#1^4-12\#1^5+\#1^6\&,\frac{\log(2+3x-\#1)\#1^2}{10768-13204\#1+3646\#1^2+148\#1^3-10\#1^4+\#1^5}\&\right]$$

input
```
Integrate[(2 + 3*x)^2/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6),x]
```

output
```
(81*RootSum[125128 + 64608*#1 - 39612*#1^2 + 7292*#1^3 + 222*#1^4 - 12*#1^
5 + #1^6 & , (Log[2 + 3*x - #1]*#1^2)/(10768 - 13204*#1 + 3646*#1^2 + 148*
#1^3 - 10*#1^4 + #1^5) & ])/2
```

Rubi [A] (verified)

Time = 2.10 (sec) , antiderivative size = 460, normalized size of antiderivative = 1.01, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2466, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(3x+2)^2}{x^6+18x^4+324x^3+108x^2+216}\,dx$$

$$\Big\downarrow \text{2466}$$

$$1259712\int\left(-\frac{(-1)^{2/3}\left(-\left(\left(6+\sqrt[3]{-3}2^{2/3}\right)x\right)+18\sqrt[3]{-3}2^{2/3}+6(-3)^{2/3}\sqrt[3]{2}+25\right)}{68024448\sqrt[3]{2}3^{2/3}\left(1+\sqrt[3]{-1}\right)^2\left(x^2-3\sqrt[3]{-3}2^{2/3}x+6\right)}+\frac{(-1)^{2/3}\left(-\left(\left(6-(-2)^{2/3}\sqrt[3]{\cdots}\right.\right.\right.}{204073344\sqrt[3]{2}\cdots}\right.$$

$$\Big\downarrow \text{2009}$$

$$1259712\left(\frac{(-1)^{2/3}\left(25-9(-2)^{2/3}\sqrt[3]{3}-3\sqrt[3]{-2}3^{2/3}\right)\arctan\left(\frac{2x+3(-2)^{2/3}\sqrt[3]{3}}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{306110016\ 2^{5/6}\sqrt[6]{3}\sqrt{4+3\sqrt[3]{-2}3^{2/3}}}+\frac{(-1)^{2/3}\left(25+3(-3)^{2/3}\sqrt[3]{2}\cdots\right.}{102036672\ 2^{5/6}\sqrt[6]{\cdots}}\right.$$

input
```
Int[(2 + 3*x)^2/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6),x]
```

output
```
1259712*(((-1)^(2/3)*(25 - 9*(-2)^(2/3)*3^(1/3) - 3*(-2)^(1/3)*3^(2/3))*Ar
cTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3))]])/(30
6110016*2^(5/6)*3^(1/6)*Sqrt[4 + 3*(-2)^(1/3)*3^(2/3)]) + ((-1)^(2/3)*(25
+ 3*(-3)^(2/3)*2^(1/3) + 9*(-3)^(1/3)*2^(2/3))*ArcTan[(2^(1/6)*(3*(-3)^(1/
3) - 2^(1/3)*x))/Sqrt[3*(4 - 3*(-3)^(2/3)*2^(1/3))]])/(102036672*2^(5/6)*3
^(1/6)*(1 + (-1)^(1/3))^2*Sqrt[4 - 3*(-3)^(2/3)*2^(1/3)]) + ((27*2^(5/6)*3
^(1/6) - 25*2^(1/6)*3^(5/6) - 9*Sqrt[6])*ArcTanh[(2^(1/6)*(3*3^(1/3) + 2^(
1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(2/3))]])/(1836660096*Sqrt[-4 + 3*2^(1/3
)*3^(2/3)]) + ((-1/6)^(2/3)*((-3)^(1/3) + 3*2^(1/3))*Log[6 - 3*(-3)^(1/3)*
2^(2/3)*x + x^2])/(68024448*(1 + (-1)^(1/3))^2) + ((-1)^(2/3)*((-2)^(2/3)
- 2*3^(2/3))*Log[6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2])/(408146688*6^(1/3)) +
((1 - 2^(1/3)*3^(2/3))*Log[6 + 3*2^(2/3)*3^(1/3)*x + x^2])/(204073344*2^(2
/3)*3^(1/3)))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2466
```
Int[(u_.)*(Q6_)^(p_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
 x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Sim
p[1/(3^(3*p)*a^(2*p))   Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*
x + b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*
(-1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d,
 0] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coef
f[Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.14 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.14

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)}\frac{\left(9\_R^2+12\_R+4\right)\ln\left(x-\_R\right)}{\_R^5+12\_R^3+162\_R^2+36\_R}\right)}{6}$	63
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)}\frac{\left(9\_R^2+12\_R+4\right)\ln\left(x-\_R\right)}{\_R^5+12\_R^3+162\_R^2+36\_R}\right)}{6}$	63

input
```
int((2+3*x)^2/(x^6+18*x^4+324*x^3+108*x^2+216),x,method=_RETURNVERBOSE)
```

output
```
1/6*sum((9*_R^2+12*_R+4)/(_R^5+12*_R^3+162*_R^2+36*_R)*ln(x-_R),_R=RootOf(
_Z^6+18*_Z^4+324*_Z^3+108*_Z^2+216))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(2+3x)^2}{216+108x^2+324x^3+18x^4+x^6}\,dx = \text{Timed out}$$

input
```
integrate((2+3*x)^2/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="fricas"
)
```

output
```
Timed out
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.14

$$\int \frac{(2+3x)^2}{216+108x^2+324x^3+18x^4+x^6}\,dx$$
$$= \text{RootSum}\left(533827938778778898432t^6 - 401275885432204480t^4 + 2296802223171072t^3 + 329841960265\right.$$

input
```
integrate((2+3*x)**2/(x**6+18*x**4+324*x**3+108*x**2+216),x)
```

output
```
RootSum(533827938778778898432*_t**6 - 401275885432204480*_t**4 + 2296802223
171072*_t**3 + 32984196026544*_t**2 + 90717389856*_t - 244640881, Lambda(_
t, _t*log(61351556677313231611493737999292l6*_t**5/307381025025457458399993
5 + 69510029598991783751411734577568*_t**4/3073810250254574583999935 - 1674
54421024100513865438958272*_t**3/236446942327274967999995 + 471474103890512
83066608263328*_t**2/3073810250254574583999935 + 994701165304699703409532667
7*_t/6147620500509149167999870 + x - 48780340025935179086785l7/245904820020
3659667199480)))
```

Maxima [F]

$$\int \frac{(2+3x)^2}{216+108x^2+324x^3+18x^4+x^6}\,dx = \int \frac{(3x+2)^2}{x^6+18x^4+324x^3+108x^2+216}\,dx$$

input
```
integrate((2+3*x)^2/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="maxima"
)
```

output
```
integrate((3*x + 2)^2/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

Giac [F]

$$\int \frac{(2+3x)^2}{216+108x^2+324x^3+18x^4+x^6}\,dx = \int \frac{(3\,x+2)^2}{x^6+18\,x^4+324\,x^3+108\,x^2+216}\,dx$$

input
```
integrate((2+3*x)^2/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="giac")
```

output
```
integrate((3*x + 2)^2/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

Mupad [B] (verification not implemented)

Time = 22.10 (sec) , antiderivative size = 338, normalized size of antiderivative = 0.74

$$\int \frac{(2+3x)^2}{216+108x^2+324x^3+18x^4+x^6}\,dx = \text{Too large to display}$$

input
```
int((3*x + 2)^2/(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216),x)
```

output
```
symsum(log(588343450752*root(z^6 - (185*z^4)/2461104 + (11579*z^3)/2691217
224 + (431011*z^2)/6975635044608 + (1296259*z)/7627856921278848 - 24464088
1/533827938778778898432, z, k)^3 - 654156*x - 228683908*root(z^6 - (185*z^
4)/2461104 + (11579*z^3)/2691217224 + (431011*z^2)/6975635044608 + (129625
9*z)/7627856921278848 - 244640881/533827938778778898432, z, k)*x - 2987691
6096*root(z^6 - (185*z^4)/2461104 + (11579*z^3)/2691217224 + (431011*z^2)/
6975635044608 + (1296259*z)/7627856921278848 - 244640881/53382793877877889
8432, z, k)^2*x - 1695645415296*root(z^6 - (185*z^4)/2461104 + (11579*z^3)
/2691217224 + (431011*z^2)/6975635044608 + (1296259*z)/7627856921278848 -
244640881/533827938778778898432, z, k)^3*x - 37159307062272*root(z^6 - (18
5*z^4)/2461104 + (11579*z^3)/2691217224 + (431011*z^2)/6975635044608 + (12
96259*z)/7627856921278848 - 244640881/533827938778778898432, z, k)^4*x - 2
89207845356544*root(z^6 - (185*z^4)/2461104 + (11579*z^3)/2691217224 + (43
1011*z^2)/6975635044608 + (1296259*z)/7627856921278848 - 244640881/5338279
38778778898432, z, k)^5*x - 3605925600*root(z^6 - (185*z^4)/2461104 + (115
79*z^3)/2691217224 + (431011*z^2)/6975635044608 + (1296259*z)/762785692127
8848 - 244640881/533827938778778898432, z, k)^2 - 170444700*root(z^6 - (18
5*z^4)/2461104 + (11579*z^3)/2691217224 + (431011*z^2)/6975635044608 + (12
96259*z)/7627856921278848 - 244640881/533827938778778898432, z, k) - 61171
801157376*root(z^6 - (185*z^4)/2461104 + (11579*z^3)/2691217224 + (4310...
```

Reduce [F]

$$\int \frac{(2+3x)^2}{216+108x^2+324x^3+18x^4+x^6}\,dx = 9\left(\int \frac{x^2}{x^6+18x^4+324x^3+108x^2+216}dx\right) + 12\left(\int \frac{x}{x^6+18x^4+324x^3+108x^2+216}dx\right) + 4\left(\int \frac{1}{x^6+18x^4+324x^3+108x^2+216}dx\right)$$

input
```
int((2+3*x)^2/(x^6+18*x^4+324*x^3+108*x^2+216),x)
```

output
```
9*int(x**2/(x**6 + 18*x**4 + 324*x**3 + 108*x**2 + 216),x) + 12*int(x/(x**
6 + 18*x**4 + 324*x**3 + 108*x**2 + 216),x) + 4*int(1/(x**6 + 18*x**4 + 32
4*x**3 + 108*x**2 + 216),x)
```

3.288 $\int \frac{2+3x}{216+108x^2+324x^3+18x^4+x^6} \, dx$

Optimal result

Integrand size = 28, antiderivative size = 441

$$\int \frac{2+3x}{216+108x^2+324x^3+18x^4+x^6}\,dx$$
$$= \frac{\left(9\sqrt[3]{-6}-2(-2)^{2/3}+6\ 3^{2/3}\right)\arctan\left(\frac{3(-2)^{2/3}\sqrt[3]{3}+2x}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{972\sqrt[6]{3}\sqrt{8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}3^{2/3}}}$$
$$+\frac{(-1)^{2/3}\left(3(-6)^{2/3}+9\sqrt[3]{-3}-2\sqrt[3]{2}\right)\arctan\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{-3}-\sqrt[3]{2}x\right)}{\sqrt{3\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{324\sqrt[6]{6}\left(1+\sqrt[3]{-1}\right)^2\sqrt{4-3(-3)^{2/3}\sqrt[3]{2}}}$$
$$+\frac{\left(2\sqrt[3]{2}+9\sqrt[3]{3}-3\ 6^{2/3}\right)\operatorname{arctanh}\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{3}+\sqrt[3]{2}x\right)}{\sqrt{3\left(-4+3\sqrt[3]{2}3^{2/3}\right)}}\right)}{972\sqrt[6]{6}\sqrt{-4+3\sqrt[3]{2}3^{2/3}}}$$
$$+\frac{(-1)^{2/3}\left(3+\sqrt[3]{-3}2^{2/3}\right)\log\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)}{216\sqrt[3]{2}3^{2/3}\left(1+\sqrt[3]{-1}\right)^2}$$
$$-\frac{\left((-6)^{2/3}+2\sqrt[3]{-2}\right)\log\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)}{1296\sqrt[3]{3}}$$
$$+\frac{\left(2^{2/3}-3^{2/3}\right)\log\left(6+3\ 2^{2/3}\sqrt[3]{3}x+x^2\right)}{648\sqrt[3]{6}}$$

output
```
1/2916*(9*(-6)^(1/3)-2*(-2)^(2/3)+6*3^(2/3))*arctan((3*(-2)^(2/3)*3^(1/3)+
2*x)/(24+18*(-2)^(1/3)*3^(2/3))^(1/2))*3^(5/6)/(8+9*I*2^(1/3)*3^(1/6)+3*2^
(1/3)*3^(2/3))^(1/2)+1/1944*(-1)^(2/3)*(3*(-6)^(2/3)+9*(-3)^(1/3)-2*2^(1/3
))*arctan(2^(1/6)*(3*(-3)^(1/3)-2^(1/3)*x)/(12-9*(-3)^(2/3)*2^(1/3))^(1/2)
)*6^(5/6)/(1+(-1)^(1/3))^2/(4-3*(-3)^(2/3)*2^(1/3))^(1/2)+1/5832*(2*2^(1/3
)+9*3^(1/3)-3*6^(2/3))*arctanh(2^(1/6)*(3*3^(1/3)+2^(1/3)*x)/(-12+9*2^(1/3
)*3^(2/3))^(1/2))*6^(5/6)/(-4+3*2^(1/3)*3^(2/3))^(1/2)+1/1296*(-1)^(2/3)*(
3+(-3)^(1/3)*2^(2/3))*ln(6-3*(-3)^(1/3)*2^(2/3)*x+x^2)*2^(2/3)*3^(1/3)/(1+
(-1)^(1/3))^2-1/3888*((-6)^(2/3)+2*(-2)^(1/3))*ln(6+3*(-2)^(2/3)*3^(1/3)*x
+x^2)*3^(2/3)+1/3888*(2^(2/3)-3^(2/3))*ln(6+3*2^(2/3)*3^(1/3)*x+x^2)*6^(2/
3)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.17

$$\int \frac{2+3x}{216+108x^2+324x^3+18x^4+x^6}\,dx = \frac{1}{6}\text{RootSum}\left[216+108\#1^2+324\#1^3+18\#1^4+\#1^6\&,\frac{2\log(x-\#1)+3\log(x-\#1)\#1}{36\#1+162\#1^2+12\#1^3+\#1^5}\&\right]$$

input
```
Integrate[(2 + 3*x)/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6),x]
```

output
```
RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^4 + #1^6 & , (2*Log[x - #1] + 3*
Log[x - #1]*#1)/(36*#1 + 162*#1^2 + 12*#1^3 + #1^5) & ]/6
```

Rubi [A] (verified)

Time = 2.14 (sec) , antiderivative size = 448, normalized size of antiderivative = 1.02, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2466, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{3x+2}{x^6+18x^4+324x^3+108x^2+216}\,dx$$

$$\Big\downarrow \text{2466}$$

$$1259712\int \left(\frac{(-1)^{2/3}\left(\left(3+\sqrt[3]{-3}2^{2/3}\right)x-9\sqrt[3]{-3}2^{2/3}-6(-3)^{2/3}\sqrt[3]{2}+2\right)}{136048896\sqrt[3]{2}3^{2/3}\left(1+\sqrt[3]{-1}\right)^2\left(x^2-3\sqrt[3]{-3}2^{2/3}x+6\right)}-\frac{(-1)^{2/3}\left(\left(3-(-2)^{2/3}\sqrt[3]{3}\right)x+6\sqrt[3]{\ldots}\right.}{408146688\sqrt[3]{2}3^{2/3}\left(x^2+3\ldots\right.}\right.$$

$$\Big\downarrow \text{2009}$$

$$1259712\left(-\frac{(-1)^{2/3}\left(4+9(-2)^{2/3}\sqrt[3]{3}+6\sqrt[3]{-2}3^{2/3}\right)\arctan\left(\frac{2x+3(-2)^{2/3}\sqrt[3]{3}}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{1224440064\ 2^{5/6}\sqrt[6]{3}\sqrt{4+3\sqrt[3]{-2}3^{2/3}}}+\frac{(-1)^{2/3}\left(3(-6)^{2/3}+9\sqrt[3]{-3}\right.}{408146688\sqrt[6]{6}\,(1}\right.$$

input
```
Int[(2 + 3*x)/(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6),x]
```

output
```
1259712*(-1/1224440064*((-1)^(2/3)*(4 + 9*(-2)^(2/3)*3^(1/3) + 6*(-2)^(1/3
)*3^(2/3))*ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^
(2/3))]])/(2^(5/6)*3^(1/6)*Sqrt[4 + 3*(-2)^(1/3)*3^(2/3)]) + ((-1)^(2/3)*(
3*(-6)^(2/3) + 9*(-3)^(1/3) - 2*2^(1/3))*ArcTan[(2^(1/6)*(3*(-3)^(1/3) - 2
^(1/3)*x))/Sqrt[3*(4 - 3*(-3)^(2/3)*2^(1/3))]])/(408146688*6^(1/6)*(1 + (-
1)^(1/3))^2*Sqrt[4 - 3*(-3)^(2/3)*2^(1/3)]) + ((2*2^(1/3) + 9*3^(1/3) - 3*
6^(2/3))*ArcTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*
3^(2/3))]])/(1224440064*6^(1/6)*Sqrt[-4 + 3*2^(1/3)*3^(2/3)]) + ((-1)^(2/3
)*(3 + (-3)^(1/3)*2^(2/3))*Log[6 - 3*(-3)^(1/3)*2^(2/3)*x + x^2])/(2720977
92*2^(1/3)*3^(2/3)*(1 + (-1)^(1/3))^2) + ((-1)^(2/3)*((-2)^(2/3) - 3^(2/3)
)*Log[6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2])/(816293376*6^(1/3)) + ((2^(2/3) -
 3^(2/3))*Log[6 + 3*2^(2/3)*3^(1/3)*x + x^2])/(816293376*6^(1/3)))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2466
```
Int[(u_.)*(Q6_)^(p_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
 x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Sim
p[1/(3^(3*p)*a^(2*p))   Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*
x + b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*
(-1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d,
 0] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coef
f[Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.04 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.13

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6}+18\_Z^{4}+324\_Z^{3}+108\_Z^{2}+216\right)}\frac{\left(2+3\_R\right)\ln\left(x-\_R\right)}{\_R^{5}+12\_R^{3}+162\_R^{2}+36\_R}\right)}{6}$	58
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6}+18\_Z^{4}+324\_Z^{3}+108\_Z^{2}+216\right)}\frac{\left(2+3\_R\right)\ln\left(x-\_R\right)}{\_R^{5}+12\_R^{3}+162\_R^{2}+36\_R}\right)}{6}$	58

input
```
int((2+3*x)/(x^6+18*x^4+324*x^3+108*x^2+216),x,method=_RETURNVERBOSE)
```

output
```
1/6*sum((2+3*_R)/(_R^5+12*_R^3+162*_R^2+36*_R)*ln(x-_R),_R=RootOf(_Z^6+18*
_Z^4+324*_Z^3+108*_Z^2+216))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{2+3x}{216+108x^2+324x^3+18x^4+x^6}\,dx=\text{Timed out}$$

input
```
integrate((2+3*x)/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.15

$$\int \frac{2+3x}{216+108x^2+324x^3+18x^4+x^6}\,dx$$
$$= \text{RootSum}\left(427062351023023118745 6t^6+101511953720146944t^4+1091967893475 84t^3-81578319264\right.$$

input
```
integrate((2+3*x)/(x**6+18*x**4+324*x**3+108*x**2+216),x)
```

output
```
RootSum(4270623510230231187456*_t**6 + 101511953720146944*_t**4 + 10919678
9347584*_t**3 - 81578319264*_t**2 - 1609632*_t - 15641, Lambda(_t, _t*log(
621953477382586648807754299964620 8*_t**5/8049794570608956253 + 95558804668
4567775465000499200*_t**4/8049794570608956253 + 13409641654345971318505496
832*_t**3/731799506418996023 + 1825730533743777545170216 32*_t**2/804979457
0608956253 - 920030330006251192317 6*_t/731799506418996023 + x - 1178150707
4505066000/8049794570608956253)))
```

Maxima [F]

$$\int \frac{2+3x}{216+108x^2+324x^3+18x^4+x^6}\,dx = \int \frac{3\,x+2}{x^6+18\,x^4+324\,x^3+108\,x^2+216}\,dx$$

input
```
integrate((2+3*x)/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="maxima")
```

output
```
integrate((3*x + 2)/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

Giac [F]

$$\int \frac{2+3x}{216+108x^2+324x^3+18x^4+x^6}\,dx = \int \frac{3\,x+2}{x^6+18\,x^4+324\,x^3+108\,x^2+216}\,dx$$

input
```
integrate((2+3*x)/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="giac")
```

output
```
integrate((3*x + 2)/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

Mupad [B] (verification not implemented)

Time = 22.33 (sec) , antiderivative size = 338, normalized size of antiderivative = 0.77

$$\int \frac{2+3x}{216+108x^2+324x^3+18x^4+x^6}\,dx = \text{Too large to display}$$

input
```
int((3*x + 2)/(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216),x)
```

output
```
symsum(log(81*x - 26640*root(z^6 + (13*z^4)/546912 + (367*z^3)/14353158528
 - (533*z^2)/27902540178432 - (23*z)/61022855370230784 - 15641/42706235102
30231187456, z, k) - 29192*root(z^6 + (13*z^4)/546912 + (367*z^3)/14353158
528 - (533*z^2)/27902540178432 - (23*z)/61022855370230784 - 15641/42706235
10230231187456, z, k)*x + 441202464*root(z^6 + (13*z^4)/546912 + (367*z^3)
/14353158528 - (533*z^2)/27902540178432 - (23*z)/61022855370230784 - 15641
/4270623510230231187456, z, k)^2*x - 775886853888*root(z^6 + (13*z^4)/5469
12 + (367*z^3)/14353158528 - (533*z^2)/27902540178432 - (23*z)/61022855370
230784 - 15641/4270623510230231187456, z, k)^3*x - 138983742784512*root(z^
6 + (13*z^4)/546912 + (367*z^3)/14353158528 - (533*z^2)/27902540178432 - (
23*z)/61022855370230784 - 15641/4270623510230231187456, z, k)^4*x - 231366
2762852352*root(z^6 + (13*z^4)/546912 + (367*z^3)/14353158528 - (533*z^2)/
27902540178432 - (23*z)/61022855370230784 - 15641/4270623510230231187456,
z, k)^5*x + 298551744*root(z^6 + (13*z^4)/546912 + (367*z^3)/14353158528 -
 (533*z^2)/27902540178432 - (23*z)/61022855370230784 - 15641/4270623510230
231187456, z, k)^2 - 563242429440*root(z^6 + (13*z^4)/546912 + (367*z^3)/1
4353158528 - (533*z^2)/27902540178432 - (23*z)/61022855370230784 - 15641/4
270623510230231187456, z, k)^3 + 6435656976384*root(z^6 + (13*z^4)/546912
+ (367*z^3)/14353158528 - (533*z^2)/27902540178432 - (23*z)/61022855370230
784 - 15641/4270623510230231187456, z, k)^4 - 56299127229407232*root(z^...
```

Reduce [F]

$$\int \frac{2+3x}{216+108x^2+324x^3+18x^4+x^6}\,dx = 3\left(\int \frac{x}{x^6+18x^4+324x^3+108x^2+216}dx\right) + 2\left(\int \frac{1}{x^6+18x^4+324x^3+108x^2+216}dx\right)$$

input
```
int((2+3*x)/(x^6+18*x^4+324*x^3+108*x^2+216),x)
```

output
```
3*int(x/(x**6 + 18*x**4 + 324*x**3 + 108*x**2 + 216),x) + 2*int(1/(x**6 +
18*x**4 + 324*x**3 + 108*x**2 + 216),x)
```

3.289 $\int \frac{1}{216+108x^2+324x^3+18x^4+x^6}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 383

$$\int \frac{1}{216+108x^2+324x^3+18x^4+x^6}\,dx = \frac{(-1)^{2/3}\left(3(-3)^{2/3}-2^{2/3}\right)\arctan\left(\frac{3\sqrt[3]{-3}2^{2/3}-2x}{\sqrt{6\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{324\sqrt[6]{3}\left(1+\sqrt[3]{-1}\right)^2\sqrt{2\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}} - \frac{\left((-2)^{2/3}-3\,3^{2/3}\right)\arctan\left(\frac{3(-2)^{2/3}\sqrt[3]{3}+2x}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{972\sqrt[6]{3}\sqrt{8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}3^{2/3}}} + \frac{\left(2^{2/3}-3\,3^{2/3}\right)\operatorname{arctanh}\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{3}+\sqrt[3]{2}x\right)}{\sqrt{3\left(-4+3\sqrt[3]{2}3^{2/3}\right)}}\right)}{972\sqrt[6]{3}\sqrt{2\left(-4+3\sqrt[3]{2}3^{2/3}\right)}} - \frac{\log\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)}{216\,2^{2/3}\sqrt[3]{3}\left(1+\sqrt[3]{-1}\right)^2} - \frac{\sqrt[3]{-\frac{1}{3}}\log\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)}{648\,2^{2/3}} + \frac{\log\left(6+3\,2^{2/3}\sqrt[3]{3}x+x^2\right)}{648\,2^{2/3}\sqrt[3]{3}}$$

output
```
1/972*(-1)^(2/3)*(3*(-3)^(2/3)-2^(2/3))*arctan((3*(-3)^(1/3)*2^(2/3)-2*x)/
(24-18*(-3)^(2/3)*2^(1/3))^(1/2))*3^(5/6)/(1+(-1)^(1/3))^2/(8-6*(-3)^(2/3)
*2^(1/3))^(1/2)-1/2916*((-2)^(2/3)-3*3^(2/3))*arctan((3*(-2)^(2/3)*3^(1/3)
+2*x)/(24+18*(-2)^(1/3)*3^(2/3))^(1/2))*3^(5/6)/(8+9*I*2^(1/3)*3^(1/6)+3*2
^(1/3)*3^(2/3))^(1/2)+1/2916*(2^(2/3)-3*3^(2/3))*arctanh(2^(1/6)*(3*3^(1/3
)+2^(1/3)*x)/(-12+9*2^(1/3)*3^(2/3))^(1/2))*3^(5/6)/(-8+6*2^(1/3)*3^(2/3))
^(1/2)-1/1296*ln(6-3*(-3)^(1/3)*2^(2/3)*x+x^2)*2^(1/3)*3^(2/3)/(1+(-1)^(1/
3))^2-1/3888*(-1)^(1/3)*3^(2/3)*ln(6+3*(-2)^(2/3)*3^(1/3)*x+x^2)*2^(1/3)+1
/3888*ln(6+3*2^(2/3)*3^(1/3)*x+x^2)*2^(1/3)*3^(2/3)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.16

$$\int \frac{1}{216+108x^2+324x^3+18x^4+x^6}\,dx = \frac{1}{6}\text{RootSum}\left[216+108\#1^2+324\#1^3+18\#1^4+\#1^6\&,\frac{\log(x-\#1)}{36\#1+162\#1^2+12\#1^3+\#1^5}\&\right]$$

input
```
Integrate[(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^(-1),x]
```

output
```
RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^4 + #1^6 & , Log[x - #1]/(36*#1
+ 162*#1^2 + 12*#1^3 + #1^5) & ]/6
```

Rubi [A] (verified)

Time = 1.82 (sec) , antiderivative size = 370, normalized size of antiderivative = 0.97, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {2466, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6+18x^4+324x^3+108x^2+216}\,dx$$

$$\Big\downarrow \text{ 2466}$$

$$1259712\int \left(\frac{\left(-\frac{1}{3}\right)^{2/3}\left(\sqrt[3]{-6}x+2^{2/3}\left(1-3(-3)^{2/3}\sqrt[3]{2}\right)\right)}{272097792\left(1+\sqrt[3]{-1}\right)^2\left(x^2-3\sqrt[3]{-3}2^{2/3}x+6\right)}-\frac{\sqrt[3]{-6}x+2^{2/3}\left((-1)^{2/3}-3\sqrt[3]{2}3^{2/3}\right)}{816293376\ 3^{2/3}\left(x^2+3(-2)^{2/3}\sqrt[3]{3}x+6\right)}+\frac{}{244\cdots}\right.$$

$$\Big\downarrow \text{ 2009}$$

$$1259712\left(\frac{\left(9-(-2)^{2/3}\sqrt[3]{3}\right)\arctan\left(\frac{2x+3(-2)^{2/3}\sqrt[3]{3}}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{1224440064\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}+\frac{(-1)^{2/3}\left(3(-3)^{2/3}-2^{2/3}\right)\arctan\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{-3}-\sqrt[3]{2}\cdots\right.}{\sqrt{3\left(4-3(-3)^{2/3}\sqrt[3]{\cdots}\right.}}\right.}{408146688\sqrt[6]{3}\left(1+\sqrt[3]{-1}\right)^2\sqrt{2\left(4-3(-3)^{2/3}\sqrt[3]{2}\cdots\right.}}\right.$$

input
```
Int[(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)^(-1),x]
```

output
```
1259712*(((9 - (-2)^(2/3)*3^(1/3))*ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sqr
t[6*(4 + 3*(-2)^(1/3)*3^(2/3))]])/(1224440064*Sqrt[6*(4 + 3*(-2)^(1/3)*3^(
2/3))]) + ((-1)^(2/3)*(3*(-3)^(2/3) - 2^(2/3))*ArcTan[(2^(1/6)*(3*(-3)^(1/
3) - 2^(1/3)*x))/Sqrt[3*(4 - 3*(-3)^(2/3)*2^(1/3))]])/(408146688*3^(1/6)*(
1 + (-1)^(1/3))^2*Sqrt[2*(4 - 3*(-3)^(2/3)*2^(1/3))]) - ((9 - 2^(2/3)*3^(1
/3))*ArcTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(2
/3))]])/(1224440064*Sqrt[6*(-4 + 3*2^(1/3)*3^(2/3))]) - Log[6 - 3*(-3)^(1/
3)*2^(2/3)*x + x^2]/(272097792*2^(2/3)*3^(1/3)*(1 + (-1)^(1/3))^2) - ((-1/
3)^(1/3)*Log[6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2])/(816293376*2^(2/3)) + Log[
6 + 3*2^(2/3)*3^(1/3)*x + x^2]/(816293376*2^(2/3)*3^(1/3)))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2466
```
Int[(u_.)*(Q6_)^(p_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
 x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Sim
p[1/(3^(3*p)*a^(2*p))   Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*
x + b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*
(-1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d,
 0] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coef
f[Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.14

method	result	size
default	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)}\frac{\ln\left(x-\_R\right)}{\_R^5+12\_R^3+162\_R^2+36\_R}\right)}{6}$	53
risch	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)}\frac{\ln\left(x-\_R\right)}{\_R^5+12\_R^3+162\_R^2+36\_R}\right)}{6}$	53

input
```
int(1/(x^6+18*x^4+324*x^3+108*x^2+216),x,method=_RETURNVERBOSE)
```

output
```
1/6*sum(1/(_R^5+12*_R^3+162*_R^2+36*_R)*ln(x-_R),_R=RootOf(_Z^6+18*_Z^4+32
4*_Z^3+108*_Z^2+216))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{216+108x^2+324x^3+18x^4+x^6}\,dx=\text{Timed out}$$

input
```
integrate(1/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.17

$$\int \frac{1}{216+108x^2+324x^3+18x^4+x^6}\,dx$$
$$=\text{RootSum}\left(34164988081841849499648t^6-3470494144278528t^4-86087932019712t^3-1530550080t^2+\right.$$

input
```
integrate(1/(x**6+18*x**4+324*x**3+108*x**2+216),x)
```

output
```
RootSum(34164988081841849499648*_t**6 - 3470494144278528*_t**4 - 860879320
19712*_t**3 - 1530550080*_t**2 + 69984*_t - 1, Lambda(_t, _t*log(185904446
699109611410573787136*_t**5/57121295165 + 63773012532679173827665592*_t**4/
57121295165 - 189046360023885643115552*_t**3/57121295165 - 4690805529151817
23968*_t**2/57121295165 - 24358640509989936*_t/57121295165 + x + 152427895
956/57121295165)))
```

Maxima [F]

$$\int \frac{1}{216+108x^2+324x^3+18x^4+x^6}\,dx=\int \frac{1}{x^6+18\,x^4+324\,x^3+108\,x^2+216}\,dx$$

input
```
integrate(1/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="maxima")
```

output
```
integrate(1/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

Giac [F]

$$\int \frac{1}{216+108x^2+324x^3+18x^4+x^6}\,dx=\int \frac{1}{x^6+18\,x^4+324\,x^3+108\,x^2+216}\,dx$$

input
```
integrate(1/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="giac")
```

output
```
integrate(1/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x)
```

Mupad [B] (verification not implemented)

Time = 22.71 (sec) , antiderivative size = 306, normalized size of antiderivative = 0.80

$$\int \frac{1}{216+108x^2+324x^3+18x^4+x^6}\,dx = \sum_{k=1}^{6} \ln\left(-\mathrm{root}\left(z^6-\frac{z^4}{9844416}-\frac{217\,z^3}{86118951168}-\frac{5\,z^2}{111610160713728}+\frac{z}{488182842961846272}-\frac{1}{34164988081841849499648},z,k\right)\,x\,6\right.$$
$$+\mathrm{root}\left(z^6-\frac{z^4}{9844416}-\frac{217\,z^3}{86118951168}-\frac{5\,z^2}{111610160713728}+\frac{z}{488182842961846272}-\frac{1}{34164988081841\cdots}\right.$$
$$-\mathrm{root}\left(z^6-\frac{z^4}{9844416}-\frac{217\,z^3}{86118951168}-\frac{5\,z^2}{111610160713728}+\frac{z}{488182842961846272}-\frac{1}{34164988081841\cdots}\right.$$
$$-\mathrm{root}\left(z^6-\frac{z^4}{9844416}-\frac{217\,z^3}{86118951168}-\frac{5\,z^2}{111610160713728}+\frac{z}{488182842961846272}-\frac{1}{34164988081841\cdots}\right.$$
$$-\mathrm{root}\left(z^6-\frac{z^4}{9844416}-\frac{217\,z^3}{86118951168}-\frac{5\,z^2}{111610160713728}+\frac{z}{488182842961846272}-\frac{1}{34164988081841\cdots}\right.$$
$$+944784\,\mathrm{root}\left(z^6-\frac{z^4}{9844416}-\frac{217\,z^3}{86118951168}-\frac{5\,z^2}{111610160713728}+\frac{z}{488182842961846272}-\frac{\cdots}{34164988\cdots}\right.$$
$$-16529940864\,\mathrm{root}\left(z^6-\frac{z^4}{9844416}-\frac{217\,z^3}{86118951168}-\frac{5\,z^2}{111610160713728}+\frac{z}{488182842961846272}-\frac{\cdots}{341\cdots}\right.$$
$$-33192121254912\,\mathrm{root}\left(z^6-\frac{z^4}{9844416}-\frac{217\,z^3}{86118951168}-\frac{5\,z^2}{111610160713728}+\frac{z}{488182842961846272}-\cdots\right.$$
$$-168897381688221696\,\mathrm{root}\left(z^6-\frac{z^4}{9844416}-\frac{217\,z^3}{86118951168}-\frac{5\,z^2}{111610160713728}+\frac{z}{48818284296184627\cdots}\right.$$
$$-\frac{z^4}{9844416}-\frac{217\,z^3}{86118951168}-\frac{5\,z^2}{111610160713728}+\frac{z}{488182842961846272}$$
$$\left.-\frac{1}{341649880818418494996648},z,k\right)$$

input
```
int(1/(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216),x)
```

output
```
symsum(log(349920*root(z^6 - z^4/9844416 - (217*z^3)/86118951168 - (5*z^2)
/111610160713728 + z/488182842961846272 - 1/34164988081841849499648, z, k)
^2*x - 6*root(z^6 - z^4/9844416 - (217*z^3)/86118951168 - (5*z^2)/11161016
0713728 + z/488182842961846272 - 1/34164988081841849499648, z, k)*x - 6122
200320*root(z^6 - z^4/9844416 - (217*z^3)/86118951168 - (5*z^2)/1116101607
13728 + z/488182842961846272 - 1/34164988081841849499648, z, k)^3*x - 2582
63796059136*root(z^6 - z^4/9844416 - (217*z^3)/86118951168 - (5*z^2)/11161
0160713728 + z/488182842961846272 - 1/34164988081841849499648, z, k)^4*x -
 6940988288557056*root(z^6 - z^4/9844416 - (217*z^3)/86118951168 - (5*z^2)
/111610160713728 + z/488182842961846272 - 1/34164988081841849499648, z, k)
^5*x + 944784*root(z^6 - z^4/9844416 - (217*z^3)/86118951168 - (5*z^2)/111
610160713728 + z/488182842961846272 - 1/34164988081841849499648, z, k)^2 -
 16529940864*root(z^6 - z^4/9844416 - (217*z^3)/86118951168 - (5*z^2)/1116
10160713728 + z/488182842961846272 - 1/34164988081841849499648, z, k)^3 -
33192121254912*root(z^6 - z^4/9844416 - (217*z^3)/86118951168 - (5*z^2)/11
1610160713728 + z/488182842961846272 - 1/34164988081841849499648, z, k)^4
- 168897381688221696*root(z^6 - z^4/9844416 - (217*z^3)/86118951168 - (5*z
^2)/111610160713728 + z/488182842961846272 - 1/34164988081841849499648, z,
 k)^5)*root(z^6 - z^4/9844416 - (217*z^3)/86118951168 - (5*z^2)/1116101607
13728 + z/488182842961846272 - 1/34164988081841849499648, z, k), k, 1, ...
```

Reduce [F]

$$\int \frac{1}{216+108x^2+324x^3+18x^4+x^6}\,dx=\int \frac{1}{x^6+18x^4+324x^3+108x^2+216}dx$$

input
```
int(1/(x^6+18*x^4+324*x^3+108*x^2+216),x)
```

output
```
int(1/(x**6 + 18*x**4 + 324*x**3 + 108*x**2 + 216),x)
```

3.290 $\int \frac{1}{(2+3x)(216+108x^2+324x^3+18x^4+x^6)}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 610

$$\int \frac{1}{(2+3x)\left(216+108x^2+324x^3+18x^4+x^6\right)}\,dx$$

$$= -\frac{(-1)^{2/3}\left(166-27(-2)^{2/3}\sqrt[3]{3}+6\sqrt[3]{-2}3^{2/3}\right)\arctan\left(\frac{3(-2)^{2/3}\sqrt[3]{3}+2x}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{648\ 2^{5/6}\sqrt[6]{3}\left(1-\sqrt[3]{-1}\right)\left(1+\sqrt[3]{-1}\right)^2\left(29-9(-2)^{2/3}\sqrt[3]{3}\right)\sqrt{4+3\sqrt[3]{-2}3^{2/3}}}$$

$$+\frac{(-1)^{2/3}\left(3(-6)^{2/3}-27\sqrt[3]{-3}-83\sqrt[3]{2}\right)\arctan\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{-3}-\sqrt[3]{2}x\right)}{\sqrt{3\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{648\sqrt[6]{6}\left(1+\sqrt[3]{-1}\right)^2\sqrt{4-3(-3)^{2/3}\sqrt[3]{2}}\left(29+9\sqrt[3]{-3}2^{2/3}\right)}$$

$$+\frac{\left(83\sqrt[3]{2}-27\sqrt[3]{3}-3\ 6^{2/3}\right)\operatorname{arctanh}\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{3}+\sqrt[3]{2}x\right)}{\sqrt{3\left(-4+3\sqrt[3]{2}3^{2/3}\right)}}\right)}{648\sqrt[6]{6}\left(1-\sqrt[3]{-1}\right)\left(1+\sqrt[3]{-1}\right)^2\left(29-9\ 2^{2/3}\sqrt[3]{3}\right)\sqrt{-4+3\sqrt[3]{2}3^{2/3}}}$$

$$+\frac{243\log(2+3x)}{125128}-\frac{\left((-6)^{2/3}+18\sqrt[3]{-3}+2\sqrt[3]{2}\right)\log\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)}{864\sqrt[3]{3}\left(1+\sqrt[3]{-1}\right)^2\left(29+9\sqrt[3]{-3}2^{2/3}\right)}$$

$$+\frac{(-1)^{2/3}\left(9\sqrt[3]{-6}+(-2)^{2/3}+3^{2/3}\right)\log\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)}{648\sqrt[3]{6}\left(58+9\ 2^{2/3}\sqrt[3]{3}-9i2^{2/3}3^{5/6}\right)}$$

$$-\frac{\left(54-6^{2/3}\left(2^{2/3}+3^{2/3}\right)\right)\log\left(6+3\ 2^{2/3}\sqrt[3]{3}x+x^2\right)}{7776\left(29-9\ 2^{2/3}\sqrt[3]{3}\right)}$$

output
```
-1/3888*(-1)^(2/3)*(166-27*(-2)^(2/3)*3^(1/3)+6*(-2)^(1/3)*3^(2/3))*arctan
((3*(-2)^(2/3)*3^(1/3)+2*x)/(24+18*(-2)^(1/3)*3^(2/3))^(1/2))*2^(1/6)*3^(5
/6)/(1-(-1)^(1/3))/(1+(-1)^(1/3))^2/(29-9*(-2)^(2/3)*3^(1/3))/(4+3*(-2)^(1
/3)*3^(2/3))^(1/2)+1/3888*(-1)^(2/3)*(3*(-6)^(2/3)-27*(-3)^(1/3)-83*2^(1/3
))*arctan(2^(1/6)*(3*(-3)^(1/3)-2^(1/3)*x)/(12-9*(-3)^(2/3)*2^(1/3))^(1/2)
)*6^(5/6)/(1+(-1)^(1/3))^2/(4-3*(-3)^(2/3)*2^(1/3))^(1/2)/(29+9*(-3)^(1/3)
*2^(2/3))+1/3888*(83*2^(1/3)-27*3^(1/3)-3*6^(2/3))*arctanh(2^(1/6)*(3*3^(1
/3)+2^(1/3)*x)/(-12+9*2^(1/3)*3^(2/3))^(1/2))*6^(5/6)/(1-(-1)^(1/3))/(1+(-
1)^(1/3))^2/(29-9*2^(2/3)*3^(1/3))/(-4+3*2^(1/3)*3^(2/3))^(1/2)+243/125128
*ln(2+3*x)-1/2592*((-6)^(2/3)+18*(-3)^(1/3)+2*2^(1/3))*ln(6-3*(-3)^(1/3)*2
^(2/3)*x+x^2)*3^(2/3)/(1+(-1)^(1/3))^2/(29+9*(-3)^(1/3)*2^(2/3))+1/3888*(-
1)^(2/3)*(9*(-6)^(1/3)+(-2)^(2/3)+3^(2/3))*ln(6+3*(-2)^(2/3)*3^(1/3)*x+x^2
)*6^(2/3)/(58+9*2^(2/3)*3^(1/3)-9*I*2^(2/3)*3^(5/6))-(54-6^(2/3)*(2^(2/3)+
3^(2/3)))*ln(6+3*2^(2/3)*3^(1/3)*x+x^2)/(225504-69984*2^(2/3)*3^(1/3))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.28

$$\int \frac{1}{(2+3x)\,(216+108x^2+324x^3+18x^4+x^6)}\,dx = \frac{243\log(2+3x)}{125128}$$
$$-\frac{81\text{RootSum}\left[125128+64608\#1-39612\#1^2+7292\#1^3+222\#1^4-12\#1^5+\#1^6\&,\frac{64608\log(2+3x-\#}{}\right.}{25}$$

input
```
Integrate[1/((2 + 3*x)*(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)),x]
```

output
```
(243*Log[2 + 3*x])/125128 - (81*RootSum[125128 + 64608*#1 - 39612*#1^2 + 7
292*#1^3 + 222*#1^4 - 12*#1^5 + #1^6 & , (64608*Log[2 + 3*x - #1] - 39612*
Log[2 + 3*x - #1]*#1 + 7292*Log[2 + 3*x - #1]*#1^2 + 222*Log[2 + 3*x - #1]
*#1^3 - 12*Log[2 + 3*x - #1]*#1^4 + Log[2 + 3*x - #1]*#1^5)/(10768 - 13204
*#1 + 3646*#1^2 + 148*#1^3 - 10*#1^4 + #1^5) & ])/250256
```

Rubi [A] (verified)

Time = 2.97 (sec) , antiderivative size = 592, normalized size of antiderivative = 0.97, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2466, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(3x+2)\left(x^6+18x^4+324x^3+108x^2+216\right)}dx$$

$$\Big\downarrow \text{2466}$$

$$1259712\int \left(\frac{(-1)^{2/3}\left(2\left(82-3(-3)^{2/3}\sqrt[3]{2}+9\sqrt[3]{-3}2^{2/3}\right)-\left(3-9(-3)^{2/3}\sqrt[3]{2}-\sqrt[3]{-3}2^{2/3}\right)x\right)}{272097792\sqrt[3]{2}3^{2/3}\left(1+\sqrt[3]{-1}\right)^2\left(29+9\sqrt[3]{-3}2^{2/3}\right)\left(x^2-3\sqrt[3]{-3}2^{2/3}x+6\right)}+\frac{1}{216221184(3x$$

$$\Big\downarrow \text{2009}$$

$$1259712\left(\frac{\sqrt[6]{-\frac{1}{3}}\left(166-27(-2)^{2/3}\sqrt[3]{3}+6\sqrt[3]{-2}3^{2/3}\right)\arctan\left(\frac{2x+3(-2)^{2/3}\sqrt[3]{3}}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{1224440064\ 2^{5/6}\sqrt{4+3\sqrt[3]{-2}3^{2/3}}\left(58i+9i2^{2/3}\sqrt[3]{3}+9\ 2^{2/3}3^{5/6}\right)}+\frac{(-1)^{2/3}\left(3(-6)^{2/3}-27\sqrt[3]{-}\right.}{816293376\sqrt[6]{6}\left(1+\sqrt[3]{-1}\right.}\right.$$

input
```
Int[1/((2 + 3*x)*(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)),x]
```

output
```
1259712*(((-1/3)^(1/6)*(166 - 27*(-2)^(2/3)*3^(1/3) + 6*(-2)^(1/3)*3^(2/3)
)*ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3))]])
/(1224440064*2^(5/6)*Sqrt[4 + 3*(-2)^(1/3)*3^(2/3)]*(58*I + (9*I)*2^(2/3)*
3^(1/3) + 9*2^(2/3)*3^(5/6))) + ((-1)^(2/3)*(3*(-6)^(2/3) - 27*(-3)^(1/3)
- 83*2^(1/3))*ArcTan[(2^(1/6)*(3*(-3)^(1/3) - 2^(1/3)*x))/Sqrt[3*(4 - 3*(-
3)^(2/3)*2^(1/3))]])/(816293376*6^(1/6)*(1 + (-1)^(1/3))^2*Sqrt[4 - 3*(-3)
^(2/3)*2^(1/3)]*(29 + 9*(-3)^(1/3)*2^(2/3))) + ((83*2^(1/3) - 27*3^(1/3) -
 3*6^(2/3))*ArcTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/
3)*3^(2/3))]])/(2448880128*6^(1/6)*(29 - 9*2^(2/3)*3^(1/3))*Sqrt[-4 + 3*2^
(1/3)*3^(2/3)]) + Log[2 + 3*x]/648663552 - ((54*(-1)^(1/3) + 3*(-2)^(2/3)*
3^(1/3) + 2*2^(1/3)*3^(2/3))*Log[6 - 3*(-3)^(1/3)*2^(2/3)*x + x^2])/(32651
73504*(1 + (-1)^(1/3))^2*(29 + 9*(-3)^(1/3)*2^(2/3))) + ((-1)^(2/3)*(9*(-6
)^(1/3) + (-2)^(2/3) + 3^(2/3))*Log[6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2])/(81
6293376*6^(1/3)*(58 + 9*2^(2/3)*3^(1/3) - (9*I)*2^(2/3)*3^(5/6))) - ((54 -
 6^(2/3)*(2^(2/3) + 3^(2/3)))*Log[6 + 3*2^(2/3)*3^(1/3)*x + x^2])/(9795520
512*(29 - 9*2^(2/3)*3^(1/3))))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2466
```
Int[(u_.)*(Q6_)^(p_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
 x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Sim
p[1/(3^(3*p)*a^(2*p))   Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*
x + b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*
(-1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d,
 0] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coef
f[Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.14 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.13

method	result
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(79206024\_Z^6+299024136\_Z^5+290189628\_Z^4+93944556\_Z^3-764154\_Z^2+1530\_Z-1\right)} \_R\ln\left(-13333991386286404702\right.\right.}{}$
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)} \frac{\left(-243\_R^5+162\_R^4-4482\_R^3-75744\_R^2+24252\_R-16168\right)\ln\left(x-\_R\right)}{\_R^5+12\_R^3+162\_R^2+36\_R}\right.}{750768}$

input
```
int(1/(2+3*x)/(x^6+18*x^4+324*x^3+108*x^2+216),x,method=_RETURNVERBOSE)
```

output
```
1/1944*sum(_R*ln(-13333991386286404702695164891 2056*_R^5-50345503479650079
4783556665488168*_R^4-48875110753810692507730937 9679144*_R^3-1583733838806
825390344811695011 20*_R^2+12155400606129270535134903 70296*_R+5885047678156
9891989955805*x-15311185049411846061713 39196),_R=RootOf(79206024*_Z^6+2990
24136*_Z^5+290189628*_Z^4+93944556*_Z^3-764154*_Z^2+1530*_Z-1))+243/125128
*ln(2+3*x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(2+3x)\left(216+108x^2+324x^3+18x^4+x^6\right)}\,dx=\text{Timed out}$$

input
```
integrate(1/(2+3*x)/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="fricas"
)
```

output
```
Timed out
```

Sympy [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.14

$$\int \frac{1}{(2+3x)\left(216+108x^2+324x^3+18x^4+x^6\right)}\,dx = \frac{243\log\left(x+\frac{2}{3}\right)}{125128} + \operatorname{RootSum}\Bigg(427499662870470694419195494 4t^6 + 83020921038875694284144 64t^5 + 414445022512084$$

input
```
integrate(1/(2+3*x)/(x**6+18*x**4+324*x**3+108*x**2+216),x)
```

output
```
243*log(x + 2/3)/125128 + RootSum(4274996628704706944191954944*_t**6 + 830
20921038875694284144 64*_t**5 + 4144450225120841023488*_t**4 + 690176868966
549504*_t**3 - 2887841890944*_t**2 + 2974320*_t - 1, Lambda(_t, _t*log(186
5499837922534525679600807491682172150890453244445108321670642511393734 6623
7534208*_t**6/575841655767724762771516233635574965835244780695944252957 5 +
 4372589027162460613842019053442868187129899840086560142288935242422884158
6688*_t**5/5758416557677247627715162336355749658352447806959442529575 - 52
2702975661454177912290454139420738467736654309175195594878675614610395234 3
04*_t**4/5758416557677247627715162336355749658352447806959442529575 - 3212
240798785473724711714709384998809672404735478426171723086471697202053 12*_t
**3/5758416557677247627715162336355749658352447806959442529575 - 586896240
2368684995985469815640275835805766608555222140897025048609100 8*_t**2/57584
16557677247627715162336355749658352447806959442529575 + 925384755274465519
6468299940585461581206375367903425714731382944*_t/230336662307089905108606
4934542299863340979122783777011 83 + x - 449582741794983102129276747942184 6
31844470423549131141927666/17275249673031742883145487009067248975057343420
878327588725)))
```

Maxima [F]

$$\int \frac{1}{(2+3x)\left(216+108x^2+324x^3+18x^4+x^6\right)}\,dx = \int \frac{1}{(x^6+18\,x^4+324\,x^3+108\,x^2+216)(3\,x+2)}\,dx$$

input
```
integrate(1/(2+3*x)/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="maxima"
)
```

output
```
-1/125128*integrate((243*x^5 - 162*x^4 + 4482*x^3 + 75744*x^2 - 24252*x +
16168)/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x) + 243/125128*log(3*x +
 2)
```

Giac [F]

$$\int \frac{1}{(2+3x)(216+108x^2+324x^3+18x^4+x^6)}\,dx = \int \frac{1}{(x^6+18\,x^4+324\,x^3+108\,x^2+216)(3\,x+2)}\,dx$$

input
```
integrate(1/(2+3*x)/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="giac")
```

output
```
integrate(1/((x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216)*(3*x + 2)), x)
```

Mupad [B] (verification not implemented)

Time = 22.15 (sec) , antiderivative size = 467, normalized size of antiderivative = 0.77

$$\int \frac{1}{(2+3x)(216+108x^2+324x^3+18x^4+x^6)}\,dx = \text{Too large to display}$$

input
```
int(1/((3*x + 2)*(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216)),x)
```

output
```
(243*log(x + 2/3))/125128 + symsum(log((2*root(z^6 + (243*z^5)/125128 + (2
98549*z^4)/307953021312 + (869857*z^3)/5387946060874752 - (4717*z^2)/69827
78094893678592 + (85*z)/122170685548259800645632 - 1/42749966287047069441
91954944, z, k))/729 + (7*root(z^6 + (243*z^5)/125128 + (298549*z^4)/307953
021312 + (869857*z^3)/5387946060874752 - (4717*z^2)/6982778094893678592 +
(85*z)/122170685548259800645632 - 1/427499662870470694419195494 4, z, k)*x)
/243 - (1445769776*root(z^6 + (243*z^5)/125128 + (298549*z^4)/307953021312
 + (869857*z^3)/5387946060874752 - (4717*z^2)/6982778094893678592 + (85*z)
/122170685548259800645632 - 1/4274996628704706944191954944, z, k)^2*x)/196
83 + (1605207476480*root(z^6 + (243*z^5)/125128 + (298549*z^4)/30795302131
2 + (869857*z^3)/5387946060874752 - (4717*z^2)/6982778094893678592 + (85*z
)/122170685548259800645632 - 1/4274996628704706944191954944, z, k)^3*x)/27
 - 11385414082473984*root(z^6 + (243*z^5)/125128 + (298549*z^4)/3079530213
12 + (869857*z^3)/5387946060874752 - (4717*z^2)/6982778094893678592 + (85*
z)/122170685548259800645632 - 1/4274996628704706944191954944, z, k)^4*x -
46778486686192041984*root(z^6 + (243*z^5)/125128 + (298549*z^4)/3079530213
12 + (869857*z^3)/5387946060874752 - (4717*z^2)/6982778094893678592 + (85*
z)/122170685548259800645632 - 1/4274996628704706944191954944, z, k)^5*x -
5076535126677026793 0624*root(z^6 + (243*z^5)/125128 + (298549*z^4)/3079530
21312 + (869857*z^3)/5387946060874752 - (4717*z^2)/6982778094893678592 ...
```

Reduce [F]

$$\int \frac{1}{(2+3x)\left(216+108x^2+324x^3+18x^4+x^6\right)}\,dx = \int \frac{1}{3x^7+2x^6+54x^5+1008x^4+972x^3+216x^2+648x+432}dx$$

input
```
int(1/(2+3*x)/(x^6+18*x^4+324*x^3+108*x^2+216),x)
```

output
```
int(1/(3*x**7 + 2*x**6 + 54*x**5 + 1008*x**4 + 972*x**3 + 216*x**2 + 648*x
 + 432),x)
```

3.291 $\int \frac{1}{432+648x+216x^2+972x^3+1008x^4+54x^5+2x^6+3x^7} \, dx$

Optimal result

Integrand size = 37, antiderivative size = 610

$$\int \frac{1}{432+648x+216x^2+972x^3+1008x^4+54x^5+2x^6+3x^7}\,dx$$
$$= -\frac{(-1)^{2/3}\left(166-27(-2)^{2/3}\sqrt[3]{3}+6\sqrt[3]{-2}3^{2/3}\right)\arctan\left(\frac{3(-2)^{2/3}\sqrt[3]{3}+2x}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{648\ 2^{5/6}\sqrt[6]{3}\left(1-\sqrt[3]{-1}\right)\left(1+\sqrt[3]{-1}\right)^2\left(29-9(-2)^{2/3}\sqrt[3]{3}\right)\sqrt{4+3\sqrt[3]{-2}3^{2/3}}}$$
$$+\frac{(-1)^{2/3}\left(3(-6)^{2/3}-27\sqrt[3]{-3}-83\sqrt[3]{2}\right)\arctan\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{-3}-\sqrt[3]{2}x\right)}{\sqrt{3\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{648\sqrt[6]{6}\left(1+\sqrt[3]{-1}\right)^2\sqrt{4-3(-3)^{2/3}\sqrt[3]{2}}\left(29+9\sqrt[3]{-3}2^{2/3}\right)}$$
$$+\frac{\left(83\sqrt[3]{2}-27\sqrt[3]{3}-3\ 6^{2/3}\right)\operatorname{arctanh}\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{3}+\sqrt[3]{2}x\right)}{\sqrt{3\left(-4+3\sqrt[3]{2}3^{2/3}\right)}}\right)}{648\sqrt[6]{6}\left(1-\sqrt[3]{-1}\right)\left(1+\sqrt[3]{-1}\right)^2\left(29-9\ 2^{2/3}\sqrt[3]{3}\right)\sqrt{-4+3\sqrt[3]{2}3^{2/3}}}$$
$$+\frac{243\log(2+3x)}{125128}-\frac{\left((-6)^{2/3}+18\sqrt[3]{-3}+2\sqrt[3]{2}\right)\log\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)}{864\sqrt[3]{3}\left(1+\sqrt[3]{-1}\right)^2\left(29+9\sqrt[3]{-3}2^{2/3}\right)}$$
$$+\frac{(-1)^{2/3}\left(9\sqrt[3]{-6}+(-2)^{2/3}+3^{2/3}\right)\log\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)}{648\sqrt[3]{6}\left(58+9\ 2^{2/3}\sqrt[3]{3}-9i2^{2/3}3^{5/6}\right)}$$
$$-\frac{\left(54-6^{2/3}\left(2^{2/3}+3^{2/3}\right)\right)\log\left(6+3\ 2^{2/3}\sqrt[3]{3}x+x^2\right)}{7776\left(29-9\ 2^{2/3}\sqrt[3]{3}\right)}$$

output
```
-1/3888*(-1)^(2/3)*(166-27*(-2)^(2/3)*3^(1/3)+6*(-2)^(1/3)*3^(2/3))*arctan
((3*(-2)^(2/3)*3^(1/3)+2*x)/(24+18*(-2)^(1/3)*3^(2/3))^(1/2))*2^(1/6)*3^(5
/6)/(1-(-1)^(1/3))/(1+(-1)^(1/3))^2/(29-9*(-2)^(2/3)*3^(1/3))/(4+3*(-2)^(1
/3)*3^(2/3))^(1/2)+1/3888*(-1)^(2/3)*(3*(-6)^(2/3)-27*(-3)^(1/3)-83*2^(1/3
))*arctan(2^(1/6)*(3*(-3)^(1/3)-2^(1/3)*x)/(12-9*(-3)^(2/3)*2^(1/3))^(1/2)
)*6^(5/6)/(1+(-1)^(1/3))^2/(4-3*(-3)^(2/3)*2^(1/3))^(1/2)/(29+9*(-3)^(1/3)
*2^(2/3))+1/3888*(83*2^(1/3)-27*3^(1/3)-3*6^(2/3))*arctanh(2^(1/6)*(3*3^(1
/3)+2^(1/3)*x)/(-12+9*2^(1/3)*3^(2/3))^(1/2))*6^(5/6)/(1-(-1)^(1/3))/(1+(-
1)^(1/3))^2/(29-9*2^(2/3)*3^(1/3))/(-4+3*2^(1/3)*3^(2/3))^(1/2)+243/125128
*ln(2+3*x)-1/2592*((-6)^(2/3)+18*(-3)^(1/3)+2*2^(1/3))*ln(6-3*(-3)^(1/3)*2
^(2/3)*x+x^2)*3^(2/3)/(1+(-1)^(1/3))^2/(29+9*(-3)^(1/3)*2^(2/3))+1/3888*(-
1)^(2/3)*(9*(-6)^(1/3)+(-2)^(2/3)+3^(2/3))*ln(6+3*(-2)^(2/3)*3^(1/3)*x+x^2
)*6^(2/3)/(58+9*2^(2/3)*3^(1/3)-9*I*2^(2/3)*3^(5/6))-(54-6^(2/3)*(2^(2/3)+
3^(2/3)))*ln(6+3*2^(2/3)*3^(1/3)*x+x^2)/(225504-69984*2^(2/3)*3^(1/3))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.23

$$\int \frac{1}{432+648x+216x^2+972x^3+1008x^4+54x^5+2x^6+3x^7}\,dx = \frac{243\log(2+3x)}{125128} - \frac{\text{RootSum}\left[216+108\#1^2+324\#1^3+18\#1^4+\#1^6\&, \frac{16168\log(x-\#1)-24252\log(x-\#1)\#1+75744\log(x-\#1)\ldots}{36\#1+16\ldots}\right.}{750768}$$

input
```
Integrate[(432 + 648*x + 216*x^2 + 972*x^3 + 1008*x^4 + 54*x^5 + 2*x^6 + 3
*x^7)^(-1),x]
```

output
```
(243*Log[2 + 3*x])/125128 - RootSum[216 + 108*#1^2 + 324*#1^3 + 18*#1^4 +
#1^6 & , (16168*Log[x - #1] - 24252*Log[x - #1]*#1 + 75744*Log[x - #1]*#1^
2 + 4482*Log[x - #1]*#1^3 - 162*Log[x - #1]*#1^4 + 243*Log[x - #1]*#1^5)/(
36*#1 + 162*#1^2 + 12*#1^3 + #1^5) & ]/750768
```

Rubi [A] (verified)

Time = 2.76 (sec) , antiderivative size = 474, normalized size of antiderivative = 0.78, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.054$, Rules used = {2462, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{3x^7+2x^6+54x^5+1008x^4+972x^3+216x^2+648x+432}dx$$

$$\Big\downarrow \text{2462}$$

$$\int \left(\frac{-243x^5+162x^4-4482x^3-75744x^2+24252x-16168}{125128\left(x^6+18x^4+324x^3+108x^2+216\right)}+\frac{729}{125128(3x+2)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(17703\sqrt[3]{-6}-51983(-2)^{2/3}-7752\ 3^{2/3}\right)\arctan\left(\frac{2x+3(-2)^{2/3}\sqrt[3]{3}}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{30406104\sqrt[6]{3}\sqrt{8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}3^{2/3}}}-\frac{(-1)^{2/3}\left(3876(-6)^{2/3}-17703\sqrt[3]{-3}+51983\sqrt[3]{2}\right)\arctan\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{-3}-\sqrt[3]{2}x\right)}{\sqrt{3\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{10135368\sqrt[6]{6}\left(1+\sqrt[3]{-1}\right)^2\sqrt{4-3(-3)^{2/3}\sqrt[3]{2}}}+\frac{\left(51983\sqrt[3]{2}+17703\sqrt[3]{3}+3876\ 6^{2/3}\right)\text{arctanh}\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{2}x+3\sqrt[3]{3}\right)}{\sqrt{3\left(3\sqrt[3]{2}3^{2/3}-4\right)}}\right)}{30406104\sqrt[6]{6}\sqrt{3\sqrt[3]{2}3^{2/3}-4}}+\frac{(-1)^{2/3}\left(10599+6561(-3)^{2/3}\sqrt[3]{2}-2750\sqrt[3]{-3}2^{2/3}\right)\log\left(x^2-3\sqrt[3]{-3}2^{2/3}x+6\right)}{6756912\sqrt[3]{2}3^{2/3}\left(1+\sqrt[3]{-1}\right)^2}-\frac{\left(39366+3^{2/3}\left(3533(-6)^{2/3}-5500\sqrt[3]{-2}\right)\right)\log\left(x^2+3(-2)^{2/3}\sqrt[3]{3}x+6\right)}{121624416}-\frac{\left(39366+6^{2/3}\left(2750\ 2^{2/3}+3533\ 3^{2/3}\right)\right)\log\left(x^2+3\ 2^{2/3}\sqrt[3]{3}x+6\right)}{121624416}+\frac{243\log(3x+2)}{125128}$$

input
```
Int[(432 + 648*x + 216*x^2 + 972*x^3 + 1008*x^4 + 54*x^5 + 2*x^6 + 3*x^7)^
(-1),x]
```

output
```
((17703*(-6)^(1/3) - 51983*(-2)^(2/3) - 7752*3^(2/3))*ArcTan[(3*(-2)^(2/3)
*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^(1/3)*3^(2/3))]])/(30406104*3^(1/6)*Sqr
t[8 + (9*I)*2^(1/3)*3^(1/6) + 3*2^(1/3)*3^(2/3)]) - ((-1)^(2/3)*(3876*(-6)
^(2/3) - 17703*(-3)^(1/3) + 51983*2^(1/3))*ArcTan[(2^(1/6)*(3*(-3)^(1/3) -
 2^(1/3)*x))/Sqrt[3*(4 - 3*(-3)^(2/3)*2^(1/3))]])/(10135368*6^(1/6)*(1 + (
-1)^(1/3))^2*Sqrt[4 - 3*(-3)^(2/3)*2^(1/3)]) + ((51983*2^(1/3) + 17703*3^(
1/3) + 3876*6^(2/3))*ArcTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4
+ 3*2^(1/3)*3^(2/3))]])/(30406104*6^(1/6)*Sqrt[-4 + 3*2^(1/3)*3^(2/3)]) +
(243*Log[2 + 3*x])/125128 + ((-1)^(2/3)*(10599 + 6561*(-3)^(2/3)*2^(1/3) -
 2750*(-3)^(1/3)*2^(2/3))*Log[6 - 3*(-3)^(1/3)*2^(2/3)*x + x^2])/(6756912*
2^(1/3)*3^(2/3)*(1 + (-1)^(1/3))^2) - ((39366 + 3^(2/3)*(3533*(-6)^(2/3) -
 5500*(-2)^(1/3)))*Log[6 + 3*(-2)^(2/3)*3^(1/3)*x + x^2])/121624416 - ((39
366 + 6^(2/3)*(2750*2^(2/3) + 3533*3^(2/3)))*Log[6 + 3*2^(2/3)*3^(1/3)*x +
 x^2])/121624416
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.13

method	result
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(79206024\_Z^6+299024136\_Z^5+290189628\_Z^4+93944556\_Z^3-764154\_Z^2+1530\_Z-1\right)}\_R\ln\left(-133339913862864047 0\right.\right.}{}$
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)}\frac{\left(-243\_R^5+162\_R^4-4482\_R^3-75744\_R^2+24252\_R-16168\right)\ln\left(x-\_R\right)}{\_R^5+12\_R^3+162\_R^2+36\_R}\right.}{750768}$

input
```
int(1/(3*x^7+2*x^6+54*x^5+1008*x^4+972*x^3+216*x^2+648*x+432),x,method=_RE
TURNVERBOSE)
```

output
```
1/1944*sum(_R*ln(-133339913862864047026951648912056*_R^5-50345503479650079
4783556665488168*_R^4-488751107538106925077309379679144*_R^3-1583733838806
825390344811695 01120*_R^2+12155400606129270535134903702 96*_R+5885047678156
9891989955805*x-153111850494118460617133919 6),_R=RootOf(79206024*_Z^6+2990
24136*_Z^5+290189628*_Z^4+93944556*_Z^3-764154*_Z^2+1530*_Z-1))+243/125128
*ln(2+3*x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{432+648x+216x^2+972x^3+1008x^4+54x^5+2x^6+3x^7}\,dx=\text{Timed out}$$

input
```
integrate(1/(3*x^7+2*x^6+54*x^5+1008*x^4+972*x^3+216*x^2+648*x+432),x, alg
orithm="fricas")
```

output
```
Timed out
```

Sympy [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.14

$$\int \frac{1}{432+648x+216x^2+972x^3+1008x^4+54x^5+2x^6+3x^7}\,dx = \frac{243\log\left(x+\frac{2}{3}\right)}{125128}$$
$$+\mathrm{RootSum}\Big(427499662870470694419195494 4t^6+83020921038875694284144 64t^5+414445022512084$$

input
```
integrate(1/(3*x**7+2*x**6+54*x**5+1008*x**4+972*x**3+216*x**2+648*x+432),
x)
```

output
```
243*log(x + 2/3)/125128 + RootSum(4274996628704706944191954944*_t**6 + 830
20921038875694284144 64*_t**5 + 414445022512084 1023488*_t**4 + 690176868966
549504*_t**3 - 2887841890944*_t**2 + 2974320*_t - 1, Lambda(_t, _t*log(186
549983792253452567960080749168217215089045324444510832167064251139373466 23
7534208*_t**6/575841655767724762771516233635574965835244780695944252 9575 +
 437258902716246061384201905344286818712989984008656014228893524242288415 8
6688*_t**5/575841655767724762771516233635574965835244780695944252 9575 - 52
270297566145417791229045413942073846773665430917519559487867561461039523 43
04*_t**4/575841655767724762771516233635574965835244780695944252 9575 - 3212
2407987854737247117147093849988096724047354784261717230864716972020531 2*_t
**3/575841655767724762771516233635574965835244780695944252 9575 - 586896240
2368684995985469815640275835805766608555222140897025048609100 8*_t**2/57584
16557677247627715162336355749658352447806959442529575 + 925384755274465519
6468299940585461581206375367903425714731382944*_t/230336662307089905108606
4934542299863340979122783777011 83 + x - 44958274179498310212927674794218 46
318444704235491311419276 66/172752496730317428831454870090672489750573434 20
878327588725)))
```

Maxima [F]

$$\int \frac{1}{432+648x+216x^2+972x^3+1008x^4+54x^5+2x^6+3x^7}\,dx$$
$$= \int \frac{1}{3\,x^7+2\,x^6+54\,x^5+1008\,x^4+972\,x^3+216\,x^2+648\,x+432}\,dx$$

input
```
integrate(1/(3*x^7+2*x^6+54*x^5+1008*x^4+972*x^3+216*x^2+648*x+432),x, alg
orithm="maxima")
```

output
```
-1/125128*integrate((243*x^5 - 162*x^4 + 4482*x^3 + 75744*x^2 - 24252*x +
16168)/(x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216), x) + 243/125128*log(3*x +
 2)
```

Giac [F]

$$\int \frac{1}{432+648x+216x^2+972x^3+1008x^4+54x^5+2x^6+3x^7}\,dx = \int \frac{1}{3\,x^7+2\,x^6+54\,x^5+1008\,x^4+972\,x^3+216\,x^2+648\,x+432}\,dx$$

input
```
integrate(1/(3*x^7+2*x^6+54*x^5+1008*x^4+972*x^3+216*x^2+648*x+432),x, alg
orithm="giac")
```

output
```
integrate(1/(3*x^7 + 2*x^6 + 54*x^5 + 1008*x^4 + 972*x^3 + 216*x^2 + 648*x
 + 432), x)
```

Mupad [B] (verification not implemented)

Time = 22.02 (sec) , antiderivative size = 467, normalized size of antiderivative = 0.77

$$\int \frac{1}{432+648x+216x^2+972x^3+1008x^4+54x^5+2x^6+3x^7}\,dx = \text{Too large to display}$$

input
```
int(1/(648*x + 216*x^2 + 972*x^3 + 1008*x^4 + 54*x^5 + 2*x^6 + 3*x^7 + 432
),x)
```

output
```
(243*log(x + 2/3))/125128 + symsum(log((2*root(z^6 + (243*z^5)/125128 + (2
98549*z^4)/307953021312 + (869857*z^3)/5387946060874752 - (4717*z^2)/69827
78094893678592 + (85*z)/122170685548259800645632 - 1/42749966287047069441919
1954944, z, k))/729 + (7*root(z^6 + (243*z^5)/125128 + (298549*z^4)/307953
021312 + (869857*z^3)/5387946060874752 - (4717*z^2)/6982778094893678592 +
(85*z)/122170685548259800645632 - 1/427499662870470694419195 4944, z, k)*x)
/243 - (1445769776*root(z^6 + (243*z^5)/125128 + (298549*z^4)/307953021312
 + (869857*z^3)/5387946060874752 - (4717*z^2)/6982778094893678592 + (85*z)
/122170685548259800645632 - 1/4274996628704706944191954944, z, k)^2*x)/196
83 + (1605207476480*root(z^6 + (243*z^5)/125128 + (298549*z^4)/30795302131
2 + (869857*z^3)/5387946060874752 - (4717*z^2)/6982778094893678592 + (85*z
)/122170685548259800645632 - 1/4274996628704706944191954944, z, k)^3*x)/27
 - 11385414082473984*root(z^6 + (243*z^5)/125128 + (298549*z^4)/3079530213
12 + (869857*z^3)/5387946060874752 - (4717*z^2)/6982778094893678592 + (85*
z)/122170685548259800645632 - 1/4274996628704706944191954944, z, k)^4*x -
46778486686192041984*root(z^6 + (243*z^5)/125128 + (298549*z^4)/3079530213
12 + (869857*z^3)/5387946060874752 - (4717*z^2)/6982778094893678592 + (85*
z)/122170685548259800645632 - 1/4274996628704706944191954944, z, k)^5*x -
50765351266770267930624*root(z^6 + (243*z^5)/125128 + (298549*z^4)/3079530
21312 + (869857*z^3)/5387946060874752 - (4717*z^2)/6982778094893678592 ...
```

Reduce [F]

$$\int \frac{1}{432+648x+216x^2+972x^3+1008x^4+54x^5+2x^6+3x^7}\,dx$$
$$=\int \frac{1}{3x^7+2x^6+54x^5+1008x^4+972x^3+216x^2+648x+432}dx$$

input
```
int(1/(3*x^7+2*x^6+54*x^5+1008*x^4+972*x^3+216*x^2+648*x+432),x)
```

output
```
int(1/(3*x**7 + 2*x**6 + 54*x**5 + 1008*x**4 + 972*x**3 + 216*x**2 + 648*x
 + 432),x)
```

3.292 $\int \frac{1}{(2+3x)^2(216+108x^2+324x^3+18x^4+x^6)} \, dx$

Optimal result

Integrand size = 30, antiderivative size = 651

$$\int \frac{1}{(2+3x)^2(216+108x^2+324x^3+18x^4+x^6)}\,dx$$
$$= -\frac{243}{125128(2+3x)}$$
$$-\frac{(-1)^{2/3}\left(274-783(-2)^{2/3}\sqrt[3]{3}-318\sqrt[3]{-2}3^{2/3}\right)\arctan\left(\frac{3(-2)^{2/3}\sqrt[3]{3}+2x}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{648\,2^{5/6}\sqrt[6]{3}\left(1-\sqrt[3]{-1}\right)\left(1+\sqrt[3]{-1}\right)^2\left(29-9(-2)^{2/3}\sqrt[3]{3}\right)^2\sqrt{4+3\sqrt[3]{-2}3^{2/3}}}$$
$$-\frac{(-1)^{2/3}\left(159(-6)^{2/3}+783\sqrt[3]{-3}+137\sqrt[3]{2}\right)\arctan\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{-3}-\sqrt[3]{2}x\right)}{\sqrt{3\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{648\sqrt[6]{6}\left(1+\sqrt[3]{-1}\right)^2\sqrt{4-3(-3)^{2/3}\sqrt[3]{2}}\left(29+9\sqrt[3]{-3}2^{2/3}\right)^2}$$
$$+\frac{\left(137\sqrt[3]{2}-783\sqrt[3]{3}+159\,6^{2/3}\right)\operatorname{arctanh}\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{3}+\sqrt[3]{2}x\right)}{\sqrt{3\left(-4+3\sqrt[3]{2}3^{2/3}\right)}}\right)}{648\sqrt[6]{6}\left(1-\sqrt[3]{-1}\right)\left(1+\sqrt[3]{-1}\right)^2\left(29-9\,2^{2/3}\sqrt[3]{3}\right)^2\sqrt{-4+3\sqrt[3]{2}3^{2/3}}}$$
$$-\frac{490617\log(2+3x)}{489281762}-\frac{\left(83(-6)^{2/3}+36\sqrt[3]{-3}-52\sqrt[3]{2}\right)\log\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)}{864\sqrt[3]{3}\left(1+\sqrt[3]{-1}\right)^2\left(29+9\sqrt[3]{-3}2^{2/3}\right)^2}$$
$$+\frac{(-1)^{2/3}\left(18\sqrt[3]{-6}-26(-2)^{2/3}+83\,3^{2/3}\right)\log\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)}{432\sqrt[3]{6}\left(1-\sqrt[3]{-1}\right)\left(1+\sqrt[3]{-1}\right)^2\left(29-9(-2)^{2/3}\sqrt[3]{3}\right)^2}$$
$$-\frac{\left(108-249\,2^{2/3}\sqrt[3]{3}+52\sqrt[3]{2}3^{2/3}\right)\log\left(6+3\,2^{2/3}\sqrt[3]{3}x+x^2\right)}{7776\left(841-522\,2^{2/3}\sqrt[3]{3}+162\sqrt[3]{2}3^{2/3}\right)}$$

output
```
-243/(250256+375384*x)-1/3888*(-1)^(2/3)*(274-783*(-2)^(2/3)*3^(1/3)-318*(
-2)^(1/3)*3^(2/3))*arctan((3*(-2)^(2/3)*3^(1/3)+2*x)/(24+18*(-2)^(1/3)*3^(
2/3))^(1/2))*2^(1/6)*3^(5/6)/(1-(-1)^(1/3))/(1+(-1)^(1/3))^2/(29-9*(-2)^(2
/3)*3^(1/3))^2/(4+3*(-2)^(1/3)*3^(2/3))^(1/2)-1/3888*(-1)^(2/3)*(159*(-6)^
(2/3)+783*(-3)^(1/3)+137*2^(1/3))*arctan(2^(1/6)*(3*(-3)^(1/3)-2^(1/3)*x)/
(12-9*(-3)^(2/3)*2^(1/3))^(1/2))*6^(5/6)/(1+(-1)^(1/3))^2/(4-3*(-3)^(2/3)*
2^(1/3))^(1/2)/(29+9*(-3)^(1/3)*2^(2/3))^2+1/3888*(137*2^(1/3)-783*3^(1/3)
+159*6^(2/3))*arctanh(2^(1/6)*(3*3^(1/3)+2^(1/3)*x)/(-12+9*2^(1/3)*3^(2/3)
)^(1/2))*6^(5/6)/(1-(-1)^(1/3))/(1+(-1)^(1/3))^2/(29-9*2^(2/3)*3^(1/3))^2/
(-4+3*2^(1/3)*3^(2/3))^(1/2)-490617/489281762*ln(2+3*x)-1/2592*(83*(-6)^(2
/3)+36*(-3)^(1/3)-52*2^(1/3))*ln(6-3*(-3)^(1/3)*2^(2/3)*x+x^2)*3^(2/3)/(1+
(-1)^(1/3))^2/(29+9*(-3)^(1/3)*2^(2/3))^2+1/2592*(-1)^(2/3)*(18*(-6)^(1/3)
-26*(-2)^(2/3)+83*3^(2/3))*ln(6+3*(-2)^(2/3)*3^(1/3)*x+x^2)*6^(2/3)/(1-(-1
)^(1/3))/(1+(-1)^(1/3))^2/(29-9*(-2)^(2/3)*3^(1/3))^2-(108-249*2^(2/3)*3^(
1/3)+52*2^(1/3)*3^(2/3))*ln(6+3*2^(2/3)*3^(1/3)*x+x^2)/(6539616-4059072*2^
(2/3)*3^(1/3)+1259712*2^(1/3)*3^(2/3))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.28

$$\int \frac{1}{(2+3x)^2(216+108x^2+324x^3+18x^4+x^6)}\,dx$$
$$= -\frac{243}{125128(2+3x)} - \frac{490617\log(2+3x)}{489281762}$$
$$+ \frac{81\text{RootSum}\left[125128+64608\#1-39612\#1^2+7292\#1^3+222\#1^4-12\#1^5+\#1^6\&, \frac{1141345500\log(2+3}{}\right.}{}$$

input
```
Integrate[1/((2 + 3*x)^2*(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)),x]
```

output
```
-243/(125128*(2 + 3*x)) - (490617*Log[2 + 3*x])/489281762 + (81*RootSum[12
5128 + 64608*#1 - 39612*#1^2 + 7292*#1^3 + 222*#1^4 - 12*#1^5 + #1^6 & , (
1141345500*Log[2 + 3*x - #1] - 433960684*Log[2 + 3*x - #1]*#1 + 55417890*L
og[2 + 3*x - #1]*#1^2 + 1980564*Log[2 + 3*x - #1]*#1^3 - 112553*Log[2 + 3*
x - #1]*#1^4 + 8076*Log[2 + 3*x - #1]*#1^5)/(10768 - 13204*#1 + 3646*#1^2
+ 148*#1^3 - 10*#1^4 + #1^5) & ])/3914254096
```

Rubi [A] (verified)

Time = 3.68 (sec) , antiderivative size = 647, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2466, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(3x+2)^2\left(x^6+18x^4+324x^3+108x^2+216\right)}\,dx$$

$$\Big\downarrow \text{2466}$$

$$1259712\int \left(\frac{(-1)^{2/3}\left(-\left(\left(249-18(-3)^{2/3}\sqrt[3]{2}+26\sqrt[3]{-3}2^{2/3}\right)x\right)+765\sqrt[3]{-3}2^{2/3}+237(-3)^{2/3}\sqrt[3]{2}+299\right)}{272097792\sqrt[3]{2}3^{2/3}\left(1+\sqrt[3]{-1}\right)^2\left(29+9\sqrt[3]{-3}2^{2/3}\right)^2\left(x^2-3\sqrt[3]{-3}2^{2/3}x+6\right)}-\frac{28\cdots}{}\right.$$

$$\Big\downarrow \text{2009}$$

$$1259712\left(-\frac{(-1)^{2/3}\left(274-783(-2)^{2/3}\sqrt[3]{3}-318\sqrt[3]{-2}3^{2/3}\right)\arctan\left(\frac{2x+3(-2)^{2/3}\sqrt[3]{3}}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{816293376\,2^{5/6}\sqrt[6]{3}\left(1-\sqrt[3]{-1}\right)\left(1+\sqrt[3]{-1}\right)^2\left(29-9(-2)^{2/3}\sqrt[3]{3}\right)^2\sqrt{4+3\sqrt[3]{-2}3^{2/3}}}-\frac{(-1)^{2/3}\left(159(-\cdots\right.}{816293376\cdots}\right.$$

input
```
Int[1/((2 + 3*x)^2*(216 + 108*x^2 + 324*x^3 + 18*x^4 + x^6)),x]
```

output

```
1259712*(-1/648663552*1/(2 + 3*x) - ((-1)^(2/3)*(274 - 783*(-2)^(2/3)*3^(1
/3) - 318*(-2)^(1/3)*3^(2/3))*ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sqrt[6*(
4 + 3*(-2)^(1/3)*3^(2/3))]])/(816293376*2^(5/6)*3^(1/6)*(1 - (-1)^(1/3))*(
1 + (-1)^(1/3))^2*(29 - 9*(-2)^(2/3)*3^(1/3))^2*Sqrt[4 + 3*(-2)^(1/3)*3^(2
/3)]) - ((-1)^(2/3)*(159*(-6)^(2/3) + 783*(-3)^(1/3) + 137*2^(1/3))*ArcTan
[(2^(1/6)*(3*(-3)^(1/3) - 2^(1/3)*x))/Sqrt[3*(4 - 3*(-3)^(2/3)*2^(1/3))]])
/(816293376*6^(1/6)*(1 + (-1)^(1/3))^2*Sqrt[4 - 3*(-3)^(2/3)*2^(1/3)]*(29
+ 9*(-3)^(1/3)*2^(2/3))^2) + ((137*2^(1/3) - 783*3^(1/3) + 159*6^(2/3))*Ar
cTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 + 3*2^(1/3)*3^(2/3))]])
/(2448880128*6^(1/6)*Sqrt[-4 + 3*2^(1/3)*3^(2/3)]*(841 - 522*2^(2/3)*3^(1/
3) + 162*2^(1/3)*3^(2/3))) - (673*Log[2 + 3*x])/845478884736 - ((108*(-1)^
(1/3) + 249*(-2)^(2/3)*3^(1/3) - 52*2^(1/3)*3^(2/3))*Log[6 - 3*(-3)^(1/3)*
2^(2/3)*x + x^2])/(3265173504*(1 + (-1)^(1/3))^2*(29 + 9*(-3)^(1/3)*2^(2/3
))^2) + ((-1)^(2/3)*(18*(-6)^(1/3) - 26*(-2)^(2/3) + 83*3^(2/3))*Log[6 + 3
*(-2)^(2/3)*3^(1/3)*x + x^2])/(544195584*6^(1/3)*(1 - (-1)^(1/3))*(1 + (-1
)^(1/3))^2*(29 - 9*(-2)^(2/3)*3^(1/3))^2) - ((108 + 6^(2/3)*(26*2^(2/3) -
83*3^(2/3)))*Log[6 + 3*2^(2/3)*3^(1/3)*x + x^2])/(9795520512*(841 - 522*2^
(2/3)*3^(1/3) + 162*2^(1/3)*3^(2/3))))
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2466

```
Int[(u_.)*(Q6_)^(p_), x_Symbol] :> With[{a = Coeff[Q6, x, 0], b = Coeff[Q6,
 x, 2], c = Coeff[Q6, x, 3], d = Coeff[Q6, x, 4], e = Coeff[Q6, x, 6]}, Sim
p[1/(3^(3*p)*a^(2*p))   Int[ExpandIntegrand[u*(3*a + 3*Rt[a, 3]^2*Rt[c, 3]*
x + b*x^2)^p*(3*a - 3*(-1)^(1/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p*(3*a + 3*
(-1)^(2/3)*Rt[a, 3]^2*Rt[c, 3]*x + b*x^2)^p, x], x], x] /; EqQ[b^2 - 3*a*d,
 0] && EqQ[b^3 - 27*a^2*e, 0]] /; ILtQ[p, 0] && PolyQ[Q6, x, 6] && EqQ[Coef
f[Q6, x, 1], 0] && EqQ[Coeff[Q6, x, 5], 0] && RationalFunctionQ[u, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.15 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.13

method	result
risch	$-\frac{81}{125128\left(x+\frac{2}{3}\right)}-\frac{490617\ln(2+3x)}{489281762}+\frac{\left(\sum_{\_R=\text{RootOf}\left(9900753\_Z^6-603729730584\_Z^5-7778441183375403\_Z^4-242862251317146570\cdots\right.}\cdots\right.}{\cdots}$
default	$\frac{\left(\sum_{\_R=\text{RootOf}\left(\_Z^6+18\_Z^4+324\_Z^3+108\_Z^2+216\right)}\frac{\left(1962468\_R^5-2575233\_R^4+37885860\_R^3+587214738\_R^2-574433916\_R+\cdots\right.}{\_R^5+12\_R^3+162\_R^2+36\_R}\cdots\right.}{11742762288}$

input
```
int(1/(2+3*x)^2/(x^6+18*x^4+324*x^3+108*x^2+216),x,method=_RETURNVERBOSE)
```

output
```
-81/125128/(x+2/3)-490617/489281762*ln(2+3*x)+1/60812208*sum(_R*ln(-581639
62462898967121550908038616382340917850 3*_R^5+354681896500547050248070950 23
880440332624355193966*_R^4+456906859878617438599177856847948959539967709 96
8421774*_R^3+142605731132969340337394235830545175757732842486715969550 8*_R
^2+124961061015846736349882751931780734837437719742418136384 47*_R+37117801
71170710248762306868718224813196689272625039439882*x-1016883406789075436 36
486542003903494388624803640301039715 24),_R=RootOf(9900753*_Z^6-60372973058
4*_Z^5-7778441183375403*_Z^4-2428622513171465709 6*_Z^3-2490748308425329890
21*_Z^2+62482523725340232084*_Z-93610072182763081420 1))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(2+3x)^2\left(216+108x^2+324x^3+18x^4+x^6\right)}\,dx=\text{Timed out}$$

input
```
integrate(1/(2+3*x)^2/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="frica
s")
```

output
```
Timed out
```

Sympy [A] (verification not implemented)

Time = 0.77 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.14

$$\int \frac{1}{(2+3x)^2 (216+108x^2+324x^3+18x^4+x^6)} \, dx = -\frac{490617 \log \left(x+\frac{2}{3}\right)}{489281762}$$
$$+\operatorname{RootSum}\Big(534921778156562570512850938232832t^6 - 536381566647968085631001690112t^5 - 11364$$
$$-\frac{243}{375384x+250256}$$

input

```
integrate(1/(2+3*x)**2/(x**6+18*x**4+324*x**3+108*x**2+216),x)
```

output

```
-490617*log(x + 2/3)/489281762 + RootSum(534921778156562570512850938232832
*_t**6 - 536381566647968085631001690112*_t**5 - 113640485675194290961317888*_t**4 - 58345820730614723051 52*_t**3 - 983985748671744*_t**2 + 4059072*_
t - 1, Lambda(_t, _t*log(-345575096043868266538272546680760901866749537221
277467418251907903386869842661563803274057343858574507122234967261272811870
06176*_t**6/265882467743308560111128055912060012355441739830233778570332881
0475949510141620376821590824383841 1 + 853869161004594421797568445431927726
629256808703577315178764539835613130871755239221659482772635333796143919723
82900224*_t**5/2658824677433085601111280559120600123554417398302337785703328
81047594951014162037682159082438384 11 + 420878873454625108135580681702917
743040121248959374155837553148505100718767048944752503788482467955027171389
7114626752512*_t**4/265882467743308560111128055912060012355441739830233778
5703328810475949510141620376821590824383841 1 + 7737433282114086877359283829
717411573483467321535165616765923328783445497409575338022682643433310072457
17174104207360*_t**3/2658824677433085601111280559120600123554417398302337785
70332881047594951014162037682159082438384 11 + 71289108234116885068004528142
442799363419454136433431932432576674957794448522581503839435152919354449612
7289856*_t**2/501665033477940679454958596060490589349890075151384487868552605
75017915285690950506077514034687 + 18770370275530568311037940905064738878658
18350246954934839146770978639840753738187334220 05751088...
```

Maxima [F]

$$\int \frac{1}{(2+3x)^2 (216+108x^2+324x^3+18x^4+x^6)} \, dx = \int \frac{1}{(x^6+18x^4+324x^3+108x^2+216)(3x+2)^2} \, dx$$

input
```
integrate(1/(2+3*x)^2/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="maxim
a")
```

output
```
-243/125128/(3*x + 2) + 1/1957127048*integrate((1962468*x^5 - 2575233*x^4
+ 37885860*x^3 + 587214738*x^2 - 574433916*x + 509397788)/(x^6 + 18*x^4 +
324*x^3 + 108*x^2 + 216), x) - 490617/489281762*log(3*x + 2)
```

Giac [F]

$$\int \frac{1}{(2+3x)^2 (216+108x^2+324x^3+18x^4+x^6)} \, dx = \int \frac{1}{(x^6+18x^4+324x^3+108x^2+216)(3x+2)^2} \, dx$$

input
```
integrate(1/(2+3*x)^2/(x^6+18*x^4+324*x^3+108*x^2+216),x, algorithm="giac"
)
```

output
```
integrate(1/((x^6 + 18*x^4 + 324*x^3 + 108*x^2 + 216)*(3*x + 2)^2), x)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 480, normalized size of antiderivative = 0.74

$$\int \frac{1}{(2+3x)^2 (216+108x^2+324x^3+18x^4+x^6)} dx = \text{Too large to display}$$

input
```
int(1/((3*x + 2)^2*(108*x^2 + 324*x^3 + 18*x^4 + x^6 + 216)),x)
```

output
```
symsum(log(x/15657016384 - (241256*root(z^6 - (490617*z^5)/489281762 - (68
2182427*z^4)/3211128804227328 - (17022113*z^3)/1560608598854481408 - (8036
21*z^2)/436870528728928107429888 + (29*z)/382174338532066308379666O224 - 1
/534921778156562570512850938232832, z, k))/733922643 - (1678771*root(z^6 -
 (490617*z^5)/489281762 - (682182427*z^4)/3211128804227328 - (17022113*z^3
)/1560608598854481408 - (803621*z^2)/436870528728928107429888 + (29*z)/382
174338532066308379666O224 - 1/534921778156562570512850938232832, z, k)*x)/
6605303787 + (305207778434812192*root(z^6 - (490617*z^5)/489281762 - (6821
82427*z^4)/3211128804227328 - (17022113*z^3)/1560608598854481408 - (803621
*z^2)/436870528728928107429888 + (29*z)/3821743385320663083796660224 - 1/5
34921778156562570512850938232832, z, k)^2*x)/4815266460723 + (244884659349
9859523392*root(z^6 - (490617*z^5)/489281762 - (682182427*z^4)/32111288042
27328 - (17022113*z^3)/1560608598854481408 - (803621*z^2)/4368705287289281
07429888 + (29*z)/3821743385320663083796660224 - 1/53492177815656257051285
0938232832, z, k)^3*x)/6605303787 + (1533157891304935213946880*root(z^6 -
(490617*z^5)/489281762 - (682182427*z^4)/3211128804227328 - (17022113*z^3)
/1560608598854481408 - (803621*z^2)/436870528728928107429888 + (29*z)/3821
743385320663083796660224 - 1/534921778156562570512850938232832, z, k)^4*x)
/244640881 + (2600474335927184107929600*root(z^6 - (490617*z^5)/489281762 -
 (682182427*z^4)/3211128804227328 - (17022113*z^3)/1560608598854481408 ...
```

Reduce [F]

$$\int \frac{1}{(2+3x)^2 (216+108x^2+324x^3+18x^4+x^6)} dx = \int \frac{1}{9x^8+12x^7+166x^6+3132x^5+4932x^4+2592x^3+2376x^2+2592x+864} dx$$

input
```
int(1/(2+3*x)^2/(x^6+18*x^4+324*x^3+108*x^2+216),x)
```

output
```
int(1/(9*x**8 + 12*x**7 + 166*x**6 + 3132*x**5 + 4932*x**4 + 2592*x**3 + 2
376*x**2 + 2592*x + 864),x)
```

3.293 $\int \frac{1}{864+2592x+2376x^2+2592x^3+4932x^4+3132x^5+166x^6+12x^7+9x^8} \, d$

Optimal result

Integrand size = 42, antiderivative size = 651

$$\int \frac{1}{864+2592x+2376x^2+2592x^3+4932x^4+3132x^5+166x^6+12x^7+9x^8}\,dx$$
$$= -\frac{243}{125128(2+3x)}$$
$$-\frac{(-1)^{2/3}\left(274-783(-2)^{2/3}\sqrt[3]{3}-318\sqrt[3]{-2}3^{2/3}\right)\arctan\left(\frac{3(-2)^{2/3}\sqrt[3]{3}+2x}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{648\ 2^{5/6}\sqrt[6]{3}\left(1-\sqrt[3]{-1}\right)\left(1+\sqrt[3]{-1}\right)^2\left(29-9(-2)^{2/3}\sqrt[3]{3}\right)^2\sqrt{4+3\sqrt[3]{-2}3^{2/3}}}$$
$$-\frac{(-1)^{2/3}\left(159(-6)^{2/3}+783\sqrt[3]{-3}+137\sqrt[3]{2}\right)\arctan\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{-3}-\sqrt[3]{2}x\right)}{\sqrt{3\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{648\sqrt[6]{6}\left(1+\sqrt[3]{-1}\right)^2\sqrt{4-3(-3)^{2/3}\sqrt[3]{2}}\left(29+9\sqrt[3]{-3}2^{2/3}\right)^2}$$
$$+\frac{\left(137\sqrt[3]{2}-783\sqrt[3]{3}+159\ 6^{2/3}\right)\operatorname{arctanh}\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{3}+\sqrt[3]{2}x\right)}{\sqrt{3\left(-4+3\sqrt[3]{2}3^{2/3}\right)}}\right)}{648\sqrt[6]{6}\left(1-\sqrt[3]{-1}\right)\left(1+\sqrt[3]{-1}\right)^2\left(29-9\ 2^{2/3}\sqrt[3]{3}\right)^2\sqrt{-4+3\sqrt[3]{2}3^{2/3}}}$$
$$-\frac{490617\log(2+3x)}{489281762}-\frac{\left(83(-6)^{2/3}+36\sqrt[3]{-3}-52\sqrt[3]{2}\right)\log\left(6-3\sqrt[3]{-3}2^{2/3}x+x^2\right)}{864\sqrt[3]{3}\left(1+\sqrt[3]{-1}\right)^2\left(29+9\sqrt[3]{-3}2^{2/3}\right)^2}$$
$$+\frac{(-1)^{2/3}\left(18\sqrt[3]{-6}-26(-2)^{2/3}+83\ 3^{2/3}\right)\log\left(6+3(-2)^{2/3}\sqrt[3]{3}x+x^2\right)}{432\sqrt[3]{6}\left(1-\sqrt[3]{-1}\right)\left(1+\sqrt[3]{-1}\right)^2\left(29-9(-2)^{2/3}\sqrt[3]{3}\right)^2}$$
$$-\frac{\left(108-249\ 2^{2/3}\sqrt[3]{3}+52\sqrt[3]{2}3^{2/3}\right)\log\left(6+3\ 2^{2/3}\sqrt[3]{3}x+x^2\right)}{7776\left(841-522\ 2^{2/3}\sqrt[3]{3}+162\sqrt[3]{2}3^{2/3}\right)}$$

output
```
-243/(250256+375384*x)-1/3888*(-1)^(2/3)*(274-783*(-2)^(2/3)*3^(1/3)-318*(
-2)^(1/3)*3^(2/3))*arctan((3*(-2)^(2/3)*3^(1/3)+2*x)/(24+18*(-2)^(1/3)*3^(
2/3))^(1/2))*2^(1/6)*3^(5/6)/(1-(-1)^(1/3))/(1+(-1)^(1/3))^2/(29-9*(-2)^(2
/3)*3^(1/3))^2/(4+3*(-2)^(1/3)*3^(2/3))^(1/2)-1/3888*(-1)^(2/3)*(159*(-6)^
(2/3)+783*(-3)^(1/3)+137*2^(1/3))*arctan(2^(1/6)*(3*(-3)^(1/3)-2^(1/3)*x)/
(12-9*(-3)^(2/3)*2^(1/3))^(1/2))*6^(5/6)/(1+(-1)^(1/3))^2/(4-3*(-3)^(2/3)*
2^(1/3))^(1/2)/(29+9*(-3)^(1/3)*2^(2/3))^2+1/3888*(137*2^(1/3)-783*3^(1/3)
+159*6^(2/3))*arctanh(2^(1/6)*(3*3^(1/3)+2^(1/3)*x)/(-12+9*2^(1/3)*3^(2/3)
)^(1/2))*6^(5/6)/(1-(-1)^(1/3))/(1+(-1)^(1/3))^2/(29-9*2^(2/3)*3^(1/3))^2/
(-4+3*2^(1/3)*3^(2/3))^(1/2)-490617/489281762*ln(2+3*x)-1/2592*(83*(-6)^(2
/3)+36*(-3)^(1/3)-52*2^(1/3))*ln(6-3*(-3)^(1/3)*2^(2/3)*x+x^2)*3^(2/3)/(1+
(-1)^(1/3))^2/(29+9*(-3)^(1/3)*2^(2/3))^2+1/2592*(-1)^(2/3)*(18*(-6)^(1/3)
-26*(-2)^(2/3)+83*3^(2/3))*ln(6+3*(-2)^(2/3)*3^(1/3)*x+x^2)*6^(2/3)/(1-(-1
)^(1/3))/(1+(-1)^(1/3))^2/(29-9*(-2)^(2/3)*3^(1/3))^2-(108-249*2^(2/3)*3^(
1/3)+52*2^(1/3)*3^(2/3))*ln(6+3*2^(2/3)*3^(1/3)*x+x^2)/(6539616-4059072*2^
(2/3)*3^(1/3)+1259712*2^(1/3)*3^(2/3))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.28

$$\int \frac{1}{864+2592x+2376x^2+2592x^3+4932x^4+3132x^5+166x^6+12x^7+9x^8}\,dx$$
$$= -\frac{243}{125128(2+3x)} - \frac{490617\log(2+3x)}{489281762}$$
$$+ \frac{81\text{RootSum}\left[125128+64608\#1-39612\#1^2+7292\#1^3+222\#1^4-12\#1^5+\#1^6\&,\frac{1141345500\log(2+3\right.}{}$$

input
```
Integrate[(864 + 2592*x + 2376*x^2 + 2592*x^3 + 4932*x^4 + 3132*x^5 + 166*
x^6 + 12*x^7 + 9*x^8)^(-1),x]
```

output
```
-243/(125128*(2 + 3*x)) - (490617*Log[2 + 3*x])/489281762 + (81*RootSum[12
5128 + 64608*#1 - 39612*#1^2 + 7292*#1^3 + 222*#1^4 - 12*#1^5 + #1^6 & , (
1141345500*Log[2 + 3*x - #1] - 433960684*Log[2 + 3*x - #1]*#1 + 55417890*L
og[2 + 3*x - #1]*#1^2 + 1980564*Log[2 + 3*x - #1]*#1^3 - 112553*Log[2 + 3*
x - #1]*#1^4 + 8076*Log[2 + 3*x - #1]*#1^5)/(10768 - 13204*#1 + 3646*#1^2
+ 148*#1^3 - 10*#1^4 + #1^5) & ])/3914254096
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 2.82 (sec) , antiderivative size = 485, normalized size of antiderivative = 0.75, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {2462, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{9x^8+12x^7+166x^6+3132x^5+4932x^4+2592x^3+2376x^2+2592x+864}dx$$

$$\Big\downarrow \text{2462}$$

$$\int \left(\frac{1962468x^5-2575233x^4+37885860x^3+587214738x^2-574433916x+509397788}{1957127048\left(x^6+18x^4+324x^3+108x^2+216\right)}-\frac{1471851}{489281762(3x+2)}+\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\left(419298975\sqrt[3]{-6}-299770033(-2)^{2/3}-312517050\ 3^{2/3}\right)\arctan\left(\frac{2x+3(-2)^{2/3}\sqrt[3]{3}}{\sqrt{6\left(4+3\sqrt[3]{-2}3^{2/3}\right)}}\right)}{475581872664\sqrt[6]{3}\sqrt{8+9i\sqrt[3]{2}\sqrt[6]{3}+3\sqrt[3]{2}3^{2/3}}}+\frac{(-1)^{2/3}\left(156258525(-6)^{2/3}-419298975\sqrt[3]{-3}+299770033\sqrt[3]{2}\right)\arctan\left(\frac{\sqrt[6]{2}\left(3\sqrt[3]{-3}-\sqrt[3]{2}x\right)}{\sqrt{3\left(4-3(-3)^{2/3}\sqrt[3]{2}\right)}}\right)}{158527290888\sqrt[6]{6}\left(1+\sqrt[3]{-1}\right)^2\sqrt{4-3(-3)^{2/3}\sqrt[3]{2}}}-\frac{\left(299770033\sqrt[3]{2}+419298975\sqrt[3]{3}+156258525\ 6^{2/3}\right)\operatorname{arctanh}\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{2}x+3\sqrt[3]{3}\right)}{\sqrt{3\left(3\sqrt[3]{2}3^{2/3}-4\right)}}\right)}{475581872664\sqrt[6]{6}\sqrt{3\sqrt[3]{2}3^{2/3}-4}}-\frac{(-1)^{2/3}\left(182775057+52986636(-3)^{2/3}\sqrt[3]{2}-75263272\sqrt[3]{-3}2^{2/3}\right)\log\left(x^2-3\sqrt[3]{-3}2^{2/3}x+6\right)}{105684860592\sqrt[3]{2}3^{2/3}\left(1+\sqrt[3]{-1}\right)^2}+\frac{\left(317919816+3^{2/3}\left(60925019(-6)^{2/3}-150526544\sqrt[3]{-2}\right)\right)\log\left(x^2+3(-2)^{2/3}\sqrt[3]{3}x+6\right)}{1902327490656}+\frac{\left(317919816+6^{2/3}\left(75263272\ 2^{2/3}+60925019\ 3^{2/3}\right)\right)\log\left(x^2+3\ 2^{2/3}\sqrt[3]{3}x+6\right)}{1902327490656}-\frac{243}{125128(3x+2)}-\frac{490617\log(3x+2)}{489281762}$$

input
```
Int[(864 + 2592*x + 2376*x^2 + 2592*x^3 + 4932*x^4 + 3132*x^5 + 166*x^6 +
12*x^7 + 9*x^8)^(-1),x]
```

output

```
-243/(125128*(2 + 3*x)) - ((419298975*(-6)^(1/3) - 299770033*(-2)^(2/3) -
312517050*3^(2/3))*ArcTan[(3*(-2)^(2/3)*3^(1/3) + 2*x)/Sqrt[6*(4 + 3*(-2)^
(1/3)*3^(2/3))]])/(475581872664*3^(1/6)*Sqrt[8 + (9*I)*2^(1/3)*3^(1/6) + 3
*2^(1/3)*3^(2/3)]) + ((-1)^(2/3)*(156258525*(-6)^(2/3) - 419298975*(-3)^(1
/3) + 299770033*2^(1/3))*ArcTan[(2^(1/6)*(3*(-3)^(1/3) - 2^(1/3)*x))/Sqrt[
3*(4 - 3*(-3)^(2/3)*2^(1/3))]])/(158527290888*6^(1/6)*(1 + (-1)^(1/3))^2*S
qrt[4 - 3*(-3)^(2/3)*2^(1/3)]) - ((299770033*2^(1/3) + 419298975*3^(1/3) +
 156258525*6^(2/3))*ArcTanh[(2^(1/6)*(3*3^(1/3) + 2^(1/3)*x))/Sqrt[3*(-4 +
 3*2^(1/3)*3^(2/3))]])/(475581872664*6^(1/6)*Sqrt[-4 + 3*2^(1/3)*3^(2/3)])
 - (490617*Log[2 + 3*x])/489281762 - ((-1)^(2/3)*(182775057 + 52986636*(-3
)^(2/3)*2^(1/3) - 75263272*(-3)^(1/3)*2^(2/3))*Log[6 - 3*(-3)^(1/3)*2^(2/3
)*x + x^2])/(105684860592*2^(1/3)*3^(2/3)*(1 + (-1)^(1/3))^2) + ((31791981
6 + 3^(2/3)*(60925019*(-6)^(2/3) - 150526544*(-2)^(1/3)))*Log[6 + 3*(-2)^(
2/3)*3^(1/3)*x + x^2])/1902327490656 + ((317919816 + 6^(2/3)*(75263272*2^(
2/3) + 60925019*3^(2/3)))*Log[6 + 3*2^(2/3)*3^(1/3)*x + x^2])/190232749065
6
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2462

```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.13

method	result
risch	$-\frac{81}{125128\left(x+\frac{2}{3}\right)}-\frac{490617\ln(2+3x)}{489281762}+\frac{\left(\sum_{\_R=\text{RootOf}\left(9900753\_Z^{6}-603729730584\_Z^{5}-7778441183375403\_Z^{4}-242862251317146570\right.}\right.}{}$
default	$\frac{\left(\sum_{\_R=\text{RootOf}\left(\_Z^{6}+18\_Z^{4}+324\_Z^{3}+108\_Z^{2}+216\right)}\frac{\left(1962468\_R^{5}-2575233\_R^{4}+37885860\_R^{3}+587214738\_R^{2}-574433916\_R+\right.}{\_R^{5}+12\_R^{3}+162\_R^{2}+36\_R}\right.}{11742762288}$

input
```
int(1/(9*x^8+12*x^7+166*x^6+3132*x^5+4932*x^4+2592*x^3+2376*x^2+2592*x+864
),x,method=_RETURNVERBOSE)
```

output
```
-81/125128/(x+2/3)-490617/489281762*ln(2+3*x)+1/60812208*sum(_R*ln(-581639
624628989671215509080386163823409178503*_R^5+354681896500547050248070950233
880440332624355193966*_R^4+45690685987861743859917785684794895953996770996
8421774*_R^3+14260573113296934033739423583054517575773284248671596955O8*_R
^2+12496106101584673634988275193178073483743771974241813638447*_R+37117801
71170710248762306868718224813196689272625039439882*x-101688340678907543636
4865420039034943886248036403010397152 4),_R=RootOf(9900753*_Z^6-60372973058
4*_Z^5-7778441183375403*_Z^4-242862251317146570 96*_Z^3-24907483084253298 90
21*_Z^2+6248252372534023208 4*_Z-9361007218276308142 01))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{864+2592x+2376x^2+2592x^3+4932x^4+3132x^5+166x^6+12x^7+9x^8}\,dx$$
$$= \text{Timed out}$$

input
```
integrate(1/(9*x^8+12*x^7+166*x^6+3132*x^5+4932*x^4+2592*x^3+2376*x^2+2592
*x+864),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [A] (verification not implemented)

Time = 0.77 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.14

$$\int \frac{1}{864 + 2592x + 2376x^2 + 2592x^3 + 4932x^4 + 3132x^5 + 166x^6 + 12x^7 + 9x^8}\,dx$$
$$= -\frac{490617\log\left(x+\frac{2}{3}\right)}{489281762}$$
$$+\mathrm{RootSum}\Big(534921778156562570512850938232832t^6 - 536381566647968085631001690112t^5 - 11364$$
$$-\frac{243}{375384x+250256}$$

input
```
integrate(1/(9*x**8+12*x**7+166*x**6+3132*x**5+4932*x**4+2592*x**3+2376*x*
*2+2592*x+864),x)
```

output
```
-490617*log(x + 2/3)/489281762 + RootSum(534921778156562570512850938232832
*_t**6 - 536381566647968085631001690112*_t**5 - 113640485675194290961317888*_t**4 - 583458207306147230515 2*_t**3 - 983985748671744*_t**2 + 4059072*_
t - 1, Lambda(_t, _t*log(-3455750960438682665382725466807609018667495372212774674182519079033868698426615638032740573438585745071222349672612728118706176*_t**6/265882467743308560111128055912060012355441739830233778570332881047594951014162037682159082438384 11 + 853869161004594421797568445431927726629256808703577315178764539835613130871755239221659482772635333796143919723829002 24*_t**5/265882467743308560111128055912060012355441739830233778570332881047594951014162037682159082438384 11 + 42087887345462510813558068170291774304012124895937415583755314850510071876704894475250378848246795502717138971146267525 12*_t**4/265882467743308560111128055912060012355441739830233778570332881047594951014162037682159082438384 11 + 773743328211408687735928382971741157348346732153516561676592332878344549740957533802268264343331007245717174104207360*_t**3/265882467743308560111128055912060012355441739830233778570332881047594951014162037682159082438384 11 + 712891082341168850680045281424427993634194541364334319324325766749577944485225815038394351529193544496127289856*_t**2/50166503347794067945495859606049058934989007515138448786855260575017915285690950506067751403468 7 + 1877037027553056831103794090506473887865818350246954934839146770978639840753738187334220057510 88...
```

Maxima [F]

$$\int \frac{1}{864 + 2592x + 2376x^2 + 2592x^3 + 4932x^4 + 3132x^5 + 166x^6 + 12x^7 + 9x^8} \, dx = \int \frac{1}{9\,x^8 + 12\,x^7 + 166\,x^6 + 3132\,x^5 + 4932\,x^4 + 2592\,x^3 + 2376\,x^2 + 2592\,x + 864} \, dx$$

input
```
integrate(1/(9*x^8+12*x^7+166*x^6+3132*x^5+4932*x^4+2592*x^3+2376*x^2+2592
*x+864),x, algorithm="maxima")
```

output
```
-243/125128/(3*x + 2) + 1/1957127048*integrate((1962468*x^5 - 2575233*x^4
+ 37885860*x^3 + 587214738*x^2 - 574433916*x + 509397788)/(x^6 + 18*x^4 +
324*x^3 + 108*x^2 + 216), x) - 490617/489281762*log(3*x + 2)
```

Giac [F]

$$\int \frac{1}{864 + 2592x + 2376x^2 + 2592x^3 + 4932x^4 + 3132x^5 + 166x^6 + 12x^7 + 9x^8} \, dx = \int \frac{1}{9\,x^8 + 12\,x^7 + 166\,x^6 + 3132\,x^5 + 4932\,x^4 + 2592\,x^3 + 2376\,x^2 + 2592\,x + 864} \, dx$$

input
```
integrate(1/(9*x^8+12*x^7+166*x^6+3132*x^5+4932*x^4+2592*x^3+2376*x^2+2592
*x+864),x, algorithm="giac")
```

output
```
integrate(1/(9*x^8 + 12*x^7 + 166*x^6 + 3132*x^5 + 4932*x^4 + 2592*x^3 + 2
376*x^2 + 2592*x + 864), x)
```

Mupad [B] (verification not implemented)

Time = 21.94 (sec) , antiderivative size = 480, normalized size of antiderivative = 0.74

$$\int \frac{1}{864+2592x+2376x^2+2592x^3+4932x^4+3132x^5+166x^6+12x^7+9x^8}\,dx$$
$$= \text{Too large to display}$$

input
```
int(1/(2592*x + 2376*x^2 + 2592*x^3 + 4932*x^4 + 3132*x^5 + 166*x^6 + 12*x
^7 + 9*x^8 + 864),x)
```

output
```
symsum(log(x/15657016384 - (241256*root(z^6 - (490617*z^5)/489281762 - (68
2182427*z^4)/3211128804227328 - (17022113*z^3)/1560608598854481408 - (8036
21*z^2)/436870528728928107429888 + (29*z)/382174338532066308379666O224 - 1
/534921778156562570512850938232832, z, k))/733922643 - (1678771*root(z^6 -
 (490617*z^5)/489281762 - (682182427*z^4)/3211128804227328 - (17022113*z^3
)/1560608598854481408 - (803621*z^2)/436870528728928107429888 + (29*z)/382
174338532066308379666O224 - 1/534921778156562570512850938232832, z, k)*x)/
6605303787 + (305207778434812192*root(z^6 - (490617*z^5)/489281762 - (6821
82427*z^4)/3211128804227328 - (17022113*z^3)/1560608598854481408 - (803621
*z^2)/436870528728928107429888 + (29*z)/382174338532066308379666O224 - 1/5
34921778156562570512850938232832, z, k)^2*x)/4815266460723 + (244884659349
9859523392*root(z^6 - (490617*z^5)/489281762 - (682182427*z^4)/32111288042
27328 - (17022113*z^3)/1560608598854481408 - (803621*z^2)/4368705287289281
07429888 + (29*z)/382174338532066308379666O224 - 1/53492177815656257051285
0938232832, z, k)^3*x)/6605303787 + (15331578913O4935213946880*root(z^6 -
(490617*z^5)/489281762 - (682182427*z^4)/3211128804227328 - (17022113*z^3)
/1560608598854481408 - (803621*z^2)/436870528728928107429888 + (29*z)/3821
74338532066308379666O224 - 1/534921778156562570512850938232832, z, k)^4*x)
/244640881 + (2600474335927184107929 60*root(z^6 - (490617*z^5)/489281762 -
 (682182427*z^4)/3211128804227328 - (17022113*z^3)/1560608598854481408 ...
```

Reduce [F]

$$\int \frac{1}{864+2592x+2376x^2+2592x^3+4932x^4+3132x^5+166x^6+12x^7+9x^8}\,dx$$
$$=\int \frac{1}{9x^8+12x^7+166x^6+3132x^5+4932x^4+2592x^3+2376x^2+2592x+864}dx$$

input
```
int(1/(9*x^8+12*x^7+166*x^6+3132*x^5+4932*x^4+2592*x^3+2376*x^2+2592*x+864
),x)
```

output
```
int(1/(9*x**8 + 12*x**7 + 166*x**6 + 3132*x**5 + 4932*x**4 + 2592*x**3 + 2
376*x**2 + 2592*x + 864),x)
```

3.294 $\int \frac{(1+x^2)^2}{ax^6+b(1+x^2)^3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 168

$$\int \frac{(1+x^2)^2}{ax^6+b(1+x^2)^3}\,dx = \frac{\arctan\left(\frac{\sqrt{\sqrt[3]{a}+\sqrt[3]{b}}\,x}{\sqrt[6]{b}}\right)}{3\sqrt{\sqrt[3]{a}+\sqrt[3]{b}}\,b^{5/6}} + \frac{\arctan\left(\frac{\sqrt{-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}}\,x}{\sqrt[6]{b}}\right)}{3\sqrt{-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}}\,b^{5/6}} + \frac{\arctan\left(\frac{\sqrt{(-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}}\,x}{\sqrt[6]{b}}\right)}{3\sqrt{(-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}}\,b^{5/6}}$$

output
```
1/3*arctan((a^(1/3)+b^(1/3))^(1/2)*x/b^(1/6))/(a^(1/3)+b^(1/3))^(1/2)/b^(5
/6)+1/3*arctan((-(-1)^(1/3)*a^(1/3)+b^(1/3))^(1/2)*x/b^(1/6))/(-(-1)^(1/3)
*a^(1/3)+b^(1/3))^(1/2)/b^(5/6)+1/3*arctan(((-1)^(2/3)*a^(1/3)+b^(1/3))^(1
/2)*x/b^(1/6))/((-1)^(2/3)*a^(1/3)+b^(1/3))^(1/2)/b^(5/6)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.06 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.57

$$\int \frac{(1+x^2)^2}{ax^6+b(1+x^2)^3}\,dx = \frac{1}{6}\text{RootSum}\left[b+3b\#1^2+3b\#1^4+a\#1^6+b\#1^6\&,\frac{\log(x-\#1)+2\log(x-\#1)\#1^2+\log(x-\#1)\#1^4}{b\#1+2b\#1^3+a\#1^5+b\#1^5}\&\right]$$

input
```
Integrate[(1 + x^2)^2/(a*x^6 + b*(1 + x^2)^3),x]
```

output
```
RootSum[b + 3*b*#1^2 + 3*b*#1^4 + a*#1^6 + b*#1^6 & , (Log[x - #1] + 2*Log
[x - #1]*#1^2 + Log[x - #1]*#1^4)/(b*#1 + 2*b*#1^3 + a*#1^5 + b*#1^5) & ]/
6
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x^2+1)^2}{ax^6+b(x^2+1)^3}\,dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(\frac{x^4}{ax^6\left(\frac{b}{a}+1\right)+3bx^4+3bx^2+b}+\frac{2x^2}{ax^6\left(\frac{b}{a}+1\right)+3bx^4+3bx^2+b}+\frac{1}{ax^6\left(\frac{b}{a}+1\right)+3bx^4+3bx^2+b}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\int \frac{1}{a\left(\frac{b}{a}+1\right)x^6+3bx^4+3bx^2+b}dx+2\int \frac{x^2}{a\left(\frac{b}{a}+1\right)x^6+3bx^4+3bx^2+b}dx+\int \frac{x^4}{a\left(\frac{b}{a}+1\right)x^6+3bx^4+3bx^2+b}dx$$

input
```
Int[(1 + x^2)^2/(a*x^6 + b*(1 + x^2)^3),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.58 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.40

method	result	size
default	$\frac{\sum_{\_R=\mathrm{RootOf}\left((a+b)\_Z^6+3b\_Z^4+3b\_Z^2+b\right)}\frac{\left(\_R^4+2\_R^2+1\right)\ln\left(x-\_R\right)}{\_R^5a+\_R^5b+2\_R^3b+b\_R}}{6}$	67
risch	$\frac{\sum_{\_R=\mathrm{RootOf}\left((a+b)\_Z^6+3b\_Z^4+3b\_Z^2+b\right)}\frac{\left(\_R^4+2\_R^2+1\right)\ln\left(x-\_R\right)}{\_R^5a+\_R^5b+2\_R^3b+b\_R}}{6}$	67

input
```
int((x^2+1)^2/(x^6*a+b*(x^2+1)^3),x,method=_RETURNVERBOSE)
```

output
```
1/6*sum((_R^4+2*_R^2+1)/(_R^5*a+_R^5*b+2*_R^3*b+_R*b)*ln(x-_R),_R=RootOf((
a+b)*_Z^6+3*b*_Z^4+3*b*_Z^2+b))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.75 (sec) , antiderivative size = 5653, normalized size of antiderivative = 33.65

$$\int \frac{(1+x^2)^2}{ax^6+b(1+x^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate((x^2+1)^2/(a*x^6+b*(x^2+1)^3),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [A] (verification not implemented)

Time = 0.98 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.25

$$\int \frac{(1+x^2)^2}{ax^6+b(1+x^2)^3}\,dx = \text{RootSum}\left(t^6\cdot\left(46656ab^5+46656b^6\right)+3888t^4b^4+108t^2b^2+1,\left(t\mapsto t\log\left(6tb+x\right)\right)\right)$$

input
```
integrate((x**2+1)**2/(a*x**6+b*(x**2+1)**3),x)
```

output
```
RootSum(_t**6*(46656*a*b**5 + 46656*b**6) + 3888*_t**4*b**4 + 108*_t**2*b*
*2 + 1, Lambda(_t, _t*log(6*_t*b + x)))
```

Maxima [F]

$$\int \frac{(1+x^2)^2}{ax^6+b(1+x^2)^3}\,dx = \int \frac{(x^2+1)^2}{ax^6+(x^2+1)^3b}\,dx$$

input
```
integrate((x^2+1)^2/(a*x^6+b*(x^2+1)^3),x, algorithm="maxima")
```

output
```
integrate((x^2 + 1)^2/(a*x^6 + (x^2 + 1)^3*b), x)
```

Giac [F]

$$\int \frac{(1+x^2)^2}{ax^6+b(1+x^2)^3}\,dx = \int \frac{(x^2+1)^2}{ax^6+(x^2+1)^3b}\,dx$$

input
```
integrate((x^2+1)^2/(a*x^6+b*(x^2+1)^3),x, algorithm="giac")
```

output
```
sage0*x
```

Mupad [B] (verification not implemented)

Time = 22.84 (sec) , antiderivative size = 504, normalized size of antiderivative = 3.00

$$\int \frac{(1+x^2)^2}{ax^6+b(1+x^2)^3}\,dx = \sum_{k=1}^{6} \ln\Big(-a^3(a+b)\Big(-\mathrm{root}\big(46656\,a\,b^5\,z^6+46656\,b^6\,z^6+3888\,b^4\,z^4+108\,b^2\,z^2+1,z,k\big)^2\,b^2\,60 - \mathrm{root}\big(46656\,a\,b^5\,z^6+46656\,b^6\,z^6+3888\,b^4\,z^4+108\,b^2\,z^2+1,z,k\big)^4\,b^4\,864 - \mathrm{root}\big(46656\,a\,b^5\,z^6+46656\,b^6\,z^6+3888\,b^4\,z^4+108\,b^2\,z^2+1,z,k\big)^4\,a\,b^3\,864 + \mathrm{root}\big(46656\,a\,b^5\,z^6+46656\,b^6\,z^6+3888\,b^4\,z^4+108\,b^2\,z^2+1,z,k\big)^3\,b^3\,x\,504 + \mathrm{root}\big(46656\,a\,b^5\,z^6+46656\,b^6\,z^6+3888\,b^4\,z^4+108\,b^2\,z^2+1,z,k\big)^5\,b^5\,x\,7776 + \mathrm{root}\big(46656\,a\,b^5\,z^6+46656\,b^6\,z^6+3888\,b^4\,z^4+108\,b^2\,z^2+1,z,k\big)\,a\,x\,2 + \mathrm{root}\big(46656\,a\,b^5\,z^6+46656\,b^6\,z^6+3888\,b^4\,z^4+108\,b^2\,z^2+1,z,k\big)\,b\,x\,8 + \mathrm{root}\big(46656\,a\,b^5\,z^6+46656\,b^6\,z^6+3888\,b^4\,z^4+108\,b^2\,z^2+1,z,k\big)^2\,a\,b\,12 - \mathrm{root}\big(46656\,a\,b^5\,z^6+46656\,b^6\,z^6+3888\,b^4\,z^4+108\,b^2\,z^2+1,z,k\big)^3\,a\,b^2\,x\,144 + \mathrm{root}\big(46656\,a\,b^5\,z^6+46656\,b^6\,z^6+3888\,b^4\,z^4+108\,b^2\,z^2+1,z,k\big)^5\,a\,b^4\,x\,7776 - 1\Big)\,3\Big)\,\mathrm{root}\big(46656\,a\,b^5\,z^6+46656\,b^6\,z^6+3888\,b^4\,z^4+108\,b^2\,z^2+1,z,k\big)$$

input
```
int((x^2 + 1)^2/(b*(x^2 + 1)^3 + a*x^6),x)
```

output
```
symsum(log(-3*a^3*(a + b)*(504*root(46656*a*b^5*z^6 + 46656*b^6*z^6 + 3888
*b^4*z^4 + 108*b^2*z^2 + 1, z, k)^3*b^3*x - 864*root(46656*a*b^5*z^6 + 466
56*b^6*z^6 + 3888*b^4*z^4 + 108*b^2*z^2 + 1, z, k)^4*b^4 - 864*root(46656*
a*b^5*z^6 + 46656*b^6*z^6 + 3888*b^4*z^4 + 108*b^2*z^2 + 1, z, k)^4*a*b^3
- 60*root(46656*a*b^5*z^6 + 46656*b^6*z^6 + 3888*b^4*z^4 + 108*b^2*z^2 + 1
, z, k)^2*b^2 + 7776*root(46656*a*b^5*z^6 + 46656*b^6*z^6 + 3888*b^4*z^4 +
 108*b^2*z^2 + 1, z, k)^5*b^5*x + 2*root(46656*a*b^5*z^6 + 46656*b^6*z^6 +
 3888*b^4*z^4 + 108*b^2*z^2 + 1, z, k)*a*x + 8*root(46656*a*b^5*z^6 + 4665
6*b^6*z^6 + 3888*b^4*z^4 + 108*b^2*z^2 + 1, z, k)*b*x + 12*root(46656*a*b^
5*z^6 + 46656*b^6*z^6 + 3888*b^4*z^4 + 108*b^2*z^2 + 1, z, k)^2*a*b - 144*
root(46656*a*b^5*z^6 + 46656*b^6*z^6 + 3888*b^4*z^4 + 108*b^2*z^2 + 1, z,
k)^3*a*b^2*x + 7776*root(46656*a*b^5*z^6 + 46656*b^6*z^6 + 3888*b^4*z^4 +
108*b^2*z^2 + 1, z, k)^5*a*b^4*x - 1))*root(46656*a*b^5*z^6 + 46656*b^6*z^
6 + 3888*b^4*z^4 + 108*b^2*z^2 + 1, z, k), k, 1, 6)
```

Reduce [F]

$$\int \frac{(1+x^2)^2}{ax^6+b(1+x^2)^3}\,dx = \int \frac{x^4}{a\,x^6+b\,x^6+3b\,x^4+3b\,x^2+b}dx + 2\left(\int \frac{x^2}{a\,x^6+b\,x^6+3b\,x^4+3b\,x^2+b}dx\right) + \int \frac{1}{a\,x^6+b\,x^6+3b\,x^4+3b\,x^2+b}dx$$

input
```
int((x^2+1)^2/(a*x^6+b*(x^2+1)^3),x)
```

output
```
int(x**4/(a*x**6 + b*x**6 + 3*b*x**4 + 3*b*x**2 + b),x) + 2*int(x**2/(a*x*
*6 + b*x**6 + 3*b*x**4 + 3*b*x**2 + b),x) + int(1/(a*x**6 + b*x**6 + 3*b*x
**4 + 3*b*x**2 + b),x)
```

3.295 $\int (a+bx^8)^p (c+dx^8)^3 \, dx$

Optimal result

Integrand size = 19, antiderivative size = 287

$$\int (a+bx^8)^p (c+dx^8)^3 \, dx$$
$$= \frac{3d(51a^2d^2-9abcd(25+8p)+b^2c^2(425+336p+64p^2))\,x(a+bx^8)^{1+p}}{b^3(9+8p)(17+8p)(25+8p)}$$
$$-\frac{d^2(17ad-3bc(25+8p))x^9(a+bx^8)^{1+p}}{b^2(17+8p)(25+8p)}+\frac{d^3x^{17}(a+bx^8)^{1+p}}{b(25+8p)}$$
$$+\frac{(b^3c^3(9+8p)\,(425+336p+64p^2)-3ad(51a^2d^2-9abcd(25+8p)+b^2c^2(425+336p+64p^2)))\,x(a+}{b^3(9+8p)(17+8p)(25+8p)}$$

output
```
3*d*(51*a^2*d^2-9*a*b*c*d*(25+8*p)+b^2*c^2*(64*p^2+336*p+425))*x*(b*x^8+a)
^(p+1)/b^3/(9+8*p)/(17+8*p)/(25+8*p)-d^2*(17*a*d-3*b*c*(25+8*p))*x^9*(b*x^
8+a)^(p+1)/b^2/(17+8*p)/(25+8*p)+d^3*x^17*(b*x^8+a)^(p+1)/b/(25+8*p)+(b^3*
c^3*(9+8*p)*(64*p^2+336*p+425)-3*a*d*(51*a^2*d^2-9*a*b*c*d*(25+8*p)+b^2*c^
2*(64*p^2+336*p+425)))*x*(b*x^8+a)^p*hypergeom([1/8, -p],[9/8],-b*x^8/a)/b
^3/(9+8*p)/(17+8*p)/(25+8*p)/((1+b*x^8/a)^p)
```

Mathematica [A] (verified)

Time = 8.30 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.48

$$\int (a+bx^8)^p (c+dx^8)^3 \, dx$$
$$= \frac{(a+bx^8)^p \left(1+\frac{bx^8}{a}\right)^{-p} \left(1275c^3x \operatorname{Hypergeometric2F1}\left(\frac{1}{8},-p,\frac{9}{8},-\frac{bx^8}{a}\right)+dx^9\left(425c^2 \operatorname{Hypergeometric2F}\right.\right.}{}$$

input
```
Integrate[(a + b*x^8)^p*(c + d*x^8)^3,x]
```

output
```
((a + b*x^8)^p*(1275*c^3*x*Hypergeometric2F1[1/8, -p, 9/8, -((b*x^8)/a)] +
 d*x^9*(425*c^2*Hypergeometric2F1[9/8, -p, 17/8, -((b*x^8)/a)] + 3*d*x^8*(
75*c*Hypergeometric2F1[17/8, -p, 25/8, -((b*x^8)/a)] + 17*d*x^8*Hypergeome
tric2F1[25/8, -p, 33/8, -((b*x^8)/a)])))/(1275*(1 + (b*x^8)/a)^p)
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {933, 25, 1025, 25, 913, 779, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx^8)^3 (a+bx^8)^p \, dx$$
$$\Big\downarrow \text{933}$$
$$\frac{\int -(bx^8+a)^p (dx^8+c) \left(d(17ad-bc(8p+41))x^8+c(ad-bc(8p+25))\right) dx}{b(8p+25)} + \frac{dx(c+dx^8)^2 (a+bx^8)^{p+1}}{b(8p+25)}$$
$$\Big\downarrow \text{25}$$

$$\frac{dx\left(c+dx^8\right)^2\left(a+bx^8\right)^{p+1}}{b(8p+25)}-\frac{\int\left(bx^8+a\right)^p\left(dx^8+c\right)\left(d(17ad-bc(8p+41))x^8+c(ad-bc(8p+25))\right)dx}{b(8p+25)}$$

$\Big\downarrow$ 1025

$$\frac{dx\left(c+dx^8\right)^2\left(a+bx^8\right)^{p+1}}{b(8p+25)}-\frac{\frac{\int-\left(bx^8+a\right)^p\left(d\left(b^2\left(64p^2+400p+753\right)c^2-2abd(40p+261)c+153a^2d^2\right)x^8+c\left(b^2\left(64p^2+336p+425\right)c^2-2abd(8p+29)c+17a^2d^2\right)\right)dx}{b(8p+17)}+\frac{dx(c+dx^8)}{}}{b(8p+25)}$$

$\Big\downarrow$ 25

$$\frac{dx\left(c+dx^8\right)^2\left(a+bx^8\right)^{p+1}}{b(8p+25)}-\frac{\frac{dx\left(c+dx^8\right)\left(a+bx^8\right)^{p+1}(17ad-bc(8p+41))}{b(8p+17)}-\frac{\int\left(bx^8+a\right)^p\left(d\left(b^2\left(64p^2+400p+753\right)c^2-2abd(40p+261)c+153a^2d^2\right)x^8+c\left(b^2\left(64p^2+336p+425\right)c^2\right.}{b(8p+17)}}{b(8p+25)}$$

$\Big\downarrow$ 913

$$\frac{dx\left(c+dx^8\right)^2\left(a+bx^8\right)^{p+1}}{b(8p+25)}-\frac{\frac{dx\left(c+dx^8\right)\left(a+bx^8\right)^{p+1}(17ad-bc(8p+41))}{b(8p+17)}-\frac{\frac{dx\left(a+bx^8\right)^{p+1}\left(153a^2d^2-2abcd(40p+261)+b^2c^2\left(64p^2+400p+753\right)\right)}{b(8p+9)}-\frac{\left(153a^3d^3-27a^2bcd^2(8p+25)+3ab^2\right.}{}}{b(8p+17)}}{b(8p+25)}$$

$\Big\downarrow$ 779

$$\frac{dx\left(c+dx^8\right)^2\left(a+bx^8\right)^{p+1}}{b(8p+25)}-\frac{\frac{dx\left(c+dx^8\right)\left(a+bx^8\right)^{p+1}(17ad-bc(8p+41))}{b(8p+17)}-\frac{\frac{dx\left(a+bx^8\right)^{p+1}\left(153a^2d^2-2abcd(40p+261)+b^2c^2\left(64p^2+400p+753\right)\right)}{b(8p+9)}-\frac{\left(a+bx^8\right)^p\left(\frac{bx^8}{a}+1\right)^{-p}\left(153a^3d^3-\right.}{}}{b(8p+1}}{b(8p+25)}$$

$\Big\downarrow$ 778

$$\frac{dx\left(c+dx^8\right)^2\left(a+bx^8\right)^{p+1}}{b(8p+25)}-\frac{\frac{dx\left(c+dx^8\right)\left(a+bx^8\right)^{p+1}(17ad-bc(8p+41))}{b(8p+17)}-\frac{\frac{dx\left(a+bx^8\right)^{p+1}\left(153a^2d^2-2abcd(40p+261)+b^2c^2\left(64p^2+400p+753\right)\right)}{b(8p+9)}-\frac{x\left(a+bx^8\right)^p\left(\frac{bx^8}{a}+1\right)^{-p}\left(153a^3d^3\right.}{}}{}}{b(8p+25)}$$

input
```
Int[(a + b*x^8)^p*(c + d*x^8)^3,x]
```

output
```
(d*x*(a + b*x^8)^(1 + p)*(c + d*x^8)^2)/(b*(25 + 8*p)) - ((d*(17*a*d - b*c
*(41 + 8*p))*x*(a + b*x^8)^(1 + p)*(c + d*x^8))/(b*(17 + 8*p)) - ((d*(153*
a^2*d^2 - 2*a*b*c*d*(261 + 40*p) + b^2*c^2*(753 + 400*p + 64*p^2))*x*(a +
b*x^8)^(1 + p))/(b*(9 + 8*p)) - ((153*a^3*d^3 - 27*a^2*b*c*d^2*(25 + 8*p)
+ 3*a*b^2*c^2*d*(425 + 336*p + 64*p^2) - b^3*c^3*(3825 + 6424*p + 3264*p^2
 + 512*p^3))*x*(a + b*x^8)^p*Hypergeometric2F1[1/8, -p, 9/8, -((b*x^8)/a)]
)/(b*(9 + 8*p)*(1 + (b*x^8)/a)^p))/(b*(17 + 8*p)))/(b*(25 + 8*p))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Si
mplify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

rule 933
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Simp[1/(b*(n*(p + q) + 1))   Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Sim
p[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q
- 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d
, 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[
a, b, c, d, n, p, q, x]
```

rule 1025
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Simp[1/(b*(n*(p + q + 1) + 1))   Int[(a + b*x
^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e
- a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[
{a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Maple [F]

$$\int \left(b\,x^{8}+a\right)^{p} \left(d\,x^{8}+c\right)^{3}d x$$

input
```
int((b*x^8+a)^p*(d*x^8+c)^3,x)
```

output
```
int((b*x^8+a)^p*(d*x^8+c)^3,x)
```

Fricas [F]

$$\int \left(a+bx^8\right)^p \left(c+dx^8\right)^3 \, dx=\int \left(dx^{8}+c\right)^{3}\left(bx^{8}+a\right)^{p}\,dx$$

input
```
integrate((b*x^8+a)^p*(d*x^8+c)^3,x, algorithm="fricas")
```

output
```
integral((d^3*x^24 + 3*c*d^2*x^16 + 3*c^2*d*x^8 + c^3)*(b*x^8 + a)^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int (a + bx^8)^p (c + dx^8)^3 \, dx = \text{Timed out}$$

input
```
integrate((b*x**8+a)**p*(d*x**8+c)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (a + bx^8)^p (c + dx^8)^3 \, dx = \int (dx^8 + c)^3 (bx^8 + a)^p \, dx$$

input
```
integrate((b*x^8+a)^p*(d*x^8+c)^3,x, algorithm="maxima")
```

output
```
integrate((d*x^8 + c)^3*(b*x^8 + a)^p, x)
```

Giac [F]

$$\int (a + bx^8)^p (c + dx^8)^3 \, dx = \int (dx^8 + c)^3 (bx^8 + a)^p \, dx$$

input
```
integrate((b*x^8+a)^p*(d*x^8+c)^3,x, algorithm="giac")
```

output
```
integrate((d*x^8 + c)^3*(b*x^8 + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+bx^8\right)^p \left(c+dx^8\right)^3 \, dx = \int \left(b\,x^8+a\right)^p \left(d\,x^8+c\right)^3 \, dx$$

input
```
int((a + b*x^8)^p*(c + d*x^8)^3,x)
```

output
```
int((a + b*x^8)^p*(c + d*x^8)^3, x)
```

Reduce [F]

$$\int \left(a+bx^8\right)^p \left(c+dx^8\right)^3 \, dx = \text{too large to display}$$

input
```
int((b*x^8+a)^p*(d*x^8+c)^3,x)
```

output

```
(1224*(a + b*x**8)**p*a**3*d**3*p*x - 1728*(a + b*x**8)**p*a**2*b*c*d**2*p
**2*x - 5400*(a + b*x**8)**p*a**2*b*c*d**2*p*x - 1088*(a + b*x**8)**p*a**2
*b*d**3*p**2*x**9 - 136*(a + b*x**8)**p*a**2*b*d**3*p*x**9 + 1536*(a + b*x
**8)**p*a*b**2*c**2*d*p**3*x + 8064*(a + b*x**8)**p*a*b**2*c**2*d*p**2*x +
 10200*(a + b*x**8)**p*a*b**2*c**2*d*p*x + 1536*(a + b*x**8)**p*a*b**2*c*d
**2*p**3*x**9 + 4992*(a + b*x**8)**p*a*b**2*c*d**2*p**2*x**9 + 600*(a + b*
x**8)**p*a*b**2*c*d**2*p*x**9 + 512*(a + b*x**8)**p*a*b**2*d**3*p**3*x**17
 + 640*(a + b*x**8)**p*a*b**2*d**3*p**2*x**17 + 72*(a + b*x**8)**p*a*b**2*
d**3*p*x**17 + 512*(a + b*x**8)**p*b**3*c**3*p**3*x + 3264*(a + b*x**8)**p
*b**3*c**3*p**2*x + 6424*(a + b*x**8)**p*b**3*c**3*p*x + 3825*(a + b*x**8)
**p*b**3*c**3*x + 1536*(a + b*x**8)**p*b**3*c**2*d*p**3*x**9 + 8256*(a + b
*x**8)**p*b**3*c**2*d*p**2*x**9 + 11208*(a + b*x**8)**p*b**3*c**2*d*p*x**9
 + 1275*(a + b*x**8)**p*b**3*c**2*d*x**9 + 1536*(a + b*x**8)**p*b**3*c*d**
2*p**3*x**17 + 6720*(a + b*x**8)**p*b**3*c*d**2*p**2*x**17 + 6216*(a + b*x
**8)**p*b**3*c*d**2*p*x**17 + 675*(a + b*x**8)**p*b**3*c*d**2*x**17 + 512*
(a + b*x**8)**p*b**3*d**3*p**3*x**25 + 1728*(a + b*x**8)**p*b**3*d**3*p**2
*x**25 + 1432*(a + b*x**8)**p*b**3*d**3*p*x**25 + 153*(a + b*x**8)**p*b**3
*d**3*x**25 - 5013504*int((a + b*x**8)**p/(4096*a*p**4 + 26624*a*p**3 + 54
656*a*p**2 + 37024*a*p + 3825*a + 4096*b*p**4*x**8 + 26624*b*p**3*x**8 + 5
4656*b*p**2*x**8 + 37024*b*p*x**8 + 3825*b*x**8),x)*a**4*d**3*p**5 - 32...
```

3.296 $\int (a+bx^8)^p (c+dx^8)^2 \, dx$

Optimal result

Integrand size = 19, antiderivative size = 171

$$\int (a+bx^8)^p (c+dx^8)^2 \, dx = -\frac{d(9ad-2bc(17+8p))x(a+bx^8)^{1+p}}{b^2(9+8p)(17+8p)} + \frac{d^2x^9(a+bx^8)^{1+p}}{b(17+8p)} + \frac{(b^2c^2(9+8p)(17+8p)+ad(9ad-2bc(17+8p)))\,x(a+bx^8)^p\left(1+\frac{bx^8}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{8},-\right.}{b^2(9+8p)(17+8p)}$$

output
```
-d*(9*a*d-2*b*c*(17+8*p))*x*(b*x^8+a)^(p+1)/b^2/(9+8*p)/(17+8*p)+d^2*x^9*(
b*x^8+a)^(p+1)/b/(17+8*p)+(b^2*c^2*(9+8*p)*(17+8*p)+a*d*(9*a*d-2*b*c*(17+8
*p)))*x*(b*x^8+a)^p*hypergeom([1/8, -p],[9/8],-b*x^8/a)/b^2/(9+8*p)/(17+8*
p)/((1+b*x^8/a)^p)
```

Mathematica [A] (verified)

Time = 5.64 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.62

$$\int (a+bx^8)^p (c+dx^8)^2 \, dx = \frac{1}{153} x (a+bx^8)^p \left(1+\frac{bx^8}{a}\right)^{-p} \left(153c^2 \text{Hypergeometric2F1}\left(\frac{1}{8},-p,\frac{9}{8},-\frac{bx^8}{a}\right) + dx^8 \left(34c \text{Hypergeometric2F1}\left(\frac{9}{8},-p,\frac{17}{8},-\frac{bx^8}{a}\right) + 9dx^8 \text{Hypergeometric2F1}\left(\frac{17}{8},-p,\frac{25}{8},-\frac{bx^8}{a}\right)\right)\right)$$

input
```
Integrate[(a + b*x^8)^p*(c + d*x^8)^2,x]
```

output
```
(x*(a + b*x^8)^p*(153*c^2*Hypergeometric2F1[1/8, -p, 9/8, -((b*x^8)/a)] +
d*x^8*(34*c*Hypergeometric2F1[9/8, -p, 17/8, -((b*x^8)/a)] + 9*d*x^8*Hyper
geometric2F1[17/8, -p, 25/8, -((b*x^8)/a)])))/(153*(1 + (b*x^8)/a)^p)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {933, 25, 913, 779, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx^8)^2 (a+bx^8)^p \, dx$$

$$\Big\downarrow \text{933}$$

$$\frac{\int -(bx^8+a)^p \left(d(9ad-bc(8p+25))x^8+c(ad-bc(8p+17))\right) dx}{b(8p+17)} + \frac{dx(c+dx^8)(a+bx^8)^{p+1}}{b(8p+17)}$$

$$\Big\downarrow \text{25}$$

$$\frac{dx(c+dx^8)(a+bx^8)^{p+1}}{b(8p+17)}-\frac{\int (bx^8+a)^p(d(9ad-bc(8p+25))x^8+c(ad-bc(8p+17)))\,dx}{b(8p+17)}$$

$$\Big\downarrow \text{913}$$

$$\frac{dx(c+dx^8)(a+bx^8)^{p+1}}{b(8p+17)}-\frac{\frac{dx(a+bx^8)^{p+1}(9ad-bc(8p+25))}{b(8p+9)}-\frac{(9a^2d^2-2abcd(8p+17)+b^2c^2(64p^2+208p+153))\int (bx^8+a)^p dx}{b(8p+9)}}{b(8p+17)}$$

$$\Big\downarrow \text{779}$$

$$\frac{dx(c+dx^8)(a+bx^8)^{p+1}}{b(8p+17)}-\frac{\frac{dx(a+bx^8)^{p+1}(9ad-bc(8p+25))}{b(8p+9)}-\frac{(a+bx^8)^p\left(\frac{bx^8}{a}+1\right)^{-p}(9a^2d^2-2abcd(8p+17)+b^2c^2(64p^2+208p+153))\int \left(\frac{bx^8}{a}+1\right)^p dx}{b(8p+9)}}{b(8p+17)}$$

$$\Big\downarrow \text{778}$$

$$\frac{dx(c+dx^8)(a+bx^8)^{p+1}}{b(8p+17)}-\frac{\frac{dx(a+bx^8)^{p+1}(9ad-bc(8p+25))}{b(8p+9)}-\frac{x(a+bx^8)^p\left(\frac{bx^8}{a}+1\right)^{-p}(9a^2d^2-2abcd(8p+17)+b^2c^2(64p^2+208p+153))\,\text{Hypergeometric2F1}\left(\frac{1}{8},-p,\frac{9}{8},-\frac{bx^8}{a}\right)}{b(8p+9)}}{b(8p+17)}$$

input
```
Int[(a + b*x^8)^p*(c + d*x^8)^2,x]
```

output
```
(d*x*(a + b*x^8)^(1 + p)*(c + d*x^8))/(b*(17 + 8*p)) - ((d*(9*a*d - b*c*(2
5 + 8*p))*x*(a + b*x^8)^(1 + p))/(b*(9 + 8*p)) - ((9*a^2*d^2 - 2*a*b*c*d*(
17 + 8*p) + b^2*c^2*(153 + 208*p + 64*p^2))*x*(a + b*x^8)^p*Hypergeometric
2F1[1/8, -p, 9/8, -((b*x^8)/a)])/(b*(9 + 8*p)*(1 + (b*x^8)/a)^p))/(b*(17 +
 8*p))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Si
mplify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

rule 933
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Simp[1/(b*(n*(p + q) + 1))   Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Sim
p[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q
- 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d
, 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[
a, b, c, d, n, p, q, x]
```

Maple [F]

$$\int \left(b\,x^{8}+a\right)^{p} \left(d\,x^{8}+c\right)^{2}d x$$

input
```
int((b*x^8+a)^p*(d*x^8+c)^2,x)
```

output
```
int((b*x^8+a)^p*(d*x^8+c)^2,x)
```

Fricas [F]

$$\int (a+bx^8)^p (c+dx^8)^2 \, dx = \int (dx^8+c)^2 (bx^8+a)^p \, dx$$

input
```
integrate((b*x^8+a)^p*(d*x^8+c)^2,x, algorithm="fricas")
```

output
```
integral((d^2*x^16 + 2*c*d*x^8 + c^2)*(b*x^8 + a)^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int (a+bx^8)^p (c+dx^8)^2 \, dx = \text{Timed out}$$

input
```
integrate((b*x**8+a)**p*(d*x**8+c)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (a+bx^8)^p (c+dx^8)^2 \, dx = \int (dx^8+c)^2 (bx^8+a)^p \, dx$$

input
```
integrate((b*x^8+a)^p*(d*x^8+c)^2,x, algorithm="maxima")
```

output
```
integrate((d*x^8 + c)^2*(b*x^8 + a)^p, x)
```

Giac [F]

$$\int \left(a+bx^8\right)^p \left(c+dx^8\right)^2 \, dx = \int \left(dx^8+c\right)^2 \left(bx^8+a\right)^p \, dx$$

input
```
integrate((b*x^8+a)^p*(d*x^8+c)^2,x, algorithm="giac")
```

output
```
integrate((d*x^8 + c)^2*(b*x^8 + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+bx^8\right)^p \left(c+dx^8\right)^2 \, dx = \int \left(b\,x^8+a\right)^p \left(d\,x^8+c\right)^2 \, dx$$

input
```
int((a + b*x^8)^p*(c + d*x^8)^2,x)
```

output
```
int((a + b*x^8)^p*(c + d*x^8)^2, x)
```

Reduce [F]

$$\int \left(a+bx^8\right)^p \left(c+dx^8\right)^2 \, dx = \text{Too large to display}$$

input
```
int((b*x^8+a)^p*(d*x^8+c)^2,x)
```

output
```
( - 72*(a + b*x**8)**p*a**2*d**2*p*x + 128*(a + b*x**8)**p*a*b*c*d*p**2*x
+ 272*(a + b*x**8)**p*a*b*c*d*p*x + 64*(a + b*x**8)**p*a*b*d**2*p**2*x**9
+ 8*(a + b*x**8)**p*a*b*d**2*p*x**9 + 64*(a + b*x**8)**p*b**2*c**2*p**2*x
+ 208*(a + b*x**8)**p*b**2*c**2*p*x + 153*(a + b*x**8)**p*b**2*c**2*x + 12
8*(a + b*x**8)**p*b**2*c*d*p**2*x**9 + 288*(a + b*x**8)**p*b**2*c*d*p*x**9
 + 34*(a + b*x**8)**p*b**2*c*d*x**9 + 64*(a + b*x**8)**p*b**2*d**2*p**2*x*
*17 + 80*(a + b*x**8)**p*b**2*d**2*p*x**17 + 9*(a + b*x**8)**p*b**2*d**2*x
**17 + 36864*int((a + b*x**8)**p/(512*a*p**3 + 1728*a*p**2 + 1432*a*p + 15
3*a + 512*b*p**3*x**8 + 1728*b*p**2*x**8 + 1432*b*p*x**8 + 153*b*x**8),x)*
a**3*d**2*p**4 + 124416*int((a + b*x**8)**p/(512*a*p**3 + 1728*a*p**2 + 14
32*a*p + 153*a + 512*b*p**3*x**8 + 1728*b*p**2*x**8 + 1432*b*p*x**8 + 153*
b*x**8),x)*a**3*d**2*p**3 + 103104*int((a + b*x**8)**p/(512*a*p**3 + 1728*
a*p**2 + 1432*a*p + 153*a + 512*b*p**3*x**8 + 1728*b*p**2*x**8 + 1432*b*p*
x**8 + 153*b*x**8),x)*a**3*d**2*p**2 + 11016*int((a + b*x**8)**p/(512*a*p*
*3 + 1728*a*p**2 + 1432*a*p + 153*a + 512*b*p**3*x**8 + 1728*b*p**2*x**8 +
 1432*b*p*x**8 + 153*b*x**8),x)*a**3*d**2*p - 65536*int((a + b*x**8)**p/(5
12*a*p**3 + 1728*a*p**2 + 1432*a*p + 153*a + 512*b*p**3*x**8 + 1728*b*p**2
*x**8 + 1432*b*p*x**8 + 153*b*x**8),x)*a**2*b*c*d*p**5 - 360448*int((a + b
*x**8)**p/(512*a*p**3 + 1728*a*p**2 + 1432*a*p + 153*a + 512*b*p**3*x**8 +
 1728*b*p**2*x**8 + 1432*b*p*x**8 + 153*b*x**8),x)*a**2*b*c*d*p**4 - 65...
```

3.297 $\int (a+bx^8)^p (c+dx^8) \, dx$

Optimal result

Integrand size = 17, antiderivative size = 93

$$\int (a+bx^8)^p (c+dx^8) \, dx = \frac{dx(a+bx^8)^{1+p}}{b(9+8p)} - \frac{(ad-bc(9+8p))x(a+bx^8)^p \left(1+\frac{bx^8}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{8},-p,\frac{9}{8},-\frac{bx^8}{a}\right)}{b(9+8p)}$$

output
```
d*x*(b*x^8+a)^(p+1)/b/(9+8*p)-(a*d-b*c*(9+8*p))*x*(b*x^8+a)^p*hypergeom([1
/8, -p],[9/8],-b*x^8/a)/b/(9+8*p)/((1+b*x^8/a)^p)
```

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.97

$$\int (a+bx^8)^p (c+dx^8) \, dx = \frac{x(a+bx^8)^p \left(1+\frac{bx^8}{a}\right)^{-p} \left(d(a+bx^8)\left(1+\frac{bx^8}{a}\right)^p + (-ad+bc(9+8p)) \text{Hypergeometric2F1}\left(\frac{1}{8},-p,\frac{9}{8},-\right.\right.}{b(9+8p)}$$

input
```
Integrate[(a + b*x^8)^p*(c + d*x^8),x]
```

output
```
(x*(a + b*x^8)^p*(d*(a + b*x^8)*(1 + (b*x^8)/a)^p + (-(a*d) + b*c*(9 + 8*p
))*Hypergeometric2F1[1/8, -p, 9/8, -((b*x^8)/a)]))/(b*(9 + 8*p)*(1 + (b*x^
8)/a)^p)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.91, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {913, 779, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c+dx^8\right) \left(a+bx^8\right)^p \, dx$$

$$\Big\downarrow \text{ 913}$$

$$\left(c-\frac{ad}{8bp+9b}\right) \int \left(bx^8+a\right)^p dx+\frac{dx\left(a+bx^8\right)^{p+1}}{b(8p+9)}$$

$$\Big\downarrow \text{ 779}$$

$$\left(a+bx^8\right)^p \left(\frac{bx^8}{a}+1\right)^{-p} \left(c-\frac{ad}{8bp+9b}\right) \int \left(\frac{bx^8}{a}+1\right)^p dx+\frac{dx\left(a+bx^8\right)^{p+1}}{b(8p+9)}$$

$$\Big\downarrow \text{ 778}$$

$$x\left(a+bx^8\right)^p \left(\frac{bx^8}{a}+1\right)^{-p} \left(c-\frac{ad}{8bp+9b}\right) \text{Hypergeometric2F1}\left(\frac{1}{8},-p,\frac{9}{8},-\frac{bx^8}{a}\right)+\frac{dx\left(a+bx^8\right)^{p+1}}{b(8p+9)}$$

input
```
Int[(a + b*x^8)^p*(c + d*x^8),x]
```

output
```
(d*x*(a + b*x^8)^(1 + p))/(b*(9 + 8*p)) + ((c - (a*d)/(9*b + 8*b*p))*x*(a
+ b*x^8)^p*Hypergeometric2F1[1/8, -p, 9/8, -((b*x^8)/a)])/(1 + (b*x^8)/a)^
p
```

Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Si
mplify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Maple [F]

$$\int \left(b\,x^{8}+a\right)^{p} \left(d\,x^{8}+c\right)dx$$

input
```
int((b*x^8+a)^p*(d*x^8+c),x)
```

output
```
int((b*x^8+a)^p*(d*x^8+c),x)
```

Fricas [F]

$$\int (a+bx^8)^p (c+dx^8) \, dx = \int (dx^8+c)(bx^8+a)^p \, dx$$

input
```
integrate((b*x^8+a)^p*(d*x^8+c),x, algorithm="fricas")
```

output
```
integral((d*x^8 + c)*(b*x^8 + a)^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int (a+bx^8)^p (c+dx^8) \, dx = \text{Timed out}$$

input
```
integrate((b*x**8+a)**p*(d*x**8+c),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (a+bx^8)^p (c+dx^8) \, dx = \int (dx^8+c)(bx^8+a)^p \, dx$$

input
```
integrate((b*x^8+a)^p*(d*x^8+c),x, algorithm="maxima")
```

output
```
integrate((d*x^8 + c)*(b*x^8 + a)^p, x)
```

Giac [F]

$$\int \left(a+bx^8\right)^p \left(c+dx^8\right)\,dx=\int \left(dx^8+c\right)\left(bx^8+a\right)^p dx$$

input
```
integrate((b*x^8+a)^p*(d*x^8+c),x, algorithm="giac")
```

output
```
integrate((d*x^8 + c)*(b*x^8 + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+bx^8\right)^p \left(c+dx^8\right)\,dx=\int \left(b\,x^8+a\right)^p \left(d\,x^8+c\right)\,dx$$

input
```
int((a + b*x^8)^p*(c + d*x^8),x)
```

output
```
int((a + b*x^8)^p*(c + d*x^8), x)
```

Reduce [F]

$$\int \left(a+bx^8\right)^p \left(c+dx^8\right)\,dx$$
$$=\frac{8(b\,x^8+a)^p\,adpx+8(b\,x^8+a)^p\,bcpx+9(b\,x^8+a)^p\,bcx+8(b\,x^8+a)^p\,bdp\,x^9+(b\,x^8+a)^p\,bd\,x^9-512\Big(}{}$$

input
```
int((b*x^8+a)^p*(d*x^8+c),x)
```

output
```
(8*(a + b*x**8)**p*a*d*p*x + 8*(a + b*x**8)**p*b*c*p*x + 9*(a + b*x**8)**p
*b*c*x + 8*(a + b*x**8)**p*b*d*p*x**9 + (a + b*x**8)**p*b*d*x**9 - 512*int
((a + b*x**8)**p/(64*a*p**2 + 80*a*p + 9*a + 64*b*p**2*x**8 + 80*b*p*x**8
+ 9*b*x**8),x)*a**2*d*p**3 - 640*int((a + b*x**8)**p/(64*a*p**2 + 80*a*p +
 9*a + 64*b*p**2*x**8 + 80*b*p*x**8 + 9*b*x**8),x)*a**2*d*p**2 - 72*int((a
 + b*x**8)**p/(64*a*p**2 + 80*a*p + 9*a + 64*b*p**2*x**8 + 80*b*p*x**8 + 9
*b*x**8),x)*a**2*d*p + 4096*int((a + b*x**8)**p/(64*a*p**2 + 80*a*p + 9*a
+ 64*b*p**2*x**8 + 80*b*p*x**8 + 9*b*x**8),x)*a*b*c*p**4 + 9728*int((a + b
*x**8)**p/(64*a*p**2 + 80*a*p + 9*a + 64*b*p**2*x**8 + 80*b*p*x**8 + 9*b*x
**8),x)*a*b*c*p**3 + 6336*int((a + b*x**8)**p/(64*a*p**2 + 80*a*p + 9*a +
64*b*p**2*x**8 + 80*b*p*x**8 + 9*b*x**8),x)*a*b*c*p**2 + 648*int((a + b*x*
*8)**p/(64*a*p**2 + 80*a*p + 9*a + 64*b*p**2*x**8 + 80*b*p*x**8 + 9*b*x**8
),x)*a*b*c*p)/(b*(64*p**2 + 80*p + 9))
```

3.298 $\int (a+bx^8)^p \, dx$

Optimal result

Integrand size = 9, antiderivative size = 44

$$\int (a+bx^8)^p \, dx = x(a+bx^8)^p \left(1+\frac{bx^8}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{8},-p,\frac{9}{8},-\frac{bx^8}{a}\right)$$

output
```
x*(b*x^8+a)^p*hypergeom([1/8, -p],[9/8],-b*x^8/a)/((1+b*x^8/a)^p)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int (a+bx^8)^p \, dx = x(a+bx^8)^p \left(1+\frac{bx^8}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{8},-p,\frac{9}{8},-\frac{bx^8}{a}\right)$$

input
```
Integrate[(a + b*x^8)^p,x]
```

output
```
(x*(a + b*x^8)^p*Hypergeometric2F1[1/8, -p, 9/8, -((b*x^8)/a)])/(1 + (b*x^
8)/a)^p
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {779, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^8)^p\,dx$$

$$\Big\downarrow \text{779}$$

$$(a+bx^8)^p\left(\frac{bx^8}{a}+1\right)^{-p}\int \left(\frac{bx^8}{a}+1\right)^p dx$$

$$\Big\downarrow \text{778}$$

$$x(a+bx^8)^p\left(\frac{bx^8}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{8},-p,\frac{9}{8},-\frac{bx^8}{a}\right)$$

input
```
Int[(a + b*x^8)^p,x]
```

output
```
(x*(a + b*x^8)^p*Hypergeometric2F1[1/8, -p, 9/8, -((b*x^8)/a)])/(1 + (b*x^
8)/a)^p
```

Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Si
mplify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \left(b\,x^{8}+a\right)^{p}dx$$

input
```
int((b*x^8+a)^p,x)
```

output
```
int((b*x^8+a)^p,x)
```

Fricas [F]

$$\int \left(a+bx^8\right)^p\,dx=\int \left(bx^8+a\right)^p\,dx$$

input
```
integrate((b*x^8+a)^p,x, algorithm="fricas")
```

output
```
integral((b*x^8 + a)^p, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 32.08 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.77

$$\int \left(a+bx^8\right)^p\,dx=\frac{a^p x\Gamma\left(\frac{1}{8}\right){}_2F_1\left(\begin{matrix}\frac{1}{8},-p\\ \frac{9}{8}\end{matrix}\middle|\frac{bx^8e^{i\pi}}{a}\right)}{8\Gamma\left(\frac{9}{8}\right)}$$

input
```
integrate((b*x**8+a)**p,x)
```

output
```
a**p*x*gamma(1/8)*hyper((1/8, -p), (9/8,), b*x**8*exp_polar(I*pi)/a)/(8*ga
mma(9/8))
```

Maxima [F]

$$\int (a+bx^8)^p \, dx = \int (bx^8+a)^p \, dx$$

input
```
integrate((b*x^8+a)^p,x, algorithm="maxima")
```

output
```
integrate((b*x^8 + a)^p, x)
```

Giac [F]

$$\int (a+bx^8)^p \, dx = \int (bx^8+a)^p \, dx$$

input
```
integrate((b*x^8+a)^p,x, algorithm="giac")
```

output
```
integrate((b*x^8 + a)^p, x)
```

Mupad [B] (verification not implemented)

Time = 22.70 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.93

$$\int (a+bx^8)^p \, dx = \frac{x\,(b\,x^8+a)^p\;{}_2\mathrm{F}_1\left(\frac{1}{8},-p;\;\frac{9}{8};\;-\frac{b\,x^8}{a}\right)}{\left(\frac{b\,x^8}{a}+1\right)^p}$$

input
```
int((a + b*x^8)^p,x)
```

output
```
(x*(a + b*x^8)^p*hypergeom([1/8, -p], 9/8, -(b*x^8)/a))/((b*x^8)/a + 1)^p
```

Reduce [F]

$$\int \left(a+bx^8\right)^p \, dx$$
$$= \frac{\left(b\,x^8+a\right)^p x+64\left(\int \frac{\left(b\,x^8+a\right)^p}{8bp\,x^8+b\,x^8+8ap+a}dx\right) a\,p^2+8\left(\int \frac{\left(b\,x^8+a\right)^p}{8bp\,x^8+b\,x^8+8ap+a}dx\right) ap}{8p+1}$$

input
```
int((b*x^8+a)^p,x)
```

output
```
((a + b*x**8)**p*x + 64*int((a + b*x**8)**p/(8*a*p + a + 8*b*p*x**8 + b*x*
*8),x)*a*p**2 + 8*int((a + b*x**8)**p/(8*a*p + a + 8*b*p*x**8 + b*x**8),x)
*a*p)/(8*p + 1)
```

3.299 $\int \frac{(a+bx^8)^p}{c+dx^8} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 57

$$\int \frac{(a+bx^8)^p}{c+dx^8} \, dx = \frac{x(a+bx^8)^p \left(1+\frac{bx^8}{a}\right)^{-p} \operatorname{AppellF1}\left(\frac{1}{8},-p,1,\frac{9}{8},-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{c}$$

output
```
x*(b*x^8+a)^p*AppellF1(1/8,-p,1,9/8,-b*x^8/a,-d*x^8/c)/c/((1+b*x^8/a)^p)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 162 vs. 2(57) = 114.

Time = 0.74 (sec) , antiderivative size = 162, normalized size of antiderivative = 2.84

$$\int \frac{(a+bx^8)^p}{c+dx^8} \, dx = -\frac{9acx(a+bx^8)^p \operatorname{AppellF1}\left(\frac{1}{8},-p,1,\frac{9}{8},-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{(c+dx^8)\left(-9ac \operatorname{AppellF1}\left(\frac{1}{8},-p,1,\frac{9}{8},-\frac{bx^8}{a},-\frac{dx^8}{c}\right)+8x^8\left(-bcp \operatorname{AppellF1}\left(\frac{9}{8},1-p,1,\frac{17}{8},-\frac{bx^8}{a},-\frac{dx^8}{c}\right)\right.\right.}$$

input
```
Integrate[(a + b*x^8)^p/(c + d*x^8),x]
```

output
```
(-9*a*c*x*(a + b*x^8)^p*AppellF1[1/8, -p, 1, 9/8, -((b*x^8)/a), -((d*x^8)/
c)])/((c + d*x^8)*(-9*a*c*AppellF1[1/8, -p, 1, 9/8, -((b*x^8)/a), -((d*x^8
)/c)] + 8*x^8*(-(b*c*p*AppellF1[9/8, 1 - p, 1, 17/8, -((b*x^8)/a), -((d*x^
8)/c)]) + a*d*AppellF1[9/8, -p, 2, 17/8, -((b*x^8)/a), -((d*x^8)/c)])))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^8)^p}{c+dx^8}dx$$

$$\Big\downarrow \text{937}$$

$$(a+bx^8)^p\left(\frac{bx^8}{a}+1\right)^{-p}\int \frac{\left(\frac{bx^8}{a}+1\right)^p}{dx^8+c}dx$$

$$\Big\downarrow \text{936}$$

$$\frac{x(a+bx^8)^p\left(\frac{bx^8}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{8},-p,1,\frac{9}{8},-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{c}$$

input
```
Int[(a + b*x^8)^p/(c + d*x^8),x]
```

output
```
(x*(a + b*x^8)^p*AppellF1[1/8, -p, 1, 9/8, -((b*x^8)/a), -((d*x^8)/c)])/(c
*(1 + (b*x^8)/a)^p)
```

Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{(b\,x^8+a)^p}{d\,x^8+c}dx$$

input
```
int((b*x^8+a)^p/(d*x^8+c),x)
```

output
```
int((b*x^8+a)^p/(d*x^8+c),x)
```

Fricas [F]

$$\int \frac{(a+bx^8)^p}{c+dx^8}\,dx=\int \frac{(bx^8+a)^p}{dx^8+c}\,dx$$

input
```
integrate((b*x^8+a)^p/(d*x^8+c),x, algorithm="fricas")
```

output
```
integral((b*x^8 + a)^p/(d*x^8 + c), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^8)^p}{c+dx^8}\,dx = \text{Timed out}$$

input
```
integrate((b*x**8+a)**p/(d*x**8+c),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^8)^p}{c+dx^8}\,dx = \int \frac{(bx^8+a)^p}{dx^8+c}\,dx$$

input
```
integrate((b*x^8+a)^p/(d*x^8+c),x, algorithm="maxima")
```

output
```
integrate((b*x^8 + a)^p/(d*x^8 + c), x)
```

Giac [F]

$$\int \frac{(a+bx^8)^p}{c+dx^8}\,dx = \int \frac{(bx^8+a)^p}{dx^8+c}\,dx$$

input
```
integrate((b*x^8+a)^p/(d*x^8+c),x, algorithm="giac")
```

output
```
integrate((b*x^8 + a)^p/(d*x^8 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^8)^p}{c+dx^8}\,dx=\int \frac{(b\,x^8+a)^p}{d\,x^8+c}\,dx$$

input
```
int((a + b*x^8)^p/(c + d*x^8),x)
```

output
```
int((a + b*x^8)^p/(c + d*x^8), x)
```

Reduce [F]

$$\int \frac{(a+bx^8)^p}{c+dx^8}\,dx=\int \frac{(b\,x^8+a)^p}{d\,x^8+c}dx$$

input
```
int((b*x^8+a)^p/(d*x^8+c),x)
```

output
```
int((a + b*x**8)**p/(c + d*x**8),x)
```

3.300 $\int \frac{(a+bx^8)^p}{(c+dx^8)^2} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 57

$$\int \frac{(a+bx^8)^p}{(c+dx^8)^2} \, dx = \frac{x(a+bx^8)^p \left(1+\frac{bx^8}{a}\right)^{-p} \operatorname{AppellF1}\left(\frac{1}{8},-p,2,\frac{9}{8},-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{c^2}$$

output
```
x*(b*x^8+a)^p*AppellF1(1/8,-p,2,9/8,-b*x^8/a,-d*x^8/c)/c^2/((1+b*x^8/a)^p)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 162 vs. $2(57) = 114$.

Time = 0.79 (sec) , antiderivative size = 162, normalized size of antiderivative = 2.84

$$\int \frac{(a+bx^8)^p}{(c+dx^8)^2} \, dx = -\frac{9acx(a+bx^8)^p \operatorname{AppellF1}\left(\frac{1}{8},-p,2,\frac{9}{8},-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{(c+dx^8)^2 \left(-9ac \operatorname{AppellF1}\left(\frac{1}{8},-p,2,\frac{9}{8},-\frac{bx^8}{a},-\frac{dx^8}{c}\right)-8x^8 \left(bcp \operatorname{AppellF1}\left(\frac{9}{8},1-p,2,\frac{17}{8},-\frac{bx^8}{a},-\frac{dx^8}{c}\right)\right.\right.}$$

input
```
Integrate[(a + b*x^8)^p/(c + d*x^8)^2,x]
```

output
```
(-9*a*c*x*(a + b*x^8)^p*AppellF1[1/8, -p, 2, 9/8, -((b*x^8)/a), -((d*x^8)/
c)])/((c + d*x^8)^2*(-9*a*c*AppellF1[1/8, -p, 2, 9/8, -((b*x^8)/a), -((d*x
^8)/c)] - 8*x^8*(b*c*p*AppellF1[9/8, 1 - p, 2, 17/8, -((b*x^8)/a), -((d*x^
8)/c)] - 2*a*d*AppellF1[9/8, -p, 3, 17/8, -((b*x^8)/a), -((d*x^8)/c)])))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^8)^p}{(c+dx^8)^2}dx$$

$$\Big\downarrow \text{ 937}$$

$$(a+bx^8)^p\left(\frac{bx^8}{a}+1\right)^{-p}\int \frac{\left(\frac{bx^8}{a}+1\right)^p}{(dx^8+c)^2}dx$$

$$\Big\downarrow \text{ 936}$$

$$\frac{x(a+bx^8)^p\left(\frac{bx^8}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{8},-p,2,\frac{9}{8},-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{c^2}$$

input
```
Int[(a + b*x^8)^p/(c + d*x^8)^2,x]
```

output
```
(x*(a + b*x^8)^p*AppellF1[1/8, -p, 2, 9/8, -((b*x^8)/a), -((d*x^8)/c)])/(c
^2*(1 + (b*x^8)/a)^p)
```

Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{(b\,x^{8}+a)^{p}}{(d\,x^{8}+c)^{2}}dx$$

input
```
int((b*x^8+a)^p/(d*x^8+c)^2,x)
```

output
```
int((b*x^8+a)^p/(d*x^8+c)^2,x)
```

Fricas [F]

$$\int \frac{(a+bx^8)^p}{(c+dx^8)^2}\,dx=\int \frac{(bx^8+a)^p}{(dx^8+c)^2}\,dx$$

input
```
integrate((b*x^8+a)^p/(d*x^8+c)^2,x, algorithm="fricas")
```

output
```
integral((b*x^8 + a)^p/(d^2*x^16 + 2*c*d*x^8 + c^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^8)^p}{(c+dx^8)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x**8+a)**p/(d*x**8+c)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^8)^p}{(c+dx^8)^2}\,dx = \int \frac{(bx^8+a)^p}{(dx^8+c)^2}\,dx$$

input
```
integrate((b*x^8+a)^p/(d*x^8+c)^2,x, algorithm="maxima")
```

output
```
integrate((b*x^8 + a)^p/(d*x^8 + c)^2, x)
```

Giac [F]

$$\int \frac{(a+bx^8)^p}{(c+dx^8)^2}\,dx = \int \frac{(bx^8+a)^p}{(dx^8+c)^2}\,dx$$

input
```
integrate((b*x^8+a)^p/(d*x^8+c)^2,x, algorithm="giac")
```

output
```
integrate((b*x^8 + a)^p/(d*x^8 + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^8)^p}{(c+dx^8)^2} \, dx = \int \frac{(b\,x^8+a)^p}{(d\,x^8+c)^2} \, dx$$

input
```
int((a + b*x^8)^p/(c + d*x^8)^2,x)
```

output
```
int((a + b*x^8)^p/(c + d*x^8)^2, x)
```

Reduce [F]

$$\int \frac{(a+bx^8)^p}{(c+dx^8)^2} \, dx = \int \frac{(b\,x^8+a)^p}{d^2x^{16}+2cd\,x^8+c^2} dx$$

input
```
int((b*x^8+a)^p/(d*x^8+c)^2,x)
```

output
```
int((a + b*x**8)**p/(c**2 + 2*c*d*x**8 + d**2*x**16),x)
```

3.301 $\int \frac{(a+bx^8)^p}{(c+dx^8)^3} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 57

$$\int \frac{(a+bx^8)^p}{(c+dx^8)^3} \, dx = \frac{x(a+bx^8)^p \left(1+\frac{bx^8}{a}\right)^{-p} \operatorname{AppellF1}\left(\frac{1}{8},-p,3,\frac{9}{8},-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{c^3}$$

output
```
x*(b*x^8+a)^p*AppellF1(1/8,-p,3,9/8,-b*x^8/a,-d*x^8/c)/c^3/((1+b*x^8/a)^p)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 162 vs. 2(57) = 114.

Time = 0.89 (sec) , antiderivative size = 162, normalized size of antiderivative = 2.84

$$\int \frac{(a+bx^8)^p}{(c+dx^8)^3} \, dx =$$

$$-\frac{9acx(a+bx^8)^p \operatorname{AppellF1}\left(\frac{1}{8},-p,3,\frac{9}{8},-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{(c+dx^8)^3 \left(-9ac \operatorname{AppellF1}\left(\frac{1}{8},-p,3,\frac{9}{8},-\frac{bx^8}{a},-\frac{dx^8}{c}\right)-8x^8 \left(bcp \operatorname{AppellF1}\left(\frac{9}{8},1-p,3,\frac{17}{8},-\frac{bx^8}{a},-\frac{dx^8}{c}\right)\right.\right.}$$

input
```
Integrate[(a + b*x^8)^p/(c + d*x^8)^3,x]
```

output
```
(-9*a*c*x*(a + b*x^8)^p*AppellF1[1/8, -p, 3, 9/8, -((b*x^8)/a), -((d*x^8)/
c)])/((c + d*x^8)^3*(-9*a*c*AppellF1[1/8, -p, 3, 9/8, -((b*x^8)/a), -((d*x
^8)/c)] - 8*x^8*(b*c*p*AppellF1[9/8, 1 - p, 3, 17/8, -((b*x^8)/a), -((d*x^
8)/c)] - 3*a*d*AppellF1[9/8, -p, 4, 17/8, -((b*x^8)/a), -((d*x^8)/c)])))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^8)^p}{(c+dx^8)^3}\,dx$$

$$\Big\downarrow \; 937$$

$$(a+bx^8)^p\left(\frac{bx^8}{a}+1\right)^{-p}\int \frac{\left(\frac{bx^8}{a}+1\right)^p}{(dx^8+c)^3}dx$$

$$\Big\downarrow \; 936$$

$$\frac{x(a+bx^8)^p\left(\frac{bx^8}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{8},-p,3,\frac{9}{8},-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{c^3}$$

input
```
Int[(a + b*x^8)^p/(c + d*x^8)^3,x]
```

output
```
(x*(a + b*x^8)^p*AppellF1[1/8, -p, 3, 9/8, -((b*x^8)/a), -((d*x^8)/c)])/(c
^3*(1 + (b*x^8)/a)^p)
```

Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{(b\,x^8+a)^p}{(d\,x^8+c)^3}dx$$

input
```
int((b*x^8+a)^p/(d*x^8+c)^3,x)
```

output
```
int((b*x^8+a)^p/(d*x^8+c)^3,x)
```

Fricas [F]

$$\int \frac{(a+bx^8)^p}{(c+dx^8)^3}\,dx = \int \frac{(bx^8+a)^p}{(dx^8+c)^3}\,dx$$

input
```
integrate((b*x^8+a)^p/(d*x^8+c)^3,x, algorithm="fricas")
```

output
```
integral((b*x^8 + a)^p/(d^3*x^24 + 3*c*d^2*x^16 + 3*c^2*d*x^8 + c^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^8)^p}{(c+dx^8)^3}\,dx = \text{Timed out}$$

input
```
integrate((b*x**8+a)**p/(d*x**8+c)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^8)^p}{(c+dx^8)^3}\,dx = \int \frac{(bx^8+a)^p}{(dx^8+c)^3}\,dx$$

input
```
integrate((b*x^8+a)^p/(d*x^8+c)^3,x, algorithm="maxima")
```

output
```
integrate((b*x^8 + a)^p/(d*x^8 + c)^3, x)
```

Giac [F]

$$\int \frac{(a+bx^8)^p}{(c+dx^8)^3}\,dx = \int \frac{(bx^8+a)^p}{(dx^8+c)^3}\,dx$$

input
```
integrate((b*x^8+a)^p/(d*x^8+c)^3,x, algorithm="giac")
```

output
```
integrate((b*x^8 + a)^p/(d*x^8 + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^8)^p}{(c+dx^8)^3}\,dx=\int \frac{{\left(b\,x^8+a\right)}^p}{{\left(d\,x^8+c\right)}^3}\,dx$$

input
```
int((a + b*x^8)^p/(c + d*x^8)^3,x)
```

output
```
int((a + b*x^8)^p/(c + d*x^8)^3, x)
```

Reduce [F]

$$\int \frac{(a+bx^8)^p}{(c+dx^8)^3}\,dx=\int \frac{\left(b\,x^{8}+a\right)^{p}}{d^{3} x^{24}+3 c\,d^{2} x^{16}+3 c^{2} d\,x^{8}+c^{3}}d x$$

input
```
int((b*x^8+a)^p/(d*x^8+c)^3,x)
```

output
```
int((a + b*x**8)**p/(c**3 + 3*c**2*d*x**8 + 3*c*d**2*x**16 + d**3*x**24),x
)
```

3.302 $\int (c+dx^4)^3 (a+bx^8)^p \, dx$

Optimal result

Integrand size = 19, antiderivative size = 209

$$\int (c+dx^4)^3 (a+bx^8)^p \, dx = \frac{3cd^2x(a+bx^8)^{1+p}}{b(9+8p)} + \frac{d^3x^5(a+bx^8)^{1+p}}{b(13+8p)} - \frac{c(3ad^2-bc^2(9+8p))\,x(a+bx^8)^p\left(1+\frac{bx^8}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{8},-p,\frac{9}{8},-\frac{bx^8}{a}\right)}{b(9+8p)} - \frac{d(5ad^2-3bc^2(13+8p))\,x^5(a+bx^8)^p\left(1+\frac{bx^8}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{5}{8},-p,\frac{13}{8},-\frac{bx^8}{a}\right)}{5b(13+8p)}$$

output

```
3*c*d^2*x*(b*x^8+a)^(p+1)/b/(9+8*p)+d^3*x^5*(b*x^8+a)^(p+1)/b/(13+8*p)-c*(
3*a*d^2-b*c^2*(9+8*p))*x*(b*x^8+a)^p*hypergeom([1/8, -p],[9/8],-b*x^8/a)/b
/(9+8*p)/((1+b*x^8/a)^p)-1/5*d*(5*a*d^2-3*b*c^2*(13+8*p))*x^5*(b*x^8+a)^p*
hypergeom([5/8, -p],[13/8],-b*x^8/a)/b/(13+8*p)/((1+b*x^8/a)^p)
```

Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.66

$$\int \left(c+dx^4\right)^3 \left(a+bx^8\right)^p \, dx = \frac{1}{195}x\left(a+bx^8\right)^p \left(1+\frac{bx^8}{a}\right)^{-p} \left(195c^3 \operatorname{Hypergeometric2F1}\left(\frac{1}{8},-p,\frac{9}{8},-\frac{bx^8}{a}\right) + dx^4\left(117c^2 \operatorname{Hypergeometric2F1}\left(\frac{5}{8},-p,\frac{13}{8},-\frac{bx^8}{a}\right) + 5dx^4\left(13c \operatorname{Hypergeometric2F1}\left(\frac{9}{8},-p,\frac{17}{8},-\frac{bx^8}{a}\right) + 3dx^4 \operatorname{Hypergeometric2F1}\left(\frac{13}{8},-p,\frac{21}{8},-\frac{bx^8}{a}\right)\right)\right)\right)$$

input
```
Integrate[(c + d*x^4)^3*(a + b*x^8)^p,x]
```

output
```
(x*(a + b*x^8)^p*(195*c^3*Hypergeometric2F1[1/8, -p, 9/8, -((b*x^8)/a)] +
d*x^4*(117*c^2*Hypergeometric2F1[5/8, -p, 13/8, -((b*x^8)/a)] + 5*d*x^4*(1
3*c*Hypergeometric2F1[9/8, -p, 17/8, -((b*x^8)/a)] + 3*d*x^4*Hypergeometri
c2F1[13/8, -p, 21/8, -((b*x^8)/a)]))))/(195*(1 + (b*x^8)/a)^p)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c+dx^4\right)^3 \left(a+bx^8\right)^p \, dx$$

$$\Big\downarrow \text{ 1770}$$

$$\int \left(c+dx^4\right)^3 \left(a+bx^8\right)^p dx$$

input
```
Int[(c + d*x^4)^3*(a + b*x^8)^p,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1770
```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol]
 :> Unintegrable[(d + e*x^n)^q*(a + c*x^(2*n))^p, x] /; FreeQ[{a, c, d, e,
n, p, q}, x] && EqQ[n2, 2*n]
```

Maple [F]

$$\int \left(x^{4} d+c\right)^{3} \left(b\,x^{8}+a\right)^{p}d x$$

input
```
int((d*x^4+c)^3*(b*x^8+a)^p,x)
```

output
```
int((d*x^4+c)^3*(b*x^8+a)^p,x)
```

Fricas [F]

$$\int \left(c+dx^4\right)^3 \left(a+bx^8\right)^p \, dx = \int (dx^4+c)^3(bx^8+a)^p \, dx$$

input
```
integrate((d*x^4+c)^3*(b*x^8+a)^p,x, algorithm="fricas")
```

output
```
integral((d^3*x^12 + 3*c*d^2*x^8 + 3*c^2*d*x^4 + c^3)*(b*x^8 + a)^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int \left(c+dx^4\right)^3 \left(a+bx^8\right)^p \, dx = \text{Timed out}$$

input
```
integrate((d*x**4+c)**3*(b*x**8+a)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \left(c+dx^4\right)^3 \left(a+bx^8\right)^p \, dx = \int \left(dx^4+c\right)^3 \left(bx^8+a\right)^p \, dx$$

input
```
integrate((d*x^4+c)^3*(b*x^8+a)^p,x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)^3*(b*x^8 + a)^p, x)
```

Giac [F]

$$\int \left(c+dx^4\right)^3 \left(a+bx^8\right)^p \, dx = \int \left(dx^4+c\right)^3 \left(bx^8+a\right)^p \, dx$$

input
```
integrate((d*x^4+c)^3*(b*x^8+a)^p,x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)^3*(b*x^8 + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(c+dx^4\right)^3 \left(a+bx^8\right)^p \, dx = \int \left(b\,x^8+a\right)^p \left(d\,x^4+c\right)^3 \, dx$$

input
```
int((a + b*x^8)^p*(c + d*x^4)^3,x)
```

output
```
int((a + b*x^8)^p*(c + d*x^4)^3, x)
```

Reduce [F]

$$\int (c+dx^4)^3 (a+bx^8)^p \, dx = \text{too large to display}$$

input
```
int((d*x^4+c)^3*(b*x^8+a)^p,x)
```

output
```
(1536*(a + b*x**8)**p*a*c*d**2*p**3*x + 3456*(a + b*x**8)**p*a*c*d**2*p**2
*x + 1560*(a + b*x**8)**p*a*c*d**2*p*x + 512*(a + b*x**8)**p*a*d**3*p**3*x
**5 + 640*(a + b*x**8)**p*a*d**3*p**2*x**5 + 72*(a + b*x**8)**p*a*d**3*p*x
**5 + 512*(a + b*x**8)**p*b*c**3*p**3*x + 1728*(a + b*x**8)**p*b*c**3*p**2
*x + 1816*(a + b*x**8)**p*b*c**3*p*x + 585*(a + b*x**8)**p*b*c**3*x + 1536
*(a + b*x**8)**p*b*c**2*d*p**3*x**5 + 4416*(a + b*x**8)**p*b*c**2*d*p**2*x
**5 + 3336*(a + b*x**8)**p*b*c**2*d*p*x**5 + 351*(a + b*x**8)**p*b*c**2*d*
x**5 + 1536*(a + b*x**8)**p*b*c*d**2*p**3*x**9 + 3648*(a + b*x**8)**p*b*c*
d**2*p**2*x**9 + 1992*(a + b*x**8)**p*b*c*d**2*p*x**9 + 195*(a + b*x**8)**
p*b*c*d**2*x**9 + 512*(a + b*x**8)**p*b*d**3*p**3*x**13 + 960*(a + b*x**8)
**p*b*d**3*p**2*x**13 + 472*(a + b*x**8)**p*b*d**3*p*x**13 + 45*(a + b*x**
8)**p*b*d**3*x**13 - 6291456*int((a + b*x**8)**p/(4096*a*p**4 + 14336*a*p*
*3 + 16256*a*p**2 + 6496*a*p + 585*a + 4096*b*p**4*x**8 + 14336*b*p**3*x**
8 + 16256*b*p**2*x**8 + 6496*b*p*x**8 + 585*b*x**8),x)*a**2*c*d**2*p**7 -
36175872*int((a + b*x**8)**p/(4096*a*p**4 + 14336*a*p**3 + 16256*a*p**2 +
6496*a*p + 585*a + 4096*b*p**4*x**8 + 14336*b*p**3*x**8 + 16256*b*p**2*x**
8 + 6496*b*p*x**8 + 585*b*x**8),x)*a**2*c*d**2*p**6 - 80904192*int((a + b*
x**8)**p/(4096*a*p**4 + 14336*a*p**3 + 16256*a*p**2 + 6496*a*p + 585*a + 4
096*b*p**4*x**8 + 14336*b*p**3*x**8 + 16256*b*p**2*x**8 + 6496*b*p*x**8 +
585*b*x**8),x)*a**2*c*d**2*p**5 - 88522752*int((a + b*x**8)**p/(4096*a*...
```

3.303 $\int (c + dx^4)^2 (a + bx^8)^p \, dx$

Optimal result

Integrand size = 19, antiderivative size = 150

$$\int (c + dx^4)^2 (a + bx^8)^p \, dx = \frac{d^2 x (a + bx^8)^{1+p}}{b(9 + 8p)} - \frac{(ad^2 - bc^2(9 + 8p))\, x (a + bx^8)^p \left(1 + \frac{bx^8}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{8}, -p, \frac{9}{8}, -\frac{bx^8}{a}\right)}{b(9 + 8p)} + \frac{2}{5} c d x^5 (a + bx^8)^p \left(1 + \frac{bx^8}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{5}{8}, -p, \frac{13}{8}, -\frac{bx^8}{a}\right)$$

output
```
d^2*x*(b*x^8+a)^(p+1)/b/(9+8*p)-(a*d^2-b*c^2*(9+8*p))*x*(b*x^8+a)^p*hyperg
eom([1/8, -p],[9/8],-b*x^8/a)/b/(9+8*p)/((1+b*x^8/a)^p)+2/5*c*d*x^5*(b*x^8
+a)^p*hypergeom([5/8, -p],[13/8],-b*x^8/a)/((1+b*x^8/a)^p)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.71

$$\int \left(c+dx^4\right)^2 \left(a+bx^8\right)^p\,dx=\frac{1}{45}x\left(a+bx^8\right)^p\left(1+\frac{bx^8}{a}\right)^{-p}\left(45c^2\,\text{Hypergeometric2F1}\left(\frac{1}{8},-p,\frac{9}{8},-\frac{bx^8}{a}\right)+dx^4\left(18c\,\text{Hypergeometric2F1}\left(\frac{5}{8},-p,\frac{13}{8},-\frac{bx^8}{a}\right)+5dx^4\,\text{Hypergeometric2F1}\left(\frac{9}{8},-p,\frac{17}{8},-\frac{bx^8}{a}\right)\right)\right)$$

input
```
Integrate[(c + d*x^4)^2*(a + b*x^8)^p,x]
```

output
```
(x*(a + b*x^8)^p*(45*c^2*Hypergeometric2F1[1/8, -p, 9/8, -((b*x^8)/a)] + d
*x^4*(18*c*Hypergeometric2F1[5/8, -p, 13/8, -((b*x^8)/a)] + 5*d*x^4*Hyperg
eometric2F1[9/8, -p, 17/8, -((b*x^8)/a)])))/(45*(1 + (b*x^8)/a)^p)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c+dx^4\right)^2 \left(a+bx^8\right)^p\,dx$$

$$\Big\downarrow \text{1770}$$

$$\int \left(c+dx^4\right)^2 \left(a+bx^8\right)^p dx$$

input
```
Int[(c + d*x^4)^2*(a + b*x^8)^p,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1770
```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol]
 :> Unintegrable[(d + e*x^n)^q*(a + c*x^(2*n))^p, x] /; FreeQ[{a, c, d, e,
n, p, q}, x] && EqQ[n2, 2*n]
```

Maple [F]

$$\int \left(x^4 d+c\right)^2 \left(b\,x^8+a\right)^p dx$$

input
```
int((d*x^4+c)^2*(b*x^8+a)^p,x)
```

output
```
int((d*x^4+c)^2*(b*x^8+a)^p,x)
```

Fricas [F]

$$\int \left(c+dx^4\right)^2 \left(a+bx^8\right)^p \,dx=\int \left(dx^4+c\right)^2 \left(bx^8+a\right)^p \,dx$$

input
```
integrate((d*x^4+c)^2*(b*x^8+a)^p,x, algorithm="fricas")
```

output
```
integral((d^2*x^8 + 2*c*d*x^4 + c^2)*(b*x^8 + a)^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int \left(c+dx^4\right)^2 \left(a+bx^8\right)^p \,dx=\text{Timed out}$$

input
```
integrate((d*x**4+c)**2*(b*x**8+a)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \left(c+dx^4\right)^2 \left(a+bx^8\right)^p \, dx = \int \left(dx^4+c\right)^2 \left(bx^8+a\right)^p \, dx$$

input
```
integrate((d*x^4+c)^2*(b*x^8+a)^p,x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)^2*(b*x^8 + a)^p, x)
```

Giac [F]

$$\int \left(c+dx^4\right)^2 \left(a+bx^8\right)^p \, dx = \int \left(dx^4+c\right)^2 \left(bx^8+a\right)^p \, dx$$

input
```
integrate((d*x^4+c)^2*(b*x^8+a)^p,x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)^2*(b*x^8 + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(c+dx^4\right)^2 \left(a+bx^8\right)^p \, dx = \int \left(b\,x^8+a\right)^p \left(d\,x^4+c\right)^2 \, dx$$

input
```
int((a + b*x^8)^p*(c + d*x^4)^2,x)
```

output
```
int((a + b*x^8)^p*(c + d*x^4)^2, x)
```

Reduce [F]

$$\int \left(c+dx^4\right)^2 \left(a+bx^8\right)^p \, dx = \text{too large to display}$$

input
```
int((d*x^4+c)^2*(b*x^8+a)^p,x)
```

output
```
(64*(a + b*x**8)**p*a*d**2*p**2*x + 40*(a + b*x**8)**p*a*d**2*p*x + 64*(a
+ b*x**8)**p*b*c**2*p**2*x + 112*(a + b*x**8)**p*b*c**2*p*x + 45*(a + b*x*
*8)**p*b*c**2*x + 128*(a + b*x**8)**p*b*c*d*p**2*x**5 + 160*(a + b*x**8)**
p*b*c*d*p*x**5 + 18*(a + b*x**8)**p*b*c*d*x**5 + 64*(a + b*x**8)**p*b*d**2
*p**2*x**9 + 48*(a + b*x**8)**p*b*d**2*p*x**9 + 5*(a + b*x**8)**p*b*d**2*x
**9 - 32768*int((a + b*x**8)**p/(512*a*p**3 + 960*a*p**2 + 472*a*p + 45*a
+ 512*b*p**3*x**8 + 960*b*p**2*x**8 + 472*b*p*x**8 + 45*b*x**8),x)*a**2*d*
*2*p**5 - 81920*int((a + b*x**8)**p/(512*a*p**3 + 960*a*p**2 + 472*a*p + 4
5*a + 512*b*p**3*x**8 + 960*b*p**2*x**8 + 472*b*p*x**8 + 45*b*x**8),x)*a**
2*d**2*p**4 - 68608*int((a + b*x**8)**p/(512*a*p**3 + 960*a*p**2 + 472*a*p
 + 45*a + 512*b*p**3*x**8 + 960*b*p**2*x**8 + 472*b*p*x**8 + 45*b*x**8),x)
*a**2*d**2*p**3 - 21760*int((a + b*x**8)**p/(512*a*p**3 + 960*a*p**2 + 472
*a*p + 45*a + 512*b*p**3*x**8 + 960*b*p**2*x**8 + 472*b*p*x**8 + 45*b*x**8
),x)*a**2*d**2*p**2 - 1800*int((a + b*x**8)**p/(512*a*p**3 + 960*a*p**2 +
472*a*p + 45*a + 512*b*p**3*x**8 + 960*b*p**2*x**8 + 472*b*p*x**8 + 45*b*x
**8),x)*a**2*d**2*p + 262144*int((a + b*x**8)**p/(512*a*p**3 + 960*a*p**2
+ 472*a*p + 45*a + 512*b*p**3*x**8 + 960*b*p**2*x**8 + 472*b*p*x**8 + 45*b
*x**8),x)*a*b*c**2*p**6 + 950272*int((a + b*x**8)**p/(512*a*p**3 + 960*a*p
**2 + 472*a*p + 45*a + 512*b*p**3*x**8 + 960*b*p**2*x**8 + 472*b*p*x**8 +
45*b*x**8),x)*a*b*c**2*p**5 + 1286144*int((a + b*x**8)**p/(512*a*p**3 +...
```

3.304 $\int (c+dx^4) (a+bx^8)^p \, dx$

Optimal result

Integrand size = 17, antiderivative size = 96

$$\int (c+dx^4) (a+bx^8)^p \, dx = cx(a+bx^8)^p \left(1+\frac{bx^8}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{8},-p,\frac{9}{8},-\frac{bx^8}{a}\right)+\frac{1}{5}dx^5(a+bx^8)^p \left(1+\frac{bx^8}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{5}{8},-p,\frac{13}{8},-\frac{bx^8}{a}\right)$$

output
```
c*x*(b*x^8+a)^p*hypergeom([1/8, -p],[9/8],-b*x^8/a)/((1+b*x^8/a)^p)+1/5*d*
x^5*(b*x^8+a)^p*hypergeom([5/8, -p],[13/8],-b*x^8/a)/((1+b*x^8/a)^p)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.78

$$\int (c+dx^4) (a+bx^8)^p \, dx = \frac{1}{5}x(a+bx^8)^p \left(1+\frac{bx^8}{a}\right)^{-p} \left(5c \, \text{Hypergeometric2F1}\left(\frac{1}{8},-p,\frac{9}{8},-\frac{bx^8}{a}\right)+dx^4 \, \text{Hypergeometric2F1}\left(\frac{5}{8},-p,\frac{13}{8},-\frac{bx^8}{a}\right)\right)$$

input
```
Integrate[(c + d*x^4)*(a + b*x^8)^p,x]
```

output
```
(x*(a + b*x^8)^p*(5*c*Hypergeometric2F1[1/8, -p, 9/8, -((b*x^8)/a)] + d*x^
4*Hypergeometric2F1[5/8, -p, 13/8, -((b*x^8)/a)]))/(5*(1 + (b*x^8)/a)^p)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1763, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c+dx^4\right) \left(a+bx^8\right)^p \, dx$$

$$\Big\downarrow \text{ 1763}$$

$$\int \left(c\left(a+bx^8\right)^p+dx^4\left(a+bx^8\right)^p\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$cx\left(a+bx^8\right)^p\left(\frac{bx^8}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{8},-p,\frac{9}{8},-\frac{bx^8}{a}\right)+\frac{1}{5}dx^5\left(a+bx^8\right)^p\left(\frac{bx^8}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{5}{8},-p,\frac{13}{8},-\frac{bx^8}{a}\right)$$

input
```
Int[(c + d*x^4)*(a + b*x^8)^p,x]
```

output
```
(c*x*(a + b*x^8)^p*Hypergeometric2F1[1/8, -p, 9/8, -((b*x^8)/a)])/(1 + (b*
x^8)/a)^p + (d*x^5*(a + b*x^8)^p*Hypergeometric2F1[5/8, -p, 13/8, -((b*x^8
)/a)])/(5*(1 + (b*x^8)/a)^p)
```

Defintions of rubi rules used

rule 1763
```
Int[((d_) + (e_.)*(x_)^(n_))*((a_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] :> I
nt[ExpandIntegrand[(d + e*x^n)*(a + c*x^(2*n))^p, x], x] /; FreeQ[{a, c, d,
 e, n}, x] && EqQ[n2, 2*n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \left(x^{4} d+c \right) \left(b \,x^{8}+a \right)^{p}d x$$

input
```
int((d*x^4+c)*(b*x^8+a)^p,x)
```

output
```
int((d*x^4+c)*(b*x^8+a)^p,x)
```

Fricas [F]

$$\int \left(c+dx^4\right) \left(a+bx^8\right)^p \, dx = \int \left(dx^{4} + c\right) {\left(bx^{8} + a\right)}^{p} \,d x$$

input
```
integrate((d*x^4+c)*(b*x^8+a)^p,x, algorithm="fricas")
```

output
```
integral((d*x^4 + c)*(b*x^8 + a)^p, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 166.52 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.78

$$\int (c+dx^4)(a+bx^8)^p\,dx = \frac{a^p cx\Gamma\left(\frac{1}{8}\right) {}_2F_1\left(\begin{matrix}\frac{1}{8},-p\\ \frac{9}{8}\end{matrix}\middle|\frac{bx^8e^{i\pi}}{a}\right)}{8\Gamma\left(\frac{9}{8}\right)} + \frac{a^p dx^5\Gamma\left(\frac{5}{8}\right) {}_2F_1\left(\begin{matrix}\frac{5}{8},-p\\ \frac{13}{8}\end{matrix}\middle|\frac{bx^8e^{i\pi}}{a}\right)}{8\Gamma\left(\frac{13}{8}\right)}$$

input
```
integrate((d*x**4+c)*(b*x**8+a)**p,x)
```

output
```
a**p*c*x*gamma(1/8)*hyper((1/8, -p), (9/8,), b*x**8*exp_polar(I*pi)/a)/(8*
gamma(9/8)) + a**p*d*x**5*gamma(5/8)*hyper((5/8, -p), (13/8,), b*x**8*exp_
polar(I*pi)/a)/(8*gamma(13/8))
```

Maxima [F]

$$\int (c+dx^4)(a+bx^8)^p\,dx = \int (dx^4+c)(bx^8+a)^p\,dx$$

input
```
integrate((d*x^4+c)*(b*x^8+a)^p,x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)*(b*x^8 + a)^p, x)
```

Giac [F]

$$\int (c+dx^4)(a+bx^8)^p\,dx = \int (dx^4+c)(bx^8+a)^p\,dx$$

input
```
integrate((d*x^4+c)*(b*x^8+a)^p,x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)*(b*x^8 + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx^4)(a+bx^8)^p\,dx = \int (b\,x^8+a)^p\,(d\,x^4+c)\,dx$$

input
```
int((a + b*x^8)^p*(c + d*x^4),x)
```

output
```
int((a + b*x^8)^p*(c + d*x^4), x)
```

Reduce [F]

$$\int (c+dx^4)(a+bx^8)^p\,dx$$
$$= \frac{8(b\,x^8+a)^p\,cpx+5(b\,x^8+a)^p\,cx+8(b\,x^8+a)^p\,dp\,x^5+(b\,x^8+a)^p\,d\,x^5+4096\left(\int \frac{(b\,x^8+a)^p}{64b\,p^2x^8+48bp\,x^8+5b\,x^8+64}\right.}{}$$

input
```
int((d*x^4+c)*(b*x^8+a)^p,x)
```

output

```
(8*(a + b*x**8)**p*c*p*x + 5*(a + b*x**8)**p*c*x + 8*(a + b*x**8)**p*d*p*x
**5 + (a + b*x**8)**p*d*x**5 + 4096*int((a + b*x**8)**p/(64*a*p**2 + 48*a*
p + 5*a + 64*b*p**2*x**8 + 48*b*p*x**8 + 5*b*x**8),x)*a*c*p**4 + 5632*int(
(a + b*x**8)**p/(64*a*p**2 + 48*a*p + 5*a + 64*b*p**2*x**8 + 48*b*p*x**8 +
 5*b*x**8),x)*a*c*p**3 + 2240*int((a + b*x**8)**p/(64*a*p**2 + 48*a*p + 5*
a + 64*b*p**2*x**8 + 48*b*p*x**8 + 5*b*x**8),x)*a*c*p**2 + 200*int((a + b*
x**8)**p/(64*a*p**2 + 48*a*p + 5*a + 64*b*p**2*x**8 + 48*b*p*x**8 + 5*b*x*
*8),x)*a*c*p + 4096*int(((a + b*x**8)**p*x**4)/(64*a*p**2 + 48*a*p + 5*a +
 64*b*p**2*x**8 + 48*b*p*x**8 + 5*b*x**8),x)*a*d*p**4 + 3584*int(((a + b*x
**8)**p*x**4)/(64*a*p**2 + 48*a*p + 5*a + 64*b*p**2*x**8 + 48*b*p*x**8 + 5
*b*x**8),x)*a*d*p**3 + 704*int(((a + b*x**8)**p*x**4)/(64*a*p**2 + 48*a*p
+ 5*a + 64*b*p**2*x**8 + 48*b*p*x**8 + 5*b*x**8),x)*a*d*p**2 + 40*int(((a
+ b*x**8)**p*x**4)/(64*a*p**2 + 48*a*p + 5*a + 64*b*p**2*x**8 + 48*b*p*x**
8 + 5*b*x**8),x)*a*d*p)/(64*p**2 + 48*p + 5)
```

3.305 $\int (a+bx^8)^p \, dx$

Optimal result

Integrand size = 9, antiderivative size = 44

$$\int (a+bx^8)^p \, dx = x(a+bx^8)^p \left(1+\frac{bx^8}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{8},-p,\frac{9}{8},-\frac{bx^8}{a}\right)$$

output
```
x*(b*x^8+a)^p*hypergeom([1/8, -p],[9/8],-b*x^8/a)/((1+b*x^8/a)^p)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int (a+bx^8)^p \, dx = x(a+bx^8)^p \left(1+\frac{bx^8}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{8},-p,\frac{9}{8},-\frac{bx^8}{a}\right)$$

input
```
Integrate[(a + b*x^8)^p,x]
```

output
```
(x*(a + b*x^8)^p*Hypergeometric2F1[1/8, -p, 9/8, -((b*x^8)/a)])/(1 + (b*x^
8)/a)^p
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {779, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^8)^p\,dx$$

$$\Big\downarrow \text{779}$$

$$(a+bx^8)^p\left(\frac{bx^8}{a}+1\right)^{-p}\int \left(\frac{bx^8}{a}+1\right)^p dx$$

$$\Big\downarrow \text{778}$$

$$x(a+bx^8)^p\left(\frac{bx^8}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{8},-p,\frac{9}{8},-\frac{bx^8}{a}\right)$$

input
```
Int[(a + b*x^8)^p,x]
```

output
```
(x*(a + b*x^8)^p*Hypergeometric2F1[1/8, -p, 9/8, -((b*x^8)/a)])/(1 + (b*x^
8)/a)^p
```

Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Si
mplify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \left(b\,x^{8}+a\right)^{p}dx$$

input
```
int((b*x^8+a)^p,x)
```

output
```
int((b*x^8+a)^p,x)
```

Fricas [F]

$$\int \left(a+bx^{8}\right)^{p}\,dx=\int \left(bx^{8}+a\right)^{p}dx$$

input
```
integrate((b*x^8+a)^p,x, algorithm="fricas")
```

output
```
integral((b*x^8 + a)^p, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 31.17 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.77

$$\int \left(a+bx^{8}\right)^{p}\,dx=\frac{a^{p}x\Gamma\left(\frac{1}{8}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{8},-p\\ \frac{9}{8}\end{matrix}\middle|\frac{bx^{8}e^{i\pi}}{a}\right)}{8\Gamma\left(\frac{9}{8}\right)}$$

input
```
integrate((b*x**8+a)**p,x)
```

output
```
a**p*x*gamma(1/8)*hyper((1/8, -p), (9/8,), b*x**8*exp_polar(I*pi)/a)/(8*ga
mma(9/8))
```

Maxima [F]

$$\int \left(a+bx^8\right)^p\,dx=\int \left(bx^8+a\right)^p\,dx$$

input
```
integrate((b*x^8+a)^p,x, algorithm="maxima")
```

output
```
integrate((b*x^8 + a)^p, x)
```

Giac [F]

$$\int \left(a+bx^8\right)^p\,dx=\int \left(bx^8+a\right)^p\,dx$$

input
```
integrate((b*x^8+a)^p,x, algorithm="giac")
```

output
```
integrate((b*x^8 + a)^p, x)
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.93

$$\int \left(a+bx^8\right)^p\,dx=\frac{x\,\left(b\,x^8+a\right)^p\,{}_2\mathrm{F}_1\left(\frac{1}{8},-p;\ \frac{9}{8};\ -\frac{b\,x^8}{a}\right)}{\left(\frac{b\,x^8}{a}+1\right)^p}$$

input
```
int((a + b*x^8)^p,x)
```

output
```
(x*(a + b*x^8)^p*hypergeom([1/8, -p], 9/8, -(b*x^8)/a))/((b*x^8)/a + 1)^p
```

Reduce [F]

$$\int \left(a+bx^8\right)^p\,dx$$
$$=\frac{\left(b\,x^{8}+a\right)^{p} x+64\left(\int \frac{\left(b\,x^{8}+a\right)^{p}}{8bp\,x^{8}+b\,x^{8}+8ap+a}dx\right) a\,p^{2}+8\left(\int \frac{\left(b\,x^{8}+a\right)^{p}}{8bp\,x^{8}+b\,x^{8}+8ap+a}dx\right) ap}{8p+1}$$

input
```
int((b*x^8+a)^p,x)
```

output
```
((a + b*x**8)**p*x + 64*int((a + b*x**8)**p/(8*a*p + a + 8*b*p*x**8 + b*x*
*8),x)*a*p**2 + 8*int((a + b*x**8)**p/(8*a*p + a + 8*b*p*x**8 + b*x**8),x)
*a*p)/(8*p + 1)
```

3.306 $\int \frac{(a+bx^8)^p}{c+dx^4}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 123

$$\int \frac{(a+bx^8)^p}{c+dx^4}\,dx = \frac{x(a+bx^8)^p\left(1+\frac{bx^8}{a}\right)^{-p}\operatorname{AppellF1}\left(\frac{1}{8},-p,1,\frac{9}{8},-\frac{bx^8}{a},\frac{d^2x^8}{c^2}\right)}{c} - \frac{dx^5(a+bx^8)^p\left(1+\frac{bx^8}{a}\right)^{-p}\operatorname{AppellF1}\left(\frac{5}{8},-p,1,\frac{13}{8},-\frac{bx^8}{a},\frac{d^2x^8}{c^2}\right)}{5c^2}$$

output
```
x*(b*x^8+a)^p*AppellF1(1/8,1,-p,9/8,d^2*x^8/c^2,-b*x^8/a)/c/((1+b*x^8/a)^p
)-1/5*d*x^5*(b*x^8+a)^p*AppellF1(5/8,1,-p,13/8,d^2*x^8/c^2,-b*x^8/a)/c^2/(
(1+b*x^8/a)^p)
```

Mathematica [F]

$$\int \frac{(a+bx^8)^p}{c+dx^4}\,dx = \int \frac{(a+bx^8)^p}{c+dx^4}\,dx$$

input
```
Integrate[(a + b*x^8)^p/(c + d*x^4),x]
```

output
```
Integrate[(a + b*x^8)^p/(c + d*x^4), x]
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1768, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^8)^p}{c+dx^4}\,dx$$

$$\Big\downarrow \text{1768}$$

$$\int \left(\frac{c(a+bx^8)^p}{c^2-d^2x^8}+\frac{dx^4(a+bx^8)^p}{d^2x^8-c^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{x(a+bx^8)^p\left(\frac{bx^8}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{8},-p,1,\frac{9}{8},-\frac{bx^8}{a},\frac{d^2x^8}{c^2}\right)}{c}-\frac{dx^5(a+bx^8)^p\left(\frac{bx^8}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{5}{8},-p,1,\frac{13}{8},-\frac{bx^8}{a},\frac{d^2x^8}{c^2}\right)}{5c^2}$$

input
```
Int[(a + b*x^8)^p/(c + d*x^4),x]
```

output
```
(x*(a + b*x^8)^p*AppellF1[1/8, -p, 1, 9/8, -((b*x^8)/a), (d^2*x^8)/c^2])/(
c*(1 + (b*x^8)/a)^p) - (d*x^5*(a + b*x^8)^p*AppellF1[5/8, -p, 1, 13/8, -((
b*x^8)/a), (d^2*x^8)/c^2])/(5*c^2*(1 + (b*x^8)/a)^p)
```

Defintions of rubi rules used

rule 1768
```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol]
 :> Int[ExpandIntegrand[(a + c*x^(2*n))^p, (d/(d^2 - e^2*x^(2*n)) - e*(x^n/
(d^2 - e^2*x^(2*n))))^(-q), x], x] /; FreeQ[{a, c, d, e, n, p}, x] && EqQ[n
2, 2*n] && NeQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && ILtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{(b\,x^8+a)^p}{x^4d+c}dx$$

input
```
int((b*x^8+a)^p/(d*x^4+c),x)
```

output
```
int((b*x^8+a)^p/(d*x^4+c),x)
```

Fricas [F]

$$\int \frac{(a+bx^8)^p}{c+dx^4}\,dx=\int \frac{(bx^8+a)^p}{dx^4+c}\,dx$$

input
```
integrate((b*x^8+a)^p/(d*x^4+c),x, algorithm="fricas")
```

output
```
integral((b*x^8 + a)^p/(d*x^4 + c), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^8)^p}{c+dx^4}\,dx=\text{Timed out}$$

input
```
integrate((b*x**8+a)**p/(d*x**4+c),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^8)^p}{c+dx^4}\,dx=\int \frac{(bx^8+a)^p}{dx^4+c}\,dx$$

input
```
integrate((b*x^8+a)^p/(d*x^4+c),x, algorithm="maxima")
```

output
```
integrate((b*x^8 + a)^p/(d*x^4 + c), x)
```

Giac [F]

$$\int \frac{(a+bx^8)^p}{c+dx^4}\,dx=\int \frac{(bx^8+a)^p}{dx^4+c}\,dx$$

input
```
integrate((b*x^8+a)^p/(d*x^4+c),x, algorithm="giac")
```

output
```
integrate((b*x^8 + a)^p/(d*x^4 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^8)^p}{c+dx^4}\,dx=\int \frac{(b\,x^8+a)^p}{d\,x^4+c}\,dx$$

input
```
int((a + b*x^8)^p/(c + d*x^4),x)
```

output
```
int((a + b*x^8)^p/(c + d*x^4), x)
```

Reduce [F]

$$\int \frac{(a+bx^8)^p}{c+dx^4}\,dx = \int \frac{(b\,x^8+a)^p}{d\,x^4+c}dx$$

input
```
int((b*x^8+a)^p/(d*x^4+c),x)
```

output
```
int((a + b*x**8)**p/(c + d*x**4),x)
```

3.307 $\int \frac{(a+bx^8)^p}{(c+dx^4)^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 189

$$\int \frac{(a+bx^8)^p}{(c+dx^4)^2}\,dx = \frac{x(a+bx^8)^p\left(1+\frac{bx^8}{a}\right)^{-p}\operatorname{AppellF1}\left(\frac{1}{8},-p,2,\frac{9}{8},-\frac{bx^8}{a},\frac{d^2x^8}{c^2}\right)}{c^2} - \frac{2dx^5(a+bx^8)^p\left(1+\frac{bx^8}{a}\right)^{-p}\operatorname{AppellF1}\left(\frac{5}{8},-p,2,\frac{13}{8},-\frac{bx^8}{a},\frac{d^2x^8}{c^2}\right)}{5c^3} + \frac{d^2x^9(a+bx^8)^p\left(1+\frac{bx^8}{a}\right)^{-p}\operatorname{AppellF1}\left(\frac{9}{8},-p,2,\frac{17}{8},-\frac{bx^8}{a},\frac{d^2x^8}{c^2}\right)}{9c^4}$$

output

```
x*(b*x^8+a)^p*AppellF1(1/8,2,-p,9/8,d^2*x^8/c^2,-b*x^8/a)/c^2/((1+b*x^8/a)
^p)-2/5*d*x^5*(b*x^8+a)^p*AppellF1(5/8,2,-p,13/8,d^2*x^8/c^2,-b*x^8/a)/c^3
/((1+b*x^8/a)^p)+1/9*d^2*x^9*(b*x^8+a)^p*AppellF1(9/8,2,-p,17/8,d^2*x^8/c^
2,-b*x^8/a)/c^4/((1+b*x^8/a)^p)
```

Mathematica [F]

$$\int \frac{(a+bx^8)^p}{(c+dx^4)^2}\,dx=\int \frac{(a+bx^8)^p}{(c+dx^4)^2}\,dx$$

input
```
Integrate[(a + b*x^8)^p/(c + d*x^4)^2,x]
```

output
```
Integrate[(a + b*x^8)^p/(c + d*x^4)^2, x]
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1768, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^8)^p}{(c+dx^4)^2}\,dx$$

$$\Big\downarrow \text{1768}$$

$$\int \left(\frac{c^2(a+bx^8)^p}{(c^2-d^2x^8)^2}+\frac{d^2x^8(a+bx^8)^p}{(d^2x^8-c^2)^2}-\frac{2cdx^4(a+bx^8)^p}{(c^2-d^2x^8)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{x(a+bx^8)^p\left(\frac{bx^8}{a}+1\right)^{-p}\operatorname{AppellF1}\left(\frac{1}{8},-p,2,\frac{9}{8},-\frac{bx^8}{a},\frac{d^2x^8}{c^2}\right)}{c^2}+\frac{d^2x^9(a+bx^8)^p\left(\frac{bx^8}{a}+1\right)^{-p}\operatorname{AppellF1}\left(\frac{9}{8},-p,2,\frac{17}{8},-\frac{bx^8}{a},\frac{d^2x^8}{c^2}\right)}{9c^4}-\frac{2dx^5(a+bx^8)^p\left(\frac{bx^8}{a}+1\right)^{-p}\operatorname{AppellF1}\left(\frac{5}{8},-p,2,\frac{13}{8},-\frac{bx^8}{a},\frac{d^2x^8}{c^2}\right)}{5c^3}$$

input
```
Int[(a + b*x^8)^p/(c + d*x^4)^2,x]
```

output
```
(x*(a + b*x^8)^p*AppellF1[1/8, -p, 2, 9/8, -((b*x^8)/a), (d^2*x^8)/c^2])/(
c^2*(1 + (b*x^8)/a)^p) - (2*d*x^5*(a + b*x^8)^p*AppellF1[5/8, -p, 2, 13/8,
 -((b*x^8)/a), (d^2*x^8)/c^2])/(5*c^3*(1 + (b*x^8)/a)^p) + (d^2*x^9*(a + b
*x^8)^p*AppellF1[9/8, -p, 2, 17/8, -((b*x^8)/a), (d^2*x^8)/c^2])/(9*c^4*(1
 + (b*x^8)/a)^p)
```

Defintions of rubi rules used

rule 1768
```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol]
 :> Int[ExpandIntegrand[(a + c*x^(2*n))^p, (d/(d^2 - e^2*x^(2*n)) - e*(x^n/
(d^2 - e^2*x^(2*n))))^(-q), x], x] /; FreeQ[{a, c, d, e, n, p}, x] && EqQ[n
2, 2*n] && NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{(b\,x^{8}+a)^{p}}{(x^{4}d+c)^{2}}dx$$

input
```
int((b*x^8+a)^p/(d*x^4+c)^2,x)
```

output
```
int((b*x^8+a)^p/(d*x^4+c)^2,x)
```

Fricas [F]

$$\int \frac{(a+bx^8)^p}{(c+dx^4)^2}\,dx=\int \frac{(bx^8+a)^p}{(dx^4+c)^2}\,dx$$

input
```
integrate((b*x^8+a)^p/(d*x^4+c)^2,x, algorithm="fricas")
```

output
```
integral((b*x^8 + a)^p/(d^2*x^8 + 2*c*d*x^4 + c^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^8)^p}{(c+dx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x**8+a)**p/(d*x**4+c)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^8)^p}{(c+dx^4)^2}\,dx = \int \frac{(bx^8+a)^p}{(dx^4+c)^2}\,dx$$

input
```
integrate((b*x^8+a)^p/(d*x^4+c)^2,x, algorithm="maxima")
```

output
```
integrate((b*x^8 + a)^p/(d*x^4 + c)^2, x)
```

Giac [F]

$$\int \frac{(a+bx^8)^p}{(c+dx^4)^2}\,dx = \int \frac{(bx^8+a)^p}{(dx^4+c)^2}\,dx$$

input
```
integrate((b*x^8+a)^p/(d*x^4+c)^2,x, algorithm="giac")
```

output
```
integrate((b*x^8 + a)^p/(d*x^4 + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^8)^p}{(c+dx^4)^2}\,dx = \int \frac{(b\,x^8+a)^p}{(d\,x^4+c)^2}\,dx$$

input
```
int((a + b*x^8)^p/(c + d*x^4)^2,x)
```

output
```
int((a + b*x^8)^p/(c + d*x^4)^2, x)
```

Reduce [F]

$$\int \frac{(a+bx^8)^p}{(c+dx^4)^2}\,dx = \int \frac{(b\,x^8+a)^p}{d^2x^8+2cd\,x^4+c^2}dx$$

input
```
int((b*x^8+a)^p/(d*x^4+c)^2,x)
```

output
```
int((a + b*x**8)**p/(c**2 + 2*c*d*x**4 + d**2*x**8),x)
```

3.308 $\int \frac{(a+bx^8)^p}{(c+dx^4)^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 255

$$\int \frac{(a+bx^8)^p}{(c+dx^4)^3}\,dx = \frac{x(a+bx^8)^p\left(1+\frac{bx^8}{a}\right)^{-p}\operatorname{AppellF1}\left(\frac{1}{8},-p,3,\frac{9}{8},-\frac{bx^8}{a},\frac{d^2x^8}{c^2}\right)}{c^3} - \frac{3dx^5(a+bx^8)^p\left(1+\frac{bx^8}{a}\right)^{-p}\operatorname{AppellF1}\left(\frac{5}{8},-p,3,\frac{13}{8},-\frac{bx^8}{a},\frac{d^2x^8}{c^2}\right)}{5c^4} + \frac{d^2x^9(a+bx^8)^p\left(1+\frac{bx^8}{a}\right)^{-p}\operatorname{AppellF1}\left(\frac{9}{8},-p,3,\frac{17}{8},-\frac{bx^8}{a},\frac{d^2x^8}{c^2}\right)}{3c^5} - \frac{d^3x^{13}(a+bx^8)^p\left(1+\frac{bx^8}{a}\right)^{-p}\operatorname{AppellF1}\left(\frac{13}{8},-p,3,\frac{21}{8},-\frac{bx^8}{a},\frac{d^2x^8}{c^2}\right)}{13c^6}$$

output
```
x*(b*x^8+a)^p*AppellF1(1/8,3,-p,9/8,d^2*x^8/c^2,-b*x^8/a)/c^3/((1+b*x^8/a)
^p)-3/5*d*x^5*(b*x^8+a)^p*AppellF1(5/8,3,-p,13/8,d^2*x^8/c^2,-b*x^8/a)/c^4
/((1+b*x^8/a)^p)+1/3*d^2*x^9*(b*x^8+a)^p*AppellF1(9/8,3,-p,17/8,d^2*x^8/c^
2,-b*x^8/a)/c^5/((1+b*x^8/a)^p)-1/13*d^3*x^13*(b*x^8+a)^p*AppellF1(13/8,3,
-p,21/8,d^2*x^8/c^2,-b*x^8/a)/c^6/((1+b*x^8/a)^p)
```

Mathematica [F]

$$\int \frac{(a+bx^8)^p}{(c+dx^4)^3}\,dx=\int \frac{(a+bx^8)^p}{(c+dx^4)^3}\,dx$$

input
```
Integrate[(a + b*x^8)^p/(c + d*x^4)^3,x]
```

output
```
Integrate[(a + b*x^8)^p/(c + d*x^4)^3, x]
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1768, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^8)^p}{(c+dx^4)^3}\,dx$$

$$\Big\downarrow \text{1768}$$

$$\int \left(\frac{3cd^2x^8(a+bx^8)^p}{(c^2-d^2x^8)^3}-\frac{3c^2dx^4(a+bx^8)^p}{(c^2-d^2x^8)^3}+\frac{d^3x^{12}(a+bx^8)^p}{(d^2x^8-c^2)^3}+\frac{c^3(a+bx^8)^p}{(c^2-d^2x^8)^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{d^3x^{13}(a+bx^8)^p\left(\frac{bx^8}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{13}{8},-p,3,\frac{21}{8},-\frac{bx^8}{a},\frac{d^2x^8}{c^2}\right)}{13c^6}+\frac{d^2x^9(a+bx^8)^p\left(\frac{bx^8}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{9}{8},-p,3,\frac{17}{8},-\frac{bx^8}{a},\frac{d^2x^8}{c^2}\right)}{3c^5}-\frac{3dx^5(a+bx^8)^p\left(\frac{bx^8}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{5}{8},-p,3,\frac{13}{8},-\frac{bx^8}{a},\frac{d^2x^8}{c^2}\right)}{5c^4}+\frac{x(a+bx^8)^p\left(\frac{bx^8}{a}+1\right)^{-p}\text{AppellF1}\left(\frac{1}{8},-p,3,\frac{9}{8},-\frac{bx^8}{a},\frac{d^2x^8}{c^2}\right)}{c^3}$$

input
```
Int[(a + b*x^8)^p/(c + d*x^4)^3,x]
```

output
```
(x*(a + b*x^8)^p*AppellF1[1/8, -p, 3, 9/8, -((b*x^8)/a), (d^2*x^8)/c^2])/(
c^3*(1 + (b*x^8)/a)^p) - (3*d*x^5*(a + b*x^8)^p*AppellF1[5/8, -p, 3, 13/8,
 -((b*x^8)/a), (d^2*x^8)/c^2])/(5*c^4*(1 + (b*x^8)/a)^p) + (d^2*x^9*(a + b
*x^8)^p*AppellF1[9/8, -p, 3, 17/8, -((b*x^8)/a), (d^2*x^8)/c^2])/(3*c^5*(1
 + (b*x^8)/a)^p) - (d^3*x^13*(a + b*x^8)^p*AppellF1[13/8, -p, 3, 21/8, -((
b*x^8)/a), (d^2*x^8)/c^2])/(13*c^6*(1 + (b*x^8)/a)^p)
```

Defintions of rubi rules used

rule 1768
```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol]
 :> Int[ExpandIntegrand[(a + c*x^(2*n))^p, (d/(d^2 - e^2*x^(2*n)) - e*(x^n/
(d^2 - e^2*x^(2*n))))^(-q), x], x] /; FreeQ[{a, c, d, e, n, p}, x] && EqQ[n
2, 2*n] && NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{(b\,x^8+a)^p}{(x^4d+c)^3}dx$$

input
```
int((b*x^8+a)^p/(d*x^4+c)^3,x)
```

output
```
int((b*x^8+a)^p/(d*x^4+c)^3,x)
```

Fricas [F]

$$\int \frac{(a+bx^8)^p}{(c+dx^4)^3}\,dx = \int \frac{(bx^8+a)^p}{(dx^4+c)^3}\,dx$$

input
```
integrate((b*x^8+a)^p/(d*x^4+c)^3,x, algorithm="fricas")
```

output
```
integral((b*x^8 + a)^p/(d^3*x^12 + 3*c*d^2*x^8 + 3*c^2*d*x^4 + c^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^8)^p}{(c+dx^4)^3}\,dx = \text{Timed out}$$

input
```
integrate((b*x**8+a)**p/(d*x**4+c)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^8)^p}{(c+dx^4)^3}\,dx = \int \frac{(bx^8+a)^p}{(dx^4+c)^3}\,dx$$

input
```
integrate((b*x^8+a)^p/(d*x^4+c)^3,x, algorithm="maxima")
```

output
```
integrate((b*x^8 + a)^p/(d*x^4 + c)^3, x)
```

Giac [F]

$$\int \frac{(a+bx^8)^p}{(c+dx^4)^3}\,dx = \int \frac{(bx^8+a)^p}{(dx^4+c)^3}\,dx$$

input
```
integrate((b*x^8+a)^p/(d*x^4+c)^3,x, algorithm="giac")
```

output
```
integrate((b*x^8 + a)^p/(d*x^4 + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^8)^p}{(c+dx^4)^3}\,dx = \int \frac{(b\,x^8+a)^p}{(d\,x^4+c)^3}\,dx$$

input
```
int((a + b*x^8)^p/(c + d*x^4)^3,x)
```

output
```
int((a + b*x^8)^p/(c + d*x^4)^3, x)
```

Reduce [F]

$$\int \frac{(a+bx^8)^p}{(c+dx^4)^3}\,dx = \int \frac{(b\,x^8+a)^p}{d^3x^{12}+3c\,d^2x^8+3c^2d\,x^4+c^3}dx$$

input
```
int((b*x^8+a)^p/(d*x^4+c)^3,x)
```

output
```
int((a + b*x**8)**p/(c**3 + 3*c**2*d*x**4 + 3*c*d**2*x**8 + d**3*x**12),x)
```

3.309 $\int (c+dx^2)^3 (a+bx^8)^p \, dx$

Optimal result

Integrand size = 19, antiderivative size = 203

$$\int (c+dx^2)^3 (a+bx^8)^p \, dx = c^3x(a+bx^8)^p \left(1+\frac{bx^8}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{8},-p,\frac{9}{8},-\frac{bx^8}{a}\right) + c^2dx^3(a+bx^8)^p \left(1+\frac{bx^8}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{3}{8},-p,\frac{11}{8},-\frac{bx^8}{a}\right) + \frac{3}{5}cd^2x^5(a+bx^8)^p \left(1+\frac{bx^8}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{5}{8},-p,\frac{13}{8},-\frac{bx^8}{a}\right) + \frac{1}{7}d^3x^7(a+bx^8)^p \left(1+\frac{bx^8}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{7}{8},-p,\frac{15}{8},-\frac{bx^8}{a}\right)$$

output
```
c^3*x*(b*x^8+a)^p*hypergeom([1/8, -p],[9/8],-b*x^8/a)/((1+b*x^8/a)^p)+c^2*
d*x^3*(b*x^8+a)^p*hypergeom([3/8, -p],[11/8],-b*x^8/a)/((1+b*x^8/a)^p)+3/5
*c*d^2*x^5*(b*x^8+a)^p*hypergeom([5/8, -p],[13/8],-b*x^8/a)/((1+b*x^8/a)^p
)+1/7*d^3*x^7*(b*x^8+a)^p*hypergeom([7/8, -p],[15/8],-b*x^8/a)/((1+b*x^8/a
)^p)
```

Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.67

$$\int (c+dx^2)^3 (a+bx^8)^p \, dx$$
$$= \frac{1}{35}x(a+bx^8)^p \left(1+\frac{bx^8}{a}\right)^{-p} \left(35c^3 \text{Hypergeometric2F1}\left(\frac{1}{8},-p,\frac{9}{8},-\frac{bx^8}{a}\right) + dx^2\left(35c^2 \text{Hypergeometric2F1}\left(\frac{3}{8},-p,\frac{11}{8},-\frac{bx^8}{a}\right) + dx^2\left(21c \text{Hypergeometric2F1}\left(\frac{5}{8},-p,\frac{13}{8},-\frac{bx^8}{a}\right)+5dx^2 \text{Hypergeometric2F1}\left(\frac{7}{8},-p,\frac{15}{8},-\frac{bx^8}{a}\right)\right)\right)\right)$$

input
```
Integrate[(c + d*x^2)^3*(a + b*x^8)^p,x]
```

output
```
(x*(a + b*x^8)^p*(35*c^3*Hypergeometric2F1[1/8, -p, 9/8, -((b*x^8)/a)] + d
*x^2*(35*c^2*Hypergeometric2F1[3/8, -p, 11/8, -((b*x^8)/a)] + d*x^2*(21*c*
Hypergeometric2F1[5/8, -p, 13/8, -((b*x^8)/a)] + 5*d*x^2*Hypergeometric2F1
[7/8, -p, 15/8, -((b*x^8)/a)]))))/(35*(1 + (b*x^8)/a)^p)
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2424, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx^2)^3 (a+bx^8)^p \, dx$$
$$\Big\downarrow \text{ 2424}$$
$$\int \left((c^3+3cd^2x^4)(a+bx^8)^p + x^2(3c^2d+d^3x^4)(a+bx^8)^p\right) dx$$
$$\Big\downarrow \text{ 2009}$$

$$c^3x(a+bx^8)^p\left(\frac{bx^8}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{8},-p,\frac{9}{8},-\frac{bx^8}{a}\right)+ c^2dx^3(a+bx^8)^p\left(\frac{bx^8}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{3}{8},-p,\frac{11}{8},-\frac{bx^8}{a}\right)+ \frac{3}{5}cd^2x^5(a+bx^8)^p\left(\frac{bx^8}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{5}{8},-p,\frac{13}{8},-\frac{bx^8}{a}\right)+ \frac{1}{7}d^3x^7(a+bx^8)^p\left(\frac{bx^8}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{7}{8},-p,\frac{15}{8},-\frac{bx^8}{a}\right)$$

input
```
Int[(c + d*x^2)^3*(a + b*x^8)^p,x]
```

output
```
(c^3*x*(a + b*x^8)^p*Hypergeometric2F1[1/8, -p, 9/8, -((b*x^8)/a)])/(1 + (
b*x^8)/a)^p + (c^2*d*x^3*(a + b*x^8)^p*Hypergeometric2F1[3/8, -p, 11/8, -(
(b*x^8)/a)])/(1 + (b*x^8)/a)^p + (3*c*d^2*x^5*(a + b*x^8)^p*Hypergeometric
2F1[5/8, -p, 13/8, -((b*x^8)/a)])/(5*(1 + (b*x^8)/a)^p) + (d^3*x^7*(a + b*
x^8)^p*Hypergeometric2F1[7/8, -p, 15/8, -((b*x^8)/a)])/(7*(1 + (b*x^8)/a)^
p)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2424
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Maple [F]

$$\int \left(d\,x^{2}+c\right)^{3}\left(b\,x^{8}+a\right)^{p}dx$$

input
```
int((d*x^2+c)^3*(b*x^8+a)^p,x)
```

output
```
int((d*x^2+c)^3*(b*x^8+a)^p,x)
```

Fricas [F]

$$\int (c + dx^2)^3 (a + bx^8)^p \, dx = \int (dx^2 + c)^3 (bx^8 + a)^p \, dx$$

input
```
integrate((d*x^2+c)^3*(b*x^8+a)^p,x, algorithm="fricas")
```

output
```
integral((d^3*x^6 + 3*c*d^2*x^4 + 3*c^2*d*x^2 + c^3)*(b*x^8 + a)^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int (c + dx^2)^3 (a + bx^8)^p \, dx = \text{Timed out}$$

input
```
integrate((d*x**2+c)**3*(b*x**8+a)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (c + dx^2)^3 (a + bx^8)^p \, dx = \int (dx^2 + c)^3 (bx^8 + a)^p \, dx$$

input
```
integrate((d*x^2+c)^3*(b*x^8+a)^p,x, algorithm="maxima")
```

output
```
integrate((d*x^2 + c)^3*(b*x^8 + a)^p, x)
```

Giac [F]

$$\int \left(c+dx^2\right)^3 \left(a+bx^8\right)^p \, dx = \int \left(dx^2+c\right)^3 \left(bx^8+a\right)^p \, dx$$

input
```
integrate((d*x^2+c)^3*(b*x^8+a)^p,x, algorithm="giac")
```

output
```
integrate((d*x^2 + c)^3*(b*x^8 + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(c+dx^2\right)^3 \left(a+bx^8\right)^p \, dx = \int \left(b\,x^8+a\right)^p \left(d\,x^2+c\right)^3 \, dx$$

input
```
int((a + b*x^8)^p*(c + d*x^2)^3,x)
```

output
```
int((a + b*x^8)^p*(c + d*x^2)^3, x)
```

Reduce [F]

$$\int \left(c+dx^2\right)^3 \left(a+bx^8\right)^p \, dx = \text{too large to display}$$

input
```
int((d*x^2+c)^3*(b*x^8+a)^p,x)
```

output

```
(512*(a + b*x**8)**p*c**3*p**3*x + 960*(a + b*x**8)**p*c**3*p**2*x + 568*(
a + b*x**8)**p*c**3*p*x + 105*(a + b*x**8)**p*c**3*x + 1536*(a + b*x**8)**
p*c**2*d*p**3*x**3 + 2496*(a + b*x**8)**p*c**2*d*p**2*x**3 + 1128*(a + b*x
**8)**p*c**2*d*p*x**3 + 105*(a + b*x**8)**p*c**2*d*x**3 + 1536*(a + b*x**8
)**p*c*d**2*p**3*x**5 + 2112*(a + b*x**8)**p*c*d**2*p**2*x**5 + 744*(a + b
*x**8)**p*c*d**2*p*x**5 + 63*(a + b*x**8)**p*c*d**2*x**5 + 512*(a + b*x**8
)**p*d**3*p**3*x**7 + 576*(a + b*x**8)**p*d**3*p**2*x**7 + 184*(a + b*x**8
)**p*d**3*p*x**7 + 15*(a + b*x**8)**p*d**3*x**7 + 16777216*int((a + b*x**8
)**p/(4096*a*p**4 + 8192*a*p**3 + 5504*a*p**2 + 1408*a*p + 105*a + 4096*b*
p**4*x**8 + 8192*b*p**3*x**8 + 5504*b*p**2*x**8 + 1408*b*p*x**8 + 105*b*x*
*8),x)*a*c**3*p**8 + 65011712*int((a + b*x**8)**p/(4096*a*p**4 + 8192*a*p*
*3 + 5504*a*p**2 + 1408*a*p + 105*a + 4096*b*p**4*x**8 + 8192*b*p**3*x**8
+ 5504*b*p**2*x**8 + 1408*b*p*x**8 + 105*b*x**8),x)*a*c**3*p**7 + 10407116
8*int((a + b*x**8)**p/(4096*a*p**4 + 8192*a*p**3 + 5504*a*p**2 + 1408*a*p
+ 105*a + 4096*b*p**4*x**8 + 8192*b*p**3*x**8 + 5504*b*p**2*x**8 + 1408*b*
p*x**8 + 105*b*x**8),x)*a*c**3*p**6 + 88702976*int((a + b*x**8)**p/(4096*a
*p**4 + 8192*a*p**3 + 5504*a*p**2 + 1408*a*p + 105*a + 4096*b*p**4*x**8 +
8192*b*p**3*x**8 + 5504*b*p**2*x**8 + 1408*b*p*x**8 + 105*b*x**8),x)*a*c**
3*p**5 + 43134976*int((a + b*x**8)**p/(4096*a*p**4 + 8192*a*p**3 + 5504*a*
p**2 + 1408*a*p + 105*a + 4096*b*p**4*x**8 + 8192*b*p**3*x**8 + 5504*b*...
```

3.310 $\int (c+dx^2)^2 (a+bx^8)^p \, dx$

Optimal result

Integrand size = 19, antiderivative size = 151

$$\int (c+dx^2)^2 (a+bx^8)^p \, dx = c^2 x (a+bx^8)^p \left(1+\frac{bx^8}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{8}, -p, \frac{9}{8}, -\frac{bx^8}{a}\right) + \frac{2}{3} c d x^3 (a+bx^8)^p \left(1+\frac{bx^8}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{3}{8}, -p, \frac{11}{8}, -\frac{bx^8}{a}\right) + \frac{1}{5} d^2 x^5 (a+bx^8)^p \left(1+\frac{bx^8}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{5}{8}, -p, \frac{13}{8}, -\frac{bx^8}{a}\right)$$

output
```
c^2*x*(b*x^8+a)^p*hypergeom([1/8, -p],[9/8],-b*x^8/a)/((1+b*x^8/a)^p)+2/3*
c*d*x^3*(b*x^8+a)^p*hypergeom([3/8, -p],[11/8],-b*x^8/a)/((1+b*x^8/a)^p)+1
/5*d^2*x^5*(b*x^8+a)^p*hypergeom([5/8, -p],[13/8],-b*x^8/a)/((1+b*x^8/a)^p
)
```

Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.70

$$\int \left(c+dx^2\right)^2 \left(a+bx^8\right)^p\,dx=\frac{1}{15}x\left(a+bx^8\right)^p\left(1+\frac{bx^8}{a}\right)^{-p}\left(15c^2\,\text{Hypergeometric2F1}\left(\frac{1}{8},-p,\frac{9}{8},-\frac{bx^8}{a}\right)+dx^2\left(10c\,\text{Hypergeometric2F1}\left(\frac{3}{8},-p,\frac{11}{8},-\frac{bx^8}{a}\right)+3dx^2\,\text{Hypergeometric2F1}\left(\frac{5}{8},-p,\frac{13}{8},-\frac{bx^8}{a}\right)\right)\right)$$

input
```
Integrate[(c + d*x^2)^2*(a + b*x^8)^p,x]
```

output
```
(x*(a + b*x^8)^p*(15*c^2*Hypergeometric2F1[1/8, -p, 9/8, -((b*x^8)/a)] + d
*x^2*(10*c*Hypergeometric2F1[3/8, -p, 11/8, -((b*x^8)/a)] + 3*d*x^2*Hyperg
eometric2F1[5/8, -p, 13/8, -((b*x^8)/a)])))/(15*(1 + (b*x^8)/a)^p)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2424, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c+dx^2\right)^2 \left(a+bx^8\right)^p\,dx$$

$$\Big\downarrow \text{2424}$$

$$\int \left(\left(c^2+d^2x^4\right)\left(a+bx^8\right)^p+2cdx^2\left(a+bx^8\right)^p\right)dx$$

$$\Big\downarrow \text{2009}$$

$$c^2x(a+bx^8)^p\left(\frac{bx^8}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{8},-p,\frac{9}{8},-\frac{bx^8}{a}\right)+\frac{2}{3}cdx^3(a+bx^8)^p\left(\frac{bx^8}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{3}{8},-p,\frac{11}{8},-\frac{bx^8}{a}\right)+\frac{1}{5}d^2x^5(a+bx^8)^p\left(\frac{bx^8}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{5}{8},-p,\frac{13}{8},-\frac{bx^8}{a}\right)$$

input
```
Int[(c + d*x^2)^2*(a + b*x^8)^p,x]
```

output
```
(c^2*x*(a + b*x^8)^p*Hypergeometric2F1[1/8, -p, 9/8, -((b*x^8)/a)])/(1 + (
b*x^8)/a)^p + (2*c*d*x^3*(a + b*x^8)^p*Hypergeometric2F1[3/8, -p, 11/8, -(
(b*x^8)/a)])/(3*(1 + (b*x^8)/a)^p) + (d^2*x^5*(a + b*x^8)^p*Hypergeometric
2F1[5/8, -p, 13/8, -((b*x^8)/a)])/(5*(1 + (b*x^8)/a)^p)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2424
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Maple [F]

$$\int \left(d\,x^{2}+c\right)^{2} \left(b\,x^{8}+a\right)^{p}d x$$

input
```
int((d*x^2+c)^2*(b*x^8+a)^p,x)
```

output
```
int((d*x^2+c)^2*(b*x^8+a)^p,x)
```

Fricas [F]

$$\int \left(c+dx^2\right)^2 \left(a+bx^8\right)^p \, dx = \int \left(dx^2+c\right)^2 \left(bx^8+a\right)^p \, dx$$

input
```
integrate((d*x^2+c)^2*(b*x^8+a)^p,x, algorithm="fricas")
```

output
```
integral((d^2*x^4 + 2*c*d*x^2 + c^2)*(b*x^8 + a)^p, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 178.63 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.79

$$\int \left(c+dx^2\right)^2 \left(a+bx^8\right)^p \, dx = \frac{a^p c^2 x \Gamma\left(\frac{1}{8}\right) {}_2F_1\left(\begin{matrix} \frac{1}{8}, -p \\ \frac{9}{8} \end{matrix} \middle| \frac{bx^8 e^{i\pi}}{a}\right)}{8\Gamma\left(\frac{9}{8}\right)} + \frac{a^p c d x^3 \Gamma\left(\frac{3}{8}\right) {}_2F_1\left(\begin{matrix} \frac{3}{8}, -p \\ \frac{11}{8} \end{matrix} \middle| \frac{bx^8 e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{11}{8}\right)} + \frac{a^p d^2 x^5 \Gamma\left(\frac{5}{8}\right) {}_2F_1\left(\begin{matrix} \frac{5}{8}, -p \\ \frac{13}{8} \end{matrix} \middle| \frac{bx^8 e^{i\pi}}{a}\right)}{8\Gamma\left(\frac{13}{8}\right)}$$

input
```
integrate((d*x**2+c)**2*(b*x**8+a)**p,x)
```

output
```
a**p*c**2*x*gamma(1/8)*hyper((1/8, -p), (9/8,), b*x**8*exp_polar(I*pi)/a)/
(8*gamma(9/8)) + a**p*c*d*x**3*gamma(3/8)*hyper((3/8, -p), (11/8,), b*x**8
*exp_polar(I*pi)/a)/(4*gamma(11/8)) + a**p*d**2*x**5*gamma(5/8)*hyper((5/8
, -p), (13/8,), b*x**8*exp_polar(I*pi)/a)/(8*gamma(13/8))
```

Maxima [F]

$$\int \left(c+dx^2\right)^2 \left(a+bx^8\right)^p \, dx = \int \left(dx^2+c\right)^2 \left(bx^8+a\right)^p \, dx$$

input
```
integrate((d*x^2+c)^2*(b*x^8+a)^p,x, algorithm="maxima")
```

output
```
integrate((d*x^2 + c)^2*(b*x^8 + a)^p, x)
```

Giac [F]

$$\int \left(c+dx^2\right)^2 \left(a+bx^8\right)^p \, dx = \int \left(dx^2+c\right)^2 \left(bx^8+a\right)^p \, dx$$

input
```
integrate((d*x^2+c)^2*(b*x^8+a)^p,x, algorithm="giac")
```

output
```
integrate((d*x^2 + c)^2*(b*x^8 + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(c+dx^2\right)^2 \left(a+bx^8\right)^p \, dx = \int \left(b\,x^8+a\right)^p \left(d\,x^2+c\right)^2 \, dx$$

input
```
int((a + b*x^8)^p*(c + d*x^2)^2,x)
```

output
```
int((a + b*x^8)^p*(c + d*x^2)^2, x)
```

Reduce [F]

$$\int \left(c+dx^2\right)^2 \left(a+bx^8\right)^p \, dx = \text{too large to display}$$

input
```
int((d*x^2+c)^2*(b*x^8+a)^p,x)
```

output
```
(64*(a + b*x**8)**p*c**2*p**2*x + 64*(a + b*x**8)**p*c**2*p*x + 15*(a + b*
x**8)**p*c**2*x + 128*(a + b*x**8)**p*c*d*p**2*x**3 + 96*(a + b*x**8)**p*c
*d*p*x**3 + 10*(a + b*x**8)**p*c*d*x**3 + 64*(a + b*x**8)**p*d**2*p**2*x**
5 + 32*(a + b*x**8)**p*d**2*p*x**5 + 3*(a + b*x**8)**p*d**2*x**5 + 262144*
int((a + b*x**8)**p/(512*a*p**3 + 576*a*p**2 + 184*a*p + 15*a + 512*b*p**3
*x**8 + 576*b*p**2*x**8 + 184*b*p*x**8 + 15*b*x**8),x)*a*c**2*p**6 + 55705
6*int((a + b*x**8)**p/(512*a*p**3 + 576*a*p**2 + 184*a*p + 15*a + 512*b*p*
*3*x**8 + 576*b*p**2*x**8 + 184*b*p*x**8 + 15*b*x**8),x)*a*c**2*p**5 + 450
560*int((a + b*x**8)**p/(512*a*p**3 + 576*a*p**2 + 184*a*p + 15*a + 512*b*
p**3*x**8 + 576*b*p**2*x**8 + 184*b*p*x**8 + 15*b*x**8),x)*a*c**2*p**4 + 1
71008*int((a + b*x**8)**p/(512*a*p**3 + 576*a*p**2 + 184*a*p + 15*a + 512*
b*p**3*x**8 + 576*b*p**2*x**8 + 184*b*p*x**8 + 15*b*x**8),x)*a*c**2*p**3 +
 29760*int((a + b*x**8)**p/(512*a*p**3 + 576*a*p**2 + 184*a*p + 15*a + 512
*b*p**3*x**8 + 576*b*p**2*x**8 + 184*b*p*x**8 + 15*b*x**8),x)*a*c**2*p**2
+ 1800*int((a + b*x**8)**p/(512*a*p**3 + 576*a*p**2 + 184*a*p + 15*a + 512
*b*p**3*x**8 + 576*b*p**2*x**8 + 184*b*p*x**8 + 15*b*x**8),x)*a*c**2*p + 2
62144*int(((a + b*x**8)**p*x**4)/(512*a*p**3 + 576*a*p**2 + 184*a*p + 15*a
 + 512*b*p**3*x**8 + 576*b*p**2*x**8 + 184*b*p*x**8 + 15*b*x**8),x)*a*d**2
*p**6 + 425984*int(((a + b*x**8)**p*x**4)/(512*a*p**3 + 576*a*p**2 + 184*a
*p + 15*a + 512*b*p**3*x**8 + 576*b*p**2*x**8 + 184*b*p*x**8 + 15*b*x**...
```

3.311 $\int (c+dx^2)(a+bx^8)^p\,dx$

Optimal result

Integrand size = 17, antiderivative size = 96

$$\int (c+dx^2)(a+bx^8)^p\,dx = cx(a+bx^8)^p\left(1+\frac{bx^8}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{8},-p,\frac{9}{8},-\frac{bx^8}{a}\right)+\frac{1}{3}dx^3(a+bx^8)^p\left(1+\frac{bx^8}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{3}{8},-p,\frac{11}{8},-\frac{bx^8}{a}\right)$$

output
```
c*x*(b*x^8+a)^p*hypergeom([1/8, -p],[9/8],-b*x^8/a)/((1+b*x^8/a)^p)+1/3*d*
x^3*(b*x^8+a)^p*hypergeom([3/8, -p],[11/8],-b*x^8/a)/((1+b*x^8/a)^p)
```

Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.78

$$\int (c+dx^2)(a+bx^8)^p\,dx = \frac{1}{3}x(a+bx^8)^p\left(1+\frac{bx^8}{a}\right)^{-p}\left(3c\,\text{Hypergeometric2F1}\left(\frac{1}{8},-p,\frac{9}{8},-\frac{bx^8}{a}\right)+dx^2\,\text{Hypergeometric2F1}\left(\frac{3}{8},-p,\frac{11}{8},-\frac{bx^8}{a}\right)\right)$$

input
```
Integrate[(c + d*x^2)*(a + b*x^8)^p,x]
```

output
```
(x*(a + b*x^8)^p*(3*c*Hypergeometric2F1[1/8, -p, 9/8, -((b*x^8)/a)] + d*x^
2*Hypergeometric2F1[3/8, -p, 11/8, -((b*x^8)/a)]))/(3*(1 + (b*x^8)/a)^p)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2424, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx^2)\,(a+bx^8)^p\,dx$$

$$\Big\downarrow \text{2424}$$

$$\int \left(c(a+bx^8)^p+dx^2(a+bx^8)^p\right)dx$$

$$\Big\downarrow \text{2009}$$

$$cx(a+bx^8)^p\left(\frac{bx^8}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{8},-p,\frac{9}{8},-\frac{bx^8}{a}\right)+\frac{1}{3}dx^3(a+bx^8)^p\left(\frac{bx^8}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{3}{8},-p,\frac{11}{8},-\frac{bx^8}{a}\right)$$

input
```
Int[(c + d*x^2)*(a + b*x^8)^p,x]
```

output
```
(c*x*(a + b*x^8)^p*Hypergeometric2F1[1/8, -p, 9/8, -((b*x^8)/a)])/(1 + (b*
x^8)/a)^p + (d*x^3*(a + b*x^8)^p*Hypergeometric2F1[3/8, -p, 11/8, -((b*x^8
)/a)])/(3*(1 + (b*x^8)/a)^p)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2424
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Maple [F]

$$\int \left(d\,x^{2}+c\right) \left(b\,x^{8}+a\right)^{p}d x$$

input
```
int((d*x^2+c)*(b*x^8+a)^p,x)
```

output
```
int((d*x^2+c)*(b*x^8+a)^p,x)
```

Fricas [F]

$$\int \left(c+dx^2\right) \left(a+bx^8\right)^p \, dx = \int \left(dx^{2} + c\right) {\left(bx^{8} + a\right)}^{p} \,d x$$

input
```
integrate((d*x^2+c)*(b*x^8+a)^p,x, algorithm="fricas")
```

output
```
integral((d*x^2 + c)*(b*x^8 + a)^p, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 130.95 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.78

$$\int (c+dx^2)(a+bx^8)^p\,dx = \frac{a^p c x \Gamma\left(\frac{1}{8}\right) {}_2F_1\left(\begin{matrix}\frac{1}{8}, -p\\ \frac{9}{8}\end{matrix}\middle|\frac{bx^8e^{i\pi}}{a}\right)}{8\Gamma\left(\frac{9}{8}\right)} + \frac{a^p d x^3 \Gamma\left(\frac{3}{8}\right) {}_2F_1\left(\begin{matrix}\frac{3}{8}, -p\\ \frac{11}{8}\end{matrix}\middle|\frac{bx^8e^{i\pi}}{a}\right)}{8\Gamma\left(\frac{11}{8}\right)}$$

input
```
integrate((d*x**2+c)*(b*x**8+a)**p,x)
```

output
```
a**p*c*x*gamma(1/8)*hyper((1/8, -p), (9/8,), b*x**8*exp_polar(I*pi)/a)/(8*
gamma(9/8)) + a**p*d*x**3*gamma(3/8)*hyper((3/8, -p), (11/8,), b*x**8*exp_
polar(I*pi)/a)/(8*gamma(11/8))
```

Maxima [F]

$$\int (c+dx^2)(a+bx^8)^p\,dx = \int (dx^2+c)(bx^8+a)^p\,dx$$

input
```
integrate((d*x^2+c)*(b*x^8+a)^p,x, algorithm="maxima")
```

output
```
integrate((d*x^2 + c)*(b*x^8 + a)^p, x)
```

Giac [F]

$$\int \left(c+dx^2\right)\left(a+bx^8\right)^p\,dx=\int \left(dx^2+c\right)\left(bx^8+a\right)^p\,dx$$

input
```
integrate((d*x^2+c)*(b*x^8+a)^p,x, algorithm="giac")
```

output
```
integrate((d*x^2 + c)*(b*x^8 + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(c+dx^2\right)\left(a+bx^8\right)^p\,dx=\int \left(b\,x^8+a\right)^p\,\left(d\,x^2+c\right)\,dx$$

input
```
int((a + b*x^8)^p*(c + d*x^2),x)
```

output
```
int((a + b*x^8)^p*(c + d*x^2), x)
```

Reduce [F]

$$\int \left(c+dx^2\right)\left(a+bx^8\right)^p\,dx$$
$$=\frac{8(b\,x^8+a)^p\,cpx+3(b\,x^8+a)^p\,cx+8(b\,x^8+a)^p\,dp\,x^3+(b\,x^8+a)^p\,d\,x^3+4096\left(\int\frac{(b\,x^8+a)^p}{64b\,p^2x^8+32bp\,x^8+3b\,x^8+64}\right.}{}$$

input
```
int((d*x^2+c)*(b*x^8+a)^p,x)
```

output

```
(8*(a + b*x**8)**p*c*p*x + 3*(a + b*x**8)**p*c*x + 8*(a + b*x**8)**p*d*p*x
**3 + (a + b*x**8)**p*d*x**3 + 4096*int((a + b*x**8)**p/(64*a*p**2 + 32*a*
p + 3*a + 64*b*p**2*x**8 + 32*b*p*x**8 + 3*b*x**8),x)*a*c*p**4 + 3584*int(
(a + b*x**8)**p/(64*a*p**2 + 32*a*p + 3*a + 64*b*p**2*x**8 + 32*b*p*x**8 +
 3*b*x**8),x)*a*c*p**3 + 960*int((a + b*x**8)**p/(64*a*p**2 + 32*a*p + 3*a
 + 64*b*p**2*x**8 + 32*b*p*x**8 + 3*b*x**8),x)*a*c*p**2 + 72*int((a + b*x*
*8)**p/(64*a*p**2 + 32*a*p + 3*a + 64*b*p**2*x**8 + 32*b*p*x**8 + 3*b*x**8
),x)*a*c*p + 4096*int(((a + b*x**8)**p*x**2)/(64*a*p**2 + 32*a*p + 3*a + 6
4*b*p**2*x**8 + 32*b*p*x**8 + 3*b*x**8),x)*a*d*p**4 + 2560*int(((a + b*x**
8)**p*x**2)/(64*a*p**2 + 32*a*p + 3*a + 64*b*p**2*x**8 + 32*b*p*x**8 + 3*b
*x**8),x)*a*d*p**3 + 448*int(((a + b*x**8)**p*x**2)/(64*a*p**2 + 32*a*p +
3*a + 64*b*p**2*x**8 + 32*b*p*x**8 + 3*b*x**8),x)*a*d*p**2 + 24*int(((a +
b*x**8)**p*x**2)/(64*a*p**2 + 32*a*p + 3*a + 64*b*p**2*x**8 + 32*b*p*x**8
+ 3*b*x**8),x)*a*d*p)/(64*p**2 + 32*p + 3)
```

3.312 $\int (a+bx^8)^p \, dx$

Optimal result

Integrand size = 9, antiderivative size = 44

$$\int (a+bx^8)^p \, dx = x(a+bx^8)^p \left(1+\frac{bx^8}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{8},-p,\frac{9}{8},-\frac{bx^8}{a}\right)$$

output
```
x*(b*x^8+a)^p*hypergeom([1/8, -p],[9/8],-b*x^8/a)/((1+b*x^8/a)^p)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int (a+bx^8)^p \, dx = x(a+bx^8)^p \left(1+\frac{bx^8}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{8},-p,\frac{9}{8},-\frac{bx^8}{a}\right)$$

input
```
Integrate[(a + b*x^8)^p,x]
```

output
```
(x*(a + b*x^8)^p*Hypergeometric2F1[1/8, -p, 9/8, -((b*x^8)/a)])/(1 + (b*x^
8)/a)^p
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {779, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^8)^p \, dx$$

$$\Big\downarrow \text{ 779}$$

$$(a+bx^8)^p \left(\frac{bx^8}{a}+1\right)^{-p} \int \left(\frac{bx^8}{a}+1\right)^p dx$$

$$\Big\downarrow \text{ 778}$$

$$x(a+bx^8)^p \left(\frac{bx^8}{a}+1\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{8},-p,\frac{9}{8},-\frac{bx^8}{a}\right)$$

input
```
Int[(a + b*x^8)^p,x]
```

output
```
(x*(a + b*x^8)^p*Hypergeometric2F1[1/8, -p, 9/8, -((b*x^8)/a)])/(1 + (b*x^
8)/a)^p
```

Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Si
mplify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \left(b\,x^{8}+a\right)^{p}dx$$

input
```
int((b*x^8+a)^p,x)
```

output
```
int((b*x^8+a)^p,x)
```

Fricas [F]

$$\int \left(a+bx^8\right)^p\,dx=\int \left(bx^{8}+a\right)^{p}\,dx$$

input
```
integrate((b*x^8+a)^p,x, algorithm="fricas")
```

output
```
integral((b*x^8 + a)^p, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 31.10 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.77

$$\int \left(a+bx^8\right)^p\,dx=\frac{a^{p}x\Gamma\left(\frac{1}{8}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{8},-p\\ \frac{9}{8}\end{matrix}\middle|\frac{bx^{8}e^{i\pi}}{a}\right)}{8\Gamma\left(\frac{9}{8}\right)}$$

input
```
integrate((b*x**8+a)**p,x)
```

output
```
a**p*x*gamma(1/8)*hyper((1/8, -p), (9/8,), b*x**8*exp_polar(I*pi)/a)/(8*ga
mma(9/8))
```

Maxima [F]

$$\int \left(a+bx^8\right)^p\,dx=\int \left(bx^8+a\right)^p\,dx$$

input
```
integrate((b*x^8+a)^p,x, algorithm="maxima")
```

output
```
integrate((b*x^8 + a)^p, x)
```

Giac [F]

$$\int \left(a+bx^8\right)^p\,dx=\int \left(bx^8+a\right)^p\,dx$$

input
```
integrate((b*x^8+a)^p,x, algorithm="giac")
```

output
```
integrate((b*x^8 + a)^p, x)
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.93

$$\int \left(a+bx^8\right)^p\,dx=\frac{x\,\left(b\,x^8+a\right)^p\,{}_2\mathrm{F}_1\left(\frac{1}{8},-p;\ \frac{9}{8};\ -\frac{b\,x^8}{a}\right)}{{\left(\frac{b\,x^8}{a}+1\right)}^p}$$

input
```
int((a + b*x^8)^p,x)
```

output
```
(x*(a + b*x^8)^p*hypergeom([1/8, -p], 9/8, -(b*x^8)/a))/((b*x^8)/a + 1)^p
```

Reduce [F]

$$\int (a+bx^8)^p \, dx = \frac{(b\,x^8+a)^p\,x+64\left(\int \frac{(b\,x^8+a)^p}{8bp\,x^8+b\,x^8+8ap+a}dx\right)a\,p^2+8\left(\int \frac{(b\,x^8+a)^p}{8bp\,x^8+b\,x^8+8ap+a}dx\right)ap}{8p+1}$$

input
```
int((b*x^8+a)^p,x)
```

output
```
((a + b*x**8)**p*x + 64*int((a + b*x**8)**p/(8*a*p + a + 8*b*p*x**8 + b*x*
*8),x)*a*p**2 + 8*int((a + b*x**8)**p/(8*a*p + a + 8*b*p*x**8 + b*x**8),x)
*a*p)/(8*p + 1)
```

3.313 $\int \frac{(a+bx^8)^p}{c+dx^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 255

$$\int \frac{(a+bx^8)^p}{c+dx^2}\,dx = \frac{x(a+bx^8)^p\left(1+\frac{bx^8}{a}\right)^{-p}\operatorname{AppellF1}\left(\frac{1}{8},-p,1,\frac{9}{8},-\frac{bx^8}{a},\frac{d^4x^8}{c^4}\right)}{c} - \frac{dx^3(a+bx^8)^p\left(1+\frac{bx^8}{a}\right)^{-p}\operatorname{AppellF1}\left(\frac{3}{8},-p,1,\frac{11}{8},-\frac{bx^8}{a},\frac{d^4x^8}{c^4}\right)}{3c^2} + \frac{d^2x^5(a+bx^8)^p\left(1+\frac{bx^8}{a}\right)^{-p}\operatorname{AppellF1}\left(\frac{5}{8},-p,1,\frac{13}{8},-\frac{bx^8}{a},\frac{d^4x^8}{c^4}\right)}{5c^3} - \frac{d^3x^7(a+bx^8)^p\left(1+\frac{bx^8}{a}\right)^{-p}\operatorname{AppellF1}\left(\frac{7}{8},-p,1,\frac{15}{8},-\frac{bx^8}{a},\frac{d^4x^8}{c^4}\right)}{7c^4}$$

output
```
x*(b*x^8+a)^p*AppellF1(1/8,1,-p,9/8,d^4*x^8/c^4,-b*x^8/a)/c/((1+b*x^8/a)^p
)-1/3*d*x^3*(b*x^8+a)^p*AppellF1(3/8,1,-p,11/8,d^4*x^8/c^4,-b*x^8/a)/c^2/(
(1+b*x^8/a)^p)+1/5*d^2*x^5*(b*x^8+a)^p*AppellF1(5/8,1,-p,13/8,d^4*x^8/c^4,
-b*x^8/a)/c^3/((1+b*x^8/a)^p)-1/7*d^3*x^7*(b*x^8+a)^p*AppellF1(7/8,1,-p,15
/8,d^4*x^8/c^4,-b*x^8/a)/c^4/((1+b*x^8/a)^p)
```

Mathematica [F]

$$\int \frac{(a+bx^8)^p}{c+dx^2}\,dx = \int \frac{(a+bx^8)^p}{c+dx^2}\,dx$$

input
```
Integrate[(a + b*x^8)^p/(c + d*x^2),x]
```

output
```
Integrate[(a + b*x^8)^p/(c + d*x^2), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^8)^p}{c+dx^2}\,dx$$
$$\Big\downarrow \text{ 2584}$$
$$\int \frac{(c-dx^2)(a+bx^8)^p}{c^2-d^2x^4}dx$$
$$\Big\downarrow \text{ 1388}$$
$$\int \frac{(a+bx^8)^p}{c+dx^2}dx$$
$$\Big\downarrow \text{ 2584}$$
$$\int \frac{(c-dx^2)(a+bx^8)^p}{c^2-d^2x^4}dx$$
$$\Big\downarrow \text{ 1388}$$
$$\int \frac{(a+bx^8)^p}{c+dx^2}dx$$
$$\Big\downarrow \text{ 2584}$$
$$\int \frac{(c-dx^2)(a+bx^8)^p}{c^2-d^2x^4}dx$$

$$\Big\downarrow \text{1388}$$

$$\int \frac{(a+bx^8)^p}{c+dx^2}dx$$

$$\Big\downarrow \text{2584}$$

$$\int \frac{(c-dx^2)(a+bx^8)^p}{c^2-d^2x^4}dx$$

$$\Big\downarrow \text{1388}$$

$$\int \frac{(a+bx^8)^p}{c+dx^2}dx$$

$$\Big\downarrow \text{2584}$$

$$\int \frac{(c-dx^2)(a+bx^8)^p}{c^2-d^2x^4}dx$$

$$\Big\downarrow \text{1388}$$

$$\int \frac{(a+bx^8)^p}{c+dx^2}dx$$

$$\Big\downarrow \text{2584}$$

$$\int \frac{(c-dx^2)(a+bx^8)^p}{c^2-d^2x^4}dx$$

$$\Big\downarrow \text{1388}$$

$$\int \frac{(a+bx^8)^p}{c+dx^2}dx$$

$$\Big\downarrow \text{2584}$$

$$\int \frac{(c-dx^2)(a+bx^8)^p}{c^2-d^2x^4}dx$$

$$\Big\downarrow \text{1388}$$

$$\int \frac{(a+bx^8)^p}{c+dx^2}dx$$

$$\Big\downarrow \text{2584}$$

$$\int \frac{(c-dx^2)(a+bx^8)^p}{c^2-d^2x^4}dx$$

$$\Big\downarrow \text{1388}$$

$$\int \frac{(a+bx^8)^p}{c+dx^2}dx$$

$$\Big\downarrow \; 2584$$

$$\int \frac{(c-dx^2)(a+bx^8)^p}{c^2-d^2x^4}dx$$

$$\Big\downarrow \; 1388$$

$$\int \frac{(a+bx^8)^p}{c+dx^2}dx$$

$$\Big\downarrow \; 2584$$

$$\int \frac{(c-dx^2)(a+bx^8)^p}{c^2-d^2x^4}dx$$

$$\Big\downarrow \; 1388$$

$$\int \frac{(a+bx^8)^p}{c+dx^2}dx$$

$$\Big\downarrow \; 2584$$

$$\int \frac{(c-dx^2)(a+bx^8)^p}{c^2-d^2x^4}dx$$

$$\Big\downarrow \; 1388$$

$$\int \frac{(a+bx^8)^p}{c+dx^2}dx$$

$$\Big\downarrow \; 2584$$

$$\int \frac{(c-dx^2)(a+bx^8)^p}{c^2-d^2x^4}dx$$

$$\Big\downarrow \; 1388$$

$$\int \frac{(a+bx^8)^p}{c+dx^2}dx$$

$$\Big\downarrow \; 2584$$

$$\int \frac{(c-dx^2)(a+bx^8)^p}{c^2-d^2x^4}dx$$

$$\Big\downarrow \; 1388$$

$$\int \frac{(a+bx^8)^p}{c+dx^2}dx$$

$$\Big\downarrow \text{ 2584}$$

$$\int \frac{(c-dx^2)\,(a+bx^8)^p}{c^2-d^2x^4}dx$$

$$\Big\downarrow \text{ 1388}$$

$$\int \frac{(a+bx^8)^p}{c+dx^2}dx$$

$$\Big\downarrow \text{ 2584}$$

$$\int \frac{(c-dx^2)\,(a+bx^8)^p}{c^2-d^2x^4}dx$$

$$\Big\downarrow \text{ 1388}$$

$$\int \frac{(a+bx^8)^p}{c+dx^2}dx$$

input
```
Int[(a + b*x^8)^p/(c + d*x^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1388
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.),
x_Symbol] :> Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p, x] /; FreeQ[{a,
 c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && (Integer
Q[p] || (GtQ[a, 0] && GtQ[d, 0]))
```

rule 2584
```
Int[((c_) + (d_.)*(x_)^(n_.))^(q_)*((a_) + (b_.)*(x_)^(nn_.))^(p_), x_Symbo
l] :> Int[ExpandToSum[(c - d*x^n)^(-q), x]*((a + b*x^nn)^p/(c^2 - d^2*x^(2*
n))^(-q)), x] /; FreeQ[{a, b, c, d, n, nn, p}, x] &&  !IntegerQ[p] && ILtQ[
q, 0] && IGtQ[Log[2, nn/n], 0]
```

Maple [F]

$$\int \frac{(b\,x^{8}+a)^{p}}{d\,x^{2}+c}dx$$

input
```
int((b*x^8+a)^p/(d*x^2+c),x)
```

output
```
int((b*x^8+a)^p/(d*x^2+c),x)
```

Fricas [F]

$$\int \frac{(a+bx^8)^p}{c+dx^2}\,dx=\int \frac{(bx^8+a)^p}{dx^2+c}\,dx$$

input
```
integrate((b*x^8+a)^p/(d*x^2+c),x, algorithm="fricas")
```

output
```
integral((b*x^8 + a)^p/(d*x^2 + c), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^8)^p}{c+dx^2}\,dx=\text{Timed out}$$

input
```
integrate((b*x**8+a)**p/(d*x**2+c),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^8)^p}{c+dx^2}\,dx=\int \frac{(bx^8+a)^p}{dx^2+c}\,dx$$

input
```
integrate((b*x^8+a)^p/(d*x^2+c),x, algorithm="maxima")
```

output
```
integrate((b*x^8 + a)^p/(d*x^2 + c), x)
```

Giac [F]

$$\int \frac{(a+bx^8)^p}{c+dx^2}\,dx=\int \frac{(bx^8+a)^p}{dx^2+c}\,dx$$

input
```
integrate((b*x^8+a)^p/(d*x^2+c),x, algorithm="giac")
```

output
```
integrate((b*x^8 + a)^p/(d*x^2 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^8)^p}{c+dx^2}\,dx=\int \frac{(b\,x^8+a)^p}{d\,x^2+c}\,dx$$

input
```
int((a + b*x^8)^p/(c + d*x^2),x)
```

output
```
int((a + b*x^8)^p/(c + d*x^2), x)
```

Reduce [F]

$$\int \frac{(a+bx^8)^p}{c+dx^2}\,dx = \int \frac{(b\,x^8+a)^p}{d\,x^2+c}dx$$

input
```
int((b*x^8+a)^p/(d*x^2+c),x)
```

output
```
int((a + b*x**8)**p/(c + d*x**2),x)
```

3.314 $\int \frac{(a+bx^8)^p}{(c+dx^2)^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 453

$$\begin{aligned}\int \frac{(a+bx^8)^p}{(c+dx^2)^2}\,dx &= \frac{x(a+bx^8)^p\left(1+\frac{bx^8}{a}\right)^{-p}\text{AppellF1}\left(\frac{1}{8},-p,2,\frac{9}{8},-\frac{bx^8}{a},\frac{d^4x^8}{c^4}\right)}{c^2}\\
&-\frac{2dx^3(a+bx^8)^p\left(1+\frac{bx^8}{a}\right)^{-p}\text{AppellF1}\left(\frac{3}{8},-p,2,\frac{11}{8},-\frac{bx^8}{a},\frac{d^4x^8}{c^4}\right)}{3c^3}\\
&+\frac{3d^2x^5(a+bx^8)^p\left(1+\frac{bx^8}{a}\right)^{-p}\text{AppellF1}\left(\frac{5}{8},-p,2,\frac{13}{8},-\frac{bx^8}{a},\frac{d^4x^8}{c^4}\right)}{5c^4}\\
&-\frac{4d^3x^7(a+bx^8)^p\left(1+\frac{bx^8}{a}\right)^{-p}\text{AppellF1}\left(\frac{7}{8},-p,2,\frac{15}{8},-\frac{bx^8}{a},\frac{d^4x^8}{c^4}\right)}{7c^5}\\
&+\frac{d^4x^9(a+bx^8)^p\left(1+\frac{bx^8}{a}\right)^{-p}\text{AppellF1}\left(\frac{9}{8},-p,2,\frac{17}{8},-\frac{bx^8}{a},\frac{d^4x^8}{c^4}\right)}{3c^6}\\
&-\frac{2d^5x^{11}(a+bx^8)^p\left(1+\frac{bx^8}{a}\right)^{-p}\text{AppellF1}\left(\frac{11}{8},-p,2,\frac{19}{8},-\frac{bx^8}{a},\frac{d^4x^8}{c^4}\right)}{11c^7}\\
&+\frac{d^6x^{13}(a+bx^8)^p\left(1+\frac{bx^8}{a}\right)^{-p}\text{AppellF1}\left(\frac{13}{8},-p,2,\frac{21}{8},-\frac{bx^8}{a},\frac{d^4x^8}{c^4}\right)}{13c^8}\end{aligned}$$

output
```
x*(b*x^8+a)^p*AppellF1(1/8,2,-p,9/8,d^4*x^8/c^4,-b*x^8/a)/c^2/((1+b*x^8/a)
^p)-2/3*d*x^3*(b*x^8+a)^p*AppellF1(3/8,2,-p,11/8,d^4*x^8/c^4,-b*x^8/a)/c^3
/((1+b*x^8/a)^p)+3/5*d^2*x^5*(b*x^8+a)^p*AppellF1(5/8,2,-p,13/8,d^4*x^8/c^
4,-b*x^8/a)/c^4/((1+b*x^8/a)^p)-4/7*d^3*x^7*(b*x^8+a)^p*AppellF1(7/8,2,-p,
15/8,d^4*x^8/c^4,-b*x^8/a)/c^5/((1+b*x^8/a)^p)+1/3*d^4*x^9*(b*x^8+a)^p*App
ellF1(9/8,2,-p,17/8,d^4*x^8/c^4,-b*x^8/a)/c^6/((1+b*x^8/a)^p)-2/11*d^5*x^1
1*(b*x^8+a)^p*AppellF1(11/8,2,-p,19/8,d^4*x^8/c^4,-b*x^8/a)/c^7/((1+b*x^8/
a)^p)+1/13*d^6*x^13*(b*x^8+a)^p*AppellF1(13/8,2,-p,21/8,d^4*x^8/c^4,-b*x^8
/a)/c^8/((1+b*x^8/a)^p)
```

Mathematica [F]

$$\int \frac{(a+bx^8)^p}{(c+dx^2)^2}\,dx=\int \frac{(a+bx^8)^p}{(c+dx^2)^2}\,dx$$

input
```
Integrate[(a + b*x^8)^p/(c + d*x^2)^2,x]
```

output
```
Integrate[(a + b*x^8)^p/(c + d*x^2)^2, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^8)^p}{(c+dx^2)^2}\,dx$$

$$\Big\downarrow \ 2584$$

$$\int \frac{(c^2-2cdx^2+d^2x^4)\,(a+bx^8)^p}{(c^2-d^2x^4)^2}dx$$

$$\Big\downarrow \ 1380$$

$$\frac{\int \frac{d^2(c-dx^2)^2(bx^8+a)^p}{(c^2-d^2x^4)^2}dx}{d^2}$$

$$\Big\downarrow \text{ 27}$$

$$\int \frac{(c-dx^2)^2 (a+bx^8)^p}{(c^2-d^2x^4)^2}dx$$

$$\Big\downarrow \text{ 1388}$$

$$\int \frac{(a+bx^8)^p}{(c+dx^2)^2}dx$$

$$\Big\downarrow \text{ 2584}$$

$$\int \frac{(c^2-2cdx^2+d^2x^4) (a+bx^8)^p}{(c^2-d^2x^4)^2}dx$$

$$\Big\downarrow \text{ 1380}$$

$$\frac{\int \frac{d^2 (c-dx^2)^2 (bx^8+a)^p}{(c^2-d^2x^4)^2}dx}{d^2}$$

$$\Big\downarrow \text{ 27}$$

$$\int \frac{(c-dx^2)^2 (a+bx^8)^p}{(c^2-d^2x^4)^2}dx$$

$$\Big\downarrow \text{ 1388}$$

$$\int \frac{(a+bx^8)^p}{(c+dx^2)^2}dx$$

$$\Big\downarrow \text{ 2584}$$

$$\int \frac{(c^2-2cdx^2+d^2x^4) (a+bx^8)^p}{(c^2-d^2x^4)^2}dx$$

$$\Big\downarrow \text{ 1380}$$

$$\frac{\int \frac{d^2 (c-dx^2)^2 (bx^8+a)^p}{(c^2-d^2x^4)^2}dx}{d^2}$$

$$\Big\downarrow \text{ 27}$$

$$\int \frac{(c-dx^2)^2 (a+bx^8)^p}{(c^2-d^2x^4)^2}dx$$

$$\Big\downarrow \text{ 1388}$$

$$\int \frac{(a+bx^8)^p}{(c+dx^2)^2}dx$$

$$\Big\downarrow \text{2584}$$

$$\int \frac{(c^2-2cdx^2+d^2x^4)(a+bx^8)^p}{(c^2-d^2x^4)^2}dx$$

$$\Big\downarrow \text{1380}$$

$$\frac{\int \frac{d^2(c-dx^2)^2(bx^8+a)^p}{(c^2-d^2x^4)^2}dx}{d^2}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{(c-dx^2)^2(a+bx^8)^p}{(c^2-d^2x^4)^2}dx$$

$$\Big\downarrow \text{1388}$$

$$\int \frac{(a+bx^8)^p}{(c+dx^2)^2}dx$$

$$\Big\downarrow \text{2584}$$

$$\int \frac{(c^2-2cdx^2+d^2x^4)(a+bx^8)^p}{(c^2-d^2x^4)^2}dx$$

$$\Big\downarrow \text{1380}$$

$$\frac{\int \frac{d^2(c-dx^2)^2(bx^8+a)^p}{(c^2-d^2x^4)^2}dx}{d^2}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{(c-dx^2)^2(a+bx^8)^p}{(c^2-d^2x^4)^2}dx$$

$$\Big\downarrow \text{1388}$$

$$\int \frac{(a+bx^8)^p}{(c+dx^2)^2}dx$$

$$\Big\downarrow \text{2584}$$

$$\int \frac{(c^2-2cdx^2+d^2x^4)(a+bx^8)^p}{(c^2-d^2x^4)^2}dx$$

$$\Big\downarrow \text{1380}$$

$$\frac{\int \frac{d^2(c-dx^2)^2(bx^8+a)^p}{(c^2-d^2x^4)^2}dx}{d^2}$$

$$\Big\downarrow \textbf{27}$$

$$\int \frac{(c-dx^2)^2(a+bx^8)^p}{(c^2-d^2x^4)^2}dx$$

$$\Big\downarrow \textbf{1388}$$

$$\int \frac{(a+bx^8)^p}{(c+dx^2)^2}dx$$

$$\Big\downarrow \textbf{2584}$$

$$\int \frac{(c^2-2cdx^2+d^2x^4)(a+bx^8)^p}{(c^2-d^2x^4)^2}dx$$

$$\Big\downarrow \textbf{1380}$$

$$\frac{\int \frac{d^2(c-dx^2)^2(bx^8+a)^p}{(c^2-d^2x^4)^2}dx}{d^2}$$

$$\Big\downarrow \textbf{27}$$

$$\int \frac{(c-dx^2)^2(a+bx^8)^p}{(c^2-d^2x^4)^2}dx$$

$$\Big\downarrow \textbf{1388}$$

$$\int \frac{(a+bx^8)^p}{(c+dx^2)^2}dx$$

$$\Big\downarrow \textbf{2584}$$

$$\int \frac{(c^2-2cdx^2+d^2x^4)(a+bx^8)^p}{(c^2-d^2x^4)^2}dx$$

$$\Big\downarrow \textbf{1380}$$

$$\frac{\int \frac{d^2(c-dx^2)^2(bx^8+a)^p}{(c^2-d^2x^4)^2}dx}{d^2}$$

input
```
Int[(a + b*x^8)^p/(c + d*x^2)^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 1388
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.),
x_Symbol] :> Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p, x] /; FreeQ[{a,
 c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && (Integer
Q[p] || (GtQ[a, 0] && GtQ[d, 0]))
```

rule 2584
```
Int[((c_) + (d_.)*(x_)^(n_.))^(q_)*((a_) + (b_.)*(x_)^(nn_.))^(p_), x_Symbo
l] :> Int[ExpandToSum[(c - d*x^n)^(-q), x]*((a + b*x^nn)^p/(c^2 - d^2*x^(2*
n))^(-q)), x] /; FreeQ[{a, b, c, d, n, nn, p}, x] &&  !IntegerQ[p] && ILtQ[
q, 0] && IGtQ[Log[2, nn/n], 0]
```

Maple [F]

$$\int \frac{(b\,x^8+a)^p}{(d\,x^2+c)^2}dx$$

input
```
int((b*x^8+a)^p/(d*x^2+c)^2,x)
```

output
```
int((b*x^8+a)^p/(d*x^2+c)^2,x)
```

Fricas [F]

$$\int \frac{(a+bx^8)^p}{(c+dx^2)^2}\,dx = \int \frac{(bx^8+a)^p}{(dx^2+c)^2}\,dx$$

input
```
integrate((b*x^8+a)^p/(d*x^2+c)^2,x, algorithm="fricas")
```

output
```
integral((b*x^8 + a)^p/(d^2*x^4 + 2*c*d*x^2 + c^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^8)^p}{(c+dx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x**8+a)**p/(d*x**2+c)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^8)^p}{(c+dx^2)^2}\,dx = \int \frac{(bx^8+a)^p}{(dx^2+c)^2}\,dx$$

input
```
integrate((b*x^8+a)^p/(d*x^2+c)^2,x, algorithm="maxima")
```

output
```
integrate((b*x^8 + a)^p/(d*x^2 + c)^2, x)
```

Giac [F]

$$\int \frac{(a+bx^8)^p}{(c+dx^2)^2}\,dx = \int \frac{(bx^8+a)^p}{(dx^2+c)^2}\,dx$$

input
```
integrate((b*x^8+a)^p/(d*x^2+c)^2,x, algorithm="giac")
```

output
```
integrate((b*x^8 + a)^p/(d*x^2 + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^8)^p}{(c+dx^2)^2}\,dx = \int \frac{(b\,x^8+a)^p}{(d\,x^2+c)^2}\,dx$$

input
```
int((a + b*x^8)^p/(c + d*x^2)^2,x)
```

output
```
int((a + b*x^8)^p/(c + d*x^2)^2, x)
```

Reduce [F]

$$\int \frac{(a+bx^8)^p}{(c+dx^2)^2}\,dx = \int \frac{(b\,x^8+a)^p}{d^2x^4+2cd\,x^2+c^2}dx$$

input
```
int((b*x^8+a)^p/(d*x^2+c)^2,x)
```

output
```
int((a + b*x**8)**p/(c**2 + 2*c*d*x**2 + d**2*x**4),x)
```

3.315 $\int (c+dx)^3 (a+bx^8)^p \, dx$

Optimal result

Integrand size = 17, antiderivative size = 203

$$\int (c+dx)^3 (a+bx^8)^p \, dx = c^3 x (a+bx^8)^p \left(1+\frac{bx^8}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{8},-p,\frac{9}{8},-\frac{bx^8}{a}\right)+\frac{3}{2}c^2 d x^2 (a+bx^8)^p \left(1+\frac{bx^8}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{4},-p,\frac{5}{4},-\frac{bx^8}{a}\right)+c d^2 x^3 (a+bx^8)^p \left(1+\frac{bx^8}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{3}{8},-p,\frac{11}{8},-\frac{bx^8}{a}\right)+\frac{1}{4}d^3 x^4 (a+bx^8)^p \left(1+\frac{bx^8}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^8}{a}\right)$$

output
```
c^3*x*(b*x^8+a)^p*hypergeom([1/8, -p],[9/8],-b*x^8/a)/((1+b*x^8/a)^p)+3/2*
c^2*d*x^2*(b*x^8+a)^p*hypergeom([1/4, -p],[5/4],-b*x^8/a)/((1+b*x^8/a)^p)+
c*d^2*x^3*(b*x^8+a)^p*hypergeom([3/8, -p],[11/8],-b*x^8/a)/((1+b*x^8/a)^p)
+1/4*d^3*x^4*(b*x^8+a)^p*hypergeom([1/2, -p],[3/2],-b*x^8/a)/((1+b*x^8/a)^
p)
```

Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.64

$$\int (c+dx)^3 \left(a+bx^8\right)^p \, dx$$
$$= \frac{1}{4} x \left(a+bx^8\right)^p \left(1+\frac{bx^8}{a}\right)^{-p} \left(4c^3 \text{Hypergeometric2F1}\left(\frac{1}{8},-p,\frac{9}{8},-\frac{bx^8}{a}\right) + dx\left(6c^2 \text{Hypergeometric2F1}\left(\frac{1}{4},-p,\frac{5}{4},-\frac{bx^8}{a}\right) + dx\left(4c\, \text{Hypergeometric2F1}\left(\frac{3}{8},-p,\frac{11}{8},-\frac{bx^8}{a}\right) + dx\, \text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^8}{a}\right)\right)\right)\right)$$

input
```
Integrate[(c + d*x)^3*(a + b*x^8)^p,x]
```

output
```
(x*(a + b*x^8)^p*(4*c^3*Hypergeometric2F1[1/8, -p, 9/8, -((b*x^8)/a)] + d*
x*(6*c^2*Hypergeometric2F1[1/4, -p, 5/4, -((b*x^8)/a)] + d*x*(4*c*Hypergeo
metric2F1[3/8, -p, 11/8, -((b*x^8)/a)] + d*x*Hypergeometric2F1[1/2, -p, 3/
2, -((b*x^8)/a)]))))/(4*(1 + (b*x^8)/a)^p)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2424, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3 \left(a+bx^8\right)^p \, dx$$
$$\Big\downarrow \; 2424$$
$$\int \left(c^3 \left(a+bx^8\right)^p + 3c^2 dx \left(a+bx^8\right)^p + 3cd^2 x^2 \left(a+bx^8\right)^p + d^3 x^3 \left(a+bx^8\right)^p\right) dx$$
$$\Big\downarrow \; 2009$$

$$c^3x\left(a+bx^8\right)^p\left(\frac{bx^8}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{8},-p,\frac{9}{8},-\frac{bx^8}{a}\right)+\frac{3}{2}c^2dx^2\left(a+bx^8\right)^p\left(\frac{bx^8}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{4},-p,\frac{5}{4},-\frac{bx^8}{a}\right)+cd^2x^3\left(a+bx^8\right)^p\left(\frac{bx^8}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{3}{8},-p,\frac{11}{8},-\frac{bx^8}{a}\right)+\frac{1}{4}d^3x^4\left(a+bx^8\right)^p\left(\frac{bx^8}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^8}{a}\right)$$

input
```
Int[(c + d*x)^3*(a + b*x^8)^p,x]
```

output
```
(c^3*x*(a + b*x^8)^p*Hypergeometric2F1[1/8, -p, 9/8, -((b*x^8)/a)])/(1 + (
b*x^8)/a)^p + (3*c^2*d*x^2*(a + b*x^8)^p*Hypergeometric2F1[1/4, -p, 5/4, -
((b*x^8)/a)])/(2*(1 + (b*x^8)/a)^p) + (c*d^2*x^3*(a + b*x^8)^p*Hypergeomet
ric2F1[3/8, -p, 11/8, -((b*x^8)/a)])/(1 + (b*x^8)/a)^p + (d^3*x^4*(a + b*x
^8)^p*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^8)/a)])/(4*(1 + (b*x^8)/a)^p)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2424
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Maple [F]

$$\int (dx+c)^3\left(b\,x^8+a\right)^p dx$$

input
```
int((d*x+c)^3*(b*x^8+a)^p,x)
```

output
```
int((d*x+c)^3*(b*x^8+a)^p,x)
```

Fricas [F]

$$\int (c+dx)^3 (a+bx^8)^p \, dx = \int (dx+c)^3 (bx^8+a)^p \, dx$$

input
```
integrate((d*x+c)^3*(b*x^8+a)^p,x, algorithm="fricas")
```

output
```
integral((d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)*(b*x^8 + a)^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int (c+dx)^3 (a+bx^8)^p \, dx = \text{Timed out}$$

input
```
integrate((d*x+c)**3*(b*x**8+a)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (c+dx)^3 (a+bx^8)^p \, dx = \int (dx+c)^3 (bx^8+a)^p \, dx$$

input
```
integrate((d*x+c)^3*(b*x^8+a)^p,x, algorithm="maxima")
```

output
```
integrate((d*x + c)^3*(b*x^8 + a)^p, x)
```

Giac [F]

$$\int (c+dx)^3 \left(a+bx^8\right)^p \, dx = \int (dx+c)^3 (bx^8+a)^p \, dx$$

input
```
integrate((d*x+c)^3*(b*x^8+a)^p,x, algorithm="giac")
```

output
```
integrate((d*x + c)^3*(b*x^8 + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^3 \left(a+bx^8\right)^p \, dx = \int (b\,x^8+a)^p\,(c+d\,x)^3 \, dx$$

input
```
int((a + b*x^8)^p*(c + d*x)^3,x)
```

output
```
int((a + b*x^8)^p*(c + d*x)^3, x)
```

Reduce [F]

$$\int (c+dx)^3 \left(a+bx^8\right)^p \, dx = \text{too large to display}$$

input
```
int((d*x+c)^3*(b*x^8+a)^p,x)
```

output

```
(256*(a + b*x**8)**p*c**3*p**3*x + 288*(a + b*x**8)**p*c**3*p**2*x + 104*(
a + b*x**8)**p*c**3*p*x + 12*(a + b*x**8)**p*c**3*x + 768*(a + b*x**8)**p*
c**2*d*p**3*x**2 + 768*(a + b*x**8)**p*c**2*d*p**2*x**2 + 228*(a + b*x**8)
**p*c**2*d*p*x**2 + 18*(a + b*x**8)**p*c**2*d*x**2 + 768*(a + b*x**8)**p*c
*d**2*p**3*x**3 + 672*(a + b*x**8)**p*c*d**2*p**2*x**3 + 168*(a + b*x**8)*
*p*c*d**2*p*x**3 + 12*(a + b*x**8)**p*c*d**2*x**3 + 256*(a + b*x**8)**p*d*
*3*p**3*x**4 + 192*(a + b*x**8)**p*d**3*p**2*x**4 + 44*(a + b*x**8)**p*d**
3*p*x**4 + 3*(a + b*x**8)**p*d**3*x**4 + 1048576*int((a + b*x**8)**p/(512*
a*p**4 + 640*a*p**3 + 280*a*p**2 + 50*a*p + 3*a + 512*b*p**4*x**8 + 640*b*
p**3*x**8 + 280*b*p**2*x**8 + 50*b*p*x**8 + 3*b*x**8),x)*a*c**3*p**8 + 249
0368*int((a + b*x**8)**p/(512*a*p**4 + 640*a*p**3 + 280*a*p**2 + 50*a*p +
3*a + 512*b*p**4*x**8 + 640*b*p**3*x**8 + 280*b*p**2*x**8 + 50*b*p*x**8 +
3*b*x**8),x)*a*c**3*p**7 + 2473984*int((a + b*x**8)**p/(512*a*p**4 + 640*a
*p**3 + 280*a*p**2 + 50*a*p + 3*a + 512*b*p**4*x**8 + 640*b*p**3*x**8 + 28
0*b*p**2*x**8 + 50*b*p*x**8 + 3*b*x**8),x)*a*c**3*p**6 + 1329152*int((a +
b*x**8)**p/(512*a*p**4 + 640*a*p**3 + 280*a*p**2 + 50*a*p + 3*a + 512*b*p*
*4*x**8 + 640*b*p**3*x**8 + 280*b*p**2*x**8 + 50*b*p*x**8 + 3*b*x**8),x)*a
*c**3*p**5 + 415744*int((a + b*x**8)**p/(512*a*p**4 + 640*a*p**3 + 280*a*p
**2 + 50*a*p + 3*a + 512*b*p**4*x**8 + 640*b*p**3*x**8 + 280*b*p**2*x**8 +
 50*b*p*x**8 + 3*b*x**8),x)*a*c**3*p**4 + 75392*int((a + b*x**8)**p/(51...
```

3.316 $\int (c+dx)^2 \left(a+bx^8\right)^p \, dx$

Optimal result

Integrand size = 17, antiderivative size = 148

$$\int (c+dx)^2 \left(a+bx^8\right)^p \, dx = c^2x\left(a+bx^8\right)^p \left(1+\frac{bx^8}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{8},-p,\frac{9}{8},-\frac{bx^8}{a}\right)+cdx^2\left(a+bx^8\right)^p \left(1+\frac{bx^8}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{4},-p,\frac{5}{4},-\frac{bx^8}{a}\right)+\frac{1}{3}d^2x^3\left(a+bx^8\right)^p \left(1+\frac{bx^8}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{3}{8},-p,\frac{11}{8},-\frac{bx^8}{a}\right)$$

output
```
c^2*x*(b*x^8+a)^p*hypergeom([1/8, -p],[9/8],-b*x^8/a)/((1+b*x^8/a)^p)+c*d*
x^2*(b*x^8+a)^p*hypergeom([1/4, -p],[5/4],-b*x^8/a)/((1+b*x^8/a)^p)+1/3*d^
2*x^3*(b*x^8+a)^p*hypergeom([3/8, -p],[11/8],-b*x^8/a)/((1+b*x^8/a)^p)
```

Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.68

$$\int (c+dx)^2 \left(a+bx^8\right)^p \, dx = \frac{1}{3}x\left(a+bx^8\right)^p \left(1+\frac{bx^8}{a}\right)^{-p} \left(3c^2 \text{Hypergeometric2F1}\left(\frac{1}{8},-p,\frac{9}{8},-\frac{bx^8}{a}\right)+dx\left(3c\,\text{Hypergeometric2F1}\left(\frac{1}{4},-p,\frac{5}{4},-\frac{bx^8}{a}\right)+dx\,\text{Hypergeometric2F1}\left(\frac{3}{8},-p,\frac{11}{8},-\frac{bx^8}{a}\right)\right)\right)$$

input
```
Integrate[(c + d*x)^2*(a + b*x^8)^p,x]
```

output
```
(x*(a + b*x^8)^p*(3*c^2*Hypergeometric2F1[1/8, -p, 9/8, -((b*x^8)/a)] + d*
x*(3*c*Hypergeometric2F1[1/4, -p, 5/4, -((b*x^8)/a)] + d*x*Hypergeometric2
F1[3/8, -p, 11/8, -((b*x^8)/a)])))/(3*(1 + (b*x^8)/a)^p)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2424, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2 \left(a+bx^8\right)^p \, dx$$

$$\Big\downarrow \; 2424$$

$$\int \left(c^2\left(a+bx^8\right)^p+2cdx\left(a+bx^8\right)^p+d^2x^2\left(a+bx^8\right)^p\right)dx$$

$$\Big\downarrow \; 2009$$

$$c^2x(a+bx^8)^p\left(\frac{bx^8}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{8},-p,\frac{9}{8},-\frac{bx^8}{a}\right)+$$
$$cdx^2(a+bx^8)^p\left(\frac{bx^8}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{4},-p,\frac{5}{4},-\frac{bx^8}{a}\right)+$$
$$\frac{1}{3}d^2x^3(a+bx^8)^p\left(\frac{bx^8}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{3}{8},-p,\frac{11}{8},-\frac{bx^8}{a}\right)$$

input
```
Int[(c + d*x)^2*(a + b*x^8)^p,x]
```

output
```
(c^2*x*(a + b*x^8)^p*Hypergeometric2F1[1/8, -p, 9/8, -((b*x^8)/a)])/(1 + (
b*x^8)/a)^p + (c*d*x^2*(a + b*x^8)^p*Hypergeometric2F1[1/4, -p, 5/4, -((b*
x^8)/a)])/(1 + (b*x^8)/a)^p + (d^2*x^3*(a + b*x^8)^p*Hypergeometric2F1[3/8
, -p, 11/8, -((b*x^8)/a)])/(3*(1 + (b*x^8)/a)^p)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2424
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Maple [F]

$$\int (dx+c)^2\left(b\,x^8+a\right)^p dx$$

input
```
int((d*x+c)^2*(b*x^8+a)^p,x)
```

output
```
int((d*x+c)^2*(b*x^8+a)^p,x)
```

Fricas [F]

$$\int (c+dx)^2 \left(a+bx^8\right)^p \, dx = \int (dx+c)^2 \left(bx^8+a\right)^p dx$$

input
```
integrate((d*x+c)^2*(b*x^8+a)^p,x, algorithm="fricas")
```

output
```
integral((d^2*x^2 + 2*c*d*x + c^2)*(b*x^8 + a)^p, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 135.24 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.80

$$\int (c+dx)^2 \left(a+bx^8\right)^p \, dx = \frac{a^p c^2 x \Gamma\left(\frac{1}{8}\right) {}_2F_1\left(\begin{matrix} \frac{1}{8}, -p \\ \frac{9}{8} \end{matrix}\middle| \frac{bx^8 e^{i\pi}}{a}\right)}{8\Gamma\left(\frac{9}{8}\right)} + \frac{a^p c d x^2 \Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\begin{matrix} \frac{1}{4}, -p \\ \frac{5}{4} \end{matrix}\middle| \frac{bx^8 e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)} + \frac{a^p d^2 x^3 \Gamma\left(\frac{3}{8}\right) {}_2F_1\left(\begin{matrix} \frac{3}{8}, -p \\ \frac{11}{8} \end{matrix}\middle| \frac{bx^8 e^{i\pi}}{a}\right)}{8\Gamma\left(\frac{11}{8}\right)}$$

input
```
integrate((d*x+c)**2*(b*x**8+a)**p,x)
```

output
```
a**p*c**2*x*gamma(1/8)*hyper((1/8, -p), (9/8,), b*x**8*exp_polar(I*pi)/a)/
(8*gamma(9/8)) + a**p*c*d*x**2*gamma(1/4)*hyper((1/4, -p), (5/4,), b*x**8*
exp_polar(I*pi)/a)/(4*gamma(5/4)) + a**p*d**2*x**3*gamma(3/8)*hyper((3/8,
-p), (11/8,), b*x**8*exp_polar(I*pi)/a)/(8*gamma(11/8))
```

Maxima [F]

$$\int (c+dx)^2 \left(a+bx^8\right)^p \, dx = \int (dx+c)^2 (bx^8+a)^p \, dx$$

input
```
integrate((d*x+c)^2*(b*x^8+a)^p,x, algorithm="maxima")
```

output
```
integrate((d*x + c)^2*(b*x^8 + a)^p, x)
```

Giac [F]

$$\int (c+dx)^2 \left(a+bx^8\right)^p \, dx = \int (dx+c)^2 (bx^8+a)^p \, dx$$

input
```
integrate((d*x+c)^2*(b*x^8+a)^p,x, algorithm="giac")
```

output
```
integrate((d*x + c)^2*(b*x^8 + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^2 \left(a+bx^8\right)^p \, dx = \int \left(b\,x^8+a\right)^p \,(c+d\,x)^2 \, dx$$

input
```
int((a + b*x^8)^p*(c + d*x)^2,x)
```

output
```
int((a + b*x^8)^p*(c + d*x)^2, x)
```

Reduce [F]

$$\int (c+dx)^2 \left(a+bx^8\right)^p \, dx = \text{Too large to display}$$

input
```
int((d*x+c)^2*(b*x^8+a)^p,x)
```

output
```
(32*(a + b*x**8)**p*c**2*p**2*x + 20*(a + b*x**8)**p*c**2*p*x + 3*(a + b*x
**8)**p*c**2*x + 64*(a + b*x**8)**p*c*d*p**2*x**2 + 32*(a + b*x**8)**p*c*d
*p*x**2 + 3*(a + b*x**8)**p*c*d*x**2 + 32*(a + b*x**8)**p*d**2*p**2*x**3 +
 12*(a + b*x**8)**p*d**2*p*x**3 + (a + b*x**8)**p*d**2*x**3 + 65536*int((a
 + b*x**8)**p/(256*a*p**3 + 192*a*p**2 + 44*a*p + 3*a + 256*b*p**3*x**8 +
192*b*p**2*x**8 + 44*b*p*x**8 + 3*b*x**8),x)*a*c**2*p**6 + 90112*int((a +
b*x**8)**p/(256*a*p**3 + 192*a*p**2 + 44*a*p + 3*a + 256*b*p**3*x**8 + 192
*b*p**2*x**8 + 44*b*p*x**8 + 3*b*x**8),x)*a*c**2*p**5 + 48128*int((a + b*x
**8)**p/(256*a*p**3 + 192*a*p**2 + 44*a*p + 3*a + 256*b*p**3*x**8 + 192*b*
p**2*x**8 + 44*b*p*x**8 + 3*b*x**8),x)*a*c**2*p**4 + 12416*int((a + b*x**8
)**p/(256*a*p**3 + 192*a*p**2 + 44*a*p + 3*a + 256*b*p**3*x**8 + 192*b*p**
2*x**8 + 44*b*p*x**8 + 3*b*x**8),x)*a*c**2*p**3 + 1536*int((a + b*x**8)**p
/(256*a*p**3 + 192*a*p**2 + 44*a*p + 3*a + 256*b*p**3*x**8 + 192*b*p**2*x*
*8 + 44*b*p*x**8 + 3*b*x**8),x)*a*c**2*p**2 + 72*int((a + b*x**8)**p/(256*
a*p**3 + 192*a*p**2 + 44*a*p + 3*a + 256*b*p**3*x**8 + 192*b*p**2*x**8 + 4
4*b*p*x**8 + 3*b*x**8),x)*a*c**2*p + 65536*int(((a + b*x**8)**p*x**2)/(256
*a*p**3 + 192*a*p**2 + 44*a*p + 3*a + 256*b*p**3*x**8 + 192*b*p**2*x**8 +
44*b*p*x**8 + 3*b*x**8),x)*a*d**2*p**6 + 73728*int(((a + b*x**8)**p*x**2)/
(256*a*p**3 + 192*a*p**2 + 44*a*p + 3*a + 256*b*p**3*x**8 + 192*b*p**2*x**
8 + 44*b*p*x**8 + 3*b*x**8),x)*a*d**2*p**5 + 31744*int(((a + b*x**8)**p...
```

3.317 $\int (c+dx) \left(a+bx^8\right)^p \, dx$

Optimal result

Integrand size = 15, antiderivative size = 96

$$\int (c+dx) \left(a+bx^8\right)^p \, dx = cx\left(a+bx^8\right)^p \left(1+\frac{bx^8}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{8},-p,\frac{9}{8},-\frac{bx^8}{a}\right)+\frac{1}{2}dx^2\left(a+bx^8\right)^p \left(1+\frac{bx^8}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{4},-p,\frac{5}{4},-\frac{bx^8}{a}\right)$$

output
```
c*x*(b*x^8+a)^p*hypergeom([1/8, -p],[9/8],-b*x^8/a)/((1+b*x^8/a)^p)+1/2*d*
x^2*(b*x^8+a)^p*hypergeom([1/4, -p],[5/4],-b*x^8/a)/((1+b*x^8/a)^p)
```

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.76

$$\int (c+dx) \left(a+bx^8\right)^p \, dx = \frac{1}{2}x\left(a+bx^8\right)^p \left(1+\frac{bx^8}{a}\right)^{-p} \left(2c \,\text{Hypergeometric2F1}\left(\frac{1}{8},-p,\frac{9}{8},-\frac{bx^8}{a}\right)+dx \,\text{Hypergeometric2F1}\left(\frac{1}{4},-p,\frac{5}{4},-\frac{bx^8}{a}\right)\right)$$

input
```
Integrate[(c + d*x)*(a + b*x^8)^p,x]
```

output
```
(x*(a + b*x^8)^p*(2*c*Hypergeometric2F1[1/8, -p, 9/8, -((b*x^8)/a)] + d*x*
Hypergeometric2F1[1/4, -p, 5/4, -((b*x^8)/a)]))/(2*(1 + (b*x^8)/a)^p)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2424, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\left(a+bx^8\right)^p\,dx$$

$$\Big\downarrow \text{2424}$$

$$\int \left(c\left(a+bx^8\right)^p+dx\left(a+bx^8\right)^p\right)dx$$

$$\Big\downarrow \text{2009}$$

$$cx\left(a+bx^8\right)^p\left(\frac{bx^8}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{8},-p,\frac{9}{8},-\frac{bx^8}{a}\right)+\frac{1}{2}dx^2\left(a+bx^8\right)^p\left(\frac{bx^8}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{4},-p,\frac{5}{4},-\frac{bx^8}{a}\right)$$

input
```
Int[(c + d*x)*(a + b*x^8)^p,x]
```

output
```
(c*x*(a + b*x^8)^p*Hypergeometric2F1[1/8, -p, 9/8, -((b*x^8)/a)])/(1 + (b*
x^8)/a)^p + (d*x^2*(a + b*x^8)^p*Hypergeometric2F1[1/4, -p, 5/4, -((b*x^8)
/a)])/(2*(1 + (b*x^8)/a)^p)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2424
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Maple [F]

$$\int (dx + c) \left(b\, x^{8} + a\right)^{p} dx$$

input
```
int((d*x+c)*(b*x^8+a)^p,x)
```

output
```
int((d*x+c)*(b*x^8+a)^p,x)
```

Fricas [F]

$$\int (c + dx) \left(a + bx^8\right)^p \, dx = \int (dx + c)\left(bx^8 + a\right)^p dx$$

input
```
integrate((d*x+c)*(b*x^8+a)^p,x, algorithm="fricas")
```

output
```
integral((d*x + c)*(b*x^8 + a)^p, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 92.31 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.78

$$\int (c+dx)\left(a+bx^8\right)^p\,dx=\frac{a^pcx\Gamma\left(\frac{1}{8}\right){}_2F_1\left(\begin{matrix}\frac{1}{8},-p\\ \frac{9}{8}\end{matrix}\middle|\frac{bx^8e^{i\pi}}{a}\right)}{8\Gamma\left(\frac{9}{8}\right)}+\frac{a^pdx^2\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},-p\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^8e^{i\pi}}{a}\right)}{8\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((d*x+c)*(b*x**8+a)**p,x)
```

output
```
a**p*c*x*gamma(1/8)*hyper((1/8, -p), (9/8,), b*x**8*exp_polar(I*pi)/a)/(8*
gamma(9/8)) + a**p*d*x**2*gamma(1/4)*hyper((1/4, -p), (5/4,), b*x**8*exp_p
olar(I*pi)/a)/(8*gamma(5/4))
```

Maxima [F]

$$\int (c+dx)\left(a+bx^8\right)^p\,dx=\int (dx+c)\left(bx^8+a\right)^p\,dx$$

input
```
integrate((d*x+c)*(b*x^8+a)^p,x, algorithm="maxima")
```

output
```
integrate((d*x + c)*(b*x^8 + a)^p, x)
```

Giac [F]

$$\int (c+dx)\left(a+bx^8\right)^p\,dx=\int (dx+c)\left(bx^8+a\right)^p\,dx$$

input
```
integrate((d*x+c)*(b*x^8+a)^p,x, algorithm="giac")
```

output
```
integrate((d*x + c)*(b*x^8 + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)\left(a+bx^8\right)^p\,dx=\int \left(b\,x^8+a\right)^p\,(c+d\,x)\,dx$$

input
```
int((a + b*x^8)^p*(c + d*x),x)
```

output
```
int((a + b*x^8)^p*(c + d*x), x)
```

Reduce [F]

$$\int (c+dx)\left(a+bx^8\right)^p\,dx = \frac{8(b\,x^8+a)^p\,cpx+2(b\,x^8+a)^p\,cx+8(b\,x^8+a)^p\,dp\,x^2+(b\,x^8+a)^p\,d\,x^2+2048\left(\int \frac{(b\,x^8+a)^p}{32b\,p^2x^8+12bp\,x^8+b\,x^8+32a}\right.}{}$$

input
```
int((d*x+c)*(b*x^8+a)^p,x)
```

output
```
(8*(a + b*x**8)**p*c*p*x + 2*(a + b*x**8)**p*c*x + 8*(a + b*x**8)**p*d*p*x
**2 + (a + b*x**8)**p*d*x**2 + 2048*int((a + b*x**8)**p/(32*a*p**2 + 12*a*
p + a + 32*b*p**2*x**8 + 12*b*p*x**8 + b*x**8),x)*a*c*p**4 + 1280*int((a +
 b*x**8)**p/(32*a*p**2 + 12*a*p + a + 32*b*p**2*x**8 + 12*b*p*x**8 + b*x**
8),x)*a*c*p**3 + 256*int((a + b*x**8)**p/(32*a*p**2 + 12*a*p + a + 32*b*p*
*2*x**8 + 12*b*p*x**8 + b*x**8),x)*a*c*p**2 + 16*int((a + b*x**8)**p/(32*a
*p**2 + 12*a*p + a + 32*b*p**2*x**8 + 12*b*p*x**8 + b*x**8),x)*a*c*p + 204
8*int(((a + b*x**8)**p*x)/(32*a*p**2 + 12*a*p + a + 32*b*p**2*x**8 + 12*b*
p*x**8 + b*x**8),x)*a*d*p**4 + 1024*int(((a + b*x**8)**p*x)/(32*a*p**2 + 1
2*a*p + a + 32*b*p**2*x**8 + 12*b*p*x**8 + b*x**8),x)*a*d*p**3 + 160*int((
(a + b*x**8)**p*x)/(32*a*p**2 + 12*a*p + a + 32*b*p**2*x**8 + 12*b*p*x**8
+ b*x**8),x)*a*d*p**2 + 8*int(((a + b*x**8)**p*x)/(32*a*p**2 + 12*a*p + a
+ 32*b*p**2*x**8 + 12*b*p*x**8 + b*x**8),x)*a*d*p)/(2*(32*p**2 + 12*p + 1)
)
```

3.318 $\int (a+bx^8)^p \, dx$

Optimal result

Integrand size = 9, antiderivative size = 44

$$\int (a+bx^8)^p \, dx = x(a+bx^8)^p \left(1+\frac{bx^8}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{8},-p,\frac{9}{8},-\frac{bx^8}{a}\right)$$

output
```
x*(b*x^8+a)^p*hypergeom([1/8, -p],[9/8],-b*x^8/a)/((1+b*x^8/a)^p)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int (a+bx^8)^p \, dx = x(a+bx^8)^p \left(1+\frac{bx^8}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{8},-p,\frac{9}{8},-\frac{bx^8}{a}\right)$$

input
```
Integrate[(a + b*x^8)^p,x]
```

output
```
(x*(a + b*x^8)^p*Hypergeometric2F1[1/8, -p, 9/8, -((b*x^8)/a)])/(1 + (b*x^
8)/a)^p
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {779, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^8\right)^p\,dx$$

$$\Big\downarrow \text{779}$$

$$\left(a+bx^8\right)^p\left(\frac{bx^8}{a}+1\right)^{-p}\int \left(\frac{bx^8}{a}+1\right)^p dx$$

$$\Big\downarrow \text{778}$$

$$x\left(a+bx^8\right)^p\left(\frac{bx^8}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{8},-p,\frac{9}{8},-\frac{bx^8}{a}\right)$$

input
```
Int[(a + b*x^8)^p,x]
```

output
```
(x*(a + b*x^8)^p*Hypergeometric2F1[1/8, -p, 9/8, -((b*x^8)/a)])/(1 + (b*x^
8)/a)^p
```

Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Si
mplify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \left(b\,x^{8}+a\right)^{p}dx$$

input
```
int((b*x^8+a)^p,x)
```

output
```
int((b*x^8+a)^p,x)
```

Fricas [F]

$$\int \left(a+bx^8\right)^p\,dx=\int \left(bx^{8}+a\right)^{p}dx$$

input
```
integrate((b*x^8+a)^p,x, algorithm="fricas")
```

output
```
integral((b*x^8 + a)^p, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 30.17 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.77

$$\int \left(a+bx^8\right)^p\,dx=\frac{a^{p}x\Gamma\left(\frac{1}{8}\right){}_{2}F_{1}\left(\left.\begin{matrix}\frac{1}{8},-p\\ \frac{9}{8}\end{matrix}\right|\frac{bx^{8}e^{i\pi}}{a}\right)}{8\Gamma\left(\frac{9}{8}\right)}$$

input
```
integrate((b*x**8+a)**p,x)
```

output
```
a**p*x*gamma(1/8)*hyper((1/8, -p), (9/8,), b*x**8*exp_polar(I*pi)/a)/(8*ga
mma(9/8))
```

Maxima [F]

$$\int \left(a+bx^8\right)^p\,dx=\int \left(bx^8+a\right)^p\,dx$$

input
```
integrate((b*x^8+a)^p,x, algorithm="maxima")
```

output
```
integrate((b*x^8 + a)^p, x)
```

Giac [F]

$$\int \left(a+bx^8\right)^p\,dx=\int \left(bx^8+a\right)^p\,dx$$

input
```
integrate((b*x^8+a)^p,x, algorithm="giac")
```

output
```
integrate((b*x^8 + a)^p, x)
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.93

$$\int \left(a+bx^8\right)^p\,dx=\frac{x\,\left(b\,x^8+a\right)^p\,{}_2\mathrm{F}_1\left(\frac{1}{8},-p;\ \frac{9}{8};\ -\frac{b\,x^8}{a}\right)}{\left(\frac{b\,x^8}{a}+1\right)^p}$$

input
```
int((a + b*x^8)^p,x)
```

output
```
(x*(a + b*x^8)^p*hypergeom([1/8, -p], 9/8, -(b*x^8)/a))/((b*x^8)/a + 1)^p
```

Reduce [F]

$$\int (a+bx^8)^p\,dx$$
$$= \frac{(b\,x^8+a)^p\,x+64\left(\int \frac{(b\,x^8+a)^p}{8bp\,x^8+b\,x^8+8ap+a}dx\right)a\,p^2+8\left(\int \frac{(b\,x^8+a)^p}{8bp\,x^8+b\,x^8+8ap+a}dx\right)ap}{8p+1}$$

input
```
int((b*x^8+a)^p,x)
```

output
```
((a + b*x**8)**p*x + 64*int((a + b*x**8)**p/(8*a*p + a + 8*b*p*x**8 + b*x*
*8),x)*a*p**2 + 8*int((a + b*x**8)**p/(8*a*p + a + 8*b*p*x**8 + b*x**8),x)
*a*p)/(8*p + 1)
```

3.319 $\int \frac{(a+bx^8)^p}{c+dx} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 521

$$\begin{aligned}
\int \frac{(a+bx^8)^p}{c+dx} \, dx &= \frac{x(a+bx^8)^p \left(1+\frac{bx^8}{a}\right)^{-p} \text{AppellF1}\left(\frac{1}{8},-p,1,\frac{9}{8},-\frac{bx^8}{a},\frac{d^8x^8}{c^8}\right)}{c} \\
&- \frac{dx^2(a+bx^8)^p \left(1+\frac{bx^8}{a}\right)^{-p} \text{AppellF1}\left(\frac{1}{4},-p,1,\frac{5}{4},-\frac{bx^8}{a},\frac{d^8x^8}{c^8}\right)}{2c^2} \\
&+ \frac{d^2x^3(a+bx^8)^p \left(1+\frac{bx^8}{a}\right)^{-p} \text{AppellF1}\left(\frac{3}{8},-p,1,\frac{11}{8},-\frac{bx^8}{a},\frac{d^8x^8}{c^8}\right)}{3c^3} \\
&- \frac{d^3x^4(a+bx^8)^p \left(1+\frac{bx^8}{a}\right)^{-p} \text{AppellF1}\left(\frac{1}{2},-p,1,\frac{3}{2},-\frac{bx^8}{a},\frac{d^8x^8}{c^8}\right)}{4c^4} \\
&+ \frac{d^4x^5(a+bx^8)^p \left(1+\frac{bx^8}{a}\right)^{-p} \text{AppellF1}\left(\frac{5}{8},-p,1,\frac{13}{8},-\frac{bx^8}{a},\frac{d^8x^8}{c^8}\right)}{5c^5} \\
&- \frac{d^5x^6(a+bx^8)^p \left(1+\frac{bx^8}{a}\right)^{-p} \text{AppellF1}\left(\frac{3}{4},-p,1,\frac{7}{4},-\frac{bx^8}{a},\frac{d^8x^8}{c^8}\right)}{6c^6} \\
&+ \frac{d^6x^7(a+bx^8)^p \left(1+\frac{bx^8}{a}\right)^{-p} \text{AppellF1}\left(\frac{7}{8},-p,1,\frac{15}{8},-\frac{bx^8}{a},\frac{d^8x^8}{c^8}\right)}{7c^7} \\
&- \frac{d^7(a+bx^8)^{1+p} \text{Hypergeometric2F1}\left(1,1+p,2+p,\frac{d^8(a+bx^8)}{bc^8+ad^8}\right)}{8(bc^8+ad^8)(1+p)}
\end{aligned}$$

output
```
x*(b*x^8+a)^p*AppellF1(1/8,1,-p,9/8,d^8*x^8/c^8,-b*x^8/a)/c/((1+b*x^8/a)^p
)-1/2*d*x^2*(b*x^8+a)^p*AppellF1(1/4,1,-p,5/4,d^8*x^8/c^8,-b*x^8/a)/c^2/((
1+b*x^8/a)^p)+1/3*d^2*x^3*(b*x^8+a)^p*AppellF1(3/8,1,-p,11/8,d^8*x^8/c^8,-
b*x^8/a)/c^3/((1+b*x^8/a)^p)-1/4*d^3*x^4*(b*x^8+a)^p*AppellF1(1/2,1,-p,3/2
,d^8*x^8/c^8,-b*x^8/a)/c^4/((1+b*x^8/a)^p)+1/5*d^4*x^5*(b*x^8+a)^p*AppellF
1(5/8,1,-p,13/8,d^8*x^8/c^8,-b*x^8/a)/c^5/((1+b*x^8/a)^p)-1/6*d^5*x^6*(b*x
^8+a)^p*AppellF1(3/4,1,-p,7/4,d^8*x^8/c^8,-b*x^8/a)/c^6/((1+b*x^8/a)^p)+1/
7*d^6*x^7*(b*x^8+a)^p*AppellF1(7/8,1,-p,15/8,d^8*x^8/c^8,-b*x^8/a)/c^7/((1
+b*x^8/a)^p)-1/8*d^7*(b*x^8+a)^(p+1)*hypergeom([1, p+1],[2+p],d^8*(b*x^8+a
)/(a*d^8+b*c^8))/(a*d^8+b*c^8)/(p+1)
```

Mathematica [F]

$$\int \frac{(a+bx^8)^p}{c+dx}\,dx=\int \frac{(a+bx^8)^p}{c+dx}\,dx$$

input
```
Integrate[(a + b*x^8)^p/(c + d*x),x]
```

output
```
Integrate[(a + b*x^8)^p/(c + d*x), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^8)^p}{c+dx}\,dx$$

$$\Big\downarrow \ 2584$$

$$\int \frac{(c-dx)(a+bx^8)^p}{c^2-d^2x^2}dx$$

$$\Big\downarrow \ 2003$$

$$\int \frac{(a+bx^8)^p}{c+dx}dx$$

$$\Big\downarrow \text{2584}$$

$$\int \frac{(c-dx)\left(a+bx^8\right)^p}{c^2-d^2x^2}dx$$

$$\Big\downarrow \text{2003}$$

$$\int \frac{\left(a+bx^8\right)^p}{c+dx}dx$$

$$\Big\downarrow \text{2584}$$

$$\int \frac{(c-dx)\left(a+bx^8\right)^p}{c^2-d^2x^2}dx$$

$$\Big\downarrow \text{2003}$$

$$\int \frac{\left(a+bx^8\right)^p}{c+dx}dx$$

$$\Big\downarrow \text{2584}$$

$$\int \frac{(c-dx)\left(a+bx^8\right)^p}{c^2-d^2x^2}dx$$

$$\Big\downarrow \text{2003}$$

$$\int \frac{\left(a+bx^8\right)^p}{c+dx}dx$$

$$\Big\downarrow \text{2584}$$

$$\int \frac{(c-dx)\left(a+bx^8\right)^p}{c^2-d^2x^2}dx$$

$$\Big\downarrow \text{2003}$$

$$\int \frac{\left(a+bx^8\right)^p}{c+dx}dx$$

$$\Big\downarrow \text{2584}$$

$$\int \frac{(c-dx)\left(a+bx^8\right)^p}{c^2-d^2x^2}dx$$

$$\Big\downarrow \text{2003}$$

$$\int \frac{\left(a+bx^8\right)^p}{c+dx}dx$$

$$\Big\downarrow \text{2584}$$

$$\int \frac{(c-dx)\left(a+bx^8\right)^p}{c^2-d^2x^2}dx$$
$$\Big\downarrow \text{2003}$$
$$\int \frac{\left(a+bx^8\right)^p}{c+dx}dx$$
$$\Big\downarrow \text{2584}$$
$$\int \frac{(c-dx)\left(a+bx^8\right)^p}{c^2-d^2x^2}dx$$
$$\Big\downarrow \text{2003}$$
$$\int \frac{\left(a+bx^8\right)^p}{c+dx}dx$$
$$\Big\downarrow \text{2584}$$
$$\int \frac{(c-dx)\left(a+bx^8\right)^p}{c^2-d^2x^2}dx$$
$$\Big\downarrow \text{2003}$$
$$\int \frac{\left(a+bx^8\right)^p}{c+dx}dx$$
$$\Big\downarrow \text{2584}$$
$$\int \frac{(c-dx)\left(a+bx^8\right)^p}{c^2-d^2x^2}dx$$
$$\Big\downarrow \text{2003}$$
$$\int \frac{\left(a+bx^8\right)^p}{c+dx}dx$$
$$\Big\downarrow \text{2584}$$
$$\int \frac{(c-dx)\left(a+bx^8\right)^p}{c^2-d^2x^2}dx$$
$$\Big\downarrow \text{2003}$$
$$\int \frac{\left(a+bx^8\right)^p}{c+dx}dx$$
$$\Big\downarrow \text{2584}$$
$$\int \frac{(c-dx)\left(a+bx^8\right)^p}{c^2-d^2x^2}dx$$

$$\Big\downarrow \text{2003}$$

$$\int \frac{(a+bx^8)^p}{c+dx}dx$$

$$\Big\downarrow \text{2584}$$

$$\int \frac{(c-dx)(a+bx^8)^p}{c^2-d^2x^2}dx$$

$$\Big\downarrow \text{2003}$$

$$\int \frac{(a+bx^8)^p}{c+dx}dx$$

$$\Big\downarrow \text{2584}$$

$$\int \frac{(c-dx)(a+bx^8)^p}{c^2-d^2x^2}dx$$

$$\Big\downarrow \text{2003}$$

$$\int \frac{(a+bx^8)^p}{c+dx}dx$$

$$\Big\downarrow \text{2584}$$

$$\int \frac{(c-dx)(a+bx^8)^p}{c^2-d^2x^2}dx$$

$$\Big\downarrow \text{2003}$$

$$\int \frac{(a+bx^8)^p}{c+dx}dx$$

input
```
Int[(a + b*x^8)^p/(c + d*x),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2003
```
Int[(u_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :
> Int[u*(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}
, x] && EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&
 !IntegerQ[n]))
```

rule 2584
```
Int[((c_) + (d_.)*(x_)^(n_.))^(q_)*((a_) + (b_.)*(x_)^(nn_.))^(p_), x_Symbo
l] :> Int[ExpandToSum[(c - d*x^n)^(-q), x]*((a + b*x^nn)^p/(c^2 - d^2*x^(2*
n))^(-q)), x] /; FreeQ[{a, b, c, d, n, nn, p}, x] &&  !IntegerQ[p] && ILtQ[
q, 0] && IGtQ[Log[2, nn/n], 0]
```

Maple [F]

$$\int \frac{(b\,x^{8}+a)^{p}}{dx+c}dx$$

input
```
int((b*x^8+a)^p/(d*x+c),x)
```

output
```
int((b*x^8+a)^p/(d*x+c),x)
```

Fricas [F]

$$\int \frac{(a+bx^8)^p}{c+dx}\,dx=\int \frac{(bx^8+a)^p}{dx+c}\,dx$$

input
```
integrate((b*x^8+a)^p/(d*x+c),x, algorithm="fricas")
```

output
```
integral((b*x^8 + a)^p/(d*x + c), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^8)^p}{c+dx}\,dx = \text{Timed out}$$

input
```
integrate((b*x**8+a)**p/(d*x+c),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^8)^p}{c+dx}\,dx = \int \frac{(bx^8+a)^p}{dx+c}\,dx$$

input
```
integrate((b*x^8+a)^p/(d*x+c),x, algorithm="maxima")
```

output
```
integrate((b*x^8 + a)^p/(d*x + c), x)
```

Giac [F]

$$\int \frac{(a+bx^8)^p}{c+dx}\,dx = \int \frac{(bx^8+a)^p}{dx+c}\,dx$$

input
```
integrate((b*x^8+a)^p/(d*x+c),x, algorithm="giac")
```

output
```
integrate((b*x^8 + a)^p/(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^8)^p}{c+dx}\,dx=\int \frac{(b\,x^8+a)^p}{c+d\,x}\,dx$$

input
```
int((a + b*x^8)^p/(c + d*x),x)
```

output
```
int((a + b*x^8)^p/(c + d*x), x)
```

Reduce [F]

$$\int \frac{(a+bx^8)^p}{c+dx}\,dx = \frac{(b\,x^8+a)^p+8\left(\int \frac{(b\,x^8+a)^p}{bd\,x^9+bc\,x^8+adx+ac}dx\right)adp-8\left(\int \frac{(b\,x^8+a)^p\,x^7}{bd\,x^9+bc\,x^8+adx+ac}dx\right)bcp}{8dp}$$

input
```
int((b*x^8+a)^p/(d*x+c),x)
```

output
```
((a + b*x**8)**p + 8*int((a + b*x**8)**p/(a*c + a*d*x + b*c*x**8 + b*d*x**
9),x)*a*d*p - 8*int(((a + b*x**8)**p*x**7)/(a*c + a*d*x + b*c*x**8 + b*d*x
**9),x)*b*c*p)/(8*d*p)
```

3.320 $\int \frac{(a+bx^8)^p}{(c+dx)^2} dx$

Optimal result

Integrand size = 17, antiderivative size = 970

$$\int \frac{(a+bx^8)^p}{(c+dx)^2} dx = \text{Too large to display}$$

output
```
x*(b*x^8+a)^p*AppellF1(1/8,2,-p,9/8,d^8*x^8/c^8,-b*x^8/a)/c^2/((1+b*x^8/a)
^p)-d*x^2*(b*x^8+a)^p*AppellF1(1/4,2,-p,5/4,d^8*x^8/c^8,-b*x^8/a)/c^3/((1+
b*x^8/a)^p)+d^2*x^3*(b*x^8+a)^p*AppellF1(3/8,2,-p,11/8,d^8*x^8/c^8,-b*x^8/
a)/c^4/((1+b*x^8/a)^p)-d^3*x^4*(b*x^8+a)^p*AppellF1(1/2,2,-p,3/2,d^8*x^8/c
^8,-b*x^8/a)/c^5/((1+b*x^8/a)^p)+d^4*x^5*(b*x^8+a)^p*AppellF1(5/8,2,-p,13/
8,d^8*x^8/c^8,-b*x^8/a)/c^6/((1+b*x^8/a)^p)-d^5*x^6*(b*x^8+a)^p*AppellF1(3
/4,2,-p,7/4,d^8*x^8/c^8,-b*x^8/a)/c^7/((1+b*x^8/a)^p)+d^6*x^7*(b*x^8+a)^p*
AppellF1(7/8,2,-p,15/8,d^8*x^8/c^8,-b*x^8/a)/c^8/((1+b*x^8/a)^p)+7/9*d^8*x
^9*(b*x^8+a)^p*AppellF1(9/8,2,-p,17/8,d^8*x^8/c^8,-b*x^8/a)/c^10/((1+b*x^8
/a)^p)-3/5*d^9*x^10*(b*x^8+a)^p*AppellF1(5/4,2,-p,9/4,d^8*x^8/c^8,-b*x^8/a
)/c^11/((1+b*x^8/a)^p)+5/11*d^10*x^11*(b*x^8+a)^p*AppellF1(11/8,2,-p,19/8,
d^8*x^8/c^8,-b*x^8/a)/c^12/((1+b*x^8/a)^p)-1/3*d^11*x^12*(b*x^8+a)^p*Appel
lF1(3/2,2,-p,5/2,d^8*x^8/c^8,-b*x^8/a)/c^13/((1+b*x^8/a)^p)+3/13*d^12*x^13
*(b*x^8+a)^p*AppellF1(13/8,2,-p,21/8,d^8*x^8/c^8,-b*x^8/a)/c^14/((1+b*x^8/
a)^p)-1/7*d^13*x^14*(b*x^8+a)^p*AppellF1(7/4,2,-p,11/4,d^8*x^8/c^8,-b*x^8/
a)/c^15/((1+b*x^8/a)^p)+1/15*d^14*x^15*(b*x^8+a)^p*AppellF1(15/8,2,-p,23/8
,d^8*x^8/c^8,-b*x^8/a)/c^16/((1+b*x^8/a)^p)-b*c^7*d^7*(b*x^8+a)^(p+1)*hype
rgeom([2, p+1],[2+p],d^8*(b*x^8+a)/(a*d^8+b*c^8))/(a*d^8+b*c^8)^2/(p+1)
```

Mathematica [F]

$$\int \frac{(a+bx^8)^p}{(c+dx)^2}\,dx=\int \frac{(a+bx^8)^p}{(c+dx)^2}\,dx$$

input
```
Integrate[(a + b*x^8)^p/(c + d*x)^2,x]
```

output
```
Integrate[(a + b*x^8)^p/(c + d*x)^2, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^8)^p}{(c+dx)^2}\,dx$$

$$\Big\downarrow \ 2584$$

$$\int \frac{(c^2-2cdx+d^2x^2)\,(a+bx^8)^p}{(c^2-d^2x^2)^2}dx$$

$$\Big\downarrow \ 2457$$

$$\int \frac{(a+bx^8)^p}{(c+dx)^2}dx$$

$$\Big\downarrow \ 2584$$

$$\int \frac{(c^2-2cdx+d^2x^2)\,(a+bx^8)^p}{(c^2-d^2x^2)^2}dx$$

$$\Big\downarrow \ 2457$$

$$\int \frac{(a+bx^8)^p}{(c+dx)^2}dx$$

$$\Big\downarrow \ 2584$$

$$\int \frac{(c^2-2cdx+d^2x^2)\,(a+bx^8)^p}{(c^2-d^2x^2)^2}dx$$

$$\Big\downarrow \text{2457}$$

$$\int \frac{(a+bx^8)^p}{(c+dx)^2}dx$$

$$\Big\downarrow \text{2584}$$

$$\int \frac{(c^2-2cdx+d^2x^2)\,(a+bx^8)^p}{(c^2-d^2x^2)^2}dx$$

$$\Big\downarrow \text{2457}$$

$$\int \frac{(a+bx^8)^p}{(c+dx)^2}dx$$

$$\Big\downarrow \text{2584}$$

$$\int \frac{(c^2-2cdx+d^2x^2)\,(a+bx^8)^p}{(c^2-d^2x^2)^2}dx$$

$$\Big\downarrow \text{2457}$$

$$\int \frac{(a+bx^8)^p}{(c+dx)^2}dx$$

$$\Big\downarrow \text{2584}$$

$$\int \frac{(c^2-2cdx+d^2x^2)\,(a+bx^8)^p}{(c^2-d^2x^2)^2}dx$$

$$\Big\downarrow \text{2457}$$

$$\int \frac{(a+bx^8)^p}{(c+dx)^2}dx$$

$$\Big\downarrow \text{2584}$$

$$\int \frac{(c^2-2cdx+d^2x^2)\,(a+bx^8)^p}{(c^2-d^2x^2)^2}dx$$

$$\Big\downarrow \text{2457}$$

$$\int \frac{(a+bx^8)^p}{(c+dx)^2}dx$$

$$\Big\downarrow \text{2584}$$

$$\int \frac{(c^2-2cdx+d^2x^2)\,(a+bx^8)^p}{(c^2-d^2x^2)^2}dx$$

$$\Big\downarrow \text{2457}$$

$$\int \frac{(a+bx^8)^p}{(c+dx)^2}dx$$

$$\Big\downarrow \text{2584}$$

$$\int \frac{(c^2-2cdx+d^2x^2)\,(a+bx^8)^p}{(c^2-d^2x^2)^2}dx$$

$$\Big\downarrow \text{2457}$$

$$\int \frac{(a+bx^8)^p}{(c+dx)^2}dx$$

$$\Big\downarrow \text{2584}$$

$$\int \frac{(c^2-2cdx+d^2x^2)\,(a+bx^8)^p}{(c^2-d^2x^2)^2}dx$$

$$\Big\downarrow \text{2457}$$

$$\int \frac{(a+bx^8)^p}{(c+dx)^2}dx$$

$$\Big\downarrow \text{2584}$$

$$\int \frac{(c^2-2cdx+d^2x^2)\,(a+bx^8)^p}{(c^2-d^2x^2)^2}dx$$

$$\Big\downarrow \text{2457}$$

$$\int \frac{(a+bx^8)^p}{(c+dx)^2}dx$$

$$\Big\downarrow \text{2584}$$

$$\int \frac{(c^2-2cdx+d^2x^2)\,(a+bx^8)^p}{(c^2-d^2x^2)^2}dx$$

$$\Big\downarrow \text{2457}$$

$$\int \frac{(a+bx^8)^p}{(c+dx)^2}dx$$

$$\Big\downarrow \text{2584}$$

$$\int \frac{(c^2-2cdx+d^2x^2)\,(a+bx^8)^p}{(c^2-d^2x^2)^2}dx$$

$$\downarrow 2457$$

$$\int \frac{(a+bx^8)^p}{(c+dx)^2}dx$$

$$\downarrow 2584$$

$$\int \frac{(c^2-2cdx+d^2x^2)(a+bx^8)^p}{(c^2-d^2x^2)^2}dx$$

$$\downarrow 2457$$

$$\int \frac{(a+bx^8)^p}{(c+dx)^2}dx$$

$$\downarrow 2584$$

$$\int \frac{(c^2-2cdx+d^2x^2)(a+bx^8)^p}{(c^2-d^2x^2)^2}dx$$

$$\downarrow 2457$$

$$\int \frac{(a+bx^8)^p}{(c+dx)^2}dx$$

input
```
Int[(a + b*x^8)^p/(c + d*x)^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2457
```
Int[(u_.)*(Px_)*(Qx_)^(q_), x_Symbol] :> Module[{Rx = PolyGCD[Px, Qx, x]},
Int[u*Rx^(q + 1)*PolynomialQuotient[Px, Rx, x]*PolynomialQuotient[Qx, Rx, x
]^q, x] /; NeQ[Rx, 1]] /; ILtQ[q, 0] && PolyQ[Px, x] && PolyQ[Qx, x]
```

rule 2584
```
Int[((c_) + (d_.)*(x_)^(n_.))^(q_)*((a_) + (b_.)*(x_)^(nn_.))^(p_), x_Symbo
l] :> Int[ExpandToSum[(c - d*x^n)^(-q), x]*((a + b*x^nn)^p/(c^2 - d^2*x^(2*
n))^(-q)), x] /; FreeQ[{a, b, c, d, n, nn, p}, x] &&  !IntegerQ[p] && ILtQ[
q, 0] && IGtQ[Log[2, nn/n], 0]
```

Maple [F]

$$\int \frac{(b\,x^8+a)^p}{(dx+c)^2}dx$$

input
```
int((b*x^8+a)^p/(d*x+c)^2,x)
```

output
```
int((b*x^8+a)^p/(d*x+c)^2,x)
```

Fricas [F]

$$\int \frac{(a+bx^8)^p}{(c+dx)^2}\,dx = \int \frac{(bx^8+a)^p}{(dx+c)^2}\,dx$$

input
```
integrate((b*x^8+a)^p/(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral((b*x^8 + a)^p/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^8)^p}{(c+dx)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x**8+a)**p/(d*x+c)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^8)^p}{(c+dx)^2}\,dx = \int \frac{(bx^8+a)^p}{(dx+c)^2}\,dx$$

input
```
integrate((b*x^8+a)^p/(d*x+c)^2,x, algorithm="maxima")
```

output
```
integrate((b*x^8 + a)^p/(d*x + c)^2, x)
```

Giac [F]

$$\int \frac{(a+bx^8)^p}{(c+dx)^2}\,dx = \int \frac{(bx^8+a)^p}{(dx+c)^2}\,dx$$

input
```
integrate((b*x^8+a)^p/(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate((b*x^8 + a)^p/(d*x + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^8)^p}{(c+dx)^2}\,dx = \int \frac{(b\,x^8+a)^p}{(c+d\,x)^2}\,dx$$

input
```
int((a + b*x^8)^p/(c + d*x)^2,x)
```

output
```
int((a + b*x^8)^p/(c + d*x)^2, x)
```

Reduce [F]

$$\int \frac{(a+bx^8)^p}{(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
int((b*x^8+a)^p/(d*x+c)^2,x)
```

output
```
((a + b*x**8)**p + 64*int((a + b*x**8)**p/(8*a*c**2*p - a*c**2 + 16*a*c*d*
p*x - 2*a*c*d*x + 8*a*d**2*p*x**2 - a*d**2*x**2 + 8*b*c**2*p*x**8 - b*c**2
*x**8 + 16*b*c*d*p*x**9 - 2*b*c*d*x**9 + 8*b*d**2*p*x**10 - b*d**2*x**10),
x)*a*c*d*p**2 - 8*int((a + b*x**8)**p/(8*a*c**2*p - a*c**2 + 16*a*c*d*p*x
- 2*a*c*d*x + 8*a*d**2*p*x**2 - a*d**2*x**2 + 8*b*c**2*p*x**8 - b*c**2*x**
8 + 16*b*c*d*p*x**9 - 2*b*c*d*x**9 + 8*b*d**2*p*x**10 - b*d**2*x**10),x)*a
*c*d*p + 64*int((a + b*x**8)**p/(8*a*c**2*p - a*c**2 + 16*a*c*d*p*x - 2*a*
c*d*x + 8*a*d**2*p*x**2 - a*d**2*x**2 + 8*b*c**2*p*x**8 - b*c**2*x**8 + 16
*b*c*d*p*x**9 - 2*b*c*d*x**9 + 8*b*d**2*p*x**10 - b*d**2*x**10),x)*a*d**2*
p**2*x - 8*int((a + b*x**8)**p/(8*a*c**2*p - a*c**2 + 16*a*c*d*p*x - 2*a*c
*d*x + 8*a*d**2*p*x**2 - a*d**2*x**2 + 8*b*c**2*p*x**8 - b*c**2*x**8 + 16*
b*c*d*p*x**9 - 2*b*c*d*x**9 + 8*b*d**2*p*x**10 - b*d**2*x**10),x)*a*d**2*p
*x - 64*int(((a + b*x**8)**p*x**7)/(8*a*c**2*p - a*c**2 + 16*a*c*d*p*x - 2
*a*c*d*x + 8*a*d**2*p*x**2 - a*d**2*x**2 + 8*b*c**2*p*x**8 - b*c**2*x**8 +
 16*b*c*d*p*x**9 - 2*b*c*d*x**9 + 8*b*d**2*p*x**10 - b*d**2*x**10),x)*b*c*
*2*p**2 + 8*int(((a + b*x**8)**p*x**7)/(8*a*c**2*p - a*c**2 + 16*a*c*d*p*x
 - 2*a*c*d*x + 8*a*d**2*p*x**2 - a*d**2*x**2 + 8*b*c**2*p*x**8 - b*c**2*x*
*8 + 16*b*c*d*p*x**9 - 2*b*c*d*x**9 + 8*b*d**2*p*x**10 - b*d**2*x**10),x)*
b*c**2*p - 64*int(((a + b*x**8)**p*x**7)/(8*a*c**2*p - a*c**2 + 16*a*c*d*p
*x - 2*a*c*d*x + 8*a*d**2*p*x**2 - a*d**2*x**2 + 8*b*c**2*p*x**8 - b*c*...
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                   convert(leaf_count_result,string)," vs. $2 (",
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
      ]
```

```
def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False

   except AttributeError as error:
      return False

def expnType(expn):
   debug=False
   if debug:
      print("expn=",expn,"type(expn)=",type(expn))

   if is_atom(expn):
     return 1
   elif isinstance(expn,list):
      return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
         return 1
      else:
         return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
   elif isinstance(expn,Pow):   #type(expn,'`^`')
      if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
         return expnType(expn.args[0])   #ExpnType(op(1,expn))
      elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
         if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
         else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
      else:
         return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),Expn
   elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)  #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#          Albert Rich to use with Sagemath. This is used to
#          grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#          'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#          issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
            'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
            'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
            'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
            'elliptic_pi','exp_integral_e','log_integral',
            'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
            'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
         print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
```

```
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expr
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file